(1) Solve $[y(1+1/x)+\cos y]dx+(x+\log x-x\sin y)dy=0$

(2) Solve $(x^3y^3+x^2y^2+xy+1)y\,dx+(x^3y^3-x^2y^2-xy+1)x\,dx=0$

(3) Solve $(y^4+2y)dx+(xy^3+2y^4-4x)dy=0$

(4) Solve $x\frac{dy}{dx}+2y-x^2\log x=0$

(5) Solve $(1-x^2)\frac{dy}{dx}+2xy-x\sqrt{1-x^2}$

(6) Solve $(x+2y^3)\frac{dy}{dx}=y.$

(7) Solve $\frac{dy}{dx}(x^2y^3+xy)=1$

(8) Solve $\sec^2 y\frac{dy}{dx}+2x\tan y=x^3$

(2) Given equ :- $(x^3y^3+x^2y^2+xy+1)\,y\,dx+(x^3y^3-x^2y^2-xy+1)\,x\,dy=0$ —(1)

comparing equ-(1) with $M\,dx+N\,dy=0$

$M=(x^3y^3+x^2y^2+xy+1)y$ $\qquad$ $N=(x^3y^3-x^2y^2-xy+1)$

$x^3y^4+x^2y^3+xy^2+y$ $\qquad$ $=x^4x^3-x^3y^2-x^2y+x$

$\frac{\partial M}{\partial y}=4x^3y^3+3x^2y^2+2xy+1$ $\qquad$ $\frac{\partial N}{\partial x}=4x^3y^3-3x^2y^2-2xy+1$

$$\frac{\partial M}{\partial y}\neq\frac{\partial N}{\partial x}.$$

equ is not exact differential equation.

Now; $Mx-Ny=(x^3y^4+x^2y^3+xy^2+y)x-(x^4y^3-x^3y^2-x^2y+x)y$

$=x^4y^4+x^3y^3+x^2y^2+xy-x^4y^4+x^3y^3+x^2y^2-xy$

$=2x^3y^3+2x^2y^2$

$=2(x^3y^3+x^2y^2)$

I.f is $\frac{1}{Mx-Ny}=\frac{1}{2(x^3y^3+x^2y^2)}=\frac{1}{2x^2y^2(xy+1)}$

from eq-(1)

$(x^3y^3+x^2y^2+xy+1)y\,dx+(x^3y^3-x^2y^2-xy+1)x\,dy=0$

$[xy(x^2y^2+1)+(x^2y^2+1)]y\,dx+[xy(x^2y^2-1)-(x^2y^2-1)]x\,dy=0$

$[(xy+1)(x^2y^2+1)]y\,dx+[(xy-1)(x^2y^2-1)]x\,dy=0$

Now, multiply above equation with I.f

$$\left[\frac{(xy+1)(x^2y^2+1)}{2x^2y^2(xy+1)}\right]dx+\left[\frac{(xy-1)(xy-1)(xy+1)x}{2x^2y^2\cdot(xy+1)}\right]dy=0$$

$$\frac{x^2y^2+1}{2x^2y}\,dx+\frac{(xy-1)^2}{2xy^2}\,dy=0$$

$$\left(\frac{x^2y^2}{2x^2y}+\frac{1}{2x^2y}\right)dx+\left(\frac{x^2y^2+1-2xy}{2xy^2}\right)dy=0$$

$$\left(\frac{y}{2}+\frac{1}{2x^2y}\right)dx+\left(\frac{x^2y^2+1-2xy}{2xy^2}\right)dy=0$$

$$\left(\frac{y}{2}+\frac{1}{2x^2y}\right)dx+\left(\frac{x^2y^2}{2xy^2}+\frac{1}{2xy^2}-\frac{2xy}{2xy^2}\right)dy=0$$

$$\left(\frac{y}{2}+\frac{1}{2x^2y}\right)dx+\left(\frac{x}{2}+\frac{1}{2xy^2}-\frac{1}{y}\right)dy=0 \quad \text{—— (2)}$$

comparing with $M_1dx+N_1dy=0$.

$$M_1=\frac{y}{2}+\frac{1}{2x^2y} \quad ; \quad N_1=\frac{x}{2}+\frac{1}{2xy^2}-\frac{1}{y}$$

$$\frac{\partial M_1}{\partial y}=\frac{1}{2}+\frac{1}{2x^2}\left(\frac{-1}{y^2}\right) \qquad \frac{\partial N_1}{\partial x}=\frac{1}{2}+\frac{1}{2y^2}\left(\frac{-1}{x^2}\right)$$

$$\frac{\partial M_1}{\partial y}=\frac{\partial N_1}{\partial x}.$$

equ-(2) is exact differential equation.

General solution is

$\int M_1dx+\int N_1dy$ (not containing terms of x) $=c$
y con

$$\int \frac{y}{2}+\frac{1}{2x^2y}\cdot dx+\int -\frac{1}{y}dy=c$$

$$\frac{y}{2}\int dx+\frac{1}{2y}\int\frac{1}{x^2}dx+\int -\frac{1}{y}dy=c$$

$$\frac{y}{2}\cdot x+\frac{1}{2y}\left(-\frac{1}{x}\right)-\log y=c$$

$$\frac{xy}{2}-\frac{1}{2xy}-\log y=c$$

$$xy-\frac{1}{xy}-2\log y=2c$$

$$xy-\frac{1}{xy}-\log y^2=2c$$

(3) Given equation $(y^4+2y)dx + (xy^3+2y^4-4x)dy = 0$ —— (1)

compare with $Mdx + Ndy = 0$

$M = y^4+2y$; $N = xy^3+2y^4-4x$

$\frac{\partial M}{\partial y} = 4y^3+2$ $\qquad$ $\frac{\partial N}{\partial x} = y^3-4$

$$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}.$$

equation -(1) is not exact differential equation.

find out ; $\frac{1}{M}\left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) = \frac{1}{y^4+2y}(y^3-4-4y^3-2)$

$= \frac{1}{y^4+2y}(-3y^3-6)$

$= \frac{1}{y(y^3+2)}[-3(y^3+2)]$

$= -3/y \Rightarrow g(y)$

Now I.f is $e^{\int g(y)dy} = e^{\int -3/y\, dy} = e^{-3\log y}$

$= e^{\log y^{-3}}$

$= y^{-3}$

$= 1/y^3$

multiply eq -(1) with I.f

$\int 1/y^3 (y^4+2y)dx + 1/y^3 (xy^3+2y^4-4x)dy = 0$

$(y + 2/y^2)dx + (x+2y-\frac{4x}{y^3})dy = 0$ —— (2)

$M_1 = y+2/y^2$; $N_1 = x+2y-\frac{4x}{y^3}$

$\frac{\partial M_1}{\partial y} = 1+2(-2y^{-3})$ $\qquad$ $\frac{\partial N_1}{\partial y} = 1-4\cancel{x}/y^3$

$= 1-4y^{-3}$ $\qquad$ $= 1-4/y^3$

$$\frac{\partial M_1}{\partial y} = \frac{\partial N_1}{\partial x}$$

equation-(2) is exact differential equation.

