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Concept Review

Filtering

Why Do We Need Filtering?

Ever listened to a song and tried to just hear one specific instrument? In order to do this your brain has to “filter” out all of the other instruments, or what is referred to in signal processing as “noise”. In a similar way, data read from sensors may have noise due to the environment they are placed in or the inherent design of the sensor. For example, in medicine, if you wanted to measure someones ECG (heart’s electronic signals), there is several noises that would need to be filtered out, from electrode motion artifacts (moving), to myoelectric control (electrical activity of muscles), even powerline interference and more! When measuring signals from motors, vibrations from the spinning motors and propellers will add noise in the accelerometer and gyroscope measurement data. Filters are super important because they allow for the removal of noise above or below a threshold frequency, or a specific frequency to process a specific signal of interest. There are many different filtering methods that exist to remove noise from a signal. This document will cover the four major filters: low-pass, high-pass, band-pass and notch filters.

Low-Pass Filters

Low-pass filters are one of the most commonly used filters. As the name implies, this filter aims to allow noise of frequencies below a certain threshold to pass through, while attenuating (removing) frequencies above the threshold. This threshold is called the cut-off frequency of the filter ω_{co} in radians/s. This behaviour is best described by the filter bode plot (figure 1), which shows low-pass filters ω_c for 10 rad/s, 100 rad/s and 1000 rad/s. All higher frequency components of the signal get attenuated by at least -3dB or around 50%.

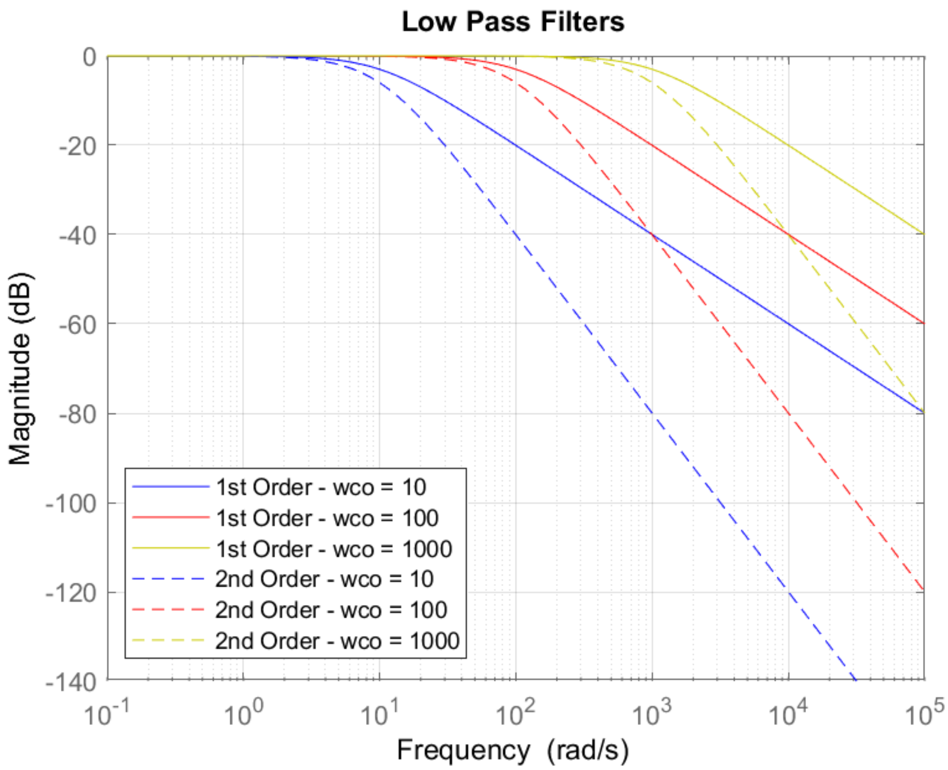

Figure 1: Bode response plot for low-pass filters of various cut-off frequencies

As seen, the magnitude of the signal above the cutoff frequency in each case rapidly decreases at a linear rate. Note that a drop of X dB corresponds to a magnitude reduction by $10^{X/10}$. The magnitude of signal noise below the cutoff frequency is left untouched. The second order filter can be seen to produce effects that are somewhat similar to a first-order low-pass filter. However, it attenuates unwanted frequencies at a faster rate.

