Discrete Mathematical Structures

- Discrete Mathematics is a branch of mathematics that is concerned with “discrete” mathematical structures instead of “continuous”.
- Discrete mathematical structures include objects with distinct values like graphs, integers, logic-based statements, set theory, recurrence relation, group theory, and graph theory.

Everyday applications of discrete mathematics

Computers run software and store files. The software and files are both stored as huge strings of 1s and 0s. Binary math is discrete mathematics.

Scheduling problems---like deciding which nurses should work which shifts, or which airline pilots should be flying which routes, or scheduling rooms for an event, or deciding timeslots for committee meetings, or which chemicals can be stored in which parts of a warehouse---are solved either using graph coloring or using combinatorial optimization, both parts of discrete mathematics. One example is scheduling games for a professional sports league.

Networks are, at base, discrete structures. The routers that run the internet are connected by long cables. People are connected to each other by social media ("following" on Twitter, "friending" on Facebook, etc.). The US highway system connects cities with roads.

An **analog clock** has gears inside, and the sizes/teeth needed for correct timekeeping are determined using discrete math.

Encryption and decryption are part of **cryptography**, which is part of discrete mathematics. For example, secure internet shopping uses public-key cryptography.

Doing web searches in multiple languages at once, and returning a summary, uses linear algebra.

Google Maps uses discrete mathematics to determine fastest driving routes and times. There is a simpler version that works with small maps and technicalities involved in adapting to large maps.

Wiring a computer network using the least amount of cable is a minimum-weight spanning tree problem.

Area codes: How do we know when we need more area codes to cover the phone numbers in a region? This is a basic combinatorics problem.

Designing **password criteria** is a counting problem: Is the space of passwords chosen large enough that a hacker can't break into accounts just by trying all the possibilities? How long do passwords need to be in order to resist such attacks? (find out here!)

Somaiya Vidyavihar University
K. J. Somaiya College of Engineering, Mumbai -77
(A Constituent College of Somaiya Vidyavihar University)

<table>
<tr><th>Course Code</th><th colspan="8">Course Title</th></tr>
<tr><td>116U01C305</td><td colspan="8">Discrete Mathematics</td></tr>
<tr><td>Teaching Scheme</td><td colspan="3">TH</td><td colspan="2">P</td><td colspan="2">TUT</td><td>Total</td></tr>
<tr><td>(Hrs./Week)</td><td colspan="3">03</td><td colspan="2">--</td><td colspan="2">01*</td><td>04</td></tr>
<tr><td>Credits Assigned</td><td colspan="3">03</td><td colspan="2">--</td><td colspan="2">01</td><td>04</td></tr>
<tr><td rowspan="4">Examination Scheme</td><td colspan="8">Marks</td></tr>
<tr><td colspan="2">CA</td><td rowspan="2">ESE</td><td rowspan="2">TW</td><td rowspan="2">O</td><td rowspan="2">P</td><td rowspan="2">P&O</td><td rowspan="2">Total</td></tr>
<tr><td>ISE</td><td>IA</td></tr>
<tr><td>30</td><td>20</td><td>50</td><td>25</td><td>--</td><td>--</td><td>--</td><td>125</td></tr>
</table>

* Batch wise Tutorial

Course prerequisites

Basic Mathematics

Course Objectives

The objective of this course is to enable students to think logically and mathematically. It will help them to solve the problems with mathematical reasoning, algorithmic thinking, and modeling.

Course Outcomes

At the end of successful completion of the course the student will be able to

CO1: Use various mathematical notations, apply various proof techniques to solve real world problems

CO2: Learn and apply core ideas of Set Theory, Relations & Functions

CO3: Use graphs and their types, to solve the practical examples

CO4: Understand the use of Algebraic Structures and lattice, to solve the problems

Module No.	Unit No.	Details of Topic	Hrs.	CO
1		**Set Theory**	03	CO1
	1.1	Sets, Venn diagrams, Operations on Sets		
	1.2	Laws of set theory, Power set and Products		
	1.3	Partitions of sets, The Principle of Inclusion and Exclusion		
2		**Logic**	04	CO1
	2.1	Propositions and logical operations, Truth tables		
	2.2	Equivalence, Implications		
	2.3	Laws of logic, Normal Forms		
	2.4	Predicates and Quantifiers		
	2.5	Mathematical Induction		
3		**Relations, Digraphs**	09	CO2
	3.1	Relations, Paths and Digraphs		
	3.2	Properties and types of binary relations		
	3.3	Manipulation of relations, Closures, Warshall's algorithm		
	3.4	Equivalence relations		
4		**Posets and Lattice**	09	CO2
	4.1	Partial ordered relations (Posets) ,Hasse diagram		
	4.2	Lattice, sublattice		
	4.3	Types of Lattice ,Boolean Algebra		
5		**Functions and Pigeon Hole Principle**	03	CO3
	5.1	Definition and types of functions: Injective, Surjective and Bijective		
	5.2	Composition, Identity and Inverse		
	5.3	Pigeon-hole principle, Extended Pigeon-hole principle		

6		**Graphs and Subgraphs**	**04**	**CO4**
	6.1	Definitions, Paths and circuits, Types of Graphs , Eulerian and Hamiltonian		
	6.2	Planer graphs		
	6.3	Isomorphism of graphs		
	6.4	Subgraph		
7		**Algebraic Structures**	**13**	**CO4**
	7.1	Algebraic structures with one binary operation: semigroup, monoids and groups		
	7.2	Cyclic groups, Normal subgroups,		
	7.3	Hamming Code ,Minimum Distance		
	7.4	Group codes ,encoding-decoding techniques		

	7.5	Parity check Matrix ,Maximum Likelihood		
	7.6	**Mathematics of Cryptography** - Modular Arithmetic, Matrices, Linear Congruence, Ring ,GF Fields		
	#**Self Learning Topic – Function Generators**			
		Total	**45**	

 # Students should prepare all Self Learning topics on their own. Self-learning topics will enable students to gain extended knowledge of the topic. Assessment of these topics may be included in IA and Laboratory Experiments.

