Derivation

Laser

1) Einstein's coefficient

At equilibrium $\frac{N_1}{N_2}$ = constant

We know,

$$N_1 = N_2 e^{(E_2-E_1)/KT} \quad -①$$

$$N_{abs} = N_{sp} + N_{st}$$

$$B_{12} N_1 Q \Delta t = A_{21} N_2 \Delta t + B_{21} N_2 \Delta t$$

According to Einstein's

$$B_{12} = B_{21}$$

$$\frac{N_1}{N_2} = 1 + \frac{A_{21}}{B_{21} Q}$$

From ①

$$e^{(E_2-E_1)/KT} = 1 + \frac{A_{21}}{B_{21} Q}$$

$$\cancel{Q = \frac{A_{21}}{B_{21}} \left[\frac{1}{1 + e^{h\nu/KT}}\right]}$$

$$Q = \frac{A_{21}}{B_{21}} \left[\frac{1}{e^{h\nu/KT} - 1}\right]$$

Compare with

$$Q = \frac{8\pi h\nu^3}{c^3} \left[\frac{1}{e^{(h\nu/KT)} - 1}\right]$$

$$\therefore \frac{A_{21}}{B_{21}} = \frac{8\pi h\nu^3}{c^3} \ J\text{-}s/m^3$$

2) Lasing Condition

For $G \geq 1 \Rightarrow \frac{I(2L)}{I_0} \geq 1$

$r_1 r_2 e^{(\gamma - \alpha_s) 2L} \geq 1$

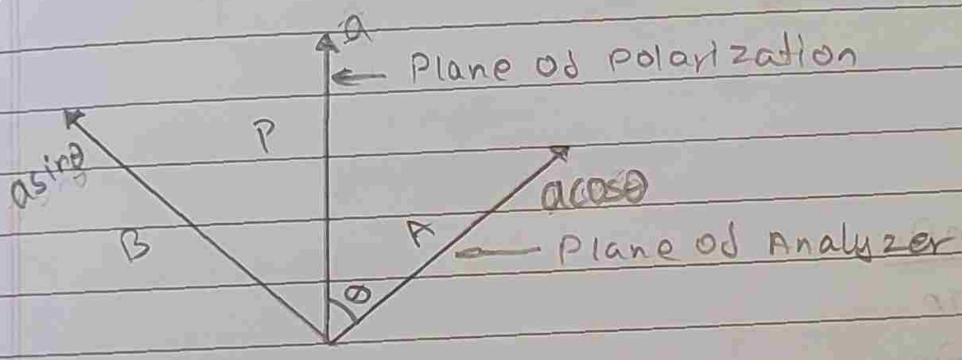
$e^{(\gamma - \alpha_s) 2L} \geq \frac{1}{r_1 r_2}$

$(\gamma - \alpha_s) 2L \geq \ln\left[\frac{1}{r_1 r_2}\right]$

$\gamma \geq \alpha_s + \frac{1}{2L} \ln\left[\frac{1}{r_1 r_2}\right]$

Polarization

3) Malus' Law



$I = K(a\cos\theta)^2$

$I = K^2 a^2 \cos^2\theta$

Where $K^2 a^2 = I_0$

$\therefore I = I_0 \cos^2\theta$

→ Case I

$\theta = 0°$ or $180°$

$\boxed{I = I_0}$

→ Case II

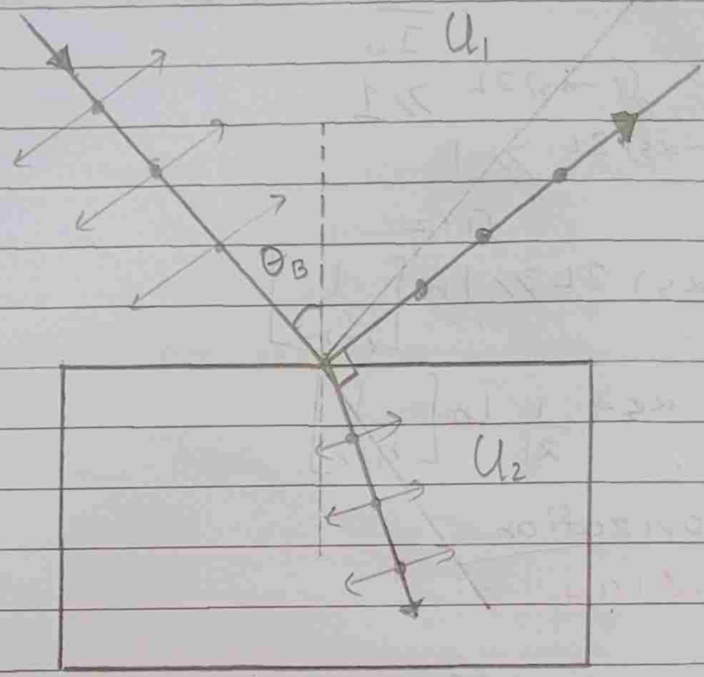
$\theta = 90°$

$\boxed{I = 0}$

→ Case III

When unpolarized light is incident on the analyzer the $\boxed{I = \frac{I_0}{2}}$

4) Brewsters Law



$\theta_B + r = 90^\circ$

$r = 90^\circ - \theta_B$

$$ {}_1\mu_2 = \frac{\sin\theta_B}{\sin r} $$

$$ {}_1\mu_2 = \frac{\sin\theta_B}{\sin(90^\circ - \theta_B)} $$

$$ {}_1\mu_2 = \frac{\sin\theta_B}{\cos\theta_B} $$

$$ {}_1\mu_2 = \tan\theta_B $$

5) Elliptically & circularly polarized light

$x = A\cos\theta \sin(\omega t + \delta) \rightarrow$ E-rays — (1)

$y = A\sin\theta \sin(\omega t) \rightarrow$ O-rays — (2)

