MTMW14: Numerical Modelling of Atmosphere and Oceans
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Project 1: The ocean recharge oscillator, a reduced coupled model for ENSO illustrating the effects of nonlinearity and forcing, based on the model of Fei-Fei Jin (1997).

The aim of this project is to make a recharge oscillator coupled model in order to illustrate the El Nino Southern Oscillation (ENSO). This model will be based on the model of Fei-Fei Jin (1997). The recharge oscillator model (ROM) is described by two differential equations:

$$\frac{dh_w}{dt} = -rh_w - \alpha bT_E - \alpha\xi_1 \tag{1}$$

$$\frac{dT_e}{dt} = RT_E + \gamma hw - e_n(hw + bT_E)^3 + \gamma\xi_1 + \xi_2 \tag{2}$$

Where T_E denotes the east Pacific SST anomaly and h_w indicates the west Pacific ocean thermocline depth. $R = \gamma b - c$ describes the Bjerknes positive feedback process. γ specifies the feedback of the thermocline gradient on the SST gradient and c is the damping rate. b is a measure of the thermocline slope which depends upon b_0 which is the coupling parameter and μ being the coupling coefficient. r represents damping of upper ocean heat content and α relates enhanced easterly wind stress to the recharge of ocean heat content. ξ_1 and ξ_2 are external forcings.

For this experiment, the Runge-Kutta fourth-order iterative time scheme was used. This model was non-dimensionalised using [T] = 7.5 K; [h] = 150 m; [t] = 2 months. All the below tasks are conducted for a time step of 1 day and five time periods of 41 months each.

Initially, the model was run without any external forcings (ξ_1 and ξ_2) and non-linearity (e_n = 0). The initial conditions were set to T = 1.125 K and h = 0. μ is the coupling coefficient, changing which changes the extent to which the cases are coupled. For a low value of μ, the system decays whereas a high value of μ causes strong coupling which results in the growth rate being too fast. But when $\mu = \mu_c$, it reaches a supercritical state where the growth rate is zero and it gives a perfect harmonic oscillation. This value was found to be $\mu = 2/3$ for a period of 41 months which we implemented in our model for Task A and ran the model. It can be seen from Fig. 1 (a) that the period for both T_E and h_w is approximately 41 months with T_E and h_w having a lag of around 6 months. Fig 1(b) shows an elliptical orbit without any significant decay. I ran this code for twenty time periods in order to check stability. The solution is stable without any decay for 820 months.

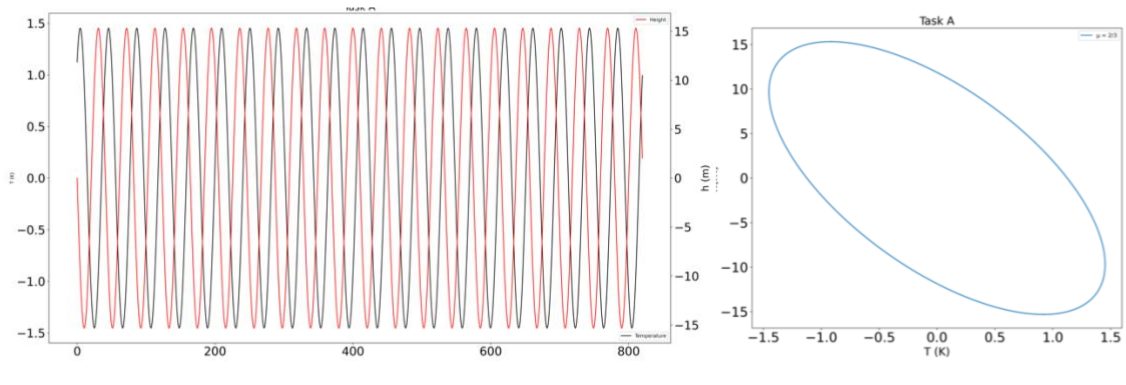

Fig. 1. Model without external forcings and non-linearity for $\mu = 2/3$. Left plot shows timeseries of Temperature (T) in K and height(h) in m vs time in months. Right plot shows trajectory plot of T (K) vs h(m)

The model was then run for 5 time periods with $\mu > 2/3$ and $\mu < 2/3$. For $\mu > 2/3$, as discussed earlier a linear solution can be seen in Fig 2(a) where the growth rate was too fast whereas, for $\mu < 2/3$ as shown in Fig. 2(b), the solution starts at $T_E = 1.125$ K and decays very quickly.

