# 2025 Wolverine Math Tournament Middle School Division: Team Round

Westview Math Club

May 17th, 2025

## 1 Instructions/Information

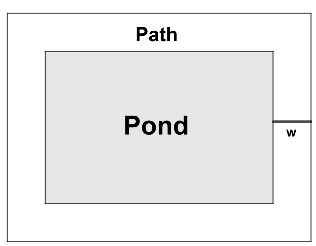
The questions are in increasing order of difficulty. Please write your answers on the answer sheets given. Simplify all fractions (write 1/2 instead of 2/4). Do not express your answer as a decimal (write 1/2, not 0.5). Write all expressions containing square roots in simplest radical form. Good luck!

## 2 Middle School Team Round Questions:

Time limit: 45 minutes

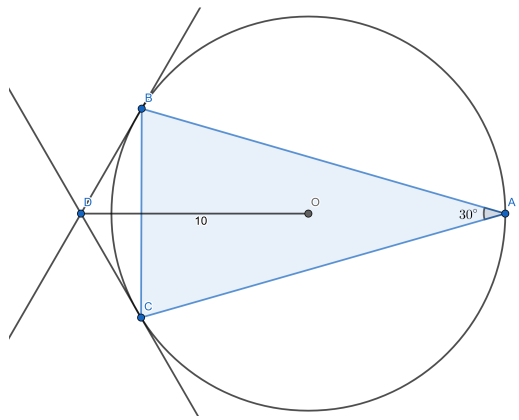
1. At the Westview cafe, a breakfast includes one drink, one main, and one pastry. The menu has 5 types of drinks, 7 types of main courses, and 4 types of pastries. However, due to supply constraints, if the customer chooses either the ”special latte” or the ”berry smoothie” as the drink, then they cannot choose the ”avocado toast” as the main course. Compute the total number of valid breakfast specials a customer can order.

2. A park’s rectangular pond measures 20 meters by 30 meters. A walking path of uniform width is built surrounding the pond so that the area of the path equals the area of the pond. Find the width (in meters) of the path.




3. Bubbykins and Luggy Puggy are 20 kilometers apart from each other on a straight road. They both start walking towards each other. Bubbykins walks at 4 kilometers per hour, while Luggy Puggy walks at 5 kilometers per hour. After a while, Bubbykins starts running at 10 kilometers per hour, while Luggy Puggy continues walking at 5 kilometers per hour. Bubbykins and Luggy Puggy stop when they meet each other. If the distance that Bubbykins walks equals the distance that he runs, find the total distance that Bubbykins traveled, in kilometers.

4. In a circle with center O, points A, B, and C lie on the circle so that the chords AB and AC satisfy $AB = AC$, and the inscribed angle ∠BAC=30 degrees. The tangents to the circle at B and C meet at D, with $\overline{OD} = 10$. Find the length of chord BC.




5. Rishi and Raunav are playing a rock-paper-scissors match. They agree to play a best-of-3 (first player to win 2 rounds wins). Rishi and Raunav are equally likely to win each round, and ties are ignored and replayed. However, if Rishi wins the best-of-3, Raunav insists on continuing the match and extending it to a best-of-5 (first to 3 wins). In this case, the scores from the best-of-3 carry over. What is the probability that Rishi wins the match?

6. Julian randomly chooses two distinct integers A and B between 1 and 217, inclusive. What is the probability that $A^2 - B^2$ is divisible by 7?

7. David, Jack, and James live along three different streets in the coordinate plane. David's house lies on the street $3x + y = 40$, Jack's on the street $x + 3y = 40$, and James's on the street $x = 3y$. They decide to meet at a location that is equidistant from all three streets. This meeting point is the center of a circle tangent to all three streets. Find the radius of this circle.

8. Find the sum of all values of x that satisfy the following system of equations, where x and y are real numbers:

$$\sqrt{9 + x - 6y} + \sqrt{25 + x - 10y} = 4$$

$$y + \sqrt{x + 6y^2 + 8y\sqrt{x}} = \sqrt{x} + \sqrt{15}y$$

9. While in math class, Jack finds 18 blocks with the letters W, E, S, T, V, I, E, W, W, O, L, V, E, R, I, N, E, S displayed on them. Using the blocks, how many distinct 4 letter sequences can he form that contain the letter E? For example, WEWW and EEOL are two possible such sequences, while SSSE is not.

10. If $N = 7^{7^{7^{7^{\cdot^{\cdot^{\cdot}}}}}}$, where there are 2025 7's, find the remainder when $N$ is divided by 1000.

