Optimal Control for Spacecraft Rendezvous

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December 10, 2021

1 Introduction

Spacecraft rendezvous consist of a chaser spacecraft conducting a set of maneuvers to come within close proximity or dock to a target spacecraft. These types of missions are common as they are performed when sending humans to the International Space Station. Other applications that require spacecraft rendezvous are emerging such as satellite servicing [1] and planetary defense against near earth objects [2]. The spacecraft rendezvous problem was posed as an optimization problem with the goal of minimizing the distance between the chaser and target spacecraft. One of the methods studied was a linear quadratic regulator controller, and the other method was formulating the problem as a constrained optimization problem that minimized fuel usage. A linear dynamics model for spacecraft relative motion was implemented and the trajectories were simulated forward. The rest of the report is formatted as follows: Section 2 describes the spacecraft dynamics and control, Section 3 depicts the results from both optimization methods, and Section 4 concludes this work and discusses future work.

2 Methods

2.1 Spacecraft Dynamics

2.1.1 Clohessy Wiltshire Model

The Clohessy Wiltshire (CW) model is a set of linear differential equations that describe the relative motion of a chaser spacecraft with respect to a target spacecraft. The target spacecraft is assumed to be in a circular orbit around a central body (Earth), and both spacecraft are considered to be point masses. Ignoring orbital perturbations in the dynamics is another assumption in the CW equations. The CW equations are derived from the nonlinear equations of relative motion by assuming the distance between the target and chaser spacecraft is significantly less than the distance between the target spacecraft and the central body (Earth). A body fixed coordinate system on the target was used to describe the relative motion of the chaser. Equation 1 depicts the CW equations of relative motion. In the CW equations, n is the target mean motion of the target spacecraft which is a function of the target orbit semi major axis (a) and the standard gravitational parameter (μ) (shown in Equation 2).

$$\begin{aligned} \ddot{x} &= 3n^2x + 2n\dot{y} \\ \ddot{y} &= -2n\dot{x} \\ \ddot{z} &= -n^2z \end{aligned} \tag{1}$$

$$n = \sqrt{\frac{\mu}{(a_t)^3}} \tag{2}$$

The continuous dynamics of the chaser spacecraft were then written in state space form. Three thrusters will provide the control in all directions for the chaser spacecraft to reach the target spacecraft. The dynamics in state space form are shown in Equation 3. Figure 1 shows the relative dynamics of the chaser spacecraft (orange point) initialized a distance r=[15, 20, 15] kilometers away from the target (green point). The goal of the project is to solve for optimized maneuvers to allow the chaser spacecraft dock with the target spacecraft.

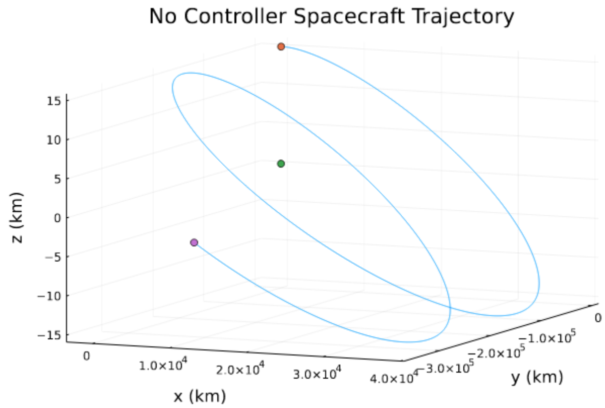


Figure 1: Chaser Spacecraft Trajectory with No Controller

$$
\begin{gathered}
\dot{x} = Ax + Bu \\
A = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 3n^2 & 0 & 0 & 0 & 2n & 0 \\ 0 & 0 & 0 & -2n & 0 & 0 \\ 0 & 0 & -n^2 & 0 & 0 & 0 \end{bmatrix} \\
B = \begin{bmatrix} 0_{3\times3} \\ I_{3\times3} \end{bmatrix} \\
x = \begin{bmatrix} x & y & z & \dot{x} & \dot{y} & \dot{z} \end{bmatrix}^T
\end{gathered} \tag{3}
$$

