Unit-3

Resistance

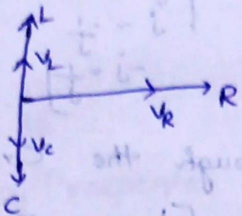
Reactance — Capacitive Reactance $X_c = \frac{1}{\omega C}$; Inductive Reactance $X_L = \omega L$

Impedance : $Z = R + jX$ → Reactance

R ↓ Resistance

Effective Resistance offered by the circuit

↓ overall



$$Z = R - jX_c$$
$$= R - j\frac{1}{\omega C}$$
$$= R + \frac{1}{j\omega C}$$

j = Imaginary

$j^2 = -1$

$j \cdot j = -1$

$j = \frac{-1}{j}$

a+ib

$$Z = R + jX_L$$
$$= R + j\omega L$$

$$\Rightarrow Z = R + jX_L - jX_C$$

Frequency Response of RC Circuit :- Low pass filter

RC circuit has low pass filter [which allows lower frequency and attenuate (block) high Frequency]

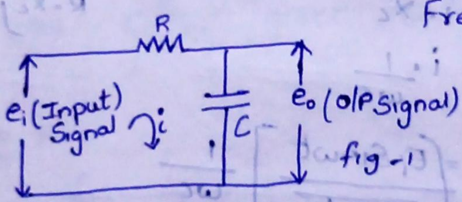


fig-1

The above figure-1 shows RC low pass filter circuit

Frequency Response :-

It is a graph, x-axis - frequency
Y-axis - Gain

$$\text{Gain} = \frac{\text{output}}{\text{Input}}$$

⇒ This circuit allows lower frequencies to the output and blocks high frequency signal.

⇒ The Capacitive Reactance is Inversely proportional to the frequency of the signal (i.e) $X_c \propto \frac{1}{f}$ $\left[X_c = \frac{1}{\omega C},\ X_c = \frac{1}{2\pi f}\right]$

⇒ for low frequency signal the Capacitive Reactance is high and hence the voltage drop across the Capacitor is high.

⇒ for high frequency signals the Capacitive Reactance is low and hence voltage drop across the Capacitor is low.

⇒ The Input Voltage is $e_i = E_i \sin\omega t \longrightarrow (1)$

the Total Impedance of the RC Circuit is

$$Z = R + X_C \Rightarrow R + jX_C$$

$$= R - jX_C$$

e_o - Voltage drop Across the Capacitor

$$= R + \frac{1}{j\omega C}$$

$$\left[\begin{array}{l} j^2 = -1 \\ j = -\frac{1}{j} \\ -j = \frac{1}{j} \end{array}\right]$$

$$|Z| = \sqrt{R^2 + \frac{1}{\omega^2 C^2}}$$

The Current passing through the Circuit is given by

$$i = \frac{e_i}{|Z|} \qquad \left[i = \frac{V}{R} = \frac{\text{Input Voltage}}{R} \right]$$

$$= \frac{E_i \sin\omega t}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}}$$

The output Voltage is given by Voltage drop Across the Capacitor

$$e_o = i \cdot X_C \qquad \left[V = IR - R \cdot X_C \right]$$

$$= i \cdot \frac{1}{\omega C}$$

$$e_o = \left(\frac{E_i \sin\omega t}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}} \right) \frac{1}{\omega C}$$

The voltage gain of the Circuit is given by -

$$\text{Voltage gain} = A_v = \frac{e_o}{e_i} = \frac{\text{output voltage}}{\text{Input voltage}}$$

$$= \frac{E_i \sin\omega t}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}} \cdot \frac{1}{\omega C} \times \frac{1}{E_i \sin\omega t}$$

$$= \frac{\frac{1}{\omega C}}{\frac{1}{\omega C}\sqrt{R^2 \omega^2 C^2 + 1}}$$

$$= \frac{1}{\sqrt{(R\omega C)^2 + 1}} = \frac{1}{\sqrt{R^2 (2\pi f)^2 C^2 + 1}}$$

$$= \frac{1}{\sqrt{(2\pi f RC)^2 + 1}}$$

$$= \frac{1}{\sqrt{\frac{f^2}{\left(\frac{1}{2\pi RC}\right)^2}+1}} = \frac{1}{\sqrt{\left(\frac{f}{f_2}\right)^2+1}} \qquad \left[\because f_2 = \frac{1}{2\pi RC}\right]$$

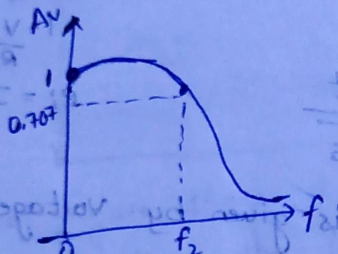
upper cut off frequency of the low pass filter

$$A_v = \frac{1}{\sqrt{\left(\frac{f}{f_2}\right)^2+1}} \Rightarrow \text{Higher}$$

If $f = f_2$

$$A_v = \frac{1}{\sqrt{2}}$$

⇒ the frequency response Curve of Low pass filter



if $f = f_2$ then 0.707

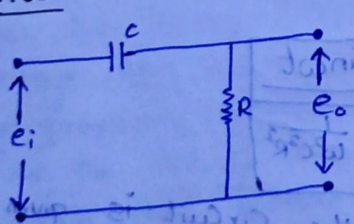
$f = 0$ then $A_v = 1$

x - axis - frequency

y - axis Voltage gain

A graph b/w frequency of the Input Signal and gain of the Circuit gives the frequency response Curve.

