Initialize: Set $\vec{u}_i = \vec{e}_i \quad i = 1, \dots, N$

Set $P = \vec{P}_0$

Starting point.

N times.

For $i = 1, \dots, N$

$\vec{P}_i$ is defined through

$\min_{\lambda} f(\vec{P}_{i-1} + \lambda \vec{u}_i)$

For $i = 1, \dots, N-1$ set $\vec{u}_{i+1} \to \vec{u}_i$

Set $\vec{P}_N = \vec{P}_0 + \vec{u}_N$.

Go from $\vec{P}_N$ in direction

$\vec{u}_N$ and call minimum $\vec{P}_0$

If f is a quadratic form, then the method converges in $N(N+1)$ one-dimensional minimizations.

Each iteration of the Powell algorithm produces a new conjugate direction

$$\left.\begin{array}{ll} 1 & \vec{u}_N \\ 2 & \vec{u}_N, \vec{u}_{N-1} \\ 3 & \vec{u}_N, \vec{u}_{N-1}, \vec{u}_{N-2} \\ \vdots & \vdots \\ N & \underbrace{\vec{u}_N, \ldots, \vec{u}_1}_{\text{Conjugate vectors}} \end{array}\right\} \text{Each iteration does } N+1 \text{ 1D minimization}$$

When f is not a quadratic form: Generation of $\vec{u}_i$ - direction may produce linear dependence:

One way to repair this: Set

$\vec{u}_i = \vec{e}_i$ every N iterations of the Powell algorithm.

Linear programming

Definition of problem (on standard form)

Maximize object function

$$z = a x \qquad = a_{01} x_1 + a_{02} x_2 + \cdots + a_{0N} x_N$$

N-dimensional vectors

$$a = (a_{01}, \ldots, a_{0N})$$

$$x = (x_1, \ldots, x_N)$$

Given the primary constraints

$$x_i \geq 0, \qquad i = 1, \ldots, N$$

And the additional constraints

- $A_i \cdot x \le b_i \quad , \quad i = 1, \dots, m_1$

N-dimensional vector

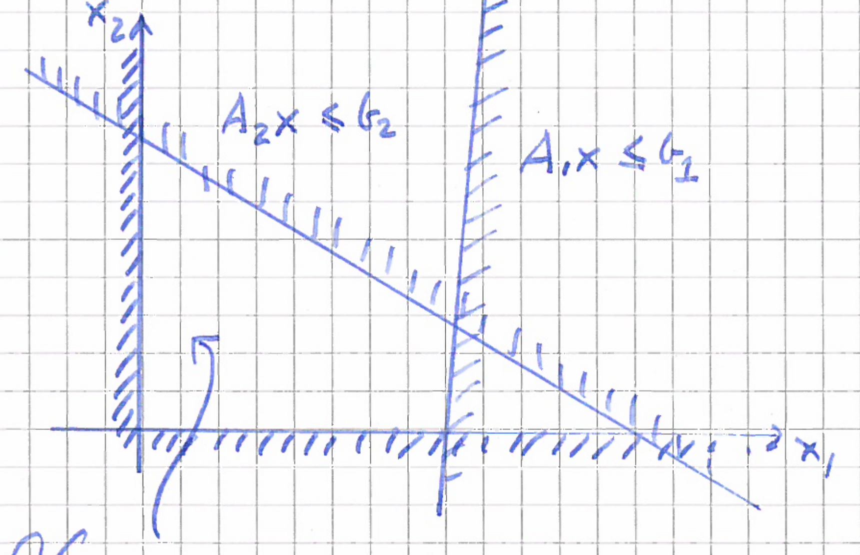
$$A_i = (a_{i1}, \dots, a_{iN})$$

- $A_i\, x \ge b_i \ge 0 \quad , \quad i = m_1+1, \dots, m_1+m_2$

- $A_i\, x = b_i \ge 0 \quad , \quad i = m_1+m_2+1, \dots, m_1+m_2+m_3 = M$

Feasible region: Those x that fulfill the constraints

$N=2$



These are the feasible vectors.