General solution is

$\int M_1 dx + \int N_1 dy$ (not containing terms of x) $= c$

y con

$$\int (y + 2/y^2)\, dx + \int 2y\, dy = c$$

y con

$$y \int dx + \frac{2}{y^2} \int dx + 2\int y\, dy = c$$

$$xy + \frac{2x}{y^2} + 2\left(\frac{y^2}{2}\right) = c$$

$$x(y + 2/y^2) + y^2 = c$$

(4) Given equation : $x\frac{dy}{dx} + 2y - x^2 \log x = 0.$

$$x\frac{dy}{dx} + 2y = x^2 \log x$$

$$\frac{dy}{dx} + \frac{2y}{x} = x \log x$$

clearly, it is in the form of $\frac{dy}{dx} + P(x)y = Q(x)$

$$P(x) = 2/x \quad ; \quad Q(x) = x \log x$$

Now $\int P(x)dx = \int 2/x\, dx = 2\log x$

$\therefore$ I.f is $e^{\int P(x)dx} = e^{2\log x} = e^{\log x^2} = x^2$

General solution $y(\text{I.f}) = \int Q(x)(\text{I.f})dx + c$

$$y(x^2) = \int (x \log x)(x^2)\, dx + c$$

$$\int x^3 \log x\, dx + c$$

$$= \frac{x^4 \log x}{4} - \frac{x^2}{16} + c.$$

$$y(x^2) = \frac{x^4}{4} \cdot (\log x - 1/4) + c$$

(5) Given equation : $\frac{dy}{dx} + \frac{2xy}{(1-x^2)} y = \frac{x\sqrt{1-x^2}}{(1-x^2)}$

$$P(x) = \frac{2x}{1-x^2} \quad ; \quad Q(x) = \frac{x\sqrt{1-x^2}}{(1-x^2)}$$

Now find out $\int P(x)dx = -\int \frac{-2x}{1-x^2} \cdot dx$

$$= -\log(1-x^2)$$

I.f is $e^{\int P(x)dx} = e^{-\log(1-x^2)} = e^{\log(1-x^2)^{-1}}$

$$= (1-x^2)^{-1}$$

$$= \frac{1}{1-x^2}$$

General solution is $y(I.f) = \int Q(x)(I.f)\,dx + c$

$$y\left(\frac{1}{1-x^2}\right) = \int \frac{x\sqrt{1-x^2}}{1-x^2} \cdot \frac{1}{1-x^2}\, dx + c$$

$$= \int \frac{x(1-x^2)^{1/2}}{(1-x^2)^2}\, dx + c$$

$$= \int \frac{x}{(1-x^2)^2 \cdot (1-x^2)^{-1/2}}\, dx + c$$

$$\int \frac{x}{(1-x^2)^{3/2}} \cdot dx + c$$

Put $1-x^2 = t$

dif. w.r.t 'x'

$$-2x\,dx = dt$$

$$x\,dx = -1/2\, dt$$

$$y\left(\frac{1}{1-x^2}\right) \Rightarrow \int \frac{-1/2\, dt}{t^{3/2}} + c \Rightarrow -\frac{1}{2}\int \frac{1}{t^{3/2}}\, dt + c \Rightarrow -\frac{1}{2}\int t^{-3/2}\, dt + c$$

$$\Rightarrow -\frac{1}{2} \cdot \frac{t^{-3/2+1}}{-3/2+1} + c \Rightarrow -\frac{1}{2} \cdot \frac{t^{-1/2}}{-1/2} + c \Rightarrow t^{-1/2} + c$$

$(1-x^2)^{-1/2}+c$

$$y\left(\frac{1}{1-x^2}\right)=\frac{1}{\sqrt{1-x^2}}+c$$

(6) Given equation :- $y\cdot\frac{dx}{dy}=x+2y^3$

$$y\frac{dx}{dy}-x=2y^3$$

$$\frac{dx}{dy}-\frac{1}{y}\cdot x=2y^2$$

Now find out $P(y)=\frac{-1}{y}$; $Q(y)=2y^2$

$$\int P(y)dy=-\int \frac{1}{y}\,dy=-\log y$$

I.f is $e^{\int P(y)dy}=e^{-\log y}=e^{\log y^{-1}}$

$$=y^{-1}=\frac{1}{y}$$

General solution is $x(\text{I.f})=\int Q(y)(\text{I.f})\,dy+c$

$$x(1/y)=\int 2y^2\cdot\frac{1}{y}\,dy+c$$

$$x/y=2\int y\,dy+c$$

$$=2\cdot\frac{y^2}{2}+c$$

$$x/y=y^2+c$$

$$x=y^3+cy.$$

(7) Given equation :- $\frac{dy}{dx}(x^2y^3+xy)=1$

$$x^2y^3+xy=\frac{dx}{dy}$$

$$x^2y^3=\frac{dx}{dy}-xy$$

$$\frac{dx}{dy}-xy=x^2y^3$$

$$\frac{dx}{dy}-y\cdot x=y^3\cdot x^2 \quad\text{— (1)}$$

Multiplying with x^{-2}

$$x^{-2}\cdot\frac{dx}{dy}-y\cdot x^{-1}=y^3$$

Put $x^{-1}=u$

$$-1\cdot x^{-2}\cdot\frac{dx}{dy}=\frac{du}{dy}$$

$$x^{-2}\cdot\frac{dx}{dy}=\frac{-du}{dy}$$

from -(1)

$$-\frac{du}{dy}-y\cdot u=y^3$$

$$\frac{du}{dy}+y\cdot u=-y^3$$

comparing with $\frac{dx}{dy}+P(y)x=Q(y)$

find out

$$P(y)=y\ ;\ Q(y)=-y^3$$

$$\int P(y)dy=\int y\,dy=y^2/2$$

I.f is $e^{\int P(y)dy}=e^{y^2/2}$

G.S is $u(\text{I.f})=\int Q(y)(\text{I.f})dy+c$

$$u(e^{y^2/2})=\int -y^3(e^{y^2/2})dy+c$$

$$=-\int y^3\cdot e^{y^2/2}dy+c$$

Put $t=y^2/2 \Rightarrow 2t=y^2$

$$dt=\frac{2y}{2}dy$$

$y\,dy = dt$

$u(e^{y^2/2}) = -\int y \cdot y^2 \cdot e^{y^2/2}\, dy + c$

$= -\int 2t \cdot e^t\, dt + c$

$= -2\int t\, e^t\, dt + c$

$-2 \cdot (e^t)(t-1)\, dt + c$

$\frac{1}{x} \cdot e^{y^2/2} = -2e^{y^2/2}\left(\frac{y^2}{2} - 1\right) + c$

(8) Given equation :- $\sec^2 y \frac{dx}{dy} + 2x \cdot \tan y = x^3$ —— (1)

