1)

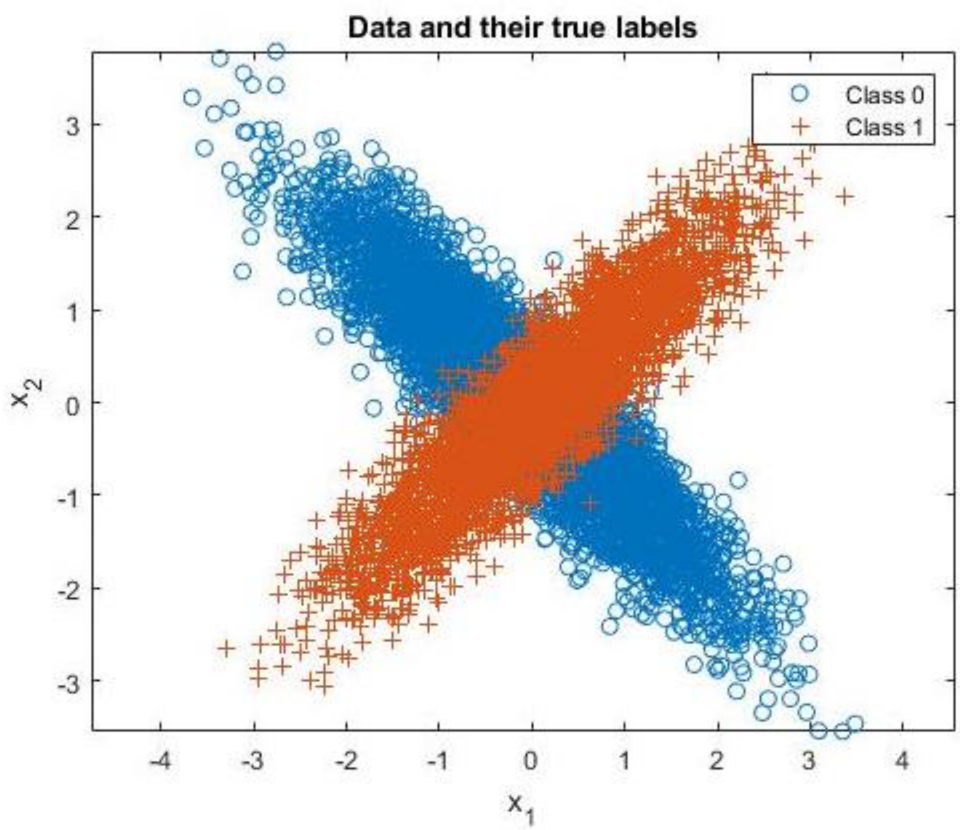
The likelihood ratio test is given by,

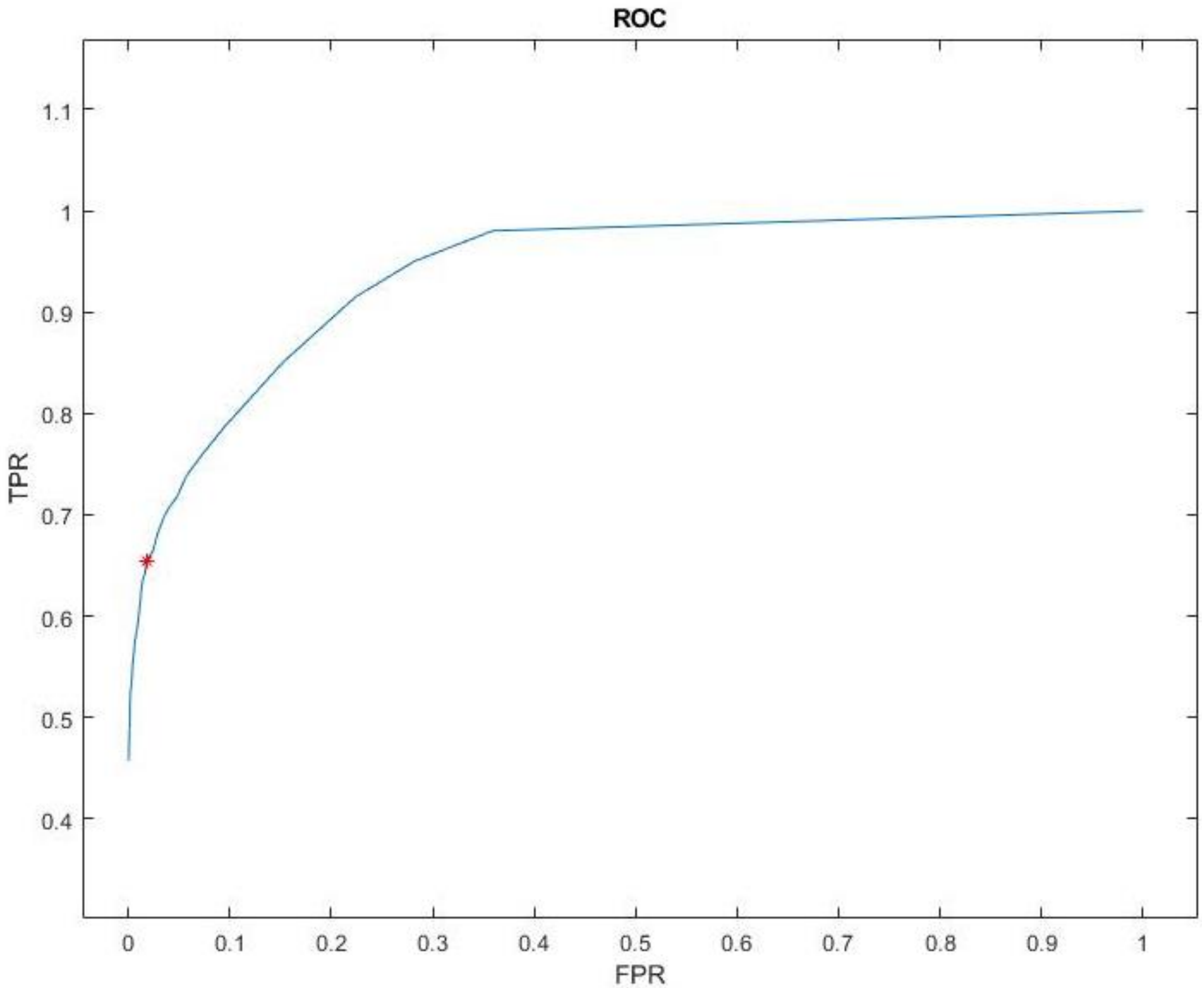
$$\frac{P(X\ |L=1)}{P(X|L=0)} \quad \underset{\boxed{D_0}}{\overset{\boxed{D_1}}{\gtrless}} \quad \frac{P(L=0)*(cost(D=1|L=0)-cost(D=0|L=0))}{P(L=1)*(cost(D=0|L=1)-cost(D=1|L=1))}$$

`

Given that, P(L = 0) = 0.8 and P(L = 1) = 0.2,

$$\frac{P(X\ |L=1)}{P(X|L=0)} \quad \underset{\boxed{D_0}}{\overset{\boxed{D_1}}{\gtrless}} \quad \frac{4\ *(cost(D=1|L=0)-cost(D=0|L=0))}{(cost(D=0|L=1)-cost(D=1|L=1))}$$





The * in above plot corresponds to the classifier with minimum probability of error.

The minimum probability of error classifier has the folowing empirically determined values,

- Gamma = 4 (ideally this should be equal to P(L = 0)/ P(L= 1), and our empirically determined matches with the ideal gamma)
- True Positive Rate (P(D =1 | L =1)) = 0.6523
- False Positive rate (P(D=1 | L=0)) = 0.0194
- Probability of Error = 0.1121