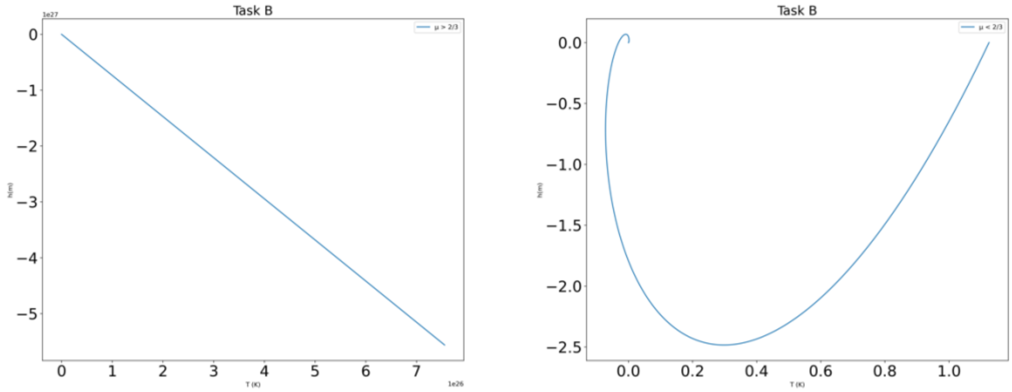

Fig. 2. Model without external forcings and non-linearity for $\mu = 1 > 2/3$ on left (a) and $\mu = 1/3 < 2/3$ on right (b). Both of them are timeseries of Temperature (T) in K and height(h) in m vs time in months

The non-linearity term e_n is then set to 0.1 for $\mu = 2/3$. When this non-linearity term is included, the time series for both temperature and height starts decaying (Fig. 3(a)) and the trajectory plot also decays strongly over 5 time periods.

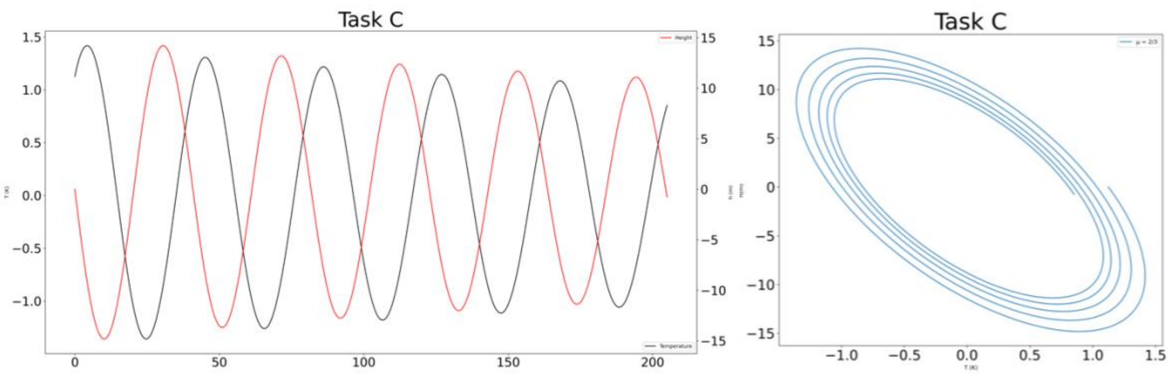

Fig. 3 Model without external forcings and non-linear coefficient en = 0.1for $\mu = 2/3$. Left plot (a) shows timeseries of Temperature (T) in K and height(h) in m vs time in months. Right plot (b) shows trajectory plot of T (K) vs h(m)

But when μ is set to 0.75 which is over its critical value, for the non-linear case, the time series starts growing but eventually stabilizes at 70 months after when it is very stable. These can be observed in Fig 4 for both the time series and trajectory plot.

