Introduction to Electromagnetic Waves

Human eyes detect these orange “sea goldie” fish swimming over a coral reef in the blue waters of the Gulf of Eilat (Red Sea) using visible light. (credit: Daviddarom, Wikimedia Commons)

The beauty of a coral reef, the warm radiance of sunshine, the sting of sunburn, the X-ray revealing a broken bone, even microwave popcorn—all are brought to us by **electromagnetic waves**. The list of the various types of electromagnetic waves, ranging from radio transmission waves to nuclear gamma-ray (γ -ray) emissions, is interesting in itself.

Even more intriguing is that all of these widely varied phenomena are different manifestations of the same thing—electromagnetic waves. (See [Figure 2].) What are electromagnetic waves? How are they created, and how do they travel? How can we understand and organize their widely varying properties? What is their relationship to electric and magnetic effects? These and other questions will be explored.

Misconception Alert: Sound Waves vs. Radio Waves

Many people confuse sound waves with **radio waves**, one type of electromagnetic (EM) wave. However, sound and radio waves are completely different phenomena. Sound creates pressure variations (waves) in matter, such as air or water, or your eardrum. Conversely, radio waves are *electromagnetic waves*, **like visible light, infrared, ultraviolet, X-rays, and gamma rays. EM waves don’t need a medium in which to propagate; they can travel through a vacuum, such as outer space.

A radio works because sound waves played by the D.J. at the radio station are converted into electromagnetic waves, then encoded and transmitted in the radio-frequency range. The radio in your car receives the radio waves, decodes the information, and uses a speaker to change it back into a sound wave, bringing sweet music to your ears.

Discovering a New Phenomenon

It is worth noting at the outset that the general phenomenon of electromagnetic waves was predicted by theory before it was realized that light is a form of electromagnetic wave. The prediction was made by James Clerk Maxwell in the mid-19th century when he formulated a single theory combining all the electric and magnetic effects known by scientists at that time. “Electromagnetic waves” was the name he gave to the phenomena his theory predicted.

Such a theoretical prediction followed by experimental verification is an indication of the power of science in general, and physics in particular. The underlying connections and unity of physics allow certain great minds to solve puzzles without having all the pieces. The prediction of electromagnetic waves is one of the most spectacular examples of this power. Certain others, such as the prediction of antimatter, will be discussed in later modules.

The electromagnetic waves sent and received by this 50-foot radar dish antenna at Kennedy Space Center in Florida are not visible, but help track expendable launch vehicles with high-definition imagery. The first use of this C-band radar dish was for the launch of the Atlas V rocket sending the New Horizons probe toward Pluto. (credit: NASA)

Maxwell’s Equations: Electromagnetic Waves Predicted and Observed

- Restate Maxwell’s equations.

The Scotsman James Clerk Maxwell (1831–1879) is regarded as the greatest theoretical physicist of the 19th century. (See [Figure 1].) Although he died young, Maxwell not only formulated a complete electromagnetic theory, represented by **Maxwell’s equations**, he also developed the kinetic theory of gases and made significant contributions to the understanding of color vision and the nature of Saturn’s rings.

James Clerk Maxwell, a 19th-century physicist, developed a theory that explained the relationship between electricity and magnetism and correctly predicted that visible light is caused by electromagnetic waves. (credit: G. J. Stodart)

Maxwell brought together all the work that had been done by brilliant physicists such as Oersted, Coulomb, Gauss, and Faraday, and added his own insights to develop the overarching theory of electromagnetism. Maxwell’s equations are paraphrased here in words because their mathematical statement is beyond the level of this text. However, the equations illustrate how apparently simple mathematical statements can elegantly unite and express a multitude of concepts—why mathematics is the language of science.

Maxwell’s Equations

1. **Electric field lines** originate on positive charges and terminate on negative charges. The electric field is defined as the force per unit charge on a test charge, and the strength of the force is related to the electric constant ϵ_0 , also known as the permittivity of free space. From Maxwell’s first equation we obtain a special form of Coulomb’s law known as Gauss’s law for electricity.
2. **Magnetic field lines** are continuous, having no beginning or end. No magnetic monopoles are known to exist. The strength of the magnetic force is related to the magnetic constant μ_0 , also known as the permeability of free space. This second of Maxwell’s equations is known as Gauss’s law for magnetism.
3. A changing magnetic field induces an electromotive force (emf) and, hence, an electric field. The direction of the emf opposes the change. This third of Maxwell’s equations is Faraday’s law of induction, and includes Lenz’s law.
4. Magnetic fields are generated by moving charges or by changing electric fields. This fourth of Maxwell’s equations encompasses Ampere’s law and adds another source of magnetism—changing electric fields.

Maxwell’s equations encompass the major laws of electricity and magnetism. What is not so apparent is the symmetry that Maxwell introduced in his mathematical framework. Especially important is his addition of the hypothesis that changing electric fields create magnetic fields. This is exactly analogous (and symmetric) to Faraday's law of induction and had been suspected for some time, but fits beautifully into Maxwell's equations.

Symmetry is apparent in nature in a wide range of situations. In contemporary research, symmetry plays a major part in the search for sub-atomic particles using massive multinational particle accelerators such as the new Large Hadron Collider at CERN.

Making Connections: Unification of Forces

Maxwell's complete and symmetric theory showed that electric and magnetic forces are not separate, but different manifestations of the same thing—the electromagnetic force. This classical unification of forces is one motivation for current attempts to unify the four basic forces in nature—the gravitational, electrical, strong, and weak nuclear forces.

Since changing electric fields create relatively weak magnetic fields, they could not be easily detected at the time of Maxwell's hypothesis. Maxwell realized, however, that oscillating charges, like those in AC circuits, produce changing electric fields. He predicted that these changing fields would propagate from the source like waves generated on a lake by a jumping fish.

The waves predicted by Maxwell would consist of oscillating electric and magnetic fields—defined to be an electromagnetic wave (EM wave). Electromagnetic waves would be capable of exerting forces on charges great distances from their source, and they might thus be detectable. Maxwell calculated that electromagnetic waves would propagate at a speed given by the equation

$$c=\frac{1}{\sqrt{\mu_0\epsilon_0}}.$$

When the values for μ_0 and ϵ_0 are entered into the equation for c , we find that

$$c=\frac{1}{\sqrt{\left(8.85\times10^{-12}\frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)\left(4\pi\times10^{-7}\frac{\text{T}\cdot\text{m}}{\text{A}}\right)}}=3.00\times10^{8}\text{m/s},$$

which is the speed of light. In fact, Maxwell concluded that light is an electromagnetic wave having such wavelengths that it can be detected by the eye.

Other wavelengths should exist—it remained to be seen if they did. If so, Maxwell's theory and remarkable predictions would be verified, the greatest triumph of physics since Newton. Experimental verification came within a few years, but not before Maxwell's death.

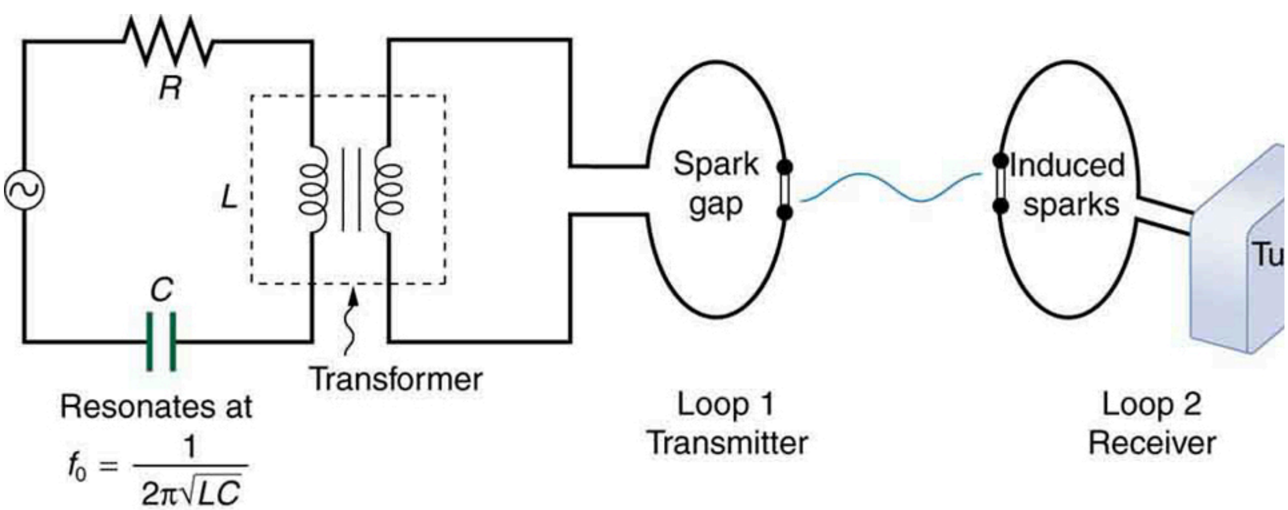
Hertz's Observations

The German physicist Heinrich Hertz (1857–1894) was the first to generate and detect certain types of electromagnetic waves in the laboratory. Starting in 1887, he performed a series of experiments that not only confirmed the existence of electromagnetic waves, but also verified that they travel at the speed of light.

Hertz used an AC RLC (resistor-inductor-capacitor) circuit that resonates at a known frequency $f_0=\frac{1}{2\pi\sqrt{LC}}$ and connected it to a loop of wire as shown in [Figure 2]. High voltages induced across the gap in the loop produced sparks that were visible evidence of the current in the circuit and that helped generate electromagnetic waves.

Across the laboratory, Hertz had another loop attached to another RLC

circuit, which could be tuned (as the dial on a radio) to the same resonant frequency as the first and could, thus, be made to receive electromagnetic waves. This loop also had a gap across which sparks were generated, giving solid evidence that electromagnetic waves had been received.



The apparatus used by Hertz in 1887 to generate and detect electromagnetic waves. An RLC circuit connected to the first loop caused sparks across a gap in the wire loop and generated electromagnetic waves. Sparks across a gap in the second loop located across the laboratory gave evidence that the waves had been received.

Hertz also studied the reflection, refraction, and interference patterns of the electromagnetic waves he generated, verifying their wave character. He was able to determine wavelength from the interference patterns, and knowing their frequency, he could calculate the propagation speed using the equation $v=f\lambda$ (velocity—or speed—equals frequency times wavelength). Hertz was thus able to prove that electromagnetic waves travel at the speed of light. The SI unit for frequency, the hertz ($1Hz=1\text{cycle/sec}$), is named in his honor.

Section Summary

- Electromagnetic waves consist of oscillating electric and magnetic fields and propagate at the speed of light c. They were predicted by Maxwell, who also showed that

  $$c=\frac{1}{\sqrt{\mu_0\epsilon_0}},$$

where μ_0 is the permeability of free space and ϵ_0 is the permittivity of free space.

- Maxwell's prediction of electromagnetic waves resulted from his formulation of a complete and symmetric theory of electricity and magnetism, known as Maxwell's equations.
- These four equations are paraphrased in this text, rather than presented numerically, and encompass the major laws of electricity and magnetism. First is Gauss's law for electricity, second is Gauss's law for magnetism, third is Faraday's law of induction, including Lenz's law, and fourth is Ampere's law in a symmetric formulation that adds another source of magnetism—changing electric fields.

Problems & Exercises

Strategy

Verify that the correct value for the speed of light c is obtained when numerical values for the permeability and permittivity of free space (μ_0 and ϵ_0) are entered into the equation $c=\frac{1}{\sqrt{\mu_0\epsilon_0}}$.

Show Solution

To verify that Maxwell's equation correctly predicts the speed of light, we need to substitute the known values of the permeability of free space μ_0 and the permittivity of free space ϵ_0 into the equation $c=\frac{1}{\sqrt{\mu_0\epsilon_0}}$ and confirm that the result equals the experimentally measured speed of light, 3.00×10^8 m/s.

Solution

The permeability of free space is:

$$\mu_0=4\pi\times10^{-7}\ \frac{\text{T}\cdot\text{m}}{\text{A}}$$

The permittivity of free space is:

$$\epsilon_0=8.85\times10^{-12}\ \frac{\text{C}^2}{\text{N}\cdot\text{m}^2}$$

First, let's calculate the product $\mu_0\epsilon_0$:

$$\mu_0\epsilon_0=\left(4\pi\times10^{-7}\ \frac{\text{T}\cdot\text{m}}{\text{A}}\right)\left(8.85\times10^{-12}\ \frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)$$
$$\mu_0\epsilon_0=(4\pi\times8.85)\times10^{-19}\ \frac{\text{T}\cdot\text{m}}{\text{A}}\cdot\frac{\text{C}^2}{\text{N}\cdot\text{m}^2}$$
$$\mu_0\epsilon_0\approx1.112\times10^{-17}\ \frac{\text{s}^2}{\text{m}^2}$$

Now we can calculate the speed of light:

$$c=\frac{1}{\sqrt{\mu_0\epsilon_0}}=\frac{1}{\sqrt{1.112\times10^{-17}\ \frac{\text{s}^2}{\text{m}^2}}}$$
$$c=\frac{1}{3.336\times10^{-9}\ \frac{\text{s}}{\text{m}}}=3.00\times10^8\ \text{m/s}$$

Discussion

This calculation demonstrates one of the most profound insights in physics: Maxwell's equations, which were formulated to describe electromagnetic phenomena, predict that electromagnetic waves travel at exactly the speed of light. This was not a coincidence—it led Maxwell to the revolutionary conclusion that light itself is an electromagnetic wave. The fact that fundamental constants from electricity (ϵ_0) and magnetism (μ_0) combine to give the speed of light reveals the deep connection between these seemingly separate phenomena. This verification was a triumph of theoretical physics, as Maxwell's prediction was later confirmed experimentally by Heinrich Hertz.

Final Answer

When the numerical values $\mu_0=4\pi\times10^{-7}$ T·m/A and $\epsilon_0=8.85\times10^{-12}\ \text{C}^2/(\text{N}\cdot\text{m}^2)$ are substituted into Maxwell's equation $c=\frac{1}{\sqrt{\mu_0\epsilon_0}}$, we obtain $c=3.00\times10^8$ m/s, which is indeed the correct experimentally measured value for the speed of light.

Strategy

Show that, when SI units for μ_0 and ϵ_0 are entered, the units given by the right-hand side of the equation in the problem above are m/s.

Show Solution

To verify that the equation $c = 1\sqrt{\mu_0 \epsilon_0}$ gives units of velocity (m/s), we need to substitute the SI units for μ_0 and ϵ_0 into the equation and perform unit analysis to show that all units cancel properly to yield m/s. We'll use the fact that SI units can be expressed in terms of fundamental units (meters, kilograms, seconds, amperes, etc.) and that the tesla and newton can be broken down into these base units.

Solution

The SI unit for the permeability of free space μ_0 is:

$$\mathrm{T \cdot mA}$$

The SI unit for the permittivity of free space ϵ_0 is:

$$\mathrm{C^2 N \cdot m^2}$$

We need to express these in terms of fundamental SI units. First, recall that:

- Tesla: $\mathrm{T = kgA \cdot s^2}$
- Newton: $\mathrm{N = kg \cdot ms^2}$
- Coulomb: $\mathrm{C = A \cdot s}$

Substituting for the tesla in μ_0:

$$\mathrm{T \cdot mA = kgA \cdot s^2 \cdot mA = kg \cdot mA^2 \cdot s^2}$$

Substituting for the newton and coulomb in ϵ_0:

$$\mathrm{C^2 N \cdot m^2 = (A \cdot s)^2 kg \cdot ms^2 \cdot m^2 = A^2 \cdot s^2 kg \cdot m^3 \cdot s^2 = A^2 \cdot s^4 kg \cdot m^3}$$

Now we can find the units of $\mu_0 \epsilon_0$:

$$\mu_0 \epsilon_0 = \mathrm{kg \cdot mA^2 \cdot s^2 \cdot A^2 \cdot s^4 kg \cdot m^3 = s^2 m^2}$$

Finally, we can find the units of $c = 1\sqrt{\mu_0 \epsilon_0}$:

$$c = 1\sqrt{\mathrm{s^2 m^2}} = 1\mathrm{sm} = \mathrm{ms}$$

Discussion

This dimensional analysis confirms that Maxwell's equation for the speed of electromagnetic waves is dimensionally consistent. The fact that the units work out to m/s (velocity) is not merely a mathematical convenience—it reflects the physical reality that c represents the propagation speed of electromagnetic waves through space. The cancellation of units also reveals how electromagnetic and mechanical quantities are interconnected. The ampere (A) cancels because it appears in both electrical and magnetic quantities, the kilogram (kg) cancels because it appears in both the definition of force (newton) and magnetic field (tesla), and what remains is purely the ratio of distance to time, which is velocity. This elegant dimensional consistency was part of what gave Maxwell confidence in his theoretical prediction.

Final Answer

When SI units for μ_0 and ϵ_0 are substituted into the equation $c = 1\sqrt{\mu_0 \epsilon_0}$, the units reduce to $\mathrm{m/s}$, confirming that the equation correctly predicts a velocity.

Glossary

electromagnetic waves
: radiation in the form of waves of electric and magnetic energy

Maxwell's equations
: a set of four equations that comprise a complete, overarching theory of electromagnetism

RLC circuit
: an electric circuit that includes a resistor, capacitor and inductor

hertz
: an SI unit denoting the frequency of an electromagnetic wave, in cycles per second

speed of light
: in a vacuum, such as space, the speed of light is a constant 3 x 10^8 m/s

electromotive force (emf)
: energy produced per unit charge, drawn from a source that produces an electrical current

electric field lines
: a pattern of imaginary lines that extend between an electric source and charged objects in the surrounding area, with arrows pointed away from positively charged objects and toward negatively charged objects. The more lines in the pattern, the stronger the electric field in that region

magnetic field lines
: a pattern of continuous, imaginary lines that emerge from and enter into opposite magnetic poles. The density of the lines indicates the magnitude of the magnetic field

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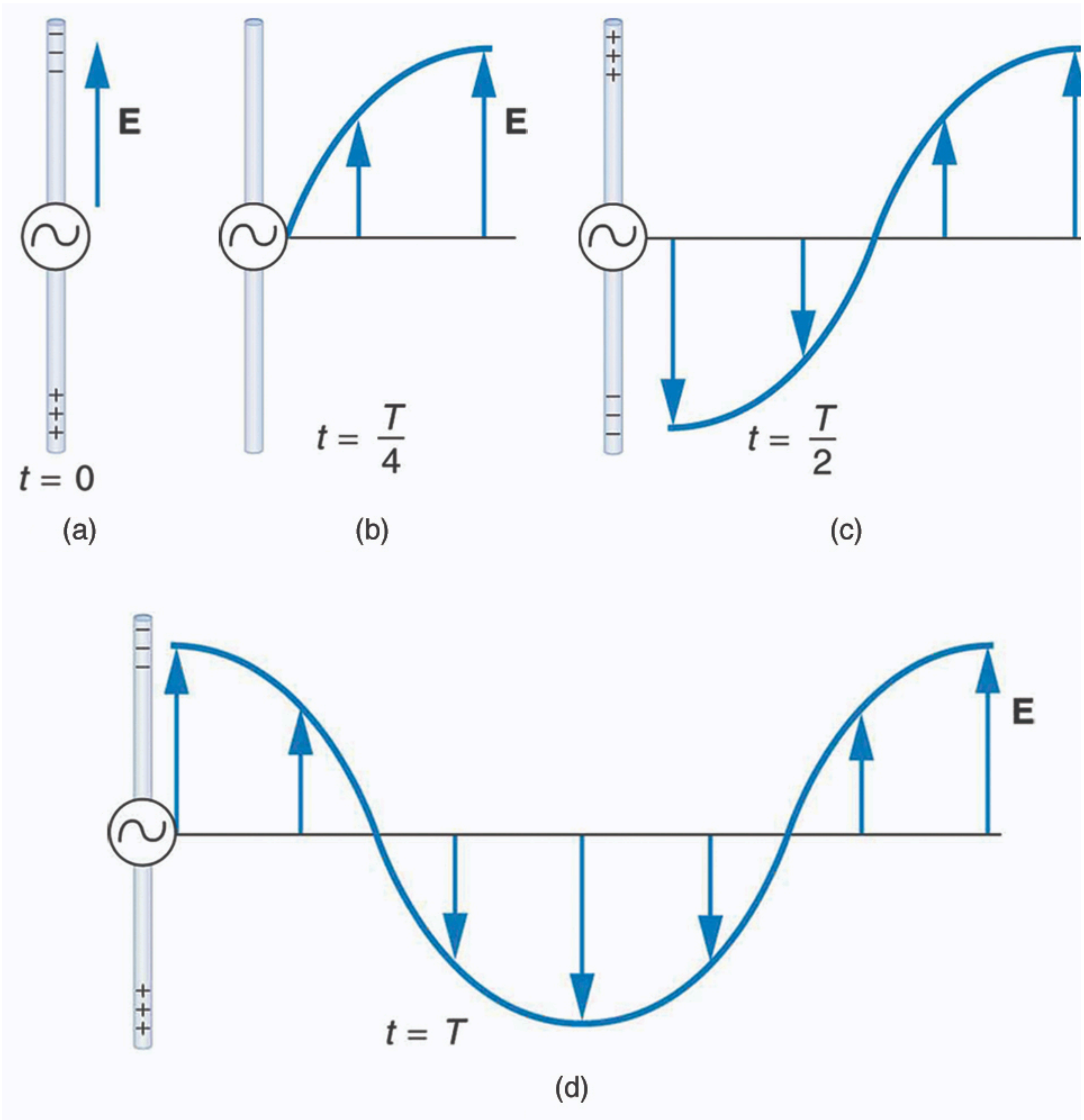
Production of Electromagnetic Waves

- Describe the electric and magnetic waves as they move out from a source, such as an AC generator.
- Explain the mathematical relationship between the magnetic field strength and the electrical field strength.
- Calculate the maximum strength of the magnetic field in an electromagnetic wave, given the maximum electric field strength.

We can get a good understanding of **electromagnetic waves** (EM) by considering how they are produced. Whenever a current varies, associated electric and magnetic fields vary, moving out from the source like waves. Perhaps the easiest situation to visualize is a varying current in a long straight wire, produced by an AC generator at its center, as illustrated in [Figure 1].



This long straight gray wire with an AC generator at its center becomes a broadcast antenna for electromagnetic waves. Shown here are the charge distributions at four different times. The electric field (E) propagates away from the antenna at the speed of light, forming part of an electromagnetic wave.

The **electric field** ($\vec{\mathbf{E}}$) shown surrounding the wire is produced by the charge distribution on the wire. Both the $\vec{\mathbf{E}}$ and the charge distribution vary as the current changes. The changing field propagates outward at the speed of light.

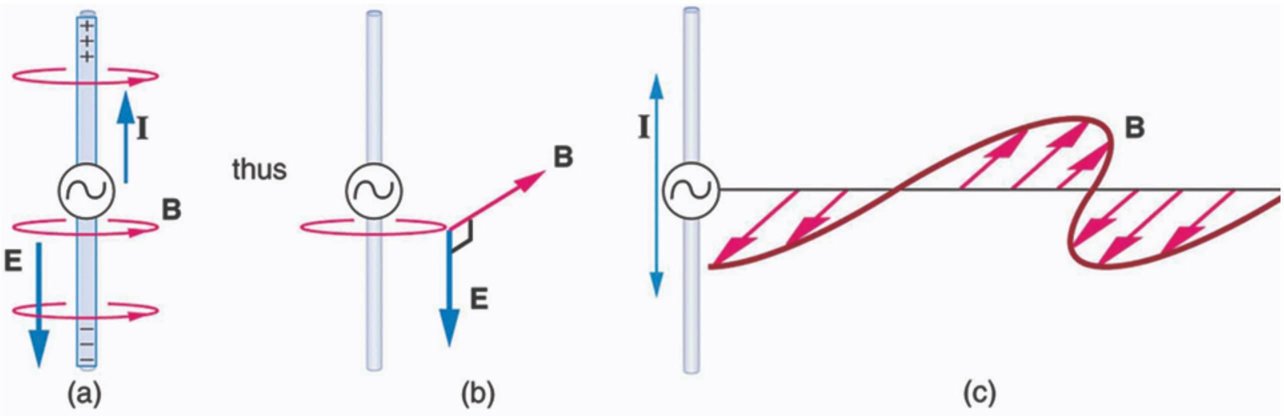
There is an associated **magnetic field** ($\vec{\mathbf{B}}$) which propagates outward as well (see [Figure 2]). The electric and magnetic fields are closely related and propagate as an electromagnetic wave. This is what happens in broadcast antennae such as those in radio and TV stations.

Closer examination of the one complete cycle shown in [Figure 1] reveals the periodic nature of the generator-driven charges oscillating up and down in the antenna and the electric field produced. At time $t = 0$, there is the maximum separation of charge, with negative charges at the top and positive charges at the bottom, producing the maximum magnitude of the electric field (or E -field) in the upward direction. One-fourth of a cycle later, there is no charge separation and the field next to the antenna is zero, while the maximum E -field has moved away at speed c.

As the process continues, the charge separation reverses and the field reaches its maximum downward value, returns to zero, and rises to its maximum upward value at the end of one complete cycle. The outgoing wave has an **amplitude** proportional to the maximum separation of charge. Its **wavelength** (λ) is proportional to the period of the oscillation and, hence, is smaller for short periods or high frequencies. (As usual, wavelength and **frequency** (f) are inversely proportional.)

Electric and Magnetic Waves: Moving Together

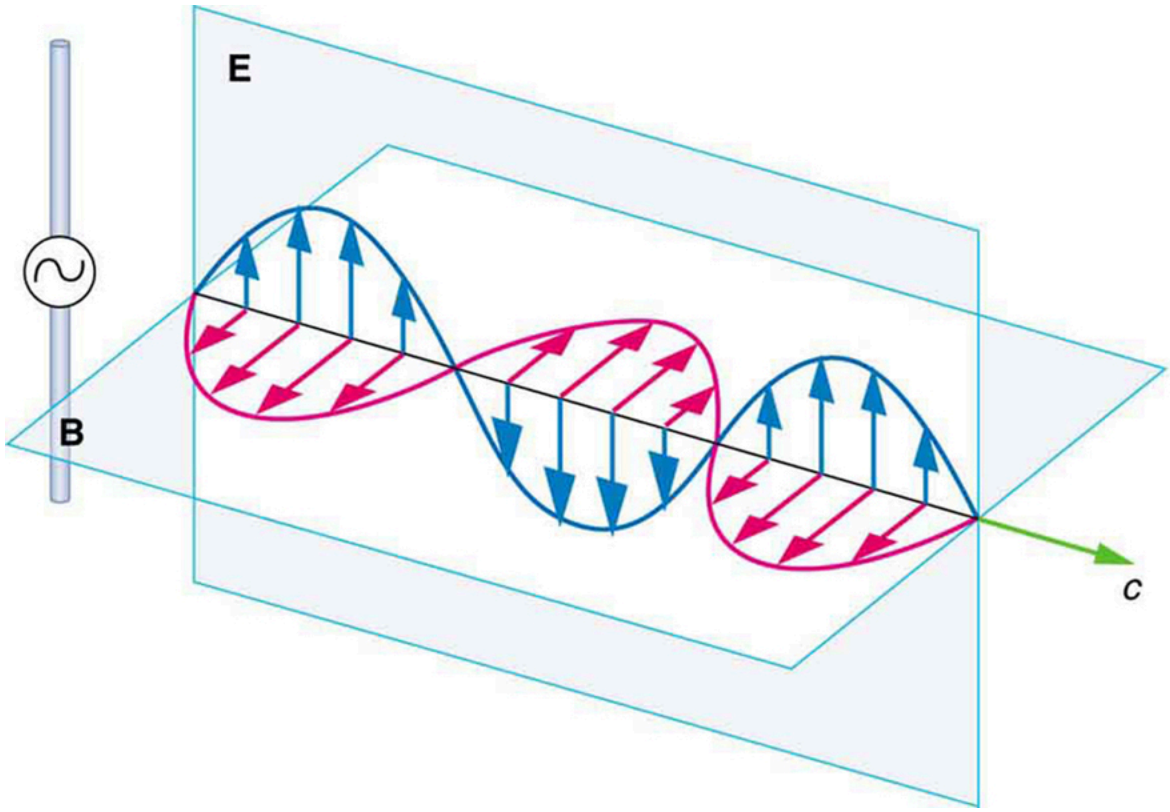
Following Ampere's law, current in the antenna produces a magnetic field, as shown in [Figure 2]. The relationship between $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$ is shown at one instant in [Figure 2] (a). As the current varies, the magnetic field varies in magnitude and direction.



(a) The current in the antenna produces the circular magnetic field lines. The current (I) produces the separation of charge along the wire, which in turn creates the electric field as shown. (b) The electric and magnetic fields (E and B) near the wire are perpendicular; they are shown here for one point in space. (c) The magnetic field varies with current and propagates away from the antenna at the speed of light.

The magnetic field lines also propagate away from the antenna at the speed of light, forming the other part of the electromagnetic wave, as seen in [Figure 2] (b). The magnetic part of the wave has the same period and wavelength as the electric part, since they are both produced by the same movement and separation of charges in the antenna.

The electric and magnetic waves are shown together at one instant in time in [Figure 3]. The electric and magnetic fields produced by a long straight wire antenna are exactly in phase. Note that they are perpendicular to one another and to the direction of propagation, making this a **transverse wave**.



A part of the electromagnetic wave sent out from the antenna at one instant in time. The electric and magnetic fields (E and B) are in phase, and they are perpendicular to one another and the direction of propagation. For clarity, the waves are shown only along one direction, but they propagate out in other directions too.

Electromagnetic waves generally propagate out from a source in all directions, sometimes forming a complex radiation pattern. A linear antenna like this one will not radiate parallel to its length, for example. The wave is shown in one direction from the antenna in [Figure 3] to illustrate its basic characteristics.

Instead of the AC generator, the antenna can also be driven by an AC circuit. In fact, charges radiate whenever they are accelerated. But while a current in a circuit needs a complete path, an antenna has a varying charge distribution forming a **standing wave**, driven by the AC. The dimensions of the antenna are critical for determining the frequency of the radiated electromagnetic waves. This is a **resonant** phenomenon and when we tune radios or TV, we vary electrical properties to achieve appropriate resonant conditions in the antenna.

Receiving Electromagnetic Waves

Electromagnetic waves carry energy away from their source, similar to a sound wave carrying energy away from a standing wave on a guitar string. An antenna for receiving EM signals works in reverse. And like antennas that produce EM waves, receiver antennas are specially designed to resonate at particular frequencies.

An incoming electromagnetic wave accelerates electrons in the antenna, setting up a standing wave. If the radio or TV is switched on, electrical components pick up and amplify the signal formed by the accelerating electrons. The signal is then converted to audio and/or video format. Sometimes big receiver dishes are used to focus the signal onto an antenna.

In fact, charges radiate whenever they are accelerated. When designing circuits, we often assume that energy does not quickly escape AC circuits, and mostly this is true. A broadcast antenna is specially designed to enhance the rate of electromagnetic radiation, and shielding is necessary to keep the radiation close to zero. Some familiar phenomena are based on the production of electromagnetic waves by varying currents. Your microwave oven, for example, sends electromagnetic waves, called microwaves, from a concealed antenna that has an oscillating current imposed on it.

Relating E

-Field and B -Field Strengths

There is a relationship between the E

- and B -field strengths in an electromagnetic wave. This can be understood by again considering the antenna just described. The stronger the E -field created by a separation of charge, the greater the current and, hence, the greater the B -field created.

Since current is directly proportional to voltage (Ohm’s law) and voltage is directly proportional to E -field strength, the two should be directly proportional. It can be shown that the magnitudes of the fields do have a constant ratio, equal to the speed of light. That is,

$$\frac{E}{B}=c$$

is the ratio of E -field strength to B -field strength in any electromagnetic wave. This is true at all times and at all locations in space. A simple and elegant result.

Calculating B -Field Strength in an Electromagnetic Wave

What is the maximum strength of the B -field in an electromagnetic wave that has a maximum E -field strength of 1000 V/m ?

Strategy

To find the B -field strength, we rearrange the above equation to solve for B, yielding

$$B=\frac{E}{c}.$$

Solution

We are given E , and c is the speed of light. Entering these into the expression for B yields

$$B=\frac{1000 \text{ V/m}}{3.00\times10^{8} \text{ m/s}}=3.33\times10^{-6} \text{ T},$$

Where T stands for Tesla, a measure of magnetic field strength.

Discussion

The B -field strength is less than a tenth of the Earth’s admittedly weak magnetic field. This means that a relatively strong electric field of 1000 V/m is accompanied by a relatively weak magnetic field. Note that as this wave spreads out, say with distance from an antenna, its field strengths become progressively weaker.

The result of this example is consistent with the statement made in the module Maxwell’s Equations: Electromagnetic Waves Predicted and Observed that changing electric fields create relatively weak magnetic fields. They can be detected in electromagnetic waves, however, by taking advantage of the phenomenon of resonance, as Hertz did. A system with the same natural frequency as the electromagnetic wave can be made to oscillate. All radio and TV receivers use this principle to pick up and then amplify weak electromagnetic waves, while rejecting all others not at their resonant frequency.

