System Dynamics with Applications - Practical Assignment - Metals Mining

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1 Introduction

We simulate metal mining through three different mine plans: 4Mt (million tonnes), 6Mt and 8Mt. The idea of simulation is to look into the future and see which mining plan to invest in. We have ready data for the mines' plans for future years and we create a simulation model through MATLAB Simulink. With the ready data variables for future years, and simulated gold price, we compute the resulting discounted net predict values (NPV) and compare the plans with them.

2 Model Structure

The model structure in MATLAB Simulink can be seen in Figure 1. Basically, we use basic product and divide blocks, as well as few MATLAB function blocks to compute mining cost inflation, discount profit, and total profit. We use all variables from our data excluding stockpile variables and waste, as they do not affect other values. The output value of our model is the discounted NPV computed for current year.

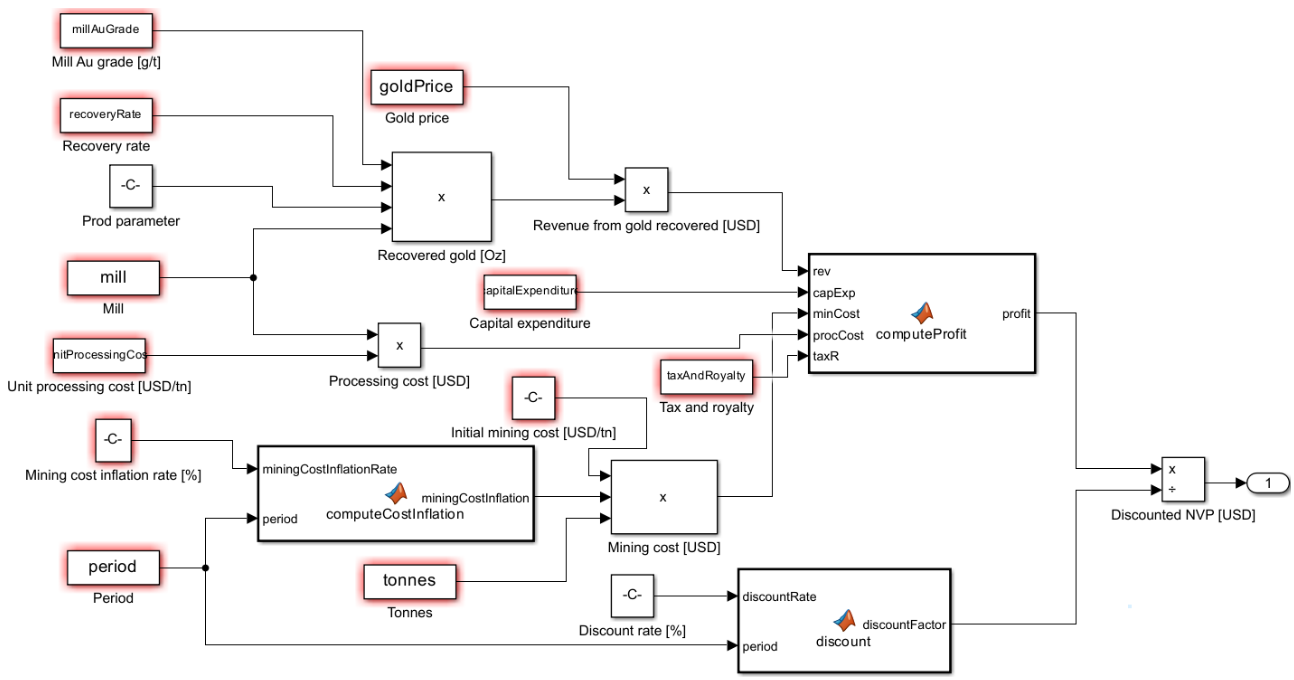

Figure 1: Mine model structure in Simulink.

3 Gold Price Estimation

We know the other parameters for future prices, but the gold price needs to be estimated based on historical data. We can assume that the gold price follows the Geometric Brownian Motion (GBM) model

$$dS_t = \mu S_t dt + \sigma S_t dW_t,$$

where μ is the trend of the gold price, and σ is the volatility of the price.

