Introduction to One-Dimensional Kinematics

The motion of a cheetah can be described by the animal's displacement, speed, velocity, and acceleration. When it runs in a straight line without any change in direction, its motion is said to be one dimensional. (credit: Mark Dumont, Wikimedia Commons)

Objects are in motion everywhere we look. Everything from a tennis game to a space-probe flyby of the planet Neptune involves motion. When you are resting, your heart moves blood through your veins. And even in inanimate objects, there is continuous motion in the vibrations of atoms and molecules. Questions about motion are interesting in and of themselves: *How long will it take for a space probe to get to Mars? Where will a football land if it is thrown at a certain angle?* But an understanding of motion is also key to understanding other concepts in physics. An understanding of acceleration, for example, is crucial to the study of force.

Our formal study of physics begins with **kinematics** which is defined as the *study of motion without considering its causes*. The word "kinematics" comes from a Greek term meaning motion and is related to other English words such as "cinema" (movies) and "kinesiology" (the study of human motion). In one-dimensional kinematics and Two-Dimensional Kinematics we will study only the *motion* of a football, for example, without worrying about what forces cause or change its motion. Such considerations come in other chapters. In this chapter, we examine the simplest type of motion—namely, motion along a straight line, or one-dimensional motion. In Two-Dimensional Kinematics, we apply concepts developed here to study motion along curved paths (two- and three-dimensional motion); for example, that of a car rounding a curve.

Displacement

- Define position, displacement, distance, and distance traveled.
- Explain the relationship between position and displacement.
- Distinguish between displacement and distance traveled.
- Calculate displacement and distance given initial position, final position, and the path between the two.

These cyclists in Vietnam can be described by their position relative to buildings and a canal. Their motion can be described by their change in position, or displacement, in the frame of reference. (credit: Suzan Black, Fotopedia)

Position

In order to describe the motion of an object, you must first be able to describe its **position** —where it is at any particular time. More precisely, you need to specify its position relative to a convenient reference frame. Earth is often used as a reference frame, and we often describe the position of an object as it relates to stationary objects in that reference frame. For example, a rocket launch would be described in terms of the position of the rocket with respect to the Earth as a whole, while a professor's position could be described in terms of where she is in relation to the nearby white board. (See Figure 2.) In other cases, we use reference frames that are not stationary but are in motion relative to the Earth. To describe the position of a person in an airplane, for example, we use the airplane, not the Earth, as the reference frame. (See Figure 3.)

Displacement

If an object moves relative to a reference frame (for example, if a professor moves to the right relative to a white board or a passenger moves toward the rear of an airplane), then the object's position changes. This change in position is known as **displacement**. The word "displacement" implies that an object has moved, or has been displaced.

Displacement

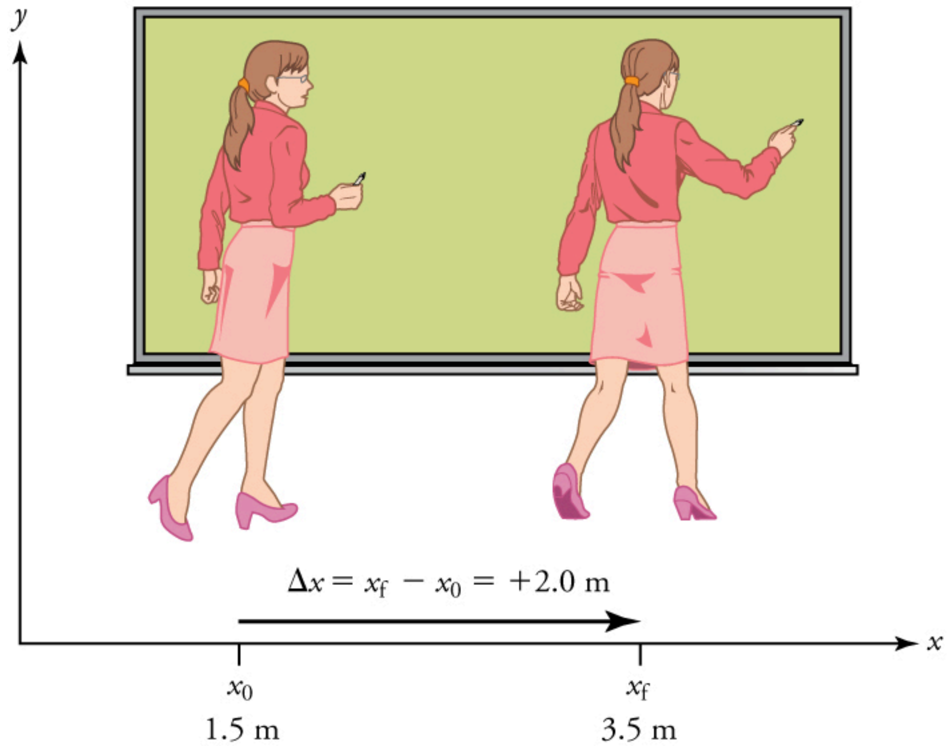
Displacement is the *change in position* of an object:

$\Delta x = x_f - x_0$,

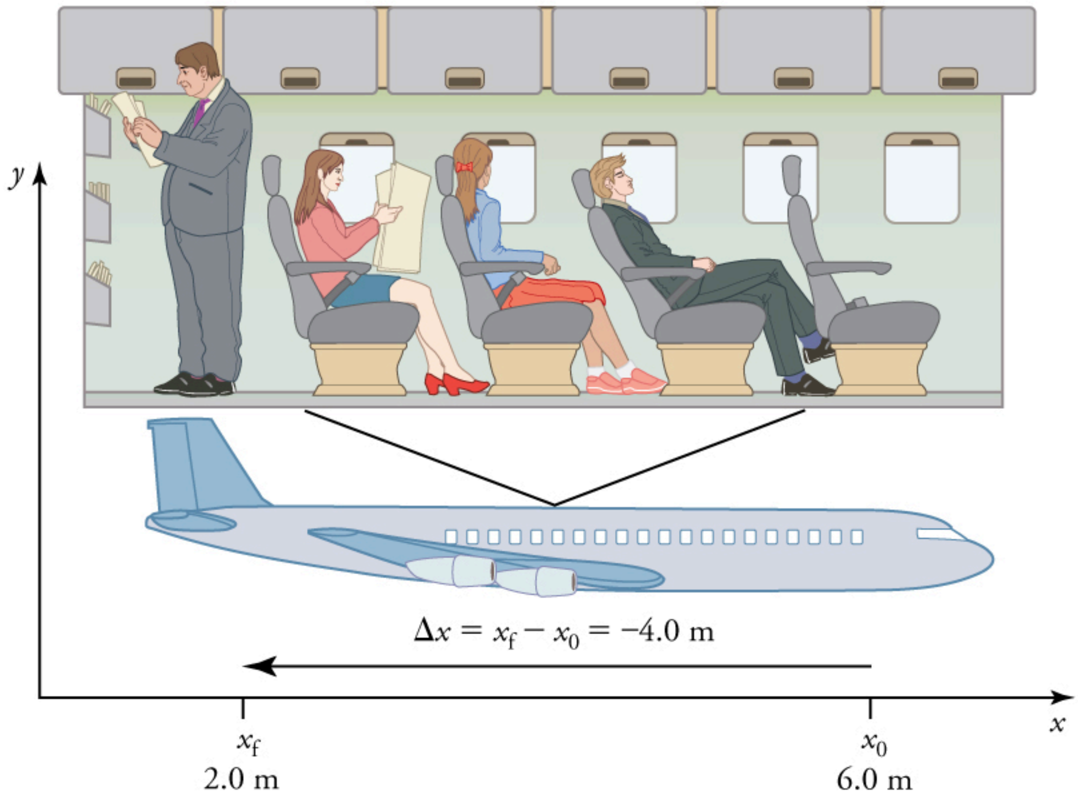
where Δx is displacement, x_f is the final position, and x_0 is the initial position.

In this text the upper case Greek letter Δ (delta) always means "change in" whatever quantity follows it; thus, Δx means *change in position*. Always solve for displacement by subtracting initial position x_0 from final position x_f.

Note that the SI unit for displacement is the meter (m) (see Physical Quantities and Units) , but sometimes kilometers, miles, feet, and other units of length are used. Keep in mind that when units other than the meter are used in a problem, you may need to convert them into meters to complete the calculation.



A professor paces left and right while lecturing. Her position relative to Earth is given by Δx . The +2.0m displacement of the professor relative to Earth is represented by an arrow pointing to the right.



A passenger moves from his seat to the back of the plane. His location relative to the airplane is given by $x_f - x_0$. The displacement of the passenger relative to the plane is represented by an arrow toward the rear of the plane. Notice that the arrow representing his displacement is twice as long as the arrow representing the displacement of the professor (he moves twice as far) in Figure 2.

Note that displacement has a direction as well as a magnitude. The professor's displacement is 2.0 m to the right, and the airline passenger's displacement is 4.0 m toward the rear. In one-dimensional motion, direction can be specified with a plus or minus sign. When you begin a problem, you should select which direction is positive (usually that will be to the right or up, but you are free to select positive as being any direction). The professor's initial position is $x_0 = 1.5\text{m}$ and her final position is $x_f = 3.5\text{m}$. Thus her displacement is

$$\Delta x = x_f - x_0 = 3.5\text{m} - 1.5\text{m} = +2.0\text{m}.$$

In this coordinate system, motion to the right is positive, whereas motion to the left is negative. Similarly, the airplane passenger's initial position is $x_0 = 6.0\text{m}$ and his final position is $x\_f = 2.0\text{m}$, so his displacement is

$$\Delta x = x_f - x_0 = 2.0\text{m} - 6.0\text{m} = -4.0\text{m}.$$

His displacement is negative because his motion is toward the rear of the plane, or in the negative x direction in our coordinate system.

Distance

Although displacement is described in terms of direction, distance is not. **Distance** is defined to be *the magnitude or size of displacement between two positions*. Note that the distance between two positions is not the same as the distance traveled between them. **Distance traveled** is *the total length of the path traveled between two positions*. Distance has no direction and, thus, no sign. For example, the distance the professor walks is 2.0 m. The distance the airplane passenger walks is 4.0 m.

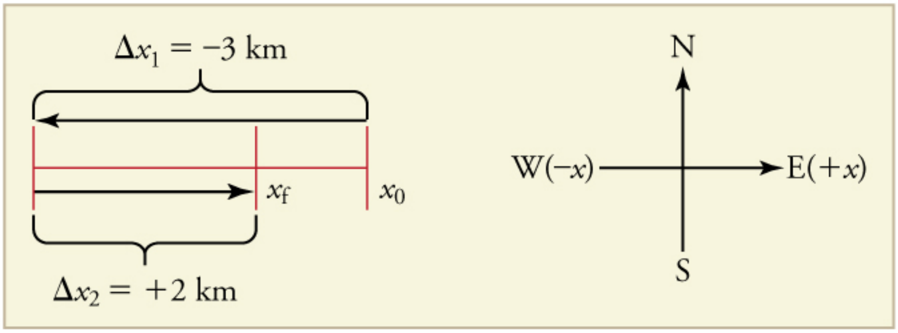
Misconception Alert: Distance Traveled vs. Magnitude of Displacement

It is important to note that the *distance traveled*, however, can be greater than the magnitude of the displacement (by magnitude, we mean just the size of the displacement without regard to its direction; that is, just a number with a unit). For example, the professor could pace back and forth many times, perhaps walking a distance of 150 m during a lecture, yet still end up only 2.0 m to the right of her starting point. In this case her displacement would be +2.0 m, the magnitude of her displacement would be 2.0 m, but the distance she traveled would be 150 m. In kinematics we nearly always deal with displacement and magnitude of displacement, and almost never with distance traveled. One way to think about this is to assume you marked the start of the motion and the end of the motion. The displacement is simply the difference in the position of the two marks and is independent of the path taken in traveling between the two marks. The distance traveled, however, is the total length of the path taken between the two marks.

Check Your Understanding

A cyclist rides 3 km west and then turns around and rides 2 km east. (a) What is her displacement? (b) What distance does she ride? (c) What is the magnitude of her displacement?

Show Solution



Displacements

(a) The rider's displacement is $\Delta x = x_f - x_0 = -1$ km. (The displacement is negative because we take east to be positive and west to be negative.)

(b) The distance traveled is 3km+2km =5km.

(c) The magnitude of the displacement is $1 km$.

Section Summary

- Kinematics is the study of motion without considering its causes. In this chapter, it is limited to motion along a straight line, called one-dimensional motion.
- Displacement is the change in position of an object.
- In symbols, displacement Δx is defined to be
  $\Delta x = x_f - x_0$,

where x_0 is the initial position and x_f is the final position. In this text, the Greek letter Δ (delta) always means "change in" whatever quantity follows it. The SI unit for displacement is the meter (m). Displacement has a direction as well as a magnitude.

- When you start a problem, assign which direction will be positive.
- Distance is the magnitude of displacement between two positions.
- Distance traveled is the total length of the path traveled between two positions.

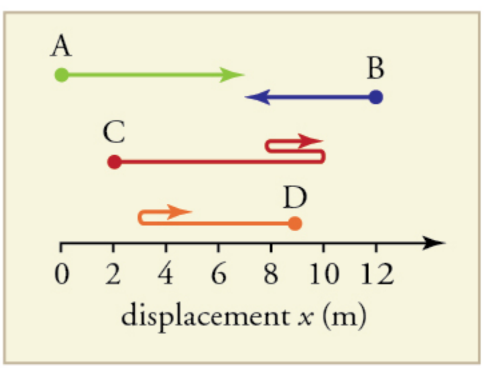
Conceptual Questions

Give an example in which there are clear distinctions among distance traveled, displacement, and magnitude of displacement. Specifically identify each quantity in your example.

Under what circumstances does distance traveled equal magnitude of displacement? What is the only case in which magnitude of displacement and displacement are exactly the same?

Bacteria move back and forth by using their flagella (structures that look like little tails). Speeds of up to $50 \mu m/s (50 \times 10^{-6} m/s)$ have been observed. The total distance traveled by a bacterium is large for its size, while its displacement is small. Why is this?

Problems & Exercises



Displacement Paths

Find the following for path A in the Figure above: (a) The distance traveled. (b) The magnitude of the displacement from start to finish. (c) The displacement from start to finish.

Show Solution

Strategy

For path A, identify the starting and ending positions, then calculate distance traveled, magnitude of displacement, and displacement.

Solution

From the figure, path A starts at $x_0 = 0\text{m}$ and ends at $x_f = 7\text{m}$. The path is a straight line with no backtracking.

(a) Distance traveled:

Since there is no backtracking, the distance traveled equals the total path length:

$$\text{Distance} = 7\text{m} - 0\text{m} = 7\text{m}$$

(b) Magnitude of displacement:

The magnitude of displacement is the absolute value of the change in position:

$$|\Delta x| = |x_f - x_0| = |7\text{m} - 0\text{m}| = 7\text{m}$$

(c) Displacement:

Displacement includes direction (sign):

$$\Delta x = x_f - x_0 = 7\text{m} - 0\text{m} = +7\text{m}$$

The positive sign indicates motion to the right.

Discussion

For straight-line motion without backtracking, the distance traveled equals the magnitude of displacement, which is exactly what we observe for path A. The object moves from 0 m to 7 m in the positive direction, traveling a total distance of 7 m. The displacement of +7 m is positive because the motion is to the right (positive x-direction). This is the simplest case of one-dimensional motion where all three quantities (distance, magnitude of displacement, and displacement) have the same numerical value.

Answer

(a) The distance traveled is 7 m.

(b) The magnitude of the displacement is 7 m.

(c) The displacement is +7 m.

Find the following for path B in the Figure above: (a) The distance traveled. (b) The magnitude of the displacement from start to finish. (c) The displacement from start to finish.

Show Solution

Strategy

For path B, identify the starting and ending positions from the figure, then calculate distance traveled, magnitude of displacement, and displacement.

Solution

From the figure, path B starts at $x_0 = 12\text{m}$ and ends at $x_f = 7\text{m}$. The path is a straight line with no backtracking.

(a) Distance traveled:

Since there is no backtracking, the distance traveled equals the total path length:

$$\text{Distance} = |x_f - x_0| = |7\text{m} - 12\text{m}| = 5\text{m}$$

(b) Magnitude of displacement:

The magnitude of displacement is the absolute value of the change in position:

$$|\Delta x| = |x_f - x_0| = |7\text{m} - 12\text{m}| = 5\text{m}$$

(c) Displacement:

Displacement includes direction (sign):

$$\Delta x = x_f - x_0 = 7\text{m} - 12\text{m} = -5\text{m}$$

The negative sign indicates motion to the left.

Discussion

For straight-line motion without backtracking, the distance traveled and magnitude of displacement are equal. The displacement is negative because the object moved in the negative x direction (left). This makes physical sense since path B clearly shows leftward motion on the diagram.

Answer

(a) The distance traveled is 5 m.

(b) The magnitude of the displacement is 5 m.

(c) The displacement is −5 m (or 5 m to the left).

Find the following for path C in the Figure above: (a) The distance traveled. (b) The magnitude of the displacement from start to finish. (c) The displacement from start to finish.

Show Solution

Strategy

For path C, trace the entire path including all direction changes to find distance traveled. For displacement, only the starting and ending positions matter.

Solution

From the figure, path C:

- Starts at $x_0 = 2\text{m}$
- Goes right to $x = 10\text{m}$ (moves 8 m)
- Turns around and goes left to $x = 8\text{m}$ (moves 2 m)
- Turns around and goes right to $x_f = 11\text{m}$ (moves 3 m)

(a) Distance traveled:

Add up all the path segments regardless of direction:

$$\text{Distance} = 8\text{m} + 2\text{m} + 3\text{m} = 13\text{m}$$

(b) Magnitude of displacement:

$$|\Delta x| = |x_f - x_0| = |11\text{m} - 2\text{m}| = 9\text{m}$$

(c) Displacement:

$$\Delta x = x_f - x_0 = 11\text{m} - 2\text{m} = +9\text{m}$$

The positive sign indicates net motion to the right.

Discussion

Path C demonstrates the important difference between distance traveled and displacement when an object changes direction. The total distance traveled (13 m) is significantly greater than the magnitude of displacement (9 m) because the object backtracked during its journey. Despite moving a total of 13 m, the object ends up only 9 m from where it started. The multiple direction changes illustrate why displacement depends only on initial and final positions, not on the path taken. This is a key concept in kinematics that distinguishes vectors (displacement) from scalars (distance).

Answer

(a) The distance traveled is 13 m.

(b) The magnitude of the displacement is 9 m.

(c) The displacement is +9 m.

Find the following for path D in the Figure above: (a) The distance traveled. (b) The magnitude of the displacement from start to finish. (c) The displacement from start to finish.

Show Solution

Strategy

For path D, trace the entire path including all direction changes to find distance traveled. For displacement, only the starting and ending positions matter.

Solution

From the figure, path D:

- Starts at $x_0 = 9 \text{m}$
- Goes left to $x = 3 \text{m}$ (moves 6 m)
- Turns around and goes right to $x_f = 5 \text{m}$ (moves 2 m)

(a) Distance traveled:

Add up all the path segments regardless of direction:

$$\text{Distance} = 6 \text{m} + 2 \text{m} = 8 \text{m}$$

(b) Magnitude of displacement:

The magnitude of displacement is the absolute value of the change in position:

$$|\Delta x| = |x_f - x_0| = |5 \text{m} - 9 \text{m}| = 4 \text{m}$$

(c) Displacement:

Displacement includes direction (sign):

$$\Delta x = x_f - x_0 = 5 \text{m} - 9 \text{m} = -4 \text{m}$$

The negative sign indicates net motion to the left.

Discussion

Notice that the distance traveled (8 m) is greater than the magnitude of displacement (4 m). This is because path D involves backtracking – the object moves left past its final position, then returns partway. The net displacement is negative because the final position (5 m) is to the left of the initial position (9 m), even though the object briefly reached as far left as 3 m. This demonstrates the important distinction between distance traveled (total path length) and displacement (net change in position).

Answer

(a) The distance traveled is 8 m.

(b) The magnitude of the displacement is 4 m.

(c) The displacement is −4 m (or 4 m to the left).

Glossary

kinematics
: the study of motion without considering its causes

position
: the location of an object at a particular time

displacement
: the change in position of an object

distance
: the magnitude of displacement between two positions

distance traveled
: the total length of the path traveled between two positions

Vectors, Scalars, and Coordinate Systems

- Define and distinguish between scalar and vector quantities.
- Assign a coordinate system for a scenario involving one-dimensional motion.

The motion of this Eclipse Concept jet can be described in terms of the distance it has traveled (a scalar quantity) or its displacement in a specific direction (a vector quantity). In order to specify the direction of motion, its displacement must be described based on a coordinate system. In this case, it may be convenient to choose motion toward the left as positive motion (it is the forward direction for the plane), although in many cases, the (x)-coordinate runs from left to right, with motion to the right as positive and motion to the left as negative. (credit: Armchair Aviator, Flickr)

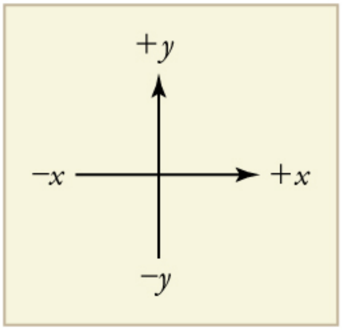
What is the difference between distance and displacement? Whereas displacement is defined by both direction and magnitude, distance is defined only by magnitude. Displacement is an example of a vector quantity. Distance is an example of a scalar quantity. A **vector** is any quantity with both *magnitude and direction*. Other examples of vectors include a velocity of 90 km/h east and a force of 500 newtons straight down.

The direction of a vector in one-dimensional motion is given simply by a plus $(+)$ or minus $(-)$ sign. Vectors are represented graphically by arrows. An arrow used to represent a vector has a length proportional to the vector's magnitude (e.g., the larger the magnitude, the longer the length of the vector) and points in the same direction as the vector.

Some physical quantities, like distance, either have no direction or none is specified. A **scalar** is any quantity that has a magnitude, but no direction. For example, a 20°C temperature, the 250 kilocalories (250 Calories) of energy in a candy bar, a 90 km/h speed limit, a person's 1.8 m height, and a distance of 2.0 m are all scalars—quantities with no specified direction. Note, however, that a scalar can be negative, such as a -20°C temperature. In this case, the minus sign indicates a point on a scale rather than a direction. Scalars are never represented by arrows.

Coordinate Systems for One-Dimensional Motion

In order to describe the direction of a vector quantity, you must designate a coordinate system within the reference frame. For one-dimensional motion, this is a simple coordinate system consisting of a one-dimensional coordinate line. In general, when describing horizontal motion, motion to the right is usually considered positive, and motion to the left is considered negative. With vertical motion, motion up is usually positive and motion down is negative. In some cases, however, as with the jet in Figure 1, it can be more convenient to switch the positive and negative directions. For example, if you are analyzing the motion of falling objects, it can be useful to define downwards as the positive direction. If people in a race are running to the left, it is useful to define left as the positive direction. It does not matter as long as the system is clear and consistent. Once you assign a positive direction and start solving a problem, you cannot change it.



It is usually convenient to consider motion upward or to the right as positive $+$and motion downward or to the left as negative (-).

Check Your Understanding

A person’s speed can stay the same as they round a corner and changes direction. Given this information, is speed a scalar or a vector quantity? Explain.

Show Solution

Speed is a scalar quantity. It does not change at all with direction changes; therefore, it has magnitude only. If it were a vector quantity, it would change as direction changes (even if its magnitude remained constant).

Section Summary

- A vector is any quantity that has magnitude and direction.
- A scalar is any quantity that has magnitude but no direction.
- Displacement and velocity are vectors, whereas distance and speed are scalars.
- In one-dimensional motion, direction is specified by a plus or minus sign to signify left or right, up or down, and the like.

Conceptual Questions

A student writes, “*A bird that is diving for prey has a speed of* -10m/s.” What is wrong with the student’s statement? What has the student actually described? Explain.

What is the speed of the bird in the previous exercise?

Acceleration is the change in velocity over time. Given this information, is acceleration a vector or a scalar quantity? Explain.

A weather forecast states that the temperature is predicted to be -5°C the following day. Is this temperature a vector or a scalar quantity? Explain.

Glossary

scalar
: a quantity that is described by magnitude, but not direction

vector
: a quantity that is described by both magnitude and direction

---

Time, Velocity, and Speed

- Explain the relationships between instantaneous velocity, average velocity, instantaneous speed, average speed, displacement, and time.
- Calculate velocity and speed given initial position, initial time, final position, and final time.
- Derive a graph of velocity vs. time given a graph of position vs. time.
- Interpret a graph of velocity vs. time.

The motion of these racing snails can be described by their speeds and their velocities. (credit: tobitasflickr, Flickr)

There is more to motion than distance and displacement. Questions such as, “How long does a foot race take?” and “What was the runner’s speed?” cannot be answered without an understanding of other concepts. In this section we add definitions of time, velocity, and speed to expand our description of motion.

Time

As discussed in Physical Quantities and Units , the most fundamental physical quantities are defined by how they are measured. This is the case with time. Every measurement of time involves measuring a change in some physical quantity. It may be a number on a digital clock, a heartbeat, or the position of the Sun in the sky. In physics, the definition of time is simple—**time** is *change*, or the interval over which change occurs. It is impossible to know that time has passed unless something changes.

The amount of time or change is calibrated by comparison with a standard. The SI unit for time is the second, abbreviated s. We might, for example, observe that a certain pendulum makes one full swing every 0.75 s. We could then use the pendulum to measure time by counting its swings or, of course, by connecting the pendulum to a clock mechanism that registers time on a dial. This allows us to not only measure the amount of time, but also to determine a sequence of events.

How does time relate to motion? We are usually interested in elapsed time for a particular motion, such as how long it takes an airplane passenger to get from his seat to the back of the plane. To find elapsed time, we note the time at the beginning and end of the motion and subtract the two. For example, a lecture may start at 11:00 A.M. and end at 11:50 A.M., so that the elapsed time would be 50 min. **Elapsed time** Δt is the difference between the ending time and beginning time,

$$\Delta t = t_f - t_0,$$

where Δt is the change in time or elapsed time, t_f is the time at the end of the motion, and t_0 is the time at the beginning of the motion. (As usual, the delta symbol, Δ, means the change in the quantity that follows it.)

Life is simpler if the beginning time t_0 is taken to be zero, as when we use a stopwatch. If we were using a stopwatch, it would simply read zero at the start of the lecture and 50 min at the end. If $t_0 = 0$, then $\Delta t = t_f \equiv t$.

In this text, for simplicity’s sake,

- motion starts at time equal to zero $(t_0=0)$ - the symbol t is used for elapsed time unless otherwise specified $(\Delta t = t_f \equiv t)$

Velocity

Your notion of velocity is probably the same as its scientific definition. You know that if you have a large displacement in a small amount of time you have a large velocity, and that velocity has units of distance divided by time, such as miles per hour or kilometers per hour.

Average Velocity

Average velocity is *displacement (change in position) divided by the time of travel*,

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_0}{t_f - t_0},$$

where $\bar{v}$ is the *average* (indicated by the bar over the v) velocity, Δx is the change in position (or displacement), and x_f and x_0 are the final and beginning positions at times t_f and t_0, respectively. If the starting time t_0 is taken to be zero, then the average velocity is simply

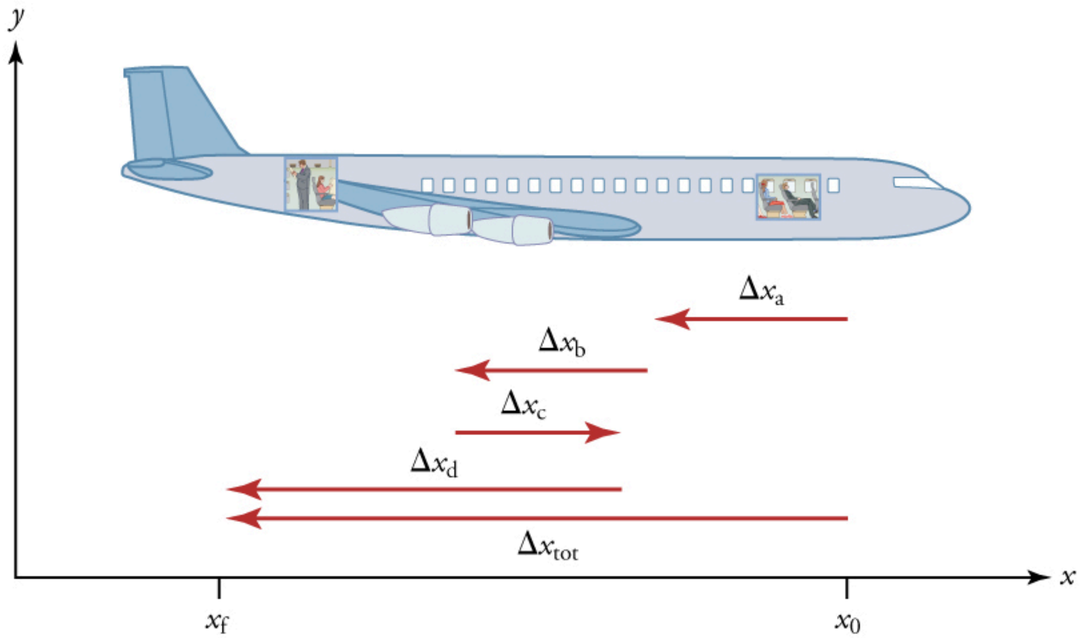
$$\bar{v}=\frac{\Delta x}{t}.$$

Notice that this definition indicates that *velocity is a vector because displacement is a vector*. It has both magnitude and direction. The SI unit for velocity is meters per second or m/s, but many other units, such as km/h, mi/h (also written as mph), and cm/s, are in common use. Suppose, for example, an airplane passenger took 5 seconds to move −4 m (the negative sign indicates that displacement is toward the back of the plane). His average velocity would be

$$\bar{v}=\frac{\Delta x}{t}=\frac{-4\text{ m}}{5\text{ s}}=-0.8\text{ m/s}.$$

The minus sign indicates the average velocity is also toward the rear of the plane.

The average velocity of an object does not tell us anything about what happens to it between the starting point and ending point, however. For example, we cannot tell from average velocity whether the airplane passenger stops momentarily or backs up before he goes to the back of the plane. To get more details, we must consider smaller segments of the trip over smaller time intervals.



A more detailed record of an airplane passenger heading toward the back of the plane, showing smaller segments of his trip.

The smaller the time intervals considered in a motion, the more detailed the information. When we carry this process to its logical conclusion, we are left with an infinitesimally small interval. Over such an interval, the average velocity becomes the *instantaneous velocity* or the *velocity at a specific instant*. A car's speedometer, for example, shows the magnitude (but not the direction) of the instantaneous velocity of the car. (Police give tickets based on instantaneous velocity, but when calculating how long it will take to get from one place to another on a road trip, you need to use average velocity.) **Instantaneous velocity** v is the average velocity at a specific instant in time (or over an infinitesimally small time interval).

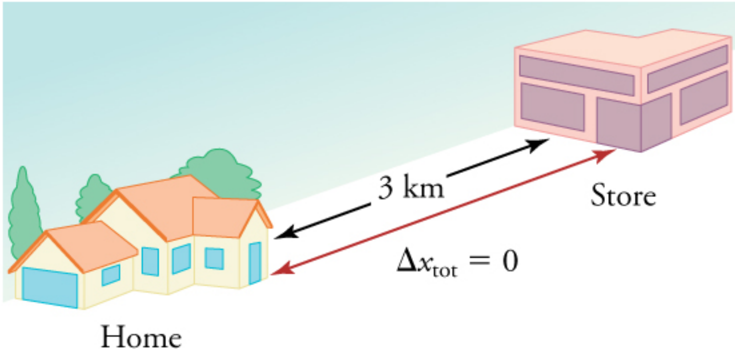
Mathematically, finding instantaneous velocity, v, at a precise instant t can involve taking a limit, a calculus operation beyond the scope of this text. However, under many circumstances, we can find precise values for instantaneous velocity without calculus.

Speed

In everyday language, most people use the terms "speed" and "velocity" interchangeably. In physics, however, they do not have the same meaning and they are distinct concepts. One major difference is that speed has no direction. Thus *speed is a scalar*. Just as we need to distinguish between instantaneous velocity and average velocity, we also need to distinguish between instantaneous speed and average speed.

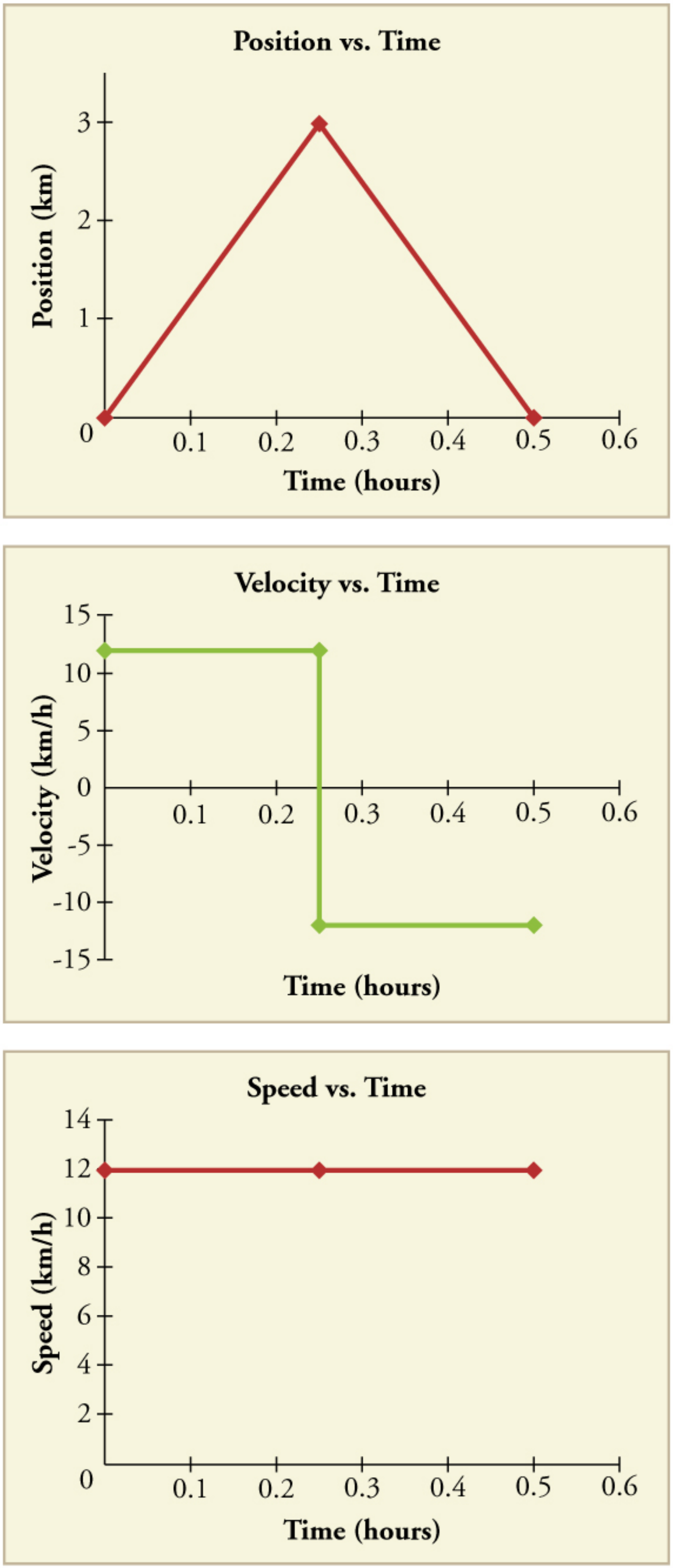
Instantaneous speed is the magnitude of instantaneous velocity. For example, suppose the airplane passenger at one instant had an instantaneous velocity of −3.0 m/s (the minus meaning toward the rear of the plane). At that same time his instantaneous speed was 3.0 m/s. Or suppose that at one time during a shopping trip your instantaneous velocity is 40 km/h due north. Your instantaneous speed at that instant would be 40 km/h—the same magnitude but without a direction. Average speed, however, is very different from average velocity. **Average speed** is the distance traveled divided by elapsed time.

We have noted that distance traveled can be greater than displacement. So average speed can be greater than average velocity, which is displacement divided by time. For example, if you drive to a store and return home in half an hour, and your car's odometer shows the total distance traveled was 6 km, then your average speed was 12 km/h. Your average velocity, however, was zero, because your displacement for the round trip is zero. (Displacement is change in position and, thus, is zero for a round trip.) Thus average speed is *not* simply the magnitude of average velocity.



During a 30-minute round trip to the store, the total distance traveled is 6 km. The average speed is 12 km/h. The displacement for the round trip is zero, since there was no net change in position. Thus the average velocity is zero.

Another way of visualizing the motion of an object is to use a graph. A plot of position or of velocity as a function of time can be very useful. For example, for this trip to the store, the position, velocity, and speed-vs.-time graphs are displayed in Figure 4. (Note that these graphs depict a very simplified **model** of the trip. We are assuming that speed is constant during the trip, which is unrealistic given that we'll probably stop at the store. But for simplicity's sake, we will model it with no stops or changes in speed. We are also assuming that the route between the store and the house is a perfectly straight line.)



Position vs. time, velocity vs. time, and speed vs. time on a trip. Note that the velocity for the return trip is negative.

Making Connections: Take-Home Investigation—Getting a Sense of Speed

If you have spent much time driving, you probably have a good sense of speeds between about 10 and 70 miles per hour. But what are these in meters per second? What do we mean when we say that something is moving at 10 m/s? To get a better sense of what these values really mean, do some observations and calculations on your own:

- calculate typical car speeds in meters per second
- estimate jogging and walking speed by timing yourself; convert the measurements into both m/s and mi/h
- determine the speed of an ant, snail, or falling leaf

Check Your Understanding

A commuter train travels from Baltimore to Washington, DC, and back in 1 hour and 45 minutes. The distance between the two stations is approximately 40 miles. What is (a) the average velocity of the train, and (b) the average speed of the train in m/s?

Show Solution

(a) The average velocity of the train is zero because $x_f = x_0$; the train ends up at the same place it starts.

(b) The average speed of the train is calculated below. Note that the train travels 40 miles one way and 40 miles back, for a total distance of 80 miles.

$$\frac{\text{distance}}{\text{time}} = \frac{80 \text{ miles}}{105 \text{ minutes}}$$

$$\frac{80 \text{ miles}}{105 \text{ minutes}} \times \frac{5280 \text{ feet}}{1 \text{ mile}} \times \frac{1 \text{ meter}}{3.28 \text{ feet}} \times \frac{1 \text{ minute}}{60 \text{ seconds}} = 20 \text{ m/s}$$

Section Summary

- Time is measured in terms of change, and its SI unit is the second (s). Elapsed time for an event is
  $$\Delta t = t_f - t_0,$$

where t_f is the final time and t_0 is the initial time. The initial time is often taken to be zero, as if measured with a stopwatch; the elapsed time is then just t.

- Average velocity $\bar{v}$ is defined as displacement divided by the travel time. In symbols, average velocity is
  $$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_0}{t_f - t_0}.$$
- The SI unit for velocity is m/s.
- Velocity is a vector and thus has a direction.
- Instantaneous velocity v is the velocity at a specific instant or the average velocity for an infinitesimal interval.
- Instantaneous speed is the magnitude of the instantaneous velocity.
- Instantaneous speed is a scalar quantity, as it has no direction specified.
- Average speed is the total distance traveled divided by the elapsed time. (Average speed is *not* the magnitude of the average velocity.) Speed is a scalar quantity; it has no direction associated with it.

Conceptual Questions

Give an example (but not one from the text) of a device used to measure time and identify what change in that device indicates a change in time.

There is a distinction between average speed and the magnitude of average velocity. Give an example that illustrates the difference between these two quantities.

Does a car's odometer measure distance or displacement? Does its speedometer measure speed or velocity?

If you divide the total distance traveled on a car trip (as determined by the odometer) by the time for the trip, are you calculating the average speed or the magnitude of the average velocity? Under what circumstances are these two quantities the same?

How are instantaneous velocity and instantaneous speed related to one another? How do they differ?

Problems & Exercises

(a) Calculate Earth's average speed relative to the Sun. (b) What is its average velocity over a period of one year?

Show Solution

Strategy

For average speed, use distance traveled divided by time. For average velocity, use displacement divided by time.

Solution

(a) Earth's average speed:

1. Earth's orbital radius is approximately $1.5 \times 10^{11} \text{m}$ (distance from Earth to Sun).
2. Calculate the circumference of Earth's orbit:

$$C = 2\pi r = 2\pi \times 1.5 \times 10^{11}\,\text{m} = 9.42 \times 10^{11}\,\text{m}$$

1. Time for one orbit (1 year) in seconds:

$$t = 365.25\text{ days} \times 24\text{ h/day} \times 3600\text{ s/h} = 3.16 \times 10^{7}\,\text{s}$$

1. Calculate average speed:

$$\text{Average speed} = \frac{C}{t} = \frac{9.42 \times 10^{11}\,\text{m}}{3.16 \times 10^{7}\,\text{s}} = 2.98 \times 10^{4}\,\text{m/s} \approx 3.0 \times 10^{4}\,\text{m/s}$$

(b) Average velocity over one year:

After one complete orbit, Earth returns to its starting position, so the displacement is zero:

$$\bar{v} = \frac{\Delta x}{t} = \frac{0\,\text{m}}{t} = 0\text{ m/s}$$

Discussion

This problem illustrates the fundamental difference between speed and velocity. Earth travels an enormous distance (9.42×10^{11} m) in one year, moving at approximately 30 km/s relative to the Sun—faster than any human-made vehicle. Yet its average velocity over a complete orbit is zero because it returns to its starting position, making the displacement zero.

This distinction is crucial in physics: speed depends only on the distance traveled (a scalar quantity), while velocity depends on displacement (a vector quantity with both magnitude and direction). For any complete orbit or round trip, average velocity is always zero regardless of how fast the object moves, while average speed depends on the total distance covered.

The calculated speed of 30,000 m/s (about 67,000 mph or 108,000 km/h) is consistent with astronomical observations and is typical for planets in our solar system. This high orbital speed is necessary to maintain Earth's orbit—it represents the balance between Earth's inertia (which would cause it to move in a straight line) and the Sun's gravitational pull (which curves its path into an ellipse, very close to circular).

Answer

(a) Earth's average speed is $3.0 \times 10^{4}\,\text{m/s}$ (or 30 km/s).

(b) Earth's average velocity over one year is 0 m/s.

A helicopter blade spins at exactly 100 revolutions per minute. Its tip is 5.00 m from the center of rotation. (a) Calculate the average speed of the blade tip in the helicopter's frame of reference. (b) What is its average velocity over one revolution?

Show Solution

Strategy

To find average speed, we need to calculate the total distance traveled by the blade tip and divide by the time taken. For one revolution, the blade tip traces out a circle. Average velocity requires finding displacement, which is zero for a complete revolution.

Solution

Given:

- Rotation rate: 100 rev/min
- Radius: $r = 5.00\,\text{m}$

(a) Average speed:

First, convert the rotation rate to revolutions per second:

$$\text{rotation rate} = 100\,\frac{\text{rev}}{\text{min}} \times \frac{1\,\text{min}}{60\,\text{s}} = \frac{5}{3}\,\frac{\text{rev}}{\text{s}}$$

The distance traveled in one revolution is the circumference of the circle:

$$d = 2\pi r = 2\pi(5.00\,\text{m}) = 31.4\,\text{m}$$

The time for one revolution:

$$t = \frac{1}{5/3\text{ rev/s}} = \frac{3}{5}\,\text{s} = 0.600\,\text{s}$$

Average speed:

$$v = \frac{d}{t} = \frac{31.4\,\text{m}}{0.600\,\text{s}} = 52.4\,\text{m/s}$$

(b) Average velocity over one revolution:

After one complete revolution, the blade tip returns to its starting position. Therefore, the displacement is zero:

$\Delta x = 0\text{m}$

Average velocity:

$\bar{v} = \frac{\Delta x}{t} = \frac{0\text{m}}{0.600\text{s}} = 0\text{m/s}$

Discussion

This problem illustrates the important distinction between speed and velocity. The blade tip is moving at a considerable speed (52.4 m/s), but because it returns to its starting point after each revolution, its average velocity is zero. This makes physical sense: velocity is a vector quantity that depends on displacement, while speed is a scalar that depends only on distance traveled. Circular motion always results in zero average velocity over complete cycles.

Answer

(a) The average speed of the blade tip is 52.4 m/s.

(b) The average velocity over one revolution is 0 m/s.

The North American and European continents are moving apart at a rate of about 3 cm/y. At this rate how long will it take them to drift 500 km farther apart than they are at present?

Show Solution

Strategy

We need to find the time required for a given displacement at constant velocity (rate). Use $t = \frac{\Delta x}{v}$, converting units appropriately.

Solution

Given:

- Rate of separation: $v = 3\text{ cm/y}$
- Displacement needed: $\Delta x = 500\text{ km}$

First, convert the displacement to centimeters:

$$\Delta x = 500\text{ km} \times \frac{1000\text{m}}{1\text{ km}} \times \frac{100\text{ cm}}{1\text{m}} = 5.0 \times 10^7\text{ cm}$$

Now calculate the time:

$$t = \frac{\Delta x}{v} = \frac{5.0 \times 10^7\text{ cm}}{3\text{ cm/y}} = 1.67 \times 10^7\text{ y} \approx 2 \times 10^7\text{ years}$$

Discussion

This result of approximately 17 million years (or 20 million years when rounded to one significant figure) represents a very long time on human scales, but it's a reasonable timescale for plate tectonics. The continents are drifting apart due to seafloor spreading at the Mid-Atlantic Ridge, where new oceanic crust is continuously created. This process is part of continental drift, first proposed by Alfred Wegener in 1912 and now explained by the theory of plate tectonics.