Put $\tan y = u$

diff. w.r.t 'x'

$\sec^2 y \frac{dy}{dx} = \frac{du}{dx}$

from —(1)

$\frac{du}{dx} + 2 \cdot x \cdot u = x^3$

It is linear differential equation in "u"

$P(x) = 2x$; $Q(x) = x^3$

find out

$\int P(x)\,dx = \int 2x\,dx = 2 \cdot x^2/2 = x^2$

$e^{\int P(x)dx} = e^{x^2}$

General solution is $u(I.f) = \int Q(x)(I.f)\,dx + c$

$u(e^{x^2}) = \int x^3 (e^{x^2})\,dx + c$

$= \int x \cdot x^2 \cdot e^{x^2}\,dx + c$

Put $x^2 = t$

$2x\,dx = dt$

$x\,dx = 1/2\,dt$. $= \frac{1}{2}[e^t(t-1)] = 1/2\,[e^{x^2}(x^2-1)]$

(1) Solve $(D^3-2D^2-5D+6)y=0$ when $x=0, y=1, y'=-7, y''=-1$

(2) Solve $(D^2-3D+2)y=\cosh x$

(3) Solve $\frac{d^2y}{dx^2}-(a+b)\frac{dy}{dx}+aby=e^{ax}+e^{bx}$

(4) Solve $(D^2+a)y=\cos^3 x.$

(5) Solve $(D^2-4D+13)y=\sin 3x\cdot\cos 2x$

(6) Solve $\frac{d^2y}{dx^2}+4y=e^x+\sin 2x+\cos 2x$

(7) Solve $(D^2-4)y=e^x+\sin 2x+\cos^2 x.$

(1) Given equation $(D^3-2D^2-5D+6)y=$

Auxiliary Equation $f(m)=0$

$m^3-2m^2-5m+6=0$

If $m=1 \Rightarrow 1-2-5+6=0$

$m=1$ is a root

$$\begin{array}{c|cccc} 1 & 1 & -2 & -5 & 6 \\ & 0 & 1 & -1 & -6 \\ \hline & 1 & -1 & -6 & \boxed{0} \end{array}$$

$m^2-m+6=0$

$m^2-3m+2m-6=0$

$m(m-3)+2(m-3)$

$(m+2)(m-3)=0$

$m=3,-2$

Roots are $1,3,-2$

$y_c = c_1e^x + c_2e^{3x} + c_3e^{-2x}$

General solution is $y = c_1e^x + c_2e^{3x} + c_3e^{-2x}$

$y' = c_1e^x + 3c_2e^{3x} - 2c_3e^{-2x}$

$y'' = c_1e^x + 9c_2e^{3x} + 4c_3e^{-2x}$

Given that $y=1$, when $x=0$

$c_1+c_2+c_3=1$ —— (1)

& $y'=-7$ when $x=0$

$c_1+3c_2-2c_3=-7$ —— (2)

And $y''=-1$ when $x=0$

$c_1+9c_2+4c_3=-1$ —— (3)

from (1) & (2)
$$\begin{array}{r} \cancel{c_1}+c_2+c_3=1 \\ -\cancel{c_1}+3c_2-2c_3=-7 \\ \hline -2c_2+3c_3=8 \end{array} \text{ —— (4)}$$

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from (1) & (3)

$$\cancel{c_1} + c_2 + c_3 = 1$$
$$\cancel{c_1} + 9c_2 + 4c_3 = -1$$
$$- \quad - \quad - \quad +$$

$$-8c_2 - 3c_3 = 2 \quad \text{--- (5)}$$

from (5) & (4)

$$-8c_2 - 3\cancel{c_3} = 2$$
$$-2c_2 + 3c_3 = 8$$

$$-10c_2 = 10$$
$$\boxed{c_2 = 1}$$

from eq-(4)

$$2(-1) + 3c_3 = 8$$
$$-2 + 3c_3 = 8.$$
$$3c_3 = 6.$$
$$\boxed{c_3 = 2}$$

from equation - (1)

$$c_1 + c_2 + c_3 = 1$$
$$c_1 - 1 + 2 = 1$$
$$c_1 + 1 = 1$$
$$\boxed{c_1 = 0}$$

$$y_c = 0.e^x + (-1)e^{3x} + (2)e^{-2x}.$$

$$y_c = -e^{3x} + 2e^{-2x}$$

G.S is $y = 2e^{-2x} - e^{3x}$

(2) Given equation :- $(D^2-3D+2)y = \cosh x$

Auxiliary equation $f(m)=0$

$$m^2-3m+2=0$$
$$m^2-2m-m+2=0$$
$$m(m-2)-1(m-2)=0$$
$$(m-1)(m-2)=0$$
$$m=1,2$$
$$y_c = c_1e^x + c_2e^{2x}.$$

$$y_p = \frac{1}{D^2-3D+2} \cdot \cosh x.$$
$$= \frac{1}{D^2-3D+2}\left[\frac{e^x+e^{-x}}{2}\right]$$
$$= \frac{1}{2}\left[\frac{1}{D^2-3D+2}\cdot e^x + \frac{1}{D^2-3D+2}e^{-x}\right]$$
$$= \frac{1}{2}\left[\frac{1}{(D-1)(D-2)}e^x + \frac{1}{(D-1)(D-2)}e^{-x}\right]$$
$$= \frac{1}{2}\left[\frac{1}{(D-1)(D-2)}e^x + \frac{1}{(-1-1)(-1-2)}e^{-x}\right]$$
$$\frac{1}{2}\left[\frac{1}{-1(D-1)}e^x + \frac{1}{6}e^{-x}\right]$$
$$= \frac{1}{2}\left[\frac{1}{-1}\cdot\frac{x^1}{1!}e^x + e^{-x}/6\right]$$
$$\frac{1}{2}\left[-xe^x + e^{-x}/6\right]$$
$$y_p = \frac{-xe^x}{2} + \frac{e^{-x}}{12}$$

General solution is $y = y_c + y_p$

$$y = c_1e^x + c_2e^{2x} - \frac{xe^x}{2} + \frac{e^{-x}}{12}$$

(3) Given equation :- $\frac{d^2y}{dx^2} - (a+b)\frac{dy}{dx} + aby = e^{ax} + e^{bx}$

Operator-form $(D^2 - (a+b)d + ab)y = e^{ax} + e^{bx}$.

