## UNITARY SUPERMULTIPLETS OF $OSp(8^*|4)$ AND THE $AdS_7/CFT_6$ DUALITY <sup>1</sup>

#### Murat Günaydin<sup>2</sup>

CERN , Theory Division 1211 Geneva 23, Switzerland and Physics Department Penn State University University Park, PA 16802 and

## Seiji Takemae <sup>3</sup>

Physics Department Penn State University University Park, PA 16802

#### Abstract

We study the unitary supermultiplets of the  $\mathcal{N}=4$  d=7 anti-de Sitter  $(AdS_7)$  superalgebra  $OSp(8^*|4)$ , with the even subalgebra  $SO(6,2)\times USp(4)$ , which is the symmetry superalgebra of M-theory on  $AdS_7\times S^4$ . We give a complete classification of the positive energy doubleton and massless supermultiplets of  $OSp(8^*|4)$ . The ultra-short doubleton supermultiplets do not have a Poincaré limit in  $AdS_7$  and correspond to superconformal field theories on the boundary of  $AdS_7$  which can be identified with d=6 Minkowski space. We show that the six dimensional Poincare mass operator  $m^2=P_\mu P^\mu$  vanishes identically for the doubleton representations. By going from the compact U(4) basis of  $SO^*(8)=SO(6,2)$  to the noncompact basis  $SU^*(4)\times \mathcal{D}$  (d=6 Lorentz group times dilatations) one can associate the positive (conformal) energy representations of  $SO^*(8)$  with conformal fields transforming covariantly under the Lorentz group in d=6. The oscillator method used for the construction of the unitary supermultiplets of  $OSp(8^*|4)$  can be given a dynamical realization in terms of chiral super-twistor fields.

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<sup>&</sup>lt;sup>2</sup>e-mail: murat@phys.psu.edu <sup>3</sup>e-mail: takemae@phys.psu.edu.

# 1 Introduction

Recently, a great deal of work has been done on AdS/CFT (anti-de Sitter/conformal field theory) dualities in various dimensions. The current interest in AdS/CFT dualities was sparked by the conjecture of Maldacena [1] relating the large N limits of certain conformal field theories in d dimensions to M-theory/string theory compactified to d+1-dimensional AdS spacetimes. Maldacena's conjecture was based on earlier work on properties of the physics of N Dp-branes in the near horizon limit [2] and the much earlier work on 10-d IIB supergravity compactified on  $AdS_5 \times S^5$  and 11-d supergravity compactified on  $AdS_7 \times S^4$  and  $AdS_4 \times S^7$  [3, 4, 5, 6, 7, 8, 9]. Maldacena's conjecture was formulated in a more precise manner in [10, 11]. The relation between Maldacena's conjecture and the dynamics of the singleton and doubleton fields that live on the boundary of AdS spacetimes was reviewed in [12, 13] and its relation to the spectra of maximal supergravities in eleven and ten dimensions in [13, 14]. For a recent review of AdS/CFT dualities and an extensive list of references on the subject see [15].

The best studied example of this AdS/CFT duality is the duality between the large N limit of the conformally invariant  $\mathcal{N}=4$  SU(N) super Yang-Mills theory in d=4 and type IIB superstring theory on  $AdS_5 \times S^5$ . The  $\mathcal{N}=4$  Yang-Mills supermultiplet is simply the unique CPT self-conjugate doubleton supermultiplet of the symmetry superalgebra SU(2,2|4) of type IIB superstring over  $AdS_5 \times S^5$  [4, 16]. Doubleton supermultiplets do not have a smooth Poincaré limit in d=5 and correspond to superconformal field theories on the boundary of  $AdS_5$ , which can be identified with the four dimensional Minkowski space. The SU(2,2|4) algebra acts as the  $\mathcal{N}=4$  extended superconformal algebra on the boundary of  $AdS_5$ . A complete list of doubleton and massless (in  $AdS_5$  sense) supermultiplets of SU(2,2|4) was given in [16, 17]. The CPT self-conjugate massless supermultiplet of SU(2,2|4) is the massless graviton supermultiplet in d=5. This massless multiplet sits at the bottom of an infinite tower of massive short supermultiplets of SU(2,2|4) corresponding to the Kaluza-Klein spectrum of IIB supergravity over  $S^5$  [4]. The oscillator construction yields not only the short multiplets but the massless and massive intermediate and long multiplets as well [4, 17, 18].

M-theory compactified on the four sphere  $S^4$  is similarly believed to be dual to (2,0) superconformal field theory in six dimensions (in a certain limit). The symmetry superalgebra of M-theory compactified to  $AdS_7$  over  $S^4$  is  $OSp(8^*|4)$  [6, 8]. The general method for the oscillator construction of unitary supermultiplets of  $OSp(8^*|4)$  was given in [8] with emphasis on short supermultiplets that appear in the Kaluza-Klein compactification of 11 dimensional supergravity theory. The entire Kaluza-Klein spectrum of the eleven dimensional supergravity over  $AdS_7 \times S^4$  can be obtained by a simple tensoring procedure from the "CPT self-conjugate" doubleton supermultiplet [8]. This "CPT self-conjugate" doubleton supermultiplet is simply the (2,0) superconformal multiplet of the dual field theory in six dimensions. The  $AdS_7/CFT_6$  duality has been studied from various points of view more recently [19].

In this paper, we shall give a complete study of the doubleton and massless (in  $AdS_7$  sense) supermultiplets of  $OSp(8^*|4)$  which we believe are relevant to the understanding of the full spectrum of M-theory over  $AdS_7 \times S^4$ . Our results extend those of [16, 17] on  $AdS_5/CFT_4$  dualities to  $AdS_7/CFT_6$  dualities and their relation to the unitary supermultiplets of  $OSp(8^*|4)$ . In particular, in section 2, we review the oscillator construction of the positive energy UIRs of SO(6,2) and show how to go from the compact  $(SU(4) \times U(1))$  basis to the noncompact  $SU^*(4) \times \mathcal{D}$  basis. Considered as the d=6 conformal group, the  $SU^*(4) \times \mathcal{D}$  subgroup of SO(6,2) can be identified with d=6 Lorentz transformations times dilatations. We show that doubleton representations of SO(6,2) are all massless in d=6. Furthermore we show that the UIR's of the lowest weight type constructed by the oscillator method are equivalent to unitary representations of the conformal

group induced by a finite dimensional representation of the Lorentz group  $SU^*(4)$  with a definite conformal dimension. Remarkably, the compact SU(4) labelling and the non-compact  $SU^*(4)$  labelling of the UIRs coincide. Furthermore, the conformal dimension, l, which is given by the eigenvalues of the dilatation operator, is simply -E, where E is the (conformal) energy. Thus, the positive energy UIRs with energy E can be identified with d=6 conformal fields with conformal dimension l=-E. In section 3, we review the superoscillator construction of the superalgebra  $OSp(8^*|4)$ . In section 4, we give a complete list of the doubleton supermultiplets of  $OSp(8^*|4)$  which correspond to massless conformal supermultiplets in d=6. In section 5, we give a complete list of massless supermultiplets of  $OSp(8^*|4)$  considered as the seven dimensional AdS supergroup. The short massless supermultiplets have spin range 2 in the d=4 sense, the long massless supermultiplets have spin range 4 and the intermediate massless supermultiplets have spin ranges between 2 and 4. We conclude with a discussion of our results as they relate to AdS/CFT dualities.

# 2 Compact $(SU(4) \times U(1))$ versus non-compact $(SU^*(4) \times \mathcal{D})$ bases for the positive energy unitary representations of the group $SO^*(8) = SO(6,2)$

The seven dimensional AdS group  $SO^*(8) = SO(6,2)$  (with the covering group  $Spin^*(8) = Spin(6,2)$ ) is isomorphic to the six dimensional conformal group (its covering group).<sup>4</sup> As the conformal group in d=6,  $SO^*(8)$  is generated by the Lorentz group (SO(5,1)) with the covering group  $SU^*(4)$ ) generators  $M_{\mu\nu}$ , the six translation generators  $P_{\mu}$ , the dilatation generator D and the generators of special conformal transformations  $K_{\mu}$  ( $\mu, \nu, \ldots = 0, 1, 2, 3, 4, 5$ ). The commutation relations of these generators are

$$[M_{\mu\nu}, M_{\rho\sigma}] = i(\eta_{\nu\rho} M_{\mu\sigma} - \eta_{\mu\rho} M_{\nu\sigma} - \eta_{\nu\sigma} M_{\mu\rho} + \eta_{\mu\sigma} M_{\nu\rho})$$

$$[P_{\mu}, M_{\rho\sigma}] = i(\eta_{\mu\rho} P_{\sigma} - \eta_{\mu\sigma} P_{\rho})$$

$$[K_{\mu}, M_{\rho\sigma}] = i(\eta_{\mu\rho} K_{\sigma} - \eta_{\mu\sigma} K_{\rho})$$

$$[D, M_{\mu\nu}] = [P_{\mu}, P_{\nu}] = [K_{\mu}, K_{\nu}] = 0$$

$$[P_{\mu}, D] = iP_{\mu}; [K_{\mu}, D] = -iK_{\mu}$$

$$[P_{\mu}, K_{\nu}] = 2i(\eta_{\mu\nu} D - M_{\mu\nu})$$

$$(2 - 1)$$

with  $\eta_{\mu\nu} = \text{diag}(+, -, -, -, -, -)$ . The rotation subgroup SO(5) ( or its covering group USp(4)) is generated by  $M_{ij}$  with i, j = 1, 2, ..., 5.

Defining

$$M_{\mu 6} = \frac{1}{2}(P_{\mu} - K_{\mu}), \quad M_{\mu 7} = \frac{1}{2}(P_{\mu} + K_{\mu}), \quad M_{67} = -D,$$
 (2 - 2)

the commutation relations of the d=6 conformal algebra can be written as

$$[M_{ab}, M_{cd}] = i(\eta_{bc}M_{ad} - \eta_{ac}M_{bd} - \eta_{bd}M_{ac} + \eta_{ad}M_{bc}), \qquad (2 - 3)$$

where  $-\eta_{66} = \eta_{77} = 1$  and a, b, ... = 0, 1, ..., 7.

Considered as the isometry group of seven dimensional anti-de Sitter space, the generators of  $SO^*(8)$  have a different physical interpretation. In particular, the rotation group becomes SO(6), with the covering group SU(4), generated by  $M_{mn}$   $(m, n, \ldots = 1, 2, ..., 6)$ . The generator  $E \equiv M_{07}$  becomes the AdS energy generating translations along the timelike Killing vector field of  $AdS_7$ ,

<sup>&</sup>lt;sup>4</sup>Below we shall sometimes denote the covering group by the same symbol as the group itself. Unless otherwise specified we shall however always be dealing with the covering groups in question.

and the non-compact generators  $M_{0m}$ ,  $M_{m7}$  correspond to "boosts" and spacelike "translations" in  $AdS_7$ .

There are two different subgroups of the d=6 conformal group SO(6,2) which play an important role in the classification of its physically relevant representations. First is the maximal compact subgroup  $SU(4) \times U(1)_E$  generated by  $M_{mn}$  and  $E \equiv M_{07}$  with the  $U(1)_E$  generator  $E = \frac{1}{2}(P_0 + K_0)$  being simply the conformal Hamiltonian. Denoting the Lie algebra of  $SU(4) \times U(1)_E$  as  $L^0$ , the conformal algebra g = SO(6,2) has a three graded decomposition with respect to  $L^0$ :

$$g = L^+ \oplus L^0 \oplus L^-, \tag{2-4}$$

where

The general construction of the positive energy unitary representations of  $SO^*(8)$  uses the realization of its generators as bilinears of an arbitrary number P (generations or colors) of pairs of bosonic annihilation  $(\mathbf{a}_i, \mathbf{b}_i)$  and creation operators  $(\mathbf{a}^i, \mathbf{b}^i)$  transforming in the fundamental representation of SU(4) and its conjugate, respectively: [8, 20, 21]

$$M_j^i = \mathbf{a}^i \cdot \mathbf{a}_j + \mathbf{b}_j \cdot \mathbf{b}^i,$$

$$A_{ij} = \mathbf{a}_i \cdot \mathbf{b}_j - \mathbf{a}_j \cdot \mathbf{b}_i,$$

$$A^{ij} = \mathbf{a}^i \cdot \mathbf{b}^j - \mathbf{a}^j \cdot \mathbf{b}^i$$
(2 - 6)

where  $\mathbf{a}_i \cdot \mathbf{b}_j = \sum_{K=1}^P a_i(K)b_j(K)$  etc.. The bosonic annihilation and creation operators  $a^i(K) = a_i(K)^{\dagger}$ ,  $b^i(K) = b_i(K)^{\dagger}$  satisfy the commutation relations:

$$[a_i(K), a^j(L)] = \delta_i^j \delta_{KL}, \quad [b_i(K), b^j(L)] = \delta_i^j \delta_{KL}$$

$$(2 - 7)$$

with i, j = 1, 2, 3, 4 and K, L = 1, ..., P.  $M_j^i$  are the generators of the maximal compact subgroup U(4). Hermitian linear combinations of the  $A_{ij}$  and  $A^{ij}$  are the non-compact generators of SO(6, 2). [20, 8]

Physically relevant representations of the conformal group are unitary irreducible representations (UIRs) of the lowest weight type in which the spectrum of the conformal Hamiltonian (resp. the AdS energy), E, is bounded from below. The natural basis for constructing them is the compact basis in which the lowest weight (positive energy) property becomes manifest. The lowest weight UIRs of SO(6,2) can then be constructed in a simple way by using the oscillator realization of the generators given above. Each lowest weight UIR is uniquely determined by the quantum numbers of a lowest weight vector,  $|\Omega\rangle$ , provided that  $|\Omega\rangle$  transforms irreducibly under  $SU(4) \times U(1)_E$  and is also annihilated by all the elements of  $L^-[8]$ .

