$f(R)$ Theories of modified gravity

UNDERGRADUATE THESIS

Submitted in partial fulfillment of the requirements of
BITS F421T Thesis

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May 2018

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"The important thing is not to stop questioning. Curiosity has its own reason for existing. One cannot help but be in awe when one contemplates the mystery of eternity, of life, of the marvelous structure of reality. It is enough if one tries to comprehend only a little of this mystery every day."

Albert Einstein

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Abstract

Bachelor of Engineering (Hons.)

$f(R)$ Theories of modified gravity

by Veerav CHEBROLU

In this report, a discussion on theories of gravity is presented in chapter 1. In chapter 2, Field equation in $f(R)$ gravity is derived by metric variation. In the first section of chapter 3, we consider a Bianchi-type I universe and calculate the modified Friedmann equations in $f(R)$ gravity. In the next section, solutions of the Friedmann equation is found. The subsequent subsections deals with different cosmological models under $f(R)$ framework. In chapter 4, observations about universe is discussed.

Acknowledgements

I would like to thank my faculty supervisor Dr. PK Sahoo for giving me the opportunity to work in the field of cosmology and for mentoring me. I would also like to thank Ms Parbati Sahoo for helping me in my thesis work.

Contents

Chapter 1

Introduction

1.1 Background

Gravity is the fundamental phenomenon which we experience everyday, yet it is so puzzling! In fact it was the first phenomenon to be studied, noticebly due to the simplicity of the set-up required for conducting variuous experiments.[1]

Galileo Galilei shaped the modern work on the theory of gravity. He found out that *acceleration due to gravity is same for all objects* [2], which was huge as it was contradictory to Aristotle's belief that *heavier objects fall faster* [3]. About a 100 years later, Sir Isaac Newton came up with *Universal gravitational law* also known as inverse square law, which successfully predicted a variety of events, both at celestial level and at small scales.

In the early 1840's, Urbain Le Verrier, when he was studying the motion of Mercury, observed a discrepancy between the *precession* of Mercury and that predicted by Newton's law. In 1859, he published that this observation cannot be explained by Newtonian physics. In fact, he tried to explain this within the framework of Newton's law by considering a planet between the Sun and Mercury, the idea which was influenced by the fact that he had earlier discovered Neptune by observing irregularities in Uranus's orbit, but this planet was never found.[1],[4]

In the 1890's, Ernst Mach came up with an idea which was loosely along the lines of *"mass out there influences inertia here"*, the idea through which Einstein found inspiration while working on General relativity. Albert Einstein interpreted his idea along the lines of *"inertia originates in a kind of interaction between bodies"*, which was a contradiction to Newton's idea that inertia was relative to absolute space.[1],[5]

In 1905, Einstein came up with special theory of relativity, which supported different physical phenomenon like Electrodynamics, but was incompatible with Newton's gravitational theory, since Newtonian gravity assumes *instantaneous action at a distance*, which is in violation with special theory of relativity that nothing can travel faster than the speed of light.[6]

In 1915, Einstein came up with general relativity in which he generalized the special relativity by incorporating non-inertial frames and gravity by introducing the concept of spacetime curvature. General relativity passed experimental tests like precession of Mercury, deflection of light by the Sun and predicted gravitational redshift of light.[7],[8]

1.2 Modern Cosmology

It has been more than 100 years since Einstein introduced general relativity and the questions regarding its limitations are becoming more apt. Before we look at the modern reasons for modifying *GR*, we will look up the historical reasons for changing it. In 1919, not more than four years since its inception, Hermann Weyl and in 1923 Arthur Eddington considered modifying GR by including higher order curvature terms to its action. In fact, Eddington himself in 1919, verified GR by measuring the deflection of light by the Sun. The early attempts to modify was a result of scientific curiosity which could provide a better understanding of GR. [9]

However, By 1960's it was becoming clear that making the action complex had its advantages. It could be showed that higher order action was renormalizable which implied *quantization* of gravity, which was not the case with GR. This motivated the scientists to consider higher order gravitational theories.

Recent experiments suggest that the universe is comprised of three different types of matter. It consists of baryons, dark matter and dark energy. Dark energy dominate the universe comprising 76%. Dark matter and baryons occupy the rest of the universe comprising 20% and4% respectively. Both, the baryons and dark matter obey the *strong energy condition* i.e $\rho c^2 + 3p > 0$. Dark energy however does not obey the strong energy condition. The fact that dark energy dominates the universe and does not follow the strong energy condition has a surprising consequence i.e *universe is undergoing an accelerating expansion.* This result is surprising because the *acceleration equation* given by $\frac{\ddot{a}}{a} = \frac{-4\pi G}{3}(\rho + \frac{3p}{c^2})$ and the belief that all matter obeyed strong energy principle, resulted in $\ddot{a} < 0$, which implied that universe is undergoing deceleration.

This phenomenon, known as late time acceleration is in addition to inflationary expansion i.e exponential expansion in the beginning of the universe. Inflation theory is necessary to explain the flatness and the horizon problem. Between these two accelerated expansion eras, universe underwent decelerated expansion to support matter domination required for galaxy formation and radiation domination required for nuclei formation of elements. [9]

The problems mentioned above, along with other problems like the *missing mass problem* and *cosmological constant problem* exists with GR. Until we find a satisfactory and suitable explanation, exploring other theories would only help enhance our understanding of the universe. In this report, we will look into *f(R)* modification of gravity.

I would like to end this chapter with the following statement :
The question that we should ask ourselves is *"To what extent does a theory explain physical phenomena?"* and not *"Is the theory correct or wrong?"*.

Chapter 2

Metric $f(R)$ gravity

2.1 $f(R)$ gravity in metric formalism

The $f(R)$ theory of gravity is the generalization of Einstein's General Relativity by relaxing the Einstein-Hilbert action from $S_{EH} = \frac{1}{2\kappa}\int \sqrt{-g}Rd^4x$ to $S = \frac{1}{2\kappa}\int \sqrt{-g}f(R)d^4x$, where the Ricci scalar R is replaced by $f(R)$.

In this section, we will derive the Field equations in $f(R)$ gravity by varying the action w.r.t metric, thus giving rise to *metric $f(R)$ gravity*[10]. In fact, we will vary the action w.r.t inverse metric g^{ij} since $g^{il}g_{lj} = \delta^i_j$, and the Kronecker delta is unchanged under any variation.[11]

The variation in action is

$$\delta S = \frac{1}{2\kappa}\int \delta(\sqrt{-g}f(R))d^4x \tag{2.1}$$

i.e

$$\delta S = \frac{1}{2\kappa}\int \delta(\sqrt{-g})f(R)d^4x + \frac{1}{2\kappa}\int (\sqrt{-g})\delta(f(R))d^4x \tag{2.2}$$

In order to calculate $\delta\sqrt{-g}$, we will first derive an identity[12]

$$\begin{aligned}
\delta log|detP| &= log|det(P+\delta P)| - log|detP| \\
&= log\Big(\frac{detP+\delta P}{detP}\Big) \\
&= log\Big(det[P^-1(P+\delta P)]\Big) \\
&= log\Big(det[\mathbf{1}+P^{-1}\delta P]\Big) \\
&= Tr\Big(log[1+P^{-1}\delta P]\Big) \\
&\approx Tr(P^{-1}\delta P)
\end{aligned}$$

where P is an invertible matrix We now apply the above equation on g_{ij} and g^{ij}

$$\begin{aligned}\delta(log\sqrt{-g}) &= \delta\Big(log\sqrt{|det(g_{ij})|}\Big)\\ &= \frac{-1}{2}\delta\Big(log|det(g^{ij})|\Big)\\ &= \frac{-1}{2}g_{ij}\delta g^{ij}\end{aligned}$$

