THE SIMPLE PENDULUM

LEARNING OUTCOMES:

- Measure acceleration due to gravity.

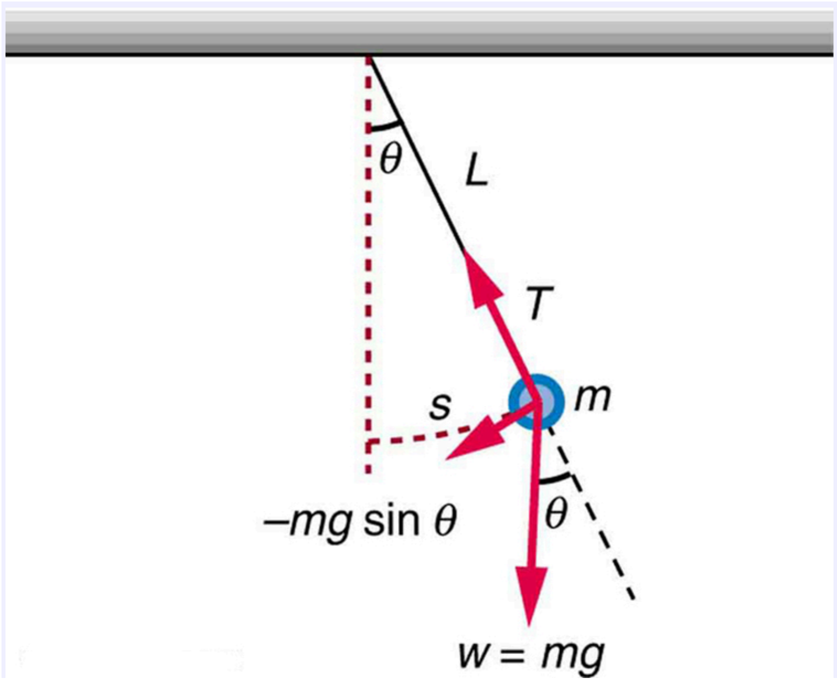


Figure 1: *A simple pendulum has a small-diameter bob and a string that has a very small mass but is strong enough not to stretch appreciably. The linear displacement from equilibrium is (s) , the length of the arc. Also shown are the forces on the bob, which result in a net force of (-mg \sin \theta) toward the equilibrium position—that is, a restoring force.*

Pendulums are in common usage. Some have crucial uses, such as in clocks; some are for fun, such as a child's swing; and some are just there, such as the sinker on a fishing line. For small displacements, a pendulum is a simple harmonic oscillator. A **simple pendulum** is defined to have an object that has a small mass, also known as the pendulum bob, which is suspended from a light wire or string, such as shown in [Figure 1]. Exploring the simple pendulum a bit further, we can discover the conditions under which it performs simple harmonic motion, and we can derive an interesting expression for its period.

We begin by defining the displacement to be the arc length s . We see from [Figure 1] that the net force on the bob is tangent to the arc and equals $-mg \sin \theta$. (The weight mg has components $mg \cos \theta$ along the string and $mg \sin \theta$ tangent to the arc.) Tension in the string exactly cancels the component $mg \cos \theta$ parallel to the string. This leaves a *net* restoring force back toward the equilibrium position at $\theta = 0$.

Now, if we can show that the restoring force is directly proportional to the displacement, then we have a simple harmonic oscillator. In trying to determine if we have a simple harmonic oscillator, we should note that for small angles (less than about 15°), $\sin \theta \approx \theta$ ($\sin \theta$ and θ differ by about 1% or less at smaller angles) . Thus, for angles less than about 15° , the restoring force F is

$$F \approx -mg\theta. \quad \text{Eq.(1)}$$

The displacement s is directly proportional to θ . When $$\theta

$is expressed in radians, the arc length in a circle is related to its radius ($L $$ in this instance) by:

$$s = L\theta, \quad \text{Eq.(2)}$$

so that

$$\theta = \frac{s}{L}. \quad \text{Eq.(3)}$$

For small angles, then, the expression for the restoring force is:

$$F \approx -\frac{mg}{L}s \quad \text{Eq.(4)}$$

This expression is of the form:

$$F = -kx, \quad \text{Eq.(5)}$$

where the force constant is given by $k = mg/L$ and the displacement is given by $x = s$. For angles less than about 15° , the restoring force is directly proportional to the displacement, and the simple pendulum is a simple harmonic oscillator.

