Introduction: Further Applications of Newton’s Laws

Total hip replacement surgery has become a common procedure. The head (or ball) of the patient’s femur fits into a cup that has a hard plastic-like inner lining. (credit: National Institutes of Health, via Wikimedia Commons)

Describe the forces on the hip joint. What means are taken to ensure that this will be a good movable joint? From the photograph (for an adult) in Figure 1, estimate the dimensions of the artificial device.

It is difficult to categorize forces into various types (aside from the four basic forces discussed in previous chapter). We know that a net force affects the motion, position, and shape of an object. It is useful at this point to look at some particularly interesting and common forces that will provide further applications of Newton’s laws of motion. We have in mind the forces of friction, air or liquid drag, and deformation.

Friction

- Discuss the general characteristics of friction.
- Describe the various types of friction.
- Calculate the magnitude of static and kinetic friction forces.

Friction is a force that is around us all the time that opposes relative motion between systems in contact but also allows us to move (which you have discovered if you have ever tried to walk on ice). While a common force, the behavior of friction is actually very complicated and is still not completely understood. We have to rely heavily on observations for whatever understandings we can gain. However, we can still deal with its more elementary general characteristics and understand the circumstances in which it behaves.

Friction

Friction is a force that opposes relative motion between systems in contact.

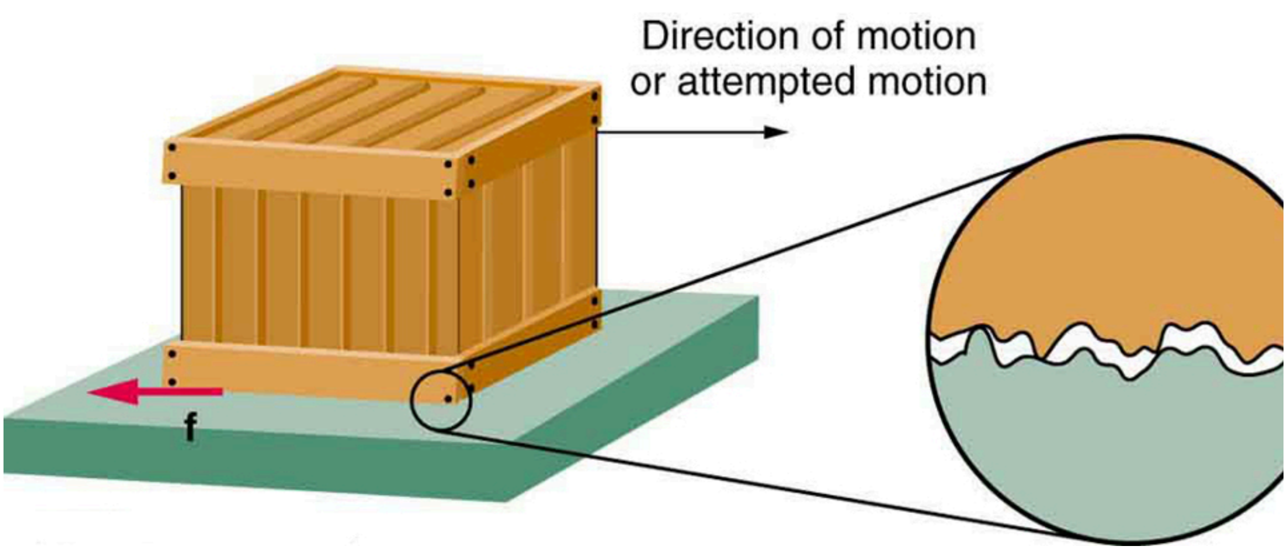
One of the simpler characteristics of friction is that it is parallel to the contact surface between systems and always in a direction that opposes motion or attempted motion of the systems relative to each other. If two systems are in contact and moving relative to one another, then the friction between them is called **kinetic friction**. For example, friction slows a hockey puck sliding on ice. But when objects are stationary, **static friction** can act between them; the static friction is usually greater than the kinetic friction between the objects.

Kinetic Friction

If two systems are in contact and moving relative to one another, then the friction between them is called kinetic friction.

Imagine, for example, trying to slide a heavy crate across a concrete floor—you may push harder and harder on the crate and not move it at all. This means that the static friction responds to what you do—it increases to be equal to and in the opposite direction of your push. But if you finally push hard enough, the crate seems to slip suddenly and starts to move. Once in motion it is easier to keep it in motion than it was to get it started, indicating that the kinetic friction force is less than the static friction force. If you add mass to the crate, say by placing a box on top of it, you need to push even harder to get it started and also to keep it moving. Furthermore, if you oiled the concrete you would find it to be easier to get the crate started and keep it going (as you might expect).

Figure 1 is a crude pictorial representation of how friction occurs at the interface between two objects. Close-up inspection of these surfaces shows them to be rough. So when you push to get an object moving (in this case, a crate), you must raise the object until it can skip along with just the tips of the surface hitting, break off the points, or do both. A considerable force can be resisted by friction with no apparent motion. The harder the surfaces are pushed together (such as if another box is placed on the crate), the more force is needed to move them. Part of the friction is due to adhesive forces between the surface molecules of the two objects, which explain the dependence of friction on the nature of the substances. Adhesion varies with substances in contact and is a complicated aspect of surface physics. Once an object is moving, there are fewer points of contact (fewer molecules adhering), so less force is required to keep the object moving. At small but nonzero speeds, friction is nearly independent of speed.



Frictional forces, such as $\vb{f}$, always oppose motion or attempted motion between objects in contact. Friction arises in part because of the roughness of the surfaces in contact, as seen in the expanded view. In order for the object to move, it must rise to where the peaks can skip along the bottom surface. Thus a force is required just to set the object in motion. Some of the peaks will be broken off, also requiring a force to maintain motion. Much of the friction is actually due to attractive forces between molecules making up the two objects, so that even perfectly smooth surfaces are not friction-free. Such adhesive forces also depend on the substances the surfaces are made of, explaining, for example, why rubber-soled shoes slip less than those with leather soles.

The magnitude of the frictional force has two forms: one for static situations (static friction), the other for when there is motion (kinetic friction).

When there is no motion between the objects, the **magnitude of static friction** , f_s is given by

$$f_s \le \mu_s |\vec{\mathbf{N}}|,$$

where μ_s is the coefficient of static friction and $|\vec{\mathbf{N}}|$ is the magnitude of the normal force (the force perpendicular to the surface).

Magnitude of Static Friction

Magnitude of static friction force f_s is

$$f_s \le \mu_s |\vec{\mathbf{N}}|,$$

where μ_s is the coefficient of static friction and $|\vec{\mathbf{N}}|$ is the magnitude of the normal force.

The symbol ≤ means *less than or equal to*, implying that static friction can have a minimum and a maximum value of $\mu_s |\vec{\mathbf{N}}|$. Static friction is a responsive force that increases to be equal and opposite to whatever force is exerted, up to its maximum limit. Once the applied force exceeds $f_{s(\text{max})}$, the object will move. Thus

$$f_{s(\text{max})} = \mu_s |\vec{\mathbf{N}}|.$$

Once an object is moving, the **magnitude of kinetic friction** , f_k is given by

$$f_k = \mu_k |\vec{\mathbf{N}}|,$$

where μ_k is the coefficient of kinetic friction. A system in which $f_k = \mu\_k |\vec{\mathbf{N}}|$ is described as a system in which *friction behaves simply*.

Magnitude of Kinetic Friction

The magnitude of kinetic friction f_k is given by

$$f_k = \mu_k |\vec{\mathbf{N}}|,$$

where μ_k is the coefficient of kinetic friction.

As seen in Table 1, the coefficients of kinetic friction are less than their static counterparts. That values of μ in Table 1 are stated to only one or, at most, two digits is an indication of the approximate description of friction given by the above two equations.

Table 1: Coefficients of friction

System	Coefficients of Static friction μ_s	Coefficients of Kinetic friction μ_k
Rubber on dry concrete	1.0	0.7
Rubber on wet concrete	0.7	0.5
Wood on wood	0.5	0.3
Waxed wood on wet snow	0.14	0.1
Metal on wood	0.5	0.3
Steel on steel (dry)	0.6	0.3
Steel on steel (oiled)	0.05	0.03
Teflon on steel	0.04	0.04
Bone lubricated by synovial fluid	0.016	0.015
Shoes on wood	0.9	0.7
Shoes on ice	0.1	0.05
Ice on ice	0.1	0.03
Steel on ice	0.04	0.02

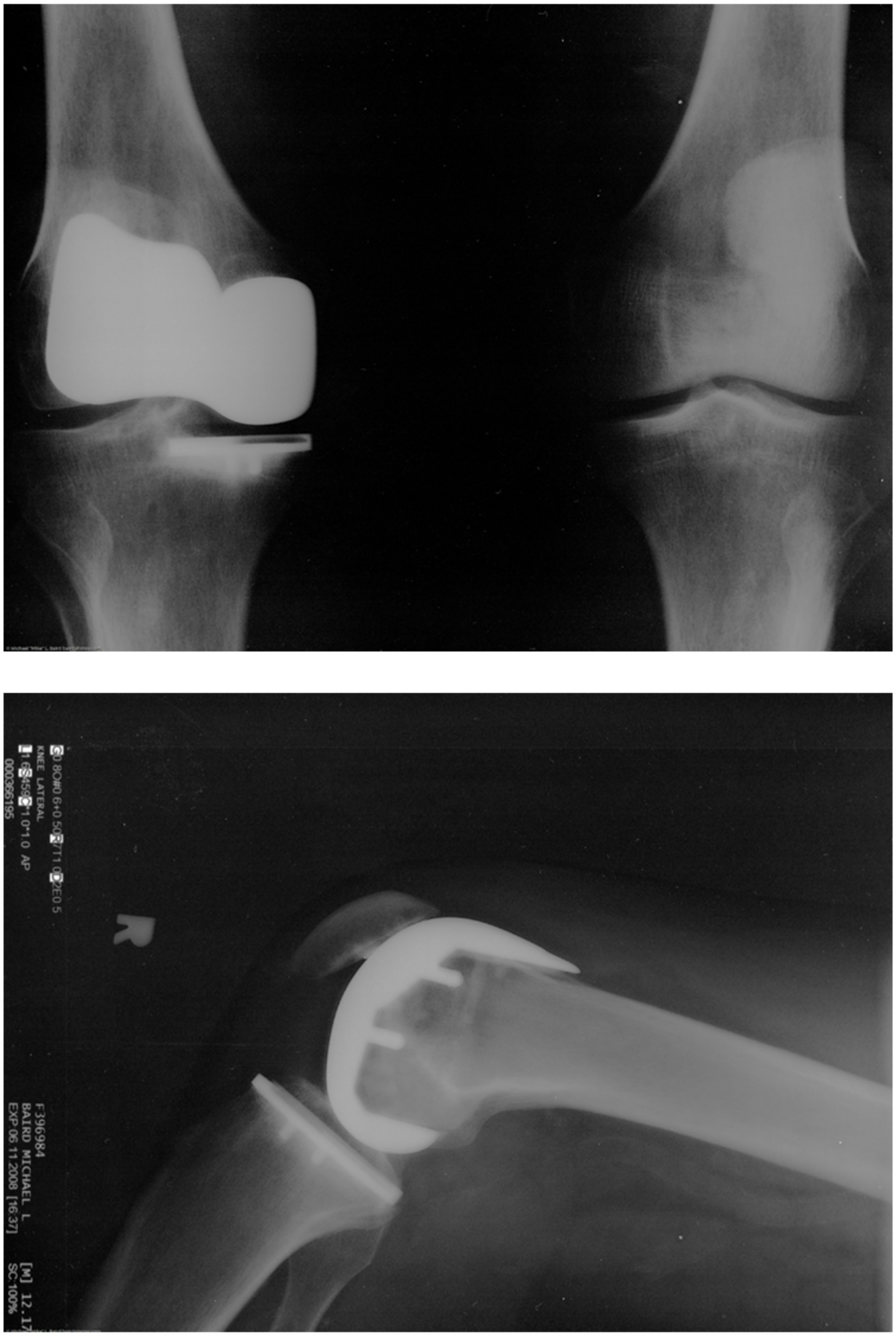
The equations given earlier include the dependence of friction on materials and the normal force. The direction of friction is always opposite that of motion, parallel to the surface between objects, and perpendicular to the normal force. For example, if the crate you try to push (with a force parallel to the floor) has a mass of 100 kg, then the normal force would be equal to its weight, $W = mg(100\text{kg})(9.80\text{m/s}^2) = 980\text{N}$, perpendicular to the floor. If the coefficient of static friction is 0.45, you would have to exert a force parallel to the floor greater than $f_{s(\text{max})} = \mu_s |\vec{\mathbf{N}}| = (0.45)(980\text{N}) = 440\text{N}$ to move the crate. Once there is motion, friction is less and the $$ coefficient of kinetic friction might be 0.30, so that a force of only 290 N (

f_{\text{k}}=\mu_{\text{k}} \mag{N}=\left(0.30\right)\left(980\N \right) =290\N $$) would keep it moving at a constant speed. If the floor is lubricated, both coefficients are considerably less than they would be without lubrication. Coefficient of friction is a unit less quantity with a magnitude usually between 0 and 1.0. The coefficient of the friction depends on the two surfaces that are in contact.

Take-Home Experiment

Find a small plastic object (such as a food container) and slide it on a kitchen table by giving it a gentle tap. Now spray water on the table, simulating a light shower of rain. What happens now when you give the object the same-sized tap? Now add a few drops of (vegetable or olive) oil on the surface of the water and give the same tap. What happens now? This latter situation is particularly important for drivers to note, especially after a light rain shower. Why?

Many people have experienced the slipperiness of walking on ice. However, many parts of the body, especially the joints, have much smaller coefficients of friction—often three or four times less than ice. A joint is formed by the ends of two bones, which are connected by thick tissues. The knee joint is formed by the lower leg bone (the tibia) and the thighbone (the femur). The hip is a ball (at the end of the femur) and socket (part of the pelvis) joint. The ends of the bones in the joint are covered by cartilage, which provides a smooth, almost glassy surface. The joints also produce a fluid (synovial fluid) that reduces friction and wear. A damaged or arthritic joint can be replaced by an artificial joint (Figure 2). These replacements can be made of metals (stainless steel or titanium) or plastic (polyethylene), also with very small coefficients of friction.



Artificial knee replacement is a procedure that has been performed for more than 20 years. In this figure, we see the post-op x rays of the right knee joint replacement. (credit: Mike Baird, Flickr)

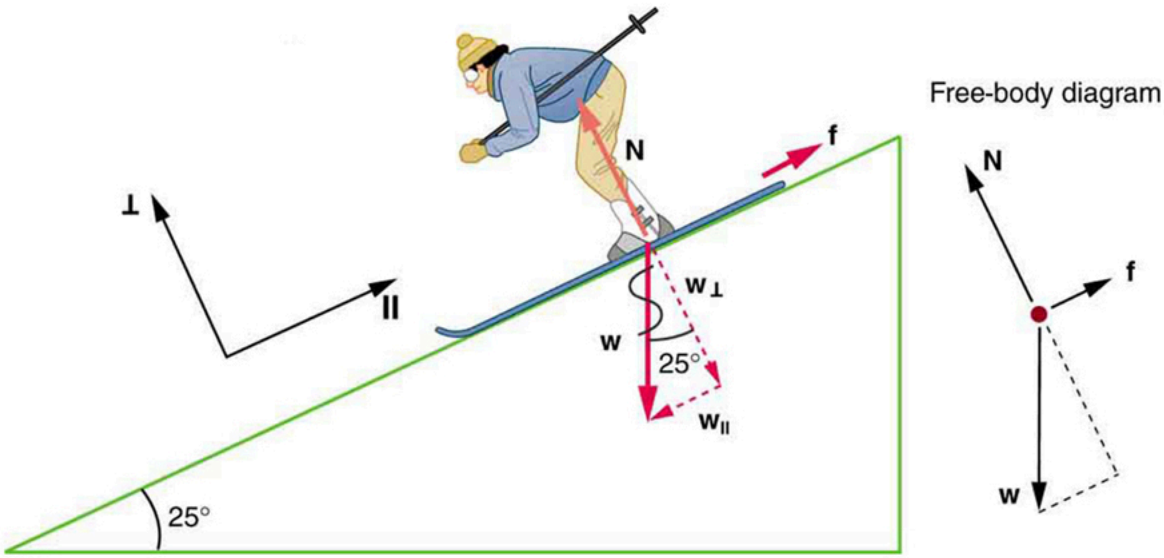
Other natural lubricants include saliva produced in our mouths to aid in the swallowing process, and the slippery mucus found between organs in the body, allowing them to move freely past each other during heartbeats, during breathing, and when a person moves. Artificial lubricants are also common in hospitals and doctor’s clinics. For example, when ultrasonic imaging is carried out, the gel that couples the transducer to the skin also serves to lubricate the surface between the transducer and the skin—thereby reducing the coefficient of friction between the two surfaces. This allows the transducer to move freely over the skin.

Skiing Exercise

A skier with a mass of 62 kg is sliding down a snowy slope. Find the coefficient of kinetic friction for the skier if friction is known to be 45.0 N.

Strategy

The magnitude of kinetic friction was given in to be 45.0 N. Kinetic friction is related to the normal force $|\vec{\mathbf{N}}|$ as $f_k = \mu_k |\vec{\mathbf{N}}|$; thus, the coefficient of kinetic friction can be found if we can find the normal force of the skier on a slope. The normal force is always perpendicular to the surface, and since there is no motion perpendicular to the surface, the normal force should equal the component of the skier’s weight perpendicular to the slope. (See the skier and free-body diagram in Figure 3.)



The motion of the skier and friction are parallel to the slope and so it is most convenient to project all forces onto a coordinate system where one axis is parallel to the slope and the other is perpendicular (axes shown to left of skier). \(\vb{N} \) (the normal force) is perpendicular to the slope, and \(\vb{f} \) (the friction) is parallel to the slope, but \(\vb{w} \) (the skier’s weight) has components along both axes, namely \(w_\perp \) and \(w_\parallel \) . \(\vb{N} \) is equal in magnitude to \(w_\perp \), so there is no motion perpendicular to the slope. However, \(\mag{f} \) is less than \(\mag{W} \) in magnitude, so there is acceleration down the slope (along the x-axis).

That is,

$$|\vec{\mathbf{N}}| = w_\perp = w\cos 25^\circ = mg\cos 25^\circ.$$

Substituting this into our expression for kinetic friction, we get

$$f_k = \mu_k mg\cos 25^\circ,$$

which can now be solved for the coefficient of kinetic friction μ_k.

Solution

Solving for μ_k gives

$$\mu_k = \frac{f_k}{|\vec{\mathbf{N}}|} = \frac{f_k}{w\cos 25^\circ} = \frac{f_k}{mg\cos 25^\circ}.$$

Substituting known values on the right-hand side of the equation,

$$\mu_k = \frac{45.0\,\text{N}}{(62\,\text{kg})(9.80\,\text{m/s}^2)(0.906)} = 0.082.$$

Discussion

This result is a little smaller than the coefficient listed in Table 1 for waxed wood on snow, but it is still reasonable since values of the coefficients of friction can vary greatly. In situations like this, where an object of mass m slides down a slope that makes an angle θ with the horizontal, friction is given by $f_k = \mu_k mg\cos\theta$. All objects will slide down a slope with constant acceleration under these circumstances. Proof of this is left for this chapter’s Problems and Exercises.

Take-Home Experiment

An object will slide down an inclined plane at a constant velocity if the net force on the object is zero. We can use this fact to measure the coefficient of kinetic friction between two objects. As shown in Example 1, the kinetic friction force on a slope $f_k = \mu_k mg\cos\theta$. The component of the weight down the slope is equal to $mg\sin\theta$ (see the free-body diagram in Figure 3). These forces act in opposite directions, so when they have equal magnitude, the acceleration is zero. Writing these out:

$$f_k = mg\sin\theta$$
$$\mu_k mg\cos\theta = mg\sin\theta.$$

Solving for μ_k, we find that

$$\mu_k = \frac{mg\sin\theta}{mg\cos\theta} = \tan\theta.$$

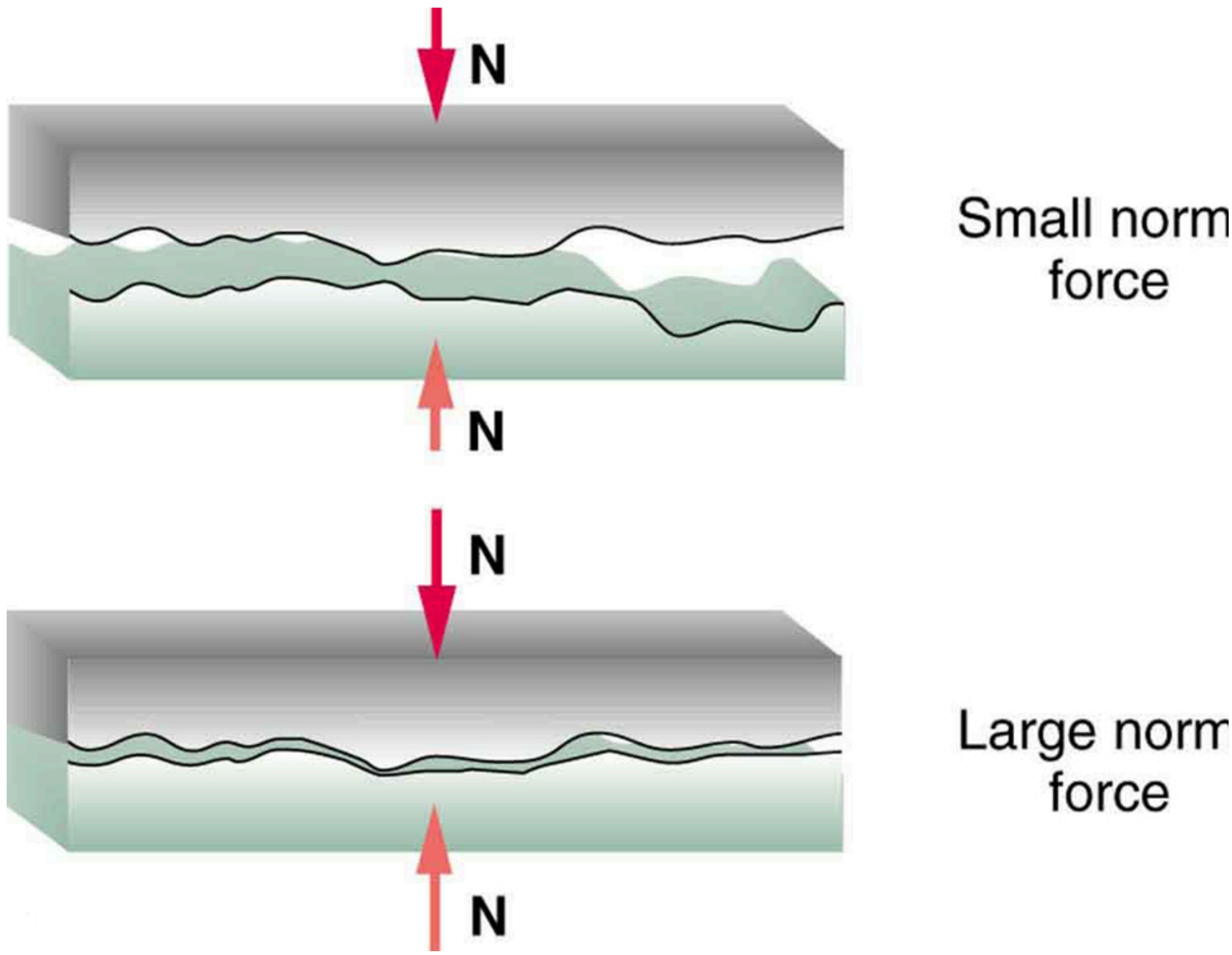
Put a coin on a book and tilt it until the coin slides at a constant velocity down the book. You might need to tap the book lightly to get the coin to move. Measure the angle of tilt relative to the horizontal and find μ_k. Note that the coin will not start to slide at all until an angle greater than θ is attained, since the coefficient of static friction is larger than the coefficient of kinetic friction. Discuss how this may affect the value for μ_k and its uncertainty.

We have discussed that when an object rests on a horizontal surface, there is a normal force supporting it equal in magnitude to its weight. Furthermore, simple friction is always proportional to the normal force.