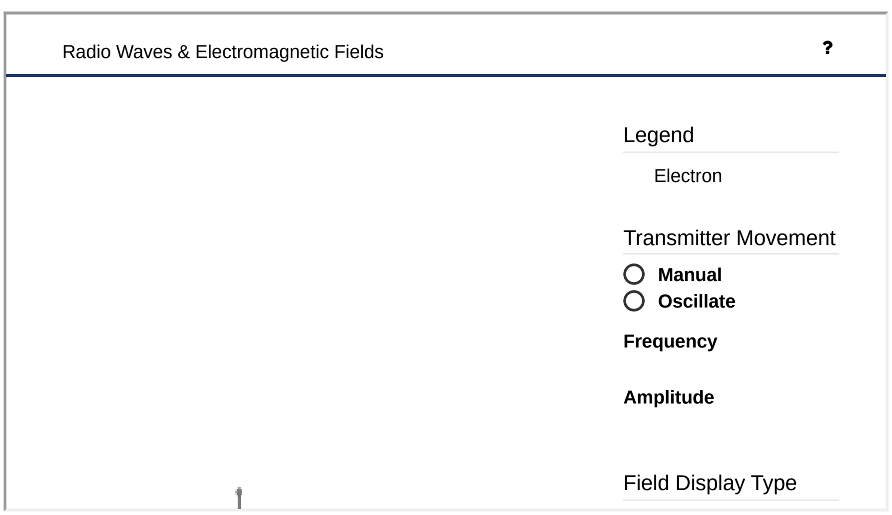
Take-Home Experiment: Antennas

For your TV or radio at home, identify the antenna, and sketch its shape. If you don’t have cable, you might have an outdoor or indoor TV antenna. Estimate its size. If the TV signal is between 60 and 216 MHz for basic channels, then what is the wavelength of those EM waves?

Try tuning the radio and note the small range of frequencies at which a reasonable signal for that station is received. (This is easier with digital readout.) If you have a car with a radio and extendable antenna, note the quality of reception as the length of the antenna is changed.

PhET Explorations: Radio Waves and Electromagnetic Fields

Broadcast radio waves from KPhET. Wiggle the transmitter electron manually or have it oscillate automatically. Display the field as a curve or vectors. The strip chart shows the electron positions at the transmitter and at the receiver.



Section Summary

- Electromagnetic waves are created by oscillating charges (which radiate whenever accelerated) and have the same frequency as the oscillation.
- Since the electric and magnetic fields in most electromagnetic waves are perpendicular to the direction in which the wave moves, it is ordinarily a transverse wave.

- The strengths of the electric and magnetic parts of the wave are related by

  $E B$=c,

which implies that the magnetic field B is very weak relative to the electric field E .

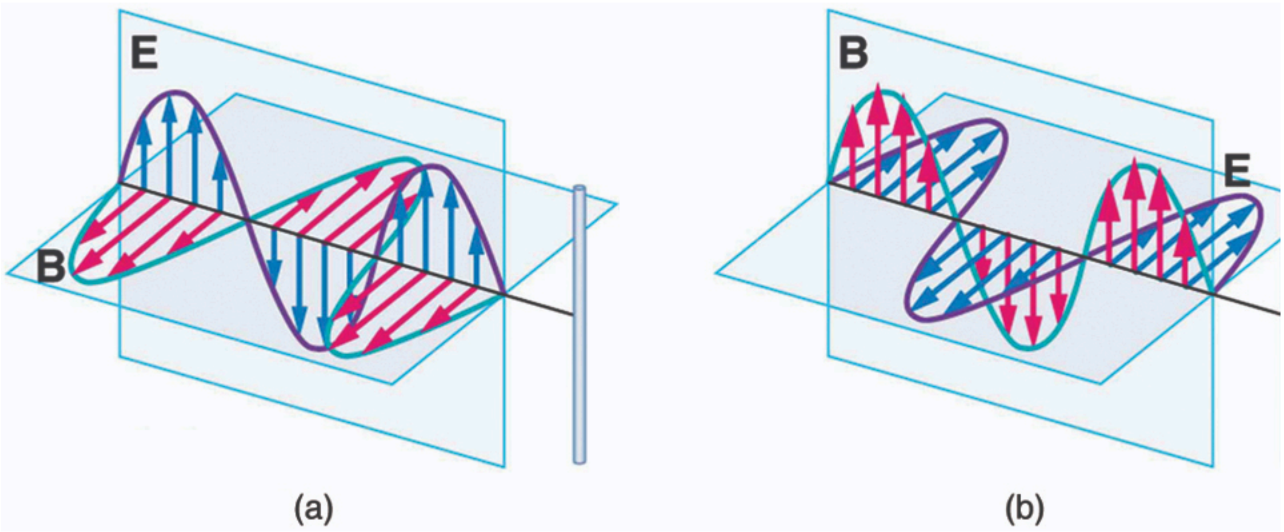
Conceptual Questions

The direction of the electric field shown in each part of [Figure 1] is that produced by the charge distribution in the wire. Justify the direction shown in each part, using the Coulomb force law and the definition of $\vec{\mathbf{E}} = \vec{\mathbf{F}}/q$, where q is a positive test charge.

Is the direction of the magnetic field shown in [Figure 2] (a) consistent with the right-hand rule for current (RHR-2) in the direction shown in the figure?

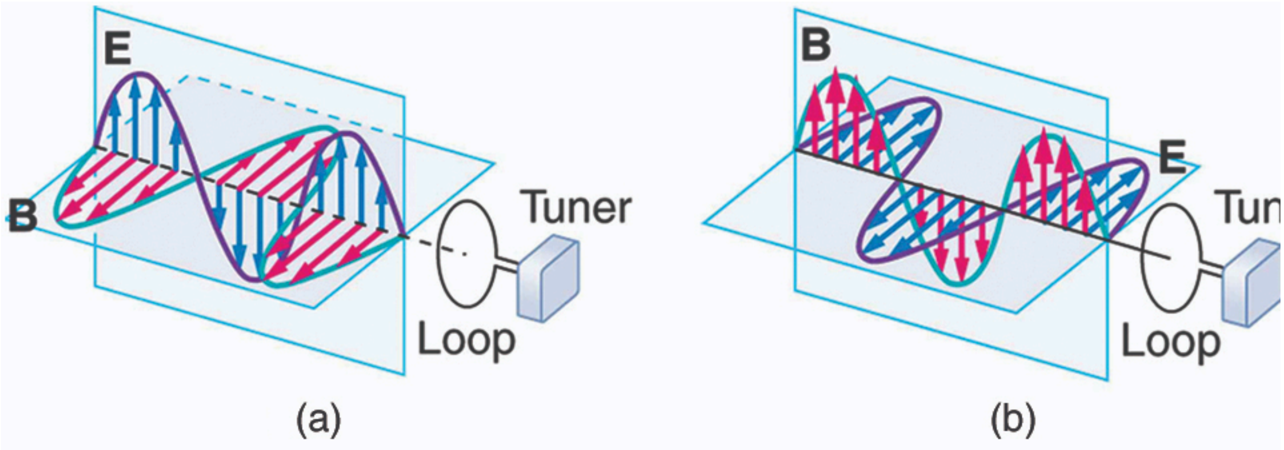
Why is the direction of the current shown in each part of [Figure 2] opposite to the electric field produced by the wire's charge separation?

In which situation shown in [Figure 4] will the electromagnetic wave be more successful in inducing a current in the wire? Explain.



Electromagnetic waves approaching long straight wires.

In which situation shown in [Figure 5] will the electromagnetic wave be more successful in inducing a current in the loop? Explain.



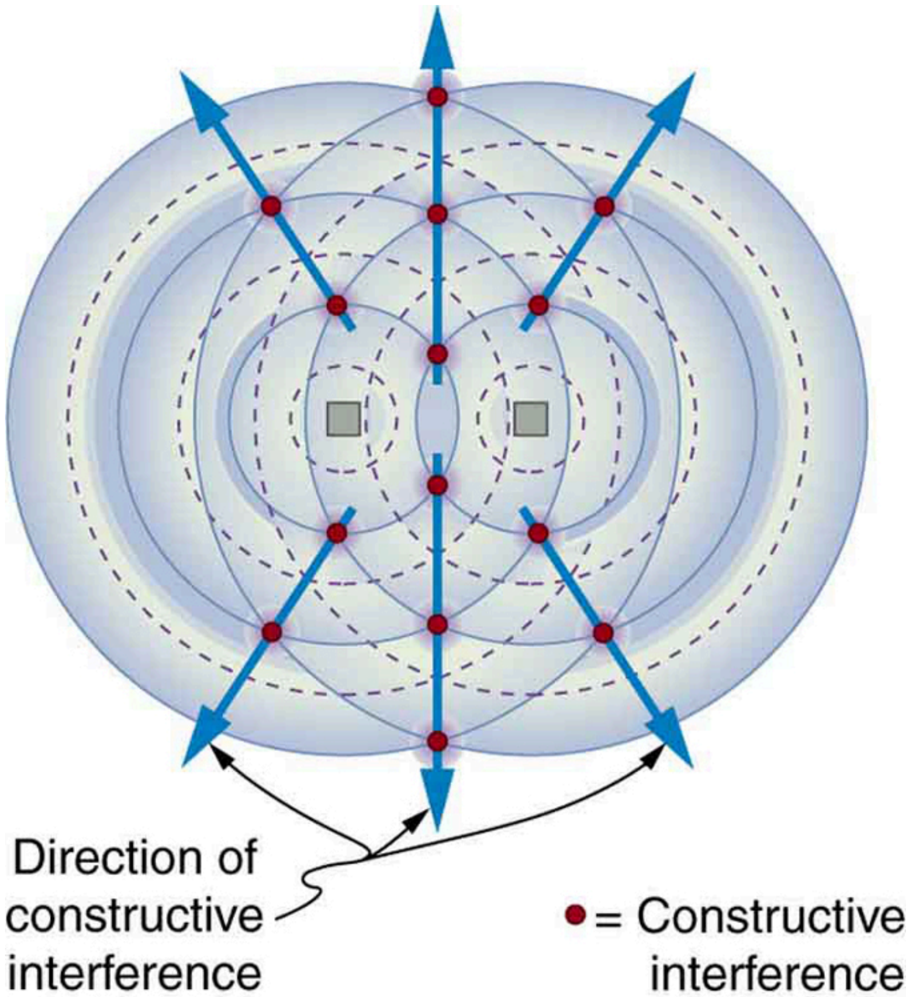
Electromagnetic waves approaching a wire loop.

Should the straight wire antenna of a radio be vertical or horizontal to best receive radio waves broadcast by a vertical transmitter antenna? How should a loop antenna be aligned to best receive the signals? (Note that the direction of the loop that produces the best reception can be used to determine the location of the source. It is used for that purpose in tracking tagged animals in nature studies, for example.)

Under what conditions might wires in a DC circuit emit electromagnetic waves?

Give an example of interference of electromagnetic waves.

[Figure 6] shows the interference pattern of two radio antennas broadcasting the same signal. Explain how this is analogous to the interference pattern for sound produced by two speakers. Could this be used to make a directional antenna system that broadcasts preferentially in certain directions? Explain.



An overhead view of two radio broadcast antennas sending the same signal, and the interference pattern they produce.

Can an antenna be any length? Explain your answer.

Problems & Exercises

Strategy

What is the maximum electric field strength in an electromagnetic wave that has a maximum magnetic field strength of 5.00×10^{-4}T (about 10 times the Earth's)?

Show Solution

To find the maximum electric field strength, we can use the fundamental relationship between electric and magnetic field strengths in an electromagnetic wave: $\frac{E}{B} = c$. Rearranging this equation to solve for E gives $E = Bc$. We are given the maximum magnetic field strength B and can use the speed of light $c = 3.00 \times 10^{8}$ m/s to calculate the corresponding maximum electric field strength.

Solution

Given:

$$B = 5.00 \times 10^{-4}\ \text{T}$$
$$c = 3.00 \times 10^{8}\ \text{m/s}$$

Using the relationship $E = Bc$:

$$E = (5.00 \times 10^{-4}\ \text{T})(3.00 \times 10^{8}\ \text{m/s})$$
$$E = 1.50 \times 10^{5}\ \text{V/m}$$

Converting to kilovolts per meter:

E=150 kV/m

Discussion

This electric field strength of 150 kV/m is quite substantial—about 15 times stronger than the breakdown threshold for air under normal conditions (approximately 10 kV/m). While a magnetic field of 5.00×10^{-4} T (ten times Earth's field) seems relatively modest, it corresponds to a very strong electric field when they oscillate together in an electromagnetic wave. This demonstrates an important characteristic of electromagnetic waves: even when the magnetic field component appears relatively weak compared to everyday magnetic fields, the associated electric field can be extremely powerful due to the large value of c in the relationship $E=Bc$. Such strong electromagnetic waves might be found near powerful radio transmitters or in natural phenomena like lightning-generated electromagnetic pulses.

Final Answer

The maximum electric field strength in an electromagnetic wave with a maximum magnetic field strength of 5.00×10^{-4} T is 1.50×10^{5} V/m or 150 kV/m.

Strategy

The maximum magnetic field strength of an electromagnetic field is 5×10^{-6}T . Calculate the maximum electric field strength if the wave is traveling in a medium in which the speed of the wave is $0.75c$.

Show Solution

The relationship $EB=c$ applies to electromagnetic waves in a vacuum. When an electromagnetic wave travels through a medium, the speed changes from c to v, where v is the speed of light in that medium. The relationship between electric and magnetic field strengths becomes $EB=v$. In this problem, $v=0.75c$, so we can use $E=Bv=B(0.75c)$ to find the maximum electric field strength.

Solution

Given:

$$B=5\times10^{-6}\text{ T}$$
$$v=0.75c=0.75(3.00\times10^{8}\text{ m/s})=2.25\times10^{8}\text{ m/s}$$

Using the relationship $E=Bv$:

$$E=(5\times10^{-6}\text{ T})(2.25\times10^{8}\text{ m/s})$$
$$E=1.125\times10^{3}\text{ V/m}$$
$$E=1.1\times10^{3}\text{ V/m}$$

or

E=1.1 kV/m

Discussion

The electric field strength in this medium is reduced compared to what it would be in a vacuum with the same magnetic field strength. In a vacuum, a magnetic field of 5×10^{-6} T would correspond to an electric field of $E=Bc=1.5\times10^{3}$ V/m, but in this medium with $v=0.75c$, the electric field is only 1.1×10^{3} V/m. This reduction is consistent with the fact that electromagnetic waves slow down when entering a denser medium—the ratio E/B equals the propagation speed of the wave, whether in vacuum or in a material medium. The refractive index of this medium would be $n=c/v=1/0.75\approx1.33$, similar to that of water. This illustrates how the properties of electromagnetic waves adapt to the medium through which they propagate.

Final Answer

The maximum electric field strength in the medium where the wave travels at $0.75c$ is 1.1×10^{3} V/m or 1.1 kV/m.

Strategy

Verify the units obtained for magnetic field strength B in [Example 1] (using the equation $B=Ec$) are in fact teslas (T).

Show Solution

To verify that the equation $B=Ec$ yields units of tesla (T), we need to substitute the SI units for electric field strength E (volts per meter, V/m) and the speed of light c (meters per second, m/s) into the equation and perform dimensional analysis. We'll then express the resulting units in terms of fundamental SI units and show that they are equivalent to the tesla, which is the SI unit for magnetic field strength.

Solution

The SI unit for electric field strength E is:

Vm

The SI unit for the speed of light c is:

ms

Substituting these into $B = Ec$:

Units of B=V/mm/s=Vm·sm=V·sm^2

Now we need to express volts in terms of fundamental units. Recall that:

- Volt: V =JC (joule per coulomb)
- Joule: J =N ·m (newton-meter)
- Newton: N =kg·ms^2

Substituting:

V=JC=N·mC=kg·ms^2·mC=kg·m^{2}C·s^2

Now substituting this expression for volts into our units for B:

Units of B=V·sm^2=kg·m^{2}C·s^2·sm^2=kgC·s

Since coulomb equals ampere-second (C =A ·s), we have:

Units of B=kgC·s=kgA·s·s=kgA·s^2

This is exactly the definition of the tesla:

T=kgA·s^2

Therefore, the units obtained for B using the equation $B = Ec$ are indeed teslas (T).

Discussion

This dimensional analysis confirms that the equation $B = Ec$ is dimensionally consistent and correctly produces magnetic field strength in teslas when electric field strength (in V/m) is divided by the speed of light (in m/s). The verification is more than just a mathematical exercise—it demonstrates the deep physical connection between electric and magnetic fields in electromagnetic waves. The fact that dividing an electric field by a velocity yields a magnetic field reflects the relativistic nature of electromagnetism: electric and magnetic fields are actually different aspects of the same electromagnetic field, related by the relative motion (characterized by c) between the observer and the source. This relationship is a direct consequence of Maxwell’s equations and special relativity.

Final Answer

The units obtained for magnetic field strength B using the equation $B = Ec$ are kgA·s^2, which is the definition of the tesla (T), confirming that the equation yields the correct SI unit for magnetic field strength.

Glossary

electric field
: a vector quantity (**E**); the lines of electric force per unit charge, moving radially outward from a positive charge and in toward a negative charge

electric field strength
: the magnitude of the electric field, denoted *E*-field

magnetic field
: a vector quantity (**B**); can be used to determine the magnetic force on a moving charged particle

magnetic field strength
: the magnitude of the magnetic field, denoted *B*-field

transverse wave
: a wave, such as an electromagnetic wave, which oscillates perpendicular to the axis along the line of travel

standing wave
: a wave that oscillates in place, with nodes where no motion happens

wavelength
: the distance from one peak to the next in a wave

amplitude
: the height, or magnitude, of an electromagnetic wave

frequency
: the number of complete wave cycles (up-down-up) passing a given point within one second (cycles/second)

resonant
: a system that displays enhanced oscillation when subjected to a periodic disturbance of the same frequency as its natural frequency

oscillate
: to fluctuate back and forth in a steady beat

The Electromagnetic Spectrum

- List three “rules of thumb” that apply to the different frequencies along the electromagnetic spectrum.
- Explain why the higher the frequency, the shorter the wavelength of an electromagnetic wave.
- Draw a simplified electromagnetic spectrum, indicating the relative positions, frequencies, and spacing of the different types of radiation bands.
- List and explain the different methods by which electromagnetic waves are produced across the spectrum.

In this module we examine how electromagnetic waves are classified into categories such as radio, infrared, ultraviolet, and so on, so that we can understand some of their similarities as well as some of their differences. We will also find that there are many connections with previously discussed topics, such as wavelength and resonance. A brief overview of the production and utilization of electromagnetic waves is found in [Table 1].

Electromagnetic Waves

Type of EM wave	Production	Applications	Life sciences aspect	Issues
Radio & TV	Accelerating charges	Communications Remote controls	MRI	Requires controls for band use
Microwaves	Accelerating charges & thermal agitation	Communications Ovens Radar	Deep heating	Cell phone use
Infrared	Thermal agitations & electronic transitions	Thermal imaging Heating	Absorbed by atmosphere	Greenhouse effect
Visible light	Thermal agitations & electronic transitions	All pervasive	Photosynthesis Human vision	
Ultraviolet	Thermal agitations & electronic transitions	Sterilization Cancer control	Vitamin D production	Ozone depletion Cancer causing
X-rays	Inner electronic transitions and fast collisions	Medical Security	Medical diagnosis Cancer therapy	Cancer causing
Gamma rays	Nuclear decay	Nuclear medicineSecurity	Medical diagnosis Cancer therapy	Cancer causing Radiation damage

Connections: Waves

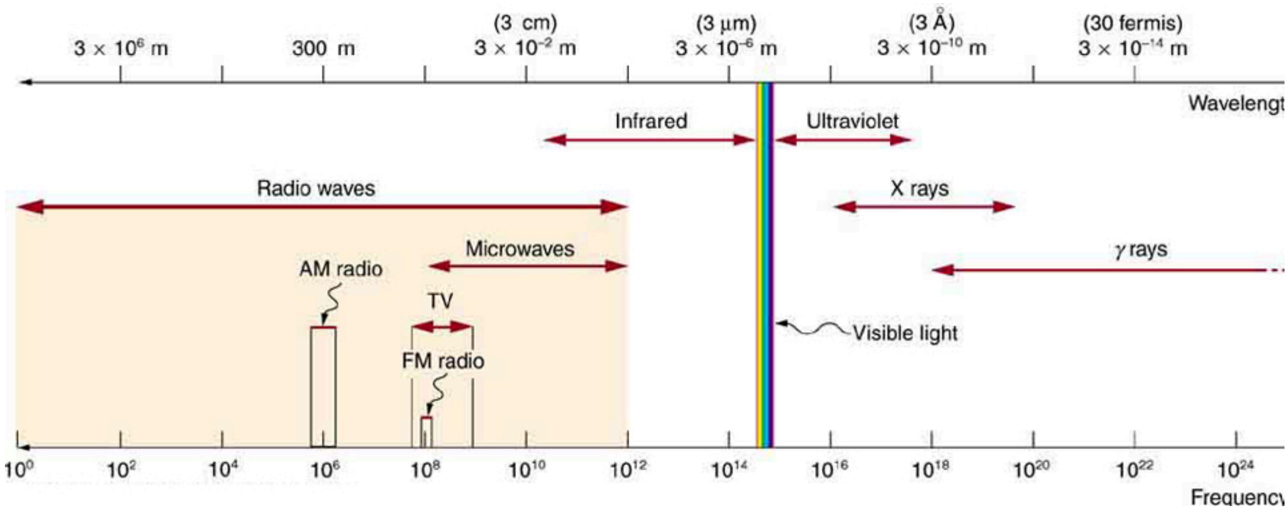
There are many types of waves, such as water waves and even earthquakes. Among the many shared attributes of waves are propagation speed, frequency, and wavelength. These are always related by the expression $v_W = f\lambda$. This module concentrates on EM waves, but other modules contain examples of all of these characteristics for sound waves and submicroscopic particles.

As noted before, an electromagnetic wave has a frequency and a wavelength associated with it and travels at the speed of light, or c . The relationship among these wave characteristics can be described by $v_W = f\lambda$, where v_W is the propagation speed of the wave, f is the frequency, and λ is the wavelength. Here $v\_W = c$, so that for all electromagnetic waves,

$$c = f\lambda.$$

Thus, for all electromagnetic waves, the greater the frequency, the smaller the wavelength.

[Figure 1] shows how the various types of electromagnetic waves are categorized according to their wavelengths and frequencies—that is, it shows the electromagnetic spectrum. Many of the characteristics of the various types of electromagnetic waves are related to their frequencies and wavelengths, as we shall see.



The electromagnetic spectrum, showing the major categories of electromagnetic waves. The range of frequencies and wavelengths is remarkable. The dividing line between some categories is distinct, whereas other categories overlap.

Electromagnetic Spectrum: Rules of Thumb

Three rules that apply to electromagnetic waves in general are as follows:

- High-frequency electromagnetic waves are more energetic and are more able to penetrate than low-frequency waves.
- High-frequency electromagnetic waves can carry more information per unit time than low-frequency waves.
- The shorter the wavelength of any electromagnetic wave probing a material, the smaller the detail it is possible to resolve.

Note that there are exceptions to these rules of thumb.

Transmission, Reflection, and Absorption

What happens when an electromagnetic wave impinges on a material? If the material is transparent to the particular frequency, then the wave can largely be transmitted. If the material is opaque to the frequency, then the wave can be totally reflected. The wave can also be absorbed by the material, indicating that there is some interaction between the wave and the material, such as the thermal agitation of molecules.

Of course it is possible to have partial transmission, reflection, and absorption. We normally associate these properties with visible light, but they do apply to all electromagnetic waves. What is not obvious is that something that is transparent to light may be opaque at other frequencies. For example, ordinary glass is transparent to visible light but largely opaque to ultraviolet radiation. Human skin is opaque to visible light—we cannot see through people—but transparent to X-rays.

Radio and TV Waves

The broad category of **radio waves** is defined to contain any electromagnetic wave produced by currents in wires and circuits. Its name derives from their most common use as a carrier of audio information (i.e., radio). The name is applied to electromagnetic waves of similar frequencies regardless of source. Radio waves from outer space, for example, do not come from alien radio stations. They are created by many astronomical phenomena, and their study has revealed much about nature on the largest scales.

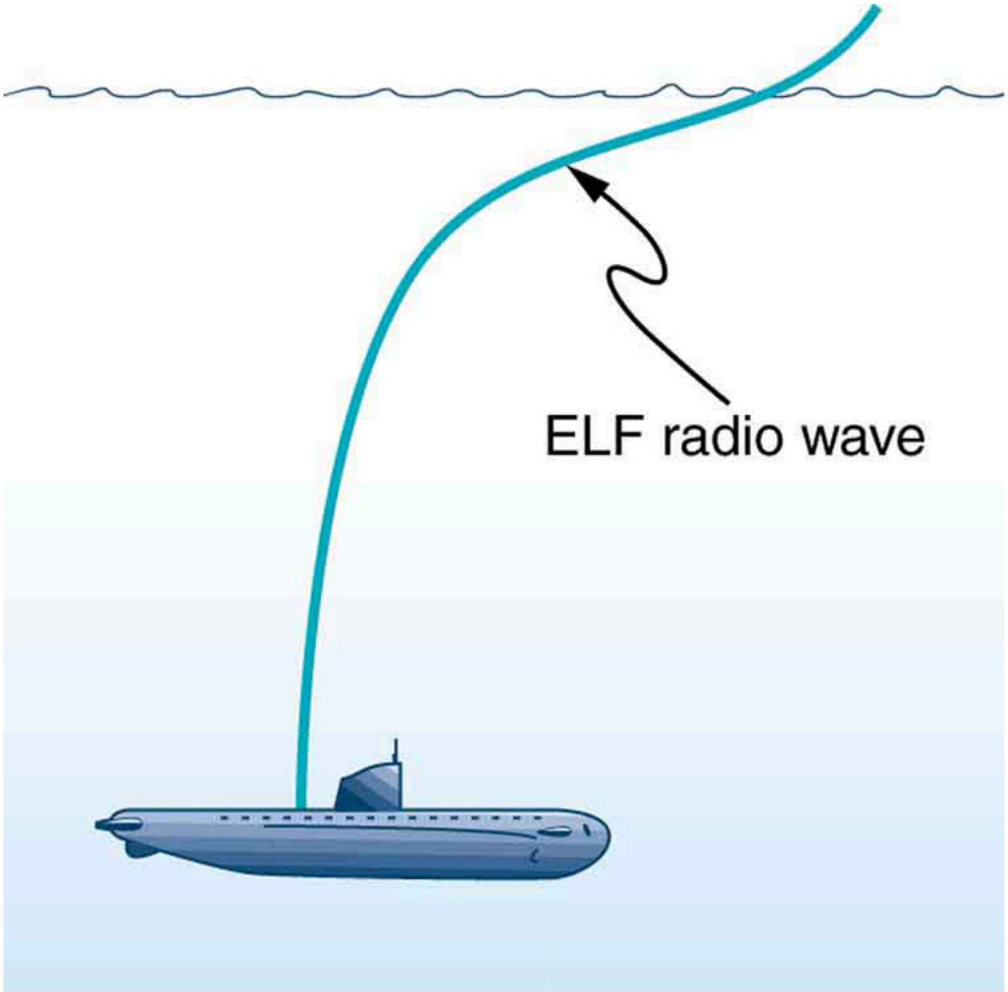
There are many uses for radio waves, and so the category is divided into many subcategories, including microwaves and those electromagnetic waves used for AM and FM radio, cellular telephones, and TV.

The lowest commonly encountered radio frequencies are produced by high-voltage AC power transmission lines at frequencies of 50 or 60 Hz. (See [Figure 2].) These extremely long wavelength electromagnetic waves (about 6000 km!) are one means of energy loss in long-distance power transmission.

This high-voltage traction power line running to Eutingen Railway Substation in Germany radiates electromagnetic waves with very long wavelengths. (credit: Zonk43, Wikimedia Commons)

There is an ongoing controversy regarding potential health hazards associated with exposure to these electromagnetic fields (E -fields). Some people suspect that living near such transmission lines may cause a variety of illnesses, including cancer. But demographic data are either inconclusive or simply do not support the hazard theory. Recent reports that have looked at many European and American epidemiological studies have found no increase in risk for cancer due to exposure to E -fields.

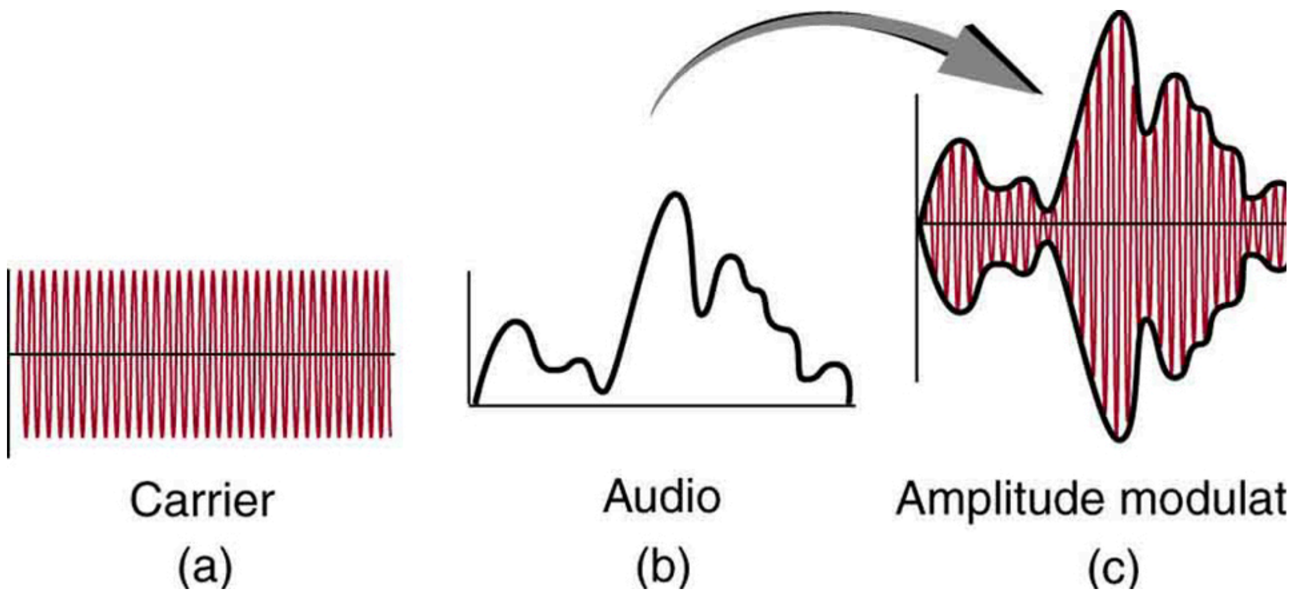
Extremely low frequency (ELF) radio waves of about 1 kHz are used to communicate with submerged submarines. The ability of radio waves to penetrate salt water is related to their wavelength (much like ultrasound penetrating tissue)—the longer the wavelength, the farther they penetrate. Since salt water is a good conductor, radio waves are strongly absorbed by it, and very long wavelengths are needed to reach a submarine under the surface. (See [Figure 3].)



Very long wavelength radio waves are needed to reach this submarine, requiring extremely low frequency signals (ELF). Shorter wavelengths do not penetrate to any significant depth.

AM radio waves are used to carry commercial radio signals in the frequency range from 540 to 1600 kHz. The abbreviation AM stands for **amplitude modulation**, which is the method for placing information on these waves. (See [Figure 4].) A **carrier wave** having the basic frequency of the radio station, say 1530 kHz, is varied or modulated in amplitude by an audio signal. The resulting wave has a constant frequency, but a varying amplitude.