Recommended Books:

Sr. No.	Name/s of Author/s	Title of Book	Name of Publisher with country	Edition and Year of Publication
1	Kenneth H. Rosen	*Discrete Mathematics and its applications*	Tata McGraw Hill	7th Edition, 2017
2	Bernard Kolman, Robert C. Busby	*Discrete Mathematical Structures*	Pearson	6th Edition, 2017
3	C. L. Liu, D. P. Mohapatra	*Elements of Discrete Mathematics West*	Tata McGraw Hill.	4th Edition, 2012
4	Douglas West	*Graph Theory*	Pearson	2nd Edition, 2017

Recommended Books

1. Bernard **Kolman**,Busby,” Discrete Mathematical Structures”,PHI.
2. Kenneth H. **Rosen**. ”Discrete Mathematics and its Applications”, Tata McGraw-Hill.
3. Seymour Lipschutz, Marc Lipson “Schaum's Outline of Discrete Mathematics”, Revised Third Edition Tata McGraw-Hill.
4. D. S. Malik and M. K. Sen, “Discrete Mathematical Structures”, Thompson.
5. C. L. Liu, D. P. Mohapatra, “Elements of Discrete Mathematics” Tata McGrawHill.
6. J. P. Trembley, R. Manohar “Discrete Mathematical Structures with Applications to Computer Science”, TataMcgraw-Hill.
7. Y N Singh, “Discrete Mathematical Structures”, Wiley-India.

SET THEORY

- **1.1 Sets, Venn diagrams, Operations on Sets**
- **1.2 Laws of set theory, Power set and Products**
- **1.3 Partitions of sets, The Principle of Inclusion and Exclusion**

SET THEORY

- SETS
- NOTATION
- SPECIAL SETS
- DEFINITION
  - SUBSET
  - EQUAL SETS
  - PROPER SUBSET
  - UNIVERSAL SET
  - NULL/EMPTY SET
  - SINGLETON SET
  - SUPER SET
  - FINITE /INFINITE SET
  - DISJOINT SET
  - CARDINALITY OF FINITE SET
- SET PROPERTIES
- VENN DIAGRAMS
- SET OPERATIONS
- LAWS OF SET THEORY
- PARTITION OF SETS
- POWER SET

SETS

- A *set* is an **unordered collection of objects.**
  - the students in this class
  - the chairs in this room
- The objects in a set are called the *elements*, or *members* of the set.
  - A set is said to *contain* its elements.
- The notation $a \in A$ denotes that a is an element of the set A.
- If a is not a member of A, write $a \notin A$

Describing a Set: **Roster Method**

- $S = \{a, b, c, d\}$
- Order not important

  $S = \{a, b, c, d\} = \{b, c, a, d\}$

- Each distinct object is either a member or not; listing more than once does not change the set.

  $S = \{a, b, c, d\} = \{a, b, c, b, c, d\}$

- Elipses (…) may be used to describe a set without listing all of the members when the pattern is clear.

  $S = \{a, b, c, d, \ldots\ldots, z\}$

Roster Method

- Set of all vowels in the English alphabet:

  $V = \{a,e,i,o,u\}$

- Set of all odd positive integers less than 10:

  $O = \{1,3,5,7,9\}$

- Set of all positive integers less than 100:

  $S = \{1,2,3,........,99\}$

- Set of all integers less than 0:

  $S = \{...., -3,-2,-1\}$

Some **Important** Sets

$\mathbf{N}$ = *natural numbers* = $\{0,1,2,3\ldots.\}$

$\mathbf{Z}$ = *integers* = $\{\ldots,-3,-2,-1,0,1,2,3,\ldots\}$

$\mathbf{Z}^{+}$ = *positive integers* = $\{1,2,3,\ldots..\}$

$\mathbf{R}$ = set of *real numbers*

$\mathbf{R}^{+}$ = set of *positive real numbers*

$\mathbf{C}$ = set of *complex numbers*.

$\mathbf{Q}$ = set of rational numbers

SET-BUILDER Notation

- Specify the property or properties that all members must satisfy:

$$S = \{x \mid x \text{ is a positive integer less than } 100\}$$

$$O = \{x \mid x \text{ is an odd positive integer less than } 10\}$$

- A predicate may be used: $S = \{x \mid \mathrm{P}(x)\}$
- Example: $S = \{x \mid \mathrm{Prime}(x)\}$

Universal Set and Empty Set

- The *universal set U* is the set containing everything **currently under consideration**.
  - Sometimes implicit/explicit/Contents depend on the context.
- Empty set is the set with no elements. Symbolized $\emptyset$, but $\{\ \}$ also used , but $\{\emptyset\}$???

"A null set is a subset of every set "

Some things to remember

- Sets can be elements of sets.

  $\{\{1,2,3\},a,\{b,c\}\}$

  $\{\mathbf{N},\mathbf{Z},\mathbf{Q},\mathbf{R}\}$

- The empty set is different from a set containing the empty set.

  $\emptyset \neq \{\emptyset\}$

Set Equality

Definition: Two sets are *equal* if and only if they have the same elements.

- Therefore if A and B are sets, then A and B are equal if and only if **A ⊆ B and B ⊆ A then A=B**
- We write *A* = *B* if *A* and *B* are equal sets.

$$\{1,3,5\} = \{3, 5, 1\}$$

$$\{1,5,5,5,3,3,1\} = \{1,3,5\}$$

Subsets

Definition: The set *A* is a *subset* of *B*, if and only if every element of *A* is also an element of *B*.

- The notation $A \subseteq B$ is used to indicate that A is a subset of the set B.

Eg: If A={1,2,3}, B={1,2,3,4,5}, C=(3,2,1}

- $\nsubseteq$ - not a subset

"Every set is a subset of itself"

Proper Subsets

Definition: **If $A \subseteq B$, but $A \neq B$**, then we say A is a *proper subset* of B, denoted by $A \subset B$.

A= {x ,y } , B= {x, y ,z}

then is $A \subset B$?