Historical gold price data includes monthly gold prices and the percentage of price change in a time period of 20 years. As we noted that the changes follow normal distribution, we simply computed the mean and variance of the change values. As we had monthly data, we multiplied the standard deviation by $\sqrt{12}$ and the mean by 12. The resulting yearly volatility estimate is $\sigma = 0.130$ and trend estimate is $\mu = 0.107$. The starting state for gold price was 1866.3 (USD/tn), as it is the last price value in the historical price dataset. We now assumed that it will be the gold price after the fist upcoming year.

We estimated the gold price for 20 years with the GBM model and previously mentioned parameters. We did the estimation in five year periods with Monte Carlo simulation. In practice, we did 10 000 simulation paths for five years. Then, we continued the paths for 5 years with updated starting states. The same volatility and trend parameters were used. We could have also used other estimation methods, such as MCMC, but GBM is enough for this kind of model. Also, we could take the 5 year periods into account and update the GBM parameters, but it was not done now. The simulated price estimates can be seen in Figure 2. We use these same simulated values for each three mine options.

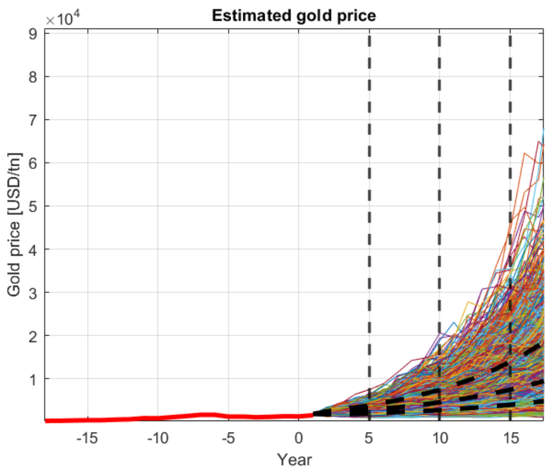

Figure 2: Simulated gold price estimates. The red line is historical gold price, and the colorful lines are the 10 000 Monte Carlo simulation paths. Black vertical dashed lines show the 5 year periods. Horizontal dashed lines show the median price of simulations, 10% quantiles, and 90% quantiles. The year values tell the price after n years, meaning that year value 1 tells the initial gold price, which is now used for the first mining year.

4 Results

We simulated the model with all 3 mine plans, and the results can be seen in Figure 3. We had data for different time periods for different plans, so the durations of the simulations differ.

We can see that in each plan, the median and 10% quantile of the discounted NPV's stay positive. In each case, the NPV's start to decrease in the last years. As NPV's stay positive, also the cumulative median NPV's increase all the time. The last column of the figures show the percentage of 10 000 simulated NPV's during each year being below the threshold value 0.

Based on the results, we can try to choose the best mine plan. Now, we will use a conservative approach and choose a plan with very low risk. It is good to note that with our data we could simulate the 4Mt plan for 20 years, 6Mt plan for 14 years, and 8Mt plan for 11 years. The 6Mt and 8Mt plans seem to be better than the 4Mt plan: we can get to higher cumulative median NPV quicker, and the risk of NPV dropping below zero is significantly smaller. We can see that the yearly cumulative median NPV growth is highest for the 8Mt plan, and I would choose it as a best very low-risk option. However, if only NPV were considered and we would want to take more risks, the best plan may be another option. The 6Mt plan can lead to higher cumulative median NPV in the end, but it takes few more years. Based on NPV value visualization we can also note that the NPV values decrease in the end of each plan. Thus, we can not expect that the cumulative NPV would increase much after the simulation period.

