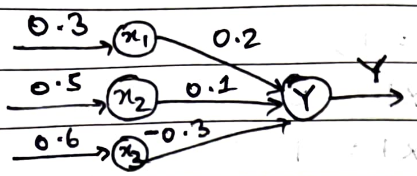
Soft Computing

Q. Calculate net input to neuron Y



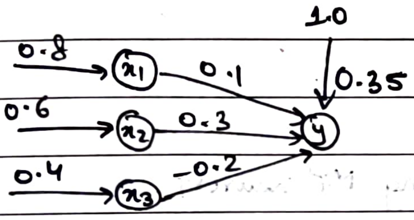
Ans. $\therefore y_{in} = x_1 w_1 + x_2 w_2 + x_3 w_3$

$= (0.3 \times 0.2) + (0.5 \times 0.1) + (0.6 \times -0.3)$

$= 0.06 + 0.05 - 0.18$

$\therefore y_{in} = -0.07$

Q. Obtain output y using activation functions
(i) binary sigmoid
(ii) bipolar sigmoid



Ans. $y_{in} = \text{bias} + x_1 w_1 + x_2 w_2 + x_3 w_3$

$= (1 \times 0.35) + (0.8 \times 0.1) + (0.6 \times 0.3) + (0.4 \times -0.2)$

$= 0.35 + 0.08 + 0.18 - 0.08$

$\therefore y_{in} = 0.53$

(i) binary sigmoid:

$y = \frac{1}{1+e^{-y_{in}}}$

$\therefore y = \frac{1}{1+e^{-0.53}} = 0.625$

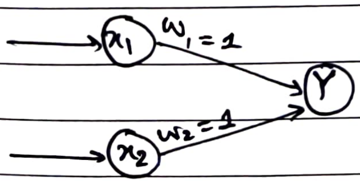
(ii) bipolar sigmoid

$y = \frac{2}{1+e^{-y_{in}}} - 1$

$\therefore y = \frac{2}{1+e^{-0.53}} - 1 = 0.259$

Q. Implement AND function using MP neuron (use binary data)

Ans.



Let $w_1 = w_2 = 1$

$\therefore 0 \times 1 + 0 \times 1 = 0$

$0 \times 1 + 1 \times 1 = 1$

$1 \times 1 + 0 \times 1 = 1$

$1 \times 1 + 1 \times 1 = 2$

x_1	x_2	y
0	0	0
0	1	0
1	0	0
1	1	1

θ (threshold) $= 2$

[Also found from formula $\theta \geq nw - p$, where n = no. of inputs, w = excitatory weight and p = inhibitory outputs]

$\therefore \theta \geq 2$

Q. Implement OR function using MP neuron.

Ans.

x_1	x_2	y
0	0	0
0	1	1
1	0	1
1	1	1

Let $w_1 = w_2 = 1$,

$\therefore 0\times1 + 0\times1 = 0$

$0\times1 + 1\times1 = 1$

$1\times1 + 0\times1 = 1$

$1\times1 + 1\times1 = 2$

$\therefore$ we can see, for OR function, threshold value ~~$\theta \geq 1$~~ $\theta = 1$

~~$\therefore \theta \geq 1$~~

Q. Implement AND NOT function using MP neuron.

Ans.

x_1	x_2	y
0	0	0
0	1	0
1	0	1
1	1	0

Here, let $w_1 = 1$ and $w_2 = -1$

$\therefore 0\times1 + 0\times(-1) = 0$

$0\times1 + 1\times(-1) = -1$

$1\times1 + 0\times(-1) = 1$

$1\times1 + 1\times(-1) = 0$

$\therefore$ we can see for AND NOT function, at $w_1 = 1$ and $w_2 = -1$ threshold value ~~$\theta \geq 1$~~ $\theta = 1$

~~Q. Implement XOR using MP neuron.~~

~~Ans.~~

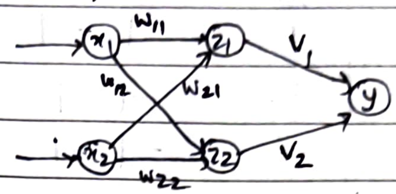
Q. Implement XOR using MP neuron.

Ans.

x_1	x_2	y
0	0	0
0	1	1
1	0	1
1	1	0

$$y = \underbrace{x_1 \cdot \bar{x}_2}_{z_1} + \underbrace{\bar{x}_1 \cdot x_2}_{z_2}$$

for XOR, we need neurons like:



Truth tables for z_1 and z_2:

x_1	x_2	z_1
0	0	0
0	1	0
1	0	1
1	1	0

x_1	x_2	z_2
0	0	0
0	1	1
1	0	0
1	1	0

Taking z_1:

Let $w_{11} = 1$ and $w_{21} = -1$

$\therefore (0 \times 1) + (0 \times -1) = 0$

$0 \times 1 + 1 \times -1 = -1$

$1 \times 1 + 0 \times -1 = 1$

$1 \times 1 + 1 \times -1 = 0$

$\therefore \theta \geq 1$ works

Taking z_2:

