

Perceptron learning rule numericals

① Use perceptron learning rule to train the network. The set of input training vectors are as follows:

$$X_1 = \begin{bmatrix} 1 \\ -2 \\ 0 \\ -1 \end{bmatrix} \quad X_2 = \begin{bmatrix} 0 \\ 1.5 \\ -0.5 \\ -1 \end{bmatrix} \quad X_3 = \begin{bmatrix} -1 \\ 1 \\ 0.5 \\ -1 \end{bmatrix}$$

and the initial weight vector W_1 is,

$$W_1 = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0.5 \end{bmatrix}. \text{ The learning constant } C = 0.1.$$

The desired response for X_1 , X_2 and X_3 are $d_1 = -1$, $d_2 = -1$ and $d_3 = 1$ respectively. Calculate the weights after one complete cycle.

Soln:-

for perceptron learning rule,

$$net_i = w_i^t x$$

$$O_i = \text{Sign}(net_i)$$

$$\Delta w_i = C(d - O_i)x$$

Step 1: Taking the first training pair $x = x_1$ and $d = d_1$

$$x = \begin{bmatrix} 1 \\ -2 \\ 0 \\ 1 \end{bmatrix} \quad d = -1$$

$$net_1 = w_1^t x = \begin{bmatrix} 1 & -1 & 0 & 0.5 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \\ 0 \\ -1 \end{bmatrix} = 2.5.$$

$$\text{Sign}(net_1) = \text{Sign}(2.5) = 1$$

$$\Delta w_1 = c(d - o_1)x = 0.1(-1-1) \begin{bmatrix} 1 \\ -2 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -0.2 \\ 0.4 \\ 0 \\ 0.2 \end{bmatrix} \quad (2)$$

$$w_2 = w_1 + \Delta w_1 = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0.5 \end{bmatrix} + \begin{bmatrix} -0.2 \\ 0.4 \\ 0 \\ 0.2 \end{bmatrix} = \begin{bmatrix} 0.8 \\ -0.6 \\ 0 \\ 0.7 \end{bmatrix}$$

Similarly.

Step 2: Taking second training pair.
Set $x = x_2$ $d = d_2$.

$$x = \begin{bmatrix} 0 \\ 1.5 \\ -0.5 \\ -1 \end{bmatrix} \quad d = 1$$

$$net_2 = w_2^t x = \begin{bmatrix} 0.8 & -0.6 & 0 & 0.7 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 0.5 \\ -0.5 \\ -1 \end{bmatrix} = -1.6$$

$$o_2 = \text{Sign}(net_2) = \text{Sign}(-1.6) = -1$$

$$d = -1 \quad o_2 = -1$$

$$\text{So } (d - o_2) = 0$$

Hence $\Delta w_2 = 0$. Thus there is no change in weight.

$$\therefore w_3 = w_2 = \begin{bmatrix} 0.8 \\ -0.6 \\ 0 \\ 0.7 \end{bmatrix}$$

Step 3: Take the third training pair
Set $x = x_3$ $d = d_3$.

$$x = \begin{bmatrix} -1 \\ 0.5 \\ -1 \end{bmatrix} \quad d = 1$$

$$\text{net}_3 = w_3^t x = \begin{bmatrix} 0.8 & -0.6 & 0 & 0.7 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 0.5 \\ -1 \end{bmatrix} = -2.1 \quad (3)$$

$$o_3 = \text{Sign}(\text{net}_3) = \text{Sign}(-2.1) = -1$$

$$d = 1 \quad o_3 = -1$$

$$\therefore \Delta w_3 = e(d - o_3) = 0.1(1 - (-1)) \begin{bmatrix} 2 \\ 1 \\ 0.5 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} -0.2 \\ 0.2 \\ 0.1 \\ -0.2 \end{bmatrix}$$

$$w_4 = w_3 + \Delta w_3 = \begin{bmatrix} 0.8 \\ -0.6 \\ 0 \\ 0.7 \end{bmatrix} + \begin{bmatrix} -0.2 \\ 0.2 \\ 0.1 \\ -0.2 \end{bmatrix} = \begin{bmatrix} 0.6 \\ -0.4 \\ 0.1 \\ 0.5 \end{bmatrix}$$

(2) Perform two training steps of the neural network using delta learning rule $d=1$ $c=0.25$.
Train the network using following data pairs.

$$x_1 = \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix} \quad d_1 = -1 \quad x_2 = \begin{bmatrix} 1 \\ -2 \\ -1 \end{bmatrix} \quad d_2 = 1. \quad \text{The}$$

initial weights are $w_1 = [1 \ 0 \ 1]^t$.

