De-Broglie wavelength of He atom

$$\lambda = \frac{h}{\sqrt{3m\,KT}}$$

Examples

Ex. 1.11.1 : The energy of photon is 5.28×10^{-19} J. Calculate frequency and wavelength.

Soln. :

$$E = h\upsilon$$

$$\upsilon = \frac{E}{h} = \frac{5.28 \times 10^{-19}\ J}{6.625 \times 10^{-34}\ J-S} = 7.96 \times 10^{14}\ HZ$$

$$\text{Again } \upsilon = \frac{c}{\lambda}$$

$$\lambda = \frac{c}{\upsilon} = \frac{3 \times 10^{8}\ m/s}{7.96 \times 10^{14}}$$

$$\lambda = 0.3768 \times 10^{-6}\ m = 3768 \times 10^{-10}\ m$$

$$= 3768\ A°$$

Ex. 1.11.2 : Electrons moving with a speed of 7.3×10^{7} m/s have wavelength of 0.1A°. Calculate Planck's constant.

Soln. :

Given : $\lambda = 0.1A° = 0.1 \times 10^{-10}$ m, $v = 7.3 \times 10^{7}$ m/s

Formula required : $\lambda = \frac{h}{mv}$

$$h = \lambda \cdot mv$$

$$= 0.1 \times 10^{-10} \times 9.1 \times 10^{-31} \times 7.3 \times 10^{7} = 6.643 \times 10^{-34}\ J.s$$

Ex. 1.11.3 : At what velocity the De Broglie wavelength of an alpha particle is equal to the wavelength of 1 KeV X-ray photons? Given mass of alpha particle is four times the mass of a proton. Mass of proton = 1.67×10^{-27} kg.

Soln. :

Given : $E = 1$ keV, $m = 1.67 \times 10^{-27}$ kg

Formula required : $\lambda = \frac{h}{mv} = \frac{hc}{E}$; $\lambda = \frac{h}{mv}$

For a photon, the energy

$$E = h\upsilon = \frac{h \cdot c}{\lambda}$$

...(1.11.3.1)

∴ Wavelength of the photon is $\lambda = \frac{hc}{E}$

The De Broglie wavelength of the alpha particle of mass m and velocity v is

$$\lambda = \frac{h}{mv} \quad ...(1.11.3.2)$$

$\therefore$ As per the data, Equation (1.11.3.1) = Equation (1.11.3.2)

$$\frac{h}{mv} = \frac{hc}{E}$$

$\therefore$ velocity of alpha particle having same energy as 1 KeV X - ray photon is

$$v = \frac{E}{mc} = \frac{10^3 \times 1.6 \times 10^{-19}}{4 \times 1.67 \times 10^{-27} \times 3 \times 10^8}$$

$$\therefore \quad v = 0.0798 \times 10^3 \text{ m/s} = 79.8 \text{ m/s}$$

Ex. 1.11.4 : (Dec. 08, 3 Marks)

Find the De Broglie wavelength of 10 KeV electrons.

Soln. :

Given : $E = 10 \text{ KeV} = 10 \times 10^3 \text{ e V} = 10 \times 10^3 \times 1.6 \times 10^{-19} \text{ J}$

Formula required : $\lambda = \frac{h}{\sqrt{2mE}}$

$$= \frac{6.625 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 10^4 \times 1.6 \times 10^{-19}}} = 1.227 \times 10^{-11} \text{ m} = 0.1227 \text{ A°}$$

Ex. 1.11.5 : De Broglie wavelength of electron in monoenergetic beam is 7.2×10^{-11} metres. Calculate the momentum and energy of electrons in the beam in electron volts.

Soln. :

Given : $\lambda = 7.2 \times 10^{-11}$ m.

Formulae required : Momentum $p = \frac{h}{\lambda}$, Energy $E = \frac{p^2}{2m}$

(i) $$p = \frac{6.63 \times 10^{-34}}{7.2 \times 10^{-11}} = 0.9208 \times 10^{-23} \text{ kg m/s}$$

(ii) $$E = \frac{(0.9208 \times 10^{-23})^2}{2 \times 9.1 \times 10^{-31}} = 0.04658 \times 10^{-15} \text{ J} = \frac{0.04658 \times 10^{-15}}{1.6 \times 10^{-19}} \text{ eV}$$

$$E = 0.0291 \times 10^4 \text{ eV}$$

Ex. 1.11.6 : Find the K.E. of a neutron which has a wavelength of 3A°. At what angle will such a neutron undergo first order Bragg reflection from a calcite crystal for which the grating space is 3.036 A°? Mass of neutron = 1.67×10^{-27} kg.

Soln. :

Given : $\lambda = 3\text{A°} = 3 \times 10^{-10}$ m

$d = 3.036 \text{ A°} = 3.036 \times 10^{-10}$ m

$m = 1.67 \times 10^{-27}$ kg.

Formulae required : Wavelength of neutron is $\lambda = \frac{h}{\sqrt{2mE}}$

$$2d \sin\theta = n\lambda$$

$$\therefore \quad \text{Neutron energy} \quad E = \frac{h^2}{2m\lambda^2}$$

$$\therefore \quad E = \frac{(6.63 \times 10^{-34})^2}{2 \times 1.67 \times 10^{-27} \times (3 \times 10^{-10})^2} = 1.449 \times 10^{-21}\ J$$

For Bragg reflection, $2\, d \sin\theta = n\lambda$

$$n = 1$$

$$\therefore \quad \sin\theta = \frac{\lambda}{2d} = \frac{3 \times 10^{-10}}{2 \times 3.036 \times 10^{-10}} = 0.4940$$

$$\therefore \quad \theta = \sin^{-1}(0.4940) = 29^\circ\, 36'$$

Ex. 1.11.7 : (Dec. 09)

Determine the velocity and kinetic energy of a neutron having De Broglie wavelength 1.0A° (mass of neutron is 1.67×10^{-27} kg)

Soln. :

We have, $\lambda = \frac{h}{mv}$, $\quad v = \frac{h}{m\lambda} = \frac{6.63 \times 10^{-34}}{1.67 \times 10^{-27} \times 10^{-10}}$

$$v = 3.97 \times 10^3 \text{ m/s}$$

Again we can write, $\lambda = \frac{h}{\sqrt{2mE}} \Rightarrow E = \frac{h^2}{2m\lambda^2}$

$$E = \frac{(6.63 \times 10^{-34})^2}{2 \times 1.67 \times 10^{-27} \times (10^{-10})^2}$$

$$= 13.16 \times 10^{-21} J = 8.225 \times 10^{-2}\ eV \quad (\because 1eV = 1.6 \times 10^{-19} J)$$

Ex.1.11.8 : Calculate the wavelength of electron of energy 291 eV.

Soln. :

Given : $E = 291\ eV = 291 \times 1.6 \times 10^{-19}\ J$

Formula required : $\lambda = \frac{h}{\sqrt{2mE}}$

The electron wavelength is,

$$\lambda = \frac{h}{\sqrt{2mE}} = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 291 \times 1.6 \times 10^{-19}}}$$

$$= 0.7202 \times 10^{-10}\ m$$

$$\therefore \quad \lambda = 0.7202\ A^\circ$$

Ex.1.11.9 : Calculate the De Broglie wavelength of a neutron having kinetic energy of 1eV.

Soln. :

De Broglie wavelength is given by $\lambda = \frac{h}{\sqrt{2mE}}$

Given : $h = 6.63 \times 10^{-34}$ J-s, $m = 1.67 \times 10^{-27}$ kg, $E = 1eV = 1 \times 1.6 \times 10^{-19}$ J

$$\lambda = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 1.67 \times 10^{-27} \times 1.6 \times 10^{-19}}}$$

$$= 2.87 \times 10^{-11} m = 0.287 A°$$

Ex.1.11.10 : If electron had existed inside the nucleus, then its De Broglie wavelength would be roughly of the order of nuclear diameter i.e. 10^{-14} m. How much momentum corresponds to this wavelength ? How much energy corresponds to this momentum ? Express this energy is 8.8 MeV and explain how this result proves that the electron cannot exist inside the nucleus. (The maximum nuclear binding energy is 8.8 MeV per nuclear particle.) Given : Planck's constant $h = 6.63 \times 10^{-34}$ Js, Mass of electron $= 9.1 \times 10^{-31}$ kg.

