Regression Project

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Group 8

Introduction

For decades, a secondary education has been considered the gold standard for ensuring a comfortable career and wage. In recent years however, it is becoming more and more difficult for recent graduates to find careers that match up with their major of study, and therefore the wages that go with it. We found this topic important to study due to the prevalence of career prospects in our own lives as well as society at large. Our data is from an inbuilt RStudio package, causaldata, and has 30401 observations, omitting NAs. Our outcome of interest is median income (earnings_med), while our predictors of interest are degree awarded (pred_degree_awarded_ipeds), working (count_working), not working (count_not_working), year, and region. Degree awarded has 3 different observations: 1 being a less-than-two-year degree, 2 being a two-year degree, and 3 being a four-year degree or more. The year variable goes from 2007 until 2014. The region variable is broken down into Northeast, North Central, South, and West.

```
# all of our packages
library(causaldata)
```

```
Warning: package 'causaldata' was built under R version 4.3.2
```

```
library(datasets)
library(tidyverse)
```

```
── Attaching core tidyverse packages ──────────────────────── tidyverse 2.0.0 ──
✔ dplyr     1.1.2     ✔ readr     2.1.4
✔ forcats   1.0.0     ✔ stringr   1.5.0
✔ ggplot2   3.4.3     ✔ tibble    3.2.1
✔ lubridate 1.9.2     ✔ tidyr     1.3.0
✔ purrr     1.0.2
── Conflicts ────────────────────────────────────────── tidyverse_conflicts() ──
✖ dplyr::filter() masks stats::filter()
✖ dplyr::lag()    masks stats::lag()
ℹ Use the conflicted package (<http://conflicted.r-lib.org/>) to force all conflicts to become
errors
```

```
library(ggfortify)
library(MASS)
```

```
Attaching package: 'MASS'

The following object is masked from 'package:dplyr':
```

```
    select
```

```r
library(pheatmap)
```

```
Warning: package 'pheatmap' was built under R version 4.3.2
```

```r
library(car)
```

```
Loading required package: carData

Attaching package: 'carData'

The following object is masked from 'package:causaldata':

    Mroz


Attaching package: 'car'

The following object is masked from 'package:dplyr':

    recode

The following object is masked from 'package:purrr':

    some
```

```r
library(lmtest)
```

```
Loading required package: zoo

Attaching package: 'zoo'

The following objects are masked from 'package:base':

    as.Date, as.Date.numeric
```

```r
library(splines)
library(reshape2)
```

```
Warning: package 'reshape2' was built under R version 4.3.2


Attaching package: 'reshape2'

The following object is masked from 'package:tidyr':
```

```
    smiths
```

```r
library(interactions)
```

```
Warning: package 'interactions' was built under R version 4.3.2
```

```r
library(boot)
```

```
Attaching package: 'boot'

The following object is masked from 'package:car':

    logit
```

```r
library(simpleboot)
```

```
Simple Bootstrap Routines (1.1-7)
```

```r
library(emmeans)

# the functions used in multiple places
predict_loo <- function(model) {
y <- model.frame(model)[,1]
loo_r <- residuals(model) / (1 - hatvalues(model))
return(y - loo_r)
}
rsq_loo <- function(model) {
y <- model.frame(model)[,1]
yhat <- predict_loo(model)
return(cor(y, yhat)^2)
}
p_print <- function(object){
  print(deparse(substitute(object)))
  print(object)
}
as.numeric.l <- function(list){
  if(is.factor(list)){
    list <- as.numeric(list)
  }
  return(list)
}

## cleaning
# making region match which region the state is in
scorecard$region <- state.region[match(scorecard$state_abbr, state.abb)]
# making the degree a factor
```

```
scorecard$degree <- as.factor(scorecard$pred_degree_awarded_ipeds)
scorecard <- na.omit(scorecard)
```

Research Questions

1. Does median income have a positive relationship with the proportion of working graduates?

- H_0: Median income will have a positive relationship with the number of working graduates.

- H_A: positive relationship and significant p-value will prove this to be true.

2. Which US region contributes most to median earnings?

- H_0: All regions do not differ significantly for median earnings

- H_A: Eastern region will be the most significant in median earnings compared to other regions.

3. Which degree length leads to higher median salary?

- H_0: Median salary does not significantly differ between degree lengths.

- H_A: People with 4 year degrees have higher median salaries compared to other degre

Data Exploration

Manipulate Data

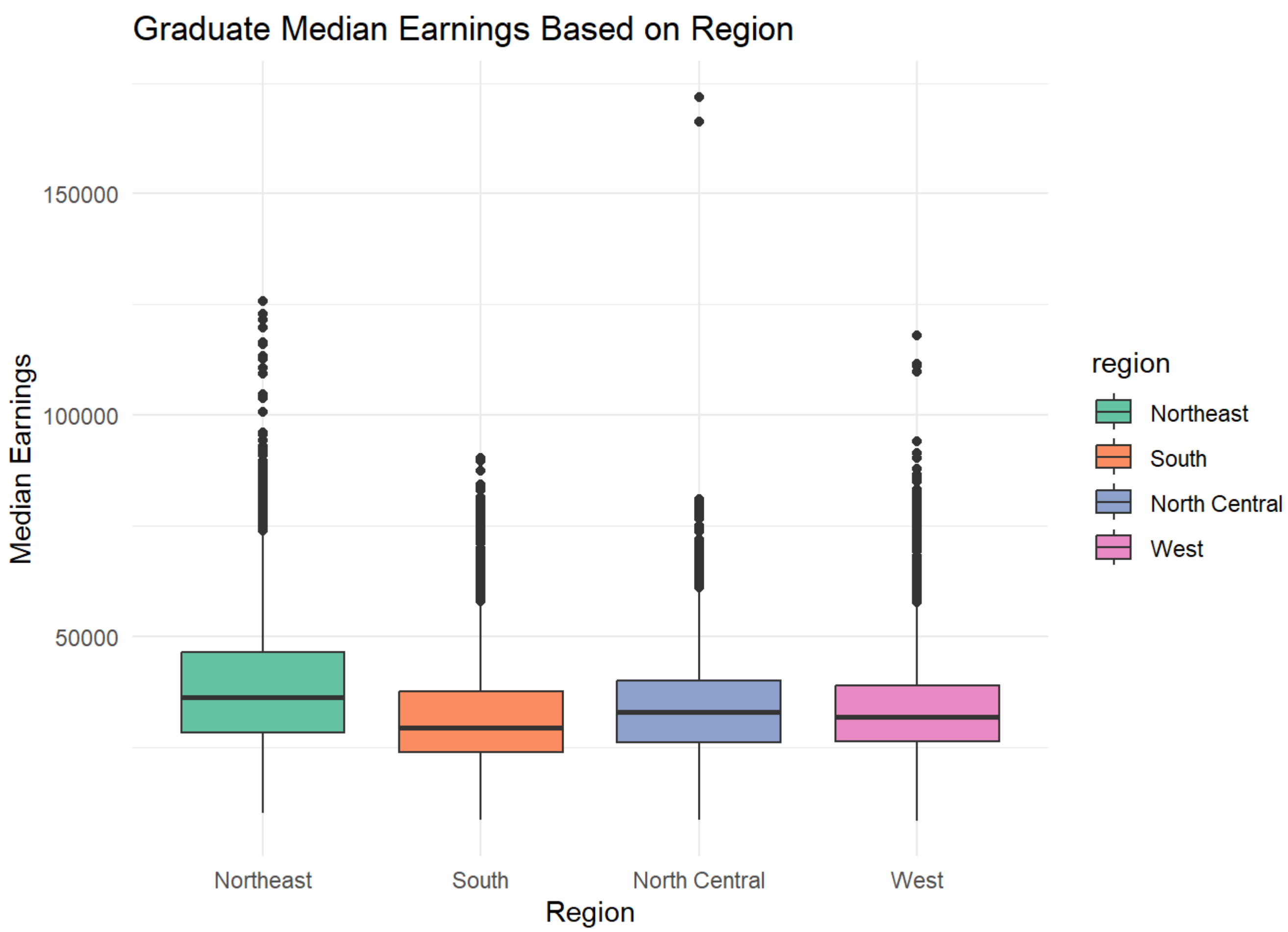
```
# Organize states into regions
scorecard$region <- state.region[match(scorecard$state_abbr, state.abb)]
scorecard <- na.omit(scorecard)

# Change variable name to 'degree'
scorecard = scorecard %>%
  mutate(degree = as.factor(pred_degree_awarded_ipeds))

glimpse(scorecard)
```

```
Rows: 30,401
Columns: 10
$ unitid                    <int> 100654, 100663, 100706, 100724, 100751, 1007…
$ inst_name                 <chr> "Alabama A & M University", "University of A…
$ state_abbr                <chr> "AL", "AL", "AL", "AL", "AL", "AL", "AL", "A…
$ pred_degree_awarded_ipeds <int> 3, 3, 3, 3, 3, 2, 3, 3, 3, 3, 2, 3, 3, 2, 2,…
$ year                      <int> 2007, 2007, 2007, 2007, 2007, 2007, 2007, 20…
$ earnings_med              <int> 36600, 40800, 49300, 30500, 46700, 28100, 41…
$ count_not_working         <int> 116, 366, 122, 210, 292, 113, 77, 193, 348, …
```

