1.0 Ant Colony Optimisation

Ants leave pheromone (a chemical) when they travel. Other ants follow the trail of pheromone. When an ant has to choose between 2 paths, it will choose the path with greater concentration of pheromones. This is the simple principle helps the ants find the shortest route between a set of stops.

If a group of ants are travelling to and fro from point A to point B and there are various different paths between the 2 points. Initially the probability of ants choosing the each path will be same, however as time passes and will be moving through the shortest path. This is because more number of ants can pass through the shorter distance in same amount of time compared to the longer distance, Hence the concentration of pheromone increases in the shorter path compared to other path. Hence the ants will gradually change to the path with shortest distance.

There are some conditions which the ants follow while moving around the route :

- Ants have memory, they do not visit any of the stops twice in their journey
- Ants know the distance from one stop to other, they choose to travel to closest city given that the pheromone levels are the same.
- If the distance of 2 paths are same, the ants choose the path with greater pheromone level

The mathematical model used to represent the pheromone level on a path is given by the formula :

$$\Delta\tau_{i,j}^{k} = \left\{ \begin{array}{ll} \frac{1}{L_k}, & \text{k ant travels on the edge i to j} \\ 0, & \text{Otherwise} \end{array} \right\} \tag{1}$$

- $\Delta\tau_{i,j}^{k}$ shows the amount of pheromone deposited by and k on path from stop i to stop j
- Pheromone level is equal to 0 if ant does not travel on the edge.
- If ant travels on the path i to j then the pheromone level is $\frac{1}{L_k}$ where L_k is the length of path from i to j. Hence the shorter the path greater the amount of pheromone deposited.

After calculating the pheromone level on each path we need to calculate the probability of ant moving onto a path. The formula for the probability P of ant moving onto a path from stop i to stop j is given by the equation:

$$P_{i,j} = \frac{(\tau_{i,j}) \cdot (\eta_{i,j})}{\Sigma((\tau_{i,j}) \cdot (\eta_{i,j}))}, \eta_{i,j} = \frac{1}{L_{i,j}} \tag{2}$$

- $\tau_{i,j}$ represents the pheromone level on the path from stop i to j.
- $\eta_{i,j}$ represents the quality of path from stop i to stop j.
- The denominator has the sum of the pheromone level of all edges times the quality of all edges.

After calculating the probability matrix we can select the stops according to their probabilities and chart out the route for the bus.

1.10 Applying ant colony optimisation

1. Step 1. To simplify the calculations, the distances measured in meters are converted to Kilometers. Here is the new distance matrix.

Stop No.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
1	0	2.7	2.5	2.7	3.2	3.2	3.2	3.6	3.4	3.5	3.8	3.6	3.4	3.4	3.5
2	2.7	0	0.29	0.75	1	0.95	0.55	0.85	0.75	0.9	1.1	0.95	1	1.1	1.4
3	2.5	0.29	0	0.35	0.75	0.7	0.65	0.85	0.8	1	1.1	1	0.95	1	1
4	2.7	0.75	0.35	0	0.4	0.45	0.5	0.75	0.7	0.85	1	0.8	0.65	0.66	0.65
5	3.2	1	0.75	0.4	0	0.085	0.4	0.65	0.45	0.6	0.85	0.6	0.45	0.5	0.5
6	3.2	0.95	0.7	0.45	0.085	0	0.3	0.55	0.4	0.55	0.75	0.55	0.45	0.45	0.55
7	3.2	0.55	0.65	0.5	0.4	0.3	0	0.3	0.15	0.3	0.5	0.35	0.45	0.45	0.6
8	3.6	0.85	0.85	0.75	0.65	0.55	0.3	0	0.17	0.19	0.3	0.4	0.55	0.55	0.7
9	3.4	0.75	0.8	0.7	0.45	0.4	0.15	0.17	0	0.07	0.35	0.24	0.35	0.4	0.5
10	3.5	0.9	1	0.85	0.6	0.55	0.3	0.19	0.07	0	0.4	0.26	0.4	0.45	0.55
11	3.8	1.1	1.1	1	0.85	0.75	0.5	0.3	0.35	0.4	0	0.5	0.7	0.7	0.65
12	3.6	0.95	1	0.8	0.6	0.55	0.35	0.4	0.24	0.26	0.5	0	0.29	0.27	0.35
13	3.4	1	0.95	0.65	0.45	0.45	0.45	0.55	0.35	0.4	0.7	0.29	0	0.018	0.15
14	3.4	1.1	1	0.66	0.5	0.45	0.45	0.55	0.4	0.45	0.7	0.27	0.018	0	0.13
15	3.5	1.4	1	0.65	0.5	0.55	0.6	0.7	0.5	0.55	0.65	0.35	0.15	0.13	0

