2025 Wolverine Math Tournament Elementary School Division: Team Round

Westview Math Club

May 17th, 2025

1 ES Team Round Questions:

The questions are in increasing order of difficulty. The last few are particularly challenging. Please write your answers on the answer sheets given. Simplify all fractions (write 1/2 instead of 2/4). Do not express your answer as a decimal (write 1/2, not 0.5). Good luck!

Time limit: 45 minutes

1. What is $\frac{1}{2+\frac{6}{1+\frac{3}{10}}}$? Express your answer as a fraction in simplest form.

2. David has a bag filled with marbles. The bag contains 3 red marbles, 5 blue marbles, and 8 green marbles. If a marble is drawn at random, what is the probability that it is blue?

3. Felix's bookshelf contains 4 times as many fiction books as nonfiction books. If 10 nonfiction books are added, there are then 3 times as many fiction books as nonfiction books. How many books were originally on the bookshelf?

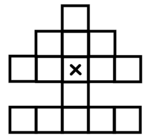
4. Rishi the racer just lost his previous race. He needs some upgrades to win his next race, but he only has 20 dollars. He currently finishes the race in 100 seconds. What will be the shortest time he can finish his next race in (in seconds), if the length of the racetrack does not change, and you cannot buy multiple of the same upgrade?

Upgrade	**Cost**	**Time it Saves**
New Wheels	\$3	10 seconds
New Engines	\$10	35 seconds
Rocket Booster	\$18	60 seconds
New Brakes	\$1	3 seconds
Oil Change	\$6	15 seconds
Stickers	\$2	5 seconds

5. Vedant finds a bag with an unknown amount of tokens in it, but he knows there are more than 5 tokens. When divided into 5 equal groups, 2 tokens remain. When divided into 7 equal groups, 2 tokens remain. What is the smallest possible number of tokens that could be in the bag?

6. The Wolverine Park is designed in the shape of an isosceles trapezoid. Its two parallel sides have lengths that differ by 20 meters, and its height is 25 meters. If the area of the trapezoid is 2025 square meters, find the length of the longest side.

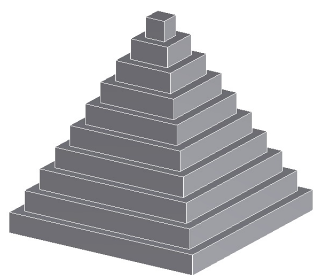
7. The Westview robotics team built a floor cleaning robot to tidy up the school quad, shaped like the grid below. The robot starts on the center tile marked X. Every 30 seconds, it can move up, down, left, or right to a neighbouring tile (not diagonally) and can revisit tiles. What is the shortest amount of time in minutes for the robot to visit every cell and clean the whole quad?



8. Hamza the hedgehog starts at the bottom-left corner of a 4×4 grid, at point $(0, 0)$, and wants to reach the top-right corner at point $(4, 4)$. He can only move one step at a time, either right or up. However, on his way to the destination, Hamza must pass through the point $(2, 2)$. How many different routes can Hamza take?

9. Philip is hiking in a forest when he realizes he needs water. A river runs in a straight line 4 miles south (directly below) of his current position running from west to east (left to right). His cabin is located 15 miles directly east (to the right) of where he is standing. He must first travel to the river to collect water before heading to his cabin. What is the shortest possible distance he can travel to complete this journey?

10. A pyramid-shaped structure is built using square-based rectangular prisms (square bases with height 1) stacked on top of each other, each with square bases and a uniform height of 1 inch. The bottom platform is 100 square inches, the next is 81, 64, so on. The pattern continues until the top rectangular prism is a square with base 1. What is the total surface area of the tower? The bottom of the tower does not count towards the surface area.



A special thanks to our sponsor Jane Street for helping make this contest possible!

Jane Street

2 MS Team Round Answer Key

1. $\frac{13}{86}$
2. $\frac{5}{16}$
3. 150
4. 35
5. 37
6. 91
7. 10
8. 36
9. 17
10. 320

3 ES Team Round Solutions

1. What is $\frac{1}{2+\frac{6}{1+\frac{3}{10}}}$? Express your answer as a fraction in simplest form.

   A lot of fractions! We can simplify step by step. Firstly, it equals $\frac{1}{2+\frac{6}{(\frac{13}{10})}}$. We can simplify this to $\frac{1}{2+\frac{60}{13}}$. This further simplifies to $\frac{1}{(\frac{86}{13})}$. After simplifying this again, we get that the answer is $\boxed{\frac{13}{86}}$.

