CS 736

Assignment - 4

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Overview

- **Objective:** Kernel Methods and Shape Analysis
- **Tasks:**
  1. Shape Analysis on Human Hand Shapes
  2. Shape Analysis on Human Cardiac Shapes
  3. Robust Shape Mean
  4. Kernel PCA to Model Variation in Object Segmentations

Task 1

Objective

Shape Analysis on Human Hand Shapes

Description

- Implement two alignment functions, first Code11 which assumes that the pointsets are pre-processed into pre-shape space (centered and scaled), so only a rotation needs to be computed and Code2 which uses the approach from [Cootes et al. 1995 CVIU] to jointly solve for rotation, scale, and translation.
- Develop two methods for computing the optimal shape mean iteratively.
- Visualizing & Plotting the required images.

Task 1

Approach

- Load the `hands2D.mat` dataset which contains multiple 2D hand shapes.
- Plot all initial raw shapes in a single graph using random colors for visual comparison.
- We used Orthogonal Procrustes Analysis to align shapes using rotation only, by solving for the optimal rotation matrix via SVD in the pre-shape space.
- Aligned shapes using full similarity transform (scale, rotation, translation) via SVD as done by Cootes et al. (1995).

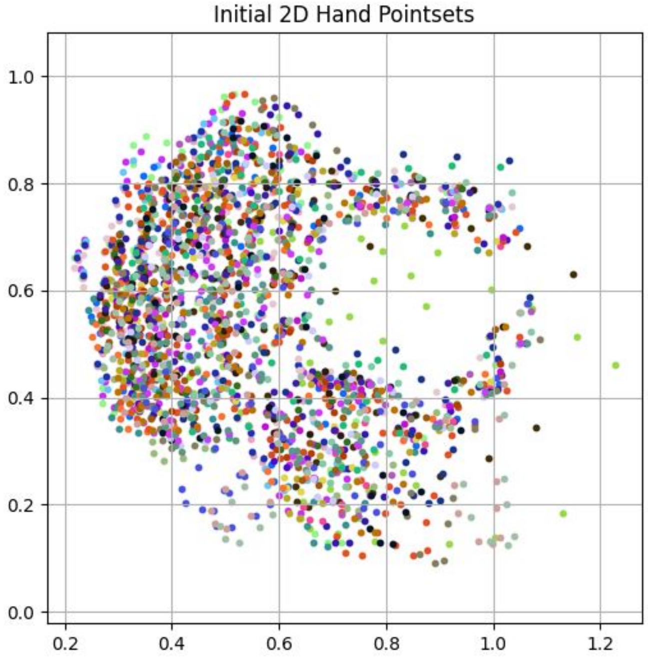
Task 1

Approach

- Computed optimal mean shape iteratively by aligning with rotation only (Code1) and updating the mean in pre-shape space.
- Estimated mean shape using full similarity alignment (Code2), updating iteratively with scale, rotation, and translation.
- Performed PCA on aligned shape vectors to extract principal modes of variation using eigen-decomposition of the covariance matrix.
- **PCA with modes**: Flatten aligned shapes, compute mean vector and covariance, then extract principal modes via eigen decomposition.
- **Visualize top shape variations** using $\pm k \times$ std $\times$ eigenvectors from PCA around the mean shape.

