UNIT-1

SINSUOBIAL ALTERNATING WAVE FORMS

Current: Flow of the charge chariers per unit time is called as current.

$$I = \frac{Q}{t} = \frac{\text{charge}}{\text{Time}}$$

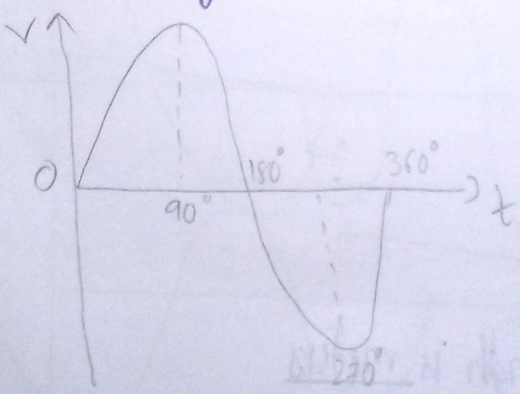
Units of current is amperes (or) Coloumbs/sec

- Both positive & negative of electrons is known as charge. units of charge 'Coloumbs'.

Voltage: Potential difference between two points in a conductive loop is called as voltage; also refered as amplitude.

- To move the electrons from one position to another position in a semi conductor material, some pressure is applied across the electrons is refered as a potential.

Units of voltage is 'volts'



Time period: Time taken for the one complete cycle or oscillation is called as time period

· Units of time period is seconds.

Frequency: No. of cycles or oscillations per second is called as frequency

Units of frequency is "hertz" (Hz)

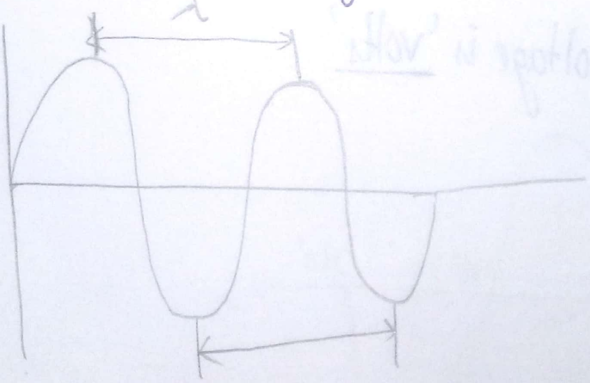
· Time period and frequency are inversly proportional to each other.

$$\boxed{T = \frac{1}{f}}$$

Amplitude: Peak to peak value of the sine wave is called as amplitude.

· Uunits of amplitude is 'volts'.

Wave length: distance between the two peak points of the sine wave (or) two depth points of the sine wave is called wavelength.



Units of wavelength is meters

Phase Difference:

- If two sine waves are in same phase are inphase then phase difference α is zero.
- If two sine waves are in out of phase the phase difference is leading (or) lagging depends upon the condition.

Sinsuodial format for voltage:

