TFY4235/FY8904 Computational Physics, Spring 2013

Solution Set 12

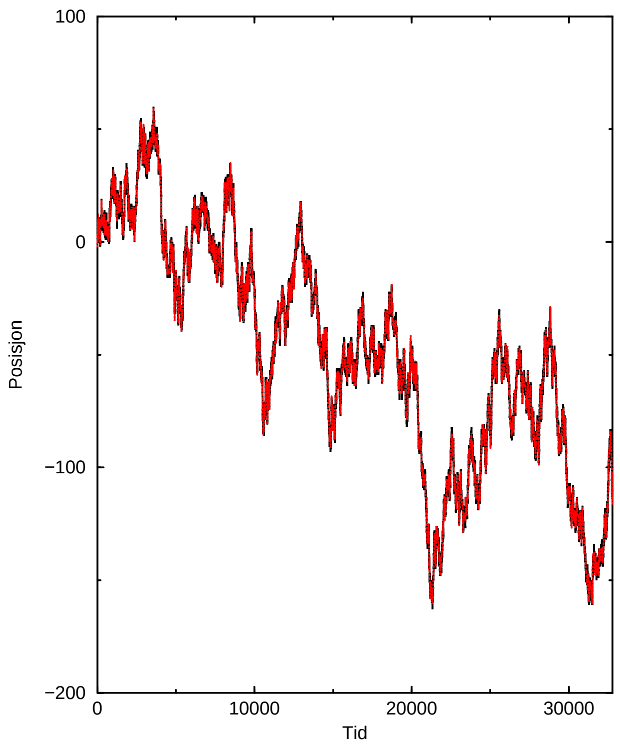
Problem 1.

I generate here a random walker consisting of 2^{15} steps. This I wavelet transform. I find the maximum absolute value of the wavelet coefficients, `amax`.
Filtering of data consists in setting wavelet coefficients whose absolute value is less than `filt*amax` where `filt` is between zero and one, to zero. I then transform the signal back into real space and compare the filtered and unfiltered signals. Here is the program:

```
program wavcom
parameter(n=32768,filt=0.001)
dimension a(n),b(n)
ibm=5251
do i=1,1000
ibm=ibm*16807
enddo
rinv=0.5/(2.**31-1.)
a(1)=0.
b(1)=0.
do i=2,n
ibm=ibm*16807
a(i)=a(i-1)+1.-2.*int(1.+ibm*rinv)
b(i)=a(i)
enddo
isign=1
call wt1(a,n,isign)
amax=-1.e+8
do i=1,n
amax=max(amax,abs(a(i)))
adel=amax*filt
enddo
do i=1,n
if(abs(a(i)).lt.adel) a(i)=0.
enddo
isign=-1
call wt1(a,n,isign)
open(unit=1,file='wavcom.dat',status='unknown')
do i=1,n
write(1,*) i,b(i),a(i)
enddo
close(1)
```

```
end
SUBROUTINE wt1(a,n,isign)
INTEGER isign,n
REAL a(n)
EXTERNAL wtstep
CU USES wtstep
INTEGER nn
if (n.lt.4) return
if (isign.ge.0) then
nn=n
1 if (nn.ge.4) then
call daub4(a,nn,isign)
nn=nn/2
goto 1
endif
else
nn=4
2 if (nn.le.n) then
call daub4(a,nn,isign)
nn=nn*2
goto 2
endif
endif
return
END
SUBROUTINE daub4(a,n,isign)
INTEGER n,isign,NMAX
REAL a(n),C3,C2,C1,C0
PARAMETER (C0=0.4829629131445341,C1=0.8365163037378079,
*C2=0.2241438680420134,C3=-0.1294095225512604,NMAX=32768)
REAL wksp(NMAX)
INTEGER nh,nh1,i,j
if(n.lt.4)return
if(n.gt.NMAX) pause 'wksp too small in daub4'
nh=n/2
nh1=nh+1
if (isign.ge.0) then
i=1
do 11 j=1,n-3,2
wksp(i)=C0*a(j)+C1*a(j+1)+C2*a(j+2)+C3*a(j+3)
wksp(i+nh)=C3*a(j)-C2*a(j+1)+C1*a(j+2)-C0*a(j+3)
i=i+1
11 continue
wksp(i)=C0*a(n-1)+C1*a(n)+C2*a(1)+C3*a(2)
wksp(i+nh)=C3*a(n-1)-C2*a(n)+C1*a(1)-C0*a(2)
else
wksp(1)=C2*a(nh)+C1*a(n)+C0*a(1)+C3*a(nh1)
wksp(2)=C3*a(nh)-C0*a(n)+C1*a(1)-C2*a(nh1)
j=3
do 12 i=1,nh-1
wksp(j)=C2*a(i)+C1*a(i+nh)+C0*a(i+1)+C3*a(i+nh1)
wksp(j+1)=C3*a(i)-C0*a(i+nh)+C1*a(i+1)-C2*a(i+nh1)
j=j+2
12 continue
endif
do 13 i=1,n
a(i)=wksp(i)
13 continue
return
END
```

I have used the wavelet routine from *Numerical Recipes.*
Here is the result:



After removing all wavelet coefficients with absolute value less than 0.001 of `amax` I get the figure above. The (black) unbroken curve is the original signal and the (red) broken curve is the signal after filtering. Only 3% of the original wavelet coefficients have survived the filtering. This gives a compression ratio of 97%!

Problem 2.

