LR DELIVERY TIME PREDICTION

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Order Delivery Time Prediction

Objectives

The objective of this assignment is to build a regression model that predicts the delivery time for orders placed through Porter. The model will use various features such as the items ordered, the restaurant location, the order protocol, and the availability of delivery partners.

The key goals are:

- Predict the delivery time for an order based on multiple input features
- Improve delivery time predictions to optimiae operational efficiency
- Understand the key factors influencing delivery time to enhance the model's accuracy

Data Pipeline

The data pipeline for this assignment will involve the following steps:

- 1. Data Loading
- 2. Data Preprocessing and Feature Engineering
- 3. Exploratory Data Analysis
- 4. Model Building
- 5. Model Inference

Data Understanding

The dataset contains information on orders placed through Porter, with the following columns:

Field	Description		
market_id	Integer ID representing the market where the restaurant is located.		
created_at	Timestamp when the order was placed.		
actual_delivery_time	Timestamp when the order was delivered.		
store_primary_category	Category of the restaurant (e.g., fast food, dine-in).		

Field	Description
order_protocol	Integer representing how the order was placed (e.g., via Porter, call to restaurant, etc.).
total_items	Total number of items in the order.
subtotal	Final price of the order.
num_distinct_items	Number of distinct items in the order.
min_item_price	Price of the cheapest item in the order.
max_item_price	Price of the most expensive item in the order.
total_onshift_dashers	Number of delivery partners on duty when the order was placed.
total_busy_dashers	Number of delivery partners already occupied with other orders.
total_outstanding_orders	Number of orders pending fulfillment at the time of the order.
distance	Total distance from the restaurant to the customer.

Importing Necessary Libraries

```
In [1]: # Import essential libraries for data manipulation and analysis
   import numpy as np
   import pandas as pd
   import matplotlib.pyplot as plt
   import seaborn as sns

import warnings
   warnings.filterwarnings('ignore')

import statsmodels
   import statsmodels.api as sm

import sklearn
   from sklearn.model_selection import train_test_split
   from sklearn.preprocessing import MinMaxScaler
```

1. Loading the data

Load 'porter_data_1.csv' as a DataFrame

```
In [2]: # Importing the file porter_data_1.csv
df = pd.read_csv('porter_data_1.csv')
df.head()
```

Out[2]:		market_id	created_at	actual_delivery_time	store_primary_category	order_prot
	0	1.0	2015-02- 06 22:24:17	2015-02-06 23:11:17	4	
	1	2.0	2015-02- 10 21:49:25	2015-02-10 22:33:25	46	
	2	2.0	2015-02- 16 00:11:35	2015-02-16 01:06:35	36	
	3	1.0	2015-02- 12 03:36:46	2015-02-12 04:35:46	38	
	4	1.0	2015-01- 27 02:12:36	2015-01-27 02:58:36	38	

In [3]: df.shape

Out[3]: (175777, 14)

In [4]: df.info()

<class 'pandas.core.frame.DataFrame'>
RangeIndex: 175777 entries, 0 to 175776

Data columns (total 14 columns): Column Non-Null Count Dtype 0 market_id 175777 non-null float64 1 created_at 175777 non-null object 2 actual_delivery_time 175777 non-null object 3 175777 non-null int64 store_primary_category 4 order_protocol 175777 non-null float64 5 total_items 175777 non-null int64 subtotal 175777 non-null int64 6 175777 non-null 7 num_distinct_items int64 175777 non-null 8 min_item_price int64 9 max_item_price 175777 non-null int64 10 total_onshift_dashers 175777 non-null float64 total_busy_dashers 175777 non-null float64 12 total_outstanding_orders 175777 non-null float64 13 distance 175777 non-null float64

dtypes: float64(6), int64(6), object(2)

memory usage: 18.8+ MB

In [5]: df.describe()

	market_id	store_primary_category	order_protocol	total_items	
count	175777.000000	175777.000000	175777.000000	175777.000000	1757
mean	2.743726	35.887949	2.911752	3.204976	2
std	1.330963	20.728254	1.513128	2.674055	18
min	1.000000	0.000000	1.000000	1.000000	
25%	2.000000	18.000000	1.000000	2.000000	14
50%	2.000000	38.000000	3.000000	3.000000	22
75%	4.000000	55.000000	4.000000	4.000000	34
max	6.000000	72.000000	7.000000	411.000000	268

2. Data Preprocessing and Feature Engineering [15 marks]

2.1 Fixing the Datatypes [5 marks]

The current timestamps are in object format and need conversion to datetime format for easier handling and intended functionality

2.1.1 [2 marks]

Out[5]:

Convert date and time fields to appropriate data type

```
In [6]: # Convert 'created_at' and 'actual_delivery_time' columns to datetime for
    df['created_at'] = pd.to_datetime(df['created_at'])
    df['actual_delivery_time'] = pd.to_datetime(df['actual_delivery_time'])

# COnfirm that the column is updated to datetime type
    df.info()
```

<class 'pandas.core.frame.DataFrame'>
RangeIndex: 175777 entries, 0 to 175776
Data columns (total 14 columns):

#	Column	Non-Null Count	Dtype						
0	market_id	175777 non-null	float64						
1	created_at	175777 non-null	<pre>datetime64[ns]</pre>						
2	actual_delivery_time	175777 non-null	<pre>datetime64[ns]</pre>						
3	store_primary_category	175777 non-null	int64						
4	order_protocol	175777 non-null	float64						
5	total_items	175777 non-null	int64						
6	subtotal	175777 non-null	int64						
7	<pre>num_distinct_items</pre>	175777 non-null	int64						
8	<pre>min_item_price</pre>	175777 non-null	int64						
9	<pre>max_item_price</pre>	175777 non-null	int64						
10	total_onshift_dashers	175777 non-null	float64						
11	total_busy_dashers	175777 non-null	float64						
12	total_outstanding_orders	175777 non-null	float64						
13	distance	175777 non-null	float64						
dtyp	es: datetime64[ns](2), flo	at64(6), int64(6)							
memo	memory usage: 18.8 MB								

2.1.2 [3 marks]

Convert categorical fields to appropriate data type

```
In [7]: # Convert categorical features to category type

# Check and review categorical columns one at a time
    df['market_id'].value_counts().sort_index()
    df['store_primary_category'].value_counts().sort_index()

df['order_protocol'].value_counts().sort_index()

# Convert categorical features to category type
    df['market_id'] = df['market_id'].astype('category')
    df['store_primary_category'] = df['store_primary_category'].astype('category')

# Confirm that the column is updated to category type
    df.info()
```

<class 'pandas.core.frame.DataFrame'>
RangeIndex: 175777 entries, 0 to 175776
Data columns (total 14 columns):

#	Column	Non-Null Count	Dtype
0	market_id	175777 non-null	category
1	created_at	175777 non-null	datetime64[ns]
2	actual_delivery_time	175777 non-null	datetime64[ns]
3	store_primary_category	175777 non-null	category
4	order_protocol	175777 non-null	category
5	total_items	175777 non-null	int64
6	subtotal	175777 non-null	int64
7	<pre>num_distinct_items</pre>	175777 non-null	int64
8	min_item_price	175777 non-null	int64
9	max_item_price	175777 non-null	int64
10	total_onshift_dashers	175777 non-null	float64
11	total_busy_dashers	175777 non-null	float64
12	total_outstanding_orders	175777 non-null	float64
13	distance	175777 non-null	float64
dtyp	es: category(3), datetime6	4[ns](2), float64	(4), int64(5)
memo	ry usage: 15.3 MB		

2.2 Feature Engineering [5 marks]

Calculate the time taken to execute the delivery as well as extract the hour and day at which the order was placed

2.2.1 [2 marks]

