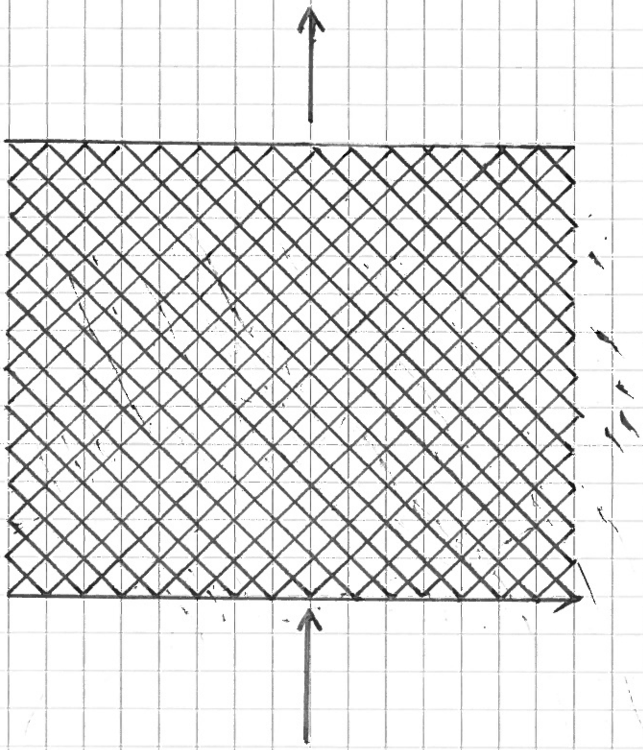


Relation between flow in tube and pressure difference across it:

$$v = M \Delta p$$

velocity — Mobility — pressure difference

Assume now a distribution of mobilities, $M \to M_{ij}$

Mobility of tube between neighbouring nodes i and j:

$$v_{ij} = M_{ij} (p_i - p_j)$$

Solve the Kirchhoff equations (flow into each node equals flow out of each node)

This gives all the v_{ij}.

Imagine now a passive tracer is added to the inlet of the flow.

It takes a time $t_{ij} \propto \frac{1}{v_{ij}}$ for the tracer to cross tube ij.

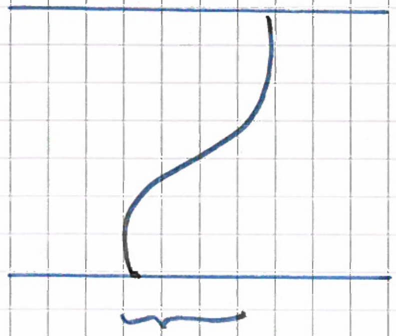
$$\boxed{\text{When do we first see the tracer at the outlet when it was added at the inlet at } t=0\text{?}}$$

This is an optimization problem!

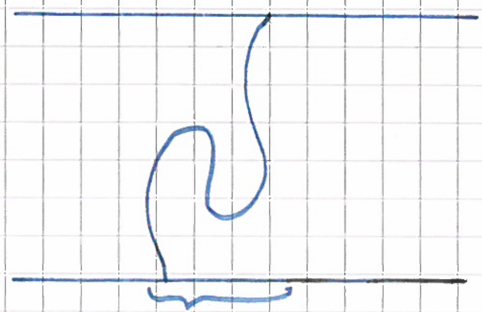
$$t_{min} = \min_{\substack{\text{all}\\ \text{paths}\\ P}} \sum_{ij \in P}' t_{ij}$$

We have to search through all paths crossing system to find the optimal path.

