University of Exeter

College of Engineering, Mathematics
and Physical Sciences

COMM510

Multi-Objective Optimisation & Decision Making

Continuous Assessment

Date Set: 8th July 2022
Date Due: 8th August 2022

This CA comprises 100% of the overall module assessment.

This is an **individual** exercise, and your attention is drawn to the guidelines on collaboration and plagiarism in the College Handbook (`https://student-harrison.emps.ex.ac.uk/`).

This referred/deferred assessment of COMM510 covers multiple aspects of research in multi-objective optimisation and decision making via a small research project. This assignment includes engaging with literature (independent reading); formulating a research programme addressing a particular question; reporting on computational experiments; analysis of results; relating results to existing literature; identifying future research directions given the results. Please ensure you read the entire document before you begin the assessment. Note, marks awarded for *referred* assessments are capped.

1 Assignment

This referred/deferred coursework requires the submission of a concise document on a research topic you have chosen from the list in Section 2. The submission must include a background and literature review, a description of the research programme you undertook to address the research topic, including aspects such as the detailed research questions you examined and experimental design, followed by a presentation and analysis of your results.

The report summarising this work should contain:

- An introduction.
- A review of the literature about the topic, describing the background to the chosen research question and what work has been done in the existing literature to investigate it.
- A reasoned plan of the empirical work to be undertaken, including (as suitable), algorithms to compare, test problems to use, quality measures to employ, experimental protocols, how the results will be evaluated, etc.
- Your prior expectations of the results, given any insights from the literature and your understanding of the task and research question. This might take the form of a research hypothesis to be investigated.
- A presentation of the results obtained in an appropriate form (e.g. tables, plots, etc.).
- An analysis of the results.
- A contextualisation of the results, relating them to the existing literature.
- A conclusion which outlines potential future research directions that lead on from your work.

2 Topics

Below is the list of research topics for the COMM510 coursework, which you will already be familiar with from CA1 and CA2, along with a couple of initial references which you should find helpful for each topic. You must select one topic for this assignment.

Behaviour coupling of archivers and optimisers. It has been known for a number of years that limiting the size of an archive or approximation set of non-dominated solutions, a practice known as *truncation*, can degrade the efficacy of a search. However recent work (Li, 2021) has highlighted that even simple artificial sequences (hand-tuned sequences of objective vectors exhibiting particular pathological properties) can cause serious problems for some archiving approaches. The aim of this project is to investigate and measure the degree of degradation that such archivers experience on sequences from optimisers on different commonly employed test problems, and ascertain if there are commonalities across e.g. front shape, archiver and optimiser pairings. Initial references:

- M. Li. 2021. *Is Our Archiving Reliable? Multiobjective Archiving Methods on "Simple" Artificial Input Sequences.* ACM Transactions Evolutionary Learning & Optimization 1, 3, Article 9 (September 2021), 19 pages. DOI:https://doi.org/10.1145/3465335
- R. Tanabe & A. Oyama. 2017. *Benchmarking MOEAs for multi-and many-objective optimization using an unbounded external archive* In Proceedings of the Genetic and Evolutionary Computation Conference (pp. 633-640)
- M. López-Ibáñez, J. Knowles J. & M. Laumanns. 2011. *On Sequential Online Archiving of Objective Vectors* In: R.H.C. Takahashi, K. Deb, E.F. Wanner & S. Greco (eds) Evolutionary Multi-Criterion Optimization. EMO 2011. Lecture Notes in Computer Science, vol 6576. p46-60. Springer, Berlin, Heidelberg. available at `https://www.researchgate.net/publication/221228538_On_Sequential_Online_Archiving_of_Objective_Vectors`

Effects of noise on multi-objective optimiser performance. Noise on the objective functions has been seen to be detrimental to the performance of standard evolutionary multi-objective optimisers, which has led to the development of bespoke optimisers for this type of problem. However, it has also been observed that small amounts of noise can actually be beneficial to some multi-objective search algorithms, even using optimisers that assume no noise is present. In this project you will investigate the combination of problem type, noise type and noise magnitude, and explore what the underlying relationships between performance of standard optimisers in noisy problems are, and the situation which led to improved performance when noise is present. Initial references:

- L.T. Bui, A. H.A. Abbass, & D. Essam. 2005. *Fitness inheritance for noisy evolutionary multi-objective optimization.* In Proceedings of the 7th annual conference on Genetic and evolutionary computation, pp. 779-785.