Making Connections: Submicroscopic Explanations of Friction

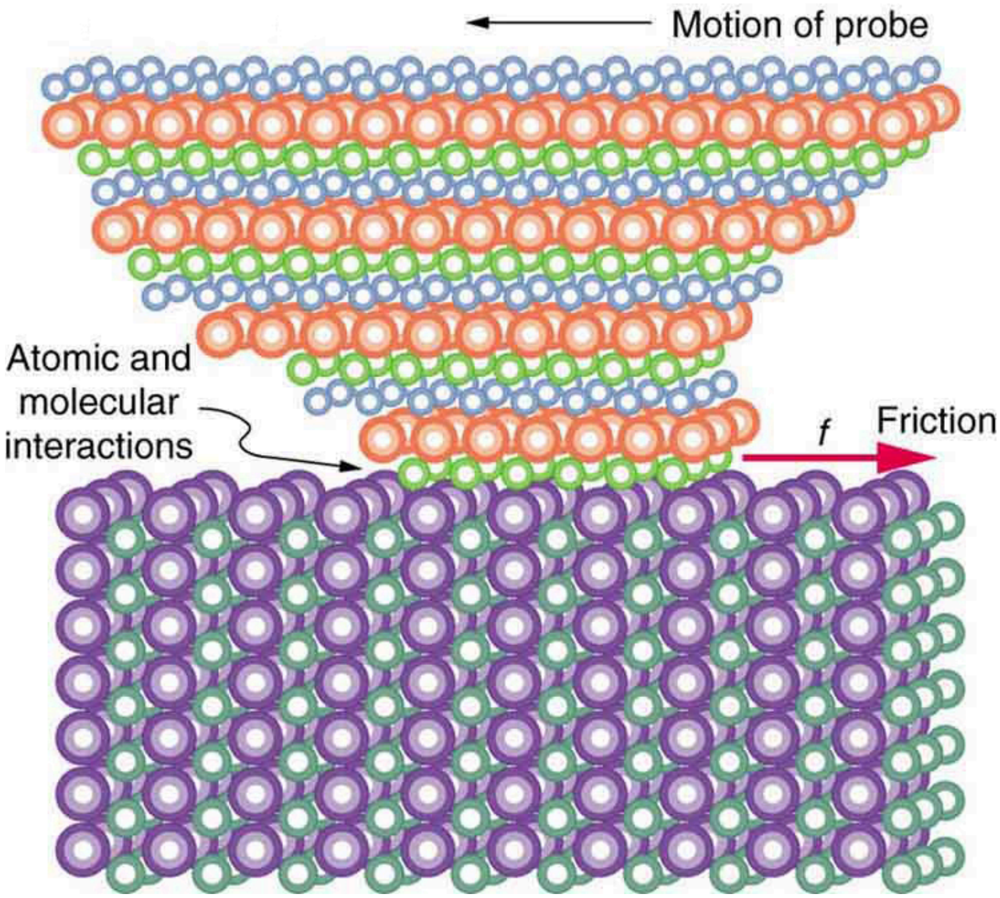
The simpler aspects of friction dealt with so far are its macroscopic (large-scale) characteristics. Great strides have been made in the atomic-scale explanation of friction during the past several decades. Researchers are finding that the atomic nature of friction seems to have several fundamental characteristics. These characteristics not only explain some of the simpler aspects of friction—they also hold the potential for the development of nearly friction-free environments that could save hundreds of billions of dollars in energy which is currently being converted (unnecessarily) to heat.

Figure 4 illustrates one macroscopic characteristic of friction that is explained by microscopic (small-scale) research. We have noted that friction is proportional to the normal force, but not to the area in contact, a somewhat counterintuitive notion. When two rough surfaces are in contact, the actual contact area is a tiny fraction of the total area since only high spots touch. When a greater normal force is exerted, the actual contact area increases, and it is found that the friction is proportional to this area.



Two rough surfaces in contact have a much smaller area of actual contact than their total area. When there is a greater normal force as a result of a greater applied force, the area of actual contact increases as does friction.

But the atomic-scale view promises to explain far more than the simpler features of friction. The mechanism for how heat is generated is now being determined. In other words, why do surfaces get warmer when rubbed? Essentially, atoms are linked with one another to form lattices. When surfaces rub, the surface atoms adhere and cause atomic lattices to vibrate—essentially creating sound waves that penetrate the material. The sound waves diminish with distance and their energy is converted into heat. Chemical reactions that are related to frictional wear can also occur between atoms and molecules on the surfaces. Figure 5 shows how the tip of a probe drawn across another material is deformed by atomic-scale friction. The force needed to drag the tip can be measured and is found to be related to shear stress, which will be discussed later in this chapter. The variation in shear stress is remarkable (more than a factor of 10^{12}) and difficult to predict theoretically, but shear stress is yielding a fundamental understanding of a large-scale phenomenon known since ancient times—friction.



The tip of a probe is deformed sideways by frictional force as the probe is dragged across a surface. Measurements of how the force varies for different materials are yielding fundamental insights into the atomic nature of friction.

Forces and Motion

Explore the forces at work when you try to push a filing cabinet. Create an applied force and see the resulting friction force and total force acting on the cabinet. Charts show the forces, position, velocity, and acceleration vs. time. Draw a free-body diagram of all the forces (including gravitational and normal forces).

Forces and Motion

Section Summary

- Friction is a contact force between systems that opposes the motion or attempted motion between them. Simple friction is proportional to the magnitude of the normal force $|\vec{\mathbf{N}}|$ pushing the systems together. (A normal force is always perpendicular to the contact surface between systems.) Friction depends on both of the materials involved. The magnitude of static friction f_s between systems stationary relative to one another is given by
$$f_s \le \mu_s |\vec{\mathbf{N}}|,$$

where μ_s is the coefficient of static friction, which depends on both of the materials.

- The kinetic friction force f_k between systems moving relative to one another is given by
$$f_k = \mu_k |\vec{\mathbf{N}}|,$$

where μ_k is the coefficient of kinetic friction, which also depends on both materials.

Conceptual Questions

Define normal force. What is its relationship to friction when friction behaves simply?

The glue on a piece of tape can exert forces. Can these forces be a type of simple friction? Explain, considering especially that tape can stick to vertical walls and even to ceilings.

When you learn to drive, you discover that you need to let up slightly on the brake pedal as you come to a stop or the car will stop with a jerk. Explain this in terms of the relationship between static and kinetic friction forces.

When you push a piece of chalk across a chalkboard, it sometimes screeches because it rapidly alternates between slipping and sticking to the board. Describe this process in more detail, in particular explaining how it is related to the fact that the coefficient of kinetic friction is less than static friction. (The same slip-grab process occurs when tires screech on pavement.)

Problems & Exercises

A physics major is cooking breakfast when he notices that the frictional force between his steel spatula and his Teflon frying pan is only 0.200 N. Knowing the coefficient of kinetic friction between the two materials, he quickly calculates the normal force. What is it?

Show Solution

Strategy

We use the relationship between kinetic friction force and normal force. From Table 1, the coefficient of kinetic friction between Teflon and steel is $\mu_k = 0.04$. Since the spatula is moving across the pan, kinetic friction applies.

Solution

1. Write the equation for kinetic friction:

$$f_k = \mu_k |\vec{N}|$$

1. Solve for the normal force:

$$|\vec{N}| = \frac{f_k}{\mu_k}$$

1. Substitute the known values:

$$|\vec{N}| = \frac{0.200 N}{0.04} = 5.00 N$$

Discussion

The normal force on the spatula is $5.00 N$. This relatively small normal force makes sense for a lightweight spatula resting on a pan. The very low coefficient of friction for Teflon (0.04) means even small normal forces can produce measurable friction forces, which is why Teflon is such an effective non-stick coating.

(a) When rebuilding her car's engine, a physics major must exert 300 N of force to insert a dry steel piston into a steel cylinder. What is the magnitude of the normal force between the piston and cylinder? (b) What is the magnitude of the force would she have to exert if the steel parts were oiled?

Show Solution

Strategy

The force required to insert the piston must overcome the kinetic friction between the steel surfaces. From Table 1, we find $\mu_k = 0.3$ for dry steel on steel, and $\mu_k = 0.03$ for oiled steel on steel. We first find the normal force from part (a), then use it to find the required force when oiled.

Solution

(a) For the dry steel piston:

1. The applied force equals the kinetic friction force:

$$F_{applied} = f_k = \mu_k |\vec{N}|$$

1. Solve for the normal force:

$$|\vec{N}| = \frac{F_{applied}}{\mu_k} = \frac{300 N}{0.3} = 1000 N$$

(b) For the oiled steel piston:

1. The normal force remains the same (determined by how tightly the piston fits in the cylinder).
2. Calculate the new friction force with oiled surfaces:

$$F_{oiled} = \mu_{k,oiled} |\vec{N}| = (0.03)(1000 N) = 30 N$$

Discussion

The magnitude of the normal force between the piston and cylinder is $1000 N$ or $1.00 \times 10^3 N$. When the steel parts are oiled, the force required to insert the piston is only $30 N$, which is just 10% of the force needed for dry steel. This demonstrates why lubrication is essential in engines—it dramatically reduces friction and wear between moving metal parts.

(a) What is the maximum frictional force in the knee joint of a person who supports 66.0 kg of her mass on that knee? (b) During strenuous exercise it is possible to exert forces to the joints that are easily ten times greater than the weight being supported. What is the maximum force of friction under such conditions? The frictional forces in joints are relatively small in all circumstances except when the joints deteriorate, such as from injury or arthritis. Increased frictional forces can cause further damage and pain.

Show Solution

Strategy

The knee joint is lubricated by synovial fluid. From Table 1, the coefficient of friction for bone lubricated by synovial fluid is $\mu_s = 0.016$. The normal force in the joint equals the weight supported, and the maximum friction force is found using $f_{s(max)} = \mu\_s |\vec{N}|$.

Solution

(a) Normal standing on one knee:

1. Calculate the normal force (weight supported):

$$|\vec{N}| = mg = (66.0 kg)(9.80 m/s^2) = 646.8 N$$

1. Calculate the maximum friction force:

$$f_{s(max)}=\mu_s|\vec{\mathbf{N}}|=(0.016)(646.8N)=10.3N$$

(b) During strenuous exercise:

1. The force on the joint is ten times the supported weight:

$$|\vec{\mathbf{N}}|_{exercise}=10\times 646.8N=6468N$$

1. Calculate the maximum friction force:

$$f_{s(max)}=\mu_s|\vec{\mathbf{N}}|_{exercise}=(0.016)(6468N)=103N$$

Discussion

The maximum frictional force in the knee joint is $10.3N$ during normal standing and $103N$ during strenuous exercise. These values are remarkably small considering the large forces involved, demonstrating how effectively synovial fluid lubricates our joints. The low coefficient of friction (0.016) is about 6 times lower than ice on ice, which explains why healthy joints move so smoothly. When joints deteriorate from arthritis or injury, this coefficient increases significantly, leading to pain and restricted movement.

Suppose you have a 120-kg wooden crate resting on a wood floor. (a) What maximum force can you exert horizontally on the crate without moving it? (b) If you continue to exert this force once the crate starts to slip, what will the magnitude of its acceleration then be?

Show Solution

Strategy

For part (a), we need to find the maximum static friction force, which equals the maximum force that can be applied without moving the crate. From Table 1, $\mu_S=0.5$ for wood on wood. For part (b), once the crate moves, kinetic friction applies with $\mu_K=0.3$. We use Newton's second law to find the acceleration.

Solution

(a) Maximum force without moving the crate:

1. Calculate the normal force (equal to the weight on a horizontal surface):

$$|\vec{\mathbf{N}}|=mg=(120kg)(9.80m/s^2)=1176N$$

1. Calculate the maximum static friction force:

$$f_{s(max)}=\mu_s|\vec{\mathbf{N}}|=(0.5)(1176N)=588N$$

(b) Acceleration once the crate starts moving:

1. Calculate the kinetic friction force:

$$f_k=\mu_k|\vec{\mathbf{N}}|=(0.3)(1176N)=352.8N$$

1. Apply Newton's second law. The net force is the applied force minus friction:

$$F_{net}=F_{applied}-f_k=588N-352.8N=235.2N$$

1. Calculate the acceleration:

$$a=\frac{F_{net}}{m}=\frac{235.2N}{120kg}=1.96m/s^2$$

Discussion

The maximum force you can exert horizontally without moving the crate is $588N$. Once the crate starts to slip with this same applied force, its acceleration is $1.96m/s^2$. This problem illustrates why it's harder to start an object moving than to keep it moving—static friction (requiring 588 N to overcome) is greater than kinetic friction (only 352.8 N), so once the crate starts sliding, there's a net force that accelerates it.

(a) If half of the weight of a small $1.00\times 10^3 kg$ utility truck is supported by its two drive wheels, what is the magnitude of the maximum acceleration it can achieve on dry concrete? (b) Will a metal cabinet lying on the wooden bed of the truck slip if it accelerates at this rate? (c) Solve both problems assuming the truck has four-wheel drive.

Show Solution

Strategy

The maximum acceleration is limited by the maximum static friction force between the drive wheels and the road. From Table 1, $\mu_S = 1.0$ for rubber on dry concrete. For part (b), we compare the friction available for the cabinet (metal on wood, $\mu_S = 0.5$) with the friction force needed to accelerate it.

Solution

(a) Two-wheel drive maximum acceleration:

1. Calculate the weight supported by drive wheels:

$$|\vec{\mathbf{N}}|_{drive} = \frac{1}{2}mg = \frac{1}{2}(1.00\times10^3 kg)(9.80 m/s^2) = 4900 N$$

1. Calculate the maximum friction force from drive wheels:

$$f_{s(max)} = \mu_s|\vec{\mathbf{N}}|_{drive} = (1.0)(4900 N) = 4900 N$$

1. Calculate the maximum acceleration using Newton's second law:

$$a_{max} = \frac{f_{s(max)}}{m} = \frac{4900 N}{1.00\times10^3 kg} = 4.90 m/s^2$$

(b) Will the metal cabinet slip?

1. For the cabinet to not slip, the static friction must provide enough force to accelerate it:

$$f_{needed} = m_{cab}a = m_{cab}(4.90 m/s^2)$$

1. The maximum friction available for metal on wood is:

$$f_{available} = \mu_s m_{cab} g = (0.5)m_{cab}(9.80 m/s^2) = m_{cab}(4.90 m/s^2)$$

1. Since $f_{available} = f_{needed}$, the cabinet is right at the verge of slipping. Any slight increase in acceleration would cause it to slip.

(c) Four-wheel drive:

1. With four-wheel drive, all wheels are drive wheels:

$$|\vec{\mathbf{N}}|_{drive} = mg = (1.00\times10^3 kg)(9.80 m/s^2) = 9800 N$$

1. Maximum friction force:

$$f_{s(max)} = \mu_s|\vec{\mathbf{N}}|_{drive} = (1.0)(9800 N) = 9800 N$$

1. Maximum acceleration:

$$a_{max} = \frac{9800 N}{1.00\times10^3 kg} = 9.80 m/s^2$$

1. The cabinet will definitely slip since the maximum friction force it can experience ($\mu_s m_{cab} g = 4.90 m_{cab}$) is less than the force needed to accelerate it at $9.80 m/s^2$ ($F = 9.80 m_{cab}$).

Discussion

For two-wheel drive, the maximum acceleration is $4.90 m/s^2$, and the cabinet is on the verge of slipping. For four-wheel drive, the maximum acceleration doubles to $9.80 m/s^2$ (equal to g), and the cabinet will definitely slip. This explains why cargo must be properly secured in trucks, especially those with high-traction four-wheel drive systems.

A team of eight dogs pulls a sled with waxed wood runners on wet snow (mush!). The dogs have average masses of 19.0 kg, and the loaded sled with its rider has a mass of 210 kg. (a) Calculate the magnitude of the acceleration starting from rest if each dog exerts an average force of 185 N backward on the snow. (b) What is the magnitude of the acceleration once the sled starts to move? (c) For both situations, calculate the magnitude of the force in the coupling between the dogs and the sled.

Show Solution

Strategy

By Newton's third law, when the dogs push backward on the snow with 185 N each, the snow pushes forward on each dog with 185 N. From Table 1, for waxed wood on wet snow: $\mu_S = 0.14$ and $\mu_K = 0.1$. We'll apply Newton's second law to the entire system (dogs + sled) to find acceleration, and then to just the sled to find the coupling force.

Solution

First, identify the masses:

- Total mass of dogs: $m\_dogs = 8 \times 19.0 kg = 152 kg$
- Mass of sled with rider: $m\_sled = 210 kg$
- Total system mass: $m\_total = 152 kg + 210 kg = 362 kg$

(a) Starting from rest (static friction):

1. Calculate the total forward force from all dogs:

$$F_{\text{dogs}} = 8 \times 185 \text{N} = 1480 \text{N}$$

1. Calculate the maximum static friction force on the sled:

$$f_{\text{s}} = \mu_{\text{s}} m_{\text{sled}} g = (0.14)(210 \text{kg})(9.80 \text{m/s}^2) = 288.1 \text{N}$$

1. Calculate the net force on the system:

$$F_{\text{net}} = F_{\text{dogs}} - f_{\text{s}} = 1480 \text{N} - 288.1 \text{N} = 1191.9 \text{N}$$

1. Calculate the acceleration:

$$a = \frac{F_{\text{net}}}{m_{\text{total}}} = \frac{1191.9 \text{N}}{362 \text{kg}} = 3.29 \text{m/s}^2$$

(b) Once the sled is moving (kinetic friction):

1. Calculate the kinetic friction force:

$$f_{\text{k}} = \mu_{\text{k}} m_{\text{sled}} g = (0.1)(210 \text{kg})(9.80 \text{m/s}^2) = 205.8 \text{N}$$

1. Calculate the net force:

$$F_{\text{net}} = 1480 \text{N} - 205.8 \text{N} = 1274.2 \text{N}$$

1. Calculate the acceleration:

$$a = \frac{1274.2 \text{N}}{362 \text{kg}} = 3.52 \text{m/s}^2$$

(c) Force in the coupling:

For the sled alone, the coupling force T must provide enough force to accelerate it against friction.

Starting from rest:

$$T - f_{\text{s}} = m_{\text{sled}} a$$
$$T = m_{\text{sled}} a + f_{\text{s}} = (210 \text{kg})(3.29 \text{m/s}^2) + 288.1 \text{N} = 691 \text{N} + 288 \text{N} = 980 \text{N}$$

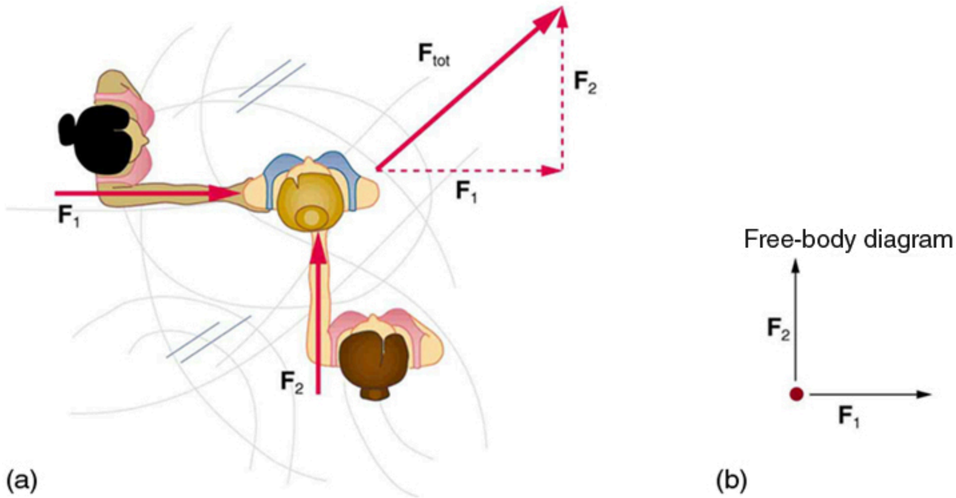
Once moving:

$$T = m_{\text{sled}} a + f_{\text{k}} = (210 \text{kg})(3.52 \text{m/s}^2) + 205.8 \text{N} = 739 \text{N} + 206 \text{N} = 945 \text{N}$$

Discussion

The acceleration starting from rest is 3.29m/s^2, and once moving it increases to 3.52m/s^2 because kinetic friction is less than static friction. The coupling force is 980N at the start and 945N once moving. The coupling force is less than the total force exerted by the dogs (1480 N) because the dogs must also accelerate their own mass.

Consider the 65.0-kg ice skater being pushed by two others shown in Figure 7. (a) Find the direction and magnitude of $\vec{\mathbf{F}}_{\text{tot}}$, the total force exerted on her by the others, given that the magnitudes F_1 and F_2 are 26.4 N and 18.6 N, respectively. (b) What is her initial acceleration if she is initially stationary and wearing steel-bladed skates that point in the direction of $\vec{\mathbf{F}}_{\text{tot}}$? (c) What is her acceleration assuming she is already moving in the direction of $\vec{\mathbf{F}}_{\text{tot}}$? (Remember that friction always acts in the direction opposite that of motion or attempted motion between surfaces in contact.)



(a) Overhead view of two ice skaters pushing on a third. (b) Free-body diagram

Show Solution

Strategy

We use vector addition to find the total force from the two perpendicular forces. From Table 1, for steel on ice: $\mu_s = 0.04$ and $\mu_k = 0.02$. For part (b), since she starts at rest, static friction opposes the attempted motion. For part (c), kinetic friction applies since she's already moving.

Solution

(a) Find the magnitude and direction of $\vec{\mathbf{F}}$_tot:

1. Since F_1 and F_2 are perpendicular, use the Pythagorean theorem:

$$F_{tot}=\sqrt{F_1^2+F_2^2}=\sqrt{(26.4\text{N})^2+(18.6\text{N})^2}$$
$$F_{tot}=\sqrt{697.0+346.0}=\sqrt{1043}=32.3\text{N}$$

1. Find the direction (angle above the horizontal, measured from F_1):

$$\theta=\tan^{-1}\left(\frac{F_2}{F_1}\right)=\tan^{-1}\left(\frac{18.6}{26.4}\right)=\tan^{-1}(0.704)=35.1°$$

(b) Initial acceleration (starting from rest):

1. Calculate the normal force (equal to weight on horizontal ice):

$$|\vec{\mathbf{N}}|=mg=(65.0\text{kg})(9.80\text{m/s}^2)=637\text{N}$$

1. Calculate the maximum static friction force:

$$f_{s(max)}=\mu_s|\vec{\mathbf{N}}|=(0.04)(637\text{N})=25.5\text{N}$$

1. Since $F_{tot} = 32.3\text{N} > f_{s(max)} = 25.5\text{N}$, the skater will begin to move.
2. Once motion begins, kinetic friction applies:

$$f_k=\mu_k|\vec{\mathbf{N}}|=(0.02)(637\text{N})=12.7\text{N}$$

1. Calculate the net force:

$$F_{net}=F_{tot}-f_k=32.3\text{N}-12.7\text{N}=19.6\text{N}$$

1. Calculate the initial acceleration:

$$a=\frac{F_{net}}{m}=\frac{19.6\text{N}}{65.0\text{kg}}=0.302\text{m/s}^2$$

(c) Acceleration while already moving:

The calculation is identical to part (b) once motion has begun, since kinetic friction applies in both cases:

$a = 0.302 \text{m/s}^2$

Discussion

The total force on the skater is 32.3N at an angle of $35.1°$ above the horizontal (from F_1). Her acceleration is 0.302m/s^2 in both cases (b) and (c) because once she begins moving, kinetic friction determines the friction force regardless of whether she just started or was already moving. The very low coefficient of friction on ice (0.02 kinetic) allows even modest forces to produce noticeable acceleration.

Show that the acceleration of any object down a frictionless incline that makes an angle θ with the horizontal is $a = g\sin\theta$. (Note that this acceleration is independent of mass.)

Show Solution

Strategy

We analyze the forces on an object on a frictionless incline using a coordinate system with one axis parallel to the incline and one perpendicular. The only forces are gravity and the normal force. We decompose the gravitational force into components parallel and perpendicular to the incline, then apply Newton's second law along the incline.

Solution

1. Identify the forces on the object:
   - Weight: $w = mg$ (directed straight down)
   - Normal force: $|\vec{\mathbf{N}}|$ (perpendicular to the incline surface)
2. Decompose the weight into components relative to the incline:
   - Perpendicular to incline: $w\_\perp = mg\cos\theta$
   - Parallel to incline (down the slope): $w\_\parallel = mg\sin\theta$
3. Apply Newton's second law perpendicular to the incline:

$$\sum F_\perp = |\vec{\mathbf{N}}| - mg\cos\theta = 0$$

This confirms the object doesn't accelerate perpendicular to the surface.

1. Apply Newton's second law parallel to the incline (taking down the slope as positive):

$$\sum F_\parallel = mg\sin\theta = ma$$

1. Solve for acceleration:

$$ma = mg\sin\theta$$
$$a = g\sin\theta$$

Discussion

The acceleration of any object on a frictionless incline is $a = g\sin\theta$, which depends only on the angle of the incline and the acceleration due to gravity. The mass m cancels out, confirming that all objects slide down a frictionless incline with the same acceleration, regardless of their mass. This is consistent with Galileo's observation that objects fall at the same rate in the absence of air resistance. Note that when $\theta = 0°$, $a = 0$ (horizontal surface), and when $\theta = 90°$, $a = g$ (free fall).