Assuming no turns,

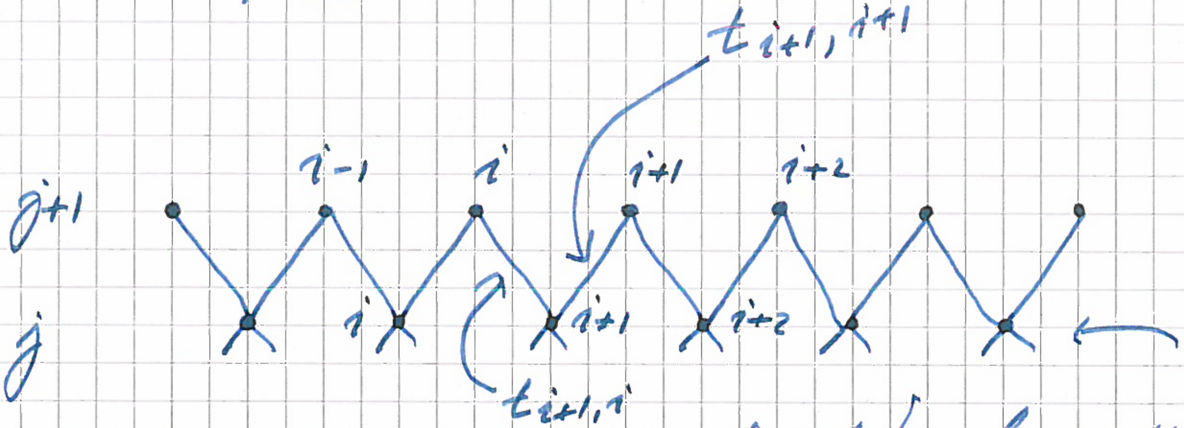


no turns



turn

We may use a <u>transfer matrix technique.</u>



We know the optimal paths at level j.

$t_{min}(i,j)$
↓
We update

$$\boxed{t_{min}(i, j+1) = \min\left(t_{min}(i,j)+t_{i,i},\ t_{min}(i+1,j)+t_{i+1,i}\right).}$$

To work out the optimal path itself:
Work backwards through system!

Generalizing to paths with turns:

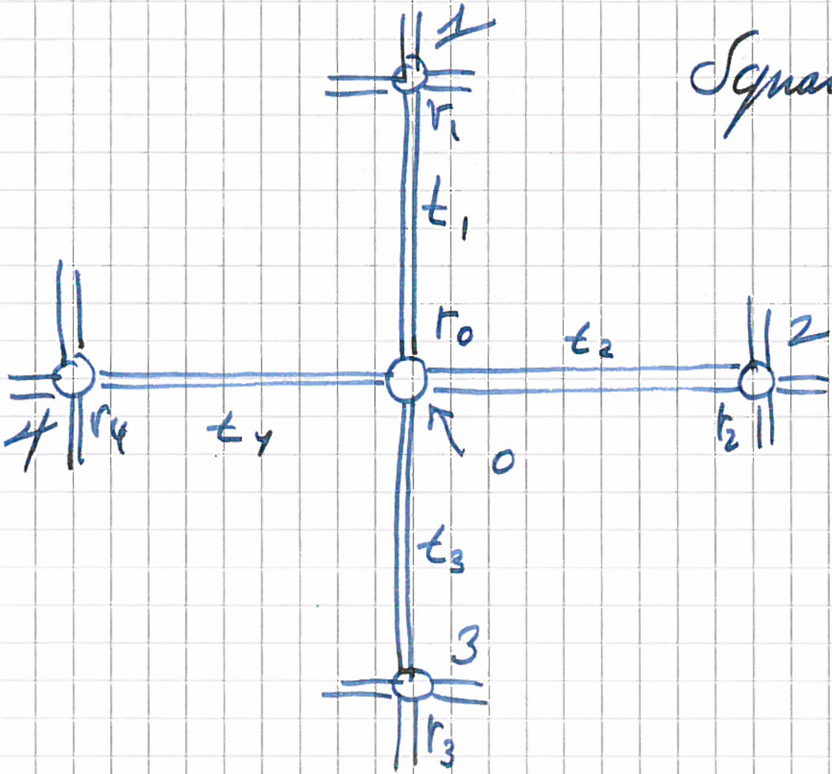
Hansen, Kertész,
Phys. Rev. Lett.
93, 040601 (2004).

A variant of the

Bellman-Ford algorithm.

This algorithm finds the optimal path between any two points in any network topology.

Square lattice.



Initialize: $r_i(0) = 0$

Then: $r_0(1) = \min(t_1 + r_1(0),$
$t_2 + r_2(0),\ t_3 + r_3(0)$
$t_4 + r_4(0))$

Repeat for all nodes.

If we want to find optimal path between node 0 and some other node N, set $r_N(0) = -1000000$

The optimal paths will then be attracted by this node and get stuck at it.

(Visualize this for yourselves.)

Some Statistical Physics

Alternative ways to sample configuration space to calculate the partition function

$$Z = \sum_{\text{configurations}} e^{-H(\text{configuration})/T}.$$

E.g. the Ising spin system.