Solu:- for delta learning rule.

$$\text{net}_i = w_i^t x$$

$$o_i = f(\text{net}_i) = \frac{2}{1 + e^{-\text{net}_i}} - 1$$

$$f'(\text{net}_i) = \frac{1}{2}(1 - o_i^2)$$

$$\Delta w_i = c(d - o_i) \left[\frac{1}{2}(1 - o_i^2) \right]$$

Step 1: 1st training pair $x=x_1$ $d=d_1$

(4)

$$x = \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix} \quad d = -1$$

$$net_1 = w_1^t x = [1 \ 0 \ 1] \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix} = 2 - 1 = 1.$$

$$o_1 = \frac{2}{1 + e^{-1}} - 1 = 0.463$$

$$f'(net_1) = \frac{1}{2} (1 - (0.463)^2) = 0.393$$

$$\Delta w_1 = 0.25 (-1 - 0.463) 0.393 \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix} \\ = -0.1437 \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} -0.287 \\ 0 \\ 0.1437 \end{bmatrix}$$

$$w_2 = w_1 + \Delta w_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} -0.287 \\ 0 \\ 0.1437 \end{bmatrix} = \begin{bmatrix} 0.713 \\ 0 \\ 1.1437 \end{bmatrix}$$

Step 2: 2nd training pair $x=x_2$ $d=d_2$

$$x = \begin{bmatrix} 1 \\ -2 \\ -1 \end{bmatrix} \quad d = 1$$

$$net_2 = w_2^t x = [0.713 \ 0 \ 1.1437] \begin{bmatrix} 1 \\ -2 \\ -1 \end{bmatrix} \\ = -0.4307$$

$$o_2 = f(net_2) = \frac{2}{1 + e^{-0.4307}} - 1 = -0.2119$$

$$f'(net_2) = \frac{1}{2} (1 - (-0.2119)^2) \\ = \frac{1}{2} (1 - 0.0449) = 0.477$$

$$\Delta W_2 = (0.25) (1 - (-0.2119)) \times (0.477) \times 2 \quad (5)$$

$$= (0.145) \begin{bmatrix} 1 \\ -2 \\ -1 \end{bmatrix} = \begin{bmatrix} 0.145 \\ -0.29 \\ -0.145 \end{bmatrix}$$

$$W_3 = W_2 + \Delta W_2 = \begin{bmatrix} 0.713 \\ 0 \\ 1.143 \end{bmatrix} + \begin{bmatrix} 0.145 \\ -0.29 \\ -0.145 \end{bmatrix}$$

$$= \begin{bmatrix} 0.858 \\ -0.29 \\ 0.998 \end{bmatrix}$$

3) Perform two training steps of windrows - Hoff learning rule

$$X_1 = \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix} \quad d_1 = -1 \quad X_2 = \begin{bmatrix} 1 \\ -2 \\ -1 \end{bmatrix} \quad d_2 = 1$$

$$W_1 = [1 \ 0 \ 1]^T \quad \lambda = 1 \quad e = 0.25$$

Soln:- windrows Hoff learning

$$O_i = W_i^T X \quad \Delta W_i = c(d_i - O_i) X$$

$$O_1 = W_1^T X_1 = [1 \ 0 \ 1] \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix} = 1$$

$$\Delta W_1 = c(d_1 - O_1) X_1 = 0.25(-1 - 1) \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 \\ 0 \\ 0.5 \end{bmatrix}$$

$$W_2 = W_1 + \Delta W_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} -1 \\ 0 \\ 0.5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1.5 \end{bmatrix}$$

$$o_2 = w_2^t x_2 = [0 \ 0 \ 1.5] \begin{bmatrix} 1 \\ -2 \\ -1 \end{bmatrix} = -1.5 \quad (6)$$

$$\Delta w_2 = c (d_2 - o_2) x_2 = 0.25 \times 2.5 \begin{bmatrix} 1 \\ -2 \\ -1 \end{bmatrix} = \begin{bmatrix} 0.625 \\ -1.25 \\ -0.625 \end{bmatrix}$$

$$\Delta w_3 = w_2 + \Delta w_2 = \begin{bmatrix} 0 \\ 0 \\ 1.5 \end{bmatrix} + \begin{bmatrix} 0.625 \\ -1.25 \\ -0.625 \end{bmatrix} = \begin{bmatrix} 0.625 \\ -1.25 \\ 0.875 \end{bmatrix}$$

(4) Consider the given input patterns P_1, P_2, P_3, P_4, P_5 & P_6 as well as initial weights

w_1, w_2 & w_3 .