Soln. :

Given : $h = 6.63 \times 10^{-34}$, $\lambda = 10^{-14}$, $m = 9.1 \times 10^{-31}$ kg

Formulae required : $p = \frac{h}{\lambda}$, $E = \frac{p^2}{2m}$

The De Broglie wavelength of a particle is $\lambda = \frac{h}{p}$

$\therefore$ The momentum of electron is

$$p = \frac{h}{\lambda} = \frac{6.63 \times 10^{-34}}{10^{-14}}$$

$$\text{or} \quad p = 6.63 \times 10^{-20} \text{ kg m/s}$$

The energy of the electron is

$$E = \frac{p^2}{2m} = \frac{(6.63 \times 10^{-20})^2}{2 \times 9.1 \times 10^{-31}} = 2.42 \times 10^{-9} \text{ J}$$

$$\text{or} \quad E = \frac{2.42 \times 10^{-9}}{1.6 \times 10^{-19}} \text{eV} = 1.5 \times 10^{10} \text{ eV.} = 15000 \text{ MeV}$$

Ex. 1.11.11 : Find the De Broglie wavelength of an electron when accelerated through a potential difference of 100 V.

Soln. :

Given : $m = 9.1 \times 10^{-31}$ kg, V = 100 volts.

Formula required : $\lambda = \frac{12.26}{\sqrt{V}}$ A°

$$\lambda = \frac{12.26}{\sqrt{100}} = 1.226 \text{ A°}$$

Ex. 1.11.12 : (May 05, 4 Marks)

A proton and an α particle are accelerated by the same potential difference. Show that the ratio of the De Broglie wavelengths associated with them is $2\sqrt{2}$. Assume the mass of alpha particle to be 4 times the mass of proton.

Soln. :

Given : $m_\alpha = 4m_p$,

m_α = mass of α - particle, m_p = mass of proton,

Let q_α = charge of particle, q_p = charge of proton

$\therefore q_\alpha = 2\,q_p$

Let $V \rightarrow$ Accelerating potential for the particles.

Formula required : $\lambda = \dfrac{h}{\sqrt{2\,m\,e\,V}}$

$$\therefore \lambda_p = \frac{h}{\sqrt{2\,m_p \cdot q_p\,V}} \quad \text{and} \quad \lambda_\alpha = \frac{h}{\sqrt{2\,m_\alpha \cdot q_\alpha\,V}}$$

$$\therefore \frac{\lambda_p}{\lambda_\alpha} = \frac{h}{\sqrt{2\,m_p \cdot q_p\,V}} \cdot \frac{\sqrt{2\,m_\alpha \cdot q_\alpha\,V}}{h} = \sqrt{\frac{m_\alpha}{m_p}} \cdot \sqrt{\frac{q_\alpha}{q_p}}$$

$$= \sqrt{4} \cdot \sqrt{2} = 2\sqrt{2}$$ ...Proved.

Ex. 1.11.13 : What accelerating potential would be required for a proton with zero velocity to acquire a velocity corresponding to De Broglie's wavelength of 10^{-14} m ?

$h = 6.62 \times 10^{-34}$ J s , Mass of proton = 1.67×10^{-27} kg, e = 1.6×10^{-19} c.

Soln. :

Given : $\lambda = 10^{-14}$ m, $H = 6.62 \times 10^{-34}$ Js,

$m_p = 1.67 \times 10^{-27}$ kg. , $e = 1.6 \times 10^{-19}$ c

Formula required : $\lambda = \dfrac{h}{\sqrt{2\,m\,e\,V}}$

$$\therefore \lambda^2 = \frac{h^2}{2\,m\,e\,V}$$

$\therefore$ Accelerating potential of proton should be

$$V = \frac{h^2}{2\,m\,e\,\lambda^2}$$

$$= \frac{(6.62 \times 10^{-34})^2}{2 \times 1.67 \times 10^{-27} \times 1.6 \times 10^{-19} \times (10^{-14})^2} = 8.199 \times 10^6 \text{ volts.}$$

$\therefore$ V = 8.199 M volts.

Ex. 1.11.14 : Find the De Broglie wavelength of

i) An electron accelerated through a potential difference of 182 volts and

ii) 1 kg object moving with a speed of 1 m/sec.

Comparing the results, explain why the wave nature of matter is not more apparent in daily observations.

Soln. :

Given : $V = 182$ V, $m = 1$ kg, $v = 1$ m/s.

Formulae required : $\lambda = \frac{12.26}{\sqrt{V}}$ A°, $\lambda = \frac{h}{mv}$

(i) De Broglie wavelength of the electron accelerated through a potential V volts is,

$$\lambda = \frac{12.26}{\sqrt{V}} A° = \frac{12.26}{\sqrt{182}} A° = 0.9087\ A°$$

(ii) The De Broglie wavelength of a body of mass m moving with velocity v is,

$$\lambda = \frac{h}{mv} = \frac{6.63 \times 10^{-34}\ J.s}{1\ kg \times 1\ m/s} = 6.63 \times 10^{-34}\ m$$

From the results (i) and (ii) we see that the wavelength for electron is measurable; But for the body of mass 1 kg the wavelength is too small to be detected. Hence the wave nature of matter is not more apparent in daily observations.

Ex. 1.11.15 : An electron initially at rest is accelerated through a potential difference of 3000 V. Calculate for the electron wave the following parameters :

(i) The momentum

(ii) The De Broglie wavelength

(iii) The wave number

(iv) The Bragg angle for its first order reflection from (1, 1, 1) planes of the nickel crystal, which are 2.04 A° apart.

Soln. :

Given : $V = 3000$ volts

Formulae Required : $\lambda = \frac{12.26}{\sqrt{V}}$ A°

$2d \sin\theta = n\lambda$, $p = \frac{h}{\lambda}$, $\bar{\lambda} = \frac{1}{\lambda}$

i) De Broglie wavelength of electron is

$$\lambda = \frac{12.26}{\sqrt{V}} A° \qquad \therefore \quad \lambda = \frac{12.26}{\sqrt{3000}} A°$$

$$\text{or} \quad \lambda = 0.2238\ A°$$

ii) Momentum of the electron is

$$p = \frac{h}{\lambda} = \frac{6.63 \times 10^{-34}}{0.2238 \times 10^{-10}} \qquad \therefore \quad p = 29.62 \times 10^{-24}\ kg.\ m/s$$

iii) The wave number is

$$\bar{\lambda} = \frac{1}{\lambda} = \frac{1}{0.2238 \times 10^{-10}}$$

$$\text{or} \quad \bar{\lambda} = 4.468 \times 10^{10}\ /m$$

iv) Bragg angle θ is given by the relation

$$2d \sin\theta = n\lambda$$

$\therefore$ for first order reflection,

$$\sin\theta = \frac{\lambda}{2d} = \frac{0.2238\ A^\circ}{2 \times 2.04\ A^\circ}$$

$$\sin\theta = 0.0548$$

$$\therefore \quad \text{Bragg angle } \theta = 3.14^\circ$$

Ex. 1.11.16 : An electron is accelerated through a potential difference of 10 kV. Calculate the De Broglie wavelength and momentum of electron.

Soln. :

Given : $V = 10\ kV$

Formula required : $\lambda = \frac{12.26}{\sqrt{V}}\ A^\circ$

The De Broglie wavelength of an electron accelerated through a p.d. of V volts is

$$\lambda = \frac{12.26}{\sqrt{V}}\ A^\circ$$

Now $\quad V = 10\ kV = 10 \times 10^3$ Volts $\quad \therefore \lambda = \frac{12.26}{\sqrt{10^4}}\ A^\circ$

$$\text{or} \quad \lambda = \frac{12.26}{10^2}\ A^\circ = 0.1226\ A^\circ$$

The momentum of the electron is given by,

$$p = \frac{h}{\lambda}$$

$$\therefore \quad p = \frac{6.625 \times 10^{-34}}{0.1226 \times 10^{-10}}$$

$$\text{or} \quad p = 54.037 \times 10^{-24}\ \text{kg. m/s}$$

Ex. 1.11.17 : Calculate the De Broglie wavelength of an α – particle accelerated through a potential difference of 200 volts.

Soln. : We have De Broglie wavelength

$$\lambda = \frac{h}{\sqrt{2meV}}$$

Given : For α-particle, charge = 2e, $m = 4 \times m_p$

$$\lambda = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 4 \times 1.67 \times 10^{-27} \times 2 \times 1.6 \times 10^{-19} \times 200}} = 7.16 \times 10^{-13}\ m$$

$$= 7.16 \times 10^{-3}\ A^\circ$$

Ex 1.11.18 : Calculate the wavelength associated with 1 MeV proton.

