Functions and Pigeon Hole Principle (04)

5.1 Definition and types of functions: Injective, Surjective and Bijective

5.2 Composition, Identity and Inverse

5.3 Pigeon-hole principle , Extended Pigeon-hole principle

Functions

Rosen 6th ed., §2.3

Definition of Functions

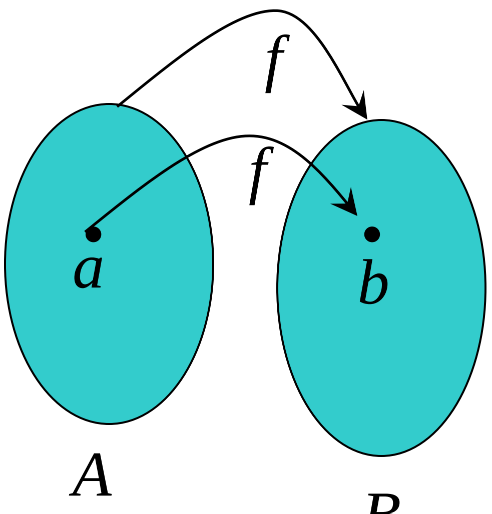
- Given any sets A, B, a function f from (or “mapping”) A to B ($f:A\rightarrow B$) is an assignment of **exactly one** element $\mathbf{f(x)\in B}$ to each element $\mathbf{x\in A}$.
- A Function assigns to each element of a set, exactly one element of a related set.
  - Functions find their application in various fields like representation of the computational complexity of algorithms, counting objects, study of sequences and strings,etc.

Generic Functions

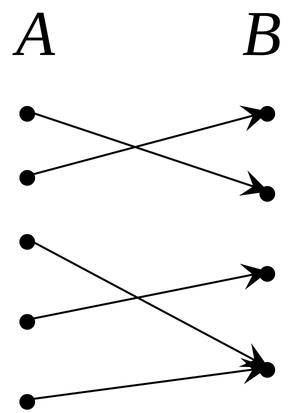
- A function f: X $\rightarrow$ Y is a relationship between elements of X to elements of Y, when each element from X is related to a unique element from Y
- X is called domain of f, range of f is a subset of Y so that for each element y of this subset there exists an element x from X such that $y = f(x)$
- Sample functions:
  - $f : R \rightarrow R$, $f(x) = x^2$
  - $f : Z \rightarrow Z$, $f(x) = x + 1$
  - $f : Q \rightarrow Z$, $f(x) = 2$

Graphical Representations

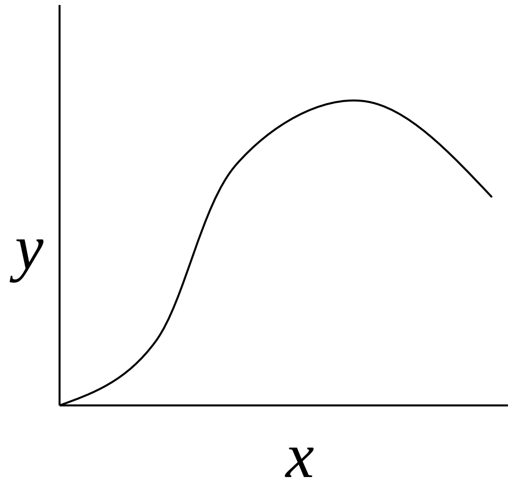
- Functions can be represented graphically in several ways:



Like Venn diagrams



Graph



Plot

Some Function Terminology

- If f:$A \rightarrow B$, and $f(a)=b$ (where $a \in A$ & $b \in B$), then:
  - A is the *domain* of f.
  - B is the *codomain* of f.
  - b is the *image* of a under f.
  - a is a *pre-image* of b under f.
    - In general, b may have more than one pre-image.
  - The *range* $R \subseteq B$ of f is $\{b \mid \exists a\ f(a)=b\}$.