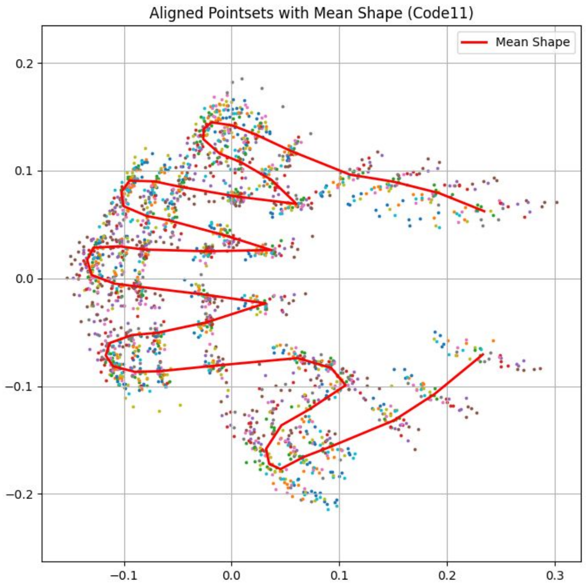
Result

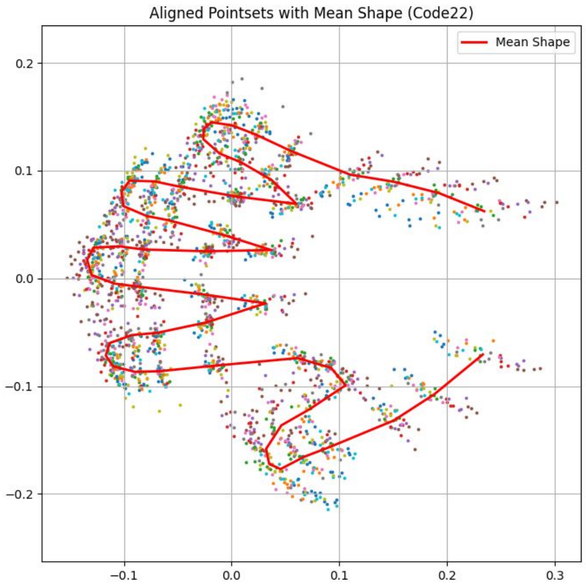
1(a) Plot of Initial Estimate



Result

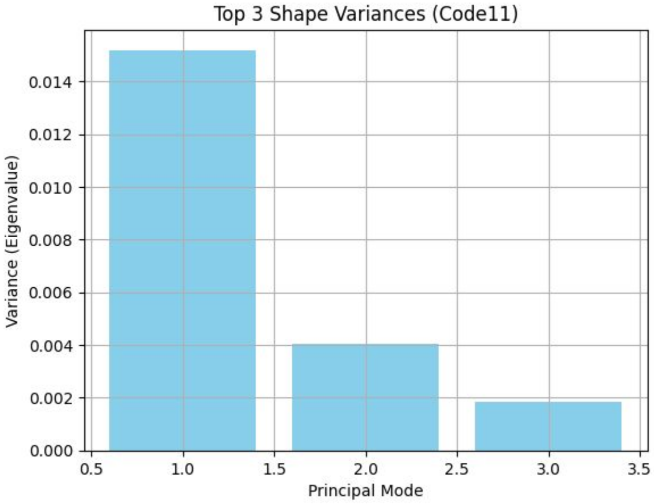
1(b) Plots of computed shape mean with aligned pointsets

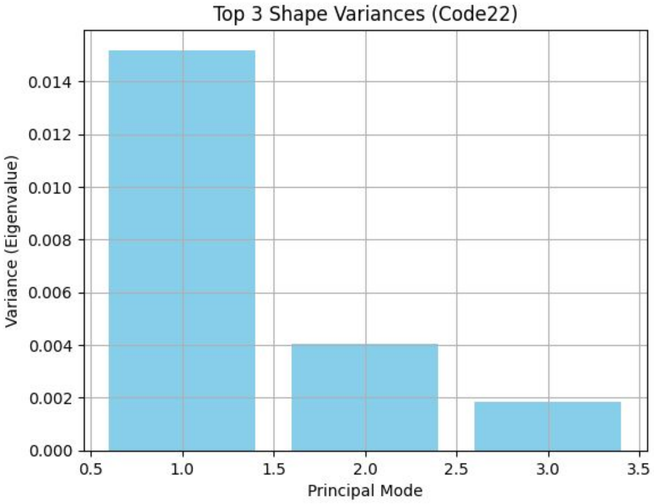




Result

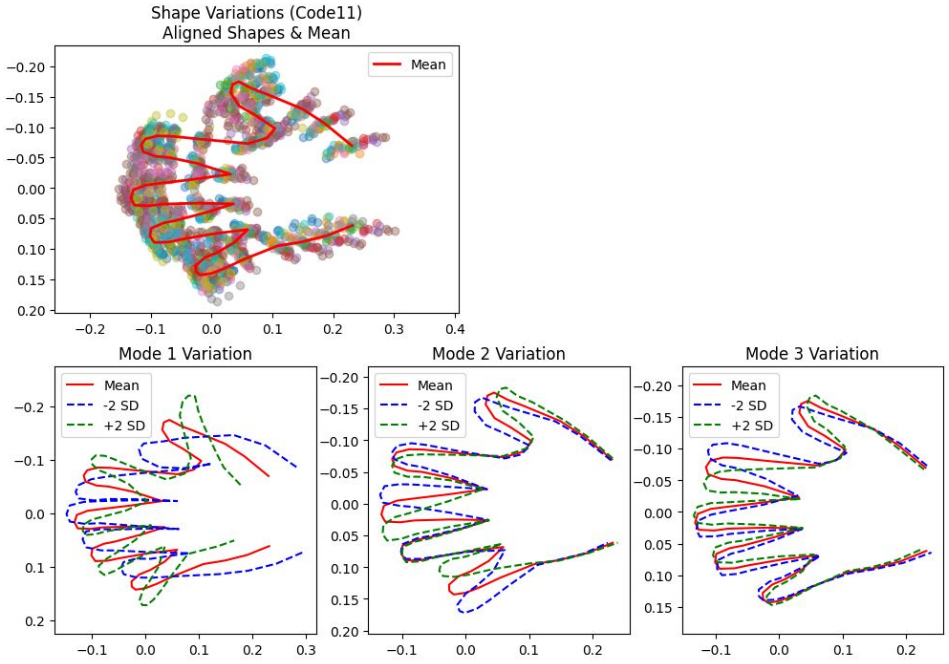
1(c) Plots of the variances





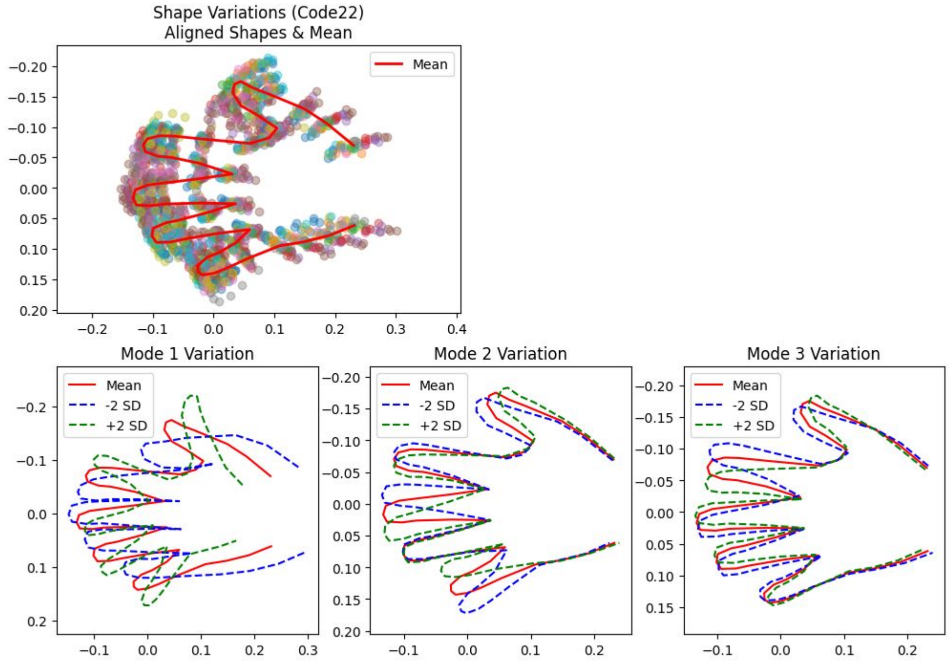
Result

1(d) Shape variation around mean (Code11)