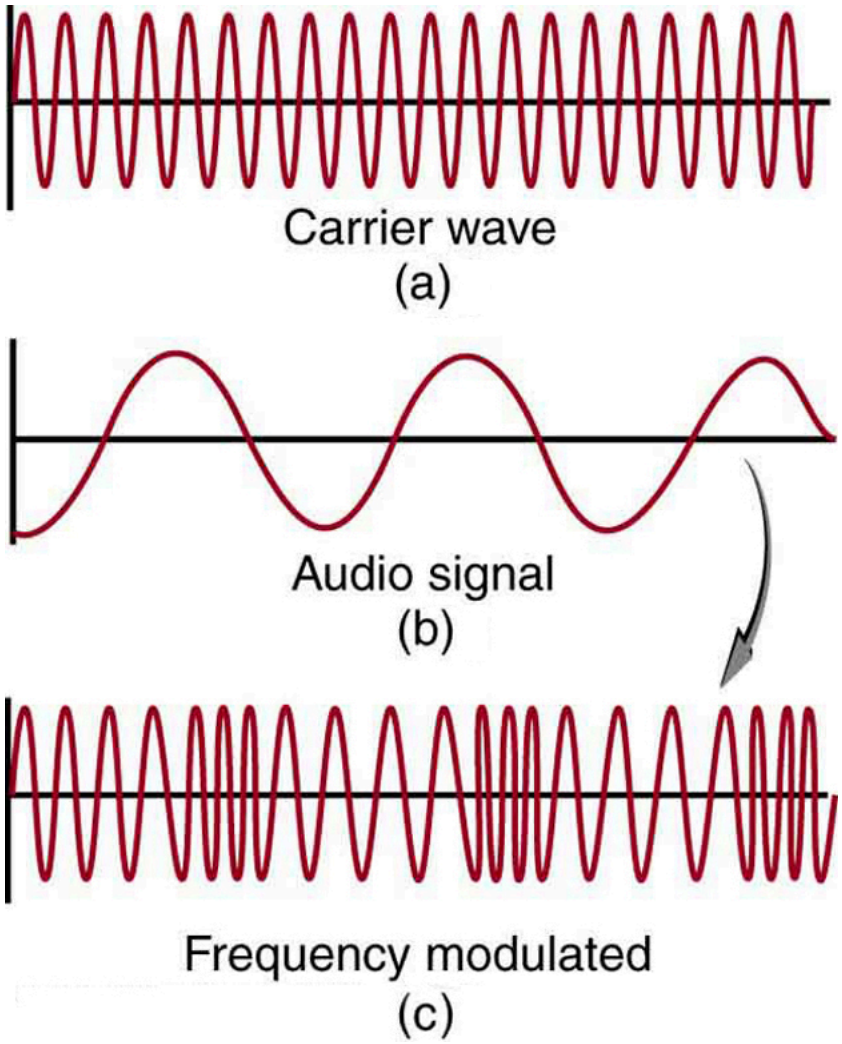
A radio receiver tuned to have the same resonant frequency as the carrier wave can pick up the signal, while rejecting the many other frequencies impinging on its antenna. The receiver’s circuitry is designed to respond to variations in amplitude of the carrier wave to replicate the original audio signal. That audio signal is amplified to drive a speaker or perhaps to be recorded.



Amplitude modulation for AM radio. (a) A carrier wave at the station's basic frequency. (b) An audio signal at much lower audible frequencies. (c) The amplitude of the carrier is modulated by the audio signal without changing its basic frequency.

FM Radio Waves

FM radio waves are also used for commercial radio transmission, but in the frequency range of 88 to 108 MHz. FM stands for **frequency modulation**, another method of carrying information. (See [Figure 5].) Here a carrier wave having the basic frequency of the radio station, perhaps 105.1 MHz, is modulated in frequency by the audio signal, producing a wave of constant amplitude but varying frequency.



Frequency modulation for FM radio. (a) A carrier wave at the station's basic frequency. (b) An audio signal at much lower audible frequencies. (c) The frequency of the carrier is modulated by the audio signal without changing its amplitude.

Since audible frequencies range up to 20 kHz (or 0.020 MHz) at most, the frequency of the FM radio wave can vary from the carrier by as much as 0.020 MHz. Thus the carrier frequencies of two different radio stations cannot be closer than 0.020 MHz. An FM receiver is tuned to resonate at the carrier frequency and has circuitry that responds to variations in frequency, reproducing the audio information.

FM radio is inherently less subject to noise from stray radio sources than AM radio. The reason is that amplitudes of waves add. So an AM receiver would interpret noise added onto the amplitude of its carrier wave as part of the information. An FM receiver can be made to reject amplitudes other than that of the basic carrier wave and only look for variations in frequency. It is thus easier to reject noise from FM, since noise produces a variation in amplitude.

Television is also broadcast on electromagnetic waves. Since the waves must carry a great deal of visual as well as audio information, each channel requires a larger range of frequencies than simple radio transmission. TV channels utilize frequencies in the range of 54 to 88 MHz and 174 to 222 MHz. (The entire FM radio band lies between channels 88 MHz and 174 MHz.) These TV channels are called VHF (for **very high frequency**). Other channels called UHF (for **ultra high frequency**) utilize an even higher frequency range of 470 to 1000 MHz.

The TV video signal is AM, while the TV audio is FM. Note that these frequencies are those of free transmission with the user utilizing an old-fashioned roof antenna. Satellite dishes and cable transmission of TV occurs at significantly higher frequencies and is rapidly evolving with the use of the high-definition or HD format.

Calculating Wavelengths of Radio Waves

Calculate the wavelengths of a 1530-kHz AM radio signal, a 105.1-MHz FM radio signal, and a 1.90-GHz cell phone signal.

Strategy

The relationship between wavelength and frequency is $c = f\lambda$, where $c = 3.00 \times 10^8 \text{ m/s}$ is the speed of light (the speed of light is only very slightly smaller in air than it is in a vacuum). We can rearrange this equation to find the wavelength for all three frequencies.

Solution

Rearranging gives

$\lambda = \frac{c}{f}$.

(a) For the $f = 1530 \text{kHz}$ AM radio signal, then,

$$\lambda = \frac{3.00 \times 10^8 \text{m/s}}{1530 \times 10^3 \text{cycles/s}} = 196 \text{m}.$$

(b) For the $f = 105.1 \text{MHz}$ FM radio signal,

$$\lambda = \frac{3.00 \times 10^8 \text{m/s}}{105.1 \times 10^6 \text{cycles/s}} = 2.85 \text{m}.$$

(c) And for the $f = 1.90 \text{GHz}$ cell phone,

$$\lambda = \frac{3.00 \times 10^8 \text{m/s}}{1.90 \times 10^9 \text{cycles/s}} = 0.158 \text{m}.$$

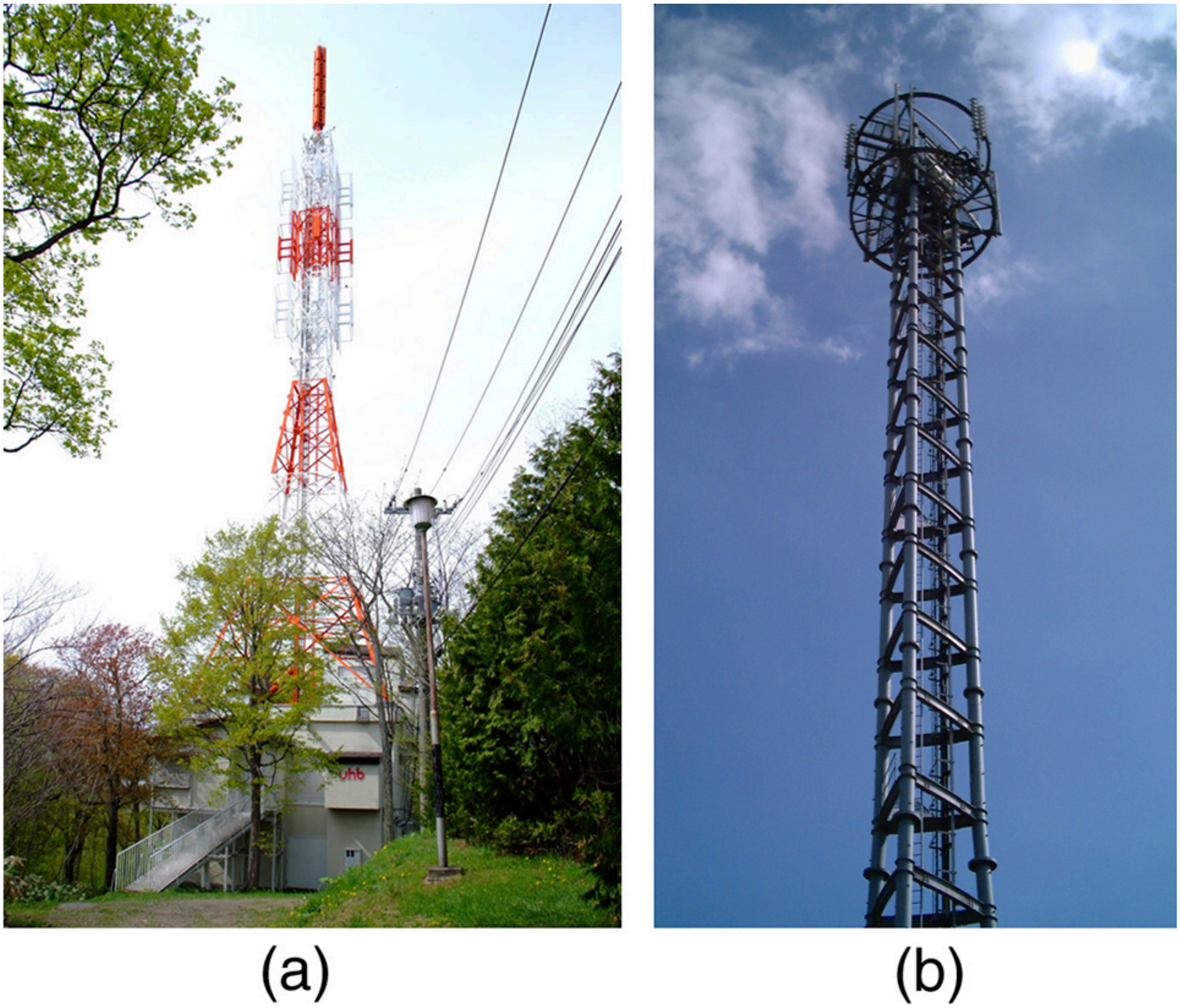
Discussion

These wavelengths are consistent with the spectrum in [Figure 1]. The wavelengths are also related to other properties of these electromagnetic waves, as we shall see.

The wavelengths found in the preceding example are representative of AM, FM, and cell phones, and account for some of the differences in how they are broadcast and how well they travel. The most efficient length for a linear antenna, such as discussed in Production of Electromagnetic Waves, is $\lambda/2$

, half the wavelength of the electromagnetic wave. Thus a very large antenna is needed to efficiently broadcast typical AM radio with its carrier wavelengths on the order of hundreds of meters.

One benefit to these long AM wavelengths is that they can go over and around rather large obstacles (like buildings and hills), just as ocean waves can go around large rocks. FM and TV are best received when there is a line of sight between the broadcast antenna and receiver, and they are often sent from very tall structures. FM, TV, and mobile phone antennas themselves are much smaller than those used for AM, but they are elevated to achieve an unobstructed line of sight. (See [Figure 6].)



(a) A large tower is used to broadcast TV signals. The actual antennas are small structures on top of the tower—they are placed at great heights to have a clear line of sight over a large broadcast area. (credit: Ozizo, Wikimedia Commons) (b) The NTT Dokomo mobile phone tower at Tokorozawa City, Japan. (credit: tokoroten, Wikimedia Commons)

Radio Wave Interference

Astronomers and astrophysicists collect signals from outer space using electromagnetic waves. A common problem for astrophysicists is the “pollution” from electromagnetic radiation pervading our surroundings from communication systems in general. Even everyday gadgets like our car keys having the facility to lock car doors remotely and being able to turn TVs on and off using remotes involve radio-wave frequencies. In order to prevent interference between all these electromagnetic signals, strict regulations are drawn up for different organizations to utilize different radio frequency bands.

One reason why we are sometimes asked to switch off our mobile phones (operating in the range of 1.9 GHz) on airplanes and in hospitals is that important communications or medical equipment often uses similar radio frequencies and their operation can be affected by frequencies used in the communication devices.

For example, radio waves used in magnetic resonance imaging (MRI) have frequencies on the order of 100 MHz, although this varies significantly depending on the strength of the magnetic field used and the nuclear type being scanned. MRI is an important medical imaging and research tool, producing highly detailed two- and three-dimensional images. Radio waves are broadcast, absorbed, and reemitted in a resonance process that is sensitive to the density of nuclei (usually protons or hydrogen nuclei).

The wavelength of 100-MHz radio waves is 3 m, yet using the sensitivity of the resonant frequency to the magnetic field strength, details smaller than a millimeter can be imaged. This is a good example of an exception to a rule of thumb (in this case, the rubric that details much smaller than the probe’s wavelength cannot be detected). The intensity of the radio waves used in MRI presents little or no hazard to human health.

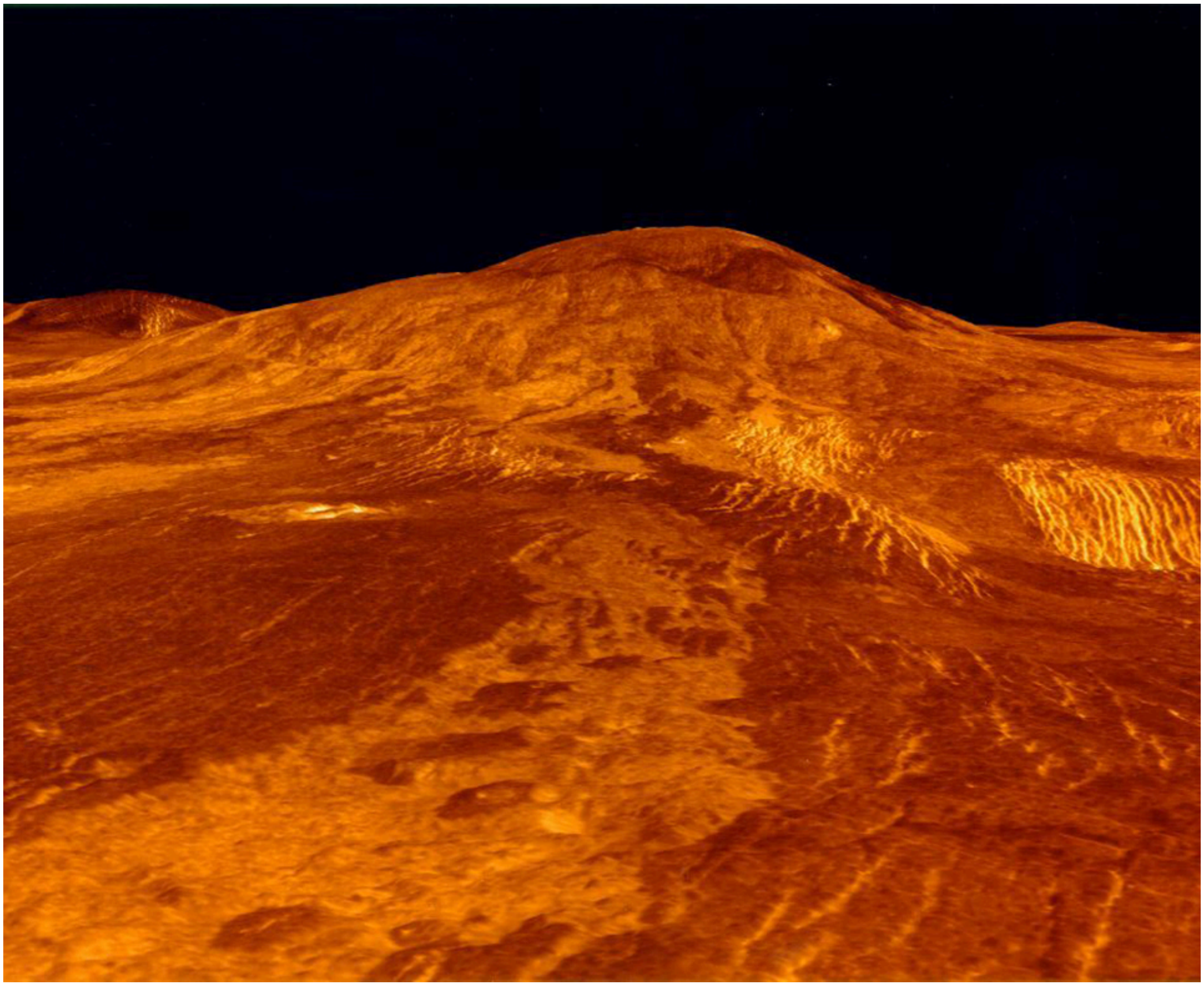
Microwaves

Microwaves are the highest-frequency electromagnetic waves that can be produced by currents in macroscopic circuits and devices. Microwave frequencies range from about 10^9Hz to the highest practical LC resonance at nearly 10^{12}Hz . Since they have high frequencies, their wavelengths are short compared with those of other radio waves—hence the name “microwave.”

Microwaves can also be produced by atoms and molecules. They are, for example, a component of electromagnetic radiation generated by **thermal agitation**. The thermal motion of atoms and molecules in any object at a temperature above absolute zero causes them to emit and absorb radiation.

Since it is possible to carry more information per unit time on high frequencies, microwaves are quite suitable for communications. Most satellite-transmitted information is carried on microwaves, as are land-based long-distance transmissions. A clear line of sight between transmitter and receiver is needed because of the short wavelengths involved.

Radar is a common application of microwaves that was first developed in World War II. By detecting and timing microwave echoes, radar systems can determine the distance to objects as diverse as clouds and aircraft. A Doppler shift in the radar echo can be used to determine the speed of a car or the intensity of a rainstorm. Sophisticated radar systems are used to map the Earth and other planets, with a resolution limited by wavelength. (See [Figure 7].) The shorter the wavelength of any probe, the smaller the detail it is possible to observe.



An image of Sif Mons with lava flows on Venus, based on Magellan synthetic aperture radar data combined with radar altimetry to produce a three-dimensional map of the surface. The Venusian atmosphere is opaque to visible light, but not to the microwaves that were used to create this image. (credit: NSSDC, NASA/JPL)

Heating with Microwaves

How does the ubiquitous microwave oven produce microwaves electronically, and why does food absorb them preferentially? Microwaves at a frequency of 2.45 GHz are produced by accelerating electrons. The microwaves are then used to induce an alternating electric field in the oven.

Water and some other constituents of food have a slightly negative charge at one end and a slightly positive charge at one end (called polar molecules). The range of microwave frequencies is specially selected so that the polar molecules, in trying to keep orienting themselves with the electric field, absorb these energies and increase their temperatures—called dielectric heating.

The energy thereby absorbed results in thermal agitation heating food and not the plate, which does not contain water. Hot spots in the food are related to constructive and destructive interference patterns. Rotating antennas and food turntables help spread out the hot spots.

Another use of microwaves for heating is within the human body. Microwaves will penetrate more than shorter wavelengths into tissue and so can accomplish “deep heating” (called microwave diathermy). This is used for treating muscular pains, spasms, tendonitis, and rheumatoid arthritis.

Making Connections: Take-Home Experiment—Microwave Ovens

1. Look at the door of a microwave oven. Describe the structure of the door. Why is there a metal grid on the door? How does the size of the holes in the grid compare with the wavelengths of microwaves used in microwave ovens? What is this wavelength?
2. Place a glass of water (about 250 ml) in the microwave and heat it for 30 seconds. Measure the temperature gain (the ΔT) . Assuming that the power output of the oven is 1000 W, calculate the efficiency of the heat-transfer process.
3. Remove the rotating turntable or moving plate and place a cup of water in several places along a line parallel with the opening. Heat for 30 seconds and measure the ΔT for each position. Do you see cases of destructive interference?

Microwaves generated by atoms and molecules far away in time and space can be received and detected by electronic circuits. Deep space acts like a blackbody with a 2.7 K temperature, radiating most of its energy in the microwave frequency range. In 1964, Penzias and Wilson detected this radiation and eventually recognized that it was the radiation of the Big Bang's cooled remnants.

Infrared Radiation

The microwave and infrared regions of the electromagnetic spectrum overlap (see [Figure 1]). **Infrared radiation** is generally produced by thermal motion and the vibration and rotation of atoms and molecules. Electronic transitions in atoms and molecules can also produce infrared radiation.

The range of infrared frequencies extends up to the lower limit of visible light, just below red. In fact, infrared means "below red." Frequencies at its upper limit are too high to be produced by accelerating electrons in circuits, but small systems, such as atoms and molecules, can vibrate fast enough to produce these waves.

Water molecules rotate and vibrate particularly well at infrared frequencies, emitting and absorbing them so efficiently that the emissivity for skin is $e = 0.97$ in the infrared. Night-vision scopes can detect the infrared emitted by various warm objects, including humans, and convert it to visible light.

We can examine radiant heat transfer from a house by using a camera capable of detecting infrared radiation. Reconnaissance satellites can detect buildings, vehicles, and even individual humans by their infrared emissions, whose power radiation is proportional to the fourth power of the absolute temperature. More mundanely, we use infrared lamps, some of which are called quartz heaters, to preferentially warm us because we absorb infrared better than our surroundings.

The Sun radiates like a nearly perfect blackbody (that is, it has $e = 1$), with a 6000 K surface temperature. About half of the solar energy arriving at the Earth is in the infrared region, with most of the rest in the visible part of the spectrum, and a relatively small amount in the ultraviolet. On average, 50 percent of the incident solar energy is absorbed by the Earth.

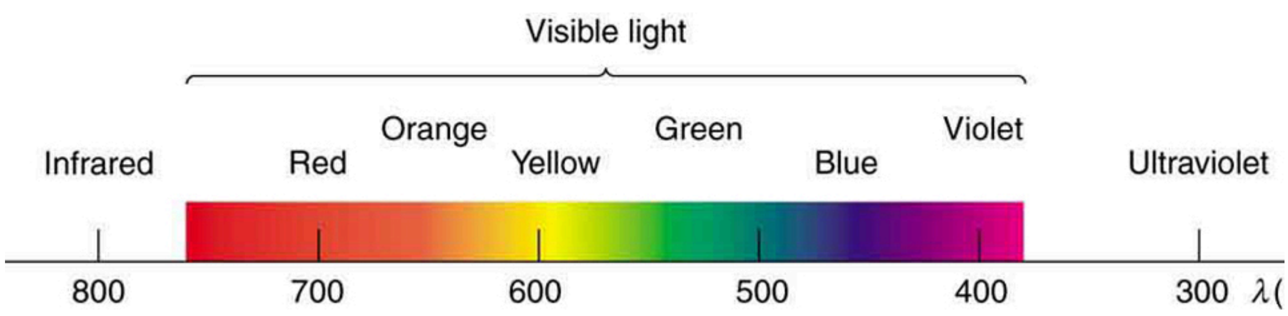
The relatively constant temperature of the Earth is a result of the energy balance between the incoming solar radiation and the energy radiated from the Earth. Most of the infrared radiation emitted from the Earth is absorbed by CO_2 and H_2O in the atmosphere and then radiated back to Earth or into outer space. This radiation back to Earth is known as the greenhouse effect, and it maintains the surface temperature of the Earth about $40^{\circ}C$ higher than it would be if there is no absorption. The increased concentration of CO_2 and other greenhouse gases in the atmosphere, resulting from increases in fossil fuel burning, has increased global average temperatures.

Visible Light

Visible light is the narrow segment of the electromagnetic spectrum to which the normal human eye responds. Visible light is produced by vibrations and rotations of atoms and molecules, as well as by electronic transitions within atoms and molecules. The receivers or detectors of light largely utilize electronic transitions. We say the atoms and molecules are excited when they absorb and relax when they emit through electronic transitions.

[Figure 8] shows this part of the spectrum, together with the colors associated with particular pure wavelengths. We usually refer to visible light as having wavelengths of between 400 nm and 750 nm. (The retina of the eye actually responds to the lowest ultraviolet frequencies, but these do not normally reach the retina because they are absorbed by the cornea and lens of the eye.)

Red light has the lowest frequencies and longest wavelengths, while violet has the highest frequencies and shortest wavelengths. Blackbody radiation from the Sun peaks in the visible part of the spectrum but is more intense in the red than in the violet, making the Sun yellowish in appearance.



A small part of the electromagnetic spectrum that includes its visible components. The divisions between infrared, visible, and ultraviolet are not perfectly distinct, nor are those between the seven rainbow colors.

Living things—plants and animals—have evolved to utilize and respond to parts of the electromagnetic spectrum they are embedded in. Visible light is the most predominant and we enjoy the beauty of nature through visible light. Plants are more selective. Photosynthesis makes use of parts of the visible spectrum to make sugars.

Integrated Concept Problem: Correcting Vision with Lasers

During laser vision correction, a brief burst of 193-nm ultraviolet light is projected onto the cornea of a patient. It makes a spot 0.80 mm in diameter and evaporates a layer of cornea $0.30\mu\text{m}$ thick. Calculate the energy absorbed, assuming the corneal tissue has the same properties as water; it is initially at 34°C . Assume the evaporated tissue leaves at a temperature of 100°C.

Strategy

The energy from the laser light goes toward raising the temperature of the tissue and also toward evaporating it. Thus we have two amounts of heat to add together. Also, we need to find the mass of corneal tissue involved.

Solution

To figure out the heat required to raise the temperature of the tissue to 100°C , we can apply concepts of thermal energy. We know that

$$\text{Q}=mc\Delta T,$$

where Q is the heat required to raise the temperature, ΔT is the desired change in temperature, m is the mass of tissue to be heated, and c is the specific heat of water equal to 4186 J/kg/K.

Without knowing the mass m at this point, we have

$$Q=m(4186\text{J/kg/K})(100^{\circ}\text{C}-34^{\circ}\text{C})=m(276276\text{J/kg})=m(276\text{kJ/kg}).$$

The latent heat of vaporization of water is 2256 kJ/kg, so that the energy needed to evaporate mass m is

$$Q_{\text{V}}=mL_{\text{V}}=m(2256\text{kJ/kg}).$$

To find the mass m , we use the equation $\rho =m/\text{V}$, where ρ is the density of the tissue and V is its volume. For this case,

$$m = \rho\text{V} = (1000\text{kg/m}^3)(\text{area}\times\text{thicknessm}^3) = (1000\text{kg/m}^3)(\pi(0.80\times10^{-3}\text{m})^2/4)(0.30\times10^{-6}\text{m}) = 0.151\times10^{-9}\text{kg}.$$

Therefore, the total energy absorbed by the tissue in the eye is the sum of Q and Q_{V} :

$$\text{Q}_{\text{tot}}=m(c\Delta\text{T}+\text{L}_{\text{V}})=(0.151\times10^{-9}\text{kg})(276\text{kJ/kg}+2256\text{kJ/kg})=382\times10^{-9}\text{kJ}.$$

Discussion

The lasers used for this eye surgery are excimer lasers, whose light is well absorbed by biological tissue. They evaporate rather than burn the tissue, and can be used for precision work. Most lasers used for this type of eye surgery have an average power rating of about one watt. For our example, if we assume that each laser burst from this pulsed laser lasts for 10 ns, and there are 400 bursts per second, then the average power is $\text{Q}_{\text{tot}} \times 400 = 150\text{mW}$.

Optics is the study of the behavior of visible light and other forms of electromagnetic waves. Optics falls into two distinct categories. When electromagnetic radiation, such as visible light, interacts with objects that are large compared with its wavelength, its motion can be represented by straight lines like rays. Ray optics is the study of such situations and includes lenses and mirrors.

When electromagnetic radiation interacts with objects about the same size as the wavelength or smaller, its wave nature becomes apparent. For example, observable detail is limited by the wavelength, and so visible light can never detect individual atoms, because they are so much smaller than its wavelength. Physical or wave optics is the study of such situations and includes all wave characteristics.

Take-Home Experiment: Colors That Match

When you light a match you see largely orange light; when you light a gas stove you see blue light. Why are the colors different? What other colors are present in these?

Ultraviolet Radiation

Ultraviolet means “above violet.” The electromagnetic frequencies of **ultraviolet radiation (UV)** extend upward from violet, the highest-frequency visible light. Ultraviolet is also produced by atomic and molecular motions and electronic transitions. The wavelengths of ultraviolet extend from 400 nm down to about 10 nm at its highest frequencies, which overlap with the lowest X-ray frequencies. It was recognized as early as 1801 by Johann Ritter that the solar spectrum had an invisible component beyond the violet range.

Solar UV radiation is broadly subdivided into three regions: UV-A (320-400 nm), UV-B (290–320 nm), and UV-C (220–290 nm), ranked from long to shorter wavelengths (from smaller to larger energies). Most UV-B and all UV-C is absorbed by ozone (O_3) molecules in the upper atmosphere. Consequently, 99% of the solar UV radiation reaching the Earth’s surface is UV-A.

One of the first illustrations of UV light’s impact on Earth occurred during the Apollo 16 mission in 1972. The mission included the first astronomical images taken from the moon, using a compact and resilient Far Ultraviolet Camera/Spectrograph designed for moon use by scientist and inventor George Robert Carruthers. Designed to capture UV images without the obscuring effects of the Earth’s atmosphere, its most famous image was of the planet itself. Carruthers, who also trained the astronauts on the device’s use, mentioned afterward that “the most immediately obvious and spectacular results were really for the Earth observations, because this was the first time that the Earth had been photographed from a distance in ultraviolet (UV) light, so that you could see the full extent of the hydrogen atmosphere, the polar auroris and what we call the tropical airglow belt.”

Human Exposure to UV Radiation

It is largely exposure to UV-B that causes skin cancer. It is estimated that as many as 20% of adults will develop skin cancer over the course of their lifetime. Again, treatment is often successful if caught early. Despite very little UV-B reaching the Earth's surface, there are substantial increases in skin-cancer rates in countries such as Australia, indicating how important it is that UV-B and UV-C continue to be absorbed by the upper atmosphere.

All UV radiation can damage collagen fibers, resulting in an acceleration of the aging process of skin and the formation of wrinkles. Because there is so little UV-B and UV-C reaching the Earth's surface, sunburn is caused by large exposures, and skin cancer from repeated exposure. Some studies indicate a link between overexposure to the Sun when young and melanoma later in life.

The tanning response is a defense mechanism in which the body produces pigments to absorb future exposures in inert skin layers above living cells. Basically UV-B radiation excites DNA molecules, distorting the DNA helix, leading to mutations and the possible formation of cancerous cells.

Repeated exposure to UV-B may also lead to the formation of cataracts in the eyes—a cause of blindness among people living in the equatorial belt where medical treatment is limited. Cataracts, clouding in the eye's lens and a loss of vision, are age related; 60% of those between the ages of 65 and 74 will develop cataracts. However, treatment is easy and successful, as one replaces the lens of the eye with a plastic lens. Prevention is important. Eye protection from UV is more effective with plastic sunglasses than those made of glass.

A major acute effect of extreme UV exposure is the suppression of the immune system, both locally and throughout the body.

Low-intensity ultraviolet is used to sterilize haircutting implements, implying that the energy associated with ultraviolet is deposited in a manner different from lower-frequency electromagnetic waves. (Actually this is true for all electromagnetic waves with frequencies greater than visible light.)

Flash photography is generally not allowed of precious artworks and colored prints because the UV radiation from the flash can cause photo-degradation in the artworks. Often artworks will have an extra-thick layer of glass in front of them, which is especially designed to absorb UV radiation.

UV Light and the Ozone Layer

If all of the Sun's ultraviolet radiation reached the Earth's surface, there would be extremely grave effects on the biosphere from the severe cell damage it causes. However, the layer of ozone (O_3) in our upper atmosphere (10 to 50 km above the Earth) protects life by absorbing most of the dangerous UV radiation.

Unfortunately, today we are observing a depletion in ozone concentrations in the upper atmosphere. This depletion has led to the formation of an "ozone hole" in the upper atmosphere. The hole is more centered over the southern hemisphere, and changes with the seasons, being largest in the spring. This depletion is attributed to the breakdown of ozone molecules by refrigerant gases called chlorofluorocarbons (CFCs).

The UV radiation helps dissociate the CFC's, releasing highly reactive chlorine (Cl) atoms, which catalyze the destruction of the ozone layer. For example, the reaction of $CFCl_3$ with a photon of light (hv) can be written as:

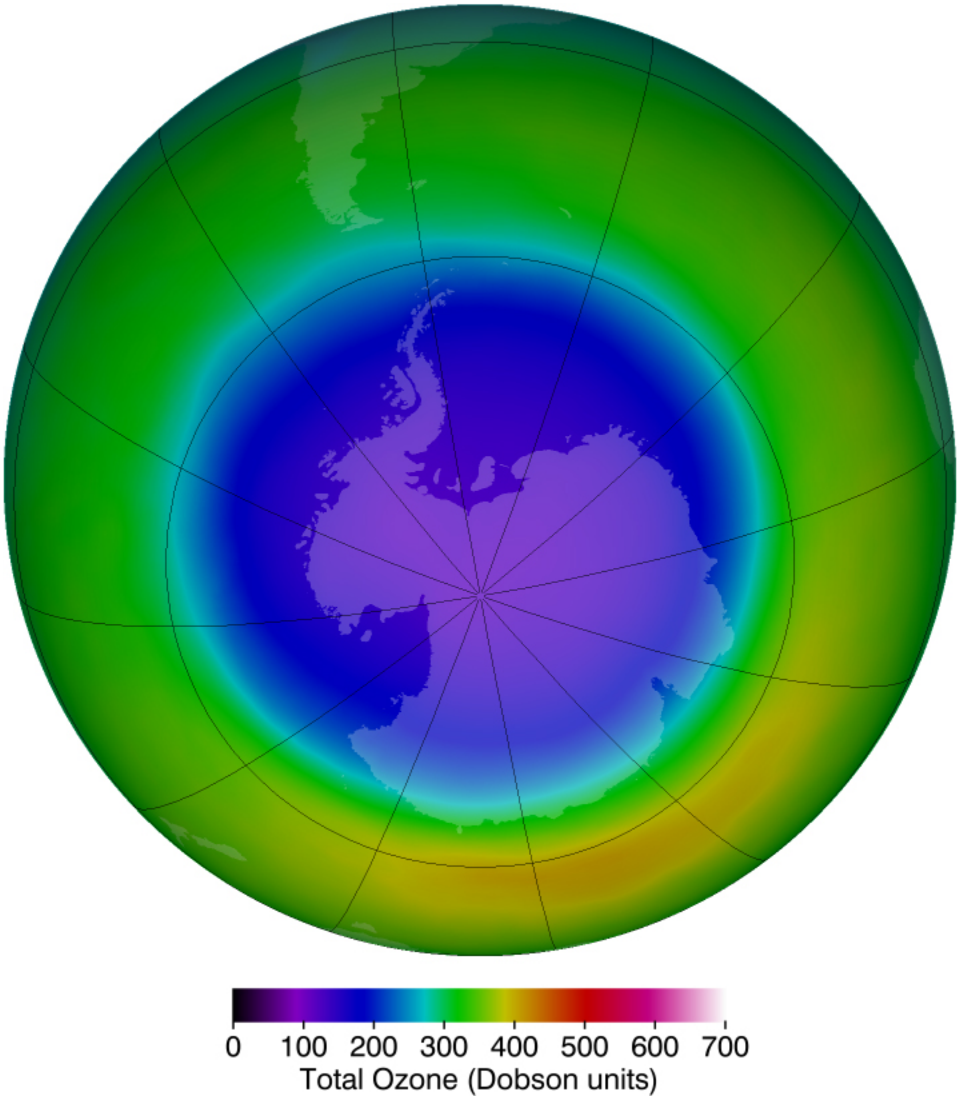
$$CFCl_3 + h\nu \rightarrow CFCl_2 + Cl.$$

The Cl atom then catalyzes the breakdown of ozone as follows:

$$Cl + O_3 \rightarrow ClO + O_2 \text{and } ClO + O_3 \rightarrow Cl + 2O_2.$$

A single chlorine atom could destroy ozone molecules for up to two years before being transported down to the surface. The CFCs are relatively stable and will contribute to ozone depletion for years to come. CFCs are found in refrigerants, air conditioning systems, foams, and aerosols.