Let, $A\cos\theta = a$ & $A\sin\theta = b$

$x = a\sin(\omega t + \delta)$ — (3)

$y = b\sin(\omega t)$ — (4)

$\sin\omega t = \frac{y}{b}$ — From (4) — (5)

$\cos\omega t = \sqrt{1 - \frac{y^2}{b^2}}$ — (6)

$\frac{x}{a} = \sin(\omega t + \delta)$ — From (3) — (7)

$\cancel{\frac{x}{a} = \frac{y}{b}\cos\delta + \sqrt{1 - \frac{y^2}{b^2}}}$

$\frac{x}{a} = \sin\omega t \cos\delta + \cos\omega t \sin\delta$

$\frac{x}{a} = \frac{y}{b}\cos\delta + \sin\delta\sqrt{1 - \frac{y^2}{b^2}}$ from (5) & (6)

$\frac{x}{a} - \frac{y}{b}\cos\delta = \sin\delta\sqrt{1 - \frac{y^2}{b^2}}$

Now squaring on bothsides

$\left(\frac{x}{a} - \frac{y}{b}\cos\delta\right)^2 = \sin^2\delta\left[1 - \frac{y^2}{b^2}\right]$

$\frac{x^2}{a^2} + \frac{y^2}{b^2}\cos^2\delta - \frac{2xy}{ab}\cos\delta = \sin^2\delta\left[1 - \frac{y^2}{b^2}\right]$

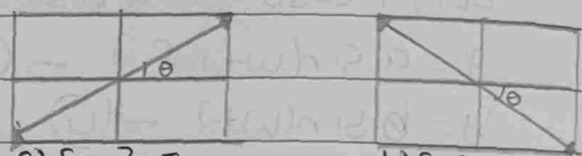
$\frac{x^2}{a^2} + \frac{y^2}{b^2}(\cos^2\delta + \sin^2\delta) - \frac{2xy}{ab}\cos\delta = \sin^2\delta$

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{2xy}{ab}\cos\delta=\sin^2\delta \quad -(7)$$

Case I

$\delta = 0$ or $2n\pi$

$y=\left(\frac{b}{a}\right)x \rightarrow$ Eq od st line

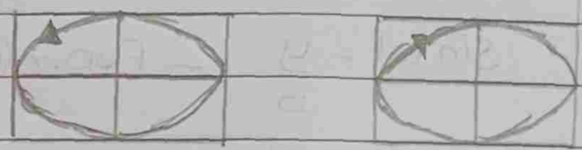


a) $\delta = 2n\pi$ b) $\delta = (2n+1)\pi$

Case II

$\delta = (2n+1)\pi$

$\therefore \frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \rightarrow$ Eq of ellipse

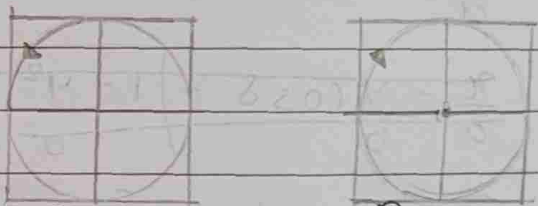


a) $\delta = \frac{\pi}{2}$ b) $\delta = \frac{3\pi}{2}$

Case III

$\delta = (2n+1)\pi$, $a=b$

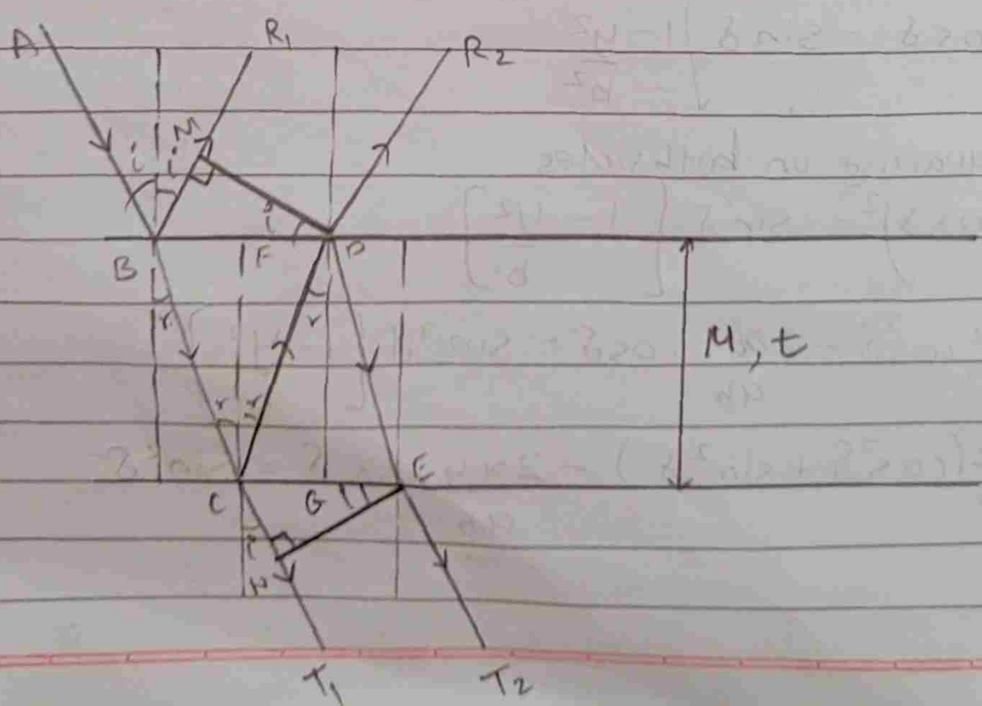
$x^2+y^2=a^2 \rightarrow$ Eq of circle



a) $\delta = \frac{\pi}{2}$ $\delta = \frac{3\pi}{2}$

Interference

6) Transmitted & Reflected (Thin flim)



i) Reflected

$Opd = \mu(BC+CD) - BN$ — ①

From fig $BC = CD$

$Opd = \mu(2BC) - BM$ — ②

consider ΔBCF,

$\cos r = \frac{CF}{CB}$

But $CF = t$

CB or $BC = \frac{t}{\cos r}$ — ③

$\tan r = \frac{BF}{CF}$

$BF = t \tan r$ — ④

From ΔBDM

$\sin i = \frac{BM}{BC} = \frac{BM}{2BF}$

$\therefore BM = 2BF \sin i$

From ④

$BM = 2t \tan r \sin i$ — ⑤

Put ③ & ⑤ in ②

$Opd = \mu \times \frac{2t}{\cos r} - \frac{\sin r}{\cos r} \times 2t \frac{\sin i}{\sin r} \times \sin r$

