BIRLA INSTITUTE OF TECHNOLOGY AND SCIENCE-PILANI, HYDERABAD CAMPUS INSTRUCTION DIVISION SECOND SEMESTER 2014-2015

Course Handout (Part II)

Date: 02-12-2015

In addition to Part I (General Handout for all courses appended to the time table) this portion gives further specific details regarding the course.

Course No. : MATH F341

Course Title : INTRODUCTION TO FUNCTIONAL ANALYSIS

Instructor-in-Charge: Debdas Ghosh

1. Scope and Objective of the Course:

Objective of the course is to present basic facts of Functional Analysis in a form that is suitable for Engineers, Scientists and Mathematicians. Ideas are not always generated through logical processes. An engineer may have a feeling for a problem which may lead to a method of solution but justifying part of that needs Analysis. Several concepts of Functional Analysis were invented as there was need from Integral and Differential Equations. Functional Analysis is also needed in Numerical Analysis and Optimization Theory. Modern theory of partial differential equations also relies heavily on the tools of functional analysis.

2. Course Description:

Normed linear spaces, Banach spaces, continuous linear transformations, open mapping theorem, closed graph theorem, uniform boundedness principle, Hahn-Banach theorem, Hilbert spaces, dual space, direct sum and orthogonal complement in Hilbert spaces, function spaces, Symmetric and self adjoint linear mapping in Hilbert spaces, finite rank and compact transformations, unbounded linear transformation, spectral theory, differential equations and linear transformations

3. Text-book: Erwin Kreyszig, Introductory Functional Analysis with Applications, Reprinted 2010, John Wiley

4.. Reference Books:

- (i) Fabian et al, Functional Analysis and infinite-dimensional geometry, Springer (2001)
- (ii) B.V. Limaye, Functional Analysis, New Age International Ltd. (1996)
- (iii) George F. Simmons, *Introduction to Topology and Modern Analysis*, Tata McGraw Hill (2004)

5. Course Plan:

Lecture no.	Learner's objective	Subject matter	Reference
1-3	Recollect some concept of linear Algebra and real Analysis	Vector spaces, dimension, finite dimensional vector spaces, Metric spaces, space of continuous functions	Chapter 1 & Chapter 2: Sec 2.1
4	Introduction to normed linear spaces and Banach spaces	Normed Linear Spaces, Banach spaces and examples such as l_p , c , c_0 , $C[a,b]$	Chapter 2 : Sec 2.2
5-7	Studying properties of normed linear spaces	Properties of normed linear spaces	Chapter 2 : Sec 2.3
8-9	All norms are equivalent on a finite dimensional normed linear space	Finite-Dimensional normed linear spaces and compact sets	Chapter 2: Sec 2.4 and 2.5

10-13	Studying continuity of linear transformations on normed linear spaces	Continuous linear transformations, linear functionals, dual spaces, reflexivity	Chapter 2: Sec 2.6 to 2.10
14-20	How concept of dot product has generalization to certain vector spaces	Inner product spaces, Hilbert spaces, orthogonal sets, direct sum, Bessel's inequality, continuous linear functionals on Hilbert space	Chapter 3: Sec 3.1 to 3.7
21-23	Dual of a Hilbert space, how transpose of a matrix has generalization to continuous linear transformations in Hilbert spaces	Riesz Representation theorem, Symmetric and self adjoint operators	Chapter 3: Sec 3.8 to 3.10
24	How a continuous linear functional defined on a subspace can be extended to whole space	Hahn-Banach Theorem	Chapter 4 Sec 4.2
25-30	When a family of Continuous linear transformations uniformly bounded, When is a continuous linear map a homeomorphism ,closed linear maps need not be bounded	Category theorem ,uniform boundedness principle, strong and weak convergence, Open Mapping theorem, Closed graph theorem	Chapter 4: Sec 4.7 to 4.13
31 - 33	Are there finite rank transformations defined on infinite dimensional spaces	Compact linear operators and their spectral properties	Chapter 8: Sec 8.1 to 8.3
34-36	Integral equations & compact linear transformations	Banach fixed point theorem , Integral equations	Chapter 5: Sec 5.1 to 5.4
37-40	Generalization of eigen values of matrices to linear transformations	Spectral theory of bounded linear transformations	Chapter 7: Sec 7.1 to 7.3

6. Evaluation Scheme:

Components	Durations	Weightage	Date & Time	Remarks
Test I	1 Hour	20%		Closed Book
Test II	1 Hour	20%		Closed Book
Assignments	-	20%	Details will be announced in the class	Open Book
Comprehensive Exam	3 Hours	40%		Closed Book

- 7. Make up Policy: Make-up will be given only for very genuine cases and prior permission has to be obtained from I/C.8. Chamber consultation hours: To be announced in class.
- **9 Notices:** The notices concerning this course will be displayed on the CMS Notice Board only.