The coupling parameter μ was then varied annually using:

$$\mu = \mu_0 \left(1 + \mu_{ann} \cos \left(2\pi t/\tau - 5\pi/6\right)\right)$$

where $e_n = 0.1$, $\mu_0 = 0.75$, $\mu_{ann} = 0.2$ and $\tau = 12$ months. τ was non-dimensionalised.

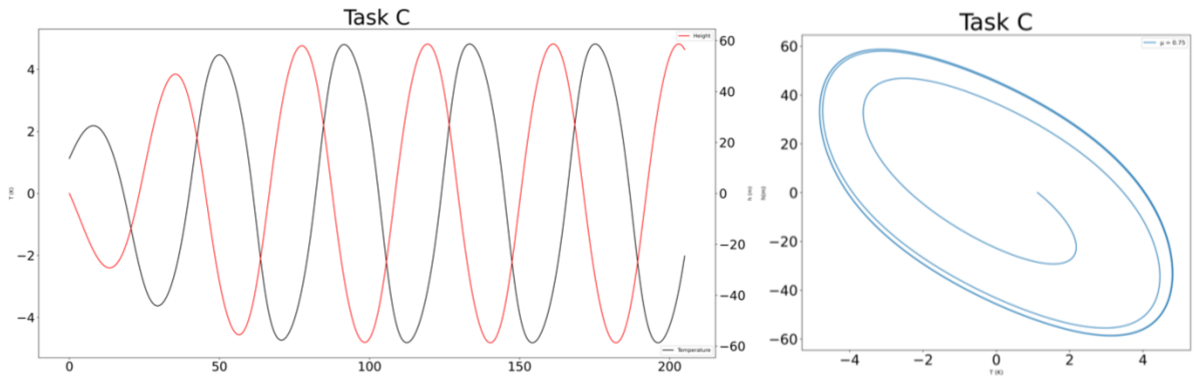

Fig. 4 Model without external forcings and non-linear coefficient en = 0.1for $\mu = 0.75$. Left plot (a) shows timeseries of Temperature (T) in K and height(h) in m vs time in months. Right plot (b) shows trajectory plot of T (K) vs h(m)

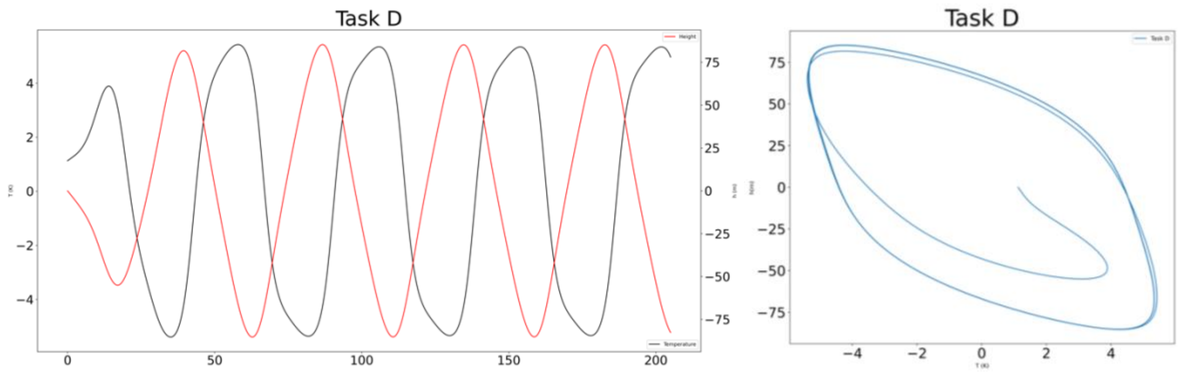

Fig. 5. Model without external forcings and non-linear coefficient en = 0.1 where μ is set to vary annually. Left plot (a) shows timeseries of Temperature (T) in K and height(h) in m vs time in months. Right plot (b) shows trajectory plot of T (K) vs h(m)

For this case, the time series for height does not have a significant impact but the temperature time series has a different irregular oscillation. Even though the number of cycles does not change, a delay in temperature peaks of around a year can be observed. For the trajectory plot, just like task C, it gets stable over time. The shape of the trajectory is not a perfect ellipse but instead looks more like a parallelogram. These results are illustrated in Fig. 5.

