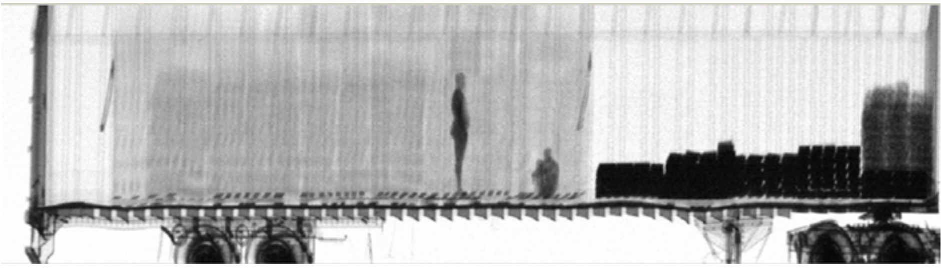
Introduction to Applications of Nuclear Physics

- Provide examples of various nuclear physics applications.

Tori Randall, Ph.D., curator for the Department of Physical Anthropology at the San Diego Museum of Man, prepares a 550-year-old Peruvian child mummy for a CT scan at Naval Medical Center San Diego. (credit: U.S. Navy photo by Mass Communication Specialist 3rd Class Samantha A. Lewis)

Applications of nuclear physics have become an integral part of modern life. From the bone scan that detects a cancer to the radioiodine treatment that cures another, nuclear radiation has diagnostic and therapeutic effects on medicine. From the fission power reactor to the hope of controlled fusion, nuclear energy is now commonplace and is a part of our plans for the future. Yet, the destructive potential of nuclear weapons haunts us, as does the possibility of nuclear reactor accidents. Certainly, several applications of nuclear physics escape our view, as seen in [Figure 2]. Not only has nuclear physics revealed secrets of nature, it has an inevitable impact based on its applications, as they are intertwined with human values. Because of its potential for alleviation of suffering, and its power as an ultimate destructor of life, nuclear physics is often viewed with ambivalence. But it provides perhaps the best example that applications can be good or evil, while knowledge itself is neither.

Customs officers inspect vehicles using neutron irradiation. Cars and trucks pass through portable x-ray machines that reveal their contents. (credit: Gerald L. Nino, CBP, U.S. Dept. of Homeland Security)



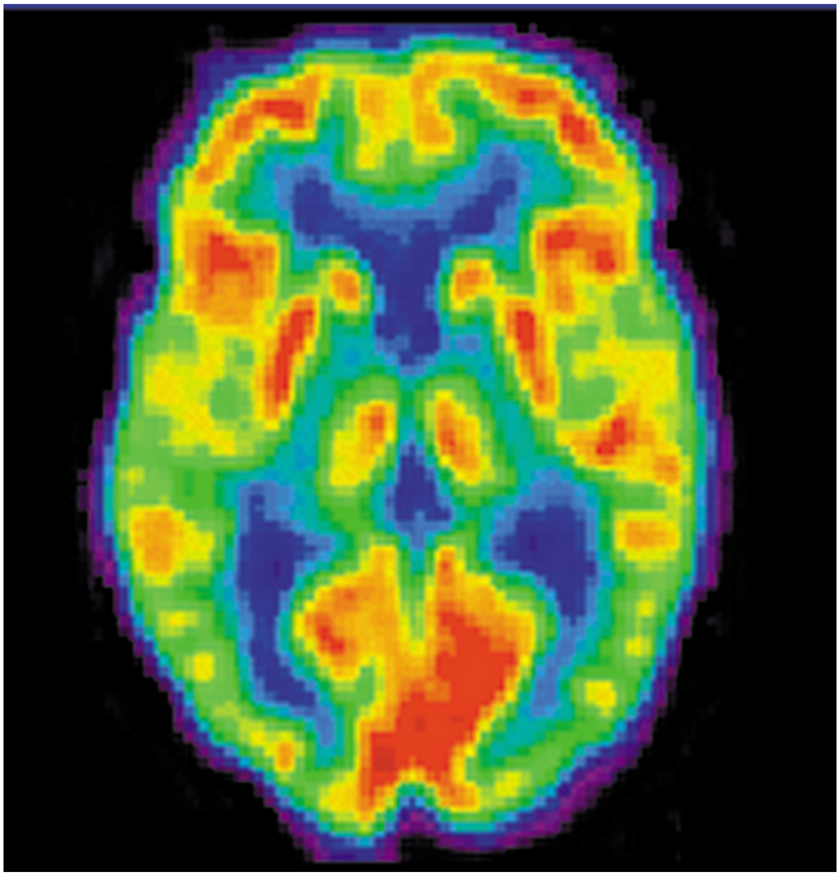
This image shows two stowaways caught illegally entering the United States from Canada. (credit: U.S. Customs and Border Protection)

Medical Imaging and Diagnostics

- Explain the working principle behind an anger camera.
- Describe the SPECT and PET imaging techniques.

Most medical and related applications of nuclear physics are driven, at their core, by the difference between a radioactive substance and a non-radioactive substance. One of the first such methods is the precision measurement and detection method known as radioimmunoassay (RIA). Developed by Rosalyn Sussman Yalow and Solomon Berson in the late 1950s, RIA relies on the principle of competitive binding. For the particular substance being measured, a sample containing a radioactive isotope is prepared. A known quantity of antibodies is then introduced. By measuring the amount of “unbound” antibodies after the reaction, technicians can detect and measure the precise amount of the target substance. Radioimmunoassay is essential in cancer screening, hepatitis diagnosis, narcotics investigation, and other analyses.

A host of medical imaging techniques employ nuclear radiation. What makes nuclear radiation so useful? First, γ radiation can easily penetrate tissue; hence, it is a useful probe to monitor conditions inside the body. Second, nuclear radiation depends on the nuclide and not on the chemical compound it is in, so that a radioactive nuclide can be put into a compound designed for specific purposes. The compound is said to be **tagged**. A tagged compound used for medical purposes is called a **radiopharmaceutical**. Radiation detectors external to the body can determine the location and concentration of a radiopharmaceutical to yield medically useful information. For example, certain drugs are concentrated in inflamed regions of the body, and this information can aid diagnosis and treatment as seen in [Figure 1]. Another application utilizes a radiopharmaceutical which the body sends to bone cells, particularly those that are most active, to detect cancerous tumors or healing points. Images can then be produced of such bone scans. Radioisotopes are also used to determine the functioning of body organs, such as blood flow, heart muscle activity, and iodine uptake in the thyroid gland.



A radiopharmaceutical is used to produce this brain image of a patient with Alzheimer’s disease. Certain features are computer enhanced. (credit: National Institutes of Health)

Medical Application

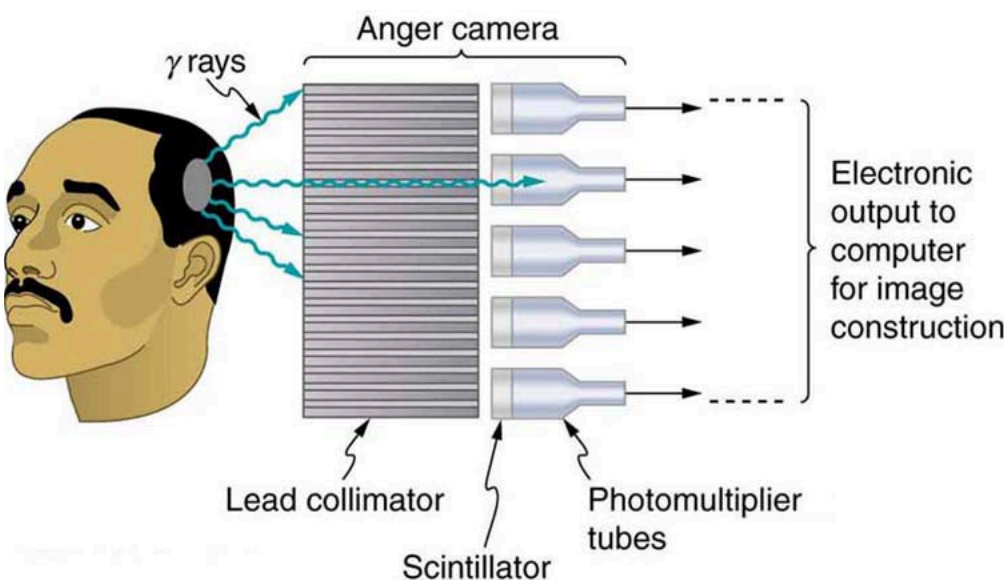
[Table 1] lists certain medical diagnostic uses of radiopharmaceuticals, including isotopes and activities that are typically administered. Many organs can be imaged with a variety of nuclear isotopes replacing a stable element by a radioactive isotope. One common diagnostic employs iodine to image the thyroid, since iodine is concentrated in that organ. The most active thyroid cells, including cancerous cells, concentrate the most iodine and, therefore, emit the most radiation. Conversely, hypothyroidism is indicated by lack of iodine uptake. Note that there is more than one isotope that can be used for several types of scans. Another common nuclear diagnostic is the thallium scan for the cardiovascular system, particularly used to evaluate blockages in the coronary arteries and examine heart activity. The salt TlCl can be used, because it acts like NaCl and follows the blood. Gallium-67 accumulates where there is rapid cell growth, such as in tumors and sites of infection. Hence, it is useful in cancer imaging. Usually, the patient receives the injection one day and has a whole body scan 3 or 4 days later because it can take several days for the gallium to build up.

Diagnostic Uses of Radiopharmaceuticals

Procedure, isotope	Typical activity (mCi), where $1 mCi = 3.7 \times 10^7$ Bq
Brain scan	
^{99m}Tc	7.5
^{113m}In	7.5
^{11}C (PET)	20
^{13}N (PET)	20
^{15}O (PET)	50
^{18}F (PET)	10
Lung scan	
^{99m}Tc	2
^{133}Xe	7.5
Cardiovascular blood pool	
^{131}I	0.2
^{99m}Tc	2
Cardiovascular arterial flow	
^{201}Tl	3
^{24}Na	7.5
Thyroid scan	
^{131}I	0.05
^{123}I	0.07
Liver scan	
^{198}Au (colloid)	0.1
^{99m}Tc (colloid)	2
Bone scan	
^{85}Sr	0.1
^{99m}Tc	10
Kidney scan	
^{197}Hg	0.1
^{99m}Tc	1.5

Note that [Table 1] lists many diagnostic uses for ^{99m}Tc , where “m” stands for a metastable state of the technetium nucleus. Perhaps 80 percent of all radiopharmaceutical procedures employ ^{99m}Tc because of its many advantages. One is that the decay of its metastable state produces a single, easily identified 0.142-MeV γ ray. Additionally, the radiation dose to the patient is limited by the short 6.0-h half-life of ^{99m}Tc . And, although its half-life is short, it is easily and continuously produced on site. The basic process for production is neutron activation of molybdenum, which quickly β decays into ^{99m}Tc . Technetium-99m can be attached to many compounds to allow the imaging of the skeleton, heart, lungs, kidneys, etc.

[Figure 2] shows one of the simpler methods of imaging the concentration of nuclear activity, employing a device called an **Anger camera** or **gamma camera**. A piece of lead with holes bored through it collimates γ rays emerging from the patient, allowing detectors to receive γ rays from specific directions only. The computer analysis of detector signals produces an image. One of the disadvantages of this detection method is that there is no depth information (i.e., it provides a two-dimensional view of the tumor as opposed to a three-dimensional view), because radiation from any location under that detector produces a signal.

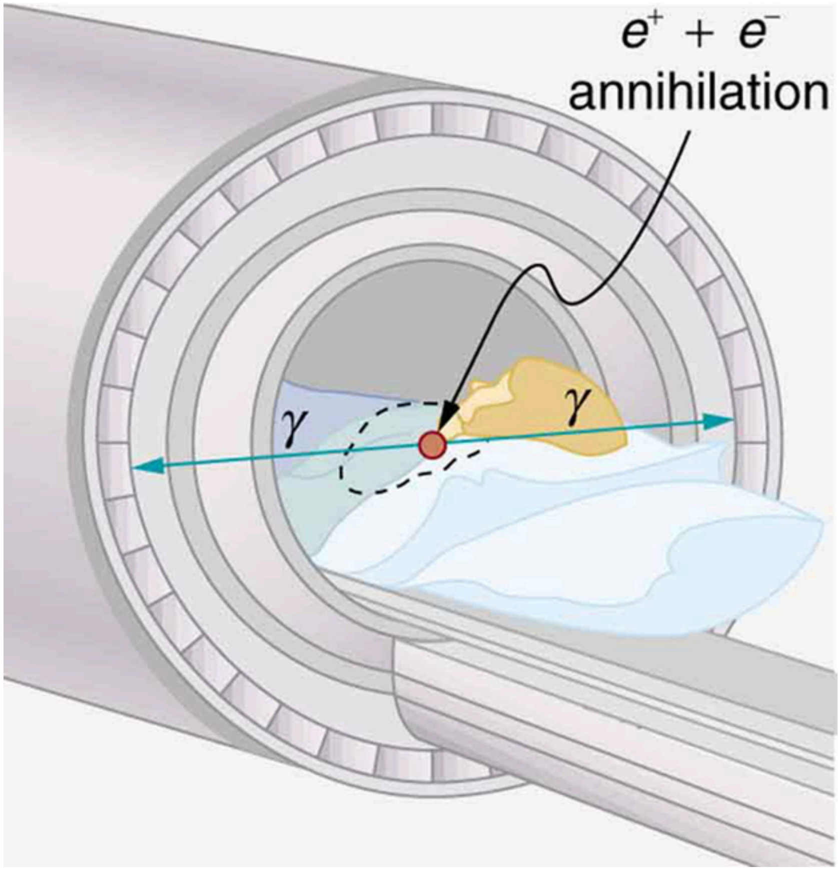


An Anger or gamma camera consists of a lead collimator and an array of detectors. Gamma rays produce light flashes in the scintillators. The light output is converted to an electrical signal by the photomultipliers. A computer constructs an image from the detector output.

