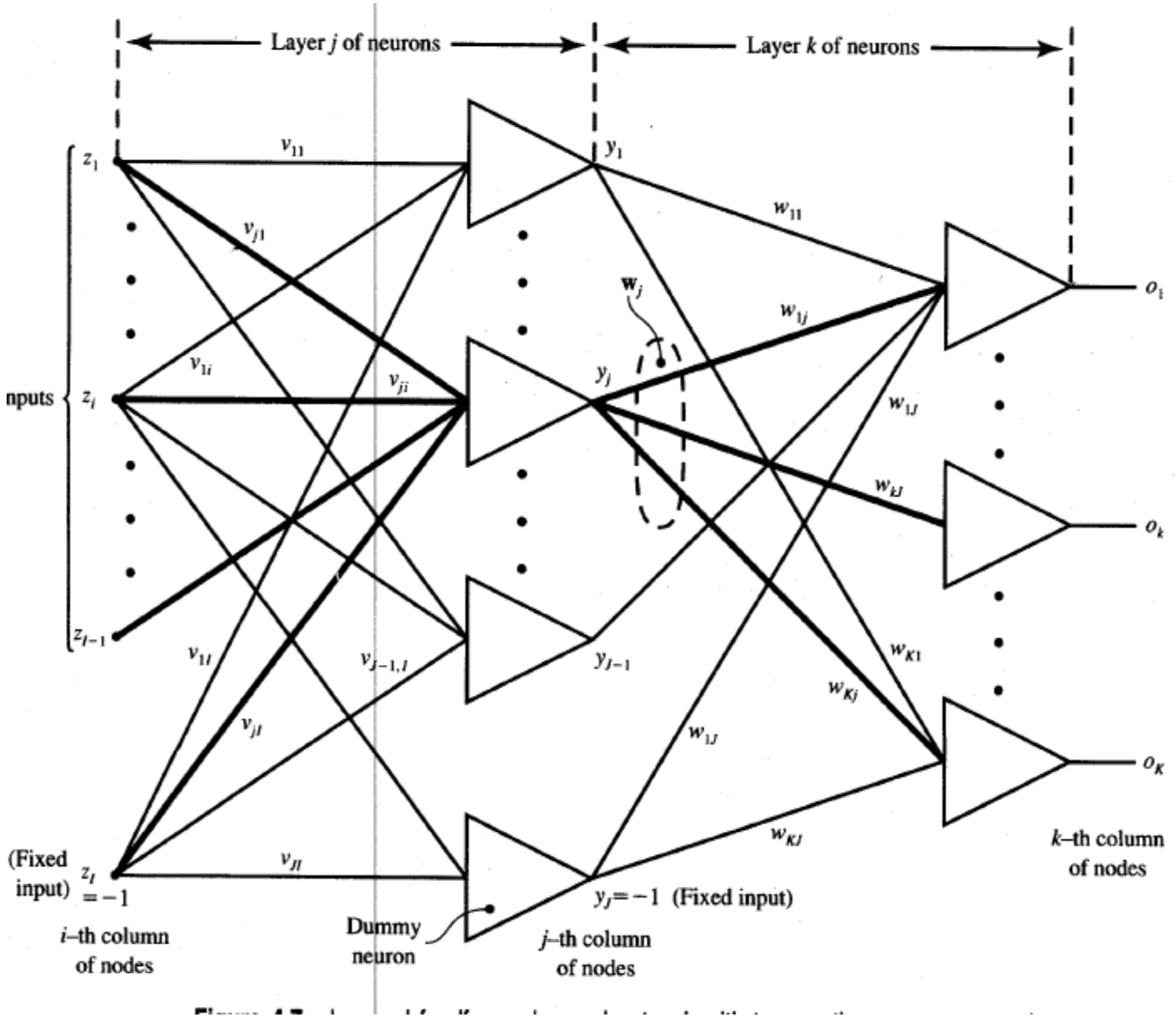
ERROR BACKPROPAGATION TRAINING



Layered feed forward network with hidden and output layers

EBPTA

Given are P training pairs

$$\{\mathbf{z}_1, \mathbf{d}_1, \mathbf{z}_2, \mathbf{d}_2, \cdots, \mathbf{z}_P, \mathbf{d}_P\},$$

where $\mathbf{z}_i$ is $(I \times 1)$, $\mathbf{d}_i$ is $(K \times 1)$, and $i = 1, 2, \ldots, P$. Note that the I'th component of each $\mathbf{z}_i$ is of value -1 since input vectors have been augmented. Size $J - 1$ of the hidden layer having outputs $\mathbf{y}$ is selected. Note that the J'th component of $\mathbf{y}$ is of value -1, since hidden layer outputs have also been augmented; $\mathbf{y}$ is $(J \times 1)$ and $\mathbf{o}$ is $(K \times 1)$.