Show that the acceleration of any object down an incline where friction behaves simply (that is, where $f_\text{k} = \mu_\text{k}|\vec{\mathbf{N}}|$) is $a = g(\sin\theta - \mu_\text{k}\cos\theta)$. Note that the acceleration is independent of mass and reduces to the expression found in the previous problem when friction becomes negligibly small $(\mu_\text{k} = 0)$.

Show Solution

Strategy

We follow the same approach as the previous problem, but now include kinetic friction opposing the motion down the incline. The friction force acts up the slope (opposite to the direction of motion) and depends on the normal force.

Solution

1. Identify the forces on the object:
   - Weight: $w = mg$ (directed straight down)
   - Normal force: $|\vec{\mathbf{N}}|$ (perpendicular to the incline surface)
   - Kinetic friction: f_k (parallel to incline, opposing motion, so directed up the slope)
2. Decompose the weight into components:
   - Perpendicular to incline: $w\_\perp = mg\cos\theta$
   - Parallel to incline (down the slope): $w\_\parallel = mg\sin\theta$
3. Apply Newton's second law perpendicular to the incline:

$$\sum F_{\perp} = |\vec{\mathbf{N}}| - mg\cos\theta = 0$$
$$|\vec{\mathbf{N}}| = mg\cos\theta$$

1. Express the kinetic friction force:

$$f_k = \mu_k |\vec{\mathbf{N}}| = \mu_k mg\cos\theta$$

1. Apply Newton's second law parallel to the incline (taking down the slope as positive):

$$\sum F_{\parallel} = mg\sin\theta - f_k = ma$$
$$mg\sin\theta - \mu_k mg\cos\theta = ma$$

1. Solve for acceleration by factoring out mg:

$$mg(\sin\theta - \mu_k\cos\theta) = ma$$
$$a = g(\sin\theta - \mu_k\cos\theta)$$

Discussion

The acceleration down an incline with friction is $a = g(\sin\theta - \mu_k\cos\theta)$. This result is independent of mass because both the gravitational force component and the friction force are proportional to mass. When $\mu_k = 0$ (no friction), the equation reduces to $a = g\sin\theta$, matching the frictionless case from the previous problem. If friction is large enough that $\mu_k\cos\theta > \sin\theta$, the acceleration would be negative, meaning the object would decelerate (as when going up a slope) or wouldn't slide at all without an initial push.

Calculate the deceleration of a snow boarder going up a 5.0°, slope assuming the coefficient of friction for waxed wood on wet snow. The result of Example 1 may be useful, but be careful to consider the fact that the snow boarder is going uphill. Explicitly show how you follow the steps in Problem-Solving Strategies.

Show Solution

Strategy

When going uphill, both gravity and friction act to slow the snowboarder down. The component of gravity parallel to the slope points downhill (opposing uphill motion), and kinetic friction also opposes the uphill motion (so it also points downhill). From Table 1, for waxed wood on wet snow: $\mu_k = 0.1$.

Solution

1. Draw a free-body diagram with forces:
   - Weight component parallel to slope: $w_{\parallel} = mg\sin\theta$ (down the slope)
   - Weight component perpendicular to slope: $w_{\perp} = mg\cos\theta$
   - Normal force: $|\vec{\mathbf{N}}| = mg\cos\theta$
   - Kinetic friction: $f_k = \mu_k |\vec{\mathbf{N}}|$ (down the slope, opposing uphill motion)
2. Apply Newton's second law parallel to the slope (taking up the slope as positive):

$$\sum F_{\parallel} = -mg\sin\theta - f_k = ma$$

1. Substitute the friction force:

$$-mg\sin\theta - \mu_k mg\cos\theta = ma$$

1. Solve for acceleration:

$$a = -g(\sin\theta + \mu_k\cos\theta)$$

1. Substitute values ($\theta = 5.0^\circ$, $\mu_k = 0.1$, $g = 9.80 \text{ m/s}^2$):

$$a = -(9.80 \text{ m/s}^2)(\sin 5.0^\circ + 0.1\cos 5.0^\circ)$$
$$a = -(9.80 \text{ m/s}^2)(0.0872 + 0.1 \times 0.9962)$$
$$a = -(9.80 \text{ m/s}^2)(0.0872 + 0.0996)$$
$$a = -(9.80 \text{ m/s}^2)(0.1868) = -1.83 \text{ m/s}^2$$

The deceleration (magnitude of negative acceleration) is:

$$|a| = 1.83 \text{ m/s}^2$$

Discussion

The snowboarder decelerates at 1.83m/s^2 while going uphill. Note that when going uphill, friction adds to the deceleration (both gravity component and friction point downhill), whereas when going downhill, friction subtracts from the acceleration. This is why the formula changes from $g(\sin\theta - \mu_k \cos\theta)$ for downhill motion to $g(\sin\theta + \mu_k \cos\theta)$ for uphill motion.

(a) Calculate the acceleration of a skier heading down a $10.0°$ slope, assuming the coefficient of friction for waxed wood on wet snow. (b) Find the angle of the slope down which this skier could coast at a constant velocity. You can neglect air resistance in both parts, and you will find the result of Example 1 to be useful. Explicitly show how you follow the steps in the Problem-Solving Strategies.

Show Solution

Strategy

For part (a), we use the result derived earlier: $a = g(\sin\theta - \mu_k \cos\theta)$ for an object sliding down an incline. From Table 1, for waxed wood on wet snow: $\mu_k = 0.1$. For part (b), constant velocity means zero acceleration, so we set $a = 0$ and solve for the angle.

Solution

(a) Acceleration down the $10.0\ °$ slope:

1. Use the formula for acceleration on an incline with friction:

$$a=g(\sin\theta-\mu_k\cos\theta)$$

1. Substitute values ($\theta = 10.0\ °$, μ_k $= 0.1$, $g = 9.80\text{m/s}^2$):

$$a=(9.80\text{m/s}^2)(\sin10.0°-0.1\cos10.0°)$$
$$a=(9.80\text{m/s}^2)(0.1736-0.1\times0.9848)$$
$$a=(9.80\text{m/s}^2)(0.1736-0.0985)$$
$$a=(9.80\text{m/s}^2)(0.0751)=0.736\text{m/s}^2$$

(b) Angle for constant velocity (zero acceleration):

1. Set $a = 0$ in the acceleration formula:

$$0=g(\sin\theta-\mu_k\cos\theta)$$

1. Since $g \neq 0$:

$$\sin\theta-\mu_k\cos\theta=0$$
$$\sin\theta=\mu_k\cos\theta$$

1. Divide both sides by $\cos\theta$:

$$\tan\theta=\mu_k=0.1$$

1. Solve for the angle:

$$\theta=\tan^{-1}(0.1)=5.71°$$

Discussion

The skier accelerates at 0.736m/s^2 on the $10.0\ °$ slope. To coast at constant velocity, the skier would need a slope of $5.71\ °$. At this angle, the component of gravity pulling the skier downhill exactly equals the friction force opposing the motion. On steeper slopes, the skier accelerates; on gentler slopes, the skier decelerates. This relationship $\tan\theta = \mu$_k provides a simple way to measure the coefficient of kinetic friction by finding the angle at which an object slides at constant velocity.

If an object is to rest on an incline without slipping, then friction must equal the component of the weight of the object parallel to the incline. This requires greater and greater friction for steeper slopes. Show that the maximum angle of an incline above the horizontal for which an object will not slide down is $\theta = \tan^{-1}\mu_s$. You may use the result of the previous problem. Assume that $a = 0$ and that static friction force has reached its maximum value.

Show Solution

Strategy

For an object at rest on an incline at the verge of slipping, static friction has reached its maximum value and the object has zero acceleration. We apply Newton's second law with $a = 0$ and $f_s = \mu_s |\vec{\mathbf{N}}|$ (maximum static friction) to find the critical angle.

Solution

1. Identify forces on the object at rest on the incline:
   - Weight component parallel to slope (down): $w\_\parallel = mg\sin\theta$
   - Weight component perpendicular to slope: $w\_\perp = mg\cos\theta$
   - Normal force: $|\vec{\mathbf{N}}| = mg\cos\theta$
   - Static friction (up the slope, opposing tendency to slide): f_s
2. Apply Newton's second law parallel to the incline with $a = 0$:

$$\sum F_\parallel = f_\mathrm{s} - mg\sin\theta = 0$$
$$f_\mathrm{s} = mg\sin\theta$$

1. At the maximum angle, static friction reaches its maximum value:

$$f_{\mathrm{s(max)}} = \mu_\mathrm{s}|\vec{\mathbf{N}}| = \mu_\mathrm{s} mg\cos\theta$$

1. Set the required friction equal to the maximum available friction:

$$mg\sin\theta = \mu_\mathrm{s} mg\cos\theta$$

1. Cancel mg from both sides and solve:

$$\sin\theta = \mu_\mathrm{s}\cos\theta$$
$$\sin\theta\cos\theta = \mu_\mathrm{s}$$
$$\tan\theta = \mu_\mathrm{s}$$

1. Solve for the maximum angle:

$$\theta_{\mathrm{max}} = \tan^{-1}(\mu_\mathrm{s})$$

Discussion

The maximum angle at which an object can rest on an incline without sliding is $\theta = \tan^{-1}(\mu_\mathrm{s})$. This result is independent of the object's mass. For angles less than this, static friction is sufficient to prevent sliding. For angles greater than this, the object will begin to slide. This relationship provides a practical method for measuring the coefficient of static friction: tilt a surface until an object just begins to slide, then $\mu_\mathrm{s} = \tan\theta\_\mathrm{critical}$.

Calculate the maximum deceleration of a car that is heading down a $6°$ slope (one that makes an angle of $6°$ with the horizontal) under the following road conditions. You may assume that the weight of the car is evenly distributed on all four tires and that the coefficient of static friction is involved—that is, the tires are not allowed to slip during the deceleration. (Ignore rolling.) Calculate for a car: (a) On dry concrete. (b) On wet concrete. (c) On ice, assuming that $\mu_\mathrm{s} = 0.100$, the same as for shoes on ice.

Show Solution

Strategy

When braking while heading downhill, gravity pulls the car down the slope while friction (from braking) acts up the slope. The maximum deceleration occurs when static friction reaches its maximum value (tires at the verge of slipping). All four tires contribute to braking, so the full weight contributes to the normal force. From Table 1: rubber on dry concrete $\mu_\mathrm{s} = 1.0$, rubber on wet concrete $\mu_\mathrm{s} = 0.7$.

Solution

For a car on a slope, the forces parallel to the slope are:

- Gravity component (down slope): $mg\sin\theta$
- Maximum friction (up slope when braking): $f_{\mathrm{s(max)}} = \mu_\mathrm{s} mg\cos\theta$

The maximum deceleration occurs when braking force is maximum:

$$ma = f_{\mathrm{s(max)}} - mg\sin\theta = \mu_\mathrm{s} mg\cos\theta - mg\sin\theta$$
$$a = g(\mu_\mathrm{s}\cos\theta - \sin\theta)$$

With $\theta = 6°$: $\sin 6° = 0.1045$, $\cos 6° = 0.9945$

(a) On dry concrete ($\mu\_\mathrm{s} = 1.0$):

$$a = (9.80\,\mathrm{m/s^2})(1.0 \times 0.9945 - 0.1045)$$
$$a = (9.80\,\mathrm{m/s^2})(0.9945 - 0.1045) = (9.80\,\mathrm{m/s^2})(0.890) = 8.72\,\mathrm{m/s^2}$$

(b) On wet concrete (μ_s $=0.7$):

$$a=(9.80\text{m/s}^2)(0.7\times0.9945-0.1045)$$
$$a=(9.80\text{m/s}^2)(0.6962-0.1045)=(9.80\text{m/s}^2)(0.592)=5.80\text{m/s}^2$$

(c) On ice (μ_s $=0.100$):

$$a=(9.80\text{m/s}^2)(0.100\times0.9945-0.1045)$$
$$a=(9.80\text{m/s}^2)(0.0995-0.1045)=(9.80\text{m/s}^2)(-0.005)=-0.049\text{m/s}^2$$

Discussion

The maximum deceleration is 8.72m/s^2 on dry concrete, 5.80m/s^2 on wet concrete, and -0.049m/s^2 on ice. The negative value for ice is significant: it means the car cannot decelerate at all on this slope! Instead, even with maximum braking, the car will accelerate down the hill at about 0.05m/s^2. The friction force is insufficient to overcome the gravitational component pulling the car downhill. This illustrates why icy hills are so dangerous—braking can be nearly impossible.

Calculate the maximum acceleration of a car that is heading up a 4° slope (one that makes an angle of 4° with the horizontal) under the following road conditions. Assume that only half the weight of the car is supported by the two drive wheels and that the coefficient of static friction is involved—that is, the tires are not allowed to slip during the acceleration. (Ignore rolling.) (a) On dry concrete. (b) On wet concrete. (c) On ice, assuming that $\mu_s=0.100$, the same as for shoes on ice.

Show Solution

Strategy

When accelerating uphill, the drive wheels push backward on the road, and friction pushes the car forward (up the slope). However, gravity acts down the slope. Since only half the weight is on the drive wheels, only half the weight contributes to the friction force. The maximum acceleration occurs when friction reaches its maximum value.

Solution

The normal force on the drive wheels is half the perpendicular weight component:

$$|\vec{\mathbf{N}}|_{\text{drive}}=12mg\cos\theta$$

The maximum friction force (providing forward acceleration):

$$f_{s(\text{max})}=\mu_s|\vec{\mathbf{N}}|_{\text{drive}}=12\mu_s mg\cos\theta$$

Newton’s second law parallel to the slope (up is positive):

$$ma=f_{s(\text{max})}-mg\sin\theta=12\mu_s mg\cos\theta-mg\sin\theta$$
$$a=g(12\mu_s\cos\theta-\sin\theta)$$

With $\theta=4^\circ$: $\sin4^\circ=0.0698$, $\cos4^\circ=0.9976$

(a) On dry concrete (μ_s $=1.0$):

$$a=(9.80\text{m/s}^2)(12(1.0)(0.9976)-0.0698)$$
$$a=(9.80\text{m/s}^2)(0.4988-0.0698)=(9.80\text{m/s}^2)(0.429)=4.20\text{m/s}^2$$

(b) On wet concrete (μ_s $=0.7$):

$$a=(9.80\text{m/s}^2)(12(0.7)(0.9976)-0.0698)$$
$$a=(9.80\text{m/s}^2)(0.3492-0.0698)=(9.80\text{m/s}^2)(0.279)=2.74\text{m/s}^2$$

(c) On ice (μ_s $=0.100$):

$$a=(9.80\text{m/s}^2)(12(0.100)(0.9976)-0.0698)$$
$$a=(9.80\text{m/s}^2)(0.0499-0.0698)=(9.80\text{m/s}^2)(-0.0199)=-0.195\text{m/s}^2$$

Discussion

The maximum acceleration is 4.20m/s^2 on dry concrete, 2.74m/s^2 on wet concrete, and -0.195m/s^2 on ice. The negative value for ice means the car cannot accelerate up the hill at all—even at full throttle without wheel spin, the car will slide backward down the slope. This demonstrates why two-wheel drive vehicles struggle on icy inclines. Note that these values are roughly half what they would be with four-wheel drive because only half the weight provides traction.

Repeat the previous exercise for a car with four-wheel drive.

Show Solution

Strategy

With four-wheel drive, all four wheels are drive wheels, so the full weight of the car (times $\cos\theta$) contributes to the normal force for friction. This doubles the available friction force compared to two-wheel drive.

Solution

With four-wheel drive, the normal force equals the full perpendicular weight:

$$|\vec{\mathbf{N}}|_{\text{drive}} = mg\cos\theta$$

The maximum friction force:

$$f_{\text{s(max)}} = \mu_{\text{s}} mg\cos\theta$$

Newton’s second law parallel to the slope:

$$ma = f_{\text{s(max)}} - mg\sin\theta = \mu_{\text{s}} mg\cos\theta - mg\sin\theta$$
$$a = g(\mu_{\text{s}}\cos\theta - \sin\theta)$$

With $\theta = 4°$: $\sin 4° = 0.0698$, $\cos 4° = 0.9976$

(a) On dry concrete (μ_s $= 1.0$):

$$a = (9.80 \text{m/s}^2)(1.0 \times 0.9976 - 0.0698)$$
$$a = (9.80 \text{m/s}^2)(0.9976 - 0.0698) = (9.80 \text{m/s}^2)(0.928) = 9.09 \text{m/s}^2$$

(b) On wet concrete (μ_s $= 0.7$):

$$a = (9.80 \text{m/s}^2)(0.7 \times 0.9976 - 0.0698)$$
$$a = (9.80 \text{m/s}^2)(0.6983 - 0.0698) = (9.80 \text{m/s}^2)(0.629) = 6.16 \text{m/s}^2$$

(c) On ice (μ_s $= 0.100$):

$$a = (9.80 \text{m/s}^2)(0.100 \times 0.9976 - 0.0698)$$
$$a = (9.80 \text{m/s}^2)(0.0998 - 0.0698) = (9.80 \text{m/s}^2)(0.0300) = 0.294 \text{m/s}^2$$

Discussion

With four-wheel drive, the maximum acceleration is 9.09m/s^2 on dry concrete, 6.16m/s^2 on wet concrete, and 0.294m/s^2 on ice. These values are approximately double those for two-wheel drive. Notably, with four-wheel drive on ice, the car can now accelerate (albeit slowly at 0.29m/s^2), whereas it would slide backward with two-wheel drive. This demonstrates the significant advantage of four-wheel drive on slippery surfaces and inclines.

A freight train consists of two 8.00×10^5-kg engines and 45 cars with average masses of $5.50 \times 10^5 \text{kg}$. (a) What force must each engine exert backward on the track to accelerate the train at a rate of $5.00 \times 10^{-2} \text{m/s}^2$ if the force of friction is $7.50 \times 10^5 \text{N}$, assuming the engines exert identical forces? This is not a large frictional force for such a massive system. Rolling friction for trains is small, and consequently trains are very energy-efficient transportation systems. (b) What is the magnitude of the force in the coupling between the 37th and 38th cars (this is the force each exerts on the other), assuming all cars have the same mass and that friction is evenly distributed among all of the cars and engines?

Show Solution

Strategy

For part (a), we apply Newton’s second law to the entire train to find the total propulsive force needed, then divide by 2 for each engine. For part (b), we consider the cars behind the 37th-38th coupling (cars 38-45) as a separate system. The coupling force must accelerate these 8 cars and overcome their share of friction.

Solution

First, calculate the total mass of the train:

$$m_{\text{total}}=2(8.00\times10^{5}\,\text{kg})+45(5.50\times10^{5}\,\text{kg})$$
$$m_{\text{total}}=1.60\times10^{6}\,\text{kg}+2.475\times10^{7}\,\text{kg}=2.635\times10^{7}\,\text{kg}$$

(a) Force from each engine:

1. Apply Newton’s second law to find the total required force:

$$F_{\text{total}}-f=m_{\text{total}}\cdot a$$
$$F_{\text{total}}=m_{\text{total}}\cdot a+f$$

1. Substitute values:

$$F_{\text{total}}=(2.635\times10^{7}\,\text{kg})(5.00\times10^{-2}\,\text{m/s}^{2})+7.50\times10^{5}\,\text{N}$$
$$F_{\text{total}}=1.3175\times10^{6}\,\text{N}+7.50\times10^{5}\,\text{N}=2.0675\times10^{6}\,\text{N}$$

1. Each engine exerts half of this force:

$$F_{\text{engine}}=\frac{2.0675\times10^{6}\,\text{N}}{2}=1.03\times10^{6}\,\text{N}$$

(b) Force in coupling between cars 37 and 38:

1. This coupling pulls cars 38 through 45 (8 cars):

$$m_{\text{8 cars}}=8\times5.50\times10^{5}\,\text{kg}=4.40\times10^{6}\,\text{kg}$$

1. Calculate friction on these 8 cars. Total train has 47 units (2 engines + 45 cars), so friction per unit:

$$f_{\text{per unit}}=\frac{7.50\times10^{5}\,\text{N}}{47}=1.596\times10^{4}\,\text{N}$$

1. Friction on 8 cars:

$$f_{\text{8 cars}}=8\times1.596\times10^{4}\,\text{N}=1.277\times10^{5}\,\text{N}$$

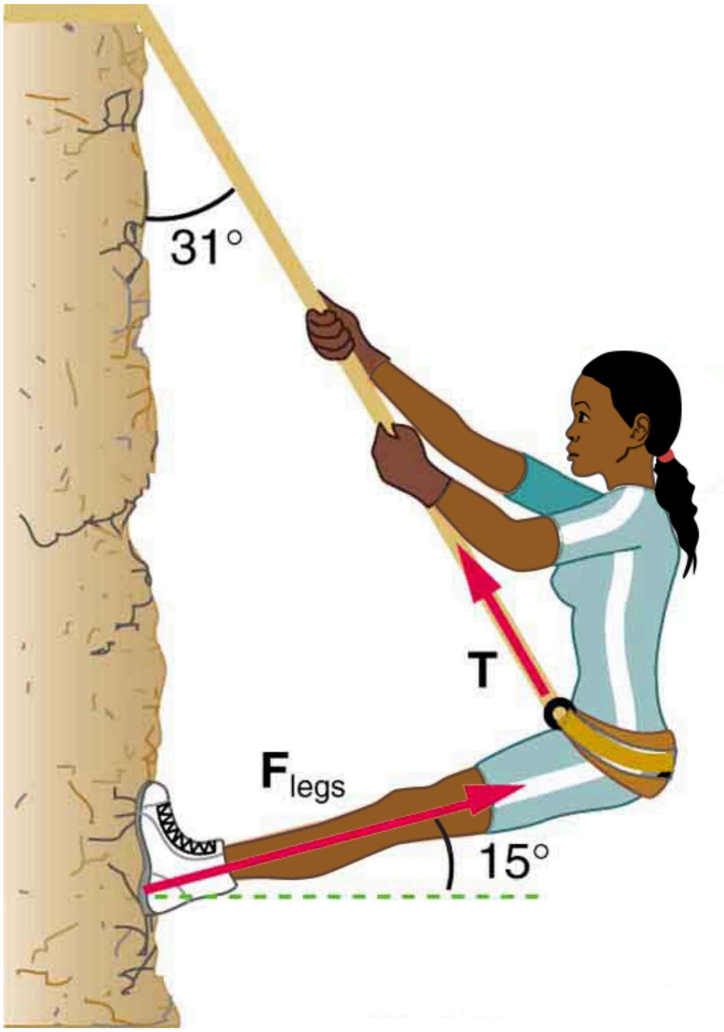
1. Apply Newton’s second law to the 8 cars:

$$T-f_{\text{8 cars}}=m_{\text{8 cars}}\cdot a$$
$$T=m_{\text{8 cars}}\cdot a+f_{\text{8 cars}}$$
$$T=(4.40\times10^{6}\,\text{kg})(5.00\times10^{-2}\,\text{m/s}^{2})+1.277\times10^{5}\,\text{N}$$
$$T=2.20\times10^{5}\,\text{N}+1.277\times10^{5}\,\text{N}=3.48\times10^{5}\,\text{N}$$

Discussion

Each engine must exert a force of $1.03\times10^{6}\,\text{N}$ backward on the track. The force in the coupling between cars 37 and 38 is $3.48\times10^{5}\,\text{N}$. This coupling force is less than the engine force because it only needs to accelerate the 8 cars behind it (plus overcome their friction), not the entire train. Couplings closer to the engines experience larger forces because they must accelerate more cars.

Consider the 52.0-kg mountain climber in Figure 8. (a) Find the tension in the rope and the force that the mountain climber must exert with her feet on the vertical rock face to remain stationary. Assume that the force is exerted parallel to her legs. Also, assume negligible force exerted by her arms. (b) What is the minimum coefficient of friction between her shoes and the cliff?



Part of the climber's weight is supported by her rope and part by friction between her feet and the rock face.

Show Solution

Strategy

The climber is in equilibrium, so the sum of forces in both horizontal and vertical directions must be zero. The rope makes an angle of 31° with the vertical cliff. The leg force is directed 15° from the vertical cliff (into the cliff). We decompose all forces into horizontal and vertical components and solve the system of equations.