In order to check the stochastic initiation hypothesis, noisy wind forcing is added to this model. ξ_1 which was defined previously as an external parameter will be the external wind forcing. It can be set as:

$$\xi_1 = f_{ann} \cos(2\pi t/\tau) + f_{ran} W \, \tau_{cor}/\Delta t$$

where W is a number randomly generated between -1 and 1 for every interval τ_{cor}. This is added in order to depict white noise. The non-linear parameter is set to 0, $f_{ann} = 0.02$, $f_{ran} = 0.2$, and $\tau_{cor} = 1$ day.

For $\mu_0 = 0.75$, this task was showing very unstable plots (Fig. 6) hence μ_0 was set to 2/3. For this external wind forcing, the time series for both temperature and height show significantly smaller amplitude compared to the previous task (Fig. 7(a)). Especially for the height, it oscillates between 20 and -20 m whereas previously it was between 75 and -75 m. Also at the peaks, small perturbations can be observed which is a result of the random forcing added. For the trajectory plot (Fig. 7(b)), the noise can be seen as the lines look very fuzzy with small vibrations.

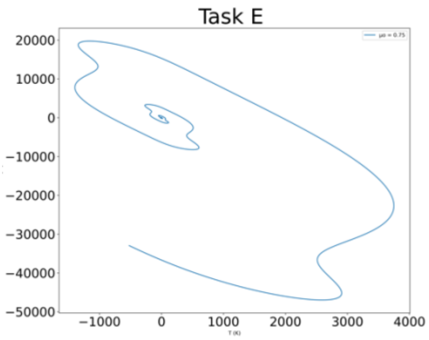

Fig. 6 Model with external forcings of wind and non-linear coefficient en = 0 where μ set to vary annually with μ_0=0.75. Plot shows trajectory plot of T (K) vs h(m)

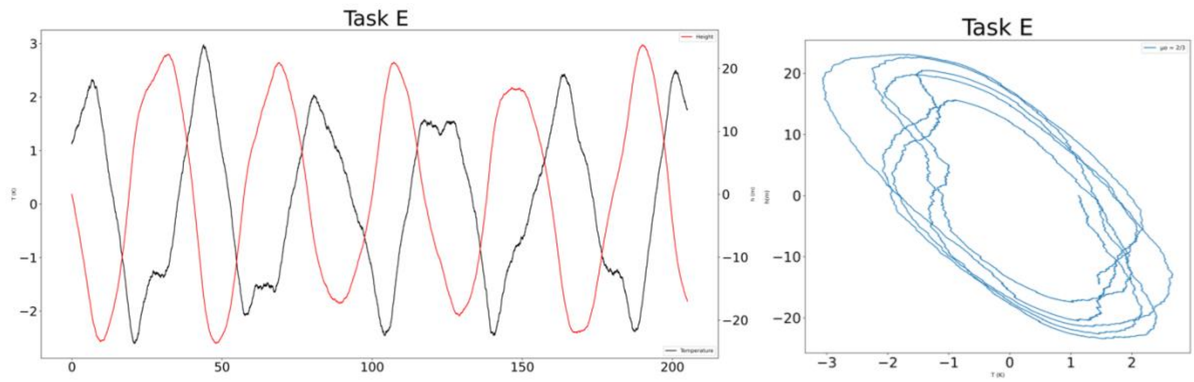

Fig. 7. Model with external wind forcing and non-linear coefficient en = 0 where μ is set to vary annually with μ_0=2/3. Left plot (a) shows timeseries of Temperature (T) in K and height(h) in m vs time in months. Right plot (b) shows trajectory plot of T (K) vs h(m)

Looking at the trajectories in Fig. 8 where only one of either f_{ann} , f_{ran} was turned on. For the annual trajectory, no vibrations can be observed whereas for the Random trajectory, the vibrations are clearly visible.

