Tag(s): Algorithms, Counting and Arrangements, Dynamic Programming, Number theory

PROBLEM

EDITORIAL

MY SUBMISSIONS

ANALYTICS

Suppose there is a grid of size n * m where each cell is either black or white in color. We need to find number of paths from (1,1) to (n,m) such that we travel only through white cells (i.e we cannot visit black cell at all). We can either go right or down from a cell.

Let TotalPath(a,e)= Total number of paths from cell a to cell $e=(a_x-e_x+a_y-e_y)!/((a_x-e_x)!*(a_y-e_y)!)$

 $m{B} = ext{Total Number of paths having at least one black cell}$

 $oldsymbol{C}=$ Total number of paths having no black cell at all

Then C = TotalPath((1,1),(n,m)) - B

In order to find B, let us assume we have a list of black cells $[b_1, b_2, b_3, \ldots, b_k]$ where each b_i is some cell (x, y). Lets sort the list in the increasing order of row and in case of same row, we will have the increasing order of column.

Let $BlackPath(b_j) =$ Number of paths ending at b_j without visiting any b_i such that i < j

Then $BlackPath(b_j) = TotalPath((1,1),b_j) - \sum_{i=1}^{j-1} BlackPath(b_i) * TotalPath(b_i,b_j)$

If (n, m) is not a black cell, then

 $C = TotalPath((1,1),(n,m)) - \sum_{i=1}^{k} BlackPath(b_i) * TotalPath(b_i,(n,m))$

CONTRIBUTOR



AUTHOR Reshab Gupta



ESTER

Amirreza Poorakhavan

THIS PROBLEM WAS ASKED IN



SOCIAL SHARE





in

Now let us solve the actual problem now.

 $X = P_1 * P_2$ where P_i is a prime number.

Let D= Number of paths having product divisible by P_1 and not by P_2

E= Number of paths having product divisible by P_2 and not by P_1

F= Number of paths having product divisible by neither P_1 nor P_2

 $\emph{G}=$ Total number of paths having product not divisible by $\emph{X}=\emph{D}+\emph{E}+\emph{F}$

We will use the above mentioned algorithm to find $oldsymbol{G}$.

Lets assume points having value divisible by P_2 are black cells and we can make a sorted list of black cells as $[b_1, b_2, b_3, \ldots, b_k]$

If (n, m) is not a black cell, then

$$C_1 = TotalPath((1,1),(n,m)) - \sum_{i=1}^k BlackPath(b_i) * TotalPath(b_i,(n,m)) = D + F$$

Like this we can also find $C_2=E+F$ and $C_3=F$

Then
$$G=C_1+C_2-C_3$$

Analysis:

- Precomputation of factorials and inverse factorials = O((n+m)*log2(MOD))
- Sorting the black cells $= O(K * log_2(K))$
- Computation of C = O(K * K) by using memoization the recurrence relation mentioned above.
- Time complexity $= O((n+m)*log2(MOD) + K^2))$
- Space complexity = O(n + m + K)
- There is a solution which has time complexity of $=O((n+m)*log2(MOD)+K^3)$ and space complexity of $O(n+m+K^2)$, but it wont be able to get 100 points for this problem. https://ideone.com/4ZPtaw

IS THIS EDITORIAL HELPFUL?



Yes, it's helpful



No, it's not helpful

17 developer(s) found this editorial helpful.

