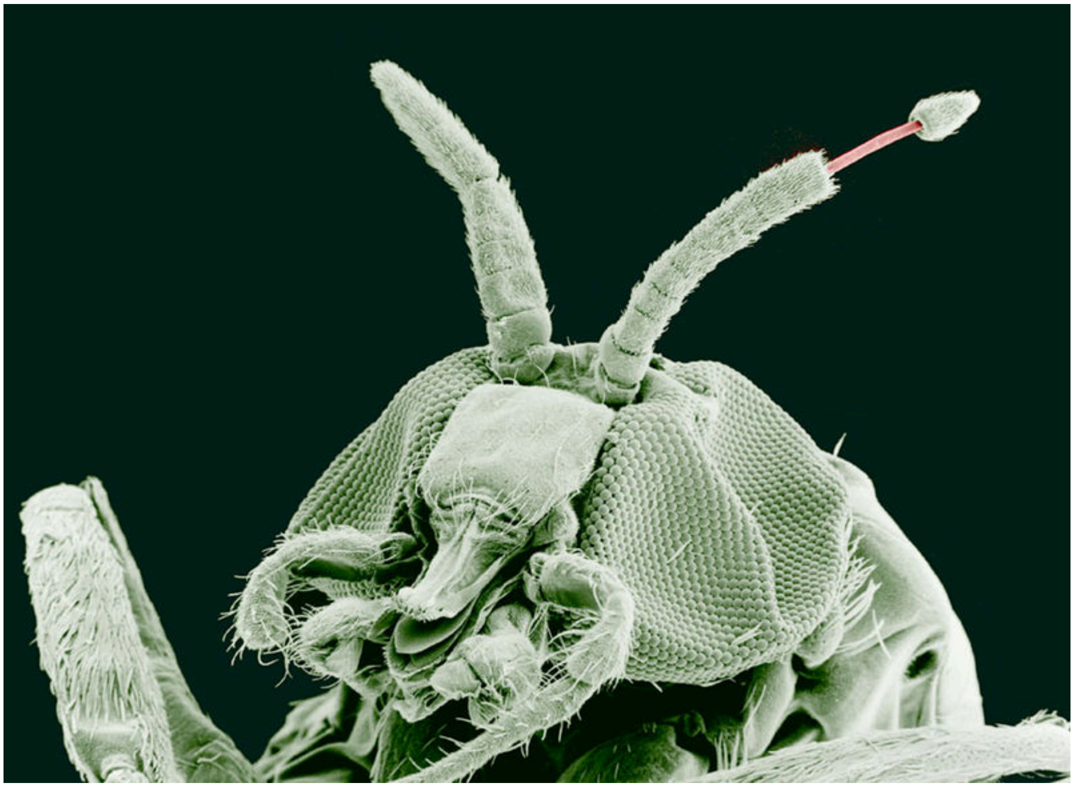
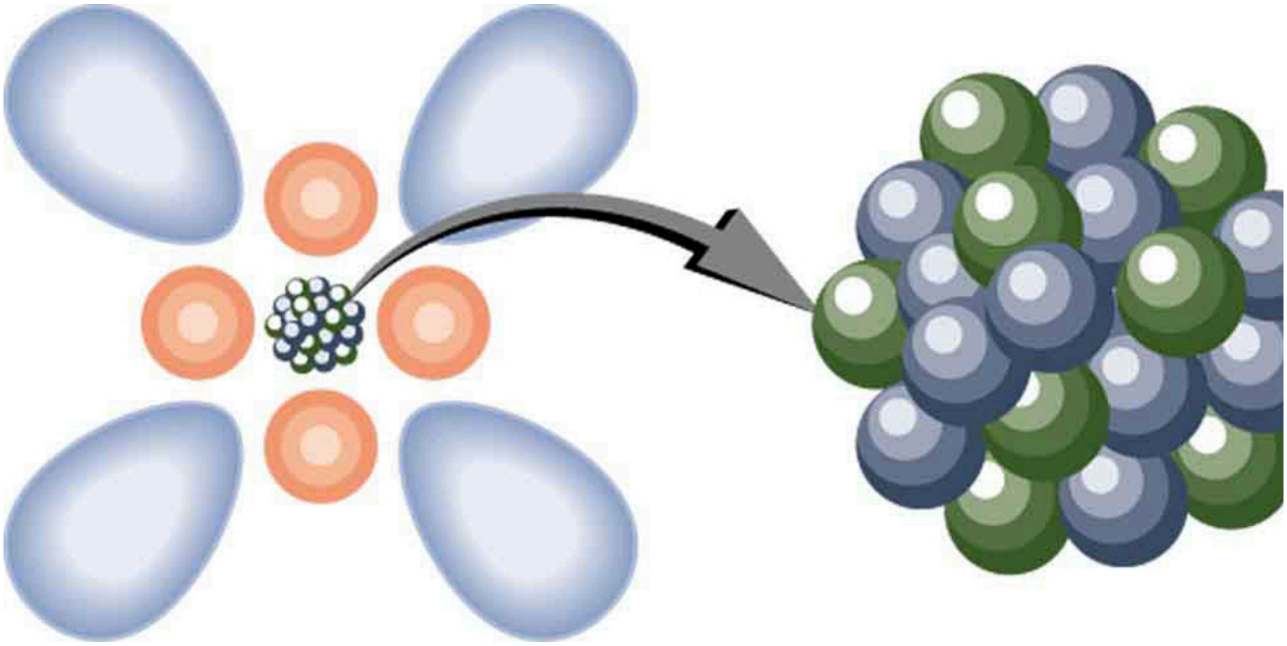
Introduction to Quantum Physics



A black fly imaged by an electron microscope is as monstrous as any science-fiction creature. (credit: U.S. Department of Agriculture via Wikimedia Commons)

Quantum mechanics is the branch of physics needed to deal with submicroscopic objects. Because these objects are smaller than we can observe directly with our senses and generally must be observed with the aid of instruments, parts of quantum mechanics seem as foreign and bizarre as parts of relativity. But, like relativity, quantum mechanics has been shown to be valid—truth is often stranger than fiction.

Certain aspects of quantum mechanics are familiar to us. We accept as fact that matter is composed of atoms, the smallest unit of an element, and that these atoms combine to form molecules, the smallest unit of a compound. (See [Figure 2].) While we cannot see the individual water molecules in a stream, for example, we are aware that this is because molecules are so small and so numerous in that stream. When introducing atoms, we commonly say that electrons orbit atoms in discrete shells around a tiny nucleus, itself composed of smaller particles called protons and neutrons. We are also aware that electric charge comes in tiny units carried almost entirely by electrons and protons. As with water molecules in a stream, we do not notice individual charges in the current through a lightbulb, because the charges are so small and so numerous in the macroscopic situations we sense directly.



Atoms and their substructure are familiar examples of objects that require quantum mechanics to be fully explained. Certain of their characteristics, such as the discrete electron shells, are classical physics explanations. In quantum mechanics we conceptualize discrete “electron clouds” around the nucleus.

Making Connections: Realms of Physics

Classical physics is a good approximation of modern physics under conditions first discussed in the The Nature of Science and Physics. Quantum mechanics is valid in general, and it must be used rather than classical physics to describe small objects, such as atoms.

Atoms, molecules, and fundamental electron and proton charges are all examples of physical entities that are **quantized**—that is, they appear only in certain discrete values and do not have every conceivable value. Quantized is the opposite of continuous. We cannot have a fraction of an atom, or part of an electron’s charge, or 14-1/3 cents, for example. Rather, everything is built of integral multiples of these substructures. Quantum physics is the branch of physics that deals with small objects and the quantization of various entities, including energy and angular momentum. Just as with classical physics, quantum physics has several subfields, such as mechanics and the study of electromagnetic forces. The **correspondence principle** states that in the classical limit (large, slow-moving objects), **quantum mechanics** becomes the same as classical physics. In this chapter, we begin the development of quantum mechanics and its description of the strange submicroscopic world. In later chapters, we will examine many areas, such as atomic and nuclear physics, in which quantum mechanics is crucial.

Glossary

quantized
: the fact that certain physical entities exist only with particular discrete values and not every conceivable value

correspondence principle
: in the classical limit (large, slow-moving objects), quantum mechanics becomes the same as classical physics

quantum mechanics
: the branch of physics that deals with small objects and with the quantization of various entities, especially energy

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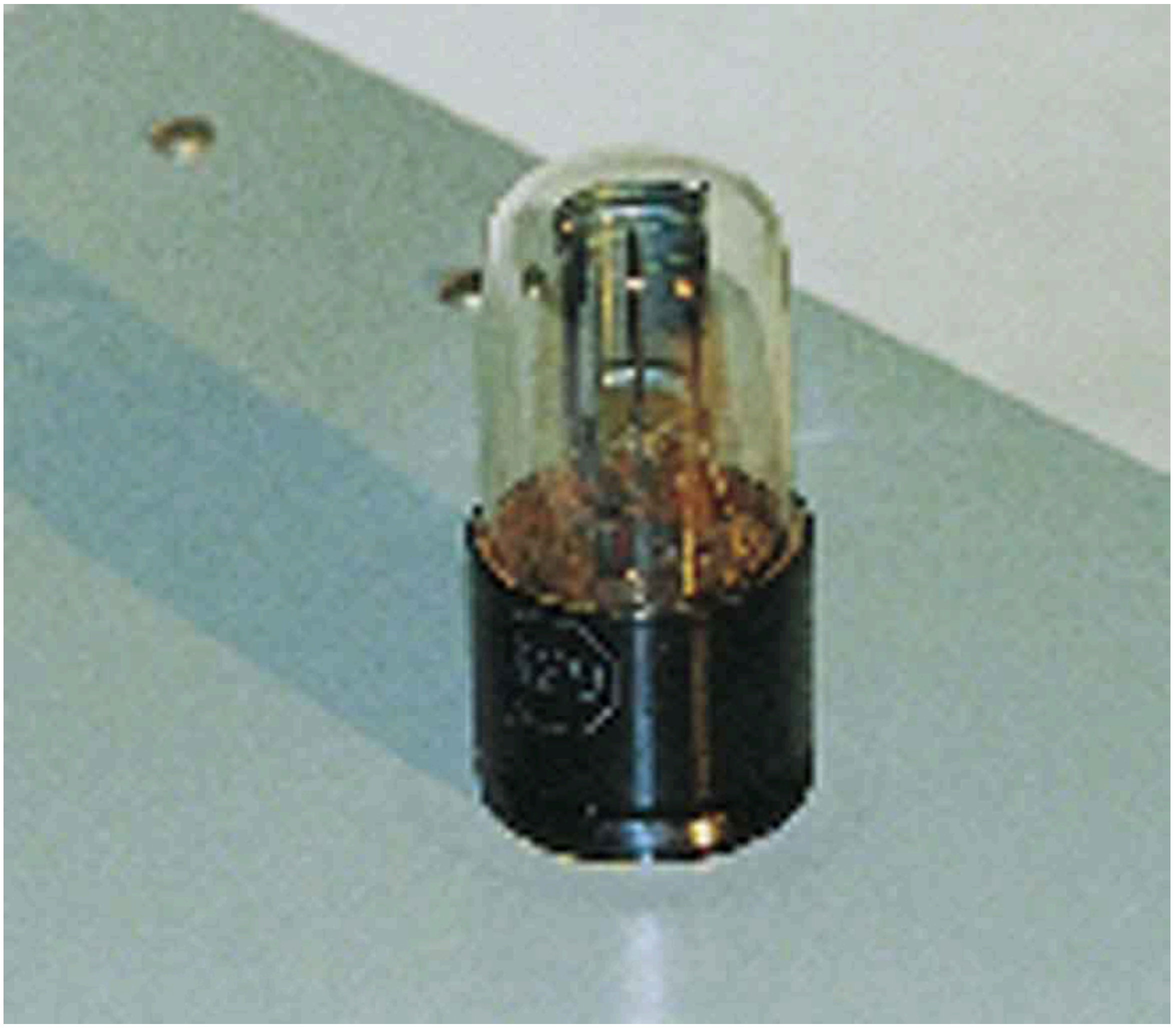
The Photoelectric Effect

- Describe a typical photoelectric-effect experiment.
- Determine the maximum kinetic energy of photoelectrons ejected by photons of one energy or wavelength, when given the maximum kinetic energy of photoelectrons for a different photon energy or wavelength.

When light strikes materials, it can eject electrons from them. This is called the **photoelectric effect**, meaning that light (*photo*) produces electricity. One common use of the photoelectric effect is in light meters, such as those that adjust the automatic iris on various types of cameras. In a similar way, another use is in solar cells, as you probably have in your calculator or have seen on a roof top or a roadside sign. These make use of the photoelectric effect to convert light into electricity for running different devices.



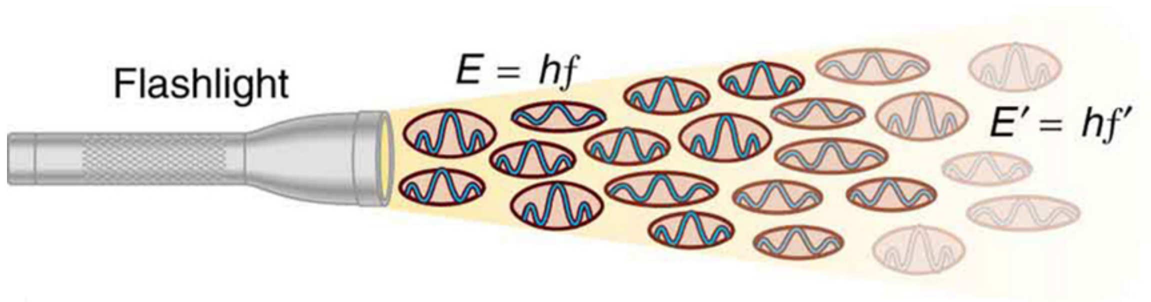
The photoelectric effect can be observed by allowing light to fall on the metal plate in this evacuated tube. Electrons ejected by the light are collected on the collector wire and measured as a current. A retarding voltage between the collector wire and plate can then be adjusted so as to determine the energy of the ejected electrons. For example, if it is sufficiently negative, no electrons will reach the wire. (credit: P.P. Urone)

This effect has been known for more than a century and can be studied using a device such as that shown in [Figure 1]. This figure shows an evacuated tube with a metal plate and a collector wire that are connected by a variable voltage source, with the collector more negative than the plate. When light (or other EM radiation) strikes the plate in the evacuated tube, it may eject electrons. If the electrons have energy in electron volts (eV) greater than the potential difference between the plate and the wire in volts, some electrons will be collected on the wire. Since the electron energy in eV is qV , where q is the electron charge and V is the potential difference, the electron energy can be measured by adjusting the retarding voltage between the wire and the plate. The voltage that stops the electrons from reaching the wire equals the energy in eV. For example, if -3.00V barely stops the electrons, their energy is 3.00 eV. The number of electrons ejected can be determined by measuring the current between the wire and plate. The more light, the more electrons; a little circuitry allows this device to be used as a light meter.

