Concept Review

Kinematic Bicycle Model

How to Describe the Motion of a Vehicle?

Depending on the mechanical and geometric constraints of a system a kinematic motion model is an important step in describing the path of a robot in space. Modern vehicles use a kinematic bicycle model to estimate the path taken given a linear velocity and a steering angle. In combination with a driving controller a kinematic model can be used to get an ideal prediction of where a vehicle will be in space. In this module a special focus will be taken in deriving the kinematic bicycle model for the purpose of motion planning.

Frame of Reference Definition

Inertial frame definition for an object in space:

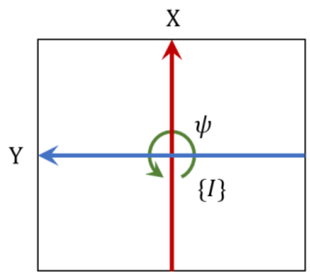


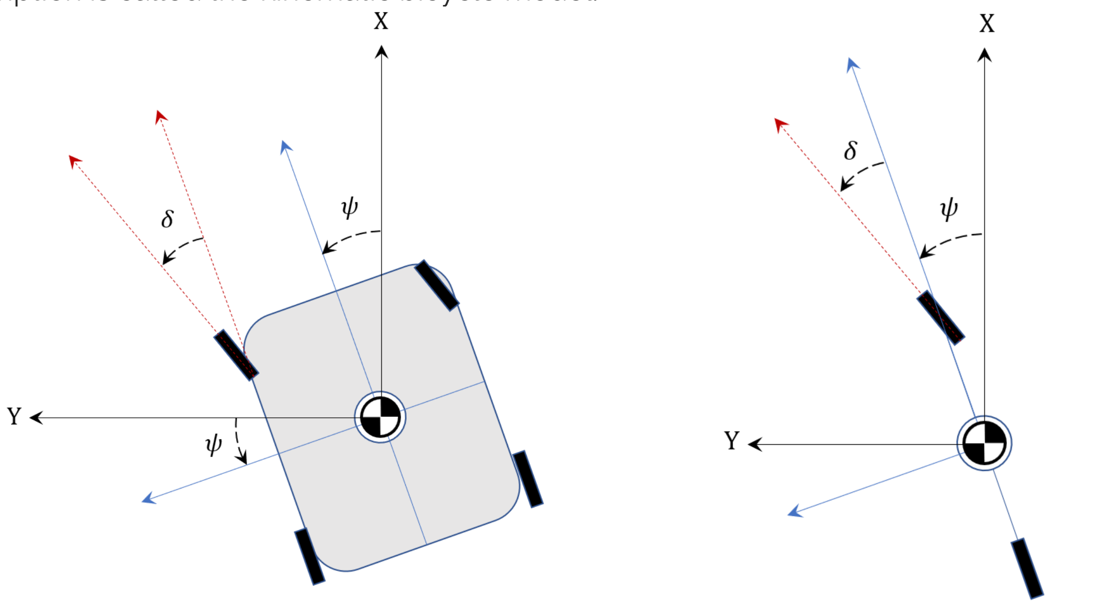
Figure 1: World Axis definition

The world definition allows us to describe the position of an object based on the kinematic properties of the system. From a 2D perspective the location of any object in this world can be uniquely described as:

$$P^I = \begin{bmatrix} X^I \\ Y^I \\ \psi^I \end{bmatrix}$$

Kinematic Modeling

The next step is to define how the system will move in the 2D world. The definition of such movement is the kinematic model. For the purposes of this document the kinematic model will assume our system uses a front steering 4-wheel platform. The first step is to pick the reference frame of the vehicle's motion called the body frame. For simplicity the wheel steering angle is assumed to be the same for both the right and the left wheel. This assumption is called the kinematic bicycle model.



a. Front steering body frame definition

b. Bicycle model definition

Figure 2: Comparison of a front steering body and bicycle model

Next the bicycle model is represented in the inertial frame $\{I\}$ and described about the center of gravity of the vehicle $\{B\}$. Note the intermediary frame $\{I'\}$, which is co-located with $\{B\}$ and co-oriented with $\{I\}$. The position of the vehicle in $\{I\}$ is determined by the vector P^I.