<u>**High pass filter**</u> :-



→ The above Circuit Shows high pass RC filter Circuit this Circuit allows the high frequency Signals to the output and blocks the low frequency Signals

⇒ The Capacitive Reactance inversely proportional to frequency of Signal.

$$X_c \propto \frac{1}{f} \qquad \left[X_c = \frac{1}{\omega C} \qquad \omega = 2\pi f\right]$$

⇒ for low frequency Signals the Capacitive Reactance is high and hence the Voltage drop across the Capacitor is high Consequently the voltage drop across Resistor (R) is low.

⇒ for high frequency signals the capacitive Reactance is low hence the voltage drop across the Resistor (R) is high

The Expression for Input Voltage is $e_i = E_i \sin\omega t$ → ①

The Total Impedance of the RC Circuit is

$$Z = R + X_C = R + jX_C$$
$$= R - j\frac{1}{\omega C}$$
$$= R + \frac{1}{j\omega C}$$

$j^2 = -1$

$j \cdot j = -1$

$j = \frac{-1}{j}$

$-j = \frac{1}{j}$

$$|Z| = \sqrt{R^2 + \frac{1}{\omega^2 C^2}}$$

The Current passing through the Circuit is given by

$V = iR$

$i = \frac{V}{R}$

e_i = Input Voltage

$$i = \frac{e_i}{|Z|}$$

$$= \frac{E_i \sin\omega t}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}}$$

The output Voltage is given by voltage drop Across the Resistor (R) is

$V = IR$

$$e_o = iR$$

$$= \left(\frac{E_i \sin\omega t}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}}\right) R = \left(\frac{E_i \sin\omega t}{\cancel{R}\sqrt{1 + \frac{1}{\omega^2 C^2 R^2}}}\right)\cancel{R}$$

$$\boxed{e_o = \frac{E_i \sin\omega t}{\sqrt{1 + \frac{1}{\omega^2 C^2 R^2}}}}$$

The Voltage gain of the Circuit is given by

$$\text{Voltage gain } A_v = \frac{e_o}{e_i}$$

$$= \frac{E_i \sin\omega t}{\sqrt{1 + \left(\frac{1}{\omega CR}\right)^2}} \times \frac{1}{E_i \sin\omega t}$$

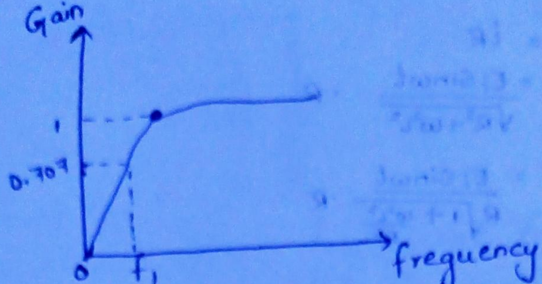
$$= \frac{1}{\sqrt{1 + \left(\frac{1}{\omega CR}\right)^2}} = \frac{1}{\sqrt{1 + \left(\frac{1}{2\pi f CR}\right)^2}}$$

$$A_v = \frac{1}{\sqrt{1 + \left(\frac{f_1}{f}\right)^2}} \qquad \left[\because f_1 = \frac{1}{2\pi RC}\right]$$

lower cut off frequency

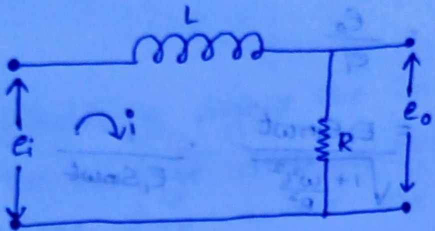
If $f_1 = f$ then $A_v = \frac{1}{\sqrt{2}}$

The frequency Response Curve of the high pass filter is given by



f_1 = Lower Cut off frequency
(f_1 Crosschesaka frequency ni allow chestundhi)

frequency Response of RL circuit as Low pass filter:-



The above figure shows that RL Low pass filter Circuit
This circuit allows the lower frequency

⇒ The Inductive Reactance directly proportional to frequency

$X_L = \omega L$
$\omega L = 2\pi f L$ $\boxed{X_L \propto f}$

→ for Low frequency Signal the Inductive Reactance is Low, So the voltage drop across inductor is Low & the voltage drop across output terminals is high.

→ for high frequency Signals the Inductive Reactance is high, So the Voltage drop across inductor is high & the Voltage drop across output terminals is low.

The Expression for Input voltage is $e_i = E_i \sin\omega t$ ⟶ ①

The Total Impedance of the RL circuit is

$$Z = R + X_L$$
$$= R + j\omega L$$
$$Z = R + j\omega L$$
$$|Z| = \sqrt{R^2 + \omega^2 L^2}$$

The Current passing through the Circuit is given by

$$i = \frac{e_i}{|Z|}$$
$$= \frac{E_i \sin\omega t}{\sqrt{R^2+\omega^2 L^2}}$$

The output Voltage is given by Voltage drop across the R is

$$e_0 = iR$$

$$= \frac{E_i \sin\omega t}{\sqrt{R^2+\omega^2 L^2}} \cdot R$$

$$= \frac{E_i \sin\omega t}{R\sqrt{1+\frac{\omega^2 L^2}{R^2}}} \cdot R$$

$$e_0 = \frac{E_i \sin\omega t}{\sqrt{1+\frac{\omega^2 L^2}{R^2}}}$$

The Voltage gain of the Circuit is given by

$$A_v = \frac{e_0}{e_i}$$

$$= \frac{E_i \sin\omega t}{\sqrt{1+\frac{\omega^2 L^2}{R^2}}} \cdot \frac{1}{E_i \sin\omega t}$$