Calculate the time taken using the features actual_delivery_time and created_at

```
In [8]: # Calculate time taken in minutes
    df['time_taken'] = (df['actual_delivery_time'] - df['created_at']).dt.tot
    df['time_taken'].value_counts().sort_index()
```

```
Out[8]: time_taken
             32.0
                           6297
              33.0
                         4092
                         4661
              34.0
              35.0
                         5241
              36.0
                         5722
             101.0
                               5
              102.0
                               1
             103.0
                                1
                                 1
              105.0
              110.0
                                 1
             Name: count, Length: 74, dtype: int64
In [9]: df.info()
           <class 'pandas.core.frame.DataFrame'>
           RangeIndex: 175777 entries, 0 to 175776
           Data columns (total 15 columns):
                 Column
                                                            Non-Null Count Dtype
                  _____
                                                             175777 non-null category
             0
                 market id
            1 created_at 175777 non-null datetime64[ns]
2 actual_delivery_time 175777 non-null datetime64[ns]
3 store_primary_category 175777 non-null category
4 order_protocol 175777 non-null int64
6 subtotal 175777 non-null int64
7 num_distinct_items 175777 non-null int64
8 min_item_price 175777 non-null int64
9 max_item_price 175777 non-null int64
10 total_onshift_dashers 175777 non-null float64
11 total_busy_dashers 175777 non-null float64
12 total_outstanding_orders 175777 non-null float64
             1 created_at
                                                            175777 non-null datetime64[ns]
             12 total_outstanding_orders 175777 non-null float64
             13 distance
                                                             175777 non-null float64
             14 time_taken
                                                             175777 non-null float64
           dtypes: category(3), datetime64[ns](2), float64(5), int64(5)
```

2.2.2 [3 marks]

memory usage: 16.6 MB

Extract the hour at which the order was placed and which day of the week it was. Drop the unnecessary columns.

```
In [10]: # Extract the hour and day of week from the 'created_at' timestamp
    df['created_at_hour'] = df['created_at'].dt.hour
    df['created_at_dayofweek'] = df['created_at'].dt.dayofweek

# Create a categorical feature 'isWeekend'
    # 5 and 6 are weekends. We will represent weekend = 1, and rest as weekend
    df['isWeekend'] = df['created_at_dayofweek'].apply(lambda x: 1 if(x in [5])

# Setting the column type as category
    df['isWeekend'] = df['isWeekend'].astype('category')
    df['created_at_hour'] = df['created_at_hour'].astype('category')
    df['created_at_dayofweek'] = df['created_at_dayofweek'].astype('category')

# Validation
    df.info()
```

```
<class 'pandas.core.frame.DataFrame'>
         RangeIndex: 175777 entries, 0 to 175776
         Data columns (total 18 columns):
               Column
                                              Non-Null Count Dtype
                                              175777 non-null category
           0
               market id
             actual_delivery_time 175777 non-null datetime@store_primary_category order_protocol 175777 non-null category total_items 175777 non-null int64 subtotal
           1
             created at
                                              175777 non-null datetime64[ns]
           2
                                              175777 non-null datetime64[ns]
           3
           4
           5
          7 num_distinct_items 175777 non-null int64
8 min_item_price 175777 non-null int64
9 max_item_price 175777 non-null int64
10 total_onshift_dashers 175777 non-null float64
11 total_busy_dashers 175777 non-null float64
12 total_outstanding_interval
             subtotal
                                             175777 non-null int64
           12 total_outstanding_orders 175777 non-null float64
           13 distance
                                             175777 non-null float64
          14 time_taken 1,5,,, ...
15 created_at_hour 175777 non-null category
16 created_at_dayofweek 175777 non-null category
175777 non-null category
         dtypes: category(6), datetime64[ns](2), float64(5), int64(5)
         memory usage: 17.1 MB
In [11]: # Drop unnecessary columns
           # As we have created the 'time_taken' column, we probably dont need 'crea
           df = df.drop(['created_at', 'actual_delivery_time'], axis=1)
In [12]: | df.info()
         <class 'pandas.core.frame.DataFrame'>
         RangeIndex: 175777 entries, 0 to 175776
         Data columns (total 16 columns):
          #
               Column
                                                                   Dtype
                                              Non-Null Count
                                              175777 non-null category
           0
             market_id
               store_primary_category
                                              175777 non-null category
           1
           2
               order_protocol
                                              175777 non-null category
           3
             total_items
                                              175777 non-null int64
                                              175777 non-null int64
           4
               subtotal
               num_distinct_items
                                              175777 non-null int64
           5
              min_item_price
                                              175777 non-null int64
           6
           7
                                              175777 non-null int64
               max_item_price
               total_onshift_dashers
total_busy_dashers
                                              175777 non-null float64
           8
                                              175777 non-null float64
           9
           10 total_outstanding_orders 175777 non-null float64
           11 distance
                                              175777 non-null float64
                                              175777 non-null float64
           12 time_taken
           13 created_at_hour
                                              175777 non-null category
           14 created_at_dayofweek
                                              175777 non-null category
                                              175777 non-null category
           15 isWeekend
         dtypes: category(6), float64(5), int64(5)
         memory usage: 14.4 MB
 In [ ]:
```

2.3.1 [2 marks]

Define target and input features

```
In [13]: # Define target variable (y) and features (X)

# Here are the dependent and the feature variables we will use
# y = 'time_taken'
# X = 'market_id', 'total_items', 'subtotal', 'num_distinct_items', 'min_
# 'total_onshift_dashers', 'total_busy_dashers', 'total_outstanding_o

y = df['time_taken']
X = df.drop('time_taken', axis=1)
In []:
```

2.3.2 [3 marks]

Split the data into training and test sets

```
In [14]: # Split data into training and testing sets
    #(df_train, df_test) = train_test_split(df, train_size=0.7, random_state=
    X_train, X_test, y_train, y_test = train_test_split(X, y, train_size=0.7,

In [15]: # Verify the train and test split data frames
    print(X_train.shape)
    print(X_test.shape)
    print(y_train.shape)
    print(y_test.shape)

(123043, 15)
    (52734, 15)
    (123043,)
    (52734,)
```

3. Exploratory Data Analysis on Training Data [20 marks]

- 1. Analyzing the correlation between variables to identify patterns and relationships
- 2. Identifying and addressing outliers to ensure the integrity of the analysis
- 3. Exploring the relationships between variables and examining the distribution of the data for better insights

3.1 Feature Distributions [7 marks]

```
In [16]: # Define numerical and categorical columns for easy EDA and data manipula
   numerical_columns = [
        'total_items', 'subtotal', 'num_distinct_items', 'min_item_price', 'm
        'total_busy_dashers','total_outstanding_orders','distance'
]

categorical_columns = [
        'market_id', 'store_primary_category', 'order_protocol',
```

```
'created_at_hour','created_at_dayofweek','isWeekend'
]
```

3.1.1 [3 marks]

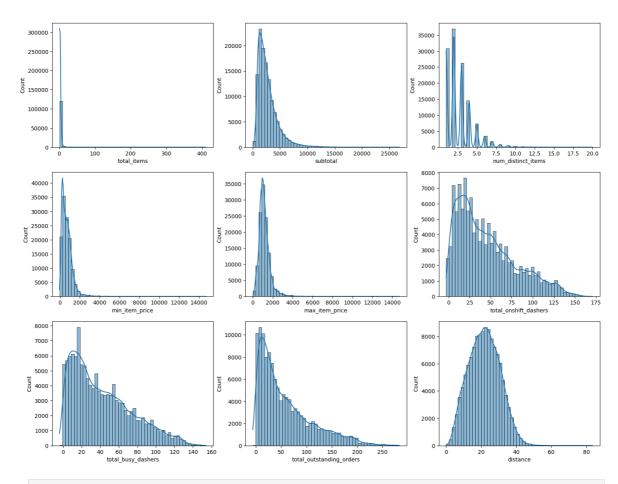
Plot distributions for numerical columns in the training set to understand their spread and any skewness

```
In [17]: # Plot distributions for all numerical columns

# General data distribution
X_train[numerical_columns].describe()
```