Therefore,

$$\delta\sqrt{-g} = \frac{-1}{2}\sqrt{-g}g_{ij}\delta g^{ij} \tag{2.3}$$

The Ricci scalar R is given by $R = g^{ij}R_{ij}$. Hence, the variation of Ricci scalar (*using product rule*) is

$$\delta R = g^{ij}\delta R_{ij} + \delta g^{ij}R_{ij} \tag{2.4}$$

Since, $\delta f = \frac{\partial f}{\partial R}\delta R$, we have :

$$\delta f = \frac{\partial f}{\partial R}g^{ij}\delta R_{ij} + \frac{\partial f}{\partial R}\delta g^{ij}R_{ij} \tag{2.5}$$

In order to calculate the variation of Ricci tensor, we first need to calculate the variation of Riemann tensor.[13] The Riemann tensor is given by

$$R^n_{imj} = \partial_m\Gamma^n_{ji} - \partial_j\Gamma^n_{mi} + \Gamma^n_{ml}\Gamma^l_{ji} - \Gamma^n_{jl}\Gamma^l_{mi} \tag{2.6}$$

The variation in Riemann tensor is given as

$$\delta R^n_{imj} = \partial_m\delta\Gamma^n_{ji} - \partial_j\delta\Gamma^n_{mi} + \delta\Gamma^n_{ml}\Gamma^l_{ji} + \Gamma^n_{ml}\delta\Gamma^l_{ji} - \delta\Gamma^n_{jl}\Gamma^l_{mi} - \Gamma^n_{jl}\delta\Gamma^l_{mi} \tag{2.7}$$

The covariant derivative of $\delta\Gamma^n_{ji}$ is given by

$$\nabla_m\Big(\delta\Gamma^n_{ji}\Big) = \partial_m\Big(\delta\Gamma^n_{ji}\Big) + \Gamma^n_{ml}\delta\Gamma^l_{ji} - \Gamma^l_{mj}\delta\Gamma^n_{li} - \Gamma^l_{mi}\delta\Gamma^n_{jl} \tag{2.8}$$

From equation 2.7 and 2.8 we can see that

$$\delta R^n_{imj} = \nabla_m\big(\Gamma^n_{ji}\big) - \nabla_j\big(\delta\Gamma^n_{mi}\big) \tag{2.9}$$

The variation of Ricci tensor can now be obtained by contracting the indices of variation of Riemann tensor.

$$\delta R_{ij} \equiv \delta R^n_{inj} = \nabla_n\big(\delta\Gamma^n_{ji}\big) - \nabla_j\big(\delta\Gamma^n_{ni}\big) \tag{2.10}$$

The equation 2.10 is also known as ***Palatini's identity***.

We will now calculate the variation of Ricci scalar using equation 2.4, by substituting the variation of Ricci tensor with *palatini's identity.*

$$\delta R = \delta g^{ij} R_{ij} + g^{ij}[\nabla_n(\delta\Gamma^n_{ji}) - \nabla_j(\delta\Gamma^n_{ni})] \tag{2.11}$$

The variation of christoffel symbol is given by

$$\delta\Gamma^l_{ij} = \frac{1}{2} g^{la}\Big(\nabla_i \delta g_{aj} + \nabla_j \delta g_{ai} - \nabla_a \delta g_{ij}\Big) \tag{2.12}$$

We will now substitute equation 2.12 in equation 2.11, and we obtain the following equation

$$\delta R = R_{ij}\delta g^{ij} + g_{ij}\Box\delta g^{ij} - \nabla_i \nabla j \delta g^{ij} \tag{2.13}$$

where $\Box$ is the *D'Alembert's opeartor*
Therefore, we can now calculate δf by using $\delta f = \dfrac{\partial f}{\partial R}\delta R$

For convenience, let us consider $\dfrac{\partial f}{\partial R} = F$

Since, now we have calculated the variation of $\sqrt{-g}$ and variation of $f(R)$, we can now get the variation of action by using equation 2.2. By substituting 2.3 and 2.13 in 2.2, we get

$$\begin{aligned}\delta S &= \frac{1}{2\kappa}\int \frac{-1}{2}\sqrt{-g} g_{ij}\delta g^{ij} f(R) d^4x + \frac{1}{2\kappa}\int \sqrt{-g} F\Big(R_{ij}\delta g^{ij} + g_{ij}\Box\delta g^{ij} - \nabla_i\nabla_j\delta g^{ij}\Big) d^4x \\ &= \frac{1}{2\kappa}\int \sqrt{-g}\delta g^{ij}\Big(F(R)R_{ij} + (g_{ij}\Box - \nabla_i\nabla_j)F - \frac{1}{2}\sqrt{-g} g_{ij} f(R)\Big) d^4x\end{aligned}$$

By using the ***principle of action*** which says that variation of action w.r.t metric is zero i.e $\delta S = 0$, we obtain the **Field equation**

$$F(R)R_{ij} - \frac{1}{2} f(R) g_{ij} + g_{ij}\Box F - \nabla_i\nabla_j F = \kappa T_{ij} \tag{2.14}$$

where

$$T_{ij} = \frac{-2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}L_m)}{\delta g^{ij}} \tag{2.15}$$

$$\Box F = \frac{1}{\sqrt{-g}}\partial_i(\sqrt{-g} g^{ij}\partial_j F) \tag{2.16}$$

Chapter 3

Cosmological model within metric f(R) gravity

3.1 Field equations in Bianchi-type I universe

The metric for Bianchi type I universe is given by

$$ds^2 = -dt^2 + A^2(t)dx^2 + B^2(t)dy^2 + C^2(t)dz^2 \tag{3.1}$$

where A,B,C are scale factors which are functions of time t. The sign convention chosen here is $(-+++)$ The metric in this universe is homogenous (i.e independent of x, y, z) and anisotropic (x, y cannot be interchanged unless $A(t) = B(t)$). [14]
The metric g_{ij} is given by

$$g_{ij} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -A^2(t) & 0 & 0 \\ 0 & 0 & -B^2(t) & 0 \\ 0 & 0 & 0 & -C^2(t) \end{bmatrix} \tag{3.2}$$

where $i, j = 0, 1, 2, 3$ where 0 represents time and $1, 2, 3$ represents spatial direction x, y, z respectively.
In this section, we will find modified Friedmann equations corresponding to $f(R)$ gravity. In order to do so, we need to calculate Ricci tensor, which requires Christoffel symbols to be calculated. Thus we will first calculate christoffel symbols and then use those to calculate Ricci tensor.

