

# PROBLEM-SOLVING STRATEGIES

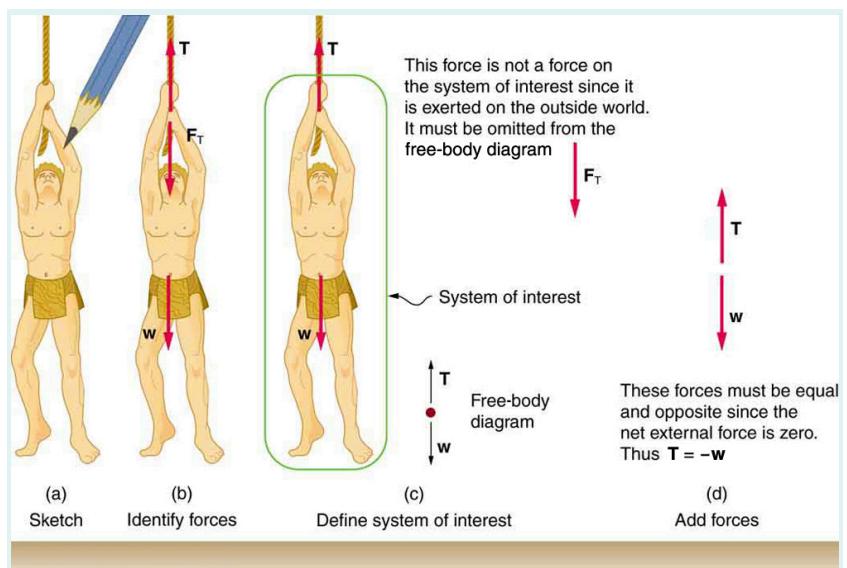
## LEARNING OUTCOMES:

- Understand and apply a problem-solving procedure to solve problems using Newton's laws of motion.

Success in problem solving is obviously necessary to understand and apply physical principles, not to mention the more immediate need of passing exams. The basics of problem solving, presented earlier in this text, are followed here, but specific strategies useful in applying Newton's laws of motion are emphasized. These techniques also reinforce concepts that are useful in many other areas of physics. Many problem-solving strategies are stated outright in the worked examples, and so the following techniques should reinforce skills you have already begun to develop.

## Problem-Solving Strategy for Newton's Laws of Motion

Step 1. As usual, it is first necessary to identify the physical principles involved. *Once it is determined that Newton's laws of motion are involved (if the problem involves forces), it is particularly important to draw a careful sketch of the situation.* Such a sketch is shown in [Figure 1\(a\)](#). Then, as in [Figure 1\(b\)](#), use arrows to represent all forces, label them carefully, and make their lengths and directions correspond to the forces they represent (whenever sufficient information exists).



**Figure 1:** (a) A sketch of Tarzan hanging from a vine. (b) Arrows are used to represent all forces.  $T$  is the tension in the vine above Tarzan,  $F_T$  is the force he exerts on the vine, and  $w$  is his weight. All other forces, such as the nudge of a breeze, are assumed negligible. (c) Suppose we are given the ape man's mass and asked to find the tension in the vine. We then define the system of interest as shown and draw a free-body diagram.  $F_T$  is no longer shown, because it is not a force acting on the system of interest; rather,  $F_T$  acts on the outside world. (d) Showing only the arrows, the head-to-tail method of addition is used. It is apparent that  $T = -w$ , if Tarzan is stationary.

Step 2. Identify what needs to be determined and what is known or can be inferred from the problem as stated. That is, make a list of knowns and unknowns. *Then carefully determine the system of interest.* This decision is a crucial step, since Newton's second law involves only external forces. Once the system of interest has been identified, it becomes possible to determine which forces are external and which are internal, a necessary step to employ Newton's second law. (See Figure 1(c).) Newton's third law may be used to identify whether forces are exerted between components of a system (internal) or between the system and something outside (external). As illustrated earlier in this chapter, the system of interest depends on what question we need to answer. This choice becomes easier with practice, eventually developing into an almost unconscious process. Skill in clearly defining systems will be beneficial in later chapters as well.

A diagram showing the system of interest and all of the external forces is called a **free-body diagram**. Only forces are shown on free-body diagrams, not acceleration or velocity. We have drawn several of these in worked examples. Figure 1(c) shows a free-body diagram for the system of interest. Note that no internal forces are shown in a free-body diagram.