$Opd = \mu \frac{2t}{\cos r} - \frac{\sin^2 r}{\cos r} \times 2t\mu$ $\quad \boxed{\because \mu = \frac{\sin i}{\sin r}}$

$Opd = \frac{2t\mu}{\cos r}\left[1 - \sin^2 r\right) = \frac{2\mu t}{\cos r} \times \cos^2 r$

$Opd = 2\mu t \cos r$ — ⑥

Now, due to reflection at surface of Denser medium,

$OPd = 2\mu t\cos r + \frac{\lambda}{2}$ — (7)

Case (I)

For maxima,

$OPd = n\lambda$

$n\lambda = 2\mu t\cos r + \frac{\lambda}{2}$

$\therefore 2\mu t\cos r = (2n-1)\frac{\lambda}{2}$, $n = 1, 2, 3 \ldots$

Case (II)

For Minima,

$OPd = (2n+1)\frac{\lambda}{2}$

$2n\lambda + \frac{\lambda}{2} = 2\mu t\cos r + \frac{\lambda}{2}$

$\therefore 2\mu t\cos r = n\lambda$, $n = 0, 1, 2, 3 \ldots$

ii) Transimitted

$OPd = \mu(CD + DE) - CN$ — (1)

From Fig

$CD = DE$

$OPd = 2\mu(CD) - CN$ — (2)

Consider ΔCDG,

$\cos r = \frac{DG}{CD}$

But $DG = t$

$CD = \frac{t}{\cos r}$ — (3)

$\tan r = \frac{CG}{DG}$

$CG = t \tan r$ — (4)

From Δ CEN,

$$\sin i = \frac{CN}{CE} = \frac{CN}{CG+GE} = \frac{CN}{2CG}$$

$\therefore CN = 2CG \sin i$

From (4)

$CN = 2t \tan r \sin i$ — (5)

Put (3) & (5) in (2)

$$Opd = \frac{2\mu t}{\cos r} - 2t \frac{\sin r}{\cos r} \frac{\sin i}{\sin r} \times \sin r$$

$$Opd = \frac{2\mu t}{\cos r}\left[1 - \sin^2 r\right]$$

$\boxed{\therefore \mu = \frac{\sin i}{\sin r}}$

$Opd = 2\mu t \cos r$ — (6)

Case (I)

For Maxima,

$Opd = n\lambda$

$n\lambda = 2\mu t \cos r$, $n = 0, 1, 2, 3 \ldots$

Case (II)

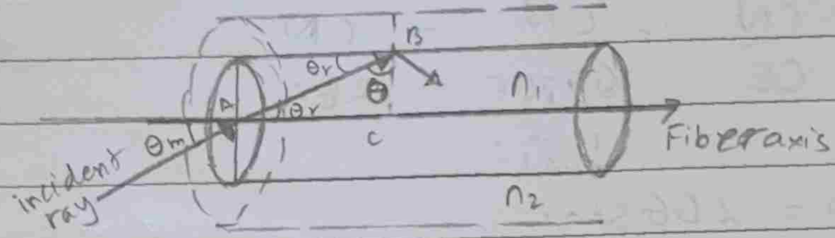
For minima

$$Opd = (2n-1)\frac{\lambda}{2}$$

$$2\mu t \cos r = (2n-1)\frac{\lambda}{2}, \quad n = 1, 2, 3 \ldots$$

Optical fibre

7) Numerical Apperture / Acceptance Angle



$n_0 \sin\theta_i = n_1 \sin\theta_r$

$\theta_r = 90 - \theta$

$n_0 \times \sin\theta_i = n_1 \sin(90-\theta)$

$\sin\theta_i = \frac{n_1}{n_0}\cos\theta$

When $\theta = \theta_c$, $\theta_i \to \theta_m$

$\sin\theta_m = \frac{n_1}{n_0}\cos\theta_c$ — ①

According to law of reflection

$n_1 \sin i = n_2 \sin r$

$r = 90°$, $i = \theta_c$

$\sin\theta_c = \frac{n_2}{n_1}$

$\cos\theta_c = \sqrt{1 - \frac{n_2^2}{n_1^2}} = \frac{\sqrt{n_1^2 - n_2^2}}{n_1}$ — ②

Put in ①

$\sin\theta_m = \frac{n_1}{n_0}\sqrt{\frac{n_1^2 - n_2^2}{n_1^2}} = \frac{\sqrt{n_1^2 - n_2^2}}{n_0}$

