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Department of Mechanical & Aerospace Engineering

Coursework Assignment Cover Sheet

***Class No:* ME528 Control Systems**

***Coursework Title:* Individual Project**

***Submission to*: Prof M Cartmell**

Submission Details

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First Name Dushyant

Degree Course Advanced Mechanical Engineering with Aerospace
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Year 5th
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I confirm that this work is my own and is the final version

Signed ......Dushyant Khatri..............................

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EXECUTIVE SUMMARY

In this report, a brief introduction to a control system and its elements is given, along with different tools and techniques used with the field of study. Along with this, a brief description of the project is given, where primary parameters are defined and a diagrammatic visualisation is provided. The project objectives, assumptions, and the scope are clearly framed to let the reader know on what he can expect from the report and what would not be included. The section following is the methodology, which elaborates on the techniques adopted and the simulation performed in order to fulfil the project objectives, with an indication to move forward to results for further justification and reasoning on the graphs obtained. The result, not only presents the numerical and graphical solutions, but also interprets the graph and explain on why such results are achieved. The first objective is fulfilled by designing seven different controllers and incorporating them into a block diagram by using Simulink, considering the mathematical equations of the controllers, and by selecting the most optimum controller for this project, where each controller has three different scenarios each. The second objective is fulfilled by adopting both the Routh-Hurwitz Stability Criterion and the Nyquist Stability Criterion to assure the reader that the results obtained are accurate and in line with what was anticipated for the most optimum controller selected. The project concludes with an indication to future development and exploring the car dynamics by applying controls engineering to it.

For the ease of access to the reader, all the figures, tables and headings, which are referred in the report are hyperlinked within the document, which implies that should the reader wish to revisit any of the headings, figures, or tables, referred to later in the report, could be reached by just a click of a button rather than scrolling all the way. The first objective results are laid out in landscape form to relate the block diagrams with the graphs plotted, or compare different scenarios alongside one another, without having to scroll back and forth. The various shades of blue used for different headings would enable the reader in distinguishing each major heading and a sub-heading with ease.

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INTRODUCTION

A control system is an interconnection of components forming a system configuration that will provide a desired system response (Mahmoud, 2018). Dorf and Bishop (2011) describe different loop arrangements of control systems such as open-loop, closed-loop, multi-loop with an inner and an outer loop, and multivariable control system, where they highlight the importance of feedback is to enable the users in controlling a desired output and improving accuracy. Understanding the control theory and establishing a robust methodology is very crucial in designing a desired control system. To study and design control systems, one relies on a set of mathematical tools, for instance, the Laplace transforms and the Z-transforms; these facilitate the engineer's work in understanding the problems he deals with as well as solving them (Paraskevopoulos, 2002). In the modern day, these mathematical tools are applied in controls engineering by using sophisticated computer technology. López (2014) explains that MATLAB offers an integrated environment in which one can design control systems and utilise a high-level platform for technical model generation, data analysis, and algorithm development, combining comprehensive engineering and mathematics functionalities with powerful visualisation and animation features. A particular add-in feature within MATLAB is devoted to control systems design called Simulink (Mathworks, 2018), a graphic interface where one can design and tune feedback loops, and use simulation models to verify control design, where it automatically generates codes for rapid prototyping and production.

In this project, we adopt these mathematical tools to perform our analysis and use MATLAB/Simulink in designing and interpreting the behaviour of the control system. We complement these with application of basic laws of Physics and Mathematics to achieve the desired design and analysis.

Project Description

The project involves analysis of an automotive cruise control design, where its functional block is shown in Figure 1. The attributes that affect the car dynamics are tabulated as follows:

TABLE 1 - TABULATED ATTRIBUTES OF CAR DYNAMICS

$m\ddot{x}$	Rate of change of momentum of the vehicle
$c\dot{x}$	Damping of the vehicle
$F(t)$	Collective propulsive force required to propel the vehicle
$mg\sin\theta$	Retarding force on the vehicle due to inclination

The equation of motion for the car negotiating the incline is given as,

$$m\ddot{x} + c\dot{x} + mg\ sin\theta = F(t) \tag{1}$$

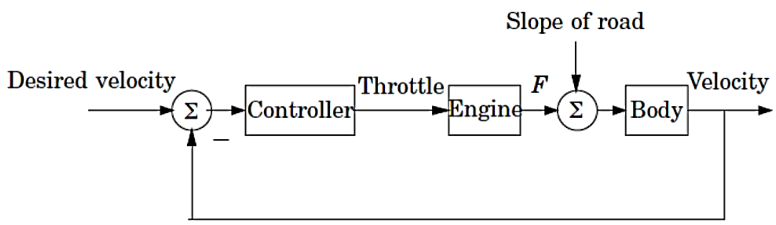


FIGURE 1 - FUNCTIONAL BLOCK OF A CRUISE CONTROL SCENARIO OF A CAR

Other parameters that would contribute to the overall analysis of car dynamics are listed as follows:

- **Damping-to-mass ratio:** The ratio is helpful in deducing the equation of motion in terms of derived parameters from the above tabulated parameters. In this problem, the ratio is considered of a typical four door saloon in top gear, which is commonly around 0.02.
- **Control Torque:** The parameter is achieved as the ratio of propulsive force to mass of the vehicle, and is mathematically denoted by $u(t)$. This is an important parameter when using during controllers for analysis. For instance, a PID controller is mathematically represented as,
$$u(t) = k_p\big(v_r - v(t)\big) + k_i \int_0^t (v_r - v(t))dt + k_d \frac{d(v_r - v(t))}{dt} \tag{2}$$
where k_p is the gain of the proportional part of the controller, k_i is the gain of the integral part, and k_d is the gain of the derivative part. $v(t)$ is the actual velocity of the car; $v = \dot{x}$.
- **Gradient Disturbance:** The parameter is mathematically denoted by $g\theta$. This disturbance is experienced due to gravitational acceleration and inclination, and is irrespective of the mass of the vehicle.

Project Objectives

The project objectives are divided into two categories:

(1) Design a mathematically formal control system, incorporating seven different controllers, the car dynamics, any disturbances that is operative, and an appropriate feedback loop.

(2) Explore the performance of the cruise controllers numerically, then selecting the best one and assessing it for stability.

Assumptions

In order to achieve an accurate result for the objectives set out, it is important to restrain the problem to certain assumptions, considering the timeline for the completion of the project and the amount of sophistication and accuracy of the results. The assumptions are indicated below:

- For simplicity in solving, and for the fact that $\sin 20° \approx 20°$ when converted to radians, equation (1) is approximated as $m\ddot{x} + c\dot{x} + mg\, sin\theta = F(t)$.
- The angle of inclination, θ, is 20°. Therefore, with a constant value of g as 9.81 m/s^2, the value of $g\theta$ becomes a product of 20° (converted to radians) and 9.81 m/s^2. The resulting value is 3.42 units. This is followed for all the scenarios. However, one of the instructions of the project emphasised to change the $g\theta$ value, this is performed as outside the scope of the project, and is discussed in the Appendix.

Project Scope

This section of the project highlights the assumptions made in successful completion of the project. This is particularly helpful in establishing that the project does not explore all the aspects of the field of study and has a restricted analysis. The project scope is listed as follows:

- It is considered that MATLAB and Simulink are efficient in performing the analyses and the scope of the project is, therefore, restricted to using these software. Any of the other software in control systems design and analysis were not explored in this project.
- It is considered that the reader has a foundation knowledge of control engineering and design aspects related to it.
- The project is restricted in its content and does not cover all the practices in controls engineering, for instance, the stability analysis is restricted to three stability criterion. Should there be other valid stability criterions, these are not used for analyses as a part of this project.

