Introduction to Electromagnetic Induction, AC Circuits and Electrical Technologies

These wind turbines in the Thames Estuary in the UK are an example of induction at work. Wind pushes the blades of the turbine, spinning a shaft attached to magnets. The magnets spin around a conductive coil, inducing an electric current in the coil, and eventually feeding the electrical grid. (credit: modification of work by Petr Kratochvil)

Nature's displays of symmetry are beautiful and alluring. A butterfly's wings exhibit an appealing symmetry in a complex system. (See [Figure 2].) The laws of physics display symmetries at the most basic level—these symmetries are a source of wonder and imply deeper meaning. Since we place a high value on symmetry, we look for it when we explore nature. The remarkable thing is that we find it.

Physics, like this butterfly, has inherent symmetries. (credit: Thomas Bresson)

The hint of symmetry between electricity and magnetism found in the preceding chapter will be elaborated upon in this chapter. Specifically, we know that a current creates a magnetic field. If nature is symmetric here, then perhaps a magnetic field can create a current. The Hall effect is a voltage caused by a magnetic force. That voltage could drive a current. Historically, it was very shortly after Oersted discovered currents cause magnetic fields that other scientists asked the following question: Can magnetic fields cause currents? The answer was soon found by experiment to be yes. In 1831, some 12 years after Oersted's discovery, the English scientist Michael Faraday (1791–1862) and the American scientist Joseph Henry (1797–1878) independently demonstrated that magnetic fields can produce currents. The basic process of generating emfs (electromotive force) and, hence, currents with magnetic fields is known as **induction**; this process is also called magnetic induction to distinguish it from charging by induction, which utilizes the Coulomb force.

Today, currents induced by magnetic fields are essential to our technological society. The ubiquitous generator—found in automobiles, on bicycles, in nuclear power plants, and so on—uses magnetism to generate current. Other devices that use magnetism to induce currents include pickup coils in electric guitars, transformers of every size, certain microphones, airport security gates, and damping mechanisms on sensitive chemical balances. Not so familiar perhaps, but important nevertheless, is that the behavior of AC circuits depends strongly on the effect of magnetic fields on currents.

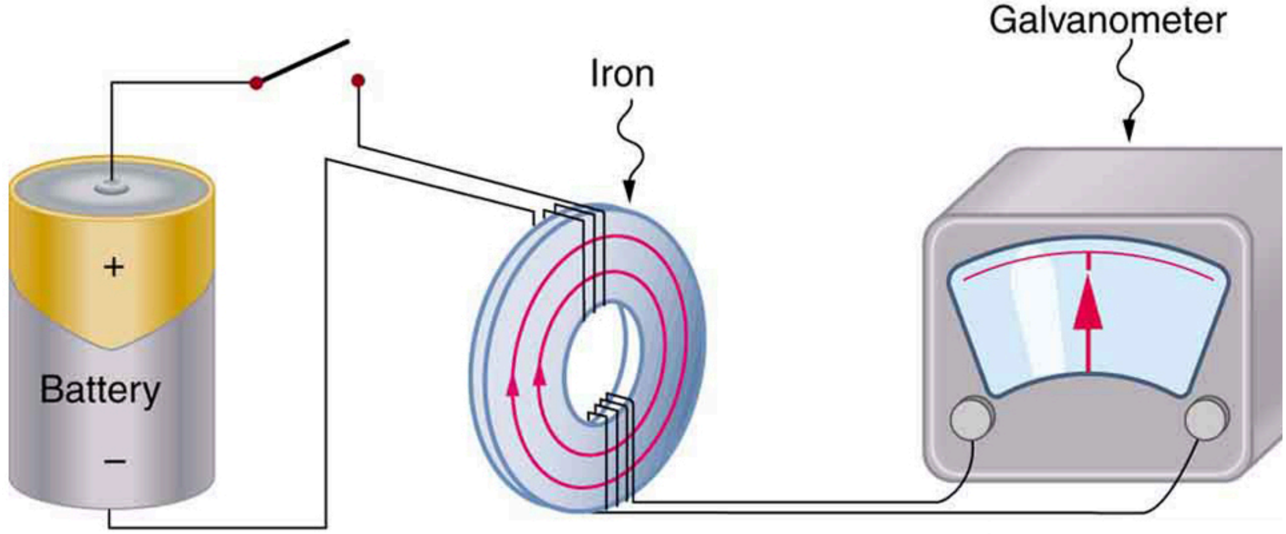
Glossary

induction
: (magnetic induction) the creation of emfs and hence currents by magnetic fields

Induced Emf and Magnetic Flux

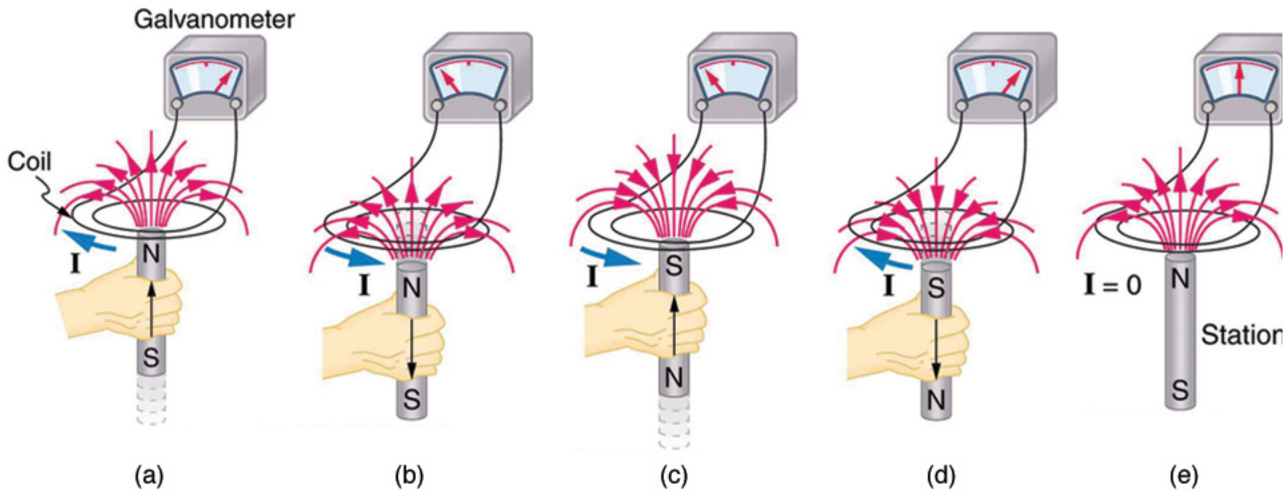
- Calculate the flux of a uniform magnetic field through a loop of arbitrary orientation.
- Describe methods to produce an electromotive force (emf) with a magnetic field or magnet and a loop of wire.

The apparatus used by Faraday to demonstrate that magnetic fields can create currents is illustrated in [Figure 1]. When the switch is closed, a magnetic field is produced in the coil on the top part of the iron ring and transmitted to the coil on the bottom part of the ring. The galvanometer is used to detect any current induced in the coil on the bottom. It was found that each time the switch is closed, the galvanometer detects a current in one direction in the coil on the bottom. (You can also observe this in a physics lab.) Each time the switch is opened, the galvanometer detects a current in the opposite direction. Interestingly, if the switch remains closed or open for any length of time, there is no current through the galvanometer. **Closing and opening the switch** induces the current. It is the **change** in magnetic field that creates the current. More basic than the current that flows is the *emf* that causes it. The current is a result of an *emf induced by a changing magnetic field*, whether or not there is a path for current to flow.



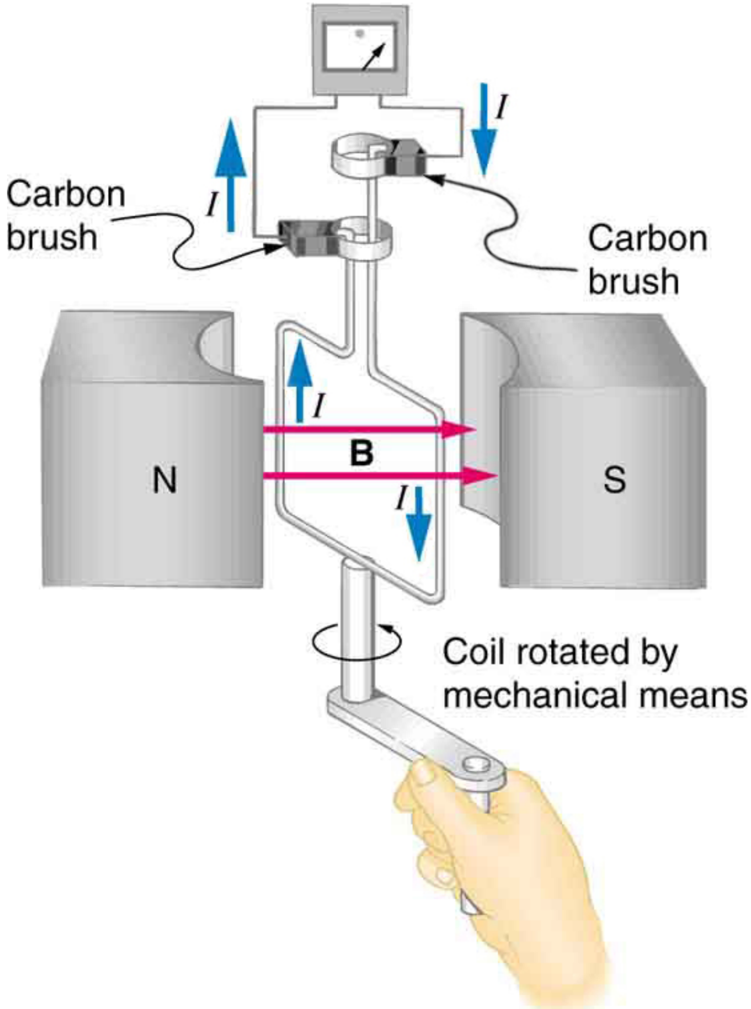
Faraday’s apparatus for demonstrating that a magnetic field can produce a current. A change in the field produced by the top coil induces an emf and, hence, a current in the bottom coil. When the switch is opened and closed, the galvanometer registers currents in opposite directions. No current flows through the galvanometer when the switch remains closed or open.

An experiment easily performed and often done in physics labs is illustrated in [Figure 2]. An emf is induced in the coil when a bar magnet is pushed in and out of it. Emfs of opposite signs are produced by motion in opposite directions, and the emfs are also reversed by reversing poles. The same results are produced if the coil is moved rather than the magnet—it is the relative motion that is important. The faster the motion, the greater the emf, and there is no emf when the magnet is stationary relative to the coil.



Movement of a magnet relative to a coil produces emfs as shown. The same emfs are produced if the coil is moved relative to the magnet. The greater the speed, the greater the magnitude of the emf, and the emf is zero when there is no motion.

The method of inducing an emf used in most electric generators is shown in [Figure 3]. A coil is rotated in a magnetic field, producing an alternating current emf, which depends on rotation rate and other factors that will be explored in later sections. Note that the generator is remarkably similar in construction to a motor (another symmetry).

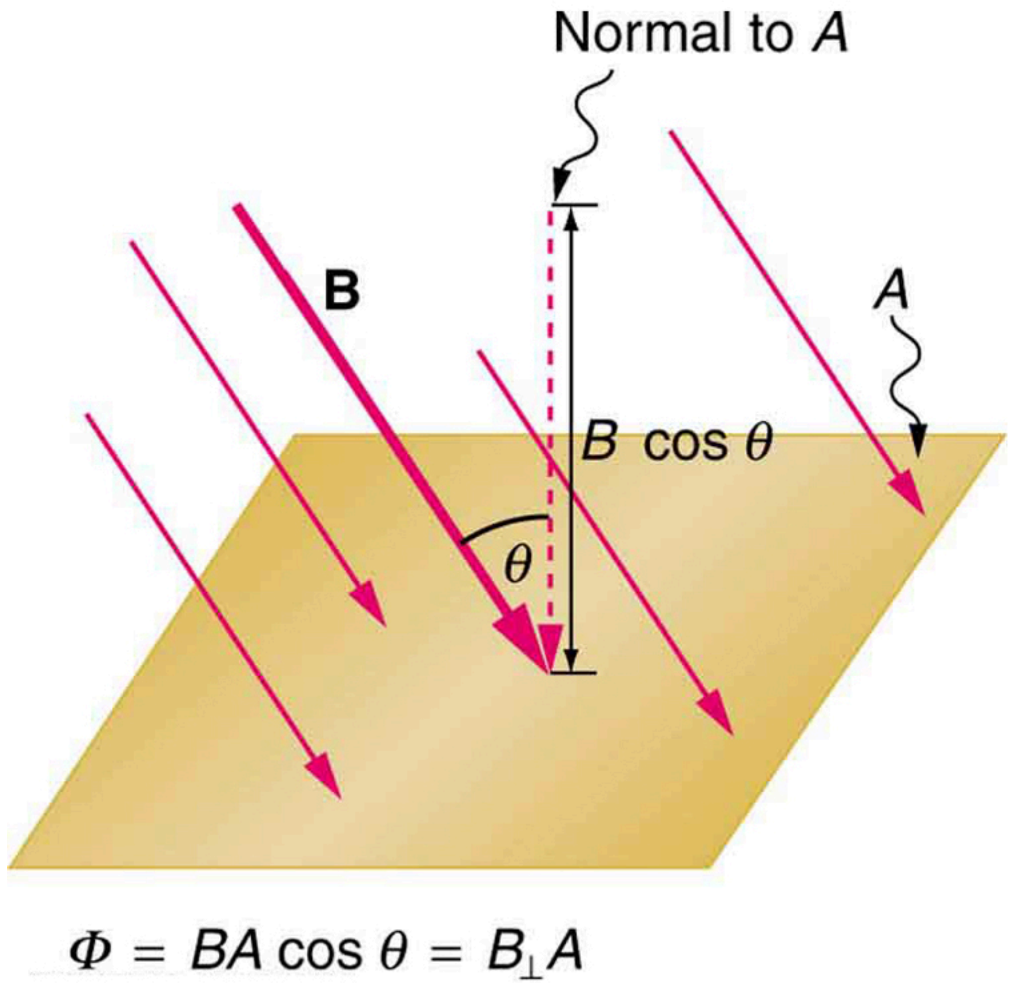


Rotation of a coil in a magnetic field produces an emf. This is the basic construction of a generator, where work done to turn the coil is converted to electric energy. Note the generator is very similar in construction to a motor.

So we see that changing the magnitude or direction of a magnetic field produces an emf. Experiments revealed that there is a crucial quantity called the **magnetic flux**, Φ, given by

$$\Phi = BA\cos\theta,$$

where B is the magnetic field strength over an area A, at an angle θ with the perpendicular to the area as shown in [Figure 4]. **Any change in magnetic flux Φ induces an emf.** This process is defined to be **electromagnetic induction**. Units of magnetic flux Φ are $\mathrm{T \cdot m^2}$. As seen in [Figure 4], $B\cos\theta = B_\perp$, which is the component of B perpendicular to the area A. Thus magnetic flux is $\Phi = B_\perp A$, the product of the area and the component of the magnetic field perpendicular to it.



Magnetic flux Φ is related to the magnetic field and the area over which it exists. The flux $\Phi = BA\cos\theta$ is related to induction; any change in Φ induces an emf.

All induction, including the examples given so far, arises from some change in magnetic flux Φ. For example, Faraday changed B and hence Φ when opening and closing the switch in his apparatus (shown in [Figure 1]). This is also true for the bar magnet and coil shown in [Figure 2]. When rotating the coil of a generator, the angle θ and, hence, Φ is changed. Just how great an emf and what direction it takes depend on the change in Φ and how rapidly the change is made, as examined in the next section.

Section Summary

- The crucial quantity in induction is magnetic flux Φ, defined to be $\Phi = BA\cos\theta$, where B is the magnetic field strength over an area A at an angle θ with the perpendicular to the area.
- Units of magnetic flux Φ are $\mathrm{T \cdot m^2}$.
- Any change in magnetic flux Φ induces an emf—the process is defined to be electromagnetic induction.

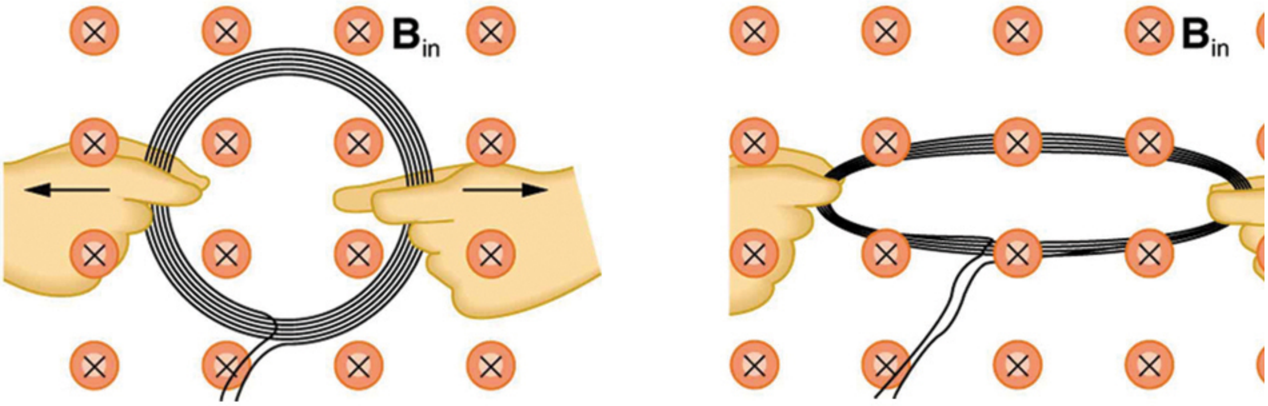
Conceptual Questions

How do the multiple-loop coils and iron ring in the version of Faraday's apparatus shown in [Figure 1] enhance the observation of induced emf?

When a magnet is thrust into a coil as in [Figure 2](a), what is the direction of the force exerted by the coil on the magnet? Draw a diagram showing the direction of the current induced in the coil and the magnetic field it produces, to justify your response. How does the magnitude of the force depend on the resistance of the galvanometer?

Explain how magnetic flux can be zero when the magnetic field is not zero.

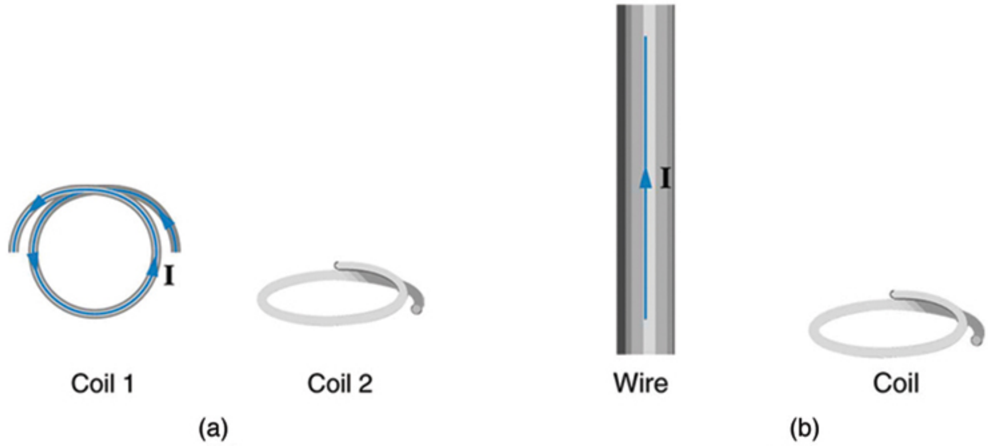
Is an emf induced in the coil in [Figure 5] when it is stretched? If so, state why and give the direction of the induced current.



A circular coil of wire is stretched in a magnetic field.

Problems & Exercises

What is the value of the magnetic flux at coil 2 in [Figure 6] due to coil 1?



(a) The planes of the two coils are perpendicular. (b) The wire is perpendicular to the plane of the coil.

Show Solution

Strategy

The two coils are perpendicular to each other. When coil 1 carries current, it produces a magnetic field perpendicular to its own plane. We need to determine the angle between this field and the normal to coil 2's area.

Solution

When current flows through coil 1, it creates a magnetic field along an axis perpendicular to coil 1's plane. Since coil 2 is positioned perpendicular to coil 1, the plane of coil 2 is parallel to the magnetic field lines from coil 1.

For the magnetic flux through coil 2, we use:

$$\Phi = BA\cos\theta$$

where θ is the angle between the magnetic field and the normal to coil 2's area.

Since the magnetic field from coil 1 is perpendicular to coil 1's plane, and coil 2's plane is perpendicular to coil 1's plane, the magnetic field from coil 1 lies in the plane of coil 2. This means the field is perpendicular to the normal of coil 2, so $\theta = 90°$.

$$\Phi = BA\cos(90°) = BA(0) = 0$$

Discussion

The perpendicular orientation of the coils is crucial. The magnetic field lines from coil 1 run parallel to the plane of coil 2, so none of them pass through coil 2's area. This is why the flux is zero despite the presence of a magnetic field.

Final Answer

The magnetic flux at coil 2 due to coil 1 is zero.

What is the value of the magnetic flux through the coil in [Figure 6](b) due to the wire?

Show Solution

Strategy

The wire is perpendicular to the plane of the coil. The magnetic field from the wire forms circles around it, and these field lines lie in the plane of the coil, not perpendicular to it.

