Number Systems

Acknowledgement

The contents (figures, concepts, graphics, texts etc.) of the slides are gathered and utilized from the books mentioned and the corresponding PPTs available online:

Books:

1. **Let Us C,** Yashawant Kanetkar, BPB Publications.
2. **The C Programming Language,** B. W. Kernighan, D. Ritchie, Pearson Education India.

Web References:

1. **Problem Solving through Programming in C,** Anupam Basu, NPTEL Video Lectures. Link: https://nptel.ac.in/courses/106/105/106105171/
2. **Compile and Execute C Online (Link:** https://www.onlinegdb.com/)

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Number System

- A set of values used to represent a different quantities is known as number system.
- A number system forms the basis of counting.
- A number system can be used to represent the number of students in a class or number of viewers watching a certain TV program etc.
- Number systems consists of symbols (like 1, 2, 3 or I, II, III etc.) to represent numbers and rules to use them.
- Many number systems came into existence for example: decimal, binary, octal etc.
- The decimal number systems has 10 digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, they are generally used for arithmetic operations.
- The total number of digits used in a number system is called its **base or radix.** Number systems can be built in many bases.

Number System

- Since counting in decimal involves 10 symbols the base or radix is 10.
- The digital computer represents all kinds of data and information in binary numbers.
- It includes audio, graphics, video, text and numbers.
- Base 2 numbers are used for computer representations. A number system with base 2 is called **binary system**.
- Octal and hexadecimal number systems are extensions of Base 2 that have 8 (0,1,…7) digits and 16(0,1,…9, A, B, …F) digits or symbols respectively.
- So octal numbers have base of 8 and the hexadecimal numbers have base of 16.
- Numbers from one system can be converted to another system.

Types of Number System

- Decimal Number System
- Binary Number System
- Octal Number System
- Hexadecimal Number System

Decimal Number System

- The number system that we use in our day-to-day life is the decimal number system.
- The word decimal is derived from the Latin root decem (ten).
- Decimal number system has base 10 as it uses 10 digits from 0 to 9.
- In decimal number system, the successive positions to the left of the decimal point represent units, tens, hundreds, thousands and so on.
- Each position represents a specific power of the base (10).
- For example, the decimal number 1234 consists of the
  - 4 in the units position
  - 3 in the tens position
  - 2 in the hundreds position
  - 1 in the thousands position, and its value can be written as:

Decimal Number System

Decimal: 1234

(1x1000)+ (2x100)+ (3x10)+ (4x1)

$(1x10^3)+ (2x10^2)+ (3x10^1)+ (4xl0^0)$

1000 + 200 + 30 + 4

1234

Binary Number System

The word binary is derived from the Latin root bini (or two by two). The characteristics of binary number system are as follows:

- Uses two digits, 0 and 1. The symbols in this system are often referred to as binary digits or bits (binary digit).
- Base 2 number system
- Each position in a binary number represents a 0 power of the base (2). Example 2^0
- Last position in a binary number represents a x power of the base (2). Example 2^x where x represents the (last position – 1).
- Calculating Decimal Equivalent of Binary Number : 10101_2

Step	Binary Number	Decimal Number
Step 1	10101_2	$((1 \times 2^4) + (0 \times 2^3) + (1 \times 2^2) + (0 \times 2^1) + (1 \times 2^0))_{10}$
Step 2	10101_2	$(16 + 0 + 4 + 0 + 1)_{10}$
Step 3	10101_2	21_{10}

Decimal Equivalent of Binary Number

Example

- 101011_2 =>

$$
\begin{array}{rcl}
1 \times 2^0 & = & 1 \\
1 \times 2^1 & = & 2 \\
0 \times 2^2 & = & 0 \\
1 \times 2^3 & = & 8 \\
0 \times 2^4 & = & 0 \\
1 \times 2^5 & = & 32 \\
\hline
 & & 43_{10}
\end{array}
$$

Octal Number System

The word octal is derived from the Latin root octo(eight). The characteristics of octal number system are as follows:

- Uses eight digits, 0,1,2,3,4,5,6,7.
- Also called base 8 number system
- Each position in an octal number represents a 0 power of the base (8). Example 8^0
- Last position in an octal number represents a x power of the base (8). Example 8^x where x represents the (last position - 1.)
- Calculating Decimal Equivalent of Octal Number : 12570_8

Step	Octal Number	Decimal Number
Step 1	12570_8	$((1 \times 8^4) + (2 \times 8^3) + (5 \times 8^2) + (7 \times 8^1) + (0 \times 8^0))_{10}$
Step 2	12570_8	$(4096 + 1024 + 320 + 56 + 0)_{10}$
Step 3	12570_8	5496_{10}

Decimal Equivalent of Octal Number

Example

- 724_8 =>

$$
\begin{aligned}
4 \times 8^0 &= 4 \\
2 \times 8^1 &= 16 \\
7 \times 8^2 &= 448 \\
\hline
& 468_{10}
\end{aligned}
$$

Hexadecimal Number System

Characteristics of octal number system are as follows:

- Uses 10 digits and 6 letters, 0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F.
- Letters represents numbers starting from 10. A = 10. B = 11, C = 12, D = 13, E = 14, F =15.
- Also called base 16 number system.
- Each position in a hexadecimal number represents a 0 power of the base (16). Example 16^0
- Last position in a hexadecimal number represents a x power of the base (16). Example 16x where x represents the (last position - 1.)

