



Strictly as per the REV- 2019'C' Scheme of MUMBAI University
w.e.f. academic year 2019-2020

ENGINEERING PHYSICS - I

(Code : FEC102)

First Year Engineering
Semester I - Common to all Branches

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Engineering Physics - I

(Code : FEC 102)

First Year Engineering

**Semester I – Common to All Branches
(Mumbai University)**

**Strictly as per the Revised course (REV- 2019 'C' Scheme)
of Mumbai University w.e.f. Academic Year 2019 – 2020**

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Engineering Physics - I (Code : FEC 102)

Dr. I. A. Shaikh

(Semester I – Common to All Branches) (Mumbai University)

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[FEC 102] (FID : MO1) (Book Code : MO1B)

(Book Code : MO1B)

*We dedicate this Publication soulfully and wholeheartedly,
in loving memory of our beloved founder director
Late. Shri. Pradeepsheth Lalchandji Lunawat,
who will always be an inspiration, a positive force and strong support
behind us.*



Lt. Shri. Pradeepji L. Lunawat

*Soulful Tribute and Gratitude for all Your
Sacrifices, Hardwork and 40 years of Strong Vision.....*

Preface

Dear Students,

I am extremely happy to come out with this book on "Engineering Physics - I" for you. The topics within the chapters have been arranged in a proper sequence to ensure smooth flow of the subject.

I present this book in the loving memory of Late Shri. Pradeepji Lunawat, our source of inspiration and a strong foundation of "TechKnowledge Publications". He will always be remembered in our heart and motivate us to achieve our milestone.

I am thankful to Shri. J. S. Katre, Mr. Shital Bhandari, Mr. Arunoday Kumar and Mr. Chandroday Kumar for the encouragement and support that they have extended.

I am also thankful to Seema Lunawat for technology enhanced reading, E-books support and the staff members of TechKnowledge Publications for their efforts to make this book as good as it is. I have made every possible efforts to eliminate all the errors in this book. However if you find any, please let me know, because that will help me to improve further.

I am also thankful to my family members and friends for patience and encouragement.

- Author

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Syllabus

Course Code	Course Name	Teaching Scheme (Contact Hours)			Credits Assigned							
		Theory	Pract.	Tut.	Theor	Tut.	Pract.	Total				
FEC102	Engineering Physics-I	2	-	-	2	-	-	2				
Course Code	Course Name	Examination Scheme										
		Theory			Term Work	Pract. /oral	Total					
		Internal Assessment		End Sem. Exam.								
		Test 1	Test 2	Avg.								
FEC102	Engineering Physics-I	15	15	15	60	2	--	75				

Rationale :

Most of the engineering branches are being off-spring of basic sciences where physics is playing a pivotal role in concept and understanding of foundation of core engineering branches. This syllabus is developed by keeping in mind, needs of all branches that we offer in University of Mumbai.

The topic distribution is being done in systematic manner and whenever required, prerequisite to the topic are mentioned for frictionless teaching-learning process. In the distribution of topics, core physics and its applied form are given priority. At the same time few modules are introduced over emerging trends in the field of technology.

For the purpose of emphasis on applied part, list of suggestive experiments is added. As per new guidelines of AICTE, a scope is kept in the syllabus for simulation technique and use of information technology to supplement laboratory practices. Further, it is ensured that these modules will cover prerequisites needed for engineering courses to be introduced in higher semesters as core subjects or as interdisciplinary subjects in respective branches.

Objectives :

1. To understand basic physics concepts and founding principles of technology.
2. To develop scientific temperament for scientific observations, recording, and inference drawing essential for technology studies.

Outcomes: Learners will be able to...

1. Illustrate the fundamentals of quantum mechanics and its application.
2. Explain peculiar properties of crystal structure and apply them in crystallography using X-ray diffraction techniques.
3. Comprehend the concepts of semiconductor physics and applications of semiconductors in electronic devices.
4. Employ the concept of interference in thin films in measurements.
5. Discuss the properties of Superconductors and Supercapacitors to apply them in novel applications.
6. Compare the properties of engineering materials for their current and futuristic frontier applications.

Module	Detailed Contents	Hrs.
01	<p>Quantum Physics</p> <p>(Prerequisites : Dual nature of radiation, Photoelectric effect Matter waves-wave nature of particles, de-Broglie relation, Davisson-Germer experiment) De Broglie hypothesis of matter waves; properties of matter waves; wave packet, phase velocity and group velocity; Wave function; Physical interpretation of wave function; Heisenberg uncertainty principle; non existence of electron in nucleus; Schrodinger's time dependent wave equation; time Independent wave equation; Particle trapped in one dimensional infinite potential well, Quantum Computing.</p> <p style="text-align: right;">(Refer chapter - 1)</p>	07
02	<p>Crystallography</p> <p>(Prerequisites : Crystal Physics (Unit cell, Space lattice, Crystal structure, Simple Cubic, Body Centered Cubic, Face Centered Cubic, Diamond Structure, Production of X-rays) Miller indices; interplanar spacing; X-ray diffraction and Bragg's law; Determination of Crystal structure using Bragg's diffractometer ;</p> <p style="text-align: right;">(Refer chapter - 2)</p>	07
03	<p>Semiconductor Physics</p> <p>(Prerequisites : Intrinsic and extrinsic semiconductors, Energy bands in conductors, semiconductors and insulators, Semiconductor diode, I-V characteristics in forward and reverse bias)</p> <p>Direct & indirect band gap semiconductor; Fermi level; Fermi dirac distribution; Fermi energy level in intrinsic & extrinsic semiconductors; effect of impurity concentration and temperature on fermi level; mobility, current density; Hall Effect; Fermi Level diagram for p-n junction (unbiased, forward bias, reverse bias);</p> <p>Applications of semiconductors: LED, Zener diode, Photovoltaic cell.</p> <p style="text-align: right;">(Refer chapter - 3)</p>	07
04	<p>Interference in Thin Film</p> <p>(Prerequisites : Wave front and Huygen's principle, reflection and refraction, Interference by division of wave front, Youngs double slit experiment)</p> <p>Interference by division of amplitude, Interference in thin film of constant thickness due to reflected and transmitted light; origin of colours in thin film; Wedge shaped film; Newton's rings.</p> <p>Applications of Interference : Determination of thickness of very thin wire or foil; determination of refractive index of liquid; wavelength of incident light; radius of curvature of lens; testing of surface flatness; Anti-reflecting films and Highly reflecting film.</p> <p style="text-align: right;">(Refer chapter - 4)</p>	07
05	<p>Superconductors And Supercapacitors</p> <p>(Prerequisites : Electric current, flow of electric charges in a metallic conductor, drift velocity, mobility and their relation with electric current, Ohm's law, electrical resistance, V-I characteristics (linear and non-linear), electrical resistivity and conductivity temperature dependence of resistance)</p> <p>Superconductors : Critical temperature, critical magnetic field, Meissner's effect, Type I and Type II and high T_c superconductors;</p> <p>Supercapacitors : Principle, construction, materials and applications, comparison with capacitor and batteries : Energy density, Power density,</p> <p style="text-align: right;">(Refer chapter - 5)</p>	07
06	<p>Engineering Materials and Applications</p> <p>(Prerequisites : Paramagnetic materials, diamagnetic materials, ferromagnetic materials, crystal physics, Conductors and insulators, free charges and bound charges inside a conductor. Dielectrics and electric polarisation, capacitors and capacitance)</p> <p>Liquid crystals : Nematic, Smectic and cholesteric phases, Liquid crystal display.</p> <p>Multiferroics : Type I & Type II multiferroics and applications,</p> <p>Magnetoresistive Oxides : Magnetoresistance, GMR and CMR materials, introduction to spintronics.</p> <p style="text-align: right;">(Refer chapter - 6)</p>	07

**Unit I****Chapter 1 : Quantum Physics**

1-1 to 1-35

Syllabus : (Prerequisites : Dual nature of radiation, Photoelectric effect, Matter waves, wave nature of particles, de-Broglie relation, Davisson-Germer experiment) de Broglie hypothesis of matter waves; properties of matter waves; wave packet, phase velocity and group velocity; Wave function; Physical Interpretation of wave function; Heisenberg uncertainty principle; non existence of electron in nucleus; Schrodinger's time dependent wave equation; time independent wave equation; Particle trapped in one dimensional infinite potential well, Quantum Computing.

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Unit III

Chapter 3 : Semiconductor Physics

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Direct & indirect band gap semiconductor; Fermi level; Fermi-Dirac distribution; Fermi energy level in intrinsic & extrinsic semiconductors; effect of impurity concentration and temperature on Fermi level; mobility, current density; Hall effect; Fermi Level diagram for p-n junction (unbiased, forward bias, reverse bias); applications of semiconductors: LED, Zener diode and Photovoltaic cell

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Syllabus : (Prerequisites : Wavefront and Huygens' principle, reflection and refraction, interference by division of wavefront, Young's double slit experiment) Interference by division of amplitude, Interference in thin film of constant thickness due to reflected and transmitted light, origin of colours in thin film, wedge-shaped film, Newton's rings Applications of interference - determination of thickness of very thin wire or foil, determination of refractive index of liquid, wavelength of incident light, radius of curvature of lens, testing of surface flatness, anti-reflecting films and highly reflecting film

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Superconductors : Critical temperature, critical magnetic field, Meissner effect, Type I and Type II and high T_c superconductors

Supercapacitors : Principle, construction, materials and applications, comparison with capacitor and batteries, energy density, power density



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Unit VI**Chapter 6 : Engineering Materials and Applications**

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Syllabus : (Prerequisites : Paramagnetic materials, diamagnetic materials, ferromagnetic materials, crystal physics, conductors and insulators, free charges and bound charges inside a conductor, dielectric and electric polarization, capacitors and capacitance) Liquid Crystals : Nematic, smectic and cholesteric phases, liquid crystal displays Multiferroic : Type I and Type II multiferroics and application Magnetoresistive oxides : Magnetoresistance, GMR and CMR materials, introduction to spintronics	
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Quantum Physics

Syllabus

(Prerequisites : Dual nature of radiation, photoelectric effect, matter waves, wave nature of particles, de-Broglie relation, Davisson-Germer experiment), de Broglie hypothesis of matter waves, properties of matter waves, wave packet, phase velocity and group velocity, wave function, physical interpretation of wave function, Heisenberg uncertainty principle, non-existence of electron in nucleus, Schrodinger's time-dependent wave equation, time-independent wave equation, particle trapped in one-dimensional infinite potential well, quantum computing

Learning Objectives :

After reading this chapter, learner should be able to

- Understand de-Broglie's hypothesis and properties of matter waves
- Describe wave function and its physical interpretation
- Understand uncertainty principle and its applications
- Derive Schrodinger's wave equation time-dependent and independent form
- Apply Schrodinger's equation to one-dimensional infinite potential well

1.1 Introduction

The topic which is discussed here is highly versatile and stores a new concept. It requires a completely new approach; therefore, we must take its introduction in detail. Let us answer the following questions :

- (a) How has the topic of mechanics evolved ?
- (b) What is the need of new mechanics ?
- (c) What forms quantum mechanics ?
- (d) What is the difference between classical mechanics and quantum mechanics ?

(a) Evolution of mechanics

- To study the motion of bodies, efforts were taken by many people starting from Galileo, Blaise Pascal up to Sir Isaac Newton.
- In *Principia Mathematica* which was published in 1687, he gave a unified theory, which accounted for all types of motions of bodies on common ground by consolidating all the ideas of previous workers in addition to his own. This was the birth of the subject Mechanics.
- By the end of the nineteenth century, a general feeling prevailed in the scientific community that knowledge regarding the subject of mechanics is complete and whatever remains to be done is only the refinement of the known ones.

**(b) Need of new mechanics**

- Towards the end of nineteenth century, the study of energy distribution in the spectrum of black body radiation came up, and Newtonian mechanics could not explain it. This was the point at which a need for new mechanics was felt.

(c) Formulation of quantum mechanics

- Towards the end of year 1900, Max Planck explained the spectrum of black body radiation by taking a completely new way i.e. he used the idea of "quanta". Hence, from year 1901, quantum mechanics came into picture, and everything before the year 1900 was labelled as classical mechanics. Bohr's theory for hydrogen atom provided a brilliant example for quantization aspects.
- In year 1924, Louis de Broglie suggested "wave-particle duality" which formed the basis for Schrodinger's work. The much-needed mechanics for the atomic and non-atomic world was shaped by the pioneering work of Schrodinger, Heisenberg, Dirac, et al. and called quantum mechanics.

(d) Difference between classical mechanics and quantum mechanics

- Both of them are fundamentally different approaches to solve problems. In classical mechanics, it is unconditionally accepted that position, mass, velocity, acceleration, etc. of a particle or a body can be measured accurately, which of course is true in our day-to-day observations.
- The explanation provided by the classical mechanics for the behaviour of a body studied in a problem of dynamics is fully valid in our usual observations. Classical mechanics predicts those magnitudes related to mass, position, etc. which agree fully with measured values.
- In contrast to the above, what quantum mechanics speaks is purely probabilistic in nature. As per the fundamental assumption of quantum mechanics, it is impossible to measure simultaneously the position and momentum of a given body. Hence, language of probability is most useful.
- For example, classical mechanics states that in the hydrogen atom, first orbit is $5.3 \times 10^{-11} \text{ m}$ whereas quantum mechanics identifies it as the most probable value of the radius one would get. If a suitable experiment is performed to measure the radius, a number of different values of radius are going to be obtained. But most of the times the values will be very close to $5.3 \times 10^{-11} \text{ m}$.
- A reader may feel that due to such uncertainties, classical mechanics is better than quantum mechanics but one must also accept that classical mechanics is simply an approximate version of quantum mechanics.
- In day-to-day life, accurate results obtained through classical mechanics are found to be true whereas in terms of quantum mechanical calculations, the departure of the observed values from the most probable value becomes totally insignificant. But once we go to atomic or subatomic level calculations, the probabilities involved in the values of various physical quantities become significant and classical mechanics fails completely.

1.2 de-Broglie Hypothesis

MU- May 19

Q. Explain de-Broglie Hypothesis of matter waves and deduce the expression for λ .**(May 19, 5 Marks)**

Although the idea of quantization of energy became indisputable, it took some time to understand its origin. The origin of quantization of energy lies in the dual behavior observed in particle and wave nature.

1.2.1 de-Broglie Wavelength

- Consider a wave of frequency ν

$$\therefore E = h\nu$$

- It can also be represented as

$$E = mc^2$$

$$\therefore h\nu = mc^2 \quad \dots(1.2.1)$$

- Now, p = Momentum associated with photon which travel in free space.

$$p = mc = \frac{mc^2}{c} = \frac{h\nu}{c} = \frac{h}{\lambda}$$

$$\therefore \lambda = \frac{h}{p} \quad \dots(1.2.2)$$

This derivation is very simple. A more rigorous derivation of de-Broglie wavelength is done on the basis of group velocity concept.

1.3 Properties of Matter Waves

MU - May 15, Dec.18

Q. What are the properties of matter waves ?

(May 15, Dec.18, 3 Marks)

The wavelength of matter waves is given by $\lambda = \frac{h}{mv}$.

Where m = mass of the particle

v = velocity of the particle

The properties of matter waves are :

(1) $\lambda \propto \frac{1}{m}$, hence lighter the particle greater is the wavelength associated with it.

(2) $\lambda \propto \frac{1}{v}$, hence greater the velocity of the particle, smaller is the associated wavelength.

(3) As $v \rightarrow \infty$, $\lambda \rightarrow 0$. But the wave becomes indeterminate when $v \rightarrow 0$ and $\lambda \rightarrow \infty$. This shows that matter waves are associated with the particles in motion. These waves do not depend on the charge of the particle, and hence matter waves are not electromagnetic in nature. The waves are not mechanical in nature.

(4) The velocity of matter waves is not a constant like electromagnetic waves but depends on the velocity of the particle generating them.

$v_p = \frac{c^2}{v}$, hence matter waves travel faster than light. ($\because v < c$)

(5) The wave velocity of matter waves depends inversely on the wavelength λ . This is the basic difference between matter waves and light waves. (Light waves have same velocity for all wavelengths).



- (6) Matter wave representation is a symbolic representation. It is a wave of probability indicating the likelihood of locating the particle.
- (7) The wave and particle duality of matter is not exhibited simultaneously.

Dissimilarities and similarities between matter wave and electromagnetic wave :

- (1) Matter waves are produced whether the particle is charged or uncharged. But electromagnetic waves are produced due to motion of charged particles only.
- (2) Velocity of matter waves is always greater than the speed of light, whereas velocity of electromagnetic waves is equal to the speed of light.
- (3) Both the waves are capable of propagation in an absolute vacuum. So they are not mechanical waves as mechanical waves cannot travel in a vacuum.

1.4 Wave Packet, Group Velocity and Phase Velocity

MU - Dec. 16 , Dec.18

Q. What do you mean by group and phase velocity? Show that the de-Broglie group velocity associated with the wave packet is equal to the velocity of the particle.

(Dec. 16, Dec.18, 5 Marks)

1.4.1 Wave Packet

- As per de Broglie's postulate, a material particle of mass m moving with velocity v is represented by a monochromatic wave of wavelength λ .
- Such a single wave representation of the particle raises some questions.
 - (i) How can a wave that spreads out over a large region of space represent a particle which is highly localized?
 - (ii) If we associate the wave with the particle, what exactly is the thing that is waving in the matter wave?
- A simple harmonic progressive wave is represented by an equation of the type.

$$y = A \sin (\omega t - kx) \quad \dots(1.4.1)$$

where, $y \rightarrow$ Displacement of the particle of the medium at time t and position x .

$\omega \rightarrow$ Angular frequency of wave

$k \rightarrow$ Propagation constant

$A \rightarrow$ Amplitude of the wave.

- The amplitude of the de Broglie waves representing a moving particle determines the probability of finding the particle at a particular place at a particular time.
- A wave of the type represented by Equation (1.4.1) is of infinite extent and is completely non-localised. Hence, a single wave cannot represent a particle which is confined to a very small volume of space. Hence, it was suggested that instead of a single wave, a combination of several waves may represent the particle.
- Superposition of waves of slightly different frequencies results in the formation of a hump at a definite place on the envelope of a smooth wave by mutual interference of the waves. This is as shown in Fig. 1.4.1.

- This hump or envelope of waves is called the **wave packet** or the **wave group** and can be made use of as a mark on the wave.
- The propagation of this distinguishing mark with time can be detected by measuring devices.

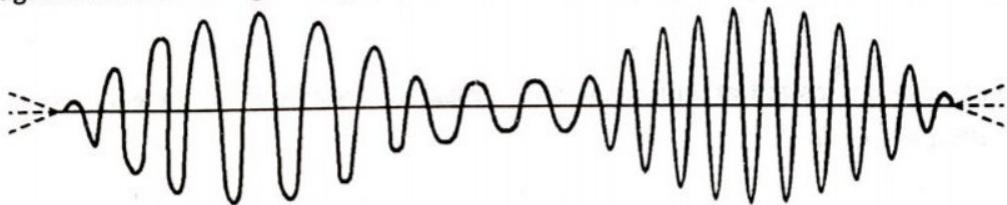


Fig. 1.4.1 : Superposition of waves

- If the number of superposed waves is increased, the hump becomes narrower and the intervening region of weaker disturbances broader.
- In the limit, if infinite number of waves of continuously varying wavelengths (or frequencies) extending over a finite range is superposed, a single hump in a narrow region results with no disturbance at any other point. This forms the wave packet. Fig. 1.4.2 shows a typical wave packet or wave group.

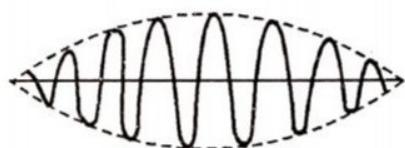


Fig. 1.4.2 : Wave packet

- Superposition of a number of waves with slightly different frequencies gives rise to a wave packet or a wave group.
- Such a wave packet possesses properties of particles as well as waves. The regular separation between successive maxima in the wave packet characterises a wave while at the same time it has particle characteristics i.e. localization in space.

1.4.2 Phase Velocity

- It is assumed here that the reader is aware about travelling wave expressed by

$$y = A \sin \omega t \quad \dots(1.4.2)$$

- To understand it in a proper way, let's consider a spring performing SHM on y axis where y is represented by Equation (1.4.2).
- Let the spring itself travel at velocity v along x-axis and let it cover distance x along x-axis in time t so that a mass suspended by spring is having same displacement y.

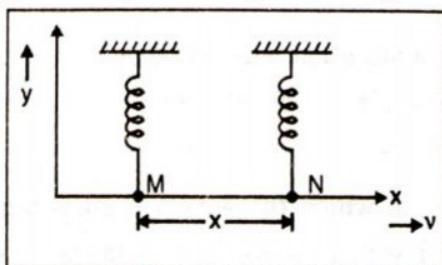


Fig. 1.4.3 : Idea of phase velocity

- Now at $t = t$, displacement on y-axis when mass is at point M is given by,

$$y = A \sin \omega t$$

- As the motion is periodic, mass will have the same displacement at N which it had at M at a time, say, t_0 earlier.
- Corresponding to the instant at which, the mass crosses the x-axis at M, the equation for displacement can be written as

$$y = A \sin (\omega (t - t_0)) \quad \dots(1.4.3)$$

as distance between M and N is x and time taken is t_0 .

$$\therefore t_0 = \frac{x}{u} \quad \dots(1.4.4)$$

$$\therefore y = A \sin \left[\omega \left(t - \frac{x}{u} \right) \right]$$

$$= A \sin \left[\omega t - \left(\frac{\omega}{u} \right) x \right]$$

$$= A \sin [\omega t - kx] \quad \dots(1.4.5)$$

where, $k = \frac{\omega}{u}$ = Wave number

$$\therefore u = \frac{\omega}{k}$$

OR

$$\frac{d}{dt} (\omega t - kx) = 0$$

$$\omega - k \frac{dx}{dt} = 0$$

$$\frac{dx}{dt} = u = \omega k$$

- Here u represents the velocity with which the wave disturbance is carried and is referred to as phase velocity.

$$\therefore u_{\text{phase}} = \frac{\omega}{k} \quad \dots(1.4.6)$$

- If a point is imagined to be marked on a travelling wave, then it becomes a representative point for a particular phase of the wave and the velocity with which it is transported owing to the motion of the wave is called **phase velocity**.

1.4.3 Group Velocity

- Here, the word group indicates a situation where two or more waves with slightly different velocities are superimposed together. The resultant pattern emerges in the shape of variation in amplitude and is called as wave packet or wave group.
- Group velocity is the velocity with which the envelope enclosing a wave group or a wave packet is transported. It is the velocity with which the energy transmission occurs in a wave.

Need to know about group velocity -

For an electromagnetic wave, phase velocity

$$u = v\lambda \quad \text{and} \quad \text{since} \quad E = hv$$

$$u = v\lambda = \frac{E}{h} \cdot \lambda$$

but,

$$\lambda = \frac{h}{mv}$$

$$\therefore u = \frac{E}{h} \cdot \frac{h}{mv} = \frac{mc^2}{mv} = \frac{c^2}{v}$$

$$\therefore u = \frac{c^2}{v^2}$$

- Since $c \gg v$, it shows that phase velocity of de Broglie wave associated with the particle moving with velocity v is greater than ' c ', the velocity of light.
- This difficulty can be overcome by assuming that each moving particle is associated with a group of waves or wave packet rather than a single wave.
- Hence, in this context, de-Broglie waves are represented by a wave packet and hence group velocity is associated with them.
- Let there be a combination of two waves represented as :

$$y_1 = a \sin(\omega_1 t - k_1 x)$$

$$y_2 = a \sin(\omega_2 t - k_2 x)$$

On superimposing

$$\begin{aligned} y &= y_1 + y_2 \\ &= a [\sin(\omega_1 t - k_1 x) + \sin(\omega_2 t - k_2 x)] \\ &= 2a \cos \left[\frac{(\omega_1 - \omega_2)}{2} t - \frac{(k_1 - k_2)}{2} x \right] \sin(\omega t - kx) \end{aligned}$$

Where,

$$\omega = \frac{\omega_1 + \omega_2}{2}, \quad k = \frac{k_1 + k_2}{2}$$

and let,

$$\Delta\omega = \omega_1 - \omega_2, \quad \Delta k = k_1 - k_2$$

$$\therefore y = 2a \cos \left[\frac{\Delta\omega t}{2} - \frac{\Delta k x}{2} \right] \sin(\omega t - kx)$$

Clearly, the equation has two components -

1. A wave of frequency ω and propagation constant k .

\therefore Its velocity

$$\frac{\omega}{k} = u = \text{phase velocity}$$

2. A wave with frequency $\frac{\Delta\omega}{2}$, propagation constant $\frac{\Delta k}{2}$

$$\therefore \text{Its velocity } \frac{\Delta\omega}{\Delta k} = G$$

- This velocity is the velocity of envelope of the group of waves hence it is called as group velocity.



1.4.4 Relation between Group Velocity and Particle Velocity

Let us start with a material particle with rest mass m_0 . At velocity v , consider its mass to be m .

$$\therefore E = mc^2 = \frac{m_0 c^2}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Momentum

$$P = mv = \frac{m_0 v^2}{\sqrt{1 - \frac{v^2}{c^2}}}$$

and frequency associated is

$$\begin{aligned} v &= \frac{E}{h} = \frac{m_0 c^2}{\sqrt{1 - \frac{v^2}{c^2}}} \frac{1}{h} \\ \omega = 2\pi v &= \frac{2\pi m_0 c^2}{h \times \sqrt{1 - \frac{v^2}{c^2}}} \\ \therefore \frac{d\omega}{dv} &= \frac{2\pi m_0 v}{h \left[1 - \left(\frac{v}{c} \right)^2 \right]^{3/2}} \end{aligned}$$

de-Broglie wavelength

$$\lambda = \frac{h}{P} = \frac{h \left[1 - \left(\frac{v}{c} \right)^2 \right]^{1/2}}{m_0 v}$$

Hence, propagation constant

$$\begin{aligned} k &= \frac{2\pi}{\lambda} = \frac{2\pi m_0 v}{h \left[1 - \left(\frac{v}{c} \right)^2 \right]^{1/2}} \\ \therefore \frac{dk}{dv} &= \frac{2\pi m_0 v}{h \left[1 - \left(\frac{v}{c} \right)^2 \right]^{3/2}} \end{aligned}$$

Since the group velocity

$$\begin{aligned} G &= \frac{d\omega}{dk} = \frac{d\omega}{dv} \cdot \frac{dv}{dk} = \frac{\frac{d\omega}{dv}}{\frac{dk}{dv}} \\ &= \frac{\frac{2\pi m_0 v}{h \left[1 - \left(\frac{v}{c} \right)^2 \right]^{3/2}}}{\frac{2\pi m_0 v}{h \left[1 - \left(\frac{v}{c} \right)^2 \right]^{3/2}}} = v \end{aligned}$$

$\therefore G = v$ = particle velocity.

\therefore The wave group associated with the moving particles travels with the same velocity as the particle.

1.5 Wave Function and Physical Interpretation of Wave Function

MU- Dec. 17. May 18

Q. What is the significance of wave function ?

(Dec. 17, May 18, 2 Marks)

In section 1.3, we have seen characteristics of matter waves. It is described that it is a new category of waves. Let us understand it in more detail.

- In classical mechanics, the square of wave amplitude associated with electromagnetic radiation is interpreted as a measure of radiation intensity. One can extend the concept to matter waves associated with electron or any particle. If we consider wave function ψ associated with a system of electrons then $|\psi|^2 d\tau$ is regarded as a measure of density of electrons. τ is a volume inside which an electron is known to be present, but where exactly the electron is situated inside τ is not known.
- In this situation, if we take ψ as wave function then $|\psi|^2 d\tau$ provides the probability of finding the electron in certain volume $d\tau$ of τ . Means $|\psi|^2$ is called the probability function. This interpretation was given by Max Born.
- Since electron must be somewhere inside the volume τ .

$$\int |\psi|^2 d\tau = 1 \quad \dots(1.5.1)$$

- It is important to know that wave function ψ has no direct physical significance but $|\psi|^2$ has.
- In quantum mechanics, it is postulated that the state of a system is completely characterized by a wave function.
- The wave functions are usually complex.

$$\text{i.e. } \psi = A + iB \quad \dots(1.5.2)$$

where, A and B are real functions.

- The integral of the wave function over entire space in the box must be equal to unity because, there is only one particle and at any given time it is present somewhere inside the box only. Therefore,

$$\int_{-\infty}^{\infty} |\psi|^2 dV = 1 \quad \dots(1.5.3)$$

A wave function which obeys Equation (1.5.3) is said to be normalized.

Ex. 1.5.1 : Calculate the de Broglie wavelength associated with an α -particle accelerated by a potential difference of 200 V. (Mass of α -particle is 6.68×10^{-27} kg).

MU - Dec. 16, May 19, 8 Marks

Soln. :

Here

$$m = 6.68 \times 10^{-27}$$

$$V = 200 \text{ Volts}$$

$$\text{Charge } q = 2e$$

$$\therefore \text{Formula } \lambda = \frac{h}{\sqrt{2mqV}}$$

$$= \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 6.68 \times 10^{-27} \times 2 \times 1.6 \times 10^{-19} \times 200}}$$

$$= 7.17 \times 10^{-13} \text{ m}$$

...Ans.

Ex. 1.5.2 : Calculate the wavelength of de-Broglie waves associated with mass 1 kg moving with a speed of 10^3 m/sec. Comment on your answer.

Soln. :**Given :**

$$m = 1 \text{ kg}$$

$$v = 1 \times 10^3 \text{ m/sec}$$

$$\therefore \lambda = \frac{h}{mv} = \frac{6.63 \times 10^{-34}}{1 \times 1 \times 10^3}$$

$$= 6.63 \times 10^{-37} \text{ m}$$

...Ans.

This wavelength is too small to have any practical significance.

Ex. 1.5.3 : A bullet of mass 40 g and an electron, both travel at the velocity of 1100 m/sec. What wavelengths can be associated with them? Why is the wave nature of the bullet not revealed through diffraction effect?

Soln. :**MU - May 13, 5 Marks****Given :**

$$\text{Mass of bullet} = 40 \times 10^{-3} \text{ kg}$$

$$\text{Velocity of bullet} = 1100 \text{ m/sec}$$

$$\text{Mass of electron} = 9.1 \times 10^{-31} \text{ kg}$$

$$\therefore \text{Let wavelength associated with bullet be } \lambda_B$$

$$\therefore \lambda_B = \frac{h}{mv}$$

$$= \frac{6.63 \times 10^{-34}}{40 \times 10^{-3} \times 1100} = 1.5 \times 10^{-35} \text{ m}$$

Let wavelength associated with an electron be λ_e

$$\therefore \lambda_e = \frac{h}{mv}$$

$$= \frac{6.63 \times 10^{-34}}{9.1 \times 10^{-31} \times 1100} = 6.62 \times 10^{-7}$$

$$m = 6620 \text{ Å}$$

...Ans.

Since the wavelength associated with a bullet is too small, it cannot be measured with the help of diffraction effect, as it requires an obstacle of the dimensions of the order of wavelength used.

Ex. 1.5.4 : Electrons accelerated through 100 V are reflected from a crystal. What is the glancing angle at which the first reflection occurs? Lattice spacing = 2.15 Å.

MU - Dec. 15, 3 Marks

Soln. :

$$V = 100 \text{ Volt}$$

∴ Wavelength associated with electron is

$$\therefore \lambda = \frac{12.26}{\sqrt{V}} = \frac{12.26}{\sqrt{100}} = 1.226 \text{ Å}$$

∴ Now, using Bragg's law for first order,

$$\begin{aligned} \therefore \lambda &= 2d \sin \theta \\ 1.226 \text{ Å} &= 2 \times 2.15 \text{ Å} \times \sin \theta \end{aligned}$$

$$\therefore \sin \theta = 0.2851$$

$$\therefore \theta = 16^\circ 33'$$

...Ans.

Ex. 1.5.5 : Find the energy of the neutron in units of electron-volt whose de-Broglie wavelength is 1 Å.

Given : $m_n = 1.674 \times 10^{-27} \text{ kg}$ and $h = 6.62 \times 10^{-34} \text{ J.sec.}$

MU - Dec. 13, 5 Marks

Soln. :

Given :

$$m_n = 1.674 \times 10^{-27} \text{ kg}$$

$$h = 6.62 \times 10^{-34} \text{ J.sec}$$

$$\lambda = 1 \text{ Å}$$

To find : Energy of neutron (in eV)

$$\begin{aligned} \text{Formula } \lambda &= \frac{h}{\sqrt{2mE}} \\ \therefore E &= \frac{h^2}{2m\lambda^2} \\ &= \frac{(6.62 \times 10^{-34})^2}{2 \times 1.674 \times 10^{-27} \times (1 \times 10^{-10})^2} \\ E &= 1.3 \times 10^{-20} \text{ J} \\ &= 0.081 \text{ eV} \end{aligned}$$

...Ans.

Ex. 1.5.6 : An electron is accelerated through 1000 volts and is reflected from a crystal. The first order reflection occurs when glancing angle is 70° . Calculate the interplanar spacing of a crystal.

MU - May 15, 5 Marks

Soln. :

$$V = 1000 \text{ volt}$$

(1) Using formula :

$$\lambda = \frac{12.26}{\sqrt{V}} = 0.388 \text{ Å}$$

(2) Using Bragg's law :

$$n\lambda = 2 d \sin \theta$$

$$d = \frac{n\lambda}{2 \times \sin \theta} = \frac{1 \times 0.388}{2 \times \sin 70}$$

$$d = 0.2065 \text{ Å}$$

...Ans.

Ex. 1.5.7 : Calculate the kinetic energy of an electron whose de-Broglie wavelength is 5000 Å.

Soln. :

Given :

$$\begin{aligned}m_e &= 9.108 \times 10^{-31} \text{ kg} \\h &= 6.625 \times 10^{-34} \text{ J.sec.} \\n &= 1, \\&\lambda = 5000 \text{ Å}\end{aligned}$$

To find : K.E. = ?

Using formula,

$$\begin{aligned}\lambda &= \frac{h}{\sqrt{2mE}} \\ \therefore E &= \frac{h^2}{2m\lambda^2} \\ &= \frac{(6.626 \times 10^{-34})^2}{2 \times 9.108 \times 10^{-31} (5000 \times 10^{-10})^2} \\ &= 9.6378 \times 10^{-25} \text{ J} = 6.023 \times 10^{-6} \text{ eV}\end{aligned}$$

...Ans

Ex. 1.5.8 : What is the wavelength of a beam of neutron having

- (1) an energy of 0.025 eV?
- (2) an electron and photon each have a wavelength of 2 Å, what are their momenta and energies?

Mass of neutron = 1.676×10^{-27} kg

Mass of electron = 9.1×10^{-31} kg

Planck's constant, $h = 6.625 \times 10^{-34}$ J.sec

MU - Dec. 14, May 17, 5 [Ans]

Soln. :

Given :

$$m_n = 1.676 \times 10^{-27} \text{ kg}$$

$$m_e = 9.1 \times 10^{-31} \text{ kg}$$

$$h = 6.625 \times 10^{-34} \text{ J.sec.}$$

Part 1 :

Given :

$$\text{Energy} = 0.025 \text{ eV} = 0.25 \times 1.6 \times 10^{-19} \text{ J}$$

To find : Neutron's wavelength

$$\begin{aligned}\lambda &= \frac{h}{\sqrt{2mE}} \\ &= \frac{6.625 \times 10^{-34}}{\sqrt{2 \times 1.676 \times 10^{-27} \times 0.025 \times 1.6 \times 10^{-19}}} \\ &= 1.709 \times 10^{-10} \text{ m} \\ \lambda &= 1.709 \text{ Å}\end{aligned}$$

...Ans

Part 2 :For an electron with wavelength 2\AA

$$\begin{aligned} E &= \frac{h^2}{\lambda^2 \times 2m} && (\text{using formula } E = \frac{p^2}{2m}) \\ &= \frac{(6.625 \times 10^{-34})^2}{(2 \times 10^{-10})^2 \times 2 \times 9.1 \times 10^{-31}} \\ &= 6.028 \times 10^{-18} \text{ J} = 37.68 \text{ eV} \end{aligned}$$

Momentum of electron

$$\begin{aligned} &= \frac{h}{\lambda} \\ &= \frac{6.625 \times 10^{-34}}{2 \times 10^{-10}} \\ &= 3.3125 \times 10^{-24} \frac{\text{kg.m}}{\text{sec}} \end{aligned}$$

...Ans.

For a photon :

$$\begin{aligned} \text{Energy} &= h\nu \\ &= \frac{hc}{\lambda} = \frac{6.625 \times 10^{-34} \times 3 \times 10^8}{2 \times 10^{-10}} \\ &= 9.9375 \times 10^{-16} \text{ J} \end{aligned}$$

...Ans.

$$\text{Momentum} = \frac{h}{\lambda} = 3.3125 \times 10^{-24} \frac{\text{kg.m}}{\text{sec}}$$

...Ans.

Ex. 1.5.9 : If the particles listed below all have the same energy, which has the shortest wavelength - electron, α -particle, neutron or proton?

Soln. : Particles with same energy α -particle; neutron; proton

$$\text{as } \lambda \propto \frac{1}{m}$$

α -particle has the highest mass among all others. It will have the shortest wavelength.

Ex. 1.5.10 : Calculate the energies in eV of an electron and a proton whose de-Broglie wavelength is 1\AA .

Soln. : Energy for electron

$$\begin{aligned} E &= \frac{h^2}{\lambda^2 \times 2m} \\ &= \frac{(6.625 \times 10^{-34})^2}{(1 \times 10^{-10})^2 \times 2 \times 9.1 \times 10^{-31}} \\ &= 2.41 \times 10^{-17} \text{ J} = 150 \text{ eV} \end{aligned}$$

...Ans.

Energy of proton

$$\begin{aligned} &= \frac{(6.625 \times 10^{-34})^2}{(1 \times 10^{-10})^2 \times 2 \times 1.66 \times 10^{-27}} = 1.32 \times 10^{-20} \text{ J} \\ &= 0.08 \text{ eV} \end{aligned}$$

...Ans.



Ex. 1.5.11 : Compare de Broglie wavelength associated with following particles :

(i) Mass of 50 microgram and travelling with the velocity of 100 cm/sec.

(ii) Mass of 9.1×10^{-31} kg and travelling with velocity of 3×10^6 m/sec. Comment on which is measurable.

Soln. :

$$\text{i) } m = 50 \times 10^{-6} \text{ gm} = 50 \times 10^{-9} \text{ kg}, \quad v = 1 \text{ m/sec}$$

$$\therefore \lambda = \frac{h}{mv} = \frac{6.625 \times 10^{-34}}{50 \times 10^{-9} \times 1}$$

$$= 1.325 \times 10^{-26} \text{ m}$$

$$\lambda = 1.325 \times 10^{-16} \text{ Å}$$

ii)

$$m = 9.1 \times 10^{-31} \text{ kg}$$

$$v = 3 \times 10^6 \text{ m/sec}$$

$$\therefore \lambda = \frac{h}{mv} = \frac{6.625 \times 10^{-34}}{9.1 \times 10^{-31} \times 3 \times 10^6}$$

$$= 2.426 \times 10^{-10} \text{ m}$$

$$\lambda = 2.40 \text{ Å}$$

...Ans.

...Ans.

\therefore Wavelength of electron is measurable.

Ex. 1.5.12 : Calculate the velocity and de-Broglie wavelength of α -particle of energy 1 keV. Given : Mass of α -particle = 6.68×10^{-27} kg.

Soln. :

MU - May 14. 5 Marks

$$E = 1 \text{ keV} = 1 \times 10^3 \times 1.6 \times 10^{-19} \text{ J}$$

$$\lambda = \frac{h}{\sqrt{2mE}}$$

$$= \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 6.68 \times 10^{-27} \times 1 \times 10^3 \times 1.6 \times 10^{-19}}}$$

$$= 4.534 \times 10^{-13} \text{ m}$$

$$\text{Now, } \lambda = \frac{h}{mv}$$

...Ans.

\therefore Velocity,

$$v = \frac{h}{m\lambda} = \frac{6.63 \times 10^{-34}}{6.68 \times 10^{-27} \times 4.534 \times 10^{-13}}$$

$$= 218.9 \times 10^3 \text{ m/sec}$$

...Ans.

Ex. 1.5.13 : An electron and a photon each have a wavelength of 2 Å. What are their momenta and energies?

Soln. :

$$\text{Mass of electron } m_e = 9.1 \times 10^{-31} \text{ kg}$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

$$\lambda = \frac{h}{p}$$

$$p = \frac{6.63 \times 10^{-34}}{2 \times 10^{-10}}$$

$$= 3.315 \times 10^{-24} \frac{\text{kg.m}}{\text{sec}}$$

...Ans.

$$\text{Energy} = \frac{p^2}{2m} = 6.038 \times 10^{-18} \text{ J}$$

...Ans.

For photon :

Momentum,

$$p = \frac{h}{\lambda}$$

$$p = \frac{6.63 \times 10^{-34}}{2 \times 10^{-10}}$$

$$= 3.315 \times 10^{-24} \frac{\text{kg.m}}{\text{sec}}$$

...Ans.

$$\text{Energy} = \frac{hc}{\lambda}$$

$$= \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{2 \times 10^{-10}}$$

$$= 9.945 \times 10^{-16} \text{ J}$$

...Ans.

Ex. 1.5.14 : Compare the energy of a photon with that of a neutron when both are associated with wavelength of 1\AA . (Mass of neutron = 1.678×10^{-27} kg) Data: Wavelength of photon $\lambda_1 = 1\text{\AA} = 1 \times 10^{-10}$ m, wavelength of neutron $\lambda_2 = 1\text{\AA}$

To find : $\frac{E_1}{E_2} = \frac{\text{Energy of photon}}{\text{Energy of neutron}}$

Soln. : For photon

$$\begin{aligned}\therefore E_1 &= h\nu = \frac{hc}{\lambda_1} \\ &= \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{1 \times 10^{-10}} \\ &= 1.989 \times 10^{-15} \text{ J} \\ &= 1.24 \times 10^4 \text{ eV}\end{aligned}$$

For neutron,

$$\begin{aligned}\lambda_2 &= \frac{h}{\sqrt{2mE_2}} \\ \therefore E_2 &= \frac{h^2}{2m\lambda_2^2} \\ &= \frac{(6.634 \times 10^{-34})^2}{2 \times 1.678 \times 10^{-27} \times (1 \times 10^{-10})^2} \\ &= 1.31 \times 10^{-20} \text{ J} = 0.082 \text{ eV} \\ \therefore \frac{E_1}{E_2} &= \frac{1.24 \times 10^4}{0.082} = 1.5 \times 10^5\end{aligned}$$

...Ans.

Ex. 1.5.15 : Calculate the de-Broglie wavelength of proton with a velocity equal to $\frac{1}{20}$ th velocity of light.
(mass of proton = 1.6×10^{-27} kg)

MU - Dec. 12, 5 Marks

Soln. :

$$\begin{aligned} h &= 6.63 \times 10^{-34} \\ m_p &= 1.6 \times 10^{-27} \text{ kg} \\ v &= \frac{1}{20} \times 3 \times 10^8 \text{ m/sec} \end{aligned}$$

Using formula,

$$\begin{aligned} \lambda &= \frac{h}{p} = \frac{h}{mv} \\ &= \frac{6.63 \times 10^{-34}}{1.6 \times 10^{-27} \times \frac{3 \times 10^8}{20}} \\ &= 2.763 \times 10^{-14} \text{ m} \\ &= 2.76 \times 10^{-4} \text{ Å} \end{aligned}$$

...Ans.

Ex. 1.5.16 : What is the wavelength of a beam of neutron having energy 0.025 eV and mass 1.676×10^{-27} kg?**Soln. :**

$$\begin{aligned} E &= 0.025 \text{ eV} = 0.025 \times 1.6 \times 10^{-19} \\ &= 0.004 \times 10^{-18} \text{ Joules.} \\ \text{As, } E &= \frac{1}{2} mv^2 = 0.004 \times 10^{-18} \end{aligned}$$

$$\therefore v = \sqrt{\frac{2 \times 0.004 \times 10^{-18}}{1.676 \times 10^{-27}}} = 2.1847 \times 10^3 \text{ m/sec}$$

$$\begin{aligned} \text{Now, } \lambda &= \frac{h}{mv} = \frac{6.634 \times 10^{-34}}{1.676 \times 10^{-27} \times 2.1847 \times 10^3} \\ &= 1.811 \text{ Å} \end{aligned}$$

...Ans.

Ex. 1.5.17 : A fast moving neutron has de-Broglie wavelength 2×10^{-12} m associated with it. Find the following :

- (a) Kinetic energy.
- (b) Phase and group velocity.

(Ignore relativity effect)

Data: $\lambda_{\text{neutron}} = \lambda_n = 2 \times 10^{-12}$ m $m_{\text{neutron}} = m_n = 1.675 \times 10^{-27}$ kg**Soln. :**

$$p = \frac{h}{\lambda} = \frac{6.63 \times 10^{-34}}{2 \times 10^{-12}} = 3.32 \times 10^{-22}$$

$$\begin{aligned} \therefore KE &= \frac{p^2}{2m} = \frac{(3.32 \times 10^{-22})^2}{2 \times 1.675 \times 10^{-27}} \\ &= 3.280 \times 10^{-17} \text{ J} \end{aligned}$$

...Ans.

Since the velocity of a particle is same as the group velocity, the group velocity can be given as

$$\begin{aligned} v_{\text{group}} &= \frac{p}{m} = \frac{3.32 \times 10^{-22}}{1.675 \times 10^{-27}} \\ &= 1.98 \times 10^5 \end{aligned}$$

....Ans.

Now using the relation

$$\begin{aligned} (v_{\text{group}})(v_{\text{phase}}) &= c^2 \\ \therefore v_{\text{phase}} &= \frac{c^2}{v_{\text{group}}} = \frac{(3 \times 10^8)^2}{1.98 \times 10^5} \\ &= 4.55 \times 10^{11} \frac{m}{sec} \end{aligned}$$

....Ans.

Ex. 1.5.18 : A particle has mass 1.157×10^{-30} kg and kinetic energy 80 eV. Find the de-Broglie wavelength, group velocity and phase velocity of de Broglie wave.

Soln. :

$$\begin{aligned} \lambda &= \frac{h}{\sqrt{2 m E}} \\ &= \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 1.157 \times 10^{-30} \times 80 \times 1.6 \times 10^{-19}}} \\ &= 1.218 \times 10^{-10} m \end{aligned}$$

...Ans.

Now,

$$\begin{aligned} v_{\text{group}} &= \frac{p}{m} = \frac{h}{\lambda \cdot m} = \frac{6.63 \times 10^{-34}}{1.218 \times 10^{-10} \times 1.157 \times 10^{-30}} \\ v_{\text{group}} &= 4.7 \times 10^6 m/sec \quad \dots \text{Ans.} \\ v_{\text{phase}} &= \frac{c^2}{v_{\text{group}}} = \frac{(3 \times 10^8)^2}{4.7 \times 10^6} \\ &= 1.913 \times 10^{10} m /sec \quad \dots \text{Ans.} \end{aligned}$$

Ex. 1.5.19 : Find the de Broglie wavelength of (i) an electron accelerated through a potential difference of 182 volts and (ii) 1 kg object moving with a speed of 1m/s. Comparing the results, explain why the wave nature of matter is not apparent in daily observations?

MU - May 16, 5 Marks

Soln. :

(i) For electron, its wavelength is given by

$$\lambda = \frac{12.26}{\sqrt{V}} \text{ Å} = \frac{12.26}{\sqrt{182}} = 0.9087 \text{ Å} \quad \dots \text{Ans.}$$

(ii) For object of 1 kg mass,

$$\begin{aligned} \lambda &= \frac{h}{mv} \\ &= \frac{6.63 \times 10^{-34}}{1 \times 10^{-3} \times 1} = 6.63 \times 10^{-31} \text{ m} \quad \dots \text{Ans.} \end{aligned}$$

This wavelength is too small, hence wave nature is not apparent in daily life. However, through diffraction it is possible to detect wave nature of electron whereas for particle of 1 kg mass it is not possible.



Ex. 1.5.20 : Calculate the frequency and wavelength of photon whose energy is 75eV.

Soln. :

Given :

$$\begin{aligned}\text{Energy} &= 75 \text{ eV} \\ &= 75 \times 1.6 \times 10^{-19} \text{ J} \\ &= 1.2 \times 10^{-17} \text{ J}\end{aligned}$$

Now for Photon

$$\begin{aligned}E &= h\nu \\ \therefore 1.2 \times 10^{-17} &= 6.62 \times 10^{-34} \times \nu \\ \nu &= 1.81 \times 10^{16} \\ \text{Now } \nu &= \frac{C}{\lambda} \\ \therefore \lambda &= \frac{C}{\nu} = \frac{3 \times 10^8}{1.81 \times 10^{16}} \\ &= 1.657 \times 10^{-8} \text{ m}\end{aligned}$$

...Ans.

1.6 Uncertainty Principle

Q. Explain Heisenberg's uncertainty principle.

- In **classical mechanics**, we describe a **particle occupying a definite place in space and having a specific momentum**. At a given instant of time, one can evaluate position and momentum simultaneously. This may appear valid only under the boundary of classical mechanics. When one steps out of classical mechanics and enters **wave mechanics** (available at atomic scale) this **idea is no more valid**.
- Heisenberg's uncertainty principle states that **quantum mechanics does not simultaneously permit the determination of position and momentum of a particle accurately**. Any effort made to make the measurement of position of the particle - such as an electron - very accurately, results in a large uncertainty in the measurement of momentum and vice versa.
- Mathematically,

$$\Delta x \cdot \Delta p_x \geq \hbar \quad \text{or}$$

$$\Delta x \cdot \Delta p_x \geq \frac{h}{2\pi}$$

where,

...(1.6.1)

Δx = Uncertainty in the measured values of position.
 Δp_x = Uncertainty in the measured values of momentum.

Important characteristics

- The uncertainty principle is a direct consequence of the wave nature of particles.
- The limit on measurement is independent of measuring procedure or sophistication of instrument.
- It is a fundamental property of nature.

- It is applicable to conjugate variables like energy and time as

$$\Delta E \cdot \Delta t \geq \hbar$$

Also to angle and angular momentum,

$$\Delta L \cdot \Delta \theta \geq \hbar$$

Physical significance of Heisenberg's uncertainty principle

- The physical significance of the above argument is that one should not think of the exact position or an accurate value for momentum of a particle. Instead, one should think of the probability of finding the particle at a certain position, or of the probable value for the momentum of the particle.
- The estimation of such probabilities is made by means of certain mathematical functions, named probability density functions in quantum mechanics.

1.7 Electron Diffraction Experiment

MU - May 12, Dec. 15

Q. With single slit electron diffraction, prove Heisenberg's uncertainty principle.

(May 12, Dec. 15, 5 Marks)

- A monochromatic light while passing through a single slit produces deviation of light, which is called diffraction.

The intensity profile is as shown in Fig. 1.7.1.

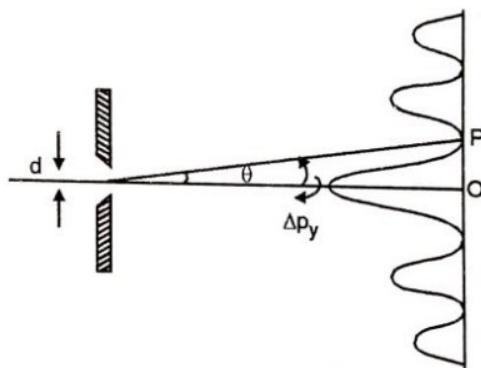


Fig. 1.7.1 : Diffraction at single slit

- Consider point P, which represents a first order minimum. Condition for first order minimum is

$$\sin \theta = \frac{\lambda}{d} \quad \dots(1.7.1)$$

- Consider a photon passing through the narrow slit. The photon has its momentum well defined before passing through the slit. After it passes through the slit, its position is known to within an uncertainty which is equal to the slit width.

$$\therefore \Delta y = d \quad \dots(1.7.2)$$

- After passing through the slit, photon has its value uncertain as it makes an angle θ with horizontal (Fig. 1.7.1).

\therefore The uncertainty in its y component of momentum is at least as large as $p \sin \theta$.

$$\therefore \Delta p_y \geq p \frac{\lambda}{d} \quad \dots(1.7.3)$$

$$\text{Now, } p = \frac{h}{\lambda} \text{ (de-Broglie hypothesis)}$$

$$\therefore \Delta p_y \geq \frac{h}{\lambda} \cdot \frac{\lambda}{d} \geq \frac{h}{d} \geq \Delta y$$

$$\therefore \Delta y \cdot \Delta p_y \geq h$$

This is in good agreement with uncertainty principle.

1.8 Gamma-ray Microscope Experiments

- This experiment is an imaginary one. Here we try to measure the position of a very small particle, say, an electron using a microscope. Since the particle is very small, let us first consider the resolving power of microscope which defines the ability to see the two particles distinctly.

The limit of resolution is given by,

$$\Delta x = \frac{\lambda}{2 \sin \theta} \quad (1.8.1)$$

where θ is semi vertical angle shown in the Fig. 1.8.1.

- For small measure of Δx , corresponding λ must be of the same order. Hence, we take gamma rays into consideration.
- If a photon of energy $h\nu$ hits the electron and the deflected photon enters the field of microscope, the momentum along x-axis will become uncertain.

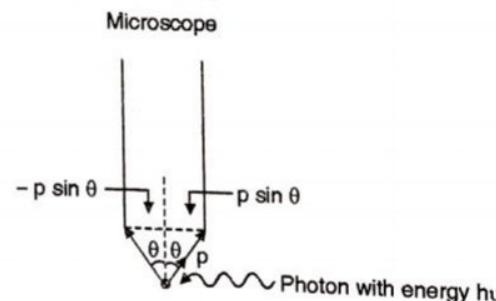


Fig. 1.8.1 : Gamma-ray experiment

Since the momentum is conserved, the uncertainty in the momentum along x-axis is given by,

$$\Delta p_x = p \sin \theta - (-p \sin \theta) = 2p \sin \theta$$

Using de-Broglie hypothesis,

$$\lambda = \frac{h}{p}$$

$$\Delta p_x = 2 \left(\frac{h}{\lambda} \right) \sin \theta \quad \dots(1.8.2)$$

\therefore The product of uncertainty in the case of simultaneous measurement of position and momentum is given by,

$$\Delta x \cdot \Delta p_x = \left(\frac{\lambda}{2 \sin \theta} \right) \cdot \left(2 \left(\frac{h}{\lambda} \right) \sin \theta \right) = h$$

or $\Delta x \cdot \Delta p_x \geq h$

This is the uncertainty principle.

1.9 Applications of Uncertainty Principle

(1) Absence of electron in nucleus

MU - Dec. 12, May 14, Dec. 14, May 15, Dec. 17, May 18

Q. Using Heisenberg's uncertainty principle, show that electrons cannot exist within the nucleus.

(Dec. 12, May 14, Dec. 14, May 15, 5 Marks)

Q. With Heisenberg's uncertainty principle prove that electron cannot survive in nucleus. (Dec. 17, May 18, 4 Marks)

- The radius of nucleus is about 5×10^{-15} m. Using uncertainty principle we can place lower limit on energy of an electron, if it is to be a part of the nucleus.

$$\text{Let } \Delta x = 2 \times 5 \times 10^{-15} \text{ m i.e. diameter.}$$

\therefore Using

$$\begin{aligned}\Delta x \cdot \Delta p &\geq \frac{h}{2\pi} \\ \therefore \Delta p &\geq \frac{h}{2\pi \cdot \Delta x} \\ &\geq \frac{6.6 \times 10^{-34}}{2 \times 2\pi \times 5 \times 10^{-15}} \\ &\geq 1 \times 10^{-20} \frac{\text{kg} \cdot \text{m}}{\text{sec}}\end{aligned}$$

- This is the uncertainty in momentum of nuclear electron. The momentum p should be at least of this order,

$$\begin{aligned}\therefore \text{K.E.} = pc &\geq 1 \times 10^{-20} \times 3 \times 10^8 \\ &\geq 3 \times 10^{-12} \text{ J} \\ &\geq \frac{3 \times 10^{-12}}{1.6 \times 10^{-19}} \text{ eV} \\ &\geq 9 \text{ MeV}\end{aligned}$$

- The KE of electron must be greater than 9 MeV if it is to remain inside the nucleus. This is practically not possible (atom becomes unstable). Hence one can say that an electron cannot survive inside the nucleus.