International concern over this problem led to the establishment of the "Montreal Protocol" agreement (1987) to phase out CFC production in most countries. However, developing-country participation is needed if worldwide production and elimination of CFCs is to be achieved. Probably the largest contributor to CFC emissions today is China. And while there are indicators that the Protocol has been a success, there is still substantial risk and variability in the ozone layer. (See [Figure 9].)



This map of ozone concentration over Antarctica in October 2011 shows severe depletion suspected to be caused by CFCs. Less dramatic but more general depletion has been observed over northern latitudes, suggesting the effect is global. With less ozone, more ultraviolet radiation from the Sun reaches the surface, causing more damage. (credit: NASA Ozone Watch)

Benefits of UV Light

Besides the adverse effects of ultraviolet radiation, there are also benefits of exposure in nature and uses in technology. Vitamin D production in the skin (epidermis) results from exposure to UVB radiation, generally from sunlight. A number of studies indicate lack of vitamin D can result in the development of a range of cancers (prostate, breast, colon), so a certain amount of UV exposure is helpful. Lack of vitamin D is also linked to osteoporosis. Exposures (with no sunscreen) of 10 minutes a day to arms, face, and legs might be sufficient to provide the accepted dietary level. However, in the winter time north of about 37° latitude, most UVB gets blocked by the atmosphere.

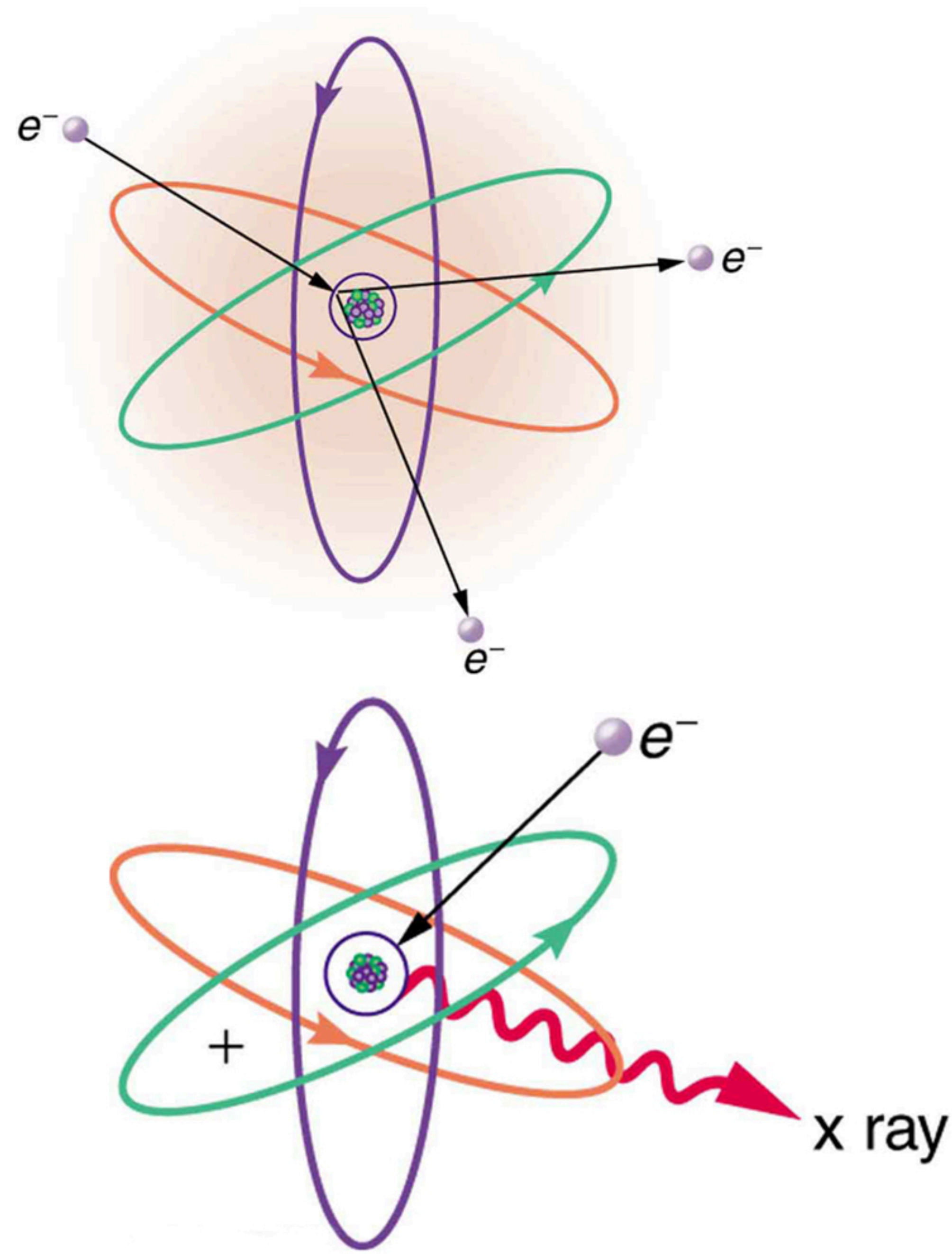
UV radiation is used in the treatment of infantile jaundice and in some skin conditions. It is also used in sterilizing workspaces and tools, and killing germs in a wide range of applications. It is also used as an analytical tool to identify substances.

When exposed to ultraviolet, some substances, such as minerals, glow in characteristic visible wavelengths, a process called fluorescence. So-called black lights emit ultraviolet to cause posters and clothing to fluoresce in the visible. Ultraviolet is also used in special microscopes to detect details smaller than those observable with longer-wavelength visible-light microscopes.

Things Great and Small: A Submicroscopic View of X-Ray Production

X-rays can be created in a high-voltage discharge. They are emitted in the material struck by electrons in the discharge current. There are two mechanisms by which the electrons create X-rays.

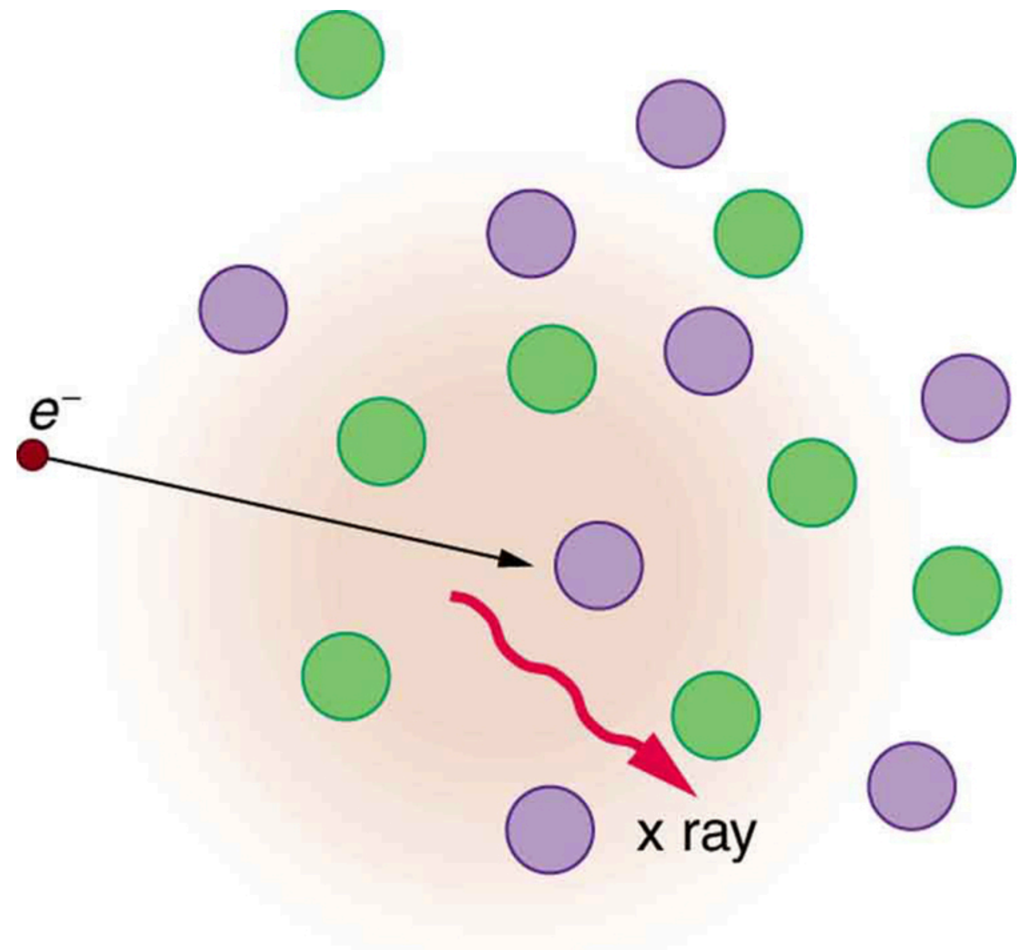
The first method is illustrated in [Figure 10]. An electron is accelerated in an evacuated tube by a high positive voltage. The electron strikes a metal plate (e.g., copper) and produces X-rays. Since this is a high-voltage discharge, the electron gains sufficient energy to ionize the atom.



Artist’s conception of an electron ionizing an atom followed by the recapture of an electron and emission of an X-ray. An energetic electron strikes an atom and knocks an electron out of one of the orbits closest to the nucleus. Later, the atom captures another electron, and the energy released by its fall into a low orbit generates a high-energy EM wave called an X-ray.

In the case shown, an inner-shell electron (one in an orbit relatively close to and tightly bound to the nucleus) is ejected. A short time later, another electron is captured and falls into the orbit in a single great plunge. The energy released by this fall is given to an EM wave known as an X-ray. Since the orbits of the atom are unique to the type of atom, the energy of the X-ray is characteristic of the atom, hence the name characteristic X-ray.

The second method by which an energetic electron creates an X-ray when it strikes a material is illustrated in [Figure 11]. The electron interacts with charges in the material as it penetrates. These collisions transfer kinetic energy from the electron to the electrons and atoms in the material.



Artist's conception of an electron being slowed by collisions in a material and emitting X-ray radiation. This energetic electron makes numerous collisions with electrons and atoms in a material it penetrates. An accelerated charge radiates EM waves, a second method by which X-rays are created.

A loss of kinetic energy implies an acceleration, in this case decreasing the electron's velocity. Whenever a charge is accelerated, it radiates EM waves. Given the high energy of the electron, these EM waves can have high energy. We call them X-rays. Since the process is random, a broad spectrum of X-ray energy is emitted that is more characteristic of the electron energy than the type of material the electron encounters. Such EM radiation is called "bremsstrahlung" (German for "braking radiation").

X-Rays

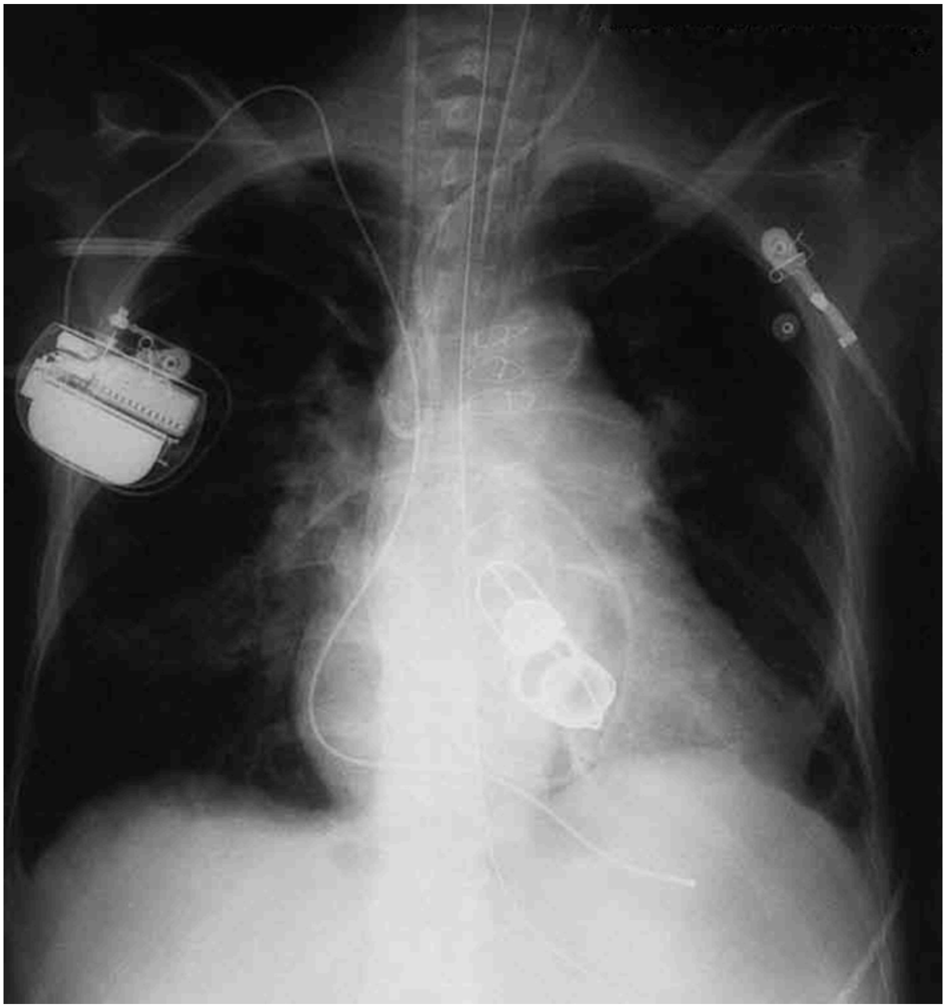
In the 1850s, scientists (such as Faraday) began experimenting with high-voltage electrical discharges in tubes filled with rarefied gases. It was later found that these discharges created an invisible, penetrating form of very high frequency electromagnetic radiation. This radiation was called an **X-ray**, because its identity and nature were unknown.

As described in Things Great and Small, there are two methods by which X-rays are created—both are submicroscopic processes and can be caused by high-voltage discharges. While the low-frequency end of the X-ray range overlaps with the ultraviolet, X-rays extend to much higher frequencies (and energies).

X-rays have adverse effects on living cells similar to those of ultraviolet radiation, and they have the additional liability of being more penetrating, affecting more than the surface layers of cells. Cancer and genetic defects can be induced by exposure to X-rays. Because of their effect on rapidly dividing cells, X-rays can also be used to treat and even cure cancer.

The widest use of X-rays is for imaging objects that are opaque to visible light, such as the human body or aircraft parts. In humans, the risk of cell damage is weighed carefully against the benefit of the diagnostic information obtained. However, questions have risen in recent years as to accidental overexposure of some people during CT scans—a mistake at least in part due to poor monitoring of radiation dose.

The ability of X-rays to penetrate matter depends on density, and so an X-ray image can reveal very detailed density information. [Figure 12] shows an example of the simplest type of X-ray image, an X-ray shadow on film. The amount of information in a simple X-ray image is impressive, but more sophisticated techniques, such as CT scans, can reveal three-dimensional information with details smaller than a millimeter.



This shadow X-ray image shows many interesting features, such as artificial heart valves, a pacemaker, and the wires used to close the sternum. (credit: P. P. Urone)

The use of X-ray technology in medicine is called radiology—an established and relatively cheap tool in comparison to more sophisticated technologies. Consequently, X-rays are widely available and used extensively in medical diagnostics. During World War I, mobile X-ray units, advocated by Madame Marie Curie, were used to diagnose soldiers.

Because they can have wavelengths less than 0.01 nm, X-rays can be scattered (a process called X-ray diffraction) to detect the shape of molecules and the structure of crystals. X-ray diffraction was crucial to Crick, Watson, and Wilkins in the determination of the shape of the double-helix DNA molecule.

X-rays are also used as a precise tool for trace-metal analysis in X-ray induced fluorescence, in which the energy of the X-ray emissions are related to the specific types of elements and amounts of materials present.

Gamma Rays

Soon after nuclear radioactivity was first detected in 1896, it was found that at least three distinct types of radiation were being emitted. The most penetrating nuclear radiation was called a **gamma ray** (γ ray) (again a name given because its identity and character were unknown), and it was later found to be an extremely high frequency electromagnetic wave.

In fact, γ rays are any electromagnetic radiation emitted by a nucleus. This can be from natural nuclear decay or induced nuclear processes in nuclear reactors and weapons. The lower end of the $\gamma\text{-ray}$ frequency range overlaps the upper end of the X-ray range, but γ rays can have the highest frequency of any electromagnetic radiation.

Gamma rays have characteristics identical to X-rays of the same frequency—they differ only in source. At higher frequencies, γ rays are more penetrating and more damaging to living tissue. They have many of the same uses as X-rays, including cancer therapy. Gamma radiation from radioactive materials is used in nuclear medicine.

[Figure 13] shows a medical image based on γ rays. Food spoilage can be greatly inhibited by exposing it to large doses of γ radiation, thereby obliterating responsible microorganisms. Damage to food cells through irradiation occurs as well, and the long-term hazards of consuming radiation-preserved food are unknown and controversial for some groups. Both X-ray and $\gamma\text{-ray}$ technologies are also used in scanning luggage at airports.

This is an image of the γ rays emitted by nuclei in a compound that is concentrated in the bones and eliminated through the kidneys. Bone cancer is evidenced by nonuniform concentration in similar structures. For example, some ribs are darker than others. (credit: P. P. Urone)

Detecting Electromagnetic Waves from Space

The entire electromagnetic spectrum is used by researchers for investigating stars, space, and time. Arthur B. C. Walker was a pioneer in X-ray and ultraviolet observations, and designed specialized telescopes and instruments to observe the Sun's atmosphere and corona. His developments significantly advanced our understanding of stars, and some of his developments are currently in use in space telescopes as well as in microchip manufacturing. As noted earlier, Penzias and Wilson detected microwaves to identify the background radiation originating from the Big Bang. Radio telescopes such as the Arecibo Radio Telescope in Puerto Rico and Parkes Observatory in Australia were designed to detect radio waves.

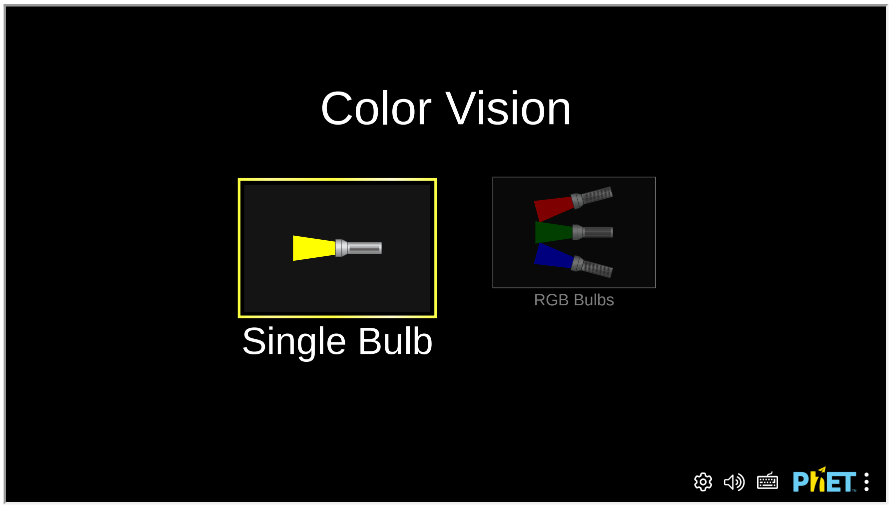
Infrared telescopes need to have their detectors cooled by liquid nitrogen to be able to gather useful signals. Since infrared radiation is predominantly from thermal agitation, if the detectors were not cooled, the vibrations of the molecules in the antenna would be stronger than the signal being collected.

The most famous of these infrared sensitive telescopes is the James Clerk Maxwell Telescope in Hawaii. The earliest telescopes, developed in the seventeenth century, were optical telescopes, collecting visible light. Telescopes in the ultraviolet, X-ray, and γ -ray regions are placed outside the atmosphere on satellites orbiting the Earth.

The Hubble Space Telescope (launched in 1990) gathers ultraviolet radiation as well as visible light. In the X-ray region, there is the Chandra X-ray Observatory (launched in 1999), and in the γ -ray region, there is the new Fermi Gamma-ray Space Telescope (launched in 2008—taking the place of the Compton Gamma Ray Observatory, 1991–2000.). The James Webb Space Telescope (JWST), launched in late 2021, observes in a lower-frequency portion of the spectrum compared to Hubble. The JWST observes in long-wavelength visible light (red) through infrared, enabling it to detect objects that are further away, older, and fainter than previous telescopes could detect.

PhET Explorations: Color Vision

Make a whole rainbow by mixing red, green, and blue light. Change the wavelength of a monochromatic beam or filter white light. View the light as a solid beam, or see the individual photons.



Section Summary

- The relationship among the speed of propagation, wavelength, and frequency for any wave is given by $v_W = f\lambda$, so that for electromagnetic waves,

  $$c = f\lambda,$$

  where f is the frequency, λ is the wavelength, and c is the speed of light.

- The electromagnetic spectrum is separated into many categories and subcategories, based on the frequency and wavelength, source, and uses of the electromagnetic waves.
- Any electromagnetic wave produced by currents in wires is classified as a radio wave, the lowest frequency electromagnetic waves. Radio waves are divided into many types, depending on their applications, ranging up to microwaves at their highest frequencies.
- Infrared radiation lies below visible light in frequency and is produced by thermal motion and the vibration and rotation of atoms and molecules. Infrared's lower frequencies overlap with the highest-frequency microwaves.
- Visible light is largely produced by electronic transitions in atoms and molecules, and is defined as being detectable by the human eye. Its colors vary with frequency, from red at the lowest to violet at the highest.
- Ultraviolet radiation starts with frequencies just above violet in the visible range and is produced primarily by electronic transitions in atoms and molecules.
- X-rays are created in high-voltage discharges and by electron bombardment of metal targets. Their lowest frequencies overlap the ultraviolet range but extend to much higher values, overlapping at the high end with gamma rays.
- Gamma rays are nuclear in origin and are defined to include the highest-frequency electromagnetic radiation of any type.

Conceptual Questions

If you live in a region that has a particular TV station, you can sometimes pick up some of its audio portion on your FM radio receiver. Explain how this is possible. Does it imply that TV audio is broadcast as FM?

Explain why people who have the lens of their eye removed because of cataracts are able to see low-frequency ultraviolet.

How do fluorescent soap residues make clothing look “brighter and whiter” in outdoor light? Would this be effective in candlelight?

Give an example of resonance in the reception of electromagnetic waves.

Illustrate that the size of details of an object that can be detected with electromagnetic waves is related to their wavelength, by comparing details observable with two different types (for example, radar and visible light or infrared and X-rays).

Why don’t buildings block radio waves as completely as they do visible light?

Make a list of some everyday objects and decide whether they are transparent or opaque to each of the types of electromagnetic waves.

Your friend says that more patterns and colors can be seen on the wings of birds if viewed in ultraviolet light. Would you agree with your friend? Explain your answer.

The rate at which information can be transmitted on an electromagnetic wave is proportional to the frequency of the wave. Is this consistent with the fact that laser telephone transmission at visible frequencies carries far more conversations per optical fiber than conventional electronic transmission in a wire? What is the implication for ELF radio communication with submarines?

Give an example of energy carried by an electromagnetic wave.

In an MRI scan, a higher magnetic field requires higher frequency radio waves to resonate with the nuclear type whose density and location is being imaged. What effect does going to a larger magnetic field have on the most efficient antenna to broadcast those radio waves? Does it favor a smaller or larger antenna?

Laser vision correction often uses an excimer laser that produces 193-nm electromagnetic radiation. This wavelength is extremely strongly absorbed by the cornea and ablates it in a manner that reshapes the cornea to correct vision defects. Explain how the strong absorption helps concentrate the energy in a thin layer and thus give greater accuracy in shaping the cornea. Also explain how this strong absorption limits damage to the lens and retina of the eye.

Problems & Exercises

Strategy

(a) Two microwave frequencies are authorized for use in microwave ovens: 900 and 2560 MHz. Calculate the wavelength of each. (b) Which frequency would produce closer together hot spots in foods due to interference effects?

Show Solution

To find the wavelengths, we use the fundamental wave equation $c = f\lambda$, which relates the speed of light c, frequency f, and wavelength λ. Rearranging gives $\lambda = \frac{c}{f}$. For part (b), we need to understand that interference patterns create hot spots separated by distances related to the wavelength —smaller wavelengths lead to more closely spaced interference maxima.

Solution

(a) Given:

- $f_1 = 900 \text{ MHz} = 900 \times 10^6 \text{ Hz} = 9.00 \times 10^8 \text{ Hz}$
- $f_2 = 2560 \text{ MHz} = 2560 \times 10^6 \text{ Hz} = 2.56 \times 10^9 \text{ Hz}$
- $c = 3.00 \times 10^8 \text{ m/s}$

For 900 MHz:

$$\lambda_1 = \frac{c}{f_1} = \frac{3.00 \times 10^8 \text{ m/s}}{9.00 \times 10^8 \text{ Hz}} = 0.333 \text{ m} = 33.3 \text{ cm}$$

For 2560 MHz:

$$\lambda_2 = \frac{c}{f_2} = \frac{3.00 \times 10^8 \text{ m/s}}{2.56 \times 10^9 \text{ Hz}} = 0.117 \text{ m} = 11.7 \text{ cm}$$

(b) The 2560 MHz microwave oven would produce closer together hot spots in foods. Since the wavelength is smaller (11.7 cm vs. 33.3 cm), the interference pattern has a smaller spatial period, resulting in hot spots that are more closely spaced.

Discussion

The wavelengths we calculated (33.3 cm and 11.7 cm) are on the order of typical food dimensions in a microwave oven, which is why uneven heating (hot spots) is a common problem. The interference effects arise because microwaves reflect off the metal walls of the oven cavity, creating standing wave patterns. The hot spots occur at the antinodes of these standing waves. While the 2560 MHz frequency creates more closely spaced hot spots, this doesn’t necessarily mean more uniform heating—it just means the pattern of non-uniformity has a finer spatial structure. This is why most modern microwave ovens include rotating turntables to help average out the hot and cold spots by moving the food through different parts of the standing wave pattern. The choice between 900 MHz and 2560 MHz also involves other considerations like penetration depth into food and manufacturing costs.

Final Answer

(a) The wavelength for 900 MHz microwaves is 33.3 cm, and the wavelength for 2560 MHz microwaves is 11.7 cm. (b) The 2560 MHz frequency produces closer together hot spots due to its smaller wavelength of 11.7 cm.

Strategy

(a) Calculate the range of wavelengths for AM radio given its frequency range is 540 to 1600 kHz. (b) Do the same for the FM frequency range of 88.0 to 108 MHz.

Show Solution

To find the wavelength range for each radio band, we use $\lambda = \frac{c}{f}$. The lowest frequency corresponds to the longest wavelength, and the highest frequency corresponds to the shortest wavelength. We’ll calculate the wavelengths at the extreme frequencies of each band.

Solution

(a) AM radio:

Given frequency range: 540 kHz to 1600 kHz

For the lowest frequency (540 kHz):

$$\lambda_{max} = \frac{c}{f_{min}} = \frac{3.00 \times 10^8 \text{ m/s}}{540 \times 10^3 \text{ Hz}} = 556 \text{ m}$$

For the highest frequency (1600 kHz):

$$\lambda_{\text{min}}=c\,f_{\text{max}}=3.00\times10^{8}\text{ m/s}1600\times10^{3}\text{ Hz}=188\text{ m}$$

Therefore, the AM wavelength range is **188 m to 556 m**.

(b) FM radio:

Given frequency range: 88.0 MHz to 108 MHz

For the lowest frequency (88.0 MHz):

$$\lambda_{\text{max}}=c\,f_{\text{min}}=3.00\times10^{8}\text{ m/s}88.0\times10^{6}\text{ Hz}=3.41\text{ m}$$

For the highest frequency (108 MHz):

$$\lambda_{\text{min}}=c\,f_{\text{max}}=3.00\times10^{8}\text{ m/s}108\times10^{6}\text{ Hz}=2.78\text{ m}$$

Therefore, the FM wavelength range is **2.78 m to 3.41 m**.

Discussion

The dramatic difference in wavelengths between AM and FM radio waves (hundreds of meters vs. a few meters) has important practical consequences. AM's longer wavelengths allow these waves to diffract around obstacles like buildings and hills more effectively, and they can also reflect off the ionosphere for long-distance transmission, especially at night. This is why AM radio stations can sometimes be heard hundreds of kilometers away. In contrast, FM's shorter wavelengths result in more line-of-sight propagation—FM signals don't diffract around obstacles as well and don't reflect off the ionosphere as efficiently. However, FM's higher frequencies allow for greater bandwidth, which is why FM provides better sound quality (stereo broadcasts with wider frequency response). The physical size of efficient antennas is also related to wavelength (typically $\lambda/2$ or $\lambda/4$), which is why AM antennas are much larger structures (tens to hundreds of meters) compared to FM antennas (around 1-2 meters).

Final Answer

(a) The AM radio wavelength range is 188 m to 556 m. (b) The FM radio wavelength range is 2.78 m to 3.41 m.
Strategy

A radio station utilizes frequencies between commercial AM and FM. What is the frequency of a 11.12-m-wavelength channel?

Show Solution

To find the frequency, we use the wave equation $c=f\lambda$ and solve for frequency: $f=c\lambda$. We're given the wavelength and know the speed of light.

Solution

Given:

- $\lambda=11.12$ m
- $c=3.00\times10^{8}$ m/s

Using $f=c\lambda$:

$$f=3.00\times10^{8}\text{ m/s}11.12\text{ m}=2.698\times10^{7}\text{ Hz}=26.98\text{ MHz}$$

Rounding to three significant figures:

$$f=27.0\text{ MHz}$$

Discussion

This frequency of approximately 27 MHz falls in the shortwave radio band, which lies between the AM band (540-1600 kHz) and the FM band (88-108 MHz). Shortwave frequencies are used for various purposes including international broadcasting, amateur radio, citizens band (CB) radio, and aviation communication. The 11.12-meter wavelength corresponds to what radio enthusiasts call the "11-meter band," which includes CB radio around 27 MHz. Radio waves at this frequency have interesting propagation characteristics—they can reflect off the ionosphere for long-distance communication, but the effectiveness varies significantly with time of day, season, and solar activity. During periods of high solar activity, the 11-meter band can support intercontinental communication, while at other times it's limited to more local use. The wavelength being just over 11 meters means efficient antennas for this frequency would be around 2.78 m ($\lambda/4$) long, which is practical for mobile and portable applications.

Final Answer

The frequency of the 11.12-m-wavelength channel is 27.0 MHz, which falls in the shortwave radio band between commercial AM and FM.

Strategy

Find the frequency range of visible light, given that it encompasses wavelengths from 380 to 760 nm.

Show Solution

Using $f = c\lambda$, we'll calculate the frequencies corresponding to the extreme wavelengths of visible light. The shortest wavelength (380 nm, violet) gives the highest frequency, and the longest wavelength (760 nm, red) gives the lowest frequency.

Solution

Given wavelength range: 380 nm to 760 nm

For the shortest wavelength (highest frequency, violet end):

$$f_{max}=c\lambda_{min}=3.00\times10^{8}\text{ m/s}380\times10^{-9}\text{ m}=7.89\times10^{14}\text{ Hz}$$

For the longest wavelength (lowest frequency, red end):

$$f_{min}=c\lambda_{max}=3.00\times10^{8}\text{ m/s}760\times10^{-9}\text{ m}=3.95\times10^{14}\text{ Hz}$$

Therefore, the frequency range of visible light is **3.95×10^{14} Hz to 7.89×10^{14} Hz**.