Soln. :

Formulae : $h = \frac{h}{\sqrt{2\ m_p E}} = \frac{h}{\sqrt{2\ m_p eV}}$

Given : $V = 10^6$ V

$$h = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 1.67 \times 10^{-27} \times 10^6 \times 1.6 \times 10^{-19}}} = 2.868 \times 10^{-14} \text{ m.}$$

...Ans.

Ex. 1.11.19 : The radius of the first Bohr orbit in hydrogen atom is 0.53×10^{-10} m. Find the velocity of the electron in that orbit using De-Broglie theory.

Soln. : The electron wave is possible around an orbit only when the circumference of an orbit is equal to an integral multiple of the wavelength, i.e.,

$$2\pi r = n\lambda$$

For the first Bohr orbit n = 1. From De-Broglie concept of matter wave $\lambda = h/mv$,

$$\text{Hence,} \quad 2\pi r = \frac{h}{mv}$$

$$v = \frac{h}{2\pi r m}$$

Given, $r = 0.53 \times 10^{-10}$ m, $h = 6.63 \times 10^{-34}$ J-sec.

and $m = 9.1 \times 10^{-31}$ kg

$$v = \frac{6.63 \times 10^{-34}}{2 \times 3.14 \times 0.53 \times 10^{-10} \times 9.1 \times 10^{-31}}$$

$$v = 2.18 \times 10^6 \text{ m/s}$$

Ex. 1.11.20 : Find the De-Broglie's wavelength of a neutron of energy 12.8 MeV.

Given : mass of neutron = 1.675×10^{-27} kg

Soln. : The De-Broglie wavelength λ for a particle of mass m is given by,

$$\lambda = \frac{h}{\sqrt{2mE}}$$

Given : $E = 12.8 \times 10^6 \times 1.6 \times 10^{-19}$ J

$$\lambda = \frac{6.62 \times 10^{-34}}{\sqrt{2 \times 1.675 \times 10^{-27} \times 12.8 \times 10^6 \times 1.6 \times 10^{-19}}}$$

$$\lambda = 8.0 \times 10^{-15} \text{ m} = 8.0 \times 10^{-5} \text{ A°}$$

Ex. 1.11.21 : What voltage must be applied to an electron microscope to produce electrons of wavelength 0.40 A°.

Soln. : If V is the applied voltage, then E = eV, then we have

$$\lambda = \frac{h}{\sqrt{2m\,eV}}$$

$$\text{or,} \quad V = \frac{h^2}{\sqrt{\lambda^2\, 2m\, e}}$$

$$V = \frac{(6.63 \times 10^{-34})^2}{[(0.40 \times 10^{-10})^2 \times 2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19}]^{1/2}}$$

$$V = \frac{4.356 \times 10^{-67}}{4.66 \times 10^{-70}}$$

$$V = 934.76 \text{ Volts.}$$

Ex. 1.11.22 : An electron and a photon each have a wavelength of 2.0 A°. Compare their as
(a) momentum (b) total energies (c) ratio of K.E.

Soln. :

(a) According to De-Broglie's formulae, the momentum of electron,

$$p_e = \frac{h}{\lambda} = \frac{6.63 \times 10^{-34}}{2.0 \times 10^{-10}}$$

$$p_e = 3.31 \times 10^{-24} \text{ kg-m/sec.}$$

The momentum of photon of frequency ν is given by,

$$p_{ph} = \frac{h\nu}{c} = \frac{h}{\lambda} = \frac{6.63 \times 10^{-34}}{2.0 \times 10^{-10}}$$

$$p_{ph} = 3.31 \times 10^{-24} \text{ kg-m/sec.}$$

(b) K. E.

$$K_e = \frac{p_e^2}{2\, m_e} = \frac{(3.31 \times 10^{-24})^2}{2 \times 9.1 \times 10^{-31}}$$

$$= 6.02 \times 10^{-18} \text{ J} / 1.6 \times 10^{-19} = 37.62 \text{ eV}$$

The rest energy of the electrons = $m_o c^2$

$$= 9.1 \times 10^{-31} \times (3 \times 10^8)^2 = 8.19 \times 10^{-14} \text{ J}$$

$$= 8.19 \times 10^{-14} / 1.6 \times 10^{-19} = 0.51 \text{ MeV}$$

Since the K. E. of the electrons (37.62 eV) is negligible compared to its rest energy, the total energy of the electron would be 0.51 MeV.

The total energy of the photon $E = h\nu = \frac{hc}{\lambda} = pc$

$$= 3.31 \times 10^{-24} \times 3.0 \times 10^8 = 9.93 \times 10^{-16} \text{ J}$$

$$= \frac{9.93 \times 10^{-16}}{1.6 \times 10^{-19}} = 6.21 \text{ KeV}$$

Since the rest energy of the photon is zero, hence its total energy would be same as its K.E. So, total energy of photon = 6.21 KeV.

(c) K. E. of electron

$$K_e = 37.62 \text{ eV}$$

and K.E. of photon

$$K_{Ph} = 6.21 \text{ KeV}$$

So,

$$\frac{K_e}{K_{Ph}} = \frac{37.62}{6.21 \times 10^3} = 6.05 \times 10^{-3}$$

Ex. 1.11.23 : (May 06, 4 Marks)

What potential difference must be applied to an electron microscope to obtain electro wavelength 0.3 A° ?

Soln. :

$$\lambda = \frac{12.27}{\sqrt{V}}, A^\circ$$

$$\lambda = 0.3\ A^\circ$$

$$\sqrt{V} = \frac{12.27}{0.3}$$

$$V = \left(\frac{12.27}{0.3}\right)^2 = 1672.81 \text{ volts}$$

Ex. 1.11.24 : (May 07, 4 Marks)

Calculate the velocity and De-Broglie Wavelength of an α-particle of energy 1 Kev.

Soln. :

$$\lambda_\alpha = \frac{h}{\sqrt{2\, m_\alpha E}}$$

$$= \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 4 \times 1.67 \times 10^{-27} \times 1 \times 10^3 \times 1.6 \times 10^{-19}}}$$

$$= 45.34 \times 10^{-14}\ \text{m} = 0.0045\ A^\circ$$

$$\lambda_\alpha = \frac{h}{m_\alpha v_\alpha}$$

$$v_\alpha = \frac{h}{m_\alpha \lambda_\alpha} = \frac{6.63 \times 10^{-34}}{4 \times 1.67 \times 10^{-27} \times 4.5 \times 10^{-13}}$$

$$v_\alpha = 0.22 \times 10^6 = 2.2 \times 10^5\ \text{m/s}$$

Ex. 1.11.25 : (Dec. 07, 4 Marks)

An electron has kinetic energy equal to its rest mass energy. Calculate De-Broglie's wavel associated with it.

Soln. :

Data : $h = 6.63 \times 10^{-34}$ JS $\quad m = 9.1 \times 10^{-31}$ kg.

$c = 3 \times 10^8$ m/sec. $\quad \lambda = ?, \quad E = m_o c^2$

Formula : $$\lambda = \frac{h}{\sqrt{2\,mE}}$$

$$\therefore \lambda = \frac{h}{\sqrt{2\, m^2 c^2}} \qquad \therefore E = mc^2.$$

$$\lambda = \frac{h}{\sqrt{2} \times mc}$$

$$= \frac{6.63 \times 10^{-34}}{\sqrt{2} \times 9.1 \times 10^{-31} \times 3 \times 10^8} = 0.1715 \times 10^{-11}\ \text{m.}$$

Ex. 1.11.26 : (May 08, 4 Marks)

In a T.V Set, electrons are accelerated by a p.d. of 10 kV. What is the wavelength associated with these electrons ?

Soln. :

Given formulae :

$$\lambda = \frac{h}{\sqrt{2\,m\,eV}} = \frac{12.3}{\sqrt{V}}\ A^\circ = \frac{12.3}{\sqrt{10^4}}\ A^\circ$$

$$\lambda = 0.12\ A^\circ$$

...Ans.

Ex. 1.11.27 : (May 09, 4 Marks)

Find the de Broglie's Wavelength Associated with Monoenergetic Electron Beam having momentum 10^{-23} kg m/s.