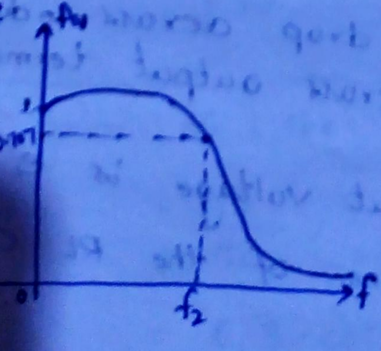
$$= \frac{1}{\sqrt{1+\frac{(2\pi f L)^2}{R^2}}} = \frac{1}{\sqrt{1+\frac{f^2}{R^2 \cdot \frac{1}{(2\pi L)^2}}}}$$

$$= \frac{1}{\sqrt{1+\left(\frac{f}{f_2}\right)^2}} \qquad f_2 = \frac{R}{2\pi L}$$

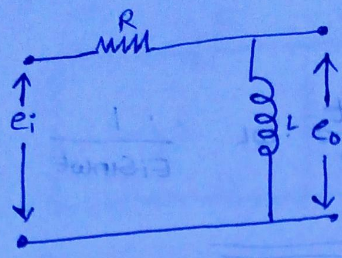
upper Cut off frequency

If $f = f_1$ $\quad A_v = \frac{1}{\sqrt{2}}$

Frequency Response Curve of Low pass filter



High pass filter :-



The above Circuit show high pass filter RL filter Circuit this Circuit allows the high frequency Signals to the output and blocks the low frequency.

⇒ Inductive Reactance directly proportional to the frequency of Signal

$X_L \propto f$

⇒ for Low frequency signal the Inductive Reactance is Low, so the voltage drop across the Inductor is low and voltage drop across the output terminal Inductor is ~~high~~ Low

⇒ for high frequency signal the Inductive Reactance is high hence the voltage drop across the Inductor is high

The Expression for Input Voltage is $e_i = E_i \sin\omega t$ → ①

The Total Impedance of the Circuit is

$$Z = R + X_L$$

$$= R + j\omega L$$

$$|Z| = \sqrt{R^2+\omega^2L^2}$$

The Current passing through the Circuit is given by

$$i = \frac{e_i}{|Z|}$$

$$= \frac{E_i \sin\omega t}{\sqrt{R^2+\omega^2L^2}}$$

The output Voltage is given by voltage drop across the Inductor is

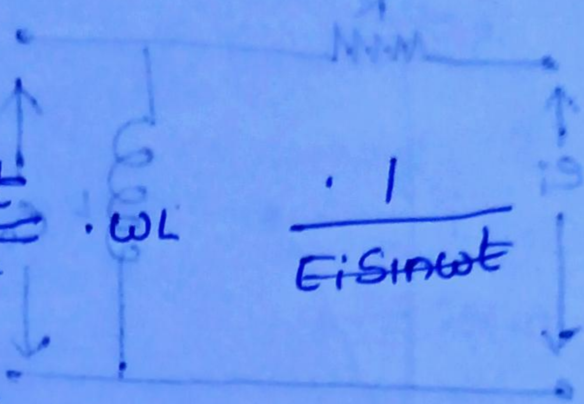
$$e_o = iX_L$$

$$= i\omega L$$

$$e_o = \frac{E_i \sin\omega t}{\sqrt{R^2+\omega^2L^2}}\,\omega L \longrightarrow ②$$

The voltage gain of the Circuit is given by

$$A_v = \frac{e_o}{e_i}$$



$$= \frac{E_i \sin\omega t}{\sqrt{R^2+\omega^2L^2}} \cdot \omega L \cdot \frac{1}{E_i \sin\omega t}$$

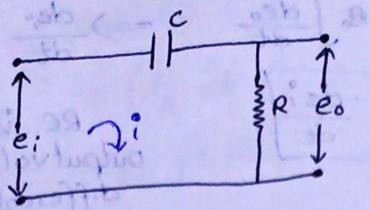
$$= \frac{\omega L}{\omega L\sqrt{1+\left(\frac{R}{\omega L}\right)^2}}$$

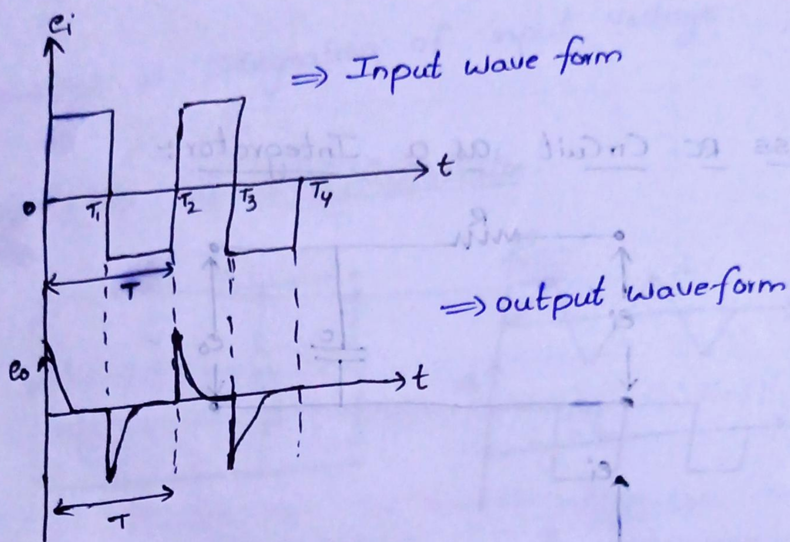
$$= \frac{1}{\sqrt{1+\left(\frac{R}{2\pi f L}\right)^2}}$$

$$= \frac{1}{\sqrt{1+\left(\frac{f_1}{f}\right)^2}}$$ here $f_1 = \frac{R}{2\pi L}$

If $\Rightarrow f = f_1$, $A_v = \frac{1}{\sqrt{2}} = 0.707$

High pass RC Circuit as a differentiator :





Condition for differentiator

The time Constant (RC) is smaller than the time period of Input (T) then high pass circuit act as the differentiator (RC << T)

