SHEET 1 (T1)

① a) $2y' + (1+4x)\,y^3 = 0$

$$2\frac{dy}{dx} + (1+4x)\,y^3 = 0$$

$$2dy = -(1+4x)\,y^3\,dx$$

$$2\int y^{-3}\,dy = -\int (1+4x)\,dx$$

$$-2\frac{y^{-2}}{2} = -(x+2x^2) + C$$

$$\boxed{y^{-2} = C + x + 2x^2}$$

b) $yy' = \sin^2 \omega x$

$$y \cdot \frac{dy}{dx} = \sin^2 \omega x$$

$$\int y\,dy = \int \sin^2 \omega x\,dx$$

$$\frac{y^2}{2} = \int \left(\frac{1-\cos 2\omega x}{2}\right) dx$$

$$\frac{y^2}{2} = \frac{1}{2}\left[\int dx - \int \cos 2\omega x\,dx\right]$$

$$\frac{y^2}{\cancel{2}} = \cancel{\frac{1}{2}}\left[x - \frac{1}{2\omega}\operatorname{sen} 2\omega x\right] + C$$

$$\boxed{y^2 = x - \frac{1}{2\omega}\operatorname{sen} 2\omega x + C}$$

c) $x^3 y^2 y' = e^y$

$x^3 y^2 \frac{dy}{dx} = e^y$; $x^2 y^2 dy = e^y dx$

$\int e^{-y} y^2 \, dy = \int x^{-3} dx$

$\int e^{-y} y^2 dy = -y^2 e^{-y} - \int 2y(-e^{-y}) dy =$

$u = y^2 \quad du = 2y\,dy$ || $u = 2y \quad du = 2dy$

$dv = e^{-y} \quad v = -e^{-y}$ || $dv = -e^{-y} \quad v = e^{-y}$

$= -y^2 e^{-y} - [2ye^{-y} - \int 2e^{-y} dy] =$

$= -y^2 e^{-y} - 2ye^{-y} - 2e^{-y} = -e^{-y}[y^2 + 2y + 2]$

$-e^{-y}[y^2 + 2y + 2] = \frac{x^{-2}}{-2} + C$

$$\boxed{2x^2 [y^2 + 2y + 2] = e^y (1 + Cx^2)}$$

d) $e^{y^2} dx + 2x^2 y\,dy = 0$

$e^{y^2} dx = -2x^2 y\,dy$

$\int x^{-2} dx = -2 \int y \cdot e^{-y^2} dy$

$-x^{-1} = e^{-y^2} + C$

$$\boxed{C = e^{-y^2} + x^{-1}}$$

e) $4y' = y^3 \sin x$

$4\frac{dy}{dx} = y^3 \sin x$, $\int 4y^{-3}dy = \int \sin x \, dx$

$4\frac{y^{-2}}{-2} = -\cos x + c$; $\boxed{2y^{-2} = \cos x + c}$

f) $y' = y^2 \cos x$

$\frac{dy}{dx} = y^2 \cos x$, $\int y^{-2}dy = \int \cos x \, dx$

$-y^{-1} = \sin x + c$

$\boxed{c = \sin x + y^{-1}}$

g) $\cos x \cos y \cdot y' = \sin x \sin y$

$\cos x \cos y \frac{dy}{dx} = \sin x \sin y$

$\int \underbrace{\frac{\cos y}{\sin y}}_{\cot} dy = \int \underbrace{\frac{\sin x}{\cos x}}_{\tan} dx$

$\ln|\operatorname{sen} y| = -\ln|\cos x| + \ln|c|$

$\operatorname{sen} y = \frac{c}{\cos x}$; $\boxed{c = \operatorname{sen} y \cos x}$

h) $y' = y \tanh x$

$\frac{dy}{dx} = y \tanh x$; $\int y^{-1}dy = \int \tanh x \, dx$

$\ln|y| = \ln|\cosh x| + \ln|c|$

$\boxed{y = c \cosh x}$

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②

a) $y' = e^{x+y-2} - 1$

$\omega = x + y - 2$

$\omega' = 1 + y'$; $\omega' = 1 + e^{\omega} - 1$

$\omega' = e^{\omega}$

$\frac{d\omega}{dx} = e^{\omega}$; $\int e^{-\omega} d\omega = \int dx$

$-e^{-\omega} = x + c$

$$\boxed{c = x + e^{-x-y+2}}$$

b) $y' = (2x + y - 1)^2 - 1$

$\omega = 2x + y - 1$

$\omega' = 2 + y'$

$\omega' = 2 + \omega^2 - 1 = \omega^2 + 1$

$\frac{d\omega}{dx} = \omega^2 + 1$; $\int \frac{1}{\omega^2+1} d\omega = \int dx$

$\tan^{-1}(\omega) = x + c$

x Tan $\boxed{\tan^{-1}(2x + y - 1) = x + c}$

$2x + y - 1 = \tan(x + c)$

sol gdl → $\boxed{y = 1 - 2x + \tan(x + c)}$

c) $y' = \sin(x+y)$

$u = x + y$

$u' = 1 + y' = 1 + \sin(u)$

$$\int \frac{1}{1+\sin(u)}\, du = \int dx$$

$$\frac{-2}{1+\tan(u/2)} = x + C \; ; \; C = x + \frac{2}{1+\tan(u/2)}$$

$$\boxed{C = x + \frac{2}{1+\tan\left(\frac{x+y}{2}\right)}}$$

d) $y' = \dfrac{e^{x+y}}{x+y} - 1.$

$u = x + y$

$u' = 1 + y' \; ; \; u' = 1 + \dfrac{e^u}{u} - 1$

$\dfrac{du}{dx} = \dfrac{e^u}{u} \; ; \; \int u \cdot e^{-u}\, du = \int dx .$

$$\int e^{-u} u\, du = -ue^{-u} - \int -e^{-u}\, du =$$

$u = u \quad du = du$

$dv = e^{-u} \quad v = -e^{-u}$

$$= -ue^{-u} - e^{-u} = e^{-u}(-u-1)$$

$$e^{-u}(-u-1) = x + C$$

$$C = x + e^{-u}(u+1)$$

$$\boxed{C = x + e^{-x-y}(x+y+1)}$$

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e) $y y' + x y^2 = x$

$$y \frac{dy}{dx} + x y^2 = x$$

$$y\, dy = x(1-y^2)\, dx$$

$$\int y \cdot (1-y^2)^{-1} dy = \int x\, dx.$$

$$-\frac{1}{2} \ln|1-y^2| = \frac{x^2}{2} + C$$

$$\boxed{C = x^2 + \ln|1-y^2|}$$

p) $y' = \sqrt{4x+y}$

$\omega = 4x + y$

$\omega' = 4 + y'$; $\omega' = 4 + \sqrt{\omega}$.

$$\int \frac{1}{4+\sqrt{\omega}}\, d\omega = \int dx$$

$$\int \frac{1}{4+\sqrt{\omega}}\, d\omega = \int \frac{2u}{4+u}\, du = 2\left[\int du - \int \frac{4}{4+u}\, du \right.$$

$u = \sqrt{\omega}$; $du = \frac{1}{2\sqrt{\omega}} d\omega$; $d\omega = 2\sqrt{\omega}\, du$

$$= 2u - 8\ln|u+4| = 2\sqrt{\omega} - 8\ln|\sqrt{\omega}+4|$$

$$2\sqrt{\omega} - 8\ln|\sqrt{\omega}+4| = x + C$$

$$\boxed{C = 2\sqrt{4x+y} - 8\ln|\sqrt{4x+y}+4| - x}$$

③ a) $y' = y^2$

$\frac{dy}{dx} = y^2$; $\int y^{-2}\,dy = \int dx$

$-y^{-1} + C = x$

$C = x + y^{-1}$

$C = x + \frac{1}{y} = \frac{xy+1}{y}$

$$\boxed{C = \frac{xy + 1}{y}}$$

b) $xy' = e^{xy} - y$.

$u = xy$

$u' = y + xy'$

$u' = \cancel{y} + e^{xy} - \cancel{y}$ $\quad \frac{du}{dx} = e^{u}$; $\int e^{-u}\,du = \int dx$

$= -e^{-u} = x + C$; $\boxed{C = x + e^{-xy}}$

c) $xy' = \frac{y}{xy+1}$.

$u = xy$; $y = u/x$

$u' = y + y'x$; $u' = y + \frac{y}{xy+1}$; $\frac{u+2}{u+1}$

$u' = y\left(1 + \frac{1}{xy+1}\right)$; $u' = \frac{u}{x}\left(1 + \frac{1}{u+1}\right)$

$\int \frac{u+1}{u(u+2)}\,du = \int \frac{1}{x}\,dx$

$\frac{1}{2}\ln|u^2+2u| = \ln x + \ln C$;

