HOJA 2:

1. Verify that $y_1(x) = \cos(3x)e^{2x}$ both satisfy $y'' - 4y' + 13y = 0$
$y_2(x) = \sin(3x)e^{2x}$
and compute the wronskian.

For $y_1(x) = \cos(3x)e^{2x}$

$y_1'(x) = -\text{sen}(3x)\cdot 3e^{2x} + \cos(3x)e^{2x}2 =$

$= e^{2x}[-3\,\text{sen}(3x) + 2\cos(3x)]$

$y_1''(x) = 2e^{2x}[-3\,\text{sen}(3x) + 2\cos(3x)] + e^{2x}[-9\cos(3x) - 6\,\text{sen}(3x)] =$

$= e^{2x}[-12\,\text{sen}(3x) - 5\cos(3x)]$

if $y'' + 4y' + 13y = 0$

$e^{2x}[-12\,\text{sen}(3x) - 5\cos(3x)] - 4\cdot e^{2x}[-3\,\text{sen}(3x) + 2\cos(3x)] +$
$+ 13\cos(3x)e^{2x} = 0$

$e^{2x}[\cancel{-12\,\text{sen}(3x)} - 5\cos(3x) + \cancel{12\,\text{sen}(3x)} - 8\cos(3x) + 13\cos(3x)] =$

$e^{2x}[0] = 0$ ✓ verified

For $y_2(x) = \sin(3x)e^{2x}$

$y_2'(x) = 3\cos(3x)e^{2x} + 2\sin(3x)e^{2x}$

$= e^{2x}[3\cos(3x) + 2\sin(3x)]$

$y_2''(x) = 2e^{2x}[3\cos(3x) + 2\sin(3x)] + e^{2x}[-9\,\text{sen}(3x) + 6\cos(3x)]$

$= e^{2x}[12\cos(3x) - 5\sin(3x)]$

if $y'' - 4y' + 13y = 0$

$e^{2x}[12\cos(3x) - 5\sin(3x)] - 4e^{2x}[3\cos(3x) + 2\sin(3x)] + 13\sin(3x)e^{2x} = 0$

$e^{2x}[\cancel{12\cos(3x)} - \cancel{5\sin(3x)} - \cancel{12\cos(3x)} - \cancel{8\sin(3x)} + \cancel{13}\sin(3x)] = 0$

$e^{2x}[0] = 0$ ✓ verified

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For the wronskian:

$$W[y_1, y_2](x) = y_1(x)\, y_2'(x) - y_2(x)\, y_1'(x)$$

$$W[y_1, y_2](x) = \cos(3x)e^{2x} \cdot e^{2x}[3\cos(3x) + 2\sin(3x)] -$$

$$\sin(3x)e^{2x} e^{2x}[-3\operatorname{sen}(3x) + 2\cos(3x)] =$$

$$= e^{4x}\,[\cos(3x)\cdot[3\cos(3x) + 2\sin(3x)]$$

$$- \sin(3x)\,[-3\operatorname{sen}(3x) + 2\cos(3x)]\,] =$$

$$= e^{4x}\,[3\cos(3x)\cos(3x) + \cancel{2\cos(3x)\sin(3x)} + 3\operatorname{sen}(3x)\operatorname{sen}(3x) -$$

$$- \cancel{2\operatorname{sen}(3x)\cos(3x)}\,] = e^{4x}\,[3\,(\underbrace{\cos^2(3x) + \operatorname{sen}^2(3x)}_{1})\,] =$$

$$\boxed{= e^{4x}\, 3}$$

2. Verify $y_1(x) = x^2$, $y_2(x) = x^{-1}$ $\rightarrow$ fundamental set of solutions.

$$W[y_1, y_2](x) = \begin{vmatrix} x^2 & x^{-1} \\ 2x & -x^{-2} \end{vmatrix} = -x^0 - 2 = -1-2 = -3 \neq 0$$

they are undependent thus they constitute a fundamental set of solutions.

3. For the given pairs $\rightarrow$ Linear differential equations

a) $y_1 = x$
$y_2 = x^3$

$$y''(x) + p(x)y'(x) + q(x)\,y(x) = 0$$

$$p(x) = -\frac{W'(x)}{W(x)}$$

$$q(x) = \frac{1}{W(x)} W[y_1', y_2']$$

$$W[y_1\, y_2](x) = \begin{vmatrix} x & x^3 \\ 1 & 3x^2 \end{vmatrix} = 3x^3 - x^3 = 2x^3$$

$$W'[y_1\, y_2](x) = 6x^2$$

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$$W[y_1', y_2'](x) = \begin{vmatrix} 1 & 3x^2 \\ 0 & 6x \end{vmatrix} = 6x$$