For air, $n_0 = 1$

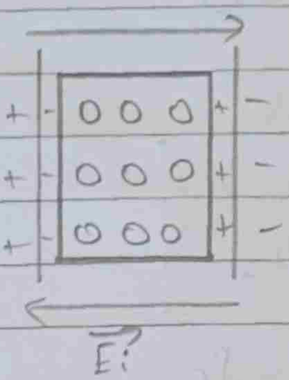
$\sin\theta_m = \sqrt{n_1^2 - n_2^2}$

$\therefore NA = \sqrt{n_1^2 - n_2^2}$

$\theta_m = \sin^{-1}[\sqrt{n_1^2 - n_2^2}]$

Dielectric

8) Relation between E, P & D



$E_0 = \frac{\sigma}{\varepsilon_0}$ — ①

$E_i = \frac{\sigma_p}{\varepsilon_0}$ — ②

$E = E_0 - E_i$

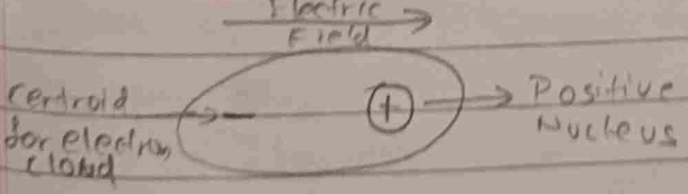
$E = \frac{\sigma - \sigma_p}{\varepsilon_0}$

$\varepsilon_0 E = \sigma - \sigma_p$

$\varepsilon_0 E = D - P$

$\boxed{\vec{D} = \varepsilon_0 \vec{E} + \vec{P}}$

9) Expression for Electronic polarization



According to Gauss's law,

Volume od sphere with Radius $x = \frac{4\pi x^3}{3}$

Volume od sphere with Radius $R = \frac{4\pi R^3}{3}$

$\therefore$ The quantity of negative charge enclosed,

$$\theta = -Ze \frac{\frac{4\pi x^3}{3}}{\frac{4\pi R^3}{3}} = -\frac{Ze\,x^3}{R^3} \quad \text{—①}$$

According to coulomb's law,

$$\frac{Ze\,x \cdot -\frac{Ze\,x^3}{R^3}}{4\pi\varepsilon_0 x^2} = \frac{-Z^2e^2 x}{4\pi\varepsilon_0 R^3} = f_c$$

At equilibrium condition, $\boxed{f_L = -ZeE}$

$$-ZeE = \frac{-Z^2e^2x}{4\pi\varepsilon_0 R^3}$$

$$E = \frac{Zex}{4\pi\varepsilon_0 R^3}$$

$$x = \frac{4\pi\varepsilon_0 E R^3}{Ze} \quad \text{—②}$$

Put ② in dipole moment

$$Ze\left[\frac{4\pi\varepsilon_0 R^3}{Ze}\right]E = (4\pi R^3\varepsilon_0)E$$

$\therefore P_e = 4\pi\varepsilon_0 R^3 N E$, where $N = \frac{\text{no. of dipole moment}}{\text{volume}}$

$\therefore P_e = \alpha N E$

$\therefore \boxed{\alpha = 4\pi\varepsilon_0 R^3}$

10) Clausius Mosotti Equation

$$\alpha_e = \frac{P}{NE_i} \quad -①$$

$$E_i = E + \frac{\gamma P}{\varepsilon_0} \quad -②$$

$$\boxed{\gamma = \frac{1}{3}}$$

Put ② in ①

$$\alpha_e = \frac{P}{N\left[E + \frac{P}{3\varepsilon_0}\right]} \quad -③$$

We know,

$$E = \frac{P}{(\varepsilon_r - 1)\varepsilon_0} \quad -④$$

Put ④ in ③

$$\alpha_e = \frac{\cancel{P}}{\frac{\cancel{P}N}{\varepsilon_0}\left[\frac{1}{\varepsilon_r - 1} + \frac{1}{3}\right]}$$

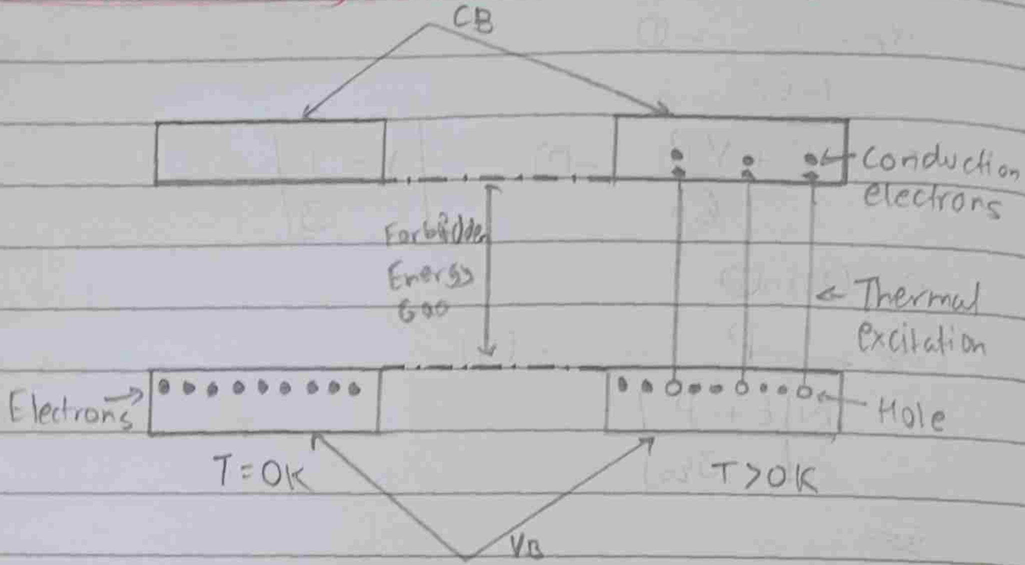
$$\alpha_e = \frac{\varepsilon_0}{N} \times \frac{1}{\left[\frac{4}{(\varepsilon_r - 1)} + \frac{1}{3}\right]}$$

$$\frac{N\alpha_e}{\varepsilon_0} = \frac{3(\varepsilon_r - 1)}{\varepsilon_r + 2}$$

$$\therefore \frac{\varepsilon_r - 1}{\varepsilon_r + 2} = \frac{N\alpha_e}{3\varepsilon_0}$$

Semiconductor

11) Fermi level is at Midway od CB & VB (Intrinsic SC)