What is really important about the photoelectric effect is what Albert Einstein deduced from it. Einstein realized that there were several characteristics of the photoelectric effect that could be explained only if *EM radiation is itself quantized*: the apparently continuous stream of energy in an EM wave is actually composed of energy quanta called photons. In his explanation of the photoelectric effect, Einstein defined a quantized unit or quantum of EM energy, which we now call a **photon**, with an energy proportional to the frequency of EM radiation. In equation form, the **photon energy** is

$$E=hf,$$

where E is the energy of a photon of frequency f and h is Planck's constant. This revolutionary idea looks similar to Planck's quantization of energy states in blackbody oscillators, but it is quite different. It is the quantization of EM radiation itself. EM waves are composed of photons and are not continuous smooth waves as described in previous chapters on optics. Their energy is absorbed and emitted in lumps, not continuously. This is exactly consistent with Planck's quantization of energy levels in blackbody oscillators, since these oscillators increase and decrease their energy in steps of hf by absorbing and emitting photons having $E=hf$. We do not observe this with our eyes, because there are so many photons in common light sources that individual photons go unnoticed. (See [Figure 2].) The next section of the text (Photon Energies and the Electromagnetic Spectrum) is devoted to a discussion of photons and some of their characteristics and implications. For now, we will use the photon concept to explain the photoelectric effect, much as Einstein did.



An EM wave of frequency f is composed of photons, or individual quanta of EM radiation. The energy of each photon is $E=hf$, where h is Planck's constant and f is the frequency of the EM radiation. Higher intensity means more photons per unit area. The flashlight emits large numbers of photons of many different frequencies, hence others have energy $E{^\prime}=hf{^\prime}$, and so on.

The photoelectric effect has the properties discussed below. All these properties are consistent with the idea that individual photons of EM radiation are absorbed by individual electrons in a material, with the electron gaining the photon's energy. Some of these properties are inconsistent with the idea that EM radiation is a simple wave. For simplicity, let us consider what happens with monochromatic EM radiation in which all photons have the same energy

$$hf$$

.

1. If we vary the frequency of the EM radiation falling on a material, we find the following: For a given material, there is a threshold frequency

$${f}_{0}$$

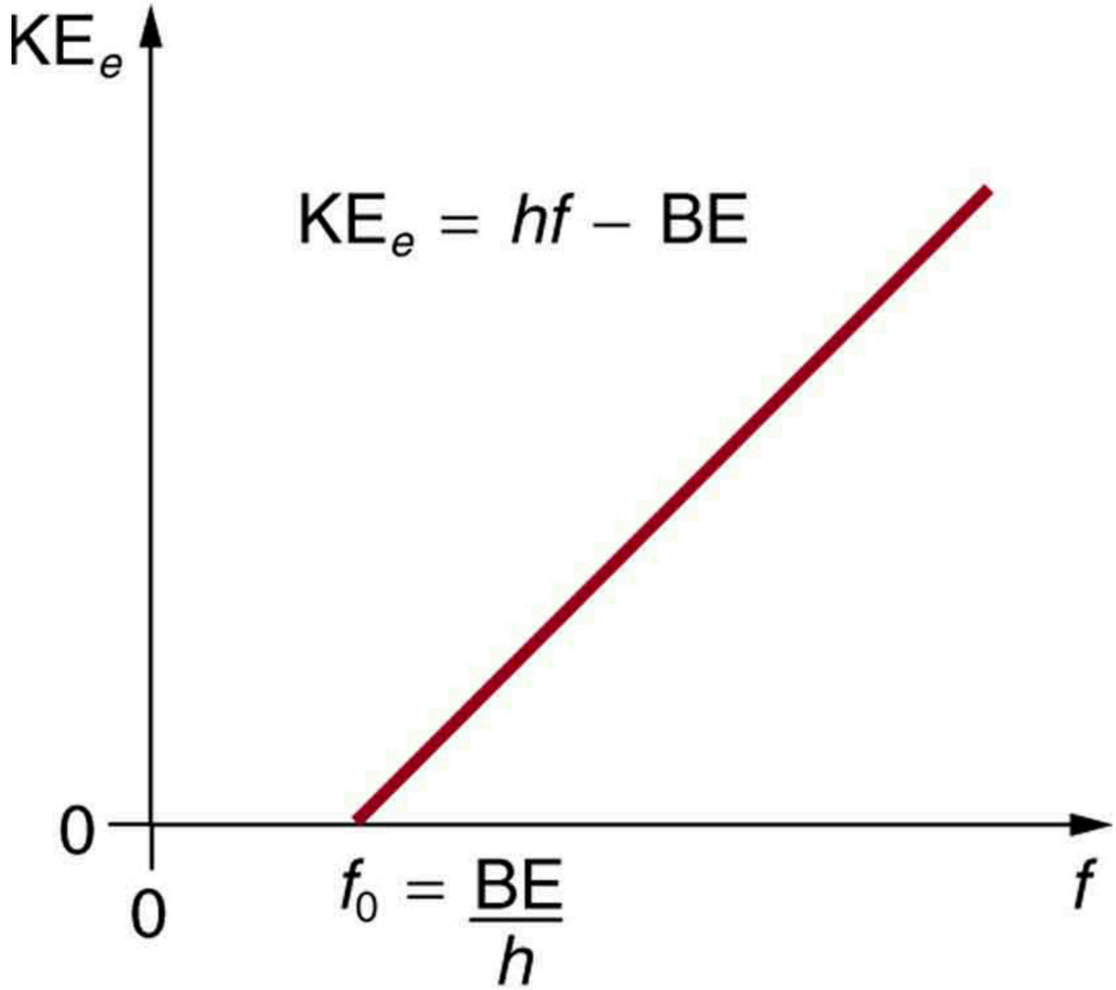
for the EM radiation below which no electrons are ejected, regardless of intensity. Individual photons interact with individual electrons. Thus if the photon energy is too small to break an electron away, no electrons will be ejected. If EM radiation was a simple wave, sufficient energy could be obtained by increasing the intensity. 2. *Once EM radiation falls on a material, electrons are ejected without delay*. As soon as an individual photon of a sufficiently high frequency is absorbed by an individual electron, the electron is ejected. If the EM radiation were a simple wave, several minutes would be required for sufficient energy to be deposited to the metal surface to eject an electron. 3. The number of electrons ejected per unit time is proportional to the intensity of the EM radiation and to no other characteristic. High-intensity EM radiation consists of large numbers of photons per unit area, with all photons having the same characteristic energy hf .

1. If we vary the intensity of the EM radiation and measure the energy of ejected electrons, we find the following: *The maximum kinetic energy of ejected electrons is independent of the intensity of the EM radiation*. Since there are so many electrons in a material, it is extremely unlikely that two photons will interact with the same electron at the same time, thereby increasing the energy given it. Instead (as noted in 3 above), increased intensity results in more electrons of the same energy being ejected. If EM radiation were a simple wave, a higher intensity could give more energy, and higher-energy electrons would be ejected.
2. The kinetic energy of an ejected electron equals the photon energy minus the binding energy of the electron in the specific material. An individual photon can give all of its energy to an electron. The photon's energy is partly used to break the electron away from the material. The remainder goes into the ejected electron's kinetic energy. In equation form, this is given by

   $$\text{KE}_e=hf-\text{BE},$$

   where KE_e is the maximum kinetic energy of the ejected electron, hf is the photon's energy, and BE is the **binding energy** of the electron to the particular material. (BE is sometimes called the *work function* of the material.) This equation, due to Einstein in 1905, explains the properties of the photoelectric effect quantitatively. An individual photon of EM radiation (it does not come any other way) interacts with an individual electron, supplying enough energy, BE, to break it away, with the remainder going to kinetic energy. The binding energy is $\text{BE}=hf_0$, where f_0 is the threshold frequency for the particular material. [Figure 3] shows a graph of maximum KE_e

   versus the frequency of incident EM radiation falling on a particular material.



Photoelectric effect. A graph of the kinetic energy of an ejected electron, KE_e, versus the frequency of EM radiation impinging on a certain material. There is a threshold frequency below which no electrons are ejected, because the individual photon interacting with an individual electron has insufficient energy to break it away. Above the threshold energy, KE_e increases linearly with f , consistent with $\text{KE}_\text{e}=hf -\text{BE}$. The slope of this line is h —the data can be used to determine Planck's constant experimentally. Einstein gave the first successful explanation of such data by proposing the idea of photons—quanta of EM radiation.

Einstein's idea that EM radiation is quantized was crucial to the beginnings of quantum mechanics. It is a far more general concept than its explanation of the photoelectric effect might imply. All EM radiation can also be modeled in the form of photons, and the characteristics of EM radiation are entirely consistent with this fact. (As we will see in the next section, many aspects of EM radiation, such as the hazards of ultraviolet (UV) radiation, can be explained _ only_ by photon properties.) More famous for modern relativity, Einstein planted an important seed for quantum mechanics in 1905, the same year he published his first paper on special relativity. His explanation of the photoelectric effect was the basis for the Nobel Prize awarded to him in 1921. Although his other contributions to theoretical physics were also noted in that award, special and general relativity were not fully recognized in spite of having been partially verified by experiment by 1921. Although hero-worshipped, this great man never received Nobel recognition for his most famous work—relativity.

Calculating Photon Energy and the Photoelectric Effect: A Violet Light

(a) What is the energy in joules and electron volts of a photon of 420-nm violet light? (b) What is the maximum kinetic energy of electrons ejected from calcium by 420-nm violet light, given that the binding energy (or work function) of electrons for calcium metal is 2.71 eV?

Strategy

To solve part (a), note that the energy of a photon is given by $E = hf$. For part (b), once the energy of the photon is calculated, it is a straightforward application of $\text{KE}_e = hf - \text{BE}$ to find the ejected electron's maximum kinetic energy, since BE is given.

Solution for (a)

Photon energy is given by

$$E = hf$$

Since we are given the wavelength rather than the frequency, we solve the familiar relationship $c = f\lambda$ for the frequency, yielding

$f = c\lambda.$

Combining these two equations gives the useful relationship

$E = hc\lambda.$

Now substituting known values yields

$E = (6.63 \times 10^{-34} \text{J} \cdot \text{s})(3.00 \times 10^{8} \text{m/s}) 420 \times 10^{-9} \text{m} = 4.74 \times 10^{-19} \text{J}.$

Converting to eV, the energy of the photon is

$E = (4.74 \times 10^{-19} \text{J}) 1 \text{eV} 1.6 \times 10^{-19} \text{J} = 2.96 \text{eV}.$

Solution for (b)

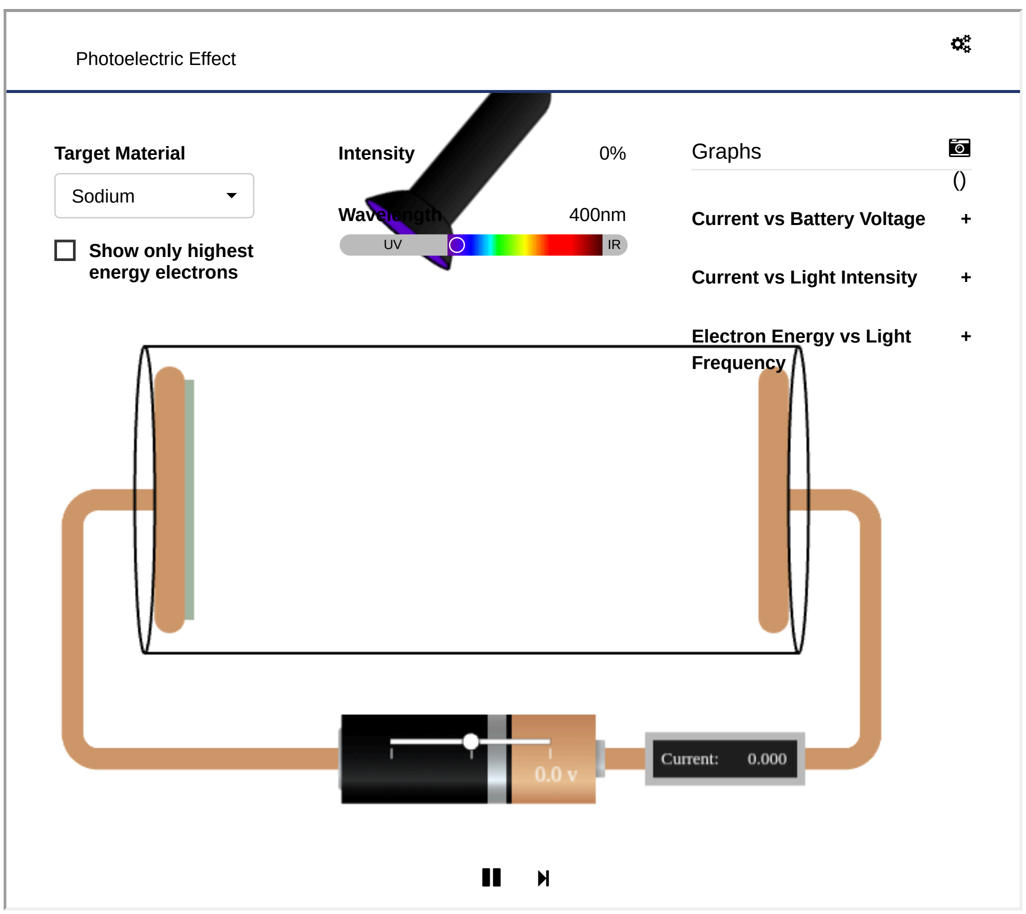
Finding the kinetic energy of the ejected electron is now a simple application of the equation $\text{KE}_e = hf - \text{BE}$. Substituting the photon energy and binding energy yields

$\text{KE}_e = hf - \text{BE} = 2.96 \text{eV} - 2.71 \text{eV} = 0.246 \text{eV}.$

Discussion

The energy of this 420-nm photon of violet light is a tiny fraction of a joule, and so it is no wonder that a single photon would be difficult for us to sense directly—humans are more attuned to energies on the order of joules. But looking at the energy in electron volts, we can see that this photon has enough energy to affect atoms and molecules. A DNA molecule can be broken with about 1 eV of energy, for example, and typical atomic and molecular energies are on the order of eV, so that the UV photon in this example could have biological effects. The ejected electron (called a *photoelectron*) has a rather low energy, and it would not travel far, except in a vacuum. The electron would be stopped by a retarding potential of but 0.26 eV. In fact, if the photon wavelength were longer and its energy less than 2.71 eV, then the formula would give a negative kinetic energy, an impossibility. This simply means that the 420-nm photons with their 2.96-eV energy are not much above the frequency threshold. You can show for yourself that the threshold wavelength is 459 nm (blue light). This means that if calcium metal is used in a light meter, the meter will be insensitive to wavelengths longer than those of blue light. Such a light meter would be completely insensitive to red light, for example.
PhET Explorations: Photoelectric Effect

See how light knocks electrons off a metal target, and recreate the experiment that spawned the field of quantum mechanics.



