FUZZY LOGIC AND FUZZY SYSTEM

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Syllabus

- **Fuzzy logic and Fuzzy system 12 CO4**
- **5.1** Introduction to Fuzzy logic, Fuzzy sets and membership functions, Operations on Fuzzy sets, Fuzzy relations, rules, propositions, implications and inferences, Defuzzification techniques,
- **5.2** Fuzzy logic controller design, Neuro Fuzzy system, Some applications of Fuzzy logic.
- #Self-Learning: Application of Fuzzy system in various appliances.

What is fuzzy?

- The word "fuzzy" means "vagueness". Fuzziness occurs when the boundary of a piece of information is not clear-cut.
- Fuzzy sets have been introduced by Lotfi A. Zadeh (1965) as an extension of the classical notion of set.
- Classical set theory allows the membership of the elements in the set in binary terms, a bivalent condition - an element either belongs or does not belong to the set.

  Fuzzy set theory permits the gradual assessment of the membership of elements in a set, described with the aid of a membership function valued in the real unit interval [0, 1].

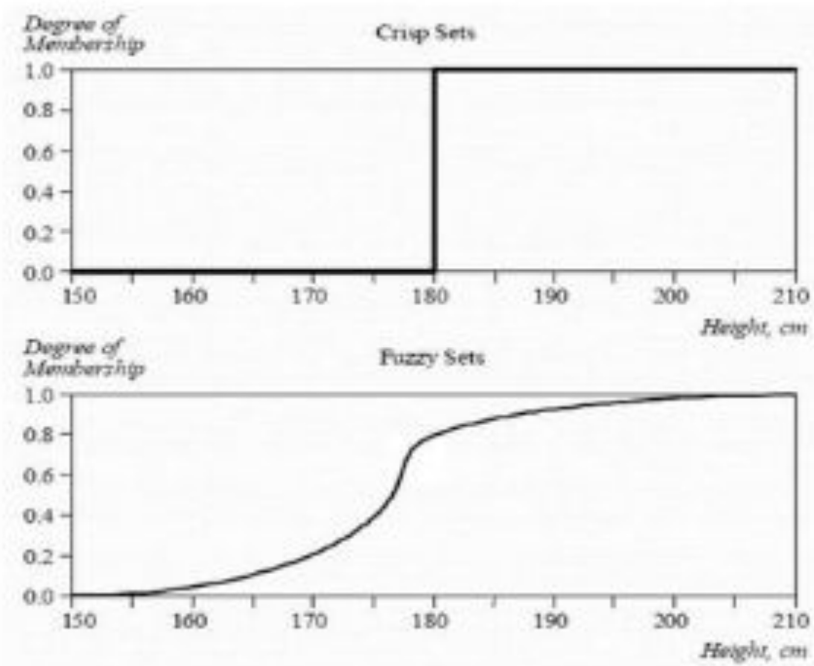
Example

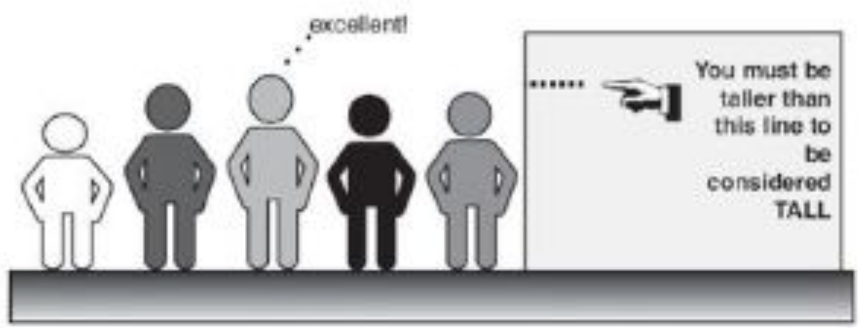
Words like young, tall, good, or high are fuzzy.

- There is no single quantitative value which defines the term young.
- For some people, age 25 is young, and for others, age 35 is young.
- The concept young has no clean boundary.
- Age 1 is definitely young and age 100 is definitely not young;
- Age 35 has some possibility of being young and usually depends on the context in which it is being considered.