Author Solution by Reshab Gupta

```
1. #include <iostream>
 2. #include <vector>
 3. #include <cstring>
 4. #include <algorithm>
 6. using namespace std;
 8. #define ll long long
 9. #define MAX 2000000
10. #define P1 1000000007
11. #define P2 100000007
12. #define MOD 1000000007
13. #define X 100000007700000049
14.
15. int fact[MAX + 5], ifact[MAX + 5];
16.
17. inline int mul(int x, int y) {
18.
       ll z = 1LL * x * y;
19.
       if (z >= MOD) {
20.
            z %= MOD;
21.
       }
22.
       return z;
23. }
24.
25. inline int add(int x, int y) {
       int z = x + y;
26.
27.
       if (z >= MOD) {
28.
            z \rightarrow MOD;
```

```
IVE EVENTS
```

```
29.
       }
30.
       return z;
31. }
32.
33. inline int sub(int x, int y) {
34.
       int z = x - y;
35.
       if (z < 0) {
36.
            z += MOD;
37.
       }
38.
       return z;
39. }
40.
41. inline int fastExp(int x, int y) {
42.
       int z = 1;
43.
       while (y) {
            if (y & 1) {
44.
                 z = mul(z, x);
45.
46.
            y >>= 1;
47.
48.
            x = mul(x, x);
49.
       }
50.
       return z;
51. }
52.
53. void preCompute(int n) {
       fact[0] = ifact[0] = 1;
54.
       for (int i = 1; i <= n; ++i) {</pre>
55.
56.
            fact[i] = mul(fact[i - 1], i);
57.
            ifact[i] = mul(ifact[i - 1], fastExp(i, MOD - 2));
58.
       }
59. }
60.
61. int getCategory(ll no) {
62.
       if (no % X == 0) {
63.
            return 2;
64.
       } else if (no % P1 == 0) {
65.
            return 1;
66.
       } else if (no % P2 == 0) {
67.
            return 0;
```

```
IVE EVENTS
```

```
68.
69.
        return -1;
70. }
71.
72. int getWays(int n, int m) {
73.
        return mul(fact[n + m], mul(ifact[n], ifact[m]));
74. }
75.
76. int solveTask(vector<pair<int, int>> points) {
77.
        int i, j, x, y, xx, yy, tot, k;
        int memoize[5005];
78.
79.
        k = points.size();
80.
        for (i = 0; i < k; ++i) {
81.
             x = points[i].first;
82.
             y = points[i].second;
83.
             tot = qetWays(x - 1, y - 1); //Total no of ways to reach point
    (x,y)
84.
             for (j = i - 1; j >= 0; --j) {
                  xx = points[j].first;
85.
                 yy = points[j].second;
86.
                  if (yy <= y) {
87.
                      //special point
88.
89.
                      tot = sub(tot, mul(memoize[j], getWays(x - xx, y - yy)));
                  }
90.
91.
92.
            //tot = total \ no \ of \ ways \ to \ reach \ point \ (x,y) \ without \ visiting
    special points
93.
             memoize[i] = tot;
94.
        return memoize[k - 1];
95.
96. }
97.
98. int main() {
99.
100.
        ios base::sync_with_stdio(0);
        cin.tie(0);
101.
          freopen("in.txt", "r", stdin);
102. //
          freopen("out1.txt", "w", stdout);
103. //
104.
```

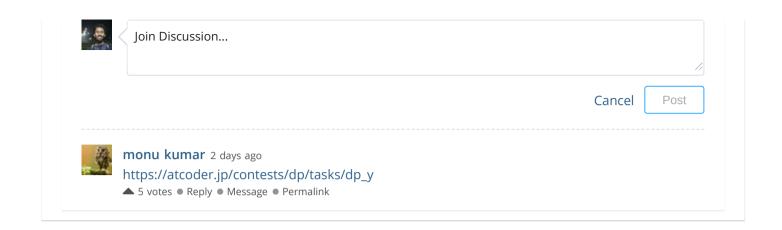
```
IVE EVENTS
```

```
int n, m, k, w, i, x, y, cat, catDest = -1, cnt[3];
105.
106.
        ll v;
107.
        vector<pair<int. int> > points[3]:
        cin >> n >> m >> k >> w:
108.
109.
        for (i = 0; i < k; ++i) {
110.
             cin >> x >> y >> y;
111.
             if ((cat = getCategory(v)) != -1) {
112.
                  if (cat != 0) {
113.
                      points[0].emplace back(x, y);
114.
                  }
                 if (cat != 1) {
115.
116.
                      points[1].emplace back(x, y);
117.
118.
                  points[2].emplace back(x, y);
119.
             }
             if (x == n \text{ and } y == m) {
120.
121.
                  catDest = getCategory(v);
122.
             }
123.
124.
        preCompute(n + m);
125.
        for (i = 0; i < 3; ++i) {
126.
             if (catDest == -1 or (catDest != 2 and catDest == i)) {
127.
                  points[i].emplace back(n, m);
128.
             sort(points[i].begin(), points[i].end());
129.
130.
131.
        cnt[2] = catDest == -1 ? solveTask(points[2]) : 0;
132.
        cnt[1] = (catDest == 1 or catDest == -1) ? solveTask(points[1]) : 0;
133.
        cnt[0] = (catDest == 0 \text{ or } catDest == -1) ? solveTask(points[0]) : 0;
        cout << sub(add(cnt[0], cnt[1]), cnt[2]);</pre>
134.
135.
136.
        return 0;
137. }
```

COMMENTS (1) 2

SORT BY: Relevance ▼

?



	For Developers	Developer Resources	For Business	Company
	Practice programming	Developers blog	Assess developers	About us
+1-650-461-4192 contact@hackerearth.cor	Complete reference to competitive	Learn to code by competitive	Conduct remote interviews	Press
	programming	programming		Careers
	Competitive coding challenges	Developers wiki	Assess university talent	Contact us
	Code Monk	How to conduct a hackathon		Technical
	Start a programming club		Organize hackathon	support
f w in				