$$p(x) = -\frac{6x^2}{2x^3} = -3x^{-1}$$

$$q(x) = \frac{1}{2x^3} \cdot 6x = 3x^{-2}$$

$$y''(x) - 3x^{-1} y'(x) + 3x^{-2} y(x) = 0 \quad (x^2)$$

$$\boxed{x^2 y'' - 3xy' + 3y = 0}$$

b) $y_1 = \sin x \quad y_2 = \cos x$

$$y''(x) + p(x) y'(x) + q(x) y(x) = 0$$

$$p(x) = \frac{-W'(x)}{W(x)} = 0$$

$$q(x) = \frac{1}{W(x)} W[y_1', y_2'] = -1(-1) = 1$$

$$W[y_1, y_2] = \begin{vmatrix} \sin x & \cos x \\ \cos x & -\operatorname{sen} x \end{vmatrix} = -\operatorname{sen}^2 x - \cos^2(x) =$$

$$= -(\underbrace{\cos^2 x + \operatorname{sen}^2 x}_{1}) = -1 \quad ; \quad W'[y_1 y_2] = 0$$

$$W[y_1', y_2'] = \begin{vmatrix} \cos x & -\operatorname{sen} x \\ -\operatorname{sen} x & -\cos x \end{vmatrix} = -\cos^2 x - \operatorname{sen}^2 x =$$

$$= -(\underbrace{\cos^2 x + \operatorname{sen}^2 x}_{1}) = -1$$

$$y''(x) + y(x) = 0$$

$$\boxed{y'' + y = 0}$$

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c) $y_1 = x^{-1}$ $\quad p(x) = \frac{-W'(x)}{W(x)} = -\frac{4}{4x} = -x^{-1}$

$y_2 = x^3$

$q(x) = \frac{1}{W(x)} W[y_1' \ y_2'] = \frac{1}{4x}(-12x^{-1}) = -3x^{-2}$

$$W[y_1 y_2](x) = \begin{vmatrix} x^{-1} & x^3 \\ -x^{-2} & 3x^2 \end{vmatrix} = 3x + x = 4x$$

$W'[y_1 y_2] = 4$

$$W[y_1', y_2'] = \begin{vmatrix} -x^{-2} & 3x^2 \\ 2x^{-3} & 6x \end{vmatrix} = -6x^{-1} - 6x^{-1} = -12x^{-1}$$

$$y''(x) - x^{-1}y'(x) - 3x^{-2}y(x) = 0 \ (x^2)$$

$$\boxed{x^2y'' - xy' - 3y = 0}$$

4. Write the general solution to the given differential equations.

a) $8y'' - 6y' + y = 0$

by constant coefficients: $8\lambda^2 - 6\lambda + 1 = 0$ $\quad \lambda_1 = \frac{1}{2}$, $\lambda_2 = \frac{1}{4}$

$$\boxed{y = c_1 e^{1/2x} + c_2 e^{1/4x}}$$

b) $y''' + y'' - 2y' = 0$

by constant coefficients: $\lambda^3 + \lambda^2 - 2\lambda = 0$, $\lambda(\lambda^2 + \lambda - 2) = 0$

$\lambda = 0$, $\lambda_1 = 1$, $\lambda_2 = -2$

$$y = c_1 e^0 + c_2 e^x + c_3 e^{-2x}$$

$$\boxed{y = c_1 + c_2 e^x + c_3 e^{-2x}}$$

c) $4y'' - 4y' - 5y = 0$

by constant coefficients $4\lambda^2 - 4\lambda - 5 = 0$

$\lambda_1 = \frac{1 + \sqrt{6}}{2}$

$\lambda_2 = \frac{1 - \sqrt{6}}{2}$

$$\boxed{y = c_1 e^{\left(\frac{1+\sqrt{6}}{2}\right)x} + c_2 e^{\left(\frac{1-\sqrt{6}}{2}\right)x}}$$

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d) $y'' + 9y = 0$

by constant coefficients

$\lambda^2 + 9 = 0 \begin{cases} \lambda_1 = 3i \\ \lambda_2 = -3i \end{cases}$

$y = c_1 e^{0x} \cos(3x) + c_2 e^{0x} \operatorname{sen}(3x) =$

$\boxed{y = c_1 \cos(3x) + c_2 \operatorname{sen}(3x)}$

e) $y'' + 6y' + 13y = 0$

by constant coefficients

$\lambda^2 + 6\lambda + 13 = 0 \begin{cases} \lambda_1 = -3+2i \\ \lambda_2 = -3-2i \end{cases}$

$\boxed{y = c_1 e^{-3x} \cos(2x) + c_2 e^{-3x} \operatorname{sen}(2x)}$

f) $y'' + 4y' + 5y = 0$

by constant coefficients

$\lambda^2 + 4\lambda + 5 = 0 \begin{cases} \lambda_1 = -2+i \\ \lambda_2 = -2-i \end{cases}$