- The standard equation of sine wave for a voltage is

$$\boxed{V(t) = V_m \sin \omega t}$$

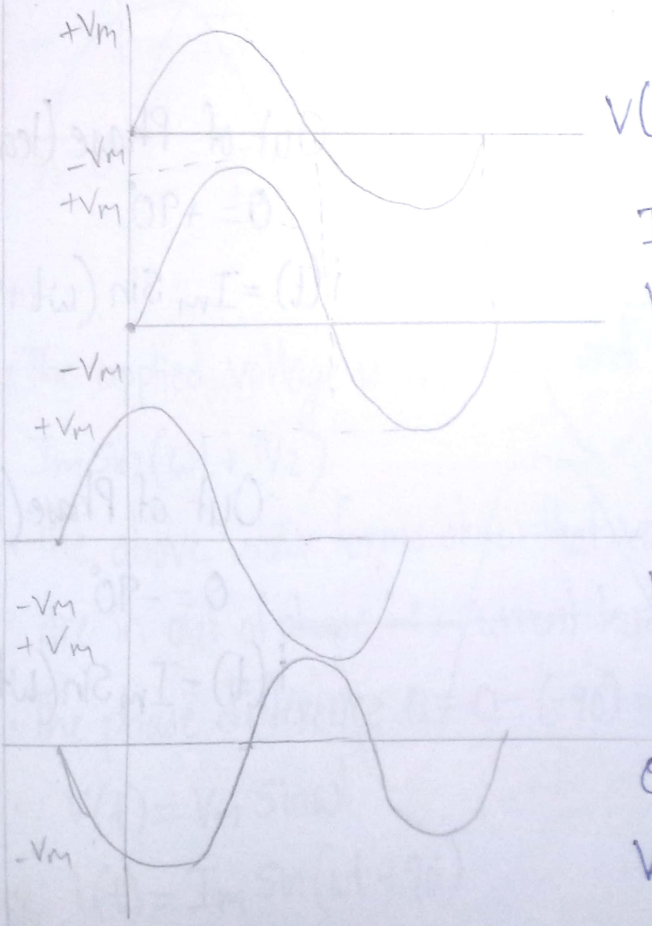
$V(t)$ = instantaneous value

$V(m)$ = maximum value (or) peak value

ω = angular frequency

$$\omega = 2\pi f \text{ (or) } \frac{2\pi}{T}$$

Comparing different waves with general eq^n of sine wave



$V(t) = V_m \sin(\omega t + \theta)$

In Phase, $\theta = 0$

$V(t) = V_m \sin \omega t$

Out of Phase

$\theta = +90°$; (leading)

$V(t) = V_m \sin(\omega t + 90°)$

Out of Phase

$\theta = -90°$; (lagging)

$V(t) = V_m \sin(\omega t - 90°)$

Sinsuodial format for current:

The standard eq^n of sine wave of current is

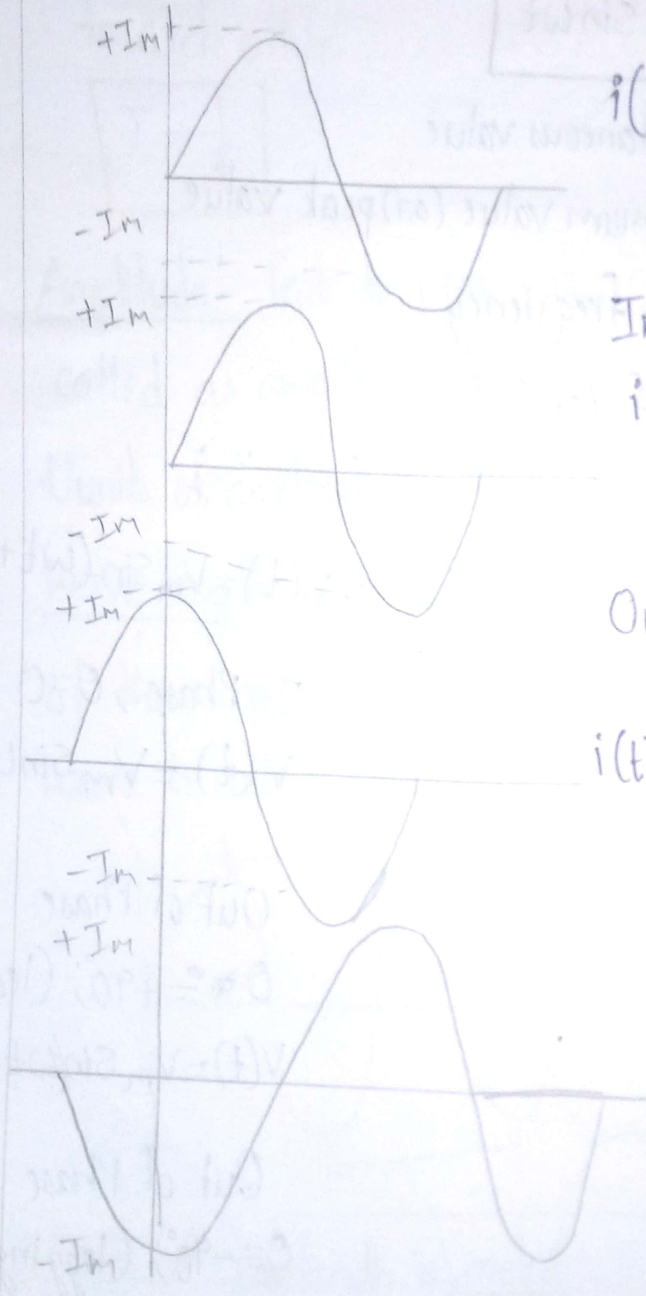
$$i(t) = I_m \sin(\omega t + \theta)$$

$i(t) \rightarrow$ instantaneous current

$I_m \rightarrow$ maximum current

$\omega \rightarrow$ angular frequency $\quad \omega = 2\pi f$ (or) $\frac{2\pi}{t}$