On the other hand, in d=6 conformal field theory, one would like to work with fields that transform covariantly under the Lorentz group,  $SU^*(4)$ , and the dilatations <sup>5</sup>. In d=4, the standard way to construct these fields is via the method of induced representations [17, 23]. In this method, a representation (typically finite dimensional) of the stability group, H, of  $x_{\mu}=0$  (where  $x_{\mu}$  is the coordinate four vector) induces a representation of the conformal group SO(4,2).

<sup>&</sup>lt;sup>5</sup>Note that the conformal group SO(6,2) has a three graded structure with respect to the subgroup  $SU^*(4) \times \mathcal{D}$  just as it has with respect to its maximal compact subgroup  $SU(4) \times U(1)$  [22]

When SO(6,2) acts in the standard way on the (conformal compactification of) 6d Minkowski spacetime, the stability group, H, of the coordinate six-vector  $x^{\mu} = 0$  becomes the semi-direct product  $(SU^*(4) \times \mathcal{D}) \odot \mathcal{K}_6$ . Furthermore,  $\mathcal{K}_6$  represents the Abelian subgroup generated by the special conformal generators,  $K_{\mu}$ . The Lie algebra of H consists of the generators,  $M_{\mu\nu}$ , of the Lorentz group,  $SU^*(4)$ , the dilatation operator, D, and the generators of the special conformal transformations,  $K_{\mu}$ . In our case the d=6 conformal group, G = SO(6,2), will be realized on fields that live on the coset space, G/H. These fields are labelled by their transformation properties under the Lorentz group,  $SU^*(4)$ , their conformal dimension, l, and certain matrices,  $\kappa_{\mu}$ . In particular, these matrices,  $\kappa_{\mu}$ , are related to their behavior under special conformal transformations,  $K_{\mu}$ , as in d = 4 [17, 23].

To establish a dictionary between the compact and non-compact viewpoints, we shall now reformulate the construction of positive energy representations of SO(6,2) [8] in terms of twistorial operators. The twistorial operators we introduce will involve eight dimensional bosonic spinors. This is analogous to the twistorial construction in the case of the d=4 conformal group [17]. A dynamical realization of the twistorial construction of the representations of SU(2,2|4) was given in [18, 24]. To this end, let  $\Gamma_{\mu}$  be the 6d gamma matrices ( $\{\Gamma_{\mu}, \Gamma_{\nu}\} = 2\eta_{\mu\nu}$ ) with  $\Gamma_{7} = -i\Gamma_{0}\Gamma_{1}\Gamma_{2}\cdots\Gamma_{5}$ . Then the matrices,

$$\Sigma(M_{\mu\nu}) := \frac{i}{4} [\Gamma_{\mu}, \Gamma_{\nu}],$$

$$\Sigma(M_{\mu6}) := \frac{i}{2} \Gamma_{\mu} \Gamma_{7},$$

$$\Sigma(M_{\mu7}) := \frac{1}{2} \Gamma_{\mu},$$

$$\Sigma(M_{67}) := \frac{1}{2} \Gamma_{7},$$

generate an eight dimensional (non-unitary) spinor representation of the conformal algebra, SO(6,2). We shall refer to this representation as the left-handed spinor representation. The right handed spinor representation is obtained by defining  $\Gamma_7$  with the opposite sign (i.e.  $\Gamma_7 = +i\Gamma_0\Gamma_1\Gamma_2\cdots\Gamma_5$ ). Unless otherwise specified we shall work with the left-handed spinor representation below. For our gamma matrices we choose

$$\Gamma_{0} = \sigma_{3} \otimes I_{2} \otimes I_{2} 
\Gamma_{1} = i\sigma_{1} \otimes \sigma_{2} \otimes I_{2} 
\Gamma_{2} = i\sigma_{1} \otimes \sigma_{1} \otimes \sigma_{2} 
\Gamma_{3} = i\sigma_{1} \otimes \sigma_{3} \otimes \sigma_{2} 
\Gamma_{4} = i\sigma_{2} \otimes I_{2} \otimes \sigma_{2} 
\Gamma_{5} = i\sigma_{2} \otimes \sigma_{2} \otimes \sigma_{1} 
\Gamma_{7} = -i\sigma_{2} \otimes \sigma_{2} \otimes \sigma_{3}$$
(2 - 8)

where  $\sigma_1, \sigma_2$  and  $\sigma_3$  are the Pauli matrices.

We now regroup these bosonic oscillators into left-handed spinorial operators,  $\Psi(K)$ , which we

define as

$$\Psi(K) := \begin{pmatrix}
a_1(K) \\
a_2(K) \\
a_3(K) \\
a_4(K) \\
b^1(K) \\
b^2(K) \\
b^3(K) \\
b^4(K),
\end{pmatrix} (2 - 9)$$

so that

$$\bar{\Psi}(K) \equiv \Psi^{\dagger}(K)\Gamma^{0} 
= \left(a^{1}(K), a^{2}(K), a^{3}(K), a^{4}(K), -b_{1}(K), -b_{2}(K), -b_{3}(K), -b_{4}(K)\right). \quad (2 - 10)$$

Consider now the bilinears of these twistorial operators involving the  $8 \times 8$  matrices  $\Sigma(M_{ab})$ :

$$\bar{\Psi}\Sigma(M_{ab})\Psi := \sum_{K=1}^{P} \bar{\Psi}(K)\Sigma(M_{ab})\Psi(K). \tag{2-11}$$

One finds that they satisfy the commutation relations of SO(6,2)

$$\left[\bar{\Psi}\Sigma(M_{ab})\Psi, \bar{\Psi}\Sigma(M_{cd})\Psi\right] = \bar{\Psi}\left[\Sigma(M_{ab}), \Sigma(M_{cd})\right]\Psi. \tag{2-12}$$

and yield an infinite dimensional unitary representation in the Fock space of the oscillators  $a^i(K)$  and  $b^i(K)$ .

We have the following relations between the generators realized as bilinears of twistorial operators and the bilinears given in terms of U(4) covariant oscillators:

$$\bar{\psi}\Gamma_{0}\psi = M_{1}^{1} + M_{2}^{2} + M_{3}^{3} + M_{4}^{4}$$

$$\bar{\psi}\Gamma_{01}\psi = A^{13} + A^{24} + A_{31} + A_{42}$$

$$\bar{\psi}\Gamma_{1}\psi = A^{13} + A^{24} + A_{13} + A_{24}$$

$$\bar{\psi}\Gamma_{02}\psi = A^{14} + A^{32} + A_{41} + A_{23}$$

$$\bar{\psi}\Gamma_{2}\psi = A^{14} + A^{32} + A_{14} + A_{32}$$

$$\bar{\psi}\Gamma_{03}\psi = A^{12} + A^{43} + A_{21} + A_{34}$$

$$\bar{\psi}\Gamma_{3}\psi = A^{12} + A^{43} + A_{12} + A_{43}$$

$$\bar{\psi}\Gamma_{04}\psi = i\left(A^{21} + A^{43} + A_{21} + A_{43}\right)$$

$$\bar{\psi}\Gamma_{4}\psi = i\left(A^{21} + A^{43} + A_{12} + A_{34}\right)$$

$$\bar{\psi}\Gamma_{5}\psi = i\left(A^{41} + A^{32} + A_{41} + A_{32}\right)$$

$$\bar{\psi}\Gamma_{5}\psi = i\left(A^{41} + A^{32} + A_{14} + A_{23}\right)$$

$$\bar{\psi}\Gamma_{7}\psi = i\left(A^{13} + A^{42} + A_{13} + A_{42}\right)$$

$$\bar{\psi}\Gamma_{7}\psi = i\left(A^{13} + A^{42} + A_{31} + A_{24}\right).$$
(2 - 13)

As stated above, the positive energy UIRs of SO(6,2) are easily obtained by constructing an irreducible representation  $|\Omega\rangle$  (lowest weight vector) of  $SU(4) \times U(1)_E$  in the Fock space of the oscillators that is annihilated by all the operators  $A_{ij}$  of  $L^-$  subspace:

$$A_{ij}|\Omega\rangle = 0. (2 - 14)$$

Acting repeatedly with the di-creation operators  $A^{ij}$  of  $L^+$  on  $|\Omega\rangle$ , one generates the basis of a positive energy UIR of the group  $SO^*(8)$ . To give concrete examples of lowest weight vectors, consider the case P=1 (doubletons). In this case the possible lowest weight vectors are of the form:

$$a^{i_1}|0\rangle = | \square \rangle$$

$$a^{i_1}a^{i_2}|0\rangle = | \square \rangle$$

$$\vdots$$

$$a^{i_1}a^{i_2}\cdots a^{i_n}|0\rangle = | \square \cdots \square \rangle. \qquad (2 - 15)$$

or of the form

This shows that the possible lowest weight vectors  $|\Omega\rangle$  of the doubleton UIR's of  $SO^*(8)$  transform in the symmetric tensor representations of SU(4).

The positive energy UIRs of  $SO^*(8)$  can be identified with conformal fields in d=6 with positive conformal energy and a definite conformal dimension transforming covariantly under the six dimensional Lorentz group. To establish this connection we need to find a mapping from the  $SU^*(4)$ - and D-covariant basis to the compact U(4) basis. For this we define the following set of generators  $J_{mn}$  (m,n, ... = 1, ..., 6) of another compact SU(4) basis in terms of the  $SU^*(4)$  generators,  $M_{\mu\nu} = \{M_{ij}, M_{i0}\}$  (i,j, ... = 1, ..., 5),

$$J_{ij} := M_{ij},$$

$$J_{i6} := iM_{i0} = -J_{6i}.$$
(2 - 17)

The two SU(4)s generated by  $J_{mn}$  and  $M_{mn}$  have as a common subgroup the rotation group USp(4) in d=6. Consider now the operator

$$U := e^{\bar{\Psi}\Sigma(M_{06} + iM_{67})\Psi}. (2 - 18)$$

It has the following important property

$$J_{mn}U = U(M_{mn} + L^{-}),$$
  
 $DU = U(-iE + L^{-}),$   
 $K_{\mu}U = U(L^{-}),$  (2 - 19)

where  $L^-$  stands for certain linear combinations of di-annihilation operators  $A_{ij}$ . Specifically, for  $J_{mn}U = U(M_{mn} + L^-)$  we find,

$$\bar{\psi}\Sigma(J_{16})\psi U = U\left(-\bar{\psi}\Sigma(M_{16})\psi + \frac{1}{2}(A_{31} + A_{42})\right) 
\bar{\psi}\Sigma(J_{26})\psi U = U\left(-\bar{\psi}\Sigma(M_{26})\psi + \frac{1}{2}(A_{41} + A_{23})\right) 
\bar{\psi}\Sigma(J_{36})\psi U = U\left(-\bar{\psi}\Sigma(M_{36})\psi + \frac{1}{2}(A_{21} + A_{34})\right) 
\bar{\psi}\Sigma(J_{46})\psi U = U\left(-\bar{\psi}\Sigma(M_{46})\psi + \frac{i}{2}(A_{21} + A_{43})\right)$$

$$\bar{\psi}\Sigma(J_{56})\psi U = U\left(-\bar{\psi}\Sigma(M_{56})\psi + \frac{i}{2}(A_{41} + A_{32})\right)$$

$$\bar{\psi}\Sigma(J_{ij})\psi U = U(\bar{\psi}\Sigma(M_{ij})\psi). \qquad (2 - 20)$$

