

Chi-Square (χ^2) Test :

χ^2 -test for Attribute:- A quality or characteristics

Eg:- Smiling, laughing, dancing, honest etc,

Working Procedure:-

Step 1:- Null hypothesis (H_0): Given Statement from problem

Step 2:- Alternative hypothesis (H_1): Opposite of null-hypothesis.

Step 3:- LOS (α): Given according to problem

Step 4:- Test statistic:

To calculate expected frequency E_i of any cell

$$E_i = \frac{(\text{Row total})(\text{column total})}{\text{Grand total}}$$

Assuming H_0 is true, test statistic

$$\chi^2 = \sum_{i=1}^n \left[\frac{(O_i - E_i)^2}{E_i} \right]$$

approx. follows χ^2 distribution with

$$df = v = (\text{no of rows} - 1)(\text{no. of columns} - 1)$$

Step 5:- Conclusion:

$$df = (\text{no of rows} - 1)(\text{no of columns} - 1)$$

Problems

Note:- First calculate expected frequency in the following way.

	A	B	Total
Columns	a	b	a+b
Rows	c	d	c+d

$$E(a) = \frac{\underbrace{(a+b)}_{\text{Row total}} \underbrace{(c+d)}_{\text{Column total}}}{\underbrace{(a+b+c+d)}_{\text{Grand total}}}$$

$$E(b) = \frac{(b+a)(b+d)}{a+b+c+d}$$

$$E(c) = \frac{(c+d)(c+a)}{a+b+c+d}$$

$$E(d) = \frac{(d+c)(d+b)}{a+b+c+d}$$

1). On the basis of info given below about treatment of 200 patients suffering from disease. State whether new treatment is comparatively superior to the conventional treatment.

	Favouable	Not Favouable	Total
New	60	30	90
Conventional	40	70	110
Total	100	100	200 (90+110/100+100)

Step 1:- Null hypothesis (H_0): New & Conventional treatments are independent.

Step 2:- Alternative hypothesis (H_1): New & conventional treatments are not independent

Step 3:- LOS (α): 5%

(2)

Step 4:- Test Statistic :

$$\chi^2 = \sum_{i=1}^n \left[\frac{(O_i - E_i)^2}{E_i} \right]$$

To calculate Expected frequency of each cell

$$E_i = \frac{\text{Row total} \times \text{Column total}}{\text{Grand total}}$$

$$E(60) = \frac{90 \times 100}{(60+30)(60+40)} = \frac{90 \times 100}{200} = 45$$

$$E(30) = \frac{90 \times 100}{200} = 45$$

$$E(40) = \frac{110 \times 100}{200} = 55$$

$$E(70) = \frac{110 \times 100}{200} = 55$$

Calculation of χ^2

O_i (Given cell values)	E_i (Calculated in the above way)	$O_i - E_i$	$(O_i - E_i)^2$	$\frac{(O_i - E_i)^2}{E_i}$
60	45	15	225	5
30	45	-15	225	5
40	55	15	225	4.09
70	55	15	225	4.09
				$\sum \left[\frac{(O_i - E_i)^2}{E_i} \right] = 18.18$

$$\chi^2_{\text{cal}} = 18.18$$

Step 5:- Conclusion :

$$df = (\text{no. of rows} - 1)(\text{no. of columns} - 1) = (2-1)(2-1) = 1$$

The tabulated value of χ^2 at 5% LOS with 1 df
= $\chi^2_{(0.05,1)} = 3.841$ (from given tables)

$$\chi^2_{\text{cal}} > \chi^2_{(0.05,1)}$$

\therefore Cal is greater than tabulated at 5% LOS with 1 df we reject null-hypothesis H_0 .

Practise problem

→ From following data, find whether there is any significant liking in the habit of taking soft drinks among categories of employees.

Soft drinks ↓	Employees ↓			Total
	clerks	teachers	officers	
Pepsi	10	25	65	100
Thumsup	15	30	65	110
Fanta	50	60	30	140
	75	115	160	350