

ENGINEERING PHYSICS –I

Rev – 2019

Course Code – FEC-102

- **Quantum Physics.**
- **Solid State Physics - Crystallography.**
- **Solid State Physics – Semiconductors.**
- **Optics-I.**
- **Super Conductors and Super Capacitors.**
- **Engineering Materials and Applications.**

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ENGINEERING PHYSICS-I (FEC 102) REV 2019

Course Code	Course Name	Teaching Scheme (Contact Hours)			Credits Assigned				
		Theory	Pract.	Tut.	Theory	Tut.	Pract.	Total	
FEC102	Engineering Physics-I	2	-	-	2	-	-	2	
Course Code	Course Name	Examination Scheme							
		Theory				End Sem. Exam.	Exam. Duration (in Hrs)	Term Work	
		Internal Assessment							
FEC102	Engineering Physics-I	Test 1	Test 2	Avg.	60	2	--	--	75

Objectives

1. To understand basic physics concepts and founding principles of technology.
2. To develop scientific temperament for scientific observations, recording, and inference drawing essential for technology studies.

Outcomes: Learner will be able to...

1. Illustrate the fundamentals of quantum mechanics and its application.
2. Illustrate the knowledge of crystal planes, X-ray diffraction and its application.
3. Illustrate the knowledge of Fermi level in semiconductors and applications of semiconductors in electronic devices.
4. Illustrate the knowledge of interference in thin films and its various applications.
5. Illustrate the basic knowledge of superconductors and super capacitors.
6. Illustrate the knowledge of engineering materials and applications.

Engineering Physics-I

Module	Contents	Hrs.
01	QUANTUM PHYSICS (Prerequisites : Dual nature of radiation, Photoelectric effect Matter waves-wave nature of particles, de-Broglie relation, Davisson-Germer experiment) De Broglie hypothesis of matter waves; properties of matter waves; wave packet, phase velocity and group velocity; Wave function; Physical interpretation of wave function; Heisenberg uncertainty principle; non existence of electron in nucleus; Schrodinger's time dependent wave equation; time independent wave equation; Particle trapped in one dimensional infinite potential well, Quantum Computing.	07
02	SOLID STATE PHYSICS - CRYSTALLOGRAPHY (Prerequisites : Crystal Physics (Unit cell, Space lattice, Crystal structure, Simple Cubic, Body Centered Cubic, Face Centered Cubic, Diamond Structure, Production of X-rays) Miller indices; interplanar spacing; X-ray diffraction and Bragg's law; Determination of Crystal structure using Bragg's diffractometer;	03
03	SOLID STATE PHYSICS - SEMICONDUCTORS (Prerequisites: Intrinsic and extrinsic semiconductors, Energy bands in conductors, semiconductors and insulators, Semiconductor diode, I-V characteristics in forward and reverse bias) Direct & indirect band gap semiconductor; Fermi level; Fermi dirac distribution; Fermi energy level in intrinsic & extrinsic semiconductors; effect of impurity concentration and temperature on fermi level; mobility, current density; Hall Effect; Fermi Level diagram for p-n junction (unbiased, forward bias, reverse bias); Applications of semiconductors: LED, Zener diode, Photovoltaic cell.	06
04	OPTICS-I (Prerequisites : Wave front and Huygens's principle, reflection and refraction, Interference by division of wave front, Young's double slit experiment) Interference by division of amplitude, Interference in thin film of constant thickness due to reflected and transmitted light; origin of colors in thin film; Wedge shaped film; Newton's rings. Applications of interference-Determination of thickness of very thin wire or foil; determination of refractive index of liquid; wavelength of incident light; radius of curvature of lens; testing of surface flatness; Anti-reflecting films and Highly reflecting film.	06
05	SUPERCONDUCTORS AND SUPERCAPACITORS (Prerequisites : Electric current, flow of electric charges in a metallic conductor, drift velocity, mobility and their relation with electric current, Ohm's law, electrical resistance, V-I characteristics (linear and non-linear), electrical resistivity and conductivity temperature dependence of resistance) Superconductors: Critical temperature, critical magnetic field, Meissner's effect, Type I and Type II and high T _c superconductors; Super capacitors: Principle, construction, types, materials and applications, comparison with capacitor and batteries: Energy density, Power density,	02

06	ENGINEERING MATERIALS AND APPLICATIONS (Prerequisites: Paramagnetic materials, diamagnetic materials, ferromagnetic materials, crystal physics, Conductors and insulators, free charges and bound charges inside a conductor. Dielectrics and electric polarization, capacitors and capacitance) Liquid crystals: Nematic, Smectic and cholesteric phases, Liquid crystal display. Multiferroics : Type I & Type II multiferroics and applications, Magneto resistive Oxides: Magneto resistance, GMR and CMR materials, Introduction to spintronics.	02
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References:

1. A text book of Engineering Physics-Avadhanulu & Kshirsagar, S. Chand
2. A textbook of Optics - N. Subramanyam and Brijlal, S.Chand
3. Fundamentals of optics by Jenkins and White, McGrawHill
4. Solid State Electronic Devices- B. G. Streetman, Prentice Hall Publisher
5. Modern Engineering Physics – Vasudeva, S.Chand
6. Concepts of Modern Physics- Arther Beiser, Tata McGraw Hill
7. A text book of Engineering Physics-Avadhanulu & Kshirsagar, S. Chand
8. A textbook of Optics - N. Subramanyam and Brijlal, S.Chand
9. Fundamentals of optics by Jenkins and White, McGrawHill
10. Solid State Electronic Devices- B. G. Streetman, Prentice Hall Publisher
11. Modern Engineering Physics – Vasudeva, S.Chand
12. Concepts of Modern Physics- Arther Beiser, Tata McGraw Hill
13. A Text Book of Engineering Physics, S.O. Pillai, New Age International Publishers.
14. Introduction to Solid State Physics- C. Kittle, John Wiley & Sons publisher
15. Ultra capacitors: The future of energy storage- R.P Deshpande, McGraw Hill
Advanced functional materials– Ashutosh Tiwari, Lokman Uzun, Scrivener Publishing LLC.

SOLID STATE PHYSICS - CRYSTALLOGRAPHY

Prerequisites:

"The study of geometrical forms and other physical properties of crystalline solid by using X-rays, neutron beam or electron beam are called crystallography"

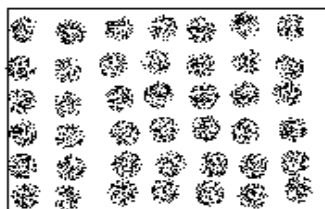
The matter always exists in three states i.e solids, liquids and gases. An atom is basic building block of matter.

- In solids molecules are closely packed, hence a solid has definite shape and volume.
- In liquids, molecules are loosely packed as compared to solid, hence liquid has definite volume but no definite shape, and it take the shapes of the container.
- In gases the molecules are at greater distances, hence they are free to move and gas has neither definite volume nor shape.

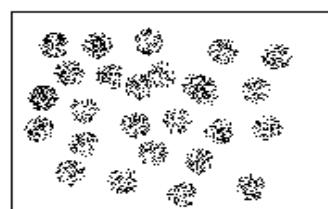
The solid materials are widely used in engineering and technology.

CLASSIFICATION OF SOLIDS:-

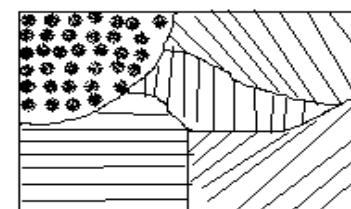
According to the arrangement of atoms, the solid materials are classified in to three types



Crystalline

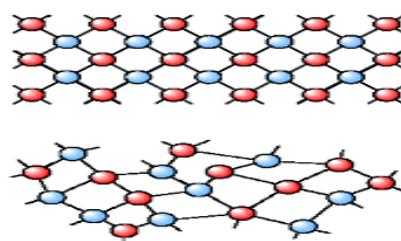


Amorphous



Polycrystalline

Crystalline has long range order of atoms while amorphous solid has short range order of atoms.



(1) CRYSTALLINE SOLIDS:-

- a) The atoms are arranged regularly and periodically in 3- dimension.
- b) Each atom is at definite distance and at definite position from the other atoms. The angular orientation of all the atoms is same. The ordered arrangement of atoms extends

over large distances corresponding to several layers of atoms. Therefore crystalline solids exhibit's long range order.

- c) They have sharp melting point.
- d) They possess directional physical properties and are also called as "anisotropic substances". The physical properties such as refractive index, electrical conductivity, elasticity etc. are different along different directions in the crystal.
- e) E.g. Quartz, alum, diamond, rock salt, copper, silver, aluminium etc.

(2) AMORPHOUS SOLIDS:-

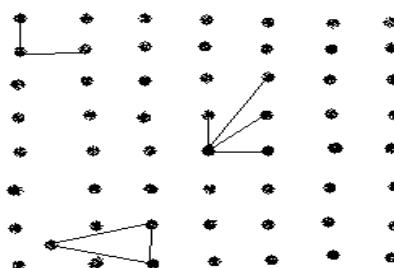
- a) In amorphous solids, the atoms or molecules are arranged irregularly and randomly in 3- dimension.
- b) They do not have regular atomic arrangement and exhibits short range order.
- c) They do not have regular structure and directional properties, hence called as "isotropic substances". They exhibit same properties in different directions.
- d) They do not exhibits a sharp melting point and have no specific electrical property but have only plasticity.
- e) E.g. plastic, glass, rubber etc.

(3) POLYCRYSTALLINE SOLIDS:-

- a) Polycrystalline solids contain smaller grains or crystallites. These grains are separated by grain boundaries. The grain size is 10^3 to 10^4 \AA^0 .
- b) In each grain, atoms are arranged regularly and periodically. The angular orientation of all the atoms in one grain is same, but in two different grains, angular orientation of the atoms is different.
- c) The crystal without grain boundary is known as single crystal.
- d) All the naturally occurring metals are polycrystalline in nature.e.g. Quartz.

The concept of space lattice and unit cell is used to study the crystal structure.

SPACE LATTICE:-



- In crystalline solids atoms are arranged regularly in ordered form. If we replace each atom by a point, then we will get set of points. These points are known as lattice points.
- The regular and periodic arrangement of the lattice points is known as space lattice.
- “The infinite array of lattice points in 3- D, in which each point has identical surroundings to that of the other point is known as space lattice”.
- To represent the crystal structure, each lattice point is associated with one or more atoms. These atoms are known as **atomic basis**.

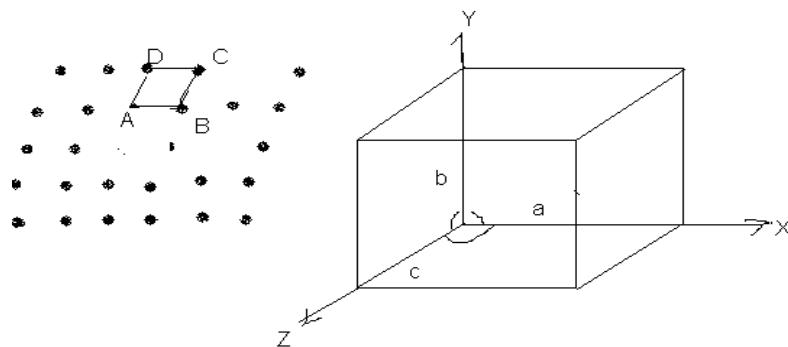


Space Lattice + Basis = Crystal Structure

Lattice + Basis = Crystal structure.

- Depending upon the number of atomic basis associated with each lattice point, the crystal may be monoatomic (Al, Cu), diatomic (NaCl, KCl) or triatomic (CaF₂).

UNIT CELL:-

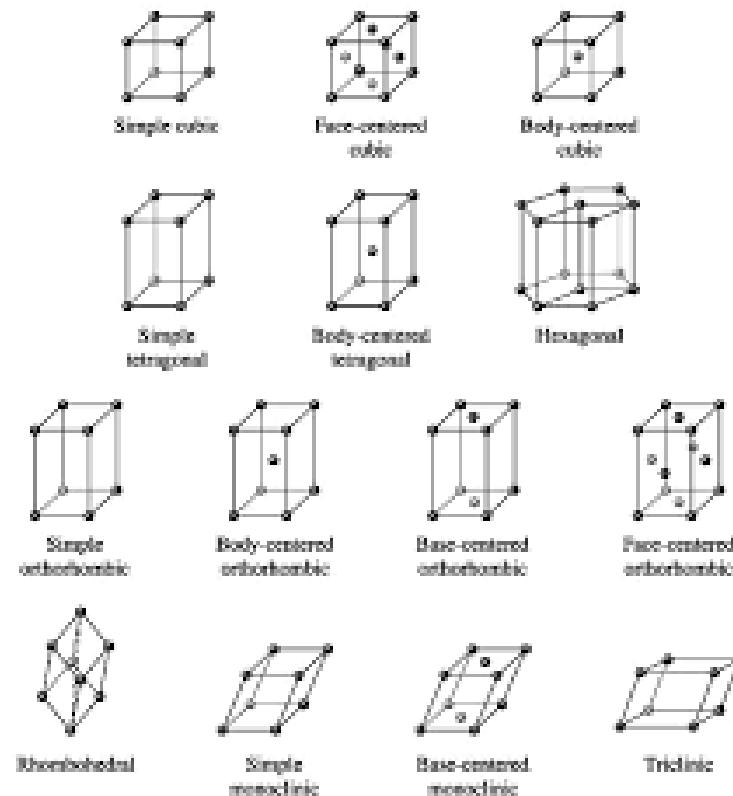


- “The smallest volume of the crystalline solid, by repetition of which the entire crystal structure is obtained is known as unit cell.”
- The intercept of the unit cell along the three crystallographic axis is known as **axial length (a, b, c)**. The angle between two axes is known as **axial angle or interfacial angle (α , β , γ)**.
- The axial lengths (a, b, c) and interfacial angles (α , β , γ) are known as **lattice parameter** of the unit cell.

- Depending upon the lattice parameter, the crystal structure is divided in to seven systems.
- The seven system of crystal structure have a total 14 unit cells based on the internal arrangement of atomic sites within the unit cell.

Crystal system:-

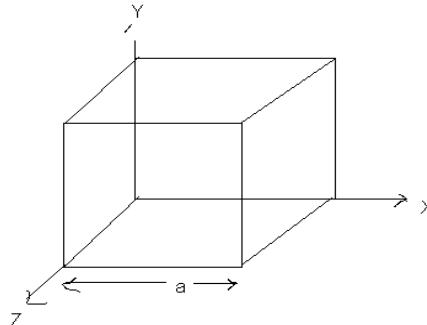
Sr.no.	Crystal system	Unit cell parameter	
1	Cubic	$a = b = c$	$\alpha = \beta = \gamma = 90$
2	Monoclinic	$a \neq b \neq c$	$\alpha = \beta = 90 \neq \gamma$
3	Triclinic	$a \neq b \neq c$	$\alpha \neq \beta \neq \gamma \neq 90$
4	Tetragonal	$a = b \neq c$	$\alpha = \beta = \gamma = 90$
5	Orthorhombic	$a \neq b \neq c$	$\alpha = \beta = \gamma = 90$
6	Rhombohedral	$a = b = c$	$\alpha = \beta = \gamma \neq 90$
7	Hexagonal	$a = b \neq c$	$\alpha = \beta = 90, \gamma = 120$



CUBIC SYSTEM:-

In cubic system, the unit cell is in the form of the cube. The axial lengths are equal in magnitude and axial angles are perpendicular to each other. ($a = b = c$, $\alpha = \beta = \gamma = 90^\circ$). The edge length of the cube is $a = b = c = 'a'$ is known as lattice constant of the unit cell.

Depending upon the position of the lattice point in the unit cell, there are two types of unit cell and three types of crystal structure in cubic system.

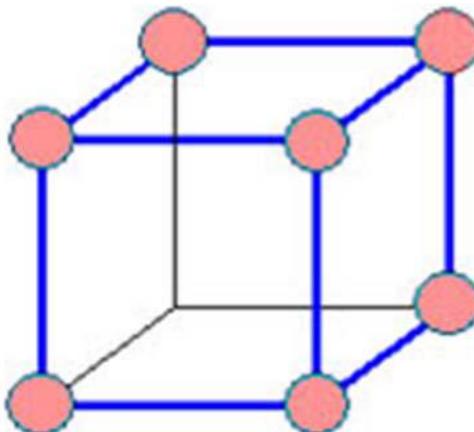


Types of unit cell:-

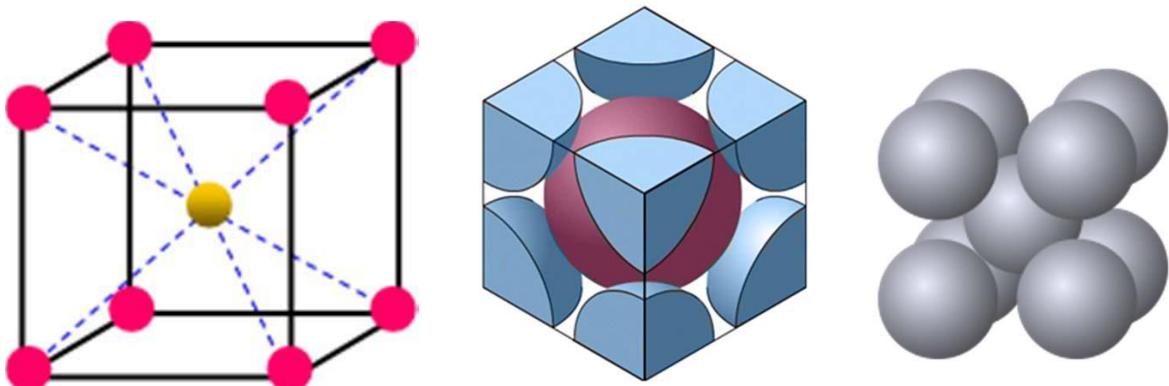
- a) Primitive unit cell: - Lattice points are located only at each corner of the unit cell.
- b) Non primitive unit cell :- In addition to corner lattice points, the lattice points are located at each face centre or edge centre or body centre of the unit cell.

Types of crystal structure:-

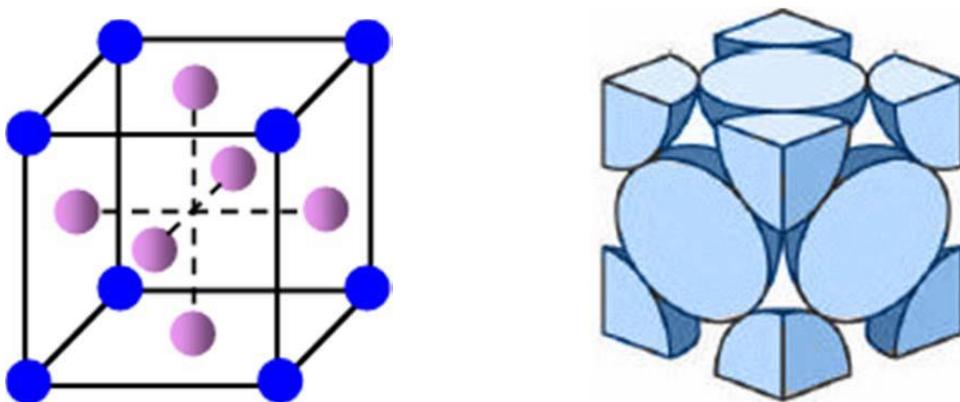
- 1) Simple cubic structure: - There is one lattice point at each corner of the unit cell.
e.g. Polonium, CsCl



- 2) Body centered cubic structure: - There is one lattice point at each corner and one lattice point at the body centered of the unit cell.
e.g. Sodium, Potassium, Chromium, molybdenum, Tungsten, Lithium etc.



- 3) Face centered cubic structure: - There is one lattice point at each corner and one lattice point at each face center of the unit cell.
 e.g. aluminium, copper, gold, silver, NaCl, KCl etc.



CHARACTRISTICS OF A CUBIC UNIT CELL:-

A) Number of atoms per unit cell(N):- For monoatomic crystal structure, in the unit cell, each corner atom is shared by 8 unit cell, each face centered atom is shared by 2 unit cell and body centered atom is itself in the unit cell.

If $N_c \rightarrow$ Total no. of corner atoms.

$N_f \rightarrow$ Total no. of face centered atoms.

$N_b \rightarrow$ total no. of body centered atoms.

Total no. of atoms per unit cell

$$N = \frac{N_c}{8} + \frac{N_f}{2} + N_b$$

B) Co – ordination number (CN):- “ The total number of equidistant nearest neighbouring atoms for any corner atom in the unit cell is called co- ordination number”.

If CN is large, then atoms in the crystal are more closely packed to each other.

C) Atomic radius (r):- “ Half the distance between the nearest neighbouring atoms in a crystal of pure elements is known as atomic radius”.

D) Atomic packing factor (APF) :- “ The ratio of volume occupied by the atoms in the unit cell to the volume of the unit cell is called atomic packing factor”.

If APF is expressed in percentage, then it is known as packing efficiency.

The APF gives closeness of the atoms in the crystal. If APF is large, then atoms are more closely pack to each other and crystal possesses stable structure.

The unoccupied or vacant space in the unit cell is known as void space.

If $N \rightarrow$ Total no. of atoms per unit cell.

$r \rightarrow$ Atomic radius

$a \rightarrow$ edge length of the cube or lattice constant

$$\text{Volume of atom} = \frac{4}{3} \pi r^3$$

$$\text{Volume of unit cell} = a^3$$

$$\text{APF} = \frac{\text{Volume of atoms per unit cell}}{\text{Volume of unit cell}}$$

$$APF = \frac{N \times \frac{4}{3} \pi r^3}{a^3}$$

$$\text{Packing efficiency} = APF \times 100 \%$$

$$\text{Void space} = [1 - APF] \times 100 \%$$

ATOMIC PACKING FACTOR (APF)

1) SIMPLE CUBIC STRUCTURE (sc):- For monoatomic crystal, there is one atom at each corner of the unit cell.

a) No. of atoms / unit cell:-

$N_c \rightarrow$ Total no. of corner atoms = 8

$N_f \rightarrow$ Total no. of face centered atoms = 0

$N_b \rightarrow$ total no. of body centered atoms = 0

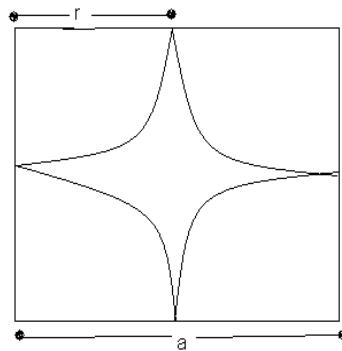
Total no. of atoms per unit cell,

$$n = \frac{N_c}{8} + \frac{N_f}{2} + N_b$$

$$n = \frac{8}{8} + \frac{0}{2} + 0$$

n = 1 atom / unit cell

b) Atomic radius (r):- In sc structure each corner atom touch to each other along the cube edge of the unit cell.



If $r \rightarrow$ atomic radius and
 $a \rightarrow$ edge length of the cube

$$a = 2r$$

$$r = \frac{a}{2}$$

c) Atomic Packing Factor:

For sc structure, $n = 1$ and $r = \frac{a}{2}$

$$APF = \frac{n \times \frac{4}{3} \pi r^3}{a^3}$$

$$APF = \frac{1 \times \frac{4}{3} \pi \left(\frac{a}{2}\right)^3}{a^3}$$

$$APF = \frac{\pi}{6} = 0.52$$

Packing efficiency = $0.52 \times 100 = 52\%$

Void space = $[1 - 0.52] \times 100 = 48\%$

- d) **Coordination number** :- For any corner atom, 4 nearest atoms in its own plane, 1 atom above and 1 atom below its plane. Therefore **CN = 6**

2) BODY CENTERED CUBIC STRUCTURE (bcc):-

For monoatomic crystal, there is one atom at each corner of the unit cell and one atom at the body centered of the unit cell.

a) No. of atoms / unit cell:-

$N_c \rightarrow$ Total no. of corner atoms = 8

$N_f \rightarrow$ Total no. of face centered atoms = 0

$N_b \rightarrow$ total no. of body centered atoms = 1

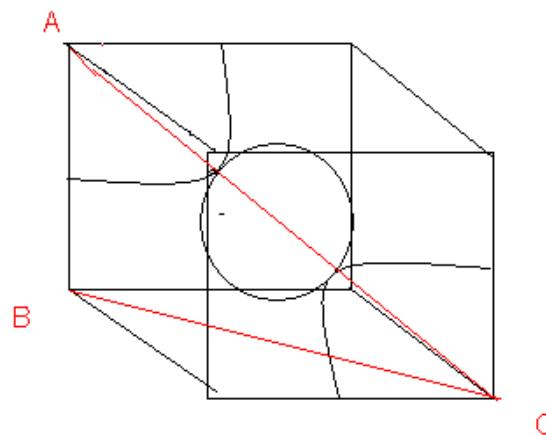
Total no. of atoms per unit cell,

$$n = \frac{N_c}{8} + \frac{N_f}{2} + N_b$$

$$n = \frac{8}{8} + \frac{0}{2} + 1$$

n = 2 atoms / unit cell

b) Atomic radius (r):- In bcc structure each corner atom is in touch with the body centered atom along the body diagonal of the unit cell.



If $r \rightarrow$ atomic radius and

$AB = a \rightarrow$ edge length of the cube

Body diagonal $AC = \sqrt{3} a$

$$r + 2r + r = \sqrt{3} a$$

$$4r = \sqrt{3} a$$

$$r = \sqrt{3} a / 4$$

c) **Atomic Packing Factor:**

For bcc structure, $n = 2$ and $r = \sqrt{3} a / 4$

$$APF = \frac{n \times \frac{4}{3} \pi r^3}{a^3}$$

$$APF = \frac{n \times \frac{4}{3} \pi (\sqrt{3} a / 4)^3}{a^3}$$

$$APF = \sqrt{3} \pi / 8 = 0.68$$

Packing efficiency = $0.68 \times 100 = 68\%$

Void space = $[1 - 0.68] \times 100 = 32\%$

- d) **Coordination number** :- For any corner atom, the body centered atom is the nearest atom. Since the corner atom is surrounded by 8 unit cell, there are 8 body centered atoms nearest to any corner atom. Therefore **CN = 8**.

3) FACE CENTERED CUBIC STRUCTURE (FCC):-

For monoatomic crystal, there is one atom at each corner of the unit cell and one atom at each face centered of the unit cell.

- a) **No. of atoms / unit cell:-**

$N_c \rightarrow$ Total no. of corner atoms = 8

$N_f \rightarrow$ Total no. of face centered atoms = 6

$N_b \rightarrow$ total no. of body centered atoms = 0

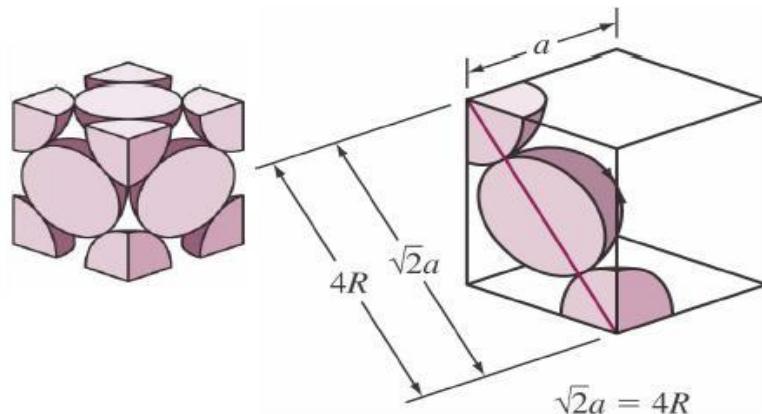
Total no. of atoms per unit cell,

$$n = \frac{N_c}{8} + \frac{N_f}{2} + N_b$$

$$n = \frac{8}{8} + \frac{6}{2} + 0$$

$n = 4$ atom / unit cell

- b) **Atomic radius (r)**:- In fcc structure each corner atom is in touch with the face centered atom along the face diagonal of the unit cell.



If $r \rightarrow$ atomic radius and
 $a \rightarrow$ edge length of the cube
 Face diagonal $AC = \sqrt{2} a$

$$\begin{aligned} r + 2r + r &= \sqrt{2} a \\ 4r &= \sqrt{2} a \\ r &= \sqrt{2} a / 4 \end{aligned}$$

c) Atomic Factor Factor:-

For fcc structure, $N = 4$ and $r = \sqrt{2} a / 4$

$$\text{APF} = \frac{N \times \frac{4}{3} \pi r^3}{a^3}$$

$$\text{APF} = \frac{4}{3} \frac{\pi (\sqrt{2} a / 4)^3}{a^3}$$

$$\text{APF} = \sqrt{2} \pi / 6 = 0.74$$

$$\text{Packing efficiency} = 0.74 \times 100 = 74 \%$$

$$\text{Void space} = [1 - 0.74] \times 100 = 26 \%$$

d) Coordination number :- For any corner atom, the face centered atom is the nearest neighboring atom. Therefore for any corner atom, 4 face centered atoms in its own plane, 4 face centered atoms above its plane and 4 face centered atoms below its plane are the nearest atoms. Therefore **CN = 4 + 4 + 4 = 12.**

The packing efficiency of fcc structure is 74 % and void space is 26 %. Therefore fcc structure is more closely packed stable structure of the crystalline solid.

RELATION BETWEEN LATTICE PARAMETER, MOLECULAR WEIGHT AND DENSITY :-

Consider a cubic lattice with lattice parameter 'a'.

$$\text{Volume of the unit cell} = a^3$$

M- Molecular weight.

N – Avogadro's number.

$$\text{Mass of each molecule} = M / N$$

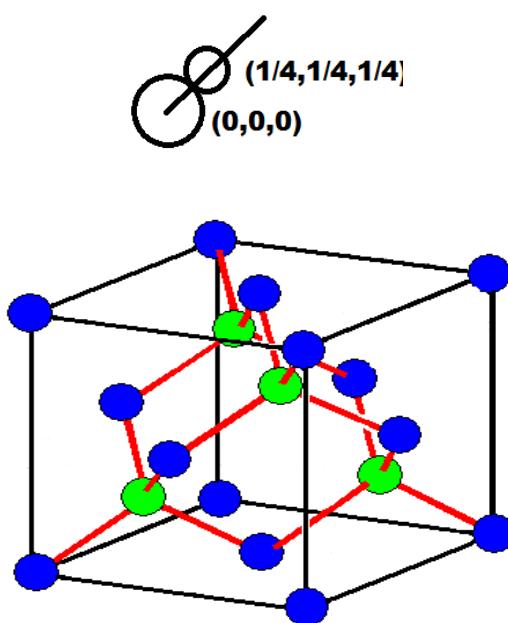
If n = no. of atoms per unit cell

ρ = density of crystal.

Density = mass / volume

$$\rho = \frac{n M / N}{a^3}$$

DIAMOND CRYSTAL STRUCTURE:-



- 1) The diamond structure is a combination of two interpenetrating fcc lattice.

- 2) The one sub lattice has its origin at (0, 0, 0) and other sub lattice has its origin at (a/4, a/4, a/4) i.e along the body diagonal at 1/4th of the edge length of the cube.
- 3) The lattice is fcc and atomic basis is 2 atoms per lattice point. The orientation of the basis is along the body diagonal of the unit cell. Thus diamond structure is fcc diatomic structure.
- 4) In the unit cell, 1 atom is at each corner and at each face centered position and 4 atoms inside the unit cell such that each atom is placed on one of the four diagonals of the unit cell at a distance of 1/4th of the body diagonal from a corner atom and it forms four covalent bond with one corner atom and three face center atom. Thus tetrahedron is formed. Therefore CN = 4.
- 5) The group IVth elements in the periodic table (Ge, Si, C) having this type of crystal structure. These are covalently bonded solid having 4 valence electrons. Each electron forms four covalent bond with four neighboring atoms. The strength of the structure depends upon the covalent bonds. As the atomic no. increases, interatomic distances increases and binding force becomes weak. The bonding in carbon (6) is strong, so it is hard and having high melting point.
- 6) No. of atoms per unit cell (n):-

$$N_c \rightarrow \text{Total no. of corner atoms} = 8$$

$$N_f \rightarrow \text{Total no. of face centered atoms} = 6$$

$$N_b \rightarrow \text{total no. of body centered atoms} = 4$$

Total no. of atoms per unit cell,

$$n = \frac{N_c}{8} + \frac{N_f}{2} + \frac{N_b}{1}$$

$$n = \frac{8}{8} + \frac{6}{2} + \frac{4}{1}$$

$$\mathbf{n = 8 \text{ atom / unit cell}}$$

- 7) Atomic radius (r):-

$$\text{Atomic diameter} = \frac{1}{4} X (\text{body diagonal of the unit cell})$$

$$2r = \frac{1}{4} \sqrt{3} a$$

$$r = \frac{\sqrt{3} a}{8}$$

- 8) Atomic packing factor (APF):-

For diamond structure, $n = 8$ and $r = \frac{\sqrt{3} a}{8}$

$$APF = \frac{\text{Volume of atoms in the unit cell}}{\text{Volume of unit cell}}$$

$$APF = \frac{n \times \frac{4}{3} \pi r^3}{a^3}$$

$$n \times \frac{4}{3} \pi (\sqrt{3} a / 8)^3$$

$$APF = \frac{3}{a^3}$$

$$\mathbf{APF = 0.34}$$

$$\mathbf{\text{Packing efficiency} = 0.34 \times 100 = 34 \%}$$

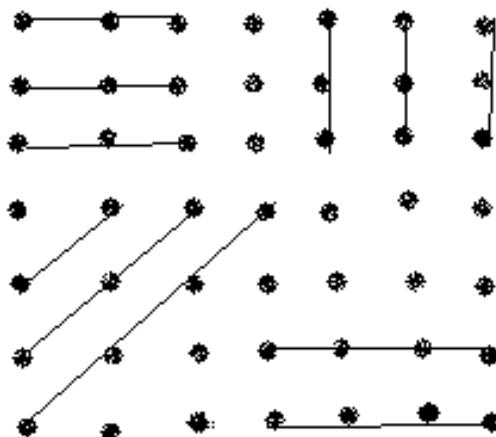
$$\mathbf{\text{Void space} = [1 - 0.34] \times 100 = 66 \%}$$

The diamond structure is loosely packed; hence elements Ge and Si are used as semiconductor element.