Using this equation, we can find the period of a pendulum for amplitudes less than about 15° . For the simple pendulum:

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m}{mg/L}}. \tag{6}$$

Thus,

$$T = 2\pi\sqrt{\frac{L}{g}} \tag{7}$$

for the period of a simple pendulum. This result is interesting because of its simplicity. The only things that affect the period of a simple pendulum are its length and the acceleration due to gravity. The period is completely independent of other factors, such as mass. As with simple harmonic oscillators, the period T for a pendulum is nearly independent of amplitude, especially if θ is less than about 15° . Even simple pendulum clocks can be finely adjusted and accurate.

Note the dependence of T on g . If the length of a pendulum is precisely known, it can actually be used to measure the acceleration due to gravity. Consider the following example.

Example 1: Measuring Acceleratio to Gravity: The Period of a Pendulum

What is the acceleration due to gravity in a region where a simple pendulum having a length 75.000 cm has a period of 1.7357 s?

Strategy

We are asked to find g given the period T and the length L of a pendulum. We can solve $T = 2\pi\sqrt{\frac{L}{g}}$ for g, assuming only that the angle of deflection is less than 15°.

Solution

1. Square $T = 2\pi\sqrt{\frac{L}{g}}$ and solve for g:

$$g = 4\pi^2 \frac{L}{T^2}. \quad \text{Eq.(8)}$$

2. Substitute known values into the new equation:

$$g = 4\pi^2 \frac{0.75000\text{m}}{(1.7357\text{s})^2}. \quad \text{Eq.(9)}$$

3. Calculate to find g:

$$g = 9.8281\text{m}/\text{s}^2. \quad \text{Eq.(10)}$$

Discussion

This method for determining g can be very accurate. This is why length and period are given to five digits in this example. For the precision of the approximation $\sin\theta \approx \theta$ to be better than the precision of the pendulum length and period, the maximum displacement angle should be kept below about 0.5°.

MAKING CAREER CONNECTIONS

Knowing g can be important in geological exploration; for example, a map of g over large geographical regions aids the study of plate tectonics and helps in the search for oil fields and large mineral deposits.

TAKE HOME EXPERIMENT: DETERMI

g

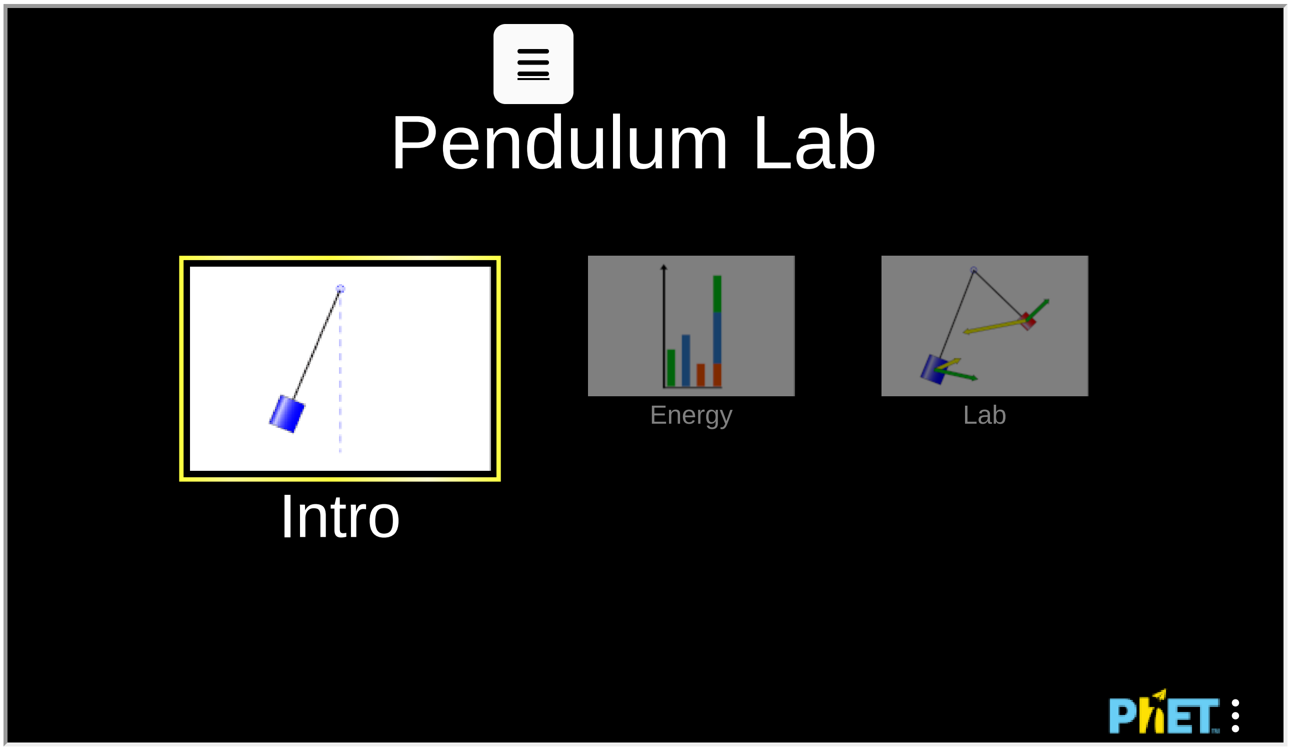
Use a simple pendulum to determine the acceleration due to gravity g in your own locale. Cut a piece of a string or dental floss so that it is about 1 m long. Attach a small object of high density to the end of the string (for example, a metal nut or a car key). Starting at an angle of less than 10°, allow the pendulum to swing and measure the pendulum's period for 10 oscillations using a stopwatch. Calculate g. How accurate is this measurement? How might it be improved?

