# A macroeconomic model with (partial) bank runs*


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**Abstract**

.



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# 1 Introduction

**Related literature.**

- Causes and consequences of bank runs: Diamond and Dybvig (1983); Allen and Gale (1998); Allen and Gale (2000); Jacklin and Bhattacharya (1988); Baron, Verner, and Xiong (2021); Jamilov, König, Müller, and Saidi (2024)
- Risk-shifting due to deposit insurance: Kareken and Wallace (1978); Keeley (1990); Hellmann, Murdock, and Stiglitz (2000); Repullo (2004)
- Quantitative models of banking / optimal bank regulation: Van den Heuvel (2008); Martinez-Miera and Suarez (2014); Begenau (2020); Corbae and D'Erasmo (2021); Elenev, Landvoigt, and Van Nieuwerburgh (2021); Begenau and Landvoigt (2022); Mendicino, Nikolov, Rubio-Ramírez, Suárez, and Supera (forthcoming)

Other literature on dynamics of depositor runs: Iyer and Puri (2012); Iyer, Puri, and Ryan (2016); Martin, Puri, and Ufier (forthcoming); Blickle, Brunnermeier, and Luck (2024)

# 2 The model

I consider an infinite-horizon economy cast in discrete time, in which dates are indexed by $t$. There is a single non-durable consumption good in every date used as the numeraire, and which can be transformed into physical capital used for production. At any given date the economy is populated by: (i) households; (ii) banks; (iii) a representative non-bank-dependent (physical-)capital-producing firm; (iv) a representative consumption-good producing firm; and (v) a deposit insurance agency (DIA).

**Households.** Each household is infinitely-lived and makes consumption-savings decisions to maximize its inter-temporal expected utility. It can invest its savings in the non-bank-dependent capital-producing firms, and hold (partially) insured demand deposits with banks. The household consists of two classes of members: workers and bankers. These agents obtain consumption insurance from the household. The workers inelastically supply one-unit of labor to the representative consumption-good producing firm and transfer their wage income to the household. The bankers manage the household's investments in banks' equity which are subject to frictions further specified below.

**Banks: island setup.** There exists a continuum of measure one of islands. In each island, there is a continuum of measure one of ex-ante identical banks. Each bank manages an investment project in physical-capital production that is operational between two consecutive dates. Its project is subject to an island-idiosyncratic shock, whose realization affects the effective-units of capital produced. Banks can exert effort in reducing the likelihood of its project being exposed to the island-shock. The island-based market segmentation only applies to the bank-dependent production sector. The deposit and equity funding of banks is freely mobile across islands.

**Non-bank-dependent firms and consumption goods production.** There is a representative non-bank-dependent (physical-)capital-producing firm that raises direct financing from households. A representative consumption-good producing firm combines non-bank-dependent physical capital, bank-dependent physical capital, and labor to produce consumption good.

**Deposit insurance agency.** The DIA is responsible for the resolution of failed-banks. It raises lump-sum taxes from the household to cover the losses on the insured portion of deposits of the banks that defaulted as a result of their previous period of operation. The taxes are subject to deadweight costs, explained below in greater detail.

**Aggregate shocks.** In addition to the island-idiosyncratic shocks, the economy faces an aggregate TFP shock and aggregate island-risk shock (modeled as a shock to the standard deviation).

The following subsections describe each of these agents, their optimization problems, and the definition of equilibrium in detail.

## 2.1 Production environment

Non-bank-dependent firms and banks produce one class of physical capital each, labeled $h$ and $b$, respectively. These classes of physical capital are not perfect substitutes and, hence, can be eventually rented to the consumption-good-producing firms at different equilibrium rental rates.[1]

**Non-bank-dependent production.** The representative firm from the non-bank de-

[1]The non-bank-dependent sector in my model can be interpreted as consisting of large firms with access to capital markets, and the bank-dependent sector as consisting of small-and-medium sized firms who primarily rely on bank financing. My modelling framework closely follows Abad, Martinez-Miera, and Suarez (2024). See Begenau (2020) or Davydiuk (2017) for alternative frameworks with similar interpretation.

pendent sector can transform $a_t^h$ consumption good units from the household in period $t$ into $k_{t+1}^h = a_t^h$ units of physical capital of class $h$ in period $t+1$. Renting this capital at $t+1$ yields a per-unit rental rate $r_{t+1}^h$ and the recovery of $1-\delta^h$ units of consumption good, where $\delta^h$ is the depreciation rate.[2] So the gross return of this class of capital is $R_{t+1}^h = 1 + r_{t+1}^h - \delta^h$.

**Bank-dependent production.** Each island is populated by a continuum of ex-ante identical banks, indexed $j \in (0,1)$. A bank $j$ can transform $a_{jt}^b$ units of consumption good into

$$k_{jt+1}^b = \begin{cases} \omega\left(a_{jt}^b - \frac{l_{jt+1}}{1-\zeta}\right), & \text{with probability } 1-m_{jt}, \\ a_{jt}^b - l_{jt+1}, & \text{with probability } m_{jt} \end{cases} \tag{1}$$

units of physical capital of class $b$ in period $t+1$. Renting this capital at $t+1$ yields a per-unit gross return $R_{t+1}^b = 1 + r_{t+1}^b - \delta^b$, where $r_{t+1}^b$ and $\delta^b$ are the corresponding rental and depreciation rates.