Section Summary

- The photoelectric effect is the process in which EM radiation ejects electrons from a material.
- Einstein proposed photons to be quanta of EM radiation having energy $E = hf$, where f is the frequency of the radiation.
- All EM radiation is composed of photons. As Einstein explained, all characteristics of the photoelectric effect are due to the interaction of individual photons with individual electrons.
- The maximum kinetic energy KE_e of ejected electrons (photoelectrons) is given by $\text{KE}_e = hf - \text{BE}$, where $$ hf is the photon energy and BE is the binding energy (or work function) of the electron to the particular material.

Conceptual Questions

Is visible light the only type of EM radiation that can cause the photoelectric effect?

Which aspects of the photoelectric effect cannot be explained without photons? Which can be explained without photons? Are the latter inconsistent with the existence of photons?

Is the photoelectric effect a direct consequence of the wave character of EM radiation or of the particle character of EM radiation? Explain briefly.

Insulators (nonmetals) have a higher BE than metals, and it is more difficult for photons to eject electrons from insulators. Discuss how this relates to the free charges in metals that make them good conductors.

If you pick up and shake a piece of metal that has electrons in it free to move as a current, no electrons fall out. Yet if you heat the metal, electrons can be boiled off. Explain both of these facts as they relate to the amount and distribution of energy involved with shaking the object as compared with heating it.

Problems & Exercises

What is the longest-wavelength EM radiation that can eject a photoelectron from silver, given that the binding energy is 4.73 eV? Is this in the visible range?

Show Solution

Strategy

The longest wavelength corresponds to the minimum photon energy needed to eject an electron, which equals the binding energy. We use $E = hf = \frac{hc}{\lambda}$ with $E = \text{BE}$.

Solution

At the threshold for photoemission, the photon energy equals the binding energy:

$$\text{BE} = \frac{hc}{\lambda}$$

Solving for wavelength:

$$\lambda = \frac{hc}{\text{BE}}$$

Using $hc = 1240 \text{ eV}\cdot\text{nm}$:

$$\lambda = \frac{1240 \text{ eV}\cdot\text{nm}}{4.73 \text{ eV}} = 262 \text{ nm}$$

Discussion

This wavelength of 262 nm is in the ultraviolet range, well below the visible spectrum which extends from approximately 380 nm (violet) to 760 nm (red). Silver requires UV radiation for the photoelectric effect because its binding energy (4.73 eV) is higher than the photon energies of visible light (1.63-3.26 eV). Photons with wavelengths shorter than 262 nm (higher energy) can eject electrons from silver, while longer wavelengths cannot. This high binding energy makes silver less suitable for photoelectric devices that operate with visible light but useful for UV detection applications.

263 nm, no this is not visible (UV radiation)

Find the longest-wavelength photon that can eject an electron from potassium, given that the binding energy is 2.24 eV. Is this visible EM radiation?

Show Solution

Strategy

The longest wavelength corresponds to the minimum photon energy needed to eject an electron, which equals the binding energy. We use $E = hf = \frac{hc}{\lambda}$ with $E = \text{BE}$.

Solution

At the threshold for photoemission, the photon energy equals the binding energy:

$$\text{BE} = \frac{hc}{\lambda}$$

Solving for wavelength:

$$\lambda=\frac{hc}{\mathrm{BE}}$$

Using hc =1240 eV·nm:

$$\lambda=\frac{1240\ \mathrm{eV\cdot nm}}{2.24\ \mathrm{eV}}=554\ \mathrm{nm}$$

Discussion

This wavelength of 554 nm is in the visible range, specifically in the green-yellow part of the spectrum. Visible light ranges from approximately 380 nm (violet) to 760 nm (red), so this threshold wavelength is indeed visible. Photons with wavelengths shorter than 554 nm (higher energy) can eject electrons from potassium, while longer wavelengths cannot. This makes potassium useful in photoelectric devices sensitive to visible light.

554 nm, yes this is visible (green-yellow light)

What is the binding energy in eV of electrons in magnesium, if the longest-wavelength photon that can eject electrons is 337 nm?

Show Solution

Strategy

The longest wavelength that can eject electrons corresponds to photons with energy equal to the binding energy. We use $\mathrm{BE}=hf=\frac{hc}{\lambda}$.

Solution

The binding energy equals the photon energy at threshold:

$$\mathrm{BE}=\frac{hc}{\lambda}$$

Using hc =1240 eV·nm:

$$\mathrm{BE}=\frac{1240\ \mathrm{eV\cdot nm}}{337\ \mathrm{nm}}=3.68\ \mathrm{eV}$$

Discussion

Magnesium has a binding energy of 3.68 eV, which falls between that of potassium (2.24 eV) and silver (4.73 eV). The threshold wavelength of 337 nm is in the ultraviolet range, just below the visible spectrum. This means magnesium requires UV light to exhibit the photoelectric effect and will not respond to visible light. This binding energy reflects the strength with which electrons are held in magnesium's outer shell and is important for applications in UV photodetectors and understanding magnesium's chemical properties.

3.69 eV

Calculate the binding energy in eV of electrons in aluminum, if the longest-wavelength photon that can eject them is 304 nm.

Show Solution

Strategy

The longest wavelength that can eject electrons corresponds to photons with energy equal to the binding energy. We use $\mathrm{BE}=hf=\frac{hc}{\lambda}$.

Solution

The binding energy equals the photon energy at threshold:

$$\mathrm{BE}=\frac{hc}{\lambda}$$

Using hc =1240 eV·nm:

$$\mathrm{BE}=\frac{1240\ \mathrm{eV\cdot nm}}{304\ \mathrm{nm}}=4.08\ \mathrm{eV}$$

Discussion

Aluminum has a binding energy of 4.08 eV, which is higher than that of potassium (2.24 eV) and sodium (2.28 eV) but lower than silver (4.73 eV). The threshold wavelength of 304 nm is in the ultraviolet range, just below the visible spectrum. This means aluminum surfaces require UV light or shorter wavelengths to exhibit the photoelectric effect, making aluminum less sensitive than alkali metals for visible-light photoelectric applications.

4.08 eV

What is the maximum kinetic energy in eV of electrons ejected from sodium metal by 450-nm EM radiation, given that the binding energy is 2.28 eV?

Show Solution

Strategy

Use the photoelectric equation $\mathrm{KE}_e = hf - \mathrm{BE}$, where the photon energy is $hf = \frac{hc}{\lambda}$. Calculate the photon energy first, then subtract the binding energy.

Solution

First, calculate the photon energy using $hc = 1240\ \mathrm{eV \cdot nm}$:

$$E_{\text{photon}} = \frac{hc}{\lambda} = \frac{1240\ \mathrm{eV \cdot nm}}{450\ \mathrm{nm}} = 2.76\ \mathrm{eV}$$

Now apply the photoelectric equation:

$$\mathrm{KE}_e = E_{\text{photon}} - \mathrm{BE} = 2.76\ \mathrm{eV} - 2.28\ \mathrm{eV} = 0.48\ \mathrm{eV}$$

Discussion

The 450-nm blue light photons have just barely enough energy (2.76 eV) to overcome sodium's binding energy (2.28 eV) and eject electrons with a small kinetic energy of 0.48 eV. This demonstrates that sodium is sensitive to visible light, making it useful for photoelectric applications. The small kinetic energy means these photoelectrons won't travel far and can be stopped by a retarding potential of only 0.48 V.

0.483 eV

UV radiation having a wavelength of 120 nm falls on gold metal, to which electrons are bound by 4.82 eV. What is the maximum kinetic energy of the ejected photoelectrons?

Show Solution

Strategy

We use the photoelectric equation $\mathrm{KE}_e = hf - \mathrm{BE}$, where the photon energy is $hf = \frac{hc}{\lambda}$. The maximum kinetic energy is the photon energy minus the binding energy.

Solution

First, calculate the photon energy using $hc = 1240\ \mathrm{eV \cdot nm}$:

$$E_{\text{photon}} = \frac{hc}{\lambda} = \frac{1240\ \mathrm{eV \cdot nm}}{120\ \mathrm{nm}} = 10.33\ \mathrm{eV}$$

Now apply the photoelectric equation:

$$\mathrm{KE}_e = E_{\text{photon}} - \mathrm{BE} = 10.33\ \mathrm{eV} - 4.82\ \mathrm{eV} = 5.51\ \mathrm{eV}$$

Discussion

The UV photon has sufficient energy (10.33 eV) to overcome the binding energy of gold (4.82 eV) and still impart 5.51 eV of kinetic energy to the ejected electron. This is a relatively high kinetic energy for a photoelectron, more than twice the binding energy. Such energetic photoelectrons could travel significant distances in a vacuum before being stopped. Gold's relatively high binding energy makes it less suitable for visible-light photoelectric applications but useful for UV detection.

5.51 eV

Violet light of wavelength 400 nm ejects electrons with a maximum kinetic energy of 0.860 eV from sodium metal. What is the binding energy of electrons to sodium metal?

Show Solution

Strategy

Use the photoelectric equation $\mathrm{KE}_e = hf - \mathrm{BE}$ and solve for the binding energy. First calculate the photon energy, then use the given kinetic energy to find BE.

Solution

Calculate the photon energy using $hc = 1240\ \mathrm{eV \cdot nm}$:

$$E_{\text{photon}} = \frac{hc}{\lambda} = \frac{1240\ \mathrm{eV \cdot nm}}{400\ \mathrm{nm}} = 3.10\ \mathrm{eV}$$

Rearrange the photoelectric equation to solve for binding energy:

$$\mathrm{BE} = E_{\text{photon}} - \mathrm{KE}_e = 3.10\ \mathrm{eV} - 0.860\ \mathrm{eV} = 2.24\ \mathrm{eV}$$

Discussion

This value of 2.24 eV matches the known work function of sodium within experimental uncertainty. The 400-nm violet photons carry 3.10 eV of energy, of which 2.24 eV is needed to overcome the binding and 0.860 eV goes into the electron's kinetic energy. This relatively low binding energy makes sodium highly sensitive to visible light, which is why sodium-based photocells are common in practical photoelectric devices. The measurement demonstrates how the photoelectric effect can be used to determine the work function of materials.

2.25 eV

UV radiation having a 300-nm wavelength falls on uranium metal, ejecting 0.500-eV electrons. What is the binding energy of electrons to uranium metal?

Show Solution

Strategy

We use the photoelectric equation $\text{KE}_e = hf - \text{BE}$ and solve for the binding energy. First calculate the photon energy, then use the given kinetic energy to find BE.

Solution

Calculate the photon energy using $hc = 1240 \text{ eV}\cdot\text{nm}$:

$$E_{\text{photon}} = \frac{hc}{\lambda} = \frac{1240 \text{ eV}\cdot\text{nm}}{300 \text{ nm}} = 4.13 \text{ eV}$$

Rearrange the photoelectric equation to solve for binding energy:

$$\text{BE} = E_{\text{photon}} - \text{KE}_e = 4.13 \text{ eV} - 0.500 \text{ eV} = 3.63 \text{ eV}$$

Discussion

Uranium metal has a binding energy of 3.63 eV for its outermost electrons. This value is intermediate between the alkali metals (around 2-2.3 eV) and more tightly bound metals like silver (4.73 eV). The 300-nm UV photon provides 4.13 eV of energy, which is sufficient to overcome the binding and give the electron 0.500 eV of kinetic energy. This binding energy indicates uranium requires UV radiation for photoelectric emission, as visible light photons lack sufficient energy.

3.63 eV

What is the wavelength of EM radiation that ejects 2.00-eV electrons from calcium metal, given that the binding energy is 2.71 eV? What type of EM radiation is this?

Show Solution

Strategy

The photon energy must equal the sum of binding energy and kinetic energy: $E_{\text{photon}} = \text{BE} + \text{KE}_e$. We then use $\lambda = \frac{hc}{E_{\text{photon}}}$ to find the wavelength.

Solution

The total photon energy needed is:

$$E_{\text{photon}} = \text{BE} + \text{KE}_e = 2.71 \text{ eV} + 2.00 \text{ eV} = 4.71 \text{ eV}$$

Now find the wavelength using $hc = 1240 \text{ eV}\cdot\text{nm}$:

$$\lambda = \frac{hc}{E_{\text{photon}}} = \frac{1240 \text{ eV}\cdot\text{nm}}{4.71 \text{ eV}} = 263 \text{ nm}$$

This wavelength is in the ultraviolet (UV) region of the electromagnetic spectrum.

Discussion

The 263-nm UV photons carry 4.71 eV of energy, with 2.71 eV used to overcome calcium’s binding energy and 2.00 eV transferred to the electron as kinetic energy. This is well into the UV range (below 400 nm), meaning calcium requires UV radiation to produce photoelectrons with this much kinetic energy. Visible light photons lack sufficient energy for this process. The relatively high photon energy requirement demonstrates why UV radiation is more effective at causing the photoelectric effect than visible light for most metals.

(a) 264 nm

(b) Ultraviolet

Find the wavelength of photons that eject 0.100-eV electrons from potassium, given that the binding energy is 2.24 eV. Are these photons visible?

Show Solution

Strategy

The photon energy must equal the sum of binding energy and kinetic energy: $E_{\text{photon}} = \text{BE} + \text{KE}_e$. We then use $\lambda = \frac{hc}{E_{\text{photon}}}$ to find the wavelength.

Solution

The total photon energy needed is:

$$E_{\text{photon}} = \text{BE} + \text{KE}_e = 2.24\ \text{eV} + 0.100\ \text{eV} = 2.34\ \text{eV}$$

Now find the wavelength using $hc = 1240\ \text{eV}\cdot\text{nm}$:

$$\lambda = \frac{hc}{E_{\text{photon}}} = \frac{1240\ \text{eV}\cdot\text{nm}}{2.34\ \text{eV}} = 530\ \text{nm}$$

Discussion

These photons have a wavelength of 530 nm, which is in the visible spectrum—specifically, green light. The visible spectrum ranges from about 380 nm (violet) to 760 nm (red), so 530 nm falls well within this range. Photons of this wavelength can eject electrons from potassium with a small kinetic energy of 0.100 eV, demonstrating that potassium is sensitive to visible light and is useful for photoelectric applications with visible radiation.

530 nm, yes these photons are visible (green light)

What is the maximum velocity of electrons ejected from a material by 80-nm photons, if they are bound to the material by 4.73 eV?

Show Solution

Strategy

First find the kinetic energy of the ejected electrons using $\text{KE}_e = hf - \text{BE}$, then use $\text{KE} = \frac{1}{2}mv^2$ to find velocity.

