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Higgs $\rightarrow \tau\tau$ Matrix Element Primer

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Abstract

This note represents a working document in which we document our progress with applying the Matrix Element method to the Higgs $\rightarrow \tau\tau$ analysis.

1 Matrix elements

In this section we describe the computation of the Matrix Elements for the Higgs $\rightarrow \tau\tau$ signal and different background processes.

1.1 Higgs $\rightarrow \tau\tau$ signal

The matrix element for the signal is given by the expression:

$$w = \frac{1}{\sigma_{acc}} \sum_{a,b} \int dx_a dx_b \frac{f(x_a)f(x_b)}{x_a x_b s} \, \delta^2((x_a P_a + x_b P_b) + \sum P_k) |\mathcal{M}|^2 \, W(\vec{y}||\vec{x}) \, d\vec{x} \, \mathcal{R} \, dP_x^{recoil} dP_y^{recoil} \tag{1}$$

σ_{acc} denotes the product of cross–section times acceptance of the analysis for the signal. The summation $\sum_{a,b}$ extends over gluons plus quark–anti quark pairs. The symbol f represents the parton distribution functions. The two–dimensional δ–function imposes conservation of energy and longitudinal momentum between initial and final state.

The phase–space element $d\vec{x}$ represents all particles in the the final state. The simplest case is the hadronic channel. In this case:

$$d\vec{x} = \frac{d^3 p_{h+}}{(2\pi)^3 2E_{h+}} \frac{d^3 p_{\bar{\nu}}}{(2\pi)^3 2E_{\bar{\nu}}} \frac{d^3 p_{h-}}{(2\pi)^3 2E_{h-}} \frac{d^3 p_{\nu}}{(2\pi)^3 2E_{\nu}},$$

where we by $h+$ ($h-$) the system of hadrons produced in the τ^+ (τ^-) decay. We use the recursive relation for phase–space elements given in the kinematics section of the PDG [1] (Eq.*43.12*) to write the phase–space element as:

$$\begin{aligned} d\vec{x} &= (2\pi)^6 \frac{d^3 p_{\tau+}}{(2\pi)^3 2E_{\tau+}} \frac{d^3 p_{h+}}{(2\pi)^3 2E_{h+}} \frac{d^3 p_{\bar{\nu}}}{(2\pi)^3 2E_{\bar{\nu}}} \, \delta(E_{\tau+} - E_{h+} - E_{\bar{n u}}) \, \delta^3(\vec{p}_{\tau+} - \vec{p}_{h+} - \vec{p}_{\bar{n u}}) \\ &\cdot \frac{d^3 p_{\tau+}}{(2\pi)^3 2E_{\tau+}} \frac{d^3 p_{h-}}{(2\pi)^3 2E_{h-}} \frac{d^3 p_{\nu}}{(2\pi)^3 2E_{\nu}} \, \delta(E_{\tau-} - E_{h-} - E_{\bar{n u}}) \, \delta^3(\vec{p}_{\tau-} - \vec{p}_{h-} - \vec{p}_{\bar{n u}}) dq_1^2 dq_2^2. \end{aligned}$$

Extending the dimension of the integral may seem like a complication at first, but will simplify the calculus later.

The actual Matrix Element $|\mathcal{M}|^2$ consists of three parts:

$$|\mathcal{M}|^2 = |\mathcal{M}_{HS}|^2 \cdot \frac{1}{(m_H \Gamma_H)^2} \delta(M_{\tau\tau}^2 - m_H^2) \frac{1}{(m_\tau \Gamma_\tau)^4} \delta(q_1^2 - m_\tau^2) \delta(q_2^2 - m_\tau^2) \cdot |\mathcal{M}_{decay}|^2.$$

The first part, $|\mathcal{M}_{HS}|^2$ denotes the Matrix Element for the "hard-scatter" interaction $pp \rightarrow Higgs \rightarrow \tau\tau$, which we intend to take from MadGraph [2]. The third part, $|\mathcal{M}_{decay}|^2$, represents the tau decays. The second part contains the Breit–Wigner functions of the propagator terms that relate the hard–scatter interaction to the tau decays.

