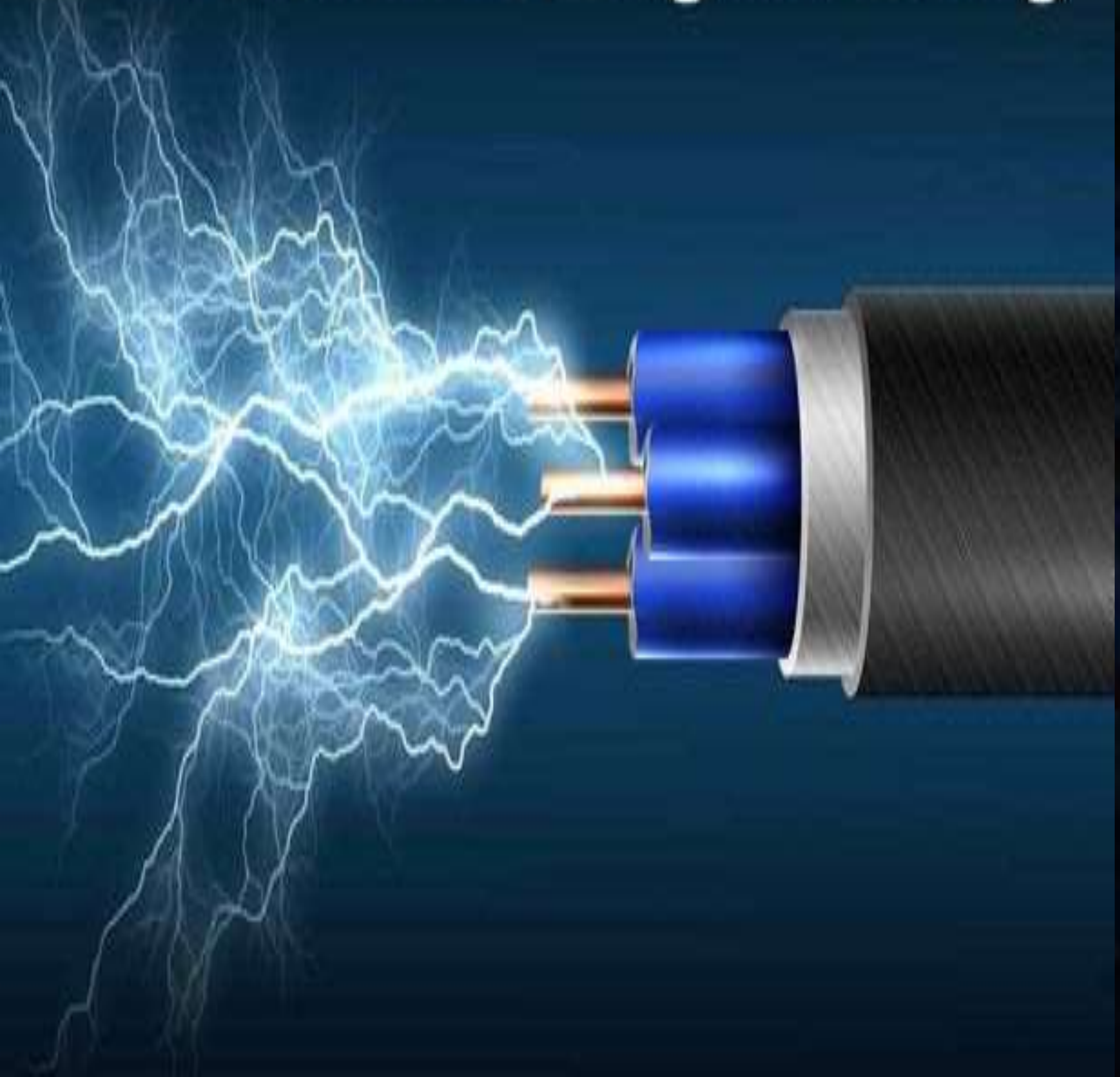
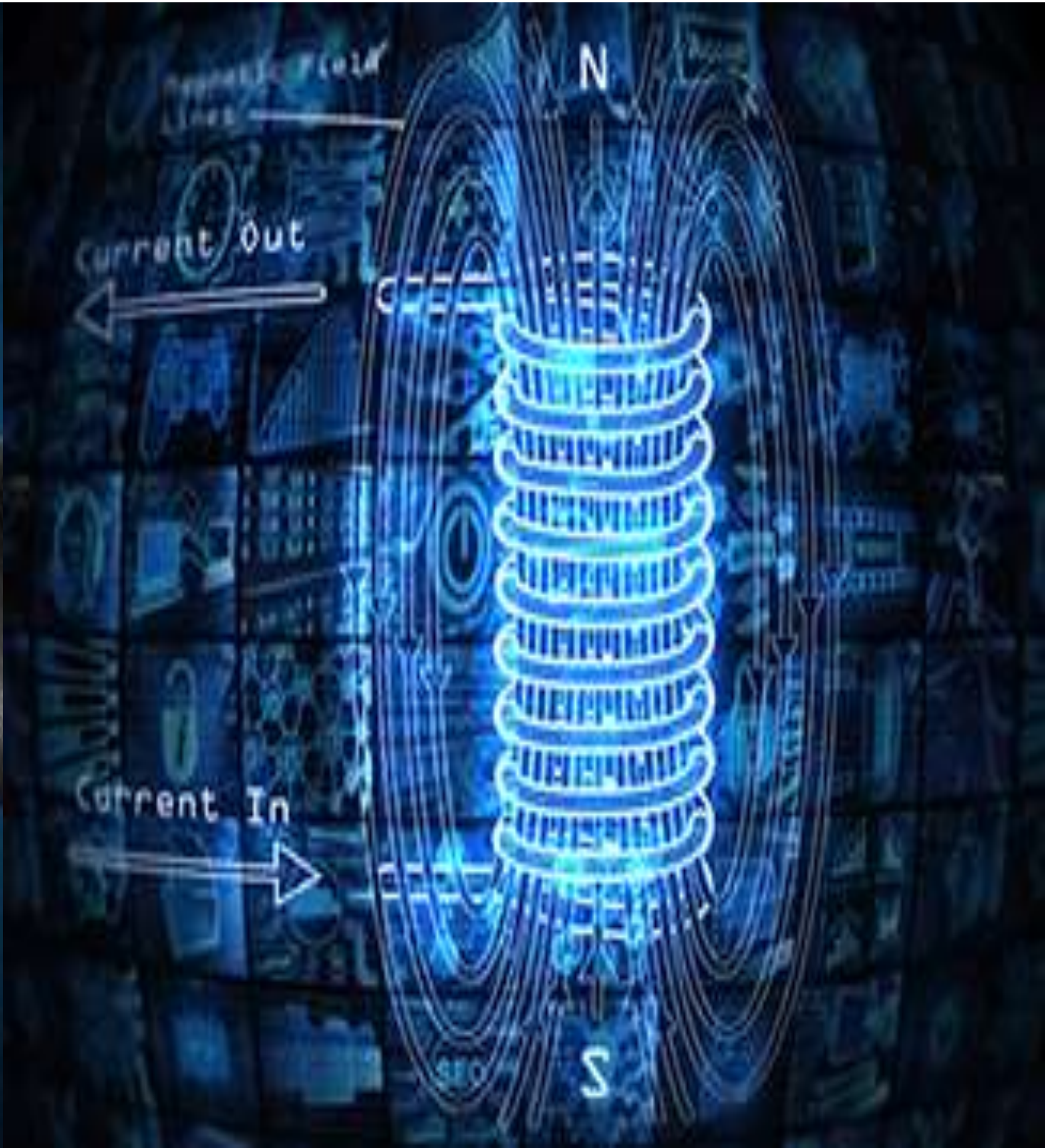
Electrical Engineering





LECTURE 4

Network Theorems

❖ Difference between circuit and Network

- An electrical network is an interconnection of electrical elements such as resistors, inductors, capacitors, transmission lines, voltage and current sources, switches etc.
- An electrical circuit is a network that has a closed loop, giving a return path for the current.
- All the circuits must be network but all the networks may or may not be circuit.

❖ Difference between Hypothesis, Theory and Law

- A hypothesis is a limited explanation of a phenomenon. It is a reasonable guess based on something that you observe in the natural world.
- A scientific theory is an in-depth empirical explanation of the observed phenomenon. A scientific theory consists of one or more hypotheses that have been supported by repeated testing.
- A theorem is a mathematical statement for the phenomena that has been proved, or can be proved.
- A law is a statement about an observed phenomenon and generally it rely on a concise mathematical equation used to describe an action under certain circumstances.

Network Theorems

➢ Superposition Theorem

- ✓ The basic principle of superposition states that if the effect produced in a system is directly proportional to the cause, then the overall effect produced in the system, due to a number of causes acting jointly, can be determined by superposing (adding) its effects of each source acting separately.

- ✓ The superposition principle is only applicable to linear networks and systems. A linear network comprises of independent sources, linear dependent sources and linear passive elements like resistor, inductor, capacitor and transformer.

  A device is called linear if it is characterized by an equation of the form $y = mx$, where m is a constant and not a function of x.

  Thus, $y = x^3$ is a nonlinear equation because in this case, $m = x^2$ is a function of the independent variable x.

  Let the sources v_1 and v_2 are applied to a linear network with zero initial conditions. If v_1 gives x_1 and v_2 gives x_2, then $(v_1 + v_2)$ gives $(x_1 + x_2)$

- ✓ In a linear network with several sources (which include the equivalent sources due to initial conditions) the overall response at any point in the network is equal to the sum of individual response of each source, considered separately, the other sources being made inoperative.

Network Theorems

➢ Superposition Theorem

✓ The two basic tests of linearity are additivity and homogeneity.

Additivity: $F(x_1 + x_2) = F(x_1) + F(x_2)$ and **Homogeneity:** $F(ax) = aF(x)$

A system that satisfies both the tests is called linear system. The superposition is only valid for linear system.

✓ Procedure of Superposition Theorem

The following steps are required to find the response in a particular branch using superposition theorem.

Step 1 − Find the response in a particular branch of a network by considering one independent source and making inoperative of other sources by

(a) short-circuiting the voltage sources and replacing them by their series impedance and

(b) open-circuiting the current sources and substituting them by their shunt impedances.

Step 2 − Repeat Step 1 for all independent sources present in the network.

Step 3 − Add all the responses in order to get the overall response in a particular branch when all independent sources are present in the network.

Network Theorems

➢ Superposition Theorem

✓ Explanation

- Let us consider any linear network containing bilateral linear impedances and energy sources as shown in **Fig. 2.1**.
- Show that the current flowing in any element is the algebraic sum of the currents that are separately caused to flow in that element by each energy source.

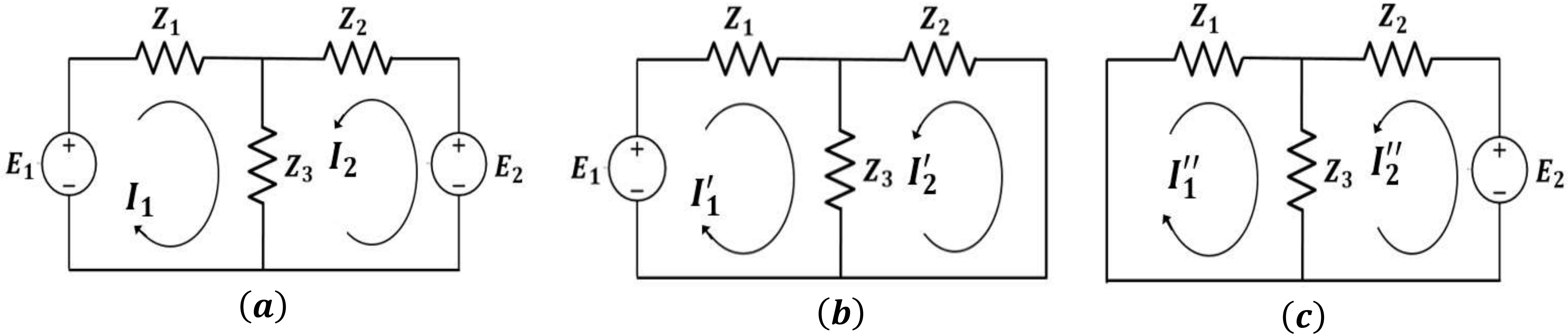


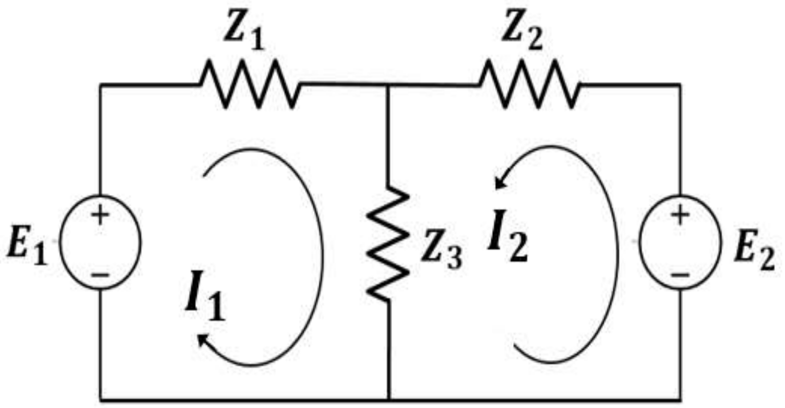
Fig. 2.1. Circuits for Superposition Theorem