Syllabus

CRYSTAL PLANES AND MILLER INDICES:-

- 1) Define crystal plane, plane direction, Miller indices of crystal plane?
- 2) Explain the procedure to determine Miller indices of the crystal plane?
- 3) What are crystallographic planes? Explain the procedure to determine the Miller indices of the plane in cubic system?



CRYSTAL PLANE:-

- 1) In crystalline solid atoms are arranged regularly and periodically in 3-D.
- 2) The regular arrangement of lattice points is known as space lattice.
- 3) The sets of parallel planes passing through the lattice points describe the space lattice.
- 4) These planes are known as lattice planes or crystal planes.
- 5) Certain atomic planes in the crystalline solid contain maximum number of atoms. The intense reflection of the X-rays from these atomic planes is used to determine the structure of crystalline solid.
- 6) The concept of miller indices is used to designate these crystal planes.

MILLER INDICES:-

- 1) A plane in the crystalline solid is identified by three integers number (h, k, l) known as Miller indices.
- 2) "The three smallest possible integer numbers which have the same ratio as the reciprocals of the intercepts of the given plane on the three coordinate axes are known as miller indices".
- 3) Each set of planes has a particular orientation with reference to the axes of the unit cell.

PROCEDURE TO DETERMINE MILLER INDICES OF THE PLANE:-

- 1) Find the intercept of the plane along the axes X, Y, Z in terms of the lattice constant a, b, c respectively e.g. p_a , q_b , r_c
- 2) Take the reciprocal of the integer parts of the intercepts. E.g. $1/p$, $1/q$, $1/r$
- 3) Reduce these reciprocals in to three integers h, k, l by taking LCM. The no. h, k, l are known as Miller indices of the plane.

Axial intercepts $\rightarrow 2a, 3b, 4c$

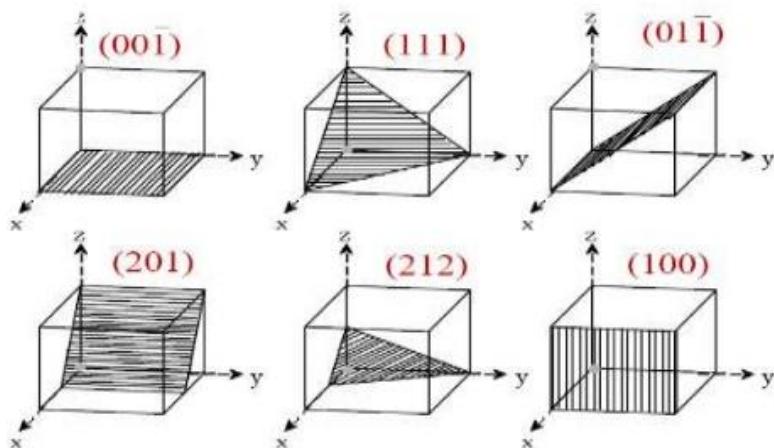
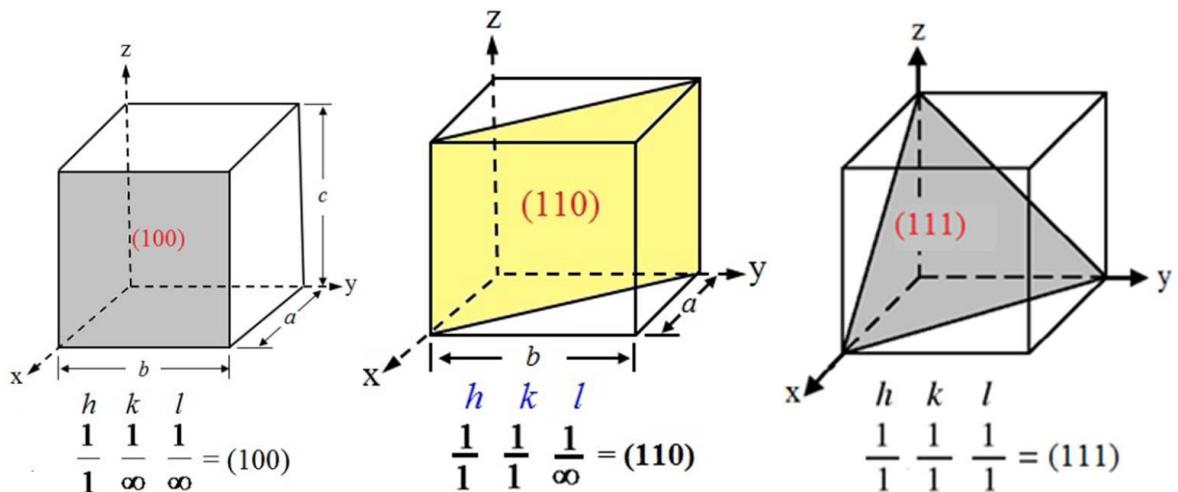
Reciprocals $\rightarrow 1/2, 1/3, 1/4$

LCM $\rightarrow 12$

$$h = \frac{1}{2} \times 12 = 6, \quad k = \frac{1}{3} \times 12 = 4, \quad l = \frac{1}{4} \times 12 = 3$$

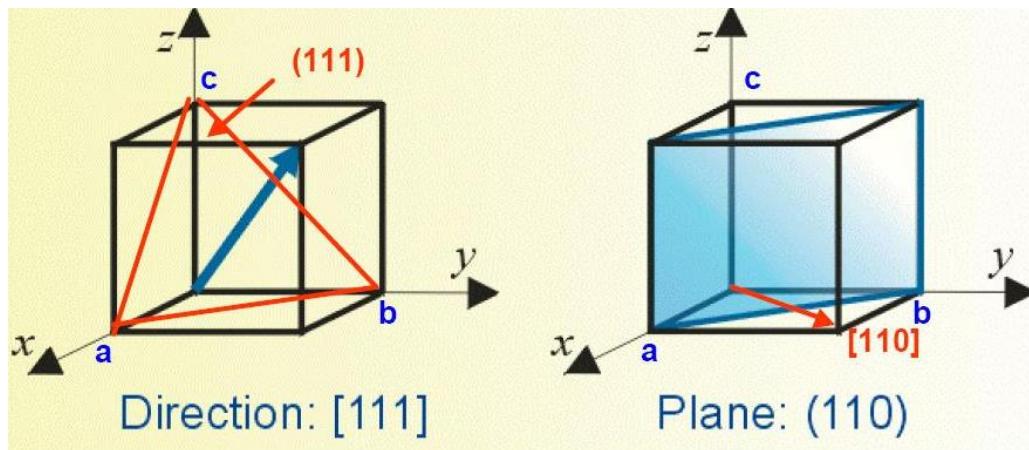
Miller indices of plane $\rightarrow 6, 4, 3$

- 4) The given plane is represented by (hkl) i.e. (643)



PROCEDURE TO REPRESENT PLANE DIRECTION:-

- 1) The line passing through the origin and perpendicular to given plane represents the plane direction.
- 2) If 'o' is origin of the unit cell and 'p' is a point of coordinate (h,k,l) , then the line 'op' represent the direction of the plane (hkl) .
- 3) The plane direction is represented by $[hkl]$.

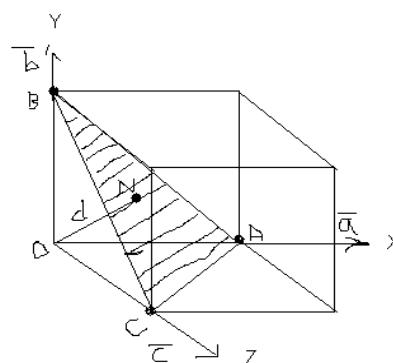


POINTS TO BE NOTED WHILE FINDING MILLER INDICES OF THE PLANE:-

- 1) The plane parallel to coordinate axis having intercept is ' ∞ ' and miller index for that axis is '0'.
- 2) The plane having negative intercept of miller index is represented by putting (-) bar sign over it.
- 3) Parallel planes having same miller indices.
- 4) A plane passing through the origin is defined in terms of a parallel plane having non zero intercepts.

INTERPLANNER SPACING:

1. **What are Miller indices of the crystal plane? Determine the expression of interplaner spacing in cubic unit cell?**



- 1) Consider a plane ABC in a cubic unit cell. If a , b , c are the lattice constant and n_1 , n_2 , and n_3 are integers along crystallographic axes X, Y, Z respectively. Then intercept of the plane are $OA = n_1a$, $OB = n_2b$ and $OC = n_3c$.
- 2) Reciprocal of integer part of the intercept are known as Miller indices.
Therefore $h = 1/n_1$, $k = 1/n_2$ and $l = 1/n_3$.
- 3) The perpendicular distance between two planes is called interplaner spacing.

ON = d, represent the interplaner spacing.

If α , β , and γ are axial angle between ON and axes X, Y, Z respectively.

$$\cos \alpha = ON / OA = d/n_1 a = dh / a$$

$$\cos \beta = ON / OB = d/n_2 b = dk / b$$

$$\cos \gamma = ON / OC = d/n_3 c = dl / c$$

The law of direction cosine is

$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$$

$$\frac{d^2 h^2}{a^2} + \frac{d^2 k^2}{b^2} + \frac{d^2 l^2}{c^2} = 1$$

But in cubic system, $a = b = c = a$

$$d^2 h^2 + d^2 k^2 + d^2 l^2$$

$$\frac{d^2 h^2 + d^2 k^2 + d^2 l^2}{a^2} = 1$$

$$\frac{d^2 [h^2 + k^2 + l^2]}{a^2} = 1$$

$$\text{Therefore, } d = \frac{a}{\sqrt{h^2 + k^2 + l^2}}$$

X – RAYS

- Q. What is X-rays? Why the X-rays are suitable to study crystalline solid?
- Q. What is X-rays? Explain the diffraction of X-rays?
- Q. State and explain Bragg's law and write its importance?
- Q. Explain Bragg's spectrometer method to determine crystal structure?

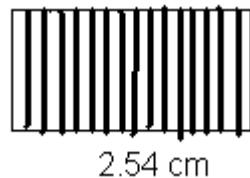
What is X – Rays ?

"X – rays are invisible, high penetrating electromagnetic radiation of very high frequency and short wavelength of the order of 0.1 \AA^0 to 100 \AA^0 ."

DIFFRACTION OF X-RAYS:-

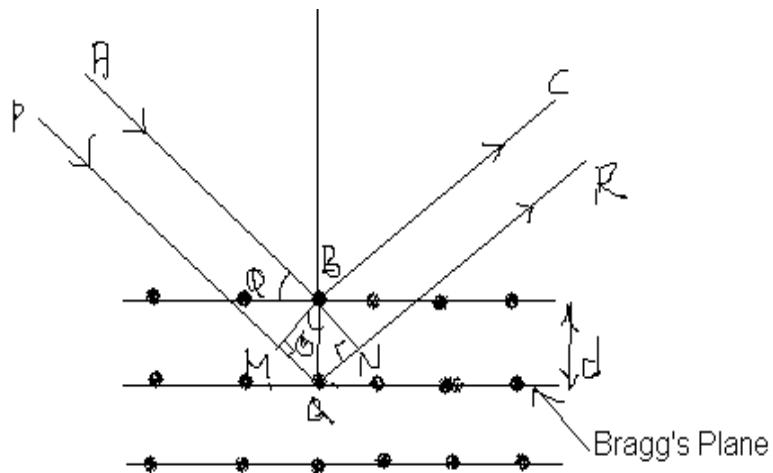
- 1) The bending of the light rays from the edges of the obstacle and formation of geometrical shadow of the obstacle on the screen is known as diffraction.
- 2) To produce diffraction, the size of the obstacle must be comparable with the wavelength of the incident light.

- 3) The number of parallel slits of equal width and separated by equal opaque portion is called diffraction grating.
- 4) On the ordinary diffraction grating 15000 slits in 1 inch. Since 1 inch = 2.54 cm
 $\text{Slit width} = 2.54 / 15000 \text{ cm} = 1.7 \times 10^{-4} \text{ cm} = 1.7 \times 10^4 \text{ AU}$
 This is comparable with the wavelength of the visible light.



- 5) The X-rays having very short wavelength 0.1 to 100 AU. Hence ordinary diffraction is not suitable to produce diffraction of the X-rays.
- 6) The interplaner distance in the crystalline solid is about 2 AU. Therefore the X-rays are diffracted by the atoms of the crystal. Since in crystalline solids, atoms are arranged in 3 – D, the crystal acts as 3-D diffraction grating for the incident X-rays.

BRAGG'S LAW :-



- 1) The X-rays are diffracted by the atoms of the crystal.
- 2) Consider 2-D space lattice. The set of parallel planes known as Bragg's plane are separated by distance 'd'.
- 3) A monochromatic beam of the X-rays of wavelength λ is incident on the crystal with an angle θ with the surface of the crystal.
- 4) The beam AB reflected from the atom at B along BC and beam PQ reflected along QR.

- 5) Reflected beams BC and QR are interfere with each other to produce either constructive or destructive interference depending upon the path difference Δ between them.
- 6) The optical path difference between the reflected rays BC and QR is

$$\Delta = MQ + QN$$

But from ΔBMQ and ΔBNQ , $MQ = QN = d \sin \theta$

$$\begin{aligned}\Delta &= d \sin \theta + d \sin \theta \\ &= 2 d \sin \theta\end{aligned}$$

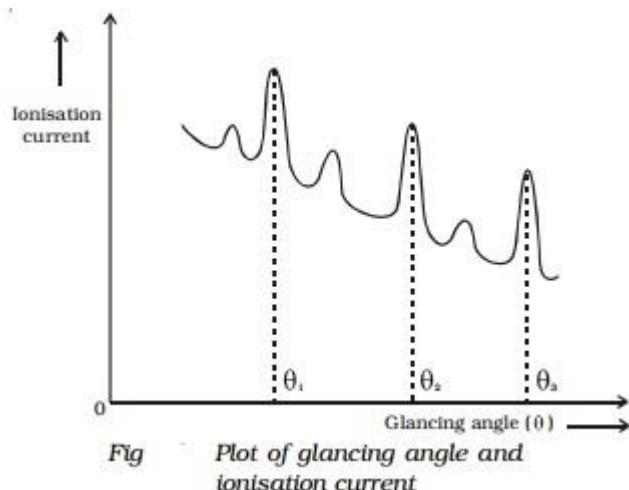
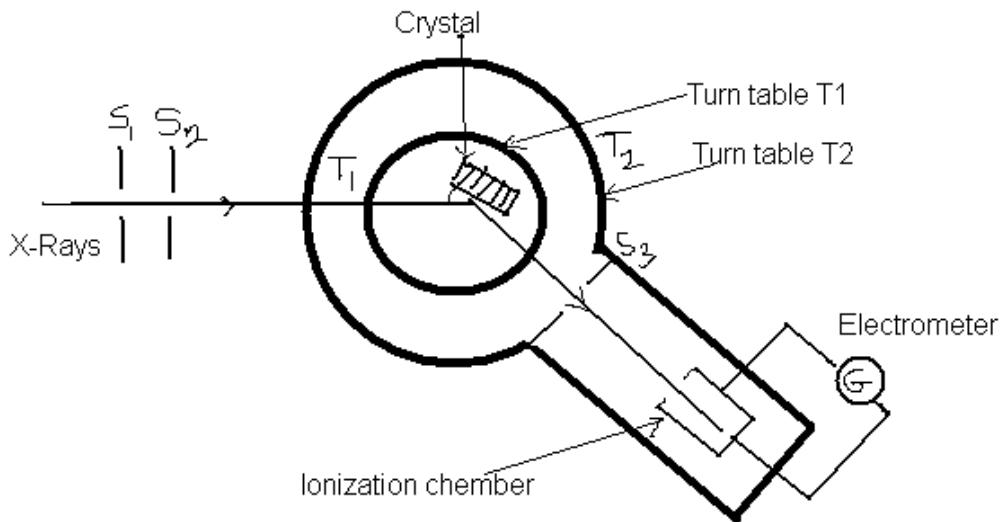
For constructive interference $\Delta = n \lambda$ where n – order of reflection.

$$\text{Therefore } \mathbf{2 d \sin \theta = n \lambda}$$

This is known as Bragg's law. It is used to study structure of crystalline solids.

Importance: 1) To determine interplaner spacing 'd'. 2) To determine wavelength of X-rays.

BRAGG'S X-RAY SPECTROMETER:-



CONSTRUCTION:-

- 1) The Bragg's X-rays spectrometer consists of a turn table T_1 and T_2 rotating about a same axis of rotation.
- 2) The crystal whose structure is to be determined is mounted on the turn table T_1 . The position of the crystal can be noted by using the scale S_1 and V_1 .
- 3) The turn table T_2 consists of ionization chamber containing known gas. This chamber is connected to the sensitive electrometer to measure ionization current.
- 4) The position of ionization chamber can be noted by using scale S_2 and V_2 .

- 5) The turn table T_1 and T_2 are linked together in such a way that when T_1 rotates through an angle θ , then turn table T_2 rotates by an angle 2θ . Therefore whatever may be the glancing angle θ of the incident X-rays, the reflected X-rays from the crystal always enter in to the ionization chamber.

WORKING:-

- 1) The X-rays from the X-ray tube are collimated in to a fine beam and incident on a crystal.
- 2) The X-rays are reflected from the atoms of the crystal and enter in to a ionization chamber and ionize the gas.
- 3) The ionization current 'I' is measured at different glancing angle θ by using electrometer.
- 4) The graph of ionization current 'I' verses glancing angle θ shows peaks at certain angles $\theta_1, \theta_2, \theta_3, \dots$ etc.
- 5) The certain atomic plane in a crystal contains maximum number of atoms. The reflected X-rays from these atomic planes are in phase to each other. Hence resultant reflection becomes maximum and ionization current increases.
- 6) The certain atomic plane contains less number of atoms. The reflected X-rays from these atomic planes are antiphase to each other. Hence resultant reflection becomes less and ionization current decreases.
- 7) The values of $\theta_1, \theta_2, \theta_3, \dots$ etc. are corresponding to 1st, 2nd, 3rd ... order reflection of X-rays.
- 8) By using Bragg's law $2 d \sin \theta = n \lambda$, we have to determine the ratio $1/d_1:1/d_2:1/d_3$.
Comparing these practical values with theoretical values, we can determine the structure of the crystal.
- 9) e.g. for KCl crystal $\theta_1 = 5.22, \theta_2 = 7.30$, and $\theta_3 = 9.05$
Bragg's law is $2d \sin \theta = n \lambda$.

$$\begin{aligned}
 1/d_1:1/d_2:1/d_3 &= \sin \theta_1 : \sin \theta_2 : \sin \theta_3 \\
 &= 1 : \sqrt{2} : \sqrt{3}
 \end{aligned}$$

These are the value for simple cubic structure. Therefore KCL has sc structure.

Similarly for,

$$\text{bcc, } 1/d_1:1/d_2:1/d_3 = 1 : \sqrt{2}/2 : \sqrt{3} \quad \&$$

$$\text{fcc, } 1/d_1:1/d_2:1/d_3 = 1 : \sqrt{2} : \sqrt{3}/2$$

Problem 1 : The spacing between the principal planes of a NaCl crystal is 2.82 \AA . It is found that the first order Bragg's reflection occur at an angle of 10°. Calculate the wavelength of X-rays.

Solution :

Formula : $2d \sin \theta = n\lambda$

$$\lambda = \frac{2d \sin \theta}{n}$$

Given : $d = 2.82\text{\AA}$

$$\theta = 10^\circ$$

$$n = 1$$

$$\therefore \lambda = \frac{2 \times 2.82 \times \sin 10^\circ}{1}$$

$$\lambda = 0.98\text{\AA}$$

Ans. $\lambda = 0.98\text{\AA}$

Problem 2 : The unit cell dimension a of NaCl lattice is 5.63 \AA . If X-ray beam of $\lambda = 1.1\text{\AA}$ fall on a family of planes with a separation of $\frac{a}{\sqrt{5}}$, how many orders of diffraction are observable?

Solution :

Formula : $2d \sin \theta = n\lambda$

$$n = \frac{2d \sin \theta}{\lambda}$$

Given : $a = 5.63\text{\AA}$

$$\lambda = 1.1\text{\AA}$$

$$d = \frac{a}{\sqrt{5}}$$

For maximum order, $\theta = 90^\circ, \sin 90^\circ = 1$

$$n = \frac{2 \times \frac{a}{\sqrt{5}} \times \sin 90^\circ}{1.1} = \frac{2 \times 5.63}{\sqrt{5} \times 1.1} = 4.585$$

Ans. Hence $n = 4$, i.e. four orders of diffraction are observable.

Problem 3 : Calculate the longest wavelength that can be analysed by a rock salt crystal of spacing $d = 2.82\text{\AA}$ in the first order.

Solution :

Formula : $2d \sin \theta = n\lambda$

Given : $d = 2.82\text{\AA}$

$$n = 1$$

For longest wavelength, $\theta = 90^\circ$, $\sin 90^\circ = 1$

$$\lambda_{\max} = \frac{2 \times 2.82}{1} = 5.64\text{\AA}$$

Ans. Longest wavelength is 5.64\AA.

Problem 4 : A beam of X-rays $\lambda = 0.842\text{\AA}$ is incident on a crystal at a glancing angle $8^\circ 35'$ when first order Bragg's reflection occurs. Calculate the glancing angle for third order reflection.

Solution :

Formula : $2d \sin \theta = n\lambda$

Given : $\lambda = 0.842\text{\AA}$

$$\theta = 8^\circ 35'$$

$$n = 1$$

For first order, $2d \sin 8^\circ 35' = 1 \times 0.842 \times 10^{-10}$

For third order ($n = 3$)

$$2d \sin \theta_3 = 3 \times 0.842 \times 10^{-10}$$

Dividing the two equations,

$$\frac{\sin \theta_3}{\sin 8^\circ 35'} = \frac{3}{1}$$

$$\sin \theta_3 = 3 \times 0.15 = 0.45$$

$$\Rightarrow \theta_3 = 26.5^\circ$$

Ans. Glancing angle for third order is 26.5° .

Problem 5 : Calculate the glancing angle on the cube (100) of a rock salt ($a = 2.814\text{\AA}$) corresponding to second order diffraction maximum for X-rays of wavelength 0.710\AA .

Solution :

Formula : $2d \sin \theta = n\lambda$

$$d = \frac{a}{\sqrt{h^2 + k^2 + l^2}}$$

Given : $(hkl) = (100)$

$$a = 2.814\text{\AA}$$

$$n = 2$$

$$\lambda = 0.710$$

$$d_{100} = \frac{2.814}{\sqrt{1^2 + 0^2 + 0^2}} = 2.814\text{\AA}$$

$$2 \times 2.814 \sin \theta = 2 \times 0.710$$

$$\sin \theta = \frac{2 \times 0.710}{2 \times 2.814} = 0.2523$$

$$\theta = 14^\circ 36' 40''$$

Ans. Glancing angle is $14^\circ 36' 40''$.

Problem 6 : The Bragg angle corresponding to the first order reflection from (111) planes of a crystal is 30° . Wavelength of X-rays is 1.75\AA . Determine Interplanar spacing and lattice constant of the crystal.

Solution :

Formula :

$$d = \frac{a}{\sqrt{h^2 + k^2 + l^2}}$$

$$2d \sin \theta = n\lambda$$

Given :

$$(hkl) = (111)$$

$$n = 1$$

$$\theta = 30^\circ, \lambda = 1.75\text{\AA}$$

$$2d \sin 30^\circ = 1 \times 1.75$$

$$d = 1.75\text{\AA}$$

$$1.75 = \frac{a}{\sqrt{1^2 + 1^2 + 1^2}}$$

$$a = 3.03\text{\AA}$$

Ans. $d = 1.75\text{\AA}$, $a = 3.03\text{\AA}$.

Problem 7 : Monochromatic X-ray beam of wavelength 5.8189\AA is reflected strongly for a glancing angle $\theta = 75.86^\circ$ in the first order diffraction by certain planes of cubic crystal of lattice constant 3\AA , find the Miller Indices of the reflecting planes.

Solution :

Formula :

$$2d \sin \theta = n\lambda, d = \frac{a}{\sqrt{h^2 + k^2 + l^2}}$$

Given :

$$\lambda = 5.8189\text{\AA}$$

$$\theta = 75.86^\circ$$

$$n = 1$$

$$a = 3\text{\AA}$$

$$2 \times d \times \sin 75.86 = 1 \times 5.8189$$

$$\Rightarrow d = 3\text{\AA}$$

$$3 = \frac{3}{\sqrt{h^2 + k^2 + l^2}}$$

$$\text{i.e. } \sqrt{h^2 + k^2 + l^2} = 1$$

Ans. The Miller indices of the possible reflecting planes are (100) , (010) , (001) , $(\bar{1}00)$, $(0\bar{1}0)$ and $(00\bar{1})$.

Problem 8 : Calculate the Bragg angle, if (200) planes of a BCC crystal with lattice parameter 2.814\AA give second order reflection with X-rays of wavelength 0.71\AA .

Solution :

Formula :

$$d = \frac{a}{\sqrt{h^2 + k^2 + l^2}}, 2d \sin \theta = n\lambda$$

Given : $(hkl) = (200)$
 $a = 2.814\text{\AA}$
 $\lambda = 0.71\text{\AA}$
 $n = 2$
 $d = \frac{2.814}{\sqrt{2^2 + 0^2 + 0^2}} = 1.407\text{\AA}$

$$2 \times 1.407 \times \sin \theta = 2 \times 0.71$$

$$\sin \theta = \frac{2 \times 0.71}{2 \times 1.407} = 30^\circ$$

Ans. Bragg's angle $\theta = 30^\circ$.

Problem 9 : X-rays of wavelength 1.4\AA is found to be reflected from the (111) plane of FCC structure. If the lattice parameter of the crystal is 5\AA , find the angle at which the X-ray is incident on the (111) plane of the crystal.

Solution :

Formula : $d = \frac{a}{\sqrt{h^2 + k^2 + l^2}}, \quad 2d \sin \theta = n\lambda$

Given : $\lambda = 1.4\text{\AA}$
 $(hkl) = (111)$
 $a = 5\text{\AA}$
 $d = \frac{5}{\sqrt{1^2 + 1^2 + 1^2}} = \frac{5}{\sqrt{3}} = 2.886\text{\AA}$
 $\sin \theta = \frac{1 \times 1.14}{2 \times 2.886}$
 $\theta = 14^\circ$

Ans. $\theta = 14^\circ$.

Problem 10 : X-rays of unknown wavelength give order Bragg's reflection at glancing angle of 20° with (212) planes of copper having FCC structure. Find the wavelength of X-rays, if the lattice constant of copper is 3.65\AA .

Solution :

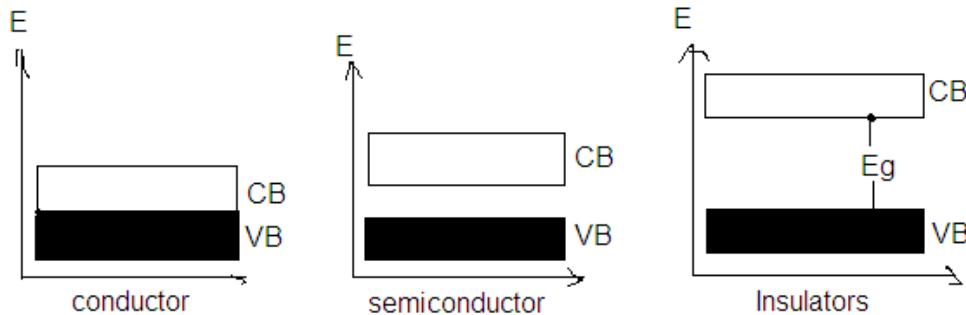
Given : $\theta = 20^\circ$
 $(hkl) = (212)$
 $a = 3.615\text{\AA}$
 $d = \frac{a}{\sqrt{h^2 + k^2 + l^2}} = \frac{3.615}{\sqrt{2^2 + 1^2 + 2^2}} = 1.2\text{\AA}$
 $2d \sin \theta = n\lambda$
 $2 \times 1.2 \times \sin 20^\circ = 1 \times \lambda$
 $\lambda = 0.823\text{\AA}$

Ans. $\lambda = 0.823\text{\AA}$.

SOLID STATE PHYSICS - SEMICONDUCTORS

Prerequisites:

CLASSIFICATION OF SOLIDS:-



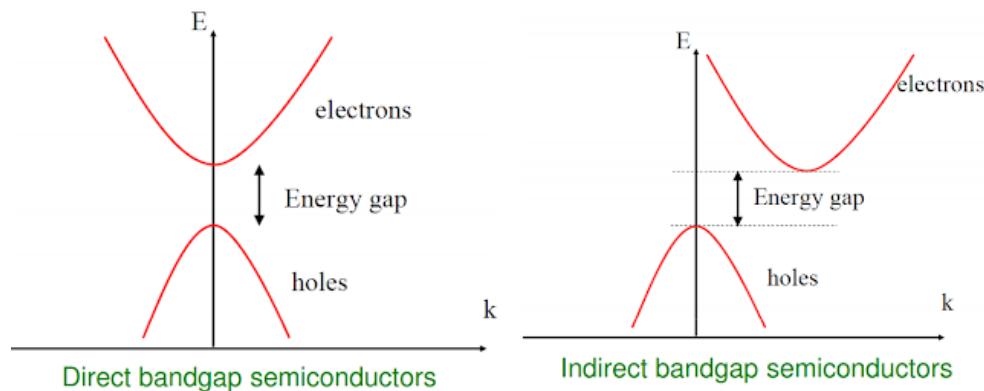
- 1) **Conductors**:- In conductors the valence band and conduction band are overlap on each other, Therefore large no. of electrons are available for conduction of electric currents.
- 2) **Insulators**:-In insulators there is wide energy gap between valence band and conduction.
- 3) **Semiconductors**:- In semiconductors, there is narrow energy gap between the valence band conduction band. For Si, $E_g = 1.1$ ev and for Ge, $E_g = 0.7$ ev.

Syllabus:

Direct and Indirect band gap semiconductor:

- 1) The solid crystals are formed when the isolated atoms are brought together. However, when two atoms are brought close to each other, it leads to intermixing of electrons in the valence shell. As a result, the number of permissible energy levels is formed, which is called an energy band.
- 2) Each band is formed due to the splitting of one or more atomic energy levels. Therefore, the minimum number of states in a band equals twice the number of atoms in the material. The reason for the factor of two is that every energy level can contain two electrons with opposite spin.
- 3) Band gap is the difference in energy between the valence band and the conduction band of a solid material that consists of the range of energy values forbidden to electrons in the material.

- 4) In semiconductor physics, the band gap of a semiconductor can be of two basic types, a **direct band gap** or an **indirect band gap**. Therefore another way of classifying semiconductors based on their band structure is
 - a) Direct band gap semiconductor b) Indirect band gap semiconductor
- 5) The difference between direct and indirect band gap semiconductors is related to their band structure. Electrons in solids have a wave like character. An electron wave is characterized by a wave vector k . Thus, for crystalline materials it is possible to plot E vs. vector k diagrams. These are related to the simple band diagrams that show the valence and conduction band.
- 6) The band gap is called "direct" if the crystal momentum (Vector k) of electrons and holes is the same in both the conduction band and the valence band. In direct band-gap (DBG) semiconductor the maximum energy level of the valence band aligns with the minimum energy level of the conduction band with respect to momentum.
- 7) The band gap is called "indirect" if the crystal momentum (Vector k) of electrons and holes is not same in both the conduction band and the valence band. In indirect band-gap (IBG) semiconductor the maximum energy level of the valence band misaligned with the minimum energy level of the conduction band with respect to momentum.



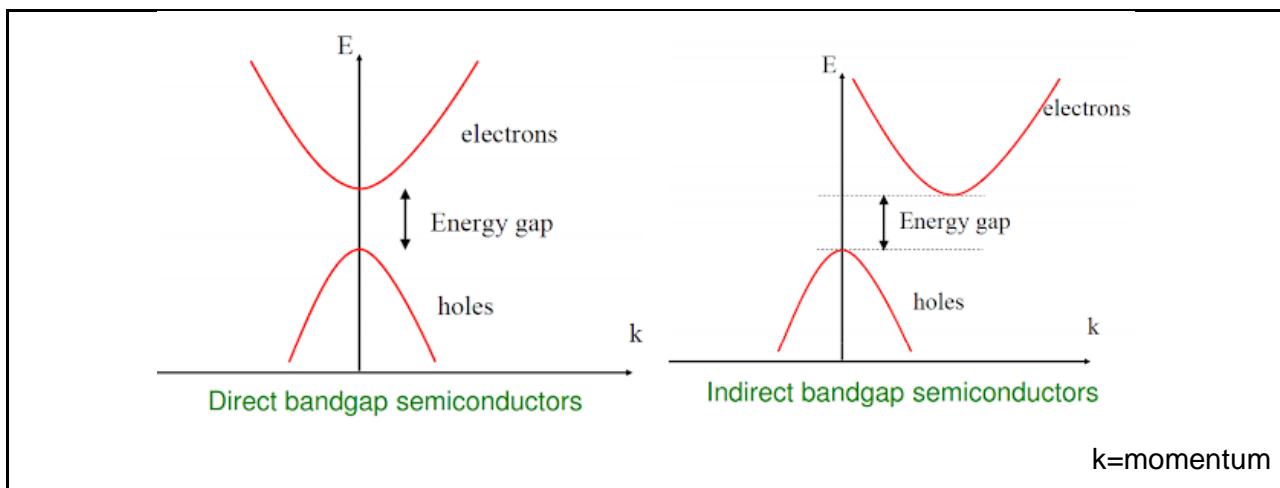
- 8) Electrons from the valence band can be excited to the conduction band by either thermal excitation or by optical absorption. When the electron returns to the valence band the energy is released either as heat or as photons.
- 9) An electron can directly emit a photon. In an "indirect" gap, a photon cannot be emitted because the electron must pass through an intermediate state and

transfer momentum to the crystal lattice.

- 10) An example of direct band gap material includes some III-V materials such as InAs, GaAs are also known as compound semiconductor. Indirect band gap materials include Si, Ge are also known as elemental semiconductor.