**A special thanks to our sponsor Jane Street for helping make this contest possible!**

Jane Street

## 3 MS Team Round Answer Key

1. 132
2. 5
3. $\frac{32}{3}$
4. $5\sqrt{3}$
5. $\frac{13}{32}$
6. $\frac{11}{42}$
7. $\frac{2\sqrt{10}}{3}$
8. 40
9. 2897
10. 343

## 4 MS Team Round Solutions

1. At the Westview cafe, a breakfast includes one drink, one main, and one pastry. The menu has 5 types of drinks, 7 types of main courses, and 4 types of pastries. However, due to supply constraints, if the customer chooses either the "special latte" or the "berry smoothie" as the drink, then they cannot choose the "avocado toast" as the main course. Compute the total number of valid breakfast specials a customer can order.

   We can use complementary counting to find the total number of possible courses and subtract the ones that are invalid. First, we proceed with counting the total number of possible cases: $5 * 7 * 4 = 140$ possible total cases. Then, we can find the number of invalid cases. The invalid cases occur if we choose the "special latte" or the "berry smoothie (2 options) along with the "avocado toast" main course (1 option). We can choose anything for the remaining pastries, leading to $2 * 1 * 4 = 8$ invalid cases. $140 - 8 = \boxed{\mathbf{132}}$.

2. A park's rectangular pond measures 20 meters by 30 meters. A walking path of uniform width is built around the pond so that the area of the path equals the area of the pond. Find the width (in meters) of the path.

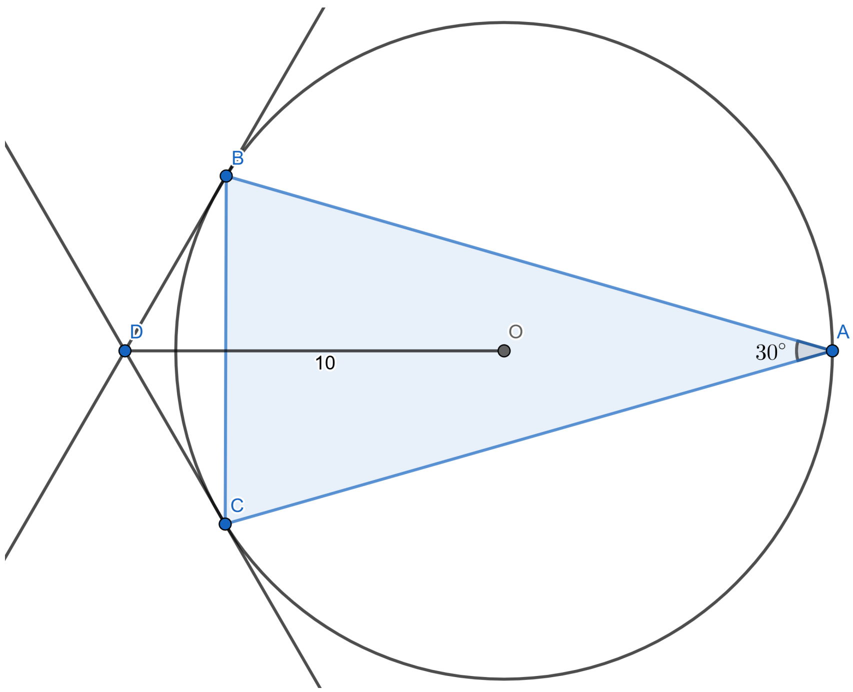
   Since the pond's area equals the path's area, both have an area of $20 * 30 = 600$, for a combined area of $600 * 2 = 1200$. Let the width of the path be x. The rectangle formed by combining the pond and the path has dimensions $2x + 20$ by $2x + 30$, and it has an area of 1200, as found earlier.

   Thus, we can find that $(2x + 20) * (2x + 30) = 1200$. Dividing everything by 4, we get that $(x + 10)(x + 15) = 300$. Expanding, we also get that $x^2 + 25x + 150 = 300$. After subtracting 300 from each side of the equation, we get that $x^2 + 25x - 150 = 0$. Factoring the quadratic, we can see that $(x + 30) * (x - 5) = 0$. This leads to two solutions: $x = 5$ and $x = -30$. The width of the path, or x, is a length, so we discard the negative solution to obtain x=$\boxed{\mathbf{5}}$.