In Equation (1), $\omega$ is an island-idiosyncratic shock realized at $t+1$. It is identically distributed across time and across islands.[3] The term $m_{jt} \in [0,1]$ is a variable that captures managerial bank-level risk choice. It is private information of the bank at date $t$. The term $l_{jt+1} > 0$ captures the possibility that at the beginning of period $t+1$, the bank $j$ may have to (early-)liquidate a portion of investment, after learning whether or not it is exposed to the island-shock, but before the realization of shocks and the completion of production process. The early liquidation process involves reversing the investment into consumption good, and is associated with proportional losses $\zeta$ conditional on the investment being exposed to the island-shock.[4]

The island-idiosyncratic shocks $\omega$ are log-normally distributed, with $\mathbb{E}(\omega) < 1$. Bank $j$'s exposure to the shock at $t+1$ is determined by its choice $m_{jt}$.[5] I assume that the choice of $m_{jt}$ is associated with a cost $\mathcal{C}(m_{jt})$ per unit of investment, where the function $\mathcal{C}(m)$ satisfies $\mathcal{C}(0) = \mathcal{C}'(0) = 0, \mathcal{C}'(m) > 0$, and $\mathcal{C}''(m) > 0$. Under this formulation, banks' risk choice can be interpreted as being complementary to their effort in *screening* investment projects.

The role of banks in my model is justified along the lines of Diamond and Rajan (2001). As I will show below, the fragility of banks' capital structure commits them to exert screening

[2] I model physical capital as fully fungible into consumption good after one period to minimize the number of state variables in the model.

[3] For an island model of banking, refer to Mendicino et al. (forthcoming).

[4] These losses can be interpreted as due-diligence costs, or costs incurred due to lack of redeployability of assets exposed to idiosyncratic risk (see, e.g., Williamson, 1988).

[5] The formulation of banks' risk taking choices in probabilistic terms is similar in spirit to Martinez-Miera and Repullo (2017).

effort. At the same time, safety-net due to deposit insurance, standard limited liability distortions, and the unobservability of $m_{jt}$ to depositors and the DIA can make risk-shifting attractive to the banks.

**Consumption goods production.** A representative consumption-good producing firm combines non-bank-dependent physical capital $K_t^h$, bank-dependent physical capital $K_t^b$, and labor $L_t$ to produce

$$Y_t = z_t G(K_t^h, K_t^b, L_t) \tag{2}$$

units of consumption good, where $G(\cdot)$ is a constant-returns-to-scale production function, and $z_t$ is an aggregate productivity shock. The firm maximizes its profits $Y_t - r_t^h K_t^h - r_t^b K_t^b - W_t L_t$ taking the rental rates $r_t^h$ and $r_t^b$ and the wage rate $W_t$ as given.

## 2.2 Households

In each period $t$, the households obtains utility from the consumption of non-durable goods. They inelastically supply one unit of labor to the representative consumption-good producing firm, remunerated with a wage $W_t$; receive net dividend payments $\Xi_t$ from bankers as further specified below; and pay lump-sum taxes $T_t$ to the DIA. Households save in the form of demand deposits at banks and invest in claims on physical capital issued by the non-bank-dependent sector.

Bank deposits held by households are insured by the DIA up to a limit $\bar{D}$. In practice, this means that even in the event of a bank failure, the household is guaranteed to recover deposits in the failing bank up to $\bar{D}$ (plus any accrued interest). Deposits in excess of the insured limit are subject to default risk. In particular, the DIA repossess the assets of a failing bank in order to first recoup what it owes to satisfy the deposit guarantees, and any residual proceeds are distributed among (partially) insured depositors on a pro-rata basis.

The problem of the household involves choosing consumption $C_t$, bank deposits $D_t$, and investment in claims issued by the non-bank-dependent sector $A_t^h$, so as to maximize its expected discounted lifetime utility

$$\mathbb{E}_t \sum_{\tau=0}^{\infty} \beta^\tau U(C_{t+\tau}), \tag{3}$$

where $\beta$ is the subjective discount rate, and $U(\cdot)$ is a standard concave, twice continuously differentiable function. In each period the household is subject to the following budget constraint

$$C_t + D_t + A^h_t \;=\; W_t + R^d_{t-1}\min\{\bar{D}, D_{t-1}\} + \tilde{R}^d_t \max\{D_{t-1} - \bar{D}, 0\} + R^h_t A^h_{t-1} + \Xi_t - T_t, \quad (4)$$

where the second term on the right hand side are the (safe) gross returns $R^d_{t-1}\min\{\bar{D}, D_{t-1}\}$ in period $t$ of the deposits held at banks up to the insured limit $\bar{D}$, and the third term are the (potentially risky) gross returns $\tilde{R}^d_t \max\{D_{t-1} - \bar{D}, 0\}$ in period $t$ of any deposits held at banks above the insured limit. I am interested in a symmetric equilibrium, hence I assume that at date $t$ the household invests its deposits symmetrically in all the (symmetric) banks in the economy.

In addition to the inter-temporal decisions above, households also face intra-temporal ones. Particularly, in between any two consecutive dates, coordination problems may trigger (some) households to withdraw their bank deposits, and they decide which banks to withdraw from. Further details on households' *run* incentives are described in Section 2.4.

## 2.3 Bankers

Each banker manages the household's investment in bank equity. Let $V_t(n^b_t)$ be the value of being a banker with net worth $n^b_t$ in period $t$. The banker can vary that net worth by $x_t$ at a pecuniary cost $\Upsilon(x^+_t)$ given by an increasing and convex function $\Upsilon(0) = \Upsilon'(0) = 0$ and $x^+_t \equiv \max\{x_t, 0\}$. So, reflecting (unmodeled) frictions in the equity raising process, equity issuance ($x_t > 0$) is costly while discretionary dividends ($x_t \in [-n^b_t, 0)$) are not.

