Neural Network Verification With Vehicle: Chapter 2 - Getting Started

VeTSS Summer School'23

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- ... introduce main building blocks of **vehicle** as a programming language

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- ... introduce main building blocks of **vehicle** as a programming language
- ... get to practice working with **vehicle**
- ... use the famous ACAS Xu benchmark to show **vehicle**'s work flow – from specification to verification
- ... identify PL problems that are resolved by **vehicle**

Recap: four PL problems

- I^O Interoperability – properties are not portable between training/counter-example search/ verification.
- I^P Interpretability – code is not easy to understand.
- $I^\int$ Integration – properties of networks cannot be linked to larger control system properties.
- E^G Embedding gap – little support for translation between problem space and input space.

Recap: ACAS Xu

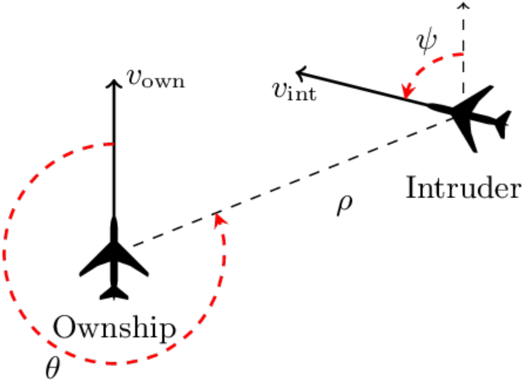
A collision avoidance system for unmanned autonomous aircraft.

Inputs:

- Distance to intruder, ρ
- Angle to intruder, θ
- Intruder heading, φ
- Speed, v_{own}
- Intruder speed, v_{int}

Outputs:

- Clear of conflict
- Strong left
- Weak left
- Weak right
- Strong right



ACAS Xu

Definition (ACAS Xu: Property 3)

If the intruder is directly ahead and is moving towards the ownship, the score for COC will not be minimal.

$$
\begin{aligned}
&1500 \leq \rho \leq 1800 \ \wedge \\
&-0.06 \leq \theta \leq 0.06 \ \wedge \\
&\psi \geq 3.10 \ \wedge \\
&v_{own} \geq 980 \ \wedge \\
&v_{int} \geq 960 \ \Rightarrow \\
&\text{the score for COC} \neq 0
\end{aligned}
$$

Types

Let us build the ACAS Xu specification.
We start with types of input and output vectors, as well as types of ACAS Xu networks

```
type InputVector = Vector Rat 5
type OutputVector = Vector Rat 5

@network
acasXu : InputVector -> OutputVector
```

The Vector type represents a mathematical vector, or in programming terms can be thought of as a fixed-length array.

Values

Types for values are automatically inferred by **vehicle**. For example, we can declare the number π and its type will be inferred as rational:

```
pi = 3.141592
```

Working with vectors

- some input or output pre-processing maybe expected when defining a neural network.

Example

It is assumed that the ACAS Xu inputs and outputs are normalised, i.e. the network does not work directly with units like m/s. However, the specifications we want to write should ideally concern the original units.

Working with vectors

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Example

It is assumed that the ACAS Xu inputs and outputs are normalised, i.e. the network does not work directly with units like m/s. However, the specifications we want to write should ideally concern the original units.

- This is an instance of "problem space / input space mismatch"
- ... that is very common in neural net verification
- Being able to reason about problem space (alongside the input space) is a feature that distinguishes **vehicle** from majority of the mainstream neural network verifiers

Vector normalisation

For clarity, we define a new type synonym for unnormalised input vectors which are in the problem space.

```
type UnnormalisedInputVector = Vector Rat 5
```

Next we define the range of the inputs that the network is designed to work over.

```
minimumInputValues : UnnormalisedInputVector
minimumInputValues = [0.0, -pi , -pi , 100.0, 0.0]

maximumInputValues : UnnormalisedInputVector
maximumInputValues = [60261.0, pi, pi, 1200.0, 1200.0]

meanScalingValues : UnnormalisedInputVector
meanScalingValues = [19791.091, 0.0, 0.0, 650.0, 600.0]
```

Vector manipulation

An alternative method to vector definition is to use the ‘foreach‘ constructor, which is used to provide a value for each ‘index i‘.

```
minimumInputValues : UnnormalisedInputVector
minimumInputValues = foreach i . 0
```

Let us see how ‘foreach‘ works with vector indexing.
We can now define the normalisation function that takes an input vector and returns the unnormalised version.

```
normalise : UnnormalisedInputVector -> InputVector
normalise x = foreach i .
  (x ! i - meanScalingValues ! i) / (maximumInputValues ! i)
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... our first acquaintance with functions!

Functions and types


```
<name> : <type>
<name> [<args>] = <expr>
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validInput : UnnormalisedInputVector -> Bool
validInput x = forall i .
  minimumInputValues ! i <= x ! i <= maximumInputValues ! i
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Our first acquaintance with predicates and quantifiers!

One of the main advantages of **vehicle** is that it can be used to state and prove specifications that describe the network's behaviour over an infinite set of values.

Functions and types

Function Composition: Exercise

What are the types of functions ‘acasXu‘ and ‘normalise‘:

```
normAcasXu : UnnormalisedInputVector -> OutputVector
normAcasXu x = acasXu (normalise x)
```

Pre-defined functions and predicates

We have already used:

`*`
`/`
`!`
`<=`

Exercise

What do they stand for?