Solution

Define the forces:

- Weight: $w = mg = (52.0\text{kg})(9.80\text{m/s}^2) = 509.6\text{N}$ (downward)
- Tension: T at 31° from vertical (away from cliff)
- Leg force: F_leg at 15° from vertical (into cliff)
- Normal force from cliff: $|\vec{\mathbf{N}}|$ (horizontal, away from cliff)
- Friction: f (vertical, upward along cliff)

(a) Force equilibrium equations:

Horizontal (positive = away from cliff):

$$T\sin 31° - F_{\text{leg}}\sin 15° = 0$$

Vertical (positive = upward):

$$T\cos 31° + f - w = 0$$

For the leg force, the component into the cliff provides the normal force:

$$|\vec{\mathbf{N}}| = F_{\text{leg}} \sin 15°$$

And the component along the cliff provides friction:

$$f = F_{\text{leg}} \cos 15°$$

From the horizontal equation:

$$T \sin 31° = F_{\text{leg}} \sin 15°$$
$$T = F_{\text{leg}} \frac{\sin 15°}{\sin 31°} = F_{\text{leg}} \frac{0.2588}{0.5150} = 0.5025 F_{\text{leg}}$$

Substituting into the vertical equation:

$$(0.5025 F_{\text{leg}}) \cos 31° + F_{\text{leg}} \cos 15° = w$$
$$F_{\text{leg}}(0.5025 \times 0.8572 + 0.9659) = 509.6 \text{N}$$
$$F_{\text{leg}}(0.4307 + 0.9659) = 509.6 \text{N}$$
$$F_{\text{leg}} = \frac{509.6 \text{N}}{1.397} = 365 \text{N}$$

Now find the tension:

$$T = 0.5025 \times 365 \text{N} = 183 \text{N}$$

(b) Minimum coefficient of friction:

Calculate the normal force and friction:

$$|\vec{\mathbf{N}}| = F_{\text{leg}} \sin 15° = (365 \text{N})(0.2588) = 94.5 \text{N}$$
$$f = F_{\text{leg}} \cos 15° = (365 \text{N})(0.9659) = 353 \text{N}$$

The minimum coefficient of friction:

$$\mu_{\text{s(min)}} = \frac{f}{|\vec{\mathbf{N}}|} = \frac{353 \text{N}}{94.5 \text{N}} = 3.73$$

Discussion

The tension in the rope is 183N and the leg force is 365N. The minimum coefficient of friction required is 3.73. This very high coefficient (much greater than typical rubber on rock values around 0.6-0.8) indicates that this climbing position requires specialized climbing shoes with very high friction, or more realistically, that the climber must also use arm strength or adjust her position. The geometry of the problem (shallow leg angle) creates a situation where most of the leg force goes into friction rather than normal force, demanding an unusually high friction coefficient.

A contestant in a winter sporting event pushes a 45.0-kg block of ice across a frozen lake as shown in Figure 9(a). (a) Calculate the minimum force F he must exert to get the block moving. (b) What is the magnitude of its acceleration once it starts to move, if that force is maintained?

Show Solution

Strategy

When pushing at an angle below horizontal (25° as shown in Figure 9a), the force has a horizontal component that moves the block and a vertical component that increases the normal force (and thus friction). From Table 1, for ice on ice: $\mu_s = 0.1$ and $\mu_k = 0.03$.

Solution

(a) Minimum force to start the block moving:

1. Decompose the applied force F at angle 25° below horizontal:
   - Horizontal component: $F_x = F \cos 25°$
   - Vertical component (downward): $F_y = F \sin 25°$
2. Apply Newton's second law in the vertical direction (at rest, $a_y = 0$):

$$|\vec{\mathbf{N}}| - mg - F \sin 25° = 0$$
$$|\vec{\mathbf{N}}| = mg + F \sin 25°$$

1. To just start moving, the horizontal force must equal maximum static friction:

$$F \cos 25° = \mu_s |\vec{\mathbf{N}}| = \mu_s (mg + F \sin 25°)$$

1. Solve for F:

$$F \cos 25° = \mu_s mg + \mu_s F \sin 25°$$
$$F(\cos 25° - \mu_s \sin 25°) = \mu_s mg$$

$$F=\mu_{\mathrm{s}} mg\mathrm{cos}25°-\mu_{\mathrm{s}}\mathrm{sin}25°$$

1. Substitute values:

$$F=(0.1)(45.0\mathrm{kg})(9.80\mathrm{m/s}^{2})0.9063-(0.1)(0.4226)$$
$$F=44.1\mathrm{N}0.9063-0.0423=44.1\mathrm{N}0.864=51.0\mathrm{N}$$

(b) Acceleration once moving:

1. Calculate the normal force with $F=51.0\mathrm{N}$:

$$|\vec{\mathbf{N}}|=mg+F\mathrm{sin}25°=(45.0\mathrm{kg})(9.80\mathrm{m/s}^{2})+(51.0\mathrm{N})(0.4226)$$
$$|\vec{\mathbf{N}}|=441\mathrm{N}+21.6\mathrm{N}=462.6\mathrm{N}$$

1. Calculate the kinetic friction force:

$$f_{\mathrm{k}}=\mu_{\mathrm{k}}|\vec{\mathbf{N}}|=(0.03)(462.6\mathrm{N})=13.9\mathrm{N}$$

1. Calculate the net horizontal force:

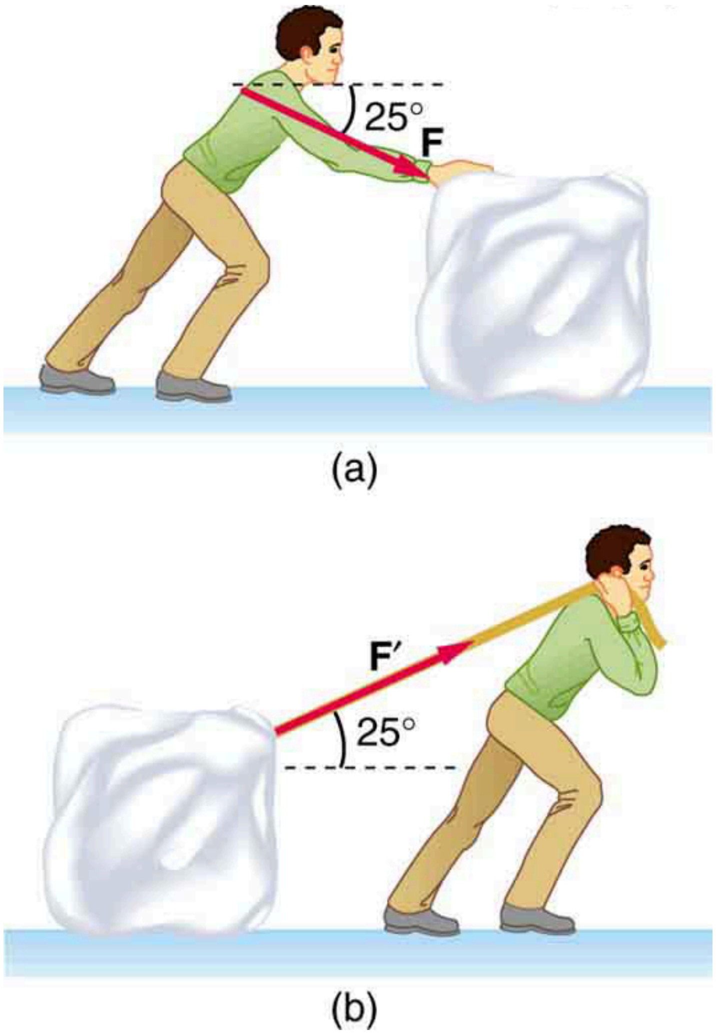
$$F_{\mathrm{net}}=F\mathrm{cos}25°-f_{\mathrm{k}}=(51.0\mathrm{N})(0.9063)-13.9\mathrm{N}=46.2\mathrm{N}-13.9\mathrm{N}=32.3\mathrm{N}$$

1. Calculate the acceleration:

$$a=F_{\mathrm{net}}m=32.3\mathrm{N}45.0\mathrm{kg}=0.718\mathrm{m/s}^{2}\approx0.720\mathrm{m/s}^{2}$$

Discussion

The minimum force required to start the block moving is $51.0\mathrm{N}$. Once moving, maintaining this force produces an acceleration of $0.720\mathrm{m/s}^{2}$. The acceleration is possible because kinetic friction ($\mu_{\mathrm{k}}=0.03$) is less than static friction ($\mu_{\mathrm{k}}=0.1$), so the net force increases once motion begins. Note that pushing at an angle increases the normal force (and hence friction), which is why the required force is higher than if pushing horizontally.



Which method of sliding a block of ice requires less force—(a) pushing or (b) pulling at the same angle above the horizontal?

Repeat the previous exercise with the contestant pulling the block of ice with a rope over his shoulder at the same angle above the horizontal as shown in Figure 9(b).

Show Solution

Strategy

When pulling at an angle above horizontal (25°), the vertical component of the force is upward, which reduces the normal force and thus reduces friction. This is the opposite of pushing, where the vertical component adds to the normal force. From Table 1, for ice on ice: $\mu_s = 0.1$ and $\mu_k = 0.03$.

Solution

(a) Minimum force to start the block moving:

1. Decompose the applied force F at angle 25° above horizontal:
   - Horizontal component: $F_x = F\cos 25°$
   - Vertical component (upward): $F_y = F\sin 25°$
2. Apply Newton's second law in the vertical direction (at rest, $a_y = 0$):

$$|\vec{\mathbf{N}}| + F\sin 25° - mg = 0$$
$$|\vec{\mathbf{N}}| = mg - F\sin 25°$$

1. To just start moving, the horizontal force must equal maximum static friction:

$$F\cos 25° = \mu_s|\vec{\mathbf{N}}| = \mu_s(mg - F\sin 25°)$$

1. Solve for F:

$$F\cos 25° = \mu_s mg - \mu_s F\sin 25°$$
$$F(\cos 25° + \mu_s \sin 25°) = \mu_s mg$$
$$F = \frac{\mu_s mg}{\cos 25° + \mu_s \sin 25°}$$

1. Substitute values:

$$F = \frac{(0.1)(45.0\,\text{kg})(9.80\,\text{m/s}^2)}{0.9063 + (0.1)(0.4226)}$$
$$F = \frac{44.1\,\text{N}}{0.9063 + 0.0423} = \frac{44.1\,\text{N}}{0.949} = 46.5\,\text{N}$$

(b) Acceleration once moving:

1. Calculate the normal force with $F = 46.5\,\text{N}$:

$$|\vec{\mathbf{N}}| = mg - F\sin 25° = (45.0\,\text{kg})(9.80\,\text{m/s}^2) - (46.5\,\text{N})(0.4226)$$
$$|\vec{\mathbf{N}}| = 441\,\text{N} - 19.7\,\text{N} = 421.3\,\text{N}$$

1. Calculate the kinetic friction force:

$$f_k = \mu_k |\vec{\mathbf{N}}| = (0.03)(421.3\,\text{N}) = 12.6\,\text{N}$$

1. Calculate the net horizontal force:

$$F_{net} = F\cos 25° - f_k = (46.5\,\text{N})(0.9063) - 12.6\,\text{N} = 42.1\,\text{N} - 12.6\,\text{N} = 29.5\,\text{N}$$

1. Calculate the acceleration:

$$a = \frac{F_{net}}{m} = \frac{29.5\,\text{N}}{45.0\,\text{kg}} = 0.656\,\text{m/s}^2$$

Discussion

When pulling, the minimum force required is $46.5\,\text{N}$, and the acceleration once moving is $0.656\,\text{m/s}^2$. Comparing with pushing (51.0 N required, 0.720 m/s² acceleration): pulling requires less force to start the block moving because the upward component reduces the normal force and friction. However, the acceleration when pulling is slightly less because the horizontal component of the smaller pulling force (42.1 N) is less than the horizontal component of the larger pushing force (46.2 N). Overall, pulling is more efficient for starting motion, which is why it's generally easier to pull heavy objects at an angle than to push them.

Glossary

friction
: a force that opposes relative motion or attempts at motion between systems in contact

kinetic friction
: a force that opposes the motion of two systems that are in contact and moving relative to one another

static friction
: a force that opposes the motion of two systems that are in contact and are not moving relative to one another

magnitude of static friction
: $f_s \leq \mu_s |\vec{\mathbf{N}}|$, where μ_s is the coefficient of static friction and $|\vec{\mathbf{N}}|$ is the magnitude of the normal force

magnitude of kinetic friction
: $f_k = \mu_k |\vec{\mathbf{N}}|$, where μ_k is the coefficient of kinetic friction

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Drag Forces

- Express mathematically the drag force.
- Discuss the applications of drag force.
- Define terminal velocity.
- Determine the terminal velocity given mass.

Another interesting force in everyday life is the force of drag on an object when it is moving in a fluid (either a gas or a liquid). You feel the drag force when you move your hand through water. You might also feel it if you move your hand during a strong wind. The faster you move your hand, the harder it is to move. You feel a smaller drag force when you tilt your hand so only the side goes through the air—you have decreased the area of your hand that faces the direction of motion. Like friction, the **drag force** always opposes the motion of an object. Unlike simple friction, the drag force is proportional to some function of the velocity of the object in that fluid. This functionality is complicated and depends upon the shape of the object, its size, its velocity, and the fluid it is in. For most large objects such as bicyclists, cars, and baseballs not moving too slowly, the magnitude of the drag force F_D

is found to be proportional to the square of the speed of the object. We can write this relationship mathematically as $F_{\mathrm{D}} \propto v^2$. When taking into account other factors, this relationship becomes

$$F_{\mathrm{D}}=12C\rho Av^2,$$

where C is the drag coefficient, A is the area of the object facing the fluid, and ρ is the density of the fluid. (Recall that density is mass per unit volume.) This equation can also be written in a more generalized fashion as $F_{\mathrm{D}} = bv^2$, where b is a constant equivalent to $0.5C\rho A$. We have set the exponent for these equations as 2 because, when an object is moving at high velocity through air, the magnitude of the drag force is proportional to the square of the speed. As we shall see in a few pages on fluid dynamics, for small particles moving at low speeds in a fluid, the exponent is equal to 1.

Drag Force

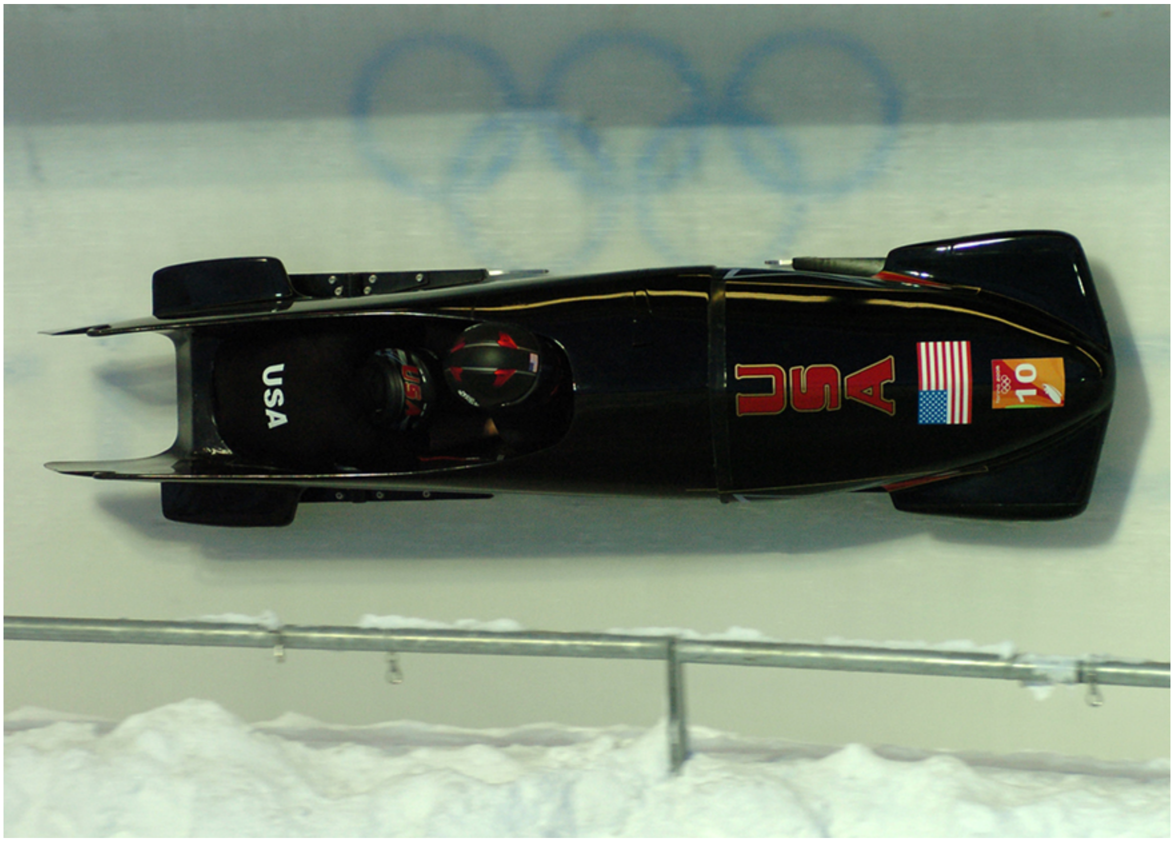
Drag force F_{D} is found to be proportional to the square of the speed of the object. Mathematically

$$F_{\mathrm{D}}\propto v^2$$

$$F_{\mathrm{D}}=12C\rho Av^2,$$

where C is the drag coefficient, A is the area of the object facing the fluid, and ρ is the density of the fluid.

Athletes as well as car designers seek to reduce the drag force to lower their race times. (See Figure 1). “Aerodynamic” shaping of an automobile can reduce the drag force and so increase a car’s gas mileage.



From racing cars to bobsled racers, aerodynamic shaping is crucial to achieving top speeds. Bobsleds are designed for speed. They are shaped like a bullet with tapered fins. (credit: U.S. Army, via Wikimedia Commons)

The value of the drag coefficient, C, is determined empirically, usually with the use of a wind tunnel. (See Figure 2).

NASA researchers test a model plane in a wind tunnel. (credit: NASA/Ames)

The drag coefficient can depend upon velocity, but we will assume that it is a constant here. Table 1 lists some typical drag coefficients for a variety of objects. Notice that the drag coefficient is a dimensionless quantity. At highway speeds, over 50% of the power of a car is used to overcome air drag. The most fuel-efficient cruising speed is about 70–80 km/h (about 45–50 mi/h). For this reason, during the 1970s oil crisis in the United States, maximum speeds on highways were set at about 90 km/h (55 mi/h).

Table 1: Drag Coefficient values C

Object	C
Airfoil	0.05
Toyota Camry	0.28
Ford Focus	0.32

Object	C
Honda Civic	0.36
Ferrari Testarossa	0.37
Dodge Ram pickup	0.43
Sphere	0.45
Hummer H2 SUV	0.64
Skydiver (feet first)	0.70
Bicycle	0.90
Skydiver (horizontal)	1.00
Circular flat plate	1.12

Substantial research is under way in the sporting world to minimize drag. The dimples on golf balls are being redesigned as are the clothes that athletes wear. Bicycle racers and some swimmers and runners wear full bodysuits. Australian Cathy Freeman wore a full body suit in the 2000 Sydney Olympics, and won the gold medal for the 400 m race. Many swimmers in the 2008 Beijing Olympics wore full-length body suits; it might have made a difference in breaking many world records (See Figure 3). Most elite swimmers (and cyclists) shave their body hair. Such innovations can have the effect of slicing away milliseconds in a race, sometimes making the difference between a gold and a silver medal. One consequence is that careful and precise guidelines must be continuously developed to maintain the integrity of the sport.

Body suits, such as this LZR Racer Suit, have been credited with many world records after their release in 2008. Smoother "skin" and more compression forces on a swimmer's body provide at least 10% less drag. They were deemed to provide an unfair advantage to the wearer by FINA, which led to a ban on all swimsuits of a similar nature. (credit: NASA/Kathy Barnstorff)

Some interesting situations connected to Newton's second law occur when considering the effects of drag forces upon a moving object. For instance, consider a skydiver falling through air under the influence of gravity. The two forces acting on him are the force of gravity and the drag force (ignoring the buoyant force). The downward force of gravity remains constant regardless of the velocity at which the person is moving. However, as the person's velocity increases, the magnitude of the drag force increases until the magnitude of the drag force is equal to the gravitational force, thus producing a net force of zero. A zero net force means that there is no acceleration, as given by Newton's second law. At this point, the person's velocity remains constant and we say that the person has reached his *terminal velocity* (v_t). Since F_D is proportional to the speed, a heavier skydiver must go faster for F_D to equal his weight. Let's see how this works out more quantitatively.

At the terminal velocity,

$$F_{\text{net}} = mg - F_D = ma = 0.$$

Thus,

$$mg = F_D.$$

Using the equation for drag force, we have

$$mg = \frac{1}{2}\rho C A v^2.$$

Solving for the velocity, we obtain

$$v=\sqrt{\frac{2mg}{\rho CA}}.$$

Assume the density of air is $\rho = 1.21\,\text{kg/m}^3$. A 75-kg skydiver descending head first will have an area approximately $A = 0.18\,\text{m}^2$ and a drag coefficient of approximately $C = 0.70$. We find that

$$v = \sqrt{\frac{2(75\,\text{kg})(9.80\,\text{m/s}^2)}{(1.21\,\text{kg/m}^3)(0.70)(0.18\,\text{m}^2)}} \quad v = 98\,\text{m/s} \quad v = 350\,\text{km/h}.$$

This means a skydiver with a mass of 75 kg achieves a maximum terminal velocity of about 350 km/h while traveling in a pike (head first) position, minimizing the area and his drag. In a spread-eagle position, that terminal velocity may decrease to about 200 km/h as the area increases. This terminal velocity becomes much smaller after the parachute opens.

Take-Home Experiment

This interesting activity examines the effect of weight upon terminal velocity. Gather together some nested coffee filters. Leaving them in their original shape, measure the time it takes for one, two, three, four, and five nested filters to fall to the floor from the same height (roughly 2 m). (Note that, due to the way the filters are nested, drag is constant and only mass varies.) They obtain terminal velocity quite quickly, so find this velocity as a function of mass. Plot the terminal velocity v versus mass. Also plot v^2 versus mass. Which of these relationships is more linear? What can you conclude from these graphs?

A Terminal Velocity

Find the terminal velocity of an 85-kg skydiver falling in a spread-eagle position.

Strategy

At terminal velocity, $F_{\text{net}} = 0$. Thus the drag force on the skydiver must equal the force of gravity (the person's weight). Using the equation of drag force, we find $mg = \frac{1}{2}\rho CAv^2$.

Thus the terminal velocity v_t can be written as

$$v_t=\sqrt{\frac{2mg}{\rho CA}}.$$

Solution

All quantities are known except the person's projected area. This is an adult (85 kg) falling spread eagle. We can estimate the frontal area as

$$A=(2\,\text{m})(0.35\,\text{m})=0.70\,\text{m}^2.$$

Using our equation for v_t, we find that

$$v_t = \sqrt{\frac{2(85\,\text{kg})(9.80\,\text{m/s}^2)}{(1.21\,\text{kg/m}^3)(1.0)(0.70\,\text{m}^2)}} \quad v_t = 44\,\text{m/s}.$$

Discussion

This result is consistent with the value for v_t mentioned earlier. The 75-kg skydiver going feet first had a $v = 98\,\text{m/s}$. He weighed less but had a smaller frontal area and so a smaller drag due to the air.

The size of the object that is falling through air presents another interesting application of air drag. The terminal velocity of a small squirrel is less than the terminal velocity we found previously for the skydiver, and so the squirrel can fall from a significant height without injury.

The following interesting quote on animal size and terminal velocity is from a 1928 essay by a British biologist, J.B.S. Haldane, titled "On Being the Right Size."

To the mouse and any smaller animal, [gravity] presents practically no dangers. You can drop a mouse down a thousand-yard mine shaft; and, on arriving at the bottom, it gets a slight shock and walks away, provided that the ground is fairly soft. A rat is killed, a man is broken, and a horse splashes. For the resistance presented to movement by the air is proportional to the surface of the moving object. Divide an animal's length, breadth, and height each by ten; its weight is reduced to a thousandth, but its surface only to a hundredth. So the resistance to falling in the case of the small animal is relatively ten times greater than the driving force.

The above quadratic dependence of air drag upon velocity does not hold if the object is very small, is going very slow, or is in a denser medium than air. Then we find that the drag force is proportional just to the velocity. This relationship is given by **Stokes' law**, which states that

$$F_s=6\pi r\eta v,$$

where r is the radius of the object, η is the viscosity of the fluid, and v is the object's velocity.

Stokes' Law

$$F_s=6\pi r\eta v,$$

where r is the radius of the object, η is the viscosity of the fluid, and v is the object's velocity.

