Book of Abstracts

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Frege's Begriffsschrift

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Frege's *Concept script*, better known by its original German name *Begriffsschrift*, is Frege's first work in logic, often overshadowed by more philosophical *Grundlagen der Arithmetik* and more mathematical *Grundgesetze der Arithmetik*. Nevertheless, it is of great importance for both philosophy and mathematics. Frege's main goal is to develop a new (formula) language that will enable a precise analysis of the concept of (natural) number. This new *ideography* must enable the investigation of arithmetic without any references to intuition and without any psychological influences.

We will present the content of *Begriffsschrift* and will try to explain the historical circumstances which lead to its creation. We will also briefly discuss its philosophical and mathematical consequences and argue why it is one of the most important works in logic.

References

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Measuring the Complexity of Isomorphisms Between Countable Structures

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In most areas of math, two structures are seen as the same if they are isomorphic. This is not the case in computable structure theory, as two copies of the same structure may be wildly different in terms of computability. We will discuss basic ideas from computable structure theory and define the notion of a categoricity ordinal, which is a measure of the complexity between copies of a given structure.

Logical induction

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With the rapid development of AI, logicians are trying to develop robust techniques for reasoning when confronted with *logical uncertainty.* As opposed to empirical uncertainty, where we're not quite sure about precise results of measurement, and therefore we must hedge our bets with respect to precise outcomes of random experiments (which is the domain of probability theory), here we are not quite sure of inner workings of a mathematical deduction system, or we simply don't have time to simulate it in every detail.

For instance, there is no sense in speaking about the **probability** of $87\,358^{\text{th}}$ digit of π being 4, since there is no empirical uncertainty we're dealing with here, but still, there is a strong sense of that claim being somehow assigned a quantity (**credence**) of 10%.

In order to get a consistent system, a lot of properties (of credence) must hold *eventually*, in the limit, whereas at every finite stage, we can have temporary inconsistencies which will get "ironed out" with time. To model the properties precisely, we have to take into account trading strategies and rational polynomial agents employing strategies in order to profit in the long run in the open market. More precisely, by the analogy with the probability being the feature of a strategy not enabling anybody to successfully bet against the agent in the long run, credence is a feature of the strategy not enabling anybody to profit from "open market exchange".

We will present some desired properties of such a system, and a "proof of concept" algorithm showing that such a notion is at least theoretically possible.

References

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Computable Type of Certain Quotient Spaces

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A closed set S in a computable topological space is said to be *computably enumerable* if it is possible to semidecide whether a basic open set intersects S. Furthermore, a compact set in a computable topological space is said to be *semicomputable* if it is possible to semidecide whether a finite union of basic open sets covers S. A set which is both computably enumerable and semicomputable is called *computable*.

Topology plays an important role in determining the relationship between semicomputability and computable enumerability. In particular, semicomputable sets with certain topological properties are necessarily computably enumarable (and, therefore, computable). This is expressed in the notion of *computable type*: a topological space A is said to have computable type if every semicomputable set homeomorphic to A must be computable. More generally, topological pair (A, B) has computable type if, whenever A is embedded in a computable topological space, semicomputability of images of A and B implies that the image of A is computable. Some known examples of spaces with computable type are topological manifolds, chainable and circularly chainable continua and finite graphs ([3, 2, 4]).

It is known that both the pair (B^n, S^{n-1}) of the unit ball and its boundary and the quotient space $B^n/S^{n-1} \cong S^n$ have computable type ([1]). Motivated by this, we consider the effect of quotients on preserving computable type. We prove the following:

Theorem 1 *Let A be a topological space and let B be a compact subset of A such that $\operatorname{Int}_A B = \emptyset$. If A/B has computable type, then (A, B) has computable type.*

However, if (A, B) has computable type, A/B generally need not have computable type even if the interior of B in A is empty and A is a compact manifold. We illustrate this with a few interesting counterexamples. Finally, we move our focus to locally Euclidean spaces and prove the following result.