Imaging techniques much like those in x-ray computed tomography (CT) scans use nuclear activity in patients to form three-dimensional images. [Figure 3] shows a patient in a circular array of detectors that may be stationary or rotated, with detector output used by a computer to construct a detailed image. This technique is called **single-photon-emission computed tomography(SPECT)** or sometimes simply SPET. The spatial resolution of this technique is poor, about 1 cm, but the contrast (i.e. the difference in visual properties that makes an object distinguishable from other objects and the background) is good.

SPECT uses a geometry similar to a CT scanner to form an image of the concentration of a radiopharmaceutical compound. (credit: Woldo, Wikimedia Commons)

Images produced by β^+ emitters have become important in recent years. When the emitted positron (β^+) encounters an electron, mutual annihilation occurs, producing two γ rays. These γ rays have identical 0.511-MeV energies (the energy comes from the destruction of an electron or positron mass) and they move directly away from one another, allowing detectors to determine their point of origin accurately, as shown in [Figure 4]. The system is called **positron emission tomography (PET)**. It requires detectors on opposite sides to simultaneously (i.e., at the same time) detect photons of 0.511-MeV energy and utilizes computer imaging techniques similar to those in SPECT and CT scans. Examples of β^+ -emitting isotopes used in PET are ^{11}C , ^{13}N , ^{15}O , and ^{18}F , as seen in [Table 1]. This list includes C, N, and O, and so they have the advantage of being able to function as tags for natural body compounds. Its resolution of 0.5 cm is better than that of SPECT; the accuracy and sensitivity of PET scans make them useful for examining the brain's anatomy and function. The brain's use of oxygen and water can be monitored with ^{15}O . PET is used extensively for diagnosing brain disorders. It can note decreased metabolism in certain regions prior to a confirmation of Alzheimer's disease. PET can locate regions in the brain that become active when a person carries out specific activities, such as speaking, closing their eyes, and so on.



A PET system takes advantage of the two identical γ-ray photons produced by positron-electron annihilation. These γ rays are emitted in opposite directions, so that the line along which each pair is emitted is determined. Various events detected by several pairs of detectors are then analyzed by the computer to form an accurate image.

PhET Explorations: Simplified MRI

Is it a tumor? Magnetic Resonance Imaging (MRI) can tell. Your head is full of tiny radio transmitters (the nuclear spins of the hydrogen nuclei of your water molecules). In an MRI unit, these little radios can be made to broadcast their positions, giving a detailed picture of the inside of your head.

Section Summary

- Radiopharmaceuticals are compounds that are used for medical imaging and therapeutics.
- The process of attaching a radioactive substance is called tagging.
- [Table 1] lists certain diagnostic uses of radiopharmaceuticals including the isotope and activity typically used in diagnostics.
- One common imaging device is the Anger camera, which consists of a lead collimator, radiation detectors, and an analysis computer.
- Tomography performed with γ-emitting radiopharmaceuticals is called SPECT and has the advantages of x-ray CT scans coupled with organ- and function-specific drugs.
- PET is a similar technique that uses β^+ emitters and detects the two annihilation γ rays, which aid to localize the source.

Conceptual Questions

In terms of radiation dose, what is the major difference between medical diagnostic uses of radiation and medical therapeutic uses?

One of the methods used to limit radiation dose to the patient in medical imaging is to employ isotopes with short half-lives. How would this limit the dose?

Problems & Exercises

A neutron generator uses an α source, such as radium, to bombard beryllium, inducing the reaction $^{4}\text{He}+^{9}\text{Be} \rightarrow ^{12}\text{C} + n$. Such neutron sources are called RaBe sources, or PuBe sources if they use plutonium to get the α s. Calculate the energy output of the reaction in MeV.

Show Solution

5.701 MeV

Neutrons from a source (perhaps the one discussed in the preceding problem) bombard natural molybdenum, which is 24 percent ^{98}Mo . What is the energy output of the reaction $^{98}\text{Mo}+n \rightarrow ^{99}\text{Mo}+\gamma$? The mass of ^{98}Mo is given in Appendix A: Atomic Masses, and that of ^{99}Mo is 98.907711 u.

Show Solution

Strategy

The energy released in this neutron capture reaction is found from the mass defect: $E = (\Delta m)c^2$, where $\Delta m = m_{\text{reactants}} - m_{\text{products}}$. We need to look up the mass of ^{98}Mo from Appendix A (97.905406 u) and use the given mass of ^{99}Mo (98.907711 u). The neutron mass is 1.008665 u.

Solution

Calculate the total mass of reactants:

$$m_{\text{reactants}}=m(^{98}\text{Mo})+m(n)=97.905406 \text{ u}+1.008665 \text{ u}=98.914071 \text{ u}$$

The mass of the product is:

$$m_{\text{products}}=m(^{99}\text{Mo})=98.907711 \text{ u}$$

Calculate the mass defect:

$$\Delta m=98.914071 \text{ u}-98.907711 \text{ u}=0.006360 \text{ u}$$

Convert to energy:

$$E=(\Delta m)c^2=(0.006360 \text{ u})(931.5 \text{ MeV}1 \text{ u})=5.93 \text{ MeV}$$

Discussion

This 5.93 MeV of energy is released as a gamma ray (or possibly multiple gamma rays that sum to this energy). Neutron capture reactions like this are exothermic because the product nucleus has greater binding energy than the sum of the reactants. This particular reaction is extremely important in nuclear medicine because it is the first step in producing ^{99m}Tc, the most widely used radiopharmaceutical. The ^{99}Mo produced undergoes beta decay with a half-life of 66 hours to form ^{99m}Tc, which has ideal properties for medical imaging: a 6-hour half-life that limits patient dose, and emission of a single easily-detected 0.142-MeV gamma ray. The fact that natural molybdenum is 24% ^{98}Mo means that natural molybdenum can be used directly as a target material in reactors without expensive isotopic enrichment. The relatively large energy release of nearly 6 MeV helps ensure a reasonable reaction probability (cross-section) when thermal neutrons are used, making this a practical production method. This reaction is performed continuously in nuclear reactors around the world to supply $^{99}\text{Mo}/^{99m}\text{Tc}$ generators to hospitals and clinics.

The purpose of producing ^{99}Mo (usually by neutron activation of natural molybdenum, as in the preceding problem) is to produce ^{99m}Tc. Using the rules, verify that the β^- decay of ^{99}Mo produces ^{99m}Tc . (Most ^{99m}Tc nuclei produced in this decay are left in a metastable excited state denoted

^{99m}Tc .)

Show Solution

$$^{99}_{42}Mo_{57} \rightarrow ^{99}_{43}Tc_{56} + \beta^- + \bar{\nu}_e$$

(a) Two annihilation γ rays in a PET scan originate at the same point and travel to detectors on either side of the patient. If the point of origin is 9.00 cm closer to one of the detectors, what is the difference in arrival times of the photons? (This could be used to give position information, but the time difference is small enough to make it difficult.)

(b) How accurately would you need to be able to measure arrival time differences to get a position resolution of 1.00 mm?

Show Solution

Strategy

Gamma rays (photons) travel at the speed of light $c = 3.00 \times 10^8$ m/s. The time difference is found from $\Delta t = \Delta d / c$, where Δd is the difference in distance traveled. For part (a), $\Delta d = 9.00$ cm. For part (b), we need $\Delta d = 1.00$ mm resolution.

Solution

(a) Convert distance to meters and calculate time difference:

$$\Delta d = 9.00 \text{ cm} = 0.0900 \text{ m}$$
$$\Delta t = \frac{\Delta d}{c} = \frac{0.0900 \text{ m}}{3.00 \times 10^8 \text{ m/s}} = 3.00 \times 10^{-10} \text{ s} = 0.300 \text{ ns}$$

(b) For 1.00 mm resolution:

$$\Delta d = 1.00 \text{ mm} = 1.00 \times 10^{-3} \text{ m}$$
$$\Delta t = \frac{1.00 \times 10^{-3} \text{ m}}{3.00 \times 10^8 \text{ m/s}} = 3.33 \times 10^{-12} \text{ s} = 3.33 \text{ ps}$$

Discussion

These incredibly short time intervals illustrate both the potential and the challenge of using time-of-flight information in PET imaging. In part (a), the time difference of 0.300 nanoseconds for a 9-cm offset is at the edge of what modern electronics can reliably measure. Current state-of-the-art PET scanners can achieve timing resolution of about 200-500 picoseconds, which would give position resolution of about 3-7.5 cm. To achieve the 1-mm resolution specified in part (b) would require measuring time differences of only 3.33 picoseconds—more than 60 times better than current technology! This is why conventional PET relies primarily on coincidence detection (detecting that two photons arrived at nearly the same time on opposite detectors) rather than precise time-of-flight measurements. The coincidence requirement determines that the photons came from somewhere along the line connecting the two detectors, and computer tomography techniques reconstruct the image from many such lines. Modern "time-of-flight PET" (TOF-PET) scanners do use the achievable ~200-500 ps timing resolution to narrow down the position along each line to about 3-7 cm, which significantly improves image quality and reduces scanning time. However, the fundamental spatial resolution of PET (about 4-6 mm) is still limited by other factors including positron range (the distance the positron travels before annihilation) and photon non-collinearity (the annihilation photons aren't exactly 180° apart due to the momentum of the positron-electron pair).

[Table 1] indicates that 7.50 mCi of ^{99m}Tc is used in a brain scan. What is the mass of technetium?

Show Solution

1.43×10^{-9} g

The activities of ^{131}I and ^{123}I used in thyroid scans are given in [Table 1] to be 50 and $70 \mu Ci$, respectively. Find and compare the masses of ^{131}I and ^{123}I in such scans, given their respective half-lives are 8.04 d and 13.2 h. The masses are so small that the radioiodine is usually mixed with stable iodine as a carrier to ensure normal chemistry and distribution in the body.

Show Solution

Strategy

For each isotope, we'll use $R = \frac{0.693 N}{t_{1/2}}$ to find the number of nuclei N, then convert to mass. First, convert activities to Bq and half-lives to seconds. Then solve for N and convert to mass using the respective molar masses.

Solution

For ^{131}I:

Convert activity:

$$R_{131} = 50 \text{ μCi} = 50 \times 10^{-6} \times 3.70 \times 10^{10} \text{ Bq} = 1.85 \times 10^6 \text{ Bq}$$

Convert half-life:

$$t_{1/2} = 8.04\ \text{d} \times \frac{24\ \text{h}}{1\ \text{d}} \times \frac{3600\ \text{s}}{1\ \text{h}} = 6.95 \times 10^{5}\ \text{s}$$

Find number of nuclei:

$$N_{131} = \frac{R \times t_{1/2}}{0.693} = \frac{(1.85 \times 10^{6})(6.95 \times 10^{5})}{0.693} = 1.86 \times 10^{12}\ \text{nuclei}$$

Convert to mass:

$$m_{131} = \frac{N_{131}}{N_A} \times M = \frac{1.86 \times 10^{12}}{6.022 \times 10^{23}} \times 131\ \text{g/mol} = 4.05 \times 10^{-10}\ \text{g} = 0.405\ \text{ng}$$

For ^{123}I:

Convert activity:

$$R_{123} = 70\ \mu\text{Ci} = 70 \times 10^{-6} \times 3.70 \times 10^{10}\ \text{Bq} = 2.59 \times 10^{6}\ \text{Bq}$$

Convert half-life:

$$t_{1/2} = 13.2\ \text{h} \times \frac{3600\ \text{s}}{1\ \text{h}} = 4.75 \times 10^{4}\ \text{s}$$

Find number of nuclei:

$$N_{123} = \frac{(2.59 \times 10^{6})(4.75 \times 10^{4})}{0.693} = 1.78 \times 10^{11}\ \text{nuclei}$$

Convert to mass:

$$m_{123} = \frac{1.78 \times 10^{11}}{6.022 \times 10^{23}} \times 123\ \text{g/mol} = 3.63 \times 10^{-11}\ \text{g} = 0.0363\ \text{ng}$$

Comparison:

$$\frac{m_{131}}{m_{123}} = \frac{0.405\ \text{ng}}{0.0363\ \text{ng}} \approx 11.2$$

The ^{131}I mass is about **11 times larger** than the ^{123}I mass.

Discussion

Despite ^{123}I having higher activity (70 μCi vs 50 μCi), it requires much less mass (only 0.036 ng compared to 0.405 ng for ^{131}I). This factor of 11 difference is primarily due to the vastly different half-lives: ^{131}I has a half-life of 8.04 days while ^{123}I has a much shorter half-life of only 13.2 hours. Shorter half-life means more decays per second for a given number of nuclei, so less mass is needed to achieve a given activity. This illustrates an important principle: to achieve high activity with less mass (and therefore potentially less total radiation dose), isotopes with shorter half-lives are preferred. Both masses are incredibly small—in the nanogram range—which is why carrier-free radiopharmaceuticals cannot be used directly. Without adding stable iodine as a carrier, these tiny amounts would stick to container walls, fail to dissolve properly, and not distribute normally in the body. The carrier ensures that the radioiodine behaves chemically like normal iodine and concentrates in the thyroid as expected. The shorter half-life of ^{123}I (13.2 h vs 8.04 d) also means the patient's radiation dose decays away much faster, making ^{123}I preferable when available, despite being more expensive and difficult to produce.

