

MTMW14: Assignment 1

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Initial conditions

```
In [78]: import numpy as np
import matplotlib.pyplot as plt
import random
from matplotlib.pyplot import figure

r = 0.25
alpha = 0.125
gamma = 0.75
b0 = 2.5
c = 1
Tnon = 7.5
hnon = 150
tnon = 2
dt1 = 1/30
tmin = 0
tmax = 41*5
dt = dt1/Tnon
tfin = tmax/Tnon
nt = int(tfin/dt)

Times=np.linspace(tmin,tfin,nt)
```

Equations

```
In [79]: def hval(r,alpha,b,T,h,E1):
    return -r*h - alpha*b*T - alpha*E1

def Tval(b,R,gamma,T,h,en,E1,E2):
    return R*T + gamma*h - en*(h + b*T)**3 + gamma*E1 + E2

def Selfexcite(mu0,muann,tau,t):
    return mu0*(1+muann*np.cos((2*np.pi*t/tau)-(5*np.pi/6)))

def noisywind(fann,fran,tau,t,W,tau_corr,delt):
    return fann*np.cos(2*np.pi*t/tau) + fran*W*tau_corr/delt
```

Runge Kutta Scheme

```
In [63]: def RungeKutta(Tinit,hinit,mu0,muann,tau,t,en,E2,fann,fran,tau_corr):
    T = (Tinit/Tnon)*np.ones(nt)
```

```

h = (hinit/hnon)*np.ones(nt)

for i in range(nt-1):

    W = np.random.uniform(-1,1)
    E1 = noisywind(fann,fran,t[i],W,tau,dt)
    b = b0*Selfexcite(mu0,muann,tau,t[i])
    R = gamma*b - c

    k1 = Tval(b,R,gamma,T[i],h[i],en,E1,E2)
    l1 = hval(r,alpha,b,T[i],h[i],E1)

    k2 = Tval(b,R,gamma,T[i]+(k1*dt/2),h[i]+(l1*dt/2),en,E1,E2)
    l2 = hval(r,alpha,b,T[i]+(k1*dt/2),h[i]+(l1*dt/2),E1)

    k3 = Tval(b,R,gamma,T[i]+(k2*dt/2),h[i]+(l2*dt/2),en,E1,E2)
    l3 = hval(r,alpha,b,T[i]+(k2*dt/2),h[i]+(l2*dt/2),E1)

    k4 = Tval(b,R,gamma,T[i]+(k3*dt),h[i]+(l3*dt),en,E1,E2)
    l4 = hval(r,alpha,b,T[i]+(k3*dt),h[i]+(l3*dt),E1)

    T[i+1]=T[i]+ 1/6 * dt*(k1 + 2*k2 + 2*k3 + k4)
    h[i+1]=h[i] + 1/6 * dt*(l1 + 2*l2 + 2*l3 + l4)

return T,h

```

Task 1

```

In [64]: T,h = RungeKutta(1.125,0,2/3,0,1,Times,0,0,0,0,0)
Tnew = T*Tnon
hnew = h*hnon

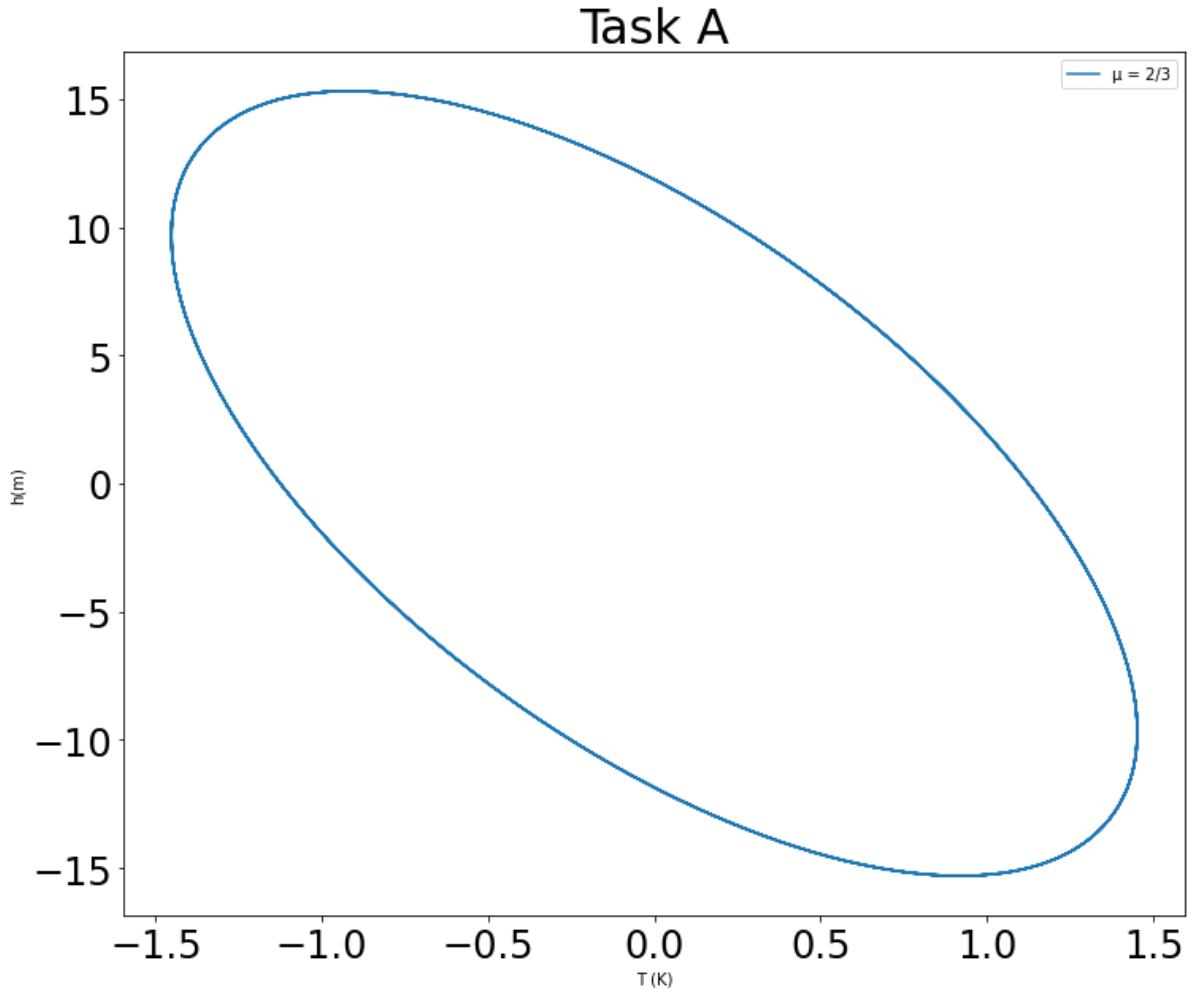
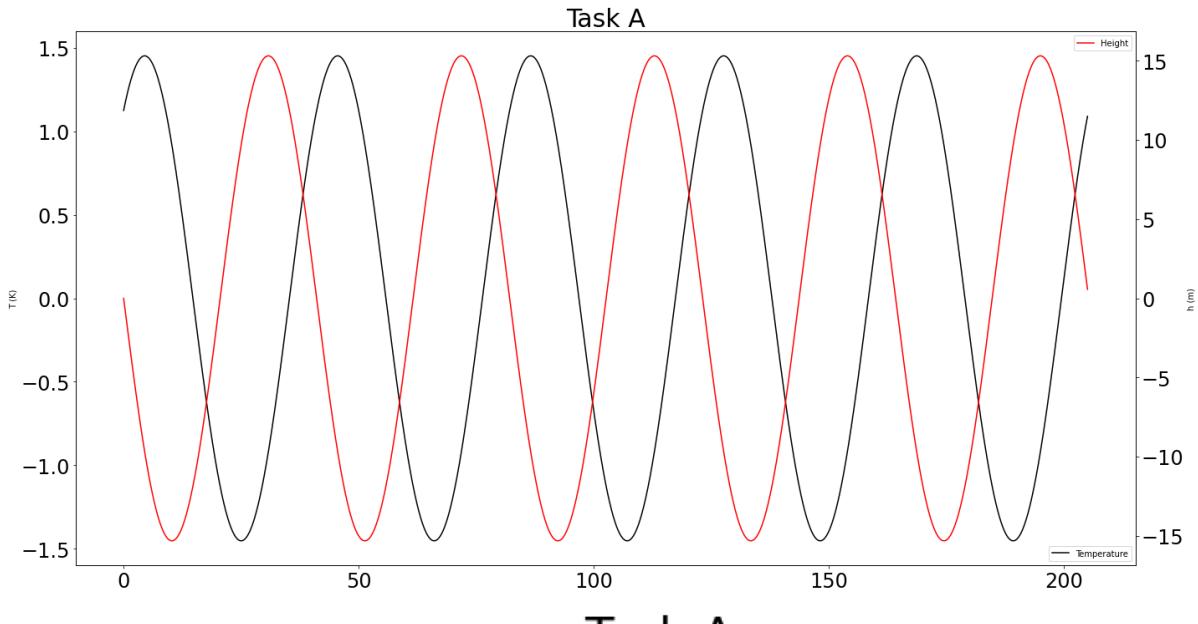
fig, ax1 = plt.subplots(figsize=(20, 10))

ax1.plot(Times*tnew,Tnew, 'k-', label='Temperature')
plt.ylabel('T (K)')
plt.legend(loc='lower right')
ax2=ax1.twinx()
ax2.plot(Times*tnew,hnew, 'r-', label='Height')
plt.ylabel('h (m)')
plt.legend(loc='best')
ax1.xaxis.set_tick_params(labelsize=24)
ax1.yaxis.set_tick_params(labelsize=24)
ax2.yaxis.set_tick_params(labelsize=24)
plt.xlabel('t (month)')
plt.title('Task A', fontsize = 30)
plt.tight_layout()
#plt.show()
filename='RK-timeseries.png'
fig.savefig(filename, dpi=300)

fig, ax1 = plt.subplots(figsize=(12, 10))
#Against each other
ax1.plot(Tnew,hnew,label='mu = 2/3')
ax1.xaxis.set_tick_params(labelsize=24)
ax1.yaxis.set_tick_params(labelsize=24)
plt.legend(loc='best')
plt.xlabel('T (K)')
plt.ylabel('h(m)')
plt.title('Task A', fontsize = 30)

```

```
#plt.show()
figname='RK-phase.png'
fig.savefig(figname, dpi=300)
```



Task B

$$\mu < 2/3$$

```
In [65]: T, h = RungeKutta(1.125, 0, 1/3, 0, 1, Times, 0, 0, 0, 0, 0)
Tnew = T*Tnon
```

```

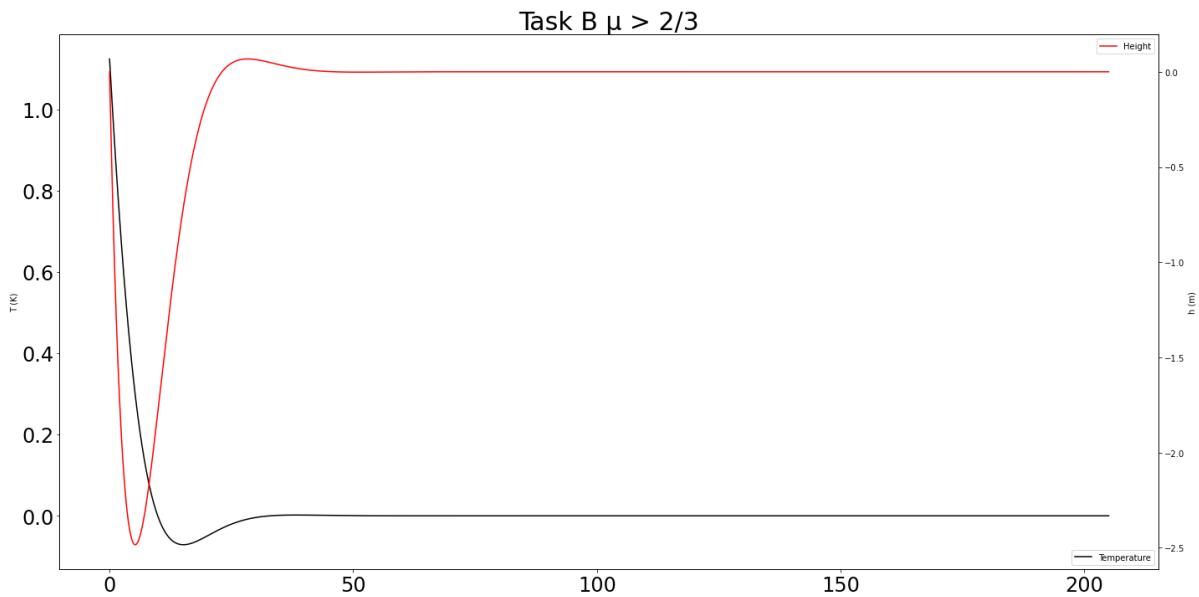
hnew = h*hnon

fig, ax1 = plt.subplots(figsize=(20, 10))

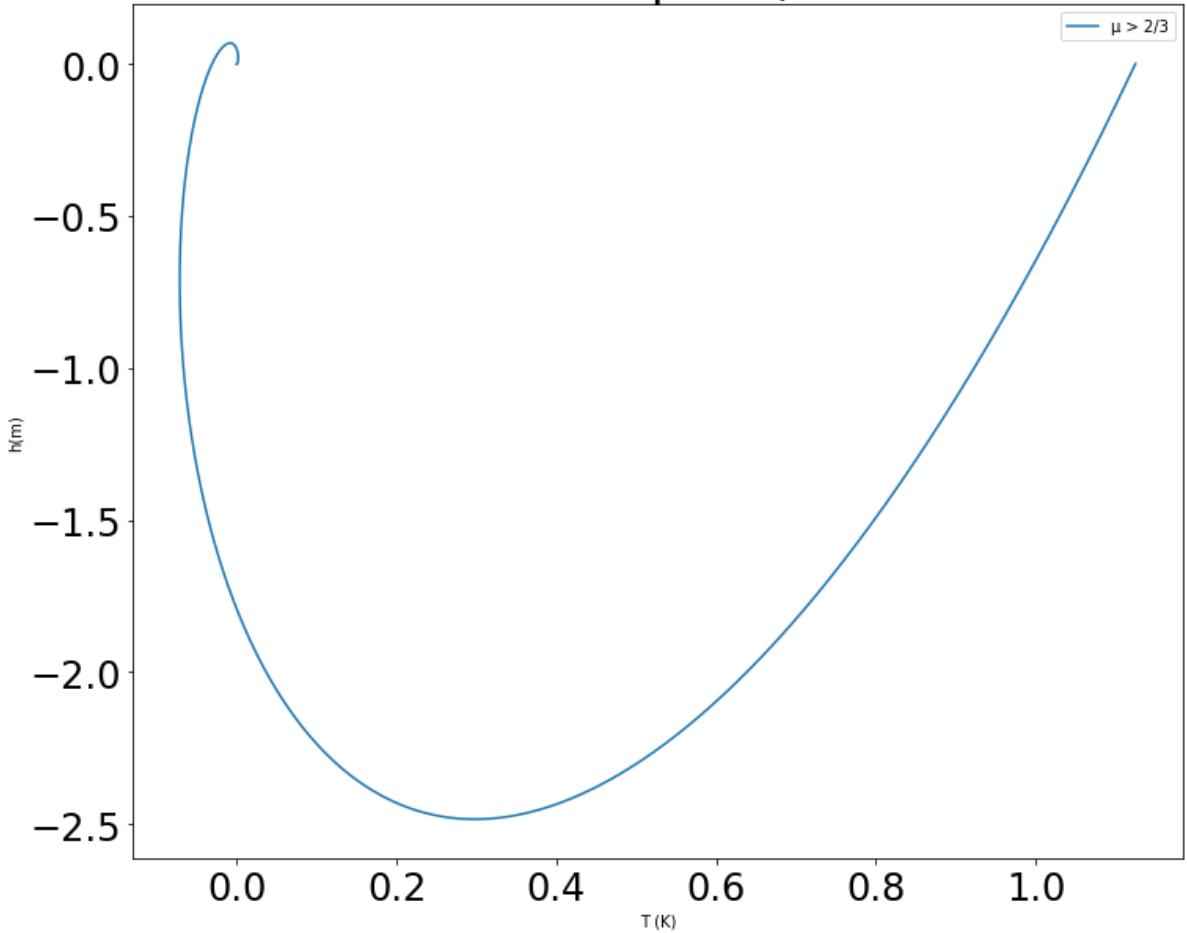
ax1.plot(Times*tnon, Tnew, 'k-', label='Temperature')
plt.ylabel('T (K)')
plt.legend(loc='lower right')
ax1.xaxis.set_tick_params(labelsize=24)
ax1.yaxis.set_tick_params(labelsize=24)
ax2.yaxis.set_tick_params(labelsize=24)
ax2=ax1.twinx()
ax2.plot(Times*tnon, hnew, 'r-', label='Height')
plt.ylabel('h (m)')
plt.legend(loc='best')
plt.xlabel('t (month)')
plt.ylabel('h (m)')
plt.title('Task B  $\mu > 2/3$ ', fontsize = 30)
plt.tight_layout()
#plt.show()
filename='RK-timeseriestask2a.png'
fig.savefig(filename, dpi=300)

fig, ax1 = plt.subplots(figsize=(12, 10))
#Against each other
ax1.plot(Tnew,hnew,label=' $\mu > 2/3$ ')
plt.legend(loc='best')
ax1.xaxis.set_tick_params(labelsize=24)
ax1.yaxis.set_tick_params(labelsize=24)
plt.xlabel('T (K)')
plt.ylabel('h(m)')
plt.title('Task B  $\mu > 2/3$ ', fontsize = 30)
#plt.show()
filename='RK-phasetask2a.png'
fig.savefig(filename, dpi=300)

