

4/11/24

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\* Time Complex & Space Complex  $\Rightarrow$  Q. 0  
 (Q, A, A) QBA only

Q. Obtain Time & Space Complexity for the above -

$\text{TC} = (\text{initialization} + \text{for loop}) \text{ or } \text{SC}$

$\text{TF}(n=5)$  — (1) ; (1)

{ — (initialization + for loop)

Statement — (1) ;

} — time

Else — (1) ;

Statement — (1)

$O \approx 1 \in O(n^0) \Leftarrow \text{if/else } \rightarrow O(n^0) \Leftarrow$   
 (either if or either else  
 will work)  $\vdash \vdash$

$O \Rightarrow \text{order}$  — i

$\text{TC} \& \text{SC} \Rightarrow$  both O are diff

Q. Add mains() ( $= n^0 + n + n^0 + n = 2n^0 + 2n$ )

{

$S = 0$  ( $n, a, A$ )  $\Rightarrow$  (1) M optA

- for ( $i=0$ ;  $i < n$ ;  $i++$ ) — (n)

{ — (initialization + for loop)

$S = S + 1$  — (n)

- i  $\exists e_a = (\text{initialization} + \text{for loop})$

Return S; — (1)

} — (initialization + for loop)

$$e_a = (\text{initialization} + \text{for loop}) = 2n^0 + 2n = O(n^0 + n)$$

Hence  $O = 2n^0$

$$= O(n)$$

(n)

$e_a = \text{initialization}$

$$\text{SC} = 3 = 3n^0 = O(n^0)$$

CS, i, m

$$(e_a)_2 = e_a + e_a + e_a + e_a = 4e_a = 4T$$

Q. Sum of two matrix

Algo Add(A, B, n)

for(i=0; i < n; i++) -  $O(n)$

{ for(j=0; j < n; j++) -  $O(n^2)$

stmt -  $O(n^2)$

SC =  $A \rightarrow n \rightarrow O(n) \rightarrow O(n)$

$B \rightarrow n \rightarrow O(n)$

$n \rightarrow 1$  (assume)

$i \rightarrow 1$

$j \rightarrow 1 \rightarrow 0 \rightarrow 1 \rightarrow 2 \rightarrow \dots \rightarrow n-1$

TC =  $n + n^2 + n^2 \Rightarrow O(n^2)$

Q. Algo Multiply(A, B, n)

for(i=0; i < n; i++) -  $O(n)$

for(j=0; j < n; j++) -  $O(n)$

{ for(k=0; k < n; k++) -  $O(n)$

stmt; -  $n^2$

{ k - 1

for(k=0; k < n; k++) -  $O(n)$

{ k - 1

stmt; -  $n^3$

SC =  $2n + 4$

=  $O(n)$

TC =  $n + n^2 + n^2 + n^3 + n^3 = O(n^3)$

Q. for ( $i=1$ ;  $i \leq n$ ;  $i = i+2$ )

{

stmt;