The banker can invest the resulting funds, $n^b_t + x_t$, in equity of any of the banks operating between $t$ and $t+1$, taking as given the distribution of equity returns $R^e_{t+1}$. Out of the gross returns earned at $t+1$, an exogenous fraction $1-\theta$ is paid out to the household and the rest is retained under the management of the banker. The optimization problem of a banker can be recursively stated as follows

$$V_t(n^b_t) = \max_{x_t \geq -n^b_t} \left\{ -x_t - \Upsilon(x^+_t) + \mathbb{E}_t\left[\Lambda_{t+1}(1-\theta)R^e_{t+1}\right](n^b_t + x_t) + \mathbb{E}_t\left[\Lambda_{t+1}V_{t+1}(n^b_{t+1})\right] \right\} \quad (5)$$

with

$$n^b_{t+1} = \theta R^e_{t+1}(n^b_t + x_t), \quad (6)$$

where the first two terms in (5) account for newly raised equity (or discretionary dividends if $x_t < 0$) and its issuance costs; the third term reflects the expected discounted value of the exogenously distributed part of the returns generated at $t+1$ by the investment in bank equity in period $t$; and the fourth term is the expected discounted value of the net worth retained under banker's management at $t+1$. Equation (6) is the law of motion of that net worth. As the household is the final receiver of all the payoffs from the wealth that bankers manage, future payments and values are discounted with the stochastic discount factor $\Lambda_{t+1} = \beta U'(C_{t+1})/U'(C_t)$. I am interested in a symmetric equilibrium, hence I assume that each banker invests symmetrically in all the (symmetric) banks in the economy.

## 2.4 Banks

Banks are constant-returns-to-scale intermediaries that operate between any two consecutive periods. They maximize the net present value of the equity that bankers invest in them. In period $t$, a bank $j$ combines this equity $e_{jt}$ with (partially) insured deposits $d_{jt}$ issued to the household in order to finance investment $a^b_{jt}$ in a physical capital project in its island. Hence, the balance sheet identity of bank $j$ imposes that

$$a^b_{jt} = e_{jt} + d_{jt}. \tag{7}$$

In addition to capital structure decisions, the bank chooses its screening intensity, $m_{jt}$, which determines its probability of being exposed to the island shock at $t+1$. Importantly, all items on the bank's balance sheet are publicly observable at date $t$, but its choice of screening intensity is not.

As under the various versions of the Basel Accord after 1988, I assume that banks are subject to a minimum regulatory capital requirement of the form

$$e_{jt} \geq \gamma a^b_{jt}, \tag{8}$$

which imposes that at least a fraction $\gamma$ of the bank's assets have to be financed with equity capital.[6]

At any date $t$, all banks in the economy face identical constraints, receive a symmetric infusion of equity from bankers, as well as symmetric investment of deposits from households. Since all banks are ex-ante identical, I focus on a symmetric equilibrium in which banks make identical decisions at $t$. For brevity, I hereafter drop the $j$ subscripts.

[6] Consistent with the assumption that banks' risk choice is not observable to depositors and the DIA, this requirement is not contingent on $m_{jt}$.

**Sunspot process and panic runs.** At the beginning of $t+1$, before the realization of the shocks and the completion of production, a fraction $\chi$ of banks' depositors – dubbed *alert* depositors – learn the identity of banks which are exposed to the island-shock. That is, the alert depositors at this stage can distinguish between banks whose screening effort at $t$ either succeeds ($s=1$) or fails ($s=0$) in insulating itself from the (yet-to-be-realized) island-shock $\omega$.

During this stage, coordination problems can trigger panic runs. In particular, a (partially-insured) alert depositor runs on a bank if (i) it perceives that other depositors will do the same, forcing the bank to early-liquidate investments, and; (ii) the forced liquidation potentially makes the bank insolvent.[7] In this situation two equilibria can exist: a "normal" equilibrium in which only the fundamentally insolvent banks (if any) fail, and a "panic" equilibrium in which alert depositors run on the banks for which conditions (i) and (ii) are satisfied (which includes fundamentally insolvent banks as well as banks that are "fundamentally solvent but vulnerable").

An aggregate sunspot process determines condition (i) in i.i.d. manner over time but perfectly correlated across banks. Specifically, let $\mathcal{S}_t$ be a binary sunspot variable that takes on a value of 1 with an exogenous probability $\varepsilon$ and a value of 0 with probability $1-\varepsilon$. When $\mathcal{S}_{t+1}=1$, alert depositors coordinate to (partially) withdraw their deposits from the vulnerable banks, i.e., banks which satisfy condition (ii). Importantly, their decisions to withdraw are based on banks' observables – $a_t^b, d_t, e_t$, and $s \in \{0,1\}$. Below, I elaborate further on alert depositors' run-incentives.

**Fundamental failures.** Let $\pi_{st+1}^{\mathcal{S}}$ denote the terminal net worth at $t+1$ of a bank conditional on its exposure to the island shock and the realization of aggregate sunspot shock. Then, when $\mathcal{S}_{t+1}=0$,

$$\pi_{1t+1}^0 = R_{t+1}^b a_t^b - R_t^d d_t - \mathcal{C}(m_t) a_t^b, \tag{9}$$

$$\pi_{0t+1}^0(\omega) = \omega R_{t+1}^b a_t^b - R_t^d d_t - \mathcal{C}(m_t) a_t^b. \tag{10}$$

A bank is fundamentally insolvent if its portfolio returns at $t+1$ are insufficient to pay the promised repayment in full, i.e., its terminal net worth is negative. To simplify the exposition,

[7] The distinction between *sleepy* and *alert* depositors has been made in the previous literature by, e.g., Hanson, Shleifer, Stein, and Vishny (2015); Jiang, Matvos, Piskorski, and Seru (2020); Drechsler, Savov, Schnabl, and Wang (2023). In a recent study on the 2023 panic runs in the US surrounding the collapse of Silicon Valley Bank, Jiang, Matvos, Piskorski, and Seru (2024) highlight the role of *sleepy vs. alert* depositors in determining the consequences of panics.