The Christoffel symbols can be calculated by the following

$$\Gamma^i_{jk} = \frac{g^{il}}{2}\Big(\frac{\partial g_{jl}}{\partial x^k} + \frac{\partial g_{kl}}{\partial x^j} - \frac{\partial g_{jk}}{\partial x^l}\Big) \tag{3.3}$$

Thus, there are a total of 64 symbols ($4 \times 4 \times 4$).
The Christoffel symbols are symmetric on lower indices i.e $\Gamma^i_{jk} = \Gamma^i_{kj}$

$$\begin{aligned}\mathbf{\Gamma^1_{01}} &= \frac{g^{1l}}{2}\Big(\frac{\partial g_{0l}}{\partial x^1} + \frac{\partial g_{1l}}{\partial x^0} - \frac{\partial g_{01}}{\partial x^l}\Big)\\ &= \frac{g^{11}}{2}\Big(\frac{\partial g_{01}}{\partial x^1} + \frac{\partial g_{11}}{\partial x^0} - \frac{\partial g_{01}}{\partial x^1}\Big)\\ &= \frac{1}{2A^2}\big(0 + \frac{\partial A^2}{\partial t} - 0\big)\\ &= \frac{\dot{\mathbf{A}}}{\mathbf{A}}\\ &= \Gamma^1_{10}\end{aligned}$$

$$\begin{aligned}\mathbf{\Gamma^2_{02}} &= \frac{g^{2l}}{2}\Big(\frac{\partial g_{0l}}{\partial x^2} + \frac{\partial g_{2l}}{\partial x^0} - \frac{\partial g_{02}}{\partial x^l}\Big)\\ &= \frac{g^{22}}{2}\Big(\frac{\partial g_{02}}{\partial x^2} + \frac{\partial g_{22}}{\partial x^0} - \frac{\partial g_{02}}{\partial x^2}\Big)\\ &= \frac{1}{2B^2}\big(0 + \frac{\partial B^2}{\partial t} - 0\big)\\ &= \frac{\dot{\mathbf{B}}}{\mathbf{B}}\\ &= \Gamma^2_{20}\end{aligned}$$

$$\begin{aligned}\mathbf{\Gamma^3_{03}} &= \frac{g^{3l}}{2}\Big(\frac{\partial g_{0l}}{\partial x^3} + \frac{\partial g_{3l}}{\partial x^0} - \frac{\partial g_{03}}{\partial x^l}\Big)\\ &= \frac{g^{33}}{2}\Big(\frac{\partial g_{03}}{\partial x^3} + \frac{\partial g_{33}}{\partial x^0} - \frac{\partial g_{03}}{\partial x^1}\Big)\\ &= \frac{1}{2C^2}\big(0 + \frac{\partial c^2}{\partial t} - 0\big)\\ &= \frac{\dot{\mathbf{C}}}{\mathbf{C}}\\ &= \Gamma^3_{30}\end{aligned}$$

$$\begin{aligned}\mathbf{\Gamma^0_{11}} &= \frac{g^{0l}}{2}\Big(\frac{\partial g_{1l}}{\partial x^1} + \frac{\partial g_{1l}}{\partial x^1} - \frac{\partial g_{11}}{\partial x^l}\Big)\\ &= \frac{g^{00}}{2}\Big(\frac{\partial g_{10}}{\partial x^1} + \frac{\partial g_{10}}{\partial x^1} - \frac{\partial g_{11}}{\partial x^0}\Big)\\ &= \frac{-1}{2}\big(0 + 0 - \frac{\partial A^2}{\partial t}\big)\\ &= \mathbf{A}\dot{\mathbf{A}}\end{aligned}$$

$$\begin{aligned}\mathbf{\Gamma^0_{22}} &= \frac{g^{0l}}{2}\Big(\frac{\partial g_{2l}}{\partial x^2} + \frac{\partial g_{2l}}{\partial x^2} - \frac{\partial g_{22}}{\partial x^l}\Big)\\ &= \frac{g^{00}}{2}\Big(\frac{\partial g_{20}}{\partial x^2} + \frac{\partial g_{20}}{\partial x^2} - \frac{\partial g_{22}}{\partial x^0}\Big)\\ &= \frac{-1}{2}\big(0 + 0 - \frac{\partial B^2}{\partial t}\big)\\ &= \mathbf{B}\dot{\mathbf{B}}\end{aligned}$$

$$\begin{aligned}\mathbf{\Gamma^0_{33}} &= \frac{g^{0l}}{2}\Big(\frac{\partial g_{3l}}{\partial x^3} + \frac{\partial g_{3l}}{\partial x^3} - \frac{\partial g_{33}}{\partial x^l}\Big)\\ &= \frac{g^{00}}{2}\Big(\frac{\partial g_{30}}{\partial x^3} + \frac{\partial g_{30}}{\partial x^3} - \frac{\partial g_{33}}{\partial x^0}\Big)\\ &= \frac{-1}{2}\big(0 + 0 - \frac{\partial C^2}{\partial t}\big)\\ &= \mathbf{C}\dot{\mathbf{C}}\end{aligned}$$

The above 9 symbols are non-zero. The rest of the 55 symbols are zero.
We, now calculate the Ricci tensor given by

$$R_{ij} = \frac{\partial \Gamma^l_{ij}}{\partial x^l} - \frac{\partial \Gamma^l_{il}}{\partial x^j} + \Gamma^m_{ij}\Gamma^l_{lm} - \Gamma^m_{il}\Gamma^l_{jm} \tag{3.4}$$

There are a total of 16 tensors (4×4).The Ricci tensor is symmetric i.e $R_{ij} = R_{ji}$.

$$
\begin{aligned}
R_{00} &= \frac{\partial \Gamma^l_{00}}{\partial x^l} - \frac{\partial \Gamma^l_{0l}}{\partial x^0} + \Gamma^m_{00}\Gamma^l_{lm} - \Gamma^m_{0l}\Gamma^l_{0m} \\
&= 0 - \Big(\frac{\partial \Gamma^1_{01}}{\partial t} + \frac{\partial \Gamma^2_{02}}{\partial t} + \frac{\partial \Gamma^3_{03}}{\partial t}\Big) + 0 - \Big(\Gamma^1_{01}\Gamma^1_{01} + \Gamma^2_{02}\Gamma^2_{02} + \Gamma^3_{03}\Gamma^3_{03}\Big) \\
&= -\Big(\frac{A\ddot{A} - \dot{A}^2}{A^2} + \frac{B\ddot{B} - \dot{B}^2}{B^2} + \frac{C\ddot{C} - \dot{C}^2}{C^2}\Big) - \Big(\frac{\dot{A}^2}{A^2} + \frac{\dot{B}^2}{B^2} + \frac{\dot{C}^2}{C^2}\Big) \\
&= -\Big(\frac{\ddot{\mathbf{A}}}{\mathbf{A}} + \frac{\ddot{\mathbf{B}}}{\mathbf{B}} + \frac{\ddot{\mathbf{C}}}{\mathbf{C}}\Big)
\end{aligned}
$$

$$
\begin{aligned}
R_{11} &= \frac{\partial \Gamma^l_{11}}{\partial x^l} - \frac{\partial \Gamma^l_{1l}}{\partial x^1} + \Gamma^m_{11}\Gamma^l_{lm} - \Gamma^m_{1l}\Gamma^l_{1m} \\
&= \frac{\partial \Gamma^0_{11}}{\partial x^0} - 0 + \Gamma^0_{11}\Gamma^l_{l0} - \Big(\Gamma^0_{11}\Gamma^1_{10} + \Gamma^1_{10}\Gamma^0_{11}\Big) \\
&= A\ddot{A} + \dot{A}^2 + A\dot{A}\Big(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\Big) - \Big(\dot{A}^2 + \dot{A}^2\Big) \\
&= A\ddot{A} + \dot{A}^2 + \dot{A}^2 + A\dot{A}\Big(\frac{\dot{B}}{B} + \frac{\dot{C}}{C}\Big) - 2\dot{A}^2 \\
&= \mathbf{A}\ddot{\mathbf{A}} + \mathbf{A}\dot{\mathbf{A}}\Big(\frac{\dot{\mathbf{B}}}{\mathbf{B}} + \frac{\dot{\mathbf{C}}}{\mathbf{C}}\Big)
\end{aligned}
$$