3. Bubbykins and Luggy Puggy are 20 kilometers apart from each other on a straight road. They both start walking towards each other. Bubbykins walks at 4 kilometers per hour, while Luggy Puggy walks at 5 kilometers per hour. After a while, Bubbykins starts running at 10 kilometers per hour, while Luggy Puggy continues walking at 5 kilometers per hour. Bubbykins and Luggy Puggy stop when they meet each other. If the distance that Bubbykins walks equals the distance that he runs, find the total distance that Bubbykins traveled, in kilometers.

   Let x and y be the time that Bubbykins walks and runs, respectively. Then, the distance that Bubbykins walks is $4x$, and the distance that he runs is $10y$. Since the distance he runs equals the distance he runs, $4x = 10y$, so $y = \frac{2x}{5}$. The total time is x+y, so the distance that Luggy Puggy travels is $5(x + y)$. Since the sum of the distances is 20, we get that $4x + 10y + 5x + 5y = 20$, so $9x + 15y = 20$. Plugging in our expression for y, we obtain $15x = 20$, so x=$\frac{20}{15}$=$\frac{4}{3}$. Thus, the distance that Bubbykins runs is $4 * \frac{4}{3} = \frac{16}{3}$. Since the distance that Bubbykins walks equals the distance that Bubbykins runs, the total distance that Bubbykins travels is $2 * \frac{16}{3} = \boxed{\mathbf{\frac{32}{3}}}$.

4. In a circle with center O, points A, B, and C lie on the circle so that the chords AB and AC satisfy $AB = AC$, and the inscribed angle ∠BAC=30 degrees. The tangents to the circle at B and C meet at D, with $\overline{OD} = 10$. Find the length of chord BC.




Draw the line segments OB and OC. Notice that since tangent lines to a circle are perpendicular to the lines drawn from the center of the circle to the tangency point, DB and DC are perpendicular to OB and OC, respectively.
By the inscribed angle theorem, the smaller arc CB has a measure of 60°. Since OB=OC (they are radii of the same circle), $\triangle ABC$ is isosceles. Let E be the intersection point of OD and BC. By symmetry, BE=EC, and since chords that are bisected by a radii are also perpendicular to that same radii, OE is the perpendicular bisector of BC. Since the perpendicular bisector from the vertex of a isosceles triangle is also the angle bisector of the vertex, OE is also the angle bisector of $\angle BOC$. Since $\angle BOC$=60°, $\angle BOE$=$\angle COE$=30°. Therefore, since OB is perpendicular to BD, $\triangle OBD$is a 30-60-90 triangle. Thus, OB=$10 * \frac{\sqrt{3}}{2}$=$5\sqrt{3}$. Noticing that $\triangle OBC$ is an equilateral triangle, we get that $\overline{BC}$=$\overline{OB}$=$\boxed{\mathbf{5\sqrt{3}}}$

5. Rishi and Raunav are playing a rock-paper-scissors match. They agree to play a best-of-3 (first player to win 2 rounds wins). Rishi and Raunav are equally likely to win each round, and ties are ignored and replayed. However, if Rishi wins the best-of-3, Raunav insists on continuing the match and extending it to a best-of-5 (first to 3 wins). In this case, the scores from the best-of-3 carry over. What is the probability that Rishi wins the match?

Since ties are ignored and replayed, there is a $\frac{1}{2}$ probability of Rishi winning any given round. Lets find all the ways for Rishi to win, and then sum their probabilities. (W means Rishi wins, L means Raunav wins)

WWW, probability $\frac{1}{8}$
WWLW, probability $\frac{1}{16}$
WWLLW, probability $\frac{1}{32}$
WLWW, probability $\frac{1}{16}$
WLWLW, probability $\frac{1}{32}$
LWWWW, probability $\frac{1}{16}$
LWWLW, probability $\frac{1}{32}$

Summing the probabilities, we get $\frac{1}{8} + 3 * \frac{1}{16} + 3 * \frac{3}{32} = \boxed{\mathbf{\frac{13}{32}}}$

6. Julian randomly chooses two distinct integers A and B between 1 and 217, inclusive. What is the probability that $A^2 - B^2$ is divisible by 7?

   We start with the observation that $A^2 - B^2 = (A+B)(A-B)$. For this product to be divisible by 7, it must be that 7 divides $(A+B)$ or $(A-B)$. This is equivalent to saying that $A \equiv B \pmod 7$ or $A \equiv -B \pmod 7$. Since 217 is divisible by 7, ($\frac{217}{7} = 31$), there are exactly 31 numbers in each residue class modulo 7. We proceed by casework.