A= {1,3}

B={1,2,3}

C={1,3,2}

A $\subset$ C ? , B $\subset$ C ?

Set Cardinality

Definition: The *cardinality* of a finite set A, denoted by $|A|$, is the **number of (distinct) elements of *A*.**

Examples:

1. $|\emptyset| = 0$
2. Let S be the letters of the English alphabet. Then $|S| = 26$
3. $|\{1,2,3\}| = 3$
4. $|\{\emptyset\}| = 1$
5. The set of integers is infinite.

SUPERSET

- If A is the subset of B then **B is the SUPERSET of A**

DISJOINT SET

- Two sets are said to be disjoint if they have **no elements in common**

SET PROPERTIES

- **Every Set A is a subset of the Universal set U**

$$\emptyset \subseteq A \subseteq U$$

- **Every set A is a subset of itself**

$$A \subseteq A$$

- **Transitivity**

$$A \subseteq B,\ B \subseteq C,\ \text{then } A \subseteq C$$

- If $A \subseteq B$ and $B \subseteq A$ then $A=B$; converse also holds true

Venn Diagrams & Set Operations

Union

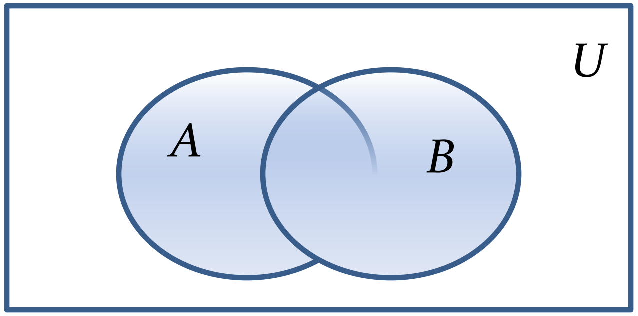
- Definition: Let *A* and *B* be sets. The *union* of the sets *A* and *B*, denoted by $A \cup B$, is the set:

$$\{x \mid x \in A \lor x \in B\}$$

- **Example**: What is $\{1,2,3\} \cup \{3, 4, 5\}$?

Solution: $\{1,2,3,4,5\}$

Venn Diagram for $A \cup B$



Intersection

- **Definition**: **The *intersection* of sets *A* and *B*, denoted by** ***A* ∩ *B*, is**

$$\{x \mid x \in A \wedge x \in B\}$$

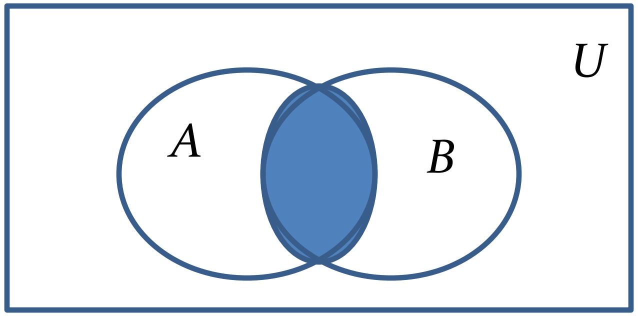
  - Note if the intersection is empty, then *A* and *B* are said to be *disjoint*.
- **Example**: What is? $\{1,2,3\} \cap \{3,4,5\}$?

  Solution: $\{3\}$
- **Example:** What is?

  $\{1,2,3\} \cap \{4,5,6\}$?

  Solution: $\emptyset$ or $\{\ \}$

Venn Diagram for $A \cap B$



Complement

Definition: **If *A* is a set, then the complement of the *A* (with respect to *U*), denoted by $\bar{A}$ is the set** ***U* - *A***

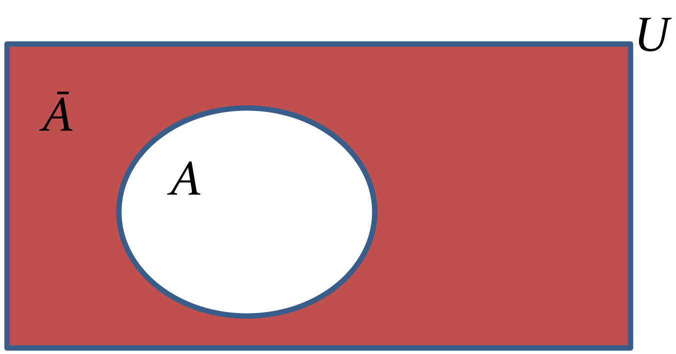
$$\bar{A} = \{x \in U \mid x \notin A\}$$

(The complement of A is sometimes denoted by A^c.)

Example: If U is the positive integers less than 100, what is the complement of $\{x \mid x > 70\}$

Solution: $\{x \mid x \leq 70\}$

Venn Diagram for Complement

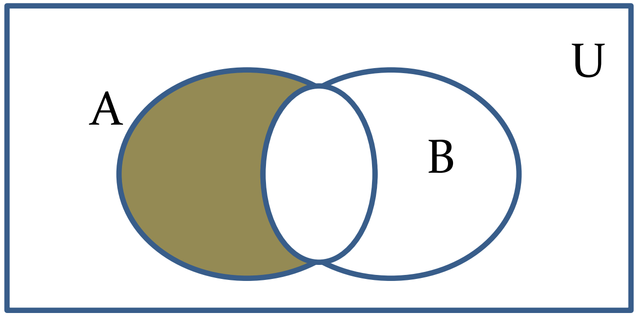


Difference

- **Definition**: Let A and B be sets. The *difference* of A and B, denoted by $\boldsymbol{A - B}$, is **the set containing the elements of $\boldsymbol{A}$ that are not in $\boldsymbol{B}$.**

$$A - B = \{x \mid x \in \mathrm{A} \wedge x \notin B\}$$

A={a , b , c } B = {b , c , d ,e } A – B = ?



