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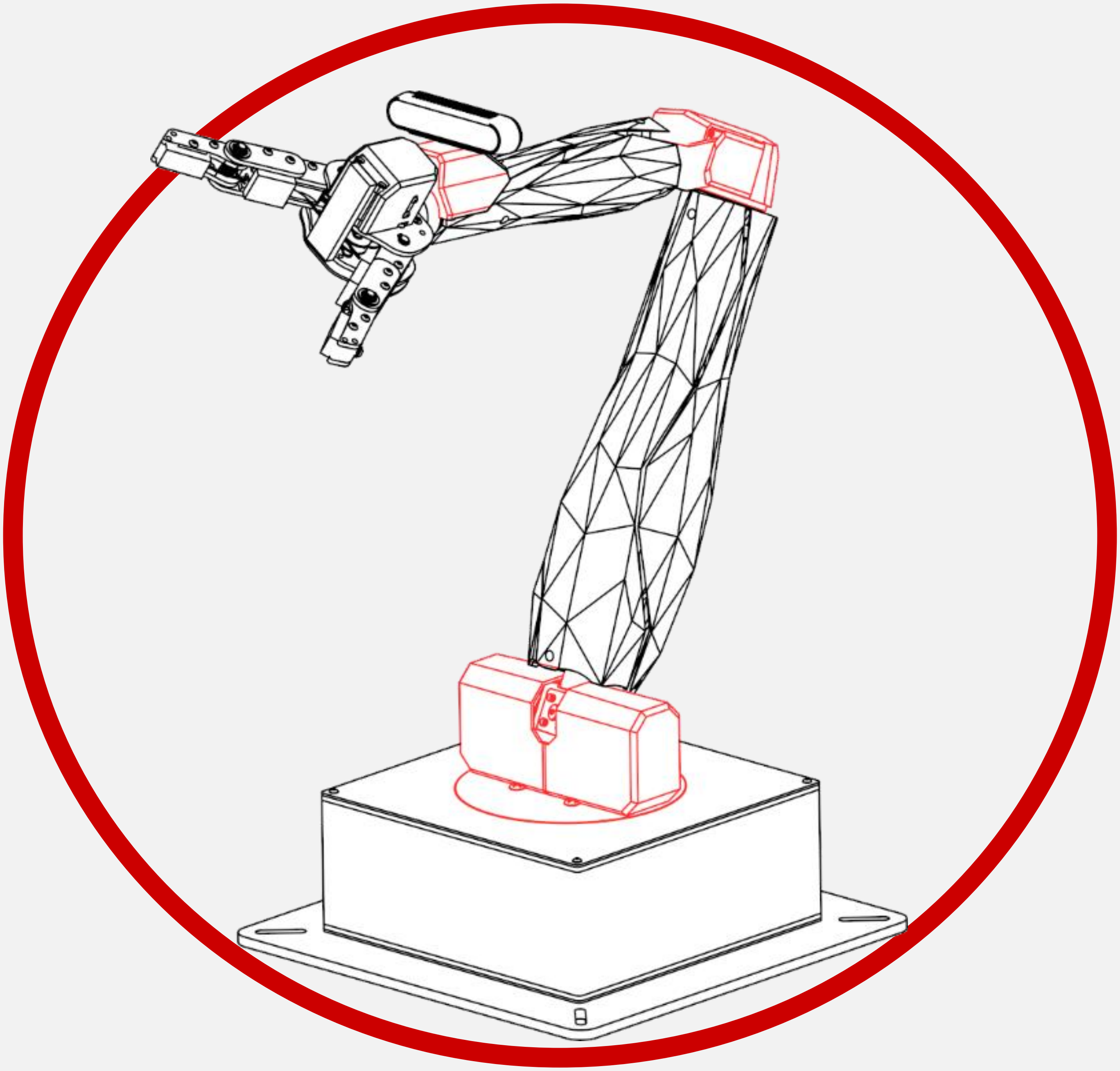

# QArm

## Teach Pendant

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Caution

**This equipment is designed to be used for educational and research purposes and is not intended for use by the public.** The user is responsible for ensuring that the equipment will be used by technically qualified personnel only. Users are responsible for certifying any modifications or additions they make to the default configuration.