Network Theorems

➢ Superposition Theorem

✓ Explanation

Consider **Fig. 1(a)**



$$E_1 = I_1(Z_1 + Z_3) + I_2 Z_3$$

$$E_2 = I_1 Z_3 + I_2(Z_2 + Z_3)$$

Solving

$$I_1 = \frac{Z_2 + Z_3}{Z_1Z_2 + Z_2Z_3 + Z_3Z_1} E_1 - \frac{Z_3}{Z_1Z_2 + Z_2Z_3 + Z_3Z_1} E_2$$

$$I_2 = \frac{-Z_3}{Z_1Z_2 + Z_2Z_3 + Z_3Z_1} E_1 + \frac{Z_1 + Z_3}{Z_1Z_2 + Z_2Z_3 + Z_3Z_1} E_2$$

Network Theorems

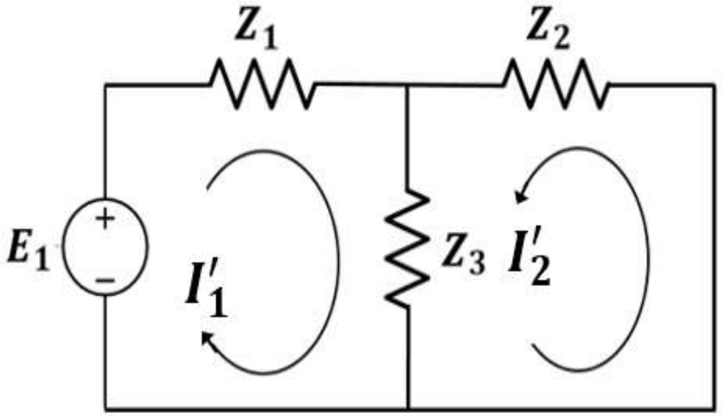
➢ Superposition Theorem

✓ Explanation

Making E_2 inoperative as in **Fig. 1(b)**

$$E_1 = I_1' \,(Z_1 + Z_3) + I_2' \, Z_3$$

$$0 = I_1' \, Z_3 + I_2' \,(Z_2 + Z_3)$$



Solving

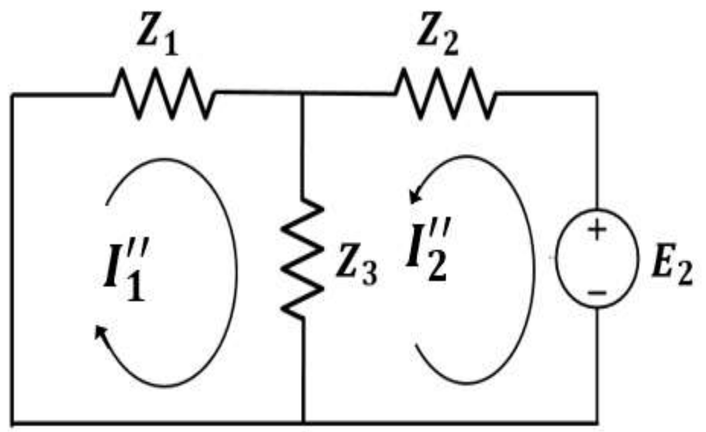
$$I_1' = \left[\frac{Z_2 + Z_3}{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}\right] E_1$$

$$I_2' = \left[\frac{-\,Z_3}{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}\right] E_1$$

Network Theorems

- **Superposition Theorem**
  - **Explanation**

Now, making E_1 inoperative as in **Fig. 1(c)**



$$0 = I_1''\,(Z_1 + Z_3) + I_2''\,Z_3$$

$$E_2 = I_1''\,Z_3 + I_2''\,(Z_2 + Z_3)$$

Solving

$$I_1'' = \left[\frac{-\,Z_3}{Z_1Z_2 + Z_2Z_3 + Z_3Z_1}\right]E_2$$

$$I_2'' = \left[\frac{Z_1 + Z_3}{Z_1Z_2 + Z_2Z_3 + Z_3Z_1}\right]E_2$$

Network Theorems

➢ Superposition Theorem

✓ Explanation

Now, $I_1 = I_1' + I_1''$ and

$$I_2 = I_2' + I_2''$$

Hence, the superposition theorem is proved.

✓ Definition

The superposition theorem for electrical circuit states that for a linear system the response (voltage or current) in any branch of a linear bilateral circuit having more than one independent source equals the algebraic sum of the responses caused by each independent source acting alone, where all the other independent sources are replaced by their internal impedances

Network Theorems

➢ Superposition Theorem

✓ Explanation

Consider a linear network N having l independent loops. The loop equations are

$$[Z]\,[I] = [E] \longrightarrow (1)$$

where the order of Z is $(l \times l)$, I is $(l \times 1)$ and E is $(l \times 1)$.

The solution of equation can be written as

$$[I] = [Z]^{-1}\,[E] = [Y]\,[E] \longrightarrow (2)$$

$$\therefore\; I_i = \sum_{j=1}^{l} Y_{ij} E_j \quad i = 1, 2, \dots \dots \dots \dots \dots . l \longrightarrow (3)$$

Network Theorems

➢ Superposition Theorem

✓ Explanation

Applying the superposition theorem making $E_j = 0$ for all j except $j = k$ and for $k = 1,2,\ldots\ldots l$ and adding the individual responses

$$I_j = \sum_{k=1}^{l} Y_{jk} E_k; \quad j = 1,2,\ldots.l \longrightarrow (4) \qquad \text{which is same as Eq. (3)}$$

First assume all E_j except E_1 to be zero. This will give the current in loop 1 as $Y_{11}E_1$, in loop 2 as $Y_{21}E_1$ etc. Similarly, when all the sources except E_2 are made inoperative, the current in loop 1 is $Y_{12}E_2$, loop 2 is $Y_{22}E_2$ and so on. The procedure is repeated in turn for the remaining sources $E_3, E_4, \ldots\ldots E_l$ and the corresponding loop responses determined. On applying the superposition principle, the individual responses are added together to give the resulting response.

For the first loop, $I_1 = Y_{11}E_1 + Y_{12}E_2 + \cdots + Y_{1l}E_l$

Which is the same as that obtained by solving the simultaneous equations.

Network Theorems

- **Superposition Theorem**

Example – P2.1

Find V_L in the circuit as shown in **Fig. P2.1** using Superposition Theorem.

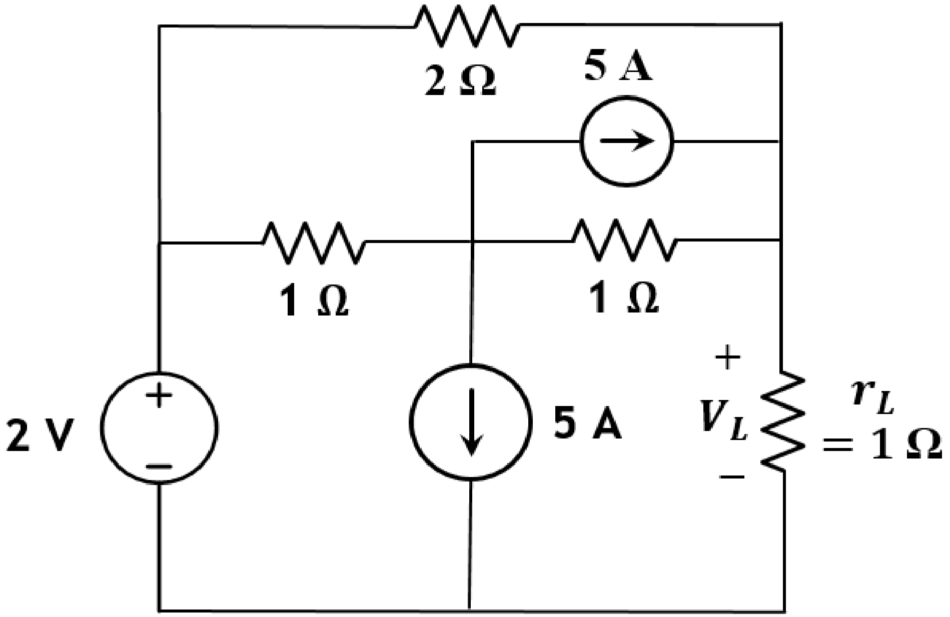


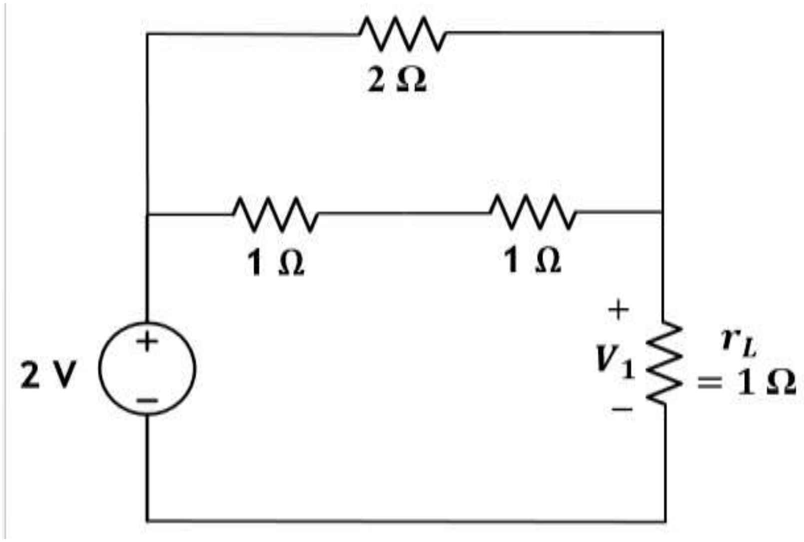
Fig. P2.1

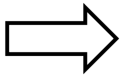
Network Theorems

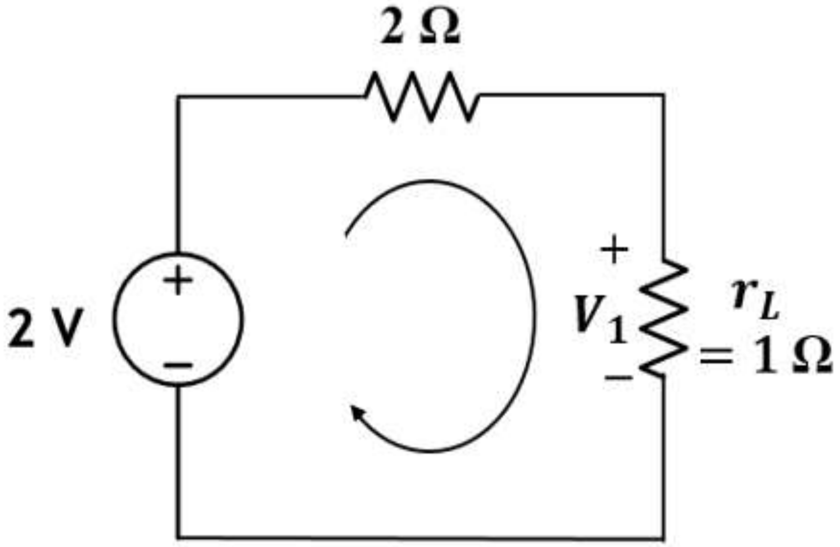
➢ Superposition Theorem

Solution of Example – P2.1

Let us first take the 2 V source deactivating the current sources.







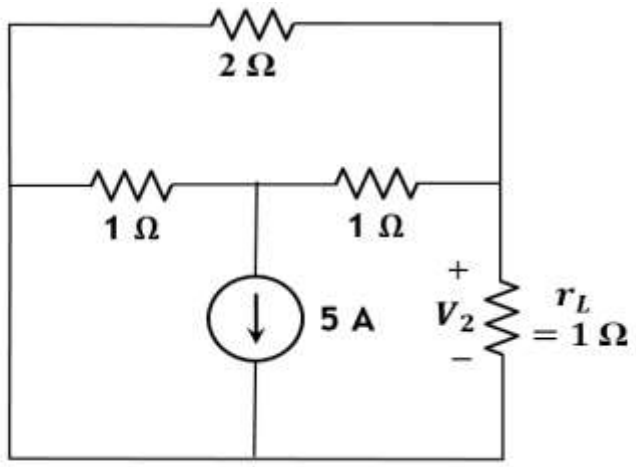
$$i_1 = \frac{2}{\frac{2 \times 2}{2+2} + 1} = 1\ A$$

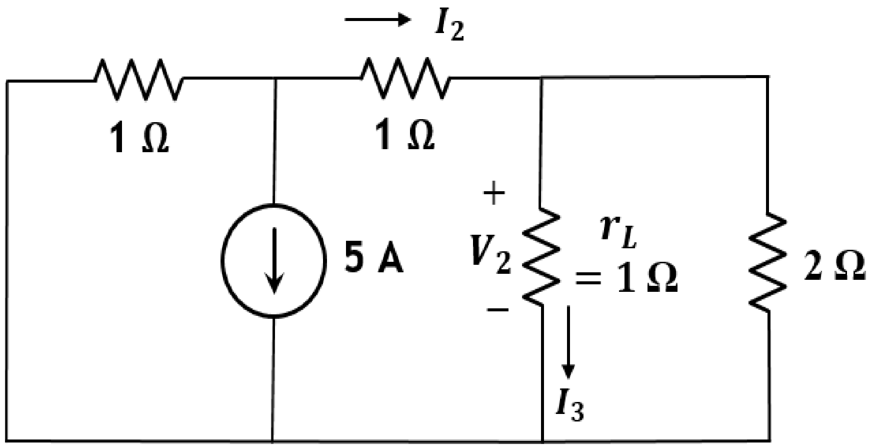
$$\therefore\ V_1(drop\ across\ r_L\ due\ to\ 2\ V\ source) = 1 \times 1\ V = 1\ V$$