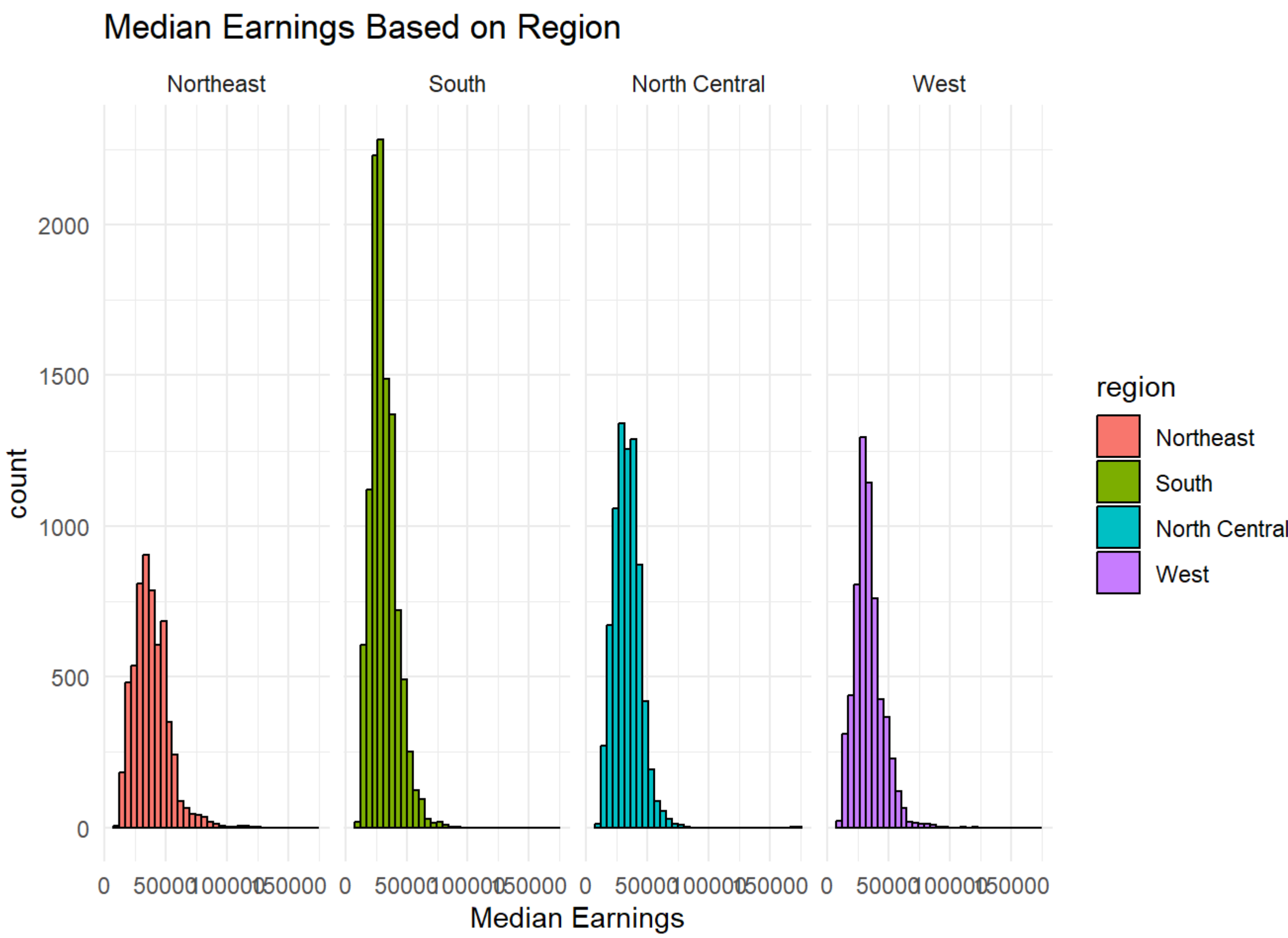
```
$ count_working            <int> 1139, 2636, 975, 1577, 2754, 590, 676, 1400,…
$ region                   <fct> South, South, South, South, South, South, So…
$ degree                   <fct> 3, 3, 3, 3, 3, 2, 3, 3, 3, 3, 2, 3, 3, 2, 2,…
```

Region Exploration

```
# Plot for Graduate Median Earnings Based on Region
scorecard %>% ggplot(aes(x = region, y = earnings_med, fill = region)) +
  geom_boxplot() +
  labs(title = "Graduate Median Earnings Based on Region",
       x = "Region",
       y = "Median Earnings" ) +
  scale_fill_brewer(palette = "Set2") +
  theme_minimal()
```



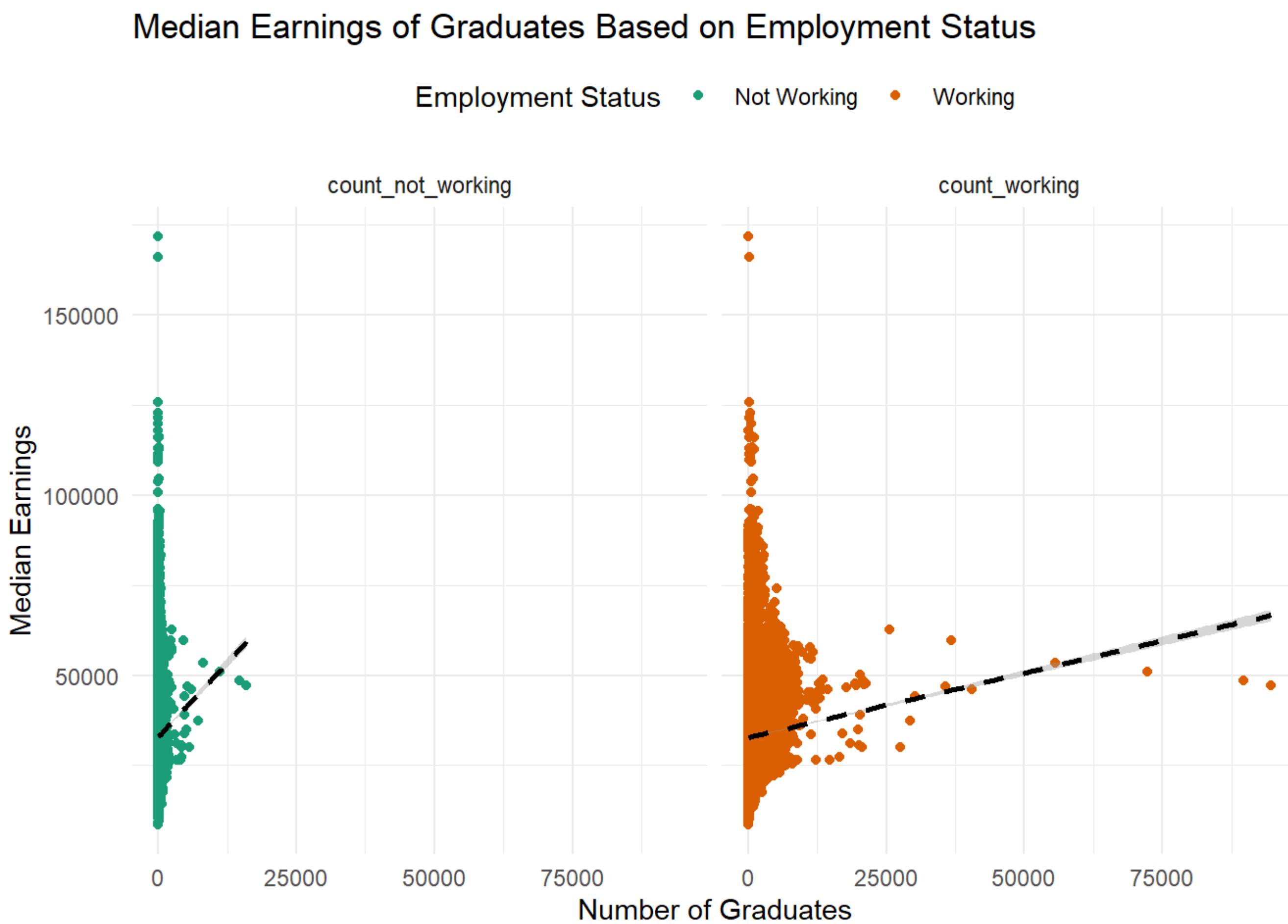

```
# Plot for Median Earnings Based on Region
scorecard %>%
  ggplot(aes(x = earnings_med, fill = region)) +
  geom_histogram(bins = 35, col = "black") + facet_grid(~region) +
  labs(title = "Median Earnings Based on Region",
       x = "Median Earnings") + facet_grid(~region) + theme_minimal()
```



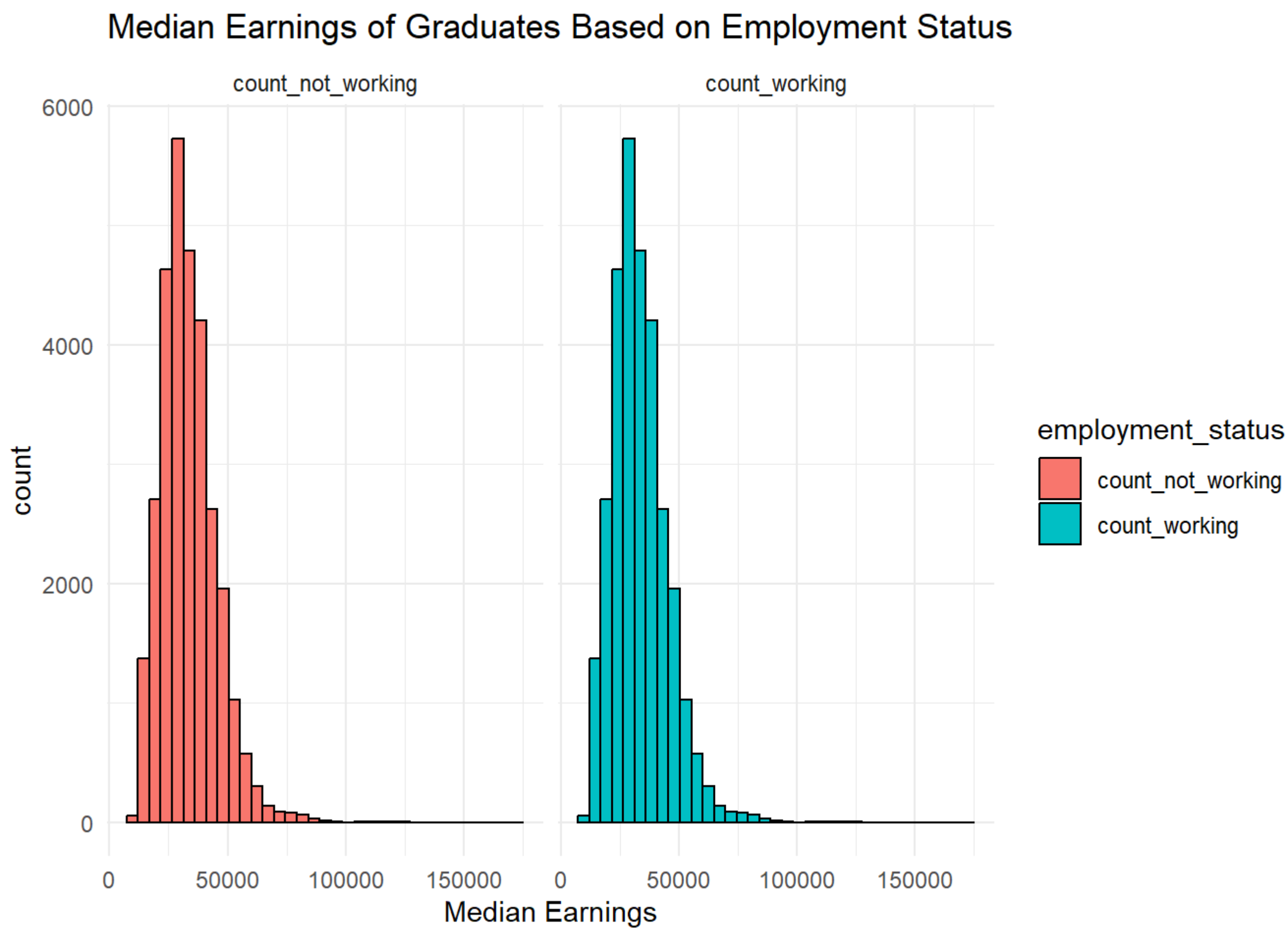
Employment Status vs. Median Earnings

```
# Create a new dataframe with tidy long performed on status of work
scorecard1 <- scorecard %>% gather(key = "employment_status", value = "working", c(count_not_work

scorecard1 <- scorecard1 %>% mutate(employment_status = as.factor(employment_status))

# Plot for Median Earnings of Graduates Based on Employment Status
scorecard1 %>%
  ggplot(aes(x = working, y = earnings_med, color = employment_status)) +
  geom_point() +   facet_grid(~employment_status) +
  labs(title = "Median Earnings of Graduates Based on Employment Status",
       x = "Number of Graduates",
       y = "Median Earnings") +
  scale_color_brewer(palette = "Dark2", name = "Employment Status", labels = c("Not Working", "Wo
  theme_minimal() +
  theme(legend.position = "top") +
  geom_smooth(method = "lm", linetype = "dashed", color = "black")
```

```
`geom_smooth()` using formula = 'y ~ x'
```

