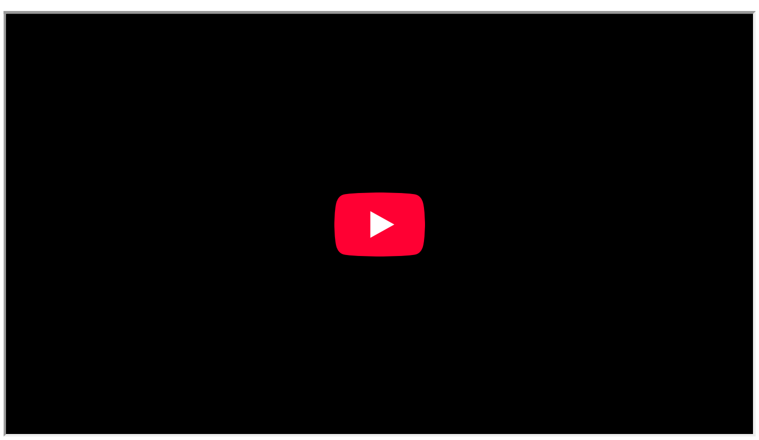
Introduction to Oscillatory Motion and Waves

There are at least four types of waves in this picture—only the water waves are evident. There are also sound waves, light waves, and waves on the guitar strings. (credit: John Norton)

What do an ocean buoy, a child in a swing, the cone inside a speaker, a guitar, atoms in a crystal, the motion of chest cavities, and the beating of hearts all have in common? They all **oscillate**—-that is, they move back and forth between two points. Many systems oscillate, and they have certain characteristics in common. All oscillations involve force and energy. You push a child in a swing to get the motion started. The energy of atoms vibrating in a crystal can be increased with heat. You put energy into a guitar string when you pluck it.

Some oscillations create **waves**. A guitar creates sound waves. You can make water waves in a swimming pool by slapping the water with your hand. You can no doubt think of other types of waves. Some, such as water waves, are visible. Some, such as sound waves, are not. But *every wave is a disturbance that moves from its source and carries energy*. Other examples of waves include earthquakes and visible light. Even subatomic particles, such as electrons, can behave like waves.

By studying oscillatory motion and waves, we shall find that a small number of underlying principles describe all of them and that wave phenomena are more common than you have ever imagined. We begin by studying the type of force that underlies the simplest oscillations and waves. We will then expand our exploration of oscillatory motion and waves to include concepts such as simple harmonic motion, uniform circular motion, and damped harmonic motion. Finally, we will explore what happens when two or more waves share the same space, in the phenomena known as superposition and interference.



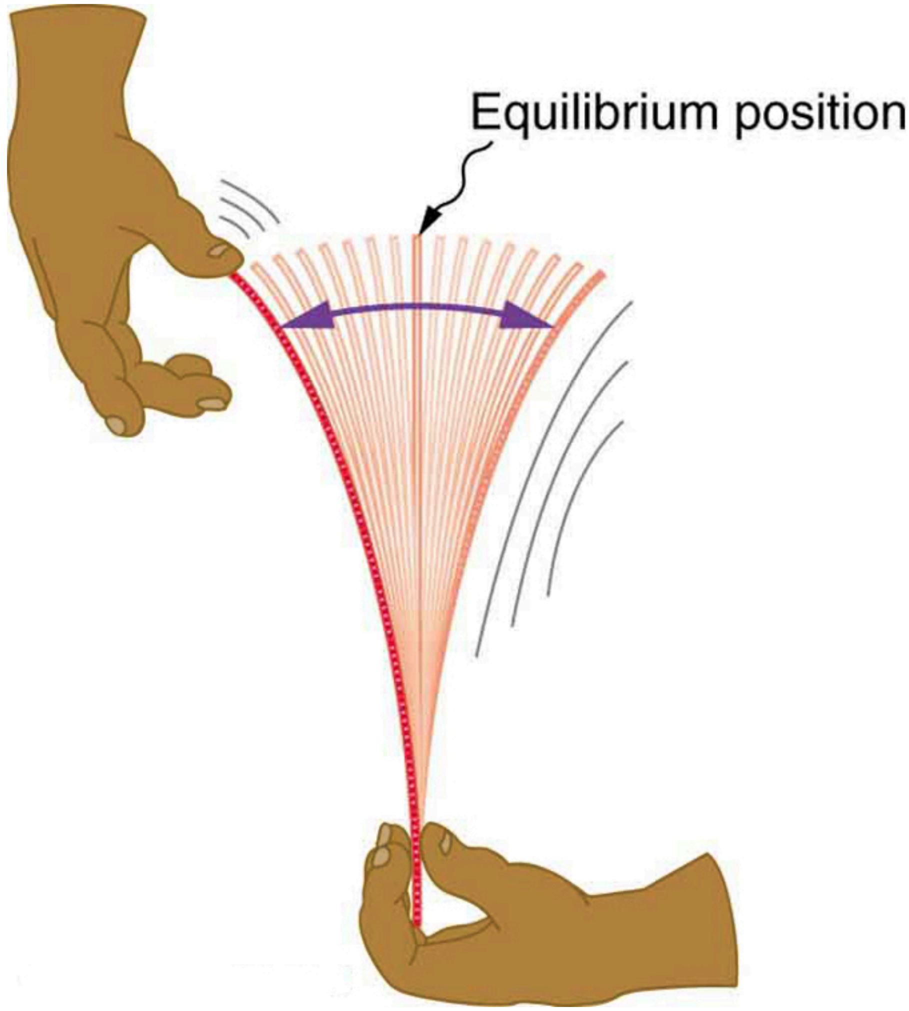
Glossary

oscillate
: moving back and forth regularly between two points

wave
: a disturbance that moves from its source and carries energy

Hooke’s Law: Stress and Strain Revisited

- Explain Newton’s third law of motion with respect to stress and deformation.
- Describe the restoration of force and displacement.
- Calculate the energy in Hook’s Law of deformation, and the stored energy in a string.



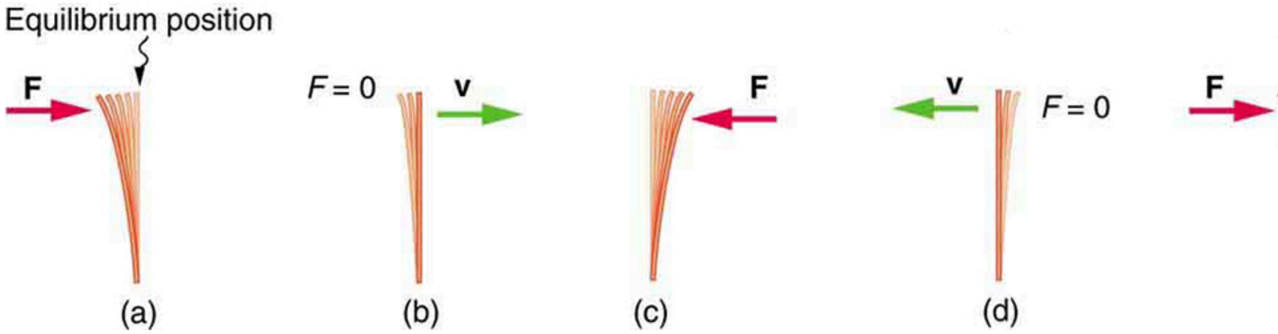
When displaced from its vertical equilibrium position, this plastic ruler oscillates back and forth because of the restoring force opposing displacement. When the ruler is on the left, there is a force to the right, and vice versa.

Newton’s first law implies that an object oscillating back and forth is experiencing forces. Without force, the object would move in a straight line at a constant speed rather than oscillate. Consider, for example, plucking a plastic ruler to the left as shown in [Figure 1]. The deformation of the ruler creates a force in the opposite direction, known as a **restoring force**. Once released, the restoring force causes the ruler to move back toward its stable equilibrium position, where the net force on it is zero. However, by the time the ruler gets there, it gains momentum and continues to move to the right, producing the opposite deformation. It is then forced to the left, back through equilibrium, and the process is repeated until dissipative forces dampen the motion. These forces remove mechanical energy from the system, gradually reducing the motion until the ruler comes to rest.

The simplest oscillations occur when the restoring force is directly proportional to displacement. When stress and strain were covered in Newton’s Third Law of Motion, the name was given to this relationship between force and displacement was Hooke’s law:

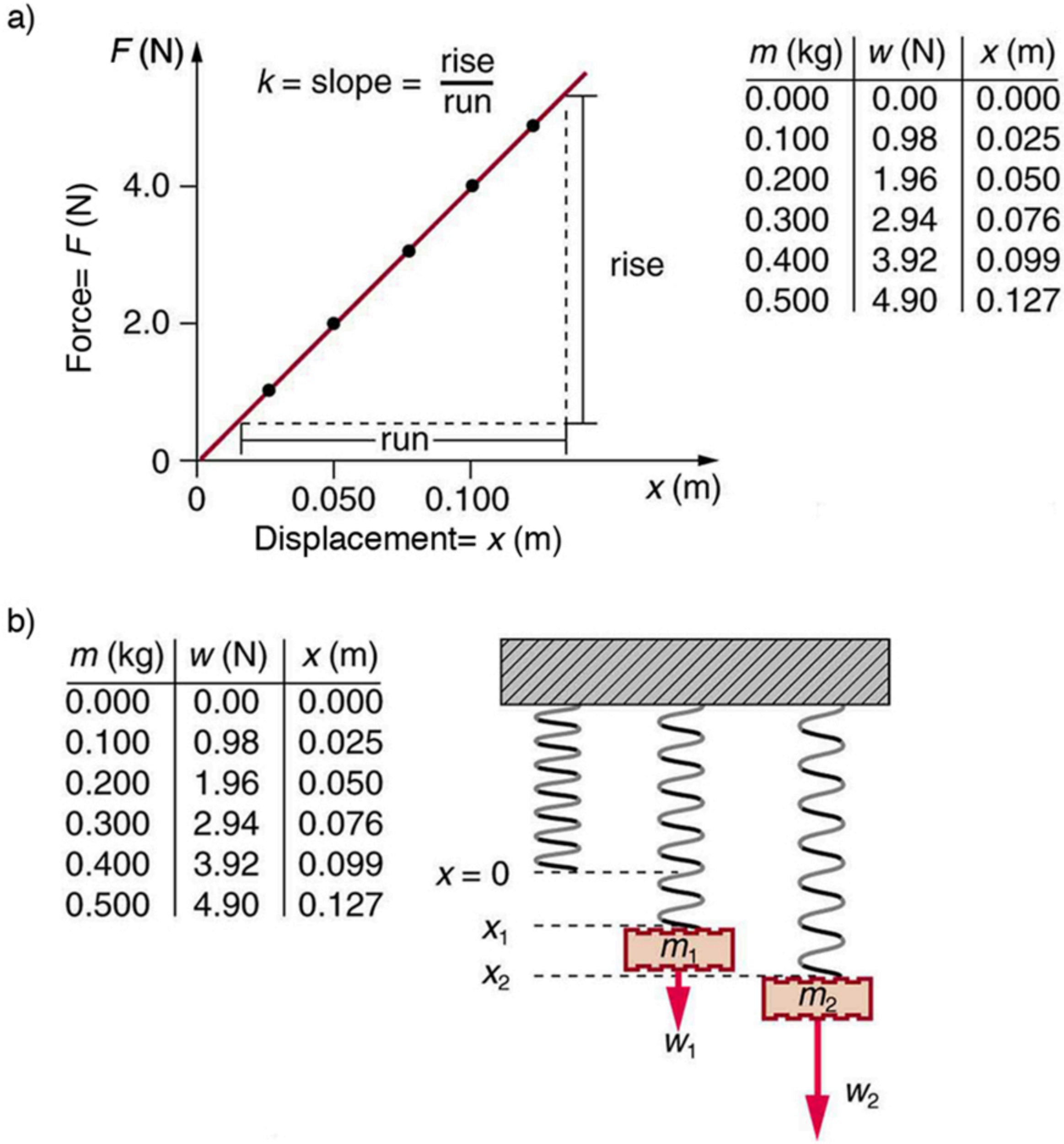
$$F=-kx.$$

Here, F is the restoring force, x is the displacement from equilibrium or **deformation**, and k is a constant related to the difficulty in deforming the system. The minus sign indicates the restoring force is in the direction opposite to the displacement.



(a) The plastic ruler has been released, and the restoring force is returning the ruler to its equilibrium position. (b) The net force is zero at the equilibrium position, but the ruler has momentum and continues to move to the right. (c) The restoring force is in the opposite direction. It stops the ruler and moves it back toward equilibrium again. (d) Now the ruler has momentum to the left. (e) In the absence of damping (caused by frictional forces), the ruler reaches its original position. From there, the motion will repeat itself.

The **force constant** k is related to the rigidity (or stiffness) of a system—the larger the force constant, the greater the restoring force, and the stiffer the system. The units of k are newtons per meter (N/m). For example, k is directly related to Young's modulus when we stretch a string. [Figure 3] shows a graph of the absolute value of the restoring force versus the displacement for a system that can be described by Hooke's law—a simple spring in this case. The slope of the graph equals the force constant k in newtons per meter. A common physics laboratory exercise is to measure restoring forces created by springs, determine if they follow Hooke's law, and calculate their force constants if they do.



(a) A graph of absolute value of the restoring force versus displacement is displayed. The fact that the graph is a straight line means that the system obeys Hooke’s law. The slope of the graph is the force constant k . (b) The data in the graph were generated by measuring the displacement of a spring from equilibrium while supporting various weights. The restoring force equals the weight supported, if the mass is stationary.

How Stiff Are Car Springs?

The mass of a car increases due to the introduction of a passenger. This affects the displacement of the car on its suspension system. (credit: exfordy on Flickr)

What is the force constant for the suspension system of a car that settles 1.20 cm when an 80.0-kg person gets in?

Strategy

Consider the car to be in its equilibrium position $x = 0$ before the person gets in. The car then settles down 1.20 cm, which means it is displaced to a position $x = -1.20 \times 10^{-2}\text{m}$. At that point, the springs supply a restoring force F equal to the person's weight $w = mg = (80.0\text{kg})(9.80\text{m/s}^2) = 784\text{N}$. We take this force to be F in Hooke's law. Knowing F and x, we can then solve the force constant k.

Solution

1. Solve Hooke's law, $F = -kx$, for k:
   $$k = -\frac{F}{x}.$$

   Substitute known values and solve k:

   $$k = -\frac{784\text{N}}{-1.20\times10^{-2}\text{m}} = 6.53\times10^{4}\text{N/m}.$$

Discussion

Note that F and x have opposite signs because they are in opposite directions—the restoring force is up, and the displacement is down. Also, note that the car would oscillate up and down when the person got in if it were not for damping (due to frictional forces) provided by shock absorbers. Bouncing cars are a sure sign of bad shock absorbers.

Energy in Hooke's Law of Deformation

In order to produce a deformation, work must be done. That is, a force must be exerted through a distance, whether you pluck a guitar string or compress a car spring. If the only result is deformation, and no work goes into thermal, sound, or kinetic energy, then all the work is initially stored in the deformed object as some form of potential energy. The potential energy stored in a spring is $\text{PE}_{\text{el}} = 12kx^2$. Here, we generalize the idea to elastic potential energy for a deformation of any system that can be described by Hooke's law. Hence,

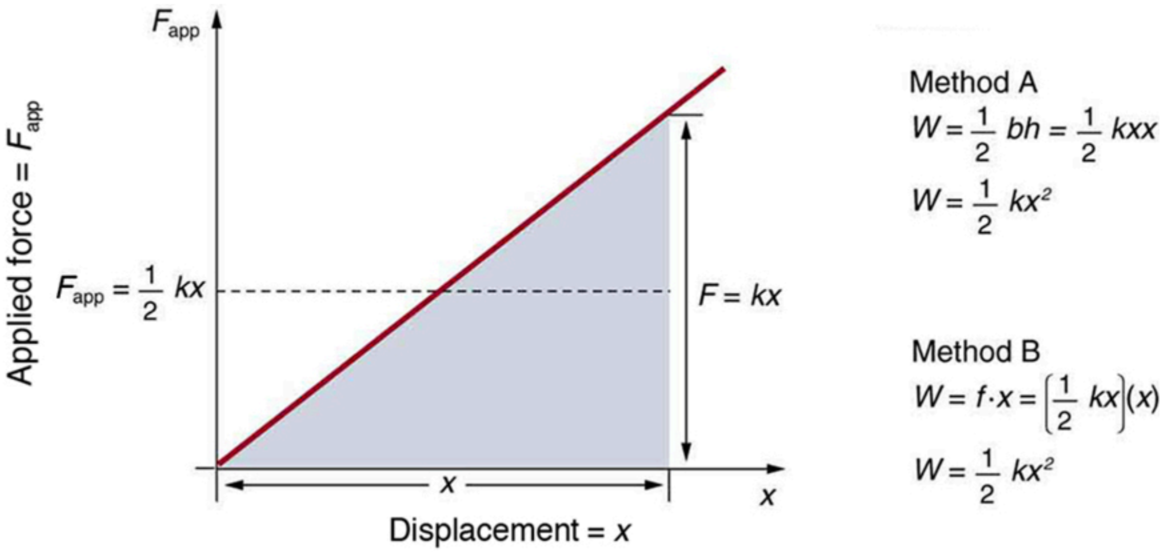
$$\text{PE}_{\text{el}} = 12kx^2,$$

where PE_{el} is the **elastic potential energy** stored in any deformed system that obeys Hooke's law and has a displacement x from equilibrium and a force constant k.