Example 1 $\{(-3,1),(0,2),(2,4)\}$

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Domain Range

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Domain

-3

0

2

Range

Example 1 $\{(-3,1),(0,2),(2,4)\}$

Domain Range

Example 1 $\{(-3,1),(0,2),(2,4)\}$

Domain

-3

0

2

Range

1

Example 1

{(-3,1),(0,2),(2,4)}

Domain

-3

0

2

Range

1

Example 1 $\{(-3,1),(0,2),(2,4)\}$

Domain Range

Example 1 $\{(-3,1),(0,2),(2,4)\}$

Domain

-3

0

2

Range

1

2

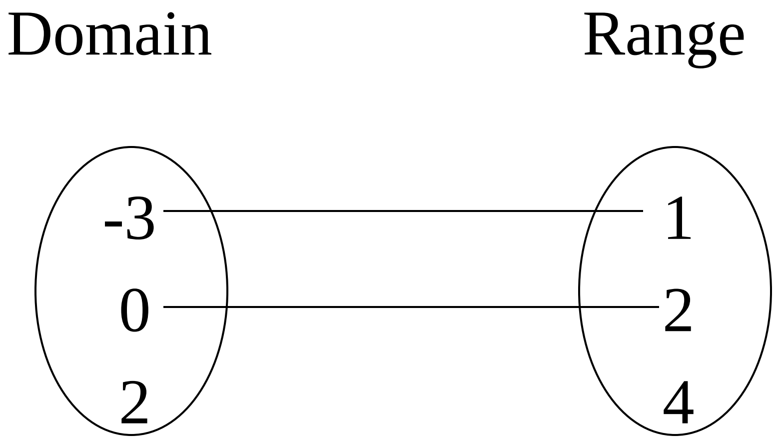
Example 1 $\{(-3,1),(0,2),(2,4)\}$

Domain	Range
-3	1
0	2
2	4

Example 1 $\{(-3,1),(0,2),(2,4)\}$

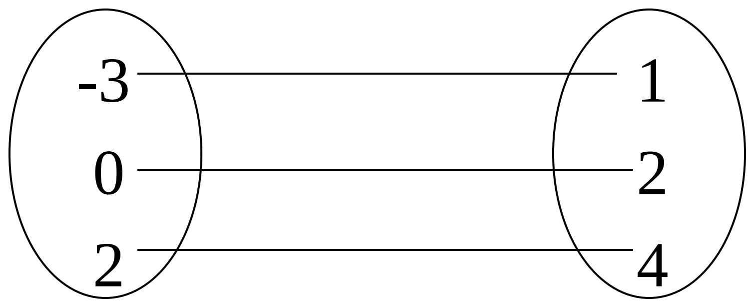
Domain Range

Example 1 $\{(-3,1),(0,2),(2,4)\}$



__Example 1__ $\{(-3,1),(0,2),(2,4)\}$

Domain Range



Example 1 {(-3,1),(0,2),(2,4)}

Domain Range

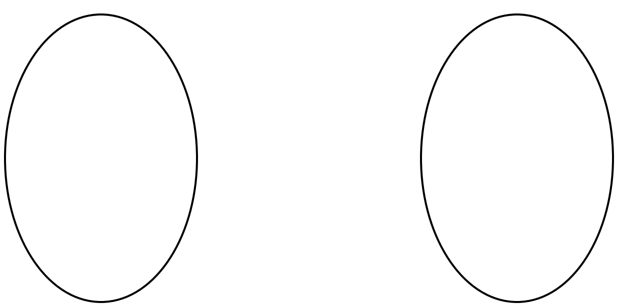
{(-1,5),(1,3),(4,5)} {(5,6),(-3,0),(1,1),(-3,6)}

Example 1 {(-3,1),(0,2),(2,4)}

Domain Range

{(-1,5),(1,3),(4,5)}

Domain Range



{(5,6),(-3,0),(1,1),(-3,6)}

Domain Range

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Domain Range

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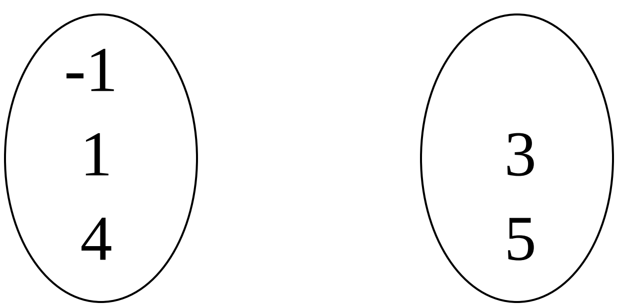

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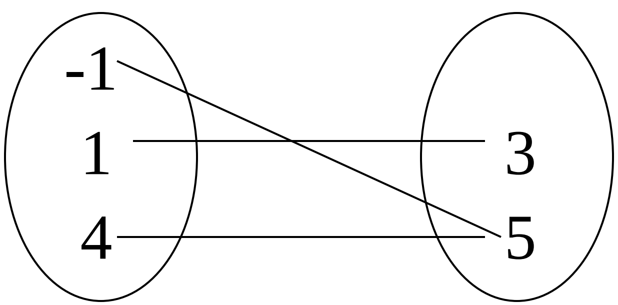

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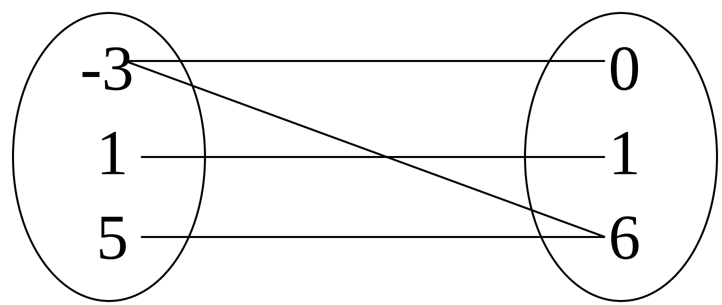
Domain Range



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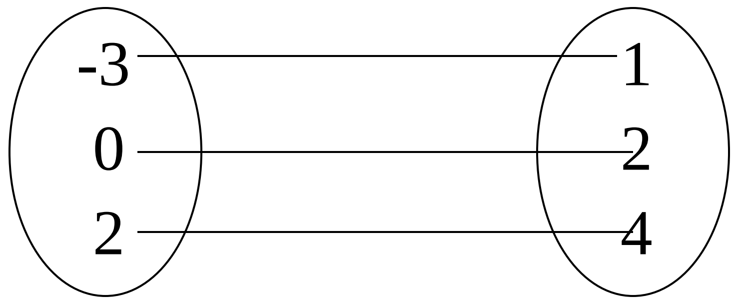
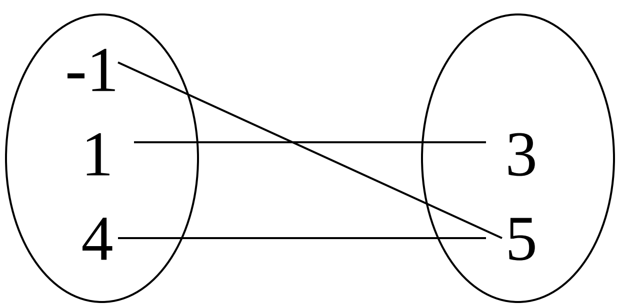


