

VEHME: A Vision-Language Model For Evaluating Handwritten Mathematics Expressions

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Abstract

Automatically assessing handwritten mathematical solutions is an important problem in educational technology with practical applications, but it remains a significant challenge due to the diverse formats, unstructured layouts, and symbolic complexity of student work. To address this challenge, we introduce VEHME—a Vision-Language Model for Evaluating Handwritten Mathematics Expressions—designed to assess open-form handwritten math responses with high accuracy and interpretable reasoning traces. VEHME integrates a two-phase training pipeline: (i) supervised fine-tuning using structured reasoning data, and (ii) reinforcement learning that aligns model outputs with multi-dimensional grading objectives, including correctness, reasoning depth, and error localization. To enhance spatial understanding, we propose an Expression-Aware Visual Prompting Module, trained on our synthesized multi-line math expressions dataset to robustly guide attention in visually heterogeneous inputs. Evaluated on AIHub and FERMAT datasets, VEHME achieves state-of-the-art performance among open-source models and approaches the accuracy of proprietary systems, demonstrating its potential as a scalable and accessible tool for automated math assessment. Our training and experiment code is publicly available at our GitHub repository.

1 Introduction

The assessment of handwritten mathematical solutions is a fundamental yet labor-intensive component of mathematics education, serving as both a diagnostic tool for educators and a critical feedback mechanism for learners (Scarlato et al., 2025; Lin, 2024; Zhang et al., 2021; Nakamoto et al., 2023). In traditional classroom settings, teachers manually

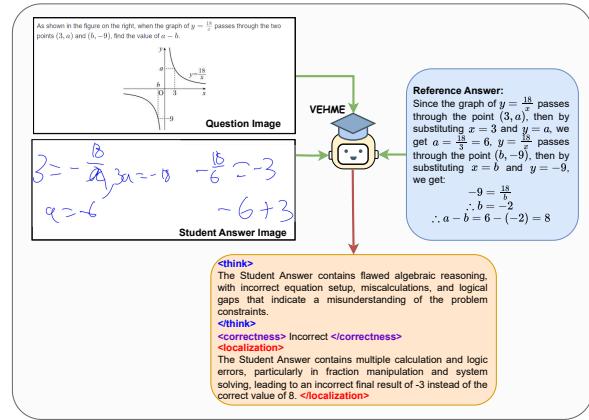


Figure 1: Our model takes a question, reference answer, and student answer image as input to predict the correctness of the student’s solution and identify any error locations, if the solution is incorrect.

evaluate student work to identify conceptual misunderstandings, procedural errors, and gaps in logical reasoning. But, the scalability of this process is severely limited by time constraints, class sizes, and the cognitive load of interpreting diverse solution strategies (Gowda and Suma, 2017; Laws et al., 2003; Callahan et al., 2021). With the rise of digital learning platforms, there is an urgent need for automated systems that can replicate the nuanced judgment of human graders while accommodating the unstructured, creative, and often messy nature of student-written content (Baral et al., 2025; Scarlato et al., 2025).

To address scalability challenges in grading, prior work has explored automated systems built around structured templates and Optical Character Recognition (OCR) pipelines (Li et al., 2019; Zhang et al., 2020, 2025; Wagstaff et al., 2019), as well as typed-text inputs processed by language models (Scarlato et al., 2025). However, these approaches rely on rigid assumptions: that student responses are syntactically clean, spatially struc-

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tured, or written within constrained templates. Furthermore, there is a notable lack of high-quality datasets that include complex, multi-expression handwritten math expressions (Yang et al., 2023), limiting the capacity to model open-form student solutions. Consequently, these systems often break down when applied to real-world data that is handwritten, unstructured, and heterogeneous, featuring symbolic math, diagrams, and natural language, all rendered in varied handwriting styles (Figure 1).

Previous approaches that leverage Large Language Models (LLMs) for grading often depend solely on the model’s end-to-end reasoning capabilities without explicitly supervising or constraining the underlying reasoning process (Zhang et al., 2025; Mok et al., 2024; Kortemeyer et al., 2024). In the text-only domain, Process Reward Models (PRMs) have been introduced to address this issue by enabling step-by-step evaluation of mathematical sequential reasoning (He et al., 2024; Wang et al., 2024; Lambert et al., 2024). More recently, PRMs have been extended into the multimodal domain (Khalifa et al., 2025; Wang et al., 2025), allowing reasoning-aware supervision in visual tasks. However, to the best of our knowledge, there is still no effective approach tailored to handwritten mathematical expressions, a domain that combines the complexities of symbolic reasoning with the challenges of visual noise and layout diversity.

While recent advances in Vision-Language Models (VLMs) have shown promise in multimodal understanding, their application to handwritten math assessment has been limited by several key challenges: (C1) the scarcity of high-quality training data that pairs handwritten expressions with detailed error annotations (Jin et al., 2025), (C2) the difficulty in robustly processing the spatial layout and visual noise characteristic of student work (Guo et al., 2025b).

In this work, we take a step toward overcoming these challenges by tackling the under-explored and difficult task of grading template-free, handwritten open-response solutions in the broader domain of K–12 mathematics. This setting requires models capable of handling significant visual heterogeneity, unstructured layouts, and sequential reasoning, where early mistakes may propagate across later steps. To this end, we present **VEHME** (a Vision-Language Model for Evaluating Handwritten Mathematics Expressions), a novel end-to-end framework for automatically grading handwritten math solutions.

Our key contributions are summarized as follows:

- To address the challenge of assessing complex open-form mathematical expressions, we propose a novel framework named **VEHME**. To the best of our knowledge, we are the first to propose an end-to-end training pipeline for grading handwritten mathematical expressions.
- We adopt a dual-phase training regime: supervised fine-tuning on our synthesized datasets, followed by reinforcement learning with a composite reward to optimize for correctness, reasoning quality, and error localization.
- To address C1, we also propose a data synthesis pipeline to construct a high-quality reasoning-augmented dataset that pairs handwritten expressions with error detection and localization reasoning traces.
- To address C2, we propose the Expression-aware Visual Prompting Module (EVPM), equipping the VLM with precise spatial localization of handwritten mathematical expressions. To train this module, we construct a complex, heterogeneous, multi-expression handwritten math expressions dataset with its corresponding spatial information.
- Our training pipeline enables small, open-source Vision-Language Models (VLMs) to surpass the performance of larger open-source counterparts and achieve results on par with state-of-the-art proprietary models.

2 Related Work

2.1 Handwritten Mathematical Expression Recognition

Handwritten Mathematical Expression Recognition (HMER) has been a long-standing and evolving research area. The inherent ambiguity and complex structure in handwritten symbols present significant challenges. Traditional methods typically employed a two-stage process: mathematical symbol recognition followed by structure analysis (Chan and Yeung, 1998, 2000).

The advent of deep learning spurred a shift towards end-to-end OCR-based models. These approaches aimed to directly predict the symbolic

representation (e.g., LaTeX) from an input handwritten mathematical expression image. Variations included models that directly output the sequence (Deng et al., 2017; Li et al., 2022; Wu et al., 2020; Zhao and Gao, 2022) and others that incorporated grammatical rules of mathematical expressions to guide and constrain the prediction process (Li et al., 2024b; Wu et al., 2022, 2021).

More recently, Vision Language Models (VLMs), which combine vision encoders with Large Language Models (LLMs), have shown strong multimodal capabilities (Liu et al., 2023; Bai et al., 2023; Chen et al., 2024c). There is emerging interest in applying VLMs to HMER, leveraging their potential for understanding complex visual and textual relationships (Guo et al., 2025c).

2.2 Automatic Grading of Handwritten Math

The labor-intensive nature of manually grading handwritten mathematical solutions has driven the demand for automated systems. Early efforts in automated grading were often limited to structured or short-answer questions due to technological constraints. A prevalent traditional method involved converting handwritten expressions to a format like LaTeX using the HMER model, followed by evaluation with rule-based system or heuristics (Li et al., 2019; Zhang et al., 2020; Chaowicharat and Dejdumrong, 2023).

With the development of LLMs, new approaches have emerged where LLMs compare a student’s formatted submission (derived via HMER) against a correct solution (Zhang et al., 2025). For finer evaluation (e.g., assessing complex solutions or providing step-level feedback), reinforcement learning (RL) has been explored to facilitate error localization and step-by-step correction (Li et al., 2025; Zheng et al., 2024). VLMs have also been investigated for direct automatic grading from handwritten input, aiming to bypass the explicit HMER formatting stage (Nath et al., 2025; Baral et al., 2025; Jin et al., 2025; Mok et al., 2024). However, the VLM-based systems show considerable sensitivity to handwriting quality, which currently restricts their robustness in practical, real-world scenarios.

2.3 Mathematical Reasoning and LLM as a Judge

The recent success of reasoning-focused models across domains such as mathematics and programming (OpenAI, 2024) draws significant attention to enhancing the reasoning capabilities of LLMs

through reinforcement learning (RL) (Zhang et al., 2024b,a; Trung et al., 2024; Chen et al., 2024b). In this context, Guo et al. (2025a) introduced a novel RL-based approach that removes the need for supervised fine-tuning while achieving strong and reliable reasoning performance. Building on this progress, recent efforts have begun exploring similar techniques in multi-modal settings, attracting increasing research interest (Liu et al., 2025; Deng et al., 2025; Zhou et al., 2025).

In tasks requiring open-ended responses, such as error localization in educational contexts or code reviews, human evaluation is often necessary to validate correctness. However, this dependence on manual judgment limits scalability. Consequently, there is growing interest in using LLMs themselves as automated judges to evaluate the quality and accuracy of responses (Zheng et al., 2023; Chen et al., 2024a; Gu et al., 2024). Recent work has started to integrate LLM-based evaluators directly into RL training pipelines for such open-ended tasks.