METHODOLOGY

Objective One

In order to achieve the first objective, the first step is to create separate block diagrams of seven different controllers considered – P, I, D, PI, PD, ID, and PID. Their mathematical representation is shown in Figure 2. Once these are designed as block diagrams, $g\theta$ is considered a constant value of 196.2 (as calculated by constant values of g and θ); other variables such as V_r, K_i, K_p, and K_d are considered to be 1, 10, 100, 0.1, 0.01, and 0.001. The total number of combinations of the four variables for the six values is 6^4, which is 1296. Therefore, analysing each and every possible combination would be considered out of scope of the project, which is why, we restrict our analysis to certain combinations to analyse the working of different controllers.

$$u(s) = k_p e(s)$$

$$u(s) = k_i \int e(s)ds$$

$$u(s) = k_d \frac{de(s)}{ds}$$

$$u(s) = k_p e(s) + k_i \int e(s)ds$$

$$u(s) = k_p e(s) + k_d \frac{de(s)}{ds}$$

$$u(s) = k_i \int e(s)ds + k_d \frac{de(s)}{ds}$$

$$u(s) = k_p e(s) + k_i \int e(s)ds + k_d \frac{de(s)}{ds}$$

FIGURE 2 - MATHEMATICAL REPRESENTATION OF P, I, D, PI, PD, ID, AND PID CONTROLLERS (FROM TOP TO BOTTOM)

The first objective was achieved by performing the following instructions:

- The block diagrams were designed in Simulink in accordance to the mathematical representation in Figure 2.
- A combination of different values described above are implemented in performing the analysis and the graphs for these are obtained. As per the requirement of the project, results for 21 different scenarios are obtained.
- The graphs are interpreted and compared in order to understand the efficiency of each controller, and selecting the two most robust controllers suitable for the problem. The results are explained in the 'Results' section.

Objective Two

Dorf and Bishop (2011) explain in their book the importance of stability in linear feedback control systems. According to Dorf and Bishop (2011), a stable system is a dynamic system with a bounded response to a bounded input, and a necessary and sufficient condition for a feedback system to be stable is that all poles of the system transfer function has negative real parts. There are three approaches given to check the stability of a control system –

a. the s-plane approach
b. the frequency plane ($j\omega$) approach
c. the time-domain approach

Among the three, two stability criterions, the s-plane approach (Routh-Hurwitz criterion) and the frequency plan approach (the Nyquist criterion), are adopted in assessing the system stability.

Routh-Hurwitz Stability Criterion

Dorf and Bishop (2014) state that the Routh-Hurwitz stability criterion is a necessary and sufficient criterion in stability analysis, investigating the characteristic equation without direct computation of the roots.

The characteristic equation is mathematically represented as $1 + G(s)H(s) = 0$. For the characteristic equation of a third-order system, the Routh-Hurwitz criterion can be formulated as represented in Figure 3.

The characteristic polynomial of a third-order system is

$$q(s) = a_3 s^3 + a_2 s^2 + a_1 s + a_0.$$

The Routh array is

$$\begin{array}{c|cc} s^3 & a_3 & a_1 \\ s^2 & a_2 & a_0 \\ s^1 & b_1 & 0 \\ s^0 & c_1 & 0 \end{array}$$

where

$$b_1 = \frac{a_2 a_1 - a_0 a_3}{a_2} \quad \text{and} \quad c_1 = \frac{b_1 a_0}{b_1} = a_0.$$

FIGURE 3 - ROUTH ARRAY REPRESENTATION (DORF AND BISHOP, 2014)

When all the numbers in the first column have the same algebraic sign, the system is said to be stable, for example, if all the numbers in the first column, i.e., a_3, a_2, b_1, and c_1 are positive, the system is said to be stable, however, if any of these have different signs than the rest, the system is said to be unstable. Also, the roots can be found out by using the MATLAB code `{roots([a₃ a₂ a₁ a₀])}` and can further be utilised for representing a pole-zero plot.

The Nyquist Stability Criterion

In order for the system stability to be verified in the frequency domain $(j\omega)$, the Nyquist stability criterion is adopted. The frequency response of a system is defined as the steady state response of the system to a sinusoidal input signal (Dorf and Bishop, 2014). In order to verify the system stability, we convert the characteristic equation from s-domain to the $j\omega$-domain. Input the numerator and the denominator in obtaining the transfer function, and calculate the Nyquist criterion by using the MATLAB code given below. Should the real axis value be less than -1, the system is stable.

```
num=1;      %numerator
den=[1 0.02];      %denominator
t=tf(num,den);      %transfer function
w=[1 2 5 10 100];      %setting the frequency
nyquist(t,w)      %nyquist functions yield the nyquist diagram
```

RESULTS

Objective One

In this section, each of the controllers are analysed and the graphs are plotted. Table 2 outlines three different scenarios considered for each of the controllers. For most of the scenarios, the block diagram is presented on the left and the output is presented on the right.

TABLE 2 - 21 SCENARIOS PLANNED AND TABULATED, WHERE 3 SCENARIOS ARE CONSIDERED FOR EACH CONTROLLER

Controller	P			I			D			PI			PD			ID			PID		
Scenario	1.1	1.2	1.3	2.1	2.2	2.3	3.1	3.2	3.3	4.1	4.2	4.3	5.1	5.2	5.3	6.1	6.2	6.3	7.1	7.2	7.3
k_i				1	0.1	10				1	100	0.1				1	10	1	10	100	10
k_p	1	10	0.1							10	10	0.1	10	1	0.1				10	10	100
k_d							1	10	0.1				1		10	1		10	0.1	1	
V_r	10																				
$g\theta$	3.42																				

Scenario 1.1

The gain, k_p, is 1, desired velocity, V_r, is 10 and disturbance, $g\theta$, is 3.42 in this scenario.

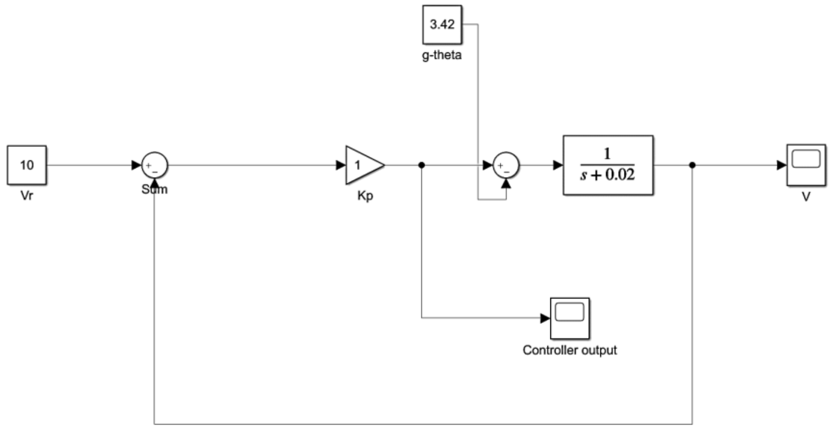


FIGURE 4 - BLOCK DIAGRAM FOR SCENARIO 1.1

From Figure 4, it can be observed that the values according to the scenario are put in and the simulation was run with time as 10. The controller output is used to relate the actual velocity in understanding on how the disturbance, the transfer function, and the proportionality gain influence the actual velocity.