FUNCTION

1-1

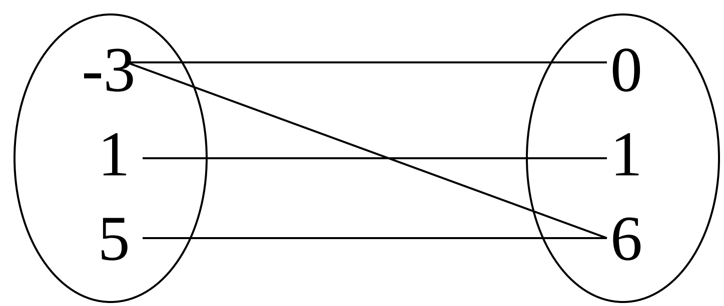
{(-1,5),(1,3),(4,5)}

Domain Range



{(5,6),(-3,0),(1,1),(-3,6)}

Domain Range



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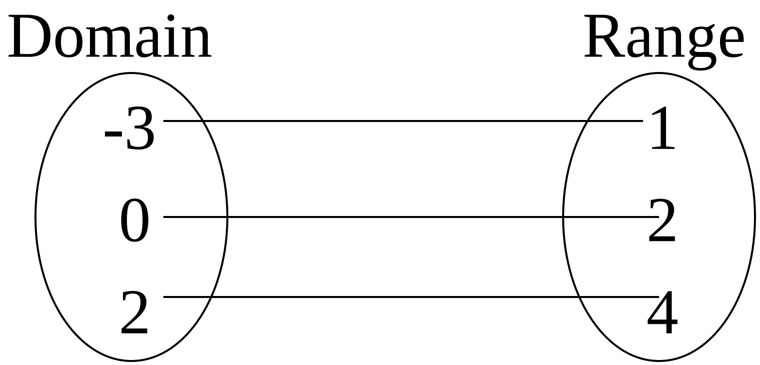

FUNCTION

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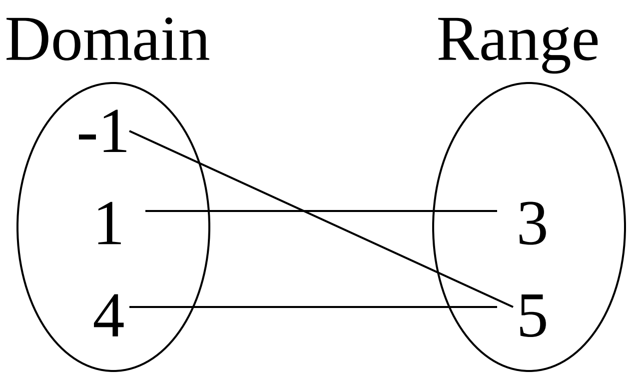
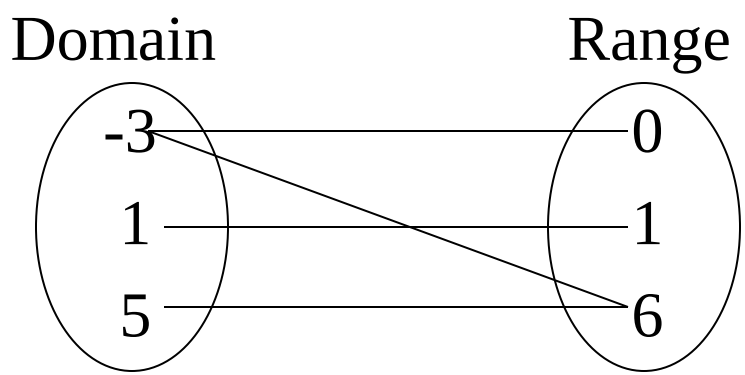
{(-1,5),(1,3),(4,5)}



FUNCTION

{(5,6),(-3,0),(1,1),(-3,6)}



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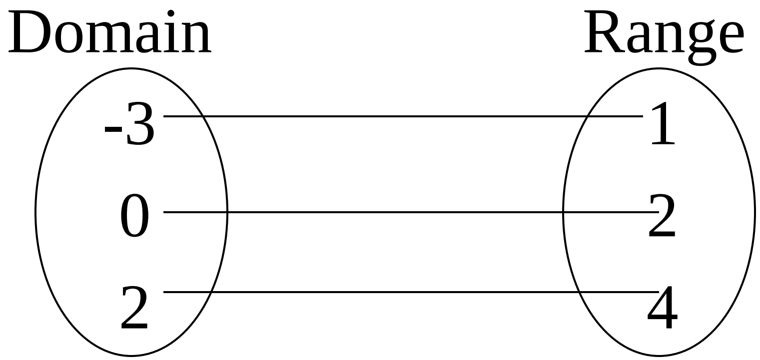

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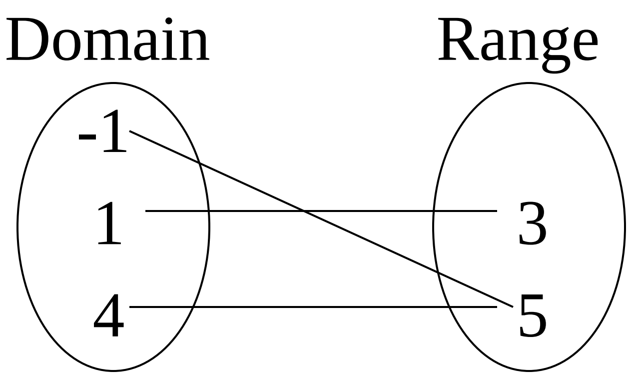