Out[17]:		total_items	subtotal	num_distinct_items	min_item_price	max_i
	count	123043.000000	123043.000000	123043.000000	123043.000000	12304
	mean	3.206082	2697.863625	2.674951	684.784506	116
	std	2.745043	1830.338637	1.625552	520.731071	5(
	min	1.000000	0.000000	1.000000	-52.000000	
	25%	2.000000	1417.000000	2.000000	299.000000	79
	50%	3.000000	2220.000000	2.000000	595.000000	109
	75%	4.000000	3405.000000	3.000000	942.000000	139
	max	411.000000	26800.000000	20.000000	14700.000000	147(

```
In [18]: # Use histplot (as distplot is deprecated). There are many columns, so we
# There are in all 9 feature variables. So we need 3x3 subplot
index = 1
plt.figure(figsize=(18,14))
for col in numerical_columns:
    plt.subplot(3, 3, index)
    sns.histplot(X_train[col], kde=True, bins=50)
    index = index + 1
plt.show()
```

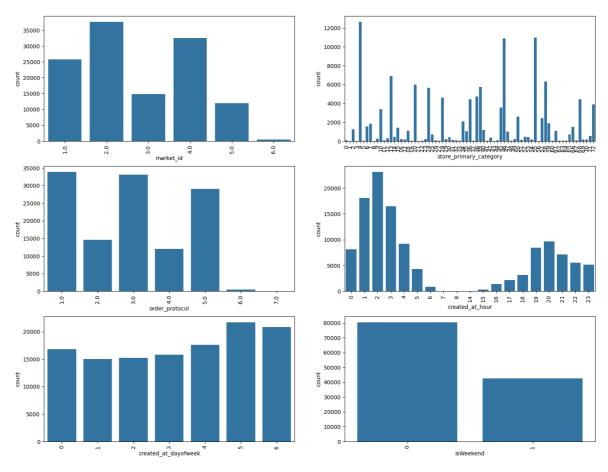


In [19]: # From above figure, there are several numeric values with outliers, exam

3.1.2 [2 marks]

Check the distribution of categorical features

```
# We cannot use histplot here as its only meant for numerical data. We wi
# There are many columns, so we will have to use a for loop
# There are in all 6 categorical feature variables. So we need 3x2 subplo
# There are too many values in 'store_primary_category', so rotating the
index = 1
plt.figure(figsize=(18,14))
for col in categorical_columns:
    plt.subplot(3, 2, index)
    sns.countplot(x=X_train[col])
    plt.xticks(rotation=90)
    index = index + 1
plt.show()
```



In [21]: # From above we can see that
 # created hours - its busy during morning and evenings
 # isWeekend - (cumulatively) More orders on weekdays than weekends
 # created at day of week - more daily orders on Saturdays and Sundays (

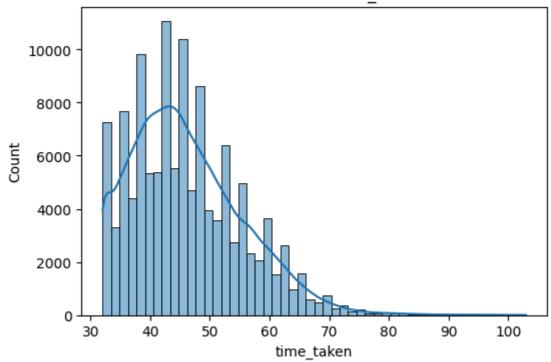
3.1.3 [2 mark]

Visualise the distribution of the target variable to understand its spread and any skewness

```
In [22]: # Distribution of time_taken

plt.figure(figsize=(6,4))
sns.histplot(y_train, kde=True, bins=50)
plt.title('Distribution of time_taken')
plt.show()
```

Distribution of time_taken



3.2 Relationships Between Features [3 marks]

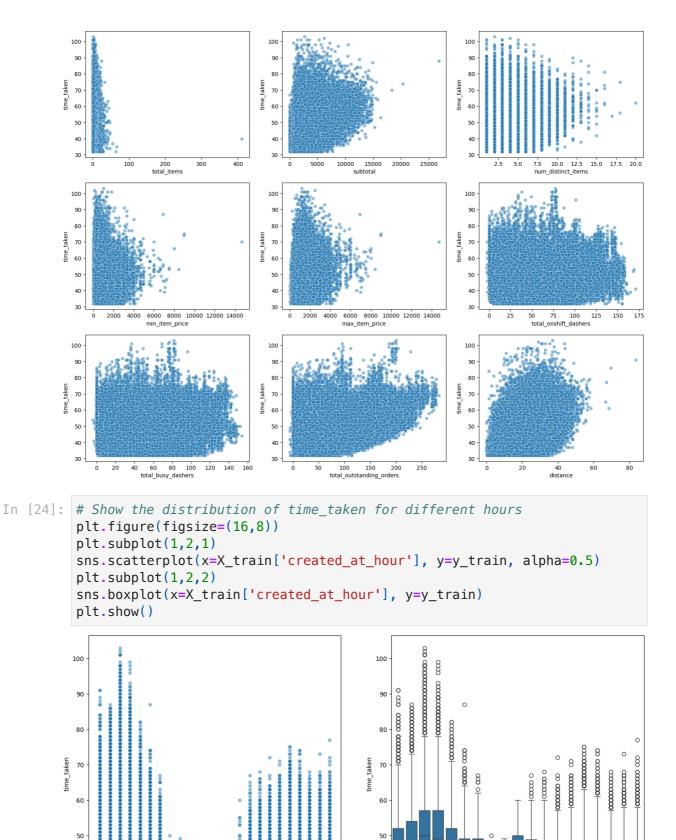
3.2.1 [3 marks]

Scatter plots for important numerical and categorical features to observe how they relate to time_taken

```
In [23]: # Scatter plot to visualise the relationship between time_taken and other

# There are many columns, so we will have to use a for loop

# There are in all 9 numeric feature variables. So we need 3x3 subplot
index = 1
plt.figure(figsize=(18,14))
for col in numerical_columns:
    plt.subplot(3, 3, index)
    sns.scatterplot(x=X_train[col], y=y_train, alpha=0.5)
    index = index + 1
plt.show()
```



3.3 Correlation Analysis [5 marks]

10 created_at_hour 15

40

30

Check correlations between numerical features to identify which variables are strongly related to time_taken

20

30

8 14 15 16 17 18 19 20 21 22 23

3.3.1 [3 marks]

Plot a heatmap to display correlations

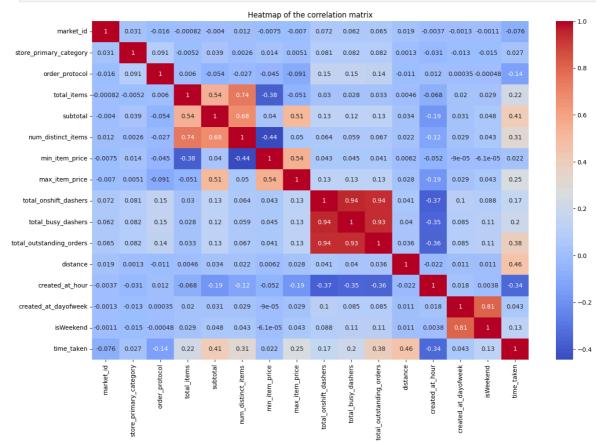
```
In [25]: # Plot the heatmap of the correlation matrix

# We need to check correlations between numerical features to identify wh
# So we need to get time_taken in the mix
# We will have to create a training data frame with time_taken

df_train = X_train.copy()

df_train['time_taken'] = y_train.copy()

plt.figure(figsize = (16, 10))
sns.heatmap(df_train.corr(), annot = True, cmap="coolwarm")
plt.title('Heatmap of the correlation matrix')
plt.show()
```



```
In [26]: # Checking correlation between numerical features to time_taken

correlations = df_train[numerical_columns + ['time_taken']].corr()['time_
correlations = correlations.sort_values(key=abs, ascending=False)
print(correlations)
```

```
1.000000
time_taken
distance
                           0.459712
subtotal
                           0.412878
total_outstanding_orders
                           0.381642
num_distinct_items
                           0.313384
max item price
                           0.254671
total_items
                          0.219104
total_busy_dashers
                         0.202562
total_onshift_dashers
                           0.166812
min_item_price
                           0.022281
Name: time_taken, dtype: float64
```

```
In []:
```

