Mathematics-II (MAC02) Problem Set

Laplace Transform

1. Evaluate:
   (i) $L\{t^5 e^{3t}\}$
   (ii) $L\{e^{2t}\cos^2 t\}$
   (iii) $L\{(\sqrt{t} - 1/\sqrt{t})^3\}$
   (iv) $L\{\sin\sqrt{t}\}$
   (v) $L\{e^{-t}\sin^2 t\}$ Ans: $\frac{3-s}{(s+1)(s^2-2s+5)}$.
   (vi) $L\{\dfrac{e^{-at} - e^{-bt}}{t}\}$ Ans: $\log(\frac{s+b}{s+a})$.
   (vii) $L\{\dfrac{1 - e^t}{t}\}$ Ans: $\log\frac{s-1}{s}$.
   (viii) $L\{\dfrac{\cos at - \cos bt}{t}\}$ Ans: $\log\sqrt{\frac{s^2+b^2}{s^2+a^2}}$.
   (ix) $L\{\dfrac{1 - \cos t}{t^2}\}$ Ans: $s\log\frac{s}{\sqrt{s^2+1}} + \cot^{-1} s$.
   (x) $L\{e^{2t}t\sin 3t\}$
   (xi) $L\{e^{-2t}\displaystyle\int_0^t t\sin 3t\,dt\}$ Ans: $\frac{6}{(s^2+4s+13)^2}$.
   (xii) $L\{t\displaystyle\int_0^t \frac{\sin u}{u}\,du\}$ Ans: $\frac{1}{s^2}\cot^{-1} s + \frac{1}{s(s^2+1)}$.
   (xiii) $L\{t\displaystyle\int_0^t \frac{e^{-t}\sin t}{t}\}\,dt$
   (xiv) $L\{t\displaystyle\int_0^t \frac{e^t\sin t}{t}\}\,dt$
   (xv) $\displaystyle\int_0^\infty \frac{\sin mt}{t}\,dt$
   (xvi) $\displaystyle\int_0^\infty \frac{e^{-t}\sin t}{t}\,dt$ Ans: $\frac{\pi}{4}$.
   (xvii) $\displaystyle\int_0^\infty e^{-2t}t\sin^2 t\,dt$ Ans: $\frac{1}{8}$.
   (xviii) $\displaystyle\int_0^\infty te^{-3t}\sin t\,dt$ Ans: $\frac{3}{50}$.
   (xix) $\displaystyle\int_0^\infty \frac{e^{-at} - e^{-bt}}{t}\,dt$ Ans: $\log\frac{b}{a}$.

2. Find the inverse Laplace transform of the following functions:
   (i) $\dfrac{4s + 5}{(s - 1)^2(s + 2)}$
   (ii) $\dfrac{1}{(s + a)(s + b)}$ Ans: $\frac{e^{-bt} - e^{-at}}{a-b}$.
   (iii) $\dfrac{1}{s^2(s + 4)}$ Ans: $\frac{t}{4} + \frac{e^{-4t}}{16} - \frac{1}{16}$.
   (iv) $\dfrac{s^2}{(s^2 + 1)^2}$ Ans: $\frac{1}{2}(\sin t + t\cos t)$.
   (v) $\log(1 + \dfrac{1}{s^2})$ Ans: $\frac{2(1 - \cos t)}{t}$.
   (vi) $\dfrac{9}{s^2(s - 3)}$ Ans: $-1 - 3t + e^{3t}$.
   (vii) $\dfrac{5}{s^2 - 2s - 3}$ Ans: $\frac{5}{2}e^{-t}\sinh 2t$.
   (viii) $\dfrac{5s^2 + 8s - 1}{(s + 3)(s^2 + 1)}$ Ans: $2e^{-3t} + 3\cos t - \sin t$.
   (ix) $\dfrac{s}{(s^2 + a^2)^2}$ Ans: $\frac{t\sin at}{2a}$.
   (x) $\log(1 + \dfrac{1}{s})$ Ans: $\frac{(1 - e^t)}{t}$.

(xi) $\dfrac{1}{s}\log(1+\dfrac{1}{s^2})$

(xii) $\log\dfrac{s+a}{s+b}$

(xiii) $\cot^{-1}(s/2)$

(xiv) $\dfrac{1}{2}\log\dfrac{s^2+b^2}{s^2+a^2}$

Ans: $2\int_0^t \dfrac{(1-\cos u)}{u}du$.

3. Use convolution theorem to find inverse Laplace transform of the following functions:

(i) $\dfrac{1}{(s^2+a^2)^2}$

(ii) $\dfrac{1}{s^2(s^2+a^2)}$

(iii) $\dfrac{1}{(s+a)(s+b)}$

(iv) $\dfrac{1}{s^2(s+1)^2}$

(v) $\dfrac{s}{(s+2)(s^2+9)}$