Lecture Notes: Implies and Biconditional

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1 Overview

Key Points

- Understanding the implies operator and its truth table
- Understanding the biconditional operator and how to use it

2 Detailed Notes

2.1 Implies

For propositions P and Q, the implication is denoted by $P \implies Q$ and read as "if P then Q".

P	Q	P $\rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

The important conclusion is that when P is true, Q cannot be false for the implication to be true.

Example: If 3 is even then $3 * 15$ is even

- P_1 = "3 is even"
- Q_1 = "3 * 15 is even"
- $P_1 \implies Q_1$ is true

P is false, so we can make any conclusion we want (the implication is vacuously true).

2.2 Biconditional

The biconditional operator, denoted by $\leftrightarrow$ or $\iff$, is read as "if and only if" (often abbreviated as "iff").

P	Q	P $\leftrightarrow Q$
T	T	T
T	F	F
F	T	F
F	F	T

The biconditional is true when both P and Q have the same truth value.
Properties of the biconditional:

- Symmetry: $P \leftrightarrow Q$ is equivalent to $Q \leftrightarrow P$
- $P \leftrightarrow Q$ is equivalent to $(P \implies Q) \wedge (Q \implies P)$

Example: $x^2 = 4 \iff x = \pm 2$ This statement is true because:

- If $x^2 = 4$, then $x = \pm 2$
- If $x = \pm 2$, then $x^2 = 4$

3 Important Formulas/Theorems/Definitions

Key Formula/Theorem

$P \leftrightarrow Q \equiv (P \implies Q) \wedge (Q \implies P)$

4 Examples

Demonstrating Implies and Biconditional

- P(x): $x^2 = 16$
- Q(x): $|x| = 4$
- $P(x) \implies Q(x)$ is true for all x
- $P(x) \leftrightarrow Q(x)$ is true for all real x

5 Questions/Topics for Further Study

- How does the biconditional relate to logical equivalence?
- Explore the use of biconditionals in mathematical definitions and theorems.