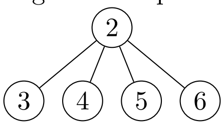
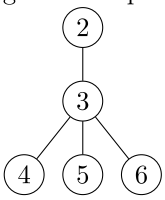
Figure 1: Caption



$$\begin{aligned} g(2,\{3,4,5,6\}) &= \min[w(2,3)+g(3,\{4,5,6\})] \quad (2.3)\\ &= \min[w(2,4)+g(4,\{3,5,6\})] \quad (2.4)\\ &= \min[w(2,5)+g(5,\{3,4,6\})] \quad (2.5)\\ &= \min[w(2,6)+g(6,\{3,4,5\})] \quad (2.6) \end{aligned}$$

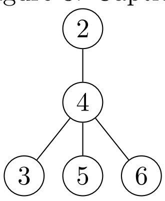
Figure 1 showcases the possibilities of bus travelling from stop 2. It can travel to stops 3,4,5 or 6. The equations representing each of these possibilities are represented towards its right. The aim is to find minimum value out of each of these equations. However each of the equation contains the function g and hence it needs to be solved. The figures and equations below represent the further solution of the equations until an integer value is found for the function g.

Figure 2: Caption



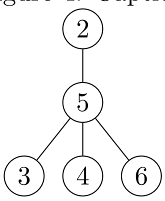
$$\begin{aligned} g(3,\{4,5,6\}) &= \min[w(3,4)+g(4,\{5,6\})] \quad (2.3.4)\\ &= \min[w(3,5)+g(5,\{4,6\})] \quad (2.3.5)\\ &= \min[w(3,6)+g(6,\{4,5\})] \quad (2.3.6) \end{aligned}$$

Figure 3: Caption



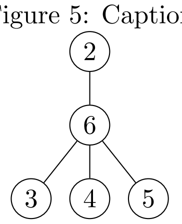
$$\begin{aligned} g(4,\{3,5,6\}) &= \min[w(4,3)+g(3,\{5,6\})] \quad (2.4.3)\\ &= \min[w(4,5)+g(5,\{3,6\})] \quad (2.4.5)\\ &= \min[w(4,6)+g(6,\{3,5\})] \quad (2.4.6) \end{aligned}$$

Figure 4: Caption



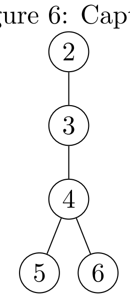
$$\begin{aligned} g(5,\{3,4,6\}) &= \min[w(5,3)+g(3,\{4,6\})] \quad (2.5.3)\\ &= \min[w(5,4)+g(4,\{3,6\})] \quad (2.5.4)\\ &= \min[w(5,6)+g(6,\{3,4\})] \quad (2.5.6) \end{aligned}$$

Figure 5: Caption



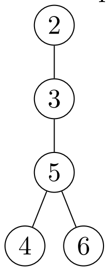
$$\begin{aligned} g(6,\{3,5,4\}) &= \min[w(6,3)+g(3,\{5,4\})] \quad (2.6.3)\\ &= \min[w(6,4)+g(4,\{3,5\})] \quad (2.6.4)\\ &= \min[w(6,5)+g(5,\{3,4\})] \quad (2.6.5) \end{aligned}$$

Figure 6: Caption



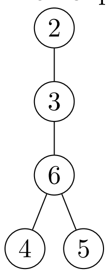
$$\begin{aligned} g(4,\{5,6\}) &= \min[w(4,5)+g(5,\{6\})] \quad (2.3.4.5)\\ &= \min[w(4,6)+g(6,\{5\})] \quad (2.3.4.6) \end{aligned}$$

Figure 7: Caption



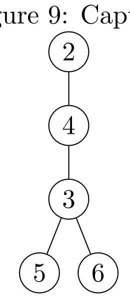
$$g(5,\{4,6\}) = \min[w(5,4) + g(4,\{6\})] \tag{2.3.5.4}$$
$$= \min[w(5,6) + g(6,\{4\})] \tag{2.3.5.6}$$

Figure 8: Caption



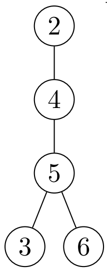
$$g(6,\{4,5\}) = \min[w(6,4) + g(4,\{5\})] \tag{2.3.6.4}$$
$$= \min[w(6,5) + g(5,\{4\})] \tag{2.3.6.5}$$