```



Task B $\mu > 2/3$



$\mu > 2/3$

```
In [66]: T,h = RungeKutta(1.125,0,1,0,1,Times,0,0,0,0,0)
Tnew = T*Tnon
hnew = h*hnon

fig, ax1 = plt.subplots(figsize=(20, 10))

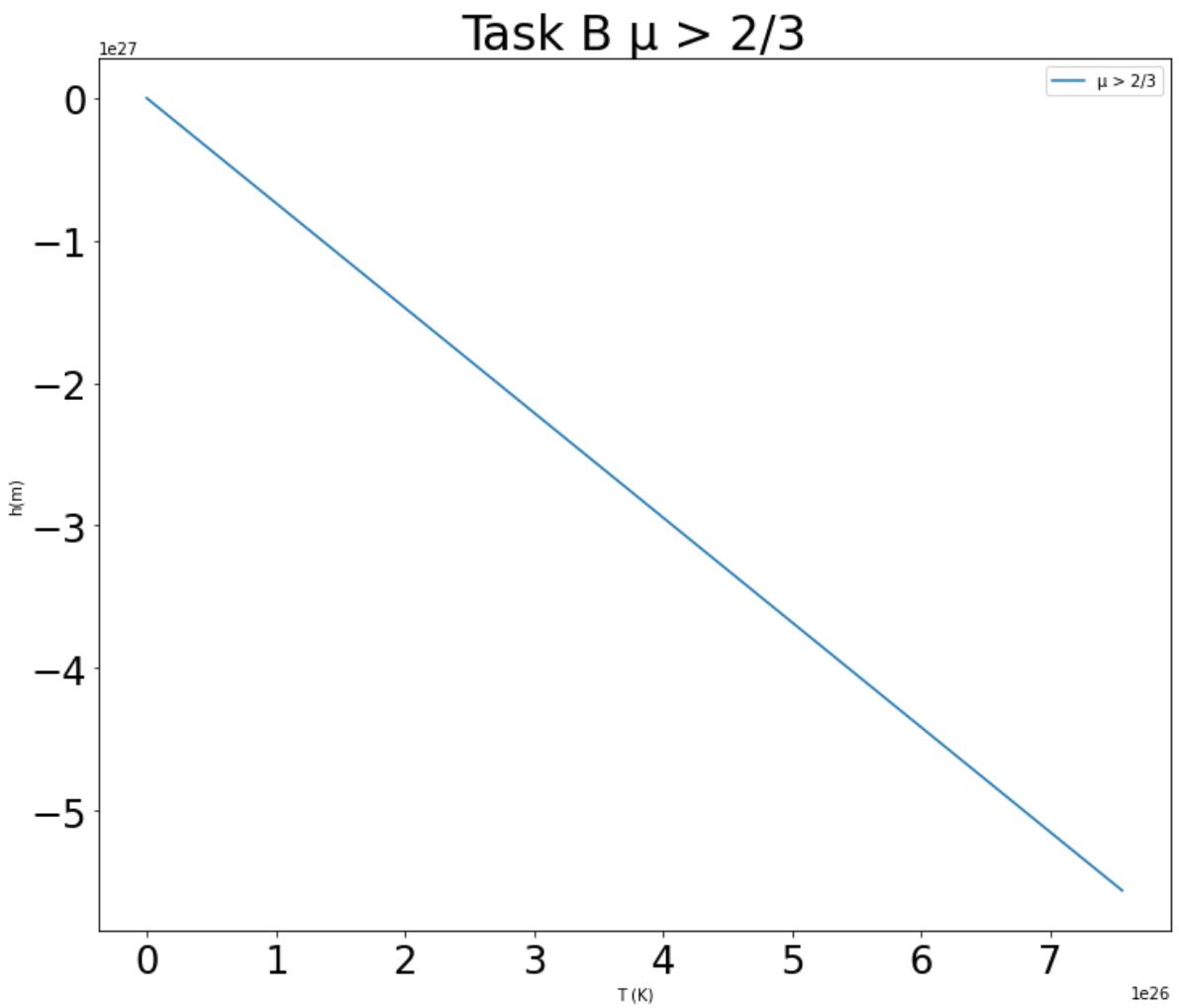
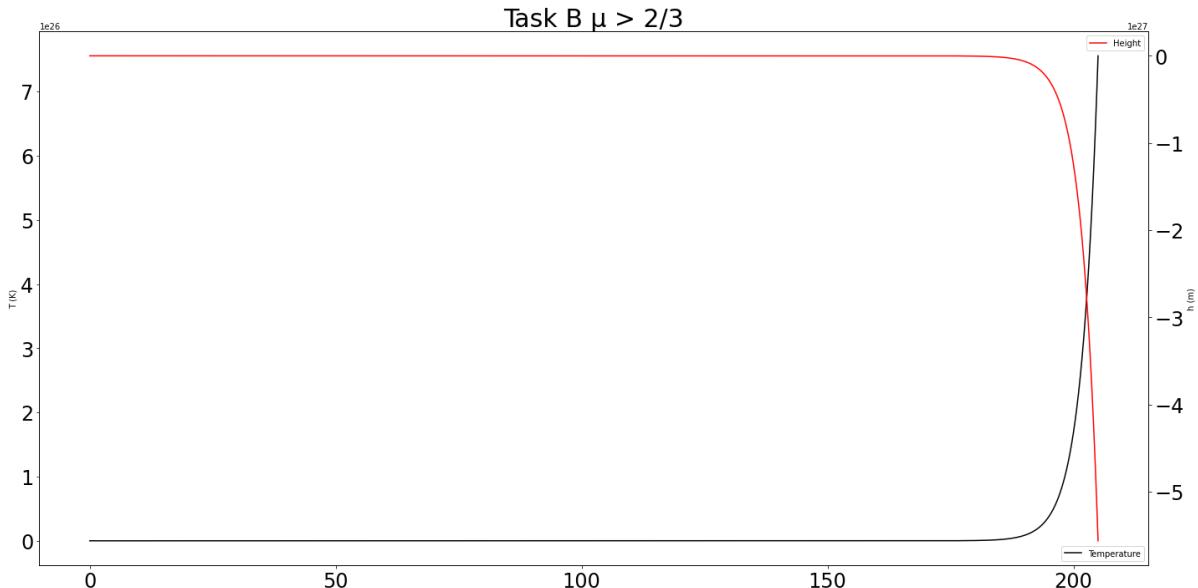
ax1.plot(Times*tnon,Tnew, 'k-', label='Temperature')
plt.ylabel('T (K)')
plt.legend(loc='lower right')
ax2=ax1.twinx()
ax2.plot(Times*tnon,hnew, 'r-', label='Height')
plt.ylabel('h (m)')
plt.legend(loc='best')
ax1.xaxis.set_tick_params(labelsize=24)
ax1.yaxis.set_tick_params(labelsize=24)
ax2.yaxis.set_tick_params(labelsize=24)
plt.xlabel('t (month)')
plt.ylabel('h (m)')
plt.title('Task B  $\mu > 2/3$ ', fontsize = 30)
plt.tight_layout()
#plt.show()
filename='RK-timeseriestask2a.png'
fig.savefig(filename, dpi=300)

fig, ax1 = plt.subplots(figsize=(12, 10))
#Against each other
ax1.plot(Tnew,hnew,label=' $\mu > 2/3$ ')
ax1.xaxis.set_tick_params(labelsize=24)
ax1.yaxis.set_tick_params(labelsize=24)
```

```

plt.legend(loc='best')
plt.xlabel('T (K)')
plt.ylabel('h(m)')
plt.title('Task B  $\mu > 2/3$ ', fontsize = 30)
#plt.show()
filename='RK-phasetask2a.png'
fig.savefig(filename, dpi=300)

```



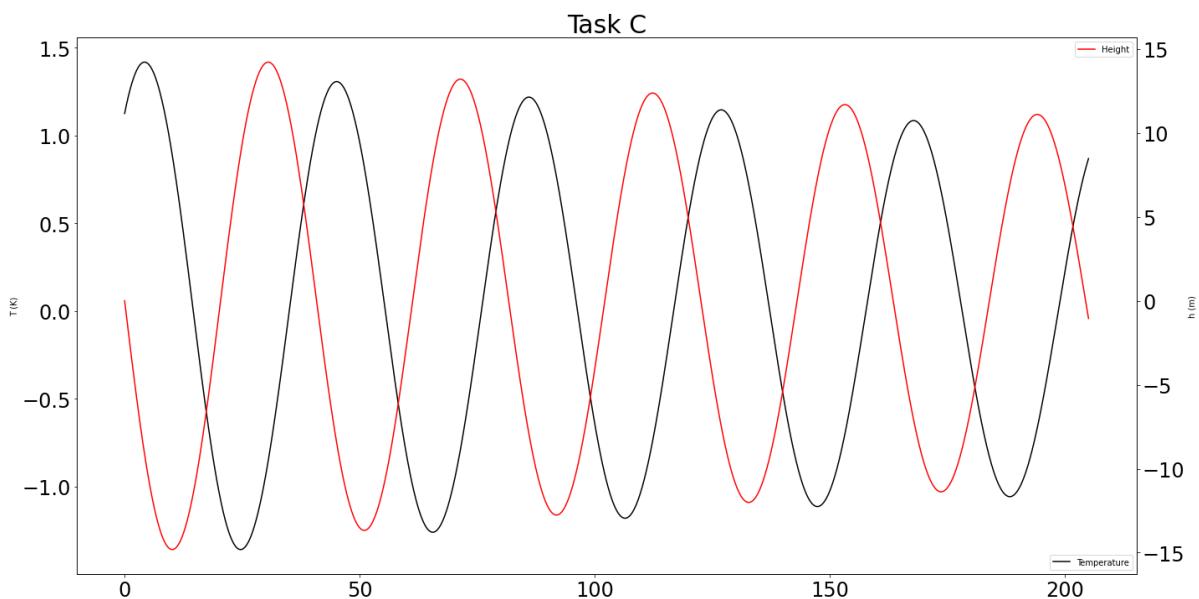
Task C

```
In [67]: T,h = RungaKutta(1.125,0,2/3,0,1,Times,0.1,0,0,0,0)
Tnew = T*Tnon
hnew = h*hnon

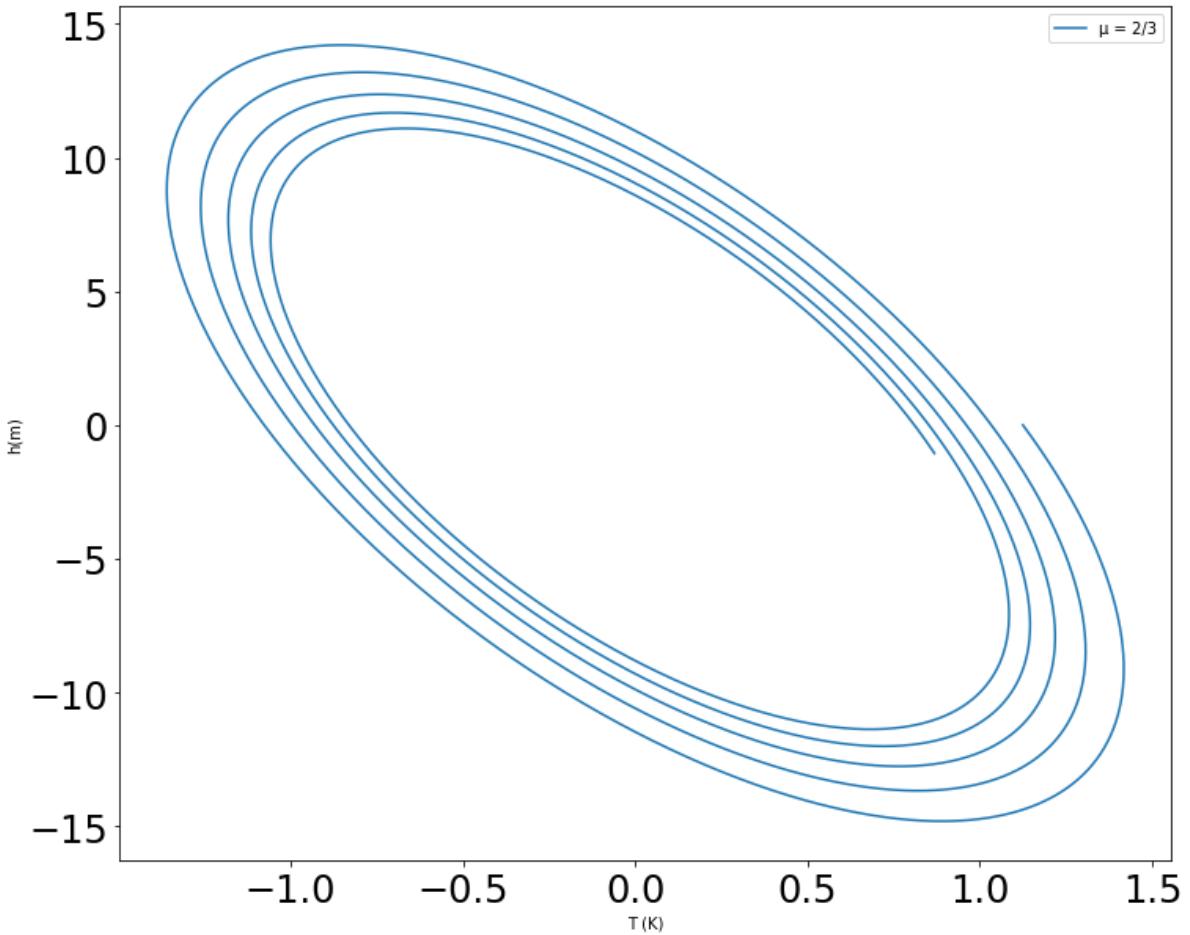
fig, ax1 = plt.subplots(figsize=(20, 10))

ax1.plot(Times*tnon,Tnew, 'k-', label='Temperature')
plt.ylabel('T (K)')
plt.legend(loc='lower right')
ax2=ax1.twinx()
ax2.plot(Times*tnon,hnew, 'r-', label='Height')
plt.ylabel('h (m)')
plt.legend(loc='best')
ax1.xaxis.set_tick_params(labelsize=24)
ax1.yaxis.set_tick_params(labelsize=24)
ax2.yaxis.set_tick_params(labelsize=24)
plt.xlabel('t (month)')
plt.ylabel('h (m)')
plt.title('Task C', fontsize = 30)
plt.tight_layout()
#plt.show()
filename='RK-timeseries.png'
fig.savefig(filename, dpi=300)

fig, ax1 = plt.subplots(figsize=(12, 10))
#Against each other
ax1.plot(Tnew,hnew,label='μ = 2/3')
ax1.xaxis.set_tick_params(labelsize=24)
ax1.yaxis.set_tick_params(labelsize=24)
plt.legend(loc='best')
plt.xlabel('T (K)')
plt.ylabel('h(m)')
plt.title('Task C', fontsize = 30)
#plt.show()
filename='RK-phase.png'
fig.savefig(filename, dpi=300)
```