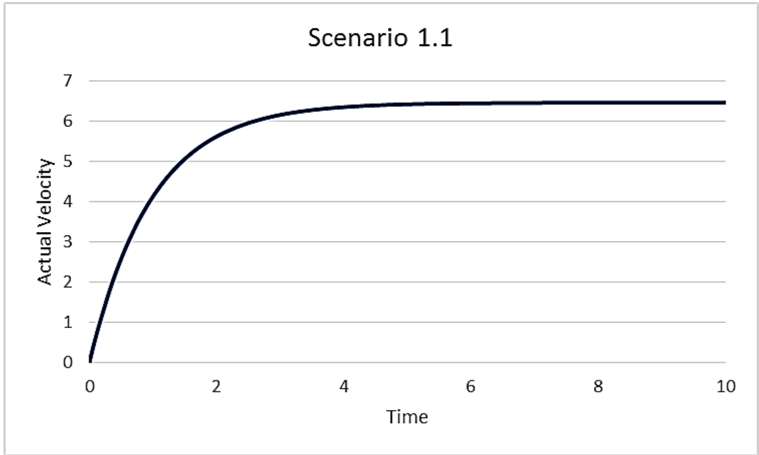


FIGURE 5 - GRAPH PLOTTED FOR SCENARIO 1.1

From Figure 5, it can be interpreted that when the desired velocity is 10, we have the actual velocity as 6.45 for a P controller. This shows that we do not achieve an accurate velocity, to an extent that we get a percentage difference of between the desired and the actual velocity.

Scenario 1.2

The gain, k_p, is 10, desired velocity, V_r, is 10 and disturbance, $g\theta$, is 3.42 in this scenario.

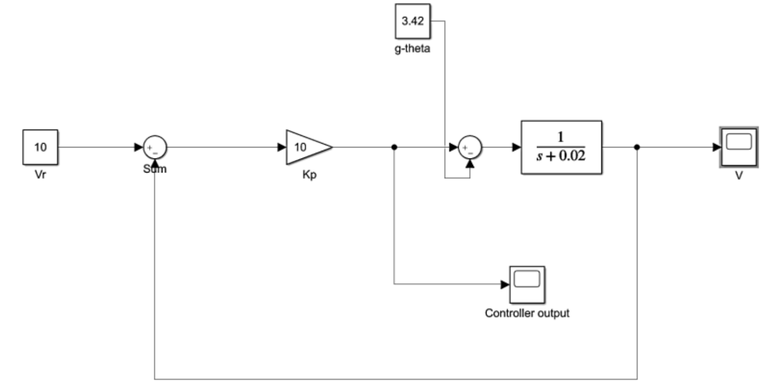


FIGURE 6 - BLOCK DIAGRAM FOR SCENARIO 1.2

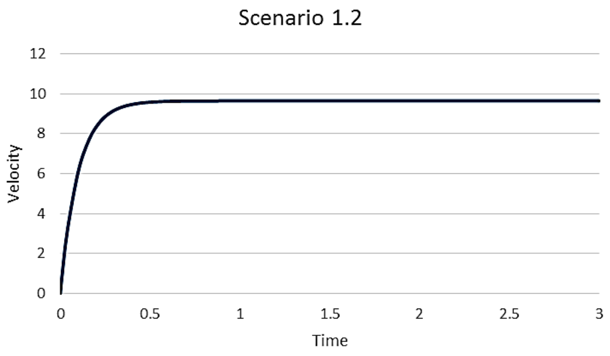


FIGURE 7 - GRAPH PLOTTED FOR SCENARIO 1.2

Figure 6 represents the block diagram for Scenario 1.2. It can be observed from Figure 7 that the actual velocity reaches 9.63 when the desired velocity is 10. When compared to Scenario 1.1, it can be indicated that by increasing the proportionality gain, we obtain an improved design for a P controller.

Scenario 1.3

The gain, k_p, is 0.1, desired velocity, V_r, is 10 and disturbance, $g\theta$, is 3.42 in this scenario.

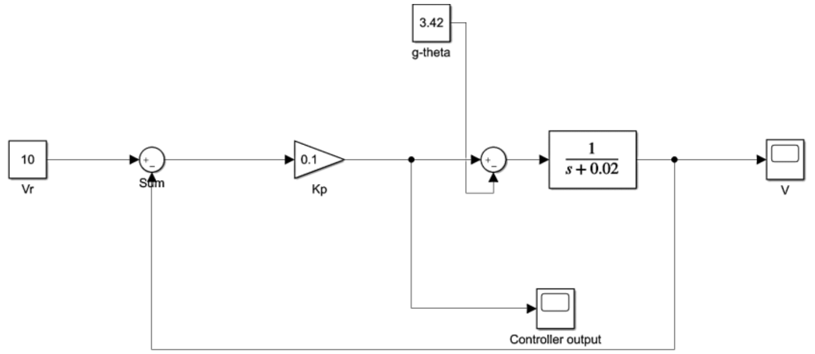


FIGURE 8 - BLOCK DIAGRAM FOR SCENARIO 1.3

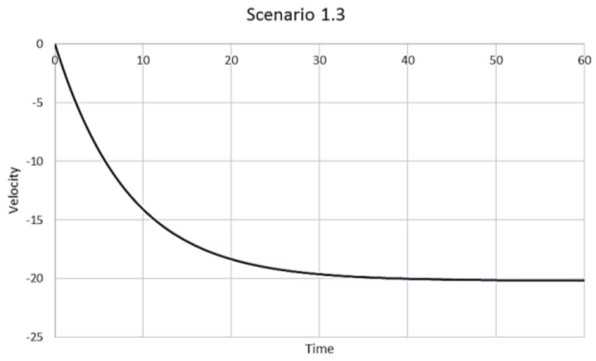


FIGURE 9 - GRAPH PLOTTED FOR SCENARIO 1.3

From Figure 9, it can be observed that the value for velocity moves on the negative axis. This means that the proportionality is inadequate, so much so that the car moves decelerates backwards. By considering the three scenarios for the P controller, it can be indicated that an increased proportionality gain for the desired velocity and the actual velocity to closely match. By simulation, it is observed that k_p is meant to be close to 50 to have an accurate actual velocity.

Scenario 2.1

The gain, k_i, is 1, desired velocity, V_r, is 10 and disturbance, $g\theta$, is 3.42 in this scenario.