Soln. :

Given : $p = 10^{-23}$ kg-m/s

Formula :

$$\lambda = \frac{h}{p} = \frac{6.63 \times 10^{-34}}{10^{-23}} = 6.63 \times 10^{-11}\text{m} = 0.663 \times 10^{-10}\text{m} = 0.663 A^\circ$$

Ex. 1.11.28 : (May 07, Dec. 11, 4 Marks)

Which has a shorter wavelength 1 eV photon or 1 eV electron ? Calculate the value and explain.

Soln. :

For photon,
$$E = h\nu = \frac{hc}{\lambda}$$

$$\lambda_{ph} = \frac{hc}{E} = \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{1.6 \times 10^{-19}} = 1.2431 \times 10^{-6}\ \text{m}$$

$$= 12431\ A^\circ$$

For electron,
$$\lambda_e = \frac{h}{\sqrt{2\,m_e\,E}} = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19}}}$$

$$\lambda_e = 1.228 \times 10^{-9}\ \text{m} = 12.28\ A^\circ$$

$\lambda_{ph} > \lambda_e$, So, frequency of electron is more, hence more effective.

Ex. 1.11.29 : (Dec. 10, 4 Marks)

At what Kinetic energy an electron will have a wavelength of 5000A° ?

Soln. :

Formula Required : $\lambda = \frac{h}{mv}$

Data Given :

$\lambda = 5000A^\circ = 5000 \times 10^{-10}$ m.

$m = 9.1 \times 10^{-31}$ kg

$h = 6.63 \times 10^{-34}$ J-S.

So,

$$v = \frac{h}{m\lambda} = \frac{6.63 \times 10^{-34}}{9.1 \times 10^{-31} \times 5 \times 10^{-7}} = 1.457 \times 10^{3}\ \text{m/s}$$

$$\text{K.E.} = \frac{1}{2}mv^2 = \frac{1}{2} \times 9.1 \times 10^{-31} \times (1.457 \times 10^{3})^2 = 9.66 \times 10^{-25}\ \text{J}$$

...Ans.

Ex. 1.11.30 : (May 11, 4 Marks)

Calculate de-Broglie wavelength of 10 ke V protons in A. U.

Soln. :

Given : $E = 10 \text{ KeV} = 10 \times 10^3 \times 1.6 \times 10^{-19} J$

$m_p = 1.67 \times 10^{-27}$ kg.

Formulae : $\lambda = \frac{h}{\sqrt{2 m_p E}} = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 1.67 \times 10^{-27} \times 10^4 \times 1.6 \times 10^{-19}}}$

$= 2.86 \times 10^{-13}$ m. $= \mathbf{2.86 \times 10^{-3}}$ **A°**

1.12 Electron Diffraction

PU - May 06

Experimental verification of matter waves :

- According to De Broglie hypothesis, a beam of particles possesses wave like characteristics. Hence it should exhibit the phenomena exhibited by waves. This conclusion was put to an experimental test and was found to be true.
- To show that De Broglie waves actually exist, an attempt was made to observe electron diffraction. To observe diffraction, the dimensions of the obstacle or slit should be comparable to the wavelength of the wave.
- The wavelength of the De Broglie waves of electrons are of the order of wavelength of X-rays. Hence crystals which can give X-ray diffraction should also give diffraction of electrons. The experimentally observed electron diffraction confirms that such matter waves actually exist.

1.13 Davison and Germer Experiment

PU - May 05, May 06, Dec. 07, May 08, Dec. 08

In 1927, Davisson and Germer performed an experiment on electron diffraction by the help of single crystal of nickel and confirmed the wave-like nature of electrons.

It also gives an experimental evidence of De Broglie hypothesis of matter waves.

1.14 Wave Packet

- As per De Broglie's postulate, a material particle of mass m moving with velocity v is represented by a monochromatic wave of wavelength λ.
- Such a single wave representation of the particle raises some questions :
  (i) How can a wave that spreads out over a large region of space can represent a particle which is highly localised ?
  (ii) If we assoicate the wave with the particle, what exactly is the thing that is waving in the matter wave ?

A simple harmonic progressive wave is represented by an equation of the type.

Revision of Important Formulae

1. **Heisenberg's uncertainty principle :**

$\Delta x \cdot \Delta p_x \geq h$ or $\Delta x \cdot \Delta p_x \geq \frac{h}{2\pi}$ (more accurately)

2. **Heisenberg's relation to pair of variables energy and time :**

$\Delta E \times \Delta t \geq \hbar$, where $\hbar = \frac{h}{2\pi}$

Examples

Ex. 1.18.1 : Calculate the minimum uncertainty in the velocity of an electron confined to a box of length 10 A°.

Soln. :

Given : $\Delta x = 10\ \text{A}° = 10 \times 10^{-10}$ m

Formula required : $\Delta x \cdot \Delta p_x = h$

$$\therefore (\Delta x)_{max} \cdot (\Delta p_x)_{min} = h$$

$$\text{or} \quad (\Delta x)_{max} \cdot (m \cdot \Delta v_x)_{min} = h$$

$$\therefore (\Delta v_x)_{min} = \frac{h}{m \cdot (\Delta x)_{max}}$$

$$= \frac{6.625 \times 10^{-34}}{9.1 \times 10^{-31} \times 10 \times 10^{-10}}$$

$$= 7.28 \times 10^5 \text{ m/s}$$

Ex. 1.18.2 : If the uncertainty in the location of the particle is equal to its De Broglie wavelength, then show that the uncertainty in its velocity is equal to its velocity.

Soln. :

Given : $\Delta x = \lambda$

Formula required : $\Delta x \cdot \Delta p_x = h$

$$\therefore \Delta x \cdot (m \cdot \Delta v_x) = h$$

$$\therefore \Delta v_x = \frac{h}{m \cdot \Delta x} = \frac{h}{m \cdot \lambda} \quad (\because \Delta x = \lambda)$$

As per De Broglie hypothesis, $\lambda = \frac{h}{mv}$

$$\therefore \Delta v_x = \frac{h}{m \cdot h/mv} = v$$

$$\therefore \Delta v = v$$

Ex. 1.18.3 : An electron has a speed of 600 m/s with an accuracy of 0.005%. Calculate the uncertainty with which we can locate the position of the electron.

Soln. :

Given : $v = 600$ m/s, $\Delta v = 0.005\%$ of v

Formula required : $\Delta x \cdot \Delta p_x = h$

$$\therefore \Delta x = \frac{h}{\Delta p_x} = \frac{h}{m \cdot \Delta v_x}$$

Now the uncertainty in velocity is

$$\Delta v_x = \frac{0.005}{100} \times 600 = 0.030 \text{ m/s}$$

$\therefore$ The uncertainty in the location of electron is

$$\Delta x = \frac{h}{m \cdot \Delta v_x} = \frac{6.63 \times 10^{-34}}{9.1 \times 10^{-31} \times 0.03}$$

$$= 24.28 \times 10^{-3} \text{ m}$$

$$\therefore \Delta x = 0.0242 \text{ m}$$

Ex. 1.18.4 : The speed of an electron is measured to be 5.00×10^3 m/s to an accuracy of 0.003%. Find the uncertainty in determining the position of this electron.

Soln. : We have, $\Delta x \cdot \Delta px \geq h$

$$\text{Momentum } p = mv = 9 \times 10^{-31} \times 5.00 \times 10^3 = 4.5 \times 10^{-27} \text{ kg.m/s}$$

$$\Delta p = \frac{0.003}{100} \times 4.5 \times 10^{-27} = 1.35 \times 10^{-31} \text{ kg-m/s}$$

So, Uncertainty in position $\Delta x = \dfrac{h}{\Delta p} = \dfrac{6.63 \times 10^{-34}}{1.35 \times 10^{-31} \times 2\pi}$

$$= 7.82 \times 10^{-4} \text{m.}$$

Ex. 1.18.5 : The position and momentum of 1 KeV electron are simultaneously measured. If its position is located to within 1 A°, find the percentage of uncertainty in its momentum.

Rest mass of electron = 9.1×10^{-31} kg

Soln. :

Given : $\Delta x = 1$ A° $= 10^{-10}$ m

Formula required : $\Delta x \cdot \Delta p_x = h$

$$\therefore \Delta p_x = \frac{h}{\Delta x} = \frac{6.62 \times 10^{-34}}{10^{-10}}$$

$$\therefore \Delta p_x = 6.62 \times 10^{-24} \text{ kg.m.s}$$

Given energy of electron is 1 KeV or 10^3 eV which is less than the rest mass energy ($m_o c^2 = 511$ KeV) of the electron.