For  $DU = U(-iE + L^{-})$ , we find,

$$\bar{\psi}\Sigma(D)\psi U = U\left(-iE + \frac{-i}{2}(A_{31} + A_{24})\right).$$
 (2 - 21)

For  $K_{\mu}U = U(L^{-})$ , we find,

$$\bar{\psi}\Sigma(K_{0})\psi U = U\left(\frac{1}{2}(A_{13} + A_{42})\right) 
\bar{\psi}\Sigma(K_{1})\psi U = U\left(\frac{1}{2}(A_{13} + A_{24})\right) 
\bar{\psi}\Sigma(K_{2})\psi U = U\left(\frac{1}{2}(A_{14} + A_{32})\right) 
\bar{\psi}\Sigma(K_{3})\psi U = U\left(\frac{1}{2}(A_{12} + A_{43})\right) 
\bar{\psi}\Sigma(K_{4})\psi U = U\left(\frac{i}{2}(A_{12} + A_{34})\right) 
\bar{\psi}\Sigma(K_{5})\psi U = U\left(\frac{i}{2}(A_{14} + A_{23})\right).$$
(2 - 22)

Acting with U on a lowest weight vector  $|\Omega\rangle$  corresponds to a (complex) rotation in the corresponding representation space of  $SO^*(8)$ :

$$U|\Omega\rangle = e^{\bar{\Psi}\Sigma(M_{06} + iM_{67})\Psi}|\Omega\rangle. \tag{2-23}$$

Since  $L^-|\Omega\rangle = 0$ , it then follows from (2 - 19) that  $\Phi(0) := U|\Omega\rangle$  is an irreducible representation of the little group H with conformal dimension  $^6 l = -E$  and trivially represented special conformal transformations  $K_\mu$  (i.e.  $\kappa_\mu = 0$ ). Acting with  $e^{-ix^\mu P_\mu}$  on  $\Phi(0)$  then translates the field in Minkowski space:

$$\Phi(x^{\mu}) = e^{-ix^{\mu}P_{\mu}} \cdot \Phi(0) = e^{-ix^{\mu}P_{\mu}}U|\Omega\rangle$$
 (2 - 24)

and generates a (group theoretically equivalent) induced representation of  $SO^*(8)$ . We should note that the state  $\Phi(x^{\mu})$  can be thought of as a coherent state labelled by  $\Omega$  and the coordinate  $x^{\mu}$ .

Since the bosonic oscillators, in terms of which we realized the generators, transform in the spinor representation of  $SO^*(8)$ , the oscillator construction can be given a dynamical realization as was done for SU(2,2) [18, 24].

We recall that the doubleton representations of  $SO^*(8)$  correspond to taking a single pair (P=1) of bosonic oscillators and they do not have a smooth Poincaré limit in d=7. Let us now show that they correspond to massless fields in d=6 with mass defined in the usual Poincaré sense,

$$m^2 = P_{\mu}P^{\mu}.\tag{2-25}$$

The translation generators  $P_{\mu}$  have the following realization in terms of the oscillators:

$$P_0 = \bar{\psi} \left( \Sigma \left( M_{06} \right) + \Sigma \left( M_{07} \right) \right) \psi$$

<sup>&</sup>lt;sup>6</sup>In our conventions, *l* has dimension of length (or inverse mass).

$$\begin{aligned}
&= \frac{1}{2}\bar{\psi}\Gamma_{0} (1 + i\Gamma_{7}) \psi \\
P_{1} &= \bar{\psi} (\Sigma (M_{16}) + \Sigma (M_{17})) \psi \\
&= \frac{1}{2}\bar{\psi}\Gamma_{1} (1 + i\Gamma_{7}) \psi \\
P_{2} &= \bar{\psi} (\Sigma (M_{26}) + \Sigma (M_{27})) \psi \\
&= \frac{1}{2}\bar{\psi}\Gamma_{2} (1 + i\Gamma_{7}) \psi \\
P_{3} &= \bar{\psi} (\Sigma (M_{36}) + \Sigma (M_{37})) \psi \\
&= \frac{1}{2}\bar{\psi}\Gamma_{3} (1 + i\Gamma_{7}) \psi \\
P_{4} &= \bar{\psi} (\Sigma (M_{46}) + \Sigma (M_{47})) \psi \\
&= \frac{1}{2}\bar{\psi}\Gamma_{4} (1 + i\Gamma_{7}) \psi \\
P_{5} &= \bar{\psi} (\Sigma (M_{56}) + \Sigma (M_{57})) \psi \\
&= \frac{1}{2}\bar{\psi}\Gamma_{5} (1 + i\Gamma_{7}) \psi.
\end{aligned} (2 - 26)$$

Substituting in the above expressions for  $P_{\mu}$  one finds after some lengthy calculation the mass operator  $m^2$  vanishes identically for P=1. Thus all the doubleton irreps of  $SO^*(8)$  are massless in d=6. For  $P \neq 1$  the mass operator is non-vanishing and the lowest weight vectors of  $SO^*(8)$  are in general not eigenstates of the mass operator. Hence they correspond to massive conformal fields in d=6. We should stress that this is in complete parallel to the situation in d=4 where the doubleton representations of SO(4,2) are all massless [25, 17].

# 3 Oscillator Construction of the UIRs of $OSp(8^*|4)$

In this section, we give the commutation relations of  $OSp(8^*|4)$  in a non-compact as well as a compact basis. Bearing in mind that it was shown above how to go from the compact basis to a non-compact basis for the UIR's of SO(6,2), we then present a realization of the  $OSp(8^*|4)$  generators in the compact basis in terms of superoscillators. For further details, we refer the reader to [8, 26].

# 3.1 The superalgebra $OSp(8^*|4)$

The symmetry group of M-theory on  $AdS_7 \times S^4$  is the supergroup  $OSp(8^*|4)$  with the even subgroup  $SO^*(8) \times USp(4)$ , where USp(4) is isomorphic to SO(5), the isometry group of the four sphere.  $OSp(8^*|4)$  can be interpreted as the  $\mathcal{N}{=}4$  extended AdS superalgebra in d=7 or as the (2,0) extended conformal superalgebra in d=6. Because of triality property of the representations of SO(8) and hence of the finite dimensional representations of SO(6,2) there exist three different forms of the superalgebra  $OSp(8^*|4)$ , according to whether the supersymmetry generators transform in the vector, left-handed or right-handed spinor representation of SO(6,2). The anti-symmetric tensor of any one of these three representations transform like the adjoint representation of SO(6,2). For M-theory on  $AdS_7 \times S^4$  the relevant form is the one in which the supersymmetry generators transform in the left-handed spinor representation of SO(6,2), such that it decomposes as  $(4+\bar{4})$  under the compact subgroup SU(4). Denoting the supersymmetry generators of  $OSp(8^*|4)$  as  $Q_{\hat{\alpha}I}$  we can write their anticommutators as [27]

$$\left\{Q_{\hat{\alpha}I}, Q_{\hat{\beta}J}\right\} = -\frac{1}{2} \left(\Omega_{IJ} M_{\hat{\alpha}\hat{\beta}} + C_{\hat{\alpha}\hat{\beta}} U_{IJ}\right). \tag{3 - 27}$$

where I, J, ... = 1, ..., 4 and  $\hat{\alpha}, \hat{\beta} = 1, ... 8$ .  $U_{IJ} = U_{JI}$  are the USp(4) generators and  $\Omega_{IJ} = -\Omega_{JI}$  is the symplectic invariant tensor. The tensor  $C_{\hat{\alpha}\hat{\beta}}$  is the charge conjugation matrix in (6,2) dimensions and is symmetric [27]. Furthermore,  $M_{\hat{\alpha}\hat{\beta}}$  are related to the SO(6,2) generators  $M_{ab}$  given in the previous section as follows:

$$M_{\hat{\alpha}\hat{\beta}} = \frac{1}{8} \left( \Gamma^{ab} \right)_{\hat{\alpha}\hat{\beta}} M_{ab} \tag{3-28}$$

The generators of USp(4) satisfy:

$$[U_{IJ}, U_{KL}] = \Omega_{I(K} U_{L)J} + \Omega_{J(K} U_{L)I}$$
(3 - 29)

The commutation relations of SO(6,2) and USp(4) with the supersymmetry generators are then of the form [27]

$$[M_{ab}, Q_{\hat{\alpha}I}] = \left(\Sigma(M^{ab})\right)_{\hat{\alpha}}^{\hat{\beta}} Q_{\hat{\beta}I},$$
  

$$[U_{IJ}, Q_{K\hat{\alpha}}] = -\Omega_{K(I}Q_{J)\hat{\alpha}}.$$
(3 - 30)

The superalgebra  $OSp(8^*|4)$  has a three graded decomposition with respect to its compact subsuperalgebra U(4|2)

$$g = L^+ \oplus L^0 \oplus L^-, \tag{3-31}$$

where

$$[L^{0}, L^{\pm}] \subseteq L^{\pm}$$
  
 $[L^{+}, L^{-}] \subseteq L^{0}$   
 $[L^{+}, L^{+}] = 0 = [L^{-}, L^{-}].$  (3 - 32)

Here  $L^0$  represents the generators of U(4|2).

Generalizing the (purely bosonic) oscillator construction for  $SO^*(8)$  in section 2, the Lie superalgebra  $OSp(8^*|4)$  can be realized in terms of bilinear combinations of bosonic and fermionic annihilation and creation operators  $\xi_A(K)$  ( $\xi^A(K) = \xi_A^{\dagger}(K)$ ) and  $\eta_M(K)$  ( $\eta^M(K) = \eta_M^{\dagger}(K)$ ) which transform covariantly and contravariantly, respectively, under the U(4|2) subsupergroup of  $OSp(8^*|4)$ 

$$\xi_{A}(K) = \begin{pmatrix} a_{i}(K) \\ \alpha_{\mu}(K) \end{pmatrix}, \quad \xi^{A}(K) = \begin{pmatrix} a^{i}(K) \\ \alpha^{\mu}(K) \end{pmatrix},$$

$$\eta_{B}(L) = \begin{pmatrix} b_{j}(L) \\ \beta_{\nu}(L) \end{pmatrix}, \quad \eta^{B}(L) = \begin{pmatrix} b^{j}(L) \\ \beta^{\nu}(L) \end{pmatrix}.$$
(3 - 33)

with i,j = 1,2,3,4;  $\mu, \nu = 1,2$  and

where { , ] represents an anti-commutator among any two fermionic oscillators and a commutator otherwise. Moreover, annihilation and creation operators are labelled by lower and upper indices,

respectively. The generators of  $OSp(8^*|4)$  are given in terms of the above superoscillators schematically as

$$A_{AB} = \boldsymbol{\xi}_{A} \cdot \boldsymbol{\eta}_{B} - \boldsymbol{\eta}_{A} \cdot \boldsymbol{\xi}_{B} = A_{ij} \oplus A_{\mu\nu} \oplus Q_{i\mu}$$

$$A^{AB} = A_{AB}^{\dagger} = \boldsymbol{\eta}^{B} \cdot \boldsymbol{\xi}^{A} - \boldsymbol{\xi}^{B} \cdot \boldsymbol{\eta}^{A} = A^{ij} \oplus A^{\mu\nu} \oplus Q^{i\mu}$$

$$M_{B}^{A} = \boldsymbol{\xi}^{A} \cdot \boldsymbol{\xi}_{B} + (-1)^{(degA)(degB)} \boldsymbol{\eta}_{B} \cdot \boldsymbol{\eta}^{A} = M_{j}^{i} \oplus M_{\nu}^{\mu} \oplus Q_{\mu}^{i} \oplus Q_{j}^{\mu}$$

$$(3 - 35)$$

where the boldfaced  $\xi$ 's and  $\eta$ 's indicate that we are taking an arbitrary number P of "generations" of superoscillators and the dot represents the summation over the internal index  $K = 1, \ldots, P$  (i.e.  $\boldsymbol{\xi}_A \cdot \boldsymbol{\eta}_M \equiv \Sigma_{K=1}^P \boldsymbol{\xi}_A(K) \eta_M(K)$ ).