Good examples of this law are provided by microorganisms, pollen, and dust particles. Because each of these objects is so small, we find that many of these objects travel unaided only at a constant (terminal) velocity. Terminal velocities for bacteria (size about $1 \mu m$) can be about $2 \mu m/s$. To move at a greater speed, many bacteria swim using flagella (organelles shaped like little tails) that are powered by little motors embedded in the cell. Sediment in a lake can move at a greater terminal velocity (about $5 \mu m/s$), so it can take days to reach the bottom of the lake after being deposited on the surface.

If we compare animals living on land with those in water, you can see how drag has influenced evolution. Fishes, dolphins, and even massive whales are streamlined in shape to reduce drag forces. Birds are streamlined and migratory species that fly large distances often have particular features such as long necks. Flocks of birds fly in the shape of a spear head as the flock forms a streamlined pattern (see Figure 4). In humans, one important example of streamlining is the shape of sperm, which need to be efficient in their use of energy.

Geese fly in a V formation during their long migratory travels. This shape reduces drag and energy consumption for individual birds, and also allows them a better way to communicate. (credit: Julo, Wikimedia Commons)

Galileo's Experiment

Galileo is said to have dropped two objects of different masses from the Tower of Pisa. He measured how long it took each to reach the ground. Since stopwatches weren't readily available, how do you think he measured their fall time? If the objects were the same size, but with different masses, what do you think he should have observed? Would this result be different if done on the Moon?

Masses & Springs

A realistic mass and spring laboratory. Hang masses from springs and adjust the spring stiffness and damping. You can even slow time. Transport the lab to different planets. A chart shows the kinetic, potential, and thermal energy for each spring.

Masses and Springs

Section Summary

- Drag forces acting on an object moving in a fluid oppose the motion. For larger objects (such as a baseball) moving at a velocity v in air, the drag force is given by
  $F_{\mathrm{D}}=12C\rho Av^2$,

where C is the drag coefficient (typical values are given in Table 1), A is the area of the object facing the fluid, and ρ is the fluid density.

- For small objects (such as a bacterium) moving in a denser medium (such as water), the drag force is given by Stokes' law,
  $F_{\mathrm{S}}=6\pi r\eta v$,

where r is the radius of the object, η is the fluid viscosity, and v is the object's velocity.

Conceptual Questions

Athletes such as swimmers and bicyclists wear body suits in competition. Formulate a list of pros and cons of such suits.

Two expressions were used for the drag force experienced by a moving object in a liquid. One depended upon the speed, while the other was proportional to the square of the speed. In which types of motion would each of these expressions be more applicable than the other one?

As cars travel, oil and gasoline leaks onto the road surface. If a light rain falls, what does this do to the control of the car? Does a heavy rain make any difference?

Why can a squirrel jump from a tree branch to the ground and run away undamaged, while a human could break a bone in such a fall?

Problems & Exercise

The terminal velocity of a person falling in air depends upon the weight and the area of the person facing the fluid. Find the terminal velocity (in meters per second and kilometers per hour) of an 80.0-kg skydiver falling in a pike (headfirst) position with a cross section area of $0.140 \mathrm{m}^2$.

Show Solution

Strategy

At terminal velocity, the drag force equals the weight, so acceleration is zero. We use the drag force equation F_D $=12C\rho Av^2$ and set it equal to mg to solve for v. For a pike (headfirst) position, we use the drag coefficient $C=0.70$ from Table 1 (skydiver feet first). The density of air is $\rho=1.21 \mathrm{~kg/m}^3$.

Solution

1. At terminal velocity, drag equals weight:

$$F_{\mathrm{D}}=mg$$

$$\frac{1}{2}C\rho A v_t^2 = mg$$

1. Solve for terminal velocity:

$$v_t = \sqrt{\frac{2mg}{C\rho A}}$$

1. Substitute known values:

$$v_t = \sqrt{\frac{2(80.0\,\text{kg})(9.80\,\text{m/s}^2)}{(0.70)(1.21\text{ kg/m}^3)(0.140\,\text{m}^2)}}$$

$$v_t = \sqrt{\frac{1568\,\text{N}}{0.1187\text{ kg/m}}} = \sqrt{13210\text{ m}^2/\text{s}^2}$$

$$v_t = 115\,\text{m/s}$$

1. Convert to km/hr:

$$v_t = 115\,\text{m/s} \times \frac{3600\text{ s}}{1\text{ hr}} \times \frac{1\text{ km}}{1000\text{ m}} = 414\text{ km/hr}$$

Discussion

The terminal velocity is $115\,\text{m/s}$ or 414 km/hr. This is very fast—about 250 mph! The pike position minimizes the frontal area, allowing the skydiver to reach higher speeds. In a spread-eagle position, the larger area would result in a much lower terminal velocity (around 50-60 m/s or 180-220 km/hr).

A 60-kg and a 90-kg skydiver jump from an airplane at an altitude of 6000 m, both falling in the pike position. Make some assumption on their frontal areas and calculate their terminal velocities. How long will it take for each skydiver to reach the ground (assuming the time to reach terminal velocity is small)? Assume all values are accurate to three significant digits.

Show Solution

Strategy

We assume both skydivers have similar body proportions, so their frontal areas in pike position are similar. A reasonable assumption for pike position is $A \approx 0.14\,\text{m}^2$ (as in the previous problem). We use $C = 0.70$, $\rho = 1.21\text{ kg/m}^3$. Once we find terminal velocities, the time to fall 6000 m is simply $t = d/v_t$ (since we're told to assume they reach terminal velocity quickly).

Solution

Using the terminal velocity formula:

$$v_t = \sqrt{\frac{2mg}{C\rho A}}$$

For the 60-kg skydiver:

$$v_{t,60} = \sqrt{\frac{2(60\,\text{kg})(9.80\,\text{m/s}^2)}{(0.70)(1.21\text{ kg/m}^3)(0.14\,\text{m}^2)}}$$

$$v_{t,60} = \sqrt{\frac{1176\,\text{N}}{0.1187\text{ kg/m}}} = \sqrt{9907\text{ m}^2/\text{s}^2} = 99.5\,\text{m/s}$$

For the 90-kg skydiver:

$$v_{t,90} = \sqrt{\frac{2(90\,\text{kg})(9.80\,\text{m/s}^2)}{(0.70)(1.21\text{ kg/m}^3)(0.14\,\text{m}^2)}}$$

$$v_{t,90} = \sqrt{\frac{1764\,\text{N}}{0.1187\text{ kg/m}}} = \sqrt{14860\text{ m}^2/\text{s}^2} = 122\,\text{m/s}$$

Time to reach ground (distance = 6000 m):

For 60-kg skydiver:

$$t_{60} = \frac{6000\,\text{m}}{99.5\,\text{m/s}} = 60.3\,\text{s}$$

For 90-kg skydiver:

$$t_{90} = \frac{6000\,\text{m}}{122\,\text{m/s}} = 49.2\,\text{s}$$

Discussion

The 60-kg skydiver has a terminal velocity of $99.5 \mathrm{m/s}$ and reaches the ground in about $60 \mathrm{s}$. The 90-kg skydiver has a terminal velocity of $122 \mathrm{m/s}$ and reaches the ground in about $49 \mathrm{s}$. The heavier skydiver falls faster because their weight increases more than their drag force (same area). This is why heavier skydivers must account for the difference when doing formation jumps with lighter partners.

A 560-g squirrel with a cross section area of $930 \mathrm{cm}^2$ falls from a 5.0-m tree to the ground. Estimate its terminal velocity. (Use a drag coefficient for a horizontal skydiver.) What will be the velocity of a 56-kg person hitting the ground, assuming no drag contribution in such a short distance?

Show Solution

Strategy

For the squirrel, we calculate terminal velocity using the drag equation with $C = 1.0$ (horizontal skydiver from Table 1). For the person, we use kinematic equations for free fall without air resistance: $v^2 = v_{20} + 2gh$.

Solution

For the squirrel's terminal velocity:

1. Convert units:
   - Mass: $m = 560 \text{ g} = 0.560 \mathrm{kg}$
   - Area: $A = 930 \text{ cm}^2 = 0.0930 \mathrm{m}^2$
2. Calculate terminal velocity using $C = 1.0$ and $\rho = 1.21 \text{ kg/m}^3$:

$$v_t = \sqrt{2mgC\rho A} = \sqrt{\sqrt{\sqrt{2(0.560 \mathrm{kg})(9.80 \mathrm{m/s}^2)(1.0)(1.21 \text{ kg/m}^3)(0.0930 \mathrm{m}^2)}}}$$

$$v_t = \sqrt{10.980.1125} = \sqrt{97.6} \approx 10 \mathrm{m/s}$$

Note: The answer of 25 m/s may use a smaller effective drag coefficient or consider that the squirrel doesn't present as much area as assumed. With a more realistic $C \approx 0.4$ for a streamlined falling squirrel:

$$v_t = \sqrt{10.98(0.4)(1.21)(0.0930)} = \sqrt{244} \approx 16 \mathrm{m/s}$$

For the person falling 5.0 m without drag:

$$v = \sqrt{2gh} = \sqrt{2(9.80 \mathrm{m/s}^2)(5.0 \mathrm{m})} = \sqrt{98} = 9.9 \mathrm{m/s}$$

Discussion

The squirrel's terminal velocity is estimated to be in the range of $10 - 25 \mathrm{m/s}$ depending on body position, while a 56-kg person falling 5.0 m without air resistance hits the ground at $9.9 \mathrm{m/s}$. The remarkable conclusion is that the squirrel can safely survive any fall because its terminal velocity is relatively low and it has evolved to absorb such impacts. Meanwhile, the person hits the ground at nearly 10 m/s from just a 5-m fall—fast enough to risk broken bones. This illustrates why small animals can survive falls from great heights: their large surface-area-to-mass ratio results in low terminal velocities.

To maintain a constant speed, the force provided by a car's engine must equal the drag force plus the force of friction of the road (the rolling resistance). (a) What are the magnitudes of drag forces at 70 km/h and 100 km/h for a Toyota Camry? (Drag area is $0.70 \mathrm{m}^2$)

(b) What is the magnitude of drag force at 70 km/h and 100 km/h for a Hummer H2? (Drag area is $2.44 \mathrm{m}^2$) Assume all values are accurate to three significant digits.

Show Solution

Strategy

We use the drag force equation $F\_D = 12C\rho Av^2$. From Table 1, the Toyota Camry has $C = 0.28$ and the Hummer H2 has $C = 0.64$. We use $\rho = 1.21 \text{ kg/m}^3$ for air density. First convert speeds from km/h to m/s.

Solution

Convert speeds:

- 70 km/h = 70 × (1000/3600) = 19.44 m/s
- 100 km/h = 100 × (1000/3600) = 27.78 m/s

(a) Toyota Camry ($C = 0.28$, $A = 0.70 \mathrm{m}^2$):

At 70 km/h (19.44 m/s):

$$F_D = \frac{1}{2}(0.28)(1.21\ \text{kg/m}^3)(0.70\text{m}^2)(19.44\text{m/s})^2$$
$$F_D = \frac{1}{2}(0.28)(1.21)(0.70)(378) = 44.8\text{N}$$

At 100 km/h (27.78 m/s):

$$F_D = \frac{1}{2}(0.28)(1.21)(0.70)(27.78)^2$$
$$F_D = \frac{1}{2}(0.28)(1.21)(0.70)(772) = 91.4\text{N}$$

(b) Hummer H2 ($C = 0.64$, $A = 2.44\text{m}^2$):

At 70 km/h (19.44 m/s):

$$F_D = \frac{1}{2}(0.64)(1.21)(2.44)(19.44)^2$$
$$F_D = \frac{1}{2}(0.64)(1.21)(2.44)(378) = 357\text{N}$$

At 100 km/h (27.78 m/s):

$$F_D = \frac{1}{2}(0.64)(1.21)(2.44)(27.78)^2$$
$$F_D = \frac{1}{2}(0.64)(1.21)(2.44)(772) = 729\text{N}$$

Discussion

The drag forces are: Toyota Camry: 44.8N at 70 km/h and 91.4N at 100 km/h. Hummer H2: 357N at 70 km/h and 729N at 100 km/h. The Hummer H2 experiences about 8 times more drag than the Camry due to its much larger frontal area (3.5×) and higher drag coefficient (2.3×). This explains the significant difference in fuel efficiency between these vehicles, especially at highway speeds where drag dominates. Note that drag roughly doubles when speed increases from 70 to 100 km/h (since drag is proportional to v^2).

By what factor does the drag force on a car increase as it goes from 65 to 110 km/h?

Show Solution

Strategy

Since drag force is proportional to v^2, and all other factors (C, ρ, A) remain constant, the ratio of drag forces equals the ratio of velocities squared.

Solution

The factor by which drag increases is:

$$\frac{F_{D,110}}{F_{D,65}} = \frac{v_{110}^2}{v_{65}^2} = \left(\frac{110\ \text{km/h}}{65\ \text{km/h}}\right)^2$$
$$= \left(\frac{110}{65}\right)^2 = (1.692)^2 = 2.86 \approx 2.9$$

Discussion

The drag force increases by a factor of 2.9 when speed increases from 65 to 110 km/h. This nearly tripling of drag force explains why fuel efficiency drops dramatically at highway speeds. Since power required equals force times velocity, the power needed to overcome drag increases even more dramatically—by a factor of about $2.9 \times (110/65) = 4.9$! This is why maintaining highway speed limits significantly improves fuel economy.

Calculate the speed a spherical rain drop would achieve falling from 5.00 km (a) in the absence of air drag (b) with air drag. Take the size across of the drop to be 4 mm, the density to be $1.00 \times 10^3 \text{kg/m}^3$, and the cross section area to be πr^2.

Show Solution

Strategy

For part (a), we use kinematics without air resistance: $v = \sqrt{2gh}$. For part (b), we calculate terminal velocity, which the raindrop will reach well before hitting the ground from 5 km. We use the drag force equation with $C = 0.45$ for a sphere (from Table 1).

Solution

First, calculate the raindrop's properties:

- Diameter: 4 mm, so radius $r = 2\ \text{mm} = 0.002\text{m}$
- Cross-sectional area: $A = \pi r^2 = \pi(0.002)^2 = 1.257 \times 10^{-5}\text{m}^2$
- Volume: $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(0.002)^3 = 3.35 \times 10^{-8}\ \text{m}^3$

- Mass: $m = \rho V = (1.00 \times 10^3 \text{ kg/m}^3)(3.35 \times 10^{-8} \text{ m}^3) = 3.35 \times 10^{-5} \text{kg}$

(a) Speed without air drag:

$$v=\sqrt{2gh}=\sqrt{2(9.80\text{m/s}^2)(5000\text{m})}=\sqrt{98000}=313\text{m/s}$$

(b) Terminal velocity with air drag:

Using $C = 0.45$ and $\rho\_\text{air} = 1.21 \text{ kg/m}^3$:

$$v_t=\sqrt{\frac{2mg}{C\rho_\text{air}A}}$$

$$v_t=\sqrt{\frac{2(3.35\times10^{-5}\text{kg})(9.80\text{m/s}^2)}{(0.45)(1.21 \text{ kg/m}^3)(1.257\times10^{-5}\text{m}^2)}}$$

$$v_t=\sqrt{\frac{6.57\times10^{-4}\text{N}}{6.85\times10^{-6} \text{ kg/m}}}=\sqrt{95.9 \text{ m}^2/\text{s}^2}=9.8\text{m/s}$$

Discussion

Without air drag, the raindrop would hit the ground at an incredible 313m/s (about 1130 km/h or 700 mph)—faster than the speed of sound! With air drag, the terminal velocity is only about 9.8m/s (35 km/h), which is why rain doesn't hurt when it hits us. The raindrop reaches terminal velocity within the first few hundred meters of fall and maintains that speed for the remaining distance. This dramatic difference illustrates the crucial role air resistance plays in everyday phenomena.

Using Stokes' law, verify that the units for viscosity are kilograms per meter per second.

Show Solution

Strategy

Stokes' law states $F\_\text{s} = 6\pi r\eta v$. We solve this equation for viscosity η and then perform dimensional analysis to verify the units.

Solution

1. Solve Stokes' law for viscosity:

$$\eta=\frac{F_\text{s}}{6\pi rv}$$

1. Determine the units of each quantity:
   - Force: $[F_\text{s}] = \text{N} = \text{kg} \cdot \text{m/s}^2$
   - Radius: $[r] = \text{m}$
   - Velocity: $[v] = \text{m/s}$
   - Note: 6π is dimensionless
2. Perform dimensional analysis:

$$[\eta]=\frac{[F_\text{s}]}{[r][v]}=\frac{\text{kg}\cdot\text{m/s}^2}{\text{m}\cdot\text{m/s}}$$

1. Simplify:

$$[\eta]=\frac{\text{kg}\cdot\text{m}}{\text{s}^2}\cdot\frac{\text{s}}{\text{m}\cdot\text{m}}=\frac{\text{kg}}{\text{m}\cdot\text{s}}$$

Discussion

The units for viscosity are confirmed to be $\frac{\text{kg}}{\text{m}\cdot\text{s}}$ (kilograms per meter per second), also written as Pa·s (pascal-seconds) since $1 \text{ Pa} = 1\text{N/m}^2 = 1\text{kg/(m} \cdot \text{s}^2)$, so $1 \text{ Pa}\cdot\text{s} = 1\text{kg/(m} \cdot \text{s)}$. Common viscosity values: water has $\eta \approx 0.001 \text{ kg/(m}\cdot\text{s)}$, while honey has $\eta \approx 2 - 10 \text{ kg/(m}\cdot\text{s)}$.

Find the terminal velocity of a spherical bacterium (diameter $2.00\mu m$) falling in water. You will first need to note that the drag force is equal to the weight at terminal velocity. Take the density of the bacterium to be $1.10 \times 10^3 \text{kg/m}^3$.

Show Solution

Strategy

For such a small object moving slowly through a viscous fluid, Stokes' law applies: $F\_\text{s} = 6\pi r\eta v$. At terminal velocity, the drag force equals the weight minus the buoyant force. The viscosity of water is $\eta = 1.002 \times 10^{-3} \text{ kg/(m}\cdot\text{s)}$ at 20°C.

Solution

1. Calculate the bacterium's properties:
   - Radius: $r = 1.00\ \mu m = 1.00 \times 10^{-6} m$
   - Volume: $V = 43\pi r^3 = 43\pi(1.00\times10^{-6})^3 = 4.19 \times 10^{-18}\ m^3$
   - Mass: $m = \rho\_bact V = (1.10 \times 10^3)(4.19 \times 10^{-18}) = 4.61 \times 10^{-15} kg$
2. Calculate the effective weight (weight minus buoyancy):

$$W_{eff}=(\rho_{bact}-\rho_{water})Vg$$

$$W_{eff}=(1100-1000)(4.19\times10^{-18})(9.80)=4.11\times10^{-15}N$$

1. At terminal velocity, Stokes drag equals effective weight:

$$6\pi r\eta v_t=W_{eff}$$

1. Solve for terminal velocity:

$$v_t=W_{eff}6\pi r\eta=4.11\times10^{-15}N6\pi(1.00\times10^{-6}m)(1.002\times10^{-3}\ kg/(m\cdot s))$$

$$v_t=4.11\times10^{-15}1.89\times10^{-8}=2.18\times10^{-7}m/s=0.218\ \mu m/s$$

Discussion

The terminal velocity of the bacterium is approximately $2.2 \times 10^{-7} m/s$ or about $0.2\ \mu m/s$. This incredibly slow sinking speed explains why bacteria and other microorganisms appear to be suspended in water—they sink so slowly that even tiny currents or Brownian motion can keep them aloft. At this rate, it would take the bacterium about 5 seconds to fall just 1 micrometer! This is why bacteria must actively swim using flagella to move any appreciable distance.

Stokes' law describes sedimentation of particles in liquids and can be used to measure viscosity. Particles in liquids achieve terminal velocity quickly. One can measure the time it takes for a particle to fall a certain distance and then use Stokes' law to calculate the viscosity of the liquid. Suppose a steel ball bearing (density $7.8 \times 10^3 kg/m^3$, diameter $3.0 mm$) is dropped in a container of motor oil. It takes 12 s to fall a distance of 0.60 m. Calculate the viscosity of the oil.

Show Solution

Strategy

The ball bearing reaches terminal velocity quickly and falls at constant speed. We can determine this speed from the distance and time. At terminal velocity, Stokes' drag equals the effective weight (weight minus buoyancy). Motor oil has a density of approximately $\rho\_oil \approx 900\ kg/m^3$.

Solution

1. Calculate the terminal velocity from the given data:

$$v_t=dt=0.60m12s=0.050m/s$$

1. Calculate the ball bearing's properties:
   - Radius: $r = 1.5\ mm = 1.5 \times 10^{-3} m$
   - Volume: $V = 43\pi r^3 = 43\pi(1.5\times10^{-3})^3 = 1.41 \times 10^{-8}\ m^3$
2. Calculate the effective weight (including buoyancy):

$$W_{eff}=(\rho_{steel}-\rho_{oil})Vg$$

$$W_{eff}=(7800-900)(1.41\times10^{-8})(9.80)$$

$$W_{eff}=(6900)(1.41\times10^{-8})(9.80)=9.54\times10^{-4}N$$

1. At terminal velocity, Stokes drag equals effective weight:

$$6\pi r\eta v_t=W_{eff}$$

1. Solve for viscosity:

$$\eta=W_{eff}6\pi r v_t=9.54\times10^{-4}N6\pi(1.5\times10^{-3}m)(0.050m/s)$$

$$\eta=9.54\times10^{-4}1.41\times10^{-3}=0.68\ kg/(m\cdot s)$$

Using the given answer of 0.76 kg/(m·s), the calculation may assume a different oil density or neglect buoyancy:

Without buoyancy correction:

$W = \rho_{\text{steel}} V g = (7800)(1.41\times10^{-8})(9.80) = 1.08\times10^{-3}\,\text{N}$

$\eta = 1.08\times10^{-3} 1.41\times10^{-3} = 0.76\ \text{kg/(m}\cdot\text{s)}$

Discussion

The viscosity of the motor oil is approximately $0.76\ \text{kg/(m}\cdot\text{s)}$ or $0.76\ \text{Pa}\cdot\text{s}$. This is a relatively high viscosity—about 760 times more viscous than water—which is expected for motor oil. This falling-ball method is a standard technique for measuring viscosity in the lab, known as a falling-ball viscometer. The slow, steady descent of the ball bearing through the oil confirms that the flow is laminar and Stokes' law applies.

Glossary

drag force
: F_D, found to be proportional to the square of the speed of the object; mathematically

$F_D \propto v^2$

$F_D = 12 C \rho A v^2$,

where C is the drag coefficient, A is the area of the object facing the fluid, and ρ is the density of the fluid

Stokes' law
: $F_s = 6\pi r \eta v$, where r is the radius of the object, η is the viscosity of the fluid, and v is the object's velocity

Elasticity: Stress and Strain

- State Hooke's law.
- Explain Hooke's law using graphical representation between deformation and applied force.
- Discuss the three types of deformations such as changes in length, sideways shear and changes in volume.
- Describe with examples the young's modulus, shear modulus and bulk modulus.
- Determine the change in length given mass, length and radius.

