EE210: ANALOG ELECTRONICS

MID SEMESTER EXAM
FEBRUARY 19, 2024

Write all your answers in the spaces provided. Do your calculations in this sheet itself. Extra sheet will be provided if you need it for rough work, but it will **NOT** be considered for evaluation. The page numbers are marked 1–13 in this question paper. Verify it before you start answering.

There are **four** places where you need to write your name and roll-number. *Fill them up before you start answering. Each unfilled instance will be penalized by 1 mark.*

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Assume $\mu_p C_{ox} = 100\mu A/V^2$ and $V_{thp} = 1V$ for all transistors in the question set.

1) : Consider the circuit in Fig. 1. Assume $V_{DD} = 5\,V$, $I_0 = 100\,\mu A$, $R = 10\,k\Omega$, $R_L = 3\,k\Omega$, $(W/L)_1 = 20$, $v_i = V_p \cos(\omega_0 t)$.

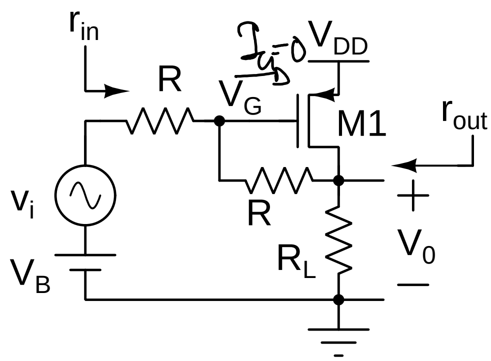


Fig. 1. Question 1

a) : Find the quiescent V_B such that the quiescent current through $V_B = 0$ **[4]**

$V_B = V_0 = V_G$ $\because$ I (through V_B) = 0 and $I_G = 0$

KCL @ V_0 $\Rightarrow$ $\frac{1}{2}\mu_p C_{ox} \frac{W}{L}(V_{DD} - V_B - V_{thp})^2 = \frac{V_B}{R_L}$

$\Rightarrow 3(4 - V_B)^2 = V_B$

$\Rightarrow 3(16 + V_B^2 - 8V_B) = V_B \Rightarrow 3V_B^2 - 25V_B + 48 = 0$

$\Rightarrow V_B = \frac{25 \pm \sqrt{625 - 4\times 3\times 48}}{6} = \frac{25 \pm 7}{6}$

$= \frac{32}{6}$ or $3\,V$.

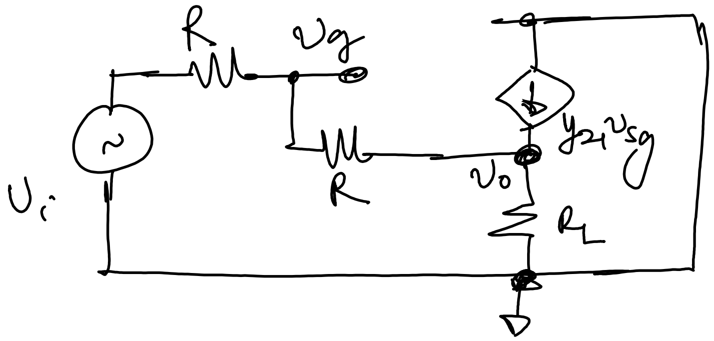
$\frac{32}{6}$ is invalid since the current through M1 = 0 in such case.

$\therefore$ $\boxed{V_B = 3V}$

b) : If $v_i = V_p \sin(\omega_0 t)$, find V_p such that $M1$ remains in saturation through the entire cycle of the input. For this value of V_p what is the minimum overdrive voltage ($V_{SG} - V_{thp}$) that $M1$ experiences in one complete cycle? **[4+4]**

$V_{GQ} = 3V \qquad V_{OQ} = 3V \qquad I_{DQ} = \frac{V_{OQ}}{R_L} = 1mA.$

$y_{21}|_{M1} = \mu_p C_{ox} \frac{W}{L}\left(V_{DD} - V_G - V_{thp}\right) = 2mS.$



$v_s = 0 \quad \therefore \quad v_{sg} = -v_g$

$v_g = \frac{v_i + v_o}{2}$

KCL @ v_o

$\Rightarrow \frac{v_o}{R_L} + \frac{v_o - v_i}{2R} = y_{21} v_{sg}$

$\Rightarrow \frac{v_o}{R_L} + \frac{v_o - v_i}{2R} = -y_{21}\left(\frac{v_i + v_o}{2}\right)$