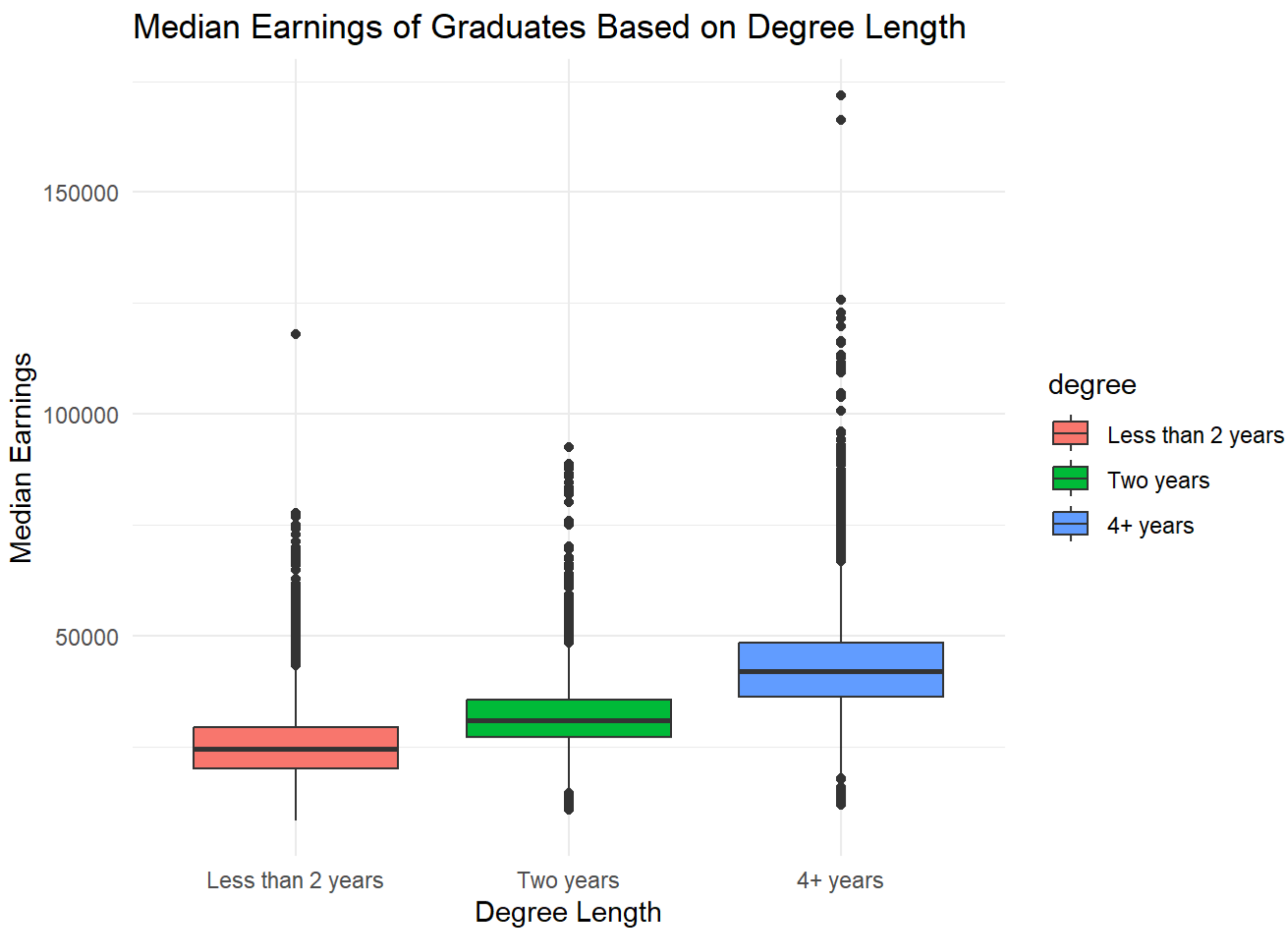



```
# Plot for Median Earnings of Graduates Based on Employment Status
scorecard1 %>%
  ggplot(aes(x =  earnings_med, fill = employment_status)) +
  geom_histogram(bins = 35, col = "black") + facet_grid(~employment_status) +
  labs(title = "Median Earnings of Graduates Based on Employment Status",
       x = "Median Earnings") +
  scale_color_brewer(palette = "Dark2", name = "Employment Status", labels = c("Not Working", "Wo
  theme_minimal()
```



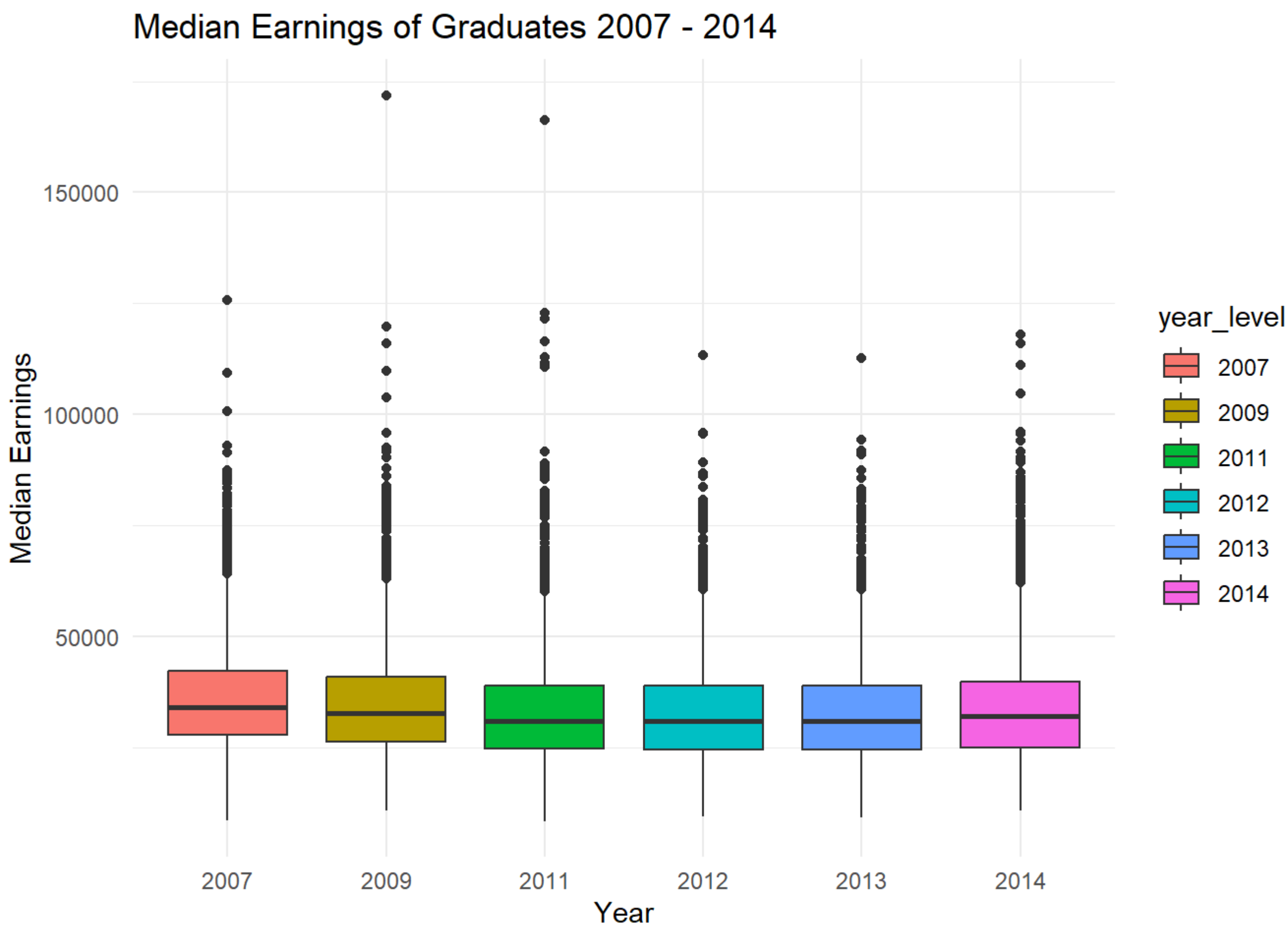
Degree Length vs. Median Earnings

```
# Plot for Median Earnings of Graduates Based on Degree Length
scorecard1 %>%
  ggplot(aes(x = degree, y = earnings_med, fill = degree)) +
  geom_boxplot() +
  labs(title = "Median Earnings of Graduates Based on Degree Length",
       x = "Degree Length", y = "Median Earnings") +  theme_minimal() + scale_fill_discrete(label
  scale_x_discrete(labels = c('Less than 2 years', 'Two years', '4+ years'))
```