}

base case

$$TC = O(C \log_2 n)$$

$$SC = i = 1 \Rightarrow O(n^0)$$

$$n = 1$$

i	$i+2$
1	2
2	4
4	8
8	16
16	32
32	64
64	128
128	256
256	512
512	1024
1024	2048
2048	4096
4096	8192
8192	16384
16384	32768
32768	65536
65536	131072
131072	262144
262144	524288
524288	1048576
1048576	2097152
2097152	4194304
4194304	8388608
8388608	16777216
16777216	33554432
33554432	67108864
67108864	134217728
134217728	268435456
268435456	536870912
536870912	1073741824
1073741824	2147483648
2147483648	4294967296
4294967296	8589934592
8589934592	17179869184
17179869184	34359738368
34359738368	68719476736
68719476736	137438953472
137438953472	274877906944
274877906944	549755813888
549755813888	1099511627776
1099511627776	219902325552
219902325552	439804651104
439804651104	879609302208
879609302208	1759218604416
1759218604416	3518437208832
3518437208832	7036874417664
7036874417664	14073748835328
14073748835328	28147497670656
28147497670656	56294995341312
56294995341312	112589990682624
112589990682624	225179981365248
225179981365248	450359962730496
450359962730496	900719925460992
900719925460992	1801439850921984
1801439850921984	3602879701843968
3602879701843968	7205759403687936
7205759403687936	14411518807375872
14411518807375872	28823037614751744
28823037614751744	57646075229503488
57646075229503488	115292150459006976
115292150459006976	230584300918013952
230584300918013952	461168601836027904
461168601836027904	922337203672055808
922337203672055808	1844674071344111616
1844674071344111616	3689348142688223232
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7378696285376446464	14757392570752892928
14757392570752892928	29514785141505785856
29514785141505785856	59029570283011571712
59029570283011571712	118059140566023143424
118059140566023143424	236118281132046286848
236118281132046286848	472236562264092573696
472236562264092573696	944473124528185147392
944473124528185147392	1888946249056370294784
1888946249056370294784	3777892498112740589568
3777892498112740589568	7555784996225481179136
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60446279969803849432988	120892559939607698865976
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241785119879215397731952	483570239758430795463904
483570239758430795463904	967140479516861590927808
967140479516861590927808	1934280959033723181855616
1934280959033723181855616	3868561918067446363711232
3868561918067446363711232	7737123836134892727422464
7737123836134892727422464	15474247672269785454844928
15474247672269785454844928	30948495344539570909689856
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39614074041010650764403008	79228148082021301528806016
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8307672698607244571186925568	16615345397214489142373851136
16615345397214489142373851136	33230690794428978284747702272
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2126764210843454610223852945408	4253528421686909220447705890816
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43556131182073950417384539025984	87112262364147900834768578051968
87112262364147900834768578051968	174224524728295801669537156103936
174224524728295801669537156103936	348449049456591603339074312207872
348449049456591603339074312207872	696898098913183206678148624415744
696898098913183206678148624415744	139379619782636641335629724883148
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557518479130546565342518899532592	111503695826109313068503779906584
111503695826109313068503779906584	223007391652218626137007559813168
223007391652218626137007559813168	446014783304437252274015119626336
446014783304437252274015119626336	892029566608874504548030239252672
892029566608874504548030239252672	178405913321774900909606467850536
178405913321774900909606467850536	356811826643549801819212935701072
356811826643549801819212935701072	713623653287099603638425871402144
713623653287099603638425871402144	1427247306574199207276857542804288
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2854494613148398414553715085608576	5708989226296796829107430171217152
5708989226296796829107430171217152	1141797845259359365821486034243408
1141797845259359365821486034243408	2283595690518718731642972068486816
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1496577271738335924379854678282496	2993154543476671848759709356564992
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1197261817390668739503883742625984	2394523634781337479007767485251968
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4789047269562674958015534970503936	9578094539125349916031069941007872
9578094539125349916031069941007872	19156189078250699832062139820015744
19156189078250699832062139820015744	38312378156501399664124279640031488
38312378156501399664124279640031488	76624756313002799328248559280062976
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245198820256009594519555779696201552	49039764051201918903911155939240304
49039764051201918903911155939240304	98079528102403837807822311878480608
98079528102403837807822311878480608	196159056204807675615646223556961216
196159056204807675615646223556961216	392318112409615351231292447113922432

$$++ \frac{n(n+1)}{2} \Rightarrow 2^{log_2 n} = 2$$

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e.g. `for(i=0; i < n; i++)`  
`for(j=0; j < i; j++)`

