Homework #3 (TAM 570, Spring 2017)

Due on 4/10/17

Consider the one-dimensional Euler equations

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u)}{\partial x} = 0, \tag{1}$$

$$\frac{\partial (\rho u)}{\partial t} + \frac{\partial (\rho u^2 + p)}{\partial x} = 0, \tag{2}$$

$$\frac{\partial E}{\partial t} + \frac{\partial ((E + p)u)}{\partial x} = 0, \tag{3}$$

in the periodic domain $0 \le x \le 2$ where $E = \frac{p}{(\gamma-1)} + \frac{1}{2}\rho u^2$ denotes the total energy, $p = \rho R T$ is the gas equation of state and $c = \sqrt{\gamma R T}$ denotes the speed of sound. The initial conditions are given by $u = u_o \sin(\pi x)$, $T = (1 + \frac{\gamma-1}{2}\frac{u}{c_o})^2$ with parameters $c_o = \sqrt{\gamma R T_o} = 1$, $u_o = 0.5$, $\gamma = 1.4$, and $T_o = T(u = 0)$. Assume the initial condition is also an isentrope.

- Determine R by requiring that

$$c_o^2 = \frac{\gamma R}{2} \int_0^2 \left(1 + \frac{\gamma - 1}{2} M_o \sin(\pi x)\right)^2 dx, \tag{4}$$

  where $M_o = u_o / c_o$.

- Develop a Galerkin and pseudo-spectral Fourier formulation of the problem. Justify your choice of time integration method.

- Integrate the equations until a discontinuity is formed (shock) and demonstrate convergence of the method up to that time.

- Estimate the time of formation of the shock.

- What happens to the numerical solution after the formation of the shock?