3 Methodology

Our approach to Handwritten Mathematics Grading integrates two key components: (1) supervised fine-tuning using distilled data from the QwQ-32B model to instill foundational reasoning capabilities, and (2) reinforcement learning via Group Relative Policy Optimization (Shao et al., 2024) to refine the model’s ability to generate accurate and explainable grading outputs. As shown in the Figure 2, this dual-phase training strategy aims to enhance both the correctness assessment and the interpretability of the model’s evaluations.

3.1 Problem Formulation

We formulate the *Handwritten Mathematics Grading* problem as a vision-language modeling task, where the goal is to assess the correctness of a student’s handwritten mathematical answer based on a given question and a reference solution.

Let each instance be represented by the tuple $x = (q, r, s)$, where: (i) $q \in \mathbb{P}^*$ is the **question**, represented as a sequence of pixels (e.g., an image of a printed or handwritten mathematical question), (ii) $r \in \mathbb{V}^*$ is the **reference answer**, expressed as a sequence of structured mathematical tokens from a vocabulary \mathbb{V} , and (iii) $s \in \mathbb{P}^*$ is the **student answer**, also represented as a sequence of pixels (i.e., a handwritten response). Here, \mathbb{P} denotes the space of pixel values (e.g., grayscale or RGB), and

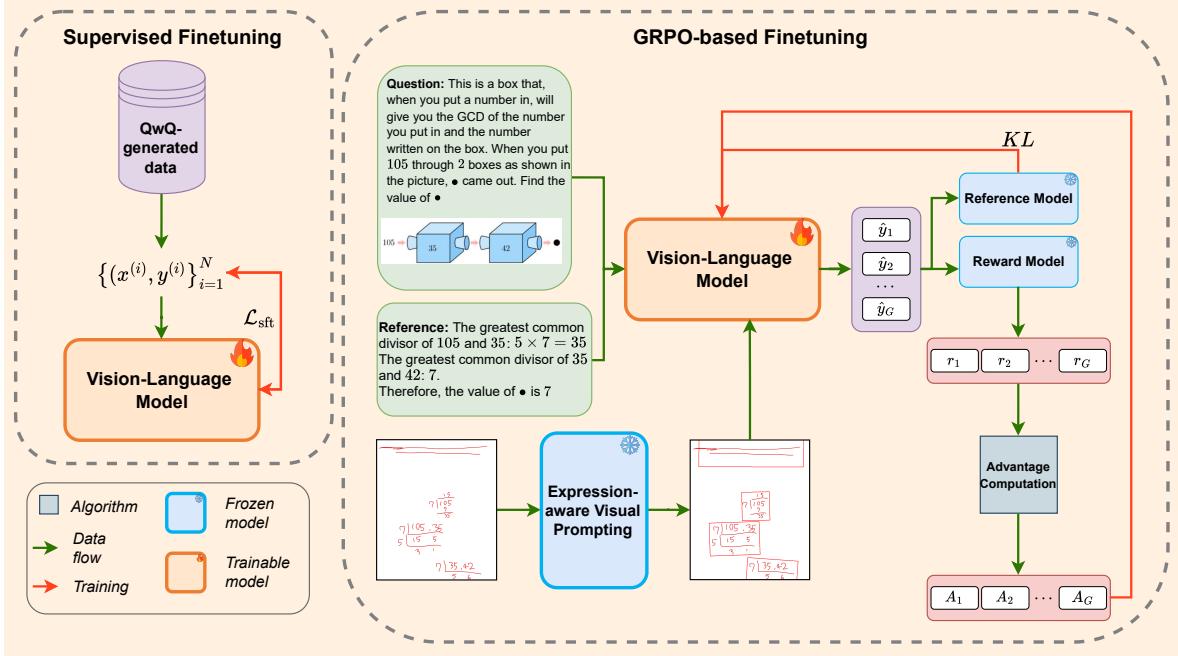


Figure 2: VEHME overview. Initially, a VLM is fine-tuned to generate outputs in the desired format using synthesized data from Sec 3.2.1. A subsequent Preference Optimization step trains the model using GRPO (Shao et al., 2024) method. This optimization is guided by rewards described in Sec 3.2.2 and takes the given problem, reference answer, and an expression-aware visual-prompted image from the student’s answer as input.

\mathbb{V} denotes the vocabulary of natural language and symbolic mathematical expressions (e.g., LaTeX or semantic tokens). Each instance is associated with a grading label $y = (c, l)$, where: (i) $c \in \{0, 1\}$ indicates the **correctness** of the student answer, and (ii) $l \in \mathbb{V}^*$ denotes the **location or description of the error**, provided as a (possibly empty) subsequence of tokens aligned to the student answer.

Let $\mathbb{X} = \mathbb{P}^* \times \mathbb{V}^* \times \mathbb{P}^*$ and $\mathbb{Y} = \{0, 1\} \times \mathbb{V}^*$ denote the input and output spaces, respectively. The goal is to learn a function $f : \mathbb{X} \rightarrow \mathbb{Y}$, that maps each input tuple $x = (q, r, s)$ to a grading label $y = (c, l)$, capturing both correctness and, if applicable, the symbolic location of the error in the student’s response. To approximate f , we train a vision-language model π_θ parameterized by θ , which defines an autoregressive conditional distribution $\pi_\theta(y_t | x, y_{<t})$ over tokens in the output space. The function f is then approximated by $f(x) \approx \arg \max_{y \in \mathbb{Y}^*} \prod_{t=1}^T \pi_\theta(y_t | x, y_{<t})$, where T is the length of the output sequence y , and $y_{<t}$ denotes the previously generated tokens.

3.2 Main Techniques

3.2.1 Data Synthesis and Supervised Fine-tuning

Data synthesis from QwQ-32B Zheng et al.

(2024) demonstrates that QwQ-32B (QwenTeam, 2025), a large language model built upon Qwen2.5-32B (Yang et al., 2024) and optimized specifically for mathematical reasoning, achieves state-of-the-art performance among open-source LLMs on step-level mathematical evaluation tasks, even surpassing fine-tuned process reward models (PRMs). This strong reasoning capability, despite QwQ-32B’s relatively modest size compared to models like GPT-4 (Achiam et al., 2023) or DeepSeek-R1 (Guo et al., 2025a), makes it a compelling teacher model for mathematical evaluation.

To capitalize on this, we perform supervised fine-tuning using data distilled from QwQ-32B as shown in the Figure 3. Given a tuple $x = (q, r, s)$ consisting of a question, a reference answer, and a student’s handwritten response, we prompt QwQ-32B to assess the correctness of the answer and, if incorrect, to identify or describe the error. The model is instructed to generate responses in a structured format composed of three fields: a detailed thinking process, a correctness label, and an error localization. We set a maximum token budget of M for efficient training, as longer models’ responses do not necessarily improve the model’s performance (Chen et al., 2025b; Yeo et al., 2025; Deng et al., 2025; Chen et al., 2025a). In cases

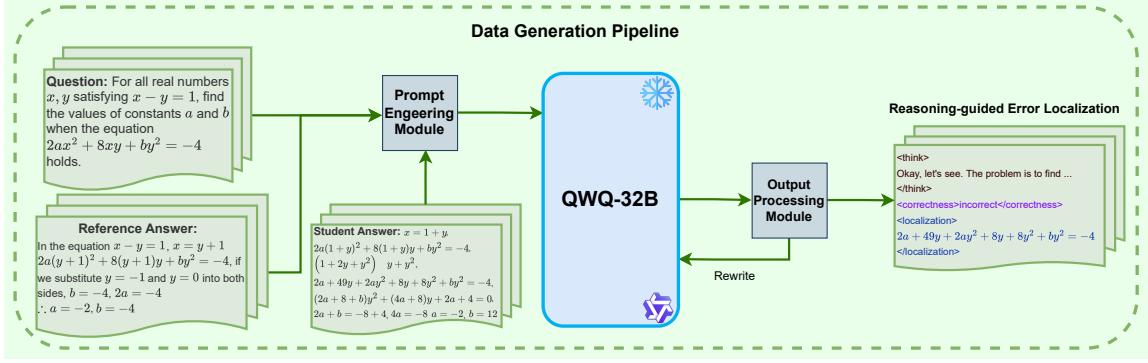


Figure 3: Process for synthesizing SFT data. QwQ-32B creates structured feedback from inputs (q, r, s) under a token budget M . Truncated outputs are repaired using a output processing module (Appendix A.2) with grammar-constrained decoding (Appendix A.2) to ensure valid SFT data.

where the model is truncated due to exceeding the token budget, we apply a structured post-processing routine that repairs truncated outputs and ensures conformance to the pre-defined format. Specifically, we truncate the response after the last complete thought from the model, append a closing "</think>" tag, and resume generation starting from "<correctness>" using grammar-constrained decoding (Geng et al., 2023). This allows the remainder of the response to be completed in a syntactically valid manner. The full post-processing procedure, the grammar, and more samples of the synthesized dataset can be found in Appendix A.2.

Supervised Fine-Tuning The distilled dataset from QwQ-32B provides high-quality demonstrations of expert-level reasoning and error localization. We use this data to initialize our model via supervised fine-tuning (SFT) (Figure 2), training it to mimic the teacher model’s structured responses through next-token prediction.

3.2.2 Reinforcement Learning with Group Relative Preference Optimization

To improve the model’s ability to produce accurate and explainable grading outputs, we adopt Group Relative Policy Optimization (GRPO) (Shao et al., 2024), a reinforcement learning technique that leverages intra-group comparisons among multiple completions generated per prompt, effectively optimizing the relative ranking of responses without relying on value function estimation. This approach has demonstrated significant improvements in mathematical reasoning tasks.