FUNCTION

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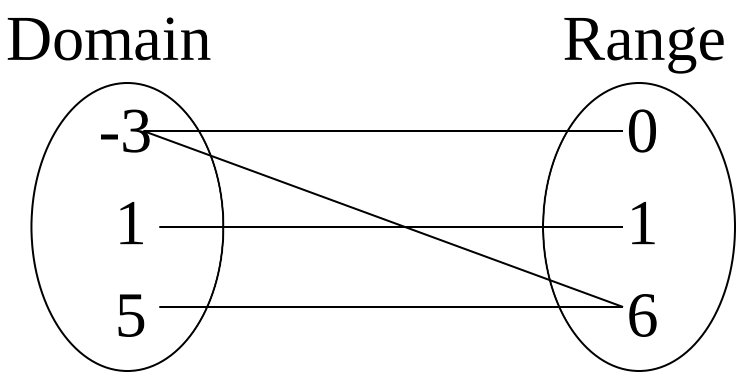

Not a Function

Example 3

Given $f(x) = 3x - 5$ and $g(x) = x^2 + 2$, find:

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(a) $f(-3)$

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(a) $f(-3)$

$$f(x) = 3x - 5$$

$$f(-3) =$$

Example 3

Given $f(x) = 3x - 5$ and $g(x) = x^2 + 2$, find:

(a) $f(-3)$

$$f(x) = 3x - 5$$

$$f(-3) = 3$$

Example 3

Given $f(x) = 3\text{x} - 5$ and $g(x) = x^2 + 2$, find:

(a) $f(-3)$

$$f(x) = \mathbf{3}\text{x} - 5$$

$$f(-3) = \mathbf{3}$$

Example 3

Given $f(x) = 3x - 5$ and $g(x) = x^2 + 2$, find:

(a) $f(-3)$

$$f(x) = 3x - 5$$

$$f(-3) = 3(-3)$$

Example 3

Given $f(x) = 3x - 5$ and $g(x) = x^2 + 2$, find:

(a) $f(-3)$

$$f(\mathbf{x}) = 3x - 5$$

$$f(\mathbf{-3}) = 3(-3)$$

Example 3

Given $f(x) = 3x - 5$ and $g(x) = x^2 + 2$, find:

(a) $f(-3)$

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$$f(\mathbf{-3}) = 3\mathbf{(-3)}$$

Example 3

Given $f(x) = 3x - 5$ and $g(x) = x^2 + 2$, find:

(a) $f(-3)$

$$f(x) = 3x - 5$$

$$f(-3) = 3(-3) - 5$$

Example 3

Given $f(x) = 3x – 5$ and $g(x) = x^2 + 2$, find:

(a) $f(-3)$

$f(x) = 3x – \mathbf{5}$

$f(-3) = 3(-3) – \mathbf{5}$

Example 3

Given $f(x) = 3x - 5$ and $g(x) = x^2 + 2$, find:

(a) $f(-3)$

$$f(x) = 3x - 5$$

$$f(-3) = 3(-3) - 5$$

$$= -9 - 5$$

Example 3

Given $f(x) = 3x – 5$ and $g(x) = x^2 + 2$, find:

(a) $f(-3)$

$$f(x) = 3x – 5$$

$$f(-3) = 3(-3) – 5$$

$$= -9 – 5 = -14$$

Example 3

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(b) $g(2z)$

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(b) $g(2z)$

$g(x) = x^2 + 2$

$g(2z)$

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(b) $g(2z)$

$$g(x) = x^2 + 2$$

$$g(2z) = (2z)$$

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(a) $f(-3)$

$$f(x) = 3x - 5$$

$$f(-3) = 3(-3) - 5$$

$$= -9 - 5 = -14$$

(b) $g(2z)$

$$g(\boldsymbol{x}) = x^2 + 2$$

$$g(\mathbf{2}\boldsymbol{z}) = (2z)$$

Example 3

Given $f(x) = 3x - 5$ and $g(x) = x^2 + 2$, find:

(a) $f(-3)$

$$f(x) = 3x - 5$$

$$f(-3) = 3(-3) - 5$$

$$= -9 - 5 = -14$$

(b) $g(2z)$

$$g(\boldsymbol{x}) = \boldsymbol{x}^2 + 2$$

$$g(\mathbf{2}\boldsymbol{z}) = \mathbf{(2}\boldsymbol{z}\mathbf{)}$$

Example 3

Given $f(x) = 3\text{x} - 5$ and $g(x) = x^2 + 2$, find:

(a) $f(-3)$

$$f(x) = 3\text{x} - 5$$

$$f(-3) = 3(-3) - 5$$

$$= -9 - 5 = -14$$

(b) $g(2z)$

$$g(x) = x^2 + 2$$

$$g(2z) = (2z)^2$$

Example 3

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(a) $f(-3)$

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$f(-3) = 3(-3) – 5$

$= -9 – 5 = -14$

(b) $g(2z)$

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$$f(x) = 3x - 5$$

$$f(-3) = 3(-3) - 5$$

$$= -9 - 5 = -14$$

(b) $g(2z)$

$$g(x) = x^2 + \mathbf{2}$$

$$g(2z) = (2z)^2 + \mathbf{2}$$

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Given $f(x) = 3x - 5$ and $g(x) = x^2 + 2$, find:

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(b) $g(2z)$

$g(x) = x^2 + 2$

$g(2z) = (2z)^2 + 2$

$= (2)^2$

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$$g(2z) = (2z)^2 + 2$$

$$= (2)^2(z)^2$$

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$$= -9 - 5 = -14$$

(b) $g(2z)$

$$g(x) = x^2 + 2$$

$$g(2z) = (2z)^2 + 2$$

$$= (2)^2(\text{z})^2 + 2$$

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$$f(-3) = 3(-3) - 5$$

$$= -9 - 5 = -14$$

(b) $g(2z)$

$$g(x) = x^2 + 2$$

$$g(2z) = (2z)^2 + 2$$

$$= (2)^2(z)^2 + 2$$

$$= 4z^2 + 2$$

TYPES OF FUNCTIONS

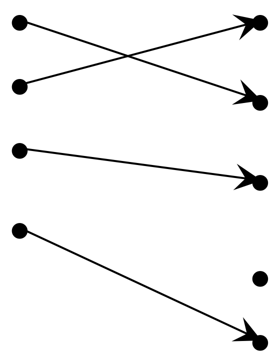
- One to One (Injective)
- Onto (Surjective)
- One to One Onto Function (Bijective)