$\boxed{y = c_1 e^{-2x} \cos(x) + c_2 e^{-2x} \operatorname{sen}(x)}$

g) $y'' - 6y' + 9y = 0$

by constant coefficients

$\lambda^2 - 6\lambda + 9 = 0 \begin{cases} \lambda_1 = 3 \\ \lambda_2 = 3 \end{cases}$] Double root → solapment

$\boxed{y = c_1 e^{3x} + c_2 x e^{3x}}$

h) $y'' - 8y' + 16y = 0$

by constant coefficients

$\lambda^2 - 8\lambda + 16 = 0 \begin{cases} \lambda_1 = 4 \\ \lambda_2 = 4 \end{cases}$] Double root → solapment

$\boxed{y = c_1 e^{4x} + c_2 x e^{4x}}$

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i) $4y'' - 4y' + 4 = 0$

by constant coefficients

$4\lambda^2 - 4\lambda + 4 = 0$ $\lambda_1 = \frac{1}{2}$, $\lambda_2 = \frac{1}{2}$

$$\boxed{y = c_1 e^{x/2} + c_2 x e^{x/2}}$$

— for the whole exercise —

$\boxed{\text{Cauchy problem } y(x_0) = y_0}$

5. Solve the initial value problems.

a) $y''' - 4y' = 0$; $y(0) = 0$; $y'(0) = 1$; $y''(0) = 8$

by constant coefficients

$\lambda^3 - 4\lambda = 0$; $\lambda(\lambda^2 - 4) = 0$ $\lambda_1 = 0$, $\lambda_2 = 2$, $\lambda_3 = -2$

$\rightarrow y = c_1 + c_2 e^{2x} + c_3 e^{-2x}$

$y' = 2c_2 e^{2x} - 2c_3 e^{-2x} = 2(c_2 e^{2x} - c_3 e^{-2x})$

$y'' = 4c_2 e^{2x} + 4c_3 e^{-2x} = 4(c_2 e^{2x} + c_3 e^{-2x})$

Cauchy problem

$y(x_0) = y_0$

- $y(0) = 0$ $c_1 + c_2 + c_3 = 0$
- $y'(0) = 1$ $2(c_2 - c_3) = 1$
- $y''(0) = 8$ $4(c_2 + c_3) = 8$

$c_1 = -2$

$c_2 = \frac{5}{4}$

$c_3 = \frac{3}{4}$

$$\boxed{y = -2 + \frac{5}{4}e^{2x} + \frac{3}{4}e^{-2x}}$$

b) $y'' - 2y' - 3y = 0$ $y(0) = 0$ $y'(0) = -4$.

by constant coefficients

$\lambda^2 - 2\lambda - 3 = 0$ $\lambda_1 = 3$, $\lambda_2 = -1$

$y = c_1 e^{3x} + c_2 e^{-x}$ $c_1 + c_2 = 0$ $c_1 = -1$

$y' = 3c_1 e^{3x} - c_2 e^{-x}$ $3c_1 - c_2 = -4$ $c_2 = 1$

$$\boxed{y = -e^{3x} + e^{-x}}$$

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c) $y'' - y' - 6y = 0$; $y(0) = 0$ $y'(0) = 5$

by constant coefficients

$$\lambda^2 - \lambda - 6 = 0 \quad \lambda_1 = 3, \quad \lambda_2 = -2$$

$$y = c_1 e^{3x} + c_2 e^{-2x}$$
$$y' = 3c_1 e^{3x} - 2c_2 e^{-2x}$$

$$c_1 + c_2 = 0$$
$$3c_1 - 2c_2 = 5$$

$$c_1 = 1$$
$$c_2 = -1$$

$$\boxed{y = e^{3x} - e^{-2x}}$$

d) $y'' + 2y' + 2y = 0$; $y(0) = 1$; $y'(0) = 1$

by constant coefficients

$$\lambda^2 + 2\lambda + 2 = 0 \quad \lambda_1 = -1 + i, \quad \lambda_2 = -1 - i$$

$$y = c_1 e^{-x} \cos(x) + c_2 \cdot e^{-x} \operatorname{sen}(x) = e^{-x}(c_1 \cos(x) + c_2 \operatorname{sen}(x))$$
$$y' = -e^{-x}(c_1 \cos(x) + c_2 \operatorname{sen}(x)) + e^{-x}(-c_1 \operatorname{sen}(x) + c_2 \cos(x))$$
$$y' = e^{-x}[\underbrace{-c_1 \cos x}_{-c_1} - \underbrace{c_2 \operatorname{sen} x}_{0} - \underbrace{c_1 \operatorname{sen} x}_{0} + \underbrace{c_2 \cos x}_{c_2}]$$

