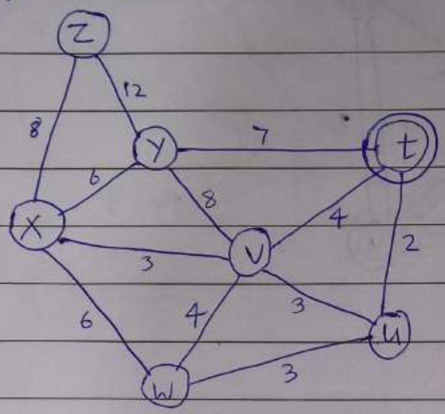
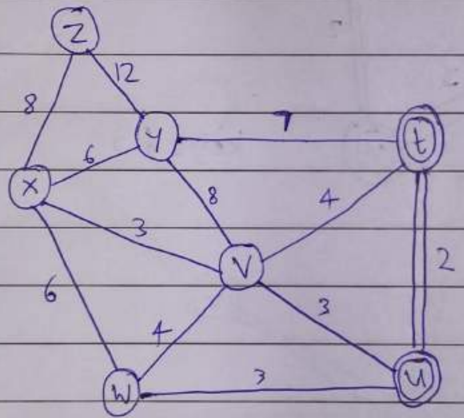
CS 575 Homework 6 Ved Ranade

Ans 1. Starting at vertex t:

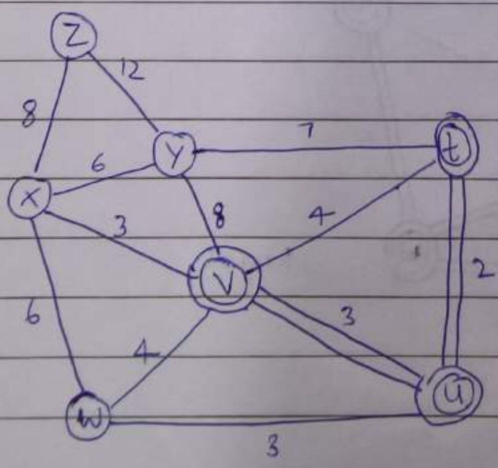
◎ = black vertices
○ = white vertices
|| = selected edge for MST