The first order transfer function for this filter looks like:

$$H_{LPF}(s) = \frac{\omega_{co}}{s + \omega_{co}} \tag{1}$$

The second order transfer function for this filter looks like:

$$H_{LPF}(s) = \frac{\omega_{co}^2}{s^2 + 2\omega_{co}s + \omega_{co}} \tag{2}$$

Here, ω_{co} represents the cut-off frequency and s is the variable that represents the relationship between the input to a system and its output in the frequency domain.

High-Pass Filters

Contrary to low-pass filters, the high-pass filter aims to allow noise of frequencies above the cut-off frequency to pass through, while attenuating (reducing or removing) frequencies below the cut-off. This behaviour is best described by the filter bode plot (figure 2), which shows three 1st order and three 2nd order high-pass filters with ω_{co} being 10 rad/s, 100 rad/s and 1000 rad/s.

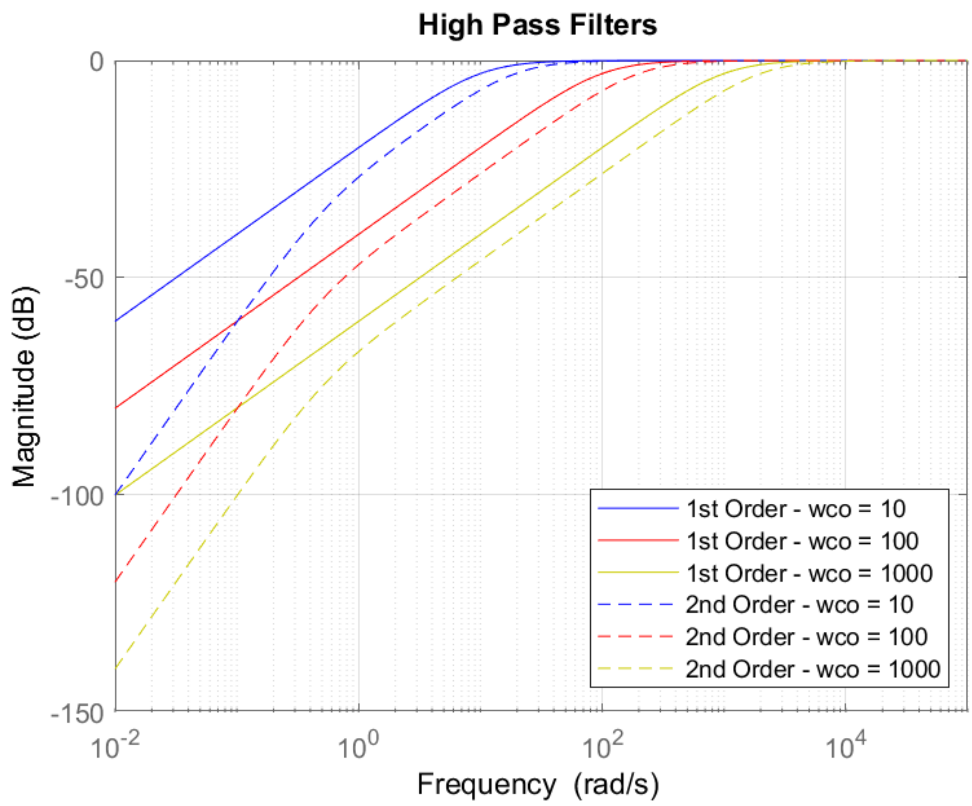

Figure 2: Bode response plot for 1st and 2nd Order high-pass filters of various cut-off frequencies

As you can see, the magnitude of signal noise rapidly increases at a linear rate up to the cutoff frequency. This shows that below the cutoff, the signal is being reduced (filtered). The magnitude of signal noise above the cutoff frequency is left untouched (0 dB). The second order filter can be seen to produce effects that are somewhat similar to a first-order high-pass filter. However, just like with low-pass filters, it attenuates unwanted frequencies at a faster rate. The transfer function of this filter looks like,

$$H_{HPF}(s) = \frac{s}{s + \omega_{co}} \tag{3}$$

The second order transfer function for this filter looks like:

$$H_{HPF}(s) = \frac{s^2}{s^2 + 2\omega_{co}s + \omega_{co}} \tag{4}$$

Here, ω_{co} represents the cut-off frequency and s is the variable that represents the relationship between the input to a system and its output in the frequency domain.