$\therefore$ Momentum of the electron can be determined as $p = \sqrt{2m \cdot E}$

$$\therefore p_x = \sqrt{2 \times 9.1 \times 10^{-31} \times 10^3 \times 1.6 \times 10^{-19}}$$

$$= 1.7 \times 10^{-23} \text{ kg.m.s}$$

$\therefore$ *Percentage of uncertainty* in the momentum of electron is,

$$\frac{\Delta p_x}{p_x} \times 100 = \frac{6.62 \times 10^{-24}}{1.7 \times 10^{-23}} \times 100 = 38.94\%$$

Ex. 1.18.6 : Compute the minimum uncertainty in the location of a 2 gm. mass moving with a speed of 1.5 m/s and the minimum uncertainty in the location of an electron moving with a speed of 0.5×10^8 m/s. Given that the uncertainty in the momentum is $\Delta p = 10^{-3}$ p for both.

Soln. :

Given : $\Delta p = 10^{-3}$ p, Speed = 1.5 m/s, mass = 2 gm

Formula required

$$\Delta x \cdot \Delta p = h$$

$$\Delta p = 10^{-3} p = 10^{-3} (mv) \qquad (\because p = m \cdot v)$$

For the body

$$\Delta x \cdot \Delta p = h$$

$$\therefore \Delta x = \frac{h}{\Delta p} = \frac{6.63 \times 10^{-34}}{10^{-3} \times 2 \times 10^{-3} \times 1.5} = 2.2 \times 10^{-28} \text{ m}$$

For the electron

$$\Delta x = \frac{h}{\Delta p} = \frac{6.63 \times 10^{-34}}{9.1 \times 10^{-31} \times 0.5 \times 10^8 \times 10^{-3}}$$

$$= 1.45 \times 10^{-8} \text{ m}$$

Ex. 1.18.7 : A bullet of mass 25 grams is moving with a speed of 400 m/s. The speed is measured accurate upto 0.02%. Calculate the certainty with which the position of the bullet can be located.

Soln. :

Given : m = 25 gm = 25×10^{-3} kg., v = 400 m/s, $\Delta v = 0.02\ \%$

Formula required : $\Delta x = \frac{h}{\Delta p_x}$

$$\Delta v = \frac{0.02}{100} \times 400 = 0.08 \text{ m}$$

$\therefore$ The certainty with which the position of bullet can be located is given by Heisenberg's principle as,

$$\Delta x = \frac{h}{\Delta p_x} = \frac{h}{m \times \Delta v} \qquad \therefore \Delta x = \frac{6.63 \times 10^{-34}}{25 \times 10^{-3} \times 0.08}$$

$$\text{or } \Delta x = 3.315 \times 10^{-31} \text{ m}$$

Ex. 1.18.8 : A hydrogen atom is 0.53 A° in radius. Use uncertainty principle to estimate the minim[um] energy an electron can have i.e. this atom.

Soln. :

We have,
$$\Delta x \cdot \Delta p_x \geq \hbar$$

Given :
$$\Delta x = 0.53 \times 10^{-10} m = 5.3 \times 10^{-11} m$$

$$\Delta p_x \geq \frac{\hbar}{\Delta x} = \frac{6.63 \times 10^{-34}}{5.3 \times 10^{-11} \times 2\pi} = 1.28 \times 10^{-24} \text{kg-m/s}$$

As p cannot be less than ΔP_x,

So, p must be atleast equal to ΔP_x.

So,
$$K.E. \geq \frac{p^2}{2m} = \frac{(1.28 \times 10^{-24})^2}{2 \times 9.1 \times 10^{-31}}$$

$$= 2.16 \times 10^{-18} J = \frac{2.36 \times 10^{-18}}{1.6 \times 10^{-19}} eV$$

$$= 13.5 \text{ eV}$$

Ex. 1.18.9 : The average time that an atom retains excess excitation energy before re-emitting it as e[lectro] magnetic radiation is 10^{-8} sec. Calculate the limit of accuracy with which th[e] excitation energy and the frequency of emitted radiation can be determined.

Soln. :

Given : $\Delta t = 10^{-8}$ sec

Formula required : $\Delta E \cdot \Delta t = h$

The photon energy is uncertain by an amount.

$$\Delta E = \frac{h}{\Delta t} = \frac{6.63 \times 10^{-34}}{10^{-8}} J$$

$$\text{or } \Delta E = 6.63 \times 10^{-26} J$$

The frequency of radiation is uncertain by

$$\Delta \nu = \frac{\Delta E}{h} \text{ (using } E = h\nu) = \frac{6.63 \times 10^{-26}}{6.63 \times 10^{-34}}$$

$$\therefore \Delta \nu = 10^8 \text{ cycles/ sec}$$

Ex. 1.18.10 : Show that the concept of Bohr orbits violates the principle of uncertainty.

Soln. : The kinetic energy of a particle of mass m and velocity v is,

$$E = \frac{p^2}{2m}$$

Where p = mv = momentum of the particle.

$$\therefore \Delta E = \frac{p \cdot \Delta p}{m}$$

But $p = mv = m \cdot \dfrac{\Delta x}{\Delta t}$

$$\therefore \frac{p}{m} = \frac{\Delta x}{\Delta t}$$

$\therefore$ We have

$$\Delta E = \frac{\Delta x}{\Delta t} \cdot \Delta p$$

or $\Delta E \cdot \Delta t = \Delta x \cdot \Delta p \geq \frac{h}{2\pi}$ as per the uncertainty principle.

As per Bohr's orbit concept, every electron revolves round the nucleus in quantised orbits and has a sharply defined energy, with no uncertainty.

$$\therefore \Delta E = 0$$

Hence $\Delta t = \infty$ which implies that all the energy states of the atom have infinite life time indicating no possibility of radiation at all due to transition between the orbits.

But actually Δt is found to be nearly 10^{-8} sec. Thus the concept of Bohr orbits violates the uncertainty principle.

Ex. 1.18.11 : An electron and a 150 gm base ball are traveling 220 m/sec. measured to an accuracy of 0.005 percent. Calculate the compare uncertainty in position of each.

Soln. :

Formula : We have $\Delta x \cdot \Delta p_x \geq h, \ \Delta x = \frac{h}{m \cdot \Delta v}$

$$\Delta v = \frac{0.005}{100} \times 220 = 0.011$$

$$\Delta x = \frac{6.63 \times 10^{-34}}{9.1 \times 10^{-31} \times 0.011} = 6.62 \times 10^{-2} \text{ m}$$ **...Ans.**

Ex. 1.18.12 : Calculate the wavelength associated with 1 MeV proton.

Soln. :

Formulae : $\lambda = \frac{h}{\sqrt{2 m_p E}} = \frac{h}{\sqrt{2 m_p eV}}$

Given : $V = 10^6$ V

$$\lambda = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 1.67 \times 10^{-27} \times 10^6 \times 1.6 \times 10^{-19}}} = 2.868 \times 10^{-14} \text{ m.}$$ **....Ans.**

Ex. 1.18.13 : The life time of an excited state of a nucleus is 10^{-12} sec. What is the uncertainty in energy of γ-ray photon emitted. (Use $\Delta E \cdot \Delta t \geq \hbar$)

Soln. : The uncertainty in energy ΔE and the time interval Δt is related as,

$$\Delta E \cdot \Delta t \geq \hbar$$

$$\text{or, } \Delta E = \frac{h}{2\pi \cdot \Delta t}$$

Hence, the minimum uncertainty in energy

$$\Delta E = \frac{6.63 \times 10^{-34}}{2 \times 3.14 \times 10^{-12}} = \frac{1.055 \times 10^{-22}}{1.6 \times 10^{-19}} = 6.598 \times 10^{-4} \text{ eV}$$

Ex. 1.18.14 : Calculate the uncertainty in the position of an electron which has been accelerated through a potential difference of $V = (1000 \pm 6)$ Volts.