}

l+statement

i	j	outer	inner
0		✓	x
1	1	✓	✓
2	1, 2	✓	✓
3	1, 2, 3	✓	✓

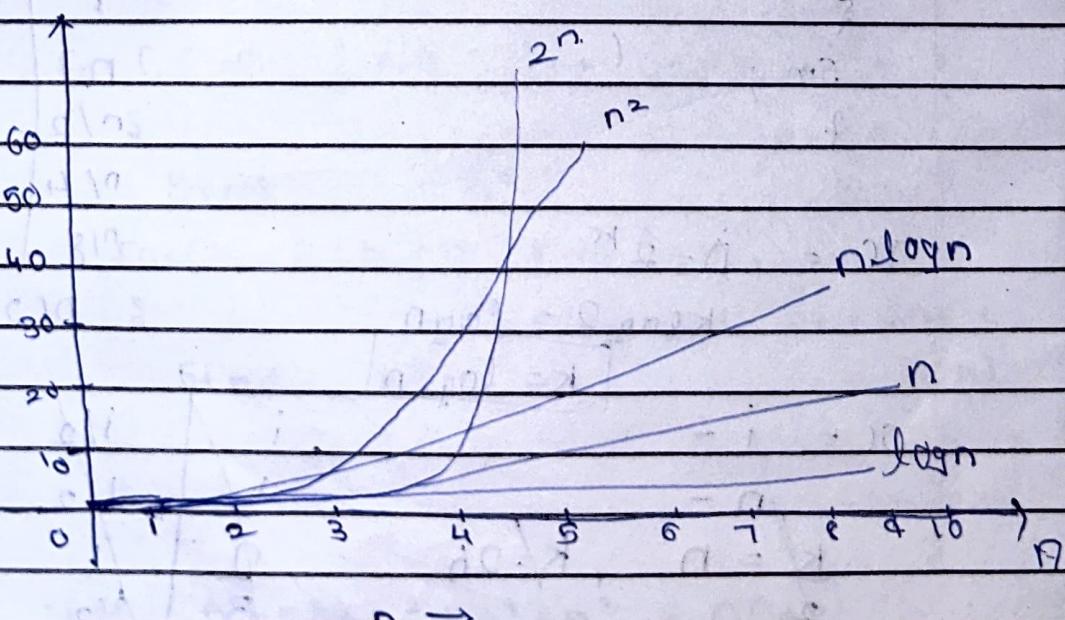
$$TC = \frac{n(n+1)}{2} = \frac{1}{2}n^2 + \frac{1}{2}n = O(n^2)$$

$$SC = O(n^2)$$

\* Growth of Function  $\Rightarrow$  (refer the table in ppt)

$$1 < \log(n) < \sqrt{n} < n < n \log(n) < n^2 < n^3 - \dots$$

$$- 2^n < n^n$$



Time complexity

\* Asymptotic Analysis  $\Rightarrow$  A method of describing limiting behavior

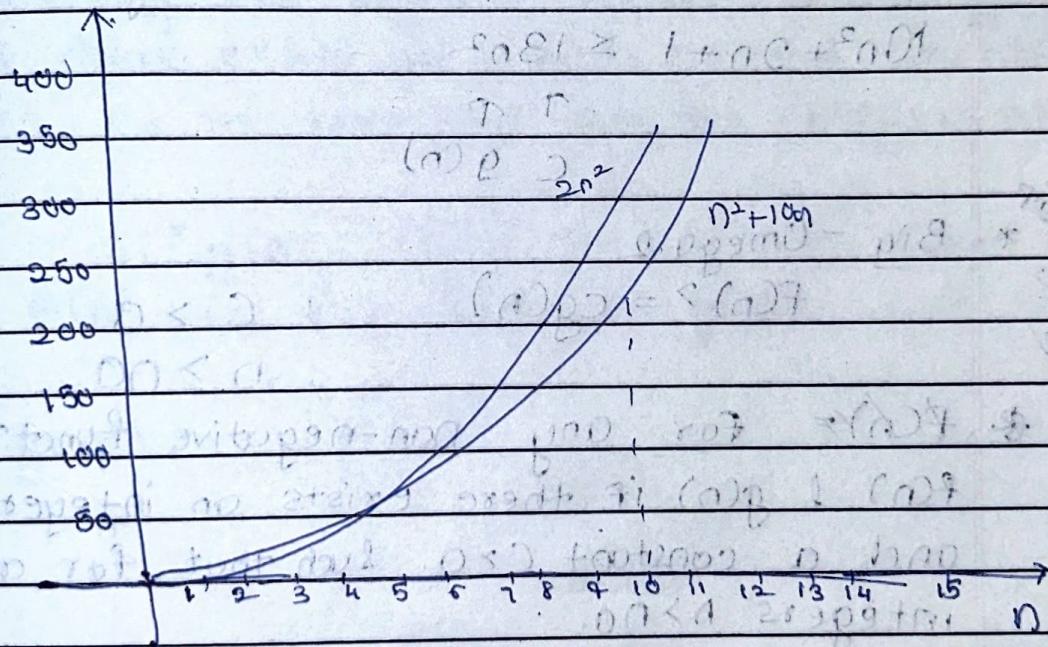
- Big O
- Big Omega
- Big Theta

(Algo Bank) - Write def of the above

$\Rightarrow$  Big O - For any non-negative funct<sup>n</sup>,  $f(n)$  &  $g(n)$ , if there exists an integer  $n_0$  and a constant  $C > 0$  such that for all integers  $n > n_0$ ,

