Digital Signal & Image Processing

I Module 1

Discrete signal is function of ~~in~~ discrete independent ~~signal~~ variable.

Discrete time signal is function of independent variable 'n' along with sampling time (T).

☆ Representation of Discrete Time Signals :-

1] Functional Representation

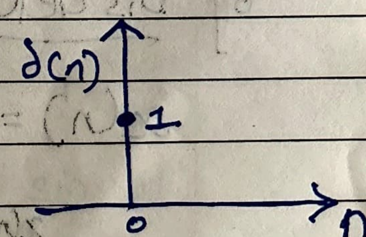
2] Graphical Representation.

3] Tabular Representation

☆ Types of Signals.

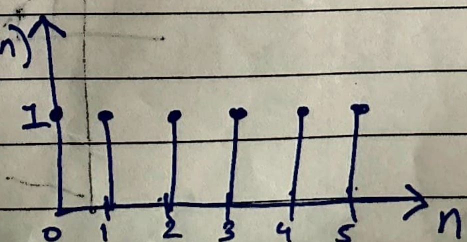
1] Unit Impulse ~~Function~~

$$\delta(n) = 1 \; ; \; n = 0$$
$$= 0 \; ; \; n \neq 0$$



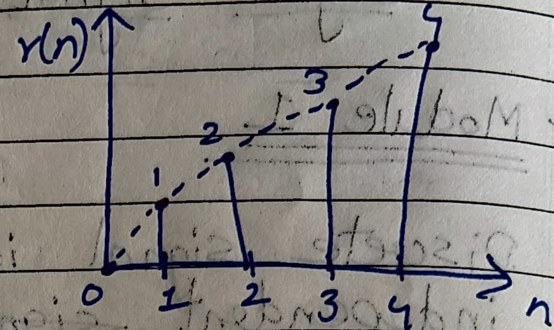
2] Unit Step Signal

$$u(n) = 1 \; ; \; n \geq 0$$
$$= 0 \; ; \; n < 0$$



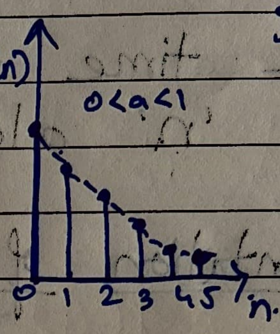
3] **Ramp Signal**

$r(n) = n \; ; \; n \geq 0$
$\quad\;\; = 0 \; ; \; n < 0$

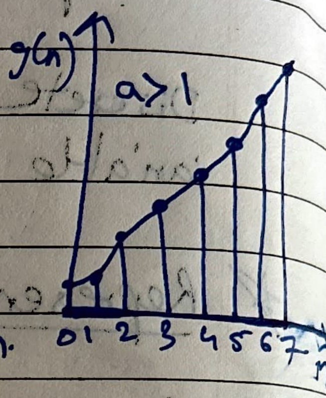


4] **Exponential Signal**

$g(n) = a^n \; ; \; n \geq 0$
$\quad\;\; = 0 \; ; \; n < 0$



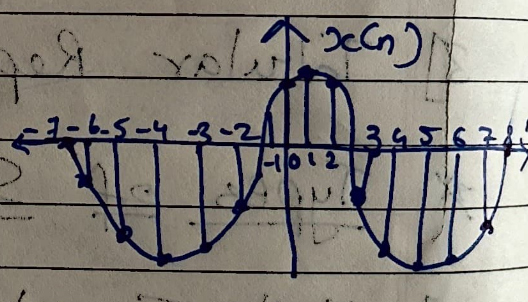
Decreasing



Increasing

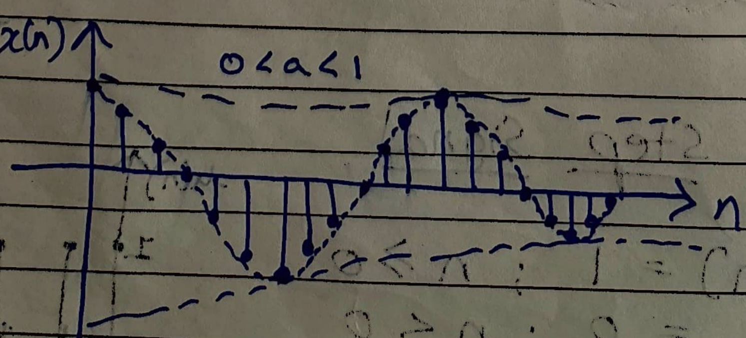
5] **Discrete Time Sinusoidal Signal**

$x(n) = A\cos(\omega_0 n + \theta)$
$x(n) = A\sin(\omega_0 n + \theta)$



6] **Discrete time complex exponential signal**

$x(n) = a^n e^{j(\omega_0 n + \theta)}$



★ Classification of Discrete Time Signals.

1] Deterministic & Non-Deterministic Signals

Signals that can be completely specified by mathematical equations are called deterministic signals. Egs:- step, ramp, exponential signals.