EBPTA

Step 1: $\eta > 0$, $E_{\max}$ chosen.

Weights $\mathbf{W}$ and $\mathbf{V}$ are initialized at small random values; $\mathbf{W}$ is $(K \times J)$, $\mathbf{V}$ is $(J \times I)$.

$$q \leftarrow 1,\ p \leftarrow 1,\ E \leftarrow 0$$

Step 2: Training step starts here (See Note 1 at end of list.)

Input is presented and the layers' outputs computed [$f(net)$ as in (2.3a) is used]:

$$\mathbf{z} \leftarrow \mathbf{z}_p,\ \mathbf{d} \leftarrow \mathbf{d}_p$$

$$y_j \leftarrow f(\mathbf{v}_j^t \mathbf{z}), \quad \text{for } j = 1, 2, \ldots, J$$

where $\mathbf{v}_j$, a column vector, is the j'th row of $\mathbf{V}$, and

$$o_k \leftarrow f(\mathbf{w}_k^t \mathbf{y}), \quad \text{for } k = 1, 2, \ldots, K$$

where $\mathbf{w}_k$, a column vector, is the k'th row of $\mathbf{W}$.

Step 3: Error value is computed:

$$E \leftarrow \frac{1}{2}(d_k - o_k)^2 + E, \quad \text{for } k = 1, 2, \ldots, K$$

Step 4: Error signal vectors $\boldsymbol{\delta}_o$ and $\boldsymbol{\delta}_y$ of both layers are computed. Vector $\boldsymbol{\delta}_o$ is $(K \times 1)$, $\boldsymbol{\delta}_y$ is $(J \times 1)$. (See Note 2 at end of list.)

The error signal terms of the output layer in this step are

$$\delta_{ok} = \frac{1}{2}(d_k - o_k)(1 - o_k^2), \quad \text{for } k = 1, 2, \ldots, K$$

The error signal terms of the hidden layer in this step are

$$\delta_{yj} = \frac{1}{2}(1 - y_j^2) \sum_{k=1}^{K} \delta_{ok} w_{kj}, \quad \text{for } j = 1, 2, \ldots, J$$

Step 5: Output layer weights are adjusted:

$$w_{kj} \leftarrow w_{kj} + \eta \delta_{ok} y_j, \quad \text{for } k = 1, 2, \ldots, K \text{ and } j = 1, 2, \ldots, J$$

Step 6: Hidden layer weights are adjusted:

$$v_{ji} \leftarrow v_{ji} + \eta \delta_{yj} z_i, \quad \text{for } j = 1, 2, \ldots, J \text{ and } i = 1, 2, \ldots, I$$

Step 7: If $p < P$ then $p \leftarrow p + 1$, $q \leftarrow q + 1$, and go to Step 2; otherwise, go to Step 8.

EBPTA

Step 8: The training cycle is completed.