Result

1(d) Shape variation around mean (code 22)



Task 2

Objective

Shape Analysis on Human Cardiac Shapes

Description

- For each image from the given data, extract pointsets for the inner and outer boundaries (the ring) using contour detection and combine them.
- Repeat the shape analysis from the first question: compute the shape mean and principal modes of variation based on these pointsets.
- Compute and display the shape mean and aligned pointsets using two methods: Code11 (pre-shape, rotation only) and Code22 (joint scale, translation, and rotation).
- Perform PCA on the aligned shapes and plot the images wherever mentioned.

Task 2

Approach

- Mostly same as Task 1
- First extract the data from the zip folder and show the plot of initial pointset with random color
- Every image is resized to a uniform 256x256 pixel grid. This step ensures all shapes are on the same scale, making it easier to compare them later and avoiding issues from different image resolutions.
- Convert each image into black and white and check the pixel brightness so be distinguish between ring and background

Task 2

Approach

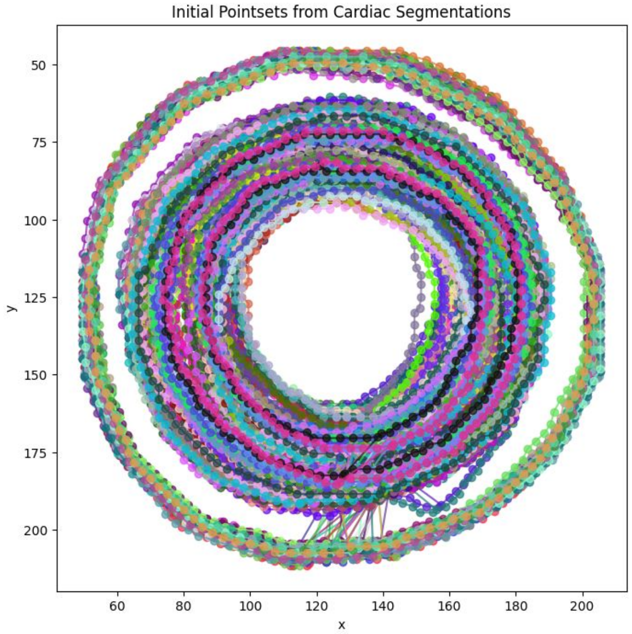
- Make a function called extract_ring_pointset to find the edge of the ring in each binary image. Look for where the image switch from black to white.
- The plot references aligned_code22, suggesting Procrustes analysis to align point sets. The alignment minimizes:

$$E = \sum_{i=1}^{M} \left\| P_i - (sRP^{\text{mean}} + t) \right\|^2$$

- Add two extra shapes for each of the top 3 r ne shape changes a few steps above and below the mean (e.g., 2-3 standard deviations).

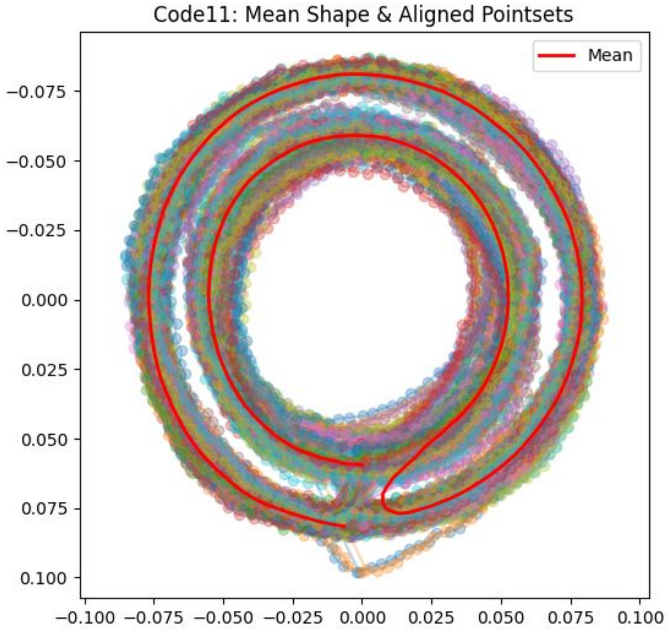
Result

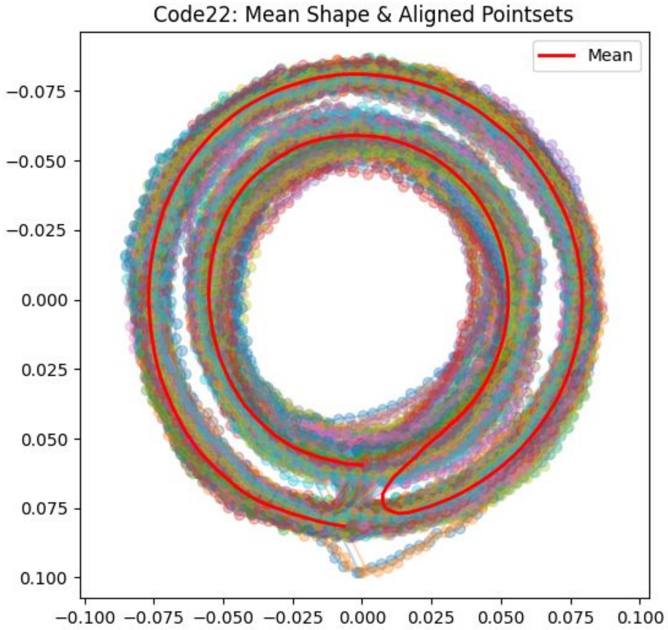
2(b) Generation of pointsets corresponding to inner and outer boundaries