$\theta \rightarrow$ phase difference



$i(t) = I_m \sin(\omega t + \theta)$

In phase, $\theta = 0$

$i(t) = I_m \sin \omega t$

Out of Phase (leading)

$\theta = +90^\circ$

$i(t) = I_m \sin(\omega t + 90^\circ)$

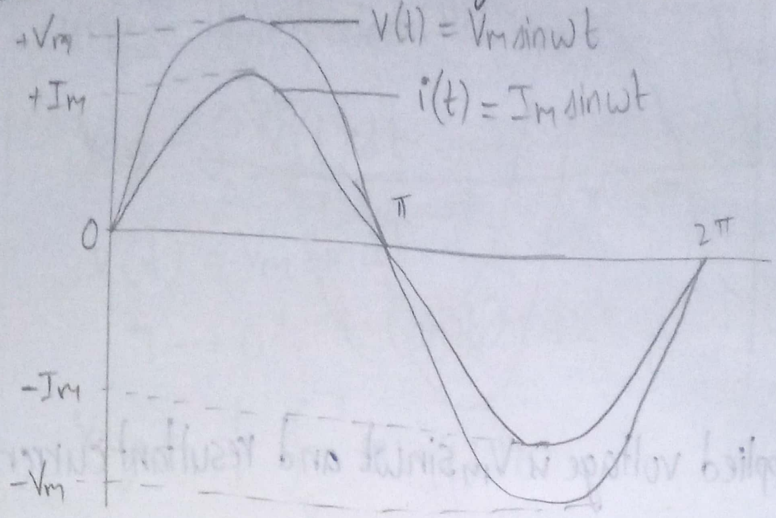
Out of Phase (lagging)

$\theta = -90^\circ$

$i(t) = I_m \sin(\omega t - 90^\circ)$

Comparing general sine wave with different waves

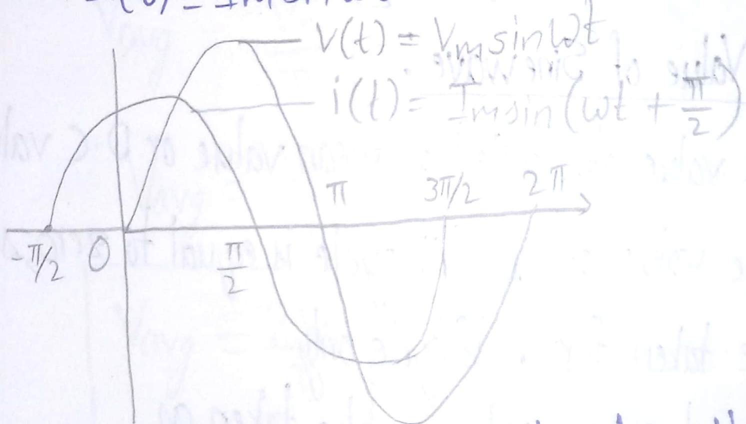
Phase relation between voltage & current:



- The applied voltage is $V_m \sin\omega t$ and resultant current is $I_m \sin\omega t$.
- The above wave form show the voltage and current are in same phase (or) inphase i.e., phase difference is zero $(\theta = 0)$

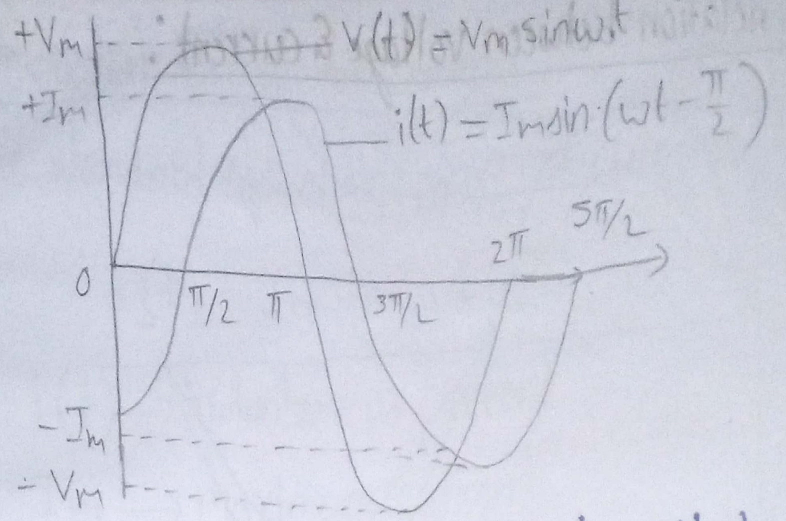
$$v(t) = V_m \sin\omega t$$

$$i(t) = I_m \sin\omega t$$



- The applied voltage is $V_m \sin\omega t$ and resultant current is $I_m \sin(\omega t + \pi/2)$
- The above wave forms show that voltage and current are in out of phase i.e., Current leads voltage by $90°$.
- The phase difference $\theta = 0 - (-90°) = 90°$
- $V(t) = V_m \sin\omega t$