I assume that $\pi^0_{1t+1} > 0$, such that a bank not exposed to the island-shock never fails in the normal equilibrium.[8] As for a bank exposed to the island-shock at $t+1$, Equation (10) can be used to define a threshold value for $\omega$ below which the bank's net worth is negative and it is therefore insolvent

$$\bar{\omega}_{t+1} = \frac{R^d_t d_t + \mathcal{C}(m_t) a^b_t}{R^b_{t+1} a^b_t}. \tag{11}$$

By the law of large numbers, for any arbitrary screening effort $m_t$ exerted at date $t$, a fraction $\mathcal{M}_{t+1} = m_t$ of banks are immune to the island-shocks $\omega$ at date $t+1$. Then, the aggregate failure rate of banks due to weak fundamentals is

$$\mathcal{D}^f_{t+1} = (1 - \mathcal{M}_{t+1}) F(\bar{\omega}_{t+1}), \tag{12}$$

where $F$ denotes the cumulative distribution function of the island-shock.

**Probability of self-fulfilling failures.** Following Dávila and Goldstein (2023), I assume that when $\mathcal{S}_{t+1} = 1$, alert depositors withdraw the uninsured portion of their deposits. Using the notation introduced in (1), and denoting $\iota_t \equiv \min\left\{\frac{\overline{D}}{D_t}, 1\right\}$ as the the insured shared of a household's deposits, the portion of investments to be liquidated by a bank subject to runs are

$$l_{t+1} = \chi(1 - \iota_t) R^d_t d_t. \tag{13}$$

First, notice that for all $\chi, \iota_t \in [0,1]$, a bank not exposed to the island-shock is immune to failure risk if subject to panic runs, as long as it is also fundamentally solvent.[9] This is because the non-exposed banks are fully liquid to satisfy any amount of depositor withdrawals. Therefore, rational depositors do not run on these banks as they do not satisfy the condition (ii) stated above.

Banks exposed to the island-shocks are illiquid. As noted in Section 2.1, satisfying withdrawals requires them to early-liquidate investments at a proportional cost $\zeta$. Further as noted in (11), these banks are subject to failure risk even in the absence of panics, therefore satisfying the condition (ii). Since the alert depositors cannot distinguish within the banks that are exposed to the island shocks, they are all subject to panic runs.[10] For a vulnerable

[8] Failure of a bank not exposed to the island-shock can only occur in a normal equilibrium for a very large negative aggregate productivity shock. In the nonstochastic steady state as well as for every combination of the parameters used in the quantitative analysis, this is never the case.

[9] This is often referred to in the literature as the state of *supersolvency* (Rochet and Vives, 2004).

[10] This is consistent with the findings in Blickle et al. (2024), who highlight depositors' inability in perfectly differentiating between banks during episodes of panic runs. In particular, they study run on the German banking system in 1931 and find no difference in total deposit outflows between failing and surviving banks.

bank to stay afloat after satisfying early withdrawals, its remaining assets should be sufficient to honor residual obligations, i.e.,

$$\omega R^b_{t+1}\left(a^b_t - \frac{\tilde{l}_{t+1}}{1-\zeta}\right) \geq [(1-\chi)(1-\iota_t)+\iota_t]R^d_t d_t + \mathcal{C}(m_t)a^b_t. \tag{14}$$

Equation (14) can be used to define the threshold value for $\omega$ below which the banks experiencing panic runs fail

$$\hat{\omega}_{t+1} = \frac{[(1-\chi)(1-\iota_t)+\iota_t]R^d_t d_t + \mathcal{C}(m_t)a^b_t}{R^b_{t+1}\left(a^b_t - \frac{l_{t+1}}{1-\zeta}\right)}. \tag{15}$$

Then, conditional on realization of the aggregate sunspot shock, the probability of self-fulfilling failures is

$$\mathcal{D}^p_{t+1} = (1-\mathcal{M}_{t+1})\left[F(\hat{\omega}_{t+1}) - F(\bar{\omega}_{t+1})\right]. \tag{16}$$

As will be seen below, a higher limit of deposit insurance has two opposing effects on the probability of self-fulfilling failures. On the one hand, it reduces depositor incentives to run on the bank (reducing the term in square brackets in (16)). On the other hand, it reduces banks' incentives to create liquidity (increasing the term in parenthesis in (16)).

## 2.5 Deposit insurance agency

The deposit insurance agency (DIA) supervises the liquidation process of failed-bank assets, which is subject to repossession costs.[11] The funds recovered are first used to meet its own obligations towards satisfying the deposit guarantees. The residual funds (if any) are distributed among all depositors on a pro-rata basis. The DIA's slack in satisfying the deposit guarantee (if any) is met by raising lump sum taxes from households to ex-post balance its budget. The DIA has to balance its budget period-by-period. I denote the losses to the DIA at time $t$ as $\Psi_t$. Following Dávila and Walther (2020), the total lump sum tax $T_t$ imposed on the households to balance the agency's budget is

$$T_t \;=\; \Psi_t + \tau(\Psi_t), \tag{17}$$

[11]The model follows Bernanke, Gertler, and Gilchrist (1999) in adopting a "costly state verification" setup, by which the DIA / banks' depositors must incur a cost that is proportional to the assets of the bank in order to observe the realization of the island-shocks $\omega$.

where $\tau(\cdot)$ is a weakly increasing and convex function that satisfies $\tau(0) = 0$ and $\lim_{\Psi\to\infty} \tau(\Psi) = \infty$. A detailed expression for $\Psi$ is provided in the Appendix.