$\Rightarrow v_o\left(\frac{1}{R_L} + \frac{1}{2R} + \frac{y_{21}}{2}\right) = v_i\left(-\frac{y_{21}}{2} + \frac{1}{2R}\right)$

$\Rightarrow v_o\left(0.33 + 0.05 + 1\right) = v_i\left(-1 + 0.05\right)$

$\Rightarrow v_o = \frac{-0.95}{1.38} v_i$ and $v_g = \frac{43}{276} v_i$

Total $V_O = V_{OQ} + v_o = 3 - \frac{95}{138} v_i$

Total $V_G = V_{GQ} + v_g = 3 + \frac{43}{276} v_i$

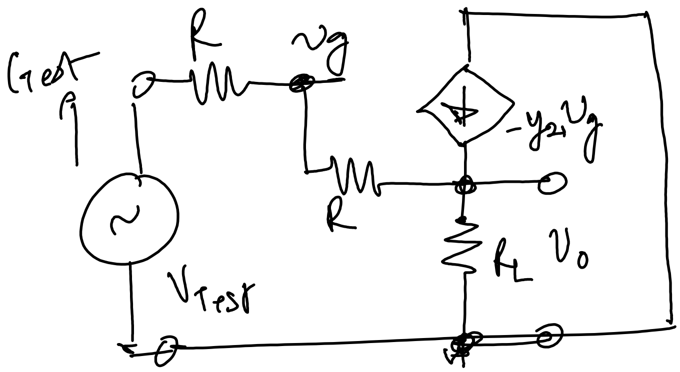
For sat: $V_O \le V_G + V_{thp} \Rightarrow \cancel{3} - \frac{95}{138} v_i \le \cancel{3} + \frac{43}{276} v_i + 1$

$\Rightarrow \frac{-233}{276} v_i \le 1$ for $v_i = -V_p$, $\boxed{V_p \le 1.18 V}$

If $v_i = +V_p \quad v_g = \frac{v_i + v_o}{2} = \frac{V_p - 0.688 V_p}{2} = 0.15 V_p$

$\therefore V_{G\,max} = 3 + 0.18 = 3.18 V \quad \therefore \boxed{V_{SG}(min) = 1.82 V}$

c) : Find the incremental input (r_{in}) and the output resistances (r_{out}) as indicated in the figure. If y_{21} of $M1$ is ∞ what are r_{in} and r_{out}? **[6+2]**



For r_{in}

$v_o = -0.688\, v_{Test}$ (From part (b))

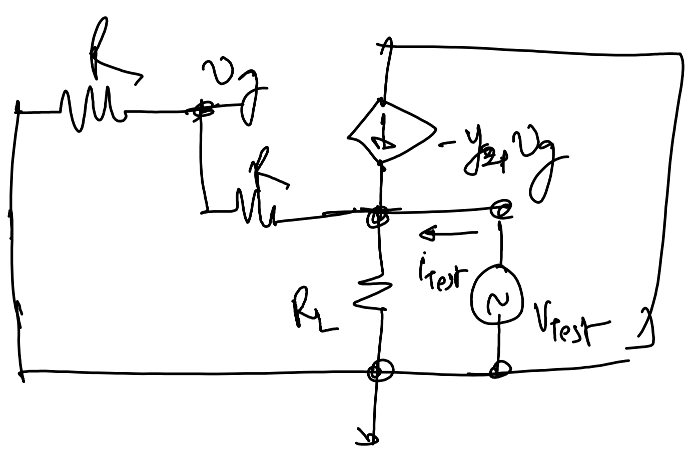
$\therefore\ i_{Test} = \dfrac{v_{Test} - v_o}{2R}$

$\Rightarrow \dfrac{v_{Test}}{i_{Test}} = r_{in} = 11.8\ k\Omega$

If $y_{21} \to \infty$ $\quad v_o = -v_i$ (From part (b))

$\therefore\ r_{in} = 10 k\Omega$

For r_{out}:



$v_g = \dfrac{v_{Test}}{2}$

$\therefore\ i_{Test} = \dfrac{v_{Test}}{R_L} + \dfrac{v_{Test}}{2R} + y_{21}\dfrac{v_{Test}}{2}$

$\Rightarrow \dfrac{i_{Test}}{v_{Test}} = \dfrac{1}{R_L} + \dfrac{1}{2R} + \dfrac{y_{21}}{2}$

$\Rightarrow r_{out} = 0.72\ k\Omega$

If $y_{21} \to \infty$

$r_{out} = 0$

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2) : Consider the circuit in Fig. 2. Assume $V_{DD} = 5\,V$, $V_B = 3\,V$, $(W/L)_1 = 20$,

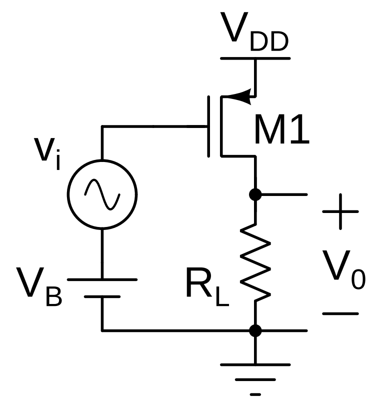


Fig. 2. Question 2

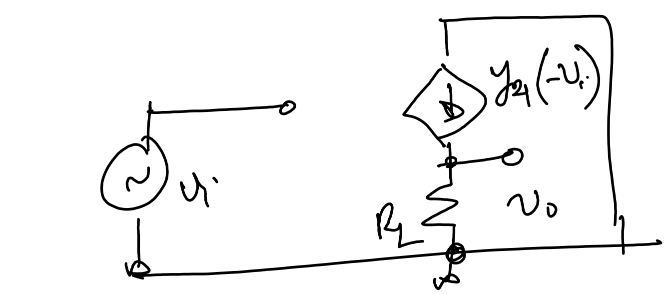
a) : Plot the small-signal gain between v_i to v_0 as you vary R_L from 0 to 100 $k\Omega$ and mark all the regions of operation clearly. **[8]**

If $V_{OQ} < V_B + V_{thp} \Rightarrow V_{OQ} < 4V$, M1 is in sat.

$$I_{SD\,sat} = \frac{1}{2}\mu_p C_{ox}\frac{W}{L}\left(V_{DD} - V_B - 1\right)^2 = 1mA.$$

$\therefore$ M1 is in sat if $R_L \le 4k\Omega$.

Small signal model for sat.



$$y_{21} = \mu_p C_{ox}\frac{W}{L}\left(V_{SGQ} - V_{thp}\right) = 2mS.$$

$$\therefore \frac{v_0}{v_i} = -2mS \times R_L$$

$$\left|\frac{v_0}{v_i}\right|_{max} = 8 \quad (\text{for } R_L = 4k\Omega)$$

contd.. For $R_L > 4k\Omega$, M1 is in linear region.

$$I_{SD} = \mu_p C_{ox} \frac{W}{L}\left[(V_{DD}-V_B-V_{thp})(V_{DD}-V_0) - \frac{1}{2}(V_{DD}-V_0)^2\right] = \frac{V_0}{R_L}$$

$$\Rightarrow 2m\left[(5-V_0) - \frac{1}{2}(5-V_0)^2\right] = \frac{V_0}{R_L(k\Omega)}$$

$$\Rightarrow R_L(10-2V_0-25-V_0^2+10V_0) = V_0$$

$$\Rightarrow R_L V_0^2 + (1-8R_L)V_0 + 15R_L = 0$$

$$\Rightarrow V_0 = \frac{8R_L-1 \pm \sqrt{(1-8R_L)^2-60R_L^2}}{2R_L} = 4-\frac{1}{2R_L} \pm \sqrt{\frac{1+4R_L^2-16R_L}{4R_L^2}}$$

$$= 4-\frac{1}{2R_L} \pm \sqrt{\frac{1}{4R_L^2}+1-\frac{4}{R_L}} \approx 4-\frac{1}{2R_L} \pm \frac{1}{2}\left(1-\frac{2}{R_L}\right)$$