One-to-One Functions

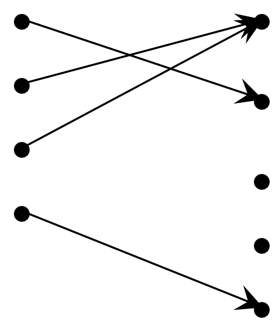
- Function f : X → Y is called one-to-one (injective) when for all elements x_1 and x_2 from X if $f(x_1) = f(x_2)$, then $x_1 = x_2$
- Iff every element of its range has **only one** pre-image.
- Only one element of the domain is mapped to any given one element of the range.
- A={1,2,3,4} and B={a,b,c,d,e} and f={(1,a),(2,e),(3,c),(4,d)}

One-to-One Illustration

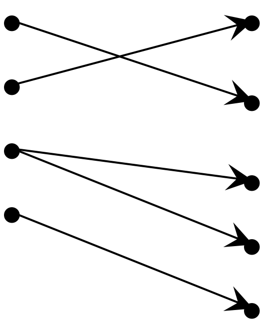
- Graph representations of functions that are (or not) one-to-one:



One-to-one



Not one-to-one



Not even a function!

- Let A = {1, 2, 3} and B = {a, b, c, d}. Which of the following is a one-to-one function?

Choices:

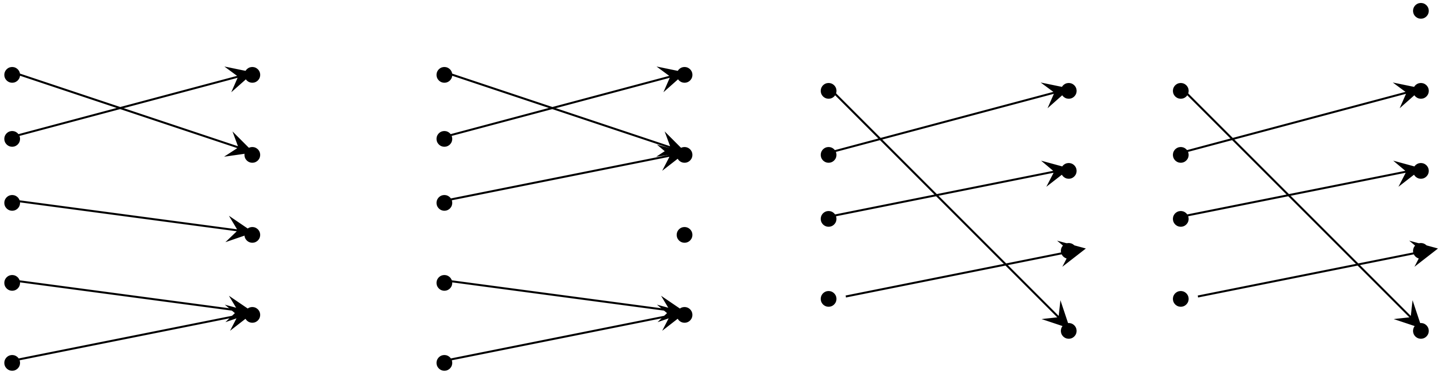
1. A. {1,a 2,c, 3,a}
2. B. {1,b 2,d 3,a}
3. C. {1,a, 2,a, 3,a}
4. D. {1,c 2,b 1,a, 3,d}
5. **Solution: B**

Onto Functions

- Function f : X → Y is called onto (surjective) when given any element y from Y, there exists x in X so that f(x) = y
- Determine whether the following functions is onto:
- A={1,2,3,4} and B={a,b,c,d} and f={(1,a),(2,a),(3,d),(4,c),(3,b)}

Illustration of Onto

- Some functions that are or are not *onto* their codomains:

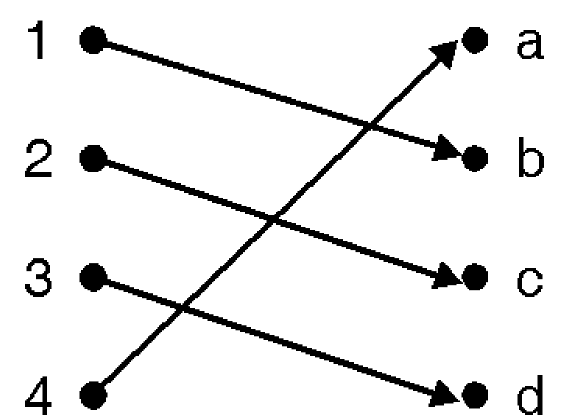


Onto (but not 1-1) | Not Onto (or 1-1) | Both 1-1 and onto | 1-1 but not onto

One to One Onto(Bijective) Functions

- Function from A to B is said to be one to one onto (bijective) if it is both one to one and onto function
- Determine whether the following functions is bijective:
- A={1,2,3,4} and B={a,b,c,d} and