Crisp set and Fuzzy set-Tall men

Name	Height, cm	Degree of Membership	
		Crisp	*Fuzzy*
Chris	208	1	1.00
Mark	205	1	1.00
John	198	1	0.98
Tom	181	1	0.82
David	179	0	0.78
Mike	172	0	0.24
Bob	167	0	0.15
Steven	158	0	0.06
Bill	155	0	0.01
Peter	152	0	0.00





Crisp set and Fuzzy set

- **Crisp set representation**

  Characteristic function

$$f_A(x): X \rightarrow 0,1$$

$$f_A(x) = \begin{cases} 1, \text{if } x \in A \\ 0, \text{if } x \notin A \end{cases}$$

- **Fuzzy set representation**

  Membership function

$$\mu_A(x): X \rightarrow [0,1]$$

$\mu_A(x) = 1$ if x is totally in A;

$\mu_A(x) = 0$ if x is not in A;

$0 < \mu_A(x) < 1$ if x is partly in A.

Fuzzy vs Probability

- A bottle of water
- 50% probability of being poisonous means 50% chance.
  - 50% water is clean.
  - 50% water is poisonous.
- 50% fuzzy membership of poisonous means that the water has poison.
  - Water is half poisonous.

Definition of fuzzy set

A **fuzzy set** **A**, defined in the universal space **X**, is a function defined in **X** which assumes values in the range **[0, 1]**.

A fuzzy set **A** is written as a set of pairs **{x, A(x)}** as

A = {{x , A(x)}} , x in the set X

where **x** is an element of the universal space **X**, and

A(x) is the value of the function **A** for this element.

The value **A(x)** is the **membership grade** of the element **x** in a fuzzy set **A**.

Definition of fuzzy set

Example : Set **SMALL** in set **X** consisting of natural numbers $\leq$ **to 12**.

Assume: **SMALL(1) = 1, SMALL(2) = 1, SMALL(3) = 0.9, SMALL(4) = 0.6,**
SMALL(5) = 0.4, SMALL(6) = 0.3, SMALL(7) = 0.2, SMALL(8) = 0.1,
SMALL(u) = 0 for u >= 9.

Then, following the notations described in the definition above :

Set SMALL = {{1, 1 }, {2, 1 }, {3, 0.9}, {4, 0.6}, {5, 0.4}, {6, 0.3}, {7, 0.2}, {8, 0.1}, {9, 0 }, {10, 0 }, {11, 0}, {12, 0}}

Graphical representation of fuzzy set -small

The fuzzy set SMALL of small numbers, defined in the universal space

$X = \{ x_i \} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$ is presented as

SetOption [FuzzySet, UniversalSpace → {1, 12, 1}]

Therefore **SetSmall** is represented as

SetSmall = FuzzySet [{{1,1},{2,1}, {3,0.9}, {4,0.6}, {5,0.4},{6,0.3}, {7,0.2}, {8, 0.1}, {9, 0}, {10, 0}, {11, 0}, {12, 0}} , **UniversalSpace** → {1, 12, 1}]

