Problem 2:

SVD(A) =>

$A = USV^T$

where U is orthonormal mxm column matrix, S is mxn diagonal matrix of singular values, V^T is nxn orthonormal matrix

Using the SVD representation of A:

$A^TA \Rightarrow (USV^T)^TUSV^T \Rightarrow VS^T(U^TU)SV^T \Rightarrow VS^2V^T$

Prove Eigen Decomposition of A^TA is VS^2V^T (Normal decomp form: $\Sigma\Lambda\Sigma^{-1}$):

Multiply both sides by V

$A^TAV = VS^2V^TV$

$A^TAV = VS^2$

V can be represented as [v1,v2,…vn] and S^2 as nxn matrix with $[s1^2, s2^2, .. sn^2]$ on the main diagonal (singular values)

$$A^TA * [v1, v2, \dots vn] = [v1, v2, \dots vn] * \begin{bmatrix} s1^2 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & s1^2 \end{bmatrix}$$

Left hand side matrix multiplication produces:

$B = [bi, \dots bn]$ column matrix where 'i' ranges from 1 to n

Right hand side matrix multiplication produces:

$B = [v1*s1^2, v2*s2^2, \dots vn*sn^2]$ column matrix

Since the column vectors of V are simply being scaled by the corresponding scalars of the S^2 matrix according to the right side multiplication, we can conclude that V must be the Eigenvector matrix and S^2 must be the diagonal eigenvalue matrix of A^TA. $V == \Sigma$ and $S^2 == \Lambda$

Conclusion:

Orthonormal matrix V of SVD(A) is the eigenvector matrix and S^2 is the eigenvalue matrix of A^TA