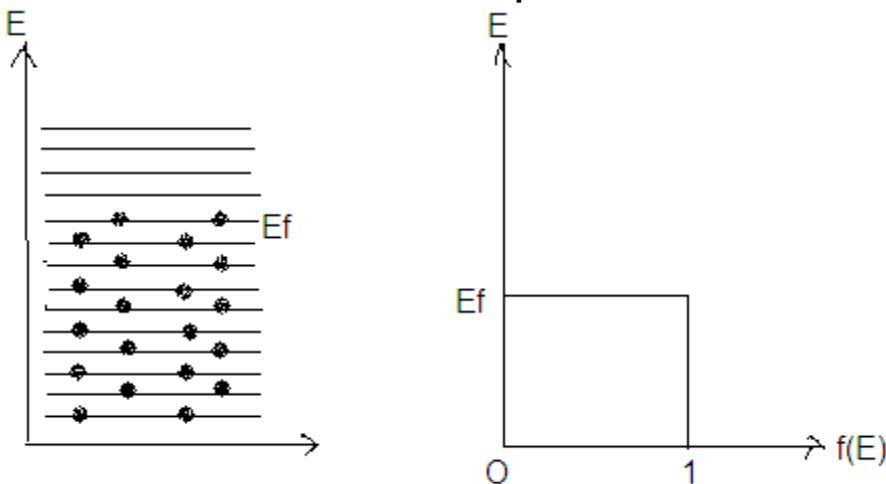
Differentiate between direct and indirect band gap semiconductors.

Direct band-gap (DBG) semiconductor	Indirect band-gap (IBG) semiconductor
<p>A direct band-gap (DBG) semiconductor is one in which the maximum energy level of the valence band aligns with the minimum energy level of the conduction band with respect to momentum.</p> <p>In a DBG semiconductor, a direct recombination takes place with the release of the energy in the form of photon of energy equal to the energy difference between the recombining particles.</p> <p>The probability of a radiative recombination is high.</p> <p>The efficiency factor of a DBG semiconductor is higher. Thus, DBG semiconductors are always preferred over IBG for making optical sources.</p> <p>Example, Gallium Arsenide (GaAs).</p>	<p>An Indirect band-gap (IBG) semiconductor is one in which the maximum energy level of the valence band and the minimum energy level of the conduction band are misaligned with respect to momentum.</p> <p>In case of a IBG semiconductor, due to a relative difference in the momentum, first, the momentum is conserved by release of energy and only after the both the momenta align themselves, a recombination occurs accompanied with the release of energy in the form of phonon (heat).</p> <p>The probability of a radiative recombination is comparatively low.</p> <p>The efficiency factor of a IBG semiconductor is lower.</p> <p>Example, Silicon and Germanium</p>



FERMI ENERGY LEVEL IN CONDUCTORS:-

- Q. What is Fermi level in metal? Write Fermi-Dirac distribution function and explain the term used in it?**
- Q. Explain the variation of Fermi level with temperature in metal?**



- 1) In conductor's valence and conduction bands are overlap on each other. Therefore continuous range of energy levels is available for the electrons.
- 2) The electrons are filling up in to the various energy levels such that first lowest energy level is filled by the pair of electrons and then second lowest energy level and so on till all the electrons occupy the particular energy level. The upper most energy level occupied by the electrons is known as Fermi level and its energy is known as Fermi energy. Thus Fermi level is allowed energy level in conductors.
- 3) The distribution of electrons of energy E in various energy levels is given by Fermi-Dirac distribution function,

$$F(E) = \frac{1}{1+e^{(E-E_f)/kT}}$$

Where, E_f – Fermi energy

K – Boltzmann constant.

T – temp. in $^{\circ}\text{K}$

Fermi function $F(E)$ gives the probability that an electrons of energy E occupying particular energy level.

4) Variation of Fermi energy level with temp:-

- At $T = 0^{\circ}\text{K}$,

When $E < E_f$, $F(E) = 1$ means all the levels below E_f are filled with electrons.

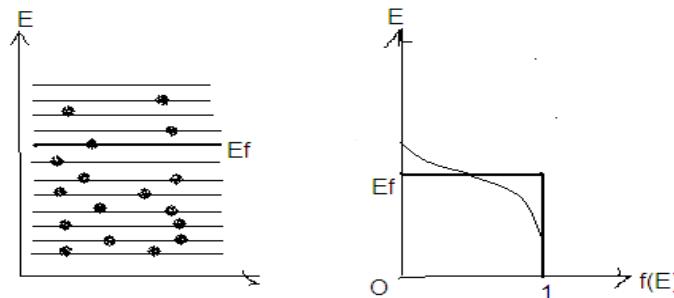
When $E > E_f$, $F(E) = 0$ means all the levels above E_f are vacant.

- At $T^{\circ}\text{K}$ temp.

Some electrons near to E_f are excited to energy levels above E_f .

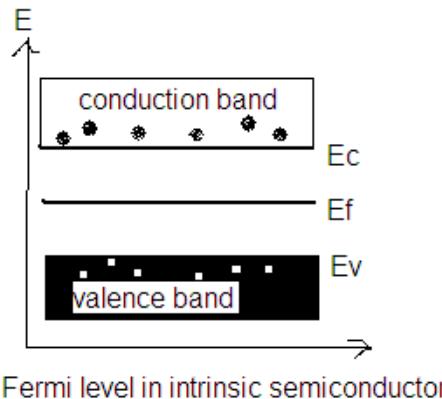
When $E < E_f$, function $F(E)$ slightly less than 1, i.e probability of electrons below E_f decreases.

When $E > E_f$, function $F(E)$ slightly greater than 1, i.e probability of electrons above E_f increases.



Q. Show that in intrinsic semiconductor, Fermi level always lies at the middle between the valence and conduction band?

FERMI LEVEL IN INTRINSIC SEMICONDUCTOR:-



- 1) In semiconductors at temp. $T = 0^{\circ}\text{K}$, the valance band is completely filled by the electrons and conduction band is empty. At temp. $T^{\circ}\text{K}$, some electrons from the valence band can jump in to the conduction band, leaving behind equal number of holes in the valence band. Thus in semiconductors the charged carriers are electrons and holes.
- 2) At temp. $T^{\circ}\text{K}$, the density of free electrons in the conduction band and density of holes in the valence band is equal. Therefore Fermi level always at the middle between the valence band and conduction band.
- 3) Thus in semiconductor Fermi level is not a allowed energy level for electrons in the semiconductor, but it is reference level between the valence band and conduction band.

If E_v – energy of upper level of valence band.

E_c – energy of bottom level of conduction band.

E_f – Fermi energy level.

E_g – energy gap between valence band and conduction band.

$$\text{Therefore, } E_f = \frac{E_c + E_v}{2} = \frac{E_g}{2}$$

Consider at temp. $T^{\circ}\text{K}$, n_c – density of free electrons in conduction band.

n_v – density of holes in the valence band.

N_c – density of states in the conduction band.

N_v – density of states in the valence band.

$$n_c = N_c e^{-(E_c - E_f) / kT}$$

$$n_v = N_v e^{-(E_f - E_v) / KT}$$

$$\text{but} \quad n_c = n_v$$

$$N_c e^{-(E_c - E_f) / KT} = N_v e^{-(E_f - E_v) / KT}$$

$$\frac{N_v}{N_c} e^{-(E_c - E_f) / KT} = \frac{N_v}{N_c} e^{-(E_f - E_v) / KT}$$

But $N_v = N_c$, therefore $N_v / N_c = 1$

$$e^{-(E_c - E_f) / KT} = e^{-(E_f - E_v) / KT}$$

Take logarithm on both sides,

$$\frac{- (E_c - E_f)}{KT} = \frac{- (E_f - E_v)}{KT}$$

$$-E_c + E_f + E_f - E_v = 0$$

$$\text{Therefore, } Ef = \frac{Ec + Ev}{2} = \frac{Eg}{2}$$

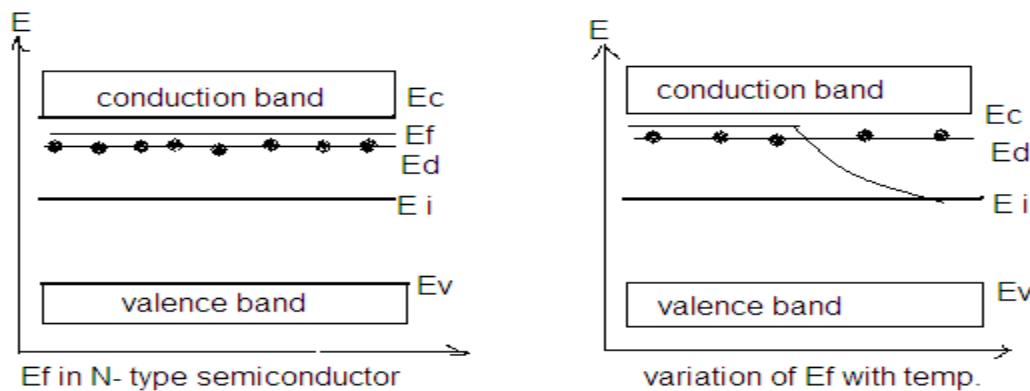
FERMI LEVEL IN EXTRINSIC SEMICONDUCTOR:-

Q. Explain the variation of Fermi level with impurity temperature & impurity concentration in N-type semiconductor?

In extrinsic semiconductor, density of free charge carriers (electrons or holes) in the conduction band and valence band increases with temperature and impurity concentration, hence Fermi level depends upon the temp. and impurity concentration.

1) **Variation of Ef with temperature:-**

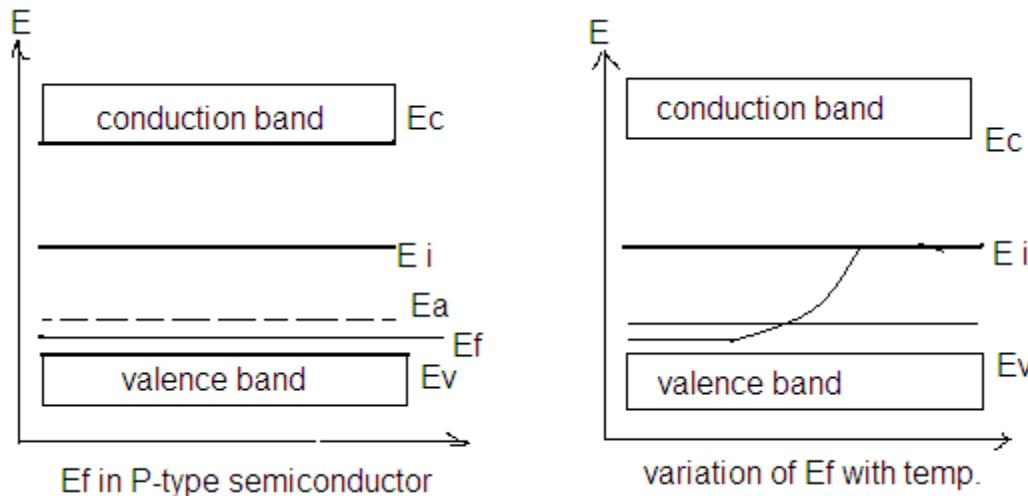
In extrinsic semiconductor the Fermi level lies in upper half or lower half of the energy gap depending upon the concentration of majority charge carriers.



In N – type semiconductor, the density of free electrons in the conduction band is greater than density of holes in the valence band. The Fermi level lies at the middle between the bottom level of conduction band Ec and donor level of impurity atoms Ed.

$$\text{Therefore, } Ef = \frac{Ec + Ed}{2}$$

As the temp. Increases, the donor level are depleted by the electrons from the valence band and density of holes in the valence band increases. Therefore Fermi level Ef go on decreasing. At particular temp. the density of free electrons in the conduction band becomes equal to the density of holes in the valence band and Fermi level reaches to intrinsic level Ei. The material loses its extrinsic nature and become intrinsic semiconductor. The Fermi level Ef becomes independent of temp.



In P – type semiconductor, the Fermi level lies at the middle between the top level of valence band E_v and acceptor level of impurity atoms E_A .

$$\text{Therefore, } E_f = \frac{E_c + E_A}{2}$$

As the temp. Increases, the Fermi level go on increasing. At particular temp., the Fermi level E_f reaches to intrinsic level E_i and material looses its extrinsic nature and become intrinsic semiconductor.

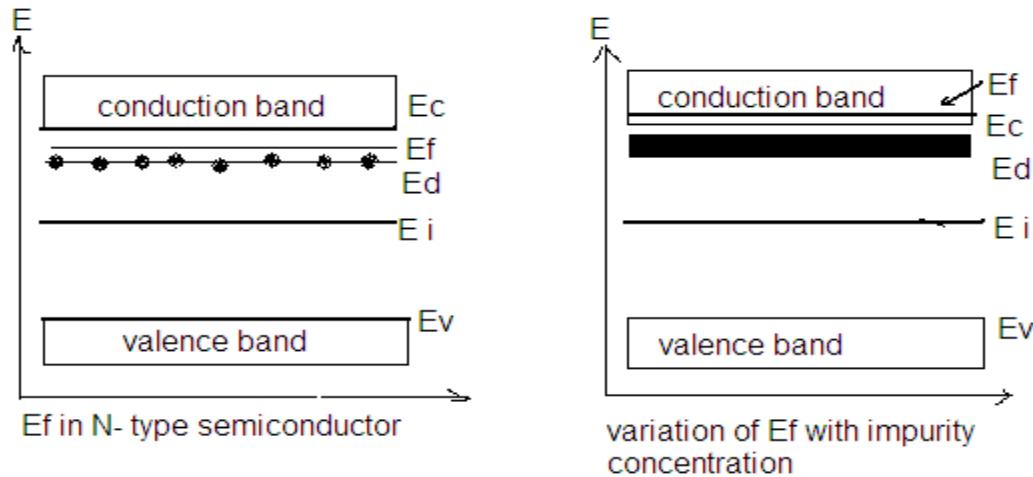
2) Variation of Ef with impurity concentration:-

In N – type semiconductor at low impurity concentration, the donor atoms are isolated from each other and therefore single donor energy level E_d is formed below the conduction band. The Fermi level lies at the middle between the bottom level of conduction band E_c and donor level of impurity atoms E_d .

$$\text{Therefore, } E_f = \frac{E_c + E_d}{2}$$

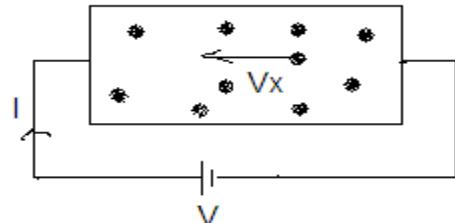
As the impurity concentration increases, the number of donor atoms in the donor level increases. These atoms are interacting with each other, therefore donor level splits in to no. of discrete energy levels and donor energy band is formed. The width of donor energy band increases with increase in impurity concentration. The Fermi level E_f shift

close to conduction band and at higher concentration E_f may enter in to the conduction band.



- Q. Write short notes on drift and diffusion current?**
Q. What is mobility of charge carriers? Give its SI unit?

DRAFT CURRENT:-



In metals large numbers of free electrons are present. There is force of repulsion between these electrons. These free electrons are colliding with each other and with the atoms of the material and rebound in various directions. The motion of electrons is equivalent to Brownian motion.

When external field E is applied, the electrons are accelerated in the direction of applied field and acquire certain velocity known as drift velocity. The applied electric field does not stop the collision of electrons with the atoms and rebounce in various directions, but the electrons are drifted in the direction of applied field and constitute an electric current. This current is known as drift current. The drift velocity per unit electric field is known as mobility of charge carriers. The SI unit of mobility is $\text{m}^2/\text{volt}\cdot\text{Second}$. The mobility of electrons is greater than holes.

If n – density of free electrons

e - electron charge.

V_e – drift velocity of electrons.

A – area of cross section

Therefore drift current, $I = n e V_e A$

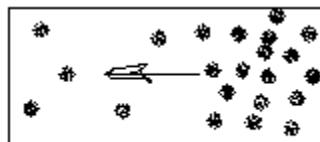
If E is applied electric field, the mobility of electrons $\mu_e = V_e / E$

$$V_e = \mu_e E$$

$$I = n e \mu_e E A$$

$$\text{Drift current density, } J_{n(\text{drift})} = I / A = n e \mu_e E$$

DIFFUSION CURRENT:-



In semiconductors, the charge carriers are electrons and holes. The concentration of charge carriers may not be same throughout the material. The change in concentration of charge carriers per unit length is called concentration gradient (dn / dx). The charge carriers which are concentrated in some region having same charge and therefore force of repulsion between them. There is tendency of charge carriers to diffuse from region of higher concentration to the region of lower concentration till all the charge carriers are distributed uniformly throughout the material. This motion of charge carriers constitute an electric current known as diffusion current.

The diffusion current density is directly proportional to the magnitude of charge and concentration gradient.

$$J_{n(\text{diffusion})} \propto (-e) (-dn / dx)$$

$$J_{n(\text{diffusion})} = D_n e dn / dx$$

Where D_n – diffusion constant for electrons.

CURRENT CONDUCTION IN SEMICONDUCTOR:-

Consider a piece of intrinsic semiconductor of length l and area of cross section A . If V is the P.D. applied across the semiconductor, then current flows through the semiconductor is due to motion of electrons and holes in opposite direction.

The current flows though the external circuit is due to electrons only. Therefore total current flowing through the semiconductor is

$$I = I_e + I_h \quad \dots \quad (1)$$

If n – density of free electrons

e - electron charge.

V_e – drift velocity of electrons.

A – area of cross section

Therefore drift current, $I = n e V_e A$

If E is applied electric field, **the mobility of electrons** $\mu_e = V_e / E$

$$V_e = \mu_e E$$

$$I_e = n e \mu_e E A$$

Similarly, $I_h = p e \mu_h E A$

Therefore, eqn.(1) $\Rightarrow I = n e \mu_e E A + p e \mu_h E A$

But in intrinsic semiconductor $n = p = n_i$ intrinsic carrier density

$$I = n_i e (\mu_e + \mu_h) E A$$

Current density $J = I / A = n_i e (\mu_e + \mu_h) E$

$$\text{But } E = V / l$$

$$I = n_i e (\mu_e + \mu_h) (V / l) A$$

$$V / l = (1 / n_i e (\mu_e + \mu_h)) I / A$$

$$R = \rho (l / A)$$

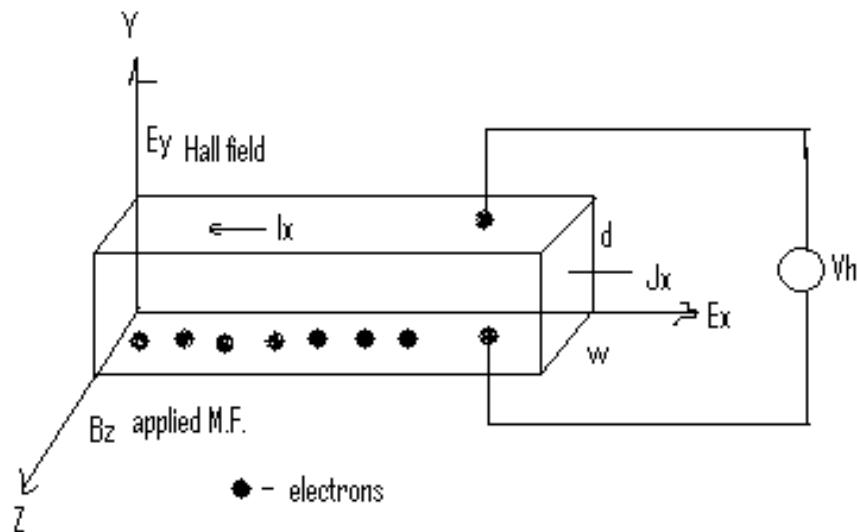
$\rho = 1 / n_i e (\mu_e + \mu_h)$ resistivity of semiconductor.

Reciprocal of resistivity is conductivity, **$\sigma = 1 / \rho = n_i e (\mu_e + \mu_h)$**

HALL EFFECT:-

Q. What is Hall Effect? Give its applications?

Q. Explain the Hall effect? How it is used to measure the concentration (density) & mobility of the charge carriers?



"If a conductors (metal or semiconductor) carrying an electric current is kept in magnetic field, then an electric field is produced in the conductor which is perpendicular to both the direction of electric current and applied magnetic field".

Consider a piece of metal carrying an electric current kept in a magnetic field.

If t – thickness and w – width of metal piece.

E_x – applied electric field along X-axis.

J_x – current density along + ve X direction.

I_x - current flowing along X- axis.

B_z – applied magnetic field along Z – axis.

V_x – drift velocity of electrons along – ve X- axis.

The force acting on the electrons due to magnetic field B_z is

$$F = B_z e V_x$$

This force is known as Lorentz force. The direction of this force is along – ve Y- axis.

Due to this force, the free electrons in the metal are collected on the bottom surface of the metal and equal deficiency of electrons will produce on the top surface of the metal piece. Thus an electric field E_y is produced along Y-axis. This field is known as Hall field.

The force acting on the electrons due to Hall field E_y is`

$$F = e E_y$$

The direction of this force is along + ve Y- axis.

At equilibrium, force due to E.F. = force due to M.F.

$$e E_y = B_z e V_x$$

$$E_y = V_x B_z \quad \dots \quad (1)$$

The current density along X- axis is $J_x = - n e V_x$

$$V_x = - J_x / n e$$

Keep this value in equation (1)

$$E_y = (-J_x / n e) B_z$$

But $R_H = - 1/ne$ known as Hall coefficient. The measurement of Hall coefficient gives the sign of the charge carriers and density of charge carriers in the material.

$$E_y = R_H J_x B_z \quad \dots \quad (2)$$

If V_h is Hall voltage across the thickness t , then $E_y = V_h / t$.

$$\text{If } w \text{ is width of sample } J_x = I_x / A = I_x / w \times t$$

$$\text{Equation (2)} \Rightarrow V_h / t = R_H (I_x / w \times t) B_z$$

$$V_h = R_H (I_x / w) B_z$$

The drift velocity per unit electric field is known as mobility of charge carriers.

$$\text{Therefore } \mu_e = V_x / E_x$$

$$V_x = \mu_e E_x$$

Keep this value in equation (1)

$$E_y = \mu_e E_x B_z \quad \dots \quad (3)$$

Equating equations (2) and (3)

$$R_H J_x B_z = \mu_e E_x B_z$$

$$\mu_e = R_H J_x / E_x$$

$$J_x / E_x = \sigma \text{ conductivity of material.}$$

$$\boxed{\mu_e = R_H \sigma}$$

This equation is used to determine the mobility of charge carriers.

From equation no. (3), $E_y / E_x = \mu_e B_z$

But $E_y / E_x = \Phi$ is known as Hall angle

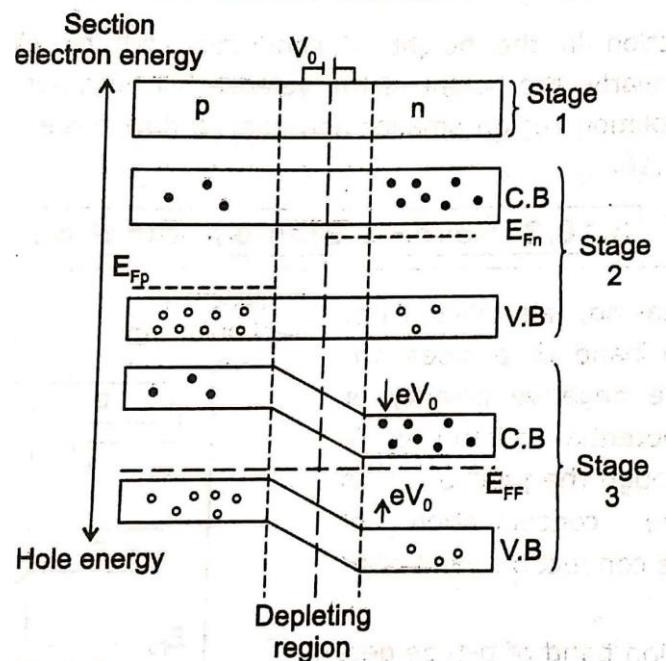
$$\Phi = \mu_e B_z$$

Applications of Hall Effect:-

- 1) To determine the sign of the charge carriers.
- 2) To determine the electronic structure of the material, whether the material is conductor, insulator or semiconductor.
- 3) To determine the concentration of the charge carriers.
- 4) To determine the mobility of the charge carriers.

Fermi level diagram of PN junction:

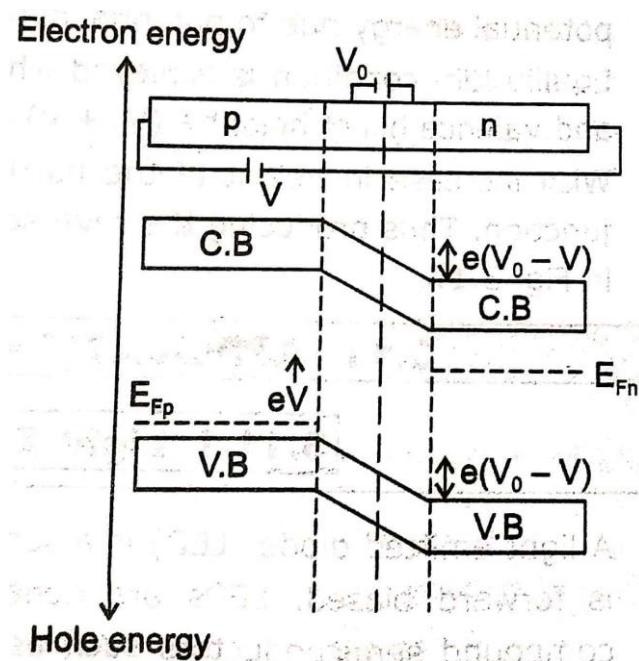
Fermi level in unbiased PN junction:



- In P-type semiconductor, there is excess of holes in valence band, therefore Fermi level E_{FP} is near to the valence band. In N-type semiconductor, there is excess of electrons in conduction band, therefore Fermi level E_{Fn} is near to the conduction band.
- When PN junction is formed, the electrons from the conduction band of N-type diffuse through the junction to P-side and holes from the valence band of P-type diffuse to N-type.
- The concentration of electron increases in the conduction band of P-type and Fermi level E_{FP} starts moving upward along with entire P-type energy band.

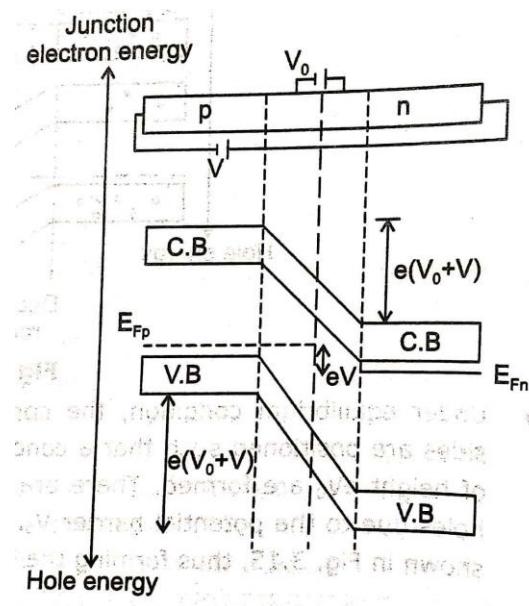
- The concentration of holes increases in the valence band of N-type and Fermi level E_{Fn} starts moving down ward along with entire N-type energy band.
- At equilibrium, both Fermi level E_{Fp} and E_{Fn} align with each other.
- A barrier potential V_0 is created at the junction and conduction hill of height $-eV_0$ and valence hill of height eV_0 are formed.

Fermi level in forward biased PN junction:



- In forward biasing, the electron density in the conduction band of the n-side increases. As a result the fermi level E_{Fn} moves upwards. Similarly due to the increase in the hole density in the valence band of the p side, the fermi level E_{Fp} moves downwards.
- The fermi levels E_{Fn} and E_{Fp} are displaced relatively by an amount eV equal to the potential energy due to the applied voltage V , which cause the displacement. The height of the conduction hill reduces by the same amount eV and becomes $e(V_0-V)$. Similarly the height of the valence hill becomes $-e(V_0-V)$.
- Reduction in height of barrier region makes the charge flow through the junction easier.

Fermi level in reverse biased PN junction:

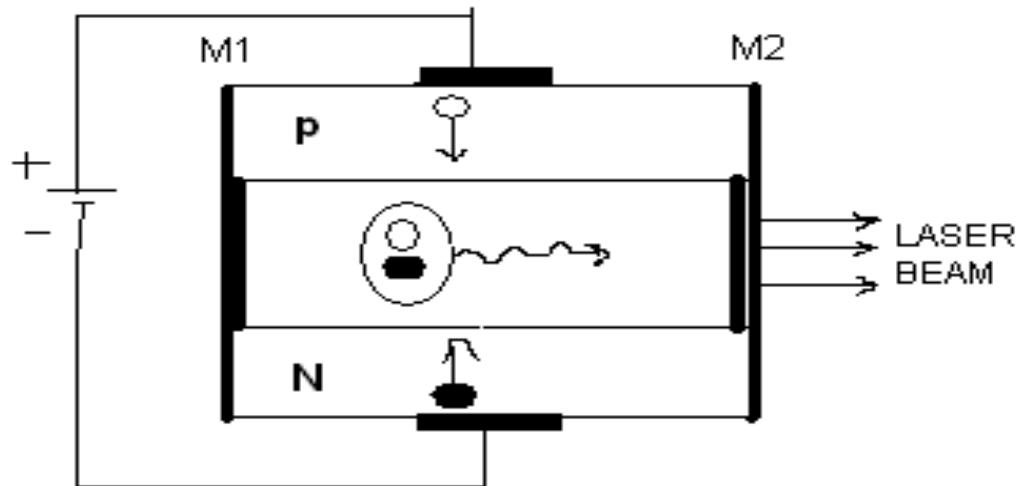


- In reverse biasing, the electron density in the conduction band of the P-side increases. As a result the fermi level E_{Fp} moves upwards. Similarly due to the increase in the hole density in the valence band of the N side, the fermi level E_{Fn} moves downwards.
- The fermi levels E_{Fn} and E_{Fp} are displaced relatively by an amount eV equal to the potential energy due to the applied voltage V , which cause the displacement.
- The height of the conduction hill increases by the same amount eV and becomes $-e(V_o+V)$. Similarly the height of the valence hill becomes $e(V_o+V)$.
- Increases in height of the barrier region, reduces the flow of current through the junction.

Light emitting diode (LED):-

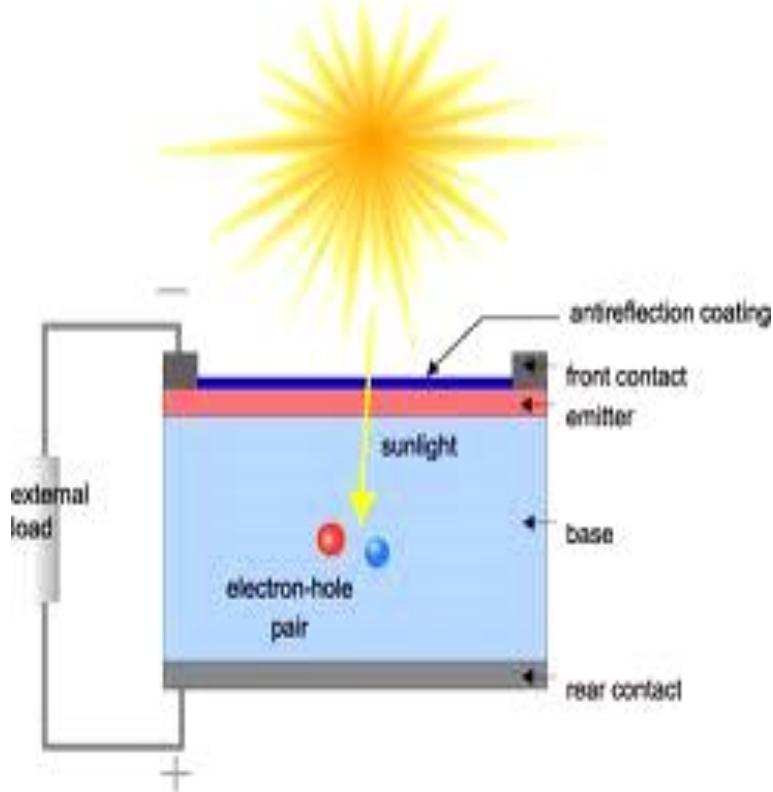
Q. Explain the use of p-n junction as Light emitting diode (LED)?

LED is p-n junction made from the direct band gap semiconductor material like GaAs, or GaASP or CdS as a base material with trivalent and pentavalent impurity material to form P-type and N-type region. P-N junction is connected in forward bias condition. The injected charge carriers from the source and the minority charge carriers are combining together to release the energy in the form of photon of energy equal to band gap energy in the semiconductor material.