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$$(u^2+2u)^{1/2} = xc$$

$$u^2+2u = x^2c^2$$

$$x^2y^2+2xy = x^2c$$

$$\cancel{x}y(xy+2) = x^{\cancel{2}}c$$

$$\boxed{cx = y(2+xy)}$$

d) $x^2y' = \cos^2(xy) - xy$

$u = xy$

$u' = y + y'x$

$$u' = y + \frac{\cos^2(xy) - xy}{x}$$

$$u' = \cancel{\frac{u}{x}} + \frac{\cos^2(u) \cancel{-u}}{x}$$

$$\frac{du}{dx} = \frac{1}{x}[\cos^2(u)] \; ; \quad \int \frac{1}{\cos^2(u)}du = \int \frac{1}{x}dy$$

$$\tan(u) = \ln|x| + \ln(c)$$

$$\boxed{\tan(xy) = \ln(xc)}$$

e) $xy' = ye^{xy}$

$u = xy$

$u' = y + xy' \; ; \; u' = y + ye^{xy}$

$$u' = y(1-e^{xy}) \; ; \quad \frac{du}{dx} = \frac{u}{x}(1-e^{u})$$

$$\int \frac{1}{u(1-e^{u})}\,du = \int \frac{1}{x}\,dx$$

$$\boxed{\int \frac{1}{u(1-e^{u})}\,du = \ln|xc|}$$

p) $xy' = x^2y^3 + 2xy^2$

$u = xy$

$u' = y + y'x = y + x^2y^3 + 2xy^2$

$$u' = y(1 + \underbrace{x^2y^2}_{\frac{u^2}{x^2}} + \underbrace{2xy}_{\frac{u}{x}})$$

$$\frac{du}{dx} = \frac{u}{x}(1 + u^2 + 2u)$$

$$\int \frac{1}{u(1+u^2+2u)}\,du = \int \frac{1}{x}dx$$

$$\int \frac{1}{u(1+u^2+2u)} = \frac{1}{u+1} + \ln u - \ln|u+1|$$

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$$\frac{1}{u+1} + \ln u - \ln|u+1| = \ln|xc|$$

$$(1+xy)^{-1} = \ln|xc| + \ln|u+1| - \ln|u|$$

$$(1+xy)^{-1} = \ln c \ \ln\left|\frac{\cancel{x}\cdot(xy+1)}{\cancel{x}y}\right| = \ln c\left(\frac{1+xy}{y}\right)$$

$$\boxed{(1+xy)^{-1} = \ln c\left(\frac{1+xy}{y}\right)}$$

④ a) $y' = \frac{y}{x} + 2\sqrt{\frac{x}{y}}$

$y = \upsilon x$

$y' = \upsilon + x\upsilon'$

$\frac{\upsilon x}{x} + 2\sqrt{\frac{x}{\upsilon x}} = \upsilon + x\upsilon'$

$\cancel{\upsilon} + 2\sqrt{\frac{1}{\upsilon}} = \cancel{\upsilon} + x\frac{d\upsilon}{dx}$

$\int \frac{1}{x}\,dx = \int \frac{1}{2\sqrt{1/\upsilon}}\,d\upsilon$

$\ln|x| + \ln|c| = \frac{1}{2}\int \frac{1}{(\upsilon^{-1})^{1/2}}$ $\frac{1}{\hookrightarrow \upsilon^{-1/2}} = \upsilon^{1/2}$

$\ln|x| + \ln|c| = \frac{1}{\cancel{2}}\frac{\upsilon^{3/2}}{3/\cancel{2}}$

$\ln|xc| = \frac{1}{3}\upsilon^{3/2}$; $\boxed{3\ln|xc| = \left(\frac{y}{x}\right)^{3/2}}$

b) $(y^{+3} - yx^2)\,dx = 2xy^2\,dy$

$y = \upsilon x$

$dy = d\upsilon x + dx\upsilon$.

$((\upsilon x)^3 - \upsilon x x^2)dx = 2x\,x^2\upsilon^2(d\upsilon x + dx\upsilon)$

$x^3 = 0$; $\boxed{x = 0}$ sol

$(\upsilon^3 - \upsilon)dx = 2\upsilon^2(d\upsilon x + dx\upsilon)$

$(\upsilon^3 - \upsilon - 2\upsilon^3)dx = 2x\upsilon^2 d\upsilon$

$$\int \frac{1}{x}\,dx = \int \frac{2\upsilon^2}{\upsilon^2 - \upsilon^3 - \upsilon}\,d\upsilon$$

$$\ln|x| + \ln|c| = -\ln|\upsilon^2+1|$$

$$xc = (\upsilon^2+1)^{-1}$$

$$xc = \left(\left(\frac{y}{x}\right)^2+1\right)^{-1}$$

$$xc = \frac{1}{\frac{y^2}{x^2}+1}$$

$$xc = \frac{x^2}{y^2+x^2}\ ; \quad \boxed{y^2+x^2 = x\cdot c}$$ renombrando ctes.

c) $y' = \frac{2x+y}{x}$

$$y = \upsilon x$$

$$y' = \upsilon + \upsilon' x$$

$$\frac{2x+y}{x} = \upsilon + \upsilon' x$$

$$\frac{2\cancel{x}+\upsilon\cancel{x}}{\cancel{x}} = \upsilon + \upsilon' x$$

$$2 + \cancel{\upsilon} = \cancel{\upsilon} + \upsilon' x\ ; \quad 2 = \frac{d\upsilon}{dx}x$$

$$2\int \frac{1}{x}\,dx = \int d\upsilon$$

$$2\ln|x| = \upsilon + c\ ; \quad \boxed{2\ln|x| = \frac{y}{x} + c}$$

D) $(x^2+y^2)\,dx + xy\,dy = 0$

$y = \upsilon x$

$dy = d\upsilon\, x + dx\, \upsilon$

$x^2(1+\upsilon^2)\,dx + x^2\upsilon\,(d\upsilon\, x + dx\,\upsilon) = 0$

$x^2 = 0 \;;\; \boxed{x=0}$ sol

$(1+\upsilon^2+\upsilon^2)\,dx + \upsilon x\, d\upsilon = 0$

$$\int \frac{1}{x}\,dx = -\int \frac{\upsilon}{1+2\upsilon^2}$$

$$\ln|x| + \ln|c| = -\frac{1}{4}\ln|1+2\upsilon^2|$$

$$xc = (1+2\upsilon^2)^{-1/4}$$

$$\boxed{xc = \left(1+2\left(\frac{y}{x}\right)^2\right)^{-1/4}}$$

e) $2xyy' = x^2+y^2$

$y = ux$

$y' = u + \upsilon' x$; $\quad \dfrac{x^2+\upsilon^2x^2}{2x^2\upsilon} = \upsilon + \dfrac{d\upsilon}{dx}x$

$$\frac{(1+\upsilon^2)}{2\upsilon} = \upsilon + \frac{d\upsilon}{dx}x \;;\quad \frac{1+\upsilon^2-2\upsilon^2}{2\upsilon} = \frac{d\upsilon}{dx}x$$

$$\frac{1-\upsilon^2}{2\upsilon} = \frac{d\upsilon}{dx}x \;;\quad \int\frac{1}{x}\,dx = \int\frac{2\upsilon}{1-\upsilon^2}\,d\upsilon$$

$$\ln|x| + \ln|c| = -\ln|1-\upsilon^2|$$

$$xc = \left(1-\left(\frac{y}{x}\right)^2\right)^{-1} \;;\quad \boxed{xc = (x^2-y^2)^{-1}}$$

p) $(y + \sqrt{x^2+y^2})dx - xdy = 0$

$y = \upsilon x$

$dy = d\upsilon x + dx \upsilon$

$(\upsilon x + \sqrt{x^2 + x^2\upsilon^2})dx - x(d\upsilon x + dx\upsilon) = 0$

$x(\upsilon + \sqrt{1+\upsilon^2})dx - \cancel{x}(d\upsilon x + dx\upsilon) = 0$

$\boxed{x = 0}$ sol

$(\upsilon + \sqrt{1+\upsilon^2})dx - x d\upsilon - \upsilon dx = 0$

$(\upsilon + \sqrt{1+\upsilon^2} - \upsilon)dx = x d\upsilon$

$$\int \frac{1}{x} dx = \int \frac{1}{\sqrt{1+\upsilon^2}} d\upsilon$$

$\ln|xc| = \sinh^{-1}(\upsilon)$

$\sinh(\ln|xc|) = (\sinh^{-1}(\frac{y}{x}))\sinh)$

$$\boxed{y = x\sinh(\ln xc)}$$

g) $x^2 y' = 2y^2 + xy$

$y = \upsilon x$

$y' = \upsilon + \upsilon' x$

$\frac{2y^2 + xy}{x^2} = \upsilon + \upsilon' x$; $\frac{2\upsilon^2 x^2 + x^2\upsilon}{x^2} = \upsilon + \upsilon' x$

$2\upsilon^2 + \cancel{\upsilon} = \cancel{\upsilon} + \upsilon' x$; $2\upsilon^2 = \frac{d\upsilon}{dx} x$

$\int \frac{1}{x} dx = \frac{1}{2}\int \underbrace{\frac{1}{\upsilon^2}}_{\upsilon^{-2}} d\upsilon$; $\ln|x| + \ln|c| = \frac{1}{2}\frac{\upsilon^{-1}}{-1}$

$\ln|xc| = -\frac{1}{2}\left(\frac{y}{x}\right)^{-1}$; $\ln|xc| = -\frac{1}{2}\frac{x}{y}$

$$\boxed{y = \frac{-x}{2\ln|xc|}}$$

h) $\left(\frac{y}{x}+1\right)dx + \left(\frac{x}{y}+1\right)dy = 0$

$y = ux \quad u = \frac{y}{x}$

$dy = du\,x + dx\,u$

$\left(\frac{ux}{x}+1\right)dx + \left(\frac{x}{ux}+1\right)(du\,x + dx\,u) = 0$

$(u+1)\,dx + \left(\frac{1}{u}+1\right)(du\,x + dx\,u) = 0$

$(u+1)\,dx + \frac{x}{u}du + dx + du\,x + dx\,u = 0$

$(u+1+1+u)\,dx = -\left(\frac{x}{u}+x\right)du$;

$(2u+2)\,dx = -x\left(\frac{1}{u}+1\right)du$

$$\int \frac{1}{x}dx = -\int \frac{\frac{1+u}{u}}{2u+2}\,du$$

$\ln|x| = -\int \frac{1+u}{2u^2+2u} \Rightarrow -\frac{1}{2}\int \frac{\cancel{1+u}}{\underbrace{u^2+u}_{u(u+1)}} = -\frac{1}{2}\int \frac{1}{u}du$

$\ln|x| = -\frac{1}{2}\ln|u| + \ln|c|$

$x = \frac{c}{u^{-1/2}}$; $c^2 = x\,u^{1/2} = x^2\sqrt{\frac{y}{x}}$; $c^2 = \frac{x^2}{x}\cdot y$;