Ex. 1.9.1 : An electron has a speed of 400 m/sec with uncertainty of 0.01%. Find the accuracy in its position.

MU - May 13, Dec. 17, May 18, Dec. 18, May 19, 5 Marks

Soln. :

$$\text{Momentum of electron} = p = mv$$

$$= 9.11 \times 10^{-31} \times 400$$

$$= 3.644 \times 10^{-28} \frac{\text{kg} \cdot \text{m}}{\text{sec}}$$

$$\therefore \Delta p = m \Delta V = mv \frac{\Delta v}{v}$$

$$= 3.644 \times 10^{-28} \times \frac{0.01}{100}$$

$$= 3.644 \times 10^{-32}$$

∴ Using Heisenberg's uncertainty formula

$$\Delta x \cdot \Delta p_x \geq \hbar$$

$$\therefore \Delta x \geq \frac{\hbar}{2\pi} \cdot \frac{1}{\Delta p_x}$$

$$\geq \frac{6.63 \times 10^{-34}}{2\pi \times 3.644 \times 10^{-32}}$$

$$\geq 2.895 \times 10^{-3} \text{ m}$$

...Ans.

Ex. 1.9.2 : An electron is confined in a box of length 10^{-8} m. Calculate minimum uncertainty in its velocity.

MU - May 17, 3 Marks

Soln. :

$$\text{As } \Delta x \cdot \Delta p_x \approx \hbar$$

$$\therefore \Delta x_{\max} \Delta p_{x\min} \approx \hbar$$

Now, box has length 10^{-8} m.

$$\therefore \Delta x_{\max} = 10^{-8} \text{ m}$$

$$\therefore \Delta p_{x\min} = \frac{\hbar}{2\pi} \cdot \frac{1}{10^{-8}} = \frac{6.63 \times 10^{-34}}{2\pi \times 10^{-8}}$$

$$= 1.055 \times 10^{-26}$$

$$\therefore \Delta p_{x\min} = m \Delta v_{x\min}$$

$$\therefore \Delta v_{x\min} = \frac{\Delta p_{x\min}}{m}$$

$$= \frac{1.055 \times 10^{-26}}{9.1 \times 10^{-31}}$$

$$= 11.595 \times 10^3 \frac{\text{m}}{\text{sec}}$$

...Ans.

Ex. 1.9.3 : The speed of an electron is measured to within an uncertainty of 2×10^4 m/sec. What is the minimum space required by the electron to be confined in an atom?

Soln. :**Data :**

$$\Delta v_x = 2 \times 10^4 \text{ m/sec}$$

To find : Δx

Using equation of uncertainty,

$$\Delta x \cdot \Delta p_x \geq \frac{\hbar}{2\pi}$$

$$\therefore \Delta x \geq \frac{\hbar}{2\pi \cdot \Delta p_x} \geq \frac{\hbar}{2\pi} \cdot \frac{1}{m \Delta v_x}$$

$$\geq \frac{6.63 \times 10^{-34}}{2\pi \times (9.1 \times 10^{-31} \times 2 \times 10^4)}$$

$$\geq 0.579 \times 10^{-10} \text{ m}$$

...Ans.

Ex. 1.9.4 : The inherent uncertainty in the measurement of time spent by a nucleus in the excited state is found to be 1.4×10^{-10} sec. Estimate the uncertainty that results in its energy in the excited state.

Data : $\Delta t = 1.4 \times 10^{-10}$ sec

To find : ΔE

Soln. : Using equation

$$\Delta E \cdot \Delta t \geq \hbar$$

$$\Delta E \cdot \Delta t \geq \hbar$$

$$\therefore \Delta E \geq \frac{\hbar}{2\pi} \cdot \frac{1}{\Delta t}$$

$$\geq \frac{6.63 \times 10^{-34}}{2\pi \times 1.4 \times 10^{-10}}$$

$$\geq 7.537 \times 10^{-25} \text{ J}$$

$$\geq \frac{7.537 \times 10^{-25}}{1.6 \times 10^{-19}} \text{ eV}$$

$$\geq 4.71 \times 10^{-6} \text{ eV}$$

...Ans.

Ex. 1.9.5 : Find the minimum energy of neutron confined to nucleus of size of the order of 10^{-14} m.

Given : Mass of neutron = 1.675×10^{-27} kg

MU - Dec 16, 5 Marks

Soln. : Using uncertainty principle

$$\therefore \Delta x \cdot \Delta p_x \geq \hbar$$

$$\Delta x \cdot \Delta p_x \geq \frac{\hbar}{2\pi}$$

$$\therefore \Delta p_x \geq \frac{\hbar}{2\pi} \cdot \frac{1}{\Delta x} = \frac{\hbar}{2\pi} \times \frac{1}{1 \times 10^{-14}}$$

$$= 1.056 \times 10^{-30}$$

$$KE = \frac{p^2}{2m} = \frac{(1.056 \times 10^{-30})^2}{2 \times 1.675 \times 10^{-27}}$$

$$= 3.328 \times 10^{-34}$$

...Ans.

1.10 One Dimensional Time-dependent Schrödinger Equation

MU - Dec. 12, Dec. 13, May 14, Dec. 14, May 16, Dec. 16, May 17, Dec. 17

Q. Derive one-dimensional time-dependent Schrödinger equation for matter waves.

(Dec.12, Dec.13, May 14, Dec.14, May 16, May 17, Dec. 17, 5 Marks)

Q. Write Schrodinger's time-dependent and time-independent wave equations of matter waves in one dimension and state physical significance of these equations. (Dec.16, 3 Marks)



- Based on de Broglie's idea of matter waves, Schrödinger developed a mathematical theory which plays the same role as Newton's laws in classical mechanics.
- Using de Broglie's hypothesis for a particle of mass m , moving with a velocity v , associated with it is a wave of wavelength.

$$\lambda = \frac{h}{p}$$

- The wave equation for a de-Broglie wave can be written as

$$\psi = A e^{-i\omega t} \quad \dots(1.10.1)$$

where, A = Amplitude

ω = Angular frequency

- For a one dimensional case, the classical wave equation has the following form

$$\frac{d^2y}{dx^2} = \frac{1}{v^2} \frac{d^2y}{dt^2} \quad \dots(1.10.2)$$

- Where, y is the displacement and v is the velocity of the wave. The solution is,

$$y(x, t) = A e^{-i(kx - \omega t)} \quad \dots(1.10.3)$$

where, $\omega = 2\pi v$

- By analogy we can write the wave equation for de-Broglie wave for the motion of a free particle as

$$\frac{d^2\psi}{dx^2} = \frac{1}{v^2} \frac{d^2\psi}{dt^2} \quad \dots(1.10.4)$$

where, $\omega = vk$

v = Phase velocity

- The solution of the above equation is,

$$\psi(x, t) = A e^{-i(Et - px)/\hbar} \quad \dots(1.10.5)$$

- There we have replaced ω and k of Equation (1.10.3) with E and p using Einstein and de-Broglie relations.

- Differentiating with respect to t ,

$$\begin{aligned} \frac{\partial\psi}{\partial t} &= \frac{\partial}{\partial t} [A e^{-i(Et - px)/\hbar}] \\ &= A e^{-i(Et - px)/\hbar} \cdot \frac{-iE}{\hbar} \\ \frac{\partial\psi}{\partial t} &= \frac{-i}{\hbar} E \psi \end{aligned} \quad \dots(1.10.6)$$

- Similarly taking double differentiation of Equation (1.10.5) with respect to x

$$\frac{\partial^2\psi}{\partial x^2} = \frac{-p^2}{\hbar^2} \psi \quad \dots(1.10.7)$$

- In classical mechanics we have energy of a free particle described as

$$E = \frac{p^2}{2m}$$

- Let there be a field where particle is present. Depending on its position in the field, the particle will possess certain potential energy V .

\therefore Total energy of particle E is given by

$$\begin{aligned} E &= \frac{p^2}{2m} + V \\ \text{or } \frac{p^2}{2m} &= E - V \end{aligned} \quad \dots(1.10.8)$$

$$\therefore \frac{p^2}{2m} \psi = E\psi - V\psi \quad \dots(1.10.9)$$

But from Equation (1.10.6),

$$E\psi = \frac{-\hbar}{i} \frac{\partial \psi}{\partial t}$$

And from equation (1.10.7)

$$p^2\psi = \hbar^2 \frac{\partial^2 \psi}{\partial x^2}$$

\therefore Equation (1.10.9) becomes,

$$\begin{aligned} \frac{-\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} &= \frac{-\hbar}{i} \frac{\partial \psi}{\partial t} - V\psi \\ \therefore \frac{-\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} + V\psi &= i\hbar \frac{\partial \psi}{\partial t} \end{aligned} \quad \dots(1.10.10)$$

Equation (1.10.10) is one-dimensional time-dependent Schrödinger equation.

1.11 Reduction of Schrodinger Equation to Time-independent Form

MU - May 15, Dec. 15, Dec. 16, Dec. 17, May 18

Q. Derive Schrodinger's time-independent wave equation. (May 15, Dec.15, Dec.16, Dec.17, May 18, 5 Marks)

- A further simplified form of Schrödinger equation is time-independent form, where the field due to which the potential energy is considered to be stationary is independent of time. Or, one can say it is a function of position only. Hence one can separate the variables of Schrödinger wave equation as

$$\psi(x, t) = \psi(x)\phi(t) \quad \dots(1.11.1)$$

\therefore Equation (1.11.1) can be modified as

$$\frac{-\hbar^2}{2m}\phi(t)\frac{\partial^2 \psi(x)}{\partial x^2} + V(x)\psi(x)\phi(t) = i\hbar\psi(x)\frac{\partial \phi(t)}{\partial t}$$

Divide both sides by $\psi(x)\phi(t)$

$$\therefore \frac{-\hbar^2}{2m}\frac{1}{\psi}\frac{\partial^2 \psi}{\partial x^2} + V(x) = i\hbar\frac{1}{\phi}\frac{\partial \phi}{\partial t} \quad \dots(1.11.2)$$

- Careful observation of Equation (1.11.2) shows that we have separated the Schrödinger equation such that on LHS is function of x only and RHS is function of t only. Since Equation (1.11.2) is valid for any x and t , both the sides must be equal to a constant say energy E .

$$\therefore \frac{-\hbar^2}{2m}\frac{1}{\psi}\frac{\partial^2 \psi}{\partial x^2} + V(x) = E$$



Or

$$\frac{-\hbar^2 \partial^2 \psi}{2m \partial x^2} + V\psi = \psi \quad \dots(1.11.3)$$

Equation (1.11.3) represents time-independent Schrödinger equation.

1.12 Eigen Functions and Eigen Values

- We have discussed about Schrödinger equation in section 1.11. On solving Schrödinger equations we get ψ . Since it is a second order differential equation, there are several solutions available. All of them may not be useful to us. Obviously, one would like to know how to get a meaningful and acceptable solution.
- Following are the postulates in quantum mechanics which must be satisfied :
 1. ψ is single valued everywhere.
 2. ψ and its first derivatives with respect to its variable are continuous everywhere.
 3. ψ is finite everywhere.
- The solutions, which are acceptable are called **Eigen functions**. These Eigen functions are used in Schrödinger equations to find energy. These energy values are known as **Eigen values**.

1.13 Application of Schrödinger Equation to Free Particle and Particle in a Box

MU.: May 13, Dec. 15, Dec. 17, May 18

- Q.** Show that the energy of an electron in the box varies as the square of natural numbers. (May 13, Dec. 15, 5 Marks)
Q. Derive the expression for energy Eigen values for free particle in one dimensional potential well.

(Dec.17, May 18, 3 Marks)

(a) Motion of free particle

- Here, free particle means that particle is not acted upon by any force. Hence, potential energy is zero and it is moving in positive x direction.
 \therefore Schrödinger equation in one dimension is

$$\begin{aligned} \frac{\partial^2 \psi}{\partial x^2} + \frac{8\pi^2 m}{h^2} E \psi &= 0 \\ \therefore \frac{\partial^2 \psi}{\partial x^2} + K^2 \psi &= 0 \end{aligned} \quad \dots(1.13.1)$$

here $K^2 = \frac{8\pi^2 m E}{h^2}$

- As particle is considered to be free i.e. without boundary conditions on K, all values of K are allowed, and hence all values of energy are allowed.
- From expression of $K^2 = \frac{8\pi^2 m E}{h^2}$

$$\therefore K = \sqrt{\frac{2mE}{\hbar^2}}$$

$$\text{Using } E = \frac{p^2}{2m}$$

$$\therefore K = \sqrt{\frac{2m}{\hbar^2} \cdot \frac{p^2}{2m}} = \frac{p}{\hbar} \approx \frac{1}{\lambda}$$

Hence, K describes the wave properties of particle and according to uncertainty principle position becomes uncertain.

$$\text{As } E = \frac{\hbar^2}{8\pi^2 m} K^2$$

$$\therefore E \propto K^2$$

(b) Particle in one dimensional potential well of infinite height (or particle in a box).

- Suppose a particle of mass m is free to move in the x-direction only in the region from $x = 0$ to $x = a$ (Fig. 1.13.1).
 - Outside this region the potential energy V is taken to be infinite, and within this region it is zero. A particle does not lose energy when it collides with walls, hence its energy remains constant.
 - Outside the box $V = \infty$ and particle cannot have infinite energy, therefore it cannot exist outside the box.
- \therefore Schrodinger's equation is written as

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{\hbar^2} (E - \infty)\psi = 0 \quad \dots(1.13.2)$$

Inside the box,

$$V = 0$$

\therefore Schrodinger's equation is written as

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{\hbar^2} E\psi = 0 \quad \dots(1.13.3)$$

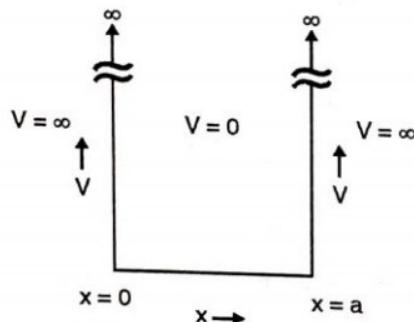


Fig. 1.13.1 : One dimensional potential well of infinite height

Equation (1.13.3) may be simplified as

$$\frac{d^2\psi}{dx^2} + K^2\psi = 0 \quad \dots(1.13.4)$$

Where,

$$K^2 = \frac{8\pi^2 m E}{\hbar^2}$$

$$\text{or } K^2 = \frac{2mE}{\hbar^2}$$

Solution of equation (1.13.4) is written as,

$$\psi = A \cos Kx + B \sin Kx \quad \dots(1.13.5)$$

When $x = 0$ at $\psi = 0$, we get,

$$0 = A \cos 0 + B \sin 0$$

(Since $\cos 0 = 1$)

$$\therefore A = 0$$

When $x = a, \psi = 0$

$$\therefore 0 = A \cos Ka + B \sin Ka$$

$$\text{But } A = 0$$

$$\therefore B \sin Ka = 0$$

...(1.13.6)

Here B need not be zero

$\therefore \sin Ka = 0$ only when

$$Ka = \frac{\sqrt{2mE}}{\hbar} a = n\pi$$

(where $n = 0, 1, 2, 3, \dots$) ... (1.13.7)

Where,

n = quantum number

$$\therefore \psi_n = B \sin \left(\frac{n\pi}{a} x \right) \quad \dots (1.13.8)$$

which represents the permitted solutions. In Equation (1.13.8), $n = 0$ is not acceptable because for $n = 0, \psi = 0$, means the electron is not present inside the box which is not true.

$$\text{as } K^2 = \frac{8\pi^2 m E}{h^2}$$

$$\text{and } K = \frac{n\pi}{a}$$

$$\therefore \frac{(n\pi)^2}{a^2} = \frac{8\pi^2 m E}{h^2}$$

$$\therefore E_n = \frac{n^2 h^2}{8ma^2}$$

($n = 1, 2, 3, \dots$)

$$\therefore E_n \propto n^2$$

This shows that the energy of the particle can have only certain values which are Eigen values.

1.14 Wave Functions, Probability Density and Energy

MU – Dec. 17, May 18

Q. What is the significance of wave function? Derive the expression for energy Eigen values for free particle in one-dimensional potential well. (Dec. 17, May 18, 5 Marks)

From Equation (1.13.5), wave function for a particle in a box with energy E_n is given by,

$$\psi_n = B \sin kx = B \sin \sqrt{\frac{2mE}{\hbar^2}} \cdot x$$

$$= B \sin \frac{n\pi x}{a}$$

(From equation (1.13.8))

$$\therefore \int_{-\infty}^{\infty} |\psi_n|^2 dx = \int_0^a |\psi_n|^2 dx \\ = B^2 \int_0^a \sin^2\left(\frac{n\pi x}{a}\right) dx$$

Using,

$$\begin{aligned} \sin^2\theta &= \frac{1 - \cos 2\theta}{2} \\ &= \frac{B^2}{2} \int_0^a \left[1 - \cos 2\left(\frac{n\pi x}{a}\right) \right] dx \\ &= \frac{B^2}{2} \left[\int_0^a dx - \int_0^a \cos \frac{2n\pi x}{a} \cdot dx \right] = \frac{B^2}{2} \\ &= \frac{B^2}{2} \left[(a - 0) - \frac{a}{2n\pi} \cdot (\sin 2n\pi - \sin 0) \right] \\ &= \frac{B^2}{2} \left[a - \frac{a}{2n\pi} \cdot (0 - 0) \right] \\ &= B^2 \cdot \left(\frac{a}{2} \right) \end{aligned} \quad \dots(1.14.1)$$

Rewriting Equation (1.14.1), for normalized wave function,

$$\int_{-\infty}^{\infty} |\psi|^2 dx = 1 \quad \dots(1.14.2)$$

\therefore From Equations (1.14.1) and (1.14.2), wave functions are normalized if,

$$B = \sqrt{\frac{2}{a}} \quad \dots(1.14.3)$$

$$\therefore \psi_n = B \sin \frac{n\pi x}{a} \text{ is given by}$$

$$\psi_n = \sqrt{\frac{2}{a}} \cdot \sin \frac{n\pi x}{a} \quad (n = 1, 2, 3, \dots)$$

Since the **particle in a box** is a problem under quantum mechanical conditions, the prime questions to be considered are, the most probable location of the particle in the box and its energies. We can write Eigen functions $\psi_1, \psi_2, \psi_3 \dots$ for particle in a box by putting

$n = 1, 2, 3 \dots$ respectively.

Case - 1 : For $n = 1$

$$\therefore \psi_1 = \sqrt{\frac{2}{a}} \cdot \sin\left(\frac{\pi}{a}\right)x$$

Here

$$\psi_1 = 0, \text{ both } x = 0 \text{ and } x = a$$

But ψ_1 has maximum value for $x = a/2$ (for $x = a/2, \sin \pi/2 = 1$)

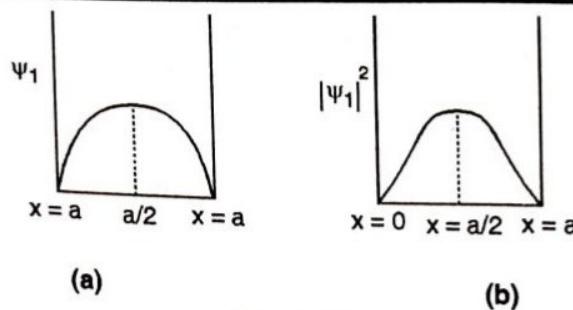


Fig. 1.14.1

Fig. 1.14.1

$$\therefore \Psi_2 = \sqrt{\frac{2}{a}} \sin\left(\frac{2\pi}{a}\right)x$$

Means $\psi_2 = 0$ at $x = 0, a/2$ and a and ψ_2 is maximum at $x = a/4$ and $\frac{3a}{4}$.
 In Fig. 1.14.2(a), we have $\psi_2 \rightarrow \infty$ as $x \rightarrow \pm \infty$.

In Fig. 1.14.2(b), we have $|\psi_2|^2 \rightarrow x$, which shows that particle cannot be observed either at the walls or at the center.

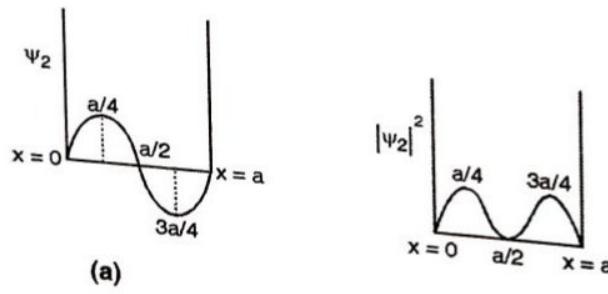


Fig. 1.14.2

Case - 3 : For n = 3

$$\Psi_3 = \sqrt{\frac{2}{a}} \cdot \sin\left(\frac{3\pi}{a}\right)x$$

For this case $\psi_3 \rightarrow a$ and $|\psi_3|^2 \rightarrow a$ are shown in Figs. 1.14.3(a) and (b) respectively.

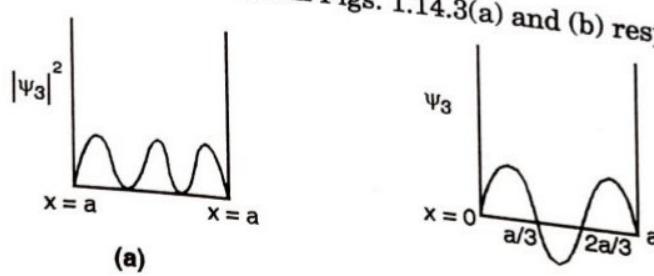


Fig. 1.14.3

1.15 Particle in a Finite Potential Well /Barrier and Tunnelling Effect

- A reader can understand this term in layman's language by just comparing a vehicle crossing the hill through a tunnel. It is not practical or may not be possible to take the vehicle all the way to the top of the hill merely to cross it. Tunnel may allow a vehicle without possessing the required energy to reach the top in order to cross it.

- Now we will take the case of classical physics. Let's consider a particle made to strike on a hard wall. It is not possible to find it on other side for obvious reasons. Quantum physics has a complete opposite solution to offer. Consider a quantum particle in the following case of potential barrier of height V_0 and thickness 'a'

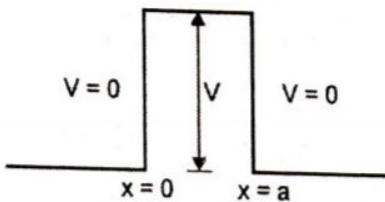


Fig. 1.15.1

- On LHS and RHS of this barrier, the potential energy of particle is zero (only KE). When a particle approaches this barrier from LHS with total energy E which is kinetic energy, if $E < V$, the particle will be reflected from the barrier. To be found inside the barrier the required condition is $E \geq V$. Here classical physics and Quantum physics offer exactly opposite solutions. Quantum physics says for $E < V$, there exists a finite chance for the particle to be found not only in the barrier but on RHS of the barrier as well. We say that the particle has tunnelled through the potential barrier. Hence this phenomenon is called tunnelling. It has been proved that electron exhibits tunnelling and based on this we have certain components, tunnel divide, etc.

One-dimensional time independent form of Schrodinger equation along 'x' axis is given by

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2m}{h^2} (E - V) \psi = 0 \quad \dots (1.15.1)$$

For LHS of the barrier let's consider $\psi = \psi_1$ and it is known that $V = 0$

$$\therefore \frac{d^2\psi_1}{dx^2} + \frac{8\pi^2m}{h^2} E \psi_1 = 0 \quad \dots (1.15.2)$$

The same situation is available on RHS of potential barrier.

Let take $\psi = \psi_3$ there

$$\therefore \frac{d^2\psi_3}{dx^2} + \frac{8\pi^2m}{h^2} E \psi_3 = 0 \quad \dots (1.15.3)$$

For $E < V$ and $V = V_0$ take $\psi = \psi_2$

$$\therefore \frac{d^2\psi_2}{dx^2} + \frac{8\pi^2m}{h^2} (E - V_0) \psi_2 = 0 \quad \dots (1.15.4)$$

Close inspection of Equations (1.15.2) and (1.15.3) it is clear that they are of $x^2 + a^2 = 0$ type hence their solutions will be of $(x - ia)(x + ia) = 0$ i.e. complex in nature. Hence their solutions are given by

$$\psi_1 = Ae^{ikx} + Be^{-ikx} \quad \dots (1.15.5)$$

$$\psi_3 = Ce^{ikx} + De^{-ikx} \quad \dots (1.15.6)$$

Where

$$K = \frac{8\pi^2mE}{h^2}$$

For Equation (1.15.4), for the condition $E < V$ the equation is modified as

$$\frac{d^2\psi_2}{dx^2} - \frac{8\pi^2m}{h^2} (V - E) \psi_2 = 0$$

it is of $(x^2 - a^2) = 0$ type

$$\therefore x = a \text{ or } x = -a$$

i.e. the solutions are of real type

hence

$$\psi_2 = Le^{kx} + Me^{-kx} \quad \dots (1.15.7)$$

- From Equation (1.15.5) it is clear that Ae^{ikx} represents de-Broglie wave travelling in $-x$ direction with amplitude A, and Be^{-ikx} represents de-Broglie wave travelling in $-x$ direction with amplitude B. Which further represents reflection. The same is true for Equation (1.15.6) only for the case $x > a$ or RHS of potential barrier, there is no reflection hence D = 0.
- For any point inside the potential barrier. We have
 - 1) Le^{kx} which shows the presence of wave in $+x$ direction in perfect contradiction with classical physics.
 - 2) Me^{-kx} representing exponentially decreasing wave.
- As a summary we can write that the wave function ψ_1 which was approaching potential barrier from left to right is successfully crossing the barrier with energy $E < V$, where V = height of potential barrier. Such penetration is called tunnel effect.

1.15.1 Tunnelling Effect Examples

- 1) **α - Decay :** When an α -particle is observed to be emitted from nucleus it is called α -decay. When α -decay takes place the energy associated with it is around 4 to 9 MeV. When such a decay takes place the forces in the nucleus set up potential barrier of height of the order of 30 MeV. The emission of α -particle or α -decay is possible only if tunneling is accepted. Means a particle with just around 4 to 9 MeV can cross the barrier height of 30 MeV.
- 2) **Tunnel Diode :** A special category diode which makes use of tunneling effect by virtue of heavily doped p-n junction with very small barrier around 100 \AA wide has a specific 'Negative Resistance', exhibited on its I-V characteristics as shown below.

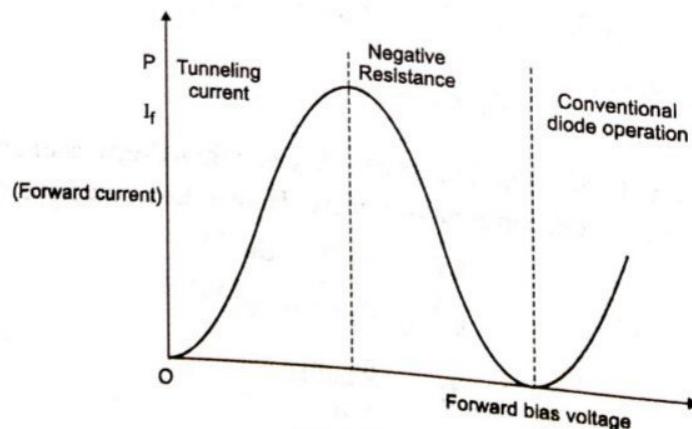


Fig. 1.15.2

Under forward biased operation, as voltage increases, electrons tunnel through the very narrow p-n junction barrier. As voltage increases further the current drops representing negative resistance. After that it enters into normal operation.

3) Scanning Tunneling Microscope (STM) :

- As the name suggests, through scanning, information of electronic configuration of surface atoms and the electron distribution around them is detected.
- A very sharp metal needle is brought very close to the surface to be imaged but not in touch with specimen.
- When a voltage is applied between tip of the needle and the specimen a tunneling current flows. Remember metal tip is not in contact with specimen hence it represents infinite resistance and results in current or does not permit any current. But it is the effect of tunneling which allows a small current.
- The small current is amplified and measured and based on this mapping of the surface is done.

Ex. 1.15.1 : An electron is bound in a one-dimensional potential well of width 2 \AA but of infinite height. Find its energy values in the ground state and in first two excited state.

Data : Width of potential well, $a = 2 \times 10^{-10} \text{ m}$

To find : E_0, E_1, E_2

MU - Dec. 12, May 13, Dec. 13, May 14, Dec. 14, May 16, 3/5 Marks

Note : A similar problem with $a = 2.5 \text{ \AA}$ was asked in Dec. 12.

Soln. : Using equation for energy of the electron in one-dimensional potential well is given by

$$E = \frac{n^2 h^2}{8m a^2}$$

For ground state $n = 1$ and E_0

$$\begin{aligned} \therefore E_0 &= \frac{1^2 (6.63 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (2 \times 10^{-10})^2} \\ &= 1.5 \times 10^{-18} \text{ J} \\ &= 9.43 \text{ eV} \end{aligned}$$

...Ans.

Similarly for first excited state, $n = 2$ and $E = E_1$

$$\begin{aligned} \therefore E_1 &= \frac{2^2 (6.63 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (2 \times 10^{-10})^2} \\ &= 6 \times 10^{-18} \text{ J} = 37.5 \text{ eV} \end{aligned}$$

Second excited state, $n = 3$ and $E = E_2$

$$\begin{aligned} \therefore E_2 &= \frac{3^2 (6.63 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (2 \times 10^{-10})^2} \\ &= 1.35 \times 10^{-17} \text{ J} \\ &= 84.375 \text{ eV} \end{aligned}$$

...Ans.

Ex. 1.15.2 : A particle is moving in one-dimensional potential well of infinite height and width 25 \AA . Calculate the probability of finding the particle with an interval of 5 \AA at a distance of $a/2, a/3$ and a where a is the width of the well assuming that particle is in least state of energy.



Data : Width of well = $a = 25 \times 10^{-10}$ m, $\Delta x = 5 \times 10^{-10}$ m, $n = 1$ (Ground state)

To find :

$$P_1 = \text{Probability at } x = a/2$$

$$P_2 = \text{Probability at } x = a/3$$

$$P_3 = \text{Probability at } x = a$$

Soln. : Use formula

$$\psi_n = \sqrt{\frac{2}{a}} \cdot \sin\left(\frac{n\pi}{a}\right)x$$

We know that probability of finding a particle over an elementary distance Δx is given by

$$P = \psi^2 \Delta x$$

$$\therefore P_1 = \left[\sqrt{\frac{2}{a}} \cdot \sin \frac{\pi}{a} \cdot \frac{a}{2} \right]^2$$

$$\Delta x = 0.4$$

...Ans.

Similarly,

And,

$$P_2 = 0.15$$

$$P_3 = 0$$

...Ans.

...Ans.

1.16 Introduction to Quantum Computing

- Quantum computing is the area of study focussed on developing computing methods based on the principle of quantum theory.
- The execution of computation is expected to be done by quantum computers, which are yet to be realized in practice.
- In classical computers, a bit is a fundamental unit of information represented as '0' or '1'.
- In quantum computing, the concept is fundamentally different. It stores information in a quantum system such as an atom or a photon. We can choose two electronic states of an atom or two different polarization orientations of light for the two states. But as per quantum mechanics, the atom apart from the two distinct states can also be prepared in a state which is said to be a coherent superposition of both the state to represent '0' and '1'. Since it follows quantum principles, it becomes a quantum system and is called a quantum bit or a qubit. It acts as fundamental unit of information in quantum computer.
- In conventional computers we have digital gates like AND, NOT, etc. Following symbol represents controlled NOT gate.



Fig.1.16.1

Similarly other gates can be created.

- A quicker computation and small sized computer can be obtained through quantum computing concept.

A Quick Revision

- Louis de Broglie put forward the dual behaviour in terms of a hypothesis which states "If the radiation behaves as particle under certain circumstances and wave under other circumstances, then one can even expect that, entities which ordinarily behave as particles will exhibit properties attributed to only waves under appropriate circumstances.
- Electromagnetic waves always travel with a constant velocity c , whereas matter waves may travel with that phase velocity which depends on mass and velocity of particle.
- Based upon the result from de-Broglie hypothesis,

$$\lambda = \frac{12.26}{\sqrt{V}} \text{ Å}$$

it was predicted that if a suitable voltage is considered then a wavelength of the order of few Å which is of the order of interplanar spacing in the crystal can be obtained.

- Experimental value which is obtained by considering electron as wave is verified with theoretical value. This confirms de Broglie's hypothesis.
- It is important to know that wave function ψ has no direct physical significance but $|\psi|^2$ has.
- Heisenberg's uncertainty principle states that quantum mechanics does not simultaneously permit the determination of position and momentum of a particle accurately.
- The KE of electron must be greater than 19 MeV if it is to remain inside the nucleus. This is practically not possible (atom becomes unstable); hence one can say that electron cannot survive inside the nucleus.
- Based on de-Broglie's idea of matter waves, Schrödinger developed a mathematical theory which plays the same role as Newton's laws in classical mechanics.





Crystallography

Syllabus

(Prerequisites : Crystal physics (unit cell, space lattice, crystal structure, simple cubic, body centered cubic, face centered cubic, diamond structure, production of x-rays), Miller indices, interplanar spacing, x-ray diffraction and Bragg's law, determination of crystal structure using Bragg's diffractometer

Learning Objectives :

After reading this chapter, learner should be able to

- Understand Miller indices for planes and direction
- Understand x-ray diffraction
- Derive Bragg's law
- Determine crystal structure using X - ray diffractometer

2.1 Introduction

Matter occurs in three states – solid, liquid and gaseous. Different solids have different structures such as crystalline and amorphous.

Table 2.1.1

Sr. No.	Crystalline solids	Amorphous solids
1.	Form a regular, repeated three dimensional pattern of atoms, ions or molecules of which they are made up	No such regular or repeated pattern or arrangement is observed
2.	Since the atoms or molecules are arranged in a regular manner over a long distance, a long range order prevails in crystals.	Atoms or molecules are not arranged in regular manner hence no such order exists. Only a short range order is exhibited.
3.	Mathematics involved to analyze is simpler	Mathematics involved is more complicated

More about crystal structure

- In a crystalline solid, each atom or molecule is fixed at a definite point in space and at a definite distance from and in a definite angular orientation to other atoms or molecules surrounding it.
- Hence, a perfect crystal is considered to be made up of infinite regular repetition of identical structural units called **unit cells**.
- Crystals can further be divided into two main categories viz. single crystal and polycrystalline.
- Single crystal is the one where orientation of atoms or molecules is uniform throughout the entire crystal.

- Polycrystalline is the one where whole crystal is made up of smaller crystallites. Each small crystallite is called **grain**. The grains form the whole crystal, for example, *quartz*.
- For a given grain, orientation of atoms is always uniform, but grains are generally oriented randomly. Due to this reason single crystals are preferred.

2.2 Miller Indices

MU - Dec. 18, May 19

Q. Draw (123) , (321) , (102) .

(Dec. 18, 3 Marks)

Q. Draw the following for a cubic unit cell – $\overline{(123)}$, (200) , $\overline{(230)}$.

(May 19, 3 Marks)

2.2.1 Crystal Planes and Miller Indices

“Identification of a plane”

- The crystal structure may be regarded as made up of an aggregate of a set of parallel equidistant planes passing through at least one lattice point or a number of lattice points. In a given crystal a plane may be selected in a number of ways.
- The position of a crystal plane can be specified in terms of three integers called **Miller indices**. Miller developed a method by which one can find three integers $(hk\bar{l})$. This method is universally employed.

The procedure is as follows:

1. Find the intercepts of the plane with the crystal axes along the basic vectors \vec{a} , \vec{b} and \vec{c} . Let the intercepts be m , n and p respectively.
2. Express m , n and p in terms of the respective basic vectors, as fractional multiples we get, $\frac{m}{a}$, $\frac{n}{b}$, $\frac{p}{c}$.
3. Take the reciprocals of the three fractions, i.e. $\frac{a}{m}$, $\frac{b}{n}$, $\frac{c}{p}$.
4. Find the LCM of the denominator by which the above three ratios are multiplied. This operation reduces them to a set of three integers h , k and l . The resultant three integers are called Miller indices of the given plane, denoted by $(hk\bar{l})$.
- While finding Miller indices of a plane, following points should be kept in mind:
 - (i) When a plane is parallel to one of the axes it is said to have intercept at ∞ and its reciprocal is zero.
 - (ii) When the intercept of a plane is on the negative part, the corresponding Miller index is distinguished by a bar over it.
 - (iii) Parallel planes have same Miller indices.
 - (iv) Miller indices, in practice do not define a particular plane but a set of parallel planes.
 - (v) A plane passing through the origin is defined in terms of a parallel plane having non-zero intercept.

Ex. 2.2.1 : Find the Miller indices of planes given in Fig. P. 2.2.1.

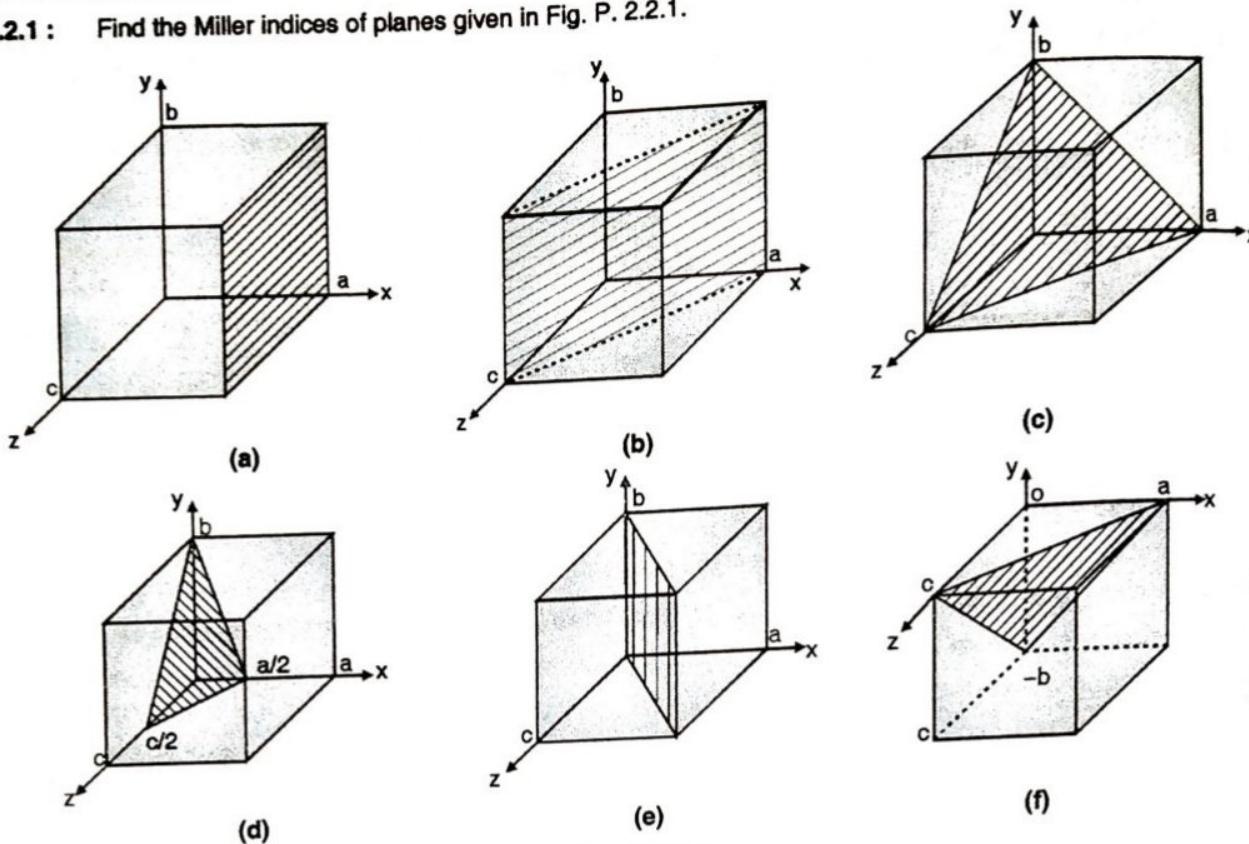


Fig. P. 2.2.1

Soln. :

(a) Here $m = a, n = \infty, p = \infty$

$$\therefore \frac{m}{a}, \frac{n}{b}, \frac{p}{c} :: \frac{a}{a}, \frac{\infty}{b}, \frac{\infty}{c}$$

\therefore Reciprocals

$$\frac{a}{m}, \frac{b}{n}, \frac{c}{p} :: \frac{a}{a}, \frac{b}{\infty}, \frac{c}{\infty} :: 1, 0, 0$$

Since they are integers, no LCM is needed. The three integers are (1, 0, 0).

$$\therefore (hkl) = (100)$$

(b) Here $m = a, n = \infty, p = c$

$$\frac{a}{m}, \frac{b}{n}, \frac{c}{p} :: \frac{a}{a}, \frac{b}{\infty}, \frac{c}{c} :: 1, 0, 1$$

$$\therefore (hkl) = (101)$$

(c) Here $m = a, n = b, p = c$

\therefore Reciprocals

$$\frac{a}{m}, \frac{b}{n}, \frac{c}{p} :: \frac{a}{a}, \frac{b}{b}, \frac{c}{c} :: 1, 1, 1$$

$$\therefore (hkl) = (111)$$

(d) Here $m = \frac{a}{2}, n = b, p = \frac{c}{2}$

\therefore Reciprocals

$$\frac{a}{m}, \frac{b}{n}, \frac{c}{p} :: \frac{a}{a/2}, \frac{b}{b}, \frac{c}{c/2} :: 2, 1, 2$$

$$\therefore (hkl) = (212)$$

- (e) Since the plane is passing through the origin, consider a plane which is parallel to it and not passing through the origin.

Here $m = -a$, $n = \infty$, $p = c$

\therefore Reciprocals

$$\frac{a}{m}, \frac{b}{n}, \frac{c}{p} :: \frac{a}{-a}, \frac{b}{\infty}, \frac{c}{c} :: -1, 0, 1$$

Taking negative intercept in consideration

$$(hkl) = (\bar{1} 0 1)$$

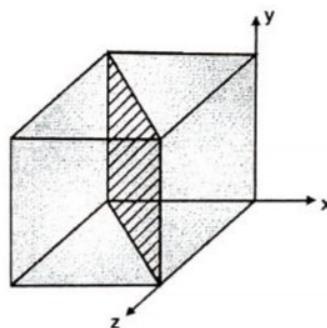


Fig. P. 2.2.1(g) : Shifting of plane

- (f) Here $m = a$, $n = -b$, $p = c$

\therefore Reciprocals

$$\frac{a}{m}, \frac{b}{n}, \frac{c}{p} :: \frac{a}{a}, \frac{b}{-b}, \frac{c}{c} :: 1, -1, 1$$

$$\therefore (hkl) = (1 \bar{1} 1)$$

Ex. 2.2.2 : Obtain the Miller indices of a plane which intercepts $a, \frac{b}{3}, 2c$ in a simple cubic unit cell.

Soln. :

$$\text{Intercepts } m:n:p :: a:\frac{b}{3}:2c$$

\therefore Intercepts in terms of respective basic vector

$$\left(\frac{m}{a}, \frac{n}{b}, \frac{p}{c}\right) = \left(\frac{a}{a}, \frac{b/3}{b}, \frac{2c}{c}\right)$$

Reciprocals

$$\left(\frac{a}{m}, \frac{b}{n}, \frac{c}{p}\right) = \left(\frac{a}{a}, \frac{b}{b/3}, \frac{c}{2c}\right)$$

$$\text{As, } a = b = c = \left(1, 3, \frac{1}{2}\right)$$

$$\text{Taking LCM} = (2 6 1)$$

$$\therefore (hkl) = (2 6 1)$$

It is also meaningful to learn how to draw a plane if Miller indices are provided.

Let us follow a simple procedure:

- Take the reciprocals of indices

- (ii) These represent intercepts on corresponding axes
 - (iii) Join the intercepts

Ex. 2.2.3 : Draw the following planes.

(i) $(1 \ 1 \ 0)$ (ii) $(1 \ \bar{1} \ 0)$ (iii) $(2 \ 3 \ 1)$ (iv) $(\bar{2} \ 3 \ \bar{1})$
 (v) $(\bar{2} \ 0 \ \bar{1})$ (vi) $(3 \ 0 \ 0)$ (vii) $(1 \ 2 \ \bar{1})$ (viii) $(1 \ \bar{2} \ 3)$ (ix) $(\bar{1} \ 1 \ \bar{1})$

Soln. :

*Only simple cubic structures where $a = b = c$ are to be considered.

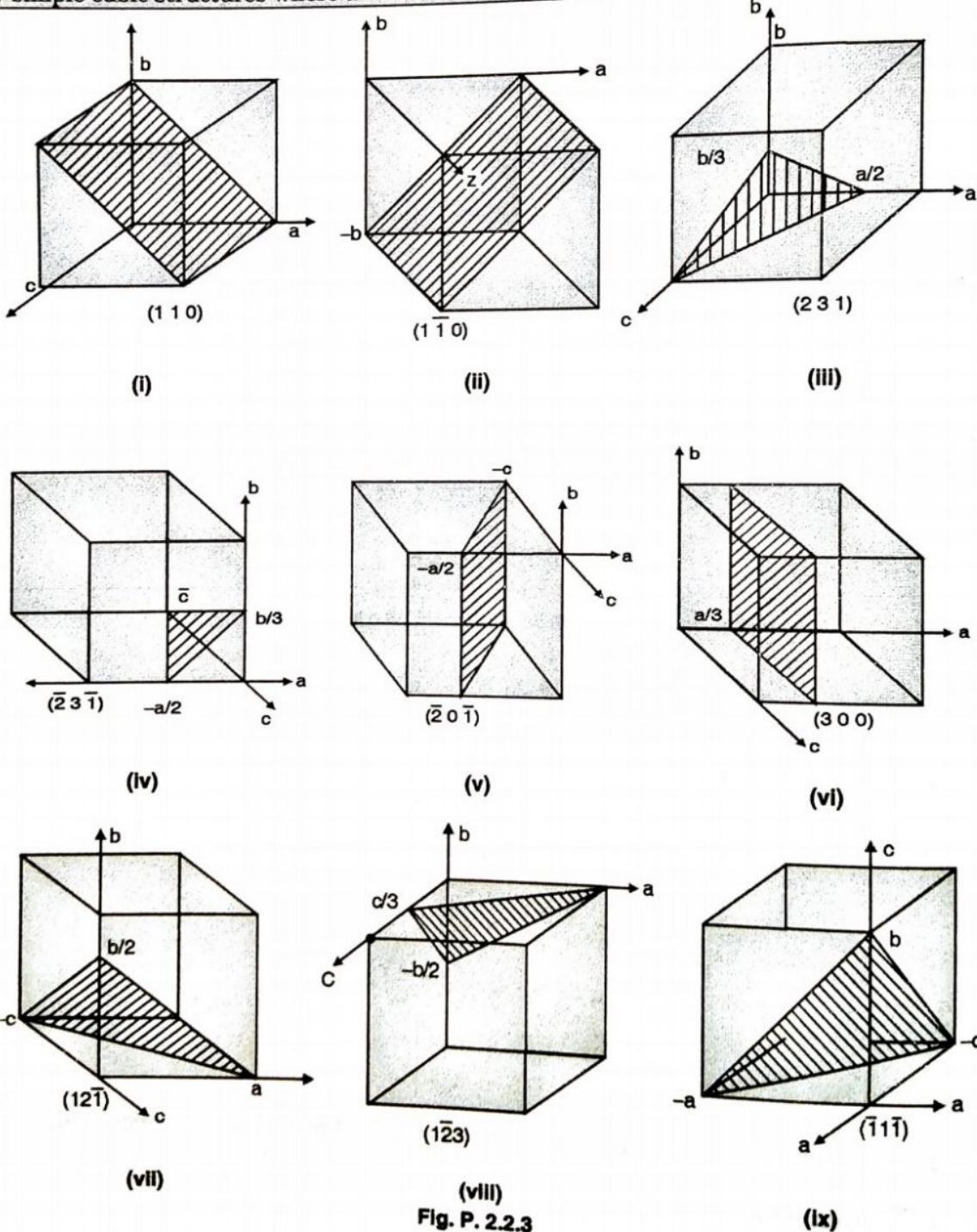


Fig. P. 2.2.3

2.2.2 Interplanar Spacing

- It is clear that parallel planes have same Miller indices.
- At the same time spacing between such parallel planes is an important parameter.
- It is denoted by $d_{hk\ell}$ i.e. the interplanar spacing between planes with same Miller indices $(hk\ell)$.
- In the Fig. 2.2.1, we have plane ABC with Miller indices $(hk\ell)$. Other plane with same Miller indices is assumed to pass through point 'O' (which is not shown in the diagram). The perpendicular spacing between these two planes is $ON = d$.

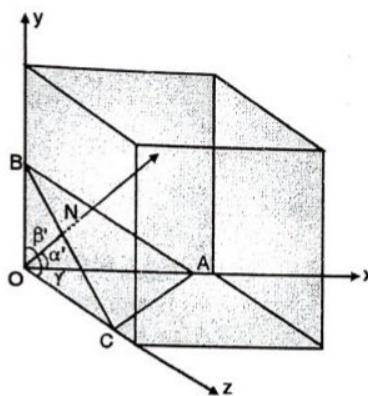


Fig. 2.2.1 : Interplanar spacing

- - Let ON make an angle α' with X axis, β' with Y axis and γ' with Z axis.

$$\therefore \cos \alpha' = \frac{ON}{OA} = \frac{d}{OA}$$

$$\cos \beta' = \frac{ON}{OB} = \frac{d}{OB}$$

$$\cos \gamma' = \frac{ON}{OC} = \frac{d}{OC}$$

$$\text{But } OA = \frac{a}{h}$$

$$OB = \frac{a}{k}$$

$$OC = \frac{a}{l}$$

$$\therefore \cos \alpha' = \frac{d}{a/h} = \frac{dh}{a}$$

$$\cos \beta' = \frac{d}{a/k} = \frac{dk}{a}$$

$$\cos \gamma' = \frac{d}{a/l} = \frac{dl}{a}$$

$$\left. \begin{aligned} \cos \alpha' &= \frac{ON}{OA} = \frac{d}{OA} \\ \cos \beta' &= \frac{ON}{OB} = \frac{d}{OB} \\ \cos \gamma' &= \frac{ON}{OC} = \frac{d}{OC} \end{aligned} \right\} \dots(2.2.1)$$

$$\left. \begin{aligned} OB &= \frac{a}{k} \\ OC &= \frac{a}{l} \end{aligned} \right\} \text{Intercepts of plane ABC} \dots(2.2.2)$$

$$\left. \begin{aligned} \cos \alpha' &= \frac{dh}{a} \\ \cos \beta' &= \frac{dk}{a} \\ \cos \gamma' &= \frac{dl}{a} \end{aligned} \right\} \dots(2.2.3)$$

Using relation of space geometry,

$$\cos^2 \alpha' + \cos^2 \beta' + \cos^2 \gamma' = 1$$

$$\therefore \left(\frac{dh}{a}\right)^2 + \left(\frac{dk}{a}\right)^2 + \left(\frac{dl}{a}\right)^2 = 1$$



$$\therefore \frac{d^2}{a^2} (h^2 + k^2 + l^2) = 1$$

$$\therefore d_{hkl} = \frac{a}{\sqrt{h^2 + k^2 + l^2}}$$

...(2.2.4)

This is the expression for interplanar spacing in terms of lattice constant a and Miller indices $(h k l)$.

Ex. 2.2.4 : Silver has FCC structure and its atomic radius is 1.414 \AA . Find the interplanar spacing for (200) and (111) planes.

Soln. :

Given : Structure is FCC.

$$r = 1.414 \text{ \AA}$$

Step 1:

$$\text{Formula } a = \frac{4r}{\sqrt{2}}$$

$$(\text{For FCC}) = \frac{4 \times 1.414}{\sqrt{2}}$$

$$= 3.999 \text{ \AA}$$

Step 2:

$$\text{Formula } d_{hkl} = \frac{a}{\sqrt{h^2 + k^2 + l^2}}$$

$$\text{For } (200) d_{200} = \frac{3.999}{\sqrt{2^2 + 0^2 + 0^2}} = 1.999 \text{ \AA}$$

...Ans.

$$\text{For } (111) d_{111} = \frac{3.999}{\sqrt{1^2 + 1^2 + 1^2}} = 2.309 \text{ \AA}$$

...Ans.

2.2.3 Directions

"A correct path to reach a plane"

- The direction in crystallography is a line joining any two points of the lattice. The indices of direction are the vector components of direction resolved along each of the axes. The vector components are again multiples of lattice constants.
- Therefore, the indices of the lattice site are simultaneously the indices of direction, assuming that the directions are through the origin. The directions are denoted by square brackets i.e. $[hkl]$. Some of the directions are shown below in Fig. 2.2.2(a).

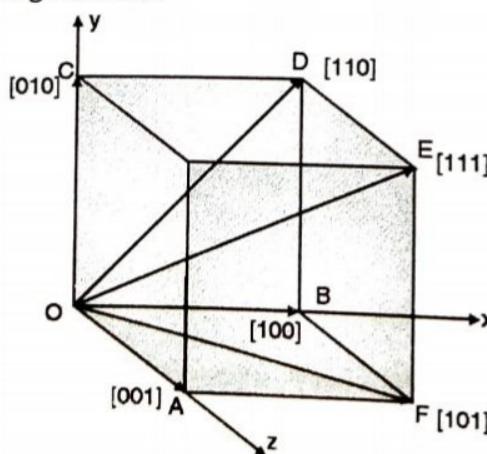


Fig. 2.2.2(a)

\rightarrow OA represents [0 0 1], OB represents [1 0 0], OC represents [0 1 0].

\rightarrow OD represents [1 1 0], OE represents [1 1 1], OF represents [1 0 1].

If there is any negative component, it is denoted by a bar placed above it.

For example, in Fig. 2.2.2(a).

\rightarrow BO represents [$\bar{1}$ 0 0], AO represents [0 0 $\bar{1}$], FO represents [$\bar{1}$ 0 $\bar{1}$], and so on.

- As far as a plane with Miller indices 0 and 1 is considered, drawing a direction is simple. But when indices are other than 0 and 1, it has been observed that students draw a plane and a normal to it from origin.
- Students draw a direction on a 2-D paper using a picture of 3-D model. This technique does not have any precision regarding the normal.
- Consider Fig. 2.2.2(b) which has direction [1 2 0] plotted on it.

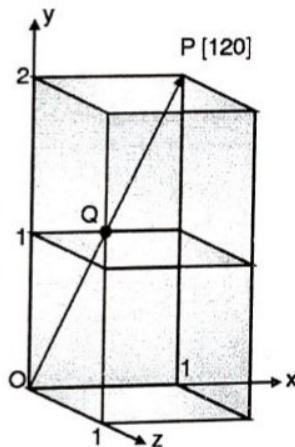


Fig. 2.2.2(b)

- Here, \overrightarrow{OP} represents [1 2 0], co-ordinates of point P are (1, 2, 0). Careful observation reveals that, co-ordinates of point Q are $(\frac{1}{2}, 1, 0)$. Using this information, we can adopt a method to scale down the co-ordinates.
- The main advantage of this method is removal of the difficulty faced by students when they draw a plane and draw a normal to it from origin to express direction. The normal in such cases is simply assumed to be just a normal, but it does not have any mathematical support.
- The procedure to draw a direction is explained through following steps (with an example [1 2 0])
 1. Divide all the indices by the highest index.