QArm – Application Guide

# Teach Pendant

## Why explore Teach Pendants?

A teach pendant is a hardware interface used to first “teach” a set of discrete points to a robotic manipulator, then initiate motion playback where the robot traverses each of the taught points. Teach pendants are commonly used to program robots to perform pick, manipulate and place tasks in an assembly line. Since the desired points that are taught to the robot are Cartesian coordinates of the end-effector, we require a formulation that determines the required joint angles that result in the desired end-effector position. This formulation is referred to as the manipulator’s inverse kinematic model. The purpose of this lab is two-fold. First, you will determine the robot’s inverse kinematics model, then use the model to implement a software-based teach pendant that simulates a robotic assembly process.

## Background

The QArm content contains 5 labs that focus on kinematic manipulation. The first one focuses on learning how to do low level control, workspace identification, lead through control, teach pendant and trajectory generation. This lab focuses on Lead Though is performed for a robotic manipulator.

Prior to starting this lab, please review the following concept reviews (should be located in Documents/Quanser/4_concept_reviews/),

- Concept Review – Forward Position Kinematics

## Getting started

The goal of this lab is to study the process for solving inverse position kinematics based on a series of desired locations specified by an operator.

Before you begin this lab, ensure that the following criteria are met.

- The QArm has been setup and tested. See the QArm Quick Start Guide for details on this step.
- You are familiar with the basics of Simulink. See the [Simulink Onramp]() for more help with getting started with Simulink.

## Inverse Kinematic Formulation

The goal of the inverse kinematics formulation $f_{IPK}$ is to calculate the joint angles from the Cartesian coordinates of the end-effector. In other words, we will develop the formulation

$$\vec{\theta} = f_{IPK}(\vec{p}) \tag{4}$$

### *End-Effector Position*

Begin with the equations describing the end-effector's $p_x$ and $p_y$.

$$\begin{aligned} p_x &= \lambda_2 c_1 c_2 - \lambda_3 c_1 s_{23} = c_1 (\lambda_2 c_2 - \lambda_3 s_{23}) \\ p_y &= \lambda_2 s_1 c_2 - \lambda_3 s_1 s_{23} = s_1 (\lambda_2 c_2 - \lambda_3 s_{23}) \end{aligned} \tag{5}$$

Next, we square and then add the equations describing $p_x$ and $p_y$, giving,

$$\begin{gathered} p_x^2 + p_y^2 = c_1^2 (\lambda_2 c_2 - \lambda_3 s_{23})^2 + s_1^2 (\lambda_2 c_2 - \lambda_3 s_{23})^2 = (\lambda_2 c_2 - \lambda_3 s_{23})^2 \\ \pm\sqrt{p_x^2 + p_y^2} = \lambda_2 c_2 - \lambda_3 s_{23} \end{gathered} \tag{6}$$

Equations 1 and 6 provide the following set of equations,

$$\pm\sqrt{p_x^2 + p_y^2} = \lambda_2 c_2 - \lambda_3 s_{23} \tag{7}$$

$$\lambda_1 - p_z = \lambda_2 s_2 + \lambda_3 c_{23}$$

This resembles the following trigonometric equations,

$$\begin{aligned} A\cos\theta_2 + B\cos(\theta_2+\theta_3) + C\sin(\theta_2+\theta_3) &= D \\ A\sin\theta_2 + B\sin(\theta_2+\theta_3) - C\cos(\theta_2+\theta_3) &= H \end{aligned} \tag{8}$$

