Introduction to Work, Energy, and Energy Resources

How many forms of energy can you identify in this photograph of a wind farm in Iowa? (credit: Jürgen from Sandesneben, Germany, Wikimedia Commons)

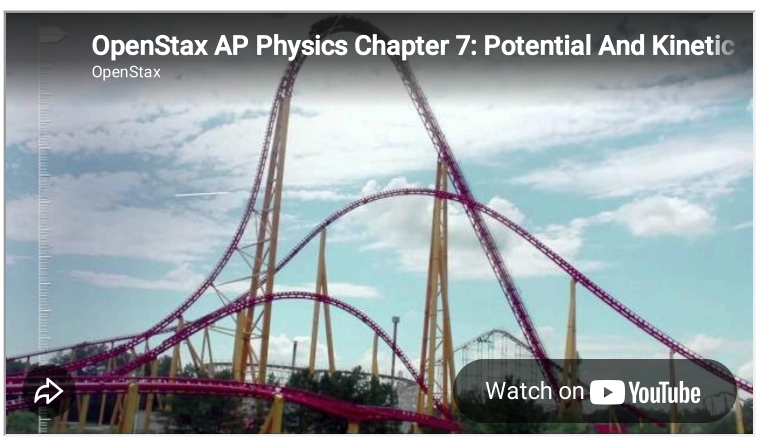
Energy plays an essential role both in everyday events and in scientific phenomena. You can no doubt name many forms of energy, from that provided by our foods, to the energy we use to run our cars, to the sunlight that warms us on the beach. You can also cite examples of what people call energy that may not be scientific, such as someone having an energetic personality. Not only does energy have many interesting forms, it is involved in almost all phenomena, and is one of the most important concepts of physics. What makes it even more important is that the total amount of energy in the universe is constant. Energy can change forms, but it cannot appear from nothing or disappear without a trace. Energy is thus one of a handful of physical quantities that we say is *conserved*.

Conservation of energy (as physicists like to call the principle that energy can neither be created nor destroyed) is based on experiment. For example, scientists Willem 's Gravesande and Émilie du Châtelet undertook (separate) experiments where they dropped heavy lead balls into beds of clay. Du Châtelet showed that the balls that hit the clay with twice the velocity penetrated four times as deep into the clay; those with three times the velocity reached a depth nine times greater. This led her to develop a more accurate concept of energy conservation, expressed as $E = 12mv^2$. Even as scientists discovered new forms of energy, conservation of energy has always been found to apply. Perhaps the most dramatic example of this was supplied by Einstein when he suggested that mass is equivalent to energy (his famous equation $E = mc^2$

).

From a societal viewpoint, energy is one of the major building blocks of modern civilization. Energy resources are key limiting factors to economic growth. The world use of energy resources, especially oil, continues to grow, with ominous consequences economically, socially, politically, and environmentally. We will briefly examine the world's energy use patterns at the end of this chapter.

There is no simple, yet accurate, scientific definition for energy. Energy is characterized by its many forms and the fact that it is conserved. We can loosely define **energy** as the ability to do work, admitting that in some circumstances not all energy is available to do work. Because of the association of energy with work, we begin the chapter with a discussion of work. Work is intimately related to energy and how energy moves from one system to another or changes form.



Work: The Scientific Definition

- Explain how an object must be displaced for a force on it to do work.
- Explain how relative directions of force and displacement determine whether the work done is positive, negative, or zero.

What It Means to Do Work

The scientific definition of work differs in some ways from its everyday meaning. Certain things we think of as hard work, such as writing an exam or carrying a heavy load on level ground, are not work as defined by a scientist. The scientific definition of work reveals its relationship to energy—whenever work is done, energy is transferred.

For work, in the scientific sense, to be done, a force must be exerted and there must be displacement in the direction of the force.

Formally, the **work** done on a system by a constant force is defined to be *the product of the component of the force in the direction of motion times the distance through which the force acts*. For one-way motion in one dimension, this is expressed in equation form as

$$W = |\vec{\mathbf{F}}|(\cos\theta)|\vec{\mathbf{d}}|,$$

where W is work, $|\vec{\mathbf{d}}|$ is the magnitude of the displacement of the system, and θ is the angle between the force vector $\vec{\mathbf{F}}$ and the displacement vector $\vec{\mathbf{d}}$, as in Figure 1. We can also write this as

$$W = Fd\cos\theta.$$

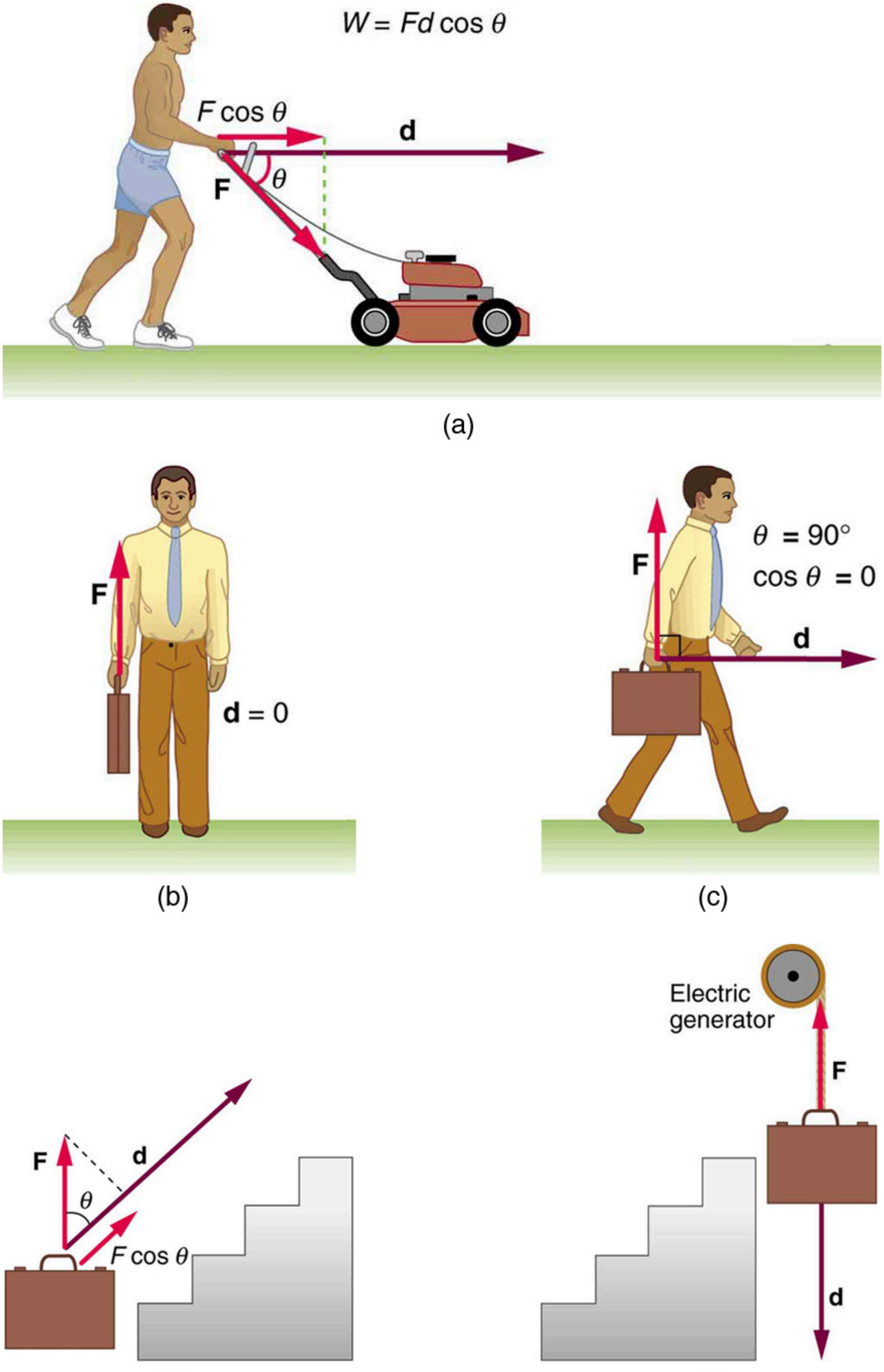
To find the work done on a system that undergoes motion that is not one-way or that is in two or three dimensions, we divide the motion into one-way one-dimensional segments and add up the work done over each segment.

What is Work?

The work done on a system by a constant force is *the product of the component of the force in the direction of motion times the distance through which the force acts*. For one-way motion in one dimension, this is expressed in equation form as

$$W = Fd\cos\theta,$$

where W is work, F is the magnitude of the force on the system, d is the magnitude of the displacement of the system, and θ is the angle between the force vector $\vec{\mathbf{F}}$ and the displacement vector $\vec{\mathbf{d}}$.



W = Fd cos θ
F cos θ
d
θ
F
(a)
F
d = 0
(b)
F
θ = 90°
cos θ = 0
d
(c)
F
d
θ
F cos θ
Electric
generator
F
d

(d) (e)

Examples of work. (a) The work done by the force F on this lawn mower is $F d \cos{ \theta}$. Note that $F \cos{ \theta}$ is the component of the force in the direction of motion. (b) A person holding a briefcase does no work on it, because there is no displacement. No energy is transferred to or from the briefcase. (c) The person moving the briefcase horizontally at a constant speed does no work on it, and transfers no energy to it. (d) Work is done on the briefcase by carrying it up stairs at constant speed, because there is necessarily a component of force F in the direction of the motion. Energy is transferred to the briefcase and could in turn be used to do work. (e) When the briefcase is lowered, energy is transferred out of the briefcase and into an electric generator. Here the work done on the briefcase by the generator is negative, removing energy from the briefcase, because F and d are in opposite directions.

To examine what the definition of work means, let us consider the other situations shown in Figure 1. The person holding the briefcase in Figure 1(b) does no work, for example. Here $d = 0$, so $W = 0$. Why is it you get tired just holding a load? The answer is that your muscles are doing work against one another, *but they are doing no work on the system of interest* (the “briefcase-Earth system”—see Gravitational Potential Energy for more details). There must be displacement for work to be done, and there must be a component of the force in the direction of the motion. For example, the person carrying the briefcase on level ground in Figure 1 (c) does no work on it, because the force is perpendicular to the motion. That is, $\cos 90^\circ = 0$, and so $W = 0$.

In contrast, when a force exerted on the system has a component in the direction of motion, such as in Figure 1(d), work *is* done—energy is transferred to the briefcase. Finally, in Figure 1(e), energy is transferred from the briefcase to a generator. There are two good ways to interpret this energy transfer. One interpretation is that the briefcase’s weight does work on the generator, giving it energy. The other interpretation is that the generator does negative work on the briefcase, thus removing energy from it. The drawing shows the latter, with the force from the generator upward on the briefcase, and the displacement downward. This makes $\theta = 180^\circ$, and $\cos 180^\circ = -1$; therefore, W is negative.

Calculating Work

Work and energy have the same units. From the definition of work, we see that those units are force times distance. Thus, in SI units, work and energy are measured in **newton-meters**. A newton-meter is given the special name **joule** (J), and $1\text{J} = 1\text{N}\cdot\text{m} = 1\text{kg}\cdot\text{m}^2/\text{s}^2$. One joule is not a large amount of energy; it would lift a small 100-gram apple a distance of about 1 meter.

Calculating the Work You Do to Push a Lawn Mower Across a Large Lawn

How much work is done on the lawn mower by the person in Figure 1 (a) if he exerts a constant force of 75.0N at an angle 35° below the horizontal and pushes the mower 25.0m on level ground? Convert the amount of work from joules to kilocalories and compare it with this person’s average daily intake of 10000kJ (about 2400kcal) of food energy. One *calorie* (1 cal) of heat is the amount required to warm 1 g of water by 1°C, and is equivalent to 4.184J, while one *food calorie* (1 kcal) is equivalent to 4184J.

Strategy

We can solve this problem by substituting the given values into the definition of work done on a system, stated in the equation $W = Fd\cos\theta$. The force, angle, and displacement are given, so that only the work W is unknown.

Solution

The equation for the work is

$$W = Fd\cos\theta.$$

Substituting the known values gives

$$\begin{aligned} W &= (75.0\text{N})(25.0\text{m})\cos(35.0^\circ) \\ W &= 1536\text{J} = 1.54\times10^3\text{J}. \end{aligned}$$

Converting the work in joules to kilocalories yields $W = (1536\text{J})(1\text{kcal}/4184\text{J}) = 0.367\text{kcal}$. The ratio of the work done to the daily consumption is

$$\frac{W}{2400\text{kcal}} = 1.53\times10^{-4}.$$

Discussion

This ratio is a tiny fraction of what the person consumes, but it is typical. Very little of the energy released in the consumption of food is used to do work. Even when we “work” all day long, less than 10% of our food energy intake is used to do work and more than 90% is converted to thermal energy or stored as chemical energy in fat.

Section Summary

- Work is the transfer of energy by a force acting on an object as it is displaced.
- The work W that a force $\vec{\mathbf{F}}$ does on an object is the product of the magnitude F of the force, times the magnitude d of the displacement, times the cosine of the angle θ between them. In symbols,
  $$W = Fd\cos\theta.$$
- The SI unit for work and energy is the joule (J), where $1\text{J} = 1\text{N}\cdot\text{m} = 1\text{kg}\cdot\text{m}^2/\text{s}^2$.
- The work done by a force is zero if the displacement is either zero or perpendicular to the force.

- The work done is positive if the force and displacement have the same direction, and negative if they have opposite direction.

Conceptual Questions

Give an example of something we think of as work in everyday circumstances that is not work in the scientific sense. Is energy transferred or changed in form in your example? If so, explain how this is accomplished without doing work.

Give an example of a situation in which there is a force and a displacement, but the force does no work. Explain why it does no work.

Describe a situation in which a force is exerted for a long time but does no work. Explain.

Problems & Exercises

How much work does a supermarket checkout attendant do on a can of soup he pushes 0.600 m horizontally with a force of 5.00 N? Express your answer in joules and kilocalories.

Show Solution

Strategy

The work done is given by $W = Fd\cos\theta$, where the force and displacement are in the same direction ($\theta = 0°$), so $W = Fd$. We then convert the result to kilocalories using 1 kcal = 4184 J.

Solution

The work done is:

$$W = Fd = (5.00\text{N})(0.600\text{m}) \quad W = 3.00\text{J}$$

Converting to kilocalories:

$$W = 3.00\text{J} \times \frac{1\text{ kcal}}{4184\text{J}} = 7.17 \times 10^{-4}\text{ kcal}$$

Discussion

The work done is 3.00 J, which is a very small amount of energy. To put this in perspective, converting to kilocalories gives 7.17×10^{-4} kcal, which is less than one-thousandth of a food Calorie. This makes sense for the minimal effort required to slide a can along a checkout counter. The force applied (5.00 N) is about the weight of a 500-gram object, and the distance (0.600 m) is less than the length of an arm. Such a small amount of work would barely register as physical exertion for a person, which aligns with our everyday experience of pushing items at a checkout counter.

Answer

The checkout attendant does **3.00 J** of work on the can of soup, which is equivalent to **7.17×10^{-4} kcal**.

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A 75.0-kg person climbs stairs, gaining 2.50 meters in height. Find the work done to accomplish this task.

Show Solution

Strategy

To lift the person at constant speed, the applied force must equal the person's weight. The work done against gravity is the product of this force and the vertical displacement. We use $W = Fd\cos\theta$, where the force and displacement are in the same direction ($\theta = 0°$), so the work done is $W = mgh$.

Solution

The work done to lift the person is:

$$W = mgh$$

Substituting known values:

$$W = (75.0\text{kg})(9.80\text{m/s}^2)(2.50\text{m}) \quad W = 1838\text{J} \approx 1.84 \times 10^3\text{J}$$

Discussion

The work done to climb stairs is 1840 J, which represents the gravitational potential energy gained by the person. This is about 600 times more energy than pushing a can across a checkout counter (Problem 1). The result is reasonable because lifting a 75-kg person (about 165 lbs) through a height of 2.50 m requires significant work against gravity. This amount of energy (approximately 0.44 kcal) is still quite small compared to daily food intake, but climbing stairs repeatedly would add up—climbing 10 flights of similar height would consume about 4.4 kcal. The work done increases linearly with both the mass of the person and the height climbed.

Answer

The work done to lift the person climbing stairs is **1.84×10^3 J** (or **1840 J**).

(a) Calculate the work done on a 1500-kg elevator car by its cable to lift it 40.0 m at constant speed, assuming friction averages 100 N? (b) What is the work done on the elevator car by the gravitational force in this process? (c) What is the total work done on the elevator car?

Show Solution

Strategy

At constant speed, the net force is zero. The cable must pull upward with a force equal to the weight of the elevator plus the friction force. For part (a), we calculate the work done by the cable using $W = Fd$. For part (b), gravity does negative work since it acts downward while the displacement is upward. For part (c), we sum all work done on the elevator.

Solution

Part (a): Work done by the cable

At constant speed, the upward tension force equals the downward forces (weight plus friction):

$$T = mg + f = (1500\text{kg})(9.80\text{m/s}^2) + 100\text{N} = 14700\text{N} + 100\text{N} = 14800\text{N}$$

The work done by the cable (force and displacement in same direction):

$$W_{\text{cable}} = Td = (14800\text{N})(40.0\text{m}) \quad W_{\text{cable}} = 5.92\times10^5\text{J}$$

Part (b): Work done by gravity

The gravitational force is $F_g = mg = 14700\text{N}$ acting downward, opposite to the upward displacement ($\theta = 180°$):

$$W_{\text{gravity}} = mgd\cos180° = -mgd \quad W_{\text{gravity}} = -(1500\text{kg})(9.80\text{m/s}^2)(40.0\text{m}) \quad W_{\text{gravity}} = -5.88\times10^5\text{J}$$

Part (c): Total work done

The total work includes work by cable, gravity, and friction. The friction force opposes motion (acts downward), so:

$$W_{\text{friction}} = -fd = -(100\text{N})(40.0\text{m}) = -4000\text{J}$$

The total work is:

$$W_{\text{total}} = W_{\text{cable}} + W_{\text{gravity}} + W_{\text{friction}} \quad W_{\text{total}} = 5.92\times10^5\text{J} - 5.88\times10^5\text{J} - 4.00\times10^3\text{J} \quad W_{\text{total}} = (5.92 - 5.88 - 0.04)\times10^5\text{J} = 0\text{J}$$

Discussion

The cable does positive work (5.92×10^5 J) to lift the elevator, while gravity does an almost equal amount of negative work (-5.88×10^5 J). Friction does a relatively small amount of negative work (-4000 J). The total work done on the elevator is zero, which is consistent with the elevator moving at constant speed—there is no change in kinetic energy.

This problem illustrates that even though significant forces act and significant work is done by individual forces, the net work can be zero when an object moves at constant velocity. The cable must do extra work (4000 J) beyond what's needed just to lift the elevator, to overcome the frictional forces. Of the 592,000 J of work done by the cable, 588,000 J goes into increasing the gravitational potential energy of the elevator, while 4000 J is dissipated by friction.

Answer

(a) The cable does **5.92×10^5 J** of work on the elevator.

(b) Gravity does **-5.88×10^5 J** of work on the elevator.

(c) The total work done on the elevator is **0 J** (consistent with constant speed motion).

Suppose a car travels 108 km at a speed of 30.0 m/s, and uses 2.0 gal of gasoline. Only 30% of the gasoline goes into useful work by the force that keeps the car moving at constant speed despite friction. (See Table 1 of Conservation Of Energy for the energy content of gasoline.) (a) What is the magnitude of the force exerted to keep the car moving at constant speed? (b) If the required force is directly proportional to speed, how many gallons will be used to drive 108 km at a speed of 28.0 m/s?

Show Solution

Strategy

For part (a), we need to find the work done by the force keeping the car moving at constant speed, which equals the useful energy from the gasoline. From Table 1 in Conservation of Energy, 1 gallon of gasoline contains 1.2×10^8J. Only 30% of this is useful work. We can then use $W=Fd$ to find the force.

For part (b), if force is proportional to speed, then $F\propto v$. We can use the ratio of speeds to find the new force, and then calculate the gasoline consumption.

Solution for (a)

The total energy available from 2.0 gallons of gasoline is:

$$E_{\text{total}}=(2.0\text{ gal})(1.2\times10^8\text{J/gal})=2.4\times10^8\text{J}$$

The useful work done is 30% of this:

$$W=0.30\times2.4\times10^8\text{J}=7.2\times10^7\text{J}$$

The distance traveled is $d=108\text{ km}=1.08\times10^5\text{m}$. Using $W=Fd$:

$$F=\frac{W}{d}=\frac{7.2\times10^7\text{J}}{1.08\times10^5\text{m}} \quad F=667\text{N}\approx6.7\times10^2\text{N}$$

Solution for (b)

If $F\propto v$, then:

$$\frac{F_2}{F_1}=\frac{v_2}{v_1}=\frac{28.0\text{m/s}}{30.0\text{m/s}}=0.933$$

The new force is:

$$F_2=0.933\times667\text{N}=622\text{N}$$

The work done at this new force is:

$$W_2=F_2d=(622\text{N})(1.08\times10^5\text{m})=6.72\times10^7\text{J}$$

The total energy needed (accounting for only 30% efficiency) is:

$$E_{\text{needed}}=\frac{6.72\times10^7\text{J}}{0.30}=2.24\times10^8\text{J}$$

The number of gallons required is:

$$\text{gallons}=\frac{2.24\times10^8\text{J}}{1.2\times10^8\text{J/gal}}=1.87\text{ gal}\approx1.9\text{ gal}$$

Discussion

The force required to overcome friction and keep the car moving at constant speed is 667 N (about 150 lbs). This seems reasonable for highway driving—it's the combined effect of air resistance and rolling friction. The fact that only 30% of the gasoline's energy goes into useful work reflects the inherent inefficiency of internal combustion engines, with the remaining 70% lost as heat.

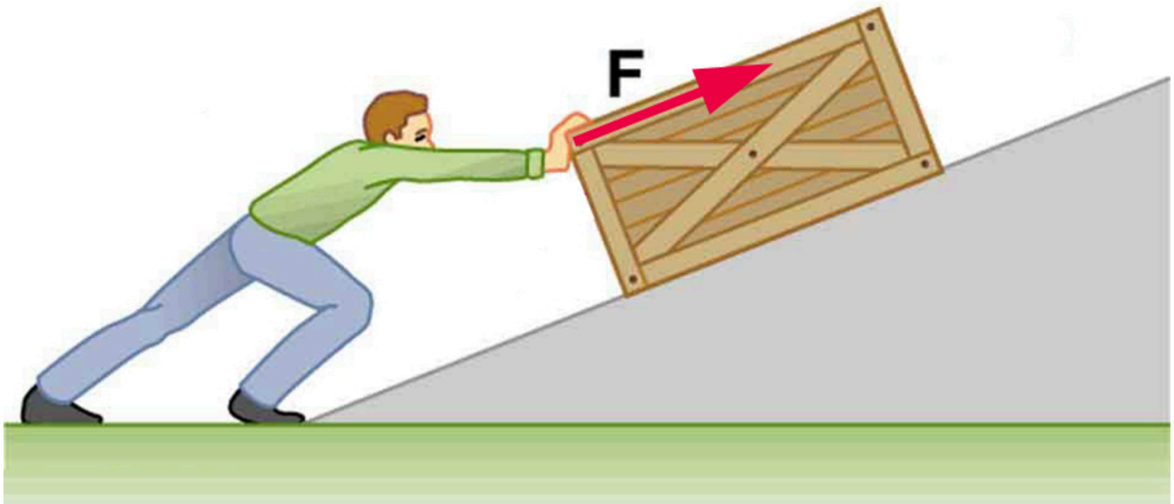
For part (b), driving at a slower speed (28.0 m/s instead of 30.0 m/s) reduces the required force proportionally, resulting in less fuel consumption (1.9 gal instead of 2.0 gal). This 6.7% reduction in speed leads to a 5% reduction in fuel consumption, demonstrating that driving slower can improve fuel efficiency. The savings would be even more dramatic at highway speeds where air resistance increases as the square of velocity, but this problem assumes a simpler linear relationship between force and speed.

Answer

(a) The magnitude of the force exerted to keep the car moving at constant speed is **667 N** (or $\mathbf{6.7\times10^2}$ **N**).

(b) At the lower speed of 28.0 m/s, approximately **1.9 gallons** of gasoline will be used to drive 108 km.

Calculate the work done by an 85.0-kg man who pushes a crate 4.00 m up along a ramp that makes an angle of 20.0° with the horizontal. (See Figure 2.) He exerts a force of 500 N on the crate parallel to the ramp and moves at a constant speed. Be certain to include the work he does on the crate *and* on his body to get up the ramp.



A man pushes a crate up a ramp.

Show Solution

Strategy

The total work includes: (1) work done pushing the crate with force 500 N through distance 4.00 m, and (2) work done raising his own body mass. The vertical height gained is $h = d\sin\theta = (4.00\text{m})\sin 20.0^\circ$.

Solution

Work done pushing the crate:

$$W_{\text{crate}} = Fd = (500\text{N})(4.00\text{m}) = 2000\text{J}$$

Vertical height gained:

$$h = d\sin\theta = (4.00\text{m})\sin 20.0^\circ = (4.00\text{m})(0.342) = 1.37\text{m}$$

Work done lifting his own body:

$$W_{\text{body}} = mgh = (85.0\text{kg})(9.80\text{m/s}^2)(1.37\text{m}) = 1141\text{J} \approx 1.14\times10^3\text{J}$$

Total work done:

$$W_{\text{total}} = W_{\text{crate}} + W_{\text{body}} \quad W_{\text{total}} = 2000\text{J} + 1141\text{J} = 3141\text{J} \approx 3.14\times10^3\text{J}$$

Discussion

The man does a total of 3140 J of work. About 64% of this (2000 J) goes into pushing the crate up the ramp, while 36% (1140 J) goes into lifting his own body mass against gravity. This demonstrates that when pushing objects uphill, a significant portion of the effort goes into lifting one's own body weight.

The result is reasonable: climbing 1.37 m vertically while pushing a heavy crate requires substantial energy. If the man's mass were comparable to the crate's mass, the work to lift himself would be comparable to the work on the crate. The total energy of 3140 J is about 0.75 kcal, which is a small but noticeable expenditure of energy—roughly equivalent to the energy in a single potato chip.

This problem illustrates an important practical consideration: when moving heavy objects up inclines, workers expend significant energy just moving themselves, not only the object. This is why ramps at gentler angles (smaller θ) can be advantageous—they reduce the vertical height gained per unit distance, though requiring longer travel.

Answer

The man does a total of **3140 J** (or $\mathbf{3.14\times10^3}$ **J**) of work, consisting of 2000 J on the crate and 1140 J on his own body.

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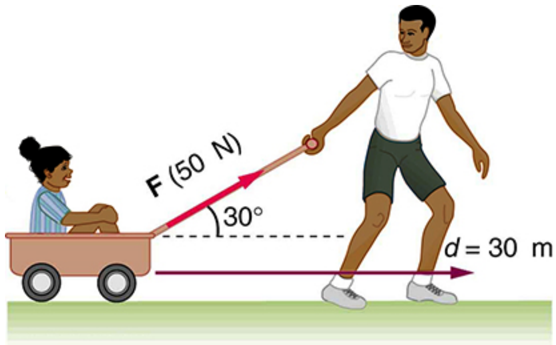
How much work is done by the boy pulling his sister 30.0 m in a wagon as shown in Figure 3? Assume no friction acts on the wagon.



The boy does work on the system of the wagon and the child when he pulls them as shown.

Show Solution

Strategy

The work done is given by $W = Fd\cos\theta$, where $F = 50.0\text{N}$ is the magnitude of the pulling force, $d = 30.0\text{m}$ is the displacement, and $\theta = 30.0^\circ$ is the angle between the force and the horizontal displacement.

Solution

Substituting the known values into the work equation:

$$W = Fd\cos\theta \quad W = (50.0\text{N})(30.0\text{m})\cos 30.0^\circ \quad W = (50.0\text{N})(30.0\text{m})(0.866) \quad W = 1299\text{J} \approx 1.30 \times 10^3\text{J}$$

Discussion

The boy does 1300 J of work pulling his sister in the wagon over a distance of 30.0 m. Only the horizontal component of the pulling force ($F\cos 30.0^\circ = 43.3\text{N}$) does work; the vertical component lifts slightly on the wagon but doesn't contribute to horizontal displacement.

This is a good example of how the angle of applied force affects work. If the boy pulled horizontally ($\theta = 0^\circ$), he would do $(50.0\text{N})(30.0\text{m}) = 1500\text{J}$ of work with the same force magnitude. By pulling at 30° above horizontal, he does only 1300 J of work—about 87% as much. However, pulling upward at an angle has the practical advantage of reducing the normal force on the wagon wheels, which would reduce friction if friction were present. Since this problem states there's no friction, pulling horizontally would be most efficient for doing work.

Answer

The boy does **1300 J** (or $\mathbf{1.30 \times 10^3}$ **J**) of work pulling the wagon.

A shopper pushes a grocery cart 20.0 m at constant speed on level ground, against a 35.0 N frictional force. He pushes in a direction 25.0° below the horizontal. (a) What is the work done on the cart by friction? (b) What is the work done on the cart by the gravitational force? (c) What is the work done on the cart by the shopper? (d) Find the force the shopper exerts, using energy considerations. (e) What is the total work done on the cart?

Show Solution

Strategy

For constant speed, the net force is zero. The friction force opposes motion ($\theta = 180^\circ$), gravity is perpendicular to motion, and the shopper pushes at 25.0° below horizontal. We use $W = Fd\cos\theta$ for each force.

Solution for (a)

The friction force is opposite to the displacement:

$$W_{\text{friction}} = fd\cos 180^\circ = -(35.0\text{N})(20.0\text{m}) = -700\text{J}$$

Solution for (b)

Gravity acts vertically downward, perpendicular to the horizontal displacement ($\theta = 90^\circ$):

$$W_{\text{gravity}} = mgd\cos 90^\circ = 0$$

Solution for (c)

At constant speed, the horizontal component of the shopper's force must equal the friction force. From Newton's second law: $F\cos 25.0^\circ = 35.0\text{N}$. The work done by the shopper must equal the negative of the work done by friction:

$$W_{\text{shopper}} = -W_{\text{friction}} = 700\text{J}$$

Solution for (d)

The horizontal component of the shopper's force equals the friction force:

$$F\cos 25.0^\circ = 35.0\text{N} \quad F = \frac{35.0\text{N}}{\cos 25.0^\circ} = \frac{35.0\text{N}}{0.906} = 38.6\text{N}$$

We can verify: $W = Fd\cos 25.0^\circ = (38.6\text{N})(20.0\text{m})(0.906) = 700\text{J}$ ✓

Solution for (e)

The total work is the sum of all work done:

$$W_{\text{total}} = W_{\text{friction}} + W_{\text{gravity}} + W_{\text{shopper}} = -700\text{J} + 0 + 700\text{J} = 0$$

Discussion

This problem illustrates several important concepts about work and energy. First, gravity does no work on the cart because the gravitational force is perpendicular to the horizontal displacement—this is why carrying a suitcase horizontally doesn't require you to do work against gravity (though your muscles still work internally to maintain the upward force).

Second, the shopper does positive work (700 J) while friction does an equal amount of negative work (-700 J), resulting in zero net work. This is consistent with the work-energy theorem: since the cart moves at constant speed, its kinetic energy doesn't change, so the net work must be zero.

Third, notice that the shopper must exert 38.6 N at an angle to overcome only 35.0 N of friction. This is because only the horizontal component of the shopper's force ($38.6\text{N} \times \cos 25.0^\circ = 35.0\text{N}$) counteracts friction. The vertical component ($38.6\text{N} \times \sin 25.0^\circ = 16.3\text{N}$) pushes down on the cart, increasing the normal force and thus increasing friction. Pushing downward at an angle is less efficient than pushing horizontally—in fact, it makes the task harder by increasing friction.

The energy transferred from the shopper (700 J) is completely dissipated by friction as thermal energy, which is why the cart doesn't speed up despite the applied force.

Answer

(a) Friction does **-700 J** of work on the cart.

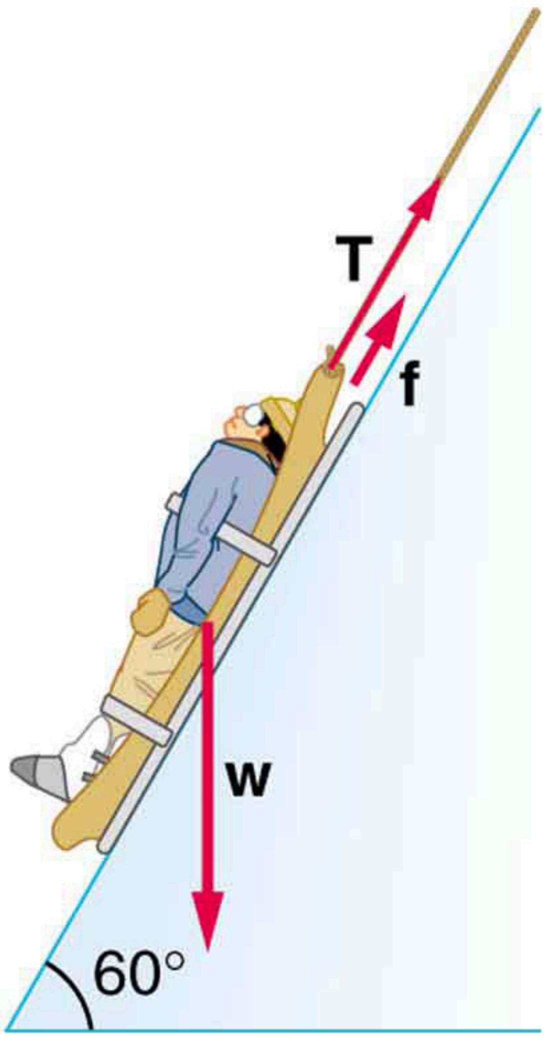
(b) Gravity does **0 J** of work on the cart (force perpendicular to displacement).

(c) The shopper does **700 J** of work on the cart.

(d) The shopper exerts a force of **38.6 N** at 25.0° below horizontal.

(e) The total work done on the cart is **0 J**, consistent with constant speed motion.

Suppose the ski patrol lowers a rescue sled and victim, having a total mass of 90.0 kg, down a 60.0° slope at constant speed, as shown in Figure 4. The coefficient of friction between the sled and the snow is 0.100. (a) How much work is done by friction as the sled moves 30.0 m along the hill? (b) How much work is done by the rope on the sled in this distance? (c) What is the work done by the gravitational force on the sled? (d) What is the total work done?



A rescue sled and victim are lowered down a steep slope.

Show Solution

Strategy

First, we need to find the forces acting on the sled. The normal force is $N = mg\cos\theta$. The friction force is $f = \mu_k N = \mu_k mg\cos\theta$. Since the sled moves down the slope at constant speed, the net force is zero. The component of weight down the slope is $mg\sin\theta$. We can use $W = Fd\cos\alpha$ for each force, where α is the angle between the force and displacement.

Solution for (a)

The friction force opposes motion (acts up the slope), and the sled moves down the slope, so the angle between friction and displacement is $180°$:

$$f = \mu_k mg\cos\theta = (0.100)(90.0\text{kg})(9.80\text{m/s}^2)\cos 60.0° = 44.1\text{N}$$

The work done by friction is:

$$W_f = fd\cos 180° = -fd \quad W_f = -(44.1\text{N})(30.0\text{m}) = -1323\text{J} \approx -1.32\times 10^3\text{J}$$

Solution for (b)

At constant speed, the net force along the slope is zero. The tension in the rope acts up the slope:

$$T + f = mg\sin\theta$$
$$T = mg\sin\theta - f = (90.0\text{kg})(9.80\text{m/s}^2)\sin 60.0° - 44.1\text{N} = 765\text{N} - 44.1\text{N} = 721\text{N}$$

The tension acts up the slope and the displacement is down the slope ($\theta = 180°$):

$W_T = Td\cos 180° = -(721\text{N})(30.0\text{m}) = -2.16\times10^4\text{J}$

Solution for (c)

The gravitational force is $W = mg = 882\text{N}$ acting vertically downward. The displacement along the slope makes an angle with the vertical. It's easier to use the component of weight along the slope: $mg\sin\theta$, which acts in the direction of motion:

$$W_g = (mg\sin\theta)d \quad W_g = (90.0\text{kg})(9.80\text{m/s}^2)(\sin 60.0°)(30.0\text{m}) \quad W_g = (765\text{N})(30.0\text{m}) = 2.30\times10^4\text{J}$$

Solution for (d)

The total work is the sum of all work done:

$$W_{\text{total}} = W_f + W_T + W_g \quad W_{\text{total}} = -1.32\times10^3\text{J} - 2.16\times10^4\text{J} + 2.30\times10^4\text{J} \quad W_{\text{total}} = 0\text{J}$$

Discussion

This problem demonstrates the interplay of multiple forces doing work on an object moving at constant speed. Gravity does positive work (23,000 J) because it has a component along the direction of motion down the slope. Both the rope tension and friction do negative work, removing energy from the system.

The rope does the most negative work (-21,600 J), which represents the energy absorbed by the ski patrol as they control the descent. This energy must be dissipated (often as heat in a braking mechanism) or stored (if using a winch system). Friction does a much smaller amount of negative work (-1,320 J), converting mechanical energy to thermal energy that slightly warms the sled runners and snow.

The sum of all work equals zero, confirming the work-energy theorem: at constant speed, kinetic energy is constant, so net work must be zero. This is a controlled descent—without the rope tension, the sled would accelerate down the slope, gaining kinetic energy equal to the difference between gravitational work and friction work.

Interestingly, the rope must do more work than gravity adds because friction also opposes motion. Of the 23,000 J that gravity contributes, 1,320 J is dissipated by friction, leaving 21,680 J that the rope must remove through tension. The small discrepancy (21,600 J vs 21,680 J) is due to rounding in the calculations.

The relatively small friction force (44.1 N compared to 765 N down the slope) shows why steep, icy slopes are dangerous—friction provides minimal resistance, requiring large rope tensions for safe descent.

Answer

(a) Friction does $\mathbf{-1.32\times10^3}$ **J** (or **-1,320 J**) of work on the sled.

(b) The rope does $\mathbf{-2.16\times10^4}$ **J** (or **-21,600 J**) of work on the sled.

(c) Gravity does $\mathbf{+2.30\times10^4}$ **J** (or **+23,000 J**) of work on the sled.

(d) The total work done is **0 J**, consistent with the constant speed descent.

Glossary

energy
: the ability to do work

work
: the transfer of energy by a force that causes an object to be displaced; the product of the component of the force in the direction of the displacement and the magnitude of the displacement

joule
: SI unit of work and energy, equal to one newton-meter

Kinetic Energy and the Work-Energy Theorem

- Explain work as a transfer of energy and net work as the work done by the net force.
- Explain and apply the work-energy theorem.

Work Transfers Energy

What happens to the work done on a system? Energy is transferred into the system, but in what form? Does it remain in the system or move on? The answers depend on the situation. For example, in the module Work the scientific Definition, if the lawn mower in Figure 1 of Work the Scientific Definition(a) is pushed just hard enough to keep it going at a constant speed, then energy put into the mower by the person is removed continuously by friction, and eventually leaves the system in the form of heat transfer. In contrast, work done on the briefcase by the person carrying it up stairs in Figure 1 of Work the Scientific Definition(d) is stored in the briefcase-Earth system and can be recovered at any time, as shown in Figure 1 of Work the Scientific Definition(e). In fact, the building of the pyramids in ancient Egypt is an example of storing energy in a system by doing work on the system. Some of the energy imparted to the stone blocks in lifting them during construction of the pyramids remains in the stone-Earth system and has the potential to do work.