## 2.6 The problem of the bank

Banks maximize the net present value of the equity that bankers invest in them. Prior to the analysis of the banks' decision problem, it is useful to first discuss the solution to bankers' optimization problem in (5)-(6).

Following Abad et al. (2024), I guess and verify that a banker's value function is an affine function of her net-worth. It involves a linear term $\nu_t n_t^b$ that implies a constant marginal shadow value $v_t$ of the net worth under the banker's management in period $t$, and an intercept $\nu_t^0$ which accounts for the extra value of the option to raise additional funds from the household (at a convex cost). The marginal shadow value of each unit of $n_t^b$ satisfies

$$\nu_t = \mathbb{E}_t \left[\Lambda_{t+1}(1 - \theta + \theta\nu_{t+1})R^e_{t+1}\right]. \tag{18}$$

Equation (18) defines the bankers' stochastic discount factor as $\Lambda^b_{t+1} = \Lambda_{t+1}(1 - \theta + \theta\nu_{t+1})$, where $\Lambda_{t+1}$ is the stochastic discount factor of the household on whose behalf the banker manages investments in bank equity, and the term $(1 - \theta + \theta\nu_{t+1})$ accounts for the marginal value $\nu_{t+1} \geq 1$ of the net worth that bankers can retain under their management at $t + 1$. Whenever $v_{t+1} > 1$, retained equity returns have extra value to bankers as they avoid incurring the equity issuance costs captured by the function $\Upsilon(\cdot)$. The remaining details of the banker's problem are provided in Appendix A.

**Banks' optimization problem.** Banks operate under limited liability, which means that the equity payoffs generated by a bank at time $t + 1$ are given by the positive part of the difference between the returns from its assets and the repayments due to its depositors, net of the monitoring costs. If the returns from the assets are greater than the repayments, the difference is paid back to the bank's equity holders. Otherwise, the bank's equity is written down to zero and its assets are repossessed by the DIA. Each bank maximizes the net present value of its shareholders' equity stakes

[Here I will setup banks' problem and provide FOC wrt screening choice
for highlighting moral hazard problem]

## 2.7 Equilibrium

A competitive equilibrium is given by the policy functions of the households, the banks, the representative non-bank-dependent firm, and the representative consumption-good producing firm, such that, given a sequence of equilibrium prices and a sequence of shocks, the sequence of each of the agents' decisions solve their corresponding problems, the sequence of prices clears all markets, and the sequence of endogenous state variables satisfies their corresponding laws of motion. A formal definition of the competitive equilibrium, together with the complete set of optimality and market clearing conditions, is provided in Appendix A.

# 3 Quantitative analysis

This section introduces the functional forms chosen for the numerical analysis, and presents the baseline parameterization. It then discusses the quantitative fit of the model by looking at its business-cycle properties, and explores the endogenous responses in the baseline economy to the realization of a sunspot shock. All data used in the quantitative analysis is publicly available. The data sources are described in Appendix B. To account for the multiplicity of equilibria due to the possibility of panic runs, I rely on a global numerical solution method. Details of the solution method are in Appendix C.

## 3.1 Functional forms and shock processes

In the quantitative analysis below, the functional form chosen for the utility function of the household is

$$U(C_t) = \frac{C_t^{1-\nu} - 1}{1-\nu}, \tag{19}$$

with constant risk-aversion parameter $\nu$. The production function of the consumption-good producing firm is Cobb-Douglas with

$$G(K_t^h, K_t^b, L_t) = \mathcal{K}(K_t^h, K_t^b)^\alpha L_t^{1-\alpha}, \tag{20}$$

where $\alpha \in (0,1)$ and $\mathcal{K}(K_t^h, K_t^b)$ is a physical capital composite

$$\mathcal{K}(K_t^h, K_t^b) = [\phi(K_t^h)^\rho + (1-\phi)(K_t^b)^\rho]^{\frac{1}{\rho}}, \tag{21}$$

with $\phi \in (0, 1)$ and $\rho > 0$, that features a constant elasticity of substitution $1/(1-\rho)$ between the physical capital produced by non-bank dependent firms $K^h$ and that produced by bank dependent firms $K^b$. The cost of screening projects is specified as

$$\mathcal{C}(m_t) = \frac{\kappa}{2}(m_t)^2, \tag{22}$$

with $\kappa > 0$, which satisfies the properties stated in the model section. Following Dávila and Goldstein (2023), the marginal cost of public funds is specified as

$$\tau(\Psi_t) = \frac{\tau_1}{\tau_2}\left(e^{\tau_2 \Psi_t} - 1\right), \tag{23}$$

for which the parameter $\tau_1 = \tau'(0)$ represents the marginal cost of public funds for a small intervention and the parameter $\tau_2 = \tau''(\Psi)/\tau'(\Psi)$ modulates how quickly the cost of public funds increases with $\Psi$. The cost of raising new equity is specified as

$$\Upsilon(x_t^+) = \frac{\upsilon}{2}\left(x_t^+\right)^2, \tag{24}$$

with $\upsilon > 0$, which satisfies the properties stated in the model section. The aggregate productivity shock obeys the following first-order process

$$z_t = 1 - \rho_z + \rho_z z_{t-1} + \sigma_z \epsilon_t, \tag{25}$$

with $\rho_z \in (0, 1), \sigma_z > 0$, and where $\epsilon_t$ is normally distributed with mean zero and variance one. The island-idiosyncratic shock is log-normally distributed

$$\log(\omega) \sim \mathcal{N}\left(\frac{-\sigma_\omega - \psi}{2}, \sigma_\omega^2\right), \tag{26}$$

with $\sigma_\omega > 0$ and $\psi > 0$.