Step 3. Once a free-body diagram is drawn, *Newton's second law can be applied to solve the problem*. This is done in [Figure 1\(d\)](#) for a particular situation. In general, once external forces are clearly identified in free-body diagrams, it should be a straightforward task to put them into equation form and solve for the unknown, as done in all previous examples. If the problem is one-dimensional—that is, if all forces are parallel—then they add like scalars. If the problem is two-dimensional, then it must be broken down into a pair of one-dimensional problems. This is done by projecting the force vectors onto a set of axes chosen for convenience. As seen in previous examples, the choice of axes can simplify the problem. For example, when an incline is involved, a set of axes with one axis parallel to the incline and one perpendicular to it is most convenient. It is almost always convenient to make one axis parallel to the direction of motion, if this is known.

### APPLYING NEWTON'S SECOND LAW

Before you write net force equations, it is critical to determine whether the system is accelerating in a particular direction. If the acceleration is zero in a particular direction, then the net force is zero in that direction. Similarly, if the acceleration is nonzero in a particular direction, then the net force is described by the equation:  $\vec{F}_{\text{net}} = m\vec{a}$ .

For example, if the system is accelerating in the horizontal direction, but it is not accelerating in the vertical direction, then you will have the following conclusions:

$$F_{\text{net},x} = ma, \quad \text{Eq.(1)}$$

$$F_{\text{net},y} = 0. \quad \text{Eq.(2)}$$

You will need this information in order to determine unknown forces acting in a system.

Step 4. As always, *check the solution to see whether it is reasonable*. In some cases, this is obvious. For example, it is reasonable to find that friction causes an object to slide down an incline more slowly than when no friction exists. In practice, intuition develops gradually through problem solving, and with experience it becomes progressively easier to judge whether an answer is reasonable. Another way to check your solution is to check the units. If you are solving for force and end up with units of m/s, then you have made a mistake.

## Section Summary

- To solve problems involving Newton's laws of motion, follow the procedure described:
  1. Draw a sketch of the problem.
  2. Identify known and unknown quantities, and identify the system of interest. Draw a free-body diagram, which is a sketch showing all of the forces acting on an object. The object is represented by a dot, and the forces are represented by vectors extending in different directions from the dot. If vectors act in directions that are not horizontal or vertical, resolve the vectors into horizontal and vertical components and draw them on the free-body diagram.
  3. Write Newton's second law in the horizontal and vertical directions and add the forces acting on the object. If the object does not accelerate in a particular direction (for example, the  $x$ -direction) then  $F_{\text{net}x} = 0$ . If the object does accelerate in that direction,  $F_{\text{net}x} = ma$ .
  4. Check your answer. Is the answer reasonable? Are the units correct?

## Problem Exercises

### Exercise 1

A  $5.00 \times 10^5$ -kg rocket is accelerating straight up. Its engines produce  $1.250 \times 10^7$ N of thrust, and air resistance is  $4.50 \times 10^6$ N. What is the rocket's acceleration? Explicitly show how you follow the steps in the Problem-Solving Strategy for Newton's laws of motion.

### Exercise 2

The wheels of a midsize car exert a force of 2100 N backward on the road to accelerate the car in the forward direction. If the force of friction including air resistance is 250 N and the acceleration of the car is  $1.80\text{m/s}^2$ , what is the mass of the car plus its occupants? Explicitly show how you follow the steps in the Problem-Solving Strategy for Newton's laws of motion. For this situation, draw a free-body diagram and write the net force equation.

**Exercise 3**

Calculate the force a 70.0-kg high jumper must exert on the ground to produce an upward acceleration 4.00 times the acceleration due to gravity. Explicitly show how you follow the steps in the Problem-Solving Strategy for Newton's laws of motion.

**Exercise 4**

When landing after a spectacular somersault, a 40.0-kg gymnast decelerates by pushing straight down on the mat. Calculate the force she must exert if her deceleration is 7.00 times the acceleration due to gravity. Explicitly show how you follow the steps in the Problem-Solving Strategy for Newton's laws of motion.