Auxiallary equation $f(m) = 0$

$(m^2 - (a+b)m + ab)y = 0$

$m = a, b.$

$y_c = c_1 e^{ax} + c_2 e^{bx}$

$y_p = \frac{1}{D^2 - (a+b)D + ab} \cdot (e^{ax} + e^{bx})$

$= \frac{1}{(D-a)(D-b)} (e^{ax} + e^{bx})$

$= \frac{1}{(D-a)(D-b)} e^{ax} + \frac{1}{(D-a)(D-b)} e^{bx}$

$= \frac{1}{a-b} \cdot \frac{x^1}{1!} e^{ax} + \frac{1}{b-a} \frac{x^1}{1!} e^{bx}$

$\frac{x}{a-b} e^{ax} + \frac{x}{b-a} e^{bx}$

$y_p = \frac{x}{a-b} (e^{ax} - e^{bx})$

General solution is

$y = y_c + y_p$

$y = c_1 e^{ax} + c_2 e^{bx} + \frac{x}{a-b} (e^{ax} - e^{bx})$

(4) Given equation :- $(D^2+9)y = \cos^3 x$.

Auxiliary equation $f(m)=0$

$$m^2+9=0$$
$$m^2=-9$$
$$m=\pm 3i$$

$$y_c = c_1 \cos 3x + c_2 \sin 3x.$$

$\left[\because \cos 3x = 4\cos^3 x - 3\cos x \right.$

$\cos 3x + 3\cos x = 4\cos^3 x$

$\left.\dfrac{\cos 3x + 3\cos x}{4} = \cos^3 x\right]$

Now

$$y_p = \frac{1}{D^2+9}\cos^3 x.$$

$$= \frac{1}{D^2+9}\left(\frac{\cos 3x + 3\cos x}{4}\right)$$

$$\frac{1}{4}\left[\frac{1}{D^2+9}\cos 3x + \frac{1}{D^2+9}3\cos x\right]$$

$$\frac{1}{4}\left[\frac{x}{2(3)}\sin 3x + \frac{3}{-1^2+9}\cos x\right]$$

$$\frac{1}{4}\left[x/6 \sin 3x + 3/8 \cos x\right]$$

$$y_p = \frac{x}{24}\sin 3x + \frac{3}{24}\cos x.$$

General solution is $y = y_c + y_p$

$$y = c_1\cos 3x + c_2 \sin 3x + \frac{x}{24}\sin 3x + 3/24 \cos x$$

(5) Given equation $(D^2-4D+3)y = \sin 3x \cdot \cos 2x$.

Auxiliary equation $f(m)=0$

$$m^2-4m+3=0$$
$$m^2-3m-m+3=0$$
$$m(m-3)-1(m-3)=0$$
$$(m-3)(m-1)=0$$
$$m=1,3$$

$$y_c = c_1e^x + c_2e^{3x}$$

Now $y_p = \dfrac{1}{D^2-4D+3} \sin 3x \cdot \cos 2x$.

$$\frac{1}{2} \cdot \frac{1}{D^2-4D+3} \cdot 2\sin 3x \cdot \cos 2x.$$

$$\frac{1}{2}\left[\frac{1}{D^2-4D+3} (\sin(3x+2x) + \sin(3x-2x))\right]$$

$$\frac{1}{2}\left[\underset{(A)}{\frac{1}{D^2-4D+3}\sin 5x} + \underset{(B)}{\frac{1}{D^2-4D+3}\sin x}\right]$$

Now (A) $\dfrac{1}{D^2-4D+3} \sin 5x$.

$$= \frac{1}{-5^2-4D+3} \sin 5x.$$

$$\frac{1}{-4D-22} \cdot \sin 5x$$

$$-\frac{1}{2} \cdot \frac{1}{2D+11} \sin 5x.$$

$$-\frac{1}{2} \frac{2D-11}{4D^2-121} \cdot \sin 5x.$$

$$-\frac{1}{2} \frac{2D-11}{4(-5)^2-121} \cdot \sin 5x.$$

$$-\frac{1}{2} \cdot \frac{2D-11}{-221} \cdot \sin 5x.$$

Now (B) $= \dfrac{1}{D^2-4D+3} \sin x$

$$= \frac{1}{-1^2-4D+3} \sin x.$$

$$\frac{1}{-4D+2} \sin x$$

$$-\frac{1}{2} \cdot \frac{1}{2D-1} \sin x.$$

$$-\frac{1}{2} \frac{2D+1}{4D^2-1} \sin x.$$

$$-\frac{1}{2} \cdot \frac{2D+1}{4(-1^2)-1} \sin x$$

$$-\frac{1}{2} \cdot \frac{2D+1}{-5} \sin x$$

(5) Given equation $(D^2-4D+3)y = \sin 3x \cdot \cos 2x$.

Auxiliary equation $f(m)=0$

$$m^2-4m+3=0$$
$$m^2-3m-m+3=0$$
$$m(m-3)-1(m-3)=0$$
$$(m-3)(m-1)=0$$
$$m=1,3$$

$$y_c = c_1e^{x} + c_2e^{3x}$$

Now $y_p = \dfrac{1}{D^2-4D+3} \sin 3x \cdot \cos 2x.$

$$\frac{1}{2} \cdot \frac{1}{D^2-4D+3} \cdot 2\sin 3x \cdot \cos 2x.$$

$$\frac{1}{2}\left[\frac{1}{D^2-4D+3}(\sin(3x+2x)+\sin(3x-2x)\right]$$

$$\frac{1}{2}\left[\underbrace{\frac{1}{D^2-4D+3}\sin 5x}_{(A)} + \underbrace{\frac{1}{D^2-4D+3}\sin x}_{(B)}\right]$$

Now (A) $\dfrac{1}{D^2-4D+3}\sin 5x.$

$$= \frac{1}{-5^2-4D+3}\sin 5x.$$

$$\frac{1}{-4D-22}\cdot\sin 5x$$

$$-\frac{1}{2}\cdot\frac{1}{2D+11}\sin 5x.$$

$$-\frac{1}{2}\frac{2D-11}{4D^2-121}\cdot\sin 5x.$$

$$-\frac{1}{2}\frac{2D-11}{4(-5)^2-121}\cdot\sin 5x.$$

$$-\frac{1}{2}\cdot\frac{2D-11}{-221}\cdot\sin 5x.$$

Now (B) $= \dfrac{1}{D^2-4D+3}\sin x$

$$= \frac{1}{-1^2-4D+3}\sin x.$$

$$\frac{1}{-4D+2}\sin x$$

$$-\frac{1}{2}\cdot\frac{1}{2D-1}\sin x.$$

$$-\frac{1}{2}\frac{2D+1}{4D^2-1}\sin x$$

$$-\frac{1}{2}\cdot\frac{2D+1}{4(-1^2)-1}\sin x$$

$$-\frac{1}{2}\cdot\frac{2D+1}{-5}\sin x.$$

$+\frac{1}{442}(2D-11)\sin 5x.$ $\qquad\qquad \frac{1}{10}(2D+1)\sin x.$

$A = \frac{1}{442}(10\cos 5x - 11\sin 5x)$ $\qquad B = \left[\frac{1}{10}(2\cos x + \sin x)\right]$

$$y_p = \frac{1}{2}\left[\frac{1}{442}(10\cos 5x - 11\sin 5x) + \frac{1}{10}(2\cos x + \sin x)\right]$$

$$\frac{1}{884}(10\cos 5x - 11\sin 5x) + \frac{1}{20}(2\cos x + \sin x)$$

General solution is $y = y_c + y_p$

$$y = c_1 e^x + c_2 e^{3x} + \frac{1}{884}(10\cos 5x - 11\sin 5x) + \frac{1}{20}(2\cos x + \sin x)$$