- J.E. Fieldsend & R.M. Everson. 2015. *The Rolling Tide Evolutionary Algorithm: A Multiobjective Optimizer for Noisy Optimization Problems* in IEEE Transactions on Evolutionary Computation, vol. 19, no. 1, pp. 103-117. doi: 10.1109/TEVC.2014.2304415

- E.J. Hughes. 2001. *Evolutionary Multi-objective Ranking with Uncertainty and Noise* In: E. Zitzler, L. Thiele, K. Deb, C.A. Coello Coello & D. Corne (eds) Evolutionary Multi-Criterion Optimization. EMO 2001. Lecture Notes in Computer Science, vol 1993. Springer, Berlin, Heidelberg. `https://www.researchgate.net/publication/221228598_Evolutionary_Multi-objective_Ranking_with_Uncertainty_and_Noise`

Performance limits of many-objective optimisation. Although there are numerous *many*-objective optimisers which have been proposed and implemented, it is still the case that the effect of particularly large numbers of objectives ($M \gg 20$), in contrast to large scale design spaces, is relatively under-explored. This project will investigate how algorithms designed for many-objective optimisation scale to very large numbers of criteria, utilising test functions that can scale their number of objectives, and investigate if some many-objective optimisers are better suited to very high objective dimensions than others. Initial references:

- Q. Zhang & H. Li. 2007. *MOEA/D: A Multiobjective Evolutionary Algorithm Based on Decomposition*, in IEEE Transactions on Evolutionary Computation, vol. 11, no. 6, pp. 712-731. doi: 10.1109/TEVC.2007.892759.

- K. Deb & H. Jain. 2014. "*An evolutionary many-objective optimization algorithm using reference-point-based nondominated sorting approach, part I: solving problems with box constraints*, IEEE Transactions on Evolutionary Computation, vol. 18, no. 4, pp. 577–601.

- S. Huband, P. Hingston, L. Barone & L. While. 2006. *A review of multiobjective test problems and a scalable test problem toolkit*, in IEEE Transactions on Evolutionary Computation, vol. 10, no. 5, pp. 477-506.

Multi-objective BORE. An exciting development in Bayesian optimisation is to be able to use classifiers in the search of where to evaluate next. The BORE (Bayesian Optimisation by Density-Ratio Estimation) method was developed for single objective optimisation. The aim of this project is to apply the BORE methodology to multi-objective problems. Initial references are:

- L.C Tiao, A. Klein, M.W. Seeger, E.V. Bonilla, C. Archambeau & F. Ramos. 2021. *BORE: Bayesian Optimization by Density-Ratio Estimation.* Proceedings of the 38th International Conference on Machine Learning, PMLR 139:10289-10300. This is the original BORE paper. See also:
  - Paper: `http://proceedings.mlr.press/v139/tiao21a.html`
  - Project website: `https://tiao.io/publication/bore-2/`
  - Presentation: `https://slideslive.com/38942425/bayesian-optimization-by-density-ratio-estimation`

- F. Gibson, R. Everson & J. Fieldsend. 2021 *Multi-Objective Bayesian Optimisation Using an Exploitative Attainment Front Acquisition.* IEEE Congress on Evolutionary Computation (CEC), 2021. doi: 10.1109/CEC45853.2021.9504899. This paper discusses some recent ways of doing mult-objective Bayesian optimisation.

- A. Rahat, R. Everson & J. Fieldsend. 2017. *Alternative Infill Strategies for Expensive Multi-Objective Optimisation.* Genetic and Evolutionary Computation Conference (GECCO 2017), 2017. More on possible ways of doing multi-objective Bayesian optimisation.