  $i(t) = I_m \sin(\omega t + 90°)$



- The applied voltage is $V_m \sin\omega t$ and resultant current is $I_m \sin(\omega t - \pi/2)$.
- The above wave form shows the voltage & current are in out of phase i.e., currents laggs voltage by $90°$.
- ∴ The phase difference $\theta = 0 - 90° = -90°$

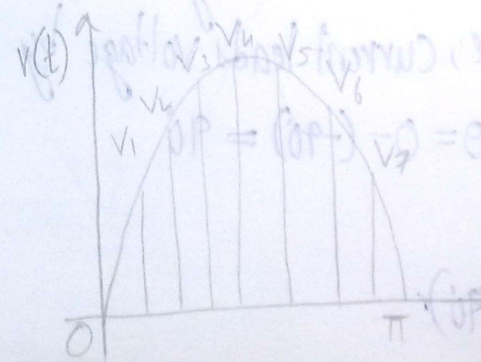
$$V(t) = V_m \sin\omega t$$

$$i(t) = I_m \sin\left(\omega t - \frac{\pi}{2}\right)$$

Average Value of Sinewave:

- Average value also called as mean value or D.C value.
- Average value of one full cycle is equal to zero, so it can be taken for half cycle only.
- In general average value can be taken as

$$\text{Avg. value} = \frac{\text{sum of observations}}{\text{no. of observations}}$$



$$\text{avg. value} = \frac{V_1+V_2+V_3+V_4+V_5+V_6+V_7}{7}$$

• Mathmedically it can be defined as ratios of area of the curve to time period

$$V_{avg} = \int_0^T \frac{v(t)\,dt}{T}$$

$$v(t) = V_m \sin\omega t$$

$$T \rightarrow 0 \text{ to } \pi \text{ (limits)}$$

$$V_{avg} = \frac{1}{T}\int_0^T v(t)\,dt$$

$$V_{avg} = \frac{1}{T}\int_0^{\pi} V_m \sin\omega t\,dt$$

$$V_{avg} = \frac{V_m}{\pi}\left[\int_0^{\pi}(\sin\omega t)\,dt\right]$$

$$V_{avg} = \frac{V_m}{\pi}\left[(-\cos\omega t)_0^{\pi}\right]$$

$$V_{avg} = \frac{V_m}{\pi}\left[-(\cos\pi) - (\cos 0)\right]$$

$$V_{avg} = \frac{V_m}{\pi}\left[-(-1) - (-1)\right]$$

$$\boxed{V_{avg} = \frac{2V_m}{\pi}}$$

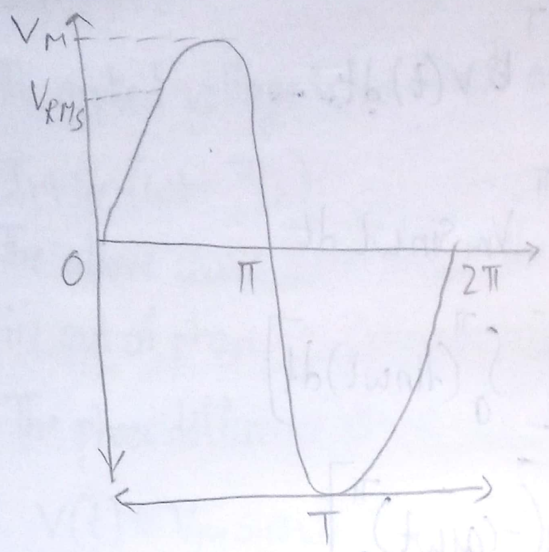
• Average value of sine wave is equal to 0.636 times of its maximum value

$$\boxed{V_{avg} = 0.636\,V_m}$$

RMS value of Sine Value :

RMS means Root Mean Square value it can also called as effective value.