Let $w_{21} = -1$ and $w_{22} = 1$

$\therefore 0 \times -1 + 0 \times 1 = 0$

$0 \times -1 + 1 \times 1 = 1$

$1 \times -1 + 0 \times 1 = \cancel{1} -1$

$1 \times -1 + 1 \times 1 = 0$

$\therefore \theta \geq 1$ works

Now, let $v_1 = 1$ and $v_2 = 1$:

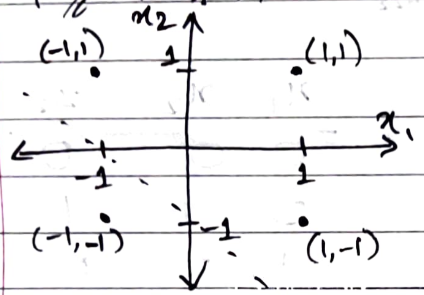
x_1	x_2	z_1	z_2	y
0	0	0	0	0
0	1	0	1	1
1	0	1	0	1
1	1	0	0	0

$\therefore \theta \geq 1$ works

Q. Using linear-separability concept, obtain the response for OR function (take bipolar inputs and targets).

Ans.

x_1	x_2	y
1	1	1
1	-1	1
-1	1	1
-1	-1	-1



[making line that separates +ve and -ve responses]

Assume coordinates as $(-1,0), (0,-1)$

$\therefore\ m = -1,\ c = -1$

$\therefore\ x_2 = -x_1 - 1$

We also know $x_2 = -\frac{w_1}{w_2} x_1 - \frac{b}{w_2}$ (b = bias)

$\Rightarrow \therefore \frac{w_1}{w_2} = 1,\ \frac{b}{w_2} = 1$

$w_1 = w_2 = b = 1$

x_1	x_2	b	$y_{in} = b + x_1w_1 + x_2w_2$
1	1	1	$= 1 + 1\times1 + 1\times1 = 3$
1	-1	1	$= 1 + 1\times1 + 1\times-1 = 1$
-1	1	1	$= 1 + 1\times-1 + 1\times1 = 1$
-1	-1	1	$= 1 + 1\times-1 + 1\times-1 = -1$

$\therefore \theta \geq 1$

$$y = \begin{cases} 1 & y_{in} \geq 1 \\ -1 & y_{in} < 1 \end{cases}$$

PAGE No. 1
DATE 11/8/23

SC - Module 2

Perceptron Model :

Q. Implement AND function using perceptron neural network (using bipolar inputs)

Ans.

X_1	X_2	Y
1	1	1
1	-1	-1
-1	1	-1
-1	-1	-1

$$y = f(y_{in}) = \begin{cases} 1, & \text{if } y_{in} > 0 \\ 0, & \text{if } y_{in} = 0 \\ -1, & \text{if } y_{in} < 0 \end{cases}$$

Assume $w_1 = 0$, $w_2 = 0$, $b = 0$ (weight of bias), $\theta = 0$, $\alpha = 1$ (learning parameter)

∴ ~~Input~~ Epoch 1

x_1	x_2	bias	t	y_{in}	y	Δw_1	Δw_2	Δb	w_1	w_2	b
									0	0	0
1	1	1	1	0	0	1	1	1	1	1	1
1	-1	1	-1	1	1	-1	1	-1	0	2	0
-1	1	1	-1	2	1	1	-1	-1	1	1	-1
-1	-1	1	-1	-3	-1	0	0	0	1	1	-1

For (1,1):

$y_{in} = b + \sum_{i=1}^{n} x_i w_i$
$= 0 + (1\times 0) + (1 \times 0) = 0$ ∴ $y = f(y_{in}) = 0$

$w_1(new) = w_1(old) + \alpha t x_1$
$= 0 + 1\times1\times1 = 1$ $\Delta w_1 = \alpha t x_1 = 1$

$w_2(new) = w_2(old) + \alpha t x_2$
$= 0 + 1\times1\times1 = 1$ $\Delta w_2 = \alpha t x_2 = 1$

$b(new) = b(old) + \alpha t$ (since bias always 1) $\Delta b = \alpha t = 1$
$= 0 + 1\times1 = 1$

For (1,-1):

$y_{in} = b + \sum_{i=1}^{n} x_i w_i$
$= 1 + (1\times1) + (-1\times1) = 1$ ∴ $y = f(y_{in}) = 1$

$w_1(new) = 1 + (1\times -1\times 1) = 0$ $\Delta w_1 = \alpha t x_1 = -1$

$w_2(new) = 1 + (1\times -1\times -1) = 2$ $\Delta w_2 = \alpha t x_2 = 1$

$b(new) = 1 + (1\times -1) = 0$ $\Delta b = \alpha t = -1$

PAGE No. 2
DATE 11/8/23

For (-1,1):