➢ Superposition Theorem

Solution of Example – P2.1

Next taking the lower current source only





$$i_2 = (-5) \times \frac{1}{1+1+\frac{2}{3}} = (-5) \times \frac{3}{8} = -\frac{15}{8}\ A$$

$$i_3 = (-5) \times \frac{1}{1+1+\frac{2}{3}} \times \frac{2}{2+1} = (-5) \times \frac{3}{8} \times \frac{2}{3} = -\frac{5}{4}\ A$$

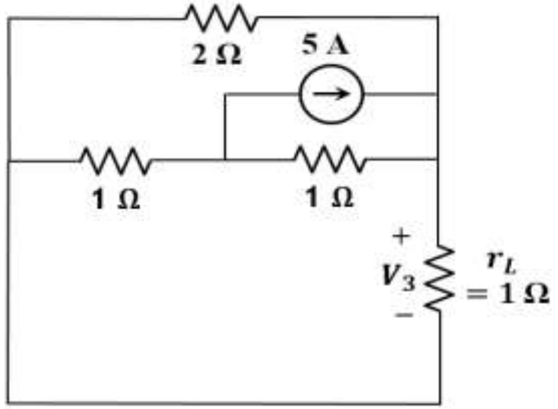
$$\therefore\ V_2 = -\frac{5}{4} \times 1 = -\frac{5}{4}\ V$$

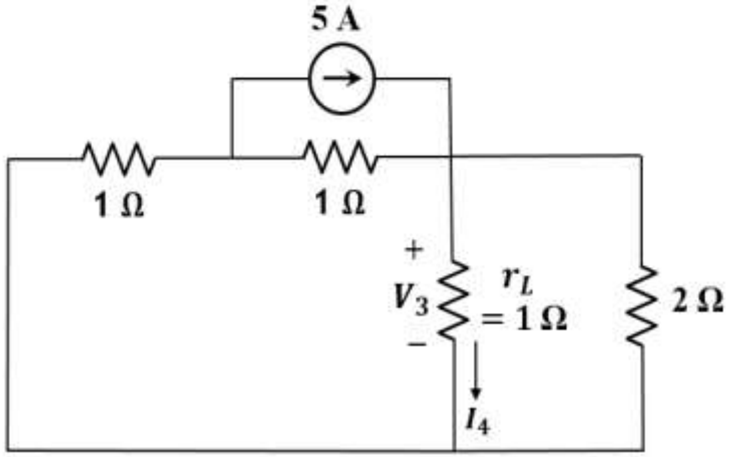
Network Theorems

- **Superposition Theorem**

Solution of Example – P2.1

Next taking the upper current source only





$$i_4 = 5 \times \frac{1}{1+1+\frac{2}{3}} \times \frac{2}{2+1} = 5 \times \frac{3}{8} \times \frac{2}{3} = \frac{5}{4} A$$

$$\therefore\ V_3 = \frac{5}{4} \times 1 = \frac{5}{4} V \qquad \therefore V = V_1 + V_2 + V_3 = 1 + \left(-\frac{5}{4}\right) + \frac{5}{4} = 1\ V$$

Network Theorems

➢ Superposition Theorem

Example – P2.2

Find i_0 and i from the circuit as shown in **Fig. P2.2** using Superposition Theorem.

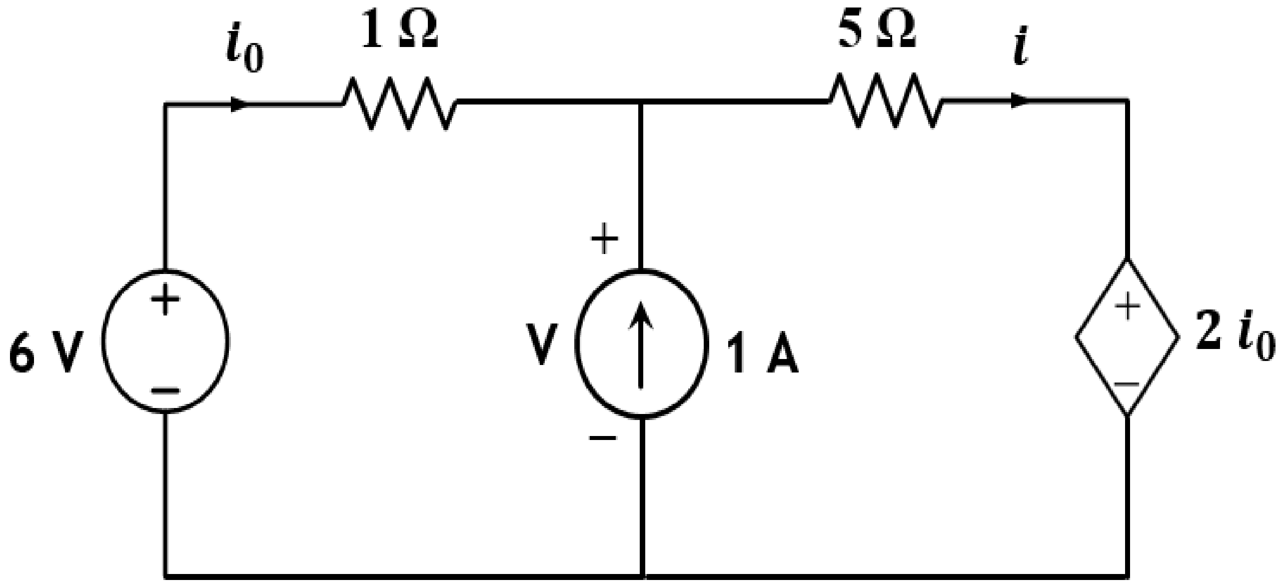


Fig. P2.2

➢ Superposition Theorem

Solution of Example – P2.2

Assuming only 6 V source to be active, with reference to **Fig. P2.2 (a)**

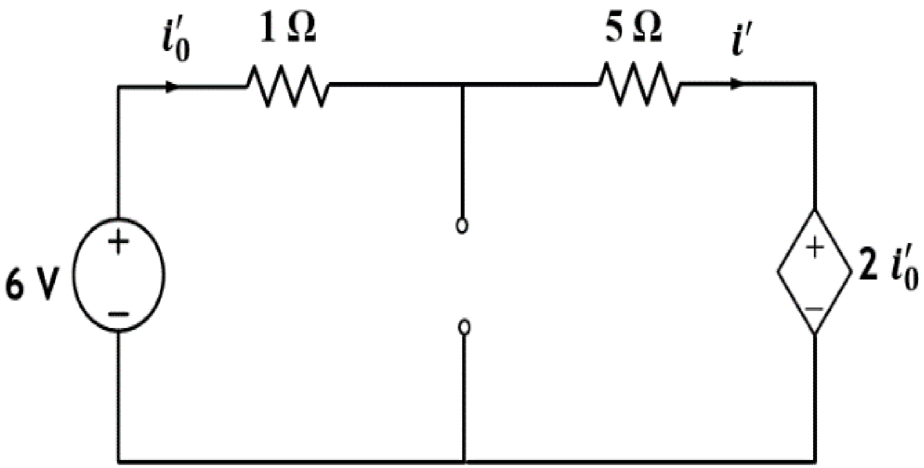


Fig. P2.2 (a)

$$-6 + (1+5)\, i_0' + 2\, i_0' = 0$$

or, $8\, i_0' = 6$

$$\therefore\ i_0' = \frac{3}{4}\ A = 0.75\ A$$

Network Theorems

➢ Superposition Theorem

Solution of Example – P2.2

Next, assuming 1 A source active only, with reference to **Fig. P2.2 (b)**

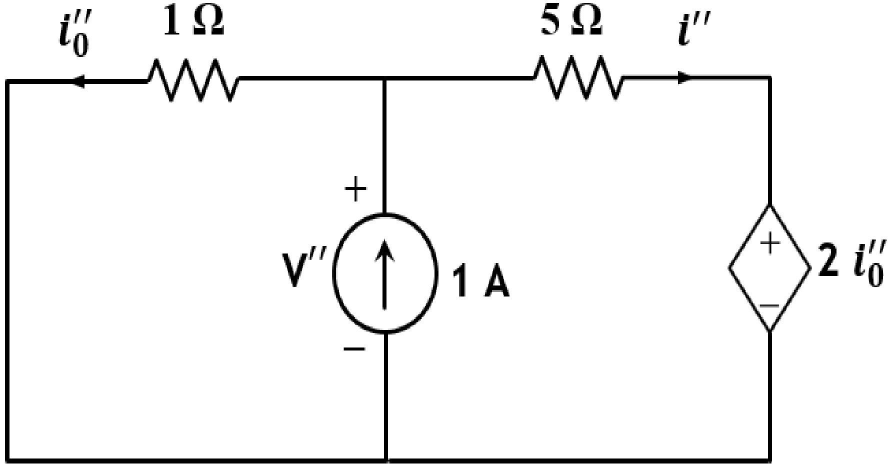


Fig. P2.2 (b)

$$1 = i_0'' + i'' = \frac{V''}{1} + \frac{V'' - 2i_0''}{5} = 1.2\,V'' - 0.4\,i_0''$$

But, $i_0'' = \frac{V''}{1}$

We finally get $1 = 1.2\,i_0'' - 0.4\,i_0'' = 0.8\,i_0''$

$$\therefore\ i_0'' = \frac{1}{0.8} = 1.25\,A$$

$$i'' = \frac{V'' - 2i_0''}{5} = \frac{-\,i_0''}{5} = -\,0.25\,A$$

Using the principle of Superposition,

$$i_0 = i_0' + i_0'' = (0.75 - 1.25)\,A = -0.5\,A$$

and $$i = i_0' + i_0'' = (0.75 - 0.25)\,A = 0.5\,A$$

Network Theorems

➢ Reciprocity Theorem

✓ A linear network is said to be reciprocal or bi-lateral, if it remains invariant due to the interchange of position of cause and effect in the network.

- Consider two loops A and B of a network N.
- An ideal voltage source E in loop A produces a current I in loop B.
- An identical source in loop B produces the same current I in loop A.
- The network is said to be reciprocal. The dual is also true.

✓ A reciprocal network comprises of linear time invariant bi-lateral passive elements.

- It is applicable to resistors, capacitors, inductors (with and without coupling) and transformers.
- Both dependent and independent sources are not permissible. The sources with zero state response are considering only.

Network Theorems

➢ Reciprocity Theorem

✓ Proof

- Let us consider a network N having only one driving voltage source $E = E_k$ in loop K. So, the current response in loop m due to the voltage source is

$$I_m = Y_{mk} E_k$$

- Now, interchange the positions of cause and effect. The same voltage source $E = E_m$ is placed in loop m and hence the current response in loop k is

$$I_k = Y_{km} E_m$$

- I_k will be equal to I_m provided $Y_{km} = Y_{mk}$.
- This is the condition for reciprocity.
- The admittance matrix Y is symmetric $as\ Y_{km} = Y_{mk}$ for all m and k.

Network Theorems

➢ Reciprocity Theorem

Example – P2.3

Show the application of reciprocity theorem in the network as shown in **Fig. P2.3**

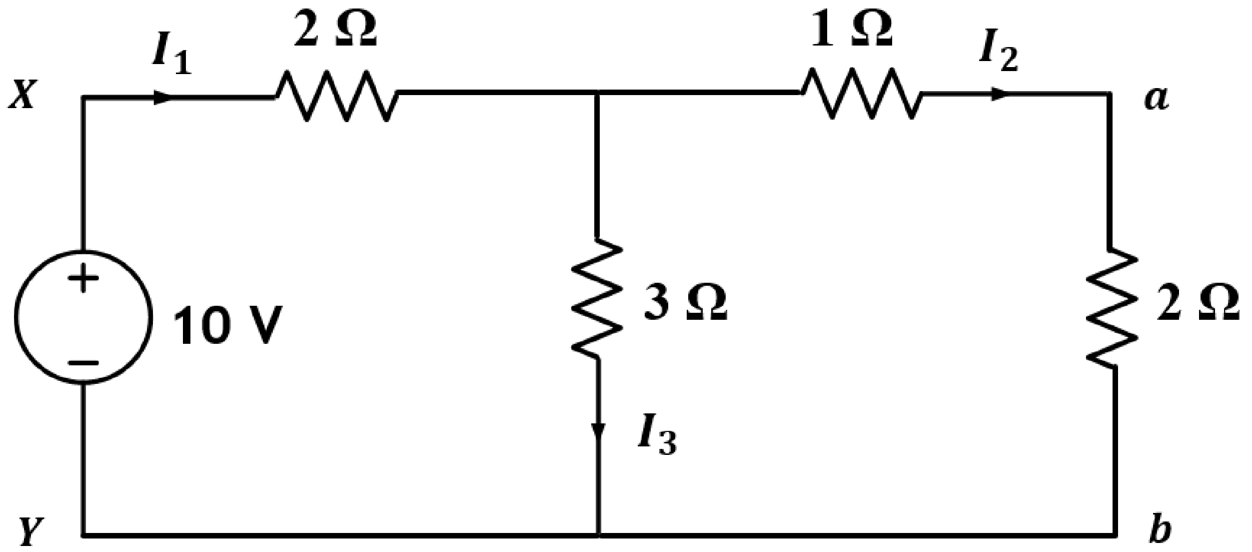


Fig. P2.3

Network Theorems

➢ Reciprocity Theorem

Solution of Example – P2.3

The equivalent resistance across X-Y of the circuit as shown in **Fig. P2.3,** is given by

$$R_{eq} = [(2+1) \parallel 3 + 2] = 3.5\ \Omega$$

$$I_1 = \frac{10}{3.5} = 2.86\ A$$

$$I_2 = 2.86 \times \frac{3}{3+3} = 1.43\ A$$

$$I_3 = 2.86 - 1.43 = 1.43\ A$$

Network Theorems

➢ Reciprocity Theorem

Solution of Example – P2.3

Now, consider the voltage source is changed in position and placed in series with 2 Ω resistance as shown in **Fig. P2.3(a)**.