Solar cell:-

Q. Explain the use of p-n junction as a solar cell?

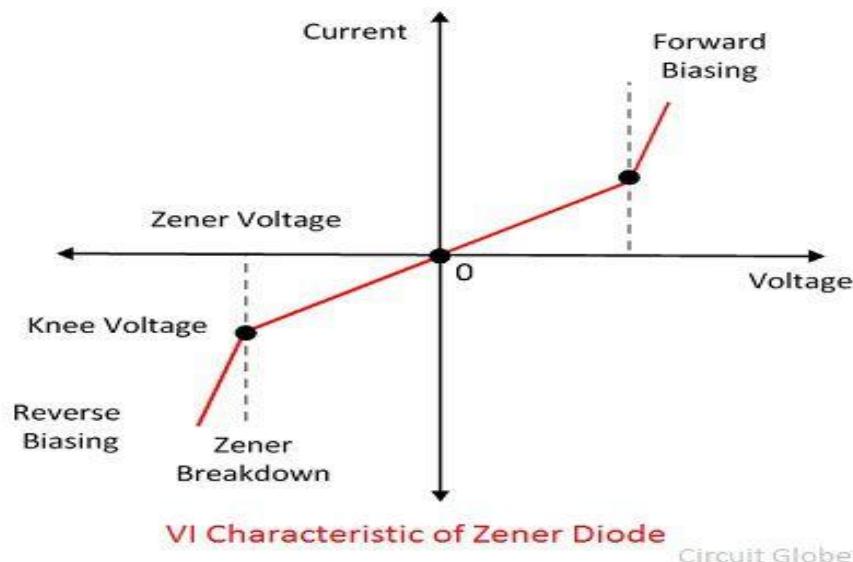
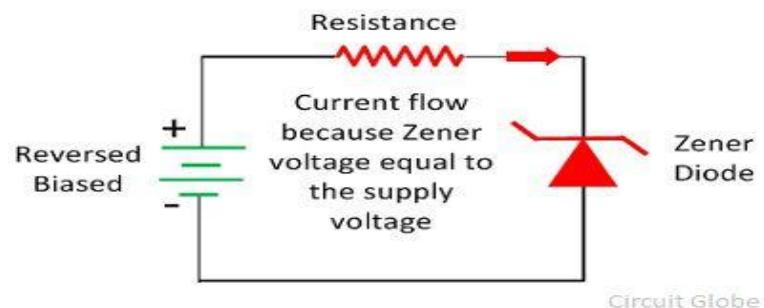
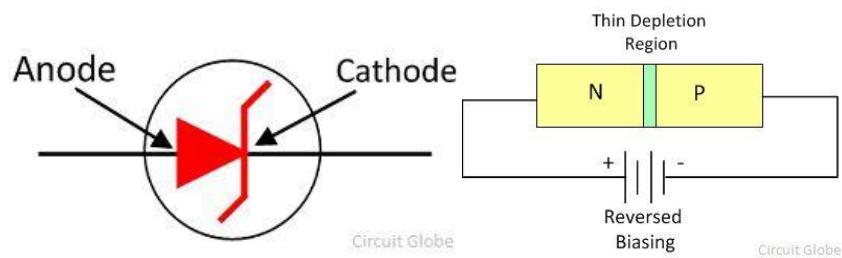


Solar cell is a device which converts light energy into electrical energy. Solar cell is a p-n junction having flat P-type and N-type region. P-type region is made thin and coated with antireflection film to avoid the reflection. When light is incident on the flat surface of p-type region, electron hole pairs are created in the p and n type region. These electron-hole pairs are separated by the barrier potential at the junction and P.D is appearing at the junction. This P.D.

is 0.5 V for silicon and 0.1V for germanium. Therefore current is flowing through the output load resistance. The number of solar cells is connected in series to increase the output voltage.

Zener Diode:

Definition: A heavily doped semiconductor diode which is designed to operate in reverse direction is known as the Zener diode.



Zener Diode Circuit Diagram

The circuit diagram of the Zener diode is shown in the figure below. The Zener diode is employed in reverse biasing. The reverse biasing means the n-type material of the diode is connected to the positive terminal of the supply and the P-type material is connected to the negative terminal of the supply. The depletion region of the diode is very thin because it is made of the heavily doped semiconductor material.

Working of Zener Diode

The Zener diode is made up of heavily doped semiconductor material. The heavily doped means the high-level impurities is added to the material for making it more conductive. The depletion region of the Zener diode is very thin because of the impurities. The heavily doping material increases the intensity of the electric field across the depletion region of the Zener diode even for the small reverse voltage.

When no biasing is applied across the Zener diode, the electrons remain in the valence band of the p-type material and no current flow through the diode. The band in which the valence electrons (outermost orbit electron) place is known as the valence band electron. The electrons of the valence band easily move from one band to another when the external energy is applied across it.

When the reverse bias applies across the diode and the supply voltage is equal to the Zener voltage then it starts conducting in the reverse bias direction. The Zener voltage is the voltage at which the depletion region completely vanishes.

The reverse bias applies across the diode increases the intensity of electric field across the depletion region. Thus, it allows the electrons to move from the valence band of P-type material to the conduction band of N-type material. This transferring of valence band electrons to the conduction band reduces the barrier between the p and n-type material. When the depletion region become completely vanishes the diode starts conducting in the reverse biased.

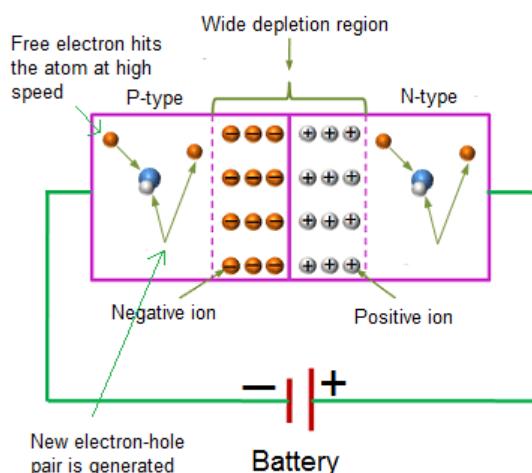
Characteristic of Zener Diode

The VI characteristic graph of the Zener diode is shown in the figure below. This curve shows that the Zener diode, when connected in forwarding bias, behaves like an ordinary diode. But when the reverse voltage applies across it and the reverse voltage rises beyond the predetermined rating, the Zener breakdown occurs in the diode. At Zener breakdown voltage the current starts flowing in the reverse direction.

Breakdown in zener diode

There are two types of reverse breakdown regions in a zener diode: avalanche breakdown and zener breakdown.

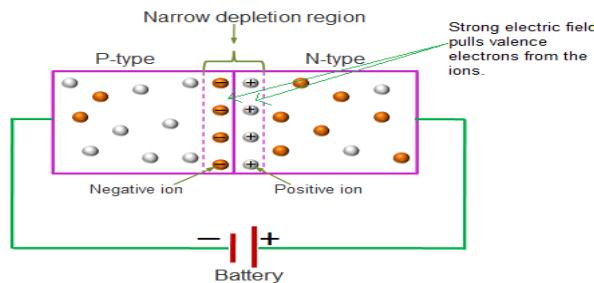
Avalanche breakdown



The avalanche breakdown occurs in both normal diodes and zener diodes at high reverse voltage. When high reverse voltage is applied to the p-n junction diode, the free electrons (minority carriers) gains large amount of energy and accelerated to greater velocities.

The free electrons moving at high speed will collides with the atoms and knock off more electrons. These electrons are again accelerated and collide with other atoms. Because of this continuous collision with the atoms, a large number of free electrons are generated. As a result, electric current in the diode increases rapidly. This sudden increase in electric current may permanently destroys the normal diode. However, avalanche diodes may not be destroyed because they are carefully designed to operate in avalanche breakdown region. Avalanche breakdown occurs in zener diodes with zener voltage (V_z) greater than 6V.

Zener breakdown



The zener breakdown occurs in heavily doped p-n junction diodes because of their narrow depletion region. When reverse biased voltage applied to the diode is increased, the narrow depletion region generates strong electric field.

When reverse biased voltage applied to the diode reaches close to zener voltage, the electric field in the depletion region is strong enough to pull electrons from their valence band. The valence electrons which gains sufficient energy from the strong electric field of depletion region will breaks bonding with the parent atom. The valance electrons which break bonding with parent atom will become free electrons. These free electrons carry electric current from one place to another place. At zener breakdown region, a small increase in voltage will rapidly increases the electric current.

- Zener breakdown occurs at low reverse voltage whereas avalanche breakdown occurs at high reverse voltage.
- Zener breakdown occurs in zener diodes because they have very thin depletion region.
- Breakdown region is the normal operating region for a zener diode.
- Zener breakdown occurs in zener diodes with zener voltage (V_z) less than 6V.

Advantages of zener diode

Power dissipation capacity is very high

High accuracy

Small size

Low cost

Applications of zener diode

It is normally used as voltage reference

Zener diodes are used in voltage stabilizers or shunt regulators.

Zener diodes are used in switching operations

Zener diodes are used in clipping and clamping circuits.

Zener diodes are used in various protection circuits.

Problem 1 : The resistivity of Cu is 1.72×10^{-8} ohm-m. Calculate the mobility of electrons in Cu. Given that the number of electrons per unit volume is $10.41 \times 10^{28}/\text{m}^3$.

Solution : Given : Resistivity of Cu (σ) = 1.72×10^{-8} ohm-m

Number of electrons per unit volume (n) = $10.41 \times 10^{28}/\text{m}^3$

Formula :

$$\sigma = \frac{1}{\rho} = \frac{1}{1.72 \times 10^{-8}}$$

$\sigma = 58.13 \times 10^6$ mho per meter

$\sigma = ne\mu_e$

$$\mu_e = \frac{\sigma}{ne} = \frac{58.13 \times 10^6}{10.41 \times 10^{28} \times 1.6 \times 10^{-19}}$$

Ans.

$$\mu_e = 3.49 \times 10^{-3} \text{ m}^2/\text{volt-sec}$$

Problem 2 : The mobility of holes is $0.025 \text{ m}^2/\text{volts sec}$. What would be the resistivity of a p-type silicon, if the Hall co-efficient of the sample is $2.25 \times 10^{-5} \text{ m}^3/\text{c}$?

Solution : Given : Mobility of holes (μ_h) = $0.025 \text{ m}^2/\text{volts sec}$.

Hall co-efficient (R_H) = $2.25 \times 10^{-5} \text{ m}^3/\text{c}$.

Formula : Conductivity of p-type semiconductor

$$\sigma_p = \sigma_e \mu_n \quad R_H = \frac{1}{pe}$$

$$\sigma_p = \frac{\mu_h}{R_H} = \frac{0.025}{2.25 \times 10^{-5}}$$

$\sigma_p = 1111.11$ mho per meter

$$\text{Resistivity } (\rho) = \frac{1}{\sigma} = \frac{1}{1111.11} = 0.0009009$$

Ans.

$$\rho = 9.00 \times 10^{-4} \text{ ohm meter}$$

Problem 5 : A sample of n-type silicon has a doner density of $10^{20}/\text{m}^3$. It is used in the Hall effect experiment. If the sample of width 4.5 mm is kept in a magnetic field of (0.55T) with current density of 500 A/m^2 , find : (1) Hall voltage developed in it. (2) Hall coefficient. (3) Hall angle of mobility of electrons is $0.17 \text{ m}^2/\text{V sec}$.

Solution : Given :

$$\text{Doner density } N_D = 10^{20}/\text{m}^3$$

$$\text{Width (W)} = 4.5 \text{ mm} \times 4.5 \times 10^{-3} \text{ m}$$

$$\text{Magnetic field (B)} = 0.55 \text{ T}$$

$$\text{Current density (J)} = 500 \text{ A/m}^2$$

$$\text{Mobility of electron } (\mu_e) = 0.17 \text{ m}^2/\text{V sec}$$

Formula : (1) Hall voltage

$$V_H = \frac{JBW}{ne}$$

$$\text{Put } n = ND$$

$$V_H = \frac{500 \times 0.55 \times 4.5 \times 10^{-3}}{10^{20} \times 1.6 \times 10^{-19}}$$

Ans.

$$V_H = 0.0773 \text{ volts}$$

(2) Hall coefficient (R_H)

$$R_H = \frac{1}{ne} = \frac{1}{NDe}$$

$$= \frac{1}{10^{20} \times 1.6 \times 10^{-19}}$$

Ans.

$$R_H = 0.0625 \text{ m}^3/\text{c}$$

(3) Hall angle

$$\tan \theta = \mu_e B$$

$$= 0.17 \times 0.55$$

$$\tan \theta = 0.0935$$

Ans.

$$\theta = 5.31^\circ$$

Problem 6 : The Hall coefficient of a specimen of doped silicon is found to be $3.66 \times 10^{-4} \text{ m}^3/\text{c}$. The resistivity of the specimen is $8.93 \times 10^{-3} \Omega \cdot \text{m}$. Find the mobility and density of the charge carrier.

Solution : Given :

$$\text{Hall coefficient } (R_H) = 3.66 \times 10^{-4} \text{ m}^3/\text{c}$$

$$\rho = 8.93 \times 10^{-3} \Omega \cdot \text{m}$$

$\mu = ?$ $n = ?$ $\rho = 8.93 \times 10^{-3} \Omega \cdot \text{m}$ $R_H = 3.66 \times 10^{-4} \text{ m}^3/\text{c}$

Formula : (1) Hall coefficient (R_H)

$$R_H = \frac{1}{ne}$$

$$n = \frac{1}{R_H e} = \frac{1}{3.66 \times 10^{-4} \times 1.6 \times 10^{-19}}$$

Ans. $n = 1.707 \times 10^{22}/m^3$

(2) Mobility of the charge carrier

$$\sigma = ne\mu$$

$$\mu = \frac{\sigma}{ne} \quad (R_H = \frac{1}{ne} \text{ and } \sigma = \frac{1}{q})$$

$$\mu = \sigma R_H$$

$$\mu = \frac{R_H}{q} = \frac{3.66 \times 10^{-4}}{8.93 \times 10^{-3}}$$

Ans. $\mu = 0.0409 \text{ m}^2/\text{volt sec}$

Problem 7 : A semiconductor crystal 12 mm long, 1 mm wide and 1 mm thick has magnetic flux density of 0.5 wb/m² applied from front to back, perpendicular to largest faces. When a current of 20 mA flows length wise through the specimen, voltage measured across its width is found to be 37 μ volts. What is the coefficient of semiconductor?

Solution : Given : Semiconducting crystal

$$\text{Length (L)} = 12 \text{ mm} = 12 \times 10^{-3} \text{ m}$$

$$\text{Thickness (t)} = 1 \text{ mm} = 1 \times 10^{-3} \text{ m}$$

$$\text{Breadth (b)} = 1 \text{ mm} = 1 \times 10^{-3} \text{ m}$$

$$\text{Current (I)} = 20 \text{ mA} = 20 \times 10^{-3} \text{ A}$$

$$\text{Hall voltage (V}_H\text{)} = 37 \mu\text{V} = 37 \times 10^{-6} \text{ V}$$

$$\text{Magnetic flux density (B)} = 0.5 \text{ Wb/m}^2$$

Formula : Hall voltage (V_H)

$$V_H = \frac{BWI}{ne} = \frac{BWI}{neA}$$

Let t be thickness, w be the width of the sample, then $A = wt$

$$V_H = \frac{BWI}{newt} = \frac{R_H BI}{t}$$

$$R_H = \frac{V_H \times t}{I \times B}$$

$$R_H = \frac{37 \times 10^{-6} \times 1 \times 10^{-3}}{20 \times 10^{-3} \times 0.5}$$

Ans. $R_H = 3.7 \times 10^{-6} \text{ m}^3/\text{c}$

Problem 8 : A bar of n type Ge of size $0.010 \text{ m} \times 0.001 \text{ m} \times 0.001 \text{ m}$ is mounted in a magnetic field of $2 \times 10^{-1} \text{ T}$. The electron density in the bar is $7 \times 10^{21}/\text{m}^3$. If one milli volt is applied across the long ends of the bar, determine the current through

the bar and the voltage between Hall electrodes placed across the short dimensions of the bar. Assume $\mu_n = 0.39 \text{ m}^2/\text{vsec}$

Solution : Given : $\mu_n = 0.39 \text{ m}^2/\text{vsec}$

$$B = 2 \times 10^{-1} \text{ T}, \quad n = 7 \times 10^{21}/\text{m}^3$$

Formula :

$$\varrho = \frac{1}{\mu_n e} = \frac{1}{0.39 \times 7 \times 10^{21} \times 1.6 \times 10^{-19}}$$

$$\varrho = 2.28 \times 10^{-3} \Omega/\text{m}$$

Now

$$R = \frac{\varrho l}{A}$$

$$= \frac{2.28 \times 10^{-3} \times 0.01}{(0.001 \times 0.001)}$$

$$R = 22.8 \Omega$$

(Here long side is used as length as instructed and other two sides are forming width and height which in turn gives cross sectional area.

$$\text{Using ohm's law } I = \frac{V}{R} = \frac{1 \times 10^{-3}}{22.8} = 4.385 \times 10^{-5} \text{ A}$$

$$I = 43.85 \mu\text{A}$$

Using formula for Hall voltage

$$V_H = R_H B J W \quad \text{As } R_H = \varrho \times \mu$$

and

$$J = \frac{I}{A} = \frac{I}{W \times t}$$

$$V_H = \varrho \times \mu \times B = \frac{I}{WL} \times t = \frac{\varrho \mu B I}{W}$$

$$= \frac{2.28 \times 10^{-3} \times 0.39 \times 0.2 \times 43.85 \times 10^{-6}}{0.001}$$

$$V_H = 7.798 \times 10^{-6} \mu\text{V}$$

Ans.

$$V_H = 7.798 \mu\text{V}$$

Problem 9 : In a semiconductor with Hall coefficient 145 cc/c having width 2 cm and thickness 0.2 cm with a magnetic field induction of 2T along the smaller dimension, a current of 150 mA is calculated the current density and Hall voltage.

Solution : Given : Hall coefficient, $R_H = 145 \text{ cc/c}$

$$\text{Width } w = 2 \text{ cm}$$

$$\text{Thickness } t = 0.2 \text{ cm}$$

Magnetic field $B = 2 \text{ T}$ along the smaller dimension

Current $I = 150 \text{ mA}$

$$\text{Current density } J = \frac{I}{A} = \frac{I}{w \times d} = \frac{150 \times 10^{-3}}{2 \times 10^{-2} \times 0.2 \times 10^{-2}}$$

$$= 3750 \text{ A/m}^2$$

OPTICS (Interference of light)

Prerequisites:

Q. What is interference?

Superposition of waves of light when two or more light waves are arriving at a point of the medium simultaneously is known as interference. Due to interference modification in the intensity of light will be takes place. The point at which light waves are interfering will appear bright (constructive interference) or dark (destructive interference) depends upon optical path difference between interfering light waves.

Q. What are the conditions for sustained or stationary interference fringes?

To obtain the sustained or stationary interference pattern light waves must be

- 1) Coherent:- The phase difference between the waves remain constant. If the light waves are not coherent, the interference pattern will change continuously.
- 2) Monochromatic: - The wavelengths of the light waves remain same so that interference pattern will be sharp and well defined. If the light waves of different wavelengths, then interference fringes of different wavelengths are overlap on each other.
- 3) Same intensity:- the light waves must have same intensity or amplitude to get good contrast of interference fringes.

There are two methods for obtaining light waves of same phase, wavelength and intensity

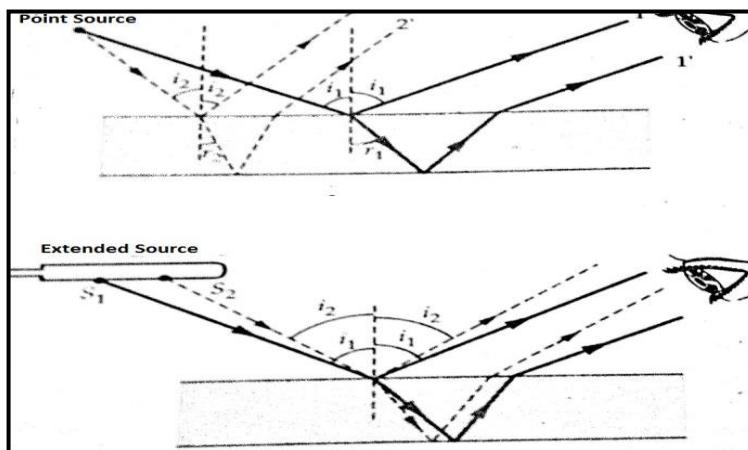
- 1) Division of wave front: The numbers of identical light rays are obtained by dividing the incident wave front of the light in to the number light rays using slits of constant width.
- 2) Division of amplitude: The incident light radiation is divided in to number of identical light rays of decreasing intensity using multiple reflections with in thin film of transparent material.

What is the need of extended source of light to observe interference pattern of thin film?

- 1) When the thin film of transparent material is illuminated by white light, then condition of

constructive interference is satisfied by different wavelengths at different angle of incidence and different colors are observed.

- 2) If the film is illuminated by point source of light, the incident light rays illuminate the film by different angle of incidence. The divergence of reflected light rays from the film is large and only small portion of the film i.e single colour will be visible at a particular angle as the field of view is small. The observer will have to change his position to observe another colour.
- 3) If the film is illuminated by extended i.e broad source of light, the incident light rays illuminate the film by different angle of incidence. The divergence of reflected light rays from the film is small and all portion of the film i.e all colours will be visible at a particular angle as the field of view is large.
- 4) If the film is illuminated by plane wave front of white light, the incident light rays are parallel and angle of incidence is same. Therefore film will reflects only one colour for which condition of constructive interference is satisfied at that particular angle of incidence.



Q. What is thin film?

A film of transparent material of thickness approximately equal to average wavelength of the visible light (i.e 5500 AU) is known as thin film.

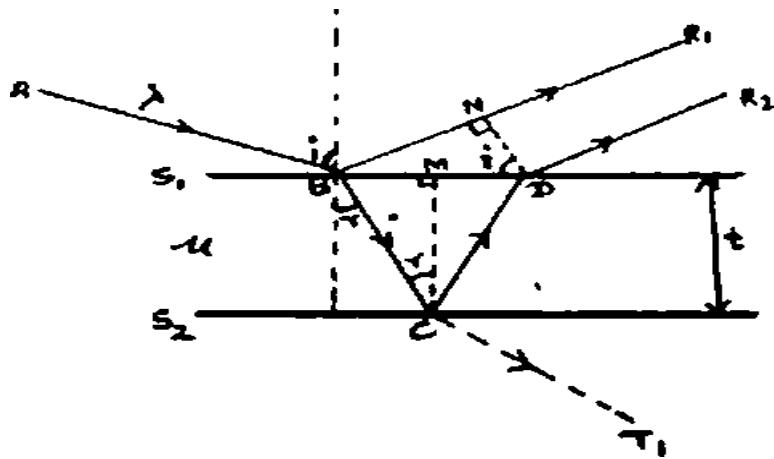
Q. What is Stokes law?

Whenever there is reflection of light from the optically denser medium, then phase change of π or path difference of $\lambda/2$ is produced between the reflected light rays.

Syllabus:

Q. Derive the conditions of interference with in thin film of transparent material of uniform thickness? Why the fringe system is complementary in nature?

Interference with in a thin film of transparent material of uniform thickness



- 1) Consider a thin film bounded between two surfaces S1 and S2.
 μ - R.I. of the film and
 t – Thickness of the film.
 The film is illuminated by the monochromatic source of light of wave length λ . Depending upon the conditions of interference, the entire film is appeared dark or bright.
- 2) The incident light beam AB make an angle i with normal to the film. Due to multiple reflections with in a thin film, we get reflected light rays R1, R2..... and transmitted light rays T1, T2.....
- 3) To determine optical path difference between reflected light rays R1 and R2 draw perpendicular CM and DN.
- 4) The optical path difference between reflected light rays R1 & R2 is

$$\Delta = \text{Path } (BC + CD) \text{ with in a film} - \text{Path } BN \text{ in air}$$

$$\Delta = (BC + CD) \mu - BN \text{ ----- (1)} \quad \text{Since } \mu = 1 \text{ for air.}$$

- 5) To determine BC + CD

From $\triangle BMC$ & $\triangle DMC$, $BC = CD$

Therefore $BC + CD = 2 BC$

From $\triangle BMC$, $\cos r = CM / BC$

But $CM = t$, Thickness of thin film

$$\cos r = t / BC$$

$$BC = t / \cos r$$

$$BC + CD = \frac{2t}{\cos r}$$

6) To determine BN

$$\text{In } \triangle BND, \sin i = BN / BD$$

$$BN = BD \sin i \text{ but by Snell's law } \sin i = \mu \sin r$$

$$BN = BD \mu \sin r$$

$$\text{But } BD = BM + MD$$

$$\text{From } \triangle BMC \text{ & } \triangle DMC, BM = MD$$

$$BD = 2 BM$$

$$\text{From } \triangle BMC, BM = CM \tan r = t \tan r$$

$$BD = 2t \tan r = \frac{2t \sin r}{\cos r}$$

$$\text{Therefore } BN = 2\mu t \frac{\sin^2 r}{\cos r}$$

7) Equation (1) =>

$$\Delta = \frac{2\mu t}{\cos r} - 2\mu t \frac{\sin^2 r}{\cos r}$$

$$\Delta = \frac{2\mu t}{\cos r} (1 - \sin^2 r), \text{ but } 1 - \sin^2 r = \cos^2 r$$

$$\Delta = 2\mu t \cos r$$

Since the reflection of the light takes place from the optically denser medium, there is phase difference of π or path difference of $\lambda/2$ takes place between the reflected light rays.

$$\text{Therefore } \Delta = 2\mu t \cos r - \lambda/2$$

- 8) For constructive interference (bright film) in reflected light

$$\Delta = n\lambda$$

$$2\mu t \cos r = (2n + 1) \lambda/2 \quad \dots \dots \dots (2)$$

For destructive interference (dark film) in reflected light

$$\Delta = (2n + 1) \lambda/2$$

$$2\mu t \cos r = n\lambda \quad \dots \dots \dots (3)$$

- 9) For transmitted light, there is no phase change or path difference change between incident and transmitted light rays, therefore

$$\Delta = 2\mu t \cos r$$

For constructive interference (bright film) in transmitted light

$$\Delta = n\lambda$$

$$2\mu t \cos r = n\lambda \quad \dots \dots \dots (4)$$

For destructive interference (dark film) in transmitted light

$$\Delta = (2n + 1) \lambda/2$$

$$2\mu t \cos r = (2n + 1) \lambda/2 \quad \dots \dots \dots (5)$$

- 10) From equation 2, 3, 4, 5, the interference pattern or fringe system is complementary in nature.**

Formation of colors with in a thin film:

Q. Explain the formation of colors when thin film of transparent material is illuminated by sunlight or white light?

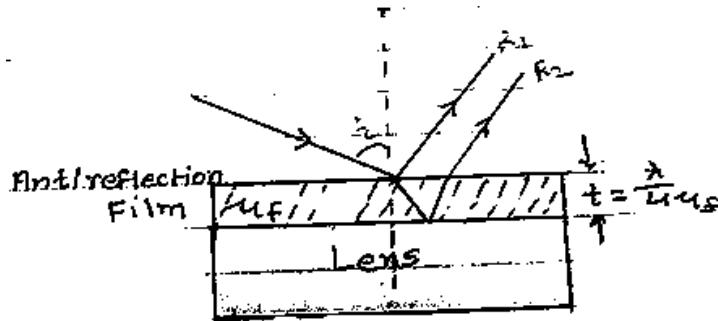
- 1) When thin film of transparent material (oil film, soap film, wings of butterfly) is illuminated by sunlight or white light different colors are observed on the film.
- 2) The variation in optical path difference will change the wavelength as well as intensity of reflected light rays and different color patches are observed on the film.
- 3) The O.P.D. between reflected light rays from the thin film is given by

$$\Delta = 2\mu t \cos r - \lambda/2$$

- 4) O.P.D , Δ varies with μ , t and r .
- 5) Refractive index of the film μ is a function of λ . The white light consists of colors of different wavelengths.
- 6) The angle of refraction varies with angle of incident i.e. angle of vision. Therefore variations of colors are observed on the thin film.

Non reflecting lens or Anti reflection film:

Q. Write short notes on antireflection film? Give its applications?



To avoid the reflection of light from the lens surface and to increase its transmittance, the surface of the lens in optical instrument like camera, telescope, binoculars, panel of solar cell etc. is coated with thin film of transparent material known as antireflection film. When light is incident on the glass surface, the intensity of reflected light is given by

$$I_r = \frac{\mu_g - \mu_a^2}{\mu_g + \mu_a} \cdot I_i$$

Since $\mu_g = 1.5$ and $\mu_a = 1$

$$I_r = 0.04 I_i$$

$$I_r = 4 \% I_i$$

Thus 4 % of incident light intensity is lost due to each reflection.

From figure, to avoid reflection, light rays R1 & R2 must interfere destructively.

There are two conditions

a) **Amplitude condition**: Amplitude of R1 = amplitude of R2

$$\left(\frac{\mu_f - \mu_a}{\mu_f + \mu_a} \right)^2 = \left(\frac{\mu_g - \mu_f}{\mu_g + \mu_f} \right)^2$$

On solving, we get $\mu_f = \sqrt{\mu_g}$

Thus R.I. of thin film material must be in between 1 & 1.5. The suitable material are MgF_2

(1.38) and Cryolite (1.36).

- b) **Phase condition**:- The phase or optical path difference between R1 & R2 must be $\lambda/2$.

$$\Delta = 2\mu f \cos r = \lambda/2$$

For normal incidence $\cos r = 1$

Therefore, required thickness of the antireflection film is $t = \frac{\lambda}{4\mu f}$

Highly reflecting film or lens:-

Q. Write short notes on highly reflection film? Give its applications?

To avoid the transmission of light from the lens surface and to increase its reflectivity, the surface of the lens in optical instrument like sun glasses, windows pane etc. is coated with thin film of transparent material known as highly reflecting film. Such types of the lenses or glasses are used in summer to avoid heat in summer.

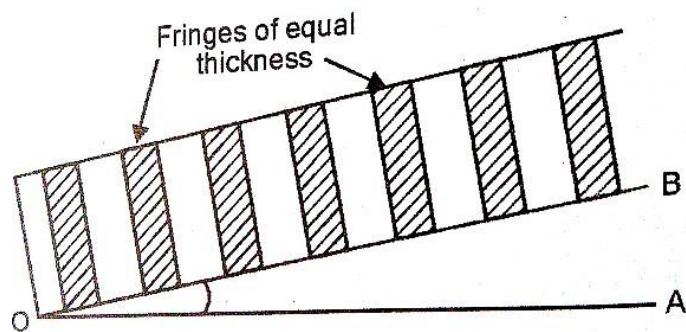
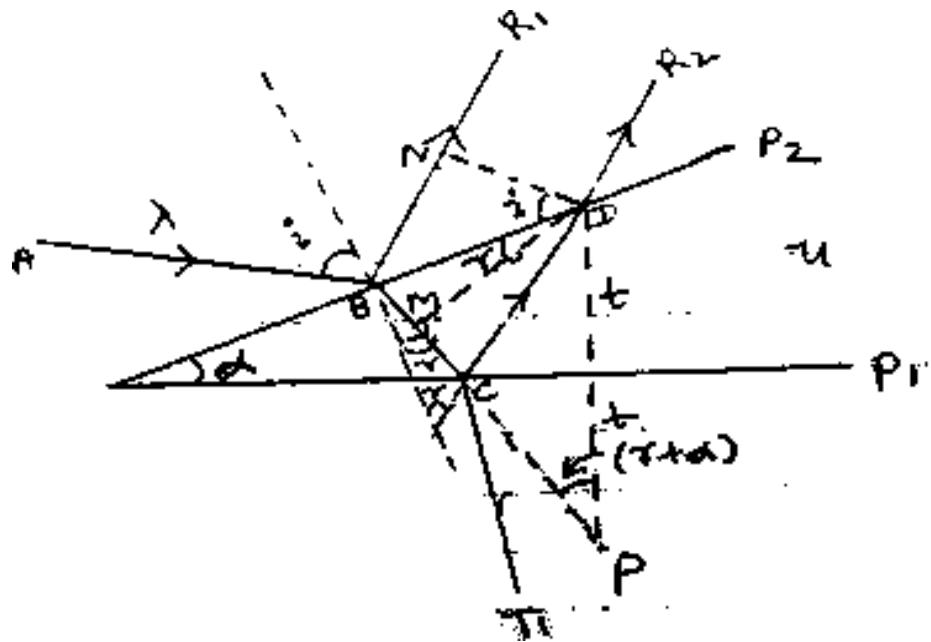
To increase the reflectivity of the lens surface, the reflected light rays R1 & R2 must interfere constructively. For constructive interference, the phase or optical path difference between R1 & R2 must be λ . $\Delta = 2\mu f \cos r = \lambda$

For normal incidence $\cos r = 1$

Therefore, required thickness of the highly reflecting film is $t = \frac{\lambda}{2\mu f}$

Interference within a thin film of wedge shape:

Q. Derive the conditions of interference within thin film of transparent material of wedge shape?



- 1) Consider two glass plates P_1 & P_2 inclined with small angle α . An air film of wedge shape is formed between the glass plates with angle of wedge α . The film is illuminated by the monochromatic source of light of wave length λ .

- 2) Due to interference with in a thin film of wedge shape film, alternate dark and bright bands of constant thickness parallel to surface of the upper glass plate are observed. These fringes of constant thickness are known as Fizeau fringes.
- 3) The incident light beam AB make an angle i with normal to the film. Due to multiple reflections with in a thin film of wedge shape, we get reflected light rays R1, R2..... and transmitted light rays T1, T2.....
- 4) To determine optical path difference between reflected light rays R1 and R2 draw perpendicular DM and DN.
- 5) The optical path difference between reflected light rays R1 & R2 is

$$\Delta = \text{Path (BC + CD) with in a film} - \text{Path BN in air}$$

$$\Delta = (\mathbf{BC + CD}) \mu - \mathbf{BN} \quad \text{----- (1)} \quad \text{Since } \mu = 1 \text{ for air.}$$

- 6) To determine BC+ CD

$$\text{But } BC = BM + MC$$

$$BC + CD = BM + MC + CD$$

$$\text{But } MC + CD = PM$$

$$\mathbf{BC + CD = BM + PM}$$

- 7) To determine BN

$$\text{From figure } \frac{\sin i}{\sin r} = \mu = \frac{BN/BD}{BM/BD} = \frac{BN}{BM}$$

Therefore, $\mathbf{BN} = \mu (\mathbf{BM})$

$$8) \text{ Therefore equation (1)} \Rightarrow \Delta = (\mathbf{BM} + \mathbf{PM}) \mu - \mu (\mathbf{BM}) \\ = \mu (\mathbf{PM})$$

$$9) \text{ From } \Delta \text{PMD, } \cos(r + \alpha) = \frac{PM}{PD}$$

$$\text{But } PD = 2t, \text{ therefore } \cos(r + \alpha) = \frac{PM}{2t}$$

$$PM = 2t \cos(r + \alpha)$$

Therefore, path difference $\Delta = 2\mu t \cos(r + \alpha)$

10) Since the reflection of the light takes place from the optically denser medium, there is phase difference of π or path difference of $\lambda/2$ takes place between the reflected light rays.

$$\text{Therefore } \Delta = 2\mu t \cos(r + \alpha) - \lambda/2$$

11) For constructive interference (bright band) in reflected light

$$\Delta = n\lambda$$

$$2\mu t \cos(r + \alpha) = (2n + 1) \lambda/2 \quad (2)$$

For destructive interference (dark band) in reflected light

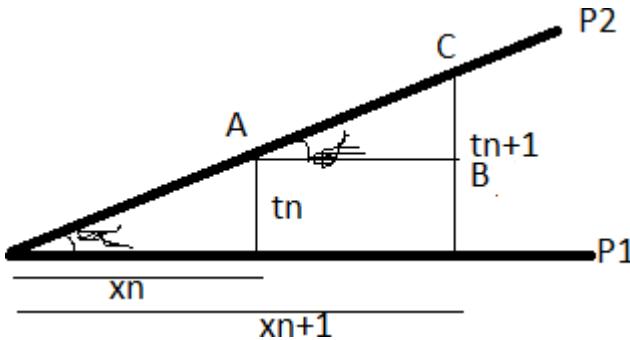
$$\Delta = (2n + 1) \lambda/2$$

$$2\mu t \cos(r + \alpha) = n\lambda \quad (3)$$

Fringe width:-

What is fringe width? Derive its expression?

