CS 736

Assignment - 1

By: Ank Kumar Gupta (22B0623) & Veenus (22B0704)

Overview

- **Objective:** Implement and analyze Bayesian denoising algorithms and dictionary learning for image denoising.

- **Tasks:**
  1. Bayesian Denoising of Phantom MRI Image
  2. Bayesian Denoising of Brain MRI Image
  3. Bayesian Denoising of RGB Microscopy Image
  4. Dictionary Learning on Image Patches for Denoising

Task 1

Objective

Bayesian Denoising of a Phantom Magnetic Resonance Image

Description

- Implementation of a MAP-based Bayesian image denoising algorithm using a suitable noise model (i.i.d. Gaussian) and an MRF prior with a 4-neighbor system
- 3 MRF priors to be used: (1) Quadratic, (2) Huber, and (3) Log-based discontinuity-adaptive.
- Parameters need to be manually tuned to minimize relative root-mean-squared error (RRMSE), using the noiseless image as reference.

Task 1

Approach

- First scaled the noisy image with min-max scaling to ensure consistent values between [0, 1]
- Defined all 3 priors with their gradients.
- Compute the gradient of the likelihood function (based on Gaussian noise).
- Utilize gradient ascent optimization technique for Image Denoising.
- Computed the update using a weighted combination of likelihood and prior gradients and also fine-tuned the weight parameter for optimal result.
- Further used a momentum-based update with velocity to smooth out updates and overcome the problem of very early convergence.

Task 1

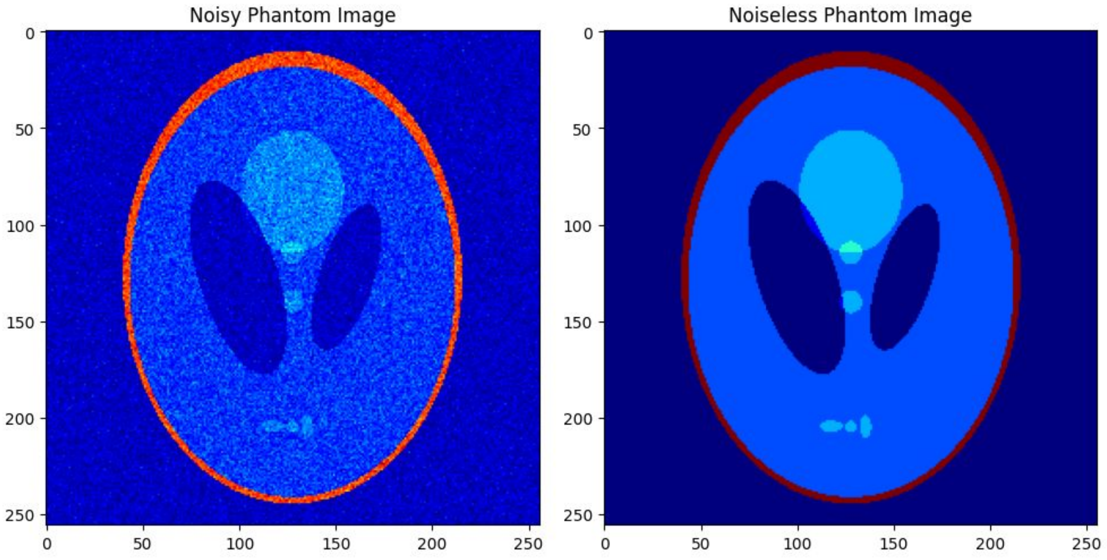
Approach

- Computed the likelihood term as the sum of squared differences from the noisy image (log likelihood).
- Computed the prior term based on the chosen prior function (Quadratic, Huber, or Discontinuity Adaptive).
- Thus, computed the log posterior as a weighted sum of the likelihood and prior terms.
- Used dynamic step size modification for effective learning.
- Fine-tuned the associated parameter for Huber and Discontinuity Adaptive priors, as well as step-size and no. of iterations to get optimal results.