For context, the Atlantic Ocean has been widening for about 180 million years, and the continents have drifted thousands of kilometers apart during that time. The rate of 3 cm/year is typical for mid-ocean ridge spreading rates.

Answer

It will take approximately **2 × 10^7 years** (20 million years, or more precisely 17 million years) for the continents to drift 500 km farther apart.

Land west of the San Andreas fault in southern California is moving at an average velocity of about 6 cm/y northwest relative to land east of the fault. Los Angeles is west of the fault and may thus someday be at the same latitude as San Francisco, which is east of the fault. How far in the future will this occur if the displacement to be made is 590 km northwest, assuming the motion remains constant?

Show Solution

Strategy

We need to find the time required for a given displacement at constant velocity. Use the relationship $t = \frac{\Delta x}{v}$, being careful to convert units consistently.

Solution

Given:

- Average velocity: $v = 6\text{ cm/y}$
- Displacement: $\Delta x = 590\text{ km northwest}$

First, convert the displacement to centimeters:

$$\Delta x = 590 \text{ km} \times \frac{1000 \text{m}}{1 \text{ km}} \times \frac{100 \text{ cm}}{1 \text{m}} = 5.90 \times 10^7 \text{ cm}$$

Now calculate the time:

$$t = \frac{\Delta x}{v} = \frac{5.90 \times 10^7 \text{ cm}}{6 \text{ cm/y}} = 9.83 \times 10^6 \text{ y}$$

Discussion

This result means it will take approximately 9.8 million years for Los Angeles to reach the same latitude as San Francisco, assuming constant motion. This is a reasonable timescale for geological processes. The calculation assumes the velocity remains constant, which is a simplification – actual tectonic motion can vary over such long periods. This problem illustrates how we can apply kinematics principles to very slow geological processes using the same equations we use for everyday motion.

Answer

Los Angeles will be at the same latitude as San Francisco in approximately 9.8×10^6 years (9.8 million years).

On May 26, 1934, a streamlined, stainless steel diesel train called the Zephyr set the world's nonstop long-distance speed record for trains. Its run from Denver to Chicago took 13 hours, 4 minutes, 58 seconds, and was witnessed by more than a million people along the route. The total distance traveled was 1633.8 km. What was its average speed in km/h and m/s?

Show Solution

Strategy

Average speed is total distance divided by total time. Calculate the time in hours first, then find the average speed in km/h and convert to m/s.

Solution

Given:

- Distance: $d = 1633.8 \text{ km}$
- Time: 13 h 4 min 58 s

Step 1: Convert time to hours

$$t = 13 \text{ h} + \frac{4 \text{ min}}{60 \text{ min/h}} + \frac{58 \text{s}}{3600 \text{ s/h}}$$
$$t = 13 + 0.06667 + 0.01611 = 13.083 \text{ h}$$

Step 2: Calculate average speed in km/h

$$\text{Average speed} = \frac{d}{t} = \frac{1633.8 \text{ km}}{13.083 \text{ h}} = 124.9 \text{ km/h}$$

Step 3: Convert to m/s

$$124.9 \frac{\text{km}}{\text{h}} \times \frac{1000 \text{m}}{1 \text{ km}} \times \frac{1 \text{ h}}{3600 \text{s}} = 34.69 \text{m/s}$$

Discussion

The Zephyr's average speed of nearly 125 km/h (about 78 mph) was remarkable for 1934, especially for a nonstop journey of over 1600 km. This was significantly faster than conventional trains of that era. The streamlined design reduced air resistance, allowing higher speeds and better fuel efficiency.

Answer

The Zephyr's average speed was **124.9 km/h** (or **34.69 m/s**).

Tidal friction is slowing the rotation of the Earth. As a result, the orbit of the Moon is increasing in radius at a rate of approximately 4 cm/year. Assuming this to be a constant rate, how many years will pass before the radius of the Moon's orbit increases by $3.84 \times 10^6 \text{m}$ (1%)?

Show Solution

Strategy

We need to find the time required for the Moon's orbital radius to increase by a given amount at a constant rate. Use $t = \frac{\Delta r}{v}$ where v is the rate of change and Δr is the change in radius.

Solution

Given:

- Rate of orbital radius increase: $v = 4 \text{ cm/year}$
- Change in radius: $\Delta r = 3.84 \times 10^6 \text{m}$

First, convert the change in radius to centimeters:

$$\Delta r = 3.84 \times 10^{6} \text{m} \times \frac{100 \text{ cm}}{1 \text{m}} = 3.84 \times 10^{8} \text{ cm}$$

Now calculate the time:

$$t = \frac{\Delta r}{v} = \frac{3.84 \times 10^{8} \text{ cm}}{4 \text{ cm/year}} = 9.6 \times 10^{7} \text{ years}$$

Discussion

This result of 96 million years is a very long time on human scales but not unreasonable for astronomical processes. The 1% increase in the Moon's orbital radius represents a significant change – the current Earth-Moon distance is approximately 384,000 km, so a 1% increase is 3,840 km. This gradual recession of the Moon is a real phenomenon caused by tidal interactions between Earth and the Moon. The energy for this comes from Earth's rotation, which is gradually slowing down as the Moon moves farther away, conserving angular momentum in the Earth-Moon system.

Answer

It will take 9.6×10^{7} years (96 million years) for the Moon's orbital radius to increase by 1%.

A student drove to the university from her home and noted that the odometer reading of her car increased by 12.0 km. The trip took 18.0 min. (a) What was her average speed? (b) If the straight-line distance from her home to the university is 10.3 km in a direction $25.0°$ south of east, what was her average velocity? (c) If she returned home by the same path 7 h 30 min after she left, what were her average speed and velocity for the entire trip?

Show Solution

Strategy

Average speed uses total distance traveled divided by time. Average velocity uses displacement (straight-line distance with direction) divided by time. For the round trip, the displacement is zero since she returns to her starting point.

Solution

(a) Average speed to university:

Given:

- Distance traveled (from odometer): $d = 12.0 \text{ km}$
- Time: $t = 18.0 \text{ min} = 0.300 \text{ h}$

$$\text{Average speed} = \frac{d}{t} = \frac{12.0 \text{ km}}{0.300 \text{ h}} = 40.0 \text{ km/h}$$

(b) Average velocity to university:

Given:

- Displacement: $\Delta x = 10.3 \text{ km at } 25.0° \text{ south of east}$
- Time: $t = 0.300 \text{ h}$

$$\bar{v} = \frac{\Delta x}{t} = \frac{10.3 \text{ km}}{0.300 \text{ h}} = 34.3 \text{ km/h at } 25.0° \text{ south of east}$$

(c) Average speed and velocity for entire round trip:

For the round trip:

- Total distance: $d\_total = 2 \times 12.0 \text{ km} = 24.0 \text{ km}$
- Total time: $t\_total = 7 \text{ h } 30 \text{ min} = 7.50 \text{ h}$
- Total displacement: $\Delta x\_total = 0$ (returns to starting point)

Average speed:

$$\text{Average speed} = \frac{24.0 \text{ km}}{7.50 \text{ h}} = 3.20 \text{ km/h}$$

Average velocity:

$$\bar{v} = \frac{0}{7.50 \text{ h}} = 0 \text{ km/h}$$

Discussion

Notice the important distinction between average speed and average velocity. The odometer reading increased by 12.0 km even though the straight-line displacement was only 10.3 km, indicating the route was not a perfectly straight line. For the round trip, the average velocity is zero because the displacement is zero (she ends where she started), but the average speed is 3.20 km/h because she did travel a total distance of 24.0 km.

Answer

(a) Her average speed to the university was **40.0 km/h**.

(b) Her average velocity to the university was **34.3 km/h at 25.0° south of east**.

(c) For the entire round trip, her average speed was **3.20 km/h** and her average velocity was **0 km/h** (or zero).

The speed of propagation of the action potential (an electrical signal) in a nerve cell depends (inversely) on the diameter of the axon (nerve fiber). If the nerve cell connecting the spinal cord to your feet is 1.1 m long, and the nerve impulse speed is 18 m/s, how long does it take for the nerve signal to travel this distance?

Show Solution

Strategy

Use the relationship between distance, speed, and time: $t = \frac{d}{v}$.

Solution

Given:

- Distance: $d = 1.1 \text{m}$
- Speed of nerve impulse: $v = 18 \text{m/s}$

Calculate the time:

$$t = \frac{d}{v} = \frac{1.1 \text{m}}{18 \text{m/s}} = 0.061 \text{s} = 61 \text{ ms}$$

Discussion

The result of 61 milliseconds (0.061 seconds) is the time it takes for a nerve signal to travel from your spinal cord to your feet. This seems quite fast, but it's actually relatively slow compared to electrical signals in wires, which travel near the speed of light. This delay is why your reaction time to step on something sharp isn't instantaneous – the signal must travel up to your brain and then back down to your muscles.

For comparison, if the signal were traveling at the speed of light ($3 \times 10^8 \text{m/s}$), it would take only about 3.7 nanoseconds to cover this distance – over 16 million times faster! The relatively slow speed of nerve impulses is due to the biological mechanism of signal propagation, which involves ion channels opening and closing sequentially along the axon rather than simple electron flow.

Answer

It takes 0.061 s (or 61 ms) for the nerve signal to travel from the spinal cord to the feet.

Conversations with astronauts on the lunar surface were characterized by a kind of echo in which the earthbound person's voice was so loud in the astronaut's space helmet that it was picked up by the astronaut's microphone and transmitted back to Earth. It is reasonable to assume that the echo time equals the time necessary for the radio wave to travel from the Earth to the Moon and back (that is, neglecting any time delays in the electronic equipment). Calculate the distance from Earth to the Moon given that the echo time was 2.56 s and that radio waves travel at the speed of light ($3.00 \times 10^8 \text{m/s}$).

Show Solution

Strategy

The echo time is the time for the radio wave to travel from Earth to Moon and back. Use the relationship $d = vt$, where the total distance traveled is twice the Earth-Moon distance.

Solution

Given:

- Speed of light (radio wave speed): $c = 3.00 \times 10^8 \text{m/s}$
- Echo time (round trip): $t\_total = 2.56 \text{s}$

The radio wave travels to the Moon and back, so:

$$d_{total} = c \times t_{total} = (3.00 \times 10^8 \text{m/s})(2.56 \text{s}) = 7.68 \times 10^8 \text{m}$$

The Earth-Moon distance is half of this:

$$d_{Earth-Moon} = \frac{d_{total}}{2} = \frac{7.68 \times 10^8 \text{m}}{2} = 3.84 \times 10^8 \text{m}$$

Convert to kilometers:

$$d_{Earth-Moon} = 3.84 \times 10^8 \text{m} \times \frac{1 \text{ km}}{1000 \text{m}} = 3.84 \times 10^5 \text{ km} = 384{,}000 \text{ km}$$

Discussion

This result of 384,000 km matches the well-known average Earth-Moon distance quite well. The echo delay of 2.56 seconds was a characteristic feature of lunar communications during the Apollo missions. This delay was noticeable in conversations and required astronauts and mission control to adapt their communication style – they couldn't have natural back-and-forth conversations but had to wait for responses.

The calculation assumes the radio waves travel at the speed of light in vacuum, which is a good approximation since most of the path is through the vacuum of space. This method of using signal travel time to measure distances is fundamental in astronomy and is how we measure distances to planets, spacecraft, and even nearby stars (using radar or laser ranging).

Answer

The distance from Earth to the Moon is **384,000 km** (or **3.84×10^5 km** or **3.84×10^8 m**).

A football quarterback runs 15.0 m straight down the playing field in 2.50 s. He is then hit and pushed 3.00 m straight backward in 1.75 s. He breaks the tackle and runs straight forward another 21.0 m in 5.20 s. Calculate his average velocity (a) for each of the three intervals and (b) for the entire motion.

Show Solution

Strategy

Average velocity is displacement divided by time. For each interval, we identify the displacement (with sign indicating direction) and time, then calculate the average velocity. For the entire motion, we sum the displacements and times.

Solution

Let's define forward (down the field) as the positive direction.

(a) Average velocity for each interval:

Interval 1: Initial run

$$\bar{v}_1 = \frac{\Delta x_1}{\Delta t_1} = \frac{+15.0\text{m}}{2.50\text{s}} = +6.00\text{m/s}$$

Interval 2: Pushed backward

$$\bar{v}_2 = \frac{\Delta x_2}{\Delta t_2} = \frac{-3.00\text{m}}{1.75\text{s}} = -1.71\text{m/s}$$

Interval 3: Breaking tackle and running forward

$$\bar{v}_3 = \frac{\Delta x_3}{\Delta t_3} = \frac{+21.0\text{m}}{5.20\text{s}} = +4.04\text{m/s}$$

(b) Average velocity for entire motion:

Total displacement:

$$\Delta x_{\text{total}} = \Delta x_1 + \Delta x_2 + \Delta x_3 = 15.0\text{m} + (-3.00\text{m}) + 21.0\text{m} = 33.0\text{m}$$

Total time:

$$\Delta t_{\text{total}} = \Delta t_1 + \Delta t_2 + \Delta t_3 = 2.50\text{s} + 1.75\text{s} + 5.20\text{s} = 9.45\text{s}$$

Average velocity for entire motion:

$$\bar{v}_{\text{total}} = \frac{\Delta x_{\text{total}}}{\Delta t_{\text{total}}} = \frac{33.0\text{m}}{9.45\text{s}} = 3.49\text{m/s}$$

Discussion

The negative velocity in interval 2 indicates backward motion. The average velocity for the entire motion (3.49 m/s) is less than the average of the three interval velocities because the quarterback spent more time in the slower interval 3. The net displacement is 33.0 m forward, which makes sense: he moved forward 15.0 m, then backward 3.00 m (net 12.0 m forward so far), then forward another 21.0 m, giving a total of 33.0 m forward. This speed of 3.49 m/s is reasonable for a quarterback navigating through defenders – it's much slower than his sprint speed but accounts for the backward motion and the time spent breaking tackles.

Answer

(a) Interval 1: 6.00 m/s forward; Interval 2: 1.71 m/s backward; Interval 3: 4.04 m/s forward.

(b) The average velocity for the entire motion is 3.49 m/s forward.

The planetary model of the atom pictures electrons orbiting the atomic nucleus much as planets orbit the Sun. In this model you can view hydrogen, the simplest atom,as having a single electron in a circular orbit $1.06 \times 10^{-10}\text{m}$ in diameter. (a) If the average speed of the electron in this orbit is known to be $2.20 \times 10^6\text{m/s}$, calculate the number of revolutions per second it makes about the nucleus. (b) What is the electron's average velocity during one revolution?

Show Solution

Strategy

The electron orbits the nucleus in a circular path. To find the number of revolutions per second, we need to find how far the electron travels in one second and divide by the circumference of one orbit. The average velocity over one complete revolution is zero because the electron returns to its starting position.

Solution

Given:

- Orbital diameter: $d = 1.06 \times 10^{-10}$ m
- Orbital radius: $r = \frac{d}{2} = 5.30 \times 10^{-11}$ m
- Average speed: $v = 2.20 \times 10^{6}$ m/s

(a) Number of revolutions per second:

1. Calculate the circumference of the orbit:

$$C = 2\pi r = 2\pi (5.30 \times 10^{-11}\text{ m}) = 3.33 \times 10^{-10}\text{ m}$$

1. The distance traveled in one second equals the speed times one second:

$$d_{\text{per second}} = v \times 1\text{s} = 2.20 \times 10^{6}\text{ m}$$

1. The number of revolutions per second is the distance traveled per second divided by the circumference of one orbit:

$$f = \frac{v}{C} = \frac{2.20 \times 10^{6}\text{ m/s}}{3.33 \times 10^{-10}\text{ m}} = 6.61 \times 10^{15}\text{ rev/s}$$

(b) Average velocity during one revolution:

After completing one full revolution, the electron returns to its starting position. Therefore:

- Displacement = 0
- Average velocity = displacement / time = 0 / t = 0 m/s

Discussion

The electron completes an astounding 6.61×10^{15} revolutions every second - that's over 6 quadrillion orbits per second! Despite this incredibly high speed (about 0.7% of the speed of light), the average velocity over one complete orbit is zero because velocity is a vector quantity that depends on displacement, not distance. The electron travels a great distance but ends up back where it started, so its net displacement is zero. This distinction between speed (a scalar based on distance) and velocity (a vector based on displacement) is fundamental to understanding motion.

Note: The planetary model of the atom, while useful for visualization, has been superseded by quantum mechanics. In reality, electrons don't orbit in fixed circular paths but exist in probability clouds called orbitals. Nevertheless, this calculation gives reasonable estimates for the characteristic speeds and frequencies involved in atomic-scale motion.

Answer

(a) The electron makes 6.61×10^{15} revolutions per second about the nucleus.

(b) The electron's average velocity during one complete revolution is **0 m/s**.

Glossary

average speed
: distance traveled divided by time during which motion occurs

average velocity
: displacement divided by time over which displacement occurs

instantaneous velocity
: velocity at a specific instant, or the average velocity over an infinitesimal time interval

instantaneous speed
: magnitude of the instantaneous velocity

time
: change, or the interval over which change occurs

model
: simplified description that contains only those elements necessary to describe the physics of a physical situation

elapsed time
: the difference between the ending time and beginning time

Acceleration

- Define and distinguish between instantaneous acceleration, average acceleration, and deceleration.
- Calculate acceleration given initial time, initial velocity, final time, and final velocity.

A plane decelerates, or slows down, as it comes in for landing in St. Maarten. Its acceleration is opposite in direction to its velocity. (credit: Steve Conry, Flickr)

In everyday conversation, to accelerate means to speed up. The accelerator in a car can in fact cause it to speed up. The greater the **acceleration**, the greater the change in velocity over a given time. The formal definition of acceleration is consistent with these notions, but more inclusive.

Average Acceleration

Average Acceleration is *the rate at which velocity changes*,

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_0}{t_f - t_0},$$

where $\bar{a}$ is average acceleration, v is velocity, and t is time. (The bar over the a means *average* acceleration.)

Because acceleration is velocity in m/s divided by time in s, the SI units for acceleration are m/s^2, meters per second squared or meters per second per second, which literally means by how many meters per second the velocity changes every second.

Recall that velocity is a vector—it has both magnitude and direction. This means that a change in velocity can be a change in magnitude (or speed), but it can also be a change in *direction*. For example, if a car turns a corner at constant speed, it is accelerating because its direction is changing. The quicker you turn, the greater the acceleration. So there is an acceleration when velocity changes either in magnitude (an increase or decrease in speed) or in direction, or both.

Acceleration as a Vector

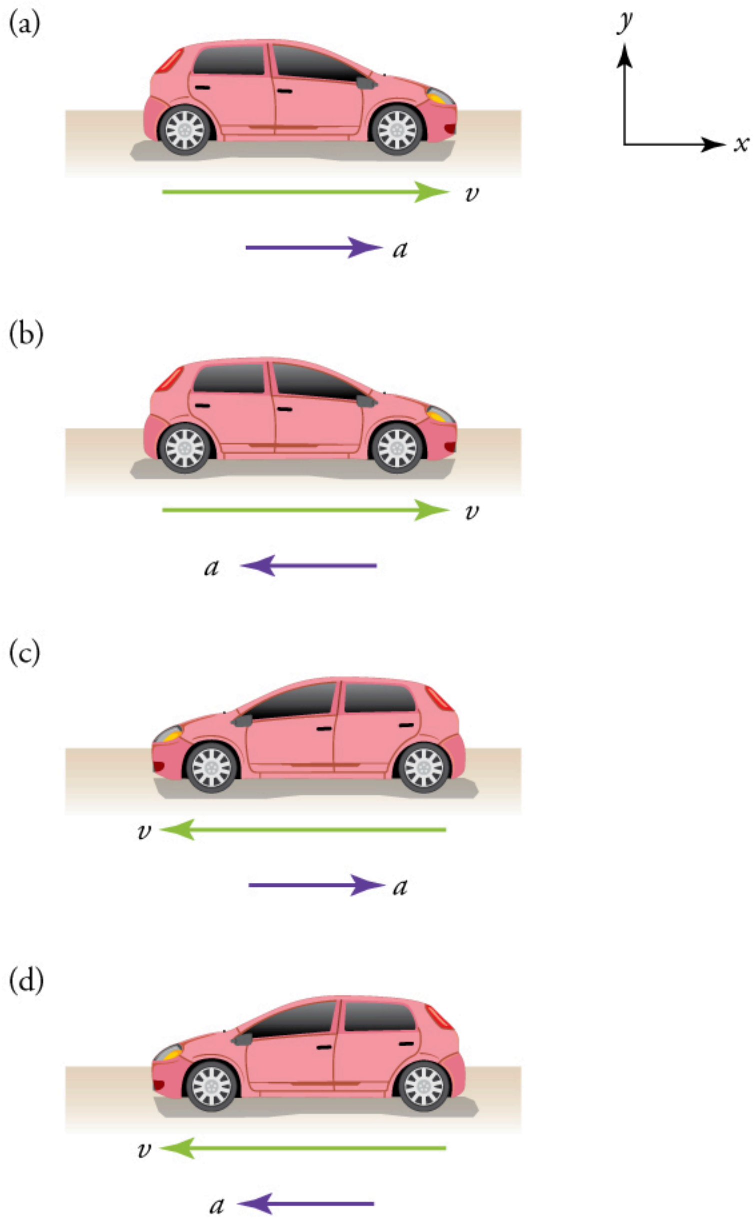
Acceleration is a vector in the same direction as the *change* in velocity, Δv. Since velocity is a vector, it can change either in magnitude or in direction. Acceleration is therefore a change in either speed or direction, or both.

Keep in mind that although acceleration is in the direction of the *change* in velocity, it is not always in the direction of *motion*. When an object slows down, its acceleration is opposite to the direction of its motion. This is known as **deceleration**.

A subway train in Sao Paulo, Brazil, decelerates as it comes into a station. It is accelerating in a direction opposite to its direction of motion. (credit: Yusuke Kawasaki, Flickr)

Misconception Alert: Deceleration vs. Negative Acceleration

Deceleration always refers to acceleration in the direction opposite to the direction of the velocity. Deceleration always reduces speed. Negative acceleration, however, is acceleration *in the negative direction in the chosen coordinate system*. Negative acceleration may or may not be deceleration, and deceleration may or may not be considered negative acceleration. For example, consider Figure 3.



(a) This car is speeding up as it moves toward the right. It therefore has positive acceleration in our coordinate system. (b) This car is slowing down as it moves toward the right. Therefore, it has negative acceleration in our coordinate system, because its acceleration is toward the left. The car is also decelerating: the direction of its acceleration is opposite to its direction of motion. (c) This car is moving toward the left, but slowing down over time. Therefore, its acceleration is positive in our coordinate system because it is toward the right. However, the car is decelerating because its acceleration is opposite to its motion. (d) This car is speeding up as it moves toward the left. It has negative acceleration because it is accelerating toward the left. However, because its acceleration is in the same direction as its motion, it is speeding up (not decelerating).

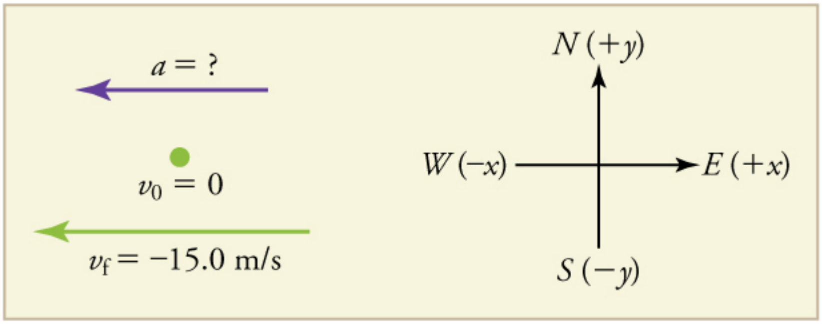
Calculating Acceleration: A Racehorse Leaves the Gate

A racehorse coming out of the gate accelerates from rest to a velocity of 15.0 m/s due west in 1.80 s. What is its average acceleration?

(credit: Jon Sullivan, PD Photo.org)

Strategy

First we draw a sketch and assign a coordinate system to the problem. This is a simple problem, but it always helps to visualize it. Notice that we assign east as positive and west as negative. Thus, in this case, we have negative velocity.



We can solve this problem by identifying Δv and Δt from the given information and then calculating the average acceleration directly from the equation $\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_0}{t_f - t_0}$.

Solution

1. Identify the knowns. $v_0 = 0$, $v_f = -15.0\,\text{m/s}$ (the negative sign indicates direction toward the west), $\Delta t = 1.80\ \text{s}$.

2. Find the change in velocity. Since the horse is going from zero to $-15.0\,\text{m/s}$, its change in velocity equals its final velocity: $\Delta v = v_f = -15.0\,\text{m/s}$.
3. Plug in the known values (Δv and Δt) and solve for the unknown $\bar{a}$.

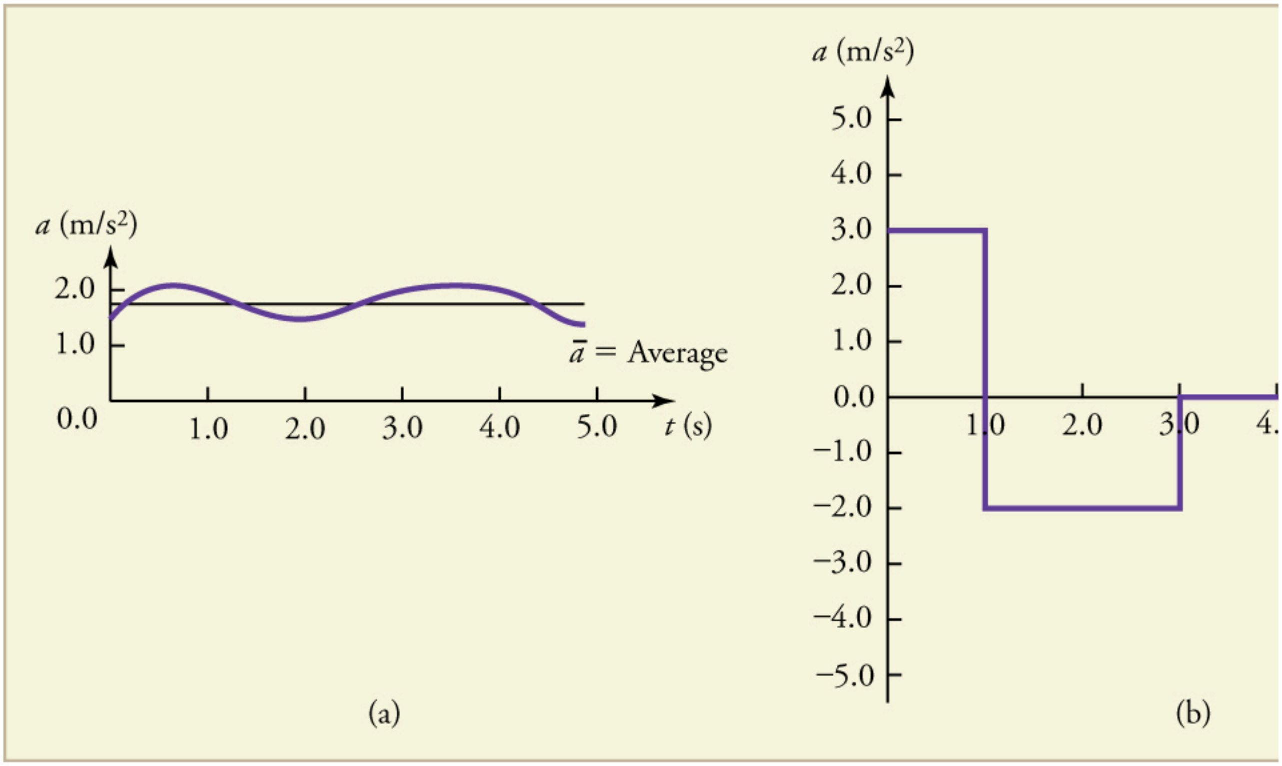
$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{-15.0\,\text{m/s}}{1.80\,\text{s}} = -8.33\,\text{m/s}^2.$$

Discussion

The negative sign for acceleration indicates that acceleration is toward the west. An acceleration of $8.33\,\text{m/s}^2$ due west means that the horse increases its velocity by 8.33 m/s due west each second, that is, 8.33 meters per second per second, which we write as $8.33\,\text{m/s}^2$. This is truly an average acceleration, because the ride is not smooth. We shall see later that an acceleration of this magnitude would require the rider to hang on with a force nearly equal to his weight.

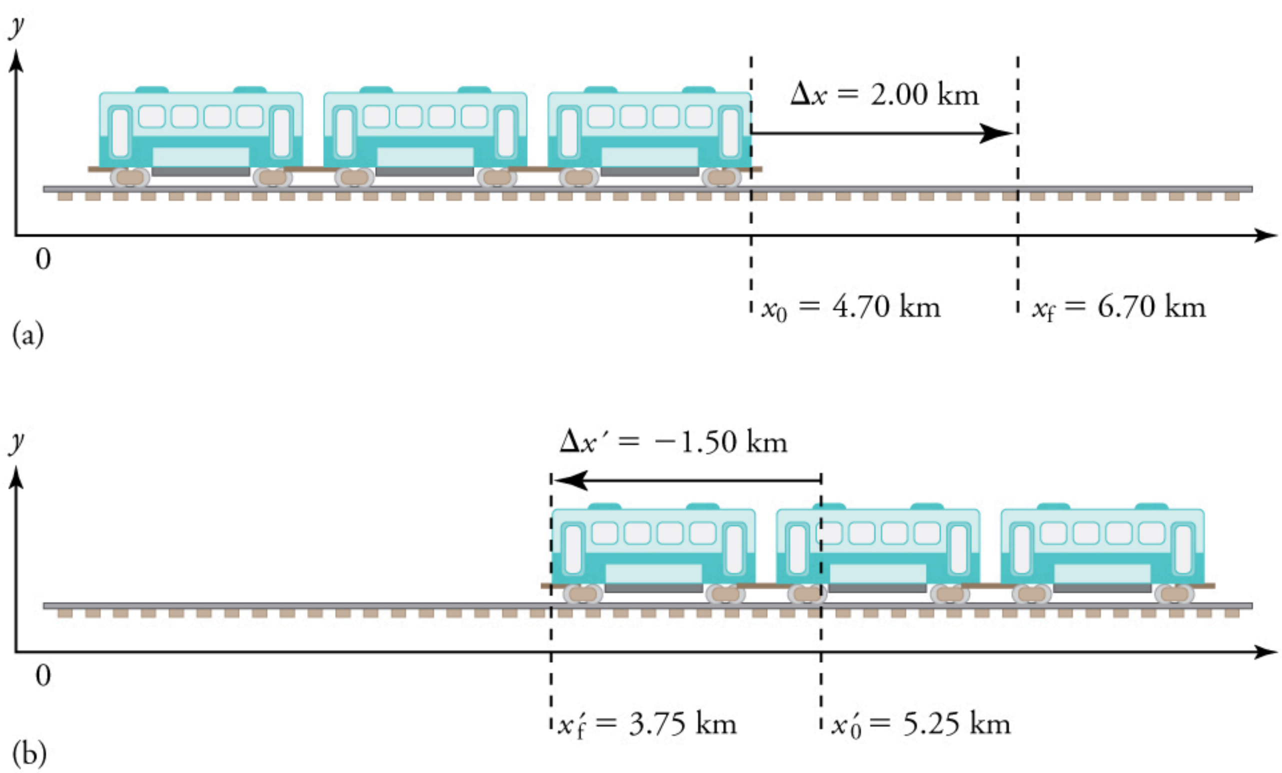
Instantaneous Acceleration

Instantaneous acceleration a, or the *acceleration at a specific instant in time*, is obtained by the same process as discussed for instantaneous velocity in Time, Velocity, and Speed —that is, by considering an infinitesimally small interval of time. How do we find instantaneous acceleration using only algebra? The answer is that we choose an average acceleration that is representative of the motion. Figure 6 shows graphs of instantaneous acceleration versus time for two very different motions. In Figure 6(a), the acceleration varies slightly and the average over the entire interval is nearly the same as the instantaneous acceleration at any time. In this case, we should treat this motion as if it had a constant acceleration equal to the average (in this case about $1.8\,\text{m/s}^2$) . In Figure 6(b), the acceleration varies drastically over time. In such situations it is best to consider smaller time intervals and choose an average acceleration for each. For example, we could consider motion over the time intervals from 0 to 1.0 s and from 1.0 to 3.0 s as separate motions with accelerations of $+3.0\,\text{m/s}^2$ and $-2.0\,\text{m/s}^2$, respectively.



Graphs of instantaneous acceleration versus time for two different one-dimensional motions. (a) Here acceleration varies only slightly and is always in the same direction, since it is positive. The average over the interval is nearly the same as the acceleration at any given time. (b) Here the acceleration varies greatly, perhaps representing a package on a post office conveyor belt that is accelerated forward and backward as it bumps along. It is necessary to consider small time intervals (such as from 0 to 1.0 s) with constant or nearly constant acceleration in such a situation.

The next several examples consider the motion of the subway train shown in Figure 7. In (a) the shuttle moves to the right, and in (b) it moves to the left. The examples are designed to further illustrate aspects of motion and to illustrate some of the reasoning that goes into solving problems.



One-dimensional motion of a subway train. Here we have chosen the x-axis so that + means to the right and - means to the left for displacements, velocities, and accelerations. (a) The subway train moves to the right from (x_0) to (x_f). Its displacement Δx is +2.0

km. (b) The train moves to the left from x'_0 to x'_f. Its displacement $\Delta x'=1.5 \text{km}$. (Note that the prime symbol (') is used simply to distinguish between displacement in the two different situations. The distances of travel and the size of the cars are on different scales to fit everything into the diagram.)

Calculating Displacement: A Subway Train

What are the magnitude and sign of displacements for the motions of the subway train shown in parts (a) and (b) of Figure 7?

Strategy

A drawing with a coordinate system is already provided, so we don't need to make a sketch, but we should analyze it to make sure we understand what it is showing. Pay particular attention to the coordinate system. To find displacement, we use the equation $\Delta x = x_f - x_0$. This is straightforward since the initial and final positions are given.

Solution

1. Identify the knowns. In the figure we see that $x_f = 6.70 \text{km}$ and $x_0 = 4.70 \text{km}$ for part (a), and $x'_f = 3.75 \text{km}$ and $x'_0 = 5.25 \text{km}$ for part (b).

2. Solve for displacement in part (a).

$$\Delta x = x_f - x_0 = 6.70 \text{km} - 4.70 \text{km} = +2.00 \text{km}$$

3. Solve for displacement in part (b).

$$\Delta x' = x'_f - x'_0 = 3.75 \text{km} - 5.25 \text{km} = -1.50 \text{km}$$

Discussion

The direction of the motion in (a) is to the right and therefore its displacement has a positive sign, whereas motion in (b) is to the left and thus has a negative sign.

Comparing Distance Traveled with Displacement: A Subway Train

What are the distances traveled for the motions shown in parts (a) and (b) of the subway train in Figure 7?

Strategy

To answer this question, think about the definitions of distance and distance traveled, and how they are related to displacement. Distance between two positions is defined to be the magnitude of displacement, which was found in Example 2. Distance traveled is the total length of the path traveled between the two positions. (See Displacement.) In the case of the subway train shown in Figure 7, the distance traveled is the same as the distance between the initial and final positions of the train.

Solution

1. The displacement for part (a) was +2.00 km. Therefore, the distance between the initial and final positions was 2.00 km, and the distance traveled was 2.00 km.

2. The displacement for part (b) was -1.5km. Therefore, the distance between the initial and final positions was 1.50 km, and the distance traveled was 1.50 km.

Discussion

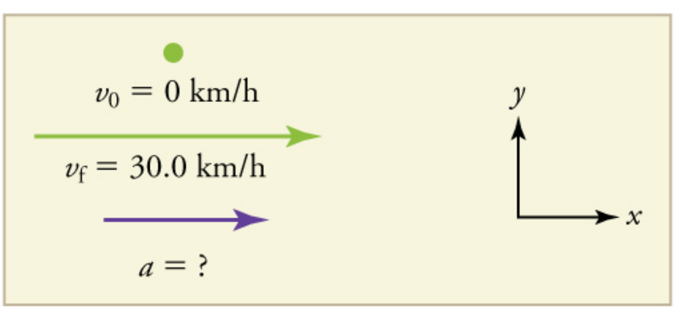
Distance is a scalar. It has magnitude but no sign to indicate direction.

Calculating Acceleration: A Subway Train Speeding Up

Suppose the train in Figure 7(a) accelerates from rest to 30.0 km/h in the first 20.0 s of its motion. What is its average acceleration during that time interval?

Strategy

It is worth it at this point to make a simple sketch:



Velocity Vectors

This problem involves three steps. First we must determine the change in velocity, then we must determine the change in time, and finally we use these values to calculate the acceleration.

Solution

1. Identify the knowns. $v_0 = 0$ (the trains starts at rest), $v_f = 30.0\text{ km/h}$, and $\Delta t = 20.0\text{ s}$.

2. Calculate Δv. Since the train starts from rest, its change in velocity is $\Delta v = +30.0\text{ km/h}$, where the plus sign means velocity to the right.

3. Plug in known values and solve for the unknown, $\bar{a}$.

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{+30.0\text{ km/h}}{20.0\text{ s}}$$

4. Since the units are mixed (we have both hours and seconds for time), we need to convert everything into SI units of meters and seconds. (See Physical Quantities and Units for more guidance.)

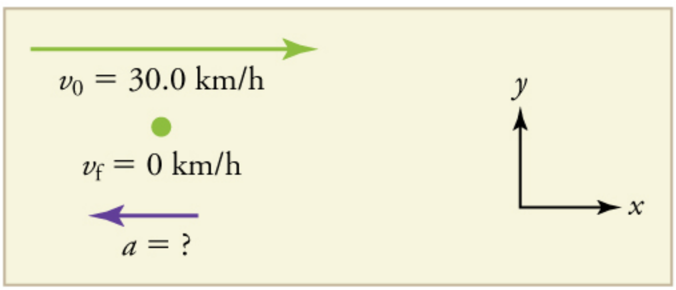
$$\bar{a} = \left(\frac{+30\text{ km/h}}{20.0\text{ s}}\right)\left(\frac{10^3\text{ m}}{1\text{ km}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 0.417\text{ m/s}^2$$

Discussion

The plus sign means that acceleration is to the right. This is reasonable because the train starts from rest and ends up with a velocity to the right (also positive). So acceleration is in the same direction as the *change* in velocity, as is always the case.
Calculate Acceleration: A Subway Train Slowing Down

Now suppose that at the end of its trip, the train in Figure 7(a) slows to a stop from a speed of 30.0 km/h in 8.00 s. What is its average acceleration while stopping?

Strategy



Velocity and Acceleration Vectors

In this case, the train is decelerating and its acceleration is negative because it is toward the left. As in the previous example, we must find the change in velocity and the change in time and then solve for acceleration.

Solution

1. Identify the knowns. $v_0 = 30.0\text{ km/h}$, $v_f = 0\text{ km/h}$ (the train is stopped, so its velocity is 0), and $\Delta t = 8.00\text{ s}$.

2. Solve for the change in velocity, Δv.

$$\Delta v = v_f - v_0 = 0 - 30.0\text{ km/h} = -30.0\text{ km/h}$$

3. Plug in the knowns, Δv and Δt, and solve for $\bar{a}$.

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{-30.0\text{ km/h}}{8.00\text{ s}}$$

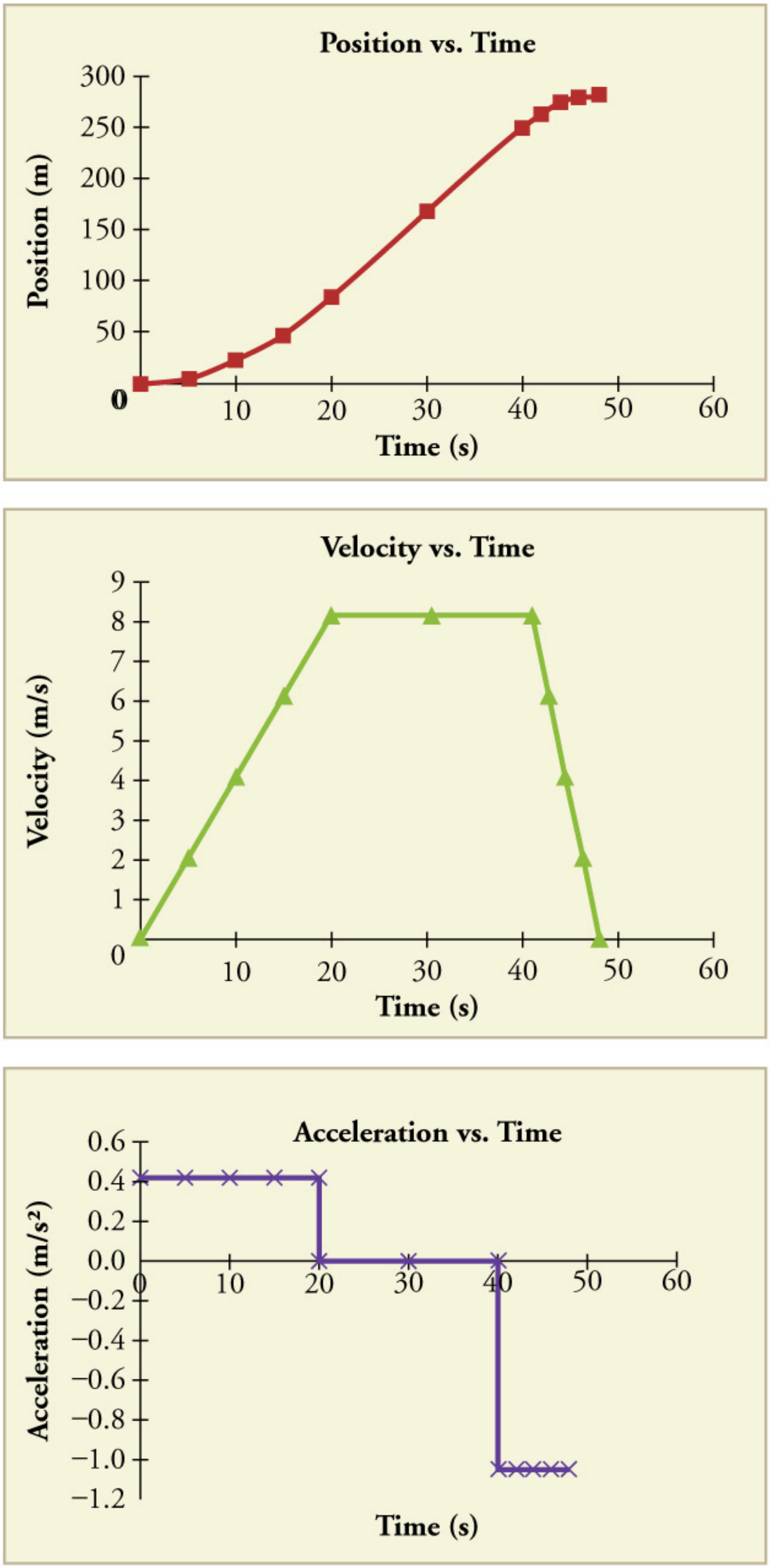
4. Convert the units to meters and seconds.

$$\bar{a} = \frac{\Delta v}{\Delta t} = \left(\frac{-30.0\text{ km/h}}{8.00\text{ s}}\right)\left(\frac{10^3\text{ m}}{1\text{ km}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right) = -1.04\text{ m/s}^2.$$

Discussion

The minus sign indicates that acceleration is to the left. This sign is reasonable because the train initially has a positive velocity in this problem, and a negative acceleration would oppose the motion. Again, acceleration is in the same direction as the *change* in velocity, which is negative here. This acceleration can be called a deceleration because it has a direction opposite to the velocity.

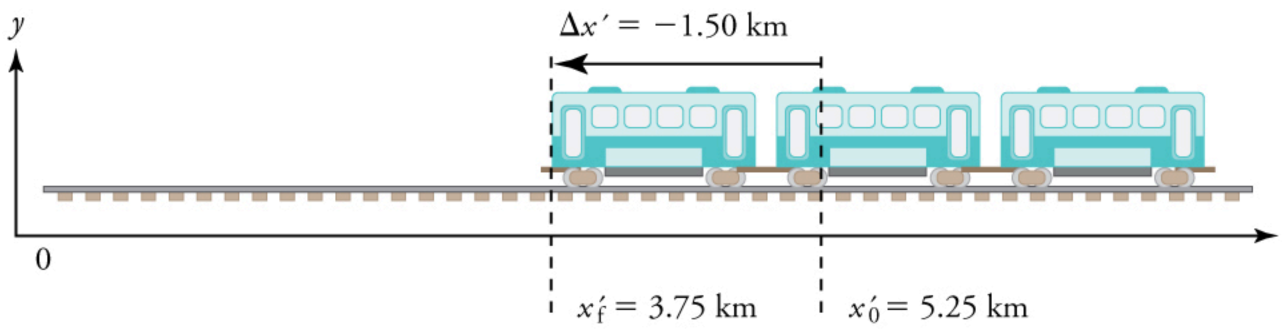
The graphs of position, velocity, and acceleration vs. time for the trains in Example 4 and Example 5 are displayed in Figure 10. (We have taken the velocity to remain constant from 20 to 40 s, after which the train decelerates.)