Figure 9: Caption



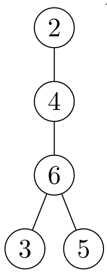
$$g(3,\{5,6\}) = \min[w(3,5) + g(5,\{6\})] \tag{2.4.3.5}$$
$$= \min[w(3,6) + g(6,\{5\})] \tag{2.4.3.6}$$

Figure 10: Caption



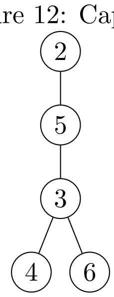
$$g(5,\{3,6\}) = \min[w(5,3) + g(3,\{6\})] \tag{2.4.5.3}$$
$$= \min[w(5,6) + g(6,\{3\})] \tag{2.4.5.6}$$

Figure 11: Caption



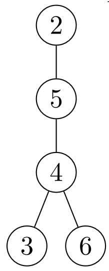
$$g(6,\{3,5\}) = \min[w(6,3) + g(3,\{5\})] \tag{2.4.6.3}$$
$$= \min[w(6,5) + g(5,\{3\})] \tag{2.4.6.5}$$

Figure 12: Caption



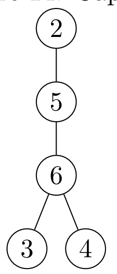
$$g(3,\{4,6\}) = \min[w(3,4) + g(4,\{6\})] \tag{2.5.3.4}$$
$$= \min[w(3,6) + g(6,\{4\})] \tag{2.5.3.6}$$

Figure 13: Caption



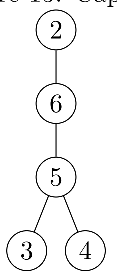
$$\begin{aligned} g(4,\{3,6\}) &= \min[w(4,3)+g(3,\{6\})] \quad (2.5.4.3)\\ &= \min[w(4,6)+g(6,\{3\})] \quad (2.5.4.6) \end{aligned}$$

Figure 14: Caption



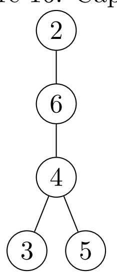
$$\begin{aligned} g(6,\{3,4\}) &= \min[w(6,3)+g(3,\{4\})] \quad (2.5.6.3)\\ &= \min[w(6,4)+g(4,\{3\})] \quad (2.5.6.4) \end{aligned}$$

Figure 15: Caption



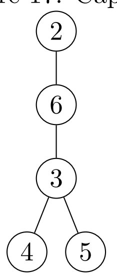
$$\begin{aligned} g(5,\{3,4\}) &= \min[w(5,3)+g(3,\{4\})] \quad (2.6.5.3)\\ &= \min[w(5,4)+g(4,\{3\})] \quad (2.6.5.4) \end{aligned}$$

Figure 16: Caption



$$\begin{aligned} g(4,\{3,5\}) &= \min[w(4,3)+g(3,\{5\})] \quad (2.6.4.3)\\ &= \min[w(4,5)+g(5,\{3\})] \quad (2.6.4.5) \end{aligned}$$

Figure 17: Caption

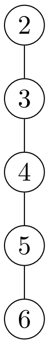


$$\begin{aligned} g(3,\{4,5\}) &= \min[w(3,4)+g(4,\{5\})] \quad (2.6.3.4)\\ &= \min[w(3,5)+g(5,\{4\})] \quad (2.6.3.5) \end{aligned}$$

Equation 2.3.4.5.6 contains the cost function of $g(6,\phi)$, this means that all the stops given is set s are visited exactly once. So now from stop 6 the bus moves back to the starting point, stop 2. Same logic is used for all the equations. The general formula for this is :

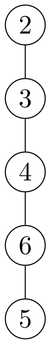
$$g(j,\phi) = w(j,i) \tag{1}$$

Figure 18: Caption



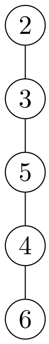
$$g(5, \{6\}) = \min[w(5, 6) + g(6, \phi)] \tag{2.3.4.5.6}$$