Let 'n' be no° od electron's
'p' be no° od Holes

At T > 0K

$n = N_c e^{-(E_c - E_F)/KT}$

$P = N_v e^{-(E_F - E_v)/KT}$

For intrinsic SC, $n = P$ —①

$\frac{N_c}{N_v} = e^{(-E_E + E_F + E_F - E_c)/KT}$

$\frac{N_v}{N_c} = e^{[2E_F - (E_c + E_v)]/KT}$ —②

$\frac{N_v}{N_c} = \frac{\cancel{2} \times \left(\frac{2\pi m_h^* \cancel{KT}}{\cancel{h^2}}\right)^{3/2}}{\cancel{2} \times \left(\frac{2\pi m_e^* \cancel{KT}}{\cancel{h^2}}\right)^{3/2}} = \left(\frac{m_h^*}{m_e^*}\right)^{3/2} \sim 1$ —③ $\boxed{\because m_e^* \sim m_h^*}$

Put ③ in ②

$e^{[2E_F - (E_c + E_v)]/KT} = 1$

$\frac{2E_F - (E_c + E_v)}{KT} = 0$

$$\boxed{E_F = \frac{E_C + E_v}{2}}$$

Quantum Mechanics

12) Debroglie hypothesis

$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$, $E = mc^2$

$E^2 = m^2c^4 \left[1 - \frac{v^2}{c^2}\right] + p^2c^2$

$E^2 = m_0^2c^4 + p^2c^2$

For photon,

$m_0 = 0$

$E = PC$

But $E = h\frac{c}{\lambda}$

$\therefore P = \frac{h}{\lambda}$

$\therefore \boxed{\lambda = \frac{h}{p}}$ — ①

For kinetic energy,

$P = \sqrt{2mK}$ — ②

$\therefore \lambda = \frac{h}{\sqrt{2mK}}$

For Potential difference,

$P = \sqrt{2mqV}$ — ③

where $q = e$

$\therefore \lambda = \frac{h}{\sqrt{2meV}}$

$\therefore \lambda = \frac{12.28 \times 10^{-10}}{\sqrt{V}} m$

13) Group velocity & phase velocity

$\Psi_1 = A\cos[(\omega t - Kx)]$

$\Psi_2 = A\cos[(\omega+\Delta\omega)t - (K+\Delta K)x]$

$\Psi = \Psi_1 + \Psi_2$

$= A[\cos(\omega t - Kx) + \cos[(\omega+\Delta\omega)t - (K+\Delta K)x]]$

$\Psi = 2A\cos\frac{1}{2}[(2\omega+\Delta\omega)t - (2K+\Delta K)x]\ \cos\frac{1}{2}[(\Delta\omega)t - (\Delta K)x]$

$\cancel{\Psi} =$

$\Delta\omega << \omega << K$

$\Psi = 2A\cos\left[\left(\frac{\Delta\omega}{2}\right)t - \left(\frac{\Delta K}{2}\right)x\right]\cos[\omega t - Kx]$

Phase velocity : $V_P = \frac{\omega}{K}$

Group velocity : $V_G = \frac{\Delta\omega}{\Delta K}$ or $\frac{d\omega}{dK}$

$\omega = 2\pi\nu = 2\pi\frac{E}{h} \cancel{= \frac{2\pi}{h}\left[m_0\sqrt{1-\frac{v^2}{c}}\right]}$

$\omega = \frac{2\pi}{h}\left[\frac{m_0c^2}{\sqrt{1-\frac{v^2}{c^2}}}\right]$ —①

$K = \frac{2\pi}{\lambda} = \frac{2\pi}{h}(mv)$

$K = \frac{2\pi}{h}\times\left(\frac{m_0v}{\sqrt{1-\frac{v^2}{c^2}}}\right)$ —②

Derivative ① & ② wrt v

$\cancel{\frac{d\omega}{dv} = \frac{2\pi}{h}\frac{m_0c^2 \times 2v/c^2}{2\sqrt{1-\frac{v^2}{c^2}}}}$

$$\frac{d\omega}{dv} = \frac{2\pi}{h} m_0 \cancel{c^2} \times \frac{+\cancel{2} v/\cancel{c^2} \times c^2}{+\cancel{2}\left[1-\frac{v^2}{c^2}\right)^{3/2}} = \frac{2\pi}{h}\left[\frac{m_0 v c^2}{(1-v^2/c^2)^{3/2}}\right] \quad -③$$

$$\frac{dk}{dv} = \frac{2\pi m_0}{h}\left[\frac{\sqrt{1-\frac{v^2}{c^2}} - \frac{\cancel{2}v^2/c^2}{(1-v^2/c^2)^{3/2}}}{(1-v^2/c^2)}\right]$$

$$\frac{dk}{dv} = \frac{2\pi m_0}{h}\left[\frac{c^2}{\sqrt{1-\frac{v^2}{c^2}}}\right] \quad -④$$

$$v_g = \frac{d\omega}{dk} = \frac{d\omega/dv}{dk/dv} = \boxed{v}$$

14) Schrödinger Equation

i) Time-Dependent

$$\psi = A e^{i[kx-\omega t]} \quad -①$$

Differentiate ① wrt dt

$$\frac{d\psi}{dt} = A e^{i[kx-\omega t]} \times -i\omega = -i\omega\psi = -i\frac{E}{\hbar}\psi$$

$$i\hbar\frac{d\psi}{dt} = E\psi \quad \cancel{②} = (K+V)\psi \quad -②$$

Differentiate ① wrt x

$$\frac{d\psi}{dx} = A e^{i(kx-\omega t)} \times ik = ik\psi \quad -③$$

Differentiate ③ wrt x

$$\frac{d^2\psi}{dx^2} = ik(ik\psi) = -\frac{p^2}{\hbar^2}\psi$$

multiply by $\hbar^2/2m$

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} = \frac{p^2}{2m}\psi = K\psi \quad -④$$

$$V(x)\,\Psi(x,t) = V\Psi \quad -⑤$$

Put ④ ~~in ②~~ and ⑤ in ②

$$\therefore\ i\hbar \frac{d\Psi}{dt} = -\frac{\hbar^2}{2m}\frac{d^2\Psi}{dx^2} + V(x)\,\Psi(x,t) \quad -⑥$$

ii) <u>Time-independent</u>

$$\Psi(x,t) = \phi(x)\, f(t) \quad -⑦$$

Put ⑦ in ⑥

$$i\hbar\,\phi\,\frac{df}{dt} = \frac{-\hbar^2}{2m} f \frac{d^2\phi}{dx^2} + V(x)\,\phi(x)\, f(t)$$

