24·12·2019 (M)

K. J. Somaiya College of Engineering, Mumbai-77

(Autonomous College Affiliated to University of Mumbai)

End Semester Exam

Nov - Dec 2019

Max. Marks: 100	**Duration: 3 Hrs.**
Class: F.Y.BTech	Semester: II
Name of the Course: Applied Mathematics-II	Branch: ALL
Course Code: 2UHC111	

Question No.			Max. Marks
Q.1	A	**Attempt the following:**	
	(a)	Examine in detail the correctness of sentence: $\int_0^\infty \sqrt[4]{x}\, e^{-\sqrt{x}}\, dx = \sqrt{\pi}$.	03
	(b)	Using Beta Function Evaluate: $\int_0^{1/2} x^3(1-4x^2)^{1/2}\, dx$.	03
	B	**Attempt any ONE of the following:**	
	(a)	Evaluate: $\int_0^{\pi/2} sin^5\theta cos^4\theta\, d\theta$. Hence deduce that $\int_0^{\pi} \theta\, sin^5\theta cos^4\theta\, d\theta = \frac{8\pi}{315}$.	07
	(b)	Using DUIS Show that $\int_0^\infty \frac{\log(1+ax^2)}{x^2} dx = \pi\sqrt{a}$ $(a > 0)$. Hence, evaluate $\int_0^\infty \frac{\log(1+x^2)}{x^2} dx$	
Q.2	A	**Attempt any ONE of the following :**	
	(a)	Find length of cardioide $r = a(1 - cos\theta)$ lying outside the circle $r = acos\theta$.	07
	(b)	Find the length of curve $x = asin2t(1 + cos2t)$, $y = acos2t(1 - cos2t)$measured from origin to $t = \pi/2$.	
Q.3	A	**Attempt the following:**	05
		Evaluate : $\int_0^1 \int_{y^2}^1 \int_0^{1-x} x\, dzdxdy$.	
	B	**Attempt any FOUR of the following:**	
	(a)	Change order of integration and evaluate: $\int_0^5 \left[\int_{2-x}^{2+x} dy\right] dx$.	28
	(b)	Change to polar coordinate and evaluate $\iint y^2 dxdy$ over the area outside $x^2 + y^2 - ax = 0$ and inside $x^2 + y^2 - 2ax = 0$.	

	(c)	Find the mass of a plate in the form of one loop of leminscate $r^2 = a^2 cos2\theta$ if density varies as the square of the distance from pole.	
	(d)	Find by double integration the Area between parabola $y = x^2 - 6x + 3$ and the line $y = 2x - 9$.	
	(e)	Evaluate the integral $\iiint xyz^2 dv$ over the region bounded by planes $x = 0, x = 1, y = -1, y = 2, z = 0 \;\&\; z = 3$.	
	(f)	Calculate $\iint r^3 dr d\theta$ over the area included between the circles $r = 2sin\theta$ and $r = 4sin\theta$.	
Q.4	A	**Attempt the following:**	
		Solve $\frac{dy}{dx} + \frac{xsinx+cosx}{xcosx} y = \frac{1}{xcosx}$	05
	B	**Attempt any FOUR of the following:**	
	(a)	Solve: $sin2x \frac{dy}{dx} = y + tanx$.	
	(b)	Solve: $(5 + 2x)^2 \frac{d^2y}{dx} - 6(5 + 2x) \frac{dy}{dx} + 8y = 6x$.	28
	(c)	Solve: $(D^2 - 4D + 3)y = 2xe^{3x} + 3e^x cos2x$.	
	(d)	Solve: $(x^4 + y^4)dx - xy^3 dy = 0$.	
	(e)	Solve : $(D^2 - 4D + 4)y = x^2 + e^x + cos2x$.	
Q:5		**Attempt any TWO of the following:**	
	(a)	If $y^{1/m} + y^{-1/m} = 2x$, prove that $(x^2 - 1)y_{n+2} + (2n + 1)xy_{n+1} + (n^2 - m^2)y_n = 0$.	
	(b)	If $y = \frac{x}{x^2+a^2}$, Prove that $y_n = (-1)^n\, n!\, a^{-n-1} sin^{n+1}\theta \cos(n + 1)\,\theta$ where $\theta = tan^{-1}\left(\frac{a}{x}\right)$.	14
	(c)	Expand $(1 + x)^{1/x}$ in powers of x, upto the term x^2.	