Figure 19: Caption



$$g(6, \{5\}) = \min[w(6, 5) + g(5, \phi)] \tag{2.3.4.6.5}$$

Figure 20: Caption

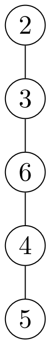


$$g(4, \{6\}) = \min[w(4, 6) + g(6, \phi)] \tag{2.3.5.4.6}$$

Figure 21: Caption

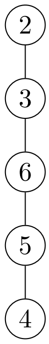
$$g(6, \{4\}) = \min[w(6, 4) + g(4, \phi)] \tag{2.3.5.6.4}$$

Figure 22: Caption



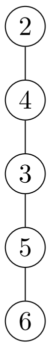
$$g(4, \{5\}) = \min[w(4,5) + g(5, \phi)] \tag{2.3.6.4.5}$$

Figure 23: Caption



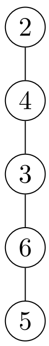
$$g(5, \{4\}) = \min[w(5,4) + g(4, \phi)] \tag{2.3.6.5.4}$$

Figure 24: Caption



$$g(5, \{6\}) = \min[w(5,6) + g(6, \phi)] \tag{2.4.3.5.6}$$

Figure 25: Caption

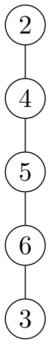


$$g(6, \{5\}) = \min[w(6,5) + g(5, \phi)] \tag{2.4.3.6.5}$$

Figure 26: Caption

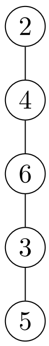
$$g(3,\{6\}) = \min[w(3,6) + g(6,\phi)] \tag{2.4.5.3.6}$$

Figure 27: Caption



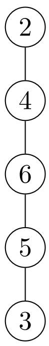
$$g(6,\{3\}) = \min[w(6,3) + g(3,\phi)] \tag{2.4.5.6.3}$$

Figure 28: Caption



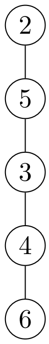
$$g(3,\{5\}) = \min[w(3,5) + g(5,\phi)] \tag{2.4.6.3.5}$$

Figure 29: Caption



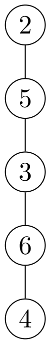
$$g(5,\{3\}) = \min[w(5,3) + g(3,\phi)] \tag{2.4.6.5.3}$$

Figure 30: Caption



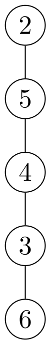
$$g(4, \{6\}) = \min[w(4, 6) + g(6, \phi)] \tag{2.5.3.4.6}$$

Figure 31: Caption



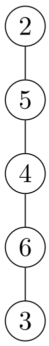
$$g(6, \{4\}) = \min[w(6, 4) + g(4, \phi)] \tag{2.5.3.6.4}$$

Figure 32: Caption



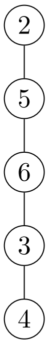
$$g(3, \{6\}) = \min[w(3, 6) + g(6, \phi)] \tag{2.5.4.3.6}$$

Figure 33: Caption



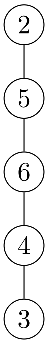
$$g(6, \{3\}) = \min[w(6, 3) + g(3, \phi)] \tag{2.5.4.6.3}$$

Figure 34: Caption



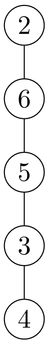
$$g(3, \{4\}) = \min[w(3,4) + g(4, \phi)] \tag{2.5.6.3.4}$$

Figure 35: Caption



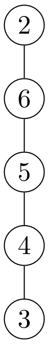
$$g(4, \{3\}) = \min[w(4,3) + g(3, \phi)] \tag{2.5.6.4.3}$$

Figure 36: Caption



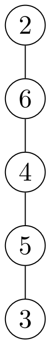
$$g(3, \{4\}) = \min[w(3,4) + g(4, \phi)] \tag{2.6.5.3.4}$$

Figure 37: Caption



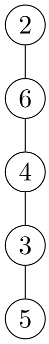
$$g(4, \{3\}) = \min[w(4,3) + g(3, \phi)] \tag{2.6.5.4.3}$$

Figure 38: Caption



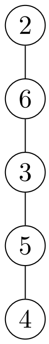
$$g(5, \{3\}) = \min[w(5,3) + g(3, \phi)] \tag{2.6.4.5.3}$$

Figure 39: Caption



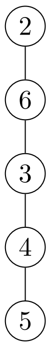
$$g(3, \{5\}) = \min[w(3,5) + g(5, \phi)] \tag{2.6.4.3.5}$$