Task C



Task C for $\mu = 0.75$

```
In [68]: T, h = RungaKutta(1.125, 0, 0.75, 0, 1, Times, 0.1, 0, 0, 0, 0)
Tnew = T*Tnon
hnew = h*hnon

fig, ax1 = plt.subplots(figsize=(20, 10))

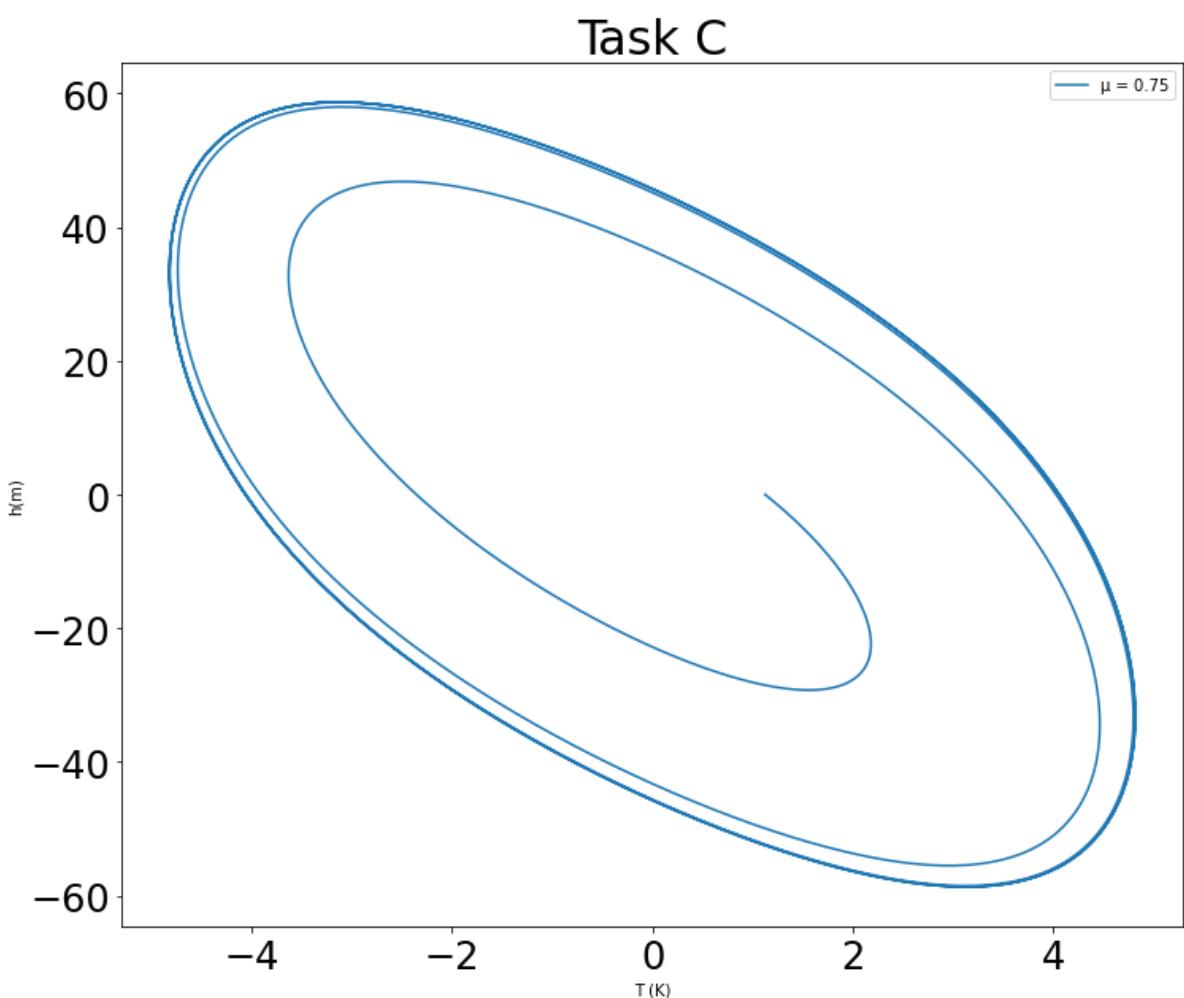
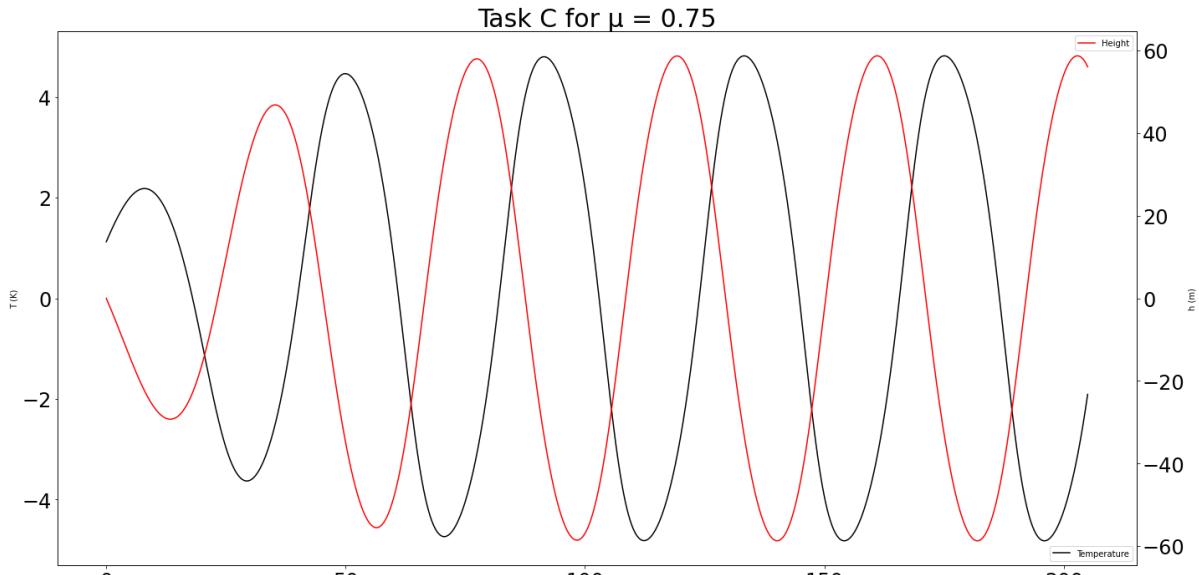
ax1.plot(Times*tnon, Tnew, 'k-', label='Temperature')
plt.ylabel('T (K)')
plt.legend(loc='lower right')
ax2=ax1.twinx()
ax2.plot(Times*tnon, hnew, 'r-', label='Height')
plt.ylabel('h (m)')
plt.legend(loc='best')
ax1.xaxis.set_tick_params(labelsize=24)
ax1.yaxis.set_tick_params(labelsize=24)
ax2.yaxis.set_tick_params(labelsize=24)
plt.xlabel('t (month)')
plt.ylabel('h (m)')
plt.title('Task C for  $\mu = 0.75$ ', fontsize = 30)
plt.tight_layout()
#plt.show()
filename='RK-timeseries.png'
fig.savefig(filename, dpi=300)

fig, ax1 = plt.subplots(figsize=(12, 10))
#Against each other
ax1.plot(Tnew, hnew,label=' $\mu = 0.75$ ')
plt.legend(loc='best')
ax1.xaxis.set_tick_params(labelsize=24)
```

```

ax1.yaxis.set_tick_params(labelsize=24)
plt.xlabel('T (K)')
plt.ylabel('h(m)')
plt.title('Task C', fontsize = 30)
#plt.show()
filename='RK-phase.png'
fig.savefig(filename, dpi=300)

```



Task D

```
In [69]: T, h = RungeKutta(1.125, 0, 0.75, 0.2, 12/tnon, Times, 0.1, 0, 0, 0, 0)
Tnew = T*tnew
```

```

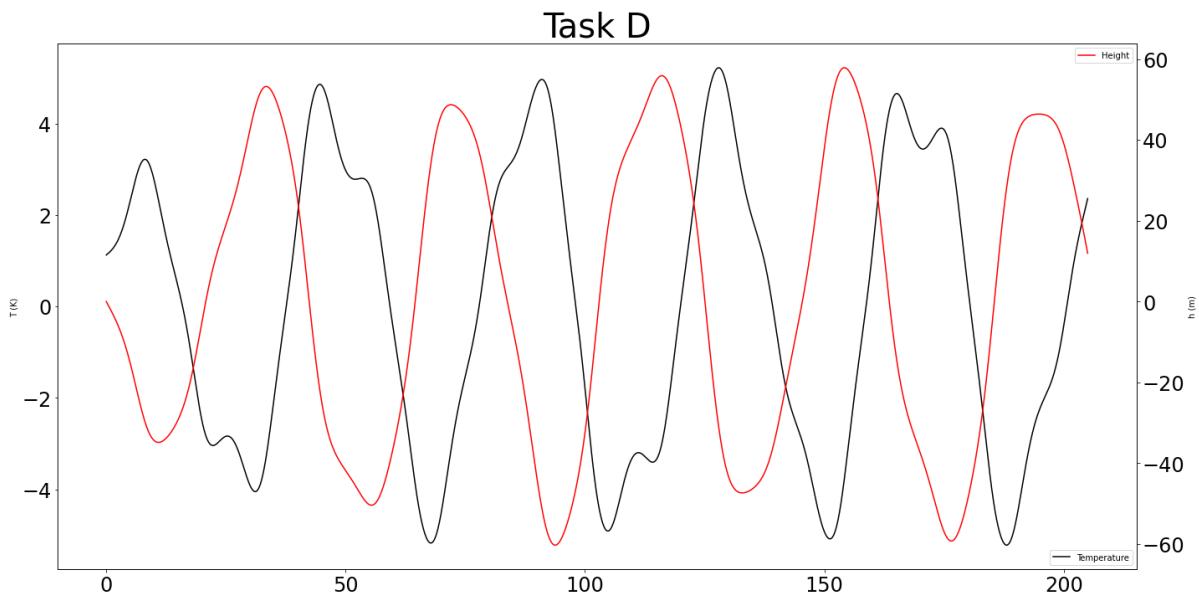
Tnew = T*Tnon
hnew = h*hnon

fig, ax1 = plt.subplots(figsize=(20, 10))

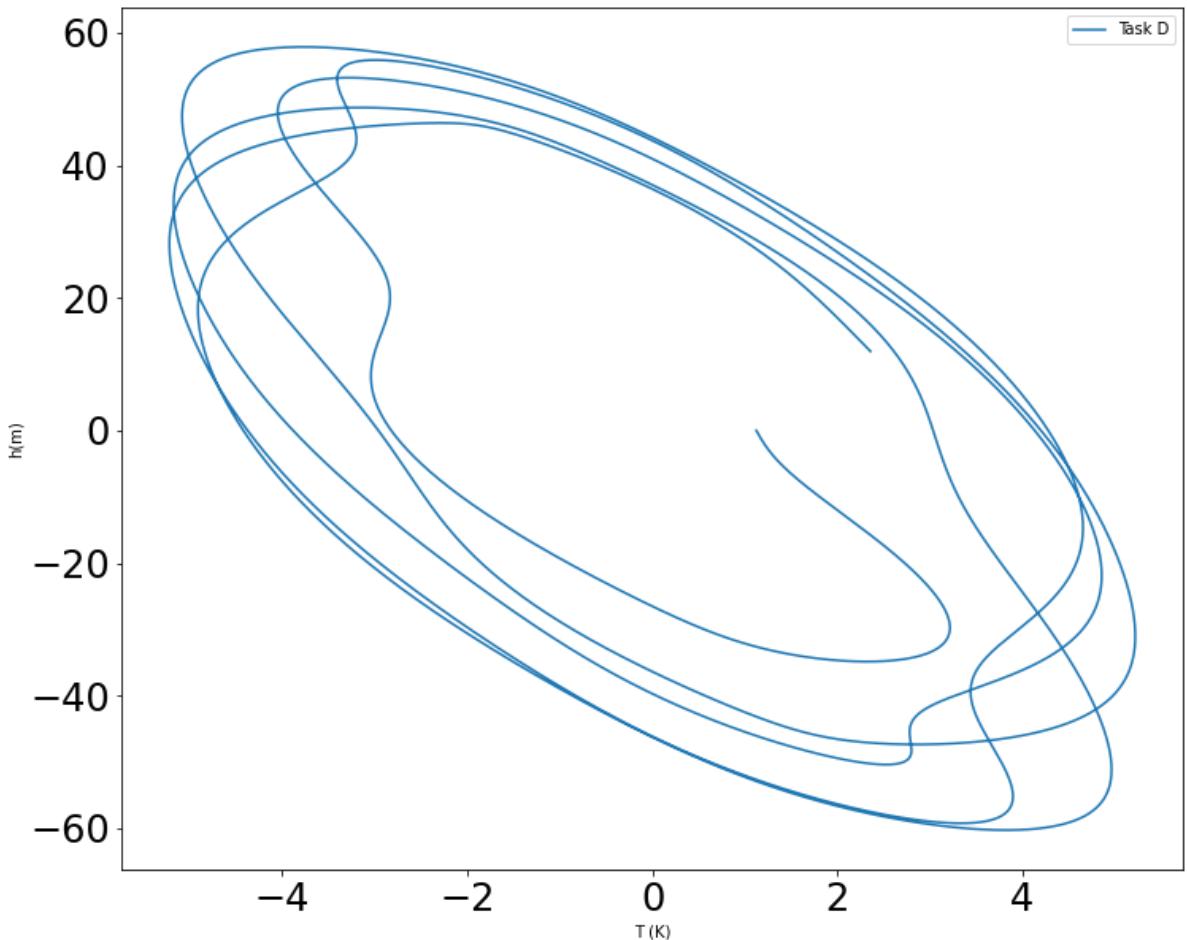
ax1.plot(Times*tnon, Tnew, 'k-', label='Temperature')
plt.ylabel('T (K)')
plt.legend(loc='lower right')
ax2=ax1.twinx()
ax2.plot(Times*tnon, hnew, 'r-', label='Height')
plt.ylabel('h (m)')
plt.legend(loc='best')
ax1.xaxis.set_tick_params(labelsize=24)
ax1.yaxis.set_tick_params(labelsize=24)
ax2.yaxis.set_tick_params(labelsize=24)
plt.xlabel('t (month)')
plt.ylabel('h (m)')
plt.title('Task D', fontsize = 40)
plt.tight_layout()
#plt.show()
filename='RK-timeseriesTaskD.png'
fig.savefig(filename, dpi=300)

fig, ax1 = plt.subplots(figsize=(12, 10))
#Against each other
ax1.plot(Tnew,hnew,label='Task D')
ax1.xaxis.set_tick_params(labelsize=24)
ax1.yaxis.set_tick_params(labelsize=24)
plt.legend(loc='best')
plt.xlabel('T (K)')
plt.ylabel('h(m)')
plt.title('Task D', fontsize = 40)
#plt.show()
filename='RK-phaseTaskD.png'
fig.savefig(filename, dpi=300)