- It can be calculated to entire full cycle.
- It is used to measure the heating effect of the wave



$$V_{RMS} = \sqrt{(V_{avg})^2}$$

$$V_{RMS}^2 = (V_{avg})^2$$

$$= \frac{1}{2\pi}\int_0^{2\pi} (V_m \sin\omega t)^2 \, dt$$

$$= \frac{V_m^2}{2\pi}\int_0^{2\pi} (\sin^2\omega t) \, dt$$

$$= \frac{V_m^2}{2\pi}\int_0^{2\pi} \left[\frac{1-\cos 2\omega t}{2}\right] dt$$

$$= \frac{V_m^2}{4\pi}\int_0^{2\pi} (1-\cos 2\omega t) \, dt$$

$$= \frac{V_m^2}{4\pi}\int_0^{2\pi} 1 dt - \int_0^{2\pi} \cos 2\omega t dt$$

$$= \frac{V_m^2}{4\pi}\left[[t]_0^{2\pi} - \left[\frac{\sin 2\omega t}{2\omega}\right]_0^{2\pi} \right]$$

$$V_{RMS}^2 = \frac{V_m^2}{4\pi}\left[(2\pi - 0) - \left(\frac{\sin 4\pi\omega}{2\omega} - \frac{\sin 0\omega}{2\omega}\right)\right]$$

$$V_{RMS}^2 = \frac{V_m^2}{4\pi}\left[(2\pi) - [0-0]\right] \quad \left[\because \begin{array}{l} \sin n\pi = 0 \\ \sin 0^\circ = 0 \end{array}\right]$$

$$V_{RMS}^2 = \frac{V_m^2}{\cancel{2}\,4\pi}(\cancel{2\pi})$$

$$V_{RMS}^2 = \frac{V_m^2}{2} \Rightarrow \boxed{V_{RMS} = \frac{V_{m}}{\sqrt{2}}}$$

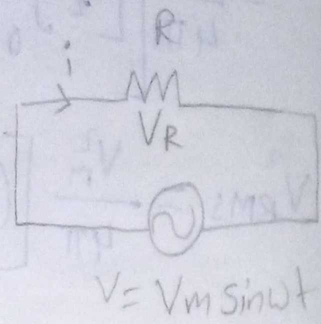
R.M.S value of the sine wave is equal to the 0.707 times of ~~th~~ its maximum value

$$V_{RMS} = \frac{V_m}{1.414} \Rightarrow \boxed{V_{RMS} = 0.707 V_m}$$

$$\boxed{I_{RMS} = \frac{I_m}{\sqrt{2}}} \Rightarrow \boxed{I_{RMS} = 0.707 I_m}$$

AC response of the pure ressistor:

- The AC voltage is applied to a pure resistor circuit then its applied voltage $V = V_m \sin\omega t$



From Ohm's law, $V = IR$ —①

applied voltage $V = V_m \sin\omega t$ —②

$$V_m \sin\omega t = IR$$

V_R = voltage across resistor

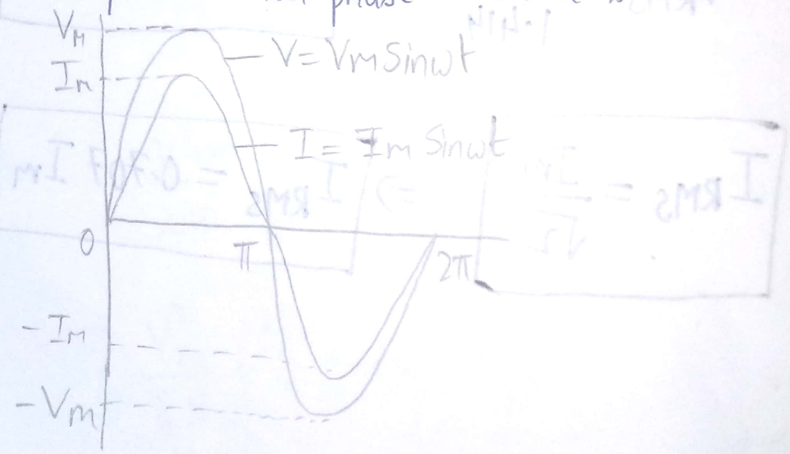
$$I = \frac{V_m \sin\omega t}{R}$$

$$I = \frac{V_m}{R} \sin\omega t$$

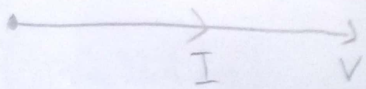
$$\boxed{I_m = \frac{V_m}{R}} \qquad \boxed{I = I_m \sin\omega t}$$

The applied voltage $V = V_m \sin\omega t$ and its resultant current is $I = I_m \sin\omega t$.