Solution

The magnetic field from a long straight wire circles the wire. At any point on the coil, the magnetic field is tangent to a circle centered on the wire. Since the coil lies in a plane that contains the wire, the magnetic field lines are parallel to the plane of the coil.

The magnetic flux is $\Phi = BA\cos\theta$ where θ is the angle between the field and the normal to the coil's area. Here, the field is parallel to the coil's plane, making it perpendicular to the normal ($\theta = 90\,°$).

$$\Phi = BA\cos(90°) = BA(0) = 0$$

Discussion

No magnetic field lines pass through the area of the coil because they all lie in the plane of the coil. This is analogous to how wind blowing parallel to a window doesn't pass through the window.

Final Answer

The magnetic flux through the coil is zero.

Glossary

magnetic flux
: the amount of magnetic field going through a particular area, calculated with $\Phi = BA\cos\theta$ where B is the magnetic field strength over an area A at an angle θ with the perpendicular to the area

electromagnetic induction
: the process of inducing an emf (voltage) with a change in magnetic flux

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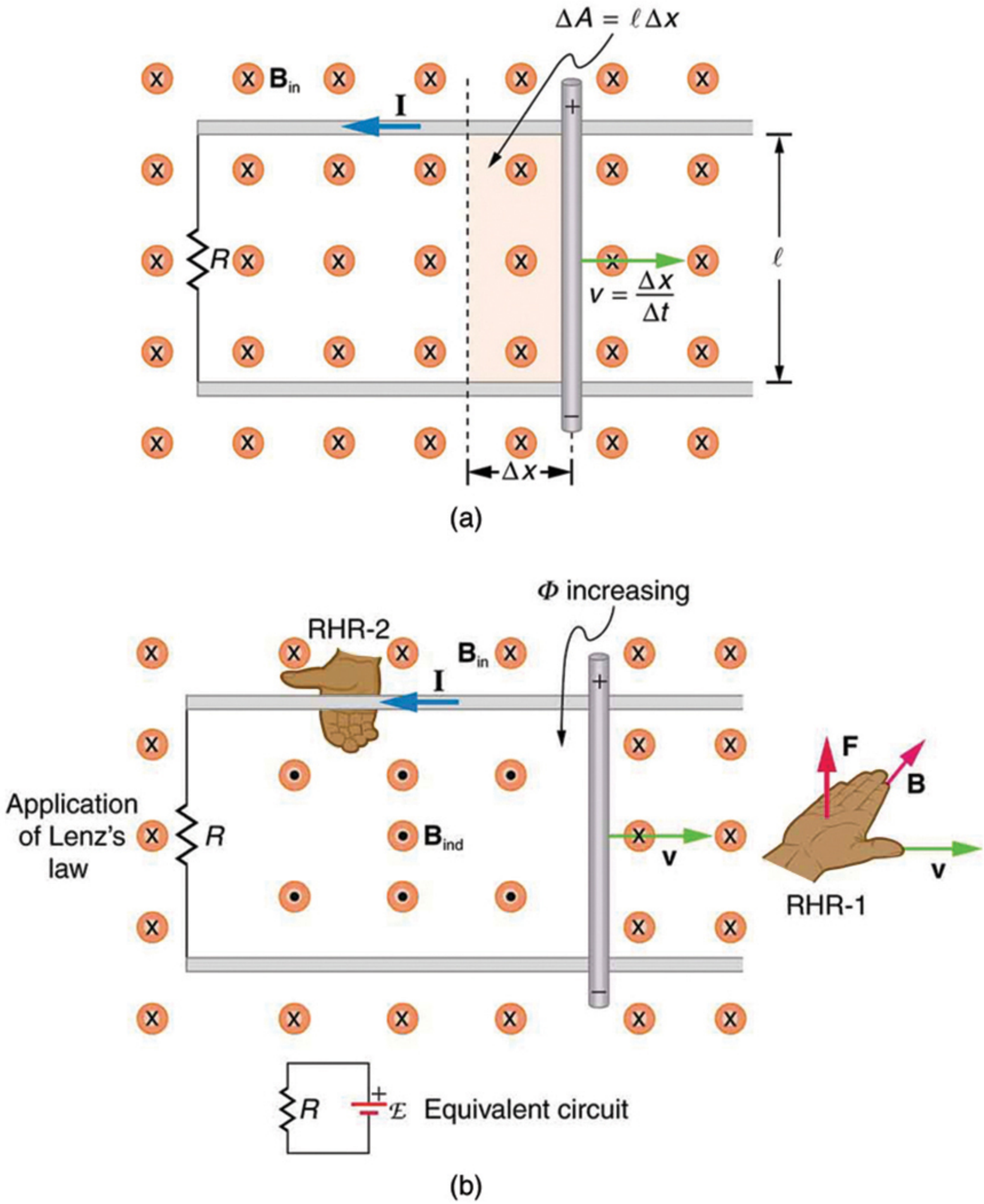
Motional Emf

- Calculate emf, force, magnetic field, and work due to the motion of an object in a magnetic field.

As we have seen, any change in magnetic flux induces an emf opposing that change—a process known as induction. Motion is one of the major causes of induction. For example, a magnet moved toward a coil induces an emf, and a coil moved toward a magnet produces a similar emf. In this section, we concentrate on motion in a magnetic field that is stationary relative to the Earth, producing what is loosely called **motional emf**.

One situation where motional emf occurs is known as the Hall effect and has already been examined. Charges moving in a magnetic field experience the magnetic force $F = qvB\sin\theta$, which moves opposite charges in opposite directions and produces an $\text{emf} = B\ell v$. We saw that the Hall effect has applications, including measurements of B and v . We will now see that the Hall effect is one aspect of the broader phenomenon of induction, and we will find that motional emf can be used as a power source.

Consider the situation shown in [Figure 1]. A rod is moved at a speed v along a pair of conducting rails separated by a distance ℓ in a uniform magnetic field B . The rails are stationary relative to B and are connected to a stationary resistor R . The resistor could be anything from a light bulb to a voltmeter. Consider the area enclosed by the moving rod, rails, and resistor. B is perpendicular to this area, and the area is increasing as the rod moves. Thus the magnetic flux enclosed by the rails, rod, and resistor is increasing. When flux changes, an *emf* is induced according to Faraday's law of induction.



(a) A motional $\text{emf} = Blv$ is induced between the rails when this rod moves to the right in the uniform magnetic field. The magnetic field B is into the page, perpendicular to the moving rod and rails and, hence, to the area enclosed by them. (b) Lenz's law gives the directions of the induced field and current, and the polarity of the induced emf. Since the flux is increasing, the induced field is in the opposite direction, or out of the page. RHR-2 gives the current direction shown, and the polarity of the rod will drive such a current. RHR-1 also indicates the same polarity for the rod. (Note that the script E symbol used in the equivalent circuit at the bottom of part (b) represents emf.)

To find the magnitude of emf induced along the moving rod, we use Faraday's law of induction without the sign:

$$\text{emf} = N\frac{\Delta\Phi}{\Delta t}.$$

Here and below, "emf" implies the magnitude of the emf. In this equation, $N = 1$ and the flux $\Phi = BA\cos\theta$. We have $\theta = 0\,^{\circ}$ and $\cos\theta = 1$, since B is perpendicular to A . Now $\Delta\Phi = \Delta(BA) = B\Delta A$, since B is uniform. Note that the area swept out by the rod is $\Delta A = \ell\Delta x$. Entering these quantities into the expression for **emf** yields

$$\text{emf} = \frac{B\Delta A}{\Delta t} = B\frac{\ell\Delta x}{\Delta t}.$$

Finally, note that $\Delta x / \Delta t = v$, ** the velocity of the rod. Entering this into the last expression shows that

$$\text{emf} = B\ell v \,(B, \ell, \text{and } v \text{ perpendicular})$$

is the motional emf. This is the same expression given for the Hall effect previously.

Making Connections: Unification of Forces

There are many connections between the electric force and the magnetic force. The fact that a moving electric field produces a magnetic field and, conversely, a moving magnetic field produces an electric field is part of why electric and magnetic forces are now considered to be different manifestations of the same force. This classic unification of electric and magnetic forces into what is called the electromagnetic force is the inspiration for contemporary efforts to unify other basic forces.

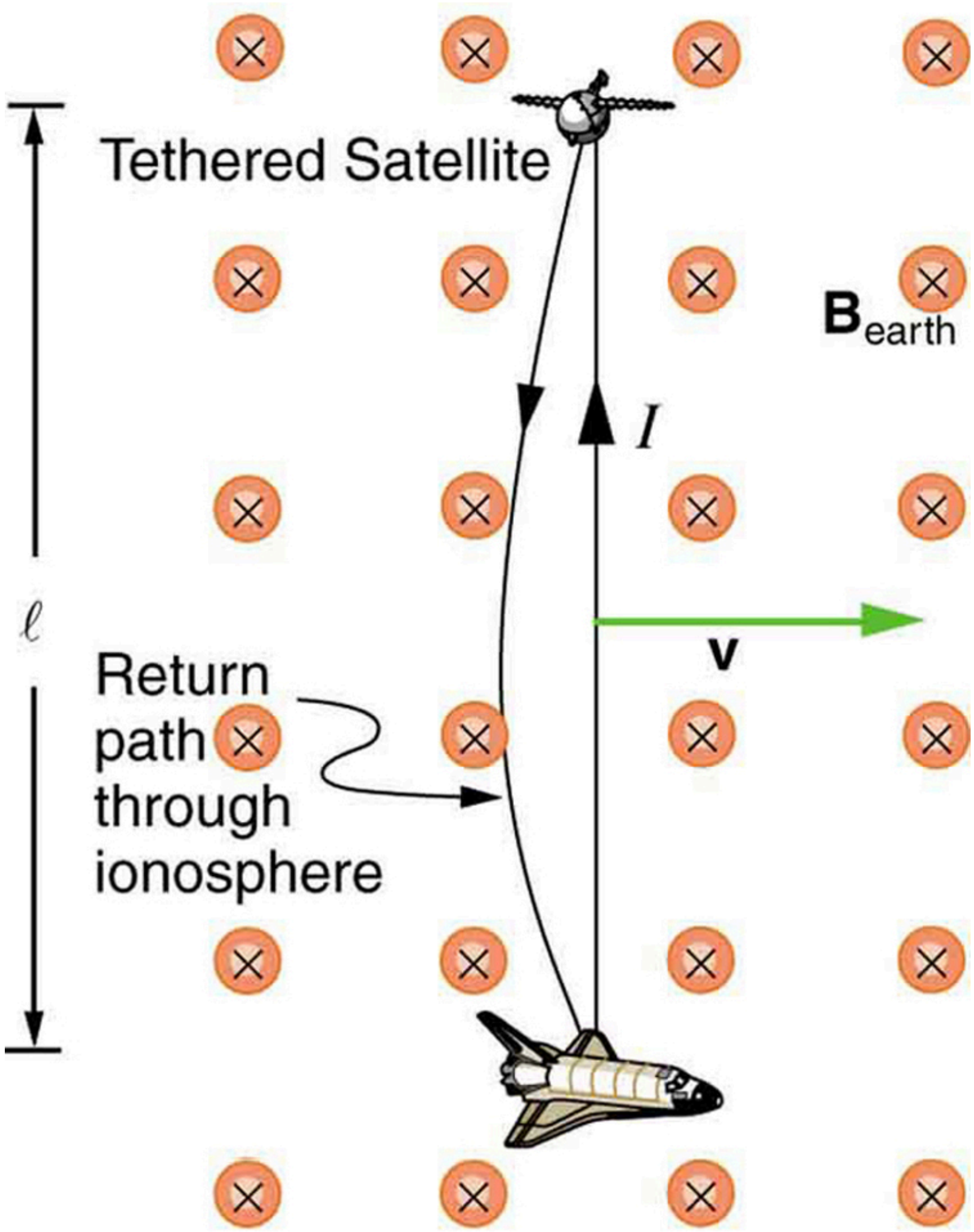
To find the direction of the induced field, the direction of the current, and the polarity of the induced emf, we apply Lenz's law as explained in [Faraday's Law of Induction: Lenz's Law](). (See [Figure 1](b).) Flux is increasing, since the area enclosed is increasing. Thus the induced field must oppose the existing one and be out of the page. And so the RHR-2 requires that *I* be counterclockwise, which in turn means the top of the rod is positive as shown.

Motional emf also occurs if the magnetic field moves and the rod (or other object) is stationary relative to the Earth (or some observer). We have seen an example of this in the situation where a moving magnet induces an emf in a stationary coil. It is the relative motion that is important. What is emerging in these observations is a connection between magnetic and electric fields. A moving magnetic field produces an electric field through its induced emf. We already have seen that a moving electric field produces a magnetic field—moving charge implies moving electric field and moving charge produces a magnetic field.

Motional emfs in the Earth's weak magnetic field are not ordinarily very large, or we would notice voltage along metal rods, such as a screwdriver, during ordinary motions. For example, a simple calculation of the motional emf of a 1 m rod moving at 3.0 m/s perpendicular to the Earth's field gives

$\text{emf} = B\ell v = (5.0\times10^{-5}\,\text{T})(1.0\,m)(3.0\,m/s) = 150\,\mu\text{V}$. This small value is consistent with experience. There is a spectacular exception, however. In 1992 and 1996, attempts were made with the space shuttle to create large motional emfs. The Tethered Satellite was to be let out on a 20 km length of wire as shown in [Figure 2], to create a 5 kV emf by moving at orbital speed through the Earth's field. This emf could be used to convert some of the shuttle's kinetic and potential energy into electrical energy if a complete circuit could be made. To complete the circuit, the stationary ionosphere was to supply a return path for the current to flow. (The ionosphere is the rarefied and partially ionized atmosphere at orbital altitudes. It conducts because of the ionization. The ionosphere serves the same function as the stationary rails and connecting resistor in [Figure 1], without which there would not be a complete circuit.) Drag on the current in the cable due to the magnetic force $F = I\ell B \sin\theta$ does the work that reduces the shuttle's kinetic and potential energy and allows it to be converted to electrical energy. The tests were both unsuccessful. In the first, the cable hung up and could only be extended a couple of hundred meters; in the second, the cable broke when almost fully extended. [Example 1] indicates feasibility in principle.

Calculating the Large Motional Emf of an Object in Orbit



Motional emf as electrical power conversion for the space shuttle is the motivation for the Tethered Satellite experiment. A 5 kV emf was predicted to be induced in the 20 km long tether while moving at orbital speed in the Earth's magnetic field. The circuit is completed by a return path through the stationary ionosphere.

Calculate the motional emf induced along a 20.0 km long conductor moving at an orbital speed of 7.80 km/s perpendicular to the Earth's $5.00 \times 10^{-5}\text{T}$ magnetic field.

Strategy

This is a straightforward application of the expression for motional emf— $\text{emf} = B\ell v$.

Solution

Entering the given values into $\text{emf} = B\ell v$ gives

$$\text{emf} = B\ell v = (5.00\times10^{-5}\text{T})(2.0\times10^{4}\text{m})(7.80\times10^{3}\text{m/s}) = 7.80\times10^{3}\text{V}.$$

Discussion

The value obtained is greater than the 5 kV measured voltage for the shuttle experiment, since the actual orbital motion of the tether is not perpendicular to the Earth's field. The 7.80 kV value is the maximum emf obtained when $\theta = 90°$ and $\sin\theta = 1$.

Section Summary

- An emf induced by motion relative to a magnetic field B is called a *motional emf* and is given by

  $$\text{emf} = B\ell v \, (B, \ell, \text{ and } v \text{ perpendicular}),$$

  where ℓ is the length of the object moving at speed v relative to the field.

Conceptual Questions

Why must part of the circuit be moving relative to other parts, to have usable motional emf? Consider, for example, that the rails in [Figure 1] are stationary relative to the magnetic field, while the rod moves.

A powerful induction cannon can be made by placing a metal cylinder inside a solenoid coil. The cylinder is forcefully expelled when solenoid current is turned on rapidly. Use Faraday's and Lenz's laws to explain how this works. Why might the cylinder get live/hot when the cannon is fired?

An induction stove heats a pot with a coil carrying an alternating current located beneath the pot (and without a hot surface). Can the stove surface be a conductor? Why won't a coil carrying a direct current work?

Explain how you could thaw out a frozen water pipe by wrapping a coil carrying an alternating current around it. Does it matter whether or not the pipe is a conductor? Explain.

Problems & Exercises

Use Faraday's law, Lenz's law, and RHR-1 to show that the magnetic force on the current in the moving rod in [Figure 1] is in the opposite direction of its velocity.

Show Solution

Strategy

We'll apply Faraday's law to find the induced emf, use Lenz's law to determine the direction of the induced current, and then apply RHR-1 to find the direction of the magnetic force on that current.

Solution

As the rod moves to the right with velocity v, the flux through the circuit increases (the enclosed area increases). By Faraday's law, this changing flux induces an emf in the circuit.

By Lenz's law, the induced current must create a magnetic field that opposes the change in flux. Since the flux into the page is increasing, the induced magnetic field must point out of the page. Using RHR-2 (right-hand rule for the magnetic field created by a current loop), the induced current must flow counterclockwise in the circuit—up through the rod.

Now we apply RHR-1 (the right-hand rule for the magnetic force on a current-carrying conductor). Point your fingers in the direction of current (upward in the rod), curl them toward the direction of the magnetic field (into the page), and your thumb points in the direction of the force. This gives a force to the left—opposite to the rod's velocity (which is to the right).

Discussion

This result makes physical sense from energy conservation. The magnetic force opposes the motion of the rod, so work must be done to keep the rod moving at constant velocity. This mechanical work is converted into electrical energy in the circuit. If the magnetic force were in the same direction as the velocity, we would have perpetual motion, violating energy conservation. The opposing force represents the mechanical "cost" of generating electrical power.

Final Answer

By Faraday's law, the moving rod induces an emf. By Lenz's law, the induced current flows upward through the rod (counterclockwise in the circuit). By RHR-1, a current flowing upward in a magnetic field directed into the page experiences a force to the left, opposite to the rod's rightward velocity.

If a current flows in the Satellite Tether shown in [Figure 2], use Faraday's law, Lenz's law, and RHR-1 to show that there is a magnetic force on the tether in the direction opposite to its velocity.

Show Solution

Strategy

We'll apply the same reasoning as in the previous problem: use Faraday's law for the induced emf, Lenz's law for the current direction, and RHR-1 for the force direction.

Solution

As the tethered satellite moves through Earth's magnetic field at orbital velocity, the tether sweeps through the field lines, creating a changing magnetic flux through the circuit formed by the tether, ionosphere, and shuttle. By Faraday's law, this induces an emf of $\text{emf} = B\ell v$.

By Lenz's law, the induced current must flow in a direction that creates a magnetic field opposing the flux change. As the system moves forward through Earth's magnetic field, the induced current flows in such a direction to oppose this motion.

Using RHR-1 (force on a current-carrying conductor in a magnetic field): Point your fingers in the direction of the current in the tether, curl them toward the direction of Earth's magnetic field, and your thumb points in the direction of the magnetic force. This force, $F = I\ell B$, is directed opposite to the satellite's velocity.

Discussion

This magnetic drag force is what enables energy conversion in the Tethered Satellite experiment. The satellite's kinetic and potential energy is converted to electrical energy through the induced emf and current. The opposing magnetic force does negative work on the satellite, slowing it down and extracting energy from its orbital motion. This is the fundamental principle behind electromagnetic braking and regenerative power generation.

Final Answer

The moving tether induces an emf by Faraday's law. The resulting current (whose direction is determined by Lenz's law to oppose the motion) experiences a magnetic force by RHR-1 that is directed opposite to the satellite's velocity, providing magnetic drag.

(a) A jet airplane with a 75.0 m wingspan is flying at 280 m/s. What emf is induced between wing tips if the vertical component of the Earth's field is $3.00 \times 10^{-5} \text{T}$? (b) Is an emf of this magnitude likely to have any consequences? Explain.

Show Solution

Strategy

(a) Use the motional emf formula $\text{emf} = B\ell v$ where ℓ is the wingspan, v is the airplane's speed, and B is the vertical component of Earth's magnetic field. (b) Assess whether 0.630 V is significant in the context of an aircraft's electrical system.

Solution

(a) Given:

- $\ell = 75.0 \text{ m}$
- $v = 280 \text{ m/s}$
- $B = 3.00 \times 10^{-5} \text{ T}$

$$\text{emf} = B\ell v = (3.00 \times 10^{-5} \text{ T})(75.0 \text{ m})(280 \text{ m/s}) = 0.630 \text{ V}$$

(b) No, this emf is very small and unlikely to have any practical consequences. Modern aircraft electrical systems operate at much higher voltages (typically 28 V DC or 115 V AC). The 0.630 V induced emf is far too small to affect aircraft electronics, cause sparking, or pose any safety hazard. Furthermore, for current to flow and dissipate energy, there would need to be a complete circuit through the aircraft, which is generally not present in a way that would utilize this emf.

Discussion

While the induced emf exists, it's insignificant compared to the voltages used in aircraft systems. This problem illustrates that motional emf in Earth's relatively weak magnetic field produces only small voltages unless the conductor is very long (as in the Tethered Satellite experiment) or moving extremely fast. The calculation confirms that pilots and passengers need not worry about electromagnetic effects from flying through Earth's magnetic field.

Final Answer

(a) 0.630 V

(b) No, this is a very small emf compared to aircraft electrical system voltages and poses no practical consequences.