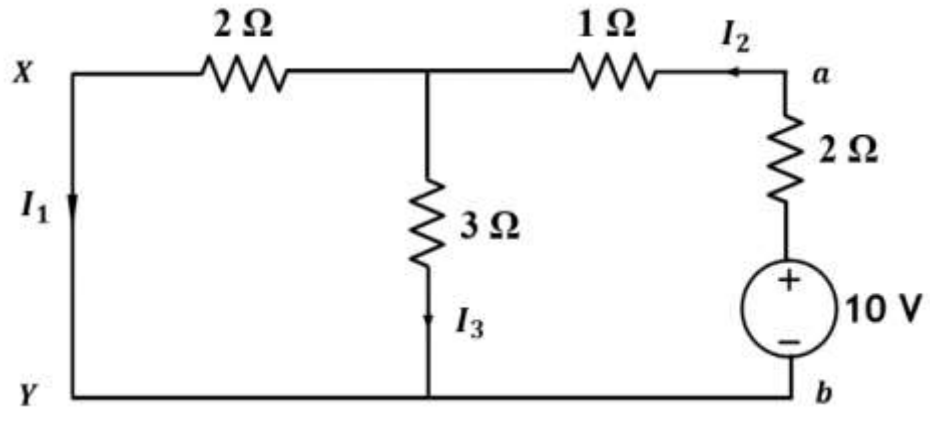


Fig. P2.3(a).

$$R_{eq} = [(2 \parallel 3) + 1 + 2] = 4.2\ \Omega$$

$$I_2 = \frac{10}{4.2} = 2.381\ A$$

$$\therefore I_1 = I_2 \times \frac{3}{3+2} = 2.381 \times \frac{3}{5} = 1.43\ A$$

Hence, we observe that when the source is in branch $X - Y$ of **Fig. P2.3**, the $a - b$ branch current is 1.43 A. Again, when the source is in branch $a - b$ of **Fig. P2.3(a),** the $X - Y$ branch current becomes 1.43 A.

Hence, this proves the reciprocity theorem.

LECTURE 5

Network Theorems

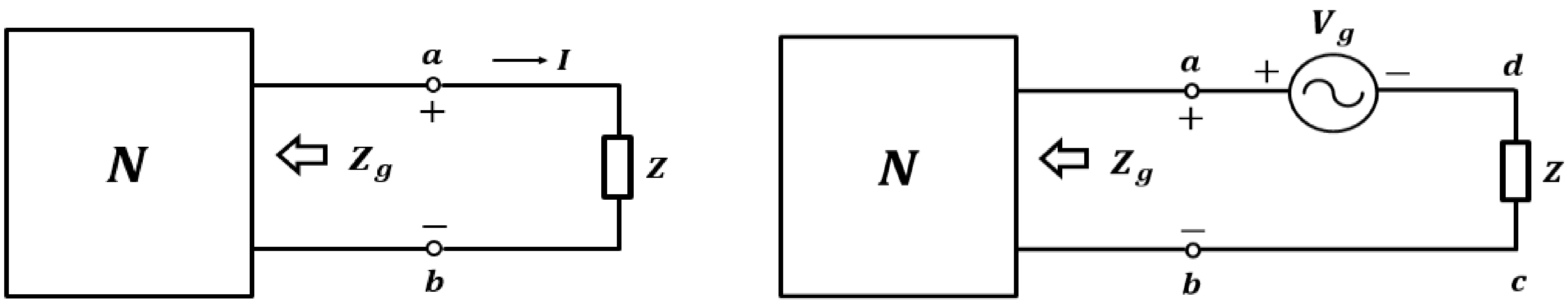
➢ Thevenin's Theorem

- ✓ **Definition:** Any two terminal linear network containing impedances and energy sources may be replaced by an independent voltage source of generated voltage V_g and internal impedance Z_g.

  V_g is the open circuit voltage at the terminals and Z_g is the impedance viewed at the terminals when all the independent energy sources are replaced by their internal impedances.

- ✓ The theorem is commonly known as Thevenin's theorem, in honor of Leon Charles Thevenin, a French telegraph engineer.

- ✓ **Proof:**



(a) (b)

Fig. 2.2. Circuits for Thevenin Theorem

Network Theorems

➢ Thevenin's Theorem

- ✓ A linear network N containing impedances and energy sources. It is connected to a load impedance Z at the terminals a, b.
- ✓ The open circuit voltage across a, b is V_g when Z is removed.
- ✓ Let the voltage source V_g be connected in series with Z with polarities as shown in **Fig. 2.2.**
- ✓ The net voltage in the loop $abcd$ will now be zero for all values of Z. The current through Z is thus zero under these conditions.
- ✓ If I is the current supplied by the network and I' is the current supplied by the added source to Z, then

$$I + I' = 0 \longrightarrow (1)$$

- ✓ By superposition theorem, the current supplied by the added source V_g is given by

$$I' = -\frac{V_g}{Z + Z_g} \longrightarrow (2)$$

where Z_g is the impedance viewed at the terminals a, b when all the independent energy sources within the network are replaced by their internal impedances.

Network Theorems

➢ Thevenin's Theorem

- ✓ We get from Eq. (2)

$$I = \frac{V_g}{Z + Z_g}$$

$$\therefore V = ZI = V_g - Z_g I \longrightarrow (3)$$

Here, V is the voltage across Z.

- ✓ Eq. (3) shows that with respect to the terminals a, b the circuit of **Fig. 2.2(a)** may be replaced by the equivalent circuit of **Fig. 2.3.**

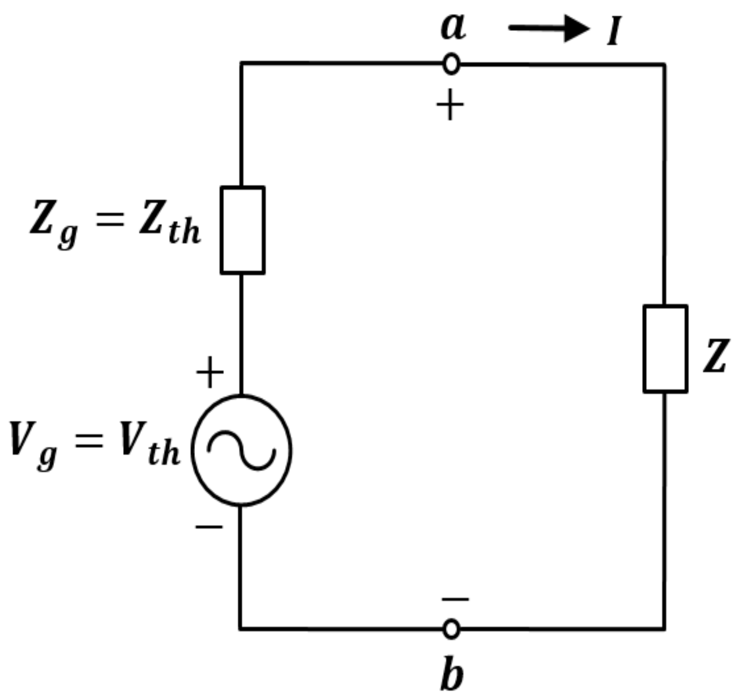


Fig. 2.3. Thevenin Equivalent Circuit

- ✓ It is seen that the network to the left of the terminals a, b is replaced by an independent voltage source of generated voltage V_g and internal impedance Z_g.

V_g is the open circuit voltage at a, b and Z_g is the impedance measured back at a, b with all the internal independent energy sources deactivated and replaced by their impedances.

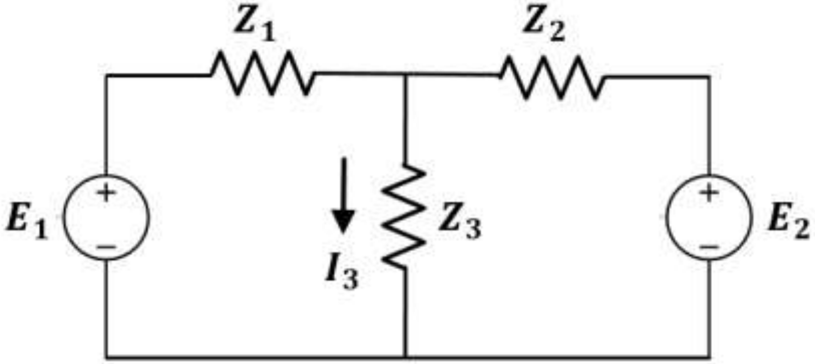
The voltage source in **Fig. 2.3** is called the equivalent Thevenin Source and impedance in **Fig. 2.3** is called the Thevenin Impedance.

Network Theorems

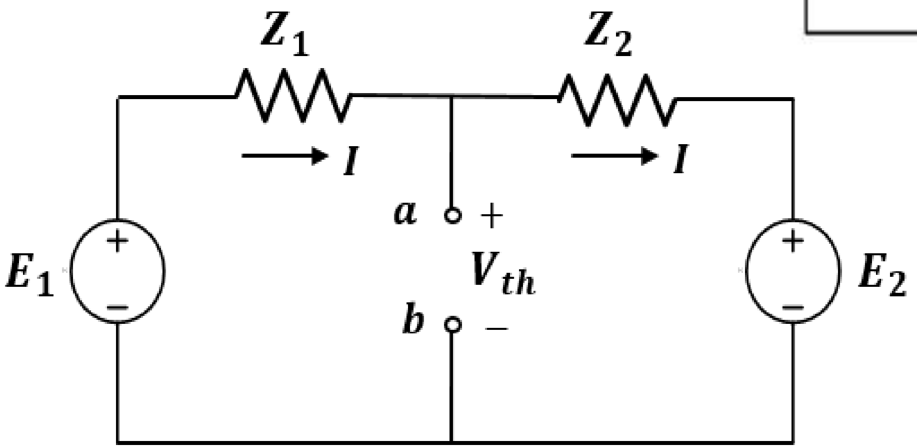
➢ Thevenin's Theorem

✓ Illustration

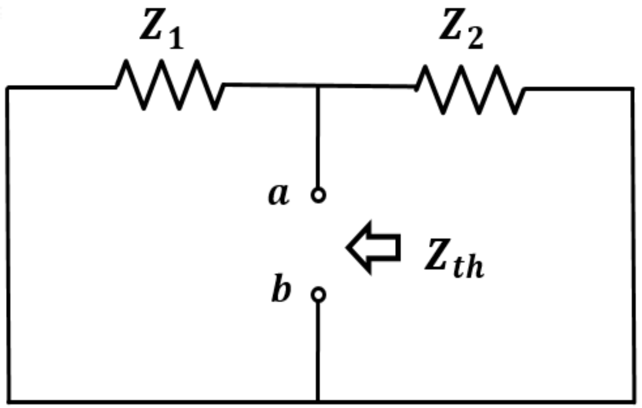
Find the current I_3, flowing through the load having impedance of Z_3 as shown in **Fig I1(a)** by Thevenin Theorem.



(a)



(b)



(c)

Fig. I1. Circuit for illustration of Thevenin Theorem

Network Theorems

- **Thevenin's Theorem**

  - **Illustration**

From **Fig. I1(b),** we get $$I = \frac{E_1 - E_2}{Z_1 + Z_2}$$

From **Fig. I1(c),** we get

$$Z_{th} = \frac{Z_1 Z_2}{Z_1 + Z_2}$$

From **Fig. I1(b),** we get

$$V_{ab} = E_1 - IZ_1$$

or, $$V_{ab} = E_1 - \frac{E_1 - E_2}{Z_1 + Z_2} . Z_1$$

$$\therefore \quad V_{th} = V_{ab} = \frac{Z_2}{Z_1 + Z_2} E_1 + \frac{Z_1}{Z_1 + Z_2} E_2$$

$$\therefore I_3 = \frac{V_{th}}{Z_{th} + Z_3}$$

$$= \frac{Z_2}{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1} E_1 + \frac{Z_1}{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1} E_2$$

Thevenin’s Theorem

Example – P2.4

Find the current in the 3 Ω resistor of the circuit as shown in **Fig. P2.4**

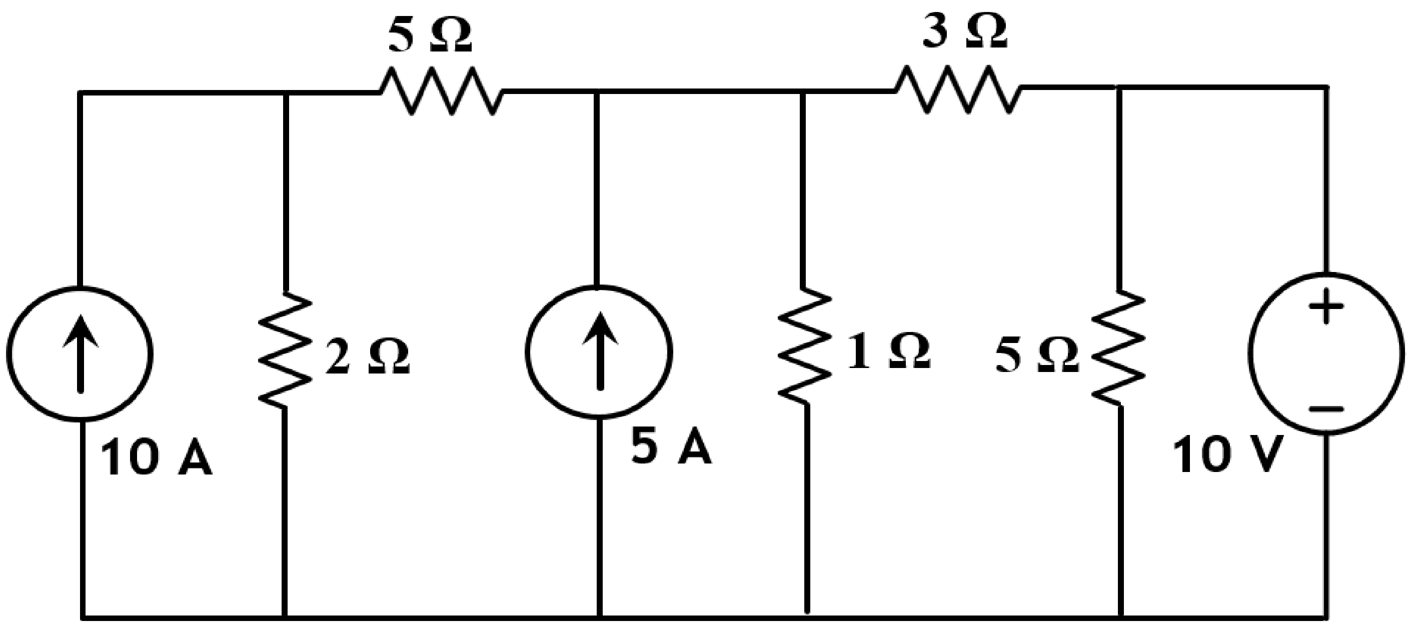


Fig. P2.4

Network Theorems

➢ Thevenin's Theorem

Solution of Example – P2.4

The circuit as shown in **Fig. P2.4(a)** is obtained after transforming current sources into voltage sources. The circuit as shown in **Fig. P2.4(b)** is obtained after removing 3 Ω resistor.