(a) Neutron activation of sodium, which is 100% ^{23}Na, produces ^{24}Na, which is used in some heart scans, as seen in [Table 1]. The equation for the reaction is $^{23}\text{Na} + n \rightarrow {}^{24}\text{Na} + \gamma$. Find its energy output, given the mass of ^{24}Na is 23.990962 u.

(b) What mass of ^{24}Na produces the needed 5.0-mCi activity, given its half-life is 15.0 h?

Show Solution

(a) 6.958 MeV

(b) $5.7 \times 10^{-10}\ \text{g}$

Glossary

Anger camera
: a common medical imaging device that uses a scintillator connected to a series of photomultipliers

gamma camera
: another name for an Anger camera

positron emission tomography (PET)
: tomography technique that uses β^+ emitters and detects the two annihilation γ rays, aiding in source localization

radiopharmaceutical
: compound used for medical imaging

single-photon-emission computed tomography (SPECT)
: tomography performed with γ -emitting radiopharmaceuticals

tagged
: process of attaching a radioactive substance to a chemical compound

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Biological Effects of Ionizing Radiation

- Define various units of radiation.
- Describe RBE.

We hear many seemingly contradictory things about the biological effects of ionizing radiation. It can cause cancer, burns, and hair loss, yet it is used to treat and even cure cancer. How do we understand these effects? Once again, there is an underlying simplicity in nature, even in complicated biological organisms. All the effects of ionizing radiation on biological tissue can be understood by knowing that **ionizing radiation affects molecules within cells, particularly DNA molecules.**

Let us take a brief look at molecules within cells and how cells operate. Cells have long, double-helical DNA molecules containing chemical codes called genetic codes that govern the function and processes undertaken by the cell. It is for unraveling the double-helical structure of DNA that James Watson, Francis Crick, and Maurice Wilkins received the Nobel Prize. Damage to DNA consists of breaks in chemical bonds or other changes in the structural features of the DNA chain, leading to changes in the genetic code. In human cells, we can have as many as a million individual instances of damage to DNA per cell per day. It is remarkable that DNA contains codes that check whether the DNA is damaged or can repair itself. It is like an auto check and repair mechanism. This repair ability of DNA is vital for maintaining the integrity of the genetic code and for the normal functioning of the entire organism. It should be constantly active and needs to respond rapidly. The rate of DNA repair depends on various factors such as the cell type and age of the cell. A cell with a damaged ability to repair DNA, which could have been induced by ionizing radiation, can do one of the following:

- The cell can go into an irreversible state of dormancy, known as senescence.
- The cell can commit suicide, known as programmed cell death.
- The cell can go into unregulated cell division leading to tumors and cancers.

Since ionizing radiation damages the DNA, which is critical in cell reproduction, it has its greatest effect on cells that rapidly reproduce, including most types of cancer. Thus, cancer cells are more sensitive to radiation than normal cells and can be killed by it easily. Cancer is characterized by a malfunction of cell reproduction, and can also be caused by ionizing radiation. Without contradiction, ionizing radiation can be both a cure and a cause.

To discuss quantitatively the biological effects of ionizing radiation, we need a radiation dose unit that is directly related to those effects. All effects of radiation are assumed to be directly proportional to the amount of ionization produced in the biological organism. The amount of ionization is in turn proportional to the amount of deposited energy. Therefore, we define a **radiation dose unit** called the **rad**, as $1/100$ of a joule of ionizing energy deposited per kilogram of tissue, which is

$$1\text{rad}=0.01\text{J/kg}.$$

For example, if a 50.0-kg person is exposed to ionizing radiation over her entire body and she absorbs 1.00 J, then her whole-body radiation dose is

$$(1.00\text{J})/(50.0\text{kg})=0.0200\text{J/kg}=2.00\text{rad}.$$

If the same 1.00 J of ionizing energy were absorbed in her 2.00-kg forearm alone, then the dose to the forearm would be

$$(1.00\text{J})/(2.00\text{kg})=0.500\text{J/kg}=50.0\text{rad},$$

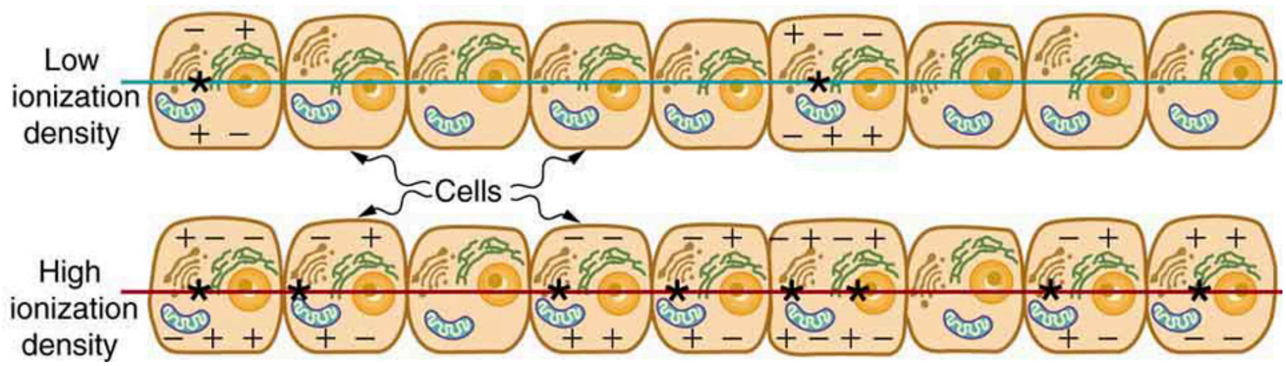
and the unaffected tissue would have a zero rad dose. While calculating radiation doses, you divide the energy absorbed by the mass of affected tissue. You must specify the affected region, such as the whole body or forearm in addition to giving the numerical dose in rads. The SI unit for radiation dose is the **gray (Gy)**, which is defined to be

$$1Gy=1\text{J/kg}=100\text{rad}.$$

However, the rad is still commonly used. Although the energy per kilogram in 1 rad is small, it has significant effects since the energy causes ionization. The energy needed for a single ionization is a few eV, or less than 10^{-18}J . Thus, 0.01 J of ionizing energy can create a huge number of ion pairs and have an effect at the cellular level.

The effects of ionizing radiation may be directly proportional to the dose in rads, but they also depend on the type of radiation and the type of tissue. That is, for a given dose in rads, the effects depend on whether the radiation is $\alpha,\beta,\gamma,$ x-ray, or some other type of ionizing radiation. In the earlier discussion of the range of ionizing radiation, it was noted that energy is deposited in a series of ionizations and not in a single interaction. Each ion pair or ionization requires a certain amount of energy, so that the number of ion pairs is directly proportional to the amount of the deposited ionizing energy. But, if the range of the radiation is small, as it is for α s, then the ionization and the damage created is more concentrated and harder for the organism to repair, as seen in [Figure 1]. Concentrated damage is more difficult for biological organisms to repair than damage that is spread out, so short-range particles have greater biological effects. The **relative biological effectiveness** (RBE) or **quality factor** (QF) is given in [Table 1] for several types of ionizing radiation —the effect of the radiation is directly proportional to the RBE. A dose unit more closely related to effects in biological tissue is called the **roentgen equivalent man** or rem and is defined to be the dose in rads multiplied by the relative biological effectiveness.

$$rem=rad\times RBE$$



The image shows ionization created in cells by α; and γ radiation. Because of its shorter range, the ionization and damage created by α is more concentrated and harder for the organism to repair. Thus, the RBE for αs is greater than the RBE for γs, even though they create the same amount of ionization at the same energy.

So, if a person had a whole-body dose of 2.00 rad of γ radiation, the dose in rem would be $(2.00 \text{rad})(1) = 2.00 \text{ rem whole body}$. If the person had a whole-body dose of 2.00 rad of α radiation, then the dose in rem would be $(2.00 \text{rad})(20) = 40.0 \text{rem whole body}$. The α s would have 20 times the effect on the person than the γ s for the same deposited energy. The SI equivalent of the rem is the **sievert** (Sv), defined to be $\text{Sv} = \text{Gy} \times \text{RBE}$, so that

$$1 Sv = 1 Gy \times \text{RBE} = 100 \text{rem}.$$

The RBEs given in [Table 1] are approximate, but they yield certain insights. For example, the eyes are more sensitive to radiation, because the cells of the lens do not repair themselves. Neutrons cause more damage than γ rays, although both are neutral and have large ranges, because neutrons often cause secondary radiation when they are captured. Note that the RBEs are 1 for higher-energy β s, γ s, and x-rays, three of the most common types of radiation. For those types of radiation, the numerical values of the dose in rem and rad are identical. For example, 1 rad of γ

radiation is also 1 rem. For that reason, rads are still widely quoted rather than rem. [Table 2] summarizes the units that are used for radiation.

Misconception Alert: Activity vs. Dose

“Activity” refers to the radioactive source while “dose” refers to the amount of energy from the radiation that is deposited in a person or object.

A high level of activity doesn’t mean much if a person is far away from the source. The activity R of a source depends upon the quantity of material (kg) as well as the half-life. A short half-life will produce many more disintegrations per second. Recall that $R = 0.693 N t_{1/2}$. Also, the activity decreases exponentially, which is seen in the equation $R = R_0 e^{-\lambda t}$.

Relative Biological Effectiveness

Type and energy of radiation	RBE[1]
X-rays	1
γ rays	1
β rays greater than 32 keV	1
βrays less than 32 keV	1.7
Neutrons, thermal to slow (<20 keV)	2–5
Neutrons, fast (1–10 MeV)	10 (body), 32 (eyes)
Protons (1–10 MeV)	10 (body), 32 (eyes)
α rays from radioactive decay	10–20
Heavy ions from accelerators	10–20

Units for Radiation

Quantity	SI unit name	Definition	Former unit	Conversion
Activity	Becquerel (bq)	decay/sec	Curie (Ci)	$1 Bq = 2.7 \times 10^{-11} \text{Ci}$
Absorbed dose	Gray (Gy)	1 J/kg	rad	Gy = 100 rad
Dose Equivalent	Sievert (Sv)	1 J/kg × RBE	rem	Sv = 100 rem

The large-scale effects of radiation on humans can be divided into two categories: immediate effects and long-term effects. [Table 3] gives the immediate effects of whole-body exposures received in less than one day. If the radiation exposure is spread out over more time, greater doses are needed to cause the effects listed. This is due to the body’s ability to partially repair the damage. Any dose less than 100 mSv (10 rem) is called a **low dose**, 0.1 Sv to 1 Sv (10 to 100 rem) is called a **moderate dose**, and anything greater than 1 Sv (100 rem) is called a **high dose**. There is no known way to determine after the fact if a person has been exposed to less than 10 mSv.

Immediate Effects of Radiation (Adults, Whole Body, Single Exposure)

Dose in Sv [2]	Effect
0–0.10	No observable effect.

Dose in Sv [2]	Effect
0.1 – 1	Slight to moderate decrease in white blood cell counts.
0.5	Temporary sterility; 0.35 for women, 0.50 for men.
1 – 2	Significant reduction in blood cell counts, brief nausea and vomiting. Rarely fatal.
2 – 5	Nausea, vomiting, hair loss, severe blood damage, hemorrhage, fatalities.
4.5	LD50/32. Lethal to 50% of the population within 32 days after exposure if not treated.
5 – 20	Worst effects due to malfunction of small intestine and blood systems. Limited survival.
>20	Fatal within hours due to collapse of central nervous system.

Immediate effects are explained by the effects of radiation on cells and the sensitivity of rapidly reproducing cells to radiation. The first clue that a person has been exposed to radiation is a change in blood count, which is not surprising since blood cells are the most rapidly reproducing cells in the body. At higher doses, nausea and hair loss are observed, which may be due to interference with cell reproduction. Cells in the lining of the digestive system also rapidly reproduce, and their destruction causes nausea. When the growth of hair cells slows, the hair follicles become thin and break off. High doses cause significant cell death in all systems, but the lowest doses that cause fatalities do so by weakening the immune system through the loss of white blood cells.

The two known long-term effects of radiation are cancer and genetic defects. Both are directly attributable to the interference of radiation with cell reproduction. For high doses of radiation, the risk of cancer is reasonably well known from studies of exposed groups. Hiroshima and Nagasaki survivors and a smaller number of people exposed by their occupation, such as radium dial painters, have been fully documented. Chernobyl victims will be studied for many decades, with some data already available. For example, a significant increase in childhood thyroid cancer has been observed. The risk of a radiation-induced cancer for low and moderate doses is generally *assumed* to be proportional to the risk known for high doses. Under this assumption, any dose of radiation, no matter how small, involves a risk to human health. This is called the **linear hypothesis** and it may be prudent, but it *is* controversial. There is some evidence that, unlike the immediate effects of radiation, the long-term effects are cumulative and there is little self-repair. This is analogous to the risk of skin cancer from UV exposure, which is known to be cumulative.