Table 1: Distance Matrix (Km)

2. Step 2. Calculating the pheromone level on each path. Using equation 1.

Stop No.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
1	0	0.37	0.40	0.37	0.31	0.31	0.31	0.28	0.29	0.29	0.26	0.28	0.29	0.29	0.29
2	0.37	0	3.45	1.33	1.00	1.05	1.82	1.18	1.33	1.11	0.91	1.05	1.00	0.91	0.71
3	0.40	3.45	0	2.86	1.33	1.43	1.54	1.18	1.25	1.00	0.91	1.00	1.05	1.00	1.00
4	0.37	1.33	2.86	0	2.50	2.22	2.00	1.33	1.43	1.18	1.00	1.25	1.54	1.52	1.54
5	0.31	1.00	1.33	2.50	0	11.76	2.50	1.54	2.22	1.67	1.18	1.67	2.22	2.00	2.00
6	0.31	1.05	1.43	2.22	11.76	0	3.33	1.82	2.50	1.82	1.33	1.82	2.22	2.22	1.82
7	0.31	1.82	1.54	2.00	2.50	3.33	0	3.33	6.67	3.33	2.00	2.86	2.22	2.22	1.67
8	0.28	1.18	1.18	1.33	1.54	1.82	3.33	0	5.88	5.26	3.33	2.50	1.82	1.82	1.43
9	0.29	1.33	1.25	1.43	2.22	2.50	6.67	5.88	0	14.29	2.86	4.17	2.86	2.50	2.00
10	0.29	1.11	1.00	1.18	1.67	1.82	3.33	5.26	14.29	0	2.50	3.85	2.50	2.22	1.82
11	0.26	0.91	0.91	1.00	1.18	1.33	2.00	3.33	2.86	2.50	0	2.00	1.43	1.43	1.54
12	0.28	1.05	1.00	1.25	1.67	1.82	2.86	2.50	4.17	3.85	2.00	0	3.45	3.70	2.86
13	0.29	1.00	1.05	1.54	2.22	2.22	2.22	1.82	2.86	2.50	1.43	3.45	0	55.56	6.67
14	0.29	0.91	1.00	1.52	2.00	2.22	2.22	1.82	2.50	2.22	1.43	3.70	55.56	0	7.69
15	0.29	0.71	1.00	1.54	2.00	1.82	1.67	1.43	2.00	1.82	1.54	2.86	6.67	7.69	0

Table 2: Pheromone Level Matrix

3. Step 3. Calculating $\tau_{i,j} \cdot \eta_{i,j}$. Below is the matrix for the same.

Stop No.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
1	0.00	0.14	0.16	0.14	0.10	0.10	0.10	0.08	0.09	0.08	0.07	0.08	0.09	0.09	0.08
2	0.14	0.00	11.89	1.78	1.00	1.11	3.31	1.38	1.78	1.23	0.83	1.11	1.00	0.83	0.51
3	0.16	11.89	0.00	8.16	1.78	2.04	2.37	1.38	1.56	1.00	0.83	1.00	1.11	1.00	1.00
4	0.14	1.78	8.16	0.00	6.25	4.94	4.00	1.78	2.04	1.38	1.00	1.56	2.37	2.30	2.37
5	0.10	1.00	1.78	6.25	0.00	138.41	6.25	2.37	4.94	2.78	1.38	2.78	4.94	4.00	4.00
6	0.10	1.11	2.04	4.94	138.41	0.00	11.1	3.31	6.25	3.31	1.78	3.31	4.94	4.94	3.31
7	0.10	3.31	2.37	4.00	6.25	11.1	0.00	11.1	44.4	11.1	4.00	8.16	4.94	4.94	2.78
8	0.08	1.38	1.38	1.78	2.37	3.31	11.1	0.00	34.60	27.70	11.1	6.25	3.31	3.31	2.04
9	0.09	1.78	1.56	2.04	4.94	6.25	44.44	34.60	0.00	204.08	8.16	17.36	8.16	6.25	4.00
10	0.08	1.23	1.00	1.38	2.78	3.31	11.1	27.70	204.08	0.00	6.25	14.79	6.25	4.94	3.31
11	0.07	0.83	0.83	1.00	1.38	1.78	4.00	11.1	8.16	6.25	0.00	4.00	2.04	2.04	2.37
12	0.08	1.11	1.00	1.56	2.78	3.31	8.16	6.25	17.36	14.79	4.00	0.00	11.8	13.7	8.16
13	0.09	1.00	1.11	2.37	4.94	4.94	4.94	3.31	8.16	6.25	2.04	11.8	0.00	3086	44.44
14	0.09	0.83	1.00	2.30	4.00	4.94	4.94	3.31	6.25	4.94	2.04	13.7	3086	0.00	59.2
15	0.08	0.51	1.00	2.37	4.00	3.31	2.78	2.04	4.00	3.31	2.37	8.16	44.4	59.2	0.00