(For [1 2 0], highest index is 2. $\therefore \left[\frac{1}{2}, 1, 0 \right]$).

2. At least one index will be unity (here it is corresponding to Y axis).
3. Select this particular axis as shown in Fig. 2.2.2(b) for better visualization.
(Advantage of this is any point taken on plane ABCD (Fig. 2.2.3) will have its Y co-ordinates unity).



4. Plot other two co-ordinates on remaining axes (draw x co-ordinate = $\frac{1}{2}$ and Z co-ordinate = 0)

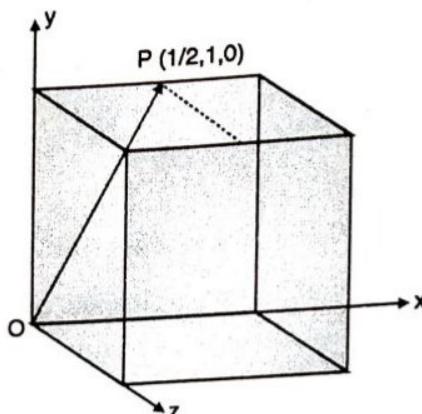


Fig. 2.2.3

5. Find the point with co-ordinates obtained in step 1 above (co-ordinates of P are $(\frac{1}{2}, 1, 0)$)
 →
 6. Join that point with the origin, the resultant vector in the desired direction OP is $[1 \ 2 \ 0]$.
 Compare Figs. 2.2.3 and 2.2.2(b).

Ex. 2.2.5 : Draw the following : (i) $[1 \ 2 \ 1]$, (ii) $[2 \ 3 \ 1]$, (iii) $[1 \ \bar{2} \ 0]$

Soln. :

(i) $[2 \ 3 \ 0]$ divide by 3, $(\frac{2}{3}, 1, 0)$ draw a point $(\frac{2}{3}, 1, 0)$

→
OQ represents [2 3 0]

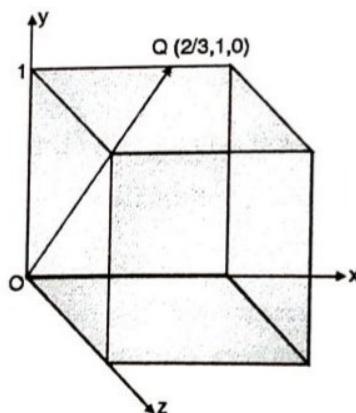


Fig. P. 2.2.5(a)

(ii) $[1 \ 2 \ 1]$ divide it by 2

$$\left(\frac{1}{2}, 1, \frac{1}{2}\right)$$

draw a point $(\frac{1}{2}, 1, \frac{1}{2})$

→
∴ OR represents [1 2 1]

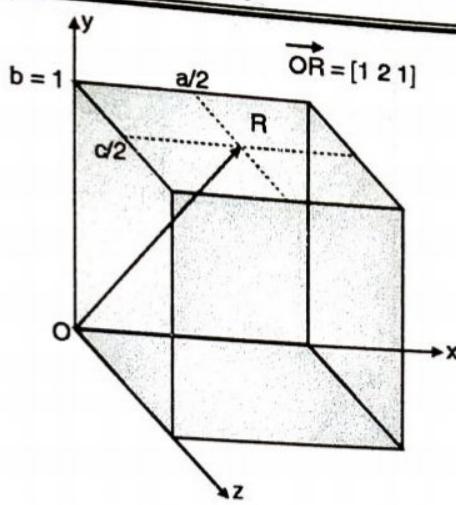


Fig. P. 2.2.5(b)

(iii) $[2 \bar{3} 1]$ divide it by 3, we get

$$\left(\frac{2}{3}, 1, \frac{1}{3}\right),$$

draw this point

\rightarrow
 $\therefore \vec{OS}$ represents $[2 \bar{3} 1]$

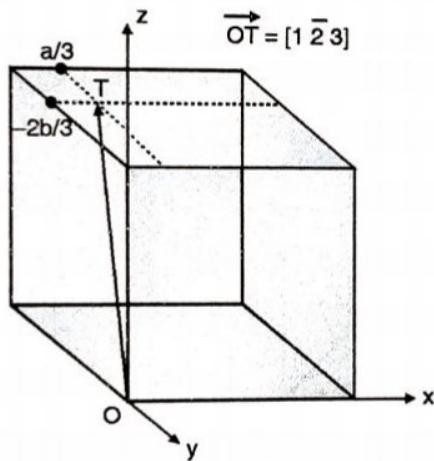


Fig. P. 2.2.5(c)

(iv) $[1 \bar{2} 3]$

(Note that one of the indices is negative.)

Divide by 3, we get $\left(\frac{1}{3}, -\frac{2}{3}, 1\right)$

Draw the point $\left(\frac{1}{3}, -\frac{2}{3}, 1\right)$.

\rightarrow
 $\therefore \vec{OT}$ represents $[1 \bar{2} 3]$

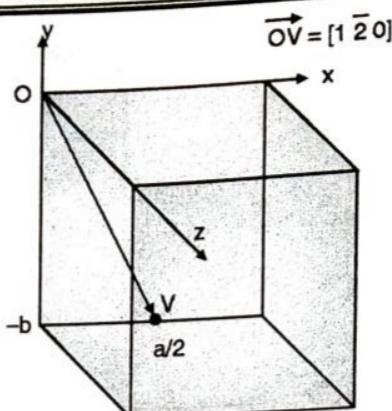


Fig. P. 2.2.5(d)

(v) $[1 \bar{2} 0]$

(Note that one of the indices is negative.)

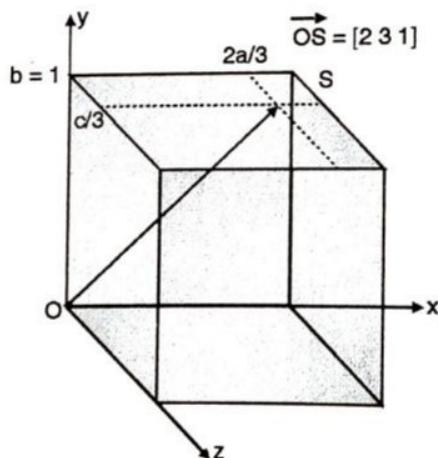
Divide by 2, we get $\left(\frac{1}{2}, -1, 0\right)$ Draw the point $\left(\frac{1}{2}, -1, 0\right)$. $\therefore \vec{OV}$ represents $[1 \bar{2} 0]$ 

Fig. P. 2.2.5(e)

2.3 X-ray Diffraction

MU - May 13, May 16, Dec. 17, May 18

- Q.** Derive Bragg's law. Explain why x-rays and not γ -rays are used for crystal structure analysis. What data about the crystal structure can be obtained from the x-ray diffraction pattern of a crystal? (May 13, 7 Marks)
- Q.** Explain the statement "crystal act as three dimensional grating with x-rays". (May 16, Dec. 17, 2 Marks)
- Q.** Why X-rays are used to study the crystal structure? (May 18, 3 Marks)

"Because of periodicity"

- Diffraction is defined as the bending of a ray of light when it encounters an object whose dimensions are of the order of the wavelength.

The explanation needs support of interference also.

Secondary wavelets originate at the location of the object and spread in all directions. The constructive and destructive interference of such waves results in increase and decrease of intensity at the corresponding regions which is known as **diffraction pattern**.

If the diffracting objects are located in random fashion, the superposition of individual diffraction effects due to various objects leads to a pattern which will not have any particular distribution of intensity. Whereas, if the diffracting objects are distributed in a regular pattern, then the diffraction pattern will also have regularity.

One essential aspect of diffraction is the availability of objects whose dimensions are extremely small, i.e. of the order of the wavelength of light incident on it.

In crystals we have seen that atoms are arranged in a perfectly ordered manner. Also the dimensions of atoms are 10^{-8} cm which is nearly of the same order of x-ray wavelength.

Hence, when x-rays are made incident upon crystals, we get an ordered, regular diffraction pattern or one can say that a **crystal act as a three-dimensional reflection grating with x-rays**.

2.4 Bragg's Law

MU - May 12, Dec. 12, May 14, May 19

- Q. Derive Bragg's equation. Explain construction and working of Bragg's spectrometer. (May 12, May 14, 4/8 Marks)
 (Dec. 12, 3 Marks)
- Q. Derive Bragg's law.
- Q. Derive Bragg's equation for x-ray diffraction in crystals. Calculate the glancing angle on a plane (1 0 0) of rock salt having lattice constant 2.814\AA corresponding to first order Bragg's diffraction maximum for x-rays of wavelength 1.541\AA . (May 19, 8 Marks)

- Before one starts the derivation of Bragg's law, it is necessary to understand Bragg's diffraction. W.L. Bragg and W.H. Bragg (father and son) put forward a novel idea for studying x-ray diffraction in crystals.
- They emphasised on the planes with same Miller indices i.e. parallel planes.
- It is possible to identify families of such planes. They addressed component plane of the said family as **Bragg planes**.
- When a monochromatic x-ray beam is made incident on them at an angle θ^* which is called as glancing angle, it is shown that constructive interference takes place between the rays scattered by the atoms only when a condition called Bragg's law as shown below is satisfied.

$$n\lambda = 2d \sin \theta$$

where, d = Interplanar spacing

n = Integer



Bragg's law

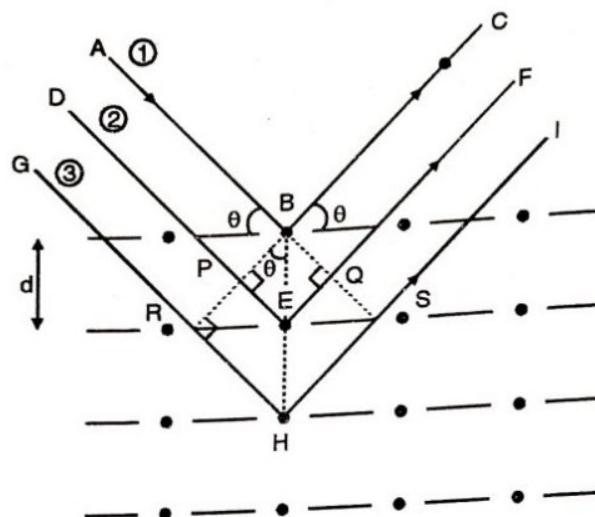


Fig. 2.4.1 : Bragg's law

- *Here, the angle θ is measured with respect to the horizontal and not from the normal as we do in optics.
 - When the condition for constructive interference is satisfied, a sudden jump in the intensity is observed. Now let's take a look at Bragg's law.
 - In the Fig. 2.4.1 we have Bragg planes with atoms shown with dots.
 - An ordered or regular arrangement of atoms has been depicted. Let the interplanar spacing be d . A monochromatic and parallel beam of x-rays at glancing angle θ is made incident on planes. Ray AB will be scattered at point B on the first plane. Rays DE and GH which are parallel to AB will also experience scattering at points E and H respectively at second and third plane. The scattering due to atoms on crystal plane is in all directions.
 - Among the scattered rays select rays BC and EF which are parallel to each other. It is assumed that they have path difference $\Delta = n\lambda$ and produce **constructive interference**. Bragg's law provides the condition at which $\Delta = n\lambda$. Let's obtain value of path difference Δ .
 - Draw perpendiculars BP and BQ to the rays DE and EF. One can say that upto BP, path covered by both the incident rays is the same. So BQ onwards parallel rays BC and EF cover the same distance.
- \therefore Path difference between rays 1 and 2 is

$$\Delta = PE + EQ$$

From ΔBPE and ΔBQE ,

$$PE = BE \sin \theta \text{ and,}$$

$$EQ = BE \sin \theta$$

$$\therefore \Delta = BE \sin \theta + BE \sin \theta$$

$$= 2 BE \sin \theta$$

$$= 2 d \sin \theta$$

$$(BE = d)$$

- As we have already assumed that constructive interference is taking place $\Delta = n\lambda$

$$\therefore n\lambda = 2d \sin \theta$$

The logic can be extended for rays 2 and 3 in the Fig. 2.4.1. Hence if rays 1 and 2 give constructive interference and rays 2 and 3 also give constructive interference, then rays 1 and 3 will also provide the same. In this case path difference between rays 1 and 3 will be

$$\Delta' = 4d \sin \theta = 2(2d \sin \theta) = 2(\Delta)$$

i.e. integral multiple of Δ .

Ex. 2.4.1 : Calculate the glancing angle on a plane (100) of rock salt having lattice constant 2.814A° corresponding to first order Bragg's diffraction maximum for X-rays of wavelength 1.641A° . May 19, 8 Marks

Soln. :

Given :

$$\text{Lattice constant } a = 2.814\text{A}^\circ$$

Crystal = Rock salt

Order of diffraction = 1

For rock salt interplanar spacing

$$D = \frac{a}{z} = \frac{2.814}{2} = 1.407\text{A}^\circ$$

Using Bragg's law

$$\begin{aligned} n\lambda &= 2d \sin \theta \\ \therefore \sin \theta &= \frac{n\lambda}{2d} \\ &= \frac{1 \times 1.541}{2 \times 1.407} \\ &= 0.5476 \\ \therefore \theta &= \sin^{-1}(0.5476) = 33.2^\circ \end{aligned}$$

... Ans.

2.5 Bragg's Spectrometer

MU - Dec. 13, Dec. 15, Dec. 16, Dec. 17, May 18, May 19

- Q. Explain with neat diagram : Construction of Bragg's x-ray spectrometer. Write the procedure to determine crystal structure. (Dec. 13, 4 Marks)
- Q. Explain analysis of crystal structure using Bragg's X-ray spectrometer. (Dec. 15, Dec. 16, Dec. 17, 5 Marks)
- Q. Explain with example how to determine crystal structure by Bragg's x-ray spectrometer. (May 18, 5 Marks)
- Q. Write a short note on Bragg's spectrometer. (May 19, 5 Marks)

- Based upon Bragg's law an instrument called as Bragg's spectrometer was designed. This is a modified form of ordinary spectrometer to suit the use of x-rays.

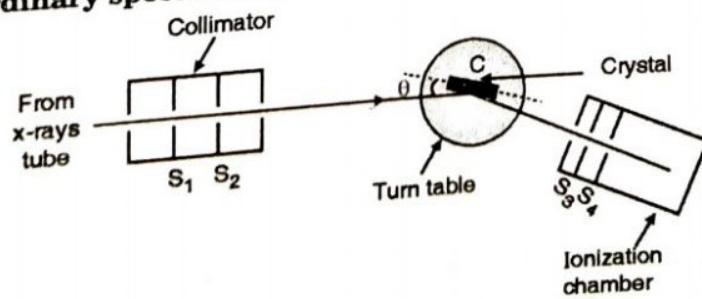


Fig. 2.5.1 : Schematic diagram of Bragg's spectrometer



- A monochromatic x-ray beam obtained from x-ray tube is made to pass through slits S_1 and S_2 which are made up of lead. The fine beam is then made to fall on the crystal C fixed on a crystal mount exactly at the centre of circular turn table.
- The x-rays reflected are collected by ionization chamber. Since ionization chamber is sturdy, the turn table is rotated till we get a sharp increase in the intensity.
- The sudden increase in the intensity of x-ray suggests that Bragg's law is satisfied at the given angle θ of the incident beam.
- The peak in ionisation current which represents the intensity occurs more than once as θ is varied because Bragg's law states $n\lambda = 2d \sin\theta$ i.e. for $n = 1, 2, 3, \dots$ we have $\theta_1, \theta_2, \theta_3, \dots$.
- If the intensity (or ionization current) is plotted against glancing angle then we get the graph as shown in Fig. 2.5.2.
- Using graph shown above we find the angles $\theta_1, \theta_2, \dots$ where the peak occurs.

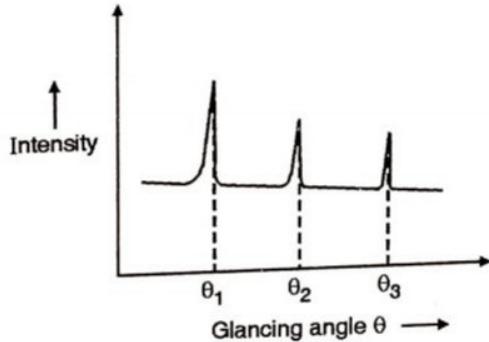


Fig. 2.5.2 : Variation of ionisation current

Determination of crystal structure (for cubic crystals)

Here the crystal face used for reflecting the x-rays can be so cut that it remains parallel to one set of planes, then to another and so on when placed at the centre of the turn table on Bragg's spectrometer with x-rays of known λ incident upon it. For a given plane used as reflecting surface, find the corresponding d using

$$n\lambda = 2d \sin \theta \quad (\text{take } n = 1)$$

Similarly, find value of d for other planes as well.

For cubic structure we select three planes viz. (100), (110), (111).

As λ is same throughout the experiment, we get,

$$\begin{aligned} \lambda &= 2d_{100} \sin \theta_1 \\ &= 2d_{110} \sin \theta_2 \\ &= 2d_{111} \sin \theta_3 \end{aligned} \quad \dots(2.5.1)$$

$$\therefore d_{100} : d_{110} : d_{111} :: \frac{1}{\sin \theta_1} : \frac{1}{\sin \theta_2} : \frac{1}{\sin \theta_3} \quad \dots(2.5.2)$$

where θ_1, θ_2 and θ_3 are obtained from the graph. Intensity $\rightarrow \theta$ i.e. where the peak occurs. The reason for selection of planes (100), (110) and (111) is that these are the planes rich enough in terms of atoms.

Studies have found out ratios of d_{100}, d_{110} and d_{111} for SC, BCC and FCC as follows,

$$\left. \begin{array}{l}
 \text{SC} \quad 1 : \frac{1}{\sqrt{2}} : \frac{1}{\sqrt{3}} \\
 \text{BCC} \quad 1 : \frac{2}{\sqrt{2}} : \frac{1}{\sqrt{3}} \\
 \text{FCC} \quad 1 : \frac{1}{\sqrt{2}} : \frac{2}{\sqrt{3}}
 \end{array} \right\} \quad \dots(2.5.3)$$

Experimentally obtained values of θ_1 , θ_2 and θ_3 will provide us d_{100} , d_{110} and d_{111} . By comparing their ratio with equation (2.5.3), one can determine crystal structure.

Ex. 2.5.1 : Find the Miller indices of a set of parallel planes which make the intercepts in the ratio $3a : 4b$ on X and Y axes and are parallel to Z axis. a, b and c are basic vectors.

MU - May 14. 3 Marks

Soln.:

Given: Intercepts of the plane are in the proportion

$3a : 4b : \infty$ (Plane is parallel to Z-axis)

As a, b and c are basic vectors, the proportion of intercepts $3 : 4 : \infty$

∴ Reciprocal

$$\frac{1}{3}, \frac{1}{4}, \frac{1}{\infty} = \frac{1}{3}, \frac{1}{4}, 0$$

Taking LCM and converting to the integers, 4, 3, 0

∴ Miller indices (430)

...Ans.

Ex. 2.5.2 : In an orthorhombic crystal lattice, a plane cuts intercepts of lengths $3a$, $-2b$ and $3c/2$ along three axes. Find Miller indices of the plane, where a, b, c are primitive vectors of the unit cell.

Soln.:

As a, b and c are primitive vectors, intercepts are $3, -2, \frac{3}{2}$

∴ Reciprocal $\frac{1}{3}, \frac{-1}{2}, \frac{2}{3}$

Taking LCM and converting them to integers 2, -3, 4

$$\therefore (hkl) = (2 \bar{3} 4)$$

...Ans.

Ex. 2.5.3 : In a simple cubic crystal, find the ratio of intercepts on the three axes by (1 2 3) plane.

Soln.:

If a plane cuts intercepts at length m, n, p on the three crystal axes, then

$$m : n : p = xa : yb : zc$$

Where a, b and c are primitive vectors of the unit cell and numbers x, y and z are related to the Miller indices (hkl) of the plane by the relation.

$$\frac{1}{x} : \frac{1}{y} : \frac{1}{z} = h : k : l$$

Since $a = b = c$ (crystal is simple cubic)

$$\text{and } (hkl) = (1 2 3)$$

$$\frac{1}{x} : \frac{1}{y} : \frac{1}{z} = 1 : 2 : 3$$

$$\therefore x:y:z = \frac{1}{1} : \frac{1}{2} : \frac{1}{3}$$

Multiply RHS by LCM = 6

$$x:y:z = 6:3:2$$

$$\therefore m:n:p = 6:3:2$$

...Ans.

Ex. 2.5.4 : A crystal whose primitive vectors are 1.2 \AA , 1.8 \AA and 2 \AA for a plane $(2\bar{3}1)$ cuts an intercept 1.2 \AA along X axis. Find the length of Y and Z intercepts along Y and Z axes.

Soln. :

Primitive vectors $a = 1.2 \text{ \AA}$, $b = 1.8 \text{ \AA}$, $c = 2 \text{ \AA}$

Miller indices of the plane $(2\bar{3}1)$

\therefore Intercepts are $\frac{a}{2}, \frac{b}{3}, \frac{c}{1}$

$$\text{i.e. } \frac{1.2}{2}, \frac{1.8}{3}, \frac{2}{1} \quad \dots(\text{A})$$

This gives the intercept along X axis as $\frac{1.2}{2} \text{ \AA} = 0.6 \text{ \AA}$. But it is given that plane cuts X-axis at 1.2 \AA . This shows that the plane under consideration is another plane which is parallel to it (to keep Miller indices same).

In the statement A,

$$\text{X intercept is } 1.2 \text{ \AA} = 2 \times 0.6 \text{ \AA}$$

\therefore Multiply other intercepts by 2

$$\therefore \text{Y intercept} = 2 \left(\frac{1.8}{3} \right) = 1.2 \text{ \AA}$$

$$\text{And, Z intercept} = 2 \left(\frac{2}{1} \right) = 4 \text{ \AA}$$

...Ans.

Ex. 2.5.5 : The interplanar spacing of $(1\bar{1}0)$ plane is 2 \AA for a FCC crystal. Find the atomic radius.

Soln. :

Given : $(hkl) = (1\bar{1}0)$, $d = 2 \text{ \AA}$

Formula :

$$d = \frac{a}{\sqrt{h^2 + k^2 + l^2}}$$

$$\therefore a = d \times \sqrt{h^2 + k^2 + l^2}$$

$$= 2 \times \sqrt{1^2 + 1^2 + 0^2} = 2.828 \text{ \AA}$$

For FCC structure,

$$r = \frac{\sqrt{2}a}{4}$$

$$= \frac{\sqrt{2} \times 2.828}{4}$$

$$\therefore \text{Radius } r = 1 \text{ \AA}$$

...Ans.

Ex. 2.5.6 : Consider the density of Cu as 8930 kg/m^3 and its atomic weight 63.546. If the average number of electrons contributed per atom is 1.23, calculate the free electron concentration in Cu.

Soln. :

Given :

$$\text{Density} = 8930 \text{ kg/m}^3,$$

$$\text{Atomic weight} = 63.546$$

$$\text{Number of } e^- / \text{atom} = 1.23$$

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$$

$$\therefore \frac{1}{\text{Volume}} = \frac{\text{Density}}{\text{Mass}}$$

Since e^- concentration is needed, let us find number of $\frac{\text{Atoms}}{\text{Volume}}$.

$$\begin{aligned} \therefore \frac{\text{Number of atoms}}{\text{Volume}} &= 6.023 \times 10^{26} \times \frac{\text{Density}}{\text{Mass}} \\ &= \frac{6.023 \times 10^{26} \times 8930}{63.546} \\ &= 8.464 \times 10^{28} \frac{\text{Atoms}}{\text{m}^3} \end{aligned}$$

Now, one atom contributes 1.23 e^-

$\therefore 8.464 \times 10^{28}$ atoms will contribute

$$1.23 \times 8.464 \times 10^{28} = 1.041 \times 10^{29}$$

\therefore Free e^- concentration

$$= 1.041 \times 10^{29} \frac{\text{electrons}}{\text{m}^3}$$

...Ans.

Ex. 2.5.7 : A crystal lattice plane (326) makes an intercept of 1.5 \AA on X-axis in a crystal having lattice constant 1.5 \AA , 2 \AA and 4 \AA on X, Y and Z axis respectively. Find Y and Z axes intercepts.

Soln. :

$$\text{Here } a = 1.5 \text{ \AA}, b = 2 \text{ \AA}, c = 4 \text{ \AA}$$

Miller indices : (326)

$$\therefore \text{Intercepts are } \frac{a}{3}, \frac{b}{2}, \frac{c}{6} \quad \text{i.e. } \frac{1.5}{3}, \frac{2}{2}, \frac{4}{6}$$

This gives the intercept along X-axis as $\frac{1.5}{3} \text{ \AA} = 0.5 \text{ \AA}$

But it is given that plane cuts X-axis at 1.5 \AA . This shows that the plane under consideration is another plane which is parallel to it and its x-intercept is $3 \times 0.5 = 1.5 \text{ \AA}$ i.e. multiple of 2

$$\therefore \text{Y-intercept} = 3 \left(\frac{2}{2} \right) = 3 \text{ \AA}$$

$$\text{Z-intercept} = 3 \left(\frac{4}{6} \right) = 2 \text{ \AA}$$

...Ans.

Ex. 2.5.8 : Sodium is a BCC crystal. Its density is $9.6 \times 10^2 \text{ kg/m}^3$ and atomic weight is 23.

Calculate the lattice constant for sodium crystal.

Soln. :

Given : $n = 2$ (BCC), $\rho = 9.6 \times 10^2 \text{ kg/m}^3$, $A = 23$

Formula :

$$a^3 \rho = \frac{nA}{N}$$

$$\therefore a = \sqrt[3]{\frac{nA}{N\rho}} = \sqrt[3]{\frac{2 \times 23}{9.6 \times 10^2 \times 6.023 \times 10^{26}}} \\ = 4.3 \times 10^{-10} \text{ m} = 4.3 \text{ \AA}$$

...Ans.

Ex. 2.5.9 : Draw the following planes (121), (100), (111).

Soln. :

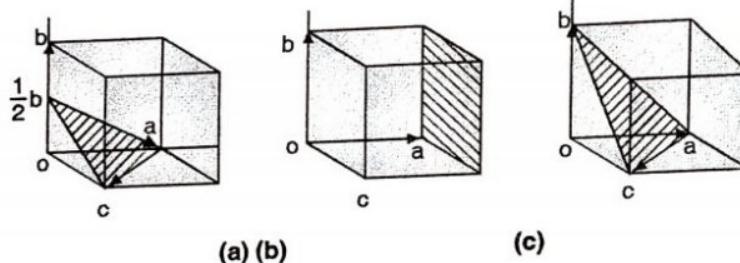


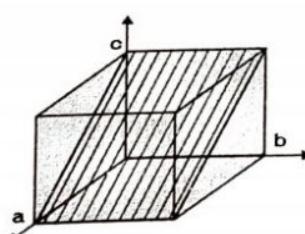
Fig. P. 2.5.9

Ex. 2.5.10 : Draw the following : (1 0 0), (2 2 0), (1 0 1)

Soln. :

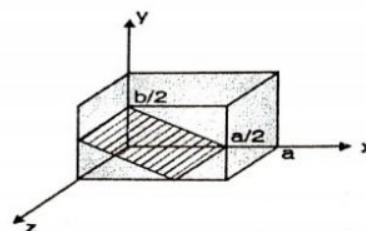
(1 0 0) → Please refer solved Fig. P. 2.5.10(b)

(1 0 1)



(a)

(2 2 0)



(b)

Fig. P. 2.5.10

Ex. 2.5.11 : Calculate the wavelength of X-rays reflected from the face of FCC crystal with lattice constant of 2.82 nm if the second order Bragg reflection occurs at a glancing angle of 17.167 deg.

Soln. :

Formulae : $2d \sin \theta = n \lambda$

Data given :

$$d = 2.82 \times 10^{-9} \text{ m}$$

$$n = 2, \quad \theta = 17.167^\circ$$

(assume reflection in 100 plane),

$$\begin{aligned}
 \text{Because, } d &= \frac{a}{\sqrt{h^2 + k^2 + l^2}} \\
 &= \frac{a}{\sqrt{1^2 + 0 + 0}} \\
 &= a \\
 \lambda &= \frac{2 \times 2.82 \times 10^{-9} \times \sin(17.167^\circ)}{2} \\
 &= 8.32 \times 10^{-10} \text{ m} \\
 &= 8.32 \text{ Å}^\circ
 \end{aligned}$$

...Ans.

Possible solutions (100), (010), (001), because all of them provide $d = a$

Ex. 2.5.12 : Find out the intercepts made by the planes (101) and (414) in a cubic unit cell. Draw [121] and [124] in a cubic unit cell. MU - May 13. 5 Marks

Soln. :

Let basic vectors be a, b, c and intercepts m, n, p respectively.

∴ On expressing m, n and p in terms of fractional multiples of a, b, c

$$\frac{m}{a}, \frac{n}{b}, \frac{p}{c}$$

As Miller indices are reciprocal of these fractions

$$(h k l) :: \left(\frac{a}{m}, \frac{b}{n}, \frac{c}{p} \right)$$

$$\frac{a}{m} = 1, \frac{b}{n} = 0, \frac{c}{p} = 1$$

∴ For (101) :

∴ w.r.t lattice parameters a, b, c the intercepts are

$$\frac{1}{1} = m = 1; \frac{1}{0} = n = \infty;$$

$$\frac{1}{1} = p = 1$$

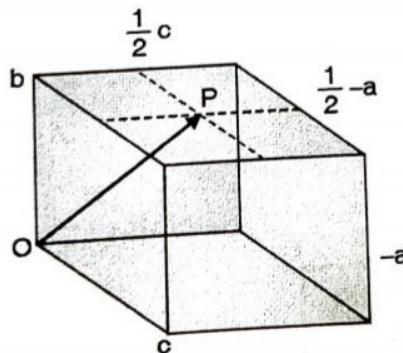


Fig. P. 2.5.12(a)

Similarly for (414)

$$m = \frac{1}{4}, n = 1, p = \frac{1}{4}$$



To draw $[\bar{1} \ 2 \ 1]$ Draw points $\left[\frac{1}{2}, 1, \frac{1}{2} \right]$

$$\vec{OP} = [\bar{1} \ 2 \ 1]$$

$[1 \ 2 \ 4]$ Draw points $\left[\frac{1}{4}, \frac{1}{2}, 1 \right]$

$$\vec{OQ} = [1 \ 2 \ 4]$$

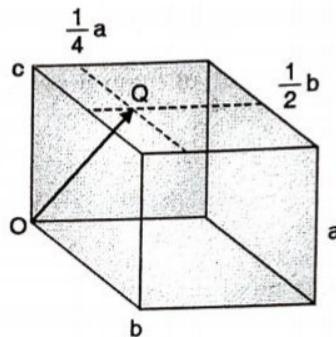


Fig. P. 2.5.12(b)

Ex. 2.5.13 : Find the interplanar spacing between the family of planes (111) in crystal of lattice constant 3 \AA .

MU - Dec. 13, 3 Marks

Soln. :**Given :**

$$a = 3,$$

$$(h k l) = (111)$$

Formula :

$$d_{hkl} = \frac{a}{\sqrt{h^2 + k^2 + l^2}}$$

$$\begin{aligned} \therefore d_{111} &= \frac{3}{\sqrt{1^2 + 1^2 + 1^2}} \\ &= \frac{3}{\sqrt{3}} = \sqrt{3} \text{ \AA} \end{aligned}$$

...Ans.

Ex. 2.5.14 : Represent the following in the unit cell. $(1 \ \bar{1} \ 2) (0 \ 0 \ 2) [1 \ 2 \ 1]$

MU - Dec. 13, 3 Marks

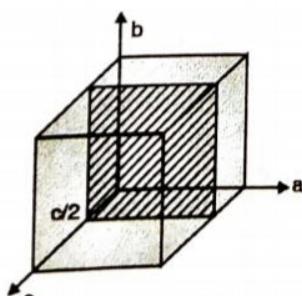
Soln. :(1) $(1 \ \bar{1} \ 2)$ 

Fig. P. 2.5.14(a)

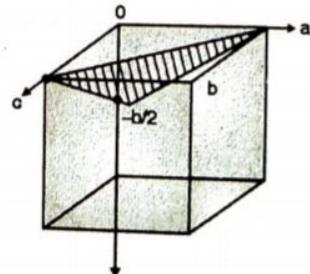
(2) $(0 \ 0 \ 2)$ 

Fig. P. 2.5.14(b)

(3) $[1 \ 2 \ 1]$ (Refer Solved Ex. 2.2.5)

Ex. 2.5.15 : Draw in cubic unit cell (021), (123), [121]

MU - May 14.3 Marks

Soln. : (a) (021)

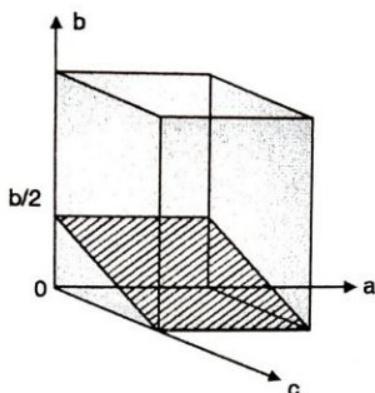


Fig. P. 2.5.15(a)

(b) (123)

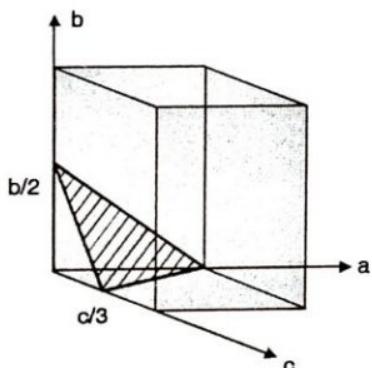


Fig. P. 2.5.15(b)

(c) [121] : Refer solved Ex. 2.2.5

Ex. 2.5.16 : Miller indices of a plane whose intercepts are, $4a$ and a where a is lattice constant. Draw (102), [201] and (040) in a cubic unit cell.

MU - Dec. 14, 5 Marks

Soln. : As intercepts area, $4a$ and a with a as lattice constant.

\therefore Actual plane intercept are 1, 4 and 1.

Take reciprocals : $\frac{1}{1}, \frac{1}{4}, \frac{1}{1}$

Taking LCM : (4 1 4)

\therefore MI are (4 1 4)

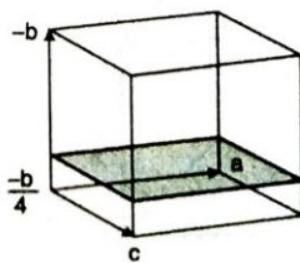
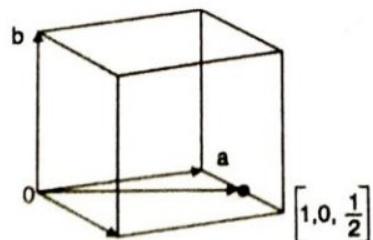
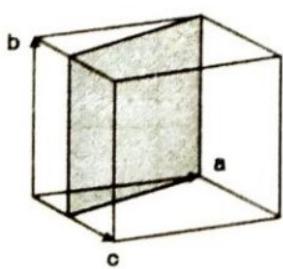
...Ans.

Draw

(1) (1 0 2)

(2) (2 0 1)

(3) (0 $\bar{4}$ 0)





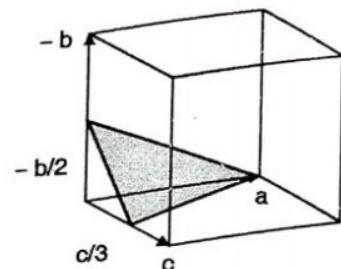
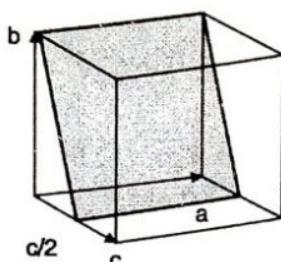
Ex. 2.5.17 : Draw the following : (a) $(0 \ 1 \ 2)$ (b) $(1 \ \bar{2} \ 3)$ (c) $[1 \ 2 \ 1]$

MU - May 15, 3 Marks

Soln. :

(a) $(0 \ 1 \ 2)$

(b) $(1 \ \bar{2}, 3)$



(c) $[1 \ 2 \ 1]$: Refer solved Ex. 2.2.5

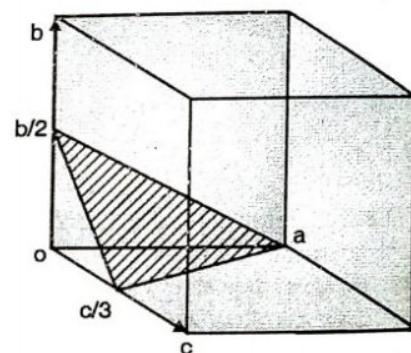
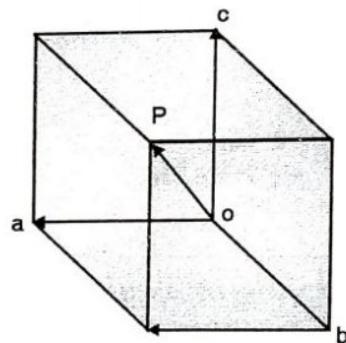
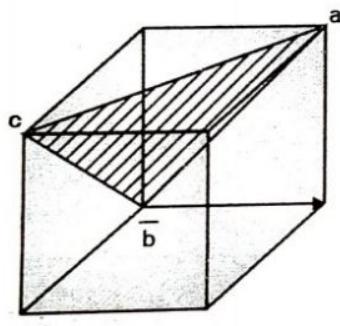
Ex. 2.5.18 : Draw the following w.r.t. a cubic unit cell : $(1 \bar{1} 1)$ $[\bar{1} \ 1 \ 1]$ (123)

MU - Dec. 15, 3 Marks

Soln. : (a) $(1 \bar{1} 1)$

(b) $[\bar{1} \ 1 \ 1]$

(C) (123)

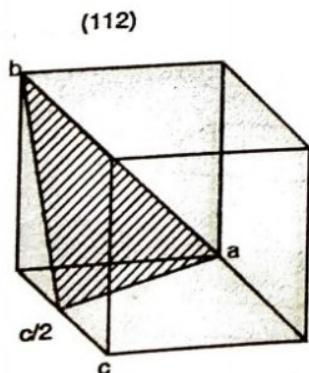


Ex. 2.5.19 : Draw (a) (112) (b) (040) (c) $[040]$ with reference to a cubic unit cell.

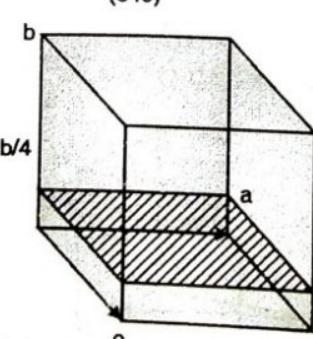
MU - May 16, 3 Marks

Soln. :

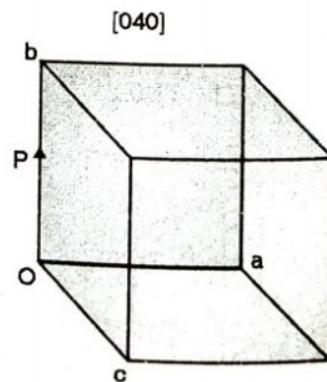
(a) (112)



(b) (040)



(c) $[040]$



2.6 Solved Problems on X-rays

Ex. 2.6.1: Calculate the smallest glancing angle at which x-ray of 1.549 \AA will be reflected from crystal having spacing of 4.255 \AA . What is the highest order of reflection that can be observed?

MU - May 14, 5 Marks

Soln. :

Consider Bragg's law,

$$n\lambda = 2d \sin \theta$$

(i) For smallest glancing angle, $n = 1$

$$\therefore \theta = \sin^{-1}\left(\frac{\lambda}{2d}\right) = 10.488^\circ \quad \dots \text{Ans.}$$

(ii) For highest order, we know that always

$$\sin \theta \leq 1$$

∴ Find n which satisfies this condition using relation

$$\sin \theta = \frac{n\lambda}{2d}$$

for $n = 1$,

$$\sin \theta = \frac{\lambda}{2d} = 0.18$$

for $n = 2$,

$$\sin \theta = 0.36$$

for $n = 3$,

$$\sin \theta = 0.54$$

for $n = 4$,

$$\sin \theta = 0.72$$

for $n = 5$,

$$\sin \theta = 0.9$$

for $n = 6$,

$$\sin \theta = 1.087 > 1$$

...Ans.

As $\sin \theta$ cannot be greater than 1, the highest order possible is $n = 5$

Other way possible to solve this is by taking maximum $\sin \theta = 1$

$$\therefore n\lambda = 2d$$

$$\therefore n = \frac{2d}{\lambda} = \frac{2 \times 4.255}{1.549} = 5.49$$

Now n is a fraction, and to convert it into integer remove fractional part.

Ex. 2.6.2: Calculate the glancing angle on the plane (100) for a crystal of rock salt ($a = 2.125 \text{ \AA}$). Consider the case of 2nd order maximum and wavelength 0.592 \AA .

MU - Dec. 12, 4 Marks

Soln. :

(i) Calculate d for rock salt

$$d = \frac{a}{2} = 1.0625 \text{ \AA}$$

(ii)

Use Bragg's law for $n = 2$

$$n\lambda = 2d \sin \theta$$

$$\therefore 2 \times 0.592 \times 10^{-10} = 2 \times 1.0625 \times 10^{-10} \times \sin \theta$$

$$\sin \theta = 0.557$$

$$\therefore \theta = 33.86^\circ$$

...Ans.

Ex. 2.6.3 : Monochromatic high energy x-rays are incident on a crystal. If first order reflection is observed at an angle 3.4° , at what angle would second order reflection be expected?

Soln. :

Data : For order

$$n = 1, \theta_1 = 3.4^\circ$$

Use Bragg's law

$$\begin{aligned} n\lambda &= 2d \sin \theta \\ \text{for } n &= 1 \\ \lambda &= 2d \sin (3.4^\circ) \\ \therefore d &= \lambda/2 \sin (3.4^\circ) \end{aligned}$$

\therefore For order $n = 2$

$$\begin{aligned} 2\lambda &= 2d \sin \theta_2 \\ \therefore \sin \theta_2 &= \frac{\lambda}{d} \quad \dots(1) \end{aligned}$$

Replace value of d from equation (1)

$$\begin{aligned} \sin \theta_2 &= \frac{\lambda}{\lambda/2 \sin (3.4)} \\ &= 2 \sin 3.4^\circ = 0.1186 \\ \therefore \theta_2 &= \sin^{-1}(0.1186) = 6.811^\circ \quad \dots\text{Ans.} \end{aligned}$$

Ex. 2.6.4 : The radiation of an x-ray tube operated at 50 kV are diffracted by a cubic KCl, FCC crystal of molecular weight 74.6 and density $1.99 \times 10^3 \text{ kg/m}^3$. Calculate:

Glancing angle for first order reflection from the reflecting planes of the crystal for wavelength, 0.248A° .

Soln. :

Given :

$$V = 50 \times 10^3 \text{ volts}, M = 74.6, \rho = 1.99 \times 10^3 \text{ kg/m}^3,$$

$$n = \text{number of atoms/unit cell} = 4 (\text{FCC}), \lambda = 0.248\text{A}^\circ$$

Step 1 :

$$\begin{aligned} \text{Now } a^3 \rho &= \frac{nM}{N} \\ \therefore a &= \sqrt[3]{\frac{nM}{N} \cdot \frac{1}{\rho}} \\ &= \sqrt[3]{\frac{4 \times 74.6}{6.023 \times 10^{26}} \times \frac{1}{1.99 \times 10^3}} \\ &= 6.29 \times 10^{-10} \text{ m} \end{aligned}$$

Since KCl is ionic crystal,

$$\therefore d = a/2 = \frac{6.29}{2} \times 10^{-10} = 3.145 \times 10^{-10} \text{ m}$$

Step 2 : Using Bragg's law

$$n\lambda = 2d \sin \theta$$

Consider

$$n = 1 \text{ for } \lambda = \lambda_{\min}$$

$$\therefore \sin \theta = \frac{1 \times \lambda}{2d} = \frac{1 \times 0.248 \times 10^{-10}}{2 \times 3.145 \times 10^{-10}} = 0.0394$$

$$\therefore \theta = \sin^{-1}(0.0394) = 2.26^\circ$$

...Ans.

Ex. 2.6.5 : Monochromatic x-rays of wavelength 0.82 \AA undergo first order Bragg reflection from a crystal of cubic lattice with lattice constant 3 \AA , at a glancing angle of $7^\circ 51' 48''$. Identify the possible planes which give rise to this reflection in terms of their Miller indices.

Soln. :

Data : $\lambda = 0.82 \times 10^{-10} \text{ m}$, $n = 1$, $\theta = 7^\circ 51' 48''$,
 $a = 3 \times 10^{-10} \text{ m}$

Use Bragg's law

$$n\lambda = 2d \sin \theta$$

$$\therefore d = \frac{n\lambda}{2 \sin \theta} = \frac{1 \times 0.82 \times 10^{-10}}{2 \times \sin(7^\circ 51' 48'')} = 3 \times 10^{-10} \text{ m}$$

$$\text{Now, } d = \frac{a}{\sqrt{h^2 + k^2 + l^2}}$$

Here we have $d = a$

$$\therefore \sqrt{h^2 + k^2 + l^2} = 1$$

\therefore Possible solutions $(100), (010), (001)$ because for all of them,

Where,

$$\sqrt{h^2 + k^2 + l^2} = 1 \quad \dots\text{Ans.}$$

Ex. 2.6.6 : The glancing angle of reflection for the first order K_α x-rays from palladium are 5.4° from (100) planes, 7.6° from (110) planes and 9.4° from (111) planes. From the above mentioned data determine the cubic lattice structure of the crystal.

Soln. :

Using Bragg's law,

$$n\lambda = 2d \sin \theta$$

$$\therefore d = \frac{\lambda}{2 \sin \theta} \quad (n=1)$$

$$\text{for } (100), d_1 = \frac{\lambda}{2 \sin 5.4^\circ} = \frac{\lambda}{0.1882}$$

$$\text{for } (110), d_2 = \frac{\lambda}{2 \sin 7.6^\circ} = \frac{\lambda}{0.2645}$$

$$\text{for } (111), d_3 = \frac{\lambda}{2 \sin 9.4^\circ} = \frac{\lambda}{0.3266}$$

$$d_1 : d_2 : d_3 = \frac{\lambda}{0.1882} : \frac{\lambda}{0.2645} : \frac{\lambda}{0.3266} [1 : 0.711 : 0.576]$$

$$d_1 : d_2 : d_3 :: [1 : \sqrt{2} : \sqrt{3}] \therefore \text{it is SC} \quad \dots\text{Ans.}$$

Ex. 2.6.7 : Bragg's reflection of the first order was observed at 21.7° for parallel planes for a crystal under test. If the wavelength of x-rays used is 1.54 \AA find the lattice constant of the crystal.

Soln. :

$$n = 1,$$

$$\theta = 21.7^\circ \text{ (Assume it is glancing angle)}$$

$$\begin{aligned}\lambda &= 1.54 \text{ \AA}, \\ \therefore n\lambda &= 2d\sin\theta \\ d &= \frac{n\lambda}{2\sin\theta} = \frac{1 \times 1.54 \times 10^{-10}}{2 \times \sin(21.7^\circ)} \\ &= 2.083 \times 10^{-10} \text{ m} \\ d &= 2.083 \text{ \AA}\end{aligned}$$

...Ans.

Ex. 2.6.8 : Calculate the glancing angle on the cube (100) of a rock salt ($a = 2.814 \text{ \AA}$) corresponding 2nd order diffraction maximum for x-rays of wavelength 0.714 \AA .

Soln. :

$$d_{nkl} = \frac{a}{\sqrt{h^2 + k^2 + l^2}} = \frac{2.814}{\sqrt{1^2 + 0^2 + 0^2}} = 2.814 \text{ \AA}$$

 For 2nd order Bragg reflection

$$\begin{aligned}n\lambda &= 2d\sin\theta \\ 2(0.714) &= 2(2.814) \sin\theta \\ \therefore \sin\theta &= 0.2537 \\ \therefore \theta &= 14^\circ 41'\end{aligned}$$

...Ans.

Ex. 2.6.9 : The spacing between the principal planes in a crystal of NaCl is 2.82 \AA . It is found that first order Bragg's reflection occurs at 10° .

- (a) What is the wavelength of x-ray?
- (b) At what angle does the second order reflection occur?
- (c) What is the highest order of reflection seen?

Soln. :

Given : $d = 2.82 \times 10^{-10} \text{ m}$, $n = 1$, $\theta = 10^\circ$

(a)

$$\begin{aligned}n\lambda &= 2d\sin\theta \\ \therefore \lambda &= 2(2.82 \times 10^{-10}) (\sin 10^\circ) \\ &= 9.79 \times 10^{-11} \text{ m} \\ \therefore \lambda &= 0.979 \text{ \AA}\end{aligned}$$

...Ans.

(b)

 For $n = 2$ find θ

$$n\lambda = 2d\sin\theta$$

$$\therefore 2 \times (0.979 \times 10^{-10}) = 2(2.82 \times 10^{-10}) \sin\theta$$

$$\therefore \sin\theta = 0.347$$

$$\therefore \theta = 20^\circ 18'$$

...Ans.

(c) For highest order

$$\begin{aligned}n\lambda &= 2d\sin\theta \\ \therefore \sin\theta &= \frac{n\lambda}{2d}\end{aligned}$$

 As $\sin\theta$ cannot exceed one,

 for various values of n i.e. $n = 1, 2, 3, \dots$ find $\sin\theta$.

 The value of n which yields

$\sin\theta > 1$ is not possible.

here for $n = 6$ we get, $\sin\theta = 1.041$

i.e. $\sin\theta > 1$

Highest order possible is 5.

...Ans.

- Ex. 2.6.10 : In comparing the wavelengths of two monochromatic x-ray lines, it is found that line A gives a 1st order Bragg reflection maximum at a glancing angle of 30° to the smooth face of a crystal. Line B of known wavelength of 0.97 Å gives a 3rd order reflection maximum at a glancing angle of 60° with the same face of the same crystal. Find the wavelength of the line A.

Soln. :

For line - A order $n = 1$

$\theta = 30^\circ, \lambda = ?$

Using Bragg's law for line - A

$$n\lambda = 2d\sin\theta$$

$$1\lambda_A = 2d\sin 30^\circ = 2d \times \frac{1}{2} = d$$

$$\therefore \lambda_A = d$$

...(1)

For line - B

$$\lambda = 0.97 \text{ \AA}$$

$$\theta = 60^\circ$$

$$n\lambda = 2d\sin\theta$$

$$3(0.97) = 2(\lambda_B) \cdot \sin 60^\circ$$

$$3(0.97) = 2(\lambda_B) \cdot \frac{\sqrt{3}}{2} = \sqrt{3}\lambda_B$$

$$\therefore \lambda_B = \frac{3 \times 0.97}{\sqrt{3}} = 1.68 \text{ \AA}$$

...Ans.

- Ex. 2.6.11 : A 10 keV electrons are passed through a thin film of a metal for which atomic spacing is 5.5×10^{-11} m. What is the angle of deviation of the first order diffraction minimum?

Soln. :

(1)

$$\text{Energy} = 10 \text{ keV} = 10 \times 10^3 \times 1.6 \times 10^{-19} \text{ J}$$

$$\therefore \lambda = \frac{h}{\sqrt{2mE}}$$

$$= \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 10^4 \times 1.6 \times 10^{-19}}} \text{ m}$$

$$= 1.23 \times 10^{-11} \text{ m}$$

Using Bragg's law (for $n = 1$)

$$n\lambda = 2d\sin\theta$$

$$\therefore \sin\theta = \frac{1 \times 1.23 \times 10^{-11}}{2 \times 5.5 \times 10^{-11}} = 0.118$$

$$\therefore \theta = 6.42^\circ$$

...Ans.

Ex. 2.6.12 : Calculate the glancing angle on cube (100) of rock salt having lattice constant 2.814 \AA corresponding to first order diffraction maximum for x-rays of wavelength 1.541 \AA .

Soln. :

Given : Plane (100)

$$a = 2.814 \text{ \AA} = 2.814 \times 10^{-10} \text{ m}$$

$$\lambda = 1.541 \text{ \AA} = 1.541 \times 10^{-10} \text{ m}$$

$$n = 1$$

Bragg's law,

$$n\lambda = 2d \sin \theta$$

For rock salt

$$d = \frac{a}{2} = 1.407 \times 10^{-10} \text{ m}$$

$$\therefore \theta = \sin^{-1}\left(\frac{n\lambda}{2d}\right)$$

$$= \sin^{-1}\left(\frac{1 \times 1.541 \times 10^{-10}}{2 \times 1.407 \times 10^{-10}}\right)$$

$$\theta = \sin^{-1}(0.5476) = 33.20^\circ$$

...Ans.

Ex. 2.6.13 : Calculate the maximum order of diffraction if x-rays of wavelength 0.819 \AA are incident on a crystal of lattice spacing 0.282 nm .

MU - Dec. 13. 4 Marks

Soln. :

Given : $\lambda = 0.819 \text{ \AA}$, $d = 0.282 \text{ nm}$

Formulae : $n\lambda = 2d \sin \theta$

for the given values of λ and d , $n \propto \sin \theta$

for n to be maximum, $\sin \theta = 1$

$$\therefore n = \frac{2d}{\lambda} = \frac{2 \times 0.282 \times 10^{-9}}{0.819 \times 10^{-10}} = 6.88$$

\therefore As order cannot be a fraction and n is maximum,

$$n = 6$$

...Ans.

Ex. 2.6.14 : Calculate Bragg angle if (200) planes of a BCC crystal with lattice parameter 2.814 \AA give second order reflection with X-rays of wavelength 0.71 \AA .

MU - Dec. 14. 4 Marks

Soln. :

$$a = 2.814$$

$$(hkl) = (200)$$

$$\begin{aligned} \therefore d &= \frac{a}{\sqrt{h^2 + k^2 + l^2}} \\ &= \frac{2.814}{\sqrt{2^2 + 0 + 0}} \\ &= 1.407 \text{ \AA} \end{aligned}$$

Using Bragg's law

$$n\lambda = 2d \sin \theta$$

$$\therefore 2 \times 0.71 = 2 \times 1.407 \times \sin \theta$$

$$\therefore \sin \theta = \frac{0.71}{1.407} = 0.5046$$

$$\therefore \theta = 30.30^\circ$$

...Ans.

Ex. 2.6.15 : X-rays of unknown wavelength give first order Bragg's reflection at glancing angle 20° with (212) planes of copper having FCC structure. Find wavelength of x-rays if the lattice constant for copper is 3.615 \AA .

MU - May 15. 7 Marks

Soln. :

Using formula

$$d = \frac{a}{\sqrt{h^2 + k^2 + l^2}}$$

$$= \frac{3.615}{\sqrt{2^2 + 1^2 + 2^2}} \text{ \AA}$$

$$= 1.205 \text{ \AA}$$

Using Bragg's law

$$n\lambda = 2d \sin \theta$$

Here,

$$n = 1$$

$$\theta = 20^\circ$$

$$\therefore 1 \times \lambda = 2 \times 1.205 \times \sin 20^\circ$$

$$\therefore \lambda = 0.824 \text{ \AA}$$

...Ans.

Ex. 2.6.16 : Monochromatic x-ray beam of wavelength $\lambda = 5.8189 \text{ \AA}$ is reflected strongly for a glancing angle of $\theta = 75.86^\circ$ in first order by certain planes of cubic of lattice constant 3 \AA . Determine Miller indices of the possible reflecting planes.