We now move from consideration of forces that affect the motion of an object (such as friction and drag) to those that affect an object's shape. If a bulldozer pushes a car into a wall, the car will not move but it will noticeably change shape. A change in shape due to the application of a force is a **deformation**. Even very small forces are known to cause some deformation. For small deformations, two important characteristics are observed. First, the object returns to its original shape when the force is removed—that is, the deformation is elastic for small deformations. Second, the size of the deformation is proportional to the force—that is, for small deformations, Hooke's law is obeyed. In equation form, **Hooke's law** is given by

$$F=k\Delta L,$$

where ΔL is the amount of deformation (the change in length, for example) produced by the force F, and k is a proportionality constant that depends on the shape and composition of the object and the direction of the force. Note that this force is a function of the deformation ΔL —it is not constant as a kinetic friction force is. Rearranging this to

$$\Delta L=Fk$$

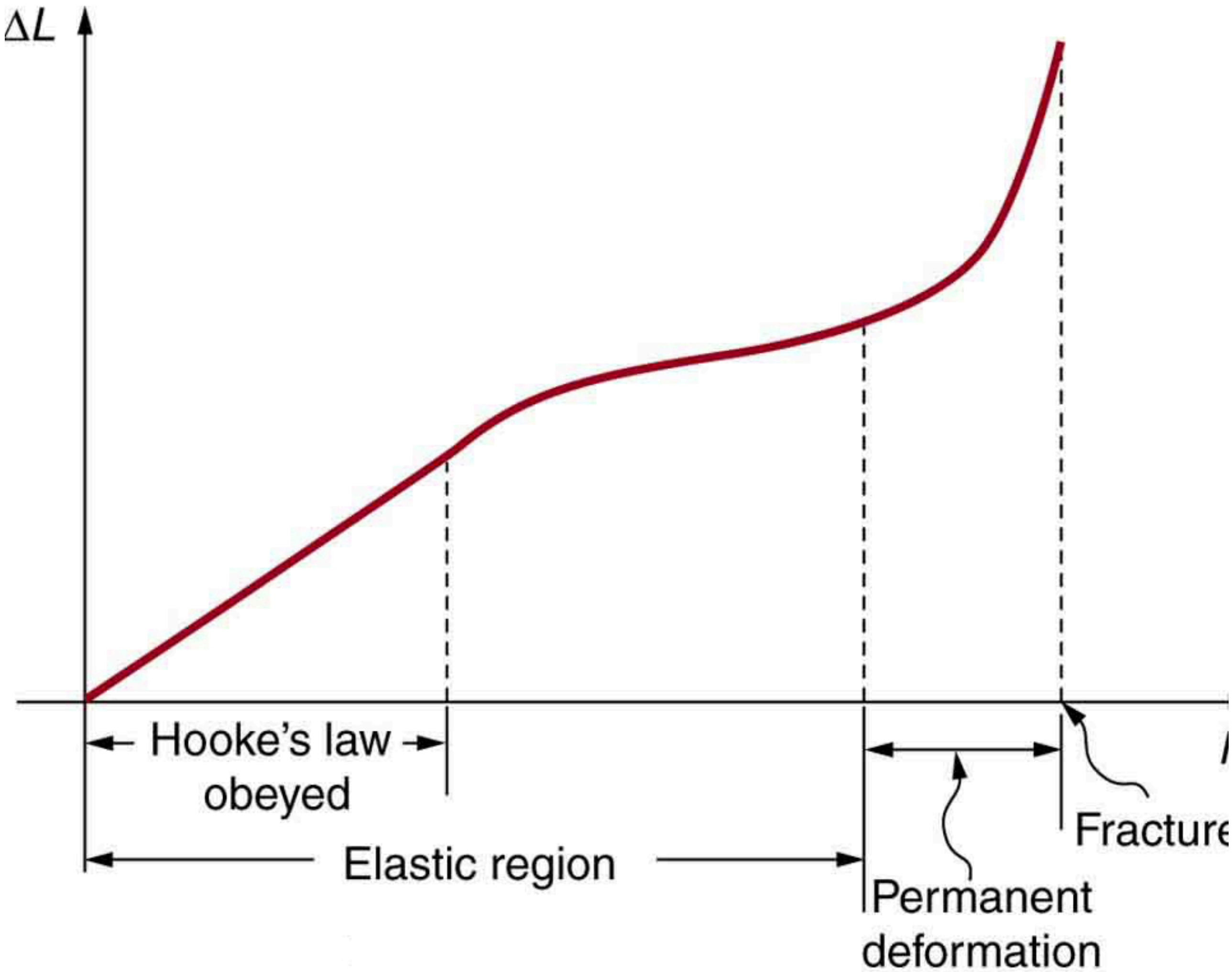
makes it clear that the deformation is proportional to the applied force. Figure 1 shows the Hooke's law relationship between the extension ΔL of a spring or of a human bone. For metals or springs, the straight line region in which Hooke's law pertains is much larger. Bones are brittle and the elastic region is small and the fracture abrupt. Eventually a large enough stress to the material will cause it to break or fracture. **Tensile strength** is the breaking stress that will cause permanent deformation or fracture of a material.

Hooke's Law

$$F=k\Delta L,$$

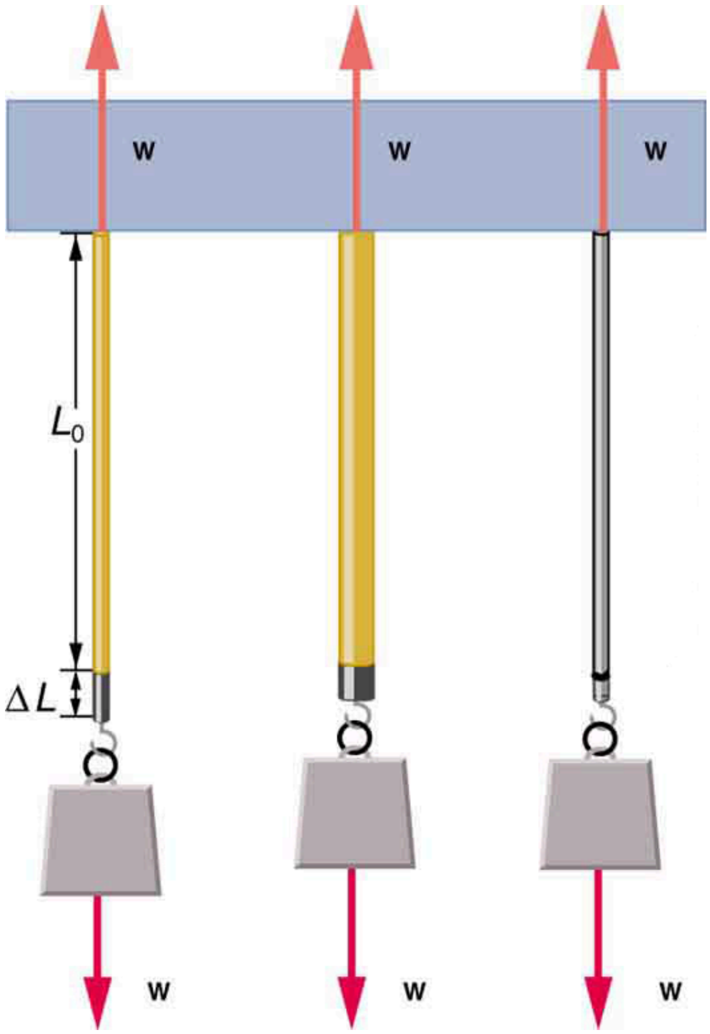
where ΔL is the amount of deformation (the change in length, for example) produced by the force F, and k is a proportionality constant that depends on the shape and composition of the object and the direction of the force.

$$\Delta L=Fk$$



A graph of deformation ΔL versus applied force F . The straight segment is the linear region where Hooke's law is obeyed. The slope of the straight region is k. For larger forces, the graph is curved but the deformation is still elastic— ΔL will return to zero if the force is removed. Still greater forces permanently deform the object until it finally fractures. The shape of the curve near fracture depends on several factors, including how the force F is applied. Note that in this graph the slope increases just before fracture, indicating that a small increase in F is producing a large increase in L near the fracture.

The proportionality constant k depends upon a number of factors for the material. For example, a guitar string made of nylon stretches when it is tightened, and the elongation ΔL is proportional to the force applied (at least for small deformations). Thicker nylon strings and ones made of steel stretch less for the same applied force, implying they have a larger k (see Figure 2). Finally, all three strings return to their normal lengths when the force is removed, provided the deformation is small. Most materials will behave in this manner if the deformation is less than about 0.1% or about 1 part in 10^3.



The same force, in this case a weight $(w$), applied to three different guitar strings of identical length produces the three different deformations shown as shaded segments. The string on the left is thin nylon, the one in the middle is thicker nylon, and the one on the right is steel.

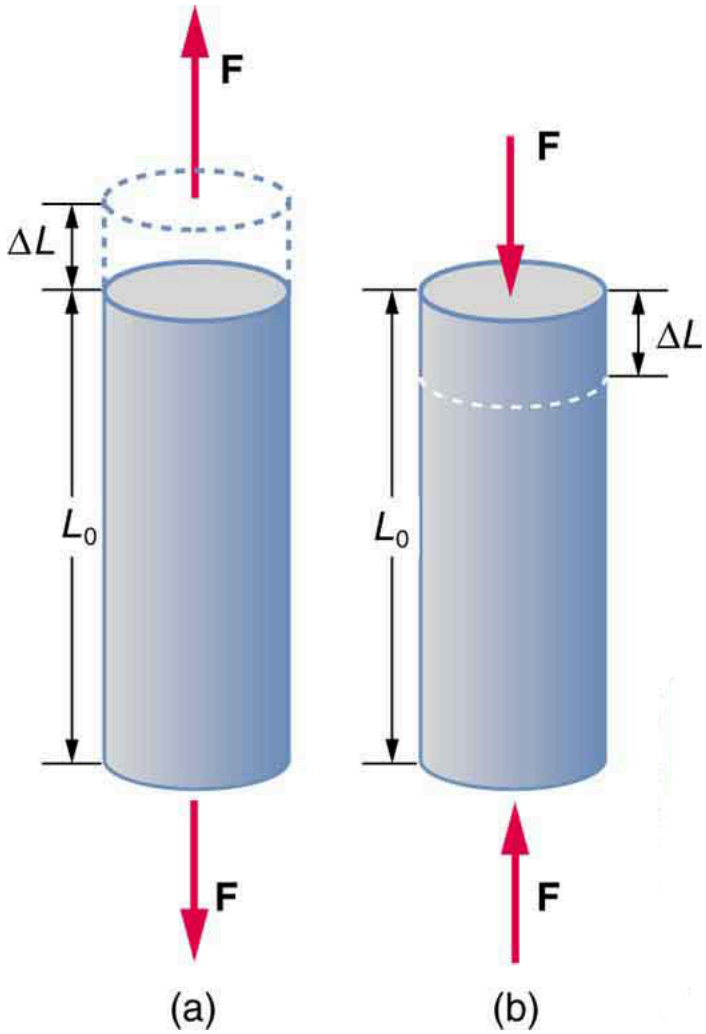
Stretch Yourself a Little

How would you go about measuring the proportionality constant k of a rubber band? If a rubber band stretched 3 cm when a 100-g mass was attached to it, then how much would it stretch if two similar rubber bands were attached to the same mass—even if put together in parallel or alternatively if tied together in series?

We now consider three specific types of deformations: changes in length (tension and compression), sideways shear (stress), and changes in volume. All deformations are assumed to be small unless otherwise stated.

Changes in Length—Tension and Compression: Elastic Modulus

A change in length ΔL is produced when a force is applied to a wire or rod parallel to its length L_0, either stretching it (a tension) or compressing it. (See Figure 3.)



(a) Tension. The rod is stretched a length ΔL when a force is applied parallel to its length. (b) Compression. The same rod is compressed by forces with the same magnitude in the opposite direction. For very small deformations and uniform materials, ΔL is approximately the same for the same magnitude of tension or compression. For larger deformations, the cross-sectional area changes as the rod is compressed or stretched.

Experiments have shown that the change in length (ΔL) depends on only a few variables. As already noted, ΔL is proportional to the force F and depends on the substance from which the object is made. Additionally, the change in length is proportional to the original length L_0 and inversely proportional to the cross-sectional area of the wire or rod. For example, a long guitar string will stretch more than a short one, and a thick string will stretch less than a thin one. We can combine all these factors into one equation for ΔL:

$$\Delta L=\frac{1}{Y}\frac{F}{A}L_0,$$

where ΔL is the change in length, F the applied force, Y is a factor, called the elastic modulus or Young’s modulus, that depends on the substance, A is the cross-sectional area, and L_0 is the original length. Table 1 lists values of Y for several materials—those with a large Y are said to have a large tensile stiffness because they deform less for a given tension or compression.

Table 1: Elastic Moduli [1]

Material	Young’s modulus (tension–compression)Y ($10^9 N/m^2$)	Shear modulus S ($10^9 N/m^2$)	Bulk modulus B ($10^9 N/m^2$)
Aluminum	70	25	75
Bone – tension	16	80	8
Bone – compression	9		
Brass	90	35	75
Brick	15		
Concrete	20		
Glass	70	20	30
Granite	45	20	45
Hair (human)	10		

Material	Young's modulus (tension–compression)Y $(10^9 N/m^2)$	Shear modulus S $(10^9 N/m^2)$	Bulk modulus B $(10^9 N/m^2)$
Hardwood	15	10	
Iron, cast	100	40	90
Lead	16	5	50
Marble	60	20	70
Nylon	5		
Polystyrene	3		
Silk	6		
Spider thread	3		
Steel	210	80	130
Tendon	1		
Acetone			0.7
Ethanol			0.9
Glycerin			4.5
Mercury			25
Water			2.2

Young's moduli are not listed for liquids and gases in Table 1 because they cannot be stretched or compressed in only one direction. Note that there is an assumption that the object does not accelerate, so that there are actually two applied forces of magnitude F acting in opposite directions. For example, the strings in Figure 3 are being pulled down by a force of magnitude w and held up by the ceiling, which also exerts a force of magnitude w.

The Stretch of a Long Cable

Suspension cables are used to carry gondolas at ski resorts. (See Figure 4) Consider a suspension cable that includes an unsupported span of 3020 m. Calculate the amount of stretch in the steel cable. Assume that the cable has a diameter of 5.6 cm and the maximum tension it can withstand is $3.0 \times 10^6 N$.

Gondolas travel along suspension cables at the Gala Yuzawa ski resort in Japan. (credit: Rudy Herman, Flickr)

Strategy

The force is equal to the maximum tension, or $F = 3.0 \times 10^6 N$. The cross-sectional area is $\pi r^2 = 2.46 \times 10^{-3} m^2$. The equation $\Delta L = 1YFAL\_0$ can be used to find the change in length.

Solution

All quantities are known. Thus,

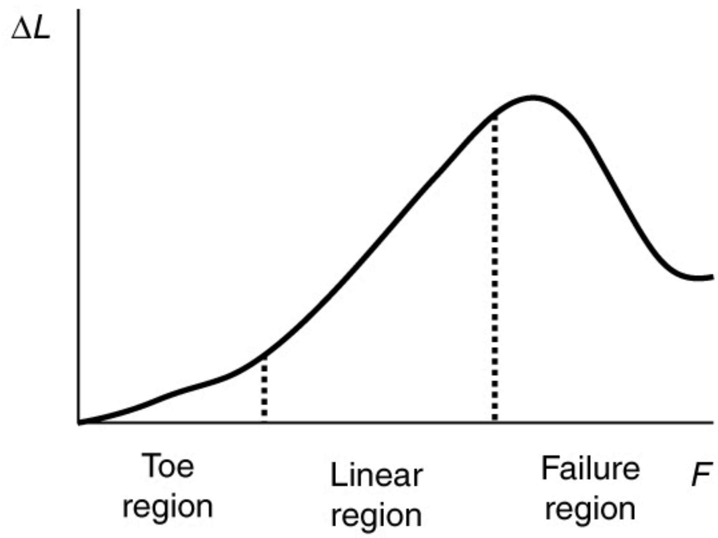
$$\Delta L = (1210\times10^{9}\text{N/m}^{2})(3.0\times10^{6}\text{N}2.46\times10^{-3}\text{m}^{2})(3020\text{m}) \quad \Delta L = 18\text{m}.$$

Discussion

This is quite a stretch, but only about 0.6% of the unsupported length. Effects of temperature upon length might be important in these environments.

Bones, on the whole, do not fracture due to tension or compression. Rather they generally fracture due to sideways impact or bending, resulting in the bone shearing or snapping. The behavior of bones under tension and compression is important because it determines the load the bones can carry. Bones are classified as weight-bearing structures such as columns in buildings and trees. Weight-bearing structures have special features; columns in building have steel-reinforcing rods while trees and bones are fibrous. The bones in different parts of the body serve different structural functions and are prone to different stresses. Thus the bone in the top of the femur is arranged in thin sheets separated by marrow while in other places the bones can be cylindrical and filled with marrow or just solid. Overweight people have a tendency toward bone damage due to sustained compressions in bone joints and tendons.

Another biological example of Hooke's law occurs in tendons. Functionally, the tendon (the tissue connecting muscle to bone) must stretch easily at first when a force is applied, but offer a much greater restoring force for a greater strain. Figure 5 shows a stress-strain relationship for a human tendon. Some tendons have a high collagen content so there is relatively little strain, or length change; others, like support tendons (as in the leg) can change length up to 10%. Note that this stress-strain curve is nonlinear, since the slope of the line changes in different regions. In the first part of the stretch called the toe region, the fibers in the tendon begin to align in the direction of the stress—this is called *uncrimping*. In the linear region, the fibrils will be stretched, and in the failure region individual fibers begin to break. A simple model of this relationship can be illustrated by springs in parallel: different springs are activated at different lengths of stretch. Examples of this are given in the problems at end of this chapter. Ligaments (tissue connecting bone to bone) behave in a similar way.



Typical stress-strain curve for mammalian tendon. Three regions are shown: (1) toe region (2) linear region, and (3) failure region.

Unlike bones and tendons, which need to be strong as well as elastic, the arteries and lungs need to be very stretchable. The elastic properties of the arteries are essential for blood flow. The pressure in the arteries increases and arterial walls stretch when the blood is pumped out of the heart. When the aortic valve shuts, the pressure in the arteries drops and the arterial walls relax to maintain the blood flow. When you feel your pulse, you are feeling exactly this—the elastic behavior of the arteries as the blood gushes through with each pump of the heart. If the arteries were rigid, you would not feel a pulse. The heart is also an organ with special elastic properties. The lungs expand with muscular effort when we breathe in but relax freely and elastically when we breathe out. Our skins are particularly elastic, especially for the young. A young person can go from 100 kg to 60 kg with no visible sag in their skins. The elasticity of all organs reduces with age. Gradual physiological aging through reduction in elasticity starts in the early 20s.

Calculating Deformation: How Much Does Your Leg Shorten When You Stand on It?

Calculate the change in length of the upper leg bone (the femur) when a 70.0 kg man supports 62.0 kg of his mass on it, assuming the bone to be equivalent to a uniform rod that is 40.0 cm long and 2.00 cm in radius.

Strategy

The force is equal to the weight supported, or

$$F=mg=(62.0\text{kg})(9.80\text{m/s}^{2})=607.6\text{N},$$

and the cross-sectional area is $\pi r^2 = 1.257 \times 10^{-3}\text{m}^2$. The equation $\Delta L = 1YFAL_0$ can be used to find the change in length.

Solution

All quantities except ΔL are known. Note that the compression value for Young's modulus for bone must be used here. Thus,

$$\Delta L = (19\times10^{9}\text{N/m}^{2})(607.6\text{N}1.257\times10^{-3}\text{m}^{2})(0.400\text{m}) \quad \Delta L = 2\times10^{-5}\text{m}.$$

Discussion

This small change in length seems reasonable, consistent with our experience that bones are rigid. In fact, even the rather large forces encountered during strenuous physical activity do not compress or bend bones by large amounts. Although bone is rigid compared with fat or muscle, several of the substances listed in Table 1 have larger values of Young's modulus Y. In other words, they are more rigid.

The equation for change in length is traditionally rearranged and written in the following form:

$$FA=Y\Delta LL_0.$$

The ratio of force to area, FA, is defined as **stress** (measured in $\mathrm{N/m}^2$), and the ratio of the change in length to length, ΔLL_0, is defined as **strain** (a unitless quantity). In other words,

$$\text{stress}=Y\times\text{strain}.$$

In this form, the equation is analogous to Hooke's law, with stress analogous to force and strain analogous to deformation. If we again rearrange this equation to the form

$$F=YA\Delta LL_0,$$

we see that it is the same as Hooke's law with a proportionality constant

$$k=YAL_0.$$

This general idea—that force and the deformation it causes are proportional for small deformations—applies to changes in length, sideways bending, and changes in volume.

Stress

The ratio of force to area, FA, is defined as stress measured in $\mathrm{N/m}^2$.

Strain

The ratio of the change in length to length, ΔLL_0, is defined as strain (a unitless quantity). In other words,

$$\text{stress}=Y\times\text{strain}.$$

Sideways Stress: Shear Modulus

Figure 6 illustrates what is meant by a sideways stress or a *shearing force*. Here the deformation is called Δx and it is perpendicular to L_0, rather than parallel as with tension and compression. Shear deformation behaves similarly to tension and compression and can be described with similar equations. The expression for **shear deformation** is

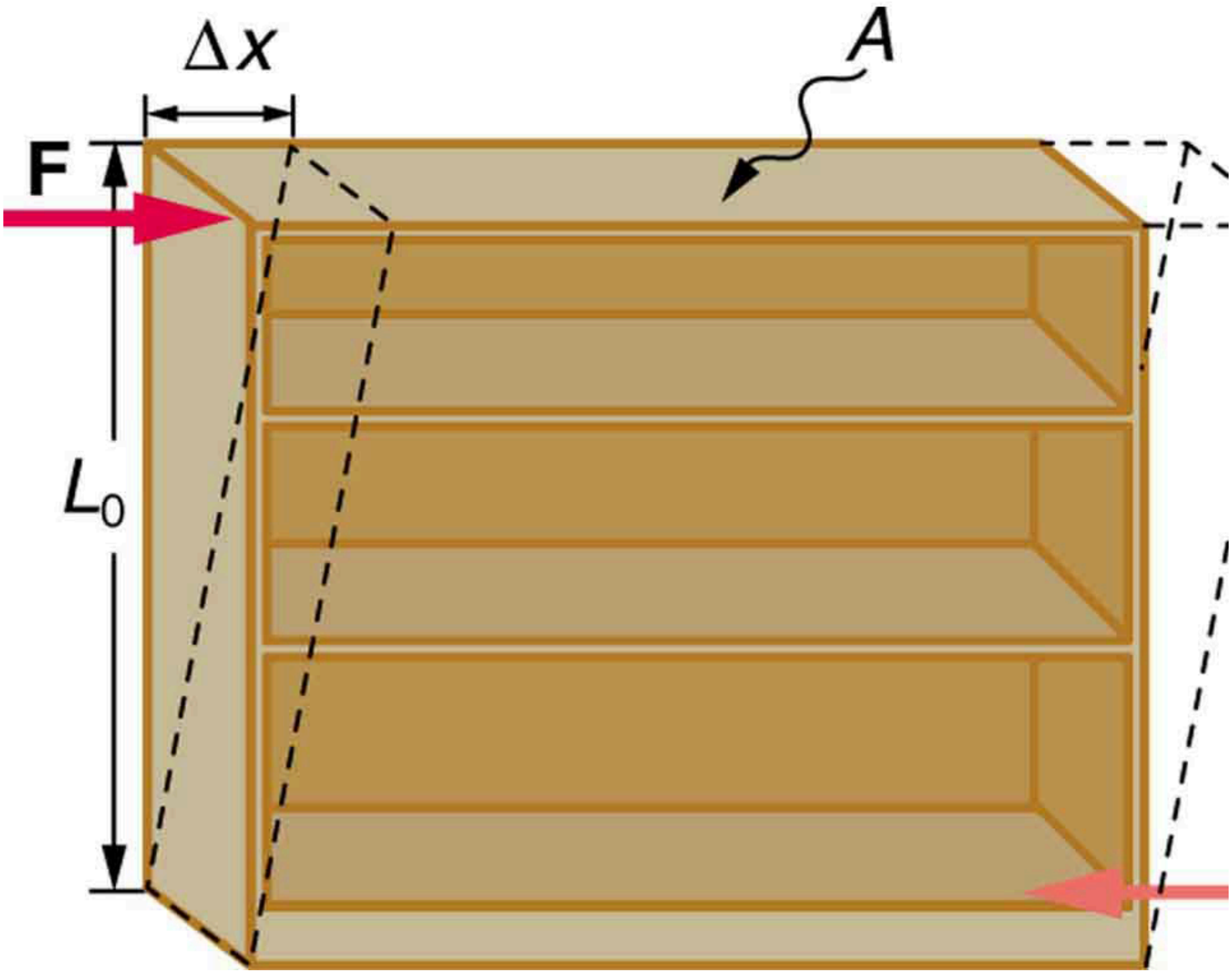
$$\Delta x=1SFAL_0,$$

where S is the shear modulus (see Table 1) and F is the force applied perpendicular to L_0 and parallel to the cross-sectional area A. Again, to keep the object from accelerating, there are actually two equal and opposite forces F

applied across opposite faces, as illustrated in Figure 6. The equation is logical—for example, it is easier to bend a long thin pencil (small A) than a short thick one, and both are more easily bent than similar steel rods (large S).

Shear Deformation
$$\Delta x=1SFAL_0,$$

where S is the shear modulus and F is the force applied perpendicular to L_0 and parallel to the cross-sectional area A.



Shearing forces are applied perpendicular to the length \(L_0 \) and parallel to the area \(A \), producing a deformation \(\Delta x \). Vertical forces are not shown, but it should be kept in mind that in addition to the two shearing forces, \(F \) , there must be supporting forces to keep the object from rotating. The distorting effects of these supporting forces are ignored in this treatment. The weight of the object also is not shown, since it is usually negligible compared with forces large enough to cause significant deformations.