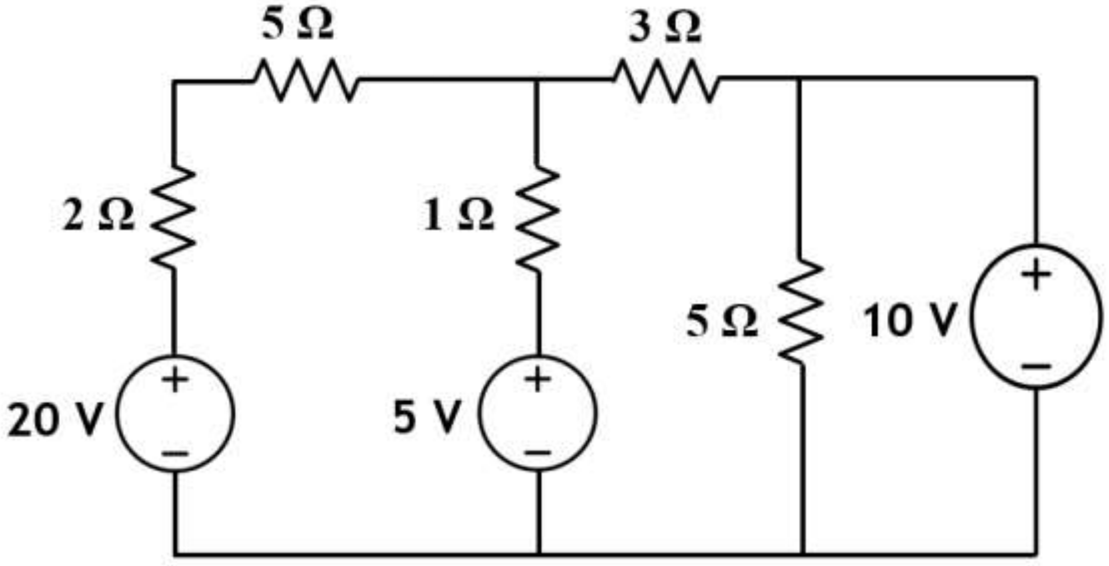


Fig. P2.4(a)

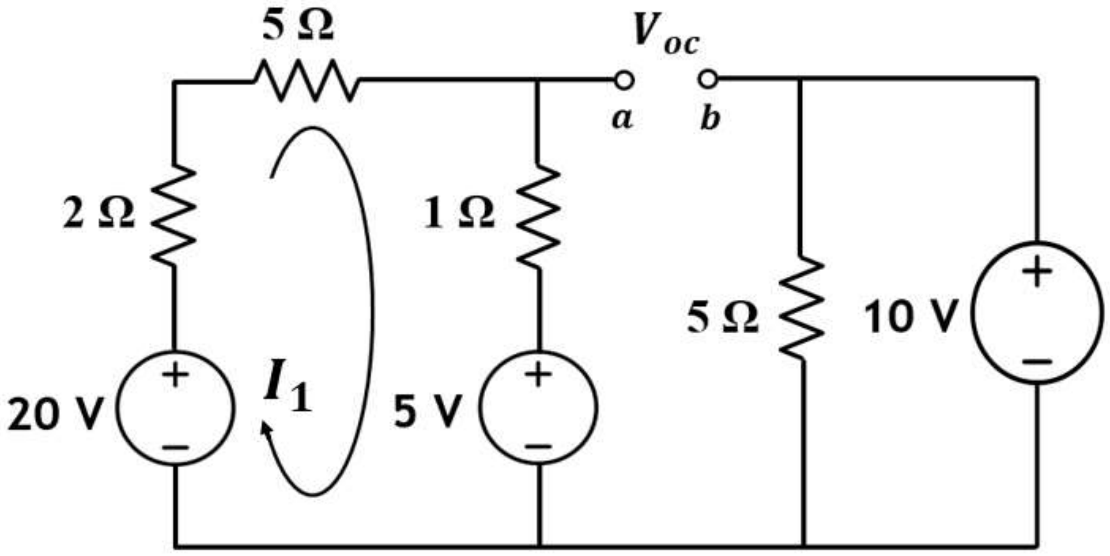


Fig. P2.4(b)

➢ Thevenin's Theorem

Solution of Example – P2.4

The circulating current (I_1) in the left most loop is then given by

$$I_1 = \frac{20-5}{2+5+1} = \frac{15}{8}\ A \qquad \therefore V_{oc} = \therefore V_{th} = 20 - \frac{15}{8}(2+5) - 10\ V = -3.125\ V$$

To find the Thevenin's resistance of the given circuit, the sources are deactivated as shown in **Fig. P2.4(a)** and **Fig. P2.4(d)** .

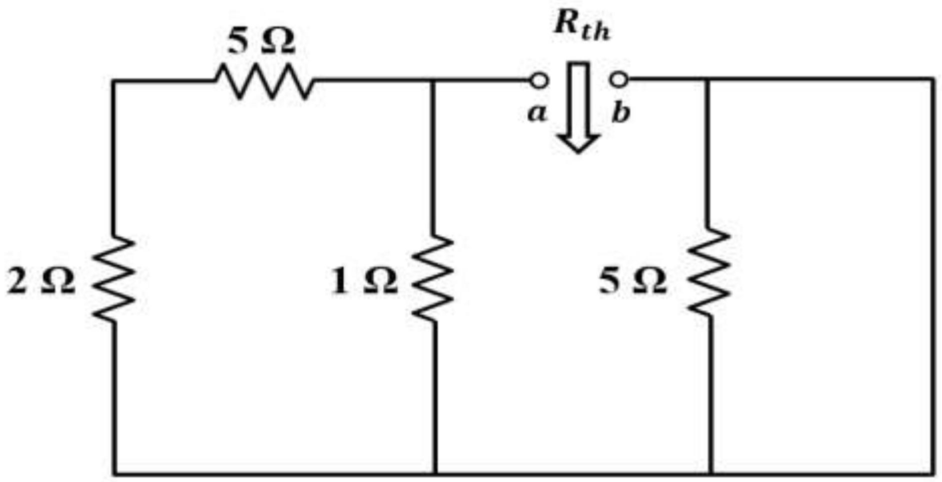


Fig. P2.4(c)

$$\therefore R_{th} = \frac{7\times1}{7+1} = \frac{7}{8}\,\Omega$$

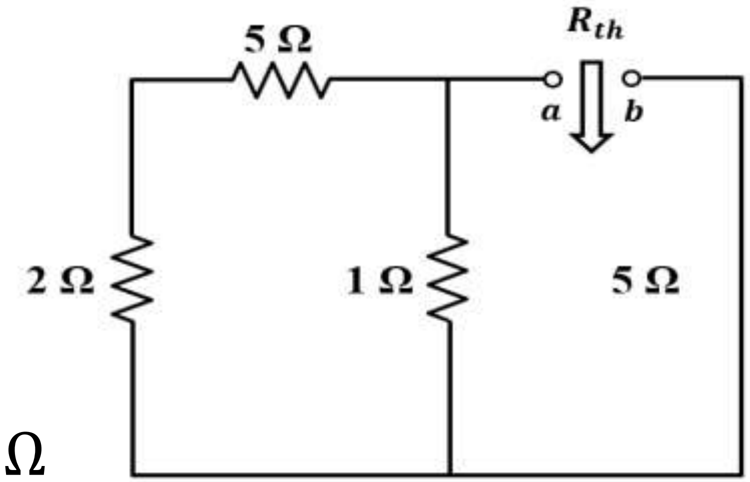


Fig. P2.4(d)

➢ Thevenin’s Theorem

Solution of Example – P2.4

The Thevenin’s Equivalent Circuit is given in **Fig. P2.4(e)**

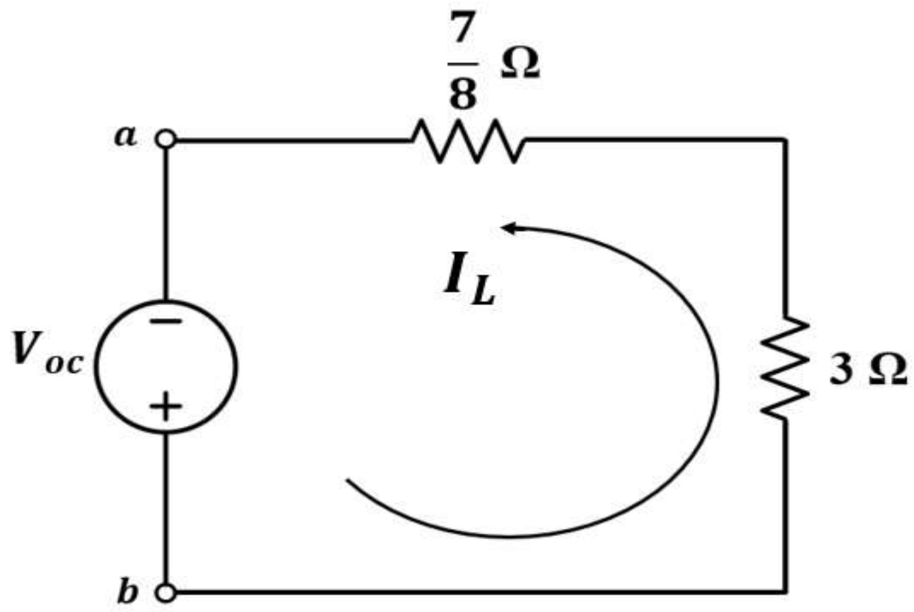


Fig. P2.4(e)

The current through 3 Ω resistor is given by

$$I_L = \frac{V_{oc}}{\frac{7}{8}+3} = \frac{3.125}{\frac{7}{8}+3} = 0.806\ A$$

Network Theorems

➢ Norton's Theorem

- ✓ **Definition:** Any two terminal linear network containing impedances and energy sources may be replaced by an independent current source I_g in parallel with an admittance Y_g.

  I_g is the short-circuit current between the terminals a, b and Y_g is the admittance viewed at the terminals a, b when all the independent energy sources are replaced by their internal admittances.

- ✓ Norton's theorem was independently derived in 1926 by Siemens & Halske researcher Hans Ferdinand Mayer (1895–1980) and Bell Labs engineer Edward Lawry Norton.

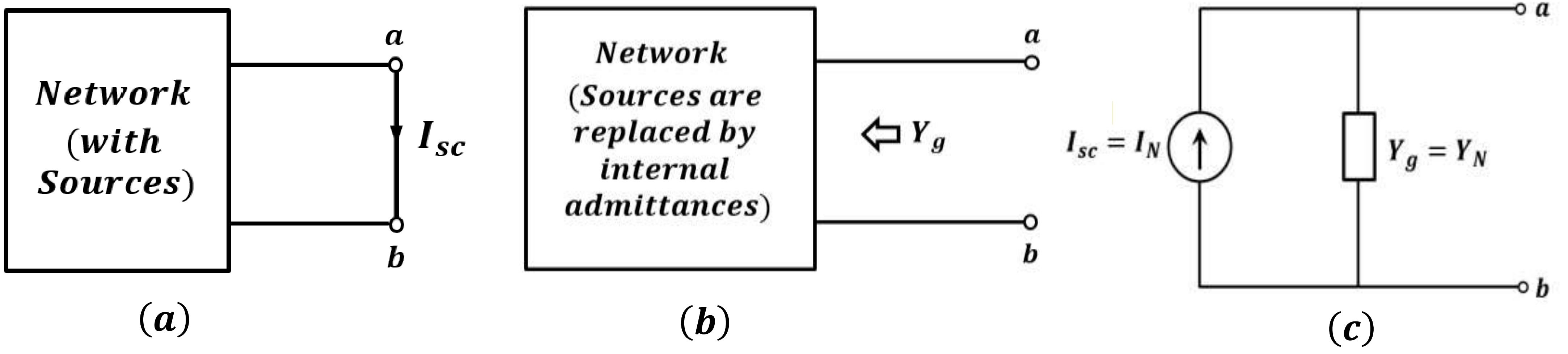


Fig. 2.3. Circuits for Norton Theorem

➢ Norton's Theorem

✓ **Definition:** Any two terminal linear network containing enery sources and resistances is equivalent to a constant current source and a parallel resistance when viewed from its output terminals.

