Introduction to Integers

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1. Basic Introduction

- Integers are numbers without decimals, like -4, 0, 5, and 1048383.
- There are three main groups of integers
    - Negative integers (integers that are less than 0, like -5, -3, and -1)
    - Zero (just 0)
    - Positive Integers (integers that are greater than 0, like 1, 5, and 72)
- Some other names are
    - **Natural Numbers**: positive integers (1, 2, 3…)
    - **Counting Numbers**: same as natural numbers
    - **Whole Numbers**: positive integers and zero
- There is a lot of repetition between these different terms, and you'll get plenty of practice using them later!
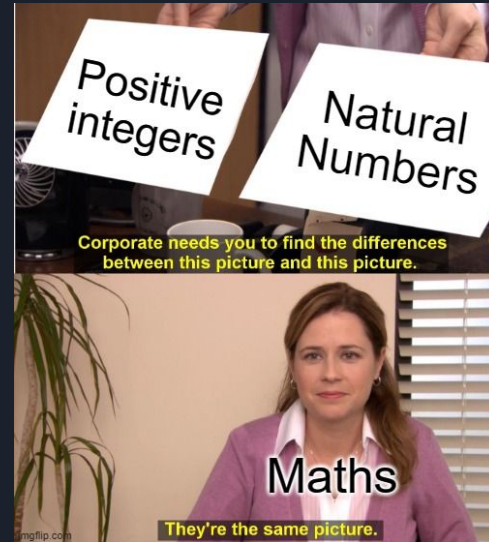
2. Integer Interactions and Multiples

- Basic Integer Operations.
    - Adding two integers results in an integer.
        - Eg: 4 + 5 = 9 and -2 + 4 = 0
    - Subtracting two integers results in an integer.
        - Eg: 4 - 5 = -1 and -6 - 2 = -8
    - Multiplying two integers results in an integer.
        - Eg: 4 x 5 = 20, (-1) x 5 = -5
    - Division is an EXCEPTION.
        - Consider ⅔ or ¾ - these are integers divided by integers, but are obviously not integers themselves.

- Perfect Powers
    - A perfect power is an integer raised to the power of an integer that greater is than or equal to two.
        - Perfect squares, like 3^2 and 4^2, are when the power is 2.
        - Perfect cubes have a power of 3.
- Multiples
    - A multiple of an integer is the product of that and any other integer.
        - Eg: 20 is a multiple of 4 as it is 4 x 5.
        - However, 21 is not a multiple of 4 since 4 can't be multiplied with another integer to give 21.

PRACTICE: Integer Multiples

1. Is 27 a multiple of 9?

2. Is 1001 a multiple of 13?

3. Is 25 a multiple of 4?

4. Is 0 a multiple of 3?

3. Divisibility and Divisors

- <u>If a/b is an integer, *a* is divisible by *b*.</u>
    - Eg: 40/8 = 5, and 5 is an integer, so 40 is divisible by 8.
        - Because 40 is divisible by 8, we call 8 a **divisor** or **factor** of 40.
        - We can also say that 8 divides 40.
        - <u>In general, if *a* is divisible by *b*, *b* divides *a* and is a factor/divisor of *a*.</u>
        - <u>A proper divisor of an integer *n* is a positive divisor of *n* apart from itself.</u>

PRACTICE

1. Is 15 a divisor of 3?

2. Is 15 divisible by 5?

3. Does 11 divide 99?

4. Which is a proper divisor of 85?
    a. 3
    b. 5
    c. 7
    d. 85

3. Divisibility and Divisors

- This concept is the same as multiples, but from a different perspective.
    - If a/b is an integer c, b x c = a, so a is a multiple of b.
    - This only is true if b ≠ 0, as that would cause a division by zero error.
- Hence, if m is divisible by n, m is a multiple of n AS LONG AS n does not equal 0.

REMEMBER: Negative integers can also be divisors or multiples, it's not just positive integers.

Some important results and notes:

1. Because 0/n = 0, n, any integer except 0, is a divisor of 0.
2. When finding the positive divisors of a positive integer n (the same can be done for a negative n as well in a similar fashion), check numbers between 0 and n.

PRACTICE

1. Find the positive divisors of 15.

2. Find the proper divisors of 24.

3. Find *all* the divisors of 6.

3. Divisibility and Divisors

Here is one basic concepts to understand.

- If a is a divisor of b and b is a divisor of c, then a is a divisor of c.
  - A simple proof is as follows;

This is the exact same as saying

- If b is divisible by a and c is divisible by b, then c is divisible by a.

or

- If b is a multiple of a and c is a multiple of b, then c is a multiple of a.

NOTATION

If a is a divisor of b, we say a | b.
If a is NOT a divisor of b, a ∤ b.

PRACTICE

1. 5 | 25?

2. 168 | 4?

3. If 124 divides x, is 4 a divisor of x?

4. Given that 143 divides 1001, is 23023 divisible by 11?

4. Summary

- Integers are either positive, negative, or zero.
- Integers can be classified as natural (counting) numbers or whole numbers.
- A perfect power is an integer raised to a power c where c is greater than or equal to 2.
    - Perfect squares and cubes are examples of this.
- A multiple of an integer is the product of that integer with any integer.
    - Because of this definition, 0 is a multiple of every integer.
- If m/n is an integer, m is divisible by n.
    - This means n is a divisor or factor of m.
        - A proper divisor of a number n is every divisor of n except itself.
    - n divides m.