$y_{in} = b + \sum_{i=1}^{n} x_i w_i$ $\qquad y = f(y_{in}) = 1$

$= 0 + (-1 \times 0) + (1 \times 2) = 2$

$w_1(new) = 0 + (1 \times -1 \times -1) = 1$ $\qquad \Delta w_1 = 1$

$w_2(new) = 2 + (1 \times -1 \times 1) = 1$ $\qquad \Delta w_2 = -1$

$b(new) = 0 + (1 \times -1) = -1$ $\qquad \Delta b = -1$

For (-1,-1):

$y_{in} = b + \sum_{i=1}^{n} x_i w_i$ $\qquad \therefore y = f(y_{in}) = -1$

$= -1 + (-1 \times 1) + (-1 \times 1) = -3$

~~w₁(new)~~ Now, since y matches with t, no change needed.

$\therefore w_1, w_2, b$ remain same, $\Delta w_1, \Delta w_2, \Delta b = 0$

Continuing with Epoch 2:

x_1	x_2	bias	t	y_{in}	y	Δw_1	Δw_2	Δb	w_1	w_2	b
1	1	1	1	1	1	0	0	0	1	1	-1
1	-1	1	-1	-1	-1	0	0	0	1	1	-1
-1	1	1	-1	-1	-1	0	0	0	1	1	-1
-1	-1	1	-1	-3	-1	0	0	0	1	1	-1

For (1,1):

$y_{in} = -1 + (1 \times 1) + (1 \times 1) = 1$ $\qquad \therefore y = f(y_{in}) = 1$

Since $y = t$, no change

For (1,-1):

$y_{in} = -1 + (1 \times 1) + (-1 \times 1) = -1$ $\qquad \therefore y = f(y_{in}) = -1$

Since $y = t$, no change

For (-1,1):

$y_{in} = -1 + (-1 \times 1) + (1 \times 1) = -1$ $\qquad \therefore y = f(y_{in}) = -1$

Since $y = t$, no change

For (-1,-1):

$y_{in} = -1 + (-1 \times 1) + (-1 \times 1) = -3$ $\qquad \therefore y = f(y_{in}) = -1$

Since $y = t$, no change

Since all 4 iterations of this epoch had no change, we can stop here.

$w_1 = 1$, $w_2 = 1$, $b = -1$

(continued)

∴ By linear separability,

$$x_2 = \frac{-w_1}{w_2} x_1 - \frac{b}{w_2}$$

$$\therefore x_2 = \frac{-1}{1} x_1 - \frac{(-1)}{1} = -x_1 + 1$$

$$\therefore x_2 = -x_1 + 1$$

Q. Implement XOR function using perceptron neural network (use binary inputs). ~~Do~~ (Try at home, most probably does not come for exam 'cause too long)

Ans.

We know $Y = X_1\bar{X_2} + \bar{X_1}X_2$

So let $Z_1 = X_1\bar{X_2}$, $Z_2 = \bar{X_1}X_2$

So, finalise for Z_1, then for Z_2 and then finally for Y.

(neglect bias, no need to use)

Q. One more type of question 7

Input vectors

$$\begin{bmatrix} -1 \\ -2 \\ 0 \\ -1 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 1.5 \\ -0.5 \\ -1 \end{bmatrix} \quad \begin{bmatrix} -1 \\ 1 \\ 0.5 \\ -1 \end{bmatrix}$$

$$W = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0.5 \end{bmatrix}$$

Learning constant $(\alpha) = 0.1$

Desired output:

$d_1 = -1$

$d_2 = -1$

$d_3 = 1$

SC – Soft Computing

Q. Use perceptron learning to train the network. The set of input training vectors are:

$$X_1 = \begin{bmatrix} 1 \\ -2 \\ 0 \\ 1 \end{bmatrix} \quad X_2 = \begin{bmatrix} 0 \\ 1.5 \\ -0.5 \\ -1 \end{bmatrix} \quad X_3 = \begin{bmatrix} -1 \\ 1 \\ 0.5 \\ -1 \end{bmatrix}$$

Initial weight vector: $W_1 = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0.5 \end{bmatrix}$

Learning constant is 0.1. Desired outputs are $d_1 = -1, d_2 = -2, d_3 = 1$. Show weights obtained after one complete cycle.

NOTE: Perceptron learning equation: $W_{new} = W_{old} + \alpha(t - o)x$
(where t = target output and o = obtained output)

Ans. Since outputs are either +1 or −1, we'll use bipolar (discrete) activation function.

