ECE 4960: Data Science & Social Networks **(Due: 05/10)**

Assignment 2

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Question 29

Erdos-Renyi graph with probability p=0.3 and number of nodes n=100

a

The degree distribution ρ_k of the random graph is shown in the figure 1.

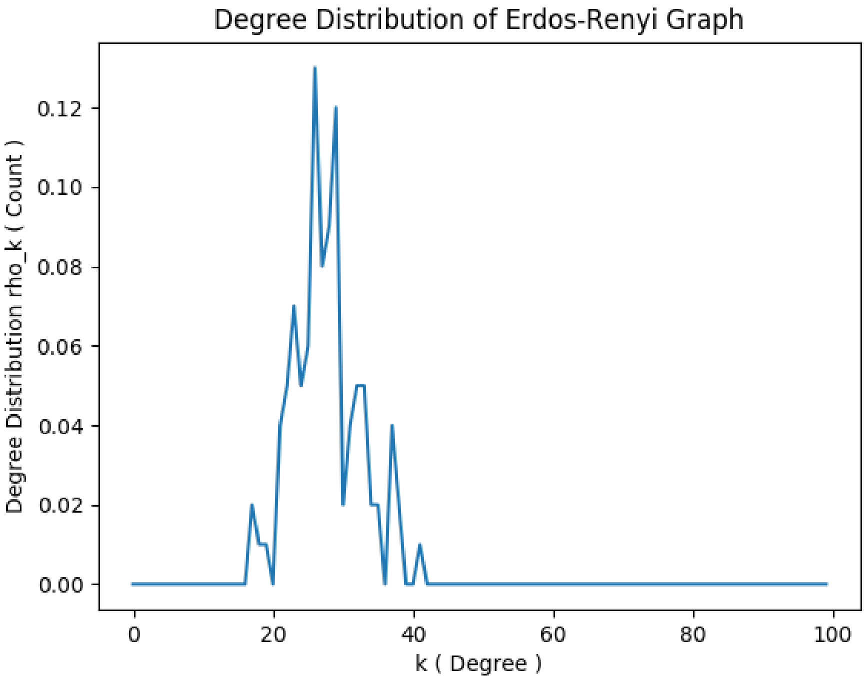


Figure 1: degree distribution ρ_k of the Erdos-Renyi random graph

b

Using least squares to fit the function $Ck - Dklogk$ to the degree distribution ρ_k. The value of C = 0.161 and D = 0.037 and a coefficient of determination R = 0.641

c

The following figure 2 shows how C, D and the coefficient of determination vary with probability p. It can be seen that as the probability increases the C,D and coefficent of determination decreases.

d

Figure 3 shows how the assortativity coefficient varies with p for different values of n. It can be seen that as the value of n increases the assortativity is close to 0 irrespective of p. This is expected from an Erdos-Renyi graph as the edges present in the graph are purely random and not dependent on any priority.

e

Figure 4 shows how Freeman's centrality.