The even subgroup  $SO^*(8) \times USp(4)$  is generated by the di-bosonic and di-fermionic generators. In particular, one recovers the  $SO^*(8)$  generators of section 2 in terms of the bosonic oscillators in their  $SU(4) \times U(1)_E$  basis:

$$A_{ij} = \mathbf{a_i} \cdot \mathbf{b_j} - \mathbf{a_j} \cdot \mathbf{b_i},$$

$$A^{ij} = \mathbf{a^i} \cdot \mathbf{b^j} - \mathbf{a^j} \cdot \mathbf{b^i},$$

$$M_j^i = \mathbf{a^i} \cdot \mathbf{a_j} + \mathbf{b_j} \cdot \mathbf{b^i},$$

$$E = Q_B = \frac{1}{2} M_i^i = \frac{1}{2} (N_B + 4P)$$
(3 - 36)

satisfying

$$[A_{ij}, A^{kl}] = \delta_i^k M_i^l + \delta_i^l M_i^k - \delta_i^k M_i^l - \delta_i^l M_i^k.$$
 (3 - 37)

Here,  $N_B = \mathbf{a^i} \cdot \mathbf{a_i} + \mathbf{b^i} \cdot \mathbf{b_i}$  is the bosonic number operator and E (which corresponds to the AdS energy) is the generator of  $U(1)_E$ .

Similarly, the USp(4) generators in their  $SU(2) \times U(1)$  basis are expressed in terms of the fermionic oscillators  $\alpha$  and  $\beta$ :

$$A_{\mu\nu} = \boldsymbol{\xi}_{\mu} \cdot \boldsymbol{\eta}_{\nu} - \boldsymbol{\eta}_{\mu} \cdot \boldsymbol{\xi}_{\nu}$$

$$A^{\mu\nu} = \boldsymbol{\eta}^{\mu} \cdot \boldsymbol{\xi}^{\nu} - \boldsymbol{\xi}^{\mu} \cdot \boldsymbol{\eta}^{\nu}$$

$$M^{\mu}_{\nu} = \boldsymbol{\xi}^{\mu} \cdot \boldsymbol{\xi}_{\nu} - \boldsymbol{\eta}_{\nu} \cdot \boldsymbol{\eta}^{\mu}$$

$$Q_{F} = \frac{1}{2} (\boldsymbol{\alpha}^{\mu} \cdot \boldsymbol{\alpha}_{\mu} - \boldsymbol{\beta}_{\mu} \cdot \boldsymbol{\beta}^{\mu}) = \frac{1}{2} (N_{F} - 2P)$$

$$(3 - 38)$$

with the closure relation

$$[A_{\mu\nu}, A^{\rho\sigma}] = -M^{\rho}_{\mu}\delta^{\sigma}_{\nu} - M^{\sigma}_{\nu}\delta^{\rho}_{\mu} - M^{\sigma}_{\mu}\delta^{\rho}_{\nu} - M^{\rho}_{\nu}\delta^{\sigma}_{\mu}. \tag{3-39}$$

Here  $N_F = \boldsymbol{\alpha}^{\mu} \cdot \boldsymbol{\alpha}_{\mu} + \boldsymbol{\beta}^{\mu} \cdot \boldsymbol{\beta}_{\mu}$  is the fermionic number operator and  $Q_F$  is the generator of U(1). Analogously, the odd generators are given by products of bosonic and fermionic oscillators  $(Q^{i\mu} = \mathbf{a}^i \cdot \boldsymbol{\beta}^{\mu} - \mathbf{b}^i \cdot \boldsymbol{\alpha}^{\mu})$  and satisfy the following closure relations

$$\begin{aligned}
\{Q^{i\mu}, Q^{j\nu}\} &= 0 \\
\{Q_{i\mu}, Q^{j\nu}\} &= \delta^{\nu}_{\mu} M^{j}_{i} - \delta^{j}_{i} M^{\nu}_{\mu}.
\end{aligned} (3 - 40)$$

## 3.2 Unitary Supermultiplets of $OSp(8^*|4)$

To construct a basis for a lowest weight UIR of  $OSp(8^*|4)$ , one starts from a set of states, collectively denoted by  $|\Omega\rangle$ , in the Fock space of the oscillators a, b,  $\alpha$ ,  $\beta$  that transforms irreducibly under U(4|2) and that is annihilated by all the generators  $A_{AB} \equiv (A_{ij} \oplus A_{\mu\nu} \oplus Q_{i\mu})$  of  $L^-$ 

$$A_{AB}|\Omega\rangle = 0. \tag{3 - 41}$$

By acting on  $|\Omega\rangle$  repeatedly with  $L^+$ , one then generates an infinite set of states that form a UIR of  $OSp(8^*|4)$ 

$$|\Omega\rangle, L^{+1}|\Omega\rangle, L^{+1}L^{+1}|\Omega\rangle, \dots$$
 (3 - 42)

The irreducibility of the resulting representation of  $OSp(8^*|4)$  follows from the irreducibility of  $|\Omega\rangle$  under U(4|2). Because of the property (3 - 41),  $|\Omega\rangle$  as a whole will be referred to as the "lowest weight vector (lwv)" of the corresponding UIR of  $OSp(8^*|4)$ .

In the restriction to the subspace involving purely bosonic oscillators, the above construction reduces to the subalgebra  $SO^*(8)$  and its positive energy UIRs as described in section 2. Similarly, when restricted to the subspace involving purely fermionic oscillators, one gets the compact internal symmetry group USp(4) (3 - 38), and the above construction yields the representations of USp(4) in its SU(2) basis.

Accordingly, a lowest weight UIR of  $OSp(8^*|4)$  decomposes into a direct sum of finitely many positive energy UIRs of  $SO^*(8)$  transforming in certain representations of the internal symmetry group USp(4). Thus, each positive energy UIR of  $OSp(8^*|4)$  corresponds to a supermultiplet of fields living in  $AdS_7$  or on its boundary. Interpreted as a UIR of the  $\mathcal{N}=2$  conformal superalgebra in d=6, each lowest weight UIR of  $OSp(8^*|4)$  corresponds to a supermultiplet of massless or massive fields.

We now briefly describe the form in which our results are presented. Consider an  $OSp(8^*|4)$  lwv  $(|\Omega\rangle)$  which transforms in an irrep of U(4|2).  $|\Omega\rangle$  can be decomposed into its irreducible components under the even subgroup  $U(4)\times U(2)$ . This is most simply done by decomposing the supertableaux of  $|\Omega\rangle$  into the Young tableaux of  $U(4)\times U(2)$  using the results of [28]. These  $U(4)\times U(2)$  irreps correspond to lwv's of UIRs of  $SO^*(8)\times USp(4)$ . In the tables given in subsequent sections, such lwv's will be marked by an asterisk adjacent to their corresponding  $U(4)\times U(2)$  Young tableaux. The additional lwv's of  $SO^*(8)\times USp(4)$  are obtained by acting on the lwv  $|\Omega\rangle$  with the supersymmetry generators  $Q^{i\mu}$  repeatedly. These lwv's are listed in the tables without an asterisk. Furthermore, the tables below have been written in "ascending order". That is, immediately following each lwv with an asterisk are its "descendant" lwv's i.e, those lwv's obtained by acting on it with supersymmetry generators. Furthermore the  $SU(4)_D$  and  $SU^*(4)_D$  transformation properties of the lwv's of  $SO^*(8)$  are indicated by their Dynkin labels. As explained above, the  $SU(4)_D$  labels coincide with the  $SU^*(4)_D$  labels.

# 4 The Doubleton Supermultiplets of $OSp(8^*|4)$

By choosing one pair of super oscillators ( $\xi$  and  $\eta$ ) in the oscillator realization of  $OSp(8^*|4)$  (i.e. for P=1), one obtains the so-called doubleton supermultiplets. These supermultiplets contain only doubleton representations of  $SO^*(8)$ , i.e. they correspond to multiplets of fields living on the boundary of  $AdS_7$  without a 7d Poincaré limit. Equivalently, they can be characterized as multiplets of massless fields in 6d Minkowski space that form a UIR of the  $\mathcal{N}=2$  superconformal algebra  $OSp(8^*|4)$ .

The doubleton supermultiplet of  $OSp(8^*|4)$ , which is defined by the lowest weight vector,  $|\Omega\rangle = |0\rangle$ , is the (2,0) conformal supermultiplet and is the analog of the  $\mathcal{N}=4$  super Yang-Mills multiplet in d=6 [8]. The content of the (2,0) supermultiplet is given in Table 1.

$SU(4) \times SU(2)$ lwv	$Q_B=E$	$Q_F$	$SU(4)_D$ or $SU^*(4)_D$	SO(5)
* 0 >	2	-1	(0,0,0)	5
	<u>5</u> 2	$\frac{-1}{2}$	(1,0,0)	4
>	3	0	(2,0,0)	1

Table 1. The doubleton supermultiplet corresponding to the lwv  $|\Omega\rangle = |0\rangle$ . The first column indicates the lwv's of  $SO^*(8) \times USp(4)$  with  $U(4) \times U(2)$  Young tableaux. The second column shows the AdS energies  $Q_B = \frac{1}{2}(N_B + 4P)$ , the third column lists the fermion U(1) charge, the fourth column shows the SU(4) Dynkin labels, and the fifth column shows the representation of SO(5) induced by the lwv.

In Table 2 we give the doubleton supermultiplet defined by the lowest weight vector  $|\Omega\rangle = |\square\rangle$ .

$SU(4) \times SU(2)$ lwv	$Q_B=E$	$Q_F$	$SU(4)_D$ or $SU^*(4)_D$	SO(5)
*	$\frac{5}{2}$	-1	(1,0,0)	5
>	3	$\frac{-1}{2}$	(2,0,0)	4
,   >	$\frac{7}{2}$	0	(3,0,0)	1
* 1,	2	$\frac{-1}{2}$	(0,0,0)	4
,   >	$\frac{5}{2}$	0	(1,0,0)	1

Table 2. Doubleton Supermultiplet defined by lwv  $|\Omega\rangle = |\square\rangle$ .

Table 3 gives the general doubleton supermultiplet whose lowest weight vector is

$$|\Omega\rangle = |\bigcap_{j=1}^{2j} \widehat{\Omega_j} \cap \widehat{\Omega_j}|$$
 for  $j > \frac{1}{2}$ .

$SU(4) \times SU(2)$ lwv	$Q_B=E$	$Q_F$	$SU(4)_D$ or $SU^*(4)_D$	SO(5)
$* $ $\overbrace{ \qquad \qquad }^{2j}$ , $1>$	j+2	-1	(2j,0,0)	5
$ $ $\longrightarrow$ $ $ $\longrightarrow$ $	$j + \frac{5}{2}$	$\frac{-1}{2}$	(2j+1,0,0)	4
$ $ $\longrightarrow$ $	j+3	0	(2j+2,0,0)	1
$* $ $\longrightarrow$ $\longrightarrow$ $\longrightarrow$ $\longrightarrow$	$j + \frac{3}{2}$	$\frac{-1}{2}$	(2j-1,0,0)	4
$ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $	j+2	0	(2j,0,0)	1
$* $ $\longrightarrow$ $\longrightarrow$ $\longrightarrow$ $\longrightarrow$ $\longrightarrow$ $\longrightarrow$	j+1	0	(2j-2,0,0)	1

Table 3. General Doubleton Supermultiplet defined by the lwv  $|\Omega\rangle = |$   $\square$   $\square$   $\square$   $\square$   $\square$  for  $j > \frac{1}{2}$  where j is an integer or half-odd integer.

# 5 The Massless Supermultiplets

The doubleton supermultiplets described in the last section are fundamental in the sense that all other lowest weight UIRs of  $OSp(8^*|4)$  occur in the tensor product of two or more doubleton supermultiplets. Instead of trying to identify these irreducible submultiplets in the (in general reducible, but not fully reducible) tensor products, one simply increases the number P of oscillator generations so that the tensoring becomes implicit while the irreducibility stays manifest.

The simplest class, corresponding to P=2, contains the supermultiplets that are known to be "massless" in the 7d AdS sense. We will therefore label all supermultiplets that are obtained by taking P=2 in the oscillator construction despite some problems with the invariant definition of the notion of "mass" in AdS spacetimes [16].