It is possible to find the work done in deforming a system in order to find the energy stored. This work is performed by an applied force F_{app} . The applied force is exactly opposite to the restoring force (action-reaction) , and so $F_{\text{app}} = kx$. [Figure 4] shows a graph of the applied force versus deformation x

for a system that can be described by Hooke's law. Work done on the system is force multiplied by distance, which equals the area under the curve or $(1/2)kx^2$ (Method A in the figure). Another way to determine the work is to note that the force increases linearly from 0 to kx , so that the average force is $(1/2)kx$, the distance moved is x , and thus $W = F_{\text{app}}d = [(1/2)kx](x) = (1/2)kx^2$

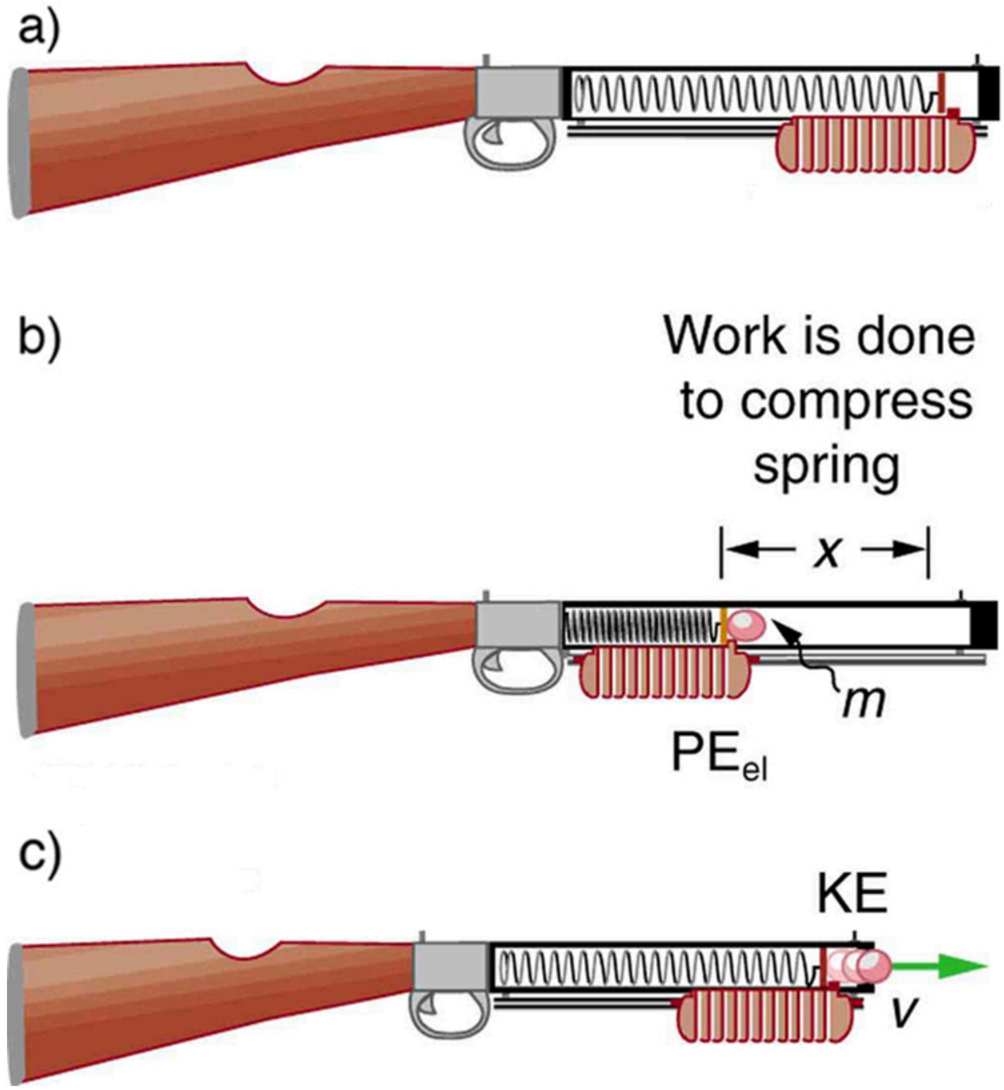
(Method B in the figure).



A graph of applied force versus distance for the deformation of a system that can be described by Hooke's law is displayed. The work done on the system equals the area under the graph or the area of the triangle, which is half its base multiplied by its height, or (W=(1/2)kx^2) .

Calculating Stored Energy: A Tranquilizer Gun Spring

We can use a toy gun's spring mechanism to ask and answer two simple questions: (a) How much energy is stored in the spring of a tranquilizer gun that has a force constant of 50.0 N/m and is compressed 0.150 m? (b) If you neglect friction and the mass of the spring, at what speed will a 2.00-g projectile be ejected from the gun?



(a) In this image of the gun, the spring is uncompressed before being cocked. (b) The spring has been compressed a distance x , and the projectile is in place. (c) When released, the spring converts elastic potential energy PE_el into kinetic energy.

Strategy for a

(a): **The energy stored in the spring can be found directly from elastic potential energy equation, because k and x are given.

Solution for a

Entering the given values for k and x yields

$$\text{PE}_\text{el} = \frac{1}{2}kx^2 = \frac{1}{2}(50.0\,\text{N/m})(0.150\,\text{m})^2 = 0.563\,\text{N}\cdot\text{m} = 0.563\,\text{J}$$

Strategy for b

Because there is no friction, the potential energy is converted entirely into kinetic energy. The expression for kinetic energy can be solved for the projectile's speed.

Solution for b

1. Identify known quantities:
   $\text{KE}_\text{f} = \text{PE}_\text{el}$ or $1/2mv^2 = (1/2)kx^2 = \text{PE}_\text{el} = 0.563\,\text{J}$
2. Solve for v :
   $v = \left[\frac{2\text{PE}_\text{el}}{m}\right]^{1/2} = \left[\frac{2(0.563\,\text{J})}{0.002\,\text{kg}}\right]^{1/2} = 23.7\,(\text{J/kg})^{1/2}$
3. Convert units: $23.7\,\text{m/s}$

Discussion

(a) and (b): This projectile speed is impressive for a tranquilizer gun (more than 80 km/h). The numbers in this problem seem reasonable. The force needed to compress the spring is small enough for an adult to manage, and the energy imparted to the dart is small enough to limit the damage it might do. Yet, the speed of the dart is great enough for it to travel an acceptable distance.
Check your Understanding

Envision holding the end of a ruler with one hand and deforming it with the other. When you let go, you can see the oscillations of the ruler. In what way could you modify this simple experiment to increase the rigidity of the system?

Show Solution
Answer

You could hold the ruler at its midpoint so that the part of the ruler that oscillates is half as long as in the original experiment.

Check your Understanding

If you apply a deforming force on an object and let it come to equilibrium, what happened to the work you did on the system?

Show Solution
Answer

It was stored in the object as potential energy.

Section Summary

- An oscillation is a back and forth motion of an object between two points of deformation.
- An oscillation may create a wave, which is a disturbance that propagates from where it was created.
- The simplest type of oscillations and waves are related to systems that can be described by Hooke's law:
  $F=-kx,$

  where F is the restoring force, x is the displacement from equilibrium or deformation, and k is the force constant of the system.
- Elastic potential energy PE_{el} stored in the deformation of a system that can be described by Hooke's law is given by
  $\text{PE}_{\text{el}}=(1/2)kx^2.$

Conceptual Questions

Describe a system in which elastic potential energy is stored.

Problems & Exercises

Fish are hung on a spring scale to determine their mass (most fishermen feel no obligation to truthfully report the mass).

(a) What is the force constant of the spring in such a scale if the spring stretches 8.00 cm for a 10.0 kg load?

(b) What is the mass of a fish that stretches the spring 5.50 cm?

(c) How far apart are the half-kilogram marks on the scale?

Show Solution

Strategy

For part (a), we use Hooke's law, recognizing that the restoring force equals the weight of the load. For part (b), we use the spring constant from part (a) with the new displacement to find the force, then calculate mass. For part (c), we find how much the spring stretches for a 0.5 kg mass using the spring constant.

Solution

(a) The weight of the load creates the restoring force:

$$F=mg=(10.0\text{ kg})(9.80\text{ m/s}^2)=98.0\text{ N}$$

The displacement is $x=8.00\text{ cm}=0.0800\text{ m}$. Using Hooke's law $F=kx$:

$$k=\frac{F}{x}=\frac{98.0\text{ N}}{0.0800\text{ m}}=1.23\times10^3\text{ N/m}$$

(b) For a displacement of $x=5.50\text{ cm}=0.0550\text{ m}$:

$$F=kx=(1.23\times10^3\text{ N/m})(0.0550\text{ m})=67.7\text{ N}$$

The mass of the fish is:

$m = \frac{F}{g} = \frac{67.7\ \text{N}}{9.80\ \text{m/s}^2} = 6.91\ \text{kg}$

Rounding to three significant figures: $6.88\ \text{kg}$

(c) For a 0.500 kg mass, the weight is:

$F = mg = (0.500\ \text{kg})(9.80\ \text{m/s}^2) = 4.90\ \text{N}$

The displacement is:

$x = \frac{F}{k} = \frac{4.90\ \text{N}}{1.23 \times 10^3\ \text{N/m}} = 0.00398\ \text{m} = 3.98\ \text{mm} \approx 4.00\ \text{mm}$

Discussion

Part (a): The spring constant of 1,230 N/m is reasonable for a fish scale. It's stiff enough that a 10 kg fish stretches the spring only 8 cm, making the scale compact and practical.

Part (b): A fish that stretches the spring 5.50 cm has mass 6.88 kg, which is 68.8% of the reference mass (10.0 kg). This ratio matches the displacement ratio (5.50/8.00 = 68.8%), confirming Hooke's law's linear relationship.

Part (c): The half-kilogram marks are only 4.00 mm apart. This close spacing means the scale needs fine gradations to be readable, which could make it challenging to use accurately in the field. Fishermen might appreciate this—it could make their catches appear larger if they misread between marks! The linear spacing is a direct consequence of Hooke's law: equal increments in mass produce equal increments in displacement.

It is weigh-in time for the local under-85-kg rugby team. The bathroom scale used to assess eligibility can be described by Hooke's law and is depressed 0.75 cm by its maximum load of 120 kg. (a) What is the spring's effective spring constant? (b) A player stands on the scales and depresses it by 0.48 cm. Is he eligible to play on this under-85 kg team?

Show Solution

Strategy

For part (a), we can use Hooke's law to find the spring constant. The maximum load creates a restoring force equal to the weight, and we know the displacement. For part (b), we use the spring constant found in part (a) along with the player's displacement to find his weight, and then determine his mass.

Solution

(a) The restoring force equals the weight of the maximum load:

$F = mg = (120\ \text{kg})(9.80\ \text{m/s}^2) = 1176\ \text{N}$

The displacement is $x = 0.75\ \text{cm} = 0.0075\ \text{m}$. Using Hooke's law $F = kx$:

$k = \frac{F}{x} = \frac{1176\ \text{N}}{0.0075\ \text{m}} = 1.57 \times 10^5\ \text{N/m}$

(b) For the player with displacement $x = 0.48\ \text{cm} = 0.0048\ \text{m}$:

$F = kx = (1.57 \times 10^5\ \text{N/m})(0.0048\ \text{m}) = 754\ \text{N}$

The player's mass is:

$m = \frac{F}{g} = \frac{754\ \text{N}}{9.80\ \text{m/s}^2} = 76.9\ \text{kg}$

Discussion

Since the player's mass (76.9 kg) is less than 85 kg, he is eligible to play on the team. The spring constant is quite large, which is appropriate for a bathroom scale that must support heavy loads without excessive compression. Note that bathroom scales are designed to be stiff enough that typical deflections are barely visible to the user.

Answer

(a) $1.57 \times 10^5\ \text{N/m}$

(b) Yes, he is eligible (mass = 76.9 kg)

One type of BB gun uses a spring-driven plunger to blow the BB from its barrel. (a) Calculate the force constant of its plunger's spring if you must compress it 0.150 m to drive the 0.0500-kg plunger to a top speed of 20.0 m/s. (b) What force must be exerted to compress the spring?

Show Solution

Strategy

For part (a), we use energy conservation. The elastic potential energy stored in the compressed spring is completely converted to kinetic energy of the plunger: $12kx^2 = 12mv^2$. We solve for the spring constant k. For part (b), we use Hooke's law $F = kx$ with the spring constant from part (a).

Solution

(a) From energy conservation:

$$12kx^2=12mv^2$$

Solving for k:

$$k=mv^2x^2=(0.0500\text{ kg})(20.0\text{ m/s})^2(0.150\text{ m})^2$$
$$k=(0.0500)(400)0.0225=20.00.0225=889\text{ N/m}$$

(b) The force required to compress the spring is:

$$F=kx=(889\text{ N/m})(0.150\text{ m})=133\text{ N}$$

Discussion

Part (a): The spring constant of 889 N/m indicates a moderately stiff spring, which is appropriate for a BB gun. The energy stored in the spring when compressed 15 cm is $12(889)(0.150)^2 = 10.0$ J, which equals the kinetic energy of the 50-gram plunger at 20 m/s: $12(0.0500)(20.0)^2 = 10.0$ J. This confirms our calculation.

Part (b): A force of 133 N (about 30 pounds-force or 13.6 kg-force) is reasonable for manually cocking a BB gun. It's manageable for most users but requires definite effort, providing appropriate resistance that makes the gun safe and controllable. The linear relationship between force and compression is a direct consequence of Hooke's law: compressing twice as far requires twice the force.

The plunger's final speed of 20.0 m/s (72 km/h or 45 mph) is quite fast, demonstrating the effectiveness of spring-powered projectile launchers. This design is safe because all the energy goes to the plunger, which then transfers it to the BB through air pressure rather than through direct spring-to-BB contact.

(a) The springs of a pickup truck act like a single spring with a force constant of $1.30 \times 10^5\text{N/m}$. By how much will the truck be depressed by its maximum load of 1000 kg?

(b) If the pickup truck has four identical springs, what is the force constant of each?

Show Solution

Strategy

For part (a), we use Hooke's law with the weight of the load as the applied force. For part (b), we need to understand how springs in parallel combine: when multiple springs share a load equally, the effective spring constant is the sum of individual spring constants.