The liklihood ratio test for Naive Bayes Classifier is,

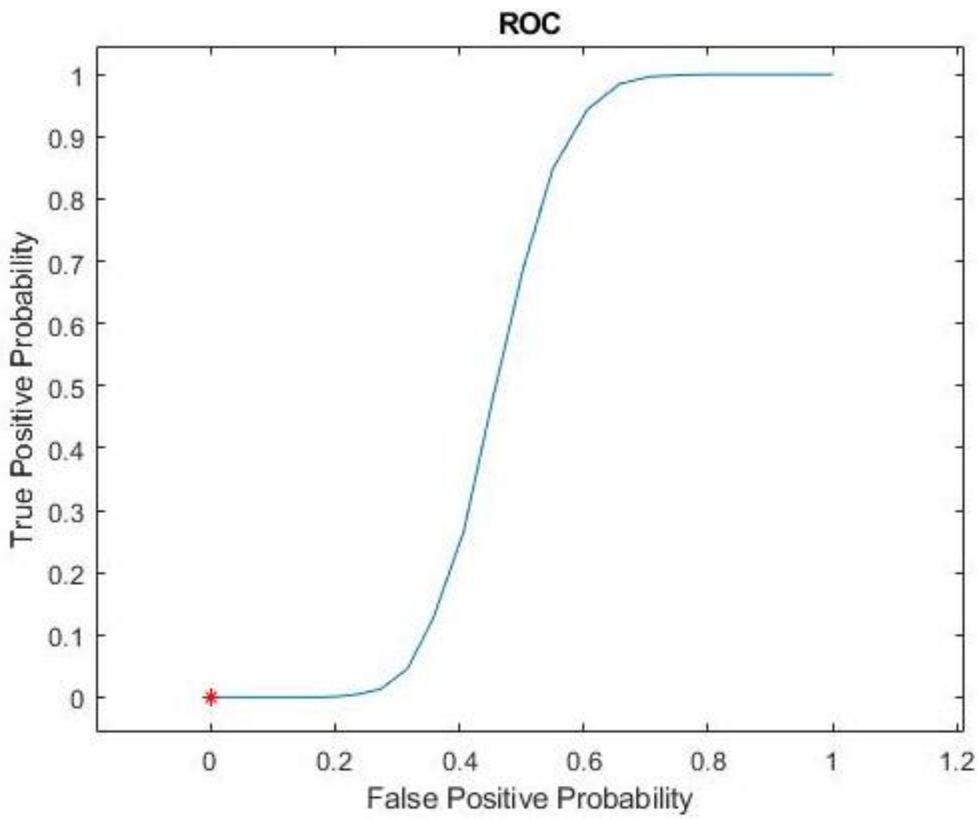
$$\frac{P(X\,|L=1)}{P(X|L=0)} \underset{\boxed{D_0}}{\overset{\boxed{D_1}}{\gtrless}} \frac{P(L=0)*(cost(D=1|L=0)-cost(D=0|L=0))}{P(L=1)*(cost(D=0|L=1)-cost(D=1|L=1))}$$

Where,

$P(X|L=1) \sim N([0.1\ \ 0]^T, I)$

$P(X|L=0) \sim N([-0.1\ \ 0]^T, I)$

I is a 2x2 Identity matrix

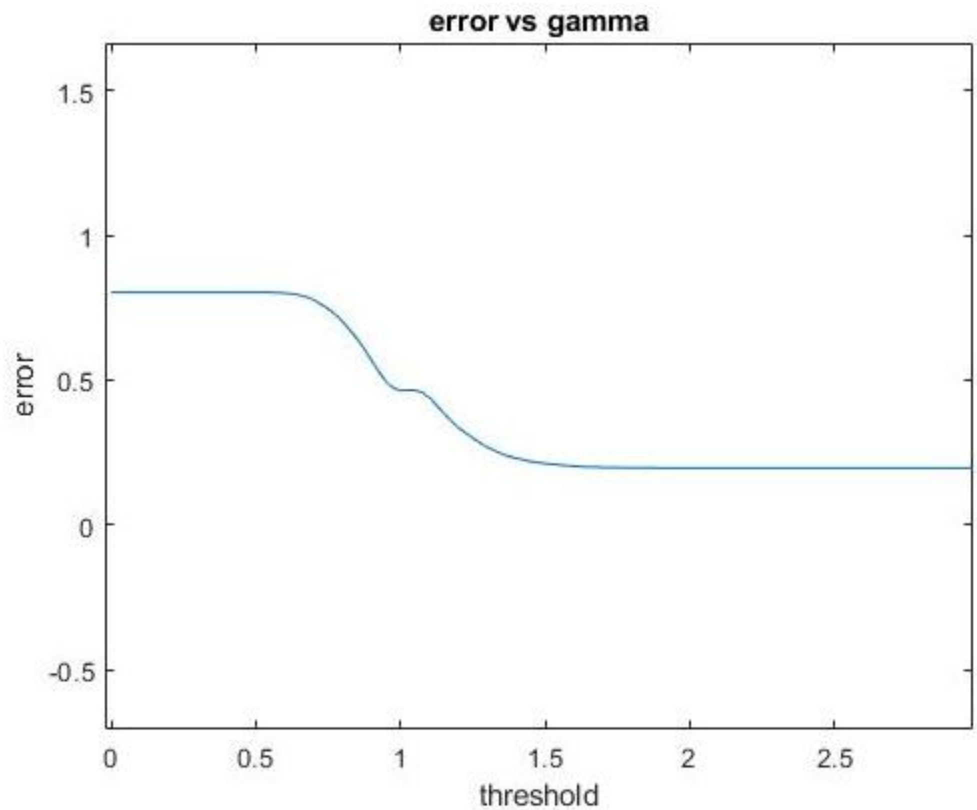


The * in above plot corresponds to the naïve bayes classifier with minimum probability of error.