**Exercise 5**

A freight train consists of two  $8.00 \times 10^4$ -kg engines and 45 cars with average masses of  $5.50 \times 10^4$  kg. (a) What force must each engine exert backward on the track to accelerate the train at a rate of  $5.00 \times 10^{-2}$  m/s<sup>2</sup> if the force of friction is  $7.50 \times 10^5$  N, assuming the engines exert identical forces? This is not a large frictional force for such a massive system. Rolling friction for trains is small, and consequently trains are very energy-efficient transportation systems. (b) What is the force in the coupling between the 37th and 38th cars (this is the force each exerts on the other), assuming all cars have the same mass and that friction is evenly distributed among all of the cars and engines?

**Exercise 6**

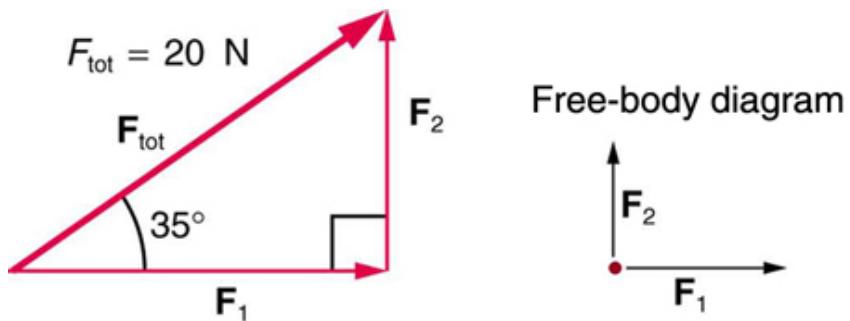
Commercial airplanes are sometimes pushed out of the passenger loading area by a tractor. (a) An 1800-kg tractor exerts a force of  $1.75 \times 10^4\text{N}$  backward on the pavement, and the system experiences forces resisting motion that total 2400 N. If the acceleration is  $0.150\text{m/s}^2$ , what is the mass of the airplane? (b) Calculate the force exerted by the tractor on the airplane, assuming 2200 N of the friction is experienced by the airplane. (c) Draw two sketches showing the systems of interest used to solve each part, including the free-body diagrams for each.

**Exercise 7**

A 1100-kg car pulls a boat on a trailer. (a) What total force resists the motion of the car, boat, and trailer, if the car exerts a 1900-N force on the road and produces an acceleration of  $0.550\text{m/s}^2$ ? The mass of the boat plus trailer is 700 kg. (b) What is the force in the hitch between the car and the trailer if 80% of the resisting forces are experienced by the boat and trailer?

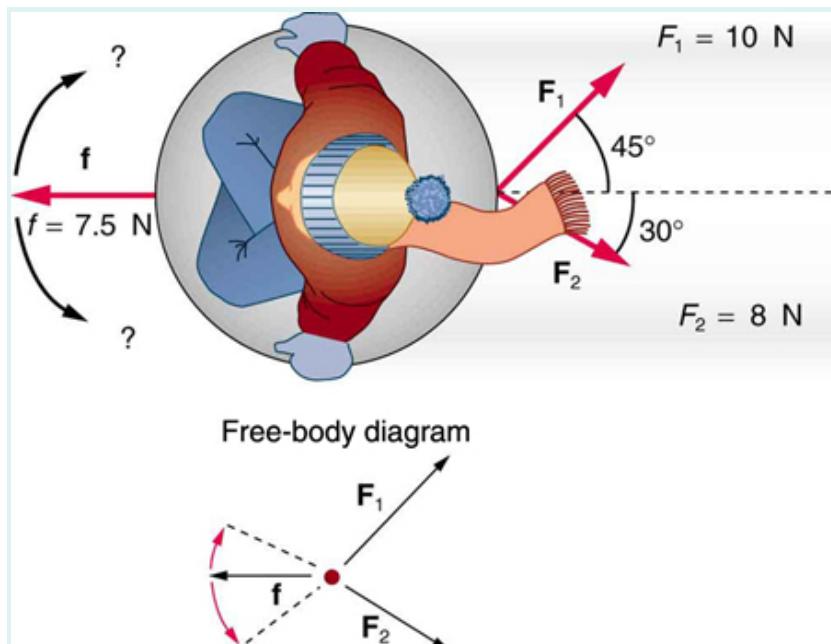
**Exercise 8**

(a) Find the magnitudes of the forces  $\vec{F}_1$  and  $\vec{F}_2$  that add to give the total force  $\vec{F}_{\text{tot}}$  shown in Figure below. This may be done either graphically or by using trigonometry. (b) Show graphically that the same total force is obtained independent of the order of addition of  $\vec{F}_1$  and  $\vec{F}_2$ . (c) Find the direction and magnitude of some other pair of vectors that add to give  $\vec{F}_{\text{tot}}$ . Draw these to scale on the same drawing used in part (b) or a similar picture.