MU - May 16. 3 Marks

Soln. :**Problem :**

$$\lambda = 5.8189 \text{ \AA}, \theta = 75.86^\circ, n = 1, a = 3 \text{ \AA}$$

$$n\lambda = 2d \sin \theta$$

$$d = \frac{n\lambda}{2 \sin \theta} = 3 \text{ \AA}$$

as $a = 3 \text{ \AA}$ (given)

$$d = \frac{a}{\sqrt{h^2 + k^2 + l^2}}$$

$$\therefore d = a$$

... Ans.

\therefore Possible planes (1 0 0), (0 1 0), (0 0 1)

Ex. 2.6.17 : An electron is accelerated through 1200 volts and is reflected from a crystal. The second order reflection occurs when glancing angle is 60° . Calculate the interplanar spacing of the crystal.

MU - May 17. 8 Marks

Soln. :Data : $V = 1200$ volt, order $n = 2$,glancing angle $= 60^\circ$ To find : Interplanar spacing d .



Using relation

$$\begin{aligned}\lambda &= \frac{h}{\sqrt{2mE}} = \frac{h}{\sqrt{2m(\text{eV})}} = 6.63 \times 10^{-34} \\ &= \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19} \times 1200}} \\ &= 3.547 \times 10^{-11} \text{ m}\end{aligned}$$

Using Bragg's law

$$\begin{aligned}n\lambda &= 2d \sin \theta \\ d &= \frac{n\lambda}{2 \sin \theta} = \frac{2 \times 3.547 \times 10^{-11}}{2 \times \sin 60^\circ} \\ &= 4.09 \times 10^{-11} \text{ m} \\ \therefore \text{Interplanar spacing} &= 4.09 \times 10^{-11} \text{ m}\end{aligned}$$

...Ans.



CHAPTER 3

Unit III

Semiconductor Physics

Syllabus

(Prerequisites : Intrinsic and extrinsic semiconductors, energy bands in conductors, semiconductors and insulators, semiconductor diode, I-V characteristics in forward and reverse bias)
Direct & indirect band gap semiconductor; Fermi level; Fermi-Dirac distribution; Fermi energy level in intrinsic & extrinsic semiconductors; effect of impurity concentration and temperature on Fermi level; mobility, current density; Hall effect; Fermi Level diagram for p-n junction (unbiased, forward bias, reverse bias); Applications of semiconductors: LED, Zener diode and Photovoltaic cell

Learning Objectives

After reading this chapter, Learner should be able to :

- Understand direct and indirect band gap semiconductors
- Apply concept of Fermi level to intrinsic and extrinsic semiconductors
- Understand the effect of impurity concentration and temperature on Fermi level.
- Understand Hall effect and its applications
- Apply knowledge of semiconductors to LED, Zener diode, photovoltaic cell

3.1 Introduction

- Semiconductors have provided immense help in development of the present digital industry due to their unique characteristics especially electrical properties. Their large scale production has reduced cost like anything.
- In their purest form i.e. intrinsic for they are not so popular as the electrical properties are not that attractive but when it is doped with selected impurities in a very limited way, a drastic change is observed.
- In this chapter we will study the success path of semiconductors with necessary mathematics, the key points ranging from mobility to Hall effect and fundamental components like p-n junctions diodes.

3.2 Direct and Indirect Band Gap

- We have already seen classification of solids on the basis of value of forbidden energy band gap. We also know that semiconductors are broadly classified in terms of intrinsic and extrinsic semiconductors. Now we will learn about classification on the basis of recombination of electrons and holes.
- When energy $E = h\nu = E_g$ (forbidden gap) is applied to an electron-hole pair in valence band, an electron will be found in conduction band and there will be a corresponding hole in the valence band. Now electron will come back to valence band by releasing the energy which was used to send it to conduction band. This process is known as recombination. (Energy conservation takes place here.)

- When an electron (or hole) of mass m travels with velocity v , the momentum, $p = mv$.
- When electron is considered so small that it can exhibit wave nature as well, then quantum momentum

$$P = \hbar k$$

Where \hbar = Planck's constant

k = wave number (momentum)

$$\text{As } E = \frac{p^2}{2m}$$

$$= \frac{(\hbar k)^2}{2m} = \frac{\hbar^2}{2m} k^2$$

as \hbar and m are constant.

$$E \propto k^2$$

i.e. It represents a parabola.

- The graph $E \rightarrow k$ which is of parabolic nature, is known an $E - k$ diagram. If we show $E - k$ diagram for conduction and valence band, it will be as shown in Fig. 3.2.1.

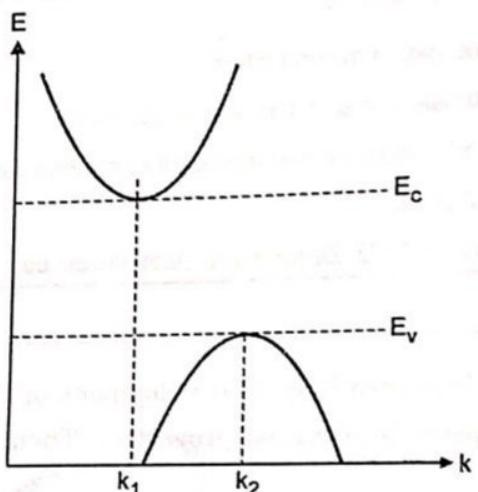


Fig. 3.2.1

- As electrons in conduction band and holes in the valence band possess different energies, they are bound to have different momentum as well ($\therefore E = \text{const } k^2$). Hence, we will have two different $E - k$ diagrams for conduction and valence bands.
- For recombination to take place, we must have momentum of hole (in valence band) and electron (in conduction band) to be equal. But as explained above, momentum of electrons and momentum of holes are different. We now have two options
 - Momentum of electrons should be made equal to that of holes.
 - Momentum of holes should be made equal to that of electrons
- Option number 2 is not possible as the mobility of holes is smaller than that of electrons.
- Hence, we will try to match momentum of electrons with holes.

We consider trapped energy states available in particular semiconductors which act as agents for recombination. This means they try to hold electron for some time to make the hole recombine with it. Trapped energy states are as shown in Fig. 3.2.2.

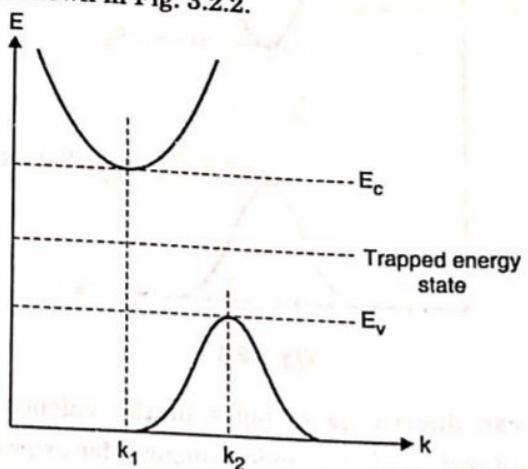


Fig. 3.2.2

As it is clear from Fig. 3.2.2 that momentum for electron is k_1 and that for hole is k_2 (i.e. momentum are different). But in order to make momentum of the same value, a transition will take place as shown in Fig. 3.2.3.

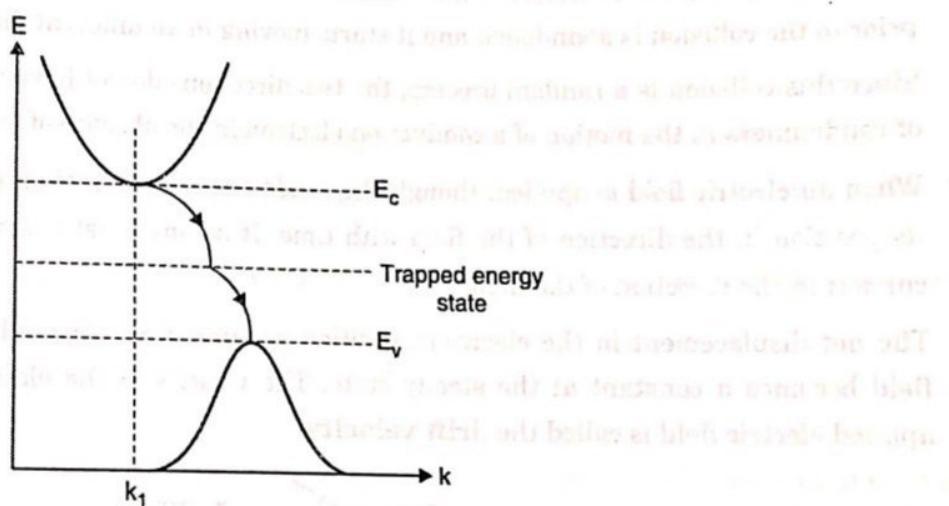


Fig. 3.2.3

i.e. the momentum becomes same not directly from conduction to valence band but indirectly through a trapped energy state available in forbidden gap.

- Such semiconductors are called indirect band gap semiconductors.
- Elemental semiconductors Si and Ge are of this type.
- Another possibility exists where the momentum of electron in conduction band and holes in valence band is the same.

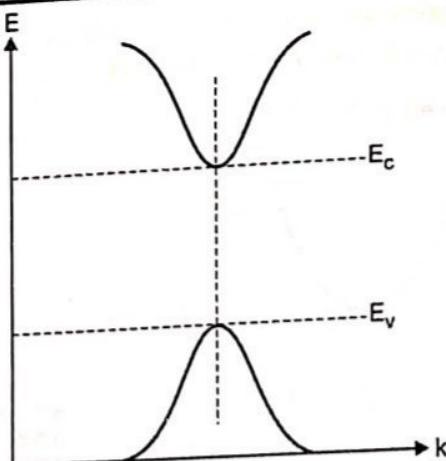


Fig. 3.2.4

- In such cases an electron can directly go to holes in the valence band for recombination. Such semiconductors are known as direct band gap semiconductors, for example GaAs.

3.3 Carrier Drift, Drift Velocity, Drift Current Mobility and Diffusion Current

MU - Dec. 13, May 15, May 16

Q. Define drift current, diffusion current and mobility of charge.

(Dec. 13, May 15, May 16, 3 Marks)

- When an electron (or a carrier) collides with the lattice, the direction to which it had confined its motion prior to the collision is abandoned and it starts moving in a different direction.
- Since this collision is a random process, the two directions do not have any interrelation. This is a kind of randomness in the motion of a conduction electron in the absence of an electric field.
- When an electric field is applied, though the randomness in its motion persists, there is overall shift in its position in the direction of the field with time. It means a net transportation of charge results in a current in the direction of the field.
- The net displacement in the electron's position per unit time caused by the application of an electric field becomes a constant at the steady state. The velocity of the electrons in the steady state in an applied electric field is called the **drift velocity**.

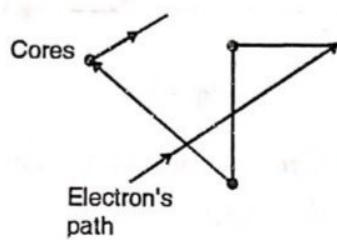


Fig. 3.3.1 : Electron random motion

- These velocities will not increase to large values because of electrical resistance due to thermal vibrations of ions (or atoms) at that temperature. For semiconductors, since we have electrons and holes, the drift velocities produce drift current in material.

Mobility of electrons

The mobility of electron is defined as the magnitude of the drift velocity acquired by the electrons in a unit field. Thus, if E is the applied electric field in which the electrons acquire a drift velocity v_d then the mobility of electrons μ is given by,

$$\mu = \frac{v_d}{E} \quad \dots(3.3.1)$$

The microscopic form of Ohm's law (Equation 3.3.1)

$$J = \sigma E$$

$$\therefore \sigma = \frac{J}{E} \quad \text{and} \quad J = \frac{I}{A}$$

$$\therefore \sigma = \frac{I}{AE}$$

Now current

$$I = ne v_d A^*$$

$$\therefore \sigma = \frac{ne v_d A}{AE} = ne \mu_e \quad (\text{for electrons})$$

$$\therefore \mu_n = \frac{\sigma}{ne}, \quad \dots(3.3.2)$$

for holes

$$\sigma_p = pe \mu_p$$

$$\sigma = \sigma_n + \sigma_p$$

\therefore For semiconductor

Typically mobility of electron = $1350 \text{ cm}^2/\text{V}\cdot\text{s}$ and of hole is $480 \text{ cm}^2/\text{V}\cdot\text{s}$

Mobility represents the ease with which the electrons could drift in a material under the influence of an electric field.

Mobility of electron is more than double that of holes, hence devices with electron as majority carriers are preferred.

$$\therefore I = ne v_d A$$

Diffusion current

Non-uniform concentration of charge carriers produces diffusion current. This kind of non-uniform concentration of charge carriers can be formed by thermal or radiation excitation of a part of the material or by injecting carriers into the material through surface.

Expression of current I

Consider a conductor of uniform area of cross section A carrying current I . If v is the velocity of the electrons, then length traversed by the electron in unit time is v . Therefore in unit time they sweep the volume vA of the conductor. If n is the number of electrons/unit volume, then the number of electrons in volume vA = (nvA) . If e is the charge on each electron, then the quantity of charge crossing any section of conductor per second is $e(nvA)$. This also represents quantity of charge crossing any section per second or simply the current $\therefore I = n e v A$. The velocity acquired by the electron due to an applied electric field is drift velocity v_d .

- Whenever concentration of electrons or holes due to excitation is noticed, these excess carriers diffuse to less concentrated places.
- The rate of diffusion is proportional to concentration gradient and forms diffusion current.
- In semiconductor, total current is due to drift and diffusion currents.
- Electron motion in solid is a complex issue. Let us understand it by considering few terminologies.

1. Current density (J)
2. Electrical field (E)
3. Conductivity (σ)
4. Resistivity (ρ)

1. Current density (J)

- It is the current per unit area of cross section of an imaginary plane held normal to the direction of current in current carrying conductor.
- If I is the current and A is the area of cross-section, then the current density J is given by

$$J = \frac{I}{A} \quad \dots(3.3.3)$$

2. Electrical field (E)

- The potential drop per unit length of a conductor gives the electric field that exists across a homogeneous conductor.
- If L is the length of a conductor of uniform cross section and of uniform material composition, and V is the potential difference between its two ends, then the electric field E at any point inside is given by

$$E = \frac{V}{L} \quad \dots(3.3.4)$$

3. Conductivity (σ)

- It is the physical property that characterizes the conducting ability of a material.
 - If R is the electrical resistance of uniform material of length L , and area of cross section A , then the electrical conductivity is given by

$$\sigma = \frac{1}{R} \left(\frac{A}{L} \right) \quad \dots(3.3.5)$$

Unit of conductivity is $(\text{ohm} - \text{m})^{-1}$

Now if we consider the product σE , then from equations (3.3.4) and (3.3.5) we get

$$\sigma E = \frac{V}{L} \frac{1}{R} \left(\frac{A}{L} \right) = \frac{V}{RA}$$

If we consider macroscopic form of Ohm's law we have

$$I = \frac{V}{R}$$

$$\therefore \sigma E = \frac{I}{A} = J$$

$$\therefore J = \sigma E$$

This is known as **point form** or **microscopic form** of Ohm's law.

...(3.3.6)

4 Resistivity (ρ)

Resistivity signifies the resistance property of the material and is given by the inverse of conductivity

$$\therefore \rho = \frac{1}{\sigma} \quad \dots(3.3.7)$$

Unit of resistivity is ohm-m.

3.3.1 Concept of Holes and Effective Mass

Q. Explain concept of hole in semiconductor.

MU - May 14, Dec. 14

Q. What is effective mass? Why is the effective mass of holes more than the effective mass of electrons?

(May 14, 3 Marks)

(Dec. 14, 4 Marks)

- An energy band which is completely filled carries no current. But if some of the electrons in a filled band are excited into a higher band, the remaining electrons in it can contribute to the current.
- The properties of a vacant state in an almost full band are exactly those of a particle with a positive charge in an otherwise empty band. This quasiparticle is known as a **hole**.
- Since a hole has positive charge, it moves in a direction opposite to that in which an electron moves when an external electrical field is applied. The motion of electrons in the valence band may be considered to be equivalent to the motion of the holes in the opposite direction.
- The electric current in a semiconductor therefore consists of two components - one due to the motion of electrons in the Conduction Band (CB) and the other due to holes in the Valence Band (VB).
- The quasiparticle which represents a vacancy needs detailed approach when we assign to it a status of particle and hence a mass.
- This mass is arrived at by band theory of solids. The motion of an electron in the solid subjected to an electric field is written as,

$$F = m_e a$$

- Provided a mass other than the rest mass m_e is used in this equation. **This mass is called the effective mass and it is denoted by m_e^*** . In order to remove the concept of negative mass in band theory, electron was withdrawn from band theory and it was replaced by a particle of positive effective mass.
- In CB energy levels are vacant. Hence electron can move freely. Whereas in VB energy levels are filled, hence holes cannot move freely. This shows effective mass of holes in VB is more than electrons in CB.
- Direct evidence of the existence of holes was furnished by the Hall effect experiment discussed later in this chapter Section 3.10.

3.4 Fermi-Dirac Statistics

MU - May 14, May 17

Q. What is Fermi level? Write Fermi-Dirac distribution function.

(May 14, 3 Marks)

Q. With the help of diagram, explain the variation of Fermi level with temperature in n-type semiconductor.

(May 17, 5 Marks)

- "Overall behavior of a system of many particles is related to the properties of the particles themselves".**
- As it is seen in formation of bands, it is now clear that there are a large number of energy levels available for occupation by large number of electrons. The branch of physics dealing with this area is called "statistical mechanics".
 - It is not concerned with the actual motions or interactions of individual particles but with what is most likely to happen.
 - While statistical mechanics cannot help us find the life history of one of the particles in a system, it is able to tell us for instance, the probability that a particle has a certain amount of energy at a certain moment.

Table 3.4.1 : Three statistical distribution functions

	Maxwell Boltzmann	Bose-Einstein	Fermi-Dirac
Applies to system of	Identical distinguishable particles	Identical indistinguishable particles	Identical indistinguishable particles
Applies to	Classical particles	Bosons	Fermions
Example	Molecules of a gas	Phonons in a solid	Free electrons in a metal.

- We will concentrate only on Fermi-Dirac statistics.
- According to Fermi-Dirac statistics, we make use of function $f(E)$ which determines the carrier occupancy of the energy. In other words, $f(E)$ governs the distribution of electrons among the energy levels as a function of temperature. It is given by

$$f(E) = \frac{1}{1 + \exp(E - E_F)/KT} \quad \dots(3.4.1)$$

Where, $f(E)$ = Probability that a particular energy level E is occupied by an electron.

E_F = Fermi energy

K = Boltzmann constant

T = Temperature in Kelvin

3.5 Fermi Energy or Fermi Level

- Q. What is Fermi level in semiconductor?**
Q. What is Fermi level? Write Fermi-Dirac distribution function.

MU - May 12, Dec. 13, May 14

(May 12, Dec. 13, 5 Marks)

(May 14, 3 Marks)

- When the filling up of electrons is undertaken, the universal rule is that the lowest energy level gets filled first.
- However, there will be many more allowed energy levels left vacant as shown in Fig. 3.5.1.

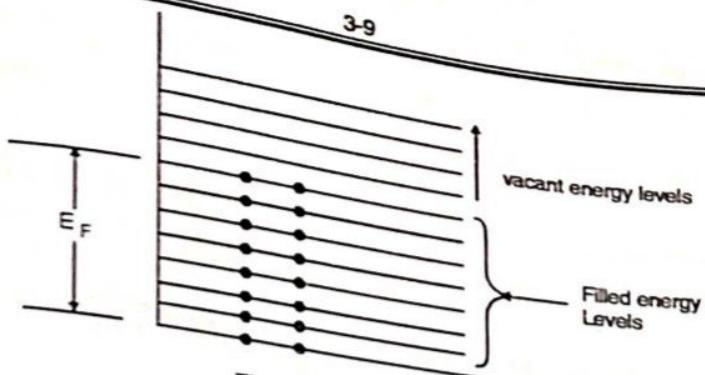


Fig. 3.5.1 : Fermi energy

Here we define Fermi energy or Fermi level as -

The energy of the highest occupied level at zero degree absolute is called the Fermi energy, and the level is referred to as the Fermi level E_F .

All the energy levels above the Fermi level at $T = 0^\circ K$ are empty and those lying below are completely filled. E_F may or may not be an allowed state. It provides a reference with which other energy levels can be compared.

3.5.1 Fermi Level in Conductor

a. What is Fermi level in semiconductor?

MU - Dec. 13, Dec. 15

(Dec. 13, Dec. 15, 5 Marks)

As mentioned in classification the conductors have many free electrons. Let us see how Fermi function helps us understand their distribution.

(a) At $T = 0^\circ K$

- At $0^\circ K$ electrons occupy the lower energy levels in the conduction band leaving upper energy levels vacant.
- The band is filled up to a certain energy level E_F , therefore Fermi level may be regarded as the uppermost filled energy level in conductor at $0^\circ K$. Let us see some important conclusions from Equation (3.4.1).

At $T = 0^\circ K$, levels below E_F have $E < E_F$

$$\begin{aligned}\therefore f(E) &= \frac{1}{1 + e^{(E-E_F)/kT}} \\ &= \frac{1}{1 + e^{-\infty}} \\ &= \frac{1}{1 + 0} = 1\end{aligned}$$

$f(E) = 1$ means all the levels below E_F
are occupied by electrons.

At $T = 0^\circ K$, levels above E_F have $E > E_F$

$$f(E) = \frac{1}{1 + e^{(E - E_F)/kT}} = \frac{1}{1 + e^0}$$

$$= \frac{1}{1 + 1} = 0$$

$\therefore f(E) = 0$ means all the levels above E_F are vacant.

At $T = 0^\circ\text{K}$ for $E = E_F$

$$f(E) = \frac{1}{1 + e^{(E - E_F)/kT}} = \frac{1}{1 + e^0}$$

$\therefore f(E)$ is indeterminate.

This is summarized in Fig. 3.5.2.

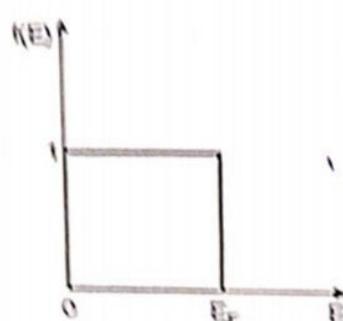


Fig. 3.5.2 : Fermi-Dirac distribution at $T = 0^\circ\text{K}$

(b) At $T > 0^\circ\text{K}$

- At temperature above 0°K , few electrons are excited to vacant levels above E_F . This happens to those electrons which are close to E_F hence probability to find an electron at $E > E_F$ will become greater than unity which was zero at $T = 0^\circ\text{K}$.
- Similarly, due to excitation of electrons, few levels just below E_F will become vacant and $f(E)$ will be slightly reduced which was unity at $T = 0^\circ\text{K}$.
- In a simple way one can understand that, what increase in $f(E)$ at $T > 0^\circ\text{K}$ above $E = E_F$ we get is equal to reduction in $f(E)$ below $E = E_F$. This is shown as below in Fig. 3.5.3.

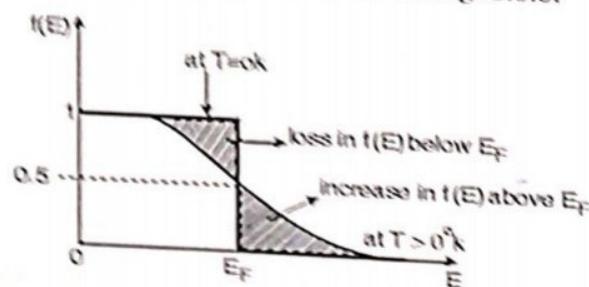


Fig. 3.5.3 : Electron occupancy at $T > 0^\circ\text{K}$

At $E = E_F$ for $T > 0^\circ\text{K}$

$$f(E) = \frac{1}{1 + e^0} = \frac{1}{1 + 1} = \frac{1}{2} = 0.5$$

3.5.2 Fermi Level In Semiconductor

- Once the concept of Fermi level is understood properly by considering conductors, it is proper to go to semiconductors.
- A semiconductor has conduction band and valence band separated by a small energy gap.
- At normal temperature, a significant number of electrons are excited to conduction band (CB) and from Valence band (VB) leaving behind same number of holes.
- Therefore $f(E)$ has non-zero probability above Fermi level and $f(E)$ reduces by same amount below E_F as shown in Fig. 3.5.4.

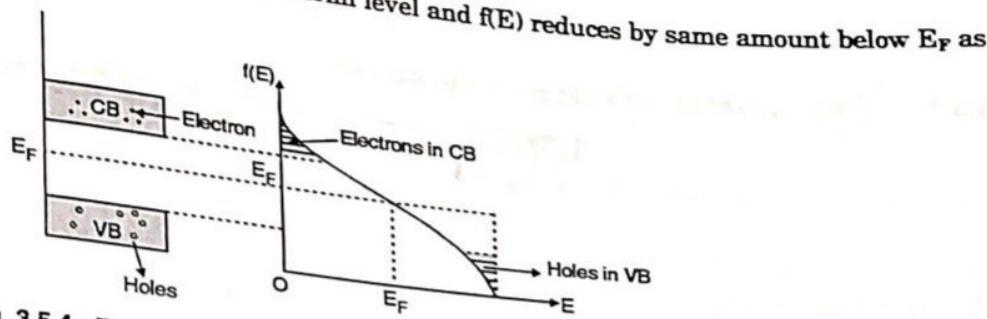


Fig. 3.5.4 : Fermi-Dirac distribution of semi conductor at $T > 0^\circ \text{K}$
Fermi level is halfway between CB and VB if it is intrinsic.

3.6 Intrinsic Semiconductors

- Pure semiconductors are called intrinsic semiconductors. In intrinsic semiconductors,

$$n_e = n_h = n_i \quad \dots(3.6.1)$$

where n_e = Number of electron per unit

volume or electron density

n_h = Hole density

n_i = Intrinsic carrier concentration

Here n_e is given by

$$n_e = N \exp(-E_g / 2KT) \quad \dots(3.6.2)$$

where N = Possible states per unit volume

E_g = Forbidden energy gap

K = Boltzmann constant

- When thermal energy is applied, a few electrons from valence band jump into conduction band by leaving hole in valence band. That is the reason $n_h = n_e$.
- These electrons will move freely when electric field is applied. The motion of holes is imagined in the direction which is opposite to that of electron.
- The conduction in intrinsic semiconductor is due to thermally excited electrons and holes, so it is very small.

- The conductivity in intrinsic semiconductor is given by,

$$\sigma_{in} = n_e e \cdot \mu_e + n_h e \mu_h$$

n_e = Number of electrons
 n_h = Number of holes
 e = Charge of electron
 μ_e = Mobility of electrons
 μ_h = Mobility of holes

Where

3.6.1 Fermi Level in Intrinsic Semiconductors

MU - Dec. 12, May 13, Dec. 13, Dec 14, May 15, Dec. 15, May 17, Dec. 17, May 19

- Q. For an intrinsic semiconductor show that the Fermi level lies in the centre of the forbidden energy gap. (Dec. 12, Dec. 15, 5 Marks, Dec. 17, 4 Marks, May 19, 3 Marks) (May 13, 8 Marks)
- Q. Derive an expression for Fermi level for an intrinsic semiconductor.
- Q. Show that in intrinsic semiconductor Fermi level always lies at the middle of forbidden energy gap. (Dec. 13, 5 Marks)
- Q. Show that for intrinsic semiconductor, the Fermi level lies midway between the conduction band and the valance band. (May 15, 8 Marks)
- Q. Explain the concept of Fermi level. Prove that Fermi level lies exactly at the centre of the Forbidden energy gap in intrinsic semiconductor. (May 17, 7 Marks)

- It can be shown for intrinsic semiconductors, Fermi energy level E_F lies midway between conduction and valence band. The proof is given below.
- At any temperature $T > 0^\circ K$,

n_e = Number of electrons in conduction band

n_v = Number of holes in valence band

$$\text{We have } n_e = N_C e^{-(E_C - E_F)/KT} \quad \dots(3.6.4)$$

Where N_C = Effective density of states in conduction band

$$\text{And } n_v = N_V e^{-(E_F - E_V)/KT} \quad \dots(3.6.5)$$

Where

For best approximation

N_C = effective density of states in valance band

$$N_C = N_V \quad \dots(3.6.6)$$

$$n_C = n_v$$

$$N_C e^{-(E_C - E_F)/KT} = N_V e^{-(E_F - E_V)/KT}$$

$$\frac{e^{-(E_C - E_F)/KT}}{e^{-(E_F - E_V)/KT}} = \frac{N_V}{N_C}$$

$$e^{-(E_C - E_F - E_F + E_V)/KT} = \frac{N_V}{N_C}$$

$$e^{-(E_C + E_V - 2E_F)/KT} = \frac{N_V}{N_C}$$

$$\text{as } N_V = N_C = 1$$

$$e^{-(E_C + E_V - 2E_F)/KT} = 1$$

$$\frac{-(E_C + E_V - 2E_F)}{KT} = 0$$

$$\therefore (E_C + E_V) = 2E_F$$

$$\therefore E_F = \frac{E_C + E_V}{2}$$

...(3.6.7)

Thus, the Fermi level in an intrinsic semiconductor lies at the center of forbidden energy gap.

3.7 Extrinsic Semiconductor

- Intrinsic semiconductors have low conductivity and serve only in limited applications. It is necessary to modify and control conductivity of intrinsic semiconductors to employ them in manufacturing useful devices.
- The conductivity of intrinsic semiconductor can be increased by adding impurities. A deliberate introduction of controlled quantities of impurities into pure semiconductor is called doping.
- The impurity added is called dopant. The doped semiconductor is called extrinsic semiconductor.
- The impurities to be used as dopants are selected from group III or group V elements because of the following reasons.
 - These atoms are nearly of the same size as Si or Ge and substitute themselves easily into the host lattice by going into the place of some of the host atoms.
 - The impurity is of substitutional type and the original crystal structure does not get distorted.
 - Extrinsic semiconductors are of two types depending upon the impurity element introduced.

3.7.1 n-Type Semiconductors

- If a pentavalent impurity is added to a pure semiconductor it becomes n-type extrinsic semiconductor. The impurity added is called donor impurity.
- As shown in Fig. 3.7.1 antimony (Sb) added as an impurity has five valence electrons. Each Sb atom forms covalent bonds with the surrounding four Si atoms with the help of four of its five electrons and the fifth valence electron remains loosely bound to the parent impurity atom which becomes available as current carrier.

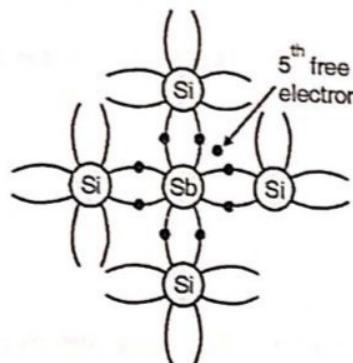


Fig. 3.7.1 : n-type semiconductor



- The energy required to remove the fifth electron is very small (0.05 eV). This energy is very small in comparison to 1.12 eV, which is the energy gap for Si, and it also represents the energy to break a covalent bond.
- So at modest temperatures the fifth electron can be detached from the impurity atom. The liberated electron is called **free electron**, and it can take part in conduction by entering into conduction band.
- Unlike the intrinsic semiconductors, by leaving fifth electron there is no hole created. Therefore at ordinary temperatures, there will be more electrons in the conduction band than holes in the valence band (holes are produced by intrinsic process). It means here electrons in conduction band come from two different ways.
 - (I) By the donor atom
 - (II) By intrinsic process
- Therefore, the majority current carriers in n-type semiconductors are electrons and minority current carriers are holes.
- The addition of an impurity adds an allowed energy level E_D at a very small distance below the conduction band as shown in Fig. 3.7.2.

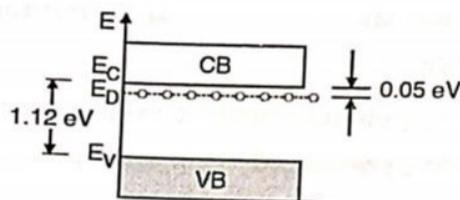


Fig. 3.7.2 : Donor level in n-type semiconductor

- This additional level lies in forbidden energy gap. An electron located at the donor level will have to acquire an energy equal to $(E_C - E_D)$ only, as against the energy $(E_C - E_V = E_g)$ required by an electron located in valence band, to go into conduction band at moderate temperatures.

As the conductivity is due to electrons, it is given by

$$\sigma_e = n \cdot e \cdot \mu_e \quad \dots(3.7.1)$$

The electron concentration will be governed by Boltzmann factor as

$$n \propto e^{-(E_C - E_D)/KT} \quad \dots(3.7.2)$$

and conductivity can be written as

$$\sigma_e = \sigma_0 e^{-(E_C - E_D)/KT} \quad \dots(3.7.3)$$

Position of Fermi level

In n-type semiconductors, as there are many free electrons in conduction band, the Fermi level gets shifted towards the conduction band. At 0°K it is between the bottom of conduction band and the level E_D .

If a trivalent impurity (Group III) is added to a pure semi-conductor, it becomes p-type extrinsic semiconductor. The impurity added is called as **acceptor impurity**. As shown in Fig. 3.7.3 boron (B) has been added as impurity which has three electrons. Each B atom tries to form covalent bonds with surrounding four Si atoms and falls short of one electron for completing four covalent bonds. As a result a vacancy is left in the bonding. This vacancy is not a hole. Originally, the environment in the crystal lattice is electrically neutral.

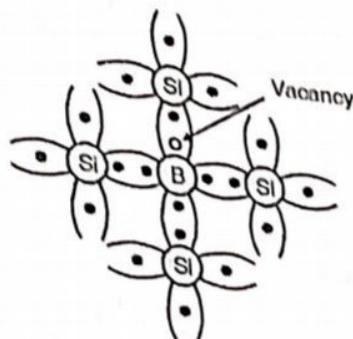


Fig. 3.7.3 : p-type semiconductor

- The introduction of impurity atom does not disturb the environment, and the vacancy arising due to the non-formation of bond is not a hole.
- However, when an electron from a neighbouring bond acquires energy and jumps into this vacancy, it leaves behind a positively charged environment in the broken bond. Therefore, a hole is generated there.
- The addition of an impurity adds an allowed level E_A at a very small distance above the top of the valence band as shown in Fig. 3.7.4

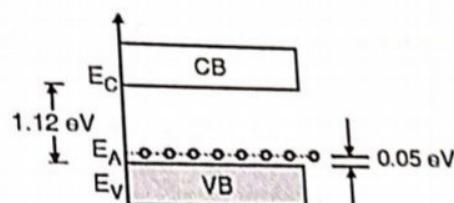


Fig. 3.7.4 : Acceptor level in p-type semiconductor

- At $T = 0^\circ\text{K}$ all acceptor levels at E_A are vacant as shown in Fig. 3.7.4 and valence band is full but the conduction band is empty.
- When the temperature increases, electrons from the valence band jump into acceptor level and leave holes behind. At moderate temperature all acceptor levels are filled and correspondingly for each acceptor atom one hole is created in the valence band.
- The generation of holes is not followed by the simultaneous generation of electrons. Therefore p-type semiconductor has holes as majority carriers and electrons as minority carriers, which results because of intrinsic process.



- The conductivity of a p-type semiconductor at ordinary temperature is given by

$$\sigma_p = p \cdot e \cdot \mu_h \quad \dots(3.7.4)$$

The hole concentration is governed by Boltzmann law as

$$n_p = e^{(E_V - E_A)/kT} \quad \dots(3.7.5)$$

The conductivity of a p-type semiconductor is given by

$$= \sigma_0 e^{(E_V - E_A)/kT} \quad \dots(3.7.6)$$

Position of Fermi level

In n-type semiconductor, as there are many free holes in valence band, the Fermi level gets shifted towards the valence band. At 0°K it is between top of valence band and the level E_A .

Table 3.7.1 : Difference between conduction for conductors and semiconductors

Sr. No.	Conduction	Semi-conduction
1.	Resistance depends upon temperature, impurity, type of material and ageing factor.	Semi-conduction depends upon temperature (for intrinsic) and on impurity atoms (for extrinsic type).
2.	Any kind of material (a) single crystal (b) poly crystalline (c) alloys or (d) amorphous material can be used for conductor.	Only single crystal is used for semi-conduction.
3.	Resistance increases with temperature.	Resistance decreases with temperature.
4.	Charge carriers are only electrons.	Charge carriers are both electrons and holes.
5.	Resistivity and hence conductivity cannot be altered to a desired value.	Conductivity (for extrinsic) can be altered by addition of impurity atoms.
6.	Active components cannot be made out of conductor.	Active components like diodes and transistors can be made of semiconductors.
7.	Conductivity is due to loosely bound electrons (metallic bonds).	Conductivity is due to covalently bonded electrons.

Table 3.7.2 : Comparison between intrinsic and extrinsic semiconductors

Sr. No.	Intrinsic semiconductors	Extrinsic semiconductors
1.	This is a pure element crystal.	This is a single crystal with impurity.
2.	The charge carriers are electrons of the parent atom of crystal and holes formed in absence of that electron.	The charge carriers are electrons or holes of the impurity atoms fitted into the crystal of parent atoms.
3.	The charge carriers, electrons and holes are equal in numbers i.e. $n_e = n_p$.	In n-type majority charge carriers are electrons and in p-type majority carriers are holes.

Intrinsic semiconductors

- No. No. 4 The charge carriers, say electrons, can be increased only by increase in temperature.
- No. 5 More energy is needed for the electrons to cross E_g (compared to extrinsic).

Extrinsic semiconductors

- The majority charge carriers can be increased by slightly increasing the percentage of impurity atoms.
- Less energy is needed for the electrons (impurity atoms) to cross E_g (compared to intrinsic).

3.7.3 Effect of Temperature on n-type Material

MU - May 17, Dec. 17, May 18

- Q. With the help of diagram, explain the variation of Fermi level with temperature in n-type semiconductor. (May 17, 5 Marks)
- Q. With energy band diagram explain the variation of Fermi energy level with temperature in extrinsic semiconductor. (Dec. 17, May 18, , 5 Marks)
- Q. Draw the energy band diagram of p-n junction diode in forward and reverse bias condition. (Dec. 17, May 18, 3 Marks)

(i) **At low temperature :** When the temperature in the semiconductor is low, only few donor atoms get ionized and electrons move from the donor level to the conduction band.

Hence, Fermi level for n-type semiconductor at low temperature lies midway between the bottom of the conduction band and donor level.

(ii) **At moderate temperature :** At moderate temperature all donor atoms are ionized. So, the concentration of electrons in conduction band is equal to the concentration of donor atoms.

When the temperature increases up to moderate value, Fermi level slowly shifts away from the conduction band and moves towards the center of the forbidden gap.

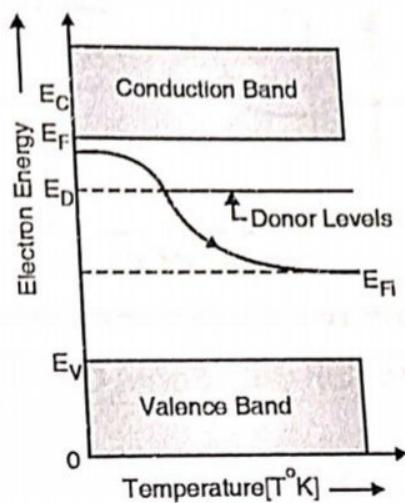


Fig. 3.7.5 Variation of E_F with temperature in n-type material

(iii) **At higher temperature :** At high temperature, the concentration of transfer of electrons from valence band to conduction band is more compared to concentration of electrons from donor atoms and Fermi level is shifted to middle of the forbidden gap.

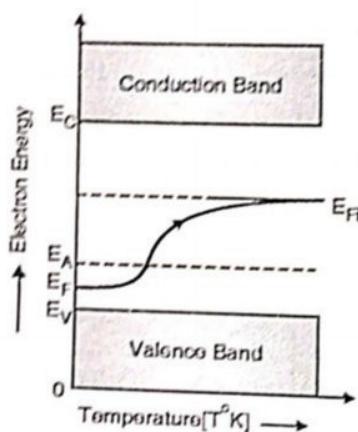
The variation of Fermi level with temperature for n-type of material is shown in Fig. 3.7.5.

3.7.4 Effect of Temperature on p-type Material

MU - Dec. 17, May 12

- Q. Draw the energy band diagram of p-n junction diode in forward and reverse bias condition. (Dec. 17, 3 Marks)
- Q. With energy band diagram explain the variation of Fermi energy level with temperature in extrinsic semiconductor. (Dec. 17, 5 Marks)
- Q. Draw the energy band diagram of p-n junction diode in forward and reverse bias condition. (May 12, 3 Marks)

- (i) **At low temperature :** At low temperature only few acceptor levels are occupied, and simultaneously holes are produced in valence band.
So, Fermi level lies in the middle of the top of the valence band and the acceptor level.
- (ii) **At moderate temperature :** At moderate temperature, all acceptor levels are filled.
So, at moderate temperature, Fermi level gradually moves up i.e. moves towards the middle of the forbidden gap.
- (iii) **At higher temperature :** At very high temperature, the contribution of conduction band for the formation of holes in the valence band is more compared to acceptor impurity.
Hence, at very higher temperature, Fermi level approaches the middle of the energy gap i.e. the position of E_F for intrinsic semiconductor. The variation of E_F with temperature in p-type material is shown in Fig. 3.7.6

Fig. 3.7.6 Variation of E_F with temperature in p-type material

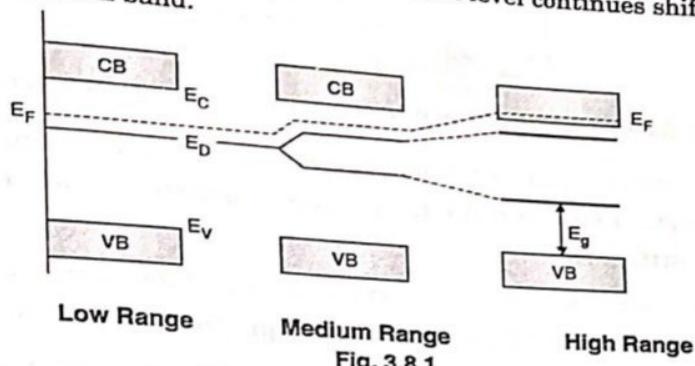
3.8 Effect of Impurity Concentration on Fermi Level

MU - Dec. 15, Dec. 16

- Q. Draw a neat labelled energy band diagram to show the variation of Fermi level with doping concentration in n-type semiconductors. (Dec. 15, 3 Marks)
- Q. How does the position of Fermi energy level change with increasing doping concentration in p-type semiconductors? Sketch the diagram. (Dec. 16, 5 Marks)

- We have seen the effect of temperature on Fermi level. The position of Fermi level is also affected by addition of impurity and by variation in the concentration of impurity.
- If a donor impurity is added to an intrinsic semiconductor, it results in n-type of semiconductor and a donor level comes into existence below the bottom of conduction band.

- At impurity concentrations, the impurity atoms are so spaced that they do not interact with each other.
- The donor levels starts splitting and forms an energy band below the conduction band. The width of this band increases with increase in the impurity concentration. At one stage it overlaps with the conduction band.
- Due to broadening of the donor levels into band the width of forbidden energy gap reduces and the Fermi level is found moved upwards.
- With increase in concentration of donor impurity, the Fermi level continues shifting towards conduction band and enters into conduction band.



- In the same way, for p-type semi conductors increase in the impurity concentration makes Fermi level shift into valence band.

3.9 P-N Junction

- The extrinsic semiconductors, individually, are of limited use. However, when a p-type and an n-type semiconductors are joined together such that crystal structure remains continuous at the metallurgical boundary, a p-n junction is formed.
- In practice it is formed by adding excessive acceptor impurities to a portion of n-type semiconductor or donor impurities to a p-type semiconductor.

3.9.1 Depletion Layer and Potential Barrier

MU = May 13, Dec 13, May 14, Dec 14, Dec 15, Dec 16

- | | |
|---|--------------------|
| Q. Draw the energy band diagram of an unbiased p-n junction and mark the barrier potential and depletion region. | (May 13, 3 Marks) |
| Q. Explain the formation of depletion layer in P-N junction. | (Dec. 13, 5 Marks) |
| Q. Explain the formation of barrier potential in P-N junction. | (May 14, 5 Marks) |
| Q. Explain the formation of depletion region in an unbiased p - n junction. | (Dec. 14, 3 Marks) |
| Q. Draw a neat labelled energy band diagram to show the variation of Fermi level with doping concentration in N-Type semi-conductors. | (Dec. 15, 3 Marks) |
| Q. How does the position of Fermi energy level change with increasing doping concentration in p-type semi-conductors? Sketch the diagram. | (Dec. 16, 5 Marks) |
| Q. Explain the formation of potential barrier across the unbiased p-n junction region. | (Dec. 16, 5 Marks) |

- In p-type of semiconductor, holes are majority carriers and its Fermi level is just above the top of acceptor level at room temperature (300°K). For n-type of semiconductor, electrons are majority carriers and its Fermi level is just below the donor level at 300°K.
- When p-n junction is being formed, both materials will find their Fermi levels non-aligned as shown in Fig. 3.9.1 and they must be aligned.

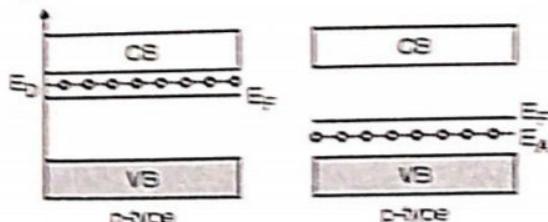


Fig. 3.9.1 : Position of Fermi-Level in n-type and p-type semiconductor

- In order to obtain alignment, holes from p-type (majority carriers for a p-type) move towards n-type and at the same time electrons from n-type (majority carriers from n-type) move towards p-type. This is called diffusion.
- As the diffusion takes place across the junction the holes and the electrons combine to neutralise each other at the junction and produce a free space called depletion layer as shown in Fig. 3.9.2.
- At the junction due to the migration of a few holes from p-type to n-type region, the negative immobile ions are produced in p-region and in the same way positive immobile ions are produced in n-region as shown in Fig. 3.9.2.
- As the name suggest these ions are immobile in nature (fixed in lattice). Under equilibrium condition it can be assumed similar to a charged parallel plate capacitor and creates an electronic potential called potential barrier or junction potential which prevents further diffusion of majority charge carriers.

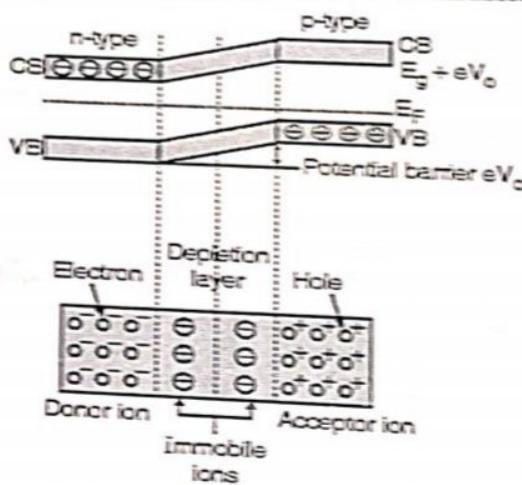


Fig. 3.9.2 : Unbiased p-n junction

3.9.2 Calculation of Potential Barrier

- The magnitude of potential barrier created can be estimated from the knowledge of the carrier densities in p and n-regions.

The concentration of electrons in the conduction band on n-side is written as

$$n_n = N_c \exp [-(E_g - E_F) / KT] \quad \dots(3.9.1)$$

Similarly the concentration on p-side is written as

$$n_p = N_c \exp [-(E_g + eV_{th} - E_F) / KT] \quad \dots(3.9.2)$$

Dividing equation (3.9.1) by equation (3.9.2),

$$\frac{n_n}{n_p} = \exp [eV_{th} / KT] \quad \dots(3.9.3)$$

Taking natural logarithm on both sides of equation (3.9.3) we get

$$V_{th} = \frac{KT}{e} \ln \left(\frac{n_n}{n_p} \right) \quad \dots(3.9.4)$$

By rewriting equation (3.9.4) as

$$V_{th} = \frac{KT}{e} \ln \left(\frac{n_n}{n_p} \cdot \frac{p_n}{p_p} \right) \quad \dots(3.9.5)$$

Where p_p is hole concentration on p-side.

At room temperature, all impurities are ionised therefore

$n_n = N_D$ = Donor impurity concentration

$p_n = N_A$ = Acceptor impurity concentration using relation,

$$n_p \cdot p_p = n_i^2$$

Equation (3.9.5) can be modified as

$$V_{th} = \frac{KT}{e} \ln \left(\frac{N_D \cdot N_A}{n_i^2} \right) \quad \dots(3.9.6)$$

$$\frac{KT}{e} = \text{Constant} = V_T$$

$$V_{th} = V_T \ln \left(\frac{N_D \cdot N_A}{n_i^2} \right) \quad \dots(3.9.7)$$

3.10 Hall Effect and its Applications

MU - Dec. 12, Dec. 13, May 15, Dec. 15, May 16, May 18, Dec. 18, May 19

- Q. What is Hall effect? Derive expression for Hall voltage with neat labelled diagram. (Dec. 12, May 16, 5 Marks)
- Q. Explain Hall effect in metal. Derive the formulae for density and mobility of electrons. (Dec. 13, 8 Marks)
- Q. State the Hall effect. Derive the expression for Hall voltage and Hall coefficient with neat diagram. (Dec. 15, 3 Marks, Dec. 17, May 18, 7 Marks)
- Q. Derive an expression for Hall voltage and Hall coefficient with neat labelled diagram. (Dec. 18, 7 Marks)
- Q. What is Hall effect? Derive an expression for Hall voltage. How can mobility be determined using Hall effect? (May 15, May 19, 3 Marks)

- If a metal or semiconductor, carrying a current I is placed in a transverse magnetic field B , an electric field E is induced in the direction perpendicular to both I and B . This phenomenon is known as Hall effect and the electric field or voltage induced is called Hall voltage (V_H). The physical process of Hall effect is as follows. Consider a specimen along positive x-direction (Fig. 3.10.1).

- The current flowing through the specimen is in the positive x-direction and the magnetic field is in the positive z-direction. The force exerted on charge carriers, that is, on electrons is downward. The electrons move downward and thus voltage V_H (Hall voltage) is developed along upper surface as positive and lower as negative

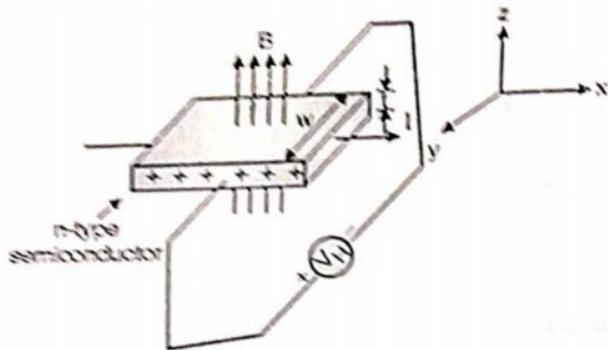


Fig. 3.10.1: Hall effect

3.10.1 Experimental Determination of Mobility

- If the specimen in Fig. 3.10.1 is assumed to be of n-type, then magnetic force experienced by electrons will be towards -Y direction as it is applied transversely
 - Magnetic force $F = e v B$
- Holes present in specimen will experience the same force but in positive Y direction. Hence, electrons and holes will be separated.

This develops potential difference between both the surfaces, denoted by V_H , called Hall voltage.

$$\therefore E_H = \frac{V_H}{w} \quad \dots(3.10.1)$$

- The current in this case is given by

$$= n A e v \quad \dots(3.10.2)$$

(Where, v = drift velocity)

- In equilibrium condition, the force due to magnetic field B and the force due to electric field E_H acting on the charge are balanced.

$$\therefore e E_H = evB \quad \dots(3.10.3)$$

or

$$E_H = vB$$

Using equation (3.10.3) and (3.10.1)

$$V_H = Bvw$$

Using equations (3.10.1) and (3.10.2).

$$v = \frac{I}{enA} = \frac{J}{en} \quad \text{where } J = \frac{I}{A}$$

Hence, Hall voltage can be written as

$$V_H = \frac{IBw}{enA} = \frac{Bw}{en} \cdot J$$

It can be also modified by using, $A = w \times t$

$$V_H = \frac{IB}{en t} \quad \dots(3.10.4)$$

By measuring V_H , I , B , and t , the charge density (n_{he} or $n_e e$) can be calculated.

Another important parameter is Hall coefficient R_H it is defined as,

$$R_H = \frac{1}{pe} \quad (\text{for p-type semiconductor})$$

$$= \frac{1}{ne} \quad (\text{for n-type semiconductor})$$

From equation (3.10.4),

$$R_H = \frac{V_H t}{BI} \quad \dots(3.10.5)$$

$$\text{As } \sigma = \mu ne, \quad \mu = \sigma R_H$$

\therefore If the conductivity and Hall coefficient are measured then the mobility and density of electron can be determined.

With increase in temperature of the semiconductor, the carrier concentration is increased and R_H decreases.

The net electric field E acting on the charge carrier is constant of the applied electric field E_x and Hall electric field E_H . The angle made by E with x -axis is called Hall angle, given by

$$\theta_H = \tan^{-1}\left(\frac{E_H}{E_x}\right)$$

Which can also be proved as,

$$\theta_H = \tan^{-1}(R_H \sigma B)$$

3.10.2 Applications

1. The Hall voltage V_H is proportional to magnetic field B , for the given current I , therefore Hall effect is used in magnetic field meter.
2. The charge carrier concentration can be determined.
3. The mobility of charge carriers can be determined.
4. The nature of semiconductor (p-type or n-type) can be determined.

3.10.3 Light Emitting Diodes (LED)

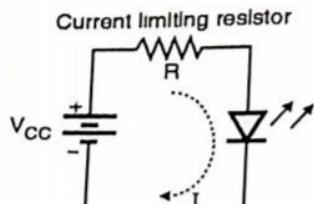
MU - Dec. 15, Dec. 18

Q. Explain the principle, construction and working of a LED.

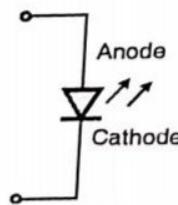
(Dec. 15, Dec. 18, 5 Marks)

- In this section, let us discuss about one of the important light sources which is the Light Emitting Diode (LED).
- An LED emits light when electrical energy is applied to it. LED is a two-terminal device. The terminals are named as anode (A) and the other as cathode (K).

- A p-n junction is formed between the anode and cathode. So, LED is basically a p-n junction diode.
- For proper operation, it is necessary to forward bias the LED as shown in Fig. 3.10.2(a) and the symbol of LED is shown in Fig. 3.10.2(b). This shows that the symbol is same as that of a p-n junction diode with two arrows indicating that it emits light.



(a) LED biasing



(b) Circuit symbol of LED

Fig. 3.10.2

Construction of LED

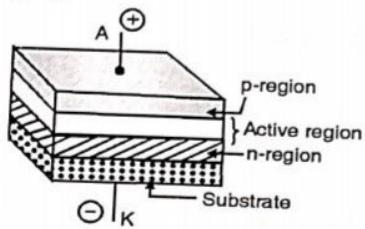
- One of the popular methods of LED construction is to deposit three semiconductor layers on the substrate as shown in Fig. 3.10.2 (c).
- The active region exists between the p and n regions. The light emerges from the active side in all the directions when electron-hole pairs recombine.
- The disadvantage of this structure is that the LED emits light in all directions. This problem can be solved by placing the basic structure of Fig. 3.10.2 (c) inside a small reflective cup, so as to focus the light in the desired direction.
- Such a structure is called as a cup-type construction and it is shown in Fig. 3.10.2(d).