$$f(n) \leq Cg(n)$$

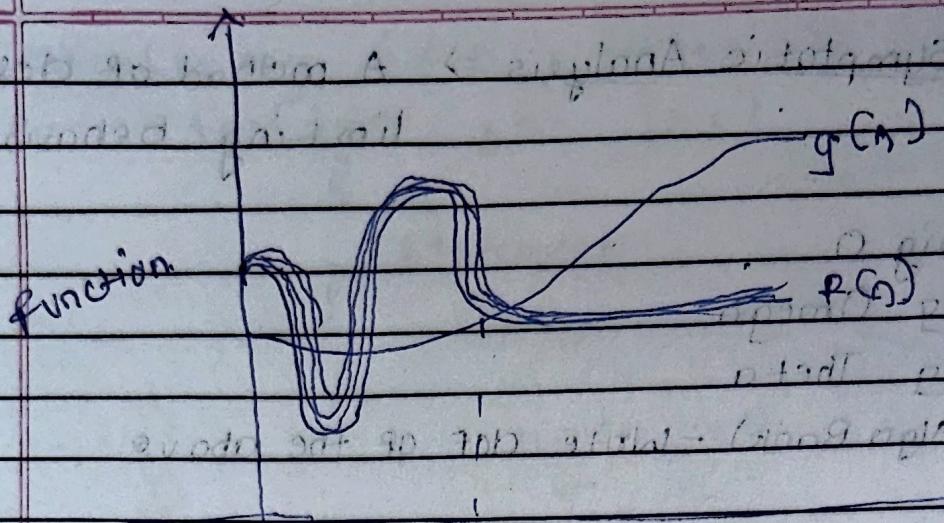
then  $f(n)$  is Big O of  $g(n)$ .



e.g.  $F(n) = 3n + 5$   $\therefore f(n) \leq cg(n)$

$(3n+5) \leq 8n \Rightarrow$  (we need to check for each cond'n & get the most appropriate one)

$$8(n) = n$$



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Q.  $f(n) = 10n^2 + 2n + 1 \geq 10n^2$

$\rightarrow f(n) \leq cg(n)$  for some constant  $c > 0$  (Big-O)

$$10n^2 + 2n + 1 \leq 13n^2$$

 $\uparrow \uparrow$  $c \ g(n)$ 

From  $\frac{f(n)}{g(n)}$   
 1.  $\frac{f(n)}{g(n)} \geq c$   
 2.  $n \geq n_0$

\* Big-Omega

$$f(n) \geq cg(n) ; c > 0$$

 $; n \geq n_0$ 

\*  $f(n) \geq cg(n)$  for any non-negative funct'

Write in algo bank  
 Write in algo bank  
 if  $f(n) \geq cg(n)$  if there exists an integer  $n_0$  and a constant  $c > 0$  such that for all integers  $n > n_0$ .

$$f(n) \geq cg(n)$$

then  $f(n)$  is Big-O of  $g(n)$ .