Band-Pass Filters

A band-pass filter allows only a specified range of frequencies, blocking frequencies above or below the range. A band-pass filter can be thought of as a low and high pass filter. This behaviour is best described by the filter bode plot (figure 3), which shows band-pass filters with ω_0 being 10 rad/s, 100 rad/s and 1000 rad/s.

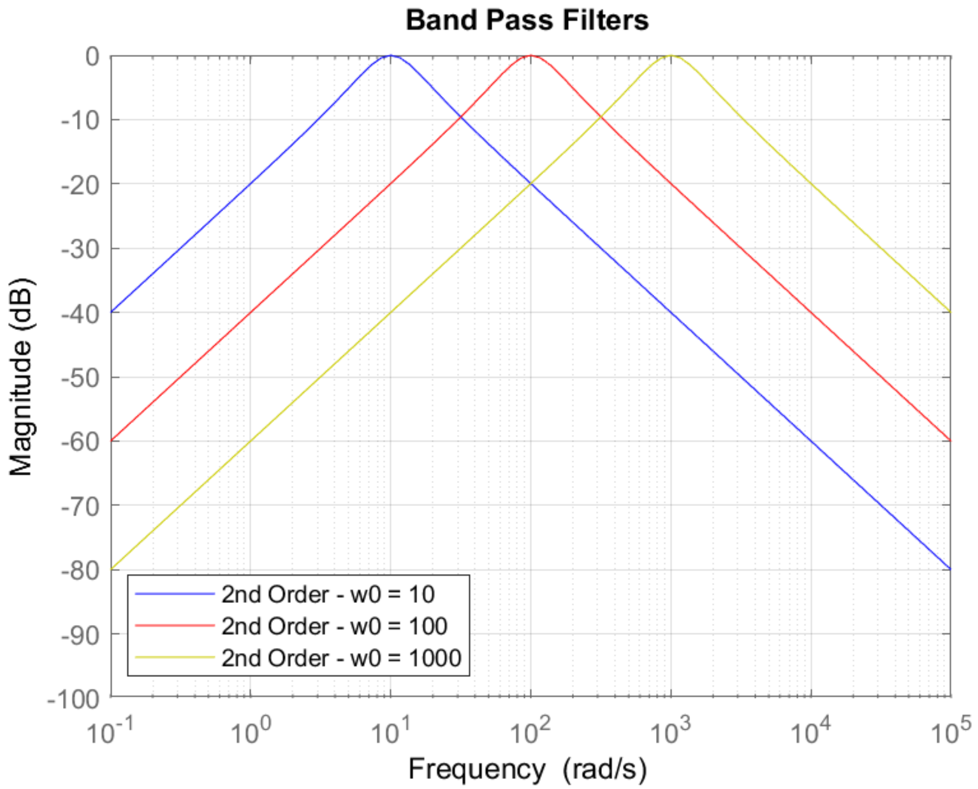

Figure 3: Bode response plot for band-pass filters of various cut-off frequencies

As you can see, the magnitude of the signal rapidly increases at a linear rate up to ½ the band width specified from the center frequency chosen, then rapidly decreases at a linear rate ½ the band width away from the center frequency chosen. This shows that below the cutoff, the signal is being reduced (filtered). The magnitude of signal noise between the two cutoff frequencies (within the band width) is left untouched (0 dB).

The transfer function for a second-order band-pass filter is the following:

$$H_{BPF}(s) = \frac{\frac{\omega_0}{\beta} s}{s^2 + \frac{\omega_0}{\beta} s + \omega_0^2} \tag{3}$$

Here, ω_0 represents the center frequency, β is the band width desired and s is the variable that represents the relationship between the input to a system and its output in the frequency domain.

In practice, no band-pass filter is ideal (or perfect). The filter will not attenuate all frequencies outside the desired frequency range completely. There is typically a region just outside the desired range where frequencies are attenuated, but not removed completely. This is known as filter roll-off and is usually expressed in dB of attenuation per octave or decade of frequency. When designing filters, the goal is typically to make the roll-off as narrow as possible around the desired signal frequency.