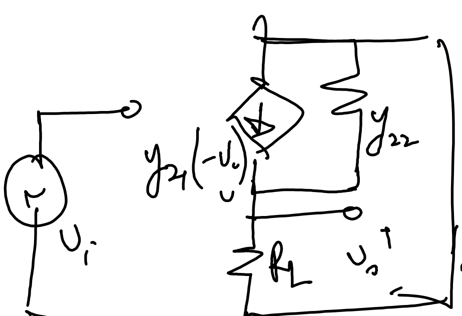
$$= 4.5-\frac{3}{2R_L} \quad \text{or} \quad 3.5+\frac{1}{2R_L}$$

In lin. $y_{21} = \mu_p C_{ox}\frac{W}{L}V_{SD} = 2m(V_{DD}-V_0) = 2m\left(5-4.5+\frac{3}{R_L}\right)$

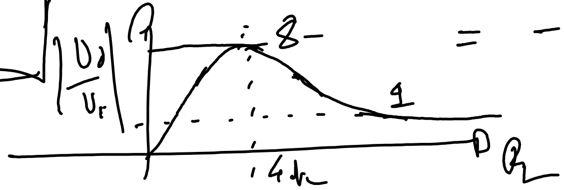
$$= \left(1m + \frac{6m}{R_L}\right)$$

$$y_{22} = \mu_p C_{ox}\frac{W}{L}(V_{SG}-V_{thp}-V_{SD}) = \mu_p C_{ox}\frac{W}{L}(-V_B-V_{thp}+V_0)$$

$$= 2m\left(-3-1+4.5-\frac{3}{2R_L}\right) = 1m - \frac{6m}{R_L}$$



$$U_0 = \frac{-y_{21}U_i}{y_{22}+G_L} \Rightarrow \frac{U_0}{U_i} = -\frac{1m+\frac{6m}{R_L}}{1m-\frac{3m}{R_L}}$$



$$= -\left(\frac{1+\frac{6}{R_L}}{1-\frac{3}{R_L}}\right) \text{ (Approximate)}$$

3) : NAME (in capital) Roll No:

Consider the circuit in Fig. 3. $V_{DD} = 5V$, $V_B = 2V$, $W/L = 20$, $R_S = 1k\Omega$, $R_L = 2k\Omega$.

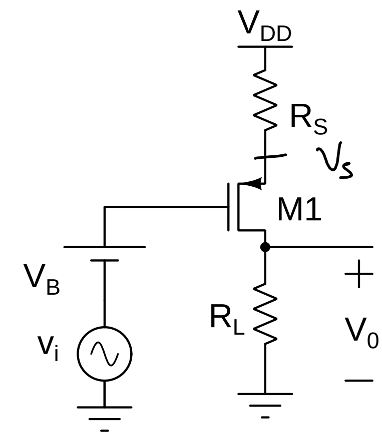


Fig. 3. Question 3

a) : Sketch the incremental Norton's equivalent network for the circuit clearly showing the short circuit current and the output resistance. **[4]**

$$I_{SD} = \frac{V_{DD} - V_S}{R_S} = \frac{1}{2}\mu_p C_{ox}\frac{W}{L}\left[V_S - V_B - V_{THP}\right]^2$$

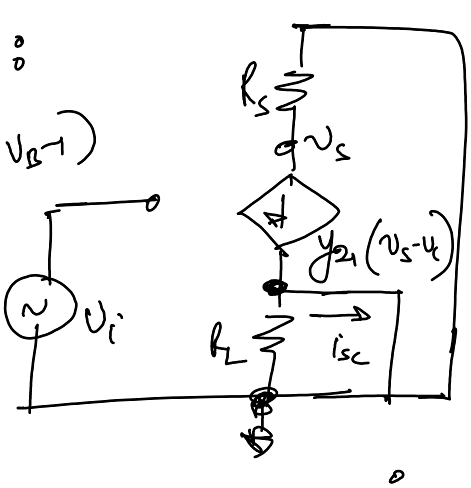
$$\Rightarrow 5 - V_S = (V_S - 3)^2$$

$$\Rightarrow V_S^2 - 5V_S + 4 = 0 \Rightarrow V_S = 4V \text{ or } 1V. \text{ (Not possible)}$$

$$\therefore I_{SD} = 1mA. \quad \therefore V_0 = 2V \Rightarrow \text{Saturation region.}$$