(a) A nonferrous screwdriver is being used in a 2.00 T magnetic field. What maximum emf can be induced along its 12.0 cm length when it moves at 6.00 m/s? (b) Is it likely that this emf will have any consequences or even be noticed?

Show Solution

Strategy

(a) Use $\text{emf} = B\ell v$ with maximum emf occurring when motion is perpendicular to the field. (b) Assess whether the calculated emf is noticeable.

Solution

(a) Given: $B = 2.00 \text{ T}$, $\ell = 12.0 \text{ cm} = 0.120 \text{ m}$, $v = 6.00 \text{ m/s}$

$$\text{emf} = B\ell v = (2.00 \text{ T})(0.120 \text{ m})(6.00 \text{ m/s}) = 1.44 \text{ V}$$

(b) This emf of 1.44 V is unlikely to be noticed or have practical consequences. It's too small to cause sparking or electrical shock. While it's larger than typical static electricity, the screwdriver's high resistance would limit current flow. A person holding the screwdriver wouldn't feel anything, and it wouldn't affect the tool's operation.

Discussion

The 1.44 V emf calculated in part (a) is significant enough to be measurable with a voltmeter but far too small to pose any safety hazard or practical concern. The strong 2.00 T field used here (typical of MRI machines or research electromagnets) produces this modest voltage even when the screwdriver moves at 6.00 m/s (about 22 km/h). This demonstrates why motional emf effects are negligible in everyday life—even in strong magnetic fields with rapid motion, the induced voltages remain small for hand-held objects.

The screwdriver is specified as “nonferrous” (non-magnetic material like brass, aluminum, or certain stainless steels) to avoid magnetic attraction forces, but the motional emf depends only on conductivity, not magnetic properties. Even if current could flow through the screwdriver, the high resistance of the tool and the person holding it would limit current to microamperes or less—completely imperceptible and harmless.

This problem illustrates the practical safety of working near strong magnetic fields. While safety protocols exist for MRI environments, the motional emf from moving metal tools is not a significant concern compared to projectile hazards from ferromagnetic materials.

Final Answer

(a) 1.44 V; (b) No practical consequences—too small to notice.

At what speed must the sliding rod in [Figure 1] move to produce an emf of 1.00 V in a 1.50 T field, given the rod’s length is 30.0 cm?

Show Solution

Strategy

Rearrange $\text{emf} = B\ell v$ to solve for v.

Solution

Given: $\text{emf} = 1.00\ \text{V}$, $B = 1.50\ \text{T}$, $\ell = 30.0\ \text{cm} = 0.300\ \text{m}$

$$v = \frac{\text{emf}}{B\ell} = \frac{1.00\ \text{V}}{(1.50\ \text{T})(0.300\ \text{m})} = \frac{1.00}{0.450} = 2.22\ \text{m/s}$$

Discussion

This is a moderate speed (about 8 km/h or 5 mph)—roughly walking pace. The relatively strong magnetic field (1.50 T, about 30,000 times Earth’s field) allows a modest velocity to generate 1.00 V, which is sufficient to drive measurable current through a load resistor.

Final Answer

2.22 m/s

The 12.0 cm long rod in [Figure 1] moves at 4.00 m/s. What is the strength of the magnetic field if a 95.0 V emf is induced?

Show Solution

Strategy

Rearrange $\text{emf} = B\ell v$ to solve for B.

Solution

Given: $\ell = 12.0\ \text{cm} = 0.120\ \text{m}$, $v = 4.00\ \text{m/s}$, $\text{emf} = 95.0\ \text{V}$

$$B = \frac{\text{emf}}{\ell v} = \frac{95.0\ \text{V}}{(0.120\ \text{m})(4.00\ \text{m/s})} = \frac{95.0}{0.480} = 198\ \text{T}$$

Discussion

This is an extremely strong magnetic field—about 4 million times stronger than Earth’s field and much stronger than typical laboratory electromagnets (usually 1-2 T). Such fields are only achievable with superconducting magnets or pulsed magnetic field facilities. This demonstrates that generating substantial voltage with a small, slow-moving conductor requires an extraordinarily strong field.

Final Answer

198 T

Prove that when B, ℓ, and v are not mutually perpendicular, motional emf is given by $\text{emf} = B\ell v \sin\theta$. If v is perpendicular to B, then θ is the angle between ℓ and B. If ℓ is perpendicular to B, then θ is the angle between v and B.

Show Solution

Strategy

Start with Faraday’s law and consider the component of either ℓ or v that is perpendicular to B.

Solution

From Faraday’s law, $\text{emf} = \frac{\Delta\Phi}{\Delta t} = B\frac{\Delta A}{\Delta t}$, where ΔA is the change in area.

Case 1: If v is perpendicular to B, then θ is the angle between ℓ and B. The effective length perpendicular to B is $\ell_\perp = \ell\sin\theta$. The area swept out in time Δt is:

$$\Delta A = \ell_\perp v\Delta t = \ell v\sin\theta \cdot \Delta t$$

Therefore:

$$\text{emf} = B\frac{\Delta A}{\Delta t} = B\ell v\sin\theta$$

Case 2: If ℓ is perpendicular to B, then θ is the angle between v and B. The effective velocity perpendicular to B is $v_\perp = v\sin\theta$. The area swept out is:

$$\Delta A = \ell v_\perp \Delta t = \ell v\sin\theta \cdot \Delta t$$

Again:

$$\text{emf} = B\frac{\Delta A}{\Delta t} = B\ell v\sin\theta$$

Discussion

This proves that motional emf depends on the sine of the angle between the vectors. Maximum emf occurs when $\theta = 90°$ (perpendicular configuration), and zero emf when $\theta = 0°$ (parallel configuration). This is consistent with the requirement that the conductor must cut through field lines to generate emf.

Final Answer

Proven: $\text{emf} = B\ell v\sin\theta$ where θ is defined as stated depending on which pair of vectors are perpendicular.

In the August 1992 space shuttle flight, only 250 m of the conducting tether considered in [Example 1] could be let out. A 40.0 V motional emf was generated in the Earth's $5.00 \times 10^{-5}\text{T}$ field, while moving at $7.80 \times 10^{3}\text{m/s}$. What was the angle between the shuttle's velocity and the Earth's field, assuming the conductor was perpendicular to the field?

Show Solution

Strategy

Use $\text{emf} = B\ell v\sin\theta$ and solve for θ, where θ is the angle between v and B.

Solution

Given:

- $\text{emf} = 40.0\ \text{V}$
- $B = 5.00\times10^{-5}\ \text{T}$
- $\ell = 250\ \text{m}$
- $v = 7.80\times10^{3}\ \text{m/s}$

$$\sin\theta = \frac{\text{emf}}{B\ell v} = \frac{40.0}{(5.00\times10^{-5})(250)(7.80\times10^{3})} = \frac{40.0}{97.5} = 0.410$$
$$\theta = \arcsin(0.410) = 24.2°$$

Discussion

The angle of 24.2° means the shuttle's velocity was not perpendicular to Earth's magnetic field, which reduced the induced emf from what it would have been at 90°. If the tether had been perpendicular to both v and B, the maximum possible emf would have been $97.5\ \text{V}$. The actual value of 40.0 V represents about 41% of this maximum, consistent with $\sin 24.2° = 0.410$.

Final Answer

24.2°

Integrated Concepts

Derive an expression for the current in a system like that in [Figure 1], under the following conditions. The resistance between the rails is R , the rails and the moving rod are identical in cross-section A and have the same resistivity ρ . The distance between the rails is l, and the rod moves at constant speed v perpendicular to the uniform field B . At time zero, the moving rod is next to the resistance R.

Show Solution

Strategy

Find the total resistance of the circuit (including the rails and rod) and use Ohm's law with the motional emf.

Solution

The motional emf is: $\text{emf} = B\ell v$

At time t, the rod has moved a distance $x = vt$ from the resistance R. The circuit consists of:

- Resistance R
- Two rail segments each of length x with resistance $R_{\text{rail}} = \frac{\rho x}{A}$ each
- The moving rod of length ℓ with resistance $R_{\text{rod}} = \frac{\rho \ell}{A}$

Total resistance:

$$R_{\text{total}} = R + \frac{2\rho x}{A} + \frac{\rho \ell}{A} = R + \frac{2\rho v t}{A} + \frac{\rho \ell}{A}$$

Current from Ohm's law:

$$I = \frac{\text{emf}}{R_{\text{total}}} = \frac{B\ell v}{R + \frac{2\rho v t}{A} + \frac{\rho \ell}{A}}$$

This can be written as:

$$I = \frac{B\ell v A}{RA + 2\rho v t + \rho \ell}$$

Discussion

The current decreases with time as the rod moves away from R, increasing the total circuit resistance. Initially (at $t = 0$), the current is maximum. As t increases, the resistance from the lengthening rail sections causes the current to decrease.

Final Answer

$I = \frac{B\ell v}{R + \frac{2\rho v t + \rho \ell}{A}}$ or equivalently $I = \frac{B\ell v A}{RA + 2\rho v t + \rho \ell}$

Integrated Concepts

The Tethered Satellite in [Figure 2] has a mass of 525 kg and is at the end of a 20.0 km long, 2.50 mm diameter cable with the tensile strength of steel. (a) How much does the cable stretch if a 100 N force is exerted to pull the satellite in? (Assume the satellite and shuttle are at the same altitude above the Earth.) (b) What is the effective force constant of the cable? (c) How much energy is stored in it when stretched by the 100 N force?

Show Solution

Strategy

(a) Use $\Delta L = \frac{F L_0}{Y A}$ where Y is Young's modulus for steel $(2.0 \times 10^{11}\ \text{N/m}^2)$. (b) Calculate $k = F/\Delta L$. (c) Use $E = \frac{1}{2}k(\Delta L)^2$.

Solution

(a) Given:

- $F = 100\ \text{N}$
- $L_0 = 20.0\ \text{km} = 2.00 \times 10^4\ \text{m}$
- $d = 2.50\ \text{mm}$, so $A = \pi r^2 = \pi(1.25 \times 10^{-3})^2 = 4.91 \times 10^{-6}\ \text{m}^2$
- $Y = 2.0 \times 10^{11}\ \text{N/m}^2$

$$\Delta L = \frac{F L_0}{Y A} = \frac{(100)(2.00 \times 10^4)}{(2.0 \times 10^{11})(4.91 \times 10^{-6})} = \frac{2.00 \times 10^6}{9.82 \times 10^5} = 2.04\ \text{m}$$

(b) <div class="equation"> $k = \frac{F}{\Delta L} = \frac{100\ \text{N}}{2.04\ \text{m}} = 49.0\ \text{N/m}$

(c)

$E = \frac{1}{2}k(\Delta L)^2 = \frac{1}{2}(49.0)(2.04)^2 = 102\ \text{J}$
Discussion The 2.04 m stretch in a 20 km cable is only 0.01% elongation, typical for steel under moderate tension. The very low spring constant (49.0 N/m) reflects the cable's length—it's much easier to stretch a long thin cable than a short thick one. The stored energy of 102 J is modest but could cause a "snap-back" hazard if the cable were suddenly released. **Final Answer** (a) 2.04 m; (b) 49.0 N/m; (c) 102 J

</div>

Integrated Concepts

The Tethered Satellite discussed in this module is producing 5.00 kV, and a current of 10.0 A flows. (a) What magnetic drag force does this produce if the system is moving at 7.80 km/s? (b) How much kinetic energy is removed from the system in 1.00 h, neglecting any change in altitude or velocity during that time? (c) What is the change in velocity if the mass of the system is 100 000 kg? (d) Discuss the long term consequences (say, a week-long mission) on the space shuttle's orbit, noting what effect a decrease in velocity has and assessing the magnitude of the effect.

Show Solution

Strategy

(a) The magnetic drag force on a current-carrying conductor in a magnetic field is $F = I\ell B$, where we need to determine ℓ from the emf and velocity. (b) Power dissipated is $P = \text{emf} \times I$, and energy over time is $E = Pt$. (c) Use the work-energy theorem: the energy removed equals the change in kinetic energy. (d) Extrapolate the velocity change over a week and discuss orbital mechanics implications.

Solution

(a) Given:

- $\text{emf}=5.00\ \text{kV}=5000\ \text{V}$
- $I=10.0\ \text{A}$
- $v=7.80\ \text{km/s}=7.80\times10^{3}\ \text{m/s}$
- From Example 1, $B = 5.00 \times 10^{-5}\ \text{T}$

First, find the length of the tether from $\text{emf} = B\ell v$:

$$\ell=\text{emf}Bv=5000(5.00\times10^{-5})(7.80\times10^{3})=50000.390=1.28\times10^{4}\ \text{m}$$

The magnetic drag force is:

$$F=I\ell B=(10.0)(1.28\times10^{4})(5.00\times10^{-5})=6.40\ \text{N}$$

Wait, let me recalculate. The problem states the tether is 20.0 km from Example 1, so:

$$F=I\ell B=(10.0\ \text{A})(2.0\times10^{4}\ \text{m})(5.00\times10^{-5}\ \text{T})=10.0\ \text{N}$$

(b) Power dissipated:

$$P=\text{emf}\times I=(5000\ \text{V})(10.0\ \text{A})=5.00\times10^{4}\ \text{W}$$

Energy removed in 1.00 hour:

$$E=Pt=(5.00\times10^{4}\ \text{W})(1.00\ \text{h})(3600\ \text{s/h})=1.80\times10^{8}\ \text{J}$$

Actually, let me recalculate using the force and distance. The work done by the drag force equals the kinetic energy removed:

$$E=F\cdot d=F\cdot v\cdot t=(10.0\ \text{N})(7.80\times10^{3}\ \text{m/s})(3600\ \text{s})=2.81\times10^{8}\ \text{J}$$

(c) Using the work-energy theorem, the energy removed equals the change in kinetic energy:

$$E=12m(v_{2i}-v_{2f})\approx mv\Delta v$$

For small changes where $\Delta v \ll v$:

$$\Delta v=Emv=2.81\times10^{8}(1.00\times10^{5})(7.80\times10^{3})=2.81\times10^{8}7.80\times10^{8}=0.360\ \text{m/s}$$

(d) For a week-long mission (168 hours):

$$\Delta v_{\text{week}}=(0.360\ \text{m/s/h})(168\ \text{h})=60.5\ \text{m/s}$$

This represents:

$$\Delta vv=60.57800=0.00776=0.776\%$$

Discussion

Part (a): The magnetic drag force of 10.0 N may seem small, but it acts continuously on the satellite system. This force results from the interaction between the 10.0 A current in the 20.0 km tether and Earth’s magnetic field. The force opposes the satellite’s motion, extracting orbital energy.

Part (b): The energy removal of 2.81×10^{8} J (281 megajoules) in just one hour is substantial—equivalent to about 78 kWh of electrical energy. This demonstrates the potential of the tethered satellite as a power source, though the “fuel” being consumed is the orbital kinetic energy of the shuttle itself. The power of 50 kW (from $P = VI$) could run several households.

Part (c): The velocity decrease of 0.36 m/s per hour seems modest, but it’s significant in orbital mechanics. This confirms that the energy accounting is consistent: the mechanical energy lost (from slowing down) equals the electrical energy generated plus any dissipated heat.

Part (d): Over a week-long mission, the cumulative velocity loss of approximately 60 m/s (about 0.8% of orbital velocity) would have measurable effects on the orbit. A decrease in orbital velocity causes the orbit to decay—the satellite drops to a lower altitude where it moves faster, but with less total energy.

The shuttle would need to fire thrusters periodically to maintain the desired orbit, effectively using rocket fuel to replace the energy extracted by the tether.

This illustrates a fundamental trade-off: the tethered satellite can generate substantial electrical power, but only by “stealing” kinetic energy from the orbit. For short experiments this is acceptable, but continuous operation would require either accepting orbital decay or using additional propulsion. The concept demonstrates electromagnetic energy harvesting but highlights why perpetual orbital power generation faces practical limitations.

Final Answer

(a) 10.0 N

(b) 2.81×10^{8} J

(c) 0.360 m/s

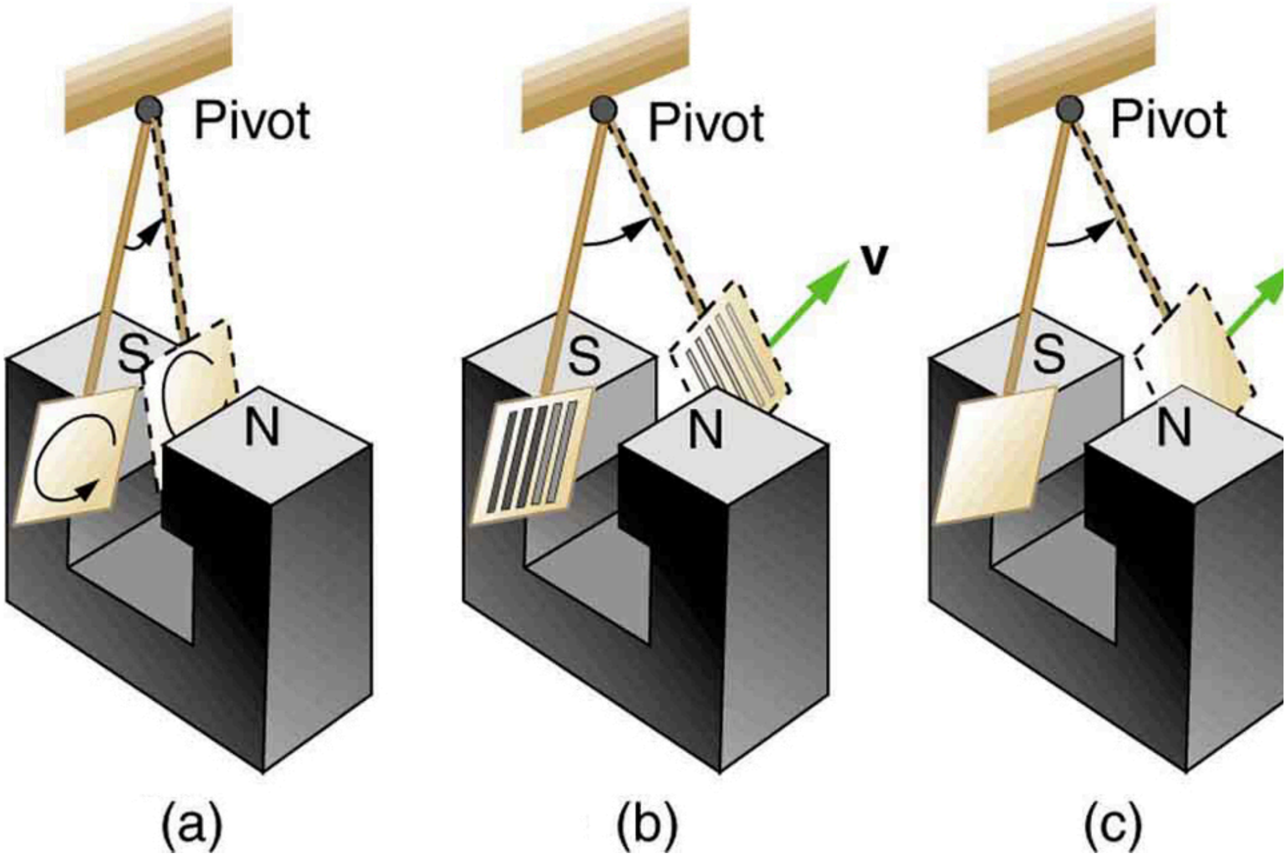
(d) For a week-long mission, $\Delta v \approx 60$ m/s (about 0.8% of orbital velocity). This velocity decrease would cause orbital decay, requiring the shuttle to use additional fuel to maintain altitude. The long-term consequence is that the electrical energy generated comes at the cost of orbital energy, making sustained operation impractical without continuous propulsion to compensate.

Eddy Currents and Magnetic Damping

- Explain the magnitude and direction of an induced eddy current, and the effect this will have on the object it is induced in.
- Describe several applications of magnetic damping.