Preference incorporation in Bayesian multi-objective optimisation. It is desired to find solutions preferable to the decision maker in the least number of expensive evaluations. The aim of this project is to use weights as preferences (a priori) and apply weighted sum, weighted Tchebycheff, and penalty boundary intersection (PBI) as scalarising functions in a Bayesian optimisation (BO) algorithm. You can use ParEGO as the BO algorithm. Assume you are the decision maker in this project. You can use standard benchmark problems (e.g. DTLZ and ZDT) for testing. You will also need to compare the performance of these scalarising functions with at least one of the preference based quality indicators. References:

- J. Hakanen & J. Knowles. 2017. *On using decision maker preferences with ParEGO.* In the proceedings of the 9th International Conference on Evolutionary Multi-Criterion Optimization - Volume 10173, Pages 282–297.
- J. Knowles. 2006. *ParEGO: a hybrid algorithm with on-line landscape approximation for expensive multiobjective optimization problems*, in IEEE Transactions on Evolutionary Computation, vol. 10, no. 1, pp. 50-66.
- Q. Zhang & H. Li. 2007. *MOEA/D: A Multiobjective Evolutionary Algorithm Based on Decomposition*, in IEEE Transactions on Evolutionary Computation, vol. 11, no. 6, pp. 712-731.
- Source code for ParEGO in python: `https://github.com/shinya-ml/Multiobj-Bayes-opt` and `https://botorch.org/v/0.3.1/tutorials/multi_objective_bo`,
- Source code for ParEGO in MATLAB: `https://github.com/zhandawei/ParEGO`

Preference incorporation in multi-objective evolutionary algorithms. It is desired to get a subset of approximated Pareto optimal solutions preferable to the decision-maker. In this project, you will use a reference point as the decision-maker's preference (a priori) and compare at least three different reference point based multi-objective evolutionary algorithms (MOEAs). You can use standard benchmark problems (e.g. DTLZ and ZDT) for testing and can assume that you are the decision-maker. References:

- S. Bechikh, M. Kessentini, L.B. Said & K. Ghédira. 2015 *Preference Incorporation in Evolutionary Multiobjective Optimization: A Survey of the State-of-the-Art*, Advances in Computers, Volume 98, Pages 141-207.
- L. Rachmawati & D. Srinivasan. 2006 *Preference incorporation in multi-objective evolutionary algorithms: A survey*, in: In IEEE Congress on Evolutionary Computation (CEC'06), IEEE, pp. 3385–3391.
- Desdeo framework (in Python) with several MOEAs: `https://desdeo.misitano.xyz/software`
- PlatEMO (in MATLAB) with several MOEAs: `https://github.com/BIMK/PlatEMO`

Comparison of reference point based quality indicators The regular quality indicators e.g. Hypervolume and Inverted Generational Distance do not provide a meaningful comparison of preference-based multi-objective optimisation algorithms. In this project, you will compare at least three reference point based quality indicators. The project aims to find if these quality indicators reflect the decision-maker's preferences. You can select a reference point based MOEA of your choice (e.g. R-NSGA-II, WASF-GA, RVEA and PIE) and assume that you are the decision-maker. You can use standard benchmark problems (e.g. DTLZ and ZDT) for testing. References:

- Kalyanmoy Deb & J. Sundar. 2006. *Reference point based multi-objective optimization using evolutionary algorithms.* In Proceedings of the 8th Annual Conference on Genetic and Evolutionary Computation, GECCO '06, 635–642. New York, NY, USA. Source code: `https://pymoo.org/algorithms/moo/rnsga2.html`

- S. Bandaru & H. Smedberg. 2019. *A parameterless performance metric for reference-point based multi-objective evolutionary algorithms.* In Proceedings of the Genetic and Evolutionary Computation Conference (GECCO '19). Association for Computing Machinery, New York, NY, USA, 499–506. DOI: https://doi.org/10.1145/3321707.3321757
- K. Li, K. Deb & X. Yao. 2018. *R-Metric: Evaluating the Performance of Preference-Based Evolutionary Multiobjective Optimization Using Reference Points*, in IEEE Transactions on Evolutionary Computation, vol. 22, no. 6, pp. 821-835. doi: 10.1109/TEVC.2017.2737781.
- Desdeo framework (in Python): `https://desdeo.misitano.xyz/software`

Robust multi-objective optimisation. In single objective optimisation a robust solution $\mathbf{x}$ is one where the objective value does not change very much as $\mathbf{x}$ is perturbed. Robust optima are advantageous because the modelled and optimised system may not exactly fit reality or conditions may change: a solution at a robust optimum will continue to show similar performance, whereas performance of a non-robust solution may degrade rapidly. This project will investigate how the single objective ideas of robust optima can be transferred to multi-objective problems. References:

- K. Deb & H. Gupta. 2006. *Introducing Robustness in Multi-Objective Optimization*, Evolutionary Computation, 14, 463-494, doi: 10.1162/evco.2006.14.4.463.
- H.-G. Beyer & B. Sendhoff. 2007. *Robust optimization – A comprehensive survey.* Computer Methods in Applied Mechanics and Engineering 196, 33, 3190 – 3218.
- T. Chatterjee, S. Chakraborty & R. Chowdhury. 2019. *A critical review of surrogate assisted robust design optimization.* Archives of Computational Methods in Engineering 26, 1, 245–274.