There is a latency period for the onset of radiation-induced cancer of about 2 years for leukemia and 15 years for most other forms. The person is at risk for at least 30 years after the latency period. Omitting many details, the overall risk of a radiation-induced cancer death per year per rem of exposure is about 10 in a million, which can be written as $10/10^6 \text{rem}\cdot\text{y}$.

If a person receives a dose of 1 rem, their risk each year of dying from radiation-induced cancer is 10 in a million and that risk continues for about 30 years. The lifetime risk is thus 300 in a million, or 0.03 percent. Since about 20 percent of all worldwide deaths are from cancer, the increase due to a 1 rem exposure is impossible to detect demographically. But 100 rem (1 Sv), which was the dose received by the average Hiroshima and Nagasaki survivor, causes a 3 percent risk, which can be observed in the presence of a 20 percent normal or natural incidence rate.

The incidence of genetic defects induced by radiation is about one-third that of cancer deaths, but is much more poorly known. The lifetime risk of a genetic defect due to a 1 rem exposure is about 100 in a million or $3.3/10^6 \text{rem}\cdot\text{y}$, but the normal incidence is 60 000 in a million. Evidence of such a small increase, tragic as it is, is nearly impossible to obtain. For example, there is no evidence of increased genetic defects among the offspring of Hiroshima and Nagasaki survivors. Animal studies do not seem to correlate well with effects on humans and are not very helpful. For both cancer and genetic defects, the approach to safety has been to use the linear hypothesis, which is likely to be an overestimate of the risks of low doses. Certain researchers even claim that low doses are *beneficial*. **Hormesis** is a term used to describe generally favorable biological responses to low exposures of toxins or radiation. Such low levels may help certain repair mechanisms to develop or enable cells to adapt to the effects of the low exposures. Positive effects may occur at low doses that could be a problem at high doses.

Even the linear hypothesis estimates of the risks are relatively small, and the average person is not exposed to large amounts of radiation. [Table 4] lists average annual background radiation doses from natural and artificial sources for Australia, the United States, Germany, and world-wide averages. Cosmic rays are partially shielded by the atmosphere, and the dose depends upon altitude and latitude, but the average is about 0.40 mSv/y. A good example of the variation of cosmic radiation dose with altitude comes from the airline industry. Monitored personnel show an average of 2 mSv/y. A 12-hour flight might give you an exposure of 0.02 to 0.03 mSv.

Doses from the Earth itself are mainly due to the isotopes of uranium, thorium, and potassium, and vary greatly by location. Some places have great natural concentrations of uranium and thorium, yielding doses ten times as high as the average value. Internal doses come from foods and liquids that we ingest. Fertilizers containing phosphates have potassium and uranium. So we are all a little radioactive. Carbon-14 has about 66 Bq/kg radioactivity whereas fertilizers may have more than 3000 Bq/kg radioactivity. Medical and dental diagnostic exposures are mostly from x-rays. It should be noted that x-ray doses tend to be localized and are becoming much smaller with improved techniques. [Table 5] shows typical doses received during various diagnostic x-ray examinations. Note the large dose from a CT scan. While CT scans only account for less than 20 percent of the x-ray procedures done today, they account for about 50 percent of the annual dose received.

Radon is usually more pronounced underground and in buildings with low air exchange with the outside world. Almost all soil contains some ^{226}Ra and ^{222}Rn , but radon is lower in mainly sedimentary soils and higher in granite soils. Thus, the exposure to the public can vary greatly, even within short distances. Radon can diffuse from the soil into homes, especially basements. The estimated exposure for ^{222}Rn is controversial. Recent studies indicate there is more radon in homes than had been realized, and it is speculated that radon may be responsible for 20 percent of lung cancers, being particularly hazardous to those who also smoke. Many countries have introduced limits on allowable radon concentrations in indoor air, often requiring the measurement of radon concentrations in a house prior to its sale. Ironically, it could be argued that the higher levels of radon exposure and their geographic variability, taken with the lack of demographic evidence of any effects, means that low-level radiation is *less* dangerous than previously thought.

Radiation Protection

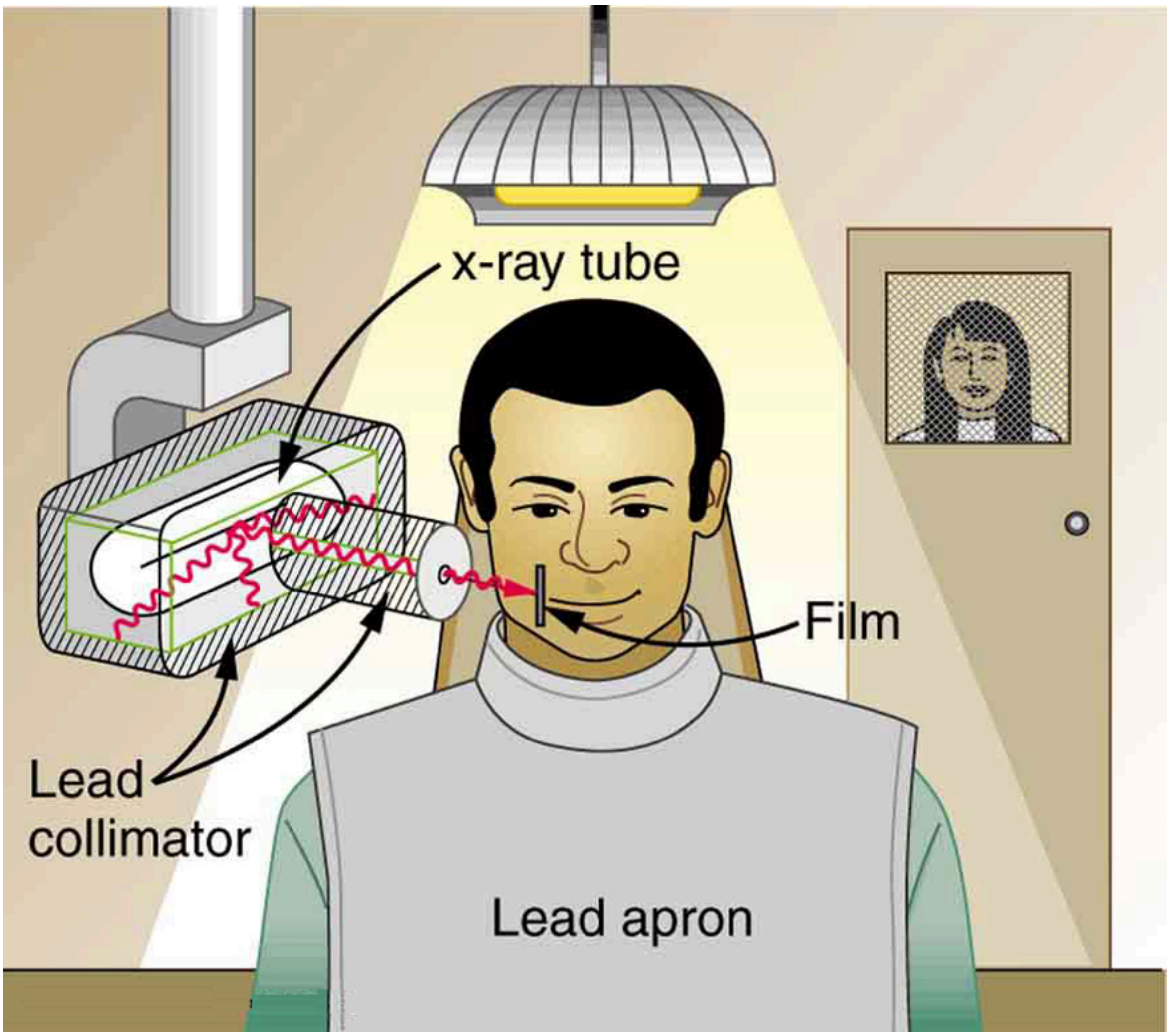
Laws regulate radiation doses to which people can be exposed. The greatest occupational whole-body dose that is allowed depends upon the country and is about 20 to 50 mSv/y and is rarely reached by medical and nuclear power workers. Higher doses are allowed for the hands. Much lower doses are permitted for the reproductive organs and the fetuses of pregnant women. Inadvertent doses to the public are limited to $1/10$ of occupational doses, except for those caused by nuclear power, which cannot legally expose the public to more than $1/1000$ of the occupational limit or 0.05 mSv/y (5 mrem/y). This has been exceeded in the United States only at the time of the Three Mile Island (TMI) accident in 1979. Chernobyl is another story. Extensive monitoring with a variety of radiation detectors is performed to assure radiation safety. Increased ventilation in uranium mines has lowered the dose there to about 1 mSv/y.

Background Radiation Sources and Average Doses

Source	**Dose (mSv/y)**[3]			
Source	Australia	Germany	United States	World
Natural Radiation - external				
Cosmic Rays	0.30	0.28	0.30	0.39
Soil, building materials	0.40	0.40	0.30	0.48
Radon gas	0.90	1.1	2.0	1.2
Natural Radiation - internal				
$^{40}K, ^{14}C, ^{226}Ra$	0.24	0.28	0.40	0.29
Medical & Dental	0.80	0.90	0.53	0.40
TOTAL	2.6	3.0	3.5	2.8

To physically limit radiation doses, we use **shielding**, increase the **distance** from a source, and limit the **time of exposure**.

[Figure 2] illustrates how these are used to protect both the patient and the dental technician when an x-ray is taken. Shielding absorbs radiation and can be provided by any material, including sufficient air. The greater the distance from the source, the more the radiation spreads out. The less time a person is exposed to a given source, the smaller is the dose received by the person. Doses from most medical diagnostics have decreased in recent years due to faster films that require less exposure time.



A lead apron is placed over the dental patient and shielding surrounds the x-ray tube to limit exposure to tissue other than the tissue that is being imaged. Fast films limit the time needed to obtain images, reducing exposure to the imaged tissue. The technician stands a few meters away behind a lead-lined door with a lead glass window, reducing her occupational exposure.

Typical Doses Received During Diagnostic X-ray Exams

Procedure	Effective dose (mSv)
Chest	0.02
Dental	0.01
Skull	0.07
Leg	0.02
Mammogram	0.40
Barium enema	7.0
Upper GI	3.0
CT head	2.0
CT abdomen	10.0

Problem-Solving Strategy

You need to follow certain steps for dose calculations, which are

Step 1. *Examine the situation to determine that a person is exposed to ionizing radiation.*

Step 2. *Identify exactly what needs to be determined in the problem (identify the unknowns).* The most straightforward problems ask for a dose calculation.

Step 3. *Make a list of what is given or can be inferred from the problem as stated (identify the knowns).* Look for information on the type of radiation, the energy per event, the activity, and the mass of tissue affected.

Step 4. *For dose calculations, you need to determine the energy deposited.* This may take one or more steps, depending on the given information.

Step 5. *Divide the deposited energy by the mass of the affected tissue.* Use units of joules for energy and kilograms for mass. If a dose in Sv is involved, use the definition that $1\text{Sv} = 1\text{J/kg}$.

Step 6. *If a dose in mSv is involved, determine the RBE (QF) of the radiation.* Recall that $1 mSv = 1 mGy \times \text{RBE}(\text{ or } 1 rem = 1 rad \times \text{RBE})$.

Step 7. *Check the answer to see if it is reasonable: Does it make sense?* The dose should be consistent with the numbers given in the text for diagnostic, occupational, and therapeutic exposures.

Dose from Inhaled Plutonium

Calculate the dose in rem/y for the lungs of a weapons plant employee who inhales and retains an activity of $1.00\mu\text{Ci of}^{239}\text{Pu}$ in an accident. The mass of affected lung tissue is 2.00 kg, the plutonium decays by emission of a 5.23-MeV α particle, and you may assume the higher value of the RBE for α s from [Table 1].

Strategy

Dose in rem is defined by $1\text{rad} = 0.01\text{J/kg}$ and $\text{rem} = \text{rad}\times\text{RBE}$. The energy deposited is divided by the mass of tissue affected and then multiplied by the RBE. The latter two quantities are given, and so the main task in this example will be to find the energy deposited in one year. Since the activity of the source is given, we can calculate the number of decays, multiply by the energy per decay, and convert MeV to joules to get the total energy.

Solution

The activity $R = 1.00\mu\text{Ci} = 3.70 \times 10^4\text{Bq} = 3.70 \times 10^4$ decays/s. So, the number of decays per year is obtained by multiplying by the number of seconds in a year:

$$(3.70\times10^4\text{decays/s})(3.16\times10^7\text{s})=1.17\times10^{12}\text{decays}.$$

Thus, the ionizing energy deposited per year is

$$E=(1.17\times10^{12}\text{decays})(5.23\text{MeV/decay})\times(1.60\times10^{-13}\text{JMeV})=0.978\text{J}.$$

Dividing by the mass of the affected tissue gives

$$E\text{mass}=0.978\text{J}2.00\text{kg}=0.489\text{J/kg}.$$

One Gray is 1.00 J/kg, and so the dose in Gy is

$$\text{dose in Gy}=0.489\text{J/kg}1.00\text{(J/kg)/Gy}=0.489Gy.$$

Now, the dose in Sv is

$$\begin{aligned}&\text{doseinSv}=\text{Gy}\times\text{RBE}\\&=(0.489Gy)(20)=9.8\text{Sv}.\end{aligned}$$

Discussion

First note that the dose is given to two digits, because the RBE is (at best) known only to two digits. By any standard, this yearly radiation dose is high and will have a devastating effect on the health of the worker. Worse yet, plutonium has a long radioactive half-life and is not readily eliminated by the body, and so it will remain in the lungs. Being an α emitter makes the effects 10 to 20 times worse than the same ionization produced by β s, γ rays, or x-rays. An activity of $1.00 \mu Ci$ is created by only $16 \mu g$ of ^{239}Pu (left as an end-of-chapter problem to verify), partly justifying claims that plutonium is the most toxic substance known. Its actual hazard depends on how likely it is to be spread out among a large population and then ingested. The Chernobyl disaster's deadly legacy, for example, has nothing to do with the plutonium it put into the environment.