- 1) The distance between two consecutive dark or bright bands in the interference fringes of wedge shaped film is known as fringe width.



- 2) Consider two glass plate P1 & P2 inclined with small angle α . An air film of wedge shape is formed between the glass plates with angle of wedge α . The film is illuminated by the monochromatic source of light of wave length λ .
- 3) Due to interference with in a thin film of wedge shape film, alternate dark and bright bands of constant thickness parallel to surface of the upper glass plate are observed. These fringes of constant thickness are known as Fizeau fringes.
- 4) For constructive interference (bright band) in reflected light

$$\Delta = n\lambda$$

$$2\mu t \cos(r + \alpha) = (2n + 1) \lambda/2$$

For destructive interference (dark band) in reflected light

$$\Delta = (2n + 1) \lambda/2$$

$$2\mu t \cos(r + \alpha) = n\lambda$$

- 5) Consider wedge shaped film of angle of wedge is α . Let n^{th} dark band is at x_n and $(n+1)^{\text{th}}$ band is at x_{n+1} from the apex of the film where thickness of the film is t_n and t_{n+1} respectively.

$x_{n+1} - x_n = \beta$ is known as fringe width

From figure, in ΔABC , $AB = x_{n+1} - x_n$ and $BC = t_{n+1} - t_n$

$$\tan \alpha = \frac{BC}{AB} = \frac{t_{n+1} - t_n}{x_{n+1} - x_n} = \frac{t_{n+1} - t_n}{\beta}$$

Angle of wedge α is very small, therefore $\tan \alpha = \alpha$

$$\text{Therefore } \alpha = \frac{t_{n+1} - t_n}{\beta}$$

$$\text{Fringe width } \beta = \frac{t_{n+1} - t_n}{\alpha} \quad \dots \dots \dots (1)$$

To determine t_{n+1} & t_n

Condition for dark band is $2\mu t \cos(r + \alpha) = n\lambda$

For air film $\mu = 1$ and for normal incidence and very thin film $\cos(r + \alpha) = 1$

$$2t = n\lambda$$

For n^{th} dark band $2t_n = n\lambda$

and for $(n+1)^{\text{th}}$ dark band $2t_{n+1} = (n+1)\lambda$

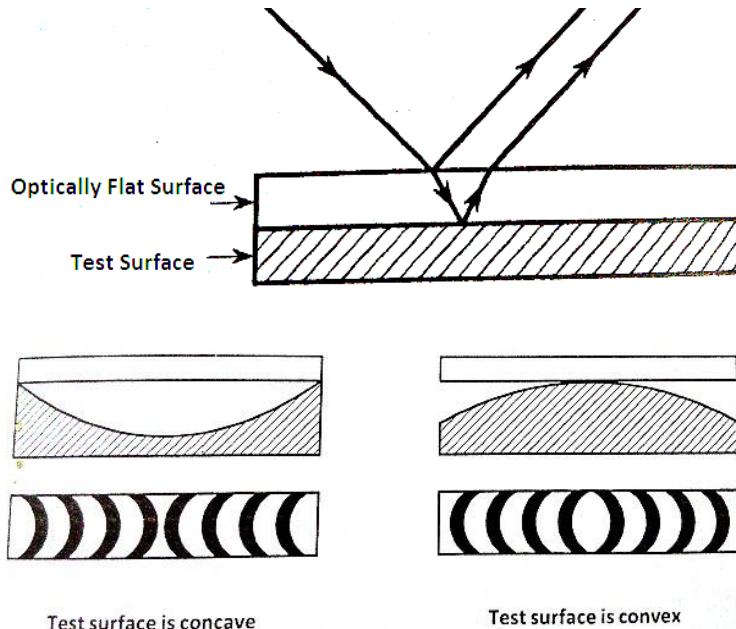
$$2t_{n+1} - 2t_n = \lambda$$

$$t_{n+1} - t_n = \lambda/2$$

Therefore equation (1) \Rightarrow , fringe width $\beta = \frac{\lambda}{2\alpha}$

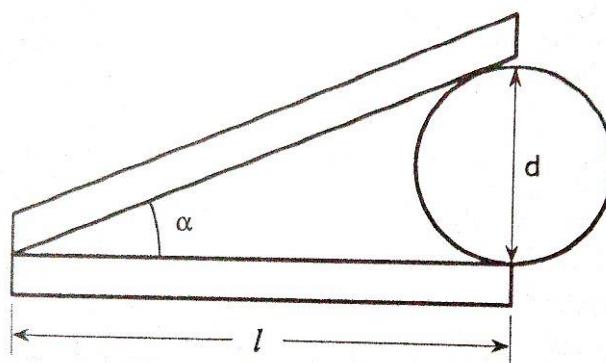
Optical flatness of glass surface or curved surface of lens:-

Q. Write short note on optical flatness?



In optical instrument, perfectly flat surface of the glass plate or smooth curved surface of the lens is required. The phenomenon of interference of light with in thin film is used to test the flat surface or curved surface of the glass plate. The glass plate or the lens under the test is kept on the reference glass plate which having perfectly flat surface to form wedge shaped thin film. The thin film is illuminated by monochromatic light and fringes of constant thickness are observed. The glass plate under the test is polished till we get the fringes of constant thickness.

To determine diameter of thin wire or thickness of thin metal foil:-



A wire whose diameter is to be measured is kept at a distance 'l' between two glass plate to form wedge shaped thin film. The thin film is illuminated by monochromatic light of known wavelength λ . The fringes of constant thickness are observed. The fringe width is measured using microscope;.

$$\text{The fringe width is given by } \beta = \frac{\lambda}{2\alpha}$$

$$\text{The angle of wedge } \alpha = \frac{\lambda}{2\beta} \text{ ----- (1)}$$

From figure $\tan \alpha = d / l$

Since angle α is very small,
 $\tan \alpha = \alpha$

$$\alpha = \frac{d}{l} \quad (2)$$

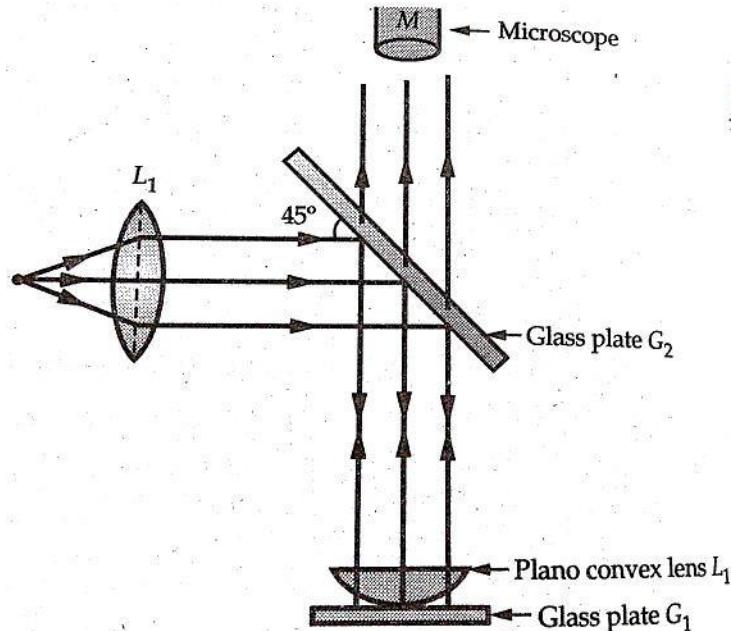
Compare equation (1) & (2)

$$\frac{\lambda}{2\beta} = \frac{d}{l}$$

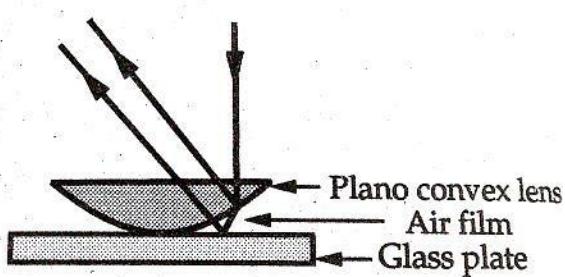
$$\text{Diameter of thin wire } d = \frac{\lambda l}{2\beta}$$

Interference with in thin film of varying thickness (Newton's rings):-

Q, Explain Newton's rings experiment?



When Plano convex lens of large radius of curvature is kept on a plane glass plate, then an air film of varying thickness is enclosed between convex surface of the lens and top surface of the glass plate. The thickness of the film is zero at the point of contact of the lens and the glass plate and thickness increases away from the point of contact along the circumference of the lens. When such a film of varying thickness is illuminated by monochromatic light, then number of dark and bright rings concentric around the point of contact of the lens and the glass plate are observed due to interference of light with in a thin of varying thickness. These rings are known as Newton's rings.

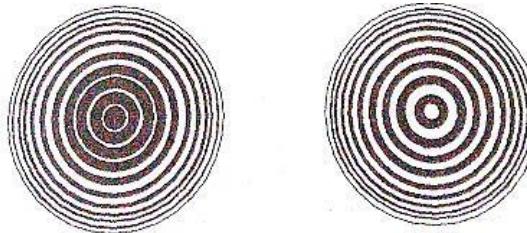


For constructive interference (bright rings) in reflected light

$$2\mu t \cos(\alpha) = (2n + 1) \lambda/2$$

For destructive interference (dark rings) in reflected light

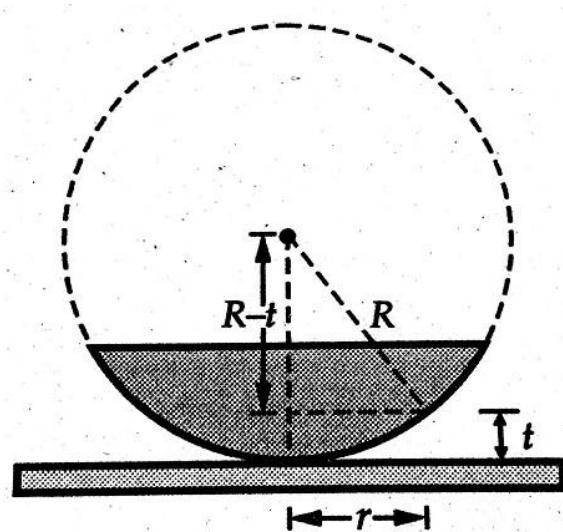
$$2\mu t \cos(\theta + \alpha) = n\lambda$$



(a) Newton's ring in reflected system (b) Newton's ring in transmitted system

Radius or diameter of Newton's ring:-

Q. Show that radius or diameter of Newton's rings is directly proportional to square root of natural number?



Consider the lens and glass plate to form thin film of varying thickness.

If R - Radius of curvature of the lens,
 r_n - Radius of n^{th} dark ring,
 t_n - Thickness of the film where n^{th} dark is obtained.

From figure, in $\triangle ABC$, $AC^2 = AB^2 + BC^2$

$$R^2 = (R - t_n)^2 + r_n^2$$

$$R^2 = R^2 - 2Rt_n + t_n^2 + r_n^2$$

Since thickness of the film is very small, neglect t_n^2

$$R^2 = R^2 - 2Rtn$$

$$+ rn^2 \ 2Rtn =$$

$$rn^2$$

$$tn = \frac{rn^2}{2R}$$

Condition for dark ring is $2\mu t \cos(r + \alpha) = n\lambda$

For normal incidence & very small angle α , $\cos(r + \alpha) = 1$

$$2\mu t_n = n\lambda$$

$$2\mu \frac{rn^2}{2R} = n\lambda$$

$$rn^2 = \frac{n\lambda R}{\mu}$$

$$\text{Therefore diameter } Dn^2 = \frac{4n\lambda R}{\mu}$$

Where λ , R and μ are constant, therefore $r_n \propto \sqrt{n}$ or $D_n \propto \sqrt{n}$

Thus radius or diameter of n^{th} dark ring is directly proportional to square root of natural number.

Condition for dark ring is $2\mu t \cos(r + \alpha) = (2n + 1)\lambda/2$

Therefore for bright ring $r_n \propto \sqrt{2n + 1}$ or $D_n \propto \sqrt{2n + 1}$

Thus radius or diameter of n^{th} bright ring is directly proportional to square root of odd the natural number.

Applications of Newton's ring's Experiment:

Q. Explain how Newton's ring experiment is used to determine radius of curvature or wavelength of light source?

1) To determine radius of curvature or wavelength of light source-

The Newton's rings are obtained using given lens and light source. The diameter of n^{th} and $(m+n)^{\text{th}}$ dark rings are measured using microscope.

Diameter of n^{th} dark ring is given by $Dn^2 = \frac{4n\lambda R}{\mu}$
 $\mu = 1$ for air film

Therefore $Dn^2 = 4n\lambda R$ ----- (1)

Similarly diameter of $(m + n)^{\text{th}}$ dark ring is given by

$(D_m + n)^2 = 4(m+n)\lambda R$ ----- (2)

(2) – (1) is given by

$$(Dm + n^2) - (Dn^2) = 4 m \lambda R$$

$$R = \frac{(Dm + n)^2 - (Dn^2)}{4 m \lambda} \quad \text{OR} \quad \lambda = \frac{(Dm + n)^2 - (Dn^2)}{4 m R}$$

These formulae are used to determine the radius of curvature or wave length of light source.

Q. Explain how Newton's ring experiment is used to determine the refractive index of any liquid or oil?

2) To determine refractive index of given liquid or oil:

The Newton's ring are obtained using air film between lens and glass plate.

The diameter of n^{th} and $(m+n)^{\text{th}}$ dark rings are measured using microscope.

$$\text{Diameter of } n^{\text{th}} \text{ dark ring is given by } Dn^2 = \frac{4n\lambda R}{\mu}$$

$\mu = 1$ for air film

$$\text{Therefore } Dn^2 = 4n\lambda R \quad \dots \dots \dots \quad (1)$$

Similarly diameter of $(m + n)^{\text{th}}$ dark ring is given by

$$(Dm + n)^2 = 4(m+n)\lambda R \quad \dots \dots \dots \quad (2)$$

(2) – (1) is given by

$$[(Dm + n)^2 - (Dn^2)] \text{air} = 4 m \lambda R \quad \dots \dots \dots \quad (3)$$

Then Newton's ring are obtained using given liquid or oil film between lens and glass plate.

The diameter of same n^{th} and $(m+n)^{\text{th}}$ dark rings are measured using microscope.

$$\text{Diameter of } n^{\text{th}} \text{ dark ring is given by } Dn^2 = \frac{4 n \lambda R}{\mu_{\text{oil}}}$$

Where μ_{oil} is R.I of oil

Similarly diameter of $(m+n)^{\text{th}}$ dark ring is given by $Dm + n^2 = \frac{4(m+n)\lambda R}{\mu}$

$$[(Dm + n^2) - (Dn^2)]oil = \frac{4m\lambda R}{\mu oil} \quad \dots \dots \dots \quad (4)$$

(3) ÷ (4) given by

$$\mu_{oil} = \frac{[(Dm+n^2) - (Dn^2)]air}{[(Dm+n^2) - (Dn^2)]oil}$$

This formula is used to determine the R.I of oil.

Numerical

Problem 1 : A parallel beam of light ($\lambda = 5870\text{\AA}$) is incident on a thin glass plate ($\mu = 1.5$), such that the angle of refraction into the plate is 60° . Calculate the smallest thickness of the glass plate which will appear dark by reflection.

Solution : Given : $\lambda = 5870 \text{\AA} = 5870 \times 10^{-10} \text{ m}$

$$\mu = 1.5, \quad r = 60^\circ$$

$$t = t_{\min} \text{ (for minimum), } n = 1$$

Condition for n^{th} order darkness (reflected system)

$$2\mu t_{\min} \cos r = n\lambda$$

$$2\mu t_{\min} \cos r = 1 \times \lambda$$

$$t_{\min} = \frac{\lambda}{2\mu \cos r}$$

$$t_{\min} = \frac{5870 \times 10^{-10}}{2 \times 1.5 \times \cos 60^\circ}$$

$$t_{\min} = 3.913 \times 10^{-7} \text{ m}$$

Ans. $t_{\min} = 3913 \text{\AA}$

Problem 2 : A soap of refractive index 1.33 is illuminated with light of different wavelengths at an angle of 45° . Calculate the smallest thickness of the film which will appear dark by reflection. Wavelength of light is used, $\lambda = 5890\text{\AA}$.

Solution : Given : $\mu = 1.33$

$$\lambda = 5890 \text{ Å} = 5890 \times 10^{-10} \text{ m}$$

$$i = 45^\circ \text{ at } \lambda_{\text{min}}$$

$$\lambda_{\text{max}} = 701 \times 10^{-10} \text{ m}, i = 60^\circ$$

$$\lambda_{\text{min}} = 4000 \text{ nm}$$

(1) Calculation of $\cos r$, $\lambda_{\text{min}} = 4000 \text{ nm}$ which is visible light reflected from air.

Using Snell's law, $\sin i / \sin r = \mu$ which is true because $\sin i / \sin r = \mu$ is the condition for reflection of light from air.

∴ $\sin r = \frac{\sin i}{\mu} = \frac{\sin 45^\circ}{1.33}$

$$\sin r = 0.5316$$

$$r = 32.11^\circ$$

Ans. $\cos r = 0.8470$

(2) Calculation of minimum thickness of film,

Condition for darkness in the reflected light

$$2\mu t \cos r = n\lambda \quad \dots n = 1, 2, 3, \dots$$

for minimum thickness film, $n = 1$

$$t_{\text{min}} = \frac{n\lambda}{2\mu \cos r}$$

$$t_{\text{min}} = \frac{1 \times 5890 \times 10^{-10}}{2 \times 1.33 \times 0.8470}$$

$$= 2.614 \times 10^{-7} \text{ m}$$

Ans. $t_{\text{min}} = 2614 \text{ Å}$

Problem 3 : White light falls normally on a film of soapy water of thickness $5 \times 10^{-5} \text{ cm}$ and refractive index 1.33 which wavelength in the visible region will be reflected most strongly?

Solution : Given : $\mu = 1.33$, $t = 5 \times 10^{-5} \text{ cm}$, $i = 0^\circ$

Condition for strong reflection is, $2\mu t \cos r = (2n+1)\lambda$

∴ $2\mu t = \frac{(2n+1)\lambda}{2}, \quad n = 0, 1, 2, \dots$

∴ $\lambda = \frac{4\mu t}{(2n+1)}$

∴ $\lambda_1 = \frac{4 \times 1.33 \times 5 \times 10^{-5}}{(2 \times 0 + 1)} = 2.66 \times 10^{-4} \text{ m}$

∴ $\lambda_2 = \frac{4 \times 1.33 \times 5 \times 10^{-5}}{(2 \times 1 + 1)} = 0.8866 \times 10^{-4} \text{ m}$

$$\text{for } n = 0, \lambda_1 = 2.66 \times 10^{-4} = 26600 \text{ Å}$$

$$\text{for } n = 1, \lambda_2 = 0.8866 \times 10^{-4} = 8866 \text{ Å}$$

$$\text{for } n = 2, \lambda_3 = 0.532 \times 10^{-4} = 5320\text{\AA}$$

$$\text{for } n = 3, \lambda_4 = 0.38 \times 10^{-4} = 3800\text{\AA}$$

Ans. Here, $\lambda_3 = 5320\text{\AA}$ lies in the visible region, ranging from 4000\AA to 7000\AA .

Ans. Here, $\lambda_3 = 5320\text{\AA}$ lies in the visible region, ranging from 4000\AA to 7000\AA .
∴ The most strongly reflected visible wavelength = 5320\AA .

Problem 4 : A soap film of refractive index 1.33 and thickness 1.5×10^{-4} cm is illuminated by white light incident at an angle of 45° . In the reflected light a dark band is observed for the wavelength 5×10^{-5} cm. Calculate the order of the interference band.

Solution : Given : $\mu = 1.33$,

$$t = 1.5 \times 10^{-4} \text{ cm}, \quad i = 45^\circ, \quad \lambda = 5 \times 10^{-5} \text{ cm}$$

Formula : For dark band

$$2\mu t \cos r = n\lambda, \quad n = 1, 2, 3, \dots$$

By Snell's law

$$\mu = \frac{\sin i}{\sin r}$$

$$\therefore \sin r = \frac{\sin i}{\mu} = \frac{\sin 45}{1.33} = 0.5316$$

$$\sin r = 0.5316$$

$$r = 32.113^\circ$$

$$\cos r = 0.8469$$

$$n = \frac{2\mu t \cos r}{\lambda}$$

$$= \frac{2 \times 1.33 \times 1.5 \times 10^{-4} \times 0.8469}{5 \times 10^{-5}}$$

Ans. $n = 6.75$

The highest possible order of the visible band is 6.

Problem 5 : A parallel beam of sodium light strikes a film of oil floating on water. When viewed at an angle of 30° from the normal 8th dark band is seen. Determine the thickness of the film. Refractive index of oil is 1.46, $\lambda = 5890\text{\AA}$.

Solution : Given : $i = 30^\circ$,

$$n = 8, \quad \mu = 1.46, \quad \lambda = 5890\text{\AA}$$

Formula : By Snell's law

$$\mu = \frac{\sin i}{\sin r}$$

$$\therefore \sin r = \frac{\sin 30}{1.46}$$

$$\sin r = 0.3424$$

$$r = \sin^{-1}(0.3424)$$

$$r = 20.027^\circ$$

problem 6 : A wedge shaped air film is illuminated by light of wavelength 4650\AA . The angle of wedge is 40 seconds. Calculate the separation between two consecutive fringes.

Solution : Given : $\mu = 1$ (air film)

$$\lambda = 4650\text{\AA} = 4650 \times 10^{-10} \text{ m}$$

$$\alpha = 40 \text{ sec}$$

$$\frac{40}{60 \times 60} \times \frac{\pi}{180} = 1.939 \times 10^{-4} \text{ radians}$$

Conversion of degree to radian

$$\pi_{\text{rad}} = 180^\circ$$

$$1^\circ = \frac{\pi}{180^\circ}$$

$\alpha (\text{radian}) = \alpha (\text{degree}) \times \frac{\pi}{180^\circ}$

$$1^\circ = 60' = 3600''$$

$$\text{Formula : } \beta = \frac{\lambda}{2\mu\alpha}$$

$$\beta = \frac{4650 \times 10^{-10}}{2 \times 1 \times 1.939 \times 10^{-4}}$$

$$\text{Ans. } \beta = 1.199 \times 10^{-3} \text{ m}$$

Problem 7 : Interference fringes are produced with monochromatic light falling normally on a wedge shaped air film of cellophane whose refractive index is 1.40. The angle of wedge is 10 seconds of an arc and distance between successive fringes is 0.5 cm. Calculate wavelength of light used.

Solution : Given : $\beta = 0.5 \text{ cm}$

$\mu = 1.40$, $\alpha = 10 \text{ sec}$

$$10 \text{ seconds} = \frac{10}{60 \times 60} \times \frac{\pi}{180}$$

$$10 \text{ seconds} = 4.848 \times 10^{-5} \text{ rad.}$$

Formula : $\beta = \frac{\lambda}{2\mu\alpha}$

$$\therefore \lambda = 2\mu\alpha\beta$$

$$\lambda = 2 \times 1.40 \times 4.848 \times 10^{-5} \times 0.5$$

Ans. $\lambda = 6.787 \times 10^{-5} \text{ cm}$

Problem 8 : Fringes of equal thickness are observed in a thin glass wedge of refractive index 1.52. The fringe spacing is 1 mm and the wavelength of light is 5893 Å. Calculate the angle of wedge in second of an arc.

Solution : Given : $\mu = 1.52$

$$\lambda = 5893 \text{ Å} = 5893 \times 10^{-10} \text{ m}$$

$$\beta = 1 \text{ mm} = 1 \times 10^{-3} \text{ m}$$

Formula : $\alpha = \frac{\lambda}{2\mu\beta} = \frac{5893 \times 10^{-10}}{2 \times 1.52 \times 1 \times 10^{-3}}$

$$\therefore \alpha = 1.938 \times 10^{-4} \text{ radians}$$

$$1 \text{ radian} = \frac{180}{\pi} \text{ degree} = \frac{180}{\pi} \times 3600 \text{ second of arc}$$

$$= 1.938 \times 10^{-4} \times \frac{180}{\pi} \times 3600 \text{ second}$$

Ans. $\alpha = 39.98 \text{ seconds of an arc.}$

Problems on Newton's Rings

Problem 9 : In Newton's ring experiment the diameter of 5th dark is 0.65 cm and that 15th dark ring is 0.95 cm. If wavelength of the source used is 6000 Å. Calculate the radius of curvature of a convex surface of the lens in contact with the glass plate.

Solution : Given : $D_5 = 0.65 \text{ cm}, D_{15} = 0.95 \text{ cm}$

$$\lambda = 6000 \text{ Å} = 6000 \times 10^{-8} \text{ cm.}$$

Formula : $\lambda = \frac{D_{n+p}^2 - D_n^2}{4PR}$

$$\therefore R = \frac{D_{n+p}^2 - D_n^2}{4P\lambda}$$

$$R = \frac{D_{15}^2 - D_5^2}{4 \times P \times \lambda}$$

$$= \frac{(0.95)^2 - (0.65)^2}{4 \times 10 \times 6000 \times 10^{-8}}$$

Ans. $R = 200 \text{ cm}$

Problem 10 : In Newton's ring experiment, the diameter of 4th and 12th dark rings are 0.40 cm and 0.70 cm. find diameter 20th dark ring.

Solution : Given : $D_4 = 0.40 \text{ cm}, D_{12} = 0.70 \text{ cm}, \mu = 1$

Formula : $D_n^2 = 4Rn\lambda$

$$R = \frac{D_{n+p}^2 - D_n^2}{4P\lambda}$$

$$R\lambda = \frac{D_{12}^2 - D_4^2}{4}$$

$$= \frac{(0.70)^2 - (0.40)^2}{4 \times 8}$$

$$R\lambda = 0.0103$$

$$D_{20}^2 = 4 \times 20 \times 0.0103 = 0.824$$

Ans.

$$D_{20} = 0.91 \text{ cm}$$

Problem 11 : In a Newton's ring experiment, the diameter of 5th dark ring was 0.336 cm. Find the radius of curvature of the plano-convex lens, if the wavelength of light used is 5880 Å. Also find the radius of 15th dark ring. $\mu = 1.5$

Solution : Given : $\mu = 1$, $D_1 = 0.336$

$$\lambda = 5880 \text{\AA} = 5880 \times 10^{-8} \text{ cm}$$

Formula : $D_p^2 = 4PR$

$$R = \frac{D_n^2}{4P_n}$$

$$= \frac{(0.336)^2}{4 \times 10 \times 5880 \times 10^{-8}}$$

Calculation of the radius of 15th dark ring (to submit end of schedule 1, pg. 8.0 end of break)

Calculation of the radius of 15th dark ring

$$D_n^2 = 4nR\lambda$$

$$r_n^2 = nR\lambda \quad (\because r_n = \frac{D_n}{2})$$

$$r_{15}^2 = 15 \times 48 \times 5880 \times 10^{-8}$$

$$r_{15} = \sqrt{15 \times 48 \times 5880 \times 10^{-8}}$$

Ans. $r_1 = 0.205 \text{ cm}$

Problem 12 : Newtons rings are observed in reflected light of wavelength 5893\AA . The diameter of 15^{th} dark ring is 0.75 cm . Find the radius of curvature of the convex lens and thickness of the air film.

Solution : Given : $\mu = 5893 \text{ \AA} = 5893 \times 10^{-8} \text{ cm}$,

$$D_{1\%} = 0.75 \text{ cm} \quad n = 15$$

Formula : $D^2 = 4PR\lambda$

$$(1) : \quad R = \frac{D_n^2}{4\pi R_0}$$

$$R = \frac{D_n^2}{4P\lambda}$$

$$= \frac{(0.75)^2}{4 \times 15 \times 5893 \times 10^{-8}}$$

Ans. $R = 159 \text{ cm}$

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Ans. : $R = 159 \text{ cm}$

$$(2) t = \frac{r_{15}^2}{2R} \quad \left(r_{15} = \frac{D_{15}}{2}, \frac{0.75}{2} = 0.375 \text{ cm} \right)$$

$$t = \frac{(0.375)^2}{2 \times 159}$$

Ans. $t = 4.42 \times 10^{-4} \text{ cm}$

Problem 13 : Newtons rings are formed with reflected light of wavelength 5896\AA with a liquid between the plane glass plate and convex lens. The diameter of the 10th dark is 0.41 cm and radius of curvature of the lens is 1 meter, Calculate refractive index of the liquid.

Solution : Given : $\mu = 5896\text{\AA} = 5896 \times 10^{-8} \text{ cm}$
 $D_{10} = 0.41 \text{ cm}, P = 10, R = 1 \text{ m} = 100 \text{ cm}$

Formula : $D_n^2 = \frac{4PR\lambda}{\mu}$

$$\therefore \mu = \frac{4P\lambda R}{D_n^2}$$

$$= \frac{4 \times 10 \times 100 \times 5893 \times 10^{-8}}{(0.41)^2}$$

Ans. $\therefore \mu = 1.40$

Problem 14 : In Newtons ring experiment if a drop of water ($\mu = 1.33$) be placed in between the convex lens and plane glass plate, the diameter of 10th dark ring is found to be 0.6 cm. Calculate the radius of curvature of the lens in contact with the glass plate and wavelength of light used is $\lambda = 6000\text{\AA}$.

Solution : Given : $\mu = 1.33, D_{10} = 0.6 \text{ cm}$

$$\lambda = 6000\text{\AA} = 6000 \times 10^{-8} \text{ cm}$$

$$P = 10$$

Formula : $D_n^2 = \frac{4PR\lambda}{\mu}$

$$R = \frac{D_n^2 \times \mu}{4P\lambda}$$

$$= \frac{(0.6)^2 \times 1.33}{4 \times 10 \times 6000 \times 10^{-8}}$$

$$\therefore R = 199.5$$

Ans. $\therefore R = 200 \text{ cm}$

Formula : $t_{\min} = \frac{\lambda}{4\mu_f} = \frac{5500}{4 \times 1.47}$ and since $\mu_f > 1$, t_{\min} will be less than $\lambda/4$

$t_{\min} = 935\text{Å}$ and this is the minimum thickness required to reflect light of wavelength 5500 Å.

Ans. Minimum thickness of film should be 935Å .

Problem 3 : In costume jewellery, rhinestones (of glass $\mu_r = 1.5$) are after coated with silicon monoxide ($\mu = 2$) to make them more reflective. How thick should be the coating to achieve strong reflection for 560nm light incident normally?

Solution : Given : $\mu_r = 1.5$, $\mu_s = 2$, $\lambda = 560\text{nm}$

$$\text{Formula : } t_{\min} = \frac{\lambda}{4\mu_f} = \frac{560 \times 10^{-9}}{4 \times 2} = 70 \text{ nm}$$

Ans. $t_{\min} = 70 \text{ nm}$

Problem 4 : What will be the order of the dark ring which will double the diameter of the 40th dark ring.

Solution : Given : $D_n = 2D_{40}$

$$\text{Formula : } D_n^2 = 4n\lambda R$$

$$D_{40}^2 = 4 \times 40 \times \lambda R$$

$$D_n^2 = 4D_{40}^2$$

$$4n\lambda R = 4 \times 4 \times 40 \times \lambda R$$

$$n = 160$$

Ans. $n = 160$

Problem 5 : The diameter of the 10th dark ring is 5 mm when light of wavelength 5500 Å is used in Newton's rings. If the space between the lens and glass plate is filled with a liquid of refractive index 1.25, what will be the diameter of 20th dark ring.

Solution : Given : $D_{10} = 5 \text{ mm} = 0.5 \text{ cm}$

$$\lambda = 5500\text{Å} = 5500 \times 10^{-8} \text{ cm}$$

$$\mu = 1.25$$

$$\text{Formula : } D_n^2 = \frac{4n\lambda R}{\mu}$$

$$D_{10}^2 = 4 \times 10\lambda R$$

$$\lambda R = \frac{D_{10}^2}{40}$$

$$\text{For liquid } \mu = 1 \quad \lambda R = \frac{D_{10}^2}{40} \quad \text{and } \lambda R = \frac{D_{20}^2}{80} \quad \text{since } n = 2 \text{ and } \mu = 1.25$$

$$\text{For liquid } \mu = 1.25 \quad \lambda R = \frac{4 \times 20 \times \lambda R}{1.25} \quad \text{and } \lambda R = \frac{4 \times 20 \times (0.5)^2}{1.25 \times 40}$$

$$= 0.4$$

$$D_{20} = 0.632 \text{ cm}$$

Ans. Diameter of 20th ring, $D_{20} = 0.632 \text{ cm}$

Problem 6 : A Newton's rings set up is illuminated by a beam of light containing two wavelengths $\lambda_1 = 6000\text{\AA}$ and $\lambda_2 = 4500\text{\AA}$. It is found that the n^{th} dark ring due to wavelength λ_1 coincides with the $(n + 1)^{\text{th}}$ dark ring due to λ_2 . If the radius of curvature of the lens, $R = 90 \text{ cm}$. Find the radius of the n^{th} dark ring.