Solution

(a) The weight of the maximum load is:

$$F=mg=(1000\text{ kg})(9.80\text{ m/s}^2)=9800\text{ N}$$

Using Hooke's law $F = kx$, we solve for displacement:

$$x=Fk=9800\text{ N}1.30\times10^5\text{ N/m}=0.0754\text{ m}=7.54\text{ cm}$$

(b) When springs act in parallel, the effective spring constant is the sum:

$$k_{\text{eff}}=k_1+k_2+k_3+k_4$$

For four identical springs:

$$k_{\text{eff}}=4k_{\text{individual}}$$

Therefore:

$$k_{\text{individual}}=k_{\text{eff}}4=1.30\times10^5\text{ N/m}4=3.25\times10^4\text{ N/m}$$

Discussion

The depression of 7.54 cm for a 1000 kg load is reasonable for a truck suspension system. This amount of compression provides a balance between comfort (too stiff would make for a harsh ride) and structural integrity (too soft would bottom out easily). The individual spring constant of 3.25×10^4

N/m for each of the four springs makes sense because springs in parallel share the load, effectively adding their spring constants together.

Answer

(a) 7.54 cm

(b) 3.25×10^4 N/m

When an 80.0-kg man stands on a pogo stick, the spring is compressed 0.120 m.

(a) What is the force constant of the spring? (b) Will the spring be compressed more when he hops down the road?

Show Solution

Strategy

For part (a), we apply Hooke's law, where the restoring force equals the man's weight at equilibrium. For part (b), we consider the dynamics of hopping: when the man lands after jumping, he has downward velocity that must be brought to zero, requiring additional compression beyond the static equilibrium position.

Solution

(a) At equilibrium, the spring force balances the man's weight:

$$F=mg=(80.0\text{ kg})(9.80\text{ m/s}^2)=784\text{ N}$$

Using Hooke's law $F=kx$ with $x=0.120$ m:

$$k=\frac{F}{x}=\frac{784\text{ N}}{0.120\text{ m}}=6.53\times10^3\text{ N/m}$$

(b) Yes, the spring will be compressed more when he hops down the road.

When hopping, the man lands with downward velocity. This kinetic energy must be absorbed by the spring, compressing it beyond the static equilibrium position. Additionally, the impact forces during landing can be several times body weight, further increasing compression.

Discussion

Part (a): The spring constant of 6,530 N/m indicates a fairly stiff spring, appropriate for supporting an 80 kg person while still providing enough "give" for bouncing. The 12 cm compression represents a reasonable balance—enough to store elastic energy for bouncing, but not so much that the user bottoms out the spring.

Part (b): The spring compression during hopping exceeds the static value for several reasons:

1. **Landing velocity**: When the person lands, they have downward kinetic energy (typically several hundred joules) that must be absorbed by additional spring compression.
2. **Impact forces**: Dynamic forces during landing can be 2-3 times static weight (1,600-2,400 N vs. 784 N static).
3. **Momentum transfer**: The abrupt deceleration when landing requires greater force than simply supporting static weight.

For example, if the man lands with a downward velocity of 2 m/s, his kinetic energy is $\frac{1}{2}(80.0)(2.0)^2=160$ J. This energy compresses the spring an additional distance Δx where $\frac{1}{2}k(\Delta x)^2=160$ J, giving $\Delta x=\sqrt{2(160)/6530}=0.22$ m. Combined with the 12 cm static compression, the total compression would be about 34 cm during landing.

This extra compression is what makes pogo sticks fun—the spring stores more energy during landing and releases it to propel the user upward on the next bounce!

A spring has a length of 0.200 m when a 0.300-kg mass hangs from it, and a length of 0.750 m when a 1.95-kg mass hangs from it. (a) What is the force constant of the spring? (b) What is the unloaded length of the spring?

Show Solution

Strategy

The total length of the spring equals the unloaded length plus the extension due to the hanging mass. We can write two equations using Hooke's law for the two different masses, then solve simultaneously for the spring constant and unloaded length.

Solution

(a) Let L_0 be the unloaded length. For the two cases:

$$L_1=L_0+x_1=0.200\text{ m}$$
$$L_2=L_0+x_2=0.750\text{ m}$$

where x_1 and x_2 are the extensions. From Hooke's law, $kx=mg$, so:

$$x_1 = \frac{m_1 g}{k} = \frac{(0.300\text{ kg})(9.80\text{ m/s}^2)}{k} = \frac{2.94\text{ N}}{k}$$
$$x_2 = \frac{m_2 g}{k} = \frac{(1.95\text{ kg})(9.80\text{ m/s}^2)}{k} = \frac{19.11\text{ N}}{k}$$

Subtracting the first length equation from the second:

$$L_2 - L_1 = x_2 - x_1$$
$$0.750 - 0.200 = \frac{19.11 - 2.94}{k}$$
$$0.550\text{ m} = \frac{16.17\text{ N}}{k}$$
$$k = \frac{16.17\text{ N}}{0.550\text{ m}} = 29.4\text{ N/m}$$

(b) Now we can find L_0 using the first case:

$$L_0 = L_1 - x_1 = 0.200\text{ m} - \frac{2.94\text{ N}}{29.4\text{ N/m}} = 0.200 - 0.100 = 0.100\text{ m}$$

Discussion

The spring constant of 29.4 N/m indicates a relatively soft spring, which makes sense given the large extension (55 cm) for a modest increase in mass (1.65 kg). We can verify our answer: with the 0.300 kg mass, the spring extends 10.0 cm from its unloaded length of 10.0 cm to reach 20.0 cm total. With the 1.95 kg mass, it extends 65.0 cm from 10.0 cm to reach 75.0 cm total. The ratio of extensions (65.0/10.0 = 6.5) equals the ratio of masses (1.95/0.300 = 6.5), confirming our solution is consistent with Hooke's law.

Answer

(a) 29.4 N/m

(b) 0.100 m or 10.0 cm

Glossary

deformation
: displacement from equilibrium

elastic potential energy
: potential energy stored as a result of deformation of an elastic object, such as the stretching of a spring

force constant
: a constant related to the rigidity of a system: the larger the force constant, the more rigid the system; the force constant is represented by *k*

restoring force
: force acting in opposition to the force caused by a deformation

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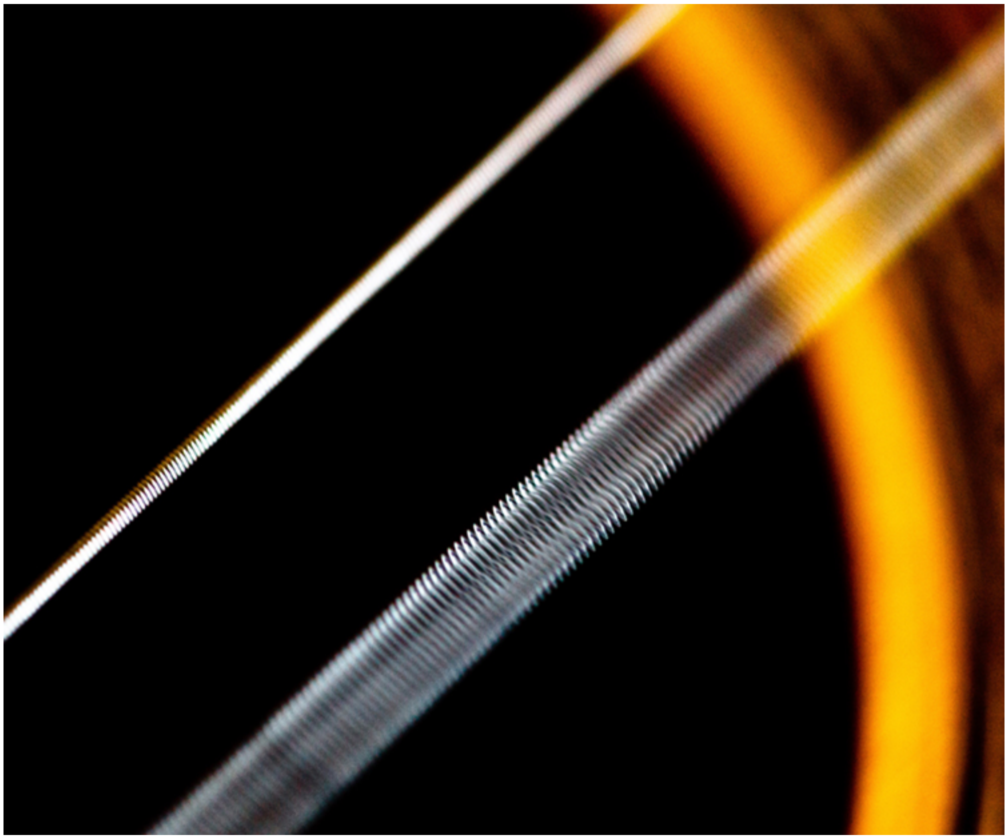
Period and Frequency in Oscillations

- Observe the vibrations of a guitar string.
- Determine the frequency of oscillations.



The strings on this guitar vibrate at regular time intervals. (credit: JAR)

When you pluck a guitar string, the resulting sound has a steady tone and lasts a long time. Each successive vibration of the string takes the same time as the previous one. We define **periodic motion** to be a motion that repeats itself at regular time intervals, such as exhibited by the guitar string or by an object on a spring moving up and down. The time to complete one oscillation remains constant and is called the **period** T . Its units are usually seconds, but may be any convenient unit of time. The word period refers to the time for some event whether repetitive or not; but we shall be primarily interested in periodic motion, which is by definition repetitive. A concept closely related to period is the frequency of an event. For example, if you get a paycheck twice a month, the frequency of payment is two per month and the period between checks is half a month. **Frequency** f is defined to be the number of events per unit time. For periodic motion, frequency is the number of oscillations per unit time. The relationship between frequency and period is

$$f=1T.$$

The SI unit for frequency is the *cycle per second*, which is defined to be a _ hertz_ (Hz):

$$1\text{Hz}=1\text{cyclesec or }1\text{Hz}=1\text{s}$$

A cycle is one complete oscillation. Note that a vibration can be a single or multiple event, whereas oscillations are usually repetitive for a significant number of cycles.

Determine the Frequency of Two Oscillations: Medical Ultrasound and the Period of Middle C

We can use the formulas presented in this module to determine both the frequency based on known oscillations and the oscillation based on a known frequency. Let’s try one example of each. (a) A medical imaging device produces ultrasound by oscillating with a period of 0.400 µs. What is the frequency of this oscillation? (b) The frequency of middle C on a typical musical instrument is 264 Hz. What is the time for one complete oscillation?

Strategy

Both questions (a) and (b) can be answered using the relationship between period and frequency. In question (a), the period T is given and we are asked to find frequency f . In question (b), the frequency f is given and we are asked to find the period T .

Solution a

1. Substitute $0.400\mu\text{s}$ for T in $f=1T$:

$f=\frac{1}{T}=\frac{1}{0.400\times10^{-6}\,s}.$

Solve to find

$f=2.50\times10^{6}\text{Hz}.$

Discussion a

The frequency of sound found in (a) is much higher than the highest frequency that humans can hear and, therefore, is called ultrasound. Appropriate oscillations at this frequency generate ultrasound used for noninvasive medical diagnoses, such as observations of a fetus in the womb.

Solution b

1. Identify the known values: The time for one complete oscillation is the period T :
   $f=\frac{1}{T}.$
2. Solve for T:
   $T=\frac{1}{f}.$
3. Substitute the given value for the frequency into the resulting expression:
   $T=\frac{1}{f}=\frac{1}{264\text{Hz}}=\frac{1}{264\text{cycles/s}}=3.79\times10^{-3}\text{s}=3.79\text{ms}.$

Discussion

The period found in (b) is the time per cycle, but this value is often quoted as simply the time in convenient units (ms or milliseconds in this case).
Check your Understanding

Identify an event in your life (such as receiving a paycheck) that occurs regularly. Identify both the period and frequency of this event.

Show Solution

I visit my parents for dinner every other Sunday. The frequency of my visits is 26 per calendar year. The period is two weeks.

Section Summary

- Periodic motion is a repetitious oscillation.
- The time for one oscillation is the period T .
- The number of oscillations per unit time is the frequency f .
- These quantities are related by

  $f=\frac{1}{T}.$

Problems & Exercises

What is the period of 60.0Hz electrical power?

Show Solution

Strategy

Period and frequency are reciprocals: $T=\frac{1}{f}$.

Solution

$T=\frac{1}{f}=\frac{1}{60.0\text{ Hz}}=0.0167\text{ s}=16.7\text{ ms}$

Discussion

The 60 Hz frequency is standard for electrical power in North America. This means the voltage oscillates with a period of 16.7 ms, completing 60 full cycles every second. In countries using 50 Hz power, the period would be 20 ms. This rapid oscillation is why incandescent lights flicker at twice this frequency (120 Hz), though too fast for the human eye to perceive.

Answer

16.7 ms

If your heart rate is 150 beats per minute during strenuous exercise, what is the time per beat in units of seconds?

Show Solution

Strategy

The heart rate is given as a frequency (150 beats per minute). We need to find the period (time per beat) in seconds. First, convert the frequency to beats per second, then use $T = \frac{1}{f}$ to find the period.

Solution

Convert the heart rate from beats per minute to beats per second:

$$f = 150 \frac{\text{beats}}{\text{min}} \times \frac{1\ \text{min}}{60\ \text{s}} = 2.50 \frac{\text{beats}}{\text{s}} = 2.50\ \text{Hz}$$

Now find the period using $T = \frac{1}{f}$:

$$T = \frac{1}{f} = \frac{1}{2.50\ \text{Hz}} = 0.400\ \text{s/beat}$$

Discussion

A period of 0.400 s per beat is reasonable for strenuous exercise. At rest, a typical heart rate might be 60-70 beats per minute (about 1 beat per second), so the higher rate during exercise corresponds to a shorter period between beats. This faster rate allows the heart to pump more blood to supply oxygen to working muscles.

Answer

0.400 s/beat

Find the frequency of a tuning fork that takes $2.50 \times 10^{-3}\text{s}$ to complete one oscillation.

Show Solution

Strategy

The period T is given as the time to complete one oscillation. Use the relationship $f = \frac{1}{T}$ to find the frequency.

Solution

Substitute the given period into $f = \frac{1}{T}$:

$$f = \frac{1}{T} = \frac{1}{2.50 \times 10^{-3}\ \text{s}} = \frac{1}{0.00250\ \text{s}} = 400\ \text{Hz}$$

Discussion

A frequency of 400 Hz falls within the range of human hearing (roughly 20 Hz to 20,000 Hz) and corresponds to a musical note near G4 or A4, depending on the tuning standard. Tuning forks are designed to produce pure tones at specific frequencies and are commonly used to tune musical instruments. The period of 2.50 ms is quite short, meaning the fork completes 400 complete vibrations every second.

Answer

400 Hz

A stroboscope is set to flash every $8.00 \times 10^{-5}\text{s}$. What is the frequency of the flashes?

Show Solution

Strategy

The time between flashes is the period T. Use $f = \frac{1}{T}$ to calculate the frequency of the flashes.

Solution

Substitute the given period into $f = \frac{1}{T}$:

$$f = \frac{1}{T} = \frac{1}{8.00 \times 10^{-5}\ \text{s}} = \frac{1}{0.0000800\ \text{s}} = 12500\ \text{Hz} = 12.5\ \text{kHz}$$

Discussion

The frequency of 12,500 Hz (12.5 kHz) is quite high and actually exceeds the upper limit of human hearing for most people. Stroboscopes are used to make rapidly moving or rotating objects appear stationary by synchronizing the flash rate with the object's motion. This high frequency allows the stroboscope to "freeze" very fast-moving objects, making 12,500 flashes per second. The period of 80 microseconds is extremely short, demonstrating the rapid nature of the flashing.

Answer

12,500 Hz or 12.5 kHz

A tire has a tread pattern with a crevice every 2.00 cm. Each crevice makes a single vibration as the tire moves. What is the frequency of these vibrations if the car moves at 30.0 m/s?

Show Solution

Strategy

The frequency is the number of vibrations per second. Since each crevice causes one vibration, we need to find how many crevices pass a given point per second. This equals the velocity divided by the spacing between crevices: $f = \frac{v}{d}$.

Solution

Convert the crevice spacing to meters:

$$d = 2.00 \text{ cm} = 0.0200 \text{ m}$$

Calculate the frequency using $f = \frac{v}{d}$:

$$f = \frac{v}{d} = \frac{30.0 \text{ m/s}}{0.0200 \text{ m}} = 1500 \text{ Hz} = 1.50 \text{ kHz}$$

Discussion

A frequency of 1,500 Hz is well within the range of human hearing and would produce an audible tone. This is why tires can make a humming or whining sound at high speeds—the regular pattern of the tread creates vibrations at specific frequencies. The speed of 30.0 m/s is about 108 km/h (67 mph), a typical highway speed. At slower speeds, the frequency would be lower and the pitch of the sound would be deeper. At faster speeds, the pitch would be higher.