Recent studies have shown that GRPO facilitates stable training and emergent reasoning behavior, often surpassing supervised fine-tuning (Chen

et al., 2025a; Deng et al., 2025; Zhou et al., 2025; Shen et al., 2025). Liu et al. (2025) further emphasizes GRPO’s robustness, while Chen et al. (2025b) shows that judgment tasks, closely related to our grading tasks, are reasoning-intensive and benefit significantly from GRPO-based optimization.

GRPO formulation Formally, for each input x , we sample a group $G = \{\hat{y}_1, \hat{y}_2, \dots, \hat{y}_{|G|}\}$ of completions from the current policy π_θ . Each response $\hat{y}_i = (\hat{c}_i, \hat{l}_i)$ is evaluated using a composite function $r(\hat{y}_i)$. The advantage is efficiently estimated using the standardization of the rewards within the group: $\hat{A}_i = \frac{r(\hat{y}_i) - \mu_G}{\sigma_G}$, where μ_G and σ_G are the mean and standard deviation of the rewards in group G , respectively. The GRPO objective is then formulated similarly to Shao et al. (2024) as:

$$\begin{aligned} \mathcal{J}_{\text{GRPO}} &= \mathbb{E}[x \sim P(X), \{y_i\}_{i=1}^{|G|} \sim \pi_{\theta_{\text{old}}} (y_i|x)] \\ &\quad \frac{1}{|G|} \sum_{i=1}^{|G|} \frac{1}{|\hat{y}_i|} \sum_{t=1}^{|\hat{y}_i|} \left[\min \left(\hat{A}_i \rho_{i,t}, \hat{A}_i \varsigma_{i,t} \right) - \beta \mathcal{D}_\theta \right], \\ \varsigma_{i,t} &= \text{clip}(\rho_{i,t}, 1 - \epsilon, 1 + \epsilon), \\ \mathcal{D}_\theta &= \mathbb{D}_{KL}[\pi_\theta || \pi_{\text{ref}}], \end{aligned}$$

where $\rho_{i,t} = \frac{\pi_\theta(\hat{y}_{i,t}|x, \hat{y}_{i,<t})}{\pi_{\theta_{\text{old}}}(\hat{y}_{i,t}|x, \hat{y}_{i,<t})}$ is the probability ratio between the current and previous policies, $\mathbb{D}_{KL}[\pi_\theta || \pi_{\text{ref}}]$ is the unbiased estimator of the KL divergence between the current and the reference policies (Schulman et al., 2017), and ϵ, β are hyperparameters controlling the clipping range and the KL penalty (Shao et al., 2024).

Reward Modeling Each response is scored using a composite reward function:

$$\begin{aligned} r(\hat{y}_i) &= r_{\text{match}}(\hat{y}_i) + r_{\text{loc}}(\hat{y}_i) + r_{\text{len}}(\hat{y}_i) \\ &\quad + r_{\text{cos}}(\hat{y}_i) + r_{\text{rep}}(\hat{y}_i), \end{aligned}$$

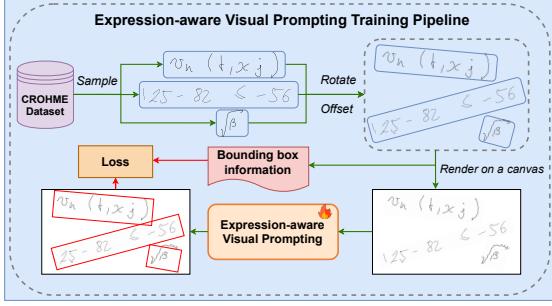


Figure 4: EVPM training pipeline. The model learns to predict ground truth bounding boxes for augmented mathematical expressions which are drawn onto a white backgrounds after augmentation.

where

$$r_{\text{match}}(\hat{y}_i) = \begin{cases} 1 & \text{if } \hat{c}_i = c_i \\ 0 & \text{otherwise} \end{cases},$$

$$r_{\text{loc}}(\hat{y}_i) = \begin{cases} 1 & \text{if } \hat{l}_i \text{ correctly describe the error} \\ 0 & \text{otherwise} \end{cases},$$

$$r_{\text{len}}(\hat{y}_i) = \begin{cases} 0.25 & \text{if } |\hat{y}_i| \geq 150 \\ 0 & \text{otherwise} \end{cases},$$

and $r_{\text{cos}}, r_{\text{rep}}$ are the Cosine Reward and the repetition penalty (Yeo et al., 2025). For r_{loc} , traditional rule-based evaluation is inadequate due to the open-ended nature of the task and the variability in valid explanations. Prior work has demonstrated that QWQ-32B achieves great results in text-based step-level error localization (Zheng et al., 2024). Thus, we opt to use QWQ-32B as an automated judge to evaluate the correctness and quality of the model’s error localization outputs for this component of the reward function.

3.2.3 Expression-aware Visual Prompting

Previous studies have shown that Vision-Language Models (VLMs) struggle to interpret multi-line handwritten mathematical expressions, which are common in students’ solutions (Guo et al., 2025c). Recognizing these complex layouts is essential for downstream tasks such as automated grading. To address this, we propose an *Expression-aware Visual Prompting Module* (EVPM), which provides the model with spatial cues by generating oriented bounding boxes around individual mathematical expressions (Li et al., 2024a).

The EVPM comprises an oriented bounding box predictor trained on a synthetic dataset of multi-line mathematical expressions (Figure 4). To train the EVPM component, we utilize Yolov11 (Jocher et al., 2023) as the backbone of the bounding box

detector. We construct the synthetic dataset by first sampling expressions from the grammar-based generator proposed by Truong et al. (2022), then rendering each expression as a handwritten image using symbol-level traces. Recognizing that students often write in non-linear or skewed patterns, particularly when unbounded by ruled lines, we simulate these realistic distortions through a stochastic layout pipeline. Each expression is randomly rotated and placed on a blank canvas with vertical spacing and padding variation, forming a more naturalistic setting for bounding box supervision. To create such expression-rich canvases, we formalize the image construction process in Appendix A.3. Each mathematical expression is first rendered into an image, then randomly rotated and positioned vertically on the canvas. The corresponding oriented bounding boxes are computed by rotating each image’s axis-aligned box by the same angle. This process supports diverse and realistic handwriting layouts during training.

By training the EVPM on these synthetic examples, the model learns to identify individual expressions even under irregular layouts. This bounding box information is provided to the downstream VLM as visual prompts, enabling more accurate understanding of complex student solutions.

4 Experiments

4.1 Evaluation Datasets and Metrics

We use two datasets for handwritten mathematical assessment: the AIHub dataset¹ and the FERMAT dataset (Nath et al., 2025). More information about datasets’ statistics and samples can be found in Appendix A.1.

AIHUB dataset is derived from a large-scale educational repository encompassing K–12 mathematics problems. It comprises 183,085 handwritten student responses corresponding to 30,050 unique problem questions, originally authored in Korean. To mitigate potential performance degradation due to cross-lingual inputs, we filtered out student answers containing non-English characters. The remaining questions and reference answers were translated into English using the Google Translate API². In the end, we obtained 81,394 training samples and 9,062 test samples for the AIHub dataset.

¹<https://aihub.or.kr>

²<https://cloud.google.com/translate/docs/reference/rest>

FERMAT dataset provides a complementary evaluation focused on error analysis, so incorrect solutions significantly outnumber correct ones at an approximate ratio of 85:15, which is different from the balanced distribution in the filtered AI-Hub dataset. It contains 2,244 manually curated solutions across eight mathematical domains from arithmetic to calculus. Each entry contains: the original question, a handwritten solution image with intentional errors, the gold-standard correct answer, and error detection labels. Due to its small size, we opt not to perform any additional filtering, as doing so could lead to an insufficient amount of data for fine-tuning. We split the dataset into 70% training and 30% test sets.

Metrics For error detection performance evaluation, we employ two key metrics—**Accuracy**: measures overall prediction correctness across both error-present and error-free samples, and **F1-Score**: balances precision and recall to handle class imbalance, particularly crucial for FERMAT’s skewed error distribution.

4.2 Implementation Details

We conduct our experiments using Qwen2.5-VL-7B-Instruct (Bai et al., 2025), a pre-trained large multimodal model. As an initial step, we perform supervised fine-tuning (SFT) using text-only data distilled from QwQ-32B (QwenTeam, 2025) to align the model with the desired output format. During reinforcement learning with GRPO, input images are resized to limit visual token overhead, capped at 50,176 pixels (224×224 image resolution) for the AIHub dataset and 501,760 pixels for FERMAT, preserving essential content while improving training efficiency. For more details, please refer to Appendix A.4.

4.3 Comparison with the State-of-the-Art

Experimental setup We conduct experiments on two subtasks of the mathematical grading task: Error Detection (ED) and Error Localization (EL), evaluating a cascaded setup where ED and EL are executed sequentially (Figure 5). The input for VLM processing consists of a problem question, a reference answer, and a student’s solution. For the ED task, the model outputs a binary classification indicating whether the student’s solution contains an error (Incorrect solution) or is error-free. Performance is evaluated based on the model’s classification accuracy. If the student’s answer contains an

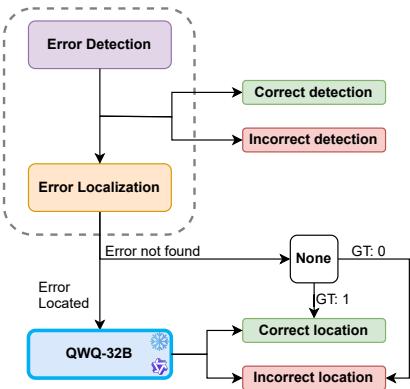


Figure 5: VLM evaluation procedure for student solutions, encompassing Error Detection (ED) and Error Localization (EL). ED serves to identify the presence of errors, while EL is responsible for determining the specific location of any detected error.

error, the model proceeds to identify the specific location of the error for the EL task. To assess localization precision, we use the critic model QwQ-32B as a Judge due to its superior performance in error localization (Zheng et al., 2024). When no error location is identified (None), we cross-check with the ground truth of error detection—if the student’s answer is correct, the localization is deemed correct; otherwise, it is marked as wrong localization. The results are reported based on a single run for consistency. Further details on datasets (e.g., AIHub, Fermat) and evaluation protocols are provided in the Appendix.