The minimum probability of error naïve bayes classifier has the folowing empirically determined values,

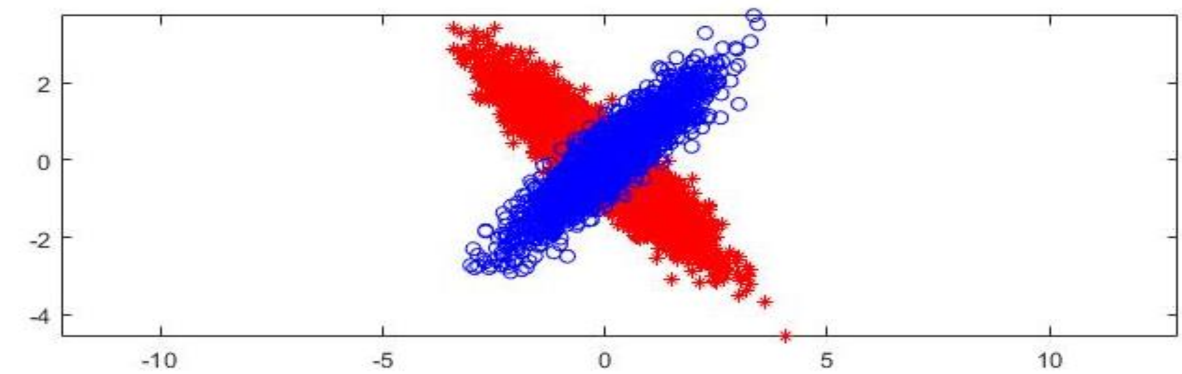
- True Positive Probability (P(D =1 | L =1)) = 0
- False Positive Probability (P(D=1 | L=0)) = 0
- From the above plot you can notice that this classifier classifies all data points as class 0.
- This classifier achieves minimum error by misclassifying all points coming from the class with minimum prior. In our case, class 1 has the minimum prior, so all class 1 data as classified as class 0. (This is the effect of having means closer to each other and identity covariance matrix)

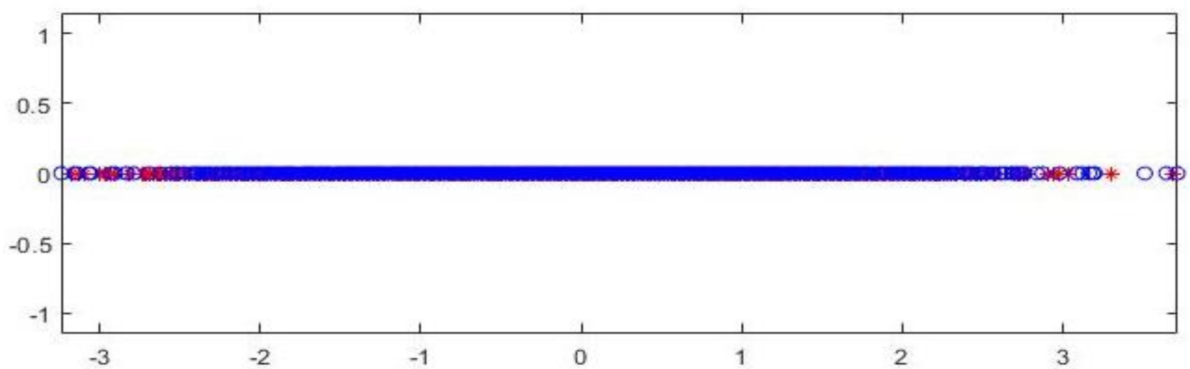
- Probability of Error = 0.1987
- From our empirical results, any gamma above 2.125 will classify all points as class 0 and will result in minimum error which equal to the prior of class 1.