Venn Diagram for $A - B$

Symmetric Difference

Definition: The *symmetric difference* of **A** and **B**, denoted by $A \oplus B$ is the set

$$(A - B) \cup (B - A)$$

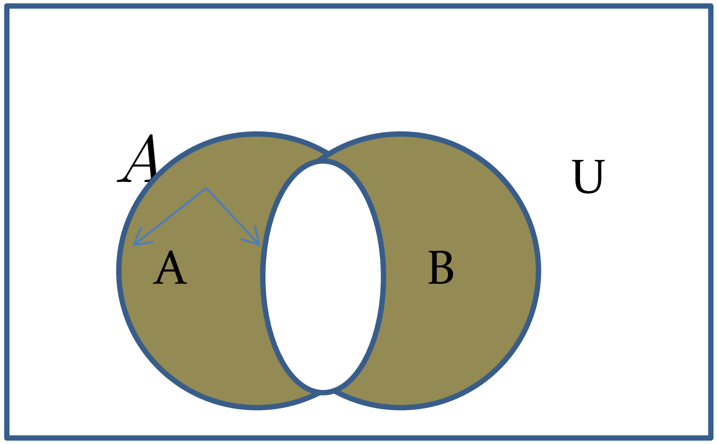
Example:

$U = \{0,1,2,3,4,5,6,7,8,9,10\}$

$A = \{1,2,3,4,5\}$ $B = \{4,5,6,7,8\}$

What is $A \oplus B$:

– **Solution**: $\{1,2,3,6,7,8\}$



Venn Diagram

Cartesian Product

Definition: The *Cartesian Product* of two sets A and B, denoted by $A \times B$ is the set of ordered pairs (a,b) where $a \in A$ and $b \in B$ (first element from A and second element from B)

Example: $A \times B = \{(a,b) | a \in A \wedge b \in B\}$

$A = \{a,b\}$ $B = \{1,2,3\}$

$A \times B = \{(a,1),(a,2),(a,3), (b,1),(b,2),(b,3)\}$

Review Questions

Example: $U = \{0,1,2,3,4,5,6,7,8,9,10\}$ $A = \{1,2,3,4,5\}$, $B = \{4,5,6,7,8\}$

1. $A \cup B$

   Solution: $\{1,2,3,4,5,6,7,8\}$

2. $A \cap B$

   Solution: $\{4,5\}$

3. $\bar{A}$

   Solution: $\{0,6,7,8,9,10\}$

4. $\bar{B}$

   Solution: $\{0,1,2,3,9,10\}$

5. $A - B$

   Solution: $\{1,2,3\}$

6. $B - A$

   Solution: $\{6,7,8\}$

PROBLEMS/ EXERCISE TO SOLVE

Consider U= {1,2,3,4,5,6,7,8,9}

A= {1,2,4,6,8}

B={2,4,5,9}

C={x|x is a positive integer and x^2<=16)

D= {7,8}

Compute following:

1. AUB
2. AUC
3. AUD
4. BUC
5. $A \cap D$
6. $B \cap C$
7. $C \cap D$
8. A-B
9. B-A
10. C-D
11. C `
12. $A \oplus B$
13. $C \oplus D$
14. $A \cap (C` \cup D)$
15. $(A \cup B) \cap D$

LAWS OF SET THEORY

- Commutative laws

$$A \cup B = B \cup A \qquad A \cap B = B \cap A$$

- Associative laws

$$A \cup (B \cup C) = (A \cup B) \cup C$$
$$A \cap (B \cap C) = (A \cap B) \cap C$$

- Distributive laws

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$
$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

LAWS OF SET THEORY

- Identity laws

$$A \cup \emptyset = A \qquad A \cap U = A$$

- Properties of Empty set

$$A \cup U = U \qquad A \cap \emptyset = \emptyset$$

- Idempotent laws

$$A \cup A = A \qquad A \cap A = A$$

- Complement law

$$(\overline{\overline{A}}) = A$$

LAWS OF SET THEORY

- De Morgan's laws

$$\overline{A \cup B} = \overline{A} \cap \overline{B} \qquad \overline{A \cap B} = \overline{A} \cup \overline{B}$$

- Absorption laws

$$A \cup (A \cap B) = A \qquad A \cap (A \cup B) = A$$

- Properties of complement law

$$A \cup \overline{A} = U \qquad A \cap \overline{A} = \emptyset$$

LAWS OF SET THEORY

Double Complement $\overline{\overline{A}} = A$

Properties of Universal Set $A \cup U = U$

$A \cap U = A$

Properties of Empty Set $A \cup \{\} = A$

$A \cap \{\} = \{\}$

* PROOF IS LEFT AS AN EXERCISE TO STUDENT

Membership Table

Example: Construct a membership table to show that the distributive law holds. $$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

Solution:

A	B	C	$B \cap C$	$A \cup (B \cap C)$	$A \cup B$	$A \cup C$	$(A \cup B) \cap (A \cup C)$
1	1	1	1	1	1	1	1
1	1	0	0	1	1	1	1
1	0	1	0	1	1	1	1
1	0	0	0	1	1	1	1
0	1	1	1	1	1	1	1
0	1	0	0	0	1	0	0
0	0	1	0	0	0	1	0
0	0	0	0	0	0	0	0

Ex. : Show that (using laws of logic)

(a) $A \cup (A^C \cap B) = A \cup B$.

(b) $A \cap (A^C \cup B) = A \cap B$.

Note : A^C means complement of A. This is another notation.

(a) $$\begin{aligned} \text{L.H.S} &= A \cup (A^C \cap B) \\ &= (A \cup A^C) \cap (A \cup B) && \text{...Distributive law} \\ &= \cup \cap (A \cup B) && \text{...Complement laws} \\ &= A \cup B \end{aligned}$$

Hence $A \cup (A^C \cap B) = A \cup B$

(b) $A \cap (A^C \cup B) = A \cap B$

$$\begin{aligned} \text{L.H.S} &= A \cap (A^C \cup B) \\ &= (A \cap A^C) \cup (A \cap B) \\ & && \text{...Distributive law} \\ &= \phi \cup (A \cap B) \\ & && \text{...Complement law} \\ &= A \cap B \end{aligned}$$

Hence $A \cap (A^C \cup B) = A \cap B$

THEOREMS

PRINCIPLE OF INCLUSION EXCLUSION

$$|A \cup B| = |A| + |B| - |A \cap B|$$

MUTUAL INCLUSION EXCLUSION PRINCIPLE

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| + |A \cap B \cap C|$$

PROBLEMS/ EXERCISE TO SOLVE

A company must hire 25 programmers to handle programming jobs & 40 programmers for application programming job of which 10 will be expected to do both kinds of roles. How many programmers must be hired?