Solution : Given : $\lambda_1 = 6000 \text{ \AA}$ for D_n^2

$$\lambda_2 = 4500 \text{ \AA} \text{ for } D_{n+1}^2$$

$$R = 90 \text{ cm}, \mu = 1$$

Formula : $D_n^2 = 4n\lambda R$

For $\lambda_1 \Rightarrow D_n^2 = 4n\lambda_1 R$

$$D_{n+1}^2 = 4(n+1)\lambda_2 R$$

$$4n\lambda_1 R = 4(n+1)\lambda_2 R$$

$$n\lambda_1 = (n+1)\lambda_2$$

$$n = \frac{\lambda_2}{\lambda_1 - \lambda_2} = \frac{4500}{(6000 - 4500)} = 3$$

$$D_n^2 = 4n\lambda_1 R$$

$$= 4 \times 3 \times 6000 \times 90 \times 10^{-8}$$

$$= 648 \times 10^{-4}$$

$$D_n = 25.45 \times 10^{-2} \text{ cm} = 2545 \text{ cm}$$

$$\text{Ans. Radius of } n^{\text{th}} \text{ dark ring, } r_n = 0.504 \text{ cm}$$

Problem 7 : In a Newton's ring experiment, the diameter of the 10th dark ring changes from 1.4 cm to 1.27 cm when a liquid is introduced between the lens and the plate. Calculate the refractive index of the liquid.

Solution : $D_n(\text{air}) = 1.4 \text{ cm}, D_n(\text{liquid}) = 1.27 \text{ cm}$

$$\mu = \frac{D_n(\text{air})}{D_n(\text{liquid})}$$

$$= \frac{(1.4)^2}{(1.27)^2}$$

$$= 1.218$$

Ans. Refractive index of liquid is 1.218.

Problem 8 : Newton's rings are formed using light of wavelength 5896\AA in reflected light with a liquid placed between plane and curved surface. The diameter of 7th

Q. 8. Bright fringe width is 0.4 cm and radius of curvature is 1 m. Find the refractive index of liquid.

Solution :

$$D_n^2 = \frac{4n\lambda R}{\lambda}$$

$$\mu = \frac{4n\lambda R}{D_n^2}$$

$$= \frac{4 \times 7 \times 1 \times 5896 \times 10^{-10}}{(4 \times 10^{-2})^2}$$

$$\mu = 1.038$$

Ans. $\mu = 1.038$

Problem 9 : Two plane rectangular pieces of glass are in contact at one edge and are separated at the other end 10 cm away by a wire to form a wedge shaped film. When the film was illuminated by light of wavelength 6000\AA , 103 fringes were observed per cm. Determine the diameter of the wire.

Solution : Given : $l = 10 \text{ cm}$, $\lambda = 6000 \text{\AA}$

No. of fringes per cm = 10

$$\therefore \beta = \frac{1}{10} = 0.1 \text{ cm}$$

From Fig. 4.19 $\tan \theta = \frac{d}{l} = \theta$... As θ is very small

$$\text{and } \beta = \frac{\lambda}{2\theta}$$

$$\therefore \frac{d}{l} = \frac{\lambda}{2\beta}$$

$$\Rightarrow d = \frac{\lambda l}{2\beta} = \frac{6000 \times 10^{-8} \times 10}{2 \times 0.1}$$

$$d = 3 \times 10^{-8} \text{ cm}$$

Ans. Diameter of wire, $d = 3 \times 10^{-8} \text{ cm}$

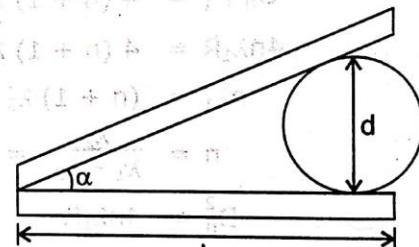


Fig. 4.19

QUANTUM PHYSICS

Quantum physics is a branch of physics which deals with the properties and interaction of the matter. According to scientist De'Broglie, matter possess dual nature i.e. wave and particle nature.

Particle:- The constituents of matter like electrons, protons, atoms, molecules are reffered as particles.

Properties:-



1. Particle has fixed mass.
2. Particle can locate at definite point.
3. Particle has ability to move from one place to another place.
4. Particle release energy when suddenly stopped.
5. Particle is specified by mass, velocity, momentum and energy.

Wave:- The oscillatory disturbance produced by a particle is known as wave.



Properties:-

1. Wave spread out over a large region of space,
2. Wave cannot locate at any particular point.
3. Wave is specified by frequency, wavelength. Amplitude and phase.

Q. State and explain De 'Broglie's hypothesis?

De'Broglies hypothesis of matter wave.:-

According to scientist De 'Broglie, a particle of mass 'm' moving with velocity 'V' is associate with a wave of wavelength $\lambda = \frac{h}{mv} = \frac{h}{p}$

Where $p = mv$ is momentum of the particle.

The wave associated with the moving particle is known as De 'Broglie's wave or matter wave and wavelength is known as De 'Broglie's wavelength.

Explanation:- The radiation i.e photons has dual nature.

The phenomenon like interference, diffraction, polarization, reflection, refraction can only be explained if the radiation is to be considered as a wave.

The phenomenon like photoelectric effect, Compton Effect can only be explained if the radiation is to be considered as a particle.

If we consider a photon as a wave of frequency γ , then its energy is given by $E = h\gamma$

If we consider a photon as a particle of mass m , then its energy is given by $E = mc^2$

$$\text{Therefore } mc^2 = h\gamma \quad \text{but } \gamma = c / \lambda$$

$$mc^2 = h \frac{c}{\lambda}$$

$$mc = \frac{h}{\lambda} \quad \text{but } mc = p \text{ momentum of photon}$$

$$p = \frac{h}{\lambda} \quad \text{therefore } \lambda = \frac{h}{p} \quad \text{this is wavelength of light wave.}$$

Similarly, if a particle of mass m moving with velocity v , then a wave of wavelength $\lambda = \frac{h}{mv}$ is associated with this particle.

Properties of matter wave:

- (1) Matter wave represents the probability of finding a particle in space.
- (2) Matter waves are not electromagnetic in nature.
- (3) de-Broglie or matter wave is independent of the charge on the material particle. It means, matter wave of de-Broglie wave is associated with every moving particle (whether charged or uncharged).
- (4) Practical observation of matter waves is possible only when the de-Broglie wavelength is of the order of the size of the particles.
- (5) Electron microscope works on the basis of de-Broglie waves.
- (6) The phase velocity of the matter waves can be greater than the speed of the light.
- (7) Matter waves can propagate in vacuum, hence they are not mechanical waves.
- (8) The number of de-Broglie waves associated with n th orbital electron is n .
- (9) Only those circular orbits around the nucleus are stable whose circumference is integral multiple of de-Broglie wavelength associated with the orbital electron.

Q. Derive De'Broglie's wavelength of electron in terms of energy?

De'Broglie's wavelength of electron in terms of energy E:-

Consider an electron of mass 'm' moving with velocity 'v', then energy of electron is

$$E = \frac{1}{2} mv^2$$

$$mv^2 = 2E \quad \text{multiply both side by } m$$

$$m^2v^2 = 2mE \quad \text{but } mv = P \text{ momentum of electron.}$$

$$P^2 = 2mE$$

$$P = \sqrt{2mE}$$

$$\text{De'Broglie wavelength is given by } \lambda = \frac{h}{p}$$

$$\text{Therefore } \lambda = \frac{h}{\sqrt{2mE}}$$

Q. Show that De'Broglie's wavelength of electron is inversely proportional to square root of accelerating potential?

De'Broglie's wavelength of electron in terms accelerating potential Va:-

If an electron of mass 'm' moving with velocity 'v' is accelerated by potential V_a , then energy of electron in terms of V_a is given by

$$E = e V_a \quad \dots \dots \dots \quad (1) \qquad \text{but } E = \frac{1}{2} mv^2 \quad \dots \dots \dots \quad (2)$$

Equating equation (1) and (2)

$$\frac{1}{2} mv^2 = eV_a$$

$$mv^2 = 2eV_a \quad \text{multiply both side by 'm'}$$

$$m^2v^2 = 2meV_a$$

$$P = \sqrt{2meV_a}$$

$$\text{De'Broglie wavelength is given by } \lambda = \frac{h}{p}$$

$$\text{Therefore } \lambda = \frac{h}{\sqrt{2meVa}}$$

Where $h = 6.63 \times 10^{-34} \text{ J-S}$

$$m = 9.1 \times 10^{-31} \text{ kg}$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

$$\text{Therefore } \lambda = \frac{12.25}{\sqrt{Va}} \text{ AU}$$

$$\lambda \propto \frac{1}{\sqrt{Va}}$$

Thus De'Broglie's wavelength is inversely proportional to square root of accelerating potential.

Q. Explain the term phase velocity and group velocity?

Phase velocity (Vph) :-

According to scientist De'Broglie, a particle of mass 'm' moving with velocity 'V' is associated with a wave of wavelength $\lambda = \frac{h}{mv} = \frac{h}{p}$

Where $p = mv$ is momentum of the particle.

The wave associated with the moving particle is known as De'Broglie's wave or matter wave and wavelength is known as De'Broglie's wavelength.

The velocity with which the De'Broglie's wave associated with the moving particle is travel is known as phase velocity.

The equation of travelling sinusoidal wave is given by

$$Y = A \sin(\omega t - kx)$$

$$\text{Phase velocity } Vph = \frac{\omega}{k}$$

But angular frequency $\omega = 2\pi\gamma$ and wave number $k = \frac{2\pi}{\lambda}$

$$\text{Therefore } Vph = \gamma \times \lambda$$

$$\text{but } E = h\gamma \quad \text{and} \quad E = mc^2$$

$$mc^2 = h \gamma$$

$$\gamma = \frac{mc^2}{h} \quad \text{and De'Broglie wavelength is given by } \lambda = \frac{h}{p} = \frac{h}{mv}$$

$$\text{Therefore, } V_{\text{ph}} = \frac{mc^2}{h} \times \frac{h}{mv} \quad \text{hence } V_{\text{ph}} = \boxed{\frac{c^2}{v}}$$

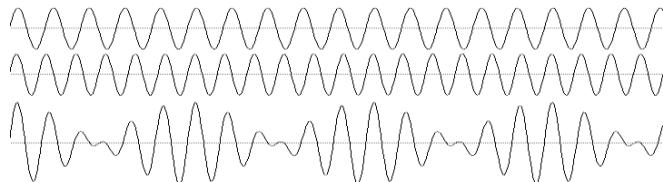
Group velocity (Vg):-

According to scientist De'Broglie, a particle of mass 'm' moving with velocity 'V' is associated with a wave of wavelength $\lambda = \frac{h}{mv} = \frac{h}{p}$

Where $p = mv$ is momentum of the particle.

The wave associated with the moving particle is known as De'Broglie's wave or matter wave and wavelength is known as De'Broglie's wavelength.

The particle is moving with velocity 'V' and wave is moving with velocity c^2/v is not possible, therefore wave associated with the moving particle is not a continuous wave, but it is a group of waves formed by superposition of individual wave of different frequency. This group of waves is known as wave group or wave packet.



The velocity with which entire wave group travel is known as group velocity. It is the velocity with which the energy transmission takes place in a wave.

The equation of travelling sinusoidal wave with slightly modulated w and k is given by

$$Y_1 = A \sin(\omega t - kx)$$

$$Y_2 = A \sin(\omega + dw)t - (k + dk)x)$$

The resultant is $Y = Y_1 + Y_2$

$$\text{The group velocity is given by } V_g = \frac{d\omega}{dk}$$

Q. Show that group velocity = particle velocity?

Expression for group velocity = particle velocity :-

The group velocity is given by $V_{group} = \frac{d\omega}{dk}$ ----- (1)

We have angular frequency $\omega = 2\pi\gamma$ and wave number $k = \frac{2\pi}{\lambda}$

$$\text{but } E = h\gamma \quad \text{and} \quad E = mc^2$$

$$mc^2 = h\gamma$$

Therefore $\gamma = \frac{mc^2}{h}$ and De'Broglie wavelength is given by $\lambda = \frac{h}{p} = \frac{h}{mv}$

$$\omega = \frac{2\pi mc^2}{h} \quad \text{and} \quad k = \frac{2\pi mv}{h}$$

The relativistic expression for mass of the particle is given by

$$\begin{aligned}
 m &= mo \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}} \\
 \omega &= \frac{2\pi m_0 c^2}{h} \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}} \quad \text{and} \quad k = \frac{2\pi mo v}{h} \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}} \\
 \frac{d\omega}{dv} &= \frac{2\pi m_0 c^2}{h} \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}} \cdot -\frac{1}{2} \left(1 - \frac{v^2}{c^2}\right)^{-\frac{3}{2}} \cdot \frac{-2v}{c^2} \\
 &= \frac{2\pi mo v}{h} \left(1 - \frac{v^2}{c^2}\right)^{-\frac{3}{2}}. \quad \text{----- (2)}
 \end{aligned}$$

$$\text{Similarly} \quad \frac{dk}{dv} = \frac{2\pi mo}{h} \left(1 - \frac{v^2}{c^2}\right)^{-\frac{3}{2}} \quad \text{----- (3)}$$

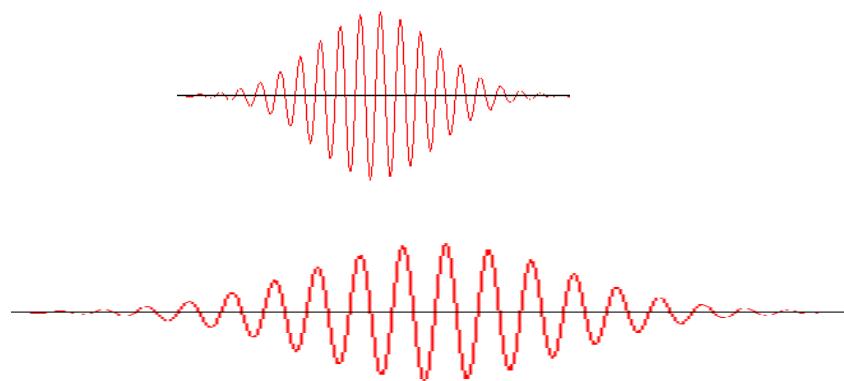
Divide eqn. (2) by eqn. (3)

$$\frac{d\omega}{dk} = v$$

Thus group velocity = particle velocity

Q. State and explain Heisenberg's uncertainty principle? Give its one illustration?

Heisenberg's uncertainty principal:-



A wave group is associated with the moving particle and energy transmission takes place in the form of wave group. When wave group is small, we can easily locate the position of the particle but since small wave group contains number of waves, it is difficult to determine the wavelength i.e momentum of the particle. When wave group is large, we can easily determine the wavelength i.e momentum of the particle but since particle may be anywhere within a large wave group, it is difficult to locate the position of the particle.

Thus certainty in the measurement of position of particle gives uncertainty in the measurement of momentum of the particle and vice versa. Thus it is not possible to measure the position and momentum of particle simultaneously and precisely.

Statement:- “. It is impossible to measure the position and momentum of particle simultaneously and precisely. The product of uncertainties in the measurement of position and momentum of the particle with in wave group is greater than or equal to planks constant”.

$$\Delta x \cdot \Delta p \geq h$$

More precisely,

$$\Delta x \cdot \Delta p \geq \hbar \quad \text{where } \hbar = \frac{h}{2\pi} = 1.055 \times 10^{-34}$$

This principle is valid for any two related physical quantities,

$$\Delta E \cdot \Delta t = \frac{h}{2\pi}$$

$$\Delta L \cdot \Delta \theta = \frac{h}{2\pi}$$

Experimental illustration of Heisenberg's uncertainty principle:-

1) Scattering of photon with in microscope:-

Consider a photon of energy $E = hu$ is incident on a particle and scatter with in a microscope. The photon is scatter along X- axis.

If Δx is uncertainty in the measurement of position of photon.

$$2 \Delta x \sin\theta = \lambda$$

$$\Delta x = \frac{\lambda}{2 \sin\theta} \quad \dots\dots\dots (1)$$

The X- component of momentum lies between $P \sin\theta$ and $-P \sin\theta$.

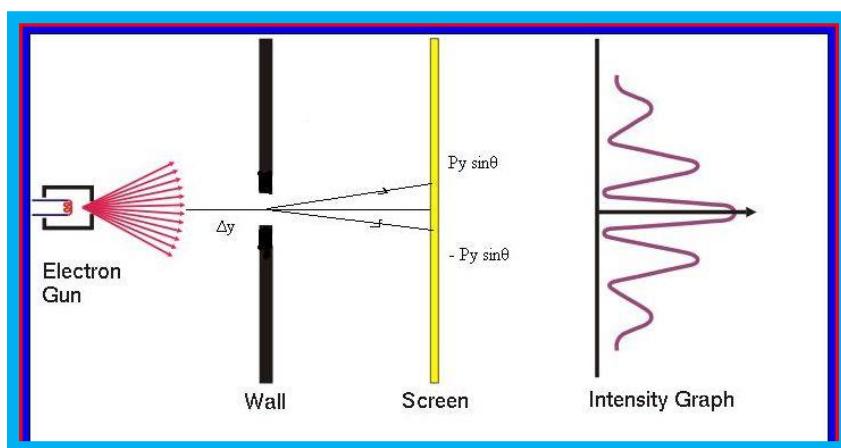
Therefore uncertainty in the measurement of momentum of photon is

$$\Delta P = P \sin\theta - (-P \sin\theta) = 2 P \sin\theta \quad \dots\dots\dots (2)$$

$$\text{From equation (1) and (2)} \quad \Delta x \Delta P = \frac{\lambda}{2 \sin\theta} \times 2 P \sin\theta = \lambda P$$

$$\text{But } \lambda = \frac{h}{p} \text{ therefore } \Delta x \Delta P = h$$

2) Diffraction of electrons by the single slit:-



Consider single slit of width ' d ' = Δy is illuminated by the electrons. The electrons are diffracted by the slit along Y-axis and produce diffraction pattern on the photographic plate. The central maximum is formed at P_0 and secondary maxima and minima are obtained on both sides of central maxima.

The uncertainty in the measurement of position of electron = slit width = Δy

$$2 \Delta y \sin\theta = \lambda$$

$$\Delta y = \frac{\lambda}{2 \sin \theta} \quad \dots \dots \dots (1)$$

The Y- component of momentum lies between $P_y \sin \theta$ and $-P_y \sin \theta$.

Therefore uncertainty in the measurement of momentum of photon is

$$\Delta P_y = P_y \sin \theta - (-P_y \sin \theta) = 2 P_y \sin \theta \quad \dots \dots \dots (2)$$

$$\text{From equation (1) and (2)} \quad \Delta y \Delta P_y = \frac{\lambda}{2 \sin \theta} \times 2 P_y \sin \theta = \lambda P_y$$

$$\text{But } \lambda = \frac{h}{p} \text{ therefore } \Delta y \Delta P_y = h$$

Q. State Heisenberg's uncertainty principle? Show that electron cannot exist in the nucleus of an atom?

Diameter of nucleus of an atom is 10^{-14} m.

Therefore uncertainty in the measurement of position of electron with in nucleus is

$$\Delta x = 10^{-14} \text{ m}$$

$$\Delta x \cdot \Delta p \geq \hbar \quad \text{Where } \hbar = \frac{h}{2\pi} = 1.055 \times 10^{-34}$$

$$\Delta p = \frac{1.055 \times 10^{-34}}{\Delta x}$$

$$\Delta p = \frac{1.055 \times 10^{-34}}{10^{-14}}$$

$$\Delta p = 10^{-20} \text{ Kg.m/sec}$$

The energy of an electron is $E = \Delta p \cdot c = 10^{-20} \cdot 3 \times 10^8 = 3 \times 10^{-12} \text{ J}$

$$E = \frac{3 \times 10^{-12}}{1.6 \times 10^{-19}} \text{ eV} = 19 \text{ MeV}$$

Thus energy required for electron to remain inside the nucleus is 19 MeV. It is not possible, therefore electron never exist inside the nucleus.

Q. What is wave function? Derive one dimensional Schrodinger time dependent equation and convert it in to time independent form?

Wave function $\psi (x,t)$:-

Wave is oscillatory disturbance travelling through the medium. For every wave there are some physical quantity that varies with position x and time t . For electromagnetic wave, there is variation in electric field and magnetic field, for sound wave there is variation in pressure.

De'Broglie's wave associated with moving particle is represented by function ψ , that varies with position x and time t . Function $\psi (x,t)$ is complex and square of this function gives the probability of locating the particle.

Schrodinger time dependent equation:-

Consider a particle of mass ' m ' moving with velocity ' v ' in a force of field i.e force is acting on the particle. The total energy of the particle is

$$E = \text{K.E.} + \text{P.E.}$$

$$E = \frac{p^2}{2m} + V \quad \dots \dots \dots (1)$$

The De'Broglie's wave associated with moving particle is given by function $\psi (x,t)$.

In classical mechanics wave equation is give by

$$\frac{\partial^2 \Psi}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 \Psi}{\partial t^2}$$

The solution of this equation is given by

$$\psi (x,t) = A e^{-i(\omega t - kx)}$$

Multiply equation (1) by $\psi (x,t)$

$$E \psi (x,t) = \frac{p^2 \psi (x,t)}{2m} + V \psi (x,t) \quad \dots \dots \dots (2)$$

To determine $E \psi (x,t)$ and $p^2 \psi (x,t)$

$$\psi (x,t) = A e^{-i(\omega t - kx)}$$

$$\omega = 2\pi\gamma \text{ and } E = h\gamma$$

$$\omega = \frac{2\pi E}{h} = \frac{E}{\hbar}$$

$$\text{And } K = \frac{2\pi}{\lambda} \text{ and } \lambda = \frac{h}{p}$$

$$k = \frac{2\pi p}{h} = \frac{p}{\hbar}$$

$$\text{Therefore } \Psi(x,t) = A e^{-\frac{i}{\hbar}(Et - px)} \quad \dots \quad (3)$$

To determine $E \psi(x,t)$ differentiate eqn (3) w.r.to t

$$\frac{\partial \psi}{\partial t} = -\frac{iE}{\hbar} \psi$$

$$E \psi = i\hbar \frac{\partial \psi}{\partial t}$$

To determine $P \psi(x,t)$ differentiate eqn (3) w.r.to x

$$\frac{\partial \psi}{\partial x} = \frac{iP\psi}{\hbar}$$

$$P\psi = \frac{\hbar}{i} \frac{\partial \psi}{\partial x}$$

$$P^2\psi = -\hbar^2 \frac{\partial^2 \psi}{\partial t^2}$$

Keep value of $E \psi(x,t)$ and $P^2\psi(x,t)$ in eqn.(2)

$$i\hbar \frac{\partial \Psi(x,t)}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \Psi(x,t)}{\partial x^2} + V\Psi(x,t)$$

This is Schrodinger time dependent equation.

Schrodinger time independent equation:-

If the motion of the particle is in free space i.e. force is not acting on the particle, then P.E. of particle is zero. We can separate the variables of function $\psi(x, t)$

$$\text{Hence } \psi(x, t) = \psi(x) \Phi(t)$$

Therefore Schrodinger time dependent eqn. can be written as

$$i\hbar \psi(x) \frac{\partial \Phi(t)}{\partial t} = -\frac{\hbar^2}{2m} \Phi(t) \frac{\partial^2 \psi(x)}{\partial x^2} + V \psi(x) \Phi(t)$$

Divide both side by $\psi(x) \Phi(t)$

$$i\hbar \frac{1}{\Phi(t)} \frac{\partial \Phi(t)}{\partial t} = -\frac{\hbar^2}{2m} \frac{1}{\psi(x)} \frac{\partial^2 \psi(x)}{\partial x^2} + V$$

RHS of this equation is only the function of 't' and LHS is only the function of 'x'. Therefore both side equal to some constant say E

$$-\frac{\hbar^2}{2m} \frac{1}{\psi(x)} \frac{\partial^2 \psi(x)}{\partial x^2} + V = E$$

Multiply both sides by $\psi(x)$

$$-\frac{\hbar^2}{2m} \frac{\partial^2 \psi(x)}{\partial x^2} + V\psi(x) = E\psi(x)$$

$$\frac{\hbar^2}{2m} \frac{\partial^2 \psi(x)}{\partial x^2} + (E - V)\psi(x) = 0$$

This is Schrodinger time independent equation.

Application of Schrodinger time independent equation:-

1) Motion of particle in a free space:-

Consider a particle of mass 'm' moving with velocity 'v' in a free space i.e. particle is not acted upon by any force. Therefore P.E. $V = 0$

The Schrodinger time independent equation is

$$\frac{\hbar^2}{2m} \frac{\partial^2 \psi(x)}{\partial x^2} + (E - V)\psi(x) = 0$$

$$\frac{\hbar^2}{2m} \frac{\partial^2 \psi(x)}{\partial x^2} + E\psi(x) = 0$$

$$\frac{\partial^2 \psi(x)}{\partial x^2} + \frac{2m}{\hbar^2} E\psi(x) = 0$$

$$\frac{\partial^2 \psi(x)}{\partial x^2} + K^2\psi(x) = 0$$

$$\text{Where } K^2 = \frac{2mE}{\hbar^2}$$

$$\text{But } E = \frac{p^2}{2m}$$

$$\text{Therefore } K = \frac{1}{\lambda}$$

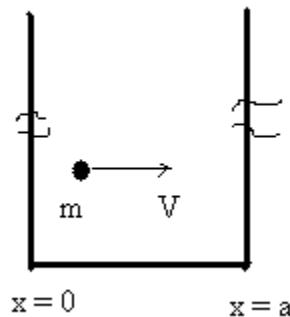
Thus K describes the wave properties of particle.

$$E = \frac{K^2 \hbar^2}{2m}$$

Thus energy values are continuous.

Q. Show that energy of an electron in the box varied as the square of natural numbers?

2) Motion of particle in a box of infinite potential well:-



Consider a particle of mass 'm' moving with velocity 'v' in a box of length 'a' and infinite height. Potential energy of the particle is constant within the box. Assume P.E. $V = 0$ within the box. P.E. $V = \infty$ at the walls and outside the box.

The Schrodinger time independent equation is

$$\frac{\hbar^2}{2m} \frac{\partial^2 \psi(x)}{\partial x^2} + (E - V)\psi(x) = 0$$

Since within box $V = 0$

$$\frac{\hbar^2}{2m} \frac{\partial^2 \psi(x)}{\partial x^2} + E\psi(x) = 0$$

$$\frac{\partial^2 \psi(x)}{\partial x^2} + \frac{2m}{\hbar^2} E\psi(x) = 0$$

$$\frac{\partial^2 \psi(x)}{\partial x^2} + K^2\psi(x) = 0 \quad \dots\dots\dots (1)$$

$$Where K^2 = \frac{2mE}{\hbar^2}$$

Solution of eqn (1) is $\psi = A \cos Kx + B \sin Kx$

The boundary conditions are at $x = 0$ and $x = a$, $\psi = 0$

$$B \sin Ka = 0$$

$$Ka = n\pi$$

$$K = \frac{n\pi}{a}$$

$$\text{But } E = \frac{k^2 \hbar^2}{2m}$$

$$E = \frac{n^2 \hbar^2 \pi^2}{2ma^2} \text{ where } n = 1, 2, 3, 4$$

Thus energy values of the particle are discrete.

Thus energy of an electron in the box varied as the square of natural numbers.

Quantum computing:

- 1) Quantum computing is the area of study focused on developing computer technology based on the principles of quantum theory, which explains the nature and behavior of energy and matter on the quantum (atomic and subatomic) level. Quantum computers can use certain phenomena from quantum mechanics, such as superposition and entanglement, to perform operations on data.
- 2) Traditional and quantum computers both store data as binary code. However, while traditional computers store information in bits as either a 1 or 0, quantum computers store data in qubits as either a 1, a 0, or both simultaneously.
- 3) Quantum computers encode information as quantum bits, or qubits, which can exist in superposition. Qubits represent atoms, ions, photons or electrons and their respective control devices that are working together to act as computer memory and a processor.
- 4) Computing capability of quantum computer is far greater than supercomputer. The quantum computer, following the laws of quantum physics, would gain enormous processing power through the ability to be in multiple states, and to perform tasks using all possible permutations simultaneously.
- 5) Modern **computers** use transistor technology for their fundamental design. Classical computing work on Boolean algebra, operating logic gate principle. Data must be processed in an exclusive binary state, that is, either 0 (off / false) or 1 (on / true). These values are binary digits, or bits. The millions of transistors and capacitors at the heart of computers can only be in one state at any point. While the time that the each transistor or capacitor need be either in 0 or 1 before switching states is now measurable in billionths of

a second, there is still a limit as to how quickly these devices can be made to switch state.

- 6) In a quantum computer, a number of elemental particles such as electrons or photons can be used, with either their charge or polarization acting as a representation of 0 and/or 1. Each of these particles is known as a quantum bit, or qubit, the nature and behavior of these particles form the basis of quantum computing.
- 7) A qubit as an electron in a magnetic field. The electron's spin may be either in alignment with the field, which is known as a *spin-up* state, or opposite to the field, which is known as a *spin-down* state. Changing the electron's spin from one state to another is achieved by using a pulse of energy.
- 8) Quantum superposition and entanglement create an enormously enhanced computing power. Where a 2-bit register in an ordinary computer can store only one of four binary configurations (00, 01, 10, or 11) at any given time, a 2-qubit register in a quantum computer can store all four numbers simultaneously, because each qubit represents two values. If more qubits are added, the increased capacity is expanded exponentially.

1. Determine the de-Broglie wavelength of an electron accelerated by a potential difference of 150V.

Given : Potential difference (V) = 150 V

Formula : The de-Broglie wavelength associated with potential difference

$$\lambda = \frac{h}{\sqrt{2meV}}$$

$$\lambda = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 150 \times 1.6 \times 10^{-19}}}$$

$$\lambda = 1.0031 \times 10^{-10} \text{ m}$$

Ans. $\lambda = 1 \times 10^{-10} \text{ m}$ or $\lambda = 1 \text{ \AA}$.

2. Calculate the de-Broglie wavelength of an α -particle accelerated through potential difference of 200V. ($m_\alpha = 6.68 \times 10^{-27} \text{ kg}$)

Given : $m_\alpha = 6.68 \times 10^{-27} \text{ kg}$

$$V = 200 \text{ V}$$

Charge of an α particle = 2 \times charge on an electron = $2 \times 1.6 \times 10^{-19} \text{ C}$

Formula :

$$\lambda = \frac{h}{\sqrt{2meV}}$$

$$\lambda = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 6.68 \times 10^{-27} \times 2 \times 1.6 \times 10^{-19} \times 200}}$$

Ans. $\lambda = 7.170 \times 10^{-13} \text{ m}$

3. Calculate the velocity of de-Broglie wavelength of a neutron of energy 10^4 eV .

Given : Mass of neutron (m) = $1.67 \times 10^{-27} \text{ kg}$

$$\begin{aligned}\text{Energy of neutron (E)} &= 10^4 \text{ eV} \\ &= 10^4 \times 1.6 \times 10^{-19} \text{ joules} \\ &= 1.6 \times 10^{-15} \text{ joules}\end{aligned}$$

Formula :

(1) Calculation of velocity of a neutron

$$E = \frac{1}{2} mv^2$$

$$v^2 = \frac{2E}{m}$$

$$v = \sqrt{\frac{2E}{m}} = \sqrt{\frac{2 \times 1.6 \times 10^{-15}}{1.67 \times 10^{-27}}}$$

$$\text{Ans. } v = 1.38 \times 10^6 \text{ m/s}$$

(2) Calculation of de-Broglie wavelength

$$\lambda = \frac{h}{mv} = \frac{6.63 \times 10^{-34}}{1.67 \times 10^{-27} \times 1.38 \times 10^6}$$

$$\text{Ans. } \lambda = 2.87 \times 10^{-13} \text{ m.}$$

4. A neutron of 1.67×10^{-27} kg mass is moving with a kinetic energy of 20 KeV. Calculate the de-Broglie wavelength associated with it.

Given : mass of neutron (m) = 1.67×10^{-27} kg

$$\text{K.E. of neutron (E)} = 20 \text{ keV} = 20 \times 10^3 \times 1.6 \times 10^{-19} = 3.2 \times 10^{-15} \text{ joules}$$

Formula :

$$\lambda = \frac{h}{\sqrt{2mE}}$$

$$\frac{6.63 \times 10^{-34}}{\sqrt{2 \times 1.67 \times 10^{-27} \times 3.2 \times 10^{-15}}}$$

$$\text{Ans. } \lambda = 2.027 \times 10^{-13} \text{ m}$$

5. A bullet of mass 40 gms and an electron both travel at velocity of 1100 m/s. What wavelengths can be associated with them?

Given : Mass of bullet = 40×10^{-3} kg

Velocity of bullet = 1100 m/s

Mass of electron = 9.1×10^{-31} kg

Formula :

$$\lambda = \frac{h}{p}$$

$$\lambda = \frac{h}{mv}$$

(1) Wavelength associated with bullet be λ_B

$$\therefore \lambda_B = \frac{h}{mv}$$

$$\therefore \lambda_B = \frac{6.63 \times 10^{-34}}{40 \times 10^{-3} \times 1100}$$

$$\text{Ans. } \lambda_B = 1.506 \times 10^{-35} \text{ m}$$

(2) Wavelength associated with an electron be λ_e

$$\therefore \lambda_e = \frac{h}{mv} = \frac{6.63 \times 10^{-34}}{9.1 \times 10^{-31} \times 1100}$$

$$\text{Ans. } \lambda_e = 6.62 \times 10^{-7} \text{ m.}$$

6. The electrons which are at rest accelerated through a potential difference of 250V. Calculate

(a) The velocity of an electron.

$$\frac{1}{2}mv^2 = qV$$

(b) Phase velocity of an electron.

$$v_p = \frac{1}{2}mv = qV$$

(c) De-Broglie's wavelength.

$$6.62 \times 10^{-34}$$

(d) Momentum.

$$6.62 \times 10^{-34}$$

(e) Wave number of an electron wave.

$$6.62 \times 10^{-34}$$

Given : Potential difference (V) = 250V

$$6.62 \times 10^{-34} = qV$$

Formula :

(a) The velocity of an electron

Kinetic energy = Potential energy

$$\frac{1}{2}mv^2 = eV$$

$$v = \sqrt{\frac{2eV}{m}} = \sqrt{\frac{2 \times 1.6 \times 10^{-19} \times 250}{9.1 \times 10^{-31}}}$$

$$\text{Ans. } v = 9.376 \times 10^6 \text{ m/s}$$

(b) Phase velocity of an electron

$$v_p = qV$$

$$V_p \cdot V_g = C^2, \quad V_g = v$$

$$\therefore V_p = \frac{C^2}{v} = \frac{(3 \times 10^8)^2}{9.376 \times 10^6}$$

$$\text{Ans. } V_p = 9.59 \times 10^9 \text{ m/sec.}$$

(c) De-Broglie's wavelength

$$\lambda = \frac{h}{p}, \quad \lambda = \frac{h}{mv}$$

$$\lambda = \frac{6.63 \times 10^{-34}}{9.1 \times 10^{-31} \times 9.376 \times 10^6}$$

$$\text{Ans. } \lambda = 7.770 \times 10^{-11} \text{ m}$$

$$(d) \text{Momentum } p = mv = 9.1 \times 10^{-31} \times 9.376 \times 10^6$$

$$\text{Ans. } p = 8.532 \times 10^{-24} \text{ kg m/sec}$$

(e) Wave number of an electron wave

$$K = \frac{2\pi}{\lambda} = \frac{2 \times 3.14}{7.77 \times 10^{-11}}$$

$$\text{Ans. } K = 8.082 \times 10^{10} \text{ m.}$$

1. If the uncertainty in position of an electron is 4×10^{-10} m, calculate the uncertainty in its momentum.

Given : Uncertainty in measurement of position (Δx) = 4×10^{-10} m

Formula : Heisenberg's uncertainty principle

$$\Delta x \cdot \Delta p \geq \frac{\hbar}{2}$$

$$\Delta x \cdot \Delta p \approx \frac{\hbar}{2}$$

$$\Delta x \cdot \Delta p = \frac{\hbar}{4\pi}$$

$$\Delta p = \frac{\hbar}{4\pi} \times \frac{1}{\Delta x}$$

$$= \frac{6.63 \times 10^{-34}}{4\pi} \times \frac{1}{4 \times 10^{-10}}$$

Ans. $\Delta p = 1.319 \times 10^{-25}$ kg-m/sec

2. If diameter of the nucleus is 8.5×10^{-14} m. Calculate minimum momentum of proton and minimum kinetic energy of proton.