The signals whose characteristics are random in nature are called non-deterministic signals. Eg:- noise signals

2] Periodic & Aperiodic Signals.

Signal is said to be periodic if it remains the same after left & right shifting operation by (K·N) or T

Shortcut: To check:- $\left(\frac{\omega}{2\pi}\right)$; ω = question (given). Denominator = No. of Samples

3] Symmetric (Even) & Antisymmetric (Odd) Signals.

Symmetry w.r.t $n=0$ then even signal

Condition:- $x(-n) = x(n)$

Antisymmetry w.r.t $n=0$ then odd signal.

$x(-n) = -x(n)$

Even Part:- $\frac{1}{2}[x(n)+x(-n)]$; Odd Part $= \frac{1}{2}[x(n) - x(-n)]$

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4] Energy & Power Signals

If energy is finite & non-zero then signal is energy signal & power is 0.

If power is finite & non-zero then signal is power signal & energy is infinite.

$$\text{Energy}, \; E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

$$\text{Power}, \; P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

$$\boxed{c^n = \frac{1}{1-c}}$$

5] Causal, Non Causal & Anticausal

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☆ Operations on Discrete Time Signals

1] Scaling

i] Amplitude Scaling is accomplished by multiplying the value of every signal sample by constant.

ii] Time Scaling

Down sampling :- n is replaced by Dn.
Upsampling :- n is replaced by $\frac{n}{I}$

where D, I are ~~integers~~ integers.

2] Folding

3] Shifting

Right shift → advance.
Left shift → Delay.

4] Addition

5] Multiplication.

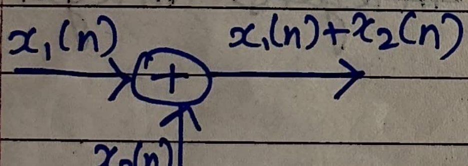
★ Discrete Time Systems-

Discrete Time systems is device or algorithm which accepts input & in discrete signal form to produce output or response in discrete time signal form.

★ Block Diagram & Signal Flow Graph.

Basic elements of Representation:

1] **Adder-**



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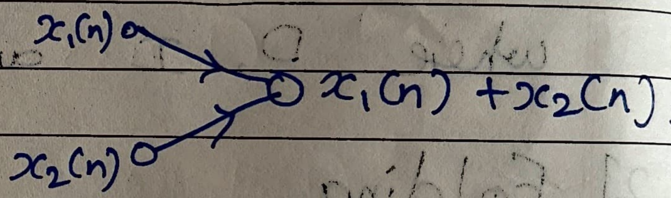

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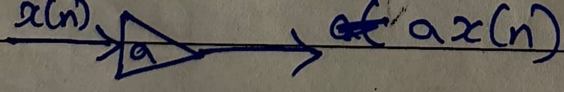
2] **Constant Multiplier**



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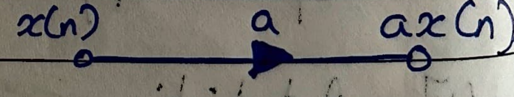

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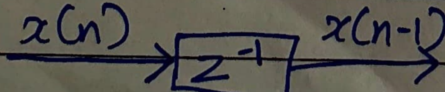
3] ~~Constant~~ **Unit Delay**



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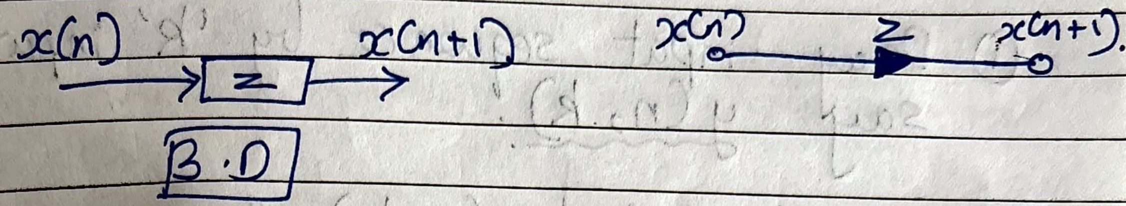
4] Unit Advance



★ Classification of Discrete Time Systems.

1] Static & Dynamic

Static :- Output depends on only current input samples but not on past or future samples. i.e. no memory.
No advance or delay.

Eg:- $y(n) = 2x^2(n)$ is Static
$y(n) = x(n) + x(n-1)$ is dynamic.

2] Time Invariant & Time Variant

System is said to be time invariant if its input output characteristics do not change with time.

Also called shift invariant & shift variant.

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Steps :-

① Delay input sequence by 'k' inputs.
say $y(n, k)$.

② Replace 'n' by $(n-k)$ & say $y(n-k)$

③ Compare ① & ②
If ① = ② → Time/Shift Invariant.
① ≠ ② → Time/Shift Variant.

Example :-

Q] $y(n) = x(n) - x(n-1)$.

① $y(n,k) = x(n-k) - x(n-k-1)$.

② $y(n-k) = x(n-k) - x(n-k-1)$.

③ ① = ② ∴ Time Invariant.

Q] $y(n) = n\,x(n)$.