(a) Position of the train over time. Notice that the train's position changes slowly at the beginning of the journey, then more and more quickly as it picks up speed. Its position then changes more slowly as it slows down at the end of the journey. In the middle of the journey, while the velocity remains constant, the position changes at a constant rate. (b) Velocity of the train over time. The train's velocity increases as it accelerates at the beginning of the journey. It remains the same in the middle of the journey (where there is no acceleration). It decreases as the train decelerates at the end of the journey. (c) The acceleration of the train over time. The train has positive acceleration as it speeds up at the beginning of the journey. It has no acceleration as it travels at constant velocity in the middle of the journey. Its acceleration is negative as it slows down at the end of the journey.

Calculating Average Velocity: The Subway Train

What is the average velocity of the train in part b of Example 2, and shown again below, if it takes 5.00 min to make its trip?



Train Diagram

Strategy

Average velocity is displacement divided by time. It will be negative here, since the train moves to the left and has a negative displacement.

Solution

1. Identify the knowns. $x'_f = 3.75\text{km}$, $x'_0 = 5.25\text{km}$, $\Delta t = 5.00\text{min}$.

2. Determine displacement, $\Delta x'$. We found $\Delta x'$ to be -1.5km in Example 2.

3. Solve for average velocity.

$$\bar{v}=\frac{\Delta x'}{\Delta t}=\frac{-1.50\text{km}}{5.00\text{min}}$$

4. Convert units.

$$\bar{v}=\frac{\Delta x'}{\Delta t}=\left(\frac{-1.50\text{km}}{5.00\text{min}}\right)\left(\frac{60\text{min}}{1h}\right)=-18.0\text{km/h}$$

Discussion

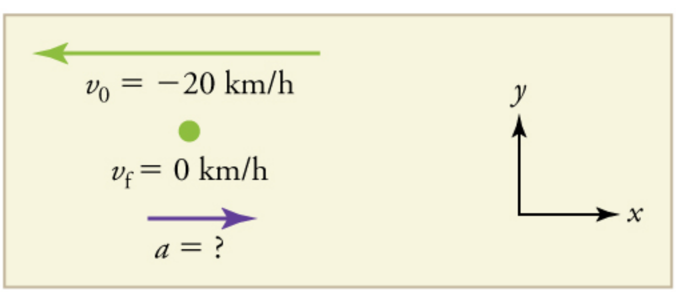
The negative velocity indicates motion to the left.

Calculating Deceleration: The Subway Train

Finally, suppose the train in Figure 11 slows to a stop from a velocity of 20.0 km/h in 10.0 s. What is its average acceleration?

Strategy

Once again, let's draw a sketch:



Vector and Acceleration Diagram

As before, we must find the change in velocity and the change in time to calculate average acceleration.

Solution

1. Identify the knowns. $v_0 = -20\text{km/h}$, $v_f = 0\text{km/h}$, $\Delta t = 10.0s$.

2. Calculate Δv. The change in velocity here is actually positive, since

$$\Delta v = v_f - v_0 = 0 - (-20 \text{ km/h}) = +20 \text{ km/h}.$$

3. Solve for $\bar{a}$.

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{+20.0 \text{ km/h}}{10.0 \text{ s}}$$

4. Convert units.

$$\bar{a} = \left(\frac{+20.0 \text{ km/h}}{10.0 \text{ s}}\right)\left(\frac{10^3 \text{ m}}{1 \text{ km}}\right)\left(\frac{1 \text{ h}}{3600 \text{ s}}\right) = +0.556 \text{ m/s}^2$$

Discussion

The plus sign means that acceleration is to the right. This is reasonable because the train initially has a negative velocity (to the left) in this problem and a positive acceleration opposes the motion (and so it is to the right). Again, acceleration is in the same direction as the *change* in velocity, which is positive here. As in Example 5, this acceleration can be called a deceleration since it is in the direction opposite to the velocity.

Sign and Direction

Perhaps the most important thing to note about these examples is the signs of the answers. In our chosen coordinate system, plus means the quantity is to the right and minus means it is to the left. This is easy to imagine for displacement and velocity. But it is a little less obvious for acceleration. Most people interpret negative acceleration as the slowing of an object. This was not the case in Example 7, where a positive acceleration slowed a negative velocity. The crucial distinction was that the acceleration was in the opposite direction from the velocity. In fact, a negative acceleration will *increase* a negative velocity. For example, the train moving to the left in Figure 11 is sped up by an acceleration to the left. In that case, both v and a are negative. The plus and minus signs give the directions of the accelerations. If acceleration has the same sign as the velocity, the object is speeding up. If acceleration has the opposite sign as the velocity, the object is slowing down.

Check Your Understanding

An airplane lands on a runway traveling east. Describe its acceleration.

Show Solution

If we take east to be positive, then the airplane has negative acceleration, as it is accelerating toward the west. It is also decelerating: its acceleration is opposite in direction to its velocity.

Moving Man Simulation

Learn about position, velocity, and acceleration graphs. Move the little man back and forth with the mouse and plot his motion. Set the position, velocity, or acceleration and let the simulation move the man for you.

Moving Man

Section Summary

- Acceleration is the rate at which velocity changes. In symbols, **average acceleration** $\bar{a}$ is $\bar{a}=\frac{\Delta v}{\Delta t}=\frac{v_f-v_0}{t_f-t_0}$.
- The SI unit for acceleration is m/s^2.
- Acceleration is a vector, and thus has a both a magnitude and direction.
- Acceleration can be caused by either a change in the magnitude or the direction of the velocity.
- Instantaneous acceleration a is the acceleration at a specific instant in time.
- Deceleration is an acceleration with a direction opposite to that of the velocity.

Conceptual Questions

Is it possible for speed to be constant while acceleration is not zero? Give an example of such a situation.

Is it possible for velocity to be constant while acceleration is not zero? Explain.

Give an example in which velocity is zero yet acceleration is not.

If a subway train is moving to the left (has a negative velocity) and then comes to a stop, what is the direction of its acceleration? Is the acceleration positive or negative?

Plus and minus signs are used in one-dimensional motion to indicate direction. What is the sign of an acceleration that reduces the magnitude of a negative velocity? Of a positive velocity?

Problems & Exercises

A cheetah can accelerate from rest to a speed of 30.0 m/s in 7.00 s. What is its acceleration?

Show Solution

Strategy

We are given initial velocity, final velocity, and time. We can use the definition of average acceleration to find the acceleration.

Solution

1. Identify the known values:
   - Initial velocity: $v_0=0$ (starts from rest)
   - Final velocity: $v_f=30.0\,\text{m/s}$
   - Time interval: $\Delta t=7.00\,\text{s}$
2. Apply the definition of average acceleration:

$$\bar{a}=\frac{\Delta v}{\Delta t}=\frac{v_f-v_0}{\Delta t}$$

1. Substitute the known values:

$$\bar{a}=\frac{30.0\,\text{m/s}-0}{7.00\,\text{s}}=\frac{30.0\,\text{m/s}}{7.00\,\text{s}}=4.29\,\text{m/s}^2$$

Discussion

The cheetah's acceleration of 4.29 m/s² is impressive—it increases its speed by 4.29 m/s every second. This is about 0.44 times the acceleration due to gravity (g = 9.80 m/s²).

The cheetah's acceleration is $4.29\,\text{m/s}^2$.

Professional Application

Dr. John Paul Stapp was U.S. Air Force officer who studied the effects of extreme deceleration on the human body. On December 10, 1954, Stapp rode a rocket sled, accelerating from rest to a top speed of 282 m/s (1015 km/h) in 5.00 s, and was brought jarringly back to rest in only 1.40 s! Calculate his (a) acceleration and (b) deceleration. Express each in multiples of g, $(9.80\,\text{m/s}^2)$ by taking its ratio to the acceleration of gravity.

Show Solution

Strategy

This problem involves two phases of motion: acceleration and deceleration. For each phase, we use the definition of average acceleration. To express the results in multiples of g, we divide each acceleration by 9.80 m/s².

Solution

(a) Acceleration phase:

1. Identify the known values:

   - Initial velocity: $v_0 = 0$ (starts from rest)
   - Final velocity: $v_f = 282 \text{m/s}$
   - Time interval: $\Delta t = 5.00 \text{s}$
2. Calculate the acceleration:

$$a = \frac{\Delta v}{\Delta t} = \frac{v_f - v_0}{\Delta t} = \frac{282 \text{m/s} - 0}{5.00 \text{s}} = 56.4 \text{m/s}^2$$

1. Express in multiples of g:

$$\frac{a}{g} = \frac{56.4 \text{m/s}^2}{9.80 \text{m/s}^2} = 5.76g$$

(b) Deceleration phase:

1. Identify the known values:
   - Initial velocity: $v_0 = 282 \text{m/s}$
   - Final velocity: $v_f = 0$ (comes to rest)
   - Time interval: $\Delta t = 1.40 \text{s}$
2. Calculate the deceleration:

$$a = \frac{\Delta v}{\Delta t} = \frac{v_f - v_0}{\Delta t} = \frac{0 - 282 \text{m/s}}{1.40 \text{s}} = -201 \text{m/s}^2$$

1. Express in multiples of g:

$$\frac{|a|}{g} = \frac{201 \text{m/s}^2}{9.80 \text{m/s}^2} = 20.5g$$

Discussion

These accelerations are extreme. During acceleration, Dr. Stapp experienced about 5.76g, which is uncomfortable but tolerable. During deceleration, he experienced about 20.5g, which is at the limit of human tolerance. Such experiments helped establish safety standards for ejection seats and crash protection.

(a) Dr. Stapp's acceleration was 56.4m/s^2, or $5.76g$.

(b) His deceleration was 201m/s^2, or $20.5g$.

A commuter backs her car out of her garage with an acceleration of 1.40m/s^2. (a) How long does it take her to reach a speed of 2.00 m/s? (b) If she then brakes to a stop in 0.800 s, what is her deceleration?

Show Solution

Strategy

For part (a), we know the acceleration and need to find time. We rearrange the acceleration equation to solve for time. For part (b), we use the definition of acceleration with the new initial velocity and final velocity (zero).

Solution

(a) Time to reach 2.00 m/s:

1. Identify the known values:
   - Initial velocity: $v_0 = 0$ (starts from rest)
   - Final velocity: $v_f = 2.00 \text{m/s}$
   - Acceleration: $a = 1.40 \text{m/s}^2$
2. Rearrange the acceleration equation to solve for time:

$$a = \frac{\Delta v}{\Delta t} \Rightarrow \Delta t = \frac{\Delta v}{a} = \frac{v_f - v_0}{a}$$

1. Substitute the known values:

$$\Delta t = \frac{2.00 \text{m/s} - 0}{1.40 \text{m/s}^2} = \frac{2.00 \text{m/s}}{1.40 \text{m/s}^2} = 1.43 \text{s}$$

(b) Deceleration while braking:

1. Identify the known values:
   - Initial velocity: $v_0 = 2.00 \text{m/s}$
   - Final velocity: $v_f = 0$ (comes to a stop)

- Time interval: $\Delta t = 0.800\text{s}$

2. Calculate the deceleration:

$$a=\frac{\Delta v}{\Delta t}=\frac{v_f-v_0}{\Delta t}=\frac{0-2.00\text{m/s}}{0.800\text{s}}=-2.50\text{m/s}^2$$

Discussion

The negative sign in part (b) indicates that the acceleration is in the opposite direction to the velocity, which is consistent with the car slowing down (decelerating).

(a) It takes the commuter 1.43s to reach a speed of 2.00 m/s.

(b) Her deceleration while braking is 2.50m/s^2 (or -2.50m/s^2 if we consider the sign).

Assume that an intercontinental ballistic missile goes from rest to a suborbital speed of 6.50 km/s in 60.0 s (the actual speed and time are classified). What is its average acceleration in m/s^2 and in multiples of g (9.80m/s^2)?

Show Solution

Strategy

We use the definition of average acceleration. First, we need to convert the final velocity from km/s to m/s to get the answer in SI units.

Solution

1. Identify the known values and convert units:
   - Initial velocity: $v_0 = 0$ (starts from rest)
   - Final velocity: $v_f = 6.50 \text{ km/s} = 6.50 \times 10^3 \text{m/s} = 6500\text{m/s}$
   - Time interval: $\Delta t = 60.0\text{s}$
2. Calculate the average acceleration:

$$\bar{a}=\frac{\Delta v}{\Delta t}=\frac{v_f-v_0}{\Delta t}=\frac{6500\text{m/s}-0}{60.0\text{s}}=108\text{m/s}^2$$

1. Express in multiples of g:

$$\frac{\bar{a}}{g}=\frac{108\text{m/s}^2}{9.80\text{m/s}^2}=11.0g$$

Discussion

This is an enormous acceleration—about 11 times the acceleration due to gravity. The missile's velocity increases by 108 m/s every second during the boost phase. Such high accelerations require robust engineering to protect the missile's components and payload.

The missile's average acceleration is 108m/s^2, which is equivalent to $11.0g$.

Glossary

acceleration
: the rate of change in velocity; the change in velocity over time

average acceleration
: the change in velocity divided by the time over which it changes

instantaneous acceleration
: acceleration at a specific point in time

deceleration
: acceleration in the direction opposite to velocity; acceleration that results in a decrease in velocity

Motion Equations for Constant Acceleration in One Dimension

- Calculate displacement of an object that is not accelerating, given initial position and velocity.
- Calculate final velocity of an accelerating object, given initial velocity, acceleration, and time.
- Calculate displacement and final position of an accelerating object, given initial position, initial velocity, time, and acceleration.

Kinematic equations can help us describe and predict the motion of moving objects such as these kayaks racing in Newbury, England. (credit: Barry Skeates, Flickr)

We might know that the greater the acceleration of, say, a car moving away from a stop sign, the greater the displacement in a given time. But we have not developed a specific equation that relates acceleration and displacement. In this section, we develop some convenient equations for kinematic relationships, starting from the definitions of displacement, velocity, and acceleration already covered.

Notation: *t*, *x*, *v*, *a*

First, let us make some simplifications in notation. Taking the initial time to be zero, as if time is measured with a stopwatch, is a great simplification. Since elapsed time is $\Delta t = t_f - t_0$, taking $t_0 = 0$ means that $\Delta t = t_f$, the final time on the stopwatch. When initial time is taken to be zero, we use the subscript 0 to denote initial values of position and velocity. That is, x_0 is the *initial position* and v_0 is the *initial velocity*. We put no subscripts on the final values. That is, t is the *final time*, x is the *final position*, and v is the *final velocity*. This gives a simpler expression for elapsed time—now, $\Delta t = t$. It also simplifies the expression for displacement, which is now $\Delta x = x - x_0$. Also, it simplifies the expression for change in velocity, which is now $\Delta v = v - v_0$. To summarize, using the simplified notation, with the initial time taken to be zero,

$$\Delta t = t \quad \Delta x = x - x_0 \quad \Delta v = v - v_0$$

where *the subscript 0 denotes an initial value and the absence of a subscript denotes a final value* in whatever motion is under consideration.

We now make the important assumption that *acceleration is constant*. This assumption allows us to avoid using calculus to find instantaneous acceleration. Since acceleration is constant, the average and instantaneous accelerations are equal. That is,

$$\bar{a} = a = \text{constant},$$

so we use the symbol a for acceleration at all times. Assuming acceleration to be constant does not seriously limit the situations we can study nor degrade the accuracy of our treatment. For one thing, acceleration *is* constant in a great number of situations. Furthermore, in many other situations we can accurately describe motion by assuming a constant acceleration equal to the average acceleration for that motion. Finally, in motions where acceleration changes drastically, such as a car accelerating to top speed and then braking to a stop, the motion can be considered in separate parts, each of which has its own constant acceleration.

Solving for Displacement (Δx) and Final Position (x) from Average Velocity when Acceleration (a) is Constant

To get our first two new equations, we start with the definition of average velocity:

$$\bar{v}=\frac{\Delta x}{\Delta t}.$$

Substituting the simplified notation for Δx and Δt yields

$$\bar{v}=\frac{x-x_0}{t}.$$

Solving for x yields

$$x=x_0+\bar{v}t,$$

where the average velocity is

$$\bar{v}=\frac{v_0+v}{2}\ (\text{constant}\ a).$$

The equation $\bar{v}=\frac{v_0+v}{2}$ reflects the fact that, when acceleration is constant, v is just the simple average of the initial and final velocities. For example, if you steadily increase your velocity (that is, with constant acceleration) from 30 to 60 km/h, then your average velocity during this steady increase is 45 km/h. Using the equation $\bar{v}=\frac{v_0+v}{2}$ to check this, we see that

$$\bar{v}=\frac{v_0+v}{2}=\frac{30\ \text{km/h}+60\ \text{km/h}}{2}=45\ \text{km/h},$$

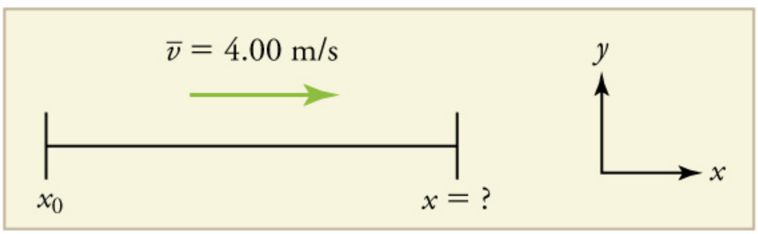
which seems logical.

Calculating Displacement: How Far does the Jogger Run?

A jogger runs down a straight stretch of road with an average velocity of 4.00 m/s for 2.00 min. What is his final position, taking his initial position to be zero?

Strategy

Draw a sketch.



The final position x is given by the equation

$$x=x_0+\bar{v}t.$$

To find x, we identify the values of x_0, $\bar{v}$, and t from the statement of the problem and substitute them into the equation.

Solution

1. Identify the knowns. $\bar{v}=4.00\ \text{m/s}$, $\Delta t=2.00\ \text{min}$, and $x_0=0\ \text{m}$.

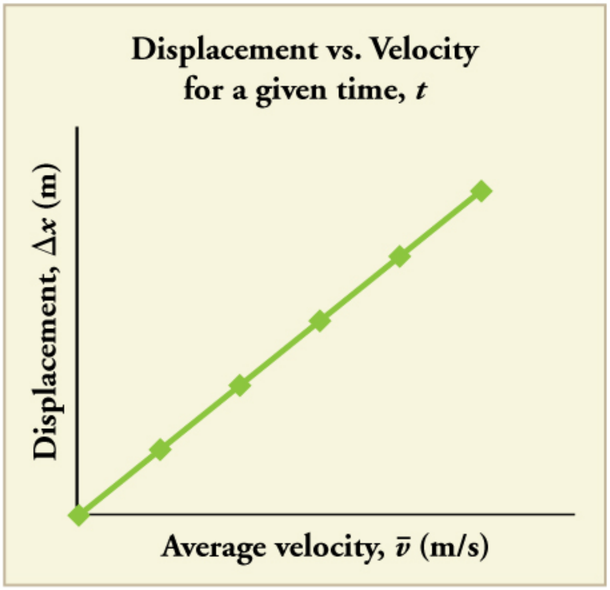
2. Enter the known values into the equation.

$$x=x_0+\bar{v}t=0+(4.00\ \text{m/s})(120\ \text{s})=480\ \text{m}$$

Discussion

Velocity and final displacement are both positive, which means they are in the same direction.

The equation $x=x_0+\bar{v}t$ gives insight into the relationship between displacement, average velocity, and time. It shows, for example, that displacement is a linear function of average velocity. (By linear function, we mean that displacement depends on $\bar{v}$ rather than on $\bar{v}$ raised to some other power, such as $\bar{v}^2$. When graphed, linear functions look like straight lines with a constant slope.) On a car trip, for example, we will get twice as far in a given time if we average 90 km/h than if we average 45 km/h.



There is a linear relationship between displacement and average velocity. For a given time (t) , an object moving twice as fast as another object will move twice as far as the other object.

Solving for Final Velocity

We can derive another useful equation by manipulating the definition of acceleration.

$a=\Delta v \Delta t$

Substituting the simplified notation for Δv and Δt gives us

$a=v-v_0 t(\text{constant}a)$.

Solving for v yields

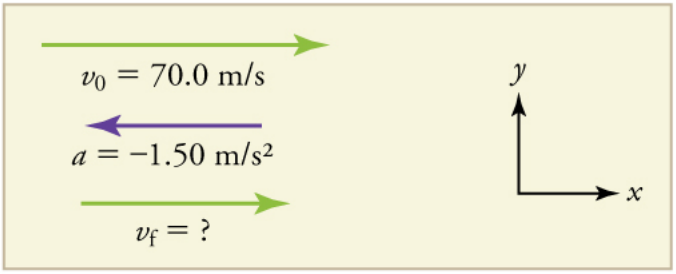
$v=v_0+at(\text{ constant } a)$.

Calculating Final Velocity: An Airplane Slowing Down after Landing

An airplane lands with an initial velocity of 70.0 m/s and then decelerates at 1.50m/s^2 for 40.0 s. What is its final velocity?

Strategy

Draw a sketch. We draw the acceleration vector in the direction opposite the velocity vector because the plane is decelerating.



Solution

1. Identify the knowns. $v\_0 = 70.0\text{m/s}$, $a = -1.50\text{m/s}^2$, $t = 40.0\text{s}$.

2. Identify the unknown. In this case, it is final velocity, $v f$.

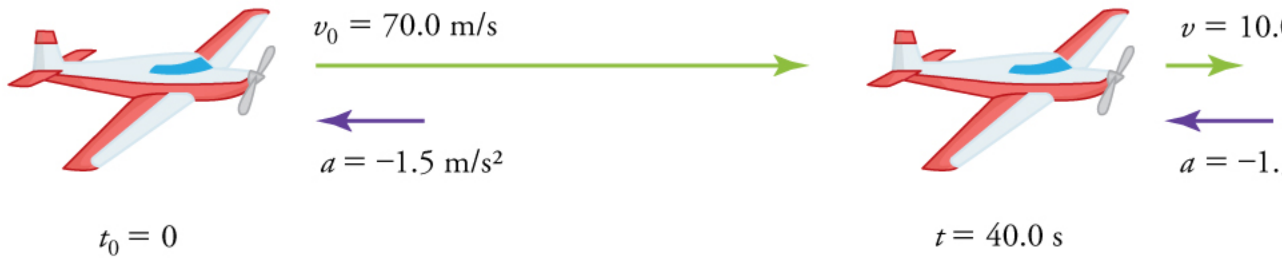
3. Determine which equation to use. We can calculate the final velocity using the equation $v = v_0 + at$.

4. Plug in the known values and solve.

$v=v_0+at=70.0\text{m/s}+(-1.50\text{m/s}^2)(40.0\text{s})=10.0\text{m/s}$

Discussion

The final velocity is much less than the initial velocity, as desired when slowing down, but still positive. With jet engines, reverse thrust could be maintained long enough to stop the plane and start moving it backward. That would be indicated by a negative final velocity, which is not the case here.



The airplane lands with an initial velocity of 70.0 m/s and slows to a final velocity of 10.0 m/s before heading for the terminal. Note that the acceleration is negative because its direction is opposite to its velocity, which is positive.

In addition to being useful in problem solving, the equation $v = v_0 + at$ gives us insight into the relationships among velocity, acceleration, and time. From it we can see, for example, that

- final velocity depends on how large the acceleration is and how long it lasts
- if the acceleration is zero, then the final velocity equals the initial velocity $(v = v_0)$, as expected (i.e., velocity is constant)
- if a is negative, then the final velocity is less than the initial velocity

(All of these observations fit our intuition, and it is always useful to examine basic equations in light of our intuition and experiences to check that they do indeed describe nature accurately.)

Making Connections: Real-World Connection

The Space Shuttle Endeavor blasts off from the Kennedy Space Center in February 2010. (credit: Matthew Simantov, Flickr)

An intercontinental ballistic missile (ICBM) has a larger average acceleration than the Space Shuttle and achieves a greater velocity in the first minute or two of flight (actual ICBM burn times are classified—short-burn-time missiles are more difficult for an enemy to destroy). But the Space Shuttle obtains a greater final velocity, so that it can orbit the earth rather than come directly back down as an ICBM does. The Space Shuttle does this by accelerating for a longer time.

Solving for Final Position When Velocity is Not Constant (i.e acceleration is not zero)

We can combine the equations above to find a third equation that allows us to calculate the final position of an object experiencing constant acceleration. We start with

$$v=v_0+at.$$

Adding v_0 to each side of this equation and dividing by 2 gives

$$\frac{v_0+v}{2}=v_0+\frac{1}{2}at.$$

Since $\frac{v_0+v}{2}=\bar{v}$ for constant acceleration, then

$$\bar{v}=v_0+\frac{1}{2}at.$$

Now we substitute this expression for $\bar{v}$ into the equation for displacement, $x=x_0+\bar{v}t$, yielding

$$x=x_0+v_0t+\frac{1}{2}at^2 \text{(constant} a\text{)}.$$

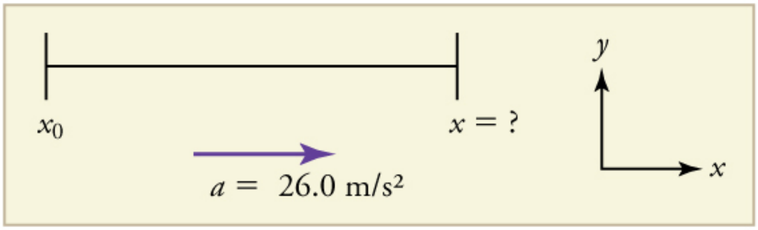
Calculating Displacement of an Accelerating Object: Dragsters

Dragsters can achieve average accelerations of 26.0m/s^2. Suppose such a dragster accelerates from rest at this rate for 5.56 s. How far does it travel in this time?

U.S. Army Top Fuel pilot Tony “The Sarge” Schumacher begins a race with a controlled burnout. (credit: Lt. Col. William Thurmond. Photo Courtesy of U.S. Army.)

Strategy

Draw a sketch.



We are asked to find displacement, which is x if we take x_0 to be zero. (Think about it like the starting line of a race. It can be anywhere, but we call it 0 and measure all other positions relative to it.) We can use the equation $x = x_0 + v_0 t + 12at^2$ once we identify v_0, a, and t from the statement of the problem.

Solution

1. Identify the knowns. Starting from rest means that $v_0 = 0$, a is given as 26.0m/s^2 and t is given as 5.56 s.

2. Plug the known values into the equation to solve for the unknown x:

$$x = x_0 + v_0 t + 12at^2.$$

Since the initial position and velocity are both zero, this simplifies to

$$x = 12at^2.$$

Substituting the identified values of a and t gives

$$x = 12(26.0\text{m/s}^2)(5.56\text{s})^2,$$

yielding

$$x = 402\text{m}.$$

Discussion

If we convert 402 m to miles, we find that the distance covered is very close to one quarter of a mile, the standard distance for drag racing. So the answer is reasonable. This is an impressive displacement in only 5.56 s, but top-notch dragsters can do a quarter mile in even less time than this.

What else can we learn by examining the equation $x = x_0 + v_0 t + 12at^2$? We see that:

- displacement depends on the square of the elapsed time when acceleration is not zero. In Example 3, the dragster covers only one fourth of the total distance in the first half of the elapsed time
- if acceleration is zero, then the initial velocity equals average velocity ($v_0 = \bar{v}$) and $x = x_0 + v_0 t + 12at^2$ becomes $x = x_0 + v_0 t$

Solving for Final Velocity when Velocity Is Not Constant (acceleration is non-zero)

A fourth useful equation can be obtained from another algebraic manipulation of previous equations.

If we solve $v = v_0 + at$ for t, we get

$$t = v - v_0 a.$$

Substituting this and $\bar{v} = v_0 + v2$ into $x = x_0 + \bar{v}t$, we get

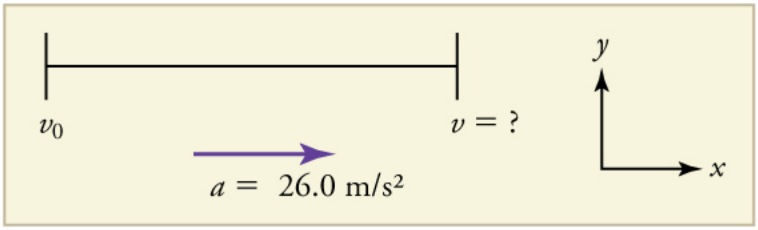
$$v^2 = v_{20} + 2a(x - x_0)(\text{ constant } a).$$

Calculating Final Velocity: Dragsters

Calculate the final velocity of the dragster in Example 3 without using information about time.

Strategy

Draw a sketch.



The equation $v^2 = v_0^2 + 2a(x - x_0)$ is ideally suited to this task because it relates velocities, acceleration, and displacement, and no time information is required.

Solution

1. Identify the known values. We know that $v_0 = 0$, since the dragster starts from rest. Then we note that $x - x_0 = 402\text{m}$ (this was the answer in Example 3). Finally, the average acceleration was given to be $a = 26.0\text{m/s}^2$.

2. Plug the knowns into the equation $v^2 = v_0^2 + 2a(x - x_0)$ and solve for v.

$$v^2 = 0 + 2(26.0\text{m/s}^2)(402\text{m}).$$

Thus

$$v^2 = 2.09\times10^4\text{m}^2/\text{s}^2.$$

To get v, we take the square root:

$$v = \sqrt{2.09\times10^4\text{m}^2/\text{s}^2} = 145\text{m/s}.$$

Discussion

145 m/s is about 522 km/h or about 324 mi/h, but even this breakneck speed is short of the record for the quarter mile. Also, note that a square root has two values; we took the positive value to indicate a velocity in the same direction as the acceleration.

An examination of the equation $v^2 = v_0^2 + 2a(x - x_0)$ can produce further insights into the general relationships among physical quantities:

- The final velocity depends on how large the acceleration is and the distance over which it acts
- For a fixed deceleration, a car that is going twice as fast doesn't simply stop in twice the distance—it takes much further to stop. (This is why we have reduced speed zones near schools.)

Putting Equations Together

In the following examples, we further explore one-dimensional motion, but in situations requiring slightly more algebraic manipulation. The examples also give insight into problem-solving techniques. The box below provides easy reference to the equations needed.

Summary of Kinematic Equations (constant acceleration)

$$x = x_0 + \bar{v}t$$

$$\bar{v} = \frac{v_0 + v}{2}$$

$$v = v_0 + at$$

$$x = x_0 + v_0t + \frac{1}{2}at^2$$

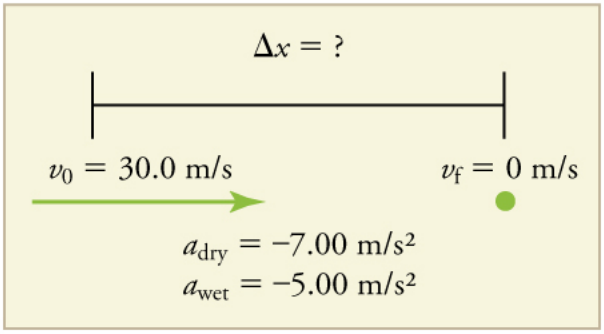
$$v^2 = v_0^2 + 2a(x - x_0)$$

Calculating Displacement: How Far Does a Car Go When Coming to a Halt?

On dry concrete, a car can decelerate at a rate of 7.00m/s^2, whereas on wet concrete it can decelerate at only 5.00m/s^2. Find the distances necessary to stop a car moving at 30.0 m/s (about 110 km/h) (a) on dry concrete and (b) on wet concrete. (c) Repeat both calculations, finding the displacement from the point where the driver sees a traffic light turn red, taking into account his reaction time of 0.500 s to get his foot on the brake.

Strategy

Draw a sketch.



In order to determine which equations are best to use, we need to list all of the known values and identify exactly what we need to solve for. We shall do this explicitly in the next several examples, using tables to set them off.

Solution for (a)

1. Identify the knowns and what we want to solve for. We know that $v_0 = 30.0 \text{m/s}$; $v = 0$; $a = -7.00 \text{m/s}^2$ (a is negative because it is in a direction opposite to velocity). We take x_0 to be 0. We are looking for displacement Δx, or $x - x_0$.

2. Identify the equation that will help up solve the problem. The best equation to use is

$$v^2 = v_{20} + 2a(x - x_0).$$

This equation is best because it includes only one unknown, x. We know the values of all the other variables in this equation. (There are other equations that would allow us to solve for x, but they require us to know the stopping time, t, which we do not know. We could use them but it would entail additional calculations.)

3. Rearrange the equation to solve for x.

$$x - x_0 = v^2 - v_{20} 2a$$

4. Enter known values.

$$x - 0 = 0^2 - (30.0 \text{m/s})^2 2(-7.00 \text{m/s}^2)$$

Thus,

$$x = 64.3 \text{m on dry concrete.}$$

Solution for (b)

This part can be solved in exactly the same manner as Part A. The only difference is that the deceleration is -5.00m/s^2. The result is

$x_{\text{wet}} = 90.0 \text{m}$ on wet concrete.

Solution for (c)

Once the driver reacts, the stopping distance is the same as it is in Parts A and B for dry and wet concrete. So to answer this question, we need to calculate how far the car travels during the reaction time, and then add that to the stopping time. It is reasonable to assume that the velocity remains constant during the driver's reaction time.

1. Identify the knowns and what we want to solve for. We know that

\overline{v}=30.0 \ms ; t_{\text{reaction}}=0.500\s ; a_ {\text{reaction}}=0 .Wetake x_{0-\text{reaction}} tobe0.Wearelookingfor x_{\text{reaction}} $$.

2. Identify the best equation to use. $x = x_0 + \bar{v}t$ works well because the only unknown value is x, which is what we want to solve for.

3. Plug in the knowns to solve the equation.

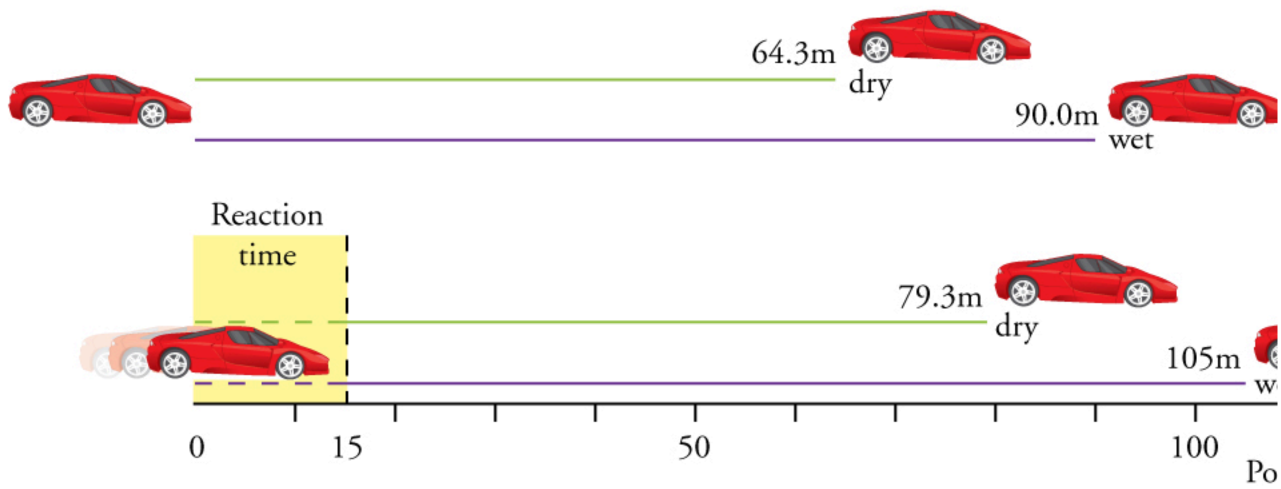
$$x = 0 + (30.0 \text{m/s})(0.500 \text{s}) = 15.0 \text{m}.$$

This means the car travels 15.0 m while the driver reacts, making the total displacements in the two cases of dry and wet concrete 15.0 m greater than if he reacted instantly.

4. Add the displacement during the reaction time to the displacement when braking.

$x_{braking} + x_{reaction} = x_{total}$

(a) 64.3 m + 15.0 m = 79.3 m when dry (b) 90.0 m + 15.0 m = 105 m when wet



The distance necessary to stop a car varies greatly, depending on road conditions and driver reaction time. Shown here are the braking distances for dry and wet pavement, as calculated in this example, for a car initially traveling at 30.0 m/s. Also shown are the total distances traveled from the point where the driver first sees a light turn red, assuming a 0.500 s reaction time.

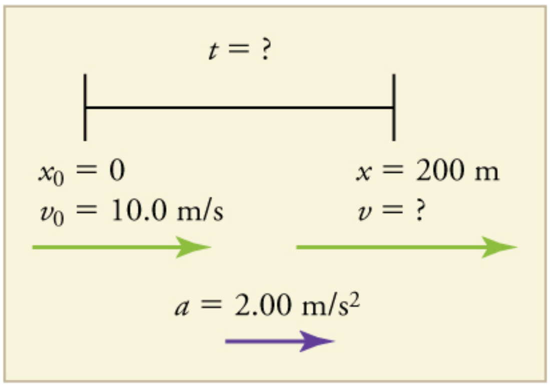
Discussion

The displacements found in this example seem reasonable for stopping a fast-moving car. It should take longer to stop a car on wet rather than dry pavement. It is interesting that reaction time adds significantly to the displacements. But more important is the general approach to solving problems. We identify the knowns and the quantities to be determined and then find an appropriate equation. There is often more than one way to solve a problem. The various parts of this example can in fact be solved by other methods, but the solutions presented above are the shortest.
Calculating Time: A Car Merges into Traffic

Suppose a car merges into freeway traffic on a 200-m-long ramp. If its initial velocity is 10.0 m/s and it accelerates at $2.00 m/s^2$, how long does it take to travel the 200 m up the ramp? (Such information might be useful to a traffic engineer.)

Strategy

Draw a sketch.



We are asked to solve for the time t. As before, we identify the known quantities in order to choose a convenient physical relationship (that is, an equation with one unknown, t).

Solution

1. Identify the knowns and what we want to solve for. We know that $v\_0 = 10 m/s$; $a = 2.00 m/s^2$; and $x = 200 m$.

2. We need to solve for t. Choose the best equation. $x = x_0 + v_0 t + 12 a t^2$ works best because the only unknown in the equation is the variable t for which we need to solve.

3. We will need to rearrange the equation to solve for t. In this case, it will be easier to plug in the knowns first.

$200\text{m}=0\text{m}+(10.0\text{m/s})t+\frac{1}{2}(2.00\text{m/s}^2)t^2$

4. Simplify the equation. The units of meters (m) cancel because they are in each term. We can get the units of seconds (s) to cancel by taking $t=t\,\text{s}$, where t is the magnitude of time and s is the unit. Doing so leaves

$200=10t+t^2.$

5. Use the quadratic formula to solve for t. (a) Rearrange the equation to get 0 on one side of the equation.

$t^2+10t-200=0$

This is a quadratic equation of the form

$at^2+bt+c=0,$

where the constants are $a=1.00$, $b=10.0$, and $c=-200$.

(b) Its solutions are given by the quadratic formula:

$t=\frac{-b\pm\sqrt{b^2-4ac}}{2a}.$

This yields two solutions for t, which are

$t=10.0 \text{ and } -20.0.$

In this case, then, the time is $t=t$ in seconds, or

$t=10.0\text{s and } -20.0\text{s}.$

A negative value for time is unreasonable, since it would mean that the event happened 20s before the motion began. We can discard that solution. Thus,

$t=10.0\text{s}.$

Discussion

Whenever an equation contains an unknown squared, there will be two solutions. In some problems both solutions are meaningful, but in others, such as the above, only one solution is reasonable. The 10.0 s answer seems reasonable for a typical freeway on-ramp.

With the basics of kinematics established, we can go on to many other interesting examples and applications. In the process of developing kinematics, we have also glimpsed a general approach to problem solving that produces both correct answers and insights into physical relationships. Problem-Solving Basics discusses problem-solving basics and outlines an approach that will help you succeed in this invaluable task.

Making Connections: Take-Home Experiment—Breaking News

We have been using SI units of meters per second squared to describe some examples of acceleration or deceleration of cars, runners, and trains. To achieve a better feel for these numbers, one can measure the braking deceleration of a car doing a slow (and safe) stop. Recall that, for average acceleration, $\bar{a}=\Delta v/\Delta t$. While traveling in a car, slowly apply the brakes as you come up to a stop sign. Have a passenger note the initial speed in miles per hour and the time taken (in seconds) to stop. From this, calculate the deceleration in miles per hour per second. Convert this to meters per second squared and compare with other decelerations mentioned in this chapter. Calculate the distance traveled in braking.

Check Your Understanding

A manned rocket accelerates at a rate of 20m/s^2 during launch. How long does it take the rocket to reach a velocity of 400 m/s?

Show Solution

To answer this, choose an equation that allows you to solve for time t, given only a, v_0, and v.

$v=v_0+at$

Rearrange to solve for t.

$t=\frac{v-v_0}{a}=\frac{400\text{m/s}-0\text{m/s}}{20\text{m/s}^2}=20\text{s}$

Section Summary

- To simplify calculations we take acceleration to be constant, so that $\bar{a}=a$ at all times.
- We also take initial time to be zero.
- Initial position and velocity are given a subscript 0; final values have no subscript. Thus,

$\Delta t = t \quad \Delta x = x - x_0 \quad \Delta v = v - v_0$

- The following kinematic equations for motion with constant a are useful:
  $x = x_0 + \bar{v} t$
  $\bar{v} = \frac{v_0 + v}{2}$
  $v = v_0 + at$
  $x = x_0 + v_0 t + \frac{1}{2} a t^2$
  $v^2 = v_0^2 + 2a(x - x_0)$
- In vertical motion, y is substituted for x.

Problems & Exercises

An Olympic-class sprinter starts a race with an acceleration of $4.50\,\text{m/s}^2$. (a) What is her speed 2.40 s later? (b) Sketch a graph of her position vs. time for this period.

Show Solution

Strategy

The sprinter starts from rest and accelerates at a constant rate. For part (a), we use the kinematic equation $v = v_0 + at$ to find the velocity after 2.40 s. For part (b), we sketch a position-time graph, which should be parabolic for constant acceleration starting from rest.

Solution

(a) Speed after 2.40 s:

1. Identify the known values:
   - Initial velocity: $v_0 = 0$ (starts from rest)
   - Acceleration: $a = 4.50\,\text{m/s}^2$
   - Time: $t = 2.40\,\text{s}$
2. Use the kinematic equation:

$$v = v_0 + at$$

1. Substitute the known values:

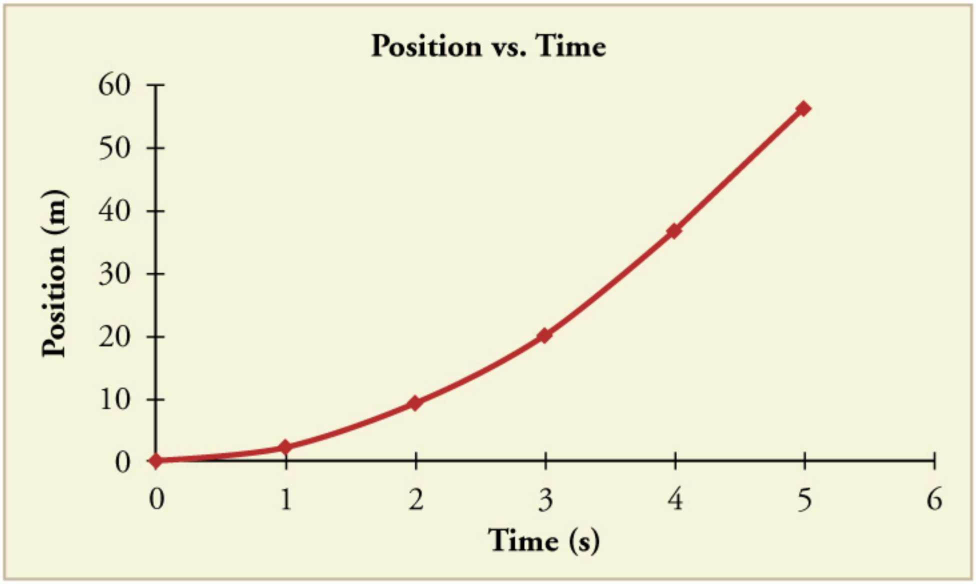
$$v = 0 + (4.50)(2.40) = 10.8\,\text{m/s}$$

(b) Position-time graph:

The position as a function of time for constant acceleration starting from rest is given by:

$$x = \frac{1}{2} a t^2 = \frac{1}{2}(4.50) t^2 = 2.25 t^2$$

This is a parabolic function, as shown in the graph below. The position increases quadratically with time, and the slope of the graph (which represents velocity) increases linearly with time.



Discussion

The sprinter reaches a speed of 10.8 m/s (about 38.9 km/h or 24.2 mph) in just 2.40 seconds with an acceleration of 4.50m/s^2, which is nearly half of gravitational acceleration. This is realistic for an Olympic-class athlete. The parabolic shape of the position-time graph is characteristic of constant acceleration - the curve gets steeper as time progresses because the velocity increases linearly. The slope at $t = 2.40 \text{s}$ corresponds to the final velocity of 10.8 m/s.

Answer

(a) The sprinter's speed 2.40 s later is 10.8 m/s.

(b) The position-time graph is parabolic, starting at the origin with increasing slope, as shown in Figure 13.

A well-thrown ball is caught in a well-padded mitt. If the deceleration of the ball is $2.10 \times 10^4 \text{m/s}^2$, and 1.85 ms $(1 \text{ms} = 10^{-3} \text{s})$ elapses from the time the ball first touches the mitt until it stops, what was the initial velocity of the ball?