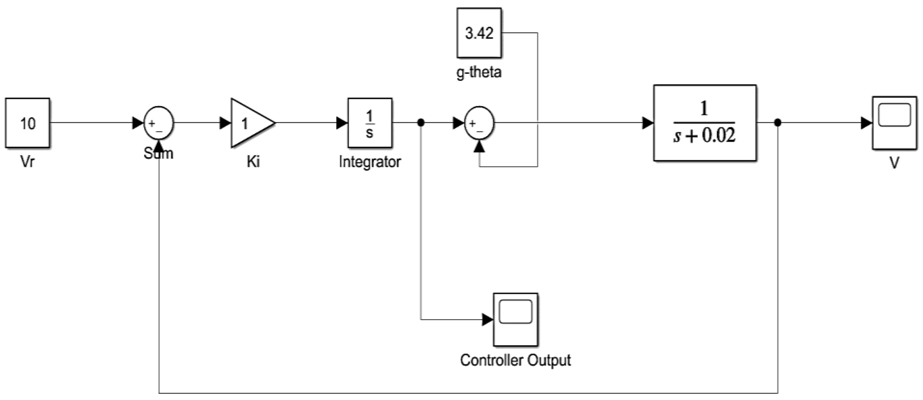


FIGURE 10 - BLOCK DIAGRAM FOR SCENARIO 2.1

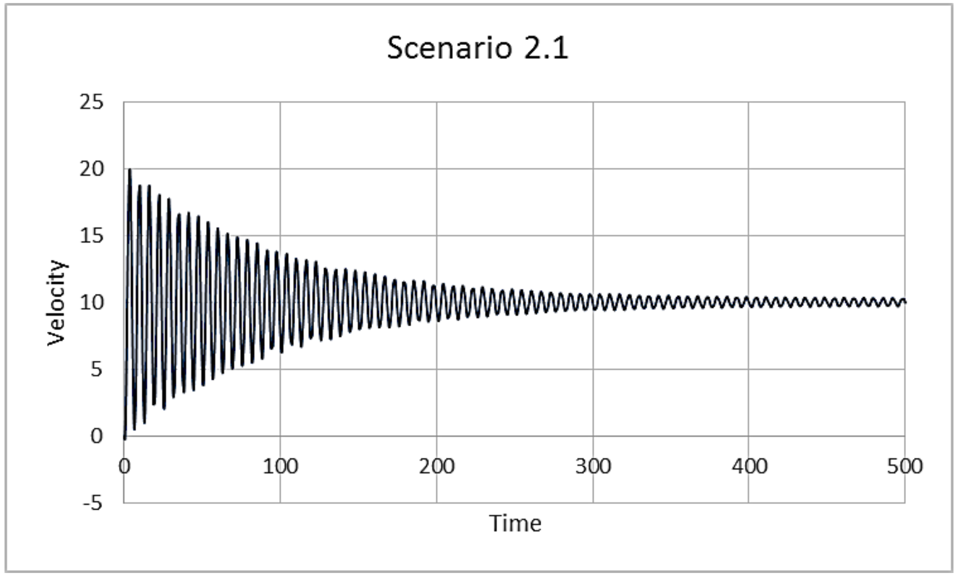


FIGURE 11 - GRAPH PLOTTED FOR SCENARIO 2.1

In comparison to the block diagram in the previous scenarios, a new component, an Integrator, is added to simulate the effects of an I-controller. Again, as used in the previous scenarios, the $g\theta$ and V_r remain the same value. The initial condition for the integrator is defined as 1 by default.

From Figure 11, it can be observed that the velocity never attains a constant value. It continues to fluctuate around 10 units. By the figure, it can also be interpreted that the fluctuations continue to reduce until 300 time units, and then fluctuates to a constant amplitude from then on.

From Scenario 2.1, it can be interpreted that because the desirable velocity is never achieved, using the I-controller is not suited for this project. Furthermore, when all the cases in Scenario 1 are compared for a P-controller with an I-controller, it is observed that an I-controller is at a drawback of not attaining equilibrium, which is not desirable for a control system. As it is deemed that an I-controller is not suitable, Scenarios 2.2 and 2.3 are not analysed.

Scenario 3.1

The gain, k_d, is 1, desired velocity, V_r, is 10 and disturbance, $g\theta$, is 3.42 in this scenario.

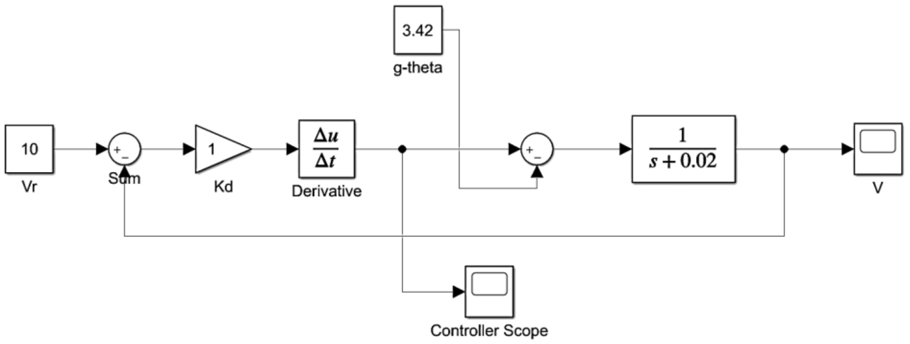


FIGURE 12 - BLOCK DIAGRAM FOR SCENARIO 3.1

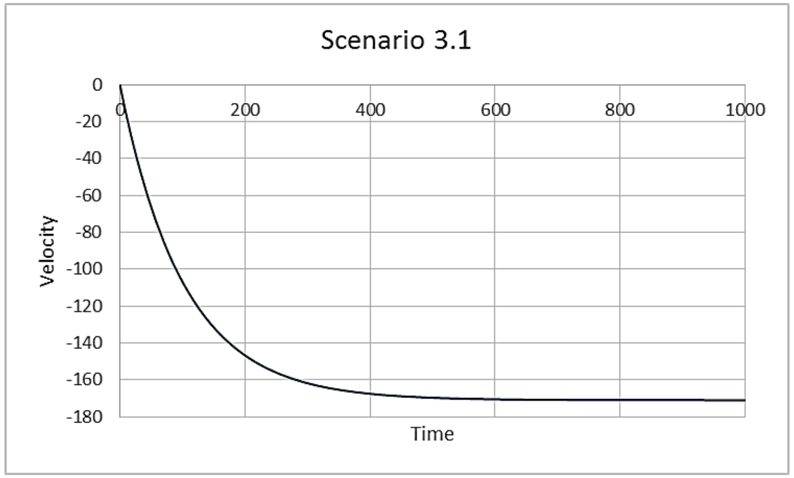


FIGURE 13 - GRAPH PLOTTED FOR SCENARIO 3.1

Figure 12 shows the block diagram for Scenario 3.1. Comparing this from Scenario 2.1, the integrator is replaced by the derivative. As the previous scenarios, the desired velocity and $g\theta$ remains the same.

From Figure 13, it can be interpreted that the car experience a negative velocity (a backward motion), similar to Scenario 1.3. This shows that the D-controller does not support the vehicle to achieve the desired velocity.

In addition to interpreting the results for Scenario 3.1, there were different values for k_d simulated, however, the value reached infinity for most of the values, which indicates that the controller is not suitable for the project. As the D-controller is deemed to be unsuitable for the project further simulation for the controller will not be considered, and therefore, Scenarios 3.2 and 3.3 do not contribute towards this report.

Scenario 4.1

For the past 3 cases, the proportional, integral, and differential controllers were considered separately. From this scenario onwards, the combination of these controllers are considered. The controller considered in this scenario is the PI controller, whose characteristics are integral gain, k_i, is 1, proportional gain, k_p, is 10, desired velocity, V_r, is 10 and disturbance, $g\theta$, is 3.42.

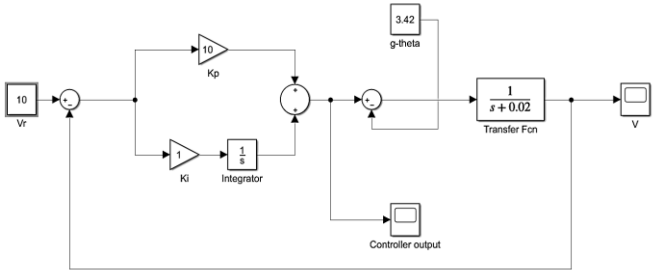


FIGURE 14 - BLOCK DIAGRAM FOR SCENARIO 4.1

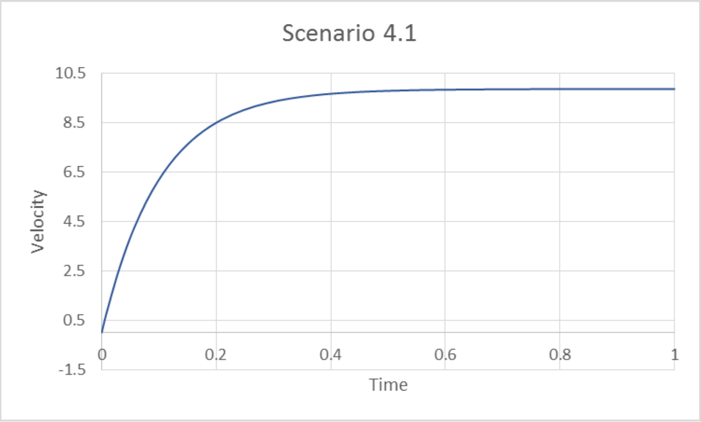


FIGURE 15 - GRAPH PLOTTED FOR SCENARIO 4.1

From Figure 15, it can be observed that the desired velocity is closely achieved rapidly (in around 0.3 sec). The velocity achieved is 9.83 m/s, which is deemed to be very accurate so far. The percentage difference between the desired velocity and the actual velocity is. Therefore, we continue with further simulation.