Solution

Calculate the photon energy using $hc = 1240\ \text{eV}\cdot\text{nm}$:

$$E_{\text{photon}} = \frac{hc}{\lambda} = \frac{1240\ \text{eV}\cdot\text{nm}}{80\ \text{nm}} = 15.5\ \text{eV}$$

Find the kinetic energy of ejected electrons:

$$\text{KE}_e = E_{\text{photon}} - \text{BE} = 15.5\ \text{eV} - 4.73\ \text{eV} = 10.77\ \text{eV}$$

Convert to joules: $\text{KE}_e = 10.77\ \text{eV} \times 1.60 \times 10^{-19}\ \text{J/eV} = 1.72 \times 10^{-18}\ \text{J}$

Use $\text{KE} = \frac{1}{2}mv^2$ to find velocity:

$$v = \sqrt{\frac{2\text{KE}}{m_e}} = \sqrt{\frac{2(1.72\times10^{-18}\ \text{J})}{9.11\times10^{-31}\ \text{kg}}} = 1.94\times10^{6}\ \text{m/s}$$

Discussion

The 80-nm UV photons carry significant energy (15.5 eV), producing fast-moving photoelectrons with velocities near 2 million m/s—about 0.6% of the speed of light. This is fast enough that we're approaching the regime where relativistic effects begin to matter (though still well within the non-relativistic approximation). The high electron velocity demonstrates the substantial energy transfer possible with short-wavelength UV radiation, which is why UV photodetectors can produce strong electrical signals.

$1.95 \times 10^{6}\text{m/s}$

Photoelectrons from a material with a binding energy of 2.71 eV are ejected by 420-nm photons. Once ejected, how long does it take these electrons to travel 2.50 cm to a detection device?

Show Solution

Strategy

First, find the kinetic energy of the ejected electrons using $\text{KE}_e = hf - \text{BE}$. Then convert this to velocity using $\text{KE}_e = \frac{1}{2}mv^2$, assuming nonrelativistic speeds. Finally, use $t = \frac{d}{v}$ to find the time.

Solution

Calculate the photon energy:

$$E_{\text{photon}} = \frac{hc}{\lambda} = \frac{1240\ \text{eV}\cdot\text{nm}}{420\ \text{nm}} = 2.95\ \text{eV}$$

Find the kinetic energy of ejected electrons:

$$\text{KE}_e = 2.95\ \text{eV} - 2.71\ \text{eV} = 0.24\ \text{eV}$$

Convert to joules:

$$KE_e = 0.24 \text{ eV} \times 1.60 \times 10^{-19} \text{ J/eV} = 3.84 \times 10^{-20} \text{ J}$$

Solve for velocity from $KE_e = \frac{1}{2}mv^2$:

$$v = \sqrt{\frac{2KE_e}{m}} = \sqrt{\frac{2(3.84 \times 10^{-20} \text{ J})}{9.11 \times 10^{-31} \text{ kg}}} = 2.90 \times 10^5 \text{ m/s}$$

Calculate the time to travel 2.50 cm:

$$t = \frac{d}{v} = \frac{0.0250 \text{ m}}{2.90 \times 10^5 \text{ m/s}} = 8.62 \times 10^{-8} \text{ s} = 86.2 \text{ ns}$$

Discussion

The photoelectrons travel at about 0.1% the speed of light (290 km/s), which is fast but well below relativistic speeds. They reach the detector in only 86.2 nanoseconds. This rapid transit time is important in photoelectric devices where quick response is essential. The relatively low kinetic energy (0.24 eV) results because the 420-nm photon energy (2.95 eV) is only slightly above the binding energy threshold (2.71 eV).

8.62×10^{-8} s or 86.2 ns

A laser with a power output of 2.00 mW at a wavelength of 400 nm is projected onto calcium metal. (a) How many electrons per second are ejected? (b) What power is carried away by the electrons, given that the binding energy is 2.71 eV?

Show Solution

Strategy

(a) Find the photon energy, then divide the total power by energy per photon to get photons/second. Assuming each photon ejects one electron gives electrons/second. (b) Find the kinetic energy per electron using the photoelectric equation, then multiply by the electron ejection rate.

Solution for (a)

Calculate the photon energy using $hc = 1240 \text{ eV·nm}$:

$$E_{\text{photon}} = \frac{hc}{\lambda} = \frac{1240 \text{ eV·nm}}{400 \text{ nm}} = 3.10 \text{ eV} = 4.96 \times 10^{-19} \text{ J}$$

Number of photons (and electrons) per second:

$$N = \frac{P}{E_{\text{photon}}} = \frac{2.00 \times 10^{-3} \text{ W}}{4.96 \times 10^{-19} \text{ J}} = 4.03 \times 10^{15} \text{ electrons/s}$$

Solution for (b)

Kinetic energy per electron:

$$KE_e = E_{\text{photon}} - BE = 3.10 \text{ eV} - 2.71 \text{ eV} = 0.39 \text{ eV} = 6.24 \times 10^{-20} \text{ J}$$

Power carried by electrons:

$$P_e = N \times KE_e = (4.03 \times 10^{15} \text{ /s})(6.24 \times 10^{-20} \text{ J}) = 2.51 \times 10^{-4} \text{ W} = 0.251 \text{ mW}$$

Discussion

The laser produces about 4 trillion electrons per second, but the electrons carry away only 0.251 mW—about 12.6% of the input power. The rest (1.75 mW, or 87.4%) goes into overcoming the binding energy and is converted to heat in the calcium metal. This illustrates the inefficiency of the photoelectric effect for power conversion: most of the photon energy is “wasted” overcoming the work function rather than creating useful kinetic energy in the photoelectrons.

(a) 4.02×10^{15}/s (b) 0.256 mW

(a) Calculate the number of photoelectrons per second ejected from a 1.00-mm ^{2} area of sodium metal by 500-nm EM radiation having an intensity of 1.30kW/m^2 (the intensity of sunlight above the Earth’s atmosphere). (b) Given that the binding energy is 2.28 eV, what power is carried away by the electrons? (c) The electrons carry away less power than brought in by the photons. Where does the other power go? How can it be recovered?

Show Solution

Strategy

(a) Find the power incident on the area, then the energy per photon, and finally the number of photons (and electrons) per second. (b) Calculate the kinetic energy per electron and multiply by the rate. (c) Compare the input and output power to account for energy conservation.

Solution for (a)

Power incident on the area:

$$P_{\text{in}}=I\times A=(1.30\times10^{3}\ \text{W/m}^2)(1.00\times10^{-6}\ \text{m}^2)=1.30\times10^{-3}\ \text{W}$$

Energy per photon:

$$E_{\text{photon}}=\frac{hc}{\lambda}=\frac{1240\ \text{eV}\cdot\text{nm}}{500\ \text{nm}}=2.48\ \text{eV}=3.97\times10^{-19}\ \text{J}$$

Number of photons (and electrons) per second:

$$N=\frac{P_{\text{in}}}{E_{\text{photon}}}=\frac{1.30\times10^{-3}\ \text{J/s}}{3.97\times10^{-19}\ \text{J}}=3.27\times10^{15}\ \text{electrons/s}$$

Solution for (b)

Kinetic energy per electron:

$$\text{KE}_e=E_{\text{photon}}-\text{BE}=2.48\ \text{eV}-2.28\ \text{eV}=0.20\ \text{eV}=3.20\times10^{-20}\ \text{J}$$

Power carried by electrons:

$$P_{\text{electrons}}=N\times\text{KE}_e=(3.27\times10^{15}\ \text{/s})(3.20\times10^{-20}\ \text{J})=0.105\ \text{W}=105\ \text{mW}$$

Solution for (c)

Power used to overcome binding energy:

$$P_{\text{binding}}=P_{\text{in}}-P_{\text{electrons}}=1.30\ \text{W}-0.105\ \text{W}=1.20\ \text{W}$$

This energy goes into heating the sodium metal. It can be recovered as thermal energy or used to keep the metal at operating temperature.

Discussion

Only about 8% of the incident photon energy is carried away as kinetic energy of the electrons; the remaining 92% goes into overcoming the binding energy and heats the metal. This demonstrates why photoelectric devices can become warm during operation and why efficient cooling may be necessary for high-intensity applications.

(a) 3.27×10^{15} electrons/s

(b) 0.105 W or 105 mW

(c) The remaining power (1.20 W) heats the sodium metal. It can be recovered as thermal energy or dissipated through cooling.

Unreasonable Results

Red light having a wavelength of 700 nm is projected onto magnesium metal to which electrons are bound by 3.68 eV. (a) Use $\text{KE}_e=hf-\text{BE}$ to calculate the kinetic energy of the ejected electrons. (b) What is unreasonable about this result? (c) Which assumptions are unreasonable or inconsistent?

Show Solution

(a) -1.90eV (b) Negative kinetic energy

(c) That the electrons would be knocked free.

Unreasonable Results

(a) What is the binding energy of electrons to a material from which 4.00-eV electrons are ejected by 400-nm EM radiation? (b) What is unreasonable about this result? (c) Which assumptions are unreasonable or inconsistent?

Show Solution

Strategy

Calculate the photon energy, then use the photoelectric equation to find the binding energy. Examine whether the result is physically reasonable.

Solution for (a)

Calculate the photon energy:

$$E_{\text{photon}}=\frac{hc}{\lambda}=\frac{1240\ \text{eV}\cdot\text{nm}}{400\ \text{nm}}=3.10\ \text{eV}$$

Using the photoelectric equation:

$$\text{BE} = E_{\text{photon}} - \text{KE}_e = 3.10\ \text{eV} - 4.00\ \text{eV} = -0.90\ \text{eV}$$

Solution for (b)

A negative binding energy is physically impossible. Binding energy represents the energy required to remove an electron from a material, which must be positive. A negative value would imply the electron is repelled by the material, contradicting the definition of binding.

Solution for (c)

The assumption that is unreasonable is that 400-nm photons can eject electrons with 4.00 eV of kinetic energy. The photon energy (3.10 eV) is less than the claimed kinetic energy of the ejected electrons (4.00 eV). By conservation of energy, this is impossible—the electron cannot have more kinetic energy than the total energy provided by the photon. Either the photon wavelength must be shorter (higher energy), or the electron kinetic energy must be less than 3.10 eV.

(a) −0.90 eV

(b) Negative binding energy is physically impossible

(c) The assumption that 400-nm photons (3.10 eV) can produce 4.00-eV electrons violates energy conservation

Glossary

photoelectric effect
: the phenomenon whereby some materials eject electrons when light is shined on them

photon
: a quantum, or particle, of electromagnetic radiation

photon energy
: the amount of energy a photon has; $E = hf$

binding energy
: also called the *work function*; the amount of energy necessary to eject an electron from a material

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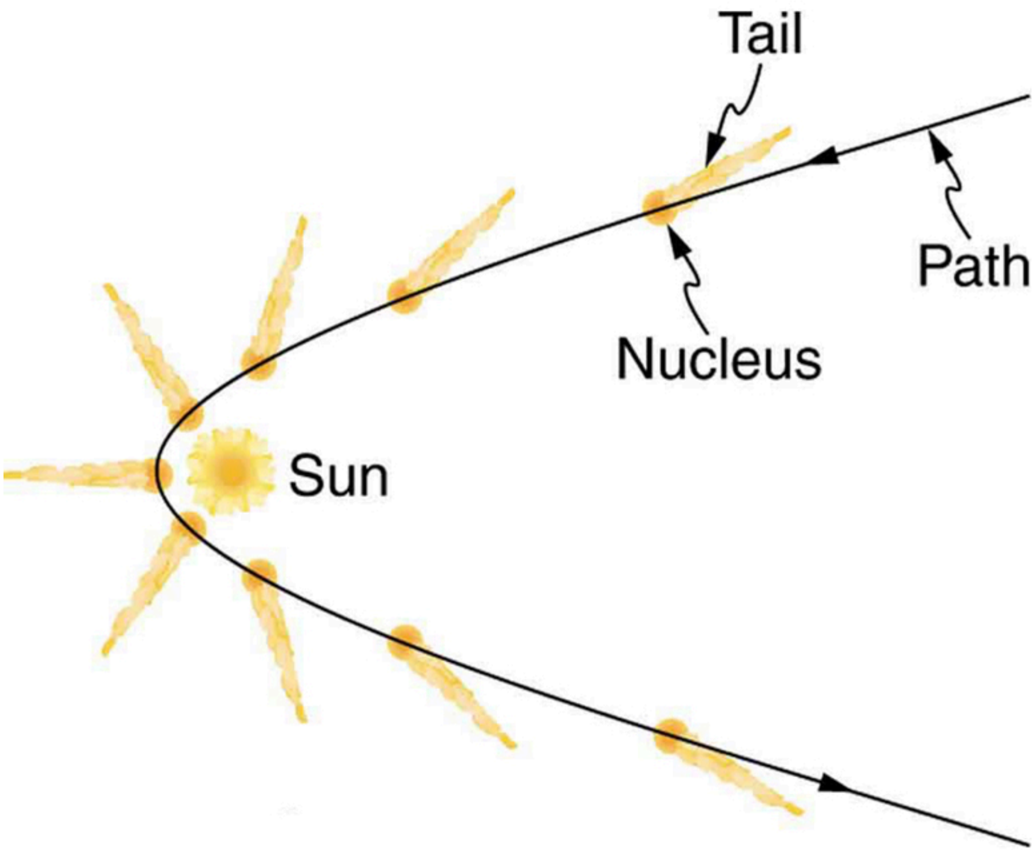
Photon Momentum

- Relate the linear momentum of a photon to its energy or wavelength, and apply linear momentum conservation to simple processes involving the emission, absorption, or reflection of photons.
- Account qualitatively for the increase of photon wavelength that is observed, and explain the significance of the Compton wavelength.

Measuring Photon Momentum

The quantum of EM radiation we call a **photon** has properties analogous to those of particles we can see, such as grains of sand. A photon interacts as a unit in collisions or when absorbed, rather than as an extensive wave. Massive quanta, like electrons, also act like macroscopic particles—something we expect, because they are the smallest units of matter. Particles carry momentum as well as energy. Despite photons having no mass, there has long been evidence that EM radiation carries momentum. (Maxwell and others who studied EM waves predicted that they would carry momentum.) It is now a well-established fact that photons **do** have momentum. In fact, photon momentum is suggested by the photoelectric effect, where photons knock electrons out of a substance. [Figure 1] shows macroscopic evidence of photon momentum.