Experimental results The experiments are conducted on two tasks—Error Detection and Error Localization, across the AIHub and FERMAT datasets. Table 1 compares our VEHME-Qwen2.5-VL-7B against four open-source baselines and four closed-source systems on the AIHub and FERMAT benchmarks. Among the open-source group, our model achieves a clear lead in both Error Detection (ED) and Error Localization (EL). On AIHub, VEHME records 73.01% ED accuracy (49.22% F1) and 61.13% EL accuracy (58.18% F1), substantially outperforming Qwen2.5-VL-7B-Instruct (46.68% Acc/ 31.48% F1 for ED, 38.00% Acc/ 33.75% F1 for EL), Pixtral-12B (52.67% / 31.69% ED, 32.20% / 38.35% EL), Phi-4-multimodal-instruct, and Llama-3.2-11B. This margin highlights the effectiveness of our data synthesis pipeline and dual-phase training in refining both expression recognition and precise localization.

Models	AIHub				FERMAT			
	ED		EL		ED		EL	
	Acc	F1	Acc	F1	Acc	F1	Acc	F1
Closed-source Models								
GPT-4o	73.00	48.67	62.88	61.01	66.62	58.57	55.04	66.07
GPT-4o-mini	57.15	32.97	38.98	49.18	81.75	56.62	47.18	62.61
Gemini-2.0-Flash	75.40	50.30	67.40	65.73	75.22	43.13	64.24	75.13
Gemini-2.5-Flash-Preview	71.01	46.91	63.00	65.68	80.12	45.68	63.35	74.72
Open-source Models								
Phi-4-multimodal-instruct	38.57	29.39	35.93	21.33	36.20	22.33	16.91	15.15
Qwen2.5-VL-7B-Instruct	46.68	31.48	38.00	33.75	44.81	26.04	24.33	32.18
Pixtral-12B	52.67	31.69	32.20	38.35	41.39	25.56	10.53	3.52
Llama-3.2-11B-Vision-Instruct	19.49	18.58	44.78	5.62	32.49	22.15	21.07	22.67
VEHME-Qwen2.5-VL-7B	73.01	49.22	61.13	58.18	62.61	29.81	31.90	44.36

Table 1: Performance comparisons of state-of-the-art Vision-Language Models on two datasets: AIHub and FERMAT (Nath et al., 2025). The evaluation metrics include Accuracy (Acc) and F1 score (F1). **ED**: Error Detection, **EL**: Error Localization. All of the reported results are in percentages (%).

On FERMAT, open-source models generally struggle, but our approach again leads the pack with 62.61% ED accuracy (29.81% F1) and 31.90% EL accuracy (44.36% F1). Notably, Pixtral-12B exhibits an unusual split—high ED accuracy (41.39%) yet very low EL performance (10.53% Acc/3.52% F1)—which echoes the observations in the original FERMAT study: its format alignment and output consistency are suboptimal for localization tasks (Nath et al., 2025). In contrast, our model’s balanced gains across both metrics demonstrate robust adherence to the expected answer format and reliable spatial grounding.

When we turn to closed-source systems, Gemini-2.0-Flash remains the top performer overall (75.40%/50.30% ED, 67.40%/65.73% EL on AIHub; 75.22%/43.13% ED, 64.24%/75.13% EL on FERMAT), followed closely by GPT-4o and its mini variant. These proprietary models, however, exceed tens or even hundreds of billions of parameters, whereas our Qwen2.5-VL-7B backbone contains only 7 billion parameters. The result demonstrates that our lightweight model surpasses other open-source approaches and nearly matches closed-source models, while operating at a fraction of the scale of closed systems, underscoring the efficiency gains enabled by our targeted data generation and training regimen. The qualitative result and analysis are presented in Appendix A.7. Additionally, we provide analysis of experimental result across different educational levels in Appendix A.5.

Models	ED		EL	
	Acc	F1	Acc	F1
Ours	73.01	49.22	61.13	58.18
-EVPM	71.98	48.87	61.70	56.59
-SFT	63.24	41.47	52.67	33.41
-RL	46.90	33.82	39.24	29.59
Baseline	46.68	31.48	38.00	33.75

Table 2: Ablation study results validating our proposed components on the AIHub dataset.

4.4 Ablation Study

Component Ablation In this section, we perform a comprehensive ablation study on the balanced AIHub dataset to quantify the impact of each major component in our architecture. Table 2 reports performance when ablating specific modules from our full model. Omitting the bounding-box visual hints (-EVPM) yields a modest decrease in ED accuracy (73.01%→71.98%) and a larger decline in EL F1 (58.18%→56.59%), underscoring the necessity of spatial grounding. Skipping supervised fine-tuning (-SFT) substantially degrades both ED and EL, indicating that task-specific supervision is crucial for aligning the vision–language backbone with the reasoning steps and expected output format. The removal of reinforcement learning (-RL) results in a dramatic collapse, particularly in EL F1 (58.18%→29.59%), highlighting RL’s indispensable role in honing expression comprehension and precise localization. Our intact model consistently outperforms all ablated variants across every metric in both tasks.

Models	ED		EL	
	Acc	F1	Acc	F1
GPT-4o	73.00	72.93	63.00	62.36
GPT-4o-mini	48.00	39.22	33.00	29.26
Gemini-2.0-Flash	71.00	71.00	65.00	64.83
Gemini-2.5-Flash*	76.00	74.78	59.00	58.80
Phi-4*	33.00	25.14	36.00	33.33
Qwen2.5-VL*	50.00	35.87	46.00	45.45
Pixtral*	41.00	25.38	29.00	28.82
Llama-3.2*	21.00	18.93	48.00	34.04
Ours	75.00	50.24	66.00	65.32
-EVPM	62.00	42.26	62.00	61.00
-SFT	61.00	39.02	58.00	53.21
-RL	43.00	31.69	44.00	41.67

Table 3: Ablation study results validating our proposed components on the “Challenging Subset of AIHub” dataset. *We use the same models as in the main experiment; we omit the suffix for brevity.

Ablation Under Heavy Rotations In our analysis, most of the standard dataset has mild rotation ($< 15^\circ$), which might explain the relatively small global improvement from EVPM. To better understand EVPM’s contribution, we constructed a “Challenging Subset of AIHub” by identifying the 100 handwritten data points with the most heavily rotated mathematical expressions (top-100; Mean: 21.81°). Examples of this subset can be found in Appendix A.6. Table 3 reports performance when the ablation study is conducted in the challenging subset. These results demonstrate that EVPM makes a significant contribution under heavy rotations. On this subset, removing EVPM causes a substantial drop in performance (ED: $75\% \rightarrow 62\%$, and EL: $66\% \rightarrow 62\%$), even with all other training and inference settings held constant. Compared with the closed-source models, our approach significantly narrows the gap in error detection and even outperforms them in error localization. Specifically, for ED, our method ($75\%/50.24\%$) comes close to the strongest closed-source baselines such as GPT-4o ($73\%/72.93\%$) and Gemini-2.5-Flash-Preview ($76\%/74.78\%$), showing that our system can achieve comparable detection accuracy under challenging rotations. More notably, in EL, our approach achieves $66\% \text{ Acc}/65.32\% \text{ F1}$, surpassing both GPT-4o ($63\%/62.36\%$) and Gemini-2.5-Flash-Preview ($59\%/58.80\%$). This indicates that while the closed-source models remain competitive in ED, our method demonstrates a clear advantage in localizing errors under heavy rotational noise.

Image Resolutions	ED		EL	
	Acc	F1	Acc	F1
224 × 224	73.01	49.22	61.13	58.18
448 × 448	75.66	50.42	63.87	56.57
768 × 768	75.29	50.11	64.58	56.50

Table 4: Ablation study results with different image resolutions on the AIHub dataset.

Image Resolution Ablation In our main experiment, we resize the input images down to 224x224 for computational efficiency. To assess the impact of image resolution on performance, we conduct an ablation study using images with varying resolutions. Table 4 shows VEHME’s performance across resolutions. Increasing the resolution generally improves both error detection and localization accuracies, with the largest gains observed when moving from 224x224 to 448x448. This improvement is likely due to better preservation of fine-grained visual details in handwritten expressions. However, further increasing the resolution to 768x768 yields only marginal improvements, with performance comparable to that at 448x448. These results suggest that while higher resolutions can capture more detail, the benefits plateau beyond a certain point, making the additional computational cost less justifiable.

5 Conclusion

We present VEHME, a novel Vision-Language Model tailored for evaluating open-form handwritten mathematical expressions with high accuracy, interpretability, and scalability. Leveraging a dual-phase training strategy, supervised fine-tuning on expert reasoning traces and reinforcement learning via GRPO, VEHME delivers fine-grained error detection and localization across visually diverse student responses. The Expression-Aware Visual Prompting Module further enhances the model’s spatial reasoning capabilities. Experiments on the AIHub and FERMAT datasets show that VEHME consistently outperforms existing open-source models and achieves performance competitive with state-of-the-art proprietary systems. These findings underscore VEHME’s promise as an effective, open-access solution for scalable, automated assessment in educational settings.

Limitations

While VEHME demonstrates promising results in evaluating handwritten mathematical expressions,

several limitations should be acknowledged. First, due to data-sharing restrictions from the source provider (AIHub), the original math problems and student responses cannot be publicly released outside Korea, limiting reproducibility. Secondly, while VEHME is robust to varied inputs, it may struggle with extremely illegible handwriting, low-quality images, or unconventional notation.