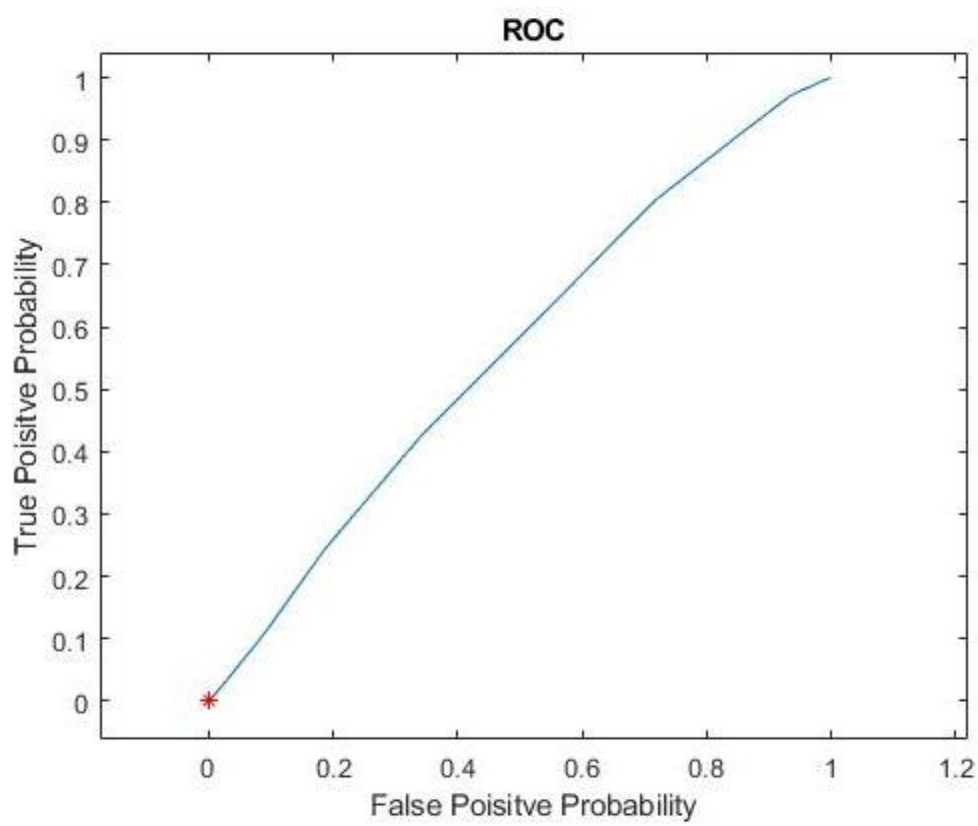


The above plot ahows the variation of probability of error with gamma. Initially the probability of error equals to the prior of class 0(all class0 points classified as class1) and finally it reaches the minimum probability of error which equals to the prior of class1(all class1 points are classified as class0)

The plot below shows how our data is getting projected onto w_{LDA} vector using Fishers LDA.