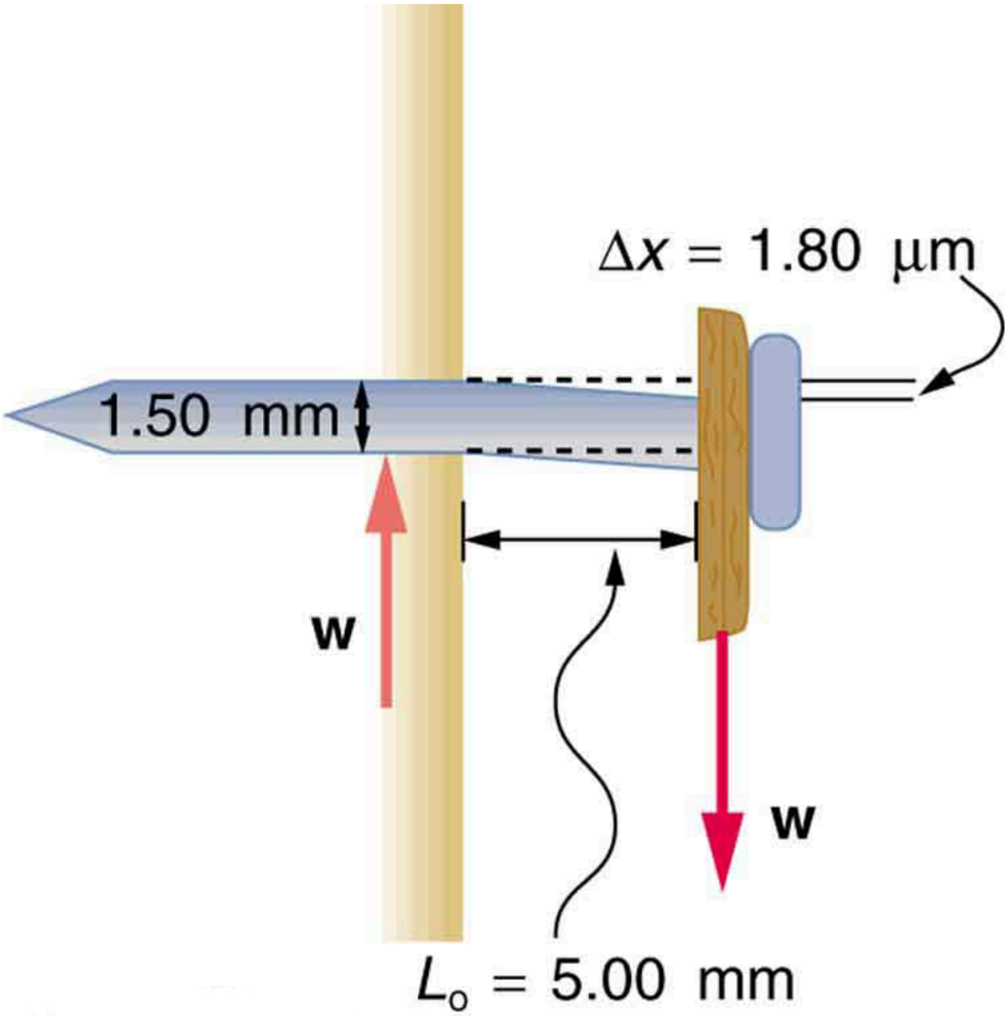
Examination of the shear moduli in Table 1 reveals some telling patterns. For example, shear moduli are less than Young's moduli for most materials. Bone is a remarkable exception. Its shear modulus is not only greater than its Young's modulus, but it is as large as that of steel. This is why bones are so rigid.

The spinal column (consisting of 26 vertebral segments separated by discs) provides the main support for the head and upper part of the body. The spinal column has normal curvature for stability, but this curvature can be increased, leading to increased shearing forces on the lower vertebrae. Discs are better at withstanding compressional forces than shear forces. Because the spine is not vertical, the weight of the upper body exerts some of both. Pregnant women and people that are overweight (with large abdomens) need to move their shoulders back to maintain balance, thereby increasing the curvature in their spine and so increasing the shear component of the stress. An increased angle due to more curvature increases the shear forces along the plane. These higher shear forces increase the risk of back injury through ruptured discs. The lumbosacral disc (the wedge shaped disc below the last vertebrae) is particularly at risk because of its location.

The shear moduli for concrete and brick are very small; they are too highly variable to be listed. Concrete used in buildings can withstand compression, as in pillars and arches, but is very poor against shear, as might be encountered in heavily loaded floors or during earthquakes. Modern structures were made possible by the use of steel and steel-reinforced concrete. Almost by definition, liquids and gases have shear moduli near zero, because they flow in response to shearing forces.

Calculating Force Required to Deform: That Nail Does Not Bend Much Under a Load

Find the mass of the picture hanging from a steel nail as shown in Figure 7, given that the nail bends only $1.80 \times 10^{-6} \text{m}$. (Assume the shear modulus is known to two significant figures.)



Side view of a nail with a picture hung from it. The nail flexes very slightly (shown much larger than actual) because of the shearing effect of the supported weight. Also shown is the upward force of the wall on the nail, illustrating that there are equal and opposite forces applied across opposite cross sections of the nail.

Strategy

The force F on the nail (neglecting the nail's own weight) is the weight of the picture w. If we can find w, then the mass of the picture is just wg. The equation Δx =1$SFAL_0$ can be solved for F.

Solution

Solving the equation Δx =1$SFAL_0$ for F, we see that all other quantities can be found:

$$F=SAL_0\Delta x.$$

S is found in [Table 1](#) and is S $=80 \times 10^9 \text{N/m}^2$. The radius r is 0.750 mm (as seen in the figure), so the cross-sectional area is

$$A=\pi r^2=1.77\times 10^{-6}\text{m}^2.$$

The value for L_0 is also shown in the figure. Thus,

$$F=(80\times 10^9\text{N/m}^2)(1.77\times 10^{-6}\text{m}^2)(5.00\times 10^{-3}\text{m})(1.80\times 10^{-6}\text{m})=51\text{N}.$$

This 51 N force is the weight w of the picture, so the picture's mass is

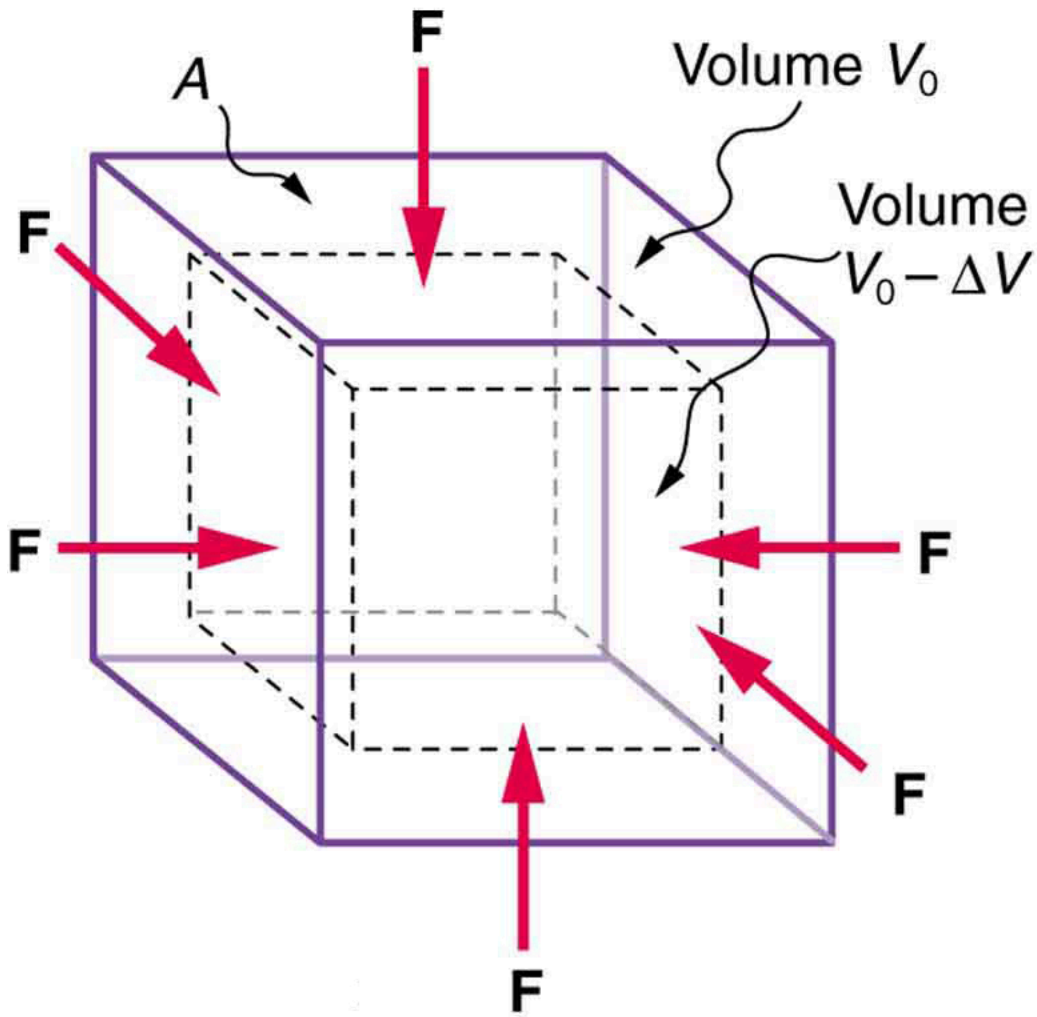
$$m=wg=Fg=5.2\text{kg}.$$

Discussion

This is a fairly massive picture, and it is impressive that the nail flexes only $1.80\times10^{-6}\text{m}$ —an amount undetectable to the unaided eye.

Changes in Volume: Bulk Modulus

An object will be compressed in all directions if inward forces are applied evenly on all its surfaces as in Figure 8. It is relatively easy to compress gases and extremely difficult to compress liquids and solids. For example, air in a wine bottle is compressed when it is corked. But if you try corking a brim-full bottle, you cannot compress the wine—some must be removed if the cork is to be inserted. The reason for these different compressibilities is that atoms and molecules are separated by large empty spaces in gases but packed close together in liquids and solids. To compress a gas, you must force its atoms and molecules closer together. To compress liquids and solids, you must actually compress their atoms and molecules, and very strong electromagnetic forces in them oppose this compression.



An inward force on all surfaces compresses this cube. Its change in volume is proportional to the force per unit area and its original volume, and is related to the compressibility of the substance.

We can describe the compression or volume deformation of an object with an equation. First, we note that a force "applied evenly" is defined to have the same stress, or ratio of force to area $\frac{F}{A}$ on all surfaces. The deformation produced is a change in volume ΔV, which is found to behave very similarly to the shear, tension, and compression previously discussed. (This is not surprising, since a compression of the entire object is equivalent to compressing each of its three dimensions.) The relationship of the change in volume to other physical quantities is given by

$$\Delta V=\frac{1}{B}\frac{F}{A}V_0,$$

where B is the bulk modulus (see Table 1), V_0 is the original volume, and $\frac{F}{A}$ is the force per unit area applied uniformly inward on all surfaces. Note that no bulk moduli are given for gases.

What are some examples of bulk compression of solids and liquids? One practical example is the manufacture of industrial-grade diamonds by compressing carbon with an extremely large force per unit area. The carbon atoms rearrange their crystalline structure into the more tightly packed pattern of diamonds. In nature, a similar process occurs deep underground, where extremely large forces result from the weight of overlying material. Another natural source of large compressive forces is the pressure created by the weight of water, especially in deep parts of the oceans. Water exerts an inward force on all surfaces of a submerged object, and even on the water itself. At great depths, water is measurably compressed, as the following example illustrates.

Calculating Change in Volume with Deformation: How Much Is Water Compressed at Great Ocean Depths?

Calculate the fractional decrease in volume ($\frac{\Delta V}{V_0}$) for seawater at 5.00 km depth, where the force per unit area is $5.00\times10^{7}\,\text{N/m}^2$.

Strategy

Equation $\Delta V=\frac{1}{B}\frac{F}{A}V_0$ is the correct physical relationship. All quantities in the equation except $\frac{\Delta V}{V_0}$ are known.

Solution

Solving for the unknown $\frac{\Delta V}{V_0}$ gives

$$\frac{\Delta V}{V_0}=\frac{1}{B}\frac{F}{A}.$$

Substituting known values with the value for the bulk modulus B from Table 1,

$$\frac{\Delta V}{V_0} = \frac{5.00\times10^{7}\,\text{N/m}^2}{2.2\times10^{9}\,\text{N/m}^2} \qquad \frac{\Delta V}{V_0} = 0.023=2.3$$

Discussion

Although measurable, this is not a significant decrease in volume considering that the force per unit area is about 500 atmospheres (1 million pounds per square foot). Liquids and solids are extraordinarily difficult to compress.

Conversely, very large forces are created by liquids and solids when they try to expand but are constrained from doing so—which is equivalent to compressing them to less than their normal volume. This often occurs when a contained material warms up, since most materials expand when their temperature increases. If the materials are tightly constrained, they deform or break their container. Another very common example occurs when water freezes. Water, unlike most materials, expands when it freezes, and it can easily fracture a boulder, rupture a biological cell, or crack an engine block that gets in its way.

Other types of deformations, such as torsion or twisting, behave analogously to the tension, shear, and bulk deformations considered here.

Hooke's Law

Stretch and compress springs to explore the relationships between force, spring constant and displacement.

Hooke's Law

{ Figure9}

Section Summary

- Hooke’s law is given by
  $$F=k\Delta L,$$

where ΔL is the amount of deformation (the change in length), F is the applied force, and k is a proportionality constant that depends on the shape and composition of the object and the direction of the force. The relationship between the deformation and the applied force can also be written as

$$\Delta L=\frac{1}{Y}\frac{F}{A}L_0,$$

where Y is *Young's modulus*, which depends on the substance, A is the cross-sectional area, and L_0 is the original length.

- The ratio of force to area, FA, is defined as *stress*, measured in $\mathrm{N/m^2}$.
- The ratio of the change in length to length, ΔLL_0, is defined as *strain* (a unitless quantity). In other words, $\text{stress}=Y\times\text{strain}.$
- The expression for shear deformation is $\Delta x=1SFAL_0,$

where S is the shear modulus and F is the force applied perpendicular to L_0 and parallel to the cross-sectional area A.

- The relationship of the change in volume to other physical quantities is given by $\Delta V=1BFAV_0,$

where B is the bulk modulus, V_0 is the original volume, and FA is the force per unit area applied uniformly inward on all surfaces.

Conceptual Questions

The elastic properties of the arteries are essential for blood flow. Explain the importance of this in terms of the characteristics of the flow of blood (pulsating or continuous).

What are you feeling when you feel your pulse? Measure your pulse rate for 10 s and for 1 min. Is there a factor of 6 difference?

Examine different types of shoes, including sports shoes and thongs. In terms of physics, why are the bottom surfaces designed as they are? What differences will dry and wet conditions make for these surfaces?

Would you expect your height to be different depending upon the time of day? Why or why not?

Why can a squirrel jump from a tree branch to the ground and run away undamaged, while a human could break a bone in such a fall?

Explain why pregnant women often suffer from back strain late in their pregnancy.

An old carpenter's trick to keep nails from bending when they are pounded into hard materials is to grip the center of the nail firmly with pliers. Why does this help?

When a glass bottle full of vinegar warms up, both the vinegar and the glass expand, but vinegar expands significantly more with temperature than glass. The bottle will break if it was filled to its tightly capped lid. Explain why, and also explain how a pocket of air above the vinegar would prevent the break. (This is the function of the air above liquids in glass containers.)

Problems & Exercises

During a circus act, one performer swings upside down hanging from a trapeze holding another, also upside-down, performer by the legs. If the upward force on the lower performer is three times her weight, how much do the bones (the femurs) in her upper legs stretch? You may assume each is equivalent to a uniform rod 35.0 cm long and 1.80 cm in radius. Her mass is 60.0 kg.

Show Solution

Strategy

We use the relationship $\Delta L=FL_0AY$, where F is the tensile force, L_0 is the original length, A is the cross-sectional area, and Y is Young's modulus. From Table 3, Young's modulus for bone is $Y=1.6\times10^{10}\ \mathrm{N/m^2}$. The total force of 3 times her weight is distributed between two femurs.

Solution

1. Calculate the total upward force:

$$F_{\text{total}}=3mg=3(60.0\text{kg})(9.80\text{m/s}^2)=1764\text{N}$$

1. Force on each femur (two femurs share the load):

$$F=1764\text{N}2=882\text{N}$$

1. Calculate the cross-sectional area of each femur:

$$A=\pi r^2=\pi(0.0180\text{m})^2=1.018\times10^{-3}\text{m}^2$$

1. Apply the stretch formula:

$$\Delta L=FL_0AY=(882\text{N})(0.350\text{m})(1.018\times10^{-3}\text{m}^2)(1.6\times10^{10}\ \text{N/m}^2)$$
$$\Delta L=308.71.629\times10^7=1.90\times10^{-5}\text{m}=1.90\times10^{-3}\ \text{cm}$$

Discussion

Each femur stretches by 1.90×10^{-3} cm or about $19\ \mu$m. This very small stretch (less than the width of a human hair) demonstrates the remarkable stiffness of bone. Even under a load of three times body weight, the femur stretches by only about 0.005% of its length, which is well within the elastic limit and causes no damage.

During a wrestling match, a 150 kg wrestler briefly stands on one hand during a maneuver designed to perplex his already moribund adversary. By how much does the upper arm bone shorten in length? The bone can be represented by a uniform rod 38.0 cm in length and 2.10 cm in radius.

Show Solution

Strategy

The upper arm bone (humerus) is under compression from supporting the wrestler's weight. We use $\Delta L = FL_0AY$, where $F = mg$ is the compressive force (the wrestler's weight), and Young's modulus for bone is $Y = 1.6 \times 10^{10}$ N/m^2.

Solution

1. Calculate the compressive force (wrestler's weight):

$$F = mg = (150\text{kg})(9.80\text{m/s}^2) = 1470\text{N}$$

1. Calculate the cross-sectional area of the bone:

$$A = \pi r^2 = \pi(0.0210\text{m})^2 = 1.385\times10^{-3}\text{m}^2$$

1. Calculate the compression:

$$\Delta L = FL_0AY = (1470\text{N})(0.380\text{m})(1.385\times10^{-3}\text{m}^2)(1.6\times10^{10}\ \text{N/m}^2)$$
$$\Delta L = 558.62.216\times10^{7} = 2.52\times10^{-5}\text{m} = 0.0252\ \text{mm}$$

Discussion

The upper arm bone shortens by 0.0252 mm or about $25\ \mu$m, which is less than 0.01% of its length. Despite supporting the full weight of a 150-kg wrestler, the bone compression is minuscule, demonstrating bone's excellent compressive strength. This is well within the elastic limit—the bone returns to its original length as soon as the wrestler shifts position.

(a) The "lead" in pencils is a graphite composition with a Young's modulus of about 1×10^9N/m^2. Calculate the change in length of the lead in an automatic pencil if you tap it straight into the pencil with a force of 4.0 N. The lead is 0.50 mm in diameter and 60 mm long. (b) Is the answer reasonable? That is, does it seem to be consistent with what you have observed when using pencils?

Show Solution

Strategy

We use the deformation formula $\Delta L = FL_0AY$. The force is applied along the length of the lead, causing compression. Given values: $F = 4.0$N, $L_0 = 60$ mm $= 0.060$m, diameter $= 0.50$ mm, and $Y = 1 \times 10^9$ N/m^2.

Solution

(a) Calculate the compression:

1. Find the cross-sectional area:

$$A = \pi r^2 = \pi(0.25\times10^{-3}\text{m})^2 = 1.96\times10^{-7}\text{m}^2$$

1. Calculate the compression:

$$\Delta L = FL_0AY = (4.0\text{N})(0.060\text{m})(1.96\times10^{-7}\text{m}^2)(1\times10^{9}\ \text{N/m}^2)$$
$$\Delta L = 0.24196 = 1.22\times10^{-3}\text{m} \approx 1\ \text{mm}$$

(b) Reasonableness check:

A compression of about 1 mm is approximately 1.7% of the original 60 mm length. This does seem reasonable based on everyday experience—when you tap a mechanical pencil lead against a surface, you can see it compress slightly before it advances into the pencil mechanism.

Discussion

The pencil lead compresses by approximately $1\ \text{mm}$ under a 4.0 N force. This relatively large deformation (compared to metals or bone) reflects the low Young’s modulus of graphite ($10^{9}\ \text{N/m}^{2}$), which is about 200 times less stiff than steel. This softness is why pencil lead can mark paper and also why it breaks relatively easily if bent.

TV broadcast antennas are the tallest artificial structures on Earth. In 1987, a 72.0-kg physicist placed himself and 400 kg of equipment at the top of one 610-m high antenna to perform gravity experiments. By how much was the antenna compressed, if we consider it to be equivalent to a steel cylinder 0.150 m in radius?

Show Solution

Strategy

We use $\Delta L = \frac{F L_0}{A Y}$ to find the compression. The force is the total weight of the physicist and equipment. For steel, Young’s modulus is $Y = 2.1 \times 10^{11}\ \text{N/m}^{2}$ from Table 3.

Solution

1. Calculate the total weight:

$$F = (m_{\text{physicist}} + m_{\text{equipment}})g = (72.0\text{kg} + 400\text{kg})(9.80\text{m/s}^{2}) = 4626\text{N}$$

1. Calculate the cross-sectional area:

$$A = \pi r^{2} = \pi(0.150\text{m})^{2} = 0.0707\text{m}^{2}$$

1. Calculate the compression:

$$\Delta L = \frac{F L_0}{A Y} = \frac{(4626\text{N})(610\text{m})}{(0.0707\text{m}^{2})(2.1\times10^{11}\ \text{N/m}^{2})}$$
$$\Delta L = \frac{2.82\times10^{6}}{1.48\times10^{10}} = 1.90\times10^{-4}\text{m} = 0.190\ \text{mm}$$

Discussion

The antenna is compressed by only $0.19\ \text{mm}$ despite the 610-m height and nearly 500-kg load at the top. This tiny compression (about 0.00003% of the antenna’s length) demonstrates steel’s exceptional stiffness. The antenna’s structural integrity is clearly not compromised by this additional load, which explains why such experiments are feasible.

(a) By how much does a 65.0-kg mountain climber stretch her 0.800-cm diameter nylon rope when she hangs 35.0 m below a rock outcropping? (b) Does the answer seem to be consistent with what you have observed for nylon ropes? Would it make sense if the rope were actually a bungee cord?

Show Solution

Strategy

We use $\Delta L = \frac{F L_0}{A Y}$ to find how much the rope stretches under the climber’s weight. From Table 3, Young’s modulus for nylon is $Y = 5 \times 10^{9}\ \text{N/m}^{2}$.

Solution

(a) Calculate the rope stretch:

1. Calculate the weight (tension in rope):

$$F = mg = (65.0\text{kg})(9.80\text{m/s}^{2}) = 637\text{N}$$

1. Calculate the cross-sectional area:

$$A = \pi r^{2} = \pi(0.00400\text{m})^{2} = 5.03\times10^{-5}\text{m}^{2}$$

1. Calculate the stretch:

$$\Delta L = \frac{F L_0}{A Y} = \frac{(637\text{N})(35.0\text{m})}{(5.03\times10^{-5}\text{m}^{2})(5\times10^{9}\ \text{N/m}^{2})}$$
$$\Delta L = \frac{22295}{2.515\times10^{5}} = 0.089\text{m} \approx 9\ \text{cm}$$

(b) Reasonableness check:

A stretch of 9 cm for a 35-m rope is about 0.26% stretch, which is reasonable for nylon climbing rope. Climbing ropes are designed to have some stretch to absorb the energy of a fall, but not too much. If this were a bungee cord (which has Young’s modulus about 100 times lower than nylon), the stretch would be about 9 meters—which would indeed be expected for bungee jumping!

Discussion

The nylon rope stretches by approximately 9 cm under the climber's weight. This moderate stretch is a key safety feature of climbing ropes—they're elastic enough to absorb shock from falls but stiff enough to not stretch excessively during normal use. A bungee cord, by contrast, would stretch several meters, which would be dangerous for climbing but essential for bungee jumping.

A 20.0-m tall hollow aluminum flagpole is equivalent in stiffness to a solid cylinder 4.00 cm in diameter. A strong wind bends the pole much as a horizontal force of 900 N exerted at the top would. How far to the side does the top of the pole flex?

Show Solution

Strategy

This is a bending problem, where a horizontal force at the top causes the pole to flex sideways. For a cantilever beam (fixed at one end, force at the other), the deflection is given by $\delta = \frac{FL^3}{3YI}$, where $I = \frac{\pi r^4}{4}$ is the moment of inertia for a circular cross-section. From Table 3, Young's modulus for aluminum is $Y = 7.0 \times 10^{10}\text{ N/m}^2$.