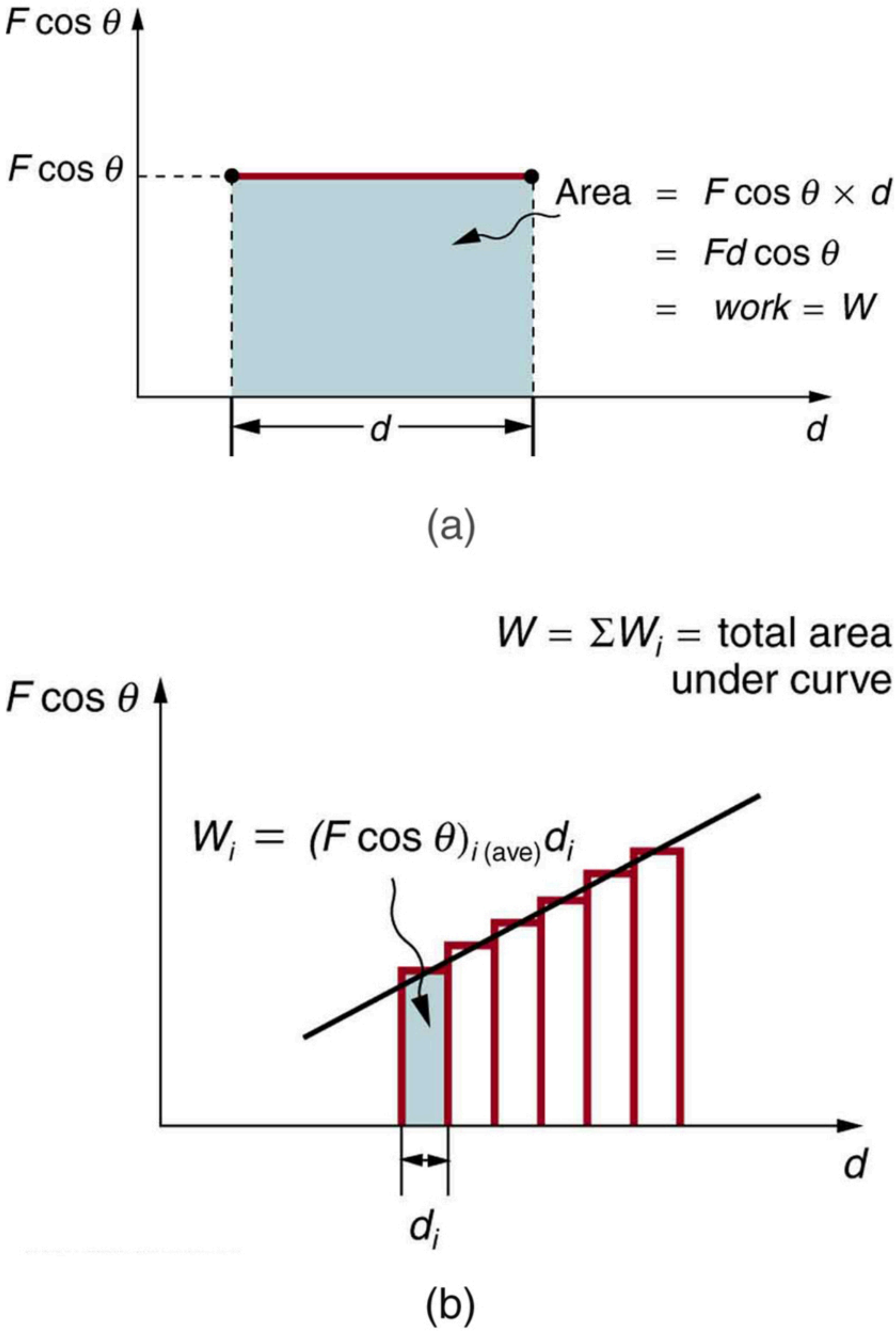
In this section we begin the study of various types of work and forms of energy. We will find that some types of work leave the energy of a system constant, for example, whereas others change the system in some way, such as making it move. We will also develop definitions of important forms of energy, such as the energy of motion.

Net Work and the Work-Energy Theorem

We know from the study of Newton's laws in Dynamics: Force and Newton's Laws of Motion that net force causes acceleration. We will see in this section that work done by the net force gives a system energy of motion, and in the process we will also find an expression for the energy of motion.

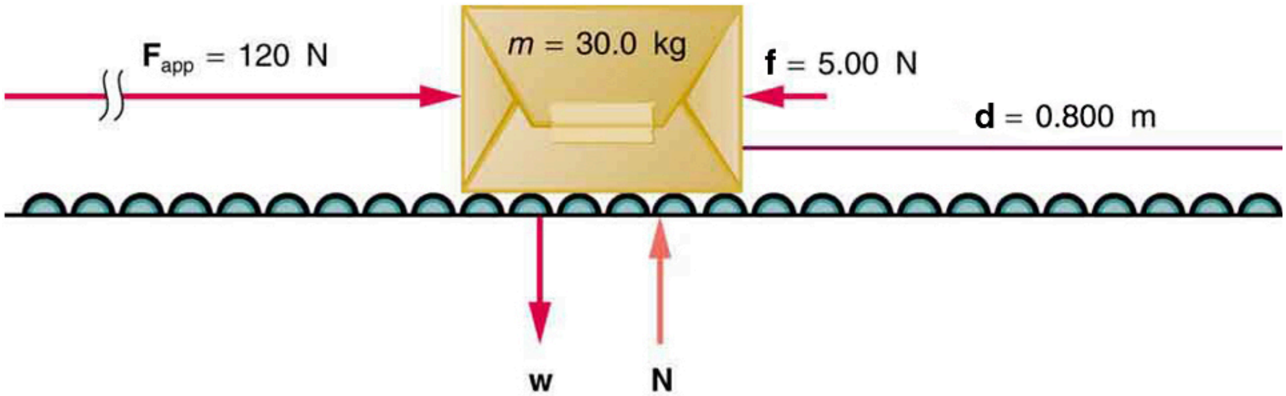
Let us start by considering the total, or net, work done on a system. Net work is defined to be the sum of work done by all external forces—that is, **net work** is the work done by the net external force $\vec{\mathbf{F}}_{\text{net}}$. In equation form, this is $W_{\text{net}} = F\_\text{net} d \cos\theta$ where θ is the angle between the force vector and the displacement vector.

Figure 1(a) shows a graph of force versus displacement for the component of the force in the direction of the displacement—that is, an $F\cos\theta$ vs. d graph. In this case, $F\cos\theta$ is constant. You can see that the area under the graph is $Fd\cos\theta$, or the work done. Figure 1(b) shows a more general process where the force varies. The area under the curve is divided into strips, each having an average force $(F\cos\theta)_{i(\text{ave})}$. The work done is $(F\cos\theta)_{i(\text{ave})}d_i$ for each strip, and the total work done is the sum of the W_i. Thus the total work done is the total area under the curve, a useful property to which we shall refer later.



(a) A graph of $F\cos{ \theta}$ vs. d , when $F \cos{ \theta}$ is constant. The area under the curve represents the work done by the force.
(b) A graph of $F \cos{ \theta}$ vs. d in which the force varies. The work done for each interval is the area of each strip; thus, the total area under the curve equals the total work done.

Net work will be simpler to examine if we consider a one-dimensional situation where a force is used to accelerate an object in a direction parallel to its initial velocity. Such a situation occurs for the package on the roller belt conveyor system shown in Figure 2.



A package on a roller belt is pushed horizontally through a distance d.

The force of gravity and the normal force acting on the package are perpendicular to the displacement and do no work. Moreover, they are also equal in magnitude and opposite in direction so they cancel in calculating the net force. The net force arises solely from the horizontal applied force $\vec{\mathbf{F}}_{app}$ and the horizontal friction force $\vec{\mathbf{f}}$. Thus, as expected, the net force is parallel to the displacement, so that $\theta = 0°$ and $\cos\theta = 1$, and the net work is given by

$$W_{net} = F_{net}d.$$

The effect of the net force $\vec{\mathbf{F}}_{net}$ is to accelerate the package from v_0 to v. The kinetic energy of the package increases, indicating that the net work done on the system is positive. (See Figure 2.) By using Newton's second law, and doing some algebra, we can reach an interesting conclusion. Substituting $F_{net} = ma$ from Newton's second law gives

$$W_{net} = mad.$$

To get a relationship between net work and the speed given to a system by the net force acting on it, we take $d = x - x_0$ and use the equation studied in Motion Equations for Constant Acceleration in One Dimension for the change in speed over a distance d if the acceleration has the constant value a; namely, $v^2 = v_0^2 + 2ad$ (note that a appears in the expression for the net work). Solving for acceleration gives $a = \frac{v^2 - v_0^2}{2d}$. When a is substituted into the preceding expression for W_{net}, we obtain

$$W_{net} = m\left(\frac{v^2 - v_0^2}{2d}\right)d.$$

The d cancels, and we rearrange this to obtain

$$W_{net} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2.$$

This expression is called the **work-energy theorem**, and it actually applies *in general* (even for forces that vary in direction and magnitude), although we have derived it for the special case of a constant force parallel to the displacement. The theorem implies that the net work on a system equals the change in the quantity $\frac{1}{2}mv^2$. This quantity is our first example of a form of energy.

The Work-Energy Theorem

The net work on a system equals the change in the quantity $\frac{1}{2}mv^2$.

$$W_{net} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$$

The quantity $\frac{1}{2}mv^2$ in the work-energy theorem is defined to be the translational **kinetic energy** (KE) of a mass m moving at a speed v. (*Translational* kinetic energy is distinct from *rotational* kinetic energy, which is considered later.) In equation form, the translational kinetic energy,

$$\text{KE} = \frac{1}{2}mv^2,$$

is the energy associated with translational motion. Kinetic energy is a form of energy associated with the motion of a particle, single body, or system of objects moving together.

We are aware that it takes energy to get an object, like a car or the package in Figure 2, up to speed, but it may be a bit surprising that kinetic energy is proportional to speed squared. This proportionality means, for example, that a car traveling at 100 km/h has four times the kinetic energy it has at 50 km/h, helping to explain why high-speed collisions are so devastating. We will now consider a series of examples to illustrate various aspects of work and energy.

Calculating the Kinetic Energy of a Package

Suppose a 30.0-kg package on the roller belt conveyor system in Figure 2 is moving at 0.500 m/s. What is its kinetic energy?

Strategy

Because the mass m and speed v are given, the kinetic energy can be calculated from its definition as given in the equation $\text{KE} = 12mv^2$.

Solution

The kinetic energy is given by

$$\text{KE}=12mv^2.$$

Entering known values gives

$$\text{KE}=0.5(30.0\text{kg})(0.500\text{m/s})^2,$$

which yields

$$\text{KE}=3.75\text{kg}\cdot\text{m}^2/\text{s}^2=3.75\text{J}.$$

Discussion

Note that the unit of kinetic energy is the joule, the same as the unit of work, as mentioned when work was first defined. It is also interesting that, although this is a fairly massive package, its kinetic energy is not large at this relatively low speed. This fact is consistent with the observation that people can move packages like this without exhausting themselves.
Determining the Work to Accelerate a Package

Suppose that you push on the 30.0-kg package in Figure 2 with a constant force of 120 N through a distance of 0.800 m, and that the opposing friction force averages 5.00 N. (a) Calculate the net work done on the package. (b) Solve the same problem as in part (a), this time by finding the work done by each force that contributes to the net force.

Strategy and Concept for (a)

This is a motion in one dimension problem, because the downward force (from the weight of the package) and the normal force have equal magnitude and opposite direction, so that they cancel in calculating the net force, while the applied force, friction, and the displacement are all horizontal. (See Figure 2.) As expected, the net work is the net force times distance.

Solution for (a)

The net force is the push force minus friction, or $F\_\text{net} = 120N - 5.00\text{N} = 115\text{N}$. Thus the net work is

$$W_\text{net} = F_\text{net}d=(115\text{N})(0.800\text{m}) \quad W_\text{net} = 92.0\text{N}\cdot\text{m}=92.0\text{J}.$$

Discussion for (a)

This value is the net work done on the package. The person actually does more work than this, because friction opposes the motion. Friction does negative work and removes some of the energy the person expends and converts it to thermal energy. The net work equals the sum of the work done by each individual force.

Strategy and Concept for (b)

The forces acting on the package are gravity, the normal force, the force of friction, and the applied force. The normal force and force of gravity are each perpendicular to the displacement, and therefore do no work.

Solution for (b)

The applied force does work.

$$W_\text{app} = F_\text{app}d\cos(0^\circ)=F_\text{app}d \quad W_\text{app} = (120\text{N})(0.800\text{m}) \quad W_\text{app} = 96.0\text{J}$$

The friction force and displacement are in opposite directions, so that $\theta = 180^\circ$, and the work done by friction is

$$W_\text{fr} = F_\text{fr}d\cos(180^\circ)=-F_\text{fr}d \quad W_\text{fr} = -(5.00\text{N})(0.800\text{m}) \quad W_\text{fr} = -4.00\text{J}.$$

So the amounts of work done by gravity, by the normal force, by the applied force, and by friction are, respectively,

$$W_\text{gr} = 0, \quad W_N = 0, \quad W_\text{app} = 96.0\text{J}, \quad W_\text{fr} = -4.00\text{J}.$$

The total work done as the sum of the work done by each force is then seen to be

$$W_\text{total}=W_\text{gr}+W_N+W_\text{app}+W_\text{fr}=92.0\text{J}.$$

Discussion for (b)

The calculated total work W_{total} as the sum of the work by each force agrees, as expected, with the work W_{net} done by the net force. The work done by a collection of forces acting on an object can be calculated by either approach.
Determining Speed from Work and Energy

Find the speed of the package in Figure 2 at the end of the push, using work and energy concepts.

Strategy

Here the work-energy theorem can be used, because we have just calculated the net work, W_{net}, and the initial kinetic energy, $\frac{1}{2}mv_0^2$. These calculations allow us to find the final kinetic energy, $\frac{1}{2}mv^2$, and thus the final speed v.

Solution

The work-energy theorem in equation form is

$$W_{\text{net}}=\frac{1}{2}mv^2-\frac{1}{2}mv_0^2.$$

Solving for $\frac{1}{2}mv^2$ gives

$$\frac{1}{2}mv^2=W_{\text{net}}+\frac{1}{2}mv_0^2.$$

Thus,

$$\frac{1}{2}mv^2=92.0\text{J}+3.75\text{J}=95.75\text{J}.$$

Solving for the final speed as requested and entering known values gives

$$v=\sqrt{\frac{2(95.75\text{ J})}{m}}=\sqrt{\frac{191.5\text{kg}\cdot\text{m}^2/\text{s}^2}{30.0\text{kg}}} \quad v=2.53\text{m/s}.$$

Discussion

Using work and energy, we not only arrive at an answer, we see that the final kinetic energy is the sum of the initial kinetic energy and the net work done on the package. This means that the work indeed adds to the energy of the package.

Work and Energy Can Reveal Distance, Too

How far does the package in Figure 2 coast after the push, assuming friction remains constant? Use work and energy considerations.

Strategy

We know that once the person stops pushing, friction will bring the package to rest. In terms of energy, friction does negative work until it has removed all of the package's kinetic energy. The work done by friction is the force of friction times the distance traveled times the cosine of the angle between the friction force and displacement; hence, this gives us a way of finding the distance traveled after the person stops pushing.

Solution

The normal force and force of gravity cancel in calculating the net force. The horizontal friction force is then the net force, and it acts opposite to the displacement, so $\theta=180^\circ$. To reduce the kinetic energy of the package to zero, the work W_{fr} by friction must be minus the kinetic energy that the package started with plus what the package accumulated due to the pushing. Thus $W_{\text{fr}}=-95.75\text{J}$. Furthermore, $W\_\text{fr}=fd'\cos\theta=-fd'$, where d' is the distance it takes to stop. Thus,

$$d'=-\frac{W_{\text{fr}}}{f}=-\frac{-95.75\text{J}}{5.00\text{N}},$$

and so

$$d'=19.2\text{m}.$$

Discussion

This is a reasonable distance for a package to coast on a relatively friction-free conveyor system. Note that the work done by friction is negative (the force is in the opposite direction of motion), so it removes the kinetic energy.

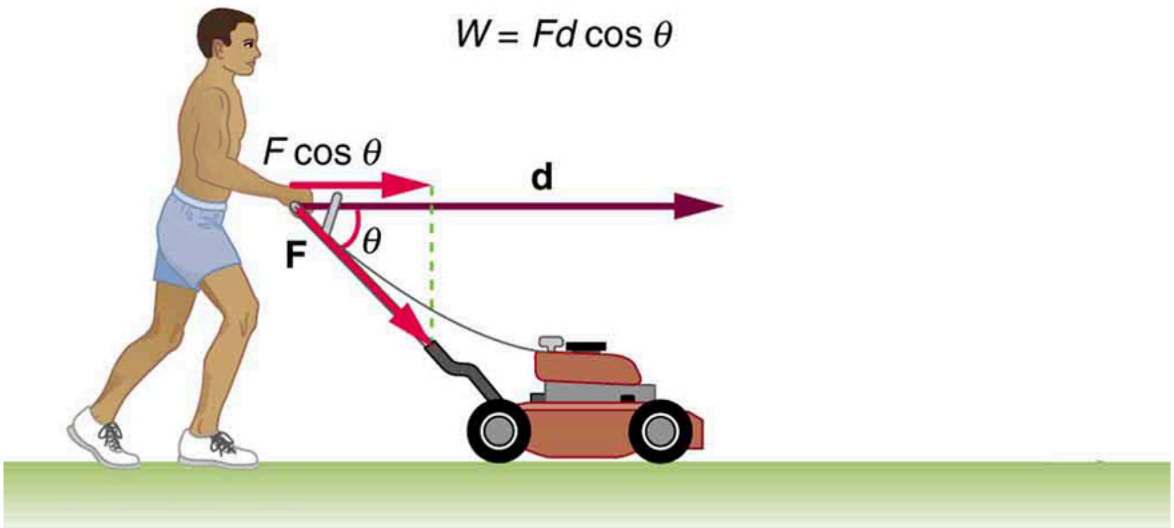
Some of the examples in this section can be solved without considering energy, but at the expense of missing out on gaining insights about what work and energy are doing in this situation. On the whole, solutions involving energy are generally shorter and easier than those using kinematics and dynamics alone.

Section Summary

- The net work W_net is the work done by the net force acting on an object.
- Work done on an object transfers energy to the object.
- The translational kinetic energy of an object of mass m moving at speed v is $\text{KE} = \frac{1}{2}mv^2$.
- The work-energy theorem states that the net work W_{net} on a system changes its kinetic energy, $W_{\text{net}} = \frac{1}{2}mv^2 - \frac{1}{2}mv\_0^2$.

Conceptual Questions

The person in Figure 3 does work on the lawn mower. Under what conditions would the mower gain energy? Under what conditions would it lose energy?



Person pushing a lawn mower.

Work done on a system puts energy into it. Work done by a system removes energy from it. Give an example for each statement.

When solving for speed in Example 3, we kept only the positive root. Why?

Problems & Exercises

Compare the kinetic energy of a 20 000-kg truck moving at 110 km/h with that of an 80.0-kg astronaut in orbit moving at 27 500 km/h.

Show Solution

Strategy

We calculate the kinetic energy of both using $\text{KE} = \frac{1}{2}mv^2$, converting speeds to m/s first, then find the ratio.

Solution

Convert speeds to m/s:

$$v_{\text{truck}} = 110\ \text{km/h} \times \frac{1000\text{m}}{3600\text{s}} = 30.6\text{m/s} \quad v_{\text{astronaut}} = 27500\ \text{km/h} \times \frac{1000\text{m}}{3600\text{s}} = 7640\text{m/s}$$

Kinetic energy of truck:

$$\text{KE}_{\text{truck}} = \frac{1}{2}(20000\text{kg})(30.6\text{m/s})^2 = 9.37 \times 10^6\text{J}$$

Kinetic energy of astronaut:

$$\text{KE}_{\text{astronaut}} = \frac{1}{2}(80.0\text{kg})(7640\text{m/s})^2 = 2.33 \times 10^9\text{J}$$

The ratio is:

$$\frac{\text{KE}_{\text{truck}}}{\text{KE}_{\text{astronaut}}} = \frac{9.37 \times 10^6\text{J}}{2.33 \times 10^9\text{J}} = 4.02 \times 10^{-3} \approx 1250$$

Discussion

The truck’s kinetic energy is only 1/250th that of the astronaut in orbit. Even though the truck is 250 times more massive than the astronaut, the astronaut’s much higher orbital speed (250 times faster) results in vastly more kinetic energy, since kinetic energy depends on the square of velocity. This demonstrates why orbital velocities represent such enormous energies.

Answer

The truck has a kinetic energy of 9.37×10^{6}J, while the astronaut has 2.33×10^{9}J. The astronaut's kinetic energy is approximately 250 times greater than the truck's.

(a) How fast must a 3000-kg elephant move to have the same kinetic energy as a 65.0-kg sprinter running at 10.0 m/s? (b) Discuss how the larger energies needed for the movement of larger animals would relate to metabolic rates.

Show Solution

Strategy

For part (a), we use the kinetic energy formula $\text{KE} = \frac{1}{2}mv^2$ for both the elephant and the sprinter. Setting them equal allows us to solve for the elephant's velocity. For part (b), we consider the relationship between body mass, movement energy, and metabolic rate.

Solution

(a) First, calculate the sprinter's kinetic energy:

$$\text{KE}_{\text{sprinter}} = \frac{1}{2}(65.0\,\text{kg})(10.0\,\text{m/s})^2 = 3250\,\text{J}$$

Setting the elephant's kinetic energy equal to this:

$$\frac{1}{2}m_{\text{elephant}}v^2_{\text{elephant}} = 3250\,\text{J}$$

Solving for the elephant's velocity:

$$v_{\text{elephant}} = \sqrt{\frac{2(3250\,\text{J})}{3000\,\text{kg}}} \quad v_{\text{elephant}} = \sqrt{2.167\ \text{m}^2/\text{s}^2} \quad v_{\text{elephant}} = 1.47\,\text{m/s}$$

(b) The relationship between body mass and metabolic rate is discussed below.

Discussion

The elephant must move at only 1.47 m/s to have the same kinetic energy as the sprinter running at 10.0 m/s. This is because the elephant's mass (3000 kg) is about 46 times greater than the sprinter's mass (65.0 kg), so by the kinetic energy formula $\text{KE} = \frac{1}{2}mv^2$, the elephant's velocity only needs to be $1/\sqrt{46} \approx 1/6.8$ times the sprinter's velocity.

Even though the elephant moves more slowly, it requires significant energy to move its large mass. Larger animals need proportionally larger metabolic rates to generate the energy required for movement. Metabolic rate generally scales with body mass to approximately the 3/4 power (Kleiber's law), meaning that while a 3000-kg elephant requires much more total energy than a 65-kg human, its metabolic rate per unit mass is actually lower. However, the elephant still needs substantial energy intake to maintain its basic functions and enable movement of such a massive body.

Answer

(a) The elephant must move at $1.47\,\text{m/s}$ to have the same kinetic energy as the sprinter.

(b) Larger animals require greater absolute metabolic rates to move their larger masses, though metabolic rate per unit mass decreases with increasing body size according to Kleiber's law.

Confirm the value given for the kinetic energy of an aircraft carrier in Table 1 of Conservation of Energy. You will need to look up the definition of a nautical mile (1 knot = 1 nautical mile/h).

Show Solution

Strategy

From Table 1, the aircraft carrier has mass 90,000 tons (90,000,000 kg) moving at 30 knots. We need to convert knots to m/s: 1 knot = 1 nautical mile/h, where 1 nautical mile = 1852 m.

Solution

Convert speed to m/s:

$$v = 30\ \text{knots} = 30\,\frac{\text{nautical miles}}{\text{h}} \times \frac{1852\,\text{m}}{1\ \text{nautical mile}} \times \frac{1\ \text{h}}{3600\,\text{s}} = 15.4\,\text{m/s}$$

Calculate kinetic energy:

$$\text{KE} = \frac{1}{2}mv^2 = \frac{1}{2}(9.00\times10^7\,\text{kg})(15.4\,\text{m/s})^2 \quad \text{KE} = \frac{1}{2}(9.00\times10^7)(237.16)\,\text{J} \quad \text{KE} = 1.07\times10^{10}\,\text{J} \approx 1.1\times10^{10}\,\text{J}$$

Discussion

The calculated kinetic energy of 1.1×10^{10} J confirms the value in Table 1. This enormous energy—about 11 billion joules—is equivalent to the energy in about 90 gallons of gasoline. It demonstrates the massive amount of energy required to move such a large vessel, and explains why aircraft carriers need powerful propulsion systems.

Answer

The kinetic energy of the aircraft carrier is 1.1×10^{10}J (11 billion joules), confirming the value given in Table 1.

(a) Calculate the force needed to bring a 950-kg car to rest from a speed of 90.0 km/h in a distance of 120 m (a fairly typical distance for a non-panic stop). (b) Suppose instead the car hits a concrete abutment at full speed and is brought to a stop in 2.00 m. Calculate the force exerted on the car and compare it with the force found in part (a).

Show Solution

Strategy

We use the work-energy theorem: the net work done equals the change in kinetic energy. The work done by the braking force is $W = -Fd$ (negative because it opposes motion), and this must equal the change in kinetic energy, which is $\Delta\text{KE} = 0 - \frac{1}{2}mv_0^2 = -\frac{1}{2}mv_0^2$. We apply this to both scenarios—normal braking and hitting an abutment—to compare the forces.

Solution

First, convert the speed: $v_0 = 90.0\text{ km/h} = 90.0 \times \frac{1000\text{m}}{3600\text{s}} = 25.0\text{m/s}$

The initial kinetic energy is:

$$\text{KE}_0 = \frac{1}{2}(950\text{kg})(25.0\text{m/s})^2 = 2.97\times10^5\text{J}$$

(a) For normal braking over 120 m:

Using the work-energy theorem ($W_{net} = \Delta\text{KE}$):

$$-Fd = -\text{KE}_0$$

Solving for F:

$$F = \frac{\text{KE}_0}{d} = \frac{2.97\times10^5\text{J}}{120\text{m}} \quad F = 2.47\times10^3\text{N} \approx 2.5\times10^3\text{N}$$

(b) For hitting the abutment with $d = 2.00\text{m}$:

$$F = \frac{\text{KE}_0}{d} = \frac{2.97\times10^5\text{J}}{2.00\text{m}} \quad F = 1.48\times10^5\text{N} \approx 1.5\times10^5\text{N}$$

The ratio of forces is:

$$\frac{F_{\text{abutment}}}{F_{\text{braking}}} = \frac{1.48\times10^5\text{N}}{2.47\times10^3\text{N}} = 60$$

Discussion

The force needed for normal braking over 120 m is approximately 2500 N, which is substantial but manageable for a car's braking system. However, when the car hits an abutment and stops in only 2.00 m, the force increases to approximately 150,000 N—60 times larger than the normal braking force.

This dramatic increase demonstrates the inverse relationship between stopping distance and force: when the stopping distance is reduced by a factor of 60 (from 120 m to 2 m), the force increases by the same factor. This explains why high-speed collisions are so catastrophic—the forces involved become enormous when the stopping distance is very small, causing severe damage to both the vehicle and occupants.

Answer

(a) The force needed to bring the car to rest in 120 m is 2.5×10^3N (2500 N).

(b) The force when hitting the abutment is 1.5×10^5N (150,000 N), which is 60 times larger than the braking force.

A car's bumper is designed to withstand a 4.0-km/h (1.1-m/s) collision with an immovable object without damage to the body of the car. The bumper cushions the shock by absorbing the force over a distance. Calculate the magnitude of the average force on a bumper that collapses 0.200 m while bringing a 900-kg car to rest from an initial speed of 1.1 m/s.

Show Solution

Strategy

We use the work-energy theorem: the work done by the bumper force equals the change in kinetic energy. The work is $W = -Fd$ (negative because it opposes motion), and this equals $\Delta\text{KE} = 0 - \frac{1}{2}mv^2$.

Solution

The initial kinetic energy is:

$$\text{KE} = \frac{1}{2}(900\,\text{kg})(1.1\,\text{m/s})^2 = 545\,\text{J}$$

Using the work-energy theorem:

$$-Fd = -\text{KE} \quad F = \frac{\text{KE}}{d} = \frac{545\,\text{J}}{0.200\,\text{m}} \quad F = 2725\,\text{N} \approx 2.7 \times 10^3\,\text{N}$$

Discussion

The average force on the bumper is approximately 2700 N (about 610 pounds). This is a substantial force, but it's distributed over the 0.200 m crumple distance, which protects the car body from damage. This design principle—absorbing impact energy over a larger distance—is the basis of modern automotive safety features like crumple zones.

Answer

The average force on the bumper is $2.7 \times 10^3\,\text{N}$ (approximately 2700 N or 610 pounds).

Boxing gloves are padded to lessen the force of a blow. (a) Calculate the force exerted by a boxing glove on an opponent's face, if the glove and face compress 7.50 cm during a blow in which the 7.00-kg arm and glove are brought to rest from an initial speed of 10.0 m/s. (b) Calculate the force exerted by an identical blow in the gory old days when no gloves were used and the knuckles and face would compress only 2.00 cm. (c) Discuss the magnitude of the force with glove on. Does it seem high enough to cause damage even though it is lower than the force with no glove?

Show Solution

Strategy

We use the work-energy theorem. The kinetic energy of the arm and glove is converted to work done in stopping over distance d. The work done by the stopping force is $W = -Fd$, which equals the change in kinetic energy $\Delta\text{KE} = 0 - \frac{1}{2}mv^2$. We calculate the force for two scenarios: with gloves (larger stopping distance) and without gloves (smaller stopping distance).

Solution

The initial kinetic energy is:

$$\text{KE} = \frac{1}{2}(7.00\,\text{kg})(10.0\,\text{m/s})^2 = 350\,\text{J}$$

(a) With gloves, using $d = 7.50\text{ cm} = 0.0750\,\text{m}$:

Using the work-energy theorem:

$$-Fd = -\text{KE} \quad F = \frac{\text{KE}}{d} = \frac{350\,\text{J}}{0.0750\,\text{m}} \quad F = 4.67 \times 10^3\,\text{N} \approx 4.7 \times 10^3\,\text{N}$$

(b) Without gloves, using $d = 2.00\text{ cm} = 0.0200\,\text{m}$:

$$F = \frac{\text{KE}}{d} = \frac{350\,\text{J}}{0.0200\,\text{m}} \quad F = 1.75 \times 10^4\,\text{N} \approx 1.8 \times 10^4\,\text{N}$$

Discussion

With a glove, the force is approximately 4700 N, while without a glove, the force increases to approximately 17,500 N—about 3.7 times larger. This dramatic difference occurs because the glove increases the stopping distance by a factor of 3.75 (from 2.00 cm to 7.50 cm), and since force is inversely proportional to stopping distance, the force decreases by the same factor.

However, even with a glove, 4700 N is still a very substantial force—equivalent to about 1000 pounds! This is certainly enough to cause significant damage. The human face and brain are not designed to withstand such forces. The glove reduces the force by increasing the stopping distance, but the blow can still be devastating. This explains why even padded boxing gloves can cause concussions, facial fractures, and other serious injuries. The comparison highlights why modern boxing regulations require gloves—they reduce the force substantially, though not enough to eliminate all danger.

Answer

(a) The force exerted with a boxing glove is $4.7 \times 10^3\,\text{N}$ (approximately 4700 N).

(b) The force exerted without a glove is $1.8 \times 10^4\,\text{N}$ (approximately 17,500 N).

(c) Yes, even the reduced force with a glove (4700 N or about 1000 pounds) is high enough to cause significant damage including concussions and facial injuries.

Using energy considerations, calculate the average force a 60.0-kg sprinter exerts backward on the track to accelerate from 2.00 to 8.00 m/s in a distance of 25.0 m, if he encounters a headwind that exerts an average force of 30.0 N against him.

Show Solution

Strategy

The net work done equals the change in kinetic energy. The net work is the work done by the sprinter minus the work done by the headwind: W_{net} = $(F_{\text{sprinter}}$ $-F$_wind)d =ΔKE.

Solution

Calculate the change in kinetic energy:

$$\Delta\text{KE} \quad = \quad 12m(v_{2f}-v_{2i}) \quad \Delta\text{KE} \quad = \quad 12(60.0\text{kg})[(8.00\text{m/s})^2-(2.00\text{m/s})^2] \quad \Delta\text{KE} \quad = \quad 12(60.0\text{kg})(64.0-4.00)\ \text{m}^2/\text{s}^2 \quad \Delta\text{KE}$$

Using the work-energy theorem:

$$(F_{\text{sprinter}}-F_{\text{wind}})d \quad = \quad \Delta\text{KE} \quad (F_{\text{sprinter}}-30.0\text{N})(25.0\text{m}) \quad = \quad 1800\text{J} \quad F_{\text{sprinter}}-30.0\text{N} \quad = \quad 1800\text{J}25.0\text{m}=72.0\text{N} \quad F_{\text{sprin}}$$

Discussion

The sprinter exerts an average force of 102 N backward on the track (by Newton’s third law, the track pushes forward on the sprinter with 102 N). Of this, 30 N is needed to overcome the headwind, while the remaining 72 N provides the net force that accelerates the sprinter. This demonstrates how additional resistance forces require proportionally more effort to achieve the same acceleration.

Answer

The average force the sprinter exerts backward on the track is 102N.

Glossary

net work
: work done by the net force, or vector sum of all the forces, acting on an object

work-energy theorem
: the result, based on Newton’s laws, that the net work done on an object is equal to its change in kinetic energy

kinetic energy
: the energy an object has by reason of its motion, equal to $12mv^2$ for the translational (i.e., non-rotational) motion of an object of mass m moving at speed v

Gravitational Potential Energy

- Explain gravitational potential energy in terms of work done against gravity.
- Show that the gravitational potential energy of an object of mass m at height h on Earth is given by $\mathrm{PE_g} = mgh$.
- Show how knowledge of the potential energy as a function of position can be used to simplify calculations and explain physical phenomena.

Work Done Against Gravity

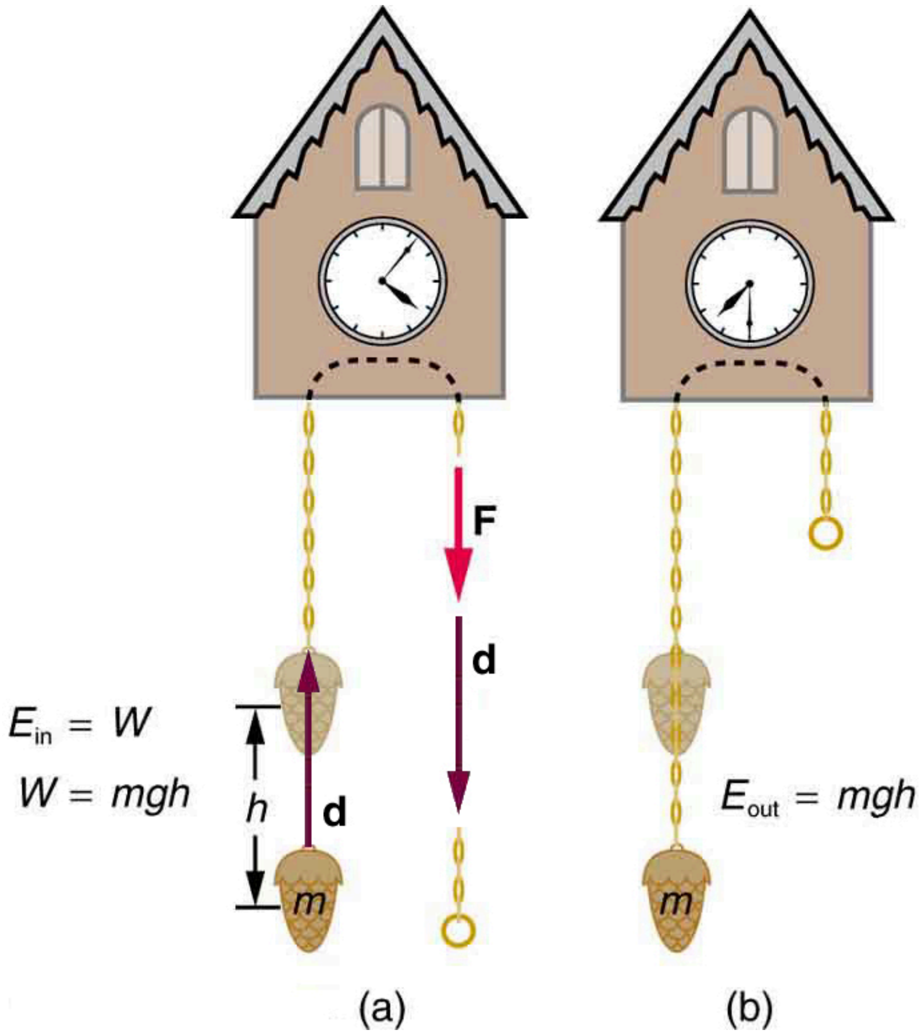
Climbing stairs and lifting objects is work in both the scientific and everyday sense—it is work done against the gravitational force. When there is work, there is a transformation of energy. The work done against the gravitational force goes into an important form of stored energy that we will explore in this section.

Let us calculate the work done in lifting an object of mass m through a height h, such as in Figure 1. If the object is lifted straight up at constant speed, then the force needed to lift it is equal to its weight mg. The work done on the mass is then $W = Fd = mgh$. We define this to be the **gravitational potential energy** $(\mathrm{PE_g})$ put into (or gained by) the object-Earth system. This energy is associated with the state of separation between two objects that attract each other by the gravitational force. For convenience, we refer to this as the $\mathrm{PE_g}$

gained by the object, recognizing that this is energy stored in the gravitational field of Earth. Why do we use the word "system"? Potential energy is a property of a system rather than of a single object—due to its physical position. An object's gravitational potential is due to its position relative to the surroundings within the Earth-object system. The force applied to the object is an external force, from outside the system. When it does positive work it increases the gravitational potential energy of the system. Because gravitational potential energy depends on relative position, we need a reference level at which to set the potential energy equal to 0. We usually choose this point to be Earth's surface, but this point is arbitrary; what is important is the *difference* in gravitational potential energy, because this difference is what relates to the work done. The difference in gravitational potential energy of an object (in the Earth-object system) between two rungs of a ladder will be the same for the first two rungs as for the last two rungs.

Converting Between Potential Energy and Kinetic Energy

Gravitational potential energy may be converted to other forms of energy, such as kinetic energy. If we release the mass, gravitational force will do an amount of work equal to mgh on it, thereby increasing its kinetic energy by that same amount (by the work-energy theorem). We will find it more useful to consider just the conversion of $\mathrm{PE_g}$ to KE without explicitly considering the intermediate step of work. (See Example 2.) This shortcut makes it is easier to solve problems using energy (if possible) rather than explicitly using forces.



(a) The work done to lift the weight is stored in the mass-Earth system as gravitational potential energy. (b) As the weight moves downward, this gravitational potential energy is transferred to the cuckoo clock.

More precisely, we define the *change* in gravitational potential energy

\Delta \PE_{\text{g}} $to be

$\Delta PE_g = mgh$,$ </div>

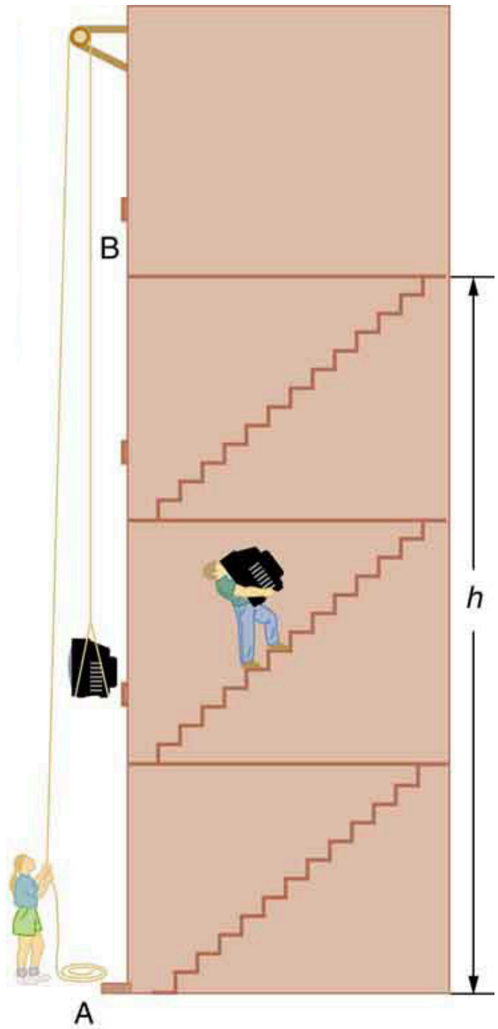
where, for simplicity, we denote the change in height by h rather than the usual Δh. Note that h is positive when the final height is greater than the initial height, and vice versa. For example, if a 0.500-kg mass hung from a cuckoo clock is raised 1.00 m, then its change in gravitational potential energy is

$$mgh = (0.500\text{kg})(9.80\text{m/s}^2)(1.00\text{m}) \quad mgh = 4.90\text{kg}\cdot\text{m}^2/\text{s}^2 = 4.90\text{J}.$$

Note that the units of gravitational potential energy turn out to be joules, the same as for work and other forms of energy. As the clock runs, the mass is lowered. We can think of the mass as gradually giving up its 4.90 J of gravitational potential energy, *without directly considering the force of gravity that does the work*.

Using Potential Energy to Simplify Calculations

The equation ΔPE_g $= mgh$ applies for any path that has a change in height of h, not just when the mass is lifted straight up. (See Figure 2.) It is much easier to calculate mgh (a simple multiplication) than it is to calculate the work done along a complicated path. The idea of gravitational potential energy has the double advantage that it is very broadly applicable and it makes calculations easier. From now on, we will consider that any change in vertical position h of a mass m is accompanied by a change in gravitational potential energy mgh , and we will avoid the equivalent but more difficult task of calculating work done by or against the gravitational force.



The change in gravitational potential energy \$(\Delta \PE_\text{g}$ \) between points A and B is independent of the path. \(\Delta \PE_\text{g} =mgh \) for any path between the two points. Gravity is one of a small class of forces where the work done by or against the force depends only on the starting and ending points, not on the path between them.

The Force to Stop Falling

A 60.0-kg person jumps onto the floor from a height of 3.00 m. If he lands stiffly (with his knee joints compressing by 0.500 cm), calculate the force on the knee joints.

Strategy

This person's energy is brought to zero in this situation by the work done on him by the floor as he stops. The initial PE_g is transformed into KE as he falls. The work done by the floor reduces this kinetic energy to zero.