Discussion

The visible light frequency range spans approximately one octave (a factor of 2 in frequency), from about 4×10^{14} Hz (red) to about 8×10^{14} Hz (violet). These extraordinarily high frequencies—nearly a million billion cycles per second—correspond to the resonant frequencies at which atoms and molecules emit and absorb electromagnetic radiation. It's remarkable that the human eye can detect only this narrow slice of the electromagnetic spectrum, which represents less than one octave out of the many decades of frequencies that electromagnetic waves can have. Our perception of different colors corresponds to our eyes' responses to different frequencies within this range: red light at the lower frequency end, progressing through orange, yellow, green, blue, and violet at the higher frequency end. The retinal cone cells in our eyes contain photopigments that are sensitive to different parts of this frequency range, with peak sensitivities roughly in the blue ($\sim6.7\times10^{14}$ Hz), green ($\sim5.4\times10^{14}$ Hz), and red ($\sim4.5\times10^{14}$ Hz) regions, allowing us to perceive millions of color combinations through the brain's processing of signals from these three types of cones.

Final Answer

The frequency range of visible light is 3.95×10^{14} Hz to 7.89×10^{14} Hz, corresponding to wavelengths from 760 nm (red) to 380 nm (violet).
Strategy

Combing your hair leads to excess electrons on the comb. How fast would you have to move the comb up and down to produce red light?

Show Solution

To produce electromagnetic radiation, charges must oscillate at the frequency of the desired radiation. Red light is at the low-frequency end of the visible spectrum, with wavelengths around 650-700 nm (middle of the red range is approximately 650 nm). We'll use $f = c\lambda$ to find the required oscillation frequency of the charged comb.

Solution

Taking a representative wavelength for red light:

$$\lambda=650\text{ nm}=650\times10^{-9}\text{ m}$$

The required frequency is:

$$f=c\lambda=3.00\times10^{8}\text{ m/s}650\times10^{-9}\text{ m}=4.62\times10^{14}\text{ Hz}$$

Rounding to two significant figures:

$$f\approx4.6\times10^{14}\text{ Hz}$$

This can also be expressed as 460 THz (terahertz).

Discussion

This frequency—about 460 trillion oscillations per second—is impossibly fast for any mechanical motion. Even the fastest vibrations we can produce mechanically (ultrasonic transducers) reach only millions of hertz, not hundreds of trillions. This illustrates why static electricity from combing your hair doesn't produce visible light, despite creating oscillating charges when you move the comb. The timescale is wrong by a factor of about 100 million!

To actually produce electromagnetic radiation at optical frequencies requires atomic or molecular processes, not mechanical motion. In atoms, electrons can transition between energy levels in timescales of nanoseconds or faster, producing oscillating electric dipoles at the frequencies needed for visible light. This is how incandescent bulbs work (thermal excitation of atoms) and how LEDs and lasers work (controlled electron transitions).

However, oscillating charges at lower frequencies can and do produce electromagnetic radiation—just not visible light. Moving a charged comb at 1000 Hz would produce 1000 Hz radio waves (extremely low frequency, or ELF). In fact, this is the principle behind radio transmission, where electrons in antennas oscillate at radio frequencies (thousands to billions of hertz) to produce radio waves.

Final Answer

To produce red light, you would need to oscillate the charged comb at a frequency of approximately 4.6×10^{14} Hz or 460 THz, which is impossibly fast for mechanical motion.
Strategy

Electromagnetic radiation having a $15.0 - \mu\text{m}$ wavelength is classified as infrared radiation. What is its frequency?

Show Solution

We'll use the wave equation $f = c \lambda$ to calculate the frequency from the given wavelength. We need to convert the wavelength from micrometers to meters first.

Solution

Given:

$$\lambda = 15.0\ \mu\text{m} = 15.0 \times 10^{-6}\ \text{m}$$

Using $f = c \lambda$:

$$f = 3.00 \times 10^{8}\ \text{m/s} 15.0 \times 10^{-6}\ \text{m} = 2.00 \times 10^{13}\ \text{Hz}$$

This can also be expressed as:

$$f = 20.0\ \text{THz}$$

Discussion

This frequency of 2.00×10^{13} Hz falls in the mid-infrared region of the electromagnetic spectrum. Infrared radiation spans from just beyond visible red light (around 760 nm or 4×10^{14} Hz) down to microwave frequencies (around 1 mm or 3×10^{11} Hz). This particular wavelength of 15.0 μm is significant because it's in the thermal infrared range—objects near room temperature (around 300 K) emit peak radiation at wavelengths around 10 μm according to Wien's displacement law. This is why thermal imaging cameras are sensitive to wavelengths in the 8-14 μm range.

This wavelength is also important in astronomy and atmospheric science because it corresponds to a relatively transparent "atmospheric window"—wavelengths where Earth's atmosphere doesn't strongly absorb infrared radiation. Water vapor and carbon dioxide in the atmosphere absorb strongly at some infrared wavelengths but less so at others, creating these windows that allow astronomers to observe cool celestial objects and that affect Earth's energy balance and climate.

Final Answer

Infrared radiation with a wavelength of 15.0 μm has a frequency of 2.00×10^{13} Hz or 20.0 THz.

Strategy

Approximately what is the smallest detail observable with a microscope that uses ultraviolet light of frequency $1.20 \times 10^{15}\text{Hz}$?

Show Solution

The resolution limit of a microscope is fundamentally limited by the wavelength of the light used—you cannot resolve details significantly smaller than the wavelength. First, we'll calculate the wavelength using $\lambda = c f$, which will give us the approximate minimum resolvable detail.

Solution

Given:

$$f = 1.20 \times 10^{15}\ \text{Hz}$$

Calculate the wavelength:

$$\lambda = c f = 3.00 \times 10^{8}\ \text{m/s} 1.20 \times 10^{15}\ \text{Hz} = 2.50 \times 10^{-7}\ \text{m}$$

Converting to nanometers:

$$\lambda = 250\ \text{nm}$$

The smallest resolvable detail is approximately equal to the wavelength, so:

$$\text{Minimum detail} \approx 250\ \text{nm} = 2.50 \times 10^{-7}\ \text{m}$$

Discussion

This UV wavelength of 250 nm provides better resolution than visible light microscopes, which are limited to details of about 400-500 nm (roughly half the wavelength of visible light, or about 200-250 nm under optimal conditions with blue light). The 250 nm UV light falls in the UV-C range and provides resolution capabilities that allow imaging of structures like large protein complexes, viruses, and subcellular organelles with greater detail than visible light microscopy.

However, there are practical challenges with UV microscopy: UV light is absorbed by glass, requiring special quartz optics; it can damage biological samples; and it's invisible to the human eye, requiring special detectors. Despite these challenges, UV microscopy is valuable in semiconductor inspection, biological research, and materials science.

For even better resolution, electron microscopes use the wave nature of electrons, which have much shorter wavelengths (on the order of picometers at typical accelerating voltages), allowing resolution down to atomic scales. This demonstrates how the fundamental diffraction limit—that you can't resolve details much smaller than the wavelength—drives technology choices in microscopy.

Final Answer

The smallest detail observable with a UV microscope using 1.20×10^{15} Hz ultraviolet light is approximately 250 nm or 2.50×10^{-7} m.

Strategy

A radar used to detect the presence of aircraft receives a pulse that has reflected off an object 6×10^{-5}s after it was transmitted. What is the distance from the radar station to the reflecting object?

Show Solution

The electromagnetic wave travels from the radar to the object and back, so the total distance traveled is twice the distance to the object. Using the relationship $\text{distance}=\text{speed}\times\text{time}$, we can find the total distance traveled by the radar pulse at the speed of light, then divide by 2 to get the distance to the object.

Solution

Given:

- Round-trip time: $t=6\times10^{-5}$ s
- Speed of light: $c=3.00\times10^{8}$ m/s

Total distance traveled by the radar pulse:

$$d_{\text{total}}=ct=(3.00\times10^{8}\text{ m/s})(6\times10^{-5}\text{ s})=1.8\times10^{4}\text{ m}$$

Distance to the object (one-way distance):

$$d=\frac{d_{\text{total}}}{2}=\frac{1.8\times10^{4}\text{ m}}{2}=9.0\times10^{3}\text{ m}=9.0\text{ km}$$

Discussion

The aircraft is 9.0 km or approximately 5.6 miles away from the radar station. This is a typical detection range for air traffic control radar. The extremely short time delay (60 microseconds) demonstrates why radar systems require precise electronic timing—the difference between an object at 8 km and one at 10 km is only about 13 microseconds in round-trip time.

Radar works because electromagnetic waves reflect off conducting or dielectric objects. Aircraft are particularly good radar targets because their metal bodies reflect radio waves efficiently. Modern radar systems can measure not only the distance (from time delay) but also the velocity (from Doppler shift of the returned frequency) and even create images of the target (using synthetic aperture radar techniques).

The speed of light being finite means there's always a time lag in radar detection. For objects at 9 km, the radar "sees" where the object was 30 microseconds ago (the one-way trip time). For most aircraft speeds (around 200-900 km/h or 55-250 m/s), the aircraft moves only about 2-8 millimeters during this time, which is negligible compared to the radar's range resolution.

Final Answer

The distance from the radar station to the reflecting object is 9.0 km or 9.0×10^{3} m.

Strategy

Some radar systems detect the size and shape of objects such as aircraft and geological terrain. Approximately what is the smallest observable detail utilizing 500-MHz radar?

Show Solution

Similar to optical microscopy, the resolution of radar is limited by the wavelength of the electromagnetic radiation used. We'll first calculate the wavelength of 500-MHz radar waves using $\lambda=\frac{c}{f}$, which gives us the approximate minimum resolvable detail.

Solution

Given:

$$f = 500\ \text{MHz} = 500 \times 10^{6}\ \text{Hz} = 5.00 \times 10^{8}\ \text{Hz}$$

Calculate the wavelength:

$$\lambda = \frac{c}{f} = \frac{3.00 \times 10^{8}\ \text{m/s}}{5.00 \times 10^{8}\ \text{Hz}} = 0.600\ \text{m}$$

The smallest observable detail is approximately equal to the wavelength:

$$\text{Minimum detail} \approx 0.600\ \text{m}$$

Discussion

A resolution of 0.600 m (about 60 cm or 2 feet) means this radar system can distinguish features on aircraft or terrain that are separated by at least this distance. This is useful for detecting the general size and shape of aircraft, distinguishing between large and small aircraft, or mapping terrain features like hills and valleys. However, it cannot resolve fine details like individual antennas or small protrusions on an aircraft.

Higher-frequency radar provides better resolution: For example, 10 GHz radar (X-band, used in weather radar and some military applications) has a wavelength of 3 cm, allowing much finer detail resolution. This is why different radar frequencies are chosen for different applications—lower frequencies (like this 500 MHz) penetrate better through weather and provide longer range, while higher frequencies provide better resolution but are more affected by atmospheric absorption and scattering.

The wavelength also affects the size of radar antennas. Efficient radar antennas are typically several wavelengths across. For 500-MHz radar with a 0.6-m wavelength, practical antennas would be a few meters in size, which is reasonable for ground-based or ship-based installations but too large for small aircraft.

Final Answer

The smallest observable detail using 500-MHz radar is approximately 0.600 m.

Strategy

Determine the amount of time it takes for X-rays of frequency $3 \times 10^{18}\text{Hz}$ to travel (a) 1 mm and (b) 1 cm.

Show Solution

X-rays, like all electromagnetic waves, travel at the speed of light $c = 3.00 \times 10^{8}$ m/s. The frequency is given but not needed for this problem—all electromagnetic waves travel at the same speed in vacuum regardless of frequency. We use $t = \frac{d}{c}$ to find the travel time for each distance.

Solution

(a) Time to travel 1 mm:

$$d = 1\ \text{mm} = 1 \times 10^{-3}\ \text{m}$$
$$t = \frac{d}{c} = \frac{1 \times 10^{-3}\ \text{m}}{3.00 \times 10^{8}\ \text{m/s}} = 3.33 \times 10^{-12}\ \text{s} = 3.33\ \text{ps}$$

(b) Time to travel 1 cm:

$$d = 1\ \text{cm} = 1 \times 10^{-2}\ \text{m}$$
$$t = \frac{d}{c} = \frac{1 \times 10^{-2}\ \text{m}}{3.00 \times 10^{8}\ \text{m/s}} = 3.33 \times 10^{-11}\ \text{s} = 33.3\ \text{ps}$$

Discussion

These incredibly short times—3.33 picoseconds for 1 mm and 33.3 picoseconds for 1 cm—illustrate how fast electromagnetic radiation travels. A picosecond (10^{-12} s) is to one second as one second is to about 32,000 years! These timescales are important in applications like X-ray crystallography, ultrafast laser physics, and high-speed electronics.

To put this in perspective, in one picosecond, light travels only 0.3 mm (about the thickness of a few sheets of paper). This is why synchronizing signals in modern computer processors (which operate at gigahertz frequencies with nanosecond clock periods) requires careful attention to the physical distances signals must travel on the chip—even centimeter-scale distances introduce measurable delays.

The frequency given in the problem (3×10^{18} Hz) corresponds to "hard" X-rays with a wavelength of about 0.1 nm ($\lambda = c/f$), which is on the order of atomic spacings. Such X-rays are used in medical imaging and materials analysis because they can penetrate matter while still interacting enough to create useful images or diffraction patterns.

Final Answer

(a) X-rays take 3.33 ps to travel 1 mm. (b) X-rays take 33.3 ps to travel 1 cm.

Strategy

If you wish to detect details of the size of atoms (about $1 \times 10^{-10}\text{m}$) with electromagnetic radiation, it must have a wavelength of about this size. (a) What is its frequency? (b) What type of electromagnetic radiation might this be?

Show Solution

To resolve atomic-scale features, we need electromagnetic radiation with a wavelength comparable to atomic dimensions (about 0.1 nm or 1×10^{-10} m). We'll use $f = c \lambda$ to find the frequency, then identify which part of the electromagnetic spectrum this corresponds to.

Solution

(a) Given:

$$\lambda = 1 \times 10^{-10} \text{ m} = 0.1 \text{ nm}$$

Calculate the frequency:

$$f = c \lambda = 3.00 \times 10^{8} \text{ m/s} 1 \times 10^{-10} \text{ m} = 3 \times 10^{18} \text{ Hz}$$

This can also be expressed as 3000 petahertz (PHz) or 3 exahertz (EHz).

(b) This frequency of 3×10^{18} Hz corresponds to **X-rays**, specifically in the hard X-ray range.

Discussion

X-rays with wavelengths around 0.1 nm are ideal for probing atomic structure because their wavelength is comparable to the spacing between atoms in crystals (typically 0.1-0.5 nm). This is the basis of X-ray crystallography, one of the most powerful techniques for determining the atomic structure of materials.

When X-rays strike a crystal, they diffract from the regular array of atoms, creating an interference pattern that can be analyzed to determine the positions of atoms in the crystal structure. This technique has been crucial for discoveries including the double-helix structure of DNA, the structures of proteins and viruses, and the atomic arrangements in countless materials. Rosalind Franklin's X-ray diffraction images were key evidence for Watson and Crick's DNA model.

The relationship between wavelength and resolvable detail is fundamental across all types of microscopy. Visible light microscopes (wavelengths 400-700 nm) can resolve features down to about 200 nm. Electron microscopes use electrons with wavelengths of picometers, allowing atomic resolution. The choice of probe—photons, electrons, neutrons, or other particles—depends on what wavelength is needed and what interactions with the sample are desired.

Final Answer

(a) The frequency required to detect atomic-scale details is 3×10^{18} Hz. (b) This electromagnetic radiation is X-rays.
Strategy

If the Sun suddenly turned off, we would not know it until its light stopped coming. How long would that be, given that the Sun is 1.50×10^{11}m away?

Show Solution

Light travels at a finite speed $c = 3.00 \times 10^{8}$ m/s, so there's a time delay for light to travel from the Sun to Earth. We use $t = d c$ to calculate this light travel time.

Solution

Given:

$$d = 1.50 \times 10^{11} \text{ m}$$

Calculate the travel time:

$$t = d c = 1.50 \times 10^{11} \text{ m} 3.00 \times 10^{8} \text{ m/s} = 500 \text{ s}$$

Converting to minutes:

$$t = 500 \text{ s} 60 \text{ s/min} = 8.33 \text{ min}$$

Discussion

We would not know the Sun had turned off for approximately 8.3 minutes (about 8 minutes and 20 seconds). This means we always see the Sun as it was 8.3 minutes ago, not as it is "now." This delay applies to all electromagnetic radiation from the Sun—visible light, infrared, ultraviolet, and radio waves all travel at the speed of light.

Interestingly, this also means that if the Sun suddenly disappeared, Earth would continue orbiting for those same 8.3 minutes before gravitational effects reached us. According to Einstein's general relativity, gravitational influences also propagate at the speed of light, not instantaneously.

This light-travel-time effect becomes even more dramatic for more distant objects. We see the nearest star (Proxima Centauri) as it was 4.2 years ago, the Andromeda galaxy as it was 2.5 million years ago, and the most distant observable galaxies as they were over 13 billion years ago. Astronomy is thus a form of time travel—we literally look back in time when we look out into space.

The distance to the Sun (1.50×10^{11} m) is defined as one Astronomical Unit (AU), a fundamental unit in astronomy for measuring distances within our solar system.

Final Answer

If the Sun suddenly turned off, we would not know for 500 seconds or approximately 8.3 minutes, the time it takes light to travel from the Sun to Earth.
Strategy

Distances in space are often quoted in units of light years, the distance light travels in one year. (a) How many meters is a light year? (b) How many meters is it to Andromeda, the nearest large galaxy, given that it is 2.00×10^{6} light years away? (c) The most distant galaxy yet discovered is 12.0×10^{9} light years away. How far is this in meters?

Show Solution

A light year is the distance light travels in one year. We'll first calculate the number of seconds in a year, then multiply by the speed of light to get the distance in meters. For parts (b) and (c), we'll multiply the distance in light years by the conversion factor from part (a).

Solution

(a) Calculate the number of seconds in one year:

$$t = 1 \text{ yr} \times \frac{365.25 \text{ days}}{1 \text{ yr}} \times \frac{24 \text{ h}}{1 \text{ day}} \times \frac{3600 \text{ s}}{1 \text{ h}} = 3.156 \times 10^{7} \text{ s}$$

Distance light travels in one year:

$$1 \text{ ly} = ct = (3.00 \times 10^{8} \text{ m/s})(3.156 \times 10^{7} \text{ s}) = 9.47 \times 10^{15} \text{ m}$$

(b) Distance to Andromeda:

$$d = (2.00 \times 10^{6} \text{ ly})(9.47 \times 10^{15} \text{ m/ly}) = 1.89 \times 10^{22} \text{ m}$$

(c) Distance to the most distant galaxy:

$$d = (12.0 \times 10^{9} \text{ ly})(9.47 \times 10^{15} \text{ m/ly}) = 1.14 \times 10^{26} \text{ m}$$

Discussion

These distances are almost incomprehensibly large. A light year (9.47×10^{15} m) is about 63,000 times the distance from Earth to the Sun. The Andromeda galaxy at 1.89×10^{22} m is so far away that light from it has been traveling for 2 million years to reach us—we see Andromeda as it was when our human ancestors were just beginning to use stone tools.

The most distant galaxies at 1.14×10^{26} m (12 billion light years) show us the universe as it was 12 billion years ago, less than 2 billion years after the Big Bang. These galaxies appear to us now as they were in their infancy, when the universe itself was much younger. The observable universe has a radius of about 46 billion light years (different from 13.8 billion because of cosmic expansion), or about 4×10^{26} m.

To put the light year in perspective: if you could travel at highway speeds (100 km/h or 28 m/s) continuously, it would take you about 11 million years to travel one light year. Even traveling at the speed of the fastest spacecraft ever built (Parker Solar Probe at about 190 km/s at perihelion), it would take about 1,600 years to cover one light year.

Final Answer

(a) One light year equals 9.47×10^{15} m. (b) The Andromeda galaxy is 1.89×10^{22} m away. (c) The most distant galaxy is 1.14×10^{26} m away.

Strategy

A certain 50.0-Hz AC power line radiates an electromagnetic wave having a maximum electric field strength of 13.0 kV/m. (a) What is the wavelength of this very low frequency electromagnetic wave? (b) What is its maximum magnetic field strength?

Show Solution

For part (a), we use $\lambda = \frac{c}{f}$ to find the wavelength from the frequency. For part (b), we use the relationship between electric and magnetic field strengths in an electromagnetic wave: $B = \frac{E}{c}$.

Solution

(a) Given:

$$f = 50.0 \text{ Hz}$$

Calculate the wavelength:

$$\lambda = \frac{c}{f} = \frac{3.00 \times 10^8 \text{ m/s}}{50.0 \text{ Hz}} = 6.00 \times 10^6 \text{ m}$$

(b) Given:

$$E_{\max} = 13.0 \text{ kV/m} = 1.30 \times 10^4 \text{ V/m}$$

Calculate the maximum magnetic field strength:

$$B_{\max} = \frac{E_{\max}}{c} = \frac{1.30 \times 10^4 \text{ V/m}}{3.00 \times 10^8 \text{ m/s}} = 4.33 \times 10^{-5} \text{ T}$$

Discussion

The wavelength of 6.00×10^6 m (6000 km) is enormous—longer than the radius of Earth! This extremely long wavelength is characteristic of extremely low frequency (ELF) electromagnetic waves. Power lines operate at 50 Hz or 60 Hz (depending on the country), and while they're designed to carry electrical power efficiently through conductors, they do radiate some electromagnetic energy at these frequencies.

These ELF waves can travel great distances and penetrate deep into seawater, which is why they've been used for communication with submarines. However, the huge wavelength means very low bandwidth—only a few bits per second can be transmitted. The magnetic field strength of 4.33×10^{-5} T (0.0433 mT or 0.433 gauss) is comparable to Earth's magnetic field (about 0.5 gauss), which is why there has been ongoing research into whether long-term exposure to power-line electromagnetic fields might have biological effects, though current scientific consensus finds no confirmed health risks at typical exposure levels.

The electric field of 13.0 kV/m is quite strong—significantly higher than the fair-weather atmospheric electric field of about 100 V/m—but these fields decrease rapidly with distance from power lines.

Final Answer

(a) The wavelength is 6.00×10^6 m or 6000 km. (b) The maximum magnetic field strength is 4.33×10^{-5} T.

Strategy

During normal beating, the heart creates a maximum 4.00-mV potential across 0.300 m of a person's chest, creating a 1.00-Hz electromagnetic wave. (a) What is the maximum electric field strength created? (b) What is the corresponding maximum magnetic field strength in the electromagnetic wave? (c) What is the wavelength of the electromagnetic wave?

Show Solution

For part (a), electric field strength is potential difference divided by distance: $E = \frac{V}{d}$. For part (b), we use $B = \frac{E}{c}$ to find the magnetic field strength. For part (c), we use $\lambda = \frac{c}{f}$ to find the wavelength.

Solution

(a) Given:

- Potential: $V = 4.00 \text{ mV} = 4.00 \times 10^{-3}$ V
- Distance: $d = 0.300$ m

Calculate the electric field strength:

$$E = \frac{V}{d} = \frac{4.00 \times 10^{-3} \text{ V}}{0.300 \text{ m}} = 1.33 \times 10^{-2} \text{ V/m} = 13.3 \text{ mV/m}$$

(b) Calculate the magnetic field strength:

$$B = \frac{E}{c} = \frac{1.33 \times 10^{-2} \text{ V/m}}{3.00 \times 10^8 \text{ m/s}} = 4.44 \times 10^{-11} \text{ T}$$

(c) Given:

- Frequency: $f = 1.00$ Hz

Calculate the wavelength:

$$\lambda = \frac{c}{f} = \frac{3.00 \times 10^8 \text{ m/s}}{1.00 \text{ Hz}} = 3.00 \times 10^8 \text{ m}$$

Discussion

This problem illustrates the bioelectric signals produced by the heart during its beating cycle—the electrical activity that electrocardiograms (ECGs or EKGs) measure. The electric field strength of 13.3 mV/m is very weak, which is why ECG electrodes must be placed directly on the skin for good electrical contact. The corresponding magnetic field of 4.44×10^{-11} T is extraordinarily weak—about one-millionth of Earth's magnetic field—yet sensitive magnetometers called SQUID (Superconducting Quantum Interference Device) detectors can measure even these tiny magnetic fields in a technique called magnetocardiography (MCG).

The wavelength of 3.00×10^8 m (300,000 km) is enormous—almost the distance from Earth to the Moon! This is because the frequency is so low (1 Hz, corresponding to a heart rate of 60 beats per minute). This extremely long wavelength means the "electromagnetic wave" is really just a quasi-static electric and magnetic field that changes slowly—the wave picture isn't very meaningful at such low frequencies and short distances. The fields are better understood as the near-field bioelectric and biomagnetic fields of the heart's electrical dipole, rather than as propagating electromagnetic waves.

Final Answer

(a) The maximum electric field strength is 1.33×10^{-2} V/m or 13.3 mV/m. (b) The maximum magnetic field strength is 4.44×10^{-11} T. (c) The wavelength is 3.00×10^8 m.
Strategy

(a) The ideal size (most efficient) for a broadcast antenna with one end on the ground is one-fourth the wavelength ($\lambda/4$) of the electromagnetic radiation being sent out. If a new radio station has such an antenna that is 50.0 m high, what frequency does it broadcast most efficiently? Is this in the AM or FM band? (b) Discuss the analogy of the fundamental resonant mode of an air column closed at one end to the resonance of currents on an antenna that is one-fourth their wavelength.

Show Solution

For part (a), if the antenna height is $\lambda/4$, we can solve for the wavelength ($\lambda = 4h$), then use $f = c\lambda$ to find the frequency. We'll then compare this frequency to the AM band (540-1600 kHz) and FM band (88-108 MHz). For part (b), we'll discuss the physical analogy between antenna resonance and acoustic resonance.

Solution

(a) Given:

- Antenna height: $h = 50.0$ m
- For optimal efficiency: $h = \lambda 4$

Solve for wavelength:

$$\lambda = 4h = 4(50.0 \text{ m}) = 200 \text{ m}$$

Calculate the frequency:

$$f = c\lambda = 3.00 \times 10^8 \text{ m/s} 200 \text{ m} = 1.50 \times 10^6 \text{ Hz} = 1.50 \text{ MHz}$$

This frequency of 1.50 MHz falls within the **AM band** (540-1600 kHz or 0.54-1.6 MHz).

(b) The analogy between a quarter-wave antenna and an air column closed at one end is quite direct. In an air column closed at one end, the fundamental resonance occurs when the column length equals one-fourth of the sound wavelength. At the closed end, there must be a pressure antinode (displacement node), and at the open end, there must be a pressure node (displacement antinode).

Similarly, in a quarter-wave antenna with one end grounded, the ground acts like the "closed end" of the air column. The current must be maximum (antinode) at the grounded end and minimum (node) at the top. The voltage distribution is opposite: minimum at the ground and maximum at the top. This standing wave pattern of current and voltage along the antenna occurs when the antenna length is one-quarter wavelength, creating a resonance that allows efficient radiation of electromagnetic waves.

In both cases—sound in an air column and electromagnetic waves on an antenna—the quarter-wavelength condition creates a resonant standing wave pattern that efficiently couples energy from the source to the surrounding medium (air for sound, free space for electromagnetic waves).

Final Answer

(a) The antenna broadcasts most efficiently at 1.50 MHz, which is in the AM band. (b) Both a quarter-wave antenna and an air column closed at one end have length $\lambda/4$ for fundamental resonance, with an antinode at the closed/grounded end and a node at the open/free end, creating efficient energy transfer through resonant standing wave patterns.

Strategy

(a) What is the wavelength of 100-MHz radio waves used in an MRI unit? (b) If the frequencies are swept over a ± 1.00 range centered on 100 MHz, what is the range of wavelengths broadcast?

Show Solution

For part (a), we use $\lambda = cf$ to find the wavelength. For part (b), we need to find the wavelengths corresponding to the minimum and maximum frequencies in the swept range. The ±1.00 likely refers to ±1.00 MHz (a 1% frequency range is typical for MRI).

Solution

(a) Given:

$$f = 100 \text{ MHz} = 1.00 \times 10^8 \text{ Hz}$$

Calculate the wavelength:

$$\lambda = \frac{c}{f} = \frac{3.00 \times 10^8 \text{ m/s}}{1.00 \times 10^8 \text{ Hz}} = 3.00 \text{ m}$$

(b) The frequency range is 100 MHz ± 1.00 MHz, so:

- Minimum frequency: $f_{\text{min}} = 99.0 \text{ MHz} = 9.90 \times 10^7 \text{ Hz}$
- Maximum frequency: $f_{\text{max}} = 101 \text{ MHz} = 1.01 \times 10^8 \text{ Hz}$

Wavelength at minimum frequency (longest wavelength):

$$\lambda_{\text{max}} = \frac{c}{f_{\text{min}}} = \frac{3.00 \times 10^8 \text{ m/s}}{9.90 \times 10^7 \text{ Hz}} = 3.03 \text{ m}$$

Wavelength at maximum frequency (shortest wavelength):

$$\lambda_{\text{min}} = \frac{c}{f_{\text{max}}} = \frac{3.00 \times 10^8 \text{ m/s}}{1.01 \times 10^8 \text{ Hz}} = 2.97 \text{ m}$$

Therefore, the wavelength range is **2.97 m to 3.03 m**.

Discussion

MRI (Magnetic Resonance Imaging) uses radio frequency electromagnetic waves to excite nuclear spins in a strong magnetic field. The specific frequency depends on the magnetic field strength and the type of nucleus being imaged (typically hydrogen in water and fat). For a 1.5-tesla MRI scanner imaging hydrogen nuclei, the resonance frequency is about 64 MHz; for a 3-tesla scanner, it's about 128 MHz. This problem's 100 MHz corresponds to about a 2.35-tesla field.

The wavelength of 3.00 m is comparable to the size of the human body and the MRI machine itself. This means the electromagnetic field doesn't propagate as a clean wave but rather fills the imaging volume quasi-statically. The frequency sweep of ±1.00 MHz allows the MRI to selectively excite spins in different locations (using magnetic field gradients) and to sample the chemical shift range—different molecular environments cause small shifts in resonance frequency, providing chemical information as well as anatomical imaging.

The relatively narrow wavelength range (2.97-3.03 m, only a 2% variation) reflects the narrow frequency sweep (±1%), demonstrating that for radio frequencies, even a substantial frequency range translates to a small wavelength variation.

Final Answer

(a) The wavelength of 100-MHz radio waves is 3.00 m. (b) The wavelength range for frequencies swept over ±1.00 MHz centered on 100 MHz is 2.97 m to 3.03 m.
Strategy

(a) What is the frequency of the 193-nm ultraviolet radiation used in laser eye surgery? (b) Assuming the accuracy with which this EM radiation can ablate the cornea is directly proportional to wavelength, how much more accurate can this UV be than the shortest visible wavelength of light?

Show Solution

For part (a), we use $f = \frac{c}{\lambda}$ to find the frequency. For part (b), we compare the UV wavelength (193 nm) to the shortest visible wavelength (approximately 380 nm, violet light), calculating the ratio to determine the improvement in accuracy.

Solution

(a) Given:

$$\lambda = 193 \text{ nm} = 193 \times 10^{-9} \text{ m}$$

Calculate the frequency:

$$f = \frac{c}{\lambda} = \frac{3.00 \times 10^8 \text{ m/s}}{193 \times 10^{-9} \text{ m}} = 1.55 \times 10^{15} \text{ Hz}$$

(b) The shortest wavelength of visible light is approximately 380 nm (violet). Calculate the ratio:

$$\frac{\lambda_{\text{visible}}}{\lambda_{\text{UV}}} = \frac{380 \text{ nm}}{193 \text{ nm}} = 1.97$$

Since accuracy is directly proportional to wavelength (smaller wavelength = better accuracy), the UV radiation is 1.97 times more accurate, or approximately **97% more accurate** than the shortest visible wavelength.

Alternatively, this can be expressed as the UV being **almost twice as accurate** as violet visible light.

Discussion

The 193-nm wavelength used in laser eye surgery (LASIK and PRK procedures) is produced by an ArF (argon fluoride) excimer laser. This deep UV wavelength is particularly well-suited for corneal surgery for several reasons beyond just the accuracy improvement demonstrated here.