Answer

1.50 kHz

Engineering Application

Each piston of an engine makes a sharp sound every other revolution of the engine. (a) How fast is a race car going if its eight-cylinder engine emits a sound of frequency 750 Hz, given that the engine makes 2000 revolutions per kilometer? (b) At how many revolutions per minute is the engine rotating?

Show Solution

Strategy

(a) The sound frequency is determined by how many piston firings occur per second. Since there are 8 cylinders and each fires once every 2 revolutions, we can find the engine's revolution rate from the sound frequency. Then, using the given relationship between revolutions and distance (2000 rev/km), we can find the car's speed.

(b) Once we know the engine's revolution rate in rev/s from part (a), we can convert this to rev/min.

Solution for (a)

First, find how many piston firings occur per engine revolution. With 8 cylinders and each firing once every 2 revolutions:

$$\text{Firings per revolution} = \frac{8 \text{ cylinders}}{2 \text{ rev}} = 4 \text{ firings/rev}$$

The sound frequency equals the number of firings per second, so we can find the engine's revolution rate:

$$f = 750 \text{ Hz} = 750 \text{ firings/s}$$
$$\text{Rev rate} = \frac{750 \text{ firings/s}}{4 \text{ firings/rev}} = 187.5 \text{ rev/s}$$

Now use the given relationship that the engine makes 2000 revolutions per kilometer to find the speed. Convert km to m:

$$2000 \text{ rev/km} = \frac{2000 \text{ rev}}{1000 \text{ m}} = 2.00 \text{ rev/m}$$

The car's speed is:

$$v = \frac{187.5 \text{ rev/s}}{2.00 \text{ rev/m}} = 93.8 \text{ m/s}$$

Solution for (b)

Convert the engine's revolution rate from rev/s to rev/min:

$$\text{Rev rate} = 187.5 \text{ rev/s} \times \frac{60 \text{ s}}{1 \text{ min}} = 11250 \text{ rev/min} = 11.3 \times 10^3 \text{ rev/min}$$

Discussion

The speed of 93.8 m/s is equivalent to about 338 km/h or 210 mph, which is reasonable for a race car at high speed. The engine speed of 11,250 rpm is also typical for a high-performance racing engine under heavy load. The high-pitched 750 Hz sound (close to the musical note G5) is characteristic of racing engines. Note that the relationship between engine revolutions and distance traveled depends on the gear ratio and wheel size—the given value of 2000 rev/km corresponds to a specific gear selection.

Answer

(a) 93.8 m/s

(b) 11.3×10^3 rev/min

Glossary

period
: time it takes to complete one oscillation

periodic motion
: motion that repeats itself at regular time intervals

frequency
: number of events per unit of time

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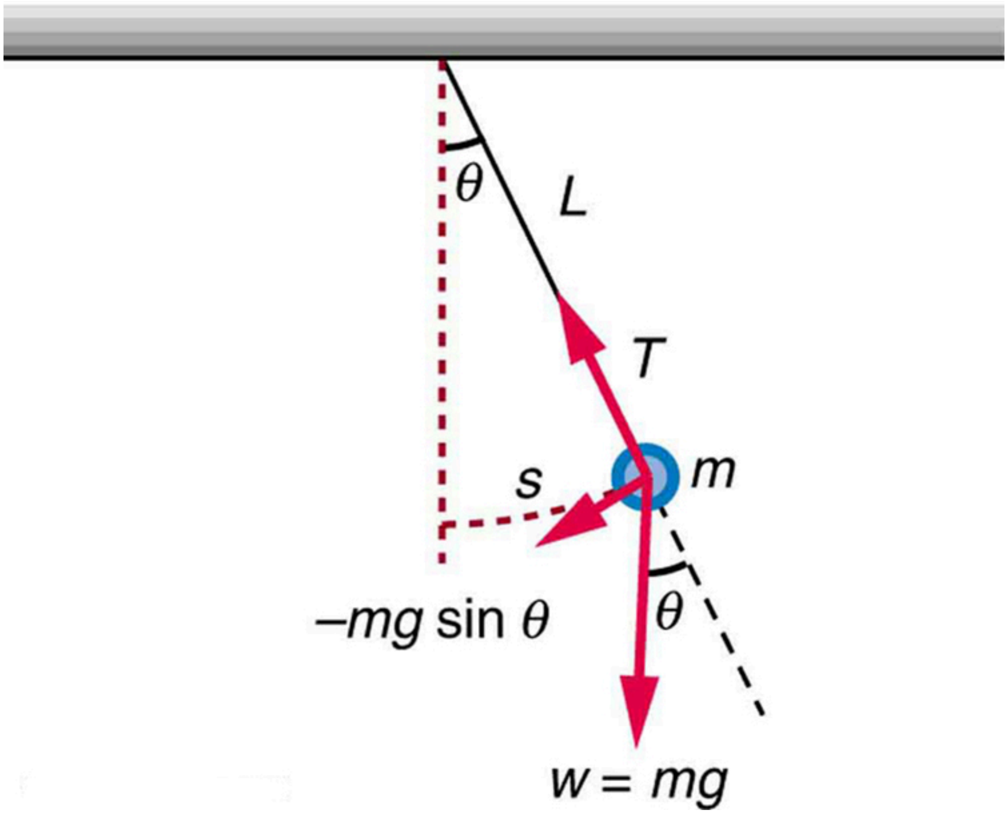
The Simple Pendulum

- Measure acceleration due to gravity.



A simple pendulum has a small-diameter bob and a string that has a very small mass but is strong enough not to stretch appreciably. The linear displacement from equilibrium is s , the length of the arc. Also shown are the forces on the bob, which result in a net force of $-mg \sin \theta$ toward the equilibrium position—that is, a restoring force.

Pendulums are in common usage. Some have crucial uses, such as in clocks; some are for fun, such as a child's swing; and some are just there, such as the sinker on a fishing line. For small displacements, a pendulum is a simple harmonic oscillator. A **simple pendulum** is defined to have an object that has a small mass, also known as the pendulum bob, which is suspended from a light wire or string, such as shown in [Figure 1]. Exploring the simple pendulum a bit further, we can discover the conditions under which it performs simple harmonic motion, and we can derive an interesting expression for its period.

We begin by defining the displacement to be the arc length s . We see from [Figure 1] that the net force on the bob is tangent to the arc and equals $-mg\sin\theta$. (The weight mg has components $mg\cos\theta$ along the string and $mg\sin\theta$ tangent to the arc.) Tension in the string exactly cancels the component $mg\cos\theta$ parallel to the string. This leaves a *net* restoring force back toward the equilibrium position at θ =0.

Now, if we can show that the restoring force is directly proportional to the displacement, then we have a simple harmonic oscillator. In trying to determine if we have a simple harmonic oscillator, we should note that for small angles (less than about 15°), $\sin\theta \approx\theta$ ($\sin\theta$ and θ differ by about 1% or less at smaller angles) . Thus, for angles less than about 15° , the restoring force F is

$$F\approx-mg\theta.$$

The displacement s is directly proportional to θ . When θ is expressed in radians, the arc length in a circle is related to its radius (L in this instance) by:

$$s=L\theta,$$

so that

$$\theta=sL.$$

For small angles, then, the expression for the restoring force is:

$$F\approx-mgLs$$

This expression is of the form:

$$F=-kx,$$

where the force constant is given by $k=mg/L$ and the displacement is given by $x=s$. For angles less than about $15°$, the restoring force is directly proportional to the displacement, and the simple pendulum is a simple harmonic oscillator.

Using this equation, we can find the period of a pendulum for amplitudes less than about $15°$. For the simple pendulum:

$$T=2\pi\sqrt{mk}=2\pi\sqrt{mmg/L}.$$

Thus,

$$T=2\pi\sqrt{Lg}$$

for the period of a simple pendulum. This result is interesting because of its simplicity. The only things that affect the period of a simple pendulum are its length and the acceleration due to gravity. The period is completely independent of other factors, such as mass. As with simple harmonic oscillators, the period T for a pendulum is nearly independent of amplitude, especially if θ is less than about $15°$. Even simple pendulum clocks can be finely adjusted and accurate.

Note the dependence of T on g. If the length of a pendulum is precisely known, it can actually be used to measure the acceleration due to gravity. Consider the following example.

Measuring Acceleration due to Gravity: The Period of a Pendulum

What is the acceleration due to gravity in a region where a simple pendulum having a length 75.000 cm has a period of 1.7357 s?

Strategy

We are asked to find g given the period T and the length L of a pendulum. We can solve $T=2\pi\sqrt{Lg}$ for g, assuming only that the angle of deflection is less than $15°$.

Solution

1. Square $T=2\pi\sqrt{Lg}$ and solve for g:

   $$g=4\pi^2LT^2.$$
2. Substitute known values into the new equation:
   $$g=4\pi^2 0.75000\text{m}(1.7357\text{s})^2.$$
3. Calculate to find g:
   $$g=9.8281\text{m/s}^2.$$

Discussion

This method for determining g can be very accurate. This is why length and period are given to five digits in this example. For the precision of the approximation $\sin\theta\approx\theta$ to be better than the precision of the pendulum length and period, the maximum displacement angle should be kept below about $0.5°$.

Making Career Connections

Knowing g can be important in geological exploration; for example, a map of g over large geographical regions aids the study of plate tectonics and helps in the search for oil fields and large mineral deposits.

Take Home Experiment: Determining g

Use a simple pendulum to determine the acceleration due to gravity g in your own locale. Cut a piece of a string or dental floss so that it is about 1 m long. Attach a small object of high density to the end of the string (for example, a metal nut or a car key). Starting at an angle of less than $10°$, allow the pendulum to swing and measure the pendulum's period for 10 oscillations using a stopwatch. Calculate g. How accurate is this measurement? How might it be improved?

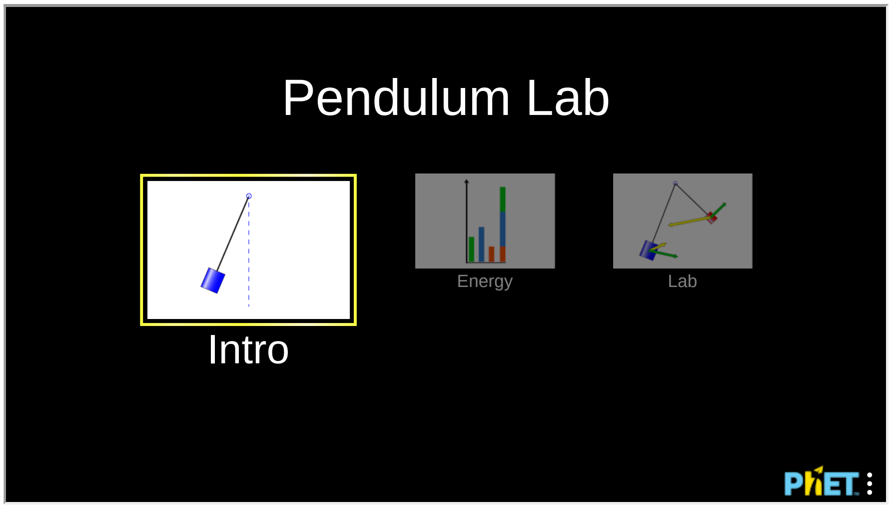
Check Your Understanding

An engineer builds two simple pendula. Both are suspended from small wires secured to the ceiling of a room. Each pendulum hovers 2 cm above the floor. Pendulum 1 has a bob with a mass of 10kg. Pendulum 2 has a bob with a mass of 100kg. Describe how the motion of the pendula will differ if the bobs are both displaced by $12°$.

Show Solution

The movement of the pendula will not differ at all because the mass of the bob has no effect on the motion of a simple pendulum. The pendula are only affected by the period (which is related to the pendulum's length) and by the acceleration due to gravity.
PhET Explorations: Pendulum Lab

Play with one or two pendulums and discover how the period of a simple pendulum depends on the length of the string, the mass of the pendulum bob, and the amplitude of the swing. It's easy to measure the period using the photogate timer. You can vary friction and the strength of gravity. Use the pendulum to find the value of g on planet X. Notice the anharmonic behavior at large amplitude.



Section Summary

- A mass m suspended by a wire of length L is a simple pendulum and undergoes simple harmonic motion for amplitudes less than about $15°$. The period of a simple pendulum is

  $$T=2\pi\sqrt{Lg},$$

where L is the length of the string and g is the acceleration due to gravity.

Conceptual Questions

Pendulum clocks are made to run at the correct rate by adjusting the pendulum's length. Suppose you move from one city to another where the acceleration due to gravity is slightly greater, taking your pendulum clock with you, will you have to lengthen or shorten the pendulum to keep the correct time, other factors remaining constant? Explain your answer.

Problems & Exercises

As usual, the acceleration due to gravity in these problems is taken to be $g = 9.80 \text{m/s}^2$, unless otherwise specified.

What is the length of a pendulum that has a period of 0.500 s?

Show Solution

Strategy

We can use the formula for the period of a simple pendulum $T = 2\pi\sqrt{Lg}$ and solve for the length L.

Solution

Squaring both sides of $T = 2\pi\sqrt{Lg}$:

$$T^2=4\pi^2 Lg$$

Solving for L:

$$L=gT^2 4\pi^2$$

Substituting the known values $g = 9.80 \text{ m/s}^2$ and $T = 0.500$ s:

$$L=(9.80)(0.500)^2 4\pi^2=(9.80)(0.250)39.48=2.4539.48=0.0621 \text{ m}$$

Converting to centimeters: $L = 6.21\text{ cm}$

Discussion

A pendulum only about 6 cm long (about 2.4 inches) has a very short period of half a second. This result demonstrates the square root relationship between length and period in the equation $T = 2\pi\sqrt{\frac{L}{g}}$. To achieve such a short period requires a very short pendulum length.

This makes physical sense: a shorter pendulum has less distance to travel in each swing, and the restoring force (which depends on the angle from vertical) acts over a shorter arc length, causing the bob to accelerate more quickly back toward equilibrium. The result is a rapid oscillation—this pendulum completes 2 full back-and-forth swings per second, or 120 swings per minute.

Such short-period pendulums have practical applications in small clocks and timing mechanisms where compact size is important. However, they're also more sensitive to friction and air resistance (which scale unfavorably at small sizes), making them less suitable for precision timekeeping compared to longer pendulums. The answer is reasonable in magnitude—a few centimeters is a practical length that could actually be constructed and used.

Answer

6.21 cm

Some people think a pendulum with a period of 1.00 s can be driven with "mental energy" or psycho kinetically, because its period is the same as an average heartbeat. True or not, what is the length of such a pendulum?

Show Solution

Strategy

We can use the formula for the period of a simple pendulum $T = 2\pi\sqrt{\frac{L}{g}}$ and solve for the length L.

Solution

Squaring both sides of $T = 2\pi\sqrt{\frac{L}{g}}$:

$$T^2 = 4\pi^2\frac{L}{g}$$

Solving for L:

$$L = \frac{gT^2}{4\pi^2} = \frac{(9.80)(1.00)^2}{4\pi^2} = \frac{9.80}{39.48} = 0.248\text{ m}$$

Discussion

A pendulum of length 24.8 cm (about 10 inches) has a period of 1.00 s, matching the average human heartbeat of about 60 beats per minute. This coincidence has led to various claims about "psychokinetic" effects, though physics provides no mechanism for mental energy to influence pendulum motion.

The result is physically reasonable and demonstrates an important property of pendulums: the period depends only on length and gravity, not on who is holding it or thinking about it. A 1-second period is very convenient for timekeeping and demonstrations, which is why "seconds pendulums" (with a half-period of 1 second, giving a full period of 2 seconds) have historically been important in clockmaking.

The 25-cm length is practical for handheld demonstrations and small experimental setups. It's also worth noting that this pendulum would complete exactly 60 full swings per minute, making it easy to observe and count. The independence of the period from mass means that different observers holding identical-length pendulums would see them swing in perfect synchrony, regardless of any purported "mental influence"—a good demonstration of the objective nature of physical laws.