Risk versus Benefit

Medical doses of radiation are also limited. Diagnostic doses are generally low and have further lowered with improved techniques and faster films. With the possible exception of routine dental x-rays, radiation is used diagnostically only when needed so that the low risk is justified by the benefit of the diagnosis. Chest x-rays give the lowest doses—about 0.1 mSv to the tissue affected, with less than 5 percent scattering into tissues that are not directly imaged. Other x-ray procedures range upward to about 10 mSv in a CT scan, and about 5 mSv (0.5 rem) per dental x-ray, again both only affecting the tissue imaged. Medical images with radiopharmaceuticals give doses ranging from 1 to 5 mSv, usually localized. One exception is the thyroid scan using ^{131}I . Because of its relatively long half-life, it exposes the thyroid to about 0.75 Sv. The isotope ^{123}I is more difficult to produce, but its short half-life limits thyroid exposure to about 15 mSv.

PhET Explorations: Alpha Decay

Watch alpha particles escape from a polonium nucleus, causing radioactive alpha decay. See how random decay times relate to the half life.

Section Summary

- The biological effects of ionizing radiation are due to two effects it has on cells: interference with cell reproduction, and destruction of cell function.
- A radiation dose unit called the rad is defined in terms of the ionizing energy deposited per kilogram of tissue:

  $$1\,rad = 0.01\,\text{J/kg}.$$

- The SI unit for radiation dose is the gray (Gy), which is defined to be $$1 Gy = 1 J/kg = 100 rad.
- To account for the effect of the type of particle creating the ionization, we use the relative biological effectiveness (RBE) or quality factor (QF) given in [Table 1] and define a unit called the roentgen equivalent man (rem) as

  $$\text{rem} = \text{rad} \times \text{RBE}.$$

- Particles that have short ranges or create large ionization densities have RBEs greater than unity. The SI equivalent of the rem is the sievert (Sv), defined to be

  $$\text{Sv} = \text{Gy} \times \text{RBE} \text{ and } 1\,\text{Sv} = 100\,\text{rem}.$$

- Whole-body, single-exposure doses of 0.1 Sv or less are low doses while those of 0.1 to 1 Sv are moderate, and those over 1 Sv are high doses. Some immediate radiation effects are given in [Table 3]. Effects due to low doses are not observed, but their risk is assumed to be directly proportional to those of high doses, an assumption known as the linear hypothesis. Long-term effects are cancer deaths at the rate of $10/10^6 \text{rem} \cdot \text{y}$ and genetic defects at roughly one-third this rate. Background radiation doses and sources are given in [Table 4]. World-wide average radiation exposure from natural sources, including radon, is about 3 mSv, or 300 mrem. Radiation protection utilizes shielding, distance, and time to limit exposure.

Conceptual Questions

Isotopes that emit α radiation are relatively safe outside the body and exceptionally hazardous inside. Yet those that emit γ radiation are hazardous outside and inside. Explain why.

Why is radon more closely associated with inducing lung cancer than other types of cancer?

The RBE for low-energy β s is 1.7, whereas that for higher-energy β s is only 1. Explain why, considering how the range of radiation depends on its energy.

Which methods of radiation protection were used in the device shown in the first photo in [Figure 3]? Which were used in the situation shown in the second photo?

(a)

(a) (b)

(a) This x-ray fluorescence machine is one of the thousands used in shoe stores to produce images of feet as a check on the fit of shoes. They are unshielded and remain on as long as the feet are in them, producing doses much greater than medical images. Children were fascinated with them. These machines were used in shoe stores until laws preventing such unwarranted radiation exposure were enacted in the 1950s. (credit: Andrew Kuchling) (b) Now that we know the effects of exposure to radioactive material, safety is a priority. (credit: U.S. Navy)

What radioisotope could be a problem in homes built of cinder blocks made from uranium mine tailings? (This is true of homes and schools in certain regions near uranium mines.)

Are some types of cancer more sensitive to radiation than others? If so, what makes them more sensitive?

Suppose a person swallows some radioactive material by accident. What information is needed to be able to assess possible damage?

Problems & Exercises

What is the dose in mSv for: (a) a 0.1 Gy x-ray? (b) 2.5 mGy of neutron exposure to the eye? (c) 1.5 mGy of α exposure?

Show Solution

(a) 100 mSv

(b) 80 mSv

(c) ~30 mSv

Find the radiation dose in Gy for: (a) A 10-mSv fluoroscopic x-ray series. (b) 50 mSv of skin exposure by an α emitter. (c) 160 mSv of β^- and γ rays from the ^{40}K in your body.

Show Solution

Strategy

To convert from dose equivalent in Sv (or mSv) to absorbed dose in Gy, we use the relationship $\text{Sv} = \text{Gy} \times \text{RBE}$ or equivalently $\text{Gy} = \frac{\text{Sv}}{\text{RBE}}$. We need to identify the appropriate RBE for each type of radiation from Table 1: x-rays have RBE = 1, alpha particles have RBE = 10-20, and beta/gamma rays have RBE = 1.

Solution

(a) For x-rays, RBE = 1:

$$\text{Dose in Gy} = \frac{10\ \text{mSv}}{1} = 10\ \text{mGy} = 0.010\ \text{Gy}$$

(b) For α particles, using RBE = 20 (the higher value for maximum biological effect):

$$\text{Dose in Gy} = \frac{50\ \text{mSv}}{20} = 2.5\ \text{mGy} = 0.0025\ \text{Gy}$$

(c) For β^- and γ rays, RBE = 1:

$$\text{Dose in Gy} = \frac{160\ \text{mSv}}{1} = 160\ \text{mGy} = 0.16\ \text{Gy}$$

Discussion

These conversions illustrate an important principle in radiation dosimetry: the absorbed dose in Gy can be much smaller than the dose equivalent in Sv for high-RBE radiation. Notice that in part (b), although the alpha exposure gives 50 mSv (five times the x-ray dose), the actual energy deposited is only 2.5 mGy (one-fourth the x-ray energy deposition of 10 mGy). This is because alpha particles, with their short range and high ionization density, cause 20 times more biological damage per unit of absorbed energy. For x-rays, beta particles, and gamma rays (RBE = 1), the numerical values in mSv and mGy are identical, which is why these units are often used interchangeably in practice for these common types of radiation. The ^{40}K in part (c) is a natural source of background radiation that exists in everyone’s body, contributing to the unavoidable radiation exposure we all experience.

How many Gy of exposure is needed to give a cancerous tumor a dose of 40 Sv if it is exposed to α activity?

Show Solution

~2 Gy

What is the dose in Sv in a cancer treatment that exposes the patient to 200 Gy of γ rays?

Show Solution

Strategy

To convert from absorbed dose in Gy to dose equivalent in Sv, we use $\text{Sv} = \text{Gy} \times \text{RBE}$. From Table 1, gamma rays have RBE = 1.

Solution

$$\text{Dose in Sv} = (200\ \text{Gy})(1) = 200\ \text{Sv}$$

Discussion

This is an extremely large dose of 200 Sv, which would be lethal if given to the whole body. However, in cancer radiotherapy, such large doses are acceptable because they are highly localized to the tumor region and delivered in fractionated treatments over several weeks rather than all at once. The treatment schedule allows normal tissue to repair itself between sessions while the more radiation-sensitive cancer cells accumulate lethal damage. Typical radiotherapy protocols deliver 200 rem (2 Sv) per treatment, 3-5 times per week, for a total cumulative dose to the tumor that can reach 50-80+ Sv as shown in Table 1. The fact that gamma rays have RBE = 1 means the numerical values in Gy and Sv are the same, simplifying dosimetry calculations for this common therapeutic radiation. The tolerance of the surrounding normal tissue ultimately limits the maximum dose that can be safely delivered to the tumor.

One half the γ rays from ^{99m}Tc are absorbed by a 0.170-mm-thick lead shielding. Half of the γ rays that pass through the first layer of lead are absorbed in a second layer of equal thickness. What thickness of lead will absorb all but one in 1000 of these γ rays?

Show Solution

1.69 mm

A plumber at a nuclear power plant receives a whole-body dose of 30 mSv in 15 minutes while repairing a crucial valve. Find the radiation-induced yearly risk of death from cancer and the chance of genetic defect from this maximum allowable exposure.

Show Solution

Strategy

The text states that the risk of radiation-induced cancer death is approximately $10/10^6\text{ rem}\cdot\text{y}$ (or 10 in a million per rem per year), and the risk of genetic defects is about one-third of this rate, $3.3/10^6\text{ rem}\cdot\text{y}$. First, we need to convert the dose from mSv to rem: $1\text{ Sv} = 100\text{ rem}$, so $30\text{ mSv} = 3.0\text{ rem}$. The yearly risk is calculated by multiplying the dose in rem by the risk factors.

Solution

First, convert to rem:

$$30\text{ mSv} = 0.030\text{ Sv} = 0.030 \times 100\text{ rem} = 3.0\text{ rem}$$

Cancer death risk per year:

$$\text{Risk}_{\text{cancer}} = (3.0\text{ rem}) \times (1010^6\text{ rem}\cdot\text{y}) = 3010^6\text{ y} = 3.0 \times 10^{-5}\text{ per year}$$

This represents a yearly risk of 30 in a million, or 0.003%.

Genetic defect risk per year:

$$\text{Risk}_{\text{genetic}} = (3.0\text{ rem}) \times (3.310^6\text{ rem}\cdot\text{y}) = 9.910^6\text{ y} \approx 1.0 \times 10^{-5}\text{ per year}$$

This represents a yearly risk of 10 in a million, or 0.001%.

Discussion

These risks must be interpreted carefully. The cancer death risk of 3.0×10^{-5} per year means that each year for approximately 30 years following exposure (the latency and risk period), the plumber has an additional 30 in a million chance of dying from radiation-induced cancer. Over the full 30-year risk period, the cumulative lifetime risk would be approximately $30 \times 30 = 900$ in a million, or about 0.09%. To put this in perspective, the normal incidence of cancer deaths is about 20% (200,000 in a million), so this radiation-induced increase would be very difficult to detect statistically in any individual case. The genetic defect risk is about one-third the cancer risk and applies to potential offspring. These risk estimates are based on the linear hypothesis, which assumes that any radiation dose, no matter how small, carries some risk proportional to the dose. While this assumption is conservative and possibly overestimates low-dose risks, it provides a prudent basis for radiation safety standards. The 30 mSv exposure is at the upper limit of allowed occupational doses and demonstrates why such exposures are carefully monitored and minimized in nuclear facilities.

In the 1980s, the term picowave was used to describe food irradiation in order to overcome public resistance by playing on the well-known safety of microwave radiation. Find the energy in MeV of a photon having a wavelength of a picometer.

Show Solution

1.24 MeV

Find the mass of ^{239}Pu that has an activity of $1.00\,\mu\text{Ci}$.

Show Solution

Strategy

We'll use the relationship between activity, number of nuclei, and half-life: $R = 0.693 N t_{1/2}$. First, convert the activity to Bq, then solve for the number of nuclei N. Finally, convert the number of nuclei to mass using the molar mass of ^{239}Pu and Avogadro's number. The half-life of ^{239}Pu is 24,120 years.

Solution

Convert activity to Bq:

$$R = 1.00\ \mu\text{Ci} = 1.00 \times 10^{-6}\ \text{Ci} \times \frac{3.70 \times 10^{10}\ \text{Bq}}{1\ \text{Ci}} = 3.70 \times 10^{4}\ \text{Bq}$$

Convert half-life to seconds:

$$t_{1/2} = 24{,}120\ \text{y} \times \frac{3.156 \times 10^{7}\ \text{s}}{1\ \text{y}} = 7.614 \times 10^{11}\ \text{s}$$

Solve for N using $R = \frac{0.693 N}{t_{1/2}}$:

$$N = \frac{R \times t_{1/2}}{0.693} = \frac{(3.70 \times 10^{4}\ \text{s}^{-1})(7.614 \times 10^{11}\ \text{s})}{0.693} = 4.06 \times 10^{16}\ \text{nuclei}$$

Convert to moles:

$$\text{moles} = \frac{4.06 \times 10^{16}}{6.022 \times 10^{23}\ \text{nuclei/mol}} = 6.74 \times 10^{-8}\ \text{mol}$$

Convert to mass (using molar mass of 239 g/mol):

$$m = (6.74 \times 10^{-8}\ \text{mol})(239\ \text{g/mol}) = 1.61 \times 10^{-5}\ \text{g} = 16.1\ \mu\text{g}$$

Discussion

This remarkably small mass of only 16.1 micrograms demonstrates the extreme radioactivity and toxicity of plutonium-239. The Discussion section of the worked example in the text mentions that $1.00\ \mu\text{Ci}$ is created by only $16\ \mu\text{g}$ of ^{239}Pu, and our calculation confirms this value. This tiny amount of material, barely visible to the naked eye, produces 37,000 disintegrations per second! The long half-life of 24,120 years means that ^{239}Pu decays slowly, but each decay releases a highly damaging 5.2-MeV alpha particle. When inhaled and retained in the lungs as discussed in the text example, even this minuscule mass delivers a devastating dose of about 10 Sv per year to lung tissue. This explains why plutonium is sometimes called "the most toxic substance known"—not because individual atoms are particularly hazardous, but because such an incredibly small mass can deliver such a large radiation dose over many years due to its alpha emissions, long half-life, and tendency to be retained in the body. This problem also illustrates why plutonium contamination is so serious: microscopic particles invisible to the eye can contain enough activity to pose significant health risks.