A computer company must hire 25 programmers to handle system programming jobs and 40 programmers for application programming of those hired 10 will be expected to perform jobs of both types. How many programmers must be hired ?

Soln. : Let, A be the set of system programmers hired.

B be the set of applications programmers hired.

We want to find $| A \cup B |$

We have, $| A | = 25$,

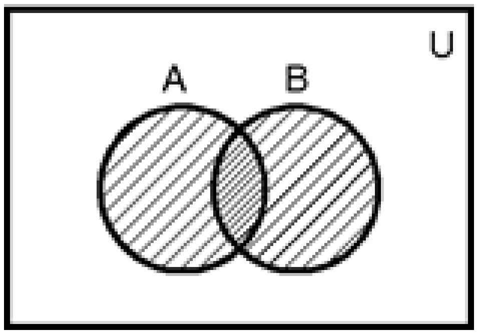
$| B | = 40$

Using the Principle of Inclusion and Exclusion,

$$| A \cup B | = | A | + | B | - | A \cap B |$$

$$= 25 + 40 - 10 = 55$$

$A \cup B = 55$



A sample of 80 people have revealed that 24 like cinema and 62 like television programmes. Find the number of people who like both cinema and television programmes.

Ans: Let A = Set of people who like cinema

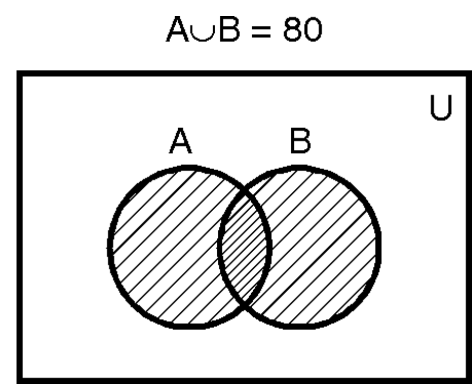
B = Set of people who like television programs

Then, we have

$| A | = 24, \qquad | B | = 62.$

Using principle of Inclusion and Exclusion

$| A \cup B | = | A | + | B | - | A \cap B |$



We obtain

$80 = 24 + 62 - | A \cap B |$

Therefore,

$| A \cap B | = 6$

Thus 6 people like both cinema and television programmes.

PROBLEMS/ EXERCISE TO SOLVE

In a survey of 260 college students , the following data were obtained:

64 had taken a maths course

94 had taken a CS course

58 had taken a Business course

28 had taken both M and B course

26 had taken both M and CS course

22 had taken both CS and B course

14 had taken all three types of courses.

(i) How many students surveyed who had taken none of the three types of courses?

(ii)Of the students surveyed how many had taken only CS course?

VENN DIAGRAM?

In a survey of 260 college students, the following data were obtained.

64 had taken a Mathematics course.
94 had taken a computer science course.
58 had taken a business course.
28 had taken both mathematics and business course.
26 had taken both a mathematics and computer science course.
22 had taken both a computer science and Business course.
14 had taken all three types of courses.

(i)How many students were surveyed who had taken none of the three types of courses ?
(ii)Of the students surveyed, how many had taken only a computer science course ?

Ans: Let A be the set of students taken Mathematics course.
B be the set of students taken Computer Science course.
C be the set of students taken Business course.

We have $|A| = 64$, $|B| = 94$, $|C| = 58$,

$|A \cap C|$ =28, taken mathematics and business course
$|A \cap B|$ =26, taken mathematics and computer science course
$|B \cap C|$ =22, taken computer science and business course
$|A \cap B \cap C|$ =14, taken mathematics, computer science and business course

VENN DIAGRAM ?.

(i) Using Principle of Inclusion and Exclusion

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| + |A \cap B \cap C|$$

$$= 64 + 94 + 58 - 28 - 26 - 22 + 14$$

$$= 154$$

Students who have not taken either of subjects,

$$|\overline{A \cup B \cup C}| = |U| - |A \cup B \cup C|$$

$$= 260 - 154 = 106$$

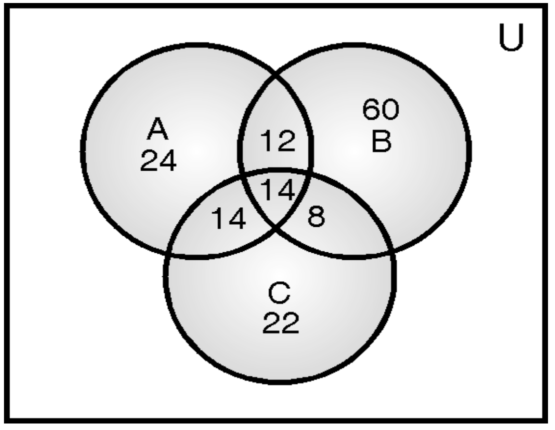
Thus 106 students had taken none of the three types of course.

(ii)Students who have taken only computer science course

$$= |B| - |B \cap A| - |B \cap C| + |A \cap B \cap C|$$

$$= 94 - 26 - 22 + 14 = 60$$

60 students had taken only computer science course.



Ex. 5 : In a survey of 60 people, it was found that 25 read Newsweek magazine, 26 read Time and 26 read Fortune. Also 9 read both Newsweek and Fortune, 11 read both Newsweek and Time, 8 read both Time and Fortune, 8 read no magazine at all.

(a)Find the number of people who read all three magazines.

(b)Fill in the correct number of people in each of eight regions of Venn diagram. Here N, T and F denote the set of people who read Newsweek, Time and Fortune respectively.

(c)Determine the number of people who read **exactly** one magazine.