Circuit symbol

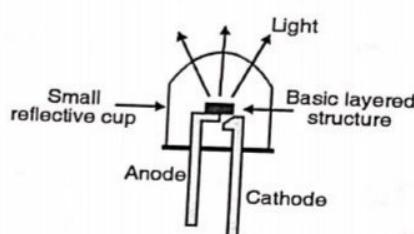
- The circuit symbol of an LED is as shown in Fig. 3.10.2(b). This shows that the symbol is identical to a p-n junction diode with two arrows indicating that it emits light.

Semiconductor materials used

- LEDs are made of Gallium Arsenide (GaAs), Gallium Arsenide Phosphide (GaAsP) and Gallium Phosphide (GaP).
- Silicon and germanium are not used because they are essentially heat producing materials and are very poor in producing light.



(c) Construction of LED

(d) Cup type construction of LED
Fig. 3.10.2

Principle of LED Operation

- When the LED is forward biased, the electrons in the n-region will cross the junction and recombine with the holes in the p-type material.
- These free electrons reside in the conduction band, and hence at a higher energy level than the holes in the valence band.
- When the recombination takes place, these electrons return back to the valence band which is at a lower energy level than the conduction band.
- While returning back, the recombining electrons give away the excess energy in the form of light. This is shown in Fig. 3.10.2(e) This process is called as electroluminescence. In this way, an LED emits light. This is the principle of operation of LED.

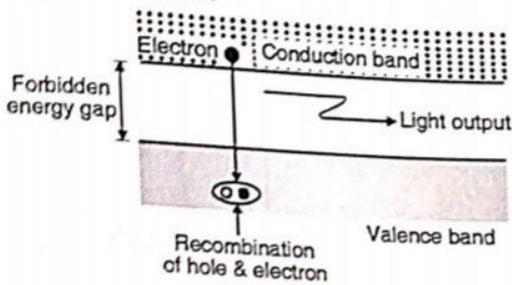
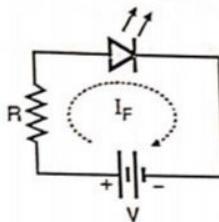


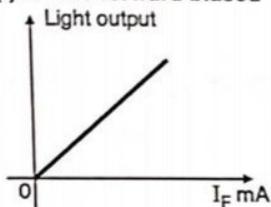
Fig. 3.10.2 (e) : Principle of operation of LED

Output Characteristics of LED

- The graph of Fig. 3.10.2 (g) shows the output characteristics of the LED. It shows the linear dependence of light output on the forward LED current.
- However, it is not possible to increase I_F indefinitely. Typically, the maximum value of I_F is 80 mA.



(f) LED is forward biased



(g) Relation between forward current and light output

Fig. 3.10.2

3.10.4 Zener Diode

- In p-n junctions, we have studied forward current but accepted that a very small reverse current is ignored.

- Under reverse-bias condition a very small current flows through diode but only up to certain voltage applied.
- Beyond that a breakdown occurs and a high current flows. This has been regarded as destructive in general.
- This current need not be destructive if the current is such that the diode does not become overheated. But suitably controlling the amount of impurity and making the p-n junction capable enough to make breakdown very distinctly sharp.
- This type diodes are called Zener diode and it is represented by the symbol shown Fig. 3.10.3(a).
- The Zener diode is used in reverse biased condition. It has its own specific application.



Fig. 3.10.3(a)

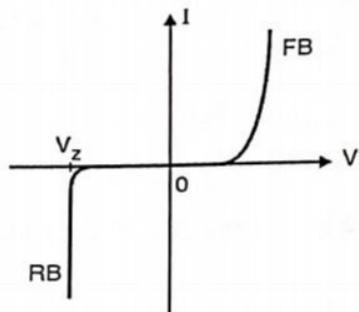


Fig. 3.10.3(b)

- Here it is seen that, once the diode has broken down, very little change in voltage across the diode results from a change in current through the diode. Thus a Zener diode of known breakdown voltage can be used as a reference voltage when operated in the reverse biased condition and can be used to regulate voltage.

3.10.5 Photovoltaic Cell

MU - May 13, Dec. 14, May 15, Dec. 17, May 18

- Q.** How is a p-n junction diode used to generate a potential difference in a photovoltaic solar cell? (May 13, 5 Marks)
- Q.** Explain how a voltage difference is generated in a p-n junction when it is used in a photovoltaic solar cell. (Dec. 14, May 15, 5 Marks)
- Q.** What is photovoltaic effect? Explain the principle and working of solar cell. (Dec. 17, May 18, 5 Marks)

- When suitable light falls on a p-n junction, a potential difference is produced across it. This voltage is capable of driving a current through an external circuit. This important phenomenon is called the 'photovoltaic effect'.
- This photovoltaic effect was first invented by E. Becquerel in 1839. He found that certain materials would produce very small amount of electric current when exposed to light. On continuous effort, Bell laboratories in 1954 constructed the first photovoltaic cell, known as a solar battery. Due to the energy crisis, presently photovoltaic technology is becoming the main source of electrical power for many industrial and domestic applications.

A device is so constructed in which solar energy is converted into electrical energy. Therefore, this device is called as photovoltaic cell or solar cell. In the solar cell, semiconductor material is used. When light energy falls on it, electrons from the semiconductor material get free from the atom in the semiconductor. Further, these electrons are made free to move through the semiconductor by applying external potential or voltage. Finally, due to this arrangement electric current is observed in the electrical circuit. The current produced is found to directly depend on light intensity falling on the semiconductor materials. The following section describes the details of solar cell.

Solar Cell

- Q. Explain with neat diagram construction and working of solar cell.
- Q. Explain the use of P-N junction as solar cell.
- Q. What is the principle of a solar cell? Write its advantages and disadvantages.

MU - Dec. 12, Dec. 13, May 14, May 17

(Dec. 12, May 14, 5 Marks)

(Dec. 13, 3 Marks)

(May 17)

(a) Principle

- A solar cell works on the principle of photovoltaic effect. It is a device that directly converts the energy in light into electrical energy.
- It was first developed by French physicist Antoine-César Becquerel. He observed the photovoltaic effect while experimenting with a solid electrode in an electrolyte solution when he saw a voltage developed when light fell upon the electrode.

(b) Construction and working

- From construction point of view, generally crystalline silicon (Si) has been used as the light-absorbing semiconductor in most solar cells, even though it is a relatively poor absorber of light and requires a considerable thickness (several hundred microns) of material.
- Presently two types of crystalline silicon are used for production of solar cell namely
 - 1) **Monocrystalline** - It is produced by slicing wafers (up to 150mm diameter and 350 microns thick) from a high-purity single crystal.
 - 2) **Multicrystalline** - It is made by sawing a cast block of silicon first into bars and then wafers.
- For both mono and multicrystalline Si, a semiconductor homojunction is formed by diffusing phosphorus (an n-type dopant) into the top surface of the boron doped (p-type) Si wafer. Construction of the solar cell is as shown in Fig. 3.10.4(a).
- Screen-printed contacts are applied to the front and rear of the cell, with the front contact pattern specially designed to allow maximum light exposure of the Si material with minimum electrical (resistive) losses in the cell.
- The **most efficient production cells** use monocrystalline Si with laser grooved and buried grid contacts for maximum light absorption and current collection.
- Each Si cell generates about 0.5V, so about 36 cells are usually soldered together in series to produce a module with an output to charge a 12V battery.
- The cells are hermetically sealed under tough, high transmission glass to produce highly reliable, weather-resistant models that may be warranted for up to 25 years.

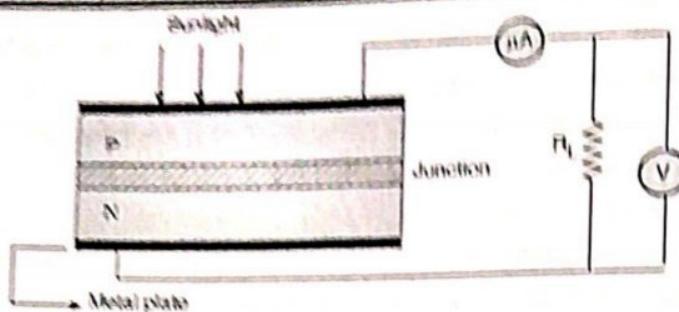


Fig. 3.10.4 : (a) Working of a solar cell

- (1) Production and development of pairs of positive and negative charges (electron-hole pairs) in the solar cell by absorbed solar radiation
- (2) Distance between positive and negative charges by a potential gradient within the cell
First it is essential to select the material or prepare the material which can absorb the energy associated with the photons of sunlight. This can be done using following relation

$$E = \frac{hc}{\lambda} \quad \dots(3.10.1)$$

Where,

and

E = Energy of a photon

λ = Wavelength of the radiation

h = Planck's constant = 6.62×10^{-27} ergs

c = Velocity of light = 3×10^8 m/s

Hence above relation becomes $E = \frac{1.24}{\lambda}$, in which E is the energy in electron-volt (eV) and λ is in nanometer.
Thus above relation is used to design a solar cell.

- The only materials suitable for absorbing the energy of the photons of sunlight are semiconductors like silicon, cadmium sulphide, gallium arsenide, etc.
- Generally in a semiconductor, the electrons occupy either valence band or the conduction band.
- The valence band has electrons at a lower energy level and is fully occupied, while the conduction band has electrons at a higher energy level and is not fully occupied.
- The difference between the energy levels of the electrons in the two bands is called the band gap energy E_g . It is necessary that photons of sunlight having energy E greater than the band gap energy E_g .
- Here, photons of sunlight having energy E which is greater than E_g is absorbed by the cell material. Therefore, excitation of the electrons takes place.
- Such excited electrons jump across the band gap from the valence band to the conduction band, but they leave behind holes in the valence band. In this way electron-hole pairs are created, it is as shown Fig. 3.10.4(b).
- Thus, electrons in the conduction band and the holes in the valence band become mobile. They can be separated and made to flow through an external electronic circuit.
- According to the photovoltaic effect, a potential gradient is developed within the cell.

In the case of silicon, the potential gradient is obtained by making the cell as a sandwich of two types of silicon, i.e. p-type and n-type. Thus, when a composite of the two types of silicon is formed, a jump in energy levels occurs at the junction interface. (See Fig. 3.10.4 (b))

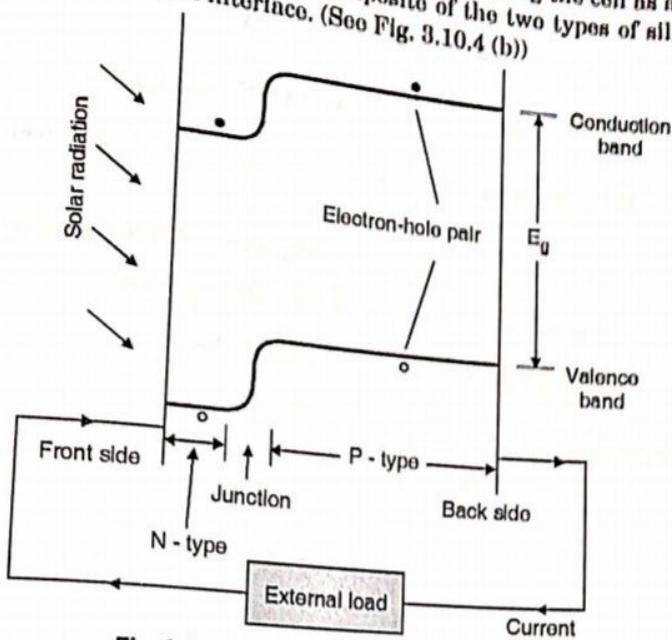


Fig. 3.10.4(b) : Band structure of solar cell

- In this way, the developed potential gradient is adequate to separate the electrons and holes, and hence produces a direct electric current to flow in the external circuit with load.

(c) Advantages of solar cell

- Raw material i.e. amount of solar energy is available at no cost for operation of the solar cell. Hence, it is more useful in satellite communication.
- It is pollution free. It is not harmful to human life.
- It is most useful in remote areas where traditional transmission may be difficult.
- They are portable as compared to other electronic energy transmission devices.

(d) Disadvantages of solar cell

- Operation in the night is not possible.
- It has low efficiency hence presently it is not economical.
- It is a dilute source.

3.11 Solved Problems on Mobility, Conductivity and Resistivity

Ex. 3.11.1 : The resistivity of Cu is 1.72×10^{-8} ohm-m. Calculate the mobility of electrons in Cu given that number of electrons per unit volume is $10.41 \times 10^{28} \text{ m}^{-3}$.

Soln. :

Given :

$$\rho = 1.72 \times 10^{-8} \Omega \cdot \text{m},$$

$$\sigma = \frac{1}{\rho} = 58.1 \times 10^6 \text{ mho/m}$$



$$n = 10.41 \times 10^{28} \text{ m}^{-3}$$

Formula :

$$\mu = \frac{\sigma}{ne} = \frac{58.1 \times 10^6}{10.42 \times 10^{28} \times 1.6 \times 10^{-19}}$$

$$\therefore \mu = 3.488 \times 10^{-3} \frac{\text{m}^2}{\text{volt sec.}}$$

...Ans.

Ex. 3.11.2 : Find the resistivity for Cu assuming that each atom contributes one free electron for conduction.

Soln. :

Given :

$$\text{Density} = 8.96 \text{ g/cm}^3, \text{ Atomic weight} = 63.5$$

$$\text{Avogadro's number} = 6.02 \times 10^{23} / \text{g-mole}$$

$$\text{Mobility of electron} = 43.3 \text{ cm}^2 / \text{V-sec.}$$

Formula :

$$\sigma = n\mu e$$

$$\text{And } \rho = \frac{1}{\sigma}$$

$$\therefore \rho = \frac{1}{n\mu e}$$

∴ Calculation of n

$$\text{Atomic density} = \frac{6.023 \times 10^{23} \times 8.96}{63.5} = 8.4985 \times 10^{22} / \text{cm}^3$$

$$\therefore n = 1 \times \text{Atomic density} = 1 \times 8.5 \times 10^{22} / \text{cm}^3$$

$$\therefore \rho = \frac{1}{8.5 \times 10^{22} \times 1.6 \times 10^{-19} \times 43.3}$$

$$\therefore \rho = 1.698 \times 10^{-6} \text{ ohm-cm}$$

...Ans.

Ex. 3.11.3 : Find the resistivity of intrinsic Ge at 300°K given the density of carriers as $2.5 \times 10^{19} \text{ m}^{-3}$.

Soln. :

Formula :

$$\sigma_{in} = n_e \cdot e \cdot \mu_e + n_h \cdot e \cdot \mu_h$$

For intrinsic semiconductor

$$n_e = n_h = n_i$$

$$\therefore \sigma = n_i \cdot e (\mu_e + \mu_h)$$

Using standard values of μ_e and μ_h

$$\therefore \sigma_{in} = 2.5 \times 10^{19} \times 1.6 \times 10^{-19} [0.39 + 0.19] = 2.32 (\text{ohm.m})^{-1}$$

$$\therefore \rho_i = \frac{1}{\sigma_i} = 0.43 \text{ ohm.m}$$

...Ans.

Ex. 3.11.4 : In an intrinsic semiconductor the energy gap E_g of an intrinsic semiconductor is 1.2 eV. Its hole mobility is much smaller than electron mobility and is independent of temperature. What is the ratio between conductivity at 600°K and that at 300°K? Comment on the result.

Soln. :

Since $\mu_e \gg \mu_h$, for intrinsic semiconductor equation (3.6.3) can be rewritten as,

$$\sigma_i = n_i \cdot e \cdot \mu_e \quad (\text{as } n_i = n_e = n_h)$$

$$\therefore n_e = N \exp [-E_g / 2KT]$$

$$\therefore \sigma_i = e \cdot \mu_e \cdot N \exp [-E_g / 2KT]$$

All the pre-exponential terms are independent of temperature. We can put a constant

$$\sigma_0 = \mu_e \cdot e \cdot N$$

$$\therefore \sigma_i = \sigma_0 \exp \left[-\frac{E_g}{2KT} \right]$$

$$= \sigma_0 \exp \left[-\frac{1.2}{2 \times K \times 600} \right]$$

$$\sigma(300^\circ\text{K}) = \sigma_0 \exp \left[-\frac{1.2}{2 \times K \times 300} \right]$$

Taking K , the Boltzmann constant in eV as $K = 8.62 \times 10^{-5}$ eV/K and solving we get

$$\frac{\sigma(600^\circ\text{K})}{\sigma(300^\circ\text{K})} = 1 \times 10^5$$

...Ans.

Comment : The conductivity of an intrinsic semiconductor is greatly influenced by temperature.

Ex 3.11.5 : Predict the effect on the electrical properties of a silicon at room temperature if every millionth silicon atom is replaced by an atom of Indium. Comment on results.

Given :

$$\text{Concentration of Si atoms} = 5 \times 10^{28} \text{ m}^{-3}$$

$$\text{Intrinsic conductivity of Si} = 4.4 \times 10^{-4} \text{ mho.m}^{-1}$$

$$\text{Mobility of holes } \mu_h = 0.048$$

Soln. :

As indium belongs to group III, holes will remain as majority carrier. Concentration of Si atoms = $5 \times 10^{28} \text{ m}^{-3}$

Concentration of impurity atoms

$$n = 5 \times 10^{28} \times 1 \times 10^{-6} = 5 \times 10^{22} \text{ m}^{-3}$$

$$\therefore \sigma_p = \mu_h \cdot e \cdot n = 0.048 \times 1.6 \times 10^{-19} \times 5 \times 10^{22}$$

$$\therefore \sigma_p = 384 \text{ mho.m}^{-1}$$

...Ans.

Comment : Intrinsic conductivity for Si at room temperature is $4.4 \times 10^{-4} \text{ mho.m}^{-1}$ and when trivalent impurity of indium at one part per million is added, the conductivity becomes 384 mho.m^{-1} which is increased by almost six orders of magnitude.

Ex.3.11.6 : An impurity of 0.01 ppm (particles per million) is added into Si. The semiconductor has a resistivity of 0.25 ohm.m⁻¹ at 300°K. Calculate the hole concentration and its mobility. Also comment on result.

Given : Atomic weight of Si = 28.1 and

$$\text{density of Si} = 2.4 \times 10^3 \text{ kg/m}^3$$

Soln. :

$$\begin{aligned} \therefore \text{Number of Si atoms / m}^3 &= \frac{\text{Avogadro number} \times \text{Density}}{\text{Atomic weight}} \\ &= \frac{6.024 \times 10^{26} \times 2.4 \times 10^3}{28.1} \end{aligned}$$

$$= 5.14 \times 10^{22} \text{ atoms/m}^3$$

Impurity level is 0.01 ppm i.e. 1 atom at every 10^8 atoms of Si

∴ Number of impurity atoms

$$= \frac{5.14 \times 10^{22}}{10^8} = 5.14 \times 10^{14}$$

Each impurity introduces one hole

$$\text{i.e. holes/m}^3 = 5.14 \times 10^{14} = n_h$$

$$\therefore \mu_p = \frac{1}{e.p.n_h}$$

$$= \frac{1}{1.6 \times 10^{-19} \times 0.25 \times 5.14 \times 10^{14}}$$

$$\therefore \mu_p = 0.0486 \text{ m}^2/\text{volt.sec}$$

...Ans.

Comment: On addition of trivalent impurity the mobility of Si remains the same but the concentration of holes increases.

Ex. 3.11.7 : The resistivity of intrinsic InSb at room temperature is $2 \times 10^{-4} \Omega \text{ cm}$. If the mobility of electron is $6 \text{ m}^2/\text{V-sec}$ and mobility of hole is $0.2 \text{ m}^2/\text{V-sec}$. Calculate its intrinsic carrier density. MU - Dec. 13, 5 Marks

Soln. :

Given

$$\rho_i = 2 \times 10^{-4} \Omega \text{ cm} = 2 \times 10^{-6} \Omega \text{ m}$$

$$\mu_n = 0.2 \text{ m}^2/\text{V-sec},$$

$$\mu_e = 6 \text{ m}^2/\text{V-sec.}$$

Formula :

$$\sigma_{in} = n_e e \mu_e + n_h e \mu_n$$

For intrinsic semiconductor

$$n_e = n_h = n_i$$

$$\therefore \sigma_{in} = n_i e (\mu_e + \mu_n)$$

Now,

$$\rho_{in} = \frac{1}{\sigma_{in}} = \frac{1}{n_i e (\mu_e + \mu_n)}$$

$$\therefore n_i = \frac{1}{\rho_{in} e (\mu_e + \mu_n)}$$

$$= \frac{1}{2 \times 10^{-6} \times 1.6 \times 10^{-19} (6 + 0.2)}$$

$$\therefore n_i = 5.04 \times 10^{23}$$

...Ans.

Ex. 3.11.8 : Calculate electron and hole concentration in intrinsic silicon at room temperature if its electrical conductivity is $4 \times 10^{-4} \text{ mho/m}$. Given that mobility of electron = $0.14 \text{ m}^2/\text{V-sec}$ and mobility of holes = $0.04 \text{ m}^2/\text{V-sec}$.

MU - May 14, May 17, Dec. 18, 3/5 Marks

Soln. :

For intrinsic semiconductor $n_e = n_h = n_i$.

and

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Semiconductor Physics

$$\sigma_{\text{in}} = n_i e (\mu_n + \mu_e)$$

$$n_i = \frac{\sigma_{\text{in}}}{e(\mu_n + \mu_e)} = \frac{4 \times 10^{-4}}{1.6 \times 10^{-19} (0.14 + 0.4)}$$

$$n_i = 1.388 \times 10^{16}$$

...Ans.

Ex 3.11.9: Determine the concentration of conduction electron in a sample of silicon if one in every million silicon atoms is replaced by a phosphorous atom. Assume every phosphorous atom to be singly ionized. Si has a molar mass of 0.028 kg/mole and density of 2300 kg/m³

MU - Dec. 14, 5 Marks

Soln.:

Given:

$$\text{Molar mass} = 0.028 \text{ kg/mole}$$

$$\text{Density} = 2300 \text{ kg/m}^3$$

As the molar mass is given, number of silicon per m³ is given by

$$\begin{aligned} &= \frac{\text{Density}}{\text{Molar mass}} \times \text{Avogadro number} \\ &= \frac{2300}{0.028} \times 6.023 \times 10^{23} = 4.947 \times 10^{25} \end{aligned}$$

As every millionth atom is replaced by phosphorous hence number of phosphorous atoms per m³ is 4.947×10^{25}

Also, phosphorous is singly ionized, each atom contributes i.e.

$$\therefore \text{Concentration of electron} = 4.947 \times 10^{25}/\text{m}^3$$

... Ans.

Ex. 3.11.10: Calculate conductivity of a germanium sample if donor impurity atoms are added to the extent of one part in 10^6 germanium atoms at room temperature.

Assume that only one electron of each atom takes part in conduction process.

Given : Avogadro's number : 6.023×10^{23} atoms / g-mole

Atomic weight of Ge = 72.6

Mobility of electrons = $3800 \text{ cm}^2/\text{volt-sec}$. Density of Ge = 5.32 g/cm^3

MU - Dec. 16, 3 Marks

Soln.:

$$\text{Atomic weight} = 72.6$$

$$\text{Density} = 5.32 \text{ g/cm}^3$$

$$\begin{aligned} \frac{5.32}{72.6} \times 6.023 \times 10^{23} &= \frac{\text{Number of atoms}}{\text{cm}^3} \\ &= 4.413 \times 10^{22} \end{aligned}$$

Since Ge is doped with a donor impurity, it will become n-type. For impurity added one part in Ge

$$\frac{\text{Number of donor atoms}}{\text{cm}^3} = 4.413 \times 10^{16}$$

Every atom provides one electron.

$$\therefore \text{Concentration of electron} = 4.413 \times 10^{16} = n$$

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Conductivity

$$\sigma = n e \mu_e$$

$$= 4.413 \times 10^{16} \times 1.6 \times 10^{-19} \times 3800$$

$$= 26.831 \text{ mho.cm}^{-1}$$

...Ans.

Ex. 3.11.11 : Calculate the current produced in a Ge sample of cross section 1 cm^2 and thickness of 0.01 m when potential difference of $2V$ is applied across it. The concentration of free electrons in Ge is $2 \times 10^{19}/\text{m}^3$ and mobilities of electron and holes are $0.36 \text{ m}^2/\text{V-sec}$ and $0.17 \text{ m}^2/\text{volt-sec}$, respectively.

MU - Dec. 16, 3 Marks

Soln. :

Ge is an intrinsic semiconductor.

$$\therefore \text{Conductivity } \sigma = ne(\mu_n + \mu_h)$$

$$= 2 \times 10^{19} \times 1.6 \times 10^{-19}$$

$$(0.36 + 0.17)$$

$$= 1.696 \text{ mho m}^{-1}$$

$$\rho = \frac{1}{\sigma} = \frac{1}{1.696} = 0.5896 \text{ ohm-m}$$

$$R = \frac{\rho l}{A} = \frac{0.5896 \times 0.01}{1 \times 10^{-4}} = 58.96 \text{ ohm.}$$

∴ Resistivity

Now

$$\therefore I = \frac{V}{R} = \frac{2}{58.96} = 0.034 \text{ Amp.}$$

3.12 Solved Problems on Fermi Level

Ex. 3.12.1 : What is the probability of an electron being thermally excited to conduction band in silicon at 27°C ? The band gap energy is 1.12 eV .

MU - Dec. 17, May 18, Dec. 18, 5/7 Marks

Soln. :

$$f(E_c) = \frac{1}{1 + \exp [(E_c - E_v) / KT]}$$

Remember

$$K = \text{Boltzmann constant}$$

$$= 1.38 \times 10^{-23} \text{ J/K}$$

In eV it is given by

$$K (\text{in eV}) = \frac{1.38 \times 10^{-23}}{1.6 \times 10^{-19}} = 86.25 \times 10^{-6} \text{ eV}$$

Also for intrinsic semiconductor

$$E_c - E_v = E_g / 2 = \frac{1.12}{2} = 0.56 \text{ eV}$$

$$\frac{E_c - E_v}{KT} = \frac{0.56}{86.25 \times 10^{-6} \times (27 + 273)}$$

$$= 21.64$$

$$\therefore f(E_c) = \frac{1}{1 + e^{21.64}}$$

$$\therefore f(E_c) = 3.99 \times 10^{-10}$$

...Ans.

Ex. 3.12.2 : In a solid the energy level lies 0.012 eV below Fermi level. What is the probability of this level not being occupied by an electron?

Given : $T = 300^\circ \text{K}$, $k = 1.38 \times 10^{-23} \text{ J/K}$.

Soln. :

Here,

$$E_F - E = 0.012 \text{ eV}$$

$$kT = \frac{1.38 \times 10^{-23} \times 300}{1.6 \times 10^{-19}}$$

$$= 0.02587 \text{ eV}$$

∴ The probability of an energy level not being occupied by an electron is $1 - f(E)$

$$\begin{aligned} 1 - f(E) &= 1 - \frac{1}{1 + \exp(E - E_F)/kT} \\ &= 1 - \frac{1}{1 + \exp\left(\frac{0.012}{0.02587}\right)} \\ &= 0.614 \end{aligned}$$

...Ans.

Ex. 3.12.3 : Explain the concept of Fermi level. What is the probability of an electron being thermally excited to conduction band in silicon at 20°C ? The band gap energy is 1.12 eV.

Given :

Boltzmann constant $1.38 \times 10^{-23} \text{ J/K}$.

Soln. :

$$f(E) = \frac{1}{[1 + \exp(E_C - E_V)/KT]}$$

$$K (\text{in eV}) = \frac{1.38 \times 10^{-23}}{1.6 \times 10^{-19}} = 86.25 \times 10^{-6} \text{ eV}$$

For intrinsic semiconductor,

$$E_C - E_V = E_g/2 = \frac{1.12}{2} = 0.56 \text{ eV}$$

$$\begin{aligned} \therefore f(E_C) &= \frac{1}{1 + \exp\left[\frac{0.56}{86.25 \times 10^{-6} \times (20 + 273)}\right]} \\ &= \frac{1}{1 + \exp\left[\frac{0.56}{22.15956}\right]} \\ &= 2.37 \times 10^{-10} \end{aligned}$$

...Ans.

Ex. 3.12.4 : Fermi level in potassium is 2.1 eV. What are the energies for which the probability of occupancy at 300 K are 0.99 and 0.01?

Soln. :

$$f(E) = \frac{1}{1 + e^{(E - E_F)/kT}}$$

$$\text{For } k = 1.38 \times 10^{-23} \text{ J/K}$$

$$\text{and } T = 300^\circ\text{K}$$

$$kT = \frac{1.38 \times 10^{-23} \times 300}{1.6 \times 10^{-19}}$$

$$= 0.026 \text{ eV}$$

Hence for $f(E) = 0.99$

$$0.99 = \frac{1}{1 + e^{(E - 2.1)/0.026}}$$

$$\therefore 1 + e^{(E - 2.1)/0.026} = \frac{1}{0.99} = 1.01$$

$$\therefore e^{(E - 2.1)/0.026} = 0.01$$

Taking \ln on both sides

$$(E - 2.1)/0.026 = e^{0.01} = 4.605$$

$$\therefore E - 2.1 = 0.1197$$

$$\therefore E = 1.98 \text{ eV}$$

...Ans.

Similarly for $f(E) = 0.01$

$$\text{We get } E = 2.218 \text{ eV}$$

...Ans.

Ex. 3.12.5 : Fermi energy of silver is 5.5 eV. Find the energy for which the probability of occupancy of 300 K is 0.9.

MU Dec. 2014, 3 Marks

Soln. :

Formula :

$$F(E) = \frac{1}{1 + e^{(E - EF)/KT}}$$

Here,

$$E_F = 5.5 \text{ eV}$$

$$T = 300 \text{ K}$$

$$F(E) = 0.9$$

$$K = 1.38 \times 10^{-23} \text{ J/K}$$

$$= 86.25 \times 10^{-6} \text{ eV}$$

$$KT = 86.25 \times 10^{-6} \times 300]$$

$$= 0.026$$

Substitute all these in formula]

$$\therefore 0.9 = \frac{1}{1 + e^{(E - 5.5)/0.026}}$$

$$\therefore e^{(E - 5.5)/0.026} = \frac{1}{0.9} - 1 = 0.111$$

$$\frac{(E_c - E_v)}{0.026} = \ln(0.111) = -2.187$$

$$E = -2.187 \times 0.026 + 0.5 \\ = 5.443 \text{ eV}$$

...Ans.

Ex 3.125: What is the probability of an electron being thermally promoted to conduction band in diamond at 27 °C if band gap is 5.6 eV?
MU - May 15, May 16, 3 Marks

Soln.:

$$f(E) = \frac{1}{1 + \exp\left[\frac{(E_c - E_v)}{2}/KT\right]} \text{ for insulator}$$

$$\text{as } t = 27^\circ\text{C}$$

$$\therefore T = 300^\circ\text{K}$$

$$\text{take } K = 1.38 \times 10^{-23} \text{ J/K}$$

$$= 86.25 \times 10^{-6} \text{ eV}$$

$$\therefore KT = 0.026 \text{ eV}$$

$$\text{also } E_g = \frac{E_c - E_v}{2} = 5.6 \text{ eV}$$

$$\therefore f(E) = \frac{1}{1 + \exp\left[\frac{5.6}{2 \times 0.026}\right]} = 1.7 \times 10^{-1} \quad \dots \text{Ans.}$$

(Since diamond is an insulator, very poor probability)

3.13 Solved Problems on Hall Effect

Ex 3.13.1: An n-type of Ge sample has a $N_D = 10^{21}/\text{m}^3$ and width 5 mm. It is arranged in a Hall effect experimental set up. If $B = 0.6 \text{ T}$, $J = 500 \text{ A/m}^2$, find Hall voltage.

Soln.:

$$\text{We know } V_H = \frac{BIw}{neA}$$

$$\text{Here } wd = A$$

$$\text{and taking } \frac{I}{A} = J$$

$$V_H = \frac{Bw}{ne} \cdot J \qquad \text{take } n = N_D = \frac{BwJ}{N_D e}$$

$$\therefore V_H = \frac{0.6 \times 5 \times 10^{-3} \times 500}{10^{21} \times 1.6 \times 10^{-19}}$$

$$= 9.3 \text{ mV}$$

...Ans.

Ex 3.13.2: A copper strip 2cm wide and 1mm thick is placed in a magnetic field $B = 1.5 \text{ wb/m}^2$. If current of 200A is set up in the strip, calculate Hall voltage that appears across the strip. (Given $R_H = 6 \times 10^{-7} \text{ m}^2/\text{C}$)

MU - May 10, 5 Marks

Soln. : Using equation (3.10.5)

$$R_H = \frac{V_{Ht}}{BI}$$

$$V_H = R_H \frac{BI}{t} = 6 \times 10^{-7} \times \frac{200 \times 1.5}{1 \times 10^{-3}}$$

$$V_H = 0.18 \text{ Volt}$$

Ex. 3.13.3 : The mobility of holes is $0.025 \text{ m}^2/\text{V}\cdot\text{sec}$. What would be the resistivity of p-type silicon if the Hall coefficient of the sample is $2.25 \times 10^{-5} \text{ m}^3/\text{C}$.

MU - Dec. 10, Dec. 12, May 17, May 19, 3 Marks
...Ans.

Soln. :

Formula :

$$\text{We know, } \sigma = ne\mu \text{ or, } \mu = \frac{1}{ne} \cdot \sigma = R_H \sigma$$

$$\text{or, } \sigma = \frac{\mu}{R_H} \text{ or, Resistivity } \rho = \frac{R_H}{\mu}$$

Data given :

$$R_H = 2.25 \times 10^{-5} \text{ m}^3/\text{C},$$

$$7\mu = 0.025 \text{ m}^2/\text{V}\cdot\text{s}$$

$$\text{So, } \rho = \frac{2.25 \times 10^{-5}}{0.025}$$

$$= 9 \times 10^{-4} \text{ ohm-m}$$

...Ans.

Ex. 3.13.4 : A sample of a n-type silicon has a donor density of $10^{20}/\text{m}^3$. It is used in the Hall effect experiment. If the sample of width 4.5 mm is kept in a magnetic field of (0.55T) with current density of 500 A/m^2 . Find

- (i) Hall voltage developed in it (ii) Hall coefficient (iii) Hall angle if mobility of electron is $0.17 \text{ m}^2/\text{V sec}$.

MU - Dec. 11, 5 Marks

Soln. :

Hall voltage developed

$$V_H = \frac{BJw}{ne} = R_H B J w$$

$$\text{Hall coefficient } R_H = \frac{1}{ne}$$

Data given :

$$n = 10^{20}/\text{m}^3, e = 1.6 \times 10^{-19} \text{ C}$$

So,

$$R_H = \frac{1}{10^{20} \times 1.6 \times 10^{-19}} = 0.0625 \text{ m}^3/\text{C}$$

(i) Hall voltage

$$V_H = R_H B J w$$

$$B = 0.55, J = 500 \text{ A/m}^2, w = 4.5 \times 10^{-3} \text{ m}$$

$$\begin{aligned} V_H &= 0.0625 \times 0.55 \times 500 \times 4.5 \times 10^{-3} \\ &= 77.3 \times 10^{-3} \text{ V} \\ &= 77.3 \text{ mV} \end{aligned}$$

...Ans.

...Ans.

- (ii) Hall coefficient
- (iii) Hall angle

$$\begin{aligned} R_H &= 0.0625 \text{ m}^3/\text{C} \\ \theta_H &= \tan^{-1}(\mu B) \\ &= \tan^{-1}(0.17 \times 0.55) \\ &= 5.3^\circ \end{aligned}$$

...Ans.

Ex. 3.13.5: The Hall coefficient of a specimen is $3.66 \times 10^{-4} \text{ m}^3/\text{C}$. Its resistivity is $8.93 \times 10^{-3} \Omega \cdot \text{m}$. Find μ and n .

MU - May 2011, 4 Marks

Soln. :

Formula :
Hall coefficient

$$\begin{aligned} R_H &= \frac{1}{n e} \\ n &= \frac{1}{R_H \cdot e} \end{aligned}$$

Data given :

$$\begin{aligned} R_H &= 3.66 \times 10^{-4} \text{ m}^3/\text{C} \\ n &= \frac{1}{3.66 \times 10^{-4} \times 1.6 \times 10^{-19}} \\ &= 1.7 \times 10^{22}/\text{m}^3 \end{aligned}$$

...Ans.

So,

Also, we know

$$\begin{aligned} \sigma &= n e \mu \\ \mu &= \frac{\sigma}{n e} = \sigma \times R_H = 1/\rho \times R_H = \frac{R_H}{\rho} \\ &= \frac{3.66 \times 10^{-4}}{8.93 \times 10^{-3}} \\ \mu &= 0.040 \text{ m}^2/\text{V} \cdot \text{s} \end{aligned}$$

...Ans.

Ex. 3.13.6: A bar of n-type Ge of size $0.010 \text{ m} \times 0.001 \text{ m} \times 0.001 \text{ m}$ is mounted in a magnetic field of $2 \times 10^{-1} \text{ T}$. The electron density in the bar is $7 \times 10^{21}/\text{m}^3$. If one millivolt is applied across the long ends of the bar, determine the current through the bar and the voltage between Hall electrodes placed across the short dimensions of the bar. Assume $\mu_e = 0.39 \text{ m}^2/\text{Vs}$.

MU - May 13, 5 Marks

Soln. :

$$\begin{aligned} \text{As } \rho &= \frac{1}{\mu_n e} = \frac{1}{0.39 \times 7 \times 10^{21} \times 1.6 \times 10^{-19}} \\ &= 2.29 \times 10^{-3} \text{ ohm/m} \\ \text{Now } R &= \frac{\rho l}{A} = \frac{2.29 \times 10^{-3} \times 0.01}{(0.001 \times 0.001)} = 22.9 \Omega \end{aligned}$$

(Here long side is used as length as instructed and other two sides are forming width and height which in turn gives cross-sectional area)

Using Ohm's law

$$I = \frac{V}{R} = \frac{1 \times 10^{-3}}{22.9} = 43.66 \mu\text{A}$$

~~Using formulae for Hall voltage~~

$$V_H = R_H B d w$$

$$\text{As, } R_H = \rho N A$$

$$\text{And, } J = \frac{I}{A} = \frac{I}{w \times t}$$

$$V_H = \rho N A B \frac{1}{w t} \times t = \frac{\rho N B I}{w}$$

$$= \frac{2.29 \times 10^{-6} \times 0.89 \times 0.2 \times 43.66 \times 10^{-6}}{0.001}$$

$$= 7.798 \mu\text{V}$$

...Ans.

Ex. 3.13.7 : In a semiconductor with Hall coefficient $145 \text{ cm}^3/\text{C}$ having width 2 cm and thickness 0.2 cm with a magnetic field induction of 2T along the smaller dimension, a current of 150 mA is calculate the current density and Hall voltage.

MU - Dec. 14, 5 Marks

Soln. :

Given :

Hall coefficient $R_H = 145 \text{ cm}^3/\text{C}$, Width $w = 2 \text{ cm}$

thickness $t = 0.2 \text{ cm}$, magnetic field $B = 2\text{T} = 200 \text{ mT} = 150 \text{ mA}$

Current density

$$\begin{aligned} J &= \frac{I}{A} = \frac{I}{W \times d} \\ &= \frac{150 \times 10^{-3}}{2 \times 10^{-2} \times 0.2 \times 10^{-2}} \\ &= 3750 \text{ A/m}^2 \end{aligned}$$

Now

Hall co-efficient

$$R_H = \frac{V_H/d}{JB}$$

$$V_H = R_H JB$$

$$\begin{aligned} \text{Now, } w &= 145 \times 10^{-6} \times 3750 \times 2 \times 0.2 \times 10^{-2} \\ &= 21.75 \text{ mV} \end{aligned}$$

...Ans.

Ex. 3.13.8 : In an n-type semiconductor the Fermi level lies 0.4 eV below the conduction band. If the concentration of donor atom is doubled, find the new position of the Fermi level w.r.t. the conduction band.

MU - May 13, 5 Marks

Note : This problem needs assumption of temperature to arrive at the answer

Soln. : Given :

$$E_C - E_F = 0.4 \text{ eV}$$

Now concentration of donor atom is doubled, hence number of electrons will be doubled.

\therefore At any temperature $T > 0^\circ \text{K}$

$n_e = N e^{-(E_C - E_F)/kT}$... (1)
 where n_e = Number of electrons in conduction band
 When donor concentration is doubled, $n'_e = 2 n_e$, and Fermi level must have been shifted to a new location say E'_F ,

$$2 n_e = N e^{-(E_C - E'_F)/kT} \quad \dots (2)$$

Divide equation (2) by (1)

$$\frac{2n_e}{n_e} = \frac{N e^{-(E_C - E'_F)/kT}}{N e^{-(E_C - E_F)/kT}}$$

$$\alpha = e^{[(E'_F - E_C) + (E_C - E_F)]/4kT}$$

For Boltzmann constant $K = 1.38 \times 10^{-23} \text{ J/K}$ and with an assumption $T = 300^\circ \text{K}$

$$\begin{aligned} KT &= \frac{1.38 \times 10^{-23} \times 300}{1.6 \times 10^{-19}} \\ &= 0.026 \text{ eV} \end{aligned}$$

\therefore Equation (3) becomes,

$$2 = e^{[(E'_F - E_C) + (E_F - E_C)]/0.026}$$

Taking \ln on both sides

$$\ln 2 = \frac{[(E'_F - E_C) + (E_F - E_C)]}{0.026}$$

$$0.026 \times \ln 2 = (E'_F - E_C) + (E_F - E_C)$$

But $E_F - E_C = 0.4 \text{ eV}$ given

$$0.01802 = (E'_F - E_C) + 0.4$$

$$E_C - E'_F = 0.4 - 0.01802 = 0.38198$$

...Ans.

\therefore Fermi level will be shifted towards conduction band.

Ex. 3.13.9 : In a Hall effect experiment a potential difference of $4.5 \mu\text{V}$ is developed across a foil of zinc of thickness 0.02 mm . When a current of 1.5 A is in a direction perpendicular to applied magnetic field of 2T , calculate

MU - Dec. 16, 7

- a) Hall coefficient of zinc b) Concentration of electrons

Marks.

Soln. : Using formula

$$R_H = \frac{V_H t}{BI} = \frac{4.5 \times 10^{-6} \times 0.02 \times 10^{-3}}{2 \times 10^5} = 3 \times 10^{-11}$$

1.

$$\text{As } R_H = \frac{1}{n_e}$$

$$\begin{aligned} \therefore n &= \frac{1}{R_H \cdot e} = \frac{1}{3 \times 10^{-11} \times 1.6 \times 10^{-19}} \\ &= 2.083 \times 10^{29} / \text{m}^3 \end{aligned}$$

... Ans.





Interference in Thin Film

Syllabus :

(Prerequisites : Wavefront and Huygens' principle, reflection and refraction, interference by division of wavefront, Young's double slit experiment)

Interference by division of amplitude, interference in thin film of constant thickness due to reflected and transmitted light, origin of colours in thin film, wedge-shaped film, Newton's rings

Applications of interference - determination of thickness of very thin wire or foil, determination of refractive index of liquid, wavelength of incident light, radius of curvature of lens, testing of surface flatness, anti-reflecting films and highly reflecting film

Learning Objectives :

- After reading this chapter, learner should be able to
- Define interference and its types
 - Discuss interference in thin film and derive path difference formula for transmitted and reflected light
 - Apply concept of interference to wedge-shaped film and Newton's rings
 - Understand the applications of interference
 - Understand anti-reflecting film

4.1 Interference in a Thin Parallel-sided Film

MU - Dec. 13, Dec. 14, May 15, Dec. 16, May 17, Dec. 17, May 18

- Q. Derive the conditions for maxima and minima due to interference of light reflected from thin film of uniform thickness. (Dec. 13, May 18, 7 Marks)
- Q. Explain the interference in thin parallel film and derive the expression for path difference between reflected rays. Hence obtain the conditions of maxima and minima for interference with monochromatic light. (Dec. 14, 7 Marks)
- Q. Derive the condition for a thin transparent film of constant thickness to appear bright and dark when viewed in reflected light. (May 15, 7 Marks)
- Q. Obtain expression for path difference between two reflected rays in thin transparent film of uniform thickness and write the conditions of maxima and minima. (Dec. 16, 4 Marks)
- Q. Derive the conditions for maxima and minima due to interference of light transmitted from thin film of uniform thickness. (May 17, 8 Marks)
- Q. Obtain the condition for maxima and minima of the light reflected from a thin transparent film of uniform thickness. Why is the visibility of the fringe much higher in the reflected system than in the transmitted system? (Dec. 17, 8 Marks)



- Consider a ray AB of monochromatic light of wavelength λ from an extended source incident at B, on the upper surface of a parallel sided thin film of thickness t and refractive index μ as shown in Fig. 4.1.1.
- Let the angle of incidence be i .
- At B, the beam is partly reflected along BR_1 and partly refracted at an angle r along BC.
- At C, it is again partly reflected along CD and partly refracted along CT_1 . Similar partial reflections and refractions occur at points D, E, etc.
- Thus we get a set of parallel reflected rays BR_1, DR_2, \dots and a set of parallel transmitted rays CT_1, ET_2, \dots
- For a thin film, the waves travelling along BR_1 and DR_2 in the reflected system will overlap.
- These waves originate from the same incident wave AB and are hence coherent.
- Hence they will interfere constructively or destructively according to if the path difference between them is an integral multiple of λ or an odd multiple of $\frac{\lambda}{2}$.

Reflected system

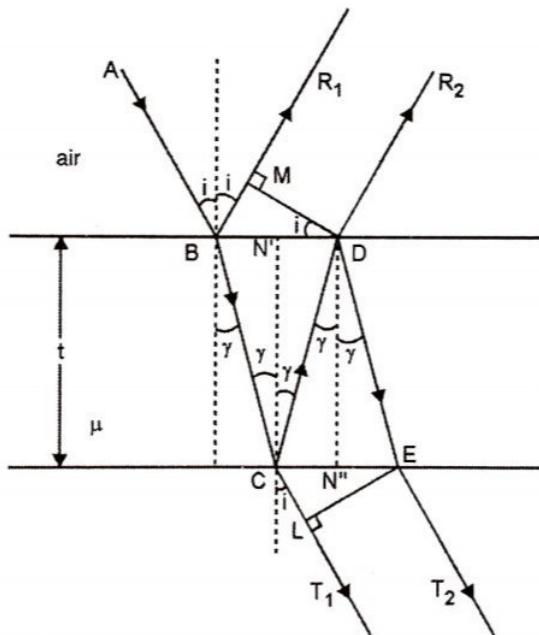


Fig. 4.1.1 : Interference In thin films

- To find the path difference between BR_1 , and DR_2 , draw DM perpendicular to BR_1 . The paths travelled by the beams beyond DM are equal. Hence the optical path difference (optical path difference is obtained by multiplying geometrical path difference by its refractive index) between them is

$$\Delta = \text{Path BCD in film} - \text{Path BM in air}$$

$$= \mu(BC + CD) - BM$$



From Fig. 4.1.1 we have $BC = CD = \frac{t}{\cos r}$

$$\therefore \mu(BC + CD) = \frac{2\mu t}{\cos r}$$

$$\text{and } BM = BD \cdot \sin i = 2BN' \sin i$$

$$(\because BD = 2BN')$$

$$= 2t \cdot \tan r \cdot \sin i$$

$$(\because BN' = CN' \tan r = t \cdot \tan r)$$

$$\therefore BM = 2t \frac{\sin r}{\cos r} \cdot \sin i$$

$$= \frac{2\mu t}{\cos r} \cdot \sin^2 r$$

$$(\because \frac{\sin i}{\sin r} = \mu)$$

\therefore The optical path difference between the rays is

$$\Delta = \frac{2\mu t}{\cos r} - \frac{2\mu t}{\cos r} \sin^2 r$$

$$= \frac{2\mu t}{\cos r} (1 - \sin^2 r)$$

$$\text{or } \Delta = 2\mu t \cos r$$

...(4.1.1)

- The film is optically denser than the surrounding air medium. Hence the ray BR_1 originating by reflection at the denser medium suffers a phase change of π or a path change of $\frac{\lambda}{2}$ due to reflection at B. (No such change of phase occurs for ray DR_2 as it is a result of reflection at C)
- Hence the effective path difference between BR_1 and DR_2 is

$$2\mu t \cos r + \frac{\lambda}{2}$$

Condition for maxima and minima in reflected light

- (i) The two rays will interfere constructively if the path difference between them is an integral multiple of λ i.e.

$$2\mu t \cos r + \frac{\lambda}{2} = n\lambda$$

or

$$2\mu t \cos r = (2n-1)\frac{\lambda}{2}, \text{ where } n = 1, 2, 3, 4, \dots \text{ (For maxima)} \dots (4.1.2)$$

$$\text{or } 2\mu t \cos r = (2n+1)\frac{\lambda}{2} \text{ when } n = 0, 1, 2, 3, \dots$$

When this condition is satisfied the film will appear bright in the reflected system.

- (ii) The two rays will interfere destructively if the path difference between them is an odd multiple of $\frac{\lambda}{2}$ i.e.

$$2\mu t \cos r + \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}$$

or

$$2\mu t \cos r = n\lambda \text{ (For minima)}$$

where $n = 0, 1, 2, 3, \dots$

...(4.1.3)

Transmitted system

- The transmitted rays CT_1 and ET_2 are also derived from the same incident ray AB and hence are coherent. (Fig. 4.1.1)
- When they interfere, they can give the interference pattern in transmitted system. To find the path difference between CT_1 and ET_2 we drop EL perpendicular to CT_1 . See Fig. 4.1.1.

$$\therefore \text{Path difference } \Delta = \mu(CD + DE) - CL$$

- It can be calculated the same way as in reflected system and it is found that path difference

$$\Delta = 2\mu t \cos r$$

- But in this case no phase change occurs due to reflection at C and D . Hence the effective path difference between CT_1 and ET_2 is $2\mu t \cos r$.
- Hence the condition for constructive interference to take place in the transmitted system is that

$$2\mu t \cos r = n\lambda \quad (\text{Maxima}) \quad \dots(4.1.4)$$

and the film appears bright in transmitted system.

- The condition for destructive interference is

$$2\mu t \cos r = (2n - 1) \frac{\lambda}{2} \quad (\text{Minima}) \quad \dots(4.1.5)$$

and the film appears dark in transmitted system.

- Comparison of equations (4.1.2), (4.1.3), (4.1.4) and (4.1.5) shows that the conditions of maxima and minima in reflected light are just opposite to those in transmitted light.
- Hence the film which appears bright in reflected light appears dark in transmitted light and vice versa.
- For the transmitted light, the intensity of maxima is about 100% and that of minima is about 85%. This results in poor contrast between bright and dark whereas in reflected light, minima has zero intensity and maxima is nearly 15% of incident energy. This results in good contrast.
- Hence visibility of fringe is much higher in reflected system.

Ex. 4.1.1 : A parallel beam of sodium light strikes a film of oil floating on water. When viewed at an angle 30° from the normal, eighth dark band is seen. Determine the thickness of the film. Refractive index of oil is 1.46 and $\lambda = 5890 \text{ \AA}$.

Soln. :

Given : $i = 30^\circ$, $\mu = 1.46$, $n = 8$, $\lambda = 5890 \text{ \AA} = 5890 \times 10^{-8} \text{ cm}$

Formula : For dark band,

$$2\mu t \cos r = n\lambda \quad n = 0, 1, 2, 3, \dots$$

$$\mu = \frac{\sin i}{\sin r}$$

$$\therefore \sin r = \frac{\sin i}{\mu} = \frac{\sin 30^\circ}{1.46} = 0.3424$$



$$\therefore \cos r = \sqrt{1 - \sin^2 r} = 0.9395$$

$$\therefore t = \frac{n\lambda}{2\mu \cdot \cos r} = \frac{8 \times 5890 \times 10^{-8}}{2 \times 1.46 \times 0.9395}$$

$$\therefore t = 1.7176 \times 10^{-4} \text{ cm.}$$

....Ans.

4.2 Thin and Thick Films

MU - May 16

Q. Why does an excessively thin film appear to be perfectly dark when illuminated by white light?

(May 16, 3 Marks)

The effective path difference between the interfering rays in reflected light is

$$2\mu t \cos r + \frac{\lambda}{2}$$

- (i) If the film is excessively thin, then its thickness being very small as compared to the wavelength of light, the term $2\mu t \cos r$ can be neglected as compared to $\frac{\lambda}{2}$. Hence the effective path difference becomes $\frac{\lambda}{2}$ which is the condition for minima. Hence every wavelength in the incident light will be absent in reflected system, and the film will appear black in reflected light.
- (ii) If the thickness of the film is large enough as compared to the wavelength of light, the path difference at any point of the film will be large. Under these conditions the same point will be a maximum for a large number of wavelengths, and the same point will be a minimum for another set of large number of wavelengths. The number of wavelengths sending maximum intensity at a point are almost equal to the number of wavelengths sending minimum intensity. Also, these wavelengths sending maximum and minimum intensity will be distributed equally over all colours in white light. Hence the **resultant effect at any point will be the sum of all colours i.e. white in thick film.**
- (iii) Hence we define 'thin film' as the film whose thickness is of the order of wavelength of the light which is used to expose it.

4.3 Production of Colours in Thin Films

MU - Dec. 12, May 15, Dec. 16, May 17

Q. Explain why we see beautiful colours in thin film when it is exposed to sunlight.

(Dec. 12, Dec. 16, 3 Marks)

Q. Comment on colours in a soap film in sunlight.

(May 15, 3 Marks)

Q. What do you mean by thin film? Comment on the colours of thin film in sunlight.

(May 17, 3 Marks)

When a thin film is exposed to white light from an extended source, it shows beautiful colours in the reflected system.

- Light is reflected from the top and bottom surfaces of a thin film, and the reflected rays interfere.
- The path difference between the interfering rays depends on the thickness of the film and the angle of refraction r and hence on the inclination of the incident ray.

- White light consists of a continuous range of wavelengths. At a particular point of the film and for a particular position of the eye (i.e. t and r constant) those wavelengths of incident light that satisfy the condition for constructive interference in the reflected system will be seen in reflected light.
- The colouration will vary with the thickness of the film and inclination of the rays (i.e. with the position of the eye with respect to the film). Hence if the same point of the film is observed with an eye in different positions or different points of the film are observed with the eye in the same position, a different set of colours is observed each time.

4.4 Necessity of the Extended Source

- In case of interference in thin films, a narrow source limits the visibility of the film.
- Consider a thin film and a narrow source of light S as shown in Fig. 4.4.1. The ray 1 produces interference fringes because rays 3 and 4 reach the eye. The ray 2 is incident on the film at some different angle and is reflected along 5 and 6. The rays 5 and 6 do not reach the eye.
- Similarly rays incident at different angles on the film do not reach the eye. Hence only the portion A of the film is visible and not the rest.

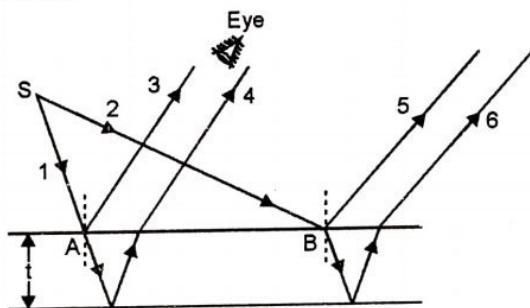


Fig. 4.4.1

- If an extended source of light is used as shown in Fig. 4.4.2, the ray 1, after reflection from the upper and lower surfaces of the film emerges as 3 and 4 which reach the eye.
- Also, the other rays incident at different angles on the film enter the eye and the field of view is large.