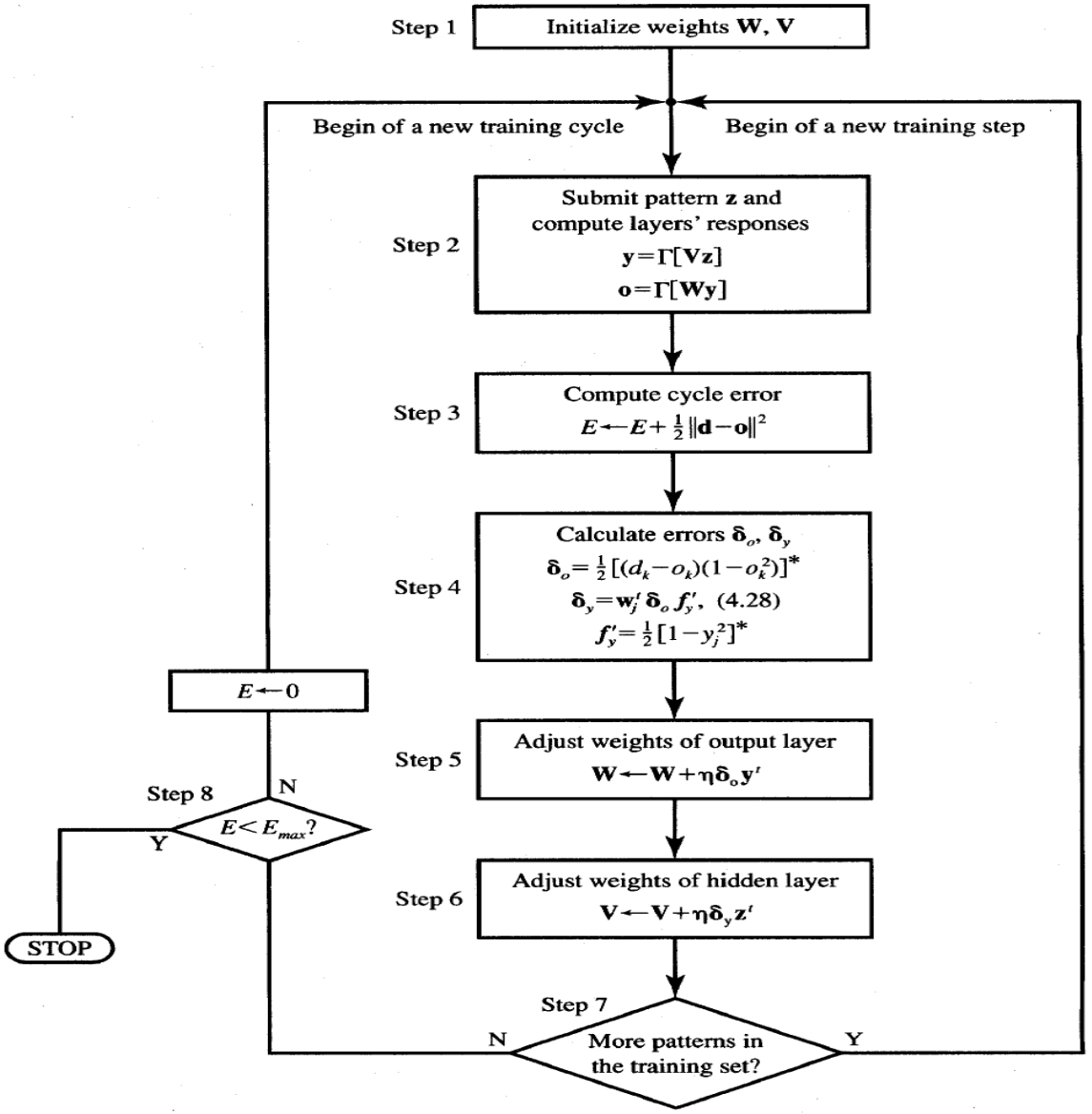
For $E < E_{\max}$ terminate the training session. Output weights $\mathbf{W}$, $\mathbf{V}$, q, and E.

If $E > E_{\max}$, then $E \leftarrow 0$, $p \leftarrow 1$, and initiate the new training cycle by going to Step 2.

- **NOTE 1** For best results, patterns should be chosen at random from the training set (justification follows in Section 4.5).
- **NOTE 2** If formula (2.4a) is used in Step 2, then the error signal terms in Step 4 are computed as follows

$$\delta_{ok} = (d_k - o_k)(1 - o_k)o_k, \quad \text{for } k = 1, 2, \ldots, K$$

$$\delta_{yj} = y_j(1 - y_j) \sum_{k=1}^{K} \delta_{ok} w_{kj}, \quad \text{for } j = 1, 2, \ldots, J$$