Result

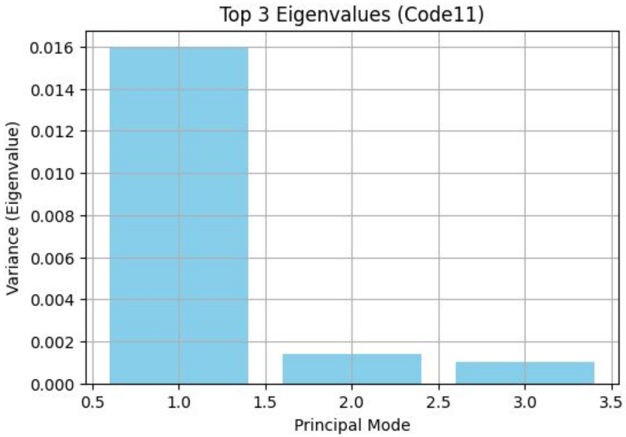
2(c) Computed shape mean, together with all the aligned pointsets

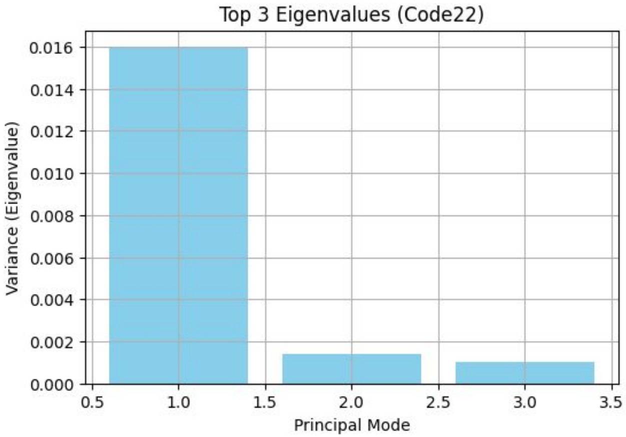




Result

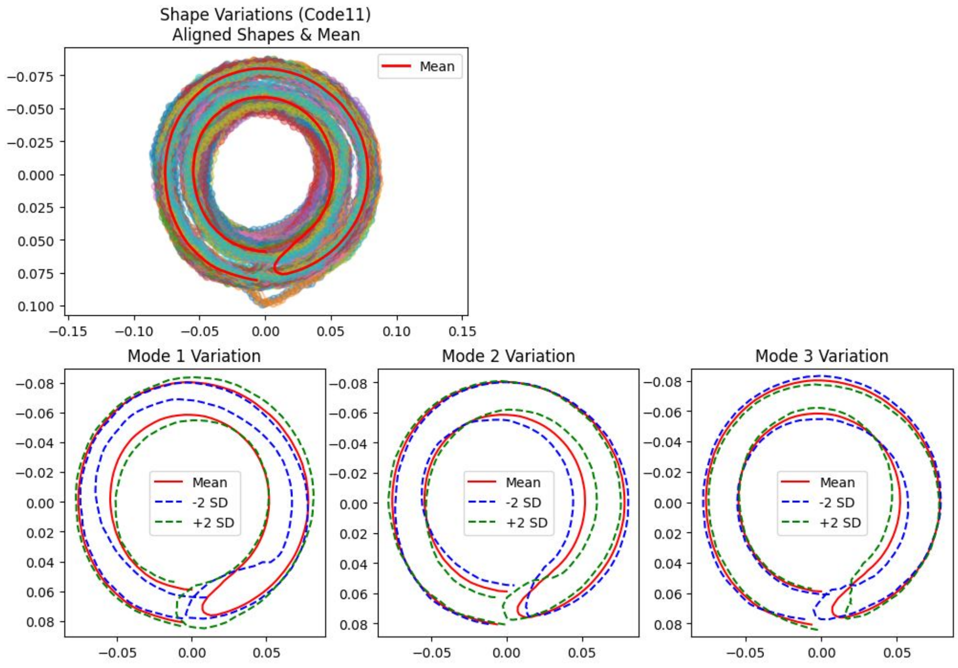
2(d) Top 3 eigenvalue:





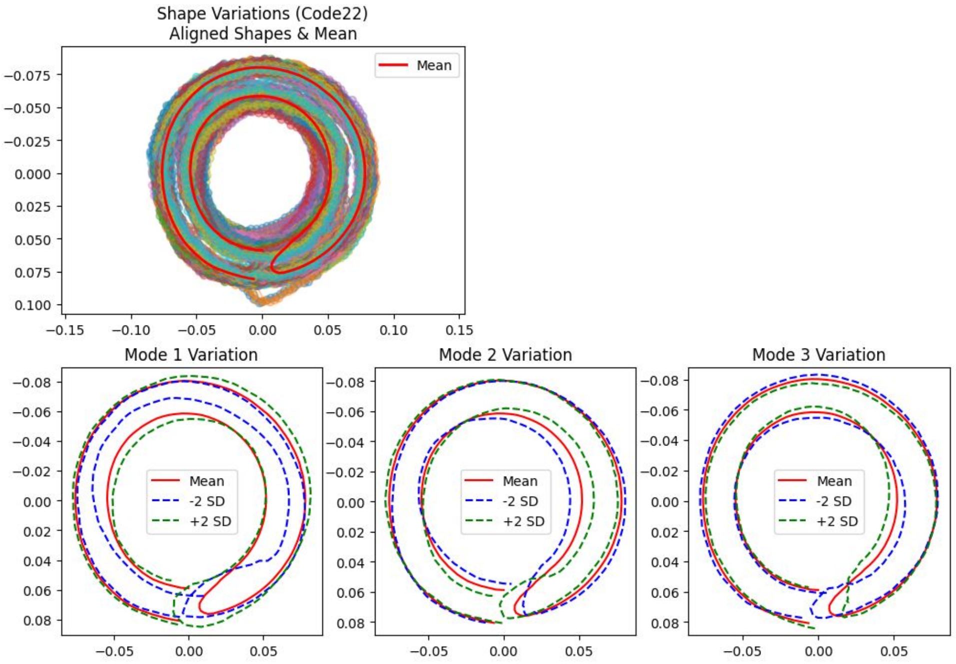
Result

2(e) Shape Variation around mean (Code11)



Result

2(e) Shape Variation around mean (Code22)



Task 3

Objective

Robust Shape Mean

Description

- Formulate an optimization problem minimizing the sum of squared Procrustes distances to compute the mean shape.
- Implement this method to compute and display the original pointsets, the estimated mean, and all pointsets aligned to the mean.
- Further, formulate a separate optimization problem minimizing the sum of (non-squared) Procrustes distances for enhanced robustness.
- Implement the robust method, then display the original pointsets, the robustly estimated mean, and all aligned pointsets.

Task 3

Approach

- Loaded the dataset and for each pointset and compute the centered and normalized version.
- Measure dissimilarity using the Procrustes distance by aligning shapes via the optimization: $\min_{s,R} \|Z - s\,X\,R\|_F^2$
  Where R is obtained by SVD of $A=X^TZ$ and the optimal scaling is $s = \mathrm{trace}(R^TA)$.
- Formulated the problem to estimate the mean shape Z by minimizing:
  $J(Z) = \sum_{m=1}^{M} \|Z - s_m\,X^{(m)}\,R_m\|_F^2$ Where $\mu(Z) = 0$ and $\|Z\|_F = 1$.
- For each iteration, aligned each shape to the current mean using rotation only (Code1), then update the mean as:

$$Z_{\text{new}} = \text{normalize}\left(\frac{1}{M}\sum_{m=1}^{M} s_m\,X^{(m)}\,R_m\right)$$

Task 3

Approach

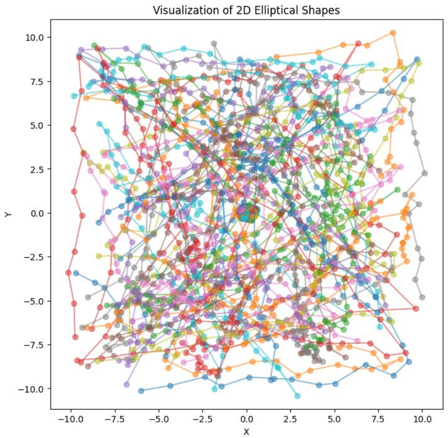
- To reduce outlier sensitivity, minimized the sum of unsquared distances:
  $$J(Z) = \sum_{m=1}^{M} \|Z - s_m X^{(m)} R_m\|_F$$
  Using weights $w_m = \frac{1}{\epsilon + \|Z - s_m X^{(m)} R_m\|_F}$ so that $Z_{\text{new}} = \text{normalize}\left(\frac{\sum_{m=1}^{M} w_m s_m X^{(m)} R_m}{\sum_{m=1}^{M} w_m}\right)$
- Plotted the original (centered & normalized) pointsets, the computed mean Z, and all aligned shapes (overlaid) for each method.

Result

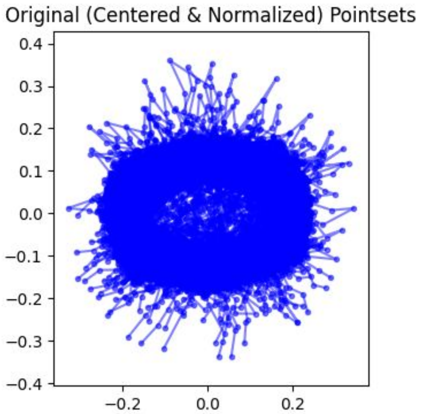
(a) Designed Optimization Problem *(sum of squared Procrustes distances)*

$$\min_{Z,\{s_m,R_m\}} \sum_{m=1}^{M} \|Z - s_m R_m X^{(m)}\|_F^2 \quad \text{Subject to} \quad \sum_{n=1}^{N} z_n = 0 \quad \text{and} \quad \|Z\|_F = 1$$

Original Pointsets:



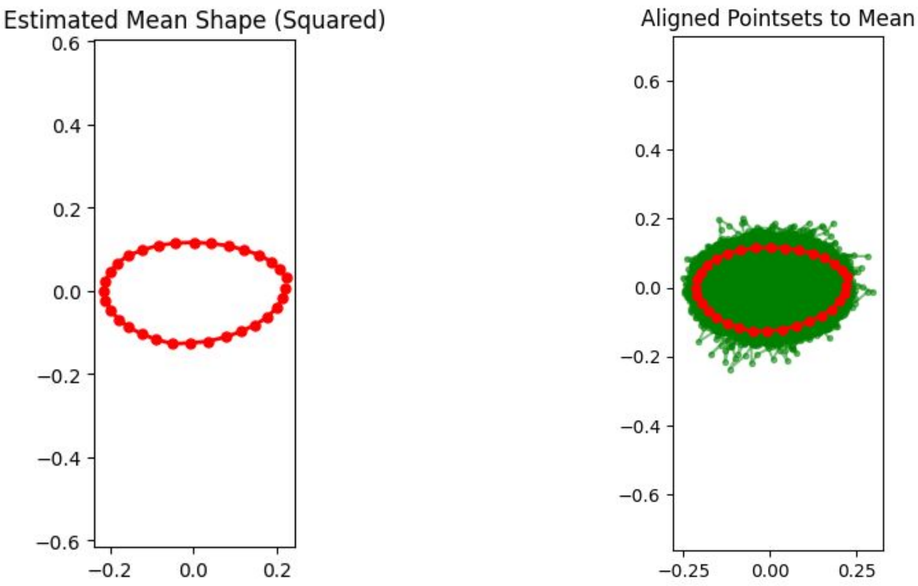
Before Centering and Normalization



After Centering and Normalization

Result

(a)

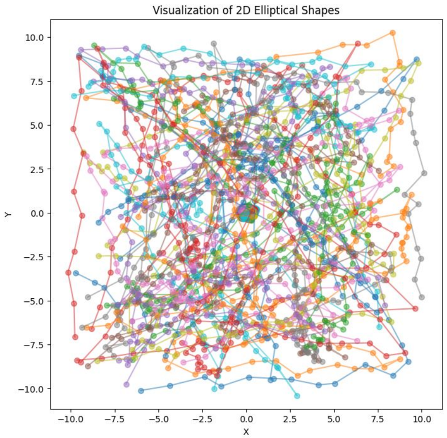

Estimated Mean Shape (Squared)
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
−0.2
0.0
0.2
Aligned Pointsets to Mean
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
−0.25
0.00
0.25

Result

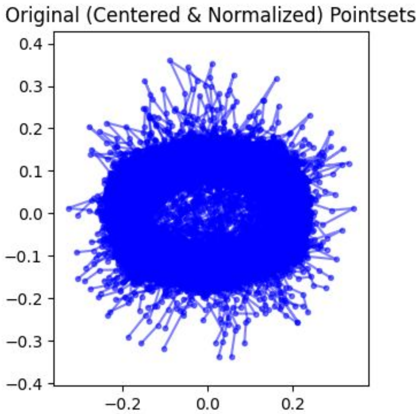
(b) Designed Optimization Problem *(sum of squared Procrustes distances)*

$$\min_{Z,\{s_m,R_m\}} \sum_{m=1}^{M} \|Z - s_m R_m X^{(m)}\|_F \text{ subject to } \sum_{n=1}^{N} z_n = 0 \text{ and } \|Z\|_F = 1.$$

Original Pointsets:



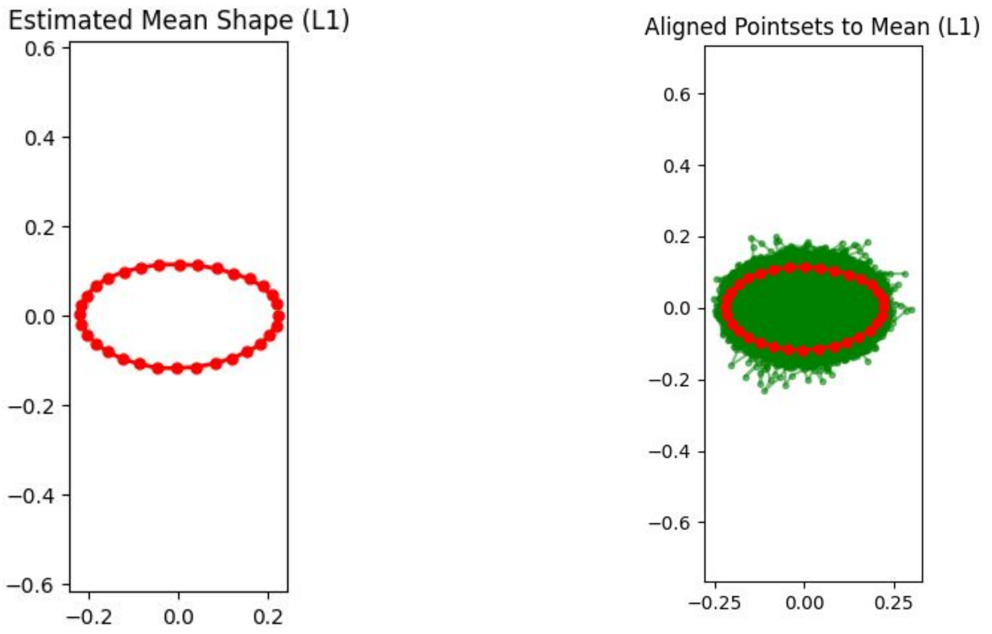
Before Centering and Normalization



After Centering and Normalization

Result

(a)


Estimated Mean Shape (L1)
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
−0.2
0.0
0.2
Aligned Pointsets to Mean (L1)
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
−0.25
0.00
0.25

Task 4

Objective

Kernel PCA to Model Variation in Object Segmentations

Description

- Perform PCA on vectorized segmentation images; display the eigen spectrum, mean image, and first 2 principal modes of variation as images.
- Implement kernel PCA using a Gaussian kernel; visualize the eigen spectrum and estimate the pre-image closest to the RKHS mean.
- For distorted segmentations, project onto the first 3 PCA modes and display the reconstructed (projected) images.
- Similarly, project distorted images into RKHS using kernel PCA, and compute their pre-images for visualization.