Small signal:

$$y_{21} = \mu_p C_{ox}\frac{W}{L}(V_S - V_B - 1) = 2mS$$



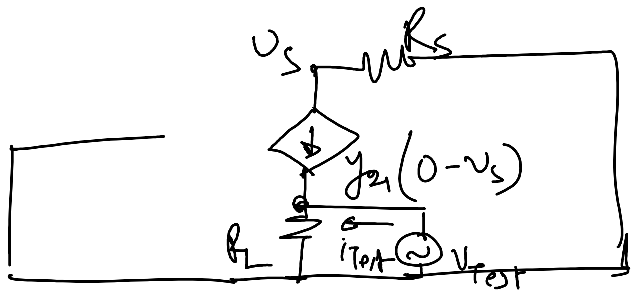
$$i_{sc} = y_{21}(v_s - v_i) = \frac{-v_s}{R_S}$$

$$\Rightarrow y_{21}v_i = v_s\left(y_{21} + \frac{1}{R_S}\right)$$

$$\Rightarrow v_s = \frac{y_{21}R_S}{1 + y_{21}R_S}v_i = \frac{2v_i}{3}$$

$$\therefore i_{sc} = \frac{-y_{21}}{1 + y_{21}R_S}v_i = -\frac{2}{3}\,mS\; v_i$$

..contd.. For r_{out}:



KCL @ v_s

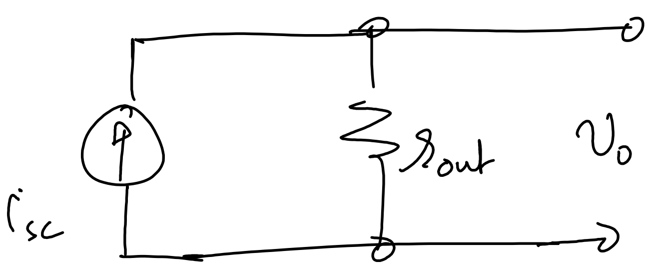
$$-y_{21} v_s + \frac{v_s}{R_s} = 0$$

$$\Rightarrow v_s = 0$$

$$i_{Test} = \frac{v_{Test}}{R_L} + y_{21} v_s$$

$$\Rightarrow r_{out} = \frac{v_{Test}}{i_{Test}} = R_L = 2k\Omega$$

b) : Find v_0/v_i. **[2]**



$$v_0 = i_{sc} \, r_{out}$$

$$= -\frac{2mS}{3} v_i \; 2k\Omega$$

$$\Rightarrow \frac{v_0}{v_i} = -\frac{4}{3}$$

c) : If $v_i = V_p \sin(\omega_0 t)$, find the maximum V_p for which $M1$ remains in saturation through the entire cycle of the input. What is the minimum overdrive voltage ($V_{SG} - V_{thp}$) that $M1$ experiences in the entire cycle of the input. **[6]**