3.3.2 [2 marks]

Drop the columns with weak correlations with the target variable

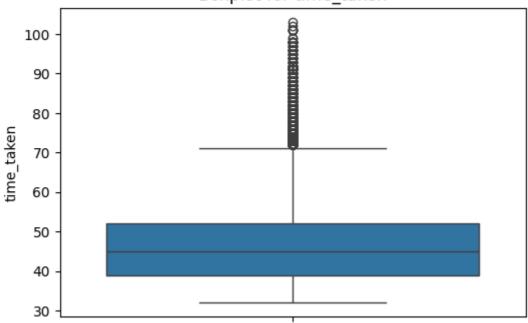
3.4 Handling the Outliers [5 marks]

3.4.1 [2 marks]

Visualise potential outliers for the target variable and other numerical features using boxplots

```
In [28]: # Boxplot for time_taken
    plt.figure(figsize=(6, 4))
    sns.boxplot(y_train)
    plt.title('Boxplot for time_taken')
    plt.show()
```

Boxplot for time_taken



```
In [29]: # Boxplot for other numerical features
            index = 1
            plt.figure(figsize=(18,14))
            for col in numerical_columns:
                 plt.subplot(3, 3, index)
                 sns.boxplot(df_train[col])
                 index = index + 1
            plt.show()
                                           25000
                                                                            17.5
            300
                                           15000
                                                                           10.0
                                           10000
                                                                            7.5
            100
           14000
                                            250
           12000
                                           ontstanding 100
           8000
           6000
                                           total
                                                                             20
                                             50
```

3.4.2 [3 marks]

Handle outliers present in all columns

```
In [30]: # Handle outliers

for column_name in numerical_columns:
    Q1 = df_train[column_name].quantile(0.25)
    Q3 = df_train[column_name].quantile(0.75)
```

```
4. Exploratory Data Analysis on Validation Data [optional]
```

Optionally, perform EDA on test data to see if the distribution match with the training data

```
In [32]: # Define numerical and categorical columns for easy EDA and data manipula

numerical_columns = [
    'total_items', 'subtotal', 'num_distinct_items', 'min_item_price', 'm
    'total_busy_dashers','total_outstanding_orders','distance'
]

categorical_columns = [
    'market_id', 'store_primary_category', 'order_protocol',
    'created_at_hour','created_at_dayofweek','isWeekend'
]
```

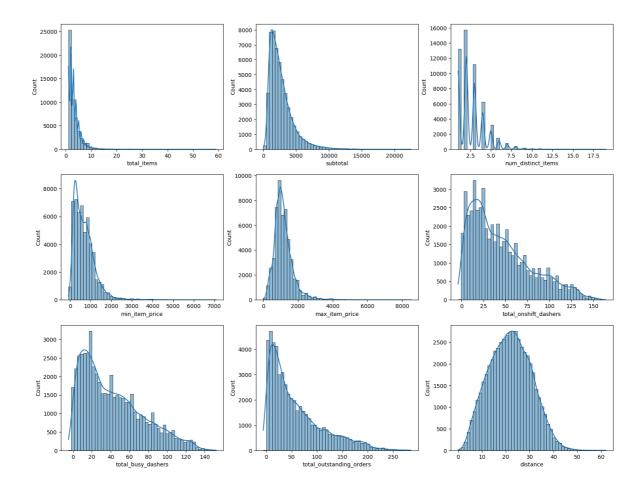
4.1 Feature Distributions

4.1.1

Plot distributions for numerical columns in the validation set to understand their spread and any skewness

```
In [33]: # Plot distributions for all numerical columns

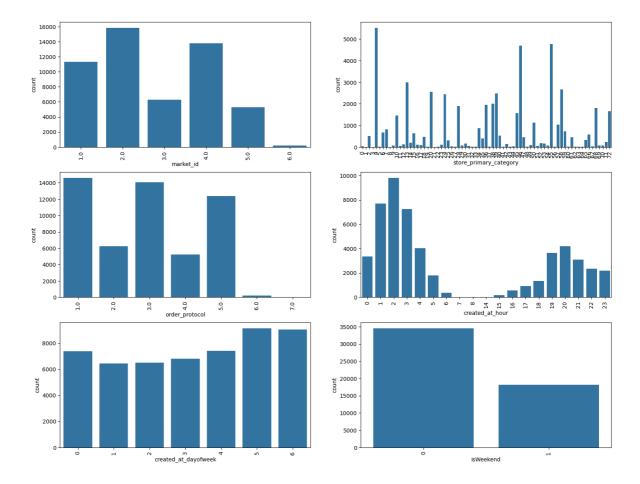
# There are in all 9 feature variables. So we need 3x3 subplot
index = 1
plt.figure(figsize=(18,14))
for col in numerical_columns:
    plt.subplot(3, 3, index)
    sns.histplot(X_test[col], kde=True, bins=50)
    index = index + 1
plt.show()
```



4.1.2 Check the distribution of categorical features

```
In [34]: # Distribution of categorical columns

# There are in all 6 categorical feature variables. So we need 3x2 subplo
# There are too many values in 'store_primary_category', so rotating the
index = 1
plt.figure(figsize=(18,14))
for col in categorical_columns:
    plt.subplot(3, 2, index)
    sns.countplot(x=X_test[col])
    plt.xticks(rotation=90)
    index = index + 1
plt.show()
```

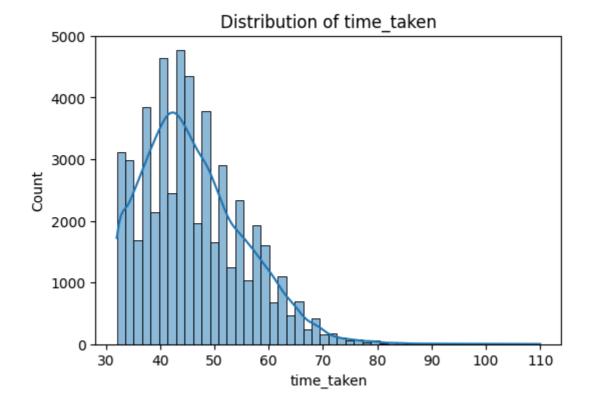


4.1.3

Visualise the distribution of the target variable to understand its spread and any skewness

```
In [35]: # Distribution of time_taken

plt.figure(figsize=(6,4))
    sns.histplot(y_test, kde=True, bins=50)
    plt.title('Distribution of time_taken')
    plt.show()
```

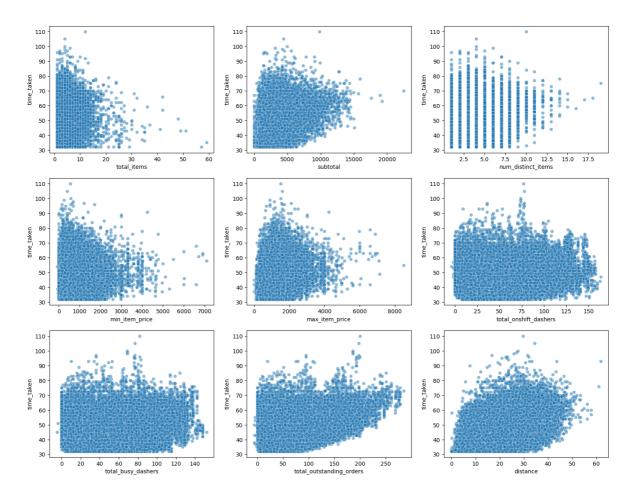


4.2 Relationships Between Features

Scatter plots for numerical features to observe how they relate to each other, especially to time_taken

```
In [36]: # Scatter plot to visualise the relationship between time_taken and other

# There are many columns, so we will have to use a for loop
# There are in all 9 numeric feature variables. So we need 3x3 subplot
index = 1
plt.figure(figsize=(18,14))
for col in numerical_columns:
    plt.subplot(3, 3, index)
    sns.scatterplot(x=X_test[col], y=y_test, alpha=0.5)
    index = index + 1
plt.show()
```