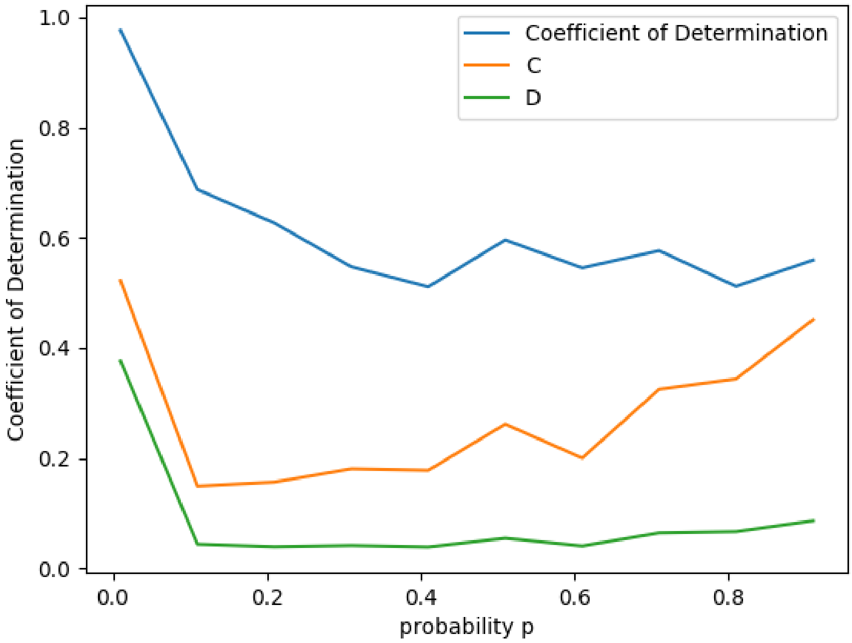


Figure 2: C, D and Coefficient of Determination varying with p

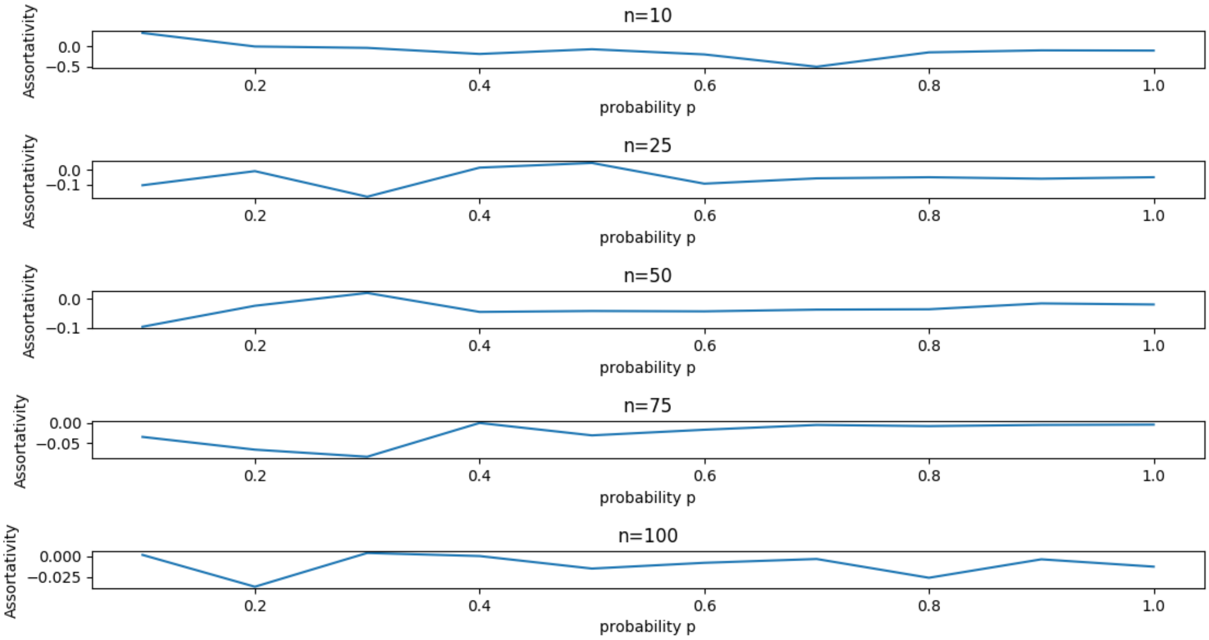


Figure 3: Assortativity as it varies with p for n=10, 25, 50 ,75, 100

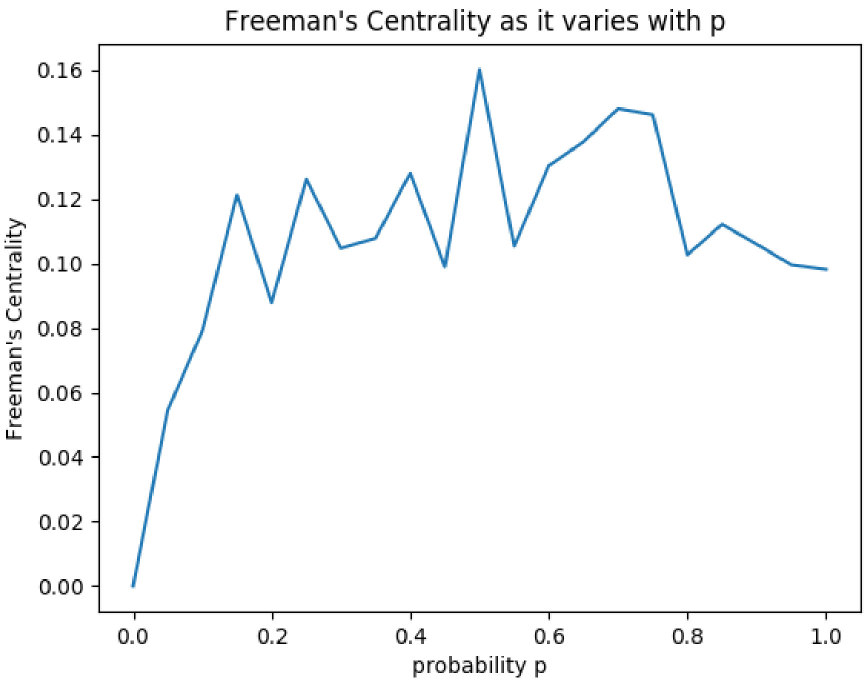


Figure 4: Freeman’s centrality as it varies with p

Question 33