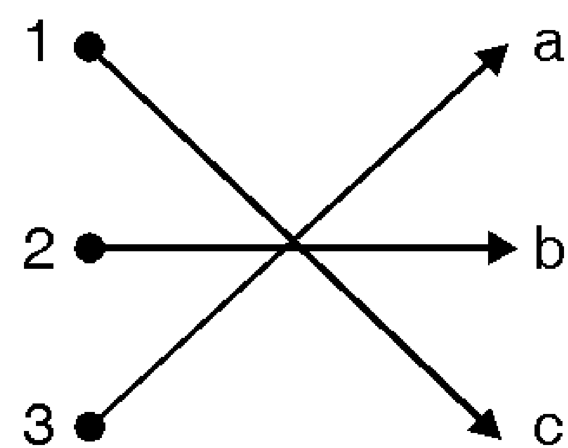
  f={(1,b),(2,c),(3,d),(4,a)}



Everywhere defined function

- A function f is said to be everywhere defined if Dom(f) = A
- Ex A= {1,2,3}, B={a,b,c}

f ={ (1,c), (2,b) , (3,a)}



Problems

Determine whether injective,surjective,bijective
A={a,b,c,d,e} and B= {x,y,z} .Determine whether R is a function,if yes , give its Range

1. R={(a,y),(b,z),(c,x),(d,z),(a,z),(b,x)}
2. R={(a,x),(b,y),(c,x)}
3. R={(a,x),(b,z),(c,y),(d,y),(e,z)}
4. R={(a,x),(b,x),(c,x),(d,x),(e,x)}
5. R={(a,z),(b,y),(c,z),(c,y),(c,x),(b,x)}

Composition of Functions

- Let f : X → Y and g : Y → Z, let range of f be a subset of the domain of g. The we can define a composition of g o f : X → Z
- Composition of two one-to-one functions is one-to-one
- Composition of two onto functions is onto

Problems

Q.1 : Let $A = \{ 1, 2, 3 \}$, $B = \{ a, b \}$ and $C = \{ 5, 6, 7 \}$.

Let $f : A \rightarrow B$ be defined by $f(1) = a$. $f(2) = a$. $f(3) = b$.

i.e. $f = \{(1, a), (2, a), (3, b)\}$.

Let $g : B \rightarrow C$ be defined by $g(a) = 5$, $g(b) = 7$

i.e. $g = \{(a, 5), (b, 7)\}$.

Find composition of f and g i.e. (g o f)

Soln. :

If $f(1) = a$ and $g(a) = 5$ then $g \circ f (1) = 5$

If $f(2) = a$ and $g(a) = 5$ then $g \circ f (2) = 5$

If $f(3) = b$ and $g(b) = 7$ then $g \circ f (3) = 7$

i.e. $(g \circ f)(1) = 5$

$(g \circ f)(2) = 5$

$(g \circ f)(3) = 7$

$g \circ f = \{(1, 5), (2, 5), (3, 7)\}$.

Solve

- Function f ,g ,h where each function maps to the set A={1,2,3,4} into itself
- f={(1,2),(2,1),(3,1),(4,4)}
- g={(1,2),(2,4),(3,1),(4,3)}
- h={(1,1),(2,3),(3,1),(4,3)}
- Find composition
  - Fog=11,24,32,41
  - h^2
  - 11,21,31,41
  - goh =12,21,32,41 f^3
  - G^2
  - 14,23,32,41
  - Fogoh

$F=\{(1,2),(2,3),(3,1)\}$

$G=\{(1,2),(2,1),(3,3)\}$

$H=\{(1,1),(2,2),(3,1)\}$

Find

FOG

GOF

FOGOH

FOHOG

G^3

$F(x)=2x+3$

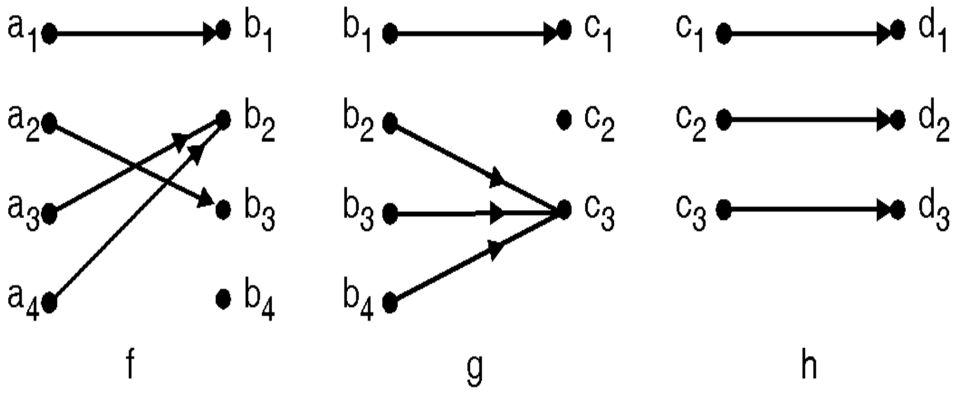
$G(x)=3x+4$

$H(x)=4x$

Find gof,fog,foh,hof,goh

problems

Q. 2 : $A = \{ a_1, a_2, a_3, a_4 \}$, $B = \{ b_1, b_2, b_3, b_4 \}$,
$C = \{c_1, c_2, c_3\}$, $D = \{ d_1, d_2, d_3 \}$.
(i) For the function f and g, determine g o f.
(ii) For the function f, g and h, determine
h o (g o f) and (h o g) o f



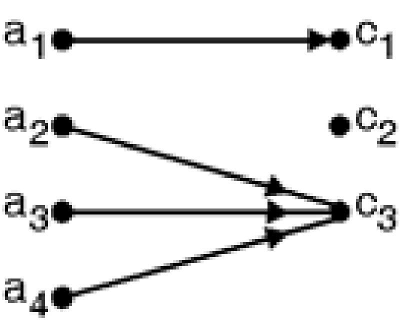
(i) $f : A \to B$ i.e. and $g : B \to C$ i.e.