$c_1 = 1$

$$-c_1 + c_2 = 1 \quad ; \quad -1 + c_2 = 1 \quad ; \quad c_2 = 2$$

$$\boxed{y = e^{-x}(\cos x + 2 \operatorname{sen} x)}$$

e) $y'' + 4y' + 5y = 0$. $y(0) = 0$ $y'(0) = 9$

By constant coefficients

$$\lambda^2 + 4\lambda + 5 = 0 \quad \lambda_1 = -2 + i, \quad \lambda_2 = -2 - i$$

$$y = e^{-2x} c_1 \cos(x) + c_2 e^{-2x} \operatorname{sen}(x) = e^{-2x}[c_1 \cos x + c_2 \operatorname{sen} x]$$
$$y' = -2e^{-2x}[c_1 \cos x + c_2 \operatorname{sen} x] + e^{-2x}[-c_1 \operatorname{sen} x + c_2 \cos x]$$
$$y' = \underbrace{e^{-2x}}_{1}[\underbrace{-2c_1 \cos x}_{-2c_1} - \underbrace{2c_2 \operatorname{sen} x}_{0} - \underbrace{c_1 \operatorname{sen} x}_{0} + \underbrace{c_2 \cos x}_{c_2}]$$

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$C_1 = 0$

$-2C_1 + C_2 = 9$; $C_2 = 9 + \cancel{2C_1} = 9$

$$\boxed{y = 9e^{-2x}\,\text{sen}(x)}$$

p) $y'' - 2y' + 2y = 0$, $y(0) = -3$ $y'(0) = 0$

by constant coefficients

$\lambda^2 - 2\lambda + 2 = 0$ $\lambda_1 = 1+i$, $\lambda_2 = 1-i$

$y = c_1 e^x \cos(x) + c_2 e^x \,\text{sen}(x) = e^x(c_1\cos x + c_2 \,\text{sen} x)$

$y' = e^x(c_1\cos x + c_2\,\text{sen} x) + e^x(-c_1\,\text{sen} x + c_2\cos x)$

$= e^x[\underbrace{c_1\cos x}_{c_1} + \underbrace{c_2\,\text{sen}}_{0} x - \underbrace{c_1\,\text{sen}}_{0} x + \underbrace{c_2\cos x}_{c_2}]$

$C_1 = -3$

$c_1 + c_2 = 0$; $c_2 = -c_1$; $c_2 = -(-3) = 3$

$y = e^x(-3\cos x + 3\,\text{sen} x)$

$$\boxed{y = 3e^x(\text{sen} x - \cos x)}$$

g) $y'' + 4y' + 4y = 0$ $y(0) = 1$ $y'(0) = 1$

by constant coefficients

$\lambda^2 + 4\lambda + 4 = 0$ $\lambda_1 = -2$, $\lambda_2 = -2$

$y = c_1 e^{-2x} + c_2 x e^{-2x} = e^{-2x}[c_1 + xc_2]$

$y' = -2e^{-2x}[c_1 + xc_2] + e^{-2x}c_2 = e^{-2x}[-2c_1 - 2xc_2 + c_2]$

$C_1 = 1$

$-2C_1 + C_2 = 1$, $C_2 = 1 + 2C_1 = 1 + 2 = 3$

$$\boxed{y = e^{-2x}[1 + 3x]}$$

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h) $4y'' - 20y' + 25y = 0 \quad y(0) = 1 \quad y'(0) = 1$
by constant coefficients

$$4\lambda^2 - 20\lambda + 25 = 0 \quad \lambda_1 = \frac{5}{2} \quad \lambda_2 = \frac{5}{2}$$

$$y = c_1 e^{5/2x} + c_2 x e^{5/2x} = e^{5/2x}[c_1 + c_2 x]$$

$$y' = \frac{5}{2} e^{5/2x}[c_1 + c_2 x] + e^{5/2x}[c_2]$$

$$= e^{5/2x}[5/2\,c_1 + 5/2\,c_2 x + c_2]$$

$$c_1 = 1$$

$$\frac{5}{2}c_1 + c_2 = 1 \;, \quad c_2 = 1 - \frac{5}{2} = -\frac{3}{2}$$

$$\boxed{y = e^{\frac{5}{2}x}[1 - \frac{3}{2}x]}$$

i) $4y'' - 4y' + y = 0 \quad y(1) = 0 \quad y'(1) = 1$

$$4\lambda^2 - 4\lambda + 1 = 0 \quad \lambda_1 = \frac{1}{2} \quad \lambda_2 = \frac{1}{2}$$