The tails of the Hale-Bopp comet point away from the Sun, evidence that light has momentum. Dust emanating from the body of the comet forms this tail. Particles of dust are pushed away from the Sun by light reflecting from them. The blue ionized gas tail is also produced by photons interacting with atoms in the comet material. (credit: Geoff Chester, U.S. Navy, via Wikimedia Commons)

[Figure 1] shows a comet with two prominent tails. What most people do not know about the tails is that they always point **away** from the Sun rather than trailing behind the comet (like the tail of Bo Peep's sheep). Comet tails are composed of gases and dust evaporated from the body of the comet and ionized gas. The dust particles recoil away from the Sun when photons scatter from them. Evidently, photons carry momentum in the direction of their motion (away from the Sun), and some of this momentum is transferred to dust particles in collisions. Gas atoms and molecules in the blue tail are most affected by other particles of radiation, such as protons and electrons emanating from the Sun, rather than by the momentum of photons.

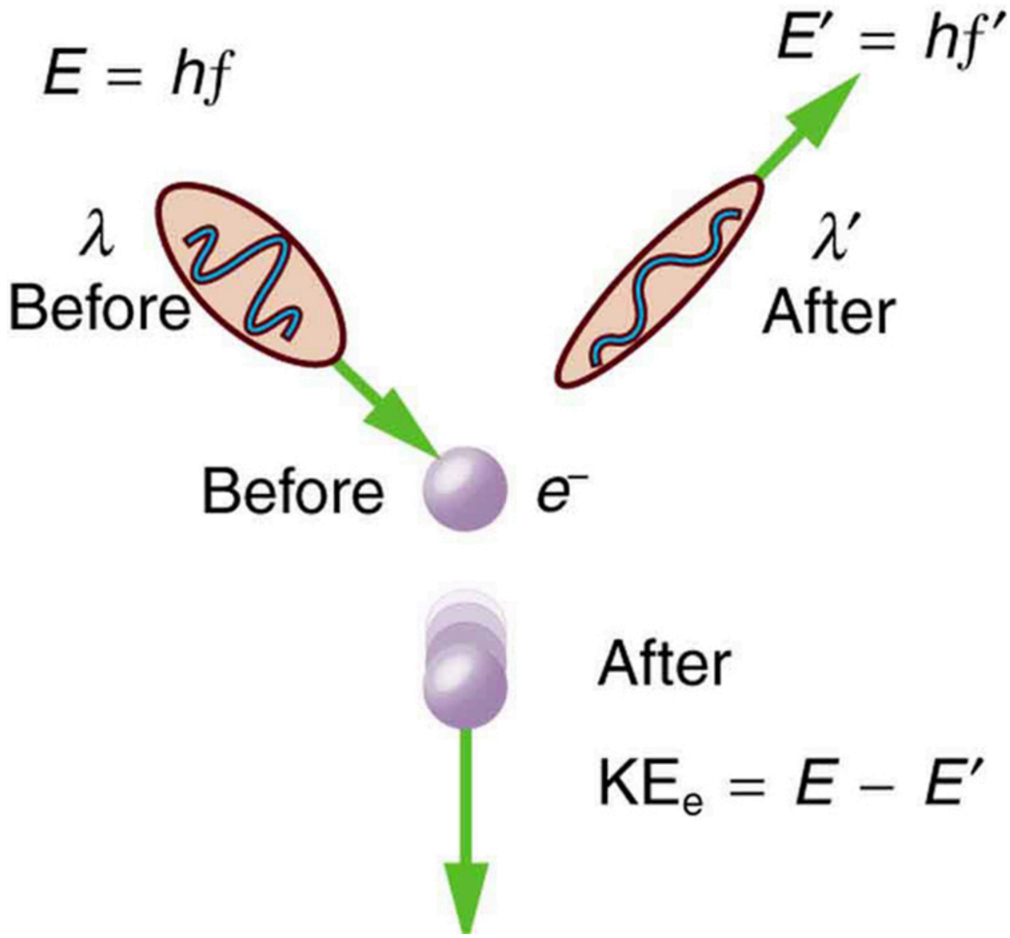
Connections: Conservation of Momentum

Not only is momentum conserved in all realms of physics, but all types of particles are found to have momentum. We expect particles with mass to have momentum, but now we see that massless particles including photons also carry momentum.

Momentum is conserved in quantum mechanics just as it is in relativity and classical physics. Some of the earliest direct experimental evidence of this came from scattering of x-ray photons by electrons in substances, named Compton scattering after the American physicist Arthur H. Compton (1892–1962). Around 1923, Compton observed that X-rays scattered from materials had a decreased energy and correctly analyzed this as being due to the scattering of photons from electrons. This phenomenon could be handled as a collision between two particles—a photon and an electron at rest in the material. Energy and momentum are conserved in the collision. (See [Figure 2]) He won a Nobel Prize in 1929 for the discovery of this scattering, now called the **Compton effect**, because it helped prove that **photon momentum** is given by

$$p=h\lambda,$$

where h is Planck's constant and λ is the photon wavelength. (Note that relativistic momentum given as $p=\gamma mu$ is valid only for particles having mass.)



The Compton effect is the name given to the scattering of a photon by an electron. Energy and momentum are conserved, resulting in a reduction of both for the scattered photon. Studying this effect, Compton verified that photons have momentum.

We can see that photon momentum is small, since $p=h/\lambda$ and h is very small. It is for this reason that we do not ordinarily observe photon momentum. Our mirrors do not recoil when light reflects from them (except perhaps in cartoons). Compton saw the effects of photon momentum because he was observing X-rays, which have a small wavelength and a relatively large momentum, interacting with the lightest of particles, the electron.

Electron and Photon Momentum Compared

(a) Calculate the momentum of a visible photon that has a wavelength of 500 nm. (b) Find the velocity of an electron having the same momentum. (c) What is the energy of the electron, and how does it compare with the energy of the photon?

Strategy

Finding the photon momentum is a straightforward application of its definition: $p = h\lambda$. If we find the photon momentum is small, then we can assume that an electron with the same momentum will be nonrelativistic, making it easy to find its velocity and kinetic energy from the classical formulas.

Solution for (a)

Photon momentum is given by the equation:

$$p=h\lambda.$$

Entering the given photon wavelength yields

$$p=6.63\times10^{-34}\text{J}\cdot\text{s}500\times10^{-9}\text{m}=1.33\times10^{-27}\text{kg}\cdot\text{m/s}.$$

Solution for (b)

Since this momentum is indeed small, we will use the classical expression $p = mv$ to find the velocity of an electron with this momentum. Solving for v and using the known value for the mass of an electron gives

$$v=pm=1.33\times10^{-27}\text{kg}\cdot\text{m/s}9.11\times10^{-31}\text{kg}=1460\text{m/s}\approx1460\text{m/s}.$$

Solution for (c)

The electron has kinetic energy, which is classically given by

$$\text{KE}_e=12mv^2.$$

Thus,

$$\text{KE}_e=12(9.11\times10^{-3}\text{kg})(1455\text{m/s})^2=9.64\times10^{-25}\text{J}.$$

Converting this to eV by multiplying by $(1\text{eV})/(1.602\times10^{-19}\text{J})$ yields

$$\text{KE}_e=6.02\times10^{-6}\text{eV}.$$

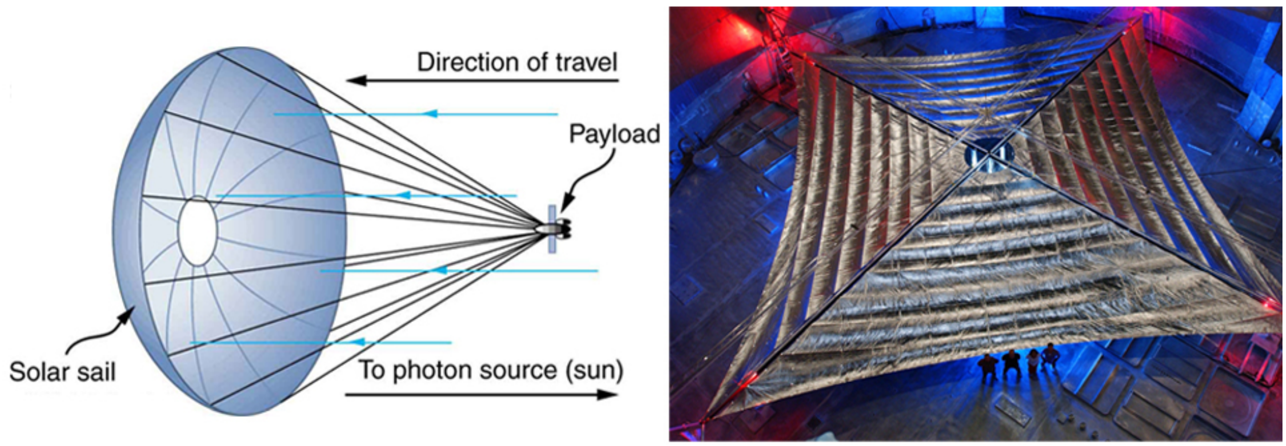
The photon energy E is

$$E=hc\lambda=1240\text{eV}\cdot\text{nm}500\text{nm}=2.48\text{eV},$$

which is about five orders of magnitude greater.

Discussion

Photon momentum is indeed small. Even if we have huge numbers of them, the total momentum they carry is small. An electron with the same momentum has a 1460 m/s velocity, which is clearly nonrelativistic. A more massive particle with the same momentum would have an even smaller velocity. This is borne out by the fact that it takes far less energy to give an electron the same momentum as a photon. But on a quantum-mechanical scale, especially for high-energy photons interacting with small masses, photon momentum is significant. Even on a large scale, photon momentum can have an effect if there are enough of them and if there is nothing to prevent the slow recoil of matter. Comet tails are one example, but there are also proposals to build space sails that use huge low-mass mirrors (made of aluminized Mylar) to reflect sunlight. In the vacuum of space, the mirrors would gradually recoil and could actually take spacecraft from place to place in the solar system. (See [Figure 3].)



(a) (b)

(a) Space sails have been proposed that use the momentum of sunlight reflecting from gigantic low-mass sails to propel spacecraft about the solar system. A Russian test model of this (the Cosmos 1) was launched in 2005, but did not make it into orbit due to a rocket failure. (b) A U.S. version of this, labeled LightSail-1, is scheduled for trial launches in the first part of this decade. It will have a 40-m2 sail. (credit: Kim Newton/NASA)

Relativistic Photon Momentum

There is a relationship between photon momentum p and photon energy E that is consistent with the relation given previously for the relativistic total energy of a particle as $E^2 = (pc)^2 + (mc^2)^2$. We know m is zero for a photon, but p is not, so that $E^2 = (pc)^2 + (mc^2)^2$ becomes <div class="equation" > $E =$pc, </div>

or

$$p = Ec(photons).$$

To check the validity of this relation, note that $E = hc/\lambda$ for a photon. Substituting this into $p = E/c$ yields

$$p = (hc/\lambda)/c = h\lambda,$$

as determined experimentally and discussed above. Thus, $p = E/c$ is equivalent to Compton's result $p = h/\lambda$. For a further verification of the relationship between photon energy and momentum, see [Example 2].

Photon Detectors

Almost all detection systems talked about thus far—eyes, photographic plates, photomultiplier tubes in microscopes, and CCD cameras—rely on particle-like properties of photons interacting with a sensitive area. A change is caused and either the change is cascaded or zillions of points are recorded to form an image we detect. These detectors are used in biomedical imaging systems, and there is ongoing research into improving the efficiency of receiving photons, particularly by cooling detection systems and reducing thermal effects.

Photon Energy and Momentum

Show that $p = E/c$ for the photon considered in the [Example 1].

Strategy

We will take the energy E found in [Example 1], divide it by the speed of light, and see if the same momentum is obtained as before.

Solution

Given that the energy of the photon is 2.48 eV and converting this to joules, we get

$$p = Ec = (2.48\text{eV})(1.60\times10^{-19}\text{J/eV})3.00\times10^{8}\text{m/s} = 1.33\times10^{-27}\text{kg}\cdot\text{m/s}.$$

Discussion

This value for momentum is the same as found before (note that unrounded values are used in all calculations to avoid even small rounding errors), an expected verification of the relationship $p = E/c$. This also means the relationship between energy, momentum, and mass given by $E^2 = (pc)^2 + (mc^2)^2$ applies to both matter and photons. Once again, note that p is not zero, even when m is.

Problem-Solving Suggestion

Note that the forms of the constants $h = 4.14 \times 10^{-15}\text{eV}\cdot\text{s}$ and $hc = 1240\text{eV}\cdot\text{nm}$ may be particularly useful for this section's Problems and Exercises.

Section Summary

- Photons have momentum, given by $p = h\lambda$, where λ is the photon wavelength.
- Photon energy and momentum are related by $p = Ec$, where $E = hf = hc/\lambda$ for a photon.

Conceptual Questions

Which formula may be used for the momentum of all particles, with or without mass?

Is there any measurable difference between the momentum of a photon and the momentum of matter?

Why don't we feel the momentum of sunlight when we are on the beach?

Problems & Exercises

(a) Find the momentum of a 4.00-cm-wavelength microwave photon. (b) Discuss why you expect the answer to (a) to be very small.

Show Solution

Strategy

(a) Use the de Broglie relation for photons: $p = h\lambda$. (b) Consider that momentum is inversely proportional to wavelength.

Solution for (a)

$$p = h\lambda = 6.63\times10^{-34}\ \text{J}\cdot\text{s}\,4.00\times10^{-2}\ \text{m} = 1.66\times10^{-32}\ \text{kg}\cdot\text{m/s}$$

Solution for (b)

The momentum is very small because the wavelength is large (4.00 cm = 0.04 m). Since photon momentum is inversely proportional to wavelength ($p = h/\lambda$), long-wavelength radiation like microwaves carries very little momentum. This is about 20 orders of magnitude smaller than the momentum of a slowly moving baseball, making the momentum of individual microwave photons essentially negligible for any macroscopic application.

Discussion

This tiny momentum (1.66×10^{-32} kg·m/s) explains why radiation pressure from microwaves is unmeasurable in everyday situations. Even though a microwave oven might emit 10^{29} photons per second, the total momentum transfer per second (force) would only be about 10^{-3} N—roughly the weight of a mosquito. This is why we don't feel pushed by microwave ovens despite their high power output.

(a) $1.66\times10^{-32}\text{kg}\cdot\text{m/s}$ (b) The wavelength of microwave photons is large, so the momentum they carry is very small.

(a) What is the momentum of a 0.0100-nm-wavelength photon that could detect details of an atom? (b) What is its energy in MeV?

Show Solution

Strategy

(a) Use $p = h\lambda$ to find the photon momentum. (b) Use $E = pc$ for a photon to find the energy, then convert to MeV.