## 3.2 Mapping the model to the data

The model is calibrated to quarterly US data from 1984 Q1 to 2006 Q4. The calibrated parameters – 21 overall – can be divided into two groups. The first group consists of twelve parameters, mainly related to household preferences, production sector, and the social costs of bank failures. They are chosen to either directly match with their data counterpart, or set to commonly agreed values in the business-cycle literature and related macro-banking

**Table 1**
**Pre-set parameters**

| | Parameter | Value | Source |
|---|---|---|---|
| $\beta$ | Impatience | 0.99 | Standard |
| $\nu$ | Risk Aversion | 2 | Standard |
| $\alpha$ | Output elasticity of capital | 0.33 | Standard |
| $\delta$ | Depreciation rate of capital | 0.025 | Standard |
| $\rho_z$ | Persistence of TFP shock | 0.95 | Standard |
| $\gamma$ | Capital requirement | 0.08 | BCBS (2004) |
| $\varepsilon$ | Sunspot Probability | 1% | Jamilov et al. (2024) |
| $\chi$ | Fraction of alert depositors | 5% | Iyer and Puri (2012) |
| $\zeta$ | Early liquidation costs | 0.45 | BCBS (2004) |
| $\mu$ | Bank default costs | 0.30 | Bennett and Unal (2015) |
| $\tau_1$ | Marginal cost of small intervention | 0.13 | Kleven and Kreiner (2006) |
| $\psi$ | Risk taking losses | 0.006 | Begenau (2020) |

papers. The remaining nine parameters in the second group are mainly those governing the behaviour of the banking sector, and hence are specific to my model. They are jointly set to match model-simulated moments to several data targets.

Table 1 and Table 2 respectively list the values assigned to all first and second group parameters in the baseline calibration, and summarize their corresponding sources or data targets. Table 3 provides the values of moments targeted in the data and compares them to their model generated counterparts. In what follows, I discuss the rationale for my choices in the calibration exercise.

**Pre-set parameters.** The subjective discount rate $\beta$ is set to a standard 0.995, delivering an annual risk-free rate around 2%. The household's risk-aversion parameter $\nu$ is set to 2, which is a value traditionally used in macroeconomics. The output share $\alpha$ of the physical capital composite $\mathcal{K}$ is set to a standard 0.33 and the depreciation rate $\delta$ of physical capital is set to a standard 0.025, delivering an annual rate around 10%. I take the persistence parameter $\rho_z$ of the aggregate productivity process from the business-cycle literature that uses a value of 0.95.

The minimum capital requirement $\gamma$ is set to 8%, consistent with the general requirement under Basel II (BCBS, 2004; part 2.I, paragraph 40) as well as its Basel I predecessor. The probability of observing a sunspot $\varepsilon$ is set at 1%, implying an average frequency of banking panics of once every 25 years, which is in line with the evidence reported in Jamilov et al. (2024) and coincides with the target for the frequency of banking panics set by Gertler, Kiyotaki, and Prestipino (2020). The proportion of alert depositors $\chi$ is set to 5%, in line

**Table 2**
**Calibrated parameters**

| | Parameter | Value | Target |
|---|---|---|---|
| $\bar{D}$ | Deposit insurance limit | 5 | Share of insured deposits |
| $\theta$ | Earnings retention rate | 0.96 | Return on bank equity |
| $\upsilon$ | Marginal cost, equity issuance | 142 | Bank equity issuance |
| $\kappa$ | Marginal cost, screening | 0.008 | Volatility, share of insured deposits |
| $\phi$ | Non-bank-dependent share in capital | 0.51 | Bank/non-bank ratio |
| $\rho$ | Substitution parameter capital composite | 0.21 | Volatility, return on equity |
| $\tau_2$ | Slope of tax function | 48 | Marginal cost of public funds |
| $\sigma_\omega$ | Mean idiosyncratic shock | 0.029 | Bank failure rate |
| $\sigma_z$ | Standard deviation of TFP shock | 0.3% | Standard deviation of output |

with the evidence in Iyer and Puri (2012) based on depositor-level data, and coincides with the value chosen for the proportion of *early depositors* in Dávila and Goldstein (2023).[12]

The value of the early-liquidation loss parameter $\zeta$ is set to 0.45, consistent with the loss-given-default (LGD) parameter of 45% that the foundation approach of Basel II (BCBS, 2004 paragraph 287) fixed for senior corporate loans without specific collateral. The value of bankruptcy parameter $\mu$ is set equal to 0.3, consistent with the 30% average discounted total resolution cost per unit of assets estimated by Bennett and Unal (2015) using FDIC data from failed banks in the period 1986-2007.[13] Following Dávila and Goldstein (2023), the marginal cost of public funds for a small intervention, captured by $\tau_1$, is set equal to 0.13 consistent with the estimate in Kleven and Kreiner (2006). Finally following Begenau (2020), the compensation for actively screening an investment project, governed by the parameter $\psi$, is set such that $1 - \mathbb{E}(\omega) = 0.32\%$ per quarter.[14]

**Calibrated parameters.** The second group of parameters are calibrated so as to simultaneously match the targets listed in Table 3. Each parameter can be mainly associated with one target, as a indicated in the last column of Table 2.