The constant current is equal to the current which flows in a short-circuit placed across the terminals a, b and parallel resistances is the resistance of the network when viewed from these open circuited terminals after all voltage and current sources have been removed and replaced by their internal resistances.

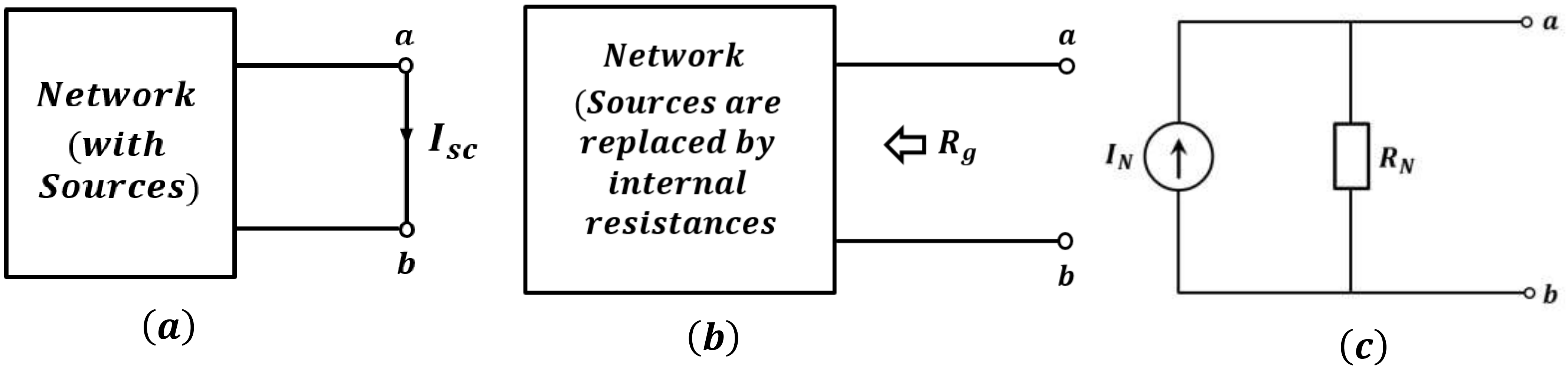


Fig. 2.4. Circuits for Norton Theorem

Network Theorems

➢ Norton's Theorem

✓ <u>How to Nortonize a given circuit</u>

The step-wise procedure of Norton theorem is given below:

1. Remove the resistance (if any) across the two given terminals and put a short-circuit across them.
2. Compute the short-circuit current I_{sc}
3. Remove all voltage sources but retain their internal resistances(if any).Similarly, remove all current sources and replace them by open circuit i.e. by infinite resistance
4. Next, find the resistance R_i (also called R_N) of the network as looked into from the given terminals. It is exactly the same as R_{th}.
5. The current source (I_{sc}) joined in parallel across R_i between the two terminals gives Norton's Equivalent Circuit.

➢ Norton's Theorem

✓ Illustration

Find the current I_3, flowing through the load having impedance of Z_3 as shown in **Fig I1(a)** by Norton Theorem.

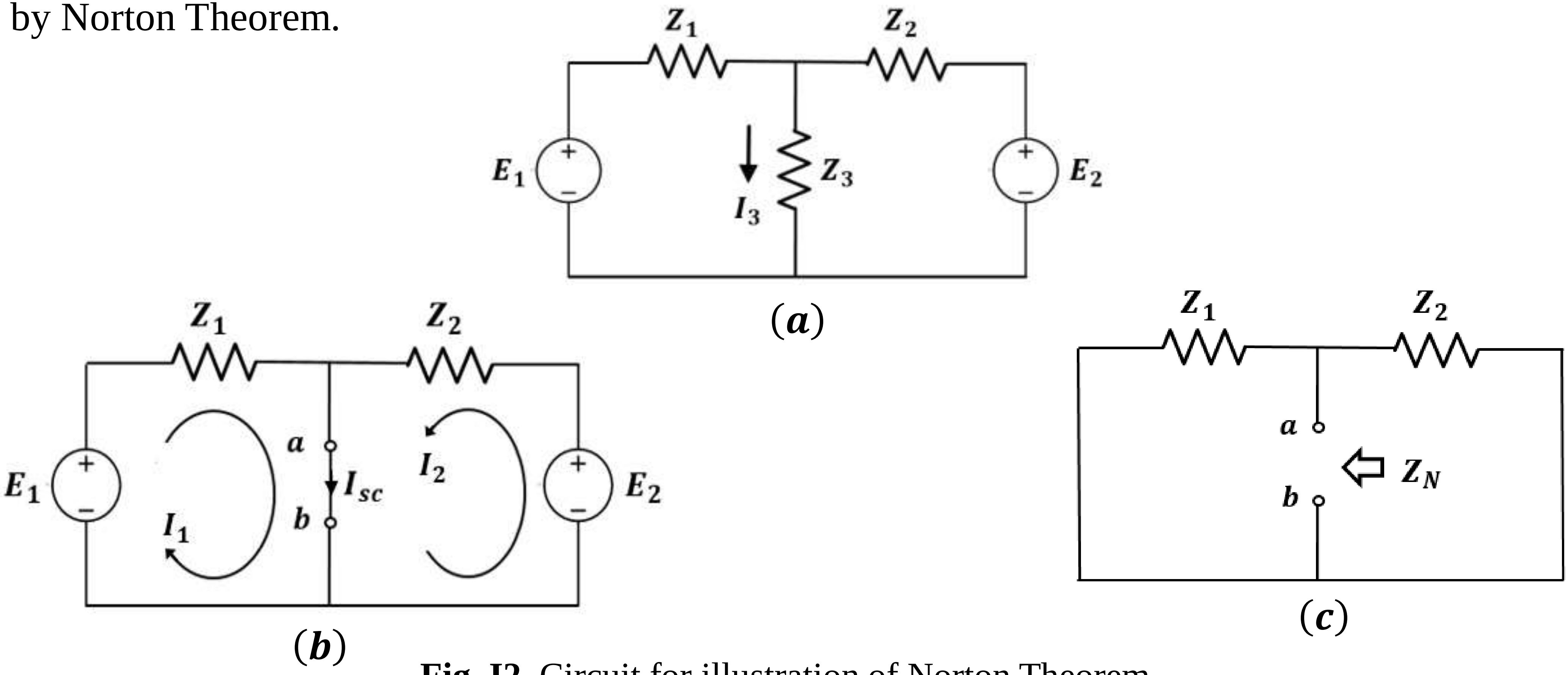


Fig. I2. Circuit for illustration of Norton Theorem

➢ Norton's Theorem

✓ Illustration

From **Fig. I2(b),** we get $$I_{sc} = \frac{E_1}{Z_1} + \frac{E_2}{Z_2}$$

From **Fig. I2(c),** we get $$Z_N = \frac{Z_1 Z_2}{Z_1 + Z_2}$$

$$\therefore I_3 = I_N \times \frac{Z_N}{Z_N + Z_3}$$

$$= \frac{Z_2}{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1} E_1 + \frac{Z_1}{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1} E_2$$

Network Theorems

➢ **Norton's Theorem**

Example – P2.5

Find the current through R_L in the circuit of **Fig. P2.5** using Norton's Theorem.

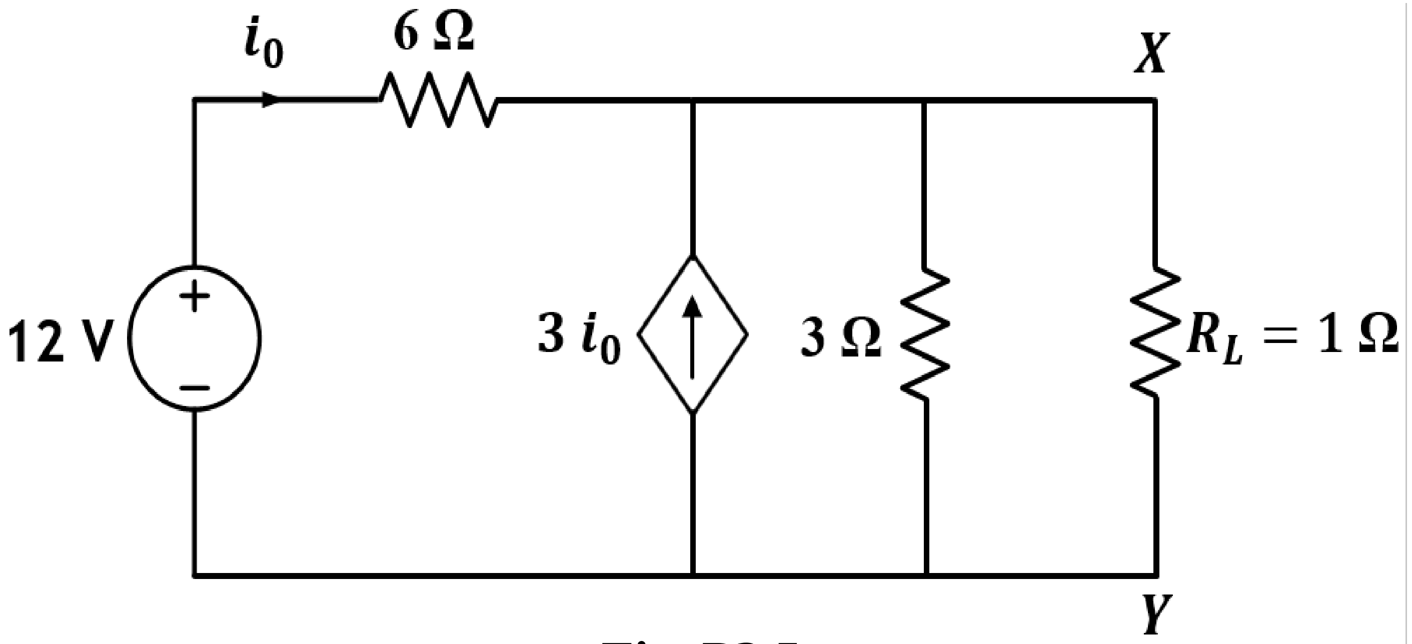


Fig. P2.5

Network Theorems

- **Norton's Theorem**

Solution of Example – P2.5

Let us first remove R_L from $X - Y$ terminals and short $X - Y$ as shown in **Fig. P2.5 (a)**

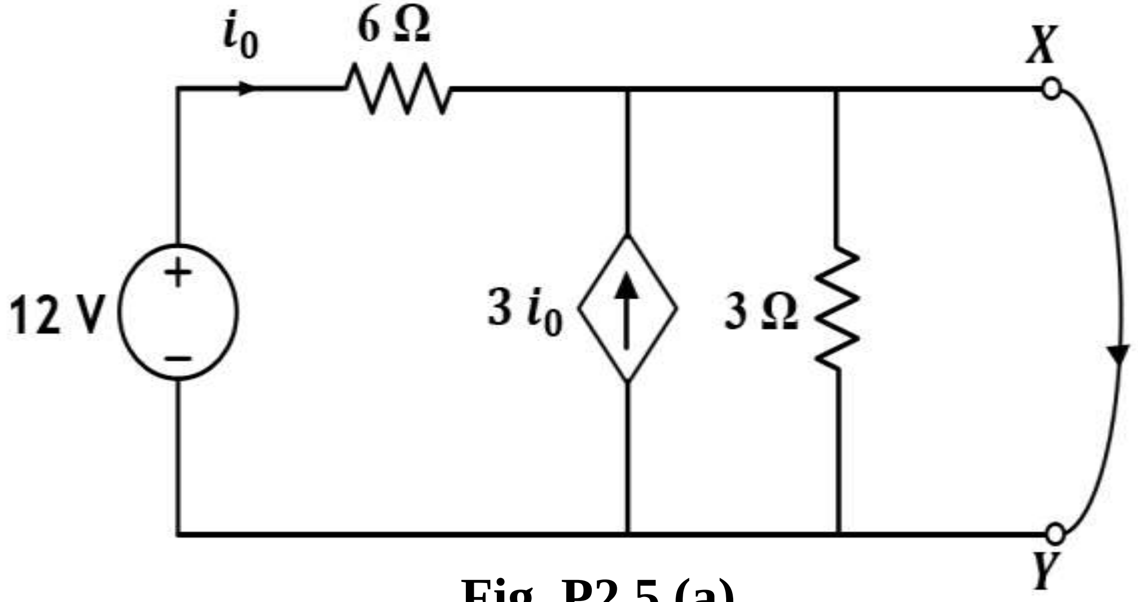


Fig. P2.5 (a)

$$I_{sc} = 3\,i_0 + i_0 = 4\,i_0$$

$$\therefore\ I_{sc} = I_N = 4 \times 2 = 8\ A \quad \left[\because\ i_0 = \frac{12}{6} = 2\ A\right]$$

➢ Norton's Theorem

Solution of Example – P2.5

Let us now remove the short circuit and the circuit is open circuited at $X-Y$ as shown in **Fig. P2.5 (b)**