4.3 Drop the columns with weak correlations with the target variable

```
In [37]: # Drop the weakly correlated columns from training dataset
         # Create test data frame
         df_test = X_test.copy()
         df_test['time_taken'] = y_test.copy()
         # Drop 3-5 weakly correlated columns from training dataset
         # Columns with weakest correlations with the target variable 'time_taken'
           # total_busy_dashers
           # total_onshift_dashers
           # min_item_price
         df_test = df_test.drop('total_busy_dashers', axis=1)
         df_test = df_test.drop('total_onshift_dashers', axis=1)
         df_test = df_test.drop('min_item_price', axis=1)
         # Update the list of numerical colums
         numerical_columns = [
             'total_items', 'subtotal', 'num_distinct_items', 'max_item_price','to
In [38]: df_test.shape
Out[38]: (52734, 13)
```

5. Model Building [15 marks]

Import Necessary Libraries

```
In [39]: # Import libraries

from sklearn.preprocessing import MinMaxScaler
from sklearn.feature_selection import RFE
from sklearn.linear_model import LinearRegression
from statsmodels.stats.outliers_influence import variance_inflation_facto
```

5.1 Feature Scaling [3 marks]

Out[41]:		time_taken	total_items	subtotal	num_distinct_items	max_it
	count	105442.000000	105442.000000	105442.000000	105442.000000	10544
	mean	0.186543	0.287593	0.377247	0.280435	
	std	0.123621	0.243330	0.199236	0.245033	
	min	0.000000	0.000000	0.000000	0.000000	
	25%	0.101449	0.166667	0.221361	0.000000	
	50%	0.173913	0.166667	0.334267	0.200000	
	75%	0.260870	0.500000	0.494478	0.400000	
	max	1.000000	1.000000	1.000000	1.000000	

Note that linear regression is agnostic to feature scaling. However, with feature scaling, we get the coefficients to be somewhat on the same scale so that it becomes easier to compare them.

5.2 Build a linear regression model [5 marks]

You can choose from the libraries statsmodels and scikit-learn to build the model.

```
In [42]: # Create/Initialise the model

y_train_scaled = df_train.pop('time_taken')
X_train_scaled = df_train
```

```
# Initialize model
X_train_scaled_with_const = sm.add_constant(X_train_scaled)
lm = sm.OLS(y_train_scaled, X_train_scaled_with_const)

In [43]: # Train the model using the training data
lm = lm.fit()
print(lm.params)
```

print(lm.summary())

const market_id store_primary_categ order_protocol total_items subtotal num_distinct_items max_item_price total_outstanding_o distance created_at_hour created_at_dayofwee isWeekend dtype: float64	-0.013670 -0.011182 0.115781 0.044993 0.008273 0.107636 0.316718 -0.002679 -0.010123 0.055806	ession Resu	lts		
====					
Dep. Variable: 0.494	time_taken	R-squar	ed:		
Model:	0LS	Adj. R-	squared:		
0.494 Method:	Least Squares	F-stati	stic:		8
577 .	·				
Date: 0.00	Tue, 09 Sep 2025) Prob (F	-statistic):		
Time: e+05	12:51:07	′ Log-Lik	elihood:	1	.0673
No. Observations:	105442	AIC:		-:	2.134
e+05 Df Residuals:	105429	BIC:		'	2.133
e+05	103 123	5101		•	-1155
Df Model:	12				
Covariance Type:	nonrobust 	:=======	========	:=======	====
=======================================					
[0.025 0.975]	coef	std err	t 	P> t	
	0.0552	0 002	24 624	0.000	
const 0.052 0.058	0.0553	0.002	34.634	0.000	
market_id	-0.0106	0.000	-52.232	0.000	
-0.011 -0.010					
store_primary_categ	ory 5.128e-05	1.31e-05	3.920	0.000	2.
56e-05 7.69e-05 order_protocol	-0.0137	0 000	-75.025	0.000	
-0.014 -0.013		0.000	-/3:023	0.000	
total_items	-0.0112	0.003	-4.042	0.000	
-0.017 -0.006					
subtotal	0.1158	0.003	43.164	0.000	
<pre>0.111 0.121 num_distinct_items</pre>	0.0450	0.002	18.552	0.000	
0.040 0.050	010.50	0.002	10.002	0.000	
<pre>max_item_price 0.004 0.013</pre>	0.0083	0.002	3.521	0.000	
0.004 0.013 total_outstanding_o	rders 0.1076	0.001	86.414	0.000	
0.105 0.110					
<pre>distance 0.314 0.320</pre>	0.3167	0.001	215.160	0.000	
created at hour	_0 0027	3 376-05	_70 408	0 000	

-0.0027 3.37e-05

-79.498

0.000

created_at_hour

-0.003 -0.003 created_at_dayofwee -0.011 -0.010 isWeekend 0.054 0.058	ek -0.0101 0.0558	0.000 0.001	-45.375 57.699	0.000
====				
Omnibus:	7157.929	Durbin-Watson:		
2.005 Prob(Omnibus):	0.000	Jarque-Bera (JB):		1335
0.865	0.000	Jai que-b	ela (JD):	1333
Skew:	0.496	<pre>Prob(JB):</pre>		
0.00				
Kurtosis:	4.434	Cond. No	•	
621.				
====				

Notes:

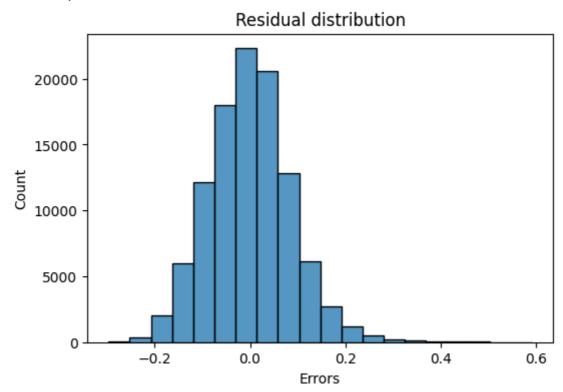
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

```
In [44]: # Make predictions
    y_train_predicted = lm.predict(X_train_scaled_with_const)
    print(y_train_predicted.shape)

# Plot the histogram of the error terms
fig = plt.figure(figsize=(6, 4))
    sns.histplot((y_train_scaled - y_train_predicted), bins = 20)
    plt.xlabel('Errors')
    plt.title('Residual distribution')
    plt.show()

# The error terms are following natural distribution and mean is 0. So th
# We need to check how to improve the R-squared value
```

(105442,)



```
In [45]: # Find results for evaluation metrics
         # Running RFE
         lm = LinearRegression()
         lm.fit(X_train_scaled, y_train_scaled)
         rfe = RFE(estimator=lm, n_features_to_select=8)
         rfe = rfe.fit(X_train_scaled, y_train_scaled)
         print('RFE results')
         rfe_results = list(zip(X_train_scaled.columns,rfe.support_,rfe.ranking_))
         for i in rfe_results:
             print(i)
         print('--
         # Select columns based on RFE recommendation
         selected_columns = X_train_scaled.columns[rfe.support_]
         print('selected_columns')
         print(selected_columns)
         print('-----
         not selected columns = X train scaled.columns[~rfe.support]
         print('not_selected_columns')
         print(not selected columns)
         print('-----
        RFE results
        ('market_id', np.True_, np.int64(1))
        ('store_primary_category', np.False_, np.int64(5))
        ('order_protocol', np.True_, np.int64(1))
        ('total_items', np.False_, np.int64(3))
        ('subtotal', np.True_, np.int64(1))
        ('num_distinct_items', np.True_, np.int64(1))
        ('max_item_price', np.True_, np.int64(1))
        ('total_outstanding_orders', np.True_, np.int64(1))
        ('distance', np.True_, np.int64(1))
        ('created_at_hour', np.False_, np.int64(4))
        ('created_at_dayofweek', np.False_, np.int64(2))
        ('isWeekend', np.True_, np.int64(1))
        selected_columns
        Index(['market_id', 'order_protocol', 'subtotal', 'num_distinct_items',
               'max_item_price', 'total_outstanding_orders', 'distance', 'isWeeken
        d'],
              dtype='object')
        not selected columns
        Index(['store_primary_category', 'total_items', 'created_at_hour',
               'created_at_dayofweek'],
              dtype='object')
```

Note that we have 12 (depending on how you select features) training features. However, not all of them would be useful. Let's say we want to take the most relevant 8 features.