⇒ When time period is large the frequency is small

⇒ When low frequency signal is applied to High pass RC (HPRC) Circuit the reactance of the Capacitor is high, Compared to $\left[X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} \right]$ the Resistor So most of the Input Voltage is appear across the Capacitor and very Very Small amount of Voltage drop across the Resistor

iR value is negligible because Vin resistor differentiator become low $V = IR$

As per KVL

$$e_i = e_c + e_R$$

$$e_i = \frac{1}{C}\int i \, d(t) + i.R$$

$i.R$ Value is Very Small, So neglect this value

$$e_i = \frac{1}{C}\int i \, dt$$

$$e_i = \frac{1}{C}\int \frac{e_o}{R} dt$$

$$e_i = \frac{1}{RC}\int e_o \, dt$$

[RC << T
T is high
$T \propto \frac{1}{f}$ f is low
X_C is high
∴ Resistor Voltage low]

$i = \frac{dq}{dt}$

$q = CV$ → Constant

$i = C \cdot \frac{dv}{dt}$

$\frac{i}{C} = \frac{dv}{dt}$

$\int \frac{i}{C} = \int \frac{dv}{dt} \Rightarrow V = \frac{1}{C}\int i \, dt$

differentiation on B.S

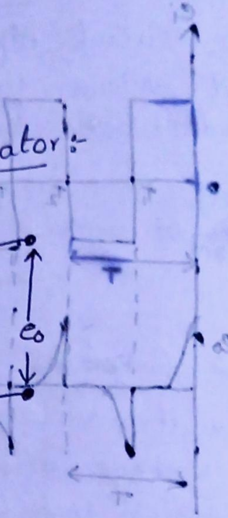
$$\frac{de_i}{dt} = \frac{1}{RC} \int \frac{de_o}{dt} \Rightarrow \frac{de_i}{dt} = \frac{1}{RC} e_o$$

Voltage across is low

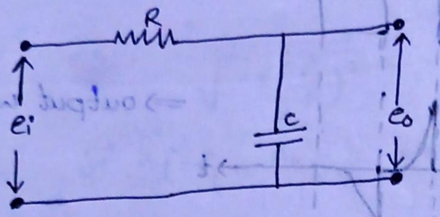
$$\boxed{e_o = RC \cdot \frac{de_i}{dt}}$$

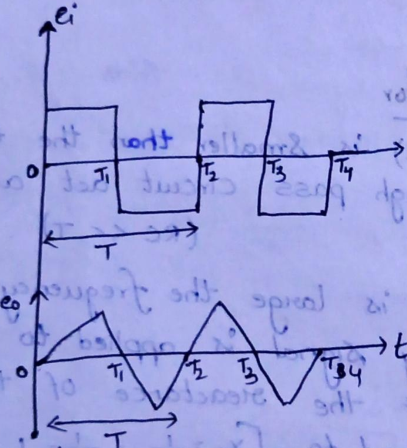
RC is time Constant
outputvoltage is differentiator of Input

th.



Low pass RC Circuit as a Integrator:-





Condition for Integrator

The time constant (RC) is higher than the time period of Input then Low pass filter act as a Integrator

⇒ when RC is greator the time period "t" is small So frequency is Large, then Reactance of Capacitor X_c is very very small (due to f high), So Voltage drop across the Capacitor is small Compared to the Resistor.

$e_i = e_c + e_R$

$e_i = e_R$ (∴ $e_c \approx$ Small or zero)

$e_i = i \cdot R$

$$\boxed{i = \frac{e_i}{R}}$$

RC >> T

$T \propto \frac{1}{f}$

frequency high

$X_c \propto \frac{1}{f}$

Capacitive Reactance low so negligible

output is taken a-cross Capacitor c

$$e_o = \frac{1}{c}\int i\,dt$$

$$= \frac{1}{c}\int \frac{e_i}{R}\,dt$$

$$= \frac{1}{RC}\int e_i\,dt$$

$i = \frac{dq}{dt}$

$q = C.V$

$i = C.\frac{dv}{dt}$

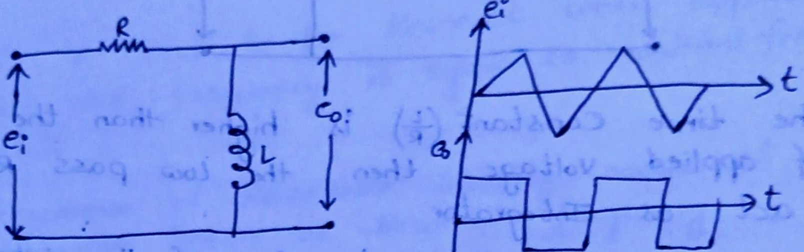
$\frac{i}{c} = \frac{dv}{dt}$

$v = \frac{1}{c}\int i.dt$

e_o

The output is Integration of input voltage

RL high pass filter as a differentiator:



The time constant $\left(\frac{L}{R}\right)$ is smaller than the time period of applied voltage, then High pass RL Circuit act as differentiator.