```



Task D



Task E

Both wind forcings

In [70]: #Task E

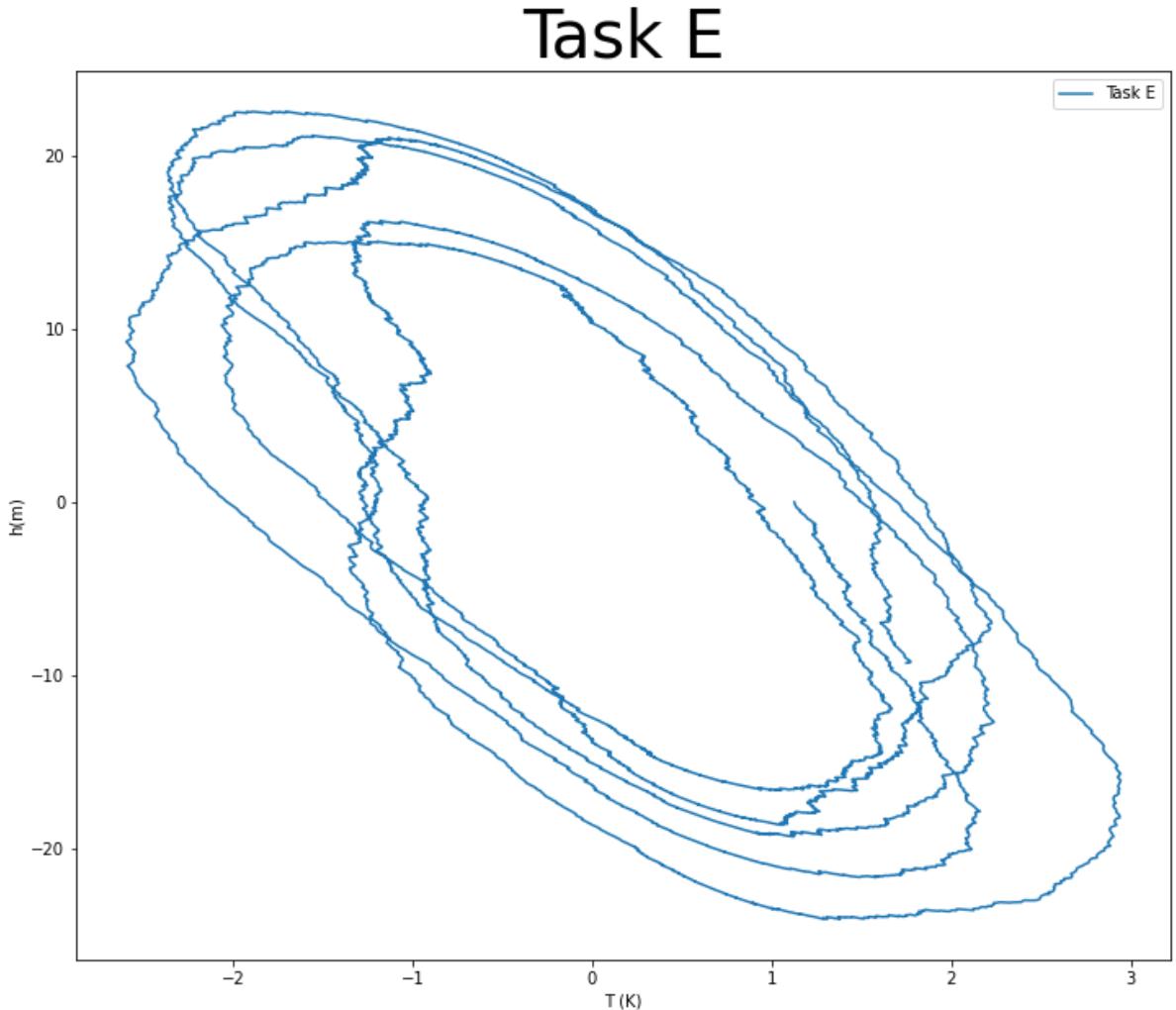
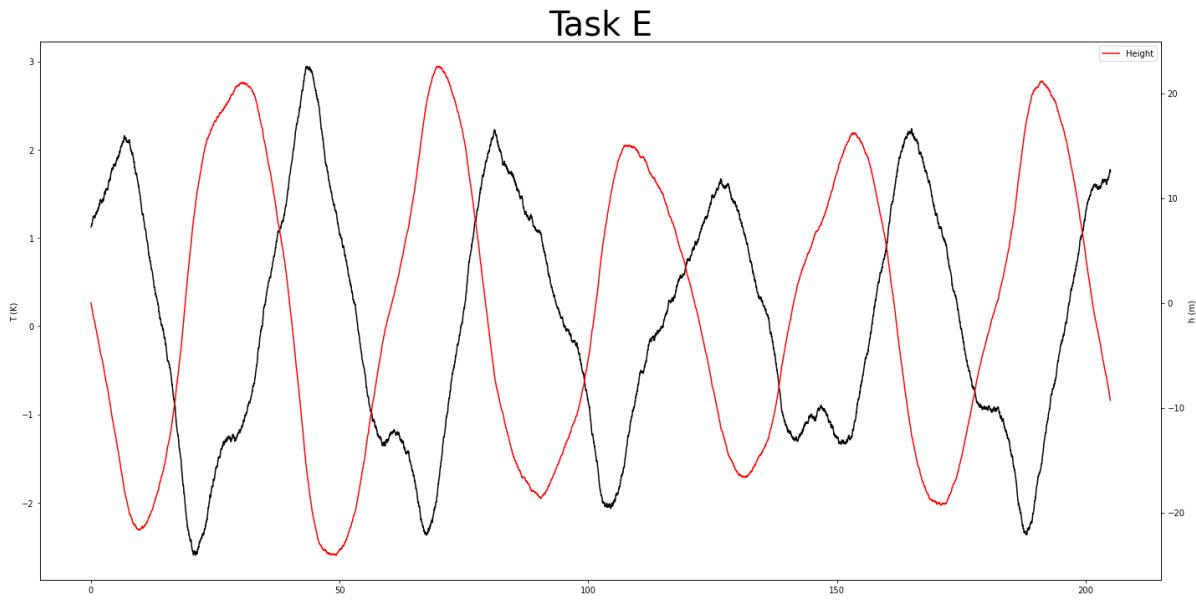
```
T,h = RungeKutta(1.125,0,2/3,0.2,12/tnew,Times,0,0,0.02,0.2,(1/30)/tnew)
Tnew = T*tnew
hnew = h*hnew

#Both
fig, ax1 = plt.subplots(figsize=(20, 10))

ax1.plot(Times*tnew,Tnew, 'k-', label='Temperature')
plt.ylabel('T (K)')
ax2=ax1.twinx()
ax2.plot(Times*tnew,hnew, 'r-', label='Height')
plt.ylabel('h (m)')
plt.legend(loc='best')
plt.xlabel('t (month)')
plt.title('Task E', fontsize = 40)
plt.tight_layout()
#plt.show()
filename='RK-timeseriesTaskE.png'
fig.savefig(filename, dpi=300)

fig, ax1 = plt.subplots(figsize=(12, 10))
```

```
#Against each other
plt.plot(Tnew,hnew,label='Task E')
plt.legend(loc='best')
plt.xlabel('T (K)')
plt.ylabel('h(m)')
plt.title('Task E', fontsize = 40)
#plt.show()
figname='RK-phaseTaskE.png'
fig.savefig(figname, dpi=300)
```



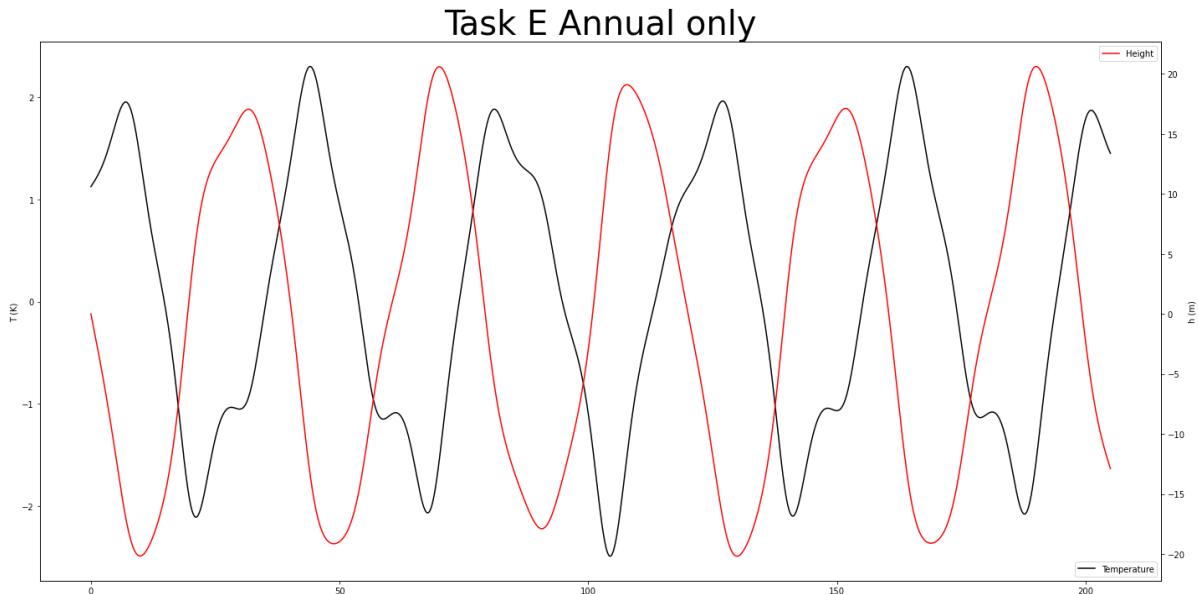
Only Annual

```
In [71]: T,h = RungeKutta(1.125,0,2/3,0.2,12/twon,Times,0,0,0.02,0,(1/30)/twon)
Tnew = T*twon
hnew = h*twon

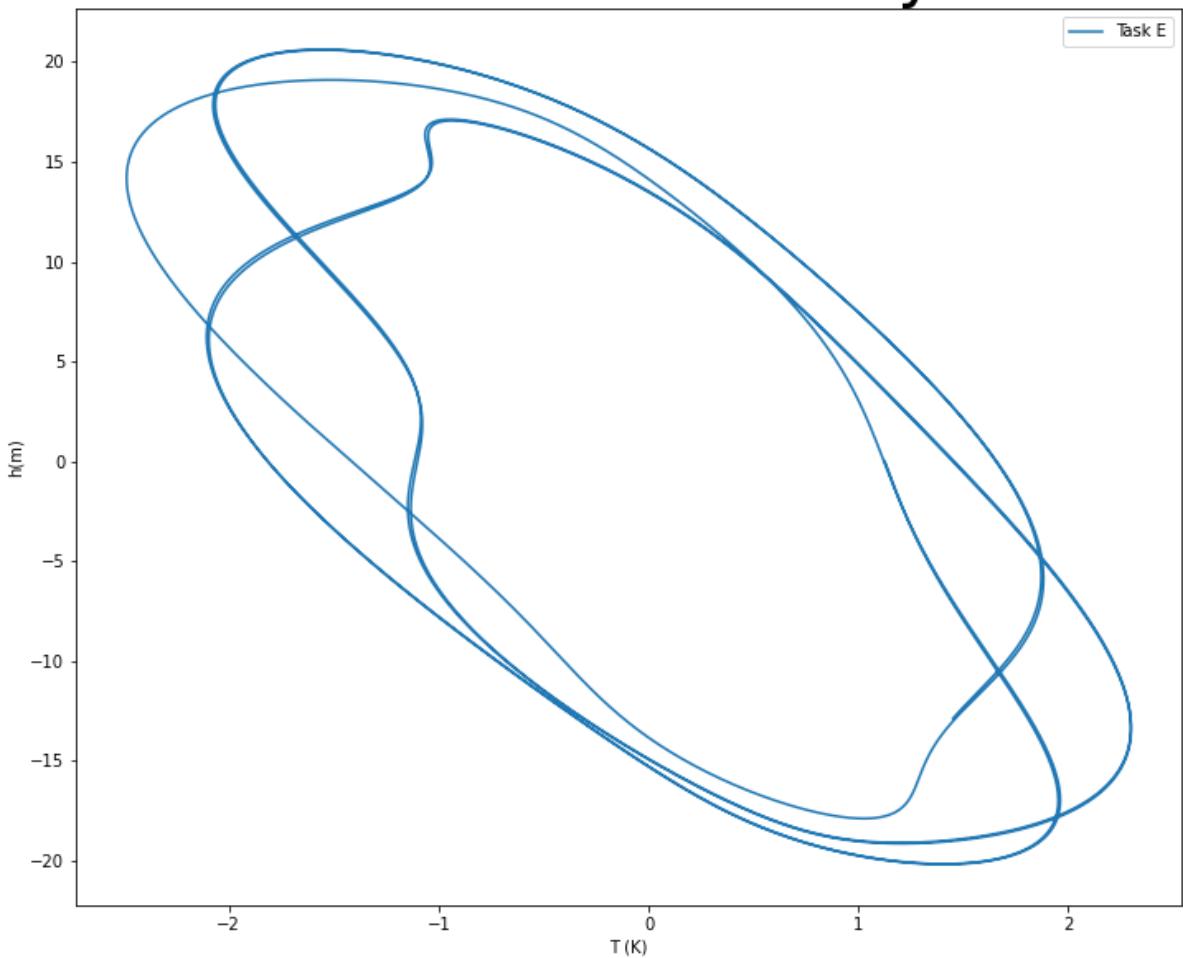
#Both
fig, ax1 = plt.subplots(figsize=(20, 10))

ax1.plot(Times*twon,Tnew, 'k-', label='Temperature')
plt.ylabel('T (K)')
plt.legend(loc='lower right')
ax2=ax1.twinx()
ax2.plot(Times*twon,hnew, 'r-', label='Height')
plt.ylabel('h (m)')
plt.legend(loc='best')
plt.xlabel('t (month)')
plt.title('Task E Annual only', fontsize = 40)
plt.tight_layout()
#plt.show()
filename='RK-timeseriesTaskEAnnual.png'
fig.savefig(filename, dpi=300)

fig, ax1 = plt.subplots(figsize=(12, 10))
#Against each other
plt.plot(Tnew,hnew,label='Task E')
plt.legend(loc='best')
plt.xlabel('T (K)')
plt.ylabel('h(m)')
plt.title('Task E Annual only', fontsize = 40)
#plt.show()
filename='RK-phaseTaskEannual.png'
fig.savefig(filename, dpi=300)
```



Task E Annual only



Random only

```
In [72]: T, h = RungeKutta(1.125, 0, 2/3, 0.2, 12/tnon, Times, 0, 0, 0, 0.2, (1/30)/tnon)
Tnew = T*tnon
hnew = h*tnon

#Both
fig, ax1 = plt.subplots(figsize=(20, 10))

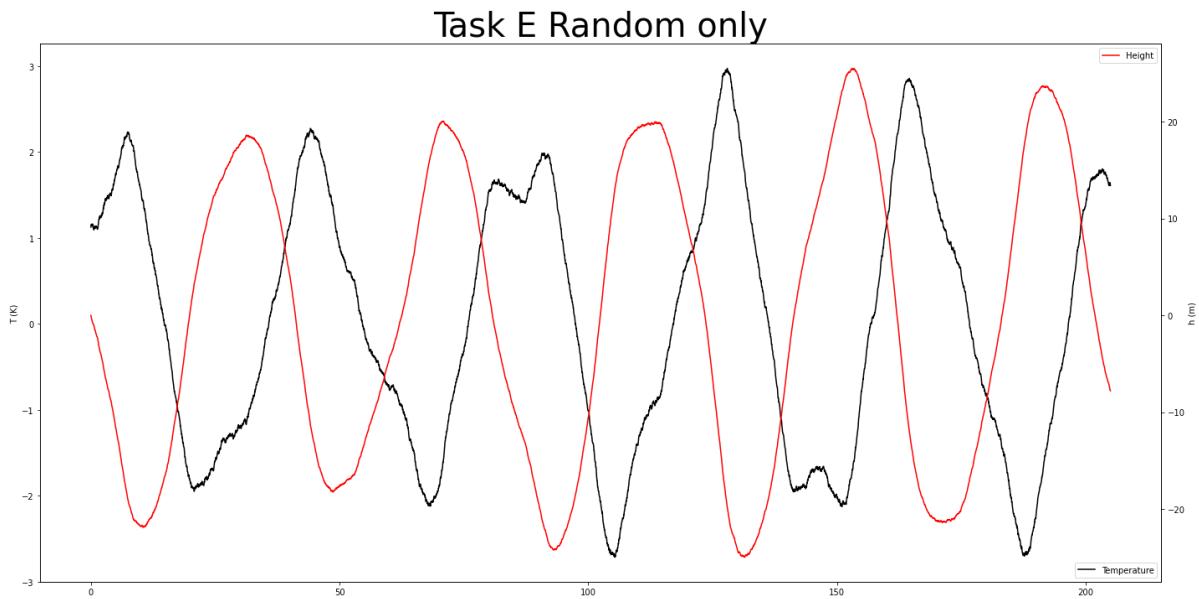
ax1.plot(Times*tnon, Tnew, 'k-', label='Temperature')
plt.ylabel('T (K)')
plt.legend(loc='lower right')
ax2=ax1.twinx()
ax2.plot(Times*tnon, hnew, 'r-', label='Height')
plt.ylabel('h (m)')
plt.legend(loc='best')
plt.xlabel('t (month)')
plt.ylabel('h (m)')
plt.title('Task E Random only', fontsize = 40)
plt.tight_layout()
#plt.show()
filename='RK-timeseriesTaskERandom.png'
fig.savefig(filename, dpi=300)

fig, ax1 = plt.subplots(figsize=(12, 10))
#Against each other
plt.plot(Tnew, hnew, label='Task E')
plt.legend(loc='best')
plt.xlabel('T (K)')
```