Answer

0.248 m or 24.8 cm

What is the period of a 1.00-m-long pendulum?

Show Solution

Strategy

We can directly use the period formula $T = 2\pi\sqrt{\frac{L}{g}}$ with the given length $L = 1.00\text{ m}$.

Solution

$$T = 2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{1.00}{9.80}}$$
$$T = 2\pi\sqrt{0.102} = 2\pi(0.319) = 2.01\text{ s}$$

Discussion

A 1-meter pendulum has a period of about 2 seconds, making it one of the most commonly used pendulum lengths in physics education and demonstrations. This result is historically significant: early proposals for defining the meter suggested using the length of a "seconds pendulum" (one with a half-period of 1 second, or full period of 2 seconds). However, this definition was ultimately rejected because g varies with latitude and altitude, meaning the required length would differ by location.

The 2-second period makes this pendulum very convenient for timing manually with a stopwatch—you can easily count "one thousand one, one thousand two" as it swings. The pendulum completes 30 full oscillations per minute, which is slow enough to observe clearly but fast enough to gather meaningful data quickly.

This length is also practical for classroom demonstrations: it requires about 1 meter of string and a ceiling height of at least 1.5-2 meters, which is available in most labs. The result demonstrates that longer pendulums have longer periods—compare this 2-second period to the 0.5-second period of the 6-cm pendulum in Problem 1. The period scales with the square root of length, so multiplying the length by 16 (from 6.21 cm to 100 cm) multiplies the period by $\sqrt{16} = 4$ (from 0.5 s to 2.0 s).

Answer

2.01 s

How long does it take a child on a swing to complete one swing if her center of gravity is 4.00 m below the pivot?

Show Solution

Strategy

The time for one complete swing is the period. We use $T = 2\pi\sqrt{\frac{L}{g}}$ where $L = 4.00$ m is the distance from the pivot to the center of gravity.

Solution

$$T = 2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{4.00}{9.80}} = 2\pi\sqrt{0.408} = 2\pi(0.639) = 4.01 \text{ s}$$

Discussion

A period of about 4 seconds is typical for a playground swing, which makes physical sense based on the swing's length. The 4-meter distance from pivot to center of gravity is realistic for a standard swing set. This means the child completes one full back-and-forth motion every 4 seconds, or about 15 complete oscillations per minute.

This result illustrates an important principle that parents intuitively discover: to effectively push a child on a swing, you must push at the swing's natural frequency (about once every 4 seconds). Pushing at any other frequency leads to poor energy transfer and small amplitudes. This is an example of resonance in simple harmonic motion.

Critically, the period depends only on the length (4 m to the center of gravity) and gravitational acceleration, not on the child's mass. Whether a small toddler or a large adult sits on the swing, the period remains 4 seconds. This mass-independence is a unique property of pendulum motion. Similarly, the period is independent of amplitude (for small angles), so whether the child swings gently or very high, the period stays constant—this is called isochronism.

The 4-second period is slow enough that parents can easily time their pushes, making swings effective and enjoyable for children. Comparing to the 2-second period of a 1-meter pendulum (Problem 3), we see that doubling the length from 1 m to 4 m doesn't double the period—it multiplies it by $\sqrt{4} = 2$, demonstrating the square root relationship in $T = 2\pi\sqrt{\frac{L}{g}}$.

Answer

4.01 s

The pendulum on a cuckoo clock is 5.00 cm long. What is its frequency?

Show Solution

Strategy

The frequency is the reciprocal of the period. We first find the period using $T = 2\pi\sqrt{\frac{L}{g}}$, then calculate $f = \frac{1}{T}$.

Solution

Convert length to meters: $L = 5.00 \text{ cm} = 0.0500 \text{ m}$

Calculate the period:

$$T = 2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{0.0500}{9.80}} = 2\pi\sqrt{0.00510} = 2\pi(0.0714) = 0.449 \text{ s}$$

Calculate the frequency:

$$f = \frac{1}{T} = \frac{1}{0.449} = 2.23 \text{ Hz}$$

Discussion

The cuckoo clock pendulum oscillates at 2.23 Hz, meaning it completes about 2.2 full swings per second, or about 134 swings per minute. This high frequency is characteristic of the short pendulum used in cuckoo clocks and creates the rapid, distinctive ticking sound these clocks are known for.

Comparing to other pendulum clocks helps establish reasonableness: a grandfather clock typically uses a pendulum about 1 meter long with a period of 2 seconds (frequency 0.5 Hz), while this 5-cm cuckoo clock pendulum has a frequency more than 4 times higher. The short pendulum allows the clock to be compact and mounted on a wall, unlike the tall grandfather clock that requires floor space.

The relationship between frequency and period ($f = 1/T$) combined with the pendulum formula gives us $f = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$. This shows that frequency is inversely proportional to the square root of length: shorter pendulums oscillate faster. The 5-cm length is practical for manufacturing small clock mechanisms and provides adequate mechanical advantage for the escapement mechanism that regulates the clock's gears.

One limitation of such short pendulums is their greater sensitivity to friction and air resistance, which can affect timing accuracy more than in longer pendulums. However, for decorative cuckoo clocks, absolute precision is less critical than for scientific instruments.

Answer

2.23 Hz

Two parakeets sit on a swing with their combined center of mass 10.0 cm below the pivot. At what frequency do they swing?

Show Solution

Strategy

The frequency is the reciprocal of the period. We first find the period using $T = 2\pi\sqrt{\frac{L}{g}}$, then calculate $f = \frac{1}{T}$.

Solution

Convert length to meters: $L = 10.0\text{ cm} = 0.100\text{ m}$

Calculate the period:

$$T = 2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{0.100}{9.80}} = 2\pi\sqrt{0.0102} = 2\pi(0.101) = 0.635\text{ s}$$

Calculate the frequency:

$$f = \frac{1}{T} = \frac{1}{0.635} = 1.57\text{ Hz}$$

Discussion

The frequency of 1.57 Hz means the parakeets complete about 1.6 full swings per second, or about 94 swings per minute. This is a fairly rapid oscillation, much faster than a playground swing (about 15 swings per minute) because the bird swing is much shorter (10 cm vs. 4 meters).

This problem beautifully illustrates a fundamental principle of pendulum motion: **the period and frequency are independent of mass**. Whether one parakeet, two parakeets, or even no parakeets sit on the swing, as long as the center of mass remains 10 cm below the pivot, the frequency stays 1.57 Hz. The formula $T = 2\pi\sqrt{\frac{L}{g}}$ contains no mass term, which is why mass doesn't affect the motion.

This mass-independence might seem counterintuitive—one might expect heavier birds to swing more slowly. However, while heavier birds do have more inertia (resistance to acceleration), they also experience proportionally more gravitational restoring force. These two effects exactly cancel, leaving the period unchanged. This is analogous to why all objects fall at the same rate in a vacuum, regardless of mass.

The 10-cm length is reasonable for a small bird toy or cage swing. The rapid 1.57 Hz frequency means the swing returns to the same position about 1.6 times per second, which birds seem to find comfortable and even enjoyable. Comparing this to the 5-cm cuckoo clock pendulum (2.23 Hz), we see that doubling the length from 5 cm to 10 cm decreases the frequency by a factor of $\sqrt{2} \approx 1.41$, from 2.23 Hz to 1.57 Hz, confirming the inverse square root relationship between length and frequency.

Answer

1.57 Hz

(a) A pendulum that has a period of 3.00000 s and that is located where the acceleration due to gravity is 9.79m/s^2 is moved to a location where the acceleration due to gravity is 9.82m/s^2 . What is its new period? (b) Explain why so many digits are needed in the value for the period, based on the relation between the period and the acceleration due to gravity.

Show Solution

Strategy

For part (a), the period of a simple pendulum is $T = 2\pi\sqrt{\frac{L}{g}}$. The length L remains constant when the pendulum is moved, so we can use the ratio of periods at different gravitational accelerations: $\frac{T_2}{T_1} = \sqrt{\frac{g_1}{g_2}}$. For part (b), we analyze how the fractional change in g affects the fractional change in T through the square root relationship.

Solution

(a) From the period formula $T = 2\pi\sqrt{\frac{L}{g}}$, we can write:

$T_2 T_1 = \surd g_1 g_2$

Solving for T_2:

$T_2 = T_1 \surd g_1 g_2$

Substituting the given values:

- T_1 =3.00000 s
- g_1 =9.79 m/s²
- g_2 =9.82 m/s²

T_2=(3.00000 s)√9.799.82
T_2=(3.00000)√0.996945
T_2=(3.00000)(0.998469)=2.99541 s

The new period is 2.99541 s.

(b) The period depends on $g^{-1/2}$, so a small change in g produces a change in T that is half as large (in fractional terms):

$\Delta T T \approx -12 \Delta g g$

In this problem:

$\Delta g g$=9.82−9.799.79=0.039.79=0.003065=0.3065%

The fractional change in period is:

$\Delta T T \approx$−12(0.003065)=−0.001533=−0.1533%

Change in period:

ΔT=(3.00000)(−0.001533)=−0.00460 s

Discussion

Part (a): The new period (2.99541 s) is slightly shorter than the original (3.00000 s) by about 0.00459 s or 0.15%. This makes physical sense: stronger gravity (9.82 vs. 9.79 m/s²) pulls the pendulum back toward equilibrium more forcefully, causing it to oscillate faster with a shorter period.

Part (b): The precision requirement stems from the square root relationship. When g changes by 0.31%, the period changes by only half that amount (0.15%). To detect a 0.15% change in a 3-second period (which is 0.0046 s), we need at least 5 significant figures, or 5 decimal places in this case.

This problem illustrates why precision pendulum clocks are sensitive to location. A pendulum clock calibrated at sea level ($g \approx 9.81$ m/s²) would run slow if moved to high altitude (lower g) or fast if moved to a location with higher g (like near the poles or in a valley). Historical timekeepers had to account for this when using pendulum clocks for navigation or scientific measurements.

The small difference in g (9.82 vs. 9.79 m/s²) might arise from:

- Different latitudes (gravity is stronger at poles than equator)
- Different elevations (gravity decreases with altitude)
- Local geological variations (dense rock formations increase g)

This high precision requirement is why atomic clocks, which don't depend on gravity, have replaced pendulums for scientific timekeeping.

A pendulum with a period of 2.00000 s in one location (g=9.80m/s^2) is moved to a new location where the period is now 1.99796 s. What is the acceleration due to gravity at its new location?

Show Solution

Strategy

Since the length of the pendulum doesn't change, we can use the relationship between period and gravity at both locations to find the new value of g. From $T = 2\pi \surd L g$, we can establish a ratio.

Solution

First, find the length using the original location data:

$L = g T^2 4\pi^2$=(9.80)(2.00000)$^2 4\pi^2$=39.239.478=0.9930 m

Now use this length with the new period to find g_{new}:

$$g_{new}=\frac{4\pi^2 L}{T_{new}^2}=\frac{4\pi^2(0.9930)}{(1.99796)^2}=\frac{39.198}{3.9918}=9.82\text{ m/s}^2$$

Alternatively, using the ratio method:

$$\frac{T_1}{T_2}=\sqrt{\frac{g_2}{g_1}}$$

$$g_2=g_1\left(\frac{T_1}{T_2}\right)^2=9.80\left(\frac{2.00000}{1.99796}\right)^2=9.80(1.00204)=9.82\text{ m/s}^2$$

Discussion

The new location has a slightly higher acceleration due to gravity (9.82 m/s² vs. 9.80 m/s²), an increase of about 0.2%. This small change in g causes a correspondingly small decrease in period (from 2.00000 s to 1.99796 s, a decrease of 0.102%). The relationship is inverse: stronger gravity pulls the pendulum back more forcefully, causing faster oscillations and a shorter period.

This problem demonstrates that pendulums can serve as precision gravimeters (instruments for measuring g). The high precision in the period measurement (5 decimal places) allows us to detect very small variations in gravitational acceleration. Such variations occur due to:

- **Latitude**: g is about 0.5% stronger at the poles than at the equator due to Earth's rotation and oblate shape
- **Altitude**: g decreases by about 0.03% per kilometer of elevation
- **Local geology**: Dense rock formations or mineral deposits can increase g locally by small amounts

The calculation is straightforward because the length doesn't change when the pendulum is moved—only g changes. Using the ratio method ($\frac{T_1}{T_2}=\sqrt{\frac{g_2}{g_1}}$) elegantly eliminates the unknown length from the calculation.

This precision measurement technique was historically important for geological surveying and mapmaking. Variations in g can reveal underground structures, and detailed gravity maps help in searching for oil deposits and mineral resources. Modern gravimeters use different technology but the principle remains: measuring how gravity affects the motion of a test mass.

Answer

9.82 m/s²

(a) What is the effect on the period of a pendulum if you double its length?

(b) What is the effect on the period of a pendulum if you decrease its length by 5.00%?

Show Solution

Strategy

Since $T=2\pi\sqrt{\frac{L}{g}}$, the period is proportional to the square root of the length. We can find the ratio of new to old period for each case.

Solution

(a) Doubling the length:

If $L_{new}=2L_{old}$, then:

$$\frac{T_{new}}{T_{old}}=\sqrt{\frac{L_{new}}{L_{old}}}=\sqrt{\frac{2L}{L}}=\sqrt{2}=1.41$$

The period increases by a factor of $\sqrt{2}$ or approximately 1.41.

(b) Decreasing the length by 5.00%:

If $L_{new}=0.950L_{old}$, then:

$$\frac{T_{new}}{T_{old}}=\sqrt{\frac{L_{new}}{L_{old}}}=\sqrt{0.950}=0.975$$

The period decreases to 97.5% of the old period (a 2.5% decrease).

Discussion

Part (a) reveals a counterintuitive result: doubling the length doesn't double the period—it only increases it by about 41% (a factor of $\sqrt{2}\approx 1.414$). This square root relationship comes directly from $T=2\pi\sqrt{\frac{L}{g}}$. If you wanted to double the period, you would need to quadruple the length (since $\sqrt{4}=2$). This non-linear relationship is a key characteristic of pendulum motion.

Part (b) shows that a 5.00% decrease in length produces only a 2.50% decrease in period (to 97.5% of the original). Notice that the fractional change in period is half the fractional change in length—this follows from the square root: if $T\propto\sqrt{L}$, then $\frac{\Delta T}{T}\approx\frac{1}{2}\frac{\Delta L}{L}$ for small changes. This means pendulums are somewhat forgiving of small length variations, which is advantageous for clock design.

The square root relationship has practical implications:

- **For clocks**: Small thermal expansion of the pendulum rod (which might change L by 0.1%) affects timing by only half that amount (0.05%), though this is still significant for precision timekeeping
- **For adjustments**: To speed up a pendulum clock, you must shorten the pendulum, but the effect is muted by the square root
- **For scaling**: You can't simply scale a pendulum design up or down linearly—a pendulum 4 times longer has a period only 2 times longer

These results are consistent with the fundamental pendulum equation and demonstrate how the square root relationship governs all pendulum behavior, making period changes less dramatic than length changes.

Answer

(a) Period increases by a factor of 1.41 ($\sqrt{2}$)

(b) Period decreases to 97.5% of old period

Find the ratio of the new/old periods of a pendulum if the pendulum were transported from Earth to the Moon, where the acceleration due to gravity is 1.63m/s^2.

Show Solution

Strategy

The length of the pendulum stays the same, so we can use the relationship $T = 2\pi\sqrt{\frac{L}{g}}$ to find the ratio of periods. Since L is constant, the ratio depends only on the ratio of gravitational accelerations.

Solution

For Earth: $T_{\text{Earth}} = 2\pi\sqrt{\frac{L}{g_{\text{Earth}}}}$

For Moon: $T_{\text{Moon}} = 2\pi\sqrt{\frac{L}{g_{\text{Moon}}}}$

Taking the ratio:

$$\frac{T_{\text{Moon}}}{T_{\text{Earth}}} = \frac{2\pi\sqrt{\frac{L}{g_{\text{Moon}}}}}{2\pi\sqrt{\frac{L}{g_{\text{Earth}}}}} = \sqrt{\frac{g_{\text{Earth}}}{g_{\text{Moon}}}}$$
$$\frac{T_{\text{Moon}}}{T_{\text{Earth}}} = \sqrt{\frac{9.80}{1.63}} = \sqrt{6.01} = 2.45$$

Discussion

The period on the Moon is 2.45 times longer than on Earth, which makes excellent physical sense. The Moon's gravity (1.63 m/s²) is about 1/6 of Earth's gravity (9.80 m/s²), so the ratio $\sqrt{\frac{9.80}{1.63}} = \sqrt{6.01} \approx 2.45$ reflects this weaker gravitational field.