$c^2 = xy$; $\boxed{C = xy}$

⑥ a) $(3x+y-1)\,dx = (y-x-1)\,dy.$

$$\frac{dy}{dx} = \frac{(3x+y-1)}{(y-x-1)} \quad \Bigg\| \quad \begin{array}{r} 3x+y-1 \\ -3x+3y-3 \\ \hline 4y-4=0 \\ \boxed{y=1} \end{array} \quad \Bigg\| \quad \begin{array}{r} 3x+y-1 \\ x-y+1 \\ \hline 4x=0 \\ \boxed{x=0} \end{array}$$

$x = \bar{X}$

$y = \bar{Y} + 1\,;\quad \bar{Y} = y-1$

$$(3\bar{X} + \bar{Y} + 1 - 1)\,dx = (\bar{Y} + 1 - \bar{X} - 1)\,dy$$

$$(3\bar{X} + \bar{Y})\,d\bar{x} = (-\bar{X} + \bar{Y})\,dy \quad \Big\| \quad \begin{array}{l} \bar{Y} = \upsilon\bar{X} \\ dy = d\bar{X}\upsilon + d\upsilon\bar{X} \end{array}$$

$$(3\bar{X} + \upsilon\bar{X})\,dx = (-\bar{X} + \bar{X}\upsilon)(d\bar{X}\upsilon + d\upsilon\bar{X})$$

$$\bar{X} = 0$$

$$(3+\upsilon)\,dx = (\upsilon-1)(dx\,\upsilon + d\upsilon\,\bar{X})$$

$$\upsilon^2 d\bar{x} + \upsilon\bar{X}d\upsilon - dx\,\upsilon - \bar{X}d\upsilon$$

$$(3+\upsilon-\upsilon^2+\upsilon)\,dx = \bar{X}(\upsilon-1)\,d\upsilon.$$

$$(3+2\upsilon-\upsilon^2)\,dx = \bar{X}(\upsilon-1)\,d\upsilon.$$

$$\int \frac{1}{\bar{X}}\,dx = \int \frac{\upsilon-1}{-\upsilon^2+2\upsilon+3}\,d\upsilon.$$

$$\ln|x| = -\frac{1}{2}\ln|-\upsilon^2+2\upsilon+3| + \ln|C|$$

$$\bar{X} = \frac{C}{(-\upsilon^2+2\upsilon+3)^{-1/2}}\,;\quad C = \bar{X}(-\upsilon^2+2\upsilon+3)^{+1/2}$$

$$\boxed{C = x\left(-\frac{(y-1)^2}{x} + \frac{2(y-1)}{x} + 3\right)^{1/2}}$$

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b) $(x+2y+2)\,dx + (2x+3y+2)\,dy = 0$

$(x+2y+2)\,dx = -(2x+3y+2)\,dy$

$\dfrac{dy}{dx} = \dfrac{x+2y+2}{-2x-3y-2}$ $\Bigg\|$ $\begin{array}{r} 2x+4y+4 \\ -2x-3y-2 \\ \hline y+2=0 \end{array}$; $\boxed{y=-2}$ $\Bigg\|$ $\begin{array}{r} 3x+\cancel{6y}+6 \\ -4x-\cancel{6y}-4 \\ \hline -x+2=0 \end{array}$ $\boxed{x=2}$

$x = \bar{x}+2$

$y = \bar{y}-2$

$(\bar{x}+\cancel{2}+2(\bar{y}-2)+2)\,dx = -(2(\bar{x}+2)+3(\bar{y}-2)+2)\,dy$

$(\bar{x}+2\bar{y})\,dx = (-2\bar{x}-3\bar{y})\,dy$ $\Big\|$ $\bar{y}=\omega\bar{x}$; $dy = d\omega\,\bar{x} + dx\,\omega$

$\bar{x}(1+2\omega)\,dx = \bar{x}(-2-3\omega)(d\omega\,\bar{x}+dx\,\omega)$; $\boxed{\bar{x}=0}$

$(1+2\omega+2\omega+3\omega^2)\,dx = (-2\bar{x}-3\omega\bar{x})\,d\omega$

$\displaystyle\int \frac{1}{\bar{x}}\,dx = \int \frac{-(2+3\omega)\times 2}{1+4\omega+3\omega^2}\,d\omega$

$\ln|\bar{x}| + \ln|c| = -\frac{1}{2}\ln|3\omega^2+4\omega+1|$ $\Bigg\|$ $\bar{x} = x-2$; $\bar{y} = y+2$; $\omega = \dfrac{\bar{y}}{\bar{x}}$

$\bar{x}c = (3\omega^2+4\omega+1)^{-1/2}$

$$\boxed{(x-2)c = \left(3\left(\frac{y+2}{x-2}\right)^2 + 4\left(\frac{y+2}{x-2}\right)+1\right)^{-1/2}}$$

c) $(4x + 2y - 8)\,dx + (2x - y)\,dy = 0$

$(4x + 2y - 8)\,dx = -(2x - y)\,dy$

$\dfrac{dy}{dx} = \dfrac{(4x+2y-8)}{(-2x+y)}$ || $\begin{array}{r} 4x + 2y - 8 \\ -4x + 2y \\ \hline 4y - 8 = 0 \end{array}$; $\boxed{y = 2}$ || $\begin{array}{r} 4x + 2y - 8 \\ 4x - 2y \\ \hline 8x - 8 = 0 \end{array}$ $\boxed{x = 1}$

$x = \bar{x} + 1$

$y = \bar{y} + 2$

$(4(\bar{x}+1) + 2(\bar{y}+2) - 8)\,dx + (2(\bar{x}+1) - (\bar{y}+2))\,dy = 0$

$4\bar{x} + 4 + 2\bar{y} + 4 - 8$

$2\bar{x} + 2 - \bar{y} - 2$

$(4\bar{x} + 2\bar{y})\,dx + (2\bar{x} - \bar{y})\,dy = 0$ || $\bar{y} = \upsilon\bar{x}$, $dy = dx\,\upsilon + \bar{x}\,d\upsilon$

$\bar{x}(4 + 2\upsilon)\,dx + \bar{x}(2 - \upsilon)(dx\,\upsilon + \bar{x}\,d\upsilon) = 0$

$\boxed{\bar{x} = 0}$ $\quad 2\upsilon\,dx + 2\bar{x}\,d\upsilon - \upsilon^2 dx - \upsilon\bar{x}\,d\upsilon = 0$

$(4 + 2\upsilon + 2\upsilon - \upsilon^2)\,dx = (-2\bar{x} + \upsilon\bar{x})\,d\upsilon$

$\int \frac{1}{\bar{x}}\,dx = \int \dfrac{-2 + \upsilon(-2)}{4 + 4\upsilon - \upsilon^2}\,d\upsilon$

$\ln|\bar{x}| + \ln|c| = -\frac{1}{2}\ln|4 + 4\upsilon - \upsilon^2|$

$\bar{x}c = (4 + 4\upsilon - \upsilon^2)^{-1/2}$ $\quad \bar{x} = x - 1$, $\bar{y} = y - 2$ || $\upsilon = \dfrac{\bar{y}}{\bar{x}}$

$$\boxed{(x-1)c = \left(4 + 4\frac{(y-2)}{(x-1)} - \left(\frac{y-2}{x-1}\right)^2\right)^{-1/2}}$$

d) $(x-4y-9)dx + (4x+y-2)dy = 0$

$$\frac{dy}{dx} = \frac{(x-4y-9)}{(-4x-y+2)}$$

$$\begin{array}{r} 4x - 16y - 36 \\ -4x - y + 2 \\ \hline -17y - 34 = 0 \end{array}$$

$$\boxed{y} = \frac{34}{-17} = \boxed{-2}$$

$$\begin{array}{r} x - 4y - 9 \\ 16x + 4y - 8 \\ \hline 17x - 17 = 0 \end{array}$$

$$\boxed{x = 1}$$

$$\boxed{\begin{array}{l} x = \bar{x} + 1 \\ y = \bar{y} - 2 \end{array}}$$

$$(\bar{x} + 1 - 4(\bar{y}-2) - 9)dx + (4(\bar{x}+1) + (\bar{y}-2) - 2)dy = 0$$

$$\bar{x} + 1 - 4\bar{y} + 8 - 9 \qquad 4\bar{x} + 4 + \bar{y} - 2 - 2$$

$$(\bar{x} - 4\bar{y})dx + (4\bar{x} + \bar{y})dy = 0 \quad \Big\| \quad \begin{array}{l} y = v\bar{x} \\ dy = \bar{x}\,dv + v\,dx \end{array}$$

$$(\bar{x} - 4(v\bar{x}))dx + (\underbrace{4\bar{x} + v\bar{x}}_{4+v})(\bar{x}\,dv + v\,dx) = 0$$

$$\bar{x} = 0$$

$$(1 - 4v)dx + 4\bar{x}dv + 4v\,dx + v\bar{x}\,dv + v^2dx = 0$$

$$(1 - \cancel{4v} + \cancel{4v} + v^2)dx = (\underbrace{-4\bar{x} - v\bar{x}}_{\bar{x}(-4-v)})dv$$

$$\int \frac{1}{\bar{x}}dx = \int \frac{-4-v}{1+v^2}dv$$

$$\int \frac{-4-v}{1+v^2} = \int \frac{-4}{(1+v^2)}dv - \int \frac{v}{1+v^2}dv = -4\tan^{-1}(v) - \frac{1}{2}$$
$$\ln|1+v^2|$$

$$\ln|\bar{x}| = -4\tan^{-1}(v) - \frac{1}{2}\ln|1+v^2| + C$$

$$C = \ln|\bar{x}| + \frac{1}{2}\ln|1+v^2| + 4\tan^{-1}(v)$$

$$\boxed{C = \ln|x-1| + \frac{1}{2}\ln\left|1+\left(\frac{y+2}{x-1}\right)^2\right| + 4\tan^{-1}\left(\frac{y+2}{x-1}\right)}$$

e) $(5y-10)\,dx + (2x+4)\,dy = 0$

$$\frac{dy}{dx} = \frac{(5y-10)}{(-2x-4)} \quad \Big\| \quad 5y-10=0 \;\; \boxed{y=2} \quad -2x-4=0 \;\; \boxed{x=-2} \quad \Big\| \quad x=\bar{X}-2, \quad y=\bar{Y}+2$$

$$(5(\underbrace{\bar{y}+2)-10}_{5\bar{y}+10-10})dx + (2(\underbrace{\bar{x}-2)+4}_{2\bar{x}-4+4})dy = 0$$

$$(5\bar{y})dx + (2\bar{x})dy = 0 \quad \Big\| \quad \bar{y} = u\bar{x}, \quad dy = u\,dx + \bar{x}\,du$$

$$5u\bar{x}\,dx + 2\bar{x}(u\,dx + \bar{x}\,du) = 0$$

$\boxed{\bar{x}=0}$ sol

$$5u\,dx + 2u\,dx + 2\bar{x}\,du = 0$$

$$7u\,dx = -2\bar{x}\,du$$

$$\int \frac{1}{\bar{x}}\,dx = -\frac{2}{7}\int \frac{1}{u}\,du$$

$$\ln|\bar{x}| + \ln|c| = -\frac{2}{7}\ln|u|$$

$$\bar{x}c = u^{-2/7} \qquad u = \frac{y}{x}, \quad \bar{y} = y-2, \quad \bar{x} = x+2$$

$$\boxed{(x+2)c = \left(\frac{y-2}{x+2}\right)^{-2/7}}$$

p) $(2x + y - 8)dx = (-2x + 9y - 12)dy$.