**Exercise 9**

Two children pull a third child on a snow saucer sled exerting forces  $\vec{F}_1$  and  $\vec{F}_2$  as shown from above in Figure below. Find the acceleration of the 49.00-kg sled and child system. Note that the direction of the frictional force is unspecified; it will be in the opposite direction of the sum of

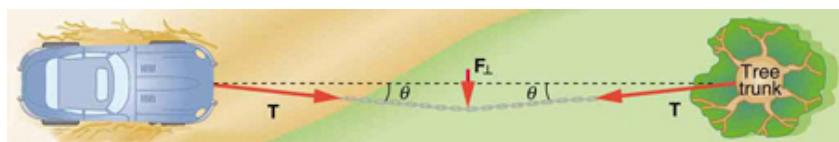
$\vec{v}_b\{\vec{F}\}\{1\}$  and  $\vec{v}_b\{\vec{F}\}\{2\}$  \$\$.



**Figure 2:** An overhead view of the horizontal forces acting on a child's snow saucer sled.

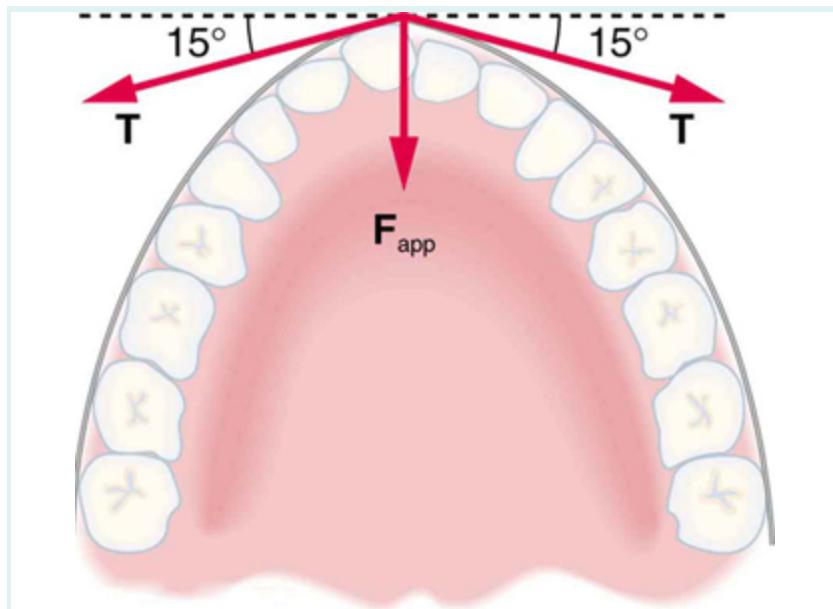
### Exercise 10

Suppose your car was mired deeply in the mud and you wanted to use the method illustrated in the [Figure below](#) to pull it out. (a) What force would you have to exert perpendicular to the center of the rope to produce a force of 12 000 N on the car if the angle is  $2.00^\circ$ ? In this part, explicitly show how you follow the steps in the Problem-Solving Strategy for Newton's laws of motion. (b) Real ropes stretch under such forces. What force would be exerted on the car if the angle increases to  $7.00^\circ$  and you still apply the force found in part (a) to its center?



**Exercise 11**

What force is exerted on the tooth in the Figure below if the tension in the wire is 25.0 N? Note that the force applied to the tooth is smaller than the tension in the wire, but this is necessitated by practical considerations of how force can be applied in the mouth. Explicitly show how you follow steps in the Problem-Solving Strategy for Newton's laws of motion.