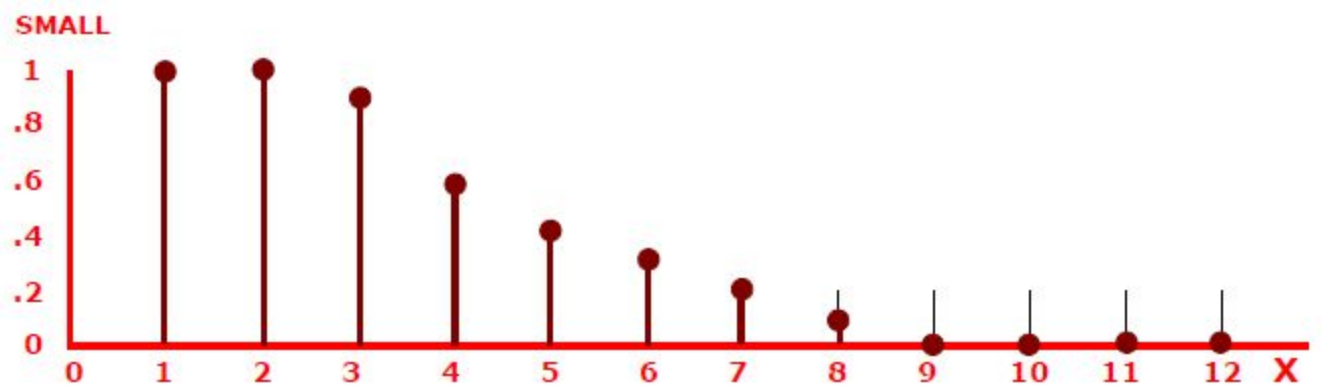


Fig Graphic Interpretation of Fuzzy Sets SMALL

Operations on fuzzy set-Union

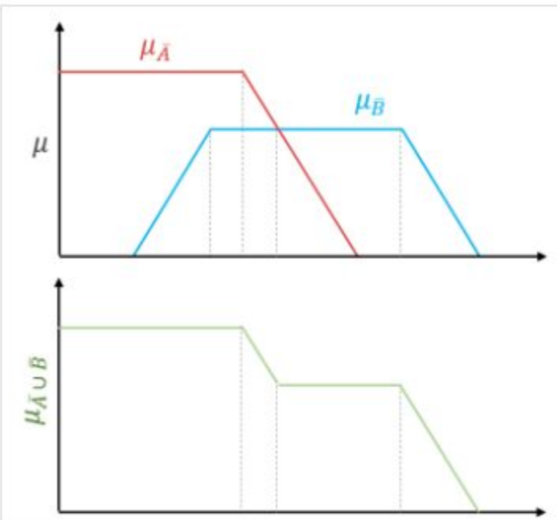
- In the case of the <u>union of crisp sets</u>, we simply have to select repeated elements only once. In the case of fuzzy sets, when there are common elements in both fuzzy sets, we should select the element with the **maximum membership value**.

The **union** of two fuzzy sets $\underline{A}$ and $\underline{B}$ is a fuzzy set $\underline{C}$, written as $\underline{C} = \underline{A} \cup \underline{B}$

$\underline{C} = \underline{A} \cup \underline{B} = \{(x, \mu_{\underline{A} \cup \underline{B}}(x)) \mid \forall x \in X\}$

$\mu_{\underline{A} \cup \underline{B}}(x) = \mu_{\underline{C}}(x) = \max(\mu_{\underline{A}}(x), \mu_{\underline{B}}(x)), \forall x \in X$

Example of Fuzzy Union:



Fuzzy union

$\underline{C} = \underline{A} \cup \underline{B} = \{(x, \mu_{\underline{A} \cup \underline{B}}(x)) \mid \forall x \in X\}$

$\underline{A} = \{ (x_1, 0.2), (x_2, 0.5), (x_3, 0.6), (x_4, 0.8), (x_5, 1.0) \}$

$\underline{B} = \{ (x_1, 0.8), (x_2, 0.6), (x_3, 0.4), (x_4, 0.2), (x_5, 0.1) \}$

$\mu_{\underline{A} \cup \underline{B}}(x_1) = \max(\mu_{\underline{A}}(x_1), \mu_{\underline{B}}(x_1)) = \max \{ 0.2, 0.8 \} = 0.8$

$\mu_{\underline{A} \cup \underline{B}}(x_2) = \max(\mu_{\underline{A}}(x_2), \mu_{\underline{B}}(x_2)) = \max \{ 0.5, 0.6 \} = 0.6$

$\mu_{\underline{A} \cup \underline{B}}(x_3) = \max(\mu_{\underline{A}}(x_3), \mu_{\underline{B}}(x_3)) = \max \{ 0.6, 0.4 \} = 0.6$

$\mu_{\underline{A} \cup \underline{B}}(x_4) = \max(\mu_{\underline{A}}(x_4), \mu_{\underline{B}}(x_4)) = \max \{ 0.8, 0.2 \} = 0.8$

$\mu_{\underline{A} \cup \underline{B}}(x_5) = \max(\mu_{\underline{A}}(x_5), \mu_{\underline{B}}(x_5)) = \max \{ 1.0, 0.1 \} = 1.0$

So, $\underline{A} \cup \underline{B} = \{ (x_1, 0.8), (x_2, 0.6), (x_3, 0.6), (x_4, 0.8), (x_5, 1.0) \}$