Dividing by $\phi(x)\, f(t)$

$$\frac{i\hbar}{f}\frac{df}{dt} = \frac{-\hbar^2}{2m\phi}\frac{d^2\phi}{dx^2} + V(x) = S$$

but $S = \hbar\omega = E$

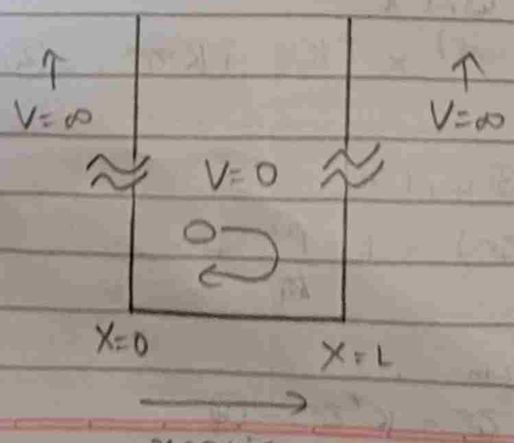
$$E = \frac{-\hbar^2}{2m\phi}\frac{d^2\phi}{dx^2} + V(x)$$

multiple by $\phi(x)$

$$E\,\phi(x) = \frac{-\hbar^2}{2m}\frac{d^2\phi}{dx} + V(x)\,\phi(x)$$

15) Infinite potential well

i) 1D



From STDE

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V(x)\,\psi = E\,\psi$$

$V = 0$ for $0 \le x \le L$

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} = E\psi$$

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}E\psi = 0$$

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{h}E\psi = 0$$

$$\frac{d^2\psi}{dx^2} + k^2\psi = 0$$

The solution,

$\psi = Ae^{ikx} + Be^{-ikx}$

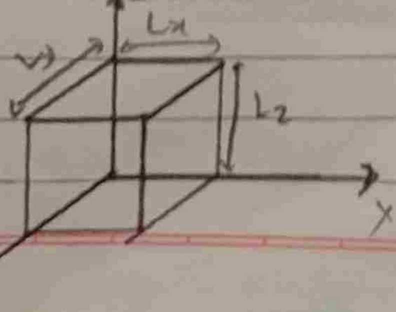
at $x=0$, $\psi = 0$

$x = L$, $\psi = L$

$$\therefore \psi_n = \sqrt{\frac{2}{L}}\sin\left(\frac{n\pi x}{L}\right) \quad -- n = 1, 2, 3$$

$$E_n = \frac{n^2h^2}{8m^2L^2} \quad --- n = 1, 2, 3$$

ii) 3D



$$\psi(x) = \sqrt{\frac{2}{L}}\sin\left(\frac{n_x \pi x}{L}\right)$$

$$\psi(y) = \sqrt{\frac{2}{L}}\sin\left(\frac{n_y \pi y}{L}\right)$$

$$\psi(z) = \sqrt{\frac{2}{L}}\sin\left(\frac{n_z \pi z}{L}\right)$$

$\psi_{(x,y,z)} = A\sin(k_x x)\sin(k_y y)\sin(k_z z)$

$k_x = n_x \pi / L_x$

$k_y = n_y \pi / L_y$

$k_z = n_z \pi / L_z$

$$E = \frac{\pi^2 \hbar^2}{2m}\left[\frac{n_x^2}{L_x^2} + \frac{n_y^2}{L_y^2} + \frac{n_z^2}{L_z^2}\right]$$

For cube,

$$E = \frac{\pi^2 h^2}{2\pi L^2}\left[n_x^2 + n_y^2 + n_z^2\right]$$

Electrodynamic

16) Gauss Law

i) Integral Form

$$\phi_E = \frac{q}{\varepsilon_0} \quad -①$$

$$\phi_E = \int_{surface} \vec{E} \cdot \vec{ds} \quad -②$$

$$q = \int_{vol} \rho \, dv \quad -③$$

From ①, ②, ③

$$\int_{surface} \vec{E} \cdot \vec{ds} = \frac{1}{\varepsilon} \int_{vol} \rho \, dv$$

$$\varepsilon \vec{E} = \vec{D}$$

$$\int_{surface} \vec{D} \, \vec{ds} = \int_{vol} \rho \, dv$$

ii) Differential form

From ~~Ga~~ divergence thm,

$$\int_{surface} \vec{D} \cdot \vec{ds} = \int_{vol} (\vec{\nabla} \cdot \vec{D}) \, dv$$

$$\int_{vol} (\vec{\nabla} \cdot \vec{D}) \, dv = \int_{vol} \rho \, dv$$

$$\therefore \vec{\nabla} \cdot \vec{D} = \rho$$

For magnetism

$$\phi_m = \oint_{surface} \vec{B} \cdot \vec{ds} = 0$$

$$\therefore \vec{\nabla} \cdot \vec{B} = 0$$

17) Faraday's Law

i) Integral form

$$e = -\frac{d\phi_m}{dt} \quad -①$$

$$e = \oint_{line} \vec{E} \cdot \vec{dl} \quad -②$$

$$\phi_m = \int_{surface} \vec{B} \cdot \vec{ds} \quad -③$$

From ①, ②, ③

$$\int \vec{E} \cdot \vec{dl} = -\frac{d}{dt} \int_{surface} \vec{B} \, \vec{ds}$$

