# INF243-Mandatory Assignment 5

## Submission Deadline: May 23rd, 2023

Instructions for the assignment:

- This assignment has 3 pages and accounts for 10 points for your final grade
- Prepare a PDF file for your answers
  - you can use Latex (see manual at this link) as the text editor which compiles to a nice PDF file
  - You are suggested to use SageMath to assist your calculations on the questions
  - make sure to properly comment your source code.
  - If you answer questions with SageMath, you can provide necessary text for each question and export it directly as a PDF file via Latex

**Q1. Reed-Muller codes** [3 pts]

Let $RM(r, m)$ be the Reed-Muller code with length $2^m$ and be associated with all monomials with degree at most $r$ in $m$ variables.

(i) Construct a generator matrix $G$ and parity-check matrix $H$ for $RM(1, 3)$

(ii) Determine the sent codeword for the following two received words for $RM(1, 3)$:

$$[1, 0, 1, 0, 1, 1, 0, 1], [0, 1, 0, 0, 1, 1, 1, 1]$$

(iii) Construct a generator matrix $G$ for $RM(2, 4)$ and encode the following two messages:

$$[1, 0, 0, 1, 1, 0, 1, 0, 1, 0, 1], \quad [1, 1, 1, 0, 0, 1, 0, 0, 1, 0, 1]$$

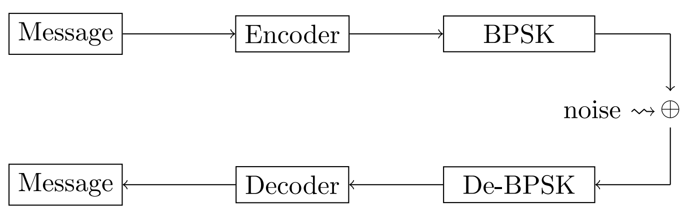
**Q2. Basics on LDPC codes** [2 pts]

(i) Follow Example 15.3 in the textbook and construct a $(w_c, w_r) = (3, 5)$ parity-check matrix $H$ for a regular LRPC code with length 20. Indicate the permutations you use in the construction

(ii) Follow Example 15.5, construct the sets $\mathcal{N}(m)$ and $\mathcal{M}(n)$ for the above parity-check matrix $H$ and construct the Tanner graph for $H$

**Q3. Decoding of LDPC codes** [1+1+2+1 pts]

Consider the following AWGN communication model




where the noise is sampled from $\mathcal{N}(0, 1)$.

Suppose an LDPC code $C$ has the following parity-check matrix

$$H = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

According to the above communication model, we encode a message $\mathbf{m} = (m_1, m_2, m_3, m_4, m_5)$ with the LRPC code $C$ defined by $H$, modulate each bit of $\mathbf{c} = (c_1, c_2, \ldots, c_{10})$ with BPSK as $\mathbf{x} = (x_1, x_2, \ldots, x_{10})$ with $x_i = (-1)^{c_i}$, and send the output $\mathbf{x} = (x_1, x_2, \ldots, x_{10})$ through the AWGN channel.

Suppose at the receiver we get a word

$$y = (-0.35, -0.43, -0.73, -0.05, 0.2, 0.98, -0.73, 0.36, -0.45, 0.58)$$

(i) **Initialization.** Construct the sets $\mathcal{N}(m)$ and $\mathcal{M}(n)$ for each $1 \leq m \leq 5$ and $1 \leq n \leq 10$. Calculate the intrinsic LLR of $\mathbf{x}$ from the channel

$$L_n := L_c(\mathbf{x}) = [L_c(x_1), L_c(x_2), \ldots, L_c(x_n)]$$

where $L_c(x_i) = \log \frac{P(y_i|x_i=1)}{P(y_i|x_i=-1)}$. (This corresponds to the initial step to create the message matrix $L_{n\to m}$ in Example 15.9)

(ii) **Update.** Consider the variable node $v_1$ connected to check nodes $C_1, C_2, C_5$, i.e., $v_1$ appears in the following three parity-check equations

$$\begin{aligned} v_1 + v_2 + v_3 + v_6 + v_7 + v_{10} &= 0 \\ v_1 + v_3 + v_5 + v_6 + v_8 + v_9 &= 0 \\ v_1 + v_2 + v_4 + v_8 + v_9 + v_{10} &= 0. \end{aligned}$$

Draw the Tanner graph for the above variable nodes and check nodes. For each of the above equations, use the information in $L_c(\mathbf{x})$ to calculate the min-sum approximation of the LLR of $x_1$, namely,

$$\begin{aligned} L_{1\to 1} &= \textstyle\prod_{j\in\{2,3,6,7,10\}} sign(L_c(x_j)) \min_{j\in\{2,3,6,7,10\}} |L_c(x_j)| \\ L_{2\to 1} &= \textstyle\prod_{j\in\{3,5,6,8,9\}} sign(L_c(x_j)) \min_{j\in\{3,5,6,8,9\}} |L_c(x_j)| \\ L_{5\to 1} &= \textstyle\prod_{j\in\{2,4,8,9,10\}} sign(L_c(x_j)) \min_{j\in\{2,4,8,9,10\}} |L_c(x_j)| \end{aligned}$$

The above LLR values of $x_1$ are considered as three repetition of LLR of $x_1$. Based on the three extrinsic LLR of $x_1$, calculate the LLR message from $x_1$ to the check nodes $C_1, C_2, C_5$ as in Algorithm 15.1

(iii) **Updating Rows and Columns.** Follow Algorithm 15.1 and Example 15.9. Construct the message matrix $L_{m\to n}$ and update it for one iteration with the following steps:

- check node to variable node step (horizontal)
- variable node check node step (vertical)

(iv) **More Iterations**. Continue the above iterations and output the message $L_{n\to \text{out}}$. For the guessed $\hat{\mathbf{c}}$, carry out the parity check of $\hat{\mathbf{c}}$ with $H$ until the correct codeword is obtained.