① $y(n,k) = n\,x(n-k)$

② $y(n-k) = n-k\ x(n-k)$

③ ① ≠ ② ∴ Time Variant.

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3] Linear & Non-Linear Systems.

A system which obeys the superposition principal & homogenity are said to be linear, else its a non-linear system.

Steps :-

1. Apply input x_1 & Record output y_1
2. Apply input x_2 & Record output y_2
3. Apply input $a_1x_1 + a_2x_2$ & Record output y_3
4. Compare :- $a_1y_1 + a_2y_2 = y_3$ $\xrightarrow{YES}$ Linear. $\xrightarrow{NO}$ Non-Linear.

Example :-

Q] $y(n) = \cancel{x}\ n\,x(n)$

1. $y_1(n) = n\,x_1(n)$
2. $y_2(n) = n\,x_2(n)$.
3. $y_3(n) = n[a_1x_1(n) + a_2x_2(n)]$.
4. $a_1\,n\,x_1(n) + a_2\,n\,x_2(n)$

$n[a_1x_1(n) + a_2x_2(n)] = y_3(n)$.

$\therefore$ System is Linear

Note :- All squares/cubes/exponential/constants are NON-LINEAR SYSTEMS.

4] Causal & Non-Causal Systems

Causal → Non-anticipating
Does not depend on future values of the input.

Non-Causal → Anticipating
Response depends on future inputs of the system.

Eg:- Weather Prediction
Stock Market Prediction
$y(n) = x(n+1)$
} Non-Causal.

~~Eg~~ Examples :-

① $y(n) = x(n)\, x(n-1)$
Causal.

② $y(n) = n\, x(n)$ → Causal

③ $y(n) = x\left(\frac{n}{2}\right)$
Causal for $n = +ve$
Non-Causal for $n = -ve$.

④ $y(n) = \sum_{k=-\infty}^{n} x(k)$ → Causal.

⑤ $y(n) = x(n) + 3x(n+a)$ → Non-Causal

⑥ $y(n) = x(n^2)$ → Non-Causal.

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5] Stable & Unstable Systems.

We define a system stable as BIBO (bounded input & bounded output) iff output sequence is bounded for every bounded input i.e.

If $x(n)$ is bounded there exists a finite value i.e M_x.

If $y(n)$ is bounded, there exists a finite value i.e M_y:

Condition for Stability :-

$$\sum_{n=-\infty}^{\infty} |h(n)| < \infty$$

Example

Q] $h(n) = 0.2^n u(n)$.

$$\sum_{n=-\infty}^{\infty} |h(n)| = \sum_{n=0}^{\infty} 0.2^n$$

$$= \frac{1}{1-0.2}$$

$$= 1.25 < \infty$$

$\therefore$ Stable

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6] FIR & IIR Systems :

FIR (Finite duration Impulse Response) system consists of finite number of samples.

IIR (Infinite) system consists of infinite number of samples.

7] Recursive & Non-Recursive Systems.

Recursive :- Output at time n depends on any number of past outputs values as well as present & past inputs.

Non-Recursive - output does not depend on past output but depends only on present & past input.

☆ CONVOLUTION.

Linear Convolution

$$y(n) = x(n) * h(n).$$

where $*$ indicates convolution.

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k) \text{ or } y(n) = \sum_{k=-\infty}^{\infty} h(k)\, x(n-k)$$

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Linear Convolution is Aperiodic.

~~Total No. of Samples.~~

~~x(n)~~ = N_1 is no. of samples in $x(n)$
N_2 is no. of samples in $h(n)$.

Total No. of Samples $= N_1 + N_2 - 1$.
starting index is $n_1 + n_2 = n$
Ende index is $m_1 + m_2 = m$.

Range :- n to m.

If $h(n-k)$ then shift Right
If $h(-n-k)$ then shift Left

Circular Convolution:-

$$y(n) = x(n) \circledast h(n)$$

$$y(n) = \sum_{k=0}^{N-1} x(k)\, h((n-k))_N.$$

OR.

$$y(n) = \sum_{k=0}^{N-1} h(k)\, x((n-k))_N.$$

$x(n)$, $h(n)$, $h(n-1)$, $h(1-n)$ → Anticlockwise shift.

$h(n+1)$, $h(-n)$ → Clockwise shift

Make sure the ~~values~~ no. of values are always symmetric i.e (4 or 8)
~~add~~
If not add 0's left or right.

★ CORRELATION

Used when two signals are to be compared
Measure of degree to which ~~sig~~ two signals are similar.

~~Cros~~ Auto-Correlation

Correlation of signal with itself

No. of samples :- $2N-1$.
Start Sequence :- $-(N-1)$
End Sequence :- $N-1$.
Range :- $-(N-1)$ to $(N-1)$

Cross-Correlation.

Correlation of two separate signals.

No. of sample :- $N_1 + N_2 - 1$
Start Sequence :- $n_1 - (n_2 + N_2 - 1)$
End Sequence :- Start Seq. $+ (N_1 + N_2 - 2)$