Solution for (a)

The photon momentum is:

$$p = h\lambda = 6.63\times10^{-34}\ \text{J}\cdot\text{s}\,0.0100\times10^{-9}\ \text{m} = 6.63\times10^{-23}\ \text{kg}\cdot\text{m/s}$$

Solution for (b)

The photon energy is:

$$E = pc = (6.63\times10^{-23}\ \text{kg}\cdot\text{m/s})(3.00\times10^{8}\ \text{m/s}) = 1.99\times10^{-14}\ \text{J}$$

Converting to MeV:

$$E = 1.99\times10^{-14}\ \text{J}\,1.60\times10^{-13}\ \text{J/MeV} = 0.124\ \text{MeV} = 124\ \text{keV}$$

Discussion

This 0.0100-nm photon is in the X-ray range with energy of 124 keV. Such photons can indeed probe atomic structure because their wavelength (0.01 nm) is comparable to atomic sizes (roughly 0.1 nm). The relatively large momentum (compared to visible light photons) means these X-ray photons can significantly disturb atomic electrons when they interact, which is why X-rays can ionize atoms and damage biological tissue.

(a) 6.63×10^{-23} kg·m/s

(b) 0.124 MeV or 124 keV

(a) What is the wavelength of a photon that has a momentum of $5.00\times10^{-29}\text{kg}\cdot\text{m/s}$? (b) Find its energy in eV.

Show Solution

Strategy

(a) Use $\lambda = hp$ to find wavelength from momentum. (b) Use $E = pc$ to find energy and convert to eV.

Solution for (a)

$$\lambda = hp = 6.63\times10^{-34}\ \text{J}\cdot\text{s}\,5.00\times10^{-29}\ \text{kg}\cdot\text{m/s} = 1.33\times10^{-5}\ \text{m} = 13.3\ \mu\text{m}$$

Solution for (b)

$$E = pc = (5.00 \times 10^{-29}\ \text{kg·m/s})(3.00 \times 10^{8}\ \text{m/s}) = 1.50 \times 10^{-20}\ \text{J}$$

Converting to eV:

$$E = \frac{1.50 \times 10^{-20}\ \text{J}}{1.60 \times 10^{-19}\ \text{J/eV}} = 0.0938\ \text{eV}$$

Discussion

This photon has a wavelength of 13.3 μm, placing it in the mid-infrared region of the spectrum. The very low momentum (5.00×10^{-29} kg·m/s) and correspondingly low energy (0.094 eV) are characteristic of IR radiation. Such photons are emitted by objects at room temperature and are used in thermal imaging cameras. The low energy means these photons cannot ionize atoms or break chemical bonds but can excite molecular vibrations.

(a) 13.3 μm

(b) 9.38×10^{-2} eV

(a) A γ -ray photon has a momentum of $8.00 \times 10^{-21}\text{kg}\cdot\text{m/s}$. What is its wavelength? (b) Calculate its energy in MeV.

Show Solution

Strategy

(a) Use $\lambda = \frac{h}{p}$ to find the wavelength. (b) Use $E = pc$ to find the energy and convert to MeV.

Solution for (a)

The wavelength is:

$$\lambda = \frac{h}{p} = \frac{6.63 \times 10^{-34}\ \text{J·s}}{8.00 \times 10^{-21}\ \text{kg·m/s}} = 8.29 \times 10^{-14}\ \text{m} = 0.0829\ \text{pm}$$

Solution for (b)

The energy is:

$$E = pc = (8.00 \times 10^{-21}\ \text{kg·m/s})(3.00 \times 10^{8}\ \text{m/s}) = 2.40 \times 10^{-12}\ \text{J}$$

Converting to MeV:

$$E = \frac{2.40 \times 10^{-12}\ \text{J}}{1.60 \times 10^{-13}\ \text{J/MeV}} = 15.0\ \text{MeV}$$

Discussion

This is a high-energy gamma-ray photon with energy of 15.0 MeV and wavelength of only 0.0829 picometers—about 1000 times smaller than an atom. Such energetic photons are produced in nuclear reactions and radioactive decay. The extremely short wavelength allows these photons to probe nuclear structure, while the high energy means they can penetrate significant amounts of matter and cause substantial ionization, making them both useful for medical imaging and hazardous to living tissue.

(a) $8.29 \times 10^{-14}\ \text{m}$ or 0.0829 pm

(b) 15.0 MeV

(a) Calculate the momentum of a photon having a wavelength of $2.50\mu\text{m}$. (b) Find the velocity of an electron having the same momentum. (c) What is the kinetic energy of the electron, and how does it compare with that of the photon?

Show Solution

(a) $2.65 \times 10^{-28}\text{kg}\cdot\text{m/s}$ (b) 291 m/s

(c) electron $3.86 \times 10^{-26}\text{J}$, photon $7.96 \times 10^{-20}\text{J}$, ratio 2.06×10^{6}

Repeat the previous problem for a 10.0-nm-wavelength photon.

Show Solution

Strategy

Follow the same procedure as the previous problem: (a) Calculate photon momentum using $p = \frac{h}{\lambda}$. (b) Find electron velocity from $p = mv$. (c) Calculate kinetic energies and compare.

Solution for (a)

Photon momentum:

$$p=\frac{h}{\lambda}=\frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{10.0\times10^{-9}\ \text{m}}=6.63\times10^{-26}\ \text{kg}\cdot\text{m/s}$$

Solution for (b)

Electron velocity with same momentum:

$$v=\frac{p}{m}=\frac{6.63\times10^{-26}\ \text{kg}\cdot\text{m/s}}{9.11\times10^{-31}\ \text{kg}}=7.28\times10^{4}\ \text{m/s}$$

Solution for (c)

Electron kinetic energy:

$$\text{KE}_e=\frac{1}{2}mv^2=\frac{1}{2}(9.11\times10^{-31}\ \text{kg})(7.28\times10^{4}\ \text{m/s})^2=2.41\times10^{-21}\ \text{J}$$

Photon energy:

$$E_{\text{photon}}=\frac{hc}{\lambda}=\frac{1240\ \text{eV}\cdot\text{nm}}{10.0\ \text{nm}}=124\ \text{eV}=1.99\times10^{-17}\ \text{J}$$

Ratio:

$$\frac{E_{\text{photon}}}{\text{KE}_e}=\frac{1.99\times10^{-17}\ \text{J}}{2.41\times10^{-21}\ \text{J}}=8.26\times10^{3}$$

Discussion

This 10-nm X-ray photon has momentum 250 times greater than the 2.50-μm IR photon from the previous problem. The electron with this momentum moves at 72.8 km/s (0.024% of light speed), which is still nonrelativistic. The photon energy exceeds the electron's kinetic energy by over 8000 times, demonstrating again that photons are far more energy-efficient momentum carriers than massive particles.

(a) 6.63×10^{-26} kg·m/s

(b) 7.28×10^{4} m/s

(c) Electron: 2.41×10^{-21} J, Photon: 1.99×10^{-17} J, Ratio: 8.26×10^{3}

(a) Calculate the wavelength of a photon that has the same momentum as a proton moving at 1.00% of the speed of light. (b) What is the energy of the photon in MeV? (c) What is the kinetic energy of the proton in MeV?

Show Solution

(a) 1.32×10^{-13} m (b) 9.39 MeV

(c) 4.70×10^{-2} MeV

(a) Find the momentum of a 100-keV x-ray photon. (b) Find the equivalent velocity of a neutron with the same momentum. (c) What is the neutron's kinetic energy in keV?

Show Solution

Strategy

(a) Use $p=\frac{E}{c}$ for a photon. (b) Use $p=mv$ to find the neutron velocity. (c) Calculate the neutron's kinetic energy using $\text{KE}=\frac{1}{2}mv^2$.

Solution for (a)

Photon momentum:

$$p=\frac{E}{c}=\frac{100\times10^{3}\ \text{eV}\times1.60\times10^{-19}\ \text{J/eV}}{3.00\times10^{8}\ \text{m/s}}=5.33\times10^{-23}\ \text{kg}\cdot\text{m/s}$$

Solution for (b)

Neutron velocity (mass = 1.675×10^{-27} kg):

$$v=\frac{p}{m}=\frac{5.33\times10^{-23}\ \text{kg}\cdot\text{m/s}}{1.675\times10^{-27}\ \text{kg}}=3.18\times10^{4}\ \text{m/s}$$

Solution for (c)

Neutron kinetic energy:

$$KE=12mv^2=12(1.675\times10^{-27}\text{ kg})(3.18\times10^4\text{ m/s})^2=8.47\times10^{-19}\text{ J}$$

Converting to keV:

$$KE=8.47\times10^{-19}\text{ J}1.60\times10^{-19}\text{ J/eV}=5.29\text{ eV}=0.00529\text{ keV}$$

Discussion

A 100-keV X-ray photon has the same momentum as a neutron moving at only 31.8 km/s, which is well below 1% of the speed of light (nonrelativistic). However, the neutron's kinetic energy is only about 5.3 eV—nearly 20,000 times less than the photon's energy! This dramatic difference illustrates that photons are extraordinarily efficient momentum carriers: they carry the same momentum as massive particles but with much more energy. This is because photon energy $E=pc$, while massive particle energy $KE=p^2 2m$.

(a) 5.33×10^{-23} kg·m/s

(b) 3.18×10^4 m/s

(c) 5.29 eV or 0.00529 keV

Take the ratio of relativistic rest energy, $E=\gamma mc^2$, to relativistic momentum, $p=\gamma\text{mu}$, and show that in the limit that mass approaches zero, you find $E/p=c$.

Show Solution

$E=\gamma mc^2$ and $P=\gamma mu$, so

Missing superscript or subscript argument

As the mass of particle approaches zero, its velocity u will approach c, so that the ratio of energy to momentum in this limit is

$$lim_{m\to0}EP=c^2c=c$$

which is consistent with the equation for photon energy.

Construct Your Own Problem

Consider a space sail such as mentioned in [Example 1]. Construct a problem in which you calculate the light pressure on the sail in N/m^2 produced by reflecting sunlight. Also calculate the force that could be produced and how much effect that would have on a spacecraft. Among the things to be considered are the intensity of sunlight, its average wavelength, the number of photons per square meter this implies, the area of the space sail, and the mass of the system being accelerated.

Unreasonable Results

A car feels a small force due to the light it sends out from its headlights, equal to the momentum of the light divided by the time in which it is emitted. (a) Calculate the power of each headlight, if they exert a total force of $2.00\times10^{-2}\text{N}$ backward on the car. (b) What is unreasonable about this result? (c) Which assumptions are unreasonable or inconsistent?

Show Solution

(a) $3.00\times10^6\text{W}$ (b) Headlights are way too bright.

(c) Force is too large.

Glossary

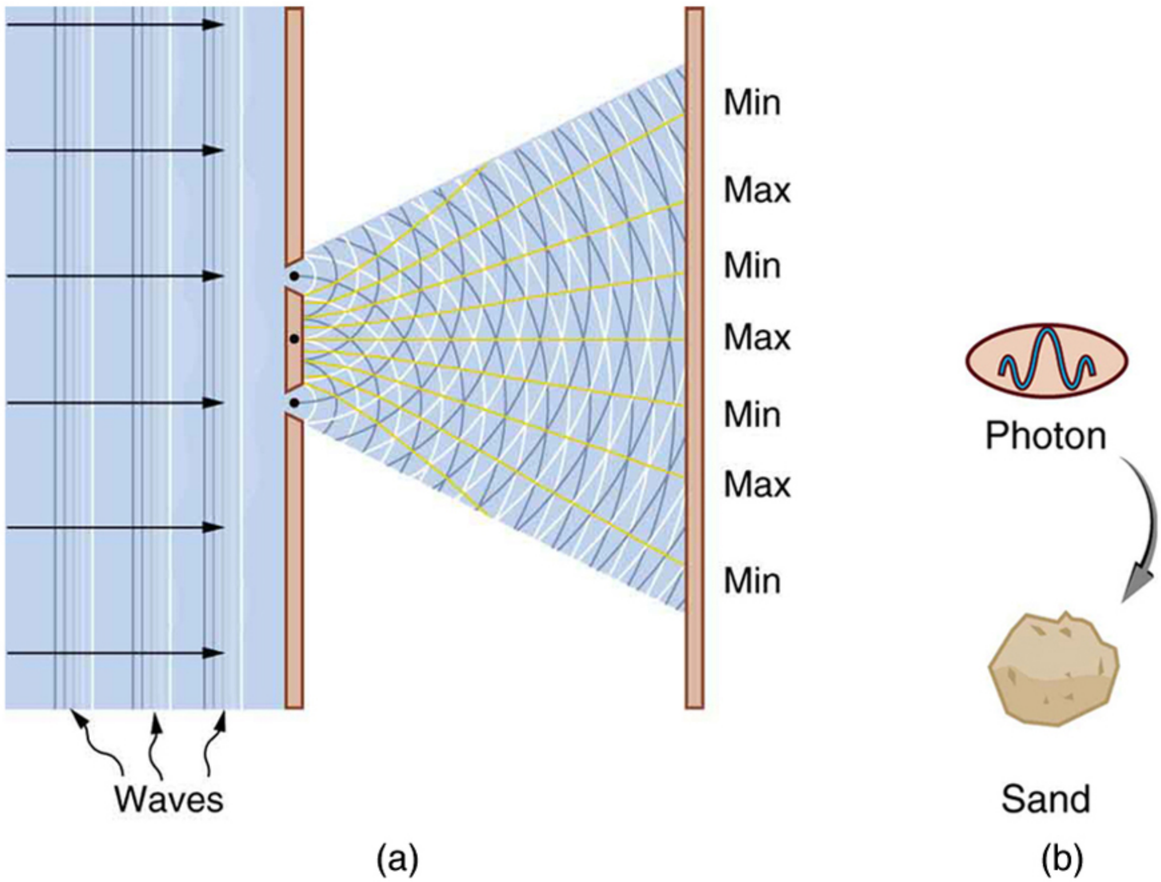
photon momentum
: the amount of momentum a photon has, calculated by $p=h\lambda=Ec$

Compton effect
: the phenomenon whereby X-rays scattered from materials have decreased energy

The Particle-Wave Duality

- Explain what the term particle-wave duality means, and why it is applied to EM radiation.

We have long known that EM radiation is a wave, capable of interference and diffraction. We now see that light can be modeled as photons, which are massless particles. This may seem contradictory, since we ordinarily deal with large objects that never act like both wave and particle. An ocean wave, for example, looks nothing like a rock. To understand small-scale phenomena, we make analogies with the large-scale phenomena we observe directly. When we say something behaves like a wave, we mean it shows interference effects analogous to those seen in overlapping water waves. (See [Figure 1].) Two examples of waves are sound and EM radiation. When we say something behaves like a particle, we mean that it interacts as a discrete unit with no interference effects. Examples of particles include electrons, atoms, and photons of EM radiation. How do we talk about a phenomenon that acts like both a particle and a wave?



(a) The interference pattern for light through a double slit is a wave property understood by analogy to water waves. (b) The properties of photons having quantized energy and momentum and acting as a concentrated unit are understood by analogy to macroscopic particles.