Physically, weaker gravity means a weaker restoring force acting on the pendulum bob. When displaced from equilibrium, the component of gravitational force pulling it back ($mg\sin\theta \approx mg\theta$) is proportionally smaller on the Moon. With less force accelerating the bob back toward equilibrium, it takes longer to complete each oscillation.

This has dramatic practical consequences: a pendulum that swings once per second on Earth would take 2.45 seconds per swing on the Moon. A pendulum clock calibrated for Earth would run extremely slowly on the Moon, losing about 59% of the time (since it would run at only 41% of the correct rate). After one Earth-day (24 hours) on the Moon, such a clock would show only about 9.8 hours had passed!

This demonstrates a fundamental limitation of pendulum clocks: they're only accurate at the gravitational acceleration for which they were calibrated. This made pendulum clocks problematic for navigation and surveying, since g varies with location. It's also why mechanical pendulum clocks would be useless on the Moon, spacecraft, or any environment with different gravity. Modern timekeeping uses atomic or quartz crystal oscillators that are independent of gravity, making them suitable for any location.

Answer

2.45

At what rate will a pendulum clock run on the Moon, where the acceleration due to gravity is 1.63m/s^2 , if it keeps time accurately on Earth? That is, find the time (in hours) it takes the clock's hour hand to make one revolution on the Moon.

Show Solution

Strategy

Since the period on the Moon is longer by a factor found in the previous problem (2.45), the clock runs slow by the same factor. When the clock's hour hand completes one revolution (which it thinks is 12 hours), more actual time will have passed.

Solution

From the previous problem, we know:

$$\frac{T_{Moon}}{T_{Earth}} = \sqrt{\frac{g_{Earth}}{g_{Moon}}} = \sqrt{\frac{9.80}{1.63}} = 2.45$$

The clock runs slow by this same factor. When the clock measures 12 hours, the actual time elapsed is:

$$t_{actual} = 2.45 \times 12 \text{ hours} = 29.4 \text{ hours}$$

Therefore, it takes 29.4 hours of actual time for the clock's hour hand to make one revolution on the Moon.

Discussion

The pendulum clock runs very slowly on the Moon—it takes 29.4 actual hours for the clock's hour hand to complete what it thinks is one 12-hour revolution. This means the clock loses about 17.4 hours per "clock cycle," running at only about 41% of its proper rate (12/29.4 ≈ 0.41).

This dramatic slowdown occurs because the Moon's gravity is only about one-sixth that of Earth's (1.63 m/s² vs. 9.80 m/s²). The restoring force on the pendulum ($F \approx -mg\theta$) is proportionally weaker, so the pendulum swings much more slowly. Each tick of the clock takes 2.45 times longer than it should.

To put this in perspective: if an astronaut set the clock to 12:00 noon upon landing on the Moon, then after one full Earth day (24 hours) had passed, the clock would only show about 9:48 (9.8 hours later). The clock would be running over 14 hours slow per day!

This makes mechanical pendulum clocks completely impractical for use on the Moon or in any environment with different gravity. Historically, this limitation affected navigation: pendulum clocks couldn't be used aboard ships due to motion and couldn't provide accurate time across different latitudes (where g varies slightly). This problem drove the development of marine chronometers using balance wheels instead of pendulums.

Modern timekeeping devices using quartz crystal oscillators or atomic transitions are fundamentally different—they rely on electromagnetic forces and quantum mechanics rather than gravity, so they work identically on the Moon, Earth, or in zero gravity. This is why spacecraft, satellites, and modern navigation systems use these technologies instead of pendulums.

Answer

29.4 hours (slow by a factor of 2.45)

Suppose the length of a clock's pendulum is changed by 1.000%, exactly at noon one day. What time will it read 24.00 hours later, assuming it the pendulum has kept perfect time before the change? Note that there are two answers, and perform the calculation to four-digit precision.

Show Solution

Strategy

A change in length changes the period. Since $T \propto \sqrt{L}$, a 1% change in length produces a 0.5% change in period. We need to consider both cases: length increased by 1% and length decreased by 1%.

Solution

The ratio of new to old period is:

$$\frac{T_{new}}{T_{old}} = \sqrt{\frac{L_{new}}{L_{old}}}$$

Case 1: Length increased by 1.000% ($L_{new} = 1.01000 L_{old}$)

$$\frac{T_{new}}{T_{old}} = \sqrt{1.01000} = 1.004988$$

The clock runs slow. In 24.00 real hours, the clock measures:

$$t_{clock} = \frac{24.00}{1.004988} = 23.88 \text{ hours}$$

Converting to hours and minutes: $23.88 \text{ h} = 23 \text{ h } 52.8 \text{ min}$

So the clock reads 11:52.8 AM (or 11:53 AM to the nearest minute).

Case 2: Length decreased by 1.000% ($L_{new} = 0.99000 L_{old}$)

$$\frac{T_{new}}{T_{old}} = \sqrt{0.99000} = 0.99499$$

The clock runs fast. In 24.00 real hours, the clock measures:

$$t_{clock} = \frac{24.00}{0.99499} = 24.12 \text{ hours}$$

Converting: $24.12 \text{ h} = 24 \text{ h } 7.2 \text{ min}$

So the clock reads 12:07.2 PM (or 12:07 PM to the nearest minute).

Discussion

A mere 1.000% change in pendulum length causes the clock to be off by about 7.2 minutes per day, which is quite significant for practical timekeeping. This demonstrates the sensitivity of pendulum clocks to length variations.

The physical mechanism is clear: if the length increases by 1%, the period increases by $\sqrt{1.01} \approx 1.005$ (about 0.5%). This means each swing takes slightly longer, so the clock runs slow. Over 24 hours, the accumulated error is substantial—the clock loses about 7 minutes. Conversely, decreasing the length makes the period shorter, so the clock runs fast and gains about 7 minutes per day.

This sensitivity has important practical implications:

Temperature effects: Most materials expand when heated. A steel pendulum rod with thermal expansion coefficient $\alpha \approx 11 \times 10^{-6} / {}^\circ C$ would change length by 0.011% per degree Celsius. A 10°C temperature increase would change the length by 0.11%, causing the clock to lose about 47 seconds per day. This is why precision pendulum clocks used temperature-compensated pendulums (like the “gridiron” pendulum using alternating brass and steel rods, or mercury-filled pendulums where the mercury rises as the rod expands, keeping the effective length constant).

Adjustment precision: To keep accurate time, pendulum lengths must be maintained to better than 0.01% precision. This requires careful mechanical design and stable operating conditions.

Calibration: The two answers (11:52.8 and 12:07.2) show that without knowing whether the length increased or decreased, we can’t determine which way the clock error goes. In practice, clock adjusters would observe whether the clock gains or loses time over several days, then adjust accordingly.

The calculation to four-digit precision (23.88 hours and 24.12 hours) is necessary to distinguish the small but significant timing errors. This problem illustrates why pendulum clocks, while historically important and accurate for their time, have been replaced by quartz and atomic clocks that are far less sensitive to environmental variations.

Answer

If length increased by 1.000%: clock reads 11:52:48 (or 11:52.8 hours)

If length decreased by 1.000%: clock reads 12:07:12 (or 12:07.2 hours)

If a pendulum-driven clock gains 5.00 s/day, what fractional change in pendulum length must be made for it to keep perfect time?

Show Solution

Strategy

If the clock gains time, it runs fast, meaning its period is too short. To correct this, we need to increase the length to increase the period. Since $T \propto \sqrt{L}$, we can relate the fractional change in period to the fractional change in length.

Solution

The clock gains 5.00 s in one day (86400 s), so the ratio of periods is:

$$\frac{T_{\text{actual}}}{T_{\text{clock}}} = \frac{86400}{86400 - 5.00} = \frac{86400}{86395} = 1.0000579$$

Since $T \propto \sqrt{L}$, we have:

$$\frac{T_{\text{new}}}{T_{\text{old}}} = \sqrt{\frac{L_{\text{new}}}{L_{\text{old}}}}$$

Therefore:

$$\frac{L_{\text{new}}}{L_{\text{old}}} = \left(\frac{T_{\text{new}}}{T_{\text{old}}}\right)^2 = (1.0000579)^2 = 1.000116$$

The fractional change in length is:

$$\frac{\Delta L}{L} = \frac{L_{\text{new}} - L_{\text{old}}}{L_{\text{old}}} = 1.000116 - 1 = 0.000116 = 0.0116\%$$

Discussion

To correct a clock that gains 5 seconds per day, the pendulum length must be increased by only 0.0116%, or about 1.16 parts in 10,000. This is an extraordinarily small adjustment that illustrates the remarkable precision required in pendulum clock design.

Scale of the adjustment: For a 1-meter pendulum, 0.0116% corresponds to 0.116 mm, which is roughly the thickness of a human hair or a sheet of paper! For a typical grandfather clock pendulum about 1 meter long, this would mean adjusting the bob’s position by little more than 0.1 mm. This demonstrates the extreme sensitivity of pendulum clocks to length variations.

Physical reasoning: The clock gains time, meaning it runs fast, which means its period is too short. From $T = 2\pi\sqrt{\frac{L}{g}}$, we see that to increase the period, we must increase the length. This makes physical sense: a longer pendulum has more distance to travel and experiences restoring force over a larger arc, both of which increase the time per oscillation.

Practical implications:

1. **Fine adjustment mechanisms**: Precision pendulum clocks include very fine adjustment screws or nuts on the bob. A typical mechanism might have threads that allow adjustments of 0.1 mm per turn, requiring multiple complete turns to make this 0.116 mm adjustment.

2. **Temperature compensation**: Temperature changes easily cause length variations exceeding 0.0116%. A 1-meter steel rod expands by about 0.011 mm per degree Celsius. A temperature swing of just 10°C would change the length by 0.11 mm, comparable to the adjustment needed here. This is why precision pendulum clocks used:
   - Invar (nickel-steel alloy) with very low thermal expansion
   - Gridiron pendulums with alternating metals that compensate for each other's expansion
   - Mercury pendulums where rising mercury compensates for rod expansion
3. **Environmental stability**: Even air currents, humidity changes affecting wooden clock cases, or building vibrations can affect pendulum length and timekeeping at this precision level.

Historical context: The fact that such tiny adjustments are needed explains why pendulum clocks required skilled clockmakers for setup and maintenance, and why atomic clocks (accurate to nanoseconds per day) have completely replaced pendulums for scientific timekeeping. Nevertheless, well-maintained pendulum clocks can achieve accuracy of a few seconds per week, which is remarkable for a purely mechanical device.

Answer

Length must increase by 0.0116%

Glossary

simple pendulum
: an object with a small mass suspended from a light wire or string

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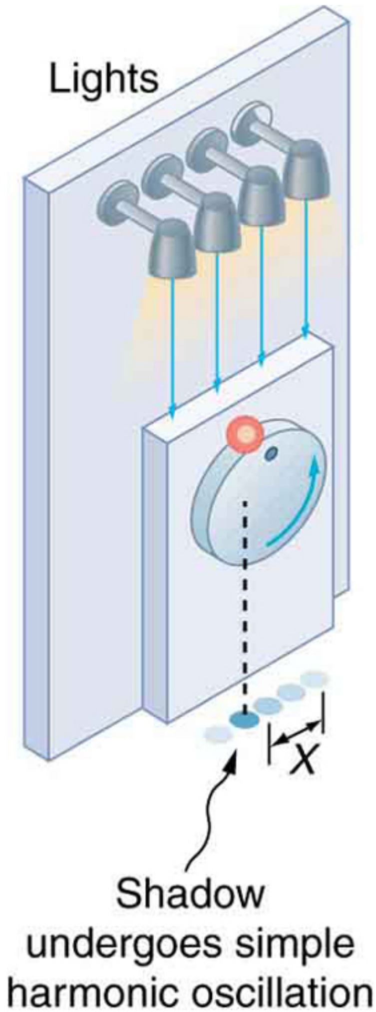
Uniform Circular Motion and Simple Harmonic Motion

- Compare simple harmonic motion with uniform circular motion.

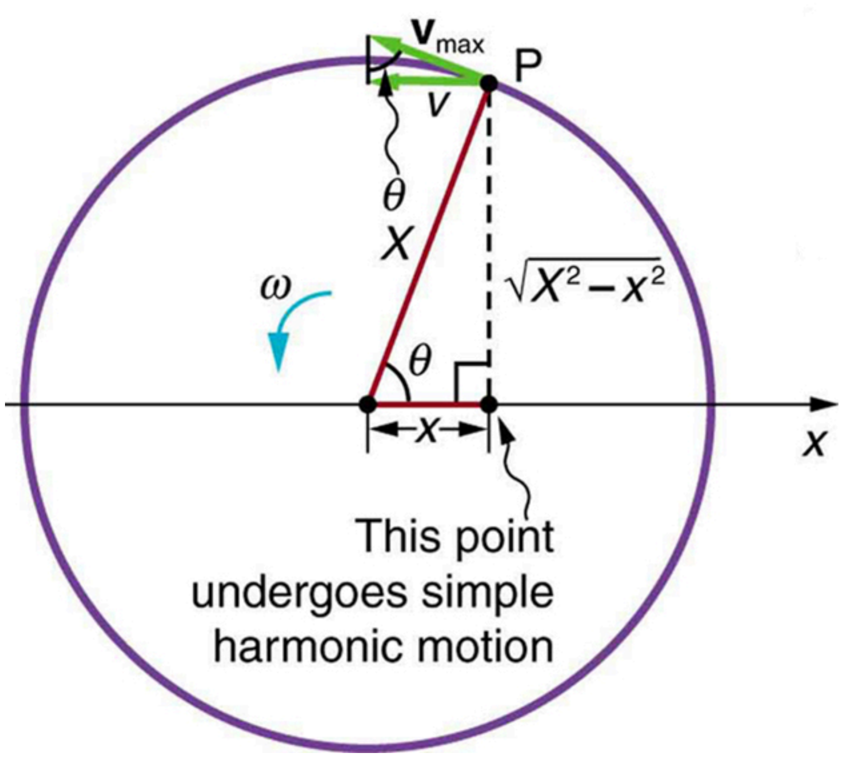
The horses on this merry-go-round exhibit uniform circular motion. (credit: Wonderlane, Flickr)

There is an easy way to produce simple harmonic motion by using uniform circular motion. [Figure 2] shows one way of using this method. A ball is attached to a uniformly rotating vertical turntable, and its shadow is projected on the floor as shown. The shadow undergoes simple harmonic motion. Hooke's law usually describes uniform circular motions (ω constant) rather than systems that have large visible displacements. So observing the projection of uniform circular motion, as in [Figure 2], is often easier than observing a precise large-scale simple harmonic oscillator. If studied in sufficient depth, simple harmonic motion produced in this manner can give considerable insight into many aspects of oscillations and waves and is very useful mathematically. In our brief treatment, we shall indicate some of the major features of this relationship and how they might be useful.



The shadow of a ball rotating at constant angular velocity ω on a turntable goes back and forth in precise simple harmonic motion.

[Figure 3] shows the basic relationship between uniform circular motion and simple harmonic motion. The point P travels around the circle at constant angular velocity ω . The point P is analogous to an object on the merry-go-round. The projection of the position of P onto a fixed axis undergoes simple harmonic motion and is analogous to the shadow of the object. At the time shown in the figure, the projection has position x and moves to the left with velocity v . The velocity of the point P around the circle equals $\bar{v}_{max}$. The projection of $\bar{v}_{max}$ on the x -axis is the velocity v of the simple harmonic motion along the x -axis.



A point P moving on a circular path with a constant angular velocity ω is undergoing uniform circular motion. Its projection on the x-axis undergoes simple harmonic motion. Also shown is the velocity of this point around the circle, $\overline{v}_{max}$, and its projection, which is v . Note that these velocities form a similar triangle to the displacement triangle.