Theorem 2 *Let $n \in \mathbb{N} \setminus \{0\}$ and let K be a compact subset of $\mathbb{R}^n$ such that $\mathbb{R}^n \setminus K$ has finitely many connected components. Then $\mathbb{R}^n/K$ has local computable type.*

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Games and tree unravelings for a new notion of bisimulations of Verbrugge semantics

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Correspondence theory systematically investigates the relationship between modal and classical logic. Bisimulations and standard translation are the two tools we need to understand modal expressivity. Van Benthem's characterization theorem (see [1]) shows that modal languages correspond to the bisimulation invariant fragment of first–order languages, which is established by classical methods of first–order model theory.

Perkov and Vuković in [4] proved that a first–order formula is equivalent to the standard first–order translation of some formula of interpretability logic with respect to Veltman models if and only if it is invariant under bisimulations between Veltman models. In order to prove that, they used bisimulation games on Veltman models for interpretability logic and an appropriate notion of model unravelling, somewhat analogous to the usual tree unravelling (see e.g. [2]).

Since for the standard definition of bisimulations (and their finite approximations called n-bisimulations) the basic result that two worlds are n-bisimilar if and only if they are n-equivalent (i.e. they satisfy the same IL-formulas of modal depth up to n) does not hold, we have defined in [3] a new notion of bisimulations for Verbrugge semantics called w-bisimulations.

In this talk we will present that new definition and show that two worlds are n-equivalent if and only if they are n-w-bisimilar. In order to do that we will define Verbrugge model comparison games called w-games and show that w-bisimulation relations may be understood as descriptions of winning strategies for one player in a w-game. Finally, we will present the appropriate notion of saturated bisimilar companion to Verbrugge models.

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Probabilistic Reasoning about Typed Combinatory Logic

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The basic idea of combinatory logic was introduced by Moses Schönfinkel in the 1920s. The foundations of combinatory logic were established by Haskell Curry in the 1930s and it has been developing ever since. In order to control application, Haskell Curry introduced types in combinatory logic. Typed combinatory logic found its application in programming languages, automated theorem provers and proof assistants, and became the topic of interest.

In this talk, we present Logic of Combinatory Logic (LCL) and its probabilistic extension (PCL). Logic of combinatory logic is a classical propositional logic for reasoning about simply typed combinatory logic, obtained by extending the simply typed combinatory logic with classical propositional connectives. We present its syntax, axiomatic system and semantics. As the main results we give the proof of soundness and completeness of the given axiomatization with respect to the proposed semantics. We use the logic LCL to develop a formal system for probabilistic reasoning about typed combinatory terms. Logic PCL, a probabilistic system for simply typed combinatory terms, is defined as a probabilistic logic over LCL. We present syntax, axiomatic system and semantics of PCL. We give the proof that the given axiomatization is sound with respect to the proposed semantics and a sketch of the proof of completeness.

Mathematical Models for Data Privacy

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For centuries, people have shared information with each other and with institutions. What has changed in the last few decades is the capacity of people to collect, store, process and share information. The development of technology in the 20th century has made it possible to manipulate large amount of data, however at the same time it has developed data privacy problems.

Technology raised new privacy concerns and technology should also help to preserve privacy. This is where the need for formal methods appears. Formal methods can help us gain a more rigorous understanding of privacy rights, threats, and violations. State machines, process algebras and game theory can be used to model the behavior of the system and its threat environment. Formal logics, like temporal logics, can be used to state privacy policies and to reason about when a model satisfies a property or policy. Also, there is a need of combining traditional formal methods with statistical methods in order to cover the statistical nature of privacy.

In this talk we will discuss initial models for data privacy, like k-anonymity, l-diversity and t-closeness. We will also give an overview of more advanced models for data privacy, like differential privacy, contextual integrity and inverse privacy.

A note on "The logic of Simpson's paradox"

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In the *Synthese* article "*The logic of Simpson's paradox*", P. S. Bandyoapdhyay and co-authors claim to have given "*the proper analysis of the paradox*" and that they have rejected "*an objection to* [theirs] *account* [which compares theirs] *with Blyth's account of the paradox*". We find their analysis incomplete and unnecessarily complicated, and their comparison with Blyth's analysis not justified. We offer a complete and simple analysis of the paradox and a justified comparison of theirs and Blyth's analysis.

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