Show numerically that the higher the assortativity coefficient, the smaller the second smallest eigenvalue of the Laplacian.
We can see from figure 5 that as the second smallest eigenvalue of the Laplacian increases the assortativity of the graph decreases.

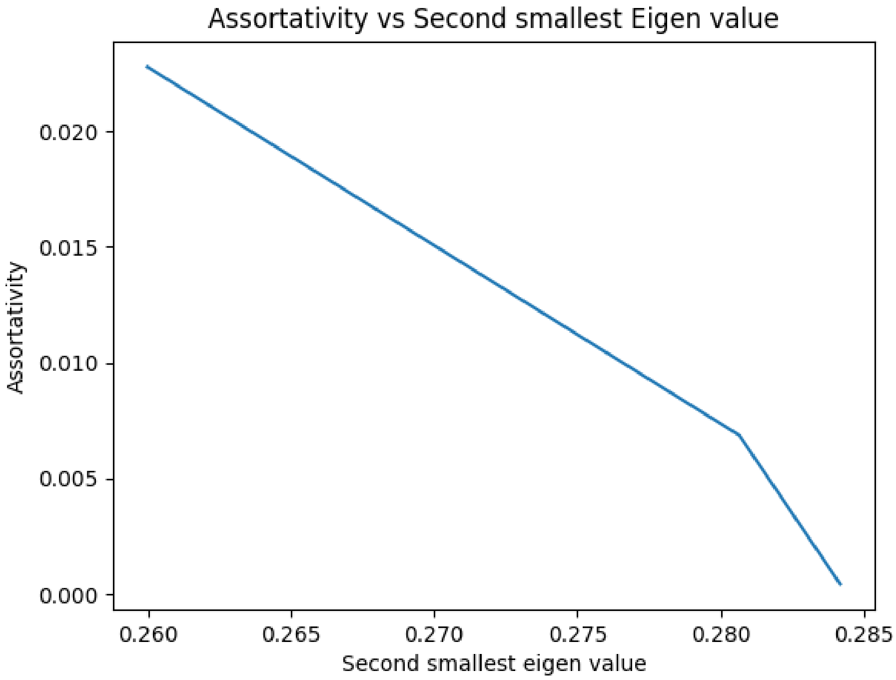


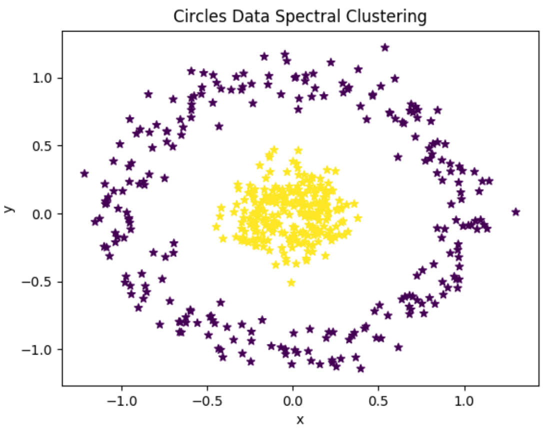
Figure 5: Assortativity vs Second smallest eigenvalue of Laplacian