We will use Recursive Feature Elimination (RFE) here.

For this, you can look at the coefficients / p-values of features from the model summary and perform feature elimination, or you can use the RFE module provided with *scikit-learn*.

5.3 Build the model and fit RFE to select the most important features [7 marks]

For RFE, we will start with all features and use the RFE method to recursively reduce the number of features one-by-one.

After analysing the results of these iterations, we select the one that has a good balance between performance and number of features.

```
In [46]: # Loop through the number of features and test the model
         # Now we need to eliminate below columns one-by-one and check the model
           # 'store_primary_category'
           # 'total_items'
           # 'created_at_hour'
           # 'created_at_dayofweek'
         # Creating dataframe with RFE selected variables
         X_train_rfe = X_train_scaled[[
                                        'market_id',
                                        'order_protocol',
                                        'subtotal',
                                        'num_distinct_items',
                                        # 'max_item_price',
                                        'total_outstanding_orders',
                                        'distance',
                                        'isWeekend',
                                        # 'store_primary_category',
                                        # 'total_items',
                                        # 'created_at_hour',
                                        'created_at_dayofweek'
         # Adding a constant variable
         X_train_rfe = sm.add_constant(X_train_rfe)
         # Running the linear model
         lm = sm.OLS(y_train_scaled, X_train_rfe).fit()
         print(lm.summary())
         # distance - R-squared: 0.272
         # total_outstanding_orders - R-squared: 0.458
         # created_at_hour - R-squared: 0.464
         # isWeekend - R-squared: 0.478
         # created_at_dayofweek - R-squared: 0.484
         # max_item_price - R-squared: 0.494
         # store_primary_category - R-squared: 0.494
         # Removing below columns makes no significant difference to the R-squared
         # Removing any other feature variable reduces the R-squared value
         # The coefficient of created_at_hour is very low, so removing created_at_
```

```
# 'max_item_price',
# 'store_primary_category',
# 'total_items',
# 'created_at_hour'
```

========	=======	==========	=======	=========	
==== Dep. Varia	nle:	time_taken	R-square	٠ ٠ .	
0.463	J (C.	cime_caken	it square		
Model:		0LS	Adj. R-s	squared:	
0.463		023	/\dj	,quar cu i	
Method:		Least Squares	F-statis	stic:	1.135
e+04		_0001 0400.00	. 5 - 5 - 5 - 5		
Date:		Tue, 09 Sep 2025	Prob (F-	-statistic):	
0.00		, ,			
Time:		12:51:07	Log-Like	elihood:	1.0358
e+05			_		
No. Observa	ations:	105442	AIC:		-2.071
e+05					
Df Residua	ls:	105433	BIC:		-2.070
e+05					
Df Model:		8			
		nonrobust			
=========		=======================================			==================================
		coef	std err	t	P> t
[0.025	0.975]				
const		0.0224	0.001	16.948	0.000
0.020	0.025				
market_id		-0.0109	0.000	-52.256	0.000
-0.011	-0.011				
order_proto	ocol	-0.0145	0.000	-78.089	0.000
-0.015	-0.014				
subtotal		0.1407	0.002	76.637	0.000
0.137					
num_distind		0.0331	0.001	22.396	0.000
	0.036				
		ers 0.1422	0.001	118.247	0.000
0.140	0.145				
distance		0.3168	0.002	208.896	0.000
0.314	0.320	0.0540	0.004	E 4 0 4 0	0.000
isWeekend	0.057	0.0546	0.001	54.849	0.000
0.053	0.057	0.0105	0 000	4E 071	0.000
<pre>created_at0.011</pre>	_dayofweek _0.010	-0.0105	0.000	-45.871	0.000
		==========	.=======	:=======	:=======
====					
Omnibus:		7312.041	Durbin-W	latson:	
2.003					
Prob(Omnibu	us):	0.000	Jarque-E	Bera (JB):	1190
5.365					
Skew:		0.547	Prob(JB)	:	
0.00					
Kurtosis:		4.231	Cond. No).	
43.3					
=========	========	==========	=======	=========	

==== Notes:

 $^{\[1\]}$ Standard Errors assume that the covariance matrix of the errors is correctly specified.

```
In [47]: # Calculate the VIFs for the new model
         vif = pd.DataFrame()
         X = X_{train_rfe}
         vif['Features'] = X.columns
         vif['VIF'] = [variance_inflation_factor(X.values, i) for i in range(X.sha
         vif['VIF'] = round(vif['VIF'], 2)
         vif = vif.sort values(by = "VIF", ascending = False)
         print(vif)
         # The multicovariance are below 5, so its acceptable
                           Features
                                      VIF
        0
                              const 22.42
        7
                          isWeekend 2.81
        8
               created_at_dayofweek 2.81
        3
                           subtotal 1.72
                 num_distinct_items 1.68
        4
        5 total_outstanding_orders 1.06
        2
                     order_protocol 1.03
        1
                          market_id 1.01
                           distance 1.00
        6
In [48]: # Build the final model with selected number of features
         # Above model is my final model
         # Selected feature sets are
           # 'market_id',
           # 'order protocol',
           # 'subtotal',
           # 'num_distinct_items',
           # 'total_outstanding_orders',
           # 'distance',
           # 'isWeekend',
           # 'created_at_dayofweek'
         # NOT selected feature sets are
          # 'max_item_price',
          # 'store_primary_category',
          # 'total_items',
          # 'created_at_hour'
         # We need to scale again on the final selected numerical features
         # We need to scale the target variable along with the numeric feature var
         final_selected_features = [
             'market_id', 'order_protocol', 'subtotal',
             'num_distinct_items', 'total_outstanding_orders',
             'distance', 'isWeekend', 'created_at_dayofweek'
         final_numerical_columns_with_target_var = [
             'time_taken', 'subtotal','num_distinct_items','total_outstanding_orde
         final_numerical_columns_without_target_var = [
             'subtotal','num_distinct_items','total_outstanding_orders','distance'
         ]
         df_train = X_train.copy()
         df_train['time_taken'] = y_train.copy()
         df_train = df_train[final_selected_features + ['time_taken']]
```

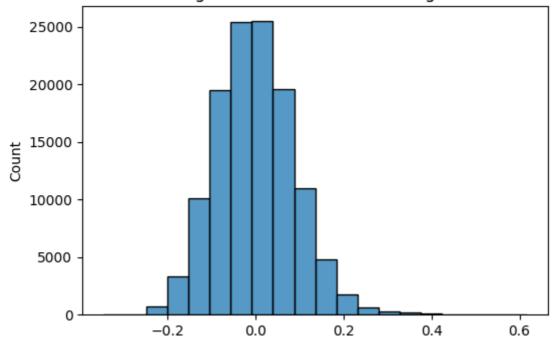
```
# Scale the training data again on the selected features
scaler = MinMaxScaler()
df_train[final_numerical_columns_with_target_var] = scaler.fit_transform(
y_train_scaled = df_train.pop('time_taken')
X_train_scaled = df_train
# Initialize model
X_train_scaled_with_const = sm.add_constant(X_train_scaled)
lm = sm.OLS(y_train_scaled, X_train_scaled_with_const).fit()
print(lm.summary())
y_train_predicted = lm.predict(X_train_scaled_with_const)
# Plot the histogram of the error terms
fig = plt.figure(figsize=(6, 4))
sns.histplot((y_train_scaled - y_train_predicted), bins = 20)
plt.title('Histogram of residuals on training data')
plt.show()
# Use the same scaler to scale the test data set
df_test = X_test.copy()
df_test['time_taken'] = y_test.copy()
df_test = df_test[final_selected_features + ['time_taken']]
df_test[final_numerical_columns_with_target_var] = scaler.transform(df_te
y_test_scaled = df_test.pop('time_taken')
X_test_scaled = df_test
# 2. Add constant
X_test_scaled = sm.add_constant(X_test_scaled)
# 3. Predict using the existing model
y_test_predicted = lm.predict(X_test_scaled)
```