2.1.2 Continuous to Discrete Dynamics

The continuous dynamics were then transformed to discrete dynamics using Equation 4 because discrete dynamics is a more accurate representation for spacecraft applications. This is because control inputs occur at distinct time events to conserve fuel. In Equation 4, dt is the time step for the discretization, while A_d and B_d are the discrete matrices that describe the dynamics of a system. A time step of dt = 1 s was used in this project.

$$
\begin{bmatrix} A_d & B_d \\ 0 & I \end{bmatrix} = exp(dt * \begin{bmatrix} A & B \\ 0 & 0 \end{bmatrix}) \tag{4}
$$

2.1.3 Controllability

Next, the controllability matrix (C) was used to validate the system being controllable. Equation 5 shows the formation of the controllabilty matrix. A full rank controllability matrix ($rank(C) = n$) proves the system is controllable which means that every state is reachable with appropriate inputs u. The variable n corresponds to the dimension of the A_d matrix which is $n \times n$.

$$
C = \begin{bmatrix} B_d & A_d B_d & (A_d)^2 B & ... & A_d^{n-1} B_d \end{bmatrix} \tag{5}
$$

The rank of the controllability matrix was 6 for this application, which proves the system is controllable. Next, optimal control techniques were used to determine the optimal control inputs for the chaser spacecraft to reach the target.

2.2 Spacecraft Control

2.2.1 Linear Quadratic Regulation

Linear quadratic regulation (LQR) is an optimal control approach for a linear approximation of a dynamical system. In this application, a discrete dynamics model is defined in the form of Equation 6.

$$x_{k+1} = A_d x_k + B_d u_k \tag{6}$$

LQR provides the optimal control strategy depending on the desired system response. A quadratic loss function that determines the stage cost is defined and minimized to obtain the optimal control strategy. The loss function (Equation 7) is quadratic to take advantage of the convexity of quadratic functions.

$$l(x,u) = \frac{1}{2}((x - x_g)^T Q(x - x_g) + u^T Ru) \tag{7}$$

In Equation 7, x is the state vector, x_g is the goal state vector, and u is the control input vector. The first term in the loss function describes a penalty on the difference between states, while the second term is a penalty on the control inputs. The matrices Q and R are the state and control weight matrices that tune the response of the system depending on the desired response. Q is a semi-positive definite matrix while R is a positive definite matrix. A fast responding system will have a high penalty on the states which yields a high Q. The LQR problem becomes to determine the trajectory (states and control inputs) that drive the system to the origin (goal state) subject to the system dynamics (Equation 8).

$$\begin{aligned} \min_{x,u} \sum_{i=0}^{\infty} l(x,u) \\ \text{s.t.} x_{k+1} = A_d x_k + B_d u_k \end{aligned} \tag{8}$$

The optimal control strategy for an LQR controller is given by a gain matrix (K) multiplied by the difference in the current state and the goal state (Equation 9). This LQR gain matrix is calculated by Equation 9 where S is the solution to the algebraic Ricatti equation.

$$\begin{aligned} u &= K(x - x_g) \\ K &= R^{-1} B_d^T S(x - x_g) \end{aligned} \tag{9}$$

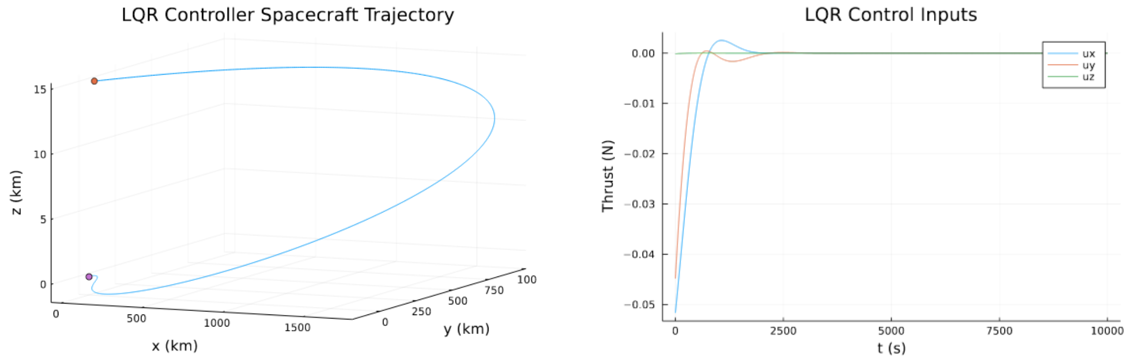
The optimal trajectories using this method are shown in Section 3.