$$
\begin{aligned}
R_{22} &= \frac{\partial \Gamma^l_{22}}{\partial x^l} - \frac{\partial \Gamma^l_{2l}}{\partial 2^2} + \Gamma^m_{22}\Gamma^l_{lm} - \Gamma^m_{2l}\Gamma^l_{2m} \\
&= \frac{\partial \Gamma^0_{22}}{\partial x^0} - 0 + \Gamma^0_{22}\Gamma^l_{l0} - \Big(\Gamma^0_{22}\Gamma^2_{20} + \Gamma^2_{20}\Gamma^0_{22}\Big) \\
&= B\ddot{B} + \dot{B}^2 + B\dot{B}\Big(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\Big) - \Big(\dot{B}^2 + \dot{B}^2\Big) \\
&= B\ddot{B} + \dot{B}^2 + \dot{B}^2 + B\dot{B}\Big(\frac{\dot{A}}{A} + \frac{\dot{C}}{C}\Big) - 2\dot{B}^2 \\
&= \mathbf{B}\ddot{\mathbf{B}} + \mathbf{B}\dot{\mathbf{B}}\Big(\frac{\dot{\mathbf{A}}}{\mathbf{A}} + \frac{\dot{\mathbf{C}}}{\mathbf{C}}\Big)
\end{aligned}
$$

$$\begin{aligned}
R_{33} &= \frac{\partial \Gamma^l_{33}}{\partial x^l} - \frac{\partial \Gamma^l_{3l}}{\partial x^3} + \Gamma^m_{33}\Gamma^l_{lm} - \Gamma^m_{3l}\Gamma^l_{3m} \\
&= \frac{\partial \Gamma^0_{33}}{\partial x^0} - 0 + \Gamma^0_{33}\Gamma^l_{l0} - \Big(\Gamma^0_{33}\Gamma^3_{30} + \Gamma^3_{30}\Gamma^0_{33}\Big) \\
&= C\ddot{C} + \dot{C}^2 + C\dot{C}\Big(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\Big) - \Big(\dot{C}^2 + \dot{C}^2\Big) \\
&= C\ddot{C} + \dot{C}^2 + \dot{C}^2 + C\dot{C}\Big(\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\Big) - 2\dot{C}^2 \\
&= \mathbf{C\ddot{C}} + \mathbf{C\dot{C}}\Big(\frac{\mathbf{\dot{A}}}{\mathbf{A}} + \frac{\mathbf{\dot{B}}}{\mathbf{B}}\Big)
\end{aligned}$$

The above are the non-zero tensors. The remaining 12 tensors are zero.

The Ricci scalar R is given by

$$R = R_{ij}g^{ij} \tag{3.5}$$

$$R = R_{00}g^{00} + R_{11}g^{11} + R_{22}g^{22} + R_{33}g^{33} \tag{3.6}$$

$$= \Big(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\Big)(-1) + \Big(A\ddot{A} + A\dot{A}\Big(\frac{\dot{B}}{B} + \frac{\dot{C}}{C}\Big)\Big)\frac{1}{A^2} + \Big(B\ddot{B} + B\dot{B}\Big(\frac{\dot{A}}{A} + \frac{\dot{C}}{C}\Big)\Big)\frac{1}{B^2} \tag{3.7}$$

$$+ \Big(C\ddot{C} + C\dot{C}\Big(\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\Big)\Big)\frac{1}{C^2} \tag{3.8}$$

$$= \mathbf{2}\Big(\frac{\mathbf{\ddot{A}}}{\mathbf{A}} + \frac{\mathbf{\ddot{B}}}{\mathbf{B}} + \frac{\mathbf{\ddot{C}}}{\mathbf{C}} + \frac{\mathbf{\dot{A}\dot{B}}}{\mathbf{AB}} + \frac{\mathbf{\dot{B}\dot{C}}}{\mathbf{BC}} + \frac{\mathbf{\dot{A}\dot{C}}}{\mathbf{AC}}\Big) \tag{3.9}$$

Energy-momentum tensor T_{ij} for a perfect fluid is given by

$$T_{ij} = \begin{bmatrix} \rho & 0 & 0 & 0 \\ 0 & -pA^2 & 0 & 0 \\ 0 & 0 & -pB^2 & 0 \\ 0 & 0 & 0 & -pC^2 \end{bmatrix} \tag{3.10}$$

T_{ij} for vacuum is zero.

Let

$$H_1 = \frac{\dot{A}}{A} \qquad H_2 = \frac{\dot{B}}{B} \qquad H_3 = \frac{\dot{C}}{C}$$

i.e

$$\dot{H}_1 = \frac{\ddot{A}}{A} - (\frac{\dot{A}}{A})^2 \qquad \dot{H}_2 = \frac{\ddot{B}}{B} - (\frac{\dot{B}}{B})^2 \qquad \dot{H}_3 = \frac{\ddot{C}}{C} - (\frac{\dot{C}}{C})^2$$

$$\frac{\ddot{A}}{A} = \dot{H}_1 + H_1^2 \qquad \frac{\ddot{B}}{B} = \dot{H}_2 + H_2^2 \qquad \frac{\ddot{C}}{C} = \dot{H}_3 + H_3^2$$

Let us consider the vacuum solution : Plugging the value of R_{ij}, g_{ij} and $T_{ij} = 0$ in the Field equation given in 2.14 , we get

For $(i, j) = (0, 0)$

$$-\Big(H_1 + H_2 + H_3\Big)\dot{F} + \Big(\dot{H}_1 + \dot{H}_2 + \dot{H}_3 + H_1^2 + H_2^2 + H_3^2\Big)F + \frac{1}{2}f = 0 \tag{3.11}$$

For $(i, j) = (1, 1)$

$$-\ddot{F} - \Big(H_2 + H_3\Big)\dot{F} + \Big(\dot{H}_1 + H_1^2 + H_1H_2 + H_1H_3\Big)F + \frac{1}{2}f = 0 \tag{3.12}$$

$(i, j) = (2, 2)$

$$-\ddot{F} - \Big(H_1 + H_3\Big)\dot{F} + \Big(\dot{H}_2 + H_2^2 + H_1H_2 + H_2H_3\Big)F + \frac{1}{2}f = 0 \tag{3.13}$$

$(i, j) = (3, 3)$

$$-\ddot{F} - \Big(H_1 + H_2\Big)\dot{F} + \Big(\dot{H}_3 + H_3^2 + H_1H_3 + H_2H_3\Big)F + \frac{1}{2}f = 0 \tag{3.14}$$

Subtracting 3.11 and 3.12 and dividing by F gives [15]

$$\dot{H}_2 + H_2^2 + \dot{H}_3 + H_3^2 - H_1H_2 - H_1H_3 - H_1\frac{\dot{F}}{F} + \frac{\ddot{F}}{F} = 0 \tag{3.15}$$

Subtracting 3.11 and 3.13 and dividing by F gives

$$\dot{H}_1 + H_1^2 + \dot{H}_3 + H_3^2 - H_1H_2 - H_2H_3 - H_2\frac{\dot{F}}{F} + \frac{\ddot{F}}{F} = 0 \tag{3.16}$$

Subtracting 3.11 and 3.14 and dividing by F gives

$$\dot{H}_1 + H_1^2 + \dot{H}_2 + H_2^2 - H_2H_3 - H_1H_3 - H_3\frac{\dot{F}}{F} + \frac{\ddot{F}}{F} = 0 \tag{3.17}$$

The above equations 3.15, 3.16, 3.17 are the modified Friedmann equation in $f(R)$ gravity in vacuum Bianchi-type I universe.