⇒ The applied input voltage is sum of voltage drop across inductor and Resistor.

$$e_i = e_L + e_R$$

$$e_i = L\frac{di}{dt} + i.R$$

Divide both side with R

$$\frac{e_i}{R} = \frac{L}{R}\frac{di}{dt} + i$$

$$i = \frac{e_i}{R} - \frac{L}{R}\frac{di}{dt}$$

from condition $\frac{e_i}{R} >> \frac{L}{R}\frac{di}{dt}$

So the term $\frac{L}{R}\frac{di}{dt}$ negligible

$$\boxed{i = \frac{e_i}{R}}$$

o/p Voltage across inductor

$$e_o = L.\frac{di}{dt}$$

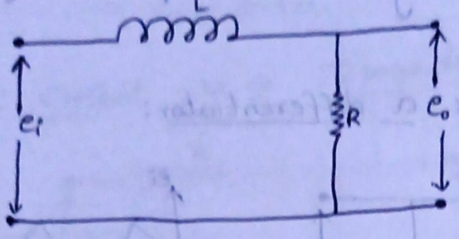
Sub: Value in above eqn

$$e_o = \frac{L}{R} \cdot \frac{de_i}{dt}$$

$$e_o \propto \frac{de_i}{dt}$$

here $\frac{L}{R}$ is time Constant

RL Low pass filter as a Integrator :-



→ The time Constant $\left(\frac{L}{R}\right)$ is higher than the time period of applied Voltage then the Low pass RL Circuit act as Integrator

⇒ The applied input Voltage is Sum of the Voltage drop across inductor and Resistor

$RC >> T$

$T \propto \frac{1}{f}$

$X_L \propto \omega L$

$$e_i = e_L + e_R$$

$$= L \cdot \frac{di}{dt} + i \cdot R$$

Divide both sides with R

$$\frac{e_i}{R} = \frac{L}{R}\frac{di}{dt} + i$$

$$i = \frac{e_i}{R} - \frac{L}{R}\frac{di}{dt}$$

from Condition $\frac{e_i}{R} << \frac{L}{R}\frac{di}{dt}$

So the term $\frac{e_i}{R}$ negligible

$$i = -\frac{L}{R}\frac{di}{dt}$$

o/p Voltage across inductor

$$e_o =$$

(or)

$$e_i = e_L + e_R$$

$$= L \cdot \frac{di}{dt} + i \cdot R$$

from Condition $\frac{e_i}{R} << \frac{L}{R}\frac{di}{dt}$

$e_i = L \cdot \frac{di}{dt}$ $\Rightarrow \int e_i dt = L \int \frac{di}{dt} dt$ — $\int e_i dt = Li$

$i = \frac{\int e_i dt}{L}$

o/p voltage across inductor

$e_o = i \cdot R$

$e_o = \int \frac{e_i}{L} \cdot R \, dt$

$e_o = \frac{R}{L} \int e_i dt$

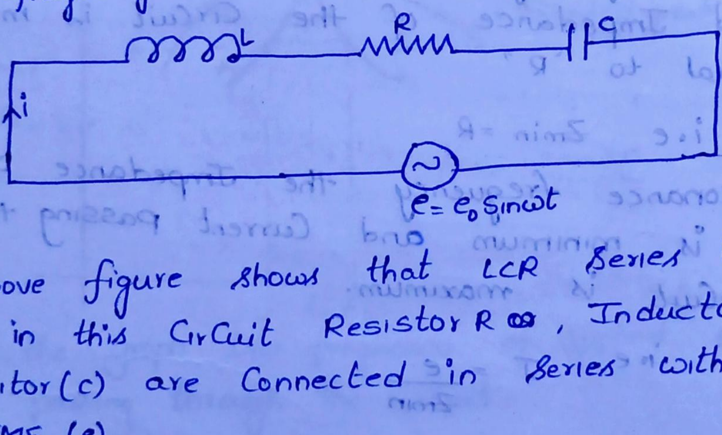
LCR Series Resonance Circuit :-

(Resonance)

the Applied frequency = Natural frequency (LCR)

⇒ A circuit is said to be Resonant when applied Sinusodial emf frequency is equal to Natural frequency of circuit

⇒ the phenomenon of Resonance usually occurs at single frequency known as Resonant frequency



$e = e_0 \sin \omega t$

⇒ The above figure shows that LCR Series Resonant Circuit in this circuit Resistor R, Inductor (L), Capacitor (C) are connected in series with a source of EMF (e)

⇒ The Applied emf is $e = e_0 \sin \omega t$

Here e_0 = peak value of voltage.

ω = Angular frequency $= 2\pi f$

f = frequency of the signal

⇒ The Impedance of the Circuit

Total opposition in Circuit

⇑

[Impedance = Resistance + Reactance]

⇓

opposition flow by "C" or L

$Z = R + X_c + X_L$

$= R + \frac{1}{j\omega C} + j\omega L$

$= R - \frac{j}{\omega C} + j\omega L$

$j \cdot j = -1$

$j = \frac{-1}{j}$

$Z = R + j\left(\omega L - \frac{1}{\omega C}\right)$

In the above eqn $j(\omega L - \frac{1}{\omega C})$ is the Reactance Component of Impedance.

⇒ In Series LCR circuit at resonance this reactance term is zero. So

$$j(\omega L - \frac{1}{\omega C}) = 0 \qquad j \neq 0,$$

$$\omega L - \frac{1}{\omega C} = 0$$

$$\omega L = \frac{1}{\omega C}$$

$$\omega^2 = \frac{1}{LC}$$

$$(2\pi f)^2 = \frac{1}{LC}$$

$$2\pi f = \frac{1}{\sqrt{LC}}$$

$$f = \frac{1}{2\pi\sqrt{LC}}$$

⇒ This is the resonance frequency of circuit at resonance frequency the reactance component is zero and Impedance of the circuit is minimum and equal to "R"

i.e $Z_{min} = R$

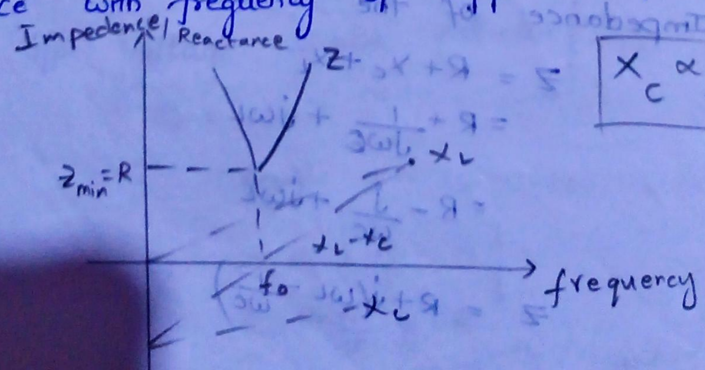
⇒ at Resonance frequency the Impedance of the circuit is minimum and current passing through the circuit is maximum.

$$I = \frac{e}{Z_{min}}$$

$$\Rightarrow I = \frac{e}{R}$$

⇒ at Resonance the circuit allows maximum and hence it called "Acceptor circuit".