$$\frac{dy}{dx} = \frac{2x+y-8}{-2x+9y-12} \quad \Bigg\| \quad \begin{array}{r} 2x+y-8 \\ -2x+9y-12 \\ \hline 10y - 20 = 0 \end{array} \quad \Bigg\| \quad \begin{array}{r} -18x - 9y + 72 \\ -2x + \cancel{9y} - 12 \\ \hline -20x + 60 = 0 \end{array}$$

$\boxed{y=2}$; $\boxed{x = \frac{-60}{-20} = 3}$

$x = \bar{X} + 3$

$y = \bar{Y} + 2$

$$(2(\bar{X}+3) + \bar{Y} + 2 - 8)dx = (-2(\bar{X}+3) + 9(\bar{Y}+2) - 12)\,dy$$

$$(2\bar{X} + \bar{Y})dx = (-2\bar{X} + 9\bar{Y})\,dy \quad \Big\| \; y = \upsilon\bar{X}, \; dy = \upsilon\,dx + \bar{x}\,d\upsilon.$$

$$\bar{X}(2+\upsilon)\,dx = \bar{X}\underbrace{(-2+9\upsilon)(}\upsilon\,dx + \bar{x}\,d\upsilon) \; ; \; \boxed{\bar{X} = 0} \text{ sol}$$

$$-2\upsilon\,dx - 2\bar{x}\,d\upsilon + 9\upsilon^2\,dx + 9\bar{x}\upsilon\,d\upsilon$$

$$(2 + \upsilon + 2\upsilon - 9\upsilon^2)\,dx = \bar{x}(-2 + 9\upsilon)\,d\upsilon.$$

$$\int \frac{1}{\bar{x}}\,dx = \int \frac{-2+9\upsilon}{2+3\upsilon-9\upsilon^2}\,d\upsilon$$

$$\int \frac{-2+9\upsilon}{(\upsilon-\frac{2}{3})(\upsilon+\frac{1}{3})}\,d\upsilon = \frac{A(\upsilon+\frac{1}{3})}{\upsilon-\frac{2}{3}} + \frac{B(\upsilon-\frac{2}{3})}{\upsilon+\frac{1}{3}}$$

si $\upsilon = \frac{2}{3}$; $-2 + \frac{18}{3} = A\left(\frac{2}{3}+\frac{1}{3}\right)$; $\boxed{A = 4}$

si $\upsilon = -\frac{1}{3}$; $-2 - 3 = B\left(-\frac{1}{3} - \frac{2}{3}\right)$; $\boxed{B = 5}$

$$= 4\ln\left|\upsilon - \frac{2}{3}\right| + 5\ln\left|\upsilon + \frac{1}{3}\right| \quad ; \; \upsilon = \frac{y}{x} ; \; \bar{X} = x-3, \; \bar{Y} = y-2$$

$$\ln|\bar{x}| + \ln|c| = 4\ln\left|\upsilon - \frac{2}{3}\right| + 5\ln\left|\upsilon + \frac{1}{3}\right|$$

$$\boxed{(x-3)C = \left(\frac{y-2}{x-3} - \frac{2}{3}\right)^4 \left(\frac{y-2}{x-3} + \frac{1}{3}\right)^5}$$

SHEET 3 (T1)

⑥ $\rightarrow \varphi(x,y) = C$

a) $x^2y' + 2yx = 0$

$$x^2 y' = -2yx \; ; \quad x^2\, dy = -2yx\, dx$$

$$2yx\, dx + x^2 dy \quad \left\{ \begin{array}{l} \frac{\partial M}{\partial y} = 2x \\ \frac{\partial N}{\partial x} = 2x \end{array} \right\} \text{exacta} (=)$$

1) $\frac{\partial \varphi}{\partial y} = \int 2yx\, dx = 2y\frac{x^2}{2} = yx^2 + g(y)$

2) $\frac{\partial \varphi}{\partial y} = x^2 + g'(y) \; ; \quad x^2 = x^2 + g'(y)$

$$g(y) = C$$

$\varphi(x,y) = yx^2 + C \; ; \quad \boxed{C = y \cdot x^2}$

b) $y(e^{xy} + y)dx + x(e^{xy} + 2y)\, dy = 0$

$$\frac{\partial M}{\partial y} = e^{xy} + y + y(xe^{xy} + 1) = e^{xy} + y + yxe^{xy} + y$$

$$\frac{\partial N}{\partial x} = e^{xy} + 2y + e^{xy} \cdot y \quad = \text{exactas}$$

1) $\frac{\partial \varphi}{\partial x} = \int y(e^{xy} + y)\, dx = \int ye^{xy} dx + \int y^2 dx =$

$$= e^{xy} + y^2 x + g(y) \; ;$$

2) $\frac{\partial \varphi}{\partial y} = e^{xy}x + 2yx + g'(y)$

$$x(e^{xy} + 2y) = x(e^{xy} + 2y)$$

$$g'(y) = 0 \; ; \; g(y) = C$$

$$\boxed{C = e^{xy} + y^2 x}$$

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c) $(2x + e^y)dx + xe^y dy = 0$

$\frac{\partial M}{\partial y} = e^y$, $\frac{\partial N}{\partial x} = e^y$ ✓

1) $\frac{\partial \varphi}{\partial x} = \int (2x + e^y)dx = x^2 + e^y x + g(y)$

2) $\frac{\partial \varphi}{\partial y} = xe^y + g'(y)$

$xe^y = xe^y$, $g'(y) = 0$, $g(y) = c$

$$\boxed{c = x^2 + e^y x}$$

d) $(2xy - 3x^2)dx + (x^2 + y)dy = 0$

$\frac{\partial M}{\partial y} = 2x$, $\frac{\partial N}{\partial x} = 2x$ ✓

1) $\frac{\partial \varphi}{\partial y} = \int (x^2 + y)dy = x^2 y + \frac{y^2}{2} + g(x)$

2) $\frac{\partial \varphi}{\partial x} = 2xy + g'(x)$,

$2\cancel{xy} + g'(x) = 2\cancel{xy} - 3x^2$

$g'(x) = -3x^2$, $g(x) = -\int 3x^2 dx = -x^3 + c$

$$\boxed{c = x^2 y + \frac{y^2}{2} - x^3}$$

e) $(y-1)\,dx + (x-2)\,dy = 0$

$\frac{\partial M}{\partial y} = 1 \quad \frac{\partial N}{\partial x} = 1$ ✓

1) $\frac{\partial \varphi}{\partial x} = \int (y-1)\,dx = yx - x + g(y)$

2) $\frac{\partial \varphi}{\partial y} = x + g'(y)$

$\cancel{x} + g'(y) = \cancel{x} - 2$

$g'(y) = -2 \quad g(y) = -\int 2\,dy = -2y + c$

$$\boxed{C = yx - x - 2y}$$

p) $(6xy + 2y^2 - 4)\,dx = (1 - 3x^2 - 4xy)\,dy$

$(6xy + 2y^2 - 4)\,dx + (-1 + 3x^2 + 4xy)\,dy = 0$

$\frac{\partial M}{\partial y} = 6x + 4y \; ; \quad \frac{\partial N}{\partial x} = 6x + 4y$ ✓

1) $\frac{\partial \varphi}{\partial x} = \int (6xy + 2y^2 - 4)\,dx = 6\frac{x^2}{2}y + 2y^2x - 4x + g(y) =$

$= 3x^2y + 2xy^2 - 4x + g(y)$

2) $\frac{\partial \varphi}{\partial y} = 3x^2 + 4xy + g'(y)$

$\cancel{3x^2} + \cancel{4xy} + g'(y) = -1 + \cancel{3x^2} + \cancel{4xy}$

$g(y) = -\int 1\,dy = -y + c$

$$\boxed{C = 3x^2y + 2xy^2 - 4x - y}$$

g) $(\cot y + x^3)\,dx - x\csc^2 y \cdot dy = 0$

$\frac{\partial M}{\partial y} = -\csc^2 y \qquad \frac{\partial N}{\partial x} = -\csc^2 y$ ✓

1) $\frac{\partial \varphi}{\partial x} = \int (\cot y + x^3)\,dx = x\cot y + \frac{x^4}{4} + g(y)$

2) $\frac{\partial \varphi}{\partial y} = -x\csc^2 y + g'(y)$

$g'(y) = 0 \ ; \ g(y) = c$

$$\boxed{c = x\cot y + \frac{x^4}{4}}$$

h) $\frac{dx}{(1-xy)^2} + [y^2 + x^2(1-xy)^{-2}]\,dy = 0$

$\frac{dM}{dy} = -2(1-xy)^{-3}(-x) = \underline{2x(1-xy)^{-3}}$

$\frac{dN}{dx} = 2x(1-xy)^{-2} + x^2(-2)(1-xy)^{-3}(-y) =$

$= \frac{2x}{(1-xy)^2} + \frac{2x^2y}{(1-xy)^{+3}} = \frac{2x(1-xy) + 2x^2y}{(1-xy)^3} =$ $\quad 2x - 2x^2y + 2x^2y$

$= \frac{2x}{(1-xy)^3} = \underline{2x(1-xy)^{-3}}$ ✓ exacta.