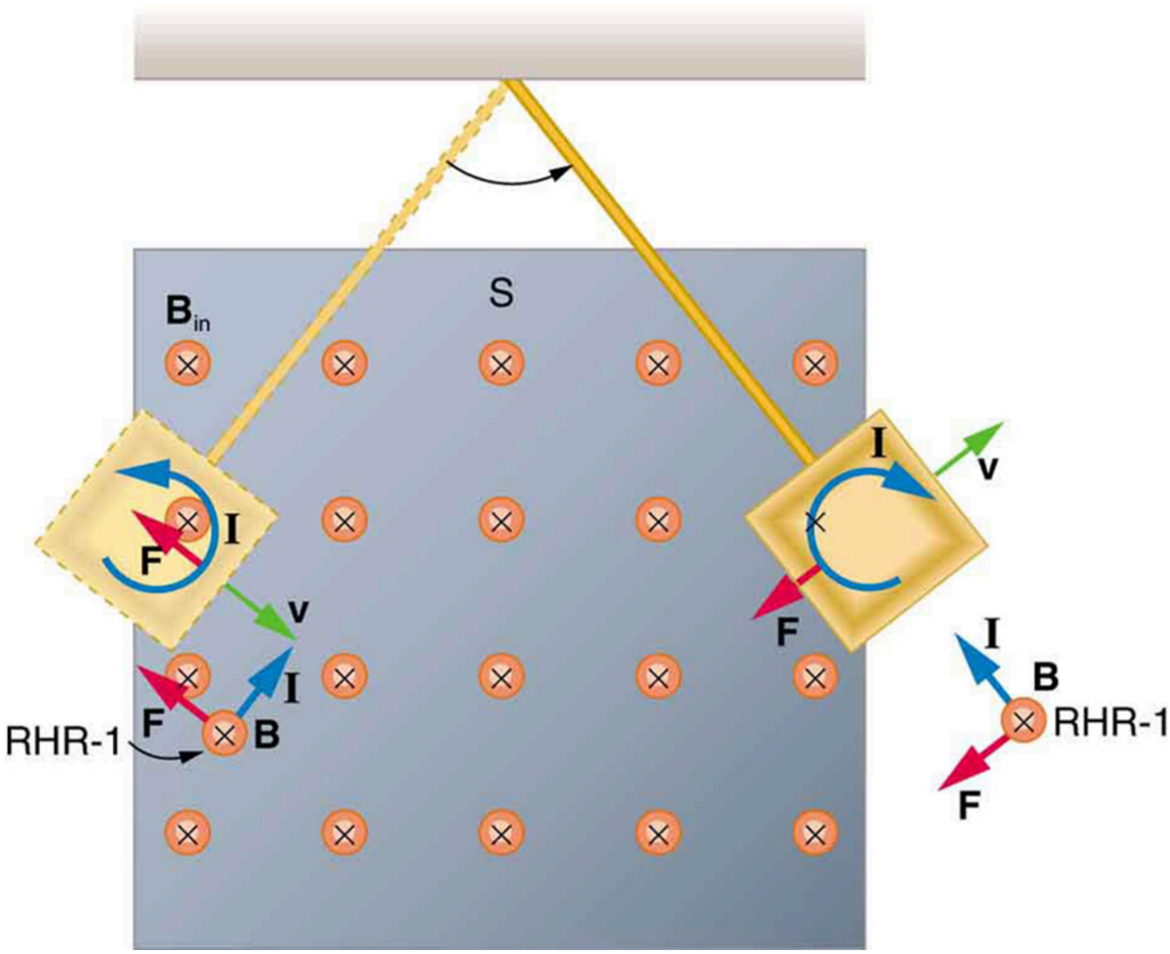
Eddy Currents and Magnetic Damping

As discussed in Motional Emf, motional emf is induced when a conductor moves in a magnetic field or when a magnetic field moves relative to a conductor. If motional emf can cause a current loop in the conductor, we refer to that current as an **eddy current**. Eddy currents can produce significant drag, called **magnetic damping**, on the motion involved. Consider the apparatus shown in [Figure 1], which swings a pendulum bob between the poles of a strong magnet. (This is another favorite physics lab activity.) If the bob is metal, there is significant drag on the bob as it enters and leaves the field, quickly damping the motion. If, however, the bob is a slotted metal plate, as shown in [Figure 1](b), there is a much smaller effect due to the magnet. There is no discernible effect on a bob made of an insulator. Why is there drag in both directions, and are there any uses for magnetic drag?



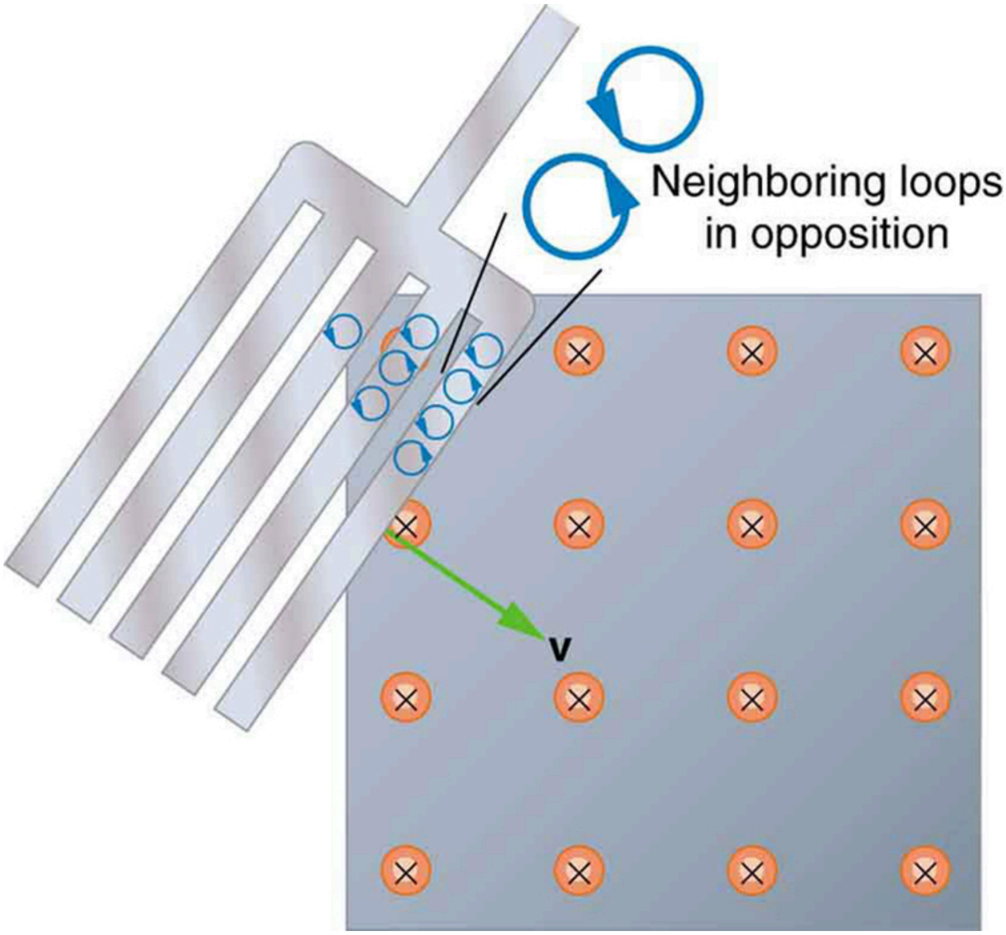
A common physics demonstration device for exploring eddy currents and magnetic damping. (a) The motion of a metal pendulum bob swinging between the poles of a magnet is quickly damped by the action of eddy currents. (b) There is little effect on the motion of a slotted metal bob, implying that eddy currents are made less effective. (c) There is also no magnetic damping on a nonconducting bob, since the eddy currents are extremely small.

[Figure 2] shows what happens to the metal plate as it enters and leaves the magnetic field. In both cases, it experiences a force opposing its motion. As it enters from the left, flux increases, and so an eddy current is set up (Faraday's law) in the counterclockwise direction (Lenz's law), as shown. Only the right-hand side of the current loop is in the field, so that there is an unopposed force on it to the left (RHR-1). When the metal plate is completely inside the field, there is no eddy current if the field is uniform, since the flux remains constant in this region. But when the plate leaves the field on the right, flux decreases, causing an eddy current in the clockwise direction that, again, experiences a force to the left, further slowing the motion. A similar analysis of what happens when the plate swings from the right toward the left shows that its motion is also damped when entering and leaving the field.



A more detailed look at the conducting plate passing between the poles of a magnet. As it enters and leaves the field, the change in flux produces an eddy current. Magnetic force on the current loop opposes the motion. There is no current and no magnetic drag when the plate is completely inside the uniform field.

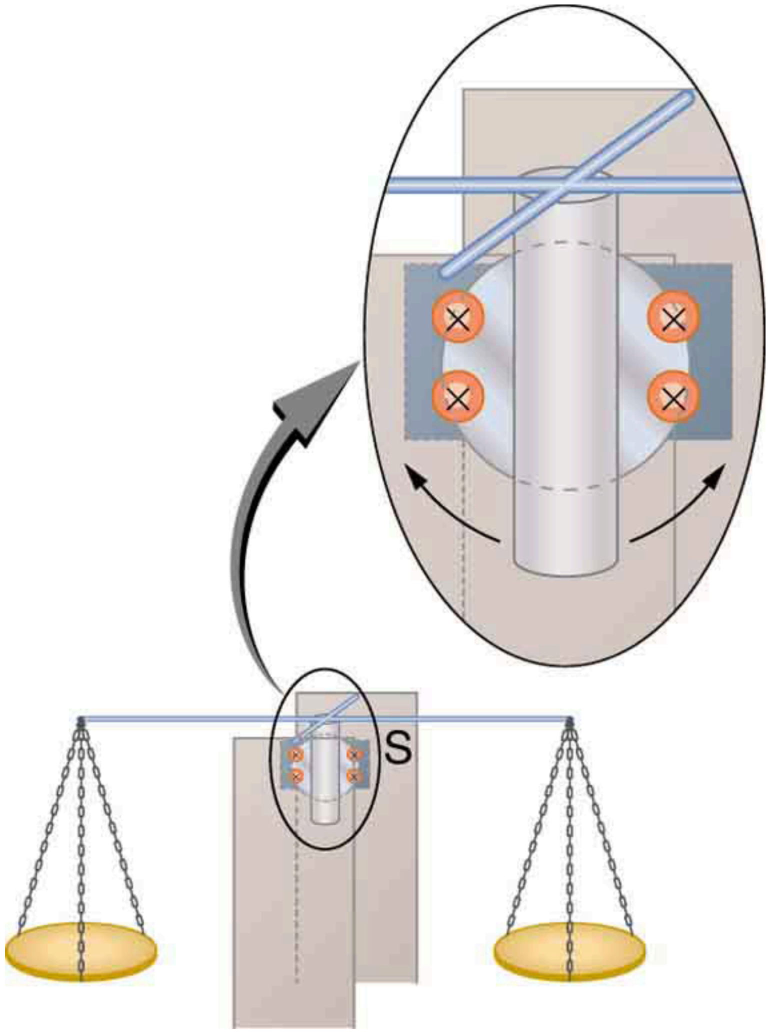
When a slotted metal plate enters the field, as shown in [Figure 3], an emf is induced by the change in flux, but it is less effective because the slots limit the size of the current loops. Moreover, adjacent loops have currents in opposite directions, and their effects cancel. When an insulating material is used, the eddy current is extremely small, and so magnetic damping on insulators is negligible. If eddy currents are to be avoided in conductors, then they can be slotted or constructed of thin layers of conducting material separated by insulating sheets.



Eddy currents induced in a slotted metal plate entering a magnetic field form small loops, and the forces on them tend to cancel, thereby making magnetic drag almost zero.

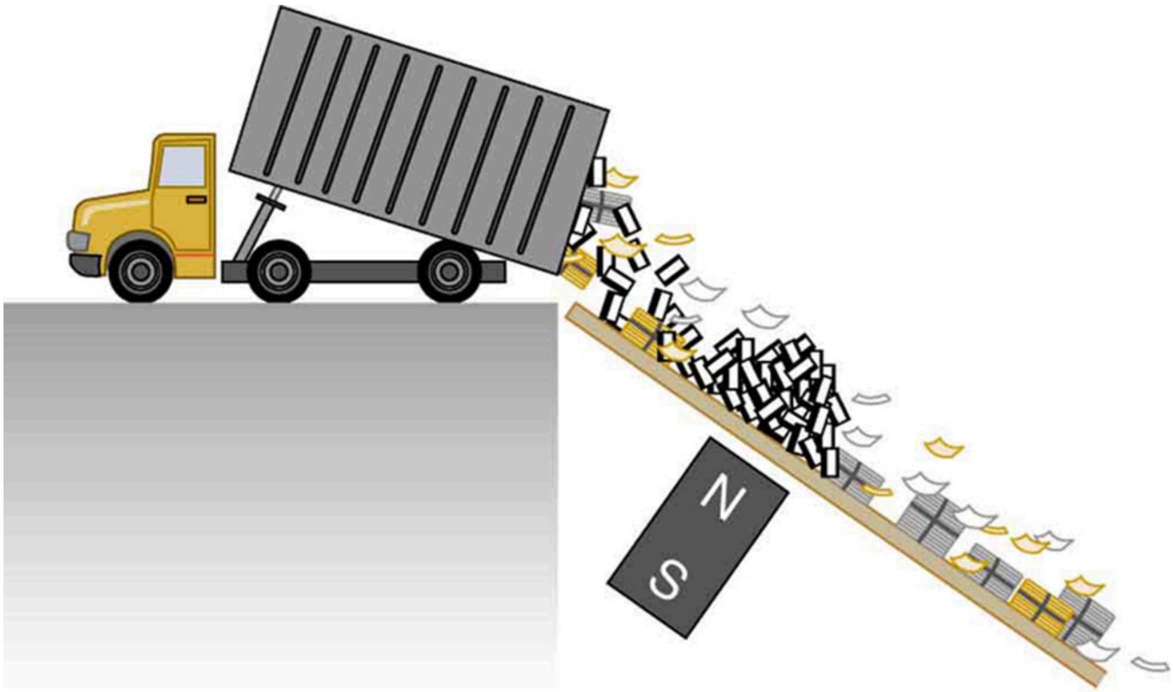
Applications of Magnetic Damping

One use of magnetic damping is found in sensitive laboratory balances. To have maximum sensitivity and accuracy, the balance must be as friction-free as possible. But if it is friction-free, then it will oscillate for a very long time. Magnetic damping is a simple and ideal solution. With magnetic damping, drag is proportional to speed and becomes zero at zero velocity. Thus the oscillations are quickly damped, after which the damping force disappears, allowing the balance to be very sensitive. (See [Figure 4].) In most balances, magnetic damping is accomplished with a conducting disc that rotates in a fixed field.



Magnetic damping of this sensitive balance slows its oscillations. Since Faraday's law of induction gives the greatest effect for the most rapid change, damping is greatest for large oscillations and goes to zero as the motion stops.

Since eddy currents and magnetic damping occur only in conductors, recycling centers can use magnets to separate metals from other materials. Trash is dumped in batches down a ramp, beneath which lies a powerful magnet. Conductors in the trash are slowed by magnetic damping while nonmetals in the trash move on, separating from the metals. (See [Figure 5].) This works for all metals, not just ferromagnetic ones. A magnet can separate out the ferromagnetic materials alone by acting on stationary trash.



Metals can be separated from other trash by magnetic drag. Eddy currents and magnetic drag are created in the metals sent down this ramp by the powerful magnet beneath it. Nonmetals move on.

Other major applications of eddy currents are in metal detectors and braking systems in trains and roller coasters. Portable metal detectors ([Figure 6]) consist of a primary coil carrying an alternating current and a secondary coil in which a current is induced. An eddy current will be induced in a piece of metal close to the detector which will cause a change in the induced current within the secondary coil, leading to some sort of signal like a shrill noise. Braking using eddy currents is safer because factors such as rain do not affect the braking and the braking is smoother. However, eddy currents cannot bring the motion to a complete stop, since the force produced decreases with speed. Thus, speed can be reduced from say 20 m/s to 5 m/s, but another form of braking is needed to completely stop the vehicle. Generally, powerful rare earth magnets such as neodymium magnets are used in roller coasters. [Figure 7] shows rows of magnets in such an application. The vehicle has metal fins (normally containing copper) which pass through the magnetic field slowing the vehicle down in much the same way as with the pendulum bob shown in [Figure 1].

![]Photograph of several soldiers in an open field. One soldier is searching for explosives by scanning the surface using a metal detector.](../resources/Figure_23_04_06.jpg ‘A soldier in Iraq uses a metal detector to search for explosives and weapons. (credit: U.S. Army)’)

The rows of rare earth magnets (protruding horizontally) are used for magnetic braking in roller coasters. (credit: Stefan Scheer, Wikimedia Commons)

Induction cooktops have electromagnets under their surface. The magnetic field is varied rapidly producing eddy currents in the base of the pot, causing the pot and its contents to increase in temperature. Induction cooktops have high efficiencies and good response times but the base of the pot needs to be ferromagnetic, iron or steel for induction to work.

Section Summary

- Current loops induced in moving conductors are called eddy currents.
- They can create significant drag, called magnetic damping.

Conceptual Questions

Explain why magnetic damping might not be effective on an object made of several thin conducting layers separated by insulation.

Explain how electromagnetic induction can be used to detect metals? This technique is particularly important in detecting buried landmines for disposal, geophysical prospecting and at airports.

Problems & Exercises

Make a drawing similar to [Figure 2], but with the pendulum moving in the opposite direction. Then use Faraday's law, Lenz's law, and RHR-1 to show that magnetic force opposes motion.

Show Solution

Strategy

We analyze the plate moving right-to-left, entering and exiting the magnetic field. We apply Faraday's law to find induced emf, Lenz's law for current direction, and RHR-1 for force direction.

Solution

Case 1: Plate entering field from right

The plate moves leftward into the field region:

1. **Flux change**: Magnetic flux into the page is increasing (Faraday's law)
2. **Induced current**: By Lenz's law, the induced current must create a field out of the page to oppose the increase. Using RHR-2, this requires a counterclockwise current (viewed from front)
3. **Force on current**: The right edge of the current loop is in the field. Using RHR-1 with current flowing upward on the right edge and field into page, the force is to the RIGHT
4. **Conclusion**: Force opposes leftward motion ✓

Case 2: Plate exiting field on left

The plate moves leftward out of the field:

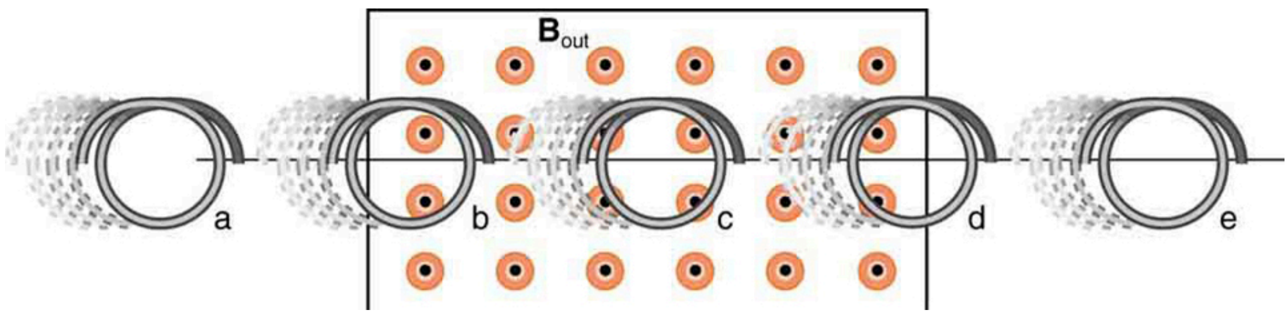
1. **Flux change**: Magnetic flux into the page is decreasing
2. **Induced current**: By Lenz's law, induced current must create a field into the page to oppose the decrease. Using RHR-2, this requires a clockwise current
3. **Force on current**: The left edge of the loop (still in field) has current flowing downward. Using RHR-1 with downward current and field into page, the force is to the RIGHT
4. **Conclusion**: Force opposes leftward motion ✓

Discussion

In both cases—entering and exiting—the magnetic force opposes the plate's motion, demonstrating magnetic damping. This confirms energy conservation: work must be done against the magnetic force to maintain motion.

Final Answer

Drawing analysis shows that whether entering or exiting the field from the right, the magnetic force always points opposite to the velocity (rightward when moving leftward), opposing the motion as required by Lenz's law and energy conservation.



A coil is moved into and out of a region of uniform magnetic field.

A coil is moved through a magnetic field as shown in [Figure 8]. The field is uniform inside the rectangle and zero outside. What is the direction of the induced current and what is the direction of the magnetic force on the coil at each position shown?

Show Solution

Strategy

We apply Lenz's law to find the direction of induced current, and RHR-1 to find the force direction. The magnetic field points out of the page.

Solution

Position (a): Coil outside field (left side)

- **Flux change**: None (flux = 0)
- **Induced current**: None
- **Magnetic force**: None

Position (b): Coil entering field (moving right)

- **Flux change**: Flux out of page is increasing
- **Induced current**: By Lenz's law, induced current creates field into page to oppose increase → **clockwise** (viewed from front)
- **Magnetic force**: Right edge of coil is in field with current flowing downward. By RHR-1 (down current, field out), force is to the **LEFT** (opposes rightward motion)

Position (c): Coil fully inside uniform field

- **Flux change**: None (constant flux)
- **Induced current**: None
- **Magnetic force**: None

Position (d): Coil exiting field (moving right)

- **Flux change**: Flux out of page is decreasing

- **Induced current**: By Lenz's law, induced current creates field out of page to oppose decrease → **counterclockwise** (viewed from front)
- **Magnetic force**: Left edge of coil (still in field) has current flowing upward. By RHR-1 (up current, field out), force is to the **LEFT** (opposes rightward motion)

Position (e): Coil outside field (right side)

- **Flux change**: None (flux = 0)
- **Induced current**: None
- **Magnetic force**: None

Discussion

The pattern demonstrates magnetic damping: forces always oppose motion, requiring work to move the coil through the field. Energy is dissipated as heat in the coil's resistance.

Final Answer

(a) No current, no force; (b) Clockwise current, force to left; (c) No current, no force; (d) Counterclockwise current, force to left; (e) No current, no force.