Result

1(a) RRMSE of noisy image and original (noiseless) image

RRMSE = 0.29311377574241915



Result

1(b) MRF Priors:

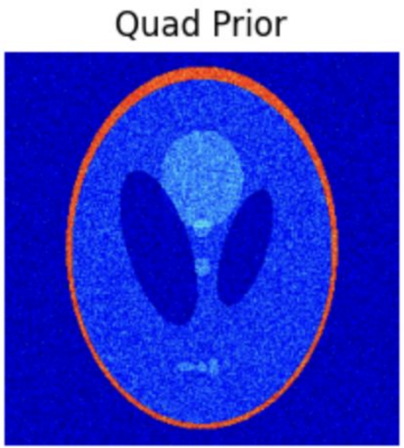
- **Quadratic function:**

  Optimal parameter, (Weight): 1.0

  Optimal RRMSE: 0.29311377574241915

  Evidence for optimality:

  Weight: 0.8   RRMSE: 0.3174088137243337



Denoised Image using Quad Prior

Result

1(b) MRF Priors:

- **Discontinuity-adaptive huber function:**

  Optimal parameter:

  Weight, Gamma: 0.1, 0.05

  Optimal RRMSE: 0.19023

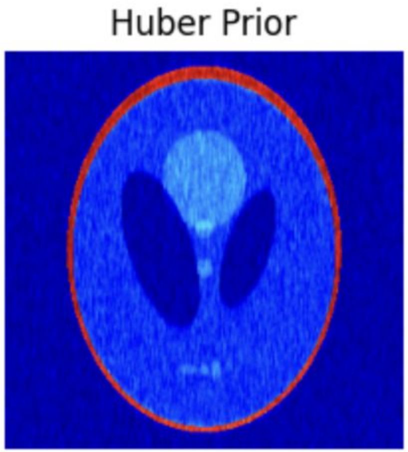
  Evidence for optimality:

  Parameters: 0.12, 0.05 RRMSE: 0.19269

  Parameters: 0.08, 0.05 RRMSE: 0.19517

  Parameters: 0.1, 0.06 RRMSE: 0.20013

  Parameters: 0.1, 0.04 RRMSE: 0.19242



Denoised Image using Huber Prior

Result

1(b) MRF Priors:

- **Discontinuity-adaptive function:**

  Optimal parameter:

  Weight, Gamma: 0.08, 0.045

  Optimal RRMSE: 0.19347

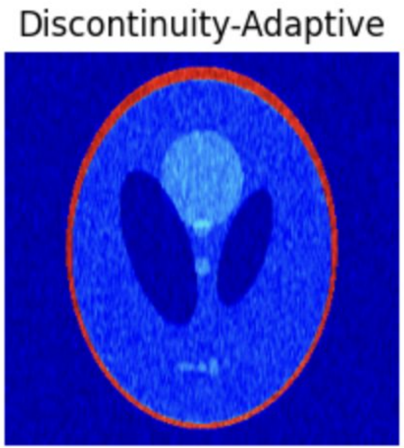
  Evidence for optimality:

  Parameters: 0.064, 0.045 RRMSE: 0.193580

  Parameters: 0.096, 0.045 RRMSE: 0.201501

  Parameters: 0.08, 0.054 RRMSE: 0.1971660

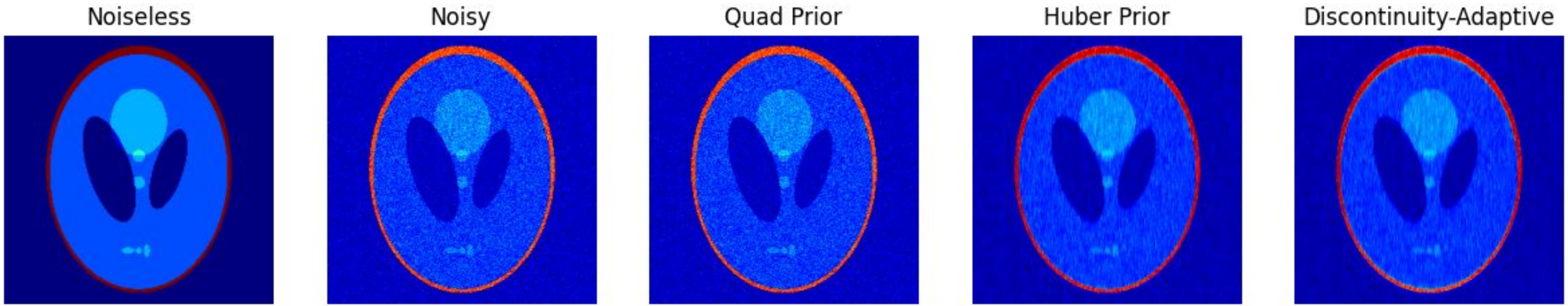
  Parameters: 0.08, 0.036 RRMSE: 0.2092144