Task 4

Approach

- Resized each segmentation image to 64×64, normalized pixel values to [0,1] and vectorized each image as $x \in R^{4096}$.
- Centered data by $\mu = \frac{1}{M}\sum_{i=1}^{M} x_i$ and computed covariance as as $C = \frac{1}{M}\sum_{i=1}^{M}(x_i - \mu)(x_i - \mu)^T$ and then performed eigen decomposition.
- Plotted the eigen spectrum, displayed the mean image (μ reshaped to 64×64), and showed the first two principal modes.
- Compute the Gaussian kernel k(x,y)=exp(−γ // x−y // ^{2}) to form the matrix K, centered it, and performed eigen-decomposition on it.
- Estimated the pre-image x^* by iteratively updating: $x^{(t+1)} = \frac{\sum_i k(x^{(t)}, x_i) x_i}{\sum_i k(x^{(t)}, x_i)}$ Starting from the arithmetic mean of the training set.

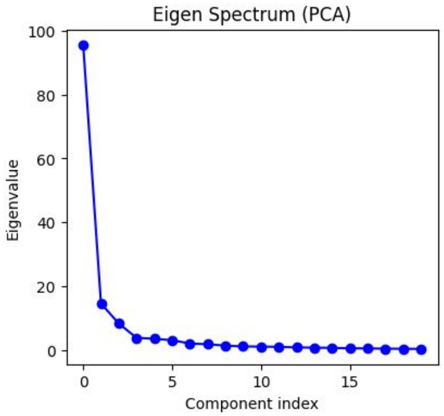
Task 4

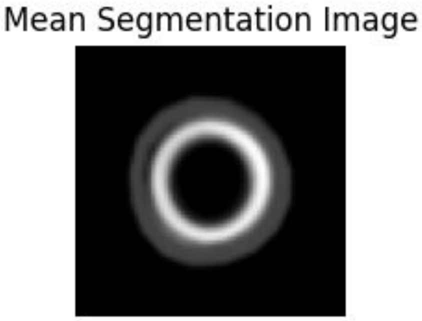
Approach

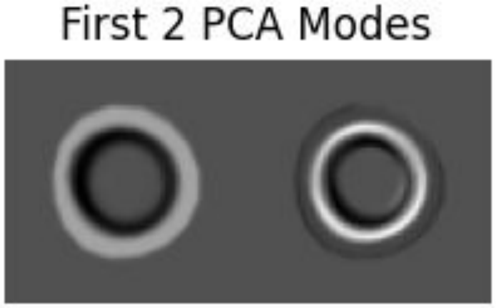
- For a test image x, computed the coefficients $c_j = v_j^T (x-\mu)$ and reconstructed as
$$x_{\text{recon}} = \mu + \sum_{j=1}^{3} c_j v_j$$
- For each test image, computed and center its kernel vector, project onto the top 3 kernel eigenvectors, then estimate its pre-image using the iterative scheme.
- Finally plotted the eigen spectra, mean images, PCA reconstructions versus originals, and kernel PCA pre-image reconstructions.

Result

(a) Eigen Spectrum, Mean Image and the first 2 modes of variation around the mean