Lets verify ACAS Xu!

```
distanceToIntruder = 0    -- measured in metres
angleToIntruder    = 1    -- measured in radians
intruderHeading    = 2    -- measured in radians
speed              = 3    -- measured in metres/second
intruderSpeed      = 4    -- measured in meters/second

clearOfConflict = 0
weakLeft        = 1
weakRight       = 2
strongLeft      = 3
strongRight     = 4
```

The fact that all vector types come annotated with their size means that it is impossible to mess up indexing into vectors, e.g. if you changed ‘distanceToIntruder = 0‘ to ‘distanceToIntruder = 5‘ the specification would fail to type-check.

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```
directlyAhead : UnnormalisedInputVector -> Bool
directlyAhead x =
  1500  <= x ! distanceToIntruder <= 1800 and
  -0.06 <= x ! angleToIntruder    <= 0.06
```

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```

Exercise!

1. Can you identify whether the specification is written in terms of input space or problem space? How do you know?
2. Can you spot another pre-defined **vehicle** function? What is it?

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movingTowards : UnnormalisedInputVector -> Bool
movingTowards x =
  x ! intruderHeading >= 3.10  and
  x ! speed           >= 980   and
  x ! intruderSpeed   >= 960
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```
minimalScore : Index 5 -> UnnormalisedInputVector -> Bool
minimalScore i x =
   forall j . i != j => normAcasXu x ! i < normAcasXu x ! j
```

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1. What kind of domain ‘forall‘ ranges over? Is it finite or infinite?

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```
@property
property3 : Bool
property3 = forall x . validInput x and
                       directlyAhead x and
                       movingTowards x =>
                       not (minimalScore clearOfConflict x)
```

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property3 : Bool
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Exercise!

1. Can you guess the purpose of the syntax

   ```
   @property
   ```

   ?

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How to run Vehicle

Checklist

1. a verifier installed (Marabou);
2. the actual network is supplied in an ONNX format
3. **vehicle** is installed.

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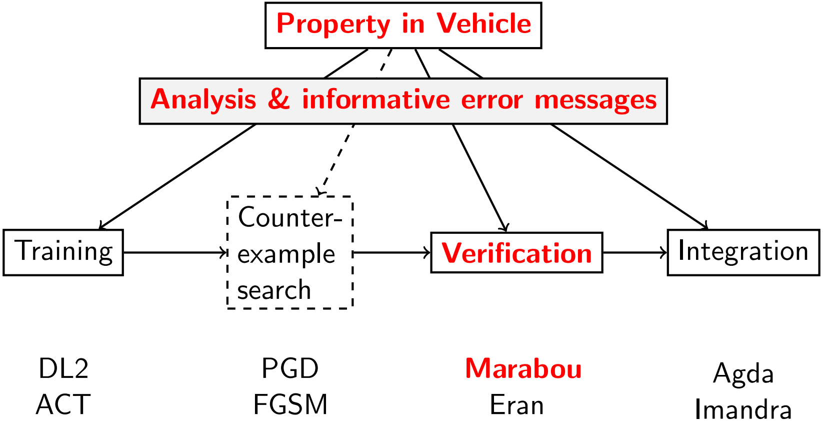
```
vehicle verify \
  --specification acasXu.vcl \
  --verifier Marabou \
  --network acasXu:acasXu_1_7.onnx \
  --property property3

Verifying properties:
property3 [========================================] 1/1 queries
  result:  counterexample found
  x: [1799.9886669999978, 1.9509286320000003e-2
```

Vehicle ...

the part that we have seen



Concluding Exercise

Which of the four PL problems we addressed?

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Harder Exercise: ACAS Xu Property 1

ACAS Xu Property 1 gives an idea how the *embedding gap* can arise not only when we reason about inputs, but also the outputs of networks!

- Can you formalise Property 1 in Vehicle?
- Can you spot the instance of the embedding gap, this time concerning the network's output?

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Definition (ACAS Xu: Property 1)

If the intruder is distant and is significantly slower than the ownship, the score of a COC advisory will always be below a certain fixed threshold:

$$(\rho \geq 55947.691) \wedge (v_{own} \geq 1145) \wedge (v_{int} \leq 60)$$
$$\Rightarrow \text{the score for COC is at most1500}$$

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Next Lecture: ϵ-ball Robustness


```
type Image = Tensor Rat [28, 28]
type Label = Index 10
validImage : Image -> Bool
validImage x = forall i j . 0 <= x ! i ! j <= 1

@network
classifier : Image -> Vector Rat 10

advises : Image -> Label -> Bool
advises x i = forall j . j != i => classifier x ! i > classifier x ! j

@parameter
epsilon : Rat

boundedByEpsilon : Image -> Bool
boundedByEpsilon x = forall i j . -epsilon <= x ! i ! j <= epsilon

robustAround : Image -> Label -> Bool
robustAround image label = forall pertubation .
  let perturbedImage = image - pertubation in
  boundedByEpsilon pertubation and validImage perturbedImage =>
    advises perturbedImage label

@dataset
trainingImages : Vector Image n
```