Show Solution

Strategy

The ball decelerates from its initial velocity to rest (final velocity = 0) in a very short time. We use the kinematic equation $v = v_0 + at$ to find the initial velocity. The deceleration (negative acceleration) and time are given.

Solution

1. Identify the known values:
   - Final velocity: $v = 0$ (ball stops)
   - Acceleration: $a = -2.10 \times 10^4 \text{m/s}^2$ (negative because it's decelerating)
   - Time: $t = 1.85 \times 10^{-3} \text{s}$ (1.85 ms)
2. Use the kinematic equation:

$$v = v_0 + at$$

1. Solve for initial velocity v_0:

$$0 = v_0 + (-2.10 \times 10^4)(1.85 \times 10^{-3})$$
$$v_0 = 2.10 \times 10^4 \times 1.85 \times 10^{-3}$$
$$v_0 = 38.85 \text{m/s} \approx 38.9 \text{m/s}$$

1. Converting to other units for context:

$$v_0 = 38.9 \text{m/s} \times \frac{3600 \text{s}}{1 \text{ hr}} \times \frac{1 \text{ km}}{1000 \text{m}} \times \frac{1 \text{ mile}}{1.609 \text{ km}} \approx 87 \text{ mph}$$

Discussion

The initial velocity of 38.9 m/s (about 87 mph) is consistent with a well-thrown baseball. The enormous deceleration of $2.10 \times 10^4 \text{m/s}^2$ (over 2,000 times gravitational acceleration) occurs because the ball stops in such a short time - only 1.85 milliseconds. Despite this huge deceleration, the well-padded mitt protects the catcher's hand by spreading the force over a larger area and slightly extending the stopping time. Without the padding, the contact time would be even shorter and the deceleration even greater, potentially causing injury. This problem illustrates why proper protective equipment is essential in sports.

Answer

The initial velocity of the ball was 38.9 m/s (approximately 87 miles per hour).

A bullet in a gun is accelerated from the firing chamber to the end of the barrel at an average rate of $6.20 \times 10^5 \text{m/s}^2$ for $8.10 \times 10^{-4} \text{s}$. What is its muzzle velocity (that is, its final velocity)?

Show Solution

Strategy

Use the kinematic equation relating final velocity, initial velocity, acceleration, and time. The bullet starts from rest in the firing chamber.

Solution

1. Identify the known values:
   - Initial velocity: $v_0 = 0$ (starts from rest)
   - Acceleration: $a = 6.20 \times 10^5 \text{m/s}^2$
   - Time: $t = 8.10 \times 10^{-4} \text{s}$
2. Apply the kinematic equation:

$$v = v_0 + at = 0 + (6.20 \times 10^5 \text{m/s}^2)(8.10 \times 10^{-4} \text{s})$$
$$v = 502 \text{m/s}$$

Discussion

This is about 1810 km/h or 1120 mph, which is supersonic (faster than the speed of sound, about 340 m/s). This is a typical muzzle velocity for a rifle bullet.

The bullet's muzzle velocity is 502m/s.

(a) A light-rail commuter train accelerates at a rate of 1.35m/s^2. How long does it take to reach its top speed of 80.0 km/h, starting from rest? (b) The same train ordinarily decelerates at a rate of 1.65m/s^2. How long does it take to come to a stop from its top speed? (c) In emergencies the train can decelerate more rapidly, coming to rest from 80.0 km/h in 8.30 s. What is its emergency deceleration in m/s^2 ?

Show Solution

Strategy

This problem involves three different scenarios of acceleration and deceleration for a train. For parts (a) and (b), we know the initial and final velocities and the acceleration, so we can use $v = v_0 + at$ to find time. For part (c), we know the velocities and time, so we can use the same equation to solve for acceleration. Remember to convert km/h to m/s first.

Solution

(a) Time to reach top speed:

1. Convert top speed to m/s:

$$v = 80.0 \text{ km/h} = 80.0 \times \frac{1000 \text{m}}{3600 \text{s}} = 22.2 \text{m/s}$$

1. Identify the known values:
   - Initial velocity: $v_0 = 0$ (starting from rest)
   - Final velocity: $v = 22.2 \text{m/s}$
   - Acceleration: $a = 1.35 \text{m/s}^2$
2. Use the kinematic equation:

$$v = v_0 + at$$

1. Solve for time:

$$t = \frac{v - v_0}{a} = \frac{22.2 - 0}{1.35} = 16.4 \text{s} \approx 16.5 \text{s}$$

(b) Time to come to a stop:

1. Identify the known values:
    - Initial velocity: $v_0 = 22.2 \text{m/s}$ (top speed)
    - Final velocity: $v = 0$ (comes to rest)
    - Deceleration: $a = -1.65 \text{m/s}^2$ (negative for deceleration)
2. Use the same equation:

$$t = \frac{v - v_0}{a} = \frac{0 - 22.2}{-1.65} = \frac{-22.2}{-1.65} = 13.5 \text{s}$$

(c) Emergency deceleration:

1. Identify the known values:
    - Initial velocity: $v_0 = 22.2 \text{m/s}$
    - Final velocity: $v = 0$
    - Time: $t = 8.30 \text{s}$
2. Solve for acceleration:

$$a = \frac{v - v_0}{t} = \frac{0 - 22.2}{8.30} = -2.67 \text{m/s}^2 \approx -2.68 \text{m/s}^2$$

The negative sign indicates deceleration.

Discussion

The train takes about 16.5 seconds to accelerate to its top speed and 13.5 seconds to decelerate normally - the deceleration is faster because the braking rate (1.65m/s^2) is greater than the acceleration rate (1.35m/s^2). In emergency situations, the train can decelerate even more rapidly at 2.68m/s^2, which is about 62% faster than normal braking. This emergency deceleration stops the train in only 8.30 seconds compared to 13.5 seconds for normal braking - significantly reducing stopping distance in critical situations. These accelerations are all relatively gentle (well under 1g), ensuring passenger comfort and safety.

Answer

(a) It takes 16.5 s to reach top speed from rest.

(b) It takes 13.5 s to come to a stop from top speed using normal deceleration.

(c) The emergency deceleration is -2.68m/s^2.

While entering a freeway, a car accelerates from rest at a rate of 2.40m/s^2 for 12.0 s. (a) Draw a sketch of the situation. (b) List the knowns in this problem. (c) How far does the car travel in those 12.0 s? To solve this part, first identify the unknown, and then discuss how you chose the appropriate equation to solve for it. After choosing the equation, show your steps in solving for the unknown, check your units, and discuss whether the answer is reasonable. (d) What is the car’s final velocity? Solve for this unknown in the same manner as in part (c), showing all steps explicitly.

Show Solution

Strategy

This problem involves constant acceleration from rest. We need to use kinematic equations to find both displacement and final velocity. Since we’re given acceleration and time, we can use the equations that relate these quantities to displacement and velocity.

Solution

(a) Sketch:

A car starts from rest (initial position $x_0 = 0$, initial velocity $v_0 = 0$) and accelerates in the positive direction with acceleration vector $a = 2.40 \text{m/s}^2$ for time $t = 12.0 \text{s}$.

(b) Knowns:

- Initial velocity: $v_0 = 0 \text{m/s}$ (starts from rest)
- Acceleration: $a = 2.40 \text{m/s}^2$
- Time: $t = 12.0 \text{s}$
- Initial position: $x_0 = 0 \text{m}$ (taking starting point as origin)

(c) Distance traveled:

Unknown: Displacement $x - x_0$ or final position x

Choosing the equation: We have v_0, a, and t, and we need to find x. The most appropriate equation is:

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$

This equation is ideal because it relates displacement to acceleration and time without requiring knowledge of final velocity.

Solving:

Since $x_0 = 0$ and $v_0 = 0$, the equation simplifies to:

$$x = \frac{1}{2}at^2$$

Substituting the known values:

$$x = \frac{1}{2}(2.40 \text{m/s}^2)(12.0 \text{s})^2$$
$$x = \frac{1}{2}(2.40)(144) \text{m} = (1.20)(144) \text{m} = 173 \text{m}$$

Units check: $\text{m/s}^2 \times \text{s}^2 = \text{m}$ ✓

Reasonableness: A distance of 173 meters (about 568 feet or 0.11 miles) seems reasonable for a car accelerating onto a freeway over 12 seconds. This is roughly the length of a typical freeway on-ramp.

(d) Final velocity:

Unknown: Final velocity v

Choosing the equation: We have v_0, a, and t, and we need to find v. The appropriate equation is:

$$v = v_0 + at$$

This equation directly relates final velocity to initial velocity, acceleration, and time.

Solving:

Since $v_0 = 0$:

$$v = at = (2.40 \text{m/s}^2)(12.0 \text{s})$$
$$v = 28.8 \text{m/s}$$

Units check: $\text{m/s}^2 \times \text{s} = \text{m/s}$ ✓

Reasonableness: Converting to more familiar units: $28.8 \text{m/s} \times \frac{3.6 \text{ km/h}}{1 \text{m/s}} = 104 \text{ km/h}$ or about 64 mph. This is a reasonable speed for merging onto a freeway.

Discussion

The results are physically reasonable. The car travels 173 m while accelerating to 28.8 m/s (about 104 km/h). We can verify our answers are consistent by using an alternative approach: using the equation $v^2 = v_0^2 + 2a(x - x_0)$:

$$v^2 = 0 + 2(2.40 \text{m/s}^2)(173 \text{m}) = 830 \text{m}^2/\text{s}^2$$
$$v = \sqrt{830} \text{m/s} = 28.8 \text{m/s}$$ ✓

This confirms our answer. The acceleration of 2.40 m/s² (about 0.24g) is typical for a family car, and the final speed is appropriate for freeway traffic.

Answer

(c) The car travels **173 m** in 12.0 seconds.

(d) The car’s final velocity is **28.8 m/s** (approximately 104 km/h or 64 mph).

At the end of a race, a runner decelerates from a velocity of 9.00 m/s at a rate of 2.00m/s^2. (a) How far does she travel in the next 5.00 s? (b) What is her final velocity? (c) Evaluate the result. Does it make sense?

Show Solution

Strategy

The runner decelerates at a constant rate. For part (a), we use the kinematic equation that relates displacement, initial velocity, time, and acceleration. For part (b), we use $v = v_0 + at$ to find her velocity after 5.00 s. For part (c), we evaluate whether the results are physically realistic by checking when the runner would actually stop.

Solution

(a) Distance traveled in 5.00 s:

1. Identify the known values:
    - Initial velocity: $v_0 = 9.00 \text{m/s}$
    - Acceleration: $a = -2.00 \text{m/s}^2$ (negative for deceleration)
    - Time: $t = 5.00 \text{s}$
2. Use the kinematic equation:

$$x = v_0 t + 12 a t^2$$

1. Substitute the known values:

$$x = (9.00)(5.00) + 12(-2.00)(5.00)^2$$
$$x = 45.0 + 12(-2.00)(25.0) = 45.0 - 25.0 = 20.0 \text{m}$$

(b) Final velocity after 5.00 s:

1. Use the kinematic equation:

$$v = v_0 + at$$

1. Substitute the known values:

$$v = 9.00 + (-2.00)(5.00) = 9.00 - 10.0 = -1.00 \text{m/s}$$

(c) Evaluation of the result:

The negative velocity in part (b) indicates a problem. Let's check when the runner actually stops:

$$t_{\text{stop}} = v - v_0 a = 0 - 9.00 - 2.00 = 4.50 \text{s}$$

The runner stops after 4.50 s, not 5.00 s. At 5.00 s, the calculation gives a velocity of -1.00 m/s, which would mean running backwards!

Discussion

This problem illustrates an important limitation of kinematic equations: they calculate mathematical results without considering physical constraints. In reality, the runner cannot continue decelerating past the point where she stops. The runner decelerates for 4.50 s and comes to rest after traveling:

$$x_{\text{actual}} = v_0 t_{\text{stop}} + 12 a t_{2\text{stop}} = (9.00)(4.50) + 12(-2.00)(4.50)^2 = 40.5 - 20.25 = 20.25 \text{m}$$

After stopping at $t = 4.50 \text{s}$, she remains at rest. So while the equation predicts $x = 20.0 \text{m}$ and $v = -1.00 \text{m/s}$, the physical answer is that she travels about 20.25 m and stops, remaining at rest thereafter. The negative velocity is unphysical in this context - a runner doesn't spontaneously start running backwards after stopping!

Answer

(a) According to the kinematic equation, the runner travels 20.0 m in 5.00 s, but this requires careful interpretation.

(b) The calculated final velocity is -1.00 m/s.

(c) This result does not make physical sense. The runner actually stops after 4.50 s, having traveled about 20.25 m. She cannot continue decelerating to achieve a negative velocity (running backwards). This demonstrates that kinematic equations must be applied with consideration of physical constraints.

Professional Application:

Blood is accelerated from rest to 30.0 cm/s in a distance of 1.80 cm by the left ventricle of the heart. (a) Make a sketch of the situation. (b) List the knowns in this problem. (c) How long does the acceleration take? To solve this part, first identify the unknown, and then discuss how you chose the appropriate equation to solve for it. After choosing the equation, show your steps in solving for the unknown, checking your units. (d) Is the answer reasonable when compared with the time for a heartbeat?

Show Solution

Strategy

Blood is accelerated over a known distance from rest to a known final velocity. To find the time, we can use the equation relating average velocity to displacement and time, since average velocity can be calculated from initial and final velocities.

Solution

(a) Sketch:

Blood starts at rest ($v_0 = 0$) in the left ventricle and is accelerated in the positive direction to a final velocity $v = 30.0$ cm/s over a distance of $x - x_0 = 1.80$ cm.

(b) Knowns:

- Initial velocity: $v_0 = 0$ (blood starts from rest)
- Final velocity: $v = 30.0\text{ cm/s} = 0.300\text{m/s}$
- Displacement: $x - x_0 = 1.80\text{ cm} = 0.0180\text{m}$
- Initial position: $x_0 = 0\text{m}$ (taking starting point as origin)

(c) Time of acceleration:

Unknown: Time t

Choosing the equation: We know v_0, v, and $x - x_0$, and we need to find t. We can use the equation for displacement with average velocity:

$$x = x_0 + \bar{v}t$$

where the average velocity for constant acceleration is:

$$\bar{v} = \frac{v_0 + v}{2}$$

This approach is best because we can calculate average velocity directly from the known initial and final velocities, then solve for time. Alternatively, we could use $v^2 = v_0^2 + 2a(x - x_0)$ to find acceleration first, then use $v = v_0 + at$, but that requires an extra step.

Solving:

First, calculate the average velocity:

$$\bar{v} = \frac{v_0 + v}{2} = \frac{0 + 0.300\text{m/s}}{2} = 0.150\text{m/s}$$

Now use the displacement equation and solve for time:

$$x - x_0 = \bar{v}t$$
$$t = \frac{x - x_0}{\bar{v}} = \frac{0.0180\text{m}}{0.150\text{m/s}} = 0.120\text{s}$$

Units check: $\frac{\text{m}}{\text{m/s}} = \text{s}$ ✓

(d) Reasonableness:

The acceleration takes 0.120 s, which is 120 milliseconds. A typical resting heart rate is about 60-100 beats per minute, which corresponds to a heartbeat duration of 0.6 to 1.0 seconds. The systolic phase (when the ventricle contracts and ejects blood) typically lasts about 0.3 seconds.

Our answer of 0.120 s is quite reasonable—it's a significant fraction of the systolic phase but short enough to allow for the complete cardiac cycle. The heart must accelerate blood very quickly to pump it efficiently through the circulatory system.

Discussion

We can verify this answer by calculating the acceleration and checking consistency. Using $v^2 = v_0^2 + 2a(x - x_0)$:

$$a = \frac{v^2 - v_0^2}{2(x - x_0)} = \frac{(0.300\text{m/s})^2 - 0}{2(0.0180\text{m})} = \frac{0.0900\text{m}^2/\text{s}^2}{0.0360\text{m}} = 2.50\text{m/s}^2$$

Now checking with $v = v_0 + at$:

$$t = \frac{v - v_0}{a} = \frac{0.300\text{m/s}}{2.50\text{m/s}^2} = 0.120\text{s} \checkmark$$

This confirms our answer. The acceleration of 2.50 m/s² (about 0.25g) may seem small, but remember that blood has significant mass and the heart must pump it continuously throughout a lifetime—truly a remarkable organ!

Answer

(c) The acceleration takes **0.120 s** (120 milliseconds).

(d) Yes, this is reasonable. It represents a significant portion of the systolic phase of the heartbeat, which typically lasts about 0.3 seconds.

In a slap shot, a hockey player accelerates the puck from a velocity of 8.00 m/s to 40.0 m/s in the same direction. If this shot takes 3.33×10^{-2} s, calculate the distance over which the puck accelerates.

Show Solution

Strategy

The puck accelerates from one velocity to another in a known time. We need to find the distance over which this acceleration occurs. We can use the equation relating distance, initial velocity, final velocity, and time, which is $x = \frac{v_0+v}{2}t$ (average velocity times time), or we can first find acceleration and then use another equation.

Solution

1. Identify the known values:
   - Initial velocity: $v_0 = 8.00 \text{m/s}$
   - Final velocity: $v = 40.0 \text{m/s}$
   - Time: $t = 3.33 \times 10^{-2} \text{s}$
2. Use the equation relating displacement to velocities and time:

$$x = \frac{v_0+v}{2}t$$

This equation works because the average velocity during constant acceleration is simply the arithmetic mean of initial and final velocities.

1. Substitute the known values:

$$x = \frac{8.00+40.0}{2} \times (3.33 \times 10^{-2})$$
$$x = \frac{48.0}{2} \times (3.33 \times 10^{-2}) = 24.0 \times 0.0333$$
$$x = 0.799 \text{m}$$

Discussion

The puck accelerates over a distance of about 80 cm (roughly 31 inches), which is reasonable for a hockey slap shot where the stick blade maintains contact with the puck during the entire shooting motion. The puck's velocity increases from 8.00 m/s to 40.0 m/s - a fivefold increase in speed! This represents an acceleration of:

$$a = \frac{v-v_0}{t} = \frac{40.0-8.00}{0.0333} = 961 \text{m/s}^2$$

This is about 98 times gravitational acceleration (98g), which demonstrates the enormous forces involved in a professional slap shot. The final speed of 40.0 m/s (144 km/h or 89 mph) is typical for a powerful slap shot in hockey.

Answer

The puck accelerates over a distance of 0.799 m (approximately 80 cm or 31 inches).

A powerful motorcycle can accelerate from rest to 26.8 m/s (100 km/h) in only 3.90 s. (a) What is its average acceleration? (b) How far does it travel in that time?

Show Solution

Strategy

Use the definition of average acceleration for part (a), then use a kinematic equation with the calculated acceleration to find distance in part (b).

Solution

(a) Average acceleration:

1. Identify the known values:
   - Initial velocity: $v_0 = 0$ (starts from rest)
   - Final velocity: $v = 26.8 \text{m/s}$
   - Time: $t = 3.90 \text{s}$
2. Calculate average acceleration:

$$a = \frac{v-v_0}{t} = \frac{26.8 \text{m/s} - 0}{3.90 \text{s}} = 6.87 \text{m/s}^2$$

(b) Distance traveled:

Use the kinematic equation:

$$x = x_0 + v_0 t + \frac{1}{2}at^2 = 0 + 0 + \frac{1}{2}(6.87 \text{m/s}^2)(3.90 \text{s})^2$$

x=12(6.87)(15.21)m=52.3m

Discussion

The acceleration of 6.87 m/s² is about 0.7g, which is quite impressive for a motorcycle. The distance of about 52 meters to reach 100 km/h is consistent with high-performance motorcycle specifications.

(a) The average acceleration is 6.87m/s^2.

(b) The motorcycle travels 52.3m in that time.

Freight trains can produce only relatively small accelerations and decelerations. (a) What is the final velocity of a freight train that accelerates at a rate of 0.0500m/s^2 for 8.00 min, starting with an initial velocity of 4.00 m/s? (b) If the train can slow down at a rate of 0.550m/s^2, how long will it take to come to a stop from this velocity? (c) How far will it travel in each case?

Show Solution

Strategy

This problem involves two phases: acceleration and deceleration of a freight train. For part (a), we use $v = v_0 + at$ to find the final velocity after acceleration. For part (b), we use the same equation to find the time to decelerate from this velocity to rest. For part (c), we calculate distances using appropriate kinematic equations. Remember to convert 8.00 minutes to seconds.

Solution

(a) Final velocity after acceleration:

1. Convert time to seconds:

t=8.00 min=8.00×60=480s

1. Identify the known values:
    - Initial velocity: $v_0 = 4.00\text{m/s}$
    - Acceleration: $a = 0.0500\text{m/s}^2$
    - Time: $t = 480\text{s}$
2. Use the kinematic equation:

v=v_0+at

1. Substitute the known values:

v=4.00+(0.0500)(480)=4.00+24.0=28.0m/s

(b) Time to decelerate to rest:

1. Identify the known values:
    - Initial velocity: $v_0 = 28.0\text{m/s}$ (from part a)
    - Final velocity: $v = 0$ (comes to rest)
    - Deceleration: $a = -0.550\text{m/s}^2$ (negative for deceleration)
2. Solve for time:

t=v−$v_0$$a$=0−28.0−0.550=−28.0−0.550=50.9s

(c) Distances traveled:

Distance during acceleration (8.00 min = 480 s):

x_{accel}=$v_0 t$+12at^2
x_{accel}=(4.00)(480)+12(0.0500)$(480)^2$
x_{accel}=1920+12(0.0500)(230400)=1920+5760=7680m=7.68 km

Distance during deceleration (50.9 s):

x_{decel}=$v_0 t$+12at^2
x_{decel}=(28.0)(50.9)+12(−0.550)$(50.9)^2$
x_{decel}=1425.2+12(−0.550)(2590.81)=1425.2−712.5=712.7m≈713m

Discussion

The freight train's small acceleration ($0.0500 m/s^2$, only about 0.5% of gravitational acceleration) is typical for heavy trains with massive inertia. Despite this tiny acceleration, over 8 minutes the train increases its velocity from 4.00 m/s to 28.0 m/s (about 14 to 101 km/h), traveling an impressive 7.68 km! The deceleration rate ($0.550 m/s^2$) is 11 times larger than the acceleration rate, so the train stops much more quickly - in less than one minute compared to 8 minutes of acceleration. However, even with this stronger braking, the train still travels 713 m (over 700 meters, or nearly half a mile) while stopping. This demonstrates why freight trains need such long distances to stop safely.

Answer

(a) The final velocity after 8.00 minutes of acceleration is 28.0 m/s.

(b) It takes 50.9 s (about 51 seconds) to come to a stop from this velocity.

(c) The train travels 7.68 km during acceleration and 713 m during deceleration.

A fireworks shell is accelerated from rest to a velocity of 65.0 m/s over a distance of 0.250 m. (a) How long did the acceleration last? (b) Calculate the acceleration.

Show Solution

Strategy

A fireworks shell is accelerated from rest to a known final velocity over a known distance. For part (a), we need to find time, and for part (b), we need to find acceleration. We can use kinematic equations that relate these quantities.

Solution

Knowns:

- Initial velocity: $v_0 = 0$ (starts from rest)
- Final velocity: $v = 65.0 m/s$
- Displacement: $x - x_0 = 0.250 m$

(a) Time of acceleration:

Unknown: Time t

Choosing the equation: We know v_0, v, and displacement. We can use the average velocity approach:

$$x - x_0 = \bar{v} t$$

where

$$\bar{v} = \frac{v_0 + v}{2} = \frac{0 + 65.0 m/s}{2} = 32.5 m/s$$

Solving:

$$t = \frac{x - x_0}{\bar{v}} = \frac{0.250 m}{32.5 m/s} = 7.69 \times 10^{-3} s = 7.69 \text{ ms}$$

(b) Acceleration:

Unknown: Acceleration a

Choosing the equation: We know v_0, v, and $x - x_0$. The best equation is:

$$v^2 = v_0^2 + 2a(x - x_0)$$

This equation relates all known quantities to acceleration without requiring time.

Solving:

Rearrange to solve for a:

$$a = \frac{v^2 - v_0^2}{2(x - x_0)}$$

Substitute the known values:

$$a = \frac{(65.0 m/s)^2 - 0}{2(0.250 m)} = \frac{4225 m^2/s^2}{0.500 m} = 8.45 \times 10^3 m/s^2$$

Units check: For time: $\frac{m}{m/s} = s$ ✓; For acceleration: $\frac{m^2/s^2}{m} = m/s^2$ ✓

Discussion

We can verify our answers are consistent using $v = v_0 + at$:

$$t = \frac{v - v_0}{a} = \frac{65.0 \text{m/s}}{8.45 \times 10^3 \text{m/s}^2} = 7.69 \times 10^{-3} \text{s} ✓$$

This confirms both answers are correct and consistent.

The acceleration is extremely large: $8.45 \times 10^3 \text{m/s}^2$ is about 862 times the acceleration due to gravity (862g)! This makes sense for a fireworks shell—the explosive charge must accelerate the shell very rapidly over the short length of the launch tube. The time of 7.69 ms (less than one-hundredth of a second) is also consistent with the explosive nature of fireworks launches.

For comparison, even the most powerful dragsters only achieve accelerations of about 3-4g during launch. The fireworks shell experiences forces hundreds of times greater, which is why they must be constructed very robustly to withstand the launch.

Answer

(a) The acceleration lasted **7.69 ms** (or $7.69 \times 10^{-3} \text{s}$).

(b) The acceleration is $\mathbf{8.45 \times 10^3 m/s^2}$ (approximately 862g).

A swan on a lake gets airborne by flapping its wings and running on top of the water. (a) If the swan must reach a velocity of 6.00 m/s to take off and it accelerates from rest at an average rate of 0.350m/s^2, how far will it travel before becoming airborne? (b) How long does this take?

Show Solution

(a) 51.4m

(b) 17.0s

Professional Application:

A woodpecker's brain is specially protected from large decelerations by tendon-like attachments inside the skull. While pecking on a tree, the woodpecker's head comes to a stop from an initial velocity of 0.600 m/s in a distance of only 2.00 mm. (a) Find the acceleration in m/s^2 and in multiples of g ($g = 9.80 \text{m/s}^2$). (b) Calculate the stopping time. (c) The tendons cradling the brain stretch, making its stopping distance 4.50 mm (greater than the head and, hence, less deceleration of the brain). What is the brain's deceleration, expressed in multiples of g ?

Show Solution

Strategy

The woodpecker's head decelerates from a known initial velocity to rest over a known distance. We can use the kinematic equation $v^2 = v_0^2 + 2a(x - x_0)$ to find the deceleration, then use $v = v_0 + at$ to find the stopping time. For part (c), we repeat the acceleration calculation with a different stopping distance.

Solution

Knowns:

- Initial velocity: $v_0 = 0.600 \text{m/s}$
- Final velocity: $v = 0$ (comes to a stop)
- Head stopping distance: $x\_head - x_0 = 2.00 \text{ mm} = 2.00 \times 10^{-3} \text{m}$
- Brain stopping distance: $x\_brain - x_0 = 4.50 \text{ mm} = 4.50 \times 10^{-3} \text{m}$
- Acceleration due to gravity: $g = 9.80 \text{m/s}^2$

(a) Head's deceleration:

Unknown: Acceleration a_head

Choosing the equation: We know v_0, v, and the stopping distance. Use:

$$v^2 = v_0^2 + 2a(x - x_0)$$

Solving:

Rearrange to solve for a:

$$a_{\text{head}}=\frac{v^2-v_0^2}{2(x_{\text{head}}-x_0)}$$

Substitute the known values:

$$a_{\text{head}}=\frac{0-(0.600\text{m/s})^2}{2(2.00\times10^{-3}\text{m})}=\frac{-0.360\text{m}^2/\text{s}^2}{4.00\times10^{-3}\text{m}}=-90.0\text{m/s}^2$$

The negative sign indicates deceleration (acceleration opposite to the direction of motion).

Express in multiples of g:

$$\frac{a_{\text{head}}}{g}=\frac{-90.0\text{m/s}^2}{9.80\text{m/s}^2}=-9.18$$

So the head experiences a deceleration of 90.0m/s^2 or $9.18g$.

(b) Stopping time:

Unknown: Time t

Choosing the equation: Use $v=v_0+at$:

$$t=\frac{v-v_0}{a_{\text{head}}}=\frac{0-0.600\text{m/s}}{-90.0\text{m/s}^2}=6.67\times10^{-3}\text{s}=6.67\text{ ms}$$

(c) Brain’s deceleration:

Unknown: Acceleration a_brain

The brain has a longer stopping distance due to the stretching tendons:

$$a_{\text{brain}}=\frac{v^2-v_0^2}{2(x_{\text{brain}}-x_0)}=\frac{0-(0.600\text{m/s})^2}{2(4.50\times10^{-3}\text{m})}$$
$$a_{\text{brain}}=\frac{-0.360\text{m}^2/\text{s}^2}{9.00\times10^{-3}\text{m}}=-40.0\text{m/s}^2$$

Express in multiples of g:

$$\frac{a_{\text{brain}}}{g}=\frac{-40.0\text{m/s}^2}{9.80\text{m/s}^2}=-4.08$$

The brain experiences a deceleration of 40.0m/s^2 or $4.08g$.

Units check: $\frac{\text{m}^2/\text{s}^2}{\text{m}}=\text{m/s}^2$ ✓; $\frac{\text{m/s}}{\text{m/s}^2}=\text{s}$ ✓

Discussion

These results reveal the remarkable biomechanical engineering of the woodpecker’s anatomy. The head decelerates at 9.18g—nearly 10 times the acceleration due to gravity! This would cause severe brain injury in most animals.

However, the tendon-like attachments allow the brain to decelerate over a distance 2.25 times greater than the head (4.50 mm vs. 2.00 mm). This reduces the brain’s deceleration to 4.08g—less than half the head’s deceleration. While still substantial, this is within a survivable range, especially with the woodpecker’s other adaptations (compact brain, minimal cerebrospinal fluid, specialized skull structure).

The stopping time of 6.67 ms is extremely brief—less than one-hundredth of a second. Woodpeckers can peck 15-20 times per second, so this rapid deceleration is essential for their feeding and territorial behaviors.

For comparison, race car drivers experience decelerations up to about 5-6g during crashes (with safety equipment), and fighter pilots can experience 9g during high-performance maneuvers. The woodpecker’s head exceeds this regularly, highlighting the extraordinary evolutionary adaptation.

Answer

(a) The head’s deceleration is $\mathbf{-90.0m/s^2}$ or **9.18g** (9.18 times the acceleration due to gravity).

(b) The stopping time is **6.67 ms** (or $6.67\times10^{-3}\text{s}$).

(c) The brain’s deceleration is **4.08g** (4.08 times the acceleration due to gravity).

An unwary football player collides with a padded goalpost while running at a velocity of 7.50 m/s and comes to a full stop after compressing the padding and his body 0.350 m. (a) What is his deceleration? (b) How long does the collision last?

Show Solution

Strategy

The football player decelerates from a known velocity to rest over a known distance. For part (a), we use the kinematic equation $v^2 = v_0^2 + 2a(x - x_0)$ to find the deceleration. For part (b), we use $v = v_0 + at$ to find the collision time. The padding helps extend the stopping distance, reducing the deceleration and impact force.

Solution

(a) Deceleration:

1. Identify the known values:
   - Initial velocity: $v_0 = 7.50 \text{m/s}$
   - Final velocity: $v = 0$ (comes to a full stop)
   - Stopping distance: $x - x_0 = 0.350 \text{m}$
2. Use the kinematic equation:

$$v^2 = v_0^2 + 2a(x - x_0)$$

1. Solve for acceleration:

$$a = \frac{v^2 - v_0^2}{2(x - x_0)}$$

1. Substitute the known values:

$$a = \frac{0 - (7.50)^2}{2(0.350)} = \frac{-56.25}{0.700} = -80.4 \text{m/s}^2$$

The negative sign indicates deceleration.

(b) Collision duration:

1. Use the kinematic equation:

$$v = v_0 + at$$

1. Solve for time:

$$t = \frac{v - v_0}{a} = \frac{0 - 7.50}{-80.4} = \frac{-7.50}{-80.4} = 0.0933 \text{s} = 9.33 \times 10^{-2} \text{s}$$

Discussion

The player experiences a deceleration of 80.4m/s^2, which is:

$$\frac{80.4}{9.80} = 8.2g$$

This is about 8.2 times gravitational acceleration - a very significant force! However, this is survivable thanks to the padding on the goalpost and the player's own protective equipment. The padding allows the player to decelerate over 0.350 m (about 14 inches) rather than stopping nearly instantly against a rigid surface. Without padding, the stopping distance might be only a few centimeters, resulting in decelerations of 50-100g or more - enough to cause serious injury.

The collision lasts about 93 milliseconds (less than one-tenth of a second), which is too fast for the player to react but slow enough (thanks to the padding) to reduce peak forces. This demonstrates the importance of proper safety equipment and padding in contact sports.

Answer

(a) The player's deceleration is -80.4m/s^2 (about 8.2 times gravitational acceleration).

(b) The collision lasts $9.33 \times 10^{-2} \text{s}$ (approximately 93 milliseconds).

In World War II, there were several reported cases of airmen who jumped from their flaming airplanes with no parachute to escape certain death. Some fell about 20 000 feet (6000 m), and some of them survived, with few life-threatening injuries. For these lucky pilots, the tree branches and snow drifts on the ground allowed their deceleration to be relatively small. If we assume that a pilot's speed upon impact was 123 mph (54 m/s), then what was his deceleration? Assume that the trees and snow stopped him over a distance of 3.0 m.

Show Solution

Strategy

The pilot decelerates from a known impact velocity to rest over a known stopping distance. We can use the kinematic equation $v^2 = v_0^2 + 2a(x - x_0)$ to find the deceleration. This equation is ideal because it relates velocities, acceleration, and distance without requiring time information.

Solution

Knowns:

- Initial velocity (impact speed): $v_0 = 54 \text{m/s}$ (or 123 mph)
- Final velocity: $v = 0$ (comes to rest)
- Stopping distance: $x - x_0 = 3.0 \text{m}$

Unknown: Acceleration a

Choosing the equation:

Use the kinematic equation:

$$v^2 = v_0^2 + 2a(x - x_0)$$

Solving:

Rearrange to solve for a:

$$a = \frac{v^2 - v_0^2}{2(x - x_0)}$$

Substitute the known values:

$$a = \frac{0 - (54 \text{m/s})^2}{2(3.0 \text{m})} = \frac{-2916 \text{m}^2/\text{s}^2}{6.0 \text{m}} = -486 \text{m/s}^2 = -4.9 \times 10^2 \text{m/s}^2$$

The negative sign indicates deceleration (acceleration opposite to the direction of motion).

Express in multiples of g (where $g = 9.80 \text{m/s}^2$):

$$\frac{a}{g} = \frac{-486 \text{m/s}^2}{9.80 \text{m/s}^2} = -49.6 \approx -50$$

The pilot experienced a deceleration of approximately 50g.

Units check: $\frac{\text{m}^2/\text{s}^2}{\text{m}} = \text{m/s}^2$ ✓

Discussion

A deceleration of 486 m/s² (about 50g) is extraordinary and would normally be fatal. For comparison:

- Modern fighter pilots can withstand up to about 9g with special suits
- Race car drivers in severe crashes experience 20-40g (often with serious injuries)
- The human body can survive very brief exposures to high g-forces if properly oriented

The survival of these airmen is remarkable and can be attributed to several factors:

1. **Gradual deceleration**: The 3.0 m stopping distance, provided by tree branches and snow, distributed the impact force over a much longer distance than hitting hard ground (which might provide only 0.1-0.2 m of deceleration).

2. **Body orientation**: If the pilot landed feet-first or in a horizontal position, the forces would be distributed more favorably than head-first impact.

3. **Sequential impacts**: Tree branches would have provided multiple smaller impacts rather than one catastrophic stop, further distributing the deceleration.

4. **Duration**: We can calculate the stopping time using $v = v_0 + at$:

$$t = \frac{v - v_0}{a} = \frac{0 - 54 \text{m/s}}{-486 \text{m/s}^2} = 0.11 \text{s}$$

The deceleration lasted about 0.11 seconds—long enough to be survivable, though still extremely dangerous.

Without the trees and snow, stopping over just 0.1 m would produce a deceleration of about 14,600 m/s² (1490g)—absolutely unsurvivable. The 3.0 m stopping distance reduced the deceleration by a factor of 30, making survival possible, though still requiring extraordinary luck.

Answer

The pilot's deceleration was $\mathbf{-4.9 \times 10^2 \text{m/s}^2}$ (or approximately 486 m/s²), which is about **50 times the acceleration due to gravity (50g)**.

Consider a grey squirrel falling out of a tree to the ground. (a) If we ignore air resistance in this case (only for the sake of this problem), determine a squirrel's velocity just before hitting the ground, assuming it fell from a height of 3.0 m. (b) If the squirrel stops in a distance of 2.0 cm through bending its limbs, compare its deceleration with that of the airman in the previous problem.

Show Solution

Strategy

This problem has two parts. For part (a), the squirrel falls freely under gravity from a known height - we use free fall equations to find the velocity just before impact. For part (b), the squirrel decelerates from this velocity to rest over a very short distance (2.0 cm) by bending its limbs - we use kinematic equations to find this deceleration and compare it to the airman's deceleration from the previous problem.

Solution

(a) Velocity just before hitting the ground:

1. Identify the known values for free fall:
   - Initial velocity: $v_0 = 0$ (starts from rest)
   - Acceleration: $a = g = 9.80 \text{m/s}^2$ (downward)
   - Distance fallen: $y = 3.0 \text{m}$
2. Use the kinematic equation:

$$v^2 = v_0^2 + 2gy$$

1. Substitute the known values:

$$v^2 = 0 + 2(9.80)(3.0) = 58.8 \text{m}^2/\text{s}^2$$
$$v = \sqrt{58.8} = 7.67 \text{m/s} \approx 7.7 \text{m/s}$$

(b) Deceleration upon landing:

1. Identify the known values:
   - Initial velocity: $v_0 = 7.7 \text{m/s}$ (from part a)
   - Final velocity: $v = 0$ (comes to rest)
   - Stopping distance: $x - x_0 = 2.0 \text{ cm} = 0.020 \text{m}$
2. Use the kinematic equation:

$$v^2 = v_0^2 + 2a(x - x_0)$$

1. Solve for acceleration:

$$a = \frac{v^2 - v_0^2}{2(x - x_0)} = \frac{0 - (7.7)^2}{2(0.020)} = \frac{-59.29}{0.040} = -1482 \text{m/s}^2 \approx -1.5 \times 10^3 \text{m/s}^2$$

1. Compare to the airman's deceleration from the previous problem (-486m/s^2):

$$\frac{1482}{486} \approx 3.0$$

The squirrel's deceleration is about 3 times greater than the airman's!

1. Express in multiples of g:

$$\frac{1482}{9.80} \approx 151g$$

Discussion

This problem reveals a surprising result: the squirrel experiences about 151g of deceleration - three times more than the airman falling from 6000 m! How can this be? The key is the stopping distance. The squirrel, though falling from only 3.0 m and hitting at "only" 7.7 m/s (compared to the airman's 54 m/s), stops in just 2.0 cm - 150 times shorter than the airman's 3.0 m stopping distance.

Small animals like squirrels can survive these enormous decelerations because of several factors:

1. **Square-cube law**: Smaller animals have proportionally stronger bones and tissues relative to their weight
2. **Flexible skeletal structure**: The squirrel bends its limbs to maximize the stopping distance for its size
3. **Lower terminal velocity**: Even without our assumption, a squirrel reaches a much lower terminal velocity in air

This is why squirrels can fall from great heights and walk away unharmed - they're naturally built to handle extreme decelerations relative to their size.

Answer

(a) The squirrel's velocity just before hitting the ground is 7.7 m/s.

(b) The squirrel's deceleration is $-1.5 \times 10^3 \text{m/s}^2$ (about 151g), which is approximately 3 times the deceleration experienced by the airman in the previous problem who was falling from thousands of meters!

An express train passes through a station. It enters with an initial velocity of 22.0 m/s and decelerates at a rate of 0.150m/s^2 as it goes through. The station is 210 m long. (a) How long did the nose of the train stay in the station? (b) How fast is it going when the nose leaves the station? (c) If the train is 130 m long, when does the end of the train leave the station? (d) What is the velocity of the end of the train as it leaves?

Show Solution

Strategy

The train decelerates at a constant rate as it passes through the station. We need to track the motion of both the nose and the end of the train. For parts (a) and (b), we analyze the nose traveling 210 m through the station. For parts (c) and (d), we analyze the end of the train, which must travel 210 m + 130 m = 340 m (the station length plus the train length) before it clears the station.

Solution

Knowns:

- Initial velocity: $v_0 = 22.0\text{m/s}$
- Acceleration: $a = -0.150\text{m/s}^2$ (negative because it's decelerating)
- Station length: L_station $= 210\text{m}$
- Train length: L_train $= 130\text{m}$

(a) Time for nose to traverse the station:

Unknown: Time t_nose

The nose travels a distance of 210 m. We know v_0, a, and $x - x_0$, and need to find t.

Choosing the equation: Use $x = x_0 + v_0 t + 12 a t^2$

Taking $x_0 = 0$:

$$210=22.0t+12(-0.150)t^2$$
$$210=22.0t-0.0750t^2$$

Rearrange to standard quadratic form:

$$0.0750t^2-22.0t+210=0$$

Using the quadratic formula with $a = 0.0750$, $b = -22.0$, $c = 210$:

$$t=-b\pm\sqrt{}b^2-4ac2a=22.0\pm\sqrt{}(-22.0)^2-4(0.0750)(210)2(0.0750)$$
$$t=22.0\pm\sqrt{}484-63.00.150=22.0\pm\sqrt{}4210.150=22.0\pm20.50.150$$

This gives two solutions:

- $t_1=22.0+20.50.150=42.50.150=283\text{s}$
- $t_2=22.0-20.50.150=1.50.150=10.0\text{s}$

The physically meaningful solution is t_nose $= 10.0\text{s}$ (the larger time would correspond to the train reversing direction, which is not realistic).

(b) Velocity when nose leaves the station:

Unknown: Final velocity v_nose

Use $v = v_0 + at$:

$$v_{\text{nose}}=22.0\text{m/s}+(-0.150\text{m/s}^2)(10.0\text{s})=22.0-1.50=20.5\text{m/s}$$

We can verify using $v^2 = v_{20} + 2a(x - x_0)$:

$$v_{2\text{nose}}=(22.0)^2+2(-0.150)(210)=484-63.0=421\text{m}^2/\text{s}^2$$
$$v_{\text{nose}}=\sqrt{}421=20.5\text{m/s} ✓$$

(c) Time for end of train to leave the station:

Unknown: Time t_end

The end of the train must travel the station length plus the train length: $210 + 130 = 340 \text{ m}$

Using $x = x_0 + v_0 t + \frac{1}{2}at^2$:

$$340 = 22.0t - 0.0750t^2$$
$$0.0750t^2 - 22.0t + 340 = 0$$

Using the quadratic formula:

$$t = \frac{22.0 \pm \sqrt{484 - 4(0.0750)(340)}}{0.150} = \frac{22.0 \pm \sqrt{484 - 102}}{0.150}$$
$$t = \frac{22.0 \pm \sqrt{382}}{0.150} = \frac{22.0 \pm 19.5}{0.150}$$

This gives:

- $t_1 = \frac{22.0 + 19.5}{0.150} = 277 \text{ s}$ (unphysical)
- $t_2 = \frac{22.0 - 19.5}{0.150} = \frac{2.5}{0.150} = 16.7 \text{ s}$

So $t\_end = 16.7 \text{ s}$.

(d) Velocity of end of train as it leaves:

Unknown: Final velocity v_end

Use $v = v_0 + at$:

$$v_{\text{end}} = 22.0 \text{ m/s} + (-0.150 \text{ m/s}^2)(16.7 \text{ s}) = 22.0 - 2.51 = 19.5 \text{ m/s}$$

Units check: All dimensional analysis confirms proper units throughout.

Discussion

The results make physical sense. The nose of the train takes 10.0 s to traverse the 210 m station, slowing from 22.0 m/s to 20.5 m/s. The end of the train, entering 130 m behind the nose, takes an additional 6.7 s to clear the station (total of 16.7 s from when the nose entered), and is moving at 19.5 m/s when it exits.

The deceleration of 0.150 m/s² is quite gentle—only about 1.5% of gravitational acceleration. This is typical for passenger trains, ensuring passenger comfort. Over the 16.7 seconds that the train occupies the station, it slows by only 2.5 m/s (from 22.0 to 19.5 m/s), which passengers would barely notice.

The average velocity during the passage can be calculated as $\bar{v} = \frac{v_0 + v}{2} = \frac{22.0 + 19.5}{2} = 20.75 \text{ m/s}$, and the total distance traveled is 340 m, giving an average time of $t = \frac{340}{20.75} = 16.4 \text{ s}$, which is consistent with our calculated 16.7 s (the small difference is due to rounding).

Answer

(a) The nose of the train stays in the station for **10.0 s**.

(b) The nose is traveling at **20.5 m/s** when it leaves the station.

(c) The end of the train leaves the station at **16.7 s** after the nose entered.

(d) The end of the train is traveling at **19.5 m/s** as it leaves the station.