We will now give a complete list of the allowed  $OSp(8^*|4)$  lowest weight vectors  $|\Omega\rangle$  for P=2. The condition  $L^-|0\rangle$  leaves the following possibilities:

- $|\Omega\rangle = |0\rangle$ . This lwv gives rise  $\mathcal{N}=4$  graviton supermultiplet in  $AdS_7$  and occurs in the tensor product of two CPT self-conjugate doubleton supermultiplets.
- $\xi^{A_1}(1)\xi^{A_2}(1)|0\rangle = | \square \rangle$ . Alternatively, one could use the states  $\xi^{A_1}(2)\xi^{A_2}(2)|0\rangle$ ,  $\eta^{B_1}(1)\eta^{B_2}(1)|0\rangle$ ,  $\eta^{B_1}(2)\eta^{B_2}(2)|0\rangle$ ,  $\xi^{(A_1}(1)\eta^{B_1}(2)|0\rangle$ , or  $\xi^{(A_1}(2)\eta^{B_1}(1)|0\rangle$ .
- $|\Omega\rangle = \xi^{A_1}(1)\xi^{A_2}(1)\cdots\xi^{A_{2j}}(1)|0\rangle = |\underbrace{\begin{array}{c}2j\\ \end{array}}$ . Equivalent lwv's are  $\xi^{A_1}(2)\xi^{A_2}(2)\cdots\xi^{A_{2j}}(2)|0\rangle, \ \eta^{B_1}(1)\eta^{B_2}(1)\dots\eta^{B_{2j}}(1)|0\rangle, \ \eta^{B_1}(2)\eta^{B_2}(2)\dots\eta^{B_{2j}}(2)|0\rangle, \ \xi^{(A_1}(1)\xi^{A_2}(1)\dots\xi^{A_r}(1)\eta^{B_{r+1}}(2)\eta^{B_{r+2}}(2)\dots\eta^{B_{2j}}(2)|0\rangle,$

or 
$$\xi^{(A_1}(2)\xi^{A_2}(2)\cdots\xi^{A_r}(2)\eta^{B_{r+1}}(1)\eta^{B_{r+2}}(1)\cdots\eta^{B_{2j}}(1)|0\rangle$$
.  
Increasing j leads to multiplets with higher and higher spins and  $AdS$  energies.

In addition to these purely (super)symmetrized lwv's, one can also anti-(super)symmetrize pairs of superoscillators, since P=2. The condition  $L^-|0\rangle$  leaves the following possibilities:

$$\begin{split} \bullet & |\Omega\rangle = \xi^{[A_1}(1)\xi^{B_1]}(2)\dots\xi^{[A_{2j}}(1)\xi^{B_{2j}]}(2)\xi^{A_{2j+1}}(1)\dots\xi^{A_{2j+n}}(1)|0\rangle \\ = & | \overbrace{\qquad \qquad } \\ \vdots \\ & Equivalent \ \text{lwv's are} \ \eta^{[A_1}(1)\eta^{B_1]}(2)\dots\eta^{[A_{2j}}(1)\eta^{B_{2j}]}(2)\eta^{A_{2j+1}}(1)\dots\eta^{A_{2j+n}}(1)|0\rangle, \\ \xi^{[A_1}(1)\eta^{B_1]}(2)\dots\xi^{[A_{2j}}(1)\eta^{B_{2j}]}(2)\xi^{A_{2j+1}}(1)\dots\xi^{A_{2j+n}}(1)|0\rangle, \\ \xi^{[A_1}(1)\eta^{B_1]}(2)\dots\xi^{[A_{2j}}(1)\eta^{B_{2j}]}(2)\xi^{B_{2j+1}}(2)\dots\xi^{B_{2j+n}}(2)|0\rangle. \end{split}$$

In the following tables we list the massless supermultiplets defined by the above lowest weight vectors<sup>7</sup>.

$SU(4) \times SU(2)$ lwv	$Q_B=E$	$Q_F$	$SU(4)_D$ or $SU^*(4)_D$	SO(5)
* 1,1>	4	-2	(0,0,0)	14
,   >	$\frac{9}{2}$	$\frac{-3}{2}$	(1,0,0)	16
>	5	-1	(0,1,0)	10
>	5	-1	(2,0,0)	5
>	$\frac{11}{2}$	$\frac{-1}{2}$	(1,1,0)	4
>	6	0	(0,2,0)	1

Table 4. Massless Graviton Supermultiplet defined by lwv  $|\Omega>=|0>$ .

 $<sup>^{7}</sup>$ I am deeply grateful to Sudarshan Fernando for having brought to my attention states which were missing from my initial calculation. Those missing states have been added to the following tables.

SU(4)⊗SU(2) lwv	$Q_B$	$Q_F$	$SU(4)_D$ or $SU^*(4)_D$	SO(5)
*	$\frac{9}{2}$	-2	(1,0,0)	14
, >	5	$\frac{-3}{2}$	(2,0,0)	16
>	5	$\frac{-3}{2}$	(0,1,0)	16
>	$\frac{11}{2}$	-1	(1,1,0)	10
>	$\frac{11}{2}$	-1	(1,1,0)	5
, _   >	$\frac{11}{2}$	-1	(3,0,0)	5
>	6	$\frac{-1}{2}$	(2,1,0)	4
>	6	$\frac{-1}{2}$	(0,2,0)	4
>	$\frac{13}{2}$	0	(1,2,0)	1
* 1,	4	$\frac{-3}{2}$	(0,0,0)	16
,   >	$\frac{9}{2}$	-1	(1,0,0)	5
,   >	$\frac{9}{2}$	-1	(1,0,0)	10
>	5	$\frac{-1}{2}$	(2,0,0)	4
>	5	$\frac{-1}{2}$	(0,1,0)	4
>	<u>11</u> 2	0	(1,1,0)	1

Table 5. Massless Supermultiplet defined by the lwv  $|\Omega>=|$   $\square>$ .

$SU(4)\otimes SU(2)$ lwv	$Q_B$	$Q_F$	$SU(4)_D$ or $SU^*(4)_D$	SO(5)
*	5	-2	(0,1,0)	14
>	11 2	$\frac{-3}{2}$	(1,1,0)	16
>	6	-1	(0,2,0)	10
>	6	-1	(2,1,0)	5
>	13 2	$\frac{-1}{2}$	(1,2,0)	4
>	7	0	(0,3,0)	1
* , >	$\frac{9}{2}$	$\frac{-3}{2}$	(1,0,0)	16
	5	-1	(2,0,0)	10
>	5	-1	(2,0,0)	5
>	5	-1	(0,1,0)	5
>	5	-1	(0,1,0)	10
>	$\frac{11}{2}$	$\frac{-1}{2}$	(3,0,0)	4
>	11 2	$\frac{-1}{2}$	(1,1,0)	4
>	6	0	(0,2,0)	1
>	6	0	(2,1,0)	1
* 1,	4	-1	(0,0,0)	10
>	9/2	$\frac{-1}{2}$	(1,0,0)	4
	5	0	(0,1,0)	1

Table 6. Massless Supermultiplet defined by lwv  $|\Omega>=|$   $\ge$  >.

SU(4)⊗SU(2) lwv	$Q_B$	$Q_F$	$SU(4)_D$ or $SU^*(4)_D$	SO(5)
* ,1 >	5	-2	(2,0,0)	14
	$\frac{11}{2}$	$\frac{-3}{2}$	(3,0,0)	16
	$\frac{11}{2}$	$\frac{-3}{2}$	(1,1,0)	16
	6	-1	(4,0,0)	5
>	6	-1	(2,1,0)	5
	6	-1	(0,2,0)	5
	6	-1	(2,1,0)	10
>	$\frac{13}{2}$	$\frac{-1}{2}$	(3,1,0)	4
>	$\frac{13}{2}$	$\frac{-1}{2}$	(1,2,0)	4
>	7	0	(2,2,0)	1
* , >	$\frac{9}{2}$	$\frac{-3}{2}$	(1,0,0)	16
>	5	-1	(0,1,0)	5
,   >	5	-1	(0,1,0)	10
,   >	5	-1	(2,0,0)	5
	5	-1	(2,0,0)	10
,	$\frac{11}{2}$	$\frac{-1}{2}$	(3,0,0)	4
>	$\frac{11}{2}$	$\frac{-1}{2}$	(1,1,0)	4
>	6	0	(2,1,0)	1
	6	0	(0,2,0)	1
* 1,	4	-1	(0,0,0)	5
,   >	9/2	$\frac{-1}{2}$	(1,0,0)	4
>	5	0	(2,0,0)	1

Table 7. Massless Supermultiplet Defined by the lwv  $|\Omega>=|$  >= >.

$SU(4)\otimes SU(2)$ lwv	$Q_B$	$Q_F$	$SU(4)_D$ or $SU^*(4)_D$	SO(5)
* ,1>	$\frac{11}{2}$	-2	(3,0,0)	14
	6	$\frac{-3}{2}$	(4,0,0)	16
	6	$\frac{-3}{2}$	(2,1,0)	16
>	13 2	-1	(5,0,0)	5
>	$\frac{13}{2}$	-1	(3,1,0)	5
	$\frac{13}{2}$	-1	(1,2,0)	5
>	$\frac{13}{2}$	-1	(3,1,0)	10
	7	$\frac{-1}{2}$	(4,1,0)	4
>	7	$\frac{-1}{2}$	(2,2,0)	4
>	15 2	0	(3,2,0)	1
* , >	5	$\frac{-3}{2}$	(2,0,0)	16
,>	$\frac{11}{2}$	-1	(3,0,0)	10
	$\frac{11}{2}$	-1	(1,1,0)	10
	11 2	-1	(3,0,0)	5
>	11 2	-1	(1,1,0)	5
	6	$\frac{-1}{2}$	(4,0,0)	4
	6	$\frac{-1}{2}$	(2,1,0)	4
	6	$\frac{-1}{2}$	(0,2,0)	4
>	13 2	0	(3,1,0)	1
>	13 2	0	(1,2,0)	1
* , >	9 2	-1	(1,0,0)	5
,   >	5	$\frac{-1}{2}$	(2,0,0)	4
,   >	5	$\frac{-1}{2}$	(0,1,0)	4
>	$\frac{11}{2}$	0	(3,0,0)	1
>	11 2	0	(1,1,0)	1

Table 8. Massless Supermultiplet Defined by the lwv  $|\Omega>=|$   $\square$  >.

$SU(4)\otimes SU(2)$ lwv	$Q_B$	$Q_F$	$SU(4)_D$ or $SU^*(4)_D$	SO(5)
*  ,1>	$\frac{11}{2}$	-2	(1,1,0)	14
	6	$\frac{-3}{2}$	(2,1,0)	16
	6	$\frac{-3}{2}$	(0,2,0)	16
>	13 2	-1	(3,1,0)	5
>	13 2	-1	(1,2,0)	5
>	$\frac{13}{2}$	-1	(1,2,0)	10
>	7	$\frac{-1}{2}$	(2,2,0)	4
>	7	$\frac{-1}{2}$	(0,3,0)	4
>	$\frac{15}{2}$	0	(1,3,0)	1
* , >	5	$\frac{-3}{2}$	(2,0,0)	16
	$\frac{11}{2}$	-1	(1,1,0)	10
,>	$\frac{11}{2}$	-1	(3,0,0)	10
,   >	$\frac{11}{2}$	-1	(3,0,0)	5
>	11 2	-1	(1,1,0)	5
>	6	$\frac{-1}{2}$	(4,0,0)	4
>	6	$\frac{-1}{2}$	(2,1,0)	4
	6	$\frac{-1}{2}$	(0,2,0)	4
>	$\frac{13}{2}$	0	(3,1,0)	1
>	13 2	0	(1,2,0)	1
* , >	9/2	-1	(1,0,0)	5
, >	5	$\frac{-1}{2}$	(2,0,0)	4
>	5	$\frac{-1}{2}$	(0,1,0)	4
, >	11 2	0	(3,0,0)	1
	11 2	0	(1,1,0)	1

Table 9. Massless Supermultiplet Defined by the lwv  $|\Omega>=|$  >(cont'd).

SU(4)⊗SU(2) lwv	$Q_B$	$Q_F$	$SU(4)_D$ or $SU^*(4)_D$	SO(5)
* 1, ->	4	$\frac{-1}{2}$	(0,0,0)	4
>	$\frac{9}{2}$	0	(1,0,0)	1
*  \[ \], \[ \] >	5	$\frac{-3}{2}$	(0,1,0)	16
* , >	$\frac{9}{2}$	-1	(1,0,0)	10

Table 9. Massless Supermultiplet defined by the lwv  $|\Omega>=|$  >.

SU(4)⊗SU(2) lwv	$Q_B$	$SU(4)_D$ or $SU^*(4)_D$	SO(5)
*  , 1 >	6	(2,1,0)	14
	$\frac{13}{2}$	(3,1,0)	16
>	13 2	(1,2,0)	16
>	7	(4,1,0)	5
>	7	(2,2,0)	5
	7	(0,3,0)	5
	7	(2,2,0)	10
>	$\frac{15}{2}$	(3,2,0)	4
>	$\frac{15}{2}$	(1,3,0)	4
	8	(2,3,0)	1
*    >	11 2	(1,1,0)	16
	6	(0,2,0)	10
	6	(2,1,0)	5
>	6	(0,2,0)	5
>	13 2	(1,2,0)	4
	13 2	(3,1,0)	4

Table 10. Massless Supermultiplet defined by the lwv  $|\Omega>=|$  >(cont'd).