Total $v_o = V_{OQ} + v_o$

$$= 2V - \frac{4v_i}{3}$$

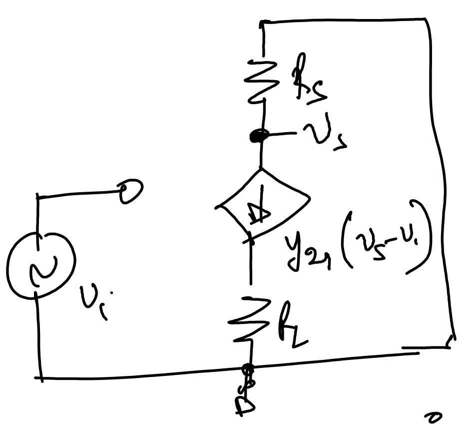
Total $v_G = V_{GQ} + v_i = 2V + v_i$

For sat $\cancel{2V} - 4v_i/3 \le \cancel{2V} + v_i + V_{thp}$

$$\Rightarrow 7v_i/3 \le V_{Thp}$$

For $v_i = -V_p$ $\quad V_p \le \frac{3}{7} V$

Min. overdrive happens when $v_i = V_p$



KCL @ $v_s \Rightarrow -\frac{v_s}{R_s} = y_{21}(v_s - v_i)$

$$\Rightarrow v_s = \frac{y_{21} R_s v_i}{1 + y_{21} R_s} = \frac{2}{3} v_i$$

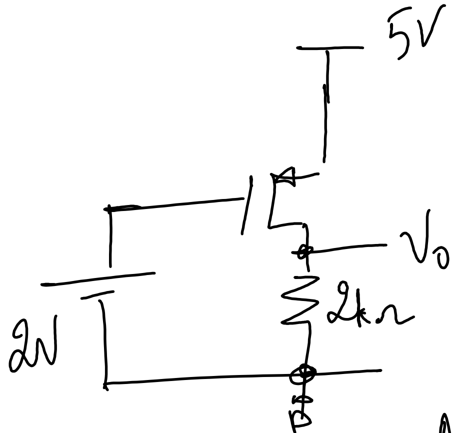
$$v_{sg} = v_s - v_i = -\frac{v_i}{3}$$

$\therefore$ Min overdrive $V_{SGQ} - V_{Thp} - \frac{V_p}{3}$

$$= 2 - 1 - \frac{1}{7}$$

$$= \frac{6}{7} V$$

d) : Now assume, $R_S = 0$. Find the quiescent drain voltage of $M1$, and the quiescent current through $M1$. **[4]**



Assume sat.

$$I_{SD} = \frac{1}{2} \times 0.1 \times 20 (3-1)^2$$

$$= 4 mA$$

$\therefore V_0 = 8V$ Not possible

Assume linear region of operation:

$$I_{SD} = \mu_p C_{ox} \frac{W}{L} \left[(V_{SG} - V_{thp}) V_{SD} - \frac{1}{2} V_{SD}^2 \right]$$

$$= 2m \left[2(5 - V_0) - \frac{1}{2}(5 - V_0)^2 \right] = \frac{V_0}{2k\Omega}$$

$$\Rightarrow 4 \left[10 - 2V_0 - \frac{1}{2}(25 + V_0^2 - 10V_0) \right] = V_0$$

$$\Rightarrow 40 - 8V_0 - 50 - 2V_0^2 + 20V_0 = V_0$$

$$\Rightarrow 2V_0^2 - 11V_0 + 10 = 0$$

$$\Rightarrow V_0 = \frac{11 \pm \sqrt{121 - 80}}{4} \Rightarrow V_0 \approx 4.2V \text{ or } 1.25V$$

$V_0 = 4.2V$ satisfies the linear region of operation

$$\therefore I_{SD} = \frac{4.2V}{2k\Omega} = 2.1 mA$$

e) : Sketch the incremental Norton's equivalent network for the circuit under the condition of the previous part (i.e. $R_s = 0$) and find the small signal gain v_0/v_i. **[4+2]**

$$I_{SD} = \mu_p C_{ox} \frac{W}{L} \left[(V_{SG} - V_{thp}) V_{SD} - \frac{1}{2} V_{SD}^2 \right]$$

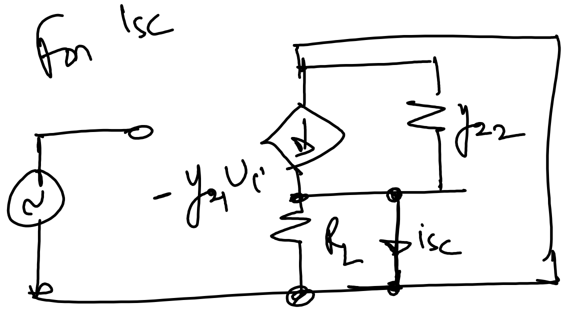
$$y_{21} = \mu_p C_{ox} \frac{W}{L} V_{SD} = 2m \times 0.8 = 1.6\ mS$$

$$y_{22} = \mu_p C_{ox} \frac{W}{L} \left[V_{SG} - V_{thp} - V_{SD} \right]$$

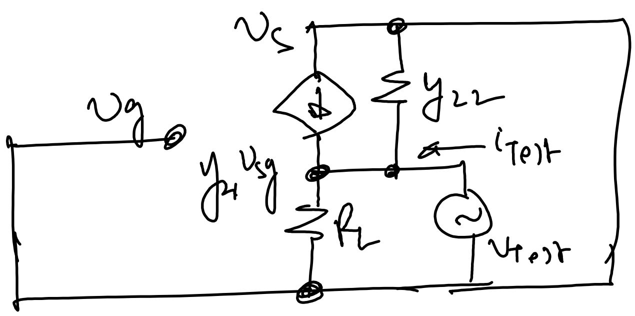
$$= 2m \left(\cancel{5} - 2 - 1 - \cancel{5} + 4.2 \right)$$