Step 1: $net_1 = W_1^t \cdot x_1$

$$= \begin{bmatrix} 1 & -1 & 0 & 0.5 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \\ 0 \\ 1 \end{bmatrix} = 3.5$$

$\therefore O_1 = sgn(net_1) = 1$

[activation function]

Since $d_1 \neq O_1$, $W_2 = W_1 + \Delta W = W_1 + \alpha \cdot (d_1 - O_1) \cdot x_1$

$$W_2 = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0.5 \end{bmatrix} + (0.1)(-1-1) \begin{bmatrix} 1 \\ -2 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.8 \\ -0.6 \\ 0 \\ 0.3 \end{bmatrix}$$

Step 2: $net_2 = w_2^t \cdot x_2$

$$= \begin{bmatrix} 0.8 & -0.6 & 0 & 0.3 \end{bmatrix} \begin{bmatrix} 0 \\ 1.5 \\ -0.5 \\ -1 \end{bmatrix} = -1.2$$

$\therefore O_2 = sgn(net_2) = -1$

since $d_2 = O_2$, no change (i.e. $w_3 = w_2$)

Step 3: $net_3 = w_3^t \cdot x_3$

$$= \begin{bmatrix} 0.8 & -0.6 & 0 & 0.3 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \\ 0.5 \\ -1 \end{bmatrix} = -1.7$$

$\therefore O_3 = sgn(net_3) = -1$

since $d_3 \neq O_3$, $w_4 = w_3 + \Delta w = w_3 + \alpha(d_3 - O_3)x_3$

$$w_4 = \begin{bmatrix} 0.8 \\ -0.6 \\ 0 \\ 0.3 \end{bmatrix} + (0.1)(1+1)\begin{bmatrix} -1 \\ 1 \\ 0.5 \\ -1 \end{bmatrix} = \begin{bmatrix} 0.6 \\ -0.4 \\ 0.1 \\ 0.1 \end{bmatrix}$$

H.W.

Q. Use Hebbian learning with binary and continuous ~~fo~~ activation functions. $w_1 = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0.5 \end{bmatrix}$ needs to be trained with

$$x_1 = \begin{bmatrix} 1 \\ -2 \\ 1.5 \\ 0 \end{bmatrix}, \quad x_2 = \begin{bmatrix} 1 \\ -0.5 \\ -2 \\ -1.5 \end{bmatrix}, \quad x_3 = \begin{bmatrix} 0 \\ 1 \\ -1 \\ 1.5 \end{bmatrix}$$

[Hebbian Learning is usually unsupervised]

Q. Given following input vectors and initial weight vector, determine final weights for Hebbian learning for single neuron network. (one cycle)

$w_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$, $c = 1$, $x_1 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$, $x_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $x_3 = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$, $x_4 = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$

Solve for: (1) Bipolar binary activation

$$o_i = sgn(net_i) \qquad \text{[sgn = sign check]}$$

(2) Bipolar continuous activation

$$o_i = \frac{2}{1 + e^{-\lambda net_i}} - 1$$

Ans. (1) First solving for bipolar binary activation

Step 1: $net_1 = w_1^t \cdot x_1 = 3$

$\therefore o_1 = sgn(net_1) = 1$

$w_2 = w_1 + \alpha \cdot sgn(net_1) \cdot x_1 = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$

Step 2: $net_2 = w_2^t \cdot x_2 = -3$

$w_3 = w_2 + \alpha \cdot sgn(net_2) \cdot x_2 = \begin{bmatrix} 2 \\ -4 \end{bmatrix}$

Step 3: $net_3 = w_3^t \cdot x_3 = -8$

$w_4 = w_3 + \alpha \cdot sgn(net_3) \cdot x_3 = \begin{bmatrix} 0 \\ -7 \end{bmatrix}$

Step 4 : $net_4 = w_4^t \cdot x_4 = 7$

$$w_5 = w_4 + \alpha\, sgn(net_4) \cdot x_4 = \begin{bmatrix} 0 \\ -8 \end{bmatrix}$$

~~Step 5~~: One Cycle done !

(2) Solving for bipolar continuous activation

Step 1 : $net_1 = w_1^t \cdot x_1 = 3$

$$w_2 = w_1 + \alpha \cdot \left(\frac{2}{1+e^{-3}} - 1 \right) \cdot x_1 = \begin{bmatrix} 1.905 \\ -2.810 \end{bmatrix}$$

Step 2 : $net_2 = w_2^t \cdot x_2 = -2.81$

$$w_3 = w_2 + \alpha \cdot \left(\frac{2}{1+e^{2.81}} - 1 \right) \cdot x_2 = \begin{bmatrix} 1.905 \\ -3.696 \end{bmatrix}$$

Step 3 : $net_3 = w_3^t \cdot x_3 = -7.278$

$$w_4 = w_3 + \alpha \cdot \left(\frac{2}{1+e^{7.278}} - 1 \right) \cdot x_3 = \begin{bmatrix} -0.091 \\ -6.69 \end{bmatrix}$$

Step 4 : $net_4 = w_4^t \cdot x_4 = 6.69$

$$w_5 = w_4 + \alpha \left(\frac{2}{1+e^{-6.69}} - 1 \right) \cdot x_4 = \cancel{\begin{bmatrix} -0.091 \\ -7.687 \end{bmatrix}} \cancel{\begin{bmatrix} -0.091 \\ -5.6925 \end{bmatrix}}$$

$$\begin{bmatrix} -0.091 \\ -7.687 \end{bmatrix}$$

Delta Learning Rule - (Supervised)