The variables P_x^{recoil} and P_y^{recoil} represent the hadronic recoil:

$$\begin{aligned} P_x^{recoil} &= -\left(E_x^{\text{miss}} + \sum_e P_x^e + \sum_\mu P_x^\mu + \sum_{\tau_h} P_x^{\tau_h} + \sum_{jets} P_x^{jet} \right) \\ P_y^{recoil} &= -\left(E_y^{\text{miss}} + \sum_e P_y^e + \sum_\mu P_y^\mu + \sum_{\tau_h} P_y^{\tau_h} + \sum_{jets} P_y^{jet} \right) \end{aligned}$$

The issue with the hadronic recoil is that the Matrix Elements generated by MadGraph are leading order. The system of particles passed to the Matrix Element are expected to be balanced in transvere momentum. The hadronic recoil spoils this balance, causing the taus produced in the Higgs decay not to be "back–to–back" and their transverse momenta to extend beyond the kinematic endpoint $P_T^{\tau_h} = \frac{1}{2} m_H$. We follow Ref. [3] and integrate over the hadronic recoil, using a transfers function $\mathcal{R}$ that constrains the integration to values compatible with the measured hadronic recoil:

$$\mathcal{R} = \frac{1}{2\pi\sqrt{\det V}} \exp\left(- \left(\vec{p}^{recoil} - \vec{p}^{recoil}_{exp} \right)^T V^{-1} \left(\vec{p}^{recoil} - \vec{p}^{recoil}_{exp} \right) \right),$$

with $\vec{p}^{recoil}_{exp} = - \left(\sum_{taus} \vec{p}_\tau \sum_{jets} \vec{p}_{jet} \right)$. Note that the sum in the expression for $\vec{p}^{recoil}_{exp}$ extend over the "true" momenta of the two taus originating from the Higgs decay plus the momenta of high P_T jets.

1.1.1 $|\mathcal{M}_{HS}|^2$

Christian suggest that we compute $|\mathcal{M}_{HS}|^2$ separately for Higgs $\to \tau\tau$ signal events with 0–jet, 1–jet and 2–jet. Jets are required to have $P_T > 30$ GeV and $|\eta| < 4.7$ in order to count towards the jet multiplicity.

The Matrix Elements generated by MadGraph can be found in the directory

```
SubProcesses/internal_MadGraph_name_of_the_MatrixElement/matrixXX.f
```

. XX is a number chosen by Madgraph in case different Feynman diagrams contribute to the same final state. The Feynman diagram for the process is stored in

```
SubProcesses/internal_MadGraph_name_of_the_MatrixElement/matrixXX.ps
```

. The Feynman diagram is neccessary to decide which parton distribution functions (gluon or quark flavor) we need for each Feynman diagram.

1.1.2 Breit–Wigner terms and tau decays

The part

$$T \equiv \frac{1}{(m_\tau \Gamma_\tau)^2} \, \delta(q^2 - m_\tau^2) \, (2\pi)^3 \, d^3p_\tau \, d^3p_h \, d^3p_\nu \, \delta(E_\tau - E_h - E_\nu) \, \delta^3(\vec{p}_\tau - \vec{p}_h - \vec{p}_{\bar{\nu}}) \, dq^2$$

has identical structure for each tau.