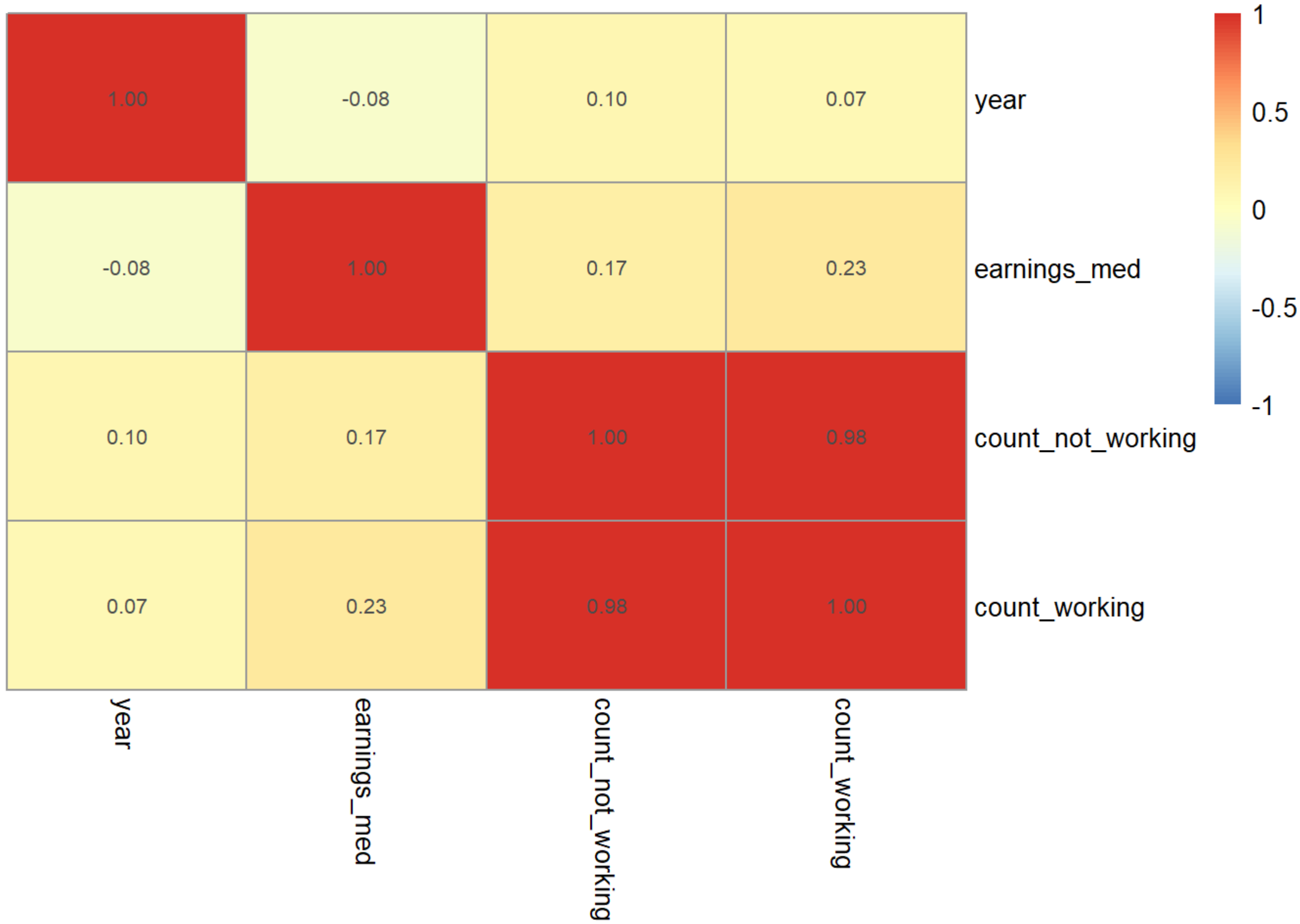
Year vs. Median Earnings

```
# changing year to a factor
scorecard1$year_level <- as.factor(scorecard1$year)

# Plot for Median Earnings of Graduates 2007 - 2014
scorecard1 %>%
  ggplot(aes(x = year_level, y = earnings_med, fill = year_level)) +
  geom_boxplot() +
  labs(title = "Median Earnings of Graduates 2007 - 2014",
       x = "Year", y = "Median Earnings") +  theme_minimal()
```



Correlations

```
# getting year as a numeric
scorecard$year_num <- as.numeric(scorecard$year)

# getting the heat map for the year, earnings_med, count_not_working, count_working
scorecard_cor <- cor(na.omit(scorecard[,c(5, 6, 7, 8)]))
pheatmap(scorecard_cor,
        treeheight_col = 0,
        treeheight_row = 0,
        display_numbers = TRUE,
        breaks = seq(-1, 1, length = 101))
```



Multiple Linear Regression Model

To assess for the presence of a predictive relationship between the median earnings of individuals graduating from colleges and universities across the United States and characteristics associated with their alma mater and post college lives, we constructed a linear model regressing median earnings on surveyed universities' regional location, the number of alumni both employed and not working (not necessarily unemployed), the primary degree awarded, and the year that each survey was conducted.

```
earnings_lm<-lm(earnings_med~region+degree+year+count_not_working+count_working, data=scorecard)
summary(earnings_lm)
```

```
Call:
lm(formula = earnings_med ~ region + degree + year + count_not_working +
    count_working, data = scorecard)

Residuals:
   Min     1Q Median     3Q    Max
-33245  -5048   -660   3946 130337

Coefficients:
```

```
                      Estimate Std. Error t value Pr(>|t|)    
(Intercept)          5.034e+05  4.173e+04   12.06   <2e-16 ***
regionSouth         -4.613e+03  1.364e+02  -33.82   <2e-16 ***
regionNorth Central -3.651e+03  1.456e+02  -25.07   <2e-16 ***
regionWest          -1.761e+03  1.552e+02  -11.35   <2e-16 ***
degree2              5.892e+03  1.220e+02   48.30   <2e-16 ***
degree3              1.567e+04  1.198e+02  130.82   <2e-16 ***
year                -2.359e+02  2.075e+01  -11.37   <2e-16 ***
count_not_working   -8.859e+00  2.397e-01  -36.97   <2e-16 ***
count_working        1.555e+00  3.766e-02   41.29   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 8373 on 30392 degrees of freedom
Multiple R-squared:  0.4928,    Adjusted R-squared:  0.4927 
F-statistic:  3691 on 8 and 30392 DF,  p-value: < 2.2e-16
```

```r
scorecard$region<-relevel(scorecard$region, ref="Northeast")
cat("Earnings median range:", range(scorecard$earnings_med))
```

```
Earnings median range: 8400 171900
```

The substantial F-statistic generated by the linear model of 3691 on 8 and 30392 degrees of freedom allowed us to reject the null hypothesis that none of the chosen variables possess any relationship to median earnings (all slopes are equal to zero) in favor of the alternative hypothesis that at least one of the predictive variables influences the earnings of American college graduates (at least one slope is not equal to zero). Given the confirmation of, at minimum, one of our independent variables' predictive power, we further explored the more nuanced ways in which each contributed to variation from the baseline predicted income of $503,340, as denoted by the intercept regression coefficient. Holding the influence of region, degrees typically awarded, year, and the number of graduates not actively employed constant, a one person increase in the number of gainfully employed graduates contributed to an institution results in a marginal $1.56 increase in predicted median earnings. Conversely, when controlling for the effect of all other predictors, the addition of a single non-working alumni unsurprisingly elicits a predicted $8.86 decline in predicted income. Assessment of the regression coefficient assigned to the year variable in the same manner revealed a slightly more impactful association between the year participants were surveyed and median earnings, with the passage of one year resulting in a loss of $235.90. Due to the categorical nature of the predominant degree awarded by collegiate study participants and the region in which each institution of higher learning resides, the analysis of their influence on predicted monetary outcomes diverged from that of aforementioned variables. As a hub for a variety of prestigious Universities, we anticipated that graduates from Northeastern schools would likely possess the highest median earnings and we accordingly designated it as the reference for our analysis of regional impacts. When controlling for the effects of all other variables and regions, prior attendance of a Southern school resulted in an average median earnings reduction of $4,613 from the Northeastern baseline. Upon similar evaluation, graduation from North Central and Western colleges comparably resulted in an average loss of $3,651 and $1,761, respectively. In considering the impact of the predominant degree awarded we identified the widest range of variation between predicted monetary outcomes, with the reference of less than 2 years differing by ampler amounts than the deviations observed between the regional categories. Controlling for all other variables and education levels, completion of a 2 year degree improved average predicted median income by $5,892, while graduation with a bachelor's degree raised income by an average of $15,670 after comparison to the baseline. Though all of the regression coefficients for both numeric and categorical variables possessed p-values significant at the zero level (p <2*10-16), the multiple R2 value of 0.498 indicates that only approximately 50% of the variation observed in median earnings for those surveyed is accounted for by the collegiate attributes analyzed above. This is reflected by the substantial residual standard error of 8373 on 30,392 degrees of freedom, meaning that the predicted values produced by the linear model deviate from actual monetary outcomes by an average of $8373. When compared to both the regression coefficients and the overall range of the actual median earnings values ($8604-$171900), the level of error observed in the estimates produced by the model is concerning and likely indicative of improper model fit through overfitting or multicollinearity.

Improving the Model

Setting up to analyze the models

```
results <- data.frame()
analyze_model <- function(model){
  # testing the if errors zero on-average
  # closer to zero is good
  resid_avg_zero_test <- t.test(resid(model), mu=0)

  # testing for constant variance
  # closer to zero is better
  heteroscedasticity_test <- bptest(model)

  # used for checking for overfitting
  #higher LOOR2 is better but R2 being much greater than LOOR2 incdicates overfitting
  LOOR2 <- rsq_loo(model)
  R2 <- summary(model)[["r.squared"]]


  # Ensure the types are compatible
  model_call_str <- as.character(paste(deparse(model$call), collapse = " "))
  resid_statistic <- as.numeric(resid_avg_zero_test$statistic)
  errors_zero_is_pass <-resid_avg_zero_test$p.value > .05
  bp_statistic <- as.numeric(heteroscedasticity_test$statistic)
  constant_variance_is_pass <- heteroscedasticity_test$p.value > .05
  loor2_value <- as.numeric(LOOR2)
  r2_value <- as.numeric(R2)

  # Append the new row
  new_row <- data.frame(model=model_call_str, t=resid_statistic, errors_zero_is_pass, bp=bp_stati

  # Making sure that the results are actually getting added
```

```
  results <<- rbind(results, new_row)
}
```

Q: Use diagnostic plots to assess whether any of the assumptions underlying the linear regression model are violated. Are the errors zero on-average for all fitted values? Do they have constant variance? Are the errors normally distributed?

```
analyze_model(earnings_lm)
print(results[NROW(results),]) # !!! TODO reformat this so it is easier to interpret
```

```
model
BP lm(formula = earnings_med ~ region + degree + year + count_not_working +      count_working,
data = scorecard)
              t errors_zero_is_pass       bp constant_variance_is_pass
BP 2.343518e-14                TRUE 855.6839                     FALSE
          R2     LOOR2
BP 0.4928242 0.4925416
```

A: According to the t.test for mean of average zero, the errors do appear to be zero on-average for all fitted values (t is extremely close to 0 which indicates a p-value close to zero and therefore is not significant evidence that the mean varies from zero). According to the Breusch-Pagan test, the errors do NOT have constant variance and are heteroskedastic (bp of 855.68 which would led to rejecting the test which indicates we reject the null hypthosesis / aka reject that it is homoskedastic). The normality is pretyy bad with the points at the higher quantiles doubling the expected ones.

Q: If there are any violations of the usual assumptions, see if you can use a transformation of either the outcome or some/all of the predictors to fix it. Then, reassess the diagnostic plots to see how well you did. (Note: it's often quite difficult to fix all off the assumptions simultaneously; just do the best you can, with the order of importance being mean-zero, then constant variance, then normality).

```
log_model <- lm(log(earnings_med) ~ region + degree + year + count_not_working +
    count_working, data = scorecard)
analyze_model(log_model)
results[NROW(results),]# !!! TODO reformat this so it is easier to interpret
```

```
model
BP1 lm(formula = log(earnings_med) ~ region + degree + year + count_not_working +
count_working, data = scorecard)
                t errors_zero_is_pass       bp constant_variance_is_pass
BP1 -3.335675e-14                TRUE 1597.033                     FALSE
           R2     LOOR2
BP1 0.5029592 0.5026963
```

A: Log dependent only transformation. According to the t.test for mean of average zero, the erros do appear to be zero on-average. According to the Breusch-Pagan test, the errors do Not have constant variance however the BP is worse than the previous model. The normality is better than the previous model with qq-plot with the residuals being nowhere as far off.