FORMULAE :

Perceptron Model ↓

$$W_{new} = W_{old} + \alpha(d_i - o_i)x_i$$

Hebbian ↓

$$W_{new} = W_{old} + \alpha o_i x_i$$

Delta ↓

$$W_{new} = W_{old} + c \cdot (d_i - o_i) \cdot f'(net) \cdot x_i$$

Q. Use bipolar continuous activation function, and delta learning rule to solve: $X_1 = \begin{bmatrix} 1 \\ -2 \\ 0 \\ -1 \end{bmatrix}$ $X_2 = \begin{bmatrix} 0 \\ 1.5 \\ -0.5 \\ -1 \end{bmatrix}$ $X_3 = \begin{bmatrix} -1 \\ 1 \\ 0.5 \\ -1 \end{bmatrix}$ $W_1 = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0.5 \end{bmatrix}$

$d_1 = -1$, $d_2 = -1$, $d_3 = 1$, $c = 0.1$, $\lambda = 1$

[one cycle]

Ans. Step 1 :

$$net^1 = W_1^T \cdot x_1 = \begin{bmatrix} 1 & -1 & 0 & 0.5 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \\ 0 \\ -1 \end{bmatrix} = 2.5$$

$$\therefore O^1 = f(net^1) = \frac{2}{1 + e^{-2.5}} - 1 = 0.848$$

$$f'(net^1) = \frac{1}{2}\left[1 - (O^1)^2\right] = \frac{1}{2}\left(1 - (0.848)^2\right) = 0.140$$

$$W_2 = W_1 + c\,(d_i - o_i) \cdot f'(net^1) \cdot x_1$$

$$= \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0.5 \end{bmatrix} + 0.1(-1 - 0.848) \cdot (0.140) \cdot \begin{bmatrix} 1 \\ -2 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 0.974 \\ -0.948 \\ 0 \\ 0.526 \end{bmatrix}$$

Step 2:

$$net^2 = w_2^T \cdot x_2 = \begin{bmatrix} 0.974 & -0.948 & 0 & 0.526 \end{bmatrix} \begin{bmatrix} 0 \\ 1.5 \\ -0.5 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ -1.422 \\ 0 \\ -0.526 \end{bmatrix}$$

(continue ahead)

Q. Perform 2 training steps using delta learning rule:

$\lambda = 1$, $c = 0.25$, $X_1 = \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix}$, $X_2 = \begin{bmatrix} 1 \\ -2 \\ -1 \end{bmatrix}$, $W = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

$d_1 = -1$, $d_2 = +1$

Ans. We know Delta Learning rule formulae:

$$W_{new} = W_{old} + c(d_i - o_i) \cdot f'(net^i) \cdot x$$

$$\rightarrow f'(net) = \frac{1}{2}\left[1 - (o)^2\right]$$

$$\rightarrow o = \frac{2}{1 + e^{-\lambda \cdot net}} - 1$$

Step 1:

$$net^1 = w^{1t} \cdot x^1 = \begin{bmatrix} 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix} = 1$$

$$o^1 = \frac{2}{1 + e^{-1}} - 1 = 0.462 \qquad f'(net^1) = \frac{1}{2}\left[1 - (o^1)^2\right] = 0.393$$

$$W_2 = W_1 + 0.25(-1 - 0.462) \times 0.393 \times X_1 = W_1 - (0.143) X_1$$

$$\therefore W_2 = \begin{bmatrix} 0.714 \\ 0 \\ 1.143 \end{bmatrix}$$

$$net^2 = w^{2t} \cdot X^2 = \begin{bmatrix} 0.714 & 0 & 1.143 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \\ -1 \end{bmatrix} = -0.430$$

$$o^2 = \frac{2}{1 + e^{0.43}} - 1 = -0.211 \qquad f'(net^2) = \frac{1}{2}\left(1 - (o^2)^2\right) = 0.478$$

$$W_3 = W_2 + 0.25(1 + 0.211) \cdot (0.478) \cdot X_2 = W_2 + (0.145) X_2$$

$$\therefore W_3 = \begin{bmatrix} 0.859 \\ -0.29 \\ 0.998 \end{bmatrix}$$

2 training steps done!