There is no doubt that EM radiation interferes and has the properties of wavelength and frequency. There is also no doubt that it behaves as particles—photons with discrete energy. We call this twofold nature the **particle-wave duality**, meaning that EM radiation has both particle and wave properties. This so-called duality is simply a term for properties of the photon analogous to phenomena we can observe directly, on a macroscopic scale. If this term seems strange, it is because we do not ordinarily observe details on the quantum level directly, and our observations yield either particle *or* wavelike properties, but never both simultaneously.

Since we have a particle-wave duality for photons, and since we have seen connections between photons and matter in that both have momentum, it is reasonable to ask whether there is a particle-wave duality for matter as well. If the EM radiation we once thought to be a pure wave has particle properties, is it possible that matter has wave properties? The answer is yes. The consequences are tremendous, as we will begin to see in the next section.

PhET Explorations: Quantum Wave Interference

When do photons, electrons, and atoms behave like particles and when do they behave like waves? Watch waves spread out and interfere as they pass through a double slit, then get detected on a screen as tiny dots. Use quantum detectors to explore how measurements change the waves and the patterns they produce on the screen.

Section Summary

- EM radiation can behave like either a particle or a wave.
- This is termed particle-wave duality.

Glossary

particle-wave duality
: the property of behaving like either a particle or a wave; the term for the phenomenon that all particles have wave characteristics

The Wave Nature of Matter

- Describe the Davisson-Germer experiment, and explain how it provides evidence for the wave nature of electrons.

De Broglie Wavelength

In 1923 a French physics graduate student named Prince Louis-Victor de Broglie (1892–1987) made a radical proposal based on the hope that nature is symmetric. If EM radiation has both particle and wave properties, then nature would be symmetric if matter also had both particle and wave properties. If what we once thought of as an unequivocal wave (EM radiation) is also a particle, then what we think of as an unequivocal particle (matter) may also be a wave. De Broglie's suggestion, made as part of his doctoral thesis, was so radical that it was greeted with some skepticism. A copy of his thesis was sent to Einstein, who said it was not only probably correct, but that it might be of fundamental importance. With the support of Einstein and a few other prominent physicists, de Broglie was awarded his doctorate.

De Broglie took both relativity and quantum mechanics into account to develop the proposal that **all particles have a wavelength**, given by

$$\lambda = \frac{h}{p} \text{(matter and photons)},$$

where h is Planck's constant and p is momentum. This is defined to be the **de Broglie wavelength**. (Note that we already have this for photons, from the equation $p = h/\lambda$.) The hallmark of a wave is interference. If matter is a wave, then it must exhibit constructive and destructive interference. Why isn't this ordinarily observed? The answer is that in order to see significant interference effects, a wave must interact with an object about the same size as its wavelength. Since h is very small, λ is also small, especially for macroscopic objects. A 3-kg bowling ball moving at 10 m/s, for example, has

$$\lambda = h/p = (6.63\times10^{-34}\,\text{J}\cdot\text{s})/[(3\,\text{kg})(10\,\text{m/s})] = 2\times10^{-35}\,\text{m}.$$

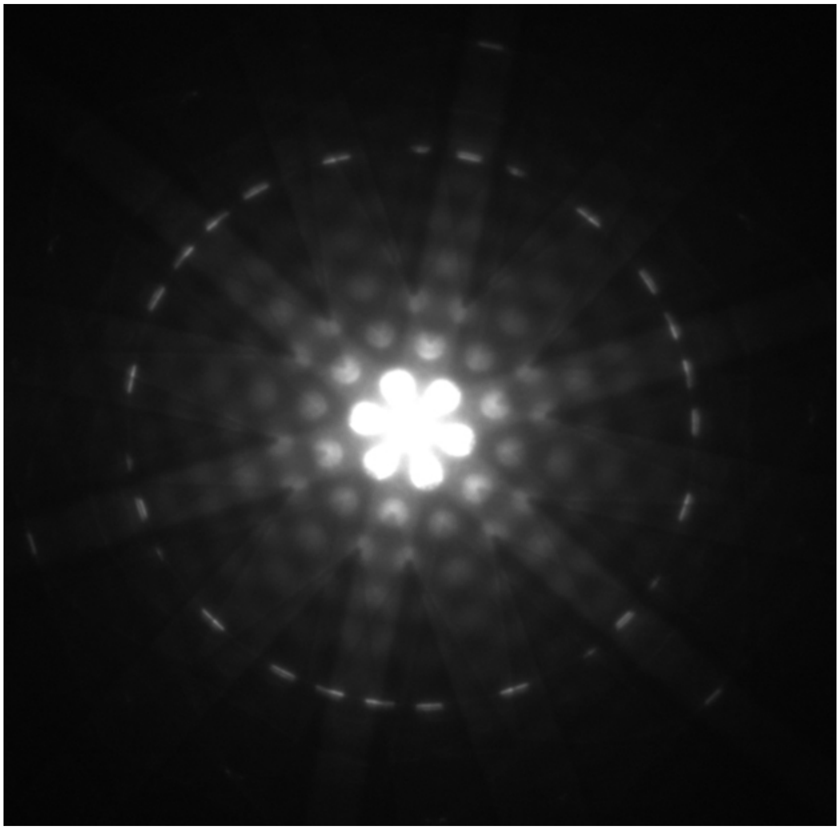
This means that to see its wave characteristics, the bowling ball would have to interact with something about $10^{-35}\,\text{m}$ in size—far smaller than anything known. When waves interact with objects much larger than their wavelength, they show negligible interference effects and move in straight lines (such as light rays in geometric optics). To get easily observed interference effects from particles of matter, the longest wavelength and hence smallest mass possible would be useful. Therefore, this effect was first observed with electrons.

American physicists Clinton J. Davisson and Lester H. Germer in 1925 and, independently, British physicist G. P. Thomson (son of J. J. Thomson, discoverer of the electron) in 1926 scattered electrons from crystals and found diffraction patterns. These patterns are exactly consistent with interference of electrons having the de Broglie wavelength and are somewhat analogous to light interacting with a diffraction grating. (See [Figure 1].)

Connections: Waves

All microscopic particles, whether massless, like photons, or having mass, like electrons, have wave properties. The relationship between momentum and wavelength is fundamental for all particles.

De Broglie's proposal of a wave nature for all particles initiated a remarkably productive era in which the foundations for quantum mechanics were laid. In 1926, the Austrian physicist Erwin Schrödinger (1887–1961) published four papers in which the wave nature of particles was treated explicitly with wave equations. At the same time, many others began important work. Among them was German physicist Werner Heisenberg (1901–1976) who, among many other contributions to quantum mechanics, formulated a mathematical treatment of the wave nature of matter that used matrices rather than wave equations. We will deal with some specifics in later sections, but it is worth noting that de Broglie's work was a watershed for the development of quantum mechanics. De Broglie was awarded the Nobel Prize in 1929 for his vision, as were Davisson and G. P. Thomson in 1937 for their experimental verification of de Broglie's hypothesis.



This diffraction pattern was obtained for electrons diffracted by crystalline silicon. Bright regions are those of constructive interference, while dark regions are those of destructive interference. (credit: Ndthe, Wikimedia Commons)

Electron Wavelength versus Velocity and Energy

For an electron having a de Broglie wavelength of 0.167 nm (appropriate for interacting with crystal lattice structures that are about this size): (a) Calculate the electron's velocity, assuming it is nonrelativistic. (b) Calculate the electron's kinetic energy in eV.

Strategy

For part (a), since the de Broglie wavelength is given, the electron's velocity can be obtained from $\lambda = h/p$ by using the nonrelativistic formula for momentum, $p = mv$. For part (b), once v is obtained (and it has been verified that v is nonrelativistic), the classical kinetic energy is simply $(1/2)mv^2$. **Solution for (a)**

Substituting the nonrelativistic formula for momentum ($p = mv$) into the de Broglie wavelength gives

$$\lambda = \frac{h}{p} = \frac{h}{mv}.$$

Solving for v gives

$$v = \frac{h}{m\lambda}.$$

Substituting known values yields

$$v = \frac{6.63 \times 10^{-34}\,\text{J}\cdot\text{s}}{(9.11 \times 10^{-31}\,\text{kg})(0.167 \times 10^{-9}\,\text{m})} = 4.36 \times 10^{6}\,\text{m/s}.$$

Solution for (b)

While fast compared with a car, this electron's speed is not highly relativistic, and so we can comfortably use the classical formula to find the electron's kinetic energy and convert it to eV as requested.

$$\text{KE} = \frac{1}{2}mv^2 = \frac{1}{2}(9.11 \times 10^{-31}\,\text{kg})(4.36 \times 10^{6}\,\text{m/s})^2 = (86.4 \times 10^{-18}\,\text{J})\left(\frac{1\,\text{eV}}{1.602 \times 10^{-19}\,\text{J}}\right) = 54.0\,\text{eV}$$

Discussion

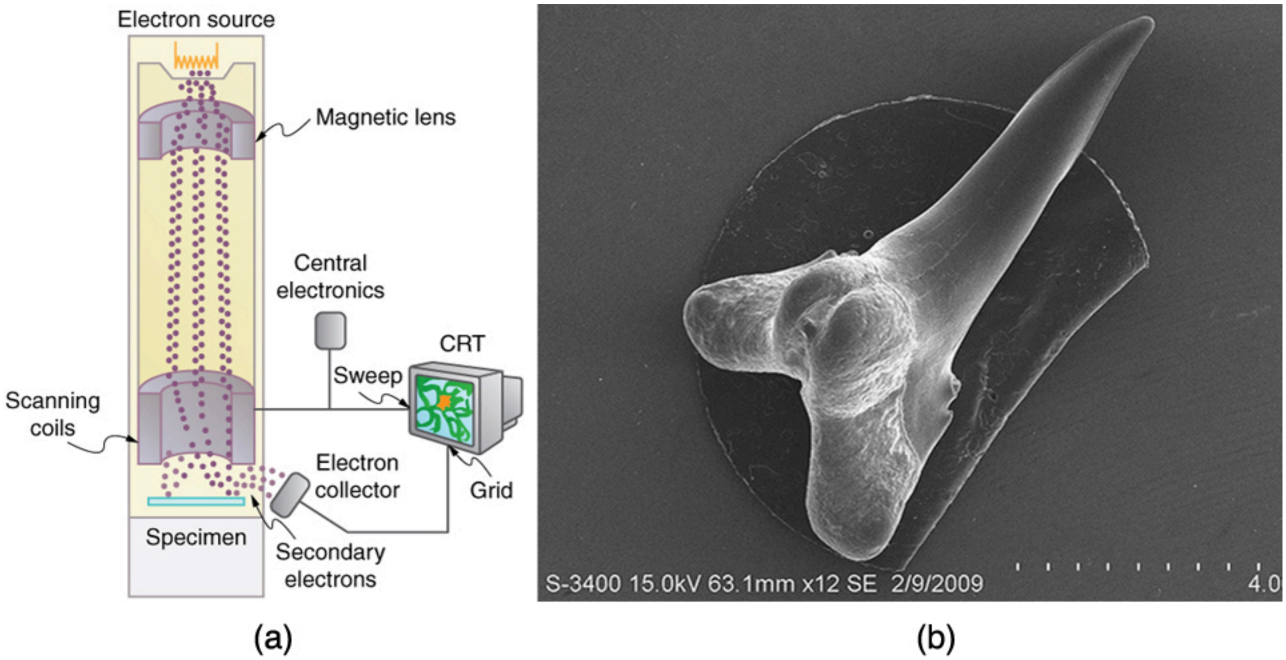
This low energy means that these 0.167-nm electrons could be obtained by accelerating them through a 54.0-V electrostatic potential, an easy task. The results also confirm the assumption that the electrons are nonrelativistic, since their velocity is just over 1% of the speed of light and the kinetic energy is about 0.01% of the rest energy of an electron (0.511 MeV). If the electrons had turned out to be relativistic, we would have had to use more involved calculations employing relativistic formulas.

Electron Microscopes

One consequence or use of the wave nature of matter is found in the electron microscope. As we have discussed, there is a limit to the detail observed with any probe having a wavelength. Resolution, or observable detail, is limited to about one wavelength. Since a potential of only 54 V can produce electrons with sub-nanometer wavelengths, it is easy to get electrons with much smaller wavelengths than those of visible light (hundreds of nanometers). Electron microscopes can, thus, be constructed to detect much smaller details than optical microscopes. (See [Figure 2].)

There are basically two types of electron microscopes. The transmission electron microscope (TEM) accelerates electrons that are emitted from a hot filament (the cathode). The beam is broadened and then passes through the sample. A magnetic lens focuses the beam image onto a fluorescent screen, a photographic plate, or (most probably) a CCD (light sensitive camera), from which it is transferred to a computer. The TEM is similar to the optical microscope, but it requires a thin sample examined in a vacuum. However it can resolve details as small as 0.1 nm ($10^{-10} m$), providing magnifications of 100 million times the size of the original object. The TEM has allowed us to see individual atoms and structure of cell nuclei.

The scanning electron microscope (SEM) provides images by using secondary electrons produced by the primary beam interacting with the surface of the sample (see [Figure 2]). The SEM also uses magnetic lenses to focus the beam onto the sample. However, it moves the beam around electrically to “scan” the sample in the *x* and *y* directions. A CCD detector is used to process the data for each electron position, producing images like the one at the beginning of this chapter. The SEM has the advantage of not requiring a thin sample and of providing a 3-D view. However, its resolution is about ten times less than a TEM.