========		:=========	=======			-==
====						
Dep. Variab	le:	time_taken	R-square	ed:		
0.527						
Model:		0LS	Adj. R−s	squared:		
0.527					4 -	
Method:		Least Squares	F-statis	stic:	1.7	12
e+04						
Date:		Tue, 09 Sep 2025	Prob (F-	-statistic):		
0.00						
Time:		12:51:07	Log-Like	elihood:	1.21	.12
e+05		400040			2	
No. Observa	tions:	123043	AIC:		-2 . 4	-22
e+05		400004	D.T.C		2	
Df Residual	S:	123034	BIC:		-2 . 4	21
e+05						
Df Model:	_	8				
		nonrobust 				
=========						
		coef	std err	t	P> t	
[0.025	0.9751				. -	
const		0.0233	0.001	19.757	0.000	
0.021	0.026					
market_id		-0.0108	0.000	-55.441	0.000	
-0.011						
order_proto		-0.0143	0.000	-82.784	0.000	
	-0.014					
subtotal		0.5513	0.005	105.903	0.000	
	0.562					
num_distinc	_	0.1199	0.004	29.103	0.000	
0.112	0.128					
total_outst		lers 0.2484	0.001	170.727	0.000	
0.246	0.251					
distance		0.5457	0.002	221.856	0.000	
0.541	0.551					
isWeekend		0.0595	0.001	64.618	0.000	
0.058	0.061					
created_at_	-	-0.0114	0.000	-53 . 235	0.000	
-0.012	-0.011					
=====	=======		=======			:==
Omnibus:		8455.358	Durbin-W	latson:		
2.001		04331330	Dui Dill V	14 (3011)		
Prob(Omnibu	c).	0.000	larque_F	Bera (JB):	148	RS
3.886	J / •	0.000	Jul que-L	,CIG (3D) I	140	, ,
Skew:		0.518	Prob(JB)			
0.00		0.010	1100(30)	•		
Kurtosis:		4.352	Cond. No) ₋		
		4.332	COHU. NO	, ı		
131.						

====

Notes: [1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

Histogram of residuals on training data

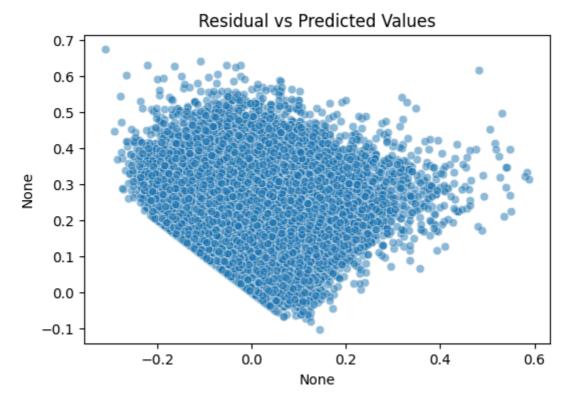


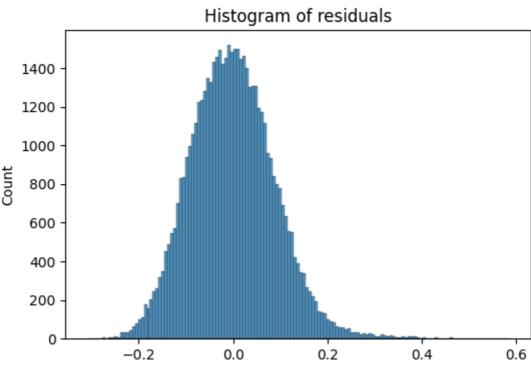
In []:

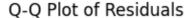
6. Results and Inference [5 marks]

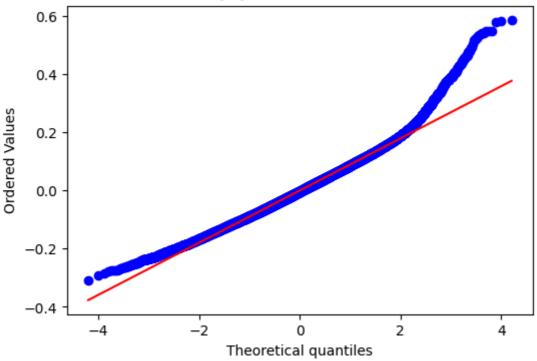
6.1 Perform Residual Analysis [3 marks]

```
In [49]: # Perform residual analysis using plots like residuals vs predicted value
         # Residuals vs Predicted values
         import scipy.stats as stats
         residuals = y_test_scaled - y_test_predicted
         fig = plt.figure(figsize=(6, 4))
         sns.scatterplot(x=residuals, y=y_test_predicted, alpha=0.5)
         plt.title('Residual vs Predicted Values')
         plt.show()
         # Plot the histogram of the error terms
         fig = plt.figure(figsize=(6, 4))
         sns.histplot(residuals)
         plt.title('Histogram of residuals')
         plt.show()
         # Q-Q plot of residuals
         fig = plt.figure(figsize=(6, 4))
         stats.probplot(residuals, dist="norm", plot=plt)
         plt.title('Q-Q Plot of Residuals')
         plt.show()
```









[Your inferences here:]

Both, the residual histogram and Q-Q plots indicate that the errors follow a normal distribution with mean as 0. The R-Squared value of 0.52 indicates that 52% of the variation can be explained. All the columns selected have p-value of 0 which means these are significant values. The VIF indicates that multicollinearity is under permissible limits.

6.2 Perform Coefficient Analysis [2 marks]

Perform coefficient analysis to find how changes in features affect the target. Also, the features were scaled, so interpret the scaled and unscaled coefficients to understand the impact of feature changes on delivery time.

```
In [50]: # Compare the scaled vs unscaled features used in the final model

# Refit model on unscaled features
X_train_unscaled = sm.add_constant(X_train[final_selected_features])
model_unscaled = sm.OLS(y_train, X_train_unscaled).fit()

# Extract coefficients
scaled_coefs = pd.Series(lm.params[1:], index=final_selected_features)
unscaled_coefs = pd.Series(model_unscaled.params[1:], index=final_selecte

print('Coefficients for scaled data set')
print(scaled_coefs)
print()
print('Coefficients for unscaled data set')
print(unscaled_coefs)
```

```
Coefficients for scaled data set
                        -0.010794
market id
order_protocol
                        -0.014290
subtotal
                         0.551312
num_distinct_items
                        0.119947
total_outstanding_orders 0.248376
distance
                        0.545700
isWeekend
                        0.059460
created_at_dayofweek
                       -0.011377
dtype: float64
Coefficients for unscaled data set
market id
                        -0.766350
                        -1.014604
order_protocol
subtotal
                        0.001461
num_distinct_items
                        0.448221
total_outstanding_orders 0.061020
distance
                        0.463898
isWeekend
                         4.221654
created_at_dayofweek -0.807791
dtype: float64
```

Additionally, we can analyse the effect of a unit change in a feature. In other words, because we have scaled the features, a unit change in the features will not translate directly to the model. Use scaled and unscaled coefficients to find how will a unit change in a feature affect the target.

```
In [51]: # Analyze the effect of a unit change in a feature, say 'total items'
         # Analyzing the effect of a unit change in 'distance' feature. This is be
         # Get the coefficient of 'distance'
         scaled coef = lm.params['distance']
         # Get the actual min and max values of 'distance' from training data
         min_val = X_train['distance'].min()
         max_val = X_train['distance'].max()
         range_val = max_val - min_val
         # Convert scaled coefficient to per-unit-change
         unit_effect = scaled_coef / range_val
         # 4. Display the results
         print(f"Coefficient = {scaled_coef}")
         print(f"min_val = {min_val}, max_val = {max_val} (range: {range_val})")
         print(f"Effect on delivery time per unit increase in distance: {unit_effe
        Coefficient = 0.5457002876618022
        min_val = 0.0, max_val = 83.52 (range: 83.52)
        Effect on delivery time per unit increase in distance: 0.00653376781204265
        1 minutes
```

Note: The coefficients on the original scale might differ greatly in magnitude from the scaled coefficients, but they both describe the same relationships between variables.