Question 35

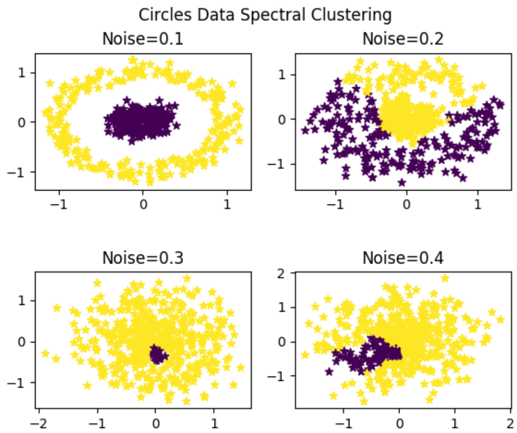
The spectral clustering of the graph is done by calculating the Fiedler vector. It gives the minimum graph cut in finding the two communities. The communities are decided using the sign of the elements of the Fielder vector.
Here for the "circles data" we first find the adjacency matrix by taking the 5 nearest neighbours. Then we find the laplacian of the adjacency matrix.

$$L = diag(A.\mathbf{1}) - A$$

Then we find the Fiedler vector of L. The clusters are then assigned on the sign of the elements.
The results are shown in the figure 6a and 6b.



(a) Spectral Clustering



(b) Spectral Clustering for various noise levels

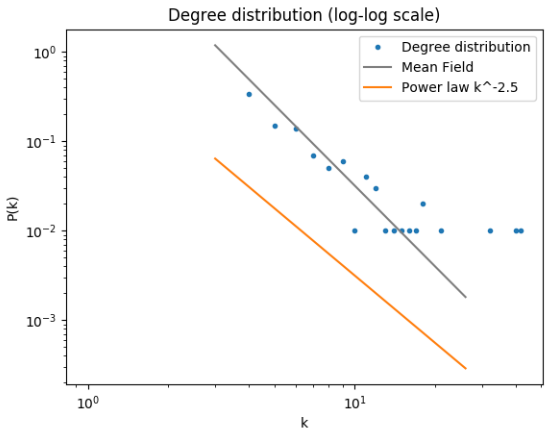
We can see that for various noise levels the spectral clustering fails.

Question 36

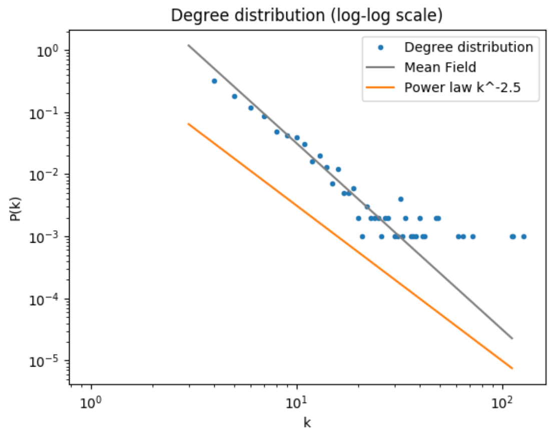
Simulate a preferential attachment model with n = 10^4 nodes and m = 4. Use as initial condition a fully connected network with 4 nodes.

a

Figure 7a and 7b shows the degree distribution for an actual Barabasi-Albert Model with 100 and 1000 nodes respectively and also the degree distribution for the mean-field model and power law model with $k^{-2.5}$. This distribution is similar to power law distribution and can be seen from the figure 7a and 7b the only difference is the offset, but we can see that the degree distribution is similar and is almost parallel to the mean-field degree distribution.



(a) Degree Distribution for a preferential attachment model with 100 nodes, mean-field distribution and power-law distribution for $k^{-2.5}$



(b) Degree Distribution for a preferential attachment model with 1000 nodes, mean-field distribution and power-law distribution for $k^{-2.5}$

b

The assortativity coefficient of a Barabasi-Albert preferential network is shown in the following table:

Nodes	Assortativity
100	-0.1076
1000	-0.045

Table 1: Nodes and Assortativity for a Barabasi-Albert preferential model

c

A preferential attachment model was formed with edge probability 0.5 in each direction and the page rank was calculated.