(6) Given equation :- $\frac{d^2y}{dx^2} + 4y = e^x + \sin 2x + \cos 2x.$

or is $(D^2+4)y = e^x + \sin 2x + \cos 2x.$

Auxiallary equation $f(m) = 0$

$$m^2 + 4 = 0$$

$$m^2 = -4$$

$$m = \pm 2i$$

Now $y_c = c_1\cos 2x + c_2 \sin 2x.$

$$y_p = \frac{1}{D^2+4}(e^x + \sin 2x + \cos 2x)$$

$$\frac{1}{D^2+4}e^x + \frac{1}{D^2+4}\sin 2x + \frac{1}{D^2+4}\cos 2x.$$

$$\frac{1}{1^2+4}e^x + \frac{1}{D^2+2^2}\sin 2x + \frac{1}{D^2+2^2}\cos 2x.$$

$$\frac{1}{5}e^x - \frac{x}{2(2)}\cos 2x + \frac{x}{2(2)}\sin 2x.$$

$$\frac{e^x}{5} - x/4\cos 2x + x/4 \sin 2x.$$

$e^x/5 - x/4 \cos 2x + \frac{x}{4} \sin 2x$

General solution is $y = y_c + y_p$

$y = c_1 \cos 2x + c_2 \sin 2x + \frac{e^x}{5} - x/4 \cos 2x + x/4 \sin 2x.$

(7) Given equation :- $(D^2-4)y = e^x + \sin 2x + \cos^2 x$

A.E is $f(m) = 0$

$m^2 - 4 = 0$

$(m+2)(m-2) = 0$

$m = 2, 2$

$y_c = c_1 e^{-2x} + c_2 e^{2x}.$

$y_p = \frac{1}{D^2-4} (e^x + \sin 2x + \cos^2 x)$

$\frac{1}{D^2-4} e^x + \frac{1}{D^2-4} \sin 2x + \frac{1}{D^2-4} \cos^2 x.$

$\frac{1}{-1^2-4} e^x + \frac{1}{-2^2-4} \sin 2x + \frac{1}{D^2-4} \left(\frac{1+\cos 2x}{2}\right)$

$\frac{-1}{-3} e^x + \frac{1}{-8} \sin 2x + \frac{1}{2} \left[\frac{1}{D^2-4} e^{0.x} + \frac{1}{D^2-4} \cos 2x\right]$

$\frac{-e^x}{3} - 1/8 \sin 2x + \frac{1}{2} \left[\frac{-1}{4} + \frac{1}{-2^2-4} \cos 2x\right]$

$y_p = \frac{-e^x}{3} - \frac{1}{8} \sin 2x - \frac{1}{8} - 1/16 \cos 2x.$

general solution is $y = y_c + y_p$

$y = c_1 e^{-2x} + c_2 e^{2x} - e^x/3 - 1/8 \sin 2x - 1/8 - 1/16 \cos 2x.$

(1) Solve $(D^2-2D+4)y = 8(x^2+e^{2x}+\sin 2x)$

(2) Solve $(D^2-2D+3)y = \cos x + x^2$

(3) Solve $(D^2-3D+2)y = e^{4x}+\sin 3x + x^2+x$

(4) Solve $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 13y = 8e^{3x}\sin 2x.$

(5) Solve $(D^2-4)y = x\sinh x$

(6) Solve $(D^2+4)y = x\sin x,$

(7) Solve $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = xe^x\sin x.$

(8) Solve $(D^2-4D+4)y = 8x^2e^{2x}\sin 2x.$

(1) Given equation :- $(D^2-2D+4)y = 8(x^2+e^{2x}+\sin 2x)$

Auxiallary Equation $f(m)=0$

$$m^2-2m+4=0$$

$$m = \frac{(-2) \pm \sqrt{(-2)^2-4(1)4}}{2(1)}$$

$$= \frac{2 \pm \sqrt{4-16}}{2}$$

$$= \frac{2 \pm \sqrt{-12}}{2} = \frac{2 \pm 2\sqrt{-3}}{2}$$

$$= 1 \pm \sqrt{3}i$$

$$y_c = (c_1 \cos \sqrt{3}x + c_2 \sin \sqrt{3}x)e^x.$$

$$y_p = \frac{1}{D^2-2D+4} \cdot 8(x^2+e^{2x}+\sin 2x)$$

$$8\left[\underset{(A)}{\frac{1}{D^2-2D+4} \cdot x^2} + \underset{(B)}{\frac{1}{D^2-2D+4} e^{2x}} + \underset{(C)}{\frac{1}{D^2-2D+4} \cdot \sin 2x}\right]$$

Soluing:-
(A)

$$\frac{1}{D^2-2D+4} x^2 = \frac{1}{4\left(\frac{D^2-2D}{4}+1\right)} \cdot x^2$$

$$= \frac{1}{4} \cdot \frac{1}{1+\frac{D^2-2D}{4}} \cdot x^2$$

$$\frac{1}{4}\left[1+\left(\frac{D^2-2D}{4}\right)\right]^{-1} \cdot x^2$$

$$\frac{1}{4}\left[1-\left(\frac{D^2-2D}{4}\right)+\left(\frac{D^2-2D}{4}\right)^2 - \cdots\right] x^2$$

$$\frac{1}{4}\left(1-\frac{D^2}{4}+\frac{2D}{4}-\frac{4D^2}{16}\cdots\right) x^2$$

$$\frac{1}{4}\left(1+\frac{D^2}{2}\right)^2 (x^2)$$

$\frac{1}{4}\left(x^2+\frac{2x}{2}\right)$

$A = \frac{1}{4}(x^2+x)$

Solving B :- $\frac{1}{D^2-2D+4}e^{2x} = \frac{1}{2^2-2(2)+4}e^{2x} = \frac{e^{2x}}{4}$