Q: For one of your numeric predictors, consider replacing the linear term $\beta j\ Xij$ with a natural spline using the ns() function from the splines library, and determine if there is evidence supporting the inclusion of a spline. (You may keep or discard to spline part of the model after doing this.)

```
for(k in 1:5){
  formula_str <- sprintf("lm(earnings_med ~ region + degree + year + ns(count_not_working, df=%d)
  spine_both_model <- eval(parse(text = formula_str))
  analyze_model(spine_both_model)

  formula_str <- sprintf("lm(earnings_med ~ region + degree + year + ns(count_not_working*count_wo
  spine_both_int_model <- eval(parse(text = formula_str))
  analyze_model(spine_both_model)


  formula_str <- sprintf("lm(log(earnings_med) ~ region + degree + year + ns(count_not_working, d
  spine_both_model <- eval(parse(text = formula_str))
  analyze_model(spine_both_model)

  formula_str <- sprintf("lm(log(earnings_med) ~ region + degree + year + ns(count_not_working*cou
  spine_both_int_model <- eval(parse(text = formula_str))
  analyze_model(spine_both_model)
}
```

A: none of the splines have a good impact on the model so we will not include one in our model !!!T ODO this is a lie??? because i was being dumb

Q: Hypothesize the existence of an interaction, then check to see if including the interaction is justified on the basis of the data by fitting a model with the interaction.

A: There is likely an interaction between count_not_working and count_working and year

Q: Interpret the interaction, regardless of its statistical significance. (You may keep or discard the interaction after doing this.)

```
count_year_int_model <- lm(earnings_med ~ region + degree + (count_not_working + count_working) *
summary(count_year_int_model)
```

```
Call:
lm(formula = earnings_med ~ region + degree + (count_not_working + 
    count_working) * year, data = scorecard)

Residuals:
   Min      1Q Median      3Q     Max
```

```
-33267  -5033   -647   3966 130287

Coefficients:
                          Estimate Std. Error t value Pr(>|t|)    
(Intercept)              5.522e+05  4.356e+04  12.676  < 2e-16 ***
regionSouth             -4.584e+03  1.367e+02 -33.542  < 2e-16 ***
regionNorth Central     -3.637e+03  1.456e+02 -24.973  < 2e-16 ***
regionWest              -1.696e+03  1.562e+02 -10.854  < 2e-16 ***
degree2                  5.923e+03  1.221e+02  48.525  < 2e-16 ***
degree3                  1.564e+04  1.204e+02 129.899  < 2e-16 ***
count_not_working       -1.252e+03  2.163e+02  -5.789 7.13e-09 ***
count_working            1.487e+02  2.953e+01   5.036 4.77e-07 ***
year                    -2.602e+02  2.166e+01 -12.009  < 2e-16 ***
count_not_working:year   6.179e-01  1.075e-01   5.747 9.15e-09 ***
count_working:year      -7.314e-02  1.468e-02  -4.981 6.35e-07 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 8368 on 30390 degrees of freedom
Multiple R-squared:  0.4934,	Adjusted R-squared:  0.4932 
F-statistic:  2960 on 10 and 30390 DF,  p-value: < 2.2e-16
```

```
analyze_model(count_year_int_model)
anova(earnings_lm, count_year_int_model)
```

```
Analysis of Variance Table

Model 1: earnings_med ~ region + degree + year + count_not_working + count_working
Model 2: earnings_med ~ region + degree + (count_not_working + count_working) * 
    year
  Res.Df        RSS Df  Sum of Sq      F   Pr(>F)    
1  30392 2.1307e+12                                  
2  30390 2.1282e+12  2 2445943783 17.463 2.63e-08 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

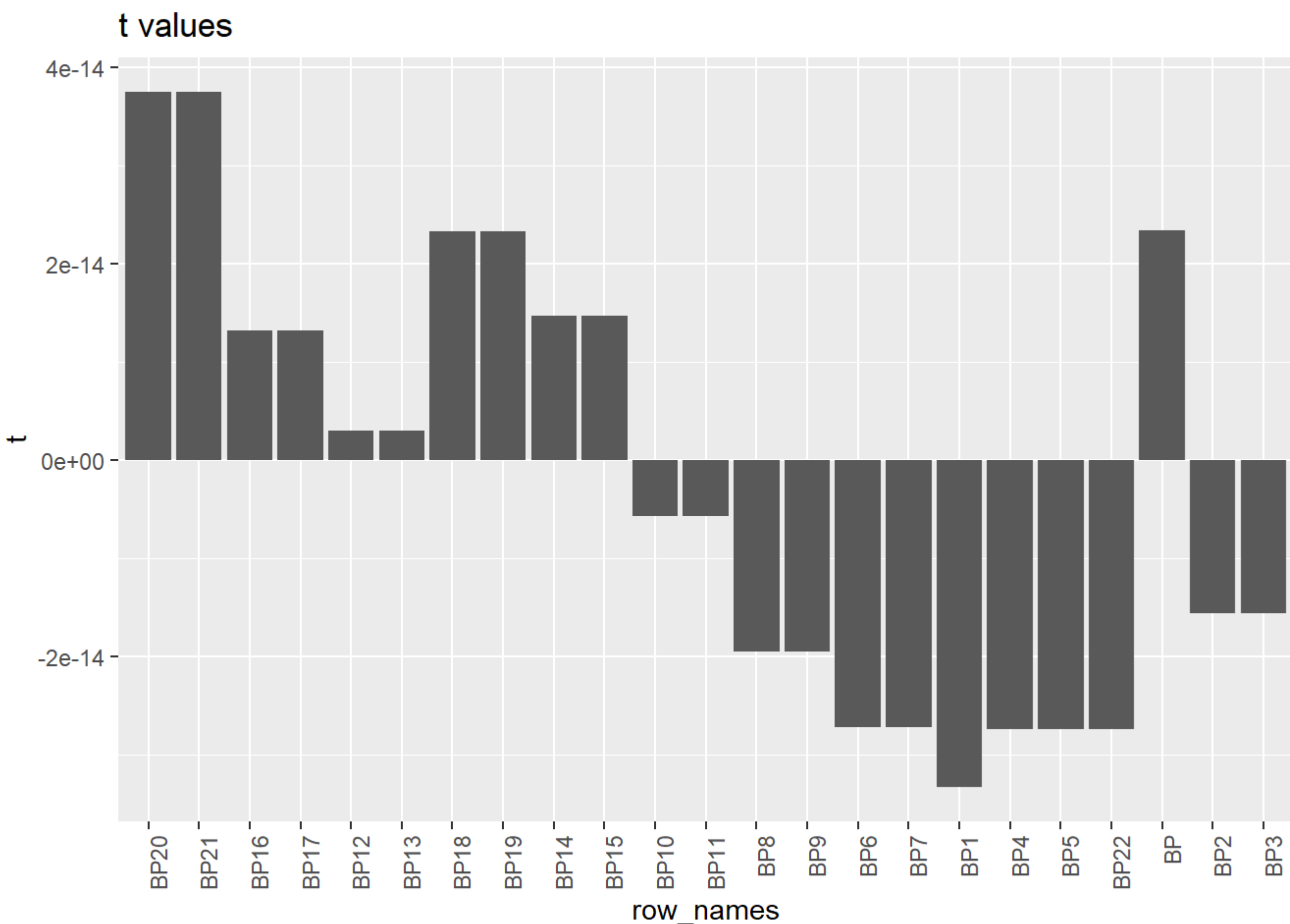
```
drop1(count_year_int_model, test = "F") # TODO FIX FORMATTING FOR THIS AREA
```

```
Single term deletions

Model:
earnings_med ~ region + degree + (count_not_working + count_working) * 
    year
                       Df  Sum of Sq        RSS    AIC  F value    Pr(>F)    
<none>                               2.1282e+12 549188                       
region                  3 9.2840e+10 2.2211e+12 550480  441.905 < 2.2e-16 ***
degree                  2 1.1864e+12 3.3146e+12 562653 8470.653 < 2.2e-16 ***
count_not_working:year  1 2.3133e+09 2.1305e+12 549219   33.033 9.149e-09 ***
count_working:year      1 1.7376e+09 2.1299e+12 549211   24.812 6.354e-07 ***
```

```
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

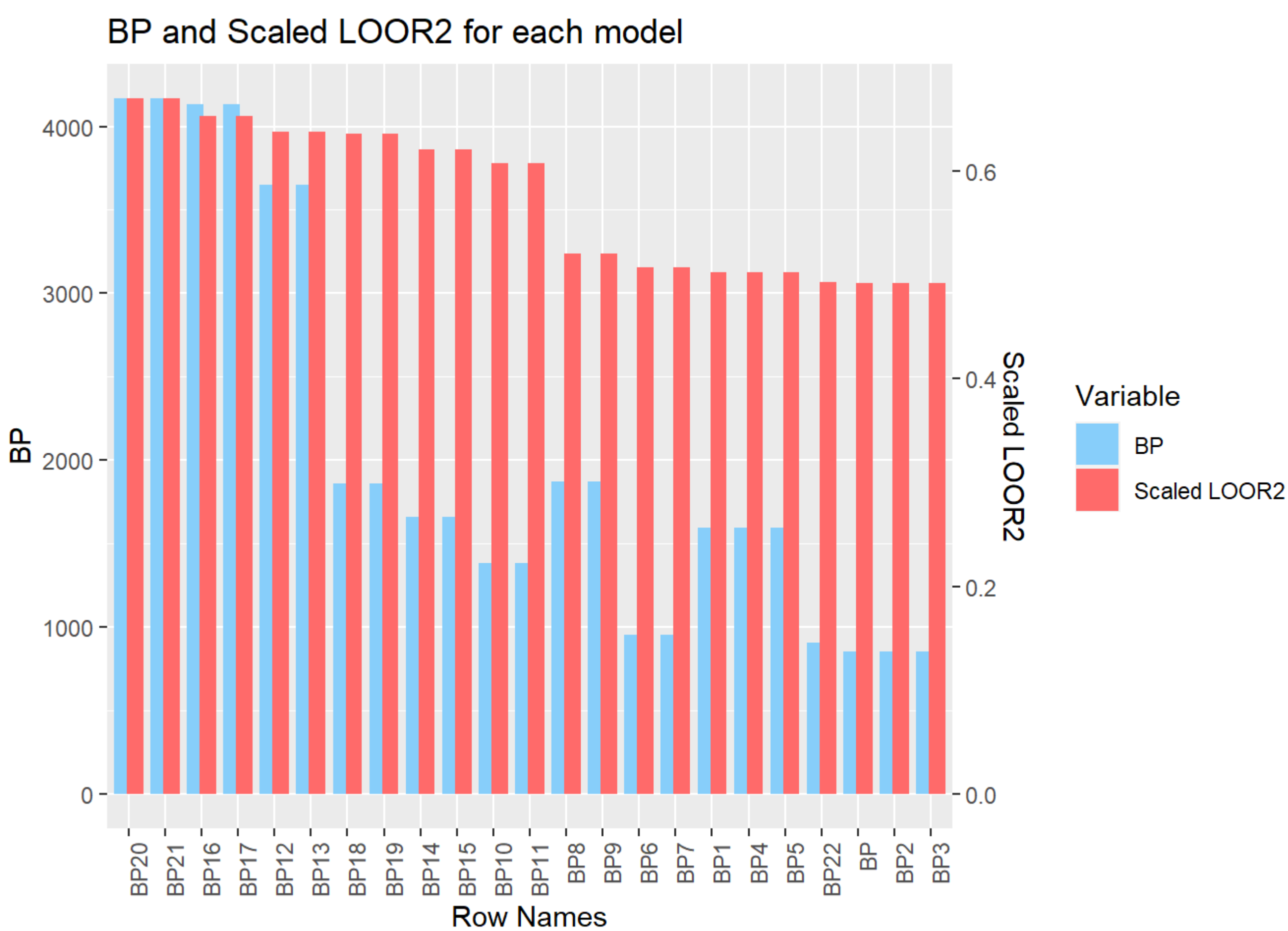
A: count_not_working:year with 6.179e-01 indicates that the model predicts for every 1 increase in count_not_working that the effect of year will increase 6.179e-01. count_working:year with -7.314e-02 indicates that the model predicts for every 1 increase in count_working that the effect of year will increase -7.314e-02.

```
results <- na.omit(results)
results$row_names <- row.names(results)
results <- results[order(-results$LOOR2), ]
results$row_names <- factor(results$row_names, levels = results$row_names)

# Plot for 't'
ggplot(results, aes(x = row_names, y = t)) +
  geom_bar(stat = "identity") +
  theme(axis.text.x = element_text(angle = 90, hjust = 1)) +
  ggtitle("t values")
```




```
# Create a transformation for 'LOOR2'
max_bp <- max(results$bp, na.rm = TRUE)
max_LOOR2 <- max(results$LOOR2, na.rm = TRUE)
scale_factor <- max_bp / max_LOOR2
```

```r
# Apply the unified scaling to both LOOR2 and R2
results$LOOR2_scaled <- results$LOOR2 * scale_factor

# Update melting to include both scaled LOOR2 and R2
long_results <- melt(results, id.vars = "row_names", measure.vars = c("bp", "LOOR2_scaled"))

# Update variable names for the legend
long_results$variable <- factor(long_results$variable, labels = c("BP", "Scaled LOOR2"))

# Plot
ggplot(long_results, aes(x = row_names, y = value, fill = variable)) +
  geom_bar(stat = "identity", position = position_dodge(width = 0.7)) +
  scale_fill_manual(values = c("BP" = "#87CEFA", "Scaled LOOR2" = "#FF6A6A")) +
  scale_y_continuous("BP", sec.axis = sec_axis(~ . / scale_factor, name = "Scaled LOOR2")) +
  theme(axis.text.x = element_text(angle = 90, hjust = 1)) +
  ggtitle("BP and Scaled LOOR2 for each model") +
  labs(x = "Row Names", y = "BP Value", fill = "Variable")
```




```r
results[1,]
```