$$y = c_1 e^{1/2x} + c_2 x e^{1/2x} = e^{1/2x}(c_1 + x c_2)$$

$$y' = \frac{1}{2} e^{1/2x}(c_1 + x c_2) + e^{1/2x}(c_2) = e^{1/2x}(\frac{1}{2}c_1 + \frac{1}{2}x c_2 + c_2)$$

$$e^{1/2}(c_1 + c_2) = 0 \;, \quad c_1 = -c_2 \;, \quad c_1 = -e^{-1/2}$$

$$e^{1/2}(\frac{1}{2}c_1 + \underbrace{\frac{1}{2}c_2 + c_2}) = 1$$

$$(\frac{1}{2}c_1 + \frac{3}{2}c_2) = e^{-1/2}$$

$$-\frac{1}{2}c_2 + \frac{3}{2}c_2 = e^{-1/2} \;, \quad c_2 = e^{-1/2}$$

$$\boxed{y = e^{x/2}(-e^{-1/2} + x e^{-1/2})}$$

Escaneado con CamScanner

6. general solution

a) $y'' + y = x^3$

$y = y_h + y_p$

y_h: by constant coeff.

$\lambda^2 + 1 = 0$ / $\lambda_1 = i$, $\lambda_2 = -i$

$y_h = C_1 \cos(x) + C_2 \operatorname{sen}(x)$

$y_p = Ax^3 + Bx^2 + Cx + D$

$y'_p = 3Ax^2 + 2Bx + C$

$y''_p = 6Ax + 2B$

$6Ax + 2B + Ax^3 + Bx^2 + Cx + D = x^3$

$Ax^3 + Bx^2 + (6Ax + Cx) + (2B + D) = x^3$

$\boxed{A = 1}$

$\boxed{Bx^2 = 0}$

$6Ax + C = 0$; $6 + C = 0$; $\boxed{C = -6}$

$\cancel{2B} + D = 0$ $\boxed{D = 0}$

$y_p = x^3 - 6x$

$$\boxed{y = C_1 \cos(x) + C_2 \operatorname{sen}(x) + x^3 - 6x}$$

b) $y'' + y' - 6y = x^2 - 2x$

$y = y_h + y_p$

y_h: constant coeff.

$\lambda^2 + \lambda - 6 = 0$ / $\lambda_1 = 2$, $\lambda_2 = -3$; $y_h = C_1 e^{2x} + C_2 e^{-3x}$

$y_p = Ax^2 + Bx + C$

$y'_p = 2Ax + B$; $y''_p = 2A$

$2A + 2Ax + B - 6(Ax^2 + Bx + C) = x^2 - 2x$

$-6Ax^2 + (2Ax - 6Bx) + (2A + B - 6C) = x^2 - 2x$

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$-6A = 1$; $A = -\frac{1}{6}$

$2A - 6B = -2$

$B = \frac{-2 - 2A}{-6} = \frac{-2}{-6}(1 + A) = \frac{1}{3}\left(1 - \frac{1}{6}\right) = \frac{5}{18}$

$2A + B - 6C = 0$; $C = \frac{\cancel{-}2A \cancel{-} B}{\cancel{-}6} = \frac{2\left(\frac{-1}{6}\right) + \frac{5}{18}}{6} = \frac{-1}{108}$

$y_P = -\frac{1}{6}x^2 + \frac{5}{18}x - \frac{1}{108}$

$$\boxed{y = C_1 e^{2x} + C_2 e^{-3x} - \frac{1}{6}x^2 + \frac{5}{18}x - \frac{1}{108}}$$

c) $y'' + 2y' - 8y = 6e^{2x}$

$y = y_h + y_p$

y_h: constant coefficients

$\lambda^2 + 2\lambda - 8 = 0$; $\lambda_1 = 2$, $\lambda_2 = -4$; $y_h = c_1 e^{2x} + c_2 e^{-4x}$

$y_P = A e^{nx} = A e^{2x}$ solapamiento (modification rule)

$y_P = Ax e^{2x}$

$y'_P = Ae^{2x} + Axe^{2x} \cdot 2 = e^{2x}(A + Ax2) =$

$= A e^{2x}(1 + 2x)$

$y''_P = 2Ae^{2x}(1 + 2x) + Ae^{2x} \cdot 2 = 2Ae^{2x}(2 + 2x) =$

$= e^{2x}(4A + 4Ax)$

$e^{2x}(4A + 4Ax) + 2 \cdot Ae^{2x}(1 + 2x) - 8Axe^{2x} = 6e^{2x}$

$e^{2x}(4A + \cancel{4Ax} + 2A + \cancel{4Ax} \cancel{-8Ax}) = 6e^{2x}$

$6A = 6$; $\boxed{A = 1}$

$y_P = xe^{2x}$

$$\boxed{y = c_1 e^{2x} + c_2 e^{-4x} + xe^{2x}}$$

Escaneado con CamScanner

d) $y'' + 5y' + 6y = 12e^x + 6x^2 + 10x$

$y = y_h + y_p$

y_h : constant coeff $\lambda^2 + 5\lambda + 6 = 0$ $\lambda_1 = -2$, $\lambda_2 = -3$

$y_h = C_1 e^{-2x} + C_2 e^{-3x}$

$y_p = y_e + y_{pol}$ → superposición de soluciones.