Parallel multi-objective optimisation. We usually measure the efficiency of optimisation algorithms by the number of times the objective function has to be evaluated. This assumes a sequential model in which one evaluation finishes before the next begins. However, in practical situations the overall elapsed or "wallclock" time is more important and distributed architectures can be used ot perform calculations in parallel. This project will investigate parallel strategies for multi-objective optimisation. Of particular interest are expensive problems where only relatively small numbers of evaluations are possible. References:

- G. De Ath, R.M. Everson & J.E. Fieldsend. 2021. *Asynchronous ϵ-Greedy Bayesian Optimisation* Uncertainty in Artificial Intelligence. arXiv:2010.07615.
- G. De Ath, R.M. Everson, J.E. Fieldsend & A.A.M. Rahat. 2020. ϵ-shotgun: ϵ- greedy batch Bayesian optimisation. In Proceedings of the 2020 Genetic and Evolutionary Computation Conference. Association for Computing Machinery, 787-795

Navigating and visualising multi-criteria sets. Being able to draw and visualise a data set often helps us understand the relationships between the entities in the dataset. This project will explore and develop new methods of visualising multi-criteria data. It will focus on low dimensional representations (that can be plotted) based on the "dominance distance" a metric that captures the relative dominance between entities in the set. We will also investigate methods for exploring the neighbourhood of a solution by moving along data-defined geodesics. References:

- D. Walker, R. Everson and J. Fieldsend. 2013. *Visualising mutually non-dominating solution sets in many-objective optimisation.* IEEE Transactions on Evolutionary Computation, 17, 165-184.

3 Software tools and packages

In planning your empirical work it is worth noting there are many existing open source packages containing implementations a number of pre-existing multi-objective optimisers. These include (but are not limited to):

- Pymoo, Python — `https://pymoo.org`
- MOEA Framework, Java — `http://moeaframework.org`

- PlatEMO, Matlab — `https://github.com/BIMK/PlatEMO`

There are often author-provided implementations of algorithms also. These will typically be highlighted in an article if available.

4 Submission

The document body of the pdf format report (excluding title page and references) should be no more than 11 pages in length. It should be typeset in LaTeX, using the style file provided on the COMM510 ELE page and be submitted by 12pm (midday) on the date specified on the cover page, using the electronic BART submission system.

If you are less familiar with LaTeX, we suggest you take advantage of the institutional Overleaf account `https://www.overleaf.com`, which you will benefit from if you register on the website with your university email address. This includes an online editor and compiler, version control, as well as 'how to' guides for typesetting using LaTeX.

Marking criteria are tabulated below in Section 5.

5 Marking Criteria

The assessment will be marked using the criteria on the following page.

Problem definition.	The degree to which the problem is concisely and clearly defined, including the distinct computer science aspects to be addressed relating to COMM510.	/5
Background literature review.	The degree to which an appropriate body of literature has been synthesised and presented, and the degree to which it contextualises and supports the research programme proposed, with no obvious omissions.	/20
Research Programme.	The degree to which the research programme presented is clear, well-aligned to the research question(s) being addressed, and has a clear and appropriate plan for the assessment of the empirical outcomes. The degree to which the expected outcomes are reasonable and well-justified given the published research in the domain.	/20
Results presentation.	The degree to which the presentation of results is clear and support the research topic being investigated, and support the reader alongside the analysis.	/20
Analysis and Contextualisation of results.	The degree to which the analysis provided of the results is appropriate, supported by the data, and provides insight into the research question, and the degree to which the results and analysis have been well-contextualised with existing results in the literature.	/20
Future research directions.	The degree to which sensible further research directions and questions have been mapped out, based upon the results and analysis provided in the report.	/5
Presentation.	The degree to which the report as a whole is clear, concise, well-written, well-structured, well-formatted and devoid of grammatical error.	/10
Length penalty.	A penalty of 10 marks will be applied for each page (or part thereof) that the document body is in excess of 11 pages (excluding references).	
	Total	/100