2. David has a bag filled with marbles. The bag contains 3 red marbles, 5 blue marbles, and 8 green marbles. If a marble is drawn at random, what is the probability that it is blue?

   The probability is $\frac{5}{3+5+8} = \boxed{\frac{5}{16}}$

3. Felix's bookshelf contains 4 times as many fiction books as nonfiction books. If 10 nonfiction books are added, the new ratio becomes 3 : 1. How many books were originally on the bookshelf?

   We can let the amount of nonfiction books at the start equal x. Thus, at first, the bookshelf contains $4x$ fiction books. After 10 nonfiction books are added, we now have $x + 10$ nonfiction books. Using the ratio we are given (3 : 1), we can get the equation $\frac{4x}{x+10} = \frac{3}{1} = 3$. Simplifying, $3x + 30 = 4x$, so $x = 30$. Because originally there were $4x$ fiction books and x nonfiction books, the total amount of books was $5x$, or $\boxed{150}$.

4. Oskar Pastry is in need of upgrades to win his next race, but he only has 20 dollars. He currently finishes the race in 100 seconds. What will be the shortest time he can finish his next race in (in seconds)? You cannot buy multiple of the same upgrade.

Upgrade	**Cost**	**Time it Saves**
New Wheels	\$3	10 seconds
New Engines	\$10	35 seconds
Rocket Booster	\$18	60 seconds
New Brakes	\$1	3 seconds
Oil Change	\$6	15 seconds
Stickers	\$2	5 seconds

   That's a lot of possibilities! Naturally, one idea comes to mind: casework. We can define the cases as follows:
   Case 1: Sets containing the rocket booster upgrade
   Case 2: Sets not containing the rocket booster upgrade

   First, we can look at case 1. If we buy the rocket booster upgrade, we are left with 2 dollars. Spending two dollars on the stickers leads to a total of 65 seconds saved, while spending one dollar on new brakes leaves one dollar behind (which can't purchase anything because there are no double purchases) and only leads to 63 seconds saved. Thus, the maximum time we can save for this case is 65 seconds.

   Now, we can look at case 2. This is where things get a little more tricky. If we do not buy the new engines, the maximum amount of time we can save (by purchasing new wheels, new brakes, oil change, and stickers (yes, there is enough money because it only costs $1 + 2 + 3 + 6 = 12$)) is $3 + 5 + 10 + 15 = 33$ seconds saved, which isn't greater than 65 seconds saved, so we must buy new engines. If we buy the new engines, we are left with 10 dollars. This means that at most, we can only buy 3 items (because the remaining 4 items cost a total of $1 + 2 + 3 + 6 = 12$ dollars, which exceeds 10. We have to either subtract the stickers, the new wheels, or the oil change. The total amount of time saved if we have the New Engines, New Wheels, New Brakes, Oil Change, and the Stickers is $3+5+10+15+35 = 68$ seconds. We must subtract an item that has greater than two dollars in value. This means that the total amount of time saved for this case would be $\leq 68 - 5 = 63$ seconds saved (subtracting the stickers subtracts the least amount of time from time saved).

Since the maximum time we can save for both cases is 65 seconds, the most amount of time we can save is $100 - 65 = \boxed{\mathbf{35}}$ seconds.

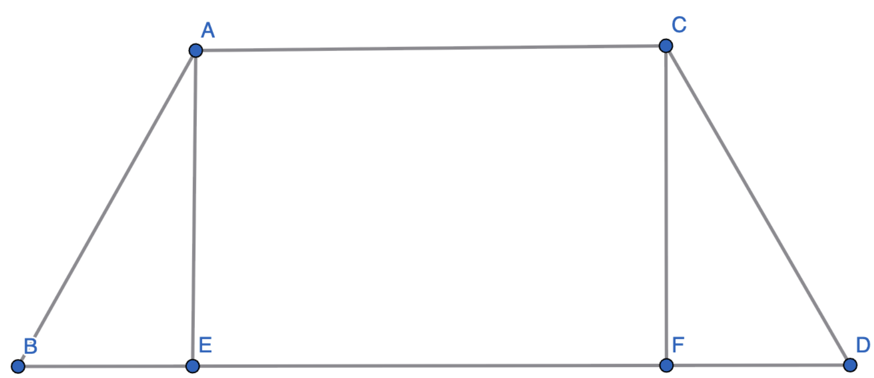
5. Vedant finds a bag of tokens. When divided into 5 equal groups, 2 tokens remain. When divided into 7 equal groups, 2 tokens remain. What is the smallest number of integral tokens that satisfies all three conditions, greater than 5?