Similarly, for a perfect fluid Bianchi-type I universe, the modified Friedmann equation in $f(R)$ gravity are [16]

For $(i,j) = (0,0)$

$$-\Big(H_1 + H_2 + H_3\Big)\dot{F} + \Big(\dot{H}_1 + \dot{H}_2 + \dot{H}_3 + H_1^2 + H_2^2 + H_3^2\Big)F + \frac{1}{2}f = -\rho \tag{3.18}$$

For $(i,j) = (1,1)$

$$-\ddot{F} - \Big(H_2 + H_3\Big)\dot{F} + \Big(\dot{H}_1 + H_1^2 + H_1H_2 + H_1H_3\Big)F + \frac{1}{2}f = p \tag{3.19}$$

$(i,j) = (2,2)$

$$-\ddot{F} - \Big(H_1 + H_3\Big)\dot{F} + \Big(\dot{H}_2 + H_2^2 + H_1H_2 + H_2H_3\Big)F + \frac{1}{2}f = p \tag{3.20}$$

$(i,j) = (3,3)$

$$-\ddot{F} - \Big(H_1 + H_2\Big)\dot{F} + \Big(\dot{H}_3 + H_3^2 + H_1H_3 + H_2H_3\Big)F + \frac{1}{2}f = p \tag{3.21}$$

Subtracting 3.19 and 3.20, 3.20 and 3.21, 3.21 and 3.19, we get

$$\frac{\ddot{F}}{F} - H_1\frac{\dot{F}}{F} - H_1H_2 - H_1H_3 + \dot{H}_2 + H_2^2 + \dot{H}_3 + H_3^2 + \frac{1}{F}\Big(\rho + p\Big) = 0 \tag{3.22}$$

$$\frac{\ddot{F}}{F} - H_2\frac{\dot{F}}{F} - H_1H_2 - H_2H_3 + \dot{H}_1 + H_1^2 + \dot{H}_3 + H_3^2 + \frac{1}{F}\Big(\rho + p\Big) = 0 \tag{3.23}$$

$$\frac{\ddot{F}}{F} - H_3\frac{\dot{F}}{F} - H_1H_3 - H_2H_3 + \dot{H}_1 + H_1^2 + \dot{H}_2 + H_2^2 + \frac{1}{F}\Big(\rho + p\Big) = 0 \tag{3.24}$$

The above equations 3.22, 3.23, 3.24 are modified Friedmann equations for a perfect fluid Bianchi type I universe.

3.2 Solutions in Bianchi type I universe

In order to find the solution of the system of equations, we will subtract 3.22 and 3.23, 3.23 and 3.24, 3.22 and 3.24

$$\left(H_2 - H_1\right)\frac{\dot{F}}{F} + H_3\left(H_2 - H_1\right) - \left(\dot{H}_1 + H_1^2\right) + \dot{H}_2 + H_2^2 = 0 \tag{3.25}$$

$$\left(H_3 - H_2\right)\frac{\dot{F}}{F} + H_1\left(H_3 - H_2\right) - \left(\dot{H}_2 + H_2^2\right) + \dot{H}_3 + H_3^2 = 0 \tag{3.26}$$

$$\left(H_3 - H_1\right)\frac{\dot{F}}{F} + H_2\left(H_3 - H_1\right) - \left(\dot{H}_1 + H_1^2\right) + \dot{H}_3 + H_3^2 = 0 \tag{3.27}$$

Writing H_1, H_2, H_3 in terms of A, B, C, and solving the system of equations we get,

$$A(t) = a(t)c_1 \mathrm{e}^{\int \frac{l_1 dt}{a^3 F}} \tag{3.28}$$

$$B(t) = a(t)c_2 \mathrm{e}^{\int \frac{l_2 dt}{a^3 F}} \tag{3.29}$$

$$C(t) = a(t)c_3 \mathrm{e}^{\int \frac{l_3 dt}{a^3 F}} \tag{3.30}$$

Where $c_1 c_2 c_3 = 1$ and $l_1 + l_2 + l_3 = 0$

3.2.1 Model 1

In order to find pressure and density as a function of time, we need to take a couple of assumptions such that the equation 3.28, 3.29, 3.30 can be solved.
First we will take a *power law* relation between F and a as shown in [17] i.e $F \propto a^n$. For simplicity of calculation, let us take $n = -2$ such that the relation becomes $F = ka^{-2}$.
The second assumption we take is that the average scale factor follows a *Hybrid expansion law* i.e $a(t) = t^n e^{mt}$ [18]

Let us find the *decelerating parameter 'q'* which will show whether the universe is undergoing an accelerated expansion. *negative* sign of q indicates an accelerated expansion of universe.
q is given by $\frac{-a\ddot{a}}{\dot{a}^2}$. On calculating q comes out to be

$$q = -1 + \frac{n}{n^2 + m^2 t^2 + 2nmt} \tag{3.31}$$

Substituting the value of F in 3.28, 3.29 and 3.30, we get

$$A(t) = a(t)c_1 e^{\int \frac{l_1 dt}{ka}}$$
$$B(t) = a(t)c_2 e^{\int \frac{l_2 dt}{ka}}$$
$$C(t) = a(t)c_3 e^{\int \frac{l_3 dt}{ka}}$$

Now, Substituting the value of $a(t)$ in the above equations,

$$A(t) = t^n e^{mt} c_1 e^{\frac{-l_1 m^{n-1}\Gamma(1-n,mt)}{k}} \tag{3.32}$$
$$B(t) = t^n e^{mt} c_2 e^{\frac{-l_2 m^{n-1}\Gamma(1-n,mt)}{k}} \tag{3.33}$$
$$C(t) = t^n e^{mt} c_3 e^{\frac{-l_3 m^{n-1}\Gamma(1-n,mt)}{k}} \tag{3.34}$$

$$\dot{A(t)} = \frac{c_1 e^{\frac{-l_1 m^{n-1}\Gamma(1-n,mt)}{k}} t^{n-1}(mt)^{-n}\left(l_1 m^n t + e^{mt} k (mt)^n (n+mt)\right)}{k}$$
$$\dot{B(t)} = \frac{c_2 e^{\frac{-l_2 m^{n-1}\Gamma(1-n,mt)}{k}} t^{n-1}(mt)^{-n}\left(l_2 m^n t + e^{mt} k (mt)^n (n+mt)\right)}{k}$$
$$\dot{C(t)} = \frac{c_3 e^{\frac{-l_3 m^{n-1}\Gamma(1-n,mt)}{k}} t^{n-1}(mt)^{-n}\left(l_3 m^n t + e^{mt} k (mt)^n (n+mt)\right)}{k}$$

Therefore, H_1, H_2, H_3 can be calculated by finding $\frac{\dot{A}}{A}$, $\frac{\dot{B}}{B}$, $\frac{\dot{C}}{C}$ respectively

$$H_1 = \frac{l_1}{t^n e^{mt}} + \frac{kn}{t} + km \tag{3.35}$$
$$H_2 = \frac{l_2}{t^n e^{mt}} + \frac{kn}{t} + km \tag{3.36}$$
$$H_3 = \frac{l_3}{t^n e^{mt}} + \frac{kn}{t} + km \tag{3.37}$$

Therefore, the average Hubble parameter given by $\frac{1}{3}(H_1 + H_2 + H_3)$ is

$$H = \frac{kn}{t} + km \tag{3.38}$$

We will Substitute H_1, H_2, H_3 in 3.18 and 3.21 to find ρ and p as a function of t. In order to do that, first we will calculate $\dot{F}$, $\ddot{F}$, $\dot{H}_1$, $\dot{H}_2$, $\dot{H}_3$

$$\dot{F} = -2kt^{-2n-1}(mt+n)\mathrm{e}^{-2mt}$$
$$\ddot{F} = kt^{-2n-2}(4m^2t^2+8mnt+4n^2+2n)\mathrm{e}^{-2mt}$$
$$\dot{H}_1 = \mathrm{e}^{-m}t^{-n-2}(ke^m nt^n + l_1 n + l_1)$$
$$\dot{H}_2 = \mathrm{e}^{-m}t^{-n-2}(ke^m nt^n + 2_1 n + l_2)$$
$$\dot{H}_3 = \mathrm{e}^{-m}t^{-n-2}(ke^m nt^n + 3_1 n + l_3)$$