Scenario 4.2

In this scenario, the integral gain, k_i, is 10, proportional gain, k_p, is 100, desired velocity, V_r, is 10 and disturbance, $g\theta$, is 3.42.

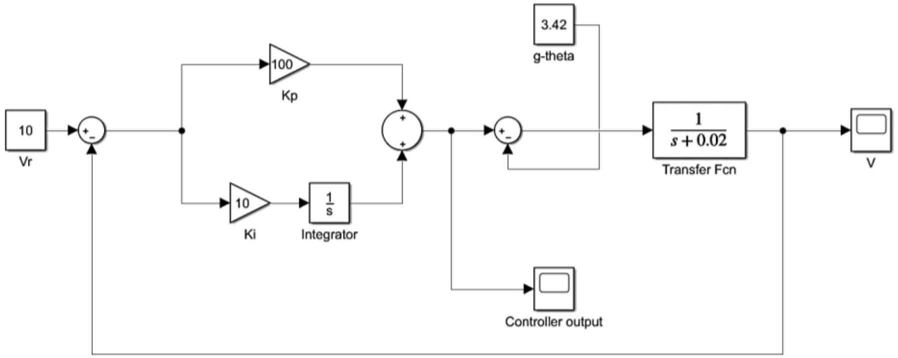


FIGURE 16 - BLOCK DIAGRAM FOR SCENARIO 4.2

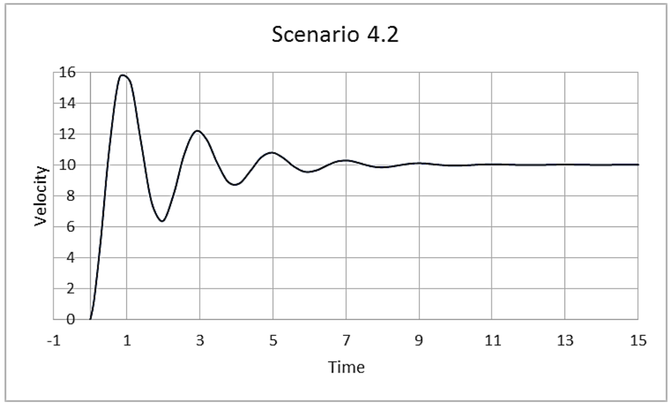


FIGURE 17 - GRAPH PLOTTED FOR SCENARIO 4.2

From Figure 17, it can be interpreted that the velocity fluctuates initially, however, in 9 seconds, the vehicle attains the desired velocity.

Scenario 4.3

In this scenario, the integral gain, k_i, is 0.1, proportional gain, k_p, is 0.1, desired velocity, V_r, is 10 and disturbance, $g\theta$, is 3.42. The worst case scenario is taken into account to verify the reliability and effective working of the controller.

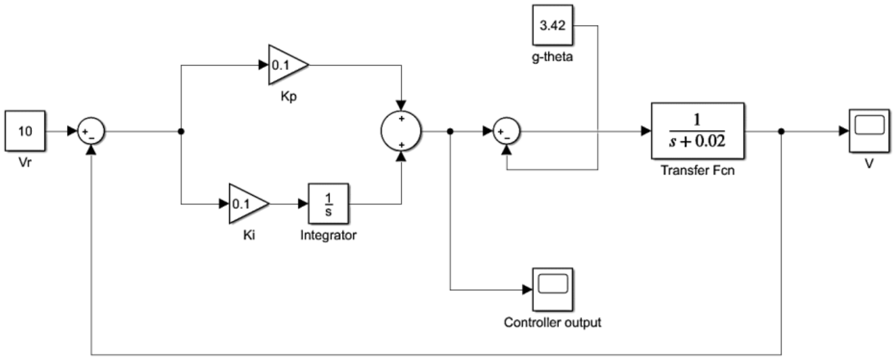


FIGURE 18 - BLOCK DIAGRAM FOR SCENARIO 4.3

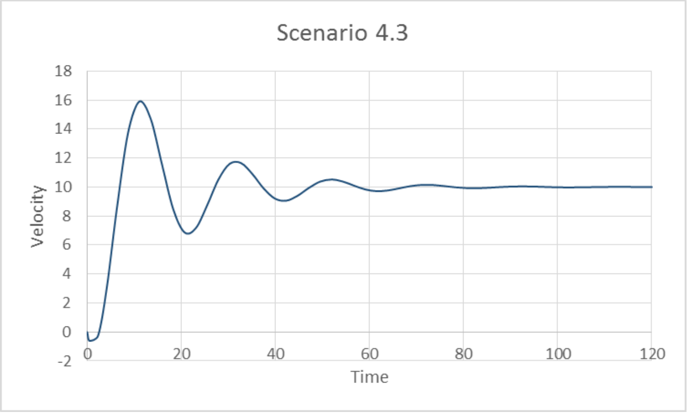


FIGURE 19 - GRAPH PLOTTED FOR SCENARIO 4.3

From Figure 19, it can be interpreted that even for the worst case scenario, the vehicle achieves the desired velocity in 60 seconds. From the above three scenarios, it can be established that the PI controller can be relied upon for design of a cruising vehicle. In fact, this controller can be considered the most suitable from all the scenarios included so far. For this reason, Scenario 4.1 will be adopted for analysis in fulfilling the second objective, further in this report.

Scenario 5.1

In this scenario, a PD controller is used, where the derivative gain, k_d, is 100, proportional gain, k_p, is 1, desired velocity, V_r, is 10 and disturbance, $g\theta$, is 3.42.

