

Exercise 3 - Linear Block Codes

Q1. Let C be the binary linear code of length 6 with generator matrix

$$G = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 \end{bmatrix}$$

- i) Determine a generator matrix for the code of the form (IA) .
- ii) Determine a parity check matrix for the code C^\perp .
- iii) Is $(1, 1, 1, 1, 1, 1)$ a parity check for the code?

Q2. Consider the linear code with the generator matrix

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 1 \end{bmatrix}.$$

- i) Determine the dimension and the minimum distance of the code and its dual.
- ii) How many errors do the two codes correct?

Q3. Let the columns of the parity check matrix of a code be h_1, h_2, h_3, h_4, h_5 , so

$$H = \begin{bmatrix} | & | & | & | & | \\ h_1 & h_2 & h_3 & h_4 & h_5 \\ | & | & | & | & | \end{bmatrix}$$

- i) What syndrome corresponds to an error at position j ?
- ii) Express $H(10011)^T$ using h_1, h_2, h_3, h_4, h_5 .
- iii) Show that if $(1, 1, 1, 1, 1)$ is a codeword, then $h_1 + h_2 + h_3 + h_4 + h_5 = 0$.

*The above questions are taken from the textbook Ch. 1.

Programming Tasks

T1. Consider the linear code C with the generator matrix as in Q2.

- i) Write a code in Python/SageMath to generate all codeword in C .

- ii) Write a code in Python/SageMath to generate the systematic generator matrix and parity-check matrix
- T2. For the binary Hamming code, write a code in Python/SageMath to conduct syndrome decoding for errors with weight one.
- T3. Let C be the code of length 9 with parity check matrix

$$H = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

- i) Write a code determine the minimum distance of C .
- ii) Write a code in Python/SageMath to decode received words 110101101 and 111111111 with syndrome decoding.