Operations on fuzzy set-Intersection

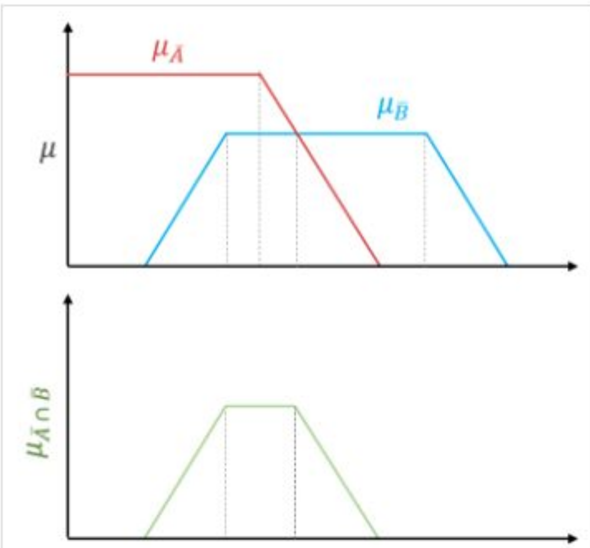
- In the case of the intersection of crisp sets, we simply have to select common elements from both sets. In the case of fuzzy sets, when there are common elements in both fuzzy sets, we should select the element with **minimum membership value**.

The **intersection** of two fuzzy sets $\underline{A}$ and $\underline{B}$ is a fuzzy set $\underline{C}$, written as $\underline{C} = \underline{A} \cap \underline{B}$

$\underline{C} = \underline{A} \cap \underline{B} = \{(x, \mu_{\underline{A} \cap \underline{B}}(x)) \mid \forall x \in X\}$

$\mu_{\underline{C}}(x) = \mu_{\underline{A} \cap \underline{B}}(x) = \min(\mu_{\underline{A}}(x), \mu_{\underline{B}}(x)), \forall x \in X$

Example of Fuzzy Intersection



Fuzzy intersection

$\underline{C} = \underline{A} \cap \underline{B} = \{(x, \mu_{\underline{A} \cap \underline{B}}(x)) \mid \forall x \in X\}$

$\underline{A} = \{ (x_1, 0.2), (x_2, 0.5), (x_3, 0.6), (x_4, 0.8), (x_5, 1.0) \}$

$\underline{B} = \{ (x_1, 0.8), (x_2, 0.6), (x_3, 0.4), (x_4, 0.2), (x_5, 0.1) \}$

$\mu_{\underline{A} \cap \underline{B}}(x_1) = \min(\mu_{\underline{A}}(x_1), \mu_{\underline{B}}(x_1)) = \min \{ 0.2, 0.8 \} = 0.2$

$\mu_{\underline{A} \cap \underline{B}}(x_2) = \min(\mu_{\underline{A}}(x_2), \mu_{\underline{B}}(x_2)) = \min \{ 0.5, 0.6 \} = 0.5$

$\mu_{\underline{A} \cap \underline{B}}(x_3) = \min(\mu_{\underline{A}}(x_3), \mu_{\underline{B}}(x_3)) = \min \{ 0.6, 0.4 \} = 0.4$

$\mu_{\underline{A} \cap \underline{B}}(x_4) = \min(\mu_{\underline{A}}(x_4), \mu_{\underline{B}}(x_4)) = \min \{ 0.8, 0.2 \} = 0.2$

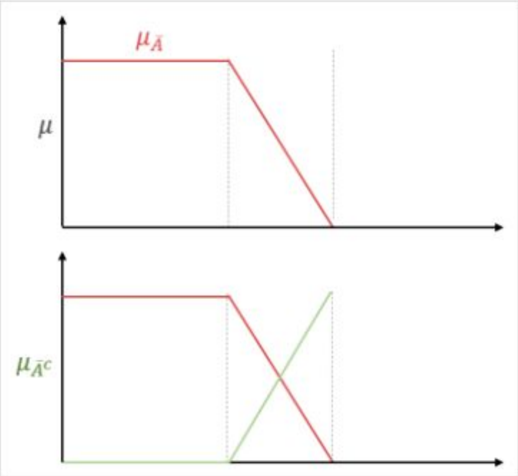
$\mu_{\underline{A} \cap \underline{B}}(x_5) = \min(\mu_{\underline{A}}(x_5), \mu_{\underline{B}}(x_5)) = \min \{ 1.0, 0.1 \} = 0.1$

So, $\underline{A} \cap \underline{B} = \{ (x_1, 0.2), (x_2, 0.5), (x_3, 0.4), (x_4, 0.2), (x_5, 0.1) \}$