$$\oint_{line} \vec{E} \cdot \vec{dl} = -\int_{surface} \frac{d\vec{B}}{dt} \cdot \vec{ds}$$

ii) Differential form

From Stoke's Law

$$\int_{line} \vec{E} \cdot dl = \int_{surface} (\vec{\nabla} \times \vec{E}) \cdot ds$$

$$\int_{surface} (\vec{\nabla} \times \vec{E}) \vec{ds} = -\int \frac{d\vec{B}}{dt} \cdot \vec{ds}$$

$$\vec{\nabla} \times \vec{E} = -\frac{d\vec{B}}{dt}$$

18) Ampere's Law

i) Integral form [Without correction]

$$\oint_{line} \vec{B} \cdot dl = \mu_0 I \quad -①$$

$$I = \int_{surface} \vec{J} \cdot ds \quad -②$$

From ① & ②

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 \int \vec{J} \cdot \vec{ds}$$

line   surface

ii) <u>Differential form</u>

By stokes thm

$$\int \vec{B}\, \vec{dl} = \int (\vec{\nabla} \cdot \vec{B}) \vec{ds}$$

line   surface

$$\int (\vec{\nabla} \times \vec{B}) \vec{ds} = \mu_0 \int \vec{J}\, \vec{ds}$$

surface   surface

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$$

$$\vec{\nabla} \times \vec{H} = \vec{J}$$

<u>With correction</u>

$$\vec{\nabla} \cdot (\vec{\nabla} \times \vec{H}) = \vec{\nabla} \cdot \vec{J}$$

$$\vec{\nabla} \cdot \vec{J} = 0$$

But $\vec{\nabla} \cdot \vec{J} \neq 0$ according to continuity eq^n

$$\therefore \vec{\nabla} \times \vec{H} = \vec{J} + \vec{J}_D$$

$$\vec{\nabla} \cdot (\vec{\nabla} \times \vec{H}) = \vec{\nabla} \vec{J} + \vec{\nabla} \vec{J}_D$$

$$0 = -\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot \vec{J}_D$$

$$\vec{\nabla} \cdot \vec{J}_D = \frac{\partial \rho}{\partial t}$$

$$\vec{\nabla} \cdot \vec{D} = \rho$$

18) <u>Maxwell's equation (Wave equation)</u>

$\rho = 0$, $\vec{J}_0 = 0$

$\vec{\nabla} \cdot \vec{D} = 0$ → From Ampere

$\vec{D} = \epsilon_0 \vec{E}$

$\vec{\nabla} \cdot \vec{E} = 0$ — ①

$\vec{\nabla} \cdot \vec{B} = 0$ — ② → From Gauss

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad -③ \quad \rightarrow \text{From Faraday}$$

$$\vec{\nabla} \times \vec{H} = \frac{\partial \vec{D}}{\partial t}$$

$$\vec{H} = \frac{\vec{B}}{\mu_0}$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \quad -④$$

Taking Curl od ③

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = -\vec{\nabla} \times \left(\frac{\partial \vec{B}}{\partial t}\right) = -\frac{\partial}{\partial t} (\vec{\nabla} \times \vec{B})$$

From ④

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = -\frac{\partial}{\partial t} \left[\mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}\right]$$

but $\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = \vec{\nabla} \cdot (\underbrace{\vec{\nabla} \cdot \vec{E}}_{0}) - \cancel{\vec{\nabla}^2} \nabla^2 \vec{E} = -\nabla^2 \vec{E}$

$$\nabla^2 \vec{E} = \frac{\partial^2 \vec{E}}{\partial t^2} [\mu_0 \varepsilon_0] \quad -⑦$$

Similary for magnetic Field

$$\nabla^2 \vec{B} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2} \quad -⑧$$

Formulae

Mod 1

1) Lasers

i) $E = nh\nu = nh\frac{c}{\lambda}$

ii) $P = \frac{E}{t}$ — P = Power

iii) $\frac{N_1}{N_2} = e^{h\nu/KT}$

iv) $\frac{A_{21}}{B_{21}} = \frac{8\pi h \nu^3}{c^3}$ Js/m^2

v) $\gamma_{th} = \frac{1}{2L}\ln\left[\frac{1}{r_1 r_2}\right] + \alpha_s$

vi) $R = \frac{1}{e^{(h\nu/KT)} - 1}$, R = Rate

2) Polarization

i) $\tan\theta_B = \frac{\mu_2}{\mu_1}$

ii) $I = I_0\cos^2\theta$

iii) $\Delta = \frac{2\pi}{\lambda}(\mu_e - \mu_o)d$

iv) $\frac{E_x^2}{a^2} + \frac{E_y^2}{b^2} - \frac{2E_xE_y}{ab}\cos\delta = \sin^2\delta$

3) Thin film

i) R-D-R or D-R-D

$$\left.\begin{array}{l} \text{a) Maxima} = 2\mu t\cos r = (2n-1)\frac{\lambda}{2} \\ \text{b) Minima} = 2\mu t\cos r = n\lambda \end{array}\right\} \text{Reflected}$$

$$\left.\begin{array}{l} \text{c) Maxima} = 2\mu t\cos r = n\lambda \\ \text{d) Minima} = 2\mu t\cos r = (2n-1)\lambda/2 \end{array}\right\} \text{transmitted}$$

ii) R - Intermidiate - D

a) maxima $= 2\mu t \cos r = n\lambda$

b) minima $= 2\mu t \cos r = (2n-1)\frac{\lambda}{2}$

iii) $t = \frac{\lambda}{4\mu_f}$ — Anti reflecting film

iv) $t = \frac{\lambda}{2\mu_f}$ — Anti transmitting film

v) $\mu_f = \sqrt{\mu_g}$

Mod 2

1) Optical fibres

i) $NA = \sin\theta_m$

ii) $\sin\theta_m = \frac{\sqrt{n_1^2 - n_2^2}}{n_0}$

iii) $NA = \sqrt{n_1^2 - n_2^2}$

iv) $\Delta = \frac{n_1 - n_2}{n_1}$

v) $NA = n_1\sqrt{2\Delta}$

vi) $V = \frac{2\pi a NA}{\lambda}$ a = radius of core

vii) $N_{max} = \frac{V^2}{2}$ [step]

viii) $N_{max} = \frac{V^2}{4}$ [Graded]