Denoised Image using Disc Prior

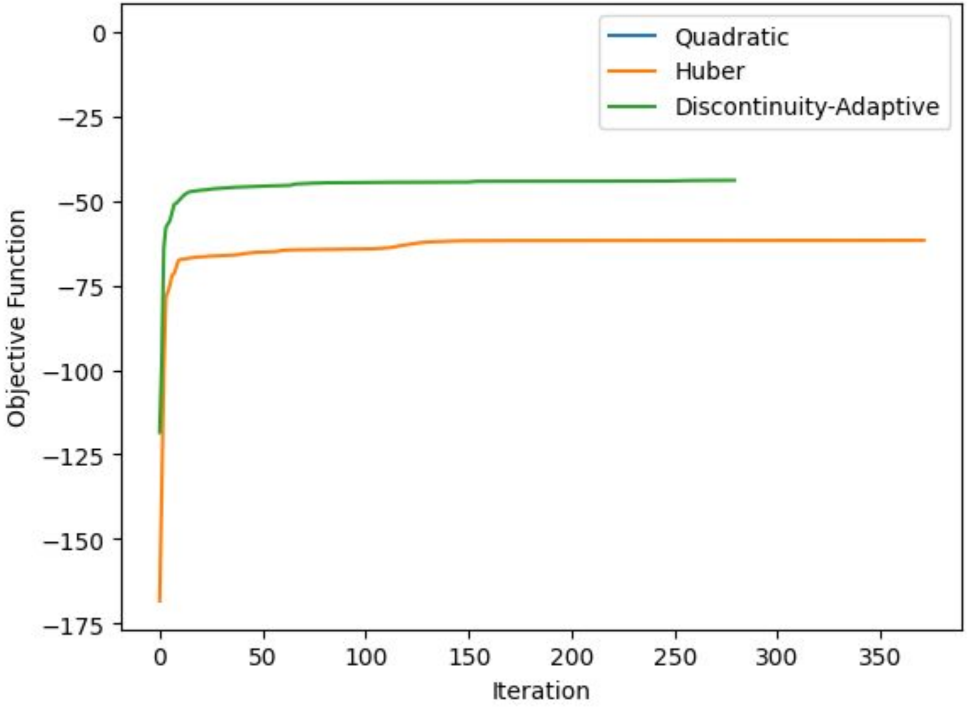
Result

1(c)


Noiseless
Noisy
Quad Prior
Huber Prior
Discontinuity-Adaptive

Result

1(d) plots of the objective-function values



Task 2

Objective

Bayesian Denoising of Brain MRI Image

Description

- Implementation of a MAP-based Bayesian image denoising algorithm using a suitable noise model (i.i.d. Gaussian) and an MRF prior with a 4-neighbor system
- 3 MRF priors to be used: (1) Quadratic, (2) Huber, and (3) Log-based discontinuity-adaptive.
- Parameters need to be manually tuned to minimize relative root-mean-squared error (RRMSE), using the noiseless image as reference.

Task 2

Approach

Similar to task 1, only image to be processed is different.

Result

MRF Prior 1:

- **Quadratic function:**

  Optimal parameter, (Weight): 0.5

  Optimal RRMSE: 0.194409

  Evidence for optimality:

  Weight: 0.6   RRMSE: 0.196301

  Weight: 0.4   RRMSE: 0.19505

Result

MRF Prior 2 & 3:

- **Huber Function:**

  Optimal parameter:

  Weight: 0.1

  Gamma: 0.03

  Optimal RRMSE: 0.1392411

- **Discontinuous Adaptive:**

  Optimal parameter:

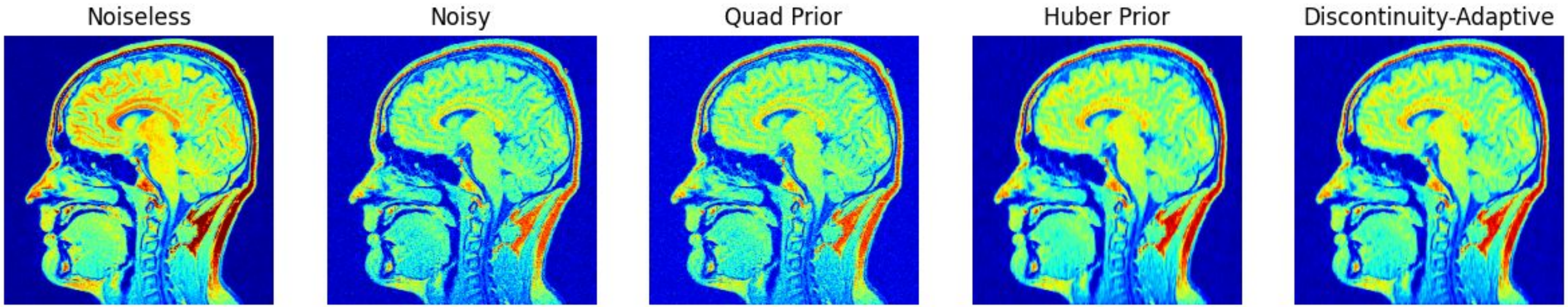
  Weight: 0.1

  Gamma: 0.02

  Optimal RRMSE: 0.143907

Results

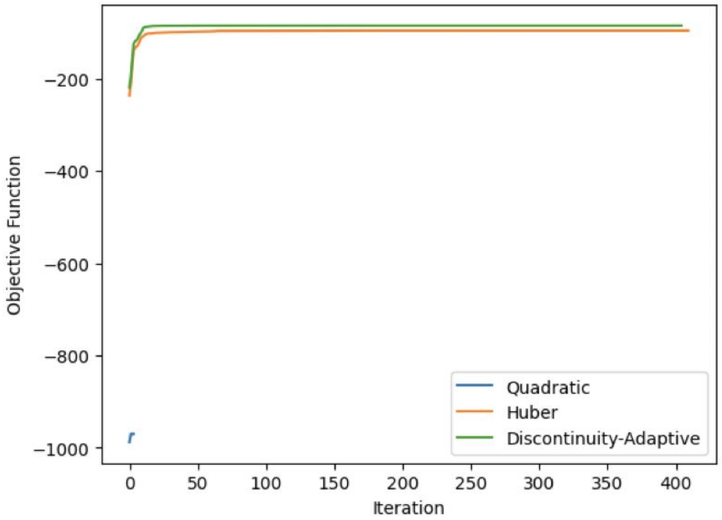
Output images for different priors



RRMSE = 0.20111204 RRMSE = 0.1944097 RRMSE = 0.1392411 RRMSE = 0.143907

Results

plots of the objective-function values



Task 3

Objective

Bayesian Denoising of RGB Microscopy Image

Description

- Implementation of a MAP-based Bayesian image denoising algorithm using a suitable noise model (i.i.d. Gaussian) and an MRF prior with a 4-neighbor system
- 3 MRF priors to be used: (1) Squared-L2-norm of vector difference, (2) L2-norm of vector difference, and (3) Huber-regularized L1-norm of vector difference..
- Parameters need to be manually tuned to minimize relative root-mean-squared error (RRMSE), using the noiseless image as reference.