Soln. : De-Broglie wavelength of an electron accelerated through a potential difference of a V volts is given by,

$$\lambda = \frac{h}{\sqrt{2meV}} \text{ and momentum } p = \frac{h}{\lambda} = \sqrt{2meV}$$

$$\therefore \quad \Delta p = \frac{m e \Delta V}{(2m eV)^{1/2}}$$

According to the uncertainty principle,

$$\Delta p \cdot \Delta x \geq h \quad \text{or, } \Delta x = \frac{h}{\Delta p}$$

$$\Delta x = \frac{h \times (2\, meV)^{1/2}}{m e \Delta V} = \frac{\sqrt{2}\, h \sqrt{V}}{\sqrt{m}\, e \Delta V} = \frac{6.63 \times 10^{-34} \times \sqrt{2} \times \sqrt{V}}{(9.1 \times 10^{-31} \times 1.6 \times 10^{-19})^{1/2} \Delta V}$$

$$= 0.245 \times 10^{-10} \cdot \frac{\sqrt{V}}{\Delta V} = 0.245 \frac{\sqrt{V}}{\Delta V} \text{ A}^\circ$$

Since the maximum uncertainty in the accelerating voltage $\Delta V = 2 \times 6 = 12$ Volts

$$\therefore \quad \Delta x = \frac{0.245 \times \sqrt{1000}}{12} = 0.645 \text{ A}^\circ$$

...Ans.

Ex. 1.18.15 : Wavelengths can be determined with accuracies of one part in 10^6. What is the uncertainty in the position of a 1 A° X-ray photon when its wavelength is simultaneously measured. (Use $\Delta x \cdot \Delta p_x \geq \hbar$)

Soln. : According to uncertainty principle

$$\Delta x \cdot \Delta p \geq \hbar \quad \text{...(1.18.15.1)}$$

and momentum of a photon $p = \dfrac{h}{\lambda}$

$$\text{or, } \quad p\lambda = h \quad \text{...(1.18.15.2)}$$

Differentiating above equation we get,

$$p\,\Delta\lambda + \lambda\,\Delta p = 0$$

$$\text{or, } \quad \Delta p = \frac{-p\,\Delta\lambda}{\lambda} = \frac{-h\,\Delta\lambda}{\lambda^2} \quad \text{...(1.18.15.3)}$$

Substitute the value of Δp in Equation (1.18.15.1)

$$\Delta x \cdot \Delta\lambda \geq \frac{\lambda^2}{2\pi}$$

The uncertainty in wavelength is given to be in the ratio $1/10^6$, so the uncertainty in a wavelength λ is given by,

$$\frac{\Delta\lambda}{\lambda} = \frac{1}{10^6} = 10^{-6}$$

Substituting this value of $\Delta\lambda$ in Equation (1.18.15.3) we get,

$$\Delta x \cdot \lambda \cdot 10^{-6} \geq \frac{\lambda^2}{2\pi}$$

$$\text{or, } \Delta x = \frac{\lambda}{2\pi \times 10^{-6}}$$

$$\text{Given } \lambda = 1A° = 10^{-10} \text{ m.}$$

$$\Delta x = \frac{10^{-10}}{2 \times 3.14 \times 10^{-6}} = 1.59 \times 10^{-5} \text{ m}$$

Ex. 1.18.16 : (Dec. 06, 4 Marks)

An electron is confined to a box of length 2A°. Calculate the minimum uncertainty in its velocity. Given : Mass of electron 9.1×10^{-31} kg, $h = 6.63 \times 10^{-34}$ J-sec.

Soln. :

$$\Delta x \cdot Ap = h$$

$$(\Delta x)_{max} \cdot (\Delta p)_{min} = h$$

$$(\Delta x)_{max} \cdot m\,(\Delta v)_{min} = h$$

$$(\Delta v)_{min} = \frac{h}{m \cdot (\Delta x)_{max}} = \frac{6.63 \times 10^{-31}}{9.1 \times 10^{-31} \times 2 \times 10^{-10}} = 0.36 \times 10^{7} \text{ m/s}$$

Summary

Newton's

- The energy levels of the particle are still discrete but there are a finite number of them. Such a limit exists because, soon the particle energy becomes equal to V_0.
- For energies higher than this the particle energy is not quantised but may have any value above V_0.
- **These predictions are unique in quantum mechanics and shows different behaviour from that expected in classical physics.**

Revision of Important Formula

1. $E_n = \frac{n^2 h^2}{8 \, mL^2}$

Examples

Ex. 2.9.1 : (May 11, 4 Marks)

Find the lowest energy level and momentum of an electron in one dimensional potential well of width 1A°.

Soln. :

Given : $L = 1A^\circ = 10^{-10}$ m.

Formulae required : $E_n = \frac{n^2h^2}{8mL^2}$, $E = \frac{p^2}{2m}$

The energy of an electron in a potential well of width L is,

$$E_n = \frac{n^2h^2}{8mL^2}$$

The lowest energy level corresponds to n = 1. Hence it is E_1

$$\text{Hence,} \quad E_1 = \frac{h^2}{8mL^2} = \frac{(6.63 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (10^{-10})^2} = 6.038 \times 10^{-18} \text{ J}$$

$$\therefore \quad E_1 = \frac{6.038 \times 10^{-18} \text{ J}}{1.6 \times 10^{-19}} = 38 \text{ ev}, \; E_2 = 4 E_1 = 152 \text{ ev}$$

$$\text{and} \quad E_2 - E_1 = 114 \text{ ev}$$

The momentum p is given as per the relation.

$$E = \frac{p^2}{2m}$$

$$\therefore \quad P = \sqrt{2mE}$$

$$\therefore \quad P_1 = \sqrt{2mE_1}$$

$$\therefore \quad P_1 = \sqrt{2mE_1} = \sqrt{2 \times 9.1 \times 10^{-31} \times 6.038 \times 10^{-18}}$$

$$\therefore \quad P_1 = 33.045 \times 10^{-25} \text{ kg.m/s}$$

Ex. 2.9.2 : An electron is bound by a potential which closely approaches an infinite square well of width 2.5 A°. Calculate the lowest three permissible quantum energies the electron can have.

Soln. :

Given : $L = 2.5\ A^\circ = 2.5 \times 10^{-10}$ m

Formula required : $E_n = \dfrac{n^2 h^2}{8mL^2}$

The energy of an electron in an infinite potential well is given by,

$$E_n = \frac{n^2 h^2}{8mL^2}$$

Putting for m, h and L we have

$$E_n = \frac{n^2 (6.63 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (2.5 \times 10^{-10})^2}$$

$$= 9.63 \times 10^{-19} \times n^2\ J = 6n^2\ e\ V$$

∴ The lowest three permissible energy levels are given for n = 1, n = 2 and n = 3 as,

$$E_1 = 6\ eV.$$

$$E_2 = 6 \times 2^2 .\ eV = 24\ e\ V.$$

$$E_3 = 6 \times 3^2\ .eV = 54\ eV.$$

Ex. 2.9.3 : Consider a marble of mass 10 gm. in a one dimensional rigid box of width 10 cm. Using the expression for the energy Eigenvalues, find E_1, E_2, E_3, E_4. Comment on these values.

Soln. :

Given : $L = 10\ cm. = 10^{-1}\ m,\quad m = 10gm = 10^{-2}\ kg.$

Formula required : $E_n = \dfrac{n^2 h^2}{8mL^2}$

The energy values of a particle in the rigid box are given by,

$$E_n = \frac{n^2 h^2}{8mL^2}$$

Putting for m, L and h we have,

$$E_n = \frac{(6.63 \times 10^{-34})^2 \times n^2}{8 \times (10^{-2}) \times (10^{-1})^2} = 5.5 \times 10^{-64} \times n^2\ J$$

The values of E_1, E_2, E_3, E_4 can be obtained by putting n = 1, 2, 3, 4 respectively.

$$\therefore\ E_1 = 5.5 \times 10^{-64} \times 1^2 = 5.5 \times 10^{-64}\ J$$

$$E_2 = 5.5 \times 10^{-64} \times 2^2 = 22 \times 10^{-64}\ J$$

$$E_3 = 5.5 \times 10^{-64} \times 3^2 = 4.95 \times 10^{-63}\ J$$

$$E_4 = 5.5 \times 10^{-64} \times 4^2 = 8.8 \times 10^{-63}\ J$$

- The difference between the consecutive energy levels is very very small.
- Hence they cannot be identified as discrete and the energy spectrum can be treated almost continuous.
- Also the lowest permissible energy corresponds to a velocity which cannot practica distinguished from zero.
- Hence the lowest energy can be treated as zero.
- Such results differ from those for microscopic particles like electrons.

Ex. 2.9.4 : An electron is bound in a one dimensional potential box which has a wi 2.5×10^{-10} m. Assuming the height of the box to be infinite, calculate the lowest permitted energy values of the electron.

