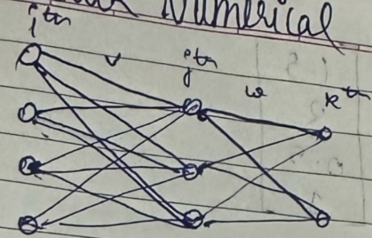
Error Back Numerical



$Z = \text{input} = [1.5, 2, 0.5, 1]$ $\begin{bmatrix} 1.5 \\ 2 \\ 0.5 \\ 1 \end{bmatrix}$ 1×4

4×1

F → unipolar sigmoid

$\lambda = 1$, $\eta = 0.8$

$$V = \begin{bmatrix} 0.2 & 0.3 & 0.1 & 0.1 \\ 0.1 & 0.2 & 0.1 & 0.2 \\ 0.2 & 0.2 & 0.3 & 0.1 \end{bmatrix} \quad 3\times 4$$

$$W = \begin{bmatrix} 0.4 & 0.2 & 0.1 \\ 0.1 & 0.2 & 0.2 \end{bmatrix} \quad 2\times 3$$

$$d = \begin{bmatrix} 1.2 \\ 0.9 \end{bmatrix}$$

FORWARD PASS

calculating net for each hidden layer neuron

at i^{th} layer,

$$\cancel{net_j = V^T \cdot Z}$$

$$\cancel{= \begin{bmatrix} 0.2 & 0.1 & 0.2 \\ 0.3 & 0.2 & 0.2 \\ 0.1 & 0.1 & 0.3 \\ 0.1 & 0.2 & 0.1 \end{bmatrix}}$$

$net_j = V \cdot Z$

$$= \begin{bmatrix} 0.2 & 0.3 & 0.1 & 0.1 \\ 0.1 & 0.2 & 0.1 & 0.2 \\ 0.2 & 0.2 & 0.3 & 0.1 \end{bmatrix} \begin{bmatrix} 1.5 \\ 2 \\ 0.5 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.3 + 0.6 + 0.05 + 0.1 \\ 0.15 + 0.4 + 0.05 + 0.2 \\ 0.3 + 0.4 + 0.15 + 0.1 \end{bmatrix}$$

$$= \begin{bmatrix} 1.05 \\ 0.8 \\ 0.95 \end{bmatrix}$$

applying activation function,

$$f(net_j) = \begin{bmatrix} 0.74 \\ 0.69 \\ 0.72 \end{bmatrix}$$

now, we'll use these outputs as inputs for k^{th} layer

at k^{th} layer,

$$net_k = W f(net_j)$$

$$= \begin{bmatrix} 0.4 & 0.2 & 0.1 \\ 0.1 & 0.2 & 0.2 \end{bmatrix} \begin{bmatrix} 0.74 \\ 0.69 \\ 0.72 \end{bmatrix}$$

$$= \begin{bmatrix} 0.296 + 0.138 + 0.072 \\ 0.074 + 0.138 + 0.144 \end{bmatrix}$$

$$= \begin{bmatrix} 0.506 \\ 0.356 \end{bmatrix}$$

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applying activation function,

$$f(net_k) = \cancel{0.602} \begin{bmatrix} 0.623 \\ 0.588 \end{bmatrix}$$

now, $f(net_k)$ is also the output O.

calculating difference d-O,

$$d - O = \begin{bmatrix} 1.2 \\ 0.9 \end{bmatrix} - \begin{bmatrix} 0.623 \\ 0.588 \end{bmatrix}$$

$$= \begin{bmatrix} 0.577 \\ 0.312 \end{bmatrix}$$

Back propagation

to calculate δ_O,

$$\delta_O = d - O \, (f'(net_k))$$

$$f'(net_k) = O(1-O) \quad \text{for unipolar sigmoid}$$

$$= \cancel{2.44} \begin{bmatrix} 0.244 \\ 0.214 \end{bmatrix}$$

$$\delta_O = \begin{bmatrix} 0.577 \\ 0.312 \end{bmatrix} \begin{pmatrix} 0.244 \\ 0.214 \end{pmatrix}$$ scaling factor

$$= \begin{bmatrix} 0.140 \\ 0.066 \end{bmatrix}$$

weight update for j^{th} layer

$$\Delta W = \eta \, \delta_O \, f(net_j)^T$$

$$= 0.8 \times \begin{bmatrix} 0.140 \\ 0.066 \end{bmatrix} \begin{bmatrix} 0.74 & 0.69 & 0.72 \end{bmatrix}$$

Teacher's Signature:..................

$$= \begin{bmatrix} 0.112 \\ 0.0528 \end{bmatrix} \begin{bmatrix} 0.74 & 0.69 & 0.72 \end{bmatrix}$$

$$= \begin{bmatrix} 0.0828 & 0.0772 & 0.08 \\ 0.039 & 0.0364 & 0.038 \end{bmatrix}$$

now, weight update for i^{th} layer,

$$\delta_y = w^t \delta_o \, \sigma'(net_j)$$

$$w^t \delta_o = \begin{bmatrix} 0.1 & 0.4 \\ 0.2 & 0.2 \\ 0.2 & 0.1 \end{bmatrix} \begin{bmatrix} 0.140 \\ 0.066 \end{bmatrix}$$

$$= \begin{bmatrix} 0.0404 \\ 0.0412 \\ 0.0346 \end{bmatrix}$$

$$\sigma'(net_j) = o(1-o) \quad (o \text{ is the } \sigma(net_j))$$

$$= \begin{bmatrix} 0.1924 \\ 0.2139 \\ 0.2016 \end{bmatrix}$$

$$\delta_y = \begin{bmatrix} 0.0404 \\ 0.0412 \\ 0.0346 \end{bmatrix} \begin{pmatrix} 0.1924 \\ 0.2136 \\ 0.2016 \end{pmatrix}$$ (scaling factor)

$$= \begin{bmatrix} 0.0077 \\ 0.0088 \\ 0.0069 \end{bmatrix}$$

$$\Delta V = \eta \, \delta_y \, Z^t$$

$$= 0.8 \begin{bmatrix} 0.0077 \\ 0.0088 \\ 0.0069 \end{bmatrix} \begin{bmatrix} 1.5 & 2 & 0.5 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.0092 & 0.0123 & 0.003 & 0.0061 \\ 0.0105 & 0.014 & 0.0035 & 0.007 \\ 0.0082 & 0.0110 & 0.0027 & 0.0055 \end{bmatrix}$$

Performing weight updation

$$W_{new} = W_{old} + \Delta W$$

$$= \begin{bmatrix} 0.4 & 0.2 & 0.1 \\ 0.1 & 0.2 & 0.2 \end{bmatrix} + \begin{bmatrix} 0.0828 & 0.0772 & 0.08 \\ 0.039 & 0.0364 & 0.038 \end{bmatrix}$$

$$\boxed{= \begin{bmatrix} 0.4828 & 0.2772 & 0.18 \\ 0.139 & 0.2364 & 0.238 \end{bmatrix}}$$

$$V_{new} = V_{old} + \Delta V$$

$$\boxed{= \begin{bmatrix} 0.2092 & 0.3123 & 0.103 & 0.1061 \\ 0.1105 & 0.214 & 0.1035 & 0.207 \\ 0.2082 & 0.2110 & 0.3027 & 0.1055 \end{bmatrix}}$$