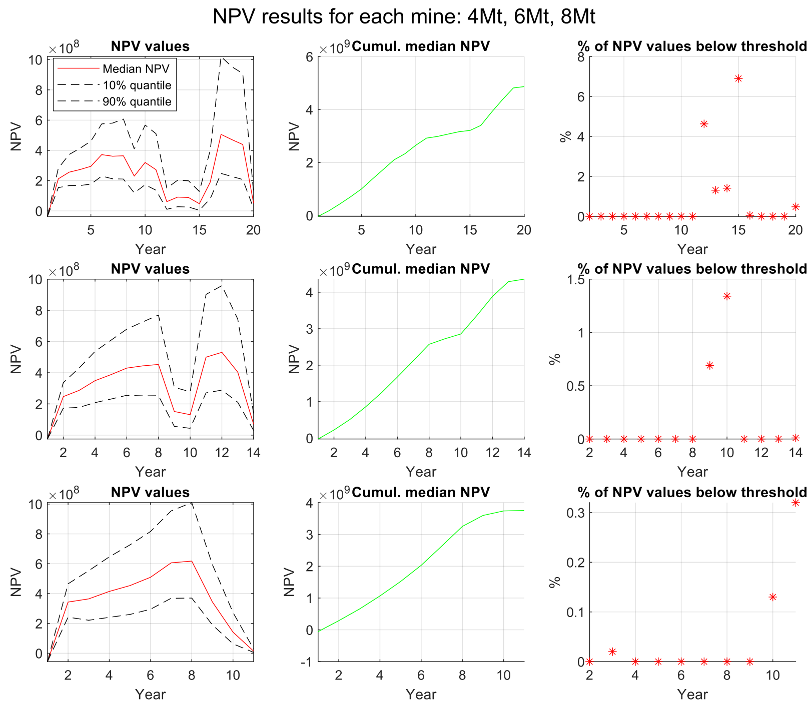

Figure 3: The simulated NPV's for all 3 mine plans. First row is the 4Mt plan, second row 6Mt, and third row 8Mt. The first column shows the median discounted NPV's computed from simulations for each year. The dotted lines show the 10% and 90% quantiles. The middle column shows the cumulative median NPV's per year. Last column shows the percentage of each years' simulated NPV's that are below the threshold value 0. Parameter year tells after how many years the following values result.

4.1 Early Mine Closing

Based on predictions, we can also think about closing the mines at some point to avoid negative NPV's. The 6Mt and 8Mt plans seem to be very low-risk options, as the probability of getting negative NPV's is very small. With these plans, I would say that we could stop after the simulation period (14 years with 6Mt plan, 11 years with 8Mt plan), as the NPV values are decreasing rapidly at that point. The 4Mt plan has higher risks, as there is a 5% chance that the NPV value is negative after 11th year. The probability of that is still very low, but the mine could be shut down after 10 years if we wouldn't want to take any risks at all. It is good to remember that all these simulations include randomness and uncertainty, and the realizations may also be totally different.

4.2 Effect of Gold Price Estimation

We evaluated the effect of the estimate of gold prices on different mining plans. Figure 4 shows the situation if the estimated trend increased massively to 0.25. In this case, the 4Mt plan is the best one because of the highest cumulative NPV values. Figure 5 shows the situation if the estimated trend decreases to 0.05. Then, the probabilities of the NPV values being below the threshold increase and the mines should be shut down at some points. The 8Mt plan is the best one in that case, as we could keep the mine open for nine years and would result in the highest cumulative median NPV. The 4Mt plan should be shut down after 11 years and the 6Mt plan after 8 years, and they would end up with smaller NPV than the 8Mt plan.

We also tested different volatility values. Figure 6 shows the results if the volatility increases to 0.25. As it adds variance to the results, the uncertainty of the plans increases. In this case, the 4Mt plan has the smallest risks (for 8 years), but the other plans have a better chance to achieve higher NPV's faster.