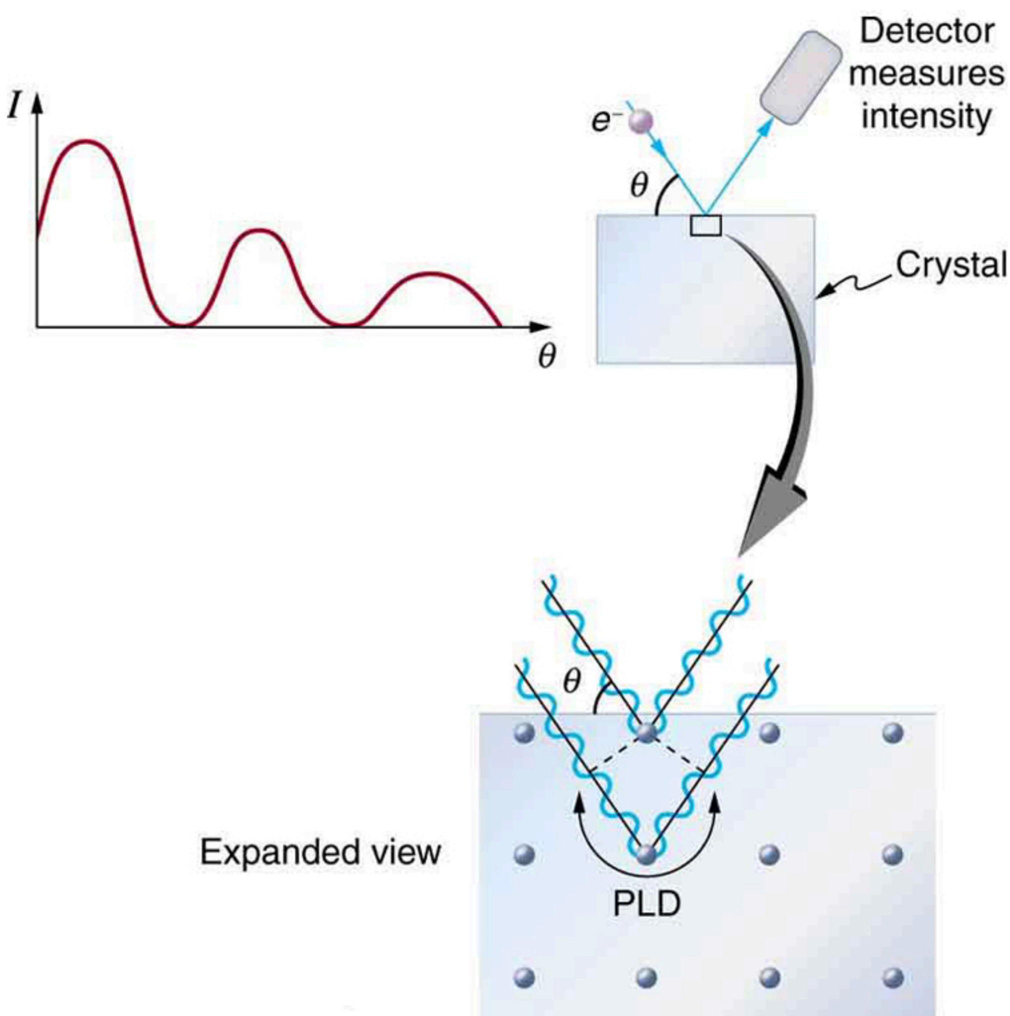
Schematic of a scanning electron microscope (SEM) (a) used to observe small details, such as those seen in this image of a tooth of a Himipristis, a type of shark (b). (credit: Dallas Krentzel, Flickr)

Electrons were the first particles with mass to be directly confirmed to have the wavelength proposed by de Broglie. Subsequently, protons, helium nuclei, neutrons, and many others have been observed to exhibit interference when they interact with objects having sizes similar to their de Broglie wavelength. The de Broglie wavelength for massless particles was well established in the 1920s for photons, and it has since been observed that all massless particles have a de Broglie wavelength $\lambda = h/p$. The wave nature of all particles is a universal characteristic of nature. We shall see in following sections that implications of the de Broglie wavelength include the quantization of energy in atoms and molecules, and an alteration of our basic view of nature on the microscopic scale. The next section, for example, shows that there are limits to the precision with which we may make predictions, regardless of how hard we try. There are even limits to the precision with which we may measure an object’s location or energy.

Making Connections: A Submicroscopic Diffraction Grating

The wave nature of matter allows it to exhibit all the characteristics of other, more familiar, waves. Diffraction gratings, for example, produce diffraction patterns for light that depend on grating spacing and the wavelength of the light. This effect, as with most wave phenomena, is most pronounced when the wave interacts with objects having a size similar to its wavelength. For gratings, this is the spacing between multiple slits.) When electrons interact with a system having a spacing similar to the electron wavelength, they show the same types of interference patterns as light does for diffraction gratings, as shown at top left in [Figure 3].

Atoms are spaced at regular intervals in a crystal as parallel planes, as shown in the bottom part of [Figure 3]. The spacings between these planes act like the openings in a diffraction grating. At certain incident angles, the paths of electrons scattering from successive planes differ by one wavelength and, thus, interfere constructively. At other angles, the path length differences are not an integral wavelength, and there is partial to total destructive interference. This type of scattering from a large crystal with well-defined lattice planes can produce dramatic interference patterns. It is called **Bragg reflection**, **for the father-and-son team who first explored and analyzed it in some detail. The expanded view also shows the path-length differences and indicates how these depend on incident angle θ in a manner similar to the diffraction patterns for X-rays reflecting from a crystal.



The diffraction pattern at top left is produced by scattering electrons from a crystal and is graphed as a function of incident angle relative to the regular array of atoms in a crystal, as shown at bottom. Electrons scattering from the second layer of atoms travel farther than those scattered from the top layer. If the path length difference (PLD) is an integral wavelength, there is constructive interference.

Let us take the spacing between parallel planes of atoms in the crystal to be d . As mentioned, if the path length difference (PLD) for the electrons is a whole number of wavelengths, there will be constructive interference—that is, $\text{PLD} = n\lambda (n=1,2,3,\ldots)$. Because $\text{AB} = \text{BC} = d\sin\theta$, we have constructive interference when $n\lambda = 2d\sin\theta$. This relationship is called the *Bragg equation* and applies not only to electrons but also to X-rays.

The wavelength of matter is a submicroscopic characteristic that explains a macroscopic phenomenon such as Bragg reflection. Similarly, the wavelength of light is a submicroscopic characteristic that explains the macroscopic phenomenon of diffraction patterns.

Section Summary

- Particles of matter also have a wavelength, called the de Broglie wavelength, given by $\lambda = hp$, where p is momentum.
- Matter is found to have the same **interference characteristics** as any other wave.

Conceptual Questions

How does the interference of water waves differ from the interference of electrons? How are they analogous?

Describe one type of evidence for the wave nature of matter.

Describe one type of evidence for the particle nature of EM radiation.

Problems & Exercises

At what velocity will an electron have a wavelength of 1.00 m?

Show Solution

$7.28 \times 10^{-4}\text{m}$

What is the wavelength of an electron moving at 3.00% of the speed of light?

Show Solution

Strategy

Use the de Broglie wavelength formula $\lambda = hp = hmv$. Calculate the electron's momentum at 3.00% of light speed, then find the wavelength.

Solution

The electron's velocity is:

$$v=0.0300c=0.0300(3.00\times10^{8}\text{ m/s})=9.00\times10^{6}\text{ m/s}$$

The de Broglie wavelength is:

$$\lambda=hmv=6.63\times10^{-34}\text{ J·s}(9.11\times10^{-31}\text{ kg})(9.00\times10^{6}\text{ m/s})=8.09\times10^{-11}\text{ m}=0.0809\text{ nm}$$

Discussion

At 3% of light speed, the electron has a wavelength of 0.0809 nm, comparable to atomic dimensions (atoms are roughly 0.1 nm). This wavelength is suitable for probing crystal structures and is typical of electrons in electron microscopes. The electron is moving fast enough that its wave nature becomes significant when interacting with matter, allowing it to exhibit diffraction and interference patterns.

0.0809 nm or 8.09×10^{-11} m

At what velocity does a proton have a 6.00-fm wavelength (about the size of a nucleus)? Assume the proton is nonrelativistic. (1 femtometer = 10^{-15}m.)

Show Solution

6.62×10^{7}m/s

What is the velocity of a 0.400-kg billiard ball if its wavelength is 7.50 cm (large enough for it to interfere with other billiard balls)?

Show Solution

Strategy

Use the de Broglie relation $\lambda = hmv$ and solve for velocity.

Solution

$$v=hm\lambda=6.63\times10^{-34}\text{ J·s}(0.400\text{ kg})(0.0750\text{ m})=2.21\times10^{-32}\text{ m/s}$$

Discussion

This velocity is absurdly small—about 10^{-32} m/s! For comparison, this is so slow that the billiard ball would take longer than the age of the universe to move the diameter of a proton. This demonstrates why we never observe wave behavior in macroscopic objects: their de Broglie wavelengths only become significant at impossibly low velocities. For reasonable velocities, macroscopic objects have wavelengths far too small to produce observable quantum effects.

2.21×10^{-32} m/s

Find the wavelength of a proton moving at 1.00% of the speed of light.

Show Solution

1.32×10^{-13}m

Experiments are performed with ultracold neutrons having velocities as small as 1.00 m/s. (a) What is the wavelength of such a neutron? (b) What is its kinetic energy in eV?

Show Solution

Strategy

(a) Use $\lambda = hmv$. (b) Calculate kinetic energy using $\text{KE} = 12mv^2$ and convert to eV.

Solution for (a)

$\lambda = hmv = 6.63\times10^{-34}\ \text{J}\cdot\text{s}(1.675\times10^{-27}\ \text{kg})(1.00\ \text{m/s}) = 3.96\times10^{-7}\ \text{m} = 396\ \text{nm}$

Solution for (b)

$\text{KE} = 12mv^{2} = 12(1.675\times10^{-27}\ \text{kg})(1.00\ \text{m/s})^{2} = 8.375\times10^{-28}\ \text{J}$

Converting to eV:

$\text{KE} = 8.375\times10^{-28}\ \text{J}1.60\times10^{-19}\ \text{J/eV} = 5.23\times10^{-9}\ \text{eV}$

Discussion

Ultracold neutrons have a remarkable wavelength of 396 nm—in the visible light range! Despite their extremely low velocity (1 m/s), their relatively large mass gives them a macroscopic wavelength. The kinetic energy is only about 5 nanoelectron volts, extraordinarily small. Such neutrons are useful for studying fundamental physics and materials science because their long wavelength allows them to interact gently with matter.

(a) 396 nm or 3.96×10^{-7} m

(b) 5.23×10^{-9} eV

(a) Find the velocity of a neutron that has a 6.00-fm wavelength (about the size of a nucleus). Assume the neutron is nonrelativistic. (b) What is the neutron's kinetic energy in MeV?

Show Solution

(a) 6.62×10^{7} m/s (b) 22.9 MeV

What is the wavelength of an electron accelerated through a 30.0-kV potential, as in a TV tube?

Show Solution

Strategy

The electron gains kinetic energy equal to $qV = eV$. Use this to find velocity, then calculate wavelength using $\lambda = hmv$.

Solution

Kinetic energy equals the potential energy:

$\text{KE} = eV = 30.0\ \text{keV} = 30.0\times10^{3}\times1.60\times10^{-19}\ \text{J} = 4.80\times10^{-15}\ \text{J}$

Find velocity from $\text{KE} = 12mv^{2}$:

$v = \sqrt{2\text{KE}m} = \sqrt{2(4.80\times10^{-15}\ \text{J})9.11\times10^{-31}\ \text{kg}} = 1.03\times10^{8}\ \text{m/s}$

Calculate wavelength:

$\lambda = hmv = 6.63\times10^{-34}\ \text{J}\cdot\text{s}(9.11\times10^{-31}\ \text{kg})(1.03\times10^{8}\ \text{m/s}) = 7.06\times10^{-12}\ \text{m} = 0.00706\ \text{nm}$

Discussion

Electrons in a 30-kV TV tube have a wavelength of 0.00706 nm, which is much smaller than atomic dimensions (0.1 nm). This short wavelength is why electron beams in old cathode ray tube TVs could create such fine resolution. The electron is moving at about 34% the speed of light, approaching relativistic speeds.

0.00706 nm or 7.06×10^{-12} m

What is the kinetic energy of an electron in a TEM having a 0.0100-nm wavelength?

Show Solution
15.1 keV

(a) Calculate the velocity of an electron that has a wavelength of $1.00\,\mu\text{m}$. (b) Through what voltage must the electron be accelerated to have this velocity?

Show Solution

Strategy

(a) Use $v = hm\lambda$. (b) The voltage accelerates the electron to kinetic energy $\text{KE} = 12mv^{2} = eV$.

Solution for (a)

$$v=\frac{h}{m\lambda}=\frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{(9.11\times10^{-31}\ \text{kg})(1.00\times10^{-6}\ \text{m})}=728\ \text{m/s}$$

Solution for (b)

$$\text{KE}=\frac{1}{2}mv^2=\frac{1}{2}(9.11\times10^{-31}\ \text{kg})(728\ \text{m/s})^2=2.42\times10^{-25}\ \text{J}$$

Converting to eV to find voltage:

$$V=\frac{\text{KE}}{e}=\frac{2.42\times10^{-25}\ \text{J}}{1.60\times10^{-19}\ \text{C}}=1.51\times10^{-6}\ \text{V}=1.51\ \mu\text{V}$$

Discussion

An electron with a 1-μm wavelength moves at only 728 m/s and requires acceleration through merely 1.51 microvolts. This demonstrates that long-wavelength (low-energy) electrons are easy to produce. Such slow electrons are useful in some surface science applications where minimal sample damage is desired.

(a) 728 m/s

(b) 1.51×10^{-6} V or 1.51 μV

The velocity of a proton emerging from a Van de Graaff accelerator is 25.0% of the speed of light. (a) What is the proton's wavelength? (b) What is its kinetic energy, assuming it is nonrelativistic? (c) What was the equivalent voltage through which it was accelerated?

Show Solution

(a) 5.29 fm

(b) 4.70×10^{-12}J (c) 29.4 MV

The kinetic energy of an electron accelerated in an x-ray tube is 100 keV. Assuming it is nonrelativistic, what is its wavelength?

Show Solution

Strategy

Find the electron's momentum from its kinetic energy, then use $\lambda=\frac{h}{p}$.

Solution

From $\text{KE}=\frac{1}{2}mv^2=\frac{p^2}{2m}$, solve for momentum:

$$p=\sqrt{2m\cdot\text{KE}}$$

Convert KE to joules:

$$\text{KE}=100\times10^3\times1.60\times10^{-19}\ \text{J}=1.60\times10^{-14}\ \text{J}$$

Calculate momentum:

$$p=\sqrt{2(9.11\times10^{-31}\ \text{kg})(1.60\times10^{-14}\ \text{J})}=5.40\times10^{-23}\ \text{kg}\cdot\text{m/s}$$

Find wavelength:

$$\lambda=\frac{h}{p}=\frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{5.40\times10^{-23}\ \text{kg}\cdot\text{m/s}}=1.23\times10^{-11}\ \text{m}=0.0123\ \text{nm}$$

Discussion

A 100-keV electron has a wavelength of 0.0123 nm, suitable for X-ray wavelengths and atomic-scale imaging. This wavelength is about 1/10 the size of an atom, allowing these electrons to probe atomic and molecular structure. Note: At 100 keV, the electron is approaching 20% of light speed, so relativistic effects become noticeable, making the nonrelativistic assumption marginally valid.

0.0123 nm or 1.23×10^{-11} m

Unreasonable Results

(a) Assuming it is nonrelativistic, calculate the velocity of an electron with a 0.100-fm wavelength (small enough to detect details of a nucleus). (b) What is unreasonable about this result? (c) Which assumptions are unreasonable or inconsistent?

Show Solution

(a) $7.28 \times 10^{12} \text{m/s}$ (b) This is thousands of times the speed of light (an impossibility).

(c) The assumption that the electron is non-relativistic is unreasonable at this wavelength.

Glossary

de Broglie wavelength
: the wavelength possessed by a particle of matter, calculated by $\lambda = h/p$

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