We also need to find f which can be calculated by finding R

$$R = 2\Big(\dot{H}_1 + H_1^2 + \dot{H}_2 + H_2^2 + \dot{H}_3 + H_3^2 + H_1H_2 + H_2H_3 + H_3H_1\Big) \tag{3.39}$$

$$\begin{aligned} R = \frac{1}{t^2}2\Big(&\mathrm{e}^{-2mt}\big(l_1^2+l_2^2+l_2l_3+l_3^2+l_1(l_2+l_3)\big)t^{2-2n} + \mathrm{e}^{-m}(l_1+l_2+l_3)(1+n)t^{-n} \\ &+ 4\mathrm{e}^{-mt}k(l_1+l_2+l_3)t^{1-n}(n+mt) + 3k(n+2kn^2+4kmnt+2km^2t^2)\Big) \end{aligned} \tag{3.40}$$

Since, $l_1+l_2+l_3=0$, R can be simplified to

$$R = \frac{1}{t^2}2\Big(\mathrm{e}^{-2mt}\big(l_1^2+l_2^2+l_2l_3+l_3^2+l_1(l_2+l_3)\big)t^{2-2n} + 3k(n+2kn^2+4kmnt+2km^2t^2)\Big) \tag{3.41}$$

f is given by $\frac{\partial F}{\partial R}$ which can be calculated by $\frac{\partial F}{\partial t}\frac{\partial t}{\partial R}$

$$\begin{aligned} f(t) = &\frac{\mathrm{e}^{-2mt}kt^{2-2n}(n+mt)}{\mathrm{e}^{-m}(l_1+l_2+l_3)(2+3n+n^2)t^{-n} + 2\mathrm{e}^{-2mt}\big(l_1^2+l_2^2+l_2l_3+l_3^2+l_1(l_2+l_3)\big)t^{2-2n}(n+mt)} \\ &+ 4\mathrm{e}^{-mt}k(l_1+l_2+l_3)t^{1-n}(n+n^2+2mnt+m^2t^2) + 6kn\big(1+2k(n+mt)\big) \end{aligned} \tag{3.42}$$

Since, $l_1+l_2+l_3=0$, f can be simplified to

$$f(t) = \frac{\mathrm{e}^{-2mt}kt^{2-2n}(n+mt)}{2\mathrm{e}^{-2mt}\big(l_1^2+l_2^2+l_2l_3+l_3^2+l_1(l_2+l_3)\big)t^{2-2n}(n+mt) + 6kn\big(1+2k(n+mt)\big)} \tag{3.43}$$

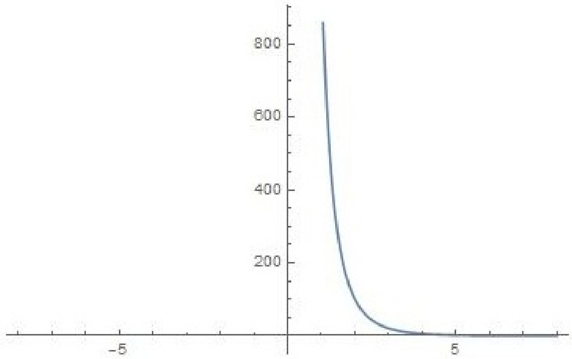
Now, we will find $\rho(t)$ by using the above values in substituting them in 3.18

$$\begin{aligned} \rho(t) = -kt^{-2-4n}\Big(&\mathrm{e}^{-m(1+4t)}\big(\mathrm{e}^m(l_1^2+l_2^2+l_3^2)t^2 + 3\mathrm{e}^{m+2mt}kt^{2n}(n+(2+k)n^2+2(2+k)mnt+(2+k)m^2t^2)\big) \\ &+ \frac{0.5\mathrm{e}^{-2mt}t^{4+2n}(n+mt)}{2\mathrm{e}^{-2mt}\big(l_1^2+l_2^2+l_2l_3+l_3^2+l_1(l_2+l_3)\big)t^{2-2n}(n+mt) + 6kn\big(1+2k(n+mt)\big)}\Big) \end{aligned} \tag{3.44}$$

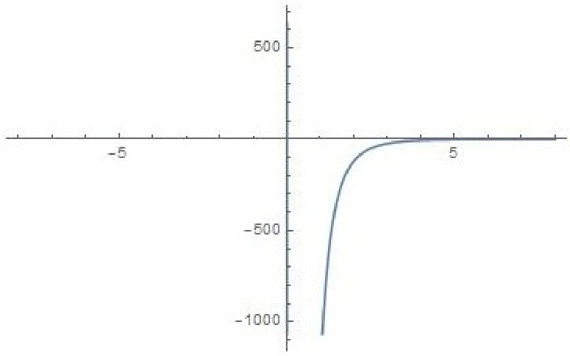
Now let us calculate $p(t)$ from 3.19

$$p(t) = kt^{-2-4n}\Big(\frac{0.5\mathrm{e}^{-2mt}t^{4+2n}(n+mt)}{2\mathrm{e}^{-2mt}\big(l_1^2+l_2^2+l_2l_3+l_3^2+l_1(l_2+l_3)\big)t^{2-2n}(n+mt)+6kn\big(1+2k(n+mt)\big)} + \mathrm{e}^{-m(1+4t)}\big(\mathrm{e}^{2mt}l_1(1+n)t^n+\mathrm{e}^{m+mt}\big(2(l_2+l_3)+k(4l_1+l_2+l_3)\big)t^{1+n}(n+mt) + \mathrm{e}^{m+2mt}t^{2n}\big((-4+4k+3k^2)n^2+(-4+4k+3k^2)m^2t^2+n(-2+k-8mt+8kmt+6k^2mt)\big)\big)\Big) \tag{3.45}$$

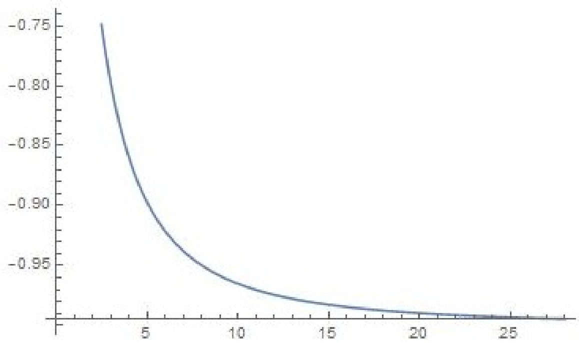
Now, let us plot these functions in a graph by considering $k=-10$, $l_1=-2$, $l_2=1$, $l_3=1$, $m=0.4$, $n=0.6$