Footnotes

- 1 Values approximate, difficult to determine.
- 2 Multiply by 100 to obtain dose in rem.
- 3 Multiply by 100 to obtain dose in mrem/y. { data-list-type="bulleted" data-bullet-style="none"}

Glossary

gray (Gy)
: the SI unit for radiation dose which is defined to be $1 Gy = 1 J/kg = 100 \text{rad}$

linear hypothesis
: assumption that risk is directly proportional to risk from high doses

rad
: the ionizing energy deposited per kilogram of tissue

sievert
: the SI equivalent of the rem

relative biological effectiveness (RBE)
: a number that expresses the relative amount of damage that a fixed amount of ionizing radiation of a given type can inflict on biological tissues

quality factor
: same as relative biological effectiveness

roentgen equivalent man (rem)
: a dose unit more closely related to effects in biological tissue

low dose
: a dose less than 100 mSv (10 rem)

moderate dose
: a dose from 0.1 Sv to 1 Sv (10 to 100 rem)

high dose
: a dose greater than 1 Sv (100 rem)

hormesis
: a term used to describe generally favorable biological responses to low exposures of toxins or radiation

shielding
: a technique to limit radiation exposure

Therapeutic Uses of Ionizing Radiation

- Explain the concept of radiotherapy and list typical doses for cancer therapy.

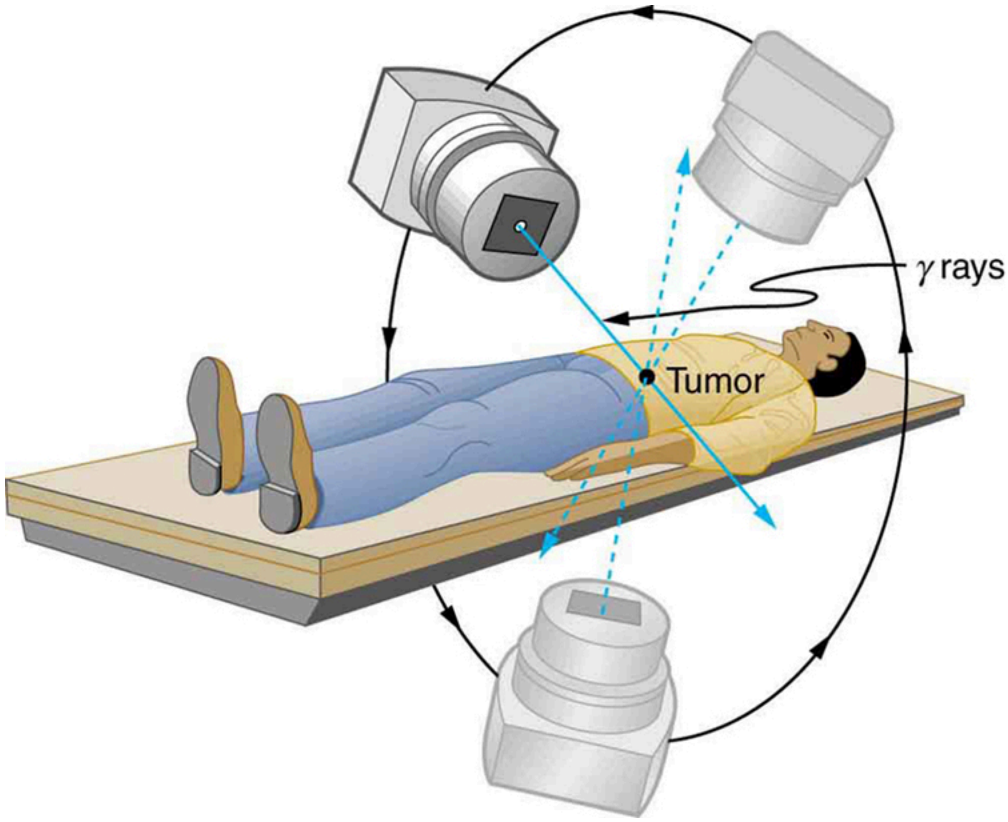
Therapeutic applications of ionizing radiation, called radiation therapy or **radiotherapy**, have existed since the discovery of X-rays and nuclear radioactivity. Today, radiotherapy is used almost exclusively for cancer therapy, where it saves thousands of lives and improves the quality of life and longevity of many it cannot save. Radiotherapy may be used alone or in combination with surgery and chemotherapy (drug treatment) depending on the type of cancer and the response of the patient. A careful examination of all available data has established that radiotherapy's beneficial effects far outweigh its long-term risks.

Medical Application

The earliest uses of ionizing radiation on humans were mostly harmful, with many at the level of snake oil as seen in [Figure 1]. Radium-doped cosmetics that glowed in the dark were used around the time of World War I. As recently as the 1950s, radon mine tours were promoted as healthful and rejuvenating—those who toured were exposed but gained no benefits. Radium salts were sold as health elixirs for many years. The gruesome death of a wealthy industrialist, who became psychologically addicted to the brew, alerted the unsuspecting to the dangers of radium salt elixirs. Most abuses finally ended after the legislation in the 1950s.

The properties of radiation were once touted for far more than its modern use in cancer therapy. Until 1932, radium was advertised for a variety of uses, often with tragic results. (credit: Struthious Bandersnatch.)

Radiotherapy is effective against cancer because cancer cells reproduce rapidly and, consequently, are more sensitive to radiation. The central problem in radiotherapy is to make the dose for cancer cells as high as possible while limiting the dose for normal cells. The ratio of abnormal cells killed to normal cells killed is called the **therapeutic ratio**, and all radiotherapy techniques are designed to enhance this ratio. Radiation can be concentrated in cancerous tissue by a number of techniques. One of the most prevalent techniques for well-defined tumors is a geometric technique shown in [Figure 2] . A narrow beam of radiation is passed through the patient from a variety of directions with a common crossing point in the tumor. This concentrates the dose in the tumor while spreading it out over a large volume of normal tissue. The external radiation can be X-rays, ^{60}Co γ rays, or ionizing-particle beams produced by accelerators. Accelerator-produced beams of neutrons, $\pi\text{-mesons}$, and heavy ions such as nitrogen nuclei have been employed, and these can be quite effective. These particles have larger QFs or RBEs and sometimes can be better localized, producing a greater therapeutic ratio. But accelerator radiotherapy is much more expensive and less frequently employed than other forms.



The 60Co source of γ-radiation is rotated around the patient so that the common crossing point is in the tumor, concentrating the dose there. This geometric technique works for well-defined tumors.

Another form of radiotherapy uses chemically inert radioactive implants. One use is for prostate cancer. Radioactive seeds (about 40 to 100 and the size of a grain of rice) are placed in the prostate region. The isotopes used are usually ^{135}I (6-month half life) or ^{103}Pd (3-month half life). Alpha emitters have the dual advantages of a large QF and a small range for better localization.

Radiopharmaceuticals are used for cancer therapy when they can be localized well enough to produce a favorable therapeutic ratio. Thyroid cancer is commonly treated utilizing radioactive iodine. Thyroid cells concentrate iodine, and cancerous thyroid cells are more aggressive in doing this. An ingenious use of radiopharmaceuticals in cancer therapy tags antibodies with radioisotopes. Antibodies produced by a patient to combat his cancer are extracted, cultured, loaded with a radioisotope, and then returned to the patient. The antibodies are concentrated almost entirely in the tissue they developed to fight, thus localizing the radiation in abnormal tissue. The therapeutic ratio can be quite high for short-range radiation. There is, however, a significant dose for organs that eliminate radiopharmaceuticals from the body, such as the liver, kidneys, and bladder. As with most radiotherapy, the technique is limited by the tolerable amount of damage to the normal tissue.

[Table 1] lists typical therapeutic doses of radiation used against certain cancers. The doses are large, but not fatal because they are localized and spread out in time. Protocols for treatment vary with the type of cancer and the condition and response of the patient. Three to five 200-rem treatments per week for a period of several weeks is typical. Time between treatments allows the body to repair normal tissue. This effect occurs because damage is concentrated in the abnormal tissue, and the abnormal tissue is more sensitive to radiation. Damage to normal tissue limits the doses. You will note that the greatest doses are given to any tissue that is not rapidly reproducing, such as in the adult brain. Lung cancer, on the other end of the scale, cannot ordinarily be cured with radiation because of the sensitivity of lung tissue and blood to radiation. But radiotherapy for lung cancer does alleviate symptoms and prolong life and is therefore justified in some cases.

Cancer Radiotherapy

Type of Cancer	Typical dose (Sv)
Lung	10–20
Hodgkin’s disease	40–45
Skin	40–50
Ovarian	50–75
Breast	50–80+
Brain	80+
Neck	80+
Bone	80+
Soft tissue	80+
Thyroid	80+

Finally, it is interesting to note that chemotherapy employs drugs that interfere with cell division and is, thus, also effective against cancer. It also has almost the same side effects, such as nausea and hair loss, and risks, such as the inducement of another cancer.

Section Summary

- Radiotherapy is the use of ionizing radiation to treat ailments, now limited to cancer therapy.
- The sensitivity of cancer cells to radiation enhances the ratio of cancer cells killed to normal cells killed, which is called the therapeutic ratio.
- Doses for various organs are limited by the tolerance of normal tissue for radiation. Treatment is localized in one region of the body and spread out in time.

Conceptual Questions

Radiotherapy is more likely to be used to treat cancer in elderly patients than in young ones. Explain why. Why is radiotherapy used to treat young people at all?

Problems & Exercises

A beam of 168-MeV nitrogen nuclei is used for cancer therapy. If this beam is directed onto a 0.200-kg tumor and gives it a 2.00-Sv dose, how many nitrogen nuclei were stopped? (Use an RBE of 20 for heavy ions.)

Show Solution

7.44×10^{8}

(a) If the average molecular mass of compounds in food is 50.0 g, how many molecules are there in 1.00 kg of food? (b) How many ion pairs are created in 1.00 kg of food, if it is exposed to 1000 Sv and it takes 32.0 eV to create an ion pair? (c) Find the ratio of ion pairs to molecules. (d) If these ion pairs recombine into a distribution of 2000 new compounds, how many parts per billion is each?

Show Solution

Strategy

(a) Find the number of moles in 1.00 kg, then multiply by Avogadro’s number. (b) Convert the dose to energy deposited, then divide by the energy per ion pair. (c) Divide the results from (b) by (a). (d) Divide the total ion pairs by 2000 to find molecules per new compound type, then find the ratio to total molecules.

Solution

(a) Number of molecules in 1.00 kg of food:

$$\text{moles} = \frac{1000\text{ g}}{50.0\text{ g/mol}} = 20.0\text{ mol}$$
$$N_{\text{molecules}} = (20.0\text{ mol})(6.022\times10^{23}\text{ molecules/mol}) = 1.20\times10^{25}\text{ molecules}$$

(b) Number of ion pairs created:

First, find energy deposited. Since $1\text{ Sv} = 1\text{ J/kg}$:

$$E = (1000\text{ Sv})(1.00\text{ kg}) = 1000\text{ J}$$

Convert to eV:

$$E = 1000\text{ J} \times \frac{1\text{ eV}}{1.602\times10^{-19}\text{ J}} = 6.24\times10^{21}\text{ eV}$$

Number of ion pairs:

$$N_{\text{ion pairs}} = \frac{6.24\times10^{21}\text{ eV}}{32.0\text{ eV/ion pair}} = 1.95\times10^{20}\text{ ion pairs}$$

(c) Ratio of ion pairs to molecules:

$$\text{Ratio} = \frac{1.95\times10^{20}}{1.20\times10^{25}} = 1.63\times10^{-5} = \frac{1}{61,500}$$

This means about 1 molecule in every 61,500 is ionized.

(d) Parts per billion for each new compound:

If 1.95×10^{20} ion pairs recombine into 2000 different new compounds:

$$\text{Molecules per compound} = \frac{1.95\times10^{20}}{2000} = 9.75\times10^{16}\text{ molecules}$$
$$\text{Parts per billion} = \frac{9.75\times10^{16}}{1.20\times10^{25}} \times 10^{9} = 8.1\times10^{0} \approx 8\text{ ppb}$$

Discussion

This problem illustrates the chemical effects of food irradiation. Even at the massive dose of 1000 Sv (which would be instantly lethal to a human), only about 1 in 61,500 molecules is ionized. This relatively small fraction nevertheless creates 1.95×10^{20} ion pairs, which can recombine to form new chemical compounds. If these reorganize into 2000 different new compounds (a reasonable estimate), each compound would be present at about 8 parts per billion. These trace amounts are generally considered safe and are similar to or less than concentrations of compounds formed during conventional food processing like cooking, canning, or freezing. The key safety consideration is whether any of these new compounds are harmful at such low concentrations. Extensive testing has shown that irradiated food is safe because: (1) the concentrations of radiolytic products are very low, (2) most such products are also formed during cooking, and (3) the compounds formed are generally simple and well-characterized. Food irradiation at much lower doses (typically 0.1-10 kGy or 0.1-10 Sv for RBE=1) is used to kill pathogens, extend shelf life, and control insects, making it a valuable food safety tool that has been approved by the FDA and WHO.