To see that the projection undergoes simple harmonic motion, note that its position x is given by

$$x = X\cos\theta,$$

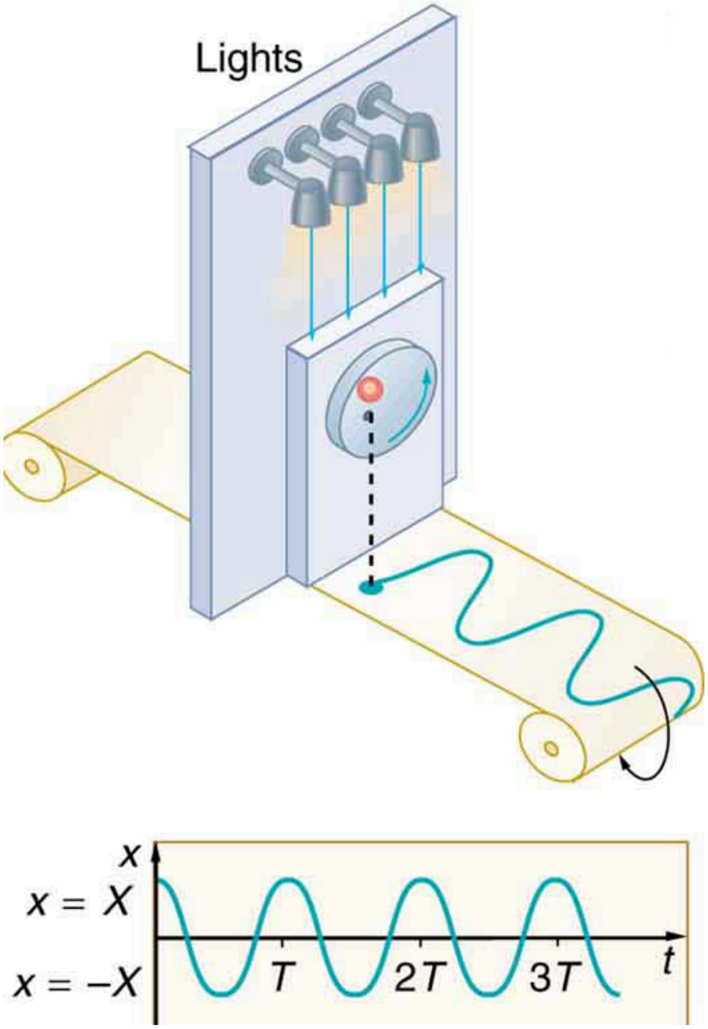
where $\theta = \omega t$, ω is the constant angular velocity, and X is the radius of the circular path. Thus,

$$x = X\cos\omega t.$$

The angular velocity ω is in radians per unit time; in this case 2π radians is the time for one revolution T . That is, $\omega = 2\pi/T$. Substituting this expression for ω, we see that the position x is given by:

$$x(t) = \cos(2\pi t T).$$

This expression is the same one we had for the position of a simple harmonic oscillator in Simple Harmonic Motion: A Special Periodic Motion . If we make a graph of position versus time as in [Figure 4], we see again the wavelike character (typical of simple harmonic motion) of the projection of uniform circular motion onto the x -axis.



The position of the projection of uniform circular motion performs simple harmonic motion, as this wavelike graph of x versus t indicates.

Now let us use [Figure 3] to do some further analysis of uniform circular motion as it relates to simple harmonic motion. The triangle formed by the velocities in the figure and the triangle formed by the displacements ($X, x,$ and $\sqrt{X^2-x^2}$) are similar right triangles. Taking ratios of similar sides, we see that

$$\frac{v}{v_{\text{max}}}=\frac{\sqrt{X^2-x^2}}{X}=\sqrt{1-\frac{x^2}{X^2}}.$$

We can solve this equation for the speed v or

$$v=v_{\text{max}}\sqrt{1-\frac{x^2}{X^2}}.$$

This expression for the speed of a simple harmonic oscillator is exactly the same as the equation obtained from conservation of energy considerations in Energy and the Simple Harmonic Oscillator. You can begin to see that it is possible to get all of the characteristics of simple harmonic motion from an analysis of the projection of uniform circular motion.

Finally, let us consider the period T of the motion of the projection. This period is the time it takes the point P to complete one revolution. That time is the circumference of the circle $2\pi X$ divided by the velocity around the circle, v_{max} . Thus, the period T is

$$T=\frac{2\pi X}{v_{\text{max}}}.$$

We know from conservation of energy considerations that

$$v_{\text{max}}=\sqrt{\frac{k}{m}}X.$$

Solving this equation for X/v_{max} gives

$$\frac{X}{v_{\text{max}}}=\sqrt{\frac{m}{k}}.$$

Substituting this expression into the equation for T yields

$$T=2\pi\sqrt{\frac{m}{k}}.$$

Thus, the period of the motion is the same as for a simple harmonic oscillator. We have determined the period for any simple harmonic oscillator using the relationship between uniform circular motion and simple harmonic motion.

Some modules occasionally refer to the connection between uniform circular motion and simple harmonic motion. Moreover, if you carry your study of physics and its applications to greater depths, you will find this relationship useful. It can, for example, help to analyze how waves add when they are superimposed.

Check Your Understanding

Identify an object that undergoes uniform circular motion. Describe how you could trace the simple harmonic motion of this object as a wave.

Show Solution

A record player undergoes uniform circular motion. You could attach dowel rod to one point on the outside edge of the turntable and attach a pen to the other end of the dowel. As the record player turns, the pen will move. You can drag a long piece of paper under the pen, capturing its motion as a wave.

Section Summary

A projection of uniform circular motion undergoes simple harmonic oscillation.

Problems & Exercises

(a)What is the maximum velocity of an 85.0-kg person bouncing on a bathroom scale having a force constant of $1.50\times10^{6}\,\text{N/m}$, if the amplitude of the bounce is 0.200 cm? (b)What is the maximum energy stored in the spring?

Show Solution

Strategy

For part (a), we use the relationship between maximum velocity, amplitude, and the spring constant for simple harmonic motion: $v_{\text{max}}=X\sqrt{\frac{k}{m}}$. For part (b), the maximum energy stored in the spring occurs at maximum displacement and equals the elastic potential energy: $\text{PE}_{\text{max}}=\frac{1}{2}kX^{2}$.

Solution

(a) First, convert the amplitude to meters:

$$X=0.200\text{ cm}=0.00200\text{ m}$$

The maximum velocity is:

$$v_{\text{max}}=X\sqrt{\frac{k}{m}}=0.00200\sqrt{\frac{1.50\times10^{6}}{85.0}}$$
$$v_{\text{max}}=0.00200\sqrt{17647}=0.00200(132.8)=0.266\text{ m/s}$$

(b) The maximum energy stored in the spring is:

$$\text{PE}_{\text{max}}=\frac{1}{2}kX^{2}=\frac{1}{2}(1.50\times10^{6})(0.00200)^{2}$$
$$\text{PE}_{\text{max}}=\frac{1}{2}(1.50\times10^{6})(4.00\times10^{-6})=3.00\text{ J}$$

Discussion

Despite the very stiff spring (force constant of 1.50×10^{6} N/m) and relatively large mass (85.0 kg), the small amplitude (only 2.00 mm) results in a modest maximum velocity of 0.266 m/s. The maximum energy of 3.00 J is also small, which makes sense given the tiny amplitude. This demonstrates that bathroom scales are designed to be very stiff to minimize deflection under a person's weight while still providing accurate measurements.

Answer

(a) 0.266 m/s

(b) 3.00 J

A novelty clock has a 0.0100-kg mass object bouncing on a spring that has a force constant of 1.25 N/m. What is the maximum velocity of the object if the object bounces 3.00 cm above and below its equilibrium position? (b) How many joules of kinetic energy does the object have at its maximum velocity?

Show Solution

Strategy

The object bounces 3.00 cm above and below equilibrium, so the amplitude is $X = 0.0300$ m. For part (a), we use the relationship $v_{\text{max}} = X\sqrt{\frac{k}{m}}$. For part (b), we calculate the kinetic energy at maximum velocity using $\text{KE} = \frac{1}{2}mv^2_{\text{max}}$.

Solution

(a) The maximum velocity is:

$$v_{\text{max}} = X\sqrt{\frac{k}{m}} = 0.0300\sqrt{\frac{1.25}{0.0100}} = 0.0300\sqrt{125} = 0.0300(11.18) = 0.335 \text{ m/s}$$

(b) The kinetic energy at maximum velocity is:

$$\text{KE} = \frac{1}{2}mv^2_{\text{max}} = \frac{1}{2}(0.0100)(0.335)^2 = 0.00500(0.112) = 5.62 \times 10^{-4} \text{ J}$$

Alternatively, we can use energy conservation. At maximum displacement, all energy is potential:

$$\text{KE}_{\text{max}} = \text{PE}_{\text{max}} = \frac{1}{2}kX^2 = \frac{1}{2}(1.25)(0.0300)^2 = 5.63 \times 10^{-4} \text{ J}$$

Discussion

The maximum velocity occurs as the object passes through the equilibrium position, where all the elastic potential energy has been converted to kinetic energy. The small kinetic energy (less than 0.001 J) is appropriate for a small bouncing object in a novelty clock. This energy oscillates between kinetic (at equilibrium) and potential (at maximum displacement) forms as the object bounces.

Answer

(a) 0.335 m/s

(b) 5.63×10^{-4} J or 0.000563 J

At what positions is the speed of a simple harmonic oscillator half its maximum? That is, what values of x/X give $v = \pm v_{\text{max}}/2$, where X is the amplitude of the motion?

Show Solution

Strategy

We use the velocity equation for simple harmonic motion: $v = v_{\text{max}}\sqrt{1 - \frac{x^2}{X^2}}$. Setting $v = \frac{v_{\text{max}}}{2}$ and solving for x/X will give us the positions where the speed is half its maximum.

Solution

Start with the velocity equation and set $v = \frac{v_{\text{max}}}{2}$:

$$\frac{v_{\text{max}}}{2} = v_{\text{max}}\sqrt{1 - \frac{x^2}{X^2}}$$

Divide both sides by v_{max}:

$$\frac{1}{2} = \sqrt{1 - \frac{x^2}{X^2}}$$

Square both sides:

$$\frac{1}{4} = 1 - \frac{x^2}{X^2}$$

Solve for x^2/X^2:

$$\frac{x^2}{X^2} = 1 - \frac{1}{4} = \frac{3}{4}$$

Take the square root:

$$\frac{x}{X} = \pm\sqrt{\frac{3}{4}} = \pm\frac{\sqrt{3}}{2} \approx \pm 0.866$$

Discussion

The speed is half the maximum at positions $x = \pm\frac{\sqrt{3}}{2}X$, which is approximately 86.6% of the amplitude on either side of equilibrium. This makes physical sense: at equilibrium ($x = 0$), the oscillator has maximum speed, and at maximum displacement ($x = \pm X$), it has zero speed. The positions where $v = v_{\text{max}}/2$ are closer to the amplitude than to equilibrium, which is consistent with the fact that the oscillator spends more time near the extremes of its motion where it moves more slowly.

Answer

$\pm\sqrt{32}$

A ladybug sits 12.0 cm from the center of a Beatles music album spinning at 33.33 rpm. What is the maximum velocity of its shadow on the wall behind the turntable, if illuminated parallel to the record by the parallel rays of the setting Sun?

Show Solution

Strategy

The shadow of the ladybug on the wall undergoes simple harmonic motion as the record spins. The maximum velocity of the shadow equals the tangential velocity of the ladybug's circular motion. We can find this using $v = r\omega$, where ω is the angular velocity in rad/s.

Solution

First, convert the rotation rate to angular velocity:

$$\omega = 33.33 \text{ rpm} = 33.33 \times \frac{2\pi \text{ rad}}{60 \text{ s}} = \frac{33.33 \times 2\pi}{60} = 3.49 \text{ rad/s}$$

The radius is $r = 12.0 \text{ cm} = 0.120 \text{ m}$

The maximum velocity of the shadow (which equals the tangential velocity) is:

$$v_{\text{max}} = r\omega = (0.120)(3.49) = 0.419 \text{ m/s}$$

Discussion

As the record spins, the ladybug moves in a circle. When illuminated by parallel rays (like sunlight), its shadow on the wall moves back and forth in simple harmonic motion. The amplitude of this motion is 12.0 cm (the radius), and the maximum velocity of the shadow is 0.419 m/s (about 42 cm/s), which occurs when the shadow passes through the center of its oscillation. This is a beautiful example of how uniform circular motion and simple harmonic motion are related—the shadow's motion is the projection of circular motion onto a line.

Answer

0.419 m/s or 41.9 cm/s

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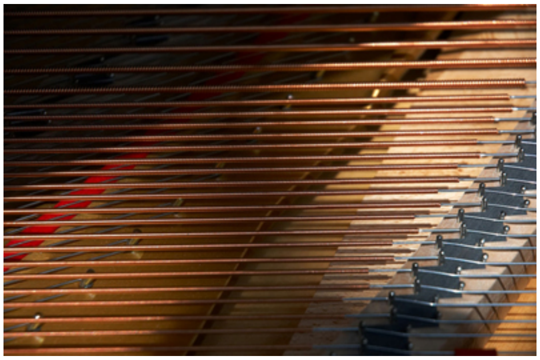
Forced Oscillations and Resonance

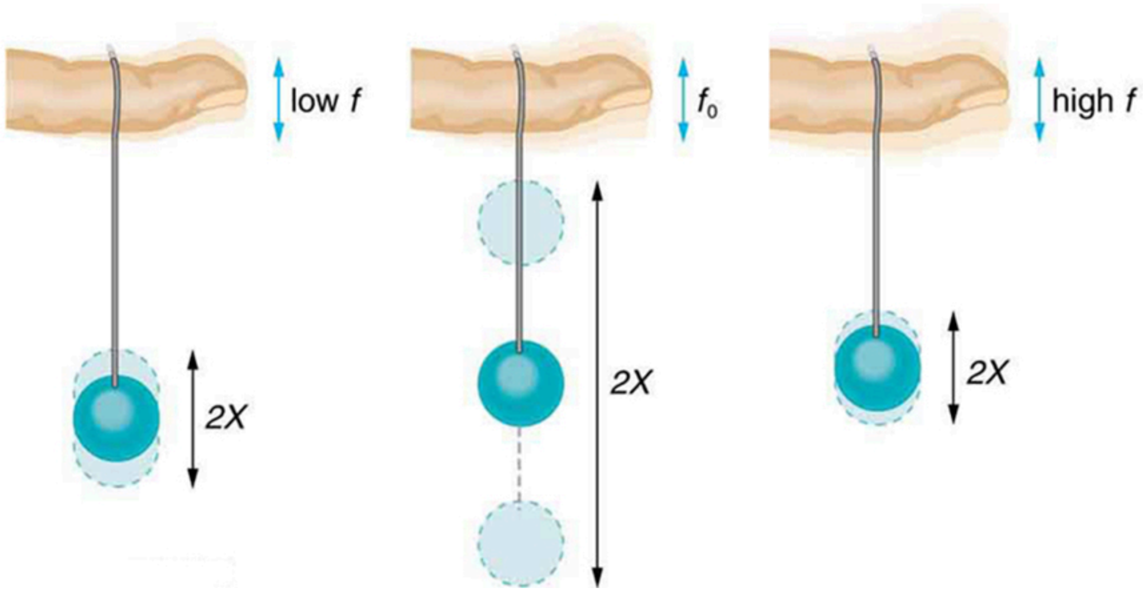
- Observe resonance of a paddle ball on a string.
- Observe amplitude of a damped harmonic oscillator.