   Case 1. $A \equiv B \pmod 7$
   For this to happen, $A$ and $B$ must be from the same residue class. For each residue r (of which there are 7), there are 31 numbers. The number of ways to choose 2 distinct numbers from the same residue class is $\binom{31}{2}$=$\frac{31*30}{2}$. Since there are 7 residue classes, the total number of pairs is $7 * \frac{31*30}{2}$=$7 * 15 * 31$

   Case 2. $A \equiv -B \pmod 7$
   Note that if $A \equiv 0 \pmod 7$, then $A \equiv -A \pmod 7$ and these pairs are already counted in Case 1. So, we only consider the nonzero residues.
   The nonzero residues modulo 7 are 1, 2, 3, 4, 5, 6, and they pair off as (1, 6), (2, 5), (3, 4). For each such pair, there are 31 numbers for one residue class and 31 in the other residue class. This gives: $31 * 31$ pairs for each residue pair. Since there are 3 such distinct pairs, the total number of pairs (A, B) for this case is $31 * 31 * 3$.

   Adding the number of pairs from both cases: $7 * 15 * 31 + 31 * 31 * 3 = 31(15 * 7 + 31 * 3) = 31 * 3(35 + 31) = 31 * 3 * 66$. The total number of possible pairs is $\binom{217}{2}$=$\frac{217*216}{2}$=$7 * 31 * 108$. Thus, the probability is $\frac{31*3*66}{7*31*108} = \frac{3*66}{7*108} = \frac{3*3*2*11}{9*2*6*7} = \boxed{\mathbf{\frac{11}{42}}}$

7. David, Jack, and James live along three different streets in the coordinate plane. David's house lies on the street $3x + y = 40$, Jack's on the street $x + 3y = 40$, and James's on the street $x = 3y$. They decide to meet at a location that is equidistant from all three streets. This meeting point is the center of a circle tangent to all three streets. Find the radius of this circle.

   We want to find the inradius of a triangle that has sides on the 3 equations x=3y, x+3y=40, and 3x+y=40. Notice that x=3y and 3x+y=40 are perpendicular, as they have opposite reciprocal slopes. Thus, the triangle is a right triangle. Solving the three equations a pair at a time, we get that the intersection points are (10, 10), (12, 4), and (20, $\frac{20}{3}$). Then, we find the side lengths of the triangle. We take the coordinates 2 at a time, and find the distances between them. We get that the side lengths are $\frac{8\sqrt{10}}{3}$, $2\sqrt{10}$, and $\frac{10\sqrt{10}}{3}$. The area is $\frac{\frac{8\sqrt{10}}{3}*2\sqrt{10}}{2} = \frac{80}{3}$. The semi-perimeter, s, is $\frac{\frac{8\sqrt{10}}{3}+2\sqrt{10}+\frac{10\sqrt{10}}{3}}{2} = 4\sqrt{3}$. Plugging this into the formula inradius=$\frac{a}{s}$, where a and s are the area and semi-perimeter, respectively, we get that the inradius is $\frac{\frac{80}{3}}{4\sqrt{10}} = \frac{20}{3\sqrt{10}} = \boxed{\mathbf{\frac{2\sqrt{10}}{3}}}$

8. Find the sum of all values of x that satisfy the following system of equations, where x and y are real numbers:

   $$\sqrt{9 + x - 6y} + \sqrt{25 + x - 10y} = 4$$

   $$y + \sqrt{x + 6y^2 + 8y\sqrt{x}} = \sqrt{x} + \sqrt{15}y$$

Lets focus on the second equation first. Isolate the large, messy square root term by moving the y to the right side. Proceed by squaring both sides and simplifying.

$\sqrt{x+6y^2+8y\sqrt{x}} = \sqrt{x} + y(\sqrt{15}-1)$

$\Rightarrow x+6y^2+8y\sqrt{x} = x+2\sqrt{x}y(\sqrt{15}-1)+y^2(16-2\sqrt{15})$

$\Rightarrow 6y^2+8y\sqrt{x} = 2\sqrt{x}y(\sqrt{15}-1)+y^2(16-2\sqrt{15}$
Notice that we can cancel y from both sides

$\Rightarrow 6y+8\sqrt{x} = 2\sqrt{x}\sqrt{15}-2\sqrt{x}+16y-2\sqrt{15}y$

$\Rightarrow \sqrt{x}(10-2\sqrt{15}) = y(10-2\sqrt{15})$

$\Rightarrow x = y^2$

Substituting this result into the first equation gives:

$\sqrt{y^2-6y+9}+\sqrt{y^2-10y+25} = 4$

$\Rightarrow |y-3|+|y-5| = 4$, which has solutions y=2 and y=6. Thus, the sum of all values of x is $2^2+6^2 = \boxed{\mathbf{40}}$

9. While in math class, Jack finds 18 blocks with the letters W, E, S, T, V, I, E, W, W, O, L, V, E, R, I, N, E, S displayed on them. Using the blocks, how many distinct 4 letter sequences can he form that contain the letter E? For example, WEWW and EEOL are two possible such sequences, while SSSE is not.