We first perform the integration over dq^2 to get rid of the Breit–Wigner term. This yields:

$$T = (2\pi)^3 \, d^3p_\tau \, d^3p_h \, d^3p_\nu \, \delta(E_\tau - E_h - E_\nu) \, \delta^3(\vec{P}_\tau - \vec{p}_h - \vec{p}_{\bar{\nu}}).$$

We then use the 3–dimensional δ–function that represents the momentum conservation in the tau decay to perform the integration over d^3P_ν:

$$T = (2\pi)^3 d^3p_\tau \, d^3p_h \, \delta(E_\tau - E_h - E_\nu(\vec{p}_\tau, \vec{p}_h)).$$

The energy of the neutrino, E_ν, is now a function of the momenta $\vec{p}_\tau$ of the tau lepton and $\vec{p}_h$ of the hadronic system: $E_\nu = P_\nu^2 = (\vec{P}_\tau - \vec{P}_h)^2 = P_\tau^2 + P_h^2 - 2P_\tau P_h \cos\theta$, where θ denotes the angle between the $\vec{p}_\tau$ and $\vec{p}_h$ vectors. We next write the phase–space element d^3p_τ in polar coordinates (*cf.* section. A.1.1). We orient the z–axis such that it coincides with the direction of the $\vec{p}_h$ vector.

$$T = (2\pi)^3\, P_\tau^2\, dP_\tau\, d\cos\theta\, d\phi\, d^3p_h\, \delta(E_\tau - E_h - E_\nu(\vec{p}_\tau, \vec{p}_h)).$$

We need to be careful when performing the integration over P_τ, as the argument of the δ–function depends on P_τ. Following the rules for δ–functions [4], $\delta(g(x)) = \sum_i \frac{1}{|g'(x_i)|}\delta(x - x_i)$, where the sum extends over all roots x_i of the function $g(x)$, we obtain:

$$\begin{aligned} T &= (2\pi)^3\, P_\tau^2\, d\cos\theta\, d\phi\, d^3p_h\, \delta(\sqrt{P_\tau^2 + m_\tau^2} - \sqrt{P_h^2 + m_h^2 - P_\tau^2 - P_h^2 + 2P_\tau P_h \cos\theta})) \\ &= (2\pi)^3\, d\cos\theta\, d\phi d^3p_h\, \left(g(x_+) + g(x_-)\right), \end{aligned} \tag{2}$$

with

$$g(x) = \frac{x}{P_\tau^2 + m_\tau^2} - \frac{P_h\cos\theta - x}{P_h^2 - 2P_h x\cos\theta + x^2}.$$

The two roots x_+ and x_- are given by:

$$x\pm = \frac{(m_\tau^2 + m_h^2)P_h\cos\theta \pm \sqrt{(P_h^2 + m_h^2)\cdot(m_\tau^2 - m_h^2)^2 - 4m_\tau^2 P_h^2 \sin^2\theta}}{4m_\tau^2 P_h^2 \sin^2\theta}. \tag{3}$$

The remaining integration over θ in Eq. 2 extends over the region for which the expression in the radical of Eq. 3 remains positive:

$$(m_\tau^2 - m_h^2)^2 - 4m_\tau^2 P_h^2 \sin^2\theta > 0 \Leftrightarrow 0 < \sin\theta < \frac{m_\tau^2 - m_h^2}{4m_\tau P_h}.$$

1.1.3 Putting it all together

Final integration variables:

- $M_{\tau\tau}$
- x_a
- $\phi_{\tau+}$
- $\phi_{\tau-}$
- P_{h+}, for energy transfer function of first hadronic tau
- P_{h-}, for energy transfer function of second hadronic tau
- P_x^{recoil}
- P_y^{recoil}

To–do:

- Perform a variable transformation from $d\theta_1 d\theta_2$ to $dM_{\tau\tau}$, the mass of the tau lepton pair, and $u = P_{z_1} + P_{z_2}$, the sum of longitudinal momenta of the two tau leptons. These transformations are neccessary in order to make the integration over the Breit–Wigner term for the Higgs and the integration over dx_b numerically stable.
- Extend the formalism for the tau decays to $\tau \to e\nu\bar{\nu}$ and $\tau \to \mu\nu\bar{\nu}$ decays.
- Check if (LO) $|\mathcal{M}_{HS}|^2$ obtained from MadGraph can handle finite transverse momentum of the Higgs. In case MadGraph requires $P_T^H = 0$, we need to replace the integration over the hadronic recoil in Eq. **??** by a two–dimensional δ–function.