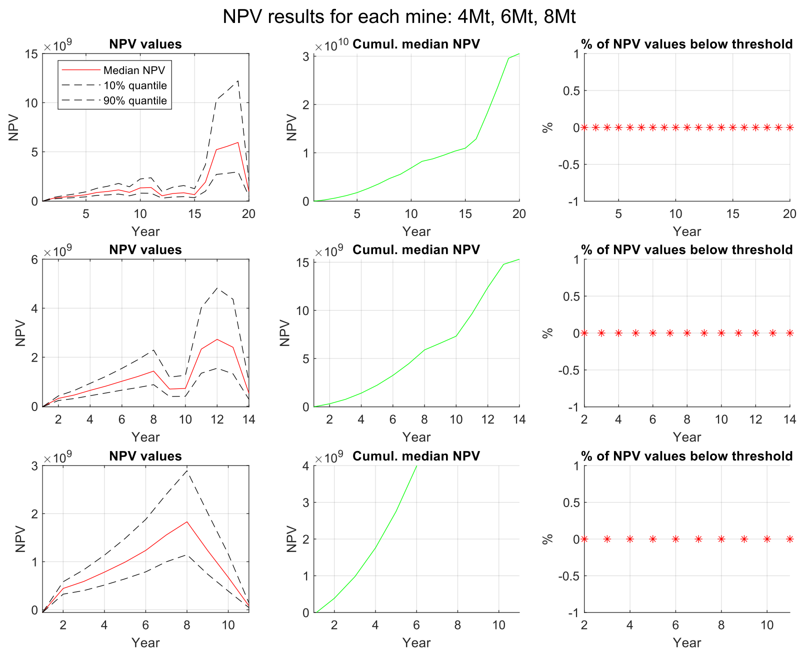

Figure 4: The simulated NPV's for lower gold price estimates.

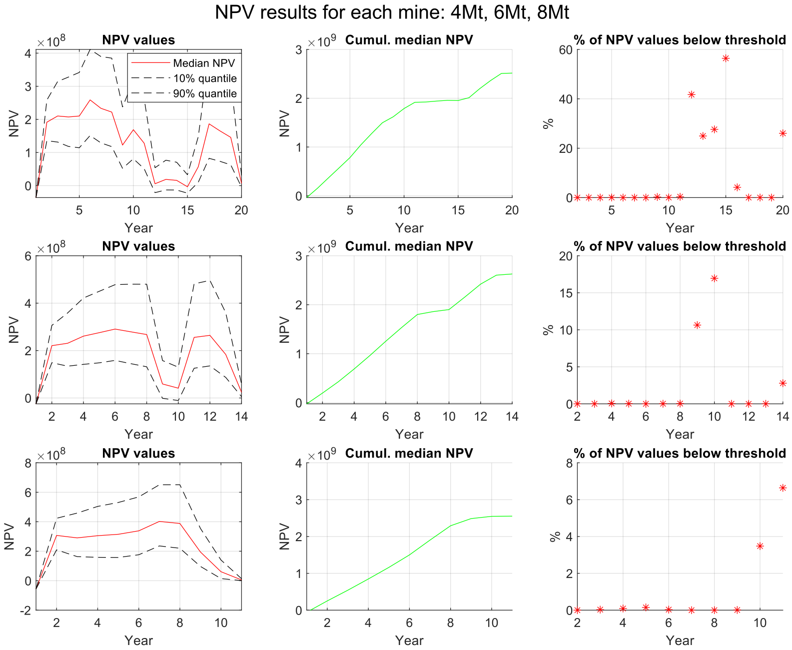

Figure 5: The simulated NPV's for higher gold price estimates.

4.3 Effect of Recovery Rate

Next, we evaluate how the recovery affects the results. Especially, we are interested to see how much the recovery rates should increase so that the worst plan (4Mt) would be the best one. Again, it is good to note that the selection of the best plan changes based on the conditions. Now, we stay with our conservative approach and want to choose a very low-risk option. We decided that a good threshold value for the percentage of NPV's being below 0 is 5%. We noted that a 7-percent increase in 4Mt plan's recovery rate would keep the percentage of NPV's being below threshold below 5, and the mine would achieve the highest NPV. However, it would take over 15 years, and 8Mt plan still has hogher yearly growth of NPV.