Calculate the dose in Sv to the chest of a patient given an x-ray under the following conditions. The x-ray beam intensity is 1.50W/m^2 , the area of the chest exposed is 0.0750m^2 , 35.0% of the X-rays are absorbed in 20.0 kg of tissue, and the exposure time is 0.250 s.

Show Solution

$4.92 \times 10^{-4} \text{Sv}$

(a) A cancer patient is exposed to γ rays from a 5000-Ci ^{60}Co transillumination unit for 32.0 s. The γ rays are collimated in such a manner that only 1.00% of them strike the patient. Of those, 20.0% are absorbed in a tumor having a mass of 1.50 kg. What is the dose in rem to the tumor, if the average γ energy per decay is 1.25 MeV? None of the β s from the decay reach the patient. (b) Is the dose consistent with stated therapeutic doses?

Show Solution

Strategy

(a) Calculate the number of decays in 32.0 s using the activity in Bq. Then find how many gamma rays are absorbed (1.00% strike × 20.0% absorbed = 0.200% total absorbed). Multiply by energy per gamma to get total energy absorbed, convert to joules, divide by tumor mass to get dose in Gy, then note that for gamma rays RBE = 1, so dose in rem equals dose in rad. (b) Compare with Table 1 therapeutic doses.

Solution

(a) Convert activity to Bq:

$$R = 5000 \text{ Ci} \times \frac{3.70 \times 10^{10} \text{ Bq}}{1 \text{ Ci}} = 1.85 \times 10^{14} \text{ Bq}$$

Number of decays in 32.0 s:

$$N_{\text{decays}} = (1.85 \times 10^{14} \text{ s}^{-1})(32.0 \text{ s}) = 5.92 \times 10^{15} \text{ decays}$$

Fraction absorbed in tumor:

$$f = (0.0100)(0.200) = 0.00200$$

Energy absorbed:

$$E = (5.92 \times 10^{15})(0.00200)(1.25 \text{ MeV}) = 1.48 \times 10^{13} \text{ MeV}$$

Convert to joules:

$$E = (1.48 \times 10^{13} \text{ MeV})(1.602 \times 10^{-13} \text{ J/MeV}) = 2.37 \text{ J}$$

Dose in Gy:

$$\text{Dose in Gy} = \frac{2.37 \text{ J}}{1.50 \text{ kg}} = 1.58 \text{ Gy} = 158 \text{ rad}$$

Since RBE = 1 for gamma rays:

$$\text{Dose in rem} = (158 \text{ rad})(1) = 158 \text{ rem} = 1.58 \text{ Sv}$$

(b) From Table 1, therapeutic doses range from 10-20 Sv for lung cancer up to 80+ Sv for brain, neck, bone, soft tissue, and thyroid cancers. A typical treatment protocol delivers about 2 Sv (200 rem) per session. Our calculated dose of 1.58 Sv (158 rem) is **consistent with a single therapeutic treatment**, slightly below the typical 200 rem per fraction. Multiple treatments would bring the total cumulative dose into the therapeutic range shown in Table 1.

Discussion

This problem demonstrates a realistic cancer radiotherapy scenario. The 5000-Ci (185 TBq) ^{60}Co source produces an enormous number of decays—nearly 6 quadrillion in just 32 seconds! However, careful collimation ensures that only 1% of the radiation strikes the patient, and only 20% of that is absorbed in the tumor, so that only 0.2% of the emitted radiation energy is actually deposited in the tumor. This geometric focusing, achieved by rotating the source around the patient as shown in Figure 2, is critical for delivering high doses to the tumor while minimizing exposure to healthy tissue. The dose of 158 rem to the tumor in a single 32-second exposure is substantial but safe when localized. This would typically be one of multiple fractionated treatments delivered over several weeks, with the tumor receiving total cumulative doses of 50-80 Sv or more depending on cancer type and location. The fact that no beta particles reach the patient is important because ^{60}Co emits both beta particles and gamma rays; the betas are easily stopped by the source housing and collimator, while the 1.17 and 1.33 MeV gamma rays penetrate to reach the tumor. Modern linear accelerators have largely replaced ^{60}Co units in developed countries, but cobalt-60 therapy remains important in developing nations due to its simplicity, reliability, and lower cost.

What is the mass of ^{60}Co in a cancer therapy transillumination unit containing 5.00 kCi of ^{60}Co ?

Show Solution

4.43 g

Large amounts of ^{65}Zn are produced in copper exposed to accelerator beams. While machining contaminated copper, a physicist ingests $50.0 \mu Ci$ of ^{65}Zn . Each ^{65}Zn decay emits an average γ -ray energy of 0.550 MeV, 40.0% of which is absorbed in the scientist's 75.0-kg body. What dose in mSv is caused by this in one day?

Show Solution

Strategy

Calculate the number of decays in one day from the activity, multiply by the energy per decay and the absorption fraction to get total energy deposited, convert to joules, and divide by body mass to get dose in Gy. Since gamma rays have RBE = 1, the dose in Sv equals the dose in Gy, then convert to mSv.

Solution

Convert activity to Bq:

$$R = 50.0\ \mu\text{Ci} = 50.0 \times 10^{-6}\ \text{Ci} \times \frac{3.70 \times 10^{10}\ \text{Bq}}{1\ \text{Ci}} = 1.85 \times 10^{6}\ \text{Bq}$$

Number of decays in one day:

$$N_{\text{decays}} = (1.85 \times 10^{6}\ \text{s}^{-1})(86{,}400\ \text{s}) = 1.60 \times 10^{11}\ \text{decays}$$

Energy absorbed (40.0% of emitted energy):

$$E = (1.60 \times 10^{11})(0.550\ \text{MeV})(0.400) = 3.52 \times 10^{10}\ \text{MeV}$$

Convert to joules:

$$E = (3.52 \times 10^{10}\ \text{MeV})(1.602 \times 10^{-13}\ \text{J/MeV}) = 5.64 \times 10^{-3}\ \text{J}$$

Dose in Gy:

$$\text{Dose in Gy} = \frac{5.64 \times 10^{-3}\ \text{J}}{75.0\ \text{kg}} = 7.52 \times 10^{-5}\ \text{Gy}$$

Since RBE = 1 for gamma rays:

$$\text{Dose in Sv} = 7.52 \times 10^{-5}\ \text{Sv} = 0.0752\ \text{mSv}$$

Discussion

The dose of 0.075 mSv in one day is relatively small but not negligible. To put this in perspective, the average annual background radiation dose is about 3 mSv, so this one-day dose from ingested ^{65}Zn represents about 2.5% of the annual background dose. However, this calculation only accounts for one day's exposure. The actual total dose depends on how long the zinc-65 remains in the body. With a radiological half-life of 244 days, ^{65}Zn decays relatively slowly, but biological elimination removes zinc from the body with a biological half-life of about 300-500 days. The effective half-life (combining radioactive decay and biological elimination) is approximately 140 days. Over this longer period, the cumulative dose would be significantly higher—roughly 150 times the one-day dose, or about 11 mSv total. This cumulative dose is still well below occupational limits (20-50 mSv/y) but represents a serious contamination incident that would require medical follow-up and investigation. This problem underscores the importance of proper safety protocols when machining or handling materials that have been exposed to radiation. Even if the copper appears "cold" (low external radiation), internal contamination from ingested or inhaled particulates can deliver significant doses. The 40% absorption fraction is realistic for gamma rays from an internally distributed source, as some gammas escape the body without interacting.

Naturally occurring ^{40}K is listed as responsible for 16 mrem/y of background radiation. Calculate the mass of ^{40}K that must be inside the 55-kg body of a woman to produce this dose. Each ^{40}K decay emits a 1.32-MeV β , and 50% of the energy is absorbed inside the body.

Show Solution

0.010 g

(a) Background radiation due to ^{226}Ra averages only 0.01 mSv/y, but it can range upward depending on where a person lives. Find the mass of ^{226}Ra in the 80.0-kg body of a man who receives a dose of 2.50-mSv/y from it, noting that each ^{226}Ra decay emits a 4.80-MeV α particle. You may neglect dose due to daughters and assume a constant amount, evenly distributed due to balanced ingestion and bodily elimination. (b) Is it surprising that such a small mass could cause a measurable radiation dose? Explain.

Show Solution

Strategy

(a) Work backwards from dose to find mass. For alpha particles, RBE = 20, so $\text{dose in Sv} = \text{dose in Gy} \times 20$. Convert dose to energy deposited per year, assuming 100% absorption since alphas have short range. Find number of decays per year, then use $R = \frac{0.693 N}{t_{1/2}}$ to find N, and convert to mass. The half-life of ^{226}Ra is 1600 years. (b) Consider the high RBE and energy of alpha particles.

Solution

(a) Convert dose in Sv to dose in Gy:

$$\text{Dose in Gy} = \frac{\text{Dose in Sv}}{\text{RBE}} = \frac{2.50\text{ mSv}}{20} = 0.125\text{ mGy} = 1.25 \times 10^{-4}\text{ Gy}$$

Energy deposited per year:

$$E = (1.25 \times 10^{-4}\text{ J/kg})(80.0\text{ kg}) = 0.0100\text{ J}$$

Convert to MeV:

$$E = \frac{0.0100\text{ J}}{1.602 \times 10^{-13}\text{ J/MeV}} = 6.24 \times 10^{10}\text{ MeV}$$

Number of decays per year (assuming 100% absorption):

$$N_{\text{decays}} = \frac{6.24 \times 10^{10}\text{ MeV}}{4.80\text{ MeV/decay}} = 1.30 \times 10^{10}\text{ decays/y}$$

Convert to activity in Bq:

$$R = \frac{1.30 \times 10^{10}\text{ decays/y}}{3.156 \times 10^{7}\text{ s/y}} = 412\text{ Bq}$$

Convert half-life to seconds:

$$t_{1/2} = 1600\text{ y} \times 3.156 \times 10^{7}\text{ s/y} = 5.05 \times 10^{10}\text{ s}$$

Find number of nuclei using $R = \frac{0.693 N}{t_{1/2}}$:

$$N = \frac{R \times t_{1/2}}{0.693} = \frac{(412)(5.05 \times 10^{10})}{0.693} = 3.00 \times 10^{13}\text{ nuclei}$$

Convert to mass:

$$m = \frac{N}{N_A} \times M = \frac{3.00 \times 10^{13}}{6.022 \times 10^{23}} \times 226\text{ g/mol} = 1.13 \times 10^{-8}\text{ g} = 11.3\text{ ng}$$

(b) Yes, it is remarkable that only 11.3 nanograms of ^{226}Ra can cause a measurable 2.50 mSv/y dose. This is not surprising when we consider: (1) Alpha particles have very high RBE = 20, causing 20 times more biological damage per unit energy than gamma rays; (2) The 4.80 MeV energy per decay is substantial; (3) Alpha particles deposit all their energy within the body due to their short range; (4) The long half-life of 1600 years means the activity persists essentially unchanged over a human lifetime.

Discussion

This problem illustrates why alpha emitters are particularly hazardous when ingested or inhaled. A mass of only 11.3 nanograms—invisible to the naked eye and containing only 30 trillion atoms—produces 412 decays per second continuously over years. Because alpha particles have a range of only about 40 micrometers in tissue, they deposit all 4.80 MeV of energy locally, and their high ionization density (RBE = 20) makes them 20 times more damaging than gamma rays for the same energy deposition. The actual average background dose from ^{226}Ra is only 0.01 mSv/y, which would correspond to a body burden of only 0.045 ng—an incredibly tiny amount. The elevated dose of 2.50 mSv/y in this problem could occur in regions with high natural radium in soil and groundwater, such as certain areas with granite geology. Radium is chemically similar to calcium and concentrates in bones when ingested, where its alpha emissions can damage bone marrow and potentially cause bone cancer or leukemia. This is what tragically occurred to the "Radium Girls" who painted watch dials with radium-containing luminous paint in the 1920s and ingested radium by licking their brushes. Their cases led to improved occupational safety standards and our modern understanding of internal alpha emitter hazards.

The annual radiation dose from ^{14}C in our bodies is 0.01 mSv/y. Each ^{14}C decay emits a β^- averaging 0.0750 MeV. Taking the fraction of ^{14}C to be $1.3 \times 10^{-12}\text{N}$ of normal ^{12}C, and assuming the body is 13% carbon, estimate the fraction of the decay energy absorbed. (The rest escapes, exposing those close to you.)

Show Solution

95%

If everyone in Australia received an extra 0.05 mSv per year of radiation, what would be the increase in the number of cancer deaths per year? (Assume that time had elapsed for the effects to become apparent.) Assume that there are 200×10^{-4} deaths per Sv of radiation per year. What percent of the actual number of cancer deaths recorded is this?

Show Solution

Strategy

Multiply Australia's population by the dose to get total person-Sv, then multiply by the death rate per Sv. Compare this to the natural cancer death rate. Australia's population is approximately 26 million (as of 2025). The natural cancer death rate is about 20% of all deaths over a lifetime, which translates to roughly 200,000 cancer deaths per million people over 70 years, or about 2850 cancer deaths per million people per year.