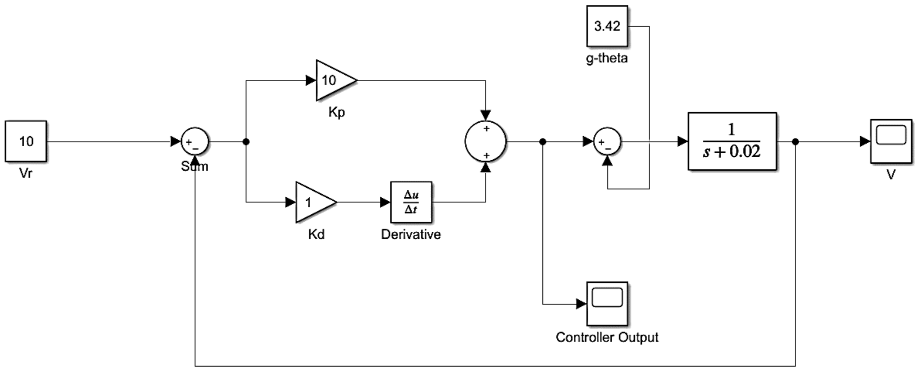


FIGURE 20 - BLOCK DIAGRAM FOR SCENARIO 5.1

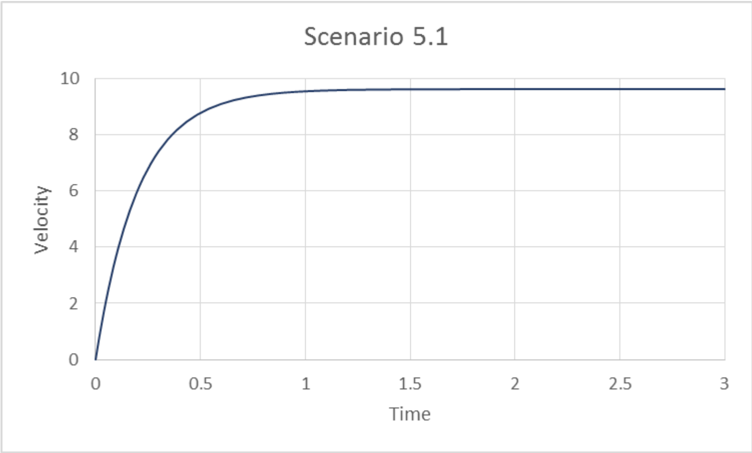


FIGURE 21 - GRAPH PLOTTED FOR SCENARIO 5.1

Figure 20 presents the block diagram of a PD controller. From Figure 21, it can be interpreted that the actual velocity is close to the desired velocity. It can also be observed that this is attained in 1 second, which can be considered as a faster response.

Scenario 5.2

In this scenario, the derivative gain, k_d, is 1, proportional gain, k_p, is 1, desired velocity, V_r, is 10 and disturbance, $g\theta$, is 3.42.

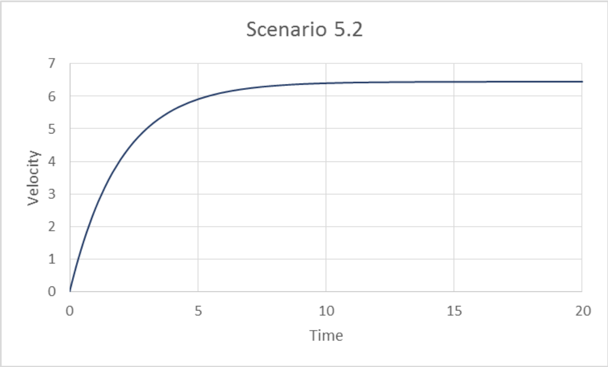


FIGURE 22 - GRAPH PLOTTED FOR SCENARIO 5.2

From Figure 22, it can be interpreted that the maximum actual velocity attained is 6.45 m/s, where the desired velocity is 10 m/s. The percentage difference is. It is, therefore, implied that this scenario is not suitable.

Scenario 5.3

Considering the attributes from Table 2 for this scenario, the only aspect which increases beyond proportion is the derivative gain (k_d). When the simulation is performed, certain errors occur. When the gain value is changed between 1 and 2, it can be seen that the simulation runs successfully for 2.1, however, indicates an error for 2.2. It can be considered that the system experiences a drastic change between 2.1 and 2.2, to reach the velocity to an infinite value. As the controller involved does not support the control system in all the scenarios, it is deemed to be unsuitable in this project.

Scenario 6.1

In this scenario, an ID controller is employed for the control systems simulation.

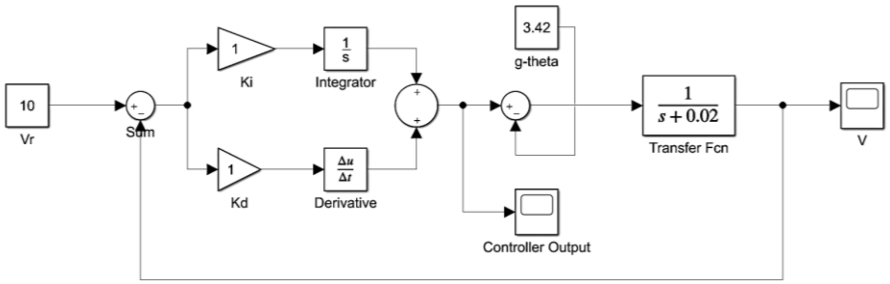


FIGURE 23 - BLOCK DIAGRAM FOR SCENARIO 6.1

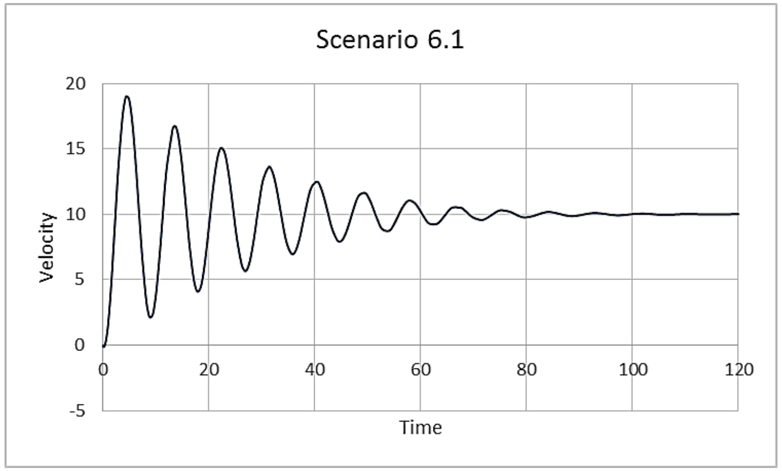


FIGURE 24 - GRAPH PLOTTED FOR SCENARIO 6.1

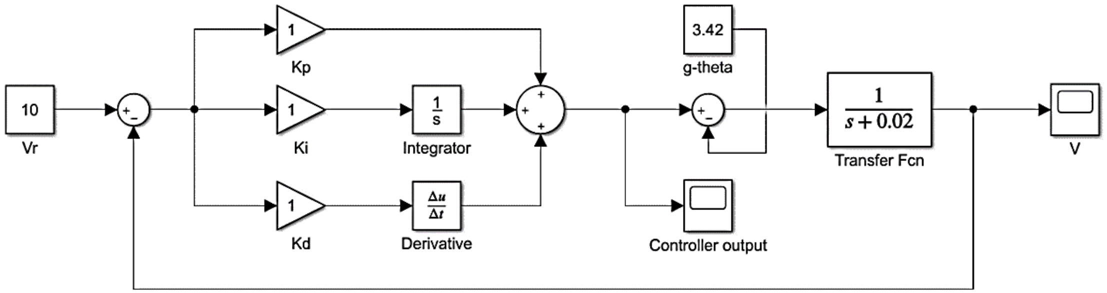
It can be observed that the derivative gain, k_d, is 1, integral gain, k_i, is 1, desired velocity, V_r, is 10 and disturbance, $g\theta$, is 3.42. This scenario would help in understanding how the controller behaves in default parameters. Figure 23 presents the block diagram for scenario 6.1.

It can be observed from Figure 24 that the velocity fluctuates to reach an actual velocity (10 m/s) over 100 seconds. This is considered a long time for the velocity fluctuation to settle down in becoming stable. A similar result was obtained in Scenario 2.1. Recognising that the scenario involved using an I-controller, it is implied that this fluctuation is caused by the integrator.

Considering the results obtained in Scenario 6.1, the integral gain were changed to analyse whether the fluctuation time can be reduced by any means. It can be interpreted that increasing the k_i decreases the fluctuation time, so much so that when the k_i is 1000, yet the actual velocity takes 4 seconds to settle down. Comparing this to the results obtained in Scenario 4.1, 4.2, and 4.3, it can be considered that a PI controller has better performance than an ID controller, and therefore, further simulation is not performed on an ID controller for the fulfilment of this project.