Soln. : We have, $E_n = \dfrac{n^2h^2}{8mL^2}$

Given : $h = 6.63 \times 10^{-34}$ J-s, $m = 9.1 \times 10^{-31}$ kg, $L = 2.5 \times 10^{-10}$ m

$$E_n = \frac{n^2(6.63 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (2.5 \times 10^{-10})^2}$$

$$= 9.66 \times 10^{-19}\, n^2 \text{Joule.}$$

$$= \frac{9.66 \times 10^{-19}}{1.6 \times 10^{-19}} = 6.04n^2 \text{eV.}$$

First lowest permitted energy level (n = 1) = 6.04 eV

Second lowest permitted energy level (n = 2) = $6.04 \times 2^2 = 24.16$ eV

Ex. 2.9.5 : (May 09, 4 Marks)

Compare the lowest three energy states for (i) an electron confined in an infinite potential well width 10A° and (ii) a grain of dust with mass 10^{-6} gm in an infinite potential well of width 0.1 m What can you conclude from this comparison ?

Soln. :

Given : L = 10A°

Formula required : $E_n = \dfrac{n^2 h^2}{8\, mL^2}$

The energy levels of the particle in an infinite potential well of width L is given by,

$$E_n = \frac{n^2 h^2}{8\, mL^2}$$

(i) For the electron, $m = 9.1 \times 10^{-31}$ kg

$$L = 10A° = 10^{-9} \text{ m}$$

$$\therefore E_n = \frac{n^2 \times (6.63 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (10^{-9})^2} \text{ J}$$

or $E_n = 0.377 \times n^2$ eV

The lowest three energy states for the electron are

$$E_1 = 0.377 \times (1)^2 = 0.377 \text{ eV}$$

$$E_2 = 0.377 \times (2)^2 = 1.508 \text{ eV}$$

$$E_3 = 0.377 \times (3)^2 = 3.393 \text{ eV}$$

(ii) For the grain of dust

$m = 10^{-6} \text{ gm} = 10^{-9} \text{ kg}$

$L = 0.1 \text{ mm} = 10^{-4} \text{ m}$

$$\therefore E_n = \frac{n^2 \times (6.63 \times 10^{-34})^2}{8 \times 10^{-9} \times (10^{-4})^2}$$

$$\therefore E_n = 5.493 \times 10^{-51} \times n^2 \text{ eV}$$

$$\text{or} \quad E_n = 3.433 \times 10^{-31} \times n^2 \text{ eV}$$

∴ The lowest three energy states for the grain of dust are

$$E_1 = 3.433 \times 10^{-31} \text{ eV}$$

$$E_2 = 3.433 \times 10^{-31} \times (2)^2 = 13.73 \times 10^{-31} \text{eV}$$

$$E_3 = 3.433 \times 10^{-31} \times (3)^2 = 30.89 \times 10^{-31} \text{eV}$$

From the energy values in cases (i) and (ii) we see that the energy levels are discrete for the electron. But for the grain of dust, the difference between the energy levels being very small, they cannot be identified as discrete and can be treated as almost continuous. Hence quantization of energy is observable for the microscopic particles only.

Ex. 2.9.6 : (May 05, 4 Marks)

Lowest energy of an electron trapped in a potential well is 38 eV. Calculate the width of the well.

Soln. :

Given : $E_1 = 38$ eV

Formula required : $E_n = \dfrac{n^2 h^2}{8mL^2}$

We have lowest energy of a electron.

$$E_n = \frac{n^2 h^2}{8 \, mL^2}$$

For lowest energy

$$n = 1$$

$$E_1 = \frac{h^2}{8mL^2}$$

$$L^2 = \frac{h^2}{8mE_1}$$

$$L^2 = \frac{(6.63 \times 10^{-38})^2}{8 \times 9.1 \times 10^{-31} \times 38 \times 1.6 \times 10^{-19}}$$

$$L^2 = 9.93 \times 10^{-21}$$

$$L = 0.996A° \approx 1A°$$

Ex. 2.9.7 : Calculate the following :

I. de Brogile wavelengths of :

(i) 150 (gm) baseball moving with 35 (m/s),

(ii) electron accelerated through 120 (V);

II. Uncertainty in position of :

(i) a 50 (gm) bullet with velocity 30 (m/s) accurate to 0.01%

(ii) an electron with velocity 30 (m/s) accurate to 0.01%.

III. First three energy levels for :

(i) a 1 micro gram dust particle moving in a one-dimension box of width 0.1

(ii) an electron confined to 10 Å .

Soln. :

(1) (i) De- Broglie wavelength for base ball :

Formulae : $$\lambda = \frac{h}{m\,v}$$

Given : $$m = 15 \text{ gm} = 0.15 \text{ kg.}$$

$$v = 35 \text{ m/s}$$

$$\lambda = \frac{6.64 \times 10^{-34}}{0.15 \times 35} = 1.264 \times 10^{-34} \text{ m.}$$

(ii) De- Broglie wavelength for electron :

Formulae : $$\lambda = \frac{12.26}{\sqrt{V}} A°$$

Given : $$V = 120 \text{ V}$$

$$\lambda = \frac{12.26}{\sqrt{120}} A° = 1.1191 A°$$

(2) (i) Uncertainty in bullet position :

Formulae : $$\Delta x = \frac{h}{m \cdot A\,v}$$

Given : $$\frac{\Delta v}{v} \times 100 = 0.01, \qquad v = 30 \text{ m/s}$$

$$\Delta v = \frac{6.64 \times 10^{-34}}{0.050 \times 0.003} = 4.42 \times 10^{-30} \text{ m}$$

(ii) **Uncertainty in electron position :**

$$\Delta x = \frac{h}{m \cdot \Delta v} = \frac{6.64 \times 10^{-34}}{9.1 \times 10^{-31} \times 0.003} = 0.2432 \text{ m}$$

(i) **Energy level for dust particle :**

Formulae :

$$E_n = \frac{n^2 h^2}{8 m L^2}$$

$$m = 1 \times 10^{-6} \text{ gm} = 1.0 \times 10^{-9} \text{ kg.}$$

$$L = 0.1 \text{ mm} = 10^{-4} \text{ m, for first level } n = 1$$

$$E_1 = \frac{1^2 \times (6.64 \times 10^{-34})^2}{8 \times 10^{-9} \times (10^{-4})^2} = 5.51 \times 10^{-51} \text{ J}$$

$$E_2 = 22.04 \times 10^{-51} \text{ J}$$

$$E_3 = 49.6 \times 10^{-51} \text{ J}$$

(ii) **Energy level for electron :**

Formulae :

$$E_n = \frac{n^2 h^2}{8 m L^2}$$

$$L = 10 \text{ A}^\circ = 10 \times 10^{-10} \text{ m}$$

for first level $n = 1$

$$E_1 = \frac{1^2 \times (6.64 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (10^{-9})^2} = 0.378 \text{ eV} = 0.38 \text{ eV}$$

$$E_2 = 1.52 \text{ eV}$$

$$E_3 = 3.42 \text{ eV}$$

Ex. 2.9.8 : A electron in confined to move between two rigid walls separated by 1 nm. Find the de-broglie wavelength representing the first two allowed energy states of the electron and the corresponding energies.

Soln. :

Formula : $E_n = \dfrac{n^2 h^2}{8 m L^2}$,

Given : $L = 1 \times 10^{-9}$ m

For first allowed state $n = 1$

$$E_1 = \frac{1^2 \times (6.63 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (10^{-9})^2} = 6.03 \times 10^{-20} \text{ J}$$

$$E_2 = 4 \times 6.03 \times 10^{-20} = 2.415 \times 10^{-19} \text{ J}$$

De-Broglie wavelength for first level.

$$\lambda_1 = \frac{h}{\sqrt{2\,m\,E_1}} = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 6.03 \times 10^{-20}}} = 2.0 \times 10^{-9}\ m$$

$$\text{So, } \lambda_2 = \frac{h}{\sqrt{2\,m\,E_2}} = \frac{\lambda_1}{2} = 1.0 \times 10^{-19}\ m$$

Ex. 2.9.9 : A particle is moving in one dimensional potential box of infinite height of wi... 25 Å, calculate the probability of finding the particle within an interval of 5 Å at centers of the box when it is in its state of least energy.