Figure 40: Caption



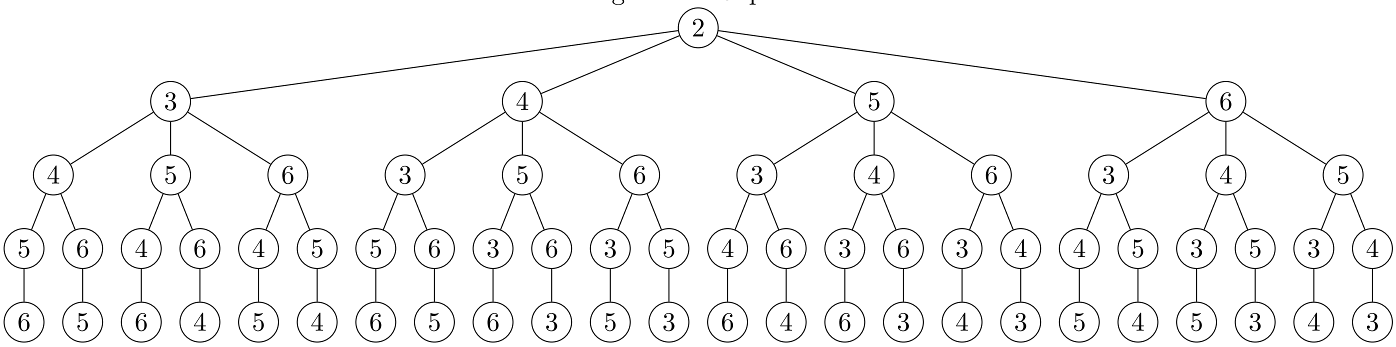
$$g(5, \{4\}) = \min[w(5,4) + g(4, \phi)] \tag{2.6.3.5.4}$$

Figure 41: Caption



$$g(4, \{5\}) = \min[w(4,5) + g(5, \phi)] \tag{2.6.3.4.5}$$

Figure 42: Caption



0.10 Solving equation 2.3

Figure 43: Expansion of equation 2.3

min(2,3)+g(3,{4,5,6})

w(3,4)+g(4,{5,6}) w(3,5)+g(5,{4,6}) w(3,6)+g(6,{4,5})

w(4,5)+g(5,{6}) w(4,6)+g(6,{5}) w(5,4)+g(4,{6}) w(5,6)+g(6,{4}) w(6,4)+g(4,{5}) w(6,5)+g(5,{4})

w(5,6)+g(6,{ϕ}) w(6,5)+g(5,{ϕ}) w(4,6)+g(6,{ϕ}) w(6,4)+g(4,{ϕ}) w(4,5)+g(5,{ϕ}) w(5,4)+g(4,{ϕ})

Solving equation 2.3.4

$$w(5,6) + g(6,\phi) = 85 + 950 = 1035 \tag{2}$$
$$w(6,5) + g(5,\phi) = 85 + 1000 = 1085 \tag{3}$$
$$w(4,5) + g(5,6) = 400 + 1035 = 1435 \tag{4}$$
$$w(4,6) + g(6,5) = 450 + 1085 = 1535 \tag{5}$$

Value of g(4,5,6) is given by the equations 4 and 5. Since objective is to minimise, the one with lowest value is chosen, which is equation 4

$$w(3,4) + g(4,5,6) = 350 + 1435 = 1785 \tag{6}$$

Solving equation 2.3.5

$$w(4,6) + g(6,\phi) = 450 + 950 = 1400 \tag{7}$$
$$w(6,4) + g(4,\phi) = 450 + 750 = 1200 \tag{8}$$
$$w(5,4) + g(4,6) = 400 + 1400 = 1800 \tag{9}$$
$$w(5,6) + g(6,4) = 85 + 1200 = 1285 \tag{10}$$

Value of g(5,4,6) is given by the equations 9 and 10. Since objective is to minimise, the one with lowest value is chosen, which is equation 10.

$$w(3,5) + g(5,4,6) = 4750 + 1285 = 2035 \tag{11}$$

solving equation 2.3.6

$$w(4,5) + g(5,\phi) = 400 + 1000 = 1400 \tag{12}$$
$$w(5,4) + g(4,\phi) = 400 + 750 = 1150 \tag{13}$$
$$w(6,4) + g(4,5) = 450 + 1400 = 1850 \tag{14}$$
$$w(6,5) + g(5,4) = 85 + 1150 = 1235 \tag{15}$$