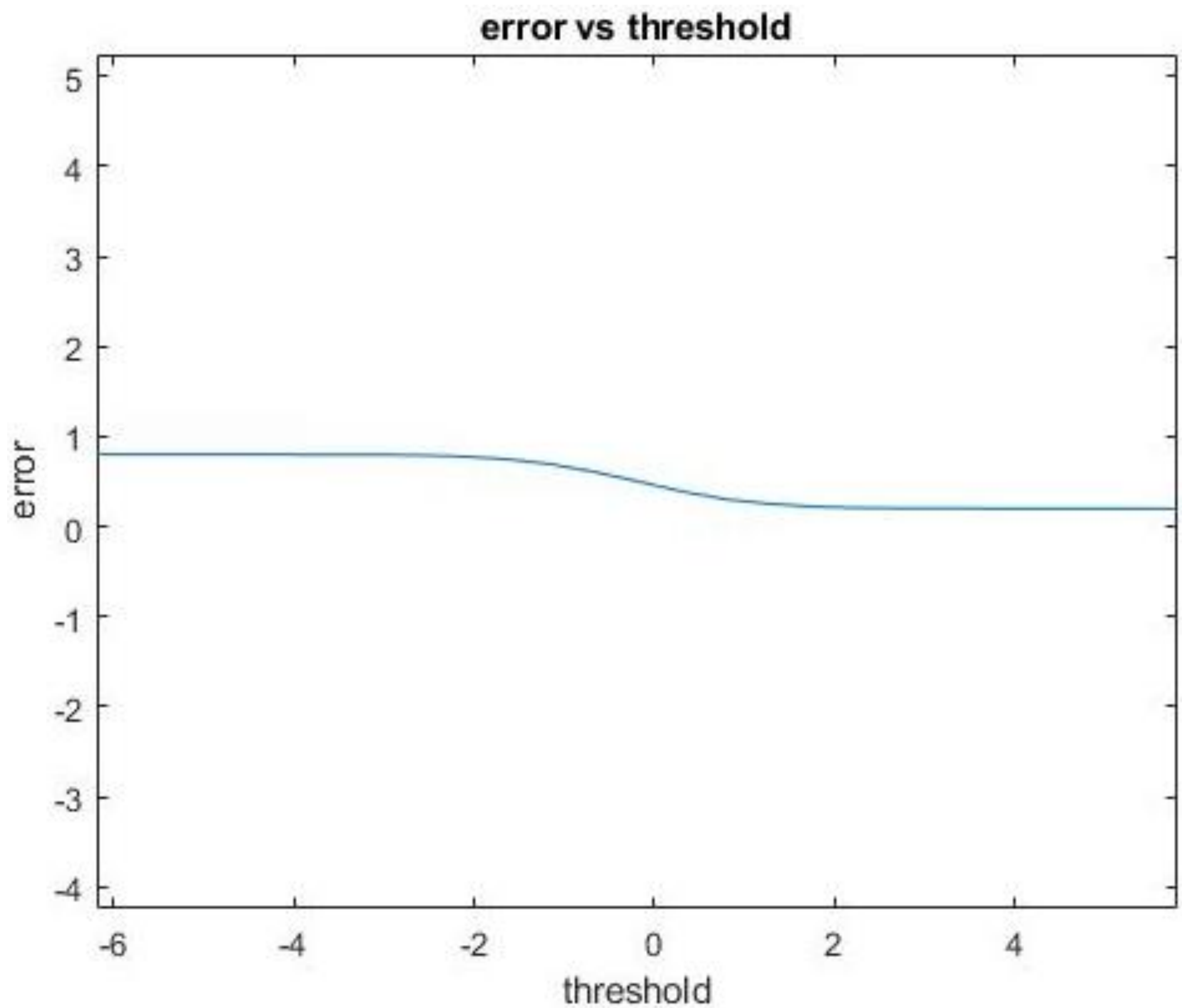




The * in above plot corresponds to the Fisher LDA classifier with minimum probability of error.

The minimum probability of error fisher LDA classifier has the folowing empirically determined values,

- True Positive Probability (P(D =1 | L =1)) = 0
- False Positive Probability (P(D=1 | L=0)) = 0
- From the above plot you can notice that this classifier classifies all data points as class 0.
- This classifier achieves minimum error by misclassifying all points coming from the class with minimum prior. In our case, class 1 has the minimum prior, so all class 1 data as classified as class 0. (This is the effect of having high overlap of ($w_{LDA}^T \cdot .x$) of both classes)
- Probability of Error = 0.2008
- From our empirical results, any gamma above 4.3 will classify all points as class 0 and will result in minimum error which equal to the prior of class 1.



The above plot ahows the variation of probability of error with threshold. Initially the probability of error equals to the prior of class 0(all class0 points classified as class1) and finally it reaches the minimum probability of error which equals to the prior of class1(all class1 points are classified as class0)

2)

The plot shows the data generated for class 0 and class 1. Each class liklihood is a gaussian mixture model.

P(L = 0) = 0.7

P(L =1) = 0.3

P(X|L=0) = (0.6 * G00) + (0.4*G01)