Scenario 7 – PID Controller

The seventh controller considered is a PID controller. Johnson (2005) explains that the PID controller is a sophisticated tool in control systems design, as it incorporates the proportional, differential, and the integral components. The author further explains the importance of the PID controller by stating three primary reasons: past record of success, wide availability, and simplicity in use. For this simulation, the PID controller is designed in parallel and the gains are tuned as per Table 2. The results generated are discussed below. From the previous scenarios, it can be interpreted that increasing the differential gain, k_d, can result in errors, however, increase in the proportional and the integral component improves the control system design. Therefore, for this simulation, k_d is 1 in Scenarios 7.2 and 7.3. However, we change the values of k_i and k_p to optimise the controller performance.



Scenario 7.1

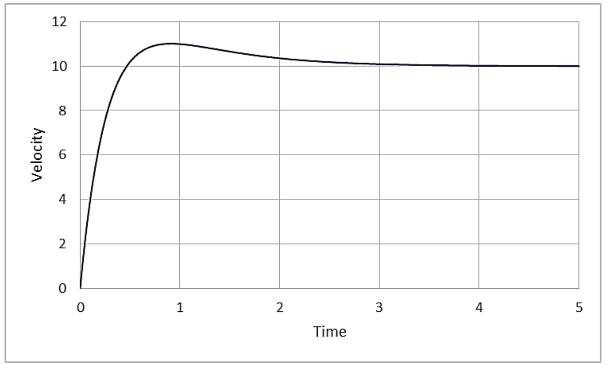


FIGURE 25 - GRAPH PLOTTED SCENARIO 7.1

As per Figure 25, it can be interpreted that the actual velocity is attained in 3 seconds when the k_i and k_p are 10, and k_d is 0.1. From Scenarios 7.2 and 7.3, the controller is seen to be optimised to explore its better performance potential.

Scenario 7.2

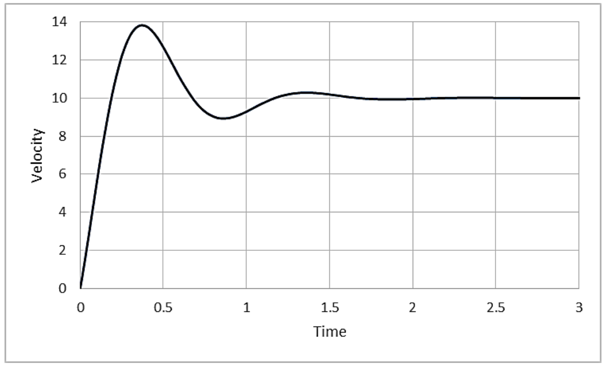


FIGURE 26 - GRAPH PLOTTED SCENARIO 7.2

As per Figure 26, it can be interpreted that the actual velocity is attained in 2 seconds when the k_i is 100, k_p is 10, and k_d is 1. It can also be observed that the overshoot reaches up to 14 m/s, however, the stability attains faster than previous.

Scenario 7.3

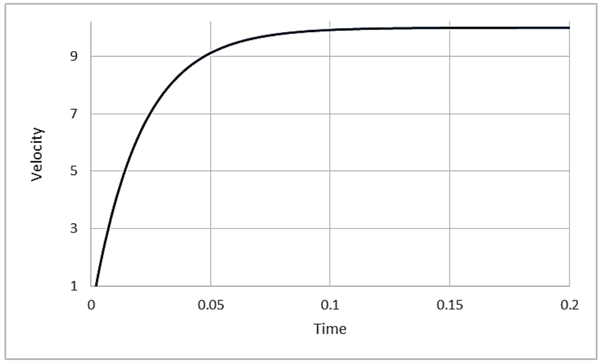


FIGURE 27 - GRAPH PLOTTED FOR SCENARIO 7.3

As per Figure 27, it can be interpreted that the actual velocity is attained in 0.1 seconds when the k_i is 10, k_p is 100, and k_d is 1. By comparing this result from the previous two, it is observed that this simulation is highly optimised and accurate.

As Scenario 7.3 is considered as one of the most optimum solutions for this problem, not just in terms of the actual velocity attained, but also in terms of how fast this is attained, this case is taken forward for further analysis in fulfilling the second objective of the project.

Objective Two

As the first objective is now fulfilled, the second objective involves considering the most optimum controller and verifying its system stability. The two scenarios that gave satisfactory results were Scenario 4.1(PI controller) and Scenario 7.3 (PID controller). As per the instructions of the project, the most optimum controller has to be selected. To select among the above two controllers, all the three scenarios were scrutinised. It is found that a PID controller attains the actual velocity in lesser time than a PI controller for different gain values. Therefore, all the three scenarios are considered in the fulfilment of the second objective.

Scenario 7.1

Routh-Hurwitz Stability Criterion

- The characteristic equation is $0.1s^3 + 10.022s^2 + 10.22s + 0.2 = 0$
- The Routh array formulated, as described in Figure 3, is

$$\begin{array}{ccc} s^3 & 0.1 & 10.22 \\ s^2 & 10.022 & 0.2 \\ s^1 & 10.218 & 0 \\ s^0 & 0.2 & 0 \end{array}$$

- As it is observed that all the values in the first column are positive, therefore, the system is said to be stable according to the Routh-Hurwitz Stability Criterion.

The Nyquist Stability Criterion

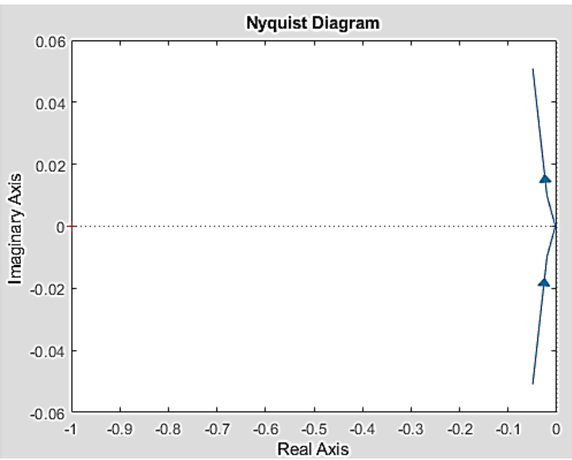


FIGURE 29 - NYQUIST DIAGRAM (FOR REFERENCE)

```
num=1;
den=[0.1 10.022 10.22 0.2];
t=tf(num,den);
w=[1 2 5 10 100];
nyquist(t,w)
```

FIGURE 28 - MATLAB CODE FOR SCENARIO 7.1

Figure 29 represents the Nyquist diagram, which indicates that the curves are drawn below -1 of the real axis. This shows that the system is stable for scenario 7.1 according to the Nyquist Stability Criterion. Figure 28 is the MATLAB code include for reference of the analysis performed on the software.

Scenario 7.2

Routh-Hurwitz Stability Criterion

- The characteristic equation is $s^3 + 10.04s^2 + 100.22s + 2 = 0$
- The Routh array formulated, as described in Figure 3, is

$$\begin{array}{ccc} s^3 & 1 & 100.22 \\ s^2 & 10.04 & 2 \\ s^1 & 0.998 & 0 \\ s^0 & 2 & 0 \end{array}$$

- As it is observed that all the values in the first column are positive, therefore, the system is said to be stable according to the Routh-Hurwitz Stability Criterion.