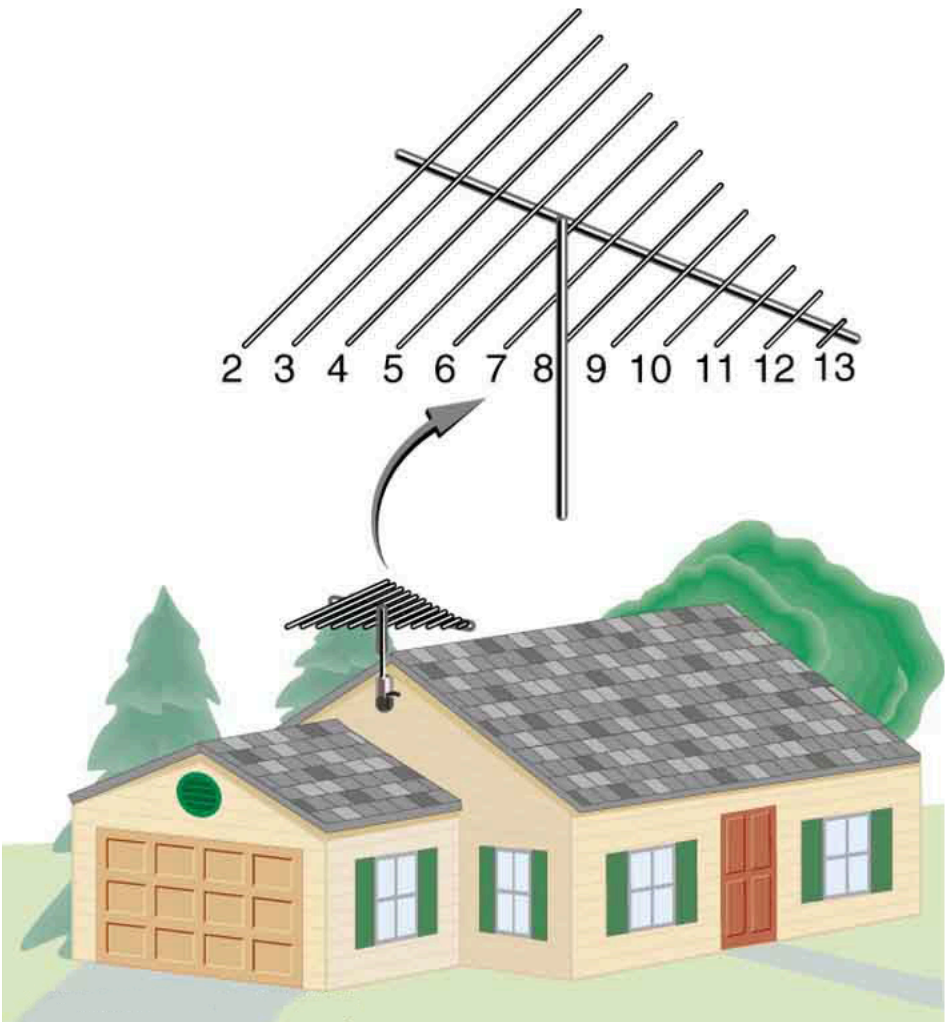
First, the shorter wavelength allows for more precise material removal—the laser can ablate tissue in layers thinner than a micrometer, allowing surgeons to reshape the cornea with great precision to correct refractive errors. Second, this wavelength is strongly absorbed by the corneal tissue (primarily by breaking molecular bonds in proteins) and doesn't penetrate deeply, so the energy is deposited in a very thin layer at the surface. This minimizes thermal damage to surrounding tissue and prevents damage to deeper structures like the lens and retina.

Third, the ablation is "cold"—it's photochemical rather than thermal, breaking chemical bonds directly rather than heating tissue. This produces clean, precise cuts with minimal collateral damage. The combination of short wavelength (high spatial precision), strong absorption (shallow penetration), and photochemical ablation (clean removal) makes 193-nm UV radiation ideal for reshaping the cornea to correct vision. millions of people worldwide have had their vision corrected using this technology.

Final Answer

(a) The frequency of 193-nm UV radiation is 1.55×10^{15} Hz. (b) The 193-nm UV radiation is 1.97 times (or 97%) more accurate than the shortest visible wavelength (380 nm), or almost twice as accurate.
Strategy

TV-reception antennas for VHF are constructed with cross wires supported at their centers, as shown in [Figure 14]. The ideal length for the cross wires is one-half the wavelength to be received, with the more expensive antennas having one for each channel. Suppose you measure the lengths of the wires for particular channels and find them to be 1.94 and 0.753 m long, respectively. What are the frequencies for these channels?



A television reception antenna has cross wires of various lengths to most efficiently receive different wavelengths.

Show Solution

If the wire length is $\lambda/2$, we can find the wavelength by multiplying the length by 2: $\lambda = 2L$. Then we use $f = c\lambda$ to find the frequency for each channel.

Solution

For the first wire (L = 1.94 m):

$$\lambda = 2L = 2(1.94 \text{ m}) = 3.88 \text{ m}$$

$$f = \frac{c}{\lambda} = \frac{3.00 \times 10^8 \text{ m/s}}{3.88 \text{ m}} = 7.73 \times 10^7 \text{ Hz} = 77.3 \text{ MHz}$$

For the second wire (L = 0.753 m):

$$\lambda = 2L = 2(0.753 \text{ m}) = 1.506 \text{ m}$$
$$f = \frac{c}{\lambda} = \frac{3.00 \times 10^8 \text{ m/s}}{1.506 \text{ m}} = 1.99 \times 10^8 \text{ Hz} = 199 \text{ MHz}$$

Discussion

These frequencies fall within the VHF (Very High Frequency) television band. The first frequency (77.3 MHz) is in the low VHF TV band (channels 2-6, 54-88 MHz), likely corresponding to channel 5 or 6. The second frequency (199 MHz) is in the high VHF TV band (channels 7-13, 174-216 MHz), likely corresponding to channel 11 or 12.

The half-wavelength dipole antenna is resonant at the frequency where its length equals half the wavelength. At resonance, the antenna efficiently couples electromagnetic waves to the receiver—the standing wave pattern on the antenna has a voltage maximum at the ends and a current maximum at the center (where it connects to the transmission line). This is analogous to a string fixed at both ends vibrating at its fundamental frequency.

Traditional outdoor TV antennas have multiple cross wires of different lengths to receive multiple channels. More expensive antennas have more elements, each tuned to a specific channel, providing better reception across the entire VHF band. Modern indoor antennas and those for digital TV (ATSC) use different designs, often including amplifiers, but the half-wavelength resonance principle remains fundamental to antenna design.

Final Answer

The frequencies for the two channels are 77.3 MHz (for the 1.94 m wire) and 199 MHz (for the 0.753 m wire).

Strategy

Conversations with astronauts on lunar walks had an echo that was used to estimate the distance to the Moon. The sound spoken by the person on Earth was transformed into a radio signal sent to the Moon, and transformed back into sound on a speaker inside the astronaut's space suit. This sound was picked up by the microphone in the space suit (intended for the astronaut's voice) and sent back to Earth as a radio echo of sorts. If the round-trip time was 2.60 s, what was the approximate distance to the Moon, neglecting any delays in the electronics?

Show Solution

Radio waves travel at the speed of light. The round-trip time includes the signal traveling from Earth to the Moon and back. Using $d = ct$ for the total distance traveled, we divide by 2 to get the one-way distance to the Moon.

Solution

Given:

- Round-trip time: $t = 2.60$ s
- Speed of light: $c = 3.00 \times 10^8$ m/s

Total distance traveled (round trip):

$$d_{\text{total}} = ct = (3.00 \times 10^8 \text{ m/s})(2.60 \text{ s}) = 7.80 \times 10^8 \text{ m}$$

One-way distance to the Moon:

$$d = \frac{d_{\text{total}}}{2} = \frac{7.80 \times 10^8 \text{ m}}{2} = 3.90 \times 10^8 \text{ m}$$

Discussion

The calculated distance of 3.90×10^8 m (390,000 km or about 242,000 miles) is close to the average Earth-Moon distance of about 384,400 km. The slight difference could be due to the Moon's elliptical orbit—its distance from Earth varies from about 356,000 km (perigee) to 407,000 km (apogee). During the Apollo missions, depending on the Moon's orbital position and the specific mission trajectory, the distance could vary.

This 2.60-second delay made conversations between Mission Control and the astronauts somewhat awkward. Each person had to wait over 2.5 seconds for a response to their question—not long enough to be completely disruptive, but long enough to require some adjustment to normal conversation patterns. Mission Control would ask a question, wait 1.3 seconds for the signal to reach the Moon, wait for the astronaut to respond, then wait another 1.3 seconds for the response to return.

This same principle is used for lunar laser ranging experiments, where laser pulses are bounced off retroreflectors placed on the Moon by Apollo astronauts. These experiments can measure the Earth-Moon distance with millimeter precision by timing the round-trip travel of laser pulses, and they've revealed that the Moon is gradually moving away from Earth at about 3.8 cm per year due to tidal effects.

Final Answer

The approximate distance to the Moon was 3.90×10^8 m or 390,000 km.

Strategy

Lunar astronauts placed a reflector on the Moon's surface, off which a laser beam is periodically reflected. The distance to the Moon is calculated from the round-trip time. (a) To what accuracy in meters can the distance to the Moon be determined, if this time can be measured to 0.100 ns? (b) What percent accuracy is this, given the average distance to the Moon is $3.84 \times 10^8 \text{m}$?

Show Solution

The uncertainty in distance is related to the uncertainty in time measurement by $\Delta d = c\Delta t$. For round-trip measurements, the one-way distance uncertainty is half the total: $\Delta d_{\text{one-way}} = \frac{c\Delta t}{2}$. For part (b), we calculate the percent accuracy as $\frac{\Delta d}{d} \times 100\%$.

Solution

(a) Given:

- Time measurement accuracy: $\Delta t = 0.100 \text{ ns} = 0.100 \times 10^{-9} \text{ s} = 1.00 \times 10^{-10} \text{ s}$

Total distance uncertainty (round trip):

$$\Delta d_{\text{total}} = c\Delta t = (3.00\times10^8 \text{ m/s})(1.00\times10^{-10} \text{ s}) = 3.00\times10^{-2} \text{ m}$$

One-way distance uncertainty:

$$\Delta d = \frac{\Delta d_{\text{total}}}{2} = \frac{3.00\times10^{-2} \text{ m}}{2} = 1.50\times10^{-2} \text{ m} = 1.50 \text{ cm}$$

(b) Given average distance: $d = 3.84 \times 10^8 \text{ m}$

Percent accuracy:

$$\text{Percent accuracy} = \frac{\Delta d}{d}\times100\% = \frac{1.50\times10^{-2} \text{ m}}{3.84\times10^8 \text{ m}}\times100\% = 3.91\times10^{-9}\%$$

Discussion

The ability to measure the Earth-Moon distance to within 1.5 cm (about half an inch) out of 384,000 km is remarkable—a precision of about 4 parts per billion! This is achieved through lunar laser ranging (LLR) experiments that use retroreflector arrays left on the Moon by Apollo 11, 14, and 15 astronauts, and by Soviet Lunokhod rovers.

These measurements require extremely precise timing (measuring round-trip times of about 2.5 seconds to nanosecond precision) and sophisticated corrections for atmospheric effects, Earth's rotation, tectonic plate motion, and relativistic effects. The 0.100 ns timing precision mentioned in this problem corresponds to modern LLR capabilities.

This extraordinary precision has enabled scientists to test general relativity, measure the Moon's orbit with unprecedented accuracy (revealing that the Moon recedes from Earth at about 3.8 cm/year due to tidal friction), study the Moon's internal structure from how its orientation wobbles, and even test whether fundamental constants like the gravitational constant might change over time (they don't, within measurement precision).

The fact that we can measure such tiny distances (centimeters) over such vast scales (hundreds of thousands of kilometers) demonstrates both the power of electromagnetic waves as precision measurement tools and the incredible advances in timing technology.

Final Answer

(a) The distance to the Moon can be determined to an accuracy of 1.50 cm or 1.50×10^{-2} m. (b) This represents a percent accuracy of $3.91 \times 10^{-9}\%$ or about 4 parts per billion.

Strategy

Radar is used to determine distances to various objects by measuring the round-trip time for an echo from the object. (a) How far away is the planet Venus if the echo time is 1000 s? (b) What is the echo time for a car 75.0 m from a Highway Police radar unit? (c) How accurately (in nanoseconds) must you be able to measure the echo time to an airplane 12.0 km away to determine its distance within 10.0 m?

Show Solution

For radar ranging, the round-trip distance is $d_{\text{total}} = ct$, and the one-way distance is $d = \frac{ct}{2}$. For part (c), if we want distance accuracy Δd, the required time accuracy is $\Delta t = \frac{2\Delta d}{c}$ (factor of 2 for round trip).

Solution

(a) Given round-trip time: $t = 1000$ s

One-way distance to Venus:

$$d = \frac{ct}{2} = \frac{(3.00\times10^8 \text{ m/s})(1000 \text{ s})}{2} = 1.50\times10^{11} \text{ m}$$

(b) Given distance: $d = 75.0$ m

Round-trip time:

$$t = \frac{2d}{c} = \frac{2(75.0 \text{ m})}{3.00\times10^8 \text{ m/s}} = 5.00\times10^{-7} \text{ s} = 0.500 \mu\text{s}$$

(c) Given:

- Distance: $d = 12.0 \text{ km} = 1.20 \times 10^4 \text{ m}$
- Required distance accuracy: $\Delta d = 10.0 \text{ m}$

Required time accuracy:

$$\Delta t = \frac{2\Delta d}{c} = \frac{2(10.0 \text{ m})}{3.00 \times 10^8 \text{ m/s}} = 6.67 \times 10^{-8} \text{ s} = 66.7 \text{ ns}$$

Discussion

These three parts illustrate radar ranging across vastly different scales. Part (a) shows planetary radar—the distance to Venus (1.50×10^{11} m or about 150 million km) is approximately the distance from Earth to the Sun (1 AU). Venus is one of the closest planets, and a 1000-second (16.7-minute) round-trip time is typical when Venus is at its closest approach to Earth (about 40 million km) to when it's on the far side of the Sun (about 260 million km). Planetary radar was first achieved in 1961 and has been used to map Venus's surface through its thick clouds and to refine our measurements of planetary orbits and the astronomical unit.

Part (b) demonstrates traffic radar. The 0.500 microsecond round-trip time for a car 75 m away is extremely short, requiring fast electronics to measure. Police radar guns typically operate at microwave frequencies (10-35 GHz) and measure both distance (from time delay) and speed (from Doppler shift in frequency). The speed measurement is actually more commonly used since it doesn't require measuring tiny time delays but instead measures the frequency shift of the returned signal.

Part (c) shows that to locate an airplane 12 km away within 10 m requires timing precision of 66.7 nanoseconds. This is challenging but achievable with modern electronics. Air traffic control radar and military radar systems can achieve such precision. The 10 m accuracy translates to knowing the aircraft's position within about 33 feet, which is important for air traffic safety, especially near airports where precise positioning is critical for preventing collisions.

Final Answer

(a) Venus is 1.50×10^{11} m or 150 million km away. (b) The echo time for a car 75.0 m away is 0.500 μs. (c) The timing accuracy must be 66.7 ns to determine the airplane's distance within 10.0 m.
Strategy

Integrated Concepts

(a) Calculate the ratio of the highest to lowest frequencies of electromagnetic waves the eye can see, given the wavelength range of visible light is from 380 to 760 nm. (b) Compare this with the ratio of highest to lowest frequencies the ear can hear.

Show Solution

For part (a), we find the frequencies corresponding to the shortest (380 nm) and longest (760 nm) visible wavelengths using $f = \frac{c}{\lambda}$, then calculate their ratio. For part (b), we use the typical human hearing range of 20 Hz to 20,000 Hz and calculate the ratio for comparison.

Solution

(a) For visible light:

Highest frequency (shortest wavelength, 380 nm):

$$f_{\text{max}} = \frac{c}{\lambda_{\text{min}}} = \frac{3.00 \times 10^8 \text{ m/s}}{380 \times 10^{-9} \text{ m}} = 7.89 \times 10^{14} \text{ Hz}$$

Lowest frequency (longest wavelength, 760 nm):

$$f_{\text{min}} = \frac{c}{\lambda_{\text{max}}} = \frac{3.00 \times 10^8 \text{ m/s}}{760 \times 10^{-9} \text{ m}} = 3.95 \times 10^{14} \text{ Hz}$$

Ratio:

$$\frac{f_{\text{max}}}{f_{\text{min}}} = \frac{7.89 \times 10^{14} \text{ Hz}}{3.95 \times 10^{14} \text{ Hz}} = 2.00$$

(b) For human hearing (typical range 20 Hz to 20,000 Hz):

$$\frac{f_{\text{max}}}{f_{\text{min}}} = \frac{20{,}000 \text{ Hz}}{20 \text{ Hz}} = 1000$$

The ear's frequency range ratio (1000:1) is **500 times larger** than the eye's frequency range ratio (2:1).

Discussion

This comparison reveals a remarkable difference in the sensory ranges of vision and hearing. The eye can detect electromagnetic waves spanning only one octave (a factor of 2 in frequency)—from about 395 THz (red) to 789 THz (violet). In musical terms, this is like being able to hear only from middle C to the C one octave above it.

In contrast, the ear can detect sound waves spanning almost 10 octaves (a factor of 1000 in frequency)—from 20 Hz (very low bass) to 20,000 Hz (high treble). This is roughly equivalent to the range from the lowest note on a pipe organ to well beyond the highest note on a piccolo. Some people, especially children, can hear even higher frequencies, up to 23,000 Hz.

Why the difference? The eye's limited range is partly because the photochemical processes in retinal photoreceptors only respond to a narrow energy range of photons. Photons with too little energy (infrared) don't trigger the molecular changes needed for vision, while photons with too much energy (ultraviolet) are absorbed by the cornea and lens before reaching the retina, protecting it from damage. The ear's wide range reflects the mechanical properties of the basilar membrane in the cochlea, which can respond to a very wide range of vibration frequencies through different regions along its length.

Interestingly, both senses use logarithmic perception—we perceive equal ratios of frequency as equal intervals, which is why octaves (2:1 ratios) sound similar in music, and why colors smoothly transition across the visible spectrum even though the frequency is doubling.

Final Answer

(a) The ratio of highest to lowest frequencies visible to the eye is 2.00 (one octave). (b) The ratio for hearing is 1000 (almost 10 octaves), which is 500 times larger than for vision.
Strategy

Integrated Concepts

(a) Calculate the rate in watts at which heat transfer through radiation occurs (almost entirely in the infrared) from 1.0m^2 of the Earth's surface at night. Assume the emissivity is 0.90, the temperature of the Earth is 15°C , and that of outer space is 2.7 K. (b) Compare the intensity of this radiation with that coming to the Earth from the Sun during the day, which averages about 800W/m^2 , only half of which is absorbed. (c) What is the maximum magnetic field strength in the outgoing radiation, assuming it is a continuous wave?

Show Solution

For part (a), we use the Stefan-Boltzmann law for net radiative heat transfer: $P_{\text{net}} = \sigma \epsilon A(T_1^4 - T_2^4)$, where $\sigma = 5.67 \times 10^{-8}\ \text{W/(m}^2 \cdot \text{K}^4)$ is the Stefan-Boltzmann constant. For part (b), we compare this intensity to the incoming solar radiation. For part (c), we use the relationship between intensity and field strengths: $I = \frac{E_{\max}^2}{2\mu_0 c}$ and $B_{\max} = \frac{E_{\max}}{c}$.

Solution

(a) Given:

- Area: $A = 1.0\ \text{m}^2$
- Emissivity: $\epsilon = 0.90$
- Earth temperature: $T_1 = 15\ ^\circ\text{C} = 288\ \text{K}$
- Space temperature: $T_2 = 2.7\ \text{K}$
- Stefan-Boltzmann constant: $\sigma = 5.67 \times 10^{-8}\ \text{W/(m}^2 \cdot \text{K}^4)$

Net power radiated:

$$P_{\text{net}} = \sigma \epsilon A(T_1^4 - T_2^4)$$
$$P_{\text{net}} = (5.67\times10^{-8})(0.90)(1.0)[(288)^4 - (2.7)^4]$$
$$P_{\text{net}} = (5.67\times10^{-8})(0.90)(6.87\times10^9 - 53) \approx (5.67\times10^{-8})(0.90)(6.87\times10^9)$$
$$P_{\text{net}} = 3.5\times10^2\ \text{W}$$

Since this is power per unit area: $I = 3.5 \times 10^2\ \text{W/m}^2$

(The negative sign in the original answer indicates heat loss from Earth.)

(b) Incoming solar intensity (absorbed): $I_{\text{sun}} = \frac{800}{2} = 400\ \text{W/m}^2$

Ratio:

$$\frac{I_{\text{Earth}}}{I_{\text{sun}}} = \frac{350}{400} = 0.875 \approx 88\%$$

The Earth's outgoing infrared radiation at night is about **88%** of the absorbed solar radiation during the day.

(c) For the outgoing radiation with $I = 350\ \text{W/m}^2$:

The relationship between intensity and electric field is:

$$I = \frac{E_{\max}^2}{2\mu_0 c}$$

Solving for $E_{\max}$:

$E_{\text{max}}=\sqrt{2\mu_0 c I}=\sqrt{2(4\pi\times10^{-7})(3.00\times10^8)(350)}$

$E_{\text{max}}=\sqrt{2.64\times10^5}=514\text{ V/m}$

Maximum magnetic field:

$B_{\text{max}}=E_{\text{max}}c=5143.00\times10^8=1.7\times10^{-6}\text{ T}=1.7\ \mu\text{T}$

Discussion

This problem illustrates Earth's energy balance. During the night, Earth radiates heat as infrared radiation at a rate of about 350 W/m². During the day, Earth absorbs about 400 W/m² from the Sun (half of the 800 W/m² incident, since about half is reflected by clouds, atmosphere, and surface). The fact that these values are similar (within about 12%) demonstrates approximate radiative equilibrium—over time, Earth must radiate away as much energy as it absorbs, or its temperature would steadily rise or fall.

The small difference suggests either that we're not in perfect equilibrium (which is true—Earth is currently warming due to greenhouse gas increases) or that the numbers are idealized. In reality, Earth's energy balance involves many factors including latitude, season, cloud cover, and greenhouse gases that trap outgoing infrared radiation.

The magnetic field strength of 1.7 μT in the thermal radiation is very weak—about 3% of Earth's magnetic field. This is characteristic of thermal radiation: it's incoherent (many different frequencies and random phases) rather than a single coherent wave, so describing it with a single "maximum field strength" is somewhat artificial. Real thermal radiation is better described statistically. The 2.7 K temperature of space refers to the cosmic microwave background radiation, the remnant thermal radiation from the Big Bang.

Final Answer

(a) The rate of heat transfer through radiation is 3.5×10^2 W/m² or 350 W/m². (b) This is 88% of the absorbed solar radiation. (c) The maximum magnetic field strength is $1.7\ \mu\text{T}$.

Glossary

electromagnetic spectrum
: the full range of wavelengths or frequencies of electromagnetic radiation

radio waves
: electromagnetic waves with wavelengths in the range from 1 mm to 100 km; they are produced by currents in wires and circuits and by astronomical phenomena

microwaves
: electromagnetic waves with wavelengths in the range from 1 mm to 1 m; they can be produced by currents in macroscopic circuits and devices

thermal agitation
: the thermal motion of atoms and molecules in any object at a temperature above absolute zero, which causes them to emit and absorb radiation

radar
: a common application of microwaves. Radar can determine the distance to objects as diverse as clouds and aircraft, as well as determine the speed of a car or the intensity of a rainstorm

infrared radiation (IR)
: a region of the electromagnetic spectrum with a frequency range that extends from just below the red region of the visible light spectrum up to the microwave region, or from $0.74\mu\text{m}$ to $300\mu\text{m}$

ultraviolet radiation (UV)
: electromagnetic radiation in the range extending upward in frequency from violet light and overlapping with the lowest X-ray frequencies, with wavelengths from 400 nm down to about 10 nm

visible light
: the narrow segment of the electromagnetic spectrum to which the normal human eye responds

amplitude modulation (AM)
: a method for placing information on electromagnetic waves by modulating the amplitude of a carrier wave with an audio signal, resulting in a wave with constant frequency but varying amplitude

extremely low frequency (ELF)
: electromagnetic radiation with wavelengths usually in the range of 0 to 300 Hz, but also about 1kHz

carrier wave
: an electromagnetic wave that carries a signal by modulation of its amplitude or frequency

frequency modulation (FM)
: a method of placing information on electromagnetic waves by modulating the frequency of a carrier wave with an audio signal, producing a wave of constant amplitude but varying frequency

TV
: video and audio signals broadcast on electromagnetic waves

very high frequency (VHF)
: TV channels utilizing frequencies in the two ranges of 54 to 88 MHz and 174 to 222 MHz

ultra-high frequency (UHF)
: TV channels in an even higher frequency range than VHF, of 470 to 1000 MHz

X-ray
: invisible, penetrating form of very high frequency electromagnetic radiation, overlapping both the ultraviolet range and the γ -ray range

gamma ray
: (γ ray); extremely high frequency electromagnetic radiation emitted by the nucleus of an atom, either from natural nuclear decay or induced nuclear processes in nuclear reactors and weapons. The lower end of the γ -ray frequency range overlaps the upper end of the X-ray range, but γ rays can have the highest frequency of any electromagnetic radiation

Energy in Electromagnetic Waves

- Explain how the energy and amplitude of an electromagnetic wave are related.
- Given its power output and the heating area, calculate the intensity of a microwave oven's electromagnetic field, as well as its peak electric and magnetic field strengths

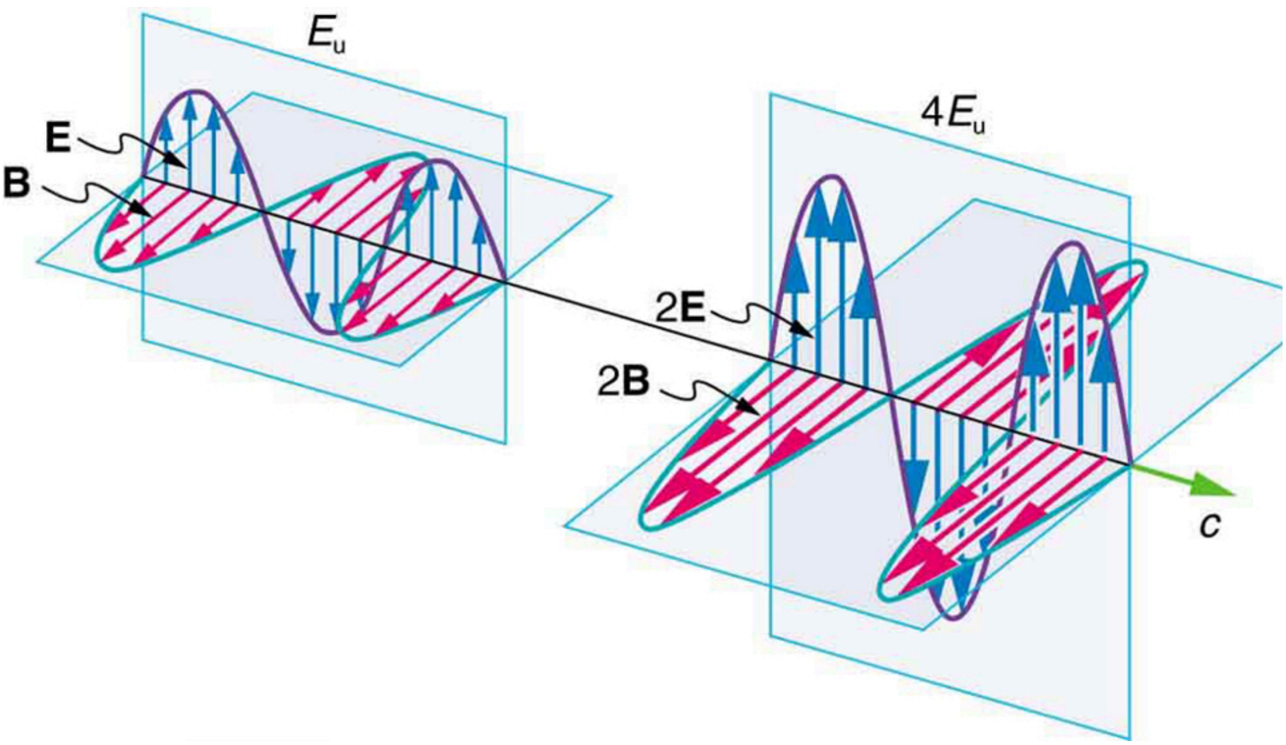
Anyone who has used a microwave oven knows there is energy in **electromagnetic waves**. Sometimes this energy is obvious, such as in the warmth of the summer sun. Other times it is subtle, such as the unfelt energy of gamma rays, which can destroy living cells.

Electromagnetic waves can bring energy into a system by virtue of their **electric and magnetic fields**. These fields can exert forces and move charges in the system and, thus, do work on them. If the frequency of the electromagnetic wave is the same as the natural frequencies of the system (such as microwaves at the resonant frequency of water molecules), the transfer of energy is much more efficient.

Connections: Waves and Particles

The behavior of electromagnetic radiation clearly exhibits wave characteristics. But we shall find in later modules that at high frequencies, electromagnetic radiation also exhibits particle characteristics. These particle characteristics will be used to explain more of the properties of the electromagnetic spectrum and to introduce the formal study of modern physics.

Another startling discovery of modern physics is that particles, such as electrons and protons, exhibit wave characteristics. This simultaneous sharing of wave and particle properties for all submicroscopic entities is one of the great symmetries in nature.



Energy carried by a wave is proportional to its amplitude squared. With electromagnetic waves, larger E -fields and B -fields exert larger forces and can do more work.

But there is energy in an electromagnetic wave, whether it is absorbed or not. Once created, the fields carry energy away from a source. If absorbed, the field strengths are diminished and anything left travels on. Clearly, the larger the strength of the electric and magnetic fields, the more work they can do and the greater the energy the electromagnetic wave carries.

A wave's energy is proportional to its **amplitude** squared (${E}^{2}$ or ${B}^{2}$). This is true for waves on guitar strings, for water waves, and for sound waves, where amplitude is proportional to pressure. In electromagnetic waves, the amplitude is the **maximum field strength** of the electric and magnetic fields. (See [Figure 1].)

Thus the energy carried and the **intensity** I of an electromagnetic wave is proportional to ${E}^{2}$ and ${B}^{2}$. In fact, for a continuous sinusoidal electromagnetic wave, the average intensity ${I}_{\text{ave}}$ is given by

$${I}_{\text{ave}}=\frac{ c\epsilon_{0}{E}_{0}^{2}}{2}, $$

where c is the speed of light, ${\epsilon }_{0}$ is the permittivity of free space, and ${E}_{0}$ is the maximum electric field strength; intensity, as always, is power per unit area (here in ${\text{W/m}}^{2}$).

The average intensity of an electromagnetic wave ${I}_{\text{ave}}$ can also be expressed in terms of the magnetic field strength by using the relationship $B=E/c$, and the fact that ${\epsilon }_{0}=1/{\mu }_{0}{c}^{2}$, where ${\mu }_{0}$ is the permeability of free space. Algebraic manipulation produces the relationship

$${I}_{\text{ave}}=\frac{ {\text{cB}}_{0}^{2}}{ {2\mu }_{0}}, $$

where ${B}_{0}$ is the maximum magnetic field strength.

One more expression for ${I}_{\text{ave}}$ in terms of both electric and magnetic field strengths is useful. Substituting the fact that $c\cdot {B}_ {0}= {E}_{0}$, the previous expression becomes

$${I}_{\text{ave}}=\frac{ {E}_{0}{B}_{0}}{ {2\mu }_{0}}. $$

Whichever of the three preceding equations is most convenient can be used, since they are really just different versions of the same principle: Energy in a wave is related to amplitude squared. Furthermore, since these equations are based on the assumption that the electromagnetic waves are sinusoidal, peak intensity is twice the average; that is, ${I}_{0}=2{I}_{\text{ave}}$.

Calculate Microwave Intensities and Fields

On its highest power setting, a certain microwave oven projects 1.00 kW of microwaves onto a 30.0 by 40.0 cm area. (a) What is the intensity in ${\text{W/m}}^{2}$? (b) Calculate the peak electric field strength ${E}_{0}$ in these waves. (c) What is the peak magnetic field strength ${B}_{0}$?

Strategy

In part (a), we can find intensity from its definition as power per unit area. Once the intensity is known, we can use the equations below to find the field strengths asked for in parts (b) and (c).

Solution for (a)

Entering the given power into the definition of intensity, and noting the area is 0.300 by 0.400 m, yields

$$I=\frac{P}{A}=\frac{1.00 \text{kW}}{0.300 \text{m} × 0.400 \text{m}}. $$

Here $I={I}_{\text{ave}}$, so that

$${I}_{\text{ave}}=\frac{1000 \text{W}}{ 0.120 {\text{m}}^{2}}= 8.33 \times 10^{3} {\text{W/m}}^{2}. $$

Note that the peak intensity is twice the average:

$${I}_{0}=2{I}_{\text{ave}}= 1.67 \times 10^{4} \text{W}/{\text{m}}^{2}. $$

Solution for (b)

To find ${E}_{0}$, we can rearrange the first equation given above for ${I}\_{\text{ave}}$ to give

$${E}_{0}={\left(\frac{2{I}_{\text{ave}}}{ c\epsilon_{0}}\right)}^{1/2}. $$

Entering known values gives

$$\begin{array}{lll}{E}_{0}& =& \sqrt{\frac{2\left(8.33 \times 10^{3} {\text{W/m}}^{2}\right)}{\left(3.00 \times 10^{8} \text{m/s}\right)\left(8.85 \times 10^{-12} {\text{C}}^{2}/\text{N}\cdot {\text{m}}^{2}\right)}}\\ & =& 2.51 \times 10^{3} \text{V/m}\text{.}\end{array} $$

Solution for (c)

Perhaps the easiest way to find magnetic field strength, now that the electric field strength is known, is to use the relationship given by

$${B}_{0}=\frac{ {E}_{0}}{c}. $$

Entering known values gives

$$\begin{array}{lll}{B}_{0}& =& \frac{ 2.51 \times 10^{3} \text{V/m}}{ 3.0 \times 10^{8} \text{m/s}}\\ & =& 8.35 \times 10^{-6} \text{T}\text{.}\end{array} $$

Discussion

As before, a relatively strong electric field is accompanied by a relatively weak magnetic field in an electromagnetic wave, since $B=E/c$, and c is a large number.