⇒ The below figure shows the variation of Impedance/reactance with frequency of applied emf.



$$\boxed{X_C \propto \frac{1}{f}}$$

⇒ 1) at low frequencies, the Inductive reactance is low and Capacitive reactance is high and hence resultant reactance is high.

i.e $\frac{1}{\omega C} > \omega L$

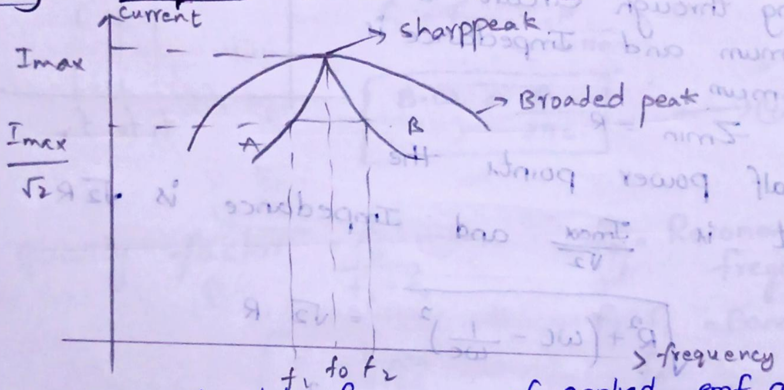
2) at high frequencies the Inductive Reactance is high and Capacitive reactance is low and hence resultant reactance is low i.e, $\frac{1}{\omega C} < \omega L$

3) At a particular frequency known as resonant frequency, both Capacitive & Inductive reactance are equal and resultant reactance is zero

4) At Resonant frequency $\frac{1}{\omega C} = \omega L$

5) At this frequency, the Circuit is pure resistive Circuit

* frequency response Curve of LCR Series Circuit



⇒ This is the graph b/w frequency of applied emf and Current passing through the Circuit.

⇒ At Resonance frequency, the Current passing through the Circuit is Maximum

⇒ A & B are the half power points. At these points the Current is $\frac{I_{max}}{\sqrt{2}}$.

⇒ The frequencies Corresponding to these half power points are known as Cut off frequencies. Here

f_1 = lower Cut off frequency.

f_2 = Higher Cut off frequency.

⇒ The Ratio of resonance frequency and band width is called quality factor(Q)

$Q = \frac{f_0}{B.W}$ where $B.W = f_2 - f_1$ ⇒ difference b/w the higher Cut off frequency and the lower Cut off frequency is B.W

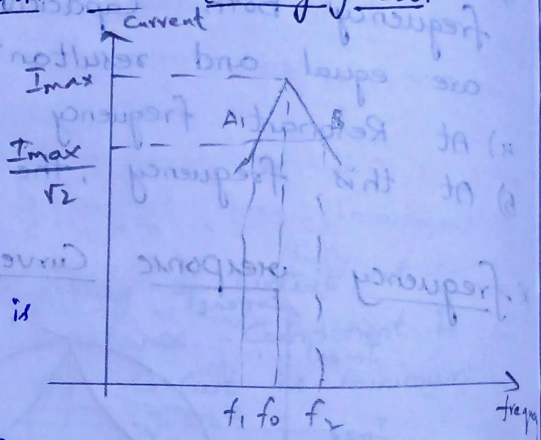
$Q = \frac{f_0}{f_2 - f_1}$

⇒ At "fo" the Current Value is Maximum (I_{max})

⇒ If LCR Circuit having low resistance and resonant Curve has sharp peak Value and high Selectivity (Q factor)

⇒ LCR Series circuit having high Resistance, the resonant Curve has broadend peak and poor Selectivity (Q factor)

Expression for Band width and Quality factor



⇒ The expression for Impedance of LCR Circuit is $Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$

at Resonance the Current passing through Circuit is Maximum and Impedance is Minimum

$$Z_{min} = R$$

at half power points the Current is $\frac{I_{max}}{\sqrt{2}}$ and Impedance is $\sqrt{2}R$

$$\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} = \sqrt{2} \cdot R$$

$$R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2 = 2R^2$$

$$\left(\omega L - \frac{1}{\omega C}\right)^2 = R^2$$

$$\omega L - \frac{1}{\omega C} = \pm R$$

At half power point ω have two Values ω_1 & ω_2

$$\omega_2 L - \frac{1}{\omega_2 C} = +R \longrightarrow (1)$$

$$\omega_1 L - \frac{1}{\omega_1 C} = -R \longrightarrow (2)$$

eq (1) + eq (2)

$$\omega_2 L + \omega_1 L - \frac{1}{\omega_2 C} - \frac{1}{\omega_1 C} = 0$$

$$(\omega_1 + \omega_2)L - \frac{1}{C}\left(\frac{1}{\omega_1} + \frac{1}{\omega_2}\right) = 0$$

$$(\omega_1 + \omega_2)L = \frac{1}{C}\left(\frac{\omega_1 + \omega_2}{\omega_1 \omega_2}\right)$$

$$\omega_1 \omega_2 = \frac{1}{LC} \longrightarrow (3)$$

eq(1) - eq(2)