Where the following mapping applies,

$$\begin{aligned} A &= \lambda_2 \\ B &= 0 \\ C &= -\lambda_3 \\ D &= \pm\sqrt{p_x^2 + p_y^2} \\ H &= \lambda_1 - p_z \\ F &= \frac{D^2+H^2-A^2-C^2}{2A} \end{aligned} \tag{9}$$

Note that there are two possible values for $D$. Regardless, the solution for $\theta_3$ involves squaring the $D$ term which clears ambiguity. The two possible solutions for $\theta_3$ are,

$$\begin{aligned} \theta_{3,1} &= 2\tan^{-1}\left(\frac{C+\sqrt{C^2+F^2}}{F}\right) = 2\,atan2\left(C+\sqrt{C^2+F^2},\ F\right) \\ \theta_{3,2} &= 2\tan^{-1}\left(\frac{C-\sqrt{C^2+F^2}}{F}\right) = 2\,atan2\left(C-\sqrt{C^2+F^2},\ F\right) \end{aligned} \tag{10}$$

Note that the function $atan2$ takes the y and x components into account separately to provide a solution in the range $[-\pi, \pi]$ as supposed to the $atan$ function, which takes a single input $p_y/p_x$ and only provides solutions in the range $[-\pi/2,\ \pi/2]$.

Before solving for $\theta_2$, some additional mapping is required,

$$\begin{aligned} M &= A + C\sin\theta_3 \\ N &= -C\cos\theta_3 \\ \cos\theta_2 &= \frac{DM+HN}{M^2+N^2} \\ \sin\theta_2 &= \frac{H-N\cos\theta_2}{M} \end{aligned} \tag{11}$$

The possible solutions for $\theta_2$ are calculated using the same equation,

$$\theta_2 = \tan^{-1}\left(\frac{\sin\theta_2}{\cos\theta_2}\right) = atan2\,(\sin\theta_2\ ,\ \cos\theta_2\ ) \tag{12}$$

The pair of terms $M$ and $N$ have 2 possible values due to the two solutions for $\theta_3$. For each of these, the $\cos\theta_2$ term has 2 possible values due to the $D$ term. This results in a total of 4 possible solutions, all found using equation 12. Knowing $\theta_2$ and $\theta_3$, return to equation 5, and take the following ratio

$$\tan\theta_1 = \frac{p_y/\lambda_2 c_2 - \lambda_3 s_{23}}{p_x/\lambda_2 c_2 - \lambda_3 s_{23}} \tag{13}$$

Using the inverse tangent, we can solve for $\theta_1$.

$$\theta_1 = \tan^{-1}\left(\frac{p_y/\lambda_2 c_2 - \lambda_3 s_{23}}{p_x/\lambda_2 c_2 - \lambda_3 s_{23}}\right) = atan2\left(p_y/\lambda_2 c_2 - \lambda_3 s_{23}\ ,\ p_x/\lambda_2 c_2 - \lambda_3 s_{23}\right) \tag{14}$$

Although the term $\lambda_2 c_2 - \lambda_3 s_{23}$ is common in the numerator and denominator of the ratio in Equation 14, note that it would be unwise to simply cancel it out. The sign of this term matters and will result in a unique $\theta_1$ solution for each set of $\theta_2$ and $\theta_3$.

### *End-Effector Orientation*

Although the wrist angle $\theta_4$ is not calculated from the position above, an end-effector orientation may also be provided in the form of a rotation matrix. Knowing the possible values for $\theta_1$, $\theta_2$ and $\theta_3$, one can solve the rotation matrix partially and match terms to derive $\theta_4$.