Hexadecimal Number System

Hexadecimal Number : $19FDE_{16}$

Step	Binary Number	Decimal Number
Step 1	$19FDE_{16}$	$((1 \times 16^4) + (9 \times 16^3) + (F \times 16^2) + (D \times 16^1) + (E \times 16^0))_{10}$
Step 2	$19FDE_{16}$	$((1 \times 16^4) + (9 \times 16^3) + (15 \times 16^2) + (13 \times 16^1) + (14 \times 16^0))_{10}$
Step 3	$19FDE_{16}$	$(65536+ 36864 + 3840 + 208 + 14)_{10}$
Step 4	$19FDE_{16}$	106462_{10}

Decimal Equivalent of Hexadecimal Number

Example

- ABC_{16}=>
-

C	x	16^0	=	12x1	=	12
B	x	16^1	=	11x16	=	176
A	x	16^2	=	10x256	=	2560
						2748_{10}

Quantities/Counting (1 of 2)

Decimal	Binary	Octal	Hexa-decimal
0	0	0	0
1	1	1	1
2	10	2	2
3	11	3	3
4	100	4	4
5	101	5	5
6	110	6	6
7	111	7	7

Quantities/Counting (2 of 2)

Decimal	Binary	Octal	Hexa-decimal
8	1000	10	8
9	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F

Conversion of Number System – Decimal to Other Base System

- Step 1 - Divide the decimal number to be converted by the value of the new base.
- Step 2 - Get the remainder from Step 1 as the rightmost digit (least significant digit) of new base number.
- Step 3 - Divide the quotient of the previous divide by the new base.
- Step 4 - Record the remainder from Step 3 as the next digit (to the left) of the new base number.
- Repeat Steps 3 and 4, getting remainders from right to left, until the quotient becomes zero in Step 3.
- The last remainder thus obtained will be the most significant digit (MSD) of the new base number.

Conversion of Number System – Decimal to Other Base System

Decimal Number : 29

Step	Operation	Result	Remainder
Step 1	29 / 2	14	1
Step 2	14 / 2	7	0
Step 3	7 / 2	3	1
Step 4	3 / 2	1	1
Step 5	1 / 2	0	1

- As mentioned in Steps 2 and 4, the remainders have to be arranged in the reverse order so that the first remainder becomes the least significant digit (LSD) and the last remainder becomes the most significant digit (MSD).
- Decimal Number : 29_{10} = Binary Number : 11101_2.

Decimal to Binary

Example

$125_{10} = ?_2$

2	125	
2	62	1
2	31	0
2	15	1
2	7	1
2	3	1
	1	1

$125_{10} = 1111101_2$

Octal to Binary

Example

$705_8 = ?_2$

7	0	5
111	000	101

$705_8 = 111000101_2$

Hexadecimal to Binary

Example

$10AF_{16} = ?_2$

1	0	A	F
0001	0000	1010	1111

$10AF_{16} = 0001000010101111_2$

Converting a Number of Another Base to Decimal number

Step 1: Determine the column (positional) value of each digit

Step 2: Multiply the obtained column values by the digits in the corresponding columns

Step 3: Calculate the sum of these products

Example

$4706_8 = ?_{10}$

Common values multiplied by the corresponding digits

$4706_8 = 4 \times 8^3 + 7 \times 8^2 + 0 \times 8^1 + 6 \times 8^0$

$= 4 \times 512 + 7 \times 64 + 0 + 6 \times 1$

$= 2048 + 448 + 0 + 6$ ← **Sum of these products**

$= 2502_{10}$

Converting a Number of Some base to Another Base

Step 1: Convert the original number to a decimal number (base 10)

Step 2: Convert the decimal number so obtained to the new base number

Example

$545_6 = ?_4$

Solution:

Step 1: Convert from base 6 to base 10

$$
\begin{aligned}
545_6 &= 5 \times 6^2 + 4 \times 6^1 + 5 \times 6^0 \\
&= 5 \times 36 + 4 \times 6 + 5 \times 1 \\
&= 180 + 24 + 5 \\
&= 209_{10}
\end{aligned}
$$

Converting a Number of Some base to Another Base

Step 2: Convert 209_{10} to base 4

4	209	Remainders
	52	1
	13	0
	3	1
	0	3

Hence, $209_{10} = 3101_4$

Decimal to Octal

Example

$1234_{10} = ?_8$

8	1234	
8	154	2
8	19	2
8	2	3
	0	2

$1234_{10} = 2322_8$

Decimal to Hexadecimal

Example

$1234_{10} = ?_{16}$

16	1234	2
16	77	13=D
16	4	

$1234_{10} = 4D2_{16}$

Binary to Octal

Example

$1011010111_2 = ?_8$

1 011 010 111

1 3 2 7

$1011010111_2 = 1327_8$

Binary to Hexadecimal

Example

$1010111011_2 = ?_{16}$

10	1011	1011
↓	↓	↓
2	B	B

$1010111011_2 = 2BB_{16}$

Signed Numbers

- Inside a computer the representation is only through 1’s and 0’s. no other symbols are available.
- Thus 1’s and 0’s are also used to represent the sign of a number.
- In a computer one of the ways a negative number may be represented is by using the most significant bit (MSB) as the sign.
- The convention is:
  - When the MSB uis 0, the number is positive
  - When the MSB is 1, the number is negative.

Signed Numbers

Binary	Unsigned	Signed	Binary	Unsigned	signed
0000	0	+0	1000	8	-0
0001	1	+1	1001	9	-1
0010	2	+2	1010	10	-2
0011	3	+3	1011	11	-3
0100	4	+4	1100	12	-4
0101	5	+5	1101	13	-5
0110	6	+6	1110	14	-6
0111	7	+7	1111	15	-7

- The MSB being allotted for holding, the total number of bit positions available value representation gets reduced by one bit.
- Notice that 0 is being represented as +0 and -0 which is an unproductive use of one value.
- This representation of sign by MSB and the number as simple binary value by rest of the bits is called signed number representation.