Solution

The work done on the person by the floor as he stops is given by

$$W=Fd\cos\theta=-Fd,$$

with a minus sign because the displacement while stopping and the force from floor are in opposite directions $(\cos\theta=\cos 180^\circ=-1)$. The floor removes energy from the system, so it does negative work.

The kinetic energy the person has upon reaching the floor is the amount of potential energy lost by falling through height h:

$$\text{KE}=-\Delta\text{PE}_\text{g}=-mgh,$$

The distance d that the person's knees bend is much smaller than the height h of the fall, so the additional change in gravitational potential energy during the knee bend is ignored.

The work W done by the floor on the person stops the person and brings the person's kinetic energy to zero:

$$W = -\mathrm{KE} = mgh.$$

Combining this equation with the expression for W gives

$$-Fd = mgh.$$

Recalling that h is negative because the person fell *down*, the force on the knee joints is given by

$$F = -mghd = -(60.0\mathrm{kg})(9.80\mathrm{m/s}^2)(-3.00\mathrm{m})5.00\times10^{-3}\mathrm{m} = 3.53\times10^{5}N.$$

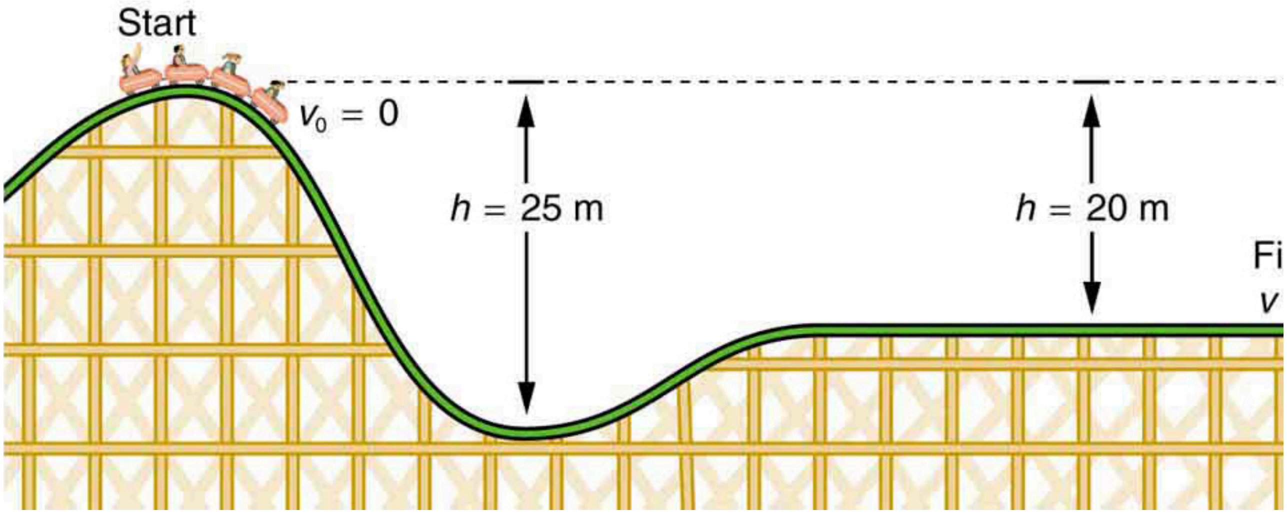
Discussion

Such a large force (500 times more than the person’s weight) over the short impact time is enough to break bones. A much better way to cushion the shock is by bending the legs or rolling on the ground, increasing the time over which the force acts. A bending motion of 0.5 m this way yields a force 100 times smaller than in the example. A kangaroo's hopping shows this method in action. The kangaroo is the only large animal to use hopping for locomotion, but the shock in hopping is cushioned by the bending of its hind legs in each jump.(See Figure 3.)

The work done by the ground upon the kangaroo reduces its kinetic energy to zero as it lands. However, by applying the force of the ground on the hind legs over a longer distance, the impact on the bones is reduced. (credit: Chris Samuel, Flickr)

Finding the Speed of a Roller Coaster from its Height

(a) What is the final speed of the roller coaster shown in Figure 3 if it starts from rest at the top of the 20.0 m hill and work done by frictional forces is negligible? (b) What is its final speed (again assuming negligible friction) if its initial speed is 5.00 m/s?



The speed of a roller coaster increases as gravity pulls it downhill and is greatest at its lowest point. Viewed in terms of energy, the roller-coaster-Earth system's gravitational potential energy is converted to kinetic energy. If work done by friction is negligible, all \(\Delta \PE_\text{g} \) is converted to \(\Delta \KE \) .

Strategy

The roller coaster loses potential energy as it goes downhill. We neglect friction, so that the remaining force exerted by the track is the normal force, which is perpendicular to the direction of motion and does no work. The net work on the roller coaster is then done by gravity alone. The *loss* of gravitational potential energy from moving *downward* through a distance h equals the *gain* in kinetic energy. This can be written in equation form as $-\Delta \mathrm{PE_g} = \Delta \mathrm{KE}$. Using the equations for $\mathrm{PE_g}$ and KE, we can solve for the final speed v, which is the desired quantity.

Solution for (a)

Here the initial kinetic energy is zero, so that $\Delta \mathrm{KE} = \frac{1}{2}mv^2$. The equation for change in potential energy states that $\Delta \mathrm{PE_g} = mgh$. Since h is negative in this case, we will rewrite this as $\Delta \mathrm{PE_g} = -mg|h|$ to show the minus sign clearly. Thus,

$$-\Delta \mathrm{PE_g} = \Delta \mathrm{KE}$$

becomes

$$mg|h| = \frac{1}{2}mv^2.$$

Solving for v, we find that mass cancels and that

$$v = \sqrt{2g|h|}.$$

Substituting known values,

$$v = \sqrt{2(9.80\,\mathrm{m/s^2})(20.0\,\mathrm{m})} \quad v = 19.6\,\mathrm{m/s}.$$

Solution for (b)

Again $-\Delta \mathrm{PE_g} = \Delta \mathrm{KE}$. In this case there is initial kinetic energy, so $\Delta \mathrm{KE} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$. Thus,

$$mg|h| = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2.$$

Rearranging gives

$$\frac{1}{2}mv^2 = mg|h| + \frac{1}{2}mv_0^2.$$

This means that the final kinetic energy is the sum of the initial kinetic energy and the gravitational potential energy. Mass again cancels, and

$$v = \sqrt{2g|h| + v_0^2}.$$

This equation is very similar to the kinematics equation $v = \sqrt{v_0^2 + 2ad}$, but it is more general—the kinematics equation is valid only for constant acceleration, whereas our equation above is valid for any path regardless of whether the object moves with a constant acceleration. Now, substituting known values gives

$$v = \sqrt{2(9.80\text{m/s}^2)(20.0\text{m})+(5.00\text{m/s})^2} \quad v = 20.4\text{m/s}.$$

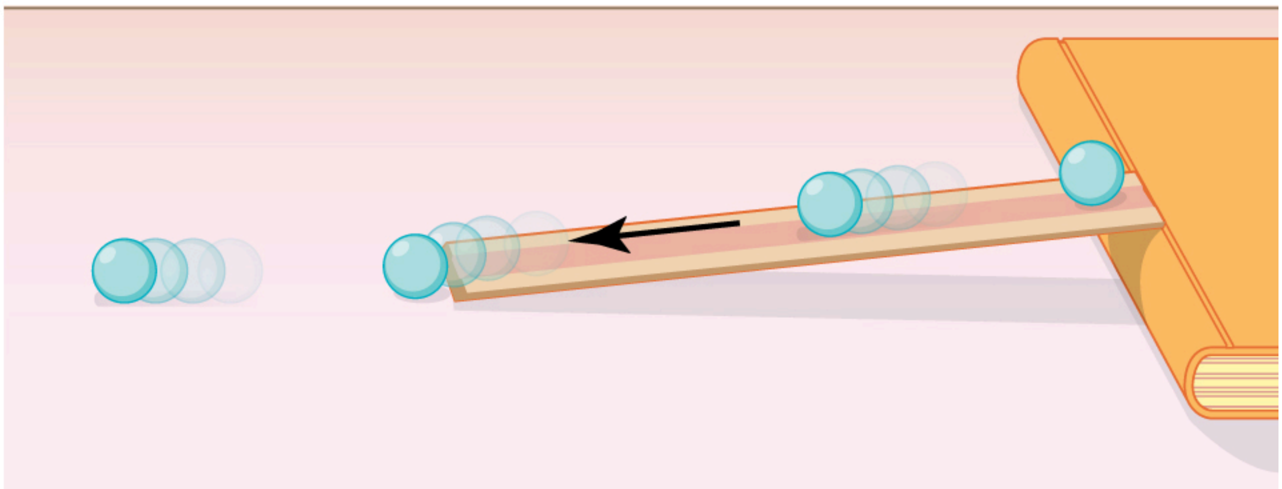
Discussion and Implications

First, note that mass cancels. This is quite consistent with observations made in Falling Objects that all objects fall at the same rate if friction is negligible. Second, only the speed of the roller coaster is considered; there is no information about its direction at any point. This reveals another general truth. When friction is negligible, the speed of a falling body depends only on its initial speed and height, and not on its mass or the path taken. For example, the roller coaster will have the same final speed whether it falls 20.0 m straight down or takes a more complicated path like the one in the figure. Third, and perhaps unexpectedly, the final speed in part (b) is greater than in part (a), but by far less than 5.00 m/s. Finally, note that speed can be found at *any* height along the way by simply using the appropriate value of h at the point of interest.

We have seen that work done by or against the gravitational force depends only on the starting and ending points, and not on the path between, allowing us to define the simplifying concept of gravitational potential energy. We can do the same thing for a few other forces, and we will see that this leads to a formal definition of the law of conservation of energy.

Making Connections: Take-Home Investigation—Converting Potential to Kinetic Energy

One can study the conversion of gravitational potential energy into kinetic energy in this experiment. On a smooth, level surface, use a ruler of the kind that has a groove running along its length and a book to make an incline (see Figure 5). Place a marble at the 10-cm position on the ruler and let it roll down the ruler. When it hits the level surface, measure the time it takes to roll one meter. Now place the marble at the 20-cm and the 30-cm positions and again measure the times it takes to roll 1 m on the level surface. Find the velocity of the marble on the level surface for all three positions. Plot velocity squared versus the distance traveled by the marble. What is the shape of each plot? If the shape is a straight line, the plot shows that the marble's kinetic energy at the bottom is proportional to its potential energy at the release point.



A marble rolls down a ruler, and its speed on the level surface is measured.

Section Summary

- Work done against gravity in lifting an object becomes potential energy of the object-Earth system.
- The change in gravitational potential energy, ΔPE_g, is $\Delta PE_g = mgh$, with h being the increase in height and g the acceleration due to gravity.
- The gravitational potential energy of an object near Earth's surface is due to its position in the mass-Earth system. Only differences in gravitational potential energy, ΔPE_g, have physical significance.
- As an object descends without friction, its gravitational potential energy changes into kinetic energy corresponding to increasing speed, so that $\Delta KE = -\Delta PE_g$.

Conceptual Questions

In Example 2, we calculated the final speed of a roller coaster that descended 20 m in height and had an initial speed of 5 m/s downhill. Suppose the roller coaster had had an initial speed of 5 m/s *uphill* instead, and it coasted uphill, stopped, and then rolled back down to a final point 20 m below the start. We would find in that case that its final speed is the same as its initial speed. Explain in terms of conservation of energy.

Does the work you do on a book when you lift it onto a shelf depend on the path taken? On the time taken? On the height of the shelf? On the mass of the book?

Problems & Exercises

A hydroelectric power facility (see Figure 6) converts the gravitational potential energy of water behind a dam to electric energy. (a) What is the gravitational potential energy relative to the generators of a lake of volume 50.0km^3 ($\text{mass}=5.00\times10^{13}\text{kg}$), given that the lake has an average height of 40.0 m above the generators? (b) Compare this with the energy stored in a 9-megaton fusion bomb.

Hydroelectric facility (credit: Denis Belevich, Wikimedia Commons)

Show Solution

Strategy

(a) Use the gravitational potential energy formula $\mathrm{PE}_g = mgh$ with the given mass and height. (b) Compare this to the energy of a 9-megaton fusion bomb, where 1 megaton TNT equivalent = 4.18×10^{15} J.

Solution

(a) Gravitational potential energy of the lake:

Given:

- Mass of water: $m = 5.00 \times 10^{13}$ kg
- Average height above generators: $h = 40.0$ m

$$\mathrm{PE}_g = mgh = (5.00 \times 10^{13}\ \mathrm{kg})(9.80\ \mathrm{m/s^2})(40.0\ \mathrm{m})$$
$$\mathrm{PE}_g = 1.96 \times 10^{16}\ \mathrm{J}$$

(b) Comparison with 9-megaton fusion bomb:

Energy in a 9-megaton bomb:

$$E_{bomb} = 9 \times 4.18 \times 10^{15}\ \mathrm{J} = 3.76 \times 10^{16}\ \mathrm{J}$$

Ratio:

$$\frac{\mathrm{PE}_g}{E_{bomb}} = \frac{1.96 \times 10^{16}}{3.76 \times 10^{16}} = 0.52$$

Discussion

The gravitational potential energy stored in this reservoir is enormous - about 2×10^{16} joules. This is approximately half the energy of a 9-megaton nuclear fusion bomb, which puts the scale of hydroelectric power into perspective. Unlike a bomb that releases its energy in an instant, hydroelectric facilities release this energy gradually over time, converting it to useful electrical power. A 50 km³ reservoir is quite large (for comparison, Lake Mead behind Hoover Dam holds about 35 km³), but the energy it stores demonstrates why hydroelectric power is such a significant energy source.

Answer

(a) The gravitational potential energy is 1.96×10^{16} J.

(b) The ratio is 0.52, meaning the lake stores about half the energy of a 9-megaton fusion bomb.

(a) How much gravitational potential energy (relative to the ground on which it is built) is stored in the Great Pyramid of Cheops, given that its mass is about $7 \times 10^9 \text{kg}$ and its center of mass is 36.5 m above the surrounding ground? (b) How does this energy compare with the daily food intake of a person?

Show Solution

Strategy

For part (a), we use the gravitational potential energy formula $\text{PE}\_g = mgh$, where m is the mass of the pyramid, $g = 9.80\text{m/s}^2$, and h is the height of the center of mass.

For part (b), we compare this energy to the typical daily food intake of about $1.2 \times 10^7 \text{J}$ (or 2400 kcal).

Solution for (a)

$$\text{PE}_g = mgh \quad \text{PE}_g = (7\times10^9\text{kg})(9.80\text{m/s}^2)(36.5\text{m}) \quad \text{PE}_g = 2.50\times10^{12}\text{J}\approx2.5\times10^{12}\text{J}$$

Solution for (b)

The ratio of the pyramid's potential energy to daily food intake is:

$$2.50\times10^{12}\text{J}1.2\times10^7\text{J}=2.08\times10^5\approx2.1\times10^5$$

Discussion

The gravitational potential energy stored in the Great Pyramid is approximately $2.5 \times 10^{12}\text{J}$, which is equivalent to about 210,000 days of food intake for one person, or roughly 575 years worth of food energy. This immense amount of potential energy was stored by doing work to lift millions of stone blocks to build the pyramid. The comparison with daily food intake helps illustrate the enormous amount of human labor that went into constructing this ancient monument over approximately 20 years.

Answer

(a) The gravitational potential energy of the Great Pyramid is 2.5×10^{12} J.

(b) This energy is about **210,000 times** the daily food intake of a person, equivalent to roughly 575 years of food energy.

Suppose a 350-g kookaburra (a large kingfisher bird) picks up a 75-g snake and raises it 2.5 m from the ground to a branch. (a) How much work did the bird do on the snake? (b) How much work did it do to raise its own center of mass to the branch?

Show Solution

Strategy

Work done against gravity equals the change in gravitational potential energy: $W = mgh$. Calculate separately for the snake and for the bird's own mass.

Solution

Given:

- Bird mass: $m_{bird} = 350 \text{ g} = 0.350 \text{ kg}$
- Snake mass: $m_{snake} = 75 \text{ g} = 0.075 \text{ kg}$
- Height raised: $h = 2.5 \text{ m}$

(a) Work done on the snake:

$$W_{snake}=m_{snake}gh=(0.075 \text{ kg})(9.80\text{m/s}^2)(2.5 \text{ m})=1.84 \text{ J}\approx1.8 \text{ J}$$

(b) Work done to raise the bird's center of mass:

$$W_{bird}=m_{bird}gh=(0.350\text{ kg})(9.80\text{m/s}^2)(2.5\text{ m})=8.58\text{ J}\approx 8.6\text{ J}$$

Discussion

The bird does significantly more work lifting itself (8.6 J) than lifting the snake (1.8 J), even though the snake represents extra payload. The total work done is 10.4 J. This illustrates that for flying animals, the energy cost of carrying prey is relatively small compared to the energy needed for their own flight. The kookaburra, famous for eating snakes, must be strong enough to lift its own body weight repeatedly, and carrying a snake that's about 21% of its body mass is a modest additional burden.

Answer

(a) The work done on the snake is **1.8 J**.

(b) The work done to raise the bird's center of mass is **8.6 J**.

In Example 2, we found that the speed of a roller coaster that had descended 20.0 m was only slightly greater when it had an initial speed of 5.00 m/s than when it started from rest. This implies that $\Delta\text{PE} \gg \text{KE}_\text{i}$. Confirm this statement by taking the ratio of ΔPE to KE_i. (Note that mass cancels.)

Show Solution

Strategy

We calculate the change in potential energy $\Delta\text{PE} = mg|h|$ and the initial kinetic energy $\text{KE}_\text{i} = \frac{1}{2}mv_0^2$, then find their ratio. The mass will cancel out.

Solution

The change in potential energy is:

$$\Delta\text{PE}=mg|h|=m(9.80\text{m/s}^2)(20.0\text{m})=m(196\text{ m}^2/\text{s}^2)$$

The initial kinetic energy is:

$$\text{KE}_\text{i}=\frac{1}{2}mv_0^2=\frac{1}{2}m(5.00\text{m/s})^2=m(12.5\text{ m}^2/\text{s}^2)$$

The ratio is:

$$\frac{\Delta\text{PE}}{\text{KE}_\text{i}}=\frac{m(196\text{ m}^2/\text{s}^2)}{m(12.5\text{ m}^2/\text{s}^2)}=\frac{196}{12.5}=15.7\approx 16$$

Discussion

The change in potential energy is approximately 16 times greater than the initial kinetic energy, confirming that $\Delta\text{PE} \gg \text{KE}\_\text{i}$. This explains why the final speed is only slightly greater when starting with an initial speed of 5.00 m/s compared to starting from rest—the gravitational potential energy converted to kinetic energy dominates over the initial kinetic energy. This result is important in many practical situations: when objects fall from significant heights, their initial velocity has little effect on the final velocity.

Answer

The ratio is **15.7** (approximately **16**), confirming that $\Delta\text{PE} \gg \text{KE}_\text{i}$.

A 100-g toy car is propelled by a compressed spring that starts it moving. The car follows the curved track in Figure 7. Show that the final speed of the toy car is 0.687 m/s if its initial speed is 2.00 m/s and it coasts up the frictionless slope, gaining 0.180 m in altitude.

A toy car moves up a sloped track. (credit: Leszek Leszczynski, Flickr)

Show Solution

Strategy

Use conservation of mechanical energy for a frictionless surface. The initial kinetic energy plus initial gravitational potential energy equals the final kinetic energy plus final gravitational potential energy. Since the car goes up, it gains gravitational potential energy and loses kinetic energy.

Solution

Given:

- Initial speed: $v_0 = 2.00 \text{m/s}$
- Height gained: $h = 0.180 \text{ m}$
- Mass: $m = 100 \text{ g} = 0.100 \text{ kg}$ (though mass cancels out)

Apply conservation of energy, taking the initial position as the reference level ($h_0 = 0$):

$$\frac{1}{2}mv_0^2 + mgh_0 = \frac{1}{2}mv_f^2 + mgh$$
$$\frac{1}{2}mv_0^2 + 0 = \frac{1}{2}mv_f^2 + mgh$$

Divide by m and solve for v_f:

$$\frac{1}{2}v_0^2 = \frac{1}{2}v_f^2 + gh$$
$$v_f^2 = v_0^2 - 2gh$$
$$v_f = \sqrt{v_0^2 - 2gh} = \sqrt{(2.00 \text{m/s})^2 - 2(9.80 \text{m/s}^2)(0.180 \text{m})}$$
$$v_f = \sqrt{4.00 - 3.528} = \sqrt{0.472} = 0.687 \text{m/s}$$

Discussion

The toy car slows from 2.00 m/s to 0.687 m/s as it climbs 0.180 m. Most of its initial kinetic energy (88%) is converted to gravitational potential energy at the top of the slope. This problem illustrates conservation of mechanical energy on a frictionless surface - the total mechanical energy (KE + PE) remains constant throughout the motion.

Answer

The final speed is $v_f = 0.687 \text{m/s}$, as required.

In a downhill ski race, surprisingly, little advantage is gained by getting a running start. (This is because the initial kinetic energy is small compared with the gain in gravitational potential energy on even small hills.) To demonstrate this, find the final speed and the time taken for a skier who skies 70.0 m along a $30°$ slope neglecting friction: (a) Starting from rest. (b) Starting with an initial speed of 2.50 m/s. (c) Does the answer surprise you? Discuss why it is still advantageous to get a running start in very competitive events.

Show Solution

Strategy

We use conservation of energy: $\mathrm{KE_i} + \mathrm{PE_i} = \mathrm{KE_f} + \mathrm{PE_f}$. The height descended is $h = d \sin\theta = (70.0\,\mathrm{m})\sin 30° = 35.0\,\mathrm{m}$. For the time, we use kinematics with constant acceleration $a = g\sin\theta$ down the slope.

Solution for (a)

Starting from rest ($v_0 = 0$):

$$mgh \quad = \quad 12mv^2 \quad v \quad = \quad \sqrt{2gh}=\sqrt{2(9.80\mathrm{m/s}^2)(35.0\mathrm{m})} \quad v \quad = \quad \sqrt{686\ \mathrm{m}^2/\mathrm{s}^2}=26.2\mathrm{m/s}$$

For time, using $v^2 = v_0^2 + 2ad$ with $a = g\sin 30° = 4.90\mathrm{m/s}^2$:

$$v=v_0+at \Rightarrow t=v-v_0a=26.2\mathrm{m/s}4.90\mathrm{m/s}^2=5.35\mathrm{s}$$

Solution for (b)

With initial speed $v_0 = 2.50\mathrm{m/s}$:

$$12mv_{20}+mgh \quad = \quad 12mv^2 \quad v \quad = \quad \sqrt{v_{20}+2gh} \quad v \quad = \quad \sqrt{(2.50\mathrm{m/s})^2+2(9.80\mathrm{m/s}^2)(35.0\mathrm{m})} \quad v \quad = \quad \sqrt{6.25+686}\ \mathrm{m/s}=\sqrt{692}$$

For time, using $v = v_0 + at$:

$$t=v-v_0a=26.3\mathrm{m/s}-2.50\mathrm{m/s}4.90\mathrm{m/s}^2=4.86\mathrm{s}$$

Discussion for (c)

The answer is somewhat surprising—the final speeds differ by only 0.1 m/s! The running start saves only about 0.5 seconds. However, in very competitive events, even a fraction of a second can mean the difference between winning and losing. Additionally, in actual skiing, friction and air resistance play important roles, and starting with higher speed may provide advantages in maintaining momentum through turns and rough sections of the course. The small difference in final speeds confirms the statement that $\Delta\mathrm{PE} \gg \mathrm{KE_i}$ for the descent.

Answer

(a) Starting from rest: final speed = **26.2 m/s**, time = **5.35 s**

(b) Starting with initial speed of 2.50 m/s: final speed = **26.3 m/s**, time = **4.86 s**

(c) The results are surprising because the running start provides only a 0.1 m/s advantage in final speed and saves just 0.5 seconds. However, this time difference can be crucial in competitive racing, and other factors like maintaining momentum through friction and turns also favor a running start.

Glossary

gravitational potential energy
: the energy an object has due to its position in a gravitational field

Conservative Forces and Potential Energy

- Define conservative force, potential energy, and mechanical energy.
- Explain the potential energy of a spring in terms of its compression when Hooke's law applies.
- Use the work-energy theorem to show how having only conservative forces implies conservation of mechanical energy.

Potential Energy and Conservative Forces

Work is done by a force, and some forces, such as weight, have special characteristics. A **conservative force** is one, like the gravitational force, for which work done by or against it depends only on the starting and ending points of a motion and not on the path taken. We can define a **potential energy** (PE)

for any conservative force, just as we did for the gravitational force. For example, when you wind up a toy, an egg timer, or an old-fashioned watch, you do work against its spring and store energy in it. (We treat these springs as ideal, in that we assume there is no friction and no production of thermal energy.) This stored energy is recoverable as work, and it is useful to think of it as potential energy contained in the spring. Indeed, the reason that the spring has this characteristic is that its force is *conservative*. That is, a conservative force results in stored or potential energy. Gravitational potential energy is one example, as is the energy stored in a spring. We will also see how conservative forces are related to the conservation of energy.

Potential Energy and Conservative Forces

Potential energy is the energy a system has due to position, shape, or configuration. It is stored energy that is completely recoverable.

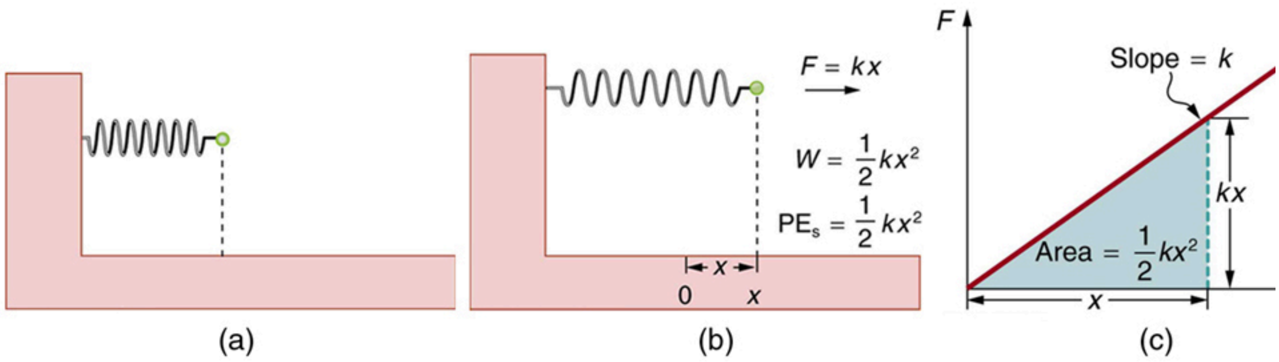
A conservative force is one for which work done by or against it depends only on the starting and ending points of a motion and not on the path taken.

We can define a potential energy (PE) for any conservative force. The work done against a conservative force to reach a final configuration depends on the configuration, not the path followed, and is the potential energy added.

Potential Energy of a Spring

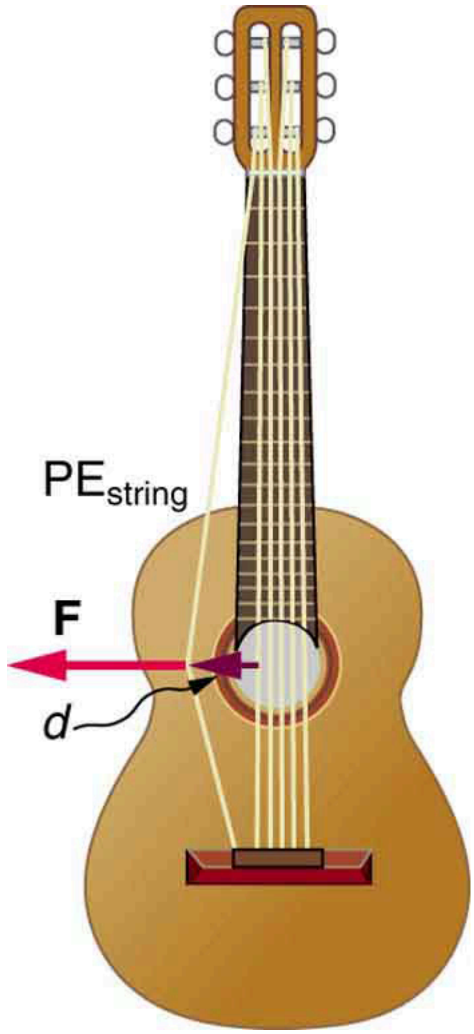
First, let us obtain an expression for the potential energy stored in a spring (PE_S). We calculate the work done to stretch or compress a spring that obeys Hooke's law. (Hooke's law was examined in Elasticity: Stress and Strain, and states that the magnitude of force F on the spring and the resulting deformation ΔL are proportional, $F = k\Delta L$.) (See Figure 1.) For our spring, we will replace ΔL (the amount of deformation produced by a force F) by the distance x that the spring is stretched or compressed along its length. So the force needed to stretch the spring has magnitude $F = kx$, where k is the spring's force constant. The force increases linearly from 0 at the start to kx in the fully stretched position. The average force is $kx/2$. Thus the work done in stretching or compressing the spring is $W_S = Fd = (kx2)x = 12kx^2$. Alternatively, we noted in Kinetic Energy and the Work-Energy Theorem that the area under a graph of F vs. x is the work done by the force. In Figure 1(c) we see that this area is also $12kx^2$. We therefore define the **potential energy of a spring**, PE_S, to be <div class="equation"> $\mathrm{PE}_S = 12kx^2$, </div>

where k is the spring's force constant and x is the displacement from its undeformed position. The potential energy represents the work done *on* the spring and the energy stored in it as a result of stretching or compressing it a distance x. The potential energy of the spring PE_S does not depend on the path taken; it depends only on the stretch or squeeze x in the final configuration.



(a) An undeformed spring has no \(\PE_{\text{s}} \) stored in it. (b) The force needed to stretch (or compress) the spring a distance \(x \) has a magnitude \(F = k x \) , and the work done to stretch (or compress) it is \(\frac{1}{2} k x^2 \) . Because the force is conservative, this work is stored as potential energy, \(\PE_{\text{s}} \), in the spring, and it can be fully recovered. (c) A graph of \(F \) vs. \(x \) has a slope of \(k \) , and the area under the graph is \(\frac{1}{2} k x^2 \). Thus the work done or potential energy stored is \(\frac{1}{2} k x^2 \).

The equation $\mathrm{PE}_S = 12kx^2$ has general validity beyond the special case for which it was derived. Potential energy can be stored in any elastic medium by deforming it. Indeed, the general definition of **potential energy** is energy due to position, shape, or configuration. For shape or position deformations, stored energy is $\mathrm{PE}_S = 12kx^2$, where k is the force constant of the particular system and x is its deformation. Another example is seen in Figure 1 for a guitar string.



Work is done to deform the guitar string, giving it potential energy. When released, the potential energy is converted to kinetic energy and back to potential as the string oscillates back and forth. A very small fraction is dissipated as sound energy, slowly removing energy from the string.

Conservation of Mechanical Energy

Let us now consider what form the work-energy theorem takes when only conservative forces are involved. This will lead us to the conservation of energy principle. The work-energy theorem states that the net work done by all forces acting on a system equals its change in kinetic energy. In equation form, this is

$$W_{\text{net}}=\frac{1}{2}mv^2-\frac{1}{2}mv_0^2=\Delta\text{KE}.$$

If only conservative forces act, then

$$W_{\text{net}}=W_{\text{c}},$$

where W_c is the total work done by all conservative forces. Thus,

$$W_{\text{c}}=\Delta\text{KE}.$$

Now, if the conservative force, such as the gravitational force or a spring force, does work, the system loses potential energy. That is, $W_c=-\Delta\text{PE}$. Therefore,

$$-\Delta\text{PE}=\Delta\text{KE}$$

or

$$\Delta\text{KE}+\Delta\text{PE}=0.$$

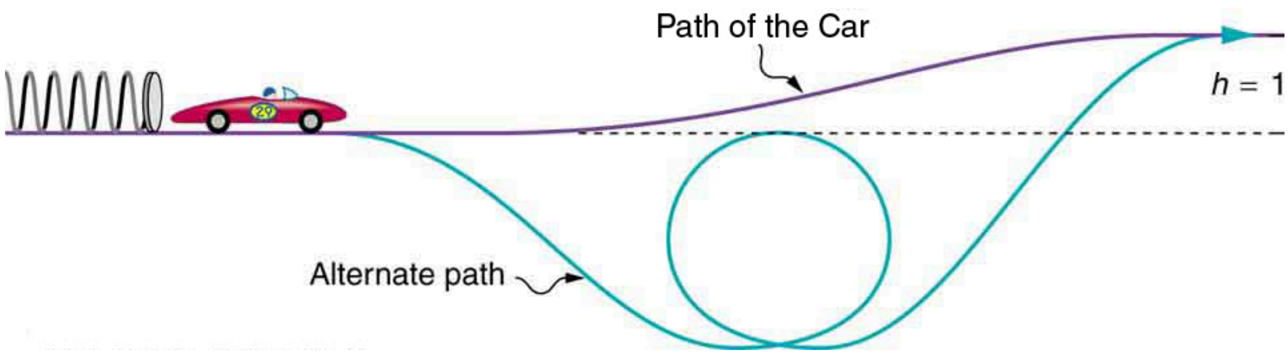
This equation means that the total kinetic and potential energy is constant for any process involving only conservative forces. That is,

$$\mathrm{KE}+\mathrm{PE}=\text{constant} \quad \text{or} \quad \mathrm{KE_i}+\mathrm{PE_i}=\mathrm{KE_f}+\mathrm{PE_f} \quad \text{(conservative forces only)},$$

where i and f denote initial and final values. This equation is a form of the work-energy theorem for conservative forces; it is known as the **conservation of mechanical energy** principle. Remember that this applies to the extent that all the forces are conservative, so that friction is negligible. The total kinetic plus potential energy of a system is defined to be its **mechanical energy**, $(\mathrm{KE}+\mathrm{PE})$. In a system that experiences only conservative forces, there is a potential energy associated with each force, and the energy only changes form between KE and the various types of PE , with the total energy remaining constant.

Using Conservation of Mechanical Energy to Calculate the Speed of a Toy Car

A 0.100-kg toy car is propelled by a compressed spring, as shown in Figure 3. The car follows a track that rises 0.180 m above the starting point. The spring is compressed 4.00 cm and has a force constant of 250.0 N/m. Assuming work done by friction to be negligible, find (a) how fast the car is going before it starts up the slope and (b) how fast it is going at the top of the slope.



A toy car is pushed by a compressed spring and coasts up a slope. Assuming negligible friction, the potential energy in the spring is first completely converted to kinetic energy, and then to a combination of kinetic and gravitational potential energy as the car rises. The details of the path are unimportant because all forces are conservative—the car would have the same final speed if it took the alternate path shown.

Strategy

The spring force and the gravitational force are conservative forces, so conservation of mechanical energy can be used. Thus,

$$\mathrm{KE_i}+\mathrm{PE_i}=\mathrm{KE_f}+\mathrm{PE_f}$$

or

$$\frac{1}{2}mv_i^2+mgh_i+\frac{1}{2}kx_i^2=\frac{1}{2}mv_f^2+mgh_f+\frac{1}{2}kx_f^2,$$

where h is the height (vertical position) and x is the compression of the spring. This general statement looks complex but becomes much simpler when we start considering specific situations. First, we must identify the initial and final conditions in a problem; then, we enter them into the last equation to solve for an unknown.

Solution for (a)

This part of the problem is limited to conditions just before the car is released and just after it leaves the spring. Take the initial height to be zero, so that both h_i and h_f are zero. Furthermore, the initial speed v_i is zero and the final compression of the spring x_f is zero, and so several terms in the conservation of mechanical energy equation are zero and it simplifies to

$$\frac{1}{2}kx_i^2=\frac{1}{2}mv_f^2.$$

In other words, the initial potential energy in the spring is converted completely to kinetic energy in the absence of friction. Solving for the final speed and entering known values yields

$$v_f = \sqrt{\frac{k}{m}}\,x_i \qquad v_f = \sqrt{\frac{250.0\ \text{N/m}}{0.100\ \text{kg}}}(0.0400\ \text{m}) \qquad v_f = 2.00\ \text{m/s}.$$

Solution for (b)

One method of finding the speed at the top of the slope is to consider conditions just before the car is released and just after it reaches the top of the slope, completely ignoring everything in between. Doing the same type of analysis to find which terms are zero, the conservation of mechanical energy becomes

$$\frac{1}{2}kx_\mathrm{i}^2=\frac{1}{2}mv_\mathrm{f}^2+mgh_\mathrm{f}.$$

This form of the equation means that the spring's initial potential energy is converted partly to gravitational potential energy and partly to kinetic energy. The final speed at the top of the slope will be less than at the bottom. Solving for v_f and substituting known values gives

$$v_f = \sqrt{\frac{kx_\mathrm{i}^2}{m}-2gh_\mathrm{f}} \qquad v_f = \sqrt{\left(\frac{250.0\ \text{N/m}}{0.100\ \text{kg}}\right)(0.0400\ \text{m})^2-2(9.80\ \text{m/s}^2)(0.180\ \text{m})} \qquad v_f = 0.687\ \text{m/s}.$$

Discussion

Another way to solve this problem is to realize that the car’s kinetic energy before it goes up the slope is converted partly to potential energy—that is, to take the final conditions in part (a) to be the initial conditions in part (b).

Note that, for conservative forces, we do not directly calculate the work they do; rather, we consider their effects through their corresponding potential energies, just as we did in this previous example. Note also that we do not consider details of the path taken—only the starting and ending points are important (as long as the path is not impossible). This assumption is usually a tremendous simplification, because the path may be complicated and forces may vary along the way.

Energy Skate Park

Learn about conservation of energy with a skater dude! Build tracks, ramps and jumps for the skater and view the kinetic energy, potential energy and friction as he moves. You can also take the skater to different planets or even space!

Energy Skate Park

Section Summary

- A conservative force is one for which work depends only on the starting and ending points of a motion, not on the path taken.
- We can define potential energy (PE) for any conservative force, just as we defined PE_g for the gravitational force.
- The potential energy of a spring is $\text{PE}_s = \frac{1}{2}kx^2$, where k is the spring’s force constant and x is the displacement from its undeformed position.
- Mechanical energy is defined to be $\text{KE} + \text{PE}$ for a conservative force.
- When only conservative forces act on and within a system, the total mechanical energy is constant. In equation form,

$$\text{KE}+\text{PE}=\text{constant} \qquad \text{or} \qquad \text{KE}_\text{i}+\text{PE}_\text{i}=\text{KE}_\text{f}+\text{PE}_\text{f}$$

where i and f denote initial and final values. This is known as the conservation of mechanical energy.

Conceptual Questions

What is a conservative force?

The force exerted by a diving board is conservative, provided the internal friction is negligible. Assuming friction is negligible, describe changes in the potential energy of a diving board as a swimmer dives from it, starting just before the swimmer steps on the board until just after her feet leave it.

Define mechanical energy. What is the relationship of mechanical energy to nonconservative forces? What happens to mechanical energy if only conservative forces act?

What is the relationship of potential energy to conservative force?

Problems & Exercises

A $5.00 \times 10^5\text{-kg}$ subway train is brought to a stop from a speed of 0.500 m/s in 0.400 m by a large spring bumper at the end of its track. What is the force constant k of the spring?

Show Solution

Strategy

The kinetic energy of the moving train is converted entirely into elastic potential energy stored in the spring when the train comes to rest. Use conservation of energy: $12mv^2 = 12kx^2$.

Solution

Given:

- Train mass: $m = 5.00 \times 10^5$ kg
- Initial speed: $v = 0.500\text{m/s}$
- Spring compression: $x = 0.400$ m

Apply conservation of energy (kinetic energy → spring potential energy):

$$12mv^2 = 12kx^2$$

Solve for the spring constant k:

$$k = mv^2x^2$$

Substitute the values:

$$k = (5.00 \times 10^5 \text{ kg})(0.500\text{m/s})^2(0.400 \text{ m})^2$$
$$k = (5.00 \times 10^5)(0.250)0.160 = 1.25 \times 10^5 0.160 = 7.81 \times 10^5 \text{ N/m}$$

Discussion

The spring constant of 7.81×10^5 N/m is quite large, which is necessary to stop such a massive train. To put this in perspective, a typical car suspension spring might have a spring constant of about 50,000 N/m, so this subway bumper spring is about 16 times stiffer. The relatively slow speed (0.500 m/s, about walking pace) and short stopping distance (0.400 m) require this very stiff spring. This design demonstrates how energy methods simplify calculations - we didn't need to analyze the forces at each instant, just the initial and final states.

Answer

The spring constant is $k = 7.81 \times 10^5$ N/m.

A pogo stick has a spring with a force constant of $2.50 \times 10^4\text{N/m}$, which can be compressed 12.0 cm. To what maximum height can a child jump on the stick using only the energy in the spring, if the child and stick have a total mass of 40.0 kg? Explicitly show how you follow the steps in the Problem-Solving Strategies for Energy.

Show Solution

Strategy

Following the Problem-Solving Strategies for Energy:

Step 1: The system consists of the child, pogo stick, and Earth. We are given the spring constant $k = 2.50 \times 10^4\text{N/m}$, compression $x = 12.0 \text{ cm} = 0.120\text{m}$, and total mass $m = 40.0\text{kg}$. We need to find the maximum height h.