A user cannot, however, specify an arbitrary position and orientation for the end-effector. There are 6 general degrees of freedom for position and orientation, however, there are only 4 degrees of freedom in the manipulator. Often, the orientation may simply be specified as a wrist angle $\gamma$, and the solution for the wrist joint angle is

$$\theta_4 = \gamma \tag{15}$$

### *Summary*

To summarize, the Inverse Kinematics formulation is given as follows,

$$\theta_3 = 2\,atan2\left(-\lambda_3 \pm \sqrt{{\lambda_3}^2 + \left(\frac{p_x^2 + p_y^2 + (\lambda_1 - p_z)^2 - \lambda_2^2 - \lambda_3^2}{2\,\lambda_2}\right)^2}\ ,\ \frac{p_x^2 + p_y^2 + (\lambda_1 - p_z)^2 - \lambda_2^2 - \lambda_3^2}{2\,\lambda_2}\right)$$

(2 solutions possible) (16)

$$\theta_2 = atan2\left(\frac{\lambda_1 - p_z - \lambda_3\cos\theta_3\left(\frac{\pm(\lambda_2 - \lambda_3\sin\theta_3)\sqrt{p_x^2 + p_y^2} + (\lambda_3\cos\theta_3)(\lambda_1 - p_z)}{(\lambda_2 - \lambda_3\sin\phi_3)^2 + (\lambda_3\cos\theta_3)^2}\right)}{M}, \frac{\pm(\lambda_2 - \lambda_3\sin\theta_3)\sqrt{p_x^2 + p_y^2} + (\lambda_3\cos\theta_3)(\lambda_1 - p_z)}{(\lambda_2 - \lambda_3\sin\theta_3)^2 + (\lambda_3\cos\theta_3)^2}\right.$$

(2 solutions possible for each $\phi_3$)

$$\phi_1 = \tan^{-1}\left(\frac{p_y/\lambda_2\cos\theta_2 - \lambda_3\sin(\theta_2+\theta_3)}{p_x/\lambda_2\cos\theta_2 - \lambda_3\sin(\theta_2+\theta_3)}\right) = atan2\left(p_y/\lambda_2\cos\theta_2 - \lambda_3\sin(\theta_2+\theta_3)\,,\ \ p_x/\lambda_2\cos\theta_2 - \lambda_3\sin(\theta_2+\theta_3)\right)$$

(1 unique solution possible for each $\theta_2$ and $\theta_3$)

$$\theta_4 = \gamma$$

(1 unique solution)

## Teach Pendant Procedure

Once the inverse kinematic model of the manipulator has been formulated, you can proceed to implement the software-based teach pendant. The teach pendant process used in this laboratory involves two steps: (a) learn and (b) follow. In the first step, illustrated in Figure 1, the robot is taught a series of end-effector coordinates as waypoints. For every waypoint, using the inverse kinematic model, the optimal joint configurations are determined and commanded to the robotic arm. These waypoints are also added to a first-in-fist-out (FIDO) queue.

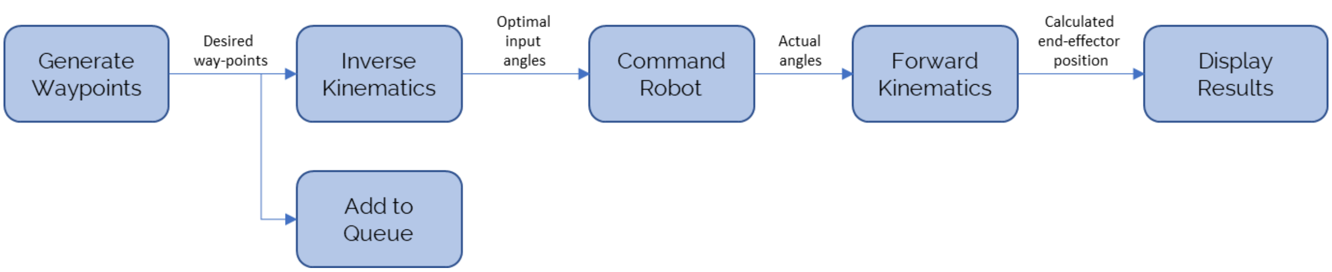


Figure 1: The “learn” procedure of the teach pendant

In the second step, illustrated in Figure 2, the taught waypoints are extracted from the queue and replayed in sequence.


Figure 2: The “follow” procedure of the teach pendant