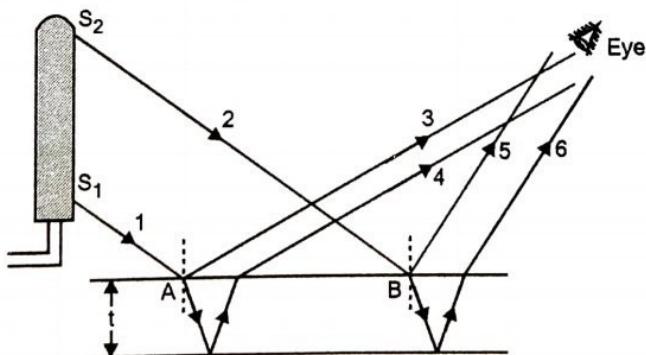


Fig. 4.4.2

- Hence to observe interference phenomenon in thin films, a broad source of light is required.



4.5 Film of Non-uniform Thickness (Wedge-shaped Film)

MU - Dec. 13, May 14, Dec. 15, Dec. 17, May 18

- Q.** Explain why an extensively thin film appears black in reflected light? **(Dec. 13, 3 Marks)**
- Q.** Obtain the conditions for maxima and minima due to interference in a wedge-shaped film observed in reflected light. **(May 14, 4 Marks)**
- Q.** Why are the fringes in wedge-shaped film straight? Derive the conditions of maxima and minima for interference in wedge-shaped films? **(Dec. 15, 7 Marks)**
- Q.** What will be the fringe pattern if wedge-shaped air film is illuminated with white light? **(Dec. 17, 3 Marks)**
- Q.** Explain how interference in wedge-shaped film is used to test optical flatness of given glass plate. **(May 18, 3 Marks)**

- Consider a film of non-uniform thickness as shown in Fig. 4.5.1. It is bound by two surfaces OX and OX' inclined at an angle θ . The thickness of the film gradually increases from O to X.
- Such a film of non-uniform thickness is known as **wedge-shaped film**. The point O at which the thickness is zero is known as the edge of the wedge.
- The angle θ between the surfaces OX and OX' is known as the angle of wedge. Let μ be the refractive index of the material of the film.
- Let a beam AB of monochromatic light of wavelength λ be incident at an angle i on the upper surface of the film. It is reflected along BR₁ and is transmitted along BC. At C also the beam suffers partial reflection, and refraction and finally we have the ray DR₂ in the reflected system.
- Thus as a result of partial reflection and refraction at the upper and lower surfaces of the film, we have two coherent rays BR₁ and DR₂ in the reflected system.

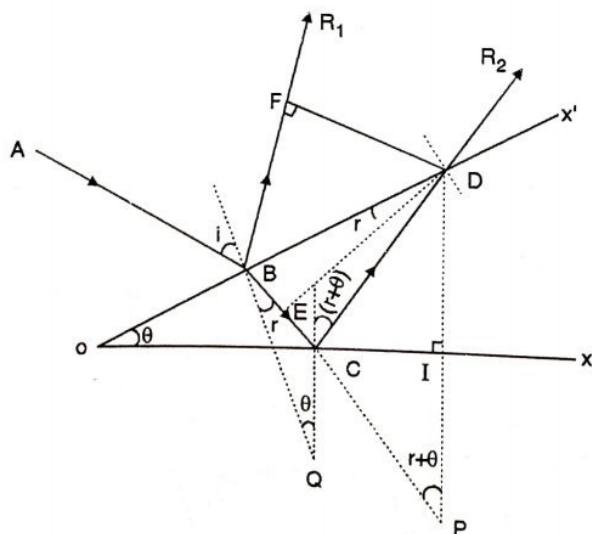


Fig. 4.5.1 : Interference in wedge-shaped film

- These rays are not parallel but diverge from each other. These rays interfere constructively or destructively according to whether the path difference between them satisfies the condition for constructive or destructive interference in the reflected system.
- To find the path difference between these two rays, draw DF perpendicular to BR₁.
- The optical path difference between the rays BR₁ and DR₂ is

$$\Delta = \mu(BC + CD) - BF \quad \dots(4.5.1)$$

- Draw a perpendicular to surface OX at point C.
- We have the perpendicular to OX' at point B. Both these perpendiculars will meet at point Q as shown in Fig. 4.5.1.

$$\therefore \angle BQC = \angle XOX' = \theta$$

- Draw a perpendicular DE on BC from D.
- As θ is small enough, BE = EC
- Also from diagram $\angle QBE = r = \angle BDE$
- Draw a perpendicular from D on OX such that it intersects OX at I and BC produced at P. Also we get CP = CD.
- Equation (4.5.1) can be written as

$$\begin{aligned} \Delta &= \mu(BC + CD) - BF \\ &= \mu(BE + EC + CP) - BF \end{aligned}$$

From diagram

$$\mu = \frac{\sin i}{\sin r} = \frac{BF}{BE} \quad \text{or} \quad BF = \mu BE$$

$$\begin{aligned} \therefore \Delta &= \mu(BE + EC + CP) - \mu BE \\ &= \mu EP \quad (\text{as } E - C - P) \end{aligned}$$

- Now consider ΔDPC

$$\text{as } CP = CD, \angle CPD = r + \theta$$

and ΔDPE is a right angle triangle

$$\therefore \cos(r + \theta) = \frac{EP}{DP}$$

$$\therefore EP = DP \cos(r + \theta) = 2t \cos(r + \theta)$$

$$\text{Where, } DP = 2DI = 2t, t$$

$$= \text{thickness of film at point D}$$

$$\therefore \Delta = \mu EP$$

$$= 2\mu t \cos(r + \theta) \quad \dots(4.5.2)$$



- Due to reflection at B, an additional path change of $\frac{\lambda}{2}$ occurs for the ray BR₁. Hence the total path difference between the interfering rays is $2\mu t \cos(r + \theta) + \frac{\lambda}{2}$.
- Hence for **maxima**, we have the condition for constructive interference

$$2\mu t \cos(r + \theta) + \frac{\lambda}{2} = n\lambda$$

or

$$2\mu t \cos(r + \theta) = (2n - 1)\frac{\lambda}{2} \quad \dots(4.5.3)$$

$$n = 1, 2, 3, 4 \dots$$

- For **minima**, we have the condition for destructive interference

$$2\mu t \cos(r + \theta) + \frac{\lambda}{2} = (2n - 1)\frac{\lambda}{2}$$

$$2\mu t \cos(r + \theta) = n\lambda \quad \dots(4.5.4)$$

∴

$$n = 0, 1, 2, 3, \dots$$

- It is clear that for a maximum or a minimum of a particular order, t must remain constant. In case of the wedge-shaped film, t remains constant along lines parallel to the thin edge of the wedge.
- Hence the maxima and minima are straight lines parallel to the thin edge of the wedge.
- At the thin edge, t = 0 hence path difference between the rays is $\frac{\lambda}{2}$, a condition for darkness. Hence the edge of the film appears dark. It is called as **zero order band**. That is the reason an extensively thin film appears dark (black) in reflected light.
- Beyond the edge for a thickness t for which path difference is λ , we obtain the first bright band. As t increases to a value for which path difference is $\frac{3\lambda}{2}$, we obtain the first dark band.
- Thus as the thickness increases we obtain alternate bright and dark bands which are equally spaced and equal in width.

- (i) For normal incidence and air film, r = 0 and $\mu = 1$

$$\text{Total path difference} = 2t \cdot \cos \theta + \frac{\lambda}{2}$$

$$2t \cos \theta = (2n - 1)\frac{\lambda}{2},$$

For maxima

$$\text{and } 2t \cos \theta = n\lambda$$

For minima

- (ii) For very small angle of the wedge,

$$\text{As } \theta \rightarrow 0, \cos \theta \rightarrow 1$$

∴ For constructive interference,

$$2t = (2n - 1)\frac{\lambda}{2}$$

and for destructive interference

$$2t = n\lambda$$

\therefore For constructive interference

$$t = (2n - 1) \frac{\lambda}{4} \quad n = 1, 2, 3, \dots$$

$$t = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \dots$$

i.e. every next bright fringe will occur for thickness interval of $\frac{\lambda}{2}$ each.

Similarly for destructive interference

$$t = \frac{n\lambda}{2} \quad n = 0, 1, 2, 3, \dots$$

$$t = 0, \frac{\lambda}{2}, \frac{2\lambda}{2}, \frac{3\lambda}{2}, \dots$$

i.e. every next dark fringe will occur for thickness interval of $\frac{\lambda}{2}$ each.

4.6 Spacing between two Consecutive Bright Bands

- For the wedge-shaped film, we have for the n^{th} maximum,

$$2\mu t \cos(r + \theta) = (2n - 1) \frac{\lambda}{2}$$

- For normal incidence and air film,

$$r = 0 \quad \text{and} \quad \mu = 1$$

$$2t \cos \theta = (2n - 1) \frac{\lambda}{2} \quad \dots(4.6.1)$$

Where, t is the thickness corresponding to n^{th} bright band.

- Consider Fig. 4.6.1. The n^{th} bright band is produced at a distance x_n from the edge of the wedge.

$$t = x_n \cdot \tan \theta \quad \dots(4.6.2)$$

\therefore Substituting for t in equation (4.6.1) we have,

$$2 \cdot x_n \tan \theta \cos \theta = (2n - 1) \frac{\lambda}{2}$$

$$\text{or } 2x_n \cdot \sin \theta = (2n - 1) \frac{\lambda}{2} \quad \dots(4.6.3)$$

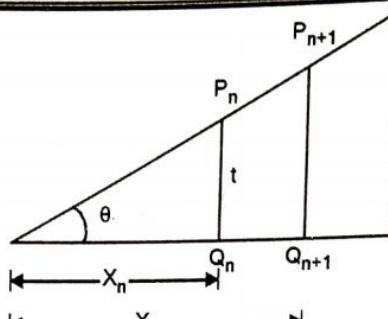


Fig. 4.6.1

- Let $(n + 1)^{\text{th}}$ maximum be obtained at a distance x_{n+1} from the thin edge. Then we have,

$$\left. \begin{aligned} 2x_{n+1} \sin \theta &= [2(n+1) - 1] \cdot \frac{\lambda}{2} \\ \text{or } 2x_{n+1} \cdot \sin \theta &= (2n+1) \frac{\lambda}{2} \end{aligned} \right\} \quad \dots(4.6.4)$$

Therefore from equations (4.6.3) and (4.6.4) we have,

$$2(x_{n+1} - x_n) \cdot \sin \theta = \lambda$$

- Therefore the spacing between two consecutive bright bands is,

$$\beta = x_{n+1} - x_n = \frac{\lambda}{2 \sin \theta}$$

$\sin \theta \rightarrow \theta$ if θ is small and measured in radians. β is called fringe width.

$$\beta = \frac{\lambda}{2\theta}$$

- For a medium of refractive index μ , we have $\beta = \frac{\lambda}{2\mu\theta}$.

as μ , λ and θ are constant, one can say that fringe width in wedge-shaped film is constant. Or **wedge-shaped fringes are of constant thickness**.

Ex. 4.6.1 : Fringes of equal thickness are observed in a thin glass wedge of refractive index 1.52. The fringe spacing is 1 mm and the wavelength of light is 5893 Å. Calculate the angle of wedge in seconds of an arc. MU - Dec. 15, 3 Marks

Soln. :

Given : $\mu = 1.52$, $\lambda = 5893 \times 10^{-8}$ cm, $\beta = 1$ mm = 0.1 cm.

Formula : The fringe spacing is

$$\beta = \frac{\lambda}{2\mu\theta}$$

The angle of wedge in radians is

$$\theta = \frac{\lambda}{2\mu \cdot \beta} = \frac{5893 \times 10^{-8}}{2 \times 1.52 \times 0.1} \text{ radians}$$

$$\therefore \theta = \frac{5893 \times 10^{-8}}{2 \times 1.52 \times 0.1} \times \frac{180}{\pi} \times 3600 \text{ seconds}$$

$$\theta = 40 \text{ seconds of an arc.}$$

...Ans.

4.7 Newton's Rings

MU - May 12, May 15, Dec. 16, May 18

- Q. Explain why Newton's rings are unequally spaced? (May 12, 3 Marks)
- Q. Show that the diameter of Newton's n^{th} dark ring is proportional to square root of ring number. (May 15, 5 Marks)
- Q. For Newton's ring, prove that diameter of n^{th} dark ring is directly proportional to the square root of natural number. (Dec. 16, May 18, 5 Marks)

- When a plano convex lens of large radius of curvature is placed on a plane glass plate, an air film is formed between the lower surface of the lens and upper surface of the plate.
- The thickness of the film gradually increases from the point of contact outwards.
- If monochromatic light is allowed to fall normally on this film, a system of alternate bright and dark concentric rings, with their centre dark, is formed in the air film.
- These rings were first studied by Newton and are hence known as **Newton's rings**. They can be seen through a low power microscope focussed on the film.

Formation of Newton's rings

- Newton's rings are formed as a result of interference between the waves reflected from the top and bottom surfaces of the air film formed between the lens and the plate.

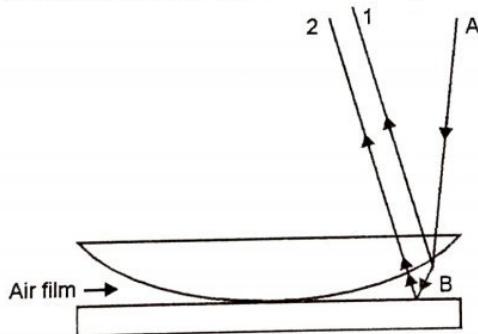


Fig. 4.7.1 : Formation of Newton's rings

- As shown in Fig. 4.7.1, let AB be a beam of monochromatic light of wavelength λ incident normally on the film. As a result of reflection at the top and bottom faces of the film, rays 1 and 2 are the coherent rays which interfere in the reflected system. For constructive interference, the path difference between them should be

$$2\mu t \cdot \cos(r + \theta) + \frac{\lambda}{2} = n\lambda$$

Where,

μ = R.I. of the film

t = Thickness at a point under consideration

r = Angle of refraction

θ = Angle of wedge



- The factor $\frac{\lambda}{2}$ accounts for a phase change of π on reflection at the lower surface of the film.
- Now for the air film $\mu = 1$
For normal incidence $r = 0$
- For a lens of large radius of curvature, $\theta = 0$ practically. This is the reason why we prefer lens with large radius.

$$\therefore \text{Path difference between rays 1 and 2 is } 2t + \frac{\lambda}{2}$$

At the point of contact of the lens and the plate, $t = 0$.

$$\therefore \text{Path difference} = \frac{\lambda}{2}$$

- This is the condition for minimum intensity. Hence the central spot is dark.
For the n^{th} maximum, we have

$$2t + \frac{\lambda}{2} = n\lambda$$

- Thus a maximum of particular order n will occur for a constant value of t . In the air film, t remains constant along a circle and hence the maximum is in the form of a circle.
- Different maxima will occur for different values of ' t '. Similarly, it can be shown that the minima are also circular in form.
- The minima occur for path difference $\frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}, \dots$ and maxima occur for path difference $\lambda, 2\lambda, 3\lambda, \dots$, the maxima and minima occur alternately.
- Each fringe is a locus of constant film thickness and hence these are fringes of constant thickness.

Diameter of dark and bright rings

- Let POQ be a plano convex lens placed on a plane glass plate AB. Let R be the radius of curvature of the lens surface in contact with the plate. Refer Fig. 4.7.2.

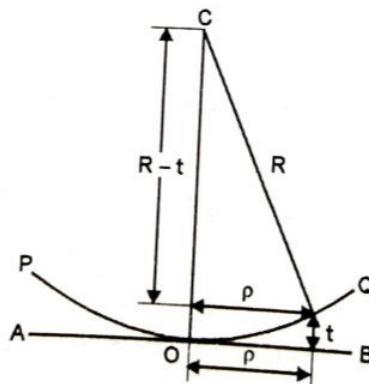


Fig. 4.7.2

- Let ρ be the radius of a Newton's ring corresponding to the constant film thickness 't'. The path difference between the two interfering rays in the reflected system is $2\mu t \cos(r + \theta) + \frac{\lambda}{2}$

Where, λ = Wavelength of incident light.

$\mu = 1$ for air film.

$r = 0$ for normal incidence

$\theta = 0$ for large R.

$$\therefore \text{Path difference} = 2t + \frac{\lambda}{2} \quad \dots(4.7.1)$$

From Fig. 4.7.2 we see that

$$R^2 = \rho^2 + (R - t)^2$$

$$\text{or } \rho^2 = R^2 - (R - t)^2$$

$$\text{or } \rho^2 = 2Rt - t^2$$

$t \ll R$ and hence we have

$$\rho^2 = 2Rt$$

$$\therefore 2t = \frac{\rho^2}{R} \quad \dots(4.7.2)$$

$$\therefore \text{Path difference between the interfering rays is } \frac{\rho^2}{R} + \frac{\lambda}{2}$$

For dark rings

The condition to get dark rings is that

$$\begin{aligned} \text{Path difference} &= \frac{\rho^2}{R} + \frac{\lambda}{2} \\ &= (2n + 1) \frac{\lambda}{2} \quad (n = 1, 2, 3, \dots) \end{aligned}$$

If D is the diameter of Newton's ring, then $\rho = \frac{D}{2}$

$$\therefore \frac{D_n^2}{4R} = n\lambda$$

Where, D_n = Diameter of n^{th} dark ring.

$$\therefore D_n^2 = 4nR\lambda$$

$$D_n = \sqrt{4nR\lambda} \quad \dots(4.7.3)$$

$$D_n \propto \sqrt{n}$$



- Hence the diameter (and hence radius) of the dark ring is proportional to the square root of natural numbers. One can say that rings are unequally spaced, because difference between two consecutive rings represents thickness of the ring.

For bright rings

The condition to get bright rings is that the path difference

$$\frac{\rho^2}{R} + \frac{\lambda}{2} = n\lambda$$

$$\text{Or, } \frac{\rho^2}{R} = (2n - 1) \frac{\lambda}{2}$$

$$\therefore \rho^2 = (2n - 1) \frac{\lambda R}{2}$$

where $n = 1, 2, 3, \dots$

Putting $\rho = \frac{D}{2}$, we have,

$$\frac{D_n^2}{4} = (2n - 1) \frac{\lambda R}{2}$$

Where, D_n = Diameter of n^{th} bright ring.

$$D_n^2 = 2\lambda R \cdot (2n - 1)$$

$$D_n = \sqrt{(2n - 1)} \sqrt{2\lambda R} \quad \dots(4.7.4)$$

$$D_n \propto \sqrt{2n - 1}$$

where $n = 1, 2, 3, \dots$

- Hence the diameter (and hence radius) of the bright ring is also proportional to the square root of odd natural numbers.

Ex. 4.7.1 : In Newton's ring experiment the diameter of 4th and 12th dark rings are 0.400 cm and 0.700 cm respectively. Deduce the diameter of 20th ring.

Soln. :

Given : $n = 4, n + p = 12,$

$$D_4 = 0.400 \text{ cm}, \quad D_{12} = 0.700 \text{ cm}$$

$$D_n^2 = 4n R\lambda, \quad D_{n+p}^2 = 4(n+p) R\lambda$$

$$D_{12}^2 - D_4^2 = 4 \times 8 \times \lambda \times R \quad \dots(1)$$

$$\text{Similarly, } D_{20}^2 - D_4^2 = 4 \times 16 \times \lambda \times R \quad \dots(2)$$

Divide equation (1) by equation (2), we get,

$$\frac{D_{12}^2 - D_4^2}{D_{20}^2 - D_4^2} = \frac{4 \times 8}{4 \times 16} = \frac{1}{2}$$

$$\begin{aligned} D_{20}^2 &= 2 D_{12}^2 - D_4^2 = 2 (0.700)^2 - (0.400)^2 \\ &= 0.98 - 0.16 = 0.82 \end{aligned}$$

$$\text{Diameter of 20}^{\text{th}} \text{ ring} = \sqrt{0.82} = 0.906 \text{ cm}$$

...Ans.



4.8 Newton's Rings by Transmitted Light

- Newton's rings can be seen in reflected light as well as in transmitted light. (As shown in Fig. 4.8.1). The ray 1' is transmitted directly through the air film while the ray 2' suffers two internal reflections (or a phase change of 2π) before emerging out.
- So, two interfering transmitted rays have a phase change of 2π or no phase difference. So, effective path difference

$$\Delta = 2\mu t, \quad (\mu = 1 \text{ for air film})$$

- For bright fringe, we can write,

$$2\mu t = n\lambda, \quad n = 0, 1, 2, 3, \dots$$

$$\therefore 2t = n\lambda \quad (\mu = 1 \text{ for air film}) \quad \dots(4.8.1)$$

And for dark fringe

$$2\mu t = (2n - 1)\lambda/2, \quad \text{where } n = 1, 2, 3, \dots$$

$$\therefore 2t = (2n - 1)\lambda/2 \quad \dots(4.8.2)$$

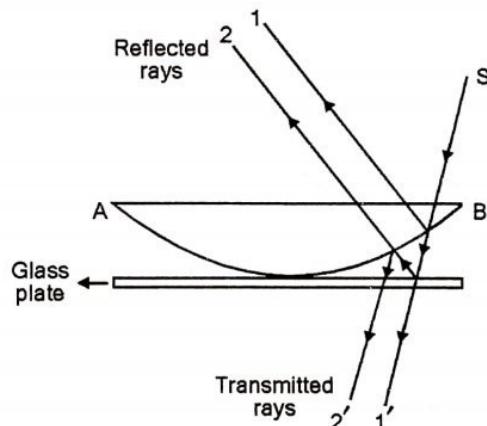


Fig. 4.8.1

- By using the property of circle, we have,

$$2t = \frac{\rho^2}{R} = \frac{D^2}{4R}, \text{ where } D \text{ is the diameter of the ring.}$$

- For bright rings, we can write,

$$2t = \frac{D^2}{4R} = n\lambda \Rightarrow D_n^2 = 4nR\lambda$$

$$D_n = \sqrt{4nR\lambda} \Rightarrow D_n \propto \sqrt{n} \quad \dots(4.8.3)$$



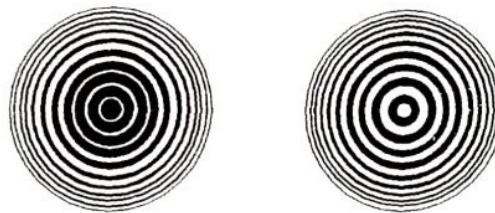
- For dark rings, we can write,

$$2t = \frac{D^2}{4R} = (2n - 1) \lambda/2$$

$$\therefore D_n^2 = 2R\lambda(2n - 1)$$

$$D_n = \sqrt{2R\lambda(2n - 1)} \Rightarrow D_n \propto \sqrt{2n - 1} \quad \dots(4.8.4)$$

- In transmitted system of light, the diameters of bright rings are proportional to the square roots of natural numbers, while the diameters of dark rings are proportional to odd natural numbers and the central ring is bright. The nature of Newton's ring in reflected and transmitted system is shown in Fig. 4.8.2.



(a) Newton's rings in reflected system (b) Newton's rings in transmitted system

Fig. 4.8.2

- We can conclude that the system of rings in transmitted light is complementary to that seen in reflected light.

4.9 Characteristics of Newton's Rings

MU - May 14, Dec. 16, May 17, Dec. 17

Q. Why are Newton's rings circular and the centre of interference pattern (reflected) dark? (May 14, Dec. 17, 3 Marks)

Q. Why are Newton's rings circular and fringes in wedge-shaped film straight? (Dec. 16, May 17, 3 Marks)

(1) Why is the centre of Newton's rings always dark?

At the point of contact the thickness of air film is zero. Consider the case of thin film at this point i.e. condition for bright spot is given by,

$$2\mu t \cos(r + \theta) = (2n + 1)\lambda/2$$

$$n = 0, 1, 2, \dots$$

and condition for dark spot is given by,

$$2\mu t \cos(r + \theta) = n\lambda$$

For Newton's ring setup $r = 0$ (large radius of lens $\theta = 0$), (for normal incidence $r = 0$)

$$\therefore \cos(r + \theta) = \cos 0 = 1$$

\therefore Condition for bright

$$2\mu t = (2n + 1)\lambda/2$$

$$n = 0, 1, 2, 3, \dots$$

\therefore Condition for dark

$$2 \mu t = n\lambda$$

At point of contact, $t = 0$ and one can see that condition for bright does not get satisfied. But condition for dark gets satisfied for $t = 0$ and $n = 0$.

(2) Why are Newton's rings always seen on the reflected side?

- Newton's rings can also be formed in transmitted system due to interference between the transmitted rays. The conditions for the bright and dark rings in the transmitted system are opposite to those in reflected system, and hence the rings have a bright centre.
- In the reflected system the intensity of the interference maxima is about 15% of the incident intensity and the intensity of minima is zero. Hence the contrast between bright and dark rings is good.
- In the transmitted system the intensity of maxima is about 100% and that of minima is about 85%. The contrast between bright and dark rings is not good. The visibility of fringes is much higher in reflected system than in transmitted system. Hence **Newton's rings are seen in reflected system only and not in transmitted system.**

(3) How does insertion of liquid affect ring structure?

- In case of reflected system, the central spot can be made bright if the space between lens surface and glass plate is filled with an oil having refractive index greater than that of lens and smaller than that of plate. Fig. 4.9.1.

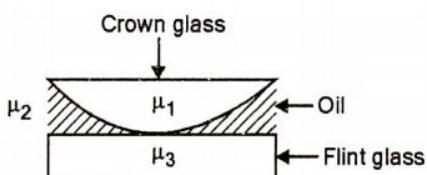


Fig. 4.9.1

- Newton's rings are formed with the lens of crown glass and glass plate of flint glass. The space with air film is then filled with an oil (like oil of sassafras) having intermediate refractive index.
- The reflections at the upper and lower surfaces of the film take place under similar conditions i.e. at the denser medium.
- Hence there is a phase change of π at both reflections. Hence the phase difference between the interfering rays at the point of contact is zero.
- This is the condition for constructive interference and hence a bright spot is produced at the centre.

(4) Why are Newton's rings circular and wedge-shaped films are straight?

- In both air-wedge film and Newton's ring experiments, each fringe is the locus of points of equal thickness of the film. In Newton's rings arrangement, the locus of points of equal thickness of air film lie on a circle with the point of contact of plano convex lens and the glass plate as centre. So, the fringes are circular in nature and concentric.



- For wedge-shaped air film, the locus of points of equal thickness are straight lines parallel to the edge of the wedge. So, fringes appear straight and parallel.

(5) What happens if plano convex lens is lifted up slowly?

- As the lens is lifted up slowly from the flat surface, the order of the ring at a given point decreases. The rings, therefore, come closer and closer until they can no longer be separately observed.
- Also it is important to know that initially as the lens is lifted up by a spacing of $\frac{\lambda}{4}$, total path difference for the ray will be $\frac{\lambda}{4} + \frac{\lambda}{4} = \frac{\lambda}{2}$, and the center will become dark to bright or bright to dark.
- For a lift of every $\lambda/4$, the center will change from dark to bright and bright to dark. Also the order of the ring will reduce by one.

4.10 Newton's Ring with White Light

MU - May 13

Q. Suppose that in an experiment on Newton's rings, first light of red colour is used and then blue light, which set of rings would have larger diameter? Justify your answer with proper expression. **(May 13, 3 Marks)**

- If a monochromatic light source is used in Newton's ring experiment, alternate dark and bright rings are obtained. If we use white light instead of monochromatic light, a few mixed coloured rings around a black centre are observed, and beyond it a uniform illumination is obtained.
- White light consists of several colours of different wavelength. Diameters of the Newton's rings are proportional to the wavelength of the different colours.
- As we know that $\lambda_r > \lambda_v$, therefore the diameter of violet ring of the same order will be smallest and those for red ring will be the largest, and the diameters of other coloured rings shall occupy the intermediate positions.
- Due to overlapping of the rings of different colours over each other, only first few coloured rings will be clearly seen while other rings cannot be observed.

4.11 Newton's Rings with Bright Centre in Reflected System

- If Newton's rings are observed in reflected system, the central spot is dark. A liquid of refractive index μ_2 is poured between lens and glass plate.
- The refractive index of the lens, liquid and glass plate μ_1, μ_2, μ_3 are such that $\mu_1 < \mu_2 < \mu_3$, then the central spot is bright. This is possible if oil of sassafras is introduced between lens of crown glass and plate of flint glass.
- Then the reflection of two interfering rays from denser to rarer medium takes place under same condition. Hence the effective path difference between both the interfering rays at the point of contact becomes zero, which is the condition of maximum intensity.
- So, the centre of Newton's ring appears bright. [Sassafras oil ($\mu = 1.57$), crown glass lens ($\mu = 1.50$), flint glass plate ($\mu = 1.65$)].

4.12 Similarities and Dissimilarities between Newton's Rings and Wedge-shaped Films

Similarities

- (1) Fringes are formed due to enclosed thin film.
- (2) Both can be explained only by the concept of division of amplitude.
- (3) Both can be used for determination of optical flatness.

Dissimilarities

Sr. No.	Newton's Rings	Wedge-shaped film
1.	We get alternate dark and bright rings	We get straight alternate dark and bright fringes
2.	Air gap has its thickness linearly increased	Air gap is non-linearly increased
3.	Popularly used for determination of unknown wavelength	Popularly used for determination of very small thickness
4.	As we go for higher orders thickness of rings reduces	Fringe width remains constant

4.13 Solved Problems

Problems on Thin Film

Ex. 4.13.1 : White light falls at an angle of 45° on a parallel soap film of refractive index 1.33. At what minimum thickness of the film will it appear bright yellow of wavelength 5896 \AA in the reflected light?

Soln. :

Given : $i = 45^\circ$, $\mu = 1.33$, $\lambda = 5896 \text{ \AA} = 5896 \times 10^{-10} \text{ m } \text{\AA}$

Formula : $2\mu t \cos r = (2n - 1)\frac{\lambda}{2}$, $n = 1, 2, 3, \dots$ For bright fringe

For minimum thickness, n is minimum i.e. $n = 1$

$$\therefore t = \frac{\lambda}{2 \times 2\mu \cos r}$$

$$= \frac{5896 \times 10^{-10}}{2 \times 2 \times 1.33 \times \cos r}$$

$$\text{Now } \mu = \frac{\sin i}{\sin r}$$

$$\sin r = \frac{\sin i}{\mu} = \frac{\sin 45^\circ}{1.33} = 0.5316$$

$$\therefore \cos r = \sqrt{1 - \sin^2 r} = 0.8469$$



$$\therefore t = \frac{5896 \times 10^{-10}}{2 \times 2 \times 1.33 \times 0.8469}$$

$$t = 1304.5 \text{ \AA}$$

....Ans..

Ex. 4.13.2 : A light of wavelength 5500 \AA incident on thin transparent denser medium has refractive index 1.45. Determine the thickness of thin medium if the angle of refraction is 45° (Consider $n = 1$).

Soln. :

Here, it is not specified whether reflected side or transmitted side is to be considered.

Let us assume reflected side.

Also it is not specified whether dark fringe or bright fringe condition is satisfied.

Let us assume dark fringe.

\therefore Condition for dark fringe on reflected side,

$$2\mu t \cos r = n\lambda$$

$$\therefore t = \frac{n\lambda}{2\mu \cos r} = \frac{1 \times 5500 \times 10^{-8}}{2 \times 1.45 \times \cos 45}$$

$$\therefore t = 2.68 \times 10^{-5} \text{ cm}$$

...Ans.

Ex. 4.13.3 : Light of wavelength 5880 \AA is incident on a thin film of glass of $\mu = 1.5$ such that the angle of refraction in the plate is 60° . Calculate the smallest thickness of the plate which will make it dark by reflection.

MU - Dec. 14, 3 Marks

Soln. :

$$\lambda = 5880 \times 10^{-8} \text{ cm}, \mu = 1.5, r = 60^\circ, t = ?$$

Condition for film to appear dark is,

$$2\mu t \cos r = n\lambda$$

The smallest thickness will be for $n = 1$.

$$2 \times 1.5 \times t \times \cos 60 = 1 \times 5880 \times 10^{-8}$$

$$t = \frac{5880 \times 10^{-8}}{2 \times 1.5 \times 0.5}$$

$$\therefore t = 3920 \times 10^{-8} \text{ cm}$$

...Ans.

Ex. 4.13.4 : A soap film of refractive index 1.43 is illuminated by white light incident at an angle 30° . The refracted light is examined by a spectroscope in which dark band corresponding to wavelength $6 \times 10^{-7} \text{ m}$ is observed. Calculate the thickness of the film.

Soln. : For the thin film, refracted light forms transmitted system and the condition of minima in transmitted system is given by,

$$2\mu t \cos r = (2n - 1)\lambda/2$$

$$t = \frac{(2n - 1)\lambda}{4\mu \cos r}$$

Given : $\mu = 1.43$, $\lambda = 6 \times 10^{-7} \text{ m}$, $i = 30^\circ$

Using Snell's law,

$$\begin{aligned}\mu &= \frac{\sin i}{\sin r} \\ \sin r &= \frac{\sin i}{\mu} \\ \cos r &= \sqrt{1 - \sin^2 r} \\ &= \sqrt{1 - \frac{\sin^2 i}{\mu^2}} \\ &= \sqrt{1 - \left(\frac{\sin 30^\circ}{1.43}\right)^2} \\ &= 0.9369 \\ \text{So, } t &= \frac{(2n - 1) \times 6 \times 10^{-7}}{4 \times 1.43 \times 0.9369} \\ &= (2n - 1) \times 1.12 \times 10^{-7} \text{ m} \\ n &= 1, 2, 3, \dots\end{aligned}$$

For minimum thickness of film, $n = 1$

Hence,

$$t = 1.12 \times 10^{-7} \text{ m} \quad \dots \text{Ans.}$$

Ex. 4.13.5 : An oil drop of volume 0.2 c.c. is dropped on the surface of a tank of water of area 1 sq. meter. The film spreads uniformly over the surface and white light which is incident normally is observed through a spectrometer. The spectrum is seen to contain one dark band coinciding with wavelength 5.5×10^{-5} cm in air. Find the refractive index of oil.

Soln. :

The oil drop of volume 0.2 c.c. spreads uniformly over 1 m^2 ; hence the thickness of the film so formed is given by,

$$t = \frac{0.2}{(100)^2} = 2 \times 10^{-5} \text{ cm.}$$

The film appears dark by reflected light.

Hence,

$$2\mu t \cos r = n\lambda$$

For normal incidence $r = 0 \therefore \cos r = 1$

$$n = 1 \text{ and } \lambda = 5.5 \times 10^{-5} \text{ cm.}$$

Refractive index of oil is

$$\mu = \frac{n\lambda}{2t \cos r} = \frac{1 \times 5.5 \times 10^{-5}}{2 \times 2 \times 10^{-5} \times 1}$$

$$\mu = \frac{5.5}{4} = 1.375 \quad \dots \text{Ans.}$$



Ex. 4.13.6 : A drop of oil of $\mu = 1.20$ floats on water with $\mu = 1.33$ surface and is observed from above by reflected light. The thickness of the oil drop at the edge is very small-almost zero and gradually increases towards the middle of the drop. Answer the following -

- Will the thinnest outer region of the drop correspond to a bright or a dark region? Give reason.
- What will be the thickness of oil drop where wavelength of 4800 \AA is intensified in reflected light for the third order?

Soln. :

(i) The thinnest region of the drop corresponds to a bright region because both the reflected rays, one from the boundary between air and oil and another from the boundary between oil and water are in phase. Hence the condition for brightness is $2\mu t \cos r = n\lambda$

At the edge, $t \approx 0$ and satisfies the condition for maximum intensity.

(ii) The condition for maximum intensity in reflected light is $2\mu t \cos r = n\lambda$

For normal incidence,

$$2\mu t = n\lambda$$

$$\therefore t = \frac{n\lambda}{2\mu}$$

The thickness when $n = 3$,

$\lambda = 4800 \text{ \AA}$ and $\mu = 1.2$ is

$$t = \frac{3 \times 4800 \text{ \AA}}{2 \times 1.2} = 6000 \text{ \AA}$$

....Ans.

Ex. 4.13.7 : Light of wavelength 5893 \AA is reflected at nearly normal incidence from a soap film of refractive index 1.42. What is the least thickness of the film that will appear

- Black
- Bright

Soln. :

(i) In reflected system, the condition of dark is

$$2\mu t \cos r = n\lambda$$

For normal incidence, $r = 0, \cos r = 1$

$$\text{So, } 2\mu t = n\lambda, t = \frac{n\lambda}{2\mu}$$

For minimum thickness of the film, $n = 1$

Hence,

$$t = \frac{\lambda}{2\mu} = \frac{5893 \times 10^{-8}}{2 \times 1.42} \text{ cm}$$

$$t = 2075 \text{ \AA}$$

(ii) In reflected system, the condition of bright is

....Ans.

$$2\mu t \cos r = (2n - 1)\lambda/2$$

For normal incidence, $r = 0$, $\cos r = 1$

For minimum thickness of the film, $n = 1$

$$\text{So, } t = \frac{\lambda}{4\mu} = \frac{5893 \times 10^{-8}}{4 \times 1.42} \text{ cm}$$

$$t = 1037.5 \text{ } \text{\AA}$$

...Ans.

Ex. 4.13.8 : Light falls normally on a soap film of thickness 5×10^{-5} cm and of refractive index 1.33. Which wavelength in the visible region will be reflected most strongly ?

MU - Dec. 12, 5 Marks

Soln. :

The condition of maxima is given by,

$$2\mu t \cos r = (2n - 1)\lambda/2 \quad \text{where } n = 1, 2, 3, \dots$$

Given :

$$t = 5 \times 10^{-5} \text{ cm}$$

$$\mu = 1.33$$

$$r = 0^\circ$$

$$\text{i.e. } \cos r = 1$$

Now,

$$\lambda = \frac{4\mu t \cos r}{(2n - 1)}$$

$$= \frac{4 \times 1.33 \times 5 \times 10^{-5}}{(2n - 1)}$$

By substituting the values of $n = 1, 2, \dots$ we get a series of wavelengths which shall be predominantly reflected by the film.

For $n = 1$,

$$\begin{aligned} \lambda_1 &= \frac{4 \times 1.33 \times 5 \times 10^{-5}}{1} \\ &= 26.66 \times 10^{-5} \text{ cm} \end{aligned}$$

Similarly,

$$\text{For } n = 2, \quad \lambda_2 = 8.866 \times 10^{-5} \text{ cm}$$

$$\text{For } n = 3, \quad \lambda_3 = 5.32 \times 10^{-5} \text{ cm}$$

$$\text{For } n = 4, \quad \lambda_4 = 3.8 \times 10^{-5} \text{ cm}$$

Out of these wavelengths 5.32×10^{-5} cm lies in the visible region (4000 Å to 7500 Å).

Hence 5320 Å is the most strongly reflected wavelength.

...Ans.

Ex. 4.13.9 : White light is incident on a soap film at an angle $\sin^{-1}(4/5)$ and the reflected light is observed with a spectroscope. It is found that two consecutive dark bands correspond to wavelengths 6.1×10^{-5} and 6.0×10^{-5} cm. If the refractive index of the film is $4/3$, calculate the thickness.

Soln. :

We have the condition for dark band in reflected system,

$$2\mu t \cos r = n\lambda$$

If n and $(n + 1)$ are the orders of consecutive dark bands for wavelengths λ_1 and λ_2 respectively, then,

$$\begin{aligned} 2\mu t \cos r &= n\lambda_1, \\ \therefore 2\mu t \cos r &= n\lambda_1 = (n + 1)\lambda_2 \\ n\lambda_1 &= (n + 1)\lambda_2 \\ \therefore n &= \frac{\lambda_2}{\lambda_1 - \lambda_2} \end{aligned} \quad \dots(1)$$

Put the value of n in equation (1), we have,

$$\begin{aligned} 2\mu t \cos r &= \frac{\lambda_1 \lambda_2}{(\lambda_1 - \lambda_2)} \\ t &= \frac{\lambda_1 \lambda_2}{(\lambda_1 - \lambda_2) 2\mu \cos r} \\ &= \frac{\lambda_1 \lambda_2}{(\lambda_1 - \lambda_2) 2\mu \cdot \sqrt{1 - \left(\frac{\sin i}{\mu}\right)^2}} \end{aligned} \quad \dots(2)$$

Given : $\mu = 4/3$, $\sin i = 4/5$

As

$$\mu = \frac{\sin i}{\sin r} \quad \text{and} \quad \cos r = \sqrt{1 - \sin^2 r}$$

$$\therefore \cos r = \sqrt{1 - \left(\frac{4/5}{4/3}\right)^2} = \sqrt{\frac{16}{25}} = \frac{4}{5}$$

Given : $\lambda_1 = 6.1 \times 10^{-5}$ cm, $\lambda_2 = 6.0 \times 10^{-5}$ cm, $\mu = 4/3$

Put all these values in equation (2),

$$t = \frac{6.1 \times 10^{-5} \times 6.0 \times 10^{-5}}{(6.1 - 6) \times 10^{-5} \times 2 \times 4/3 \times 4/5}$$

$$t = 0.0017 \text{ cm}$$

...Ans.

Ex. 4.13.10 : A soap film of refractive index $\frac{4}{3}$ and thickness 1.5×10^{-4} cm is illuminated by white light incident at an angle of 45° . The light reflected by it is examined by a spectroscope in which is found a dark band corresponding to a wavelength of 5×10^{-5} cm. Calculate the order of interference band.

Soln. :

Given : $\mu = \frac{4}{3}$, $t = 1.5 \times 10^{-4}$ cm., $i = 45^\circ$, $\lambda = 5 \times 10^{-5}$ cm

Formula : For dark band,

$$2\mu t \cos r = n\lambda; \quad 2\mu t \cos r = n\lambda$$

$$\text{Now } \mu = \frac{\sin i}{\sin r}$$

$$\sin r = \frac{\sin i}{\mu} = \frac{\sin 45^\circ}{4/3}$$

$$\therefore r = \sin^{-1} \left(\frac{\sin 45^\circ}{4/3} \right) \\ = \sin^{-1} \left(\frac{1}{\sqrt{2}} \times \frac{3}{4} \right) = 32.02^\circ$$

$$\therefore \cos r = 0.8478$$

\therefore Order of dark band is

$$n = \frac{2\mu t \cos r}{\lambda} \\ = \frac{2 \times 1.33 \times 1.5 \times 10^{-4} \times 0.8478}{5 \times 10^{-5}}$$

$$\text{or } n = 6.7$$

\therefore Order of dark band is 6.

...Ans.

Ex. 4.13.11 : A film of refractive index μ is illuminated by white light at an angle of incidence i . In reflected light two consecutive bright fringes of wavelength λ_1 and λ_2 are found overlapping. Obtain expression for thickness of film.

Soln. :

Say, n^{th} bright fringe of λ_1 overlaps with $(n+1)^{\text{th}}$ fringe of λ_2

For maxima of λ_1

$$2\mu t \cos r = (2n-1) \frac{\lambda_1}{2} \quad \dots(1)$$

And for λ_2

$$2\mu t \cos r = \{ 2(n+1)-1 \} \frac{\lambda_2}{2} \quad \dots(2)$$

$$\text{So, } (2n-1) \frac{\lambda_1}{2} = \{ 2(n+1)-1 \} \frac{\lambda_2}{2}$$

$$\therefore 2n(\lambda_1 - \lambda_2) = \lambda_1 - \lambda_2$$

$$\therefore 2n = \frac{\lambda_1 + \lambda_2}{\lambda_1 - \lambda_2}$$

Thickness of the film t , we get from equation (1)

$$2\mu t \cos r = \left\{ \frac{\lambda_1 + \lambda_2}{\lambda_1 - \lambda_2} - 1 \right\} \frac{\lambda_1}{2} = \frac{\lambda_1 \lambda_2}{\lambda_1 - \lambda_2}$$

$$t = \frac{\lambda_1 \lambda_2}{2\mu \cos r (\lambda_1 - \lambda_2)} \quad \dots\text{Ans.}$$

Ex. 4.13.12 : White light is incident at an angle of 45° on a soap film 4×10^{-5} cm thick. Find the wavelength of light in the visible spectrum which will be absent in the reflected light ($\mu = 1.2$).

Soln. :



Here white light is made incident, and on reflected side it is expected to find the absent wavelength i.e. the one which will satisfy the condition for dark.

$$2\mu t \cos r = n\lambda$$

$$\text{Here, } t = 4 \times 10^{-5} \text{ cm}$$

$$i = 45^\circ; \mu = 1.2$$

$$\therefore 1.2 = \frac{\sin 45}{\sin r}$$

$$\therefore r = \sin^{-1} \left(\frac{\sin 45}{1.2} \right)$$

$$= 36.104^\circ$$

$$\therefore \cos r = 0.8079$$

\therefore Now find λ for various order n such that it remains between 4000 \AA to 8000 \AA i.e. visible spectrum.

\therefore For $n = 1$, condition for dark fringe

$$1 \times \lambda = 2 \times 1.2 \times 4 \times 10^{-5} \times 0.8079$$

$$\lambda = 7755 \times 10^{-8} \text{ cm}$$

This is in visible range and it will remain absent

Similarly for $n = 2$

$$\lambda = \frac{2 \times 1.2 \times 4 \times 10^{-5} \times 0.8079}{2}$$

$$= 3877 \times 10^{-8} \text{ cm}$$

This is not in visible range.

$\therefore 7755 \text{ \AA}$ will remain absent.

...Ans.

Ex. 4.13.13 : A plane wave of monochromatic light falls normally on a uniform thin film of oil, which covers a glass plate. The wavelength of the source can be varied continuously. Complete destructive interference is obtained only for wavelengths 5000 \AA and 7000 \AA . Find the thickness of the oil layer.
Given R.I. of oil = 1.3 and R.I. of glass = 1.5.

MU - May 13, 7 Marks

Soln. :

Here the path is from air to oil and oil to glass.

(1) For air to oil :

For destructive interference the condition is, (for normal incidence)

$$2\mu_{\text{oil}} t_{\text{oil}} = (2n + 1) \frac{\lambda}{2} \quad \dots(1)$$

As the arrangement remains same for both the wavelengths, μ_{oil} and t_{oil} will be the same or constant.

$$\therefore \text{const.} = (2n + 1) \frac{\lambda}{2}$$



∴ Order of destructive interference is inversely proportional to wavelength.

∴ For $\lambda_1 = 7000 \text{ \AA}$ take order n and for next wavelength $\lambda_2 = 5000 \text{ \AA}$ take next order i.e. n + 1

∴ For $\lambda_1 = 7000 \text{ \AA}^{\circ}$

$$2 \mu_{\text{oil}} t_{\text{oil}} = (2n + 1) \times \frac{7000}{2} \text{ \AA} \quad \dots(2)$$

For $\lambda_2 = 5000 \text{ \AA}$

$$2 \mu_{\text{oil}} t_{\text{oil}} = (2(n + 1) + 1) \times \frac{5000}{2} \text{ \AA} \quad \dots(3)$$

$$\therefore (2n + 1) \times \frac{7000}{2} = (2(n + 1) + 1) \times \frac{5000}{2}$$

$$\therefore \frac{(2n + 1)}{(2n + 3)} = \frac{5000}{7000}$$

On solving, n = 2

∴ Substitute in equation (2)

$$\begin{aligned} t_{\text{oil}} &= \frac{(2(2) + 1) \times 7000}{2 \times 1.3 \times 2} \\ &= 6730.769 \text{ \AA} \end{aligned} \quad \dots\text{Ans.}$$

Problems on Wedge-shaped Film

Ex. 4.13.14 : A wedge shaped air film having an angle of 40 seconds is illuminated by monochromatic light and fringes are observed vertically through a microscope. The distance measured between consecutive bright fringes is 0.12 cm. Calculate the wavelength of light used.

MU - Dec. 17, May 18, 5 Marks

Soln. :

Given : $\theta = 40 \text{ seconds} = \frac{40}{3600} \text{ degrees}$

$$= \frac{40}{3600} \times \frac{\pi}{180} \text{ radians,}$$

$$\beta = 0.12 \text{ cm.}$$

Formula : Spacing between the consecutive bright fringes is,

$$\beta = \frac{\lambda}{2\theta} \quad (\text{For air film})$$

The wavelength is,

$$\begin{aligned} \lambda &= 2\beta \cdot \theta = 2 \times 0.12 \times \frac{40 \times \pi}{3600 \times 180} \\ &= 4654 \times 10^{-8} \text{ cm} \end{aligned}$$

$$\lambda = 4654 \text{ \AA}^{\circ}$$

....Ans.

Ex. 4.13.15 : Light of wavelength 5500 \AA° falls normally on a thin wedge-shaped film of refractive index 1.4 forming fringes that are 2.5 mm apart. Find the angle of wedge in seconds.

**Soln. :**

We have fringe width,

$$\beta = \frac{\lambda}{2\mu\theta}; \quad \theta = \frac{\lambda}{2\mu\beta}$$

Given :

$$\lambda = 5500 \times 10^{-8} \text{ cm}, \mu = 1.4, \beta = 0.25 \text{ cm}$$

$$\theta = \frac{5500 \times 10^{-8}}{2 \times 1.4 \times 0.25} = 7.86 \times 10^{-5} \text{ radian}$$

$$\theta = 7.86 \times 10^{-5} \times \frac{180^\circ}{\pi} = 0.0045^\circ$$

$$= 0.0045^\circ \times 3600 = 16.2 \text{ sec.}$$

...Ans.

Problems on Newton's Rings

Ex. 4.13.16 : Newton's rings are obtained with reflected light of wavelength 5500 \AA . The diameter of 10^{th} dark ring is 5 mm. Now the space between the lens and the plate is filled with a liquid of refractive index 1.25. What is the diameter of the 10^{th} ring now ?

Soln. :**Given :** $\lambda = 5500 \text{ \AA} = 5500 \times 10^{-8} \text{ cm}, D_{10} = 5 \text{ mm} = 0.5 \text{ cm}, \mu = 1.25$ **Formula :** Diameter of n^{th} dark ring is given by,

$$D_n^2 = \frac{4nR\lambda}{\mu}$$

For the air film, $\mu = 1$ Hence diameter of 10^{th} dark ring is,

$$(0.5)^2 = D_{10}^2 = \frac{4 \times 10 \times R \times 5500 \times 10^{-8}}{1}$$

For the liquid film, the diameter of the 10^{th} dark ring is,

$$D'_{10}^2 = \frac{4 \times 10 \times R \times 5500 \times 10^{-8}}{1.25}$$

$$\frac{D'_{10}^2}{D_{10}^2} = \frac{1}{1.25}$$

$$\therefore D'_{10}^2 = \frac{D_{10}^2}{1.25}$$

$$D'_{10} = \frac{D_{10}}{\sqrt{1.25}} = \frac{D_{10}}{1.1180} = \frac{0.5}{1.118}$$

$$D'_{10} = 0.4472 \text{ cm.}$$

Diameter of 10^{th} dark ring for the liquid film

$$D'_{10} = 4.472 \text{ mm}$$

...Ans.

Ex. 4.13.17 : Newton's rings formed with sodium light between a flat glass plate and a convex lens are viewed normally. What will be the order of the dark ring which will have double the diameter of that of the 40th dark ring?

MU - May 18, 3 Marks
Soln. :

Let the diameter of the nth dark ring be double the diameter of 40th dark ring.

$$\therefore D_n = 2 D_{40}$$

Now the diameter of nth dark ring is given by the expression

$$D_n^2 = 4 n R \lambda \quad \dots(1)$$

Where, R = Radius of curvature of lens.

λ = Wavelength of light.

Hence for the 40th dark ring.

$$D_{40}^2 = 4 \times 40 \times R \lambda \quad \dots(2)$$

From equations (1) and (2) we have

$$D_n^2 = 4 \times D_{40}^2$$

$$\therefore 4 n R \lambda = 4 \times 4 \times 40 \times R \lambda$$

$$n = 160$$

...Ans.

Ex. 4.13.18 : The diameter of 5th dark ring in Newton's ring experiment was found to be 0.42 cm. Determine the diameter of 10th dark ring.

MU - May 16, 4 Marks
Soln. :

$$D_n^2 = 4 n R \lambda$$

As, diameter of 5th dark ring = 0.42 cm

Now, diameter of 10th dark ring = ?

$$\therefore \frac{D_5^2}{D_{10}^2} = \frac{4(5) R \lambda}{4(10) R \lambda}$$

$$\therefore 2(D_5^2) = D_{10}^2$$

$$D_{10} = \sqrt{2} (D_5) = \sqrt{2} (0.42)$$

$$= 0.594 \text{ cm}$$

\therefore Diameter of 10th dark ring = 0.594 cm

...Ans.

Ex. 4.13.19 : A Newton's ring arrangement is used with a source emitting two wavelengths $\lambda_1 = 6 \times 10^{-5}$ cm and $\lambda_2 = 4.5 \times 10^{-5}$ cm. It is found that the nth dark ring due to λ_1 coincides with (n + 1)th dark ring for λ_2 . If the radius of the curved surface is 90 cm, find the diameter of 3rd dark ring for λ_1 .

**Soln. :**Given : $\lambda_1 = 6 \times 10^{-5}$ cm, $\lambda_2 = 4.5 \times 10^{-5}$ cm,

$$R = 90 \text{ cm}$$

$$[D_n]_{\lambda_2} = [D_{n+1}]_{\lambda_1}$$

Formula : Diameter of n^{th} dark ring is,

$$D_n^2 = 4 n R \lambda$$

 $(\mu = 1)$ ∴ For the n^{th} dark ring λ_1

$$[D_n^2]_{\lambda_1} = 4 n R \lambda_1$$

...(1)

and for the $(n + 1)^{\text{th}}$ dark ring λ_2

$$[D_{n+1}^2]_{\lambda_2} = 4(n + 1) \cdot R \cdot \lambda_2$$

...(2)

$$\therefore 4 n R \lambda_1 = 4(n + 1) R \cdot \lambda_2$$

$$\therefore n \lambda_1 = (n + 1) \lambda_2$$

$$\therefore n = \frac{\lambda_2}{\lambda_1 - \lambda_2}$$

$$= \frac{0.45 \times 10^{-5}}{(6 - 0.45) \times 10^{-5}}$$

$$\therefore n = 3$$

∴ Using equation (1), the diameter of 3rd dark ring for λ_1 is

$$[D_3^2] = 4 \times 3 \times 90 \times 6 \times 10^{-5}$$

$$\therefore [D_3]_{\lambda_1} = \sqrt{4 \times 3 \times 90 \times 6 \times 10^{-5}}$$

$$= 0.2545 \text{ cm.}$$

∴ Diameter of third dark ring for λ_1 is 0.2545 cm.

...Ans.

Ex. 4.13.20 : Light containing two wavelengths λ_1 and λ_2 falls normally on a convex lens of radius of curvature R , resting on a glass plate. Now if the n^{th} dark ring due to λ_1 coincides with $(n + 1)^{\text{th}}$ dark ring due to λ_2 , then prove that the radius of the n^{th} dark ring due to λ_1 is $\sqrt{\frac{\lambda_1 \lambda_2}{\lambda_1 - \lambda_2}} \cdot R$.

Soln. :The diameter of n^{th} dark ring is given by a relation

$$D_n^2 = 4 n R \lambda$$

∴ For n^{th} dark ring of λ_1 ,

$$D_n^2 = 4 n R \lambda_1$$

...(1)

and $(n + 1)^{th}$ dark ring of λ_2 ,

$$D_{n+1}^2 = 4(n + 1) R \lambda_2 \quad \dots(2)$$

$$\text{Now } D_n^2 = D_{n+1}^2 \text{ as per data}$$

$$\therefore 4nR\lambda_1 = 4(n + 1)R\lambda_2$$

$$\therefore n = \frac{\lambda_2}{\lambda_1 - \lambda_2}$$

As per equation (1), the diameter of n^{th} dark ring of λ_1 is,

$$D_n = 2\sqrt{nR\lambda_1}$$

\therefore Radius of n^{th} dark ring of λ_1 is,

$$r_n = \frac{D_n}{2} = \sqrt{nR\lambda_1}$$

$$\begin{aligned} \text{or } r_n &= \sqrt{\frac{\lambda_2}{\lambda_1 - \lambda_2} \cdot R\lambda_1} \\ &= \sqrt{\frac{\lambda_1 \lambda_2}{\lambda_1 - \lambda_2} \cdot R} \end{aligned}$$

Hence proved.