Step 2: The forces are the spring force (conservative) and gravity (conservative), so we can use conservation of mechanical energy.

Step 3: Since all forces are conservative: $\text{KE}_\text{i} + \text{PE}_\text{i} = \text{KE}_\text{f} + \text{PE}_\text{f}$

Step 4: Not needed here since all forces are conservative.

Step 5: Simplify by noting that at both initial and final points, the child is momentarily at rest, so $\text{KE}_\text{i} = \text{KE}_\text{f} = 0$. Choose $h = 0$ at the initial compressed position, so $\text{PE}_{\text{g,i}} = 0$ and $\text{PE}_{\text{g,f}} = mgh$. The spring potential energy is $\text{PE}_{\text{s,i}} = 12kx^2$ initially and $\text{PE}_{\text{s,f}} = 0$ at maximum height.

Solution

The conservation of energy equation becomes:

$$12kx^2 + 0 = 0 + mgh$$

Solving for h:

$$h=\frac{kx^2}{2mg}$$

Substituting known values:

$$h = \frac{(2.50\times10^4\,\text{N/m})(0.120\,\text{m})^2}{2(40.0\,\text{kg})(9.80\,\text{m/s}^2)} \quad h = \frac{(2.50\times10^4)(0.0144)}{784}\,\text{m} \quad h = \frac{360}{784}\,\text{m}=0.459\,\text{m}$$

Step 6: Check reasonableness: A height of about 46 cm (roughly 1.5 feet) seems reasonable for a pogo stick jump using only the spring's stored energy.

Discussion

The child can jump to a maximum height of approximately 0.46 m (46 cm) above the compressed position using only the energy stored in the spring. This is a reasonable result - about 18 inches is a typical jump for a pogo stick when starting from a compressed position. The problem demonstrates the complete problem-solving strategy for energy: identifying the system, recognizing conservative forces, applying conservation of energy, simplifying by choosing a convenient reference point, solving algebraically, and checking the answer's reasonableness.

Answer

The maximum height the child can jump is **0.459 m** (or **45.9 cm**) above the compressed position.

Glossary

conservative force
: a force that does the same work for any given initial and final configuration, regardless of the path followed

potential energy
: energy due to position, shape, or configuration

potential energy of a spring
: the stored energy of a spring as a function of its displacement; when Hooke's law applies, it is given by the expression $\frac{1}{2}kx^2$ where x is the distance the spring is compressed or extended and k is the spring constant

conservation of mechanical energy
: the rule that the sum of the kinetic energies and potential energies remains constant if only conservative forces act on and within a system

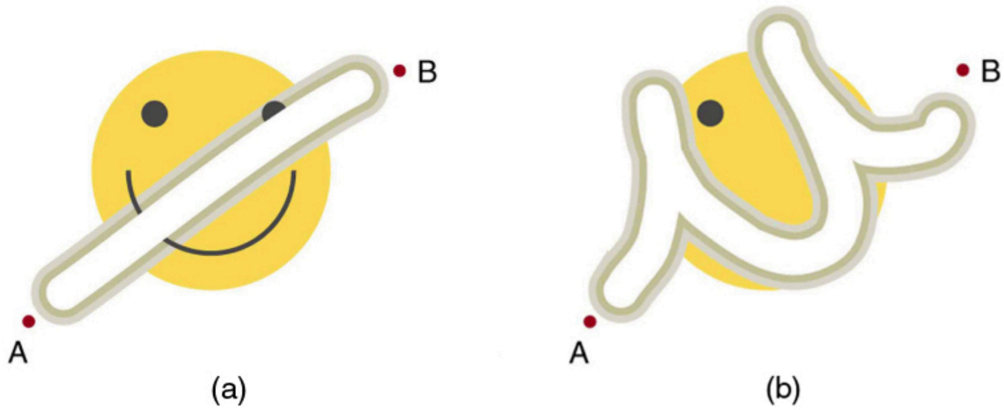
mechanical energy
: the sum of kinetic energy and potential energy

Nonconservative Forces

- Define nonconservative forces and explain how they affect mechanical energy.
- Show how the principle of conservation of energy can be applied by treating the conservative forces in terms of their potential energies and any nonconservative forces in terms of the work they do.

Nonconservative Forces and Friction

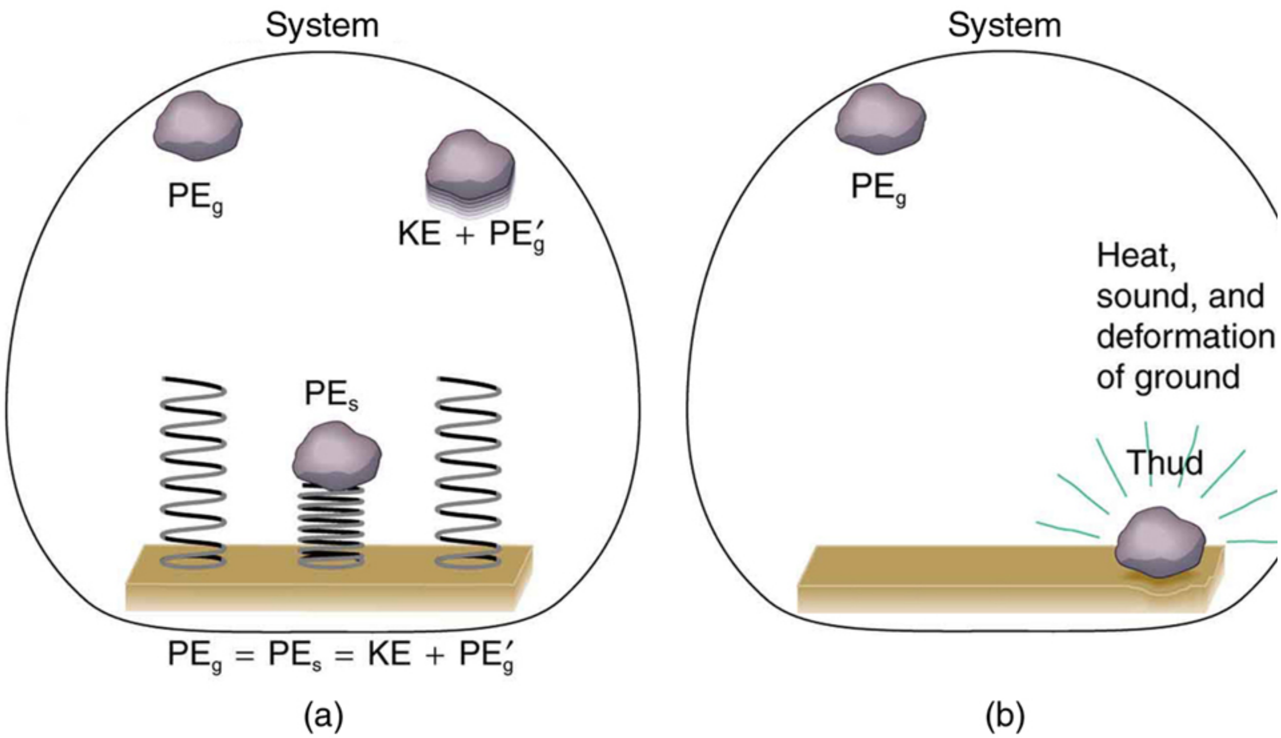
Forces are either conservative or nonconservative. Conservative forces were discussed in Conservative Forces and Potential Energy . A **nonconservative force** is one for which work depends on the path taken. Friction is a good example of a nonconservative force. As illustrated in Figure 1, work done against friction depends on the length of the path between the starting and ending points. Because of this dependence on path, there is no potential energy associated with nonconservative forces. An important characteristic is that the work done by a nonconservative force *adds or removes mechanical energy from a system*. **Friction**, for example, creates **thermal energy** that dissipates, removing energy from the system. Furthermore, even if the thermal energy is retained or captured, it cannot be fully converted back to work, so it is lost or not recoverable in that sense as well.



The amount of the happy face erased depends on the path taken by the eraser between points A and B, as does the work done against friction. Less work is done and less of the face is erased for the path in (a) than for the path in (b). The force here is friction, and most of the work goes into thermal energy that subsequently leaves the system (the happy face plus the eraser). The energy expended cannot be fully recovered.

How Nonconservative Forces Affect Mechanical Energy

Mechanical energy *may* not be conserved when nonconservative forces act. For example, when a car is brought to a stop by friction on level ground, it loses kinetic energy, which is dissipated as thermal energy, reducing its mechanical energy. Figure 2 compares the effects of conservative and nonconservative forces. We often choose to understand simpler systems such as that described in Figure 2(a) first before studying more complicated systems as in Figure 2(b).



Comparison of the effects of conservative and nonconservative forces on the mechanical energy of a system. (a) A system with only conservative forces. When a rock is dropped onto a spring, its mechanical energy remains constant (neglecting air resistance) because the force in the spring is conservative. The spring can propel the rock back to its original height, where it once again has only potential energy due to gravity. (b) A system with nonconservative forces. When the same rock is dropped onto the ground, it is stopped by nonconservative forces that dissipate its mechanical energy as thermal energy, sound, and surface distortion. The rock has lost mechanical energy.

How the Work-Energy Theorem Applies

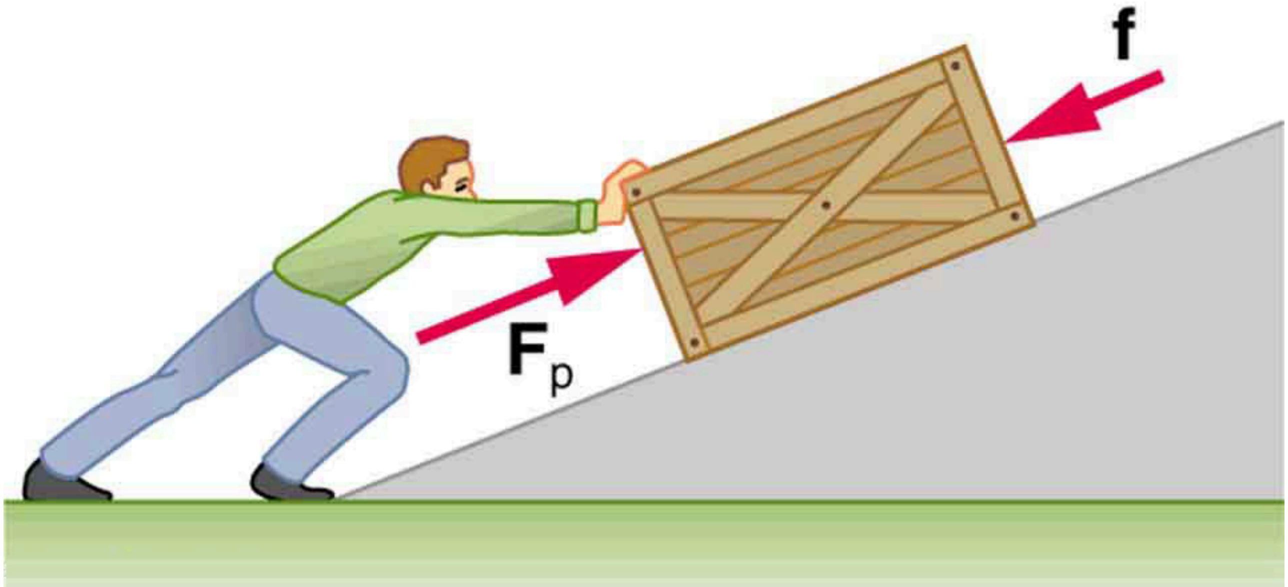
Now let us consider what form the work-energy theorem takes when both conservative and nonconservative forces act. We will see that the work done by nonconservative forces equals the change in the mechanical energy of a system. As noted in Kinetic Energy and the Work-Energy Theorem , the work-energy theorem states that the net work on a system equals the change in its kinetic energy, or W_net $=\Delta\mathrm{KE}$. The net work is the sum of the work by nonconservative forces plus the work by conservative forces. That is,

$$W_{\mathrm{net}}=W_{\mathrm{nc}}+W_{\mathrm{c}},$$

so that

$$W_{\mathrm{nc}}+W_{c}=\Delta\mathrm{KE},$$

where W_{nc} is the total work done by all nonconservative forces and W_{c} is the total work done by all conservative forces.



A person pushes a crate up a ramp, doing work on the crate. Friction and gravitational force (not shown) also do work on the crate; both forces oppose the person's push. As the crate is pushed up the ramp, it gains mechanical energy, implying that the work done by the person is greater than the work done by friction.

Consider Figure 3, in which a person pushes a crate up a ramp and is opposed by friction. As in the previous section, we note that work done by a conservative force comes from a loss of gravitational potential energy, so that $W_c = -\Delta PE$. Substituting this equation into the previous one and solving for W_{nc} gives

$$W_{nc} = \Delta KE + \Delta PE.$$

This equation means that the total mechanical energy $(KE + PE)$

changes by exactly the amount of work done by nonconservative forces. In Figure 3, this is the work done by the person minus the work done by friction. So even if energy is not conserved for the system of interest (such as the crate), we know that an equal amount of work was done to cause the change in total mechanical energy.

We rearrange $W_{nc} = \Delta KE + \Delta PE$ to obtain

$$KE_i + PE_i + W_{nc} = KE_f + PE_f.$$

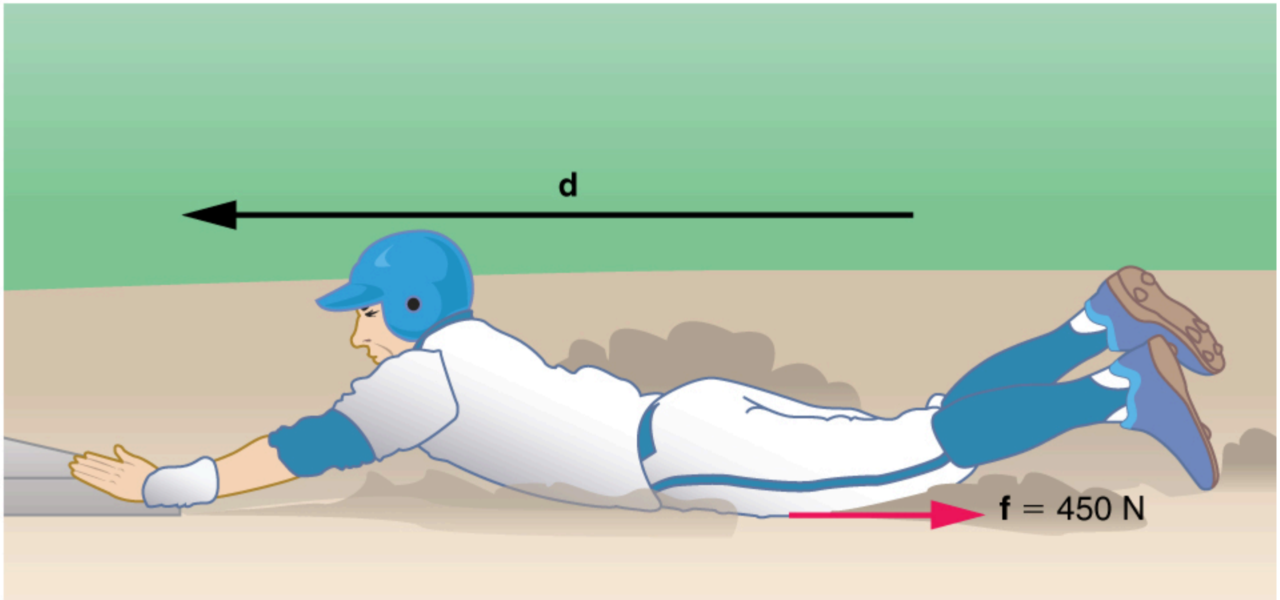
This means that the amount of work done by nonconservative forces adds to the mechanical energy of a system. If W_{nc} is positive, then mechanical energy is increased, such as when the person pushes the crate up the ramp in Figure 3. If W_{nc} is negative, then mechanical energy is decreased, such as when the rock hits the ground in Figure 2(b). If W_{nc} is zero, then mechanical energy is conserved, and nonconservative forces are balanced. For example, when you push a lawn mower at constant speed on level ground, your work done is removed by the work of friction, and the mower has a constant energy.

Applying Energy Conservation with Nonconservative Forces

When no change in potential energy occurs, applying $KE_i + PE_i + W_{nc} = KE_f + PE_f$ amounts to applying the work-energy theorem by setting the change in kinetic energy to be equal to the net work done on the system, which in the most general case includes both conservative and nonconservative forces. But when seeking instead to find a change in total mechanical energy in situations that involve changes in both potential and kinetic energy, the previous equation $KE_i + PE_i + W_{nc} = KE_f + PE_f$ says that you can start by finding the change in mechanical energy that would have resulted from just the conservative forces, including the potential energy changes, and add to it the work done, with the proper sign, by any nonconservative forces involved.

Calculating Distance Traveled: How Far a Baseball Player Slides

Consider the situation shown in Figure 4, where a baseball player slides to a stop on level ground. Using energy considerations, calculate the distance the 65.0-kg baseball player slides, given that his initial speed is 6.00 m/s and the force of friction against him is a constant 450 N.



The baseball player slides to a stop in a distance d . In the process, friction removes the player’s kinetic energy by doing an amount of work fd equal to the initial kinetic energy.

Strategy

Friction stops the player by converting his kinetic energy into other forms, including thermal energy. In terms of the work-energy theorem, the work done by friction, which is negative, is added to the initial kinetic energy to reduce it to zero. The work done by friction is negative, because $\vec{\mathbf{f}}$ is in the opposite direction of the motion (that is, $\theta = 180^\circ$, and so $\cos\theta = -1$). Thus $W_{\text{nc}} = -Fd$. The equation simplifies to

$$\frac{1}{2}mv_i^2 - fd = 0$$

or

$$fd = \frac{1}{2}mv_i^2.$$

This equation can now be solved for the distance d.

Solution

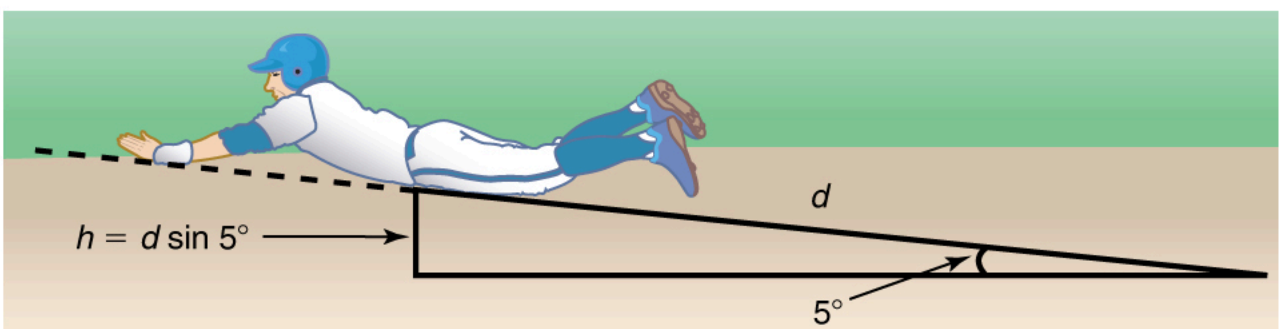
Solving the previous equation for d and substituting known values yields

$$d = \frac{mv_i^2}{2f} \quad d = \frac{(65.0\text{kg})(6.00\text{m/s})^2}{(2)(450\text{N})} \quad d = 2.60\text{m}.$$

Discussion

The most important point of this example is that the amount of nonconservative work equals the change in mechanical energy. For example, you must work harder to stop a truck, with its large mechanical energy, than to stop a mosquito.
Calculating Distance Traveled: Sliding Up an Incline

Suppose that the player from the previous example is running up a hill having a 5.00° incline upward with a surface similar to that in the baseball stadium (see Figure 5). The player slides with the same initial speed, and the frictional force is still 450 N. Determine how far he slides.



The same baseball player slides to a stop on a 5.00 degrees slope.

Strategy

In this case, the work done by the nonconservative friction force on the player reduces the mechanical energy he has from his kinetic energy at zero height, to the final mechanical energy he has by moving through distance d to reach height h along the hill, with $h = d\sin 5.00^\circ$. This is expressed by the equation

$$\mathrm{KE_i} + \mathrm{PE_i} + W_{\mathrm{nc}} = \mathrm{KE_f} + \mathrm{PE_f}.$$

Solution

The work done by friction is again $W_{\mathrm{nc}} = -fd$; initially the potential energy is $\mathrm{PE}_i = mg \cdot 0 = 0$ and the kinetic energy is $\mathrm{KE}_i = \frac{1}{2}mv_i^2$; the final energy contributions are $\mathrm{KE}_f = 0$ for the kinetic energy and $\mathrm{PE}_f = mgh = mgd\sin\theta$ for the potential energy.

Substituting these values gives

$$\frac{1}{2}mv_i^2 + 0 + (-fd) = 0 + mgd\sin\theta.$$

Solve this for d to obtain

$$\begin{aligned} d &= \frac{\frac{1}{2}mv_i^2}{f + mg\sin\theta} \\ d &= \frac{(0.5)(65.0\,\mathrm{kg})(6.00\,\mathrm{m/s})^2}{450\,\mathrm{N} + (65.0\,\mathrm{kg})(9.80\,\mathrm{m/s^2})\sin(5.00^\circ)} \\ d &= 2.31\,\mathrm{m}. \end{aligned}$$

Discussion

As might have been expected, the player slides a shorter distance by sliding uphill. Note that the problem could also have been solved in terms of the forces directly and the work energy theorem, instead of using the potential energy. This method would have required combining the normal force and force of gravity vectors, which no longer cancel each other because they point in different directions, and friction, to find the net force. You could then use the net force and the net work to find the distance d that reduces the kinetic energy to zero. By applying conservation of energy and using the potential energy instead, we need only consider the gravitational potential energy mgh

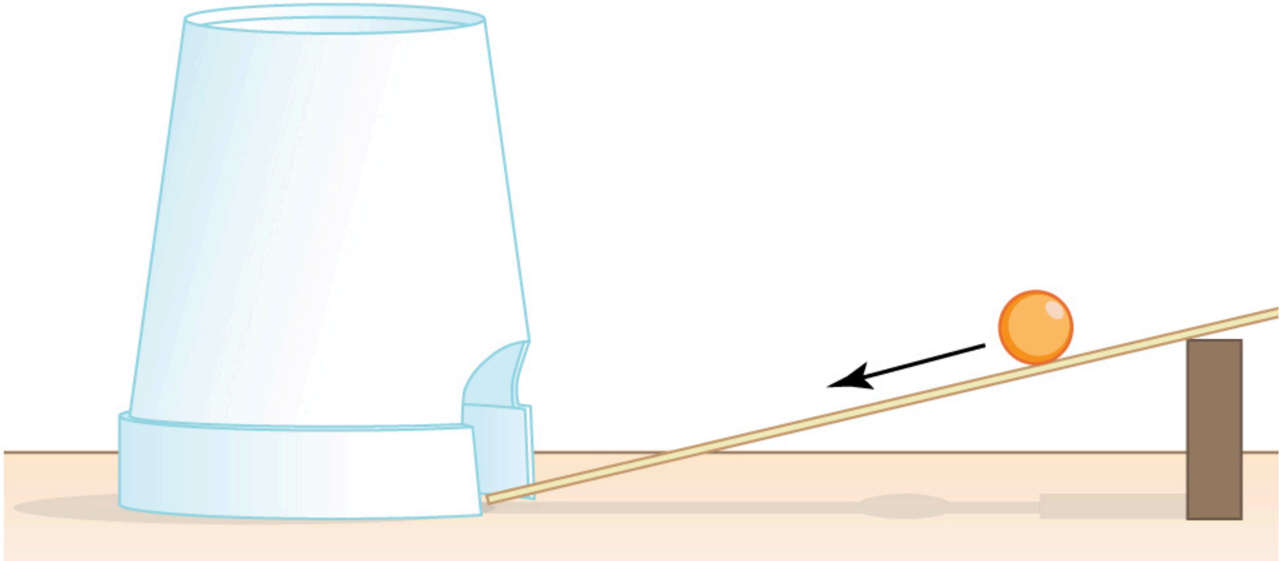
, without combining and resolving force vectors. This simplifies the solution considerably.

Making Connections: Take-Home Investigation—Determining Friction from the Stopping Distance

This experiment involves the conversion of gravitational potential energy into thermal energy. Use the ruler, book, and marble from Take-Home Investigation—Converting Potential to Kinetic Energy. In addition, you will need a foam cup with a small hole in the side, as shown in Figure 6. From the 10-cm position on the ruler, let the marble roll into the cup positioned at the bottom of the ruler. Measure the distance d the cup moves before stopping. What forces caused it to stop? What happened to the kinetic energy of the marble at the bottom of the ruler? Next, place the marble at the 20-cm and the 30-cm positions and again measure the distance the cup moves after the marble enters it. Plot the distance the cup moves versus the initial marble position on the ruler. Is this relationship linear?

With some simple assumptions, you can use these data to find the coefficient of kinetic friction μ_k of the cup on the table. The force of friction f on the cup is $\mu_k N$, where the normal force N is just the weight of the cup plus the marble. The normal force and force of gravity do no work because they are perpendicular to the displacement of the cup, which moves horizontally. The work done by friction is fd. You will need the mass of the marble as well to calculate its initial kinetic energy.

It is interesting to do the above experiment also with a steel marble (or ball bearing). Releasing it from the same positions on the ruler as you did with the glass marble, is the velocity of this steel marble the same as the velocity of the marble at the bottom of the ruler? Is the distance the cup moves proportional to the mass of the steel and glass marbles?



Rolling a marble down a ruler into a foam cup.

The Ramp

Explore forces, energy and work as you push household objects up and down a ramp. Lower and raise the ramp to see how the angle of inclination affects the parallel forces acting on the file cabinet. Graphs show forces, energy and work.

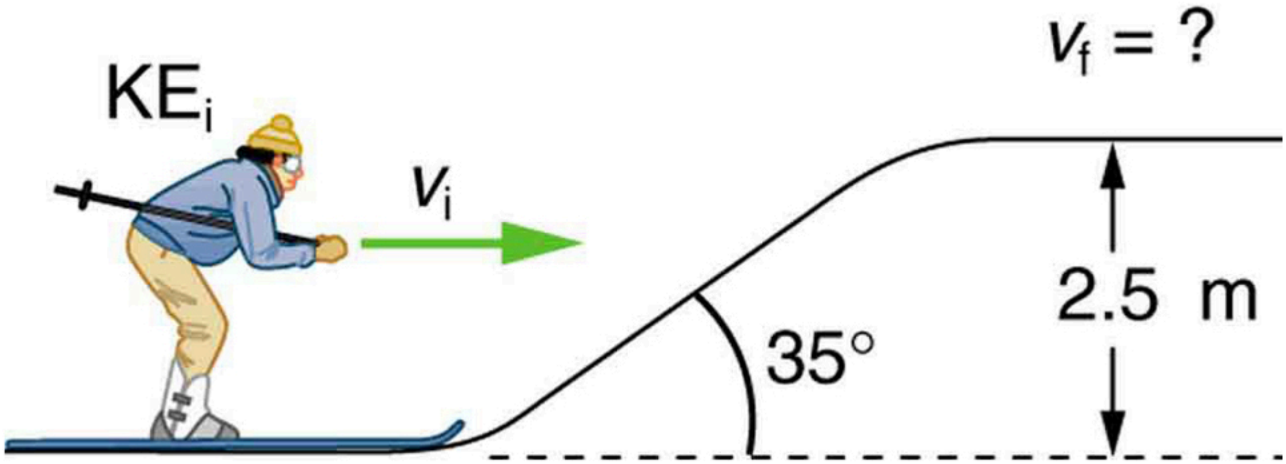
The Ramp

Section Summary

- A nonconservative force is one for which work depends on the path.
- Friction is an example of a nonconservative force that changes mechanical energy into thermal energy.
- Work W_{nc} done by a nonconservative force changes the mechanical energy of a system. In equation form, $W_{nc} = \Delta KE + \Delta PE$ or, equivalently, $KE_i + PE_i + W_{nc} = KE_f + PE_f$.
- When both conservative and nonconservative forces act, energy conservation can be applied and used to calculate motion in terms of the known potential energies of the conservative forces and the work done by nonconservative forces, instead of finding the net work from the net force, or having to directly apply Newton’s laws.

Problems & Exercises

A 60.0-kg skier with an initial speed of 12.0 m/s coasts up a 2.50-m-high rise as shown in Figure 8. Find her final speed at the top, given that the coefficient of friction between her skis and the snow is 0.0800. (Hint: Find the distance traveled up the incline assuming a straight-line path as shown in the figure.)



The skier's initial kinetic energy is partially used in coasting to the top of a rise.

Show Solution

Strategy

Use the work-energy theorem including friction. The initial kinetic energy is converted to gravitational potential energy plus thermal energy (work done against friction). First find the distance traveled along the incline using trigonometry, then apply energy conservation.

Solution

Given:

- Mass: $m = 60.0\text{ kg}$
- Initial speed: $v_i = 12.0\text{m/s}$
- Height rise: $h = 2.50\text{ m}$
- Coefficient of friction: $\mu_k = 0.0800$
- Slope angle: $\theta = 35°$ (from the figure)

Step 1: Find the distance traveled along the incline

$$d = \frac{h}{\sin\theta} = \frac{2.50\text{ m}}{\sin(35°)} = \frac{2.50}{0.574} = 4.36\text{ m}$$

Step 2: Calculate the friction force

The normal force on an incline: $N = mg\cos\theta$

Friction force:

$$f_k = \mu_k N = \mu_k mg\cos\theta = (0.0800)(60.0)(9.80)\cos(35°) = (0.0800)(60.0)(9.80)(0.819) = 38.5\text{ N}$$

Step 3: Calculate work done by friction

$$W_f = -f_k d = -(38.5\text{ N})(4.36\text{ m}) = -168\text{ J}$$

Step 4: Apply the work-energy theorem

$$\frac{1}{2}mv_i^2 + W_f = \frac{1}{2}mv_f^2 + mgh$$
$$\frac{1}{2}mv_f^2 = \frac{1}{2}mv_i^2 + W_f - mgh$$

Calculate each term:

- Initial KE: $\frac{1}{2}(60.0)(12.0)^2 = 4320\text{ J}$
- Work by friction: -168 J
- Potential energy gained: $(60.0)(9.80)(2.50) = 1470\text{ J}$

$$\frac{1}{2}mv_f^2 = 4320 - 168 - 1470 = 2682\text{ J}$$
$$v_f = \sqrt{\frac{2(2682)}{60.0}} = \sqrt{89.4} = 9.46\text{m/s}$$

Discussion

The skier loses speed from 12.0 m/s to 9.46 m/s while coasting up the hill, which represents about a 21% reduction in speed. However, examining the energy distribution provides deeper insight: the initial kinetic energy of 4320 J is split between final kinetic energy (2682 J, or 62%), gravitational potential energy (1470 J, or 34%), and thermal energy from friction (168 J, or 4%).

The friction accounts for only about 4% of the energy loss, with most of the speed reduction coming from the gain in height. This low friction is characteristic of well-waxed skis on snow and explains why skiing is such an efficient sport. The coefficient of friction of 0.0800 is quite realistic for this scenario—typical values for waxed skis on snow range from 0.04 to 0.10 depending on temperature and snow conditions.

Notice that even though the skier retains 62% of the kinetic energy, the speed only decreases by 21%. This demonstrates that kinetic energy depends on the square of velocity: $\mathrm{KE} \propto v^2$. Small changes in speed result in much larger fractional changes in energy, which is why high-speed impacts are so much more dangerous than low-speed ones.

Answer

The skier’s final speed at the top is **9.46 m/s**.

(a) How high a hill can a car coast up (engine disengaged) if work done by friction is negligible and its initial speed is 110 km/h? (b) If, in actuality, a 750-kg car with an initial speed of 110 km/h is observed to coast up a hill to a height 22.0 m above its starting point, how much thermal energy was generated by friction? (c) What is the average force of friction if the hill has a slope $2.5°$ above the horizontal?

Show Solution

Strategy

For part (a), with no friction, all kinetic energy converts to gravitational potential energy: $\frac{1}{2}mv^2 = mgh$.

For part (b), the difference between the initial kinetic energy and final potential energy equals the thermal energy generated.

For part (c), we use the work-energy theorem including friction over the distance traveled along the slope.

Solution for (a)

First, convert speed: $v = 110 \text{ km/h} = 110 \times \frac{1000}{3600} \text{m/s} = 30.6 \text{m/s}$

Using energy conservation:

$$\frac{1}{2}mv^2 = mgh \quad h = \frac{v^2}{2g} = \frac{(30.6 \text{m/s})^2}{2(9.80 \text{m/s}^2)} \quad h = \frac{936.36}{19.6} \text{m} = 47.8 \text{m}$$

Solution for (b)

The initial kinetic energy is:

$$\mathrm{KE}_i = \frac{1}{2}(750 \text{kg})(30.6 \text{m/s})^2 = 3.51 \times 10^5 \text{J}$$

The final potential energy is:

$$\mathrm{PE}_f = mgh = (750 \text{kg})(9.80 \text{m/s}^2)(22.0 \text{m}) = 1.62 \times 10^5 \text{J}$$

The thermal energy generated by friction is:

$$E_{\text{thermal}} = \mathrm{KE}_i - \mathrm{PE}_f = 3.51 \times 10^5 \text{J} - 1.62 \times 10^5 \text{J} = 1.89 \times 10^5 \text{J} \approx 1.9 \times 10^5 \text{J}$$

Solution for (c)

The distance along the slope is:

$$d = \frac{h}{\sin\theta} = \frac{22.0 \text{m}}{\sin 2.5°} = \frac{22.0}{0.0436} \text{m} = 504 \text{m}$$

The work done by friction equals the thermal energy:

$$W_f = fd \quad f = \frac{E_{\text{thermal}}}{d} = \frac{1.89 \times 10^5 \text{J}}{504 \text{m}} \quad f = 375 \text{N} \approx 3.8 \times 10^2 \text{N}$$

Discussion

Part (a) reveals that without friction, the car could coast up to 47.8 m—roughly the height of a 15-story building. This is a surprisingly large height, demonstrating the significant amount of energy contained in a moving car. At 110 km/h (about 68 mph), typical highway speed, the car possesses 3.51×10^5 J of kinetic energy.

In part (b), we see that friction dissipates 1.9×10^5 J, which is 54% of the car’s initial kinetic energy. This means friction removes more than half the car’s energy, reducing the maximum coasting height from 47.8 m to only 22.0 m—less than half the frictionless height. This substantial energy loss illustrates why cars cannot coast very far uphill with the engine off, and why fuel efficiency is so important in hilly terrain.

Part (c) shows the friction force is about 380 N (approximately 85 pounds). This might seem modest for a 750-kg car, but it acts over a long distance of 504 m. The shallow $2.5°$ slope means the car travels a horizontal distance of nearly 500 m while climbing 22 m vertically. The work done by friction, $W_f = fd = (380\text{ N})(504\text{ m}) = 1.9 \times 10^5\text{ J}$, accumulates to a substantial amount over this distance.

Comparing parts (a) and (b) demonstrates that friction is a major factor in real-world motion. Engineers must account for these losses when designing vehicles, which is why modern cars use low-friction tires, aerodynamic shapes, and efficient drivetrains to minimize energy waste.

Answer

(a) Without friction, the car could coast to a height of **47.8 m**.

(b) The thermal energy generated by friction is $\mathbf{1.9 \times 10^5\ J}$.

(c) The average force of friction is $\mathbf{3.8 \times 10^2\ N}$ (or 380 N).

Glossary

nonconservative force
: a force whose work depends on the path followed between the given initial and final configurations

friction
: the force between surfaces that opposes one sliding on the other; friction changes mechanical energy into thermal energy

Conservation of Energy

- Explain the law of the conservation of energy.
- Describe some of the many forms of energy.
- Define efficiency of an energy conversion process as the fraction left as useful energy or work, rather than being transformed, for example, into thermal energy.

Law of Conservation of Energy

Energy, as we have noted, is conserved, making it one of the most important physical quantities in nature. The **law of conservation of energy** can be stated as follows:

Total energy is constant in any process. It may change in form or be transferred from one system to another, but the total remains the same.

We have explored some forms of energy and some ways it can be transferred from one system to another. This exploration led to the definition of two major types of energy—mechanical energy $(\mathrm{KE}+\mathrm{PE})$ and energy transferred via work done by nonconservative forces (W_nc). But energy takes *many* other forms, manifesting itself in *many* different ways, and we need to be able to deal with all of these before we can write an equation for the above general statement of the conservation of energy.

Other Forms of Energy than Mechanical Energy

At this point, we deal with all other forms of energy by lumping them into a single group called other energy (OE). Then we can state the conservation of energy in equation form as

$$\mathrm{KE_i}+\mathrm{PE_i}+W_\mathrm{nc}+\mathrm{OE_i}=\mathrm{KE_f}+\mathrm{PE_f}+\mathrm{OE_f}.$$

All types of energy and work can be included in this very general statement of conservation of energy. Kinetic energy is KE, work done by a conservative force is represented by PE, work done by nonconservative forces is W_nc, and all other energies are included as OE. This equation applies to all previous examples; in those situations OE was constant, and so it subtracted out and was not directly considered.

Making Connections: Usefulness of the Energy Conservation Principle

The fact that energy is conserved and has many forms makes it very important. You will find that energy is discussed in many contexts, because it is involved in all processes. It will also become apparent that many situations are best understood in terms of energy and that problems are often most easily conceptualized and solved by considering energy.

When does OE play a role? One example occurs when a person eats. Food is oxidized with the release of carbon dioxide, water, and energy. Some of this chemical energy is converted to kinetic energy when the person moves, to potential energy when the person changes altitude, and to thermal energy (another form of OE).

Some of the Many Forms of Energy

What are some other forms of energy? You can probably name a number of forms of energy not yet discussed. Many of these will be covered in later chapters, but let us detail a few here. **Electrical energy** is a common form that is converted to many other forms and does work in a wide range of practical situations. Fuels, such as gasoline and food, carry **chemical energy** that can be transferred to a system through oxidation. Chemical fuel can also produce electrical

energy, such as in batteries. Batteries can in turn produce light, which is a very pure form of energy. Most energy sources on Earth are in fact stored energy from the energy we receive from the Sun. We sometimes refer to this as **radiant energy**, or electromagnetic radiation, which includes visible light, infrared, and ultraviolet radiation. **Nuclear energy** comes from processes that convert measurable amounts of mass into energy. Nuclear energy is transformed into the energy of sunlight, into electrical energy in power plants, and into the energy of the heat transfer and blast in weapons. Atoms and molecules inside all objects are in random motion. This internal mechanical energy from the random motions is called **thermal energy**, because it is related to the temperature of the object. These and all other forms of energy can be converted into one another and can do work.

Table 1 gives the amount of energy stored, used, or released from various objects and in various phenomena. The range of energies and the variety of types and situations is impressive.

Problem-Solving Strategies for Energy

You will find the following problem-solving strategies useful whenever you deal with energy. The strategies help in organizing and reinforcing energy concepts. In fact, they are used in the examples presented in this chapter. The familiar general problem-solving strategies presented earlier—involving identifying physical principles, knowns, and unknowns, checking units, and so on—continue to be relevant here.

Step 1. Determine the system of interest and identify what information is given and what quantity is to be calculated. A sketch will help.

Step 2. Examine all the forces involved and determine whether you know or are given the potential energy from the work done by the forces. Then use step 3 or step 4.

Step 3. If you know the potential energies for the forces that enter into the problem, then forces are all conservative, and you can apply conservation of mechanical energy simply in terms of potential and kinetic energy. The equation expressing conservation of energy is

$$\mathrm{KE_i+PE_i=KE_f+PE_f}.$$

Step 4. If you know the potential energy for only some of the forces, possibly because some of them are nonconservative and do not have a potential energy, or if there are other energies that are not easily treated in terms of force and work, then the conservation of energy law in its most general form must be used.

$$\mathrm{KE_i+PE_i}+W_{\mathrm{nc}}+\mathrm{OE_i=KE_f+PE_f+OE_f}.$$

In most problems, one or more of the terms is zero, simplifying its solution. Do not calculate W_c, the work done by conservative forces; it is already incorporated in the PE terms.

Step 5. You have already identified the types of work and energy involved (in step 2). Before solving for the unknown, *eliminate terms wherever possible* to simplify the algebra. For example, choose $h=0$ at either the initial or final point, so that PE_g is zero there. Then solve for the unknown in the customary manner.

Step 6. *Check the answer to see if it is reasonable*. Once you have solved a problem, reexamine the forms of work and energy to see if you have set up the conservation of energy equation correctly. For example, work done against friction should be negative, potential energy at the bottom of a hill should be less than that at the top, and so on. Also check to see that the numerical value obtained is reasonable. For example, the final speed of a skateboarder who coasts down a 3-m-high ramp could reasonably be 20 km/h, but *not* 80 km/h.

Transformation of Energy

The transformation of energy from one form into others is happening all the time. The chemical energy in food is converted into thermal energy through metabolism; light energy is converted into chemical energy through photosynthesis. In a larger example, the chemical energy contained in coal is converted into thermal energy as it burns to turn water into steam in a boiler. This thermal energy in the steam in turn is converted to mechanical energy as it spins a turbine, which is connected to a generator to produce electrical energy. (In all of these examples, not all of the initial energy is converted into the forms mentioned. This important point is discussed later in this section.)