2) Dielectric

i) $C = \frac{K\varepsilon_0 A}{d}$

ii) $E = \frac{Q}{K\varepsilon_0 A}$

iii) $V = \frac{Qd}{K\varepsilon_0 A}$

iv) $\chi_e = \varepsilon_r - 1$

v) $\mu = qd$ – dipole

vi) $P = \frac{\mu}{V} = N\mu$ – Polarization

vii) $\alpha = \frac{3\varepsilon_0(\varepsilon_r - 1)}{N(\varepsilon_r + 2)}$

viii) $\alpha_e = 4\pi\varepsilon_0 R^3$

ix) $E_i = \frac{\gamma P}{\varepsilon_0}$

3) Semiconductor

i) $\frac{1}{\rho} = \sigma = e(n\mu_e + p\mu_h)$

ii) $\sigma = e \times n_i(\mu_e + \mu_h)$ → Intrinsic

iii) $n_i = \sqrt{N_c N_v}\ \exp\left[\frac{-E_g}{2K_BT}\right]$

iv) $J_{drift\ (el)} = neV_e$

v) $J_{(drift)\ (Hole)} = peV_h$

vi) $V_d = \mu E$ → Mobility ← Electric field

vii) $J_{(diff)\ (el)} = eD_n \frac{dn}{dx}$

viii) $J_{(diff)\ Holes} = -eD_p \frac{dp}{dx}$

ix) $m^* = \frac{h^2}{(d^2E/dk^2)}$

x) $f(E) = \frac{1}{1 + e^{(E-E_f)/kT}}$

xi) $E_F = \frac{E_c + E_v}{2}$ → Intrinsic

Mod 3

i) $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}} = \frac{12.428 \times 10^{-10}}{\sqrt{V}} m$

ii) $E^2 = m_0^2 c^4 + p^2 c^2$

iii) $m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$

iv) $\lambda = \frac{h}{\sqrt{2 m_0 q V \left(1 + \frac{qV}{2 m_0 c^2}\right)}}$

v) $\Delta x \times \Delta p_x \geq \frac{\hbar}{2}$ → 3D or $\hbar$ → 2D

vi) $\Delta E \times \Delta t \geq \hbar$

vii) $\psi_n = \sqrt{\frac{2}{L}} \sin\left(\frac{n \pi x}{L}\right)$

viii) $E = \frac{n^2 h^2}{8 m L^2}$

ix) $V_g = \frac{d\omega}{dk}$

x) $V_p = \frac{\omega}{k}$

xi) $2d \sin\theta = n\lambda$

Mod 4

i) $\vec{\nabla} \cdot \phi = \hat{i} \frac{\partial \phi}{\partial x} + \hat{j} \frac{\partial \phi}{\partial y} + \hat{k} \frac{\partial \phi}{\partial z}$

ii) $\vec{\nabla} \cdot \vec{v} = \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z}$

iii) $\vec{\nabla} \times \vec{V} = \hat{i}\left[\frac{\partial V_z}{\partial y} - \frac{\partial V_y}{\partial z}\right] + \hat{j}\left[\frac{\partial V_x}{\partial z} - \frac{\partial V_z}{\partial x}\right] + \hat{k}\left[\frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y}\right]$

iv) $\vec{\nabla}\cdot\vec{J} + \dfrac{\partial \rho}{\partial t} = 0$

v) $E = \left[\dfrac{1}{4\pi\varepsilon_0\varepsilon_r}\right]\dfrac{q}{r^2}$

vi) <u>Integral form</u>

a) $\int_{surface} \vec{D}\,\vec{ds} = \int_{Vol} \rho\, dv$ – Gauss

b) $\oint_{line} \vec{B}\,\vec{dl} = \mu_0 \int_{surface} \vec{J}\,\vec{ds}$ – Ampere

c) $\oint_{line} \vec{E}\cdot\vec{dl} = -\int_{surface} \dfrac{\partial \vec{B}}{\partial t}\,\vec{ds}$ – Faraday

d) $\oint_{surface} \vec{B}\,\vec{ds} = 0$ – Gauss mag.

vii) <u>Differential form</u>

a) $\vec{\nabla}\cdot\vec{D} = \rho$

b) $\vec{\nabla}\times\vec{B} = \vec{J}$ or $\vec{\nabla}\times\vec{H} = \vec{J} + \vec{J}_D$

c) $\vec{\nabla}\times\vec{E} = -\dfrac{\partial \vec{B}}{\partial t}$

d) $\vec{\nabla}\cdot\vec{B} = 0$

viii) $\nabla^2\vec{E} = \mu_0\varepsilon_0 \dfrac{\partial^2 \vec{E}}{\partial t^2}$

ix) $\nabla^2\vec{B} = \mu_0\varepsilon_0 \dfrac{\partial^2 \vec{B}}{\partial t^2}$

<u>Mods</u>

i) $f = \dfrac{1}{2L}\sqrt{\dfrac{Y}{\rho}}$

ii) $T_N = \dfrac{T_C + T_F}{2}$

iii) $E = at + \dfrac{1}{2}bt^2$

iv) $V_H = R_H \dfrac{IBd}{A}$

v) $R_H = \frac{1}{qp}$ or $\frac{1}{qn}$

vi) $R = R_0 \left[1 + \frac{\Delta R}{R} \cos^2 \alpha \right]$