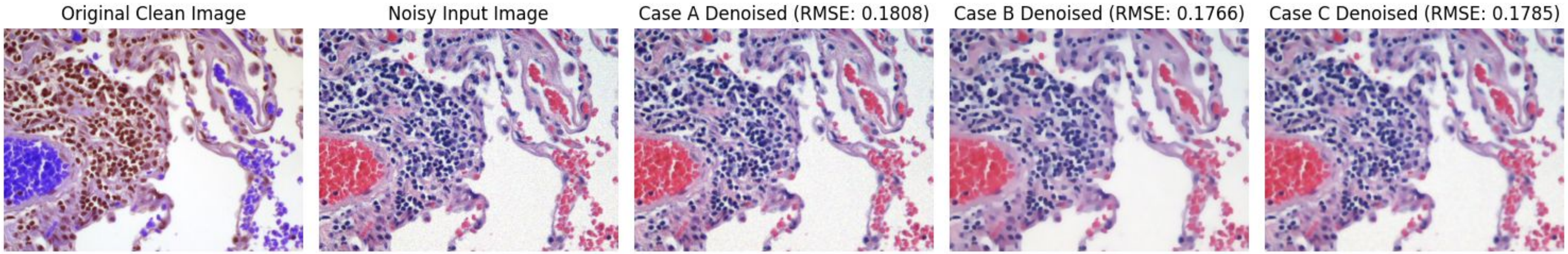
Task 3

Approach

- Used an optimization-based approach, by refining the noisy image iteratively by minimizing a objective function consisting of a data fidelity term and a prior-based regularization term.
- Used Adam optimizer for effective optimization.
- Implemented three different prior functions named as Squared L2 Norm, L2 Norm and Huber Regularized L1 Norm.
- Further, in each iteration, the denoised image gets updated by computing gradients of the total energy function with respect to the image pixels, thus, balancing noise suppression and detail preservation.

Results

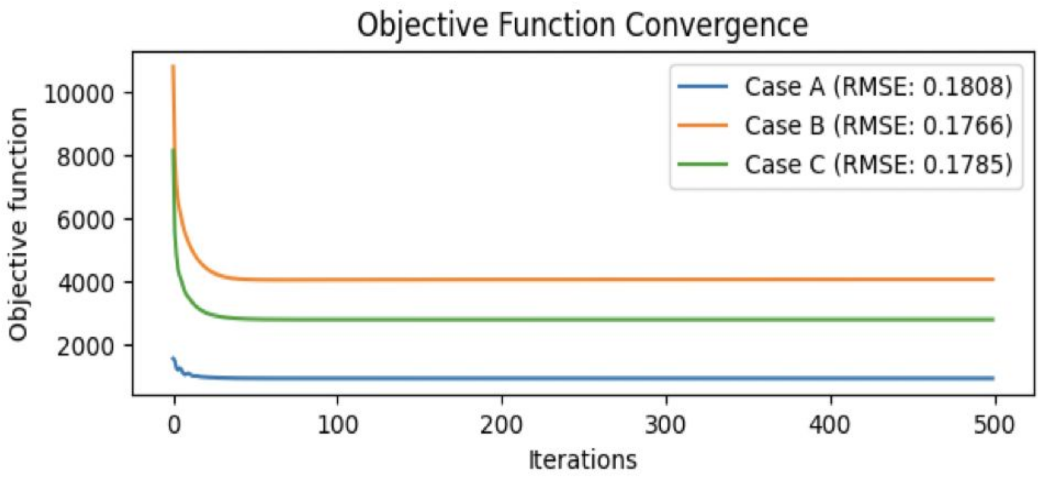
Output images for priors



Case C i.e. L1 norm provides better edge preservation, because it incorporate sparsity, leading to major differences in pixels (sharp transitions) across edges.

Results

plots of the objective-function values



Task 4

Objective

Dictionary Learning on Image Patches, Followed by Image Denoising

Description

- Implementing a function to learn the dictionary D for 8×8 image patches
- Experimenting with Different p Values and interpreting graph of objective function v/s iterations
- Visualizing dictionary atoms before and after optimization.
- Denoising a simulated noisy 2D Chest-CT Image Using Learned Dictionary
- Visualizing the results

Task 4

Approach

- Extracted 8*8 overlapping patches, then selected patches having the variance in top 20%.
- Estimated sparse coefficients r for patches x by minimizing reconstruction error while enforcing sparsity using soft-thresholding with p-norm regularization.
- Improved dictionary D by computing the gradient of reconstruction error and updating D using a learning rate, followed by column-wise normalization to maintain unit-norm atoms.
- Then, Optimized Σ $\|X - DR\|_F^2 + \lambda \sum |r|^p$ by fine-tuning λ for different p values.