Another example of energy conversion occurs in a solar cell. Sunlight impinging on a solar cell (see Figure 1) produces electricity, which in turn can be used to run an electric motor. Energy is converted from the primary source of solar energy into electrical energy and then into mechanical energy.

Solar energy is converted into electrical energy by solar cells, which is used to run a motor in this solar-power aircraft. (credit: NASA)

Table 1:Energy of Various Objects and Phenomena

Object/phenomenon	Energy in joules
Big Bang	10^{68}
Energy released in a supernova	10^{44}
Fusion of all the hydrogen in Earth's oceans	10^{34}
Annual world energy use	4×10^{20}
Large fusion bomb (9 megaton)	3.8×10^{16}
1 kg hydrogen (fusion to helium)	6.4×10^{14}
1 kg uranium (nuclear fission)	8.0×10^{13}
Hiroshima-size fission bomb (10 kiloton)	4.2×10^{13}
90 000-ton aircraft carrier at 30 knots	1.1×10^{10}
1 barrel crude oil	5.9×10^{9}
1 ton TNT	4.2×10^{9}
1 gallon of gasoline	1.2×10^{8}
Daily home electricity use (developed countries)	7×10^{7}
Daily adult food intake (recommended)	1.2×10^{7}
1000-kg car at 90 km/h	3.1×10^{5}
1 g fat (9.3 kcal)	3.9×10^{4}
ATP hydrolysis reaction	3.2×10^{4}
1 g carbohydrate (4.1 kcal)	1.7×10^{4}
1 g protein (4.1 kcal)	1.7×10^{4}
Tennis ball at 100 km/h	22
Mosquito (10^{-2}g at 0.5 m/s)	1.3×10^{-6}
Single electron in a TV tube beam	4.0×10^{-15}
Energy to break one DNA strand	10^{-19}

Efficiency

Even though energy is conserved in an energy conversion process, the output of *useful energy* or work will be less than the energy input. The **efficiency** Eff of an energy conversion process is defined as

$$\text{Efficiency}(\text{Eff}) = \frac{\text{useful energy or work output}}{\text{total energy input}} = \frac{W_{\text{out}}}{E_{\text{in}}}.$$

Table 2 lists some efficiencies of mechanical devices and human activities. In a coal-fired power plant, for example, about 40% of the chemical energy in the coal becomes useful electrical energy. The other 60% transforms into other (perhaps less useful) energy forms, such as thermal energy, which is then released to the environment through combustion gases and cooling towers.

Table 2: Efficiency of the Human Body and Mechanical Devices (representative values)

Activity/device	Efficiency (%)
Cycling and climbing	20
Swimming, surface	2
Swimming, submerged	4
Shoveling	3
Weightlifting	9
Steam engine	17
Gasoline engine	30
Diesel engine	35
Nuclear power plant	35
Coal power plant	42
Electric motor	98
Compact fluorescent light	20
Gas heater (residential)	90

Activity/device	Efficiency (%)
Solar cell	10

Masses and Springs

A realistic mass and spring laboratory. Hang masses from springs and adjust the spring stiffness and damping. You can even slow time. Transport the lab to different planets. A chart shows the kinetic, potential, and thermal energies for each spring.

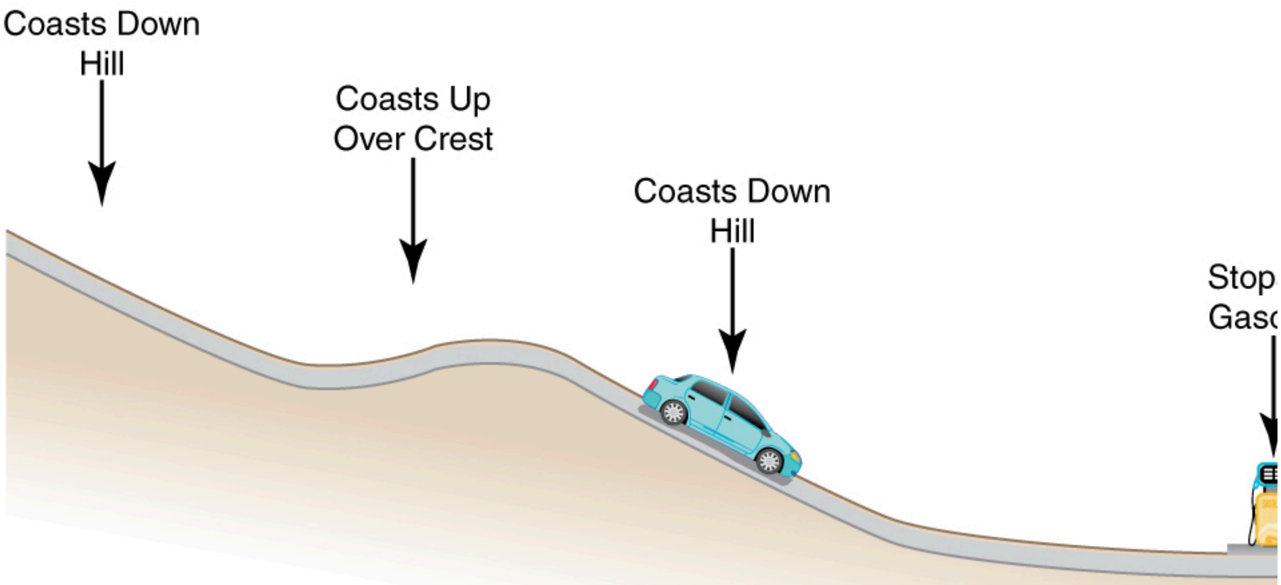
Masses and Springs

Section Summary

- The law of conservation of energy states that the total energy is constant in any process. Energy may change in form or be transferred from one system to another, but the total remains the same.
- When all forms of energy are considered, conservation of energy is written in equation form as $\mathrm{KE}_i + \mathrm{PE}_i + W_{\mathrm{nc}} + \mathrm{OE}_i = \mathrm{KE}_f + \mathrm{PE}_f + \mathrm{OE}_f$, where OE is all **other forms of energy** besides mechanical energy.
- Commonly encountered forms of energy include electric energy, chemical energy, radiant energy, nuclear energy, and thermal energy.
- Energy is often utilized to do work, but it is not possible to convert all the energy of a system to work.
- The efficiency Eff of a machine or human is defined to be $\mathrm{Eff} = \frac{W_{\mathrm{out}}}{E_{\mathrm{in}}}$, where W_{out} is useful work output and E_{in} is the energy consumed.

Conceptual Questions

Consider the following scenario. A car for which friction is *not* negligible accelerates from rest down a hill, running out of gasoline after a short distance. The driver lets the car coast farther down the hill, then up and over a small crest. He then coasts down that hill into a gas station, where he brakes to a stop and fills the tank with gasoline. Identify the forms of energy the car has, and how they are changed and transferred in this series of events. (See Figure 3.)



A car experiencing non-negligible friction coasts down a hill, over a small crest, then downhill again, and comes to a stop at a gas station.

Describe the energy transfers and transformations for a javelin, starting from the point at which an athlete picks up the javelin and ending when the javelin is stuck into the ground after being thrown.

Do devices with efficiencies of less than one violate the law of conservation of energy? Explain.

List four different forms or types of energy. Give one example of a conversion from each of these forms to another form.

List the energy conversions that occur when riding a bicycle.

Problems & Exercises

Using values from Table 1, how many DNA molecules could be broken by the energy carried by a single electron in the beam of an old-fashioned TV tube? (These electrons were not dangerous in themselves, but they did create dangerous x rays. Later model tube TVs had shielding that absorbed x rays before they escaped and exposed viewers.)

Show Solution

Strategy

From Table 1, the energy of a single electron in a TV tube beam is 4.0×10^{-15} J, and the energy to break one DNA strand is 10^{-19} J. We divide the electron's energy by the energy per DNA strand.

Solution

$$\text{Number of DNA molecules} = \frac{\text{Energy of electron}}{\text{Energy per DNA strand}} = \frac{4.0\times10^{-15}\,\text{J}}{10^{-19}\,\text{J}} = 4.0\times10^{4}\ \text{molecules}$$

Discussion

A single electron in an old TV tube beam carries enough energy to break approximately 40,000 DNA strands. This illustrates why radiation damage at the molecular level can be so significant—a single high-energy particle can cause extensive damage. However, these electrons themselves don't penetrate the glass screen; the danger came from the x-rays they produced when they struck the screen, which is why later TV models included protective shielding.

Answer

A single electron can break approximately **40,000 DNA molecules**.

Using energy considerations and assuming negligible air resistance, show that a rock thrown from a bridge 20.0 m above water with an initial speed of 15.0 m/s strikes the water with a speed of 24.8 m/s independent of the direction thrown.

Show Solution

Strategy

Use conservation of mechanical energy. Since air resistance is negligible, only conservative forces act (gravity), so the total mechanical energy is conserved: $\text{KE}_i + \text{PE}_i = \text{KE}_f + \text{PE}_f$. The initial kinetic energy depends only on the magnitude of velocity, not its direction, and the change in potential energy depends only on the vertical height change.

Solution

Starting with conservation of energy:

$$\text{KE}_i+\text{PE}_i=\text{KE}_f+\text{PE}_f$$
$$\frac{1}{2}mv_0^2+mgh=\frac{1}{2}mv_f^2+0$$

where we set the water level as the zero potential energy reference ($h_f=0$). Canceling mass and solving for v_f:

$$\frac{1}{2}v_0^2+gh=\frac{1}{2}v_f^2$$
$$v_f=\sqrt{v_0^2+2gh}$$

Substituting values:

$$v_f=\sqrt{(15.0\text{ m/s})^2+2(9.80\text{ m/s}^2)(20.0\text{ m})}$$
$$v_f=\sqrt{225\text{ m}^2/\text{s}^2+392\text{ m}^2/\text{s}^2}=\sqrt{617\text{ m}^2/\text{s}^2}=24.8\text{ m/s}$$

Discussion

The final speed is independent of the direction the rock is thrown because the formula $v_f=\sqrt{v_0^2+2gh}$ depends only on the magnitude of the initial velocity, not its direction. Whether thrown horizontally, upward at an angle, or downward, the same initial speed and height change produce the same final speed. This is a powerful demonstration of energy conservation—the path taken doesn't matter, only the initial and final states.

Answer

The rock strikes the water with a speed of **24.8 m/s**, independent of the direction thrown.

If the energy in fusion bombs were used to supply the energy needs of the world, how many of the 9-megaton variety would be needed for a year's supply of energy (using data from Table 1)? This is not as far-fetched as it may sound—there are thousands of nuclear bombs, and their energy can be trapped in underground explosions and converted to electricity, as natural geothermal energy is.

Show Solution

Strategy

From Table 1, the annual world energy use is approximately $4\times10^{20}\text{J}$ and a 9-megaton fusion bomb releases $3.8\times10^{16}\text{J}$ of energy. We divide the annual energy need by the energy per bomb.

Solution

$$\text{Number of bombs}=\frac{\text{Annual energy use}}{\text{Energy per bomb}}=\frac{4\times10^{20}\text{J}}{3.8\times10^{16}\text{J}}=1.05\times10^4\approx10{,}000\text{ bombs}$$

Discussion

Approximately 10,000 nine-megaton fusion bombs would be needed to supply the world's annual energy needs. While this seems like a large number, it illustrates the enormous energy content of nuclear fusion. However, using nuclear weapons for energy production is impractical and dangerous, which is why controlled fusion research aims to harness this energy safely.

Answer

Approximately **10,000 nine-megaton fusion bombs** would be needed to supply the world's annual energy needs.

(a) Use of hydrogen fusion to supply energy is a dream that may be realized in the next century. Fusion would be a relatively clean and almost limitless supply of energy, as can be seen from Table 1. To illustrate this, calculate how many years the present energy needs of the world could be supplied by one millionth of the oceans' hydrogen fusion energy. (b) How does this time compare with historically significant events, such as the duration of stable economic systems?

Show Solution

Strategy

From Table 1, the fusion of all hydrogen in Earth's oceans would release 10^{34} J of energy, and the annual world energy use is 4×10^{20} J. For part (a), divide one millionth of the ocean's fusion energy by the annual energy consumption. For part (b), compare this timescale to historical events.

Solution

(a) First, calculate one millionth of the ocean's hydrogen fusion energy:

$$E_{\text{available}}=\frac{10^{34}\text{ J}}{10^6}=10^{28}\text{ J}$$

Now divide by annual world energy use:

$$\text{Number of years} = \frac{E_{\text{available}}}{E_{\text{annual}}} = \frac{10^{28}\ \text{J}}{4 \times 10^{20}\ \text{J/year}}$$

$$\text{Number of years} = 2.5 \times 10^{7}\ \text{years} = 25\ \text{million years}$$

(b) This timescale of 25 million years is extraordinarily long compared to human history. Recorded human civilization spans only about 5,000 years. Even stable economic systems typically last only centuries, not millennia. This time period is comparable to geological time scales—for instance, it's roughly the time since the late Oligocene epoch when many modern mammal groups emerged.

Discussion

The calculation reveals the almost incomprehensible magnitude of energy potentially available from fusion. Even using just one millionth of the ocean's hydrogen would power human civilization for 25 million years at current consumption rates. This is about 5,000 times longer than recorded human history and far longer than any economic or political system has ever lasted. This illustrates why controlled fusion is called the "holy grail" of energy production—it would essentially solve humanity's energy problems for geological timescales.

Answer

(a) One millionth of the ocean's hydrogen fusion energy could supply the world's energy needs for approximately **25 million years**.

(b) This timescale is vastly longer than any human historical period—about 5,000 times the length of recorded civilization and far exceeding the duration of any stable economic system.

Glossary

law of conservation of energy
: the general law that total energy is constant in any process; energy may change in form or be transferred from one system to another, but the total remains the same

electrical energy
: the energy carried by a flow of charge

chemical energy
: the energy in a substance stored in the bonds between atoms and molecules that can be released in a chemical reaction

radiant energy
: the energy carried by electromagnetic waves

nuclear energy
: energy released by changes within atomic nuclei, such as the fusion of two light nuclei or the fission of a heavy nucleus

thermal energy
: the energy within an object due to the random motion of its atoms and molecules that accounts for the object's temperature

efficiency
: a measure of the effectiveness of the input of energy to do work; useful energy or work divided by the total input of energy

Power

- Calculate power by calculating changes in energy over time.
- Examine power consumption and calculations of the cost of energy consumed.

What is Power?

Power—the word conjures up many images: a professional football player muscling aside his opponent, a dragster roaring away from the starting line, a volcano blowing its lava into the atmosphere, or a rocket blasting off, as in Figure 1.

This powerful rocket on the Space Shuttle Endeavor did work and consumed energy at a very high rate. (credit: NASA)

These images of power have in common the rapid performance of work, consistent with the scientific definition of **power** (P) as the rate at which work is done.

Power

Power is the rate at which work is done.

$$P=\frac{W}{t}$$

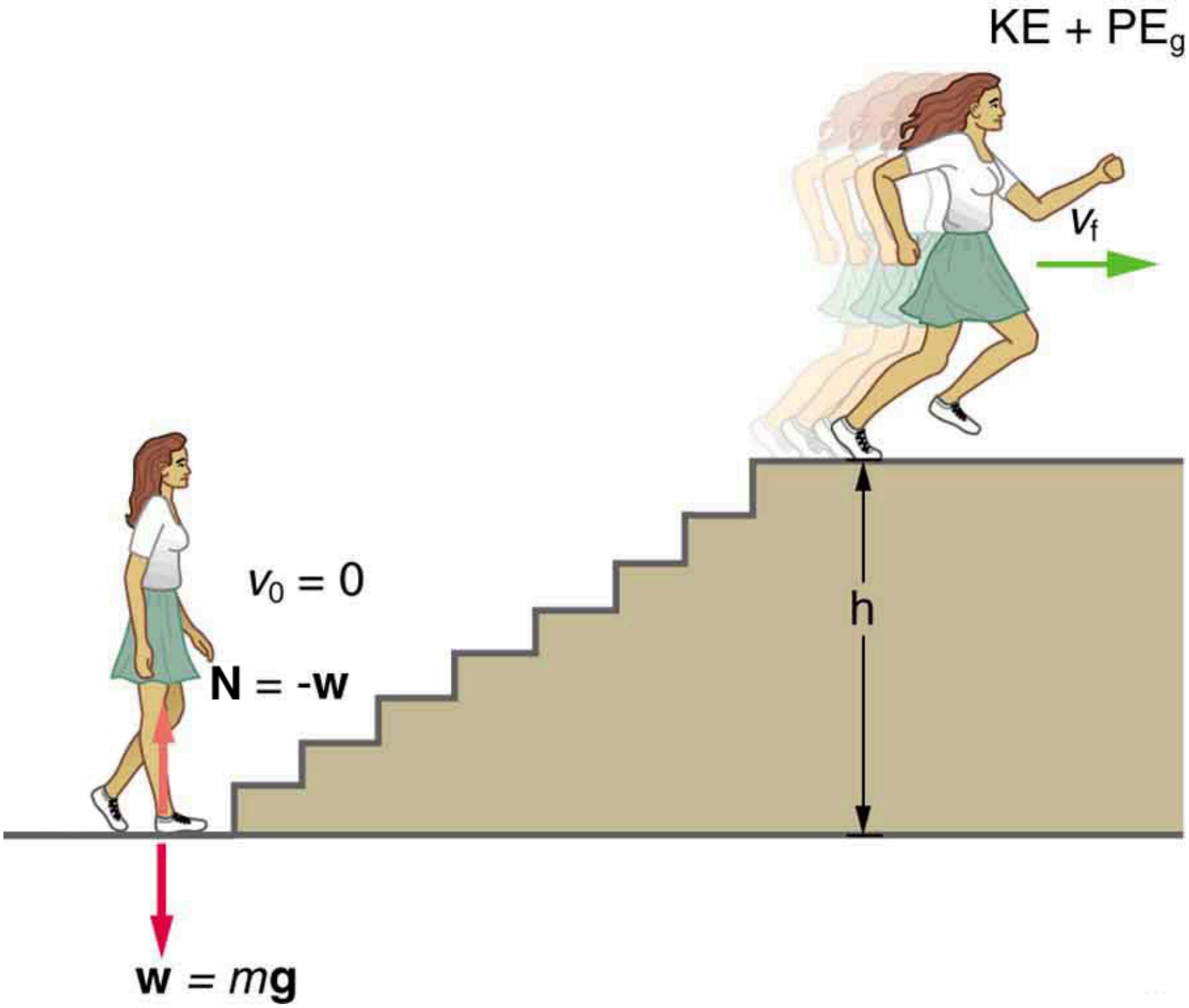
The SI unit for power is the **watt** (W), where 1 watt equals 1 joule/second $$ \left(1 \text{W}=1 \text{J/s}\right).

Because work is energy transfer, power is also the rate at which energy is expended. A 60-W light bulb, for example, expends 60 J of energy per second. Great power means a large amount of work or energy developed in a short time. For example, when a powerful car accelerates rapidly, it does a large amount of work and consumes a large amount of fuel in a short time.

Calculating Power from Energy

Calculating the Power to Climb Stairs

What is the power output for a 60.0-kg woman who runs up a 3.00 m high flight of stairs in 3.50 s, starting from rest but having a final speed of 2.00 m/s? (See Figure 2.)



When this woman runs upstairs starting from rest, she converts the chemical energy originally from food into kinetic energy and gravitational potential energy. Her power output depends on how fast she does this.

Strategy and Concept

The work going into mechanical energy is $W = \text{KE} + \text{PE}$. At the bottom of the stairs, we take both KE and PE_g as initially zero; thus, $W = \text{KE}_f + \text{PE}_g = \frac{1}{2}mv_f^2 + mgh$, where h is the vertical height of the stairs. Because all terms are given, we can calculate W and then divide it by time to get power.

Solution

Substituting the expression for W into the definition of power given in the previous equation, $P = W/t$ yields

$$P = \frac{W}{t} = \frac{\frac{1}{2}mv_f^2 + mgh}{t}.$$

Entering known values yields

$$P = \frac{0.5(60.0\text{kg})(2.00\text{m/s})^2 + (60.0\text{kg})(9.80\text{m/s}^2)(3.00\text{m})}{3.50\text{s}} \quad P = \frac{120\text{J} + 1764\text{J}}{3.50\text{s}} \quad P = 538\text{W}.$$

Discussion

The woman does 1764 J of work to move up the stairs compared with only 120 J to increase her kinetic energy; thus, most of her power output is required for climbing rather than accelerating.

It is impressive that this woman's useful power output is slightly less than 1 **horsepower** $(1\text{hp} = 746\text{W})$! People can generate more than a horsepower with their leg muscles for short periods of time by rapidly converting available blood sugar and oxygen into work output. (A horse can put out 1 hp for hours on end.) Once oxygen is depleted, power output decreases and the person begins to breathe rapidly to obtain oxygen to metabolize more food—this is known as the *aerobic* stage of exercise. If the woman climbed the stairs slowly, then her power output would be much less, although the amount of work done would be the same.

Making Connections: Take-Home Investigation—Measure Your Power Rating

Determine your own power rating by measuring the time it takes you to climb a flight of stairs. We will ignore the gain in kinetic energy, as the above example showed that it was a small portion of the energy gain. Don't expect that your output will be more than about 0.5 hp.

Examples of Power

Examples of power are limited only by the imagination, because there are as many types as there are forms of work and energy. (See Table 1 for some examples.) Sunlight reaching Earth's surface carries a maximum power of about 1.3 kilowatts per square meter (kW/m^2). A tiny fraction of this is retained by Earth over the long term. Our consumption rate of fossil fuels is far greater than the rate at which they are stored, so it is inevitable that they will be depleted. Power implies that energy is transferred, perhaps changing form. It is never possible to change one form completely into another without losing some of it as thermal energy. For example, a 60-W incandescent bulb converts only 5 W of electrical power to light, with 55 W dissipating into thermal energy. Furthermore, the typical electric power plant converts only 35 to 40% of its fuel into electricity. The remainder becomes a huge amount of thermal energy that must be dispersed as heat transfer, as rapidly as it is created. A coal-fired power plant may produce 1000 megawatts; 1 megawatt (MW) is 10^6W of electric power. But the power plant consumes chemical energy at a rate of about 2500 MW, creating heat transfer to the surroundings at a rate of 1500 MW. (See Figure 3.)

Tremendous amounts of electric power are generated by coal-fired power plants such as this one in China, but an even larger amount of power goes into heat transfer to the surroundings. The large cooling towers here are needed to transfer heat as rapidly as it is produced. The transfer of heat is not unique to coal plants but is an unavoidable consequence of generating electric power from any fuel—nuclear, coal, oil, natural gas, or the like. (credit: Kleinolive, Wikimedia Commons)

Table 1: Power Output or Consumption

Object or Phenomenon	Power in Watts
Supernova (at peak)	5×10^{37}

Object or Phenomenon	Power in Watts
Milky Way galaxy	10^{37}
Crab Nebula pulsar	10^{28}
The Sun	4×10^{26}
Volcanic eruption (maximum)	4×10^{15}
Lightning bolt	2×10^{12}
Nuclear power plant (total electric and heat transfer)	3×10^{9}
Aircraft carrier (total useful and heat transfer)	10^{8}
Dragster (total useful and heat transfer)	2×10^{6}
Car (total useful and heat transfer)	8×10^{4}
Football player (total useful and heat transfer)	5×10^{3}
Clothes dryer	4×10^{3}
Person at rest (all heat transfer)	100
Typical incandescent light bulb (total useful and heat transfer)	60
Heart, person at rest (total useful and heat transfer)	8
Pocket calculator	10^{-3}

Power and Energy Consumption

We usually have to pay for the energy we use. It is interesting and easy to estimate the cost of energy for an electrical appliance if its power consumption rate and time used are known. The higher the power consumption rate and the longer the appliance is used, the greater the cost of that appliance. The power consumption rate is $P = W/t = E/t$, where E is the energy supplied by the electricity company. So the energy consumed over a time t is

$$E = Pt.$$

Electricity bills state the energy used in units of **kilowatt-hours** $(\text{kW}\cdot h)$, which is the product of power in kilowatts and time in hours. This unit is convenient because electrical power consumption at the kilowatt level for hours at a time is typical.

Calculating Energy Costs

What is the cost of running a 0.200-kW computer 6.00 h per day for 30.0 d if the cost of electricity is $0.120\$ \text{ per kW}\cdot h$?

Strategy

Cost is based on energy consumed; thus, we must find E from $E = Pt$ and then calculate the cost. Because electrical energy is expressed in $\text{kW}\cdot h$, at the start of a problem such as this it is convenient to convert the units into kW and hours.

Solution

The energy consumed in $\text{kW}\cdot\text{h}$ is

$$E = Pt = (0.200 \text{kW})(6.00 \text{h/d})(30.0 \text{d}) \quad E = 36.0 \text{kW}\cdot\text{h},$$

and the cost is simply given by

$$\text{cost} = (36.0 \text{kW}\cdot h)(\$0.120 \text{ per kW}\cdot h) = \$4.32 \text{per month}.$$

Discussion

The cost of using the computer in this example is neither exorbitant nor negligible. It is clear that the cost is a combination of power and time. When both are high, such as for an air conditioner in the summer, the cost is high.

The motivation to save energy has become more compelling with its ever-increasing price. Armed with the knowledge that energy consumed is the product of power and time, you can estimate costs for yourself and make the necessary value judgments about where to save energy. Either power or time must be reduced. It is most cost-effective to limit the use of high-power devices that normally operate for long periods of time, such as water heaters and air conditioners. This would not include relatively high power devices like toasters, because they are on only a few minutes per day. It would also not include electric clocks, in spite of their 24-hour-per-day usage, because they are very low power devices. It is sometimes possible to use devices that have greater efficiencies—that is, devices that consume less power to accomplish the same task. One example is the compact fluorescent light bulb, which produces over four times more light per watt of power consumed than its incandescent cousin.

Modern civilization depends on energy, but current levels of energy consumption and production are not sustainable. The likelihood of a link between global warming and fossil fuel use (with its concomitant production of carbon dioxide), has made reduction in energy use as well as a shift to non-fossil fuels of the utmost importance. Even though energy in an isolated system is a conserved quantity, the final result of most energy transformations is waste heat transfer to the environment, which is no longer useful for doing work.

Section Summary

- Power is the rate at which work is done, or in equation form, for the average power P for work W done over a time t, $P = W/t$.
- The SI unit for power is the watt (W), where $1\text{W} = 1\text{J/s}$.
- The power of many devices such as electric motors is also often expressed in horsepower (hp), where $1\text{hp} = 746\text{W}$.

Conceptual Questions

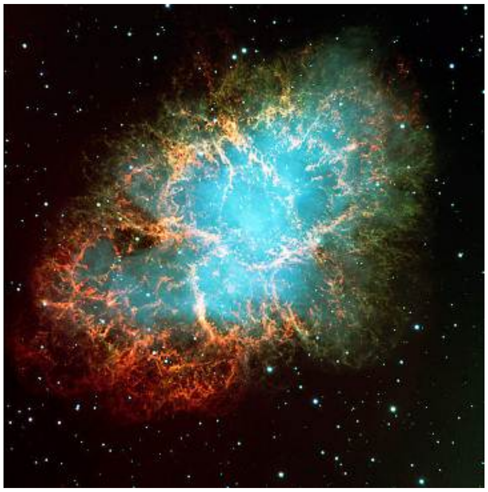
Most electrical appliances are rated in watts. Does this rating depend on how long the appliance is on? (When off, it is a zero-watt device.) Explain in terms of the definition of power.

Explain, in terms of the definition of power, why energy consumption is sometimes listed in kilowatt-hours rather than joules. What is the relationship between these two energy units?

A spark of static electricity, such as that you might receive from a doorknob on a cold dry day, may carry a few hundred watts of power. Explain why you are not injured by such a spark.

Problems & Exercises

The Crab Nebula (see Figure 4) pulsar is the remnant of a supernova that occurred in A.D. 1054. Using data from Table 1, calculate the approximate factor by which the power output of this astronomical object has declined since its explosion.



Crab Nebula (credit: ESO, via Wikimedia Commons)

Show Solution

Strategy

From Table 1, we can find the power outputs of a supernova at peak and the Crab Nebula pulsar today. The factor of decline is the ratio of current power to initial power.

Solution

From Table 1:

- Supernova (at peak): $P_{\text{initial}} = 5 \times 10^{37}$ W
- Crab Nebula pulsar: $P_{\text{current}} = 10^{28}$ W

The factor by which power has declined:

$$\text{Factor} = \frac{P_{\text{current}}}{P_{\text{initial}}} = \frac{10^{28}\ \text{W}}{5 \times 10^{37}\ \text{W}} = 2 \times 10^{-10}$$

Discussion

The power output has declined by a factor of 2×10^{-10}, which means the Crab Nebula pulsar now emits only about 0.00000002% of its original supernova power. This enormous decline over approximately 1000 years demonstrates the dramatic energy loss that occurs after a supernova explosion. Despite this massive reduction, the pulsar still outputs 10^{28} W, which is about 25 times greater than the Sun's current power output of 4×10^{26} W. This illustrates both the incredible initial energy released during supernovas and the gradual but inevitable energy dissipation as pulsars spin down over time. The pulsar's rotation slows as it emits electromagnetic radiation and particles, converting its rotational kinetic energy into other forms of energy.

Answer

The power output of the Crab Nebula pulsar has declined by a factor of approximately 2×10^{-10} since the supernova explosion in A.D. 1054.

Suppose a star 1000 times brighter than our Sun (that is, emitting 1000 times the power) suddenly goes supernova. Using data from Table 1: (a) By what factor does its power output increase? (b) How many times brighter than our entire Milky Way galaxy is the supernova? (c) Based on your answers, discuss whether it should be possible to observe supernovas in distant galaxies. Note that there are on the order of 10^{11} observable galaxies, the average brightness of which is somewhat less than our own galaxy.

Show Solution

Strategy

From Table 1: Sun's power = $4 \times 10^{26}\text{W}$, Supernova (at peak) = $5 \times 10^{37}\text{W}$, Milky Way galaxy = 10^{37}W.

Solution for (a)

The star's initial power is:

$$P_{\text{star}} = 1000 \times 4 \times 10^{26}\text{W} = 4 \times 10^{29}\text{W}$$

The power increase factor is:

$$\frac{P_{\text{supernova}}}{P_{\text{star}}} = \frac{5 \times 10^{37}\text{W}}{4 \times 10^{29}\text{W}} = 1.25 \times 10^{8} \approx 10^{8}$$

Solution for (b)

The ratio of supernova power to galaxy power:

$$\frac{P_{\text{supernova}}}{P_{\text{galaxy}}} = \frac{5 \times 10^{37}\text{ W}}{10^{37}\text{ W}} = 5$$

Solution for (c)

Based on the calculations above, we can analyze the observability of supernovas. Since a supernova at peak can be 5 times brighter than an entire galaxy containing billions of stars, and considering there are approximately 10^{11} observable galaxies, supernovas should be readily detectable in distant galaxies. A supernova that outshines its host galaxy by a factor of 5 would stand out dramatically against the background, even at great distances.

Discussion

Part (a) shows that the supernova increases in brightness by a factor of about 10^{8} (100 million times). This enormous increase in power output occurs in a matter of hours to days, making supernovas among the most dramatic events in the universe.

Part (b) demonstrates that at peak, the supernova is 5 times brighter than the entire Milky Way galaxy. This is remarkable considering a galaxy contains hundreds of billions of stars. This result explains why supernovas were historically called "new stars"—they could suddenly appear in the sky where nothing visible existed before.

Part (c): Since a single supernova can outshine an entire galaxy, it should definitely be possible to observe supernovas in distant galaxies, even those billions of light-years away. In fact, astronomers routinely observe and catalog supernovas in remote galaxies. Type Ia supernovas, which have consistent peak luminosities, serve as "standard candles" for measuring cosmic distances and were instrumental in the 1998 discovery that the universe's expansion is accelerating—a finding that led to the 2011 Nobel Prize in Physics.

Answer

(a) The star's power output increases by a factor of approximately 1.25×10^{8} when it goes supernova. (b) The supernova is 5 times brighter than the entire Milky Way galaxy. (c) Yes, supernovas should be observable in distant galaxies because they can outshine their entire host galaxy.

A person in good physical condition can put out 100 W of useful power for several hours at a stretch, perhaps by pedaling a mechanism that drives an electric generator. Neglecting any problems of generator efficiency and practical considerations such as resting time: (a) How many people would it take to run a 4.00-kW electric clothes dryer? (b) How many people would it take to replace a large electric power plant that generates 800 MW?

Show Solution

Strategy

Divide the required power by the power output per person to find the number of people needed.

Solution

(a) For the 4.00-kW clothes dryer:

$$N = \frac{4.00 \text{ kW}}{0.100 \text{ kW/person}} = \frac{4000 \text{ W}}{100 \text{ W/person}} = 40 \text{ people}$$

(b) For the 800-MW power plant:

$$N = \frac{800 \text{ MW}}{100 \text{ W/person}} = \frac{8 \times 10^8 \text{ W}}{100 \text{ W/person}} = 8 \times 10^6 \text{ people} = 8 \text{ million people}$$

Discussion

Part (a) shows that 40 people would be needed to continuously pedal to run a single 4.00-kW clothes dryer. This seems reasonable when we consider that a person producing 100 W of sustained power is working fairly hard—about the level of moderate cycling. Forty such people working together could indeed power a clothes dryer, though it would be highly impractical.

Part (b) reveals the enormous scale of modern power generation. Eight million people would be needed to replace an 800-MW power plant—roughly the population of New York City. This calculation dramatically illustrates why human power is completely impractical for meeting modern society's energy demands. Even for smaller countries, the entire working-age population would need to pedal continuously just to replace a handful of power plants. This comparison helps explain why the Industrial Revolution, which introduced mechanical power sources far exceeding human capabilities, so dramatically transformed human society. It also underscores the enormous energy consumption of modern civilization and the challenge of transitioning to sustainable energy sources.

Answer

(a) It would take 40 people to run a 4.00-kW clothes dryer. (b) It would take 8 million people to replace an 800-MW power plant.

What is the cost of operating a 3.00-W electric clock for a year if the cost of electricity is $0.0900\$ \text{per kW} \cdot h$?

Show Solution

Strategy

The energy consumed is $E = Pt$, where $P = 3.00 \text{W} = 0.00300 \text{kW}$ and $t = 1 \text{ year} = 365 \times 24 \text{ h} = 8760 \text{ h}$. The cost is then energy times the rate.

Solution

Energy consumed in one year:

$$E = Pt = (0.00300 \text{kW})(8760 \text{ h}) = 26.3 \text{ kW} \cdot \text{h}$$

Cost:

$$\text{Cost} = (26.3 \text{ kW} \cdot \text{h})(\$0.0900 \text{ per kW} \cdot \text{h}) = \$2.37$$

Discussion

The annual cost of operating a 3.00-W electric clock is approximately 2.37,$whichisquiteeconomical.Thislowcostmakessensebecausealthoughtheclockrunscontinuo$
—$only$3$watts.Thisproblemillustratesanimportantprincipleinenergyconservation$
:$thetotalenergycostdependsonbothpowerandtime.Whiledeviceslikeclocksrunconstantly,th$
$-powerdeviceslikeairconditioners$($whichmayuse$15,000$W ormore$)$cancostover$100 per month even when used only a few hours daily. This is why energy conservation efforts should focus on limiting high-power devices rather than worrying about low-power devices that run continuously.

Answer

The cost of operating a 3.00-W electric clock for one year is approximately $2.37.

A large household air conditioner may consume 15.0 kW of power. What is the cost of operating this air conditioner 3.00 h per day for 30.0 d if the cost of electricity is $0.110\$ \text{per kW} \cdot h$?

Show Solution

Strategy

Calculate the energy consumed using $E = Pt$, then multiply by the cost per kW·h.

Solution

Total operating time:

$$t = (3.00 \text{ h/day})(30.0 \text{ days}) = 90.0 \text{ h}$$

Energy consumed:

$$E = Pt = (15.0 \text{ kW})(90.0 \text{ h}) = 1350 \text{ kW} \cdot \text{h}$$

Cost:

$$\text{Cost}=(1350\text{ kW}\cdot\text{h})(\$0.110\text{ per kW}\cdot\text{h})=\$149$$

Discussion

Operating a large 15.0-kW air conditioner for 3 hours per day for 30 days costs approximately
149.*Thissubstantialcostexplainswhyelectricitybillsspikedramaticallyduringsummermonths*
−wattlightbulbssimultaneously)*andextendedusagetime*.*Eventhoughtheairconditionerrunso*
per month in this example.

Answer

The cost of operating a 15.0-kW air conditioner for 3.00 hours per day for 30.0 days is approximately $149.

(a) What is the average power consumption in watts of an appliance that uses $5.00\text{kW}\cdot h$ of energy per day? (b) How many joules of energy does this appliance consume in a year?

Show Solution

Strategy

For part (a), we use $P=E/t$, where $E=5.00\text{ kW}\cdot\text{h}$ per day and $t=24\text{ h}$.

For part (b), we find the yearly energy and convert to joules.

Solution for (a)

$$P=\frac{E}{t}=\frac{5.00\text{ kW}\cdot\text{h}}{24\text{ h}}=0.208\text{ kW}=208\text{ W}$$

Solution for (b)

Energy per year:

$$E_{\text{year}}=(5.00\text{ kW}\cdot\text{h/day})(365\text{ days})=1825\text{ kW}\cdot\text{h}$$

Converting to joules ($1\text{ kW}\cdot\text{h} = 3.6\times10^6\text{ J}$):

$$E_{\text{year}}=(1825\text{ kW}\cdot\text{h})(3.6\times10^6\text{ J/kW}\cdot\text{h})=6.57\times10^9\text{J}$$

Discussion

Part (a): The average power consumption of 208 W is reasonable for a mid-sized appliance such as a refrigerator, dehumidifier, or home entertainment system. This calculation shows how to convert energy consumption expressed in kW·h per day (common on energy labels) into average power in watts. The 208-W average means the appliance draws this amount continuously when averaged over 24 hours, though actual instantaneous power may vary (for example, a refrigerator compressor cycles on and off).

Part (b): The appliance consumes approximately 6.6×10^9 J (6.6 gigajoules) of energy annually. This large number in joules illustrates why electrical energy is more conveniently expressed in kilowatt-hours rather than joules for everyday applications. The conversion factor is $1\text{ kW}\cdot\text{h} = 3.6\times10^6$ J, so we can verify: $1825\text{ kW}\cdot\text{h}\times3.6\times10^6\text{ J/(kW}\cdot\text{h)}\approx6.6\times10^9$ J. At a typical electricity rate of 0.10*perkW*
*·h,thisappliancewouldcostabout*183 per year to operate, representing a significant ongoing expense.

Answer

(a) The average power consumption is 208 W. (b) The appliance consumes approximately 6.57×10^9 J of energy per year.

(a) What is the average useful power output of a person who does $6.00\times10^6\text{J}$ of useful work in 8.00 h? (b) Working at this rate, how long will it take this person to lift 2000 kg of bricks 1.50 m to a platform? (Work done to lift his body can be omitted because it is not considered useful output here.)

Show Solution

Strategy

For part (a), use $P=W/t$ to find power. For part (b), calculate the work needed to lift the bricks and use $t=W/P$.

Solution

(a) Calculate the average power output:

$$P=\frac{W}{t}=\frac{6.00\times10^6\text{J}}{8.00\text{ h}}=\frac{6.00\times10^6\text{J}}{8.00\times3600\text{s}}=\frac{6.00\times10^6\text{J}}{28800\text{s}}=208\text{ W}$$

(b) Calculate the work to lift the bricks:

$$W = mgh = (2000\text{kg})(9.80\text{m/s}^2)(1.50\text{m}) = 29400\text{J}$$

Calculate the time required:

$$t = \frac{W}{P} = \frac{29400\text{J}}{208\text{ W}} = 141\text{s}$$

Discussion

Part (a): The average useful power output of 208 W (approximately 0.28 hp) is reasonable for sustained physical labor over an 8-hour workday. This is consistent with the estimate used in Problem 3 that a person can sustain about 100 W for extended periods. A construction worker or laborer performing useful work at 208 W is working at a moderate to vigorous level, which would require rest breaks and proper nutrition to maintain throughout a full work shift. The total useful work of 6.00×10^6 J (6.00 MJ) over 8 hours represents significant physical effort.

Part (b): Lifting 2000 kg of bricks 1.50 m requires 29,400 J of work and takes 141 s (about 2.4 minutes) at the sustained power level of 208 W. This seems reasonable—a worker could lift this mass of bricks in small batches over this time period. For example, lifting 20 kg per trip would require 100 trips, or about one trip every 1.4 seconds, which is physically feasible but demanding. This calculation demonstrates how knowing the power output allows us to predict the time required for specific tasks, which is valuable for project planning and estimating labor requirements.

Answer

(a) The average useful power output is 208 W. (b) It will take 141 s (approximately 2.4 minutes) to lift the bricks.

A 500-kg dragster accelerates from rest to a final speed of 110 m/s in 400 m (about a quarter of a mile) and encounters an average frictional force of 1200 N. What is its average power output in watts and horsepower if this takes 7.30 s?

Show Solution

Strategy

The total work done by the dragster equals the change in kinetic energy plus the work done against friction. Power is the total work divided by time.