Dragsters can actually reach a top speed of 145 m/s in only 4.45 s—considerably less time than given in Example 3 and Example 4. (a) Calculate the average acceleration for such a dragster. (b) Find the final velocity of this dragster starting from rest and accelerating at the rate found in (a) for 402 m (a quarter mile) without using any information on time. (c) Why is the final velocity greater than that used to find the average acceleration? *Hint*: Consider whether the assumption of constant acceleration is valid for a dragster. If not, discuss whether the acceleration would be greater at the beginning or end of the run and what effect that would have on the final velocity.

Show Solution

Strategy

For part (a), we calculate average acceleration using the change in velocity and time. For part (b), we use this acceleration with the distance to find final velocity, assuming constant acceleration. For part (c), we discuss why constant acceleration is not a valid assumption for dragsters and how this affects our calculated values.

Solution

(a) Average acceleration:

1. Identify the known values:
   - Initial velocity: $v_0 = 0$ (starts from rest)
   - Final velocity: $v = 145 \text{ m/s}$

- Time: $t = 4.45\text{s}$

2. Calculate average acceleration:

$$a=\frac{v-v_0}{t}=\frac{145-0}{4.45}=32.6\text{m/s}^2$$

(b) Final velocity over a quarter mile (402 m):

1. Identify the known values:
   - Initial velocity: $v_0 = 0$
   - Acceleration: $a = 32.6\text{m/s}^2$ (from part a)
   - Distance: $x = 402\text{m}$
2. Use the kinematic equation (without time):

$$v^2=v_0^2+2ax$$

1. Substitute the known values:

$$v^2=0+2(32.6)(402)=26{,}210\text{m}^2/\text{s}^2$$
$$v=\sqrt{26{,}210}=162\text{m/s}$$

(c) Why is the final velocity greater?

The calculated final velocity of 162 m/s is greater than the actual top speed of 145 m/s used to find the average acceleration. This discrepancy reveals that the assumption of constant acceleration is not valid for a dragster.

A dragster's acceleration is not constant because:

1. **Gear changes**: The dragster shifts through multiple gears during the run, and acceleration is greatest in first gear
2. **Decreasing force-to-resistance ratio**: As velocity increases, air resistance increases dramatically (proportional to v^2)
3. **Tire grip limitations**: Maximum traction force varies with speed and tire heating
4. **Engine power curve**: The engine delivers different power outputs at different RPMs

The acceleration would be greatest at the beginning of the run (possibly 40-50 m/s² in first gear) and would decrease significantly toward the end (perhaps only 10-15 m/s² in high gear at top speed). Therefore, the dragster is not accelerating at 32.6 m/s² during the last few meters—it's accelerating substantially less. This means the actual final velocity is less than the 162 m/s we calculated assuming constant acceleration.

Discussion

This problem illustrates an important principle: kinematic equations assume constant acceleration. When applied to situations with varying acceleration, they give average values that may not match actual instantaneous values. The 32.6 m/s² represents the average acceleration over the entire 4.45 s run, not the acceleration at any particular moment.

For comparison, 32.6 m/s² is about 3.3 times gravitational acceleration (3.3g), which is similar to what fighter pilots experience. Top fuel dragsters can actually achieve peak accelerations exceeding 4-5g at launch, far beyond what most people ever experience!

Answer

(a) The average acceleration is 32.6 m/s².

(b) If the dragster maintained this constant acceleration over 402 m, the final velocity would be 162 m/s.

(c) This calculated velocity (162 m/s) is greater than the actual top speed (145 m/s) because the assumption of constant acceleration is not valid for a dragster. Dragsters change gears and experience increasing air resistance, so acceleration is greatest at the beginning and decreases throughout the run. The actual acceleration during the final meters is substantially less than 32.6 m/s², resulting in a final velocity less than 162 m/s.

A bicycle racer sprints at the end of a race to clinch a victory. The racer has an initial velocity of 11.5 m/s and accelerates at the rate of 0.500m/s^2 for 7.00 s. (a) What is his final velocity? (b) The racer continues at this velocity to the finish line. If he was 300 m from the finish line when he started to accelerate, how much time did he save? (c) One other racer was 5.00 m ahead when the winner started to accelerate, but he was unable to accelerate, and traveled at 11.8 m/s until the finish line. How far ahead of him (in meters and in seconds) did the winner finish?

Show Solution

Strategy

This problem involves multiple phases of motion. The winner accelerates for 7.00 s, then continues at constant velocity. We need to find the final velocity after acceleration, calculate the time saved compared to non-accelerated motion, and compare the winner's finish with the other racer who maintains constant velocity.

Solution

Knowns:

- Winner's initial velocity: $v_0 = 11.5 \text{m/s}$
- Winner's acceleration: $a = 0.500 \text{m/s}^2$
- Acceleration time: t_accel $= 7.00 \text{s}$
- Distance to finish line: d_total $= 300 \text{m}$
- Other racer's velocity: v_other $= 11.8 \text{m/s}$
- Other racer's initial advantage: 5.00m ahead

(a) Final velocity after acceleration:

Unknown: Final velocity v

Use $v = v_0 + at$:

$$v = 11.5 \text{m/s} + (0.500 \text{m/s}^2)(7.00 \text{s}) = 11.5 + 3.50 = 15.0 \text{m/s}$$

(b) Time saved by accelerating:

Unknown: Time saved

First, calculate distance covered during acceleration using $x = x_0 + v_0 t + 12 a t^2$:

$$x_{\text{accel}} = (11.5 \text{m/s})(7.00 \text{s}) + 12(0.500 \text{m/s}^2)(7.00 \text{s})^2$$
$$x_{\text{accel}} = 80.5 \text{m} + 12(0.500)(49.0) \text{m} = 80.5 + 12.25 = 92.75 \text{m}$$

Remaining distance to finish line:

$$x_{\text{remaining}} = 300 - 92.75 = 207.25 \text{m}$$

Time to cover remaining distance at constant velocity of 15.0 m/s:

$$t_{\text{remaining}} = 207.25 \text{m} 15.0 \text{m/s} = 13.8 \text{s}$$

Total time with acceleration:

$$t_{\text{total, accel}} = 7.00 + 13.8 = 20.8 \text{s}$$

Time if no acceleration (maintaining 11.5 m/s for entire 300 m):

$$t_{\text{no accel}} = 300 \text{m} 11.5 \text{m/s} = 26.1 \text{s}$$

Time saved:

$$\Delta t = 26.1 - 20.8 = 5.3 \text{s}$$

(c) Comparison with other racer:

The other racer starts 5.00 m ahead (at position -5.00 m if finish is at 300 m), so must travel $300 + 5.00 = 305 \text{m}$ to reach the finish.

Other racer's time:

$$t_{\text{other}} = 305 \text{m} 11.8 \text{m/s} = 25.8 \text{s}$$

Winner finishes in: 20.8s

Time difference:

$$\Delta t = 25.8 - 20.8 = 5.0 \text{s}$$

Distance ahead: When the winner finishes (at $t = 20.8 \text{s}$), the other racer has traveled:

$$x_{\text{other}} = (11.8 \text{m/s})(20.8 \text{s}) = 245 \text{m}$$

The other racer still needs to travel $305 - 245 = 60 \text{m}$ to reach the finish line.

So the winner finishes **60 m ahead** and **5.0 s ahead**.

Units check: All dimensional analysis confirms proper units throughout.

Discussion

The acceleration phase was crucial to victory. By accelerating from 11.5 m/s to 15.0 m/s over 7 seconds, the winner increased his speed by 30%. This saved 5.3 seconds compared to maintaining the initial pace—a significant advantage in competitive cycling.

The comparison with the other racer is particularly interesting. Despite starting 5.00 m behind, the winner's acceleration allowed him to finish 60 m ahead—a complete reversal of position! The other racer's slight initial speed advantage (11.8 m/s vs. 11.5 m/s) was completely overwhelmed by the winner's acceleration.

In real bicycle racing, this represents a classic sprint finish strategy. The ability to accelerate at the end of a race—when other competitors are fatigued—is a decisive competitive advantage. An acceleration of 0.500 m/s^2 sustained for 7 seconds is impressive for a cyclist, especially near the end of a race.

The winner's average velocity during acceleration was $\bar{v} = \frac{11.5+15.0}{2} = 13.25 \text{m/s}$, which we can verify: $(13.25)(7.00) = 92.75 \text{m}$ ✓

Answer

(a) The racer's final velocity is **15.0 m/s**.

(b) By accelerating, the racer saved **5.3 s** compared to maintaining constant velocity.

(c) The winner finished **60 m ahead** and **5.0 s ahead** of the other racer.

In 1967, New Zealander Burt Munro set the world record for an Indian motorcycle, on the Bonneville Salt Flats in Utah, with a maximum speed of 183.58 mi/h. The one-way course was 5.00 mi long. Acceleration rates are often described by the time it takes to reach 60.0 mi/h from rest. If this time was 4.00 s, and Burt accelerated at this rate until he reached his maximum speed, how long did it take Burt to complete the course?

Show Solution

104 s

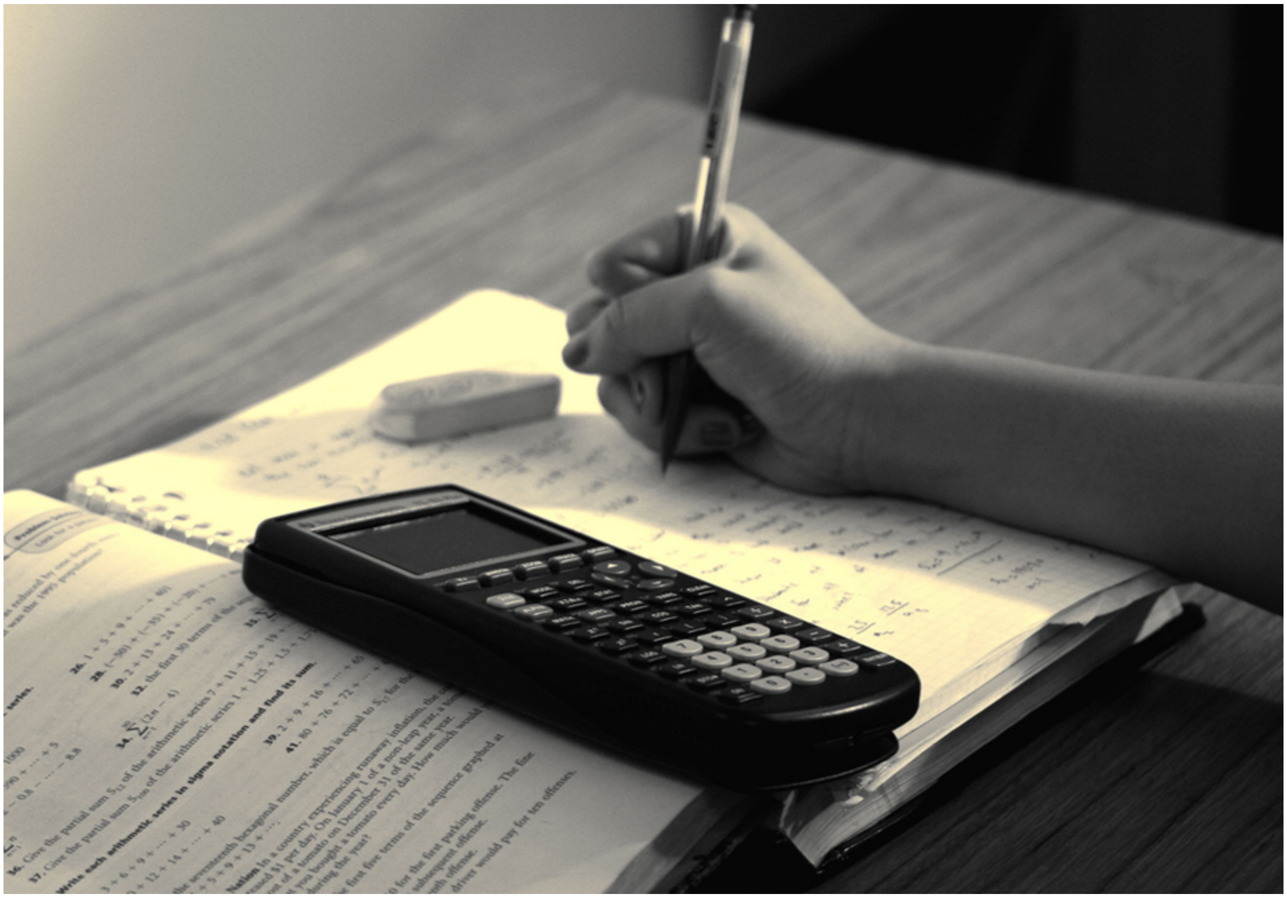
(a) A world record was set for the men's 100-m dash in the 2008 Olympic Games in Beijing by Usain Bolt of Jamaica. Bolt "coasted" across the finish line with a time of 9.69 s. If we assume that Bolt accelerated for 3.00 s to reach his maximum speed, and maintained that speed for the rest of the race, calculate his maximum speed and his acceleration. (b) During the same Olympics, Bolt also set the world record in the 200-m dash with a time of 19.30 s. Using the same assumptions as for the 100-m dash, what was his maximum speed for this race?

Show Solution

(a) $v = 12.2 \text{m/s}$; $a = 4.07 \text{m/s}^2$ (b) $v = 11.2 \text{m/s}$

Problem-Solving Basics for One-Dimensional Kinematics

- Apply problem-solving steps and strategies to solve problems of one-dimensional kinematics.
- Apply strategies to determine whether or not the result of a problem is reasonable, and if not, determine the cause.



Problem-solving skills are essential to your success in Physics. (credit: scui3asteveo, Flickr)

Problem-solving skills are obviously essential to success in a quantitative course in physics. More importantly, the ability to apply broad physical principles, usually represented by equations, to specific situations is a very powerful form of knowledge. It is much more powerful than memorizing a list of facts. Analytical skills and problem-solving abilities can be applied to new situations, whereas a list of facts cannot be made long enough to contain every possible circumstance. Such analytical skills are useful both for solving problems in this text and for applying physics in everyday and professional life.

Problem-Solving Steps

While there is no simple step-by-step method that works for every problem, the following general procedures facilitate problem solving and make it more meaningful. A certain amount of creativity and insight is required as well.

Step 1

Examine the situation to determine which physical principles are involved. It often helps to *draw a simple sketch* at the outset. You will also need to decide which direction is positive and note that on your sketch. Once you have identified the physical principles, it is much easier to find and apply the equations representing those principles. Although finding the correct equation is essential, keep in mind that equations represent physical principles, laws of nature, and relationships among physical quantities. Without a conceptual understanding of a problem, a numerical solution is meaningless.

Step 2

Make a list of what is given or can be inferred from the problem as stated (identify the knowns). Many problems are stated very succinctly and require some inspection to determine what is known. A sketch can also be very useful at this point. Formally identifying the knowns is of particular importance in applying physics to real-world situations. Remember, “stopped” means velocity is zero, and we often can take initial time and position as zero.

Step 3

Identify exactly what needs to be determined in the problem (identify the unknowns). In complex problems, especially, it is not always obvious what needs to be found or in what sequence. Making a list can help.

Step 4

Find an equation or set of equations that can help you solve the problem. Your list of knowns and unknowns can help here. It is easiest if you can find equations that contain only one unknown—that is, all of the other variables are known, so you can easily solve for the unknown. If the equation contains more than one unknown, then an additional equation is needed to solve the problem. In some problems, several unknowns must be determined to get at the one needed most. In such problems it is especially important to keep physical principles in mind to avoid going astray in a sea of equations. You may have to use two (or more) different equations to get the final answer.

Step 5

Substitute the knowns along with their units into the appropriate equation, and obtain numerical solutions complete with units. This step produces the numerical answer; it also provides a check on units that can help you find errors. If the units of the answer are incorrect, then an error has been made. However, be warned that correct units do not guarantee that the numerical part of the answer is also correct.

Step 6

Check the answer to see if it is reasonable: Does it make sense? This final step is extremely important—the goal of physics is to accurately describe nature. To see if the answer is reasonable, check both its magnitude and its sign, in addition to its units. Your judgment will improve as you solve more and more physics problems, and it will become possible for you to make finer and finer judgments regarding whether nature is adequately described by the answer to a problem. This step brings the problem back to its conceptual meaning. If you can judge whether the answer is reasonable, you have a deeper understanding of physics than just being able to mechanically solve a problem.

When solving problems, we often perform these steps in different order, and we also tend to do several steps simultaneously. There is no rigid procedure that will work every time. Creativity and insight grow with experience, and the basics of problem solving become almost automatic. One way to get practice is to work out the text's examples for yourself as you read. Another is to work as many end-of-section problems as possible, starting with the easiest to build confidence and progressing to the more difficult. Once you become involved in physics, you will see it all around you, and you can begin to apply it to situations you encounter outside the classroom, just as is done in many of the applications in this text.

Unreasonable Results

Physics must describe nature accurately. Some problems have results that are unreasonable because one premise is unreasonable or because certain premises are inconsistent with one another. The physical principle applied correctly then produces an unreasonable result. For example, if a person starting a foot race accelerates at 0.40m/s^2 for 100 s, their final speed will be 40 m/s (about 150 km/h)—clearly unreasonable because the time of 100 s is an unreasonable premise. The physics is correct in a sense, but there is more to describing nature than just manipulating equations correctly. Checking the result of a problem to see if it is reasonable does more than help uncover errors in problem solving—it also builds intuition in judging whether nature is being accurately described.

Use the following strategies to determine whether an answer is reasonable and, if it is not, to determine what is the cause.

Step 1

Solve the problem using strategies as outlined and in the format followed in the worked examples in the text. In the example given in the preceding paragraph, you would identify the givens as the acceleration and time and use the equation below to find the unknown final velocity. That is,

$$v=v_0+at=0+(0.40\text{m/s}^2)(100\text{s})=40\text{m/s}.$$

Step 2

Check to see if the answer is reasonable. Is it too large or too small, or does it have the wrong sign, improper units, …? In this case, you may need to convert meters per second into a more familiar unit, such as miles per hour.

$$\left(\frac{40\text{m}}{\text{s}}\right)\left(\frac{3.28\text{ft}}{\text{m}}\right)\left(\frac{1\text{mi}}{5280\text{ft}}\right)\left(\frac{60\text{s}}{\text{min}}\right)\left(\frac{60\text{min}}{1\text{h}}\right)=89\text{mph}$$

This velocity is about four times greater than a person can run—so it is too large.

Step 3

If the answer is unreasonable, look for what specifically could cause the identified difficulty. In the example of the runner, there are only two assumptions that are suspect. The acceleration could be too great or the time too long. First look at the acceleration and think about what the number means. If someone accelerates at 0.40m/s^2, their velocity is increasing by 0.4 m/s each second. Does this seem reasonable? If so, the time must be too long. It is not possible for someone to accelerate at a constant rate of 0.40m/s^2 for 100 s (almost two minutes).

Section Summary

- *The six basic problem solving steps for physics are:*

  Step 1. Examine the situation to determine which physical principles are involved.

  Step 2. Make a list of what is given or can be inferred from the problem as stated (identify the knowns).

Step 3. Identify exactly what needs to be determined in the problem (identify the unknowns).

Step 4. Find an equation or set of equations that can help you solve the problem.

Step 5. Substitute the knowns along with their units into the appropriate equation, and obtain numerical solutions complete with units.

Step 6. Check the answer to see if it is reasonable: Does it make sense?

Conceptual Questions

What information do you need in order to choose which equation or equations to use to solve a problem? Explain.

What is the last thing you should do when solving a problem? Explain.

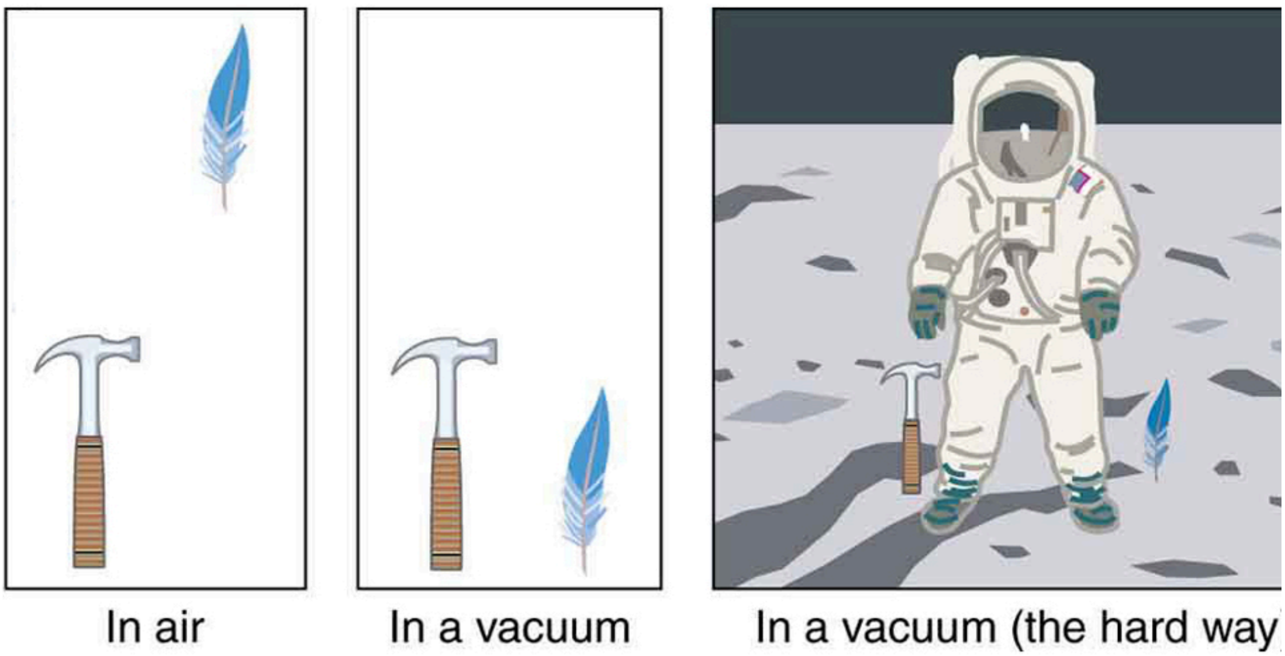
Falling Objects

- Describe the effects of gravity on objects in motion.
- Describe the motion of objects that are in free fall.
- Calculate the position and velocity of objects in free fall.

Falling objects form an interesting class of motion problems. For example, we can estimate the depth of a vertical mine shaft by dropping a rock into it and listening for the rock to hit the bottom. By applying the kinematics developed so far to falling objects, we can examine some interesting situations and learn much about gravity in the process.

Gravity

The most remarkable and unexpected fact about falling objects is that, if air resistance and friction are negligible, then in a given location all objects fall toward the center of Earth with the *same constant acceleration, independent of their mass*. This experimentally determined fact is unexpected, because we are so accustomed to the effects of air resistance and friction that we expect light objects to fall slower than heavy ones.



A hammer and a feather will fall with the same constant acceleration if air resistance is considered negligible. This is a general characteristic of gravity not unique to Earth, as astronaut David R. Scott demonstrated on the Moon in 1971, where the acceleration due to gravity is only $1.67 \mss$

In the real world, air resistance can cause a lighter object to fall slower than a heavier object of the same size. A tennis ball will reach the ground after a hard baseball dropped at the same time. (It might be difficult to observe the difference if the height is not large.) Air resistance opposes the motion of an object through the air, while friction between objects—such as between clothes and a laundry chute or between a stone and a pool into which it is dropped—also opposes motion between them. For the ideal situations of these first few chapters, an object *falling without air resistance or friction* is defined to be in **free-fall**.

The force of gravity causes objects to fall toward the center of Earth. The acceleration of free-falling objects is therefore called the **acceleration due to gravity**. The acceleration due to gravity is *constant*, which means we can apply the kinematics equations to any falling object where air resistance and friction are negligible. This opens a broad class of interesting situations to us. The acceleration due to gravity is so important that its magnitude is given its own symbol, g. It is constant at any given location on Earth and has the average value

$$g = 9.80 \text{m/s}^2.$$

Although g varies from 9.78m/s^2 to 9.83m/s^2, depending on latitude, altitude, underlying geological formations, and local topography, the average value of 9.80m/s^2 will be used in this text unless otherwise specified. The direction of the acceleration due to gravity is *downward (towards the center of Earth)*. In fact, its direction *defines* what we call vertical. Note that whether the acceleration a in the kinematic equations has the value $+g$ or $-g$ depends on how we define our coordinate system. If we define the upward direction as positive, then $a = -g = -9.80 \text{m/s}^2$ $,and if we define the downward direction as positive, then$ a=g=9.80 \mss

.

One-Dimensional Motion Involving Gravity

The best way to see the basic features of motion involving gravity is to start with the simplest situations and then progress toward more complex ones. So we start by considering straight up and down motion with no air resistance or friction. These assumptions mean that the velocity (if there is any) is vertical. If the object is dropped, we know the initial velocity is zero. Once the object has left contact with whatever held or threw it, the object is in free-fall. Under these circumstances, the motion is one-dimensional and has constant acceleration of magnitude g. We will also represent vertical displacement with the symbol y and use x for horizontal displacement.

Kinematic Equations for Objects in Free-Fall where Acceleration = -*g*

$v=v_0-gt$

$y=y_0+v_0t-12gt^2$

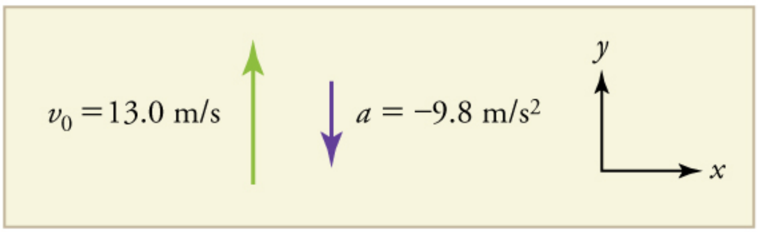
$v^2=v_{20}-2g(y-y_0)$

Calculating Position and Velocity of a Falling Object: A Rock Thrown Upward

A person standing on the edge of a high cliff throws a rock straight up with an initial velocity of 13.0 m/s. The rock misses the edge of the cliff as it falls back to earth. Calculate the position and velocity of the rock 1.00 s, 2.00 s, and 3.00 s after it is thrown, neglecting the effects of air resistance.

Strategy

Draw a sketch.



Velocity and Acceleration Sketch

We are asked to determine the position y at various times. It is reasonable to take the initial position y_0 to be zero. This problem involves one-dimensional motion in the vertical direction. We use plus and minus signs to indicate direction, with up being positive and down negative. Since up is positive, and the rock is thrown upward, the initial velocity must be positive too. The acceleration due to gravity is downward, so a is negative. It is crucial that the initial velocity and the acceleration due to gravity have opposite signs. Opposite signs indicate that the acceleration due to gravity opposes the initial motion and will slow and eventually reverse it.

Since we are asked for values of position and velocity at three times, we will refer to these as y_1 and v_1; y_2 and v_2; and y_3 and v_3.

Solution for Position y_1

1. Identify the knowns. We know that $y_0 = 0$; $v_0 = 13.0 \text{m/s}$; $a = -g = -9.80 \text{m/s}^2$; and $t = 1.00 \text{s}$.

2. Identify the best equation to use. We will use $y = y_0 + v_0 t + 12 a t^2$ because it includes only one unknown, y (or y_1, here), which is the value we want to find.

3. Plug in the known values and solve for y_1.

$$y_1=0+(13.0\text{m/s})(1.00\text{s})+12(-9.80\text{m/s}^2)(1.00\text{s})^2=8.10\text{m}$$

Discussion

The rock is 8.10 m above its starting point at $t = 1.00 \text{s}$, since $y_1 > y_0$. It could be *moving* up or down; the only way to tell is to calculate v_1 and find out if it is positive or negative. **Solution for Velocity v_1**

1. Identify the knowns. We know that $y_0 = 0$; $v_0 = 13.0 \text{m/s}$; $a = -g = -9.80 \text{m/s}^2$; and $t = 1.00 \text{s}$. We also know from the solution above that $y_1 = 8.10 \text{m}$.

2. Identify the best equation to use. The most straightforward is $v = v_0 - gt$ (from $v = v\_0 + at$, where $a = \text{gravitational acceleration} = -g$).

3. Plug in the knowns and solve.

$$v_1=v_0-gt=13.0\text{m/s}-(9.80\text{m/s}^2)(1.00\text{s})=3.20\text{m/s}$$

Discussion

The positive value for v_1 means that the rock is still heading upward at $t = 1.00 \text{s}$. However, it has slowed from its original 13.0** m/s, as expected.

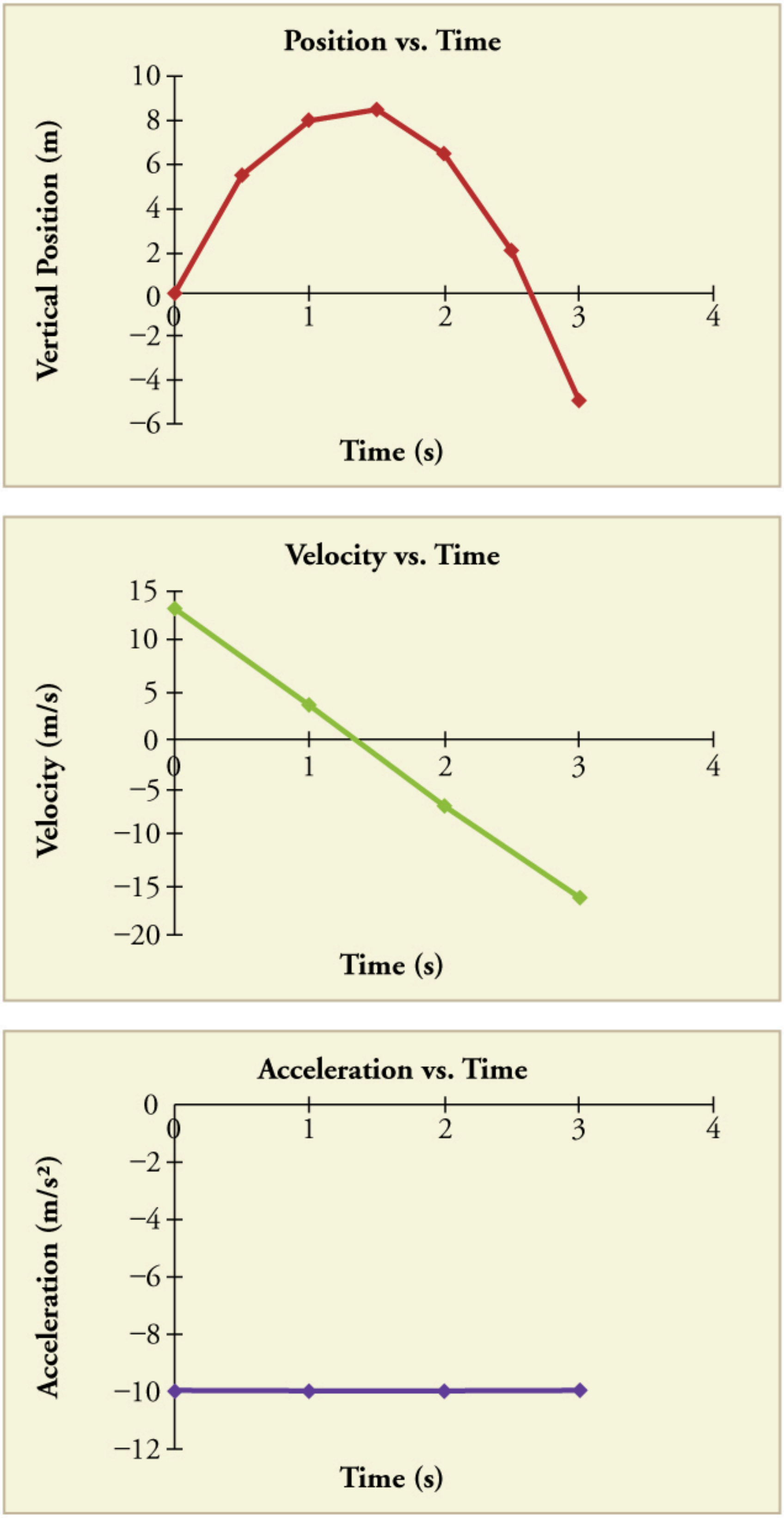
Solution for Remaining Times

The procedures for calculating the position and velocity at $t = 2.00\text{s}$ and 3.00s are the same as those above. The results are summarized in Table 1 and illustrated in Figure 3.

Table: Results

Time, t	Position, y	Velocity, v	Acceleration, a
1.00s	8.10m	3.20m/s	-9.80m/s^2
2.00s	6.40m	−6.60m/s	-9.80m/s^2
3.00s	−5.10m	−16.4m/s	-9.80m/s^2

Graphing the data helps us understand it more clearly.



Vertical position, vertical velocity, and vertical acceleration vs. time for a rock thrown vertically up at the edge of a cliff. Notice that velocity changes linearly with time and that acceleration is constant. Misconception Alert! Notice that the position vs. time graph shows vertical position only. It is easy to get the impression that the graph shows some horizontal motion;the shape of the graph looks like the path of a projectile. But this is not the case; the horizontal axis is time, not space. The actual path of the rock in space is straight up, and straight down.

Discussion

The interpretation of these results is important. At 1.00 s the rock is above its starting point and heading upward, since y_1 and v_1 are both positive. At 2.00 s, the rock is still above its starting point, but the negative velocity means it is moving downward. At 3.00 s, both y_3 and v_3 are negative, meaning the rock is below its starting point and continuing to move downward. Notice that when the rock is at its highest point (at 1.3 s), its velocity is zero, but its acceleration is still -9.80m/s^2. Its acceleration is -9.80m/s^2 for the whole trip—while it is moving up and while it is moving down. Note that the values for y are the positions (or displacements) of the rock, not the total distances traveled. Finally, note that free-fall applies to upward motion as well as downward. Both have the same acceleration—the acceleration due to gravity, which remains constant the entire time. Astronauts training in the famous Vomit Comet, for example, experience free-fall while arcing up as well as down, as we will discuss in more detail later.
Making Connections: Take-Home Experiment—Reaction Time

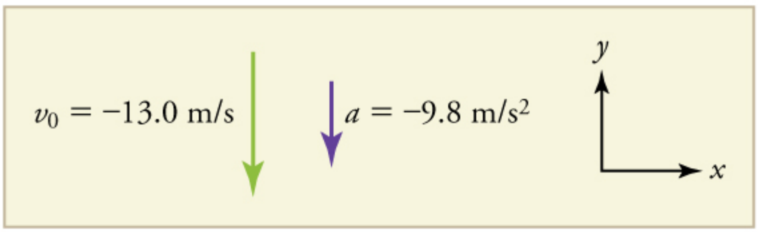
A simple experiment can be done to determine your reaction time. Have a friend hold a ruler between your thumb and index finger, separated by about 1 cm. Note the mark on the ruler that is right between your fingers. Have your friend drop the ruler unexpectedly, and try to catch it between your two fingers. Note the new reading on the ruler. Assuming acceleration is that due to gravity, calculate your reaction time. How far would you travel in a car (moving at 30 m/s) if the time it took your foot to go from the gas pedal to the brake was twice this reaction time?

Calculating Velocity of a Falling Object: A Rock Thrown Down

What happens if the person on the cliff throws the rock straight down, instead of straight up? To explore this question, calculate the velocity of the rock when it is 5.10 m below the starting point, and has been thrown downward with an initial speed of 13.0 m/s.

Strategy

Draw a sketch.



Velocity and Accelerator Sketch

Since up is positive, the final position of the rock will be negative because it finishes below the starting point at $y\_0 = 0$. Similarly, the initial velocity is downward and therefore negative, as is the acceleration due to gravity. We expect the final velocity to be negative since the rock will continue to move downward.

Solution

1. Identify the knowns. $y_0 = 0$; $y_1 = -5.10 \text{m}$; $v\_0 = -13.0 \text{m/s}$; $a = -g = -9.80 \text{m/s}^2$.

2. Choose the kinematic equation that makes it easiest to solve the problem. The equation $v^2 = v_{20} + 2a(y - y_0)$ works well because the only unknown in it is v. (We will plug y_1 in for y.)

3. Enter the known values

$$v^2 = (-13.0 \text{m/s})^2 + 2(-9.80 \text{m/s}^2)(-5.10 \text{m} - 0 \text{m}) = 268.96 \text{m}^2/\text{s}^2,$$

where we have retained extra significant figures because this is an intermediate result.

Taking the square root, and noting that a square root can be positive or negative, gives

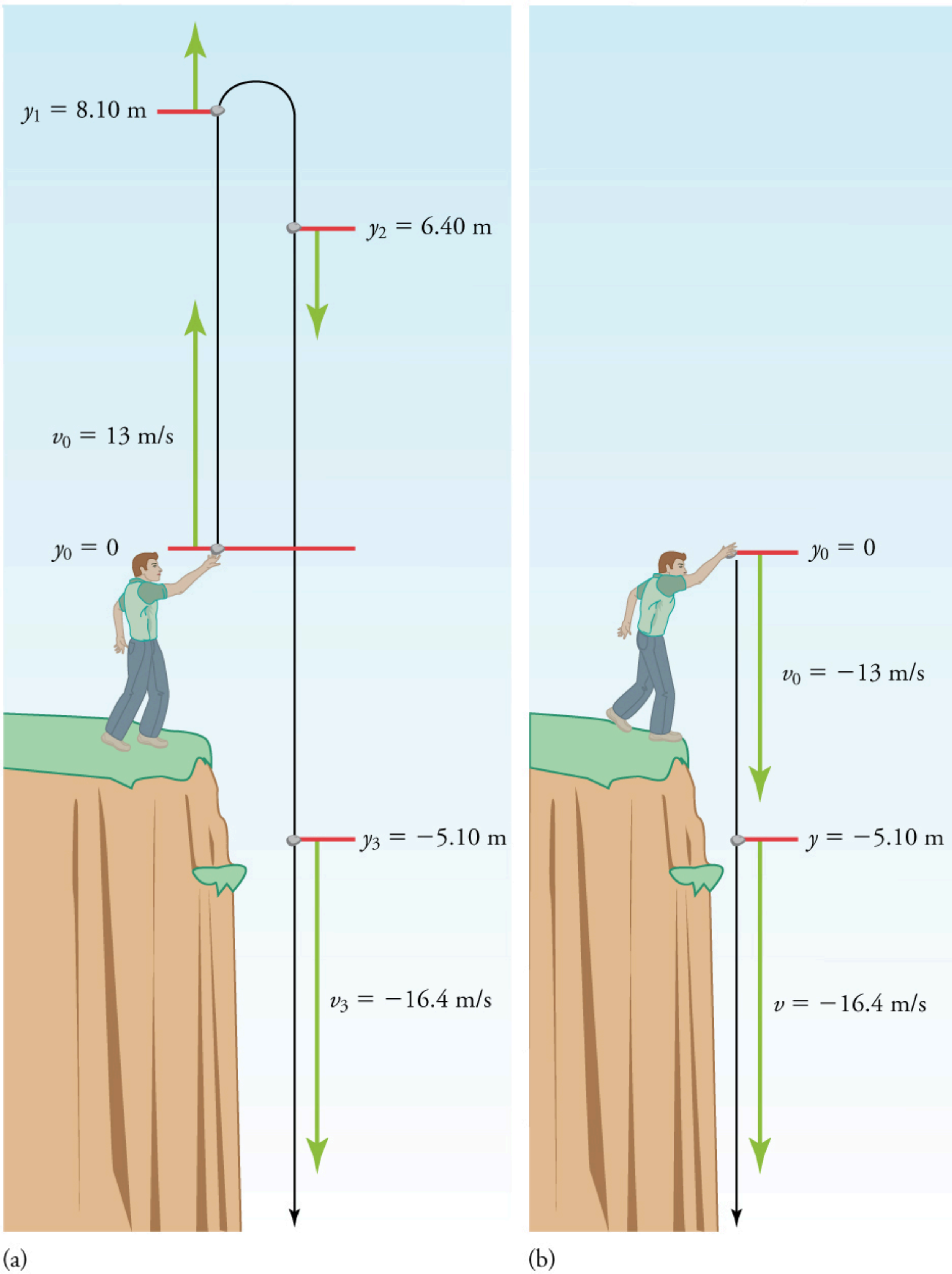
$$v = \pm 16.4 \text{m/s}.$$

The negative root is chosen to indicate that the rock is still heading down. Thus,

$$v = -16.4 \text{m/s}.$$

Discussion

Note that *this is exactly the same velocity the rock had at this position when it was thrown straight upward with the same initial speed*. (See Example 1 and Figure 5(a).) This is not a coincidental result. Because we only consider the acceleration due to gravity in this problem, the *speed* of a falling object depends only on its initial speed and its vertical position relative to the starting point. For example, if the velocity of the rock is calculated at a height of 8.10 m above the starting point (using the method from Example 1) when the initial velocity is 13.0 m/s straight up, a result of $\pm 3.20 \text{m/s}$ is obtained. Here both signs are meaningful; the positive value occurs when the rock is at 8.10 m and heading up, and the negative value occurs when the rock is at 8.10 m and heading back down. It has the same *speed* but the opposite direction.

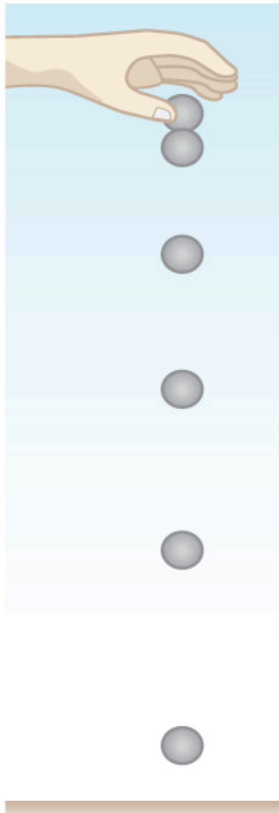


(a) A person throws a rock straight up, as explored in Example 1. The arrows are velocity vectors at 0, 1.00, 2.00, and 3.00 s. (b) A person throws a rock straight down from a cliff with the same initial speed as before, as in Example 2. Note that at the same distance below the point of release, the rock has the same velocity in both cases.

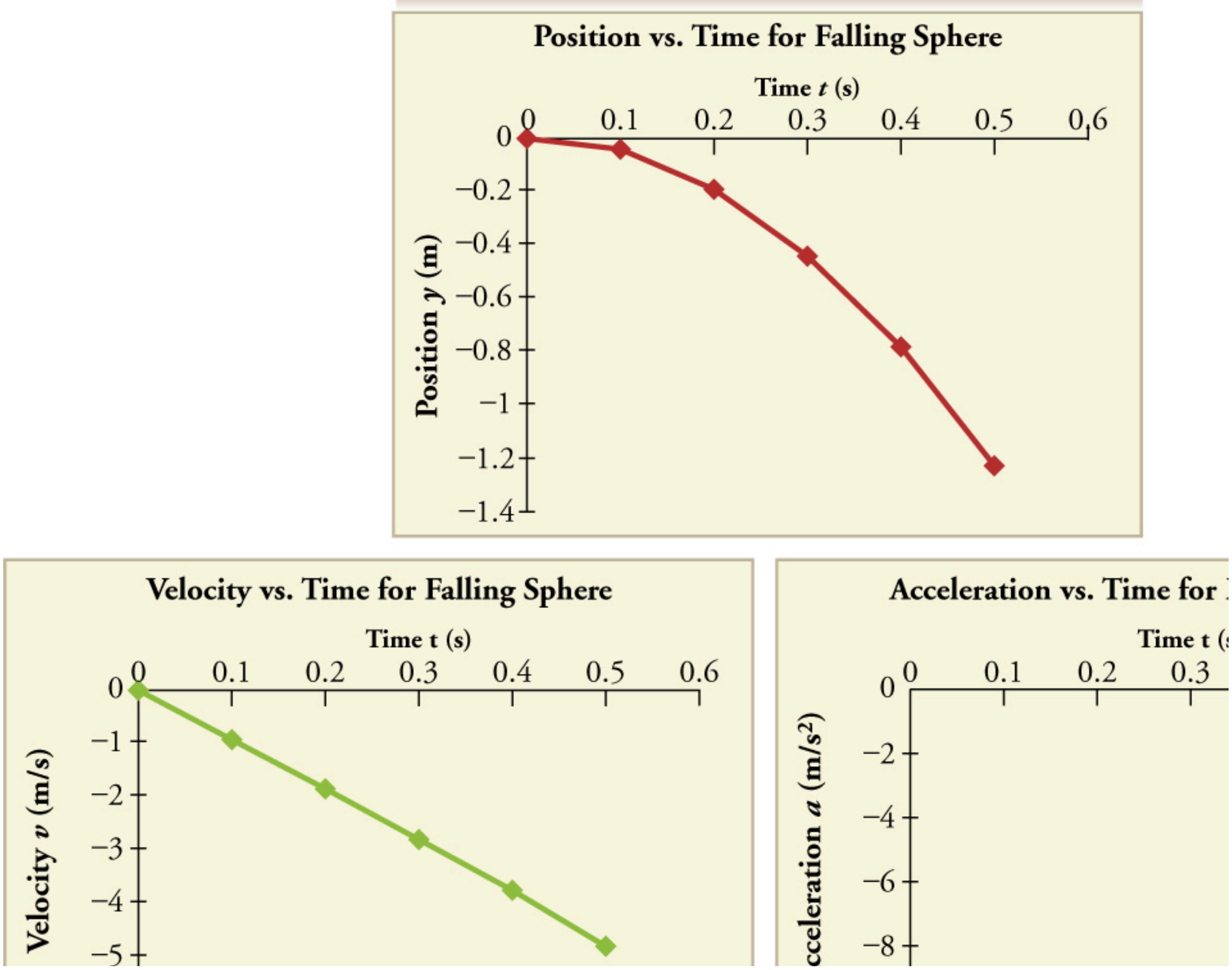
Another way to look at it is this: In Example 1, the rock is thrown up with an initial velocity of 13.0m/s. It rises and then falls back down. When its position is $y=0$ on its way back down, its velocity is -13.0m/s. That is, it has the same speed on its way down as on its way up. We would then expect

its velocity at a position of $y=-5.10\text{m}$ to be the same whether we have thrown it upwards at $+13.0\text{m/s}$ or thrown it downwards at -13.0m/s. The velocity of the rock on its way down from $y=0$ is the same whether we have thrown it up or down to start with, as long as the speed with which it was initially thrown is the same.
Find *g* from Data on a Falling Object

The acceleration due to gravity on Earth differs slightly from place to place, depending on topography (e.g., whether you are on a hill or in a valley) and subsurface geology (whether there is dense rock like iron ore as opposed to light rock like salt beneath you.) The precise acceleration due to gravity can be calculated from data taken in an introductory physics laboratory course. An object, usually a metal ball for which air resistance is negligible, is dropped and the time it takes to fall a known distance is measured. See, for example, Figure 6. Very precise results can be produced with this method if sufficient care is taken in measuring the distance fallen and the elapsed time.



y (m)	v (m/s)	t (s)
0	0	0
−0.049	−0.98	0.1
−0.196	−1.96	0.2
−0.441	−2.94	0.3
−0.784	−3.92	0.4
−1.225	−4.90	0.5

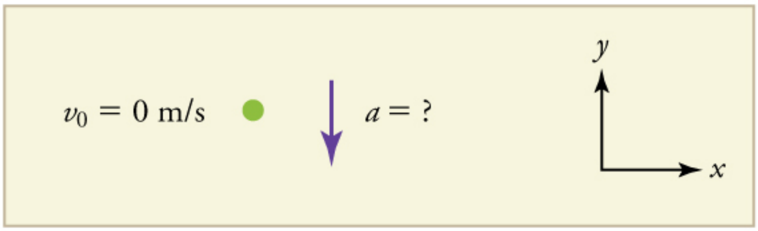


Positions and velocities of a metal ball released from rest when air resistance is negligible. Velocity is seen to increase linearly with time while displacement increases with time squared. Acceleration is a constant and is equal to gravitational acceleration.