Given : Diameter of the nucleus (Δx) = 8.5×10^{-14} m

Formula :

- (a) To calculate the minimum momentum of proton :

According to Heisenberg's uncertainty principle,

$$\Delta x \cdot \Delta p \geq \frac{\hbar}{2}$$

$$\Delta x \cdot \Delta p \approx \frac{\hbar}{2}$$

$$(\Delta x)_{\text{max}} \cdot (\Delta p)_{\text{min}} = \frac{\hbar}{4\pi}$$

$$(\Delta p)_{\text{min}} = \frac{\hbar}{4\pi} \times \frac{1}{(\Delta x)_{\text{max}}} = \frac{6.63 \times 10^{-34}}{4\pi} \times \frac{1}{8.5 \times 10^{-14}}$$

Ans. $(\Delta p) = 6.210 \times 10^{-22}$ kg-m/sec.

Minimum momentum $(\Delta p)_{\text{min}} = 6.20 \times 10^{-22}$ kgm/sec.

- (b) To calculate the minimum energy of proton :

$$E = \frac{mv^2}{2}$$

$$E = \frac{m^2v^2}{2m}$$

$$(E)_{\text{min}} = \frac{(p^2)_{\text{min}}}{2m}$$

$$= \frac{(6.20 \times 10^{-22})^2}{2 \times 1.67 \times 10^{-27}}$$

$$(E)_{\min} = 1.150 \times 10^{-16} \text{ joules}$$

$$(E)_{\min} = \frac{1.150 \times 10^{-16}}{1.6 \times 10^{-19}}$$

Ans. $(E)_{\min} = 718.75 \text{ eV}$

Minimum energy of proton in $(E)_{\min} = 718 \text{ eV}$.

3. A hydrogen atom has diameter of 0.53 \AA . Estimate the minimum energy an electron can have in this atom.

Given : $\Delta x = 0.53 \text{ \AA} = 0.53 \times 10^{-10} \text{ m}$

- (a) Calculation of minimum momentum of an electron

Formula :

$$\Delta x \cdot \Delta p \geq \frac{\hbar}{2}$$

$$\Delta x \cdot \Delta p \approx \frac{\hbar}{2}$$

$$(\Delta x)_{\max} \cdot (\Delta p)_{\min} = \frac{\hbar}{4\pi}$$

$$(\Delta p)_{\min} = \frac{\hbar}{4\pi} \times \frac{1}{(\Delta x)_{\max}}$$

$$(\Delta p)_{\min} = \frac{6.63 \times 10^{-34}}{4\pi} \times \frac{1}{0.53 \times 10^{-10}}$$

Ans. $(\Delta p)_{\min} = 9.959 \times 10^{-25} \text{ kg m/sec.}$

- (b) Calculation of minimum energy of an electron

$$\text{Kinetic energy} = \frac{1}{2} mv^2 = \frac{m^2 v^2}{2m} = \frac{P^2}{2m} \quad (\because (p)_{\min} = (\Delta p)_{\min})$$

$$\text{K.E. } (E)_{\min} = \frac{(P^2)_{\min}}{2m}$$

$$= \frac{(9.959 \times 10^{-25})^2}{2 \times 9.1 \times 10^{-31}}$$

$$= 5.44 \times 10^{-19} \text{ joules}$$

Ans. Kinetic energy $(E)_{\min} = 5.44 \times 10^{-19} \text{ joules.}$

Minimum energy of an electron = $5.44 \times 10^{-19} \text{ joules.}$

1. Find out the lowest energy of an electron in a one dimensional box width of 4 \AA .
Given : Width of box (a) = $4 \text{ \AA} = 4 \times 10^{-10} \text{ m}$

Formula : Electron in one dimensional potential well,

$$E_n = \frac{n^2 h^2}{8ma^2}$$

Lowest energy of an electron can be calculated by putting $n = 1$,

$$E_1 = \frac{1^2 \times (6.63 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (4 \times 10^{-10})^2}$$

$$E_1 = 3.77 \times 10^{-19} \text{ joules}$$

$$E_1 = \frac{3.77 \times 10^{-19}}{1.6 \times 10^{-19}}$$

$$E_1 = 2.35 \text{ eV}$$

Ans. The lowest energy of an electron volt = 2.35eV.

2. An electron is bound in a one-dimensional potential well of width 2 \AA , but of infinite height. Find its energy values in the ground state and first two excited states.

Given : Width of potential well (a) = $2 \text{ \AA} = 2 \times 10^{-10} \text{ m}$

Formula : Energy of an electron in one dimensional potential well,

$$E_n = \frac{n^2 h^2}{8ma^2}$$

(a) Calculation of energy of an electron in the ground state [put $n = 1$].

$$E_1 = \frac{1^2 \times (6.63 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (2 \times 10^{-10})^2}$$

$$= 1.50 \times 10^{-18} \text{ joules}$$

$$E_1 = \frac{1.50 \times 10^{-18}}{1.6 \times 10^{-19}}$$

$$E_1 = 9.375 \text{ eV}$$

(b) Calculation of energy of an electron in the first excited state [put $n = 2$].

$$E_2 = \frac{2^2 \times (6.63 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (2 \times 10^{-10})^2}$$

$$E_2 = 6.03 \times 10^{-18} \text{ joules}$$

$$E_2 = 37.68 \text{ eV}$$

(c) Calculation of energy of an electron in the second excited state [put n = 3].

$$E_3 = \frac{3^2 \times (6.63 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (2 \times 10^{-10})^2}$$

$$E_3 = 1.35 \times 10^{-17} \text{ joules}$$

$$E_3 = \frac{1.35 \times 10^{-17}}{1.6 \times 10^{-19}}$$

$$E_3 = 84.37 \text{ eV}$$

Ans. $E_1 = 9.375 \text{ eV}$, $E_2 = 37.68 \text{ eV}$, $E_3 = 84.37 \text{ eV}$.

3. An electron is confined to move between two right walls separated by 1nm. Find the De-Broglie wavelength representing the first two allowed energy states of the electron and the corresponding energies.

Formula : $E_n = \frac{n^2 h^2}{8mL^2}$

Given : L = $1 \times 10^{-9} \text{ m}$

For first allowed state, n = 1

$$E_1 = \frac{1^2 \times (6.63 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (10^{-9})^2} = 6.03 \times 10^{-20} \text{ joules}$$

$$E_2 = 4 \times 6.03 \times 10^{-20} = 2.415 \times 10^{-19} \text{ joules}$$

De-Broglie wavelength for the first level

$$\lambda_1 = \frac{h}{\sqrt{2mE_1}} = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 6.03 \times 10^{-20}}} = 2.0 \times 10^{-9} \text{ m}$$

$$\lambda_2 = \frac{h}{\sqrt{2mE_2}} = \frac{\lambda_1}{2} = 1.0 \times 10^{-19} \text{ m}$$

Ans. $E_1 = 6.03 \times 10^{-20} \text{ joules}$, $\lambda_1 = 2.0 \times 10^{-9} \text{ m}$

$E_2 = 2.415 \times 10^{-19} \text{ joules}$, $\lambda_2 = 1.0 \times 10^{-19} \text{ m}$

4. Lowest energy of an electron trapped in a potential well is 38eV. Calculate the width of the well.

Given : $E_1 = 38 \text{ eV}$

Formula : $E_n = \frac{n^2 h^2}{8mL^2}$

For lowest energy, n = 1

$$\therefore E_1 = \frac{h^2}{8mL^2} \rightarrow L^2 = \frac{h^2}{8mE_1}$$

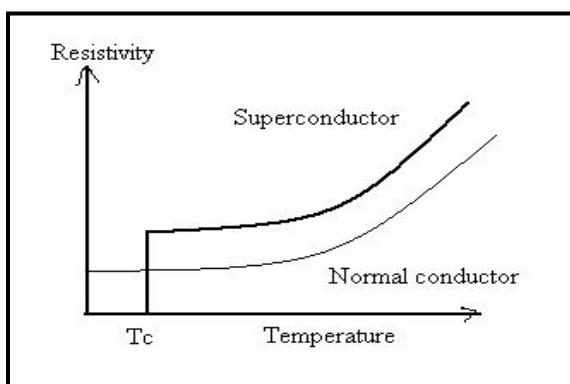
$$L^2 = \frac{(6.63 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times 38 \times 1.6 \times 10^{-19}}$$

$$L^2 = 9.93 \times 10^{-21}$$

Ans. $\therefore L = 0.996 \text{ \AA} \approx 1 \text{ \AA}$.

SUPERCONDUCTORS AND SUPERCAPACITORS

Metals and semiconductor are the conductors. Metals are PTC of resistor and semiconductors are NTC of resistor. Metals consist of large number of free electrons. Due to the vibrational motion of electros, metals offer resistance to the flow of electric current. The scientist Kammerlingh Onnes in 1957 was studied the variation of resistance of certain metals at low temperature. He used helium gas as coolant and measured the resistivity of Hg at low temperature. He observed that resistivity of mercury drops suddenly to zero at liquid helium temperature 4.2^0K . Above temperature 4.2^0K , mercury is in the state of normal conductor, but below 4.2^0K , the mercury transforms in to super conducting state.



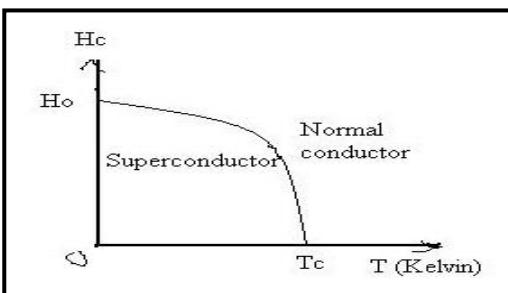
Superconductivity :-

"The phenomenon in which the resistivity of material drops suddenly to zero when cooled to sufficiently low temperature is called superconductivity".

Critical temperature (T_c):-

"The minimum temperature at which the material is transform from the state of normal conductor to the superconducting state is called critical temperature or transition temperature T_c ". The critical temperature is different for different superconductors.

Critical magnetic field (H_c):-



Superconductivity of material depends upon magnetic field. When the super conductors are cooled below its critical temperature in presence of external magnetic field, then up to certain minimum magnetic field material remains in the superconducting state and after that material becomes normal conductor.

'The minimum magnetic field up to which the material remains in the superconducting state and after that material becomes normal conductor s called critical magnetic field.'

$$H_c = H_0 \left[1 - \left(\frac{T}{T_c} \right)^2 \right]$$

Where H_0 is magnetic field at 0°K temp.

The critical magnetic field depends upon the critical temperature and temperature of the surroundings. The variation of critical magnetic field with respect to critical temperature is as shown in fig.

Critical current density (Ic):-

When voltage is applied to the superconductor, the minimum current density up to which the material remains in the superconducting state is called critical current density.

Persistent current:-

If a current is initiated in a coil of superconductor, that current remains circulating in the coil with out any external emf up to several years and coil becomes a permanent magnet. This phenomenon is called persistent current. This phenomenon is used to construct a powerful electromagnet.

BCS theory of superconductivity:-

- 1) All the metals are good conductor of electricity. Metals consist of large number of free electrons. The free electrons are in vibrational motion.
- 2) There is force of repulsion between these electrons. The electrons are taking collision with each other as well as with the crystal lattice. Therefore metal possess resistivity and power loss I^2R occur when current flows through the metal.
- 3) When the metal cools at sufficiently lower temperature, vibrational motion of electrons decreases and exchange of force takes place.
- 4) When the electrons have opposite spin and momenta, then electrostatic force of attraction becomes maximum as compare to electrostatic force of repulsion between them.

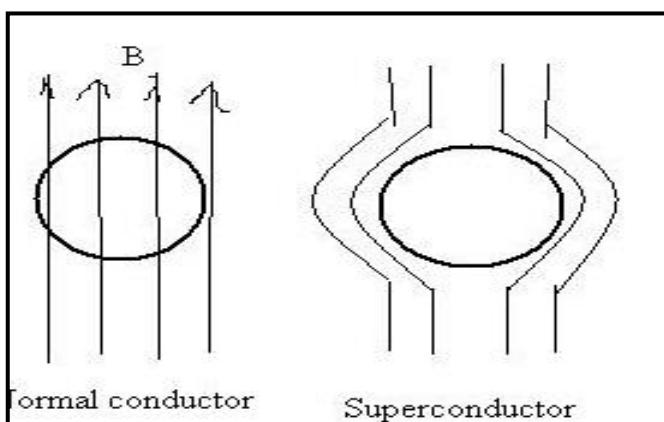
- 5) All the free electrons in the metal become a bound system and electron pairs are formed in the material. These electron pairs are known as Cooper's pair. Formation of Cooper pairs decreases the resistivity of metal and resistivity of metal becomes zero at critical temperature.
- 6) "The minimum temperature at which the material is transform from the state of normal conductor to the superconducting state is called critical temperature or transition temperature T_c ".

The critical temperature is different for different superconductors.

e.g. Pb, Sn, Zn, Nb_3Sn etc.

Meissner effect in superconductors:-

"When superconductors are cooled below its critical temperature in magnetic field, then in superconducting state, material repels the magnetic lines of force". This effect is known as Meissner effect.



Explanation:- Consider superconductor in the form of cylinder is kept in a magnetic field of induction B . Above the critical temperature, when the material is in the state of normal conductor, then magnetic lines of force are penetrates through the material. But below the critical temperature, when the material is in

Superconducting state, the magnetic lines of force are repels out from the material and material acts as perfectly diamagnetic.

Above critical temperature T_c , the material is normal conductor, therefore total flux density in the material is

$$B = \mu_0 H + \mu_0 M$$

Where $\mu_0 H \rightarrow$ flux density due to applied magnetic field H .

$\mu_0 M \rightarrow$ Flux density due to magnetization of the material.

Below critical temperature T_c , the material becomes superconductor, and total flux density in the material becomes zero.

$$0 = \mu_0 H + \mu_0 M$$

$$\mu_0 M = -\mu_0 H$$

$$\frac{M}{H} = -1$$

$\frac{M}{H}$ is known as magnetic susceptibility (χ)

$$\text{Therefore } \chi = -1$$

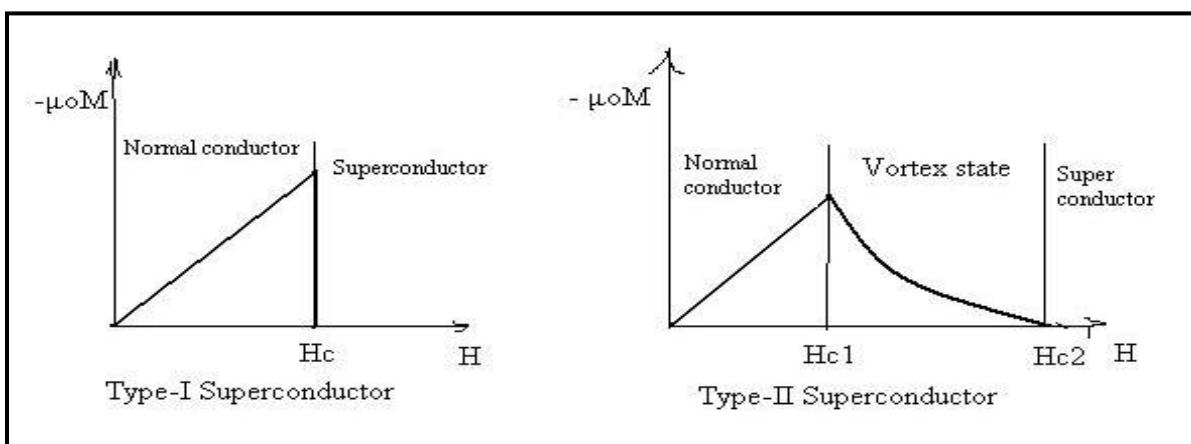
Magnetic susceptibility for superconductors is – Ve, hence superconductors are perfectly diamagnetic material. The magnetization in the superconductor is negative.

If a small magnet is kept on the surface of superconductors, then it floats on the surface of the superconductors is known as magnetic levitation.

Type- I and Type – II superconductors:-

When the superconductors are kept in magnetic field, the magnetic field can penetrate through the small region on the surface of the material. The length up to which the magnetic field can penetrate through the surface of the superconductors is called London penetration depth.

Depending upon the magnetization of material with applied magnetic field, the superconductors are classified in to two types.



Type – I Superconductors:-

- 1) When superconductors are kept in magnetic field, the magnetization in the material increases linearly in proportion with applied magnetic field up to critical field H_c . At

critical field H_c , the magnetization in the material suddenly drops to zero. Such materials are known as type – I superconductors. The magnetization is negative.

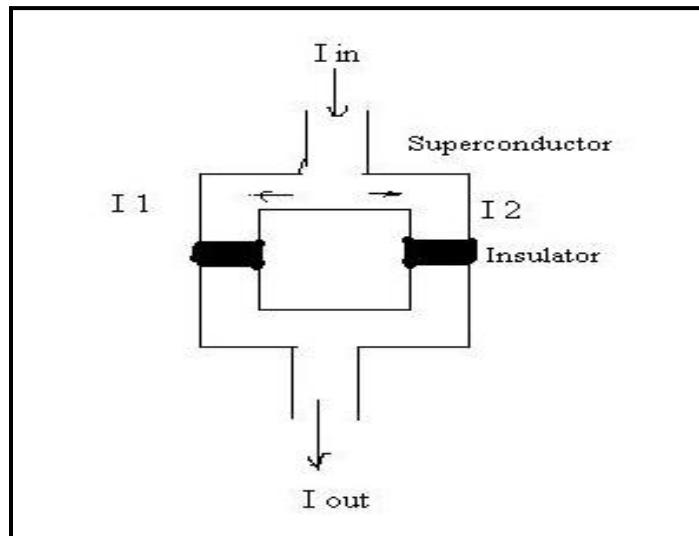
- 2) Below H_c , the material is in superconducting state and above critical field, the material becomes normal conductor. Therefore complete Meissner effect takes place.
- 3) Critical field is low.
- 4) Critical temperature is low.
- 5) Type-I superconductors are also known as soft superconductor.
- 6) It is used for small magnetic field applications.
- 7) Type-I superconductivity is explained by BCS theory.
- 8) E.g. Pb, Hg, Al, Sn etc.

Type-II Superconductors:-

- 1) When superconductors are kept in magnetic field, the magnetization in the material increases linearly in proportion with applied magnetic field up to lower critical field H_{c1} . From critical field H_{c1} , the magnetization in the material starts to decrease and completely becomes zero at higher critical field H_{c2} . Such materials are known as type – II superconductors. The magnetization is negative.
- 2) Below H_{c1} , the material is in superconducting state and above critical field H_{c2} , the material becomes normal conductor. Therefore complete Meissner effect will not takes place.
- 3) Critical field is high.
- 4) Critical temperature is high.
- 5) Type-I superconductors are also known as hard superconductor.
- 6) It is used to construct powerful electromagnet.
- 7) Type-II superconductivity cannot be explained by BCS theory.
- 8) In between critical field H_{c1} and H_{c2} , the material is electrically superconductor but magnetically it is normal conductor. The state of superconductor in between H_{c1} and H_{c2} is called vortex state.
- 9) E.g. alloy niobium, vanadium, silicon, transition metals etc.

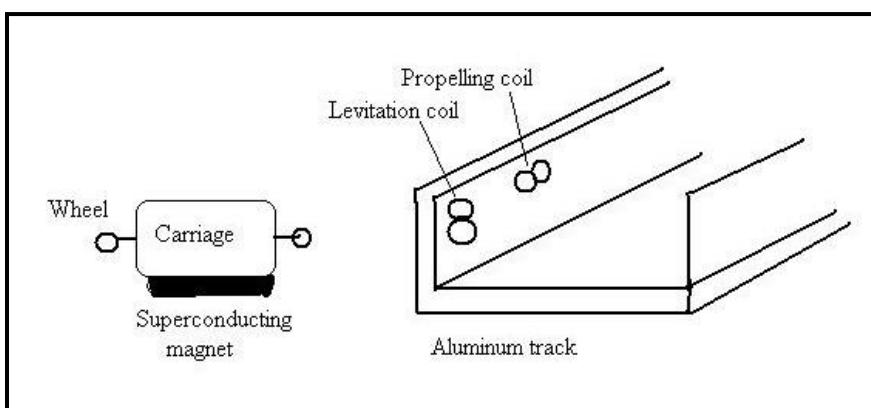
Application of superconductor:

SQUID (Superconducting Quantum Interface Device):



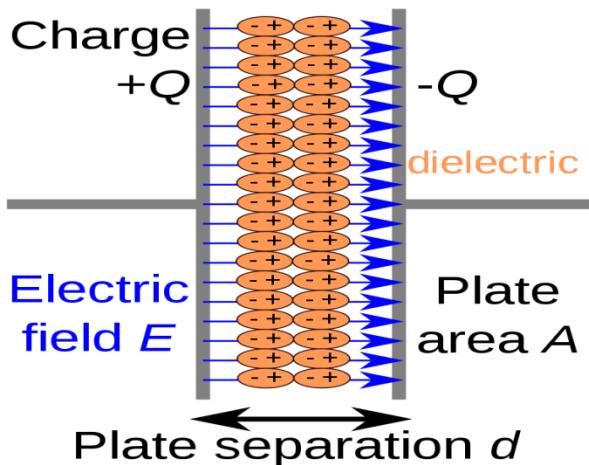
- 1) SQUID is Superconducting Quantum Interface Device used to measure very small magnetic field of the order of 10^{-15} Tesla.
- 2) It consists of two Josephson junction in a superconducting loop.
- 3) The input current flowing through the device is I_{in} and output current is I_{out} . The current I_{in} is divided into two parts I_1 and I_2 passing through the Josephson junction.
- 4) The applied magnetic changes the phases of currents I_1 and I_2 , according to this output current I_{out} changes.
- 5) By detecting the changes in output current, we can measure the very small magnetic field.
- 6) SQUID is used to measure small magnetic pulses produced by human heart and brain.

Magnetic levitation or Maglev:-



- 1) Maglev means magnetic levitation. Magnetic susceptibility for superconductors is $-Ve$, hence superconductors are perfectly diamagnetic material. The magnetization in the superconductor is negative.
- 2) If a small magnet is kept on the surface of superconductors, then it floats on the surface of the superconductors is known as magnetic levitation.
- 3) If a current is initiated in a coil of superconductor, that current remains circulating in the coil without any external emf up to several years and coil becomes a permanent magnet. This phenomenon is called persistent current. This phenomenon is used to construct a powerful electromagnet.
- 4) The carriage and aluminum track of the maglev train is as show in figure. A strong superconducting electromagnet is fitted at the bottom of the carriage.
- 5) The walls of the track consist of series of horizontal and vertical coils of shape like '8' made of ordinary material. Vertical coil is known as levitation coil and horizontal coil is known as propelling coil.
- 6) When carriage starts to run over a track, the motion of carriage induces electric current in the levitation and propelling coil. Therefore these coils became electromagnets. The levitation coil lift the carriage above the track by the distance 5 to 10 cm and propelling coil push the carriage in forward direction.
- 7) Due to frictionless contact between wheels of the carriage and the track, the maglev train can attain very high speed of the order of 500 km/h.

Super capacitor:



Conventional capacitors consist of two conducting electrodes separated by an insulating dielectric material. When a voltage is applied to a capacitor, opposite charges accumulate on the surfaces of each electrode. The charges are kept separate by the dielectric, thus producing an electric field that allows the capacitor to store energy.

Capacitance C is defined as the ratio of stored (positive) charge Q to the applied voltage V.

$$C = Q/V$$

For a conventional capacitor, C is directly proportional to the surface area A of each electrode and inversely proportional to the distance D between the electrodes: $C \propto A / D$

$$C = \epsilon_0 \epsilon_r A / D .$$

The product of the first two factors on the right hand side of the last equation is a constant of proportionality wherein ϵ_0 is the dielectric constant (or “permittivity”) of free space and ϵ_r is the dielectric constant of the insulating material between the electrodes.

Thus Capacitance depends upon three main factors.

- Surface Area – the surface area, A of the two conductive plates which make up the capacitor, the larger the area the greater the capacitance.
- Distance – the distance, d between the two plates, the smaller the distance the greater the capacitance.
- Dielectric Material – the type of material which separates the two plates called the “dielectric”, the higher the permittivity of the dielectric the greater the capacitance.

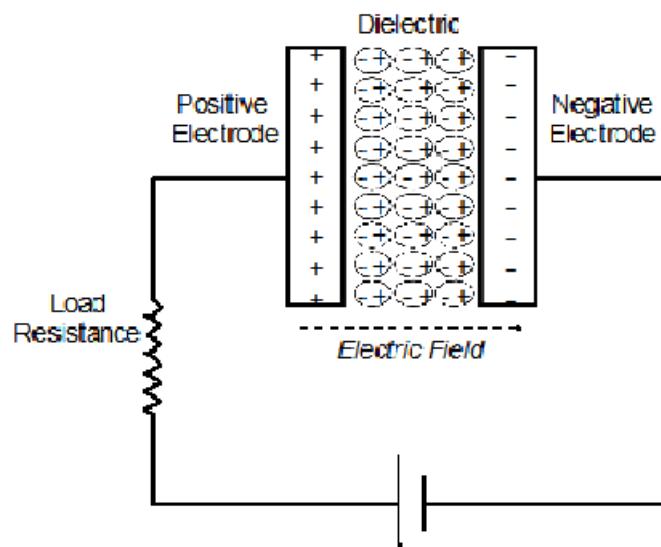
The two primary attributes of a capacitor are its energy density and power density.

Energy density

The energy density can be calculated as a quantity per unit mass or per unit volume. The energy E stored in a capacitor is directly proportional to its capacitance: $E = 1/2 CV^2$.

Power density

The power density P is the energy expended per unit time. To determine P for a capacitor, though, one must consider that capacitors are generally represented as a circuit in series with an external "load" resistance R .



The maximum power P_{max} for a capacitor is given by:

$$P_{max} = V^2 / 4 R \quad \text{Where } R \text{ is equivalent series resistance.}$$

- . The ESR can limit the maximum power of a capacitor.

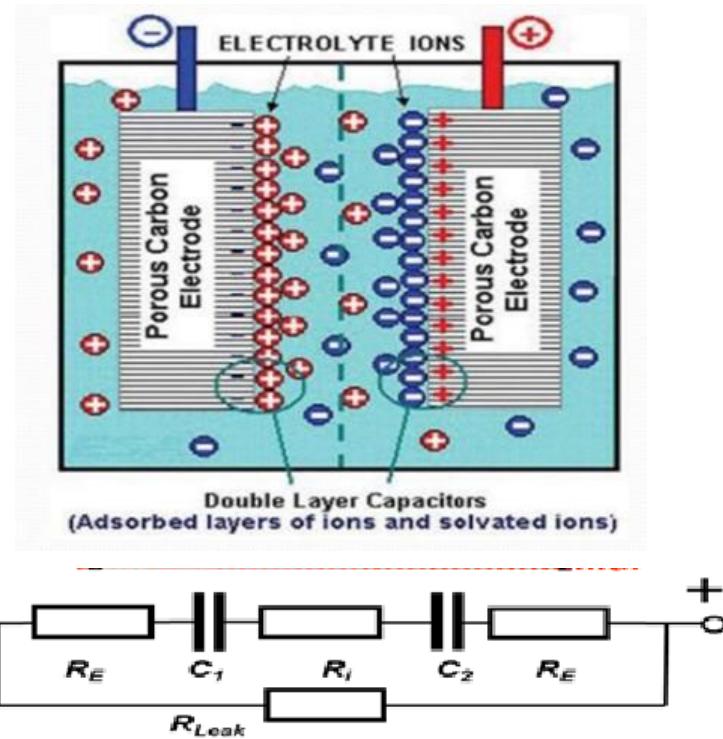
Thus energy density depends upon capacitance of capacitor and power density depends upon equivalent series resistance.

Super capacitors:

A super capacitor is an electrochemical capacitor that has a very high energy density. Super capacitor has high capacitance and stores high amount of energy.

Super capacitors, also known as ultra-capacitors or electrochemical capacitors, utilize high surface area electrode materials and thin electrolytic dielectrics to achieve capacitances several orders of magnitude larger than conventional capacitors.

Storage principles



Super capacitors have two metal plates. These plates are coated with activated carbon which is a sponge-like porous material. These plates are immersed in an electrolyte which contains positive and negative ions. One carbon-coated plate, or electrode, is positive, and the other is negative. During charging, ions from the electrolyte accumulate on the surface of each carbon-coated plate.

Basic super capacitor is double layer capacitor cell

Energy storage occurs within the double-layers of both electrodes as a mixture of a double-layer capacitance and pseudo capacitance.

The distance between the two charge layers at the electrode is exceedingly small and this means that very high capacitance levels are achievable. Also as there are charge layers at each electrode, in effect a super capacitor is actually two capacitors in series, one at each electrode

Electrical energy is stored in super capacitors via two storage principles, static double-layer

capacitance and electrochemical pseudo capacitance. The distribution of the two types of capacitance depends on the material and structure of the electrodes.

Super capacitors are classified into double layer capacitors, pseudo capacitors and hybrid capacitors based on their charge storage methods. Double-layer capacitors use electrostatic charge storage, Pseudo capacitors use electrochemical and

There are two types of double layer capacitor resulting from different charge storage mechanisms:

- ***Electrical double-layer capacitor:*** Double-layer capacitors use electrostatic charge storage. An EDLC stores energy in the double-layer at the electrode/electrolyte interface. In this type of capacitor, the electrode material used for the construction of the cell for the former is mainly carbon material.
- ***Electrochemical double layer capacitor or Pseudo-capacitor:*** In this type of super capacitor the electrode material consists of either transition metal oxides or mixtures of carbon and metal oxides/polymers. Pseudo capacitors use electrochemical storage.
- ***Hybrid capacitor:*** Hybrid capacitors use both electrostatic and electrochemical charge storage.

The electrolytes can be either aqueous or non-aqueous depending on the mode of construction of capacitor cell.

Comparison of super capacitor with capacitor and batteries:

Benefits or advantages of Super capacitor

Following are the benefits or **advantages of Super capacitor:**

- It offers high energy density and high power density compare to common capacitor.
- It offers high capacitance (From 1 mF to >10,000F) .
- It offers fast charging ability.
- It offers superior low temperature performance (from -40°C to 70°C).
- It offers longer Service and long life (about 10 to 15 years compare to 5-10 years of Li-ion battery) . It offers virtually unlimited cycle life and can be cycled millions of time.
- It offers higher reliability of performance.

- It reduces size of the battery, its weight and consecutively cost.
- Super capacitors meet environmental standards. Hence they are eco-friendly.

Drawbacks or disadvantages of Super capacitor

Following are the drawbacks or **disadvantages of Super capacitor**:

- They have higher self-discharge rate. This is considerably high compare to battery.
- Individual cells have low voltages. Hence series connections are required in order to achieve higher voltages.
- Amount of energy stored per unit weight is considerably lower compare to electrochemical battery. This is about 3 to 5 W.h/Kg for an ultra-capacitor than 30 to 40 W.h/Kg of a battery.
- It offers low energy density compare to battery. This is about $(1/5)^{\text{th}}$ to $(1/10)^{\text{th}}$ the energy of the battery.
- It cannot be used in AC and higher frequency circuits.

ENGINEERING MATERIALS AND APPLICATIONS

Magnetic material

Prerequisites:

Magnetic material: - The material which when kept in the magnetic field, either strongly attracted or repelled by the magnetic field are called magnetic materials.

Magnetic field: - The space surrounding the given magnet in which other magnetic material experience a force of attraction or repulsion is called magnetic field.

Magnetic flux (ϕ): - The total number of magnetic lines of force near a magnetic source is called magnetic flux (ϕ). (SI unit – weber)

Magnetic induction (B):- Magnetic flux passing per unit area is called magnetic flux density or magnetic induction. (SI unit – weber / m² or Tesla)

$$B = \frac{\phi}{A} \quad wb/m^2$$

Magnetic field strength or magnetizing force (H) : - The force experienced by the unit north pole when kept in the magnetic field is called magnetic field strength or magnetic field intensity. (SI unit – ampere / m)

Magnetic dipole: - The two opposite poles (north and south) separated by certain distance is called magnetic dipole.

Magnetic dipole moment (μ_m) : - The product of pole strength of one pole and distance between two poles is called magnetic dipole moment.

The magnetic dipole moment of a current loop is $\mu_m = I \times A \quad \text{Amp} - \text{m}^2$.