Glossary

eddy current
: a current loop in a conductor caused by motional emf

magnetic damping
: the drag produced by eddy currents

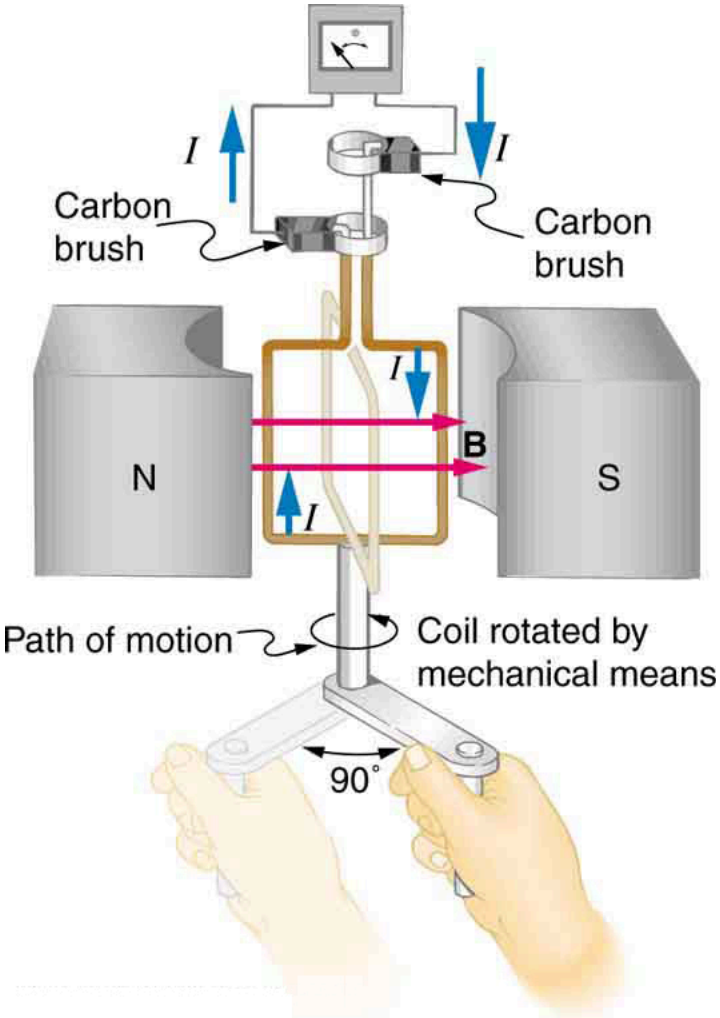
Electric Generators

- Calculate the emf induced in a generator.
- Calculate the peak emf which can be induced in a particular generator system.

Electric generators induce an emf by rotating a coil in a magnetic field, as briefly discussed in Induced Emf and Magnetic Flux. We will now explore generators in more detail. Consider the following example.

Calculating the Emf Induced in a Generator Coil

The generator coil shown in [Figure 1] is rotated through one-fourth of a revolution (from $\theta = 0°$ to $\theta = 90°$) in 15.0 ms. The 200-turn circular coil has a 5.00 cm radius and is in a uniform 1.25 T magnetic field. What is the average emf induced?



When this generator coil is rotated through one-fourth of a revolution, the magnetic flux Φ changes from its maximum to zero, inducing an emf.

Strategy

We use Faraday's law of induction to find the average emf induced over a time Δt :

$$\text{emf} = -N\frac{\Delta\Phi}{\Delta t}.$$

We know that $N = 200$ and $\Delta t = 15.0 \text{ ms}$, and so we must determine the change in flux $\Delta\Phi$ to find emf.

Solution

Since the area of the loop and the magnetic field strength are constant, we see that

$$\Delta\Phi = \Delta(BA\cos\theta) = AB\Delta(\cos\theta).$$

Now, $\Delta(\cos\theta) = -1.0$, since it was given that θ goes from 0° to 90° . Thus $\Delta\Phi = -AB$, and

$$\text{emf} = NAB\Delta t.$$

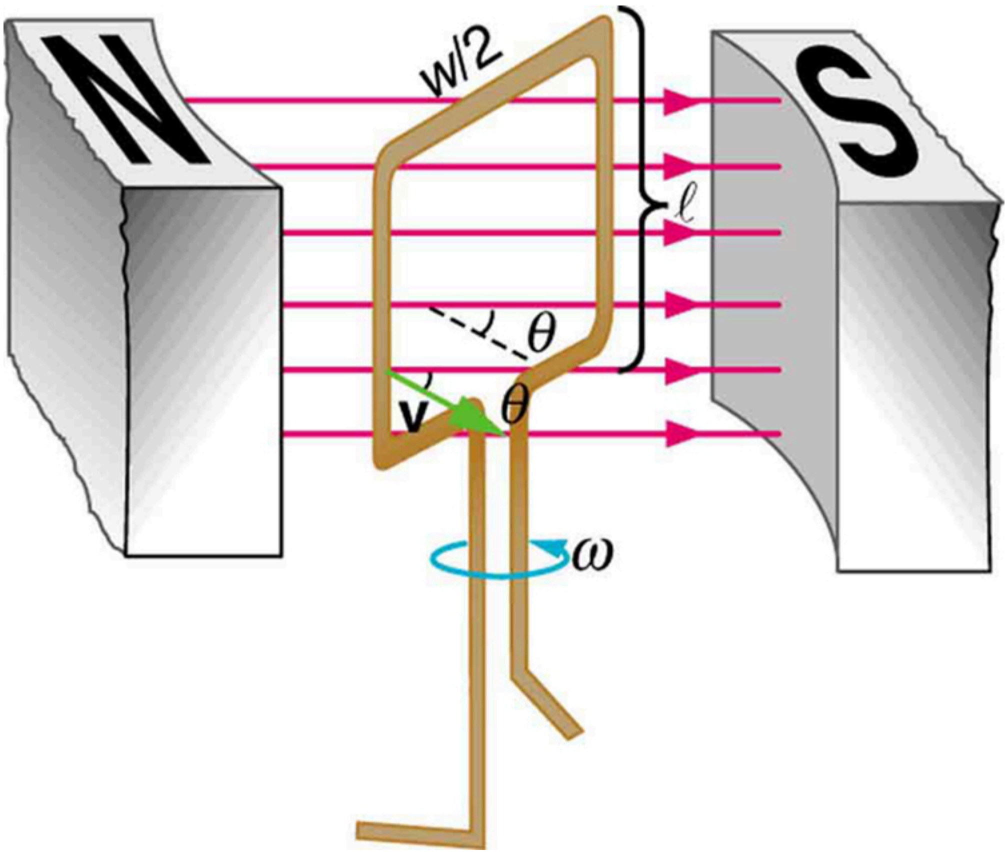
The area of the loop is $A = \pi r^2 = (3.14...)(0.0500\text{m})^2 = 7.85 \times 10^{-3}\text{m}^2$. Entering this value gives

$$\text{emf} = 200(7.85 \times 10^{-3}\text{m}^2)(1.25\text{T})15.0 \times 10^{-3}\text{s} = 131\text{V}.$$

Discussion

This is a practical average value, similar to the 120 V used in household power.

The emf calculated in Example 1 is the average over one-fourth of a revolution. What is the emf at any given instant? It varies with the angle between the magnetic field and a perpendicular to the coil. We can get an expression for emf **as a function of time by considering the motional emf on a rotating rectangular coil of width w and height ℓ in a uniform magnetic field, as illustrated in [Figure 2].



A generator with a single rectangular coil rotated at constant angular velocity in a uniform magnetic field produces an emf that varies sinusoidally in time. Note the generator is similar to a motor, except the shaft is rotated to produce a current rather than the other way around.

Charges in the wires of the loop experience the magnetic force, because they are moving in a magnetic field. Charges in the vertical wires experience forces parallel to the wire, causing currents. But those in the top and bottom segments feel a force perpendicular to the wire, which does not cause a current. We can thus find the induced emf by considering only the side wires. Motional emf is given to be $\text{emf} = B\ell v$, where the velocity *v* is perpendicular to the magnetic field B . Here the velocity is at an angle θ with B , so that its component perpendicular to B is $v\sin\theta$ (see [Figure 2]). Thus in this case the emf induced on each side is $\text{emf} = B\ell v\sin\theta$, and they are in the same direction. The total emf around the loop is then

$$\text{emf} = 2B\ell v\sin\theta.$$

This expression is valid, but it does not give emf as a function of time. To find the time dependence of emf, we assume the coil rotates at a constant angular velocity ω . The angle θ is related to angular velocity by $\theta = \omega t$, so that

$$\text{emf} = 2B\ell v\sin\omega t.$$

Now, linear velocity v is related to angular velocity ω by $v = r\omega$. Here $r = w/2$, so that $v = (w/2)\omega$, and

$$\text{emf} = 2B\ell w2\omega\sin\omega t = (\ell w)B\omega\sin\omega t.$$

Noting that the area of the loop is $A = \ell w$, and allowing for N loops, we find that

$\text{emf} = NAB\omega \sin \omega t$

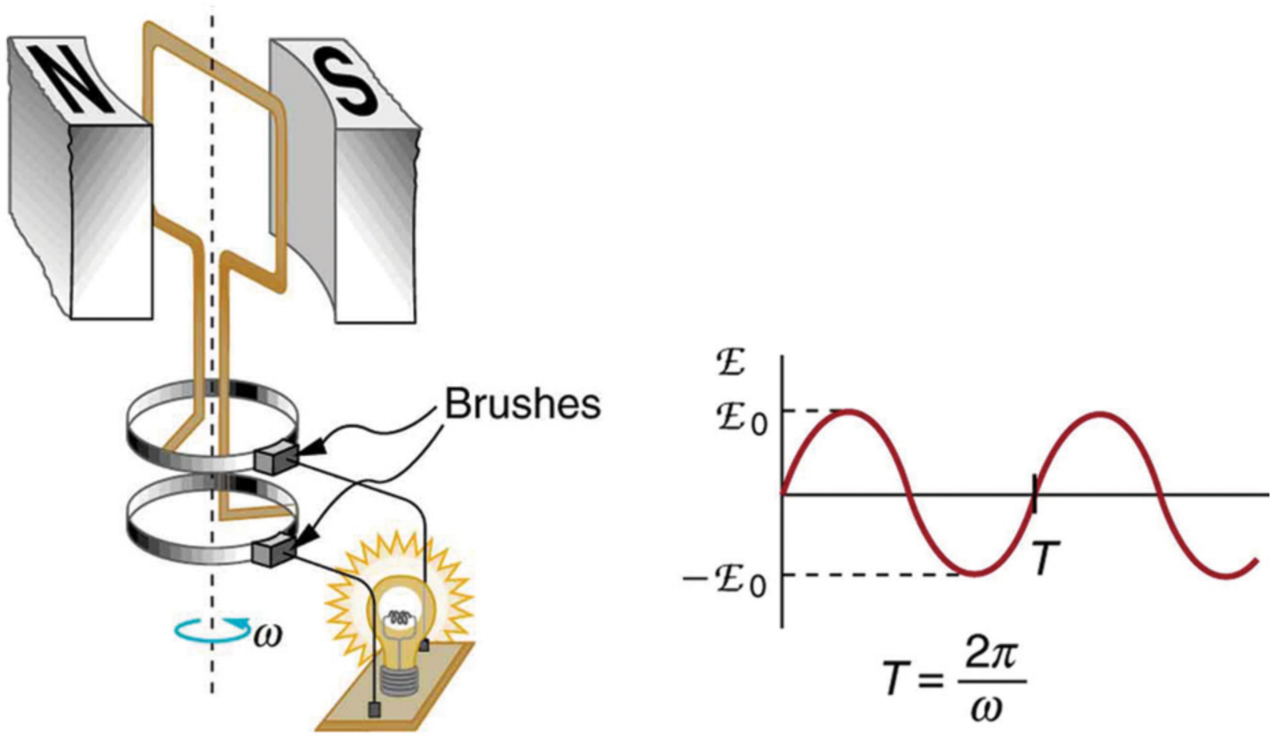
is the **emf induced in a generator coil** of N turns and area A rotating at a constant angular velocity ω in a uniform magnetic field B . This can also be expressed as

$\text{emf} = \text{emf}_0 \sin \omega t,$

where

$\text{emf}_0 = NAB\omega$

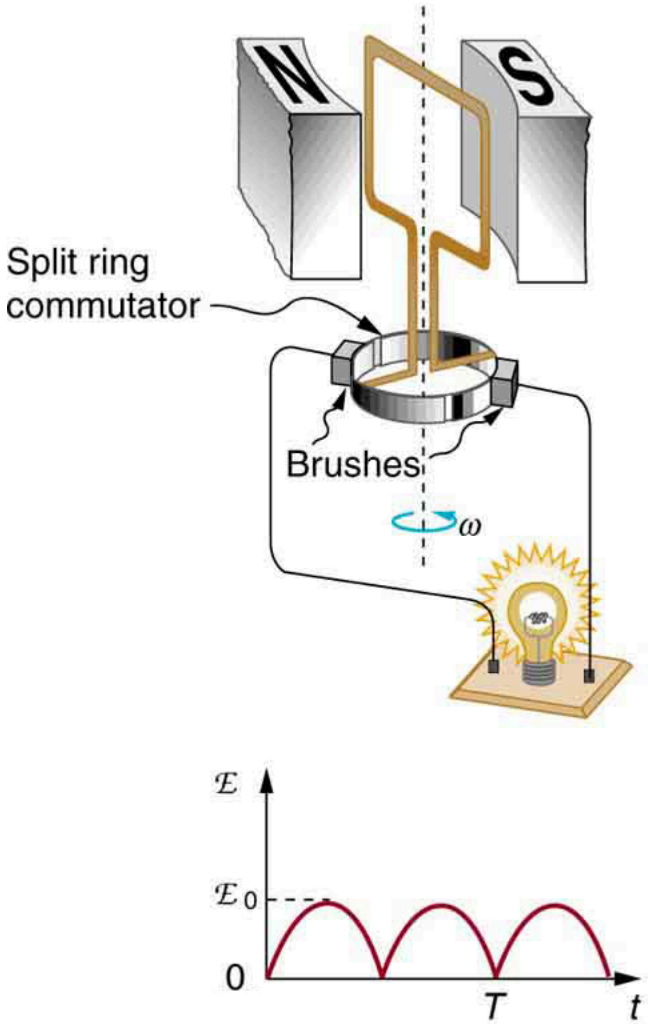
is the maximum **(peak) emf**. Note that the frequency of the oscillation is $f = \omega / 2\pi$, and the period is $T = 1/f = 2\pi / \omega$. [Figure 3] shows a graph of emf as a function of time, and it now seems reasonable that AC voltage is sinusoidal.



The emf of a generator is sent to a light bulb with the system of rings and brushes shown. The graph gives the emf of the generator as a function of time. emf_0 is the peak emf. The period is $T=1/f=2 \pi/\omega$, where f is the frequency. Note that the script E stands for emf.

The fact that the peak emf, $\text{emf}_0 = NAB\omega$, makes good sense. The greater the number of coils, the larger their area, and the stronger the field, the greater the output voltage. It is interesting that the faster the generator is spun (greater ω), the greater the emf. This is noticeable on bicycle generators—at least the cheaper varieties. One of the authors as a juvenile found it amusing to ride his bicycle fast enough to burn out his lights, until he had to ride home lightless one dark night.

[Figure 4] shows a scheme by which a generator can be made to produce pulsed DC. More elaborate arrangements of multiple coils and split rings can produce smoother DC, although electronic rather than mechanical means are usually used to make ripple-free DC.



Split rings, called commutators, produce a pulsed DC emf output in this configuration.

Calculating the Maximum Emf of a Generator

Calculate the maximum emf, emf_0 , of the generator that was the subject of [Example 1].

Strategy

Once ω , the angular velocity, is determined, $\text{emf}_0 = NAB\omega$ can be used to find emf_0 . All other quantities are known.

Solution

Angular velocity is defined to be the change in angle per unit time:

$$\omega = \frac{\Delta\theta}{\Delta t}.$$

One-fourth of a revolution is $\pi/2$ radians, and the time is 0.0150 s; thus,

$$\omega = \frac{\pi/2 \text{ rad}}{0.0150 \text{ s}} = 104.7 \text{ rad/s}.$$

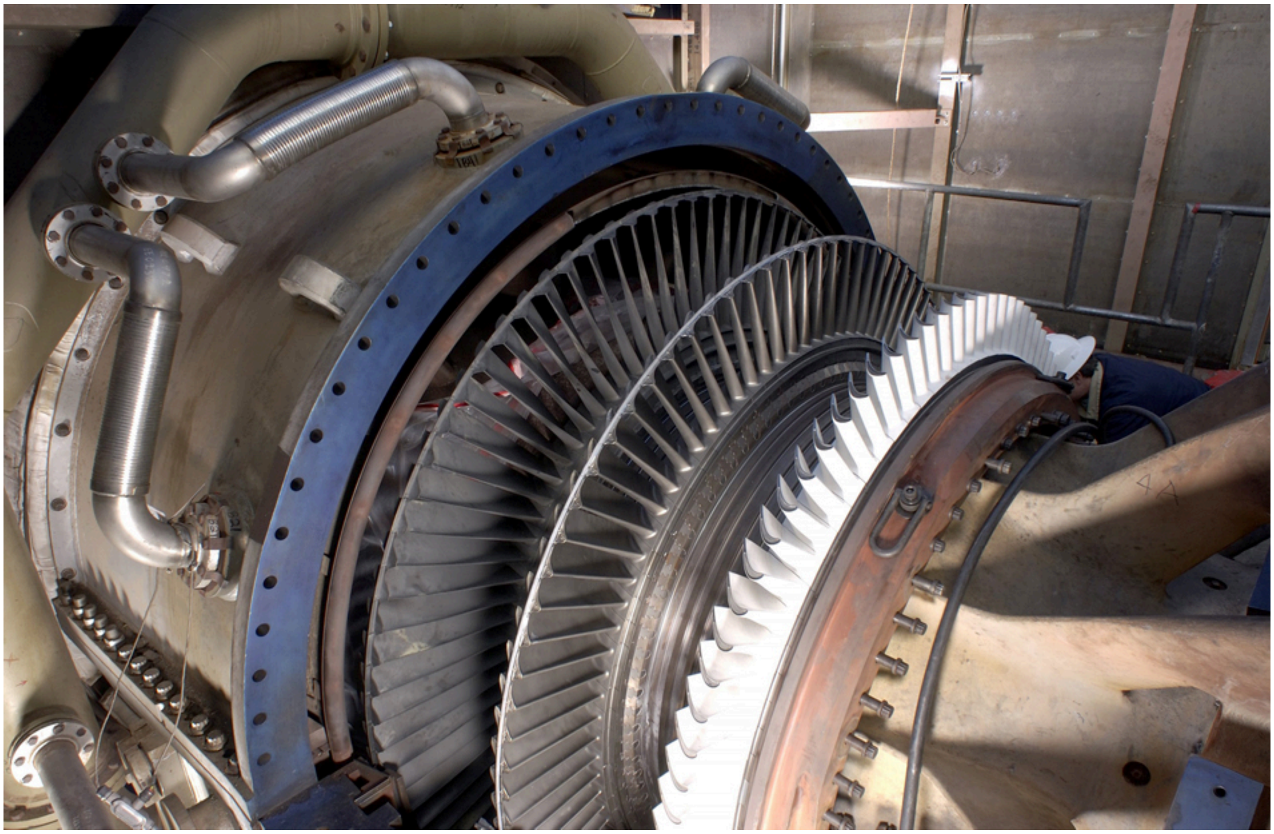
104.7 rad/s is exactly 1000 rpm. We substitute this value for ω and the information from the previous example into $\text{emf}_0 = NAB\omega$, yielding

$$\text{emf}_0 = NAB\omega = 200(7.85\times10^{-3}\text{ m}^2)(1.25\text{ T})(104.7\text{ rad/s}) = 206\text{ V}.$$

Discussion

The maximum emf is greater than the average emf of 131 V found in the previous example, as it should be.

In real life, electric generators look a lot different than the figures in this section, but the principles are the same. The source of mechanical energy that turns the coil can be falling water (hydropower), steam produced by the burning of fossil fuels, or the kinetic energy of wind. [Figure 5] shows a cutaway view of a steam turbine; steam moves over the blades connected to the shaft, which rotates the coil within the generator.



Steam turbine/generator. The steam produced by burning coal impacts the turbine blades, turning the shaft which is connected to the generator. (credit: Nabonaco, Wikimedia Commons)

Generators illustrated in this section look very much like the motors illustrated previously. This is not coincidental. In fact, a motor becomes a generator when its shaft rotates. Certain early automobiles used their starter motor as a generator. In Back Emf, we shall further explore the action of a motor as a generator.

Section Summary

- An electric generator rotates a coil in a magnetic field, inducing an emf given as a function of time by

$$\text{emf} = NAB\omega \sin \omega t,$$

where A is the area of an N -turn coil rotated at a constant angular velocity ω in a uniform magnetic field B .

- The peak emf emf_0 of a generator is

$$\text{emf}_0 = NAB\omega.$$

Conceptual Questions

Using RHR-1, show that the emfs in the sides of the generator loop in [Figure 4] are in the same sense and thus add.

The source of a generator's electrical energy output is the work done to turn its coils. How is the work needed to turn the generator related to Lenz's law?

Problems & Exercises

Calculate the peak voltage of a generator that rotates its 200-turn, 0.100 m diameter coil at 3600 rpm in a 0.800 T field.

Show Solution

Strategy

We use the peak emf formula $\text{emf}_0 = NAB\omega$. First, we need to find the area of the circular coil and convert the angular velocity from rpm to rad/s.