The Nyquist Stability Criterion

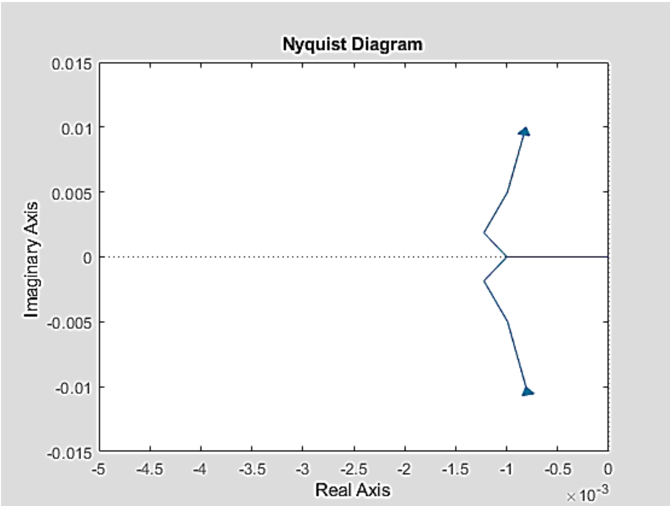


FIGURE 30 - NYQUIST DIAGRAM FOR SCENARIO 7.2

As per **Error! Reference source not found.**, the real axis value is slightly more than 10^{-3}, which indicates a control system with highly stable properties according to the Nyquist Stability Criterion.

Scenario 7.3

Routh-Hurwitz Stability Criterion

- The characteristic equation is $s^3 + 100.022s^2 + 12.02s + 0.2 = 0$
- The Routh array formulated, as described in Figure 3, is

$$\begin{array}{ccc} s^3 & 1 & 12.02 \\ s^2 & 10.022 & 0.2 \\ s^1 & 144.45 & 0 \\ s^0 & 0.2 & 0 \end{array}$$

- As it is observed that all the values in the first column are positive, therefore, the system is said to be stable according to the Routh-Hurwitz Stability Criterion.

The Nyquist Stability Criterion

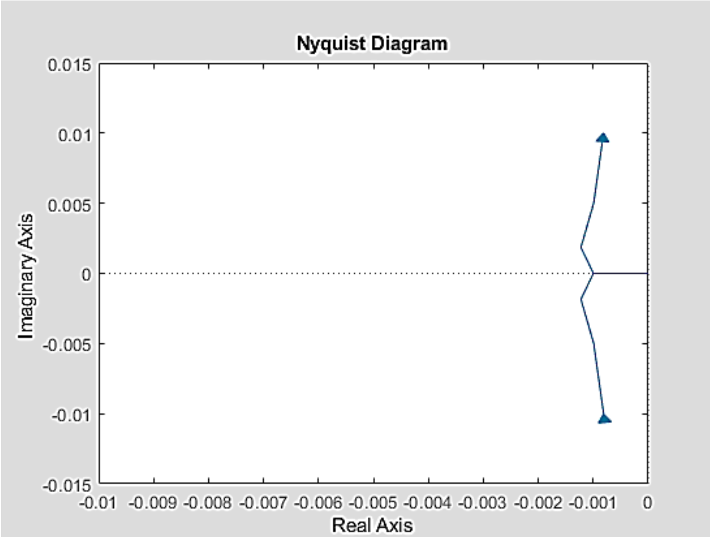


FIGURE 31 - NYQUIST DIAGRAM FOR SCENARIO 7.3

As per Figure 31, the real axis value is slightly more than -0.001, which indicates a control system with highly stable properties according to the Nyquist Stability Criterion.

CONCLUSION

From the results, it can be concluded that among the seven controllers, the PI controller and the PID controller are the most robust controllers for this project. These controllers were particularly verified for a range of gain values, and the worst case scenarios are considered to check their reliability. Although both of the controllers gave satisfactory results, the stability analysis was performed on all the three scenarios on a PID controller. On performing the stability analysis by adopting the Routh-Hurwitz Stability Criterion and the Nyquist Stability Criterion, the PID controller is found to be highly stable for all the three scenarios. This fulfils both the project objectives and indicates that the controller selected would deliver high performance and stability when used for a cruising automotive.

In addition to the fulfilment of the objectives, the report addresses the theoretical aspects of various control system principles and enables combining these to practice. The report also adds on considering two stability criterion to ensure that the stability aspect thoroughly addressed, along with verifying the validity of the criteria itself. The project has been helpful in broadening the technical understanding of controls engineering and various mathematical tools used within the field of study. In conclusion, the project may restrict itself to the objectives in this report, however, it has future development aspects, for instance, incorporating other automobile aspects within the control system and develop a holistic control system design.

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APPENDIX

Influence of V_r and $g\theta$ in attaining the actual velocity

Although it is assumed in the Assumptions section that the V_r and $g\theta$ are considered the same value for all the scenario to compare each controller fairly and with simplicity, it is also indicated that this is one of the instructions of the project. To present this, the most optimum (PID) controller is considered, specifically Scenario 7.2.

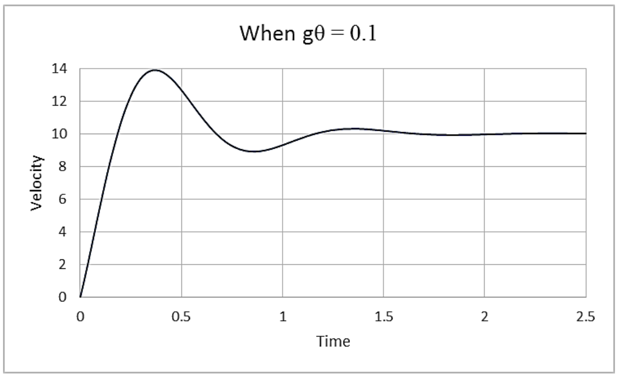

FIGURE 33 - G-THETA IS 0.1

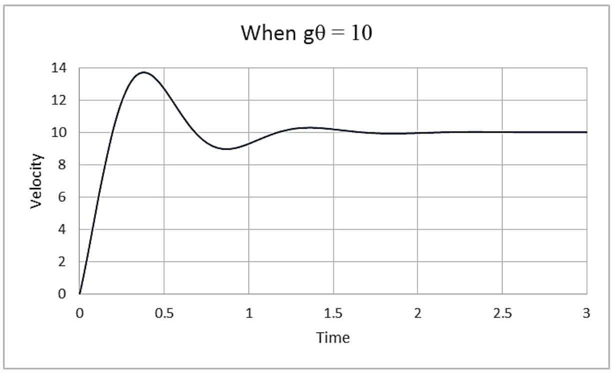

FIGURE 32 - G-THETA IS 10

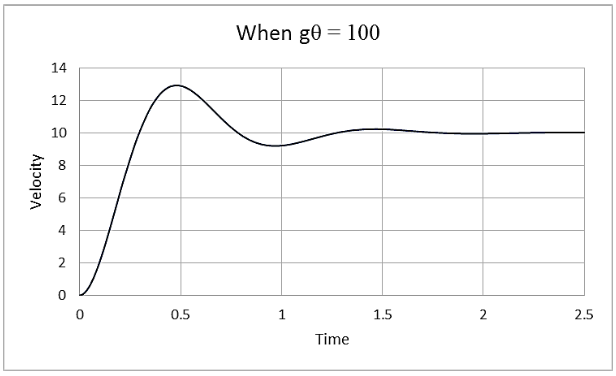

FIGURE 34 - G-THETA IS 100

Considering all the figures (Figure 32, 33, and 34), where the value of $g\theta$ changes as 0.1, 10, 100, respectively, it can be observed that there is no significant change in the value. Therefore, it can be established that the $g\theta$ would have lesser influence on the overall actual velocity attained of the vehicle.

V_r is not changed in any of the cases because it can be interpreted that because it is the desired value, this will enable us to relate the actual velocity, and that the actual velocity changes to the value of V_r when this is changed.

TABLE OF FIGURES