Thirdly, QwQ-32B may propagate biases in our evaluation pipeline, stemming from length-based tendencies or anchoring effects introduced during training and reward design. While output length bias was explicitly controlled, addressing deeper preference and anchoring biases remains an open challenge for future work.

Lastly, the reinforcement learning setup, though effective, can be sensitive to reward design and may introduce training instability. Future work will explore more accessible datasets, improved robustness, and interpretable learning signals.

Acknowledgments

This work was supported by the National Research Foundation of Korea (NRF) grant funded by the Korea government (MSIT) (No. RS-2024-00456247, No. RS-2023-00218913, No. RS-2023-00219959) and by Institute of Information & communications Technology Planning & Evaluation (IITP) grant funded by the Korea government(MSIT) (No. RS-2025-25443718, Next-HCAI Project, No.RS-2022-II220608/2022-0-00608, Artificial intelligence research about multimodal interactions for empathetic conversations with humans, No.IITP-2025-RS-2024-00360227, Leading Generative AI Human Resources Development, No.RS-2025-25442824, AI Star Fellowship Program(Ulsan National Institute of Science and Technology), No.RS-2020-II201336, Artificial Intelligence graduate school support(UNIST) and No.RS-2019-II191906, Artificial Intelligence Graduate School Program(POSTECH).

This work utilized datasets provided by the Open AI Dataset Project (AIHub, South Korea), which are publicly available through AIHub (www.aihub.or.kr).

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A Appendix

A.1 Datasets

Figure 6 shows representative samples from the AIHub dataset and the FERMAT dataset. Both contain math problems paired with student solutions, but they differ in format and collection style.

AIHub comprises 183,085 handwritten student responses corresponding to 30,050 unique problem questions, originally authored in Korean. To mitigate potential performance degradation due to cross-lingual inputs, we filtered out student answers containing non-English characters. Given that many questions include figures, after translating to English, we further rendered the translated content into LaTeX and converted it into images to ensure compatibility with visual input processing.

As shown in Figure 7, the original AIHub dataset exhibits a significant class imbalance, with approximately 72% of answers being correct and 28% incorrect. To mitigate this skew, we curate the training data as follows: **Step 1:** We discard problems that had only correct or only incorrect student answers, leaving approximately 150,000 problem-

answer pairs. **Step 2:** For each question, we sample an equal number of correct and incorrect answers to form a balanced training set. In the end, we obtained a balanced dataset of correct and incorrect solutions from students with 81,394 training samples and 9,062 test samples.

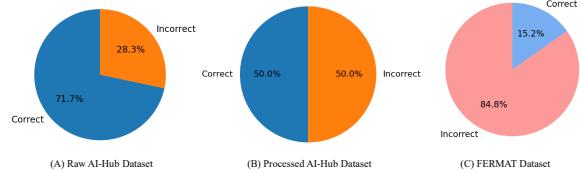


Figure 7: Dataset distribution for correct/incorrect answer.

FERMAT provides a complementary evaluation focused on error analysis, so incorrect solutions significantly outnumber correct ones at an approximate ratio of 85:15, which is different from the balanced distribution in the filtered AIHub dataset. It contains 2,244 manually curated solutions across eight mathematical domains from arithmetic to calculus. Each entry contains: the original question, a handwritten solution image with intentional er-

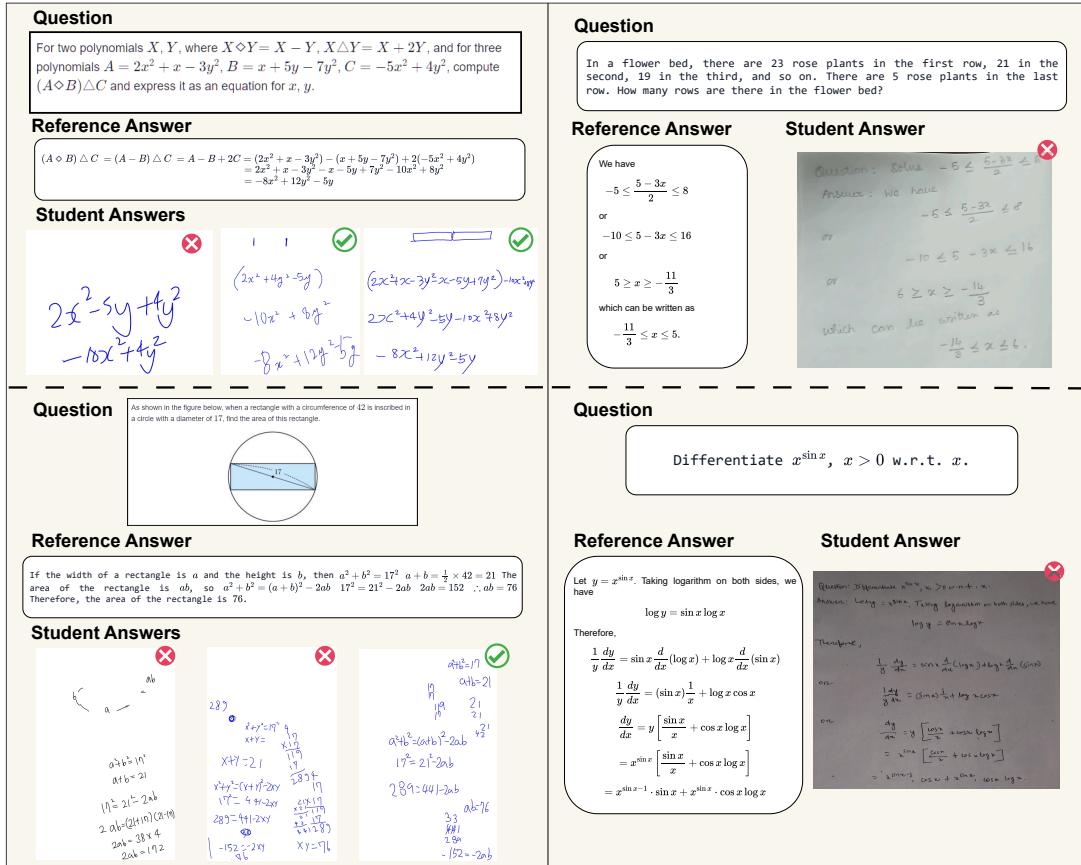


Figure 6: Data samples of AIHub (left) and FERMAT (Right) Datasets.

rors, the gold-standard correct answer, and error detection labels. Due to its small size, we opt not to perform any additional filtering, as doing so could lead to an insufficient amount of data for fine-tuning. We split the dataset into 70% training and 30% test sets.

A.2 Data synthesis from QwQ-32B for SFT

Figure 8 illustrates some data synthesized from QwQ-32B, showcasing its reasoning process for assessing correctness and localizing errors in students’ handwritten responses.

Please refer to Algorithm 1 for a detailed view of the data synthesis procedure.

The grammar G used in the decoding process is as follows:

$$\begin{aligned} \text{root} &\rightarrow C \text{ “</correctness>” } L \\ C &\rightarrow \text{“correct” | “incorrect”} \\ L &\rightarrow \text{“<localization>” } E \text{ “</localization>”} \\ E &\rightarrow \text{“None” | } \Sigma_{\text{math}}^* \\ &\quad | \text{“Lack of intermediate steps”} \end{aligned}$$

where Σ_{math} is the set of valid characters repre-

Algorithm 1 Post-Processing for Generated Data

Input: Output text T , Prompt P , Maximum token limit M , Grammar constraint G , Language model LLM

Output: Final response with correct format

```

1: if TokenCount( $T$ ) <  $M$  then
2:   return  $T$ 
3: end if
4:  $T' \leftarrow \text{TRUNCATELASTTHOUGHT}(T)$ 
5:  $T'' \leftarrow T' \parallel \text{“</think>\n\n<correctness>”}$ 
6:  $P' \leftarrow P \parallel T''$ 
7: Configure  $LLM$  with grammar constraint  $G$ 
8:  $R \leftarrow LLM.\text{GENERATE}(P')$ 
9: return  $P' \parallel R$ 

```

Figure 8 displays two examples of synthesized data from QwQ-32B for SFT. Each example includes a 'Question' box, a 'Reference Answer' box, a 'Student Answer' box with a red 'X' icon, and a 'Reasoning Trace from QwQ' box.

Example 1 (Top):