$y_e = Ae^x$

$y'_e = Ae^x$

$y''_e = Ae^x$

$Ae^x + 5Ae^x + 6Ae^x = 12e^x$

$e^x(A + 5A + 6A) = 12e^x$

$12A = 12$; $A = 1$

$y_e = e^x$

$y_{pol} = Ax^2 + Bx + C$

$y'_{pol} = 2Ax + B$

$y''_{pol} = 2A$

$2A + 5(2Ax + B) + 6(Ax^2 + Bx + C) =$

$= 6x^2 + 10x$

$2A + 10Ax + 5B + 6Ax^2 + 6Bx + 6C$

$= 6x^2 + 10x$

$6A = 6$; $A = 1$

$10A + 6B = 10$; $B = \frac{10 - 10A}{6} =$

$= \frac{10 - 10}{6} = 0$

$2A + 6C = 0$

$C = -\frac{2}{6} = -\frac{1}{3}$

$y_{pol} = x^2$

$y_p = e^x + x^2 - \frac{1}{3}$

$$\boxed{y = C_1 e^{-2x} + C_2 e^{-3x} + e^x + x^2 - \frac{1}{3}}$$

Escaneado con CamScanner

e) $y'' - 2y' + 2y = 2e^{x}\cos x$

$y = y_h + y_p$

y_h : constant coefficients

$\lambda^2 - 2\lambda + 2 = 0$ $\quad \lambda_1 = 1+i$, $\quad \lambda_2 = 1-i$

$y_h = C_1 e^{x}\cos(x) + C_2 e^{x}\operatorname{sen}(x)$

$y_p = e^{x}(C_1\cos x + C_2\sin x)$ ↑ solapamiento de soluciones

$y_p = xe^{x}(C_1\cos x + C_2\sin x)$

$y'_p = (e^{x} + xe^{x})(C_1\cos x + C_2\operatorname{sen}x) + xe^{x}(-C_1\operatorname{sen}x + C_2\cos x)$

$y''_p = (e^{x} + e^{x} + xe^{x})(C_1\cos x + C_2\operatorname{sen}x) + (e^{x} + xe^{x})(-C_1\operatorname{sen}x + C_2\cos x)$
$+ (e^{x} + xe^{x})(-C_1\operatorname{sen}x + C_2\cos x) + xe^{x}(-C_1\cos x - C_2\operatorname{sen}x)$

$y''_p = (2e^{x} + \cancel{xe^{x}} - \cancel{xe^{x}})(C_1\cos x + C_2\operatorname{sen}x) + (e^{x} + xe^{x} + e^{x} + xe^{x})$
$(-C_1\operatorname{sen}x + C_2\cos x)$

$y''_p = 2e^{x}(C_1\cos x + C_2\operatorname{sen}x) + (\overbrace{2e^{x} + 2xe^{x}}^{1+x})(-C_1\operatorname{sen}x + C_2\cos x)$

$y''_p = 2e^{x}[C_1\cos x + C_2\operatorname{sen}x - C_1\operatorname{sen}x + C_2\cos x - xC_1\operatorname{sen}x + xC_2\cos x]$

$y''_p = 2e^{x}[((C_1 + C_2 + xC_2)\cos x + (C_2 - C_1 - xC_1)\operatorname{sen}x]$

$2e^{x}[((C_1 + C_2 + xC_2)\cos x + (C_2 - C_1 - xC_1)\operatorname{sen}x] - 2e^{x}[C_1\cos x + C_2\operatorname{sen}x$
$+ xC_1\cos x + xC_2\operatorname{sen}x - xC_1\operatorname{sen}x + xC_2\cos x] + 2e^{x}[xC_1\cos x +$
$xC_2\operatorname{sen}x] = 2e^{x}\cos x$

$(C_1 + C_2 + xC_2) - C_1\cos - xC_1\cos x - xC_2\cos x + xC_1\cos x = \cos x$

$\cancel{C_1} + C_2 + \cancel{xC_2} - \cancel{C_1} - \cancel{xC_1} - \cancel{xC_2} + \cancel{xC_1} = 1$

$\boxed{C_2 = 1}$

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$\cancel{C_2} - C_1 - \cancel{xC_1} - \cancel{C_2} - \cancel{xC_2} + \cancel{xC_1} + \cancel{xC_2} = 0$