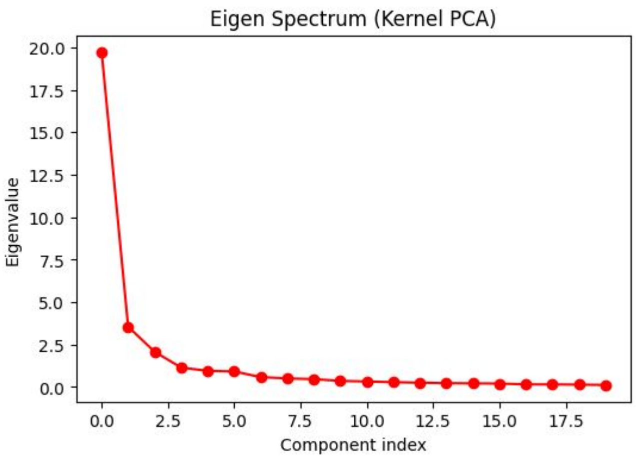


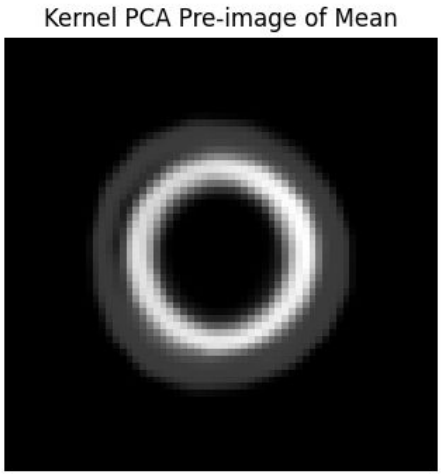




Result

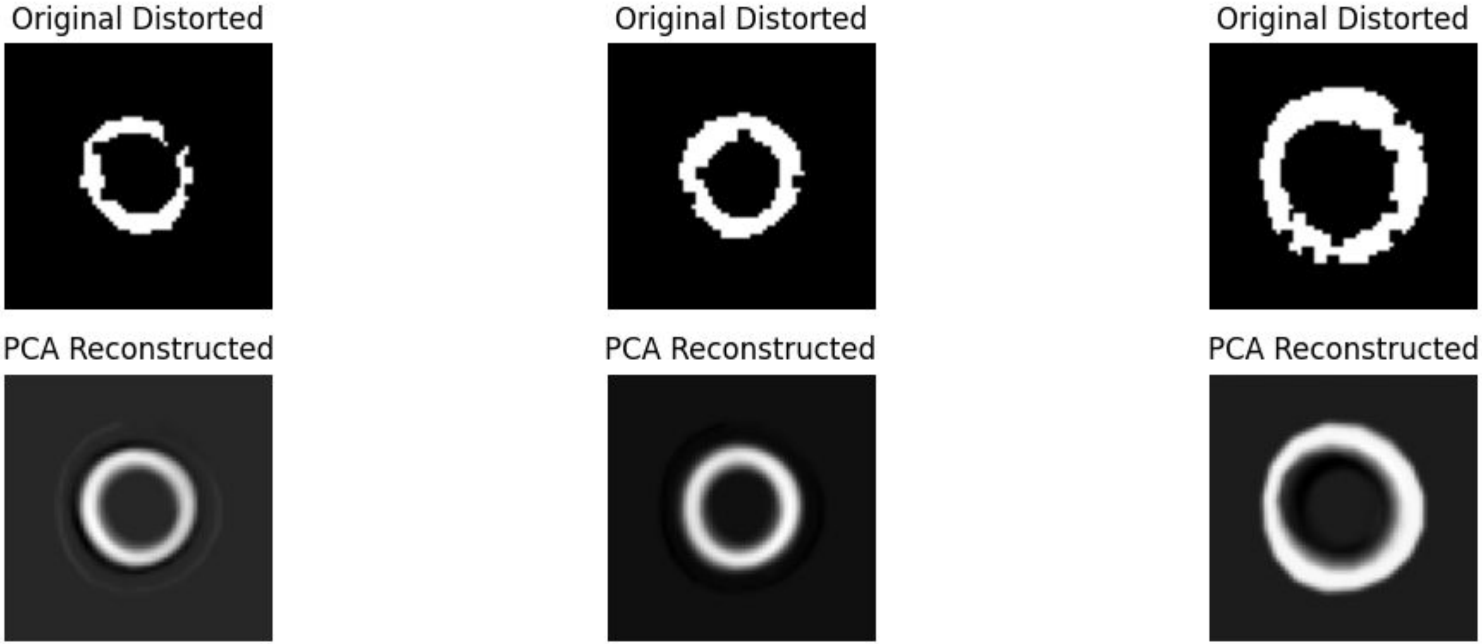
(b) Eigen Spectrum, Kernel PCA pre-image of the mean





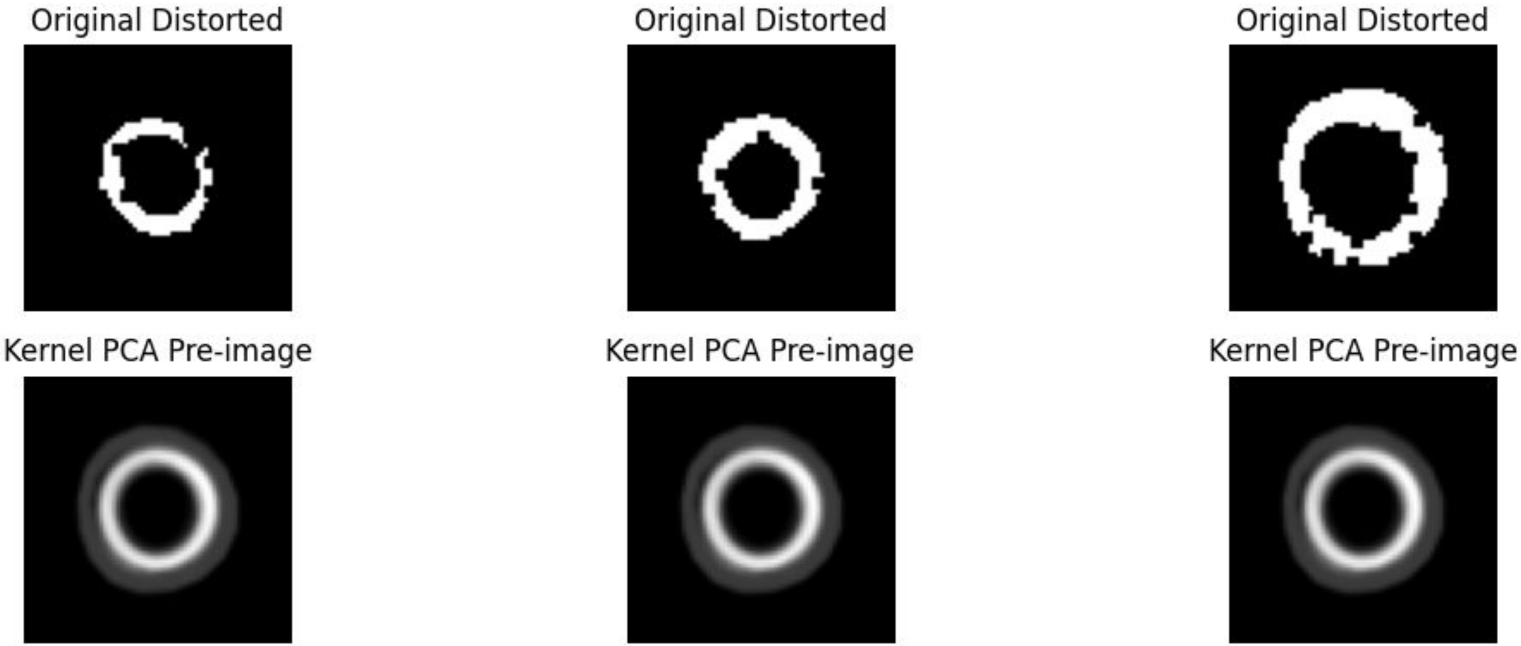
Result

(c) PCA and projected segmentation image



Result

(c) Kernel PCA



Thank You!