Multiple substitutions

- The model assumes that point mutations alter one nucleotide at a time, hence most of the instantaneous rates (3134/3761 or 84.2% in the case of the universal genetic code) are 0.
- This restriction, however, does not mean that the model disallows any substitutions that involve multiple nucleotides (e.g., ACT → AGG).
 - This can be further relaxed with models supporting multiple nucleotide changes.
- Such substitutions must simply be realized via several single nucleotide steps, e.g.,
 ACT
 —AGT
 —AGG
- In fact the (i,j) element of $T(t) = \exp(Qt)$ sums the probabilities of all such possible pathways of duration t, including reversions
- Compare this to the naive NG86 parsimony approach.

Alignment-wide estimates

- Using standard MLE approaches it is straightforward to obtain point estimates of $dN/dS := \beta/\alpha$
- Can also easily test whether or not dN/dS > 1, or < 1 using the likelihood ratio test (LRT)
- Codon models also support the concepts of synonymous and nonsynonymous distances between sequences using standard properties of Markov processes (exponentially distributed waiting times)

$$E[subs] = -\sum_{i} \pi_{i} \hat{q}_{ii}, \quad E[subs] = E[syn] + E[nonsyn] = -\sum_{i} \pi_{i} \hat{q}_{ii}^{s} - \sum_{i} \pi_{i} \hat{q}_{ii}^{ns}.$$