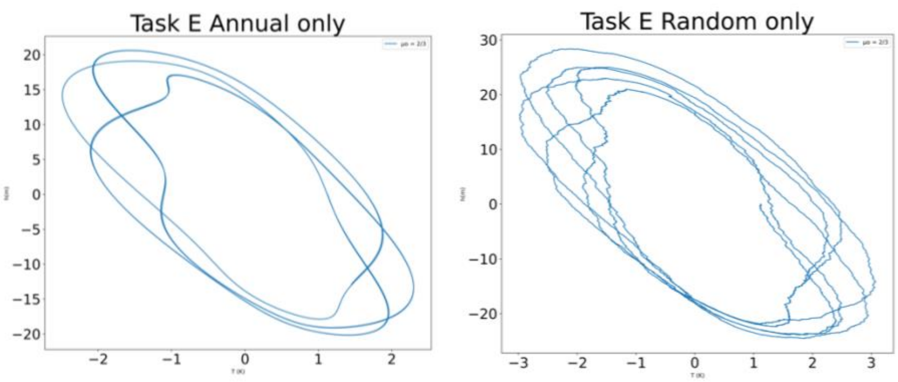

Fig. 8. Model with external wind forcing and non-linear coefficient en = 0 where μ is set to vary annually with μ_0=2/3. Left plot (a) shows trajectory plot of T (K) vs h(m) with fran set to 0. Right plot (b) shows trajectory plot of T (K) vs h(m) with fann set to 0.

When non-linearity is turned on for the above task i.e. e_n = 0.1 and μ_0 is again set to 0.75, the trajectory looks very stable compared to the linear model with stochastic forcing (Fig. 9). In this case, despite the noise parameter being turned on, the vibrations are less noticeable compared to the previous task. However, the maximum oscillation amplitude has grown back to 60 m for the height and 5 K for the temperature.

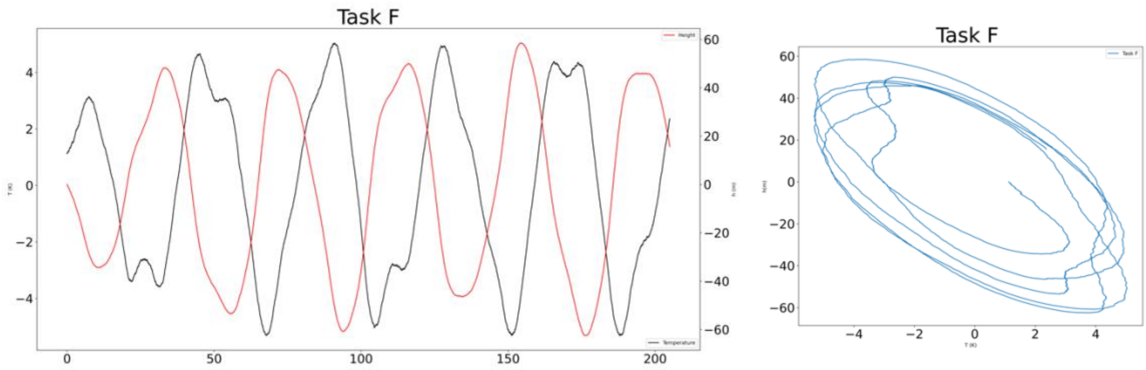

Fig. 9. Model with external wind forcing and non-linear coefficient en = 0.1 where μ is set to vary annually with μ_0=0.75. Left plot (a) shows timeseries of Temperature (T) in K and height(h) in m vs time in months. Right plot (b) shows trajectory plot of T (K) vs h(m)

For the next task ensembles were generated to study chaotic behavior. Chaos was first observed by Edward Lorenz in the 1950s when he accidentally rounded off the temperature to three decimal points instead of six and observed a significant difference in the time series of the two temperatures. Climate models usually simulate chaos when mathematical equations are used and since Fei-Fei Jin's model that we have used is based on mathematical equations, it is important to test if this model is chaotic.