You can cause the strings in a piano to vibrate simply by producing sound waves from your voice. (credit: Matt Billings, Flickr)

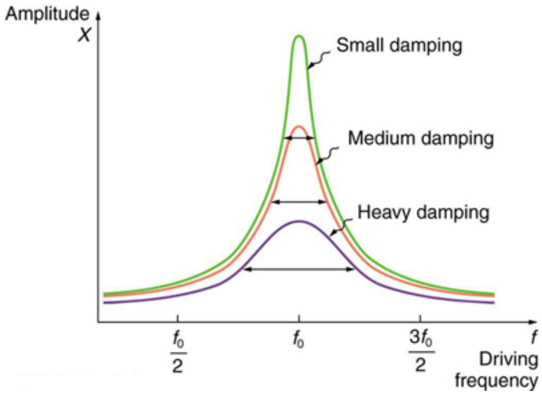
Sit in front of a piano sometime and sing a loud brief note at it with the dampers off its strings. It will sing the same note back at you—the strings, having the same frequencies as your voice, are resonating in response to the forces from the sound waves that you sent to them. Your voice and a piano's strings is a good example of the fact that objects—in this case, piano strings—can be forced to oscillate but oscillate best at their natural frequency. In this section, we shall briefly explore applying a *periodic driving force* acting on a simple harmonic oscillator. The driving force puts energy into the system at a certain frequency, not necessarily the same as the natural frequency of the system. The **natural frequency** is the frequency at which a system would oscillate if there were no driving and no damping force.

Most of us have played with toys involving an object supported on an elastic band, something like the paddle ball suspended from a finger in [Figure 2]. Imagine the finger in the figure is your finger. At first you hold your finger steady, and the ball bounces up and down with a small amount of damping. If you move your finger up and down slowly, the ball will follow along without bouncing much on its own. As you increase the frequency at which you move your finger up and down, the ball will respond by oscillating with increasing amplitude. When you drive the ball at its natural frequency, the ball's oscillations increase in amplitude with each oscillation for as long as you drive it. The phenomenon of driving a system with a frequency equal to its natural frequency is called **resonance**. A system being driven at its natural frequency is said to **resonate**. As the driving frequency gets progressively higher than the resonant or natural frequency, the amplitude of the oscillations becomes smaller, until the oscillations nearly disappear and your finger simply moves up and down with little effect on the ball.



The paddle ball on its rubber band moves in response to the finger supporting it. If the finger moves with the natural frequency f_{0} of the ball on the rubber band, then a resonance is achieved, and the amplitude of the ball's oscillations increases dramatically. At higher and lower driving frequencies, energy is transferred to the ball less efficiently, and it responds with lower-amplitude oscillations.

[Figure 3] shows a graph of the amplitude of a damped harmonic oscillator as a function of the frequency of the periodic force driving it. There are three curves on the graph, each representing a different amount of damping. All three curves peak at the point where the frequency of the driving force equals the natural frequency of the harmonic oscillator. The highest peak, or greatest response, is for the least amount of damping, because less energy is removed by the damping force.



Amplitude of a harmonic oscillator as a function of the frequency of the driving force. The curves represent the same oscillator with the same natural frequency but with different amounts of damping. Resonance occurs when the driving frequency equals the natural frequency, and the greatest response is for the least amount of damping. The narrowest response is also for the least damping.

It is interesting that the widths of the resonance curves shown in [Figure 3] depend on damping: the less the damping, the narrower the resonance. The message is that if you want a driven oscillator to resonate at a very specific frequency, you need as little damping as possible. Little damping is the case for piano strings and many other musical instruments. Conversely, if you want small-amplitude oscillations, such as in a car's suspension system, then you want heavy damping. Heavy damping reduces the amplitude, but the tradeoff is that the system responds at more frequencies.

These features of driven harmonic oscillators apply to a huge variety of systems. When you tune a radio, for example, you are adjusting its resonant frequency so that it only oscillates to the desired station's broadcast (driving) frequency. The more selective the radio is in discriminating between stations, the smaller its damping. Magnetic resonance imaging (MRI) is a widely used medical diagnostic tool in which atomic nuclei (mostly hydrogen nuclei) are made to resonate by incoming radio waves (on the order of 100 MHz). A child on a swing is driven by a parent at the swing's natural frequency to achieve maximum amplitude. In all of these cases, the efficiency of energy transfer from the driving force into the oscillator is best at resonance. Speed bumps and gravel roads prove that even a car's suspension system is not immune to resonance. In spite of finely engineered shock absorbers, which ordinarily convert mechanical energy to thermal energy almost as fast as it comes in, speed bumps still cause a large-amplitude oscillation. On gravel roads that are corrugated, you may have noticed that if you travel at the "wrong" speed, the bumps are very noticeable whereas at other speeds you may hardly feel the bumps at all. [Figure 4] shows a photograph of a famous example (the Tacoma Narrows Bridge) of the destructive effects of a driven harmonic oscillation. The Millennium Bridge in London was closed for a short period of time for the same reason while inspections were carried out.

In our bodies, the chest cavity is a clear example of a system at resonance. The diaphragm and chest wall drive the oscillations of the chest cavity which result in the lungs inflating and deflating. The system is critically damped and the muscular diaphragm oscillates at the resonant value for the system, making it highly efficient.

In 1940, the Tacoma Narrows Bridge in Washington state collapsed. Heavy cross winds drove the bridge into oscillations at its resonant frequency. Damping decreased when support cables broke loose and started to slip over the towers, allowing increasingly greater amplitudes until the structure failed (credit: PRI's Studio 360, via Flickr)

Check Your Understanding

A famous magic trick involves a performer singing a note toward a crystal glass until the glass shatters. Explain why the trick works in terms of resonance and natural frequency.

Show Solution

The performer must be singing a note that corresponds to the natural frequency of the glass. As the sound wave is directed at the glass, the glass responds by resonating at the same frequency as the sound wave. With enough energy introduced into the system, the glass begins to vibrate and eventually shatters.

Section Summary

- A system's natural frequency is the frequency at which the system will oscillate if not affected by driving or damping forces.
- A periodic force driving a harmonic oscillator at its natural frequency produces resonance. The system is said to resonate.
- The less damping a system has, the higher the amplitude of the forced oscillations near resonance. The more damping a system has, the broader response it has to varying driving frequencies.

Conceptual Questions

Why are soldiers in general ordered to “route step” (walk out of step) across a bridge?

Problems & Exercises

How much energy must the shock absorbers of a 1200-kg car dissipate in order to damp a bounce that initially has a velocity of 0.800 m/s at the equilibrium position? Assume the car returns to its original vertical position.

Show Solution

Strategy

The car is bouncing at the equilibrium position with an initial velocity of 0.800 m/s. All the energy at this point is kinetic energy. To damp the bounce completely, the shock absorbers must dissipate all this kinetic energy.

Solution

The kinetic energy of the car at equilibrium is:

$$\text{KE}=12mv^2$$

Substituting the given values:

$$E=12(1200)(0.800)^2=12(1200)(0.640)$$
$$E=384\text{ J}$$

Discussion

The shock absorbers must dissipate 384 J of energy to completely damp the bounce. This energy is converted to thermal energy through the viscous damping mechanism in the shock absorbers, typically involving oil forced through small orifices. The energy calculation is straightforward because at the equilibrium position, all the oscillation energy is in the form of kinetic energy (none is stored as potential energy). Good shock absorbers dissipate this energy quickly, preventing the car from continuing to bounce after hitting a bump. The relatively large mass of the car (1200 kg) combined with even a modest velocity (0.800 m/s) results in a significant amount of energy that must be absorbed.

Answer

384 J

If a car has a suspension system with a force constant of $5.00\times10^4\text{N/m}$, how much energy must the car’s shocks remove to dampen an oscillation starting with a maximum displacement of 0.0750 m?

Show Solution

Strategy

The oscillation starts with maximum displacement, so all the energy is initially stored as elastic potential energy in the suspension. To dampen the oscillation completely (bring it to rest), the shocks must remove all this energy. We use $\text{PE}_{\text{el}}=12kx^2$.

Solution

The initial energy stored in the suspension is:

$$E=12kx^2=12(5.00\times10^4)(0.0750)^2$$
$$E=12(5.00\times10^4)(5.625\times10^{-3})=141\text{ J}$$

Discussion

The shocks must remove 141 J of energy to completely dampen the oscillation. Without shock absorbers, this energy would cause the car to bounce up and down repeatedly. Good shock absorbers dissipate this energy as heat through viscous damping (oil flowing through small orifices), bringing the car quickly back to equilibrium after hitting a bump. The relatively large energy (141 J) for a modest displacement (7.5 cm) reflects the stiff suspension needed to support a car’s weight.

Answer

141 J

(a) How much will a spring that has a force constant of 40.0 N/m be stretched by an object with a mass of 0.500 kg when hung motionless from the spring? (b) Calculate the decrease in gravitational potential energy of the 0.500-kg object when it descends this distance. (c) Part of this gravitational energy goes into the spring. Calculate the energy stored in the spring by this stretch, and compare it with the gravitational potential energy. Explain where the rest of the energy might go.

Show Solution

Strategy

For part (a), when the object hangs motionless, the spring force balances the weight: $kx = mg$. For part (b), we calculate the change in gravitational potential energy using $\Delta PE_g = mgh$ (negative because the object descends). For part (c), we find the elastic potential energy stored in the spring using $PE_{el} = 12kx^2$ and compare it to the gravitational energy change.

Solution

(a) At equilibrium, $kx = mg$:

$$x = mgk = (0.500\ \text{kg})(9.80\ \text{m/s}^2)40.0\ \text{N/m}$$
$$x = 4.90\ \text{N}40.0\ \text{N/m} = 0.1225\ \text{m} \approx 0.123\ \text{m}$$

(b) The gravitational PE decreases as the object descends:

$$\Delta PE_g = -mgh = -(0.500\ \text{kg})(9.80\ \text{m/s}^2)(0.123\ \text{m})$$
$$\Delta PE_g = -0.602\ \text{J} \approx -0.600\ \text{J}$$

(c) Energy stored in the spring:

$$PE_{el} = 12kx^2 = 12(40.0\ \text{N/m})(0.123\ \text{m})^2$$
$$PE_{el} = 12(40.0)(0.01513) = 0.303\ \text{J} \approx 0.300\ \text{J}$$

Comparing energies:

- Gravitational PE lost: 0.600 J
- Elastic PE stored: 0.300 J
- Energy unaccounted for: 0.300 J

The spring stores only half the gravitational potential energy lost. The remaining 0.300 J is dissipated as heat through damping forces (air resistance, internal friction in the spring).

Discussion

The energy analysis reveals an interesting result: exactly half the gravitational potential energy goes into elastic potential energy, and half is dissipated. This 50-50 split is not a coincidence but arises from the work-energy relationship.

As the object descends distance x, the spring force increases linearly from 0 to kx. The average force during descent is $12kx$. The work done against this average force is $W = (12kx)(x) = 12kx^2$, which is exactly the elastic PE stored.

Meanwhile, gravity does work $W_g = mgx = (kx)(x) = kx^2$ (since $mg = kx$ at equilibrium). Notice that $kx^2 = 2(12kx^2)$—gravity's work is exactly twice the elastic energy stored.

The "missing" energy ($kx^2 - 12kx^2 = 12kx^2$) must have gone into kinetic energy that was then dissipated by damping. If we gently lowered the mass with our hand (providing an external damping force), we would feel warmth in our muscles—this is the 0.300 J being dissipated.

In a real scenario where the mass is simply released, it would initially overshoot the equilibrium position, oscillate, and gradually settle as damping dissipates energy. The final state has 0.300 J in the spring and 0.300 J dissipated as heat during the oscillations.

Suppose you have a 0.750-kg object on a horizontal surface connected to a spring that has a force constant of 150 N/m. There is simple friction between the object and surface with a static coefficient of friction $\mu_s = 0.100$. (a) How far can the spring be stretched without moving the mass? (b) If the object is set into oscillation with an amplitude twice the distance found in part (a), and the kinetic coefficient of friction is $\mu_k = 0.0850$, what total distance does it travel before stopping? Assume it starts at the maximum amplitude.

Show Solution

Strategy

For part (a), the maximum stretch occurs when the spring force equals the maximum static friction force. For part (b), we use energy conservation: the initial elastic potential energy equals the work done by kinetic friction over the total distance traveled.

Solution

(a) At the maximum stretch without moving, the spring force equals the static friction force:

$$kx = \mu_s mg$$
$$x = \mu_s mgk = (0.100)(0.750)(9.80)150 = 0.735150 = 0.00490\ \text{m} = 4.90\ \text{mm}$$

(b) The initial amplitude is $X = 2x = 2(0.00490) = 0.00980$ m

Initial elastic potential energy:

$$E_i = \frac{1}{2}kX^2 = \frac{1}{2}(150)(0.00980)^2 = 7.20 \times 10^{-3} \text{ J}$$

Work done by kinetic friction over total distance d:

$$W_f = \mu_k mg \cdot d$$

Setting the initial energy equal to the work done by friction:

$$\frac{1}{2}kX^2 = \mu_k mg \cdot d$$
$$d = \frac{kX^2}{2\mu_k mg} = \frac{(150)(0.00980)^2}{2(0.0850)(0.750)(9.80)}$$
$$d = \frac{0.0144}{1.249} = 0.0115 \text{ m} = 11.5 \text{ mm}$$

Discussion

In part (a), the spring can only stretch about 5 mm before overcoming static friction—a very small distance. In part (b), with initial amplitude of 9.8 mm, the object travels a total distance of 11.5 mm before friction brings it to rest. This is only slightly more than one complete oscillation because kinetic friction continuously removes energy. The object doesn't oscillate many times like an undamped system would; instead, it quickly comes to rest due to the significant frictional damping.

Answer

(a) 4.90 mm or 0.00490 m

(b) 11.5 mm or 0.0115 m

Engineering Application: A suspension bridge oscillates with an effective force constant of 1.00×10^8 N/m . (a) How much energy is needed to make it oscillate with an amplitude of 0.100 m? (b) If soldiers march across the bridge with a cadence equal to the bridge's natural frequency and impart 1.00×10^4 J of energy each second, how long does it take for the bridge's oscillations to go from 0.100 m to 0.500 m amplitude?

Show Solution

Strategy

For part (a), the energy in a harmonic oscillator at maximum displacement is purely elastic potential energy: $E = \frac{1}{2}kx^2$. For part (b), we first calculate the energy needed to increase amplitude from 0.100 m to 0.500 m, then divide by the power input to find the time required.

Solution

(a) Energy needed for 0.100 m amplitude:

$$E_1 = \frac{1}{2}kx_1^2 = \frac{1}{2}(1.00 \times 10^8)(0.100)^2$$
$$E_1 = \frac{1}{2}(1.00 \times 10^8)(0.0100) = 5.00 \times 10^5 \text{ J}$$

(b) Energy at 0.500 m amplitude:

$$E_2 = \frac{1}{2}kx_2^2 = \frac{1}{2}(1.00 \times 10^8)(0.500)^2$$
$$E_2 = \frac{1}{2}(1.00 \times 10^8)(0.250) = 1.25 \times 10^7 \text{ J}$$

Additional energy needed:

$$\Delta E = E_2 - E_1 = 1.25 \times 10^7 - 5.00 \times 10^5 = 1.20 \times 10^7 \text{ J}$$

Time to add this energy at a rate of 1.00×10^4 J/s:

$$t = \frac{\Delta E}{P} = \frac{1.20 \times 10^7}{1.00 \times 10^4} = 1.20 \times 10^3 \text{ s}$$

Discussion

Part (a) shows that even a 10 cm oscillation of a massive bridge requires 500,000 J of energy due to the enormous force constant (10^8 N/m). In part (b), the soldiers marching at the bridge's natural frequency cause resonance, allowing energy to accumulate efficiently. The time of 1200 seconds (20 minutes) is relatively short considering the enormous energy involved (12 million joules). This demonstrates the danger of resonance: sustained forcing at the natural frequency can build up large-amplitude oscillations that may exceed the structure's design limits. This is why soldiers are instructed to break step when crossing bridges—to avoid inadvertently matching the bridge’s natural frequency and triggering resonance. The Tacoma Narrows Bridge collapse mentioned in the chapter is a dramatic example of this phenomenon.

Answer

(a) 5.00×10^5 J

(b) 1.20×10^3 s

Glossary

natural frequency
: the frequency at which a system would oscillate if there were no driving and no damping forces

resonance
: the phenomenon of driving a system with a frequency equal to the system's natural frequency

resonate
: a system being driven at its natural frequency

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