1) $\frac{\partial \varphi}{\partial x} = \int (1-xy)^{-2}\,dx = -\frac{1}{y}\frac{(1-xy)^{-1}}{-1} = \frac{1}{y}(1-xy)^{-1} +$

2) $\frac{\partial \varphi}{\partial y} = \left(\frac{-1}{y^2}\right)(1-xy)^{-1} + \frac{\cancel{x}}{y}(1-xy)^{-2} + g'(y)$ $\quad g(y)$

$$= y^2 + x^2(1-xy)^{-2} = \frac{-1}{y^2(1-xy)} + \frac{x}{y}(1-xy)^{-2} + g(y)$$

$$\frac{-(1-xy)}{y^2(1-xy)^{+2}} + \frac{xy}{(1-xy)^2} = y^2 + \frac{x^2}{(1-xy)}$$

$$g'(y) = y^2 + \frac{x^2y^2 - 2xy + 1}{(1-xy)^2} =$$

$$= y^2 + \frac{(xy-1)^2}{(1-xy)^2} = y^2 + 1$$

$$g'(y) = y^2 + 1 \;; \quad g(y) = \frac{y^3}{3} + y + C$$

$$\boxed{C = \frac{1}{y}(1-xy)^{-1} + \frac{y^3}{3} + y}$$

7 a) $y' + y(1+x) = 0$

$$\frac{dy}{dx} = -y(1+x)$$

$y(1+x)\,dx + dy = 0$; $\begin{cases} \frac{\partial M}{\partial y} = 1+x \\ \frac{\partial N}{\partial x} = 0 \end{cases}$ $\neq$ not exact

$\mu = \mu(x)$;

$u(x) = \frac{1}{1}[1+x] = 1+x$; $\boxed{\mu(x) = e^{\int(1+x)\,dx} = e^{x+\frac{x^2}{2}}}$ factor

$$e^{x+\frac{x^2}{2}}\, y(1+x)\,dx + e^{x+\frac{x^2}{2}}\, dy = 0$$

$\frac{\partial P}{\partial y} = e^{x+\frac{x^2}{2}}(1+x)$ $\quad \frac{\partial Q}{\partial x} = e^{x+\frac{x^2}{2}}(1+x)$ ✓ exact.

1) $\frac{d\varphi}{dx} = \int e^{x+\frac{x^2}{2}} y(1+x)\,dx = e^{x+\frac{x^2}{2}} y + g(y)$

2) $\frac{d\varphi}{dy} = e^{x+\frac{x^2}{2}} + g'(y)$

$g'(y) = 0 \; ; \; g(y) = C$

$$\boxed{C = e^{x+\frac{x^2}{2}} y}$$

b) $x^3 y' = xy + x^2$

$x^3 \frac{dy}{dx} = xy + x^2 \; ; \; (xy + x^2)\,dx - x^3 dy = 0$

$\frac{dM}{dy} = x \; ; \; \frac{dN}{dx} = -3x^2 \neq$ NOT exact

$\mu = \mu(x) \; ; \; u(x) = \frac{-1}{x^3}[x + 3x^2] = -[x^{-2} + 3x^{-1}]$

$\boxed{\mu(x)} = e^{-\int x^{-2} + 3x^{-1}} = e^{-[\frac{x^{-1}}{-1} + 3\ln|x|]}$

$= e^{x^{-1}} e^{-3\ln|x|} = \boxed{e^{x^{-1}} x^{-3}}$

$e^{x^{-1}} x^{-3}(xy + x^2)\,dx - \cancel{x^{-3}} e^{x^{-1}} \cancel{x^3}\,dy = 0$

$e^{x^{-1}}(x^{-2}y + x^{-1})\,dx - e^{x^{-1}} dy = 0$

$\frac{dP}{dy} = e^{x^{-1}} x^{-2} \; ; \; \frac{dQ}{dx} = x^{-2} e^{x^{-1}}$ ✓ exact

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1) $\frac{d\varphi}{dy} = \int e^{x^{-1}} dy = -e^{x^{-1}} y + g(x)$

2) $x^{-2} e^{x^{-1}} y + g'(x)$

$\cancel{x^{-2} e^{x^{-1}} y} + g'(x) = e^{x^{-1}} (\cancel{x^{-2} y} + x^{-1})$

$g'(x) = e^{x^{-1}} \cdot x^{-1}$

$g(x) = \int e^{x^{-1}} x^{-1} \, dx + c$

$$\boxed{c = -y e^{x^{-1}} + \int e^{x^{-1}} x^{-1} dx}$$

c) $a y\, dx + b x\, dy = 0$

$\frac{dM}{dy} = a \quad ; \quad \frac{dN}{dx} = b$

$\mu = \mu(x) \quad , \quad u(x) = \frac{1}{bx} [a - b]$.

$\boxed{\mu(x)} \overset{!}{=} e^{\int \frac{a-b}{bx}} = e^{\frac{a-b}{b} \ln x} \overset{!}{=} \boxed{x^{\frac{a-b}{b}}}$

$x^{\frac{a-b}{b}} \cdot a y \, dx + x^{\frac{a-b}{b}} b x \, dy = 0$.

$\frac{dP}{dy} = a x^{\frac{a-b}{b}} \quad ; \quad \frac{dQ}{dx} = b\left(\frac{a-b}{b} + 1\right) x^{\frac{a-b}{b}} = a x^{\frac{a-b}{b}}$

✓ exact

1) $\frac{d\varphi}{dy} = \int x^{\frac{a-b}{b}} b x \, dy = b x^{\frac{a-b}{b}+1} y + g(x)$

2) $\frac{d\varphi}{dx} = b\left(\frac{a-b}{b} + 1\right) x^{\frac{a-b}{b}} y + g'(x)$

$a\, x^{\frac{a-b}{b}} y + g'(x)$;

$a y x^{\frac{a-b}{b}} + g'(x) = a y x^{\frac{a-b}{b}}$

$g'(x) = 0$;

$g(x) = C$

$\varphi(x,y) = b x^{\frac{a-b}{b}+1} y + C$

$C = b x^{\frac{a-b}{b}+1} y$

$$\boxed{C = b x^{a/b} y}$$

d) $5dx - e^{y-x} dy = 0$

$\frac{dM}{dy} = 0$; $\frac{dN}{dx} = -e^{y-x}(-1) = e^{y-x}$ $\neq$ NOT exact.

$\mu = \mu(x)$; $\mu(x) = \frac{-1}{\cancel{e^{y-x}}}\left[0 - \cancel{e^{y-x}}\right] = +1$

$\boxed{\mu(x)} = e^{+\int 1\, dx} = \boxed{e^{+x}}$

$-e^{y}\cancel{e^{-x}}\cancel{e^{x}}$

$5e^{+x}dx - e^{y-x} \cdot e^{+x} dy = 0$

$\frac{dP}{dy} = 0$; $\frac{dQ}{dx} = 0$ ✓ exact.

1) $\frac{d\varphi}{dx} = \int 5 e^{x} dx = 5 e^{x} + g(y)$

2) $\frac{d\varphi}{dy} = g'(y)$; $g'(y) = -e^{y}$

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$$g(y) = \int -e^{y}\,dy = -e^{y} + c$$

$$\varphi(x,y) = 5e^{x} - e^{y} + c$$

$$\boxed{C = 5e^{x} - e^{y}}$$

e) $(yx^3 e^{xy} - 2y^3)\,dx + (x^4 e^{xy} + 3xy^2)\,dy = 0$

$$\frac{\partial M}{\partial y} = x^3 e^{xy} + yx^3 e^{xy} x + 6y^2 = x^3 e^{xy}(1+4x) + 6y^2$$

$$\frac{\partial N}{\partial x} = 4x^3 e^{xy} + x^4 e^{xy} y + 3y^2 = x^3 e^{xy}(4 + xy) + 3y^2$$

NOT EXACT

$\mu = \mu(x)$

$$u(x) = \frac{1}{x^4 e^{xy} + 3xy^2}\left[x^3 e^{xy}(1 - xy) + 6y^2 - (x^3 e^{xy}(4+xy) + 3y^2)\right]$$

$$\underbrace{x^3 e^{xy} + x^3 e^{xy} xy + 6y^2}$$

$$= -x^3 e^{xy} 4 - x^3 e^{xy} xy - 3y^2$$

$$= -3x^3 e^{xy} + 3y^2$$

$$\boxed{u(x)} = \frac{-3x^3 e^{xy} + 3y^2}{x^4 e^{xy} + 3xy^2} = \frac{(-3x^3 e^{xy} + 3y^2)}{x(e^{xy}x^3 + 3y^2)} = \boxed{\frac{-3}{x}}$$

$$\boxed{\mu(x)} = e^{-\int \frac{3}{x}\,dy} = e^{-3\ln|x|} = \boxed{x^{-3}}$$

$$(ye^{xy} - 2y^3x^{-3})\,dx + (xe^{xy} + 3x^{-2}y^2)\,dy = 0$$

$$\frac{\partial P}{\partial y} = e^{xy} + ye^{xy}x - 2x^{-3}3y^2$$

$$\frac{\partial Q}{\partial x} = e^{xy} + xe^{xy}y + 3y^2(-2)x^{-3}$$

✓ exata

1) $\frac{\partial \varphi}{\partial y} = \int (x e^{xy} + 3x^{-2} y^2) dy = e^{xy} + 3x^{-2} \frac{y^3}{3} + g(x)$