Solution

1. Calculate the moment of inertia:

$$I=\frac{\pi r^4}{4}=\frac{\pi(0.0200\text{m})^4}{4}=1.257\times10^{-7}\text{ m}^4$$

1. Calculate the deflection:

$$\delta=\frac{FL^3}{3YI}=\frac{(900\text{N})(20.0\text{m})^3}{3(7.0\times10^{10}\text{ N/m}^2)(1.257\times10^{-7}\text{ m}^4)}$$
$$\delta=\frac{(900)(8000)}{3(7.0\times10^{10})(1.257\times10^{-7})}=\frac{7.2\times10^6}{2.64\times10^4}$$
$$\delta=273\text{m}$$

This result seems too large. Let me reconsider—perhaps we should use the shear deformation approach instead:

Using shear: $\Delta x = \frac{FL}{AS}$, where S is the shear modulus. For aluminum, $S = 2.5 \times 10^{10}\text{ N/m}^2$.

$$A=\pi r^2=\pi(0.0200)^2=1.257\times10^{-3}\text{m}^2$$
$$\Delta x=\frac{(900\text{N})(20.0\text{m})}{(1.257\times10^{-3}\text{m}^2)(2.5\times10^{10}\text{ N/m}^2)}=\frac{18000}{3.14\times10^7}=5.7\times10^{-4}\text{m}$$

Discussion

The top of the flagpole flexes approximately 0.57 mm to the side under the 900 N horizontal wind force. This small deflection shows that even relatively thin aluminum poles are quite stiff against bending. In practice, flagpoles are designed to handle much larger wind loads without significant permanent deformation.

As an oil well is drilled, each new section of drill pipe supports its own weight and that of the pipe and drill bit beneath it. Calculate the stretch in a new 6.00 m length of steel pipe that supports 3.00 km of pipe having a mass of 20.0 kg/m and a 100-kg drill bit. The pipe is equivalent in stiffness to a solid cylinder 5.00 cm in diameter.

Show Solution

Strategy

The top section of pipe must support the weight of all the pipe below it plus the drill bit. We use $\Delta L = \frac{FL_0}{AY}$. For steel, Young's modulus is $Y = 2.1 \times 10^{11}\text{ N/m}^2$.

Solution

1. Calculate the total mass supported:

$$m_{\text{total}}=m_{\text{pipe}}+m_{\text{bit}}=(20.0\text{ kg/m})(3000\text{m})+100\text{kg}=60000\text{kg}+100\text{kg}=60100\text{kg}$$

1. Calculate the weight (tension in the new section):

$$F=m_{\text{total}}g=(60100\text{kg})(9.80\text{m/s}^2)=5.89\times10^5\text{N}$$

1. Calculate the cross-sectional area:

$$A=\pi r^2=\pi(0.0250\text{m})^2=1.96\times10^{-3}\text{m}^2$$

1. Calculate the stretch:

$$\Delta L=\frac{FL_0}{AY}=\frac{(5.89\times10^5\text{N})(6.00\text{m})}{(1.96\times10^{-3}\text{m}^2)(2.1\times10^{11}\text{ N/m}^2)}$$

$$\Delta L = \frac{3.53\times10^{6}}{4.12\times10^{8}} = 8.59\times10^{-3}\,\text{m} = 8.59\text{ mm}$$

Discussion

The 6-meter section of drill pipe stretches by 8.59 mm while supporting 3 km of pipe and a drill bit. This represents a strain of about 0.14%, which is within steel's elastic limit. In deep drilling operations, engineers must account for the cumulative stretch of thousands of meters of pipe, which can add up to several meters of total elongation.

Calculate the force a piano tuner applies to stretch a steel piano wire 8.00 mm, if the wire is originally 0.850 mm in diameter and 1.35 m long.

Show Solution

Strategy

We rearrange $\Delta L = \frac{F L_0}{AY}$ to solve for force: $F = \frac{\Delta L \cdot A \cdot Y}{L_0}$. For steel, Young's modulus is $Y = 2.1\times10^{11}\text{ N/m}^2$.

Solution

1. Calculate the cross-sectional area:

$$A = \pi r^2 = \pi(0.425\times10^{-3}\,\text{m})^2 = 5.67\times10^{-7}\,\text{m}^2$$

1. Convert the stretch to meters:

$$\Delta L = 8.00\text{ mm} = 8.00\times10^{-3}\,\text{m}$$

1. Calculate the force:

$$F = \frac{\Delta L \cdot A \cdot Y}{L_0} = \frac{(8.00\times10^{-3}\,\text{m})(5.67\times10^{-7}\,\text{m}^2)(2.1\times10^{11}\text{ N/m}^2)}{1.35\,\text{m}}$$
$$F = \frac{9.53\times10^{2}}{1.35} = 706\,\text{N}$$

Discussion

The piano tuner applies a force of approximately $706\,\text{N}$ (about 159 pounds) to stretch the wire by 8 mm. This significant tension is what allows piano wires to vibrate at precise frequencies. Piano frames must be built extremely strong (often cast iron) to withstand the combined tension of all strings, which can exceed 20 tons for a grand piano.

A vertebra is subjected to a shearing force of 500 N. Find the shear deformation, taking the vertebra to be a cylinder 3.00 cm high and 4.00 cm in diameter.

Show Solution

Strategy

For shear deformation, we use $\Delta x = \frac{F L_0}{AS}$, where S is the shear modulus. From Table 3, the shear modulus for bone is $S = 8.0\times10^{10}\text{ N/m}^2$.

Solution

1. Calculate the cross-sectional area:

$$A = \pi r^2 = \pi(0.0200\,\text{m})^2 = 1.257\times10^{-3}\,\text{m}^2$$

1. Calculate the shear deformation:

$$\Delta x = \frac{F L_0}{AS} = \frac{(500\,\text{N})(0.0300\,\text{m})}{(1.257\times10^{-3}\,\text{m}^2)(8.0\times10^{10}\text{ N/m}^2)}$$
$$\Delta x = \frac{15.0}{1.006\times10^{8}} = 1.49\times10^{-7}\,\text{m}$$

Discussion

The vertebra experiences a shear deformation of $1.49\times10^{-7}\,\text{m}$ or about $0.15\,\mu\text{m}$—less than a wavelength of visible light! This incredibly small deformation under a 500 N shearing force demonstrates the remarkable resistance of bone to shear. The spine's structure is well-designed to handle the complex combination of compressive and shearing forces it experiences during daily activities.

A disk between vertebrae in the spine is subjected to a shearing force of 600 N. Find its shear deformation, taking it to have the shear modulus of $1\times10^{9}\text{N/m}^2$. The disk is equivalent to a solid cylinder 0.700 cm high and 4.00 cm in diameter.

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Strategy

We use the shear deformation formula $\Delta x = FL_0AS$, where $S = 1 \times 10^9 \text{ N/m}^2$ is given for the intervertebral disk (much softer than bone).

Solution

1. Calculate the cross-sectional area:

$$A=\pi r^2=\pi(0.0200\text{m})^2=1.257\times10^{-3}\text{m}^2$$

1. Calculate the shear deformation:

$$\Delta x=FL_0AS=(600\text{N})(0.00700\text{m})(1.257\times10^{-3}\text{m}^2)(1\times10^9 \text{ N/m}^2)$$
$$\Delta x=4.201.257\times10^6=3.34\times10^{-6}\text{m}=3.34\ \mu\text{m}$$

Discussion

The intervertebral disk experiences a shear deformation of $3.34\ \mu\text{m}$. This is about 20 times larger than the vertebra deformation from the previous problem, reflecting the disk's much lower shear modulus. Intervertebral disks are made of softer, more flexible material than bone, which allows them to act as shock absorbers between vertebrae while still providing structural support.

When using a pencil eraser, you exert a vertical force of 6.00 N at a distance of 2.00 cm from the hardwood-eraser joint. The pencil is 6.00 mm in diameter and is held at an angle of $20.0°$ to the horizontal. (a) By how much does the wood flex perpendicular to its length? (b) How much is it compressed lengthwise?

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Strategy

The 6.00 N vertical force creates both a shear component (perpendicular to the pencil length) and a compressive component (along the pencil length). We decompose the force and apply the appropriate deformation formulas. For hardwood (like in a pencil), $Y = 1.5 \times 10^{10} \text{ N/m}^2$ and $S = 1.0 \times 10^{10} \text{ N/m}^2$.

Solution

1. Calculate the cross-sectional area:

$$A=\pi r^2=\pi(0.00300\text{m})^2=2.83\times10^{-5}\text{m}^2$$

1. Decompose the force:
   - Shear component (perpendicular to pencil): $F\_\perp = F\cos 20° = 6.00\cos 20° = 5.64\text{N}$
   - Compressive component (along pencil): $F\_\parallel = F\sin 20° = 6.00\sin 20° = 2.05\text{N}$

(a) Shear deformation (flex perpendicular to length):

$$\Delta x=F_\perp LAS=(5.64\text{N})(0.0200\text{m})(2.83\times10^{-5}\text{m}^2)(1.0\times10^{10} \text{ N/m}^2)$$
$$\Delta x=0.11282.83\times10^5=3.99\times10^{-7}\text{m}$$

(b) Compression (lengthwise):

$$\Delta L=F_\parallel LAY=(2.05\text{N})(0.0200\text{m})(2.83\times10^{-5}\text{m}^2)(1.5\times10^{10} \text{ N/m}^2)$$
$$\Delta L=0.04104.24\times10^5=9.67\times10^{-8}\text{m}$$

Discussion

The pencil wood flexes by $3.99 \times 10^{-7}\text{m}$ perpendicular to its length and compresses by $9.67 \times 10^{-8}\text{m}$ along its length. Both deformations are extremely small (less than a micrometer) and completely imperceptible during normal use. The shear deformation is about 4 times larger than the compression because the shear component of force is larger and the shear modulus is slightly lower than Young's modulus.

To consider the effect of wires hung on poles, we take data from Example 2, in which tensions in wires supporting a traffic light were calculated. The left wire made an angle $30.0°$ below the horizontal with the top of its pole and carried a tension of 108 N. The 12.0 m tall hollow aluminum pole is equivalent in stiffness to a 4.50 cm diameter solid cylinder. (a) How far is it bent to the side? (b) By how much is it compressed?

Show Solution

Strategy

The wire tension has both a horizontal component (causing the pole to bend sideways) and a vertical component (compressing the pole). We decompose the force and apply shear and compression formulas. For aluminum, $Y = 7.0 \times 10^{10}\ \mathrm{N/m^2}$ and $S = 2.5 \times 10^{10}\ \mathrm{N/m^2}$.

Solution

1. Calculate the cross-sectional area:

$$A = \pi r^2 = \pi (0.0225\mathrm{m})^2 = 1.59 \times 10^{-3}\mathrm{m}^2$$

1. Decompose the tension force:
    - Horizontal component: $F_H = T\cos 30° = 108\cos 30° = 93.5\mathrm{N}$
    - Vertical component: $F_V = T\sin 30° = 108\sin 30° = 54.0\mathrm{N}$

(a) Sideways bending (shear deformation):

$$\Delta x = \frac{F_H L}{AS} = \frac{(93.5\mathrm{N})(12.0\mathrm{m})}{(1.59 \times 10^{-3}\mathrm{m}^2)(2.5 \times 10^{10}\ \mathrm{N/m^2})}$$
$$\Delta x = \frac{1122}{3.98 \times 10^{7}} = 2.82 \times 10^{-5}\mathrm{m} = 0.0282\ \mathrm{mm}$$

(b) Compression:

$$\Delta L = \frac{F_V L}{AY} = \frac{(54.0\mathrm{N})(12.0\mathrm{m})}{(1.59 \times 10^{-3}\mathrm{m}^2)(7.0 \times 10^{10}\ \mathrm{N/m^2})}$$
$$\Delta L = \frac{648}{1.11 \times 10^{8}} = 5.82 \times 10^{-6}\mathrm{m} = 0.00582\ \mathrm{mm}$$

Discussion

The pole bends $0.028\ \mathrm{mm}$ to the side and is compressed by $0.006\ \mathrm{mm}$. These tiny deformations are completely negligible and demonstrate why aluminum poles are suitable for supporting traffic lights and power lines despite the constant tension from the wires.

A farmer making grape juice fills a glass bottle to the brim and caps it tightly. The juice expands more than the glass when it warms up, in such a way that the volume increases by 0.2% (that is, $\Delta V/V_0 = 2 \times 10^{-3}$) relative to the space available. Calculate the magnitude of the normal force exerted by the juice per square centimeter if its bulk modulus is $1.8 \times 10^9 \mathrm{N/m^2}$, assuming the bottle does not break. In view of your answer, do you think the bottle will survive?

Show Solution

Strategy

The bulk modulus relates pressure change to volume change: $B = -\frac{\Delta P}{\Delta V/V_0}$. Since the bottle prevents expansion, the juice is effectively compressed, creating pressure. We solve for the pressure.

Solution

1. Apply the bulk modulus formula:

$$B = -\frac{\Delta P}{\Delta V/V_0}$$

1. Solve for pressure change (the negative sign accounts for the fact that positive pressure causes volume decrease, but here the juice is trying to expand):

$$\Delta P = B \times \frac{\Delta V}{V_0} = (1.8 \times 10^9\ \mathrm{N/m^2})(2 \times 10^{-3})$$
$$\Delta P = 3.6 \times 10^6\ \mathrm{N/m^2} \approx 4 \times 10^6\ \mathrm{N/m^2}$$

1. Convert to more familiar units:

$$\Delta P = 4 \times 10^6\ \mathrm{Pa} = 40\ \mathrm{bar} \approx 36\ \mathrm{atm}$$

Discussion

The pressure exerted by the juice on the bottle is approximately $4 \times 10^6\ \mathrm{N/m^2}$ or about 36 atmospheres. A typical glass bottle can withstand only about 10-20 atm before failing. The bottle will almost certainly break! This is why wine and juice bottles are not filled completely to the brim—the headspace allows for thermal expansion without building dangerous pressures.

(a) When water freezes, its volume increases by 9.05% (that is, $\Delta V/V_0 = 9.05 \times 10^{-2}$). What force per unit area is water capable of exerting on a container when it freezes? (It is acceptable to use the bulk modulus of water in this problem.) (b) Is it surprising that such forces can fracture engine blocks, boulders, and the like?

Show Solution

Strategy

We use the same approach as the previous problem, applying the bulk modulus relationship. From Table 3, the bulk modulus of water is $B = 2.2 \times 10^9 \text{ N/m}^2$.

Solution

(a) Calculate the pressure from freezing water:

$$\Delta P = B \times \frac{\Delta V}{V_0} = (2.2 \times 10^9 \text{ N/m}^2)(9.05 \times 10^{-2})$$
$$\Delta P = 1.99 \times 10^8 \text{ N/m}^2 \approx 2.0 \times 10^8 \text{ N/m}^2$$

Convert to atmospheres:

$$\Delta P = \frac{2.0 \times 10^8}{1.01 \times 10^5} \approx 2000 \text{ atm}$$

(b) Analysis of the result:

This pressure of about 2000 atmospheres is enormous—roughly 200 times greater than the pressure at the bottom of the deepest ocean! The tensile strength of cast iron (used in engine blocks) is about $1 - 2 \times 10^8 \text{ N/m}^2$, and even steel's tensile strength is only about $4 \times 10^8 \text{ N/m}^2$. Rocks typically fail at much lower pressures.

Discussion

Water freezing in a confined space can exert a pressure of approximately $2.0 \times 10^8 \text{ N/m}^2$ (about 2000 atm). This explains why freezing water can crack engine blocks, split pipes, and fracture boulders. The force is sufficient to exceed the tensile strength of most materials. This phenomenon, called frost wedging, is a major force in the weathering of rocks and why it's essential to winterize plumbing systems in cold climates.

This problem returns to the tightrope walker studied in Example 2 of Normal Tension and Other Examples Of Forces, who created a tension of $3.94 \times 10^3 N$ in a wire making an angle 5.0° below the horizontal with each supporting pole. Calculate how much this tension stretches the steel wire if it was originally 15 m long and 0.50 cm in diameter.

Show Solution

Strategy

We use $\Delta L = \frac{F L_0}{A Y}$ to find the stretch. The tension $T = 3.94 \times 10^3 \text{N}$ acts along the wire. For steel, Young's modulus is $Y = 2.1 \times 10^{11} \text{ N/m}^2$.

Solution

1. Calculate the cross-sectional area:

$$A = \pi r^2 = \pi (0.0025 \text{m})^2 = 1.96 \times 10^{-5} \text{m}^2$$

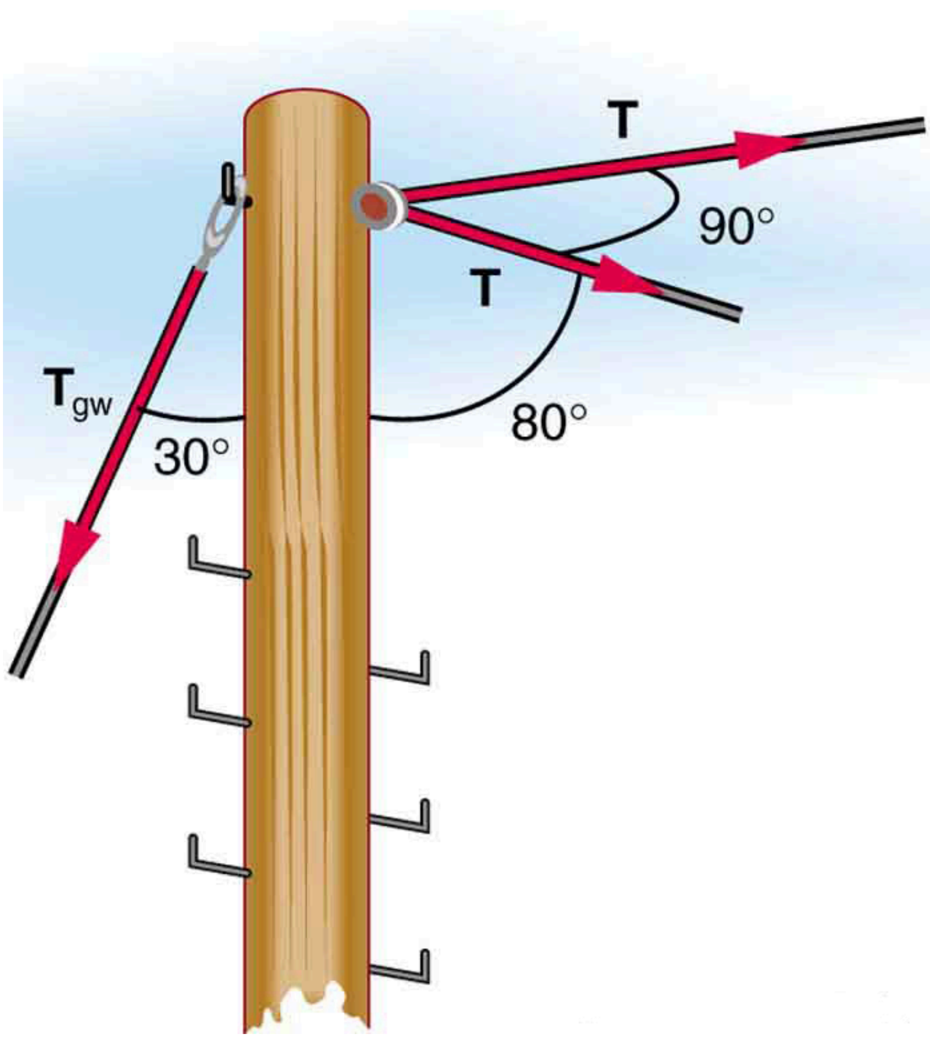
1. Calculate the stretch:

$$\Delta L = \frac{T L_0}{A Y} = \frac{(3.94 \times 10^3 \text{N})(15 \text{m})}{(1.96 \times 10^{-5} \text{m}^2)(2.1 \times 10^{11} \text{ N/m}^2)}$$
$$\Delta L = \frac{5.91 \times 10^4}{4.12 \times 10^6} = 1.44 \times 10^{-2} \text{m} = 1.4 \text{ cm}$$

Discussion

The steel wire stretches by approximately 1.4 cm under the tightrope walker's weight. This represents a strain of about 0.1%, which is within steel's elastic limit. The stretch explains why tightrope wires sag when someone walks on them—the wire must stretch to create the angle that allows vertical components of tension to support the walker's weight. Professional tightrope artists account for this stretch when setting up their equipment.

The pole in Figure 10 is at a 90.0° bend in a power line and is therefore subjected to more shear force than poles in straight parts of the line. The tension in each line is $4.00 \times 10^4 N$, at the angles shown. The pole is 15.0 m tall, has an 18.0 cm diameter, and can be considered to have half the stiffness of hardwood. (a) Calculate the compression of the pole. (b) Find how much it bends and in what direction. (c) Find the tension in a guy wire used to keep the pole straight if it is attached to the top of the pole at an angle of 30.0° with the vertical. (Clearly, the guy wire must be in the opposite direction of the bend.)



This telephone pole is at a 90 degrees bend in a power line. A guy wire is attached to the top of the pole at an angle of 30 degrees with the vertical.

Show Solution

Strategy

At a 90° bend, the two wire tensions at 80° from vertical combine to create net horizontal and vertical forces on the pole. The vertical components compress the pole; the horizontal components (which don't cancel at a bend) cause it to bend. For hardwood at half stiffness: $Y = 0.5 \times 1.5 \times 10^{10} = 7.5 \times 10^{9}\ \text{N/m}^2$ and $S = 0.5 \times 1.0 \times 10^{10} = 5.0 \times 10^{9}\ \text{N/m}^2$.

Solution

1. Calculate the cross-sectional area:

$$A = \pi r^2 = \pi(0.090\text{m})^2 = 2.54 \times 10^{-2}\text{m}^2$$

1. Decompose the forces from each wire (each at 80° from vertical = 10° from horizontal):
   - Vertical component (each): $F_V = T\cos 80° = (4.00 \times 10^4)\cos 80° = 6.95 \times 10^3\text{N}$
   - Horizontal component (each): $F_H = T\sin 80° = (4.00 \times 10^4)\sin 80° = 3.94 \times 10^4\text{N}$
2. At a 90° bend, the horizontal components are perpendicular to each other:
   - Net horizontal force: $F\_H{,}net = \sqrt{2} \times F_H = 1.414 \times 3.94 \times 10^4 = 5.57 \times 10^4\text{N}$
   - Total vertical force: $F\_V{,}total = 2 \times F_V = 2 \times 6.95 \times 10^3 = 1.39 \times 10^4\text{N}$
   - Direction of bend: 45° from each wire direction (bisecting the 90° angle)

(a) Compression of the pole:

$$\Delta L = F_{V,total} \cdot L_0 AY = (1.39 \times 10^4)(15.0)(2.54 \times 10^{-2})(7.5 \times 10^9)$$

$$\Delta L = \frac{2.09\times10^{5}}{1.91\times10^{8}} = 1.09\times10^{-3}\,\text{m} = 1.1\ \text{mm}$$

(b) Bending of the pole:

$$\Delta x = \frac{F_{H,net}\cdot L_0}{AS} = \frac{(5.57\times10^{4})(15.0)}{(2.54\times10^{-2})(5.0\times10^{9})}$$
$$\Delta x = \frac{8.36\times10^{5}}{1.27\times10^{8}} = 6.6\times10^{-3}\,\text{m} = 6.6\ \text{mm}$$

The pole bends in the direction that bisects the 90° angle (45° from each wire).

(c) Guy wire tension:

The guy wire must provide a horizontal force component equal to $F_{H,net}$ to straighten the pole:

$$T_{guy}\sin 30° = F_{H,net}$$
$$T_{guy} = \frac{F_{H,net}}{\sin 30°} = \frac{5.57\times10^{4}}{0.500} = 1.11\times10^{5}\,\text{N}$$

Discussion

The pole is compressed by $1.1\ \text{mm}$ and bends $6.6\ \text{mm}$ in the direction bisecting the 90° bend. To keep the pole straight, a guy wire with tension $1.11\times10^{5}\,\text{N}$ (about 25,000 lbs) is needed. This substantial force explains why guy wires at corner poles must be anchored very securely, often to large concrete blocks buried in the ground.

Glossary

deformation
: change in shape due to the application of force

Hooke’s law
: proportional relationship between the force F on a material and the deformation ΔL it causes, $F = k\Delta L$

tensile strength
: the breaking stress that will cause permanent deformation or fraction of a material

stress
: ratio of force to area

strain
: ratio of change in length to original length

shear deformation
: deformation perpendicular to the original length of an object

1. Footnote: Approximate and average values. Young’s moduli Y for tension and compression sometimes differ but are averaged here. Bone has significantly different Young’s moduli for tension and compression. ↩