Ans: $|N| = 25$, $|T| = 26$, $|F|$ 26,

And $|N \cap T| = 11$, $|N \cap F| = 9$, $|T \cap F| = 8$.

8 read no magazine at all.

$$\underline{\mathbf{|N \cup T \cup F| = 60 - 8 = 52}}$$

By the principle of inclusion and exclusion we have,

$$|N \cup T \cup F| = |N| + |T| + |F| - |N \cap T| - |T \cap F| - |N \cap F| + |N \cap T \cap F|$$

$$52 = 25 + 26 + 26 - 11 - 9 - 8 + |N \cap T \cap F|$$

$$|N \cap T \cap F| = 3$$

Hence, 3 people read all three magazines.

VENN DIAGRAM ?

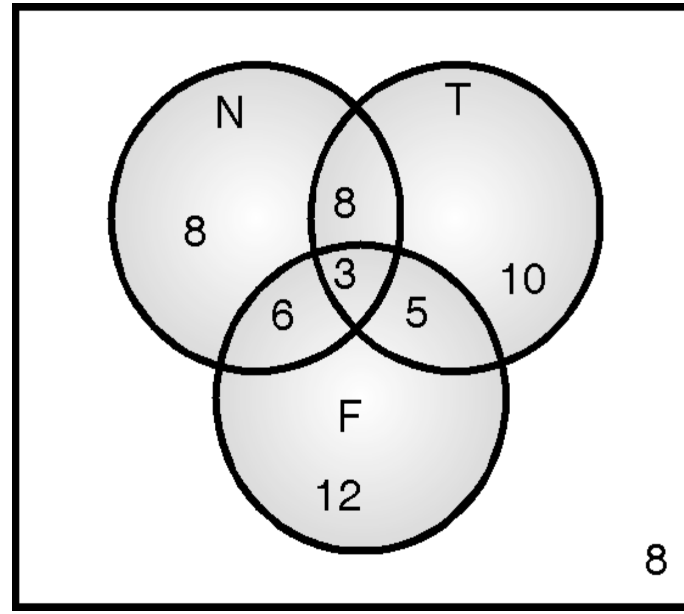
(b)To draw Venn Diagram,

3 read all three magazines

$|N \cap T| - |N \cap T \cap F| = 11 - 3 = 8$ read Newsweek and Time but not all three magazines.

$|N \cap F| - |N \cap T \cap F| = 9 - 3 = 6$ read Newsweek and Fortune but not all three magazines.

$|T \cap F| - |N \cap T \cap F| = 8 - 3 = 5$ read Time and Fortune but not all three magazines.



$|N| - |N \cap T| - |N \cap F| + |N \cap T \cap F| =$

$25 - 11 - 9 + 3 = 8$ **read only Newsweek**

$|T| - |T \cap F| - |N \cap T| + |N \cap T \cap F| =$

$26 - 8 - 11 + 3 = 10$ **read only Time**

$|F| - |T \cap F| - |N \cap F| + |N \cap T \cap F| =$

$26 - 8 - 9 + 3 = 12$ **read only Fortune**

(c)People who read only one magazine is $8 + 10 + 12 = 30$

PROBLEMS/ EXERCISE TO SOLVE

Suppose that 100 of 120 students at a college take at least one of the languages French , German and Russian . Also suppose

65 study French

20 study French and German

45 study German

25 study French and Russian

42 study Russian

15 study German and Russian

(i) How many students study all three languages ?

(ii)Fill in the correct regions in Venn Diagram

(iii)Hence find out the number of students 'k' who study

* **Exactly** 1 language ***Exactly** 2 languages

Ex. 6

Suppose that 100 of the 120 mathematics students at a college take at least one of the languages French, German and Russian. Also suppose 65 study French,

20 study French and German,

45 study German,

25 study French and Russian,

42 study Russian,

15 study German and Russian,

(a) Find the number of students who study all three languages.

(b) Fill in the correct number of students in each of the eight regions of Venn diagram.

Here F, G and R denote the sets of students studying French, German and Russian respectively.

(c) Determine the number k of students who study

(i) exactly one language

(ii) exactly two languages

(a) Since 100 of the students study at least one of languages.

$$\therefore \quad |F \cup G \cup R| = 100$$

Again, from given data

$$|F| = 65 \qquad |G| = 45$$

$$|R| = 42 \qquad |F \cap G| = 20$$

$$|F \cap R| = 25 \qquad |G \cap R| = 15$$

Using formula of inclusion and exclusion, we have

$$|F \cup G \cup R| = |F| + |G| + |R| - |F \cap G| - |F \cap R| - |G \cap R| + |F \cap G \cap R|$$

$$100 = 65 + 45 + 42 - 20 - 25 - 15 + |F \cap G \cap R|$$

$$|F \cap G \cap R| = 8$$

Eight students study all three languages.

(b) Using the above result, we draw the Venn Diagram

8 study all three languages

$\therefore \quad |F \cap G| - |F \cap G \cap R| = 20 - 8 = 12$

study French and German but not Russian.

$\therefore \quad |F \cap R| - |F \cap G \cap R| = 25 - 8 = 17$

study French and Russian but not German.

$\therefore \quad |G \cap R| - |F \cap G \cap R| = 15 - 8 = 7$

study German and Russian but not French

Only French $= |F| - |F \cap G| - |F \cap R| + |F \cap G \cap R|$

$= 65 - 20 - 25 + 8 = 28$

$\therefore$ 28 student study only French language

Only German $= |G| - |G \cap R| - |F \cap G| + |F \cap G \cap R|$

$= 45 - 15 - 20 + 8 = 18$

$\therefore$ 18 students study only German language.

Only Russian $= |R| - |F \cap R| - |G \cap R| + |F \cap G \cap R|$

$= 42 - 25 - 15 + 8 = 10$

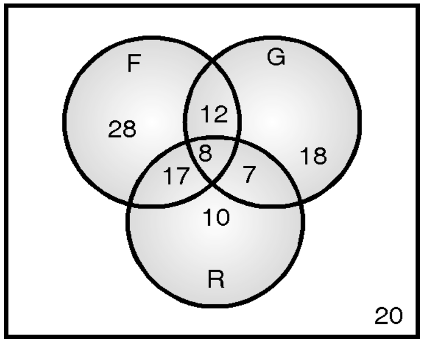
$\therefore$ 10 students study only Russian language.

$\therefore \ |U| - |F \cup G \cup R|$

$= 120 - 100$

$= 20$

do not study any of the languages.



(c) From Venn diagram

(i) $k = 28 + 18 + 10 = 56$

Thus 56 mathematics students study exactly one language.

(ii) $k = 12 + 17 + 7 = 36$

Thus 36 mathematics students study exactly two languages.

PROBLEMS/ EXERCISE TO SOLVE

1) How many integers between 1 and 60 are not divisible by 2,not by 3,nor by 5 ?

2) How many integers between 1 and 300 are divisible by 3,5, or 7 and are not divisible by 3 nor 5 nor 7 ?

3) Determine number of integers between 1 & 1000 that are divisible and not divisible by 2 ,3 & 5 ?

Ex. 6 : Find how many integers between 1 and 60 are not divisible by 2 nor by 3 and nor by 5 ?

Soln. : Let A_1, A_2 and A_3 be the set of integers between 1 and 60 divisible by 2, 3 and 5 respectively.

$$\therefore \quad |A_1| = \left|\frac{60}{2}\right| = 30$$

$$|A_2| = \left|\frac{60}{3}\right| = 20$$

$$|A_3| = \left|\frac{60}{5}\right| = 12$$

and $$|A_1 \cap A_2| = \left|\frac{60}{2 \times 3}\right| = 10$$

$$|A_1 \cap A_3| = \left|\frac{60}{2 \times 5}\right| = 6$$

$$|A_2 \cap A_3| = \left|\frac{60}{3 \times 5}\right| = 4$$

and $$|A_1 \cap A_2 \cap A_3| = \left|\frac{60}{2 \times 3 \times 5}\right| = 2$$

Number of integers between 1 and 60 which are divisible by 2, 3 or 5 are

$$= |A_1 \cup A_2 \cup A_3|$$

$$= |A_1| + |A_2| + |A_3| - |A_1 \cap A_2|$$

$$- |A_1 \cap A_3| - |A_2 \cap A_3| + |A_1 \cap A_2 \cap A_3|$$

$$= 30 + 20 + 12 - 10 - 6 - 4 + 2 = 44$$

Hence the number of integers between 1 and 60 are not divisible by 2, 3 or 5 = 60 – 44 = 16

Ex. 7 : Among the integers 1 and 300, how many of them are divisible by 3, 5 or 7 and are not divisible by 3, nor by 5, nor by 7? How many of them are divisible by '3' but not by '5' nor by '7' ?

Soln. : Let A_1 denote the set of integers between 1 and 300 divisible by '3'. Similarly A_2 and A_3 be the sets of integers divisible by 5 and 7 respectively.

Then $$|A_1| = \left|\frac{300}{3}\right| = 100$$

$$|A_2| = \left|\frac{300}{5}\right| = 60$$

$$|A_3| = \left|\frac{300}{7}\right| = 42$$

And $|A_1 \cap A_2|$ = Number of integers divisible by 3 and 5

$$= \left|\frac{300}{3 \times 5}\right| = 20$$

$$|A_1 \cap A_3| = \left|\frac{300}{3 \times 7}\right| = 14$$

$$|A_2 \cap A_3| = \left|\frac{300}{5 \times 7}\right| = 8$$

And $$|A_1 \cap A_2 \cap A_3| = \left|\frac{300}{3 \times 5 \times 7}\right| = 2$$

Hence, using the principle of inclusion and exclusion, we have number of integers which are divisible by 3 or 5 or 7.

$$= |A_1 \cup A_2 \cup A_3|$$

$$= |A_1| + |A_2| + |A_3| - |A_1 \cap A_2| - |A_2 \cap A_3| - |A_1 \cap A_3| + |A_1 \cap A_2 \cap A_3|$$

$$= 100 + 60 + 42 - 20 - 14 - 8 + 2$$

$$= 162$$

Hence, there are 162 numbers between 1 and 300 divisible by 3, 5 or 7.

$\therefore$ Number of integers which are not divisible by 3, nor by 5, and nor by 7 = 300 – 162 = 138.

Again number of integers between 1 and 300 which are divisible by 3 but not by 5, nor by 7 = $|A_1| - |A_1 \cap A_2| - |A_1 \cap A_3| + |A_1 \cap A_2 \cap A_3|$ = 100 – 20 – 14 + 2 = 68.

Power Sets

Definition: **The set of all subsets of a set *A***, denoted $\mathcal{P}(A)$, is called the *power set* of *A*.

Example: If $A = \{a,b\}$ then

$$\mathcal{P}(A) = \{\emptyset, \{a\},\{b\},\{a,b\}\}$$

- If a set has *n* elements, then the cardinality of the power set is 2^n.

PROBLEMS/ EXERCISE TO SOLVE

1. Let A={1 , 2 ,3 }.Determine the power set of A

1. Let A={a , b , c , d }. Determine the power set of A

Partition of Sets

- If A is a set, a partition of A is any set of non empty subset $A_1, A_2, A_3 \ldots\ldots$ Of A such that

$$\mathbf{A_1 \cup A_2 \cup A_3 \ldots\ldots\ldots = A}$$

&

$$\mathbf{A_i \cap A_j = \emptyset}$$ for $i \neq j$ (subsets are mutually disjoint)

Example A = { a , b , c}

Verify whether { { a } , { b , c } } is a partition of A or not

PROBLEMS/ EXERCISE TO SOLVE

1. S={1,2,3,4,5,6,7,8,9}.Determine whether each is a partition or not

   (i) {{1,3,5},{2,6},{4,8,9,}}

   (ii) {{1,3,5},{2,4,6,8},{5,7,9}}

   (iii) {{1,3,5},{2,4,6,8},{7,9}}

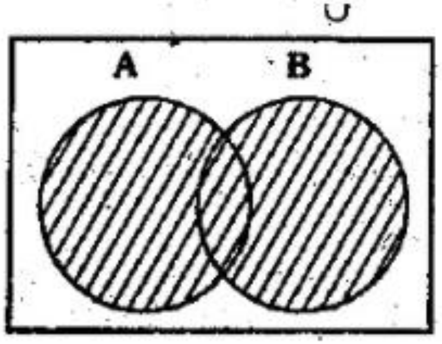
2. Let A={a,b,c,d,e,f,g,h }.Consider the following subsets of A

   A1={a,b,c,d} A3={a,c,e,g}

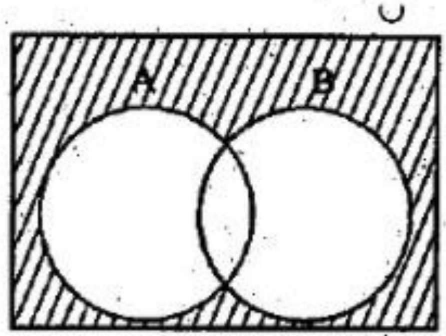
   A2={a,c,e,g,h} A4={b,d} A5={f,h}

   Determine whether following is a partition of A or not. Justify

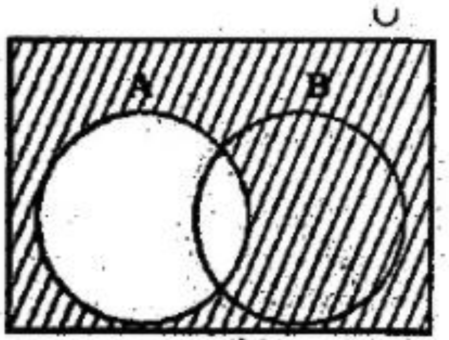
   (i) {A1,A2} (ii) {A1, A5} (iii) {A3,A4,A5}



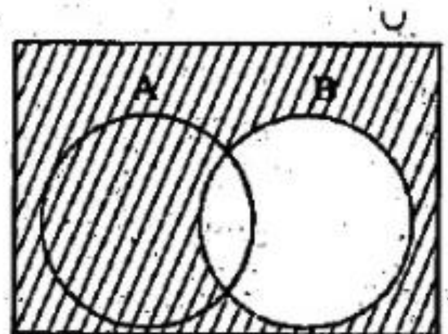
$A \cup B$



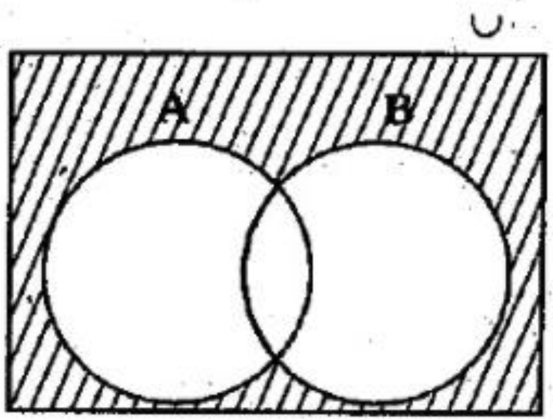
$(A \cup B)'$ ... (1)



A'



B'

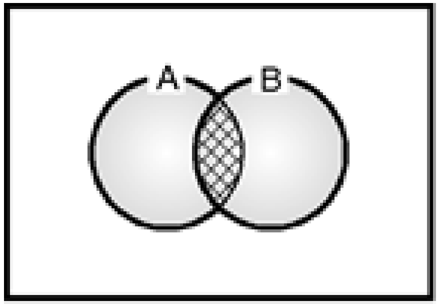


... (2)

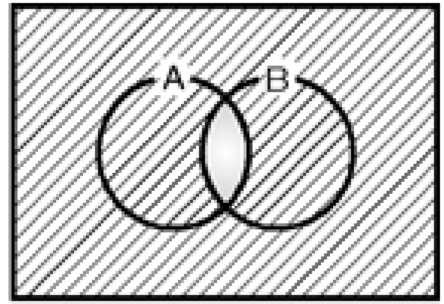
$A' \cup B'$

Ex. : Use Venn diagram to illustrate De Morgan's law for sets, viz.

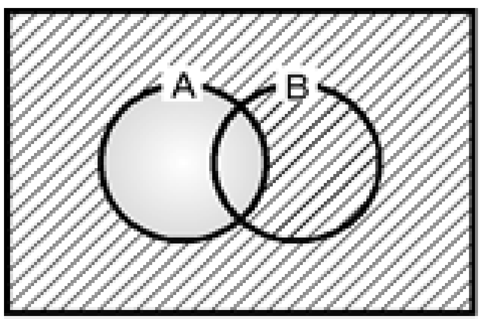
$$\overline{(A \cap B)} = \bar{A} \cup \bar{B}$$



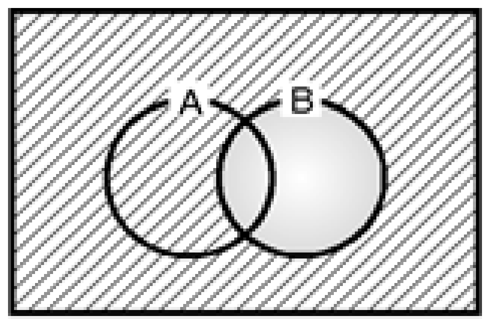
$A \cap B$



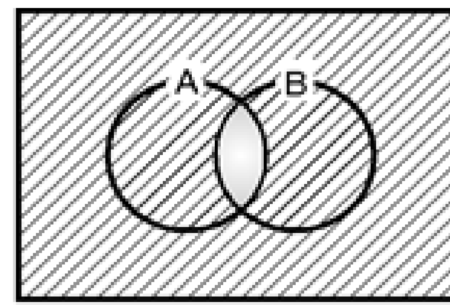
$\overline{A \cap B}$



$\bar{A}$



$\bar{B}$



$\bar{A} \cap \bar{B}$