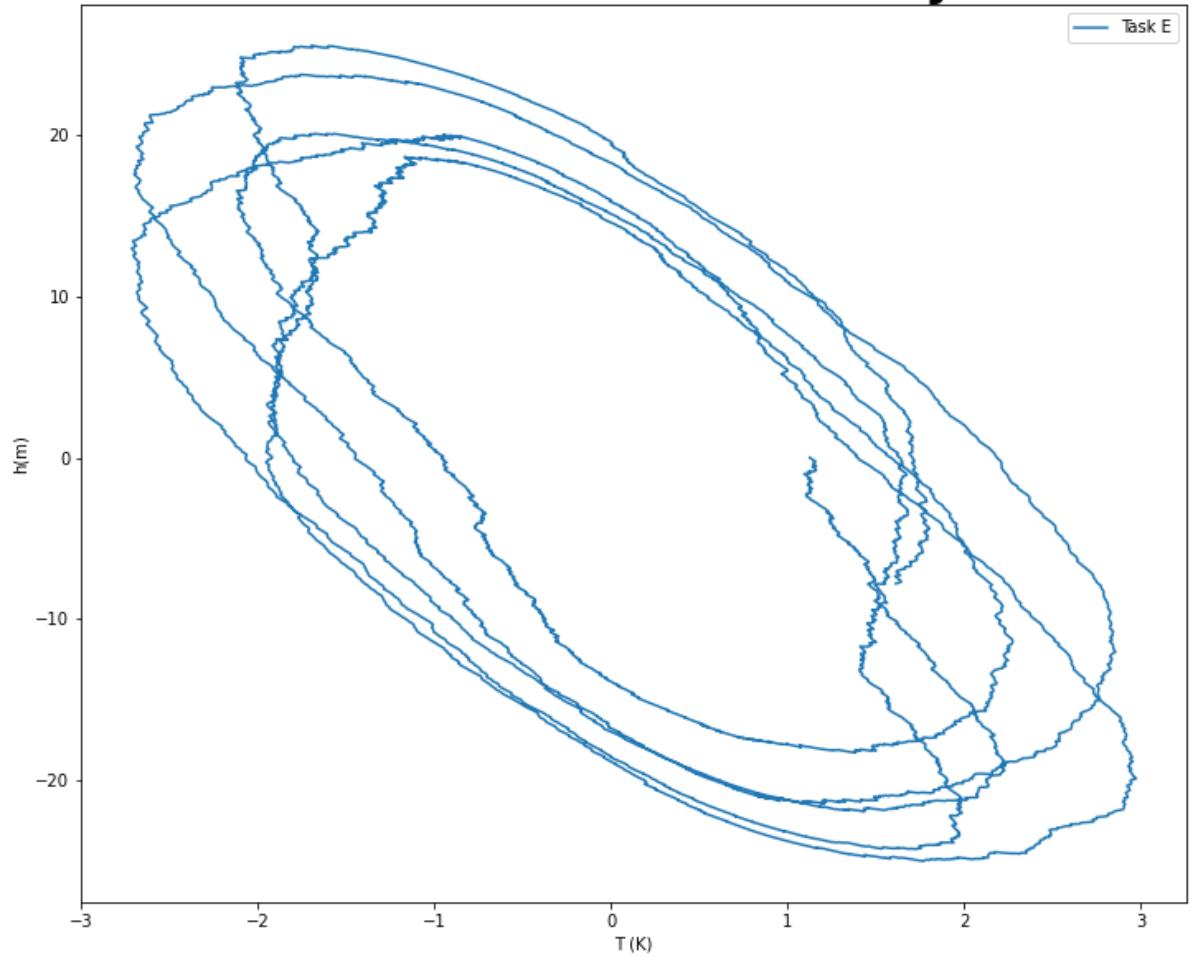
```

plt.ylabel('h(m)')
plt.title('Task E Random only', fontsize = 40)
# plt.show()
filename='RK-phaseTaskERandom.png'
fig.savefig(filename, dpi=300)

```



Task E Random only



Task F

```
In [73]: T,h = RungeKutta(1.125,0,0.75,0.2,12/tnon,Times,0.1,0,0.02,0.2,(1/30)/tnon)
Tnew = T*tnon
```

```

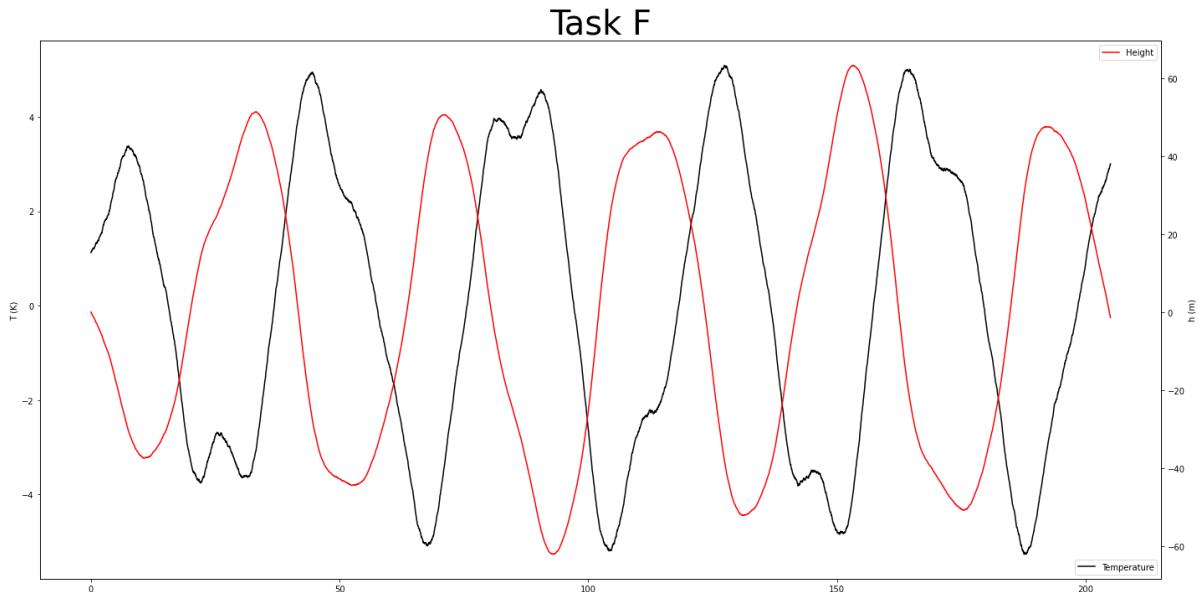
hnew = h*hnon

fig, ax1 = plt.subplots(figsize=(20, 10))

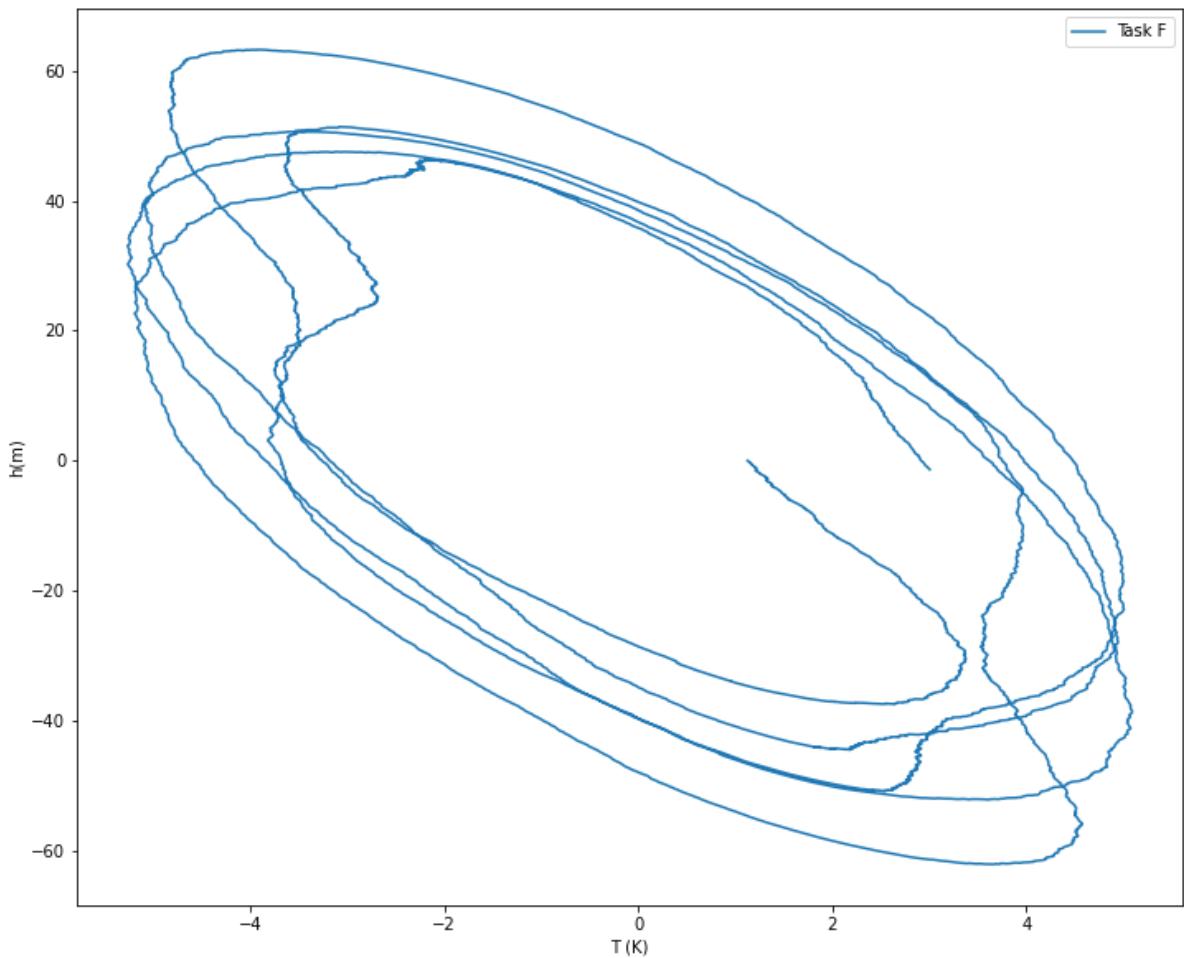
ax1.plot(Times*tnon, Tnew, 'k-', label='Temperature')
plt.ylabel('T (K)')
plt.legend(loc='lower right')
ax2=ax1.twinx()
ax2.plot(Times*tnon,hnew, 'r-', label='Height')
plt.ylabel('h (m)')
plt.legend(loc='best')
plt.xlabel('t (month)')
plt.xlabel('h (m)')
plt.title('Task F', fontsize = 40)
plt.tight_layout()
#plt.show()
filename='RK-timeseriesTaskF.png'
fig.savefig(filename, dpi=300)

fig, ax1 = plt.subplots(figsize=(12, 10))
#Against each other
plt.plot(Tnew,hnew,label='Task F')
plt.legend(loc='best')
plt.xlabel('T (K)')
plt.ylabel('h(m)')
plt.title('Task F ', fontsize = 40)
#plt.show()
filename='RK-phaseTaskF.png'
fig.savefig(filename, dpi=300)

```



Task F



Task G Ensembles

I made 12 ensembles in this case

```
In [80]: def Runge_Kutta(Tinit,hinit,μ₀,μann,τ,t,en,fann,fran,τcorr,E2):

    Tfin = []
    hfin = []
    Tfinl = []
    hfinl = []

    for i in range(12):
        Tarray = (Tinit/Tnon)*np.ones((nt,1))
        harray = (hinit/hnon)*np.ones((nt,1))
        WT = np.random.uniform(-0.2,0.2,(nt,1))
        Wh = np.random.uniform(-2,2,(nt,1))
        Tnew = Tarray + WT
        hnew = harray + Wh
        Tfin.append(Tnew)
        hfin.append(hnew)

    for j in range(12):
        T = Tfin[j]
        h = hfin[j]

        for i in range(nt-1):
```

```

W = np.random.uniform(-1,1)
E1 = noisywind(fann,fran,t[i],W,taucorr,dtnon)
b = b0*Selfexcite(mu0,muann,tau,t[i])
R = gamma*b - c

k1 = Tval(b,R,gamma,T[i],h[i],en,E1,E2)
l1 = hval(r,alpha,b,T[i],h[i],E1)

k2 = Tval(b,R,gamma,T[i]+(k1*dtnon/2),h[i]+(l1*dtnon/2),en,E1,E2)
l2 = hval(r,alpha,b,T[i]+(k1*dtnon/2),h[i]+(l1*dtnon/2),E1)

k3 = Tval(b,R,gamma,T[i]+(k2*dtnon/2),h[i]+(l2*dtnon/2),en,E1,E2)
l3 = hval(r,alpha,b,T[i]+(k2*dtnon/2),h[i]+(l2*dtnon/2),E1)

k4 = Tval(b,R,gamma,T[i]+(k3*dtnon),h[i]+(l3*dtnon),en,E1,E2)
l4 = hval(r,alpha,b,T[i]+(k3*dtnon),h[i]+(l3*dtnon),E1)

T[i+1]=T[i]+ 1/6 * dtnon*(k1 + 2*k2 + 2*k3 + k4)
h[i+1]=h[i] + 1/6 * dtnon*(l1 + 2*l2 + 2*l3 + l4)

Tfin1.append(T)
hfin1.append(h)

return Tfin1,hfin1

```

Tplot,hplot = Runge_Kutta(1.125,0,0.75,0.2,12/dtnon,Times,0.1,0.02,0.2,(1/30)


```

for i in range(12):
    Tplotvar = Tplot[i] * Tnon
    hplotvar = hplot[i] * hnon

    fig, ax1 = plt.subplots(figsize=(20,10))
    ax1.plot(Times*dtnon,Tplotvar, 'k-', label='Temperature')
    plt.ylabel('T(k)')
    plt.legend(loc='lower right')
    ax2=ax1.twinx()
    ax2.plot(Times*dtnon,hplotvar, 'r-', label='Height')
    plt.ylabel('h(m)')
    plt.legend(loc='best',bbox_to_anchor=(1,1))
    plt.xlabel('t (month)')
    plt.ylabel('h (m)')
    plt.title('Ensemble %i' %(i+1), fontsize = 40)
    plt.tight_layout()
    plt.show()
    figname='RK-timeseriesEnsemble %i' %(i+1)
    fig.savefig(figname, dpi=300)

#Against each other
fig, ax3 = plt.subplots(figsize=(12,10))
plt.plot(Tplotvar,hplotvar,'k-', label='Ensemble %i' %(i+1))
plt.legend(loc='best')
plt.xlabel('T (K)')
plt.ylabel('h(m)')
plt.show()
figname1='RK-phaseEnsemble %i' %(i+1)
fig.savefig(figname1, dpi=300)
plt.figure(figsize=(20,10))
for i in range(12):
    Tplotvar = Tplot[i] * Tnon
    hplotvar = hplot[i] * hnon