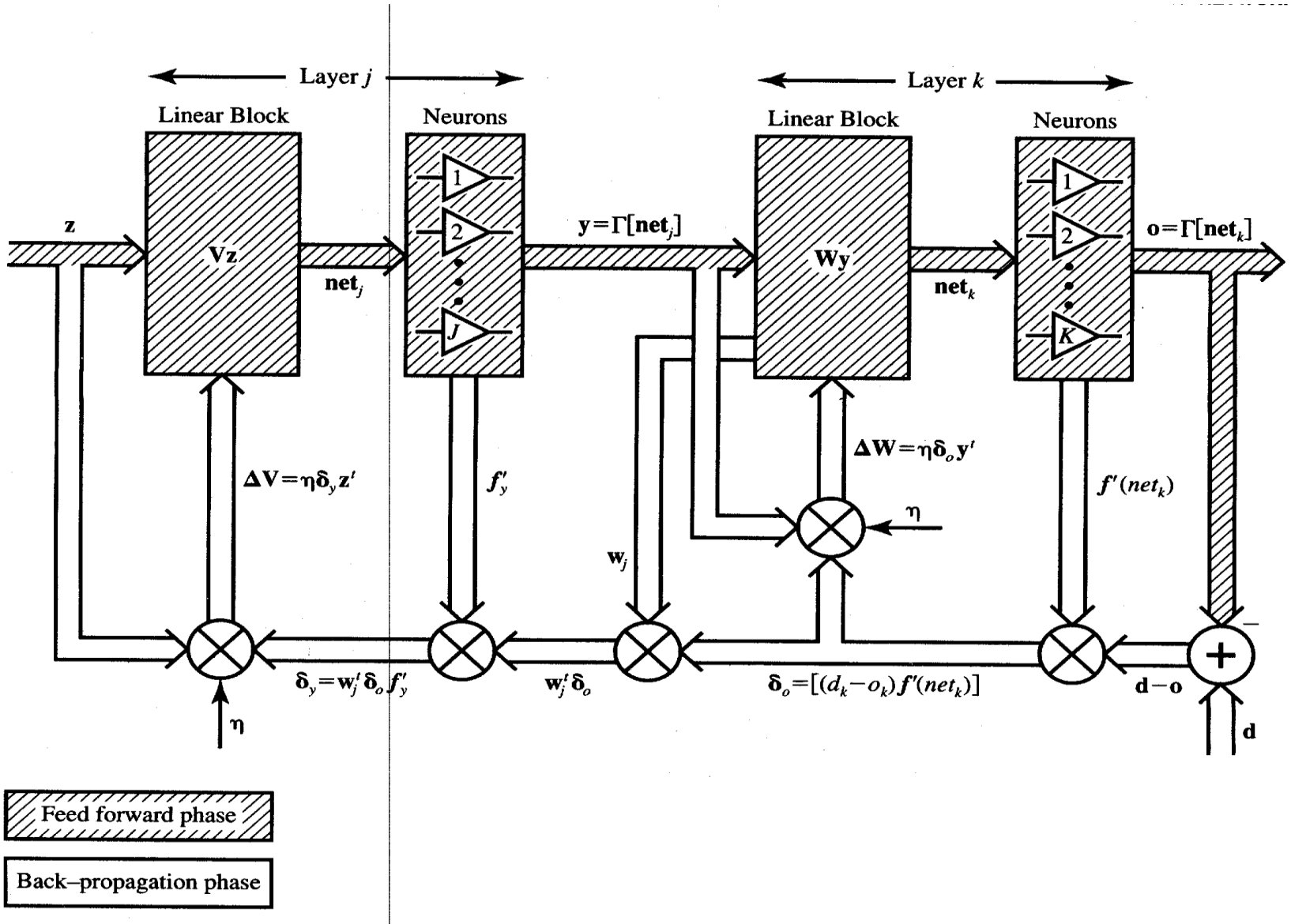


*If $f(net)$ given by (2.4a) is used in Step 2, then in Step 4 use

$$\delta_o=[(d_k-o_k)(1-o_k)o_k],\quad f_y'=[(1-y_j)y_j]$$

(a)

Figure 4.8a Error back-propagation training (EBPT algorithm): (a) algorithm flowchart.



(b)

Figure 4.8b Error back-propagation training (EBPT algorithm) *(continued):* (b) block diagram illustrating forward and backward signal flow.