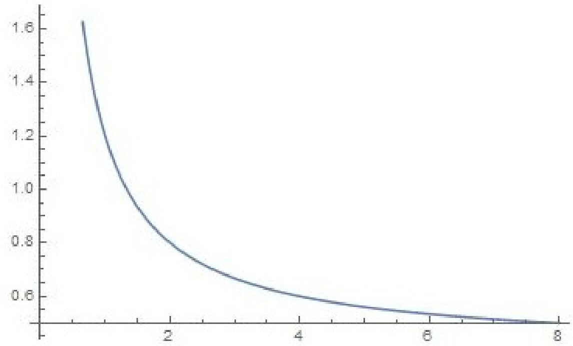
density vs *time*



pressure vs *time*



q (deceleration parameter) vs *time*



Hubble parameter vs *time*

3.2.2 Model 2

Let us consider $f(R) = R + bR^x$. We will assume *hybrid expansion* for scale factor such that $a(t) = t^n\mathrm{e}^{mt}$ To make our calculations simplified let us assume $l_1 = l_2 = l_3 = 0$
$F(R)$ given by $\frac{\partial F}{\partial R}$ is

$$F(R) = 1 + xbR^{x-1} \tag{3.46}$$

The directional Hubble parameter reduces to $H_1 = H_2 = H_3 = \frac{\dot{a}}{a}$ Substituting the value of of $a(t)$ we get

$$H_1 = H_2 = H_3 = m + \frac{n}{t} \tag{3.47}$$

From this we get $\dot{H}_1 = \dot{H}_2 = \dot{H}_3 = \frac{-n}{t^2}$ Using these values we will calculate Ricci scalar. It simplifies to

$$R = \frac{-6n}{t^2} + 12\big(m + \frac{n}{t}\big)^2 \tag{3.48}$$

Now we will find and

$$\dot{F} = b(x-1)x\big(12(\frac{n}{t} + m)^2 - \frac{6n}{t^2}\big)^{x-2}\big(\frac{12n}{t^3} - \frac{24n(\frac{n}{t} + m)}{t^2}\big) \tag{3.49}$$

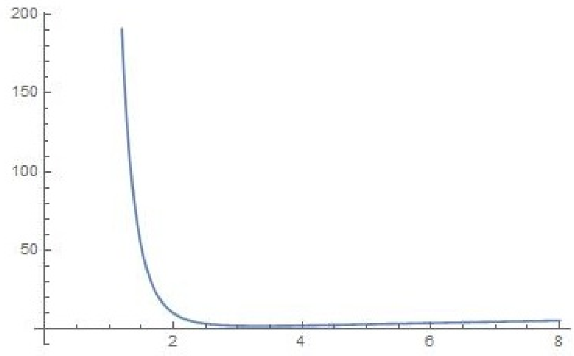
$$\begin{aligned}\ddot{F} &= b\,(x-2)\,(x-1)\,x\left(12\left(\frac{n}{t}+m\right)^2 - \frac{6n}{t^2}\right)^{x-3}\left(\frac{12n}{t^3} - \frac{24n\left(\frac{n}{t}+m\right)}{t^2}\right)^2 \\ &+ b \quad (x-1)\,x\left(\frac{48n\left(\frac{n}{t}+m\right)}{t^3} + \frac{24n^2}{t^4} - \frac{36n}{t^4}\right)\left(12\left(\frac{n}{t}+m\right)^2 - \frac{6n}{t^2}\right)^{x-2}\end{aligned} \tag{3.50}$$

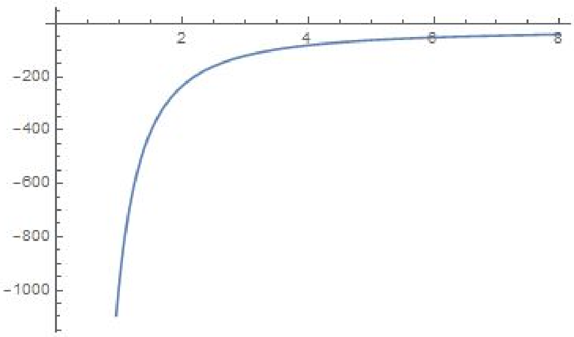
Now we will substitute the above values into 3.18 to obtain $\rho(t)$

$$\begin{aligned}\rho(t) = -\frac{1}{2}b\left(12\left(m+\frac{n}{t}\right)^2 - \frac{6n}{t^2}\right)^x + \frac{3\left(n-(mt+n)^2\right)\left(bx\left(12\left(m+\frac{n}{t}\right)^2 - \frac{6n}{t^2}\right)^{x-1}+1\right)}{t^2} + \\ \frac{bn6^x(1-x)x(n-2n(mt+n))\left(-\frac{n-2(mt+n)^2}{t^2}\right)^{x-2}}{t^5} - \frac{6(mt+n)^2}{t^2} + \frac{3n}{t^2}\end{aligned} \tag{3.51}$$

Similarly, we will use 3.19 to calculate $p(t)$

$$\begin{aligned}p(t) = \frac{-1}{2t^6}\Big(&t^4(6n-12(n+mt)^2 - 6^x bt^2(-\frac{n-2(n+mt)^2}{t^2})^x) \\ &- 48b\big(12(m+\frac{n}{t})^2 - \frac{6n}{t^2}\big)^{-2+x} t^2(n+mt)(n-2n(n+mt))(1-x)x \\ &+ 2^{1+x}3^{-1+x}bnt^2(3-6n-4mt)\big(\frac{-n-2(n+mt)^2}{t^2}\big)^{-2+x}(1-x)x \\ &+ 288b\big(12(m+\frac{n}{t})^2 - \frac{6n}{t^2}\big)^{-3+x}(n-2n(n+mt))^2(1-x)(2-x)x \\ &+ 2t^4(n-3(n+mt)^2)\big(1+b\big(12(m+\frac{n}{t})^2 - \frac{6n}{t^2}\big)^{1+x}x\big)\Big)\end{aligned} \tag{3.52}$$





density vs *time*
$(x = 2,\ b = -2,\ m = 0.5,\ n = -1)$

pressure vs *time*
$(x = 2,\ b = -2,\ m = 0.5\ ,n = 1)$

Chapter 4

Universe as we know

4.1 Background

Latest observations of our universe suggest that there are *three* types of matter present in it namely *baryons*, which are protons and neutrons and consists of about 4% of the universe. The other type of matter is called *dark matter* and consist of about 20% of the universe. These do not interact with electromagnetic radiation and their presence is suggested from astronomical observations such as *gravitational lensing* and motion of galaxies. The third type of matter present is *dark energy* and consists of about 76% of the universe. This is an *hypothesized* form of energy to explain the accelerated expansion of the universe. We will look about this in more detail in the later section.

Recent observations suggest that the universe is very close to a flat geometry. This also has a theoretical support from cosmological inflation, a theory which resolves several cosmological issues. We will look about this in detail in the next section.

Based on the observations of WMAP in 2012, the universe is 13.772 billion years old with an uncertainty of 59 million years. In 2013, plank spacecraft predicted the age of universe to be 13.82 billion years old from its observations .

According to our current observations and understanding of density parameter and expansion rate, the universe will continue to expand forever which can result the universe to approach absolute zero eventually ceasing star formation leading to a darker universe.

4.2 Cosmological Inflation

Inflation is an extra addition to Big bang theory which is applied at the early stage of the universe. The theory was introduced by Alan Guth in 1981 to explain few of the problems present in the Big bang theory like the *Horizon problem* and the *Flatness problem*. Inflation says that $a(\ddot{t}) > 0$ at the beginning of the universe.