Check Your Understanding

An engineer builds two simple pendula. Both are suspended from small wires secured to the ceiling of a room. Each pendulum hovers 2 cm above the floor. Pendulum 1 has a bob with a mass of 10kg. Pendulum 2 has a bob with a mass of 100kg. Describe how the motion of the pendula will differ if the bobs are both displaced by 12°.

PhET Explorations: Pendulum Lab

Play with one or two pendulums and discover how the period of a simple pendulum depends on the length of the string, the mass of the pendulum bob, and the amplitude of the swing. It's easy to measure the period using the photogate timer. You can vary friction and the strength of gravity. Use the pendulum to find the value of g on planet X. Notice the anharmonic behavior at large amplitude.



Section Summary

- A mass m suspended by a wire of length L is a simple pendulum and undergoes simple harmonic motion for amplitudes less than about 15°. The period of a simple pendulum is

$$T = 2\pi\sqrt{\frac{L}{g}}, \quad \text{Eq.(11)}$$

where L is the length of the string and g is the acceleration due to gravity.

Conceptual Questions

Exercise 1

> Pendulum clocks are made to run at the correct rate by adjusting the pendulum's length. Suppose you move from one city to another where the acceleration due to gravity is slightly greater, taking your pendulum clock with you, will you have to lengthen or shorten the pendulum to keep the correct time, other factors remaining constant? Explain your answer.

Problems & Exercises

As usual, the acceleration due to gravity in these problems is taken to be $g = 9.80 \mathrm{m/s}^2$, unless otherwise specified.

Exercise 2

What is the length of a pendulum that has a period of 0.500 s?

Exercise 3

Some people think a pendulum with a period of 1.00 s can be driven with "mental energy" or psycho kinetically, because its period is the same as an average heartbeat. True or not, what is the length of such a pendulum?

Exercise 4

What is the period of a 1.00-m-long pendulum?

Exercise 5

How long does it take a child on a swing to complete one swing if her center of gravity is 4.00 m below the pivot?

Exercise 6

The pendulum on a cuckoo clock is 5.00 cm long. What is its frequency?

Exercise 7

Two parakeets sit on a swing with their combined center of mass 10.0 cm below the pivot. At what frequency do they swing?

Exercise 8

(a) A pendulum that has a period of 3.00000 s and that is located where the acceleration due to gravity is $9.79 \mathrm{m/s}^2$ is moved to a location where the acceleration due to gravity is $9.82 \mathrm{m/s}^2$. What is its new period? (b) Explain why so many digits are needed in the value for the period, based on the relation between the period and the acceleration due to gravity.

Exercise 9

A pendulum with a period of 2.00000 s in one location $\left(g = 9.80 \mathrm{m/s}^2\right)$ is moved to a new location where the period is now 1.99796 s. What is the acceleration due to gravity at its new location?

Exercise 10

(a) What is the effect on the period of a pendulum if you double its length?

(b) What is the effect on the period of a pendulum if you decrease its length by 5.00%?

Exercise 11

Find the ratio of the new/old periods of a pendulum if the pendulum were transported from Earth to the Moon, where the acceleration due to gravity is $1.63 \mathrm{m/s}^2$.

Exercise 12

At what rate will a pendulum clock run on the Moon, where the acceleration due to gravity is $1.63 \mathrm{m/s}^2$, if it keeps time accurately on Earth? That is, find the time (in hours) it takes the clock's hour hand to make one revolution on the Moon.

Exercise 13

Suppose the length of a clock's pendulum is changed by 1.000%, exactly at noon one day. What time will it read 24.00 hours later, assuming it the pendulum has kept perfect time before the change? Note that there are two answers, and perform the calculation to four-digit precision.

Exercise 14

If a pendulum-driven clock gains 5.00 s/day, what fractional change in pendulum length must be made for it to keep perfect time?

GLOSSARY

simple pendulum

an object with a small mass suspended from a light wire or string