Learning Factors

Initial Weights

- Initialization strongly affects the ultimate solution
- Initial weights with equal weight values →if the solution requires unequal weights to be developed→ wrong training
- Network disturbance by random factors or the random character of input patterns during training→ result in symmetric weights
- Continuing training beyond a certain low-level results in the undesirable drift of weights-
  - network learning should be restarted with other random weights

Cumulative weight Adjustment versus incremental Updating

- Cumulative weight adjustment after each completed training cycle
- Advisable to use the incremental weight updating after each pattern presentation, but choose patterns in a random sequence from a training set

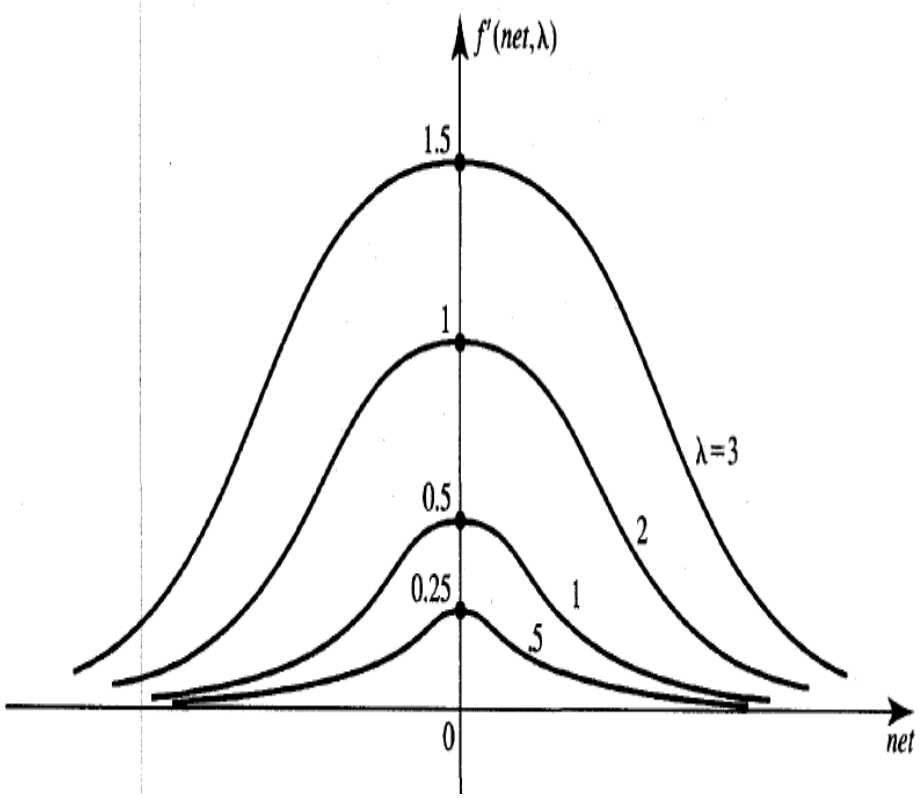
Steepness of the Activation Function (Steepness factor)

- Determines slope of activation function
- Depends on value of net
- multiplying factor in building components of the error signal vectors δ_o, and δ_y
- both the choice and shape of the activation function strongly affect the speed of network learning
- derivative of the activation function-

$$f'(net) = \frac{2\lambda \exp(-\lambda net)}{[1 + \exp(-\lambda net)]^2}$$

- reaches a maximum value of $1/2\lambda$ at net = **0**

Steepness of the Activation Function



- For a fixed learning constant all adjustments of weights in proportion to the steepness coefficient **λ**
- Using activation functions with large **λ** may yield results similar as in the case of large learning constant
- Keep **λ** at a standard value of 1, and control the learning speed using solely the learning constant η, rather than controlling both **λ and η**

Learning Constant

- choice of the learning constant → depends strongly on the class of the learning problem and on the network architecture
- values ranging from 10^{-3} to 10 reported as successful
- For large learning constants, the learning speed can be drastically increased

Momentum Method (Factor)

- To accelerate the convergence of the error back-propagation learning algorithm

$$\Delta w(t) = -\eta \nabla E(t) + \alpha \Delta w(t-1)$$

momentum term

- α is chosen between 0.1 and 0.8