$$= 2.4\ mS.$$

For i_{sc}



$$i_{sc} = -y_{21} v_i = -1.6\ mS\ v_i.$$



For R_{out}

For R_{out} $\quad v_g = 0 \quad v_s = 0$

$\therefore\ y_{21} v_{sg} = 0$

$$\therefore\ i_{test} = \frac{v_{test}}{R_L} + y_{22} v_{test}$$

$$= 2.9\ mS\ v_{test}$$

$$\Rightarrow R_{out} = \frac{1}{2.9\ mS}$$

$$\therefore\ v_0 = i_{sc} R_{out} = 1.6m\, v_i \times \frac{1}{2.9\ mS}$$

$$\Rightarrow \boxed{\frac{v_o}{v_i} = \frac{-1.6}{2.9}}$$

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4) : Fig.4 shows a nonlinear two port network with the current voltage characteristics as follows. $I_1 = 0.1mS \times V_1$, and $I_2 = 0.5mA/V^2 \times V_1^2 + 0.08mS \times V_2 + 0.2mA$.

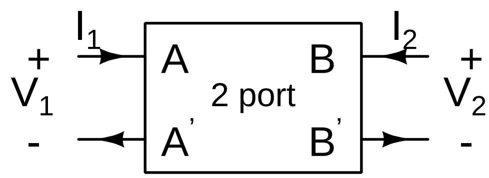


Fig. 4. Problem 4

a) : Is the network active or passive? Justify your answer. **[2]**

If $V_1 = 0$ and $V_2 = 0$

$I_1 = 0$ and $I_2 = 0.2mA$

There's a current source inside.

Hence active.

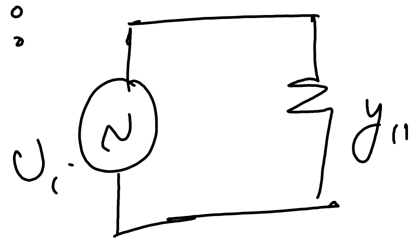
b) : You have a sinusoidal voltage source $V_p \cos(\omega_0 t)$ and a load resistance $R_L = 50k\Omega$ across which you want to develop a linear incremental voltage of $10V_p \cos(\omega_0 t)$ (i.e., |small signal gain| = 10). Find the quiescent voltages and currents in the above network that will enable you to achieve the requirement. **[4]**

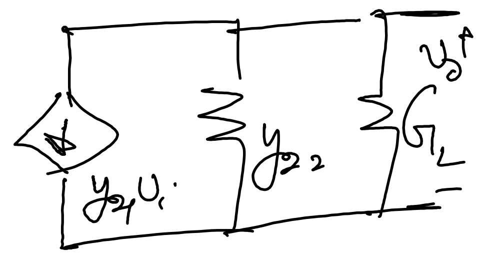
$I_1 = 0.1m V_1$ $\quad\quad$ $I_2 = 0.5m\, V_1^2 + 0.08m\, V_2 + 0.2m$

$y_{11} = 0.1m$ $\quad\quad$ $y_{12} = 0$

$y_{21} = 1m\, V_{1Q}$ $\quad\quad$ $y_{22} = 0.08m$

Incremental model:





$$\frac{v_o}{v_i} = \frac{-y_{21}}{G_L + y_{22}} = -10 V_{1Q}$$

$$\therefore \left|\frac{v_o}{v_i}\right| = 0 \quad \Rightarrow \quad V_{1Q} = 1V$$

$$\Rightarrow \quad I_{1Q} = 0.1mA$$

$$V_{2Q} = \text{Anything}$$

$$I_{2Q} = 0.7mA + 0.08m\, V_{2Q}$$

c) : How will you apply the bias, the input voltage and the load resistance to the network to realize a small signal gain of 10. Sketch and show your connection. There are no additional constraints. (The internal circuitry of the network with controlled sources is not needed) **[4]**