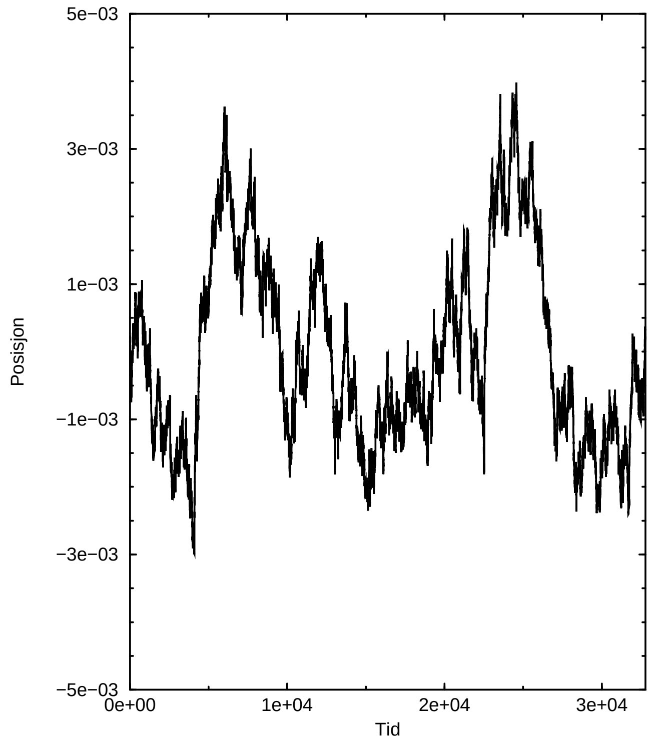
The key to using wavelets to generate random walks lays in the scaling properties of the wavelet coefficients. If a curve $x(t)$ with statistical properties $\langle x\rangle = 0$ and $\langle x^2\rangle^{1/2} \propto \sqrt{t}$, the wavelet coefficients will scale as

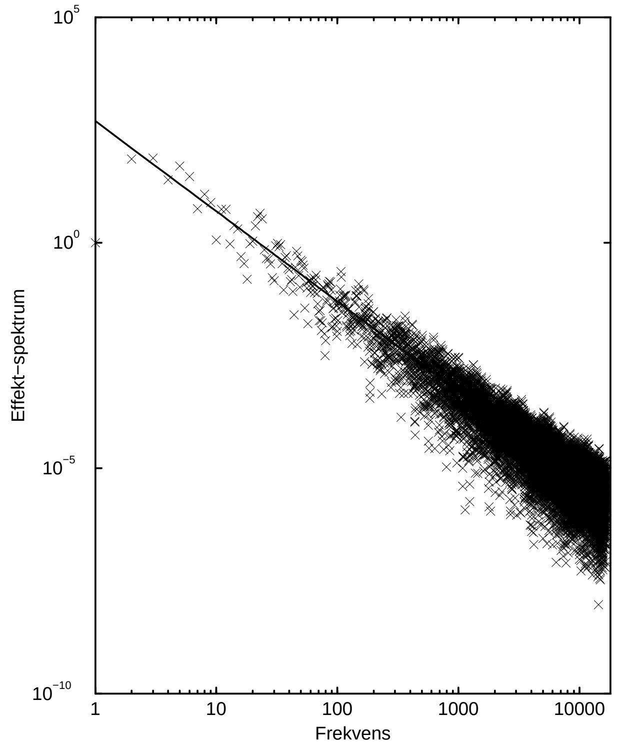
$$A_{a,\lambda b} = \lambda A_{a,b} \tag{1}$$

where a is position and b is scale — see Simonsen et al. Phys. Rev. E **58**, 2779–2787 (1998). This means that $A_{a,b} \propto b$. furthermore, there are no correlations between wavelet coefficients with same b but different a. Hence, one generates random wavelet coefficients drawn from a flat distribution centered around zero and width b. We then wavelet transform the signal "backwards" into real space — and the random walk has been generated. Here is the program:

```
program wavspe
parameter(npk=32768,np2=npk/2)
dimension b(npk),c(2,npk),p(np2)
ibm=4711
do i=1,1000
ibm=ibm*16807
enddo
rinv=0.5/(2.**31-1.)
c Genererer virrevandringen
k=1
amp=1.
1600 continue
if(2**k.eq.npk) goto 1700
amp=amp*0.5
do i=2**k+1,2**(k+1)
ibm=ibm*16807
ran=ibm*rinv
b(i)=ibm*rinv*amp
enddo
k=k+1
goto 1600
1700 continue
```

```
call wt1(b,npk,-1)
open(unit=1,file='wavsp1.dat',status='unknown')
do i=1,npk
write(1,*) i,b(i)
c(1,i)=b(i)
c(2,i)=0.
enddo
close(1)
c Analyserer virrevandringen
call four1(c,npk,1)
do i=1,np2
p(i)=0.5*(abs(c(1,i)*c(2,i))+abs(c(1,npk+1-i)*c(2,npk+1-i)))
enddo
open(unit=2,file='wavsp2.dat',status='unknown')
do i=1,np2
write(2,*) i,p(i)
enddo
close(2)
end
SUBROUTINE wt1(a,n,isign)
INTEGER isign,n
REAL a(n)
EXTERNAL wtstep
INTEGER nn
if (n.lt.4) return
if (isign.ge.0) then
nn=n
1 if (nn.ge.4) then
call daub4(a,nn,isign)
nn=nn/2
goto 1
endif
else
nn=4
2 if (nn.le.n) then
call daub4(a,nn,isign)
nn=nn*2
goto 2
endif
endif
return
END
```





The random walk that this program generates looks as follows:

We calculate the power spectrum (in the program above). The power spectrum scales as

$$P(f) \propto \frac{1}{f^2}\ . \tag{2}$$

This is true for all free random walkers (why?). I have plotted the power spectrum from the random walk on log-log scale in the figure the bottom of the previous page. I have in this figure added a line following Eq. (2).