Value of g(6,5,4) is given by the equations 14 and 15. Since objective is to minimise, the one with lowest value is chosen, which is equation 15

$$w(3,6) + g(6,4,5) = 700 + 1235 = 1935 \tag{16}$$

Value of g(3,{4,5,6}) is given by the equations 6, 11 and 16. Since objective is to minimise, the one with lowest value is chosen,
which is equation 6.

$$w(2,3) + g(3,4,5,6) = 290 + 1785 = 2075 \tag{17}$$

0.20 Solving Equation 2.4

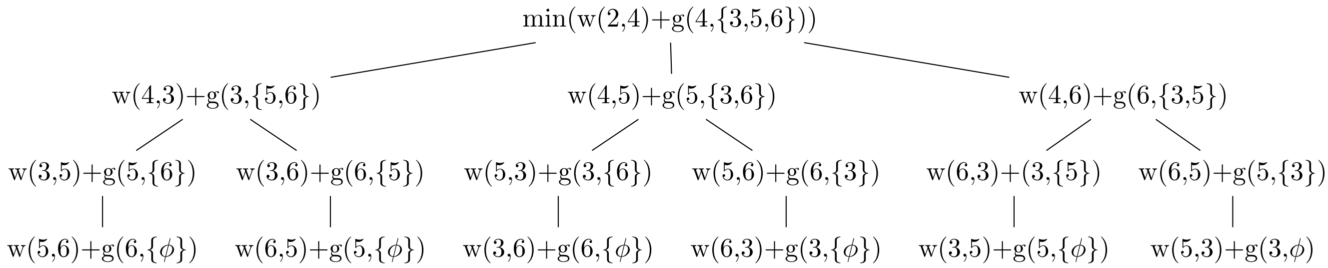


Figure 44: Caption

Solving equation 2.4.3

$$w(5,6) + g(6,\phi) = 85 + 950 = 1035 \tag{18}$$
$$w(6,5) + g(5,\phi) = 85 + 1000 = 1085 \tag{19}$$
$$w(3,5) + g(5,6) = 750 + 1035 = 1785 \tag{20}$$
$$w(3,6) + g(6,5) = 700 + 1085 = 1785 \tag{21}$$

Value of g(6,5,4) is given by the equations 20 and 21. Since objective is to minimise, the one with lowest value is chosen, since both of the equations have same value, both the equations are applicable.

$$w(4,3) + g(3,5,6) = 350 + 1785 = 2135 \tag{22}$$

Solving equation 2.4.5

$$w(3,6) + g(6,\phi) = 700 + 950 = 1650 \tag{23}$$
$$w(6,3) + g(3,\phi) = 700 + 290 = 990 \tag{24}$$
$$w(5,3) + g(3,6) = 750 + 1650 = 2400 \tag{25}$$
$$w(5,6) + g(6,3) = 85 + 990 = 1075 \tag{26}$$

Value of g(5,3,6) is given by the equations 25 and 26. Since objective is to minimise, the one with lowest value is chosen, which is equation 26.

$$w(4,5) + g(5,3,6) = 400 + 1075 = 1475 \tag{27}$$

Solving equation 2.4.6

$$w(3,5) + g(5,\phi) = 750 + 1000 = 1750 \tag{28}$$
$$w(5,3) + g(3,\phi) = 750 + 290 = 1040 \tag{29}$$
$$w(6,3) + (3,5) = 700 + 1750 = 2450 \tag{30}$$
$$w(6,5) + g(5,3) = 85 + 1040 = 1125 \tag{31}$$

Value of g(6,3,5) is given by the equations 30 and 31. Since objective is to minimise, the one with lowest value is chosen, which is equation 31.

$$w(4,6) + g(6,3,5) = 450 + 1125 = 1575 \tag{32}$$

Value of g(4,{3,5,6}) is given by the equations 22, 27 and 32. Since objective is to minimise, the one with lowest value is chosen, which is equation 27.