The initial conditions were perturbed in order to check if the model is sensitive to them. Different random arrays were added to both Temperature and height arrays for every ensemble and then passed through the Range Kutta function. Temperature perturbation was set to ±0.2 K as the temperature ranges between 4 to -4 K for most cases. This number was chosen for uncertainty as it is under 10% of the range. The same was done for height and the uncertainty was set to ±2 m as height mostly varies between 30 to -30 m on average. The perturbations were not rounded off so by default they reach up to 8 significant digits after the decimal. This was done in order to check if a change of 8 significant digits will give a significantly different output. Twelve ensembles were generated in order to compare them with the previous case as these ensembles are based on the initial conditions given in Task F. The results are shown in Figures 10 and 11. We can observe from Fig. 10 that for height, even though we perturbed the initial conditions by only ± 2 m, for ensembles 4 and 11, the heights reach 300m and -250m which is 4 times what we observed in task F. Even though the uncertainty was so small the output exceeded by more than 100 times the uncertainty. Whereas for temperature, the uncertainty was ±0.2 K, for most of the ensembles, the temperature does not vary much unlike height. From Fig. 11 we can observe that for height, it usually starts with an extreme value but then stabilizes and oscillates in the range observed in Task F. Also, both variables are out of phase but the phase difference between them for all the ensembles is almost the same. When it comes to the noise generated from the wind forcing, there isn't much difference in the impact when compared with the previous case. Also looking at the plume diagram (Fig. 12) and comparing it with the SST plot for ENSO by (Risbey et al., 2021), we can observe that for temperatures, some of them start with a positive value whereas some with a negative. This should not be the case as for ensembles, the values must be different by a small magnitude but not of a different sign for the same magnitude. For the SST multi-model ensemble mean, the ensembles are averaged and then plotted but for our case, it will have very small variations as half of the ensembles are out of phase from the other half. It is due to chaos. Hence overall, when it comes to the height as well as temperature, the system shows some chaotic properties. Therefore, Fei-Fei Jin's model can be said to be chaotic.

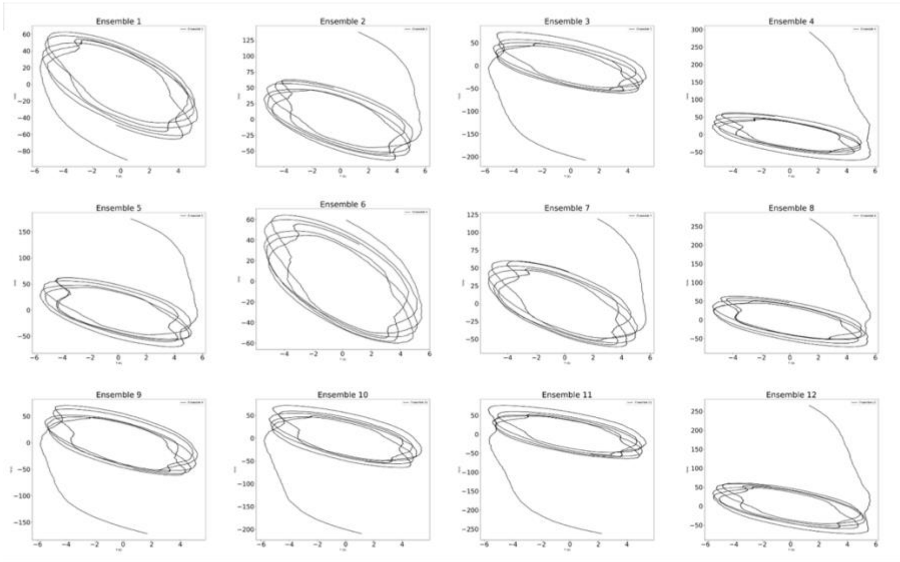

Fig. 10. Model with external wind forcing and non-linear coefficient en = 0.1 where μ is set to vary annually with μ_0=0.75. Trajectory plot s of T (K) vs h(m) of ensembles 1-12 with perturbations added to initial conditions