Solution

The radius of the coil is:

$$r=\frac{d}{2}=\frac{0.100}{2}=0.0500\text{ m}$$

The area is:

$$A=\pi r^2=\pi(0.0500)^2=7.85\times10^{-3}\text{ m}^2$$

Convert angular velocity from rpm to rad/s:

$$\omega=3600\text{ rpm}\times\frac{2\pi\text{ rad}}{1\text{ rev}}\times\frac{1\text{ min}}{60\text{ s}}=\frac{3600\times2\pi}{60}=377\text{ rad/s}$$

Now calculate the peak emf:

$$\text{emf}_0 = NAB\omega = (200)(7.85\times10^{-3})(0.800)(377) = 474\text{ V}$$

Discussion

This is a substantial voltage, typical of industrial generators. The high rotation rate (3600 rpm is 60 Hz, a standard power frequency) and strong magnetic field combine to produce a practical output voltage.

Answer

474 V

At what angular velocity in rpm will the peak voltage of a generator be 480 V, if its 500-turn, 8.00 cm diameter coil rotates in a 0.250 T field?

Show Solution

Strategy

We use $\text{emf}_0 = NAB\omega$ and solve for ω, then convert from rad/s to rpm.

Solution

The area of the coil is:

$$A=\pi r^2=\pi(0.0400)^2=5.03\times10^{-3}\text{ m}^2$$

From $\text{emf}_0 = NAB\omega$:

$$\omega=\frac{\text{emf}_0}{NAB}=\frac{480}{(500)(5.03\times10^{-3})(0.250)}=\frac{480}{0.629}=763\text{ rad/s}$$

Converting to rpm:

$$\omega=763\text{ rad/s}\times\frac{60\text{ s}}{2\pi\text{ rad/rev}}=\frac{45780}{6.283}=7.29\times10^3\text{ rpm}$$

Discussion

This is a very high rotation rate (over 7000 rpm), typical of some high-speed generators but requiring careful engineering to handle the mechanical stresses.

Final Answer

The required angular velocity is 7290 rpm or 7.29×10^3 rpm.

What is the peak emf generated by rotating a 1000-turn, 20.0 cm diameter coil in the Earth's $5.00\times10^{-5}\text{T}$ magnetic field, given the plane of the coil is originally perpendicular to the Earth's field and is rotated to be parallel to the field in 10.0 ms?

Show Solution

Strategy

We need to find ω from the rotation information (perpendicular to parallel in 10.0 ms is one-quarter revolution), then use $\text{emf}_0 = NAB\omega$.

Solution

The radius of the coil is:

$$r=\frac{d}{2}=\frac{0.200}{2}=0.100\text{ m}$$

The area is:

$$A=\pi r^2=\pi(0.100)^2=3.14\times10^{-2}\text{ m}^2$$

One-quarter revolution is $\pi/2$ radians in 10.0 ms:

$$\omega=\frac{\Delta\theta}{\Delta t}=\frac{\pi/2}{10.0\times10^{-3}}=\frac{1.571}{0.0100}=157 \text{ rad/s}$$

Calculate the peak emf:

$$\text{emf}_0 = NAB\omega = (1000)(3.14\times10^{-2})(5.00\times10^{-5})(157) = 0.247 \text{ V}$$

Discussion

The Earth's magnetic field is very weak compared to fields in typical generators, so even with 1000 turns and a relatively large coil, the output voltage is only about a quarter volt. This demonstrates why practical generators require much stronger magnetic fields.

Answer

0.247 V

What is the peak emf generated by a 0.250 m radius, 500-turn coil is rotated one-fourth of a revolution in 4.17 ms, originally having its plane perpendicular to a uniform magnetic field. (This is 60 rev/s.)

Show Solution

Strategy

This problem refers to a coil setup from a previous problem in the Faraday's law chapter where a similar coil with B = 0.425 T was used. We'll use $\text{emf}_0 = NAB\omega$ with that field strength. First, we calculate the area and angular velocity.

Solution

The area of the coil is:

$$A=\pi r^2=\pi(0.250)^2=0.196 \text{ m}^2$$

The angular velocity can be found from the frequency. Since one-fourth revolution occurs in 4.17 ms, one complete revolution takes 4 × 4.17 = 16.68 ms, giving f = 60.0 Hz:

$$\omega=2\pi f=2\pi(60.0)=377 \text{ rad/s}$$

Using the magnetic field from the related problem (B = 0.425 T):

$$\text{emf}_0 = NAB\omega = (500)(0.196)(0.425)(377) = 1.57\times10^4 \text{ V} = 15.7 \text{ kV}$$

Discussion

This is a very high voltage, appropriate for power generation at 60 Hz. The peak emf is $\pi/2$ times larger than the average emf during a quarter-revolution (which was 10.0 kV in the related problem), consistent with the sinusoidal variation of the induced emf.

Answer

15.7 kV

(a) A bicycle generator rotates at 1875 rad/s, producing an 18.0 V peak emf. It has a 1.00 by 3.00 cm rectangular coil in a 0.640 T field. How many turns are in the coil? (b) Is this number of turns of wire practical for a 1.00 by 3.00 cm coil?

Show Solution

Strategy

We use $\text{emf}_0 = NAB\omega$ and solve for N. The area of the rectangular coil is length times width.

Solution for (a)

The area of the rectangular coil is:

$$A=(0.0100)(0.0300)=3.00\times10^{-4} \text{ m}^2$$

From $\text{emf}_0 = NAB\omega$, we solve for N:

$$N=\frac{\text{emf}_0}{AB\omega}=\frac{18.0}{(3.00\times10^{-4})(0.640)(1875)}$$
$$N=\frac{18.0}{0.360}=50 \text{ turns}$$

Solution for (b)

For a rectangular coil of dimensions 1.00 cm by 3.00 cm, the perimeter is:

$$P=2(1.00+3.00)=8.00\text{ cm}$$

With 50 turns, the total length of wire is:

$$L=50\times0.0800=4.00\text{ m}$$

This is a reasonable length of wire. If we use 0.5 mm diameter wire (typical for small coils), 50 turns would fit in a thickness of 25 mm = 2.5 cm, which could be accommodated in a compact bicycle generator housing.

Discussion

Part (a) shows that 50 turns is needed to produce 18 V at this rotation rate. Part (b) confirms this is practical - 50 turns of thin wire in an 8 cm perimeter coil requires only about 4 meters of wire and would fit in a small space. Bicycle generators commonly use coils with this number of turns.

Answer

(a) 50 turns; (b) Yes, this is practical for a compact bicycle generator.

Integrated Concepts

This problem refers to the bicycle generator considered in the previous problem. It is driven by a 1.60 cm diameter wheel that rolls on the outside rim of the bicycle tire. (a) What is the velocity of the bicycle if the generator's angular velocity is 1875 rad/s? (b) What is the maximum emf of the generator when the bicycle moves at 10.0 m/s, noting that it was 18.0 V under the original conditions? (c) If the sophisticated generator can vary its own magnetic field, what field strength will it need at 5.00 m/s to produce a 9.00 V maximum emf?

Show Solution

Strategy

The generator wheel rolls on the bicycle tire, so $v=r\omega$ where r is the wheel radius. The emf is proportional to ω, which is proportional to bicycle speed.

Solution for (a)

$$v=r\omega=\frac{d}{2}\omega=\frac{0.0160}{2}(1875)=(0.00800)(1875)=15.0\text{ m/s}$$

Solution for (b)

Since emf is proportional to ω and ω is proportional to v:

$$\frac{\text{emf}_{\text{new}}}{\text{emf}_{\text{old}}}=\frac{v_{\text{new}}}{v_{\text{old}}}$$
$$\text{emf}_{\text{new}}=\text{emf}_{\text{old}}\times\frac{v_{\text{new}}}{v_{\text{old}}}=18.0\times\frac{10.0}{15.0}=12.0\text{ V}$$

Solution for (c)

At 5.00 m/s, the emf without field adjustment would be:

$$\text{emf}_{5.00}=18.0\times\frac{5.00}{15.0}=6.00\text{ V}$$

To get 9.00 V, the field must be increased by:

$$\frac{B_{\text{new}}}{B_{\text{old}}}=\frac{9.00}{6.00}=1.50$$

From the previous problem, $B_{\text{old}}=0.640\text{ T}$:

$$B_{\text{new}}=1.50\times0.640=0.960\text{ T}$$

Discussion

Part (a) shows the bicycle moves at 15.0 m/s (54 km/h) when the generator spins at 1875 rad/s. Part (b) shows slower speeds produce lower emf. Part (c) demonstrates that increasing the magnetic field can compensate for lower rotation speeds.

Final Answer

(a) 15.0 m/s; (b) 12.0 V; (c) 0.960 T.

(a) A car generator turns at 400 rpm when the engine is idling. Its 300-turn, 5.00 by 8.00 cm rectangular coil rotates in an adjustable magnetic field so that it can produce sufficient voltage even at low rpms. What is the field strength needed to produce a 24.0 V peak emf? (b) Discuss how this required field strength compares to those available in permanent and electromagnets.

Show Solution

Strategy

We use $\text{emf}_0 = NAB\omega$ and solve for B. First, we need to convert rpm to rad/s and calculate the coil area.

Solution for (a)

The area of the rectangular coil is:

$$A=(0.0500)(0.0800)=4.00\times10^{-3}\text{ m}^2$$

Convert angular velocity from rpm to rad/s:

$$\omega=400\text{ rpm}\times\frac{2\pi\text{ rad}}{1\text{ rev}}\times\frac{1\text{ min}}{60\text{ s}}=\frac{800\pi}{60}=41.9\text{ rad/s}$$

From $\text{emf}_0 = NAB\omega$, solve for B:

$$B=\frac{\text{emf}_0}{NA\omega}=\frac{24.0}{(300)(4.00\times10^{-3})(41.9)}$$
$$B=\frac{24.0}{50.3}=0.477\text{ T}$$

Solution for (b)

Modern permanent magnets made from neodymium-iron-boron (NdFeB) or samarium-cobalt can produce fields of 1.0-1.4 T. Electromagnets can easily produce fields from 0.5 T to over 2 T with moderate current. The required field of 0.477 T is well within the capability of both types of magnets.

Discussion

The relatively low field strength needed (about 0.5 T) makes this practical for automotive applications. Car alternators typically use electromagnets (with field windings) rather than permanent magnets, because the field strength can be adjusted electronically to regulate output voltage as engine speed varies. At higher rpms, the field can be reduced to maintain constant voltage output.

Answer

(a) 0.477 T; (b) This field strength is readily achievable with either permanent magnets or electromagnets, making it practical for automotive generators.

Show that if a coil rotates at an angular velocity ω, the period of its AC output is $2\pi/\omega$.

Show Solution

Strategy

The emf varies as $\text{emf} = \text{emf}_0 \sin(\omega t)$. The period T is the time for one complete cycle, when the argument of sine increases by 2π.

Solution

The emf from a rotating coil is:

$$\text{emf}(t)=\text{emf}_0\sin(\omega t)$$

For one complete cycle, the sine function must go through 2π radians:

$$\omega T=2\pi$$

Solving for T:

$$T=\frac{2\pi}{\omega}$$

Alternatively, the coil rotates through angle $\theta = \omega t$. For one complete revolution, $\theta = 2\pi$:

$$2\pi=\omega T$$
$$T=\frac{2\pi}{\omega}$$

Discussion

This result connects angular velocity to period. Since frequency $f = 1/T$, we also have $f = \omega/(2\pi)$ or $\omega = 2\pi f$, the familiar relationship between angular frequency and frequency.

Final Answer

Proven: The period is $T = 2\pi/\omega$.

A 75-turn, 10.0 cm diameter coil rotates at an angular velocity of 8.00 rad/s in a 1.25 T field, starting with the plane of the coil parallel to the field. (a) What is the peak emf? (b) At what time is the peak emf first reached? (c) At what time is the emf first at its most negative? (d) What is the period of the AC voltage output?

Show Solution

Strategy

We use $\text{emf}_0 = NAB\omega$ for peak emf. The emf varies as $\text{emf} = \text{emf}_0 \sin(\omega t)$ when starting with the plane parallel to the field (maximum flux change rate at t = 0). The period is $T = 2\pi/\omega$.

Solution for (a)

The radius and area of the coil are:

$$r = \frac{d}{2} = \frac{0.100}{2} = 0.0500\text{ m}$$
$$A = \pi r^2 = \pi(0.0500)^2 = 7.85\times10^{-3}\text{ m}^2$$

The peak emf is:

$$\text{emf}_0 = NAB\omega = (75)(7.85\times10^{-3})(1.25)(8.00) = 5.89\text{ V}$$

Solution for (b)

Since the coil starts with its plane parallel to the field, the flux through it is zero and the rate of flux change is maximum. This means the emf starts at its maximum value at t = 0.

Solution for (c)

Since the coil starts with its plane parallel to the field and has maximum emf at t = 0, the emf varies as $\text{emf} = \text{emf}_0 \cos(\omega t)$. The most negative value occurs when $\cos(\omega t) = -1$, which happens when $\omega t = \pi$:

$$t = \frac{\pi}{\omega} = \frac{\pi}{8.00} = \frac{3.142}{8.00} = 0.393\text{ s}$$

Solution for (d)

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{8.00} = \frac{6.283}{8.00} = 0.785\text{ s}$$

Discussion

Starting with the plane parallel to the field means maximum flux change rate at t = 0, giving maximum emf immediately. The emf follows $\text{emf} = \text{emf}_0 \cos(\omega t)$, reaching its most negative value after half a period. The relatively slow rotation (8 rad/s) gives a period of nearly 0.8 seconds.

Answer

(a) 5.89 V; (b) t = 0; (c) 0.393 s; (d) 0.785 s

(a) If the emf of a coil rotating in a magnetic field is zero at $t = 0$, and increases to its first peak at $t = 0.100\text{ms}$, what is the angular velocity of the coil? (b) At what time will its next maximum occur? (c) What is the period of the output? (d) When is the output first one-fourth of its maximum? (e) When is it next one-fourth of its maximum?

Show Solution

Strategy

The emf is $\text{emf} = \text{emf}_0 \sin(\omega t)$. Starting at zero, it reaches maximum when $\sin(\omega t) = 1$, which occurs when $\omega t = \pi/2$.

Solution for (a)

At the first peak, $\omega t = \pi/2$:

$$\omega = \frac{\pi/2}{t} = \frac{\pi/2}{0.100\times10^{-3}} = \frac{1.571}{10^{-4}} = 1.57\times10^{4}\text{ rad/s}$$

Solution for (b)

The next maximum occurs when $\omega t = \pi/2 + 2\pi = 5\pi/2$:

$$t = \frac{5\pi/2}{\omega} = \frac{5\pi/2}{1.57\times10^{4}} = \frac{7.854}{1.57\times10^{4}} = 5.00\times10^{-4}\text{ s} = 0.500\text{ ms}$$

Solution for (c)

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{1.57\times10^{4}} = 4.00\times10^{-4}\text{ s} = 0.400\text{ ms}$$

Solution for (d)

When $\text{emf} = \text{emf}_0/4$:

$\sin(\omega t)=14$
$\omega t=\sin^{-1}(0.25)=0.2527\text{ rad}$
$t=0.25271.57\times10^{4}=1.61\times10^{-5}\text{ s}=0.0161\text{ ms}=16.1\text{ μs}$

Solution for (e)

The next time occurs on the downslope at $\omega t=\pi-0.2527=2.889\text{ rad}$:

$t=2.8891.57\times10^{4}=1.84\times10^{-4}\text{ s}=0.184\text{ ms}$

Discussion

The coil rotates very rapidly at 15,700 rad/s (about 2500 Hz). The periodic nature of the output is clearly shown by these calculations.

Final Answer

(a) 1.57×10^{4} rad/s; (b) 0.500 ms; (c) 0.400 ms; (d) 16.1 μs; (e) 0.184 ms.

Unreasonable Results

A 500-turn coil with a 0.250m^2 area is spun in the Earth's $5.00\times10^{-5}\text{T}$ field, producing a 12.0 kV maximum emf. (a) At what angular velocity must the coil be spun? (b) What is unreasonable about this result? (c) Which assumption or premise is responsible?

Show Solution

Strategy

We use $\text{emf}_0=NAB\omega$ and solve for ω. Then we evaluate whether this angular velocity is physically reasonable for a mechanical system.

Solution for (a)

From $\text{emf}_0=NAB\omega$, solve for ω:

$\omega=\text{emf}_0 NAB=12.0\times10^{3}(500)(0.250)(5.00\times10^{-5})$
$\omega=120006.25\times10^{-3}=1.92\times10^{6}\text{ rad/s}$

Solution for (b)

This angular velocity is extremely unreasonable. Converting to rpm:

$\omega=1.92\times10^{6}\text{ rad/s}\times60\text{ s/min}2\pi\text{ rad/rev}=1.83\times10^{7}\text{ rpm}$

This is over 18 million rpm! For comparison:

- High-speed turbines operate at ~30,000 rpm
- Jet engine turbines reach ~50,000 rpm
- Even ultracentrifuges only reach ~150,000 rpm
- At the rim of a 0.5 m diameter coil, the linear velocity would be $v=r\omega=(0.25)(1.92\times10^{6})=4.8\times10^{5}$ m/s, which exceeds the speed of light divided by 600!

This is far beyond what any mechanical system can achieve without disintegrating due to centrifugal forces.

Solution for (c)

The unreasonable assumption is that 12.0 kV can be generated using the Earth's weak magnetic field. The Earth's field (5.00×10^{-5} T) is about 10,000 times weaker than typical generator fields (0.5-1.5 T). To produce significant voltages with such a weak field would require impossibly high rotation rates or an impractically large coil with many turns.

Discussion

This problem illustrates why practical generators use strong artificial magnetic fields rather than the Earth's field. The Earth's field is useful for compasses and navigation but far too weak for power generation. Realistic generators with fields of ~1 T operating at ~3600 rpm (60 Hz) produce hundreds of volts - a reasonable balance of mechanical and electromagnetic parameters.

Answer

(a) 1.92×10^{6} rad/s; (b) This is unreasonably high - over 18 million rpm, which would destroy any mechanical system and give rim speeds approaching the speed of light; (c) The assumption that 12.0 kV can be obtained from Earth's weak magnetic field is unreasonable.

Glossary

electric generator
: a device for converting mechanical work into electric energy; it induces an emf by rotating a coil in a magnetic field

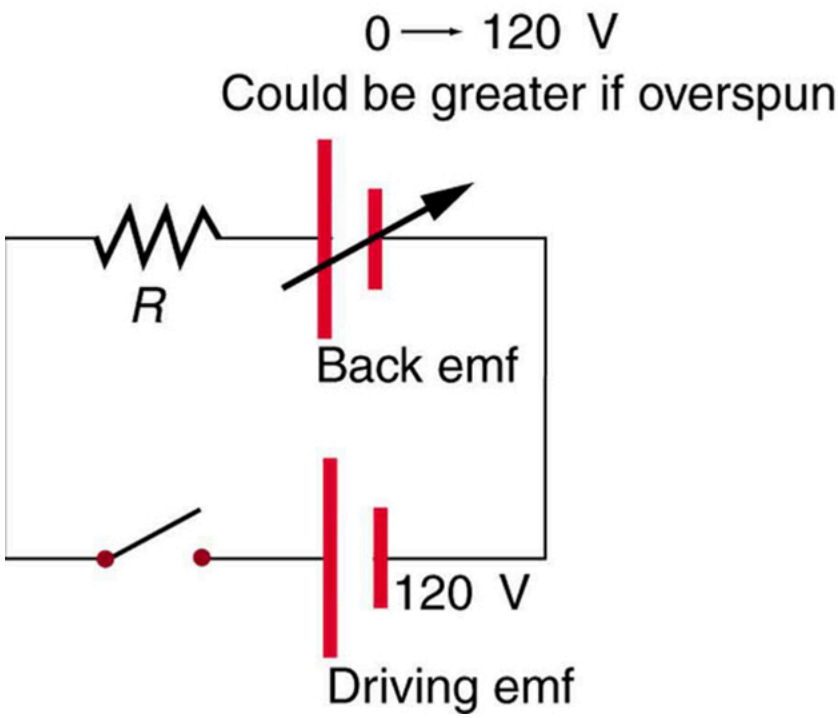
emf induced in a generator coil
: $\text{emf} = NAB\omega \sin \omega t$, where A is the area of an N -turn coil rotated at a constant angular velocity ω in a uniform magnetic field B , over a period of time t

peak emf
: $\text{emf}_0 = NAB\omega$

Back Emf

- Explain what back emf is and how it is induced.

It has been noted that motors and generators are very similar. Generators convert mechanical energy into electrical energy, whereas motors convert electrical energy into mechanical energy. Furthermore, motors and generators have the same construction. When the coil of a motor is turned, magnetic flux changes, and an emf (consistent with Faraday's law of induction) is induced. The motor thus acts as a generator whenever its coil rotates. This will happen whether the shaft is turned by an external input, like a belt drive, or by the action of the motor itself. That is, when a motor is doing work and its shaft is turning, an emf is generated. Lenz's law tells us the emf opposes any change, so that the input emf that powers the motor will be opposed by the motor's self-generated emf, called the **back emf** of the motor. (See [Figure 1].)



The coil of a DC motor is represented as a resistor in this schematic. The back emf is represented as a variable emf that opposes the one driving the motor. Back emf is zero when the motor is not turning, and it increases proportionally to the motor's angular velocity.