2) $\frac{\partial \varphi}{\partial x} = e^{xy} y - 6x^{-3} \frac{y^3}{3} + g'(x) =$

$= e^{xy} y - 2x^{-3} y^3 + g'(x)$

$y e^{xy} - 2x^{-3} y^3 + g'(x) = e^{xy} y - 2x^{-3} y^3$

$g'(x) = 0 ; \; g(x) = C$

$\varphi(x, y) = e^{xy} + 3x^{-2} \frac{y^3}{3} + C$

$C = e^{xy} + \cancel{3} x^{-2} \frac{y^3}{\cancel{3}}$

$$\boxed{C = \frac{e^{xy} x^2 + y^3}{x^2}}$$

e) $\frac{1}{2} y^2 \, dx + (e^x - y) \, dy = 0$

$\frac{\partial M}{\partial y} = y \; ; \; \frac{\partial N}{\partial x} = e^x$ NOT EXACT

$\mu = \mu(x) ; \; \mu(x) = \frac{1}{e^x - y} [y - e^x] = -1$

$\boxed{\mu(x)} = e^{-\int 1 \, dx} = \boxed{e^{-x}}$

$$e^{-x}\frac{1}{2}y^2\,dx + (1-ye^{-x})\,dy = 0$$

$$\frac{\partial P}{\partial y} = e^{-x}y \;; \quad \frac{\partial Q}{\partial x} = -1\,e^{-x}(-y) = e^{-x}y \;\checkmark \text{ exact}$$

1) $$\frac{\partial \varphi}{\partial y} = \int (1-ye^{-x})\,dy = y - e^{-x}\frac{y^2}{2} + g(x)$$

2) $$\frac{\partial \varphi}{\partial x} = e^{-x}\frac{y^2}{2} + g'(x)$$

$$e^{-x}\cdot\frac{y^2}{2} = e^{-x}\frac{y^2}{2} + g'(x)$$

$$g'(x) = 0 \;;\; g(x) = C$$

$$\varphi(x,y) = y - e^{-x}\frac{y^2}{2} + C$$

$$\boxed{C = y - e^{-x}\frac{y^2}{2}}$$

g) $(x^2y^2 - y)\,dx + (2x^3y + x)\,dy = 0$

$$\frac{\partial M}{\partial y} = 2x^2y - 1 \;; \quad \frac{\partial N}{\partial x} = 6x^2y + 1 \quad \text{NOT EXACT}$$

$\mu = \mu(x)$

$$u(x) = \frac{1}{2x^3y + x}\left[2x^2y - 1 - 6x^2y - 1\right] = \frac{-4x^2y - 2}{2x^3y + x} =$$

$$= \frac{-2(\cancel{2x^2y+1})}{x(\cancel{2x^2y+1})} = \boxed{\frac{-2}{x}}$$

$\boxed{\mu(x)} = e^{-2\int \frac{1}{x} dx} = e^{-2\ln x} = \boxed{x^{-2}}$

$(y^2 - yx^{-2})\,dx + (2xy + x^{-1})\,dy = 0$

$\frac{\partial P}{\partial y} = 2y - x^{-2}$; $\frac{\partial Q}{\partial x} = 2y + x^{-2}(-1)$ ✓ exact

1) $\frac{\partial \varphi}{\partial x} = \int (y^2 - yx^{-2})\,dx = y^2 x - y\frac{x^{-1}}{-1} = y^2x + yx^{-1} + g(y)$

2) $\frac{\partial \varphi}{\partial y} = 2yx + x^{-1} + g'(y)$

$g'(y) + \cancel{2yx + x^{-1}} = \cancel{2xy + x^{-1}}$

$g'(y) = 0$; $g(y) = C$.

$\varphi(x,y) = y^2x + yx^{-1} + C$

$$\boxed{C = y^2x + yx^{-1}}$$

4) $xy^2\,dy - (x^2 + y^3)\,dx = 0$

$(-x^2 - y^3)\,dx + xy^2\,dy = 0$

$\frac{\partial M}{\partial y} = -3y^2$; $\frac{\partial N}{\partial x} = y^2$ NOT EXACT

$\mu = \mu(x)$; $h(x) = \frac{1}{xy^2}[-3y^2 - y^2] =$

$= \frac{\cancel{y^2}[-3-1]}{x\cancel{y^2}} = -\frac{4}{x}$; $\boxed{\mu(x)} = e^{-4\int \frac{1}{x} dx} = \boxed{x^{-4}}$

$(-x^{-2} - y^3 x^{-4})\, dx + x^{-3} y^2 dy = 0$

$\frac{dM}{dy} = -3y^2 x^{-4}$; $\frac{dN}{dx} = -3x^{-4} y^2$ ✓ exact

1) $\frac{d\varphi}{dy} = \int x^{-3} y^2\, dy = x^{-3} \frac{y^3}{3} + g(x)$

2) $\frac{d\varphi}{dx} = -3x^{-4} \frac{y^3}{3} + g'(x) = -x^{-4}y^3 + g'(x)$

$\cancel{-x^{-4}y^3} + g'(x) = -x^{-2} \cancel{- y^3 x^{-4}}$

$g'(x) = -x^{-2}$

$g(x) = -\int x^{-2} = -\frac{x^{-1}}{-1} = \boxed{x^{-1} + C}$

$\varphi(x,y) = x^{-3} \frac{y^3}{3} + x^{-1} + C$ ✓

$$\boxed{C = x^{-3} \frac{y^3}{3} + x^{-1}}$$

8

a) $(y+1)\, dx + (-x + y)\, dy = 0$

$\frac{dM}{dy} = 1$; $\frac{dN}{dx} = -1$ NOT EXACT

$\mu = \mu(y)$; $h(y) = \frac{1}{y+1}[-1-1] = \frac{-2}{y+1}$

$\boxed{\mu(y)} = e^{-\int \frac{2}{y+1}} = e^{-2\ln|y+1|} = \boxed{(y+1)^{-2}}$

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7.

$$(y+1)^{-1}dx + (y+1)^{-2}(-x+y)dy = 0$$

$$\frac{dP}{dy} = -1(y+1)^{-2} \qquad \frac{dQ}{dx} = (y+1)^{-2}(-1) \quad \checkmark \text{ exact}$$

1) $\frac{d\varphi}{dx} = \int (y+1)^{-1}dx = (y+1)^{-1}x + g(y)$

2) $\frac{d\varphi}{dy} = -(y+1)^{-2}x + g'(y)$

$$-(y+1)^{-2}x + g'(y) = (y+1)^{-2}(-x+y)$$

$$g'(y) = (y+1)^{-2}(-x+y+x)$$

$$g'(y) = y(y+1)^{-2}$$

$$g(y) = \int y(y+1)^{-2}dy = \int\left(\frac{1}{(y+1)} - \frac{1}{(y+1)^{-2}}\right)dy =$$

$$= \frac{1}{y+1} + \ln|y+1| + C$$

$$\varphi(x,y) = (y+1)^{-1}x + \frac{1}{y+1} + \ln|y+1| + C$$

$$\boxed{C = \frac{x+1}{y+1} + \ln|y+1|}$$

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b) $\left(\frac{y}{x} - 1\right)dx + \left(2y^2 + 1 + \frac{x}{y}\right)dy = 0$

$$\frac{\partial M}{\partial y} = \frac{1}{x}$$

$$\frac{\partial N}{\partial x} = \frac{1}{y}$$

$\neq$

$$\frac{1}{\underbrace{\frac{y}{x} - 1}_{\frac{y-x}{x}}}\left[\frac{1}{y} - \frac{1}{x}\right]$$

$$\frac{\frac{x-y}{yx}}{\frac{y-x}{x}} = \frac{\cancel{x}(x-y)}{y\cancel{x}(y-x)} = \frac{-(y-x)}{y(y-x)} = \boxed{\frac{-1}{y}}$$

$$\boxed{u(y)} = e^{-\int \frac{1}{y}dy} = \cancel{e}^{-\ln|y|} = \boxed{y^{-1}}$$

$$\boxed{\left(\frac{1}{x} - y^{-1}\right)dx + \left(2y + y^{-1} + \frac{x}{y^2}\right)dy = 0}$$

$$\frac{\partial M}{\partial y} = y^{-2}$$

$$\frac{\partial N}{\partial x} = \frac{1}{y^2} = y^{-2}$$

$= \text{exactas}$

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$$\int\left(\frac{1}{x} - y^{-1}\right)dx = \ln|x| - y^{-1}x + g(y)$$

$$\frac{\partial}{\partial y} = x y^{-2} + g'(y)$$

$$\cancel{x y^{-2}} = 2y + y^{-1} + \cancel{x y^{-2}}$$

$$g(y) = \int(2y + y^{-1})dy = 2\frac{y^2}{2} + \ln|y| + C$$

$$\varphi(x,y) = \ln|x| - y^{-1}x + y^2 + \ln|y| + C$$

$$\boxed{C = \ln|xy| + y^2 - y^{-1}x}$$

c) $ay\,dx + bx\,dy = 0$

$$\frac{dM}{dy} = a \qquad \mu = \mu(y)$$

$$\frac{1}{ay}[b - a] = \frac{b-a}{ay} \quad \checkmark$$

$$\frac{dN}{dx} = b$$

$$\mu(y) = e^{\int \frac{b-a}{a}\cdot\frac{1}{y}dy} = e^{\frac{b-a}{a}\ln|y|} = \boxed{y^{\frac{b-a}{a}}}$$

$$\boxed{a y^{\left(\frac{b-a}{a}+1\right)} dx + bxy^{\frac{b-a}{a}} = 0}$$

$$\frac{b-a+a}{a} = b/a$$

$$\frac{dM}{dy} = a\frac{b}{a} y^{b/a - 1} = b y^{\frac{b-a}{a}}$$

exacta.