$$P_1 = \begin{bmatrix} -0.1961 \\ 0.9806 \end{bmatrix}$$

$$P_2 = \begin{bmatrix} 0.1961 \\ 0.9806 \end{bmatrix}$$

$$P_3 = \begin{bmatrix} 0.9806 \\ 0.1966 \end{bmatrix}$$

$$P_4 = \begin{bmatrix} 0.9806 \\ -0.1961 \end{bmatrix}$$

$$P_5 = \begin{bmatrix} -0.5812 \\ -0.8173 \end{bmatrix}$$

$$P_6 = \begin{bmatrix} -0.8137 \\ -0.5812 \end{bmatrix}$$

$$w_1 = \begin{bmatrix} 0.7071 \\ -0.7071 \end{bmatrix}$$

$$w_2 = \begin{bmatrix} 0.7071 \\ 0.7071 \end{bmatrix}$$

$$w_3 = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$$

Consider $\alpha = 0.5$, perform clustering using winner take all algorithm (Competitive learning)
Inputs patterns presented in following sequence
 $P_4, P_3, P_2, P_1, P_6, P_5$

Soln:-

$$W^t = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = \begin{bmatrix} 0.7071 & -0.7071 \\ 0.7071 & 0.7071 \\ -1 & 0 \end{bmatrix}$$

(7)

Step 1:- compete w^t by

$$= \begin{bmatrix} 0.7071 & -0.7071 \\ 0.7071 & 0.7071 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 0.9806 \\ -0.1961 \end{bmatrix}$$

$$= \begin{bmatrix} 0.832 \\ 0.5547 \\ -0.9806 \end{bmatrix}$$

w_1 win's So update w_1

$$w_{1 \text{ new}} = w_{1 \text{ old}} + \alpha (P_4 - w_{1 \text{ old}})$$

$$= \begin{bmatrix} 0.7071 \\ -0.7071 \end{bmatrix} + 0.5 \left(\begin{bmatrix} 0.9806 \\ -0.1961 \end{bmatrix} - \begin{bmatrix} 0.7071 \\ -0.7071 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 0.844 \\ -0.452 \end{bmatrix}$$

updated weights

$$w^t = \begin{bmatrix} 0.844 & -0.452 \\ 0.7071 & 0.7071 \\ -1 & 0 \end{bmatrix}$$

Step 2:- compete w^t by P_3

$$w^t P_3 = \begin{bmatrix} 0.739 \\ 0.832 \\ -0.9806 \end{bmatrix}$$

w_2 win's So update only w_2

$$W_{2\text{new}} = W_{2\text{old}} + 2 (P_3 - W_{2\text{old}})$$

$$= \begin{bmatrix} 0.844 \\ 0.452 \end{bmatrix}$$

hence $wt = \begin{bmatrix} 0.844 & -0.452 \\ 0.844 & 0.452 \\ -1 & 0 \end{bmatrix}$

Step 3: compute $W^t P_2$

$$= \begin{bmatrix} -0.2777 \\ 0.6087 \\ -0.1961 \end{bmatrix}$$

$$W_{2\text{new}} = W_{2\text{new}} + 2 (P_2 - W_{2\text{old}})$$

$$= \begin{bmatrix} 0.52 \\ 0.7163 \end{bmatrix}$$

$$wt = \begin{bmatrix} 0.844 & -0.452 \\ 0.52 & 0.7163 \\ -1 & 0 \end{bmatrix}$$

Step 4 :- compute $\begin{bmatrix} -0.6087 \\ 0.6 \\ 0.1961 \end{bmatrix}$

again W_2 being:

$$W_{2\text{new}} = \begin{bmatrix} 0.1619 \\ 0.8484 \end{bmatrix}$$

~~$W_1 (0.844 \ -0.452)$~~
 ~~$(-0.740, 0.15) P_5$~~

updated weights are

$$w^t = \begin{bmatrix} 0.844 & -0.452 \\ 0.1619 & 0.8484 \\ -1 & 0 \end{bmatrix}$$

⑨

Step 5: compete $w^t p_6$.

$$\text{compete} \begin{bmatrix} -0.424 \\ -0.624 \\ 0.8137 \end{bmatrix}$$

w_3 wins

$$w_{\text{new}} = \begin{bmatrix} -0.9 \\ -0.3 \end{bmatrix}$$

$$w^t = \begin{bmatrix} 0.844 & -0.452 \\ 0.1619 & 0.844 \\ -0.9 & -0.3 \end{bmatrix}$$