WINNER-TAKE-ALL LEARNING RULE :

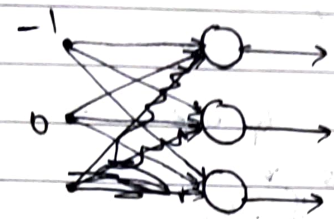
Q. Given below are 3 input vectors: $\alpha = 0.5$

$P_1 = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$, $P_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $P_3 = \begin{bmatrix} 0.709 \\ 0.709 \end{bmatrix}$

$w_1 = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$, $w_2 = \begin{bmatrix} -0.894 \\ 0.447 \end{bmatrix}$, $w_3 = \begin{bmatrix} -0.447 \\ 0.894 \end{bmatrix}$

Input Pattern :
$P_1\ P_2\ P_3\ P_1\ P_2\ P_3$

Ans.



$$W = \begin{bmatrix} 0 & -0.894 & -0.447 \\ -1 & 0.447 & 0.894 \end{bmatrix}$$

Step 1 :

$net^1 = \cancel{w^t} \; w^t . P_1$

$$\therefore net^1 = \begin{bmatrix} 0 & -1 \\ -0.894 & 0.447 \\ -0.447 & 0.894 \end{bmatrix} \begin{bmatrix} -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0.894 \\ 0.447 \end{bmatrix}$$ → winner (highest value)

$\therefore$ only updating w_2 :

$$w_{2new} = w_{2old} + \alpha (P_1 - w_{2old})$$

$$= \begin{bmatrix} -0.894 \\ 0.447 \end{bmatrix} + 0.5 \left[\begin{bmatrix} -1 \\ 0 \end{bmatrix} - \begin{bmatrix} -0.894 \\ 0.447 \end{bmatrix} \right]$$

$$w_{2new} = \begin{bmatrix} -0.947 \\ +0.223 \end{bmatrix}$$

$$\therefore w = \begin{bmatrix} 0 & -0.947 & -0.447 \\ -1 & 0.223 & 0.894 \end{bmatrix}$$

Step 2 :

$net^2 = w^t . P_2$

$$\therefore net^2 = \begin{bmatrix} 0 & -1 \\ -0.947 & 0.223 \\ -0.447 & 0.894 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 0.223 \\ 0.894 \end{bmatrix}$$ → winner (highest value)

$\therefore$ only updating w_3 :

$$w_{3new} = w_{3old} + \alpha (P_2 - w_{3old})$$

(Continued) →

$$W_{3new} = \begin{bmatrix} -0.447 \\ 0.894 \end{bmatrix} + 0.5\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix} - \begin{bmatrix} -0.447 \\ 0.894 \end{bmatrix}\right)$$

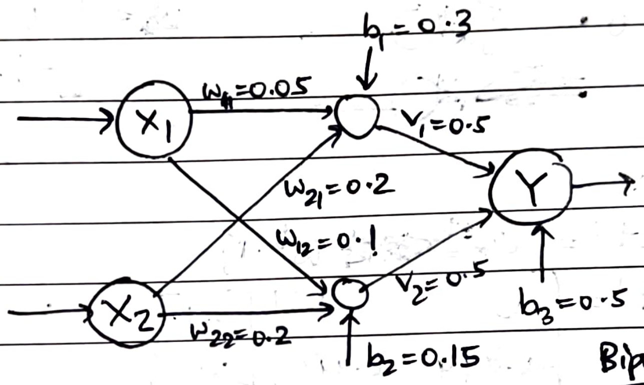
$$= \begin{bmatrix} -0.2235 \\ 0.947 \end{bmatrix}$$

$$\therefore W = \begin{bmatrix} 0 & -0.947 & -0.223 \\ -1 & 0.223 & 0.947 \end{bmatrix}$$

MADALINE

Q.

x_1	x_2	t
1	1	-1
1	-1	1
-1	1	1
-1	-1	-1



$\alpha = 0.5$

Bipolar binary activation

Ans. $z_{in_1} = b_1 + x_1 w_{11} + x_2 w_{21}$
$= 0.3 + 1(0.05) + 1(0.2) = 0.55$ $\quad \therefore z_1 = f(z_{in_1}) = 1$

$z_{in_2} = b_2 + x_1 w_{12} + x_2 w_{22}$
$= 0.15 + 1(0.1) + 1(0.2) = 0.45$ $\quad z_2 = f(z_{in_2}) = 1$

$Y_{in} = b_3 + z_1 v_1 + z_2 v_2$
$= 0.5 + 1(0.5) + 1(0.5) = 1.5$ $\quad \therefore Y = f(Y_{in}) = 1$

$\because Y \neq t$, updating weights

$w_{11}new = w_{11}old + \alpha(t - z_{in_1})x_1 = 0.05 + 0.5(-1 - 0.55)1 = -0.725$

$w_{12}new = w_{12}old + \alpha(t - z_{in_2})x_1 = 0.1 + 0.5(-1 - 0.45)1 = -0.625$

$w_{21}new = w_{21}old + \alpha(t - z_{in_1})x_2 = 0.2 + 0.5(-1 - 0.55)1 = -0.575$

$w_{22}new = w_{22}old + \alpha(t - z_{in_2})x_2 = 0.2 + 0.5(-1 - 0.45)1 = -0.525$