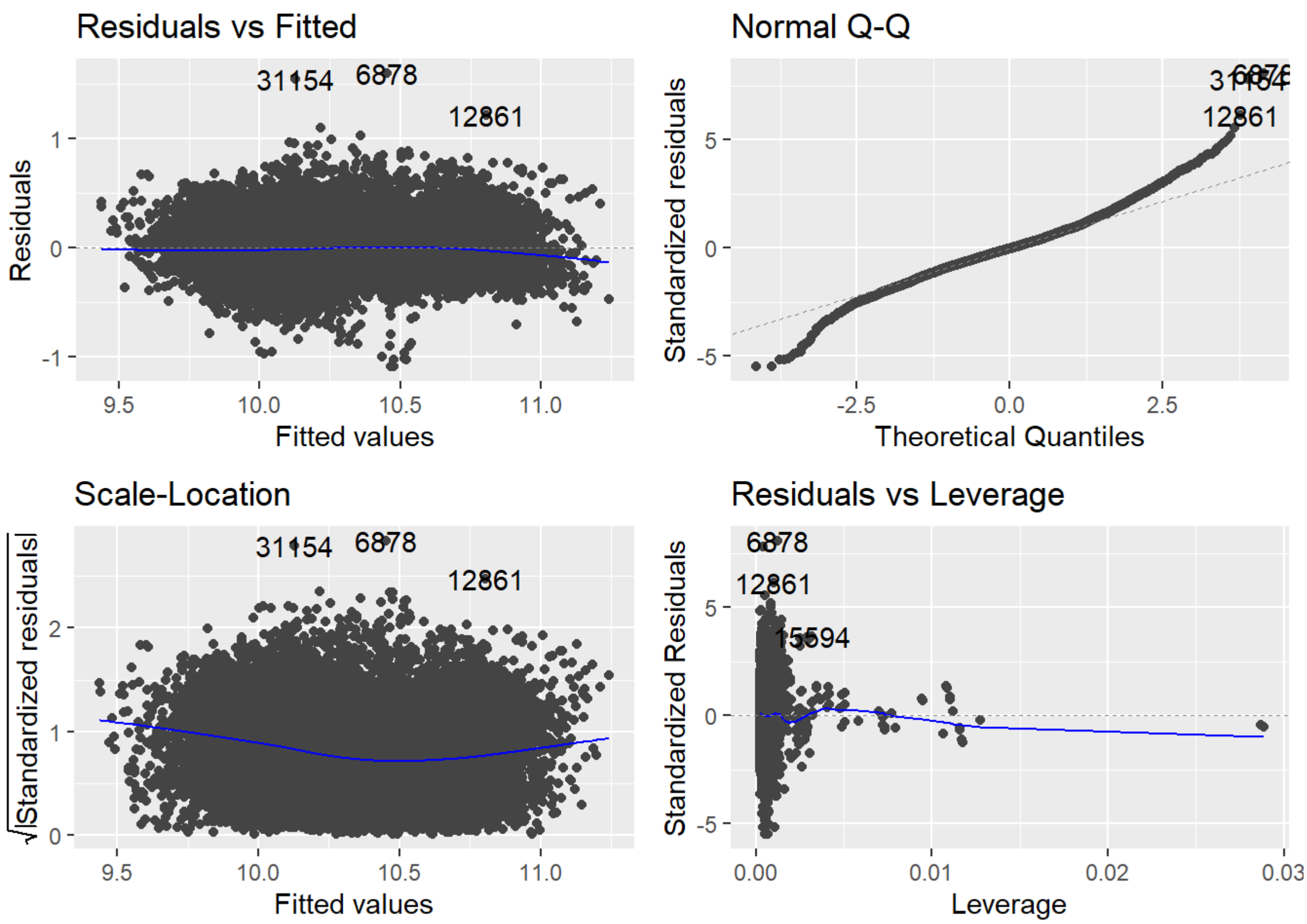
```
model
BP20 lm(formula = log(earnings_med) ~ region + degree + year + ns(count_not_working,      df = 5)
```

```
+ ns(count_working, df = 5), data = scorecard)
                t errors_zero_is_pass       bp constant_variance_is_pass
BP20 3.750755e-14                TRUE 4167.365                     FALSE
            R2     LOOR2 row_names LOOR2_scaled
BP20 0.6705142 0.6701226      BP20     4167.365
```

```
best_model <- eval(parse(text=results[1,"model"]))
summary(best_model)
```

```
Call:
lm(formula = log(earnings_med) ~ region + degree + year + ns(count_not_working,
    df = 5) + ns(count_working, df = 5), data = scorecard)

Residuals:
     Min       1Q   Median       3Q      Max
-1.09080 -0.12070 -0.00725  0.11245  1.60130

Coefficients:
                                 Estimate Std. Error t value Pr(>|t|)
(Intercept)                    15.0553770  1.0270091  14.659  < 2e-16 ***
regionSouth                    -0.0862461  0.0032675 -26.395  < 2e-16 ***
regionNorth Central            -0.1021554  0.0034527 -29.587  < 2e-16 ***
regionWest                      0.0285273  0.0037829   7.541 4.79e-14 ***
degree2                         0.0993290  0.0031403  31.631  < 2e-16 ***
degree3                         0.2253786  0.0036532  61.694  < 2e-16 ***
year                           -0.0023121  0.0005115  -4.520 6.20e-06 ***
ns(count_not_working, df = 5)1 -1.0130723  0.0112104 -90.369  < 2e-16 ***
ns(count_not_working, df = 5)2 -1.3584820  0.0137695 -98.659  < 2e-16 ***
ns(count_not_working, df = 5)3 -3.3317829  0.0516781 -64.472  < 2e-16 ***
ns(count_not_working, df = 5)4 -3.2037915  0.0966333 -33.154  < 2e-16 ***
ns(count_not_working, df = 5)5 -0.8452659  0.1873692  -4.511 6.47e-06 ***
ns(count_working, df = 5)1      0.9135139  0.0085746 106.537  < 2e-16 ***
ns(count_working, df = 5)2      1.3030282  0.0111868 116.479  < 2e-16 ***
ns(count_working, df = 5)3      3.6453484  0.0528845  68.930  < 2e-16 ***
ns(count_working, df = 5)4      3.4197801  0.0847634  40.345  < 2e-16 ***
ns(count_working, df = 5)5      1.2464874  0.1778226   7.010 2.44e-12 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.1982 on 30384 degrees of freedom
Multiple R-squared:  0.6705,    Adjusted R-squared:  0.6703
F-statistic:  3865 on 16 and 30384 DF,  p-value: < 2.2e-16
```

```
autoplot(best_model)
```



Formal Hypothesis Tests

To begin the project, we wanted to investigate how the number of working graduates (`count_working`), region of the university (`region`), and degree type (`degree`) relate to the median income of graduates (`earnings_med`). In this section, we tested if each of these three variables are significant in predicting the median income. Firstly, we used the following equation to represent the relationship between median earnings and the chosen predictors:

$$Y = \beta_0 + \beta_{r_1}X_{r_1} + \beta_{r_2}X_{r_2} + \beta_{r_3}X_{r_3} + \beta_{d_1}X_{d_2} + \beta_{d_2}X_{d_2} + \beta_y X_y + \beta_n X_n + \beta_w X_w + \epsilon$$

Where: Y = `earnings_med` , X_r = `region`, X_d = `degree`, X_y = `year`, X_n = `count_not_working`, and X_w = `count_working` Using our final model, `earnings_lm`, we performed the following hypotheses testing:

For `region`:

- H_0: $\beta_{r_1} = \beta_{r_2} = \beta_{r_3} = 0$
- H_a: $\beta_{r1} \neq \beta_{r2} \neq \beta_{r3} \neq 0$

For `degree`:

- H_0: $\beta_{d_1} = \beta_{d_2} = 0$

- H_a: $\beta_{d_1} \neq \beta_{d_2} \neq 0$

For `count_working`:

- H_0: $\beta_w = 0$
- H_a: $\beta_w \neq 0$

Using the p-values from the `drop1` function, we see that β_r, β_g, and β_w are all significant predictors of `earnings_med`.

```
drop1(earnings_lm, test = "F")
```

```
Single term deletions

Model:
earnings_med ~ region + degree + year + count_not_working + count_working
                  Df  Sum of Sq        RSS    AIC F value    Pr(>F)    
<none>                          2.1307e+12 549219                      
region             3 9.3190e+10 2.2238e+12 550514  443.09 < 2.2e-16 ***
degree             2 1.2069e+12 3.3375e+12 562859 8607.45 < 2.2e-16 ***
year               1 9.0561e+09 2.1397e+12 549346  129.18 < 2.2e-16 ***
count_not_working  1 9.5798e+10 2.2265e+12 550554 1366.48 < 2.2e-16 ***
count_working      1 1.1952e+11 2.2502e+12 550876 1704.88 < 2.2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Furthermore, using the `summary` functions we see that β_{r1}, β_{r2}, β_{r3}, β_{d1}, and β_{d2} are all significant predictors of Y. We therefore reject H_0 for both `region` and `degree` and conclude that median income changes based on the regional location of the college and the type of degree the college offers. Also, we see that there is significant evidence that β_w is positive (which confirms our hypothesis in Part 1). We therefore reject H_0 for `count_working` variable and conclude that median earnings tend to increase as the number of working graduates increases.