Table 3: $\tau_{i,j} \cdot \eta_{i,j}$

4. Step 4. Calculating the probabilities of each path. Sample calculation is shown for path from stop to to 3. The values are multiplied by hundred to get values between 0 and 100.

$$P_{2,3} = \frac{\tau_{2,3} \cdot \eta_{2,3}}{\Sigma((\tau_{2,j})\eta_{2,j})} \times 100 \tag{3}$$

$$= \frac{11.89}{0.14 + 0 + 11.89 + 1.78 + ... + 0.08} \times 100 \qquad = 42.64 \tag{4}$$

Here is the probability matrix :

Stop No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	0.00	0.49	0.45	0.34	0.05	0.05	0.08	0.07	0.03	0.03	0.15	0.08	0.00	0.00	0.06
2	9.99	0.00	33.70	4.44	0.55	0.59	2.79	1.26	0.52	0.43	1.80	1.18	0.03	0.03	0.37
3	11.65	42.64	0.00	20.38	0.98	1.08	2.00	1.26	0.45	0.35	1.80	1.06	0.03	0.03	0.73
4	9.99	6.37	23.14	0.00	3.45	2.62	3.37	1.62	0.59	0.48	2.18	1.66	0.07	0.07	1.72
5	7.11	3.59	5.04	15.60	0.00	73.30	5.27	2.16	1.44	0.96	3.02	2.95	0.16	0.13	2.91
6	7.11	3.97	5.78	12.33	76.48	0.00	9.37	3.01	1.82	1.15	3.88	3.51	0.16	0.15	2.40
7	7.11	11.85	6.71	9.98	3.45	5.88	0.00	10.13	12.93	3.86	8.72	8.67	0.16	0.15	2.02
8	5.62	4.96	3.92	4.44	1.31	1.75	9.37	0.00	10.07	9.61	24.23	6.64	0.10	0.10	1.48
9	6.30	6.37	4.43	5.09	2.73	3.31	37.47	31.54	0.00	70.81	17.80	18.44	0.26	0.20	2.91
10	5.94	4.43	2.83	3.45	1.53	1.75	9.37	25.25	59.37	0.00	13.63	15.71	0.20	0.15	2.40
11	5.04	2.96	2.34	2.50	0.76	0.94	3.37	10.13	2.37	2.17	0.00	4.25	0.06	0.06	1.72
12	5.62	3.97	2.83	3.90	1.53	1.75	6.88	5.70	5.05	5.13	8.72	0.00	0.37	0.43	5.94
13	6.30	3.59	3.14	5.91	2.73	2.62	4.16	3.01	2.37	2.17	4.45	12.63	0.00	96.63	32.31
14	6.30	2.96	2.83	5.73	2.21	2.62	4.16	3.01	1.82	1.71	4.45	14.57	97.00	0.00	43.02
15	5.94	1.83	2.83	5.91	2.21	1.75	2.34	1.86	1.16	1.15	5.16	8.67	1.40	1.85	0.00

Table 4: Probability Matrix

5. Step 5. Begin with stop 1 and move on to the stop with the highest probability. From the next stop, see which stop has the highest probability and move on to that stop. Like this the optimum route can be obtained. The optimum route obtained is shown in the table given :

Stop No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1															
2			33.70												
3	11.65														
4				0.00		2.62									
5															2.91
6					76.48										
7		11.85													
8										9.61					
9							37.47								
10									59.37						
11								10.13							
12											8.72				
13														96.63	
14												14.57			
15													1.40		

Table 5: Ant Colonisation - Optimum Route

Therefore the optimum route given by ant colonisation method is :

$$1 \longrightarrow 3 \longrightarrow 2 \longrightarrow 7 \longrightarrow 9 \longrightarrow 10 \longrightarrow 8 \longrightarrow 11 \longrightarrow 12 \longrightarrow 14 \longrightarrow 13 \longrightarrow 15 \longrightarrow 5 \longrightarrow 6 \longrightarrow 4 \longrightarrow 1$$

The total distance of the journey is 8723m.