

# GRAVITATIONAL POTENTIAL ENERGY

## LEARNING OUTCOMES:

- Explain gravitational potential energy in terms of work done against gravity.
- Show that the gravitational potential energy of an object of mass  $m$  at height  $h$  on Earth is given by  $PE_g = mgh$ .
- Show how knowledge of the potential energy as a function of position can be used to simplify calculations and explain physical phenomena.

## Work Done Against Gravity

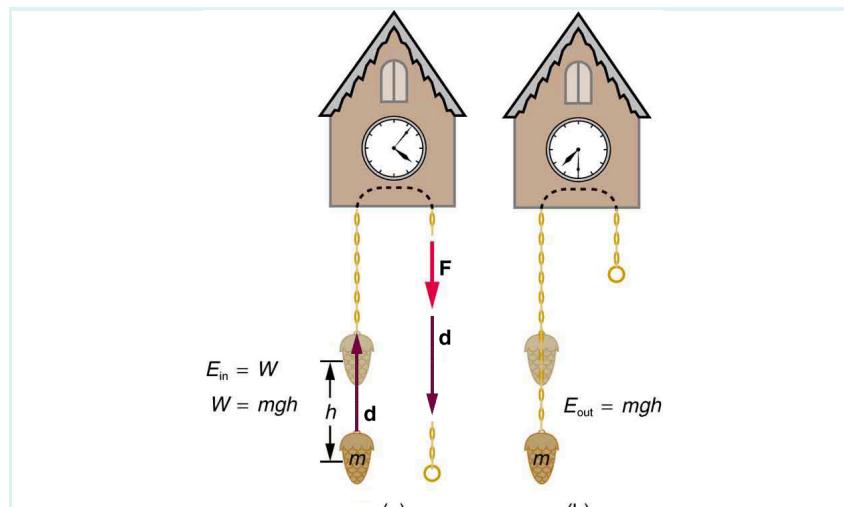
Climbing stairs and lifting objects is work in both the scientific and everyday sense—it is work done against the gravitational force. When there is work, there is a transformation of energy. The work done against the gravitational force goes into an important form of stored energy that we will explore in this section.

Let us calculate the work done in lifting an object of mass  $m$  through a height  $h$ , such as in [Figure 1](#). If the object is lifted straight up at constant speed, then the force needed to lift it is equal to its weight  $mg$ . The work done on the mass is then  $W = Fd = mgh$ . We define this to be the **gravitational potential energy** ( $PE_g$ ) put into (or gained by) the object-Earth system. This energy is associated with the state of separation between two objects that attract each other by the gravitational force. For convenience, we refer to this as the  $PE_g$

gained by the object, recognizing that this is energy stored in the gravitational field of Earth. Why do we use the word “system”? Potential energy is a property of a system rather than of a single object—due to its physical position. An object’s gravitational potential is due to its position relative to the surroundings within the Earth-object system. The force applied to the object is an external force, from outside the system. When it does positive work it increases the gravitational potential energy of the system. Because gravitational potential energy depends on relative position, we need a reference level at which to set the potential energy equal to 0. We usually choose this point to be Earth’s surface, but this point is arbitrary; what is important is the *difference* in gravitational potential energy, because this difference is what relates to the work done. The difference in gravitational potential energy of an object (in the Earth-object system) between two rungs of a ladder will be the same for the first two rungs as for the last two rungs.

## Converting Between Potential Energy and Kinetic Energy

Gravitational potential energy may be converted to other forms of energy, such as kinetic energy. If we release the mass, gravitational force will do an amount of work equal to  $mgh$  on it, thereby increasing its kinetic energy by that same amount (by the work-energy theorem). We will find it more useful to consider just the conversion of  $PE_g$  to KE without explicitly considering the intermediate step of work. (See [Example 2](#).) This shortcut makes it easier to solve problems using energy (if possible) rather than explicitly using forces.



**Figure 1:** (a) The work done to lift the weight is stored in the mass-Earth system as gravitational potential energy. (b) As the weight moves downward, this gravitational potential energy is transferred to the cuckoo clock.

More precisely, we define the *change* in gravitational potential energy

$\Delta PE_g$  to be

$\Delta PE_g = mgh$  where, for simplicity, we denote the change in height by  $h$  rather than the usual  $\Delta h$ .

Note that  $h$  is positive when the final height is greater than the initial height, and vice versa.

For example, if a 0.500-kg mass hung from a cuckoo clock is raised 1.00 m, then its change in gravitational potential energy is

Note that the units of gravitational potential energy turn out to be joules, the same as for work and other forms of energy. As the clock runs, the mass is lowered. We can think of the mass as gradually giving up its 4.90 J of gravitational potential energy, without directly considering the force of gravity that does the work.

The equation  $\Delta PE_g = mgh$  applies for any path that has a change in height of  $h$ , not just when the mass is lifted straight up. (See Figure 2.) It is much easier to calculate  $mgh$  (a simple multiplication) than it is to calculate the work done along a complicated path. The idea of gravitational potential energy has the double advantage that it is very broadly applicable and it makes calculations easier. From now on, we will consider that any change in vertical position  $h$  of a mass  $m$  is accompanied by a change in gravitational potential energy  $mgh$ , and we will avoid the equivalent but more difficult task of calculating work done by or against the gravitational force.



**Figure 2:** The change in gravitational potential energy  $(\Delta PE_{\text{g}})$  between points A and B is independent of the path. Gravity is one of a small class of forces where the work done by or against the force depends only on the starting and ending points, not on the path between them.

### Example 1: The Force

A 60.0-kg person jumps onto the floor. If he lands stiffly (with his knee joints compressed by 5.00 cm), calculate the force

**Strategy**

This person's energy is brought to work done on him by the floor as transformed into KE as he falls. This reduces this kinetic energy.

**Solution**

The work done on the person by the floor is

$$W = Fd \cos \theta = -$$

with a minus sign because the displacement and the force from floor are  $(\cos \theta = \cos 180^\circ = -1)$ . The floor is a system, so it does negative work.

The kinetic energy the person has is the amount of potential energy lost.

$$KE = -\Delta PE_g = -$$

The distance  $d$  that the person's knee joints compress is less than the height  $h$  of the fall, so the gravitational potential energy during the jump is less than  $mgh$ .

The work  $W$  done by the floor on the person and brings the person's kinetic energy to zero.

$$W = -KE = mgd$$

Combining this equation with the definition of work gives

$$-Fd = mgh.$$

Recalling that  $h$  is negative because the force on the knee joints is compressive, we have

$$F = -\frac{mgh}{d} = -\frac{(60.0\text{kg})(9.80\text{m/s}^2)}{5.00 \times 10^{-3}}$$

**Discussion**

Such a large force (500 times more than the weight of the person) over the short impact time is enormous. There is a better way to cushion the shock of landing: roll over, rolling on the ground, increasing the time of impact. A bending motion of 0.5 m is 10 times smaller than in the example, and shows this method in action. The

$$\begin{aligned} mgh &= (0.500\text{kg}) (9.80\text{m/s}^2) (1.00\text{m}) \\ mgh &= 4.90\text{kg} \cdot \text{m}^2/\text{s}^2 = 4.90\text{J.} \end{aligned}$$

### Using Potential Energy to Simplify Calculations

animal to use hopping for locc  
hopping is cushioned by the benefi  
jump.( See Ei