Ex. 4.13.21 : Show, with clear examples, that separation between two consecutive similar rings, in Newton's rings experiment goes on reducing as the serial number of ring increases.

Soln. : According to theory of Newton's ring the diameter of the n^{th} dark ring is given by,

$$D_n = \sqrt{4nR\lambda}$$

$\therefore D_n \propto \sqrt{n}$, where n is the serial number of the ring.

For example, first calculate the separation of 5^{th} and 4^{th} dark ring,

$$\therefore D_5 \propto \sqrt{5} = 2.236 \text{ and}$$

$$D_4 \propto \sqrt{4} = 2$$

Hence the separation between 5^{th} and 4^{th} ring is

$$D_5 - D_4 \propto 0.236 \text{ in SI unit.}$$

Similarly, the separation between 79^{th} and 80^{th} ring is

$$D_{80} \propto \sqrt{80} = 8.9442$$

and

$$D_{79} \propto \sqrt{79} = 8.8881$$

$$D_{80} - D_{79} \propto 0.0560 \text{ in SI unit.}$$

Hence, we can conclude that,

$$D_{80} - D_{79} < D_5 - D_4 \text{ hence proved.}$$



4.14 Determination of Thickness of Very Thin Wire or Foil

- There are many applications wherein we need to know the thickness of a very thin wire or an equivalent for example "contact lens". As mentioned in introduction, concept of inference can lead us to an experimental arrangement which gives us accurate measure up to one tenth of a micrometer.

- Measurement of thickness of very thin wire or foil :

As seen in the case of "spacing between two consecutive bright bands", we have derived fringe width $\beta = \frac{\lambda}{2\theta}$

- The same concept is used for determination of thickness of thin wire. We take two glass slides (optically flat) and put them in touch at one end and at the other we put the wire or foil whose thickness is to be determined. Hence we prepare a wedge with very small angle ' θ '.
- Here we make the use of same setup which is used for Newton's rings. In place of plano convex lens we take the above mentioned wedge.
- On viewing through microscope when illuminated by a monochromatic light of wavelength λ at normal incidence, we get alternate dark and bright lines.
- We take readings of dark lines at the spacing of some interval say 'P' number of lines.
- We draw a graph for Reading \rightarrow order of dark line and find the fringe width ' β '.
- From experimental set up measure 'l'

$$\therefore \tan \theta = \frac{t}{l} \approx \theta$$

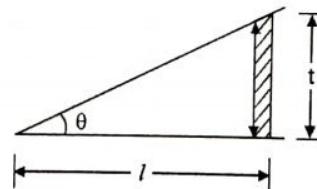
(as θ is very small)

Fig. 4.14.1

$$\text{as } \beta = \frac{\lambda}{2\theta}$$

$$\therefore \frac{t}{l} = \frac{\lambda}{2\beta}$$

$$\therefore t = \frac{\lambda l}{2\beta}$$

...(4.14.1)

Since λ , l and β are known we can calculate thickness of a foil or thin wire.

Ex. 4.14.1 : Two plane glass surfaces in contact along one edge are separated at the opposite edge by a thin wire. If 20 interference fringes are observed between these edges in sodium light at normal incidence, what is the thickness of the wire? (Given $\lambda = 5893 \text{ \AA}$)

Soln. :

$$\text{Fringe width } \beta = \frac{\lambda}{2\mu\theta} \dots (\text{For normal incidence})$$

$$\text{For air film } \mu = 1, \beta = \frac{\lambda}{2\theta}$$

$$\text{Thickness of wire } t = x_n \tan \theta$$

$$= x_n \cdot \theta$$

(for small θ , $\tan \theta \approx \theta$)

$$= 20 \cdot \beta \cdot \theta$$

As there are 20 fringes between two edge points

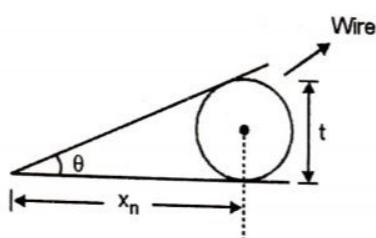


Fig. P. 4.14.1

Putting the value of β , we get,

$$\begin{aligned} t &= 20 \left(\frac{\lambda}{2\theta} \right) \cdot \theta = 10 \lambda \\ &= 10 \times 5893 \times 10^{-10} \\ &= 5.893 \times 10^{-6} \text{ m} \\ &= 5.893 \mu\text{m} \end{aligned}$$

...Ans.

4.15 Determination of Wavelength of Monochromatic Light or Radius of Curvature of Lens by Newton's Rings Method

MU - May 13, Dec. 14, May 17, Dec. 18

- Q.** With the help of proper diagram and necessary expressions, explain how Newton's ring experiment is useful to determine the radius of curvature of a plano convex lens. (May 13, Dec. 18, 5 Marks)
- Q.** With proper diagram and necessary expressions explain how Newton's ring experiment is useful to determine the radius of curvature of plano-convex lens. (Dec. 14, 8 Marks)
- Q.** With Newton's ring experiment explain how to determine the refractive index of liquid? (May 17, 5 Marks)

4.15.1 Experimental Arrangement

- A carefully cleaned convex surface of a plano convex lens L of large radius of curvature is placed on a plane glass plate P. (Fig. 4.15.1).
- Another glass plate G is held at a suitable distance above at an angle 45° with the vertical (to make the normal incidence on the film).



- With a condensing lens, maximum light from source (sodium lamp) is allowed to fall on G. The inclined plate G reflects light onto the air film between the lens L and plate P.

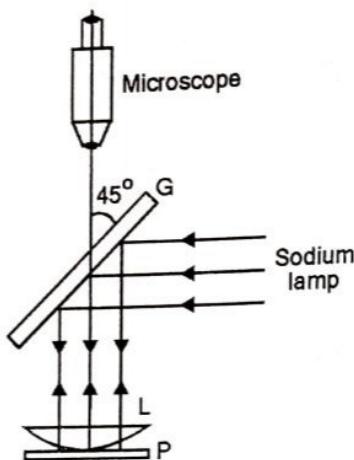


Fig. 4.15.1

- Newton's rings are formed as a result of interference between the rays reflected from the top and bottom faces of the air film.
- They are seen through a low power microscope focussed on the air film where the rings are formed.

Theory

- The effective path difference between the interfering rays is,

$$2\mu t \cos(r + \theta) + \frac{\lambda}{2}$$

Where, μ = R.I. of film.

θ = Angle of film at any point

λ = Wavelength of light

- If D_n is the diameter of n^{th} dark ring then as per the theory of Newton's rings described in equation below

$$D_n^2 = 4 n R \lambda$$

where R = Radius of curvature of lower surface of lens.

$$\text{Let } x D_{n+p}^2 = 4(n+p)R\lambda \quad (\text{for } (n+p)^{\text{th}} \text{ ring}) \quad \dots(4.15.1)$$

$$\therefore D_{n+p}^2 - D_n^2 = 4pR\lambda$$

$$\lambda = \frac{D_{n+p}^2 - D_n^2}{4pR} \quad \dots(4.15.2)$$

- Alternately if wavelength of incident monochromatic ray is known, using equation (4.15.2), we can find R i.e. the radius of curvature.
- For a more accurate approach we plot a graph of D_n^2 vs n as shown in Fig. 4.15.2.

- Thus by measuring the diameter of n^{th} and $(n + p)^{\text{th}}$ dark rings and the radius of curvature R , the wavelength λ can be calculated.
- The diameter of the rings is measured with the travelling microscope and the radius of curvature can be determined by using lens equation or by a spherometer.

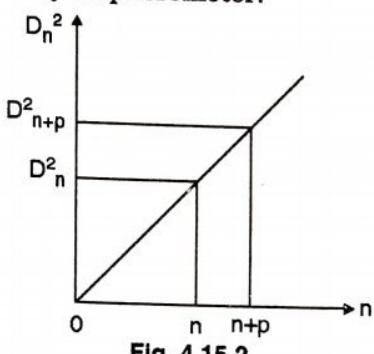


Fig. 4.15.2

4.15.2 Determination of Radius of Curvature of Lens

- This can be done easily with the help of spherometer and the formula

$$R = \frac{l^2}{6h} + \frac{h}{2} \quad \dots(4.15.3)$$

Where, l = Distance between two legs of spherometer

h = Difference in the reading when placed on lens as well as when placed on surface

Ex. 4.15.1 : In Newton's rings experiment the diameter of 5th ring was 0.336 cm and the diameter of 15th ring was 0.590 cm. Find the radius of curvature of plano convex lens if the wavelength of light used is 5890 Å.

MU - May 15, Dec. 18, 5 Marks

Soln. :

Given : $D_5 = 0.336 \text{ cm}$, $D_{15} = 0.590 \text{ cm}$,

$$\lambda = 5890 \text{ \AA} = 5890 \times 10^{-8} \text{ cm}$$

Formula : $D_{n+p}^2 - D_n^2 = 4pR\lambda$

$$\begin{aligned} R &= \frac{D_{n+p}^2 - D_n^2}{4 \cdot p \cdot \lambda} \\ &= \frac{(0.590)^2 - (0.336)^2}{4 \times 10 \times 5890 \times 10^{-8}} \\ &= 99.91 \text{ cm} \end{aligned}$$

$$\therefore R = 99.91 \text{ cm}$$

...Ans.

4.16 Determination of Refractive Index of a Liquid by Newton's Rings

MU - May 14, May 16

- Q. With Newton's ring experiment explain how to determine the refractive index of liquid. (May 14, 4 Marks)
- Q. How is Newton's ring experiment used to determine refractive index of liquid medium? (May 16, 4 Marks)



- There is a popular branch of engineering known as "hydraulics" in which liquid is in motion and transmits energy, viscosity and refractive index are some of the parameters which can be used to decide the suitability of the oil (called grade of the oil). In such applications, the correct measurement of the R.I. is thus an important issue.
- Consider that a transparent liquid whose refractive index is to be determined is placed between the lens L and plate P of the Newton's rings arrangement.
- If the liquid is rarer than glass, a phase change of π will occur at the reflection from the lower surface of liquid film.
- If the liquid is denser than glass, then a phase change of π will occur due to reflection at the upper surface of the film.
- Hence in either case, a path difference of $\lambda/2$ will be introduced between the interfering rays in the reflected system, and hence the effective path difference between them will be

$$2\mu t \cos(r + \theta) + \frac{\lambda}{2}$$

Now $r = 0$ for normal incidence

$$\theta = 0 \text{ for large } R$$

$$\therefore \text{Path difference} = 2\mu t + \frac{\lambda}{2}$$

$$2t = \frac{\rho^2}{2}$$

\therefore For n^{th} dark ring, we have,

$$\frac{\mu \rho^2}{R} + \frac{\lambda}{2} = (2n+1)\lambda/2 \quad \therefore \frac{\mu \cdot D_n^2}{4R} = n\lambda$$

$$\therefore D_n^2 = \frac{4nR\lambda}{\mu}$$

Similarly for the $(n+p)^{\text{th}}$ dark ring we have,

$$D_{n+p}^2 = \frac{4(n+p)R\lambda}{\mu} \quad \dots(4.16.1)$$

$$D_{n+p}^2 - D_n^2 = \frac{4pR\lambda}{\mu}$$

$$D_{n+p} = \text{Diameter of } (n+p)^{\text{th}} \text{ dark ring, then} \quad \text{we have}$$

$$\mu = \frac{4pR\lambda}{(D_{n+p}^2 - D_n^2)} \quad \text{liquid} \quad \dots(4.16.2)$$

$$\therefore \mu = \frac{4pR\lambda}{[D_{n+p}^2 - D_n^2]} = 1 \quad (\because \mu = 1 \text{ for air}) \quad \dots(4.16.3)$$

For more accurate approach we plot a graph of D_n^2 vs n as shown below.

It is similar to Fig. 4.15.2.

∴ From equations (4.16.2) and (4.16.3) we have,

$$\mu = \frac{[D_{n+p}^2 - D_n^2]_{\text{air}}}{[D_{n+p}^2 - D_n^2]_{\text{liquid}}}$$

- If we obtain ring structure using air as the medium then diameter is given by $D_n^2(\text{air}) = 4nR\lambda$. Now slowly if we insert a liquid with R.I. μ then $D_n^2(\text{liquid}) = \frac{4nR\lambda}{\mu}$. Take ratio of these two.

$$\mu = \frac{D_n^2(\text{air})}{D_n^2(\text{liq})} \quad \dots(4.16.4)$$

- Thus to find μ , Newton's rings are formed with air film. The diameters of n^{th} and $(n + p)^{\text{th}}$ dark rings are measured for air film.
- Then the transparent liquid is introduced between lens L and plate P.
- The diameters of n^{th} and $(n + p)^{\text{th}}$ dark rings are measured with liquid film. Hence using equation (4.16.4), μ can be calculated.

Ex. 4.16.1 : Newton's rings are formed in reflected light of wavelength 6000 A° with a liquid between the plane and curved surfaces. If the diameter of the 6^{th} bright ring is 3.1 mm and the radius of curvature of the curved surface is 100 cm , calculate the refractive index of the liquid.

Soln. :

The diameter of n^{th} bright ring is,

$$D_n^2 = \frac{2(2n-1)\lambda R}{\mu}$$

$$\mu = \frac{2(2n-1)\lambda R}{D_n^2}$$

Given : $n = 6$, $\lambda = 6000 \times 10^{-8} \text{ cm}$, $R = 100 \text{ cm}$, $D_6 = 0.31 \text{ cm}$

$$\mu = \frac{2(2 \times 6 - 1) 6000 \times 10^{-8} \times 100}{(0.31)^2}$$

$$= \frac{2 \times 11 \times 6 \times 10^{-3}}{(0.31)^2}$$

$$= 1.373$$

...Ans.

4.17 Applications of Interference

- Reader can easily understand one fact that if a thin film has its thickness altered by $\frac{\lambda}{4}$ where λ = wavelength of monochromatic ray used to illuminate, then total path difference is $\frac{\lambda}{4} + \frac{\lambda}{4} = \frac{\lambda}{2}$ for reflected ray.



- Now $\frac{\lambda}{2}$ is the path difference which converts interference fringes from dark to bright or bright to dark. This is easily observable. Hence if we consider any thin film which is illuminated by Na lamp (5896 \AA), then a change of

$$\frac{1}{4} \times 5896 = 1474 \text{ \AA}$$

can be detected by change in interference dark to bright (or bright to dark). It is important to note that $1474 \text{ \AA} = 0.1474$ micrometer.

- Now coming to the area of surface finish, it is essential to know that surface finish of parts can significantly affect their friction, wear, fatigue, corrosion, tightness of contact joints, position accuracy and so on. Surface finish has always been considered an important factor for manufacturing process monitoring and quality control inspection.
- Surface finish is a representation of the vertical deviations of a measured from its ideal form. If the deviations are substantial then the surface is rough and if these deviations are minor the surface is smooth. For many engineering applications, the finish on the surface can have a big effect on the performance or durability of parts. Hence, a quick measurement of flatness is an important area and optics through interference is considered useful. As mentioned above, interference is capable of detecting a variation of 0.1474 micrometres using Na lamp.
- The smoothness of surface can be detected by using an optical flat which is precisely polished flat surface usually within a few tens of nanometres used as a reference against which the flatness of an unknown surface may be compared.
- An optical flat is usually placed upon a surface under investigation. If a monochromatic light (Na lamp) is used to illuminate the workpiece, a series of dark and bright interference fringes are formed. These interference fringes determine the flatness of the work piece relative to the optical flat up to the accuracy of few micrometres as discussed above. By using a wedge as shown in Fig. 4.17.1, fringes are formed.

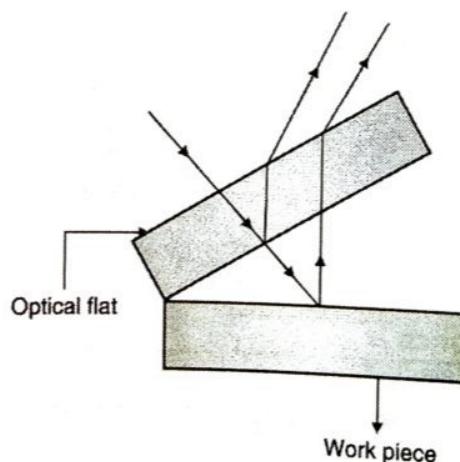


Fig. 4.17.1

- If the workpiece is perfectly flat then straight parallel interference fringes will form.



Fig. 4.17.2

- More and thinner fringes indicate a steeper wedge while fewer but wider fringes indicate smaller wedge.
- If the workpiece is concave or convex, the fringes will be as shown in Fig. 4.17.3.

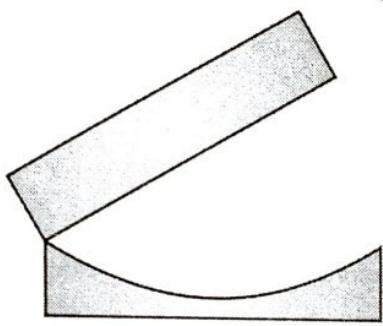


Fig. 4.17.3 (a)



Fig. 4.17.3 (b)

- If the fringes are curved towards the contact edge, the workpiece surface is concave (Fig. 4.17.3 (a)) and if the fringes are curved away the work piece surface is convex Fig. 4.17.3 (b).

4.17.1 Testing the Optical Flatness of Surfaces

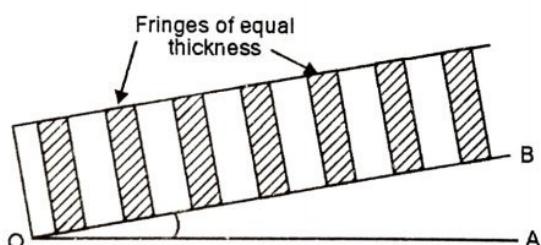


Fig. 4.17.4

- The phenomenon of interference is also used in testing the plainness of the surfaces.
- If two surfaces OA and OB (Fig. 4.17.4) are perfectly plane, the air film between them gradually varies in thickness from O to A. The fringes are of equal thickness as each fringe is the locus of the points at which the thickness of the film has a constant value.
- If the fringes are not of equal thickness, it means that the surfaces are not plane.
- To test the optical flatness of a surface, the specimen surface to be tested (OB) is placed over an optically plane surface (OA).
- The fringes are observed in the field of view. If they are of equal thickness the surface OB is plane. If not, then surface OB is not plane.



- The surface OB is polished and the process is repeated. When fringes observed are of equal width, it means the surface OB is plane.
- The accuracy in this method is far more superior compared to any other technique adopted. The accuracy level is of the order of fraction a of micrometer.

4.18 Concept of Anti-reflecting Coating (Non-reflecting Films)

MU - May 12, Dec. 13, Dec. 17, Dec. 18

- Q. Describe in detail the concept of anti-reflecting film with a proper ray diagram of thin film interference. Which condition the material should satisfy to act as anti-reflecting film? (May 12, 8 Marks)
- Q. What do you understand by anti-reflecting coating? Derive the conditions with proper diagram. (Dec. 13, 8 Marks)
- Q. Describe in detail the concept of anti-reflecting film with a proper ray diagram. (Dec. 17, 5 Marks)
- Q. What is antireflection coating? What should be the refractive index and minimum thickness of the coating? (Dec. 18, 3 Marks)

- We are aware that compound microscope, telescope, camera lenses, etc. use a combination of lenses.
- When the light enters the optical instrument at the glass-air interface, around 4% of light (for air with $n_1 = 1$ and glass with $n_2 = 1.5$) that too at single reflection is lost by reflection which is highly undesirable. For advanced telescopes the total loss comes out to be nearly 30% and cannot be tolerated if working under low intensity applications.
- In order to reduce the reflection loss, a transparent film of proper thickness is deposited on the surface. This film is known as "non-reflecting film".
- Popular material used is MgF_2 because its refractive index is 1.38 (i.e. between air and glass). Cryolite ($n_1 = 1.36$) is also used.
- Thickness of the film may be obtained for given purpose as shown in Fig. 4.18.1.

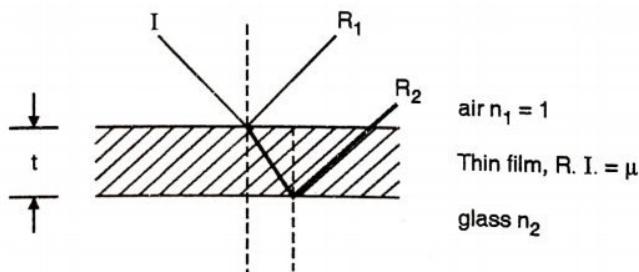


Fig. 4.18.1 : Thin film coating

- Let a ray I be incident upon thin film of MgF_2 coated on glass. This ray is reflected from upper surface as R_1 and from lower surface as R_2 . The optical path difference between these two rays is $n_1(2t)$, as the incident ray enters from rarer to denser twice i.e. at air to film and film to glass. If both the rays R_1 and R_2 interfere with each other and path difference is $(2n + 1)\lambda/2$ (for $n = 0, 1, 2, \dots$) then destructive interference will take place.

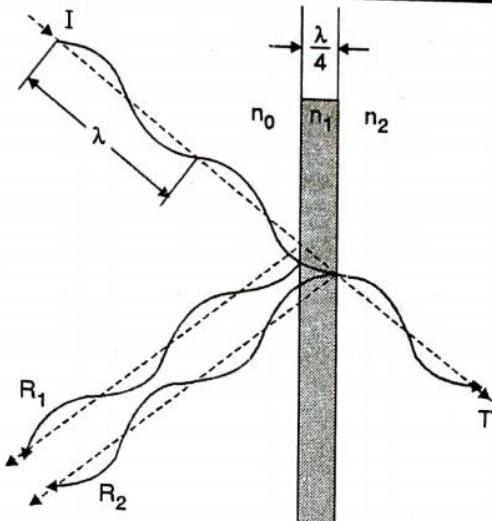


Fig. 4.18.2

$$\therefore 2n_1 t = \frac{\lambda}{2} \quad (\text{for } n = 0)$$

$$\therefore n_1 t = \frac{\lambda}{4\mu}$$

- It means, in order to have destructive interference a layer of $n_1 t = \frac{\lambda}{4}$ is coated on glass plate.

Amplitude condition :

- The amplitude condition requires that the amplitudes of reflected rays, ray 1 and ray 2 are equal. That is,

$$I_1 = I_2 \quad \therefore E_1 = E_2$$

- For complete destructive condition, intensities of two reflected beams should be equal.
- It requires that,

$$\left[\frac{\mu_f - \mu_a}{\mu_f + \mu_a} \right]^2 = \left[\frac{\mu_g - \mu_f}{\mu_g + \mu_f} \right]^2$$

- Where μ_a , μ_f and μ_g are the refractive indices of air, thin film and glass substrate respectively. As $\mu_a = 1$, the above expression may be rewritten as

$$\left[\frac{\mu_f - 1}{\mu_f + 1} \right]^2 = \left[\frac{\mu_g - \mu_f}{\mu_g + \mu_f} \right]^2$$

- Take square root on both sides to compare their amplitudes.

$$\frac{\mu_f - 1}{\mu_f + 1} = \frac{\mu_g - \mu_f}{\mu_g + \mu_f}$$

$$\begin{aligned} (\mu_f - 1)(\mu_g + \mu_f) &= (\mu_g - \mu_f)(\mu_f + 1) \\ &= \mu_f \mu_g + \mu_f^2 - \mu_g - \mu_f \\ &= \mu_g \mu_f + \mu_g - \mu_f^2 - \mu_f \end{aligned}$$



$$2\mu_f^2 - 2\mu_g = 0$$

$$\mu_f^2 - \mu_g = 0$$

$$\therefore \mu_f = \sqrt{\mu_g}$$

4.19 Highly Reflecting Film

- As shown in the case of non-reflecting film, we have seen that a thin film of thickness $\lambda/4$ will create additional path difference of $\lambda/4 + \lambda/4 = \lambda/2$ or additional phase difference $\frac{\pi}{2} + \frac{\pi}{2} = \pi$.

This creates destructive interference.

- The same logic is extended by considering a film of thickness $\lambda/2$.
- In this case the total path difference is $\frac{\lambda}{2} + \frac{\lambda}{2} = \lambda$ or phase difference of $\pi + \pi = 2\pi$
- Thus by the condition of complete constructive interference is satisfied. With this we can make majority of light reflected back from the surface of the glass.

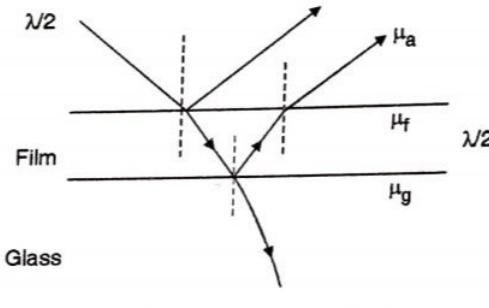


Fig. 4.19.1

- This principle is used in all kind of sun control films used on car, sunglasses, etc.

4.20 Solved Problems on Application

Problems on Application of Newton's Rings

Ex. 4.20.1 : In a Newton's rings experiment, the diameter of the 5th ring was 0.336 cm and that of 15th ring was 0.59 cm. If the radius of curvature of the plano convex lens 100 cm, calculate the wavelength of light.

MU : May 13, May 14, 3 Marks

Soln. :

Given : R = 100 cm, D₁₅ = 0.59 cm, D₅ = 0.336 cm

$$\text{Formula : } \lambda = \frac{D_{15}^2 - D_5^2}{4 \times n \times R}$$

$$= \frac{(0.59)^2 - (0.336)^2}{4 \times 10 \times 100}$$

$$= \frac{0.2352}{4000} = 5.88 \times 10^{-5} \text{ cm}$$

...Ans.

Ex. 4.20.2 : Newton's rings are observed by keeping a spherical surface of 100 cm radius on a plane glass plate. If the diameter of the 15th bright ring is 0.590 cm and the diameter of the 5th bright ring is 0.336 cm, what is the wavelength of light used?

Soln. :

$$\lambda = \frac{D_{n+p}^2 - D_n^2}{4PR} = \frac{(0.590)^2 - (0.336)^2}{4 \times 10 \times 100}$$

$$= 5.880 \times 10^{-5} \text{ cm}$$

$$= 5880 \text{ Å}$$

...Ans.

Ex. 4.20.3 : In a Newton's ring experiment the diameter of the 10th dark ring changes from 1.4 cm to 1.27 cm when a liquid is introduced between the lens and the plate. Calculate the refractive index of the liquid. MU - Dec. 12, 5 Marks

Soln. :

$$D_n(\text{air}) = 1.4 \text{ cm}$$

$$D_n(\text{liquid}) = 1.27 \text{ cm}$$

$$\mu = \frac{D_n^2(\text{air})}{D_n^2(\text{liquid})} = \frac{1.4^2}{1.27^2} = 1.215$$

...Ans.

Ex. 4.20.4 : Newton's rings are formed using light of wavelength 5896 Å in reflected light with a liquid placed between plane and curved surface. The diameter of 7th bright fringe is 0.4 cm and radius of curvature is 1 m. Find the refractive index of liquid. MU - Dec. 12, 5 Marks

Soln. :

For Newton's rings diameter of nth dark ring with air as medium

$$D_n^2 = 4nR\lambda$$

and with liquid having R.I. μ is given by

$$D_n^2 = \frac{4nR\lambda}{\mu}$$

$$\mu = \frac{4nR\lambda}{D_n^2}$$

$$= \frac{4 \times 7 \times 1 \times 5896 \times 10^{-10}}{(0.4 \times 10^{-2})^2}$$

$$= 1.038$$

...Ans.

Ex. 4.20.5 : Newton's rings are formed by light reflected normally from a convex lens of radius of curvature 90 cm and a glass plate with a liquid in between them. The diameter of nth dark ring is 2.25 mm and that of (n + 9)th dark ring is 4.5 mm. Calculate the refractive index of the liquid. Given : $\lambda = 6000 \text{ Å}$.

Soln. :

Given : $R = 90 \text{ cm}$, $D_n = 2.25 \text{ mm}$,

$$D_{n+9} = 4.5 \text{ mm}$$



$$\lambda = 6000 \text{ \AA} = 6000 \times 10^{-8} \text{ cm}$$

Formula :

$$\begin{aligned}\lambda &= \frac{\mu(D_{n+p}^2 - D_n^2)}{4pR} \\ \mu &= \frac{4pR\lambda}{(D_{n+p}^2 - D_n^2)} \\ &= \frac{4 \times 9 \times 6000 \times 10^{-8}}{(0.45)^2 - (0.225)^2} \\ &= 1.28\end{aligned}$$

...Ans.

Ex. 4.20.6 : In a Newton's rings arrangement if a drop of water ($\mu = 4/3$) is placed in between the lens and the plate, the diameter of 10th ring is found to be 0.6 cm. Obtain the radius of curvature of the face of the contact with the plate. The wavelength of light used is 6000 Å.

Soln. :

Given : $\mu = 4/3$, $D_{10}^2 = 0.6 \text{ cm.}$,

$$\lambda = 6000 \text{ \AA} = 6000 \times 10^{-8} \text{ cm}$$

Formula : Diameter of nth dark ring is

$$\begin{aligned}D_n^2 &= \frac{4nR\lambda}{\mu} \\ R &= \frac{\mu \cdot D_n^2}{4n\lambda} \\ \frac{4 \times (0.6)^2}{3 \times 4 \times 10 \times 6000 \times 10^{-8}} &= 200 \text{ cm.}\end{aligned}$$

$$\therefore R = 200 \text{ cm.}$$

...Ans.

Ex. 4.20.7 : Newton's rings are observed in reflected light of wavelength 6000 Å. The diameter of the 10th dark ring is 0.5 cm. Find the radius of curvature of the lens and the thickness of the corresponding air film.

Soln. :

We have, the diameter of dark ring,

$$D_n^2 = 4nR\lambda, \quad R = \frac{D_n^2}{4n\lambda}$$

Given :

$$n = 10, \quad D_n = 0.5 \times 10^{-2} \text{ m}, \quad \lambda = 6 \times 10^{-7} \text{ m}$$

$$\begin{aligned}R &= \text{Radius of curvature} \\ \therefore R &= \frac{(0.5 \times 10^{-2})^2}{4 \times 10 \times 6 \times 10^{-7}} = \frac{25}{24} \\ &= 1.04 \text{ m} = 104 \text{ cm}\end{aligned}$$

Thickness of air film,

$$t = \frac{D^2}{8R}$$

$$D = 0.5 \times 10^{-2} \text{ m}, R = 1.04 \text{ m}$$

$$t = \frac{(0.5 \times 10^{-2})^2}{8 \times 1.04} = 3.0 \times 10^{-6} \text{ m}$$

$$= 3.0 \mu\text{m}$$

...Ans.

Ex. 4.20.8 : In a Newton's ring arrangement with a film observed with light of wavelength $6 \times 10^{-5} \text{ cm}$, the difference of square of diameters of successive rings are 0.125 cm^2 . What will happen to this quantity if :

- (i) Wavelength of light changed to $4.5 \times 10^{-5} \text{ cm}$.
- (ii) A liquid of refractive index is 1.33 introduced between the lens and the plate.
- (iii) The radius of curvature of convex surface of plano-convex lens is doubled.

Soln. :

Using equation (4.15.2) with $\mu \neq 1$.

(i) We have,

$$D_{n+1}^2 - D_n^2 = \frac{4pR\lambda}{\mu}$$

For successive rings $p = 1$, so,

$$D_{n+1}^2 - D_n^2 = \frac{4\lambda R}{\mu} \quad \dots(1)$$

When wavelength changes to λ' , we have,

$$D_{n+1}'^2 - D_n'^2 = \frac{4\lambda' R}{\mu} \quad \dots(2)$$

Divide equation (2) by equation (1), we get,

$$\frac{D_{n+1}'^2 - D_n'^2}{D_{n+1}^2 - D_n^2} = \frac{\lambda'}{\lambda}$$

Given : $\lambda' = 4.5 \times 10^{-5} \text{ cm}$, $\lambda = 6 \times 10^{-5} \text{ cm}$,

$$D_{n+1}^2 - D_n^2 = 0.125 \text{ cm}$$

$$\text{So, } D_{n+1}'^2 - D_n'^2 = \frac{4.5 \times 10^{-5}}{6.0 \times 10^{-5}} \times 0.125 \\ = 0.0937 \text{ cm}^2$$

...Ans.

(ii) When liquid introduced between the lens and plate, we have,

$$D_{n+1}'^2 - D_n'^2 = \frac{4\lambda R}{\mu'} \quad \dots(3)$$

Divide equation (3) by equation (1), we get,



$$\frac{D_{n+1}'^2 - D_n'^2}{D_{n+1}^2 - D_n^2} = \frac{\mu}{\mu'}$$

Given : $\mu = 1, \mu' = 1.33,$

$$D_{n+1}'^2 - D_n'^2 = \frac{1}{1.33} \times 0.125 = 0.094 \text{ cm}^2$$

...Ans.

(iii) When radius curvature (R) is made doubled, we can write,

$$D_{n+1}'^2 - D_n'^2 = \frac{4\lambda R'}{\mu} \quad \dots(4)$$

where $R' = 2R$

Divide equation (4) by equation (1),

$$\begin{aligned} \frac{D_{n+1}'^2 - D_n'^2}{D_{n+1}^2 - D_n^2} &= \frac{R'}{R} = \frac{2R}{R} = 2 \\ D_{n+1}'^2 - D_n'^2 &= 2 \times 0.125 \\ &= 0.250 \text{ cm}^2 \end{aligned}$$

...Ans.

Ex. 4.20.9 : In Newton's ring experiment the diameter of n^{th} and $(n + 8)^{\text{th}}$ bright rings are 4.2 mm and 7 mm respectively. Radius of curvature of lower surface of lens is 2 m. Determine the wavelength of light used.

MU - Dec. 16. 5 Marks

Soln. :

Given :

- (1) Diameter of n^{th} bright ring = 4.2 mm
- (2) Diameter of $(n + 8)^{\text{th}}$ bright ring = 7 mm
- (3) Radius of curvature of plane convex lens = 2 m

To find : Wavelength of monochromatic light.

Formula :

$$D_n^2 = 2\lambda R (2n - 1)$$

For $(n + 8)^{\text{th}}$ ring

$$\begin{aligned} D_{n+8}^2 &= 2\lambda R (2(n + 8) - 1) \\ &= (7 \times 10^{-3})^2 \end{aligned}$$

...(1)

$$\begin{aligned} D_n^2 &= 2\lambda R (2n - 1) \\ &= (4.2 \times 10^{-3})^2 \end{aligned}$$

...(2)

$$\frac{2n + 15}{2n - 1} = \left(\frac{7}{4.2}\right)^2 = 2.7778$$

$$\therefore n = 5$$

...(3)

Using this in equation (2)

$$D_n^2 = 2 \lambda R (2 n - 1) = (4.2 \times 10^{-3})^2$$

$$\therefore \lambda = 4.9 \times 10^{-7} \text{ m}$$

...Ans.

Problems on Application of Wedge-shaped Film

Ex. 4.20.10 : Interference fringes are produced by monochromatic light falling normally on a wedge shaped film of cellophane whose refractive index is 1.4. The angle of wedge is 20 sec of arc and the distance between successive fringes is 0.25 cm. Calculate the wavelength of light.

Soln. :

Given : $\beta = 0.25 \text{ cm}$, $\mu = 1.4$,

$$\theta = 20 \text{ sec} = \frac{20}{60 \times 60} \times \frac{\pi}{180} \text{ radians}$$

Formula :

$$\beta = \frac{\lambda}{2\mu\theta}$$

$$\lambda = 2\mu\theta\beta$$

$$= 2 \times 1.4 \times \frac{\pi}{180 \times 180} \times 0.25$$

$$\lambda = 6.7873 \times 10^{-5} \text{ cm}$$

...Ans.

Ex. 4.20.11 : Two optically plane glass strips of length 10 cm are placed one over the other. A thin foil of thickness 0.01 mm is introduced between them at one end to form an air film. If the light used has wavelength 5900 Å, find the separation between consecutive bright fringes. MU - May 14, May 17. 5 Marks

Soln. :

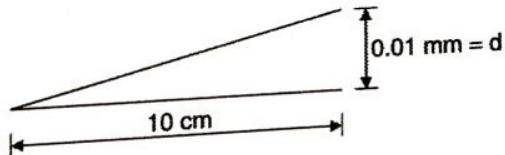


Fig. P. 4.20.11

Let $l = 10 \text{ cm}$, $d = 0.01 \text{ mm}$, $\lambda = 5900 \text{ \AA}$

$$\tan \theta = \frac{0.01 \times 10^{-3}}{10 \times 10^{-2}} = 1 \times 10^{-4}$$

here,

For very small angles, $\tan \theta \approx \theta$

Now fringe width

$$\beta = \frac{\lambda}{2\mu\theta}$$

For air film, $\mu = 1$

$$\therefore \beta = \frac{5900 \times 10^{-8}}{2 \times 1 \times 10^{-4}} = 0.295 \text{ cm}$$

...Ans.

∴ Separation between two consecutive bright fringes is 0.295 cm



Ex. 4.20.12 : Two plane rectangular pieces of glass are in contact at one edge and are separated at other end 10 cm away by a wire to form a wedge-shaped film. When the film was illuminated by light of wavelength 6000 Å, 10 fringes were observed per cm. Determine the diameter of the wire.

Soln. :

Given : $l = 10 \text{ cm}$; $\lambda = 6000 \text{ \AA}$

No. of fringes per cm = 10

$$\therefore \beta = \frac{1}{10} = 0.1 \text{ cm}$$

To find : Diameter of wire

- Assuming that (i) wedge angle is very small,
(ii) the medium as air (iii) incidence of light is normal.

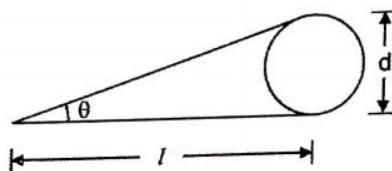


Fig. P. 4.20.12

$$\tan \theta = \frac{d}{l} = \theta \quad (\text{for very small } \theta)$$

$$\text{and } \beta = \frac{\lambda}{2\theta}$$

$$\therefore \frac{d}{l} = \frac{\lambda}{2\beta}$$

$$\therefore d = \frac{\lambda l}{2\beta}$$

$$= \frac{6000 \times 10^{-8} \times 10}{2 \times 0.1}$$

$$\text{Diameter} = 3 \times 10^{-3} \text{ cm}$$

...Ans.

Important Formulae

1. Intensity in interference pattern where phase difference between two waves is δ

$$I = a_1^2 + a_2^2 + 2a_1 a_2 \cos \delta$$

2. For maxima, $\delta = 2n\pi$ or path difference

$$\Delta = n\lambda$$

$$I_{\max} = (a_1 + a_2)^2$$

3. For minima, $\delta = (2n - 1)\pi$, or path difference

$$\Delta = (2n - 1)\lambda/2$$

$$I_{\min} = (a_1 - a_2)^2$$

4. Interference in thin parallel films

Reflected light

(i)

$$2\mu t \cos r = (2n - 1) \frac{\lambda}{2} \text{ (For maxima or bright fringe)}$$

where $n = 1, 2, 3, 4, \dots$

(ii)

$$2\mu t \cos r = n \lambda \text{ (For minima or dark fringe)}$$

Transmitted light

(i)

$$2\mu t \cos r = n \lambda \quad (\text{For maxima or bright fringe})$$

(ii)

$$2\mu t \cos r = (2n - 1) \frac{\lambda}{2} \quad (\text{For minima or dark fringe})$$

$$\mu = \frac{\sin i}{\sin r}$$

$$\text{The fringe spacing } \beta = \frac{\lambda}{2\mu \theta} \text{ (For wedge-shaped film)}$$

5. Wedge-shaped film

Reflected system

$$\text{Condition of maxima, } 2\mu t \cos(r + \theta) = (2n - 1) \lambda/2$$

$$\text{Condition of minima, } 2\mu t \cos(r + \theta) = n\lambda$$

6. Newton's ring in reflected light

$$\text{For } n^{\text{th}} \text{ bright ring } D_n^2 = 2\lambda R \cdot (2n - 1)$$

$$\text{For } n^{\text{th}} \text{ dark ring } D_n^2 = \frac{4nR\lambda}{\mu}$$

Diameter of $(n + p)^{\text{th}}$ dark ring is

$$D_{n+p}^2 = \frac{4(n+p)R\lambda}{\mu}$$

$$\lambda = \frac{\mu(D_{n+p}^2 - D_n^2)}{4pR}$$

$$R = \frac{D_{n+p}^2 \cdot \mu}{4(n+p)\lambda}$$

$$\mu = \frac{4pR\lambda}{(D_{n+p}^2 - D_n^2)}$$

Interference in thin parallel films

Wedge-shaped films

$$1. \text{ The fringe spacing } \beta = \frac{\lambda}{2\mu \theta}$$

$$2. \text{ Wedge-shaped film}$$



Reflected system

Condition of maxima,

$$2\mu t \cos(r + \theta) = (2n - 1)\lambda/2$$

$$\text{Condition of minima, } 2\mu t \cos(r + \theta) = n\lambda$$

Newton's Rings

$$\lambda = \frac{\mu(D_{n+p}^2 - D_n^2)}{4pR}$$

$$R = \frac{D_{n+p}^2 \cdot \mu}{4(n+p)\lambda}$$

$$\mu = \frac{4pR\lambda}{(D_{n+p}^2 - D_n^2)}$$

Anti-reflecting film

Condition for anti-reflecting film : Thickness of the film = $\lambda/4$.

A Quick Revision

- The sources having constant initial phase are called coherent sources.
- Interference of light is the redistribution of intensity in the region of superposition.
- Interference is constructive when the actual path difference between the rays is integral multiple of wavelength λ .
- Interference is destructive when the actual path difference between the rays is odd integral multiple of half the wavelength.
- Interference in thin films is due to division of amplitude of incident beam.
- A path change of λ corresponds to phase change of 2π .
- When reflection occurs at the boundary of denser medium a path change of $\frac{\lambda}{2}$ occurs and there is no path change for reflection at a rarer medium.
- When a beam of light travels a thickness t of a medium of R.I. μ , the equivalent path is $\mu \cdot t$.
- The conditions for constructive and destructive interference in reflected and transmitted systems of a thin film are complementary.
- Production of colours in thin films is a result of interference of light.
- To observe the interference in thin films an extended source of light is required.
- Newton's rings are produced as a result of interferences at the wedge-shaped film.
- Diameters of dark rings are proportional to the square roots of natural numbers and diameters of bright rings are proportional to square roots of odd natural numbers.

- The centre of the ring system is dark in reflected system (usually).
- The separation between diameters of Newton's rings decreases with increase of order.
- Newton's rings are produced as a result of interferences at the wedge-shaped film.
- Newton's rings method can be used to find the wavelength of monochromatic light, R.I. of liquids.

$$\lambda = \frac{\mu (D_{n+p}^2 - D_n^2)}{4pR} \text{ for air R.I. is } 1$$

If wavelength λ is known one can find radius of curvature R ,

$$\mu = \frac{4pR\lambda}{(D_{n+p}^2 - D_n^2)}$$

- When a drop of water is introduced between plano convex lens and glass plate, the rings contract.
- The centre of Newton's ring may be made bright by introducing a drop of sassafras oil between crown glass lens and flint glass plate.
- Condition for anti-reflecting film : Thickness of the film = $\lambda/4$





Superconductors and Supercapacitors

Syllabus :

(Prerequisites : Electric current, flow of electric charges in a metallic conductor, drift velocity, mobility and their relation with electric current, Ohm's law, electrical resistance, V-I characteristics (linear and non-linear), electrical resistivity and conductivity, temperature dependence of resistance)

Superconductors : Critical temperature, critical magnetic field, Meissner effect, Type I and Type II and high T_c superconductors

Supercapacitors : Principle, construction, materials and applications, comparison with capacitor and batteries, energy density, power density

Learning Objectives

After reading this chapter, learner should be able to

- Understand superconductivity
- Explain Meissner effect
- Classify superconductors
- Understand principle, construction and types of supercapacitors
- Compare supercapacitor with batteries

Superconductors

5.1 Introduction

MU - May 14, May 15

Q. What is superconductivity?

(May 14, 2 Marks)

Q. Define the term superconductivity.

(May 15, 2 Marks)

- Superconductivity is one of the most exciting phenomena in physics, both because of the very nature of the phenomenon and also of its prospective applications of immense potential. Kamerlingh Onnes had discovered the phenomenon saying that "electrical resistance of certain metals (like mercury) is completely wiped out without any trace at very low temperatures" (very near to absolute zero). He received the Nobel prize for this in the year 1913. The state of zero electrical resistance was termed as **superconductivity**.
- In the year 1933, Meissner and Ochsenfeld revealed that magnetic flux is expelled when a material attains superconductivity. A reasonably successful theory called BCS (Bardeen, Cooper and Schrieffer) was provided to explain superconductivity. The trio was awarded Nobel prize in 1972. There are many discoveries which have the potential to change the picture on world level right from power sector, transport, up to medical facilities.

5.2 Temperature Dependence of Resistance of Metal

- All metals are good conductors of electricity. It is because they have loosely bound electrons in their outermost shells. These electrons called free electrons, can readily move under the influence of electric field.
- During the flow of current in a metal, the electrons leave the atom to which they were originally bound and move in a general direction which follows the field's direction.
- Because of the loss of electrons, the atoms become positive ion cores.
- Due to thermal excitation, the ions will always be oscillating about fixed positions in the framework of the metal. These vibrations are known as **lattice vibrations**.
- The resistance of the metal to the flow of current is caused by the scattering of the conduction electrons by lattice vibrations.
- When the temperature increases, the amplitude of lattice vibrations also increases thereby increasing the resistance. Refer Fig. 5.2.1.

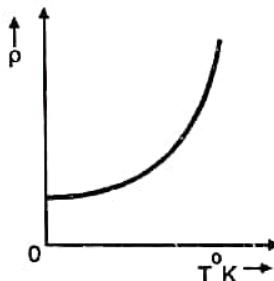


Fig. 5.2.1 : Resistivity → Temperature In non-superconducting state

- The resistance decreases with temperature and reaches a minimum value at $T = 0^{\circ}\text{K}$. The residual resistance at $T = 0$ is due to impurities in metal.

Mathematically it is expressed as,

$$\rho = \rho_0 + \rho(T) \quad \dots(5.2.1)$$

Where, ρ_0 = Residual resistivity

$\rho(T)$ = Temperature dependent part of resistivity

5.3 Superconductivity and its Properties

MU - May 12, May 13, May 16, Dec. 16., May 17

- | | |
|---|----------------------------------|
| Q. Define superconductors. | (May 12, May 13, May 16, 1 Mark) |
| Q. Define superconductivity and critical temperature. Plot the variation of resistance with temperature in case of superconducting state of the material. | (Dec. 16, 3 Marks) |
| Q. Define the term critical temperature. | (May 17, 2 Marks) |
- Kamerlingh Onnes observed that for few of the metals/materials electrical resistance drops to zero below a certain temperature. These materials are called **superconductors**.
 - The drop of electrical resistance is shown in Fig. 5.3.1.

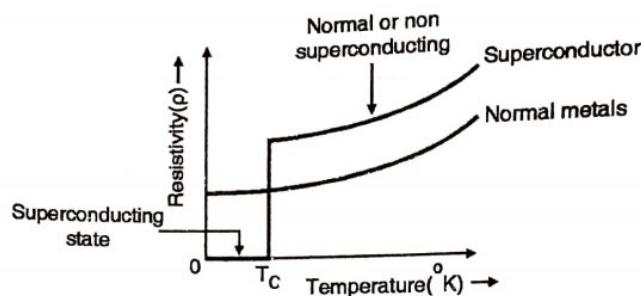


Fig. 5.3.1 : Drop of resistance

- The resistance of a superconductor in the non-superconducting state decreases with decrease in temperature. This is similar to a normal metal. At a particular temperature T_c , the resistance abruptly drops to zero and then onwards metal passes into the superconducting state.
- The temperature at which a normal material turns into superconducting state is called **critical temperature**.
- The transition from normal to superconducting state is sharp for pure and structurally perfect specimen otherwise it is slightly broad.
- The critical temperature is different for different superconductors. For mercury it is 4.2°K .

Properties of superconductors

- Almost all superconductors' physical properties vary from material to material, such as heat capacity and the critical temperature at which superconductivity is found to be destroyed.
- On the other hand, there is another group of properties that are independent on its material. For example, all superconductors have *exactly* zero resistance to low applied current when there is no application of magnetic field.
- The existence of these universal properties suggests that superconductivity is a *thermodynamic phase*, and thus they possess certain distinguishing properties.
- The next section describes the details of such properties of superconductors which are independent of microscopic details.

5.3.1 Magnetic Field Effect (Critical Field H_c)

Q. Define critical temperature and critical magnetic field.

Q. Explain critical magnetic field and critical temperature of a superconductor.

MU - May 12, May 13, May 16

(May 12, May 16, 2 Marks)

(May 13, 2 Marks)

- We know that the superconducting state of a superconductor is mainly dependent upon :

(i) Temperature, and

(ii) Strength of applied magnetic field

- Superconductivity vanishes if the temperature of it is increased above the critical temeperature T_c or very strong sufficient magnetic field \mathbf{H} is applied to it.

Consider that for a superconductor, if sufficiently strong applied magnetic field \mathbf{H} is increased at any temperature T below critical temperature T_c , then superconductor is found to be converted into normal resistive conductor. That is, transition of superconductor to normal conductor takes place under the applied sufficient magnetic field. This is as shown in Fig. 5.3.2.

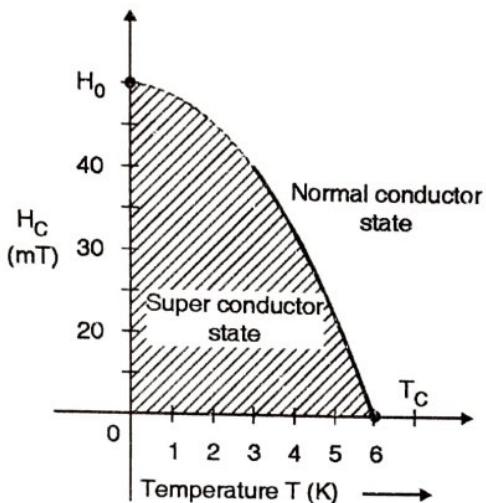


Fig. 5.3.2 : Effect of magnetic field on superconductor

- This minimum value of the magnetic field required to destroy the superconducting phase completely is known as **critical magnetic field**, and is denoted by H_c .
- Further it is clear that critical magnetic field H_c is the function of temperature T , which is expressed as,

$$H_c = H_0 \left[1 - \frac{T^2}{T_c^2} \right] \quad \dots(5.3.1)$$

Where,

H_c = Critical field at any temperature T

H_0 = Critical field at $0^\circ K$

T_c = Critical temperature of the superconductor

- From the Fig. 5.3.2 is found that within the curve the material is in superconducting state and outside the curve it is in normal conducting state.
- In practice typical value of H_0 (critical field at absolute zero or $0 K$) i.e. 5000 A/m is considered.
- Table 5.3.1 shows some superconductor materials and their critical temperatures.

Table 5.3.1: Critical temperatures and critical magnetic fields

Superconductor	T_c in K	$H_c (0)$ in Tesla
Al	1.18	0.0105
Hg	4.15	0.0411



Superconductor	T_c in K	$H_c(0)$ in Tesla
Ln	3.41	0.0281
Pb	7.19	0.0803
Sn	3.72	0.0305
Zn	0.85	0.0054

5.3.2 Persistent Current

- According to theory of electromagnetic induction, it is clear that current can be induced in a superconductor.
- If specimen is in the normal state then current reduces quickly because of the resistance of the superconductor specium.
- Since the superconductor has got zero resistance, the current flows indefinitely (for an unlimited time) without any change; it persists without any applied voltage.
- "A superconductor can carry current for a long time (several years) without any applied field (voltage). The current is called persistent current".

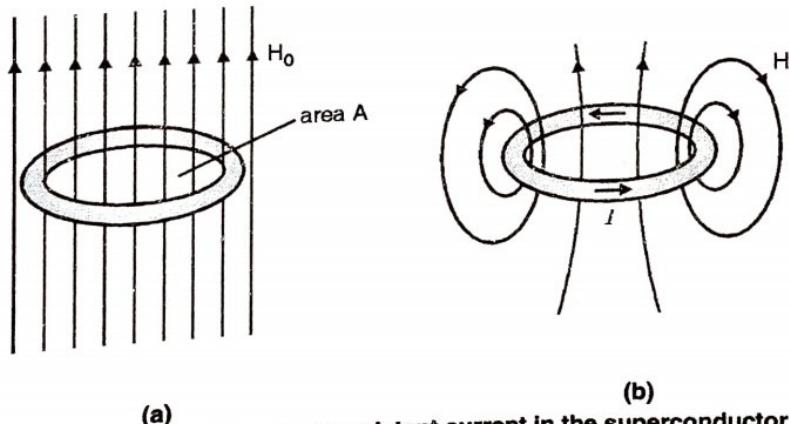


Fig. 5.3.3 : Phenomenon of persistent current in the superconductor

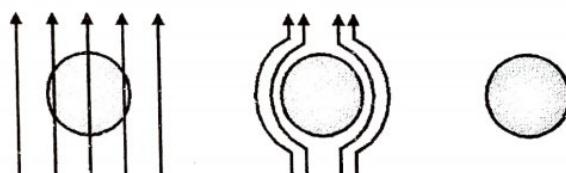
- This is one of the most important properties of the superconductor. It was first observed in 1983 by *Makus Buttiker and Yoseph Imry*. Coils of superconductor with persistent current produce magnetic field.
- Therefore they can be used as magnets which do not require power supply to maintain its magnetic field. Such magnets are known as **superconductor magnets**. They have ten times greater magnetic power than the best normal magnet.

5.4 Meissner Effect

MU - May 12, Dec. 14, May 15, May 17, Dec. 17, May 18, Dec. 18, May 19

- Q. What is Meissner effect? (May 12, 4 Marks)
- Q. Why is superconductor termed as 'perfect diamagnet'? (Dec. 14, 3 Marks)
- Q. Show that in the superconducting state the material is perfectly diamagnetic. (May 15, May 17, 3 Marks)
- Q. "Superconductor is a perfect diamagnet". Explain. (Dec. 17, May 18, 3 Marks)
- Q. Explain Meissner effect with the help of diagram. (Dec. 18, May 19, 3 Marks)

- A superconducting material kept in a **magnetic field expels** the magnetic flux out of its body when cooled below the **critical temperature** and exhibits perfect diamagnetism. This effect is called **Meissner effect**.
- Refer Fig. 5.4.1(a), where a specimen is subjected to a magnetic field. The specimen is in **normal state**. We find that **magnetic field penetrates the specimen**.
- Refer Fig. 5.4.1(b). now the specimen is cooled below its T_c , the **superconductor expels field lines from its body**. This is **Meissner effect**.
- Refer Fig. 5.4.1(c), when the field is switched off magnetic field will not be trapped by the superconductor cooled below T_c .



(a) Magnetic flux lines (b) Expulsion (c) Magnetic field not trapped

Fig. 5.4.1 : Meissner effect

- As specimen expels the magnetic flux, it is exhibition of perfect diamagnetism, **susceptibility** is found out to be - 1. Let's see it mathematically.