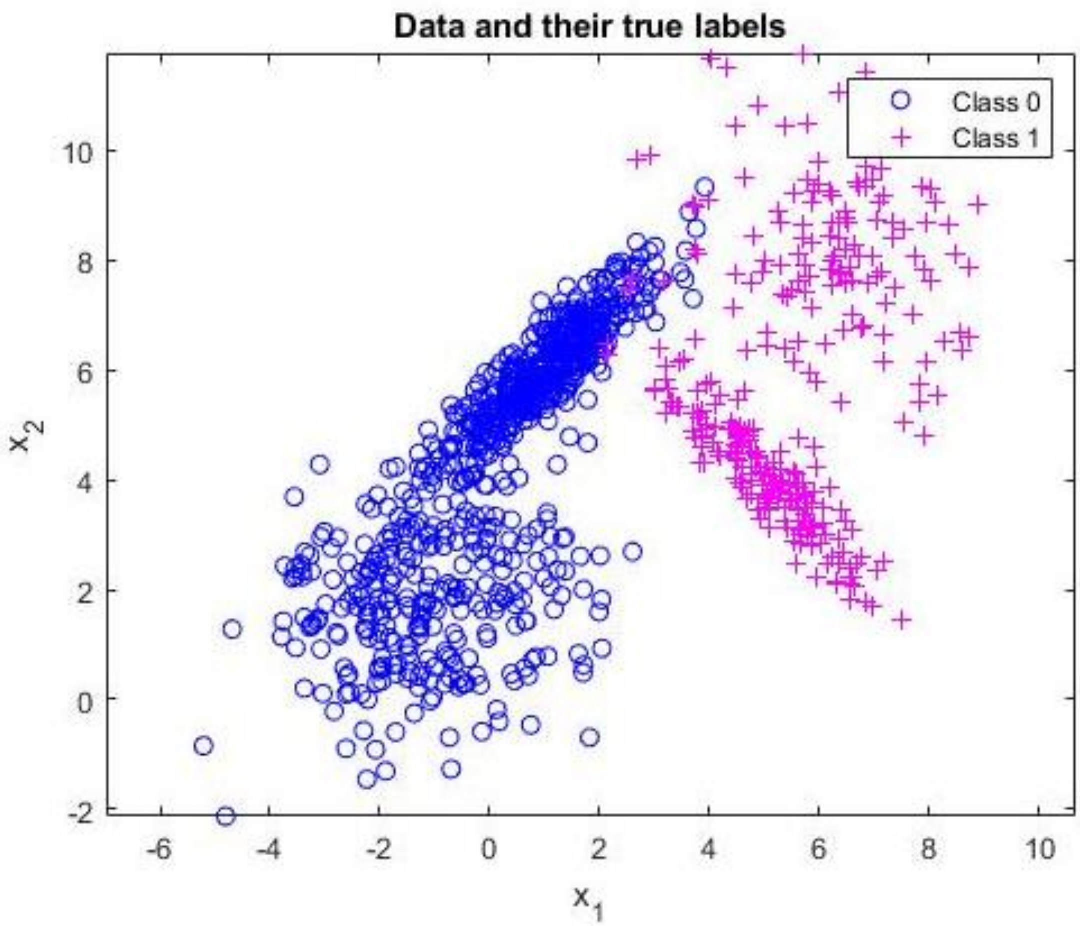
P(X|L=1) = (0.55 * G10) + (0.45*G11)

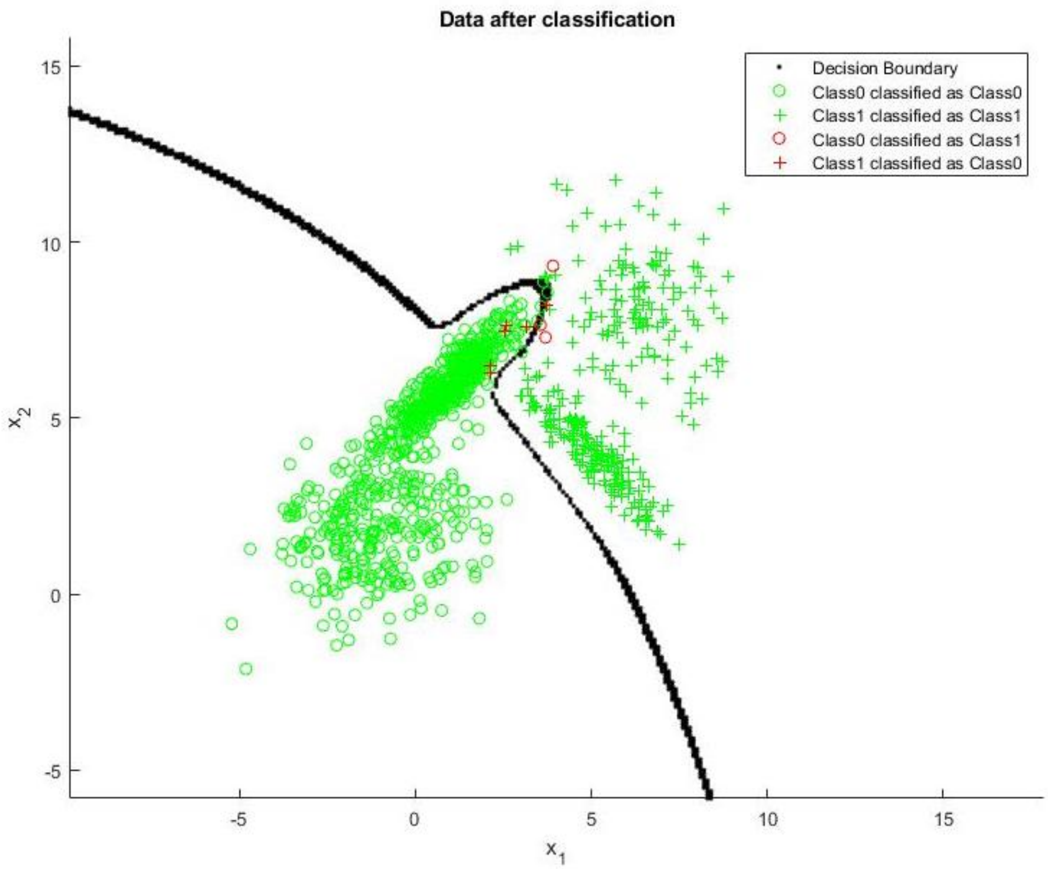
G00 ~ N($[1\ 6]^T$, [1 0.9; 0.9 1])