```

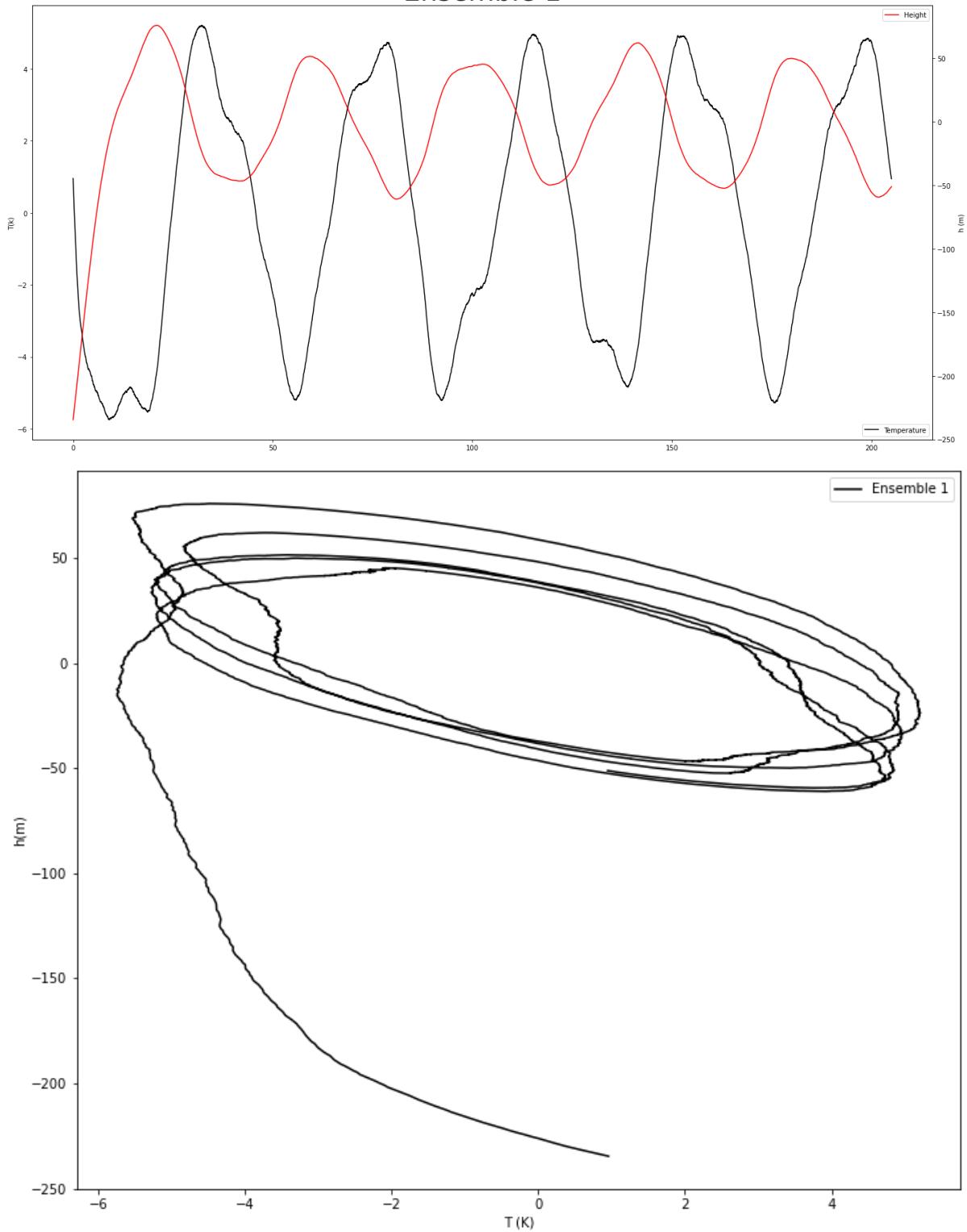
```

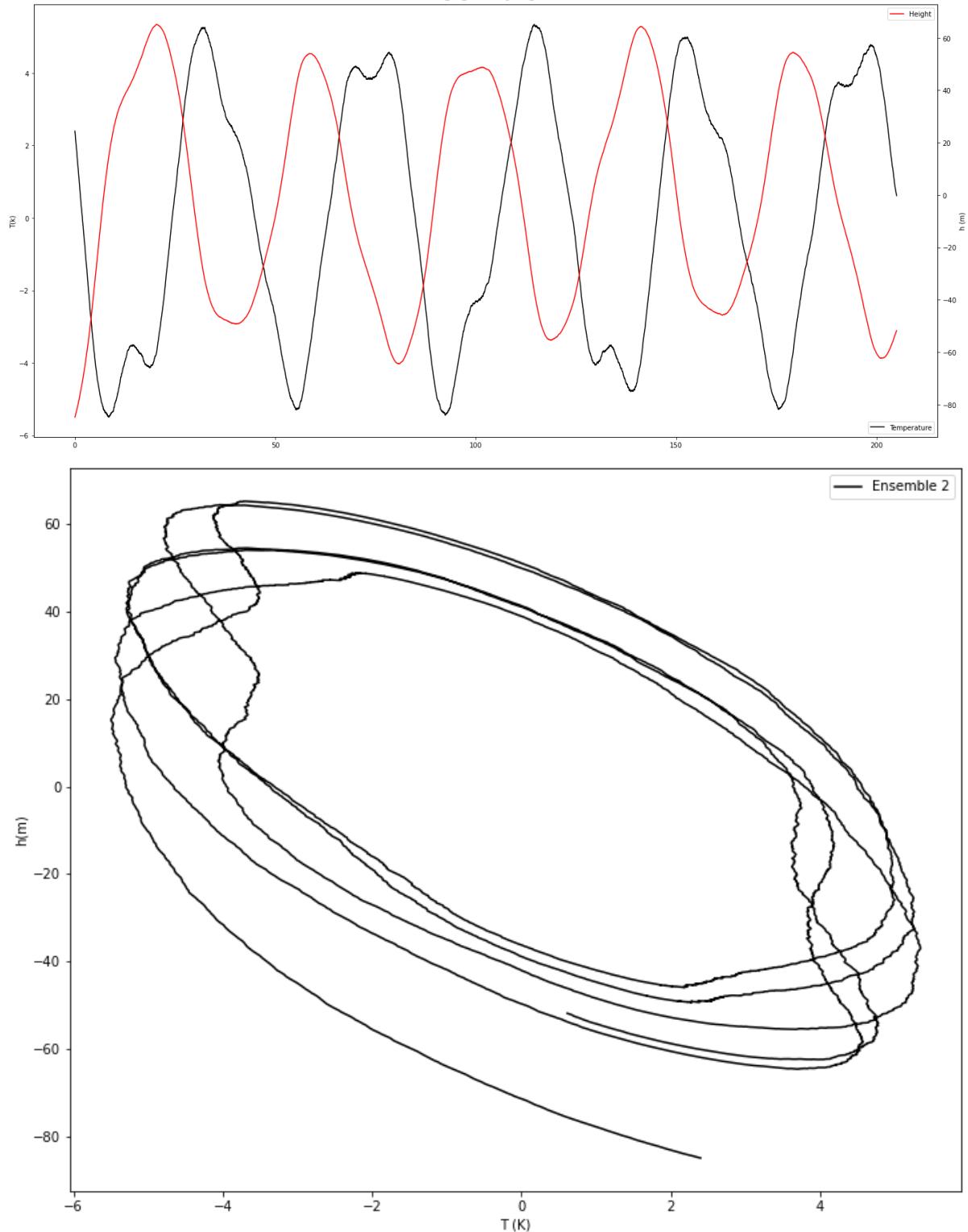
plt.plot(Times*tnon,Tplotvar,label='Ensemble %i' %(i+1))

plt.ylabel('T(k)')
plt.legend(loc='best', bbox_to_anchor=(1,1))
plt.xlabel('t (month)')
plt.title('Ensemble plume', fontsize = 20)
plt.tight_layout()
plt.show()

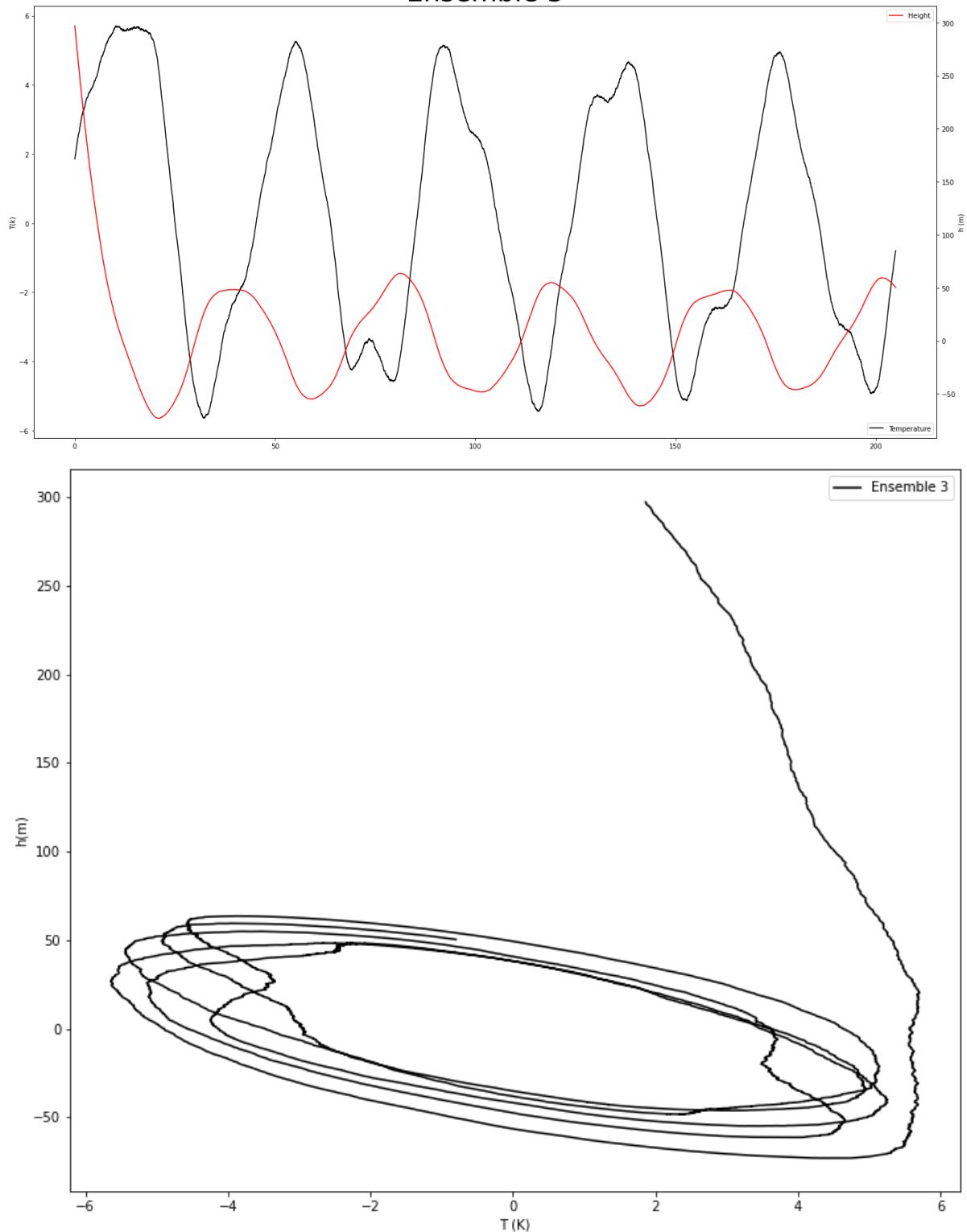