Where I is current flowing through the loop and A is area enclosed by the loop.

Magnetization (M) : - The magnetic dipole moment induced in the material per unit volume is called magnetization. (A / m)

Magnetic susceptibility (x) : - The ratio of magnetization in the material to the applied magnetic field strength is called magnetic susceptibility.

$$X = \frac{M}{H}$$

Permeability or absolute permeability (μ_a) : - The ability of the material to conduct the magnetic flux through it is called permeability.

The ratio of magnetic flux density to the magnetic field strength is called magnetic permeability.

$$\mu_a = \frac{B}{H} \quad \text{therefore} \quad B = \mu_a H$$

Permeability of free space is constant and has a value of $\mu_0 = 4\pi \times 10^{-7} \text{ H/m}$.

Relative permeability (μ_r) : - The ratio of permeability of the medium (μ_a) to the permeability of free space (μ_0) is called relative permeability of medium.

$$\mu_r = \frac{\mu_a}{\mu_0} \quad \text{therefore} \quad \mu_a = \mu_0 \mu_r$$

Atomic theory of origin of magnetization:

1. In the atoms of the material, electrons are revolving around the nucleus in various orbits.
2. The revolving electron has orbital as well as spin motion.
3. The orbit of an electron is like a current carrying loop. The magnetic dipole moment associated with the current carrying loop is given by

$$\mu = IA$$

If r is radius of orbit and f is frequency of revolution of an electron, then

$$I = -ef \quad \text{and} \quad A = \pi r^2$$

Therefore, orbital magnetic dipole moment of electron is

$$\mu (\text{orbital}) = -ef \pi r^2$$

But $f = \frac{1}{T}$ and $T = \frac{2\pi r}{v}$ where v is velocity of electron.

$$\text{Therefore } \mu = -\frac{evr}{2}$$

The orbital angular momentum of electron is $L = mvr$

$$\text{Therefore } vr = \frac{L}{m}$$

$$\text{Hence } \mu (\text{orbital}) = -\frac{e}{2m} L$$

Similarly magnetic dipole moment due to spin motion of an electron is

$$\mu (\text{spin}) = - \frac{e}{m} S$$

Where S is spin angular momentum of electron.

4. The resultant magnetic dipole moment of an electron is the vector sum of the orbital and spin magnetic dipole moment. The magnetic dipole moment of an atom is the vector sum of the magnetic dipole moment due to all the electrons. The magnetic dipole moment of all the atoms in the material gives the resultant magnetic dipole moment of that material.
5. If the resultant magnetic dipole moment produces the magnetic field, then material behaves like magnet.
6. When material is kept in the magnetic field, the magnetic dipole moments of electrons are modified. According to the behavior of the material in the magnetic field, the material is classified in to diamagnetic, paramagnetic and ferromagnetic material.

Types of magnetic materials:-

Depending upon the magnetization in the material with applied magnetic field, the magnetic material are classified in to three basic type as diamagnetic, paramagnetic and ferromagnetic material.

Diamagnetic material:-

- 1) The material which when kept in the magnetic field acquire feeble magnetism in the opposite direction to the direction of applied magnetic field are called diamagnetic material.
- 2) Do not have permanent magnetic dipole moment.
- 3) Magnetization is in opposite direction to the applied magnetic field.
- 4) Magnetic susceptibility is – ve.
- 5) Susceptibility does not depend upon temperature.
- 6) They reduce the magnetic flux density.
- 7) Relative permeability is less than 1.
- 8) They weakly repelled by magnetic field.
- 9) They can exist in any phase of matter.

E.g. gold, silver, water, argon, alcohol, mercury, copper, lead, bismuth etc.

Paramagnetic material:-

- 1) The material which when kept in the magnetic field acquire feeble magnetism in the direction of applied magnetic field are called paramagnetic material.
- 2) They have permanent magnetic dipole moment but since magnetic dipoles are oriented randomly. The magnetization is isotropic.
- 3) Magnetization is in direction of applied magnetic field and varies linearly with applied magnetic field.
- 4) Magnetic susceptibility is + ve but very small.
- 5) Susceptibility depends upon temperature.
- 6) They concentrate the magnetic flux density and weakly magnetized.
- 7) Relative permeability is greater than 1.
- 8) They weakly attracted by magnetic field and cannot be permanently magnetized.
- 9) They can exist in any phase of matter.

E.g. aluminium, platinum, oxygen, calcium, magnesium, tungsten etc.

Ferromagnetic material:-

- 1) The material which when kept in the magnetic field acquire strong magnetism in the direction of applied magnetic field are called ferromagnetic material.
- 2) They have permanent magnetic dipole moment and magnetic dipoles are distributed in domains. The magnetization is anisotropic.
- 3) Magnetization is in direction of applied magnetic field and varies non linearly with applied magnetic field. Therefore exhibits hysteresis effect.
- 4) Magnetic susceptibility is + ve but very large.
- 5) Susceptibility depends upon temperature.
- 6) They concentrate the magnetic flux density and strongly magnetized.
- 7) Relative permeability is greater than 1.
- 8) They strongly attracted by magnetic field and can be permanently magnetized.
- 9) They exist only in solid state of matter.

E.g. iron, cobalt, nickel, gadolinium etc.

Depending upon the magnitude and alignment of magnetic dipoles, the ferromagnetic materials are classified in to two types a) Antiferromagnetic magnetic. b) Ferrimagnetic material.

Antiferromagnetic material:-

- 1) In antiferromagnetic material, all the magnetic dipoles are equal in magnitude and interaction between adjacent dipoles is such that they align antiparallel to each other.
- 2) The opposite alignment of adjacent dipole is due to an exchange of interaction.
- 3) All the dipoles are equal in magnitudes, therefore resultant magnetization is zero.
- 4) Magnetic susceptibility is small, positive and temperature dependent.
- 5) E.g Mn, Cr, MnO₂, Cr₂O₃ etc.

Ferrimagnetic material or Ferrites:-

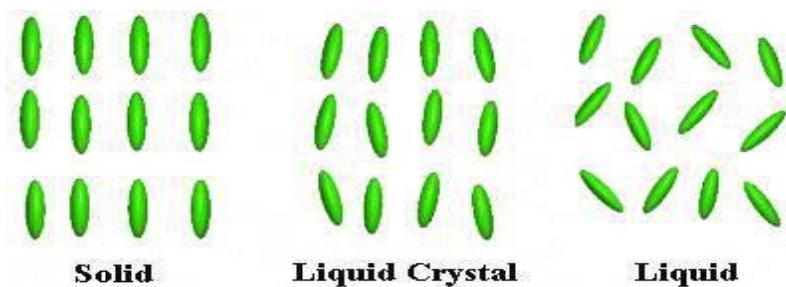
- 1) A group of magnetic alloys exhibits the property of magnetization are called ferrites or ferrimagnetic material. The magnetic alloys is given by XOF₂O₃, where X = Co, Ni, Zn, Cd, Mn etc.
- 2) In ferrimagnetic material, all the magnetic dipoles are not equal in magnitude and interaction between adjacent dipoles is such that they align antiparallel to each other.
- 3) The opposite alignment of adjacent dipole is due to an exchange of interaction.
- 4) All the dipoles are not equal in magnitudes; therefore resultant magnetization is not equal to zero.
- 5) Magnetic susceptibility is large, positive and temperature dependent.
- 6) The small value of applied magnetic field produces large value of magnetization.
- 7) Resistivity is very high and ferromagnetic materials are good electrical insulator.

Syllabus:

Liquid Crystal

- In crystalline solid atoms or molecules are in their positional order as well as long range orientational order. Therefore crystalline solid are anisotropic substance.
- In liquid state of matter, the atoms or molecules do not have positional as well as orientational order. Therefore it is called isotropic substance.
- In liquid crystal state of matter molecules do not have positional order like crystals but have long range orientational order (fluidity) like liquid. The liquid crystal state is a distinct phase of matter between the crystalline (solid) and isotropic (liquid) states known as mesomorphic state.
- Liquid crystal consists of long and rod like organic molecules. These molecules are binding together by strong lateral forces of attraction between them. All molecules do not have the same orientation, but they tend to orient in one direction. It is called director.
- On application of very small voltage, the liquid crystal molecules rearrange their orientation to produce the display.

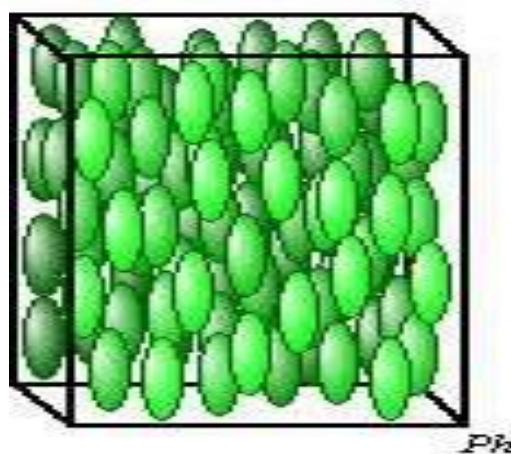
MESOMORPHIC PHASE:-



SOLID STATE heat → LIQUID CRYSTAL heat → LIQUID STATE

There are three types of liquid crystals depending upon the positional order and orientational order of the molecules in this phase.

Nematic Phase:-



- 1) The nematic liquid crystal consists of long thread like molecules. The shape of the molecules is elongated ellipsoid.
- 2) The molecules are aligned in one direction but do not have positional order. The molecules are distributed randomly.
- 3) The physical properties of the material vary with the average alignment of molecules with the director.
- 4) If the alignment is large, the material is very anisotropic. Similarly, if the alignment is small, the material is almost isotropic.

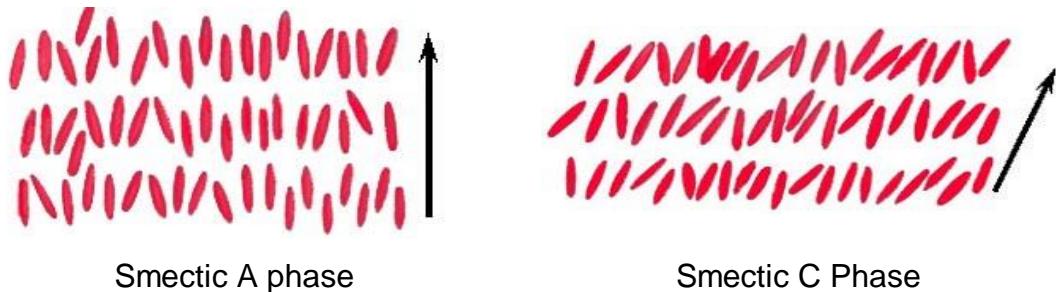
Smectic Phases

- 1) The word "smectic" is derived from the Greek word for soap. In the smectic state, the molecules maintain the general orientational order, but also tend to align themselves in layers or planes.
- 2) Motion is restricted to within these planes, and separate planes are observed to flow past each other. The increased order means that the smectic state is more "solid-like" than the nematic. There are three types in this phase

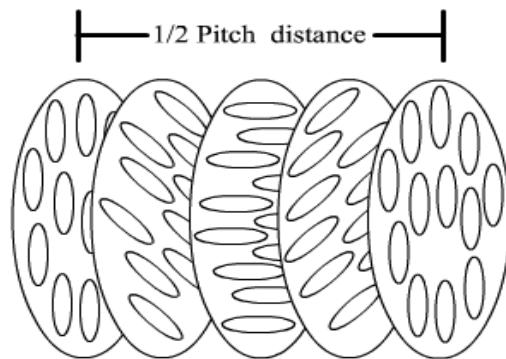
Smectic-A-- The director is perpendicular to the smectic plane, and there is no particular positional order in the layer.

Smectic-B -- The director is perpendicular to the smectic plane, but the molecules are arranged into a network of hexagons within the layer.

Smectic-C -- The director is at a constant tilt angle measured normally to the smectic plane.



Cholesteric Phases

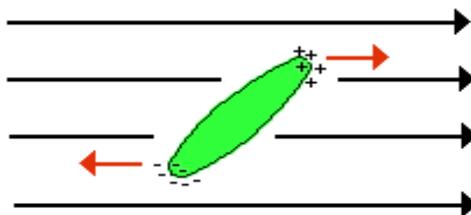


- 1) In cholesteric liquid crystal, the molecules are arranged in parallel planes. In every plane the molecules are aligned in particular direction, but those in adjacent planes, molecules are slightly rotated with respect to each other.
- 2) The molecules shown are merely representations of the many chiral nematic mesogens lying in the slabs of infinitesimal thickness with a distribution of orientation around the director.
- 3) An important characteristic of the cholesteric mesophase is the **pitch**. The pitch, p , is defined as the distance taken by the director to rotate one full turn in the helix. The crystal reflects the light of wavelength equal to pitch length. Any change in temperature will change the director orientation between successive layers, which modifies the pitch length. Hence that color will be reflected when the pitch is equal to the corresponding wavelength of light in the visible spectrum.

- 4) Increase in temperature, increase the angle of orientation of the director and decreases the pitch length. Therefore these liquid crystals are used in a temperature sensor to build a liquid crystal thermometer that displays the temperature of its environment by the reflected color.

Electric and Magnetic Field Effects

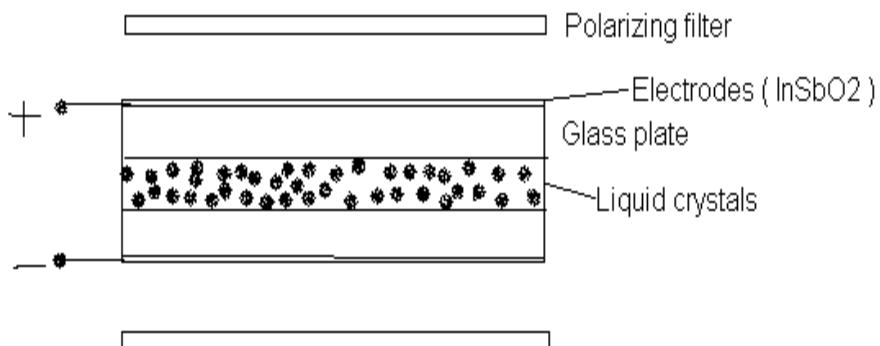
The response of liquid crystal molecules to an electric field is the major characteristic utilized in industrial applications. The ability of the director to align along an external field is caused by the electric nature of the molecules. *Permanent electric dipoles* result when one end of a molecule has a net positive charge while the other end has a net negative charge. When an external electric field is applied to the liquid crystal, the dipole molecules tend to orient themselves along the direction of the field



PROPERTIES OF LIQUID CRYSTAL:-

- 1) The colour of cholesteric materials is sensitive to temperature.
- 2) Nematics and cholesteric materials are sensitive to external electric and magnetic fields.
- 3) They can be stimulated by low voltage and electric power.
- 4) They are sensitive to pressure and chemical contamination.
- 5) Nematic materials are birefringent (They possess two different refractive indices. (One corresponds to light polarized along the director and other due to light polarized perpendicular to the director.)
- 6) Cholesteric materials show the property of circular birefringence and it is highly wavelength dependent.

LIQUID CRYSTAL DISPLAY (LCD):



- 1) Displays are of two types, active and passive. An active display emits light while passive display reflects the incidents light falls on it.
- 2) LCD display is a passive display. In LCD, a thin layer of liquid crystal is sandwiched between two glass plates. The glass plates are separated by a distance of 8 to 10 μm . These plates are coated with transparent conductive films made of indium tin oxide (InSbO_2) which acts as electrodes. The liquid crystal film is divided in to smaller areas known as **pixels**. The electric field is applied to these pixels of the liquid crystal film by these electrodes.
- 3) The polarizing coloured filters are placed on one or both sides of the glass plates. The red, blue and green filters are used on group of 3 pixels called **triads** to obtain coloured displays.
- 4) The display property of LCD based on polarization of light.
- 5) The application of small electric field to the pixels causes the crystal molecules to rearrange their orientation to produce the display. Thus liquid crystal modifies the polarization of light and based upon switching of electric field in to ON or OFF state, the liquid crystal pixels are activated or neutralized. There are various logics systems used to control this feature.

SPECIFICATION OF LCD:-

- 1) Threshold voltage: - It is the minimum voltage applied across the liquid crystal pixels to produce any response.
- 2) Resolution: - The number of rows and columns of liquid crystal pixels to create display.
- 3) Dot pitch: - The size of triads plus distance between two triads.
- 4) Viewable size: - The diagonal length of LCD.

- 5) Response time: - Time to produce required display.
- 6) Viewing angle: - Maximum angle at which display can be viewed with comfort.
- 7) Brightness: - Amount of light emitted from each pixel.
- 8) Contrast: - Ratio of light intensity between brightest and darkest objects.
- 9) Aspect ratio: - Ratio of longest dimension to the shortest dimension in 2-dimensional picture.

PROPERTIES OF LIQUID CRYSTAL:-

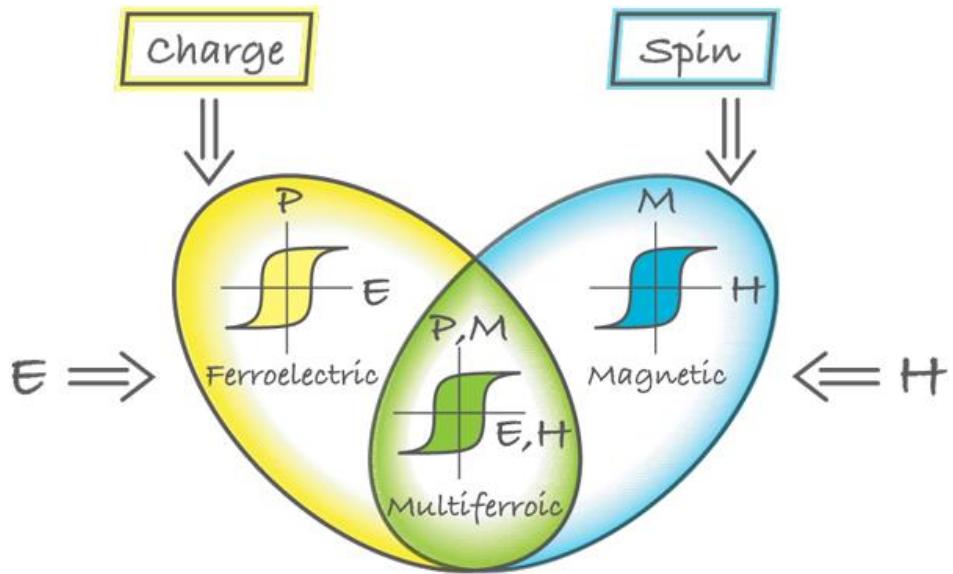
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APPLICATIONS OF LIQUID CRYSTAL:-

- 1) To make display for television, computers, oscilloscopes etc.
- 2) Liquid crystal films are used to modulate light and maintain a good contrast.
- 3) They are used to detect temperature of hot points and also to detect the fracture of bones and tumors.
- 4) They are used in detectors to detect pressure, density changes and chemical contamination in liquid and gases.

Multiferroics material

- 1) Multiferroic materials are magneto electric materials in which ferromagnetic or anti ferromagnetic and ferroelectric properties occur simultaneously.
- 2) **Multiferroics** are defined as materials that exhibit more than one of the primary ferroic properties:
 - ferromagnetism—a magnetization changes by an applied magnetic field,
 - ferroelectricity—an electric polarization changes by an applied electric field, and
 - ferroelasticity—a deformation changes by an applied stress.
- 3) These three types of ferroics are called primary ferroics. Ferroic materials exhibit a hysteresis effect. Ferroics are functional materials whose physical properties are sensitive to changes in external conditions such as temperature, pressure, electric and magnetic fields. A ferroic material exhibits ferroelectricity, ferromagnetism or ferroelastic ordering.
- 4) Multiferroics are single phase materials which simultaneously possess two or more primary ferroic properties. Multiferroics are materials that inherently exhibit both magnetic and electric polarizations. Multiferroicity typically derives from asymmetry in a material's crystal structure. One of the most appealing aspects of multiferroics is magnetoelectric coupling. In a magnetoelectric effect, a magnetic field can tune the electric polarization and an electric field can tune the magnetization.
- 5) In a multiferroic material, the magnetic polarization is usually produced by transition metal ions (like iron and nickel), while the electric polarization (or ferroelectric effect) often relies on ions that shift position through interactions with their neighbors. These shifts can produce an electric polarization, but only if the crystal structure lacks an inversion center around which the crystal is symmetric.
- 6) Schematic illustration of multiferroic defined from the combination of ferroelectric and magnetic properties



- 7) Multiferroics usually belong to the group of the perovskite transition metal oxides. They also include rare-earth manganites and -ferrites (e.g. TbMnO₃, HoMn₂O₅, LuFe₂O₄ and recently, PZTFT). Other examples are bismuth compounds (such as BiFeO₃ and BiMnO₃), and non-oxides such as BaNiF₄ and spinel chalcogenides (ZnCr₂Se₄). Nickel iodine boracite (Ni₃B₇O₁₃I) was the first discovered multiferroic material that was simultaneously ferroelectric and ferromagnetic
- 8) **Application:** Due to the coupling between ferroelectric and ferromagnetic ordering, multiferroics have a wide range of applications in spintronics, multiple state memory, data storage media, sensor, tunable microwave devices.

CLASSIFICATION OF MULTIFERROICS:

According to the origin of magnetic and ferroelectric ordering multiferroics are classified into:

(i) Type I multiferroics:

In this group of multiferroics ferroelectricity and magnetism have different sources and appear largely independently of one another. A weak coupling between ferroelectricity and magnetism is observed. In these materials, ferroelectricity is usually observed at higher temperatures than magnetism and the spontaneous polarization (P) is quite large.

As different ions are involved in magnetism and ferroelectricity the magnetoelectric coupling becomes weak. Examples of such type of materials are BiFeO₃ or BiCrO₃ etc.

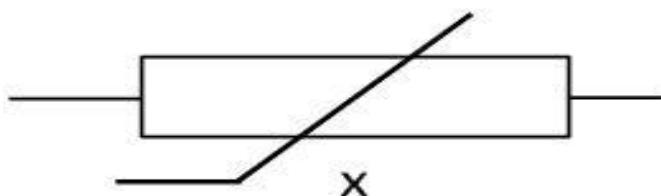
(ii) Type-II multiferroics:

In this type of multiferroics a particular type of magnetic spiral or even for collinear magnetic structures causes ferroelectricity. This implies a strong coupling

between the two. But the polarization in these materials is usually very small and the ferroelectricity usually appears at low temperatures. Examples of such type of materials are TbMnO₃, Ni₃V₂O₈, MnWO₄ etc

Magneto resistor

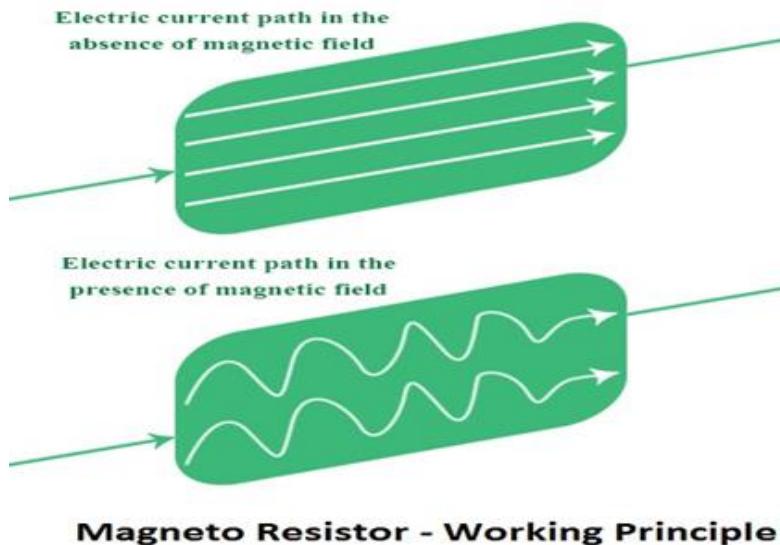
- **Definition:** The resistance of some of the metal and the semiconductor material varies in the presence of the magnetic field. This effect is called the magneto resistance effect.
- The element which has these effects is known as the **magneto resistor**. In other words, the magneto resistor is a type of resistor whose resistance varies with the magnetic field.
- The flow of electric current through the magneto resistor changes when an external magnetic field is applied to it.
- The magneto resistor is used for determining the presence of a magnetic field their strength and the direction of the force.
- The deflection of the magneto resistor electrons depends on their mobility. It is more in the semiconductor material as compared to the metals.
- It is made of the indium antimonide or indium arsenide semiconductor material.



Magnetoresistors

Circuit Globe

How magneto resistors work?

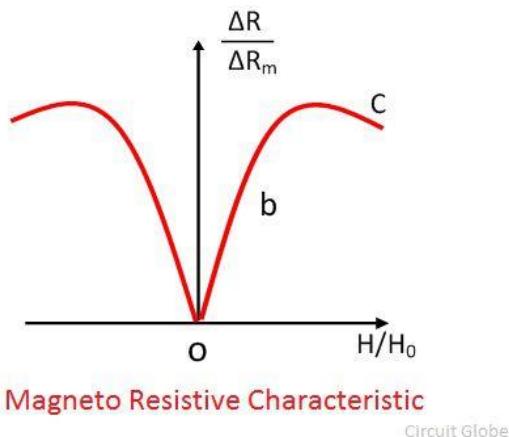


- The magneto resistors that are placed in the magnetic field will experience a change in resistance. When the strength of the magnetic field is increased, the resistance of magneto resistors also increases. On the other hand, when the strength of magnetic field is reduced, the resistance of magneto resistors decreases. This change in resistance is caused by the magneto resistive effect.
- In the absence of magnetic field, the charges carriers in the material move in a straight path. Therefore, electric current flows in a straight path. When the magnetic field is applied to the material, the magnetic forces cause the mobile charge carriers (free electrons) to change their direction from direct path to indirect path. This increases the length of electric current path.
- Hence, large number of free electrons collides with the atoms and loses their energy in the form of heat and only a small number of free electrons flow through the conductive path. The small number of free electrons moving from one place to another place carries the electric current. Therefore, the resistance of the material increases with increasing magnetic field.

- This magneto resistive effect occurs in materials such as semiconductors, non-magnetic metals, and magnetic metals.
- An Irish mathematical physicist and engineer William Thomson first discovered this magneto resistive effect in 1856. He observed that resistance of the pieces of iron increased when the electric current is flowing in the same direction as the magnetic force or magnetic field and the resistance is decreased when the electric current is flowing at 90° to the magnetic field or magnetic force.
- After that, he performed the same experiment with nickel and he found that the resistance of the nickel is affected in the same manner but the magnitude of this magnetic field was much greater than before. This effect is called Anisotropic Magneto Resistance (AMR).

Characteristic of Magneto resistor:

- The sensitivity of the magneto resistor depends on the strength of the magnetic field. The characteristic curve of the magneto resistor is shown in the figure below.



- In the absence of the magnetic field, the magnetization of the element becomes zero. When the magnetic field slightly increases.
- The resistance of the material reaches near to b. the magneto resistor element moves by an angle of 450 , because of the presence of a magnetic field.

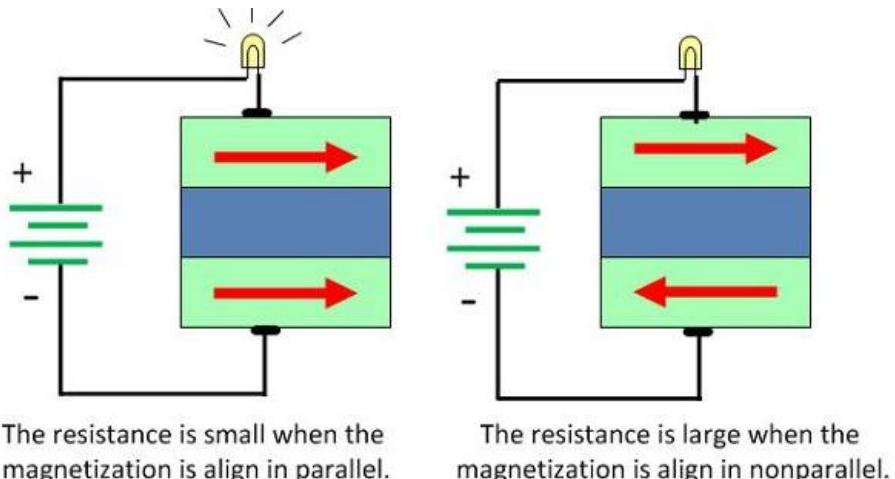
- The further increase in the magnetic field make the curve saturates, which is represented by the point C. the magneto resistive element either operates at O or at b. it gives linear characteristics when operates at b.

Types of magneto resistive effects:

Magneto resistive effects are of three types:

- Giant Magneto Resistance (GMR)
- Extraordinary Magneto Resistance (EMR)
- Tunnel Magneto Resistance (TMR)
- Colossal magneto Resistance (CMR)

Giant Magneto Resistance (GMR)



Giant Magnetoresistor

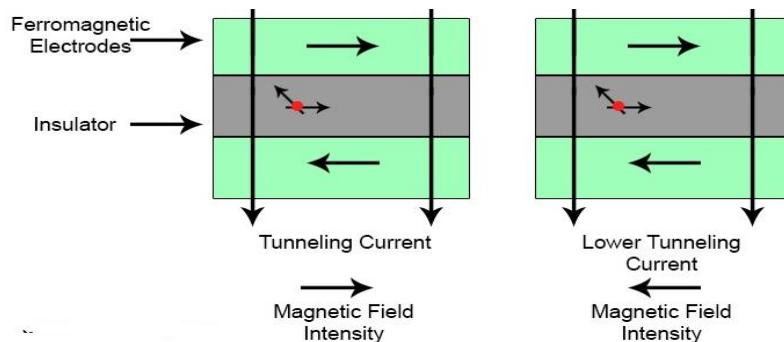
- Albert Fert and Peter Grunberg discovered the Giant Magneto Resistance (GMR) in 1988.
- This effect is observed in the ferromagnetic materials.

- The resistance of the ferromagnetic material depends on whether the magnetization of adjacent ferromagnetic layers are aligned parallel or anti-parallel.
- The resistance is high for the ferromagnetic layers with anti-parallel alignment whereas the resistance is low for the ferromagnetic layers with parallel alignment.

Extraordinary Magneto Resistance (EMR)

- The magnetic field effect of EMR (Extraordinary Magneto Resistance) is much greater than (GMR) Giant Magneto Resistance.
- The EMR effect occurs in semiconductor metal hybrid systems when a transverse magnetic field is applied.
- In the absence of magnetic field, the resistance of semiconductor metal hybrid system is very low whereas in the presence of strong magnetic field, the resistance of semiconductor metal hybrid system is high.

Tunnel Magneto Resistance (TMR)



Tunnel Magneto Resistor

- M. Julliere discovered the Tunnel Magneto Resistance (TMR) in 1975.
- The Tunnel Magneto Resistance (TMR) occurs in Magnetic Tunnel Junction (MTJ). Magnetic Tunnel Junction (MTJ) is a component consists of two ferromagnets separated by an insulator.

- The electrons will flow from one ferromagnet to another ferromagnet through the tunnel barrier or insulator.
- The amount of electric current or charge carriers flowing through the tunnel is depends on orientation of magnetizations.
- If the magnetic field is applied to Magnetic Tunnel Junction (MTJ) in such a way that the direction of magnetizations of ferromagnet is aligned in a parallel manner, a large numbers of electrons flow easily. As a result, electric current increases and resistance decreases.
- On the other hand, if the magnetic field is applied in such as way that the direction of magnetizations of ferromagnet is aligned in an anti-parallel manner, only a small number of charges carriers (free electrons) flows through the tunnel and large number of free electrons are blocked. As a result, electric current decreases and resistance increases.

Colossal magneto resistance (CMR)

- It is a property of some materials, mostly manganese-based perovskite oxides, that enables them to dramatically change their electrical resistance in the presence of a magnetic field.
- This arises because of strong mutual coupling of spin, charge and lattice degrees of freedom.
- Magneto resistive oxides exhibit the properties of high temperature superconductivity, ferroelectricity and ferromagnetism.
- The magneto resistance of conventional materials enables changes in resistance of up to 5%, but some of CMR oxides shows 100% magneto resistance.
- Most of the perovskite oxides exhibit the following types of ordering. They are, charge ordering orbital ordering, and spin ordering.

Advantages of magneto resistors

The magneto resistor operates without physical contacts which is their major advantage.

Applications of magneto resistors

The various applications of magneto resistors include:

- Bio-sensors
- Hard disk drives
- Magnetic field sensors
- Magneto resistors are used in electronic compass for measuring earth's magnetic field.
- Magneto resistors are used for measuring electric current.

Spintronics

- **Definition:** Spintronics is "Spin based electronics". Spintronics is a field of electronics concerned with the detection and manipulation of electron spin in solid-state physics.
- In spintronics, in addition to electron charge, the electron spin is taken into account which increases the efficiency of data storage and transfer.
- Spintronics is a fundamental application in quantum computing.
- Spintronics is also known as spin electronics and fluxtronics.
- The spintronics is a field in nanoscale electronics that involves the detection and control of the electron's spin.
- The electron spin is detected as a magnetic field that has one of two orientations, simply known as up or down. Spintronics provides two extra binary states to the electron's conventional high and low charge states.

- With this, there are now four different states that an electron can be in, namely:
 - Down-high
 - Down-low
 - Up-high
 - Up-low
- The above states represent quantum bits, called qubits. Because of this doubling in the number states, exponential growth in storage capacity, transfer speeds, greater memory density and processing power may be possible.
- With the application of spintronic technology to hard drives, positive results are already being seen. Provided that newer technology can fully control the spin of the electron, then it can be applied to more practical purposes, especially in quantum computing and even in consumer electronics.
- Spintronic device do not need an electric current to retain their "spin". Spin is more "reliable", and such devices will operate better in high temperature or radiation environments. Theoretically Spintronic devices will be smaller, faster and more powerful than electronic ones.
- There are many possible usages of Spintronics. MRAM is based on Spintronics, and it is fast, small and non-volatile memory. Spin-Valves, which are used in Hard-Disks and sensors, There are several Spintronics - based sensors.
- Spintronics based transistors, which will allow us to replace electronics circuit boards with Spintronics ones.