Q.  $f(n) = 3n + 5$

$$f(n) \geq cg(n)$$

$$3n + 5 \geq cn$$

$$f(n) \geq c(1)$$

$\Rightarrow$  any value below  
 satisfies will always

\* Big - Theta

$$10(13)^2 + 2(13^2) + 1$$

Q.  $10n^2 + 2n + 1 \geq cg(n)$

$$10n^2 + 2n + 1 \geq 10n^2$$

$$\begin{matrix} \uparrow & \uparrow \\ c & g(n) \end{matrix}$$

$$10 + 2 + 1$$

\* Big - Theta Q. →

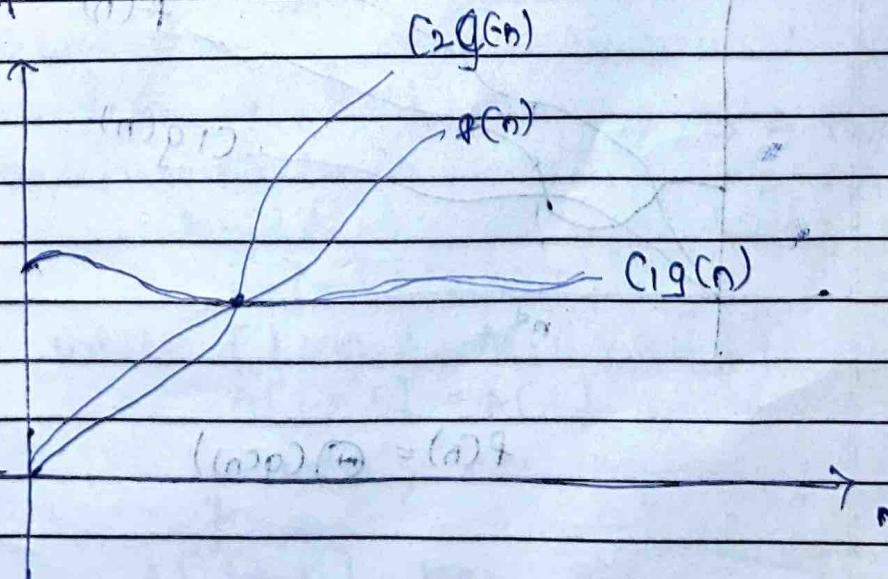
When Big - Oh & Ω meet, we indicate this by using  $\Theta$  notation.

→  $\Theta(f(n))$ , if it is in  $O(f(n))$  and it is in  $\Omega(f(n))$ .

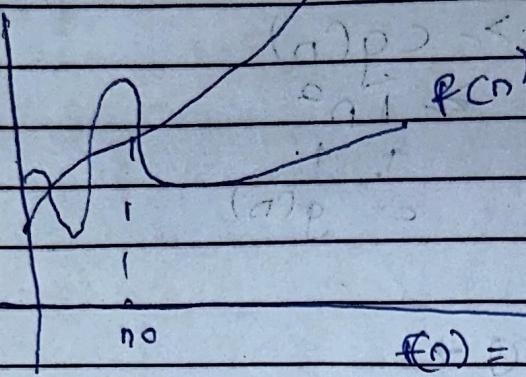
→ For any non-negative funct<sup>n</sup>,  $f(n) \leq g(n)$  if there exists an integer  $n_0$  and constant  $c_1, c_2 > 0$  such that all the integers  $n \geq n_0$

$c_1 * g(n) \leq f(n) \leq c_2 g(n)$ . Then  
 $f(n)$  is Big Theta of  $g(n)$ .