$C_2 \quad -C_1 = 0 \; ; \; \boxed{C_1 = 0}$

$y_p = xe^x \operatorname{sen} x$

$y = C_1 e^x \cos x + C_2 e^x \sin x + xe^x \operatorname{sen} x$

$$\boxed{y = e^x [C_1 \cos x + C_2 \sin x + x \sin x]}$$

p) $(D^2 + D - 6) y = 5e^{2x} \qquad 5 = 5x^0$

Ecuación homogenea - método de coeficientes constantes

$\lambda^2 + \lambda - 6 = 0 \quad \lambda_1 = 2$

$\lambda_2 = -3 \qquad y_h = C_1 e^{2x} + C_2 e^{-3x}$

y_p - anuladores

anulador exp

$(D-2)(D^2+D-6) y = (D-2)(5e^{2x})$

$(D-2)(D^2+D-6) y = 0$

$(\lambda - 2) = 0 \; ; \; \lambda_1 = 2 \qquad y_p = \underbrace{xA e^{2x}}_{y_p} + \underbrace{Be^{2x} + Ce^{-3x}}_{y_h}$

$(\lambda^2 + \lambda - 6) = 0 \quad \lambda_2 = 2$

$\lambda_3 = -3$

$y'' + y' - 6y = 5e^{2x} \qquad y_p = Axe^{2x}$

$y_p' = Ae^{2x} + Ax2e^{2x}$

$= e^{2x}[A + 2Ax]$

$y_p'' = 2e^{2x}[A + 2Ax] + e^{2x}[2A]$

$e^{2x}[2A + 4Ax + 2A]$

$e^{2x}[4Ax + 4A]$

$e^{2x}[4Ax + 4A] + e^{2x}[A + 2Ax] -$

$6Axe^{2x} = 5e^{2x}$

$\cancel{4Ax} + 4A + A + \cancel{2Ax} - \cancel{6Ax} = 5$

$5A = 5 \; ; \; A = 1$

$y_p = xe^{2x}$

$$\boxed{y = xe^{2x} + C_1 e^{2x} + C_2 e^{-3x}}$$

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7 solve by the variation of parameters method

a) $y'' + y = 3\csc x$.

$y_h = c_1 \cos x + c_2 \operatorname{sen} x$

$\lambda^2 + 1 = 0$; $\lambda_1 = i$
$\lambda_2 = -i$

$y_p = c_1(x)\cos x + c_2(x)\operatorname{sen} x$

$$c_1 = -\int \frac{y_2(x)\, f(x)}{W[y_1\, y_2](x)}\, dx$$

$$c_2 = \int \frac{y_1(x)\, f(x)}{W[y_1, y_2](x)}\, dx$$

$$W[y_1 y_2] = \begin{vmatrix} \cos x & \operatorname{sen} x \\ -\operatorname{sen} x & \cos x \end{vmatrix} = \cos^2 x + \operatorname{sen}^2 x = 1$$

$$c_1 = -\int \operatorname{sen} x \cdot 3\csc x\, dx = -3\int \operatorname{sen} x \csc x\, dx = -3\int \frac{\operatorname{sen} x}{\operatorname{sen} x}\, dx$$

$= \boxed{-3x}$

$$c_2 = \int \cos x\; 3\csc x\, dx = 3\int \frac{\cos x}{\operatorname{sen} x}\, dx = \boxed{3 \cdot \ln|\sin x|}$$

$tg^{-1} = \cot g$

$y_p = -3x\cos x + 3\ln|\sin x| \cdot \sin x$

$$\boxed{y = c_1 \cos x + c_2 \operatorname{sen} x - 3x\cos x + 3\ln|\sin x|\sin x}$$

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b) $y'' + 4y' + 4y = \dfrac{2e^{-2x}}{x^2+1}$

y_h.

$\lambda^2 + 4\lambda + 4 = 0 \quad \lambda_1 = -2, \quad \lambda_2 = -2 \qquad y_h = C_1 e^{-2x} + x C_2 e^{-2x}$

$y_p = c_1(x) e^{-2x} + c_2(x) x e^{-2x}$

$W[y_1 y_2](x) = \begin{vmatrix} e^{-2x} & xe^{-2x} \\ -2e^{-2x} & e^{-2x} - 2xe^{-2x} \end{vmatrix} = (e^{-2x})^2 \begin{vmatrix} 1 & x \\ -2 & 1-2x \end{vmatrix} =$

$= e^{-4x} [1 - 2x + 2x] = e^{-4x}$

$C_1(x) = -\int \dfrac{x\cancel{e^{-2x}} \cdot 2\cancel{e^{-2x}}}{\cancel{e^{-4x}} \cdot (x^2+1)} dx = -\int \dfrac{2x}{x^2+1} dx =$