Interpretation is key: Focus on the direction and magnitude of the coefficients on the original scale to understand the impact of each variable on the response variable in the original units.

Include conclusions in your report document.

Subjective Questions [20 marks]

Answer the following questions only in the notebook. Include the visualisations/methodologies/insights/outcomes from all the above steps in your report.

Subjective Questions based on Assignment

Question 1. [2 marks]

Are there any categorical variables in the data? From your analysis of the categorical variables from the dataset, what could you infer about their effect on the dependent variable?

Answer:

Yes, the dataset contains several categorical columns. Some of these are derived from existing fields such as created_at, actual_delivery_time, market_id, store_primary_category, order_protocol, created_at_hour, created_at_dayofweek, and isWeekend.

In our analysis of the relationship between target variables and categories, we identified the most influential categorical variables affecting the dependent variable as market_id, order_protocol, isWeekend, and created_at_dayofweek.

It is important to note that categorical variables were excluded from the scaling process.

Analysis

Histograms of categorical columns reveal distinct patterns. For instance, created_at_hour shows higher order counts in the early mornings and late evenings, while created_at_dayofweek highlights increased orders on Saturdays and Sundays.

The Variance Inflation Factor (VIF) for all categorical variables is below 2.

Model coefficients remain consistent and comparable between scaled and unscaled versions.

Among these, market_id and order_protocol exhibit higher coefficients relative to the other variables.

Question 2. [1 marks]

What does test_size = 0.2 refer to during splitting the data into training and test sets?

Answer:

test_size = 0.2 represents an 80-20 split of the dataset into training and testing sets.

80% of the data is allocated for training the model (TRAIN set).

The remaining 20% is reserved for validating/evaluating the model (TEST set).

Question 3. [1 marks]

Looking at the heatmap, which one has the highest correlation with the target variable?

Answer:

From the training dataset, distance shows the strongest correlation with time_taken at 0.459712.

distance: 0.459712

subtotal: 0.412878

total_outstanding_orders: 0.381642

Question 4. [2 marks]

What was your approach to detect the outliers? How did you address them?

Answer:

I applied the Interquartile Range (IQR) method. First, Q1 (25th percentile) and Q3 (75th percentile) were calculated. The IQR was then determined as (Q3 – Q1).

Next, the lower and upper limits were established as:

lower_ceiling = $Q1 - (1.5 \times IQR)$

higher_ceiling = $Q3 + (1.5 \times IQR)$

Finally, all numeric columns were iterated through, and the dataframe was filtered to retain only the values falling within these limits.

Question 5. [2 marks]

Based on the final model, which are the top 3 features significantly affecting the delivery time?

Answer:

Based on the coefficients from the scaled data in the final model, the following are the top three features with the greatest influence on time_taken per unit change:

subtotal: 0.5513

distance: 0.5457

total_outstanding_orders: 0.2484

General Subjective Questions

Question 6. [3 marks]

Explain the linear regression algorithm in detail

Answer:

A linear regression model seeks to describe the relationship between a dependent variable and one or more independent variables using a straight line. When there is just one independent variable, it is termed simple linear regression; with multiple independent variables, it is referred to as multiple linear regression.

At least one predictor must have a linear relationship with the target variable. The main goal of linear regression is to predict values of the target variable using a line that minimizes the Residual Sum of Squares (RSS), also known as the best-fitting line or least cost function.

The target variable is expressed through the equation of the best-fit line: $y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 \dots$

 β_0 represents the intercept

 βx are the coefficients of the predictor variables

Model performance is assessed using metrics such as p-value, R-squared, and the F-statistic.

Key considerations include overfitting, multicollinearity, and feature selection.

Additionally, the dataset should satisfy assumptions such as normally distributed

error terms, mean equal to zero, constant variance, and independence among predictor variables.

Question 7. [2 marks]

Explain the difference between simple linear regression and multiple linear regression

Answer:

Simple Linear Regression (SLR)

In simple linear regression, there is only one independent variable used to predict the dependent variable.

The relationship is represented by the equation: $y = \beta_0 + \beta_1 X_1$

It is straightforward, as it only measures how one predictor impacts the target variable.

Multiple Linear Regression (MLR)

In multiple linear regression, two or more independent variables are used to explain the dependent variable.

The equation looks like this: $y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 \dots$

Because multiple predictors are involved, additional steps are required:

RFE (Recursive Feature Elimination): To identify and retain only the most significant variables.

VIF (Variance Inflation Factor): To check for multicollinearity and ensure that independent variables do not strongly affect each other.

Scaling: To bring all features to comparable ranges so that coefficients are not biased due to differing units.

Common Assumptions (Apply to Both SLR & MLR)

Error terms should follow a normal distribution.

Mean of error terms should be 0.

Variance of errors should remain constant (homoscedasticity).

Predictor variables should be independent of each other.

In summary, SLR is simpler with one predictor, while MLR uses multiple predictors but requires additional checks (RFE, VIF, scaling) to ensure accuracy and validity.

Question 8. [2 marks]

What is the role of the cost function in linear regression, and how is it minimized?

Answer:

Minimizing the cost function is essential to determine the best-fitting line. One common approach is to minimize the Residual Sum of Squares (RSS).

If we define the error as:

```
error = y(actual) - y(predicted)
```

then RSS = error1^2 + error2^2 + error3^2 + error4^2

The goal is to identify the optimal intercept and coefficients such that RSS is minimized. Libraries like statsmodels or scikit-learn are commonly used to calculate these values.

To evaluate the model, we rely on certain metrics:

R-Squared (R2):

R-Square = 1 – (RSS/TSS)

where TSS is total sum of squares

RSE = SQRT(RSS/(n-2)) MSE = RSS/n

Question 9. [2 marks]

Explain the difference between overfitting and underfitting.

Answer:

Overfitting

Occurs when the model becomes too complex by adding too many variables or features.

Instead of learning general patterns, the model starts memorizing the training data.

This results in very high training accuracy but poor test accuracy, since the model cannot generalize to unseen data.

An overfit model often shows a very high R-squared value on the training set, which is misleading because it does not actually perform well in prediction.

Underfitting

The opposite of overfitting, underfitting happens when the model is too simple to capture the underlying patterns in the data.

As a result, it fails to achieve good accuracy on both the training set and the test set.

The R-squared value remains low for both, showing that the model is not explaining much variance in the target variable.

Question 10. [3 marks]

How do residual plots help in diagnosing a linear regression model?

Answer:

1. Linearity Check

In linear regression, we assume a linear relationship between predictors and the target variable.

If the residual plot shows random scatter around zero, this assumption holds.

If there's a clear curve/pattern, it suggests the relationship is not linear \rightarrow the model is misspecified.

2. Constant Variance (Homoscedasticity)

Another assumption is that residuals have constant variance across all fitted values.

If the spread of residuals increases or decreases (funnel shape), it indicates heteroscedasticity, meaning errors vary across predictions.

3. Independence of Errors

Residuals should not display any systematic pattern (like trends or clustering).

Patterns may indicate autocorrelation or that some important variable is missing.

4. Normality of Errors

While not directly shown in a residual vs. fitted plot, residual plots combined with histograms or Q-Q plots help check if residuals are normally distributed, which is an assumption for hypothesis testing in regression.