Example:

$$Z = x_1 + x_2 + x_3 - \frac{1}{4} x_4$$

$$\left.\begin{aligned} x_1 + 2x_3 &\leq 740 \\ 2x_2 - 7x_4 &\leq 0 \end{aligned}\right\} m_1 = 2$$

$$x_2 - x_3 + 2x_4 \geq \frac{1}{2} \quad \} \; m_2 = 1$$

$$x_1 + x_2 + x_3 + x_4 = 9 \quad \} \; m_3 = 1$$

$$M = m_1 + m_2 + m_3 = 4.$$

Fundamental theorem in linear programming.

Feasible vector: fulfills all constraints

Feasible basic vector:

Fulfills N constraints as equalities and fulfills all constraints

If $N > M$, then a feasible basic vector has at least $N-M$ of its components equal to zero since we need to pick from the primary constraints.

Theorem:

If an optimal feasible vector exists, then there is a feasible basic vector that is optimal.

This theorem reduces the optimization problem to identifying which N constraints among the $N+M$ that should be satisfied by the optimal feasible basic vector.

Simplex algorithm

Normal form :

$$x_i \geq 0 \ , \quad i = 1, \dots, N$$

$$\sum_{i=1}^{N} a_{ki} x_i = b_k \qquad k = 1, \dots, M$$

All equalities

Restricted normal form:

At least one $a_{ki} \geq 0$

for each k.

Corresponding x_i appears

only in the constraint.

Keeps these on the left hand side of

the constraints.

Move everything else on the right

hand side of 2. Normalize.

Example:

$$z = 2x_2 - 4x_3 \quad \leftarrow \text{ No } x_1 \text{ and } x_4 \text{ here.}$$

$$x_1 = 2 - 6x_2 + x_3$$

$$x_4 = 8 + 3x_2 - 4x_3$$

$$\begin{pmatrix} x_1 \\ x_4 \end{pmatrix} \qquad \begin{pmatrix} x_2 \\ x_3 \end{pmatrix}$$

Right hand variables.

left hand variables

z is written using only right hand variables.

→ No constraints except from right hand variables always must be positive.

Can always find a feasible basic vector by setting right hand variables to zero.

The <u>simplex method</u> is a method to systematic interchange left and right hand variables.

Our example:

$$\underbrace{\begin{matrix} x_2 = 0 \\ x_3 = 0 \end{matrix}}_{r.h.} \Longrightarrow \underbrace{\begin{matrix} x_1 = 2 \\ x_4 = 8 \end{matrix}}_{l.h.} \qquad z = 0$$

$$\left.\begin{matrix} x_1 = 2 \\ x_2 = 0 \\ x_3 = 0 \\ x_4 = 8 \end{matrix}\right\}$$ Feasible basic vector.

Using a <u>tableau</u>:

		x_2	x_3
z	0	2	-4
x_1	2	-6	1
x_4	8	3	-4

left hand variables

right hand variables.

(1) Increase x_2, keep $x_3 = 0 \Rightarrow$

<u>z increases.</u>

Keep $x_2 = 0$, increase $x_3 \Rightarrow$

<u>z decreases</u>

Only columns with z-row component which is positive, is of interest.

(2) How much can we increase x_2 before x_1 or x_4 become negative?

$x_1 = 2 - 6x_2 = 0 \Rightarrow x_2 = \frac{1}{3}$

In general, search for <u>negative components.</u>

Components with the largest negative ratio between right hand column and constant column is the <u>pivot element.</u>

In our example:

$x_1 \leftrightarrows x_2$

$x_1 = 2 - 6x_2 + x_3 \rightarrow x_2 = \frac{1}{3} - \frac{1}{6}x_1 + \frac{1}{6}x_3$

$z = 2x_2 - 4x_3 = \frac{2}{3} - \frac{1}{3}x_1 - \frac{11}{3}x_3$

$x_4 = 8 + 3x_2 - 4x_3 = 9 - \frac{1}{2}x_1 - \frac{7}{2}x_3$

New tableau:

		x_1	x_3
z	$2/3$	$-1/3$	$-11/3$
x_2	$1/3$	$-1/6$	$1/6$
x_4	9	$-1/2$	$-7/2$

Should we continue to pivot?

All are negative $\Rightarrow$ No!

We are done!

Solution is: (Reading from constant column)

$x_1 = 0$

$x_2 = 1/3$

$z = 2/3$

$x_3 = 0$

$x_4 = 9$