2.2.2 Fuel Usage Optimization

The same problem can be formulated as a constrained optimization problem. LQR does not allow for multiple constraints and may not optimize for important variables. For this approach, the fuel of the spacecraft will be optimized to reduce operation costs and the weight of the spacecraft. Therefore, the L1 norm of the control input (proportional to the fuel) is minimized over a certain amount of time. The constraints of the problem are the initial position of the chaser spacecraft at the zero time step, the final position of the chaser spacecraft equal to the target spacecraft position at the final time step, and the L2 norm of the control input is less than or equal to 0.06 N for each time step (Equation 10).

$$\begin{aligned} l(x,u) &= ||u_k||_1 \\ \min_{x,u} &\sum_{i=0}^{\infty} l(x,u) \\ \text{s.t.} x_0 &= x_{initial} \\ x_N &= [0,0,0]^T \\ |u_k|_2 &\leq 0.06 \end{aligned} \tag{10}$$

This constrained optimization problem was formatted in the Julia programming language and solved using the ECOS convex optimization solver. The trajectory of the constrained optimization problem approach is shown in the Section 3.



(a) LQR Trajectory

(b) LQR Controls

Figure 2: LQR Trajectory and Controls

3 Results

The fourth order Runga-Kutta integrator was used to generate the trajectories using the optimal control inputs from both methods and the spacecraft dynamics described in Section 2.1. For both methods, the chaser spacecraft was a initialized a vector $\mathbf{r_0}$ away from target spacecraft where $\mathbf{r_0} = [15, 20, 15]$ and an initial velocity vector $\mathbf{v_0} = [10, 10, 0]$ (measurements in kilometers). The overall goal was to minimize the distance between the chaser spacecraft and the target spacecraft which was placed at the origin [0,0,0]. For the LQR method, Q was defined as the identity matrix multiplied by 5, while the R matrix was the identity matrix multiplied by 5e10. The results of the LQR controller are shown in Figures 2 and 3, and the fuel optimization method results are shown Figures 4 and 5. For the trajectory plots (Figure 2a and Figure 4a), the orange dot corresponds to the initial position of the chaser spacecraft and the purple dot is the position of the target spacecraft. The control inputs for the LQR case are continuous and it takes around 40 minutes for the chaser spacecraft to reach the target spacecraft. However, the fuel optimization method provides a thrust command at discrete time steps which saves 2 units of fuel over the 2500 time steps. A high cost was imposed on the LQR control input to not generate aggressive maneuvers and conserve fuel. Similar behaviours are seen from the states of both methods (Figures 3 and 5), however, the constrained optimization method has more advantages due to the ability to add additional constraints (such hardware limits or approaching the target from a certain direction).

4 Conclusion and Future Work

Overall, this project shows two optimal control strategies for spacecraft rendezvous. The dynamics of the spacecraft were linearized using the CW equations and the trajectories were plotted using a numerical integrator. Both methods led to a successful rendezvous, however they produced different trajectories. The constrained optimization approach is more realistic as an infinite number of constraints can be added such as the direction to approach the target vehicle once it is within proximity, and constraints on the thrust are necessary because the thrusters have a limit. Future work consists of adding more constraints to the problem as well as implementing a model predictive controller to compare the generated trajectory to previous methods.