Step 6: - compete $w^t p_5$

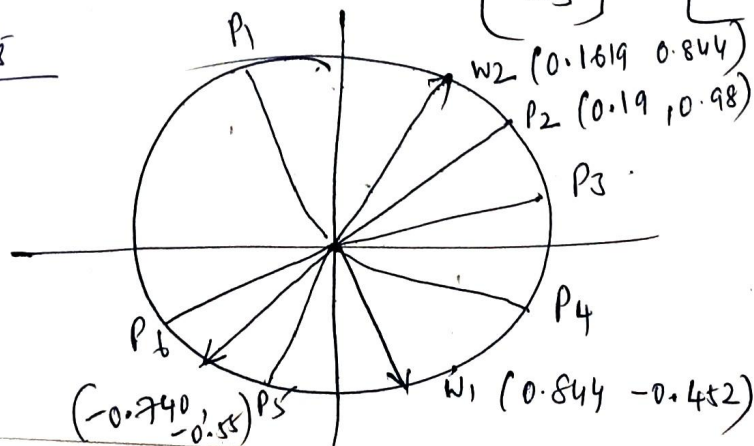
$$\text{compete} \begin{bmatrix} -0.1227 \\ -0.7808 \\ 0.7671 \end{bmatrix}$$

w_3 wins

$$w_{\text{new}} = \begin{bmatrix} -0.7406 \\ -0.5569 \end{bmatrix}$$

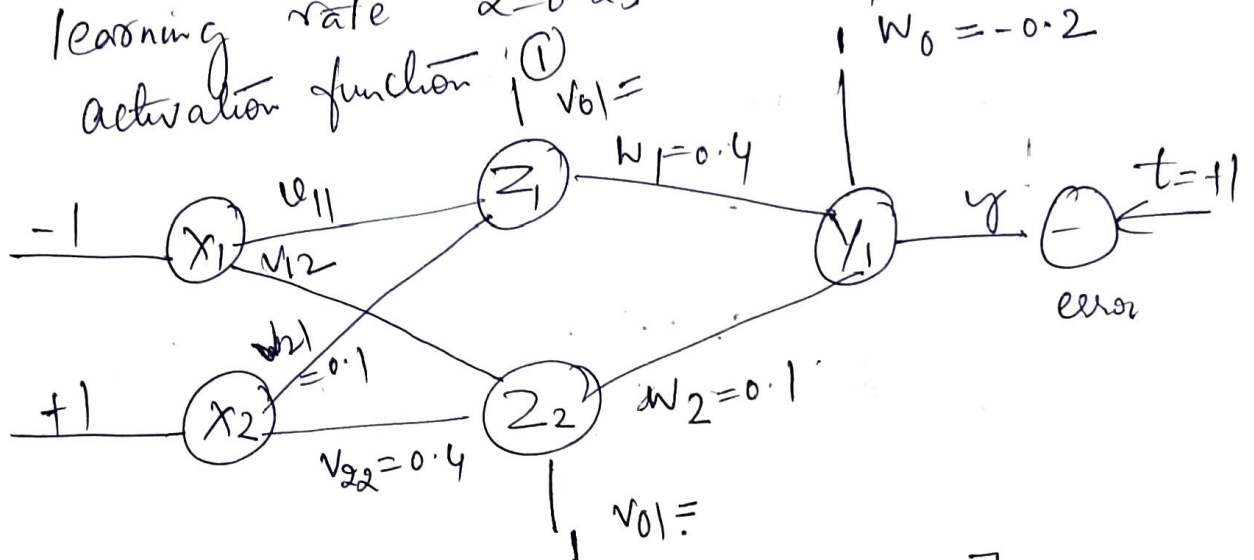
Thus final weights $w^t = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = \begin{bmatrix} 0.844 & -0.452 \\ 0.1619 & 0.844 \\ -0.7406 & -0.5569 \end{bmatrix}$

final wts



w_2 is related towards pattern p_1, p_2 ... (10)
 Similarly w_3 is related towards p_5 & p_6 forming
 cluster centers for respective clusters.

- Error
 (5) Back propagation algorithm - (EBPTA)
 find the new weights - using EBPTA to find
 the new weights for the following network.
 The network is presented using input pattern
 $[-1, 1]$ & target output 1. Use a
 learning rate $\alpha = 0.25$ & bipolar sigmoidal
 activation function (1)



Soln:

$$\begin{bmatrix} v_{01} & v_{11} & v_{12} \end{bmatrix} = [0.3 \quad 0.6 \quad -0.1]$$

$$\begin{bmatrix} v_{02} & v_{12} & v_{21} \end{bmatrix} = [0.5, -0.3, 0.4]$$

$$\begin{bmatrix} w_0 & w_1 & w_2 \end{bmatrix} = [-0.2, 0.4, 0.1]$$

input $[x_1, x_2] = [-1, +1]$

target o/p $t = +1$ $\alpha = 0.25$ -

Step 6 - change in weight b/w hidden & (11)