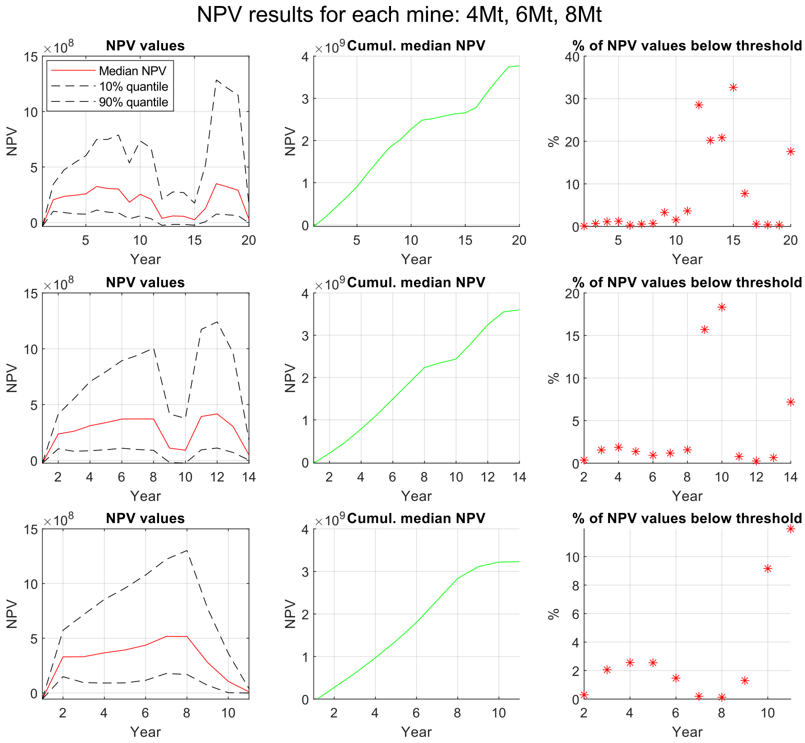

Figure 6: The simulated NPV's for higher volatility in gold prices.

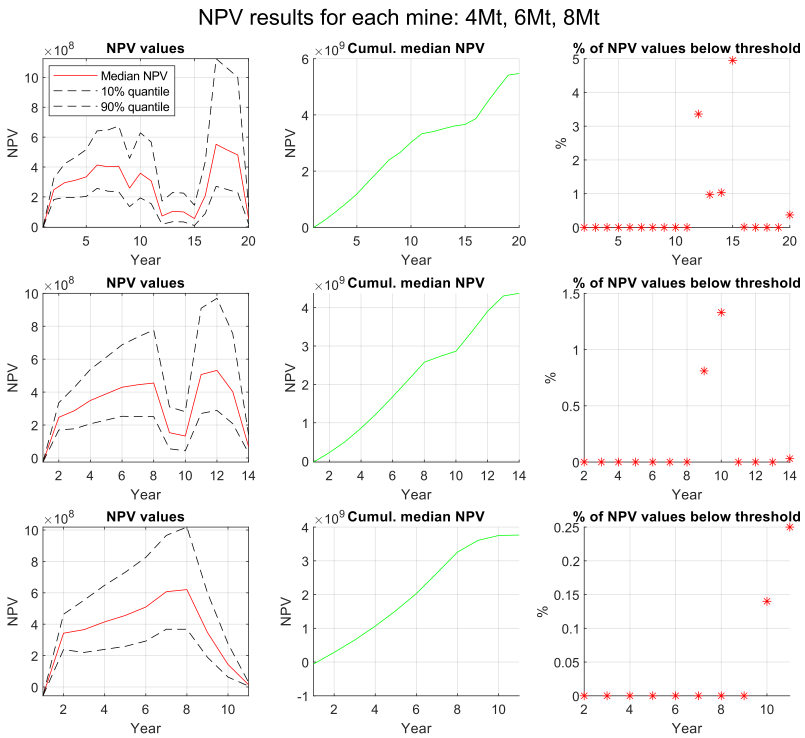

Figure 7: The simulated NPV values with higher recovery rate for 4Mt plan.