5 References

[1] https://www.northropgrumman.com/space/space-logistics-services/
[2] https://www.nasa.gov/planetarydefense/dart
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[4] 16-745 Lecture Notes
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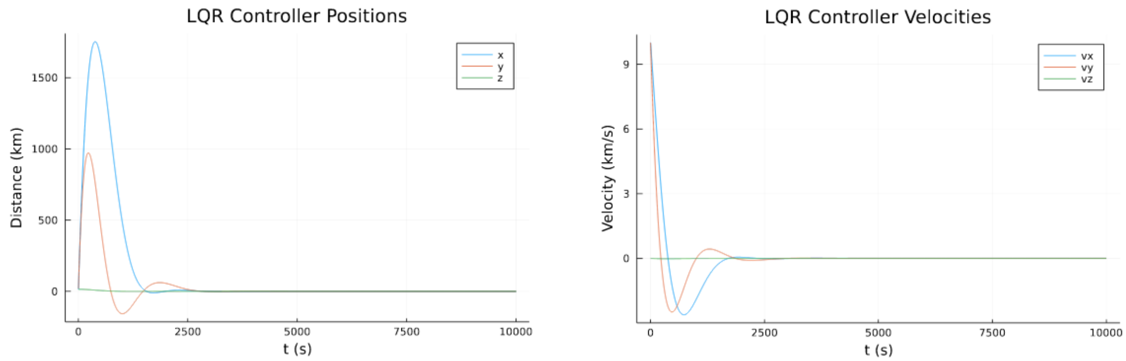
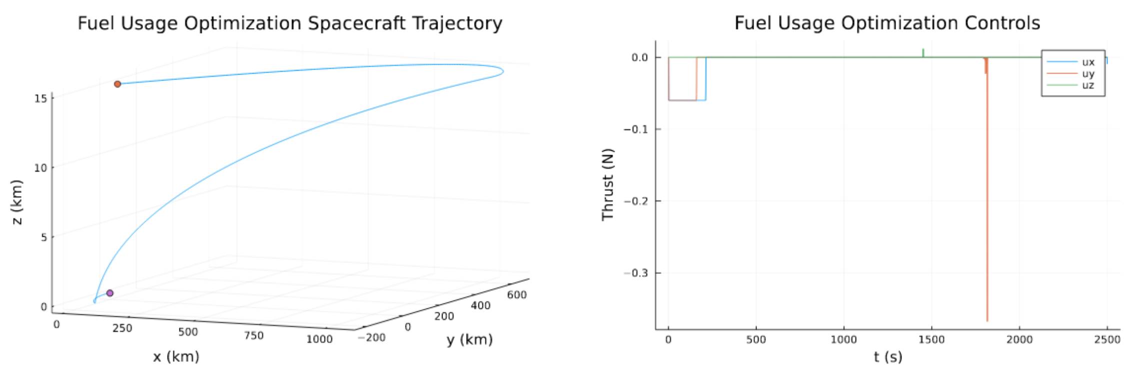


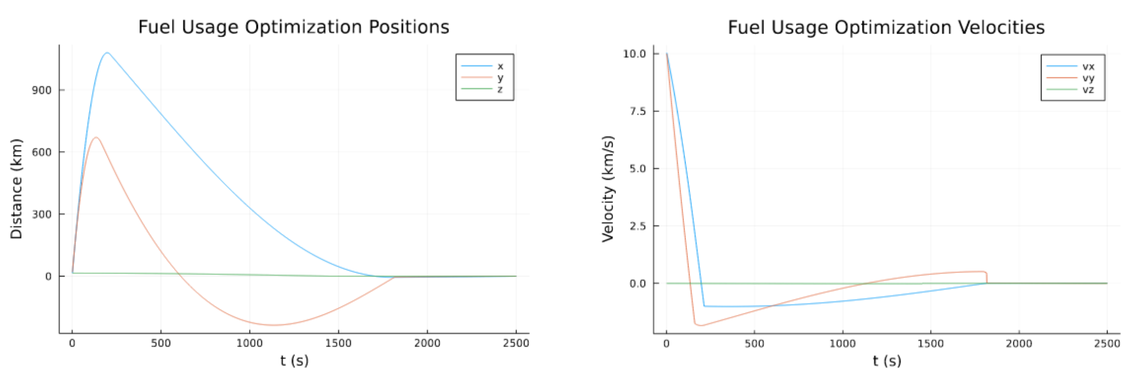
(a) LQR Positions (b) LQR Velocities

Figure 3: LQR States



(a) Fuel Optimization Trajectory (b) Fuel Optimization Controls

Figure 4: Fuel Optimization Trajectory and Controls



(a) Fuel Optimization Positions (b) Fuel Optimization Velocities

Figure 5: Fuel Optimization States