CS-736 Assignment:
Mathematical Imaging, Image Reconstruction

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1. **(20 points) X-Ray Computed Tomography: Radon Transform.**

   Generate an image of the Shepp-Logan phantom using the Matlab (or an equivalent) function call "phantom (128)". Call this image as $f(x, y)$. Associate (logically) a coordinate frame with the image with the origin located at the center pixel.

   (a) (5 points)

   Given an image, such as $f(\cdot)$, implement a function (named **myXrayIntegration()**) to perform integration of the image intensities along lines $L_{t,\theta}$ that are parametrized by t and θ, where $t \in (-\infty, \infty)$ is the (signed) distance of the line from the origin and $\theta \in [0, 180)$ is the angle (in degrees) made by the upward-pointing normal to the line with the $+X$ axis. For this purpose, you may use the "interp2()" function in Matlab (or an equivalent function).

   • Choose a suitable value for the step size Δs for the discrete/summation approximation of the integral; justify your choice.

   • Choose a suitable image-interpolation scheme; justify your choice.

   (b) (5 points)

   Use the function you implemented in the previous question to implement another function (named **myXrayCTRadonTransform()**) that computes the Radon transform $Rf(t, \theta)$ for the discrete set of values $t = -90, -85, \cdots, 85, 90$ ($\Delta t = 5$ pixel-width unit) and the discrete set of values $\theta = 0, 5, 10, \cdots, 175$ ($\Delta\theta = 5$ degrees).

   (c) (5 points)

   Compare Radon transforms computed with different choices of parameters (during the integration). Specifically, compute and show the Radon-transform images with the following 3 Δs values: $\Delta s = 0.5$ pixel-width unit, $\Delta s = 1$ pixel-width unit, $\Delta s = 3$ pixel-width units.

   • Show the 1D function plots for the Radon-transform image values for the following values θ: 0 degrees and 90 degrees. Which 1D plot appears the smoothest/roughest, and why ?

   • Which Radon-transform image (one for each Δs) appears the smoothest/roughest, and why ?

   (d) (2.5 points)

   If you were to design an Xray CT scanner, what parameter settings (for $\Delta t, \Delta\theta$) would you use or *not* use ? Justify your choice (using suitable images / plots) and describe the underlying trade-offs.

   (e) (2.5 points)

   If you were to design a CT reconstruction software relying on the algebraic reconstruction technique (ART), then:

   • How would you choose the number of pixels, and the pixel size, in the scene grid ?

   • For a chosen scene-image pixel grid, describe the effects of choosing $\Delta s >> 1$ pixel width and $\Delta s << 1$ pixel width.

2. **(20 points) X-Ray Computed Tomography: Reconstruction by Filtered Backprojection (FBP).**

Generate an image of the Shepp-Logan phantom using the Matlab (or an equivalent) function call "phantom (128)". Call this image as $f(x,y)$. Associate (logically) a coordinate frame with the image with the origin located at the center pixel.

(a) (8 points)

Use the Matlab (or an equivalent) function "radon()" to compute a Radon transform of $f(x,y)$ with the values of $\theta = 0, 3, 6, \cdots, 177$ degrees.

• Now, implement the Ram-Lak filter, the Shepp-Logan filter, and the Cosine filter (in the Fourier domain; use Matlab (or equivalent) functions "fft()" and "ifft()"), where L is a user-controlled parameter. Name the function as **myFilter()**.

Use your function to filter (in all 3 aforementioned ways) the Radon-transform data for 2 different values of L, i.e., $L = w_{\text{max}}$ and $L = w_{\text{max}}/2$, where w_{max} is the highest frequency in the (discrete) Fourier representation.

Use the Matlab (or an equivalent) function "iradon()", with suitable parameters, to compute the (unfiltered) backprojection associated with the Radon transform.

• Show the resulting images and justify the similarities and the differences observed between the different combinations of filters and parameter values.

(b) (8 points)

Generate **blurred versions of the Shepp-Logan image** by convolving it with a Gaussian, as follows.

Use the Matlab (or equivalent) function calls "mask = fspecial ('gaussian', 11, 1)" to create a convolution mask and then use "conv2 (image, mask, 'same')" for the smoothing.

Generate another blurred image by using "mask = fspecial ('gaussian', 51, 5)".

• Show the 3 different versions of the Shepp-Logan images, say S_0 (unblurred), S_1, and S_5.

For these 3 versions S_0, S_1, S_5, first compute the radon transform with $\theta = 0, 3, 6, \cdots, 177$ degrees, then apply the Ram-Lak filter with $L = w_{\text{max}}$, followed by the (unfiltered) backprojection.

• Show the 3 Ram-Lak-filtered backprojections, say, R_0, R_1, R_5.

• Also, compute the relative root-mean-squared errors (RRMSE) RRMSE(S_i, R_i), where the RRMSE for 2 images A and B is defined as:

RRMSE$(A,B) = \sqrt{\sum_p (A(p) - B(p))^2} / \sqrt{\sum_p A(p)^2}$, where the summation is over all pixels p. In which of the 3 cases is the RRMSE the highest and the lowest ? Justify theoretically.

(c) (4 points)

For each of the 3 examples (i.e., Shepp-Logan phantoms S_0, S_1, S_5 and their Ram-Lak-filtered backprojections R_0, R_1, R_5), plot the RRMSE values as a function of L with $L = w_{\text{max}}/50, 2w_{\text{max}}/50, \cdots, w_{\text{max}}$. Explain your findings.

3. **(10 points) X-Ray Computed Tomography: Incomplete Data.**

   Load ground-truth images from `chestCT.mat` and `myPhantom.mat`

   Consider a CT imaging scenario where data can be acquired over 151 angles than span over 150 degrees (not 180 degrees), in increments of 1 degree. That is, you can acquire data at the angles $[\theta, \theta+1, \cdots, \theta+150]$ degrees, for any given/fixed θ.

   (a) (5 points)

   For each dataset, plot the RRMSE values between the ground-truth images and the reconstructed images for $\theta \in [0, 1, \cdots, 180]$.

   (b) (5 points)

   For each dataset, show the reconstructed image for that value of θ that gives the least RRMSE.

   For each of the given two datasets, which contiguous set of 151 angles will you acquire the data to minimize the error in reconstruction ?

4. **(12 points) Algebraic Reconstruction Technique (ART).**

   Load ground-truth images from `chestCT.mat`

   Consider a CT imaging scenario where data is acquired over 180 angles that span over 180 degrees, in increments of 1 degree. That is, you can acquire data at the angles $[0, 1, \cdots, 179]$ degrees.

   Consider that the acquired Radon-transform data is corrupted with noise that is approximately additive i.i.d. zero-mean Gaussian noise with a standard deviation that is 5% of the intensity range in the data.

   (a) (7 points)

   Implement an ART algorithm for reconstruction, starting with an initial estimate of the reconstructed image as having all zero values. Call your function as **myART()**. Your function should be able to follow various orderings of using the projection data, which is specified as an argument (as specified in the next question).

   You function should construct the imaging (radon-transform / integration) matrix from scratch, i.e., without using a library function for the radon transform, similar what you did in first question in this assignment.

   (b) (5 points: 2 + 3)

   For any specific ordering and a choice of step-length λ (as described in the lecture slides), across iterations i, write code to plot the graph of (i) the RRMSE between the reconstructed image and the ground truth for iteration i versus (ii) the iteration number i.

   For a fixed ordering, plot the aforementioned graphs for λ varying from 0.1 to 1 in steps of 0.1.