$SU(4)\otimes SU(2)$ lwv	$Q_B$	$SU(4)_D$ or $SU^*(4)_D$	SO(5)
>	7	(0,3,0)	1
* , >	$\frac{11}{2}$	(3,0,0)	16
>	6	(2,1,0)	10
	6	(4,0,0)	10
>	6	(4,0,0)	5
	13 2	(5,0,0)	4
>	7	(4,1,0)	1
	7	(2,2,0)	1
*    ,   >	5	(0,1,0)	5
	<u>11</u> 2	(1,1,0)	4
	6	(2,1,0)	1
* , >	5	(2,0,0)	10
>	$\frac{11}{2}$	(3,0,0)	4
* , >	5	(2,0,0)	5
	6	(0,2,0)	1
>	6	(4,0,0)	1
* , >	$\frac{9}{2}$	(1,0,0)	4
	5	(2,0,0)	1
>	5	(0,1,0)	1

Table 10. Massless Supermultiplet defined by the lwv  $|\Omega>=|$  >.

$SU(4)\otimes SU(2)$ lwv	$Q_B$	$SU(4)_D$ or $SU^*(4)_D$	SO(5)
*   1 >	13 2	(3,1,0)	14
	7	(4,1,0)	16
	7	(2,2,0)	16
	$\frac{15}{2}$	(5,1,0)	5
	1 <u>5</u>	(3,2,0)	5
	$\frac{15}{2}$	(1,3,0)	5
	$\frac{15}{2}$	(3,2,0)	10
>	8	(4,2,0)	4
>	8	(2,3,0)	4
	$\frac{17}{2}$	(3,3,0)	1
*	6	(2,1,0)	16
	13 2	(3,1,0)	10
	13 2	(3,1,0)	5
>	$\frac{13}{2}$	(1,2,0)	10
>	$\frac{13}{2}$	(1,2,0)	5
>	$\frac{15}{2}$	(3,2,0)	1
>	7	(4,1,0)	4
	7	(0,3,0)	4
>	7	(2,2,0)	4
>	$\frac{15}{2}$	(1,3,0)	1
*	$\frac{11}{2}$	(1,1,0)	5

Table 11. Massless Supermultiplet Defined by the lwv  $|\Omega>=|$  >= > (cont'd).

$SU(4)\otimes SU(2)$ lwv	$Q_B$	$SU(4)_D$ or $SU^*(4)_D$	SO(5)
>	6	(2,1,0)	4
	6	(0,2,0)	4
	$\frac{13}{2}$	(3,1,0)	1
* , >	$\frac{11}{2}$	(3,0,0)	10
>	6	(4,0,0)	4
* , >	5	(2,0,0)	4
	11/2	(3,0,0)	1
	11 2	(1,1,0)	1
*	6	(4,0,0)	16
>	$\frac{13}{2}$	(5,0,0)	10
>	$\frac{13}{2}$	(5,0,0)	5
>	7	(6,0,0)	4
>	15 2	(5,1,0)	1
* , >	11/2	(3,0,0)	5
	13 2	(5,0,0)	1
>	13 2	(1,2,0)	1

Table 11. Massless Supermultiplet Defined by the lwv  $|\Omega>=|$  >.

$SU(4)\otimes SU(2)$ lwv	$Q_B$	$Q_F$	$SU(4)_D$ or $SU^*(4)_D$	SO(5)
* , 1 >	6	-2	(0,2,0)	14
>	$\frac{13}{2}$	$\frac{-3}{2}$	(1,2,0)	16
>	7	-1	(2,2,0)	5
>	7	-1	(0,3,0)	10
>	$\frac{15}{2}$	$\frac{-1}{2}$	(1,3,0)	4
>	8	0	(0,4,0)	1
*    >	11 2	$\frac{-3}{2}$	(1,1,0)	16
	6	-1	(0,2,0)	10
>	6	-1	(2,1,0)	5
>	6	-1	(0,2,0)	5
	6	-1	(2,1,0)	10
>	$\frac{13}{2}$	$\frac{-1}{2}$	(1,2,0)	4
>	$\frac{13}{2}$	$\frac{-1}{2}$	(3,1,0)	4
>	7	0	(2,2,0)	1
>	7	0	(0,3,0)	1
* , >	5	-1	(2,0,0)	5
	$\frac{11}{2}$	$\frac{-1}{2}$	(3,0,0)	4
	$\frac{11}{2}$	$\frac{-1}{2}$	(1,1,0)	4
>	6	0	(4,0,0)	1
	6	0	(0,2,0)	1
	6	0	(2,1,0)	1
*	5	-1	(0,1,0)	10

Table 12. Massless Supermultiplet defined by the lwv  $|\Omega>=|$  >= > (cont'd).

$SU(4)\otimes SU(2)$ lwv	$Q_B$	$Q_F$	$SU(4)_D$ or $SU^*(4)_D$	SO(5)
* , >	$\frac{9}{2}$	$\frac{-1}{2}$	(1,0,0)	4
>	5	0	(0,1,0)	1
>	5	0	(2,0,0)	1
* 1, ==>	4	0	(0,0,0)	1

Table 12. Massless Supermultiplet defined by the lwv  $|\Omega>=|$  >.

$SU(4)\otimes SU(2)$ lwv	$Q_B$	$Q_F$	$SU(4)_D$ or $SU^*(4)_D$	SO(5)
*  \(\frac{2j}{\ldots}\), 1 >	j+4	-2	(2j,0,0)	14
$ $ $\longrightarrow$ $ $ $\longrightarrow$ $ $ $\longrightarrow$ $ $ $>$	$j + \frac{9}{2}$	$\frac{-3}{2}$	(2j+1,0,0)	16
$ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $	$j + \frac{9}{2}$	$\frac{-3}{2}$	(2j-1,1,0)	16
$ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $	j+5	-1	(2j+2,0,0)	5
$ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $	j+5	-1	(2j,1,0)	5
	j+5	-1	(2j-2,2,0)	5
2j+1	j+5	-1	(2j,1,0)	10
2j+2	$j + \frac{11}{2}$	$\frac{-1}{2}$	(2j+1,1,0)	4
2j+1	$j + \frac{11}{2}$	$\frac{-1}{2}$	(2j-1,2,0)	4
2j+2	j+6	0	(2j,2,0)	1
	$j + \frac{7}{2}$	$\frac{-3}{2}$	(2j-1,0,0)	16
	j+4	-1	(2j,0,0)	10
$ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $	j+4	-1	(2j-2,1,0)	10
$ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $	j+4	-1	(2j,0,0)	5
$ $ $\bigcap_{j=1}^{2j-1}$ $ $ $ $ $ $ $ $ $ $	j+4	-1	(2j-2,1,0)	5

Table 13. Massless Supermultiplet Defined by lwv  $|\Omega>=|$   $\sum_{i=1}^{2j}\cdots\sum_{i=1}^{2j}>$  (cont'd).

SU(4)⊗SU(2) lwv	$Q_B$	$Q_F$	$SU(4)_D$ or $SU^*(4)_D$	SO(5)
$ \overbrace{\qquad \qquad }^{2j+1}, \qquad  >$	$j + \frac{9}{2}$	$\frac{-1}{2}$	(2j+1,0,0)	4
$ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $	$j + \frac{9}{2}$	$\frac{-1}{2}$	(2j-1,1,0)	4
$ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $	$j + \frac{9}{2}$	$\frac{-1}{2}$	(2j-3,2,0)	4
2j+1	j+5	0	(2j,1,0)	1
	j+5	0	(2j-2,2,0)	1
$ \begin{array}{c c} \hline & 2j-2 \\ *   &                                  $	j+3	-1	(2j-2,0,0)	5
$ $ $\longrightarrow$ $ $ $\longrightarrow$ $ $ $>$	$j + \frac{7}{2}$	$\frac{-1}{2}$	(2j-1,0,0)	4
$ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $	$j + \frac{7}{2}$	$\frac{-1}{2}$	(2j-3,1,0)	4
$ $ $\longrightarrow$ $ $ $\longrightarrow$ $ $ $>$	j+4	0	(2j,0,0)	1
$ $ $\longrightarrow$ $ $ $\longrightarrow$ $ $ $\longrightarrow$ $ $ $>$	j+4	0	(2j-2,1,0)	1
$ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $	j+4	0	(2j-4,2,0)	1

Table 13. General Doubleton Supermultiplet defined by the lwv  $|\Omega>=|$   $\overbrace{\hspace{1cm}}^{2j}$  > .

$SU(4)\otimes SU(2)$ lwv	$Q_B$	$Q_F$	$SU(4)_D$ or $SU^*(4)_D$	SO(5)
*  \( \frac{2j}{\ldots} \), 1 >	2j+4	-2	(0,2j,0)	14
$ $ $\longrightarrow$ $ $ $\longrightarrow$ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $	$2j + \frac{9}{2}$	$\frac{-3}{2}$	(1,2j,0)	16
$ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $	2j+5	-1	(2,2j,0)	5
$ $ $\longrightarrow$ $ $ $\longrightarrow$ $ $ $\longrightarrow$ $ $ $>$	2j+5	-1	(0,2j+1,0)	10
$ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $	2j+6	0	(0,2j+2,0)	1
$ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $	$2j + \frac{11}{2}$	$\frac{-1}{2}$	(1,2j+1,0)	4
$ *  \overbrace{ \qquad \qquad }^{2j-1},  \boxed{ \qquad } >$	2j+3	-1	(0,2j-1,0)	10
	$2j + \frac{7}{2}$	$\frac{-1}{2}$	(1,2j-1,0)	4
	2j+4	0	(0,2j,0)	1
*  \( \frac{2j}{\ldots} \), \( \propto > \)	$2j + \frac{7}{2}$	$\frac{-3}{2}$	(1,2j-1,0)	16
	2j+4	-1	(0,2j,0)	10
$ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $	2j+4	-1	(2,2j-1,0)	10
$ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $	2j+4	-1	(2,2j-1,0)	5

Table 14. Massless Supermultiplet Defined by the lwv  $|\Omega>=|$   $\sum_{i=1}^{2j}$   $\sum_{i=1}^{2j}$   $\sum_{i=1}^{2j}$   $\sum_{i=1}^{2j}$ 

$SU(4)\otimes SU(2)$ lwv	$Q_B$	$Q_F$	$SU(4)_D$ or $SU^*(4)_D$	SO(5)
$ $ $\longrightarrow$ $ $ $\longrightarrow$ $ $ $>$	2j+4	-1	(0,2j,0)	5
$ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $	$2j + \frac{9}{2}$	$\frac{-1}{2}$	(3,2j-1,0)	4
$ $ $\longrightarrow$	$2j + \frac{9}{2}$	$\frac{-1}{2}$	(1,2j,0)	4
	2j+5	0	(2,2j,0)	1
$ $ $\longrightarrow$ $ $	2j+5	0	(0,2j+1,0)	1
*  \( \frac{2j-2}{\ldots} \), \( \frac{1}{\ldots} \) >	2j+2	0	(0,2j-2,0)	1
$* $ $\longrightarrow$ $\longrightarrow$ $\longrightarrow$ $\longrightarrow$	$2j + \frac{5}{2}$	$\frac{-1}{2}$	(1,2j-2,0)	4
$ $ $\longrightarrow$ $ $ $\longrightarrow$ $ $ $\longrightarrow$ $ $ $>$	2j+3	0	(0,2j-1,0)	4
	2j+3	0	(2,2j-2,0)	1
*  \( \frac{2j}{\ldots} \), \( \begin{picture}(100,0) \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\	2j+3	-1	(2,2j-2,0)	5
$ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $	$2j+\frac{7}{2}$	$\frac{-1}{2}$	(3,2j-2,0)	4
$ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $	2j+4	0	(4,2j-2,0)	1
$2j+1$ $\rightarrow$ $\rightarrow$ $\rightarrow$ $\rightarrow$ $\rightarrow$	2j+4	0	(2,2j-1,0)	1

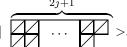
Table 14. Massless Supermultiplet Defined by the lwv  $|\Omega>=|$ 