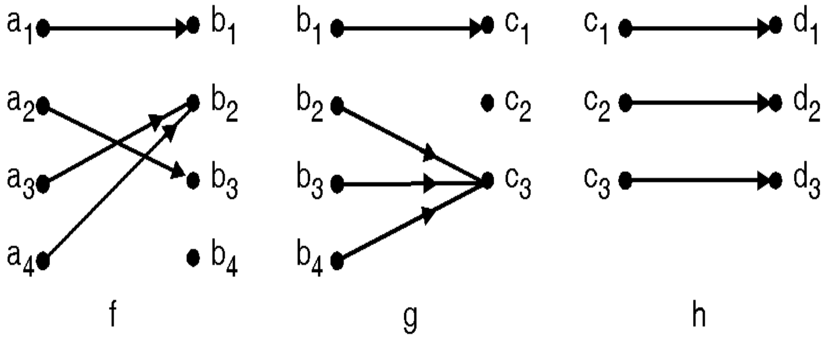
$$
\begin{aligned}
f(a_1) &= b_1, & \qquad g(b_1) &= c_1, \\
f(a_2) &= b_3, & \qquad g(b_2) &= c_3, \\
f(a_3) &= b_2, & \qquad g(b_3) &= c_3, \\
f(a_4) &= b_2. & \qquad g(b_4) &= c_3
\end{aligned}
$$

$$
\begin{aligned}
g \circ f &= g(f(a_1)) = g(b_1) = c_1 \\
&= g(f(a_2)) = g(b_3) = c_3 \\
&= g(f(a_3)) = g(b_2) = c_3 \\
&= g(f(a_4)) = g(b_2) = c_3
\end{aligned}
$$

$\therefore$

$$
\begin{aligned}
g \circ f(a_1) &= c_1 \\
g \circ f(a_2) &= c_3 \\
g \circ f(a_3) &= c_3 \\
g \circ f(a_4) &= c_3
\end{aligned}
$$





(ii) h o (g o f) $= h\,(g \text{ o } f\,(a_1))$
$= h\,(c_1)$
$= d_1.$
$= h\,(g \text{ o } f\,(a_2))$
$= h\,(c_3)$
$= d_3.$
$= h\,(g \text{ o } f\,(a_3))$
$= h\,(c_3)$
$= d_3.$
$= h\,(g \text{ o } f\,(a_4))$
$= h\,(c_3)$
$= d_3.$

∴

For (h o g) o f

h o g $= h\,(g\,(b_1))$
$=$ $\mathbf{h\,(c_1)=d_1}$
$= h\,(g\,(b_2))$
$=$ $\mathbf{h\,(c_3)=d_3}$
$= h\,(g\,(b_3))$
$=$ $\mathbf{h\,(c_3)=d_3}$
$= h\,(g\,(b_4))$
$=$ $\mathbf{h\,(c_3)=d_3}$

h o g $(b_1) = d_1$
h o g $(b_2) = d_3$
h o g $(b_3) = d_3$
h o g $(b_4) = d_3$

(h o g) o f $= (h \text{ o } g)\,(f\,(a_1))$
$=$ hog (b_1)
$= d_1$
$= (h \text{ o } g)\,(f\,(a_2))$
$=$ hog (b_3)
$= d_3$
$= (h \text{ o } g)\,(f\,(a_3))$
$=$ hog (b_2)
$= d_3$
$= (h \text{ o } g)\,(f\,(a_4))$
$=$ hog (b_2)
$= d_3$

Q. 3 : Consider the above function $f(x) = 2x - 3$. Find a formula for the composition functions (i) $f^2 = f \circ f$ and (ii) $f^3 = f \circ f \circ f$.

IDENTITY FUNCTION

- Let A be a non empty set, then we can define f: A → A(i.e B = A) as f(a)=a for all a belongs to A
- Eg A ={1,2,3} and f: A→A is an identity function since f(1)=1

  f(2)=2

  f(3)=3

Inverse Functions

- If $f : X \rightarrow Y$ is a bijective function, then it is possible to define an inverse function $f^{-1}: Y \rightarrow X$ so that $f^{-1}(y) = x$ whenever $f(x) = y$
- Find an inverse for the following functions:
  - $f(1)=2, f(2)=3, f(3)=1$
    - $f^{-1}(1) = \{3\}$, …

A FUNCTION f FOR WHICH f^{-1} EXISTS IS CALLED INVERTIBLE

Ex. : Let f be a function, from A = { 1, 2, 3, 4 } to B = { a, b, c, d }. Determine whether f^{-1} is a function.

(i) f = {(1, a), (2, a), (3, c), (4, d)}.

(ii) f = {(1, a), (2, c), (3, b), (4, d)}.

Soln. : $f(1) = \{a\}$ $\quad f(2) = \{a\}$.

$f(3) = \{c\}$ $\quad f(4) = \{d\}$.

$f^{-1}(a) = \{1, 2\}$ $\quad f^{-1}(c) = \{3\}$.

$f^{-1}(d) = \{4\}$.

f^{-1} is not a function, since $f^{-1}(a) = \{1, 2\}$. Hence f is not invertable.

(ii) $f(1) = \{a\}$ $\quad f(2) = \{c\}$.

$f(3) = \{b\}$ $\quad f(4) = \{d\}$.

$f^{-1}(a) = \{1\}$ $\quad f^{-1}(c) = \{2\}$.

$f^{-1}(b) = \{3\}$ $\quad f^{-1}(d) = \{4\}$.

f^{-1} is a function. Hence f is invertible.

Ex. : Let $f : Z \rightarrow Z$ where $f(x) = x^2 - 1$. Is f Invertible?