```
summary(earnings_lm)
```

```

Call:
lm(formula = earnings_med ~ region + degree + year + count_not_working + 
    count_working, data = scorecard)

Residuals:
   Min     1Q Median     3Q    Max 
-33245  -5048   -660   3946 130337 

Coefficients:
                    Estimate Std. Error t value Pr(>|t|)    
(Intercept)        5.034e+05  4.173e+04   12.06   <2e-16 ***
regionSouth       -4.613e+03  1.364e+02  -33.82   <2e-16 ***
```

```
regionNorth Central -3.651e+03  1.456e+02  -25.07   <2e-16 ***
regionWest          -1.761e+03  1.552e+02  -11.35   <2e-16 ***
degree2              5.892e+03  1.220e+02   48.30   <2e-16 ***
degree3              1.567e+04  1.198e+02  130.82   <2e-16 ***
year                -2.359e+02  2.075e+01  -11.37   <2e-16 ***
count_not_working   -8.859e+00  2.397e-01  -36.97   <2e-16 ***
count_working        1.555e+00  3.766e-02   41.29   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 8373 on 30392 degrees of freedom
Multiple R-squared:  0.4928,    Adjusted R-squared:  0.4927
F-statistic:  3691 on 8 and 30392 DF,  p-value: < 2.2e-16
```

To further investigate the `region` and `degree` variables, we ran the respective `constrast` functions and found that income significantly varies between all regions as well as between all degree types which confirms our initial hypothesis stated in Part 1.

```r
cat("Comparing median income between regions:", "\n")
```

```
Comparing median income between regions:
```

```r
contrast(emmeans(earnings_lm, ~ region), method = "pairwise", adjust = "none")
```

```
 contrast                  estimate  SE    df t.ratio p.value
 Northeast - South             4613 136 30392  33.816  <.0001
 Northeast - North Central     3651 146 30392  25.072  <.0001
 Northeast - West              1761 155 30392  11.348  <.0001
 South - North Central         -962 126 30392  -7.644  <.0001
 South - West                 -2852 135 30392 -21.172  <.0001
 North Central - West         -1890 145 30392 -12.998  <.0001

Results are averaged over the levels of: degree
```

```r
cat("\n","Comparing median income between degrees:", "\n", sep = "")
```

```

Comparing median income between degrees:
```

```r
contrast(emmeans(earnings_lm, ~ degree), method = "pairwise", adjust = "none")
```

```
 contrast          estimate  SE    df  t.ratio p.value
 degree1 - degree2    -5892 122 30392  -48.304  <.0001
 degree1 - degree3   -15673 120 30392 -130.823  <.0001
 degree2 - degree3    -9781 128 30392  -76.311  <.0001

Results are averaged over the levels of: region
```

In conclusion, based on our findings, all of our initial hypotheses seem to be confirmed. The median earnings do seem to increase with the number of graduates that are able to find a job. The earnings also vary based on degree type the graduate received and the geographic region of the US where the college is located. These conclusions do have serious limitations though. Firstly, our model contained only 5 predictors all of which were found to be significant. However, the inclusion of more predictors can affect the trends of the model and change the significance of each of the original 5 predictors. Also, we need to consider the possibility of existence of confounding variables. For example, it is possible that graduates who go to elite colleges are more likely to both find a job and earn a higher wage. Along with this, some regions in the US, like the Northeast, tend to have many states with a significantly higher cost of living which can explain the difference in median earnings. It is also important to account for the fact that we performed multiple tests in this section, hence we adjusted our p-values using the Bonferroni correction. Firstly, for both region and degree variables, we repeated the pairwise comparisons using the Bonferroni adjusted p-values. In both cases, our conclusions did not change.

```
cat("Comparing median income between regions:", "\n")
```

```
Comparing median income between regions:
```

```
contrast(emmeans(earnings_lm, ~ degree), method = "pairwise", adjust = "bonferroni")
```

```
 contrast          estimate  SE    df  t.ratio p.value
 degree1 - degree2    -5892 122 30392  -48.304  <.0001
 degree1 - degree3   -15673 120 30392 -130.823  <.0001
 degree2 - degree3    -9781 128 30392  -76.311  <.0001

Results are averaged over the levels of: region
P value adjustment: bonferroni method for 3 tests
```

```
cat("\n", "Comparing median income between degrees:", "\n", sep="")
```

```

Comparing median income between degrees:
```

```
contrast(emmeans(earnings_lm, ~ region), method = "pairwise", adjust = "bonferroni")
```

```
 contrast                  estimate  SE    df t.ratio p.value
 Northeast - South             4613 136 30392  33.816  <.0001
 Northeast - North Central     3651 146 30392  25.072  <.0001
 Northeast - West              1761 155 30392  11.348  <.0001
 South - North Central         -962 126 30392  -7.644  <.0001
 South - West                 -2852 135 30392 -21.172  <.0001
 North Central - West         -1890 145 30392 -12.998  <.0001

Results are averaged over the levels of: degree
P value adjustment: bonferroni method for 6 tests
```

Then, since we tested three separate sets of hypotheses, the resulting p-values had to be multiplied by a factor of 3 to perform the Bonferroni correction. However, in all three cases we ended up with a $p - value < 2 * 10^{-16}$ so it follows that we still must reject H_0 in all three cases.

Robustness of Results

• Use the function lm.boot in the simpleboot package to compute the bootstrap standard

errors of the regression coefficients. How do these compare in magnitude to the standard

errors from the output of summary? Are there any problems suggested by these standard

errors?

```
earnings_boot <- lm.boot(earnings_lm, R=500,000)
summary(earnings_boot)
```

```
BOOTSTRAP OF LINEAR MODEL  (method = residuals)

Original Model Fit
------------------
Call:
lm(formula = earnings_med ~ region + degree + year + count_not_working +
    count_working, data = scorecard)

Coefficients:
        (Intercept)          regionSouth  regionNorth Central
         503400.046            -4613.457            -3651.164
         regionWest              degree2              degree3
          -1761.305             5891.914            15673.119
               year    count_not_working        count_working
           -235.889               -8.859                1.555

Bootstrap SD's:
        (Intercept)          regionSouth  regionNorth Central
       4.262499e+04         1.349504e+02         1.391159e+02
         regionWest              degree2              degree3
       1.489281e+02         1.144888e+02         1.182471e+02
               year    count_not_working        count_working
       2.119831e+01         2.355493e-01         3.749265e-02
```

```
## Compute the T-statistic
t_boot <- coef(earnings_lm) / summary(earnings_boot)[["stdev.params"]]
t_orig <- coef(earnings_lm) / summary(earnings_lm)$coefficients[,2]

## Compute the P-values
p_boot <- 2 * pt(abs(t_boot), df = earnings_lm$df.residual, lower.tail = FALSE)
p_orig <- 2 * pt(abs(t_orig), df = earnings_lm$df.residual, lower.tail = FALSE)
```

```
## Print T statistics and P-values
print(cbind(`sd-orig` = summary(earnings_lm)$coefficients[,2], `sd-boot` = summary(earnings_boot)
```

```
                       sd-orig      sd-boot   boot - orig diff / o (%)
(Intercept)         4.173481e+04 4.262499e+04  8.901807e+02    2.1329454
regionSouth         1.364273e+02 1.349504e+02 -1.476865e+00   -1.0825287
regionNorth Central 1.456266e+02 1.391159e+02 -6.510685e+00   -4.4708065
regionWest          1.552075e+02 1.489281e+02 -6.279390e+00   -4.0458027
degree2             1.219756e+02 1.144888e+02 -7.486779e+00   -6.1379333
degree3             1.198041e+02 1.182471e+02 -1.556984e+00   -1.2996087
year                2.075459e+01 2.119831e+01  4.437168e-01    2.1379211
count_not_working   2.396662e-01 2.355493e-01 -4.116875e-03   -1.7177540
count_working       3.766204e-02 3.749265e-02 -1.693894e-04   -0.4497616
```

```
print(cbind(`t-orig` = t_orig, `P-orig` = p_orig, `t-boot` = t_boot, `P-boot` = p_boot))
```

```
                      t-orig         P-orig    t-boot         P-boot
(Intercept)          12.06187  2.002191e-33  11.80997  4.075569e-32
regionSouth         -33.81623 4.189453e-246 -34.18630 2.254844e-251
regionNorth Central -25.07209 2.501101e-137 -26.24547 3.806822e-150
regionWest          -11.34806  8.705499e-30 -11.82654  3.349198e-32
degree2              48.30405  0.000000e+00  51.46280  0.000000e+00
degree3             130.82291  0.000000e+00 132.54548  0.000000e+00
year                -11.36565  7.125026e-30 -11.12774  1.044681e-28
count_not_working   -36.96596 1.238241e-292 -37.61204 1.203590e-302
count_working        41.29013  0.000000e+00  41.47668  0.000000e+00
```

All of the coeficents are very closely related and not outside of a magnitude of each other. With the largest change being in degree 2 with a percent change of -2.83. Due to them being closely related, there appear to be no issues suggested by these.

• Use cross-validation to estimate the leave-one-out prediction error.