Back emf is the generator output of a motor, and so it is proportional to the motor's angular velocity ω . It is zero when the motor is first turned on, meaning that the coil receives the full driving voltage and the motor draws maximum current when it is on but not turning. As the motor turns faster and faster, the back emf grows, always opposing the driving emf, and reduces the voltage across the coil and the amount of current it draws. This effect is noticeable in a number of situations. When a vacuum cleaner, refrigerator, or washing machine is first turned on, lights in the same circuit dim briefly due to the IR drop produced in feeder lines by the large current drawn by the motor. When a motor first comes on, it draws more current than when it runs at its normal operating speed. When a mechanical load is placed on the motor, like an electric wheelchair going up a hill, the motor slows, the back emf drops, more current flows, and more work can be done. If the motor runs at too low a speed, the larger current can overheat it (via resistive power in the coil, $P = I^2R$), perhaps even burning it out. On the other hand, if there is no mechanical load on the motor, it will increase its angular velocity ω until the back emf is nearly equal to the driving emf. Then the motor uses only enough energy to overcome friction.

Consider, for example, the motor coils represented in [Figure 1]. The coils have a 0.400Ω equivalent resistance and are driven by a 48.0 V emf. Shortly after being turned on, they draw a current $I = V/R = (48.0\text{V})/(0.400\Omega) = 120\text{A}$ and, thus, dissipate $P = I^2R = 5.76\text{kW}$ of energy as heat transfer. Under normal operating conditions for this motor, suppose the back emf is 40.0 V. Then at operating speed, the total voltage across the coils is 8.0 V (48.0 V minus the 40.0 V back emf), and the current drawn is $I = V/R = (8.0\text{V})/(0.400\Omega) = 20\text{A}$. Under normal load, then, the power dissipated is $P = IV = (20\text{A})/(8.0\text{V}) = 160\text{W}$. The latter will not cause a problem for this motor, whereas the former 5.76 kW would burn out the coils if sustained.

Section Summary

- Any rotating coil will have an induced emf—in motors, this is called back emf, since it opposes the emf input to the motor.

Conceptual Questions

Suppose you find that the belt drive connecting a powerful motor to an air conditioning unit is broken and the motor is running freely. Should you be worried that the motor is consuming a great deal of energy for no useful purpose? Explain why or why not.

Problems & Exercises

Suppose a motor connected to a 120 V source draws 10.0 A when it first starts. (a) What is its resistance? (b) What current does it draw at its normal operating speed when it develops a 100 V back emf?

Show Solution

Strategy

(a) When first starting, back emf is zero, so use Ohm's law with full voltage. (b) At operating speed, the effective voltage is the supply voltage minus back emf.

Solution

(a) At startup, back emf = 0, so the full 120 V appears across the resistance:

$$R=\frac{V}{I}=\frac{120\ \text{V}}{10.0\ \text{A}}=12.0\ \Omega$$

(b) At operating speed, the effective voltage across the resistance is:

$$V_{\text{eff}}=V_{\text{supply}}-V_{\text{back emf}}=120\ \text{V}-100\ \text{V}=20\ \text{V}$$

The current is:

$$I=\frac{V_{\text{eff}}}{R}=\frac{20\ \text{V}}{12.0\ \Omega}=1.67\ \text{A}$$

Discussion

The current drops from 10.0 A at startup to 1.67 A at operating speed—a factor of 6 reduction. This is why motors draw large "inrush" currents when starting and why circuit breakers must be rated to handle these transient currents. The back emf acts as a self-regulating mechanism that limits the motor's current draw during normal operation.

Final Answer

(a) 12.0 Ω; (b) 1.67 A

A motor operating on 240 V electricity has a 180 V back emf at operating speed and draws a 12.0 A current. (a) What is its resistance? (b) What current does it draw when it is first started?

Show Solution

Strategy

(a) At operating speed, find resistance using the effective voltage (supply minus back emf). (b) At startup, use the full supply voltage with the resistance from part (a).

Solution

(a) At operating speed, the effective voltage is:

$$V_{\text{eff}}=240\ \text{V}-180\ \text{V}=60\ \text{V}$$

The resistance is:

$$R=\frac{V_{\text{eff}}}{I}=\frac{60\ \text{V}}{12.0\ \text{A}}=5.00\ \Omega$$

(b) At startup, back emf = 0, so the current is:

$$I=\frac{V_{\text{supply}}}{R}=\frac{240\ \text{V}}{5.00\ \Omega}=48.0\ \text{A}$$

Discussion

The startup current of 48.0 A is four times the operating current of 12.0 A. This large inrush current explains why large motors often require special starting circuits (like soft starters or star-delta starters) to limit the current surge and prevent tripping circuit breakers or causing voltage sags in the power system.

Final Answer

(a) 5.00 Ω; (b) 48.0 A

What is the back emf of a 120 V motor that draws 8.00 A at its normal speed and 20.0 A when first starting?

Show Solution

Strategy

First find the resistance using startup conditions (no back emf), then use it to find back emf at operating speed.

Solution

At startup, back emf = 0, so:

$$R=\frac{V}{I_{start}}=\frac{120\text{ V}}{20.0\text{ A}}=6.00\ \Omega$$

At operating speed, the effective voltage across the resistance is:

$$V_{eff}=IR=(8.00\text{ A})(6.00\ \Omega)=48.0\text{ V}$$

The back emf is:

$$V_{\text{back emf}}=V_{supply}-V_{eff}=120\text{ V}-48.0\text{ V}=72.0\text{ V}$$

Discussion

The back emf of 72.0 V is 60% of the supply voltage, which is typical for motors at normal operating speed. This substantial back emf reduces the current from 20.0 A to 8.00 A, preventing overheating and allowing efficient operation. The back emf represents the motor's energy conversion from electrical to mechanical form.

Final Answer

72.0 V

The motor in a toy car operates on 6.00 V, developing a 4.50 V back emf at normal speed. If it draws 3.00 A at normal speed, what current does it draw when starting?

Show Solution

Strategy

Find the resistance using operating conditions, then calculate startup current with full voltage and no back emf.

Solution

At operating speed, the effective voltage is:

$$V_{eff}=6.00\text{ V}-4.50\text{ V}=1.50\text{ V}$$

The resistance is:

$$R=\frac{V_{eff}}{I}=\frac{1.50\text{ V}}{3.00\text{ A}}=0.500\ \Omega$$

At startup, back emf = 0, so:

$$I_{start}=\frac{V_{supply}}{R}=\frac{6.00\text{ V}}{0.500\ \Omega}=12.0\text{ A}$$

Discussion

The startup current of 12.0 A is four times the operating current of 3.00 A. For a toy car motor, this surge is brief and manageable, but it illustrates why small batteries can sometimes struggle to start motors—the internal resistance of depleted batteries limits the current they can supply during startup.

Final Answer

12.0 A

Integrated Concepts

The motor in a toy car is powered by four batteries in series, which produce a total emf of 6.00 V. The motor draws 3.00 A and develops a 4.50 V back emf at normal speed. Each battery has a 0.100Ω internal resistance. What is the resistance of the motor?

Show Solution

Strategy

Account for the internal resistance of all four batteries in series, then find the motor resistance using the voltage distribution in the circuit.

Solution

The total internal resistance of four batteries is:

$$R_{internal}=4\times0.100\ \Omega=0.400\ \Omega$$

The total resistance in the circuit includes the motor resistance R_{motor} and internal resistance. The effective voltage across the motor resistance is the supply voltage minus the back emf and voltage drop across internal resistance:

$$V_{motor}=V_{supply}-V_{\text{back emf}}-IR_{internal}$$

$$V_{\text{motor}} = 6.00\text{ V} - 4.50\text{ V} - (3.00\text{ A})(0.400\ \Omega) = 1.50\text{ V} - 1.20\text{ V} = 0.300\text{ V}$$

The motor resistance is:

$$R_{\text{motor}} = \frac{V_{\text{motor}}}{I} = \frac{0.300\text{ V}}{3.00\text{ A}} = 0.100\ \Omega$$

Discussion

The motor resistance (0.100 Ω) is quite small, comparable to the internal resistance of a single battery. Most of the voltage drop in this circuit occurs across the back emf (4.50 V) and the battery internal resistance (1.20 V), with only 0.300 V dissipated as heat in the motor resistance. This is characteristic of an efficient motor at operating speed.

Final Answer

0.100 Ω

Glossary

back emf
: the emf generated by a running motor, because it consists of a coil turning in a magnetic field; it opposes the voltage powering the motor

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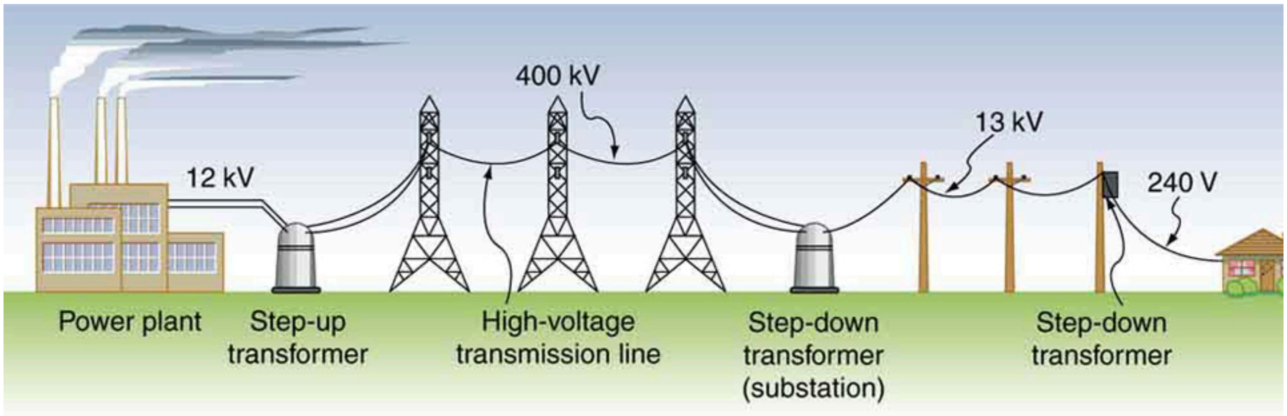
Transformers

- Explain how a transformer works.
- Calculate voltage, current, and/or number of turns given the other quantities.

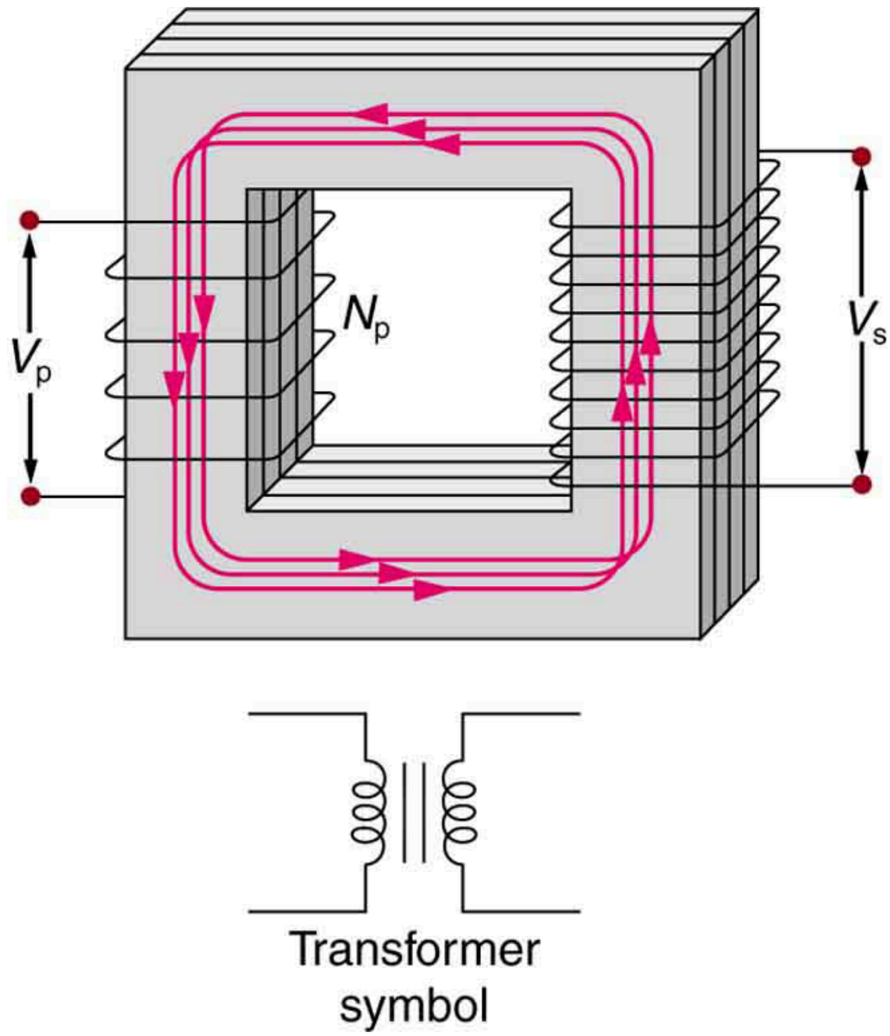
Transformers do what their name implies—they transform voltages from one value to another (The term voltage is used rather than emf, because transformers have internal resistance). For example, many cell phones, laptops, video games, and power tools and small appliances have a transformer built into their plug-in unit (like that in [Figure 1]) that changes 120 V or 240 V AC into whatever voltage the device uses. Transformers are also used at several points in the power distribution systems, such as illustrated in [Figure 2] . Power is sent long distances at high voltages, because less current is required for a given amount of power, and this means less line loss, as was discussed previously. But high voltages pose greater hazards, so that transformers are employed to produce lower voltage at the user's location.

The plug-in transformer has become increasingly familiar with the proliferation of electronic devices that operate on voltages other than common 120 V AC. Most are in the 3 to 12 V range. (credit: Shop Xtreme)



Transformers change voltages at several points in a power distribution system. Electric power is usually generated at greater than 10 kV, and transmitted long distances at voltages over 200 kV—sometimes as great as 700 kV—to limit energy losses. Local power distribution to neighborhoods or industries goes through a substation and is sent short distances at voltages ranging from 5 to 13 kV. This is reduced to 120, 240, or 480 V for safety at the individual user site.

The type of transformer considered in this text—see [Figure 3]—is based on Faraday's law of induction and is very similar in construction to the apparatus Faraday used to demonstrate magnetic fields could cause currents. The two coils are called the **primary** and **secondary coils**. In normal use, the input voltage is placed on the primary, and the secondary produces the transformed output voltage. Not only does the iron core trap the magnetic field created by the primary coil, its magnetization increases the field strength. Since the input voltage is AC, a time-varying magnetic flux is sent to the secondary, inducing its AC output voltage.



A typical construction of a simple transformer has two coils wound on a ferromagnetic core that is laminated to minimize eddy currents. The magnetic field created by the primary is mostly confined to and increased by the core, which transmits it to the secondary coil. Any change in current in the primary induces a current in the secondary.

For the simple transformer shown in [Figure 3], the output voltage V_S depends almost entirely on the input voltage V_P and the ratio of the number of loops in the primary and secondary coils. Faraday's law of induction for the secondary coil gives its induced output voltage V_S to be

$$V_s = -N_s \frac{\Delta\Phi}{\Delta t},$$

where N_S is the number of loops in the secondary coil and $\Delta\Phi / \Delta t$ is the rate of change of magnetic flux. Note that the output voltage equals the induced emf ($V_S = \text{emf}_S$), provided coil resistance is small (a reasonable assumption for transformers). The cross-sectional area of the coils is the same on either side, as is the magnetic field strength, and so $\Delta\Phi/\Delta t$ is the same on either side. The input primary voltage V_P is also related to changing flux by

$$V_p = -N_p \frac{\Delta\Phi}{\Delta t}.$$

The reason for this is a little more subtle. Lenz's law tells us that the primary coil opposes the change in flux caused by the input voltage V_P , hence the minus sign (This is an example of **self-inductance**, a topic to be explored in some detail in later sections). Assuming negligible coil resistance, Kirchhoff's loop rule tells us that the induced emf exactly equals the input voltage. Taking the ratio of these last two equations yields a useful relationship:

$$\frac{V_s}{V_p} = \frac{N_s}{N_p}.$$

This is known as the **transformer equation**, and it simply states that the ratio of the secondary to primary voltages in a transformer equals the ratio of the number of loops in their coils.

The output voltage of a transformer can be less than, greater than, or equal to the input voltage, depending on the ratio of the number of loops in their coils. Some transformers even provide a variable output by allowing connection to be made at different points on the secondary coil. A **step-up transformer** is one that increases voltage, whereas a **step-down transformer** decreases voltage. Assuming, as we have, that resistance is negligible, the electrical power output of a transformer equals its input. This is nearly true in practice—transformer efficiency often exceeds 99%. Equating the power input and output,

$$P_p = I_p V_p = I_s V_s = P_s.$$

Rearranging terms gives

$$V_s V_p = I_p I_s.$$

Combining this with $V_s V_p = N_s N_p$, we find that

$$I_s I_p = N_p N_s$$

is the relationship between the output and input currents of a transformer. So if voltage increases, current decreases. Conversely, if voltage decreases, current increases.

Calculating Characteristics of a Step-Up Transformer

A portable x-ray unit has a step-up transformer, the 120 V input of which is transformed to the 100 kV output needed by the x-ray tube. The primary has 50 loops and draws a current of 10.00 A when in use. (a) What is the number of loops in the secondary? (b) Find the current output of the secondary.

Strategy and Solution for (a)

We solve $V_s V_p = N_s N_p$ for N_s , the number of loops in the secondary, and enter the known values. This gives

$$N_s = N_p V_s V_p = (50) 100000 \text{V} 120 \text{V} = 4.17 \times 10^4.$$

Discussion for (a)

A large number of loops in the secondary (compared with the primary) is required to produce such a large voltage. This would be true for neon sign transformers and those supplying high voltage inside TVs and CRTs.

Strategy and Solution for (b)

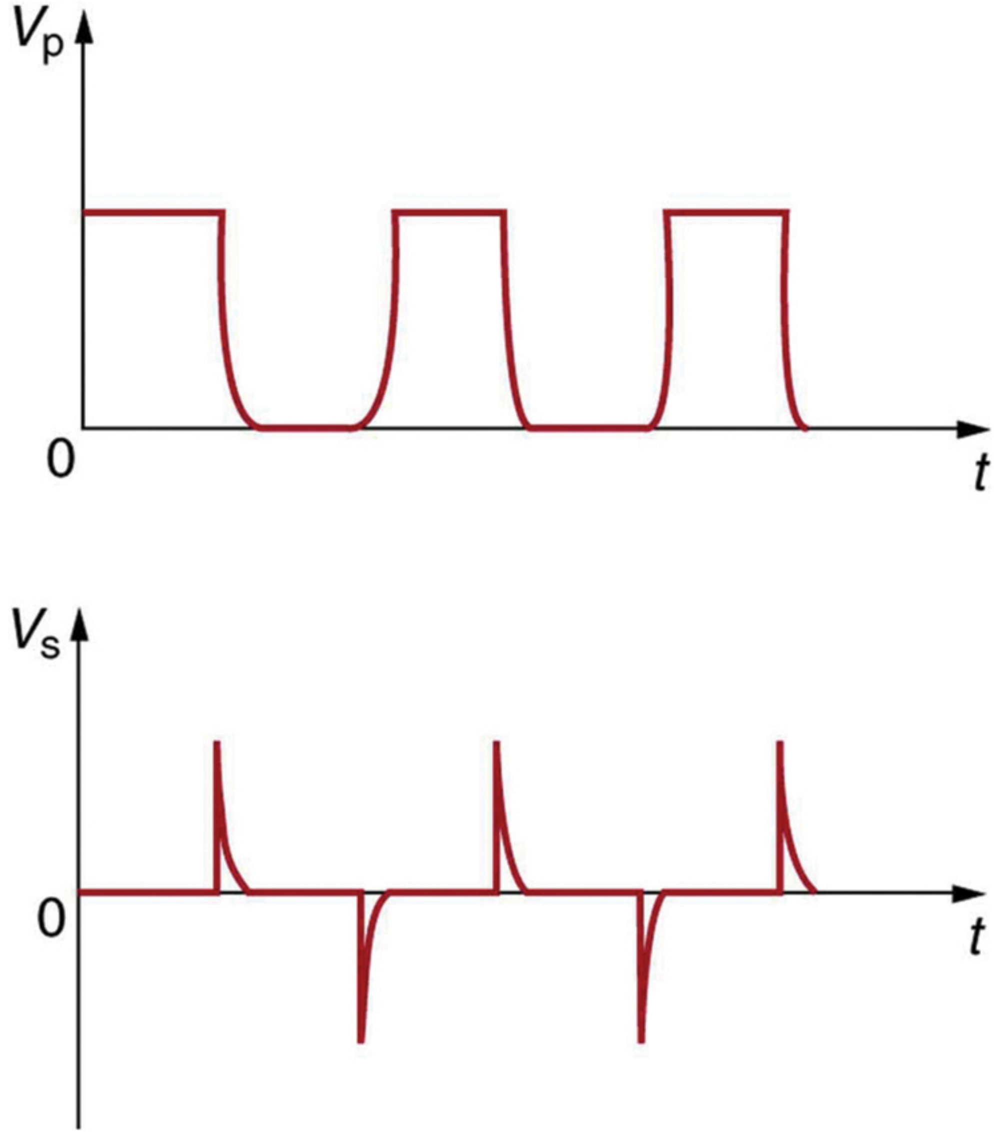
We can similarly find the output current of the secondary by solving $I_s I_p = N_p N_s$ for I_s and entering known values. This gives

$$I_s = I_p N_p N_s = (10.00 \text{A}) 50 4.17 \times 10^4 = 12.0 \text{mA}.$$

Discussion for (b)

As expected, the current output is significantly less than the input. In certain spectacular demonstrations, very large voltages are used to produce long arcs, but they are relatively safe because the transformer output does not supply a large current. Note that the power input here is $P_p = I_p V_p = (10.00 \text{A})(120 \text{V}) = 1.20 \text{kW}$. This equals the power output $P_p = I_s V_s = (12.0 \text{mA})(100 \text{kV}) = 1.20 \text{kW}$, as we assumed in the derivation of the equations used.