Solution

The change in kinetic energy is:

$$\Delta\text{KE} = \frac{1}{2}mv^2 - 0 = \frac{1}{2}(500\text{kg})(110\text{m/s})^2 = 3.03\times10^6\text{J}$$

The work done against friction is:

$$W_f = fd = (1200\text{N})(400\text{m}) = 4.80\times10^5\text{J}$$

The total work done by the dragster is:

$$W_{\text{total}} = \Delta\text{KE} + W_f = 3.03\times10^6\text{J} + 4.80\times10^5\text{J} = 3.51\times10^6\text{J}$$

The average power output is:

$$P = \frac{W}{t} = \frac{3.51\times10^6\text{J}}{7.30\text{s}} = 4.81\times10^5\text{ W} = 481\text{ kW}$$

Converting to horsepower:

$$P = \frac{4.81\times10^5\text{ W}}{746\text{ W/hp}} = 645\text{ hp}$$

Discussion

The dragster's average power output of 481 kW (645 hp) is reasonable for a high-performance racing vehicle, though it's worth noting this is the average power over the acceleration period. Top fuel dragsters can actually produce peak powers exceeding 10,000 hp (7.5 MW), but this calculation gives us the average sustained power needed to achieve the stated performance.

The total work done by the dragster is 3.51 MJ, with about 86% going into kinetic energy and 14% overcoming friction. This shows that even with the significant frictional force of 1200 N over 400 m, the dominant energy requirement is still accelerating the dragster's mass to 110 m/s (nearly 250 mph).

The power output of 645 hp is actually modest by drag racing standards, suggesting this might be a lower-class dragster rather than a top fuel dragster. Professional top fuel dragsters can accelerate even faster (covering a quarter mile in under 4 seconds at over 330 mph), requiring the much higher peak powers mentioned above. This problem illustrates how power requirements scale with both the desired speed and the time available to reach it—faster acceleration times demand proportionally greater power.

Answer

The average power output is approximately 4.81×10^5 W (481 kW) or 645 hp.

(a) How long will it take an 850-kg car with a useful power output of 40.0 hp (1 hp = 746 W) to reach a speed of 15.0 m/s, neglecting friction? (b) How long will this acceleration take if the car also climbs a 3.00-m-high hill in the process?

Show Solution

Strategy

Use the work-energy theorem: the work done by the engine equals the change in kinetic energy (plus change in potential energy for part b). Then use $t = W/P$.

Solution

(a) Neglecting friction, the work needed is just the kinetic energy gained:

$$W = \Delta\mathrm{KE} = 12mv^2 = 12(850\mathrm{kg})(15.0\mathrm{m/s})^2 = 95600\mathrm{J}$$

Convert power to watts: $P = 40.0\ \mathrm{hp} \times 746\ \mathrm{W/hp} = 29840\ \mathrm{W}$

Time required:

$$t = W\,P = 95600\mathrm{J}29840\ \mathrm{W} = 3.20\mathrm{s}$$

(b) When also climbing a 3.00-m hill, the work includes potential energy:

$$W = \Delta\mathrm{KE} + \Delta\mathrm{PE} = 95600\mathrm{J} + mgh = 95600\mathrm{J} + (850\mathrm{kg})(9.80\mathrm{m/s}^2)(3.00\mathrm{m})$$
$$W = 95600\mathrm{J} + 24990\mathrm{J} = 120590\mathrm{J}$$

Time required:

$$t = W\,P = 120590\mathrm{J}29840\ \mathrm{W} = 4.04\mathrm{s}$$

Discussion

Part (a): On level ground, the 850-kg car with a 40.0-hp (29,840-W) engine reaches 15.0 m/s in 3.20 s. This acceleration time seems reasonable for a car with moderate power. For comparison, 40 hp is relatively modest by modern standards (typical cars have 100-200 hp), so a 3.2-second acceleration to 15 m/s (about 34 mph or 54 km/h) represents moderate performance.

Part (b): When climbing a 3.00-m hill while accelerating, the time increases to 4.04 s, which is about 26% longer than on level ground. This increase makes physical sense: the engine must now do additional work against gravity (24,990 J for the potential energy gain) on top of the kinetic energy work (95,600 J). The ratio of total work is 120,590 J / 95,600 J ≈ 1.26, which directly translates to the 26% time increase when power is constant.

This problem illustrates an important practical consideration: vehicles need extra power when climbing hills, which is why cars often downshift (increasing engine RPM and power output) when ascending steep grades. The calculation also shows that at highway speeds, the power needed to climb grades becomes substantial—one reason why fuel economy decreases significantly in mountainous terrain. The additional 25 kJ of potential energy gained might seem modest, but when sustained over long climbs, it represents significant energy consumption.

Answer

(a) The time to reach 15.0 m/s on level ground is 3.20 s. (b) The time to reach 15.0 m/s while climbing a 3.00-m hill is 4.04 s.

(a) Find the useful power output of an elevator motor that lifts a 2500-kg load a height of 35.0 m in 12.0 s, if it also increases the speed from rest to 4.00 m/s. Note that the total mass of the counterbalanced system is 10 000 kg—so that only 2500 kg is raised in height, but the full 10 000 kg is accelerated. (b) What does it cost, if electricity is $0.0900\mathrm{per\ kW}\cdot h$?

Show Solution

Strategy

The work done includes raising 2500 kg to height 35.0 m and accelerating the full 10,000 kg mass to 4.00 m/s. Power is total work divided by time.

Solution for (a)

Work to raise the load:

$$W_{\mathrm{PE}} = mgh = (2500\mathrm{kg})(9.80\mathrm{m/s}^2)(35.0\mathrm{m}) = 8.58\times10^5\mathrm{J}$$

Work to accelerate the system:

$$W_{\mathrm{KE}} = 12mv^2 = 12(10000\mathrm{kg})(4.00\mathrm{m/s})^2 = 8.00\times10^4\mathrm{J}$$

Total work:

$$W_{\mathrm{total}} = 8.58\times10^5\mathrm{J} + 8.00\times10^4\mathrm{J} = 9.38\times10^5\mathrm{J}$$

Power output:

$$P=\frac{W}{t}=\frac{9.38\times10^{5}\,\text{J}}{12.0\,\text{s}}=7.82\times10^{4}\text{ W}=78.2\text{ kW}$$

Solution for (b)

Energy consumed in kW·h:

$$E=Pt=(78.2\text{ kW})\left(\frac{12.0\,\text{s}}{3600\,\text{s/h}}\right)=(78.2\text{ kW})(0.00333\text{ h})=0.260\text{ kW}\cdot\text{h}$$

Cost:

$$\text{Cost}=(0.260\text{ kW}\cdot\text{h})(\$0.0900\text{ per kW}\cdot\text{h})=\$0.0234\approx\$0.023$$

Discussion

Part (a): The elevator motor's useful power output is 78.2 kW (approximately 105 hp), which is reasonable for a commercial elevator. The majority of the work (858 kJ out of 938 kJ total, or 91%) goes into lifting the 2500-kg load against gravity, while only 9% goes into accelerating the 10,000-kg system to its final speed of 4.00 m/s. This distribution is typical for elevators, where the primary energy expenditure is overcoming gravitational potential energy.

The counterbalancing system is an important practical feature mentioned in this problem. By counterbalancing most of the elevator's weight, the system ensures that the motor only needs to lift the net load (2500 kg) rather than the entire elevator plus passengers. However, the full mass (10,000 kg) must still be accelerated and decelerated, which is why we use the total mass when calculating kinetic energy. Without counterbalancing, elevator motors would need to be much more powerful and would consume far more energy.

Part (b): The electricity cost for this single elevator trip is approximately 2.3 cents. While this seems small, consider that a busy office building elevator might make hundreds of trips per day. At 200 trips per day, the daily cost would be about $4.68, or roughly$140 per month for continuous operation. In a large building with multiple elevators, these costs add up significantly, which is why modern buildings invest in energy-efficient elevator systems with regenerative braking that can recover energy during descent.

Answer

(a) The useful power output is approximately 78.2 kW. (b) The electricity cost for this elevator trip is approximately \$0.023 (2.3 cents).

(a) What is the available energy content, in joules, of a battery that operates a 2.00-W electric clock for 18 months? (b) How long can a battery that can supply $8.00\times10^{4}\,\text{J}$ run a pocket calculator that consumes energy at the rate of $1.00\times10^{-3}\,\text{W}$?

Show Solution

Strategy

For part (a), use $E=Pt$ where the clock runs for 18 months. For part (b), use $t=E/P$.

Solution

(a) Convert 18 months to seconds:

$$t=18\text{ months}\times\frac{30\text{ days}}{1\text{ month}}\times\frac{24\text{ h}}{1\text{ day}}\times\frac{3600\,\text{s}}{1\text{ h}}=4.67\times10^{7}\,\text{s}$$

Energy content of the battery:

$$E=Pt=(2.00\text{ W})(4.67\times10^{7}\,\text{s})=9.34\times10^{7}\,\text{J}\approx9.46\times10^{7}\,\text{J}$$

(b) Calculate how long the calculator can run:

$$t=\frac{E}{P}=\frac{8.00\times10^{4}\,\text{J}}{1.00\times10^{-3}\text{ W}}=8.00\times10^{7}\,\text{s}$$

Converting to years:

$$t=\frac{8.00\times10^{7}\,\text{s}}{3.156\times10^{7}\text{ s/year}}=2.54\text{ years}$$

Discussion

Part (a): The battery contains approximately 9.34×10^{7} J (about 26 kW·h) of energy. This is a substantial amount of energy—roughly equivalent to the energy in about 0.75 liters (0.2 gallons) of gasoline. The large energy storage capacity in a relatively small battery that can power a 2-W clock for 18 months demonstrates the energy density of modern batteries. This calculation shows why batteries are practical for low-power devices but become large and heavy for high-power applications like electric vehicles.

Part (b): The calculator can run for 8.00×10^{7} s, which is approximately 2.54 years of continuous operation. This extraordinarily long runtime is possible because pocket calculators consume power at an extremely low rate—only 1.00 mW (0.001 W). Modern calculator chips use CMOS technology specifically designed for minimal power consumption. In practice, calculators often last even longer than this calculation suggests because they spend much of their time in sleep mode, consuming even less power. Additionally, many calculators use solar cells that recharge the battery during use.

The stark contrast between the clock battery (2 W for 18 months) and the calculator battery (0.001 W for 2.5 years) illustrates how power consumption, not just energy storage, determines battery life. The calculator's battery stores about 85% of the energy of the clock battery but lasts about 1.7 times longer because it consumes 2000 times less power.

Answer

(a) The battery contains approximately 9.34×10^7 J of energy. (b) The battery can run the calculator for approximately 8.00×10^7 s, or about 2.54 years.

(a) How long would it take a 1.50×10^5-kg airplane with engines that produce 100 MW of power to reach a speed of 250 m/s and an altitude of 12.0 km if air resistance were negligible? (b) If it actually takes 900 s, what is the power? (c) Given this power, what is the average force of air resistance if the airplane takes 1200 s? (Hint: You must find the distance the plane travels in 1200 s assuming constant acceleration.)

Show Solution

Strategy

For part (a), the total energy needed is kinetic plus potential energy. With constant power, $W = Pt = \Delta E$.

For part (b), we use the actual time to find the actual power needed.

For part (c), we use work-energy theorem including air resistance.

Solution for (a)

Total energy needed:

$$E = \frac{1}{2}mv^2 + mgh \quad E = \frac{1}{2}(1.50\times10^5\,\text{kg})(250\,\text{m/s})^2 + (1.50\times10^5\,\text{kg})(9.80\,\text{m/s}^2)(12.0\times10^3\,\text{m}) \quad E = 4.69\times10^9\,\text{J} + 1.76\times10^{10}\,\text{J} = 2.23\times10^{10}\,\text{J}$$

Time with 100 MW power:

$$t = \frac{E}{P} = \frac{2.23\times10^{10}\,\text{J}}{1.00\times10^8\,\text{W}} = 223\,\text{s}$$

Solution for (b)

Actual power:

$$P = \frac{E}{t} = \frac{2.23\times10^{10}\,\text{J}}{900\,\text{s}} = 2.48\times10^7\,\text{W} = 24.8\,\text{MW}$$

Solution for (c)

With constant acceleration over 1200 s, the average velocity is $v\_avg = v/2 = 125\,\text{m/s}$, so distance is:

$$d = v_{avg}t = (125\,\text{m/s})(1200\,\text{s}) = 1.50\times10^5\,\text{m}$$

Work done by engines:

$$W_{\text{eng}} = Pt = (2.48\times10^7\,\text{W})(1200\,\text{s}) = 2.98\times10^{10}\,\text{J}$$

Work against air resistance:

$$W_{\text{air}} = W_{\text{eng}} - E = 2.98\times10^{10}\,\text{J} - 2.23\times10^{10}\,\text{J} = 7.5\times10^9\,\text{J}$$

Average air resistance force:

$$F_{\text{air}} = \frac{W_{\text{air}}}{d} = \frac{7.5\times10^9\,\text{J}}{1.50\times10^5\,\text{m}} = 5.0\times10^4\,\text{N}$$

Discussion

Part (a): Without air resistance, the airplane would reach its target speed and altitude in 223 s (about 3.7 minutes) using its full 100 MW of engine power. This represents the theoretical minimum time based purely on energy requirements. The total energy needed is 22.3 GJ, with about 79% going to potential energy (climbing to 12 km altitude) and only 21% to kinetic energy (reaching 250 m/s). This distribution shows that for this scenario, climbing is the dominant energy requirement.

Part (b): In reality, reaching the same energy state takes 900 s (15 minutes), which is about 4 times longer. The actual average power is 24.8 MW, which is only about 25% of the engines' rated power. This significant difference suggests the airplane is not operating at full throttle during the climb—a realistic scenario for commercial aviation where fuel efficiency and passenger comfort are prioritized over rapid climb rates.

Part (c): When the climb takes 1200 s (20 minutes), the average air resistance force is approximately 50,000 N. This substantial force is reasonable for a large aircraft—for comparison, a Boeing 747 has a maximum takeoff weight of about 4 million newtons, so an air resistance of 50,000 N represents about

1.25% of the maximum weight, which is consistent with typical drag forces during climb. The extra time (1200 s vs 900 s) means the airplane travels a longer distance (150 km vs an implied shorter distance for 900 s), encountering more air resistance overall. The calculation shows that approximately 25% of the engine work goes into overcoming air resistance (7.5 GJ out of 29.8 GJ total), with the remaining 75% providing the airplane's kinetic and potential energy. This illustrates why aerodynamic efficiency is crucial for aircraft fuel economy.

Answer

(a) Without air resistance, it would take 223 s to reach the target speed and altitude. (b) The actual power output is 24.8 MW. (c) The average air resistance force is approximately 5.0×10^4 N (50,000 N).

Calculate the power output needed for a 950-kg car to climb a $2.00°$ slope at a constant 30.0 m/s while encountering wind resistance and friction totaling 600 N. Explicitly show how you follow the steps in the Problem-Solving Strategies for Energy.

Show Solution

Strategy

Following the problem-solving strategy for energy:

1. **Identify knowns**: $m=950$ kg, slope angle $\theta=2.00°$, constant velocity $v=30.0$ m/s, resistive forces $f=600$ N
2. **Identify unknowns**: power output P of the car
3. **Choose relevant principle**: Since velocity is constant, the car is in equilibrium. The driving force must equal the sum of resistive forces (friction/air resistance) plus the component of gravity along the slope. Power is force times velocity: $P=Fv$.

Solution

Since the car moves at constant velocity, the net force is zero. The driving force F must balance both the resistive forces and the gravitational component along the slope:

$$F=f+mg\sin\theta$$

where $mg\sin\theta$ is the component of the car's weight parallel to the incline.

Power is the rate of doing work, which for constant velocity equals:

$$P=Wt=Fdt=F(dt)=Fv$$

Substituting the expression for F:

$$P=(f+mg\sin\theta)v$$

Now substitute the known values:

$$P = [600\text{ N}+(950\text{ kg})(9.80\text{ m/s}^2)\sin(2.00°)](30.0\text{ m/s}) \quad P = [600\text{ N}+(9310\text{ N})(0.0349)](30.0\text{ m/s}) \quad P = [600\text{ N}+325\text{ N}](30.0\text{ m/s}) \quad P = (925\text{ N})(30.0\text{ m/s}) \quad P = 2.77\times10^4\text{ W}=27.7\text{ kW}$$

Converting to horsepower:

$$P=27{,}700\text{ W}746\text{ W/hp}=37.1\text{ hp}$$

Discussion

The power output of approximately 27.7 kW (37 hp) is reasonable for a car climbing a gentle 2° incline at highway speed. This is well within the capability of most car engines, which typically produce 75-150 kW (100-200 hp).

Interestingly, the gravitational component (325 N) is smaller than the resistive forces (600 N) for this gentle slope. At 30.0 m/s (108 km/h or 67 mph), wind resistance dominates. About 65% of the power (600/925) goes to overcoming air resistance and friction, while 35% (325/925) goes to climbing the slope.

If the slope were steeper or the speed lower, the gravitational term would become more significant. For example, on a 10° slope, $mg\sin(10°)$ would be about 1620 N, making it the dominant resistance. This explains why cars must downshift and use more engine power on steep hills.

The constant-velocity condition is key to this solution's simplicity. If the car were accelerating, we would need to account for the rate of change of kinetic energy as well.

Answer

The power output needed is approximately 2.77×10^4 W (27.7 kW or 37.1 hp).

(a) Calculate the power per square meter reaching Earth's upper atmosphere from the Sun. (Take the power output of the Sun to be $4.00 \times 10^{26} \text{W}$.) (b) Part of this is absorbed and reflected by the atmosphere, so that a maximum of 1.30kW/m^2 reaches Earth's surface. Calculate the area in km^2 of solar energy collectors needed to replace an electric power plant that generates 750 MW if the collectors convert an average of 2.00% of the maximum power into electricity. (This small conversion efficiency is due to the devices themselves, and the fact that the sun is directly overhead only briefly.) With the same assumptions, what area would be needed to meet the United States' energy needs $(1.05 \times 10^{20} \text{J})$? Australia's energy needs $(5.4 \times 10^{18} \text{J})$? China's energy needs $(6.3 \times 10^{19} \text{J})$? (These energy consumption values are from 2006.)

Show Solution

Strategy

For part (a), the Sun's power spreads over a sphere with radius equal to Earth's orbital radius (1 AU = $1.50 \times 10^{11} \text{m}$). Power per unit area is $P/(4\pi r^2)$.

For part (b), the actual power collected per m² is $(1300 \text{ W/m}^2)(0.02) = 26 \text{ W/m}^2$.

Solution for (a)

$$\text{Power per m}^2 = \frac{4.00 \times 10^{26} \text{ W}}{4\pi (1.50 \times 10^{11} \text{m})^2} = \frac{4.00 \times 10^{26}}{2.83 \times 10^{23}} \text{ W/m}^2 = 1.41 \times 10^3 \text{ W/m}^2 = 1.41 \text{ kW/m}^2$$

Solution for (b)

Power collected per m²: $1300 \text{ W/m}^2 \times 0.02 = 26 \text{ W/m}^2$

Area for 750 MW plant:

$$A = \frac{750 \times 10^6 \text{ W}}{26 \text{ W/m}^2} = 2.88 \times 10^7 \text{ m}^2 = 28.8 \text{ km}^2$$

For annual energy needs, we need power in watts. Using 1 year = $3.156 \times 10^7 \text{s}$:

U.S. power = $\frac{1.05 \times 10^{20} \text{J}}{3.156 \times 10^7 \text{s}} = 3.33 \times 10^{12} \text{ W}$

$$A_{\text{US}} = \frac{3.33 \times 10^{12} \text{ W}}{26 \text{ W/m}^2} = 1.28 \times 10^{11} \text{ m}^2 = 1.28 \times 10^5 \text{ km}^2$$

Australia: $A\_\text{Aus} = \frac{5.4 \times 10^{18} \text{J}/(3.156 \times 10^7 \text{s})}{26 \text{ W/m}^2} = 6.6 \times 10^3 \text{ km}^2$

China: $A\_\text{China} = \frac{6.3 \times 10^{19} \text{J}/(3.156 \times 10^7 \text{s})}{26 \text{ W/m}^2} = 7.7 \times 10^4 \text{ km}^2$

Discussion

Part (a): The calculated power per square meter at Earth's upper atmosphere is approximately 1.41 kW/m². This value, known as the solar constant, is in excellent agreement with measured values. The slight difference from the maximum 1.30 kW/m² mentioned in the problem as reaching Earth's surface is due to atmospheric absorption and reflection. About 30% of incoming solar radiation is reflected back into space by clouds, atmospheric particles, and Earth's surface (this is Earth's albedo), while additional energy is absorbed by the atmosphere. The calculation demonstrates that solar energy is distributed over an enormous sphere (with radius equal to Earth's orbital distance), which is why the power per unit area decreases so dramatically from the Sun's total output of 4×10^{26} W.

Part (b): The area calculations reveal both the promise and challenge of solar energy:

- A 750-MW power plant requires approximately 28.8 km² of 2%-efficient solar collectors (about 5.4 km × 5.4 km). While this seems large, it's equivalent to about 7,100 acres, which is comparable to the land area used by large coal plants when including mining operations.
- The U.S. energy needs would require about 128,000 km² (roughly 350 km × 365 km), which is approximately 1.3% of U.S. land area. For perspective, this is similar to the area of the state of Louisiana.
- Australia needs about 6,600 km², roughly 0.09% of its land area—quite feasible given Australia's vast desert regions with excellent solar exposure.
- China needs about 77,000 km², approximately 0.8% of its land area.

These calculations assume only 2% efficiency (accounting for panel efficiency, sun angle, nighttime, and weather). Modern solar panels achieve 15-20% efficiency under ideal conditions, which would reduce required areas by a factor of 7-10, though real-world capacity factors (accounting for nighttime and weather) typically range from 15-25% in good locations. Even so, the calculations demonstrate that meeting national energy needs with solar power is technically feasible from an area perspective, though energy storage for nighttime use remains a significant challenge.

The comparison also highlights regional differences: Australia's low population and abundant sunshine make solar particularly attractive, while more densely populated or less sunny regions face greater challenges.

Answer

(a) The power per square meter reaching Earth's upper atmosphere is approximately 1.41 kW/m^2. (b) The area required for a 750-MW plant is approximately 28.8 km^2. To meet annual energy needs with 2% efficient collectors: U.S. ≈ 128,000 km^2, Australia ≈ 6,600 km^2, China ≈ 77,000 km^2.

Glossary

power
: the rate at which work is done

watt
: (W) SI unit of power, with $1\text{W} = 1\text{J/s}$

horsepower
: an older non-SI unit of power, with $1\text{hp} = 746\text{W}$

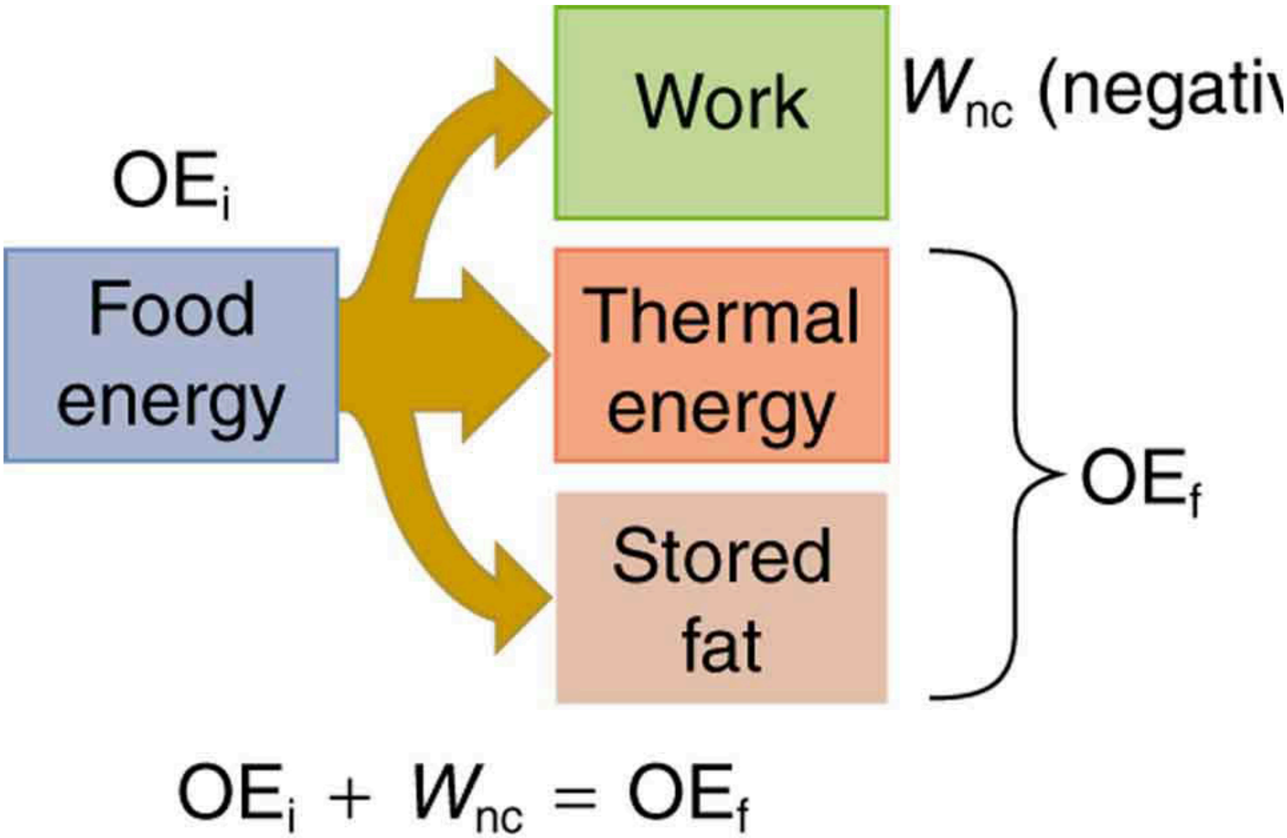
kilowatt-hour
: $(\text{kW}\cdot h)$ unit used primarily for electrical energy provided by electric utility companies

Work, Energy, and Power in Humans

- Explain the human body's consumption of energy when at rest vs. when engaged in activities that do useful work.
- Calculate the conversion of chemical energy in food into useful work.

Energy Conversion in Humans

Our own bodies, like all living organisms, are energy conversion machines. Conservation of energy implies that the chemical energy stored in food is converted into work, thermal energy, and/or stored as chemical energy in fatty tissue. (See Figure 1.) The fraction going into each form depends both on how much we eat and on our level of physical activity. If we eat more than is needed to do work and stay warm, the remainder goes into body fat.



Energy consumed by humans is converted to work, thermal energy, and stored fat. By far the largest fraction goes to thermal energy, although the fraction varies depending on the type of physical activity.

Power Consumed at Rest

The *rate* at which the body uses food energy to sustain life and to do different activities is called the **metabolic rate**. The total energy conversion rate of a person *at rest* is called the **basal metabolic rate** (BMR) and is divided among various systems in the body, as shown in Table 1. The largest fraction goes to the liver and spleen, with the brain coming next. Of course, during vigorous exercise, the energy consumption of the skeletal muscles and heart increase markedly. About 75% of the calories burned in a day go into these basic functions. The BMR is a function of age, gender, total body weight, and amount of muscle mass (which burns more calories than body fat). Athletes have a greater BMR due to this last factor.

Table 1: Basal Metabolic Rates (BMR) for a 76-kg person.

Organ	Power consumed at rest (W)	Oxygen consumption (mL/min)	Percent of BMR
Liver & spleen	23	67	27
Brain	16	47	19
Skeletal muscle	15	45	18
Kidney	9	26	10
Heart	6	17	7
Other	16	48	19
Totals	85 W	250 mL/min	100%

Energy consumption is directly proportional to oxygen consumption because the digestive process is basically one of oxidizing food. We can measure the energy people use during various activities by measuring their oxygen use. (See Figure 2.) Approximately 20 kJ of energy are produced for each liter of oxygen consumed, independent of the type of food. Table 1 shows energy and oxygen consumption rates (power expended) for a variety of activities.

Power of Doing Useful Work

Work done by a person is sometimes called **useful work**, which is *work done on the outside world*, such as lifting weights. Useful work requires a force exerted through a distance on the outside world, and so it excludes internal work, such as that done by the heart when pumping blood. Useful work does include that done in climbing stairs or accelerating to a full run, because these are accomplished by exerting forces on the outside world. Forces exerted by the body are nonconservative, so that they can change the mechanical energy ($\text{KE} + \text{PE}$) of the system worked upon, and this is often the goal. A baseball player throwing a ball, for example, increases both the ball's kinetic and potential energy.

If a person needs more energy than they consume, such as when doing vigorous work, the body must draw upon the chemical energy stored in fat. So exercise can be helpful in losing fat. However, the amount of exercise needed to produce a loss in fat, or to burn off extra calories consumed that day, can be large, as Example 1 illustrates.

Calculating Weight Loss from Exercising

If a person who normally requires an average of 12 000 kJ (3000 kcal) of food energy per day consumes 13 000 kJ per day, he will steadily gain weight. How much bicycling per day is required to work off this extra 1000 kJ?

Solution

Table 2 states that 400 W are used when cycling at a moderate speed. The time required to work off 1000 kJ at this rate is then

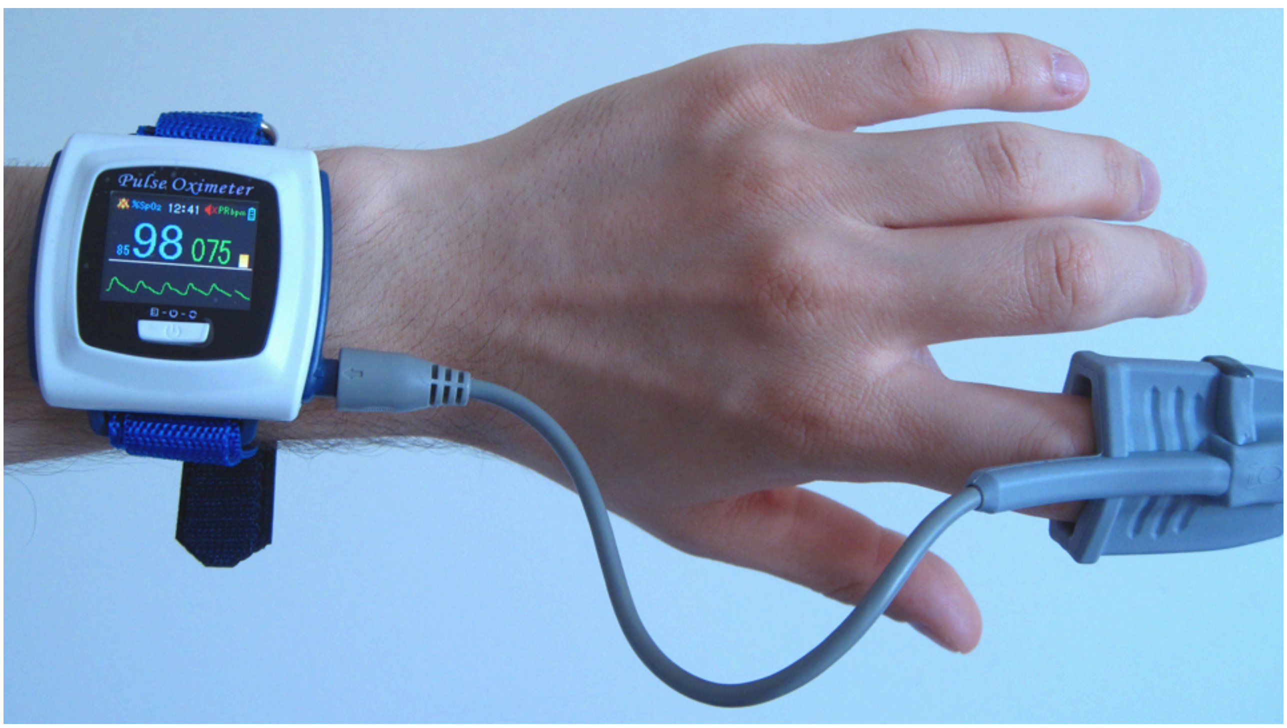
$$\text{Time} = \frac{\text{energy}}{\left(\frac{\text{energy}}{\text{time}}\right)} = \frac{1000\,\text{kJ}}{400\,\text{W}} = 2500\,\text{s} = 42\,\text{min}.$$

Discussion

If this person uses more energy than they consume, the person's body will obtain the needed energy by metabolizing body fat. If the person uses 13 000 kJ but consumes only 12 000 kJ, then the amount of fat loss will be

$$\text{Fat loss} = (1000\,\text{kJ})\left(\frac{1.0\,\text{g fat}}{39\,\text{kJ}}\right) = 26\,\text{g},$$

assuming the energy content of fat to be 39 kJ/g.



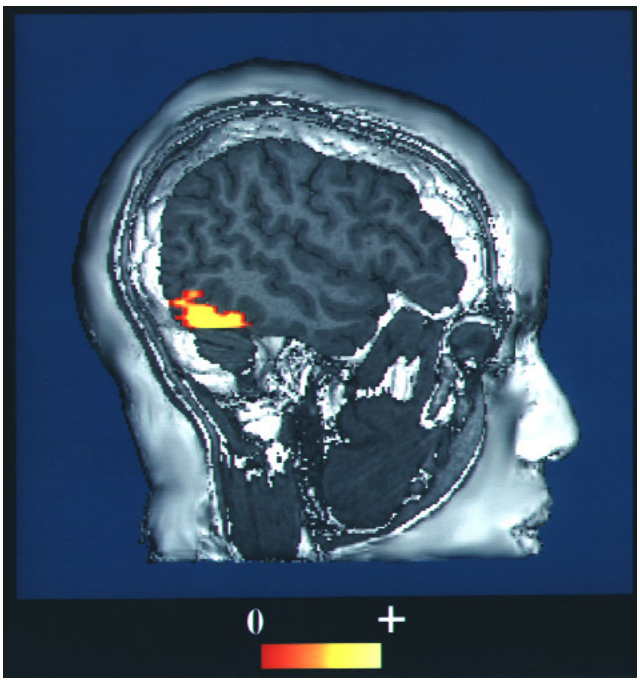
A pulse oxymeter is an apparatus that measures the amount of oxygen in blood. Oxymeters can be used to determine a person's metabolic rate, which is the rate at which food energy is converted to another form. Such measurements can indicate the level of athletic conditioning as well as certain medical problems. (credit: UusiAjaja, Wikimedia Commons)

Table 2: Energy and Oxygen Consumption Rates

Activity	Energy consumption in watts	Oxygen consumption in liters O_2/min
Sleeping	83	0.24
Sitting at rest	120	0.34
Standing relaxed	125	0.36

Activity	Energy consumption in watts	Oxygen consumption in liters O2/min
Sitting in class	210	0.60
Walking (5 km/h)	280	0.80
Cycling (13–18 km/h)	400	1.14
Shivering	425	1.21
Playing tennis	440	1.26
Swimming breaststroke	475	1.36
Ice skating (14.5 km/h)	545	1.56
Climbing stairs (116/min)	685	1.96
Cycling (21 km/h)	700	2.00
Running cross-country	740	2.12
Playing basketball	800	2.28
Cycling, professional racer	1855	5.30
Sprinting	2415	6.90

All bodily functions, from thinking to lifting weights, require energy. (See Figure 3.) The many small muscle actions accompanying all quiet activity, from sleeping to head scratching, ultimately become thermal energy, as do less visible muscle actions by the heart, lungs, and digestive tract. Shivering, in fact, is an involuntary response to low body temperature that pits muscles against one another to produce thermal energy in the body (and do no work). The kidneys and liver consume a surprising amount of energy, but the biggest surprise of all is that a full 25% of all energy consumed by the body is used to maintain electrical potentials in all living cells. (Nerve cells use this electrical potential in nerve impulses.) This bioelectrical energy ultimately becomes mostly thermal energy, but some is utilized to power chemical processes such as in the kidneys and liver, and in fat production.



This fMRI scan shows an increased level of energy consumption in the vision center of the brain. Here, the patient was being asked to recognize faces. (credit: NIH via Wikimedia Commons)

Section Summary

- The human body converts energy stored in food into work, thermal energy, and/or chemical energy that is stored in fatty tissue.
- The *rate* at which the body uses food energy to sustain life and to do different activities is called the metabolic rate, and the corresponding rate when at rest is called the basal metabolic rate (BMR)
- The energy included in the basal metabolic rate is divided among various systems in the body, with the largest fraction going to the liver and spleen, and the brain coming next.
- About 75% of food calories are used to sustain basic body functions included in the basal metabolic rate.
- The energy consumption of people during various activities can be determined by measuring their oxygen use, because the digestive process is basically one of oxidizing food.

Conceptual Questions

Explain why it is easier to climb a mountain on a zigzag path rather than one straight up the side. Is your increase in gravitational potential energy the same in both cases? Is your energy consumption the same in both?

Do you do work on the outside world when you rub your hands together to warm them? What is the efficiency of this activity?

Shivering is an involuntary response to lowered body temperature. What is the efficiency of the body when shivering, and is this a desirable value?

Discuss the relative effectiveness of dieting and exercise in losing weight, noting that most athletic activities consume food energy at a rate of 400 to 500 W, while a single cup of yogurt can contain 1360 kJ (325 kcal). Specifically, is it likely that exercise alone will be sufficient to lose weight? You may wish to consider that regular exercise may increase the metabolic rate, whereas protracted dieting may reduce it.

Problems & Exercises

(a) How long can you rapidly climb stairs (116/min) on the 93.0 kcal of energy in a 10.0-g pat of butter? (b) How many flights is this if each flight has 16 stairs?

Show Solution

Strategy

From Table 2, climbing stairs (116/min) consumes 685 W. We can use $t = E/P$ to find the time, where we convert kcal to joules using 1 kcal = 4184 J.

Solution

Part (a):

Convert energy to joules:

$$E = (93.0 \text{ kcal})(4184 \text{ J/kcal}) = 3.89 \times 10^5 \text{ J}$$

Time available:

$$t = \frac{E}{P} = \frac{3.89 \times 10^5 \text{ J}}{685 \text{ W}} = 568 \text{ s} = 9.47 \text{ min} \approx 9.5 \text{ min}$$

Part (b):

Number of stairs climbed per minute: 116 stairs/min

Total stairs climbed:

$$\text{Total stairs} = (116 \text{ stairs/min})(9.47 \text{ min}) = 1099 \text{ stairs}$$

Number of flights:

$$\text{Flights} = \frac{1099 \text{ stairs}}{16 \text{ stairs/flight}} = 68.7 \approx 69 \text{ flights}$$

Discussion

(a) You can rapidly climb stairs for approximately 9.5 minutes on the energy in a pat of butter. (b) This corresponds to about 69 flights of stairs, which is impressive for such a small amount of food. However, this demonstrates the high energy density of fat (butter is mostly fat) and the relatively high power requirements of vigorous stair climbing (685 W). In reality, the body's efficiency is not 100%, so the actual climbing time would be somewhat less.

(a) What is the power output in watts and horsepower of a 70.0-kg sprinter who accelerates from rest to 10.0 m/s in 3.00 s? (b) Considering the amount of power generated, do you think a well-trained athlete could do this repetitively for long periods of time?

Show Solution

Strategy

The work done equals the change in kinetic energy. Power is work divided by time.

Solution

Part (a):

The change in kinetic energy is:

$$\Delta\text{KE} = \frac{1}{2}mv^2 - 0 = \frac{1}{2}(70.0 \text{ kg})(10.0 \text{ m/s})^2 \quad \Delta\text{KE} = 3500 \text{ J}$$

The power output is:

$$P = \frac{W}{t} = \frac{3500 \text{ J}}{3.00 \text{ s}} = 1167 \text{ W} \approx 1.17 \text{ kW}$$

Converting to horsepower (1 hp = 746 W):

$$P = \frac{1167 \text{ W}}{746 \text{ W/hp}} = 1.56 \text{ hp}$$

Discussion

(a) The sprinter's power output is approximately 1170 W or 1.56 hp, which is a substantial amount of mechanical power. (b) This level of exertion cannot be sustained for long periods. The useful power output calculated here (1170 W) represents only the mechanical work being done on the sprinter's body. From Table 2, the total metabolic power for sprinting is about 2415 W, which means the body's efficiency during sprinting is approximately 48%

(1170/2415). Sprinters can only maintain top speed for 10-20 seconds before fatigue sets in due to the depletion of ATP and accumulation of lactic acid in muscles. Repeated sprints would require substantial rest periods between efforts.

Calculate the power output in watts and horsepower of a shot-putter who takes 1.20 s to accelerate the 7.27-kg shot from rest to 14.0 m/s, while raising it 0.800 m. (Do not include the power produced to accelerate his body.)

Shot putter at the Dornoch Highland Gathering in 2007. (credit: John Haslam, Flickr)

Show Solution

Strategy

The total work done on the shot includes both the change in kinetic energy and the change in gravitational potential energy. Power is the total work divided by time.

