Spectral Element Method

Abstract

Spectral Element Method (SEM) has emerged as a powerful numerical technique for solving partial differential equations (PDEs) by combining the accuracy of spectral methods with the flexibility of finite element methods. It offers high-order accuracy, geometric flexibility, and excellent resolution of boundary layers and discontinuities. It has application in various fields, including computational fluid dynamics, structural mechanics, electromagnetics, and geophysics. The method is particularly known for it's higher order spectral basis functions which are employed in each element to approximate the solution resulting in rapid convergence. The most important property of the SEM is that the mass matrix is exactly diagonal by construction, which drastically simplifies the implementation and reduces the computational cost because one can use an explicit time integration scheme without having to invert a linear system.

This study aims to provide an overview of the Spectral Element Method such as method outline, its comparison with traditional Finite Element Methods (FEM), advantages, and applications in various domains such as fluid dynamics, computational sciences, seismology, etc.

References:

1. Patera, A.T. (1984). A spectral element method for fluid dynamics: Laminar flow in a channel expansion. *Journal of Computational Physics*, 54(3), pp.468–488. doi:https://doi.org/10.1016/0021-9991(84)90128-1.
2. Karniadakis, G. and Sherwin, S.J. (1999). *Spectral/hp element methods for CFD*. New York: Oxford University Press.
3. Kopriva, D.A. and Springerlink (Online Service (2009). *Implementing Spectral Methods for Partial Differential Equations : Algorithms for Scientists and Engineers*. Dordrecht: Springer Netherlands.
4. Xu, H., Cantwell, C.D., Monteserin, C., Eskilsson, C., Engsig-Karup, A.P. and Sherwin, S.J. (2018). Spectral/hp element methods: Recent developments, applications, and perspectives. *Journal of Hydrodynamics*, 30(1), pp.1–22. doi:https://doi.org/10.1007/s42241-018-0001-1.
5. Muhammad Bilal Hafeez and Krawczuk, M. (2023). A Review: Applications of the Spectral Finite Element Method. *Archives of Computational Methods in Engineering*. doi:https://doi.org/10.1007/s11831-023-09911-2.

6. Dimitri Komatitsch and Tromp, J. (1999). Introduction to the spectral element method for three-dimensional seismic wave propagation. *Geophysical Journal International*, 139(3), pp.806–822. doi:https://doi.org/10.1046/j.1365-246x.1999.00967.x.
7. C. Meza-Fajardo, K. and S. Papageorgiou, A. The Spectral Element Method (SEM): Formulation and Implementation of the method for Engineering Seismology Problems. The 14th World Conference on Earthquake Engineering.

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