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Class: F.Y.BTech	Semester: II
Name of the Course: Applied Mathematics-II	Branch: ALL
Course Code: USHC201 (KJSCE2014)	

Instructions:
(1) All Questions are Compulsory
(2) Figures to right indicate full marks. Each sub-question has equal marks.

Question No.			Max. Marks
Q.1	A	Evaluate $\int_0^\infty e^{-x^4}dx$	04
	B	**Solve any THREE of the following:**	
	(i)	Prove that $\int_0^1 \frac{\tan^{-1}ax}{x(1+x^2)}dx = \frac{\pi}{2}\log(1+a)$, using rule of DUIS	
	(ii)	Prove that $\int_0^\infty xe^{-x^8}dx.\int_0^\infty x^2e^{-x^4}dx = \frac{\pi}{16\sqrt{2}}$	
	(iii)	Prove that: $\int_{-\pi/6}^{\pi/3}\left(\sqrt{3}\,sin\theta + cos\theta\right)^{1/4}d\theta = 2^{-3/4}\,B\left(\frac{1}{2},\frac{5}{8}\right)$.	18
	(iv)	Prove that: $\int_3^7 \sqrt[4]{(x-3)(7-x)}dx = \frac{2\left(\Gamma 1/4\right)^2}{3\sqrt{\pi}}$.	
	(v)	Using DUIS Show that $\int_0^\infty \frac{\log(1+ax^2)}{x^2}dx = \pi\sqrt{a}$ $(a>0)$. Hence, evaluate $\int_0^\infty \frac{\log(1+x^2)}{x^2}dx$	
Q.2	A	Solve: $(D^2+3D+2)y = 0$	04
	B	**Solve any FOUR of the following:**	
	(i)	Solve: $(D^2-5D+6)y = e^x + sin2x$.	
	(ii)	Solve: $(D^2+2)y = x^2e^{3x}$.	
	(iii)	Solve : $x\frac{dy}{dx} + y = x^3y^6$	24
	(iv)	Use method of variation of parameters, Solve $(D^2-6D+9)y = \frac{e^{3x}}{x^2}$.	
	(v)	Solve $y(x+y)dx - x(y-x)dy = 0$	
	(vi)	Solve: $(x^2D^2+3xD+1)y = x^2logx$.	

Q.3	A	Using Euler's method, find the approximate value of y at x=1 in five steps i.e. $h = 0.2$ when $\frac{dy}{dx} = x^2 + y^2$ and $y = 2$ when $x = 0$	04
	B	**Solve any TWO of the following:**	12
	(i)	In a circuit containing inductance L, resistance R, and voltage E, the current i is given by $L\frac{di}{dt} + Ri = E$. Find the current i at time t if at $t = 0, i = 0$ and L, R, E are constants.	
	(ii)	Using Taylor's series method, solve $\frac{dy}{dx} = x + y$ with $x_0 = 1, y_0 = 0$. Find y when = 1.2 . Compare the result with exact solution.	
	(iii)	Apply Runga-Kutta of fourth order to find y at $x = 0.2$. Given $\frac{dy}{dx} + y + xy^2 = 0,\ y(0) = 1,\ h = 0.1$.	
Q.4		**Solve any ONE of the following:**	06
	(i)	Find the total length of the curve $y = (x/2)^{2/3}$ from $x = 0$ to $x = 2$	
	(ii)	Find the length of one arc of the cycloid $x = a(\theta + \sin\theta),\ y = a(1 - cos\theta)$	
Q.5	A	Evaluate the integral $\int_0^1 \int_0^{x^2} e^{\frac{y}{x}}\, dy\, dx$	04
	B	**Solve any FOUR of the following:**	24
	(i)	Evaluate $\int_0^{\pi/2} \int_0^{acos\theta} r\sqrt{a^2 - r^2} dr\, d\theta$	
	(ii)	Evaluate: $\int_0^1 \int_{y^2}^1 \int_0^{1-x} x\, dzdxdy.$	
	(iii)	Find by double integration the area bounded by the lines $y = a + x, y = a - x$ and $x = a\, , (a > 0)$	
	(iv)	Find the area outside the circle $r = a$ and inside the cardioids $r = a(1 + cos\theta)$	
	(v)	Change order of integration and evaluate: $\int_0^5 \left[\int_{2-x}^{2+x} dy\right] dx.$	