- The applied voltage & resultant current are "inphase" or "same phase" then phase difference is zero.



Phasor Diagram:



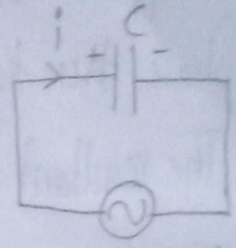
I & V are in same phase

AC response of the pure capacitor.

• The AC voltage is applied to a pure capacitors or pur circuit then its applied voltage is $V = V_m \sin\omega t$

The charge of capacitor is $Q = CV$

current $i = \frac{Q}{t}$



$V = V_m \sin\omega t$

differentiate the charge of capacitor w.r.t. 't'

$$\frac{dQ}{dt} = \frac{d}{dt}(CV)$$

$$i = C\frac{dV}{dt}$$

$$i = C\frac{d}{dt}(V_m \sin\omega t)$$

$$i = V_m C \frac{d}{dt}(\sin\omega t)$$

$$i = V_m C (\cos\omega t)\,\omega$$

$$i = V_m C\omega (\cos\omega t)$$

$$\boxed{X_c = \frac{1}{2\pi f C} \qquad X_c = \frac{1}{\omega C}}$$

X_c = capacitive reactance

$$i = \frac{V_m}{X_c}(\cos\omega t)$$

$$\cancel{i = I_m \cos\omega t}$$

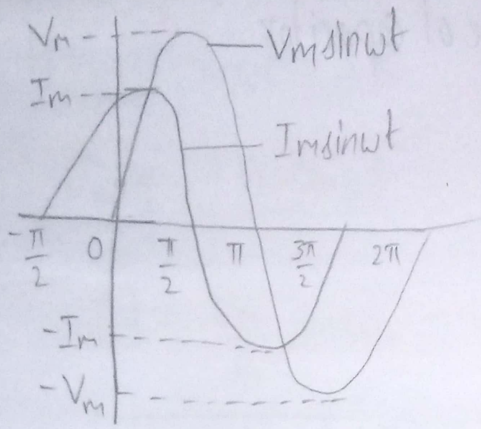
$$i = \boxed{I_m = \frac{V_m}{X_c}}$$

$$i = I_m \cos\omega t$$

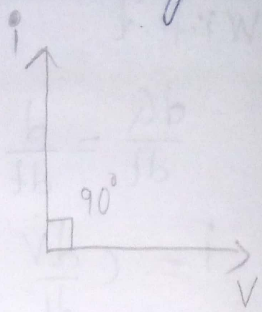
$$i = I_m \sin(90 + \omega t)$$

$$i = I_m \sin(\omega t + 90^\circ)$$

- The applied voltage $V = V_m \sin\omega t$ and its resultant current
  $i = I_m \sin(\omega t + \frac{\pi}{2})$
- The applied voltage and resultant current are in out of phase then the phase difference is $90°$ (leading)
- The resultant current leads applied voltage by $90°$.

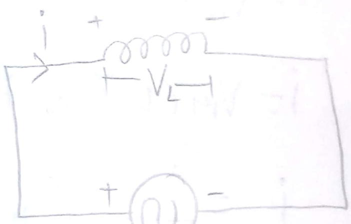


Phasor diagram



AC response of the pure inductor:

- The AC voltage is applied to the pure inductor circuit then its applied voltage is $V = V_m \sin\omega t$
- The applied voltage $\Rightarrow V = V_m \sin\omega t$