$= -\ln|x^2+1|$

$C_2(x) = \int \dfrac{\cancel{e^{-2x}} \cdot 2\cancel{e^{-2x}}}{\cancel{e^{-4x}}(x^2+1)} dx = \int \dfrac{2}{x^2+1} dx = 2\int \dfrac{1}{x^2+1} dx =$

$= 2 \cdot \operatorname{arctg}(x)$

$y_p = -\ln|x^2+1| e^{-2x} + 2\operatorname{arctg}(x) x e^{-2x}$

$y = c_1 e^{-2x} + x c_2 e^{-2x} - \ln|x^2+1| e^{-2x} + 2\operatorname{arctg}(x) x e^{-2x}$

$\boxed{y = e^{-2x} [c_1 + x c_2 - \ln|x^2+1| + 2x \operatorname{arctg}(x)]}$

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c) $y'' + 2y' + y = 4e^x \ln x$

$y_h = c_1 e^{x} + c_2 x e^{x}$

$\lambda^2 + 2\lambda + 1 = 0 \quad \lambda_1 = 1, \ \lambda_2 = 1$

$y_p = c_1(x)\, e^{x} + c_2(x) \cdot x e^{x}$

$$W[y_1 y_2](x) = \begin{vmatrix} e^{x} & xe^{x} \\ e^{x} & e^{x} + xe^{x} \end{vmatrix} = (e^{x})^2 \begin{vmatrix} 1 & x \\ 1 & 1+x \end{vmatrix} =$$

$$= e^{2x}(1 - x + x) = e^{2x}$$

$$c_1(x) = -\int \frac{x\cancel{e^{x}}\, 4\cancel{e^{x}} \ln x}{\cancel{e^{2x}}} dx = -4\int x \cdot \ln x \, dx =$$

$u = \ln x \quad du = \frac{1}{x} dx$

$dv = x \quad v = \frac{x^2}{2}$

$$= -4\left[\ln x \frac{x^2}{2} - \int \frac{1}{\cancel{x}} \frac{x^{\cancel{2}}}{2} dx\right] =$$

$$= -4\left[\ln x \frac{x^2}{2} - \frac{1}{2}\frac{x^2}{2}\right] = -2\ln x \cdot x^2 + x^2 = \boxed{x^2(-2\ln x + 1)}$$

$$c_2(x) = \int \frac{\cancel{e^{x}}\, 4\cancel{e^{x}} \ln x}{\cancel{e^{2x}}} dx = 4\int \ln x\, dx = 4\left[\ln x \cdot x - \int \frac{1}{\cancel{x}} \cancel{x}\, dx\right]$$

$u = \ln x \quad du = \frac{1}{x}$

$dv = dx \quad v = x$

$$= 4[\ln x \cdot x - x] = \boxed{4x[\ln x - 1]}$$

$$y_p = x^2(-2\ln x + 1)\, e^{x} + 4x(\ln|x| - 1)\, x e^{x}$$

$$\boxed{y = e^{x}[c_1 + c_2 x + x^2(1 - 2\ln|x|) + 4x^2(\ln|x| - 1)]}$$

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P d) $y'' - 6y' + 9y = x^{-2} e^{3x}$

$y_h = C_1 e^{3x} + C_2 x e^{3x}$

$\lambda^2 - 6\lambda + 9 = 0$ $\quad \lambda_1 = 3$, $\lambda_2 = 3$

$y_p = C_1(x) e^{3x} + C_2(x) \cdot x e^{3x}$

$$W[y_1 y_2](x) = \begin{vmatrix} e^{3x} & x e^{3x} \\ 3e^{3x} & e^{3x} + x 3 \cdot e^{3x} \end{vmatrix} = (e^{3x})^2 \begin{vmatrix} 1 & x \\ 3 & 1+3x \end{vmatrix} =$$

$$= e^{6x} [1 + 3x - 3x] = e^{6x}$$

$$C_1(x) = -\int \frac{x \cancel{e^{3x}} \cdot x^{-2} \cancel{e^{3x}}}{\cancel{e^{6x}}} dx = -\int x^{-1} dx =$$

$$\boxed{= -\ln|x|}$$

$$C_2(x) = \int \frac{\cancel{e^{3x}} \cdot x^{-2} \cancel{e^{3x}}}{\cancel{e^{6x}}} dx = \int x^{-2} dx \boxed{= -x^{-1}}$$

$$y_p = -\ln|x| e^{3x} - \cancel{x^{-1}} \cancel{x} e^{3x}$$

$$y = C_1 e^{3x} + C_2 x e^{3x} - \ln|x| e^{3x} - e^{3x}$$

$$\boxed{y = e^{3x} [C_1 + C_2 x - \ln|x| - 1]}$$

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