Operations on fuzzy set- Complement

- Fuzzy complement is identical to crisp complement operation. The membership value of every element in the fuzzy set is complemented with respect to 1, i.e. it is subtracted from 1.
- The **complement** of fuzzy set $\underline{A}$, denoted by $\underline{A}^C$, is defined as
- $\underline{A}^C = \{(x, \mu_{\underline{A}C}(x)) \mid \forall x \in X\}$
- $\underline{A}^C(x) = 1 - \mu_{\underline{A}}(x)$

Example of Fuzzy Complement



Fuzzy complement

$\underline{A}^C(x) = 1 - \mu_{\underline{A}}(x)$

$\underline{A} = \{ (x_1, 0.2), (x_2, 0.5), (x_3, 0.6), (x_4, 0.8), (x_5, 1.0) \}$

$\underline{A}^C = \{ (x_1, 0.8), (x_2, 0.5), (x_3, 0.4), (x_4, 0.2), (x_5, 0.0) \}$

$\underline{A} \cup \underline{A}^C = \{ (x_1, 0.8), (x_2, 0.5), (x_3, 0.6), (x_4, 0.8), (x_5, 1.0) \} \neq X$

$\underline{A} \cap \underline{A}^C = \{ (x_1, 0.2), (x_2, 0.5), (x_3, 0.4), (x_4, 0.2), (x_5, 0.0) \} \neq \Phi$

Cartesian product

- Let $\underline{A}$ be a fuzzy set on universe X and $\underline{B}$ be a fuzzy set on universe Y, then the Cartesian product between fuzzy sets $\underline{A}$ and $\underline{B}$ will result in a fuzzy relation $\underline{R}$ which is contained with the full Cartesian product space or it is a subset of the cartesian product of fuzzy subsets. Formally, we can define fuzzy relation as,

$\underline{R} = \underline{A} \times \underline{B}$

and

$\underline{R} \subset (X \times Y)$

where the relation $\underline{R}$ has a membership function,

$\mu_{\underline{R}}(x, y) = \mu_{\underline{A} \times \underline{B}}(x, y) = \min(\mu_{\underline{A}}(x), \mu_{\underline{B}}(y))$

Cartesian product

- Let $\underline{A} = \{a_1, a_2, \ldots, a_n\}$ and $\underline{B} = \{b_1, b_2, .., b_m\}$, then the fuzzy relation between $\underline{A}$ and $\underline{B}$ is described by the **fuzzy relation matrix** as,

$$\begin{bmatrix} \mu_{R(a_1,b_1)} & \mu_{R(a_1,b_2)} & \cdot\ \cdot & \mu_{R(a_1,b_m)} \\ \mu_{R(a_2,b_1)} & \mu_{R(a_2,b_2)} & \cdot\ \cdot & \mu_{R(a_2,b_m)} \\ \cdot & \cdot & \cdot\ \cdot & \cdot \\ \cdot & \cdot & \cdot\ \cdot & \cdot \\ \mu_{R(a_n,b_1)} & \mu_{R(a_n,b_2)} & \cdot\ \cdot & \mu_{R(a_n,b_m)} \end{bmatrix}$$

Fuzzy relation matrix

Example:

Given $\underline{A} = \{ (a_1, 0.2), (a_2, 0.7), (a_3, 0.4) \}$ and $\underline{B} = \{ (b_1, 0.5), (b_2, 0.6)\}$, find the relation over $\underline{A} \times \underline{B}$

Cartesian product

$$\bar{R} = \bar{A} \times \bar{B} = \begin{array}{c} \\ a_1 \\ a_2 \\ a_3 \end{array} \begin{array}{c} \begin{array}{cc} b_1 & b_2 \end{array} \\ \begin{bmatrix} 0.2 & 0.2 \\ 0.5 & 0.6 \\ 0.4 & 0.4 \end{bmatrix} \end{array}$$

Cartesian product

Fuzzy Relation

Fuzzy relations are very important because they can describe interactions between variables.