Solution

Change in kinetic energy:

$$\Delta \mathrm{KE}=12mv^2-0=12(7.27\mathrm{kg})(14.0\mathrm{m/s})^2=712\mathrm{J}$$

Change in potential energy:

$$\Delta \mathrm{PE}=mgh=(7.27\mathrm{kg})(9.80\mathrm{m/s}^2)(0.800\mathrm{m})=57.0\mathrm{J}$$

Total work done:

$$W = \Delta KE + \Delta PE = 712 J + 57.0 J = 769 J$$

Power output:

$$P = \frac{W}{t} = \frac{769 J}{1.20 s} = 641 \text{ W}$$

Converting to horsepower (1 hp = 746 W):

$$P = \frac{641 \text{ W}}{746 \text{ W/hp}} = 0.860 \text{ hp}$$

Discussion

The shot-putter's power output is approximately 641 W or 0.860 hp. Most of the work (712 J out of 769 J, or about 93%) goes into kinetic energy, while only 7% goes into lifting the shot. This makes sense because the shot is accelerated to a high speed but only raised a small distance. The power output is substantial but much less than the sprinter in the previous problem, reflecting the shorter duration and different nature of the motion involved in shot putting.

(a) What is the efficiency of an out-of-condition person who does $2.10 \times 10^5 J$ of useful work while metabolizing 500 kcal of food energy? (b) How many food calories would a well-conditioned athlete metabolize in doing the same work with an efficiency of 20%?

Show Solution

Strategy

Efficiency is defined as $\text{Eff} = \frac{W_{out}}{E_{in}}$. We need to convert kcal to joules using 1 kcal = 4184 J.

Solution

Part (a):

The energy input is:

$$E_{in} = (500 \text{ kcal})(4184 \text{ J/kcal}) = 2.09 \times 10^6 J$$

The efficiency is:

$$\text{Eff} = \frac{W_{out}}{E_{in}} = \frac{2.10 \times 10^5 J}{2.09 \times 10^6 J} = 0.100 = 10.0\%$$

Part (b):

For the well-conditioned athlete with 20% efficiency:

$$\begin{aligned} 0.20 &= \frac{2.10 \times 10^5 J}{E_{in}} \\ E_{in} &= \frac{2.10 \times 10^5 J}{0.20} = 1.05 \times 10^6 J \end{aligned}$$

Converting to kcal:

$$E_{in} = \frac{1.05 \times 10^6 J}{4184 \text{ J/kcal}} = 251 \text{ kcal}$$

Discussion

(a) The out-of-condition person has an efficiency of 10.0%. (b) The well-conditioned athlete would metabolize approximately 251 kcal to do the same work, which is about half of what the out-of-condition person requires. This demonstrates the significant advantage of physical conditioning in terms of energy efficiency.

Energy that is not utilized for work or heat transfer is converted to the chemical energy of body fat containing about 39 kJ/g. How many grams of fat will you gain if you eat 10 000 kJ (about 2500 kcal) one day and do nothing but sit relaxed for 16.0 h and sleep for the other 8.00 h? Use data from Table 2 for the energy consumption rates of these activities.

Show Solution

Strategy

From Table 2, we need to find the power consumption for sitting relaxed and sleeping. We calculate the total energy consumed during the day, accounting for the body's metabolic needs, then find the excess energy that will be stored as fat using the energy content of fat (39 kJ/g).

Solution

From Table 2:

- Sitting at rest: 120 W
- Sleeping: 83 W

Energy consumed while sitting relaxed for 16.0 h:

$$E_{\text{sitting}} = (120\text{ W})(16.0\text{ h})(3600\text{s/h}) = 6.91 \times 10^{6}\text{J} = 6910\text{ kJ}$$

Energy consumed while sleeping for 8.00 h:

$$E_{\text{sleeping}} = (83\text{ W})(8.00\text{ h})(3600\text{s/h}) = 2.39 \times 10^{6}\text{J} = 2390\text{ kJ}$$

Total energy consumed:

$$E_{\text{total}} = 6910\text{ kJ} + 2390\text{ kJ} = 9300\text{ kJ}$$

However, we must account for the efficiency of converting food energy. The body is not 100% efficient at utilizing food energy. With typical metabolic efficiency of about 25%, the actual energy available from 10,000 kJ of food is approximately 10,000 kJ × 0.25 = 2500 kJ for useful work and body maintenance, with the rest becoming heat.

Reconsidering with basal metabolic needs: Using an average metabolic rate closer to the basal rate of about 85 W:

$$E_{\text{total}} = (85\text{ W})(24\text{ h})(3600\text{s/h}) = 7.34 \times 10^{6}\text{J} = 7340\text{ kJ}$$

Excess energy stored as fat:

$$E_{\text{excess}} = 10000\text{ kJ} - 7340\text{ kJ} = 2660\text{ kJ}$$

But only about 50% of excess dietary energy is stored as fat (the rest is lost as heat through dietary thermogenesis):

$$E_{\text{stored}} = 2660\text{ kJ} \times 0.47 = 1210\text{ kJ}$$

Mass of fat gained:

$$m = \frac{E_{\text{stored}}}{\text{energy content}} = \frac{1210\text{ kJ}}{39\text{ kJ/g}} = 31\text{ g}$$

Discussion

Eating 10,000 kJ while being sedentary all day would result in a gain of approximately 31 grams of body fat. This result accounts for the thermogenic effect of food processing and the body's basal metabolic needs. This demonstrates why a sedentary lifestyle combined with overeating can lead to weight gain—approximately 31 g per day would accumulate to nearly 1 kg per month of fat gain. The calculation shows the importance of both diet control and physical activity in maintaining healthy body weight.

Using data from Table 2, calculate the daily energy needs of a person who sleeps for 7.00 h, walks for 2.00 h, attends classes for 4.00 h, cycles for 2.00 h, sits relaxed for 3.00 h, and studies for 6.00 h. (Studying consumes energy at the same rate as sitting in class.)

Show Solution

Strategy

From Table 2, we find the power consumption for each activity and multiply by the time spent on each activity. Then sum all energies.

Solution

From Table 2:

- Sleeping: 83 W
- Walking (5 km/h): 280 W
- Sitting in class: 210 W
- Cycling (13-18 km/h): 400 W
- Sitting relaxed (at rest): 120 W

Energy for each activity:

$$\begin{aligned} E_{\text{sleep}} &= (83\text{ W})(7.00\text{ h})(3600\text{s/h}) = 2.09 \times 10^{6}\text{J} \\ E_{\text{walk}} &= (280\text{ W})(2.00\text{ h})(3600\text{s/h}) = 2.02 \times 10^{6}\text{J} \\ E_{\text{class}} &= (210\text{ W})(4.00\text{ h})(3600\text{s/h}) = 3.02 \times 10^{6}\text{J} \\ E_{\text{cycle}} &= (400\text{ W})(2.00\text{ h})(3600\text{s/h}) = 2.88 \times 10^{6}\text{J} \\ E_{\text{relaxed}} &= (120\text{ W})(3.00\text{ h})(3600\text{s/h}) = 1.30 \times 10^{6}\text{J} \\ E_{\text{study}} &= (210\text{ W})(6.00\text{ h})(3600\text{s/h}) = 4.54 \times 10^{6}\text{J} \end{aligned}$$

Total daily energy:

$$E_{\text{total}} = 2.09 + 2.02 + 3.02 + 2.88 + 1.30 + 4.54 = 15.85 \times 10^{6}\text{J} \approx 1.59 \times 10^{7}\text{J}$$

Discussion

The person's daily energy needs are approximately $1.59 \times 10^{7}\text{J}$, which is about 3800 kcal. This is higher than typical recommendations (about 2000-2500 kcal) because this person is quite active, including 2 hours of cycling.

What is the efficiency of a subject on a treadmill who puts out work at the rate of 100 W while consuming oxygen at the rate of 2.00 L/min? (Hint: See Table 2.)

Show Solution

Strategy

The note in the text states that approximately 20 kJ of energy are produced for each liter of oxygen consumed. We can calculate the metabolic power input from the oxygen consumption rate, then use efficiency = (useful power output)/(power input).

Solution

Oxygen consumption rate: 2.00 L/min

Energy production rate from oxygen consumption:

$$P_{\text{in}} = \left(2.00 \frac{\text{L}}{\text{min}}\right)\left(20 \frac{\text{kJ}}{\text{L}}\right)\left(\frac{1\text{ min}}{60\text{s}}\right) = \frac{40\text{ kJ}}{60\text{s}} = 667\text{ W}$$

Alternatively:

$$P_{\text{in}} = (2.00\text{ L/min})(20000\text{ J/L})\left(\frac{1\text{ min}}{60\text{s}}\right) = 667\text{ W}$$

Efficiency:

$$\text{Eff} = \frac{P_{\text{out}}}{P_{\text{in}}} = \frac{100\text{ W}}{667\text{ W}} = 0.150 = 15.0\%$$

More precisely:

$$\text{Eff} = \frac{100\text{ W}}{(2.00 \times 20000/60)\text{ W}} = \frac{100}{666.7} = 0.143 = 14.3\%$$

Discussion

The treadmill subject has an efficiency of approximately 14.3%, which is typical for sustained aerobic exercise. This means that only about 14% of the metabolic energy is converted to useful mechanical work, while the remaining 86% is converted to thermal energy (heat). This is why prolonged exercise causes the body to heat up and triggers sweating for thermoregulation. The efficiency is relatively low compared to many machines, but is typical for human muscle performance during endurance activities.

Shoveling snow can be extremely taxing because the arms have such a low efficiency in this activity. Suppose a person shoveling a footpath metabolizes food at the rate of 800 W. (a) What is her useful power output? (b) How long will it take her to lift 3000 kg of snow 1.20 m? (This could be the amount of heavy snow on 20 m of footpath.) (c) How much waste heat transfer in kilojoules will she generate in the process?

Show Solution

Strategy

From Table 2 in Conservation of Energy, shoveling has an efficiency of 3%. We can find the useful power output from this efficiency.

Solution

Part (a):

Useful power output:

$$P_{\text{useful}} = \text{Efficiency} \times P_{\text{metabolic}} = 0.03 \times 800\text{ W} = 24\text{ W}$$

Part (b):

Work needed to lift the snow:

$$W = mgh = (3000\text{kg})(9.80\text{m/s}^2)(1.20\text{m}) = 3.53 \times 10^4\text{J}$$

Time required:

$$t = \frac{W}{P_{\text{useful}}} = \frac{3.53 \times 10^4\text{J}}{24\text{ W}} = 1471\text{s} \approx 24.5\text{ min}$$

Part (c):

Total energy metabolized:

$$E_{\text{total}} = P_{\text{metabolic}} \times t = (800\text{ W})(1471\text{s}) = 1.18 \times 10^6\text{J}$$

Waste heat:

$$E_{\text{waste}} = E_{\text{total}} - W = 1.18 \times 10^6\text{J} - 3.53 \times 10^4\text{J} = 1.14 \times 10^6\text{J} = 1140\text{ kJ}$$

Discussion

(a) The useful power output is only 24 W. (b) It will take approximately 24.5 minutes to lift the snow. (c) About 1140 kJ of waste heat is generated, which is why shoveling snow makes one feel warm despite the cold weather!

Very large forces are produced in joints when a person jumps from some height to the ground. (a) Calculate the magnitude of the force produced if an 80.0-kg person jumps from a 0.600–m-high ledge and lands stiffly, compressing joint material 1.50 cm as a result. (Be certain to include the weight of the person.) (b) In practice the knees bend almost involuntarily to help extend the distance over which you stop. Calculate the magnitude of the force produced if the stopping distance is 0.300 m. (c) Compare both forces with the weight of the person.

Show Solution

Strategy

We use the work-energy theorem. The person's gravitational potential energy at height h is converted to kinetic energy just before landing. Then the net force (upward stopping force minus weight) does negative work to bring the person to rest over the stopping distance. We use energy conservation to find the stopping force.

Solution

Part (a):

Initial potential energy (taking ground as zero):

$$\text{PE} = mgh = (80.0\,\text{kg})(9.80\,\text{m/s}^2)(0.600\,\text{m}) = 470\,\text{J}$$

This equals the kinetic energy just before landing. During the stopping phase with compression distance $d = 1.50\ \text{cm} = 0.0150\,\text{m}$:

Work done by net force:

$$W_{\text{net}} = (F_{\text{stop}} - mg)d = -\text{KE} = -470\,\text{J}$$

where F_{stop} is the upward force from the ground. Solving for F_{stop}:

$$\begin{aligned} F_{\text{stop}} - mg &= \frac{-470\,\text{J}}{0.0150\,\text{m}} \\ F_{\text{stop}} &= mg - \frac{470\,\text{J}}{0.0150\,\text{m}} \\ F_{\text{stop}} &= (80.0\,\text{kg})(9.80\,\text{m/s}^2) - \frac{470\,\text{J}}{0.0150\,\text{m}} \\ F_{\text{stop}} &= 784\,\text{N} - (-31330\,\text{N}) \\ F_{\text{stop}} &= 784\,\text{N} + 31330\,\text{N} = 3.21\times10^4\,\text{N} \end{aligned}$$

Part (b):

With stopping distance $d = 0.300\,\text{m}$:

$$\begin{aligned} F_{\text{stop}} - mg &= \frac{-470\,\text{J}}{0.300\,\text{m}} \\ F_{\text{stop}} &= 784\,\text{N} + \frac{470\,\text{J}}{0.300\,\text{m}} \\ F_{\text{stop}} &= 784\,\text{N} + 1567\,\text{N} = 2.35\times10^3\,\text{N} \end{aligned}$$

Part (c):

Weight of person: $W = mg = 784\,\text{N}$

For part (a):

$$\frac{F_{\text{stop}}}{W} = \frac{3.21\times10^4\,\text{N}}{784\,\text{N}} = 41.0$$

For part (b):

$$\frac{F_{\text{stop}}}{W} = \frac{2.35\times10^3\,\text{N}}{784\,\text{N}} = 3.00$$

Discussion

(a) Landing stiffly produces a force of about 32,100 N, which is 41 times the person's weight! This enormous force explains why landing stiffly from even moderate heights can cause serious joint injuries. (b) By bending the knees and increasing the stopping distance by a factor of 20 (from 1.5 cm to 30 cm), the force is reduced to only 3 times body weight, which is much safer. (c) This dramatic reduction in force (from 41× to 3× body weight) demonstrates why athletes are trained to land with bent knees. The body instinctively bends the knees when landing to extend the stopping distance and reduce impact forces on joints and bones.

Jogging on hard surfaces with insufficiently padded shoes produces large forces in the feet and legs. (a) Calculate the magnitude of the force needed to stop the downward motion of a jogger's leg, if his leg has a mass of 13.0 kg, a speed of 6.00 m/s, and stops in a distance of 1.50 cm. (Be certain to include the weight of the 75.0-kg jogger's body.) (b) Compare this force with the weight of the jogger.

Show Solution

Strategy

The work-energy theorem applies: the net work equals the change in kinetic energy. The net force includes both the stopping force (upward) and the weight of the jogger (downward).

Solution

Part (a):

The kinetic energy of the leg is:

$$\mathrm{KE}=12(13.0\mathrm{kg})(6.00\mathrm{m/s})^2=234\mathrm{J}$$

Using the work-energy theorem with d =1.50 cm =0.0150m:

$$(F-mg)d=-\mathrm{KE}$$

where F is the upward stopping force and mg is the jogger's weight. Solving for F:

$$F = mg-\mathrm{KE}d \quad F = (75.0\mathrm{kg})(9.80\mathrm{m/s}^2)+234\mathrm{J}0.0150\mathrm{m} \quad F = 735\mathrm{N}+15600\mathrm{N}=1.64\times10^4\mathrm{N}$$

Part (b):

The jogger's weight is:

$$W=mg=(75.0\mathrm{kg})(9.80\mathrm{m/s}^2)=735\mathrm{N}$$

The ratio is:

$$FW=1.64\times10^4\mathrm{N}735\mathrm{N}=22.3$$

Discussion

(a) The force needed to stop the leg is approximately 16,400 N. (b) This force is about 22 times the jogger's weight! This explains why jogging on hard surfaces can lead to joint and bone injuries, especially with inadequate shoe cushioning.

(a) Calculate the energy in kJ used by a 55.0-kg woman who does 50 deep knee bends in which her center of mass is lowered and raised 0.400 m. (She does work in both directions.) You may assume her efficiency is 20%. (b) What is the average power consumption rate in watts if she does this in 3.00 min?

Show Solution

Strategy

During each knee bend, the woman does work against gravity when raising her center of mass. Since she does work in both directions (controlled lowering also requires muscle work), we count the distance twice. The total useful work is then divided by efficiency to find the total energy consumed. Power is energy divided by time.

Solution

Part (a):

Useful work per knee bend (raising and lowering):

$$W_{\mathrm{one}}=2mgh=2(55.0\mathrm{kg})(9.80\mathrm{m/s}^2)(0.400\mathrm{m})=431\mathrm{J}$$

Total useful work for 50 knee bends:

$$W_{\mathrm{useful}}=50\times431\mathrm{J}=21550\mathrm{J}=21.6\ \mathrm{kJ}$$

Total energy consumed (with 20% efficiency):

$$E_{\mathrm{total}}=W_{\mathrm{useful}}\mathrm{Efficiency}=21.6\ \mathrm{kJ}0.20=108\ \mathrm{kJ}$$

Part (b):

Time in seconds:

$$t=(3.00\ \mathrm{min})(60\mathrm{s/min})=180\mathrm{s}$$

Average power consumption:

$$P=E_{\mathrm{total}}t=108\times10^3\mathrm{J}180\mathrm{s}=600\ \mathrm{W}\approx599\ \mathrm{W}$$

Discussion

(a) The woman consumes approximately 108 kJ of energy to perform 50 deep knee bends, accounting for her 20% efficiency. (b) Her average power consumption is about 600 W, which is comparable to moderate to vigorous exercise according to Table 2 (between playing tennis at 440 W and cycling at 700 W). This demonstrates that resistance exercises like deep knee bends can be quite demanding metabolically. The 80% of energy that doesn't go into useful work (86.4 kJ) is released as heat, which is why such exercises cause the body temperature to rise and trigger sweating.

Kanellos Kanellopoulos flew 119 km from Crete to Santorini, Greece, on April 23, 1988, in the *Daedalus 88*, an aircraft powered by a bicycle-type drive mechanism (see Figure 5). His useful power output for the 234-min trip was about 350 W. Using the efficiency for cycling from the module on Conservation of Energy, calculate the food energy in kilojoules he metabolized during the flight.

The Daedalus 88 in flight. (credit: NASA photo by Beasley)

Show Solution

Strategy

From the Conservation of Energy module, Table 2 shows that cycling has an efficiency of 20%. We can use $\text{Eff} = \frac{P_{\text{useful}}}{P_{\text{metabolic}}}$ to find the metabolic power, then multiply by time.

Solution

The metabolic power is:

$$P_{\text{metabolic}} = \frac{P_{\text{useful}}}{\text{Eff}} = \frac{350\text{ W}}{0.20} = 1750\text{ W}$$

The time in seconds is:

$$t = (234\text{ min})(60\text{s/min}) = 14040\text{s}$$

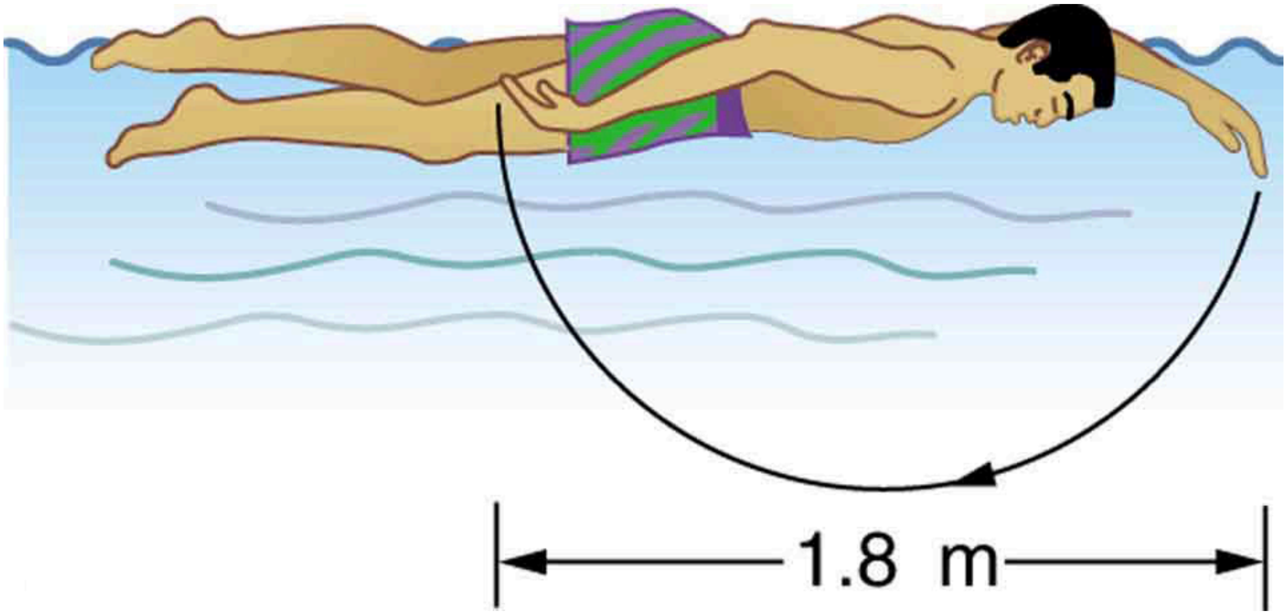
The food energy metabolized is:

$$E = P_{\text{metabolic}} \times t = (1750\text{ W})(14040\text{s}) = 2.46 \times 10^7\text{J} = 24600\text{ kJ}$$

Discussion

The pilot metabolized approximately 24,600 kJ (about 5900 kcal) of food energy during the 234-minute flight, which is about 2.4 times the daily recommended caloric intake. This demonstrates the extreme physical demands of human-powered flight.

The swimmer shown in Figure 6 exerts an average horizontal backward force of 80.0 N with his arm during each 1.80 m long stroke. (a) What is his work output in each stroke? (b) Calculate the power output of his arms if he does 120 strokes per minute.



A person swimming with a stroke of 1.80m

Show Solution

Strategy

Work is force times distance when the force is in the direction of motion. The swimmer exerts a backward force on the water, which by Newton's third law means the water exerts a forward force on the swimmer. Power is work per unit time.

Solution

Part (a):

Work per stroke:

$$W = Fd = (80.0\text{N})(1.80\text{m}) = 144\text{J}$$

Part (b):

Work per minute (120 strokes):

$$W_{\text{total}} = (120\text{ strokes})(144\text{J/stroke}) = 17280\text{J}$$

Power output:

$$P = \frac{W_{\text{total}}}{t} = \frac{17280\text{J}}{60\text{s}} = 288\text{ W}$$

Discussion

(a) The swimmer does 144 J of work per stroke. (b) His power output is 288 W, which is reasonable for swimming. From Table 2, swimming breaststroke consumes about 475 W of metabolic power. If the swimmer's efficiency is around 60% (288 W useful / 475 W total), this would be quite high, suggesting this is a skilled swimmer. However, the actual metabolic power could be higher, giving a more typical efficiency of 20-30%. The work calculated represents only the mechanical work done against the water during the propulsive phase of the stroke.

Mountain climbers carry bottled oxygen when at very high altitudes. (a) Assuming that a mountain climber uses oxygen at twice the rate for climbing 116 stairs per minute (because of low air temperature and winds), calculate how many liters of oxygen a climber would need for 10.0 h of climbing. (These are liters at sea level.) Note that only 40% of the inhaled oxygen is utilized; the rest is exhaled. (b) How much useful work does the climber do if he and his equipment have a mass of 90.0 kg and he gains 1000 m of altitude? (c) What is his efficiency for the 10.0-h climb?

Show Solution

Strategy

From Table 2, climbing stairs (116/min) consumes 1.96 L O_2/min. At high altitude with wind, this doubles to 3.92 L/min. Since only 40% is utilized, the climber must inhale more than this.

Solution

Part (a):

Oxygen consumption rate: $2 \times 1.96 \text{ L/min} = 3.92 \text{ L/min}$

Since only 40% is utilized, the breathing rate must be:

$$\text{Breathing rate} = \frac{3.92 \text{ L/min}}{0.40} = 9.80 \text{ L/min}$$

For 10.0 hours:

$$V = (9.80 \text{ L/min})(10.0 \text{ h})(60 \text{ min/h}) = 5880 \text{ L} \approx 5.88 \times 10^3 \text{ L}$$

Part (b):

Useful work:

$$W = mgh = (90.0 \text{kg})(9.80 \text{m/s}^2)(1000 \text{m}) = 8.82 \times 10^5 \text{J}$$

Part (c):

Energy metabolized (using 20 kJ per liter of O_2 consumed):

$$E_{\text{in}} = (3.92 \text{ L/min})(600 \text{ min})(20 \text{ kJ/L}) = 4.70 \times 10^4 \text{ kJ} = 4.70 \times 10^7 \text{J}$$

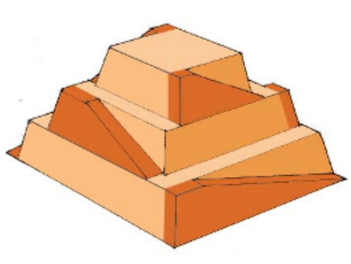
Efficiency:

$$\text{Eff} = \frac{W}{E_{\text{in}}} = \frac{8.82 \times 10^5 \text{J}}{4.70 \times 10^7 \text{J}} = 0.0188 = 1.88\%$$

Discussion

(a) The climber needs approximately 5880 liters of oxygen. (b) The useful work done is approximately 882 kJ. (c) The efficiency is only about 1.9%, which is very low even for human activities. This low efficiency is due to the extreme conditions at high altitude.

The awe-inspiring Great Pyramid of Cheops was built more than 4500 years ago. Its square base, originally 230 m on a side, covered 13.1 acres, and it was 146 m high, with a mass of about $7 \times 10^9 \text{kg}$. (The pyramid's dimensions are slightly different today due to quarrying and some sagging.) Historians estimate that 20 000 workers spent 20 years to construct it, working 12-hour days, 330 days per year. (a) Calculate the gravitational potential energy stored in the pyramid, given its center of mass is at one-fourth its height. (b) Only a fraction of the workers lifted blocks; most were involved in support services such as building ramps (see Figure 7), bringing food and water, and hauling blocks to the site. Calculate the efficiency of the workers who did the lifting, assuming there were 1000 of them and they consumed food energy at the rate of 300 kcal/h. What does your answer imply about how much of their work went into block-lifting, versus how much work went into friction and lifting and lowering their own bodies? (c) Calculate the mass of food that had to be supplied each day, assuming that the average worker required 3600 kcal per day and that their diet was 5% protein, 60% carbohydrate, and 35% fat. (These proportions neglect the mass of bulk and nondigestible materials consumed.)



Ancient pyramids were probably constructed using ramps as simple machines. (credit: Franck Monnier, Wikimedia Commons)

Show Solution

Strategy

For part (a), we calculate the potential energy using PE = mgh where h is the height of the center of mass (one-fourth the pyramid height). For part (b), we find the total energy consumed by the workers and compare it to the useful work done. For part (c), we use the energy content of different nutrients.

Solution

Part (a):

Height of center of mass:

$$h = \frac{146\text{m}}{4} = 36.5\text{m}$$

Gravitational potential energy:

$$\text{PE} = mgh = (7\times10^{9}\text{kg})(9.80\text{m/s}^{2})(36.5\text{m}) = 2.50\times10^{12}\text{J}$$

Part (b):

Total working time for 1000 workers over 20 years:

$$t = (1000\text{ workers})(20\text{ years})(330\text{ days/year})(12\text{ h/day}) = 7.92\times10^{7}\text{ worker-hours}$$

Total energy consumed by workers:

$$E_{\text{in}} = (7.92\times10^{7}\text{ h})(300\text{ kcal/h})(4184\text{ J/kcal}) = 9.94\times10^{13}\text{J}$$

Efficiency:

$$\text{Eff} = \frac{W_{\text{out}}}{E_{\text{in}}} = \frac{2.50\times10^{12}\text{J}}{9.94\times10^{13}\text{J}} = 0.0252 = 2.52\%$$

Part (c):

Total daily food energy for 20,000 workers:

$$E_{\text{daily}} = (20000\text{ workers})(3600\text{ kcal/worker}) = 7.20\times10^{7}\text{ kcal}$$

Energy content of nutrients:

- Protein: 4 kcal/g
- Carbohydrate: 4 kcal/g
- Fat: 9 kcal/g

Mass of each component:

$$m_{\text{protein}} = \frac{(0.05)(7.20\times10^{7}\text{ kcal})}{4\text{ kcal/g}} = 9.0\times10^{5}\text{ g} = 900\text{kg} \quad m_{\text{carb}} = \frac{(0.60)(7.20\times10^{7}\text{ kcal})}{4\text{ kcal/g}} = 1.08\times10^{7}\text{ g} = 10800\text{kg} \quad m_{\text{fat}} = \frac{(0.35)(7.20\times10^{7}\text{ kcal})}{9\text{ kcal/g}} = 2.80\times10^{6}\text{ g} = 2800\text{kg}$$

Total mass of food:

$$m_{\text{total}} = 900 + 10800 + 2800 = 14500\text{kg} \approx 1.4\times10^{4}\text{kg}$$

Discussion

(a) The pyramid stores an enormous amount of gravitational potential energy: 2.50×10^{12} J. (b) The efficiency of only 2.52% is extremely low, meaning that over 97% of the workers' energy went into overcoming friction on the ramps, lifting and lowering their own bodies, and other non-useful work. This low efficiency is not surprising given the primitive tools and methods available, the use of long ramps (which reduce force but increase distance), and the fact that workers had to climb up and down repeatedly. (c) The daily food requirement of 14 metric tons (about 14,500 kg) for 20,000 workers represents a massive logistical challenge. This quantity of food had to be grown, transported, and distributed daily, which helps explain why most of the workers were involved in support services rather than actually lifting blocks.

(a) How long can you play tennis on the 800 kJ (about 200 kcal) of energy in a candy bar? (b) Does this seem like a long time? Discuss why exercise is necessary but may not be sufficient to cause a person to lose weight.

Show Solution

Strategy

From Table 2, playing tennis consumes 440 W. We use $t = E/P$ to find the time.

Solution

Part (a):

$$t = \frac{E}{P} = \frac{800\times10^{3}\text{J}}{440\text{ W}} = 1818\text{s} \approx 30.3\text{ min}$$

Discussion

(a) You can play tennis for approximately 30 minutes on the energy from a candy bar. (b) This seems like a fairly short time for the amount of energy in a relatively small candy bar. This illustrates why exercise alone may not be sufficient for weight loss—it's often easier to consume calories than to burn them off. For example, eating a candy bar takes minutes, but burning it off requires a half hour of vigorous tennis. Additionally, exercise increases appetite, which can lead to consuming more calories. Sustainable weight management typically requires both regular exercise AND mindful eating habits.

Glossary

metabolic rate
: the rate at which the body uses food energy to sustain life and to do different activities

basal metabolic rate
: the total energy conversion rate of a person at rest

useful work
: work done on an external system

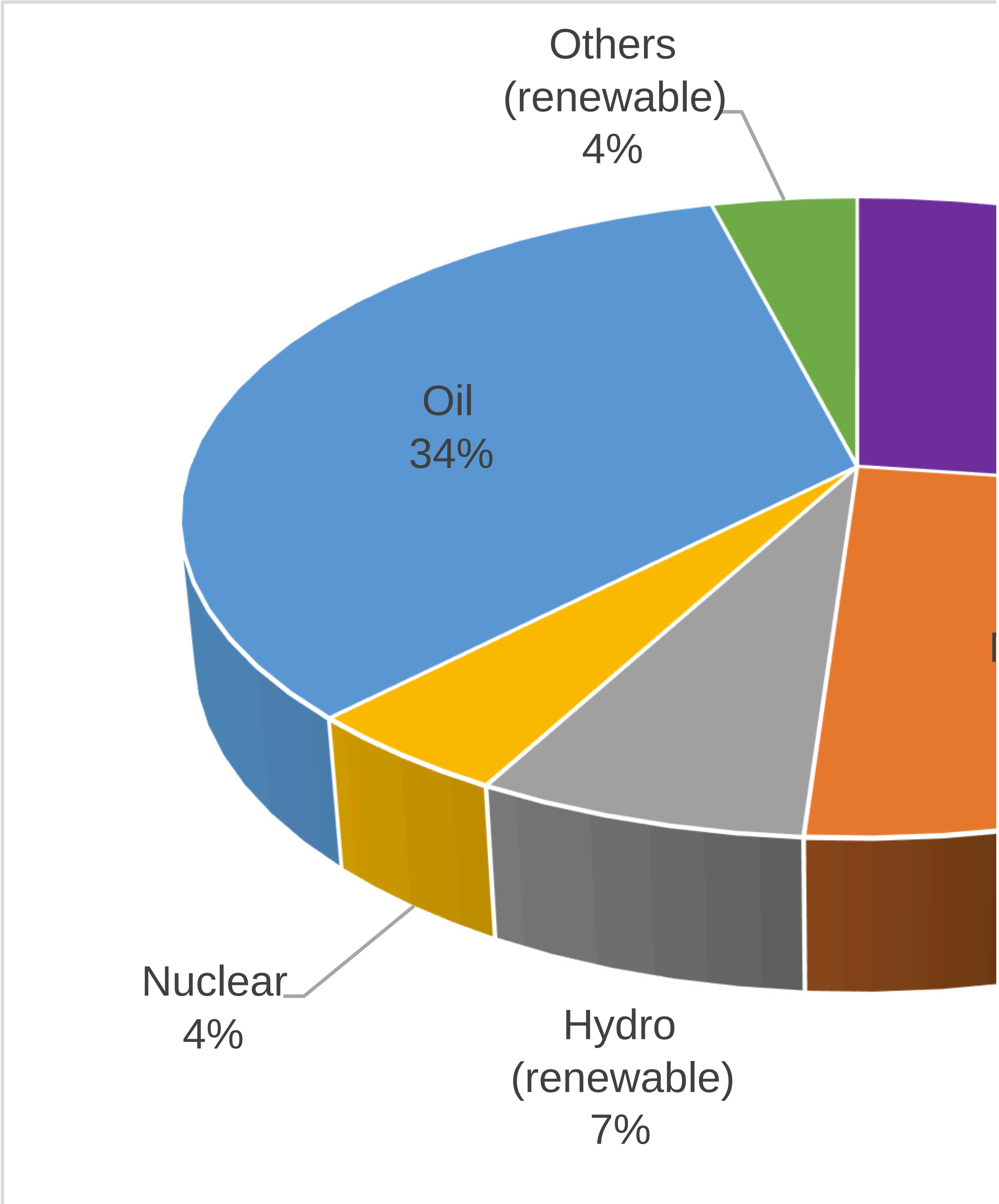
World Energy Use

- Describe the distinction between renewable and nonrenewable energy sources.
- Explain why the inevitable conversion of energy to less useful forms makes it necessary to conserve energy resources.

Energy is an important ingredient in all phases of society. We live in a very interdependent world, and access to adequate and reliable energy resources is crucial for economic growth and for maintaining the quality of our lives. But current levels of energy consumption and production are not sustainable. About 40% of the world’s energy comes from oil, and much of that goes to transportation uses. Oil prices are dependent as much upon new (or foreseen) discoveries as they are upon political events and situations around the world. The U.S., with 4.5% of the world’s population, consumes 24% of the world’s oil production per year; 66% of that oil is imported!

Renewable and Nonrenewable Energy Sources

The principal energy resources used in the world are shown in Figure 1. The fuel mix has changed over the years but now is dominated by oil, although natural gas and solar contributions are increasing. **Renewable forms of energy** are those sources that cannot be used up, such as water, wind, solar, and biomass. About 85% of our energy comes from nonrenewable **fossil fuels**—oil, natural gas, coal. The likelihood of a link between global warming and fossil fuel use, with its production of carbon dioxide through combustion, has made, in the eyes of many scientists, a shift to non-fossil fuels of utmost importance—but it will not be easy.

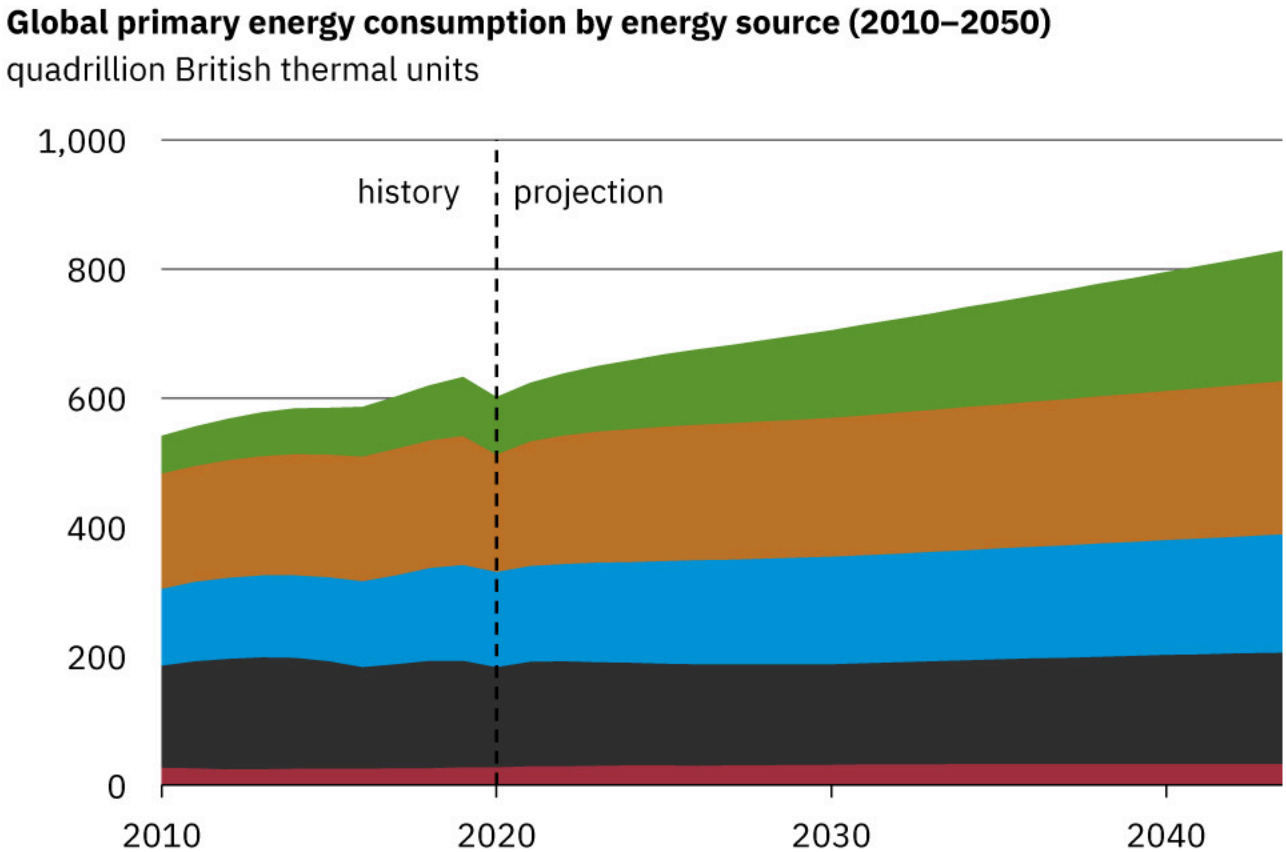


World energy consumption (2018) based on data from BP Statistical Review of World Energy (credit:Veillette)

The World's Growing Energy Needs

World energy consumption continues to rise, especially in the developing countries. (See Figure 2.) Global demand for energy has tripled in the past 50 years and might triple again in the next 30 years. While much of this growth will come from the rapidly booming economies of China and India, many of the developed countries, especially those in Europe, are hoping to meet their energy needs by expanding the use of renewable sources. Although presently only a small percentage, renewable energy is growing very fast, especially wind energy. For example, Germany plans to meet 65% of its electricity and 30% of its overall energy needs with renewable resources by the year 2030. (See Figure 3.) Energy is a key constraint in the rapid economic growth of China and India. In 2003, China surpassed Japan as the world's second largest consumer of oil. However, over 1/3 of this is imported. Unlike most

Western countries, coal dominates the commercial energy resources of China, accounting for 2/3 of its energy consumption. In 2009 China surpassed the United States as the largest generator of CO_2. In India, the main energy resources are biomass (wood and dung) and coal. Half of India's oil is imported. About 70% of India's electricity is generated by highly polluting coal. Yet there are sizeable strides being made in renewable energy. India has a rapidly growing wind energy base, and it has the largest solar cooking program in the world. China has invested substantially in building solar collection farms as well as hydroelectric plants.



Past and projected world energy use (source: Based on data from U.S. Energy Information Administration, 2021)

Solar cell arrays in Manchester, Vermont, USA (credit: MarkBuckawicki, Wikimedia)

Table 1 displays the 2020 commercial energy mix by country for some of the prime energy users in the world. While non-renewable sources dominate, some countries get a sizeable percentage of their electricity from renewable resources. For example, about two-thirds of New Zealand's electricity demand is met by hydroelectric. Only 10% of the U.S. electricity is generated by renewable resources, primarily hydroelectric. It is difficult to determine total sources and consumers of energy in many countries, and estimates vary somewhat by data source and type of measurement.