Notch Filters

Unlike the band-pass filters which filter everything outside of a band width, notch filters, also known as band stop filters or band rejection filters, attenuate (reduce or remove) all signals within a specified band width. This band width is called the stop band. This behaviour is best described by the filter bode plot (figure 4), which shows notch filters with ω_0 being 10 rad/s, 100 rad/s and 1000 rad/s and a bandwidth of 2 Hz.

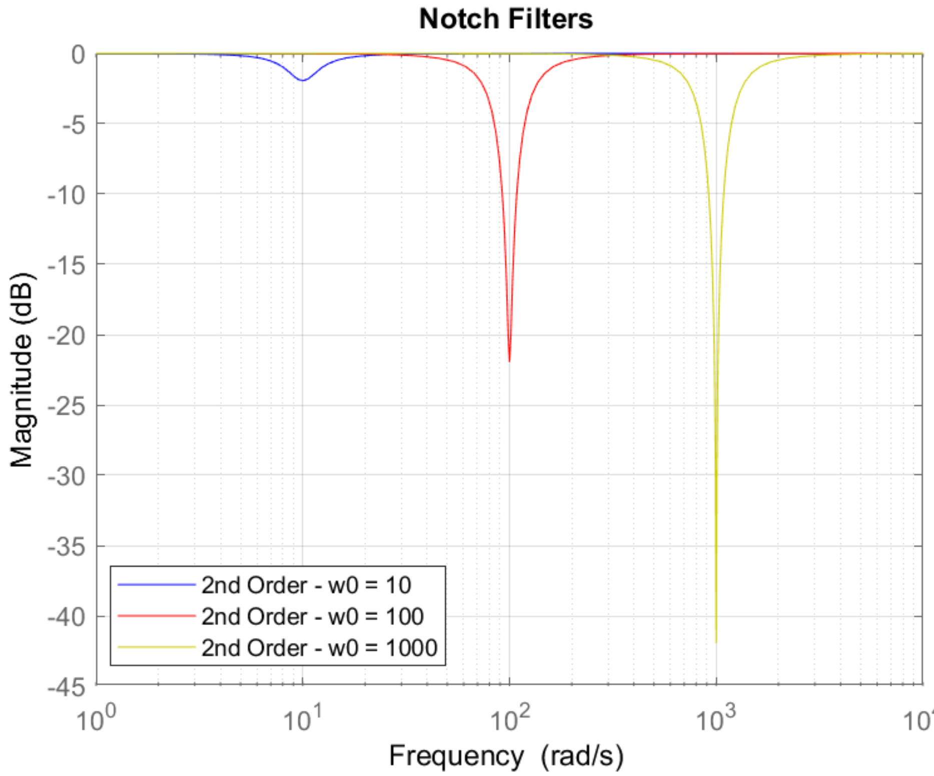

Figure 4: Bode response plot for notch filters of various cut-off frequencies at a bandwidth of 2 Hz.

Since there is two frequencies needed in order to calculate the The transfer function for a second-order notch filter is the following:

$$H_{NF}(s) = \frac{s^2 + 2\beta s + \omega_0^2}{s^2 + \frac{\omega_0}{\beta} s + \omega_0^2} \tag{4}$$

Here, ω_0 represents the center frequency, β is the bandwidth desired to be removed and s is the variable that represents the relationship between the input to a system and its output in the frequency domain.

Complementary Filters

Combining our understanding of low-pass and high-pass filters we can now take a look at complementary filters.

If a noisy signal was sent through a high-pass filter and a low-pass filter with the same cut-off frequency in parallel, and the filtered signals were added together, one would expect to achieve the same signal again. This can be illustrated through the following equation

$$C_{filtered} = H(s)x + G(s)x = \left(\frac{s}{s+\omega_n}\right)x + \left(\frac{\omega_n}{s+\omega_n}\right)x = x$$

The high-pass and low-pass filters with the same cut-off frequency are complementary in nature, a fact that makes complementary filters very useful in fusing sensor data. For more information check out the Concept Review - Sensor Fusion.

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