Soln. :

The wave function ψ of a particle moving in an infinite potential well is given by,

$$\psi_n(x) = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L},$$

where L is the width of the well

For lowest energy state, n = 1

So, $$\psi(x) = \sqrt{\frac{2}{L}} \cdot \sin \frac{\pi x}{L}$$

At the center of box, $$x = \frac{L}{2}$$

So, the probability of finding the particle at the center of the box is given by

$$|\psi(x)|^2 = \psi(x) \cdot \psi^*(x) = \left| \sqrt{\frac{2}{L}} \cdot \sin \frac{\pi \cdot \frac{L}{2}}{L} \right|^2$$

$$= \frac{2}{L} \cdot \sin^2 \frac{\pi}{2} = \frac{2}{L}$$

Probability in the interval Δx is given by

$$P = |\psi(x)|^2 \cdot \Delta x = \frac{2}{L} \cdot \Delta x$$

$$L = 25 \times 10^{-10}\ m, \qquad \Delta x = 5 \times 10^{-10}\ m$$

$$\therefore \quad P = \frac{2 \times 5 \times 10^{-10}}{25 \times 10^{-10}} = 0.4$$

Ex. 2.9.10 : A particle is in motion along a line between x = 0 and x = a with zero potential energy. At points for which x < 0 and x > a, the potential energy is infinite. The wave function for the particle the n^{th} state is given by

$$\psi_n = A \sin \frac{n\pi x}{a}$$

Find the expression for the normalized wave function.

Soln. :

The wave function of the particle in the n^{th} state is given by

$$\psi_n = A \sin \frac{n\pi x}{a}$$

Where a is the length of the line along which particle is moving.

To find the value of A we apply condition of normalization as

$$\int_{-\infty}^{+\infty} |\psi_n(x)|^2 \, dx = 1$$

or,
$$\int_{-\infty}^{+\infty} \left| A \sin\left(\frac{n\pi x}{a}\right) \right|^2 dx = 1,$$
or,
$$A^2 \int_0^a \sin^2 \frac{n\pi x}{a} \, dx = 1$$

or,
$$\frac{A^2}{2} \int_0^a \left[1 - \cos 2\left(\frac{\pi n x}{a}\right)\right] dx = 1$$

or,
$$\frac{A^2}{2} \left[x - \frac{\sin(2\pi n x / a)}{(2\pi n x / a)}\right]_0^a = 1$$

This gives,

$$\frac{A^2}{2}(a) = 1$$

or,
$$A = \sqrt{\frac{2}{a}}$$

Therefore, the normalized wave function of the particle is

$$\psi_n = \sqrt{\frac{2}{a}} \cdot \sin\left(\frac{n\pi x}{a}\right)$$

Ex. 2.9.11 : Find the probabilities of finding a particle trapped in a box of length L in the region from 0.45 L to 0.55 L for the ground state and the first excited state.

Soln. :

The eigen functions of a particle trapped in a box of length L are :

$$\psi_n = \sqrt{\frac{2}{L}} \cdot \sin \frac{n\pi x}{L}$$

The probability of finding the particle between x_1 and x_2, when it is in the n^{th} state, is

$$P = \int_{x_1}^{x_2} |\psi_n|^2 \, dx = \frac{2}{L} \int_{x_1}^{x_2} \sin^2\left(\frac{n\pi x}{L}\right) dx$$

$$P = \frac{2}{L}\int_{x_1}^{x_2} \frac{1}{2}\left(1 - \cos\frac{2\pi nx}{L}\right) dx$$

$$P = \frac{1}{L}\left[x - \frac{L}{2\pi n}\sin\frac{2\pi nx}{L}\right]_{x_1}^{x_2}$$

Here $x_1 = 0.45L$, $x_2 = 0.55L$ and for ground state, $n = 1$

$$\therefore \quad P = \frac{1}{L}\left[\left(0.55L - \frac{L}{2\pi}\sin(1.1\,\pi)\right) - \left(0.45\,L - \frac{L}{2\pi}\sin(0.90\,\pi)\right)\right]$$

$$P = \left[\left(0.55 - \frac{1}{2\pi}\sin 198°\right) - \left(0.45 - \frac{1}{2\pi}\sin 162°\right)\right]$$

$$P = (0.55 - 0.45) - \frac{1}{2\pi}(\sin 198° - \sin 162°)$$

$$P = 0.10 - \frac{1}{2\pi}(2\cos 180° \sin 18°)$$

$$\ldots\left[\because \sin A - \sin B = 2\cos\frac{A+B}{2}\cdot\sin\frac{A-B}{2}\right]$$

$$P = 0.10 + \frac{0.3090}{3.14}$$

$$P = 19.8\%$$

Similarly, for the first excited state ($n = 2$) repeat the above process and get $P = 0.65\%$.

Ex. 2.9.12 : (Dec. 08, 4 Marks)

☞ Calculate first two energy eigen values of an electron in eV which is confined to a box of length 2 A. U.

Soln. :

Given : $L = 2A° = 2 \times 10^{-10}$ m.

Formula : $E_n = \frac{n^2h^2}{8mL^2}$

$$E_n = \frac{n^2 (6.63 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (2 \times 10^{-10})^2} = 1.50 \times 10^{-18} n^2 J = 9.43\, n^2\, eV$$

First two given values are,

$$E_1 = 9.43 eV$$

$$E_2 = 9.43 \times 4\ eV = 37.72\ eV$$

Ex. 2.9.13 : (Dec. 05, 4 Marks)

☞ An electron is bound by a potential which closely approaches an infinite square well of width 1 A°. Calculate the lowest three permissible energies (in electron volts) the electron can have.

Soln. :

Formula $E = \frac{n^2 h^2}{8m L^2}$, Given $L = 1\ A° = 10^{-10}$ m

$$= \frac{(6.63 \times 10^{-34})^2 \times n^2}{8 \times 9.1 \times 10^{-31} \times (10^{-10})^2\ J} = \frac{0.6037 \times 10^{-17}\ n^2}{1.6 \times 10^{-19}}\ eV$$

The lowest three permissible energies of electron correspond to n = 1,2, 3 respectively.

$$E_1 = 38 \times (1)^2\ eV = 38\ eV$$

$$E_2 = 38 \times (2)^2\ eV = 152\ eV$$

$$E_3 = 38 \times (3)^2\ eV = 342\ eV$$

Ex. 2.9.14 : (May 06, 4 Marks)

An electron is trapped in a rigid box of width 2A°. Find its lowest energy level and momentum. Hence find energy of the 3rd energy level.

Soln. :

Formulae :

$$E_n = \frac{n^2 h^2}{8m L^2}$$

For lowest energy level $n = 1$

$$L = 2\ A° = 2 \times 10^{-10}\ m$$

$$E_1 = \frac{(6.63 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (2 \times 10^{-10})^2}$$

$$= \frac{43.95 \times 10^{-68}}{291 \times 10^{-51}} = 1.5105 \times 10^{-18}\ J$$

$$= \frac{15.105 \times 10^{-19}}{1.6 \times 10^{-19}}\ eV = 9.44\ eV$$

$$F_1 = 9.5\ eV$$

$$P_1 = \sqrt{2\ m\ E_1} = \sqrt{2 \times 9.1 \times 10^{-31} \times 9.5 \times 1.6 \times 10^{-19}}$$

$$= 1.663 \times 10^{-24}\ kg\text{-}m/s$$

and $E_3 = 9 \times 9.5\ eV = 85.5\ eV$ (as n = 3)

Ex. 2.9.15 : (Dec. 11, 5 Marks)

Assuming Atomic Nucleus to be a rigid box (infinite potential well), calculate the ground state energy of an electron if it existed inside the nucleus.

(Given Planck's Constant = 6.63×10^{-34} j – s, Mass of the Electron = 9.1×10^{-31} kg. and size of the nucleus ~ 10^{-15} m. Using this result, argue that electron cannot exist inside the nucleus. Given, maximum binding energy per nucleon = 8.8 Me V)

Soln. :

Formula $E_n = \dfrac{n^2 h^2}{8\, m\, L^2}$

Data given $h = 6.63 \times 10^{-34}$ J–s

$m_e = 9.1 \times 10^{-31}$ kg.

$L = 10^{-15}$ m.

So, $E_1 = \dfrac{1^2 \times (6.63 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (10^{-15})^2}$

$= 6.038 \times 10^{-8}$ J/1.6×10^{-13} meu $= 3.77 \times 10^5$ meu

If an electron exists inside the nucleus, then its ground state energy $= 3.7 \times 10^5$ meu, but maximum binding energy per nucleon = 8.8 meu, so, electron can not exist inside the nucleus.

Summary