Example: A simple example of a binary fuzzy relation on X = {1, 2, 3}, called ”approximately equal” can be defined as

$\underline{R}(1, 1) = \underline{R}(2, 2) = \underline{R}(3, 3) = 1$

$\underline{R}(1, 2) = \underline{R}(2, 1) = \underline{R}(2, 3) = \underline{R}(3, 2) = 0.7$

$\underline{R}(1, 3) = \underline{R}(3, 1) = 0.3$

The membership function and relation matrix of $\underline{R}$ are given by

$$\bar{R}(x, y) = \begin{cases} 1, & if\, x = y \\ 0.7, & if\, |x - y| = 1 \\ 0.3, & if\, |x - y| = 2 \end{cases}$$

$$\bar{R} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 1.0 & 0.7 & 0.3 \\ 0.7 & 1.0 & 0.7 \\ 0.3 & 0.7 & 1.0 \end{bmatrix} \end{array}$$

Operations on fuzzy relation:

$$\bar{R} = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{c} \begin{array}{cccc} y_1 & y_2 & y_3 & y_4 \end{array} \\ \begin{bmatrix} 0.8 & 0.1 & 0.1 & 0.7 \\ 0.0 & 0.8 & 0.0 & 0.0 \\ 0.9 & 1.0 & 0.7 & 0.8 \end{bmatrix} \end{array}$$

$$\bar{S} = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{c} \begin{array}{cccc} y_1 & y_2 & y_3 & y_4 \end{array} \\ \begin{bmatrix} 0.4 & 0.0 & 0.9 & 0.6 \\ 0.9 & 0.4 & 0.5 & 0.7 \\ 0.3 & 0.0 & 0.8 & 0.5 \end{bmatrix} \end{array}$$

$$\bar{R} \cup \bar{S} = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{c} \begin{array}{cccc} y_1 & y_2 & y_3 & y_4 \end{array} \\ \begin{bmatrix} 0.8 & 0.1 & 0.9 & 0.7 \\ 0.9 & 0.8 & 0.5 & 0.7 \\ 0.9 & 1.0 & 0.8 & 0.8 \end{bmatrix} \end{array}$$

Union of fuzzy relations

$$\bar{R} \cap \bar{S} = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{c} \begin{array}{cccc} y_1 & y_2 & y_3 & y_4 \end{array} \\ \begin{bmatrix} 0.4 & 0.0 & 0.1 & 0.6 \\ 0.0 & 0.4 & 0.0 & 0.0 \\ 0.3 & 0.0 & 0.7 & 0.5 \end{bmatrix} \end{array}$$

Intersection of relation

$$\bar{R}^c = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{c} \begin{array}{cccc} y_1 & y_2 & y_3 & y_4 \end{array} \\ \begin{bmatrix} 0.2 & 0.9 & 0.9 & 0.3 \\ 1.0 & 0.2 & 1.0 & 1.0 \\ 0.1 & 0.0 & 0.3 & 0.2 \end{bmatrix} \end{array}$$

Complement of fuzzy relation

Fuzzy Proposition

- The proposition value for classical proposition is either true or false but in case of fuzzy proposition the range is not confined to only two values it varies from 2 to n.
- For example speed may be fast, very fast, medium, slow, and very slow.
- In fuzzy logic the truth value of fuzzy proposition also depends on an additional factor known as degree of truth whose value is varies between 0 and 1.

p: Speed is Slow

$T(p) = 0.8$, if p is partly true

$T(p) = 1$, if p is absolutely true

$T(p) = 0$, if p is totally false

- So, we can say that fuzzy proposition is a statement p which acquires a fuzzy truth value T(p) ranges from(0 to1).

Fuzzy Proposition

P : Ram is honest

1. T(P) = 0.0 : Absolutely false
2. T(P) = 0.2 : Partially false
3. T(P) = 0.4 : May be false or not false
4. T(P) = 0.6 : May be true or not true
5. T(P) = 0.8 : Partially true
6. T(P) = 1.0 : Absolutely true.

Fuzzy Connectives

The fuzzy logic is similar to crisp logic supported by connectives.

Table below illustrates the definitions of fuzzy connectives.

Table : Fuzzy Connectves

Connective	Symbols	Usage	Definition
Nagation	$\neg$	$\neg$ P	1 – T(P)
Disjuction	$\vee$	P $\vee$ Q	Max[T(P) , T(Q)]
Conjuction	$\wedge$	P $\wedge$ Q	min[T(P) , T(Q)]
Implication	$\Rightarrow$	P $\Rightarrow$ Q	$\neg$P $\vee$ Q = max (1-T(P), T(Q)]

Here P , Q are fuzzy proposition and T(P) , T(Q) are their truth values.