$SU(4)\otimes SU(2)$ lwv	$Q_B$	$SU(4)_D$ or $SU^*(4)_D$	SO(5)
$*  \overbrace{ \cdots }, 1 >$	$2j + \frac{9}{2}$	(1,2j,0)	14
$ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $	2j+5	(2,2j,0)	16
$ \overbrace{\prod_{i=1}^{2j+1}}, \square >$	2j+5	(0,2j+1,0)	16
$ $ $\longrightarrow$ $ $ $\longrightarrow$ $	$2j + \frac{11}{2}$	(1,2j+1,0)	10
$ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $	$2j + \frac{11}{2}$	(3,2j,0)	5
$ $ $\longrightarrow$ $ $ $\longrightarrow$ $ $ $\longrightarrow$ $ $ $>$	$2j + \frac{11}{2}$	(1,2j+1,0)	5
$ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $	2j+6	(2,2j+1,0)	4
$ $ $\longrightarrow$ $ $ $\longrightarrow$ $ $ $\longrightarrow$ $ $ $>$	2j+6	(0,2j+2,0)	4
$ $ $\longrightarrow$ $ $ $\longrightarrow$ $ $ $\longrightarrow$ $ $ $>$	$2j + \frac{13}{2}$	(1,2j+2,0)	1
*  \( \frac{2j}{\ldots} \), \( \square > \)	2j+4	(0,2j,0)	16
$ $ $\longrightarrow$ $ $ $\longrightarrow$ $ $ $\longrightarrow$ $ $ $ $ $\longrightarrow$ $	$2j + \frac{9}{2}$	(1,2j,0)	10
$ $ $\longrightarrow$ $ $ $\longrightarrow$ $ $ $\longrightarrow$ $ $ $>$	$2j + \frac{9}{2}$	(1,2j,0)	5
$ $ $\longrightarrow$ $ $ $\longrightarrow$ $ $ $\longrightarrow$ $ $ $>$	2j+5	(0,2j+1,0)	4
$ $ $\longrightarrow$ $ $ $\longrightarrow$ $ $ $\longrightarrow$ $ $ $>$	2j+5	(2,2j,0)	4

$SU(4)\otimes SU(2)$ lwv	$Q_B$	$SU(4)_D$ or $SU^*(4)_D$	SO(5)
2j+2			, ,
	$2j + \frac{11}{2}$	(1,2j+1,0)	1
$\frac{2j}{2}$			
*	$2j + \frac{7}{2}$	(1,2j-1,0)	10
2j+1			
	2j+4	(2,2j-1,0)	4
2 <i>j</i>			
	2j+4	(0,2j,0)	4
2 <i>j</i> +1			
	$2j + \frac{9}{2}$	(1,2j,0)	1
2j-1			
*  \( \bigcup_{\cdots}, \bigcup_{\cdots} >	2j+3	(0,2j-1,0)	4
2j+1			
*	2j+4	(2,2j-1,0)	16
2j+2			
	$2j + \frac{9}{2}$	(3,2j-1,0)	10
2j+2			
	$2j + \frac{9}{2}$	(3,2j-1,0)	5
2j+3			
<del>                                  </del>	$2j + \frac{9}{2}$	(4,2j-1,0)	4
2j+3			
	$2j + \frac{11}{2}$	(3,2j,0)	1

$SU(4)\otimes SU(2)$ lwv	$Q_B$	$SU(4)_D$ or $SU^*(4)_D$	SO(5)
*  \( \frac{2j}{\ldots} \), \( \begin{picture}(100,0) \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\	$2j + \frac{7}{2}$	(1,2j-1,0)	5
$* $ $\longrightarrow$ $\longrightarrow$ $\longrightarrow$ $\longrightarrow$ $\longrightarrow$ $\longrightarrow$	$2j + \frac{5}{2}$	(1,2j-2,0)	1
*  \( \frac{2j}{\ldots} \), \( \frac{1}{\ldots} \) >	2j+3	(2,2j-2,0)	4
$ $ $\longrightarrow$ $ $ $\longrightarrow$ $ $ $\longrightarrow$ $ $ $>$	$2j + \frac{7}{2}$	(1,2j-1,0)	1
$ $ $\longrightarrow$ $ $	$2j + \frac{7}{2}$	(3,2j-2,0)	1
*  \( \frac{2j+1}{\ldots} \), \( \subseteq > \)	$2j + \frac{7}{2}$	(3,2j-2,0)	5
$ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $	2j+4	(4,2j-2,0)	4
$ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $	$2j + \frac{9}{2}$	(5,2j-2,0)	1
$ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $	$2j + \frac{9}{2}$	(3,2j-1,0)	1

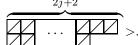
Table 15. Massless Supermultiplet Defined by the lwv  $|\Omega>=|$   $\sum_{i=1}^{2j+1}$   $\sum_{i=1}^{2j+1}$   $\sum_{i=1}^{2j+1}$ 



$SU(4)\otimes SU(2)$ lwv	$Q_B$	$SU(4)_D$ or $SU^*(4)_D$	SO(5)
2j+2			
* ,1>	2j+5	(2,2j,0)	14
2j+3			
	$2j + \frac{11}{2}$	(3,2j,0)	16
$ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $	$2j + \frac{11}{2}$	(1,2j+1,0)	16
2 <i>j</i> +4			
	2j+6	(4,2j,0)	5
2j+3			
	2j+6	(2,2j+1,0)	5
2j+2			
	2j+6	(0,2j+2,0)	5
2j+3			
<del>                                  </del>	2j+6	(2,2j+1,0)	10
2j+4			
	$2j + \frac{13}{2}$	(3,2j+1,0)	4
2j+3	19	(, , , , , , , , )	
	$2j + \frac{13}{2}$	(1,2j+2,0)	4
$ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $	2j+7	(2,2j+2,0)	1
2j+1			
*	$2j + \frac{9}{2}$	(1,2j,0)	16
2j+2			
	2j+5	(2,2j,0)	10
2j+2	0:   5	(2.2:0)	۲
	2j+5	(2,2j,0)	5
$2j+1$ $\rightarrow$ $\rightarrow$ $\rightarrow$ $\rightarrow$	2j+5	(0,2j+1,0)	10
	J , -	( ) 0 ' )-/	_

$SU(4)\otimes SU(2)$ lwv	$Q_B$	$SU(4)_D$ or $SU^*(4)_D$	SO(5)
$ $ $\longrightarrow$ $ $ $\longrightarrow$ $ $ $>$	2j+5	(0,2j+1,0)	5
$ \overbrace{\qquad \qquad \cdots \qquad \qquad }^{2j+3}, \   \longrightarrow >$	$2j + \frac{11}{2}$	(3,2j,0)	4
$ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $	$2j + \frac{11}{2}$	(1,2j+1,0)	4
$ $ $\longrightarrow$ $ $ $\longrightarrow$ $ $ $>$	2j+6	(0,2j+2,0)	1
$ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $	2j+6	(2,2j+1,0)	1
*	2j+4	(0,2j,0)	5
*  \( \frac{2j+1}{\top} \), \( \sum_{>} \)	2j+4	(2,2j-1,0)	10
$ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $	2j+5	(2,2j,0)	1
$ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $	$2j + \frac{9}{2}$	(3,2j-1,0)	4
$ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $	$2j + \frac{9}{2}$	(1,2j,0)	4
*  \( \frac{2j}{\ldots} \), \( \frac{1}{\ldots} \) >	$2j + \frac{7}{2}$	(1,2j-1,0)	4
$ $ $\longrightarrow$ $ $ $\longrightarrow$ $ $ $\longrightarrow$ $ $ $>$	2j+4	(2,2j-1,0)	1
$ $ $\longrightarrow$ $ $ $\longrightarrow$ $ $ $>$	2j+4	(0,2j,0)	1
$* $ $\longrightarrow$ $\longrightarrow$ $\longrightarrow$ $\longrightarrow$ $\longrightarrow$ $\longrightarrow$ $\longrightarrow$ $\longrightarrow$	$2j + \frac{9}{2}$	(3,2j-1,0)	16

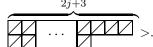
$SU(4)\otimes SU(2)$ lwv	$Q_B$	$SU(4)_D$ or $SU^*(4)_D$	SO(5)
	2j+5	(4,2j-1,0)	10
$ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $	2j+5	(4,2j-1,0)	5
$ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $	$2j + \frac{11}{2}$	(5,2j-1,0)	4
$2j+4$ $\rightarrow$ $\rightarrow$ $\rightarrow$ $\rightarrow$ $\rightarrow$	2j+6	(4,2j,0)	1
*	2j+4	(2,2j-1,0)	5
$ $ $\longrightarrow$ $ $ $\longrightarrow$ $ $ $\longrightarrow$ $ $ $>$	2j+5	(2,2j,0)	1
$2j+3$ $\rightarrow$	2j+5	(4,2j-1,0)	1
$ $ $\longrightarrow$ $ $ $\longrightarrow$ $ $ $\longrightarrow$ $ $ $>$	2j+5	(0,2j+1,0)	1
*  \( \frac{2j}{\ldots} \), \( \frac{1}{\ldots} \) >	2j+3	(2,2j-2,0)	1
*  \( \frac{2j+1}{\ldots} \), \( \frac{1}{\ldots} \) >	$2j + \frac{7}{2}$	(3,2j-2,0)	4
$ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $	2j+4	(4,2j-2,0)	1
$* $ $\longrightarrow$ $\longrightarrow$ $\longrightarrow$ $\longrightarrow$ $\longrightarrow$ $\longrightarrow$ $\longrightarrow$ $\longrightarrow$ $\longrightarrow$	2j+4	(4,2j-2,0)	5
$ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $	$2j + \frac{9}{2}$	(5,2j-2,0)	4
$ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $	2j+5	(6,2j-2,0)	1



$SU(4)\otimes SU(2)$ lwv	$Q_B$	$SU(4)_D$ or $SU^*(4)_D$	SO(5)
*  \( \frac{2j+3}{\top}, 1 > \)	$2j + \frac{11}{2}$	(3,2j,0)	14
$ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $	2j+6	(4,2j,0)	16
$ $ $\longrightarrow$ $ $ $\longrightarrow$ $	2j+6	(2,2j+1,0)	16
$  \overbrace{ \qquad \qquad }^{2j+5}, \   >$	$2j + \frac{13}{2}$	(5,2j,0)	5
$ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $	$2j + \frac{13}{2}$	(3,2j+1,0)	5
$ $ $\longrightarrow$ $ $ $\longrightarrow$ $ $ $\longrightarrow$ $ $ $>$	$2j + \frac{13}{2}$	(1,2j+2,0)	5
$ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $	$2j + \frac{13}{2}$	(3,2j+1,0)	10
$ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $	2j+7	(4,2j+1,0)	4
$ $ $\longrightarrow$ $ $	2j+7	(2,2j+2,0)	4
$ $ $\longrightarrow$ $ $	$2j + \frac{15}{2}$	(3,2j+2,0)	1
*  \( \frac{2j+2}{\ldots} \), \( \subseteq > \)	2j+5	(2,2j,0)	16
$ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $	$2j + \frac{11}{2}$	(3,2j,0)	10
$2j+3$ $\longrightarrow$ $\longrightarrow$ $\longrightarrow$ $\longrightarrow$ $\longrightarrow$ $\longrightarrow$	$2j + \frac{11}{2}$	(3,2j,0)	5
$ $ $\longrightarrow$ $ $ $\longrightarrow$ $ $ $\longrightarrow$ $ $ $>$	$2j + \frac{11}{2}$	(1,2j+1,0)	10

$SU(4)\otimes SU(2)$ lwv	$Q_B$	$SU(4)_D$ or $SU^*(4)_D$	SO(5)
$ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $	$2j + \frac{13}{2}$	(3,2j+1,0)	1
$ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $	$2j + \frac{11}{2}$	(1,2j+1,0)	5
$ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $	2j+6	(4,2j,0)	4
$ $ $\longrightarrow$ $ $ $\longrightarrow$ $ $ $\longrightarrow$ $ $ $>$	2j+6	(0,2j+2,0)	4
$ $ $\longrightarrow$ $ $ $\longrightarrow$ $ $ $\longrightarrow$ $ $ $>$	2j+6	(2,2j+1,0)	4
$ $ $\longrightarrow$ $ $ $\longrightarrow$ $ $ $\longrightarrow$ $ $ $>$	$2j + \frac{13}{2}$	(1,2j+2,0)	1
$* $ $\longrightarrow$ $\longrightarrow$ $\longrightarrow$ $\longrightarrow$ $\longrightarrow$ $\longrightarrow$	$2j + \frac{9}{2}$	(1,2j,0)	5
$ $ $\longrightarrow$ $ $ $\longrightarrow$ $ $ $\longrightarrow$ $ $ $>$	2j+5	(2,2j,0)	4
$ $ $\longrightarrow$ $ $ $\longrightarrow$ $ $ $\longrightarrow$ $ $ $>$	2j+5	(0,2j+1,0)	4
$ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $	$2j + \frac{11}{2}$	(3,2j,0)	1
$ $ $\longrightarrow$ $ $ $\longrightarrow$ $ $ $>$	$2j + \frac{11}{2}$	(1,2j+1,0)	1
$* $ $\longrightarrow$ $\longrightarrow$ $\longrightarrow$ $\longrightarrow$ $\longrightarrow$ $\longrightarrow$	$2j + \frac{9}{2}$	(3,2j-1,0)	10
$2j+3$ $\rightarrow$ $\rightarrow$ $\rightarrow$ $\rightarrow$	2j+5	(4,2j-1,0)	4
$* $ $\longrightarrow$ $\longrightarrow$ $\longrightarrow$ $\longrightarrow$ $\longrightarrow$	2j+4	(2,2j-1,0)	4
$ $ $\longrightarrow$ $ $ $\longrightarrow$ $ $ $\longrightarrow$ $ $ $>$	$2j + \frac{9}{2}$	(3,2j-1,0)	1