Suppose the ball falls 1.0000 m in 0.45173 s. Assuming the ball is not affected by air resistance, what is the precise acceleration due to gravity at this location?

Strategy

Draw a sketch.



We need to solve for acceleration a. Note that in this case, displacement is downward and therefore negative, as is acceleration.

Solution

1. Identify the knowns. $y_0 = 0$; $y = -1.0000\text{m}$; $t = 0.45173\text{s}$; $v_0 = 0$.

2. Choose the equation that allows you to solve for a using the known values.

$$y=y_0+v_0t+12at^2$$

3. Substitute 0 for v_0 and rearrange the equation to solve for a. Substituting 0 for v_0 yields

$$y=y_0+12at^2.$$

Solving for a gives

$$a=2(y-y_0)t^2.$$

4. Substitute known values yields

$$a=2(-1.0000\text{m}-0)(0.45173\text{s})^2=-9.8010\text{m/s}^2,$$

so, because $a = -g$ with the directions we have chosen,

$$g=9.8010\text{m/s}^2.$$

Discussion

The negative value for a indicates that the gravitational acceleration is downward, as expected. We expect the value to be somewhere around the average value of 9.80m/s^2, so 9.8010m/s^2 makes sense. Since the data going into the calculation are relatively precise, this value for g is more precise than the average value of 9.80m/s^2; it represents the local value for the acceleration due to gravity.
Check Your Understanding

A chunk of ice breaks off a glacier and falls 30.0 meters before it hits the water. Assuming it falls freely (there is no air resistance), how long does it take to hit the water?

Show Solution

We know that initial position $y_0 = 0$, final position $y = -30.0\text{m}$, and $a = -g = -9.80\text{m/s}^2$. We can then use the equation $y = y_0 + v_0t + 12at^2$ to solve for t. Inserting $a = -g$, we obtain <div class="equation">

$$y = 0+0-12gt^2 \quad t^2 = 2y-g \quad t = \pm\sqrt{2y-g}=\pm\sqrt{2(-30.0\text{m})-9.80\text{m/s}^2}=\pm\sqrt{6.12\text{s}^2}=2.47\text{s}\approx 2.5\text{s}$$

where we take the positive value as the physically relevant answer. Thus, it takes about 2.5 seconds for the piece of ice to hit the water.

</div>

Kinematics of an Elevator

The three kinematic variables, position, velocity, and acceleration are all related. See how they evolve graphically.

Kinematics of an Elevator

Section Summary

- An object in free-fall experiences constant acceleration if air resistance is negligible.
- On Earth, all free-falling objects have an acceleration due to gravity g

  , which averages

  $g = 9.80 \text{m/s}^2.$

- Whether the acceleration a should be taken as $+g$ or $-g$ is determined by your choice of coordinate system. If you choose the upward direction as positive, $a=-g=-9.80\text{m/s}^2$ is negative. In the opposite case, $a=+g=9.80\text{m/s}^2$ is positive. Since acceleration is constant, the kinematic equations above can be applied with the appropriate $+g$ or $-g$ substituted for a.
- For objects in free-fall, up is normally taken as positive for displacement, velocity, and acceleration.

Conceptual Questions

What is the acceleration of a rock thrown straight upward on the way up? At the top of its flight? On the way down?

An object that is thrown straight up falls back to Earth. This is one-dimensional motion. (a) When is its velocity zero? (b) Does its velocity change direction? (c) Does the acceleration due to gravity have the same sign on the way up as on the way down?

Suppose you throw a rock nearly straight up at a coconut in a palm tree, and the rock misses on the way up but hits the coconut on the way down. Neglecting air resistance, how does the speed of the rock when it hits the coconut on the way down compare with what it would have been if it had hit the coconut on the way up? Is it more likely to dislodge the coconut on the way up or down? Explain.

If an object is thrown straight up and air resistance is negligible, then its speed when it returns to the starting point is the same as when it was released. If air resistance were not negligible, how would its speed upon return compare with its initial speed? How would the maximum height to which it rises be affected?

The severity of a fall depends on your speed when you strike the ground. All factors but the acceleration due to gravity being the same, how many times higher could a safe fall on the Moon be than on Earth (gravitational acceleration on the Moon is about 1/6 that of the Earth)?

How many times higher could an astronaut jump on the Moon than on Earth if his takeoff speed is the same in both locations (gravitational acceleration on the Moon is about 1/6 of g on Earth)?

Problems & Exercises

Assume air resistance is negligible unless otherwise stated.

Calculate the displacement and velocity at times of (a) 0.500, (b) 1.00, (c) 1.50, and (d) 2.00 s for a ball thrown straight up with an initial velocity of 15.0 m/s. Take the point of release to be $y_0=0$.

Show Solution

Strategy

Use the kinematic equations for free-fall with upward as positive. Since the ball is thrown upward, the initial velocity is positive, while the acceleration due to gravity is negative ($a=-g=-9.80\text{m/s}^2$).

Solution

For position: $y=y_0+v_0t+12at^2=v_0t-12gt^2$

For velocity: $v=v_0+at=v_0-gt$

(a) At t = 0.500 s:

$$y_1=(15.0\text{m/s})(0.500\text{s})-12(9.80\text{m/s}^2)(0.500\text{s})^2=7.50\text{m}-1.23\text{m}=6.28\text{m}$$
$$v_1=15.0\text{m/s}-(9.80\text{m/s}^2)(0.500\text{s})=15.0\text{m/s}-4.90\text{m/s}=10.1\text{m/s}$$

(b) At t = 1.00 s:

$$y_2=(15.0\text{m/s})(1.00\text{s})-12(9.80\text{m/s}^2)(1.00\text{s})^2=15.0\text{m}-4.90\text{m}=10.1\text{m}$$
$$v_2=15.0\text{m/s}-(9.80\text{m/s}^2)(1.00\text{s})=5.20\text{m/s}$$

(c) At t = 1.50 s:

$$y_3=(15.0\text{m/s})(1.50\text{s})-12(9.80\text{m/s}^2)(1.50\text{s})^2=22.5\text{m}-11.0\text{m}=11.5\text{m}$$
$$v_3=15.0\text{m/s}-(9.80\text{m/s}^2)(1.50\text{s})=0.300\text{m/s}$$

(d) At t = 2.00 s:

$$y_4=(15.0\text{m/s})(2.00\text{s})-12(9.80\text{m/s}^2)(2.00\text{s})^2=30.0\text{m}-19.6\text{m}=10.4\text{m}$$
$$v_4=15.0\text{m/s}-(9.80\text{m/s}^2)(2.00\text{s})=-4.60\text{m/s}$$

Discussion

At t = 1.50 s, the velocity is nearly zero, indicating the ball is near its maximum height. The negative velocity at t = 2.00 s confirms the ball is now moving downward.

(a) At 0.500 s, the ball is at $y_1 = 6.28\text{m}$ with velocity $v_1 = 10.1\text{m/s}$ upward.

(b) At 1.00 s, the ball is at $y_2 = 10.1\text{m}$ with velocity $v_2 = 5.20\text{m/s}$ upward.

(c) At 1.50 s, the ball is at $y_3 = 11.5\text{m}$ with velocity $v_3 = 0.300\text{m/s}$ upward.

(d) At 2.00 s, the ball is at $y_4 = 10.4\text{m}$ with velocity $v_4 = 4.60\text{m/s}$ downward.

Calculate the displacement and velocity at times of (a) 0.500, (b) 1.00, (c) 1.50, (d) 2.00, and (e) 2.50 s for a rock thrown straight down with an initial velocity of 14.0 m/s from the Verrazano Narrows Bridge in New York City. The roadway of this bridge is 70.0 m above the water.

Show Solution

Strategy

Take downward as positive since the rock is thrown downward. This means the initial velocity $v_0 = +14.0\text{m/s}$ and $a = +g = +9.80\text{m/s}^2$. Set $y_0 = 0$ at the bridge.

Solution

For position: $y = y_0 + v_0 t + 12gt^2 = v_0 t + 12gt^2$

For velocity: $v = v_0 + gt$

(a) At t = 0.500 s:

$$y_1 = (14.0\text{m/s})(0.500\text{s}) + 12(9.80\text{m/s}^2)(0.500\text{s})^2 = 7.00\text{m} + 1.23\text{m} = 8.23\text{m}$$
$$v_1 = 14.0\text{m/s} + (9.80\text{m/s}^2)(0.500\text{s}) = 18.9\text{m/s}$$

(b) At t = 1.00 s:

$$y_2 = (14.0\text{m/s})(1.00\text{s}) + 12(9.80\text{m/s}^2)(1.00\text{s})^2 = 14.0\text{m} + 4.90\text{m} = 18.9\text{m}$$
$$v_2 = 14.0\text{m/s} + (9.80\text{m/s}^2)(1.00\text{s}) = 23.8\text{m/s}$$

(c) At t = 1.50 s:

$$y_3 = (14.0\text{m/s})(1.50\text{s}) + 12(9.80\text{m/s}^2)(1.50\text{s})^2 = 21.0\text{m} + 11.0\text{m} = 32.0\text{m}$$
$$v_3 = 14.0\text{m/s} + (9.80\text{m/s}^2)(1.50\text{s}) = 28.7\text{m/s}$$

(d) At t = 2.00 s:

$$y_4 = (14.0\text{m/s})(2.00\text{s}) + 12(9.80\text{m/s}^2)(2.00\text{s})^2 = 28.0\text{m} + 19.6\text{m} = 47.6\text{m}$$
$$v_4 = 14.0\text{m/s} + (9.80\text{m/s}^2)(2.00\text{s}) = 33.6\text{m/s}$$

(e) At t = 2.50 s:

$$y_5 = (14.0\text{m/s})(2.50\text{s}) + 12(9.80\text{m/s}^2)(2.50\text{s})^2 = 35.0\text{m} + 30.6\text{m} = 65.6\text{m}$$
$$v_5 = 14.0\text{m/s} + (9.80\text{m/s}^2)(2.50\text{s}) = 38.5\text{m/s}$$

Discussion

At 2.50 s, the rock has fallen 65.6 m, which is still above the water (70.0 m below the bridge). The rock continues to accelerate as it falls.

(a) At 0.500 s: displacement = 8.23 m, velocity = 18.9 m/s downward.

(b) At 1.00 s: displacement = 18.9 m, velocity = 23.8 m/s downward.

(c) At 1.50 s: displacement = 32.0 m, velocity = 28.7 m/s downward.

(d) At 2.00 s: displacement = 47.6 m, velocity = 33.6 m/s downward.

(e) At 2.50 s: displacement = 65.6 m, velocity = 38.5 m/s downward.

A basketball referee tosses the ball straight up for the starting tip-off. At what minimum velocity must a basketball player leave the ground to rise 1.25 m above the floor in an attempt to get the ball?

Show Solution

Strategy

At the maximum height, the player's velocity is zero. We can use the kinematic equation relating velocity, displacement, and acceleration to find the initial velocity needed to reach 1.25 m.

Solution

1. Identify the known values:
    - Maximum height: $y = 1.25 \text{m}$
    - Final velocity at maximum height: $v = 0$
    - Initial position: $y_0 = 0$
    - Acceleration: $a = -g = -9.80 \text{m/s}^2$ (taking upward as positive)
2. Choose the appropriate kinematic equation:

$$v^2 = v_{20} + 2a(y - y_0)$$

1. Solve for initial velocity v_0:

$$0 = v_{20} + 2(-9.80 \text{m/s}^2)(1.25 \text{m} - 0)$$
$$v_{20} = 2(9.80 \text{m/s}^2)(1.25 \text{m}) = 24.5 \text{m}^2/\text{s}^2$$
$$v_0 = \sqrt{24.5 \text{m}^2/\text{s}^2} = 4.95 \text{m/s}$$

Discussion

This is a reasonable jumping speed for an athlete. The calculation shows that to reach a height of 1.25 m, a player must leave the ground with a velocity of about 5 m/s (approximately 18 km/h).

The basketball player must leave the ground with a minimum velocity of 4.95m/s (about 5.0 m/s) to rise 1.25 m above the floor.

A rescue helicopter is hovering over a person whose boat has sunk. One of the rescuers throws a life preserver straight down to the victim with an initial velocity of 1.40 m/s and observes that it takes 1.8 s to reach the water. (a) List the knowns in this problem. (b) How high above the water was the preserver released? Note that the downdraft of the helicopter reduces the effects of air resistance on the falling life preserver, so that an acceleration equal to that of gravity is reasonable.

Show Solution

Strategy

Take downward as positive since the preserver is thrown downward. Use the kinematic equation for displacement.

Solution

(a) Known values:

- Initial velocity: $v_0 = 1.40 \text{m/s}$ (downward, positive)
- Time: $t = 1.8 \text{s}$
- Acceleration: $a = g = 9.80 \text{m/s}^2$ (downward, positive)
- Initial position: $y_0 = 0$

(b) Height above water:

Use the kinematic equation:

$$y = y_0 + v_0 t + 12 g t^2$$

Substitute the known values:

$$y = 0 + (1.40 \text{m/s})(1.8 \text{s}) + 12(9.80 \text{m/s}^2)(1.8 \text{s})^2$$
$$y = 2.52 \text{m} + 12(9.80 \text{m/s}^2)(3.24 \text{s}^2)$$
$$y = 2.52 \text{m} + 15.9 \text{m} = 18.4 \text{m}$$

Discussion

The helicopter was hovering about 18 meters above the water. This is a reasonable height for a rescue operation, allowing the crew to see the victim while staying clear of the water.

(a) The knowns are: $v_0 = 1.40 \text{m/s}$, $t = 1.8 \text{s}$, $a = g = 9.80 \text{m/s}^2$, and $y_0 = 0$.

(b) The life preserver was released from a height of approximately 18m above the water.

A dolphin in an aquatic show jumps straight up out of the water at a velocity of 13.0 m/s. (a) List the knowns in this problem. (b) How high does his body rise above the water? To solve this part, first note that the final velocity is now a known and identify its value. Then identify the unknown, and discuss how you chose the appropriate equation to solve for it. After choosing the equation, show your steps in solving for the unknown, checking units, and discuss whether the answer is reasonable. (c) How long is the dolphin in the air? Neglect any effects due to his size or orientation.

Show Solution

(a) $a = -9.80 \text{m/s}^2$; $v_0 = 13.0 \text{m/s}$; $y_0 = 0 \text{m}$ (b) $v = 0 \text{m/s}$. Unknown is distance y to top of trajectory, where velocity is zero. Use equation $v^2 = v_0^2 + 2a(y - y_0)$ because it contains all known values except for y, so we can solve for y. Solving for y gives

$$v^2 - v_0^2 = 2a(y - y_0) \quad \frac{v^2 - v_0^2}{2a} = y - y_0 \quad y = y_0 + \frac{v^2 - v_0^2}{2a} = 0 m + \frac{(0 \text{m/s})^2 - (13.0 \text{m/s})^2}{2(-9.80 \text{m/s}^2)} = 8.62 \text{m}$$

Dolphins measure about 2 meters long and can jump several times their length out of the water, so this is a reasonable result.

(c) 2.65s

A swimmer bounces straight up from a diving board and falls feet first into a pool. She starts with a velocity of 4.00 m/s, and her takeoff point is 1.80 m above the pool. (a) How long are her feet in the air? (b) What is her highest point above the board? (c) What is her velocity when her feet hit the water?

Show Solution

Strategy

Take upward as positive. The swimmer launches upward from the diving board with initial velocity $v_0 = 4.00 \text{m/s}$, reaches a maximum height, then falls through the air until her feet hit the water 1.80 m below the starting point. Use $g = 9.80 \text{m/s}^2$ for the acceleration due to gravity.

Solution

(a) Time in the air:

1. Identify the known values:
   - Initial velocity: $v_0 = +4.00 \text{m/s}$ (upward)
   - Initial position: $y_0 = 0$ (at the board)
   - Final position: $y = -1.80 \text{m}$ (at water level, below the board)
   - Acceleration: $a = -g = -9.80 \text{m/s}^2$
2. Use the kinematic equation for displacement:

$$y = y_0 + v_0 t + \frac{1}{2} a t^2$$

1. Substitute the known values:

$$-1.80 = 0 + (4.00)t + \frac{1}{2}(-9.80)t^2$$
$$-1.80 = 4.00t - 4.90t^2$$

Rearranging into standard quadratic form:

$$4.90t^2 - 4.00t - 1.80 = 0$$

1. Apply the quadratic formula $t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$:

$$t = \frac{4.00 \pm \sqrt{(-4.00)^2 + 4(4.90)(1.80)}}{2(4.90)} = \frac{4.00 \pm \sqrt{16.0 + 35.28}}{9.80}$$
$$t = \frac{4.00 \pm \sqrt{51.28}}{9.80} = \frac{4.00 \pm 7.16}{9.80}$$

Taking the positive root:

$$t = \frac{4.00 + 7.16}{9.80} = \frac{11.16}{9.80} = 1.14 \text{s}$$

(b) Highest point above the board:

1. At maximum height, velocity equals zero: $v=0$
2. Use the kinematic equation:

$$v^2=v_{20}+2a(y-y_0)$$

1. Solve for the height y:

$$0=(4.00)^2+2(-9.80)(y-0)$$
$$0=16.0-19.6y$$
$$y=16.019.6=0.816\text{m}$$

(c) Velocity when feet hit the water:

Use the kinematic equation relating velocity, displacement, and acceleration:

$$v^2=v_{20}+2a(y-y_0)$$

Substitute known values:

$$v^2=(4.00)^2+2(-9.80)(-1.80-0)$$
$$v^2=16.0+2(-9.80)(-1.80)=16.0+35.28=51.28\text{m}^2/\text{s}^2$$
$$v=\pm\sqrt{51.28}=\pm 7.16\text{m/s}$$

Taking the negative root (downward motion):

$$v=-7.16\text{m/s}$$

Discussion

The swimmer is in the air for about 1.1 seconds, which is reasonable for a dive. She rises 0.816 m (about 82 cm) above the board before falling. Her final velocity of 7.16 m/s downward is greater in magnitude than her initial upward velocity because she falls through a greater distance (1.80 m) than she rises (0.82 m). The symmetry of projectile motion would give her a velocity of -4.00 m/s when passing the board on the way down; the additional 1.80 m of fall increases her speed further.

Answer

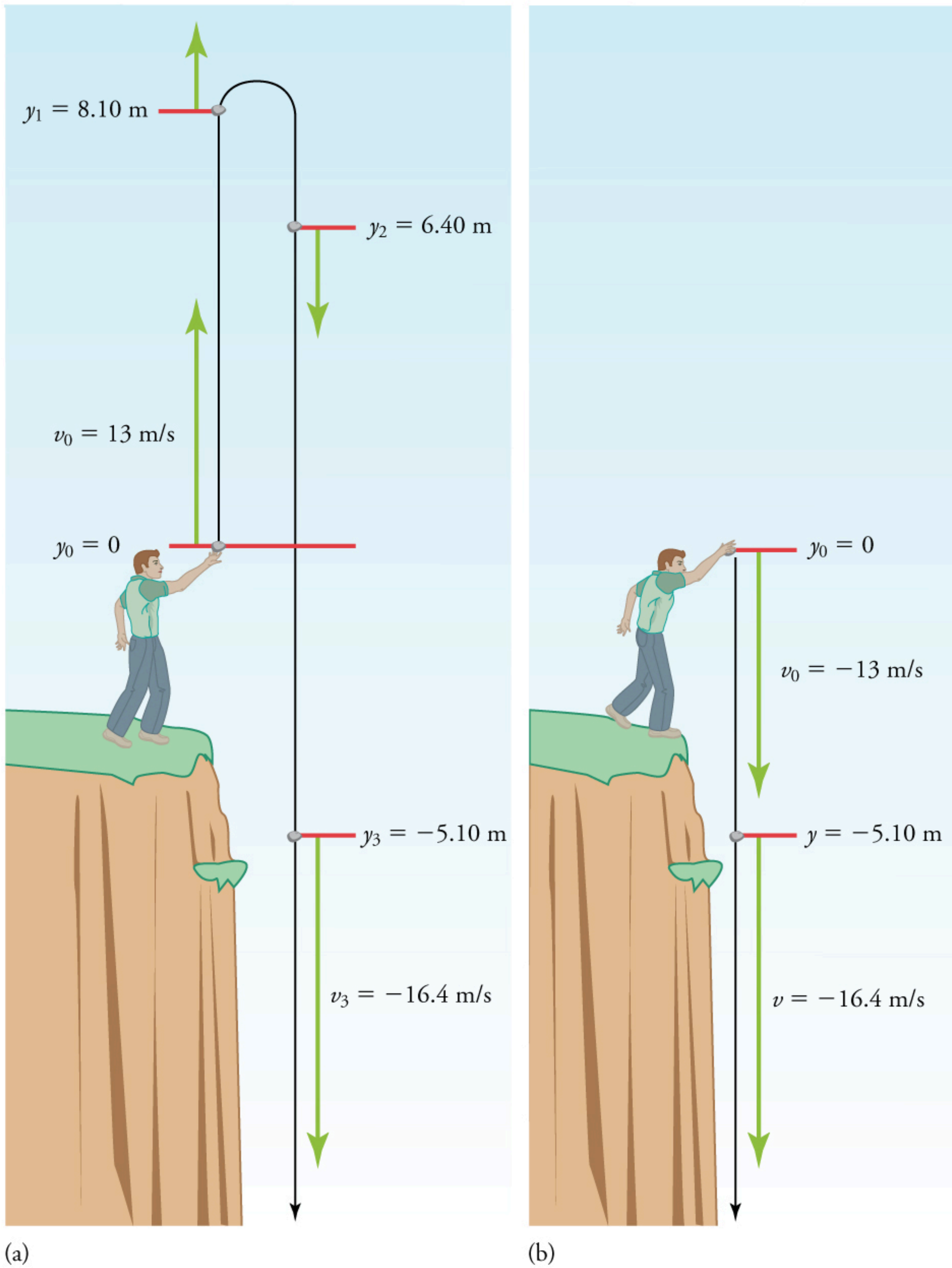
(a) The swimmer's feet are in the air for 1.14s.

(b) The highest point above the board is 0.816m (or about 81.6 cm).

(c) Her velocity when her feet hit the water is 7.16m/s downward.

(a) Calculate the height of a cliff if it takes 2.35 s for a rock to hit the ground when it is thrown straight up from the cliff with an initial velocity of 8.00 m/s. (b) How long would it take to reach the ground if it is thrown straight down with the same speed?

Show Solution



Strategy

Take upward as positive. The rock is thrown upward, goes up, then comes back down past the cliff and hits the ground below. The final position is negative (below the starting point).

Solution

(a) Height of the cliff:

1. Identify the known values:
   - Initial velocity: $v_0 = +8.00 \text{m/s}$ (upward)
   - Time: $t = 2.35 \text{s}$
   - Acceleration: $a = -g = -9.80 \text{m/s}^2$
   - Initial position: $y_0 = 0$ (at cliff top)
2. Use the kinematic equation for displacement:

$$y = y_0 + v_0 t + \frac{1}{2} a t^2$$

1. Substitute the known values:

$$y = 0 + (8.00 \text{m/s})(2.35 \text{s}) + \frac{1}{2}(-9.80 \text{m/s}^2)(2.35 \text{s})^2$$
$$y = 18.8 \text{m} - \frac{1}{2}(9.80 \text{m/s}^2)(5.52 \text{s}^2)$$
$$y = 18.8 \text{m} - 27.1 \text{m} = -8.26 \text{m}$$

The negative sign indicates the ground is 8.26 m below the cliff top.

(b) Time if thrown straight down:

1. Identify the known values:
   - Initial velocity: $v_0 = -8.00 \text{m/s}$ (downward)
   - Final position: $y = -8.26 \text{m}$
   - Acceleration: $a = -9.80 \text{m/s}^2$
2. Use the kinematic equation and solve for time:

$$y = y_0 + v_0 t + \frac{1}{2} a t^2$$
$$-8.26 = 0 + (-8.00)t + \frac{1}{2}(-9.80)t^2$$
$$-8.26 = -8.00t - 4.90t^2$$

Rearranging: $4.90t^2 + 8.00t - 8.26 = 0$

1. Apply the quadratic formula:

$$t = \frac{-8.00 \pm \sqrt{(8.00)^2 + 4(4.90)(8.26)}}{2(4.90)} = \frac{-8.00 \pm \sqrt{64.0 + 161.9}}{9.80}$$
$$t = \frac{-8.00 \pm 15.03}{9.80}$$

Taking the positive root: $t = \frac{-8.00 + 15.03}{9.80} = 0.717 \text{s}$

Discussion

When thrown downward, the rock takes much less time (0.717 s vs. 2.35 s) because it doesn't have to go up first and then come back down.

(a) The cliff is 8.26m high.

(b) If thrown straight down with the same speed, it takes 0.717s to reach the ground.

A very strong, but inept, shot putter puts the shot straight up vertically with an initial velocity of 11.0 m/s. How long does he have to get out of the way if the shot was released at a height of 2.20 m, and he is 1.80 m tall?

Show Solution

Strategy

Take upward as positive. The shot is released at height 2.20 m with initial upward velocity of 11.0 m/s. The shot putter is 1.80 m tall, so his head is at 1.80 m. We need to find when the shot returns to this height on its way back down. Use $g = 9.80 \text{m/s}^2$ for the acceleration due to gravity.

Solution

1. Identify the known values:
   - Initial position: $y_0 = 2.20 \text{m}$ (release height)
   - Initial velocity: $v_0 = +11.0 \text{m/s}$ (upward)
   - Final position: $y = 1.80 \text{m}$ (height of his head)

- Acceleration: $a = -g = -9.80 \text{m/s}^2$

2. Use the kinematic equation for displacement:

$$y = y_0 + v_0 t + \frac{1}{2} a t^2$$

1. Substitute the known values:

$$1.80 = 2.20 + (11.0)t + \frac{1}{2}(-9.80)t^2$$
$$1.80 = 2.20 + 11.0t - 4.90t^2$$

Rearranging:

$$1.80 - 2.20 = 11.0t - 4.90t^2$$
$$-0.40 = 11.0t - 4.90t^2$$

Rearranging into standard quadratic form:

$$4.90t^2 - 11.0t - 0.40 = 0$$

1. Apply the quadratic formula $t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$:

$$t = \frac{11.0 \pm \sqrt{(-11.0)^2 + 4(4.90)(0.40)}}{2(4.90)} = \frac{11.0 \pm \sqrt{121 + 7.84}}{9.80}$$
$$t = \frac{11.0 \pm \sqrt{128.84}}{9.80} = \frac{11.0 \pm 11.35}{9.80}$$

This gives two solutions:

$$t_1 = \frac{11.0 - 11.35}{9.80} = \frac{-0.35}{9.80} = -0.036 \text{s} \text{ (not physical)}$$
$$t_2 = \frac{11.0 + 11.35}{9.80} = \frac{22.35}{9.80} = 2.28 \text{s}$$

Discussion

The negative time solution represents when the shot would have been at head height before being released (if we extended the trajectory backward in time), which is not physically relevant. The positive solution of 2.28 s is the time when the shot passes his head height on the way back down. This gives him just over 2 seconds to move out of the way after releasing the shot - not much time! The shot goes up, reaches a maximum height above his release point, then comes back down and passes head level at 2.28 s.

Answer

The shot putter has 2.28s (about 2.3 seconds) to get out of the way before the shot comes back down to head level.

You throw a ball straight up with an initial velocity of 15.0 m/s. It passes a tree branch on the way up at a height of 7.00 m. How much additional time will pass before the ball passes the tree branch on the way back down?

Show Solution

Strategy

Take upward as positive. The ball passes the tree branch twice: once on the way up and once on the way down. We need to find both times when the ball is at height 7.00 m, then calculate the difference between these times.

Solution

Given:

- Initial velocity: $v_0 = 15.0 \text{m/s}$
- Initial position: $y_0 = 0$ (ground level)
- Height of tree branch: $y = 7.00 \text{m}$
- Acceleration: $a = -g = -9.80 \text{m/s}^2$

1. Use the kinematic equation for position:

$$y = y_0 + v_0 t + \frac{1}{2} a t^2$$

1. Substitute the known values:

$$7.00 = 0 + (15.0)t + \frac{1}{2}(-9.80)t^2$$
$$7.00 = 15.0t - 4.90t^2$$

1. Rearrange into standard quadratic form:

$$4.90t^2 - 15.0t + 7.00 = 0$$

1. Apply the quadratic formula $t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$:

$$t = \frac{15.0 \pm \sqrt{(15.0)^2 - 4(4.90)(7.00)}}{2(4.90)}$$
$$t = \frac{15.0 \pm \sqrt{225 - 137.2}}{9.80} = \frac{15.0 \pm \sqrt{87.8}}{9.80}$$
$$t = \frac{15.0 \pm 9.37}{9.80}$$

1. This gives two solutions:

$t_1 = \frac{15.0 - 9.37}{9.80} = \frac{5.63}{9.80} = 0.575\,\text{s}$ (on the way up)
$t_2 = \frac{15.0 + 9.37}{9.80} = \frac{24.37}{9.80} = 2.49\,\text{s}$ (on the way down)

1. The additional time between passing the branch on the way up and on the way down:

$$\Delta t = t_2 - t_1 = 2.49\,\text{s} - 0.575\,\text{s} = 1.91\,\text{s}$$

Discussion

The ball passes the tree branch at 0.575 s on its way up and again at 2.49 s on its way down, giving an additional time of 1.91 s. This result makes sense because of the symmetry of projectile motion: the ball takes the same amount of time to go from the branch to its maximum height as it does to fall from maximum height back to the branch. Since the ball reaches its maximum height at $t = \frac{v_0}{g} = \frac{15.0}{9.80} = 1.53\,\text{s}$, the time from the first branch crossing to the peak is $1.53 - 0.575 = 0.96\,\text{s}$, and from the peak back to the branch is another 0.96 s, giving a total of 1.91 s. This symmetry is a fundamental property of motion under constant acceleration.

Answer

The ball will pass the tree branch on the way down **1.91 s** after passing it on the way up.

A kangaroo can jump over an object 2.50 m high. (a) Calculate its vertical speed when it leaves the ground. (b) How long is it in the air?

Show Solution

Strategy

Take upward as positive. The kangaroo leaves the ground with some initial velocity, reaches a maximum height of 2.50 m where its velocity becomes zero, then returns to the ground. Use $g = 9.80\,\text{m/s}^2$ for the acceleration due to gravity.

Solution

(a) Vertical speed when leaving the ground:

1. Identify the known values:
    - Initial position: $y_0 = 0$ (ground level)
    - Maximum height: $y = 2.50\,\text{m}$
    - Velocity at maximum height: $v = 0\,\text{m/s}$
    - Acceleration: $a = -g = -9.80\,\text{m/s}^2$
2. Use the kinematic equation relating velocity, displacement, and acceleration:

$$v^2 = v_0^2 + 2a(y - y_0)$$

1. Solve for initial velocity v_0:

$$0 = v_0^2 + 2(-9.80)(2.50 - 0)$$
$$0 = v_0^2 - 2(9.80)(2.50)$$
$$v_0^2 = 2(9.80)(2.50) = 49.0\,\text{m}^2/\text{s}^2$$
$$v_0 = \sqrt{49.0} = 7.00\,\text{m/s}$$

(b) Time in the air:

Method 1: Use the velocity equation to find time to reach maximum height, then double it.

1. Use the kinematic equation:

$$v = v_0 + at$$

1. At maximum height, $v=0$:

$$0=7.00+(-9.80)t$$
$$9.80t=7.00$$
$$t=\frac{7.00}{9.80}=0.714\,\text{s}$$

1. This is the time to reach maximum height. By symmetry, the total time in the air is:

$$t_{total}=2\times0.714=1.43\,\text{s}$$

Discussion

A takeoff speed of 7.00 m/s (about 25 km/h) is quite impressive for an animal jump. The kangaroo can clear a 2.50 m obstacle, which is taller than most humans. The total air time of 1.43 seconds is reasonable - the kangaroo spends about 0.71 seconds going up and 0.71 seconds coming down. This symmetric motion is characteristic of free-fall under constant gravitational acceleration.

Answer

(a) The kangaroo leaves the ground with a vertical speed of $7.00\,\text{m/s}$.

(b) The kangaroo is in the air for $1.43\,\text{s}$ (about 1.4 seconds).

Standing at the base of one of the cliffs of Mt. Arapiles in Victoria, Australia, a hiker hears a rock break loose from a height of 105 m. He can’t see the rock right away but then does, 1.50 s later. (a) How far above the hiker is the rock when he can see it? (b) How much time does he have to move before the rock hits his head?

Show Solution

Strategy

Take downward as positive. The rock falls from rest at height 105 m. After 1.50 s, the hiker can see it. We need to find where the rock is at that time, and then how long it takes to fall the remaining distance.

Solution

(a) Height of rock when hiker sees it:

1. Identify the known values:
   - Initial position: $y_0=0$
   - Initial velocity: $v_0=0$ (breaks loose from rest)
   - Time: $t=1.50\,\text{s}$
   - Acceleration: $a=g=9.80\,\text{m/s}^2$ (downward, positive)
2. Find the distance fallen in 1.50 s:

$$y=y_0+v_0t+\frac{1}{2}gt^2=0+0+\frac{1}{2}(9.80)(1.50)^2$$
$$y=4.90(2.25)=11.0\,\text{m}$$

1. The rock is above the hiker by:

$$h=105-11.0=94.0\,\text{m}$$

(b) Time remaining before impact:

1. The rock still needs to fall 94.0 m. Its velocity after 1.50 s is:

$$v=v_0+gt=0+(9.80)(1.50)=14.7\,\text{m/s}$$

1. Use the kinematic equation for the remaining fall:

$$y=v_0t+\frac{1}{2}gt^2$$
$$94.0=(14.7)t+\frac{1}{2}(9.80)t^2$$
$$4.90t^2+14.7t-94.0=0$$

1. Using the quadratic formula:

$$t=\frac{-14.7\pm\sqrt{(14.7)^2+4(4.90)(94.0)}}{2(4.90)}$$
$$t=\frac{-14.7\pm\sqrt{216.1+1842.4}}{9.80}=\frac{-14.7\pm45.4}{9.80}$$

Taking the positive root:

$t=30.79.80=3.13\text{s}$

Discussion

The rock falls 11.0 m in the first 1.50 seconds, leaving it 94.0 m above the hiker when he first sees it. This is reasonable since falling objects don't travel very far in the first second or two. The hiker then has 3.13 seconds to move out of the way - enough time to react if he's alert, but not much margin for error! The total fall time is $1.50+3.13=4.63\text{s}$. This problem illustrates the importance of staying alert near cliffs and the accelerating nature of falling objects.

Answer

(a) When the hiker first sees the rock, it is 94.0 m above him.

(b) He has 3.13 s to move before the rock hits his head.

An object is dropped from a height of 75.0 m above ground level. (a) Determine the distance traveled during the first second. (b) Determine the final velocity at which the object hits the ground. (c) Determine the distance traveled during the last second of motion before hitting the ground.

Show Solution

Strategy

Take downward as positive since the object is dropped and falls downward. The initial velocity is zero (dropped, not thrown). Use $g=9.80\text{m/s}^2$ for the acceleration due to gravity.

Solution

(a) Distance traveled during the first second:

1. Identify the known values:
    - Initial position: $y_0=0$
    - Initial velocity: $v_0=0$ (dropped from rest)
    - Time: $t=1.00\text{s}$
    - Acceleration: $a=g=9.80\text{m/s}^2$ (downward, positive)
2. Use the kinematic equation for displacement:

$$y=y_0+v_0t+12gt^2$$

1. Substitute the known values:

$$y=0+0+12(9.80)(1.00)^2$$
$$y=12(9.80)(1.00)=4.90\text{m}$$

(b) Final velocity when hitting the ground:

1. Identify the known values:
    - Initial velocity: $v_0=0$
    - Total displacement: $y=75.0\text{m}$
    - Acceleration: $a=g=9.80\text{m/s}^2$
2. Use the kinematic equation:

$$v^2=v_{20}+2ay$$

1. Substitute the known values:

$$v^2=0+2(9.80)(75.0)$$
$$v^2=1470\text{m}^2/\text{s}^2$$
$$v=\sqrt{1470}=38.3\text{m/s}$$

(c) Distance traveled during the last second:

First, find the total time to fall 75.0 m:

1. Use the kinematic equation:

$$y=y_0+v_0t+12gt^2$$
$$75.0=0+0+12(9.80)t^2$$

$t^2 = \frac{2(75.0)}{9.80} = 15.31 s^2$
$t = 3.91 s$

1. Now find the distance fallen at $t = 2.91 s$ (one second before hitting the ground):

$$y_{2.91} = \frac{1}{2}(9.80)(2.91)^2 = \frac{1}{2}(9.80)(8.468) = 41.5 m$$

1. The distance traveled during the last second is:

$$\Delta y = 75.0 - 41.5 = 33.5 m$$

Discussion

The object falls only 4.90 m in the first second but travels 33.5 m in the last second - nearly 7 times as far! This illustrates how falling objects accelerate continuously. The final velocity of 38.3 m/s (about 138 km/h or 86 mph) is quite fast, which explains why falling from such heights is dangerous. The asymmetry between the first and last seconds demonstrates that the object is continuously gaining speed as it falls.

Answer

(a) The object travels $4.90 m$ during the first second.

(b) The final velocity when hitting the ground is $38.3 m/s$ (downward).

(c) The object travels $33.5 m$ during the last second of motion before hitting the ground.

There is a 250-m-high cliff at Half Dome in Yosemite National Park in California. Suppose a boulder breaks loose from the top of this cliff. (a) How fast will it be going when it strikes the ground? (b) Assuming a reaction time of 0.300 s, how long will a tourist at the bottom have to get out of the way after hearing the sound of the rock breaking loose (neglecting the height of the tourist, which would become negligible anyway if hit)? The speed of sound is 335 m/s on this day.

Show Solution

Strategy

Take downward as positive. The boulder falls from rest from a height of 250 m. For part (a), we find the impact velocity. For part (b), we must account for the time it takes for the boulder to fall, plus the time for sound to travel back up, plus the reaction time. Use $g = 9.80 m/s^2$ for the acceleration due to gravity.

Solution

(a) Velocity when striking the ground:

1. Identify the known values:
   - Initial position: $y_0 = 0$
   - Final position: $y = 250 m$ (ground level, 250 m below starting point)
   - Initial velocity: $v_0 = 0$ (breaks loose from rest)
   - Acceleration: $a = g = 9.80 m/s^2$ (downward, positive)
2. Use the kinematic equation:

$$v^2 = v_0^2 + 2ay$$
$$v^2 = 0 + 2(9.80)(250)$$
$$v^2 = 4900 m^2/s^2$$
$$v = 70.0 m/s$$

Taking the coordinate system with upward as positive (more conventional):

$v = -70.0 m/s$ (downward)

(b) Time available to move:

1. First, find the time for the boulder to fall:

$$y = y_0 + v_0 t + \frac{1}{2} g t^2$$
$$250 = 0 + 0 + \frac{1}{2}(9.80) t^2_{fall}$$
$$t_{fall} = \sqrt{\frac{2(250)}{9.80}} = \sqrt{51.02} = 7.14 s$$

1. Time for sound to travel up 250 m:

$$t_{sound} = \frac{distance}{speed} = \frac{250 \text{ m}}{335 \text{ m/s}} = 0.746 \text{ s}$$

1. Total time from when boulder breaks loose until tourist hears it:

$$t_{hear} = t_{fall} + t_{sound} = 7.14 + 0.746 = 7.89 \text{ s}$$

1. After hearing the sound, the tourist needs 0.300 s to react, but the boulder has already hit the ground at $t = 7.14 \text{ s}$. The time available to move is:

$$t_{available} = t_{fall} - t_{sound} - t_{reaction} = 7.14 - 0.746 - 0.300 = 6.09 \text{ s} \approx 6.10 \text{ s}$$

Discussion

The boulder strikes the ground at 70.0 m/s (about 252 km/h or 157 mph), which is extremely fast and dangerous. For part (b), the tourist hears the sound 0.746 seconds after the boulder hits, meaning they must move before hearing the impact! By the time sound travels from the top of the cliff to the bottom (0.746 s) and the tourist reacts (0.300 s), about 1.05 seconds have elapsed. Since the boulder takes 7.14 s to fall, the tourist actually has about 6.10 seconds from when the rock breaks to when it hits. This problem illustrates an important safety consideration: at cliffs, you cannot rely on hearing falling rocks as a warning system since sound takes time to travel.

Answer

(a) The boulder will be traveling at 70.0 m/s downward (or -70.0 m/s with upward positive) when it strikes the ground.

(b) The tourist has 6.10 s to get out of the way.

A ball is thrown straight up. It passes a 2.00-m-high window 7.50 m off the ground on its path up and takes 0.312 s to go past the window. What was the ball's initial velocity? Hint: First consider only the distance along the window, and solve for the ball's velocity at the bottom of the window. Next, consider only the distance from the ground to the bottom of the window, and solve for the initial velocity using the velocity at the bottom of the window as the final velocity.

Show Solution

Strategy

Take upward as positive. We'll solve this in two steps as suggested by the hint:

1. First, analyze the motion through the window to find the velocity at the bottom of the window
2. Then, analyze the motion from ground to the bottom of the window to find the initial velocity

Solution

Step 1: Find velocity at the bottom of the window

The window is 2.00 m high and the ball takes 0.312 s to pass through it.

1. Identify the known values for motion through the window:
    - Displacement through window: $\Delta y = 2.00 \text{ m}$
    - Time through window: $t = 0.312 \text{ s}$
    - Acceleration: $a = -g = -9.80 \text{ m/s}^2$
2. Use the kinematic equation:

$$\Delta y = v_{bottom} t + \frac{1}{2} a t^2$$

1. Substitute and solve for v_bottom:

$$2.00 = v_{bottom}(0.312) + \frac{1}{2}(-9.80)(0.312)^2$$
$$2.00 = v_{bottom}(0.312) - 4.90(0.0973)$$
$$2.00 = v_{bottom}(0.312) - 0.477$$
$$v_{bottom}(0.312) = 2.00 + 0.477 = 2.477$$
$$v_{bottom} = \frac{2.477}{0.312} = 7.94 \text{ m/s}$$

Step 2: Find initial velocity from ground to bottom of window

The bottom of the window is 7.50 m above the ground.

1. Identify the known values:
    - Initial position: $y_0 = 0$
    - Final position (bottom of window): $y = 7.50 \text{ m}$
    - Final velocity (at bottom of window): $v = 7.94 \text{ m/s}$
    - Acceleration: $a = -g = -9.80 \text{ m/s}^2$
2. Use the kinematic equation:

$$v^2 = v_0^2 + 2a(y - y_0)$$

1. Solve for initial velocity v_0:

$$(7.94)^2 = v_0^2 + 2(-9.80)(7.50 - 0)$$
$$63.0 = v_0^2 - 147$$
$$v_0^2 = 63.0 + 147 = 210\,\text{m}^2/\text{s}^2$$
$$v_0 = \sqrt{210} = 14.5\,\text{m/s}$$

Discussion

The ball was thrown with an initial velocity of 14.5 m/s. By the time it reaches the bottom of the window at 7.50 m, it has slowed to 7.94 m/s due to gravity. This is reasonable - the ball is still moving upward but has lost significant speed. The two-step approach allows us to work backwards from the observable motion (passing the window) to find the initial condition.

Answer

The ball's initial velocity was $14.5\,\text{m/s}$ upward.