Task 4

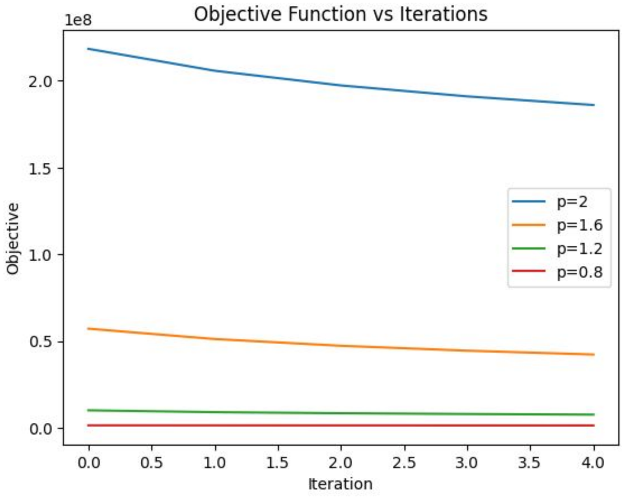
Approach

- Further plotted Initial v/s Learned Dictionary and histogram of coefficients for different values of p.
- Simulated a noisy chest-CT image by adding Gaussian noise with a standard deviation of 10% of the intensity range
- Given the learned dictionary D, estimated sparse coefficients r for noisy patches using gradient descent and soft-thresholding, by focusing on minimizing reconstruction error while enforcing sparsity.
- Monitored and plotted the objective function across iterations.

Results

Graph of the objective function v/s iterations for Dictionary Learning.

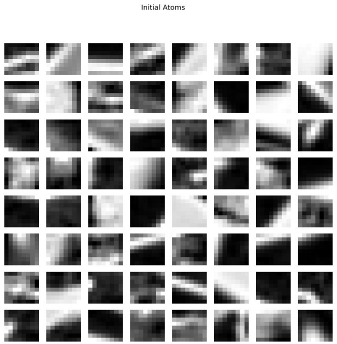
Inference: A tradeoff between reconstruction error and sparsity can be observed, where larger p value encourage less sparsity and allow a better approximation of the patches while smaller p value encourages sparsity but also compromises reconstruction quality.



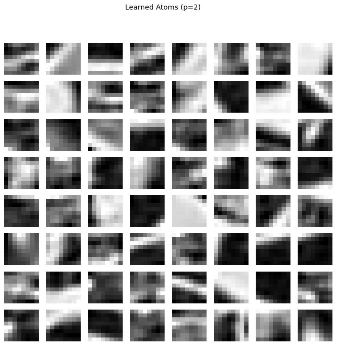
Results

Atoms used (Before Learning v/s After Learning)

Initial

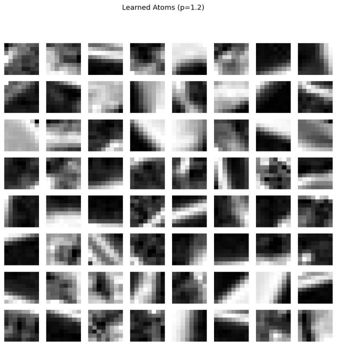


p = 2

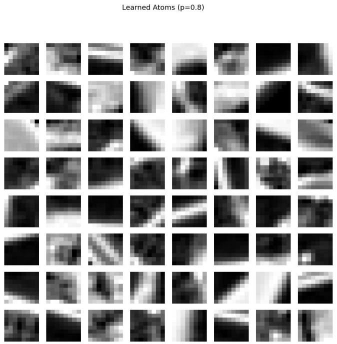


p = 1.6

p = 1.2



p = 0.8

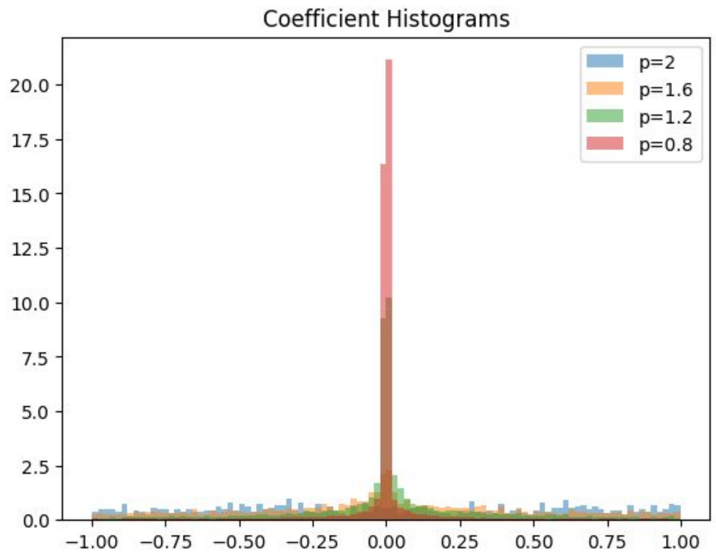


Observation: With decreasing value of p, detailing is getting reduced and sparsity is increasing.