- Question:** When I converted the fraction $\frac{2}{75}$ to a decimal using a calculator, I got the following result. Find the value of the natural number x . $0.2666666\dots = 0.2\overline{6} = \frac{26}{99} = \frac{13}{49} = \frac{1}{75} = \frac{1}{75} \times 75 = 20$
- Reference Answer:** $0.2666666\dots = 0.2\overline{6} = \frac{26}{99} = \frac{13}{49} = \frac{1}{75} = \frac{1}{75} \times 75 = 20$
- Student Answer:** $\frac{1}{75} = \frac{2}{75}$
- Reasoning Trace from QwQ:**
 - <think>**: Okay, let's see here. The problem is to find the natural number x such that $x/75$ equals $0.266666\dots$, right? The reference solution is doing the conversion by repeating decimal to a fraction and solving ending up with $x=20$.
 - <incorrect>**: Now, the student's solution is written in mathematical expressions. Let me parse their steps. The first expression they have is $(24/900)$ simplified as $26/2$ over 900 . Wait, no. Wait, the student's solution is written as $\frac{24}{900} = 2/75$.
 - <incorrect>**: Hm, maybe I need to look more carefully. The student's solution is written as **<inThe** fraction with a big overset structure. Let me break it down. The numerator has an overset with 2 (from cancel 4), then cancel 12, then cancel 24. The denominator has underset with 75, cancel 150, cancel 450, cancel 900. Then it equals 2/75.
 - <incorrect>**: Wait, that seems confusing. Let me think step by step.
 - <incorrect>**: The student's expression is presented as a single equation: $(24/900)$ reduced step by step? Let me see. The way the cancels are written might be an attempt to simplify the fraction step by step. The numerator starts as 24, and the denominator as 900. Then they cancel factors step by step. InWai, perhaps the student is trying to simplify $24/900$. Let's see: 24 divided by 900. Let's see the steps:
 - <incorrect>**: First, 24 and 900 can both be divided by 12? Let me check. $24 + 12 = 2$, and $900 - 12 = 75$. So $24/900 = 2/75$. So maybe the student is showing cross cancellation steps.
 - <incorrect>**: But how did they get 24/900 in the first place? The problem is $x/75$ equals the decimal $0.26666\dots$ which the reference solution converts to 24/900, then simplifies to 4/15, leading to $x=20$.
 - <think>**: Okay, let me try to figure out where the student went wrong. The problem is to find the length of BC in triangle ABC given sin B, sin C, and AB = 5 cm. The reference solution uses some trigonometric steps, and the student's solution is a bunch of expressions. I need to check each step for errors.
 - <incorrect>**: First, looking at the student's expressions. They start with $\sqrt{10}$, $2\sqrt{5}$, $\sqrt{5}$, etc. Let's try to parse them step by step.
 - <incorrect>**: The reference solution uses AH (the height from A to BC) calculated as $AB \sin B$. Since AB = 5 cm and sin B is $\sqrt{5}/5$, that gives $AH = 5 * (\sqrt{5}/5) = \sqrt{5}$. That part seems correct. Then BH is $\sqrt{AB^2 - AH^2}$ which is $\sqrt{25 - 5} = \sqrt{20} = 2\sqrt{5}$.
 - <incorrect>**: So that's where the $2\sqrt{5}$ comes from. Then in triangle AHC, they calculate AC using $AH \sin C$? Wait, the reference solution uses $AC = AH / \sin C$? Wait, since sin C is $\sqrt{2}/2$, so $\sqrt{5}/(\sqrt{2}/2)$ would be $2\sqrt{5}/\sqrt{2}$ which simplifies to $\sqrt{10}$. That makes sense because AH squared minus AH squared would be CH squared. So CH would be $\sqrt{(\sqrt{10})^2 - (\sqrt{5})^2} = \sqrt{10 - 5} = \sqrt{5}$. Then adding BH ($2\sqrt{5}$) and CH ($\sqrt{5}$) gives $3\sqrt{5}$.
 - <incorrect>**: Now looking at the student's expressions. The first thing written is $\sqrt{10}$. That's the AC length from the reference solution, so that's correct if they got that properly. Then $2\sqrt{5}$. That's BH, so that's correct. Then $\sqrt{5}$, which is AH, so that's also correct. Then another $\sqrt{5}$. Maybe that's CH? Which is correct as $\sqrt{5}$.
 - <incorrect>**: $\langle\text{correctness}\rangle$
 - <incorrect>**: $\langle\text{incorrectness}\rangle$
 - <localization>**: $5 \times \frac{\sqrt{2}}{2}$
 - <localization>**: $\frac{5\sqrt{2}}{2}$

Figure 8: Examples of data synthesized from QwQ-32B for SFT. The model inputs—questions, reference answers, and student responses—are provided in LaTeX format. The figure shown here is for illustrative purposes only; no images are given to the model.

senting mathematical expressions

$$\Sigma_{\text{math}} = [\text{a-zA-Z0-9}\{\}_\cdot^\cdot \$\(\), ; \''=-<>+|*/]*$$

A.3 Expression-aware Visual Prompting

Please refer to Algorithm 2 for a formal view of how we synthesize multi-line mathematical expressions.

Algorithm 2 Multi-Line Expression Canvas Synthesis

Input: Set of HMES files \mathcal{H} , padding bound P , rotation bound θ

Output: Synthesized canvas \mathcal{C} and list of oriented bounding boxes \mathcal{B}

- 1: Render each expression $h_i \in \mathcal{H}$ to image I_i
 - 2: Compute canvas size from maximum image width and height
 - 3: Initialize blank canvas \mathcal{C} and vertical offset $y \leftarrow 0$
 - 4: **for all** images I_i **do**
 - 5: Sample rotation angle $\alpha \sim \mathcal{U}(-\theta, \theta)$
 - 6: Rotate image with α to obtain I_i^{rot}
 - 7: Sample horizontal offset
 - $x \sim \mathcal{U}(0, W - \text{width}(I_i^{\text{rot}}))$
 - 8: Compute rotated bounding box corners around image center
 - 9: Add (x, y) -shifted rotated corners to \mathcal{B}
 - 10: Paste I_i^{rot} onto canvas \mathcal{C} at (x, y)
 - 11: Update $y \leftarrow y + h_i^{\text{rot}} + \text{randint}(-P, P)$
 - 12: **end for**
 - 13: **return** Cropped canvas \mathcal{C} and bounding box list \mathcal{B}
-

A.4 Implementation Details

To train our model, we apply LoRA (Hu et al., 2022) to all MLP layers with a rank of 8 and an α -scaling factor of 32. Optimization is performed using the AdamW optimizer (Loshchilov and Hutter, 2017) with a learning rate of 5e-5, cosine learning rate decay, and a warm-up ratio of 0.05. We set a global batch size of 192 and apply a weight decay of 0.1 to mitigate overfitting. Training is conducted over a single epoch, requiring approximately 6 GPU-days on NVIDIA A100s.

To train our EVPM component, we utilize Yolov11 (Jocher et al., 2023) as the backbone of the bounding box detector. We synthesize 10,000 training data, and an additional 1000 validation data. Optimization is performed using the AdamW

optimizer (Loshchilov and Hutter, 2017) with a learning rate of 0.01. We set a global batch size of 16. Training the EVPM is conducted over 200 epochs, requiring only a few hours on a single NVIDIA RTX 3090.

To ensure efficiency during both supervised and reinforcement learning stages, we adopt the ms-swift framework (Zhao et al., 2024), which provides optimized memory utilization and high-throughput training for vision-language models. For inference, we employ the v11m framework (Kwon et al., 2023), enabling fast, memory-efficient evaluation across all open-source models, including our own. The prompts for inference are presented in Figure 9 and Figure 10.

A.5 Quantitative Experiment on Different Education Levels

Educational datasets naturally vary in complexity depending on the target grade level: primary school problems tend to be more straightforward and involve simpler arithmetic or reasoning steps, whereas middle and high school problems increasingly incorporate multi-step reasoning, algebraic manipulation, and abstract logic. Since error detection (ED) and error localization (EL) are reasoning-intensive tasks, it is important to evaluate whether models maintain consistent performance across different difficulty levels. Table 5 reports results on the AIHub dataset split by education levels (Primary, Middle, and High School). The evaluation again covers Error Detection (ED) and Error Localization (EL).

Gemini-2.0-Flash emerges as the strongest proprietary baseline across all education levels. It records peak accuracy at the High School level (76.40% ED / 68.96% EL), showing consistent gains as the problem difficulty increases. GPT-4o also performs robustly, particularly on Middle School data (75.24% ED / 65.71% EL). The open-source VLMs generally struggle to match closed-source performance, with accuracies ranging between 18–53%. Pixtral-12B achieves stable ED accuracy ($\approx 52\%$) across levels but remains weak in EL ($\approx 32\%$). Llama-3.2-11B shows relatively higher EL than other open-source peers ($\approx 45\%$), though at the cost of very poor ED performance ($\approx 19\%$). Phi-4 and Qwen2.5-VL-7B-Instruct sit in the middle range, with incremental improvements from Primary to High School, but failing to scale effectively. VEHME–Qwen2.5-VL-7B consistently outperforms other open-source competitors across

System Prompt
You are a helpful assistant. You first think about the reasoning process in the mind and then provide the user with the answer.
User Prompt
The following contains a math problem consisting of both a Question and a Student Answer. Your task is to analyze the Student Answer to determine whether there is any error. Begin by providing a detailed reasoning for your analysis, explaining where and why you believe an error is present or absent in the Student Answer. After the reasoning, provide a final output indicating whether the Student Answer is correct or incorrect. Please follow the exact format below without adding any extra information:
<think> Detailed reasoning for your analysis, explaining where and why you believe an error is present or absent in the Student Answer. </think>
<correctness> Correct or Incorrect </correctness>
<localization> The localization of the error in the Student Answer. Write None if there is no error. </localization>
Few-shot Prompt
<think> The Student Answer correctly identifies the slope and y-intercept of the original line as 2 and 3, respectively. The equation of the translated line is also correctly written as $y = 2x + 3 + a$. The substitution of the point (5, 6) into the equation is also done correctly: $6 = 2 \times 5 + 3 + a$, which simplifies to $6 = 13 + a$. The final step of solving for a is correct as well: $a = -7$. The Student Answer follows the same logical steps as the Reference Answer, and no errors are present in the calculations or reasoning. </think>
<correctness> Correct </correctness>
<localization> None </localization>
Input
Question: <image> Reference answer: reference answer Student answer : <image>

Figure 9: Prompt examples for the inferencing on AIHUB.