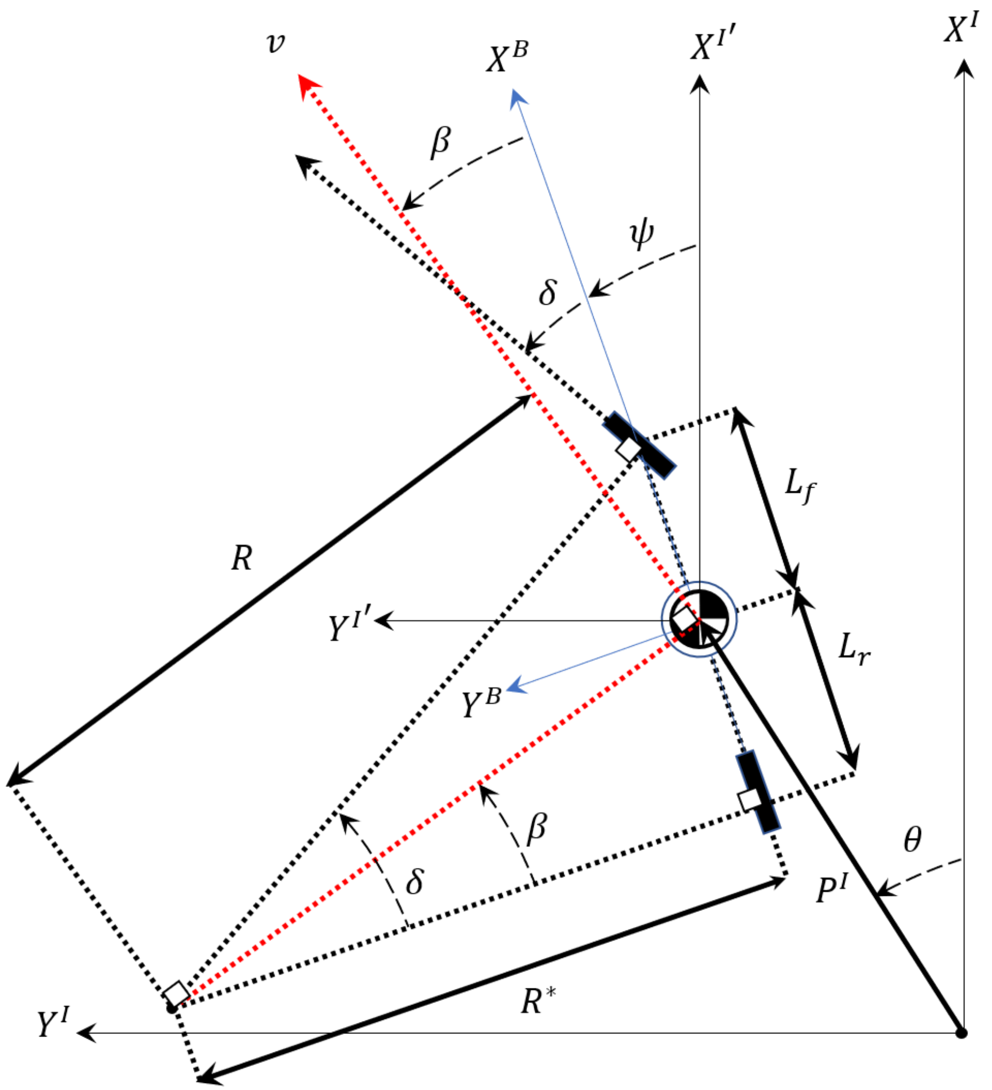


Figure 3: kinematic bicycle model

The body reference frame of the QCar is picked at the geometric center which changes how the linear and angular velocity are calculated. Using the relationship in Figure 3, the 3DOF bicycle model can provide the value of β (vehicle sideslip angle) using geometric relationships and the input steering angle δ (vehicle steering).

$$\tan\delta = \frac{L_f + L_r}{R^*} \tag{1}$$

$$\tan\beta = \frac{L_r}{R^*} \tag{2}$$

Combining equations (1) and (2) yields the following relationship,

$$\frac{1}{R^*} = \frac{\tan\delta}{L_f + L_r} = \frac{\tan\beta}{L_r} \tag{3}$$

where R is the turning radius of the vehicle with respect to the rear axle. Solving for β gives the following sideslip angle calculation,

$$\beta = \tan^{-1}\left(\frac{L_r \tan\delta}{L_f + L_r}\right) \tag{4}$$

Knowing the sideslip angle is important to understand the true turning radius of a vehicle R at any given steering angle δ. Using Figure 3, write the turning radius using the following relationship:

$$R = \frac{R^*}{\cos\beta} \tag{5}$$

Use (2) in (5) to solve for R

$$R = \frac{L_r \ / \tan\ \beta}{\cos\beta} = {}^{L_r}/_{\sin\beta} \tag{6}$$

Understanding the turning radius is important as this defines how tight a turn a vehicle can make when traversing an environment. The sideslip angle is also important to get an estimate of the inertial frame velocity of a vehicle:

$$v^I = \begin{bmatrix} \dot{X}^I \\ \dot{Y}^I \\ \dot{\psi}^I \end{bmatrix} = \begin{bmatrix} v\cos(\beta + \psi) \\ v\sin(\beta + \psi) \\ \dot{\psi} \end{bmatrix}$$

Solutions for teaching and research. Made in Canada.