For normal state, magnetic induction inside the specimen is given by,

$$B = \mu_0 (H + M) \quad \dots(5.4.1)$$

Where,

μ_0 = Absolute permeability

H = External field applied

M = Magnetization produced within specimen

At $T < T_c$, $B = 0$ i.e. superconducting state

$$\therefore \mu_0 (H + M) = 0$$



$$\text{Susceptibility, } \chi = \frac{M}{H} = -1$$

...(5.4.2)

- It is the diamagnetism which brings strong repulsion to external magnets. This has given us **levitation effect** and MAGLEV trains.

5.5 Types of Superconductors

MU - May 12, Dec. 13

Q. Explain type-I and type-II superconductors.

(May 12, 4 Marks)

Q. What are type I and II superconductors ?

(Dec.13, 5 Marks)

- Based upon magnetic behavior, superconductors are classified into two types viz. Type-I superconductors and Type - II superconductors.

Type - I superconductors

- Type-I superconductors exhibit complete Meissner effect. In the presence of an external magnetic field $H < H_c$, the material in superconducting state is a perfect diamagnet.
- Since it is a diamagnet, it possesses negative magnetization ($-\mu_0 M$).
- Here the transition from superconducting state to normal state in the presence of a magnetic field occurs sharply at the critical value H_c as shown in Fig. 5.5.1.

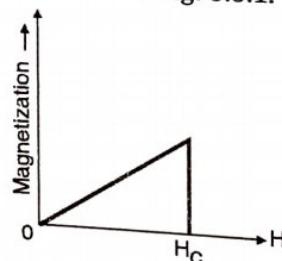


Fig. 5.5.1 : Type-I superconductor

- Aluminium, indium and lead are examples of Type-I superconductors. Since H_c is just around 0.2 wb/m^2 , it is not useful for any industrial application.

Type-II superconductors

Q. What is the vortex state of a superconductor ?

MU - May 14

- Type-II superconductors are characterized by two critical magnetic fields, H_{c1} and H_{c2} . For any applied field strength less than H_{c1} , it expels the magnetic field from its body completely and behaves as a perfect diamagnet.
- When $H > H_{c1}$, the flux penetrates and fills partially in the body of the material. With further increase in H , the flux filling also increases and diamagnetic property decreases.
- At $H = H_{c2}$ material turns into a normal conductor.

(May 14, 3 Marks)

- Between H_{C1} and H_{C2} , specimen is magnetically in mixed state but electrically it is a superconductor. Also H_{C2} is around 50 Wb/m^2 . This shows that even for relatively high values of magnetic field superconductivity is retained. Hence it is very useful.

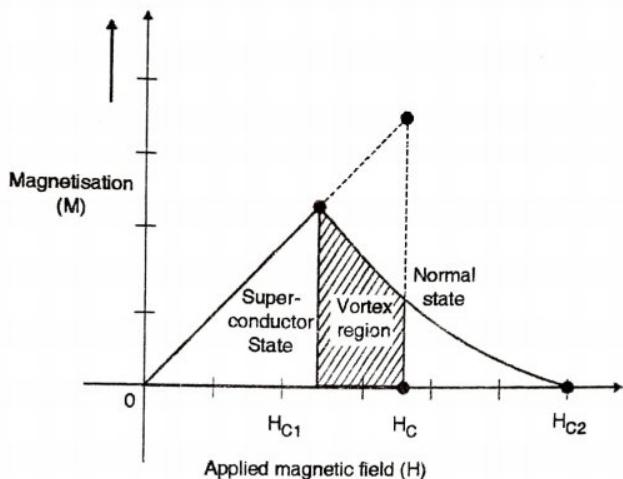


Fig. 5.5.2 : Type-II superconductor

- Now it is observed that specimen is in **mixed state between H_{C1} and H_{C2}** . And above H_{C2} specimen returns to its **normal state**. It is concluded that Meissner effect is found to be incomplete in the region between H_{C1} and H_{C2} . This region is known as **vortex or mixed region**. In this state the specimen has both superconducting and normal regions and can possess zero resistance and partial penetrating flux.
- Due to the vortex or mixed area, some penetration by an external magnetic field (H) into its surface will be allowed. Therefore new microscopic phenomena like superconducting '**stripes**' and '**flux-lattice vortices**' can be observed. Such a partial penetration given the applied magnetic field is responsible for breaking the superconductivity state (critical magnetic field H_C).
- Thus in Type-II superconductors, applied magnetic field and temperature are the main variables of the phase diagram.
- For Type-II superconductors, **lattice structure** plays an important role. There is no complete model to explain Type-II superconductors like BCS theory explains Type-I superconductor.
- Transition metals and alloys consisting of niobium, aluminium, silicon and vanadium exhibit, Type-II superconductivity.

5.5.1 Comparison between Type I and Type II Superconductors

MU - May 13, May 14, Dec. 14, Dec. 15, May 16, May 17, Dec. 18, May 19

- Q. Distinguish between type I and II superconductors.
Q. Differentiate between Type-I and Type II superconductors.

(May 13, 5 Marks)

(May 14, Dec. 14, Dec. 15, May 16, May 17, Dec. 18, May 19, 5 Marks)



Type - I superconductor	Type - II superconductor
Commercially these superconductors are called as <i>soft superconductors</i> .	These superconductors are called as <i>hard superconductors</i> .
These superconductors exhibit only one critical field (H_c).	These superconductors exhibit two critical fields namely <i>lower critical field</i> (H_{c1}) and <i>higher critical field</i> (H_{c2}).
The critical magnetic field value is very low.	The critical magnetic field value is high.
These are explained on the basis of BCS theory.	There is no fixed theory developed to explain it.
These superconductors exhibit perfect and complete Meissner effect.	These do not exhibit a perfect and complete Meissner effect.
These materials have limited technical applications because of very low field strength value.	These materials have wider technological applications because of very high field strength value.
<i>Examples :</i> Pb, Hg, Zn, etc.	<i>Examples :</i> Nb_3Ge , Nb_3Si , $Y_1Ba_2 Cu_3 O_7$, etc.

5.6 High T_c Superconductivity

"Revival of hope"

- In year 1986, J. Bednorz an K. A. Muller* discovered superconductivity in ceramics (earlier it was in metals).
- The most important point was superconductivity at $30^\circ K$, which is very high compared to Mercury ($4.2^\circ K$). This discovery marked the beginning of new era.
- Transition temperature saw a jump to $92^\circ K$ by the work of C. W. Chu and M. K. Wu.
- The point which must be appreciated is that the success broke the barrier of liquid nitrogen temperature of $77^\circ K$.
- Reader must know that, despite having tremendous potential for great applications, very low T_c was a big hurdle.
- Liquid nitrogen is a product which is manufactured easily, available at many places and is inexpensive.
- This means superconductivity at a cheaper rate and with great comfort.

Note : *Both were awarded Nobel price in year 1987.

- The material used to get superconductivity at liquid nitrogen temperature was in the form $\text{YBa}_2\text{Cu}_3\text{O}_{7-x}$ ($x = .01, 0.02, \dots, 0.1$) popularly referred as Y - 123 compound. Even other oxides of Cu have reported high T_c . They all bear perovskite type of crystal structure.
- By and large experimental results are in conformity with BCS theory, but a more authentic and foolproof theory to explain the basic mechanism to explain high T_c superconductivity is yet to be worked out.
- Other technical challenges involved are
 - (1) To obtain high T_c materials in the form of wire or ribbon so that one can facilitate the flow of electricity over a long range.
 - (2) Increase in transition temperature i.e. towards room temperature.
 - (3) A proper theory which can explain all possible aspects of high T_c superconductivity.

5.7 Josephson Effect

- Prerequisite knowledge of some basic aspects related with tunnelling effect studied through quantum mechanics :
- This is one of the important effect for a superconductor invented by British physicist Brian David **Josephson**, in 1962. The **Josephson effect** is the phenomenon of electric current across two weakly coupled superconductors, which are separated by a very thin insulating barrier. In this arrangement, two superconductors are linked by a non-conducting barrier which is known as a **Josephson junction**.
- The current that crosses the barrier is called **Josephson current**. It has great applications in quantum mechanical circuits, such as SQUIDs (Super conducting Quantum Interference Devices) or RSFQ(Rapid Single-Flux-Quantum) digital electronics.

Josephson effect

- The Josephson effect is explained with the help of two metals (conductor) which are separated by thin insulator (thickness 20 \AA). Such insulating layer acts as a **potential barrier** for flow of electrons (current) from one metal to another through a junction.
- Barrier is very small (thin), therefore quantum mechanically electrons can **tunnel** through from higher potential to lower potential. This process continues until the chemical (potential) equilibrium takes place.

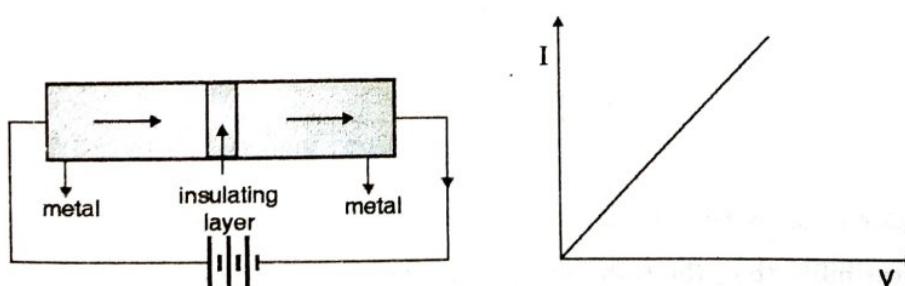


Fig. 5.7.1 : Illustration of Josephson effect with applied field



- Let us see application of potential difference (battery) across the potential barrier, as shown in the Fig. 5.7.1.
- Here more electrons are found to tunnel through the insulating layer. Current-voltage relation obeys Ohm's law at low voltage.
- Now consider that in the above experiment, instead of conductor if superconductor is used and voltage is applied across them, as shown in Fig. 5.7.2.

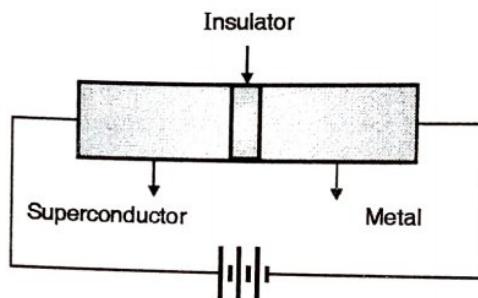


Fig. 5.7.2 : A thin insulating layer sandwiched between metal and superconductor

- Here it is observed that no current flows across the junction until the potential reaches a threshold value. It is seen that the value of threshold voltage is equal to the energy gap in the superconductor state.
- Thus it is important to note that the value of threshold potential or voltage is nothing but the energy gap of superconductor. Further, this threshold voltage becomes a function of temperature below T_c .
- As the temperature is increased towards the critical temperature (T_c) of the superconductor, more and more thermally excited electrons are generated, and they require less energy to tunnel.
- Therefore, threshold voltage decreases. The current-voltage relations at different temperatures are recorded in the Fig. 5.7.3.

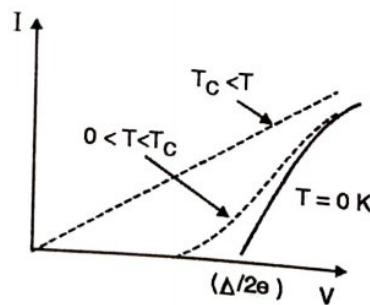


Fig.5.7.3 : Current voltage relationship at the junction with respective temperature

- Consider the possibility that, the thin insulator is sandwiched between two superconductors as shown in Fig. 5.7.4.

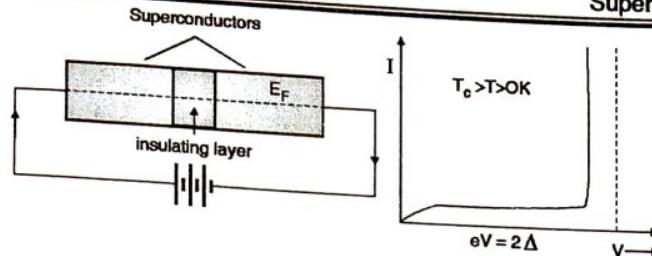


Fig. 5.7.4 : Tunneling in a superconductor -insulator-superconductor and Current-voltage characteristics at $T = 0 \text{ K}$ and $T_c > T > 0 \text{ K}$

- Here in addition to single electrons, the superelectrons i.e. Cooper pairs also tunnel through the junction from one superconductor to another even at zero potential difference across the junction.
- The wave function (ϕ) is highly correlated to both the sides. Thus phenomenon of getting small current due to tunnelling of superelectrons of the superconductor is known as Josephson effect.
- The current-voltage relation at different temperatures is shown in the Fig. 5.7.4. The tunnelling current across the junction is very small. In practice two Josephson effects are used, namely D.C. Josephson effect and A.C. Josephson effect.

5.8 Applications of Superconductivity

5.8.1 MAGLEV Trains

MU - Dec. 13, Dec. 15, May 17, Dec. 17, May 19

Q. What is MAGLEV ?	(Dec. 13, 3 Marks)
Q. How can the MAGLEV train have very high speed ?	(Dec. 15, 3 Marks)
Q. What is MAGLEV? How it can have very high speed ?	(May 17, 3 Marks)
Q. What is working principle of MAGLEV ? Explain how it can acquire high speed ?	(Dec. 17, 5 Marks)
Q. Write short note on MAGLEV.	(May 19, 5 Marks)

- Based on the **Meissner effect** we have a superb application of **frictionless bearings** that is used in **MAGLEV** trains.
- MAGLEV stands for Magnetically Levitated Vehicles.
- They are used in transportation by **being set afloat** above a guideway.
- The utility of such levitation is that in the absence of contact between moving and stationary systems the friction is eliminated. This brings great speeds with low energy consumption.

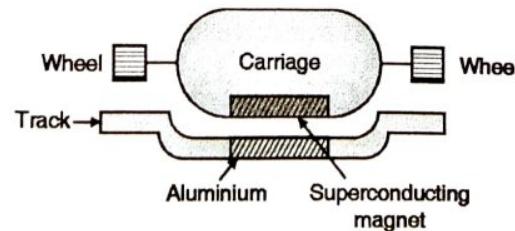


Fig. 5.8.1 : MAGLEV



- The train has a superconducting magnet built into its base.
 - There is an aluminium guideway over which the vehicle will set afloat by magnetic levitation.
 - The magnetic levitation is brought about by enormous repulsion between two highly powerful magnetic field by superconducting magnet and aluminium guideway. Wheels have no role to play when vehicle is lifted up.
 - The track is made up of large number of segments and the flow of currents through the coils could be related to the position and speed of the vehicle.
 - Prototype of such vehicle has achieved speed of 400 kmph.
- 5.8.2 Superconducting Magnets (Electromagnets)**
- For production of heavy duty superconducting magnets type-II superconductor wires are used. They are wound in the form of solenoids to generate strong magnetic fields of induction about 22 Tesla. The size of these magnets is very small as compared to normal magnet.
 - They are very economic.
 - They are used in the transformers in which magnetic core material is not required.

5.8.3 Bearings (Superconducting)

- The Meissner effect of superconductor is used in production of bearings. The mutual repulsion between two superconductors is used to reduce the friction inside the bearings. Therefore it can be used without any power loss.
- The life of the superconductor bearing is more than the ordinary mechanical type bearings.

5.8.4 Superconducting Quantum Interference Device (SQUID)

MU - May 18

Q. - What is the working principle of SQUID ? Explain how it is used to detect the magnetic field ? (May 18, 5 Marks)

- The Superconducting Quantum Interference Device (SQUID) is based on the principle of Josephson effect of superconductor.
- It consists of two superconductors separated by thin insulating layers to form a Josephson junction which is as shown in the Fig. 5.8.2.
- The great sensitivity of the SQUID devices is associated with measuring changes in magnetic field associated with one flux quantization in the superconducting ring. In this case the total magnetic flux through the ring is quantized.
- This device is basically used to measure the small magnetic fields of living organisms. Sample measurements of the field are : magnetic field of heart is 10^{-10} T and brain is 10^{-13} .

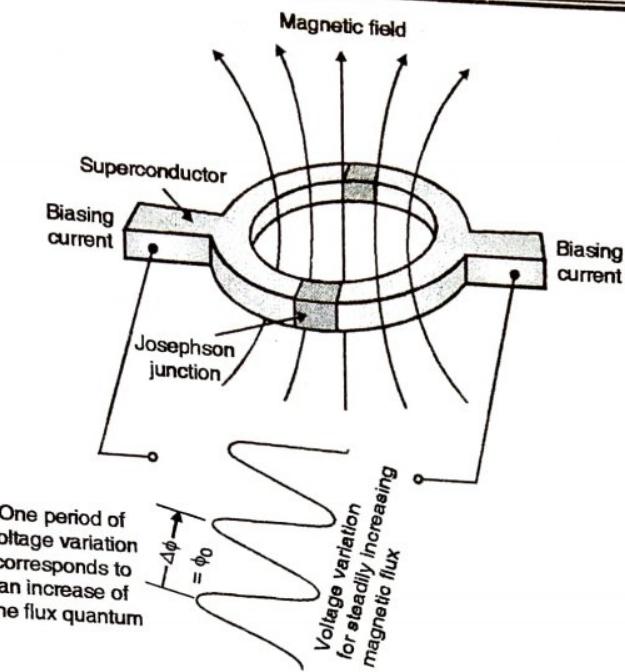


Fig.5.8.2 : Structure of SQUID

- SQUID is able to sense very minute magnetic signals. They are used to study the minute magnetic pulses from the brain and heart. Due to extra high sensitive measurement system in the SQUID, it is quite useful to measure accurate magnetic field inside the human body, hence it helps to diagnose the other medical disorders in the body.
- For example SQUID detectors are used to measure the levels of iron in liver (here magnetic property of iron is used), so that precaution can be taken; iron built up can be treated before much harm is done to the body.

5.8.5 Fast Electrical Switching

- The magnetic penetration depth or length is one of the important characteristics of the superconductors. Therefore Type-II superconductors can be used as very fast electronic switches (there are no moving parts), in which a magnetic field can penetrate into the superconductor.
- Here application of magnetic field greater than the H_c can change the state of material from superconductor to normal conductor. This can reverse the process. This principle is used to develop switching element known as **cryotron**. The structure of **cryotron** is as shown in the Fig. 5.8.3.

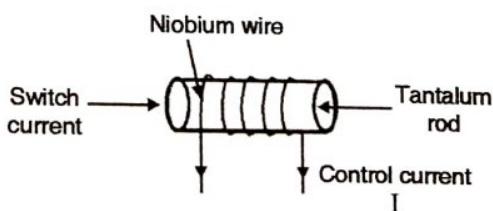


Fig. 5.8.3 : Structure of crytron (fast superconducting switches)



- This consists of tantalum core around which niobium wire is wound. These both are used as superconductors in which superconducting current is controlled with respect to temperature and switching action is achieved faster than the ordinary switches.
- We know that the Josephson junction is a super fast switching device. Josephson junctions can perform switching functions such as switching voltages approximately ten times faster than ordinary semi-conducting circuits.
- This is of distinguished advantage in a computer, which depends on short, on-off electrical pulses. As the computer speed is dependent on fast switching action so that less time is required to transmit signal pulses. The semiconductor electronic junction devices have exceptional switching speed, that it why they are ideal for use in high speed super fast and much compact computers.

5.9 Solved Problems

Ex. 5.9.1 : The critical field of niobium is 1×10^5 A/m at 8°K and 2×10^5 A/m at 0°K. Calculate critical temperature of the element.

Soln. :

Using Equation (5.3.1)

$$H_C = H_0 \left[1 - \left(\frac{T}{T_c} \right)^2 \right]$$

$$\therefore T_c = \frac{1}{\left[1 - \frac{H_C}{H_0} \right]^{1/2}} = \frac{8}{\left[1 - \frac{1 \times 10^5}{2 \times 10^5} \right]^{1/2}} = 11.3^\circ K$$

[Ans. : critical temperature $T_{C2} = 11.3^\circ K$]

Ex. 5.9.2 : Determine the transition temperature and critical field at 4.2 K for a given specimen of a superconductor if the critical fields are 1.41×10^5 A/m and 4.205×10^5 A/m at 1.41 K and 12.9 K, respectively.

Soln. :

Given : $H_{C1} = 1.41 \times 10^5$ A/m, $T_1 = 14.1$ K,

$H_{C2} = 4.205 \times 10^5$ A/m, $T_2 = 12.9$ K.

Formula :

$$(1) \quad H_C = H_0 \left[1 - \left(\frac{T}{T_c} \right)^2 \right]$$

Thus, the critical fields at temperatures T_1 and T_2 can be written

$$(2) \quad H_{C1} = H_0 \left[1 - \left(\frac{T_1}{T_c} \right)^2 \right]$$

and

$$(3) \quad H_{C2} = H_0 \left[1 - \left(\frac{T_2}{T_c} \right)^2 \right]$$

Taking the ratio of formula (2) and (3) we get,

$$(4) \quad \frac{H_{c1}}{H_{c2}} = \frac{T_c^2 - T_1^2}{T_c^2 - T_2^2} \text{ or } \frac{1.41 \times 10^5}{4.205 \times 10^5} = \frac{T_c^2 - (14.1)^2}{T_c^2 - (12.9)^2}$$

Substituting the values, we get

$$H_c = 1.41 \times 10^5 \text{ A/m},$$

$$T_1 = 14.1 \text{ K and } T_c = 14.67 \text{ K}$$

$$H_c = H_0 \left[1 - \left(\frac{T}{T_c} \right)^2 \right]$$

$$1.41 \times 10^5 = H_0 \left[1 - \left(\frac{14.1}{14.67} \right)^2 \right]$$

$$\text{or } H_0 = 18.504 \times 10^5 \text{ A/m}$$

The critical field at $T = 4.2 \text{ K}$ and $T_c = 14.67 \text{ K}$

$$H_c = H_0 \left[1 - \left(\frac{T}{T_c} \right)^2 \right]$$

$$= 18.504 \times 10^5 \times \left[1 - \left(\frac{4.2}{14.67} \right)^2 \right]$$

$$H_c = 16.99 \times 10^5 \text{ A/m}$$

$$H_c \approx 16.0 \times 10^5 \text{ A/m.}$$

[Ans. : $T_c = 14.67 \text{ K}$ and $H_c = 16.00 \times 10^5 \text{ A/m.}$]

Ex. 5.9.3 : The critical temperature T_C for Hg with isotopic mass 199.5 is 4.185 K. What will be its critical temperature when its isotopic mass is increased to 203.4?

Soln. :

Given :

$$T_{C_1} = 4.185 \text{ K,}$$

$$M_1 = 199.5,$$

$$M_2 = 203$$

$$T_{C_2} = \text{For increased mass}$$

Formula :

$$T_C M^{0.5} = \text{constant}$$

Let T_{C_1} is for M_1 mass and T_{C_2} is for M_2 mass (increased).

$$\therefore T_{C_1} M_1^{0.5} = \text{constant} \quad \dots(1)$$

$$T_{C_2} M_2^{0.5} = \text{constant} \quad \dots(2)$$

\therefore From equation (1) and equation (2) we get,

$$T_{C_1} M_1^{0.5} = T_{C_2} M_2^{0.5} \quad \dots(3)$$



$$(4) \quad \frac{H_{c1}}{H_{c2}} = \frac{T_c^2 - T_1^2}{T_c^2 - T_2^2} \text{ or } \frac{1.41 \times 10^5}{4.205 \times 10^5} = \frac{T_c^2 - (14.1)^2}{T_c^2 - (12.9)^2}$$

Substituting the values, we get

$$H_c = 1.41 \times 10^5 \text{ A/m},$$

$$T_1 = 14.1 \text{ K and } T_c = 14.67 \text{ K}$$

$$H_c = H_0 \left[1 - \left(\frac{T}{T_c} \right)^2 \right]$$

$$1.41 \times 10^5 = H_0 \left[1 - \left(\frac{14.1}{14.67} \right)^2 \right]$$

$$\text{or } H_0 = 18.504 \times 10^5 \text{ A/m}$$

The critical field at $T = 4.2 \text{ K}$ and $T_c = 14.67 \text{ K}$

$$H_c = H_0 \left[1 - \left(\frac{T}{T_c} \right)^2 \right]$$

$$= 18.504 \times 10^5 \times \left[1 - \left(\frac{4.2}{14.67} \right)^2 \right]$$

$$H_c = 16.99 \times 10^5 \text{ A/m}$$

$$H_c \approx 16.0 \times 10^5 \text{ A/m.}$$

[Ans. : $T_c = 14.67 \text{ K}$ and $H_c = 16.00 \times 10^5 \text{ A/m.}$]

Ex. 5.9.3 : The critical temperature T_C for Hg with isotopic mass 199.5 is 4.185 K. What will be its critical temperature when its isotopic mass is increased to 203.4?

Soln. :

Given :

$$T_{C_1} = 4.185 \text{ K,}$$

$$M_1 = 199.5,$$

$$M_2 = 203$$

$$T_{C_2} = \text{For increased mass}$$

Formula :

$$T_C M^{0.5} = \text{constant}$$

Let T_{C_1} is for M_1 mass and T_{C_2} is for M_2 mass (increased).

$$\therefore T_{C_1} M_1^{0.5} = \text{constant} \quad \dots(1)$$

$$T_{C_2} M_2^{0.5} = \text{constant} \quad \dots(2)$$

\therefore From equation (1) and equation (2) we get,

$$T_{C_1} M_1^{0.5} = T_{C_2} M_2^{0.5} \quad \dots(3)$$



Substituting the values, we get

$$4.185 \times (199.5)^{0.5} = T_{C_2} \times (203)^{0.5}$$

$$4.185 \times 14.12 = T_{C_2} \times 14.247$$

$$\frac{59.1108}{14.242} = T_{C_2}$$

$$T_{C_2} = 4.15 \text{ K}$$

[Ans. : Critical temperature T_{C_2} at increased mass m_2 is 4.15 K]

Ex. 5.9.4 : The critical temperature of a given superconducting sample is 1.19 K with mass 26.91. Determine the critical temperature when the isotope mass changes to 32.13.

Soln. :

Given : $T_{C_1} = 1.19 \text{ K}$, $M_1 = 26.91$ and $M_2 = 32.13$, $T_{C_2} = ?$

Formula used is,

$$T_{C_1} M_1^{1/2} = T_{C_2} M_2^{1/2}$$

$$T_{C_2} = \frac{T_{C_1} M_1^{1/2}}{M_2^{1/2}} = \frac{1.19 \times (26.91)^{1/2}}{(32.13)^{1/2}}$$

$$= \frac{1.19 \times 5.127}{5.668} = \frac{6.173}{5.668} = 1.089$$

[Ans. : Critical temperature $T_{C_2} = 1.089 \text{ K}$]

Ex. 5.9.5 : A voltage of $5.9 \mu\text{V}$ is applied across a Josephson junction. What is the frequency of the radiation emitted by the junction?

Soln. :

Given : $V = 5.9 \times 10^{-6} \text{ V}$; $v = ?$

Formula : $v = \frac{2eV}{h}$

Substituting the values we get,

$$= \frac{2 \times 1.6 \times 10^{-19} \times 5.9 \times 10^{-6}}{6.62 \times 10^{-34}}$$

$$v = 2.851 \times 10^9 \text{ Hz.}$$

[Ans. : Frequency of the radiation $v = 2.851 \times 10^9 \text{ Hz.}$]

Ex. 5.9.6 : Determine the critical current through superconducting wire of diameter of 1.0 mm, given that critical field is $7.26 \times 10^3 \text{ A/m}$.

Soln. :

Given : $d = 1.0 \text{ mm}$

$$\therefore r = 0.5 \text{ mm} = 0.5 \times 10^{-3} \text{ m}$$

$$H_C = 7.26 \times 10^8 \text{ A/m}$$

$$I_C = 2\pi r H_C$$

Formula :

Substituting the values we get,

$$\begin{aligned} I_C &= 2 \times 3.14 \times 0.5 \times 10^{-3} \times 7.26 \times 10^8 \\ &= 2 \times 3.14 \times 0.5 \times 7.26 \\ I_C &= 22.608 \text{ A} \end{aligned}$$

[Ans. : Critical current flowing through the wire is $I_C = 22.608 \text{ A}$]

- Ex. 5.9.7 : If the voltage $6.2 \mu\text{V}$ is applied across a Josephson junction what will be the frequency of radiation emitted by the junction ?

Soln. :

Given :

$$V = 6.2 \mu\text{V} = 6.2 \times 10^{-6} \text{ V}$$

$$\nu(\text{frequency}) = \dots? \text{ Hz}$$

Formula :

$$\nu = \frac{2eV}{h}$$

Substituting the values we get,

$$\begin{aligned} \nu &= \frac{2 \times 1.6 \times 10^{-19} \times 6.2 \times 10^{-6}}{6.62 \times 10^{-34}} \\ &= \frac{19.84}{6.62} \times 10^9 \end{aligned}$$

$$\nu = 2.99 \times 10^9 \text{ Hz}$$

[Ans. : Frequency emitted by Josephson junction is

$$\nu = 2.99 \times 10^9 \text{ Hz}]$$

- Ex. 5.9.8 : In the case of lead superconductor to find the maximum value of magnetic field which will allow retaining its superconductivity, if critical temperature is 7.2 K and at 4 K it loses the superconducting property if subjected to a magnetic of $3.33 \times 10^4 \text{ A/m}$.

Soln. :

Given :

$$T_C = 7.2 \text{ K};$$

$$H_C = 3.33 \text{ A/m};$$

$$T = 4 \text{ K}$$

$$H_C(0) = \dots? \text{ at } 0 \text{ K}$$

**Formula :**

$$H_c(T) = H_c(0) \left[1 - \left(\frac{T}{T_c} \right)^2 \right]$$

Where, $H_c(0)$ is the critical magnetic field at 0 K.

$$\text{or } H_c(0) = \frac{H_c(T)}{1 - \frac{T^2}{T_c^2}}$$

Substituting the values we get,

$$\begin{aligned} H_c(0) &= \frac{3.33 \times 10^4}{1 - \frac{16}{51.84}} \\ &= \frac{3.33 \times 10^4}{0.69} = 4.82 \times 10^4 \text{ A/m} \end{aligned}$$

[Ans. : Critical field at 0 K is 4.82×10^4 A/m]

Ex. 5.9.9 : A superconductor has a critical temperature 3.7°K at zero magnetic field. At 0°K the critical magnetic field is 0.0306 Tesla. What is the critical magnetic field at temperature 2.0°K ?

MU - May 15, 3 Marks

Soln. :

Here $H_0 = 0.0306 \text{ T}$, $T = 2.0^\circ\text{K}$, $T_0 = 3.7^\circ\text{K}$

$$\begin{aligned} H_c &= \left[1 - \left(\frac{T}{T_c} \right)^2 \right] = 0.0306 \left[1 - \left(\frac{2}{3.7} \right)^2 \right] \\ &= 0.271 \text{ T} \end{aligned}$$

[Ans. : Critical magnetic field at temperature 2.0°K = 0.271 T]

Supercapacitors

5.10 Introduction

Q. Explain the term (a) conventional capacitor (b) supercapacitor.

- Few of the currently burning topics.
- Let's have a recap of ordinary capacitor also called as parallel plate capacitor. Using a dielectric medium between two plates, accumulation of charges takes place. This stores electrical energy and provides potential difference.
- Advanced version electrolytic capacitors have fixed polarity for anode and cathode.
- These capacitors are occasionally abbreviated as e-cap, whose anode is made of a metal that forms an insulating oxide layer through anodization. Applying a reverse polarity voltage or exceeding the maximum rated working voltage may invite a capacitor failure, and that can be hazardous.
- Despite all efforts, the capacity of capacitors always remains low. Hence, to store electrical charge, batteries were given preference.

- Now we have supercapacitors or ultracapacitors (or Electrical Double Layer Capacitors (EDLCs) or pseudo capacitors or power capacitors or power catches) which are electrochemical energy storage devices.

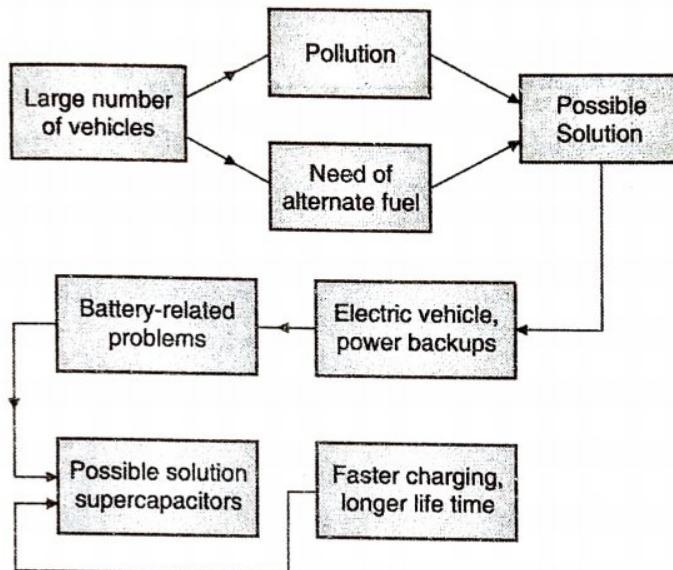


Fig. 5.10.1

5.10.1 Principle

Q. Explain the function of supercapacitors in detail.

- They store and release energy by reversible absorption and desorption of ions at the interfaces between electrode materials and electrolytes.
- Compared to common rechargeable batteries like lead-acid and Li-ion batteries they have -
 - (1) Considerably high specific power
 - (2) Longer cycle life times.

5.10.2 Construction

Q. Explain in detail construction of supercapacitors.

- Ordinary capacitor makes use of two parallel plates separated by a solid dielectric material.
- Supercapacitors, in principle, make use of
 - (1) Double layered capacitance
 - (2) Electro chemical pseudeo capacitance
- This can be further explained by stating that it consists of two electrodes separated by an ion permeable membrane which acts as the layer separator of electrolyte having positive and negative ions. When voltage is applied to the electrodes, polarization takes place. Ions in the electrolyte form an electric double layer as shown in the Fig. 5.10.2. Positive terminal will have negative ions from electrolyte forming a layer, and similarly electrode connected to negative terminal will have positive ions from electrolyte.

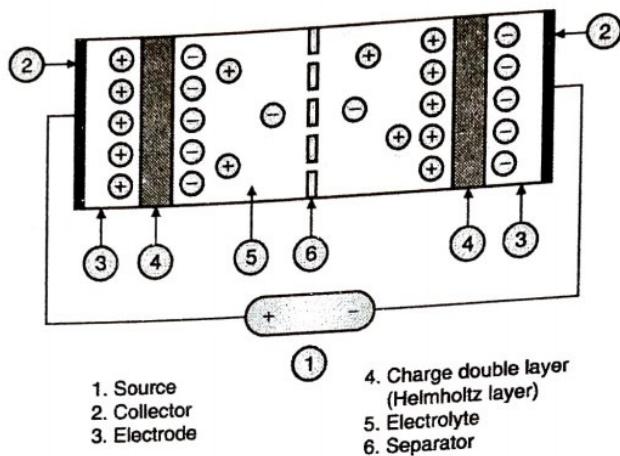


Fig. 5.10.2

1. Source
2. Collector
3. Electrode
4. Charge double layer (Helmholtz layer)
5. Electrolyte
6. Separator

- As shown in the Fig. 5.10.2, there are two capacitors formed at each end by accumulation of charges.

5.10.3 Types or Classification of Supercapacitors

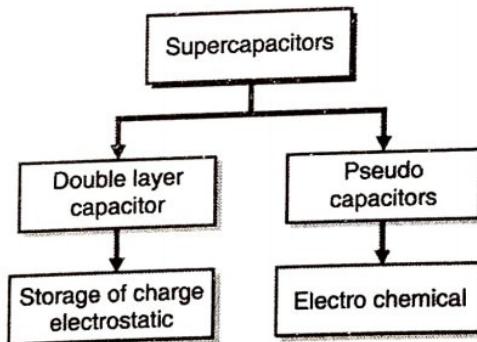


Fig. 5.10.3

5.10.4 Applications

Q. Explain at least three applications of supercapacitors.

Supercapacitors have their applications mainly due to the fact that they are useful when -

1. Fast charging is required for batteries. Hence supercapacitors are used for LED flash lights, digital cameras, laptop batteries, etc.
2. Supercapacitors can stabilize the power supply and are hence useful as portable media players, laptops, or similar hand-held machines.
3. As supercapacitors are capable to correct current, power fluctuations and harmonics, they are used as an interface between the load and grid to act as buffer in between.

4. On cards of computers, microcontrollers or similar digital devices act as emergency power backup in low power equipments.
5. One brilliant application of supercapacitors is in energy storage devices for energy harvesting systems. In energy harvesting systems we collect energy from renewable sources, for example electromagnetic fields, light, regenerative breaking system in vehicles or any mechanical movements. Such energy is stored in capacitor and provided back to the system when it is needed. Like in railway engines, energy while applying brakes is stored in supercapacitors and when acceleration is needed, this energy is provided back to system.

5.11 Comparison with Capacitor and Batteries, Energy Density, Power Density

Q. What is energy density and power density? Compare conventional batteries and supercapacitors on these points.

- When we consider conventional demand of electricity, especially for application of batteries the main problem is with charging. It takes longer time to charge our regular alkaline or Li-ion batteries. Reader can understand the process of cell phone battery charging. Imagine if we go for advanced industrial requirements like electrical vehicles, the charging time is one of the big hurdles.
- Recently the local transport (like BEST) has introduced electric buses, but the vehicle remains out of operation just due to charging issues for a longer time. Hence they can't go on a longer route. The other problem is, how many times the cycle of charging-discharging can take place. It is further observed that after a certain number of cycles of charging and discharging, the battery develops problems and needs a replacement. This adds to its cost.
- As per syllabus we will compare it with capacitor and batteries in terms of energy density and power density. But before that, let's understand these two terms. Since we are aware about the fact that conventional capacitor is energy storage device and energy stored is given by

$$E = \frac{1}{2} CV^2$$

- Energy stored per unit mass is described as specific energy which is represented in terms of Wh/kg. This is also referred to as gravimetric measure. If the capacitor energy is represented in terms of energy stored in capacitor it is represented in terms of per unit volume of that capacitor, it is called energy density. It is described in the units of $\frac{\text{watt Hours}}{\text{lit.}}$ or simply $\frac{\text{Wh}}{\text{t}}$
- Specific power is a term which describes the speed at which energy can be delivered to or absorbed from the load. If it is measured gravimetrically i.e. per mass, it is said to be representing specific power (unit kW/kg).
- Similarly, if measured volumetrically i.e. per volume of capacitor, it is known as power density (unit kW/l)
- On the scale of comparison specific energy of electrolyte capacitor is approximately in the range of 0.01 to 0.3 Wh/kg.
- Whereas, for lead acid battery it is in the range of 30 to 40 Wh/kg. Li - ion battery offers it around 100 - 200 Wh/kg.

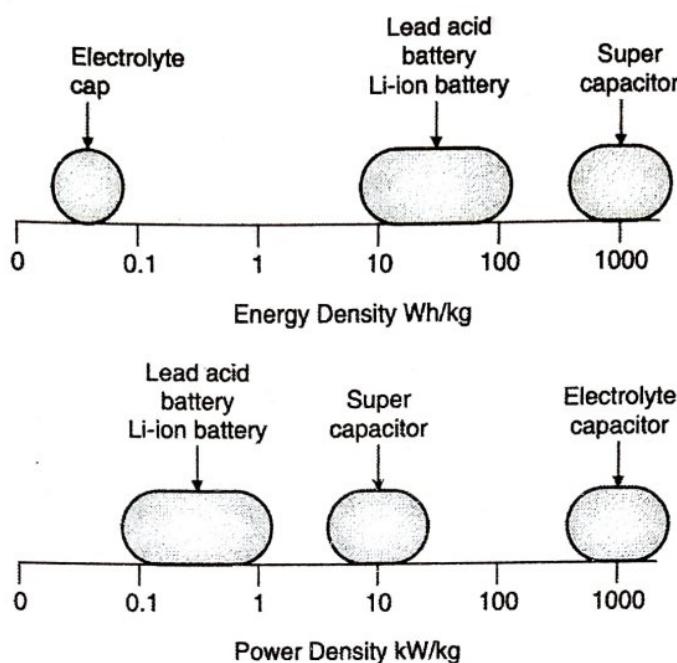


Fig. 5.11.1 : Energy and power density (kW/kg)

- If we consider their comparison, it is clear that specific energy supercapacitor which is inferior with respect to batteries, has an advantage when specific power is considered.

A Quick Revision

- The temperature at which a normal material turns into superconducting state is called critical temperature.
- The critical temperature is known to be inversely proportional to the square root of the atomic mass.
- The critical or transition temperature (T_c) of superconductor is found to vary with its isotopic mass (M), this phenomenon is called Isotopic Effect.
- Minimum value of the magnetic field required to destroy completely the superconducting phase is known as critical magnetic field, and is denoted by H_c .
- Critical magnetic field H_c is the function of temperature T, which is expressed as -

$$H_c = H_0 \left[1 - \frac{T^2}{T_c^2} \right]$$

- A superconducting material kept in a magnetic field expels the magnetic flux out of its body when cooled below the critical temperature and exhibits perfect diamagnetism. This effect is called 'Meissner effect'.
- Thus in a superconductor, the current is generated due to Cooper pairs, instead of individual electrons.
- Type-I superconductor exhibit complete Meissner effect. In the presence of an external magnetic $H < H_c$, the material in superconducting state is a perfectly diamagnetic.

- Type-II superconductors are characterized by two critical magnetic fields, H_{C1} and H_{C2} .
- Specimen is in ***mixed state between H_{C1} and H_{C2}*** . And above H_{C2} specimen returns to its ***normal state***. This region is known as ***vortex or mixed region***.
- Thus phenomenon of getting small current due to tunnelling of superelectrons of the superconductor is known as Josephson effect.
- The great sensitivity of the SQUID devices is associated with measuring changes in magnetic field associated with one flux quantization in the superconducting ring.





Engineering Materials and Applications

Unit VI

Syllabus

(Prerequisites : Paramagnetic materials, diamagnetic materials, ferromagnetic materials, crystal physics, conductors and insulators, free charges and bound charges inside a conductor, dielectric and electric polarization, capacitors and capacitance)

Liquid Crystals : Nematic, smectic and cholesteric phases, liquid crystal displays

Multiferroic : Type I and Type II multiferroics and applications

Magnetoresistive oxides : Magnetoresistance, GMR and CMR materials, introduction to spintronics

Learning Objectives :

After reading this chapter, learner should be able to :

- Understand types of liquid crystals
- Explain function of LCD
- Describe multiferroics and its types
- Define magnetoresistance, GMR, CMR
- Introduce spintronics

6.1 Liquid Crystal and Phases

MU - May 12, May 13

Q. Explain different phases of liquid crystal.

(May 12, 3 Marks)

Q. Explain the differences between three different liquid crystal phases w.r.t. the order in the arrangement of molecules, with the help of diagram. Which property of the liquid crystal is used for display?

(May 13, 5 Marks)

Q. Describe various phases of liquid crystals.

6.1.1 Mesomorphic Phase

MU - Dec. 13, May 14

Q. What is mesomorphic state of matter?

(Dec.13, 2 Marks)

Q. What is liquid crystal state of matter? Draw the diagram to describe molecular arrangement in their different phases.

(May 14, 5 Marks)

"Altering the externally generated illumination"

- In some substances, the tendency toward an ordered arrangement of molecules is so great that crystalline solid does not melt directly into liquid state but passes first through an intermediate stage called liquid crystal (or mesomorphic phase) before changing to liquid state on further application of heat.

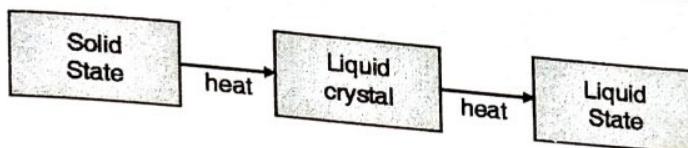


Fig. 6.1.1

Liquid crystal is one of the homogeneous phases with properties distinct from either of the traditional states. The liquid crystals show some of the properties of solid state and some of the properties of liquid state.

Properties related to solid state	Properties related to liquid state
Exhibit properties like double refraction of light and interference patterns in polarized light	Exhibit properties like surface tension, flow and viscosity
Ordered arrangement of atoms or molecules is still found	Ordered arrangement of atoms or molecules is of shorter order

- The temperature at the transition point provides enough energy to disrupt the binding between some of the molecules but the energy is not sufficient to break the strong lateral force of attraction between the long molecules.
- The important property in liquid crystals for practical application is that the arrangement of molecules can be upset by very slight changes in their surrounding i.e. molecules in liquid crystal rearrange themselves when a small electric field is applied and the change from an isotropic to more isotropic arrangement changes the way the crystal absorbs light.
- A small electric field can disturb the alignment of molecules while the large electric field induces turbulence in the liquid crystal with light scattering.

Types of liquid crystals

- (a) Smectic or soap-like liquid crystals
- (b) Nematic or thread-like liquid crystals
- (c) Cholesteric liquid crystals

(a) Smectic or soap-like liquid crystals

- Crystalline state, the orientation and periodicity – all these characteristics are retained.
- On heating, the crystal loses periodicity within the plane, but retains the orientation and arrangement in equispaced planes.

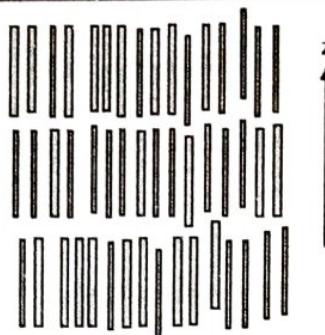


Fig. 6.1.2 : Smectic phase

(b) Nematic or thread-like liquid crystals

- These crystals on heating lose their planar or layered structure but retain a parallel alignment. Thus they retain orientation but lose periodicity.
- Hence molecules lie parallel to each other but can move up and down or sideways or can rotate along their axes.



Fig. 6.1.3 : Nematic phase

(c) Cholesteric liquid crystals

MU - Dec. 13

Q. Explain the cholesteric phase with neat diagram.

(Dec.13, 3 Marks)

- These liquid crystals have the same arrangement of molecules as in nematic type, but their optical activity is many times higher than that of its solid crystalline variety.
- **The cholesteric phase :** The cholesteric phase of liquid crystals has molecules parallel to each other, but the direction of alignment twists gradually and results in a helical structure. Therefore the substance consists of parallel layers. Molecules are aligned parallel in each layer. The helical structure of cholesteric substance is responsible for optical activity (optical rotation).

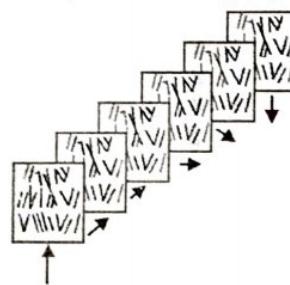


Fig. 6.1.4 : Cholesteric phase

6.1.2 Discotic Phase

They are disc-shaped rather than elongated. They form thermotropic liquid crystal phases.

6.1.3 Applications of Liquid Crystals

1. Liquid crystals are used in Liquid Crystal Displays (LCD) like digital watches and pocket calculators because of the fact that their orientation can be easily changed by electric field or pressure. The changed orientations have different light transmission and reflection. This is achieved by the application of electric field with the help of electrodes.
2. For smectic liquid crystals, the distance between planes having molecules that too with same orientation is considered to be the main feature. When a beam of light strikes these planes, the properties of reflected beam depend upon the distance in planes. This distance between the planes is highly temperature sensitive, and therefore colour of the reflected light depends upon temperature. Arrangement based upon this principle can have sensitivity as good as 0.01°C .
3. Liquid crystals are used in gas-liquid chromatography because they have electrical properties lying between crystalline solids and isotropic liquids.

6.2 Liquid Crystal Display

Q. How do liquid crystals display works?

- Liquid crystal display has *rotation of modules when a small electric voltage is applied across the liquid crystal* as the principle.
- Liquid crystals are basically organic compounds consisting of long rod-like molecules. In their natural state they arrange themselves with their long axes roughly parallel. However when they come into contact with a grooved surface they line up parallel along the grooves.
- When two finely grooved surfaces have their grooves 90° to each other, the liquid crystal sandwiched between them will twist. When light is made incident, it follows the alignment of molecules and therefore light is also twisted by 90° while passing through liquid crystal. When a small voltage is applied to the liquid crystal, the molecules rotates and rearrange themselves vertically and permit light to pass through untwisted.
- The use of two polarizers to control the brightness is the core principle behind LCD monitors. In LCD, the brightness of each pixel needs to be independently controlled. Since there are millions of pixels available on monitor, mechanically it is impractical to rotate polarizers within each one of the pixels.
- But using for any colour monitor, each pixel is divided into three smaller subpixels. These subpixels are covered with red, green and blue filters and display brightness to these components of light. Each of the subpixel acts as an independent pixel, but due to their small size and proximity with each other, viewer's eyes fail to distinguish them as separate pixels. All the colours that can be sensed by us are produced using different combinations of red, green and blue light.

6.3 Multiferroics

- Combination of electricity and magnetism has provided us with electromagnetism.
- Through Maxwell's equations, we have gone through an ocean of applications which have changed human life drastically. But electric and magnetic ordering in solids is usually considered separately. The reason behind this is very clear - the electrons and ions are responsible for charge effects whereas electron's spin is responsible for magnetic properties.



- A reader can start with piezoelectric effect wherein application of mechanical stress produces static electricity due to polarization of charges in some natural crystals. The applications were limited as piezoelectric constant was small.
- A new class of manmade materials with increased piezoelectric constants was discovered. They were described as ferroelectrics.
- Along the same lines the concept of piezomagnetism is developed wherein one can induce magnetic moment by applying physical stress or a physical deformation by applying a magnetic field. Some antiferromagnetic crystals exhibit this effect.
- A new possibility was coined when two independent phenomena
 - (1) The appearance of magnetization M in an electric field E
 - (2) Appearance of electric polarization P due to magnetic field coexists, but also two types of ordering i.e.
 - (a) Spontaneous ordering of orbital and spin magnetic moments i.e. ferromagnetism
 - (b) Spontaneous ordering of electric dipole moments i.e. ferroelectricity

coexist in a material without applying any external electric or magnetic fields. These materials are called multiferroics.

6.3.1 Classification of Multiferroics

To understand multiferroics further, they are classified into two main categories

- (1) Type – I multiferroics
- (2) Type – II multiferroics

Sr. No.	Type – I Multiferroic	Type – II Multiferroic
1.	Ferroelectricity and magnetism occur at different temperatures.	Ferroelectricity and magnetism occur at same temperature.
2.	Ferroelectricity and magnetism occur due to different mechanism.	Ferroelectricity and magnetism occur due to same mechanism.
3.	In such materials, ferroelectricity is the result of structural distortion which occurs at high temperature denoted by T_L .	In such materials inversion symmetry is broken through magnetic ordering and directly ferroelectricity is observed.
4.	Magnetic ordering sets in at low temperature denoted by T_N	Ferroelectricity and magnetism occur at same temperature.
5.	Example : For BiFeO_3 , $T_C = 1100^\circ \text{ k}$ and $T_N = 643^\circ \text{ K}$	Example TbMnO_3

6.3.2 Applications of Multiferroics

- Multiferroics have offered a completely new technological avenue for us i.e. to control magnetism by using electric field using a novel way of magnetoelectric coupling.
- This has the potential to reduce energy in conventional devices working on electromagnetism.
- New devices which work on magnetoelectricity like TMR sensor (Tunnel Magneto Resistance), also high sensitivity AC magnetic field sensors and tunable microwave devices are conceptualized.

6.4 Magnetoresistive Oxides

6.4.1 Magnetoresistance

Q. Explain the term magnetoresistance. Describe GMR and CMR materials in detail.

It is the characteristic exhibited by a material preferably ferromagnetic to cause the variation in the value of electrical resistance when external magnetic field is applied to it.

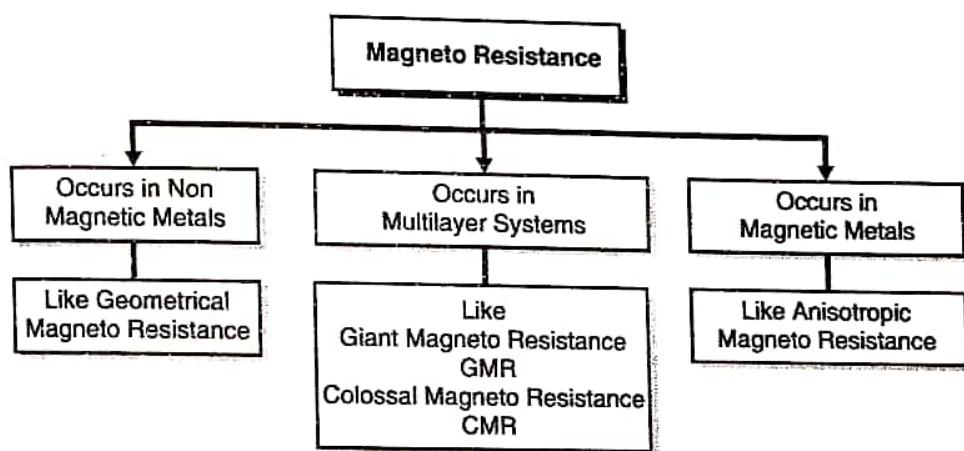


Fig. 6.4.1

1. Giant Magnetooresistance (GMR)

Let's consider

$$R(H) = \text{Electrical resistance in magnetic field (H)}$$

$$R(0) = \text{Electrical resistance when } H = 0$$

$$\therefore \delta_H = \frac{R(H) - R(0)}{R(0)}$$

- Here δ_H represents magnetoresistance. The term giant magnetoresistance is used here because δ_H is very high.
- GMR is basically a quantum mechanical effect observed in alternative ferromagnetic and non-magnetic conductive layers. If $R(H)/R(0) \rightarrow H$ graph is plotted it appears as shown below.

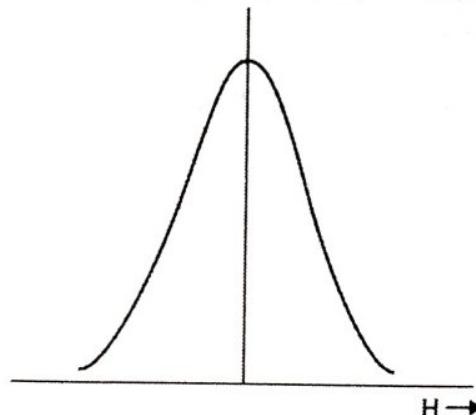


Fig. 6.4.2

Applications :

1. Magnetic field sensors
2. To read data hard disk drives and bio-sensors
2. **Colossal Magneto Resistance (CMR)**

- CMR is relatively a very advance topic. Complete understanding is still not so clear. It can be said honestly that it is a matter of current research activities.
- It is a property observed in manganese-based perovskite oxides.
- Like GMR, CMR also offers change in resistance with respect to change in external magnetic field.
- CMR provides interesting scope for the strong coupling of magnetic properties to the lattice structure and ordering of spin charge and angular momentum.

6.5 Introduction to Spintronics

Q. What is spintronics? Why is it considered so important for the future?

- As the name suggests, it is spin-based electronics. It is a nanotechnology which deals with spin-dependent properties of an electron instead of charge-dependent properties.
- It is the focus of attention because in contrast to electronics where electron charge is used for storage and transfer of information, spintronics uses spin in addition to or in place of the electron charge.
- Spin is represented by clockwise or anticlockwise i.e. two orientations $\pm\hbar/2$.
- Directional and coherent motion of electron spin circulates a spin current which can be used to carry information controlled by quantum spin in spintronic devices.

Significance :

- Size of transistors or other components has already reached such a small level that very soon we will be unable to reduce it further. Spintronics is one of the effective solutions expected in the future.

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