Results

Histogram of Coeff within each ri, pooled across all ri

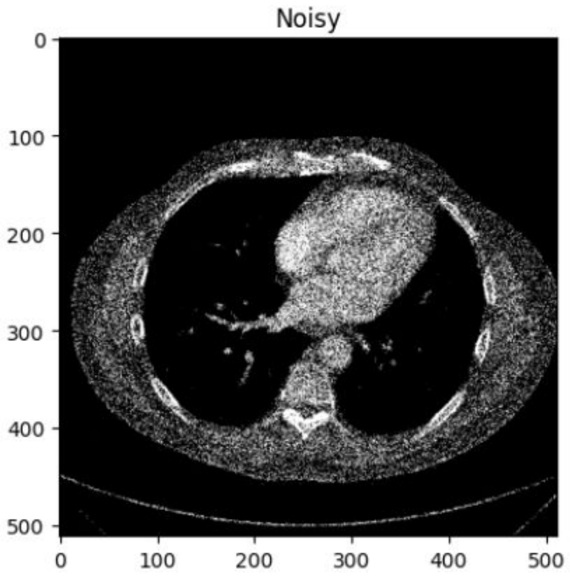
Observation & Inference: All cases are exhibiting bimodal distributions and as the p decreases, the graph gets narrower and peak increases, thus making histogram narrower and sharper, which confirms the earlier observation that lower p values enforce stronger sparsity, leading to fewer large coefficients and more very small or negligible ones.



Results

Simulated Noisy Version of Image

Simulated by introducing i.i.d. Gaussian zero-mean noise of standard deviation equaling 10% of the intensity range in the given image



Results

Optimization Problem for the denoising of simulated noisy image:

$$\min_{X,R} \quad \frac{1}{2}\|X - DR\|_F^2 + \lambda \sum_i \|R_i\|_p^p + \frac{\mu}{2}\|X - Y\|_F^2$$

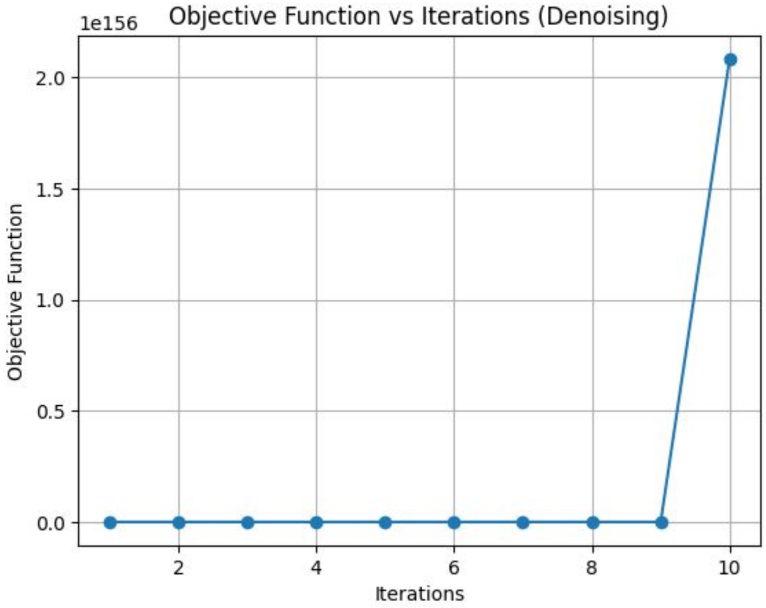
Reasoning for choosing this optimization problem:

- Here, the first term Ensures that the reconstructed patches X closely match their dictionary-based representation D&R.
- The other regularization term ensures sparse representation of the image, making the image compact and suppressing noise while the parameter λ balances both the detailing and noise.
- Last term ensures that the denoised image remains close to the noisy input, therefore, controlling how much noise is to be removed.