↑ graph

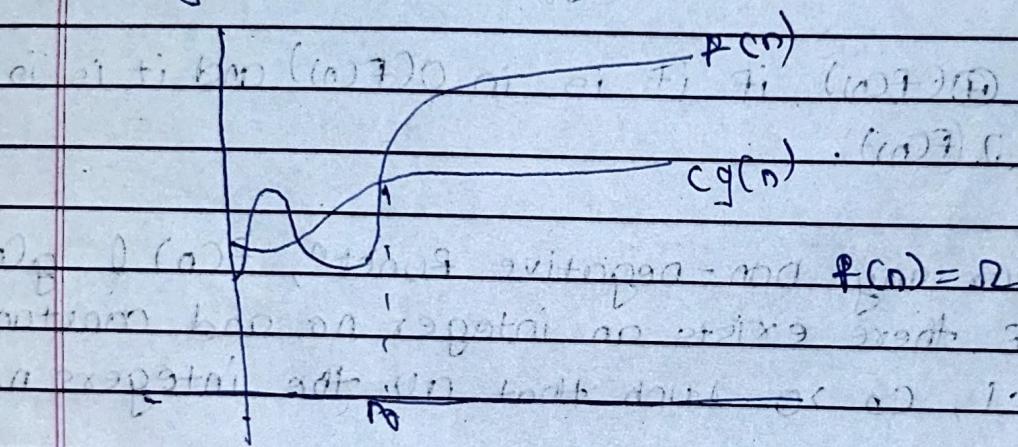


\* Big O graph



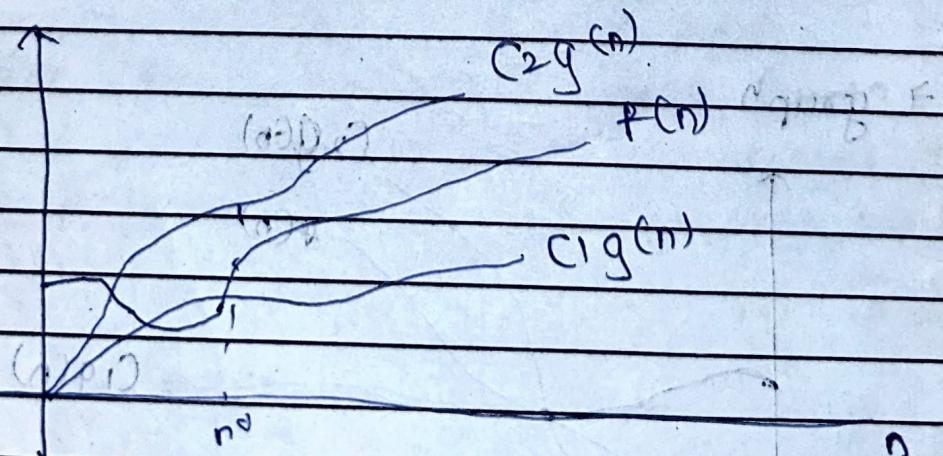
$$f(n) = \Theta(g(n))$$

\* Big - Ω graph



$$f(n) = \Omega(g(n))$$

\* Big - Theta graph



$$f(n) = \Theta(g(n))$$

$$\textcircled{1} \quad f(n) = 3n + 5$$

$$\rightarrow 3n + 5$$

Big Theta of  $f(n)$

$$= \Theta(n)$$

$$c_1 g(n) \leq f(n) \leq c_2 g(n)$$

$$c_1 n \leq 3n + 5 \leq c_2 n \quad \text{if } n \geq 5$$

$$c_1 = 1, c_2 = 8, g(n) = n \quad \text{obv if } n \geq 5$$

$$\textcircled{2} \quad f(n) = 10n^2 + 2n + 1$$

$$c_1 g(n) \leq f(n) \leq c_2 g(n)$$

$$n^2 \leq 10n^2 + 2n + 1 \leq 13n^2$$

both sides

( $n^2$ ) is slow

$$\textcircled{3} \quad f(n) = n!$$

$$c_1 g(n) \leq n! \leq c_2 g(n)$$

Can't predict (because nothing can be less than one)

(so) Can't be able to find  $c_1$  &  $c_2$  &  $g(n)$ . Draw  $n!$  graph

\* Insertion Sorting  $\Rightarrow$

$$\text{eg} - 5 12 4 6 1 3 \rightarrow B \Theta(n)$$

n for  $j=2$  to length

$$\text{key} = A[j] \rightarrow R \Theta(n)$$

$$i=j-1; \rightarrow C \Theta(n)$$

n while  $[i > 0 \& A[i] > \text{key}] \{$

$$A[i+1] = A[i] \rightarrow D \Theta(n)$$

$$i = i - 1; \rightarrow E \Theta(n)$$

$$A[i+1] = \text{key} \rightarrow F \Theta(n)$$

$$f(n) = 4n + 3n^2 \\ = O(n^2)$$

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Date \_\_\_\_\_  
Page \_\_\_\_\_\* Recursion  $\Rightarrow$  Recursion

$$\text{fac}(n) = n * \text{fac}(n-1)$$

~~\* → Funct<sup>n</sup> must call itself & it must terminate properly~~

\* Fibonacci Series  $\Rightarrow$ \* 3 methods to identify recursion  $\Rightarrow$ 

(1) Iterative Method

(2) Binary Tree Method

(3) Master Method

void Test( $\star$ )