G01 ~ N($[-1\ 2]^T$, [2 0.5; 0.5 2])

G10 ~ N($[5\ 4]^T$, [1 -0.9; - 0.9 1])

G11 ~ N($[6\ 8\ ]^T$, [2 0; 0 2])





The likelihood ratio test is given by,

$$\frac{P(X\,|L=1)}{P(X|L=0)} \underset{\boxed{D_0}}{\overset{\boxed{D_1}}{\gtrless}} \frac{P(L=0)*(cost(D=1|L=0)-cost(D=0|L=0))}{P(L=1)*(cost(D=0|L=1)-cost(D=1|L=1))}$$

Minimum probability of error classification rule can be attained by using 1-0 cost in likelihood ratio test. (All correct classifications do not cost anything, and all misclassifications cost 1)

The minimum probability of error classification rule is,

$$\frac{P(X\,|L=1)}{P(X|L=0)} \underset{\boxed{D_0}}{\overset{\boxed{D_1}}{\gtrless}} \frac{P(L=0))}{P(L=1)}$$

3) $P(X|L=0) = \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{(x+2)^2}{2}} \sim N(-2,1)$

$P(X|L=1) = \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{(x-2)^2}{2}} \sim N(2,1)$

$P(L=0) = P(L=1) = 0.5$

The liklihood ratio test for minimum probability of error is given by,

$$\frac{P(X|L=1)}{P(X|L=0)} \underset{D_0}{\overset{D_1}{\gtrless}} \frac{P(L=0)}{P(L=1)}$$

$$\ln\left(\frac{P(X|L=1)}{P(X|L=0)}\right) \underset{D_0}{\overset{D_1}{\gtrless}} \ln\left(\frac{0.5}{0.5}\right)$$

$$\ln(P(X|L=1)) - \ln(P(X|L=0)) \underset{D_0}{\overset{D_1}{\gtrless}} 0$$

$$\ln\left(\frac{1}{\sqrt{2\pi}}\right) + \ln\left(e^{(x-2)^2/2}\right) - \ln\left(\frac{1}{\sqrt{2\pi}}\right) - \ln\left(e^{-(x+2)^2/2}\right) \underset{D_0}{\overset{D_1}{\gtrless}} 0$$

$$-\frac{(x-2)^2}{2} + \frac{(x+2)^2}{2} \underset{D_0}{\overset{D_1}{\gtrless}} 0$$

$$\frac{-(x^2-2x+4) + (x^2+2x+4)}{2} \underset{D_0}{\overset{D_1}{\gtrless}} 0$$

$$\frac{4x}{2} \underset{D_0}{\overset{D_1}{\gtrless}} 0$$

$$\boxed{x \underset{D_0}{\overset{D_1}{\gtrless}} 0}$$

The ~~minimum~~ probability of error achievable by this classifier is given by,

$$P(\text{error}) = P(D=1|L=0) \cdot P(L=0) + P(D=0|L=1) \cdot P(L=1)$$

$$P(\text{error}) = 0.5\left[\int_0^{\infty} P(X|L=0) \cdot dx + \int_{-\infty}^{0} P(X|L=1) \cdot dx\right]$$

All Matlab code can be found in,

https://github.com/veeraragav/EECE5644_Into_To_ML