Solving C :- $\frac{1}{D^2-2D+4}\sin 2x = \frac{1}{-2^2-2D+4}\sin 2x$

$= \frac{1}{-2D}\sin 2x = \frac{-1}{2}\left(\frac{-\cos 2x}{2}\right)$

$= \frac{\cos 2x}{4}$

$y_p = 8\left[\frac{1}{4}(x^2+x)+\frac{e^{2x}}{4}+\frac{1}{4}\cos 2x\right]$

$2(x^2+x)+2e^{2x}+2\cos 2x.$

General solution is $y = y_c+y_p$

$y = (c_1\cos\sqrt{3}x + c_2\sin\sqrt{3}x)e^{x} + 2(x^2+x)+2e^{2x}$
$+ 2\cos 2x.$

(2) Given equation :- $(D^2-2D+3)y = \cos x + x^2$

Auxiliary equation $f(m)=0$

$m^2-2m+3=0$

$m = \frac{-(-2) \pm \sqrt{(-2)^2-4(1)(3)}}{2(1)}$

$= \frac{2 \pm \sqrt{4-12}}{2}$

$= \frac{2 \pm \sqrt{8}i}{2}$

$= 1 \pm \sqrt{2}i$

$y_c = (c_1 \cos\sqrt{2}x + c_2 \sin\sqrt{2}x)e^x.$

$y_p = \frac{1}{D^2-2D+3}(\cos x + x^2)$

$= \frac{1}{D^2-2D+3} \cos x + \frac{1}{D^2-2D+3} \cdot x^2$

Solving (A) :-

$\frac{1}{D^2-2D+3} \cos x = \frac{1}{-1^2-2D+3} \cos x.$

$= \frac{1}{-2D+2} \cos x. = \frac{1}{-2(D+1)} \cos x.$

$-\frac{1}{2} \frac{D+1}{D^2-1} \cos x.$

$= -\frac{1}{2} \frac{D+1}{-1^2-1} \cos x.$

$-\frac{1}{2} \frac{D+1}{-2} \cos x.$

$\frac{1}{4}(D+1)\cos x.$

$A = \frac{1}{4}(-\sin x + \cos x)$

Solving B :- $\frac{1}{D^2-2D+3}x^2 = \frac{1}{3\left(1+\frac{D^2-2D}{3}\right)} x^2$

$$= \frac{1}{3}\left(1+\frac{D^2-2D}{3}\right)^{-1}(x^2)$$

$$= \frac{1}{3}\left[1-\frac{(D^2-2D)}{3}+\frac{(D^2-2D)^2}{3}\cdots\right](x^2)$$

$$\frac{1}{3}\left[1-\frac{D^2}{3}+\frac{2D}{3}+\frac{4D^2}{9}\right](x^2)$$

$$\frac{1}{3}\left[1+\frac{2D}{3}+\frac{D^2}{9}\right](x^2)$$

$$\frac{1}{3}\left[x^2+2/3\,(2x)+1/9\,(2)\right]$$

$$x^2/3+\frac{4x}{3}+\frac{2}{27}$$

$$\Rightarrow y_p = \frac{1}{4}(\cos x-\sin x)+\frac{x^2}{3}+\frac{4x}{3}+\frac{2}{27}$$

General solution is $y = y_c + y_p$

$$y = (c_1\cos\sqrt{2}x + c_2\sin\sqrt{2}x)e^x + \frac{1}{4}(\cos x-\sin x)+\frac{x^2}{3}+\frac{4x}{3}+\frac{2}{27}$$

(3) Given equation :- $(D^2-3D+2)y = e^{4x} + \sin 3x + x^2+x$.

Auxiallary Equation $f(m)=0$

$$m^2-3m+2=0$$

$$m^2-2m-m+2=0$$

$$m(m-2)-1(m-2)=0$$

$$(m-1)(m-2)$$

$$m=1,2$$

$$y_c = c_1e^{x}+c_2e^{2x}.$$

Now $y_p = \dfrac{1}{D^2-3D+2}(e^{4x}+\sin 3x + x^2+x)$

$$\Rightarrow \underset{(A)}{\frac{1}{D^2-3D+2}e^{4x}} + \underset{(B)}{\frac{1}{D^2-3D+2}\cdot \sin 3x} + \underset{(C)}{\frac{1}{D^2-3D+2}(x^2+x)} ---$$

Solving - (A) :- $\dfrac{1}{D^2-3D+2}e^{4x} = \dfrac{1}{4^2-3(4)+2}\cdot e^{4x} = \dfrac{1}{6}e^{4x}$.

Solving - (B) :- $\dfrac{1}{D^2-3D+2}\cdot \sin 3x = \dfrac{1}{-3^2-3D+2}\cdot \sin 3x$

$$= \frac{1}{-3D-7}\sin 3x$$

$$= \frac{1}{-3(D+7)}\sin 3x.$$

$$-\frac{(3D+7)}{9D^2-49} = \frac{-3(D+7)}{9(-3)^2-49}\sin 3x.$$

$$-\frac{(3D-7)}{-81-49}\sin 3x.$$

$$= -\frac{(3D-7)}{-130}\sin 3x.$$

$$\frac{1}{130}[3(3\cos 3x-7\sin 3x)]$$

3) Given equation :- $(D^2-3D+2)y = e^{4x} + \sin 3x + x^2 + x$.

Auxiliary Equation $f(m)=0$

$$m^2-3m+2=0$$

$$m^2-2m-m+2=0$$

$$m(m-2)-1(m-2)=0$$

$$(m-1)(m-2)$$

$$m=1,2$$

$$y_c = c_1e^x + c_2e^{2x}.$$

Now $y_p = \dfrac{1}{D^2-3D+2}(e^{4x} + \sin 3x + x^2 + x)$

$$\Rightarrow \underset{(A)}{\frac{1}{D^2-3D+2}e^{4x}} + \underset{(B)}{\frac{1}{D^2-3D+2}\cdot \sin 3x} + \underset{(C)}{\frac{1}{D^2-3D+2}(x^2+x)} \text{ ---}$$

Solving - (A) :-

$$\frac{1}{D^2-3D+2}e^{4x} = \frac{1}{4^2-3(4)+2}\cdot e^{4x} = \frac{1}{6}e^{4x}.$$

Solving - (B) :

$$\frac{1}{D^2-3D+2}\cdot \sin 3x = \frac{1}{-3^2-3D+2}\cdot \sin 3x$$

$$= \frac{1}{-3D-7}\sin 3x$$

$$= \frac{1}{-3(D+7)}\sin 3x.$$

$$= \frac{(3D+7)}{9D^2-49} = \frac{-3(D+7)}{9(-3)^2-49}\sin 3x.$$

$$\frac{-(3D-7)}{-81-49}\sin 3x.$$

$$= \frac{-(3D-7)}{-130}\sin 3x.$$

$$\frac{1}{130}\left[3(3\cos 3x - 7\sin 3x)\right]$$

$\frac{1}{130}(9\cos 3x - 7\sin 3x)$

Solving (C) :- $\frac{1}{D^2-3D+2}(x^2+x) = \frac{1}{2\left(\frac{D^2-3D}{2}+1\right)}(x^2+x)$

