Exercise 3 - Linear Block Codes

Q1. Let C be the binary linear code of length 6 with generator matrix

$$G = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 \end{bmatrix}$$

i) Determine a generator matrix for the code of the form (IA).

ii) Determine a parity check matrix for the code $C^{\perp}$.

iii) Is $(1, 1, 1, 1, 1, 1)$ a parity check for the code?

Q2. Consider the linear code with the generator matrix

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 1 \end{bmatrix}.$$

i) Determine the dimension and the minimum distance of the code and its dual.

ii) How many errors do the two codes correct?

Q3. Let the columns of the parity check matrix of a code be h_1, h_2, h_3, h_4, h_5, so

$$H = \begin{bmatrix} | & | & | & | & | \\ h_1 & h_2 & h_3 & h_4 & h_5 \\ | & | & | & | & | \end{bmatrix}$$

i) What syndrome corresponds to an error at position j ?

ii) Express $H(10011)^T$ using h_1, h_2, h_3, h_4, h_5.

iii) Show that if $(1, 1, 1, 1, 1)$ is a codeword, then $h_1+h_2+h_3+h_4+h_5 = 0$.

*The above questions are taken from the textbook Ch. 1.

Programming Tasks

You can use Python, Sagemath, or Matlab for this task.

T1. Consider the linear code C with the generator matrix as in Q2. With such a generator matrix, write a code that complete the following functions:

i) to generate the systematic generator matrix and parity-check matrix

ii) to generate all codeword in C

iii) to get the weight enumerator of the code C

iv) to generate the standard array (as in Table 3.1, Page 101) for complete decoding

v) to generate a simplified array for syndrome decoding

T2. For the binary Hamming code, write a code in Python/SageMath to conduct syndrome decoding for errors with weight one.

T3. Let C be the code of length 9 with parity check matrix

$$H = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

i) Write a code determine the minimum distance of C.

ii) Write a code in Python/SageMath to decode received words 110101101 and 111111111 with syndrome decoding.