Results

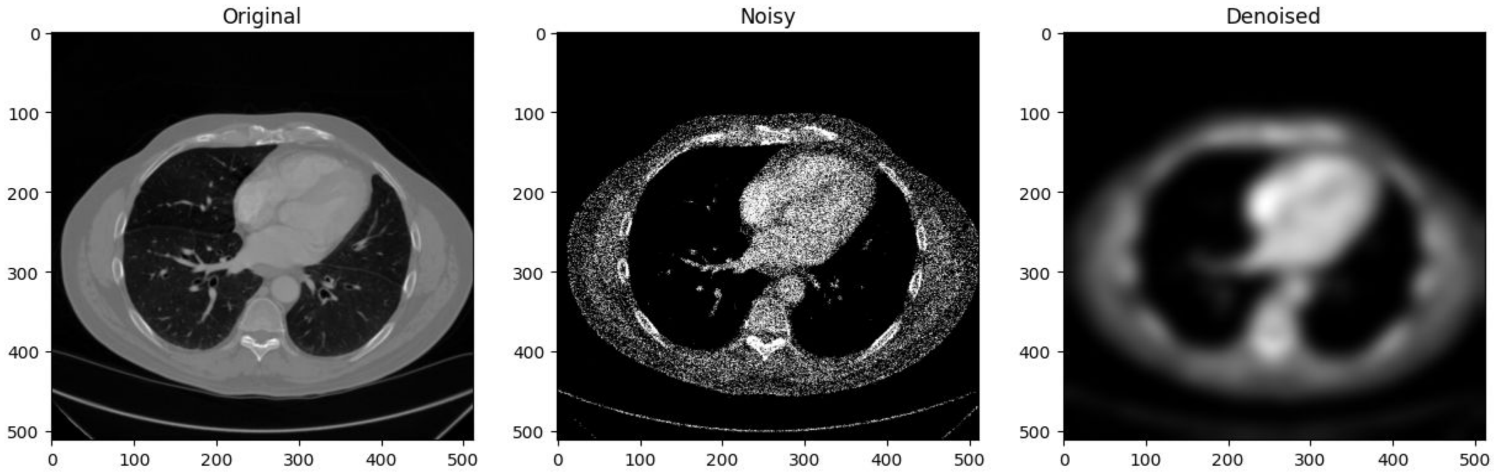
Graph of Optimization function v/s iterations:

Inference: The curve does not look good as it remains stable for most iterations but then faces a sharp increment, suggesting instability in the process, which could be due to non-enough parameter tuning or the dictionary itself be poorly adapted to the noisy image.

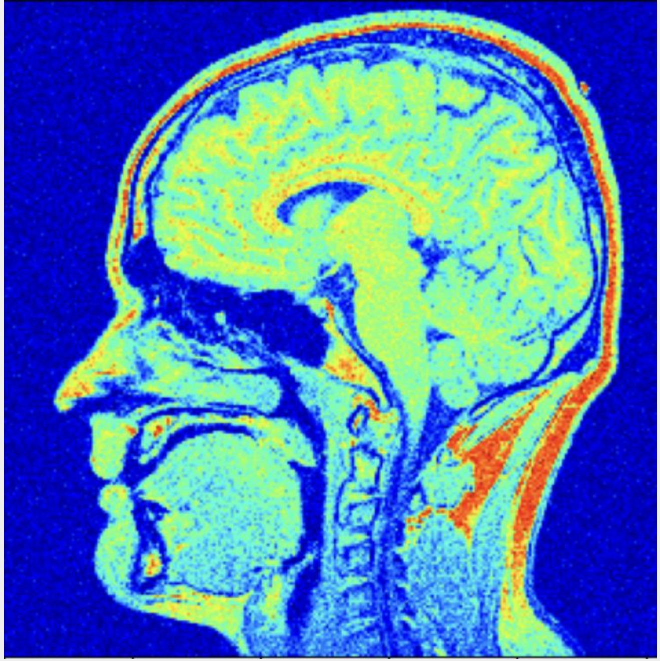


Results

Simulated Noisy, Original & Denoised Image



Observation: Too blurred denoised image indicates Reconstructional inconsistencies are visible in the denoised images, thus have a major scope for improvement.



Thank You!