$\cancel{v_1new = v_1old + \alpha(t - Y_{in})z_{in}}$

$b_1new = b_1old + \alpha(t - z_{in_1}) = 0.3 + 0.5(-1 - 0.55) = -0.475$

$b_2new = b_2old + \alpha(t - z_{in_2}) = 0.15 + 0.5(-1 - 0.45) = -0.575$

1 Iteration Done

SC

→ **EBPTA : (Error Back-Propagation Training Algorithm)**

Binary Continuous ↴

$$f(net) = \frac{1}{1+e^{-\lambda net}}$$

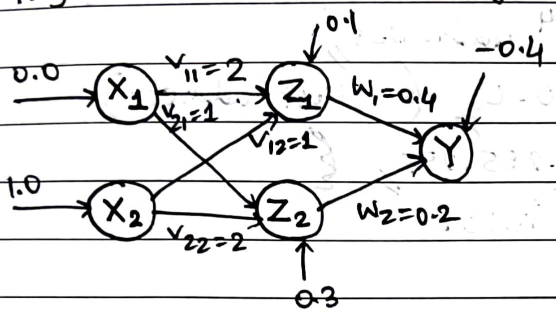
Bipolar Continuous ↴

$$f(net) = \frac{2}{1+e^{-\lambda net}} - 1$$

Q. Apply EBPTA on the following: Use bipolar activation

$$\therefore f(net) = \frac{2}{1+e^{-\lambda \cdot net}} - 1$$

$[\lambda = 1]$



desired output = 1.0

Ans. **Forward Pass** ↴

$net_{z_1} = 0.1 + (2\times0) + (1\times1) = 1.1$

$\therefore f(net_{z_1}) = 0.50$ (using formula)

$net_{z_2} = 0.3 + (0\times1) + (1\times2) = 2.3$

$\therefore f(net_{z_2}) = 0.818$ (using formula → $\frac{2}{1+e^{-1\cdot(2.3)}} - 1$)

$net_Y = -0.4 + (0.50\times0.4) + (0.818\times0.2) = -0.0364$

$f(net_Y) = -0.0182$

Now,

$E = E + \frac{1}{2}(d-y)^2$ (d = desired, y = obtained)

$= 0 + \frac{1}{2}(1-(-0.0182))^2$

$\therefore E = 0.518$

Backward Pass ↴ (Calculating errors δ_o and δ_y)

Since we're using bipolar activation,

$\delta_o = \frac{1}{2}[(d_k - O_k)\cdot(1-O_k^2)]$

$\delta_y = \frac{1}{2}[(d-y)\cdot(1-y^2)]$

$= \frac{1}{2}[(1-(-0.0182))\cdot(1-(-0.0182)^2)]$

$\therefore \delta_y = 0.5089$

(continued) →

$$\delta z_1 = \frac{1}{2}[1-z_1]^2 \cdot \delta_y \cdot w_1$$
$$= \frac{1}{2}[1-0.5]^2 \cdot (0.5089) \cdot 0.4$$
$$= 0.025$$

$$\delta z_2 = \frac{1}{2}[1-z_2]^2 \cdot \delta_y \cdot w_2$$
$$= \frac{1}{2}[1-0.818]^2 \cdot (0.5089) \cdot 0.2$$
$$= 0.00169$$

Updating weights :

$$W_{1\,new} = W_{1\,old} + \delta_y \cdot z_1$$
$$= 0.4 + (0.5089) \cdot (0.5)$$
$$= 0.654$$

$$W_{2\,new} = W_{2\,old} + \delta_y \cdot z_2$$
$$= 0.2 + (0.5089) \cdot (0.818)$$
$$= 0.616$$

~~$W_{(0)new} = W_{(0)old}$~~ $W_{b\,new} = W_{b\,old} + \delta_y \cdot 1$
$$= -0.4 + 0.5089$$
$$= 0.1089$$

$$V_{11\,new} = V_{11\,old} + \delta z_1 \cdot x_1$$
$$= 2 + (0.025) \cdot 0$$
$$= 2$$

$$V_{12\,new} = V_{12\,old} + \delta z_1 \cdot x_2$$
$$= 1 + (0.025) \cdot 1$$
$$= 1.025$$

$$V_{21\,new} = V_{21\,old} + \delta z_2 \cdot x_1$$
$$= 1 + (0.00169 \times 0)$$
$$= 1$$

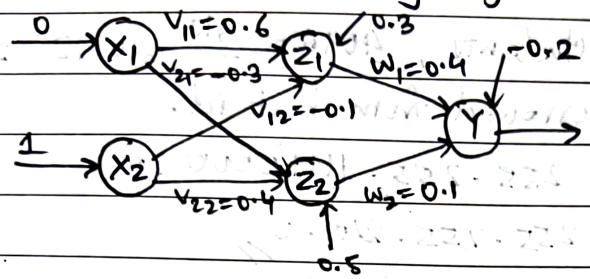
$$V_{22\,new} = V_{22\,old} + \delta z_2 \cdot x_2$$
$$= 2 + (0.00169) \cdot 1$$
$$= 2.00169$$

$$V_{b_1\,new} = V_{b_1\,old} + \delta z_1 = 0.1 + 0.025 = 0.125$$
$$V_{b_2\,new} = V_{b_2\,old} + \delta z_2 = 0.3 + 0.00169 = 0.30169$$

SC

Q. Using backpropagation network, find new weights for the net shown in the figure. It is presented with input pattern (0, 1) and the target output is 1. Learning constant is 0.25, and activation function is binary sigmoidal.