$$w(2,4) + g(4,3,5,6) = 750 + 1475 = 2225 \tag{33}$$

0.30 Solving equation 2.5

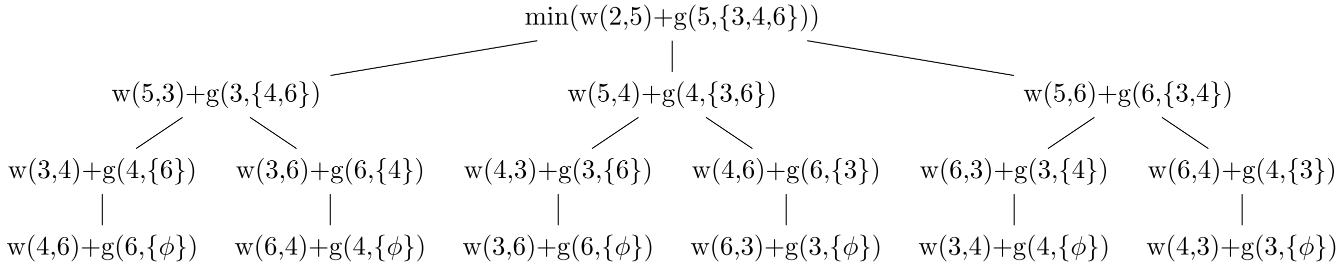


Figure 45: Caption

Solving equation 2.5.3

$$w(4,6) + g(6, \phi = 450 + 950 = 1400 \tag{34}$$
$$w(6,4) + g(4, \phi = 450 + 750 = 1200 \tag{35}$$
$$w(3,4) + g(4, 6 = 350 + 1400 = 1750 \tag{36}$$
$$w(3,6) + g(6,4) = 700 + 1200 = 1900 \tag{37}$$
$$\tag{38}$$

Value of g(3,4,6) is given by the equations 37 and 36. Since objective is to minimise, the one with lowest value is chosen, which is equation 36.

$$w(5,3) + g(3,4,6) = 750 + 1750 = 2500 \tag{39}$$

Solving equation 2.5.4

$$w(3,6) + g(6, \phi = 700 + 950 = 1650 \tag{40}$$
$$w(6,3) + g(3, \phi = 700 + 290 = 990 \tag{41}$$
$$w(4,3) + g(3,6) = 350 + 1650 = 2000 \tag{42}$$
$$w(4,6) + g(6,3) = 450 + 990 = 1440 \tag{43}$$

Value of g(4,3,6) is given by the equations 42 and 43. Since objective is to minimise, the one with lowest value is chosen, which is equation 43.

$$w(5,4) + g(4,3,6) = 400 + 1440 = 1840 \tag{44}$$

Solving equation2.5.6

$$w(3,4) + g(4, \phi) = 350 + 750 = 1100 \tag{45}$$
$$w(4,3) + g(3, \phi) = 350 + 290 = 640 \tag{46}$$
$$w(6,3) + g(3,4) = 700 + 1100 = 1800 \tag{47}$$
$$w(6,4) + g(4,3) = 450 + 640 = 1090 \tag{48}$$

Value of g(6,3,4) is given by the equations 47 and 48. Since objective is to minimise, the one with lowest value is chosen, which is equation 48.

$$w(5,6) + g(6,3,4) = 85 + 1090 = 1175 \tag{49}$$

Value of g(5,{3,4,6}) is given by the equations 39, 44 and 49. Since objective is to minimise, the one with lowest value is chosen, which is equation 49.