The limit on deposit insurance $\bar{D}$ is set to match an average share of insured deposits of about 67% at US banks in the period 1984-2006. The bankers' wealth retention rate $\theta$ is set to 0.956 to match the about 13% average real return on average equity. The marginal cost

[12]See also Kelly and Gráda (2000) who document withdrawals by a similar fraction of depositors during the bank run on Emigrants Industrial Savings Bank that occurred in 1854.

[13]See also Granja, Matvos, and Seru (2017), who estimate a recovery rate on bank assets after failure of 72%.

[14]In particular, Begenau (2020) obtains this spread by computing the average pretax excess return on the aggregate loan portfolio of banks relative to a maturity and credit-matched replicating portfolio based on investment-grade corporate bonds from Vanguard.

**Table 3**
**Calibration targets and model fit**

| Target | Description | Data | Model |
|---|---|---|---|
| $\mathbb{E}(\iota)$ | Share of insured deposits (%) | 66.8 | 66.7 |
| $\mathbb{E}(A^b/A^h)$ | Bank/non-bank ratio (%) | 85.9 | 85.1 |
| $\mathbb{E}(R^e)$ | Return on bank equity (%) | 12.98 | 13.04 |
| $\mathbb{E}(x/N^b)$ | Bank equity issuance (%) | 5.36 | 5.42 |
| $\mathbb{E}(\mathcal{D}^f)$ | Bank failure rate (%) | 0.76 | 0.81 |
| $\mathbb{E}(\tau(\cdot))$ | Marginal cost of public funds | 0.15 | 0.15 |
| std$(R^e)$ | Volatility, return on equity (p.p.) | 1.22 | 0.59 |
| std$(\iota)$ | Volatility insured shared of deposits (p.p.) | 0.4 | 0.38 |
| std$(Y)$ | Volatility output | 0.98 | 0.94 |

Notes:

of equity issuance $\upsilon$ is set to 142. It targets US banks' average annual real equity issuance of about 5.36% of pre-existing equity.

The share of non-bank-dependent physical capital in the physical capital aggregator, $\phi$, is fixed to match the about 86% bank to non-bank financing ratio in the economy, which is obtained following the same procedure as in De Fiore and Uhlig (2011).[15] The value of the elasticity of substitution parameter in the physical capital aggregator, $\rho$, and the marginal cost of screening effort, $\kappa$, are jointly set to match (i) volatility of return on bank equity; (ii) volatility of the share of insured deposits. The standard deviation of the aggregate productivity shock $\sigma_z$ is set to match the volatility of real GDP.

### 3.2.1 Business-cycle properties

In this subsection, I discuss the quantitative performance of the model by looking at its business-cycle properties.

Banks' choice of screening intensity is an important source of endogenous risk-taking in my model. As it is private information of the bank, there is no direct counterpart in the data to draw comparisons. Nonetheless, to study the business-cycle behaviour of banks' risk-taking, I rely on an empirical proxy based on the "Senior loan officer opinion survey on bank lending practices" published by the Federal Reserve Board.[16] The two panels of Figure

[15] In particular, I identify such a ratio with the ratio of corporate loans to corporate securities, which is calculated using the balance sheet of non-financial corporate businesses reported in the US Flow of Funds Accounts (Table B.103). Securities are the sum of commercial paper, municipal securities and corporate bonds. Loans are the sum of bank loans, mortgages and other loans and advances.

[16] The survey is administered by senior staff at the Federal Reserve Banks with knowledge of bank lending

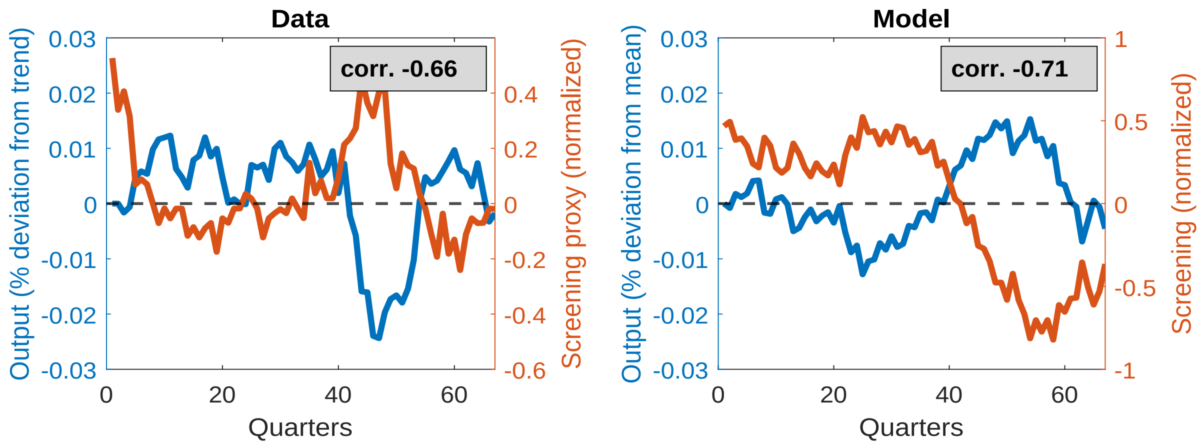


**Figure 1. Screening in the data and model**

Notes:

1 provide an account of the dynamics of banks' lending practices over the business-cycle in the data (left panel) and in the model (right panel).