   We first note that there are 4 E's, 3 W's, 2 S's, 2 V's, 2 I's, and 1 each of T, O, L, R, and N. We proceed with casework, with the cases being the repetition pattern in the word (the number of times a letter appears)

   All letters distinct (1, 1, 1, 1):
   There needs to be an E. There are 9 other letters, and $\binom{9}{3}$=210 ways to choose 3 of them. There are $4! = 24$ different ways to order them, for a total of $84 * 24 = 2016$ possible words for this case.

   Two of one letter, the other two distinct (2, 1, 1):
   There are 2 subcases: 1 E or 2E's.
   1 E: 4 choices for the letter that appears twice, 8 choices for the other letter than appears once, $8 * 4 = 32$ cases.
   2 E's: $\binom{9}{2}$= 36 ways to choose the 2 that only appear once.
   There are 12 permutations, so there are $12(36+32) = 12 * 68 = 816$ words for this case.

   Three of one letter, one other (3, 1)
   There are only two letters that appear at least 3 times: E and W. If the letter that appears 3 times is W, since there has to be an E, the other letter is E. This is one possibility. If the letter that appears 3 times is E, then there are 9 ways to choose the letter that appears once. There are 4 ways to rearrange the letters after they have been chosen, for a total of $4(9+1) = 40$ words for this case.

   Two of one letter, Two of another letter

Since there must be an E, one of the two distinct letters has to be E. There are 4 other letters that appear at least twice, so there are 4 ways to choose the other letter. Once the letters have been chosen, there are $\binom{4}{2}$=6 ways to rearrange the word. There are $4 * 6 = 24$ total words for this case.

All letters the same
There is one case: E is used 4 times.

Summing up all of the cases, we get that the total number of distinct 4-letter sequences containing the letter E is $2016 + 816 + 40 + 24 + 1 = \boxed{\mathbf{2897}}$.

10. If $N = 7^{7^{7^{7^{\cdot^{\cdot^{\cdot}}}}}}$, where there are 2025 7's, find the remainder when $N$ is divided by 1000.

Because $\gcd(7, 1000) = 1$, Euler's theorem applies. $\phi(1000) = 1000(1 - \frac{1}{5})(1 - \frac{1}{2}) = 400$. By Euler's theorem, $7^{400} \equiv 1(\text{mod } 1000)$. Let the given expression be written as $7^E$, where $E$ is $7^{7^{\cdot^{\cdot^{\cdot}}}}$, where there are 2024 7's. We want to find $E$ mod 400.

Lets work our way down from the top of the exponent tower. Notice that $7^4 = 2401 \equiv 1$ mod 400. Therefore, $7^7 \equiv 7^3 \equiv 343$ mod 400. However, this becomes the exponent of the 7 beneath it, so we can take this mod 4 (because $7^4 \equiv 1$ mod 400) to get $343 \equiv 3$ mod 4. This process keeps on repeating, all the way down until we get to $E \equiv 7^{7^7} \equiv 7^{7^3} \equiv 7^{343} \equiv 7^3 \equiv 343$ mod 400.

All that remains is to calculate $7^E = 7^{343}$ mod 1000. One way to do this is to find the multiplicative order of 7 mod 1000. $7^2 \equiv 1$ mod 8, so the multiplicative order of 7 mod 8 is 2. Now for 125. $7^{10} \equiv (7^4)^2 * 7^2 \equiv (26)^2 * 49 \equiv 51 * 49 = 50^2 - 1 \equiv -1$ mod 125. Therefore, $7^{20} \equiv (7^{10})^2 \equiv (-1)^2 \equiv 1$ mod 125. Combining this with the result for 8, we find that $7^{20} \equiv 1$ mod 1000. Thus, $7^{343} \equiv (7^{20})^{17} * 7^3 \equiv 7^3 \equiv \boxed{\mathbf{343}}$ mod 1000.