$$\text{for } i=0, i < n, i++ \text{ do } f(n) = n + f(n-1)$$

$$f(n-1) = f(n-2) + (n-1)$$

A) Iterative Method

Delay the eq<sup>n</sup> ① by 1

$$f(n) = f(n-2) + f(n-1) - ②$$

put ② in ①

$$f(n) = f(n-2) + 2n - 1$$

$$f(n) = f(n-2) + 2n - 3 - ③$$

delay eq<sup>n</sup> ③ by 1

$$f(n-2) = f(n-3) + (n-2) - ④$$

$$f(n) = f(n-1) + n$$

$$f(0) = 1 \quad \text{Date: } \quad \text{Page: }$$

$$\text{put eqn ④ in ②} : (1-a)^2 = (a)$$

$$f(n) = f(n-3) + 3n \quad \text{or} \quad (1-a) +$$

$$f(0) = f(n-k) + k_n$$

$$(k=n)$$

$$: (a) = f(0) + n^2 \quad (a-a)^2 = (a)$$

$$f(n) = 1+n^2 \rightarrow O(n^2)$$

b) Binary Tree

$$(a+a) \cdot f(n) = a + a^2$$

$$(1-a)^2 + (a-a) \cdot f(n-1) = (a)^2$$

$$f(n) = \dots + ((n-1)^2 + n^2) + (n-1) + \dots + n-2$$

$$1+2+3+\dots+(n-2)+(n-1)+\dots+n-2+(a-1)^2$$

$$f(n) = a(n+1) = n^2 + 5 \rightarrow O(n^2)$$

$$n-2 \quad f(n-3)$$

$$f(n-k)$$

$$(a-a)(a-a) \dots (a-a) \cdot f(0)$$

eg- void Test(n)

for (i=0; i>n; i=i+2)

Statement

}

Test(n-1)

}

$$f(n) = f(n-1) + \log_2(n) \quad \text{--- (1)}$$

delay 'say' ① by 1

$$f(n-1) = f(n-2) + \log_2(n-1) \quad \text{--- (2)}$$

Put ② in ①

$$f(n) = f(n-2) + \log_2(n-1) + \log_2(n) \quad \text{--- (3)}$$

$n = k$

$$f(n) = f$$

$$f(n-2) = f(n-3) + \log_2(n-2)$$

$$f(n) = f(n-3) + \log_2(n-2) + \log_2(n-1) + \log_2(n)$$

$n = k$

$$f(n) = f(n-k) + \log_2(n-(k+1)) + \dots +$$

$$\log_2(n-2) + \log_2(n-1) + \log_2(n)$$

$n = k$

$$f(n) = f(0) + \log_2(1) + \dots + \log_2(n-2) + \log_2(n-1) + \log_2 n$$

$$= \log_2(1 \cdot 2 \cdot 3 \cdots (n-2)(n-1)n)$$

$$= \log_2(n!)$$

$$f(n) = \log_2(n) + f(n-1)$$

$$= \log_2(n) + \log_2(n-1) + f(n-2)$$

$$= \log_2(n-1) + \log_2(n-2) + f(n-3)$$

$$\log_2(n!)$$

# DAC

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\* Masters Method - in Book Book

ex-  $T(n) = 2T\left(\frac{n}{2}\right) + n^2$

$$\begin{aligned} T(n) &= aT\left(\frac{n}{b}\right) + f(n) & a = 2 \\ &= aT\left(\frac{n}{b}\right) + O(n^2) & b = 2 \\ &\quad \cancel{\text{Left}} & c = 2 \end{aligned}$$

$$\frac{\log b}{\log b} = \frac{\log_2 2}{\log_2 2} = \frac{\log_2 2}{\log_2 2} = 1$$

$$\begin{aligned} c &= 2 & c > \log_b b & \therefore T(n) = O(f(n)) \\ && &= O(n^2) \end{aligned}$$

ex-  $T(n) = 8T\left(\frac{n}{4}\right) + n^2$

$$a = 8 \quad b = 4 \quad c = 2$$

$$\frac{\log 2^3}{\log 2^2}$$

$$\frac{\log b}{\log b} = \frac{\log_2 4}{\log_2 4} = \frac{\log_2 4}{\log_2 4} = 1.5 = \frac{3}{2} \cancel{\log}$$

$$c = \cancel{1.5} 2 \quad c > \log_b b \quad O(n^2)$$

ex-  $T(n) = 8T\left(\frac{n}{2}\right) + 100n^2$

$$a = 8, b = 2, c = \cancel{n^2} 2$$

$$\frac{\log b}{\log b} = \frac{\log_2 2}{\log_2 2} = \frac{\log_2 2}{\log_2 2} = 1 \cancel{\log} 2 = 3$$

$$T(n) = O(n^3)$$

Q.F. =

$$c < \log_b a \quad - T(n) = O(n \cancel{\log_2 2})$$