4.2.1 Flatness Problem

Flatness problem basically says that the value of *density parameter* is too specific. Let us try to understand this statement.
Let us consider the Friedmann equation in general relativity which is given as :

$$H^2 = \frac{8\pi G}{3}\rho - \frac{\kappa}{a^2} \tag{4.1}$$

Let us consider *critical density* ρ_c, which is the density such as the universe is flat i.e $\kappa = 0$

Therefore, $\rho_c = \frac{3H^2}{8\pi G}$ and let the *density parameter* $\Omega = \frac{\rho}{\rho_c}$. Using ρ_c and Ω in 4.1,

$$H^2 = \frac{8\pi G}{3}\Omega\rho_c - \frac{\kappa}{a^2}$$
$$H^2 = H^2\Omega - \frac{\kappa}{a^2}$$
$$\Omega - 1 = \frac{\kappa}{a^2H^2} \tag{4.2}$$

4.2 implies that for a flat universe i.e $\kappa = 0$, $\Omega = 1$.
Suppose we consider an Universe that is matter dominated or radiation dominated, and neglect the influence of cosmological constant and curvature, we have

$$a^2H^2 \propto \frac{1}{t^{2/3}} \qquad \textit{matter dominated} \tag{4.3}$$
$$a^2H^2 \propto \frac{1}{t} \qquad \textit{radiation dominated} \tag{4.4}$$

From 4.2 and 4.3, we have $\Omega - 1 \propto t^{2/3}$ for a matter dominated universe, and from 4.2 and 4.4, $\Omega - 1 \propto t$ in a radiation dominated universe. In both the cases, $\Omega - 1$ is an *increasing function of time*.
This shows that any deviation of the universe from flatness i.e Ω shifting away from 1 is an *unstable solution*. This is because if Ω deviates from 1 in the positive direction, then as time increases density increases, and eventually the universe would have been so dense that it would have collapsed. If Ω deviates from 1 in the negative direction, density will decrease with time, and it would eventually become so low that the universe could not have supported galaxy formation. By calculating the density parameter we find that at $t = 1sec$, $1 - 10^{-18} < \Omega < 1 + 10^{-18}$ which is a very specific value. Any different value of Ω would have lead to a different fate of universe.

4.2.2 Horizon Problem

Horizon problem arises from the observation that *cosmic microwave background* is isotropic and has a temperature of $2.725K$. The fact that it is isotropic is the reason for the problem. This is because in order to achieve isotropy, radiation should interact with each other.
If for example, we take two galaxies on the opposite side of the universe and consider our galaxy between them such that the two galaxies are not in the same cosmic bubble and our galaxy is present in the cosmic bubble of both the galaxies, then the radiation reaching us from these two galaxies should not have same temperature as they have not interacted with each other. But we find that they have the same temperature. This is known as the *horizon problem.*

4.3 Present day accelerated expansion

As we have mentioned above, the universe is undergoing an accelerated expansion. For this to be possible, we require matter which violates *strong energy principle* i.e $\rho c^2 + 3p < 0$.This is because according to GR, we have $\frac{\ddot{a}}{a} = \frac{-4\pi G}{3}(\rho + \frac{3p}{c^2})$, and accelerated expansion is implied by $\ddot{a(t)} > 0$ which is only possible by violating strong energy condition.To support this, currently we have a theory that the universe is dominated by *dark energy* which violates strong energy principle.

4.4 Conclusion

Einstein's *general relativity* is the most successful theory in explaining gravity. However, it fails to explain several cosmological issues like flatness problem, horizon problem, monopole problem. This had led the scientists to search for alternate theories. In this report, we have discussed one such theory known as *f(R) theory of gravity* which is an extension to GR in which the Ricci scalar R in Einstein-Hilbert action is replaced by a general function $f(R)$. The theory gained a lots of attention due to its simplicity and its ability to explain inflation and late time acceleration.
In this report we have derived Field equations under $f(R)$ gravity by a method known as variation of metric from the modified Einstein-Hilbert action. We have found out the corresponding Friedmann equations in Bianchi type I universe, and considered a couple of cosmological models to understand different parameters of universe like pressure, density and acceleration.
I would like to end this report by reiterating what I had mentioned at the end of the *first chapter.*

The question that we should ask ourselves is *"To what extent does a theory explain physical phenomena?"* and not *"Is the theory correct or wrong?"*.

Bibliography

[1] Thomas P. Sotiriou. “Modified Actions for Gravity: Theory and Phenomenology”. PhD thesis. SISSA, Trieste, 2007. arXiv: 0710.4438 [gr-qc]. URL: https://inspirehep.net/record/765250/files/arXiv:0710.4438.pdf.

[2] Wikipedia contributors. *Galileo Galilei — Wikipedia, The Free Encyclopedia.* [Online; accessed 13-March-2018]. 2018. URL: https://en.wikipedia.org/w/index.php?title=Galileo_Galilei&oldid=829209134.

[3] Wikipedia contributors. *Aristotle — Wikipedia, The Free Encyclopedia.* [Online; accessed 13-March-2018]. 2018. URL: https://en.wikipedia.org/w/index.php?title=Aristotle&oldid=830029745.

[4] Wikipedia contributors. *Urbain Le Verrier — Wikipedia, The Free Encyclopedia.* [Online; accessed 13-March-2018]. 2018. URL: https://en.wikipedia.org/w/index.php?title=Urbain_Le_Verrier&oldid=821604182.

[5] Wikipedia contributors. *Mach’s principle — Wikipedia, The Free Encyclopedia.* [Online; accessed 13-March-2018]. 2018. URL: https://en.wikipedia.org/w/index.php?title=Mach%27s_principle&oldid=825313601.

[6] Wikipedia contributors. *Action at a distance — Wikipedia, The Free Encyclopedia.* [Online; accessed 13-March-2018]. 2018. URL: https://en.wikipedia.org/w/index.php?title=Action_at_a_distance&oldid=822820241.

[7] Wikipedia contributors. *Inertia — Wikipedia, The Free Encyclopedia.* [Online; accessed 13-March-2018]. 2018. URL: https://en.wikipedia.org/w/index.php?title=Inertia&oldid=829495312.

[8] Wikipedia contributors. *Tests of general relativity — Wikipedia, The Free Encyclopedia.* [Online; accessed 13-March-2018]. 2018. URL: https://en.wikipedia.org/w/index.php?title=Tests_of_general_relativity&oldid=827986035.

[9] Thomas P Sotiriou and Valerio Faraoni. “f (R) theories of gravity”. In: *Reviews of Modern Physics* 82.1 (2010), p. 451.

[10] S. Capozziello and M. De Laurentis. “F(R) theories of gravitation”. In: *Scholarpedia* 10.2 (2015). revision #147843, p. 31422. DOI: 10.4249/scholarpedia.31422.

[11] Sean Carroll. *Spacetime and Geometry: An Introduction to General Relativity.* 1,Pearson New International edition. Pearson Education Limited, 2014. ISBN: 1292026634,9781292026633, 9781292039015. URL: http://gen.lib.rus.ec/book/index.php?md5=c95d46f8f1c9772e1f8f32ff5a3be

[12] Emil T. Akhmedov. "Lectures on General Theory of Relativity". In: (2016). arXiv: 1601.04996 [gr-qc].

[13] Wikipedia contributors. *Einstein–Hilbert action — Wikipedia, The Free Encyclopedia.* [Online; accessed 11-March-2018]. 2018. URL: https://en.wikipedia.org/w/index.php?title=Einstein%E2%80%93Hilbert_action&oldid=820526487.

[14] A. Pontzen. "Bianchi universes". In: *Scholarpedia* 11.4 (2016). revision #182483, p. 32340. DOI: 10.4249/scholarpedia.32340.

[15] M Sharif and M Farasat Shamir. "Exact solutions of Bianchi-type I and V spacetimes in the f (R) theory of gravity". In: *Classical and Quantum Gravity* 26.23 (2009), p. 235020.

[16] M Sharif and M Farasat Shamir. "Non-vacuum Bianchi types I and V in f (R) gravity". In: *General Relativity and Gravitation* 42.11 (2010), pp. 2643–2655.

[17] Kotub Uddin, James E Lidsey, and Reza Tavakol. "Cosmological perturbations in Palatini-modified gravity". In: *Classical and Quantum Gravity* 24.15 (2007), p. 3951.

[18] Pedro Moraes and Pradyumn Sahoo. "The simplest non-minimal matter-geometry coupling in the f(R,T) cosmology". In: 77 (July 2017), p. 480.