System Prompt
You are a helpful assistant. You first think about the reasoning process in the mind and then provide the user with the answer.
User Prompt
The following is a single image containing both a Question and a Student Answer. Your task is to analyze the Student Answer to determine whether there is any error. Begin by providing a detailed reasoning for your analysis, explaining where and why you believe an error is present or absent in the Student Answer. After the reasoning, provide a final output indicating whether the Student Answer is correct or incorrect. The error location must be a specific mathematical expression in the Student Answer. Please follow the exact format below without adding any extra information:
<think> Detailed reasoning for your analysis, explaining where and why you believe an error is present or absent in the Student Answer. </think>
<correctness> Correct or Incorrect </correctness>
<localization> The localization of the error in the Student Answer. Write None if there is no error. </localization>
Few-shot Prompt
<think> The Student Answer correctly identifies the slope and y-intercept of the original line as 2 and 3, respectively. The equation of the translated line is also correctly written as $y = 2x + 3 + a$. The substitution of the point (5, 6) into the equation is also done correctly: $6 = 2 \times 5 + 3 + a$, which simplifies to $6 = 13 + a$. The final step of solving for a is correct as well: $a = -7$. The Student Answer follows the same logical steps as the Reference Answer, and no errors are present in the calculations or reasoning. </think>
<correctness> Correct </correctness>
<localization> None </localization>
Input
Question and Student answer: <image> Reference answer: reference answer

Figure 10: Prompt examples for the inferencing on FERMAT.

Models	AIHub – Primary School				AIHub – Middle School				AIHub – High School			
	ED		EL		ED		EL		ED		EL	
	Acc	F1	Acc	F1	Acc	F1	Acc	F1	Acc	F1	Acc	F1
Closed-source Models												
GPT-4o	74.00	49.35	64.81	64.51	75.24	75.17	65.71	65.68	75.34	75.34	66.25	66.04
GPT-4o-mini	58.60	34.83	40.45	38.65	56.10	31.45	37.42	34.34	55.90	31.52	39.65	35.65
Gemini-2.0-Flash	75.59	50.43	67.58	67.52	74.98	50.01	66.84	66.76	76.40	50.95	68.96	68.85
Gemini-2.5-Flash-Preview	71.20	46.97	62.94	62.67	70.91	46.86	62.56	62.35	70.70	70.23	64.99	64.82
Open-source Models												
Phi-4-multimodal-instruct	38.01	28.91	35.27	32.79	39.84	30.23	36.50	34.32	35.59	27.44	36.17	34.13
Qwen2.5-VL-7B-Instruct	46.66	32.58	39.01	38.69	47.53	33.35	39.33	39.26	48.65	33.65	37.72	37.71
Pixtral-12B	52.70	31.90	32.03	31.52	52.76	31.41	32.19	31.31	52.71	32.14	32.88	32.25
Llama-3.2-11B-Vision-Instruct	19.40	18.34	44.58	33.29	19.77	18.78	44.55	33.40	18.67	18.15	46.42	32.77
VEHME-Qwen2.5-VL-7B	72.10	48.60	60.04	59.85	73.77	49.76	61.50	61.33	73.40	49.35	63.83	63.45

Table 5: Performance comparisons of state-of-the-art Vision-Language Models on different education levels in the AIHub dataset. The evaluation metrics include Accuracy (Acc) and F1 score (F1). **ED**: Error Detection, **EL**: Error Localization. All of the reported results are in percentages (%).

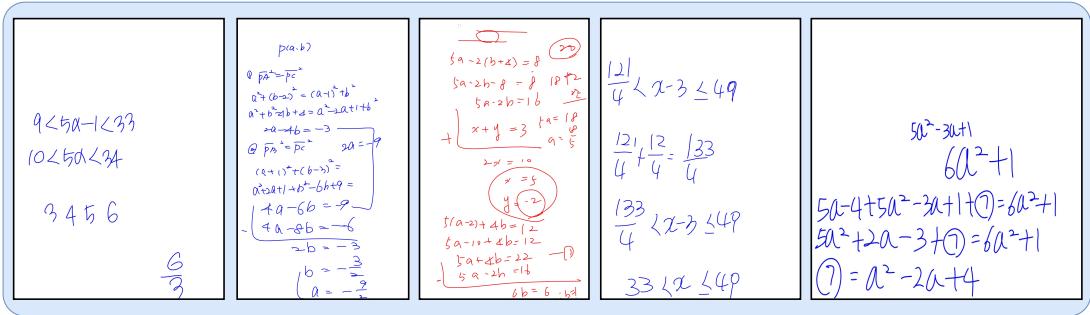


Figure 11: Sample data from the test set of the AIHub dataset.

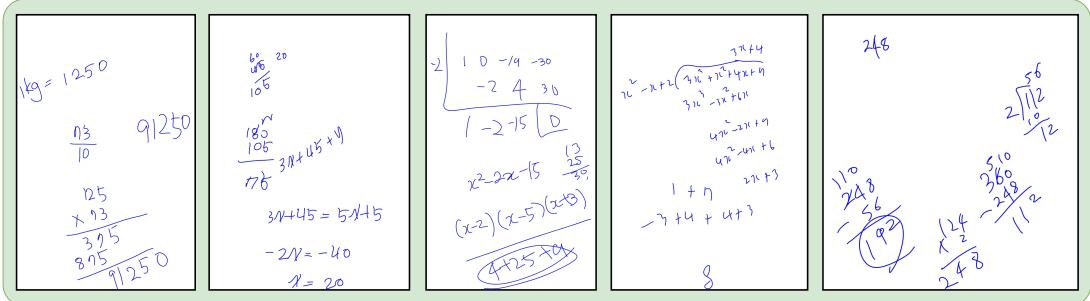


Figure 12: Examples containing heavily rotated mathematical expressions from Challenging Subset of AIHub.

all three education levels. It achieves 72.10% / 48.60% (ED) and 60.04% / 59.85% (EL) at the Primary School level, climbing to 73.40% / 49.35% (ED) and 63.83% / 63.45% (EL) at the High School level. This trend demonstrates that VEHME not only generalizes well across increasing difficulty but also narrows the gap with closed-source systems in EL, where precise localization becomes more challenging.

Overall, our findings suggest that performance is largely consistent across grade levels, with no significant degradation or sensitivity to problem complexity. This indicates that the models' ability to detect and localize errors is not strongly dependent on the education level of the task.

A.6 Challenging Subset of AIHub

In our analysis, most of the standard dataset has mild rotation (< 15°) (as shown in Figure 11), which might explain the relatively small global improvement from EVPM. To better understand EVPM's contribution, we constructed a “*Challenging Subset of AIHub*” by identifying the 100 handwritten data points with the most heavily rotated mathematical expressions (top-100; Mean: 21.81°, Std: 3.40°, Min: 17.56°, Max: 33.20°) as shown in the Figure 12.

A.7 Qualitative Result

We further compare our qualitative results against other baselines, as shown in Figure 13 and Figure 14.

Pixtral GPT-4o Phi-4 Qwen2.5 Llama3.2 Gemini VHEME

Find the maximum value of real number a when there are 3 negative integers x that satisfy the simultaneous inequalities $\begin{cases} 5x+2 < 3(x+2) \\ 9-x < 4a \end{cases}$.

Question: <image1>

Reference Answer:

$$\begin{aligned} 5x+2 &< 3(x+2), \quad 5x+2 < 3x+6, \quad 2x < 4 \quad \therefore x < 2 \\ 9-x &< 4a, \quad -x < 4a-9 \quad \therefore x > -4a+9 \end{aligned}$$

In order for the number of negative integers x to satisfy the given system of inequalities to be 3, it must be as shown in the figure, so $-4 \leq 4a+9 < -3$

$$\therefore 3 < a \leq \frac{13}{4}$$

Therefore, the maximum value of real number a is $\frac{13}{4}$.

Student Answer: <image2>

Answer: Correct

Think: The Student Answer provides a visualization and step-by-step solution for the given system of inequalities. Here's a breakdown of the solution: (1.) $5x + 2 < 3(x + 2)$ - Expand the right side: $5x + 2 < 3x + 6$ - Subtract $3x$ and 2 from both sides: $2x < 4$ - Divide by 2: $x < 2$ (2.) $9 - x < 4a$: - Add x to both sides: $9 - x < 4a$ - Multiply by -1 (and flip the inequality): $x > -4a + 9$ (3.) For the number of negative integers x to be 3, the interval $-4 \leq -4a + 9 < -3$ must hold true: - Subtract 9 from all sides: $-13 \leq -4a < -12$ - Divide by -4 (and flip the inequality): $\frac{13}{4} < a \leq \frac{13}{3}$. (4.) The maximum value of real number a is $\frac{13}{3}$ (since $\frac{13}{3} > \frac{13}{4}$). The Student Answer contains the correct steps to solve the system of inequalities and determine the maximum value of a .

Correctness: Correct ✓

Think: The Student Answer begins by rewriting the inequalities but makes an initial error in the transformation. The original inequality $5x + 2 < 3(x + 2)$ simplifies correctly to $5x + 2 < 3x + 6$, and then to $2x < 4$, which implies $x < 2$. However, the student does not follow this transformation correctly. Instead, they write $x - x < 6 - 2$, which is not a valid manipulation of the inequality and leads to $x < 2$ but through an incorrect method. Furthermore, while the student attempts to establish the conditions for x based on the inequalities $9 - x < 4a$ and $-x < 4a - 9$, they do not derive $x > -4a + 9$ accurately as shown in the Reference Answer. They derive the numerical bounds for a but make mistakes in the algebraic steps leading to those bounds. Finally, while the conclusion $3 < a \leq \frac{13}{4}$ seems correct, the path to arrive there is flawed. Thus, the inconsistencies in reasoning and algebra suggest that the Student Answer contains errors in both the understanding and manipulation of the inequalities.

Correctness: Incorrect ✗

Think: The student answer provided appears to have a few calculation steps that are missing or incorrectly written. **The reference solution seems to be dealing with solving a system of inequalities, but the student answer provided seems to be about finding the maximum value of a variable, not solving inequalities.** The reference answer is a sequence of algebraic manipulations to solve inequalities step by step, whereas the student answer does not clearly follow a similar process. Additionally, the student answer does not provide the required system of inequalities but instead presents an unrelated inequality manipulation. There are no algebraic steps that can be directly compared with the reference answer. The reference answer involves finding an interval for a variable 'a', not solving for 'x' directly. The reference answer and the student answer seem to address different problems. The reference answer solves a series of inequalities, while the student's response seems to be an attempt to solve another set of inequalities not provided in the question. Without the complete and correct inequalities that need to be solved, it is impossible to determine if the student's answer is correct or incorrect based on the given information.

