

a)

$$\begin{aligned}x(t) &= x_1 t + x_0 \\y(t) &= y_1 t + y_0 \\z(t) &= z_1 t + z_0\end{aligned}$$

b)

Using perspective projection camera model:

u is equivalent to image plane x coordinate, and v is equivalent to image plane y coordinate

$$u = \frac{f(x_1 t + x_0)}{z(t) = z_1 t + z_0} \quad v = \frac{f(y_1 t + y_0)}{z(t) = z_1 t + z_0}$$

c)

Rewrite each equation to equal 't' and equate to each other

$$\begin{aligned}uz_1 t + uz_0 &= fx_1 t + fx_0 \\t &= \frac{fx_0 - uz_0}{uz_1 - fx_1}\end{aligned} \tag{1}$$

$$\begin{aligned}vz_1 t + vz_0 &= fy_1 t + fy_0 \\t &= \frac{fy_0 - vz_0}{vz_1 - fy_1}\end{aligned}$$

$$\frac{fx_0 - uz_0}{uz_1 - fx_1} = \frac{fy_0 - vz_0}{vz_1 - fy_1} \tag{2}$$

Cross multiply, bring right side to left side, and move terms around to get implicit form of a line equation:

$$(fx_0 - uz_0)(vz_1 - fy_1) = (uz_1 - fx_1)(fy_0 - vz_0) \tag{3}$$

$$(fz_0 y_1 - fz_1 y_0) u + (fz_1 x_0 - fz_0 x_1) v - f^2 x_0 y_1 + f^2 x_1 y_0 = 0$$

$$(fz_0 y_1 - fz_1 y_0) u + (fz_1 x_0 - fz_0 x_1) v + f^2 x_1 y_0 - f^2 x_0 y_1 = 0 \tag{4}$$

d)

When the line is directly parallel to the z axis or axis which spans out of the camera, the fixing of x and y axis allows for the line to continually extend in the z direction which give the illusion of a single point being projected onto the image plane