~~Ans~~.



Ans. $\because$ $[V_{11}\ V_{12}\ V_{01}] = [0.6, -0.1, 0.3]$

$[V_{21}\ V_{22}\ V_{02}] = [-0.3, 0.4, 0.5]$

$[W_1\ W_2\ W_0] = [0.4, 0.1, -0.2]$

for Unipolar Continuous,

$$f(net) = \frac{1}{1+e^{-\lambda net}}$$

Forward Pass : $(\lambda = 1)$

~~net~~ ! $net_{z_1} = 0.3 + (0 \times 0.6) + (1 \times -0.1) = 0.2$

$\therefore f(net_{z_1}) = \cancel{\frac{1}{1+e^{-\lambda net}}}\ \frac{1}{1+e^{-\cancel{(\lambda net)}\times 0.2}} = \cancel{0.550} \ \cancel{0.512}\ 0.550$

$net_{z_2} = 0.5 + (0 \times -0.3) + (1 \times 0.4) = 0.9$

$\therefore f(net\ z_2) = \cancel{\frac{1}{1+e^{-(0.25)\times 0.9}}}\ \frac{1}{1+e^{-0.9}} = 0.711$

$net_y = -0.2 + (0.55 \times 0.4) + (0.711 \times 0.1) = 0.0911$

$\therefore f(net_y) = \frac{1}{1+e^{-0.0911}} = 0.523$

Now, $E = E_{old} + \frac{1}{2}(d-y)^2$

$= 0 + \frac{1}{2}(1-0.523)^2$

$\therefore E = 0.114$

Backward Pass :

for bipolar continuous → $\delta_o = (d-o)(1-o).o$

$\therefore \delta_y = (d-y)(1-y).y = (1-0.523)(1-0.523)(0.523)$

$\therefore \delta_y = 0.119$

Updating 'w' weights ↴

$w_{1\,new} = w_{1\,old} + \eta \cdot \delta_y \cdot z_1$

$\therefore w_{1\,new} = 0.4 + (0.25)\cdot(0.119)\cdot(0.550)$

$w_{1\,new} = 0.416$ //

$w_{2\,new} = w_{2\,old} + \eta \cdot \delta_y \cdot z_2$

$= 0.1 + (0.25)(0.119)(0.711)$

$\therefore w_{2\,new} = 0.121$ //

$w_{0\,new} = w_{0\,old} + \eta \cdot \delta_y$

$= -0.2 + (0.25)(0.119)$

$\therefore w_{0\,new} = -0.170$ //

Now,

$\delta_{z_1} = z_1 \cdot (1-z_1) \cdot \delta_y \cdot w_1$

$= 0.55 \cdot (1-0.55) \cdot (0.119) \cdot (0.4)$

$\therefore \delta_{z_1} = 0.012$ //

$\delta_{z_2} = z_2 \cdot (1-z_2) \cdot \delta_y \cdot w_2$

$= 0.711 \cdot (1-0.711) \cdot (0.119) \cdot (0.1)$

$\therefore \delta_{z_2} = 0.0024$ //

Updating 'v' weights ↴

$v_{11\,new} = v_{11\,old} + \eta \cdot \delta z_1 \cdot x_1$ (But $x_1 = 0$)

$\therefore v_{11\,new} = v_{11\,old} = 0.6$ //

$v_{12\,new} = v_{12\,old} + \eta \cdot \delta z_1 \cdot x_2 = -0.1 + (0.25)\cdot(\cancel{0.0024}\ \cancel{0.119}\ 0.012)\cdot 1$

$\therefore v_{12\,new} = \cancel{-0.0994}, \cancel{-0.070}, -0.097$ //

$v_{01\,new} = v_{01\,old} + \eta \cdot \delta z_1 = 0.3 + (0.25)\cdot(0.012)$

$\therefore v_{01\,new} = 0.303$ //

$v_{21\,new} = v_{21\,old} + \eta \cdot \delta z_2 \cdot x_1$ (But $x_1 = 0$)

$\therefore v_{21\,new} = v_{21\,old} = -0.3$ //

$v_{22\,new} = v_{22\,old} + \eta \cdot \delta z_2 \cdot x_2 = 0.4 + (0.25)\cdot(0.0024)\cdot 1$

$\therefore v_{22\,new} = 0.4006$ //

$v_{02\,new} = v_{02\,old} + \eta \cdot \delta_{z_2} = 0.5 + (0.25)\cdot(0.0024)$

$\therefore v_{02\,new} = 0.5006$ //