The fact that transformers are based on Faraday’s law of induction makes it clear why we cannot use transformers to change DC voltages. If there is no change in primary voltage, there is no voltage induced in the secondary. One possibility is to connect DC to the primary coil through a switch. As the switch is opened and closed, the secondary produces a voltage like that in [Figure 4]. This is not really a practical alternative, and AC is in common use wherever it is necessary to increase or decrease voltages.



Transformers do not work for pure DC voltage input, but if it is switched on and off as on the top graph, the output will look something like that on the bottom graph. This is not the sinusoidal AC most AC appliances need.

Calculating Characteristics of a Step-Down Transformer

A battery charger meant for a series connection of ten nickel-cadmium batteries (total emf of 12.5 V DC) needs to have a 15.0 V output to charge the batteries. It uses a step-down transformer with a 200-loop primary and a 120 V input. (a) How many loops should there be in the secondary coil? (b) If the charging current is 16.0 A, what is the input current?

Strategy and Solution for (a)

You would expect the secondary to have a small number of loops. Solving $\frac{V_s}{V_p}=\frac{N_s}{N_p}$ for N_s and entering known values gives

$$N_s = N_p\frac{V_s}{V_p} = (200)\frac{15.0\text{ V}}{120\text{ V}} = 25.$$

Strategy and Solution for (b)

The current input can be obtained by solving $\frac{I_s}{I_p}=\frac{N_p}{N_s}$ for I_p and entering known values. This gives

$$I_p = I_s\frac{N_s}{N_p} = (16.0\text{ A})\frac{25}{200} = 2.00\text{ A}.$$

Discussion

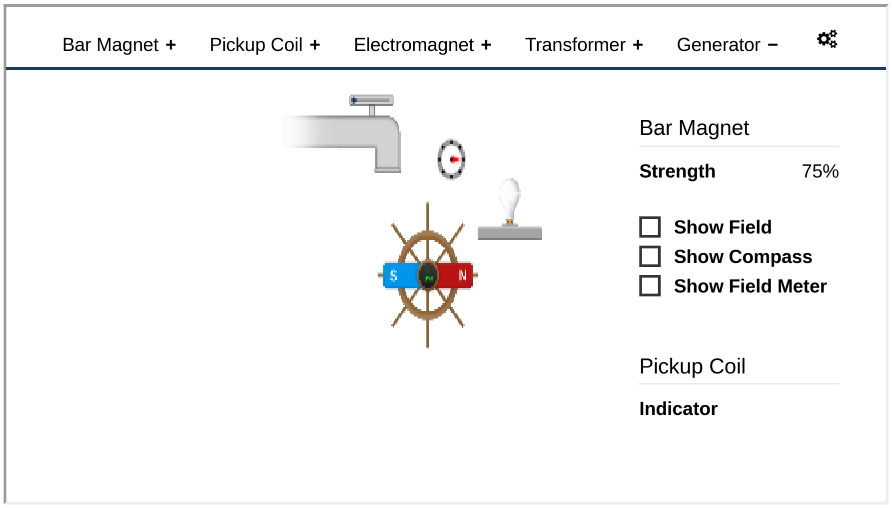
The number of loops in the secondary is small, as expected for a step-down transformer. We also see that a small input current produces a larger output current in a step-down transformer. When transformers are used to operate large magnets, they sometimes have a small number of very heavy loops in the secondary. This allows the secondary to have low internal resistance and produce large currents. Note again that this solution is based on the assumption of 100% efficiency—or power out equals power in ($P_p=P_s$) —reasonable for good transformers. In this case the primary and secondary power is 240 W.

(Verify this for yourself as a consistency check.) Note that the Ni-Cd batteries need to be charged from a DC power source (as would a 12 V battery). So the AC output of the secondary coil needs to be converted into DC. This is done using something called a rectifier, which uses devices called diodes that allow only a one-way flow of current.

Transformers have many applications in electrical safety systems, which are discussed in Electrical Safety: Systems and Devices .

PhET Explorations: Generator

Generate electricity with a bar magnet! Discover the physics behind the phenomena by exploring magnets and how you can use them to make a bulb light.



Section Summary

- Transformers use induction to transform voltages from one value to another.
- For a transformer, the voltages across the primary and secondary coils are related by

  $$\frac{V_s}{V_p}=\frac{N_s}{N_p},$$

  where V_p and V_s are the voltages across primary and secondary coils having N_p and N_s turns.
- The currents I_p and I_s in the primary and secondary coils are related by $\frac{I_s}{I_p}=\frac{N_p}{N_s}$.
- A step-up transformer increases voltage and decreases current, whereas a step-down transformer decreases voltage and increases current.

Conceptual Questions

Explain what causes physical vibrations in transformers at twice the frequency of the AC power involved.

Problems & Exercises

A plug-in transformer, like that in [Figure 4], supplies 9.00 V to a video game system. (a) How many turns are in its secondary coil, if its input voltage is 120 V and the primary coil has 400 turns? (b) What is its input current when its output is 1.30 A?

Show Solution

Strategy

This is a step-down transformer since the output voltage (9.00 V) is less than the input voltage (120 V). We use the transformer equation $\frac{V_s}{V_p}=\frac{N_s}{N_p}$ for part (a) and the current relationship $\frac{I_s}{I_p}=\frac{N_p}{N_s}$ for part (b).

Solution for (a)

Solving for N_s:

$$N_s=N_p\times\frac{V_s}{V_p}=400\times\frac{9.00\text{ V}}{120\text{ V}}=400\times0.0750=30.0\text{ turns}$$

Solution for (b)

Solving for I_p:

$$I_p = I_s \times \frac{N_s}{N_p} = 1.30\text{ A} \times \frac{30.0}{400} = 1.30 \times 0.0750 = 9.75 \times 10^{-2}\text{ A}$$

Discussion

As expected for a step-down transformer, the number of turns in the secondary is much less than in the primary (30 vs 400). The input current is also much smaller than the output current, which is characteristic of step-down transformers. We can verify energy conservation: $P_p = V_p I_p = (120)(0.0975) = 11.7\text{ W}$ and $P_s = V_s I_s = (9.00)(1.30) = 11.7\text{ W}$, confirming our calculations.

Answer

(a) 30.0 turns

(b) $9.75 \times 10^{-2}\text{ A}$ or 97.5 mA

An American traveler in New Zealand carries a transformer to convert New Zealand's standard 240 V to 120 V so that she can use some small appliances on her trip. (a) What is the ratio of turns in the primary and secondary coils of her transformer? (b) What is the ratio of input to output current? (c) How could a New Zealander traveling in the United States use this same transformer to power her 240 V appliances from 120 V?

Show Solution

Strategy

We use the transformer equations: $\frac{V_s}{V_p} = \frac{N_s}{N_p}$ and $\frac{I_s}{I_p} = \frac{N_p}{N_s}$.

Solution for (a)

$$\frac{N_p}{N_s} = \frac{V_p}{V_s} = \frac{240\text{ V}}{120\text{ V}} = 2.00$$

Solution for (b)

$$\frac{I_p}{I_s} = \frac{V_s}{V_p} = \frac{120}{240} = 0.500$$

Or equivalently, the input current is half the output current.

Solution for (c)

The New Zealander would reverse the connections: use the 120 V-side (originally secondary) as the primary connected to the 120 V US supply, and the 240 V-side (originally primary) as the secondary to power the 240 V appliance. The same transformer works in reverse because the turns ratio is the same.

Discussion

The 2:1 turns ratio makes this a step-down transformer when used to convert 240 V to 120 V. Reversing the connections makes it step-up from 120 V to 240 V.

Final Answer

(a) $N_p/N_s = 2.00$; (b) $I_p/I_s = 0.500$; (c) Reverse the connections, using the 120 V coil as primary and 240 V coil as secondary.

A cassette recorder uses a plug-in transformer to convert 120 V to 12.0 V, with a maximum current output of 200 mA. (a) What is the current input? (b) What is the power input? (c) Is this amount of power reasonable for a small appliance?

Show Solution

Strategy

This is a step-down transformer (120 V to 12.0 V). For an ideal transformer, power is conserved: $P_p = P_s$, which means $V_p I_p = V_s I_s$. We can use this to find the input current, then calculate the power.

Solution for (a)

Using power conservation:

$$I_p = I_s \times \frac{V_s}{V_p} = 200\text{ mA} \times \frac{12.0\text{ V}}{120\text{ V}} = 200 \times 0.100 = 20.0\text{ mA}$$

Solution for (b)

The power input is:

$$P_p = V_p I_p = (120\text{ V})(20.0 \times 10^{-3}\text{ A}) = 2.40\text{ W}$$

We can verify with the output power: $P_s = V_s I_s = (12.0)(0.200) = 2.40\text{ W}$ ✓

Solution for (c)

Yes, 2.40 W is quite reasonable for a small appliance like a cassette recorder. This is a very modest power consumption, typical of portable electronic devices that use batteries or small power supplies.

Discussion

The voltage is reduced by a factor of 10, and correspondingly the input current is 1/10 of the output current, maintaining power conservation. This small power requirement explains why such devices can run on batteries for extended periods.

Answer

(a) 20.0 mA

(b) 2.40 W

(c) Yes, this amount of power is quite reasonable for a small appliance.

(a) What is the voltage output of a transformer used for rechargeable flashlight batteries, if its primary has 500 turns, its secondary 4 turns, and the input voltage is 120 V? (b) What input current is required to produce a 4.00 A output? (c) What is the power input?

Show Solution

Strategy

We use $\frac{V_S}{V_P} = \frac{N_S}{N_P}$ for voltage, $\frac{I_S}{I_P} = \frac{N_P}{N_S}$ for current, and $P = IV$ for power.

Solution for (a)

$$V_S = V_P \times \frac{N_S}{N_P} = 120 \times \frac{4}{500} = 120 \times 0.00800 = 0.960 \text{ V}$$

Solution for (b)

$$I_P = I_S \times \frac{N_S}{N_P} = 4.00 \times \frac{4}{500} = 4.00 \times 0.00800 = 0.0320 \text{ A} = 32.0 \text{ mA}$$

Solution for (c)

$$P = V_P I_P = (120)(0.0320) = 3.84 \text{ W}$$

Or check with output: $P = V_S I_S = (0.960)(4.00) = 3.84 \text{ W}$ ✓

Discussion

This step-down transformer reduces voltage by a factor of 125, suitable for charging batteries. The power is conserved (assuming 100% efficiency).

Final Answer

(a) 0.960 V; (b) 32.0 mA; (c) 3.84 W.

(a) The plug-in transformer for a laptop computer puts out 7.50 V and can supply a maximum current of 2.00 A. What is the maximum input current if the input voltage is 240 V? Assume 100% efficiency. (b) If the actual efficiency is less than 100%, would the input current need to be greater or smaller? Explain.

Show Solution

Strategy

For part (a), we use power conservation for an ideal transformer: $P_P = P_S$, which gives $V_P I_P = V_S I_S$. For part (b), we consider what happens when some power is lost to heat in a real transformer.

Solution for (a)

Using power conservation:

$$I_P = I_S \times \frac{V_S}{V_P} = 2.00 \text{ A} \times \frac{7.50 \text{ V}}{240 \text{ V}} = 2.00 \times 0.03125 = 0.0625 \text{ A}$$

Rounding to two significant figures: $I_P = 0.063 \text{ A}$ or 63 mA.

Solution for (b)

If the actual efficiency is less than 100%, the input current would need to be **greater**.

Here's why: In a real transformer, some energy is lost to heat due to resistance in the coils and eddy currents in the core. If the output power must remain $P_S = V_S I_S = (7.50)(2.00) = 15.0 \text{ W}$, but the transformer is only, say, 90% efficient, then the input power must be:

$$P_P = \frac{P_S}{\text{efficiency}} = \frac{15.0 \text{ W}}{0.90} = 16.7 \text{ W}$$

This requires a larger input current: $I_P = P_P / V_P = 16.7/240 = 0.070\text{ A}$, which is greater than 0.063 A.

Discussion

This problem illustrates why transformer efficiency matters. Even a small loss in efficiency requires noticeably more input current (and therefore more input power), which increases operating costs and heat generation. Real transformers typically have efficiencies of 95-99%, so the difference is usually small but still significant for high-power applications.

Answer

(a) 0.063 A (or 63 mA)

(b) Greater input current needed, because energy losses must be compensated by additional input power.

A multipurpose transformer has a secondary coil with several points at which a voltage can be extracted, giving outputs of 5.60, 12.0, and 480 V. (a) The input voltage is 240 V to a primary coil of 280 turns. What are the numbers of turns in the parts of the secondary used to produce the output voltages? (b) If the maximum input current is 5.00 A, what are the maximum output currents (each used alone)?

Show Solution

Strategy

We use $N_S = N_P \times \frac{V_S}{V_P}$ for turns and $I_S = I_P \times \frac{V_P}{V_S}$ for currents.

Solution for (a)

For 5.60 V output:

$$N_S = 280 \times \frac{5.60}{240} = 280 \times 0.0233 = 6.53 \approx 6.5\text{ turns}$$

For 12.0 V output:

$$N_S = 280 \times \frac{12.0}{240} = 280 \times 0.0500 = 14.0\text{ turns}$$

For 480 V output:

$$N_S = 280 \times \frac{480}{240} = 280 \times 2.00 = 560\text{ turns}$$

Solution for (b)

For 5.60 V output:

$$I_S = 5.00 \times \frac{240}{5.60} = 5.00 \times 42.86 = 214\text{ A}$$

For 12.0 V output:

$$I_S = 5.00 \times \frac{240}{12.0} = 5.00 \times 20.0 = 100\text{ A}$$

For 480 V output:

$$I_S = 5.00 \times \frac{240}{480} = 5.00 \times 0.500 = 2.50\text{ A}$$

Discussion

The low-voltage taps can deliver very high currents, suitable for applications like welding. The high-voltage tap delivers low current but high voltage.

Final Answer

(a) 6.5, 14.0, and 560 turns; (b) 214 A, 100 A, and 2.50 A respectively.

A large power plant generates electricity at 12.0 kV. Its old transformer once converted the voltage to 335 kV. The secondary of this transformer is being replaced so that its output can be 750 kV for more efficient cross-country transmission on upgraded transmission lines. (a) What is the ratio of turns in the new secondary compared with the old secondary? (b) What is the ratio of new current output to old output (at 335 kV) for the same power? (c) If the upgraded transmission lines have the same resistance, what is the ratio of new line power loss to old?

Show Solution

Strategy

For part (a), we use the voltage-turns relationship. For part (b), we use power conservation to relate currents at different voltages. For part (c), we use $P_{\text{loss}} = I^2 R$ to compare line losses.

Solution for (a)

Both transformers have the same primary (12.0 kV input), so the ratio of secondary turns is the same as the ratio of secondary voltages:

$$\frac{N_{s,\text{new}}}{N_{s,\text{old}}}=\frac{V_{s,\text{new}}}{V_{s,\text{old}}}=\frac{750\text{ kV}}{335\text{ kV}}=2.24\approx 2.2$$

Solution for (b)

For the same power transmitted, $P=VI$, so:

$$\frac{I_\text{new}}{I_\text{old}}=\frac{P/V_\text{new}}{P/V_\text{old}}=\frac{V_\text{old}}{V_\text{new}}=\frac{335}{750}=0.447\approx 0.45$$

Solution for (c)

Line power loss is $P_\text{loss}=I^2R$. Since the resistance is the same:

$$\frac{P_{\text{loss,new}}}{P_{\text{loss,old}}}=\frac{I^2_{\text{new}}R}{I^2_{\text{old}}R}=\left(\frac{I_\text{new}}{I_\text{old}}\right)^2=(0.447)^2=0.200$$

This is 0.20 or 20.0% of the old loss.

Discussion

By increasing the transmission voltage from 335 kV to 750 kV (a factor of 2.24), the current is reduced by the same factor. Since power loss depends on the square of the current, the losses are reduced to 20% of their previous value—an 80% reduction! This dramatic improvement in efficiency justifies the cost of upgrading transformers and transmission lines. The new secondary coil needs 2.24 times as many turns, making it larger and more expensive, but the energy savings over time make this economically beneficial.

Answer

(a) 2.2 (new secondary has 2.24 times as many turns as old secondary)

(b) 0.45 (new current is 44.7% of old current)

(c) 0.20, or 20.0% (new losses are only 20% of old losses)

If the power output in the previous problem is 1000 MW and line resistance is $2.00\,\Omega$, what were the old and new line losses?

Show Solution

Strategy

From the previous problem, the old voltage was 335 kV and new voltage is 750 kV. We find the current required for 1000 MW at each voltage using $P=VI$, then calculate line loss using $P_\text{loss}=I^2R$.

Solution

Old system (335 kV):

$$I_\text{old}=\frac{P}{V}=\frac{1000\times10^6}{335\times10^3}=2985\text{ A}$$
$$P_\text{loss,old}=I^2R=(2985)^2(2.00)=17.8\times10^6\text{ W}=17.8\text{ MW}$$

New system (750 kV):

$$I_\text{new}=\frac{1000\times10^6}{750\times10^3}=1333\text{ A}$$
$$P_\text{loss,new}=(1333)^2(2.00)=3.56\times10^6\text{ W}=3.56\text{ MW}$$

Discussion

The new system reduces line losses from 17.8 MW to 3.56 MW, saving 14.2 MW (80% reduction). This demonstrates why high-voltage transmission is economically beneficial despite the cost of transformers.

Final Answer

Old line loss: 17.8 MW; New line loss: 3.56 MW.

Unreasonable Results

The 335 kV AC electricity from a power transmission line is fed into the primary coil of a transformer. The ratio of the number of turns in the secondary to the number in the primary is $N_s/N_p=1000$. (a) What voltage is induced in the secondary? (b) What is unreasonable about this result? (c) Which assumption or premise is responsible?

Show Solution

Strategy

This is an “unreasonable results” problem, where we apply the transformer equation correctly but arrive at a physically unrealistic result. We use $\frac{V_S}{V_P}=\frac{N_S}{N_P}$ to find the secondary voltage, then evaluate whether the result makes physical sense.

Solution for (a)

Using the transformer equation:

$$V_S=V_P\times\frac{N_S}{N_P}=335\text{ kV}\times1000=335{,}000\text{ kV}=335\text{ MV}$$

Solution for (b)

This voltage of 335 MV (335 million volts) is unreasonably high. The breakdown voltage of air is approximately 3 MV per meter, which means this voltage would cause electrical breakdown and arcing through air over distances of more than 100 meters. Such extreme voltages cannot be contained by any practical insulation system. Additionally, no real transformer could be built to handle such voltages—the insulation requirements would be impossibly large and expensive. Even the highest transmission voltages used in practice are less than 1.2 MV.

Solution for (c)

The unreasonable assumption is the combination of a very high turns ratio (1000:1) with an already high input voltage (335 kV). In practice:

- Transmission line voltages of 335 kV are appropriate and commonly used
- A turns ratio of 1000:1 would be reasonable for stepping up from low voltages (e.g., 240 V to 240 kV)
- But combining both creates an impossibly high output voltage

The responsible premise is either: (1) using a step-up transformer ($N_S/N_P>1$) when the input voltage is already at transmission-line levels, or (2) having such an extreme turns ratio (1000:1) for high-voltage applications.

Discussion

This problem illustrates that transformers at transmission-line voltages are typically step-down transformers (to reduce voltage for local distribution) or have turns ratios close to 1:1 (for voltage regulation). Massive step-up ratios like 1000:1 are only used when starting from relatively low voltages, such as power plant generators that produce electricity at 12-25 kV.

Answer

(a) 335 MV (335 million volts)

(b) Way too high—well beyond the breakdown voltage of air over reasonable distances, and impossible to insulate practically

(c) The unreasonable assumption is applying a 1000:1 step-up ratio to an already high input voltage of 335 kV. In reality, transformers operating at transmission-line voltages are step-down or have modest turns ratios.

Construct Your Own Problem

Consider a double transformer to be used to create very large voltages. The device consists of two stages. The first is a transformer that produces a much larger output voltage than its input. The output of the first transformer is used as input to a second transformer that further increases the voltage. Construct a problem in which you calculate the output voltage of the final stage based on the input voltage of the first stage and the number of turns or loops in both parts of both transformers (four coils in all). Also calculate the maximum output current of the final stage based on the input current. Discuss the possibility of power losses in the devices and the effect on the output current and power.

Glossary

transformer
: a device that transforms voltages from one value to another using induction

transformer equation
: the equation showing that the ratio of the secondary to primary voltages in a transformer equals the ratio of the number of loops in their coils; $\frac{V_\text{s}}{V_\text{p}}=\frac{N_\text{s}}{N_\text{p}}$

step-up transformer
: a transformer that increases voltage

step-down transformer
: a transformer that decreases voltage

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