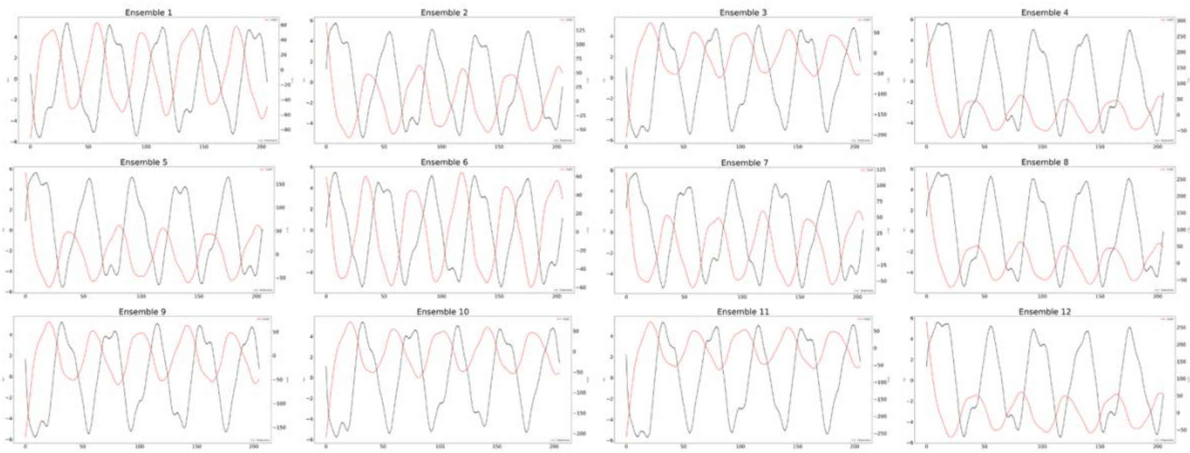

Fig. 11. Model with external wind forcing and non-linear coefficient en = 0.1 where μ is set to vary annually with μ_0=0.75. Timeseries of Temperature (T) in K and height(h) in m vs time in months of ensembles 1-12 with perturbations added to initial conditions

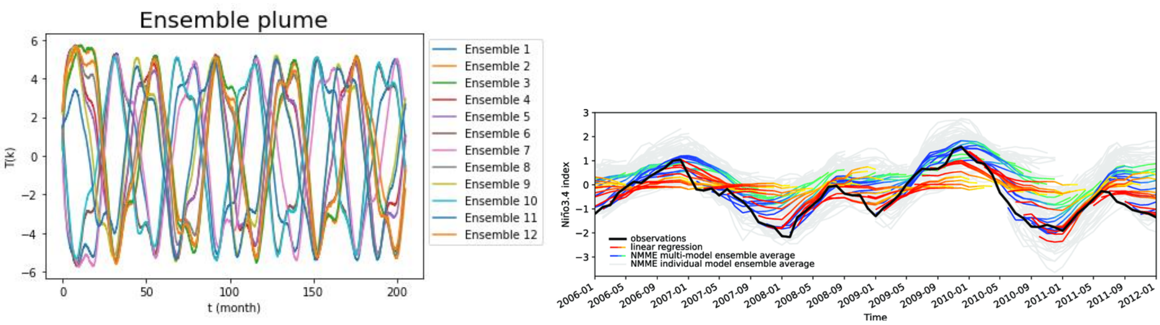

Fig. 12. Plume diagram of temperatures for all ensembles plotted vs time in months (left). El Niño Southern Oscillation (ENSO) time series and forecasts. The black line shows the observed value of Niño3.4. The blue/green lines are 12-month forecasts of Niño3.4 for the North American Multimodel Ensemble (NMME) and Copernicus Climate Change Service (C3S) multimodel ensemble mean. The shading for these forecast lines runs from dark blue for the first month of the forecast through green for long lead times. The red/yellow lines are 12-month forecasts of Niño3.4 for the linear regression model, where the first month is shaded red and longer leads are yellow. The grey lines are ensemble mean forecasts from individual NMME and C3S models. The fair method is used to calculate anomalies (Risbey et al., 2021).

References:

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