```
LOOR2 <- rsq_loo(earnings_lm)
R2 <- summary(earnings_lm)[["r.squared"]]

print(cbind(`LOOR2`=LOOR2, `R2`=R2))
```

```
         LOOR2        R2
[1,] 0.4925416 0.4928242
```

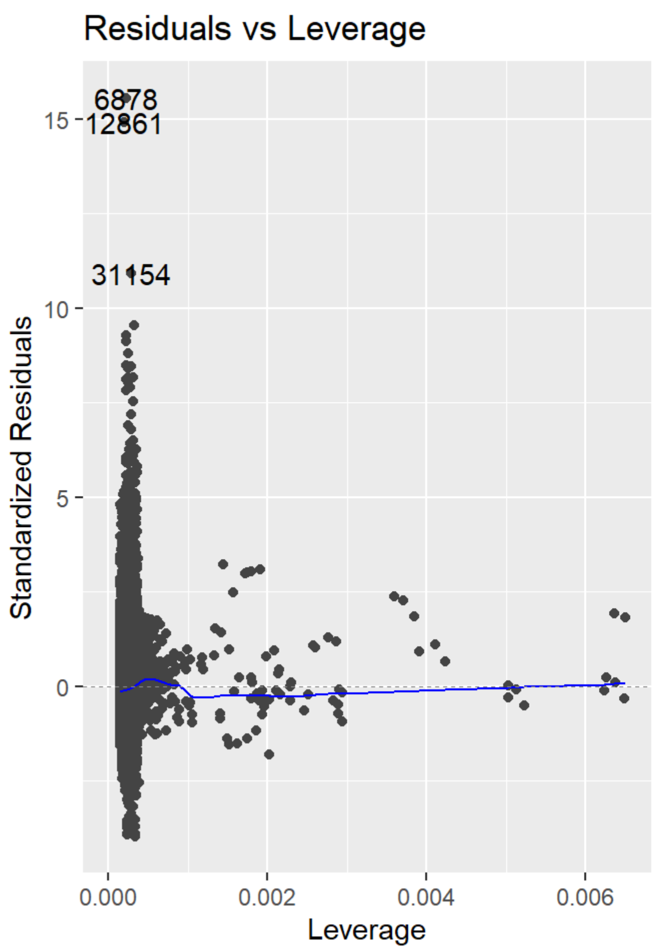
• Comment if there is (or is not) evidence that your model is overfit.

There does not appear to be evidence that the model is overfit. Especially sin

• Revisit the output of autoplot to check for any highly influential points. If you find

any, what features of these observations lead to them having high leverage, and what influence does deleting these observations have on your results?

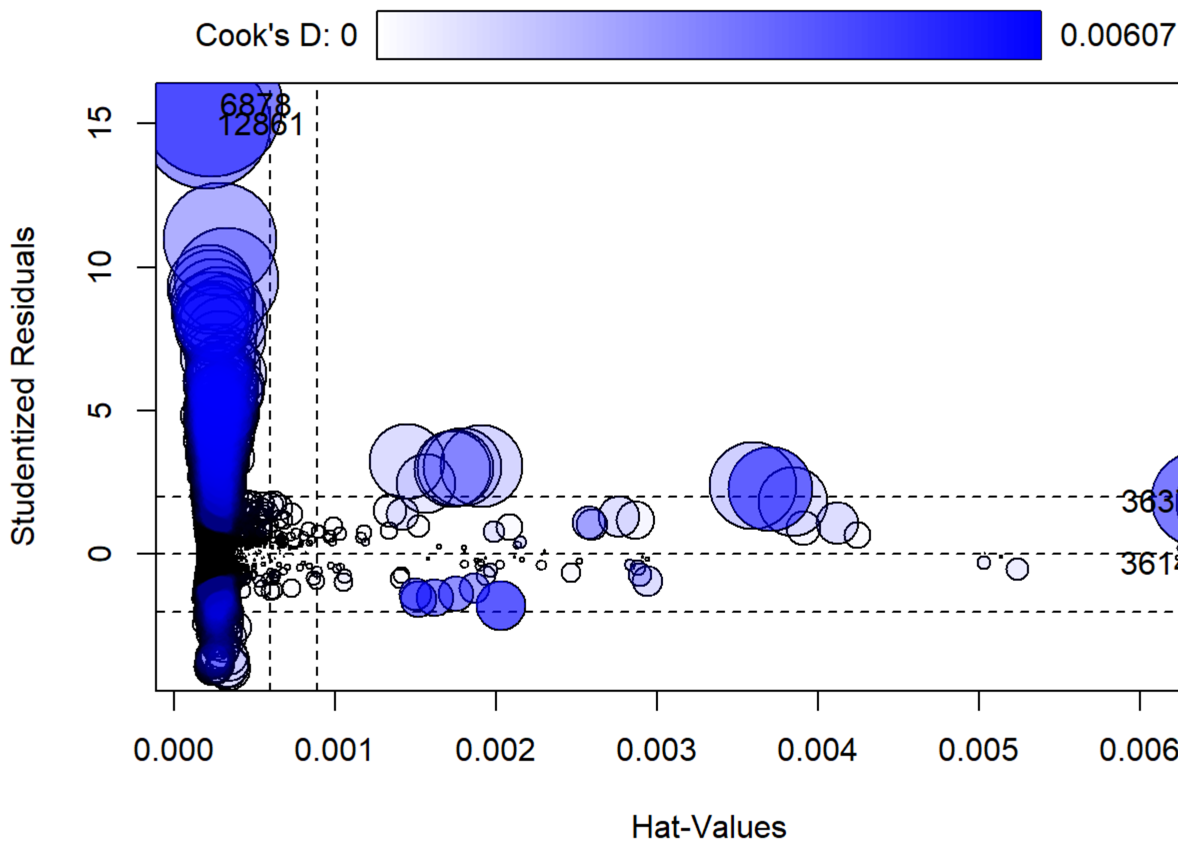
```
ap <- autoplot(earnings_lm)
ap[4]
```




```
# using
influencePlot(earnings_lm)
```

```
        StudRes          Hat        CookD
6878  15.630475 0.0002254133 6.071784e-03
12861 14.978881 0.0001986742 4.917719e-03
36141 -0.335153 0.0064934965 8.157635e-05
36374  1.804679 0.0065014905 2.367940e-03
```

```
ip <- influencePlot(earnings_lm)
```

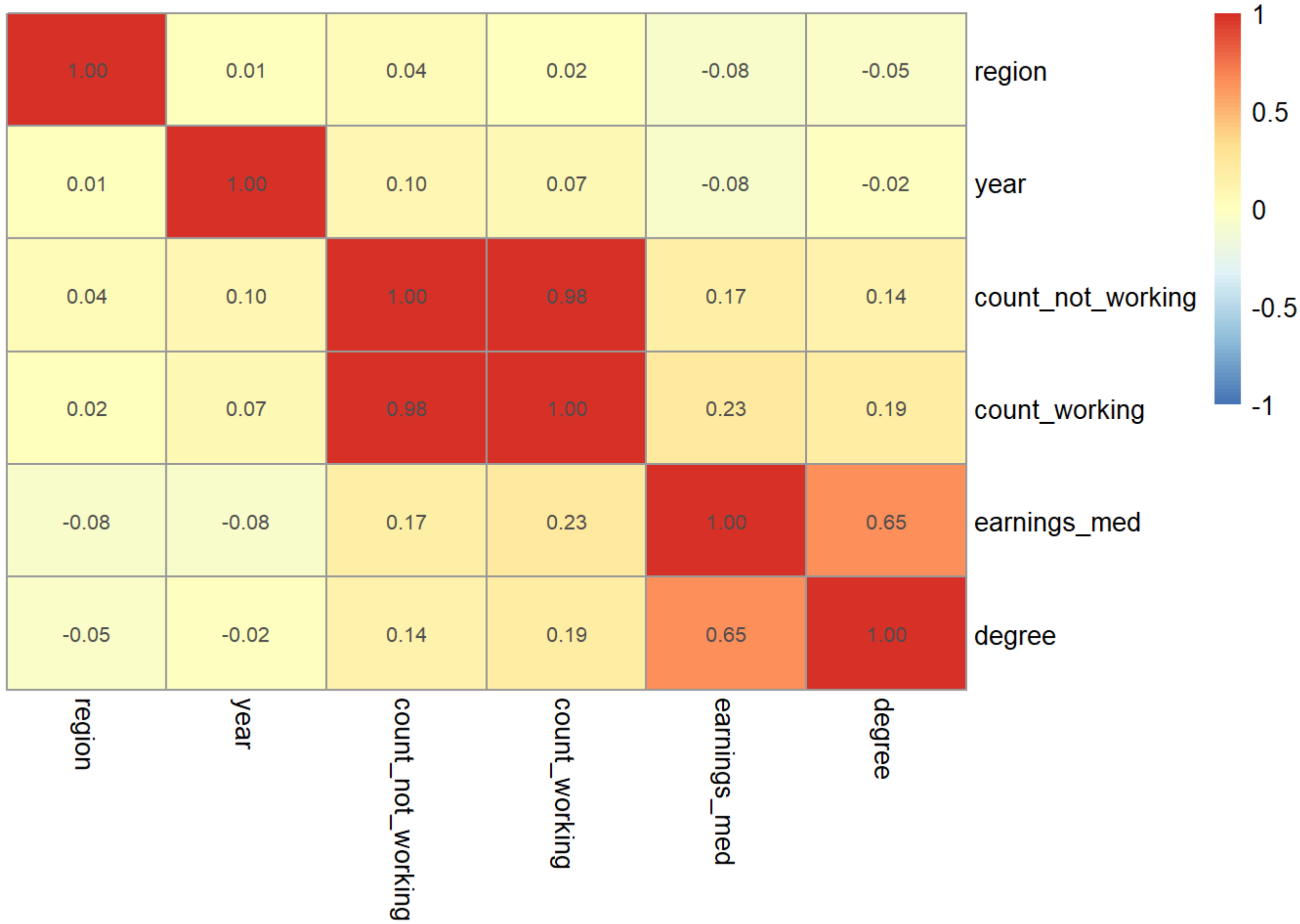



```
hip <- as.numeric(row.names(ip))
earnings_lm$model[hip,]
```

```
      earnings_med    region degree year count_not_working count_working
8100         45400 Northeast      3 2009                 54           399
15075        31100 Northeast      2 2011                151           478
NA              NA      <NA>   <NA>   NA                 NA            NA
NA.1            NA      <NA>   <NA>   NA                 NA            NA
```

- Are there any potential issues with multicolinearity of predictors in your data? Compute variance inflation factors for each of your predictors to assess this.

```
numeric_df <- do.call(data.frame, lapply(earnings_lm$model, FUN = as.numeric.l))
pheatmap(cor(numeric_df), treeheight_col = 0, treeheight_row = 0, display_numbers = TRUE, breaks =
```




```
vif_values <- vif(earnings_lm)
print(vif_values)
```

```
                       GVIF Df GVIF^(1/(2*Df))
region             1.036907  3        1.006059
degree             1.167530  2        1.039482
year               1.033172  1        1.016451
count_not_working 36.115252  1        6.009597
count_working     36.561674  1        6.046625
```

```
# redoing with better lm

earnings_lm <- lm(earnings_med~region+degree+year+count_not_working, data=scorecard)
numeric_df <- do.call(data.frame, lapply(earnings_lm$model, FUN = as.numeric.l))
pheatmap(cor(numeric_df), treeheight_col = 0, treeheight_row = 0, display_numbers = TRUE, breaks =
```


```
vif_values <- vif(earnings_lm)
print(vif_values)
```

```
                      GVIF Df GVIF^(1/(2*Df))
region            1.020995  3        1.003469
degree            1.039036  2        1.009619
year              1.010920  1        1.005445
count_not_working 1.034970  1        1.017335
```

- high multicolinearity between count_working and count_not_working which is also indicated by the extremely high vif for both of them
- once you remove one of the variables the vifs become a lot more balanced and within 1 of each other

Conclusions

(Your text here)