- the P and Q are related by the $\Rightarrow$ operator are known as antecedents and consequent respectively.
- as crisp logic, here in fuzzy logic also the operator $\Rightarrow$ represents IF-THEN statement like,

  IF x is A THEN y is B, is equivalent to

  R = (A x B) U ($\neg$ A x Y)

Fuzzy Connectives

the membership function of **R** is given by

$$\mu_R(x, y) = \max[\min(\mu_A(x), \mu_B(y)), 1 - \mu_A(x)]$$

– For the compound implication statement like

IF x is A THEN y is B, ELSE y is C is equivalent to

$$R = (A \times B) \cup (\neg A \times C)$$

the membership function of **R** is given by

$$\mu_R(x, y) = \max[\min(\mu_A(x), \mu_B(y)), \min(1 - \mu_A(x), \mu_C(y))]$$

Fuzzy Proposition

Example1:

P : Mary is efficient ; T(P) = 0.8;

Q : Ram is efficient ; T(Q) = 0.6

Mary is not efficient.

$T(\neg P) = 1 - T(P) = 0.2$

Mary is efficient and so is Ram.

$T(P \wedge Q) = min\{T(P), T(Q)\} = 0.6$

Either Mary or Ram is efficient

$T(P \vee Q) = max\ T(P), T(Q) = 0.8$

If Mary is efficient then so is Ram

$T(P \Longrightarrow Q) = max\ \{1 - T(P), T(Q)\} = 0.6$

Fuzzy Proposition

Example 2 : (Ref : Previous slide on fuzzy connective)

Let X = {a, b, c, d} ,

A = {(a, 0) (b, 0.8) (c, 0.6) (d, 1)}

B = {(1, 0.2) (2, 1) (3, 0.8) (4, 0)}

C = {(1, 0) (2, 0.4) (3, 1) (4, 0.8)}

Y = { 1, 2, 3, 4} the universe of discourse could be viewed as

{ (1, 1) (2, 1) (3, 1) (4, 1) }

i.e., a fuzzy set all of whose elements x have $\mu(x) = 1$

Determine the implication relations

(i) If x is A THEN y is B

(ii) If x is A THEN y is B Else y is C

Solution 1: IF x is A THEN y is B, is equivalent to R = (A x B) U (¬ A x Y) and the membership function R is given by

$$\mu_R(x, y) = \max[\min(\mu_A(x), \mu_B(y)), 1 - \mu_A(x)]$$

Fuzzy Intersection A x B is defined as :

for all x in the set X,

$(A \cap B)(x) = \min[A(x), B(x)]$,

Fuzzy Intersection ¬A x Y is defined as :

for all x in the set X

$(\neg A \cap Y)(x) = \min[A(x), Y(x)]$,

A x B =

A \ B	1	2	3	4
a	0	0	0	0
b	0.2	0.8	0.8	0
c	0.2	0.6	0.6	0
d	0.2	1	0.8	0

¬A x Y =

A \ y	1	2	3	4
a	1	1	1	1
b	0.2	0.2	0.2	0.2
c	0.4	0.4	0.4	0.4
d	0	0	0	0

Fuzzy Union is defined as $(A \cup B)(x) = \max[A(x), B(x)]$ for all $x \in X$

Therefore R = (A x B) U (¬ A x Y) gives

R =

x \ y	1	2	3	4
a	1	1	1	1
b	0.2	0.8	0.8	0
c	0.4	0.6	0.6	0.4
d	0.2	1	0.8	0

This represents If x is A THEN y is B ie $T(A \Rightarrow B) = \max(1 - T(A), T(B))$

Solution 2:

IF x is A THEN y is B Else y is C, is equivalent to

R = (A x B) U (¬ A x C) and

the membership function of R is given by

$$\mu_R(x, y) = \max[\min(\mu_A(x), \mu_B(y)), \min(1 - \mu_A(x), \mu_C(y)]$$

Fuzzy Intersection A x B is defined as :
for all x in the set X,
$(A \cap B)(x) = \min[A(x), B(x)]$,

A \ B	1	2	3	4
a	0	0	0	0
b	0.2	0.8	0.8	0
c	0.2	0.6	0.6	0
d	0.2	1	0.8	0

A x B =

Fuzzy Intersection ¬A x Y is defined as :
for all x in the set X
$(\neg A \cap C)(x) = \min[A(x), C(x)]$,

