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Concept Review

Frames of Reference

Why are Frames of Reference Used?

With multiple moving parts in a complex system of agents, whether it is stationary or dynamic robots or environmental objects, it is useful to localize the pose of each robot or agent. Establishing a common standard with respect to which the position and orientation of these agents is defined is vital for tracking. Frames of Reference serve as the base upon which modern mathematics and equations of motion are implemented. This document will cover three popular frames of reference – inertial, body and horizon.

Frames of Reference

A frame of reference in the space $\mathbb{R}^3$ is defined using three mutually independent basis vectors. If the three vectors are also mutually perpendicular to each other, they are referred to as orthogonal basis vectors. It is also useful to select a set of unit vectors to specify direction easily without amplification. If the set of basis vectors picked are both mutually perpendicular and of unit length, they are called orthonormal basis vectors. The unit vectors $\hat{x}$, $\hat{y}$ and $\hat{z}$ in 3D space are common orthonormal basis vectors of $\mathbb{R}^3$. A representation of these vectors in $\mathbb{R}^3$ is,

$$\hat{x} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \hat{y} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \hat{z} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

These vectors follow the following properties,

1. Orthogonality

$$\hat{x} \cdot \hat{y} = 0,\ \hat{y} \cdot \hat{z} = 0 \text{ and } \hat{z} \cdot \hat{x} = 0$$

2. Unit length

$$\|\hat{x}\| = \|\hat{y}\| = \|\hat{z}\| = 1$$

3. Right-hand-convention

$$\hat{x} \times \hat{y} = \hat{z},\ \hat{y} \times \hat{z} = \hat{x} \text{ and } \hat{z} \times \hat{x} = \hat{y}$$

Left-handed frames of reference do exist and have common applications in Computer Vision. However, for most dynamic formulations, a right-handed convention is adopted.

Inertial Reference Frame $\{I\}$

This is a reference frame attached to the inertial center of a system, one, that for most intents and purposes can be considered to be non-accelerating (such as the earth for applications on the surface of the earth or the sun for the orbital motion of the planets etc.). All other frames of reference are described as moving with respect to this reference frame. The basis vectors of this frame are commonly denoted as $\hat{x}_I$, $\hat{y}_I$ and $\hat{z}_I$.

Body Reference Frame $\{B\}$

This is a reference frame attached to the body of interest and rigidly translating and rotating with the body. Tracking this reference frame with respect to the Inertial Frame is often the key goal of a localization problem. The basis vectors of this frame are commonly denoted as $\hat{x}_B$, $\hat{y}_B$ and $\hat{z}_B$.

Horizon Reference Frame $\{H\}$

This is a reference frame attached to the body of interest but not rigidly moving with it completely. The Horizon Frame's origin translates with the body itself and also rotates about the inertial yaw axis $\hat{z}_I$. However, it does not roll or pitch with the body. The basis vectors of this frame are commonly denoted as $\hat{x}_H$, $\hat{y}_H$ and $\hat{z}_H$.

This frame of reference finds a use case in GPS/compass-based car/drone navigation, where the body-frame 6-axis IMU (gyroscope/accelerometer) data can be used to estimate the body roll and pitch, but cannot be used to estimate the yaw angle. This lets you map the states of the robotic agent from the Body Frame to the Horizon Frame. A combination of GPS/compass can provide estimates for the 3 DOF position as well as heading (yaw). In essence, the GPS/compass data can help one map the robot from the Inertial Frame to the Horizon Frame. With all estimates in the Horizon Frame, sensor fusion techniques can safely be used in a single frame of reference.

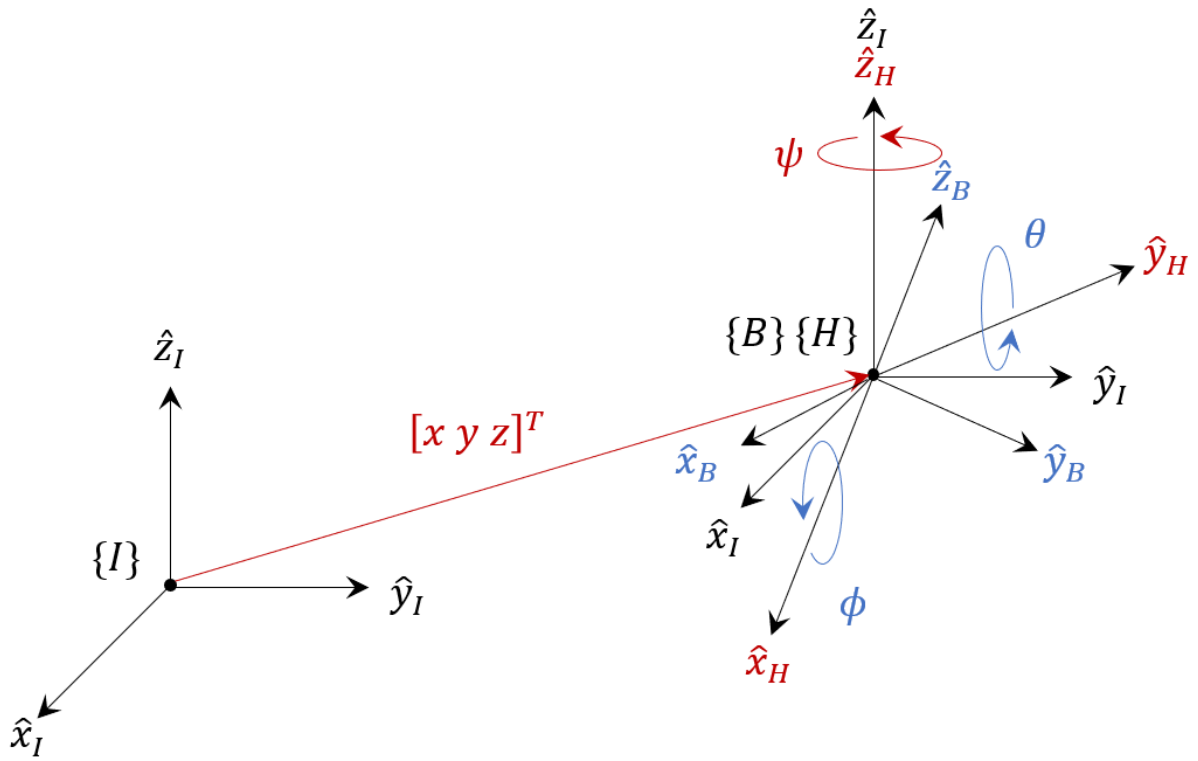

Figure 1. Inertial, Horizon and Body Reference Frames

Figure 1 shows the visual transformation from the Inertial to the Horizon frame that involves a translation by $[x\ y\ z]^T$ and a rotation about the $\hat{z}_I$ frame by a yaw angle ψ. Following this, rotations of roll ϕ and pitch θ about the $\hat{x}_I$ and $\hat{y}_I$ axis, respectively, yield the Body frame. For more information on rotations and how they are represented in detail, please refer to the concept review on Orientation and Position.

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