The left vertical axis in the data panel reports % deviation of real GDP from its (HP-filtered) trend. The right vertical axis reports a standardized measure of the net share of banks that tighten their landing standards – normalized around 0 and ranging between $[-1, 1]$ – published each quarter by the Fed Board. Data corresponds to the period between 1990 Q2 - 2006 Q4. The left and right vertical axes in the model panel respectively report % deviation of output and banks' screening intensity from their respective means.

While the empirical proxy does not allow for a quantitative comparison of magnitudes, the model captures the strong countercyclicality of banks' screening effort. This means that banks relax lending standards during booms and tighten them during a recession, consistent with findings on banks' risk taking behaviour in Martinez-Miera and Suarez (2014).

Table 4 summarizes the business-cycle correlations of the model's banking sector and compares them to the data. The model generates realistic business-cycle correlations of banks' capital structure, the return on equity they generate, and their share of financing the production of physical capital in the economy. Without being a calibration target, the model captures well the countercyclicality of the share of insured bank deposits, as well as banks' equity issuance.

practices. As stated on the Board's website, "the purpose of the survey is to provide qualitative and limited quantitative information on bank credit availability and loan demand, as well as on evolving developments and lending practices in the U.S. loan markets."

**Table 4**
**Business-cycle correlations**
**(untargeted)**

| Variable | Data | Model |
|---|---|---|
| Total deposits | 0.395 | 0.84 |
| Insured deposits | 0.06 | 0 |
| Share of insured deposits | -0.57 | -0.84 |
| Equity | 0.15 | 0.84 |
| Equity issuance | -0.11 | -0.31 |
| Return on equity | 0.50 | 0.36 |
| Bank/non-bank ratio | 0.42 | 0.71 |

Notes:

### 3.3 Banking panics in the baseline economy

## 4 The effects of deposit insurance

## 5 Conclusion

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# Appendices

# A Model Details

## A.1 Producers

Total physical capital at $t+1$ is given by

$$K_{t+1} = \left[\phi \left(K^h_{t+1}\right)^\rho + (1-\phi)\left(K^b_{t+1}\right)^\rho\right]^{\frac{1}{\rho}}. \tag{A.1}$$

The consumption-good producer combines physical capital and labor to produce the final output (GDP)

$$Y_{t+1} = z_{t+1} K^\alpha_{t+1} H^{1-\alpha}_{t+1}. \tag{A.2}$$

The consumption-good producer's first order conditions for physical capital and labor yield

$$R^h_{t+1} = \phi\alpha \left(K^h_{t+1}\right)^{\rho-1} \frac{Y_{t+1}}{K^\rho_{t+1}} + 1 - \delta^h, \tag{A.3}$$

$$R^b_{t+1} = (1-\phi)\alpha \left(K^b_{t+1}\right)^{\rho-1} \frac{Y_{t+1}}{K^\rho_{t+1}} + 1 - \delta^b, \tag{A.4}$$

$$W_{t+1} = z_{t+1}(1-\alpha)\left(\frac{K_{t+1}}{H_{t+1}}\right)^\alpha. \tag{A.5}$$

## A.2 Household

I modify the household's maximization problem in the text by allowing for a riskless nominal bond $B_t$ which will be in zero net supply. I do so to be able to pin down the riskless nominal rate $R^s_t$. The first order conditions for riskless bonds, deposits, and investments in physical capital, are as follows:

$$\mathbb{E}_t \left[\Lambda_{t+1} R^s_t\right] = 1, \tag{A.6}$$

$$\mathbb{E}_t \left[\Lambda_{t+1} \tilde{R}^d_{t+1}\right] = 1, \tag{A.7}$$

$$\mathbb{E}_t \left[\Lambda_{t+1} R^h_{t+1}\right] = 1, \tag{A.8}$$

where the stochastic discount factor of the household can be defined as

$$\Lambda_{t+1} \equiv \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma},$$

and the realized (gross) return on deposits is

$$\tilde{R}^d_{t+1} \;=\; \iota_t R^d_t + (1-\iota_t)(R^d_t - \Omega_{t+1}),$$

with $\iota_t \equiv \min\{\frac{\overline{D}}{D_t}, 1\}$ being the household's insured share of deposits, and $\Omega_{t+1}$ the losses per unit of deposits defined below. The household's budget constraint is given by

$$C_t + D_t + A^h_t = W_t + \tilde{R}^d_t D_{t-1} + R^h_t A^h_{t-1} + \Xi_t - T_t, \tag{A.9}$$

and the household's net worth $N^h_{t+1}$ evolves according to the following law of motion

$$N^h_{t+1} = W_{t+1} + \tilde{R}^d_{t+1} D_t + R^h_{t+1} K^h_{t+1} + \Xi_{t+1} - T_{t+1}. \tag{A.10}$$

## A.3 Bankers

The bankers' first order condition for net equity issuance $x_t$ is

$$\nu_t = 1 + \Upsilon'(x^+_t), \tag{A.11}$$

where $\nu_t$, the marginal value of one unit of net worth of the banker, is

$$\nu_t = \mathbb{E}_t \left[ \Lambda_{t+1}(1-\theta+\theta\nu_{t+1}) R^e_{t+1} \right], \tag{A.12}$$

and the stochastic discount factor of the banker is defined as

$$\Lambda^b_{t+1} = \Lambda_{t+1}(1-\theta+\theta\nu_{t+1}).$$

The net worth of bankers $n^b_{t+1}$ evolves according to the following law of motion

$$n^b_{t+1} = \theta R^e_{t+1}(n^b_t + x_t). \tag{A.13}$$

### A.4 Banks

### A.5 Deposit insurance agency

### A.6 Market clearing and aggregation

### A.7 Equilibrium

## B Data sources

## C Solution Method