Triangular Configuration

The three Phase differnces are,

$$\theta_{0c} = K\ \cos\phi \tag{1.1}$$

$$\theta_{c1} = -K\ \cos\left(\frac{\pi}{3} + \phi\right) \tag{1.2}$$

$$\theta_{c2} = -K\cos\left(\frac{\pi}{3} - \phi\right) \tag{1.3}$$

where ,

$$K = \left(\frac{2\pi R}{\lambda}\right)\cos\alpha \tag{1.4}$$

Let us define delta,

$$\Delta = \theta_{c2} - \theta_{c1} \tag{1.5}$$

$$\Delta = -K\left[\cos\left(\frac{\pi}{3} - \phi\right) - \cos\left(\frac{\pi}{3} + \phi\right)\right]$$

$$\Delta = -K\left[\cos\left(\frac{\pi}{3}\right)\cos\phi + \sin\left(\frac{\pi}{3}\right)\sin\phi - \cos\left(\frac{\pi}{3}\right)\cos\phi + \sin\left(\frac{\pi}{3}\right)\sin\phi\right]$$

$$\Delta = -K\left[2\sin\left(\frac{\pi}{3}\right)\sin\phi\right]$$

$$\Delta = -K\left[2\ \frac{\sqrt{3}}{2}\ \sin\phi\right]$$

$$\Delta = -\sqrt{3}\,K\ \sin\phi \tag{1.6}$$

The AoA can be derived as,

from equation (1.1),

$$\cos\phi = \left(\frac{\theta_{0c}}{K}\right) \tag{1.7}$$

from equation (1.6),

$$\sin\phi = -\left(\frac{\Delta}{\sqrt{3}\,K}\right) \tag{1.8}$$

$$\tan\phi = \frac{\sin\phi}{\cos\phi} = \left[-\left(\frac{\Delta}{\sqrt{3}\,K}\right)\left(\frac{K}{\theta_{0c}}\right)\right]$$

$$\phi = \tan^{-1}\left(\frac{-\Delta}{\sqrt{3}\,\theta_{0c}}\right) \tag{1.9}$$

The expression for elevation angle can be derived by expanding the below expression,

$$\begin{aligned}
\left(\frac{\Delta}{\sqrt{3}}\right)^2+\theta_{0c}^2 &= \left(\frac{-\sqrt{3}\,K\sin\phi}{\sqrt{3}}\right)^2 + \left(K\cos\phi\right)^2 \\
&= K^2\sin^2\phi + K^2\cos^2\phi \\
&= K^2\left(\sin^2\phi + \cos^2\phi\right) \\
\left(\frac{\Delta}{\sqrt{3}}\right)^2+\theta_{0c}^2 &= K^2 \qquad (1.10)
\end{aligned}$$

and from equation (1.4),

$$K^2 = \left(\frac{2\pi R}{\lambda}\cos\alpha\right)^2$$

therefore,

$$\alpha = \cos^{-1}\left[\left(\frac{\lambda}{2\pi R}\right)\sqrt{\left(\left(\frac{\Delta}{\sqrt{3}}\right)^2+\theta_{0c}^2\right)}\right] \qquad (1.11)$$

Quad Configuration

The four phase differences are

$$\theta_{0c} = K\cos\phi \qquad (1.12)$$

$$\theta_{1c} = K\sin\phi \qquad (1.13)$$

$$\theta_{c3} = -K\sin\phi \qquad (1.14)$$

$$\theta_{c2} = -K\cos\phi \qquad (1.15)$$

The AoA can be derived as,

$$\tan\phi = \frac{\sin\phi}{\cos\phi} = \frac{\theta_{1c}}{\theta_{0c}} = \frac{\theta_{c3}}{\theta_{c2}}$$

$$\phi = \tan^{-1}\left(\frac{\theta_{1c}}{\theta_{0c}}\right) \qquad (1.16)$$

$$\phi = \tan^{-1}\left(\frac{\theta_{c3}}{\theta_{c2}}\right) \qquad (1.17)$$

The elevation can be derived as,

$$\theta_{0c}^2 + \theta_{1c}^2 = K^2\left(\sin^2\phi + \cos^2\phi\right)$$

$$\theta_{0c}^2 + \theta_{1c}^2 = K^2 \tag{1.18}$$

and from equation (1.4),

$$K^2 = \left(\frac{2\pi R}{\lambda}\cos\alpha\right)^2$$

therfore,

$$\alpha = \cos^{-1}\left[\left(\frac{\lambda}{2\pi R}\right)\sqrt{\left(\theta_{0c}^2 + \theta_{1c}^2\right)}\right] \tag{1.19}$$

similarly,

$$\alpha = \cos^{-1}\left[\left(\frac{\lambda}{2\pi R}\right)\sqrt{\left(\theta_{c2}^2 + \theta_{c3}^2\right)}\right] \tag{1.20}$$