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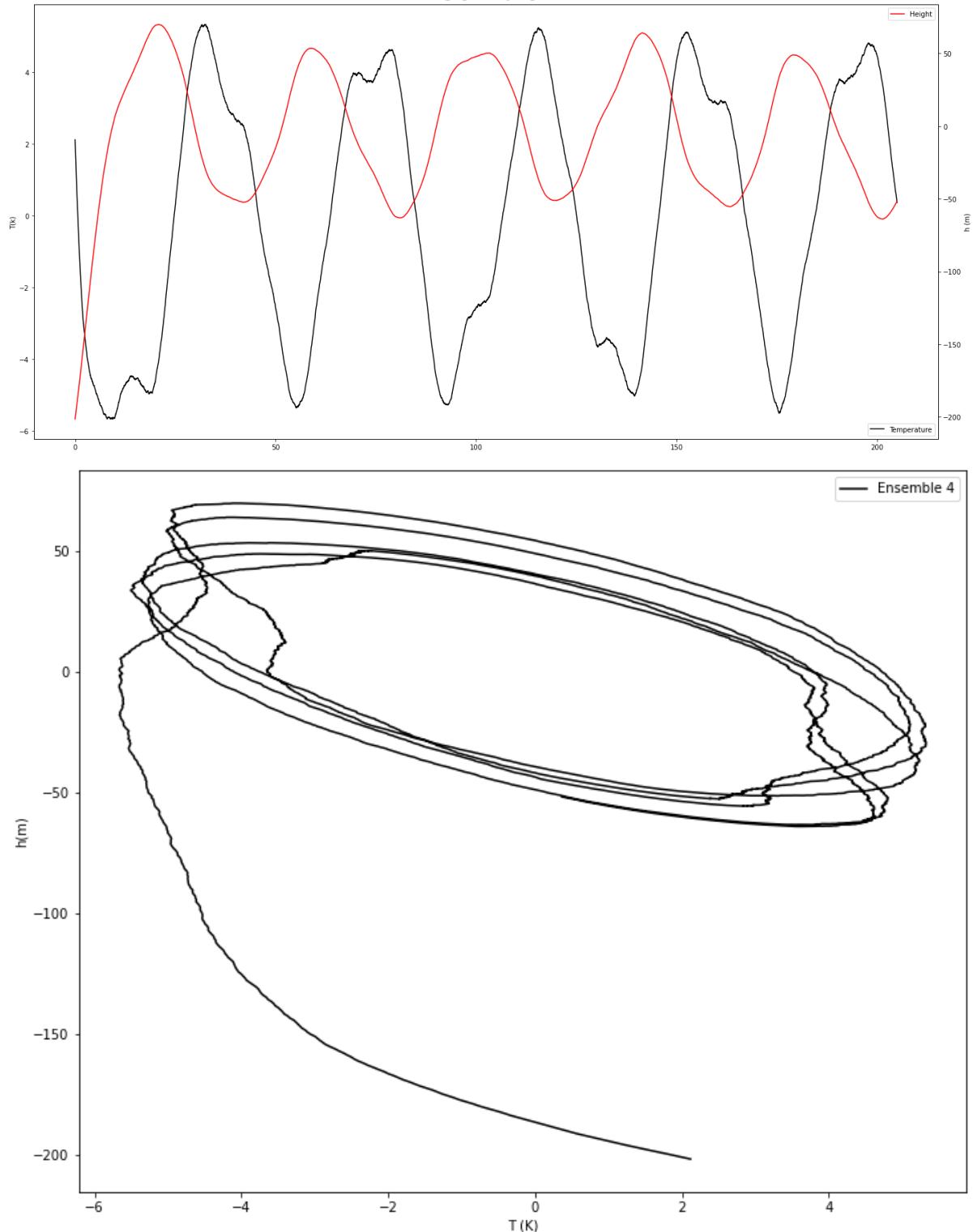
Ensemble 1

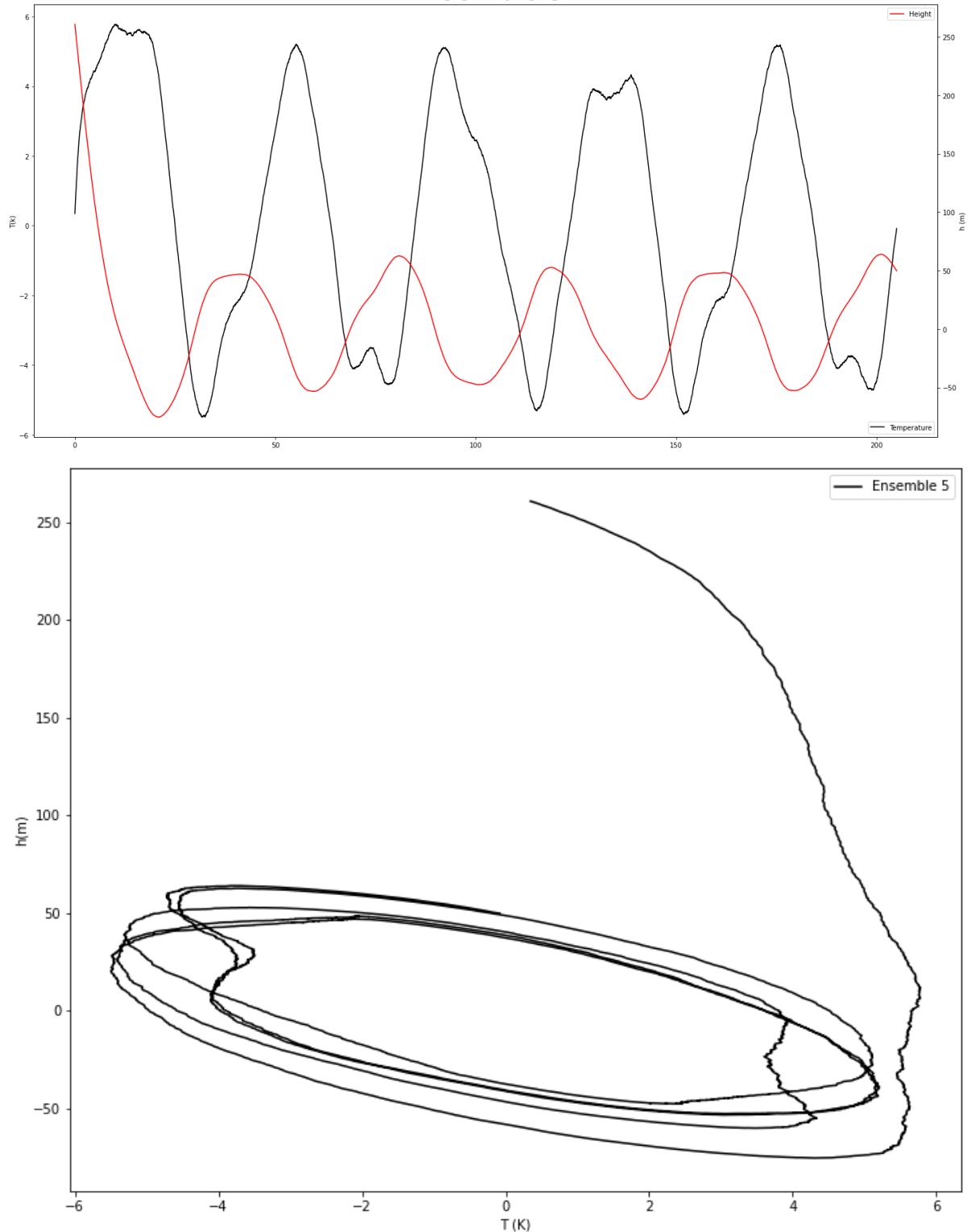


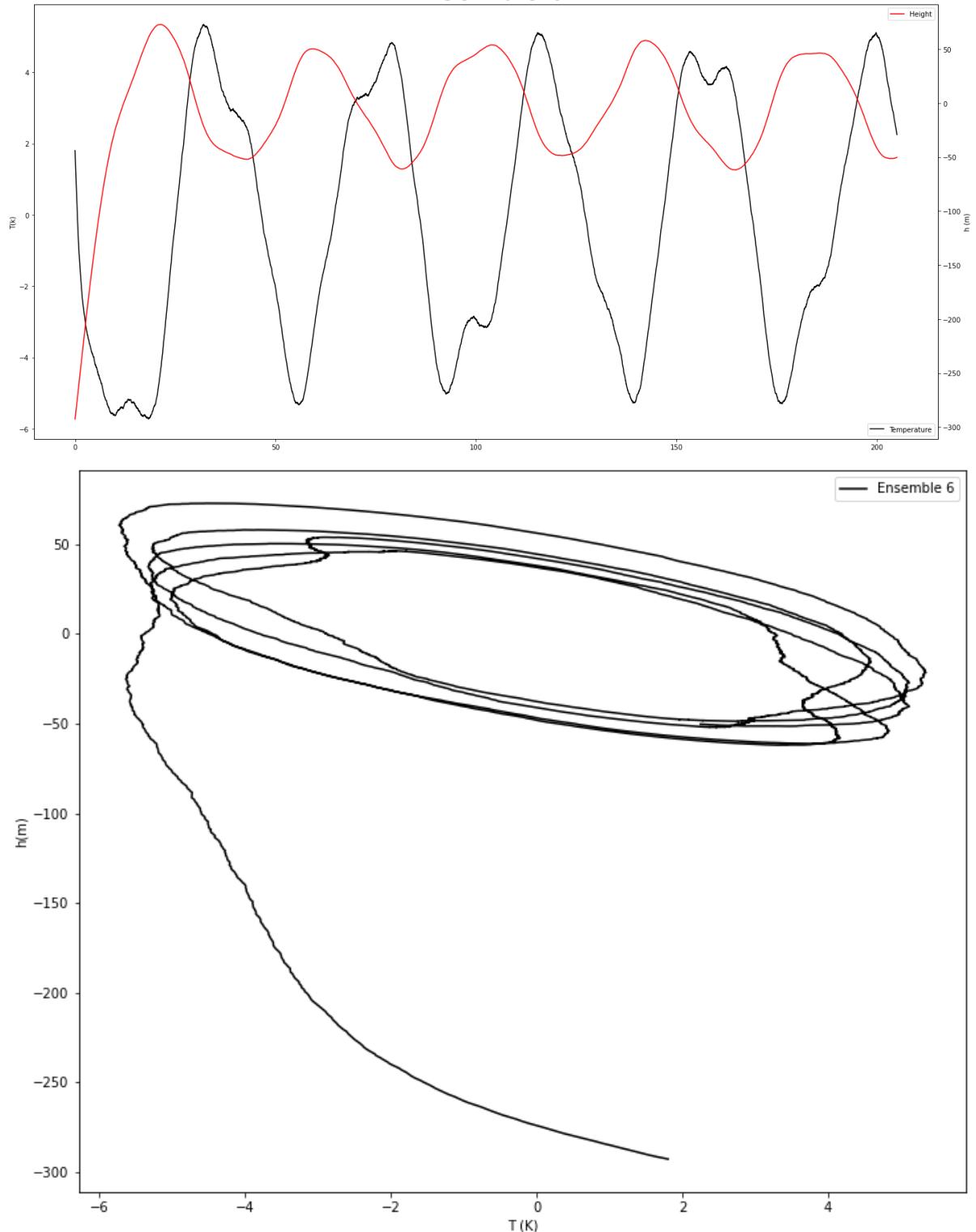
Ensemble 2

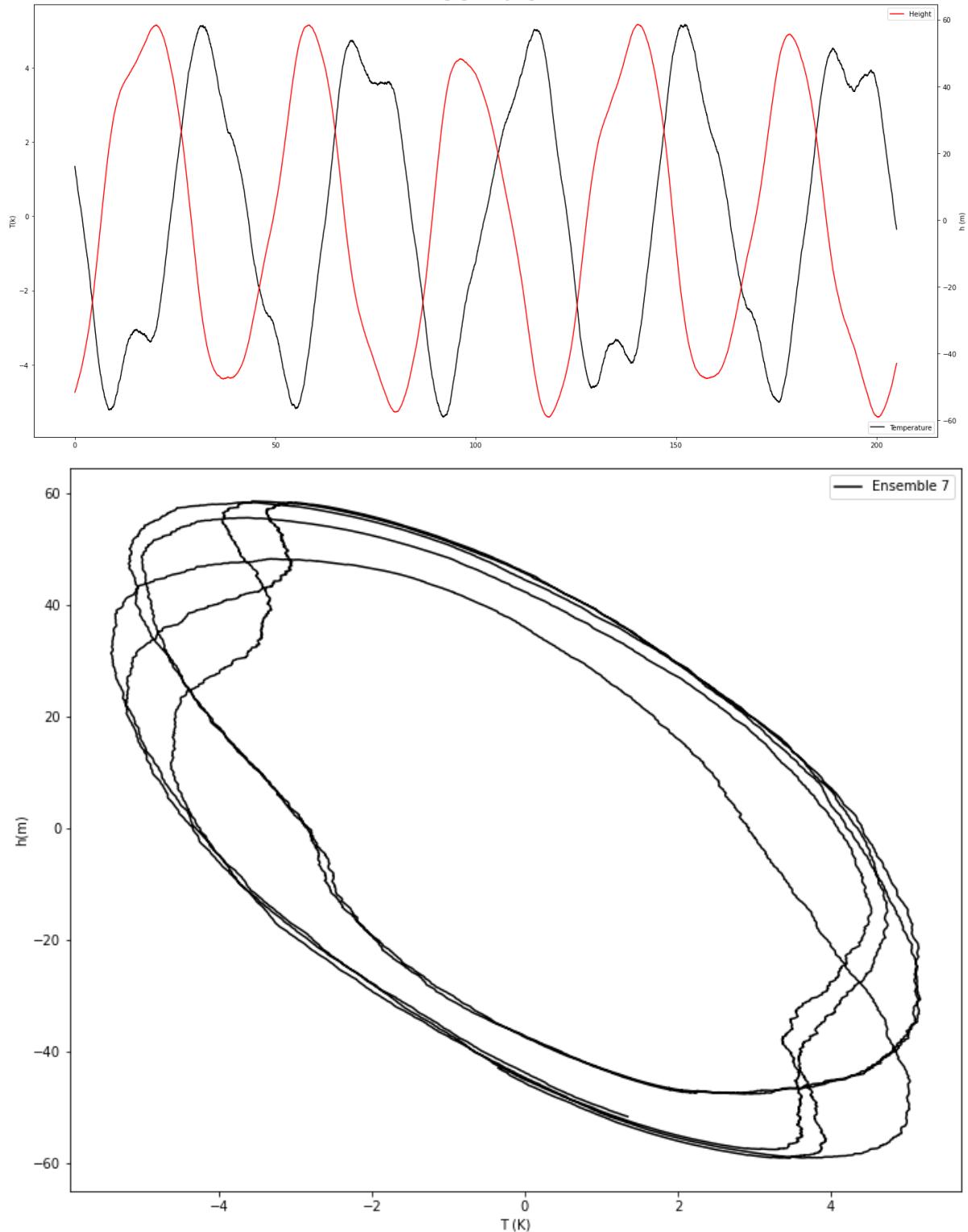
Ensemble 3

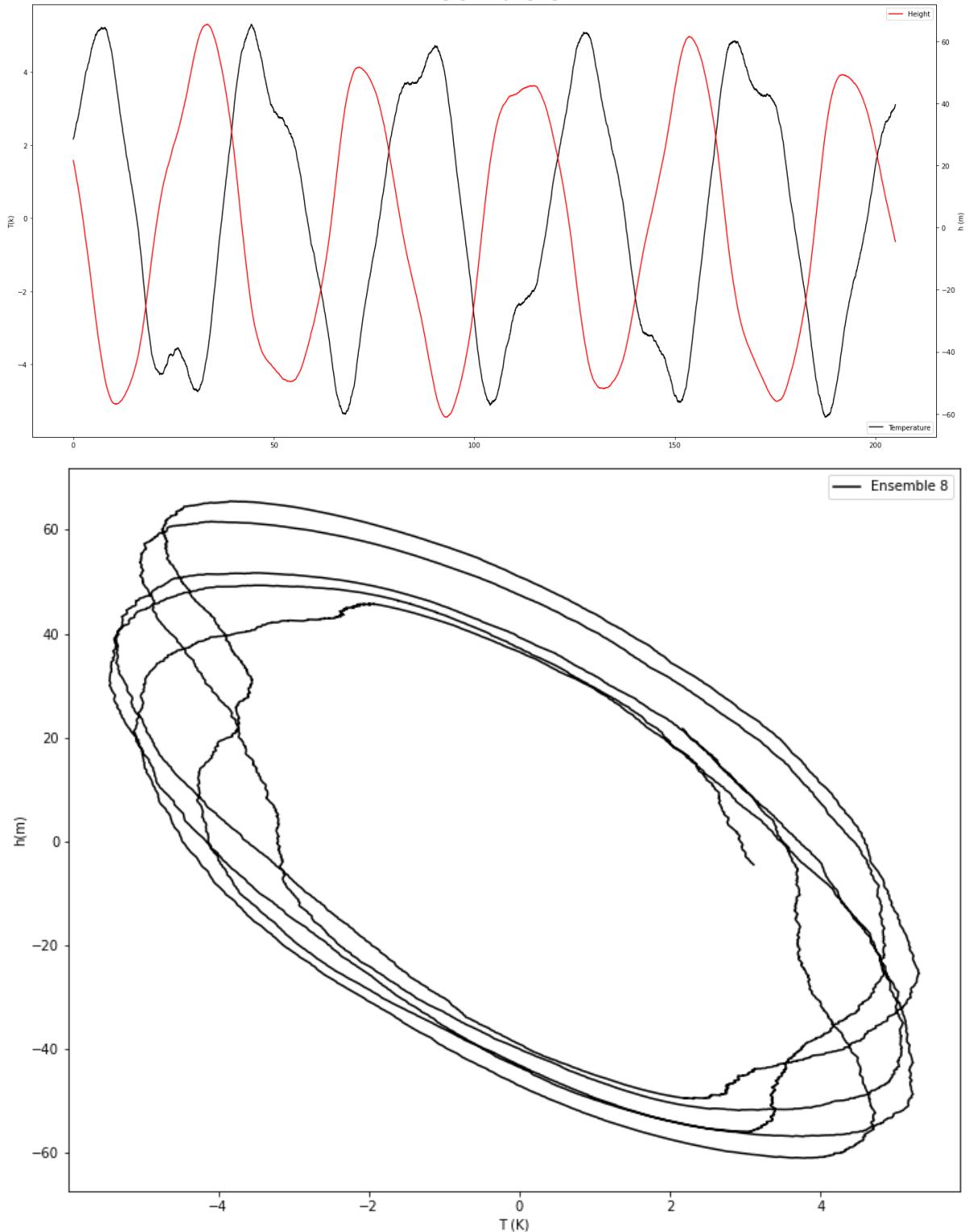


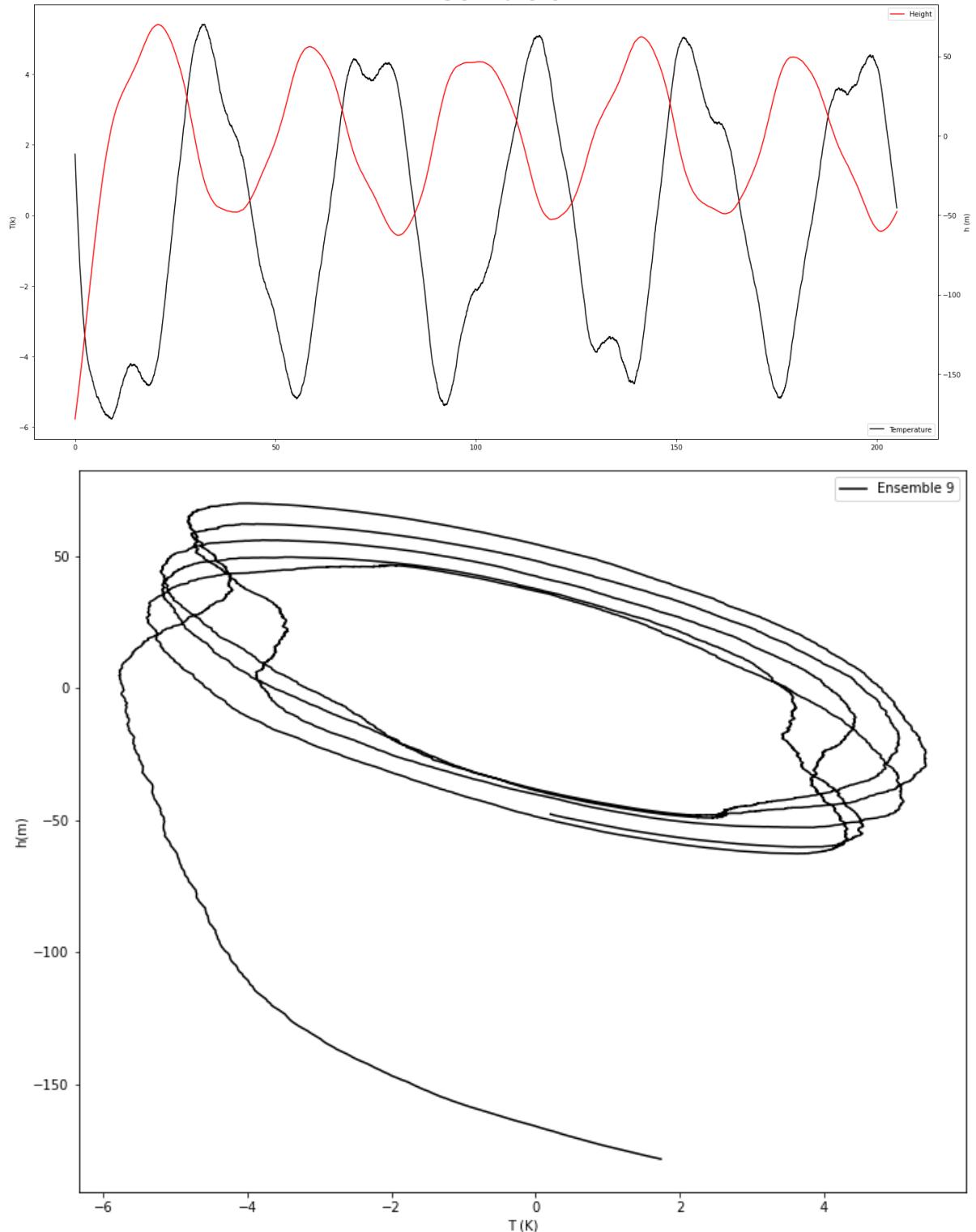