Suppose you drop a rock into a dark well and, using precision equipment, you measure the time for the sound of a splash to return. (a) Neglecting the time required for sound to travel up the well, calculate the distance to the water if the sound returns in 2.0000 s. (b) Now calculate the distance taking into account the time for sound to travel up the well. The speed of sound is 332.00 m/s in this well.

Show Solution

Strategy

Take downward as positive. For part (a), we neglect the time for sound to travel up and assume all 2.0000 s is the time for the rock to fall. For part (b), we must account for the fact that the total time includes both the fall time and the sound travel time. Use $g = 9.80\,\text{m/s}^2$ for the acceleration due to gravity and a sound speed of 332.00 m/s.

Solution

(a) Neglecting sound travel time:

1. Identify the known values:
   - Initial position: $y_0 = 0$
   - Initial velocity: $v_0 = 0$ (dropped from rest)
   - Acceleration: $a = g = 9.80\,\text{m/s}^2$ (taking down as positive)
   - Time: $t = 2.0000\,\text{s}$
2. Use the kinematic equation:

$$y = y_0 + v_0 t + \frac{1}{2}at^2$$

1. Substitute the known values:

$$y = 0 + 0 + \frac{1}{2}(9.80)(2.0000)^2$$
$$y = \frac{1}{2}(9.80)(4.0000) = 19.6\,\text{m}$$

(b) Accounting for sound travel time:

The total time is the sum of the fall time t_{fall} and the sound travel time t_{sound}:

$$t_{\text{total}} = t_{\text{fall}} + t_{\text{sound}} = 2.0000\,\text{s}$$

1. For the falling rock (distance d):

$$d = \frac{1}{2}g t_{\text{fall}}^2$$

1. For the sound traveling up:

$$d = v_{\text{sound}} \times t_{\text{sound}}$$
$$t_{\text{sound}} = \frac{d}{v_{\text{sound}}} = \frac{d}{332.00}$$

1. Substitute into the total time equation:

$$t_{\text{fall}} + \frac{d}{332.00} = 2.0000$$

1. But $d = \frac{1}{2} g t^2_{\text{fall}} = 4.90 t^2_{\text{fall}}$, so:

$$t_{\text{fall}} + \frac{4.90 t^2_{\text{fall}}}{332.00} = 2.0000$$
$$t_{\text{fall}} + 0.01476 t^2_{\text{fall}} = 2.0000$$
$$0.01476 t^2_{\text{fall}} + t_{\text{fall}} - 2.0000 = 0$$

1. Using the quadratic formula:

$$t_{\text{fall}} = \frac{-1 \pm \sqrt{1 + 4(0.01476)(2.0000)}}{2(0.01476)}$$
$$t_{\text{fall}} = \frac{-1 \pm \sqrt{1 + 0.11808}}{0.02952} = \frac{-1 \pm \sqrt{1.11808}}{0.02952}$$
$$t_{\text{fall}} = \frac{-1 \pm 1.0574}{0.02952}$$

Taking the positive root:

$$t_{\text{fall}} = \frac{0.0574}{0.02952} = 1.944 \text{s}$$

1. Calculate the distance:

$$d = 4.90 \times (1.944)^2 = 4.90 \times 3.779 = 18.5 \text{m}$$

Discussion

When we account for the time it takes sound to travel up the well, the calculated depth is less than when we ignore it. This makes sense: if the total time is 2.0000 s and some of that time is used for sound to travel up, less time is available for the rock to fall, so it doesn't fall as far. The sound travel time is $t\_\text{sound} = 2.0000 - 1.944 = 0.056 \text{s}$, which is about 2.8% of the total time. The difference in calculated depth is about 1.1 m, or about 6% - relatively small but not negligible for precision measurements. This problem illustrates that for accurate measurements, we must consider all relevant physical processes, not just the primary one.

Answer

(a) Neglecting the time for sound to travel up the well, the distance to the water is 19.6 m.

(b) Accounting for the time sound takes to travel up the well, the distance to the water is 18.5 m.

A steel ball is dropped onto a hard floor from a height of 1.50 m and rebounds to a height of 1.45 m. (a) Calculate its velocity just before it strikes the floor. (b) Calculate its velocity just after it leaves the floor on its way back up. (c) Calculate its acceleration during contact with the floor if that contact lasts 0.0800 ms $(8.00 \times 10^{-5} \text{s})$. (d) How much did the ball compress during its collision with the floor, assuming the floor is absolutely rigid?

Show Solution

Strategy

Take upward as positive. The ball falls from rest, hits the floor, and rebounds. We'll analyze the motion in stages: before impact, during collision, and after rebound. Use $g = 9.80 \text{m/s}^2$ for the acceleration due to gravity.

Solution

(a) Velocity just before striking the floor:

1. Identify the known values for the downward fall:
   - Initial position: $y_0 = 1.50 \text{m}$
   - Final position: $y = 0$ (floor level)
   - Initial velocity: $v_0 = 0$ (dropped from rest)
   - Acceleration: $a = -g = -9.80 \text{m/s}^2$
2. Use the kinematic equation:

$$v^2 = v_0^2 + 2a(y - y_0)$$

1. Substitute the known values:

$$v^2 = 0 + 2(-9.80)(0 - 1.50)$$
$$v^2 = 2(-9.80)(-1.50) = 29.4 \text{m}^2/\text{s}^2$$
$$v = \pm\sqrt{29.4} = \pm 5.42 \text{m/s}$$

Taking the negative root (downward motion):

$$v = -5.42 \text{m/s}$$

(b) Velocity just after leaving the floor:

1. Identify the known values for the upward rebound:
   - Initial position: $y_0 = 0$ (floor level)
   - Final position: $y = 1.45\text{m}$ (maximum rebound height)
   - Final velocity at max height: $v = 0$
   - Acceleration: $a = -g = -9.80\text{m/s}^2$
2. Use the kinematic equation:

$$v^2 = v_{20} + 2a(y - y_0)$$

1. Solve for initial velocity after rebound v_0:

$$0 = v_{20} + 2(-9.80)(1.45 - 0)$$
$$v_{20} = 2(9.80)(1.45) = 28.42\text{m}^2/\text{s}^2$$
$$v_0 = \sqrt{28.42} = +5.33\text{m/s}$$

(c) Acceleration during contact with the floor:

During the collision, the ball’s velocity changes from -5.42 m/s to +5.33 m/s.

1. Identify the known values:
   - Initial velocity: $v_0 = -5.42\text{m/s}$
   - Final velocity: $v = +5.33\text{m/s}$
   - Time of contact: $t = 8.00 \times 10^{-5}\text{s}$
2. Use the kinematic equation:

$$v = v_0 + at$$

1. Solve for acceleration:

$$a = v - v_0 t = 5.33 - (-5.42) 8.00 \times 10^{-5}$$
$$a = 10.75 8.00 \times 10^{-5} = 1.34 \times 10^5 \text{m/s}^2$$

(d) Distance the ball compressed during collision:

1. Use the kinematic equation with average velocity:

$$y = y_0 + v_0 + v 2 t$$

1. Substitute the known values:

$$\Delta y = -5.42 + 5.33 2 \times (8.00 \times 10^{-5})$$
$$\Delta y = -0.092 \times (8.00 \times 10^{-5}) = -3.6 \times 10^{-6}\text{m}$$

The magnitude of compression is:

$$|\Delta y| = 3.6 \times 10^{-6}\text{m} = 3.6\ \mu\text{m}$$

Alternatively, using $y = y_0 + v_0 t + 12 a t^2$:

$$\Delta y = (-5.42)(8.00 \times 10^{-5}) + 12(1.34 \times 10^5)(8.00 \times 10^{-5})^2$$
$$\Delta y = -4.34 \times 10^{-4} + 4.30 \times 10^{-4} = -4.0 \times 10^{-6}\text{m}$$
$$|\Delta y| \approx 4.0 \times 10^{-6}\text{m} = 4.0\ \mu\text{m}$$

Discussion

The ball hits the floor at 5.42 m/s and rebounds at 5.33 m/s - nearly the same speed, indicating an almost elastic collision. The acceleration during contact is enormous - about 13,700 times the acceleration due to gravity (about 13,700 g’s)! Despite this huge acceleration, the ball compresses only about 4 micrometers because the contact time is so brief (0.08 milliseconds). These values are reasonable for a hard steel ball on a hard floor. The small loss in rebound height (from 1.50 m to 1.45 m) represents energy lost to heat and sound during the collision.

Answer

(a) The velocity just before striking the floor is 5.42 m/s downward.

(b) The velocity just after leaving the floor is 5.33 m/s upward.

(c) The acceleration during contact with the floor is $1.34 \times 10^5 \text{ m/s}^2$ (upward, or about 13,700 g's).

(d) The ball compressed approximately $4.0 \times 10^{-6} \text{ m}$ or $4.0\ \mu\text{m}$ during its collision with the floor.

A coin is dropped from a hot-air balloon that is 300 m above the ground and rising at 10.0 m/s upward. For the coin, find (a) the maximum height reached, (b) its position and velocity 4.00 s after being released, and (c) the time before it hits the ground.

Show Solution

Strategy

Take upward as positive. The coin is released from a balloon that is already moving upward at 10.0 m/s, so the coin inherits this initial velocity. After release, the coin decelerates, reaches a maximum height, then falls back down. We'll analyze the motion in stages using $g = 9.80 \text{ m/s}^2$.

Solution

(a) Maximum height reached:

1. Identify the known values:
   - Initial position: $y_0 = 300 \text{ m}$ (above ground)
   - Initial velocity: $v_0 = +10.0 \text{ m/s}$ (upward, same as balloon)
   - Acceleration: $a = -g = -9.80 \text{ m/s}^2$
   - Final velocity at max height: $v = 0$
2. Use the kinematic equation:

$$v^2 = v_0^2 + 2a(y - y_0)$$

1. Solve for the maximum height y:

$$\begin{aligned}
0 &= (10.0)^2 + 2(-9.80)(y - 300) \\
0 &= 100 - 19.6(y - 300) \\
19.6(y - 300) &= 100 \\
y - 300 &= \frac{100}{19.6} = 5.10 \text{ m} \\
y &= 305 \text{ m}
\end{aligned}$$

(b) Position and velocity at t = 4.00 s:

1. Position after 4.00 s:

$$\begin{aligned}
y &= y_0 + v_0 t + \frac{1}{2} a t^2 \\
y &= 300 + (10.0)(4.00) + \frac{1}{2}(-9.80)(4.00)^2 \\
y &= 300 + 40.0 + \frac{1}{2}(-9.80)(16.0) \\
y &= 300 + 40.0 - 78.4 = 262 \text{ m}
\end{aligned}$$

1. Velocity after 4.00 s:

$$\begin{aligned}
v &= v_0 + at \\
v &= 10.0 + (-9.80)(4.00) \\
v &= 10.0 - 39.2 = -29.2 \text{ m/s}
\end{aligned}$$

The negative sign indicates the coin is moving downward.

(c) Time before hitting the ground:

1. When the coin hits the ground, $y = 0$. Use:

$$\begin{aligned}
y &= y_0 + v_0 t + \frac{1}{2} a t^2 \\
0 &= 300 + 10.0t + \frac{1}{2}(-9.80)t^2 \\
0 &= 300 + 10.0t - 4.90t^2 \\
4.90t^2 &- 10.0t - 300 = 0
\end{aligned}$$

1. Using the quadratic formula:

$$t=\frac{10.0\pm\sqrt{(10.0)^2+4(4.90)(300)}}{2(4.90)}$$
$$t=\frac{10.0\pm\sqrt{100+5880}}{9.80}=\frac{10.0\pm\sqrt{5980}}{9.80}$$
$$t=\frac{10.0\pm77.33}{9.80}$$

Taking the positive root:

$$t=\frac{10.0+77.33}{9.80}=\frac{87.33}{9.80}=8.91\text{s}$$

Discussion

This problem demonstrates motion with an initial upward velocity. The coin doesn't immediately fall - it first continues upward for $\frac{v_0}{g}=\frac{10.0}{9.80}=1.02\text{s}$, rising an additional 5.10 m to reach 305 m. At $t=4.00\text{s}$, the coin has passed through its maximum height and is falling downward at 29.2 m/s. The total time to hit the ground (8.91 s) is longer than it would be if dropped from rest at 300 m (which would take $\sqrt{\frac{2(300)}{9.80}}=7.82\text{s}$) because the initial upward velocity adds extra time. The coin actually falls from a maximum height of 305 m, not 300 m.

Answer

(a) The maximum height reached by the coin is 305 m above the ground.

(b) At $t=4.00\text{s}$ after release, the coin is at a position of 262 m above the ground and has a velocity of 29.2m/s downward.

(c) The coin hits the ground 8.91 s after being released from the balloon.

A soft tennis ball is dropped onto a hard floor from a height of 1.50 m and rebounds to a height of 1.10 m. (a) Calculate its velocity just before it strikes the floor. (b) Calculate its velocity just after it leaves the floor on its way back up. (c) Calculate its acceleration during contact with the floor if that contact lasts 3.50 ms $(3.50\times10^{-3}\text{s})$. (d) How much did the ball compress during its collision with the floor, assuming the floor is absolutely rigid?

Show Solution

Strategy

Take upward as positive. The tennis ball falls from rest, hits the floor, compresses, and rebounds. We'll analyze the motion in stages: before impact, during collision, and after rebound. Note that the longer contact time and lower rebound height indicate a less elastic collision compared to the steel ball. Use $g=9.80\text{m/s}^2$ for the acceleration due to gravity.

Solution

(a) Velocity just before striking the floor:

1. Identify the known values for the downward fall:
   - Initial position: $y_0=1.50\text{m}$
   - Final position: $y=0$ (floor level)
   - Initial velocity: $v_0=0$ (dropped from rest)
   - Acceleration: $a=-g=-9.80\text{m/s}^2$
2. Use the kinematic equation:

$$v^2=v_0^2+2a(y-y_0)$$

1. Substitute the known values:

$$v^2=0+2(-9.80)(0-1.50)$$
$$v^2=2(-9.80)(-1.50)=29.4\text{m}^2/\text{s}^2$$
$$v=\pm\sqrt{29.4}=\pm5.42\text{m/s}$$

Taking the negative root (downward motion):

$$v=-5.42\text{m/s}$$

(b) Velocity just after leaving the floor:

1. Identify the known values for the upward rebound:
   - Initial position: $y_0=0$ (floor level)
   - Final position: $y=1.10\text{m}$ (maximum rebound height)
   - Final velocity at max height: $v=0$
   - Acceleration: $a=-g=-9.80\text{m/s}^2$

2. Use the kinematic equation:

$$v^2 = v_{20} + 2a(y - y_0)$$

1. Solve for initial velocity after rebound v_0:

$$0 = v_{20} + 2(-9.80)(1.10 - 0)$$
$$v_{20} = 2(9.80)(1.10) = 21.56 \text{m}^2/\text{s}^2$$
$$v_0 = \sqrt{21.56} = +4.64 \text{m/s}$$

(c) Acceleration during contact with the floor:

During the collision, the ball's velocity changes from -5.42 m/s to +4.64 m/s.

1. Identify the known values:
   - Initial velocity: $v_0 = -5.42 \text{m/s}$
   - Final velocity: $v = +4.64 \text{m/s}$
   - Time of contact: $t = 3.50 \times 10^{-3} \text{s}$
2. Use the kinematic equation:

$$v = v_0 + at$$

1. Solve for acceleration:

$$a = v - v_0 t = 4.64 - (-5.42) 3.50 \times 10^{-3}$$
$$a = 10.06 3.50 \times 10^{-3} = 2.87 \times 10^3 \text{m/s}^2$$

(d) Distance the ball compressed during collision:

1. Use the kinematic equation:

$$y = y_0 + v_0 t + 12 a t^2$$

1. Substitute the known values:

$$\Delta y = (-5.42)(3.50 \times 10^{-3}) + 12(2.87 \times 10^3)(3.50 \times 10^{-3})^2$$
$$\Delta y = -1.90 \times 10^{-2} + 12(2.87 \times 10^3)(1.225 \times 10^{-5})$$
$$\Delta y = -1.90 \times 10^{-2} + 1.76 \times 10^{-2} = -1.4 \times 10^{-3} \text{m}$$

The magnitude of compression is:

$$|\Delta y| = 1.4 \times 10^{-3} \text{m} = 1.4 \text{ mm}$$

Alternatively, using average velocity:

$$\Delta y = v_0 + v 2 t = -5.42 + 4.64 2 \times (3.50 \times 10^{-3})$$
$$\Delta y = -0.782 \times (3.50 \times 10^{-3}) = -1.37 \times 10^{-3} \text{m}$$
$$|\Delta y| \approx 1.4 \times 10^{-3} \text{m} = 1.4 \text{ mm}$$

Discussion

The soft tennis ball shows markedly different behavior from the steel ball. It hits the floor at the same speed (5.42 m/s) since both are dropped from 1.50 m, but rebounds at only 4.64 m/s compared to the steel ball's 5.33 m/s. This represents greater energy loss during the collision. The contact time is much longer (3.50 ms vs. 0.08 ms), resulting in a much smaller acceleration (about 293 g's vs. 13,700 g's for the steel ball). The soft tennis ball compresses about 1.4 mm, which is over 300 times more than the steel ball (4 micrometers). This greater compression and longer contact time are characteristic of softer materials and less elastic collisions. The significant loss in rebound height (from 1.50 m to 1.10 m, a 27% loss) shows that much more energy was dissipated as heat, sound, and permanent deformation compared to the steel ball.

Answer

(a) The velocity just before striking the floor is 5.42m/s downward.

(b) The velocity just after leaving the floor is 4.64m/s upward.

(c) The acceleration during contact with the floor is $2.87 \times 10^3 \text{m/s}^2$ (upward, or about 293 g's).

(d) The ball compressed approximately $1.4 \times 10^{-3} \text{m}$ or 1.4 mm during its collision with the floor.

Glossary

free-fall
: the state of movement that results from gravitational force only

acceleration due to gravity
: acceleration of an object as a result of gravity

Graphical Analysis of One-Dimensional Motion

- Describe a straight-line graph in terms of its slope and y-intercept.
- Determine average velocity or instantaneous velocity from a graph of position vs. time.
- Determine average or instantaneous acceleration from a graph of velocity vs. time.
- Derive a graph of velocity vs. time from a graph of position vs. time.
- Derive a graph of acceleration vs. time from a graph of velocity vs. time.

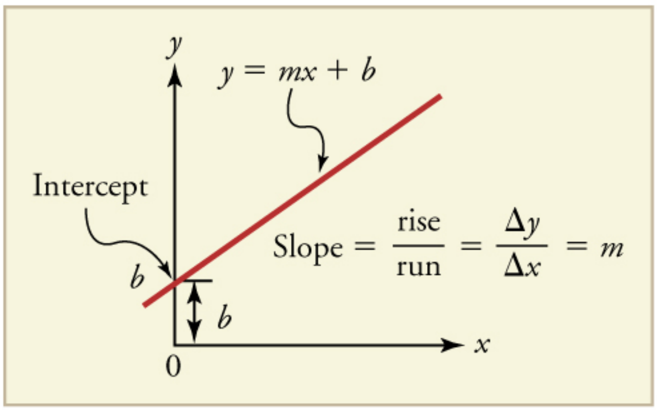
A graph, like a picture, is worth a thousand words. Graphs not only contain numerical information; they also reveal relationships between physical quantities. This section uses graphs of position, velocity, and acceleration versus time to illustrate one-dimensional kinematics.

Slopes and General Relationships

First note that graphs in this text have perpendicular axes, one horizontal and the other vertical. When two physical quantities are plotted against one another in such a graph, the horizontal axis is usually considered to be an **independent variable** and the vertical axis a **dependent variable**. If we call the horizontal axis the x -axis and the vertical axis the y -axis, as in Figure 1, a straight-line graph has the general form

$$y=mx+b.$$

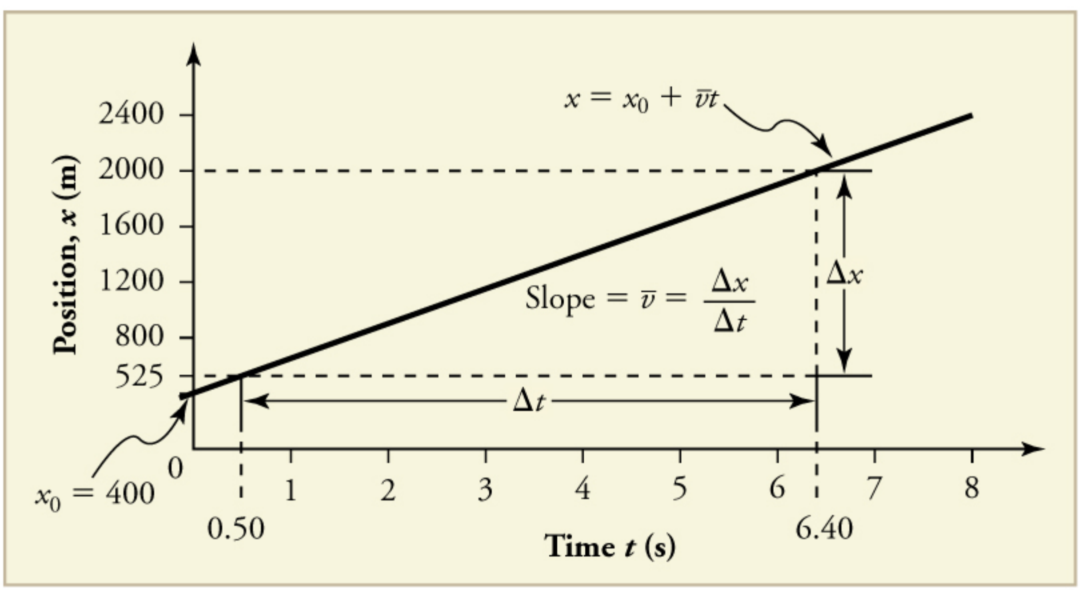
Here m is the **slope**, defined to be the rise divided by the run (as seen in the figure) of the straight line. The letter b is used for the **y-intercept**, which is the point at which the line crosses the vertical axis.



A straight-line graph. The equation for a straight line is $y= mx + b$

Graph of Position vs. Time (*a* = 0, so *v* is constant)

Time is usually an independent variable that other quantities, such as position, depend upon. A graph of position versus time would, thus, have x on the vertical axis and t on the horizontal axis. Figure 2 is just such a straight-line graph. It shows a graph of position versus time for a jet-powered car on a very flat dry lake bed in Nevada.



Graph of position versus time for a jet-powered car on the Bonneville Salt Flats.

Using the relationship between dependent and independent variables, we see that the slope in the graph above is average velocity $\bar{v}$ and the intercept is position at time zero—that is, x_0. Substituting these symbols into $y = mx + b$ gives

$$x = \bar{v}t + x_0$$

or

$$x = x_0 + \bar{v}t.$$

Thus a graph of position versus time gives a general relationship among displacement(change in position), velocity, and time, as well as giving detailed numerical information about a specific situation.

The Slope of *x* vs. *t*

The slope of the graph of position x vs. time t is velocity v.

$$\text{slope} = \frac{\Delta x}{\Delta t} = v$$

Notice that this equation is the same as that derived algebraically from other motion equations in Motion Equations for Constant Acceleration in One Dimension.

From the figure we can see that the car has a position of 25 m at 0.50 s and 2000 m at 6.40 s. Its position at other times can be read from the graph; furthermore, information about its velocity and acceleration can also be obtained from the graph.

Determining Average Velocity from a Graph of Position versus Time: Jet Car

Find the average velocity of the car whose position is graphed in Figure 2.

Strategy

The slope of a graph of x vs. t is average velocity, since slope equals rise over run. In this case, rise = change in position and run = change in time, so that

$$\text{slope} = \frac{\Delta x}{\Delta t} = \bar{v}.$$

Since the slope is constant here, any two points on the graph can be used to find the slope. (Generally speaking, it is most accurate to use two widely separated points on the straight line. This is because any error in reading data from the graph is proportionally smaller if the interval is larger.)

Solution

1. Choose two points on the line. In this case, we choose the points labeled on the graph: (6.4 s, 2000 m) and (0.50 s, 525 m). (Note, however, that you could choose any two points.)

2. Substitute the x and t values of the chosen points into the equation. Remember in calculating change (Δ) we always use final value minus initial value.

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{2000\text{m} - 525\text{m}}{6.4\text{s} - 0.50\text{s}},$$

yielding

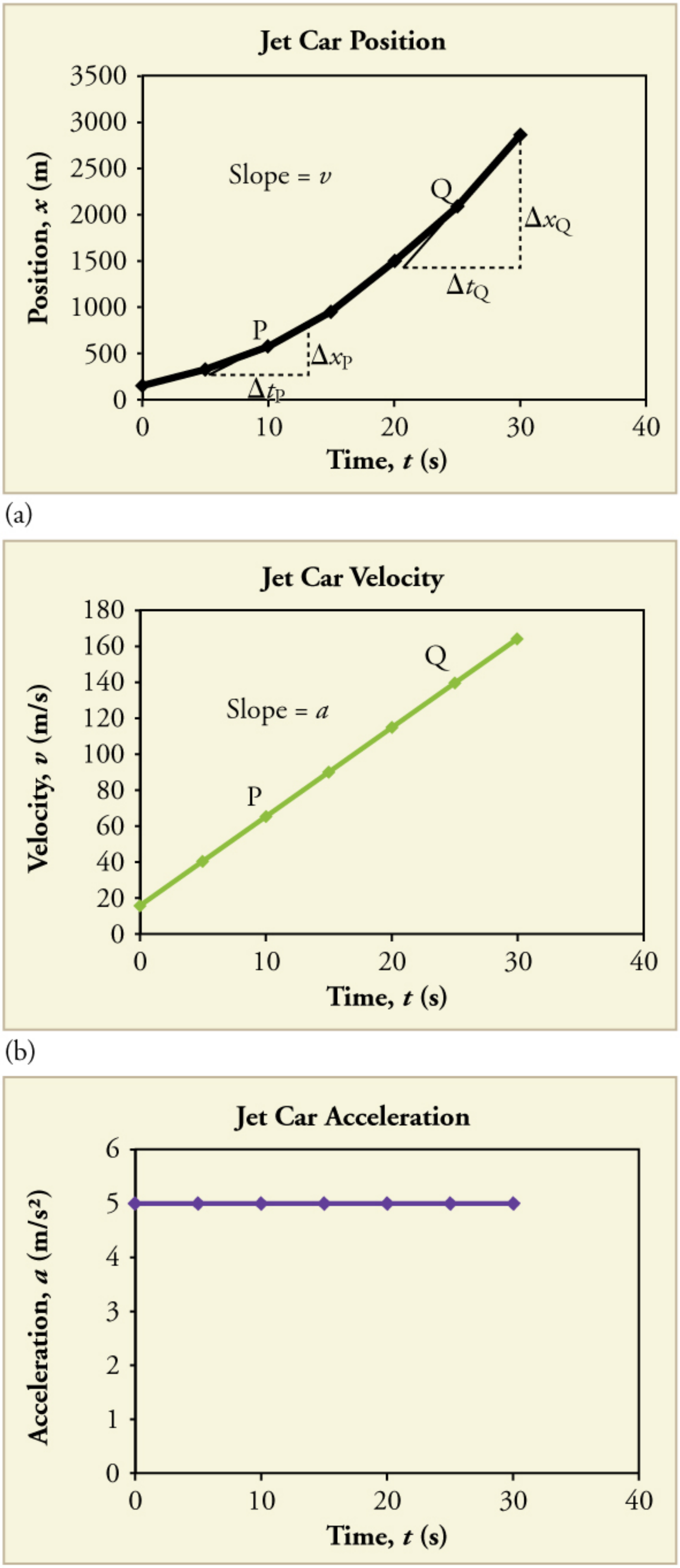
$$\bar{v} = 250\text{m/s}.$$

Discussion

This is an impressively large land speed (900 km/h, or about 560 mi/h): much greater than the typical highway speed limit of 60 mi/h (27 m/s or 96 km/h), but considerably shy of the record of 343 m/s (1234 km/h or 766 mi/h) set in 1997.

Graphs of Motion when a is constant but $a \neq 0$

The graphs in Figure 3 below represent the motion of the jet-powered car as it accelerates toward its top speed, but only during the time when its acceleration is constant. Time starts at zero for this motion (as if measured with a stopwatch), and the position and velocity are initially 200 m and 15 m/s, respectively.


Jet Car Position
Slope = v
Position, x (m)
Time, t (s)
P
Q
Δx_P
Δt_P
Δx_Q
Δt_Q
Jet Car Velocity
Slope = a
Velocity, v (m/s)
Jet Car Acceleration
Acceleration, a (m/s²)

(a)

(b)

(c)

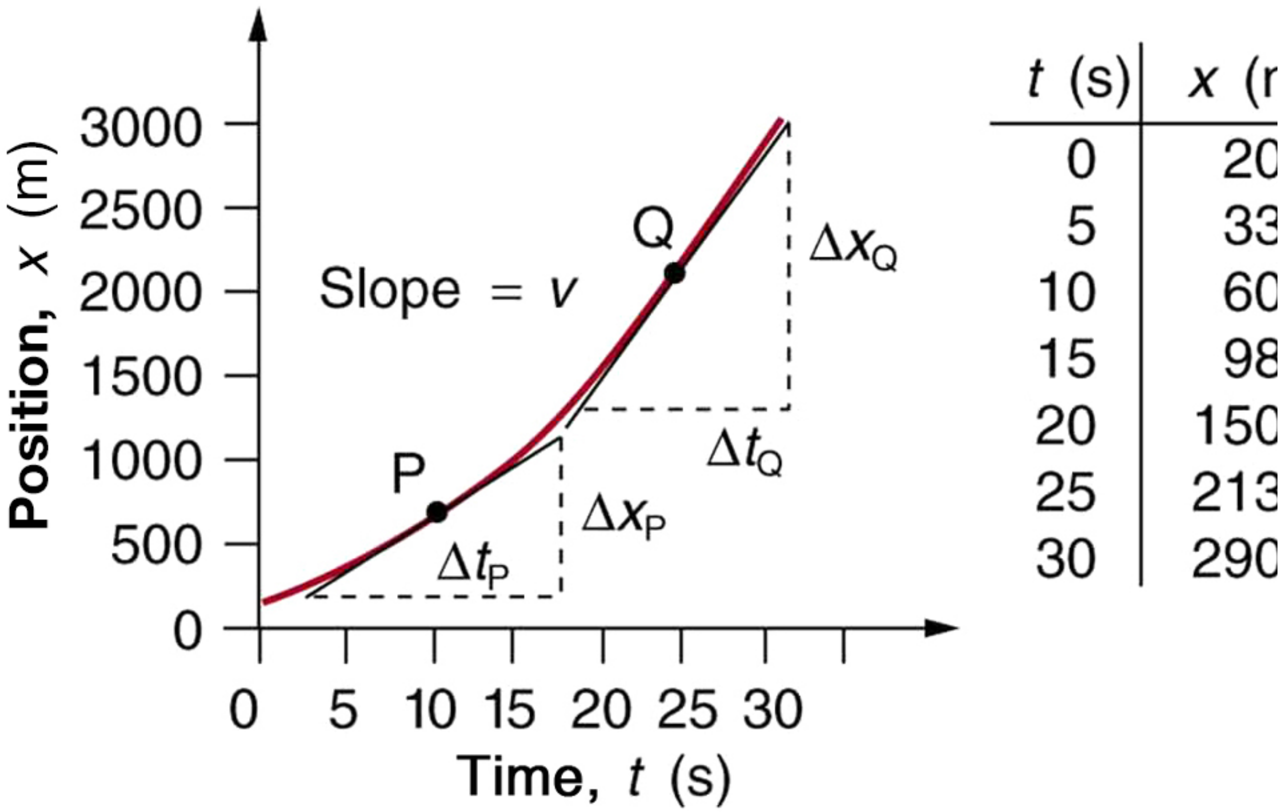
Graphs of motion of a jet-powered car during the time span when its acceleration is constant. (a) The slope of an (x) vs. (t) graph is velocity. This is shown at two points, and the instantaneous velocities obtained are plotted in the next graph. Instantaneous velocity at any point is the slope of the tangent at that point. (b) The slope of the (v) vs. (t) graph is constant for this part of the motion, indicating constant acceleration. (c) Acceleration has the constant value of (5.0 \mss) over the time interval plotted.

A U.S. Air Force jet car speeds down a track. (credit: Matt Trostle, Flickr)

The graph of position versus time in Figure 3(a) is a curve rather than a straight line. The slope of the curve becomes steeper as time progresses, showing that the velocity is increasing over time. The slope at any point on a position-versus-time graph is the instantaneous velocity at that point. It is found by drawing a straight line tangent to the curve at the point of interest and taking the slope of this straight line. Tangent lines are shown for two points in Figure 3(a). If this is done at every point on the curve and the values are plotted against time, then the graph of velocity versus time shown in Figure 3(b) is obtained. Furthermore, the slope of the graph of velocity versus time is acceleration, which is shown in Figure 3(c).

Determining Instantaneous Velocity from the Slope at a Point: Jet Car

Calculate the velocity of the jet car at a time of 25 s by finding the slope of the x vs. t graph in the graph below.



The slope of an x vs. t graph is velocity. This is shown at two points. Instantaneous velocity at any point is the slope of the tangent at that point.

Strategy

The slope of a curve at a point is equal to the slope of a straight line tangent to the curve at that point. This principle is illustrated in Figure 5, where Q is the point at $t = 25 s$.

Solution

1. Find the tangent line to the curve at $t = 25 s$.

2. Determine the endpoints of the tangent. These correspond to a position of 1300 m at time 19 s and a position of 3120 m at time 32 s.

3. Plug these endpoints into the equation to solve for the slope, v.

$$\text{slope} = v_Q = \frac{\Delta x_Q}{\Delta t_Q} = \frac{(3120 m - 1300 m)}{(32 s - 19 s)}$$

Thus,

$$v_Q = \frac{1820 m}{13 s} = 140 m/s.$$

Discussion

This is the value given in this figure’s table for v at $t = 25 s$. The value of 140 m/s for v_Q is plotted in Figure 5. The entire graph of v vs. t can be obtained in this fashion.

Carrying this one step further, we note that the slope of a velocity versus time graph is acceleration. Slope is rise divided by run; on a v vs. t graph, rise = change in velocity Δv and run = change in time Δt.

The Slope of v vs. t

The slope of a graph of velocity v vs. time t is acceleration a.

$$\text{slope} = \frac{\Delta v}{\Delta t} = a$$

Since the velocity versus time graph in Figure 3(b) is a straight line, its slope is the same everywhere, implying that acceleration is constant. Acceleration versus time is graphed in Figure 3(c).

Additional general information can be obtained from Figure 5 and the expression for a straight line, $y = mx + b$.

In this case, the vertical axis y is V, the intercept b is v_0, the slope m is a, and the horizontal axis x is t . Substituting these symbols yields

$$v = v_0 + at.$$

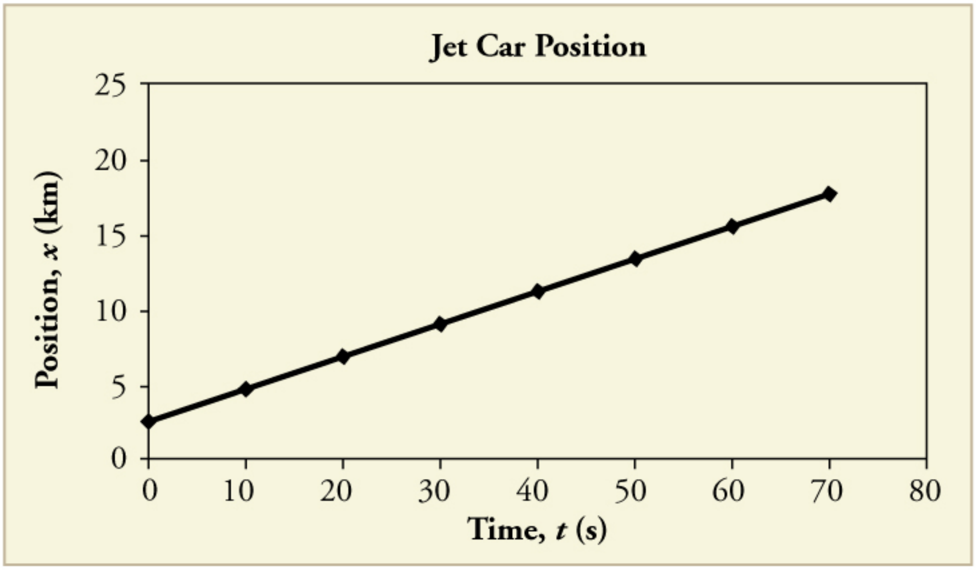
A general relationship for velocity, acceleration, and time has again been obtained from a graph. Notice that this equation was also derived algebraically from other motion equations in Motion Equations for Constant Acceleration in One Dimension .

It is not accidental that the same equations are obtained by graphical analysis as by algebraic techniques. In fact, an important way to *discover* physical relationships is to measure various physical quantities and then make graphs of one quantity against another to see if they are correlated in any way. Correlations imply physical relationships and might be shown by smooth graphs such as those above. From such graphs, mathematical relationships can sometimes be postulated. Further experiments are then performed to determine the validity of the hypothesized relationships.

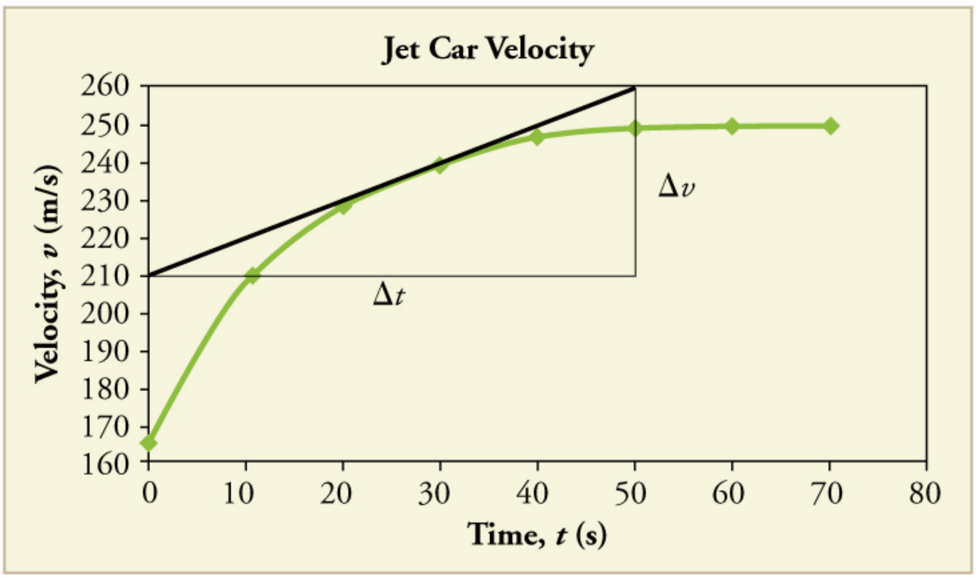
Graphs of Motion Where Acceleration is Not Constant

Now consider the motion of the jet car as it goes from 165 m/s to its top velocity of 250 m/s, graphed in Figure 6. Time again starts at zero, and the initial position and velocity are 2900 m and 165 m/s, respectively. (These were the final position and velocity of the car in the motion graphed in Figure 3.)

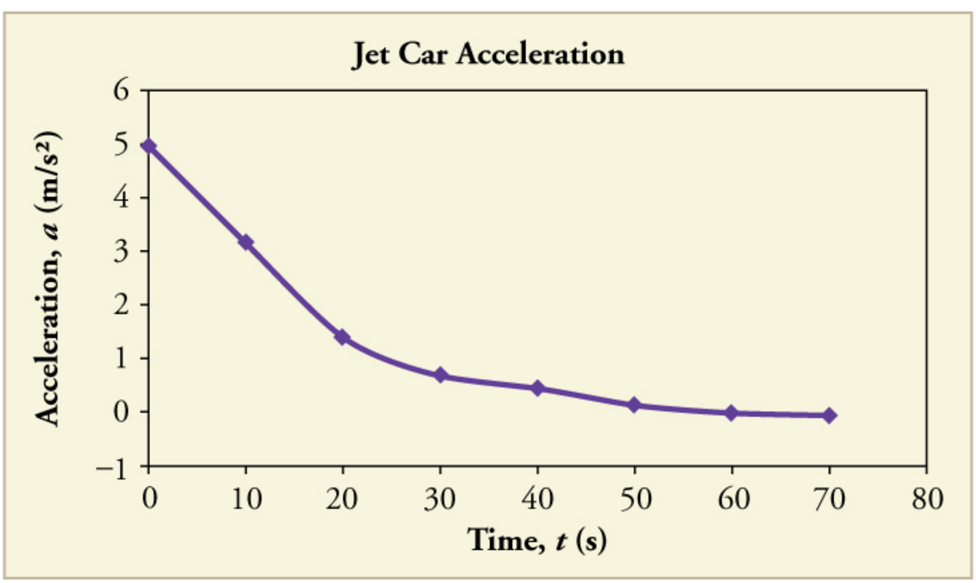
Acceleration gradually decreases from $5.0\,\text{m/s}^2$ to zero when the car hits 250 m/s. The slope of the x vs. t graph increases until $t = 55\,\text{s}$, after which time the slope is constant. Similarly, velocity increases until 55 s and then becomes constant, since acceleration decreases to zero at 55 s and remains zero afterward.


Jet Car Position
Position, x (km)
25
20
15
10
5
0
0
10
20
30
40
50
60
70
80
Time, t (s)

(a)


Jet Car Velocity
Velocity, v (m/s)
260
250
240
230
220
210
200
190
180
170
160
Δv
Δt
0
10
20
30
40
50
60
70
80
Time, t (s)

(b)


Jet Car Acceleration
Acceleration, a (m/s²)
6
5
4
3
2
1
0
−1
0
10
20
30
40
50
60
70
80
Time, t (s)

(c)

Graphs of motion of a jet-powered car as it reaches its top velocity. This motion begins where the motion in Figure 3 ends. (a) The slope of this graph is velocity; it is plotted in the next graph. (b) The velocity gradually approaches its top value. The slope of this graph is acceleration; it is plotted in the final graph. (c) Acceleration gradually declines to zero when velocity becomes constant.

Calculating Acceleration from a Graph of Velocity versus Time

Calculate the acceleration of the jet car at a time of 25 s by finding the slope of the v vs. t graph in Figure 6(b).

Strategy

The slope of the curve at $t = 25 \text{ s}$ is equal to the slope of the line tangent at that point, as illustrated in Figure 6(b).

Solution

Determine endpoints of the tangent line from the figure, and then plug them into the equation to solve for slope, a.

$$\text{slope} = \frac{\Delta v}{\Delta t} = \frac{(260 \text{ m/s} - 210 \text{ m/s})}{(51 \text{ s} - 1.0 \text{ s})}$$

$$a = \frac{50 \text{ m/s}}{50 \text{ s}} = 1.0 \text{ m/s}^2.$$

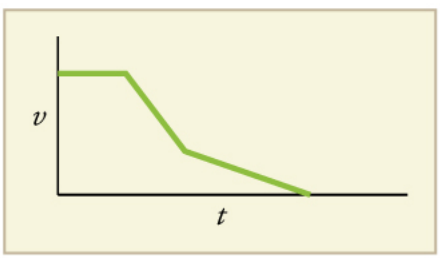
Discussion

Note that this value for a is consistent with the value plotted in Figure 6(c) at $t = 25 \text{ s}$.

A graph of position versus time can be used to generate a graph of velocity versus time, and a graph of velocity versus time can be used to generate a graph of acceleration versus time. We do this by finding the slope of the graphs at every point. If the graph is linear (i.e., a line with a constant slope), it is easy to find the slope at any point and you have the slope for every point. Graphical analysis of motion can be used to describe both specific and general characteristics of kinematics. Graphs can also be used for other topics in physics. An important aspect of exploring physical relationships is to graph them and look for underlying relationships.

Check Your Understanding

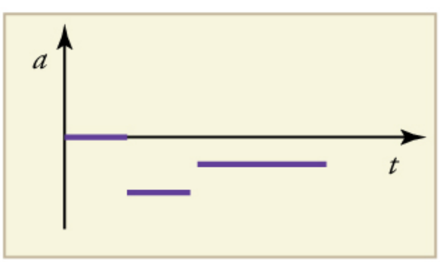
A graph of velocity vs. time of a ship coming into a harbor is shown below. (a) Describe the motion of the ship based on the graph. (b) What would a graph of the ship's acceleration look like?



Show Solution

(a) The ship moves at constant velocity and then begins to decelerate at a constant rate. At some point, its deceleration rate decreases. It maintains this lower deceleration rate until it stops moving.

(b) A graph of acceleration vs. time would show zero acceleration in the first leg, large and constant negative acceleration in the second leg, and constant negative acceleration.