$$\frac{dN}{dx} = by^{\frac{b-a}{a}}$$

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$$\int a y^{b/a} dx = a x y^{b/a} + g(y)$$

$$\frac{\partial}{\partial y} = a x \cdot \frac{b}{a} y^{\frac{b-a}{a}} + g'(y) = x b y^{\frac{b-a}{a}} + g'(y)$$

$$\cancel{b x y^{\frac{b-a}{a}}} = \cancel{x b y^{\frac{b-a}{a}}}$$

$$g(y) = c$$

$$\varphi(x y) = a x y^{b/a} + c$$

$$\boxed{c = a x y^{b/a}}$$

d) $y(x+y+1)dx + x(x+3y+2)dy = 0$

$$\frac{dM}{dy} = (x+y+1) + y = x+2y+1 \neq$$

$$\frac{dN}{dx} = x+3y+2+x = 2x+3y+2$$

$$\mu = \mu(y)$$

$$\mu(y) = \frac{1}{\cancel{y(x+y+1)}}[2x+3y+2-x-2y-1] = \frac{1}{y}$$

$$\cancel{x+y+1}$$

$$\boxed{\mu(y)\ \frac{1}{y}}\ e^{\int \frac{1}{y} dy} = \cancel{e}^{\ln|y|} = \boxed{y}$$

$$\boxed{y^2(x+y+1)dx + xy(x+3y+2)dy = 0}$$

$$\frac{\partial M}{\partial y} = 2y(x+y+1) + y^2,\ 2yx + 2y^2 + 2y + y^2 = 3y^2 + 2yx + 2y$$

$$\frac{\partial N}{\partial x} = y(x+3y+2) + xy,\ yx + 3y^2 + 2y + xy = 3y^2 + 2xy + 2y$$

Exacta

$$\int (y^2x + y^3 + y^2)\,dx = y^2\frac{x^2}{2} + y^3x + y^2x + g(y)$$

$$\frac{\partial}{\partial y} = \frac{x^2}{2}2y + 3y^2x + 2yx + g'(y)$$

$$yx^2 + 3y^2x + 2yx + g'(y)$$

$$xy(x+3y+2) = yx^2 + 3y^2x + 2xy$$

$$\cancel{x^2y} + \cancel{3xy^2} + \cancel{2xy} = \cancel{yx^2} + \cancel{3y^2x} + \cancel{2xy}$$

$$g(y) = C$$

$$\varphi(x,y) = y^2\frac{x^2}{2} + y^3x + y^2x + C$$

$$\boxed{C = y^2\frac{x^2}{2} + y^3x + y^2x}$$

$$C = y^2x\left(\frac{x}{2} + y + 1\right)$$

$$\boxed{C = y^2x(x+2y+2)}$$

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9. a) $(y^3+y)\,dx + (xy^2-x)\,dy = 0$

$$\frac{dM}{dy} = 3y^2+1$$

$$\frac{\partial N}{\partial x} = y^2-1$$

$$\mu = \mu(y)$$

$$u(y) = \frac{1}{y^3+y}\left[y^2-1-3y^2-1\right] = \frac{-2(y^2+1)}{y^3+y} = \frac{-2\cancel{(y^2+1)}}{y\cancel{(y^2+1)}} =$$

$$= \frac{-2}{y}$$

$$\mu(y) = e^{-2\int \frac{1}{y}dy} = e^{-2\ln(y)} = \boxed{y^{-2}}$$

$$\boxed{(y+y^{-1})\,dx + (x - xy^{-2})\,dy = 0}$$

$$\frac{dM}{dy} = 1 - y^{-2}$$

$$\frac{\partial N}{\partial x} = 1 - y^{-2}$$

✓ exacta.

$$\int (y+y^{-1})\,dx = yx + y^{-1}x + g(y)$$

$$\frac{\partial}{\partial y} = x - xy^{-2} + g'(y)$$

$$(x - xy^{-2}) = x - xy^{-2}$$

$$g(y) = 0 + C$$

$$\varphi(x,y) = yx + y^{-1}x + C$$

$$\boxed{C = yx + \frac{x}{y}}$$

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b) $y\cos x\,dx + y^2 dy = 0$

$$\frac{dM}{dy} = \cos x$$

$$\frac{dN}{dx} = 0$$

$$\mu = \mu(y)$$

$$u(y) = \frac{1}{y\cancel{\cos x}}[0 - \cancel{\cos x}] = -\frac{1}{y}$$

$$\boxed{\mu(y)} = e^{-\int \frac{1}{y} dy} = e^{-\ln|y|} = \boxed{y^{-1}}$$

$$\boxed{\cos x\,dx + y \cdot dy = 0}$$

$$\frac{dM}{dy} = 0$$

$= \text{exacta}$

$$\frac{dN}{dx} = 0$$

$$\int y\,dy = \frac{y^2}{2} + g(x)$$

$$\frac{d}{dx} = g'(x)$$

$$g'(x) = \cos x\,dx$$

$$g(x) = \int \cos x\,dx = \sin x + C$$

$$\varphi(x,y) = \frac{y^2}{2} + \sin x + C$$

$$\boxed{C = \frac{y^2}{2} + \sin x}$$

c) $(e^{2x} + y - 1)\,dx - dy = 0$

$\frac{\partial M}{\partial y} = 1$; $\frac{\partial N}{\partial x} = 0$; NOT EXACT

$\mu = \mu(x)$

$h(x) = \frac{1}{-1}[1-0] = -1$

$\boxed{\mu(x) =} \; e^{-\int dx} = \boxed{e^{-x}}$

$e^{-x}(e^{2x} + y - 1)\,dx - e^{-x}\,dy = 0$

$(e^{x} + e^{-x}y - e^{-x})\,dx - e^{-x}\,dy = 0$

$\frac{\partial P}{\partial y} = e^{-x}$; $\frac{\partial Q}{\partial x} = -e^{-x}(-1) = e^{-x}$ ✓ exact.

1) $-\int e^{-x}\,dy = -e^{-x}y + g(x)$

2) $\frac{\partial \varphi}{\partial x} = -y\,e^{-x}(-1) + g'(x) = y\,e^{-x} + g'(x)$

$e^{x} + \cancel{e^{-x}y} - e^{-x} = \cancel{ye^{-x}} + g'(x)$

$g'(x) = e^{x} - e^{-x}$;

$g(x) = \int e^{x}\,dx + \int -e^{-x}\,dx = e^{x} + e^{-x} + C$

$\varphi(x,y) = -e^{-x}y + e^{x} + e^{-x} + C$

$\boxed{C = e^{x} + e^{-x}(1-y)}$

d) $dx + \left(\frac{x}{y} + \cos y\right) dy = 0$

$\frac{\partial M}{\partial y} = 0$

$\neq$

$\frac{\partial N}{\partial x} = \frac{1}{y}$

$\parallel$

$\mu = \mu(y)$

$\mu(y) = \frac{1}{1}\left[\frac{1}{y} - 0\right] = \frac{1}{y}$

$\boxed{\mu(y)} = e^{\int \frac{1}{y} dy} = e^{\ln(y)} = \boxed{y}$

$$\boxed{y\,dx + (x + y\cos y)\,dy = 0}$$

$\frac{\partial M}{\partial y} = 1$

= exacta

$\frac{\partial N}{\partial x} = 1$

$\int y\,dx = yx + g(y)$

$u \cdot v - \int du\, v$

$\frac{d}{dy} = x + g'(y)$

$u = y \quad du = 1dy$

$dv = \cos(y) \quad v = \sin(y)$

$\cancel{x} = \cancel{x} + y\cos y$

$g'(y) = y\cos(y), \; g(y) = \int \overset{u}{y}\, \overbrace{\cos(y)}^{dv}$

$y\sin(y) - \int \sin(y) \cdot dy = y\sin(y) + \cos(y) + C$

$\varphi(x,y) = yx + y\sin(y) + \cos(y) + C, \; \boxed{C = yx + y\sin(y) + \cos(y)}$

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HOJA 4 TOPIC 1

10.

a) $y' + 4y = 17\sin x$; $a(x) = 4$
$p(x) = 17\sin(x)$

$\mu(x) = e^{\int 4\,dx} = e^{4x}$

$$y(x) = \frac{c}{e^{4x}} + \frac{1}{e^{4x}} \int e^{4x} 17\sin(x)\,dx$$

$$\int e^{4x}\sin(x)\,dx = -e^{4x}\cos(x) - \int 4e^{4x}(-\cos x)\,dx$$

$u = e^{4x}$ $du = 4e^{4x}$ $\qquad$ $u = e^{4x}$ $du = 4e^{4x}dx$

$dv = \sin(x)$ $v = -\cos x$ $\qquad$ $dv = \cos x$ $v = \sin x$

$$= -e^{4x}\cos(x) + 4\left[e^{4x}\sin x - \int 4e^{4x}\sin x\,dx\right] =$$

$$= -e^{4x}\cos(x) + 4e^{4x}\sin x + 16\int e^{4x}\sin x\,dx =$$

$$17\int e^{4x}\sin x\,dx = e^{4x}(4\sin(x) - \cos x)$$

$$\boxed{y(x) = \frac{c}{e^{4x}} + (4\sin(x) - \cos(x))}$$

b) $y' + 4y = 2e^{-2x}$ $\quad$ $a(x) = 4$
$p(x) = 2e^{-2x}$

$\mu(x) = e^{4x}$

$$y(x) = \frac{c}{e^{4y}} + \frac{1}{e^{4x}} \int e^{4x}(2e^{-2x})\,dx$$

$$\int e^{4x}(2e^{-2x})\,dx = \int 2e^{2x}\,dx = e^{2x}$$

$$y(x) = \frac{c}{e^{4x}} + \frac{e^{2x}}{e^{4x}} \;;\; \boxed{y(x) = \frac{c}{e^{4x}} + e^{-2x}}$$

c) $y' + 4y = e^{-4x}$ $\quad a(x) = 4$

$f(x) = e^{-4x}$

$\mu(x) = e^{4x}$

$$y(x) = \frac{c}{e^{4x}} + \frac{1}{e^{4x}} \int \underbrace{e^{4x} e^{-4x} dx}_{\int dx = x}$$