Voltage across inductor is

$$V_L = L \cdot \frac{di}{dt}$$

apply KVL to to above circuit,

$$\forall (t) \quad V - V_L = 0$$

$$\boxed{V = V_L}$$

$V = V_L$

$$V_m \sin\omega t = L \cdot \frac{di}{dt}$$

$$\frac{di}{dt} = \frac{V_m \sin\omega t}{L}$$

$$\frac{di}{dt} = \frac{V_m}{L} \sin\omega t$$

Apply $\int$ on both sides

$$\int \frac{di}{dt} = \frac{V_m}{L} \int \sin\omega t \cdot dt$$

$$i = \frac{V_m}{L} \int \sin\omega t \cdot dt$$

$$i = \frac{V_m}{L} \left[-\frac{\cos\omega t}{\omega} \right]$$

$$i = \frac{V_m}{\omega L} (-\cos\omega t)$$

$$\boxed{\begin{gathered} X_L = 2\pi f L \\ X_L = \omega L \end{gathered}}$$

$$i = \frac{V_m}{X_L} (-\cos\omega t)$$

$$I_m = \frac{V_m}{X_L}$$

$$i = I_m (-\cos\omega t)$$

$$i = I_m (-\sin(90 - \omega t))$$

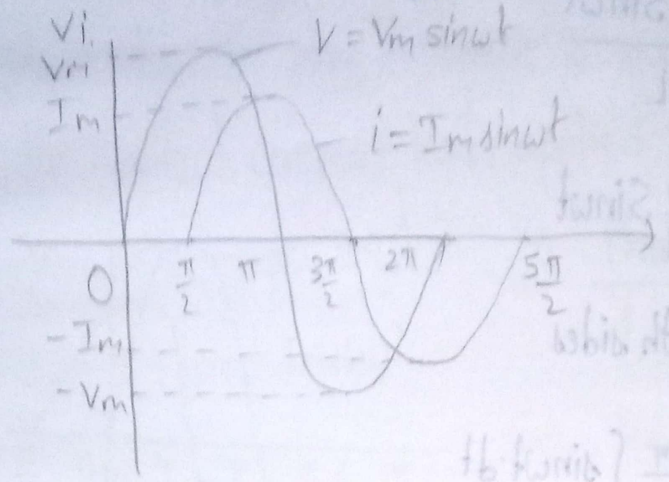
$$i = I_m \sin(\omega t - 90^\circ)$$

$$i = I_m \sin\left(\omega t - \frac{\pi}{2}\right)$$

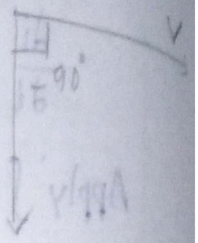
- The applied voltage $V = V_m \sin\omega t$ and resultant current $i = I_m \sin\left(\omega t - \frac{\pi}{2}\right)$

• The applied voltage & resultant current are in out of phase then phase difference = -90°

• The resultant current laggs applied voltage by 90°.



Phasor diagram



Pb: An alternating current is represented by $i = 220\sin\left(157t + \frac{\pi}{2}\right)$ calculate the (a) frequency (b) Timeperiod (c) peak value (d) mean value (e) RMS value

sol: $i = 220\sin\left(157 + \frac{\pi}{2}\right)$

$I_m = 220 \quad \omega = 157 \quad \theta = \frac{\pi}{2}$

(a) $\omega = 2\pi f$

$157 = 2\pi f$

$f = \frac{157}{2\times\frac{22}{7}} = 24.97 \approx 25\,Hz \quad \boxed{\therefore f \Rightarrow 25\,Hz}$

(b) Time period $\Rightarrow f = \frac{1}{T}$

$\Rightarrow T = \frac{1}{f}$

$= \frac{1}{25}$

$\boxed{T = 0.04\,sec}$

(c) peak value (I_m)

$I_m = 220$

(d) Mean value

$$I_{avg} = \frac{2I_m}{\pi} = \frac{2\times220}{\pi} = \frac{440}{\pi} = 140.12$$

$I_{avg} = 140.12$

(e) R.M.S value

$$I_{R.M.S} = \frac{I_m}{\sqrt{2}}$$

$$I_{R.M.S} = \frac{220}{1.414}$$

$$\boxed{I_{RMS} = 155.58 \text{ amp}}$$

A sinusodial wave form is given by $I = 10\sin(6284t + 10^\circ)$ find it's (a) peak value (b) R.M.S value (c) frequency (d) Time period.

sol: (a) peakvalue $\Rightarrow$ 10

$I = 10\sin(6284t + 10^\circ)$

$I = I_m \sin(\omega t + \theta)$

$I_m = 10 \quad \omega = 6284 \quad \theta = 10^\circ$

(a) peak value (I_m) $= 10$

(b) R.M.S value

$$I_{R.M.S} = \frac{I_m}{\sqrt{2}} = \frac{10}{1.414} = 7.07 \text{ amp}$$

(c) Frequency

$\omega = 6284$

$2\pi f = 6284$

$f = \frac{6284}{2\pi} = \frac{6284}{6.28} = 1000\,Hz = 1\,kHz$

(d) Time period

$f = \frac{1}{t} \Rightarrow f = \frac{1}{1\times10^{3}} = 1\times10^{-3}\,ms$

• An alternating current of frequency 50 Hz has the maximum value of 100 amp. Write down the equation for its instantaneous value.