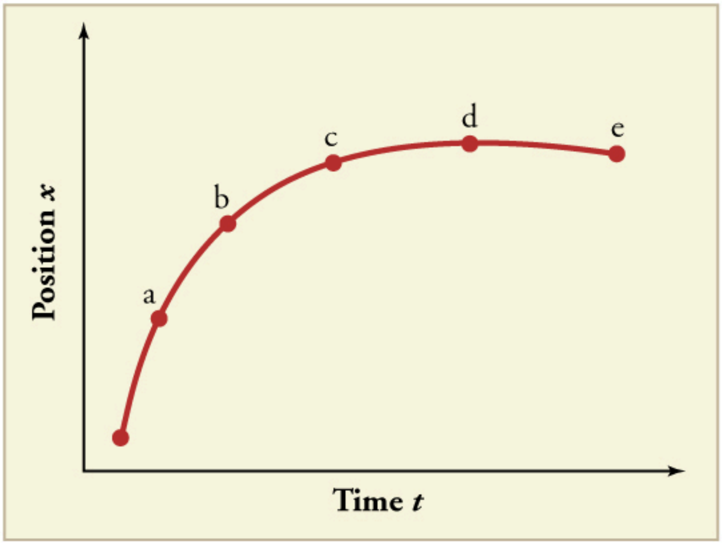


Section Summary

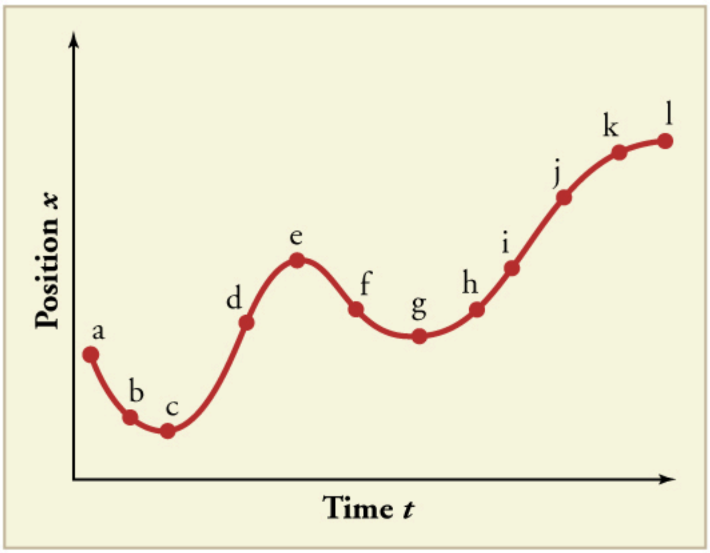
- Graphs of motion can be used to analyze motion.
- Graphical solutions yield identical solutions to mathematical methods for deriving motion equations.
- The slope of a graph of displacement x vs. time t is velocity v.
- The slope of a graph of velocity v vs. time t graph is acceleration a.
- Average velocity, instantaneous velocity, and acceleration can all be obtained by analyzing graphs.

Conceptual Questions

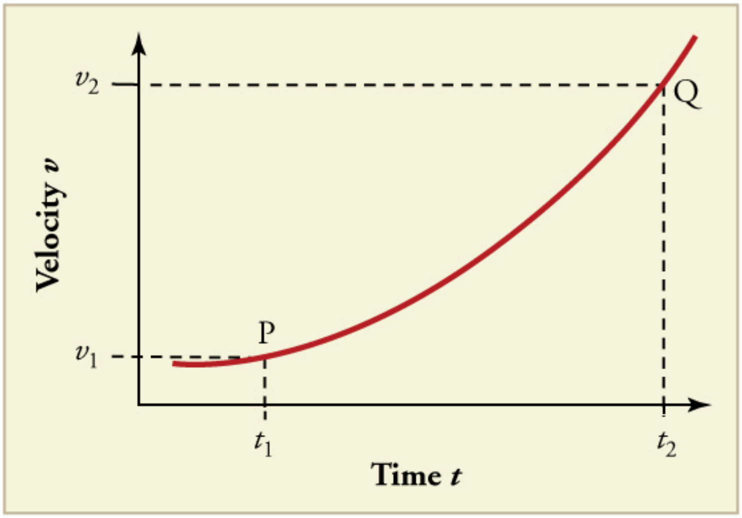
(a) Explain how you can use the graph of position versus time in Figure 9 to describe the change in velocity over time. Identify (b) the time (t_a, t_b, t_c, t_d, or t_e) at which the instantaneous velocity is greatest, (c) the time at which it is zero, and (d) the time at which it is negative.



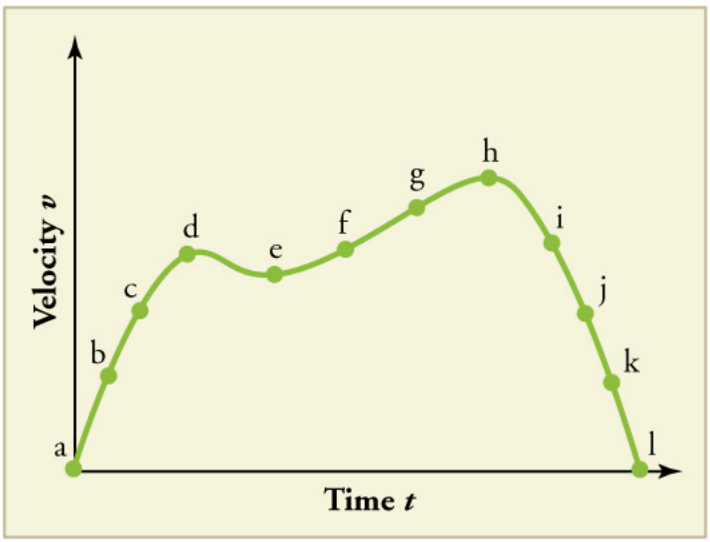
(a) Sketch a graph of velocity versus time corresponding to the graph of position versus time given in Figure 10. (b) Identify the time or times (t_a, t_b, t_c, etc.) at which the instantaneous velocity is greatest. (c) At which times is it zero? (d) At which times is it negative?



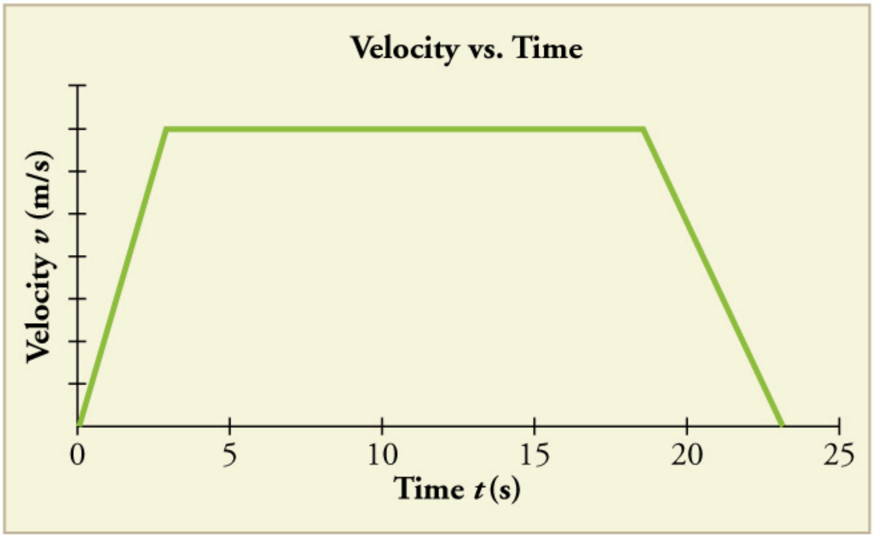
(a) Explain how you can determine the acceleration over time from a velocity versus time graph such as the one in Figure 11. (b) Based on the graph, how does acceleration change over time?



(a) Sketch a graph of acceleration versus time corresponding to the graph of velocity versus time given in Figure 12. (b) Identify the time or times (t_a, t_b, t_c, etc.) at which the acceleration is greatest. (c) At which times is it zero? (d) At which times is it negative?



Consider the velocity vs. time graph of a person in an elevator shown in Figure 13. Suppose the elevator is initially at rest. It then accelerates for 3 seconds, maintains that velocity for 15 seconds, then decelerates for 5 seconds until it stops. The acceleration for the entire trip is not constant so we cannot use the equations of motion from Motion Equations for Constant Acceleration in One Dimension for the complete trip. (We could, however, use them in the three individual sections where acceleration is a constant.) Sketch graphs of (a) position vs. time and (b) acceleration vs. time for this trip.

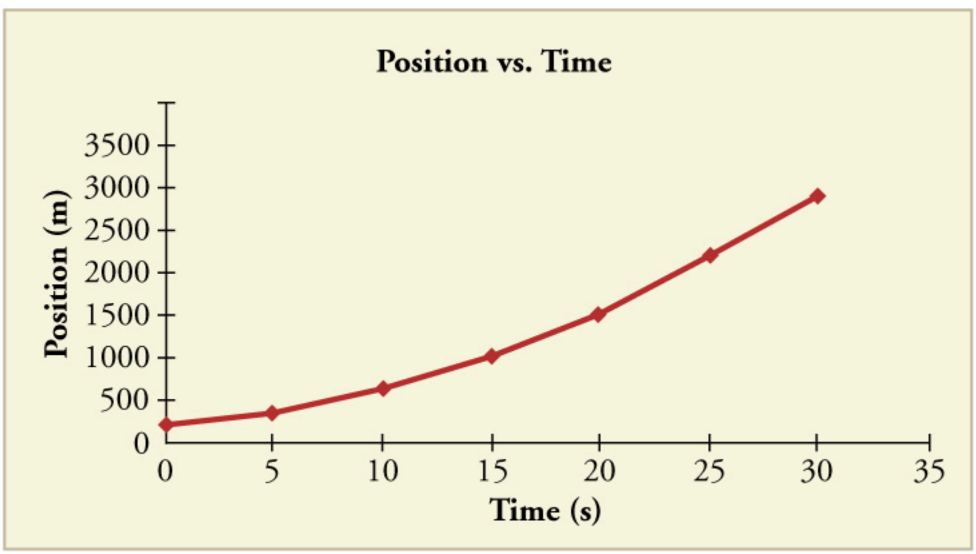


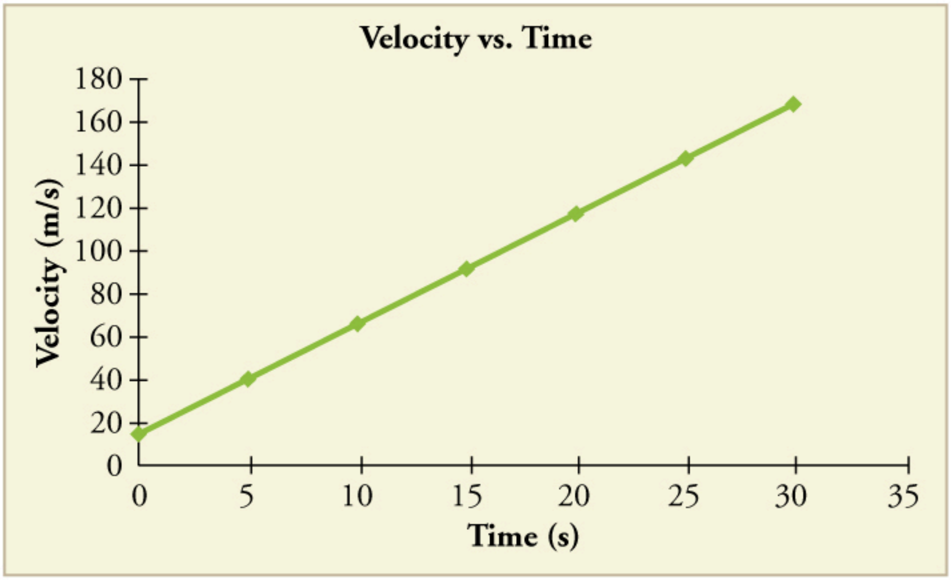
A cylinder is given a push and then rolls up an inclined plane. If the origin is the starting point, sketch the position, velocity, and acceleration of the cylinder vs. time as it goes up and then down the plane.

Problems & Exercises

Note: There is always uncertainty in numbers taken from graphs. If your answers differ from expected values, examine them to see if they are within data extraction uncertainties estimated by you.

(a) By taking the slope of the curve in Figure 14, verify that the velocity of the jet car is 115 m/s at $t = 20\text{s}$. (b) By taking the slope of the curve at any point in Figure 15, verify that the jet car's acceleration is 5.0m/s^2.





Show Solution

Strategy

For part (a), we find the velocity by calculating the slope of the position-time graph at $t = 20\text{s}$. For part (b), we find the acceleration by calculating the slope of the velocity-time graph, which should give a constant value since Figure 15 appears to be a straight line. The slope is calculated using $\text{slope} = \frac{\Delta y}{\Delta x}$, where the axes represent position and time for velocity, and velocity and time for acceleration.

Solution

(a) Finding velocity from the position-time graph:

1. To find the instantaneous velocity at $t = 20\text{s}$, we need to find the slope of the position-time curve in Figure 14 at that point.

2. From Figure 14, we can estimate the slope by drawing a tangent line or by choosing two points on the curve near $t = 20\text{s}$. Looking at the graph, we can estimate points approximately:
   - At $t_1 \approx 15\text{s}$: $x_1 \approx 1000\text{m}$
   - At $t_2 \approx 25\text{s}$: $x_2 \approx 2150\text{m}$
3. Calculate the slope:

$$v = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{2150 - 1000}{25 - 15} = \frac{1150}{10} = 115\text{m/s}$$

(b) Finding acceleration from the velocity-time graph:

1. The acceleration is the slope of the velocity-time graph in Figure 15.

2. Since Figure 15 is a straight line, the slope is constant. From the graph, we can read approximate values:
   - At $t_1 = 0\text{s}$: $v_1 = 0\text{m/s}$
   - At $t_2 = 20\text{s}$: $v_2 = 100\text{m/s}$
3. Calculate the slope:

$$a = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{100 - 0}{20 - 0} = \frac{100}{20} = 5.0\text{m/s}^2$$

Discussion

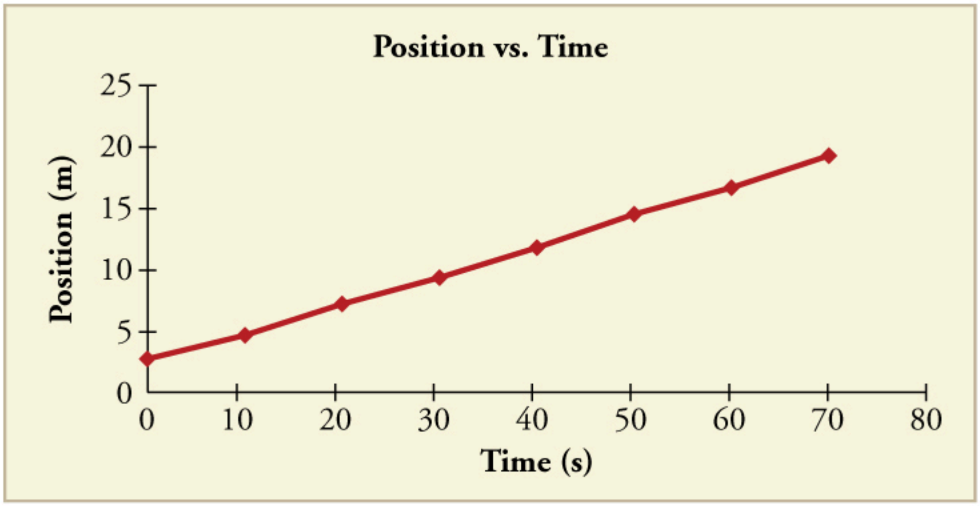
These calculations demonstrate the fundamental relationships in kinematics: velocity is the slope of the position-time graph, and acceleration is the slope of the velocity-time graph. The jet car's velocity of 115 m/s at $t = 20\text{s}$ represents an extremely high speed (about 410 km/h or 257 mph), consistent with a jet-powered vehicle. The constant acceleration of 5.0m/s^2 shown in the straight-line velocity graph indicates that the jet car is accelerating uniformly. This acceleration is about half of gravitational acceleration, which is substantial for maintaining over an extended period. The parabolic shape of the position-time curve in Figure 14 is characteristic of constant acceleration motion.

Answer

(a) The velocity of the jet car at $t = 20\text{s}$ is 115 m/s, verified by taking the slope of the position-time curve in Figure 14.

(b) The jet car's acceleration is 5.0m/s^2, verified by taking the slope of the velocity-time graph in Figure 15.

Using approximate values, calculate the slope of the curve in Figure 16 to verify that the velocity at $t = 10.0 s$ is 0.208 m/s. Assume all values are known to 3 significant figures.



Show Solution

Strategy

The slope of a position vs. time graph gives the velocity. To find the instantaneous velocity at $t = 10.0 s$, we need to find the slope of the curve at that point. Since this appears to be a straight line, we can choose two points on the line (preferably well-separated for accuracy) and calculate the slope using $v = \Delta x \Delta t$.

Solution

From Figure 16, we can read approximate values from the graph. Let's choose two points that are widely separated:

At $t_1 = 0.0 s$, the position is approximately $x_1 = 0.50 \text{ km} = 500 m$

At $t_2 = 20.0 s$, the position is approximately $x_2 = 4.65 \text{ km} = 4650 m$

Now calculate the slope:

$$v = \Delta x \Delta t = x_2 - x_1 t_2 - t_1 = 4650 m - 500 m 20.0 s - 0.0 s$$
$$v = 4150 m 20.0 s = 207.5 m/s \approx 208 m/s$$

To verify at $t = 10.0 s$ specifically, we can also use points around that time:

At $t_1 = 5.0 s$, $x_1 \approx 1.54 \text{ km} = 1540 m$

At $t_2 = 15.0 s$, $x_2 \approx 3.62 \text{ km} = 3620 m$

$$v = 3620 m - 1540 m 15.0 s - 5.0 s = 2080 m 10.0 s = 208 m/s$$

Discussion

The velocity is constant throughout the motion because the position-time graph is a straight line. The positive slope indicates the object is moving in the positive direction. This value of 208 m/s (or 0.208 km/s) is consistent across different time intervals, confirming that the motion has zero acceleration.

Answer

The velocity at $t = 10.0 s$ is 208 m/s (or 0.208 km/s), which verifies the given value.

Using approximate values, calculate the slope of the curve in Figure 16 to verify that the velocity at $t = 30.0 s$ is approximately 0.24 m/s.

Show Solution

Strategy

To find the instantaneous velocity at $t = 30.0 s$, we need to find the slope of the position-time graph at that point. Since Figure 16 shows a curve (not a straight line), we draw a tangent line at $t = 30.0 s$ or use nearby points to estimate the slope. The velocity is calculated using $v = \Delta x \Delta t$.

Solution

1. To find the velocity at $t = 30.0\text{s}$, examine Figure 16 near that time and identify two points on the curve that we can use to estimate the slope.
2. From the graph, choosing points symmetrically around $t = 30.0\text{s}$:
   - At $t_1 = 20.0\text{s}$: $x_1 \approx 6.95 \text{ km} = 6.95 \times 10^3\text{m}$
   - At $t_2 = 40.0\text{s}$: $x_2 \approx 11.7 \text{ km} = 11.7 \times 10^3\text{m}$
3. Calculate the slope:

$$v = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{(11.7 - 6.95) \times 10^3\text{m}}{40.0 - 20.0\text{s}}$$

$$v = \frac{4.75 \times 10^3\text{m}}{20.0\text{s}} = 238\text{m/s}$$

1. Converting to km/s (which appears to be the units intended in the problem):

$$v = 238\text{m/s} = 0.238 \text{ km/s} \approx 0.24 \text{ km/s}$$

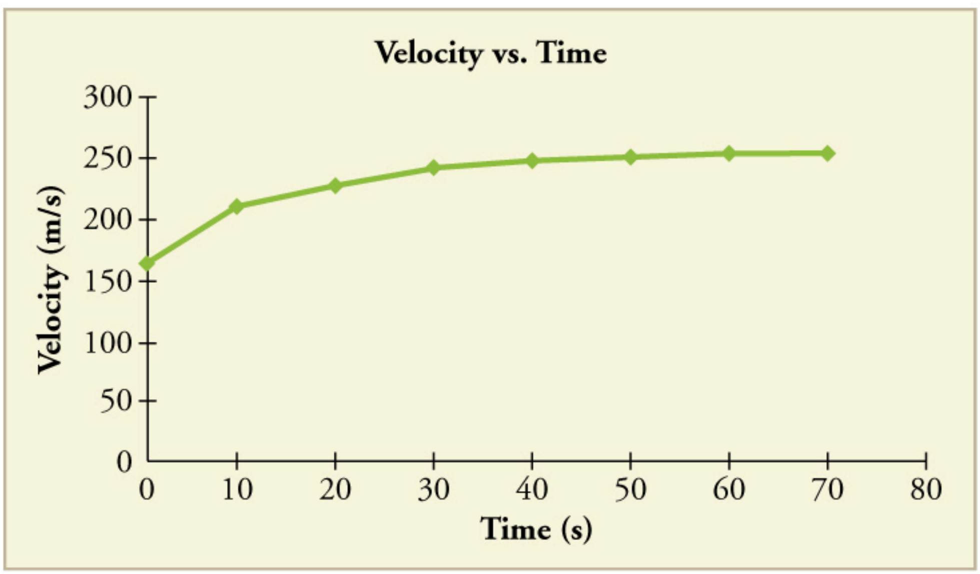
Discussion

The calculated velocity of 238 m/s (or approximately 0.24 km/s) is the instantaneous velocity at $t = 30.0\text{s}$. This high velocity is consistent with Figure 16, which appears to show a spacecraft or high-speed vehicle's position measured in kilometers. The velocity of 238 m/s is approximately 856 km/h or 532 mph. The graph shows increasing position with time, indicating positive velocity throughout the motion. Note that when working with graphs, the choice of units is critical - the problem statement likely intended the answer in km/s (0.24 km/s) rather than m/s to match the position units shown in kilometers.

Answer

The velocity at $t = 30.0\text{s}$ is approximately 238 m/s (or 0.24 km/s), verified by calculating the slope of the position-time curve in Figure 16.

By taking the slope of the curve in Figure 17, verify that the acceleration is 3.2m/s^2 at $t = 10\text{s}$.



Show Solution

Strategy

The slope of a velocity vs. time graph gives the acceleration. Since the curve in Figure 17 is not a straight line, the acceleration varies with time. To find the instantaneous acceleration at $t = 10\text{s}$, we need to draw a tangent line to the curve at that point and calculate the slope of the tangent line using $a = \frac{\Delta v}{\Delta t}$.

Solution

To find the acceleration at $t = 10\text{s}$, we draw a tangent line to the curve at that point.

From the tangent line at $t = 10\text{s}$, we can identify two points on the tangent:

At $t_1 \approx 5\text{s}$, the tangent line gives $v_1 \approx 100\text{m/s}$

At $t_2 \approx 20\text{s}$, the tangent line gives $v_2 \approx 148\text{m/s}$

Now calculate the slope of the tangent line:

$$a = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{148 \text{m/s} - 100 \text{m/s}}{20 \text{s} - 5 \text{s}}$$
$$a = \frac{48 \text{m/s}}{15 \text{s}} = 3.2 \text{m/s}^2$$

Discussion

The acceleration at $t = 10 \text{s}$ is positive, indicating that the velocity is increasing at this time. However, the curve shows that the slope (acceleration) is decreasing as time progresses, eventually approaching zero when the velocity becomes constant. This represents a situation where an object is speeding up but the rate at which it speeds up is decreasing over time. This could represent a car accelerating but approaching its maximum speed, or an object experiencing increasing air resistance as it speeds up.

Answer

The acceleration at $t = 10 \text{s}$ is 3.2m/s^2, which verifies the given value.

Construct the position graph for the subway shuttle train as shown in the Acceleration module (a). Your graph should show the position of the train, in kilometers, from t = 0 to 20 s. You will need to use the information on acceleration and velocity given in the examples for this figure.

Show Solution

Strategy

To construct a position-time graph from velocity and acceleration information, we use the relationships between these quantities. The position changes according to the area under the velocity-time curve. We need to refer to the Acceleration module example to get the specific values for the subway train's motion, then integrate the velocity over time to find position. The graph should show two distinct phases of motion based on the changing velocity.

Solution

From the Acceleration module example, the subway train's motion has two phases:

Phase 1 (0 to 10 s): Constant velocity of 5.0 m/s

1. Initial position: $x_0 = 0$ km at $t = 0$
2. Position increases linearly: $x = v \cdot t = (5.0 \text{m/s})(t)$
3. At $t = 10 \text{s}$: $x = 5.0 \times 10 = 50 \text{m} = 0.050$ km

Phase 2 (10 to 20 s): Constant acceleration of 1.3m/s^2

1. Initial velocity at $t = 10 \text{s}$: $v_0 = 5.0 \text{m/s}$
2. Initial position at $t = 10 \text{s}$: $x_0 = 50 \text{m}$
3. For constant acceleration starting at $t = 10 \text{s}$:

$$x = x_0 + v_0(t - 10) + \frac{1}{2}a(t - 10)^2$$

1. At $t = 20 \text{s}$ (i.e., 10 s after acceleration begins):

$$x = 50 + (5.0)(10) + \frac{1}{2}(1.3)(10)^2$$
$$x = 50 + 50 + 65 = 165 \text{m} = 0.165 \text{ km}$$

Constructing the graph:

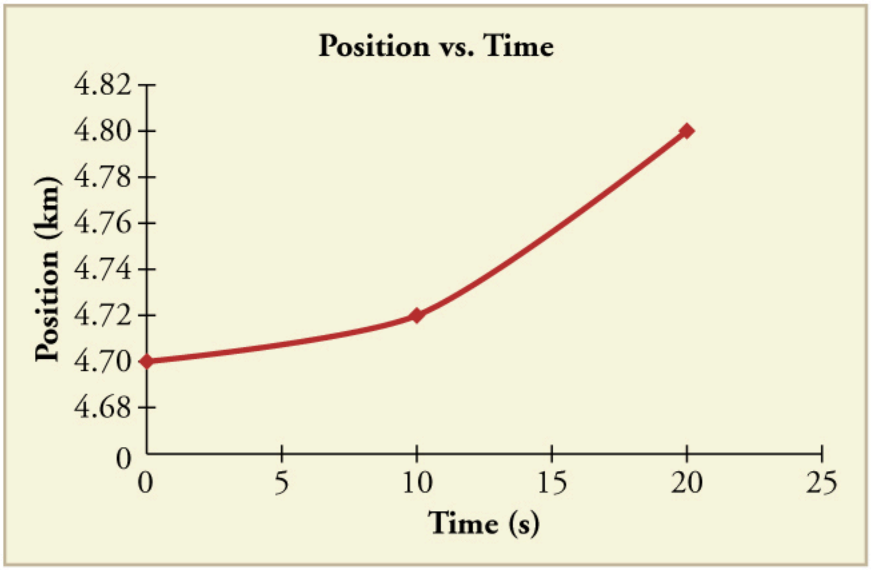
- From $t = 0$ to 10s: Draw a straight line from (0, 0) to (10 s, 0.050 km) with constant slope
- From $t = 10 \text{s}$ to 20s: Draw a parabolic curve from (10 s, 0.050 km) to (20 s, 0.165 km) with increasing slope
- The curve should show a "kink" at $t = 10 \text{s}$ where the motion transitions from constant velocity to constant acceleration

Discussion

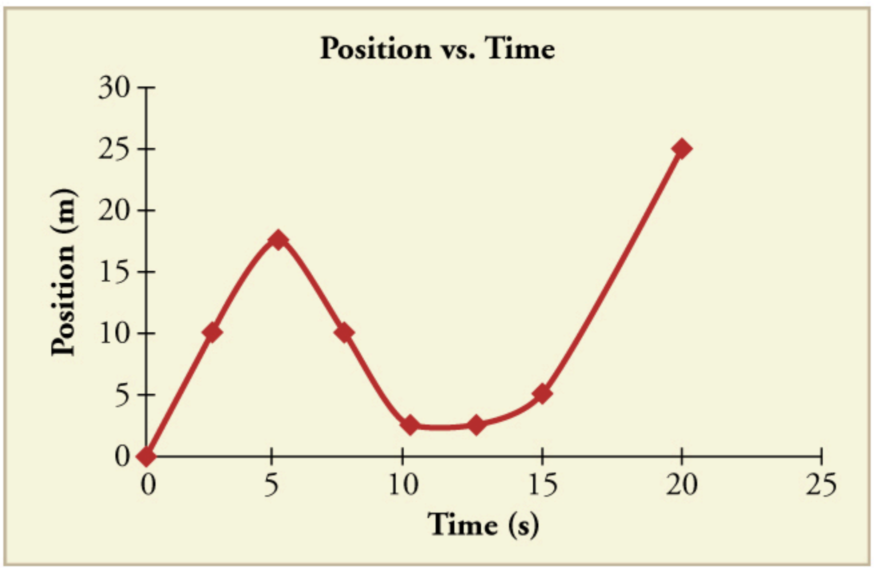
The position-time graph clearly shows two distinct motion phases. In the first 10 seconds, the train moves at constant velocity, producing a straight line with constant slope (5.0 m/s). At $t = 10 \text{s}$, the train begins accelerating, which creates a parabolic curve with increasing slope. The kink at $t = 10 \text{s}$ represents the instant the acceleration begins. The steeper slope in the second phase reflects the increasing velocity due to acceleration. This type of piecewise motion is common in real transportation systems, where vehicles may cruise at constant speed before accelerating to reach their destination more quickly.

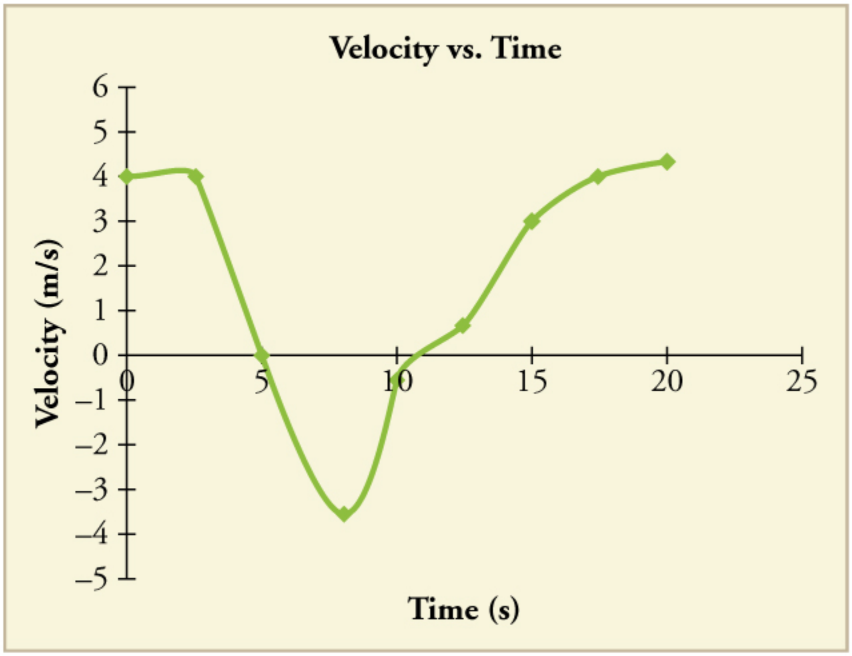
Answer

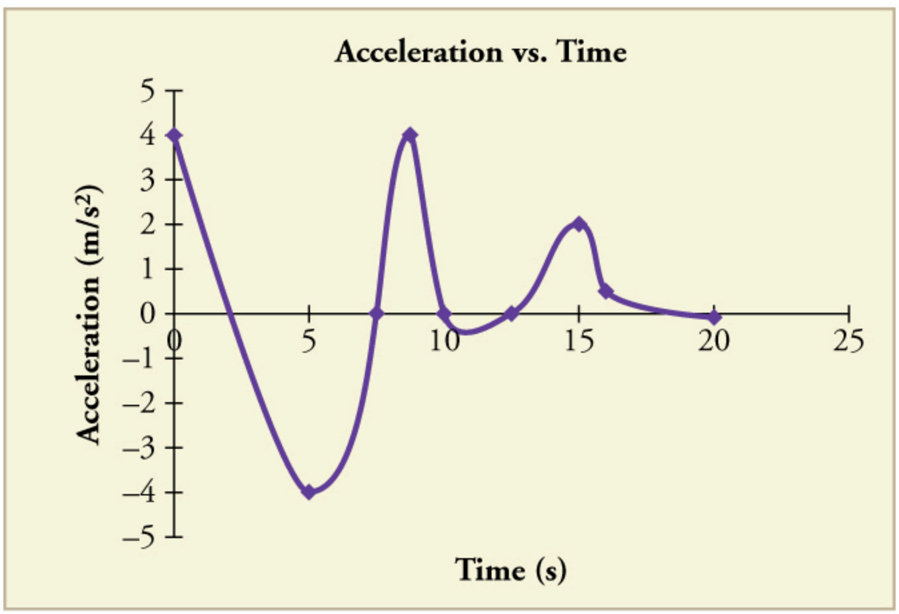
The position-time graph shows the train starting at the origin and moving with constant velocity (5.0 m/s) for the first 10 seconds, reaching 0.050 km. At $t = 10 \text{s}$, the train begins accelerating at 1.3m/s^2, following a parabolic path to reach approximately 0.165 km at $t = 20 \text{s}$.



(a) Take the slope of the curve in Figure 19 to find the jogger's velocity at $t = 2.5 \text{s}$. (b) Repeat at 7.5 s. These values must be consistent with the graph in Figure 20.







Show Solution

Strategy

The slope of a position vs. time graph at any point gives the instantaneous velocity at that time. Since Figure 19 shows a curved line with changing slope, we need to draw tangent lines at t =2.5s and t =7.5s and calculate their slopes. The results should be consistent with the corresponding velocity values shown in Figure 20.

Solution

(a) Velocity at t =2.5s:

Draw a tangent line to the position curve at t =2.5s. From the tangent line, we can identify two points:

At $t_1 \approx 0$s, the tangent gives $x_1 \approx 0$m

At $t_2 \approx 5$s, the tangent gives $x_2 \approx 20$m

Calculate the slope:

$v = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{20\text{m} - 0\text{m}}{5\text{s} - 0\text{s}} = \frac{20\text{m}}{5\text{s}} = 4\text{m/s}$

Checking Figure 20, at $t = 2.5\text{s}$, the velocity graph shows approximately 4m/s. ✓

(b) Velocity at $t = 7.5\text{s}$:

Draw a tangent line to the position curve at $t = 7.5\text{s}$. This section of the curve slopes downward, indicating negative velocity. From the tangent line:

At $t_1 \approx 5\text{s}$, the tangent gives $x_1 \approx 40\text{m}$

At $t_2 \approx 10\text{s}$, the tangent gives $x_2 \approx 20\text{m}$

Calculate the slope:

$v = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{20\text{m} - 40\text{m}}{10\text{s} - 5\text{s}} = \frac{-20\text{m}}{5\text{s}} = -4\text{m/s}$

Checking Figure 20, at $t = 7.5\text{s}$, the velocity graph shows approximately -4m/s. ✓

Discussion

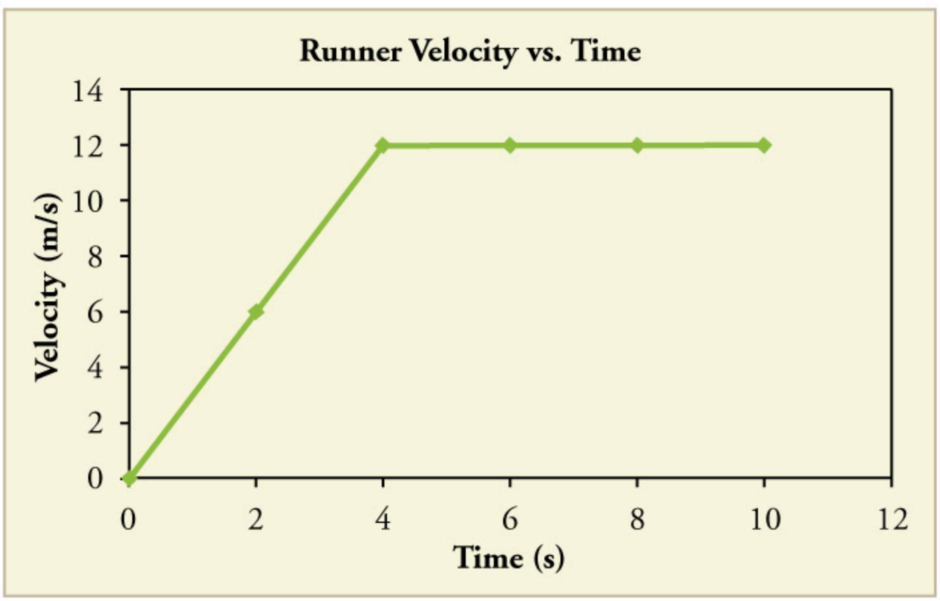
The results demonstrate the fundamental relationship between position and velocity graphs. At $t = 2.5\text{s}$, the position graph has a positive slope, meaning the jogger is moving in the positive direction at 4m/s. At $t = 7.5\text{s}$, the position graph has a negative slope, indicating the jogger is moving in the negative direction (perhaps running back) at -4m/s. The magnitudes are equal but opposite in sign, showing the jogger runs backward at the same speed as forward. The values extracted from the position-time graph are perfectly consistent with the velocity-time graph in Figure 20.

Answer

(a) The jogger's velocity at $t = 2.5\text{s}$ is approximately 4m/s.

(b) The jogger's velocity at $t = 7.5\text{s}$ is approximately -4m/s.

A graph of $v(t)$ is shown for a world-class track sprinter in a 100-m race. (See Figure 22). (a) What is his average velocity for the first 4 s? (b) What is his instantaneous velocity at $t = 5\text{s}$? (c) What is his average acceleration between 0 and 4 s? (d) What is his time for the race?



Show Solution

Strategy

Figure 22 shows a velocity-time graph with two distinct segments: an accelerating phase and a constant velocity phase. For part (a), average velocity equals displacement divided by time, which is the area under the velocity curve divided by time. For part (b), instantaneous velocity is read directly from the graph. For part (c), average acceleration is the slope of the velocity curve. For part (d), the race time is when the total displacement (area under curve) equals 100 m.

Solution

(a) Average velocity for the first 4 s:

The average velocity is:

$$\bar{v}=\frac{\Delta x}{\Delta t}$$

From Figure 22, the velocity increases linearly from 0 to 12 m/s during the first 4 seconds. The displacement is the area under the velocity curve:

$$\Delta x=\frac{1}{2}(v_0+v)(t)=\frac{1}{2}(0+12)(4)=24\text{m}$$

Therefore:

$$\bar{v}=\frac{24\text{m}}{4\text{s}}=6\text{m/s}$$

(b) Instantaneous velocity at $t=5\text{s}$:

From Figure 22, at $t=5\text{s}$, the sprinter has reached the constant velocity phase. Reading directly from the graph:

$$v(5\text{s})=12\text{m/s}$$

(c) Average acceleration between 0 and 4 s:

The average acceleration is:

$$\bar{a}=\frac{\Delta v}{\Delta t}=\frac{v_f-v_0}{t_f-t_0}$$

$$\bar{a}=\frac{12\text{m/s}-0\text{m/s}}{4\text{s}-0\text{s}}=\frac{12}{4}=3\text{m/s}^2$$

(d) Time for the 100-m race:

The total displacement must equal 100 m. The graph shows two phases:

Phase 1 (0 to 4 s): Acceleration

- Displacement: $x_1=\frac{1}{2}(12)(4)=24\text{m}$

Phase 2 (4 s to t_f): Constant velocity of 12 m/s

- Remaining distance: $100-24=76\text{m}$
- Time for remaining distance: $t_2=\frac{76\text{m}}{12\text{m/s}}=6.33\text{s}$

Total time:

$$t_{\text{total}}=4+6.33=10.33\text{s}\approx 10\text{s}$$

Discussion

This velocity-time graph illustrates a world-class sprinter's race strategy. The sprinter accelerates at 3m/s^2 for 4 seconds, reaching a maximum velocity of 12 m/s (about 43 km/h or 27 mph). During acceleration, the average velocity is only 6 m/s because the sprinter starts from rest. After reaching maximum velocity, the sprinter maintains it for the remainder of the race. The total race time of approximately 10 seconds is consistent with world-class 100-m sprint times. The acceleration phase covers only 24 m (less than a quarter of the race), while the constant velocity phase covers 76 m. This demonstrates that even elite sprinters spend most of the race at constant maximum velocity rather than accelerating.

Answer

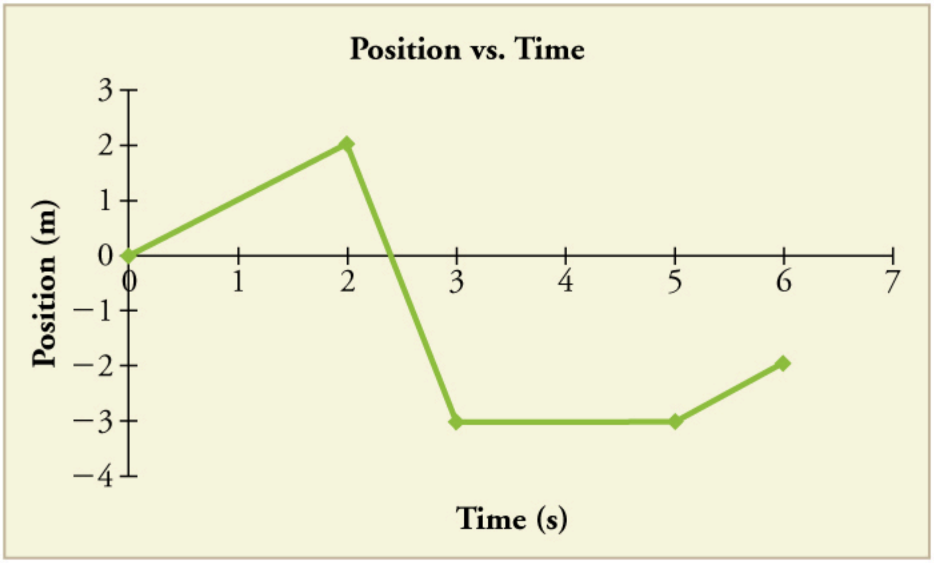
(a) The average velocity for the first 4 s is 6 m/s.

(b) The instantaneous velocity at $t=5\text{s}$ is 12 m/s.

(c) The average acceleration between 0 and 4 s is 3m/s^2.

(d) The time for the 100-m race is approximately 10 s.

Figure 23 shows the position graph for a particle for 5 s. Draw the corresponding velocity and acceleration graphs.



Show Solution

Strategy

To construct the velocity graph from the position graph, we calculate the slope of each segment, since velocity equals the slope of the position-time graph. To construct the acceleration graph from the velocity graph, we calculate the slope of the velocity-time graph, since acceleration equals the slope of velocity vs. time. We analyze each distinct segment where the slope is constant.

Solution

Analyzing the Position Graph (Figure 23):

The position graph has four distinct linear segments:

Segment 1 (0 to 1 s): Positive slope

- From the graph: $\Delta x \approx 2\text{m}$, $\Delta t = 1\text{s}$
- Velocity: $v_1 = \frac{2\text{m}}{1\text{s}} = 2\text{m/s}$

Segment 2 (1 to 2 s): Negative slope

- From the graph: $\Delta x \approx -4\text{m}$, $\Delta t = 1\text{s}$
- Velocity: $v_2 = \frac{-4\text{m}}{1\text{s}} = -4\text{m/s}$

Segment 3 (2 to 4 s): Zero slope (horizontal line)

- The position is constant
- Velocity: $v_3 = 0\text{m/s}$

Segment 4 (4 to 5 s): Positive slope

- From the graph: $\Delta x \approx 1\text{m}$, $\Delta t = 1\text{s}$
- Velocity: $v_4 = \frac{1\text{m}}{1\text{s}} = 1\text{m/s}$

Velocity Graph:

The velocity graph is a series of horizontal line segments at:

- $v = 2\text{m/s}$ from $t = 0$ to 1s
- $v = -4\text{m/s}$ from $t = 1$ to 2s
- $v = 0\text{m/s}$ from $t = 2$ to 4s
- $v = 1\text{m/s}$ from $t = 4$ to 5s

Analyzing the Velocity Graph for Acceleration:

The acceleration is the slope of the velocity graph. At transition points, there are instantaneous changes (vertical jumps) in velocity, which represent infinite acceleration. In reality, these would be very large but finite accelerations:

- **0 to 1 s:** Constant velocity, so $a = 0$
- **At t = 1 s:** Velocity changes from 2m/s to -4m/s (vertical jump)

- **1 to 2 s:** Constant velocity, so $a = 0$
- **At t = 2 s:** Velocity changes from -4m/s to 0m/s (vertical jump)
- **2 to 4 s:** Constant velocity, so $a = 0$
- **At t = 4 s:** Velocity changes from 0m/s to 1m/s (vertical jump)
- **4 to 5 s:** Constant velocity, so $a = 0$

Acceleration Graph:

The acceleration graph shows $a = 0$ for all segments between transitions, with vertical spikes (impulses) at $t = 1 \text{s}$, $t = 2 \text{s}$, and $t = 4 \text{s}$ where the velocity changes abruptly.

Discussion

The position graph shows a particle that moves forward, then backward, then stops for a period, then moves forward again. Each segment has constant velocity (straight line segments), so acceleration is zero during those times. The sharp corners in the position graph correspond to abrupt changes in velocity, which would require very large (theoretically infinite) accelerations. In real physical situations, these corners would be slightly rounded, representing large but finite accelerations over very short time intervals.

Answer

The velocity graph consists of horizontal segments at 2 m/s (0-1 s), -4 m/s (1-2 s), 0 m/s (2-4 s), and 1 m/s (4-5 s). The acceleration graph is zero throughout except for vertical spikes at t = 1 s, 2 s, and 4 s where the velocity changes instantaneously.

Glossary

independent variable
: the variable that the dependent variable is measured with respect to; usually plotted along the x -axis

dependent variable
: the variable that is being measured; usually plotted along the y -axis

slope
: the difference in y -value (the rise) divided by the difference in x-value (the run) of two points on a straight line

y-intercept
: the y- value when $x = 0$, or when the graph crosses the y -axis