$= \frac{1}{2}\left[1+\left(\frac{D^2-3D}{2}\right)\right]^{-1}(x^2+x)$

$\Rightarrow \frac{1}{2}\left[1-\left(\frac{D^2-3D}{2}\right)+\left(\frac{D^2-3D}{2}\right)^2 \ldots\right](x^2+x)$

$\Rightarrow \frac{1}{2}\left[1-\left(\frac{D^2-3D}{2}\right)+\left(\frac{D^2-3D}{2}\right)^2 \ldots\right](x^2+x)$

$\Rightarrow \frac{1}{2}\left[1-D^2/_2+3D/_2+\frac{9D^2}{4}\right](x^2+x)$

$\Rightarrow \frac{1}{2}\left[1+\frac{3D}{2}+\frac{9D^2}{4}-\frac{D^2}{2}\right](x^2+x)$

$\Rightarrow \frac{1}{2}\left[1+\frac{3D}{2}+\frac{7D^2}{4}\right](x^2+x)$

$\frac{1}{2}\left(x^2+\frac{3}{2}(2x)+\frac{7}{4}(2)+x+3/_2\right]$

$\frac{1}{2}\left(x^2+3x+7/_2+x+3/_2\right)$

$\frac{1}{2}(x^2+4x+5)$

$y_p = \frac{e^{4x}}{6}+1/_{130}(9\cos 3x-7\sin 3x)+1/_2(x^2+4x+5)$

General solution is $y = y_c + y_p$

$y = c_1e^x + c_2e^{2x} + \frac{e^{4x}}{6} + 1/_{130}(9\cos 3x - 7\sin 3x) + 1/_2$
(x^2+4x+5)

(4) Given equation:-

$$(D^2-6D+13)y = 8e^{3x}\sin 2x.$$

Auxiliary equation $f(m)=0$

$$m^2-6m+13=0$$

$$m = \frac{6 \pm \sqrt{36-52}}{2}$$

$$= \frac{6 \pm \sqrt{-16}}{2}$$

$$= \frac{6 \pm 4i}{2}$$

$$= 3 \pm 2i$$

$$y_c = (c_1\cos 2x + c_2 \sin 2x)e^{3x}.$$

Now $y_p = \frac{1}{D^2-6D+13} 8e^{3x} \cdot \sin 2x.$

$$8e^{3x} \cdot \frac{1}{(D+3)^2-6(D+3)+13} \sin 2x.$$

$$8e^{3x} \cdot \frac{1}{D^2+9+6D-6D-18+13} \sin 2x.$$

$$8e^{3x} \cdot \frac{1}{D^2+4} \sin 2x.$$

$$8e^{3x} \cdot \frac{1}{D^2+2^2} \sin 2x.$$

$$8e^{3x} \cdot \left(\frac{-x}{2(2)} \cos 2x\right)$$

$$= -2xe^{3x} \cdot \cos 2x.$$

$$y = (c_1\cos 2x + c_2 \sin 2x)e^{3x} - 2xe^{3x} \cdot \cos 2x.$$

(5) Given equation :- $(D^2-4)y = x\sinh x$

Auxiliary Equation $f(m)=0$

$m^2-4=0$

$m=4$

$m=\pm 2.$

$y_c = c_1e^{2x}+c_2e^{-2x}.$

Now $y_p = \frac{1}{D^2-4}\, x\sinh x.$

$\frac{1}{D^2-4}\cdot x\cdot\left(\frac{e^x-e^{-x}}{2}\right)$

$\frac{1}{2}\Bigg[\underbrace{\frac{1}{D^2-4}\cdot xe^x}_{(A)} - \underbrace{\frac{1}{D^2-4}\,xe^{-x}}_{(B)}\Bigg]$

Solving (A) :- $\frac{1}{D^2-4}\,xe^x.$

$e^x\cdot\frac{1}{D^2-4}\,x.$

$e^x\cdot\frac{1}{(D+1)^2-4}\cdot x.$

$e^x\cdot\frac{1}{D^2+2D+1-4}\cdot x$

$e^x\cdot\frac{1}{D^2+2D-3}\,x.$

$\frac{e^x}{-3}\left(\frac{1}{\frac{D^2+2D}{3}+1}\right)x.$

$-\frac{e^x}{3}\left(1-\frac{D^2+2D}{3}\right)^{-1}x.$

$-\frac{e^x}{3}\left(1+\frac{D^2+2D}{3}\right)x.$

Solving (B) :- $\frac{1}{D^2-4}\,xe^{-x}.$

$e^{-x}\,\frac{1}{(D-1)^2-4}\cdot x$

$e^{-x}\,\frac{1}{D^2-2D+1-4}\,x$

$e^{-x}\,\frac{1}{D^2-2D-3}\cdot x$

$\frac{-e^{-x}}{3}\left[\frac{1}{\frac{D^2-2D}{3}-1}\right]x.$

$\frac{-e^{-x}}{3}\left(1-\frac{D^2-2D}{3}\right)^{-1}(x)$

$\frac{-e^{-x}}{3}\left(1+\frac{D^2-2D}{3}\right)(x)$

$-e^{-x}/3\,(1-2D/3)(x)$

$\frac{-e^{-x}}{3}\left(1+\frac{2D}{3}\right)(x)$ $\qquad$ $\frac{-e^{-x}}{3}(x-2/3)$

$A = \frac{-e^{-x}}{3}(1+2/3)$ $\qquad$ $B = \frac{-e^{-x}}{9}(3x-2)$

$y_p = \frac{1}{2}\left[\frac{-e^{-x}}{9}(3x+2) - \left(\frac{-e^{-x}}{9}(3x-2)\right)\right]$

$\frac{1}{2}\left[\frac{-3x}{9}(-e^{x}+e^{-x}) + \frac{2}{9}(-e^{x}-e^{-x})\right]$

$-\frac{3x}{9}\left(\frac{e^{x}-e^{-x}}{2}\right) - 2/9\left(\frac{e^{x}-e^{-x}}{2}\right)$