A \ y	1	2	3	4
a	0	0.4	1	0.8
b	0.2	0.2	0.2	0.2
c	0.4	0.4	0.4	0.4
d	0	0	0	0

¬A x C =

Fuzzy Union is defined as $(A \cup B)(x) = \max[A(x), B(x)]$ for all $x \in X$

Therefore R = (A x B) U (¬ A x C) **gives**

R =

x \ y	1	2	3	4
a	1	1	1	1
b	0.2	0.8	0.8	0
c	0.4	0.6	0.6	0.4
d	0.2	1	0.8	0

This represents **If x is A THEN y is B Else y is C**

Fuzzy Quantifiers

In crisp logic, the predicates are quantified by quantifiers.

Similarly, in fuzzy logic the propositions are quantified by quantifiers.

There are two classes of fuzzy quantifiers :

- Absolute quantifiers and
- Relative quantifiers

Examples :

Absolute quantifiers	**Relative quantifiers**
round about 250	almost
much greater than 6	about
some where around 20	most

Fuzzification

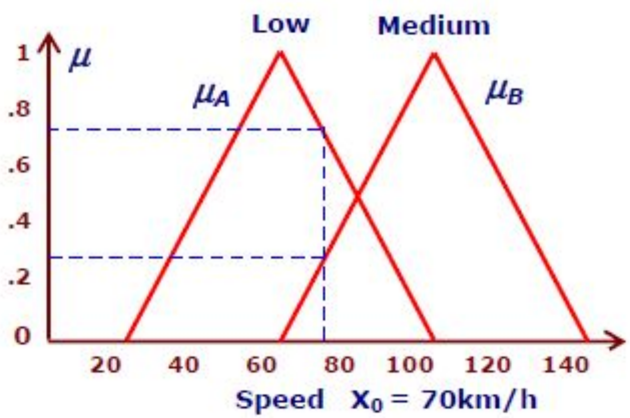
The fuzzification is a process of transforming crisp values into grades of membership for linguistic terms of fuzzy sets.

The purpose is to allow a fuzzy condition in a rule to be interpreted.

- **Fuzzification of the car speed**

  Example 1 : Speed X_0 = 70km/h

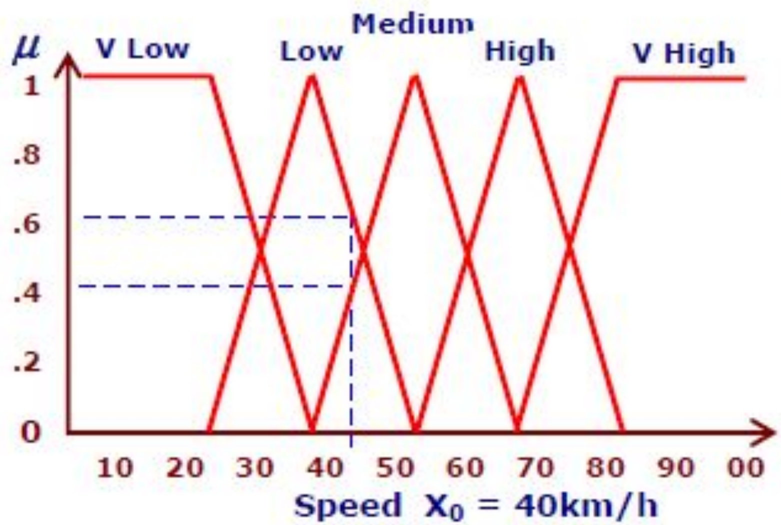
  Fig below shows the fuzzification of the car speed to characterize a low and a medium speed fuzzy set.



Given car speed value X_0=70km/h : grade $\mu_A(x_0)$ = **0.75** belongs to fuzzy low, and grade $\mu_B(x_0)$ = **0.25** belongs to fuzzy medium

Fuzzification

Example 2 : Speed X_0 = 40km/h



Characterizing five grades, Very low, low, medium, high and very high speed fuzzy set

Given car speed value **X_0=40km/h** :

grade **$\mu_A(x_0)$ = 0.6** belongs to fuzzy low, and grade **$\mu_B(x_0)$ = 0.4** belongs to fuzzy medium.

CONCLUSION

- Fuzzy logic provides an alternative way to represent linguistic and subjective attributes of the real world in computing.
- It is able to be applied to control systems and other applications in order to improve the efficiency and simplicity of the design process.