Section Summary

- The energy carried by any wave is proportional to its amplitude squared. For electromagnetic waves, this means intensity can be expressed as

  $${I}_{\text{ave}}=\frac{ c\epsilon_{0}{E}_{0}^{2}}{2}, $$

where ${I}_{\text{ave}}$ is the average intensity in ${\text{W/m}}^{2}$, and ${E}_{0}$ is the maximum electric field strength of a continuous sinusoidal wave.

- This can also be expressed in terms of the maximum magnetic field strength ${B}_{0}$ as

  $${I}_{\text{ave}}=\frac{ {\text{cB}}_{0}^{2}}{ {2\mu }_{0}} $$

and in terms of both electric and magnetic fields as

$${I}_{\text{ave}}=\frac{ {E}_{0}{B}_{0}}{ {2\mu }_{0}}. $$

- The three expressions for ${I}_{\text{ave}}$ are all equivalent.

Problems & Exercises

Strategy

What is the intensity of an electromagnetic wave with a peak electric field strength of 125 V/m?

Show Solution

The intensity of an electromagnetic wave is related to the peak electric field strength by $I=\frac{c\epsilon _{0}E_{0}^{2}}{2}$, where c is the speed of light and $\epsilon _{0}$ is the permittivity of free space.

Solution

Given:

- Peak electric field: $E_{0}=125 \text{ V/m}$
- Speed of light: $c=3.00 \times 10^{8} \text{ m/s}$
- Permittivity of free space: $\epsilon _{0}=8.85 \times 10^{-12} \text{ C}^{2}/(\text{N}\cdot \text{m}^{2})$

Calculate the intensity:

$$I=\frac{c\epsilon _{0}E_{0}^{2}}{2}=\frac{\left(3.00 \times 10^{8} \text{ m/s}\right)\left(8.85 \times 10^{-12} \text{ C}^{2}/(\text{N}\cdot \text{m}^{2})\right)(125 \text{ V/m})^{2}}{2}$$
$$I=\frac{\left(3.00 \times 10^{8}\right)\left(8.85 \times 10^{-12}\right)(15{,}625)}{2}=\frac{41.44}{2}=20.7 \text{ W/m}^{2}$$
Discussion

An intensity of 20.7 W/m² is relatively modest compared to sunlight (which averages about 1000 W/m² at Earth's surface on a clear day). This level of electromagnetic radiation is typical of what you might experience near a moderately powered radio transmitter or microwave communication device. The calculation demonstrates how the energy flux in an electromagnetic wave scales with the square of the electric field—doubling the electric field quadruples the intensity. This is because intensity represents power per unit area, and the power delivered by an electromagnetic wave depends on both the electric and magnetic fields, each of which is proportional to the amplitude, giving an overall E² dependence.

Final Answer

The intensity of the electromagnetic wave is 20.7 W/m².

Strategy

Find the intensity of an electromagnetic wave having a peak magnetic field strength of $4.00 \times 10^{-9} \text{T}$.

Show Solution

The intensity of an electromagnetic wave can be calculated from the peak magnetic field strength using $I=\frac{cB_{0}^{2}}{2\mu_{0}}$, where c is the speed of light and μ_{0} is the permeability of free space.

Solution

Given:

- Peak magnetic field: $B_{0}=4.00 \times 10^{-9} \text{ T}$
- Speed of light: $c=3.00 \times 10^{8} \text{ m/s}$
- Permeability of free space: $\mu_{0}=4\pi \times 10^{-7} \text{ T}\cdot\text{m/A}$

Calculate the intensity:

$$I=\frac{cB_{0}^{2}}{2\mu_{0}}=\frac{\left(3.00 \times 10^{8} \text{ m/s}\right)\left(4.00 \times 10^{-9} \text{ T}\right)^{2}}{2\left(4\pi \times 10^{-7} \text{ T}\cdot\text{m/A}\right)}$$
$$I=\frac{\left(3.00 \times 10^{8}\right)\left(1.60 \times 10^{-17}\right)}{2\left(1.257 \times 10^{-6}\right)}=\frac{4.80 \times 10^{-9}}{2.513 \times 10^{-6}}=1.91 \times 10^{-3} \text{ W/m}^{2}$$
Discussion

This intensity of 1.91 mW/m² is extremely weak—about 500,000 times weaker than sunlight. Such weak electromagnetic radiation is typical of what might be received from distant radio sources or weak broadcast signals. The very small magnetic field strength of 4 nanotesla (about 100,000 times weaker than Earth's magnetic field) corresponds to this low intensity. This demonstrates an important point: electromagnetic waves can have very weak field strengths yet still be detectable with sensitive instruments. Radio telescopes, for example, routinely detect signals with intensities millions of times weaker than this.

Final Answer

The intensity of the electromagnetic wave is $1.91 \times 10^{-3} \text{ W/m}^{2}$ or 1.91 mW/m².

Strategy

Assume the helium-neon lasers commonly used in student physics laboratories have power outputs of 0.250 mW. (a) If such a laser beam is projected onto a circular spot 1.00 mm in diameter, what is its intensity? (b) Find the peak magnetic field strength. (c) Find the peak electric field strength.

Show Solution

For part (a), intensity is power per unit area: $I=\frac{P}{A}$. For a circular spot, $A=\pi r^{2}$. For part (b), we use $I=\frac{cB_{0}^{2}}{2\mu_{0}}$ and solve for B_{0}. For part (c), we use $E_{0}=cB_{0}$.

Solution

(a) Given:

- Power: $P=0.250 \text{ mW}=0.250 \times 10^{-3} \text{ W}$
- Diameter: $d=1.00 \text{ mm}$, so radius $r=0.500 \text{ mm}=0.500 \times 10^{-3} \text{ m}$

$$I=\frac{P}{A}=\frac{P}{\pi r^{2}}=\frac{0.250 \times 10^{-3} \text{ W}}{\pi \left(0.500 \times 10^{-3} \text{ m}\right)^{2}}=\frac{0.250 \times 10^{-3}}{\pi \times 0.250 \times 10^{-6}}=318 \text{ W/m}^{2}$$

(b) From $I=\frac{cB_{0}^{2}}{2\mu_{0}}$, solve for B_{0}:

$$B_{0}=\sqrt{\frac{2\mu_{0}I}{c}}=\sqrt{\frac{2\left(4\pi \times 10^{-7} \text{ T}\cdot\text{m/A}\right)\left(318 \text{ W/m}^{2}\right)}{3.00 \times 10^{8} \text{ m/s}}}$$
$$B_{0}=\sqrt{\frac{8.00 \times 10^{-4}}{3.00 \times 10^{8}}}=\sqrt{2.67 \times 10^{-12}}=1.63 \times 10^{-6} \text{ T}$$

(c) Using $E_{0}=cB_{0}$:

$$E_{0}=\left(3.00 \times 10^{8} \text{ m/s}\right)\left(1.63 \times 10^{-6} \text{ T}\right)=490 \text{ V/m}$$
Discussion

The intensity of 318 W/m^2 from this small laser is actually quite significant—about one-third the intensity of sunlight. This high intensity is achieved by concentrating a modest power (0.250 mW) into a very small area (less than 1 mm^2 diameter spot). This is why laser pointers, despite their low power, can be dangerous to eyes—the concentrated beam delivers substantial intensity to the small area of the retina.

The electric field strength of 490 V/m is modest compared to the breakdown field of air (about 3 MV/m), which is why the laser doesn't ionize air. The magnetic field of 1.63 μT is very weak—about 30 times weaker than Earth's magnetic field—demonstrating again that electromagnetic waves have relatively weak magnetic components compared to their electric components when measured in everyday units.

Helium-neon lasers produce coherent red light at 632.8 nm and are commonly used in physics labs for interference and diffraction experiments, as well as in barcode scanners and surveying equipment.

Final Answer

(a) The intensity is 318 W/m^2. (b) The peak magnetic field strength is 1.63×10^{-6} T or 1.63 μT. (c) The peak electric field strength is 490 V/m.
Strategy

An AM radio transmitter broadcasts 50.0 kW of power uniformly in all directions. (a) Assuming all of the radio waves that strike the ground are completely absorbed, and that there is no absorption by the atmosphere or other objects, what is the intensity 30.0 km away? (Hint: Half the power will be spread over the area of a hemisphere.) (b) What is the maximum electric field strength at this distance?

Show Solution

For part (a), since the ground absorbs the downward radiation, only half the power spreads over a hemisphere. The intensity at distance r is $I=\frac{P/2}{2\pi r^{2}}=\frac{P}{4\pi r^{2}}$. For part (b), we use $I=\frac{c\epsilon_{0}E_{0}^{2}}{2}$ and solve for E_{0}.

Solution

(a) Given:

- Power: $P=50.0 \text{ kW}=5.00 \times 10^{4} \text{ W}$
- Distance: $r=30.0 \text{ km}=3.00 \times 10^{4} \text{ m}$

The power spreads over a hemisphere (half a sphere), so:

$$I=\frac{P}{2\pi r^{2}}=\frac{5.00 \times 10^{4} \text{ W}}{2\pi \left(3.00 \times 10^{4} \text{ m}\right)^{2}}=\frac{5.00 \times 10^{4}}{2\pi \times 9.00 \times 10^{8}}$$
$$I=\frac{5.00 \times 10^{4}}{5.655 \times 10^{9}}=8.84 \times 10^{-6} \text{ W/m}^{2}=8.84 \text{ }\mu\text{W/m}^{2}$$

(b) From $I=\frac{c\epsilon_{0}E_{0}^{2}}{2}$, solve for E_{0}:

$$E_{0}=\sqrt{\frac{2I}{c\epsilon_{0}}}=\sqrt{\frac{2\left(8.84 \times 10^{-6} \text{ W/m}^{2}\right)}{\left(3.00 \times 10^{8} \text{ m/s}\right)\left(8.85 \times 10^{-12} \text{ C}^{2}/(\text{N}\cdot\text{m}^{2})\right)}}$$
$$E_{0}=\sqrt{\frac{1.768 \times 10^{-5}}{2.655 \times 10^{-3}}}=\sqrt{6.66 \times 10^{-3}}=0.0816 \text{ V/m}=81.6 \text{ mV/m}$$
Discussion

The intensity of 8.84 μW/m^2 at 30 km is very weak—more than 100 million times weaker than sunlight. Yet AM radios can easily detect this signal because they're specifically tuned to resonate at the transmission frequency, amplifying the weak signal. The electric field of 81.6 mV/m is tiny compared to everyday electric fields, but the radio's antenna and circuitry are designed to pick up these weak oscillating fields.

The inverse-square law is evident here: doubling the distance reduces the intensity by a factor of 4. This is why AM radio stations need powerful transmitters (often 50 kW or more) to cover large areas. The hemisphere model (rather than a full sphere) is reasonable because the ground reflects some radiation upward but also absorbs much of it, and we're interested in the intensity above ground level. In reality, the ground's conductivity, atmospheric conditions, and time of day (affecting ionospheric reflection) all influence actual signal strength.

Final Answer

(a) The intensity 30.0 km away is 8.84×10^{-6} W/m^2 or 8.84 μW/m^2. (b) The maximum electric field strength at this distance is 81.6 mV/m.
Strategy

Suppose the maximum safe intensity of microwaves for human exposure is taken to be $1.00 \text{W}{\text{/m}}^{2}$. (a) If a radar unit leaks 10.0 W of microwaves (other than those sent by its antenna) uniformly in all directions, how far away must you be to be exposed to an intensity considered to be safe? Assume that the power spreads uniformly over the area of a sphere with no complications from absorption or reflection. (b) What is the maximum electric field strength at the safe intensity? (Note that early radar units leaked more than modern ones do. This caused identifiable health problems, such as cataracts, for people who worked near them.)

Show Solution

For part (a), power spreads over a spherical surface, so $I=\frac{P}{4\pi r^{2}}$. We solve for r when $I=1.00 \text{ W/m}^{2}$. For part (b), we use $I=\frac{c\epsilon_{0}E_{0}^{2}}{2}$ to find E_{0}.

Solution

(a) Given:

- Leaked power: $P=10.0 \text{ W}$
- Safe intensity: $I=1.00 \text{ W/m}^{2}$

From $I=\frac{P}{4\pi r^{2}}$, solve for r:

$$r=\sqrt{\frac{P}{4\pi I}}=\sqrt{\frac{10.0 \text{ W}}{4\pi \left(1.00 \text{ W/m}^{2}\right)}}=\sqrt{\frac{10.0}{12.566}}=\sqrt{0.796}=0.892 \text{ m}=89.2 \text{ cm}$$

(b) From $I=\frac{c\epsilon_{0}E_{0}^{2}}{2}$:

$$E_{0}=\sqrt{\frac{2I}{c\epsilon_{0}}}=\sqrt{\frac{2\left(1.00 \text{ W/m}^{2}\right)}{\left(3.00 \times 10^{8} \text{ m/s}\right)\left(8.85 \times 10^{-12} \text{ C}^{2}/(\text{N}\cdot\text{m}^{2})\right)}}$$
$$E_{0}=\sqrt{\frac{2.00}{2.655 \times 10^{-3}}}=\sqrt{753.3}=27.4 \text{ V/m}$$
Discussion

The safe distance of 89.2 cm (about 3 feet) is quite close to the leaking radar unit. This illustrates why early radar technicians who worked in close proximity to equipment developed health problems—the leaked microwave radiation was intense enough at typical working distances to cause tissue heating and damage, particularly to the eyes (causing cataracts) because the lens has poor blood circulation and can't dissipate heat effectively.

Modern radar units have much better shielding and leak far less power, making them safer. The safe intensity limit of 1.00 W/m^2 is based on thermal effects—microwaves at this intensity can cause measurable tissue heating. The electric field of 27.4 V/m at the safe intensity seems modest, but remember that this oscillates at microwave frequencies (typically GHz), and the energy is efficiently absorbed by water molecules in tissue, causing rotation and heating.

The inverse-square law means that moving just twice as far away (to about 1.8 m) would reduce the intensity to 0.25 W/m^2, well below the safety threshold. This is why maintaining distance from microwave sources is an effective safety measure.

Final Answer

(a) You must be at least 89.2 cm away to be exposed to the safe intensity. (b) The maximum electric field strength at the safe intensity is 27.4 V/m.

Strategy

A 2.50-m-diameter university communications satellite dish receives TV signals that have a maximum electric field strength (for one channel) of $7.50 \mu V/m$. (See [Figure 2].) (a) What is the intensity of this wave? (b) What is the power received by the antenna? (c) If the orbiting satellite broadcasts uniformly over an area of $1.50 \times 10^{13} {\text{m}}^{2}$ (a large fraction of North America), how much power does it radiate?

Satellite dishes receive TV signals sent from orbit. Although the signals are quite weak, the receiver can detect them by being tuned to resonate at their frequency.

Show Solution

For part (a), use $I=\frac{c\epsilon_{0}E_{0}^{2}}{2}$. For part (b), power received is $P=IA$, where $A=\pi r^{2}$ is the dish area. For part (c), if the satellite broadcasts uniformly over area $A_{\text{broadcast}}$, then $I=\frac{P_{\text{satellite}}}{A_{\text{broadcast}}}$, so we can solve for $P_{\text{satellite}}$.

Solution

(a) Given: $E_{0}=7.50 \text{ }\mu\text{V/m}=7.50 \times 10^{-6} \text{ V/m}$

$$I=\frac{c\epsilon_{0}E_{0}^{2}}{2}=\frac{\left(3.00 \times 10^{8}\right)\left(8.85 \times 10^{-12}\right)\left(7.50 \times 10^{-6}\right)^{2}}{2}$$
$$I=\frac{\left(3.00 \times 10^{8}\right)\left(8.85 \times 10^{-12}\right)\left(5.625 \times 10^{-11}\right)}{2}=\frac{1.493 \times 10^{-13}}{2}=7.47 \times 10^{-14} \text{ W/m}^{2}$$

(b) Dish diameter = 2.50 m, so radius $r=1.25 \text{ m}$

$$A=\pi r^{2}=\pi \left(1.25\right)^{2}=4.91 \text{ m}^{2}$$
$$P=IA=\left(7.47 \times 10^{-14} \text{ W/m}^{2}\right)\left(4.91 \text{ m}^{2}\right)=3.67 \times 10^{-13} \text{ W}$$

(c) Given: $A_{\text{broadcast}}=1.50 \times 10^{13} \text{ m}^{2}$

From $I=\frac{P_{\text{satellite}}}{A_{\text{broadcast}}}$:

$$P_{\text{satellite}}=IA_{\text{broadcast}}=\left(7.47 \times 10^{-14} \text{ W/m}^{2}\right)\left(1.50 \times 10^{13} \text{ m}^{2}\right)=1.12 \text{ kW}$$
Discussion

The intensity of 7.47×10^{-14} W/m^2 is incredibly weak—about 10 trillion times weaker than sunlight! Yet satellite dishes can receive clear TV signals at this intensity because: (1) the large dish area (4.91 m^2) collects enough power (367 femtowatts) for detection, (2) the receiver is tuned to resonate at the specific broadcast frequency, amplifying the signal, and (3) modern low-noise amplifiers can detect extraordinarily weak signals.

The satellite's transmitted power of 1.12 kW seems modest for broadcasting over such a large area (1.50×10^{13} m^2, roughly the area of North America). This is possible because: (1) satellites use directional antennas that focus power toward Earth rather than radiating uniformly in all directions, (2) the signal is digital and error-correcting codes allow recovery of information even from weak, noisy signals, and (3) geosynchronous satellites at 36,000 km altitude have a clear line-of-sight to ground receivers with minimal atmospheric absorption.

This problem beautifully illustrates how modern communications can work with incredibly weak signals through clever engineering—sensitive receivers, large collecting areas, frequency tuning, and signal processing.

Final Answer

(a) The intensity is 7.47×10^{-14} W/m^2. (b) The power received is 3.67×10^{-13} W or 367 femtowatts. (c) The satellite radiates 1.12 kW.
Strategy

Lasers can be constructed that produce an extremely high intensity electromagnetic wave for a brief time—called pulsed lasers. They are used to ignite nuclear fusion, for example. Such a laser may produce an electromagnetic wave with a maximum electric field strength of $1.00 \times 10^{11} \text{V}/\text{m}$ for a time of 1.00 ns. (a) What is the maximum magnetic field strength in the wave? (b) What is the intensity of the beam? (c) What energy does it deliver on a $1.00{\text{-mm}}^{2}$ area?

Show Solution

For part (a), use $B_{0}=\frac{E_{0}}{c}$. For part (b), use $I=\frac{c\epsilon_{0}E_{0}^{2}}{2}$. For part (c), energy delivered is $E=P \cdot t=I \cdot A \cdot t$.

Solution

(a) Given: $E_{0}=1.00 \times 10^{11} \text{ V/m}$

$$B_{0}=\frac{E_{0}}{c}=\frac{1.00 \times 10^{11} \text{ V/m}}{3.00 \times 10^{8} \text{ m/s}}=333 \text{ T}$$

(b) Calculate intensity:

$$I=\frac{c\epsilon_{0}E_{0}^{2}}{2}=\frac{\left(3.00 \times 10^{8}\right)\left(8.85 \times 10^{-12}\right)\left(1.00 \times 10^{11}\right)^{2}}{2}$$
$$I=\frac{\left(3.00 \times 10^{8}\right)\left(8.85 \times 10^{-12}\right)\left(1.00 \times 10^{22}\right)}{2}=\frac{2.655 \times 10^{19}}{2}=1.33 \times 10^{19} \text{ W/m}^{2}$$

(c) Given:

- Area: $A=1.00 \text{ mm}^{2}=1.00 \times 10^{-6} \text{ m}^{2}$
- Time: $t=1.00 \text{ ns}=1.00 \times 10^{-9} \text{ s}$

$$E=I \cdot A \cdot t=\left(1.33 \times 10^{19} \text{ W/m}^{2}\right)\left(1.00 \times 10^{-6} \text{ m}^{2}\right)\left(1.00 \times 10^{-9} \text{ s}\right)$$
$$E=1.33 \times 10^{4} \text{ J}=13.3 \text{ kJ}$$
Discussion

These numbers are extraordinary! The magnetic field of 333 T is about 7 million times stronger than Earth's magnetic field and rivals the strongest magnetic fields ever produced in laboratories (which can damage equipment and are dangerous to humans). The intensity of 1.33×10^{19} W/m^2 is incomprehensibly large—about 10 trillion times the intensity of sunlight!

Such extreme intensities are achieved by concentrating enormous power into a tiny area for a very brief time. This particular laser delivers 13.3 kJ of energy to a 1 mm^2 area in just 1 nanosecond, corresponding to an instantaneous power of 1.33×10^{13} W or 13.3 terawatts—more than the total electrical generating capacity of the entire United States, concentrated in a pinpoint!

These ultra-high-intensity pulsed lasers are used in inertial confinement fusion experiments, where the intense electromagnetic fields compress and heat fusion fuel pellets to conditions similar to those in the core of stars. The National Ignition Facility (NIF) uses 192 such laser beams focused on a tiny capsule. When such intense fields interact with matter, non-linear effects dominate, electrons are stripped from atoms almost instantly, and the material becomes a plasma. The electric fields are strong enough to accelerate electrons to near-relativistic speeds within the wavelength of the light itself.

Final Answer

(a) The maximum magnetic field strength is 333 T. (b) The intensity is 1.33×10^{19} W/m^2. (c) The energy delivered to a 1.00-mm^2 area is 13.3 kJ.

Strategy

Show that for a continuous sinusoidal electromagnetic wave, the peak intensity is twice the average intensity (${I}_{0}={2I}_{\text{ave}}$), using either the fact that ${E}_{0}=\sqrt{2}{E}_{\text{rms}}$, or ${B}_{0}=\sqrt{2}{B}_{\text{rms}}$, where rms means average (actually root mean square, a type of average).

Show Solution

We'll use the relationship between peak and rms values for sinusoidal waves, and the fact that intensity is proportional to the square of the field amplitude. We can use either electric or magnetic fields; the result is the same.

Solution

The intensity of an electromagnetic wave is proportional to the square of the electric field:

$$I \propto E^{2}$$

For a sinusoidal wave, the instantaneous electric field is $E(t)=E_{0}\sin(\omega t)$, where E_{0} is the peak (maximum) value.

The peak intensity occurs when $E=E_{0}$:

$$I_{0} \propto E_{0}^{2}$$

The average intensity is related to the rms (root mean square) electric field:

$$I_{\text{ave}} \propto E_{\text{rms}}^{2}$$

For a sinusoidal wave, the relationship between peak and rms values is:

$$E_{0}=\sqrt{2}E_{\text{rms}}$$

Squaring both sides:

$$E_{0}^{2}=2E_{\text{rms}}^{2}$$

Therefore:

$$\frac{I_{0}}{I_{\text{ave}}}=\frac{E_{0}^{2}}{E_{\text{rms}}^{2}}=2$$

Thus:

$$I_{0}=2I_{\text{ave}}$$

Discussion

This result makes physical sense: for a sinusoidal wave, the instantaneous intensity varies from zero (when the field passes through zero) to a maximum value I_{0} (when the field is at its peak). The time-average of $\sin^{2}(\omega t)$ is $\frac{1}{2}$, so the average intensity is half the peak intensity.

This factor of 2 is important in practical applications. When we quote the intensity of a laser beam or the power output of a radio transmitter, we typically mean the average intensity. The peak intensity (which occurs twice per oscillation cycle) is twice this value. For low-power continuous-wave lasers, this distinction doesn’t matter much, but for pulsed lasers or when considering nonlinear optical effects (which depend on peak intensity), the difference between peak and average intensity is crucial.

The same relationship holds for any sinusoidal quantity—electrical power in AC circuits, sound intensity from pure tones, etc. The rms value is used because it relates directly to average power, which is the physically meaningful quantity for energy transfer.

Final Answer

For a continuous sinusoidal electromagnetic wave, the peak intensity is twice the average intensity: $I_{0}=2I_{\text{ave}}$, which follows from $E_{0}=\sqrt{2}E_{\text{rms}}$ and $I \propto E^{2}$.

Strategy

Suppose a source of electromagnetic waves radiates uniformly in all directions in empty space where there are no absorption or interference effects. (a) Show that the intensity is inversely proportional to ${r}^{2}$, the distance from the source squared. (b) Show that the magnitudes of the electric and magnetic fields are inversely proportional to r.

Show Solution

For part (a), we use conservation of energy: the total power radiated must pass through any spherical surface centered on the source. For part (b), we use the relationship between intensity and field strength.

Solution

(a) A source radiating power P uniformly in all directions emits energy that spreads over spherical surfaces. At distance r from the source, the power is distributed over a sphere of area $A=4\pi r^{2}$.

Intensity is power per unit area:

$$I=\frac{P}{A}=\frac{P}{4\pi r^{2}}$$

Since P and 4π are constants:

$$I \propto \frac{1}{r^{2}}$$

This is the inverse-square law for electromagnetic radiation.

(b) The intensity is related to the field strengths by:

$$I \propto E_{0}^{2}$$

and

$$I \propto B_{0}^{2}$$

Since $I \propto \frac{1}{r^{2}}$, we have:

$$E_{0}^{2} \propto \frac{1}{r^{2}}$$

Taking the square root:

$$E_{0} \propto \frac{1}{r}$$

Similarly:

$$B_{0} \propto \frac{1}{r}$$

Therefore, the electric and magnetic field amplitudes decrease inversely with distance.

Discussion

The inverse-square law is fundamental to understanding radiation from point sources. It follows directly from geometry—as radiation spreads out from a point, it must cover larger and larger spherical surfaces, diluting the intensity. Doubling the distance reduces the intensity by a factor of 4, tripling the distance reduces it by a factor of 9, and so on.

This has profound practical implications. Radio stations must use much higher power to reach distant listeners. The intensity of sunlight on Mars (1.5 times farther from the Sun than Earth) is only $(1/1.5)^{2} \approx 0.44$ times the intensity on Earth. The faint signals from distant spacecraft (like Voyager, now billions of kilometers away) require enormous receiving antennas to collect enough power for communication.

The $1/r$ dependence of field strength (compared to $1/r^{2}$ for intensity) reflects the fact that intensity depends on the square of the field. This means that while field strength drops relatively slowly with distance, the power (which depends on field squared) drops much faster. This is why electromagnetic radiation can propagate over enormous distances—weak fields can still be detected—but the power received decreases rapidly, making long-distance communication challenging.

The inverse-square law applies to any form of radiation spreading from a point source, including light, radio waves, gamma rays, and even gravitational waves.

Final Answer

(a) For uniform radiation, $I=\frac{P}{4\pi r^{2}} \propto \frac{1}{r^{2}}$, demonstrating the inverse-square law. (b) Since $I \propto E_{0}^{2}$ and $I \propto B_{0}^{2}$, and $I \propto \frac{1}{r^{2}}$, we have $E_{0} \propto \frac{1}{r}$ and $B_{0} \propto \frac{1}{r}$.
Strategy

Integrated Concepts

An LC circuit with a 5.00-pF capacitor oscillates in such a manner as to radiate at a wavelength of 3.30 m. (a) What is the resonant frequency? (b) What inductance is in series with the capacitor?

Show Solution

For part (a), an LC circuit that radiates electromagnetic waves does so at its resonant frequency. The wavelength is related to frequency by $c=f\lambda$. For part (b), the resonant frequency of an LC circuit is $f=\frac{1}{2\pi\sqrt{LC}}$, which we can solve for L.

Solution

(a) Given:

- Wavelength: $\lambda=3.30 \text{ m}$
- Speed of light: $c=3.00 \times 10^{8} \text{ m/s}$

From $c=f\lambda$, solve for frequency:

$$f=\frac{c}{\lambda}=\frac{3.00 \times 10^{8} \text{ m/s}}{3.30 \text{ m}}=9.09 \times 10^{7} \text{ Hz}=90.9 \text{ MHz}$$

(b) Given:

- Capacitance: $C=5.00 \text{ pF}=5.00 \times 10^{-12} \text{ F}$
- Frequency: $f=9.09 \times 10^{7} \text{ Hz}$

From the resonant frequency formula $f=\frac{1}{2\pi\sqrt{LC}}$, solve for L:

$$\sqrt{LC}=\frac{1}{2\pi f}$$
$$L=\frac{1}{(2\pi f)^{2}C}=\frac{1}{4\pi^{2}f^{2}C}$$
$$L=\frac{1}{4\pi^{2}\left(9.09 \times 10^{7}\right)^{2}\left(5.00 \times 10^{-12}\right)}$$
$$L=\frac{1}{4\pi^{2} \times 8.26 \times 10^{15} \times 5.00 \times 10^{-12}}=\frac{1}{1.632 \times 10^{6}}=6.13 \times 10^{-7} \text{ H}=613 \text{ nH}$$
Discussion

A wavelength of 3.30 m corresponds to the FM radio band (around 91 MHz), which is in the VHF (Very High Frequency) range. The small capacitance (5 pF) and small inductance (613 nH) are typical of high-frequency LC oscillator circuits.

LC circuits work by exchanging energy between the electric field in the capacitor and the magnetic field in the inductor. When oscillating, they naturally radiate electromagnetic waves at their resonant frequency. In practical radio transmitters, the LC circuit is deliberately coupled to an antenna to efficiently radiate this energy into space.

The inductance of 613 nH is quite small—it could be just a few turns of wire or even the stray inductance of circuit traces on a printed circuit board. At these frequencies, careful circuit design is essential because even small parasitic capacitances and inductances can shift the resonant frequency. Modern FM transmitters and receivers use precisely manufactured components or tunable elements to maintain the exact frequency required by broadcast standards.

The relationship $f=\frac{1}{2\pi\sqrt{LC}}$ shows that to increase frequency, you need to decrease either L or C (or both). This is why high-frequency circuits use small capacitors and inductors, while low-frequency circuits (like 60 Hz power systems) use large values.

Final Answer

(a) The resonant frequency is 9.09×10^{7} Hz or 90.9 MHz. (b) The inductance is 6.13×10^{-7} H or 613 nH.

Integrated Concepts

What capacitance is needed in series with an $800-\mu \text{H}$ inductor to form a circuit that radiates a wavelength of 196 m?

Show Solution

13.5 pF

Strategy

Integrated Concepts

Police radar determines the speed of motor vehicles using the same Doppler-shift technique employed for ultrasound in medical diagnostics. Beats are produced by mixing the double Doppler-shifted echo with the original frequency. If $1.50 \times 10^{9} \text{-Hz}$ microwaves are used and a beat frequency of 150 Hz is produced, what is the speed of the vehicle? (Assume the same Doppler-shift formulas are valid with the speed of sound replaced by the speed of light.)

Show Solution

The radar signal undergoes a double Doppler shift: first when the moving vehicle receives the signal (shift due to approach), and second when the vehicle reflects it back to the radar (another shift). For speeds much less than c, the beat frequency is $f_{\text{beat}}=\frac{2vf_{0}}{c}$, where v is the vehicle speed, f_{0} is the original frequency, and c is the speed of light.

Solution

Given:

- Original frequency: $f_{0}=1.50 \times 10^{9} \text{ Hz}$
- Beat frequency: $f_{\text{beat}}=150 \text{ Hz}$
- Speed of light: $c=3.00 \times 10^{8} \text{ m/s}$

From the double Doppler shift formula for $v \ll c$:

$$f_{\text{beat}}=\frac{2vf_{0}}{c}$$

Solving for vehicle speed v:

$$v=\frac{cf_{\text{beat}}}{2f_{0}}=\frac{\left(3.00 \times 10^{8} \text{ m/s}\right)(150 \text{ Hz})}{2\left(1.50 \times 10^{9} \text{ Hz}\right)}$$
$$v=\frac{4.50 \times 10^{10}}{3.00 \times 10^{9}}=15.0 \text{ m/s}$$

Convert to more familiar units:

$$v=15.0 \text{ m/s} \times \frac{3600 \text{ s}}{1000 \text{ m}} \times \frac{1 \text{ km}}{1} = 54.0 \text{ km/h}$$

Or in miles per hour:

$$v=15.0 \text{ m/s} \times \frac{3600 \text{ s}}{1609 \text{ m}} = 33.6 \text{ mph}$$
Discussion

The vehicle speed of 15.0 m/s (54 km/h or about 34 mph) is a reasonable speed on city streets or residential areas. Police radar works on the principle of the Doppler effect: when a vehicle approaches the radar unit, it receives microwaves at a slightly higher frequency than transmitted (first Doppler shift). The vehicle then reflects these microwaves back, and since it's a moving source, the reflected signal undergoes a second Doppler shift in the same direction. The result is a double Doppler shift.

The radar unit mixes the returned signal with the original transmitted signal to produce a beat frequency—the difference between the two frequencies. This beat frequency is proportional to the vehicle's speed and is in the audio range (150 Hz in this case), making it easy to measure electronically.