$$\omega_2 L - \omega_1 L - \frac{1}{\omega_2 C} + \frac{1}{\omega_1 C} = 2R$$

$$L(\omega_2 - \omega_1) + \frac{1}{C}\left(\frac{1}{\omega_1} - \frac{1}{\omega_2}\right) = 2R$$

$$L(\omega_2 - \omega_1) + \frac{1}{C}\left(\frac{\omega_2 - \omega_1}{\omega_1 \omega_2}\right) = 2R$$

from (3)

$$L(\omega_2 - \omega_1) + \frac{1}{C}\left(\frac{\omega_2 - \omega_1}{1/LC}\right) = 2R$$

$$L(\omega_2 - \omega_1) + \frac{LC}{C}(\omega_2 - \omega_1) = 2R$$

$$2L(\omega_2 - \omega_1) = 2R$$

$$\omega_2 - \omega_1 = \frac{R}{L}$$

$$2\pi f_2 - 2\pi f_1 = \frac{R}{L}$$

$$f_2 - f_1 = \frac{R}{2\pi L}$$

$$\boxed{B.\omega = \frac{R}{2\pi L}} \longrightarrow (4)$$

Quality factor $Q = \frac{f_0}{f_2 - f_1}$ f_0 = Resonant frequency

$f_2 - f_1$ = Band width

$$= \frac{\frac{1}{2\pi\sqrt{LC}}}{\frac{R}{2\pi L}} \qquad \frac{f_0}{\frac{R}{2\pi f_0 L}}$$

$$= \frac{2\pi L}{2\pi\sqrt{LC}\, R} \qquad = \frac{2\pi f_0 L}{R} = \frac{\omega_0 L}{R}$$

At Resonance $\omega_0 L = \frac{1}{\omega_0 C}$

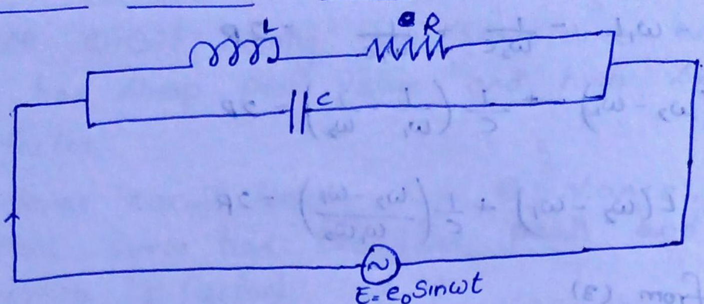
$$\boxed{Q = \frac{1}{\omega_0 C R}}$$ If Capacitance Given

→ Sharpness of Resonance is the Ratio of Band width and Resonant frequency.

Q = quality factor

$$SR = \frac{B.\omega}{f_0} = \frac{1}{Q}$$

LCR parallel Resonance CirCuit :-



In above figure shows LCR parallel Resonance CirCuit in which the first branch Consist of the Inductor(L) Resistor(R) and Second branch Consist Capacitor(C)

=> The Applied AC Voltage is $E = e_0 \sin\omega t$

Here $\omega = 2\pi f$ f = frequency of applied voltage

Z_1 = Impedance of first branch

$$Z_1 = R + X_L$$

$$\boxed{Z_1 = R + j\omega L} \longrightarrow ①$$

Z_2 = Impedance of Second branch

$$Z_2 = X_C$$

$$\boxed{Z_2 = \frac{1}{j\omega C}} \longrightarrow ②$$

Let us Assume the resonant Impedance of LCR parallel CirCuit is Z

$$\frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2} \quad \text{(parallel)}$$

$$= \frac{1}{R+j\omega L} + \frac{1}{1/j\omega C}$$

$$= \frac{1}{R+j\omega L} + j\omega C$$

$$= \frac{R - j\omega L}{R^2+\omega^2L^2} + j\omega C \qquad j^2 = -1$$

$$\boxed{\frac{1}{Z} = \underbrace{\frac{R}{R^2+\omega^2L^2}}_{\text{Realpart}} + j\underbrace{\left(\omega C - \frac{\omega L}{R^2+\omega^2L^2}\right)}_{\text{Imaginary}}}$$

We Know that admittance of the CirCuit is Reciprocal to Impedance.

$$y = \frac{1}{Z}$$

$$y = \frac{R}{R^2+\omega^2L^2} + j\left(\omega C - \frac{\omega L}{R^2+\omega^2L^2}\right) \longrightarrow ③$$

In the above eqn the first term is real quantity and 2nd term is Imaginary quantity (Reactance Component)

⇒ At Reasonance the Reactance Component is 0.

So $j\left(\omega C - \frac{\omega L}{R^2+\omega^2L^2}\right) = 0$ Hence j is $\neq 0$

$$\omega C = \frac{\omega L}{R^2+\omega^2 L^2} \Rightarrow C = \frac{L}{R^2+\omega^2L^2}$$

$$R^2+\omega^2L^2 = \frac{L}{C}$$

$$\omega^2L^2 = \frac{L}{C} - R^2$$

$$\omega^2 = \frac{\frac{L}{C} - R^2}{L^2} = \frac{1}{LC} - \frac{R^2}{L^2}$$

$$\omega = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

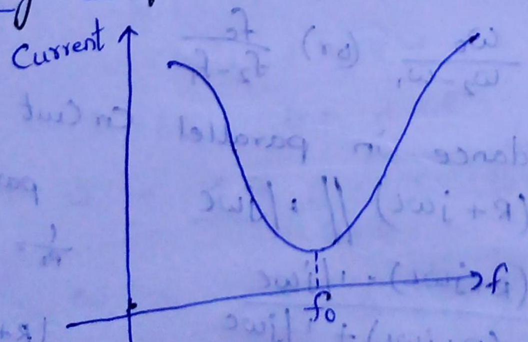
$$2\pi f = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$$\boxed{f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}}$$ f_0 = Resonant frequency

f_0 is Resonant frequency of LCR parallel CirCut.