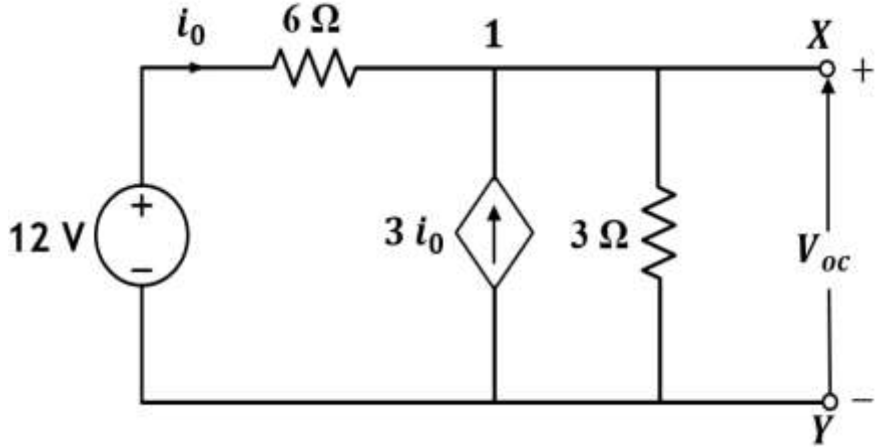


Fig. P2.5 (b)

Nodal analysis at node 1 gives

$$i_0 + 3\,i_0 - \frac{V_{oc}}{3} = 0$$

or, $$4i_0 - \frac{V_{oc}}{3} = 0$$

or, $$4 \times \left(\frac{12 - V_{oc}}{6}\right) - \frac{V_{oc}}{3} = 0$$

or, $$8 - \frac{2\,V_{oc}}{3} - \frac{V_{oc}}{3} = 0$$

$\therefore$ $V_{oc} = 8\ V$

$$\therefore \quad R_{int} = \frac{V_{oc}}{I_{sc}} = \frac{8}{8} = 1\ \Omega$$

➢ Norton's Theorem

Solution of Example – P2.5

Norton's equivalent circuit is shown in **Fig. P2.5 (c).**

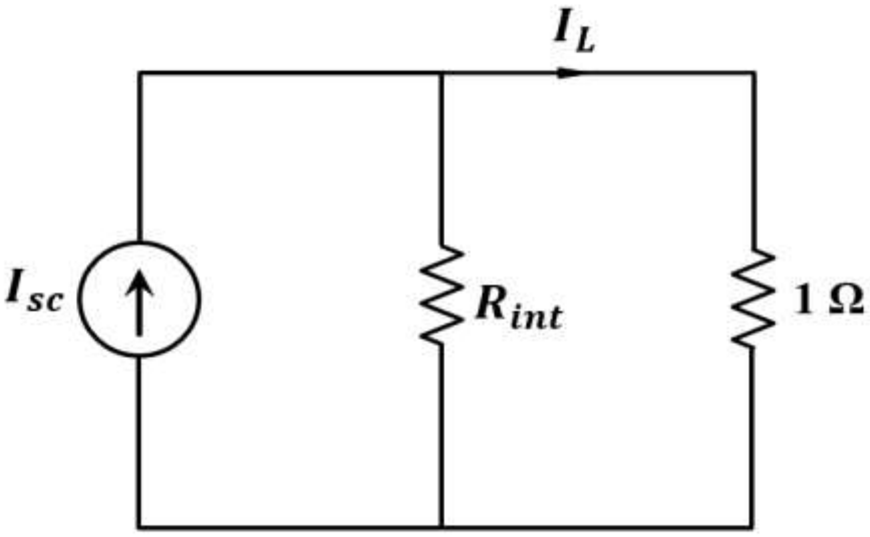


Fig. P2.5 (c)

$$I_L = I_{sc}\frac{R_{int}}{R_{int}+R_L} = 8\times\frac{1}{1+1} = 4\ A$$

LECTURE 6

Network Theorems

➢ Maximum Power Transfer Theorem

✓ The goal of 'Maximum Power Transfer Theorem' is either to receive or transmit maximum power but the overall efficiency of a network supplying maximum power is only 50%. The power output (P_{max}) and efficiency (η) for maximum power condition is shown in **Fig. 2.5**.

- The application of the theorem for power transmission and distribution networks is limited.

- Although the efficiency is reduced, the theorem is particularly useful for analysing electronic and communication networks when power involved is only a few milliwatts or microwatts.

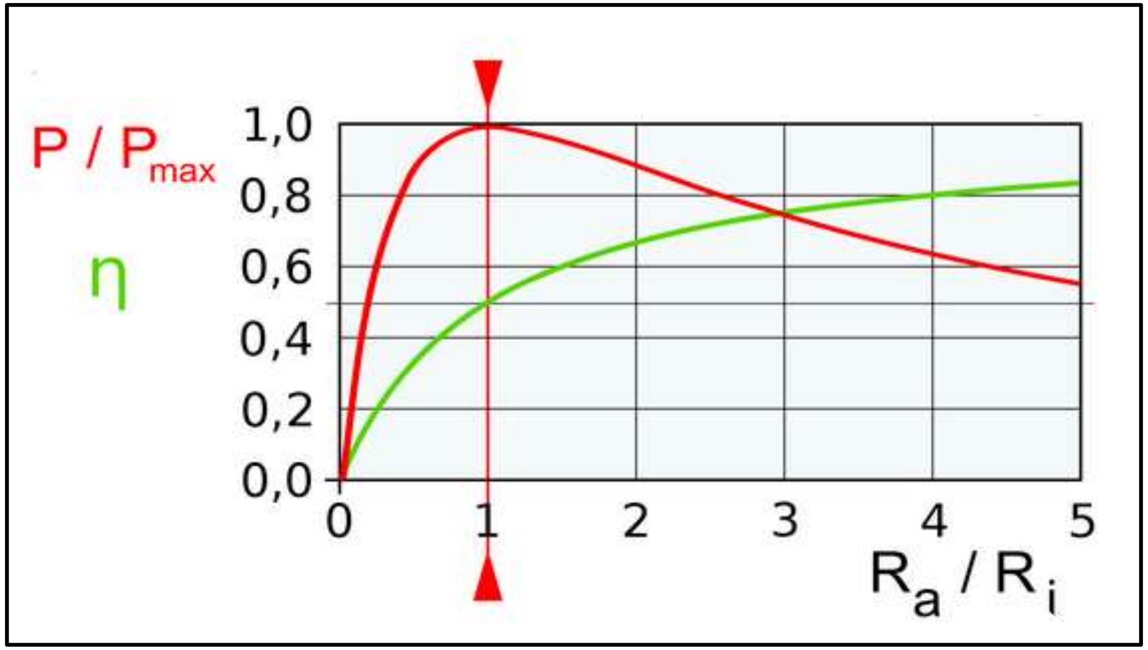


Fig. 2.5. Maximum output and efficiency

Network Theorems

➢ Maximum Power Transfer Theorem

✓ Condition of Maximum Power Transfer for different networks

1. **Purely resistive circuit, the load resistance is varied:** Maximum power will be delivered from a voltage source to a load when the load resistance is equal to the internal resistance of the source.

2. **Reactance present, load resistance and reactance are independently varied:** Maximum power will be delivered from a voltage source to a load when the load impedance is the complex conjugate of the source impedance.

3. **Reactance present, the magnitude but not the angle of the load impedance is varied:** Maximum power is delivered from a voltage source to the load impedance when the magnitude of the load impedance is equal to the magnitude of source impedance.

Network Theorems

➢ Maximum Power Transfer Theorem

✓ Condition of Maximum Power Transfer for different networks

i) Purely resistive circuit and the load resistance is variable

Proof:

A voltage source of generated voltage V_g and internal resistance R_g is connected to a load resistance R as shown in **Fig. 2.6.**

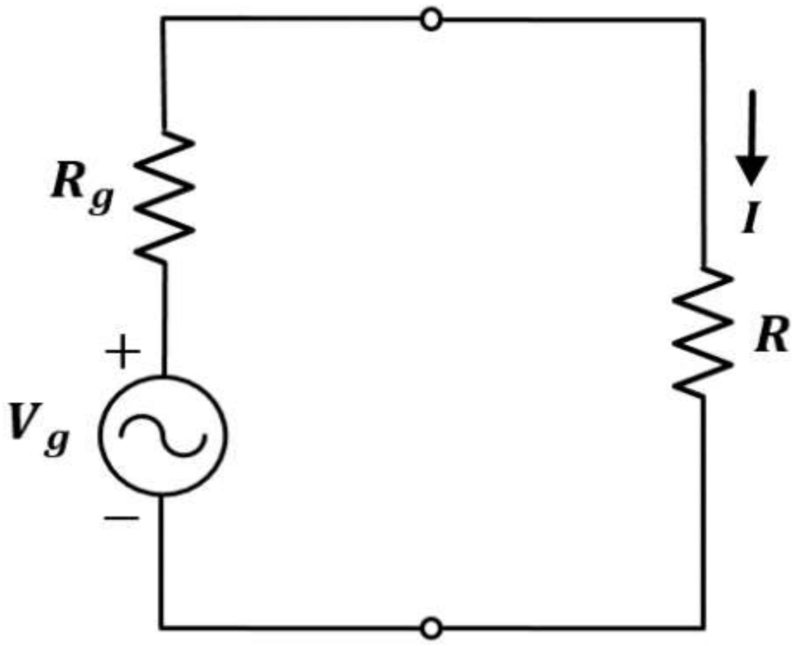


Fig. 2.6. Circuit having resistive load

The load current is

$$I = \frac{V_g}{R+R_g}$$

The power delivered to the load is

$$P = I^2R = \left(\frac{V_g}{R+R_g}\right)^2 R$$

Network Theorems

➢ Maximum Power Transfer Theorem

✓ Condition of Maximum Power Transfer for different networks

i) Purely resistive circuit and the load resistance is variable

Proof:

The power is maximized by changing R. So,

$$\frac{dP}{dR} = 0$$

or, $$\frac{V_g^2\left[\left(R+R_g\right)^2 - 2R\left(R+R_g\right)\right]}{\left(R+R_g\right)^4} = 0$$

or, $$\left(R+R_g\right)^2 = 2R\left(R+R_g\right)$$

$\therefore$ $$R = R_g$$

This is the condition for transmitting maximum power from source to load.

Network Theorems

➢ Maximum Power Transfer Theorem

✓ Condition of Maximum Power Transfer for different networks

i) Purely resistive circuit and the load resistance is variable

The output power for the condition of $R = R_g$ is

$$P_{out} = \left(\frac{V_g}{R + R_g}\right)^2 R = \left(\frac{V_g}{R + R}\right)^2 R = \frac{V_g^2}{4R}$$

and the input power for the same condition is

$$P_{in} = V_g \times I = V_g \times \frac{V_g}{R + R_g} = \frac{V_g^2}{2R}$$

So, the efficiency under this condition is

$$\eta = \frac{P_{out}}{P_{in}} \times 100\ \% = 50\ \%$$

The efficiency, 50% means one-half of the total generated power is dissipated within the source and the remaining other-half is utilized by the load.

Network Theorems

➢ Maximum Power Transfer Theorem

✓ **Condition of Maximum Power Transfer for different networks**

2. Reactance present, load resistance and reactance are independently varied:

Proof:

A voltage source of generated voltage V_g and internal impedance Z_g is connected to a load impedance Z as shown in **Fig. 2.7.**

where $Z_g = R_g + jX_g$, internal impedance of the source and Z $= R + jX$, load impedance.

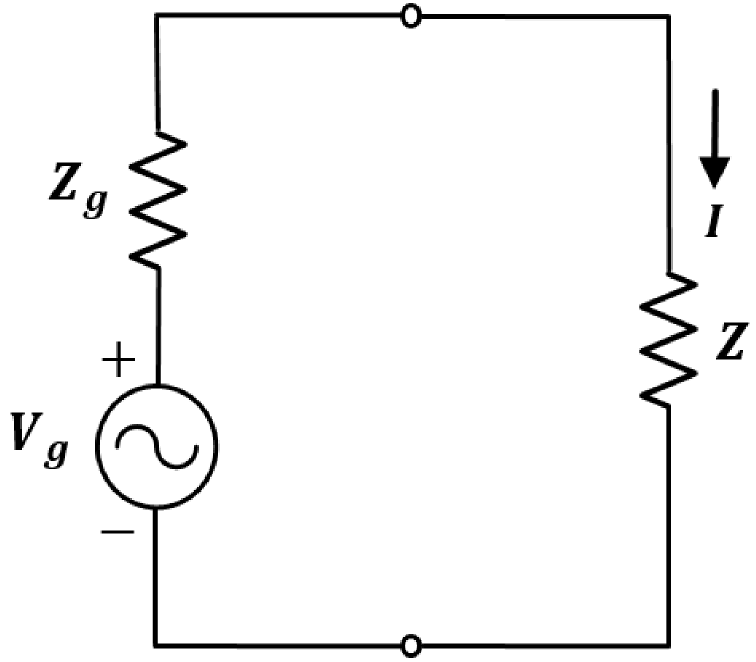


Fig. 2.7. Circuit having load with resistance and reactance.

The load current is

$$I = \frac{V_g}{(R+R_g)+j(X+X_g)}$$

Network Theorems

➢ Maximum Power Transfer Theorem

✓ Condition of Maximum Power Transfer for different networks

2. Reactance present, load resistance and reactance are independently varied:

Proof:

The power delivered to the load is

$$P = I^2 R = \frac{{V_g}^2\, R}{\left(R + R_g\right)^2 + \left(X + X_g\right)^2}$$

The power is maximized by changing X, we must have

$$\frac{dP}{dX} = 0$$

$$\therefore\ X = -X_g$$

Network Theorems

➢ Maximum Power Transfer Theorem

✓ Condition of Maximum Power Transfer for different networks

2. Reactance present, load resistance and reactance are independently varied:

Proof:

Clearly, P is maximum with respect to reactance variation when $X = -X_g$. Under this condition

$$P = \frac{\left(V_g\right)^2 R}{\left(R + R_g\right)^2}$$

Maximum power transfer, P to the load will be maximum when $Z = R_g - jXg = Z_g^*$.

So, the load impedance is the complex conjugate of the internal impedance of the source.