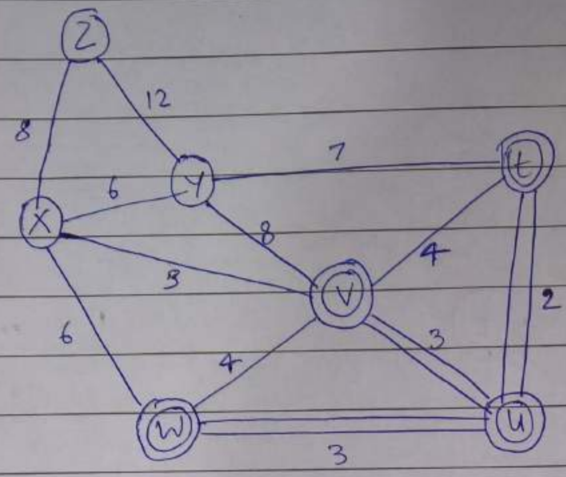
key = 0



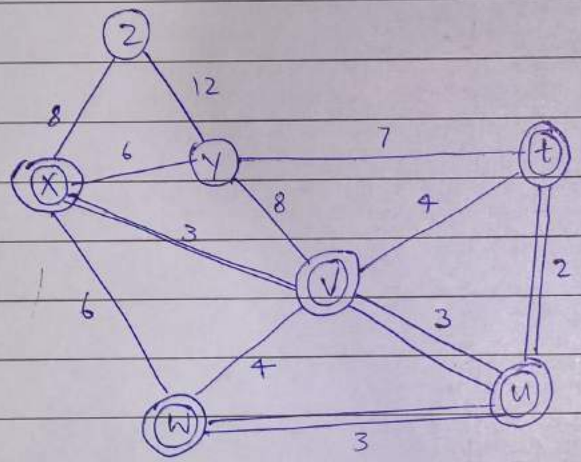
key = 2



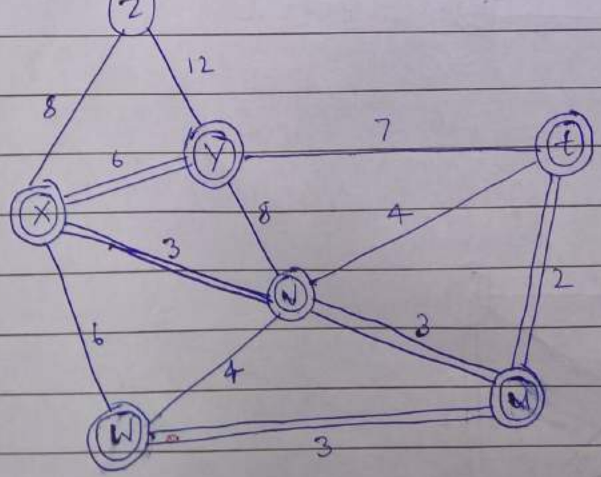
Key = 3



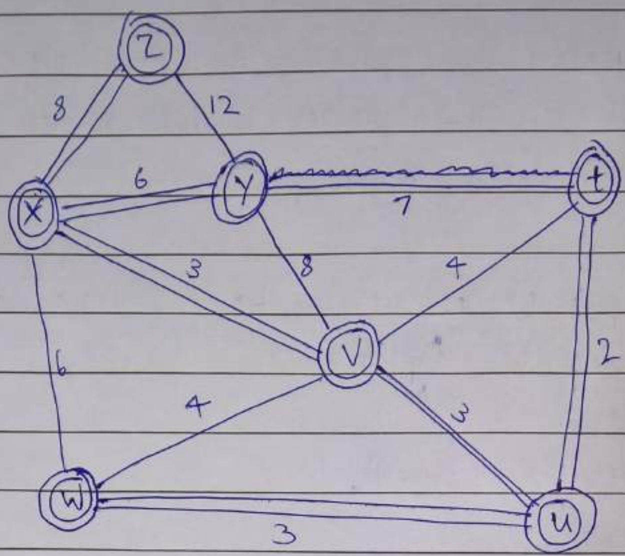
key=3



key= ~~6~~ 3

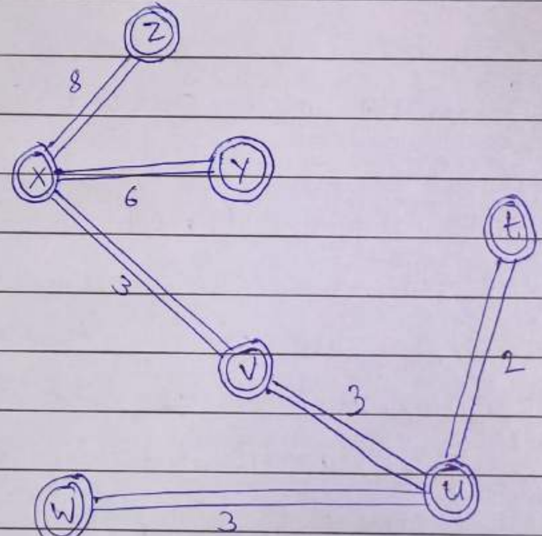


key=6.



Key=8

MST :



Ans. 2. Proof:

We know that the vertex cover problem is NP-complete.
Hence, we will prove that the given problem is NP-complete by reducing the vertex cover problem to the given problem.

Let L be the given problem and let L' be the vertex cover problem.
~~To prove~~ To prove L to be NP complete:

1. $L \in NP$
2. $L' \leq_p L$ need to be proved

Consider: $L' \leq_p L$:
Proof: Let G be a graph having V vertices and E edges.
Let i be an instance of the vertex cover problem.
Now, we can create an instance of L as follows:
Let $U = E$
Also, let S_u be a set for each u in V, such that S_u contains edges adjacent to u.
U can be covered by n sets if and only if graph G has a vertex cover of size $\leq i$, where n is also $\leq i$
This is because if sets $S_{u_1}, S_{u_2}, \ldots S_{u_i}$ cover U then every edge is adjacent to at least one of the vertices $u_1, u_2, \ldots u_i$, which gives a vertex cover of size i, If $u_1, u_2, \ldots u_i$ is a vertex cover, then sets $S_{u_1}, S_{u_2}, \ldots S_{u_i}$ cover U. Hence, we show that $L' \leq_p L$

Now consider: $L \in NP$
In this case, the certificate provides a list of k sets from the given collection.
We can check whether they cover all of U in polynomial time.
Since the certificate can be checked in polynomial time, we can say that $L \in NP$.

Hence, the given problem is proved to be NP complete.