$$w(2,5) + g(5,3,4,6) = 1000 + 1175 = 2175 \tag{50}$$

0.40 Solving equation 2.6

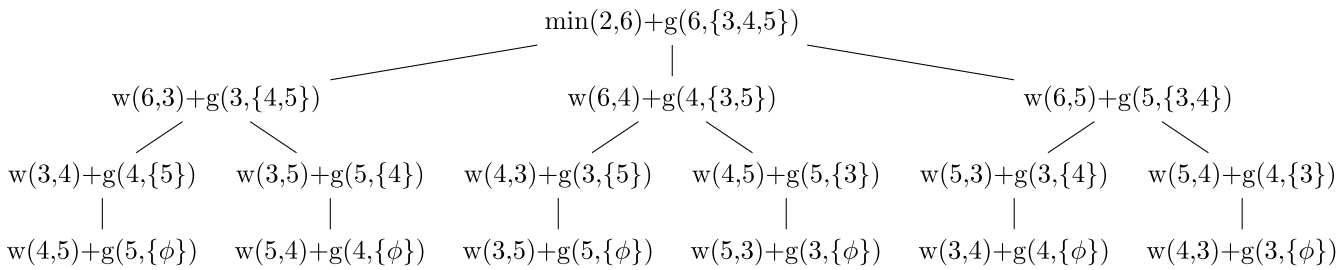


Figure 46: Caption

Solving equation 2.6.3

$$w(4,5) + g(5,\phi) = 400 + 1000 = 1400 \tag{51}$$
$$w(5,4) + g(4,\phi) = 400 + 750 = 1150 \tag{52}$$
$$w(3,4) + g(4,5) = 350 + 1400 = 1750 \tag{53}$$
$$w(3,5) + g(3,4,5) = 750 + 1150 = 1900 \tag{54}$$

Value of g(3,4,5) is given by the equations 53 and 54. Since objective is to minimise, the one with lowest value is chosen, which is equation 53.

$$w(6,3) + g(3,4,5) = 700 + 1750 = 2450 \tag{55}$$

Solving equation 2.6.4

$$w(3,5) + g(5,\phi) = 750 + 1000 = 1750 \tag{56}$$
$$w(5,3) + g(3,\phi) = 750 + 290 = 1040 \tag{57}$$
$$w(4,3) + g(3,\{5\}) = 350 + 1750 = 2100 \tag{58}$$
$$w(4,5) + g(5,\{3\}) = 400 + 1040 = 1440 \tag{59}$$

Value of g(4,3,5) is given by the equations 58 and 59. Since objective is to minimise, the one with lowest value is chosen, which is equation 59.

$$w(6,4) + g(4,\{3,5\}) = 450 + 1440 = 1890 \tag{60}$$

Solving equation 2.6.5

$$w(3,4) + g(4,phi) = 350 + 750 = 1100 \tag{61}$$
$$w(4,3) + g(3,phi) = 350 + 290 = 640 \tag{62}$$
$$w(5,3) + g(3,\{4\}) = 750 + 1100 = 1850 \tag{63}$$
$$w(5,4) + g(4,\{3\}) = 400 + 640 = 1040 \tag{64}$$

Value of g(5,3,4) is given by the equations 63 and 64. Since objective is to minimise, the one with lowest value is chosen, which is equation 64.

$$w(6,5) + g(5,3,4) = 85 + 1040 = 1125 \tag{65}$$

Value of g(6,{3,4,5}) is given by the equations 55, **??** and 65. Since objective is to minimise, the one with lowest value is chosen, which is equation 49.

$$w(2,6) + g(6,3,4,5) = 950 + 1125 = 2075 \tag{66}$$

0.50 Solving the problem

Value of g(2,{3,4,5,6}) is given by the equations 17, 33, 50 and 66. Since objective is to minimise, the one with lowest value is chosen, which are equation 17 and 66.
Since 17 and 66 both have the lowest and the same value, the dynamic programming gives us 2 optimum routes which are :

$$2 \longrightarrow 3 \longrightarrow 4 \longrightarrow 5 \longrightarrow 6 \longrightarrow 2$$

$$2 \longrightarrow 6 \longrightarrow 5 \longrightarrow 4 \longrightarrow 3 \longrightarrow 2$$

In both the cases the net distance travelled by the bus is 2075m.
To find the optimum route for the second cluster which are stops 7 to 15, a code was created which used dynamic programming algorithm to find the optimum route.
The optimum route given by it is

$$7 \longrightarrow 12 \longrightarrow 14 \longrightarrow 13 \longrightarrow 15 \longrightarrow 11 \longrightarrow 8 \longrightarrow 10 \longrightarrow 9 \longrightarrow 7$$

Now if we combine both the optimum routes and add stop 1 at the start and the end of the route we get the route :

$$1 \longrightarrow 2 \longrightarrow 3 \longrightarrow 4 \longrightarrow 5 \longrightarrow 6 \longrightarrow 7 \longrightarrow 12 \longrightarrow 14 \longrightarrow 13 \longrightarrow 15 \longrightarrow 11 \longrightarrow 8 \longrightarrow 10 \longrightarrow 9 \longrightarrow 1$$

The distance bus needs to travel during this route is : 9523m.