Correctness: Incorrect ✗

Think: The Student Answer appears to have solved the inequalities $5x + 2 < 3(x + 2)$ and $9 - x < 4a$ separately and then combined the results to find the range of a . The process seems correct in terms of isolating x in both inequalities, but the final step of combining the inequalities is not shown correctly. The student has not derived the range for a from the combined inequalities. **The inequalities $-4 \leq -4a + 9 < -3$ should be combined to find the range for a , but the student has not done this.** Therefore, the student's answer is incomplete and does not provide the correct maximum value for a .

Correctness: Incorrect ✗

Think: The Student Answer is correct. The correct answer is $\frac{13}{4}$.

Correctness: Incorrect ✗

Think: The student correctly solves the first inequality to get $x < 2$. For the second inequality, they write $9 - 4a < x$. Since the question asks for 3 negative integers, $x = -1, -2, -3$ must satisfy the inequality. Thus, -3 must be greater than $9 - 4a$, and -4 must be less than or equal to $9 - 4a$. Therefore, $-4 \leq 9 - 4a < -3$. Solving the inequalities: $-4 \leq 9 - 4a \rightarrow 4a \leq 13 \rightarrow a \leq \frac{13}{4}$, and $9 - 4a < -3 \rightarrow 12 < 4a \rightarrow 3 < a$. Thus, $3 < a \leq \frac{13}{4}$. **The student makes an error when writing $-4 < 9 - 4a < -3$. It should be $-4 \leq 9 - 4a < -3$.** They also make an error by directly writing $12 \leq 4a$. After subtracting 9 from $-4 < 9 - 4a \leq -3$, we get $-13 \leq -4a \leq -12$. Dividing by -4 and flipping signs, we get $3 \leq a \leq \frac{13}{4}$. So the student made some mistakes in handling inequalities.

Correctness: Incorrect ✗

Think: To verify the Student's solution, I'll carefully examine their algebraic manipulations and logical steps. From the image, the inequalities are simplified correctly, leading to bounds on a . The intervals drawn also align properly with their final inequality. Their working matches the reference method, showing each transformation accurately. The key part, finding $-4 \leq$ expression < -3 , is derived correctly, ensuring three integer solutions between those. This results in the right conclusion about $3 < a \leq \frac{13}{4}$. The intermediate equations show proper intermediate values, maintaining chain of thought. No visible mistakes regarding arithmetic operations or inequality sign handling; all transformations appear valid. The equality check confirms correctness. Thus, the problem solve accurately mirrors expected steps.

Correctness: Correct ✓

Figure 13: Qualitative example for Error Detection (ED). Among the 7 models, VEHME provides the most correct error detection. The model's error is colored in red. The incorrect response from the models can be categorized into 3 main types: “recognition error”, “problem misunderstanding”, and “hallucination”. Pixtral, GPT, Qwen2.5, and Gemini all suffer from incorrect OCR of the student’s handwritten mathematical expressions. While Pixtral still produces the correct assessment, its thought was wrong due to incorrect OCR. Phi-4 misinterprets the problem as solving the inequality, while the problem is asking to find the maximum value of a . Llama3.2 suffers from hallucination where its thought is correct, but the final assessment is *incorrect*.

There is a rectangle whose length is 5 cm longer than its width. If the perimeter of this rectangle is no longer than 110 cm, find how much less than its width must be.
<image1>

Question: **<image1>**

Reference Answer: If the width is x cm, the height is $(x + 5)$ cm, so

$$\begin{aligned} 2(x + (x + 5)) &\leq 110 \\ 2(2x + 5) &\leq 110 \\ 4x + 10 &\leq 110 \\ 4x &\leq 100 \end{aligned}$$

$\therefore x \leq 25$. Therefore, the width must be less than 25 cm.

Student Answer: **<image2>**

Answer: Incorrect

$$\begin{aligned} &x \\ &x+5 \\ &2x+5 \\ &4x \leq 110 \\ &4x \leq 100 \end{aligned}$$

<image2>



Think: The Student Answer is providing a sketch of the rectangle with its width as x and height as $x + 5$. **The inequality provided is $2x + 5 \leq 110$** , which is not the correct formulation for the problem described. The reference solution correctly states the problem as $2(x + (x + 5)) \leq 110$, which simplifies to $2(2x + 5) \leq 110$. The Student Answer misses the step of including the total perimeter calculation and simplifies incorrectly to $2x + 5 \leq 110$. The correct approach should be $2(2x + 5) \leq 110 \rightarrow 4x + 10 \leq 110$, then $4x \leq 100$, and finally $x \leq 25$. However, **the Student Answer incorrectly simplifies to $4x \leq 10$, leading to $x \leq 2.5$** . This is not correct based on the given problem. The localization of the error is that the student missed the correct step in including the complete perimeter calculation and incorrectly simplified the inequality.

Correctness: Incorrect

Error Localization: The inequality was **incorrectly simplified to $4x \leq 10$** instead of $4x \leq 100$. **X**



Think: The student begins by correctly identifying the expressions for the width x and the length $x + 5$ of the rectangle. The expression for the perimeter, $2(x + (x + 5))$, simplifies to $4x + 10$. **The student writes $2x + 5$, which is incorrect for finding the perimeter**. The correct inequality should be $4x + 10 \leq 110$, simplifying to $4x \leq 100$. The student incorrectly writes $4x \leq 120$. Thus, they made an error in simplifying and setting up the inequality.

Correctness: Incorrect

Error Localization: **Incorrect perimeter calculation** and inequality setup. **X**



Think: The student incorrectly solved the inequality. They correctly started with the perimeter formula $2(x + (x + 5))$ and wrote $2(2x + 5)$, but then made an error in simplifying the inequality. The reference answer shows the correct simplification: $4x + 10 \leq 110$. The student **did not proceed beyond the initial simplification step**, leaving their solution incomplete and incorrect.

Their final conclusion also lacks the necessary steps to derive the correct width constraint, which is $x \leq 25$. Without this step, their answer does not confirm the problem's solution.

The student did not include the confirmation step about the perimeter being less than the given value if $x < 25$, which is a part of the complete solution process.

Correctness: Incorrect

Error Localization: The error occurs in the simplification step of the inequality and the conclusion step. The **student stopped after the correct formula $2(2x + 5)$** without solving for x , and they did not mention the confirmation step involving the perimeter. **X**



Think: The student's equation simplifies to $4x \leq 120$, but there is a recognizable error in the addition of the constants. The correct expression for the perimeter should be $2(x + (x + 5)) = 2(2x + 5)$. The student **mistakenly wrote $2x + 5$ instead of $2(2x + 5)$** . This is a mistake in the representation of the perimeter, which changes the inequality solution.

Correctness: Incorrect

Error Localization: The error is localized in the simplification step where the student **writes $2x + 5$ instead of $2(2x + 5)$** in the perimeter expression. **X**



Think: To determine if the Student Answer is correct, we need to analyze the reasoning and calculations presented. The problem states that the width of a rectangle is less than 25 cm and the perimeter is less than 110 cm. The Student Answer has determined the width and perimeter, **but it has not determined the number of sides**. Therefore, the Student Answer is incorrect.

Correctness: Incorrect

Error Localization: The localization of the error in the Student Answer is in the determination of the number of sides. The Student Answer has not determined the number of sides, which is a crucial aspect of the problem. The Student Answer should have determined the number of sides, but it has not done so. **X**



Think: The student starts correctly by representing the width by x and the length by $x + 5$. The student sets up the inequality correctly. However, there is a calculation error when subtracting 10 from both sides, **they incorrectly get 20 instead of 100**.

Correctness: Incorrect

Error Localization: The student made an error while isolating x on the left-hand side. They **incorrectly wrote $4x \leq 20$ instead of $4x \leq 100$** . **X**



Think: I'll review the handwritten notes for clarity and correctness. The student seems to have sketched the problem diagrammatically but has missing steps. Numerically, their work indicates setting up equations like " $4x \leq 120$ ", which is incorrect, as the previous correct solution was solving " $4x \leq 100$ " right off. There's also a visible typo where " $x + 5$ " might be simplified prematurely or incorrectly written. The layout shows calculations cut short, ending with a flawed outcome. The perimeter inequality conversion isn't visibly sound. Without seeing intermediate workings, it's speculative, but errors can involve algebraic mistakes or skipped necessary conditions checks. The result here would imply miscalculation, leading me to conclude potential miscalculation or misunderstanding key steps.

Correctness: Incorrect

Error Localization: The equations show partial results (" $2x + 5$," " $4x + 10 \leq 110$ ") but terminate abruptly at an incorrect culmination (" $4x \leq 120$ "). Upon comparison with standard methods, the solution should focus on isolating x after forming the inequality, ensuring all terms align logically. Here, the presented process skips important transpositions and lacks proper derivation checks, indicating an arithmetic mistake. **✓**

Figure 14: Qualitative example for Error Localization (EL). Among the 7 models, VEHME provides the most correct localization. The model's error is colored in red. The incorrect response from the models can be categorized into 2 main types: “recognition error” and “student’s answer misunderstanding”. Pixtral, Phi-4, and Gemini all suffer from incorrect OCR of the student’s handwritten mathematical expressions. Pixtral and Gemini misread the inequality $4x \leq 120$ while Phi-4 cannot read beyond the inequality $2x + 5$. GPT, Qwen2.5, and Llama3.2 misinterpret the student’s answer. GPT and Qwen2.5 think that the student writes $2x + 5$ as the perimeter, while Llama3.2 thinks that the student must determine the number of sides.