- We need to determine the product of signal cross–section times acceptance, σ_{acc}, in Eq. 1.1.3. Christian suggest we approximate this by the Standard Model Higgs production cross–section obtained from the LHC Higgs cross section working–group [] times the signal acceptance as function of mass from the MSSM Higgs $\rightarrow \tau\tau$ analysis [5].
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For debugging:

- We appreciate that the matrix element formalism is not trivial and we anticipate that we may make mistakes that we need to debug. The idea is to compute Eq. 1.1.3 for a series of test Higgs masses $\{m_H^i\}$ in simulated Higgs $\rightarrow \tau\tau$ signal events with $m_H^{true} = 125$ GeV. We expect that for most signal events the maximum weight w is attained for a test mass m_H^i close to the true Higgs mass m_H^{true}, modulo a typical resolution of $\mathcal{O}(20\%)$ that we plan to compare with SVfit **??**.

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A Collection of formulas

A.1 Transformations of phase–space element from Cartesian to polar coordinates

We will assume that the direction of electrons, muons, hadronic taus and jets is measured precisely.

With this assumption the transfer functions that related the measured four–vector to the true four–vector (used in the Matrix Element):

$$W(\vec{x}', \vec{x}) = W(E', E)\delta(\theta' - \theta)\delta(\phi' - \phi).$$

The use of polar coordinates (P, θ, ϕ) or equivalently (P, η, ϕ) is hence advantageous for the computation of transfer functions integrals.

A.1.1 Transformation of d^3p to $dPd\theta d\phi$

The transformation to (P, θ, ϕ) is given by the expressions:

$$\begin{aligned} P_x &= P\cos\phi\sin\theta P_y & = \\ P\sin\phi\sin\theta P_z &= P\cos\theta. \end{aligned}$$

The Jacobi determinant associated with the transformation is:

$$\det M = P^2 \sin\theta$$

The phase–space element can hence be written:

$$\frac{d^3p}{2E} = \frac{P^2 \sin\theta \, dP \, d\theta \, d\phi}{2E} = \frac{P^2 \, dP \, d\cos\theta \, d\phi}{2E} = \frac{P}{2} dE \, d\cos\theta \, d\phi.$$

The last step follows as $E = \sqrt{P^2 + m^2}$ implies $\frac{dE}{dp} = \frac{P}{E} \Leftrightarrow PdP = EdE$.

A.2 Transformation of d^3p to $dP_T d\eta d\phi$

The transformation to (P_T, η, ϕ) is given by the following expressions:

$$\begin{aligned} P_x &= P_T\cos\phi P_y & = \\ P_T\sin\phi P_z &= P_T\sinh\eta. \end{aligned}$$

The Jacobi determinant associated with this transformation is:

$$\det M = P_T^2 \cosh\eta$$

The phase–space element can hence be written:

$$\frac{d^3p}{2E} = \frac{P_T^2 \, \cosh\eta \, dP_T \, d\eta \, d\phi}{2E}.$$

A.3 Transformation to resonance mass

In case the phase–space integral extends over daughter particles of a narrow resonance, numerical stability of the computation requires to perform a transformation of the integration variables such that one integration variable corresponds to the mass of the resonance [6].

Starting from the variables (P_T, η, ϕ) of two daughters, the mass of the resonance can be expressed by the transverse momentum of the second daughter by:

$$P_{T_2} = \frac{M}{2P_{T_1}\cos(\phi_1 - \phi_2) + 2P_{T_1}\sinh\eta_1\sinh\eta_2}.$$

The Jacobi determinant associated with the transformation is:

$$\det M = \frac{1}{2P_{T_1}\cos(\phi_1 - \phi_2) + 2P_{T_1}\sinh\eta_1\sinh\eta_2}.$$