   Method 1: We can see that if we ignore the greater than 5 condition, 2 is the smallest positive number that satisfies this. The question asks for a number greater than 5, so we have to think a little bit more. We can see that 5 and 7 are relatively prime, meaning that $5 * 7 = 35$ is the smallest positive integer that is divisible by 5 and 7. Thus, adding 35 has no effect on the remainder when divided into equal groups of 5 and 7. The answer is $35 + 2 = \boxed{\mathbf{37}}$

   Method 2: We are trying to find the least positive integer that is greater than 5 and is $\equiv 2 \pmod 5$ and $\equiv 2 \pmod 7$. By CRT, since 5 and 7 are relatively prime, there is a unique integer that satisfies these conditions $\pmod{35}$. We can see that this solution is $\equiv 2 \pmod{35}$, so the least integer that is $\equiv 2 \pmod{35}$ and greater than 5 is $35 + 2 = \boxed{\mathbf{37}}$

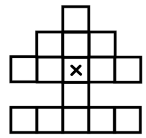
6. The Wolverine Park is designed in the shape of an isosceles trapezoid. Its two parallel sides have lengths that differ by 20 meters, and its height is 25 meters. If the area of the trapezoid is 2025 square meters, find the length of the longest side.

   We can let the longer base of the trapezoid equal x. Thus, the smaller base will be $x - 20$ because they differ by 20 and the smaller base's length is less than the long base's length. The area of a trapezoid is (longer base + shorter base)/2*height. Plugging in, we get that $(x+x-20)/2*25 = 2025$. Thus, $(x-10)*25 = 2025$. Simplifying, we get $x-10 = 2025/25 = 81$. Thus, $x = 91$. We still need to verify that this indeed is the longest side, so let's find the length of the legs of the trapezoid. We can look at a quick diagram:



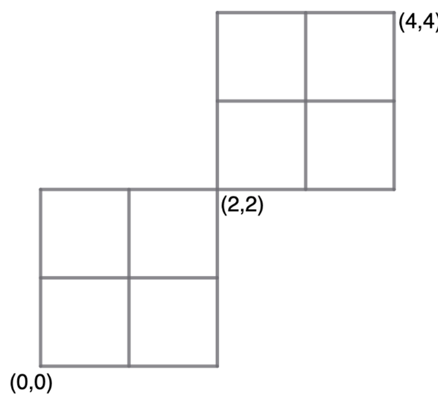
   We can draw altitudes to BD from A and C, and call the feet of these altitudes E and F respectively. We know that $\overline{AC} = 71$, and since it's $ABCD$ is an isosceles trapezoid, we also know that $\overline{EF} = 71$ and $\overline{BE} = \overline{DF} = (91 - 71)/2 = 10$. We also know that the height $= \overline{AE} = \overline{CF} = 15$. By Pythagorean theorem, $\overline{AB} = \overline{CD}$ equals $\sqrt{10^2 + 15^2} = \sqrt{325}$. We can figure out that $\sqrt{325}^2 = 325 < 91^2 = 8281$, meaning that $\overline{AB} = \overline{CD} < x$, or $\overline{BD}$. Thus, $\overline{BD}$ is the longest side, and it equals x, which equals $\boxed{\mathbf{91}}$.

7. The Westview robotics team built a floor cleaning robot to tidy up the school quad, shaped like the grid below. The robot starts on the center tile marked X. Every 30 seconds, it can move up, down, left, or right to a neighbouring tile (not diagonally) and can revisit tiles. What is the shortest amount of time in minutes for the robot to visit every cell and clean the whole quad?



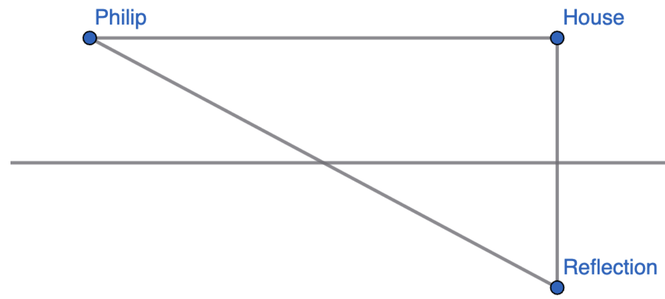
Testing possible paths, you will quickly find that the shortest path has a length of 20 moves, which will take $20 * \frac{1}{2} = \boxed{\mathbf{10}}$ minutes.