Network Theorems

➢ Maximum Power Transfer Theorem

✓ Condition of Maximum Power Transfer for different networks

3. Reactance present, the magnitude but not the angle of the load impedance is varied:

Proof:

A voltage source of generated voltage V_g and internal impedance Z_g is connected to a load impedance Z as shown in **Fig. 2.8.**

where $Z_g = R_g + jX_g$, internal impedance of the source and $Z = |Z|\cos\theta + j|Z|\sin\theta$, load impedance.

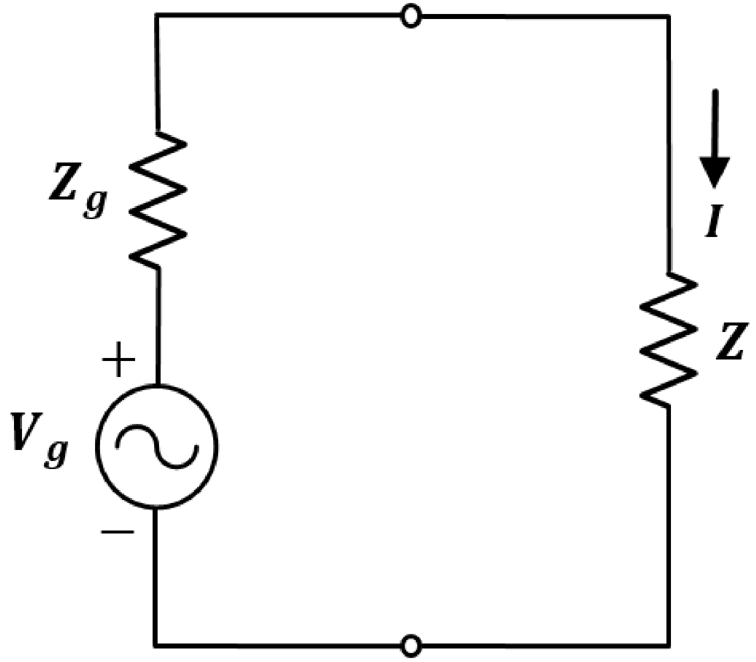


Fig. 2.8. Circuit having load with resistance and reactance.

The load current is

$$I = \frac{V_g}{\left(|Z|\cos\theta + R_g\right) + j\left(|Z|\sin\theta + X_g\right)}$$

Network Theorems

➢ Maximum Power Transfer Theorem

✓ Condition of Maximum Power Transfer for different networks

3. Reactance present, the magnitude but not the angle of the load impedance is varied:

Proof:

The power delivered to the load is

$$P = \frac{\left(V_g\right)^2 |Z| \cos\theta}{\left(R_g + |Z|\cos\theta\right)^2 + \left(X_g + |Z|\sin\theta\right)^2}$$

For maximum power transfer we must have

$$\frac{dP}{d|Z|} = 0$$

Under this condition and simplifying, we get $\left(R_g\right)^2 + \left(X_g\right)^2 = |Z|^2$

$$\therefore \left|Z_g\right| = |Z|$$

This is the condition of maximum power transfer and it holds for a transformer

➢ Maximum Power Transfer Theorem

Example – P2.6

Find the value of R_L for maximizing power transfer and also find the value of maximum power for the circuit as shown in **Fig. P2.6.**

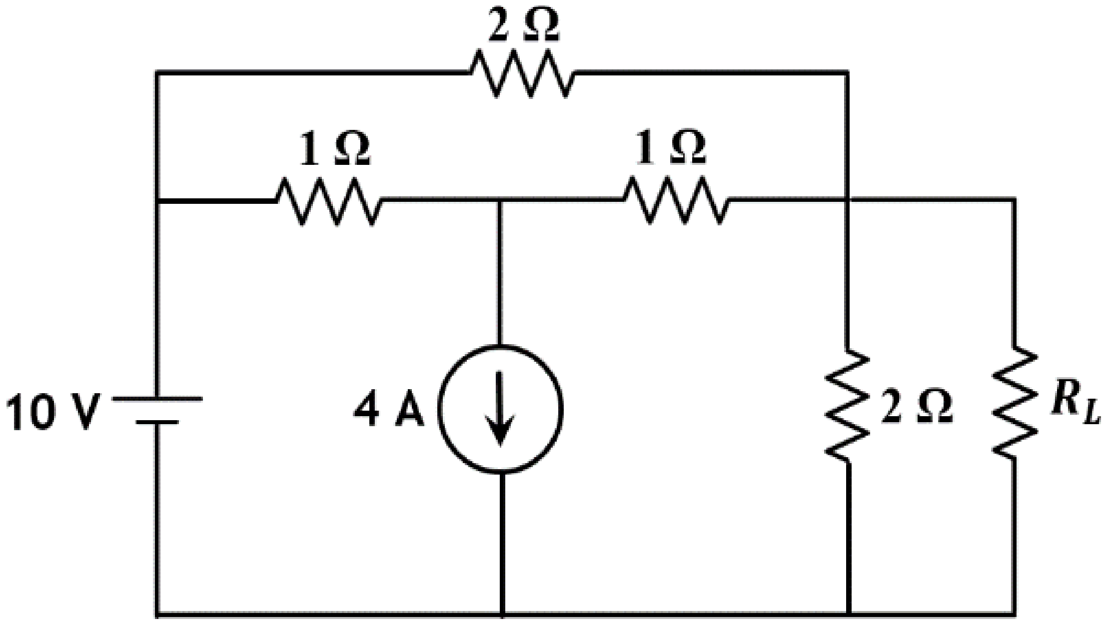


Fig. P2.6

Network Theorems

➢ Maximum Power Transfer Theorem

Solution of Example – P2.6

The circuit for finding Thevenin Voltage is as shown in **Fig. P2.6 (a).**

Apply KCL at node A, we get

$$\frac{V_A - 10}{1} + \frac{V_A - V_B}{1} + 4 = 0$$

or, $V_A - 10 + V_A - V_B + 4 = 0$

$\therefore \quad 2\,V_A - V_B = 6 \longrightarrow (1)$

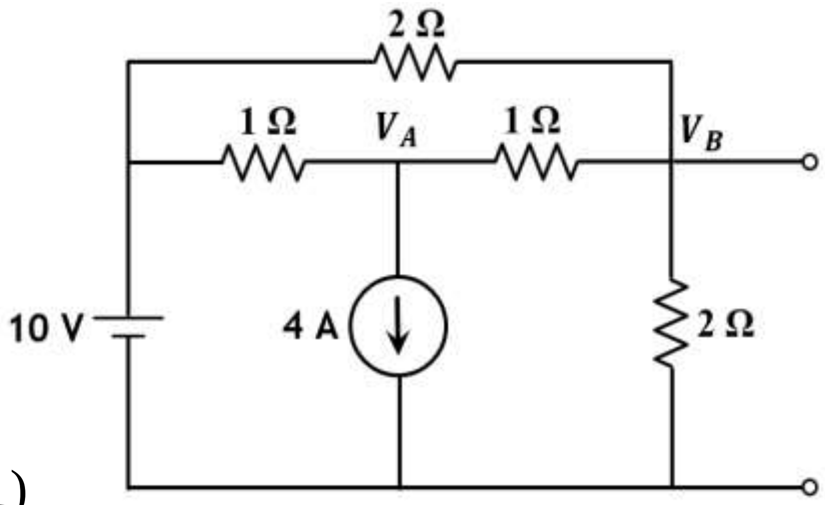


Fig. P2.6 (a)

Apply KCL at node B, we get

$$\frac{V_B - V_A}{1} + \frac{V_B - 10}{2} + \frac{V_B}{2} = 0$$

or, $2\,V_B - 2\,V_A + V_B - 10 + V_B = 0$

or, $4\,V_B - 2V_A = 10$

$\therefore \quad 2\,V_B - V_A = 5$

Multiply Eq. (1) by 1 and Eq. (2) by 2, then add and we get

$$\therefore\; V_B = V_{Th} = \frac{16}{3}\ V$$

Maximum Power Transfer Theorem

Solution of Example – P2.6

The circuit for finding Thevenin Resistance is as shown in **Fig. P2.6 (b).**

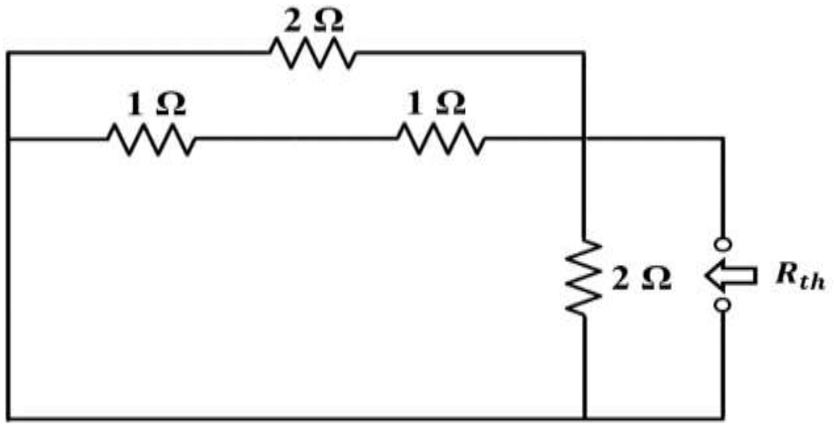


Fig. P2.6 (b)

$$R_{th} = [2 \parallel (1+1)] \parallel 2 = (1 \parallel 2) = \frac{2}{3}\,\Omega$$

The Thevenin equivalent circuit along with load resistance is shown in **Fig. P2.6 (c)**

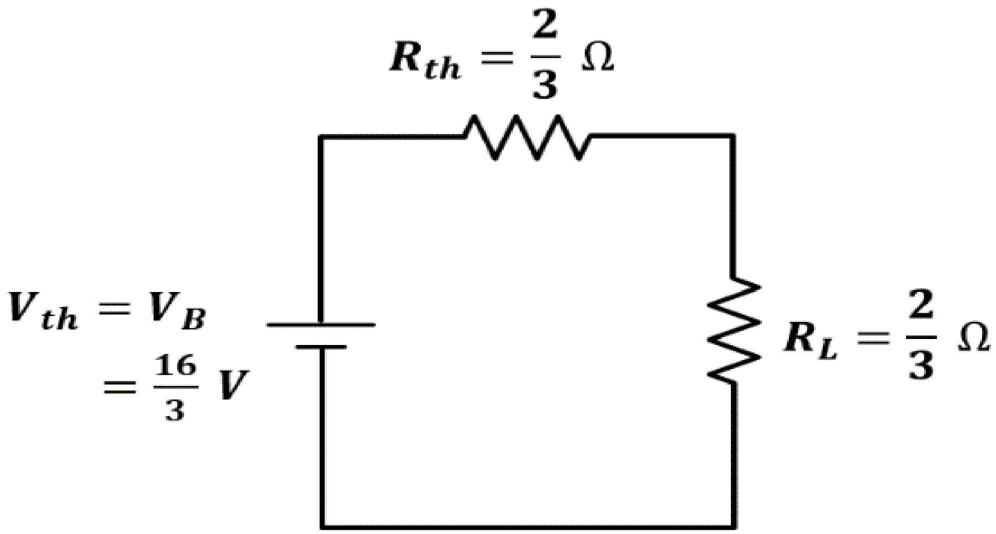


Fig. P2.6 (c)

$$\therefore \text{Maximum power transfer, } P_L = \frac{(V_{th})^2}{4\,R_L} = \left(\frac{16}{3}\right)^2 \times \frac{1}{4 \times \frac{2}{3}} = \frac{16 \times 16}{3 \times 3} \times \frac{3}{8} = \frac{32}{3} = 10.67\ watt$$

Network Theorems

➢ Maximum Power Transfer Theorem

Example – P2.7

Find the value of load resistance R_L to receive maximum power from the source and also find the maximum power delivered to the load in the circuit as shown in **Fig. P2.7.**

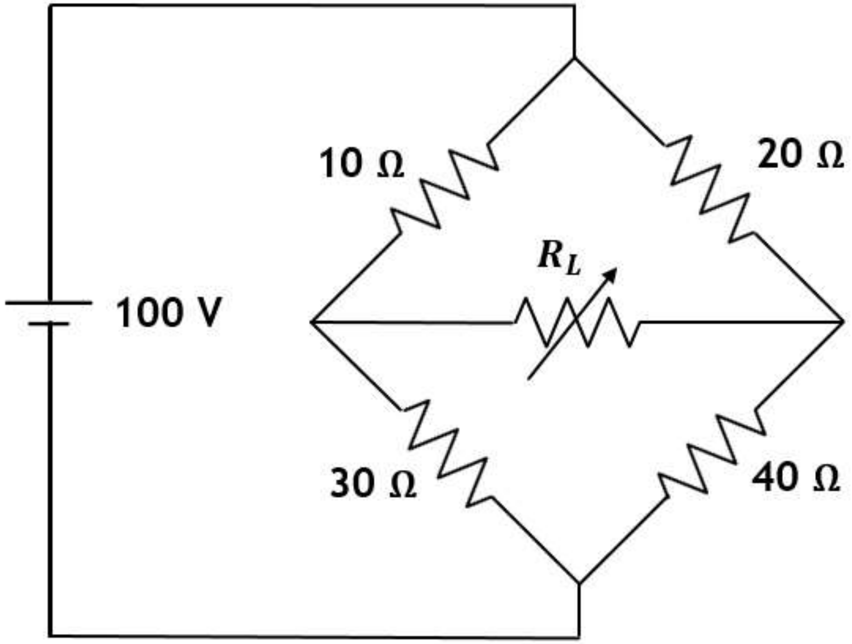


Fig. P2.7


Thank you