sol: $f = 50\,Hz$

$I_m = 100\,Amp$

$i = I_m \sin(\omega t + \theta)$

$\theta = 0$

$\omega = 2\pi f$

$\omega = 2\pi\times50 = 100\pi$

$\omega = 100\times3.14 = 314$

$\therefore\ i = 100\sin 314t$

• A sinusoidal voltage $V = 200\sin 314t$ is applied to a resistor of 10Ω resistance. Calculate the (a) R.M.S value of the voltage (b) R.M.S value of the current (c) power dissipated as heat in Watts

sol: a Given $V = 200\sin 314t$

Here $V_m = 200$ $R = 10\Omega$

(i) RMS value of voltage

$$V_{R.M.S} = \frac{V_m}{\sqrt{2}} = \frac{200}{\sqrt{2}} = 141.4 \text{ volts}$$

(ii) R.M.S value of current

$$I_{R.M.S} \quad I_m = \frac{V_m}{R} = \frac{200}{10} = 20$$

$$I_{R.M.S} = \frac{I_m}{\sqrt{2}} = \frac{20}{\sqrt{2}} = 14.14 \text{ amp}$$

(iii) Power dissipated

$P = VI$
P = power
V = voltage
I = current

$$P = V_{R.M.S} \times I_{R.M.S}$$
$$= 141.4 \times 14.14 = 1999.396 = 2000 \text{ watts}$$

- An alternating current is given by $I = 141.4 \sin 314t$ find the (a) Maximum value of current (b) frequency (c) time period

sol:- $I = I_m \sin(\omega t + \theta)$

$$I = 141.4 \sin 314t$$

$$\therefore I_m = 141.4 \quad \omega = 314 \quad \theta = 0^\circ$$

(a) Maximum value of current $I_m = 141.4$

(b) frequency

$$\omega = 2\pi f \Rightarrow f = \frac{\omega}{2\pi} = \frac{314}{2 \times 3.14} = \frac{314}{2} \times \frac{100}{314} = 50 \text{ Hz}$$

(c) Time Period

$$T = \frac{1}{f} = \frac{1}{50} = 0.02 \text{ sec}$$

• Find the peak value, average value and RMS value of an alternating current is represented by $I = 28.4 \sin 628t$

sol: $I = I_m \sin(\omega t + \theta)$

$I_0 = 28.4 \sin 628t$

$I_m = 28.4 \quad \omega = 628$

(i) Peak value $= I_m = 28.4$

(ii) Average value

$$I_{avg} = \frac{2I_m}{\pi} = \frac{2 \times 28.4}{3.14} = 18 \text{ Amp}$$

(iii) R.M.S value

$$I_{R.M.S} = \frac{I_m}{\sqrt{2}} = \frac{28.4}{1.414} = 20 \text{ amp}$$

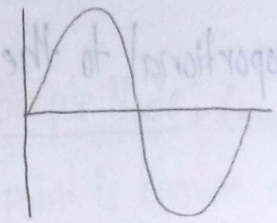
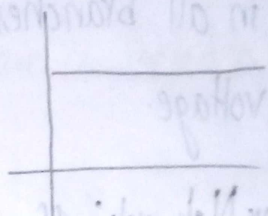
• If AC main supply is 220 volts then find the average e.m.f during a positive half cycle?

$$V_{R.M.S} = \frac{V_m}{\sqrt{2}}$$

$$V_m = V_{RMS} \times \sqrt{2} = 220 \times 1.414 = 311 \text{ Amp Volts}$$

$$V_{avg} = \frac{2V_m}{\pi} = \frac{2 \times 311}{3.14} = 198 \text{ Amp Volts}$$

Difference b/w AC and DC :

A.C	D.C
• A.C means alternating current.	• D.C means direct current.
• It is a signal which changes magnitude and polarity.	• It is a signal which changes magnitude but not polarity.
• Frequency is 50 Hz	• Frequency is '0' Hz.
• It is a bidirectional.	• It is a unidirectional
	
• It provides long distance transmission.	• It provides short distance transmission.
• It is produced from the powerstation.	• It is produced by battery.
• Distribution efficiency is high	• Distribution efficiency is low.
• Power consumption is low.	• Power consumption is high.
• Power factor is lies between 0 to 1.	• Power factor is '1'
• Use of transformer is possible	
• AC motors, domestic and industrial supply applications.	• DC machines, HBV DC system applications.