Ensemble 4

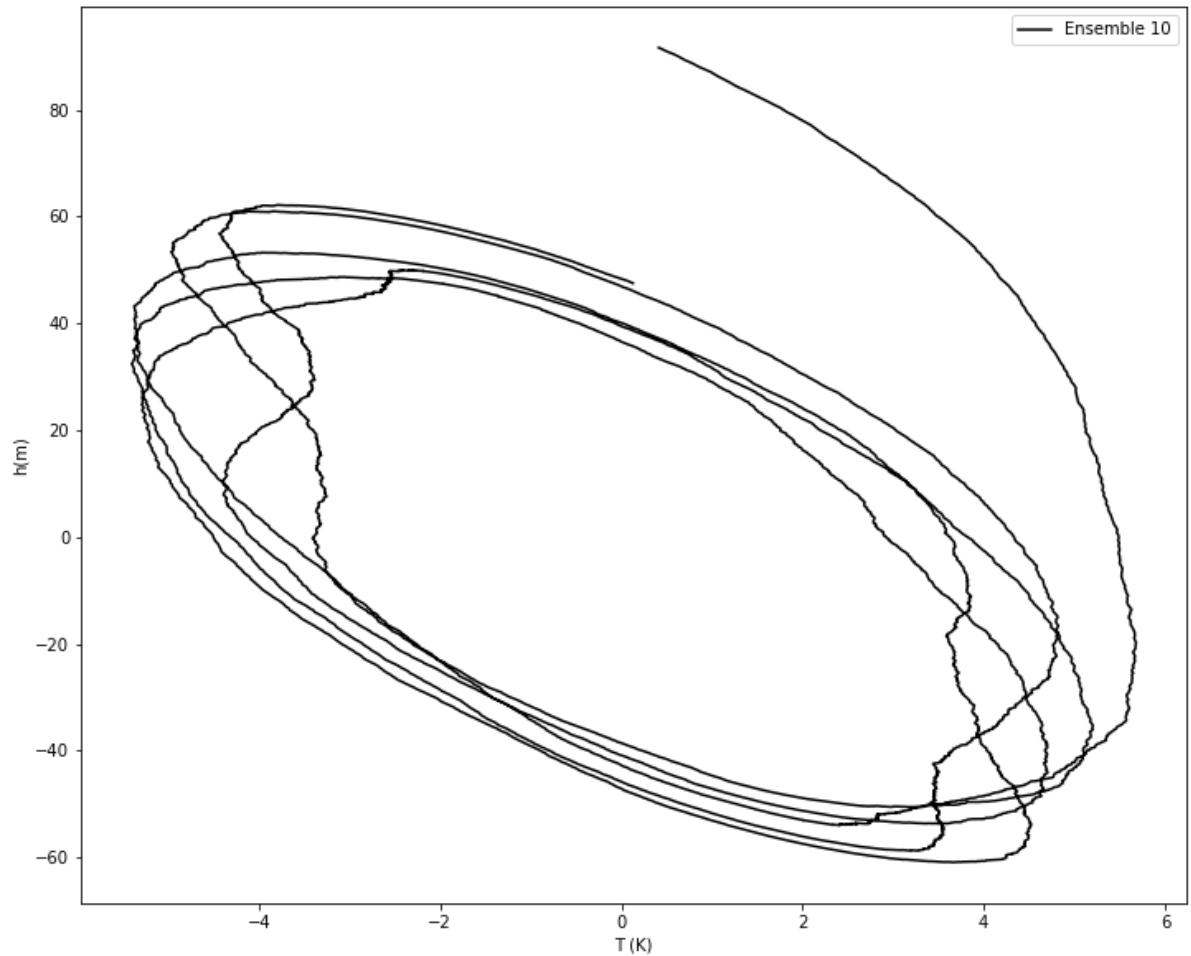
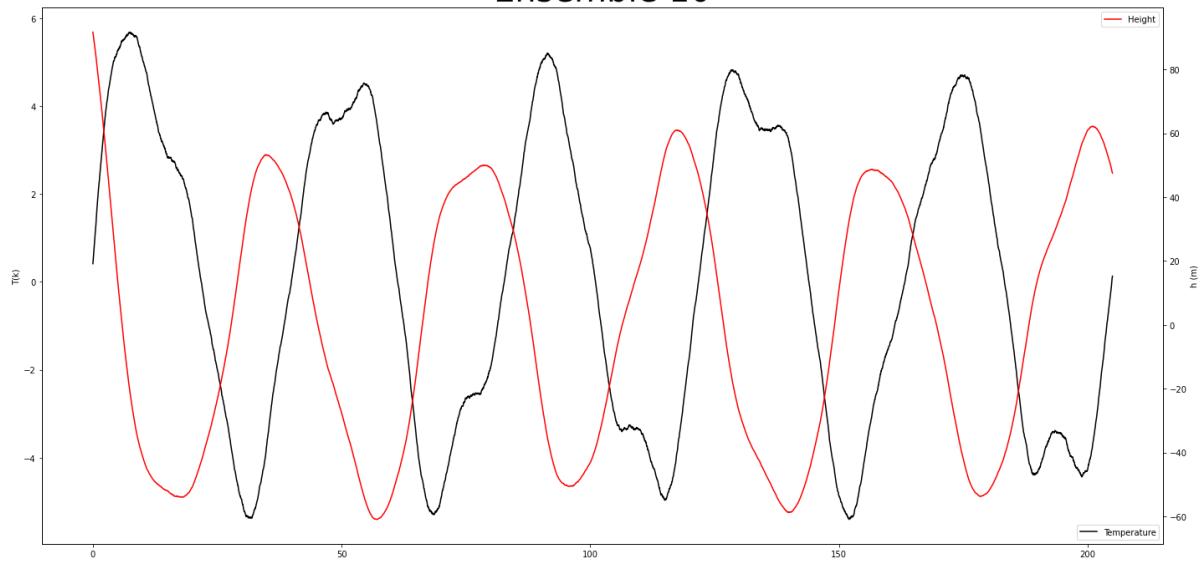
Ensemble 5

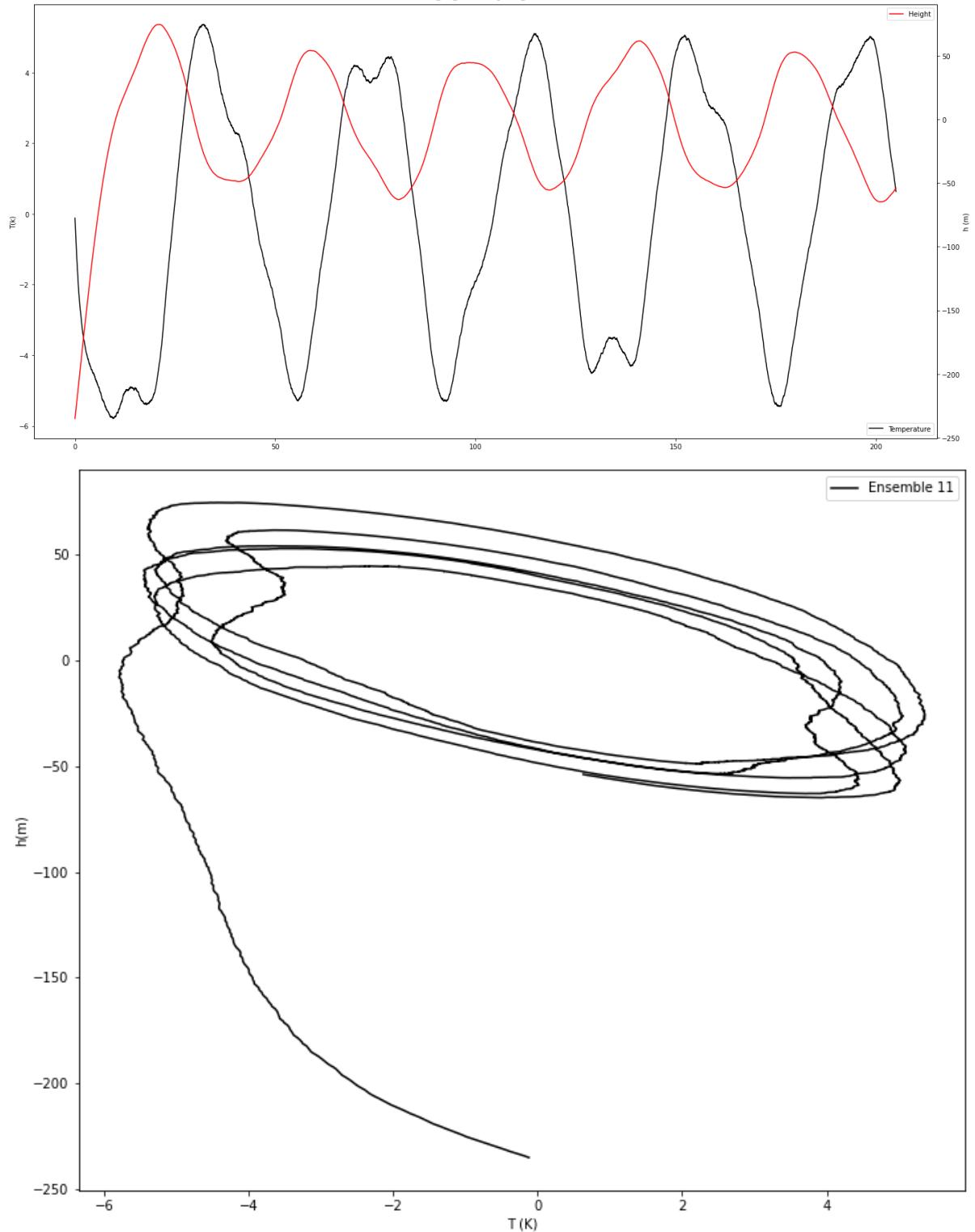
Ensemble 6

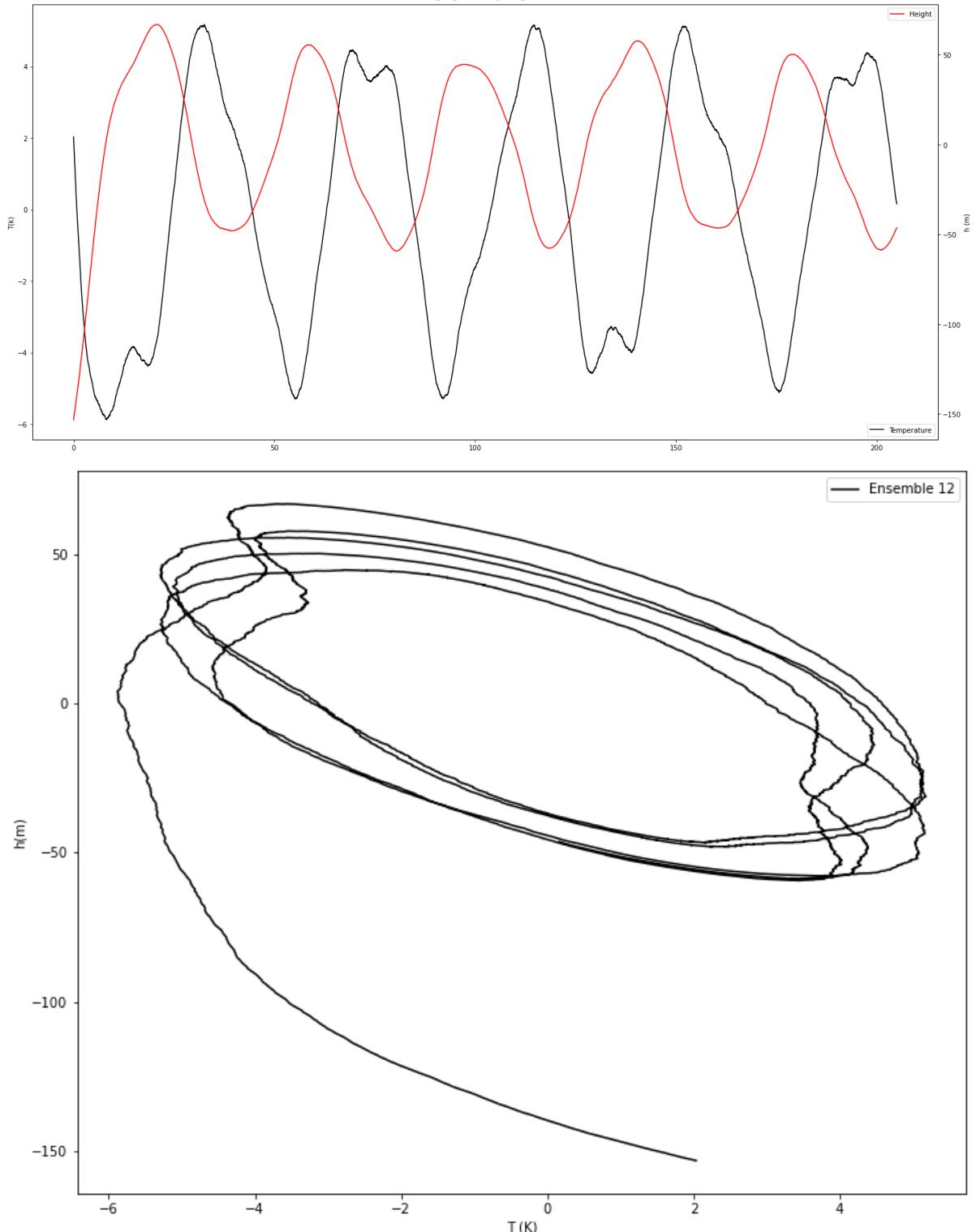
Ensemble 7

Ensemble 8

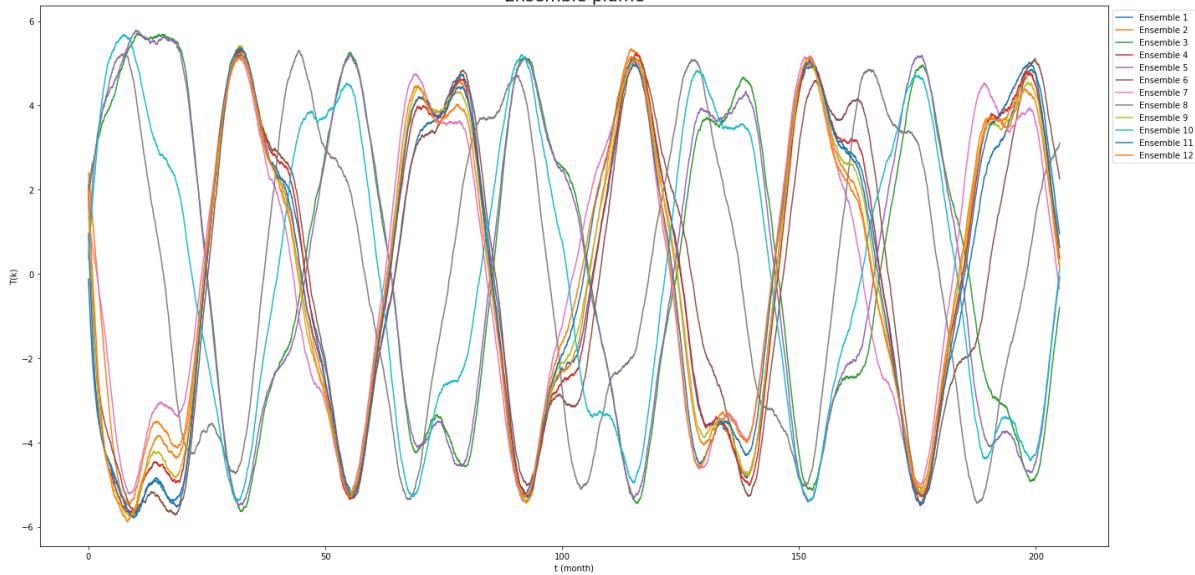
Ensemble 9

Ensemble 10

Ensemble 11

Ensemble 12

Ensemble plume



In []: