

04-09-2023

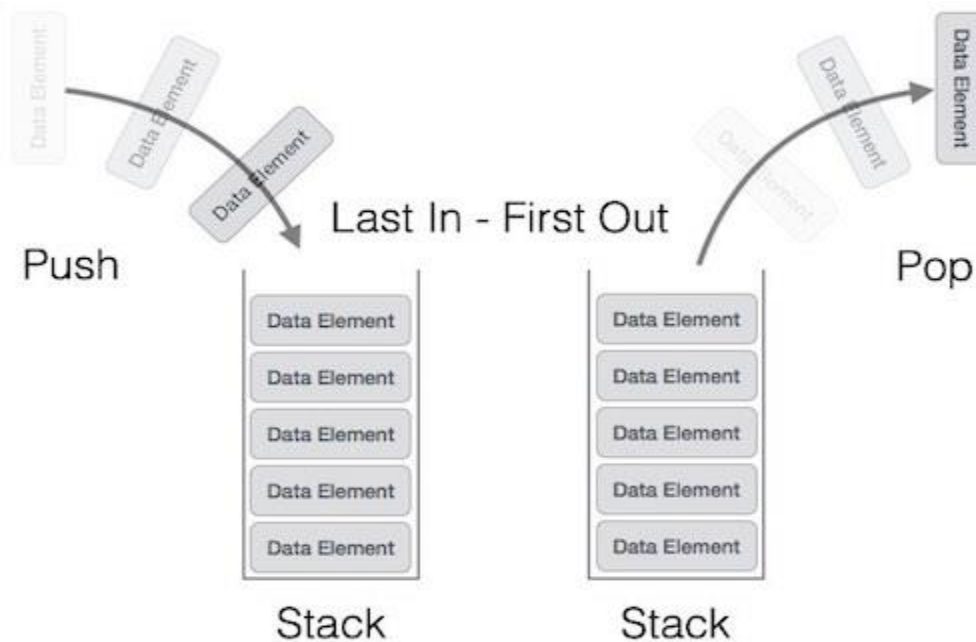
# Stacks

# Stacks

2

04-09-2023

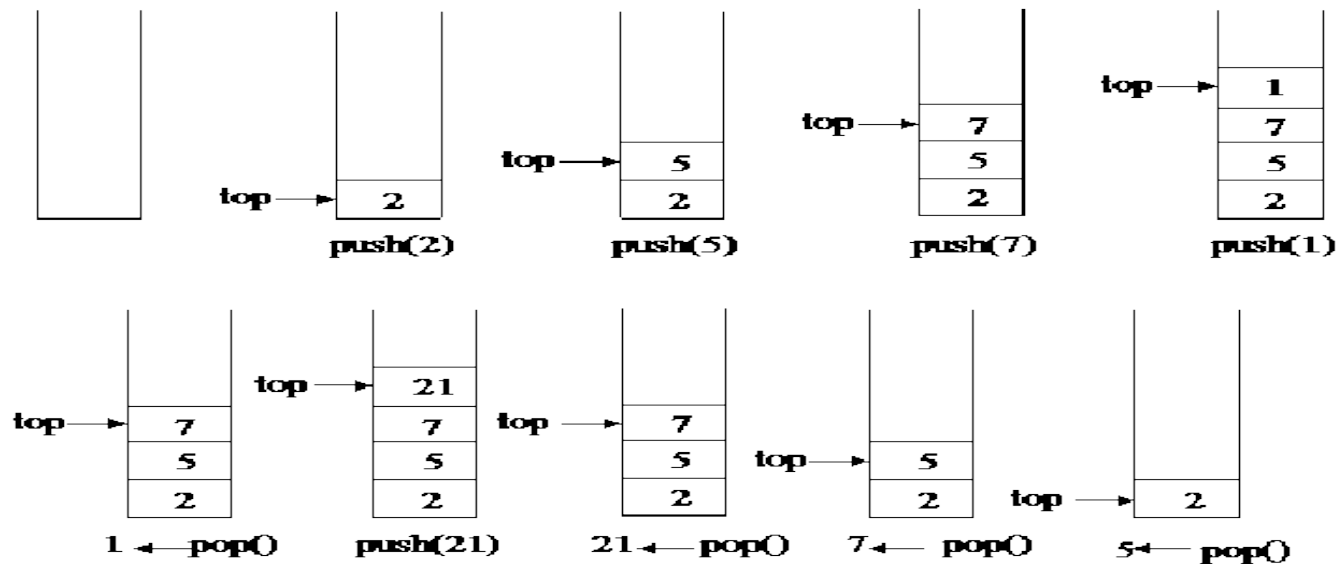
- Stack is a linear data structure which follows a particular order in which the operations are performed.
- The order may be LIFO (Last In First Out) or FILO (First In Last Out).



# Operation on Stacks<sup>3</sup>

04-09-2023

- **Push operation**-The insertion of an element into stack
- **Pop operation** - Deletion of an element from the stack
- In stack we always keep track of the last element present in the list with a pointer called top.



# Array Implementation of Stack

`top`      Index of element at the top of stack

Stack is empty  $\rightarrow$  `top` is -1

Push operation

`top` is increased by 1

New element is placed at index `top`

Pop operation

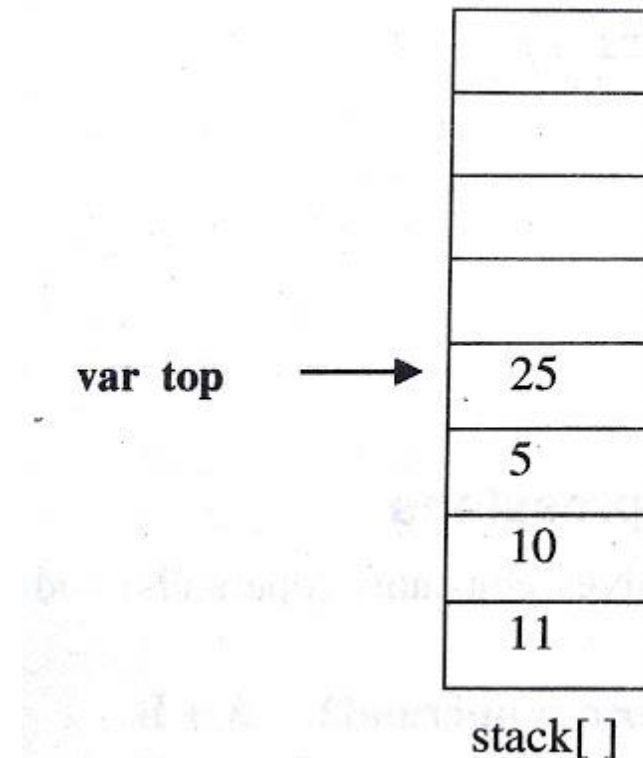
Element at index `top` is taken out

`top` is decreased by 1

# Array Implementation of Stack

# Array Implementation of Stack

- Push element one by one onto stack from 0<sup>th</sup> position to n-1<sup>th</sup> position.
- Variable top=position of the top element in the array
- If there is no element in the stack, value of top will be -1



# Stack Overflow /FullCondition

If( $\text{top} == \text{MAX} - 1$ )

Then print stack is full or overflowing

# Stack Underflow /Empty Condition

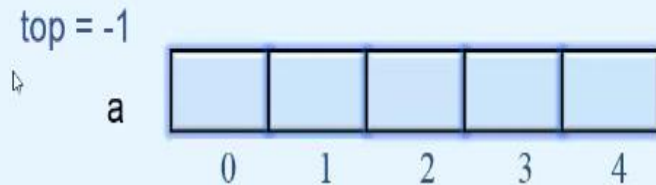
If( $\text{top} == -1$ )

Then print stack is empty or underflowing

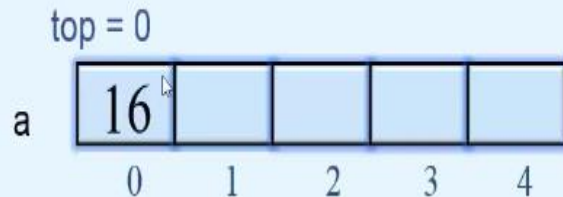


# PUSH Operation on Stack

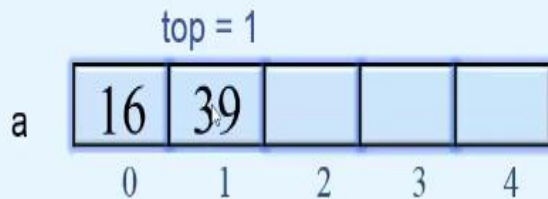
Empty stack



Push 16



Push 39



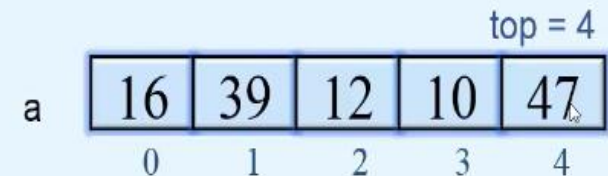
Push 12



Push 10



Push 47



# Algorithm for PUSH Operation

- 1) Check Stack Full condition  
If( $\text{top} == \text{MAX} - 1$ )  
Then print stack is full or overflow
- 2) Otherwise increase the top value by 1  
 $\text{top} = \text{top} + 1$
- 3) Input the value
- 4) Assign the item at top position  
 $\text{stack\_arr}[\text{top}] = \text{pushed\_item}$

Try writing the code for Push Operation function

# PUSH Operation

```
push()
{
    int pushed_item;
    if(top == (MAX-1))
        printf("Stack Overflow\n");
    else
    {
        printf("Enter the item to be pushed in stack : ");
        scanf("%d",&pushed_item);
        top=top+1;
        stack_arr[top] = pushed_item;
    }
}/*End of push()*/
```

# POP Operation on Stack

Empty stack

top = -1



a



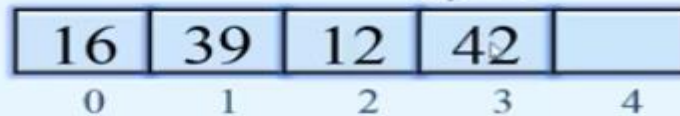
Stack Underflow  
condition

If(top == -1)

Push 42

top = 3

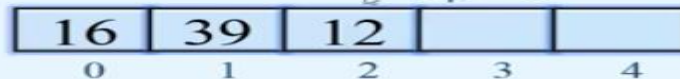
a



Pop

top = 3

a

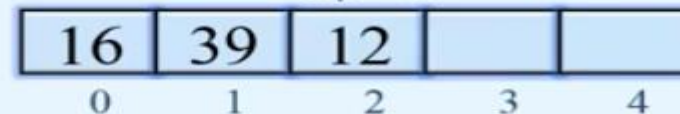


Popped item = 42

Pop

top = 2

a



# Algorithm for POP Operation

- 1) Check Stack Underflow condition  
If( $\text{top} == -1$ )  
Then print stack is underflow
- 2) Otherwise, delete the top position element  
 $\text{Popped\_item} = \text{stack\_arr}[\text{top}]$
- 3) Decrease the position of top  
 $\text{top} = \text{top} - 1$
- 4) Print the popped\_item

Try writing the code for Pop Operation  
function

# POP Operation

```
pop()
{
    if(top == -1)
        printf("Stack Underflow\n");
    else
    {
        printf("Popped element is :
%d\n",stack_arr[top]);
        top=top-1;
    }
}/*End of pop()*/
```



Try writing the code for Display  
Operation function

# Display fn

```
display()
{
    int i;
    if(top == -1)
        printf("Stack is empty\n");
    else
    {
        printf("Stack elements :\n");
        for(i = top; i >=0; i--)
            printf("%d\n", stack_arr[i] );
    }
}/*End of display()*/
```

## Peek Operation

- An operation on certain abstract data types, specifically sequential collections such as stacks and queues,
  - which returns the value of the top ("front") of the collection
  - without removing the element from the collection.
- It thus returns the same value as operations such as "pop" or "dequeue", but does not modify the data.

## Peek Operation

- The name "peek"
  - the name for this operation varies depending on data type and language.
- Peek is generally considered an inessential operation,
  - not included in the basic definition of these data types.

## Peek Operation

- Sequential types for which peek is often implemented include:
  - Stack
  - Queue
  - Priority queue (such as a heap)
  - Double-ended queue (deque)
  - Double-ended priority queue (DEPQ)

## Peek Operation

- ◉ Single-ended types, such as stack,
  - ◉ generally only admit a single peek, at the end that is modified.
- ◉ Double-ended types, such as dequeues,
  - ◉ admit two peeks, one at each end.
- ◉ Names for peek vary.
  - ◉ For queues "front" is common.
  - ◉ Dequeues have varied names, often "front" and "back" or "first" and "last".
  - ◉ The name "peak" is also occasionally found

04-09-2023

# Linked List Representation of Stack

# Linked List Representation of Stack

- The info field of the node holds the elements of the stack
  - Link fields hold pointers to the neighbouring element in the stack
- **The start pointer behaves as the TOP pointer variable of the stack**
- **Null pointer of the last node in the linked list signals the bottom of the stack**



# Linked List Representation of Stack

```
struct node
{
    int info;
    struct node *link;
} *top=NULL;
```

# Push Operation

- Inserting a node into the front or start of the list. i.e. at the top of the stack
- Stack after Before Push Operation,



- Stack after Push Operation, **Push www into stack, inserted at Top**



# Linked List Representation of Stack

Beginning of linked list                      top of the stack

top      Point to first node of the list

Stack is empty  $\rightarrow$  top is NULL

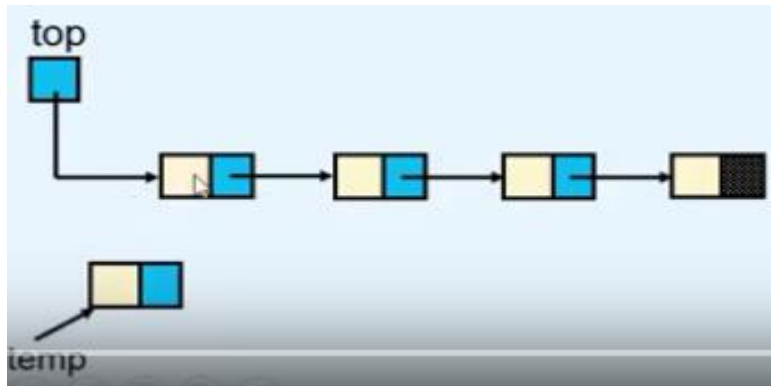
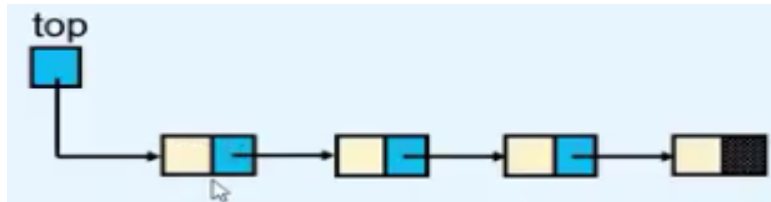
Push operation

Node is inserted in the beginning of the list

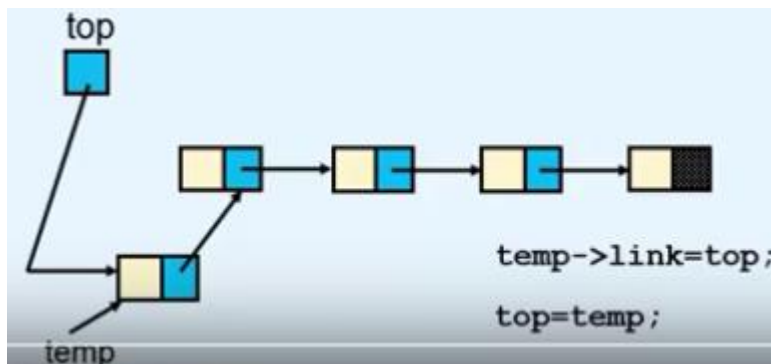
Pop operation

First node of the list is deleted

# PUSH Operation on Stack



```
tmp = (struct node
*)malloc(sizeof(struct node));
```



```
tmp->link=top;
top=tmp;
```

# Push Operation

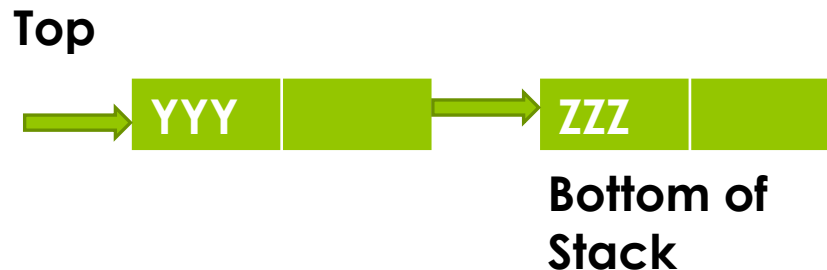
```
push()
{
    struct node *tmp;
    int pushed_item;
    tmp = (struct node *)malloc(sizeof(struct node));
    printf("Input the new value to be pushed on the stack : ");
    scanf("%d",&pushed_item);
    tmp->info=pushed_item;
    tmp->link=top;
    top=tmp;
}/*End of push()*/
```

# Pop Operation

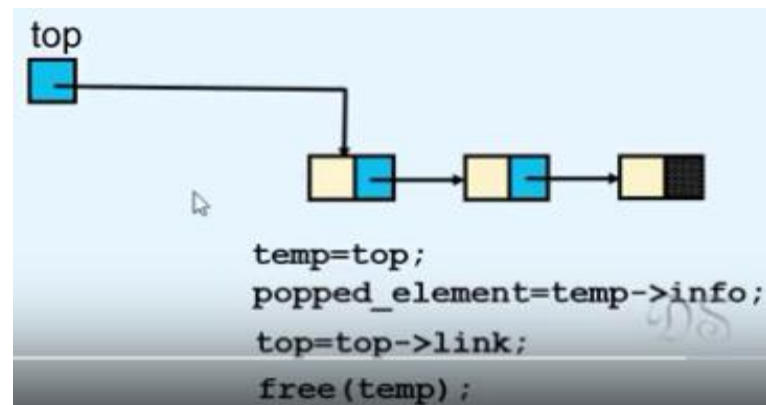
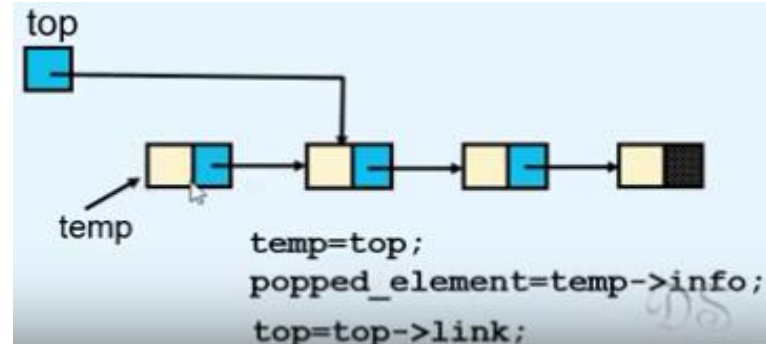
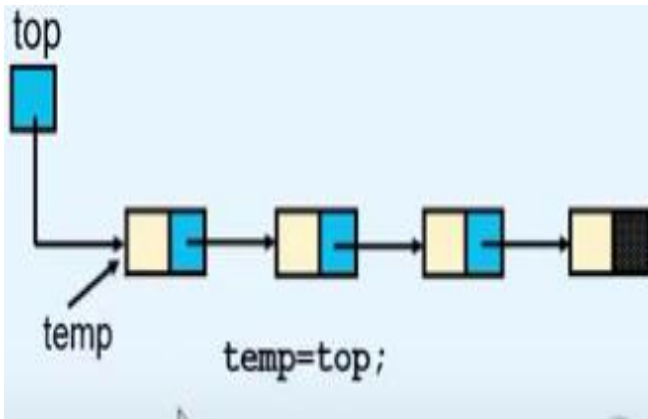
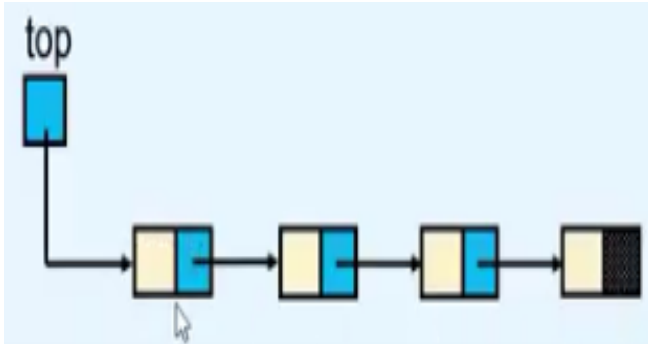
- Deleting a node pointed to by start pointer of the list.  
i.e. at the top of the stack
- Stack after Before Pop Operation



- Stack after Pop Operation, element at top deleted



# POP Operation on Stack



# POP Operation

```
pop()
{
    struct node *tmp;
    if(top == NULL)
        printf("Stack is empty\n");
    else
    {
        tmp=top;
        printf("Popped item is %d\n",tmp->info);
        top=top->link;
        free(tmp);
    }
}

/*End of pop()*/
```



# Display Operation

```
display()
{
    struct node *ptr;
    ptr=top;
    if(top==NULL)
        printf("Stack is empty\n");
    else
    {
        printf("Stack elements :\n");
        while(ptr!= NULL)
        {
            printf("%d\n",ptr->info);
            ptr = ptr->link;
        }
        /*End of while */
    }
    /*End of else*/
}
/*End of display()*/
```

04-09-2023

# Application of Stack

## Application of Stack –Reversal of a String

- Push each character of the string on the stack.
- When whole string is pushed on the stack
- Pop the characters from the stack to get the reversed string

## Application of Stack –Reversal of a String

- Try Writing the Code snippet for it.....

# Application of Stack –Polish Notation

- Polish notation (PN), also known as
  - normal Polish notation (NPN)
  - Łukasiewicz notation
  - Warsaw notation,
  - Polish prefix notation or
  - simply prefix notation,

## Polish Notation-Prefix Notation

- Is a mathematical notation in which operators precede their operands,
- The description "Polish" refers to the nationality of logician Jan Łukasiewicz who invented Polish notation in 1924.

# Reverse Polish Notation-Postfix Notation

- Reverse Polish notation (RPN), in which operators follow their operands.
- **The term Polish notation is sometimes taken (as the opposite of infix notation) to also include reverse Polish notation.**

# Expression Representation

- An expression is defined as a number of operands or data items combined using several operators.
- 3 popular methods for representation:
  - Infix Notation
  - Prefix Notation
  - Postfix Notation



# Infix Notation

- Used in General Mathematics
- Operator is written in between the operands
- Eg-  $a+b$ ,  $x+y*z$
- Called infix because of the position of the operator in expression.

## Evaluation-

- Evaluated left to right but operator precedence must be taken into consideration
- **Not used inside computer, due to additional complexity of handling of precedence**

# Prefix Notation

- Operator is written before the operands
- Also called Polish Notation
- Eg-  $+ab$ ,  $+x*yz$

# Postfix Notation

- Operator is written after the operands
- Also called Reverse Polish or Suffix Notation
- Eg-  $ab+$ ,  $xyz^*+$

# Polish Notation

- Prefix and Postfix are free from any precedence
- Computers use postfix form

# Notation Conversion

- Scan the expression from left to right
- **Operator Precedence-**
  - Paranthesis evaluated first
  - After that evaluation is on the basis of operator precedence
- Logical Not
  - Exponential Operator
  - Multiplication/division/modulus
  - Addition/Subtraction
  - Left shift, Right Shift
  - Relational
  - Logical And
  - Logical Or



# Operator Precedence Table

46

04-09-2023

Description	Operator	Associativity
Function expression	( )	Left to Right
Array Expression	[ ]	Left to Right
Structure operator	>	Left to Right
Structure operator	.	Left to Right
Unary minus	-	Right to left
Increment/Decrement	++      --	Right to Left
One's compliment	~	Right to left
Negation	!	Right to Left
Address of	&	Right to left
Value of address	*	Right to left
Type cast	( type )	Right to left
Size in bytes	sizeof	Right to left
Multiplication	*	Left to right
Division	/	Left to right
Modulus	%	Left to right
Addition	+	Left to right
Subtraction	-	Left to right
Left shift	<<	Left to right
Right shift	>>	Left to right
Less than	<	Left to right
Less than or equal to	<=	Left to right
Greater than	>	Left to right
Greater than or equal to	>=	Left to right
Equal to	==	Left to right
Not equal to	!=	Left to right
Description	Operator	Associativity
Bitwise AND	&	Left to right
Bitwise exclusive OR	^	Left to right
Bitwise inclusive OR		Left to right
Logical AND	&&	Left to right
Logical OR		Left to right
Conditional	? :	Right to left
Assignment	=	Right to left
	*=      /=      %=	Right to left
	+=      -=      &=	Right to left
	^=       =	Right to left
	<<=      >>=	Right to left
Comma	,	Right to left

## Manual Conversion -Infix to Postfix

- $A + B * C$                       Given infix form
- Left to right scan, Brackets first then Operator Precedence
- $A + \underline{B C *}$                       Convert the multiplication
- $A B C * +$                       Convert the addition

## Manual Conversion -Infix to Postfix

- $A + [(B + C) + (D + E) * F] / G$



## Manual Conversion -Infix to Postfix

- $A + [(B + C) + (D + E) * F] / G$
- Left to right scan, Brackets first then Operator Precedence
- $A + [(\underline{BC}+) + (D + E) * F] / G$
- $A + [(\underline{BC}+) + (\underline{DE}+) * F] / G$
- $A + [(\underline{BC}+) + (\underline{DE}+) F *] / G$
- $A + [(BC+) (DE+) F * +] / G$
- $A + [(BC+) (DE+) F * +] G /$
- $A [(BC+) (DE+) F * +] G / +$
- $ABC + DE + F * + G / +$

## Manual Conversion -Infix to Postfix

Convert the following expressions from Infix to Postfix;

- 1)  $A + B + C + D$
- 2)  $(A + B) / (C - D)$
- 3)  $(A + B) * C - (D - E) * (F + G)$
- 4)  $((A+B) - C * (D/E)) + F$
- 5)  $((A + B) * (C - D) + E) / (F + G)$

## Manual Conversion -Infix to Postfix

Convert the following expressions from Infix to Postfix;

1)  $A + B + C + D = A B + C + D +$

$= \underline{AB} + C + D$

$= \underline{AB+C} + D$

$= AB+C+D +$

## Manual Conversion -Infix to Postfix

Convert the following expressions from Infix to Postfix;

1)  $A + B + C + D = A B + C + D +$

2)  $(A + B) / (C - D) = A B + C D - /$

## Manual Conversion -Infix to Postfix

Convert the following expressions from Infix to Postfix;

- 1)  $A + B + C + D = A B + C + D +$
- 2)  $(A + B) / (C - D) = A B + C D - /$
- 3)  $(A + B) * C - (D - E) * (F + G) = AB + C * DE - FG + * -$

## Manual Conversion -Infix to Postfix

Convert the following expressions from Infix to Postfix;

- 1)  $A + B + C + D = A B + C + D +$
- 2)  $(A + B) / (C - D) = A B + C D - /$
- 3)  $(A + B) * C - (D - E) * (F + G) = AB + C * DE - FG + * -$
- 4)  $((A + B) - C * (D / E)) + F = A B + C D E / * - F +$

## Manual Conversion -Infix to Postfix

Convert the following expressions from Infix to Postfix;

- 1)  $A + B + C + D = A B + C + D +$
- 2)  $(A + B) / (C - D) = A B + C D - /$
- 3)  $(A + B) * C - (D - E) * (F + G) = AB + C * DE - FG + * -$
- 4)  $((A + B) - C * (D / E)) + F = A B + C D E / * - F +$
- 5)  $((A + B) * (C - D) + E) / (F + G) = A B + C D - * E + F G + /$

## Manual Conversion -Infix to Prefix

- $A/B^C+D$
- Left to right scan, Brackets first then Operator Precedence
- $A/^BC+D$
- $/A^BC+D$
- $+/A^BCD$



## Manual Conversion -Infix to Prefix

Convert the following expressions from Infix to Prefix;

- 1)  $(P * Q \wedge R + S)$
- 2)  $(A - B / C) * (D * E - F)$
- 3)  $(A * B + (C / D)) - F$
- 4)  $A + B * C - (D / E \wedge F) * G * H$
- 5)  $(A + B) * C / D + E \wedge F / G$

# Manual Conversion - Infix to Prefix

Convert the following expressions from Infix to Prefix;

1)  $(P * Q \wedge R + S) = + * P \wedge Q R S$

# Manual Conversion - Infix to Prefix

Convert the following expressions from Infix to Prefix;

1)  $(P * Q^R + S) = + * P^R Q R S$

2)  $(A - B / C) * (D * E - F) = * (- A / B C) (- * D E F)$

# Manual Conversion - Infix to Prefix

Convert the following expressions from Infix to Prefix;

1)  $(P * Q^R + S) = + * P^R Q R S$

2)  $(A - B / C) * (D * E - F) = * (- A / B C) (- * D E F)$

3)  $(A * B + (C / D)) - F = - + * A B / C D F$

# Manual Conversion - Infix to Prefix

Convert the following expressions from Infix to Prefix;

1)  $(P * Q \wedge R + S) = + * P \wedge Q R S$

2)  $(A - B / C) * (D * E - F) = * (- A / B C) (- * D E F)$

3)  $(A * B + (C / D)) - F = - + * A B / C D F$

4)  $A + B * C - (D / E \wedge F) * G * H$

$$= A + \underline{* B C} - (\underline{/ D \wedge E F}) * G * H$$

$$= A + * B C - \underline{* / D \wedge E F G} * H$$

$$= A + \underline{* B C} - \underline{** / D \wedge E F G H}$$

$$= \underline{+ A * B C} - \underline{** / D \wedge E F G H}$$

$$= - + A * B C ** / D \wedge E F G H$$

# Manual Conversion - Infix to Prefix

Convert the following expressions from Infix to Prefix;

1)  $(P * Q \wedge R + S) = + * P \wedge Q R S$

2)  $(A - B / C) * (D * E - F) = * (- A / B C) (- * D E F)$

3)  $(A * B + (C / D)) - F = - + * A B / C D F$

4)  $A + B * C - (D / E \wedge F) * G * H$

$$= A + * B C - (/ D \wedge E F) * G * H$$

$$= A + * B C - * / D \wedge E F G * H$$

$$= A + * B C - ** / D \wedge E F G H$$

$$= + A * B C - ** / D \wedge E F G H$$

$$= - + A * B C ** / D \wedge E F G H$$

5)  $(A + B) * C / D + E \wedge F / G$

$$= + \underline{A B} * C / D + E \wedge F / G$$

$$= + \underline{A B} * C / D + \underline{\wedge E F} / G$$

$$= * + \underline{A B C} / D + \underline{\wedge E F} / G$$

$$= / * + \underline{A B C D} + \underline{\wedge E F} / G$$

$$= / * + \underline{A B C D} + / \underline{\wedge E F G}$$

$$= + / * + \underline{A B C D} / \underline{\wedge E F G}$$

# Algorithm for Infix to Postfix using stack

Step 1-Put the opening bracket '(' on the start of the expression and add the closing bracket ')' at the end of the input expression 'P'

Step 2-Scan the Expression P from left to right and repeat steps 3 to 6 for each element of P until the stack is empty

Step 3-If an operand is encountered, add it to the output expression 'Q'

Step 4-If a left bracket is encountered push it onto the stack

# Algorithm for Infix to Postfix using stack

Step 5-If an operator is encountered then

5a)Repeatedly pop from the stack and add to the output expression Q, **each operator which has the same or higher priority** compared to the operator just scanned.

5b) then Add the operator just scanned to the stack

Step 6-If a right bracket ')' is encountered then repeatedly pop from the stack and add to the output expression Q until a left bracket is encountered.

Pop the left bracket also



# Infix to Postfix Conversion using stack

Eg-  $P = (A + B) * C$

# Infix to Postfix Conversion using stack

Eg-  $P = (A+B)*C$

$P = ( (A+B) * C )$

Scan	Stack	Expression Q
(	(	
(	((	
A	((	A
+	((+	A
B	((+	AB
)	(	AB+
*	(*	AB+
C	(*	AB+C
)	NULL	AB+C*

## Infix to Postfix Conversion using stack

Convert  $P/(Q-R)*S+T$

# Infix to Postfix Conversion using stack

Convert  $P/(Q-R)*S+T=(P/(Q-R)*S+T)$

Scan	Stack	Expression Q
(	(	
P	(	P
/	( /	P
(	( / (	P
Q	( / (	PQ
-	( / (-	PQ
R	( / (-	PQR
)	( /	PQR-
*	( *	PQR-/
S	( *	PQR-/S
+	( +	PQR-/S*
T	( +	PQR-/S*T
)	NULL	PQR-/S*T+

# Infix to Postfix Conversion using stack

Convert  $(P/(Q-R)*S+T)$

$(P/(Q-R)*S+T)$

Symbol	Stack	Expression
(	(	—
P	(	P
/	(/	P
(	(/(	P
Q	(/(	PQ
—	(/(—	PQ
R	(/(—	PQR
)	(/	PQR—
*	(*	PQR—/
S	(*	PQR—/S
+	(+	PQR—/S*
T	(+	PQR—/S*T
	null	PQR—/S*T+

## Infix to Postfix Conversion using stack

Convert  $P = (a \ \&\& \ b \ || \ c \ || \ ! \ ( \ e \ > \ f \ ) \ )$

# Notation Conversion

- Scan the expression from left to right
- **Operator Precedence-**
  - Paranthesis evaluated first
  - After that evaluation is on the basis of operator precedence
- Logical Not
- Exponential Operator
- Multiplication/division/modulus
- Addition/Subtraction
- Left shift, Right Shift
- Relational
- Logical And
- Logical Or



# Infix to Postfix Conversion using stack

Convert P=(a && b || c || ! ( e > f ) )

- Operator Precedence
- Logical Not
- Exponential Operator
- Multiplication/division/modulus
- Addition/Subtraction
- Left shift, Right Shift
- Relational
- Logical And &&
- Logical Or ||

Scan	Stack	Expression
(	(	
a	(	a
&&	(&&	a
b	(&&	ab
	(	ab&&
c	(	ab&&c
	(	ab&&c
!	(  !	ab&&c
(	(  !(	ab&&c
e	(  !(	ab&&c    e
>	(  !(>	ab&&c    e
f	(  !(>	ab&&c    ef
)	(  !	ab&&c    ef>
)	NULL	ab&&c    ef>



## Infix to Postfix Conversion using stack

Convert  $P = (A + (B * C - (D / E \wedge F) * G * H))$

# Infix to Postfix Conversion using stack

Convert  $P = (A + (B * C - (D / E \wedge F) * G * H))$

Scan	Stack	Expression Q
(	(	
A	(	A
+	( +	A
(	( + (	AB
B	( + (	AB
*	( + (*	AB
C	( + (*	ABC
-	( + (-	ABC*
(	( + (- (	ABC*
D	( + (- (	ABC*D
/	( + (- ( /	ABC*D
E	( + (- ( /	ABC*DE
^	( + (- ( / ^	ABC*DE
F	( + (- (/ ^	ABC*DEF
)	( + (-	ABC*DEF^/
*	( + (-*	ABC*DEF^/
G	( + (-*	ABC*DEF^/G
*	( + (-*	ABC*DEF^/G*
H	( + (-*	ABC*DEF^/G*H
)	( +	ABC*DEF^/G*H*-
)	null	ABC*DEF^/G*H*-+

Convert  $P=(A+(B*C-(D/E^F)*G*H))$

Scan	Stack	Expression Q
(	(	
A	(	A
+	( +	A
(	( + (	AB
B	( + (	AB
*	( + (*	AB
C	( + (*	ABC
-	( + (-	ABC*
(	( + (- (	ABC*
D	( + (- (	ABC*D
/	( + (- (/	ABC*D
E	( + (- (/	ABC*DE
^	( + (- (/^	ABC*DE
F	( + (- (/^	ABC*DEF
)	( + (-	ABC*DEFA^/
*	( + (-*	ABC*DEFA^/
G	( + (-*	ABC*DEFA^/G
*	( + (-*	ABC*DEFA^/G*
H	( + (-*	ABC*DEFA^/G*H
)	( +	ABC*DEFA^/G*H*-
)	null	ABC*DEFA^/G*H*-+

## Manual Evaluation of a Prefix notation

- Lets take an eg:  $+5*32$
- Find an operator from left to right having 2 operands after it
- Multiplication of 3 and 2 is carried out
- Expression becomes  $+56$
- Now  $+$  has two operands so evaluated
- $\text{Exp}=11$

## Manual Evaluation of a Postfix notation

- Lets take an eg:  $532^*+$
- Find first operator from left to right having 2 operands before it
- perform the operation
- Multiplication of 3 and 2 is carried out
- Expression becomes  $56+$
- Now  $+$  is evaluated
- $\text{Exp}=11$

# Algorithm for Evaluation of Postfix using stack

Step 1-Scan the Expression P from left to right and repeat steps 2 and 3 for each element of 'P' until the last element

Step 2- If an operand is encountered, push it on the stack

**1<sup>st</sup> operand say 'a' is pushed**

**2<sup>nd</sup> operand say 'b' is pushed**

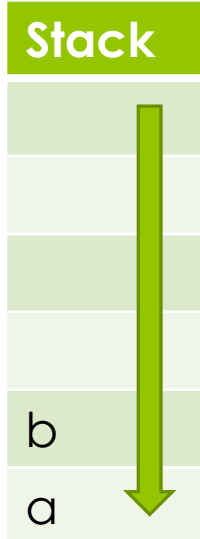
Step 3-If an operator is encountered then

3a) Remove the top two elements of stack and perform

**a operator b**

(where b is the top element and a is the next element to top element)

3b)Push the result onto the stack



# Algorithm for Evaluation of Postfix using stack

$$P = 653 + 9^* +$$
$$P = 53 + 9 * +$$
$$P = 3 + 9 * +$$

P=+9\*+

Evaluate  $5+3=8$

Push 8

$$P=9^{*}+$$
$$P = * +$$

## Evaluate

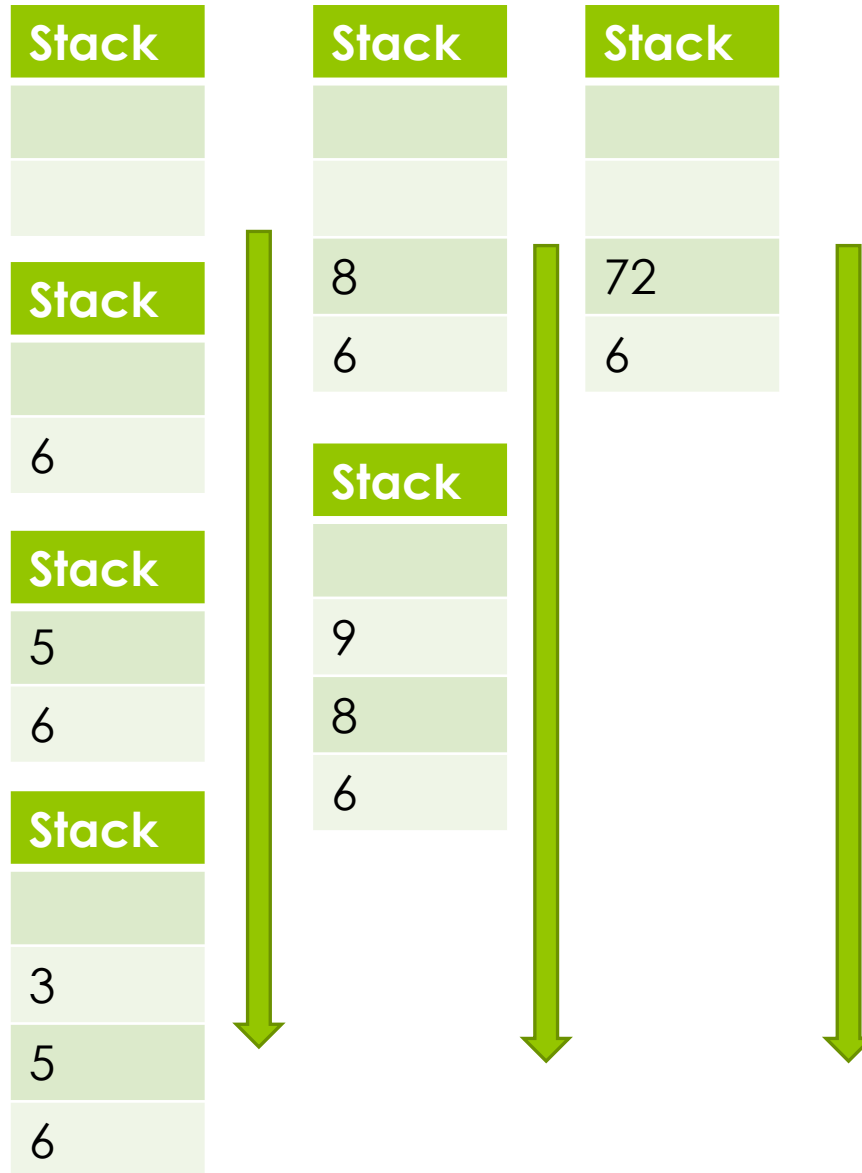
$$8 \times 9 = 72$$

## Push 72

$$P=+$$

Evaluate  $6+72=78$

Ans=78



# Algorithm for Evaluation of Postfix using stack

Evaluate the following expression-

- 1)  $432^*+5-$
- 2)  $532^*+4-5+$
- 3)  $53+82-*$
- 4) Evaluate  $562+*(12)4/-$   
taking 12 as a single number



## Evaluate $432^*+5-$

Postfix  $\rightarrow$  a b c \* + d - Let, a = 4, b = 3, c = 2, d = 5

Postfix  $\rightarrow$  4 3 2 \* + 5 -

Operator/Operand	Action	Stack
4	Push	4
3	Push	4, 3
2	Push	4, 3, 2
*	Pop (2, 3) and $3*2 = 6$ then Push 6	4, 6
+	Pop (6, 4) and $4+6 = 10$ then Push 10	10
5	Push	10, 5
-	Pop (5, 10) and $10-5 = 5$ then Push 5	5

WVA

**Example**

Evaluate the postfix expression  
 $5 \ 3 \ 2 \ * \ + \ 4 \ - \ 5 \ +$

(a) Input so far (shaded):

$5 \ 3 \ 2 \ * \ + \ 4 \ - \ 5 \ +$



(b) Input so far (shaded):

$5 \ 3 \ 2 \ * \ + \ 4 \ - \ 5 \ +$



(c) Input so far (shaded):

$5 \ 3 \ 2 \ * \ + \ 4 \ - \ 5 \ +$



(d) Input so far (shaded):

$5 \ 3 \ 2 \ * \ + \ 4 \ - \ 5 \ +$



(e) Input so far (shaded):

$5 \ 3 \ 2 \ * \ + \ 4 \ - \ 5 \ +$



(f) Input so far (shaded):

$5 \ 3 \ 2 \ * \ + \ 4 \ - \ 5 \ +$



(g) Input so far (shaded):

$5 \ 3 \ 2 \ * \ + \ 4 \ - \ 5 \ +$



(h) Input so far (shaded):

$5 \ 3 \ 2 \ * \ + \ 4 \ - \ 5 \ +$



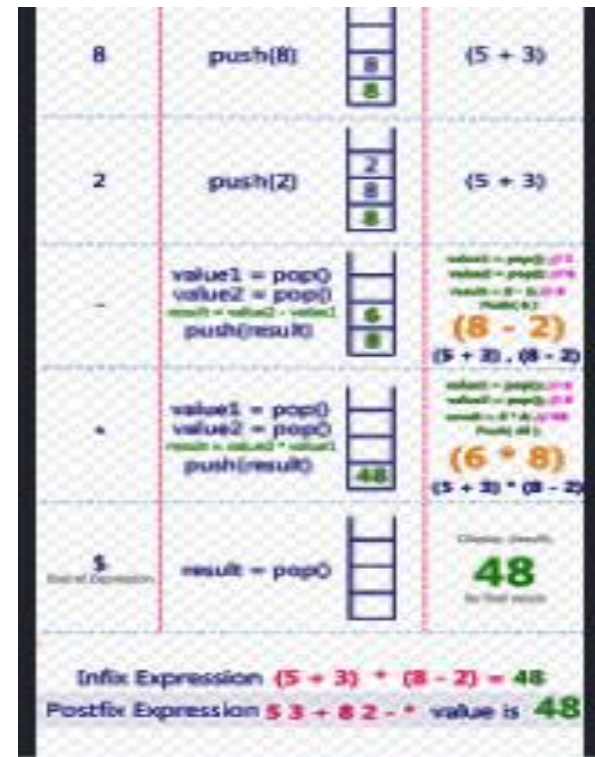
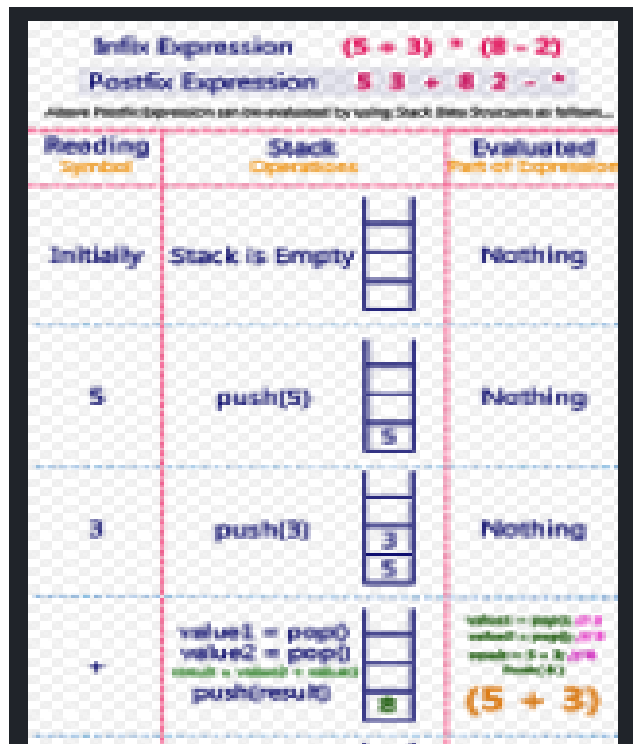
(i) Input so far (shaded):

$5 \ 3 \ 2 \ * \ + \ 4 \ - \ 5 \ +$



The result of the computation is 12.

# Postfix Expression 5 3 + 8 2 - \*



Courtesy: [http://btechsmartclass.com/data\\_structures/postfix-evaluation.html](http://btechsmartclass.com/data_structures/postfix-evaluation.html)

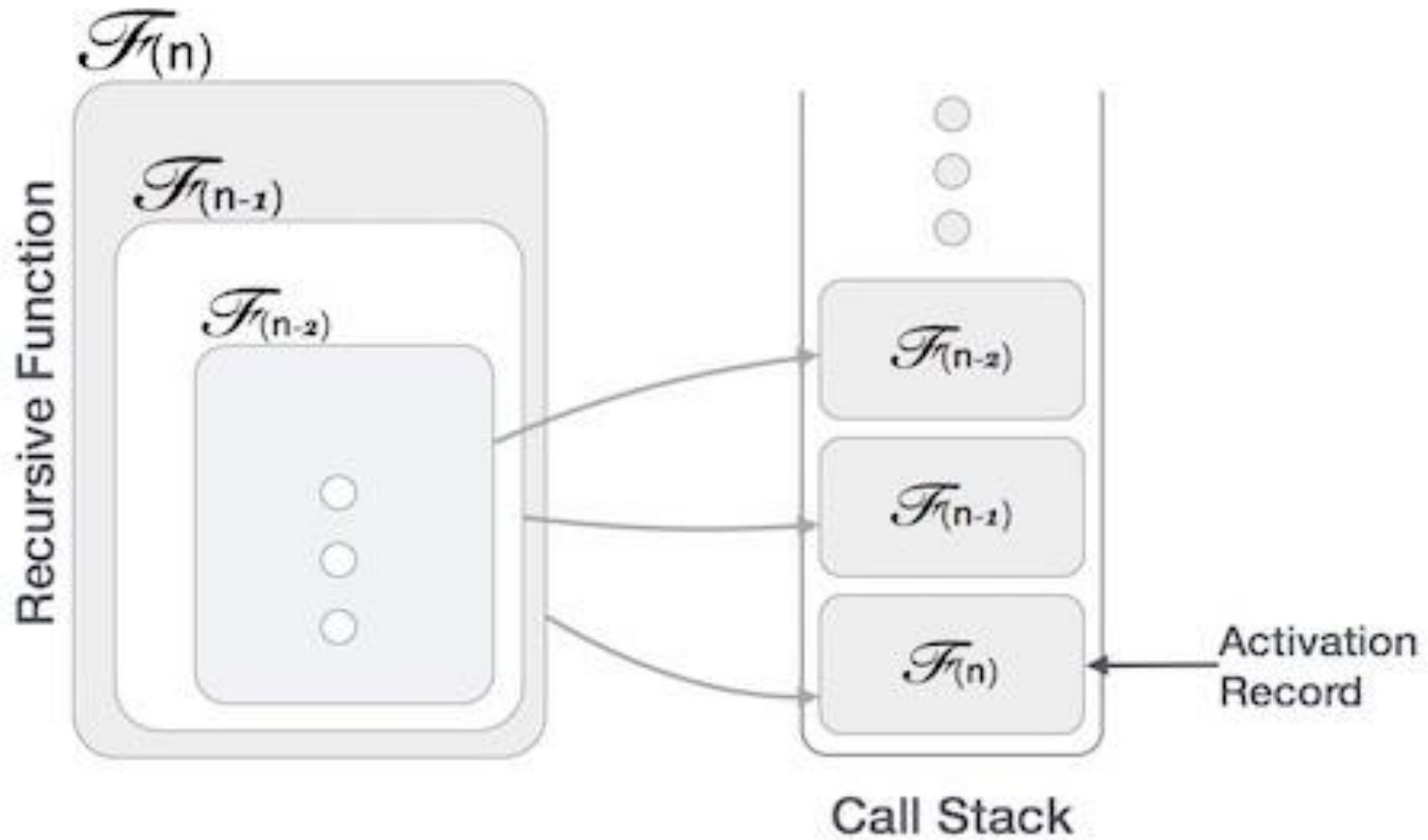
Evaluate  $562+*(12)4/-$   
taking 12 as a single number

$562+*124/-$

Step	Input Symbol/Element	Stack	Intermediate Calculations Output
1	5 Push	5	
2	6 Push	5, 6	
3	2 Push	5, 6, 2	
4	+ Pop 2 elements and evaluate	5	$6 + 2 = 8$
5	Push result 8	5, 8	
6	* Pop 2 elements and evaluate	# empty	$5 \times 8 = 40$
7	Push result 40	40	
8	12 Push	40, 12	
9	4 Push	40, 12, 4	
10	/ Pop 2 elements and evaluate	40	$12 / 4 = 3$
11	Push result 3	40, 3	
12	- Pop 2 elements and evaluate	# empty	$40 - 3 = 37$
13	Push result 37	37	
14	No more elements		<b>37</b>

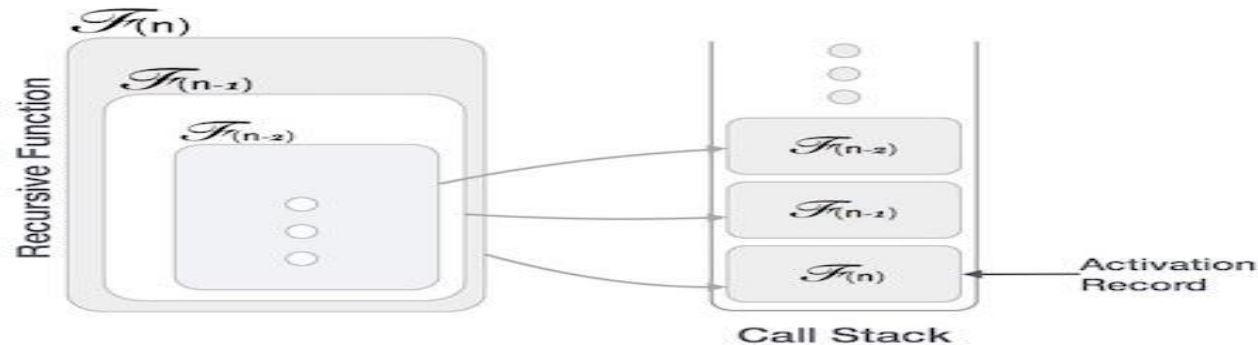
Courtesy: <https://unacademy.com/lesson/evaluation-of-a-postfix-expression-in-tabular-form/8Z5PDFL5>

# Application of Stack – Recursion



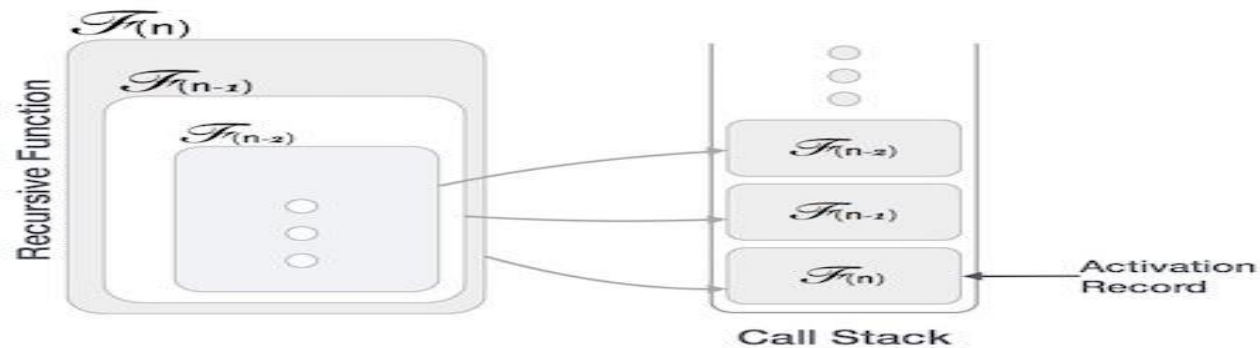
# Application of Stack – Recursion

- Many programming languages implement recursion by means of **stacks**.
- A function (**caller**) calls another function (**callee**) or itself as callee,
  - The caller function transfers execution control to the callee.
  - This transfer process may also involve some data to be passed from the caller to the callee.



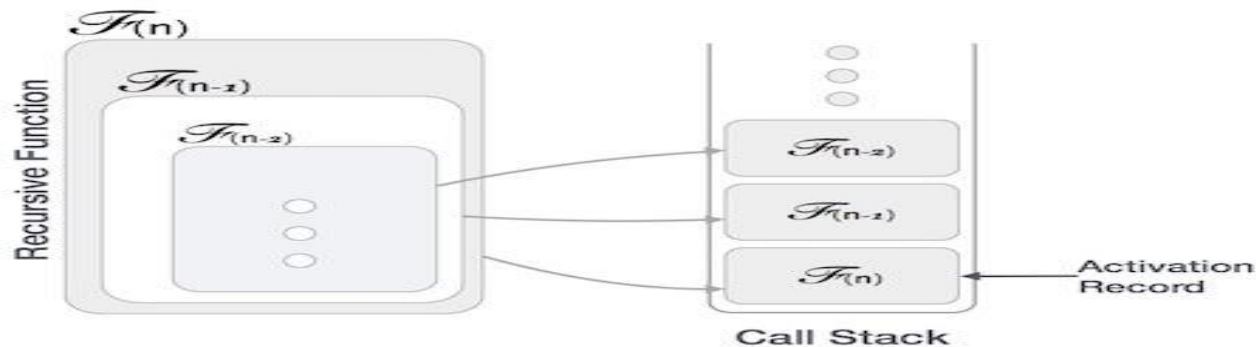
# Application of Stack – Recursion

- The caller function has
  - to suspend its execution temporarily and
  - resume later
  - when the execution control returns from the callee function.



# Application of Stack – Recursion

- Here, the caller function needs to start exactly from the point of execution where it puts itself on hold.
  - It also needs the exact same data values it was working on.
  - So, an activation record (or stack frame) is created for the caller function.
  - Activation record keeps the information about
    - local variables,
    - formal parameters,
    - return address and
    - all information passed to the callee function.





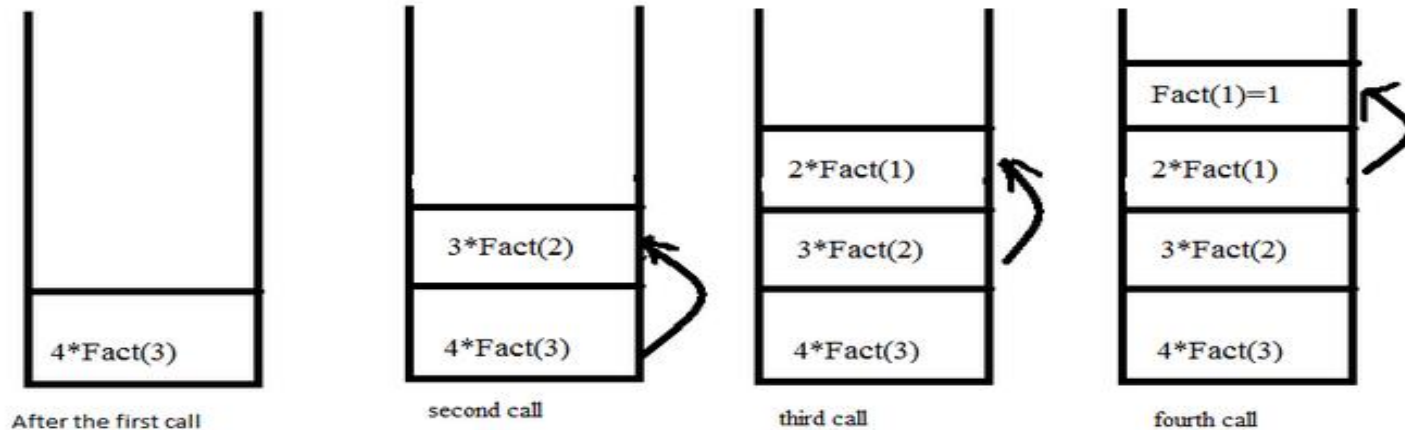
# Application of Stack – Recursion

A recursive function to find the factorial of a positive whole number

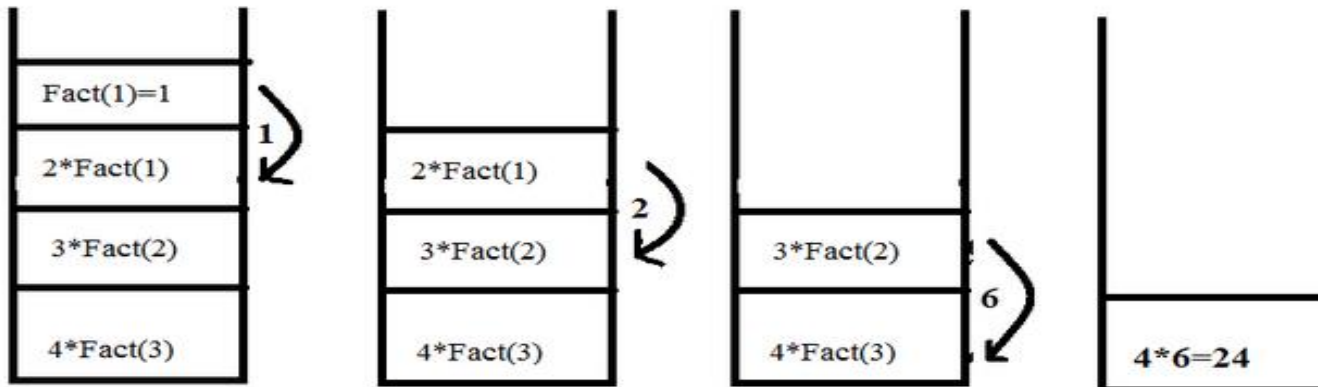
```
int factorial(n)
{
    if (n == 1)
    {
        // base case
        return 1;
    }
    else
    {
        return n * factorial(n - 1);
        // function calls itself
    }
}
```

# Application of Stack – Recursion

**When function call happens previous variables gets stored in stack**



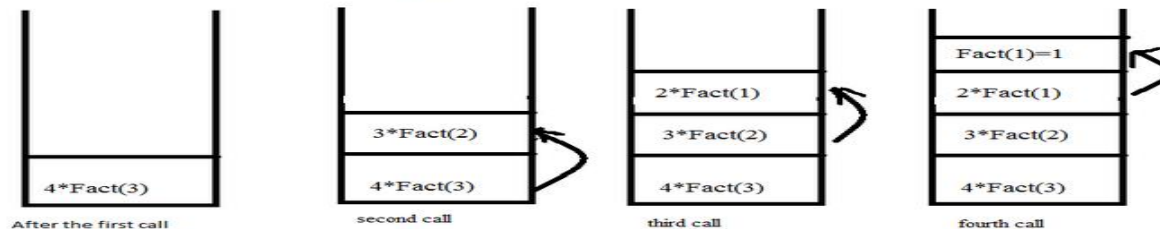
**Returning values from base case to caller function**



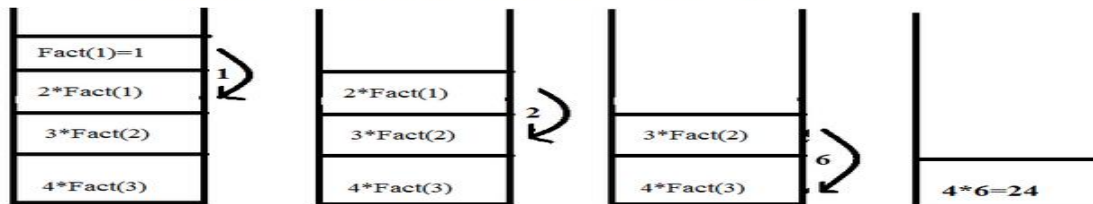
# Application of Stack – Recursion

- Functions calls are 'stacked' one on top of the other,
- This is called the call stack (or execution stack)
- The call stack operates on a "Last In, First Out" basis. An item is "pushed" onto a stack on function call, and an item is "popped" off the stack when that function returns a value.

**When function call happens previous variables gets stored in stack**



**Returning values from base case to caller function**



# Application of Stack – Recursion

## Time Complexity

- In iterations,
  - we take number of iterations to count the time complexity.
- In recursion,
  - assuming everything is constant,
  - count the number of times a recursive call is being made.

## Space Complexity

- Space complexity is counted as
  - what amount of extra space is required for a module to execute.
- In iterations,
  - the compiler hardly requires any extra space.
  - The compiler keeps updating the values of variables used in the iterations.

# Application of Stack – Recursion

## Time Complexity

- A call made to a function is  $O(1)$ ,
- hence the  $(n)$  number of times a recursive call is made
- makes the recursive function  $O(n)$ .

## Space Complexity

- In recursion,
  - the system needs to store activation record each time a recursive call is made.
- Thus, space complexity of recursive function
  - may go higher than that of a function with iteration.