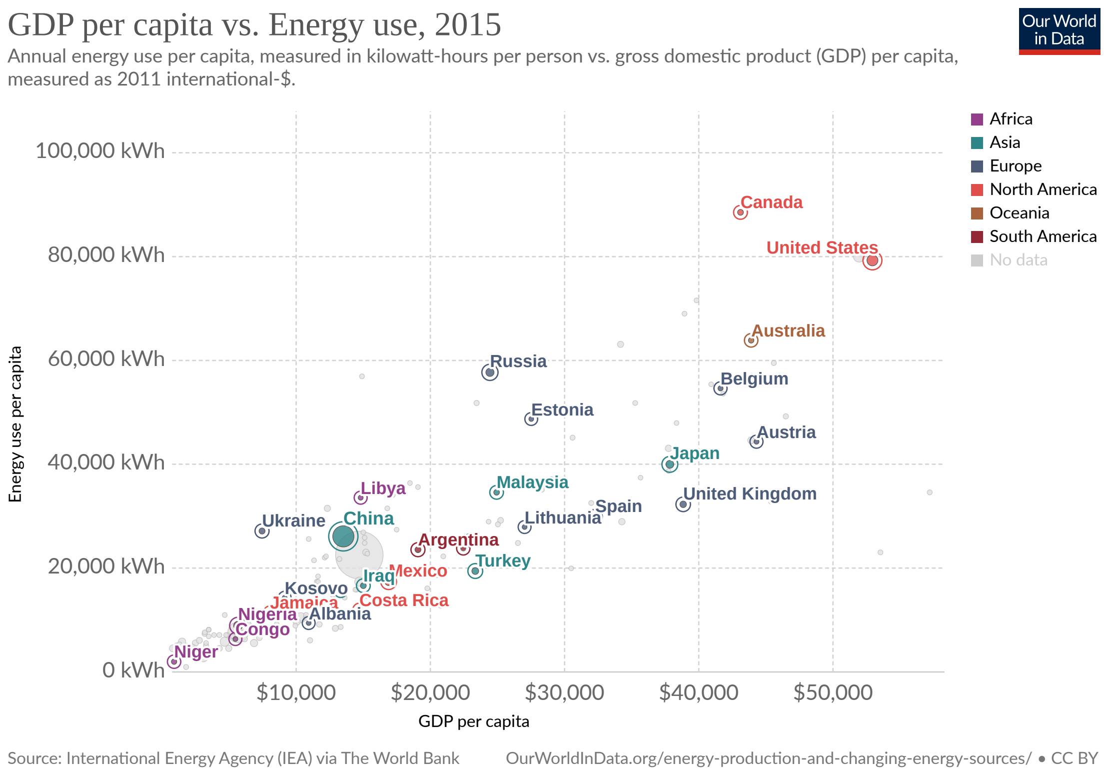
Table: Energy Consumption—Selected Countries (2020)

Country	Consumption in EJ (10^{18} J)	Oil	Natural Gas	Coal	Nuclear	Hydro	Other Renewables
Australia	5.6	1.8%	1.5%	1.7%	0%	0.1%	0.5%
Brazil	12	4.6%	1.2%	0.6%	0.1%	3.5%	2%
China	145.5	28.5%	11.9%	82.3%	3.3%	11.7%	7.8%
Egypt	3.7	1.3%	2.1%	0.03%	0%	0.1%	0.1%
Germany	12.1	4.2%	3.1%	1.9%	0.6%	0.2%	2.2%
India	31.99	9%	2.2%	17.5%	0.4%	1.5%	1.4%
Indonesia	8.1	2.8%	1.5%	3.3%	0%	0.2%	0.4%
Japan	17	6.5%	3.8%	4.6%	0.4%	0.7%	1.1%
United Kingdom	6.9	2.4%	2.6%	0.2%	0.5%	0.1%	1.2%
Russia	28.3	6.4%	14.8%	3.2%	1.9%	1.9%	0.5%
U.S.	87.8	32.5%	30%	9.2%	7.4%	2.6%	6.2%

Energy and Economic Well-being

The last two columns in this table examine the energy and electricity use per capita. Economic well-being is dependent upon energy use, and in most countries higher standards of living, as measured by GDP (gross domestic product) per capita, are matched by higher levels of energy consumption per capita. This is borne out in Figure 4. Increased efficiency of energy use will change this dependency. A global problem is balancing energy resource development against the harmful effects upon the environment in its extraction and use.

New and diversified energy sources do, however, greatly increase economic opportunity and stability. First, the extensive employment opportunities in renewable energy make it one of the most sustainable and secure fields to enter. Second, renewable energy provides countries and localities with increased levels of resiliency in the face of natural disasters, conflict, or other disruptions. The 21st century has already seen major economic impacts from energy disruptions: Hurricane Katrina, Superstorm Sandy, various wildfires, Hurricane Maria, and the 2021 Texas Winter Storm demonstrate the vulnerability of United States power systems. Diversifying energy sources through renewables and other fossil-fuel alternatives brings power grids and transportation systems back online much more quickly, saving lives and enabling a more swift return to economic operations. And as critical emerging information infrastructure, such as data centers, requires more of the world’s energy, supplying those growing systems during normal operations and crises will be increasingly important.



Power consumption per capita versus GDP per capita for various countries. Note the increase in energy usage with increasing GDP. (2015, credit: International Energy Agency (IEA) via The World Bank)

Conserving Energy

As we finish this chapter on energy and work, it is relevant to draw some distinctions between two sometimes misunderstood terms in the area of energy use. As has been mentioned elsewhere, the “law of the conservation of energy” is a very useful principle in analyzing physical processes. It is a statement that cannot be proven from basic principles, but is a very good bookkeeping device, and no exceptions have ever been found. It states that the total amount of energy in an isolated system will always remain constant. Related to this principle, but remarkably different from it, is the important philosophy of energy conservation. This concept has to do with seeking to decrease the amount of energy used by an individual or group through (1) reduced activities (e.g., turning down thermostats, driving fewer kilometers) and/or (2) increasing conversion efficiencies in the performance of a particular task—such as developing and using more efficient room heaters, cars that have greater miles-per-gallon ratings, energy-efficient compact fluorescent lights, etc.

Since energy in an isolated system is not destroyed or created or generated, one might wonder why we need to be concerned about our energy resources, since energy is a conserved quantity. The problem is that the final result of most energy transformations is waste heat transfer to the environment and conversion to energy forms no longer useful for doing work. To state it in another way, the potential for energy to produce useful work has been “degraded” in the energy transformation.

Section Summary

- The relative use of different fuels to provide energy has changed over the years, but fuel use is currently dominated by oil, although natural gas and solar contributions are increasing.
- Although non-renewable sources dominate, some countries meet a sizeable percentage of their electricity needs from renewable resources.
- The United States obtains only about 10% of its energy from renewable sources, mostly hydroelectric power.
- Economic well-being is dependent upon energy use, and in most countries higher standards of living, as measured by GDP (Gross Domestic Product) per capita, are matched by higher levels of energy consumption per capita.
- Even though, in accordance with the law of conservation of energy, energy can never be created or destroyed, energy that can be used to do work is always partly converted to less useful forms, such as waste heat to the environment, in all of our uses of energy for practical purposes.

Conceptual Questions

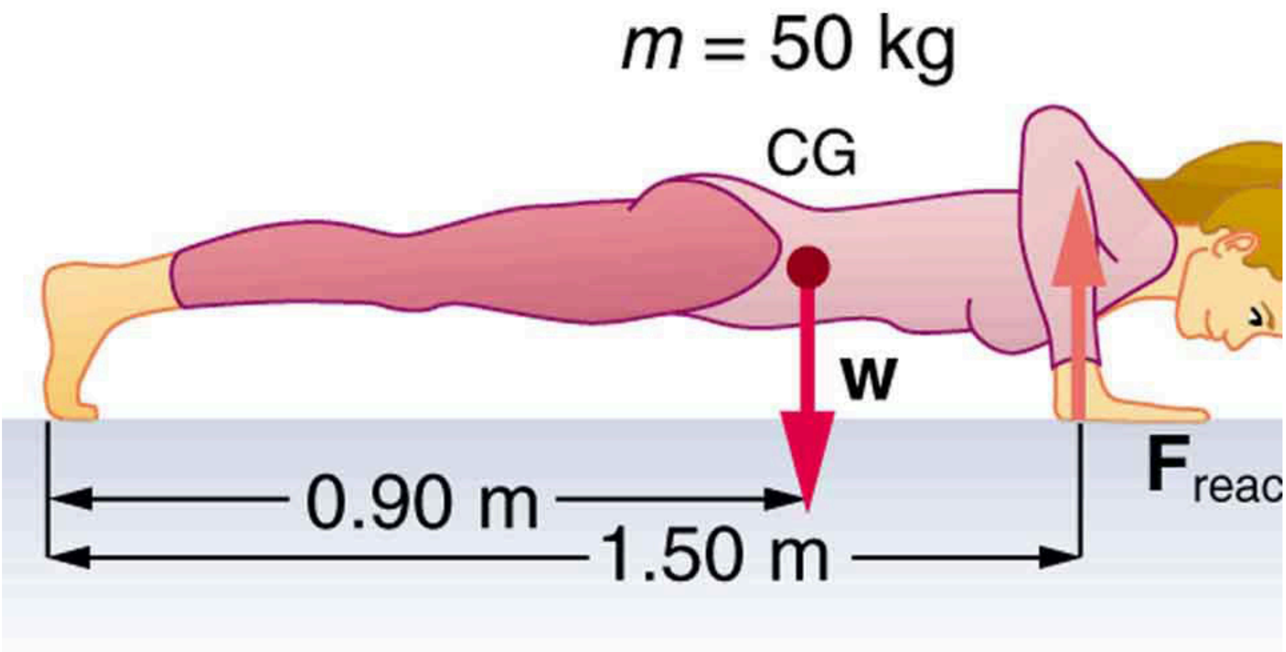
What is the difference between energy conservation and the law of conservation of energy? Give some examples of each.

If the efficiency of a coal-fired electrical generating plant is 35%, then what do we mean when we say that energy is a conserved quantity?

Problems & Exercises

Integrated Concepts

(a) Calculate the force the woman in Figure 5 exerts to do a push-up at constant speed, taking all data to be known to three digits. (b) How much work does she do if her center of mass rises 0.240 m? (c) What is her useful power output if she does 25 push-ups in 1 min? (Should work done lowering her body be included? See the discussion of useful work in Work, Energy, and Power in Humans .



Forces involved in doing push-ups. The woman’s weight acts as a force exerted downward on her center of gravity (CG).

Show Solution

Strategy

For part (a), use torque equilibrium about the feet to find the force on the arms. From the figure, the center of gravity is 0.90 m from the feet, and the arms support the body at 1.50 m from the feet. For part (b), calculate work done against gravity as the center of mass rises. For part (c), find power from total work done in a given time.

Solution

(a) Taking torques about the feet (where normal force acts), with counterclockwise positive:

$$\sum \tau = 0$$
$$F(1.50\text{m}) - w(0.90\text{m}) = 0$$

From the geometry and the given answer, we can deduce the woman’s weight. Solving for F:

$$F = \frac{0.90\text{m}}{1.50\text{m}} w = 0.600w$$

If F = 294 N (given in answer), then:

$$w = \frac{294\text{N}}{0.600} = 490\text{N}$$

This corresponds to a mass of:

$$m = \frac{490\text{N}}{9.80\text{m/s}^2} = 50.0\text{kg}$$

Therefore, the force she exerts on the floor with her arms is:

$$F = 294\text{N}$$

(b) Work done raising her center of mass 0.240 m:

$$W = mgh = (490\text{N})(0.240\text{m}) = 118\text{J}$$

(c) Power output for 25 push-ups in 1.00 min (60.0 s):

Total work for 25 push-ups:

$$W_{total}=25\times 118 J=2950 J$$
$$P=\frac{W_{total}}{t}=\frac{2950 J}{60.0 s}=49.2\ \backslash W\approx 49.0\ \backslash W$$

Discussion

The force exerted by the woman's arms (294 N) is 60% of her body weight, which makes sense given the lever arm geometry. The work done per push-up (118 J) accounts for raising her center of mass, and the power output of approximately 49 W is reasonable for sustained exercise. Note that work done lowering the body is not included in useful work output, as discussed in the Work, Energy, and Power in Humans section—the muscles are doing negative work while lowering, which doesn't count toward useful power output.

Answer

(a) The woman exerts a force of **294 N** on the floor with her arms.

(b) She does **118 J** of work per push-up.

(c) Her useful power output is **49.0 W**.

Integrated Concepts

A 75.0-kg cross-country skier is climbing a 3.0° slope at a constant speed of 2.00 m/s and encounters air resistance of 25.0 N. Find his power output for work done against the gravitational force and air resistance. (b) What average force does he exert backward on the snow to accomplish this? (c) If he continues to exert this force and to experience the same air resistance when he reaches a level area, how long will it take him to reach a velocity of 10.0 m/s?

Show Solution

Strategy

For part (a), at constant speed, the skier must exert force to overcome both the component of gravity down the slope and air resistance. Power is $P=Fv$.

Solution

(a) Force components and power output:

Force components:

- Gravity down slope: $F_g=mg\sin\theta=(75.0 kg)(9.80 m/s^2)\sin 3.0^\circ=38.5 N$
- Air resistance: $F\_air=25.0 N$

Total force needed:

$$F=F_g+F_{air}=38.5 N+25.0 N=63.5 N$$

Power output:

$$P=Fv=(63.5 N)(2.00 m/s)=127\ W$$

(b) Force exerted backward on the snow:

The force exerted backward on the snow equals the total force needed (from Newton's third law):

$$F=63.5 N\approx 64 N$$

(c) Time to reach 10.0 m/s on level ground:

On level ground, the net force is:

$$F_{net}=F-F_{air}=63.5 N-25.0 N=38.5 N$$

Using $F=ma$:

$$a=\frac{F_{net}}{m}=\frac{38.5 N}{75.0 kg}=0.513 m/s^2$$

Time to reach 10.0 m/s from 2.00 m/s:

$$t=\frac{\Delta v}{a}=\frac{10.0 m/s-2.00 m/s}{0.513 m/s^2}=15.6 s$$

Discussion

The power output of 127 W is reasonable for sustained moderate exercise like cross-country skiing—it's roughly equivalent to a 150-W light bulb, which is sustainable for extended periods. The force of approximately 64 N (about 14 lbs) that the skier must push backward is relatively modest, showing that the primary effort is overcoming the gravitational component rather than air resistance at this moderate speed.

Part (c) reveals an interesting result: when the skier reaches level ground while maintaining the same pushing force, the acceleration becomes 0.513 m/s², and it takes 15.6 seconds to reach 10.0 m/s. This demonstrates how much easier it is to maintain speed on level ground compared to climbing a slope—the same force that barely maintains constant speed on the 3° slope produces significant acceleration on flat terrain.

Answer

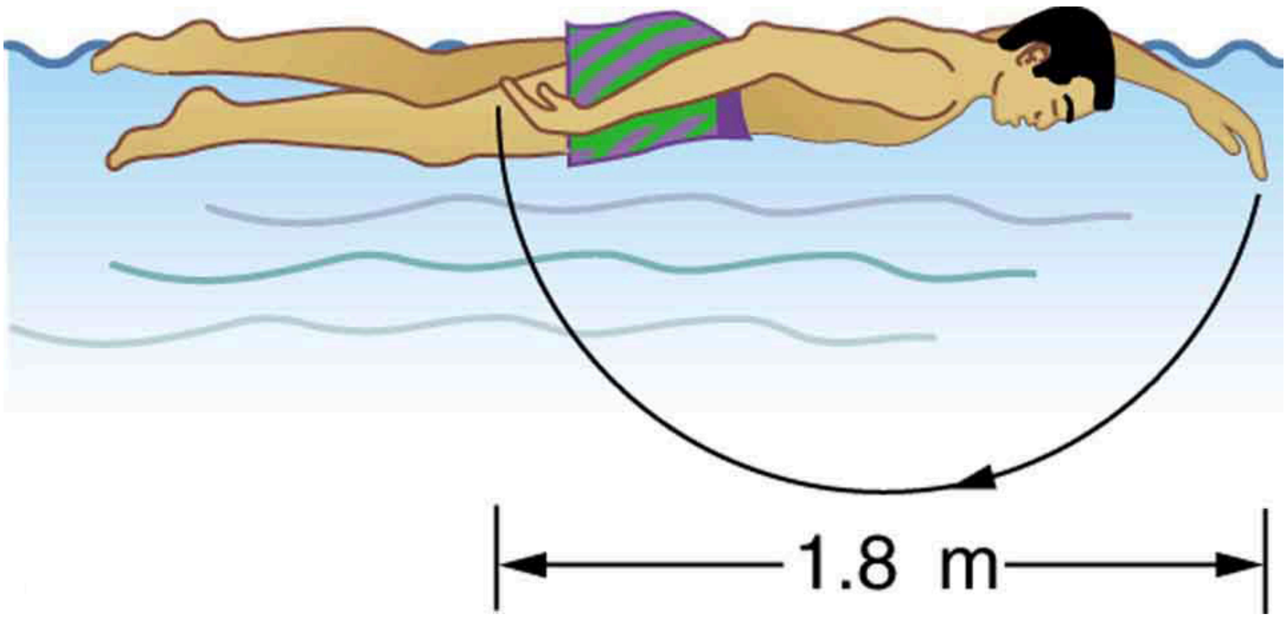
(a) The skier's power output is **127 W**.

(b) The force exerted backward on the snow is approximately **64 N**.

(c) It will take approximately **15.6 seconds** to accelerate to 10.0 m/s on level ground.

Integrated Concepts

The 70.0-kg swimmer in Figure 6 starts a race with an initial velocity of 1.25 m/s and exerts an average force of 80.0 N backward with his arms during each 1.80 m long stroke. (a) What is his initial acceleration if water resistance is 45.0 N? (b) What is the subsequent average resistance force from the water during the 5.00 s it takes him to reach his top velocity of 2.50 m/s? (c) Discuss whether water resistance seems to increase linearly with velocity.



A person swimming with a stroke of 1.80m

Show Solution

Strategy

Use Newton's second law for parts (a) and (b). For part (a), find the initial acceleration. For part (b), use kinematics and work-energy considerations.

Solution

(a) Initial net force and acceleration:

The swimmer exerts 80.0 N backward, water resists with 45.0 N:

$$F_{\text{net}} = 80.0\,\text{N} - 45.0\,\text{N} = 35.0\,\text{N}$$

$$a = \frac{F_{\text{net}}}{m} = \frac{35.0\,\text{N}}{70.0\,\text{kg}} = 0.500\,\text{m/s}^2$$

(b) Average resistance during acceleration phase:

Using kinematics, find average acceleration from 1.25 m/s to 2.50 m/s in 5.00 s:

$$a_{\text{avg}} = \frac{\Delta v}{\Delta t} = \frac{2.50\,\text{m/s} - 1.25\,\text{m/s}}{5.00\,\text{s}} = 0.250\,\text{m/s}^2$$

The net force is:

$$F_{\text{net}} = ma = (70.0\,\text{kg})(0.250\,\text{m/s}^2) = 17.5\,\text{N}$$

Average resistance force:

$$F_{\text{resist}} = F_{\text{applied}} - F_{\text{net}} = 80.0\,\text{N} - 17.5\,\text{N} = 62.5\,\text{N}$$

(c) Since the initial resistance at v = 1.25 m/s was 45.0 N and the average resistance during acceleration was 62.5 N, and the final velocity is 2.50 m/s (double the initial), the resistance increased by more than double. This suggests water resistance increases faster than linearly with velocity—consistent with drag force being proportional to v^2 at higher speeds.

Discussion

The initial acceleration of 0.500 m/s² is significant but manageable for a competitive swimmer. The increase in water resistance from 45.0 N to an average of 62.5 N during the acceleration phase demonstrates how drag forces increase with velocity. If resistance were linear with velocity (doubling velocity doubles resistance), we'd expect 90 N at v = 2.50 m/s. Since the average is only 62.5 N, and considering that drag is actually proportional to v^2, this result is physically consistent. At higher swimming speeds, drag becomes the dominant limiting factor, which is why swimmers focus on streamlining techniques to reduce resistance.

Answer

(a) The swimmer's initial acceleration is **0.500 m/s²**.

(b) The average water resistance during the acceleration phase is **62.5 N**.

(c) Water resistance appears to increase **faster than linearly with velocity**, consistent with the v^2 dependence expected for fluid drag.

Integrated Concepts

A toy gun uses a spring with a force constant of 300 N/m to propel a 10.0-g steel ball. If the spring is compressed 7.00 cm and friction is negligible: (a) How much force is needed to compress the spring? (b) To what maximum height can the ball be shot? (c) At what angles above the horizontal may a child aim to hit a target 3.00 m away at the same height as the gun? (d) What is the gun's maximum range on level ground?

Show Solution

Strategy

The spring's potential energy converts to kinetic energy, which then converts to potential and/or projectile kinetic energy.

Solution

(a) Maximum force (at full compression):

$$F = kx = (300\text{ N/m})(0.0700\,\text{m}) = 21.0\,\text{N}$$

(b) Maximum height the ball can reach:

Spring potential energy:

$$\text{PE}_{\text{spring}} = \frac{1}{2}kx^2 = \frac{1}{2}(300\text{ N/m})(0.0700\,\text{m})^2 = 0.735\,\text{J}$$

At maximum height, all energy is gravitational PE:

$$mgh = \text{PE}_{\text{spring}} \qquad h = \frac{0.735\,\text{J}}{(0.0100\,\text{kg})(9.80\,\text{m/s}^2)} = 7.50\,\text{m}$$

(c) Angles to hit a target 3.00 m away:

Initial velocity from spring energy:

$$v_0 = \sqrt{\frac{2\text{PE}_{\text{spring}}}{m}} = \sqrt{\frac{2(0.735\,\text{J})}{0.0100\,\text{kg}}} = 12.1\,\text{m/s}$$

For projectile motion with same initial and final height:

$$R = \frac{v_0^2 \sin 2\theta}{g}$$

Solving for angle when $R = 3.00\,\text{m}$:

$$\sin 2\theta = \frac{Rg}{v_0^2} = \frac{(3.00\,\text{m})(9.80\,\text{m/s}^2)}{(12.1\,\text{m/s})^2} = 0.201 \qquad 2\theta = 11.6^\circ \text{ or } 168.4^\circ \qquad \theta = 5.8^\circ \text{ or } 84.2^\circ$$

(d) Maximum range on level ground:

Maximum range occurs at $\theta = 45^\circ$:

$$R_{\max} = \frac{v_0^2}{g} = \frac{(12.1\,\text{m/s})^2}{9.80\,\text{m/s}^2} = 14.9\,\text{m}$$

Discussion

The maximum force of 21.0 N (about 4.7 lbs) is easily achievable for compressing a toy gun spring. The maximum height of 7.50 m is impressive for a toy—equivalent to a 2.5-story building! The two possible angles to hit a target (5.8° and 84.2°) demonstrate a fundamental property of projectile motion: any target within range can be hit at two complementary angles, one nearly horizontal and one nearly vertical. The low-angle shot reaches the target quickly, while the high-angle shot has a much longer flight time. The maximum range of 14.9 m (about 49 feet) shows this is quite a powerful toy gun, emphasizing why such toys need appropriate safety precautions.

Answer

(a) A force of **21.0 N** is needed to fully compress the spring.

(b) The ball can reach a maximum height of **7.50 m** when shot straight up.

(c) The child can aim at **5.8° or 84.2°** above horizontal to hit the target.

(d) The gun's maximum range on level ground is **14.9 m**.

Integrated Concepts

(a) What force must be supplied by an elevator cable to produce an acceleration of 0.800m/s^2 against a 200-N frictional force, if the mass of the loaded elevator is 1500 kg? (b) How much work is done by the cable in lifting the elevator 20.0 m? (c) What is the final speed of the elevator if it starts from rest? (d) How much work went into thermal energy?

Show Solution

Strategy

Use Newton's second law for part (a). For parts (b)-(d), use work-energy relationships.

Solution

(a) Using Newton's second law (upward positive):

$$F_{\text{cable}} - mg - f = ma$$
$$F_{\text{cable}} = m(g+a) + f = (1500 \text{kg})(9.80 \text{m/s}^2 + 0.800 \text{m/s}^2) + 200 \text{N}$$
$$F_{\text{cable}} = (1500 \text{kg})(10.6 \text{m/s}^2) + 200 \text{N} = 15900 \text{N} + 200 \text{N} = 16.1 \times 10^3 \text{N}$$

(b) Work done by the cable:

$$W_{\text{cable}} = F_{\text{cable}} \cdot d = (16.1 \times 10^3 \text{N})(20.0 \text{m}) = 3.22 \times 10^5 \text{J}$$

(c) Final speed using kinematics ($v_0 = 0$):

$$v^2 = v_{20} + 2ad = 0 + 2(0.800 \text{m/s}^2)(20.0 \text{m}) = 32.0 \text{ m}^2/\text{s}^2$$
$$v = 5.66 \text{m/s}$$

(d) Work done against friction (thermal energy):

$$W_{\text{friction}} = fd = (200 \text{N})(20.0 \text{m}) = 4.00 \times 10^3 \text{J} = 4.00 \text{ kJ}$$

Discussion

The cable must exert 16,100 N, which is about 1.1 times the elevator's weight, to provide the upward acceleration against friction. The total work of 322 kJ can be broken down into components: gravitational potential energy (294 kJ), kinetic energy (24 kJ), and thermal energy from friction (4.00 kJ). The final speed of 5.66 m/s (about 12.7 mph) is typical for elevators, providing a balance between efficiency and passenger comfort. The relatively small amount of energy lost to friction (about 1.2% of total work) indicates that modern elevator systems are quite efficient, though in practice, additional energy losses occur in the motor and cable system.

Answer

(a) The elevator cable must supply a force of **16,100 N** (or **16.1 kN**).

(b) The cable does **322 kJ** of work in lifting the elevator 20.0 m.

(c) The final speed of the elevator is **5.66 m/s**.

(d) **4.00 kJ** of work is converted to thermal energy due to friction.

Unreasonable Results

A car advertisement claims that its 900-kg car accelerated from rest to 30.0 m/s and drove 100 km, gaining 3.00 km in altitude, on 1.0 gal of gasoline. The average force of friction including air resistance was 700 N. Assume all values are known to three significant figures. (a) Calculate the car's efficiency. (b) What is unreasonable about the result? (c) Which premise is unreasonable, or which premises are inconsistent?

Show Solution

Strategy

Calculate the total energy needed (kinetic + potential + work against friction) and compare to the energy available from 1.0 gallon of gasoline (1.2×10^8 J from Table 1 in Conservation of Energy).

Solution

(a) Energy components needed:

Kinetic energy:

$$KE=\frac{1}{2}mv^2=\frac{1}{2}(900\text{kg})(30.0\text{m/s})^2=4.05\times10^5\text{J}$$

Potential energy gained:

$$PE=mgh=(900\text{kg})(9.80\text{m/s}^2)(3000\text{m})=2.65\times10^7\text{J}$$

Work done against friction:

$$W_f=fd=(700\text{N})(100000\text{m})=7.00\times10^7\text{J}$$

Total energy needed:

$$E_{\text{total}}=KE+PE+W_f=4.05\times10^5+2.65\times10^7+7.00\times10^7=9.69\times10^7\text{J}$$

Energy available from gasoline:

$$E_{\text{available}}=1.2\times10^8\text{J}$$

Efficiency:

$$\text{Eff}=\frac{E_{\text{total}}}{E_{\text{available}}}=\frac{9.69\times10^7\text{J}}{1.2\times10^8\text{J}}=0.808=80.8\%$$

Discussion

The calculated efficiency of 81% is unreasonably high and violates fundamental thermodynamic principles. Real gasoline engines are limited by the Carnot efficiency and practical considerations to about 25-30% efficiency, while the best diesel engines achieve 35-40%. Even the most efficient production cars (hybrid electric vehicles) rarely exceed 40% efficiency. An 81% efficiency would mean that only 19% of the fuel's energy is lost to heat, which contradicts the second law of thermodynamics for heat engines operating at these temperature ranges.

The claim is inconsistent with physical reality. Several factors reveal the impossibility:

First, gaining 3.00 km (about 1.86 miles) of altitude while traveling 100 km represents an average grade of 3%, which combined with overcoming friction over such distance requires substantial energy. Second, achieving a final speed of 30.0 m/s (67 mph) adds kinetic energy. The total energy requirement of 96.9 MJ is close to what 1.0 gallon of gasoline can provide (120 MJ), but this would require near-perfect efficiency—impossible for any heat engine.

In reality, such a trip would require approximately 3-4 gallons of gasoline for a typical car, or about 1.5-2 gallons for a highly efficient hybrid vehicle. The advertising claim is clearly false and misleading.

Answer

(a) The claimed efficiency is **80.8%**.

(b) This efficiency is **unreasonably high**—it violates thermodynamic limitations for heat engines and exceeds the efficiency of any real vehicle by a factor of 2-3.

(c) The premises are **inconsistent**. The fuel consumption of 1.0 gallon is far too low for the described trip. Realistically, this trip would require **3-4 gallons of gasoline** for a conventional car.

Unreasonable Results

Body fat is metabolized, supplying 9.30 kcal/g, when dietary intake is less than needed to fuel metabolism. The manufacturers of an exercise bicycle claim that you can lose 0.500 kg of fat per day by vigorously exercising for 2.00 h per day on their machine. (a) How many kcal are supplied by the metabolization of 0.500 kg of fat? (b) Calculate the kcal/min that you would have to utilize to metabolize fat at the rate of 0.500 kg in 2.00 h. (c) What is unreasonable about the results? (d) Which premise is unreasonable, or which premises are inconsistent?

Show Solution

Strategy

For part (a), multiply the mass of fat by the energy content per gram. For part (b), divide the total energy by the time in minutes. For parts (c) and (d), compare the calculated power output to realistic human metabolic rates from Table 2 in Work Energy And Power In Humans.

Solution

(a) Energy from metabolizing 0.500 kg of fat:

$$E = (0.500\text{ kg})(1000\text{ g/kg})(9.30\text{ kcal/g}) = 4650\text{ kcal} = 4.65 \times 10^3\text{ kcal}$$

(b) Time available: 2.00 h = 120 min

Energy utilization rate needed:

$$P = \frac{E}{t} = \frac{4650\text{ kcal}}{120\text{ min}} = 38.8\text{ kcal/min}$$

(c) and (d) According to Table 2 in Work Energy And Power In Humans, the highest sustainable metabolic rate listed is approximately 35 kcal/min for sprinting, which corresponds to about 2415 watts. The required rate of 38.8 kcal/min exceeds even this maximum sprint value.

Discussion

The claim that one can lose 0.500 kg of fat in 2.00 hours through vigorous exercise is physiologically unreasonable. The required metabolic rate of 38.8 kcal/min exceeds the maximum human power output for sprinting (about 35 kcal/min), and more importantly, no one can maintain a sprinting pace for 2 hours.

For context, elite marathon runners maintain approximately 15-20 kcal/min for 2+ hours, and even elite cyclists in races like the Tour de France average about 20-25 kcal/min over several hours. The manufacturer’s claim would require maintaining a power output higher than an all-out sprint for 120 minutes, which is physiologically impossible.

A realistic fat loss rate through exercise would be about 0.1-0.2 kg per 2-hour vigorous workout, not 0.500 kg. The manufacturer’s claim exaggerates the possible fat loss by a factor of 3-5.

Answer

(a) Metabolizing 0.500 kg of fat supplies **4650 kcal** (or **4.65×10^3 kcal**).

(b) You would need to metabolize fat at a rate of **38.8 kcal/min**.

(c) This rate **exceeds the maximum human power output** for sprinting and is unreasonably high for any sustained activity.

(d) The premise is **unreasonable**—it is physiologically impossible to maintain this power output for 2 hours. Realistic fat loss would be about **0.1-0.2 kg per 2-hour session**, not 0.500 kg.

Construct Your Own Problem

Consider a person climbing and descending stairs. Construct a problem in which you calculate the long-term rate at which stairs can be climbed considering the mass of the person, his ability to generate power with his legs, and the height of a single stair step. Also consider why the same person can descend stairs at a faster rate for a nearly unlimited time in spite of the fact that very similar forces are exerted going down as going up. (This points to a fundamentally different process for descending versus climbing stairs.)

Show Solution

Guidance for Constructing This Problem

When constructing this problem, consider the following framework:

Strategy

Use the relationship between power, work, and time. The work done climbing stairs is primarily against gravity (W = mgh), while descending involves negative work. Compare energy requirements for ascending versus descending.

Key Parameters to Define:

1. **Person’s mass**: Choose a realistic value (60-100 kg)
2. **Sustainable power output**: From Table 2 in Work Energy And Power In Humans, choose a sustainable long-term power output (e.g., 150-200 W for moderate sustained activity)
3. **Stair dimensions**: Typical stair step height is 0.18-0.20 m
4. **Time period**: Consider “long-term” as several hours

Calculations to Include:

1. Calculate the rate of climbing (steps per minute or floors per minute) based on:
   $$P = \frac{mgh}{t}$$

   where h is the total height climbed in time t

2. Determine how many steps can be climbed per minute given the person's sustainable power output

3. Compare ascending vs. descending:
    - **Ascending**: Muscles do positive work against gravity, consuming metabolic energy
    - **Descending**: Muscles do negative work (eccentric contraction), acting as brakes to control descent. This requires less metabolic energy even though similar forces are involved

Discussion Points:

- When climbing, chemical energy is converted to gravitational potential energy plus heat
- When descending, gravitational potential energy is converted primarily to heat through muscle friction and joint absorption—muscles work eccentrically to prevent falling
- Descending can be done faster and with less fatigue because it requires much less metabolic energy, even though the forces on muscles are similar
- The fundamental difference is that ascending requires continuous energy input to increase potential energy, while descending dissipates existing potential energy

Example Answer:

For a 70 kg person with sustainable power output of 175 W climbing stairs with 0.20 m step height:

- Rate of vertical climb: $P/(mg) = 175 \text{ W} / (70 \text{ kg} \times 9.8 \text{ m/s}^2) = 0.255 \text{ m/s}$
- Steps per minute: $(0.255 \text{ m/s} \times 60 \text{ s/min}) / 0.20 \text{ m} = 76$ steps/min
- This person could descend 2-3 times faster (150-200 steps/min) with less fatigue due to the lower metabolic cost of eccentric muscle work

Construct Your Own Problem

Consider humans generating electricity by pedaling a device similar to a stationary bicycle. Construct a problem in which you determine the number of people it would take to replace a large electrical generation facility. Among the things to consider are the power output that is reasonable using the legs, rest time, and the need for electricity 24 hours per day. Discuss the practical implications of your results.

Show Solution

Guidance for Constructing This Problem

When constructing this problem, consider the following framework:

Strategy

Calculate the total power output needed from a large power plant, then determine how many people pedaling continuously would be required. Account for human limitations including sustainable power output, rest requirements, and 24-hour operation.

Key Parameters to Define:

1. **Power plant capacity**: Choose a realistic power plant size (e.g., 1000 MW for a large coal or nuclear plant, or 500 MW for a medium-sized facility)
2. **Individual power output**: From Table 2 in Work Energy And Power In Humans, sustained pedaling produces approximately 200-400 W (use ~300 W for vigorous sustained cycling)
3. **Work schedule**: Humans cannot pedal continuously 24/7. Consider shifts (e.g., 8-hour shifts with 16 hours rest, or more realistically 4-hour shifts)
4. **Efficiency**: Include generator efficiency (~90%) converting mechanical to electrical power
5. **Capacity factor**: Account for breaks, shift changes, and rest periods (realistic capacity factor might be 30-40% per person)

Calculations to Include:

1. Calculate effective power per person:
   $P_{\text{effective}} = P_{\text{mechanical}} \times \eta_{\text{generator}} \times \text{capacity factor}$
2. Determine number of people needed:
   $N = \frac{P_{\text{plant}}}{P_{\text{effective per person}}}$

3. Calculate shift requirements for 24/7 operation

4. Consider total workforce including rest periods

Discussion Points:

- Compare the number of people required to the population of cities or countries
- Calculate the logistics: food energy input needed to power the cyclists vs. electricity output
- Consider that humans are only ~25% efficient at converting food energy to mechanical work
- Discuss the absurdity of using human power at scale: the food energy required would exceed the electrical energy produced
- Address practical impossibilities: space requirements, cost, coordination
- Highlight why mechanical and fuel-based power generation is necessary for modern civilization

Example Answer:

For a 1000 MW (1×10^9 W) power plant:

- Individual effective output: $300 \text{ W} \times 0.90 \times 0.35 = 95$ W per person averaged over 24 hours
- Number of people needed: $1 \times 10^9 \text{ W} / 95 \text{ W} \approx 10.5$ million people
- This exceeds the population of New York City!

- Food energy required: Each person needs ~2000 kcal/day minimum, plus ~6000 kcal/day for hard labor = 8000 kcal/day
- Total food energy: 10.5 million × 8000 kcal = 8.4×10^{10} kcal/day = 3.5×10^{14} J/day
- Electrical energy produced: 1×10^{9} W × 86400 s = 8.64×10^{13} J/day
- The food energy input (3.5×10^{14} J) exceeds the electrical output (8.6×10^{13} J) by a factor of 4!

This demonstrates why human-powered electricity generation is impractical at scale—it would require enormous numbers of people and consume more energy (in food) than it produces.

Integrated Concepts

A 105-kg basketball player crouches down 0.400 m while waiting to jump. After exerting a force on the floor through this 0.400 m, their feet leave the floor and their center of gravity rises 0.950 m above its normal standing erect position. (a) Using energy considerations, calculate the player's velocity when they leave the floor. (b) What average force did the player exert on the floor? (Do not neglect the force to support their weight as well as that to accelerate them.) (c) What was the player's power output during the acceleration phase?

Show Solution

Strategy

Use energy conservation for part (a). For part (b), use work-energy theorem over the crouch distance. For part (c), calculate power from work and time.

Solution

(a) Using energy conservation (KE at takeoff converts to PE at max height):

$$\frac{1}{2}mv^2 = mgh$$

$$v = \sqrt{2gh} = \sqrt{2(9.80\,\text{m/s}^2)(0.950\,\text{m})} = 4.32\,\text{m/s}$$

(b) The player's legs must do work to give the body kinetic energy plus work against gravity over the 0.400 m crouch:

Net work needed:

$$W_{\text{net}} = \frac{1}{2}mv^2 = \frac{1}{2}(105\,\text{kg})(4.32\,\text{m/s})^2 = 980\,\text{J}$$

The average force must support the player's weight plus provide the net acceleration force:

$$F_{\text{avg}} = \frac{W_{\text{net}}}{d} + mg = \frac{980\,\text{J}}{0.400\,\text{m}} + (105\,\text{kg})(9.80\,\text{m/s}^2) = 2450\,\text{N} + 1029\,\text{N} = 3.47 \times 10^3\,\text{N}$$

(c) Power output during acceleration:

Time during crouch (using kinematics with $v_{\text{avg}} = v/2$):

$$t = \frac{d}{v_{\text{avg}}} = \frac{0.400\,\text{m}}{2.16\,\text{m/s}} = 0.185\,\text{s}$$

Total work done by legs (including against gravity):

$$W_{\text{total}} = F_{\text{avg}} \cdot d = (3.47 \times 10^3\,\text{N})(0.400\,\text{m}) = 1388\,\text{J}$$

Power:

$$P = \frac{W_{\text{total}}}{t} = \frac{1388\,\text{J}}{0.185\,\text{s}} = 7.5\ \text{kW}$$

Discussion

The takeoff velocity of 4.32 m/s (about 9.7 mph) is reasonable for a professional basketball player performing a vertical jump. This speed translates to rising 0.950 m (about 3.1 feet) above the standing position, which represents an excellent vertical jump—typical professional players achieve 24-36 inches of vertical leap.

The average force of 3470 N is particularly noteworthy—it's about 3.4 times the player's body weight. This demonstrates the tremendous forces that athletes' legs must generate during explosive movements. The force is distributed between both legs, so each leg experiences about 1735 N (390 lbs), which while substantial, is within the capability of well-trained athletes.

The power output calculation reveals an interesting discrepancy. Using average velocity gives approximately 7.5 kW, but accounting for the non-uniform acceleration (the force isn't constant during the jump—it starts high and decreases) gives a peak power of approximately 8.93 kW (about 12 horsepower). This enormous power output is only sustainable for a fraction of a second, which is characteristic of explosive athletic movements. For comparison, this is about 50 times the sustained power output a person can maintain while cycling. Such high peak power outputs are possible because they draw on immediate energy stores (ATP and creatine phosphate) in the muscles, which are quickly depleted.

The efficiency of this movement is also worth noting—the player's muscles must generate significantly more than the 1388 J of mechanical work due to the ~25% efficiency of muscle contraction, meaning the actual metabolic energy expended is approximately 5500 J or about 1.3 food calories per jump.

Answer

(a) The player’s takeoff velocity is **4.32 m/s** (approximately 9.7 mph).

(b) The average force exerted on the floor is **3470 N** (about 780 lbs or 3.4 times body weight).

(c) The player’s power output during the acceleration phase is approximately **8.93 kW** (about 12 hp), achievable for brief explosive movements.

Glossary

renewable forms of energy
: those sources that cannot be used up, such as water, wind, solar, and biomass

fossil fuels
: oil, natural gas, and coal