⇒ At Reasonance the admittance of the CirCuit is Minimum and Impedance of the CirCut is Maximum. Hence it allows minimum Current at Resonant frequency. So it is Called Rejected CirCuit.

Frequency Response of LCR parallel CirCut :-



f_0 daggara Current Min

⇒ At Reasonance the admittance of the CirCut is Min and Impedance of the CirCut is Max. Hence it allows Min Current at Resonant frequency.

Dynamic Resistance (R_d):

The dynamic Resistance of LCR parallel circuit is defined as it is the Impedance of the circuit at Reasonance.

At Resonance Component

$$j\left(\omega C - \frac{\omega L}{R^2+\omega^2L^2}\right) = 0 \quad \rightarrow j \neq 0$$

$$\omega C = \frac{\omega L}{R^2+\omega^2L^2}$$

$$R^2+\omega^2L^2 = \frac{\omega L}{\omega C} = \frac{L}{C}$$

$$R^2+\omega^2L^2 = \frac{L}{C} \longrightarrow (1)$$

$$Y_{min} = \frac{1}{Z_{max}} = \frac{R}{R^2+\omega^2L^2}$$

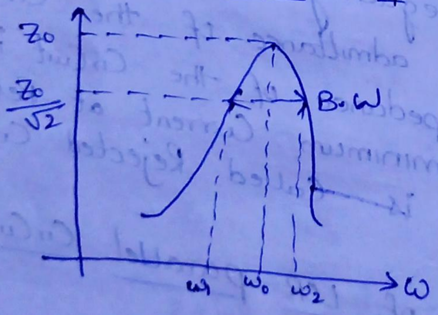
y minimum ay the z maximum

$$Z_{max} = \frac{R^2+\omega^2L^2}{R}$$

from (1)

$$Z_{max} = \frac{L}{RC}$$

Selectivity and Band width of parallel Circuit :



$$Q = \frac{\omega_0}{\omega_2-\omega_1} \text{ (or) } \frac{f_0}{f_2-f_1}$$

⇒ The Total Impedance in parallel Circuit is given by

$$Z = (R+j\omega L) \parallel 1/j\omega C$$

$$= \frac{(R+j\omega L)\cdot 1/j\omega C}{(R+j\omega L)+1/j\omega C}$$

$\omega L >> R$ R is neglected

only numerator R can be neglected

$$Z = \frac{j\omega L/j\omega C}{j\omega CR-\omega^2LC+1}$$

parallel

$$\frac{1}{R_t} = \frac{R_1R_2}{R_1+R_2}$$

$$(R+j\omega L)+\frac{1}{j\omega C}$$

$$R+j\omega L+\frac{1}{j\omega C}$$

$$\frac{Rj\omega C + j^2\omega^2CL+1}{j\omega C}$$

$$Z = \frac{-j\omega L}{(\omega^2 LC - 1) - j\omega CR}$$

$$|Z| = \frac{\omega L}{\sqrt{(\omega^2 LC - 1)^2 + (\omega CR)^2}}$$

Imaginary part only Squaring $\sqrt{a^2+b^2}$

Multiply both numerator & denominator with $\frac{1}{\omega CR}$

$$Z = \frac{\omega L / \omega CR}{\frac{1}{\omega CR}\sqrt{(\omega^2 LC - 1)^2 + (\omega CR)^2}}$$

$$Z = \frac{L / CR}{\frac{1}{\omega CR}\sqrt{(\omega^2 LC - 1)^2 + (\omega CR)^2}}$$

$$= \frac{Z_0}{\frac{1}{\omega CR}\sqrt{(\omega^2 LC - 1)^2 + (\omega CR)^2}} \longrightarrow (1)$$

At half power points $Z = \frac{Z_0}{\sqrt{2}} \longrightarrow (2)$ Impedance

Given by from 1 & 2

$$\sqrt{2} = \frac{1}{\omega CR}\sqrt{(\omega^2 LC - 1)^2 + (\omega CR)^2}$$

$$\sqrt{2}\,\omega CR = \sqrt{(\omega^2 LC - 1)^2 + (\omega CR)^2}$$

$$2(\omega CR)^2 = (\omega^2 LC - 1)^2 + (\omega CR)^2$$

$$(\omega CR)^2 = (\omega^2 LC - 1)^2$$

$$\omega^2 LC - 1 = \pm \omega CR$$

$$\omega^2 LC \pm \omega CR - 1 = 0$$

$a = LC$, $b = \pm CR$, $c = -1$

$$\omega = \frac{\pm CR \pm \sqrt{(CR)^2 + 4LC}}{2LC} \qquad \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\omega = \frac{\pm R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

Here $\frac{R}{2L}$ is very small as compared to $\frac{1}{LC}$

$$\omega = \pm \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

$$\omega = \omega_0 \pm \frac{1}{\sqrt{LC}}$$

Take the ~~-ve~~ +ve values of ω_0 (ω_0 = frequency)

$$\omega_1 = \omega_0 - \frac{1}{\sqrt{LC}}$$

$$\omega_2 = \omega_0 + \frac{1}{\sqrt{LC}}$$

$$\text{Band width} = \omega_2 - \omega_1$$

$$= \omega_0 + \frac{1}{\sqrt{LC}} - \omega_0 + \frac{1}{\sqrt{LC}}$$

$$= +\frac{2}{\sqrt{LC}}$$

$$Q = \frac{\omega_0}{\omega_2 - \omega_1} = \frac{\omega_0}{\frac{2}{\sqrt{LC}}} = \frac{\omega_0\sqrt{LC}}{2}$$