$SU(4)\otimes SU(2)$ lwv	$Q_B$	$SU(4)_D$ or $SU^*(4)_D$	SO(5)
2j+1			
	$2j + \frac{9}{2}$	(1,2j,0)	1
$* $ $\longrightarrow$ $\longrightarrow$ $\longrightarrow$ $\longrightarrow$ $\longrightarrow$ $\longrightarrow$ $\longrightarrow$ $\longrightarrow$	2j+5	(4,2j-1,0)	16
2j+4	$2j + \frac{11}{2}$	(5,2j-1,0)	10
$ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $	$2j + \frac{11}{2}$	(5,2j-1,0)	5
	2j+6	(6,2j-1,0)	4
2j+5   >	$2j + \frac{13}{2}$	(5,2j,0)	1
$* $ $\longrightarrow$ $\longrightarrow$ $\longrightarrow$ $\longrightarrow$ $\longrightarrow$ $\longrightarrow$	$2j + \frac{9}{2}$	(3,2j-1,0)	5
$2j+4$ $\rightarrow$ $\rightarrow$ $\rightarrow$ $\rightarrow$ $\rightarrow$ $\rightarrow$	$2j + \frac{11}{2}$	(5,2j-1,0)	1
*	$2j + \frac{7}{2}$	(3,2j-2,0)	1
*  \( \frac{2j+2}{ } \)	2j+4	(4,2j-2,0)	4
$  \overbrace{ \qquad \qquad }^{2j+3}, \   >$	$2j + \frac{9}{2}$	(5,2j-2,0)	1
$* $ $\longrightarrow$ $\longrightarrow$ $\longrightarrow$ $\longrightarrow$ $\longrightarrow$ $\longrightarrow$ $\longrightarrow$ $\longrightarrow$	$2j + \frac{9}{2}$	(5,2j-2,0)	5
$2j+4$ $\rightarrow$ $\rightarrow$ $\rightarrow$ $\rightarrow$ $\rightarrow$ $\rightarrow$	2j+5	(6,2j-2,0)	4
$  \overbrace{ \qquad \qquad }^{2j+5} \rangle$	$2j + \frac{11}{2}$	(7,2j-2,0)	1



$SU(4)\otimes SU(2)$ lwv	$Q_B$	$SU(4)_D$ or $SU^*(4)_D$	SO(5)
2j $n$			
*  \( \frac{1}{1} \cdots \cdots \),1 >	$\frac{1}{2}(4j+n+8)$	(n,2j,0)	14
2j $n+1$			
	$\frac{1}{2}(4j+n+9)$	(n+1,2j,0)	16
2j+1 $n-1$			
, >	$\frac{1}{2}(4j+n+9)$	(n-1,2j+1,0)	16
2j $n+2$			
	$\frac{1}{2}(4j+n+10)$	(n+2,2j,0)	5
2j+1 n			
	$\frac{1}{2}(4j+n+10)$	(n,2j+1,0)	5
2j+2 $n-2$			
	$\frac{1}{2}(4j+n+10)$	(n-2,2j+2,0)	5
2j+1 $n$			
	$\frac{1}{2}(4j+n+10)$	(n,2j+1,0)	10
2j+1 $n+1$			
	$\frac{1}{2}(4j+n+11)$	(n+1,2j+1,0)	4
2j+2 $n-1$			
<del>                                  </del>	$\frac{1}{2}(4j+n+11)$	(n-1,2j+2,0)	4
2j+2 $n$			
	$\frac{1}{2}(4j+n+12)$	(n,2j+2,0)	1
2j $n-1$			
*	$\frac{1}{2}(4j+n+7)$	(n-1,2j,0)	16
2 <i>j n</i>			
	$\frac{1}{2}(4j+n+8)$	(n,2j,0)	10
2j $n$			
	$\frac{1}{2}(4j+n+8)$	(n,2j,0)	5

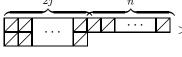
Table 18. General Massless Supermultiplet defined by the lwv  $|\Omega>=|$   $\frac{2j}{1}$   $\frac{n}{1}$   $\frac{n}{1}$  where j is an integer or half-odd integer such that  $j>\frac{1}{2}$  and n is a positive integer such that n>3 (cont'd).

SU(4)⊗SU(2) lwv	$Q_B$	$SU(4)_D$ or $SU^*(4)_D$	SO(5)
$ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $	$\frac{1}{2}(4j+n+8)$	(n-2, 2j+1,0)	10
$ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $	$\frac{1}{2}(4j+n+8)$	(n-2,2j+1,0)	5
$ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $	$\frac{1}{2}(4j+n+9)$	(n-3,2j+2,0)	4
$ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $	$\frac{1}{2}(4j+n+9)$	(n-1,2j+1,0)	4
$ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $	$\frac{1}{2}(4j+n+10)$	(n-2,2j+2,0)	1
$ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $	$\frac{1}{2}(4j+n+10)$	(n,2j+1,0)	1
$*  \overbrace{ \begin{array}{c} 2j \\ \\ \end{array}} \underbrace{ \begin{array}{c} n-2 \\ \\ \end{array}}, \begin{array}{c} \\ \\ \end{array}} >$	$\frac{1}{2}(4j+n+6)$	(n-2,2j,0)	5
$  \overbrace{ \qquad \qquad }^{2j+1} \underbrace{ \qquad \qquad }^{n-3}, \   \longrightarrow \rangle$	$\frac{1}{2}(4j+n+7)$	(n-3,2j+1,0)	4
$ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $	$\frac{1}{2}(4j+n+8)$	(n-2,2j+1,0)	1
$ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $	$\frac{1}{2}(4j+n+8)$	(n-4,2j+2,0)	1
$*  \overbrace{ \begin{array}{c} 2j-1 \\ \\ \end{array}} \underbrace{ \begin{array}{c} n \\ \\ \end{array}} >$	$\frac{1}{2}(4j+n+6)$	(n,2j-1,0)	10
$  \overbrace{ \begin{array}{c} 2j-1 \\ \hline \\ \end{array} }, \ \underset{>}{ \begin{array}{c} \\ \\ \end{array} } >$	$\frac{1}{2}(4j+n+7)$	(n+1,2j-1,0)	4
$ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $	$\frac{1}{2}(4j+n+7)$	(n-1,2j,0)	4

Table 18. General Massless Supermultiplet defined by the lwv  $|\Omega>=|$ (cont'd).

SU(4)⊗SU(2) lwv	$Q_B$	$SU(4)_D$ or $SU^*(4)_D$	SO(5)
2j n			
	$\frac{1}{2}(4j+n+8)$	(n,2j,0)	1
2j-1 $n-1$			
*	$\frac{1}{2}(4j+n+5)$	(n-1,2j-1,0)	4
2j-1 $n$			
	$\frac{1}{2}(4j+n+6)$	(n, 2j-1, 0)	1
2j $n-2$			
<del>                                  </del>	$\frac{1}{2}(4j+n+6)$	(n-2,2j,0)	1
2j-1 $n+1$			
*  , >	$\frac{1}{2}(4j+n+7)$	(n+1,2j-1,0)	16
2j-1 $n+2$			
	$\frac{1}{2}(4j+n+8)$	(n+2,2j-1,0)	10
2j-1 $n+2$			
	$\frac{1}{2}(4j+n+8)$	(n+2,2j-1,0)	5
2j-1 $n+3$			
	$\frac{1}{2}(4j+n+9)$	(n+3,2j-1,0)	4
2j n+1			
	$\frac{1}{2}(4j+n+9)$	(n+1,2j,0)	4
2j n+2			
	$\frac{1}{2}(4j+n+10)$	(n+2,2j,0)	1
2j-1 $n$			
*  ,	$\frac{1}{2}(4j+n+6)$	(n,2j-1,0)	5
2j-1 $n+2$			
<del>                                  </del>	$\frac{1}{2}(4j+n+8)$	(n+2,2j-1,0)	1
2j-2 n			
*	$\frac{1}{2}(4j+n+4)$	(n,2j-2,0)	1
2j-2 $n+1$			
* ,	$\frac{1}{2}(4j+n+5)$	(n+1,2j-2,0)	4

Table 18. General Massless Supermultiplet defined by the lwv  $|\Omega>=|$  (cont'd).



$SU(4)\otimes SU(2)$ lwv	$Q_B$	$SU(4)_D$ or $SU^*(4)_D$	SO(5)
2j-2 $n+2$			
	$\frac{1}{2}(4j+n+6)$	(n+2,2j-2,0)	1
2j-2 $n+2$			
* , ->	$\frac{1}{2}(4j+n+6)$	(n+2,2j-2,0)	5
2j-2 $n+3$			
	$\frac{1}{2}(4j+n+7)$	(n+3,2j-2,0)	4
2j-2 $n+4$			
	$\frac{1}{2}(4j+n+8)$	(n+4,2j-2,0)	1

Table 18. General Massless Supermultiplet defined by the lwv  $|\Omega>=|$ 

## 6 Discussion and Conclusions

We saw in section [4] that there exist infinitely many doubleton supermultiplets of  $OSp(8^*|4)$ . The CPT-"self-conjugate" irreducible doubleton supermultiplet appears as gauge modes in the Kaluza-Klein spectrum of 11-d SUGRA over  $S^4$  and decouples from the spectrum. It is the supermultiplet of the (2,0) superconformal theory in d=6 which is believed to be dual to M-theory over  $AdS_7 \times S^4$ . This is in complete parallel to the duality between the large N limit of  $\mathcal{N}=4$  supersymmetric SU(N) Yang-Mills theory and the IIB superstring theory over  $AdS_5 \times S^5$ . The other non-CPT self-conjugate doubleton supermultiplets of SU(2,2|4) with non-zero central charge [17]. In the case of SU(2,2|4), it was conjectured [17] that these non-CPT self-conjugate doubleton supermultiplets are related to (p,q) superstrings [29]. Hence, we expect the non-CPT self-conjugate doubleton supermultiplets of  $OSp(8^*|4)$  to be related to the sector of M-theory that is dual to (p,q) superstrings.

Since the  $\Psi$  are bosonic spinors one can give a dynamical realization of the oscillator construction of the UIR's of  $OSp(8^*|4)$  in the language of six dimensional twistors as was suggested for the d=4 conformal group in [17] and explicitly realized in [18, 24]. Based on results in [18, 24], we expect that such a dynamical (twistorial) realization of the oscillator construction of the UIRs of  $OSp(8^*|4)$  to correspond to some extreme limit of M-theory over  $AdS_7 \times S^4$ .

In section [5] we gave a complete list of massless supermultiplets of  $OSp(8^*|4)$  considered as  $\mathcal{N}=4$  supersymmetry algebra in  $AdS_7$ . The CPT self-conjugate massless supermultiplet is simply the graviton supermultiplet. The other massless supermultiplets have spin range between 2 and 4.

One can easily extend the above construction to build all the positive energy massive supermultiplets of  $OSp(8^*|4)$  by taking P > 2. The short massive supermultiplets of spin range 2 (vacuum supermultiplets) correspond to the massive Kaluza-Klein spectrum of 11-d SUGRA over  $S^4$  [8].

We should stress that there are three different forms of the superalgebra  $OSp(8^*|4)$  related via triality. The form considered in this paper as well as in [8] satisfies the standard spin and statistics connection and is the one relevant for the sector of M-theory over  $AdS_7 \times S^4$  containing the 11 dimensional supergravity. It is an open question whether or not the other forms of  $OSp(8^*|4)$  with exotic spin and statistics connection are relevant to M-theory. One would have naively expected the form of  $OSp(8^*|4)$  in which the supersymmetry generators transform in the right-handed spinor representation of SO(6,2) to satisfy the usual spin and statistics connection. However, since the right handed spinor representation decomposes as  $(1+6+\bar{1})$  under SU(4) subgroup it is evident that the "supersymmetry generators" in this case can not satisfy the usual spin and statistics connection.

Acknowledgements: We would like to thank Marco Zagermann for useful discussions.

Note added: This version of the manuscript in the arXiv hep-th incorporates the erratum sent to Nucl. Phys. B about missing states in the tables of some of the supermultiplets. We are grateful to Sudarshan Fernando for alerting us to the fact that some states were missing in the tables of the original published version.

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