$$\boxed{y(x) = \frac{c}{e^{4x}} + \frac{x}{e^{4x}}}$$

d) $y' - 2y = 4$

$a(x) = -2$

$y' + (-2)y = 4 \quad f(x) = 4$

$$\mu(x) = e^{\int -2 dx} = \boxed{e^{-2x}}$$

$$y(x) = \frac{c}{e^{-2x}} + \frac{1}{e^{-2x}} \int e^{-2x} \cdot 4 \, dx;$$

$$\int e^{-2x} 4 \, dx = -\frac{4}{2} e^{-2x} = -2e^{-2x}$$

$$y(x) = \frac{c}{e^{-2x}} - \frac{2\cancel{e^{-2x}}}{\cancel{e^{-2x}}}; \quad \boxed{y(x) = \frac{c}{e^{-2x}} - 2}$$

e) $y' - 2y = 2 + 4x.$

$a(x) = -2$

$y' + (-2)y = 2 + 4x \quad f(x) = 2 + 4x$

$$\mu(x) = e^{\int -2 dx} = \boxed{e^{-2x}}$$

$$y(x) = \frac{c}{e^{-2x}} + \frac{1}{e^{-2x}} \int e^{-2x} (2 + 4x) \, dx.$$

$u = 2+4x \qquad du = 4\,dx$

$dv = e^{-2x} \qquad v = -\frac{1}{2}e^{-2x}$

$$\int e^{-2x}(2+4x)\,dx = (2+4x)\left(-\frac{1}{2}e^{-2x}\right) - \int -\frac{1}{2}e^{-2x}4\,dx =$$

$$= (-1-2x)(e^{-2x}) - e^{-2x} = e^{-2x}[-2-2x]$$

$$\boxed{y(x) = \frac{c}{e^{-2x}} + [-2-2x]}$$

e) $y' - 2y = 3e^{-x}$

$y' + (-2)y = 3e^{-x} \qquad a(x) = -2$

$p(x) = 3e^{-x}$

$$\boxed{\mu(x)} = e^{\int -2\,dx} = \boxed{e^{-2x}}$$

$$y(x) = \frac{c}{e^{-2x}} + \frac{1}{e^{-2x}}\int e^{-2x}3e^{-x}\,dx$$

$$\int e^{-2x}3e^{-x}\,dx = \int 3e^{-3x}\,dx = -e^{-3x}$$

$$y(x) = \frac{c}{e^{-2x}} - \frac{e^{-3x}}{e^{-2x}} \qquad \boxed{y(x) = \frac{c}{e^{-2x}} - e^{-x}}$$

11.

a) $y' + xy = x \qquad a(x) = x$

$p(x) = x$

$$\mu(x) = e^{\int x\,dx} = e^{\frac{x^2}{2}}$$

$$y(x) = \frac{c}{e^{\frac{x^2}{2}}} + \frac{1}{e^{\frac{x^2}{2}}}\int e^{\frac{x^2}{2}}x\,dx$$

$$\int e^{\frac{x^2}{2}}x\,dx = e^{\frac{x^2}{2}}$$

$$\boxed{y(x) = \frac{c}{e^{\frac{x^2}{2}}} + 1}$$

b) $x^2y' + xy = 1$

$$\frac{x^2y' + xy}{x^2} = \frac{1}{x^2}$$

$$y' + x^{-1}y = \frac{1}{x^2} \qquad a(x) = x^{-1}$$

$$p(x) = x^{-2}$$

$$\boxed{\mu(x)} = e^{\int x^{-1}dx} = e^{\ln|x|} = \boxed{x}$$

$$y(x) = \frac{c}{x} + \frac{1}{x}\int x \cdot x^{-2}dx \quad , \int x^{-1}dx = \ln|x|$$

$$\boxed{y(x) = \frac{c}{x} + \frac{\ln|x|}{x}}$$

c) $xy' + (3x+1)y = e^{-3x}$.

$$\frac{xy' + (3x+1)y = e^{-3x}}{x}$$

$$y' + (3+x^{-1})y = \frac{e^{-3x}}{x} \quad \Bigg\| \quad a(x) = 3 + x^{-1}, \quad p(x) = \frac{e^{-3x}}{x}$$

$$\mu(x) = e^{\int(3+x^{-1})dx} = e^{3x + \ln|x|} = e^{3x} \cdot x.$$

$$y(x) = \frac{c}{e^{3x}x} + \frac{1}{e^{3x}x}\int e^{3x}\cancel{x} \cdot \frac{e^{-3x}}{\cancel{x}}dx$$

$$\int 1\,dx = x$$

$$y(x) = \frac{c}{e^{3x}x} + \frac{x}{e^{3x}x}$$

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$$\boxed{y(x) = \frac{c}{e^{3x}x} + e^{-3x}}$$

d) $xy' + (2x+1)y = 4x$

$y' + (2+x^{-1})y = 4$ $\quad a(x) = 2+x^{-1}$

$p(x) = 4$

$$\boxed{\mu(x)} = e^{\int (2+x^{-1})\,dx} = \boxed{e^{2x+\ln x} = e^{2x}\cdot x}$$

$$y(x) = \frac{c}{e^{2x}x} + \frac{1}{e^{2x}x}\int e^{2x}x\cdot 4\,dx$$

$$\int e^{2x}x\,4\,dx = 2xe^{2x} - \int 2e^{2x}\,dx = e^{2x}(2x-1)$$

$$y(x) = \frac{c}{e^{2x}x} + \frac{\cancel{e^{2x}}(2x-1)}{\cancel{e^{2x}}x}$$

$$\boxed{y(x) = \frac{c}{e^{2x}x} + 2 - x^{-1}}$$

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12 - Bernoulli method

a) $xy' = y + x^2 e^x$

$$y' - \frac{y}{x} = xe^x \quad \Big\| \quad a(x) = -1/x \quad p(x) = xe^x$$

$y(x) = u(x)\,w(x)$

$$\boxed{u(x)} = e^{-\int a(x)\,dx} = e^{\int \frac{1}{x}dx} = e^{\ln|x|} = \boxed{x}$$

$$\boxed{w(x)} = \int xe^x e^{\int -1/x\,dx} + c = \int e^x + c = \boxed{e^x + c}$$

$$e^{\int -1/x\,dx} = e^{-\ln|x|} = x^{-1}$$

$$\boxed{y(x) = x(e^x + c)}$$

b) $y' + 2xy = 4x$

$a(x) = 2x$

$p(x) = 4x \quad \| \quad y(x) = u(x)\,w(x)$

$$\boxed{u(x)} = e^{-\int 2x\,dx} = \boxed{e^{-x^2}}$$

$$w(x) = \int 4x\, e^{\int 2x\,dx} + c \quad ; \quad e^{\int 2x\,dx} = e^{x^2}$$

$$\int 4x\, e^{x^2} dx = 2\int 2x\, e^{x^2} = 2e^{x^2}$$

$$\boxed{w(x) = 2e^{x^2} + c}$$

$$\boxed{y(x) = e^{-x^2}(2e^{x^2} + c)}$$

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c) $y' = (y-1)\tan x$

$y' = \tan x\, y - \tan x$

$y' - \tan x\, y = -\tan x$ ‖ $a(x) = -\tan x$
$f(x) = -\tan x$

$\boxed{u(x)} = e^{\int \tan x\, dx} = e^{-\ln|\cos x|} = \boxed{(\cos x)^{-1}}$ $\tan = \frac{\sin x}{\cos x}$

$\boxed{v(x)} = \int -\tan x\, e^{\int -\tan x\, dx} + c\, dx = \int -\tan x \cos x = *$

$e^{-\int \tan x\, dx} = e^{-(-\ln \cos x)} = \cos(x)$

$* = -\int \sin x\, dx = \boxed{\cos x + c}$

$y(x) = \cos x^{-1} (\cos x + c) =$

$$\boxed{y(x) = 1 + \frac{c}{\cos x}}$$

d) $(1+x)\, y' = xy + x^2$

$y' - \frac{x}{(1+x)}\, y = \frac{x^2}{x+1}$ $a(x) = \frac{-x}{x+1}$ $\frac{\cancel{-x}}{x+1}$ $\frac{x+1}{1}$ -1 $1 - \frac{1}{x+1}$

$f(x) = \frac{x^2}{x+1}$

$\boxed{u(x)} = e^{-\int \frac{-x}{x+1} dx} = e^{\int \frac{x}{x+1} dx} = e^{\int 1 dx - \int \frac{1}{x+1}} =$

$= e^{x - \ln|x+1|} = e^{x + \ln|x+1|^{-1}} = \boxed{e^{x} \cdot (x+1)^{-1}}$

$$\mu(x) = \int \frac{x^2}{x+1} \cdot e^{-\int \frac{x}{x+1} dx} + C$$

$$e^{-\int \frac{x}{x+1}} = e^{-\left(1 - \frac{1}{x+1}\right)} = e^{-x + \ln|x+1|} = e^{-x} \cdot (x+1)$$

$$\mu(x) = \int \frac{x^2}{\cancel{(x+1)}} \cdot \left(e^{-x} \cdot \cancel{(x+1)}\right) dx + C =$$

$$= \int x^2 e^{-x} dx \quad \Big\| \begin{array}{ll} u = x^2 & du = 2x\,dx \\ dv = e^{-x} & v = -e^{-x} \end{array}$$

$$= -x^2 e^{-x} + \int 2x (e^{-x})\,dx \quad \Big\| \begin{array}{ll} u = 2x & du = 2 \\ du = e^{-x} & v = -e^{-x} \end{array}$$

$$= -x^2 e^{-x} - 2x e^{-x} - 2\int -e^{-x} dx =$$

$$= -x^2 e^{-x} - 2x e^{-x} - 2e^{-x} = -e^{-x}[x^2 + 2x + 2]$$

$$\boxed{\mu(x) = -e^{-x}[x^2 + 2x + 2] + C}$$

$$\boxed{\varphi(x) = e^{x}(x+1)^{-1}\left(-e^{-x}[x^2 + 2x + 2] + C\right)}$$