Bipolar activation function $f(x) = \frac{1 - e^{-x}}{1 + e^{-x}}$

first derivative $f'(x) = \frac{1}{2} (1 - f(x)^2)$

Step 1 - Calculate the hidden layer z_1, z_2

$$z_{in1} = 0.3 \times 1 + (-1 \times 0.6) + (1 \times -0.1) \\ = 0.3 - 0.6 - 0.1 = -0.4$$

$$z_{in2} = 0.5 \times 1 + (-1 \times 0.3) + (1 \times 0.4) \\ = 1.2$$

$$z_1 = f(z_{in1}) = \frac{1 - e^{+0.4}}{1 + e^{0.4}} = -0.1974$$

$$z_2 = f(z_{in2}) = \frac{1 - e^{-1.2}}{1 + e^{1.2}} = 0.537$$

Step 2 Calculate the output layer (y)

$$y_{in} = z_1 w_1 + z_2 w_2 + 1 \times w_0 \\ = -0.1974 \times 0.4 + 0.537 \times 0.1 + 1 \times -0.2 \\ = -0.22526$$

$$y = f(y_{in}) = \frac{1 - e^{+0.22526}}{1 + e^{0.22526}} = -0.1122$$

Step 3 - Compute the error b/w output & hidden layer

(12)

$$\text{error } (\delta_{oh}) = (t - y) f'(y)$$

$$\delta_{oh} = -(1 + 0.1122) \frac{1}{2} (1 - 0.1122^2)$$

$$= \underline{0.5491}$$

Step 4: Change the weight b/w output layer & hidden layer

$$\Delta w_1 = 2 \delta_{oh} z_1 = 0.25 \times 0.5491 \times (-0.1974)$$

$$\Delta w_1 = -0.0291$$

$$\Delta w_2 = 2 \delta_{oh} z_2 = 0.25 \times 0.5491 \times (0.537)$$

$$\Delta w_2 = 0.0737$$

$$\Delta w_0 = 2 \delta_{oh} 1 = 0.25 \times 0.5491$$

$$= 0.1373$$

Step 4: Compute error b/w hidden & i/p layer

$$\text{error } (\delta_{ihz_1}) = \delta_{oh} \times w_1 \times f'(z_1)$$

$$= 0.5491 \times 0.4 \times \frac{1}{2} (1 - z_1^2)$$

$$= 0.1056$$

$$\text{error } (\delta_{ihz_2}) = \delta_{oh} \times w_2 \times f'(z_2)$$

$$= 0.0195$$

Step 6 - Change in weight b/w hidden & i/p layer (13)

$$\Delta v_{01} = \alpha \delta_{ihz_1} x_1 = 0.25 \times 0.1056 \times 1 = 0.0264$$

$$\Delta v_{11} = -0.0264 \quad (\alpha \delta_{ihz_1} z_1)$$

$$\Delta v_{12} = 0.0264 \quad (\alpha \delta_{ihz_1} x_2)$$

$$\Delta v_{02} = 0.0049 \quad (\alpha \delta_{ihz_2} x_1)$$

$$\Delta v_{21} = -0.0049 \quad (\alpha \delta_{ihz_2} x_2)$$

$$\Delta v_{22} = +0.0049 \quad (\alpha \delta_{ihz_2} x_1)$$

Step 7: Calculation of new weights.

$$v_{01}(\text{new}) = v_{01}(\text{old}) + \Delta v_{01} = 0.3 + 0.0264 = \underline{\underline{0.3264}}$$

$$v_{11}(\text{new}) = v_{11}(\text{old}) + \Delta v_{11} = 0.6 + (-0.0264) = \underline{\underline{0.5736}}$$

$$v_{12}(\text{new}) = -0.1 + 0.0264 = \underline{\underline{-0.0736}}$$

$$v_{02}(\text{new}) = 0.5 + 0.0049 = \underline{\underline{0.5049}}$$

$$v_{21}(\text{new}) = -0.3 + 0.0049 = \underline{\underline{-0.2951}}$$

$$v_{22}(\text{new}) = 0.4 + 0.0049 = \underline{\underline{0.4049}}$$

$$w_0(\text{new}) = -0.2 + 0.0271 = \underline{\underline{-0.1729}}$$

$$w_1(\text{new}) = 0.4 + 0.0271 = \underline{\underline{0.4271}}$$

$$w_2(\text{new}) = 0.1 + 0.0737 = \underline{\underline{0.1737}}$$