Techniques for Solving Equations

Nomenclature

These are just some of the common terms used in algebra, and this lesson in particular. Ask in the comments for more!

- expression: combination of terms
- equation: equality of two expressions
- inequality: inequality of two expressions
- constant: fixed numerical value
- variable: symbol that can take different values
- term: combinations of constants and variables through multiplication and division
- coefficient: number in front of an expression

Table of contents

1

Isolation

What is Isolation?

Isolation involves isolating a variable in an equation by using the properties of equality (on both sides).

Essentially, the variable to be solved for is left on one side and the constants are on the other.

Isolation is usable for single variable equations and inequalities.

- This is the most basic method of solving an equation
- Isolation works with a single variable; the other methods are needed for other cases.
- Isolation can be used for equations and inequalities

Examples: Isolation

Here are some examples of equations and inequalities being solved with isolation:

	Equations	Reasoning
1.	$4x + 5 = 33$	Original equation
	$4x = 28$	Subtracting 5
	$x = 7$	Dividing by 4
2.	$3x + 8 = 4x - 7$	Original equation
	$-x = -15$	Subtracting 4x and 8
	$x = 15$	Multiplying by -1

	Inequalities	Reasoning
1.	$2x + 5 > 35 - x$	Original inequality
	$2x > 30 - x$	Subtracting 5
	$3x > 30$	Adding x
	$x > 10$	Dividing by 3
2.	$1x - 4 > 4x - 9$	Original inequality
	$-3x - 4 > -9$	Subtracting 4x
	$-3x > -5$	Adding 4
	$x < 5/3$	Dividing by -3

Remember to flip the inequality sign!

2

Substitution

What is Substitution?

Substitution deals with two or more variables in two or more equations. This is essentially an extension of isolation.

- Extension of isolation
- Handles more than one variable

Explanation

Done on video.

Examples: Substitution

Here are some examples of equations being solved with substitution:

$4x + y = 5$ $x - 2y = -1$	Original system of equations
$x = 2y - 1$	Isolating x
$4(2y - 1) + y = 5$	Substituting x
$8y - 4 + y = 5$	Distributing
$8y + y = 9$	Adding 4
$9y = 9$	Combining Like Terms
$\boldsymbol{y = 1}$	Dividing by 9
$x = 2(1) - 1$	Plugging in y
$\boldsymbol{x = 1}$	Solving for x
$(1, 1)$	Final solution

$x + y = 5$ $x = y - x + 4$	Original system of equations
$y = 2x - 4$	Isolating y
$x + (2x - 4) = 5$	Substituting y
$3x - 4 = 5$	Combining Like Terms
$3x = 9$	Adding 4
$\boldsymbol{x = 3}$	Dividing by 3
$y = 2(3) - 4$	Plugging in x
$\boldsymbol{y = 2}$	Solving for y
$(3, 2)$	Final solution

3

Elimination

What is Elimination?

In elimination, equations are combined so that variables may be eliminated, resulting in simpler equations with fewer variables.

This method is useful for larger equations and can help reduce time and work significantly.

- Eliminates/removes a variable
- Fast and short method

Explanation

Done on video.

Examples: Elimination

Here is an example of a word problem being solved with elimination.

John is 7 times as old as his son. 3 years from now, he will be 5 times as old as his son. How old was John 5 years ago?

Sol: Let John's age be *j* and his son's be *s*. Then we have:

(I) $j = 7s$
(II) $j + 3 = 5(s + 3)$

(II) - (I) →

$3 = 5(s + 3) - 7s$
$3 = 5s + 15 - 7s$
$-12 = -2s$
$s = 6$
$j = 7(6)$
$j = 42$
Since John is 42 now, 5 years ago, he was ***37***.

Remember to read the question carefully and answer only what it asks for.

4

Larger Systems

What are Larger Systems?

Larger systems can be solved the same way as two variable equations.

The larger the systems get, the more tedious work can get.

However, it is important to keep in mind that larger systems often have tricks, as shown in the next slide.

- Solvable with previous methods
- Can often become tedious

More about Larger Systems

There are important terms to keep in mind when dealing with larger systems of equations, described here.

- When there is at least one solution, the system is **consistent**. Otherwise, it is **inconsistent**.

- When any equation in the system is a linear combination of the others, and provides no new information, the system is considered **dependent** or **redundant**. If all equations are independent of each other, the system is considered **independent.**

5

Review + Practice

Review

Isolation is the most basic method for solving equations, but only works for single variable equations. Extensions of isolation are **substitution** and **elimination**, which can be used to solve **larger systems**. Depending on the equations within them, larger systems are classified as consistent/inconsistent and dependent/independent.

TIPS:

- Factoring can be useful in questions, as you will see in the practice problem set.
- Remember to focus on what the question is asking; it is not always necessary to solve a system of equations.

Thanks

Practice problems and solutions are available on my website (linked in the description below).