$-\frac{x}{3}\sinh x - 2/9 \cosh x.$

General solution is $y = y_c + y_p$

$y = c_1e^{2x} + c_2e^{-2x} - x/3 \sinh x - 2/9 \cosh x.$

(6) Given equation :- $(D^2+4)y = x\sin x.$

A.E is $f(m)=0$

$m^2+4 = 0$

$m^2 = -4$

$m = \pm 2i$

$y_c = c_1\cos 2x + c_2 \sin 2x.$

$y_p = \frac{1}{D^2+4} x \sin x.$

$\left[\frac{1}{f(D)}(xV) = x\cdot\frac{1}{f(D)}V - \frac{f'(D)}{(f(D))^2}\cdot V\right]$

$\frac{1}{D^2+4} x\sin x = x\cdot\frac{1}{D^2+4}\sin x - \frac{2D}{(D^2+4)^2}\sin x.$

$x\cdot\frac{1}{-1^2+4}\sin x - \frac{2D}{(-1^2+4)^2}\sin x.$

$x \cdot \frac{1}{3} \sin x - \frac{2}{9} \cos x.$

$\frac{x}{3} \sin x - \frac{2}{9} \cos x.$

General solution $y = y_c + y_p$

$y = c_1 \cos 2x + c_2 \sin 2x + \frac{x}{3} \sin x - \frac{2}{9} \cos x.$

(b) Given equation:-

O.f is $(D^2+3D+2)y = xe^x \sin x.$

A.E is $f(m) = 0$

$(m^2+3m+2) = 0$

$m^2+2m+m+2 = 0$

$m(m+2)+1(m+2) = 0.$

$(m+1)(m+2) = 0.$

$m = -1, -2$

$y_c = c_1 e^{-x} + c_2 e^{-2x}.$

Now $y_p = \frac{1}{D^2+3D+2} xe^x \sin x.$

$e^x. \frac{1}{(D+1)^2+3(D+1)+2} x \sin x.$

$e^x. \frac{1}{D^2+2D+1+3D+3+2} x \sin x.$

$e^x. \frac{1}{D^2+5D+6} x \sin x.$

$e^x \left[x. \frac{1}{D^2+5D+6} \sin x - \frac{2D+5}{(D^2+5D+6)^2} \sin x\right]$

$e^x \left[x. \frac{1}{-1^2+5D+6} \sin x - \frac{2D+5}{(-1^2+5D+6)^2} \sin x\right]$

$$e^{x}\left[x\,\frac{1}{5(D+1)}\sin x-\frac{2D+5}{25(D+1)^2}\sin x\right]$$

$$e^{x}\left[\frac{x}{5}\cdot\frac{D-1}{D^2-1}\sin x-\frac{2D+5}{25(D^2+2D+1)}\sin x\right]$$

$$e^{x}\left[\frac{x}{5}\,\frac{D-1}{D^2-1}\sin x-\frac{2D+5}{25(-1^2+2D+1)}\sin x\right]$$

$$e^{x}\left[\frac{-x}{10}(\cos x-\sin x)-\frac{1}{25}\left(1+\frac{5}{2D}\right)\sin x\right]$$

$$e^{x}\left[\frac{-x}{10}(\cos x-\sin x)-\frac{1}{25}\left(\sin x+\frac{5}{2}(-\cos x)\right)\right]$$

$$e^{x}\left[-\frac{x}{10}(\cos x-\sin x)-\frac{1}{25}\sin x+\frac{5}{2}\cdot\frac{1}{25}\cos x\right]$$

$$e^{x}\left[-\frac{x}{10}(\cos x\cdot\sin x)-\frac{1}{25}\sin x+\frac{1}{10}\cos x\right]$$

General solution is $y=y_c+y_p$

$$y=c_1e^{-x}+c_2e^{-2x}+e^{x}\left[-\frac{x}{10}(\cos x-\sin x)-\frac{1}{25}\sin x+\frac{1}{10}\cos x\right]$$

(8) Given equation:- $(D^2-4D+4)y=8x^2e^{2x}\sin 2x$.

A.E is $f(m)=0$

$m^2-4m+4=0$

$(m-2)^2=0$

$(m-2)(m-2)=0$

$m=2,2$

$y_c=(c_1+c_2x)e^{2x}$.

Now $y_p=\dfrac{1}{D^2-4D+4}\,8x^2e^{2x}\sin 2x$.

$8e^{2x} \cdot \frac{1}{(D+2)^2 - 4(D+2)+4} x^2 \sin 2x.$

$8e^{2x} \frac{1}{D^2+4D+4-4D-8+4} \cdot x^2 \sin 2x$

$8e^{2x} \cdot \frac{1}{D^2} x^2 \sin 2x.$

$8e^{2x}$ I.p of $\frac{1}{D^2} x^2 e^{i \cdot 2x}.$

$8e^{2x}$ I.p of $e^{2x} \cdot \frac{1}{(D+2i)^2} x^2$

$8e^{2x}$ I.p of $e^{i \cdot 2x} \cdot \frac{1}{D^2+4Di-4} x^2$

$8e^{2x}$. I.p of $e^{i \cdot 2x} \frac{1}{-4\left(\frac{D^2+4Di}{-4}+1\right)} \cdot x^2$

$\frac{8}{-4} e^{2x}$. I.p of $e^{i \cdot 2x} \cdot \frac{1}{1-\left(\frac{D^2+4Di}{4}\right)} x^2$

$\frac{8}{-4} e^{2x}$. I.p of $e^{i \cdot 2x} \left[1 - \frac{D^2+4Di}{4}\right]^{-1} \cdot x^2$

$-2e^{2x}$. I.p of $e^{i \cdot 2x} \left[1+\left(\frac{D^2+4Di}{4}\right)+\left(\frac{D^2+4Di}{4}\right)^2+ \cdots\right](x^2)$

$-2e^{2x}$ I.p of $e^{i \cdot 2x} \left[1+ \frac{D^2}{4} + \frac{4Di}{4} - \frac{16D^2}{16}\right)(x^2)$

$-2e^{2x}$. I.p of $e^{i \cdot 2x} \cdot \left[1+ \frac{D^2}{4} + Di - D^2\right)(x^2)$

$-2e^{2x}$ I.p of $e^{i \cdot 2x} \left(1+Di - \frac{3D^2}{4}\right)(x^2)$

$-2e^{2x}$ I.p of $e^{i \cdot 2x} \left(x^2+2xi - \frac{3}{4}\right)(2)\right]$

$-2e^{2x}$ I.p of $e^{i \cdot 2x} \left(x^2+2xi - \frac{3}{2}\right)\right]$

$-2e^{2x}$ I.p of $\left[(\cos 2x + i \sin 2x)\left(x^2+2xi - \frac{3}{2}\right)\right]$

$-2e^{2x}\left[2x\cos 2x + \left(x^2 - 3/2\right)\sin 2x\right]$

Now

General solution is $y = y_c + y_p$

$$y = (c_1 + c_2 x)e^{2x} - 2e^{2x}\left[2x\cos 2x + \left(x^2 - 3/2\right)\sin 2x\right]$$