8. Hamza the hedgehog starts at the bottom-left corner of a 4×4 grid, at point $(0, 0)$, and wants to reach the top-right corner at point $(4, 4)$. He can only move one step at a time, either right or up. However, on his way to the destination, Hamza must pass through the point $(2, 2)$. How many different routes can Hamza take?



Hamza can only take the paths available in the diagram if he is to pass through $(2, 2)$. We can look at the bottom left 2x2 square and the top right 2x2 square. If we look at just one of these squares, there must be two points where Hamza goes to the right and two ways where Hamza goes up because he starts at the bottom left of these squares and ends up at the top right point of these squares. Let U=up and R=right. This means that we are actually counting the sequences consisting of 2 U's and 2 R's in some order. This is just $\binom{4}{2}$ (to select where the 2 U's or 2 R's go), which is just 6. Because there are 2 2x2 squares, the answer is $6^2 = \boxed{\mathbf{36}}$ ways.

9. Philip is hiking in a forest when he realizes he needs water. A river runs in a straight line 4 miles south (directly below) of his current position running from west to east (left to right). His cabin is located 15 miles directly east (to the right) of where he is standing. He must first travel to the river to collect water before heading to his cabin. What is the shortest possible distance he can travel to complete this journey?



The diagram consists of the original scene with Philip and his house reflected across the river (the line in the middle that bisects the house and its reflection). The distance Philip travels is minimized when he travels from where he is on a straight line to the reflection of his house. This is because the distance from the point he gets water from the river to the reflection of his house is the same as the distance from the point he gets water to his house, because the diagram is a reflection. This occurs when he walks in a straight line toward the reflection of his house, forming a right triangle. The legs of this right triangle are $2 \cdot 4$ and 15, or 8 and 15. Thus, the minimum is $\sqrt{8^2+15^2} = \boxed{\mathbf{17}}$ miles.

10. A pyramid-shaped structure is built using square-based rectangular prisms (square bases with height 1) stacked on top of each other, each with square bases and a uniform height of 1 inch. The bottom platform is 100 square inches, the next is 81, 64, so on. The pattern continues until the top rectangular prism is a square with base 1. What is the total surface area of the tower? The bottom of the tower does not count towards the surface area.

    Method 1: We can calculate the total surface area by calculating the sum of the vertical sides' surface area and the horizontal sides' surface area.

    First, if we look at the pyramid from the bottom up, we can see that the surface area of the bottom of the pyramid has $10^2 = 100$. Looking from the top down, we can also see that the squares being stacked have no effect on the total surface area - they still have a cumulative total of $10^2 = 100$.

    Now, we can calculate the total vertical areas on the pyramid. If we look at each layer, if the rectangular prism is k by k by 1, its sides have a total area of $k\cdot1+k\cdot1+k\cdot1+k\cdot1 = 4k$. This, we can see that the total sum of the vertical sides is $4(10+9+8+7+6+5+4+3+2+1) = 4*11/2*10$. This equals 220.
    Now we can add up 220 and 200 to get 420. Since the bottom isn't included, we have to subtract 100 to get $\boxed{\mathbf{320}}$

    Method 2: We can calculate the total surface area of the ten square-based rectangular prisms and then subtract the faces that are not shown. The total surface area of one of the square-based rectangular prisms with side length k is $2k^2+4k$, or $2(k)(k+2)$, which becomes $2((k+1)^2-1)$. Thus, the sum of the total surface areas of the ten rectangular prisms is $2(11^2-1+10^2-1+ \ldots 2^2-1)$, or $2(11^2-1+\ldots 2^2-1+1^2-1) = 2(11^2+\ldots 1^2)-22$. This equals $2*(11*12*23/6)-22$, or 990. Now, we have to subtract the overlaps or the faces that aren't showing. This occurs when a square base covers another portion of area. For example, the bottom base of the rectangular prism has area 81, so we have to subtract $2*81$ (because we are subtracting the surface area from the top of the rectangular prism with side length 10 and subtracting the surface area from the bottom of the rectangular prism with side length 9). We have to subtract $2*(9^2+8^2+\ldots 1^2)$, which is $2*(9*10*19/6)$, or 570. $990-570 = 420$. We still have to subtract 100 because the bottom isn't counted, so we get that the answer is $\boxed{\mathbf{320}}$.