The beauty of this technique is its precision: a beat frequency of 150 Hz measured from a 1.5 GHz carrier translates to a very accurate speed measurement. The frequency of 1.50 GHz (1500 MHz) is typical of X-band police radar, though K-band (24 GHz) and Ka-band (35 GHz) are also common in modern systems.

The approximation $v \ll c$ is extremely valid here: the vehicle speed is only 5×10^{-8} times the speed of light, so relativistic effects are completely negligible. This same Doppler technique is used in weather radar (to measure wind speeds), astronomical observations (to measure stellar velocities), and medical ultrasound (to measure blood flow).

Final Answer

The speed of the vehicle is 15.0 m/s, which is equivalent to 54.0 km/h or 33.6 mph.

Integrated Concepts

Assume the mostly infrared radiation from a heat lamp acts like a continuous wave with wavelength $1.50 \mu \text{m}$. (a) If the lamp’s 200-W output is focused on a person’s shoulder, over a circular area 25.0 cm in diameter, what is the intensity in ${\text{W/m}}^{2}$

? (b) What is the peak electric field strength? (c) Find the peak magnetic field strength. (d) How long will it take to increase the temperature of the 4.00-kg shoulder by $2.00º C$, assuming no other heat transfer and given that its specific heat is $3.47 \times 10^{3} \text{J/kg}\cdot \text{ºC}$?

Show Solution

(a) $4.07 \text{k}{\text{W/m}}^{2}$ (b) 1.75 kV/m

(c) $5.84 \mu \text{T}$ (d) 2 min 19 s

Strategy

Integrated Concepts

On its highest power setting, a microwave oven increases the temperature of 0.400 kg of spaghetti by 45.0ºC in 120 s. (a) What was the rate of power absorption by the spaghetti, given that its specific heat is $3.76 \times 10^{3} \text{J/kg}\cdot \text{ºC}$? (b) Find the average intensity of the microwaves, given that they are absorbed over a circular area 20.0 cm in diameter. (c) What is the peak electric field strength of the microwave? (d) What is its peak magnetic field strength?

Show Solution

For part (a), use $Q=mc\Delta T$ to find the energy absorbed, then divide by time to get power. For part (b), intensity is power per unit area. For part (c), use $I=\frac{c\epsilon_{0}E_{0}^{2}}{2}$. For part (d), use $B_{0}=\frac{E_{0}}{c}$.

Solution

(a) Given:

- Mass: $m=0.400 \text{ kg}$
- Temperature change: $\Delta T=45.0\text{ºC}$
- Time: $t=120 \text{ s}$
- Specific heat: $c=3.76 \times 10^{3} \text{ J/(kg·ºC)}$

Energy absorbed:

$$Q=mc\Delta T=\left(0.400 \text{ kg}\right)\left(3.76 \times 10^{3} \text{ J/(kg·ºC)}\right)\left(45.0\text{ºC}\right)$$
$$Q=6.77 \times 10^{4} \text{ J}=67.7 \text{ kJ}$$

Power absorbed:

$$P=\frac{Q}{t}=\frac{6.77 \times 10^{4} \text{ J}}{120 \text{ s}}=564 \text{ W}$$

(b) Given diameter $d=20.0 \text{ cm}=0.200 \text{ m}$, radius $r=0.100 \text{ m}$:

$$A=\pi r^{2}=\pi\left(0.100\right)^{2}=0.0314 \text{ m}^{2}$$
$$I=\frac{P}{A}=\frac{564 \text{ W}}{0.0314 \text{ m}^{2}}=1.80 \times 10^{4} \text{ W/m}^{2}=18.0 \text{ kW/m}^{2}$$

(c) From $I=\frac{c\epsilon_{0}E_{0}^{2}}{2}$:

$$E_{0}=\sqrt{\frac{2I}{c\epsilon_{0}}}=\sqrt{\frac{2\left(1.80 \times 10^{4}\right)}{\left(3.00 \times 10^{8}\right)\left(8.85 \times 10^{-12}\right)}}$$
$$E_{0}=\sqrt{\frac{3.60 \times 10^{4}}{2.655 \times 10^{-3}}}=\sqrt{1.356 \times 10^{7}}=3.68 \times 10^{3} \text{ V/m}=3.68 \text{ kV/m}$$

(d) Using $B_{0}=\frac{E_{0}}{c}$:

$$B_{0}=\frac{3.68 \times 10^{3} \text{ V/m}}{3.00 \times 10^{8} \text{ m/s}}=1.23 \times 10^{-5} \text{ T}=12.3 \text{ }\mu\text{T}$$
Discussion

The power absorbed by the spaghetti (564 W) is less than the typical microwave oven output (often 800-1200 W) because not all the microwave energy is absorbed by the food—some is reflected, some heats the container, and the microwave distribution isn’t perfectly uniform. The efficiency here is roughly 564/1000 ≈ 56%, which is reasonable for microwave heating.

The intensity of 18.0 kW/m² is quite high—about 18 times the intensity of sunlight. This explains why microwave ovens can heat food so quickly. The electric field strength of 3.68 kV/m is substantial but well below the breakdown field of air (about 3 MV/m), which is why the air inside the microwave doesn’t ionize.

Microwave ovens operate at 2.45 GHz because this frequency is: (1) efficiently absorbed by water molecules (causing them to rotate and heat up), (2) assigned by regulatory authorities for industrial, scientific, and medical (ISM) use, avoiding interference with communications, and (3) penetrates a few centimeters into food, allowing internal heating rather than just surface heating.

The specific heat of the spaghetti (3.76 kJ/(kg·°C)) is close to that of water (4.18 kJ/(kg·°C)), which makes sense since cooked spaghetti is mostly water. The temperature rise of 45°C (from perhaps 20°C to 65°C) is typical for reheating food to a comfortable eating temperature.

The magnetic field of 12.3 μT is weak—about one-quarter the strength of Earth’s magnetic field—showing again that electromagnetic waves carry most of their energy in the electric field component from a practical standpoint, though both components carry equal energy densities in the wave.

Final Answer

(a) The power absorption rate is 564 W. (b) The average intensity is 1.80×10^{4} W/m^2 or 18.0 kW/m^2. (c) The peak electric field strength is 3.68 kV/m. (d) The peak magnetic field strength is 1.23×10^{-5} T or 12.3 μT.

Integrated Concepts

Electromagnetic radiation from a 5.00-mW laser is concentrated on a

1.00{\text{-mm}}^{2} $area. (a) What is the intensity in${\text{W/m}}^{2} $$? (b) Suppose a 2.00-nC static charge is in the beam. What is the maximum electric force it experiences? (c) If the static charge moves at 400 m/s, what maximum magnetic force can it feel?

Show Solution

(a) $5.00 \times 10^{3} {\text{W/m}}^{2}$ (b) $3.88 \times 10^{-6} \text{N}$ (c) $5.18 \times 10^{-12} \text{N}$

Strategy

Integrated Concepts

A 200-turn flat coil of wire 30.0 cm in diameter acts as an antenna for FM radio at a frequency of 100 MHz. The magnetic field of the incoming electromagnetic wave is perpendicular to the coil and has a maximum strength of $1.00 \times 10^{-12} \text{T}$. (a) What power is incident on the coil? (b) What average emf is induced in the coil over one-fourth of a cycle? (c) If the radio receiver has an inductance of $2.50 \mu \text{H}$, what capacitance must it have to resonate at 100 MHz?

Show Solution

For part (a), calculate intensity from $I=\frac{cB_{0}^{2}}{2\mu_{0}}$, then multiply by the coil area to get power. For part (b), use Faraday’s law: the induced emf equals the rate of change of magnetic flux. For part (c), use the resonance condition $f=\frac{1}{2\pi\sqrt{LC}}$.

Solution

(a) Given:

- Magnetic field: $B_{0}=1.00 \times 10^{-12} \text{ T}$
- Diameter: $d=30.0 \text{ cm}=0.300 \text{ m}$, radius $r=0.150 \text{ m}$

Calculate intensity:

$$I=\frac{cB_{0}^{2}}{2\mu_{0}}=\frac{\left(3.00 \times 10^{8}\right)\left(1.00 \times 10^{-12}\right)^{2}}{2\left(4\pi \times 10^{-7}\right)}$$
$$I=\frac{\left(3.00 \times 10^{8}\right)\left(1.00 \times 10^{-24}\right)}{2.513 \times 10^{-6}}=\frac{3.00 \times 10^{-16}}{2.513 \times 10^{-6}}=1.19 \times 10^{-10} \text{ W/m}^{2}$$

Calculate coil area and power:

$$A=\pi r^{2}=\pi\left(0.150\right)^{2}=0.0707 \text{ m}^{2}$$
$$P=IA=\left(1.19 \times 10^{-10} \text{ W/m}^{2}\right)\left(0.0707 \text{ m}^{2}\right)=8.43 \times 10^{-12} \text{ W}=8.43 \text{ pW}$$

(b) The magnetic flux through the coil is $\Phi=NBA\cos(\omega t)$ where $N=200$ turns.

The induced emf is:

$$\mathcal{E}=-\frac{d\Phi}{dt}=NBA\omega\sin(\omega t)$$

The maximum emf is $\mathcal{E}_{0}=NBA\omega=NBA(2\pi f)$.

Over one-fourth cycle (from $t=0$ to $t=T/4$), the average emf is:

$$\mathcal{E}_{\text{avg}}=\frac{\Delta\Phi}{\Delta t}=\frac{NBA-0}{T/4}=\frac{4NBA}{T}=4NBAf$$

Given $f=100 \text{ MHz}=1.00 \times 10^{8} \text{ Hz}$:

$$\mathcal{E}_{\text{avg}}=4\left(200\right)\left(1.00 \times 10^{-12} \text{ T}\right)\left(0.0707 \text{ m}^{2}\right)\left(1.00 \times 10^{8} \text{ Hz}\right)$$
$$\mathcal{E}_{\text{avg}}=4 \times 200 \times 1.00 \times 10^{-12} \times 0.0707 \times 1.00 \times 10^{8}=5.66 \times 10^{-3} \text{ V}=5.66 \text{ mV}$$

(c) From $f=\frac{1}{2\pi\sqrt{LC}}$, solve for C:

$$C=\frac{1}{(2\pi f)^{2}L}=\frac{1}{4\pi^{2}f^{2}L}$$

Given $L=2.50 \text{ }\mu\text{H}=2.50 \times 10^{-6} \text{ H}$:

$$C=\frac{1}{4\pi^{2}\left(1.00 \times 10^{8}\right)^{2}\left(2.50 \times 10^{-6}\right)}$$
$$C=\frac{1}{4\pi^{2} \times 1.00 \times 10^{16} \times 2.50 \times 10^{-6}}=\frac{1}{9.87 \times 10^{11}}=1.01 \times 10^{-12} \text{ F}=1.01 \text{ pF}$$

Discussion

The incident power of 8.43 picowatts is incredibly weak—about a trillion times less than a typical LED! Yet FM radios can receive clear signals at this power level because: (1) the receiver is tuned to resonate at exactly 100 MHz, amplifying signals at that frequency, (2) the multi-turn coil (200 turns) multiplies the induced voltage, and (3) modern low-noise amplifiers can detect microvolts of signal.

The average induced emf of 5.66 mV over a quarter cycle is small but workable for radio reception. The factor of 200 from the number of turns is crucial —without multiple turns, the signal would be 200 times weaker and much harder to detect.

The capacitance of 1.01 pF needed for resonance is very small—typical of high-frequency tuning circuits. At 100 MHz (in the FM broadcast band of 88-108 MHz), the LC circuit must be precisely tuned. Variable capacitors (varactors or tuning capacitors) allow the radio to select different stations by adjusting the resonant frequency.

The resonance condition $f=\frac{1}{2\pi\sqrt{LC}}$ shows that at resonance, the impedances of the inductor and capacitor cancel, allowing maximum current flow and maximum signal detection. This is how a radio “tunes in” to a specific station—by adjusting C (or L) until the circuit resonates at the desired frequency.

FM radio uses frequencies around 100 MHz because: (1) these frequencies can carry high-fidelity audio with frequency modulation, (2) they propagate by line-of-sight (unlike AM which can bounce off the ionosphere), giving reliable local coverage, and (3) the wavelength (about 3 m) is small enough for practical antennas yet large enough to avoid excessive atmospheric absorption.

Final Answer

(a) The power incident on the coil is 8.43×10^{-12} W or 8.43 pW. (b) The average induced emf over one-fourth of a cycle is 5.66 mV. (c) The required capacitance is 1.01×10^{-12} F or 1.01 pF.

Integrated Concepts

If electric and magnetic field strengths vary sinusoidally in time, being zero at $t=0$, then $E={E}_{0} \sin 2\pi \text{ft}$ and $B={B}_{0} \sin 2\pi \text{ft}$. Let $f=1.00 \text{GHz}$ here. (a) When are the field strengths first zero? (b) When do they reach their most negative value? (c) How much time is needed for them to complete one cycle?

Show Solution

(a) $t=0$ (b) $7.50 \times 10^{-10} \text{s}$ (c) $1.00 \times 10^{-9} \text{s}$

Strategy

Unreasonable Results

A researcher measures the wavelength of a 1.20-GHz electromagnetic wave to be 0.500 m. (a) Calculate the speed at which this wave propagates. (b) What is unreasonable about this result? (c) Which assumptions are unreasonable or inconsistent?

Show Solution

For part (a), use the wave equation $c=f\lambda$ to calculate the wave speed. For part (b), compare the result to the known speed of electromagnetic waves in vacuum. For part (c), identify which measurement or assumption is incorrect.

Solution

(a) Given:

- Frequency: $f=1.20 \text{ GHz}=1.20 \times 10^{9} \text{ Hz}$
- Wavelength: $\lambda=0.500 \text{ m}$

Using the wave equation:

$$v=f\lambda=\left(1.20 \times 10^{9} \text{ Hz}\right)\left(0.500 \text{ m}\right)=6.00 \times 10^{8} \text{ m/s}$$

(b) This result is unreasonable because the calculated speed is 6.00×10^{8} m/s, which is twice the speed of light in vacuum ($c=3.00 \times 10^{8}$ m/s). According to special relativity, electromagnetic waves in vacuum always propagate at exactly c, and nothing can travel faster than the speed of light. Even in material media, electromagnetic waves travel slower than c, never faster.

(c) Either the frequency measurement or the wavelength measurement (or both) is incorrect. The two measurements are inconsistent with each other for an electromagnetic wave.

If the frequency is correctly measured at 1.20 GHz, then the wavelength in vacuum should be:

$$\lambda=\frac{c}{f}=\frac{3.00 \times 10^{8} \text{ m/s}}{1.20 \times 10^{9} \text{ Hz}}=0.250 \text{ m}=25.0 \text{ cm}$$

Alternatively, if the wavelength is correctly measured at 0.500 m, then the frequency should be:

$$f=\frac{c}{\lambda}=\frac{3.00 \times 10^{8} \text{ m/s}}{0.500 \text{ m}}=6.00 \times 10^{8} \text{ Hz}=600 \text{ MHz}$$

The most likely error is in the wavelength measurement—it's probably 0.250 m, not 0.500 m. Alternatively, the frequency might be 600 MHz rather than 1.20 GHz.

Discussion

This problem illustrates an important consistency check in electromagnetic wave measurements. The relationship $c=f\lambda$ is fundamental and inviolable for electromagnetic waves in vacuum. Any pair of measurements that violates this relationship indicates measurement error.

In this case, 1.20 GHz is in the microwave range (used for some satellite communications and radar), where the correct wavelength is 25 cm. A wavelength of 50 cm would correspond to 600 MHz, which is in the UHF television band.

Common sources of error in such measurements include:

1. **Frequency errors**: Misreading the oscillator frequency, having the oscillator operating at a harmonic (half or double the intended frequency)
2. **Wavelength errors**: Measuring the wrong distance (perhaps measuring a half-wavelength as a full wavelength, or including phase shifts in transmission lines)
3. **Medium effects**: Forgetting that the wave is propagating in a material medium rather than vacuum (though no known material has a refractive index less than 1, which would be required for faster-than-light propagation)

The fact that the calculated speed is exactly twice c suggests a systematic error, perhaps measuring every other node in a standing wave pattern or a factor-of-2 error in the frequency setting.

This type of "unreasonable results" problem is valuable for developing physical intuition—recognizing when calculated results violate fundamental physical principles and then tracing back to find the error.

Final Answer

(a) The calculated wave speed is 6.00×10^{8} m/s. (b) This is unreasonable because it's twice the speed of light, violating special relativity—electromagnetic waves cannot exceed c in vacuum. (c) The frequency and wavelength measurements are inconsistent; either the frequency is actually 600 MHz, or the wavelength is actually 0.250 m (25.0 cm).

Unreasonable Results

The peak magnetic field strength in a residential microwave oven is $9.20 \times 10^{-5} \text{T}$. (a) What is the intensity of the microwave? (b) What is unreasonable about this result? (c) What is wrong about the premise?

Show Solution

(a) $1.01 \times 10^{6} {\text{W/m}}^{2}$ (b) Much too great for an oven.

(c) The assumed magnetic field is unreasonably large.

Strategy

Unreasonable Results

An LC circuit containing a 2.00-H inductor oscillates at such a frequency that it radiates at a 1.00-m wavelength. (a) What is the capacitance of the circuit? (b) What is unreasonable about this result? (c) Which assumptions are unreasonable or inconsistent?

Show Solution

For part (a), first find the frequency from $f=\frac{c}{\lambda}$, then use the LC resonance formula $f=\frac{1}{2\pi\sqrt{LC}}$ to solve for capacitance. For part (b), examine whether the calculated capacitance is physically reasonable. For part (c), identify which parameter is inconsistent with practical LC circuits.

Solution

(a) Given:

- Wavelength: $\lambda=1.00 \text{ m}$
- Inductance: $L=2.00 \text{ H}$
- Speed of light: $c=3.00 \times 10^{8} \text{ m/s}$

Find the frequency:

$$f=\frac{c}{\lambda}=\frac{3.00 \times 10^{8} \text{ m/s}}{1.00 \text{ m}}=3.00 \times 10^{8} \text{ Hz}=300 \text{ MHz}$$

From the resonance formula $f=\frac{1}{2\pi\sqrt{LC}}$, solve for C:

$$C=\frac{1}{(2\pi f)^{2}L}=\frac{1}{4\pi^{2}f^{2}L}$$
$$C=\frac{1}{4\pi^{2}\left(3.00 \times 10^{8}\right)^{2}\left(2.00\right)}$$
$$C=\frac{1}{4\pi^{2} \times 9.00 \times 10^{16} \times 2.00}=\frac{1}{7.10 \times 10^{18}}=1.41 \times 10^{-19} \text{ F}$$

(b) This capacitance is unreasonably small. At 1.41×10^{-19} F (0.141 attofarads), it's far smaller than any practical capacitor. For comparison:

- Typical small capacitors: picofarads (10^{-12} F) to microfarads (10^{-6} F)

- Minimum practical capacitor: ~0.1 pF (10^{-13} F)
- This result: 10^{-19} F, which is a million times smaller than the smallest practical capacitor

Such a small capacitance is comparable to the stray capacitance between individual atoms and is impossible to construct as a discrete circuit element.

(c) The inductance of 2.00 H is unreasonably large for a circuit operating at 300 MHz. Large inductances (in henries) are used in low-frequency applications like power supplies (50-60 Hz) or audio circuits (kHz range), not in radio-frequency circuits.

At 300 MHz, practical inductors are in the nanohenry (nH) to microhenry (μH) range:

- For a 1-m wavelength (300 MHz) with a reasonable capacitance of, say, 1 pF:

$$L=\frac{1}{4\pi^{2}f^{2}C}=\frac{1}{4\pi^{2}\left(3.00 \times 10^{8}\right)^{2}\left(1.00 \times 10^{-12}\right)}=2.81 \times 10^{-7} \text{ H}=281 \text{ nH}$$

A 2.00-H inductor at 300 MHz would have enormous inductive reactance ($X_L=2\pi fL=3.77 \times 10^{9}$ Ω or 3.77 GΩ), making it impractical. Additionally, the large physical size and parasitic capacitance of a 2-H inductor would prevent it from operating effectively at such high frequencies.

The unreasonable assumption is using a large inductance (appropriate for low frequencies) in a high-frequency circuit. For an LC circuit to radiate efficiently at 300 MHz, both L and C must be small.

Discussion

This problem illustrates a fundamental principle in RF circuit design: as frequency increases, both L and C must decrease. The product LC determines the resonant frequency through $f=\frac{1}{2\pi\sqrt{LC}}$, so to achieve high frequency, you need small LC products.

The relationship between component size and frequency is not arbitrary—it's dictated by:

1. **Physical size**: At high frequencies, wavelengths are comparable to component dimensions, making lumped-element analysis (treating L and C as discrete components) invalid for large components
2. **Parasitic effects**: Large inductors have significant parasitic capacitance between windings, and large capacitors have parasitic inductance in their leads
3. **Skin effect and losses**: AC resistance increases with frequency, making large inductors lossy at high frequencies
4. **Radiation**: Large loops act as antennas, radiating away energy rather than storing it in the magnetic field

In practice, circuits operating at 300 MHz (VHF band, used for FM radio and some television) use:

- Inductances: 10 nH to 1 μH (microwave circuits use even smaller values)
- Capacitances: 0.5 pF to 100 pF
- Often stripline or microstrip transmission line resonators instead of discrete L and C

This "unreasonable results" problem teaches that not all mathematical solutions to the resonance equation correspond to physically realizable circuits.

Final Answer

(a) The capacitance is 1.41×10^{-19} F or 0.141 attofarads. (b) This is unreasonably small—about a million times smaller than the smallest practical capacitor. (c) The assumption that a 2.00-H inductor can operate effectively at 300 MHz is unreasonable; such large inductances are only suitable for low-frequency circuits (Hz to kHz range), not VHF radio frequencies.

Unreasonable Results

An LC circuit containing a 1.00-pF capacitor oscillates at such a frequency that it radiates at a 300-nm wavelength. (a) What is the inductance of the circuit? (b) What is unreasonable about this result? (c) Which assumptions are unreasonable or inconsistent?

Show Solution

(a) $2.53 \times 10^{-20} \text{H}$ (b) L is much too small.

(c) The wavelength is unreasonably small.

Strategy

Create Your Own Problem

Consider electromagnetic fields produced by high voltage power lines. Construct a problem in which you calculate the intensity of this electromagnetic radiation in ${\text{W/m}}^{2}$ based on the measured magnetic field strength of the radiation in a home near the power lines. Assume these magnetic field strengths are known to average less than a μT . The intensity is small enough that it is difficult to imagine mechanisms for biological damage due to it. Discuss how much energy may be radiating from a section of power line several hundred meters long and compare this to the power likely to be carried by the lines. An idea of how much power this is can be obtained by calculating the approximate current responsible for μT fields at distances of tens of meters.

Show Solution

This is an open-ended problem where you design your own scenario. A good approach: (1) Choose a specific magnetic field strength (less than 1 μT), (2) Calculate the EM radiation intensity using $I=\frac{cB_{0}^{2}}{2\mu_{0}}$, (3) Estimate radiated power from a power line section, (4) Compare to transmitted power, (5) Calculate the current creating the field using Ampere's law.

Example Solution

Problem Setup: A home is located 50 m from a high-voltage transmission line. The measured 60-Hz magnetic field at the home is 0.50 μT. A 500-m section of power line is considered. The line carries three-phase AC power at 500 kV.

Part 1: Intensity of EM Radiation

Given: $B_{0}=0.50 \text{ }\mu\text{T}=5.0 \times 10^{-7} \text{ T}$

$$I=\frac{cB_{0}^{2}}{2\mu_{0}}=\frac{\left(3.00 \times 10^{8}\right)\left(5.0 \times 10^{-7}\right)^{2}}{2\left(4\pi \times 10^{-7}\right)}=\frac{7.5 \times 10^{-5}}{2.513 \times 10^{-6}}=29.8 \text{ W/m}^{2}$$

Part 2: Radiated Power from 500-m Section

If we model the radiation as spreading cylindrically from the line, over a cylindrical surface at radius $r=50 \text{ m}$:

$$A=2\pi rL=2\pi(50)(500)=1.57 \times 10^{5} \text{ m}^{2}$$
$$P_{\text{rad}}=IA=\left(29.8 \text{ W/m}^{2}\right)\left(1.57 \times 10^{5} \text{ m}^{2}\right)=4.68 \times 10^{6} \text{ W}=4.68 \text{ MW}$$

Part 3: Current Estimate

For a long straight wire, the magnetic field at distance r is $B=\frac{\mu_{0}I}{2\pi r}$. Solving for current:

$$I=\frac{2\pi rB}{\mu_{0}}=\frac{2\pi(50)\left(5.0 \times 10^{-7}\right)}{4\pi \times 10^{-7}}=\frac{1.57 \times 10^{-4}}{1.257 \times 10^{-6}}=125 \text{ A}$$

Part 4: Transmitted Power

For a three-phase line at 500 kV with 125 A per phase:

$$P_{\text{trans}}=\sqrt{3}VI\cos\phi \approx \sqrt{3}(500{,}000)(125)(0.95)=103 \text{ MW}$$

(assuming power factor $\cos\phi=0.95$)

Comparison: The radiated power (4.68 MW) is about 4.5% of the transmitted power (103 MW).

Discussion

Important caveat: This calculation vastly overestimates the actual radiated power because 60-Hz electromagnetic waves don't propagate efficiently. At 60 Hz, $\lambda=c/f=5000 \text{ km}$, which is enormous. Power lines are much shorter than a wavelength, making them extremely inefficient antennas. The formula $I=\frac{cB_{0}^{2}}{2\mu_{0}}$ applies to propagating electromagnetic waves, not to the near-field region of a 60-Hz source.

In reality, most of the measured magnetic field near power lines is **quasi-static** (varying slowly at 60 Hz) rather than radiating electromagnetic waves. The electric and magnetic fields exist primarily in the near field, storing energy that oscillates back and forth rather than propagating away. True radiation from power lines is negligible—typical radiated power is measured in watts, not megawatts.

The actual health concerns from power lines relate to the quasi-static fields themselves (if any effects exist), not to EM radiation intensity. Extensive research has found no consistent evidence that 60-Hz magnetic fields at these levels (< 1 μT) cause biological harm, though studies continue.

Key learning points from this problem:

1. Not all oscillating fields are radiating waves—frequency and antenna size matter
2. The intensity formula $I=\frac{cB_{0}^{2}}{2\mu_{0}}$ applies to propagating waves, not near fields
3. Power lines operate in the near-field regime where energy is stored, not radiated
4. Efficient radiation requires antennas comparable to wavelength (λ/4 or λ/2)
5. The measured field strength tells us about current (125 A is reasonable for transmission lines)

Alternative scenarios you could explore:

- Different distances (10 m vs 100 m)
- Different field strengths (0.1 μT vs 0.9 μT)
- Different line lengths (100 m vs 1000 m)
- Different voltages and power levels
- Comparison with other radiation sources (cell phones, Wi-Fi)

Final Answer

This is a student-designed problem. The example shows that while naive application of the intensity formula gives 29.8 W/m² and suggests MW of radiated power, this is incorrect for 60-Hz quasi-static fields. Real power line radiation is negligible. The exercise illustrates the distinction between near-field energy storage and far-field radiation, and why frequency and antenna size are critical for EM wave propagation.
Strategy

Create Your Own Problem

Consider the most recent generation of residential satellite dishes that are a little less than half a meter in diameter. Construct a problem in which you calculate the power received by the dish and the maximum electric field strength of the microwave signals for a single channel received by the dish. Among the things to be considered are the power broadcast by the satellite and the area over which the power is spread, as well as the area of the receiving dish.

Show Solution

This is an open-ended problem where you design your own scenario. A good approach: (1) Choose realistic dish diameter (< 0.5 m), (2) Estimate satellite broadcast power and coverage area, (3) Calculate intensity at Earth's surface, (4) Find power received by dish, (5) Calculate electric field strength from intensity.

Example Solution

Problem Setup: A residential satellite TV dish has a diameter of 45 cm. A geosynchronous satellite broadcasting at 36,000 km altitude transmits 200 W of power for a single channel over a coverage area (footprint) of $2.0 \times 10^{13} \text{ m}^{2}$ (covering roughly the continental United States).

Part 1: Intensity at Ground Level

Given:

- Satellite power (one channel): $P_{\text{sat}}=200 \text{ W}$
- Coverage area: $A_{\text{coverage}}=2.0 \times 10^{13} \text{ m}^{2}$

$$I=\frac{P_{\text{sat}}}{A_{\text{coverage}}}=\frac{200 \text{ W}}{2.0 \times 10^{13} \text{ m}^{2}}=1.0 \times 10^{-11} \text{ W/m}^{2}=10 \text{ pW/m}^{2}$$

Part 2: Power Received by Dish

Dish diameter $d=45 \text{ cm}=0.45 \text{ m}$, radius $r=0.225 \text{ m}$:

$$A_{\text{dish}}=\pi r^{2}=\pi(0.225)^{2}=0.159 \text{ m}^{2}$$
$$P_{\text{received}}=I \times A_{\text{dish}}=\left(1.0 \times 10^{-11} \text{ W/m}^{2}\right)(0.159 \text{ m}^{2})=1.59 \times 10^{-12} \text{ W}=1.59 \text{ pW}$$

Part 3: Maximum Electric Field Strength

From $I=\frac{c\epsilon_{0}E_{0}^{2}}{2}$:

$$E_{0}=\sqrt{\frac{2I}{c\epsilon_{0}}}=\sqrt{\frac{2\left(1.0 \times 10^{-11}\right)}{\left(3.00 \times 10^{8}\right)\left(8.85 \times 10^{-12}\right)}}$$
$$E_{0}=\sqrt{\frac{2.0 \times 10^{-11}}{2.655 \times 10^{-3}}}=\sqrt{7.53 \times 10^{-9}}=8.68 \times 10^{-5} \text{ V/m}=86.8 \text{ }\mu\text{V/m}$$

Part 4: Maximum Magnetic Field Strength

$$B_{0}=\frac{E_{0}}{c}=\frac{8.68 \times 10^{-5} \text{ V/m}}{3.00 \times 10^{8} \text{ m/s}}=2.89 \times 10^{-13} \text{ T}=289 \text{ fT}$$
Discussion

The received power of 1.59 picowatts is incredibly small—about 12 orders of magnitude less than a typical LED! Yet satellite TV works because:

1. **Large dish area**: Even a modest 45-cm dish collects signal from 0.159 m² of wavefront
2. **Sensitive receivers**: Low-noise block downconverters (LNBs) at the dish focus can detect signals down to femtowatt levels
3. **Digital modulation**: Modern DVB-S2 encoding with error correction allows signal recovery from very noisy signals
4. **Frequency allocation**: Ku-band (12-18 GHz) and Ka-band (26-40 GHz) satellites use dedicated frequencies with minimal interference
5. **Directional transmission**: Satellites use phased array or shaped reflector antennas to concentrate power toward specific regions, not radiating uniformly

The electric field of 86.8 μV/m is extremely weak—about 10 billion times weaker than the electric field that breaks down air. The magnetic field of 289 femtoteslas is about 100 million times weaker than Earth's magnetic field.

Why 200 W is sufficient:

- Each transponder on a satellite typically transmits 50-200 W
- Modern satellites have 24-72 transponders
- Total satellite power: 5-15 kW
- Powered by large solar panel arrays (20-30 m² in orbit)
- Signal is concentrated into specific coverage zones (footprints)

Comparison with earlier satellite systems:

- 1970s-1980s: Large dishes (2-3 m diameter) needed because satellites had lower power (5-10 W/transponder) and receivers were less sensitive
- 1990s: Introduction of high-power DBS (Direct Broadcast Satellite) with 120-240 W/transponder enabled smaller dishes (60-75 cm)
- 2000s-present: Advanced coding and modulation allow even smaller dishes (45 cm or less)

Alternative scenarios to explore:

- Smaller dish (30 cm) for mobile satellite receivers
- Larger dish (60 cm) for better signal in rain (rain fade mitigation)
- Different satellite power (100 W vs 300 W per channel)
- Different coverage areas (spot beam vs continental coverage)
- Different frequencies (C-band at 4-8 GHz vs Ka-band at 26-40 GHz)
- Calculate signal-to-noise ratio needed for reception
- Account for atmospheric absorption (clear sky vs rain)

Physical insights:

- The inverse-square law means doubling satellite altitude requires 4× more power or 4× larger dish
- Geosynchronous orbit (36,000 km) is optimal for fixed dishes but has 0.25-second round-trip delay
- Lower orbit satellites (like Starlink at 550 km) have stronger signals but require tracking dishes

Final Answer

This is a student-designed problem. The example shows that a 45-cm dish receiving from a 200-W satellite transponder covering 2.0×10^{13} m^2 receives 1.59 pW of power with an electric field strength of 86.8 μV/m. Despite these incredibly weak signals, modern receiver technology makes reliable satellite TV reception possible through sensitive electronics, large collecting areas, and sophisticated digital signal processing.

Glossary

maximum field strength
: the maximum amplitude an electromagnetic wave can reach, representing the maximum amount of electric force and/or magnetic flux that the wave can exert

intensity
: the power of an electric or magnetic field per unit area, for example, Watts per square meter

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