Soln.:

$$\begin{aligned} \text{We have } f(x) &= x^2 - 1 \,; \\ \text{For} \qquad x &= 1 \text{ and } -1 \\ f(1) &= 0 \text{ and } f(-1) = 0 \\ \therefore \qquad f^{-1}(0) &= \{1, -1\}. \end{aligned}$$

$f^{-1}(n)$ is not a single value function. Hence f is not invertible.

Ex. : Function $f(x) = (4x + 3) / (5x - 2)$. Find f^{-1}.

Soln.: To find f^{-1}

Set $y = f(x)$

and then interchange x and y as follow.

$$y = (4x + 3) / (5x - 2)$$

One interchanging x and y.

$$x = (4y + 3) / (5y - 2)$$

$$5xy - 2x = 4y + 3$$

$$5xy - 4y = 2x + 3$$

$$y\,(5x - 4) = (2x + 3)$$

$$y = \frac{(2x + 3)}{(5x - 4)}$$

$$f^{-1}(x) = \frac{(2x + 3)}{(5x - 4)}$$

- Steps

  Set y = f (x)

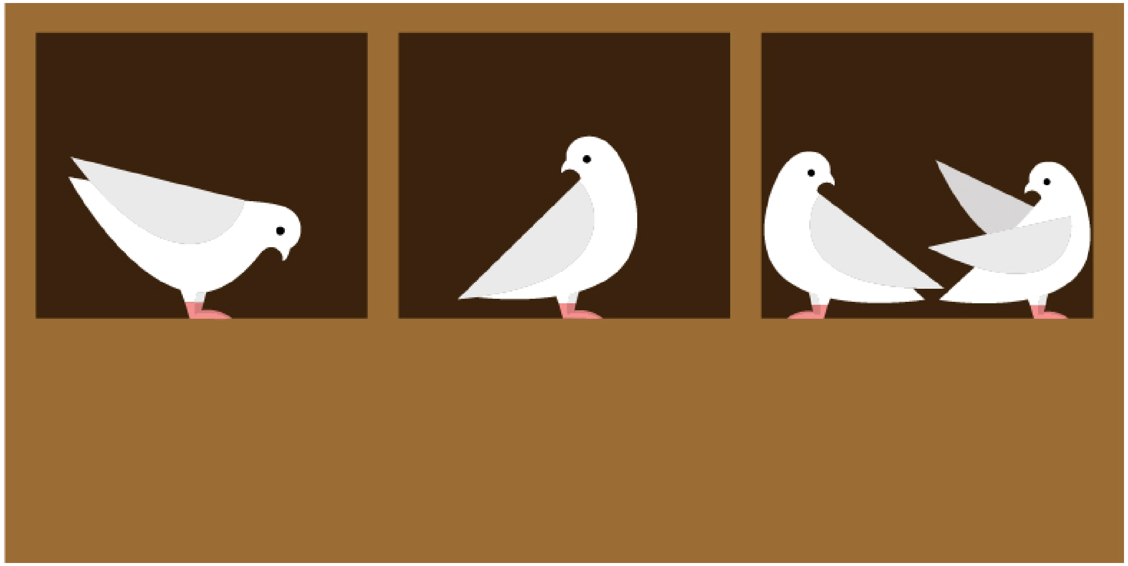
  Interchange x and y

  Solve for y which is $f^{-1}(x)$

1) $f(x) = 1 / (x - 2)$
2) $f (x) = (x + 1) / x$
3) $f(x)= x^3 + 2$
4) $f(x)= 5x – 7$
5) $f(x)= 8 / (9 – 3x)$
6) $f(x)= (4x + 3) / (5x – 2)$
7) $f(x)=(7 + 4x) / (6 – 5x)$

Pigeonhole Principle

If n pigeons are assigned to m pigeonholes, and m < n, then at least one pigeonhole contains two or more pigeons.



Pigeonhole Principle

- **If n pigeons fly into m pigeonholes and n > m, then at least one hole must contain two or more pigeons**
- In a group of 8 people chosen anyway there must be at least two who will be born on the same day of the week?
  - 7 days in a week and 8 people
- Show that if any 5 numbers are chosen from 1 to 8, two of them will add up to 9

A1 ={1,8}, A2={2,7},A3={3,6},A4={4,5}

Extended Pigeonhole Principle:

- It states that if n pigeons are assigned to m pigeonholes (The number of pigeons is very large than the number of pigeonholes), then one of the pigeonholes must contain at least **[(n-1)/m]+1** pigeons.

Example 1

Show that if 30 dictionaries in a library contain a total of 61,327 pages, then one of the dictionaries must have atleast have 2045 pages

Solution

- Let pages be pigeons and dictionaries the pigeonholes
- Using extended hole pigeonhole principle
  - **[(n-1)/m]+1**
  - **[(61327-1)/30]+1**
  - **2045**

Example 2

- Six friends discover that they have a total of 2161 Rs with them. Show that one or more of them must be have at least 361 Rs

- **[(n-1)/m]+1**
- **[(2160-1)/6]+1**
- **361**

1) **Golf:** Let us suppose that there are 8 balls and 7 holes. If balls are to be put in different holes, then at least one hole must have more than one ball.

2) **Handshake:** If a number of people does handshake with one another, then according to pigeonhole principle, there must exist two people who shake hanks with same people.

3) **Birthday:** Let us consider that n people are chosen at random from a group of people. Then, in order to find the probability of having same birthday, pigeonhole principle is applied. It says that at least two people will have same birthday.

4) **Marble picking:** Consider that we have a mixture of different color marbles in a jar. In order to find at least how many marbles will be picked before two same color marbles are guaranteed. It can be calculated using pigeonhole principle assuming one pigeonhole per color will be assumed.