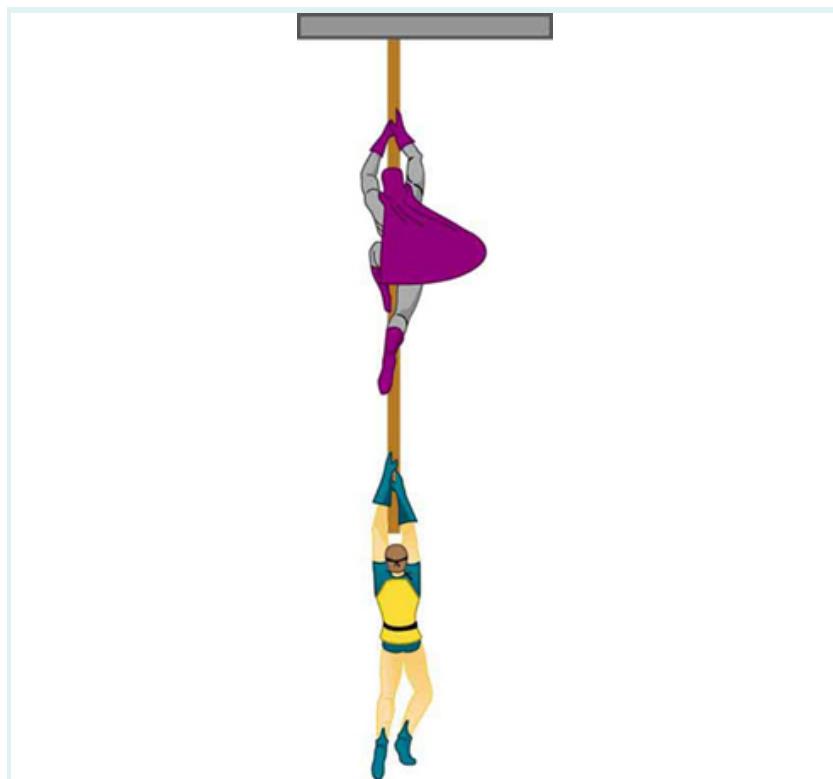


**Figure 3:** Braces are used to apply forces to teeth to realign them.

Shown in this figure are the tensions applied by the wire to the protruding tooth. The total force applied to the tooth by the wire,  $F_{\text{app}}$ , points straight toward the back of the mouth.

**Exercise 12**

The Figure below shows Superhero and Trusty Sidekick hanging motionless from a rope. Superhero's mass is 90.0 kg, while Trusty Sidekick's is 55.0 kg, and the mass of the rope is negligible. (a) Draw a free-body diagram of the situation showing all forces acting on Superhero, Trusty Sidekick, and the rope. (b) Find the tension in the rope above Superhero. (c) Find the tension in the rope between Superhero and Trusty Sidekick. Indicate on your free-body diagram the system of interest used to solve each part.



**Figure 4:** Superhero and Trusty Sidekick hang motionless on a rope as they try to figure out what to do next. Will the tension be the same everywhere in the rope?

### Exercise 13

A nurse pushes a cart by exerting a force on the handle at a downward angle  $35.0^\circ$  below the horizontal. The loaded cart has a mass of 28.0 kg, and the force of friction is 60.0 N. (a) Draw a free-body diagram for the system of interest. (b) What force must the nurse exert to move at a constant velocity?

**Exercise 14****Construct Your Own Problem**

Consider the tension in an elevator cable during the time the elevator starts from rest and accelerates its load upward to some cruising velocity. Taking the elevator and its load to be the system of interest, draw a free-body diagram. Then calculate the tension in the cable. Among the things to consider are the mass of the elevator and its load, the final velocity, and the time taken to reach that velocity.

**Exercise 15****Construct Your Own Problem**

Consider two people pushing a toboggan with four children on it up a snow-covered slope. Construct a problem in which you calculate the acceleration of the toboggan and its load. Things to be considered are the forces exerted by those pushing, the angle of the slope, and the masses of the toboggan and children.

**Exercise 16****Unreasonable Results**

(a) Repeat [Exercise 7](#), but assume an acceleration of  $1.20\text{m/s}^2$

is produced. (b) What is unreasonable about the result? (c) Which premise is unreasonable, and why is it unreasonable?

**Exercise 17****Unreasonable Results**

(a) What is the initial acceleration of a rocket that has a mass of  $1.50 \times 10^6 \text{ kg}$  at takeoff, the engines of which produce a thrust of  $2.00 \times 10^6 \text{ N}$ ? Do not neglect gravity. (b) What is unreasonable about the result? (This result has been unintentionally achieved by several real rockets.) (c) Which premise is unreasonable, or which premises are inconsistent? (You may find it useful to compare this problem to the rocket problem earlier in this section.)



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