Solution

Total collective dose for Australia:

$$\text{Collective dose} = (26 \times 10^6 \text{ people})(0.05 \text{ mSv}) = 1.3 \times 10^6 \text{ mSv} = 1300 \text{ person-Sv}$$

Increase in cancer deaths per year using the given rate of $200 \times 10^{-4} = 0.0200$ deaths per Sv per year:

$$\Delta\text{deaths} = (1300 \text{ person-Sv})(0.0200 \text{ deaths/Sv}\cdot\text{y}) = 26 \text{ deaths/y}$$

Expected cancer deaths per year in Australia without additional radiation:

Australia has about 26 million people. With a crude cancer death rate of approximately 110 deaths per 100,000 people per year:

$$\text{Natural cancer deaths} = (26 \times 10^6)\left(\frac{110}{100{,}000}\right) \approx 28{,}600 \text{ deaths/y}$$

Percentage increase:

$$\text{Percent increase} = \frac{26}{28{,}600} \times 100\% = 0.091\% \approx 0.09\%$$

Discussion

An additional 26 cancer deaths per year from an extra 0.05 mSv annual dose across Australia's entire population would represent only about 0.09% of the natural cancer death rate. This increase would be completely undetectable statistically, lost in the natural variations in cancer rates that occur from year to year and region to region. This calculation demonstrates several important points about low-dose radiation risk: (1) Even a population-wide exposure produces effects that are statistically invisible against the natural cancer rate; (2) The linear hypothesis predicts that collective doses matter—26 million people each receiving a tiny dose has the same predicted effect as 26 people each receiving a large dose; (3) Individual risk remains very small—each person's additional risk is only $0.05 \text{ mSv} \times 0.0200 \text{ deaths/Sv} = 10^{-6}$ or one in a million. This level of dose and risk is comparable to natural variations in background radiation between locations (for example, moving from sea level to a mile-high city increases annual dose by about 0.5 mSv). The inability to detect such small increases in cancer rates is one reason why the health effects of low-dose radiation remain controversial—we must rely on extrapolation from high-dose data using the linear hypothesis, which may overestimate risks at very low doses. Nevertheless, radiation protection standards use this conservative approach to minimize all unnecessary exposures following the ALARA principle (As Low As Reasonably Achievable).

Glossary

radiotherapy
: the use of ionizing radiation to treat ailments

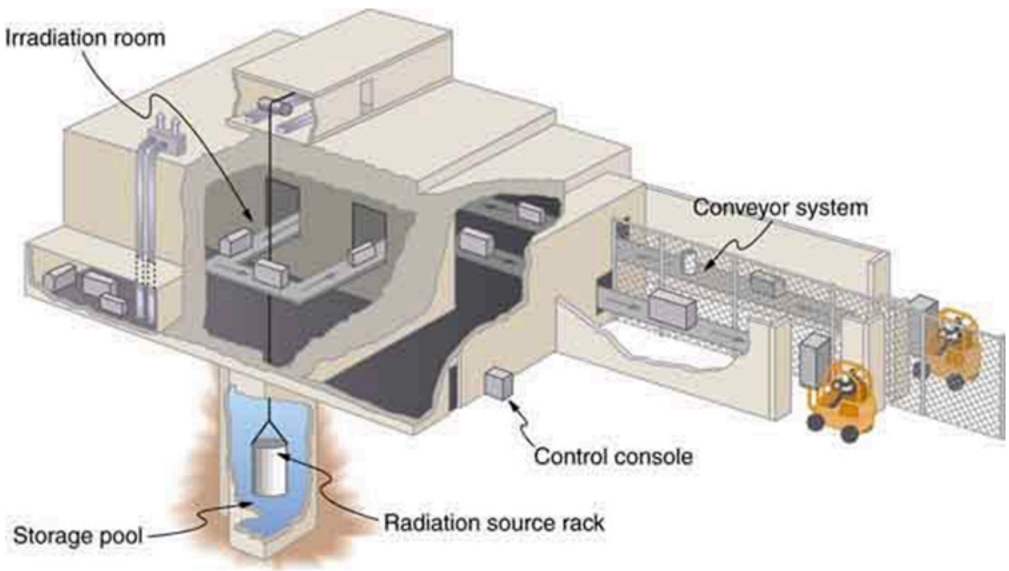
therapeutic ratio
: the ratio of abnormal cells killed to normal cells killed

Food Irradiation

- Define food irradiation low dose, and free radicals.

Ionizing radiation is widely used to sterilize medical supplies, such as bandages, and consumer products, such as tampons. Worldwide, it is also used to irradiate food, an application that promises to grow in the future. **Food irradiation** is the treatment of food with ionizing radiation. It is used to reduce pest infestation and to delay spoilage and prevent illness caused by microorganisms. Food irradiation is controversial. Proponents see it as superior to pasteurization, preservatives, and insecticides, supplanting dangerous chemicals with a more effective process. Opponents see its safety as unproven, perhaps leaving worse toxic residues as well as presenting an environmental hazard at treatment sites. In developing countries, food irradiation might increase crop production by 25.0% or more, and reduce food spoilage by a similar amount. It is used chiefly to treat spices and some fruits, and in some countries, red meat, poultry, and vegetables. Over 40 countries have approved food irradiation at some level.

Food irradiation exposes food to large doses of γ rays, X-rays, or electrons. These photons and electrons induce no nuclear reactions and thus create *no residual radioactivity*. (Some forms of ionizing radiation, such as neutron irradiation, cause residual radioactivity. These are not used for food irradiation.) The γ source is usually ^{60}Co or ^{137}Cs , the latter isotope being a major by-product of nuclear power. Cobalt-60 γ rays average 1.25 MeV, while those of ^{137}Cs are 0.67 MeV and are less penetrating. X-rays used for food irradiation are created with voltages of up to 5 million volts and, thus, have photon energies up to 5 MeV. Electrons used for food irradiation are accelerated to energies up to 10 MeV. The higher the energy per particle, the more penetrating the radiation is and the more ionization it can create. [Figure 1] shows a typical γ -irradiation plant.



A food irradiation plant has a conveyor system to pass items through an intense radiation field behind thick shielding walls. The γ source is lowered into a deep pool of water for safe storage when not in use. Exposure times of up to an hour expose food to doses up to 10^4Gy.

Owing to the fact that food irradiation seeks to destroy organisms such as insects and bacteria, much larger doses than those fatal to humans must be applied. Generally, the simpler the organism, the more radiation it can tolerate. (Cancer cells are a partial exception, because they are rapidly reproducing and, thus, more sensitive.) Current licensing allows up to 1000 Gy to be applied to fresh fruits and vegetables, called a *low dose* in food irradiation. Such a dose is enough to prevent or reduce the growth of many microorganisms, but about 10 000 Gy is needed to kill salmonella, and even more is needed to kill fungi. Doses greater than 10 000 Gy are considered to be high doses in food irradiation and product sterilization.

The effectiveness of food irradiation varies with the type of food. Spices and many fruits and vegetables have dramatically longer shelf lives. These also show no degradation in taste and no loss of food value or vitamins. If not for the mandatory labeling, such foods subjected to low-level irradiation (up to 1000 Gy) could not be distinguished from untreated foods in quality. However, some foods actually spoil faster after irradiation, particularly those with high water content like lettuce and peaches. Others, such as milk, are given a noticeably unpleasant taste. High-level irradiation produces significant and chemically measurable changes in foods. It produces about a 15% loss of nutrients and a 25% loss of vitamins, as well as some change in taste. Such losses are similar to those that occur in ordinary freezing and cooking.

How does food irradiation work? Ionization produces a random assortment of broken molecules and ions, some with unstable oxygen- or hydrogen-containing molecules known as **free radicals**. These undergo rapid chemical reactions, producing perhaps four or five thousand different compounds called **radiolytic products**, some of which make cell function impossible by breaking cell membranes, fracturing DNA, and so on. How safe is the food afterward? Critics argue that the radiolytic products present a lasting hazard, perhaps being carcinogenic. However, the safety of irradiated food is not known precisely. We do know that low-level food irradiation produces no compounds in amounts that can be measured chemically. This is not surprising, since trace amounts of several thousand compounds may be created. We also know that there have been no observable negative short-term effects on consumers. Long-term effects may show up if large number of people consume large quantities of irradiated food, but no effects have appeared due to the small amounts of irradiated food that are consumed regularly. The case for safety is supported by testing of animal diets that were irradiated; no transmitted genetic effects have been observed. Food irradiation (at least up to a million rad) has been endorsed by the World Health Organization and the UN Food and Agricultural Organization. Finally, the hazard to consumers, if it exists, must be weighed against the benefits in food production and preservation. It must also be weighed against the very real hazards of existing insecticides and food preservatives.

Section Summary

- Food irradiation is the treatment of food with ionizing radiation.
- Irradiating food can destroy insects and bacteria by creating free radicals and radiolytic products that can break apart cell membranes.
- Food irradiation has produced no observable negative short-term effects for humans, but its long-term effects are unknown.

Conceptual Questions

Does food irradiation leave the food radioactive? To what extent is the food altered chemically for low and high doses in food irradiation?

Compare a low dose of radiation to a human with a low dose of radiation used in food treatment.

Suppose one food irradiation plant uses a ^{137}Cs source while another uses an equal activity of ^{60}Co . Assuming equal fractions of the γ rays from the sources are absorbed, why is more time needed to get the same dose using the ^{137}Cs source?

Glossary

food irradiation
: treatment of food with ionizing radiation

free radicals
: ions with unstable oxygen- or hydrogen-containing molecules

radiolytic products
: compounds produced due to chemical reactions of free radicals

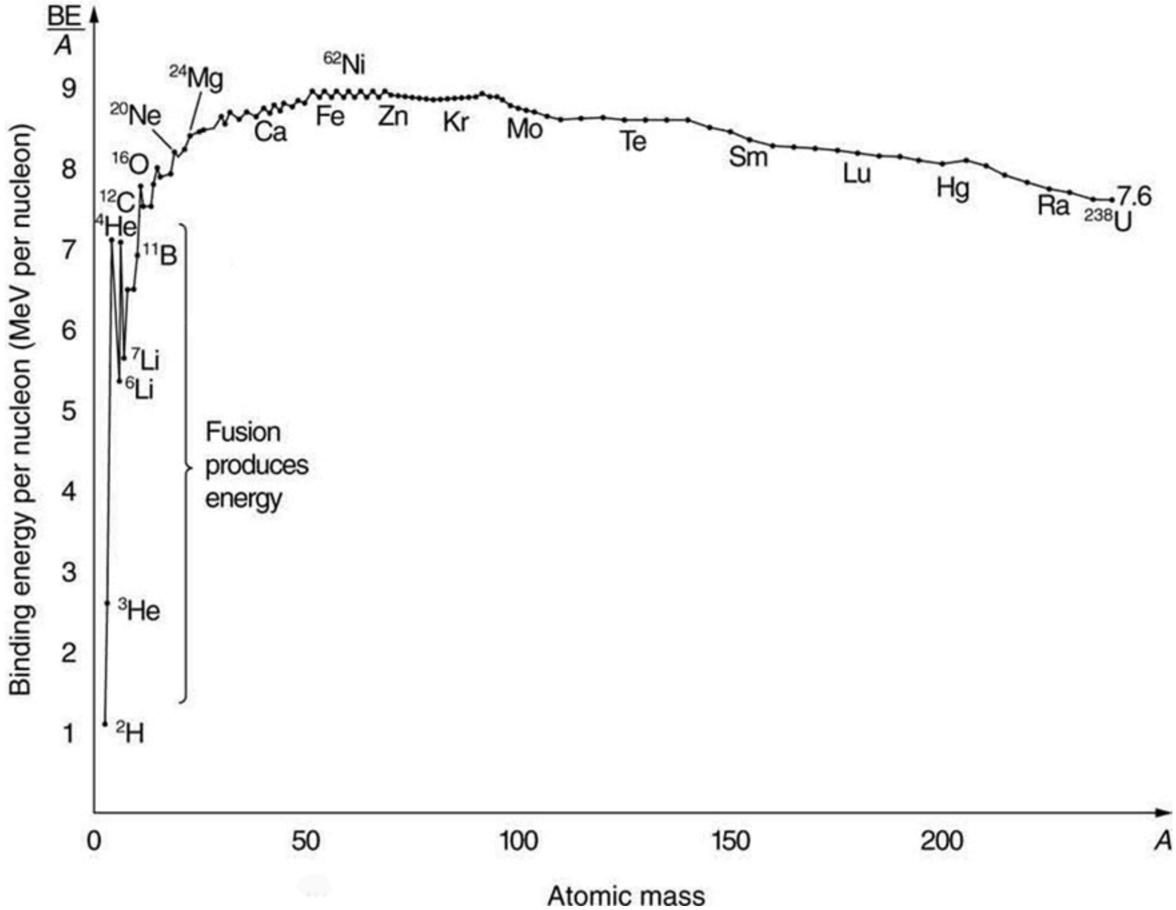
Fusion

- Define nuclear fusion.
- Discuss processes to achieve practical fusion energy generation.

While basking in the warmth of the summer sun, a student reads of the latest breakthrough in achieving sustained thermonuclear power and vaguely recalls hearing about the cold fusion controversy. The three are connected. The Sun's energy is produced by nuclear fusion (see [Figure 1]). Thermonuclear power is the name given to the use of controlled nuclear fusion as an energy source. While research in the area of thermonuclear power is progressing, high temperatures and containment difficulties remain. The cold fusion controversy centered around unsubstantiated claims of practical fusion power at room temperatures.

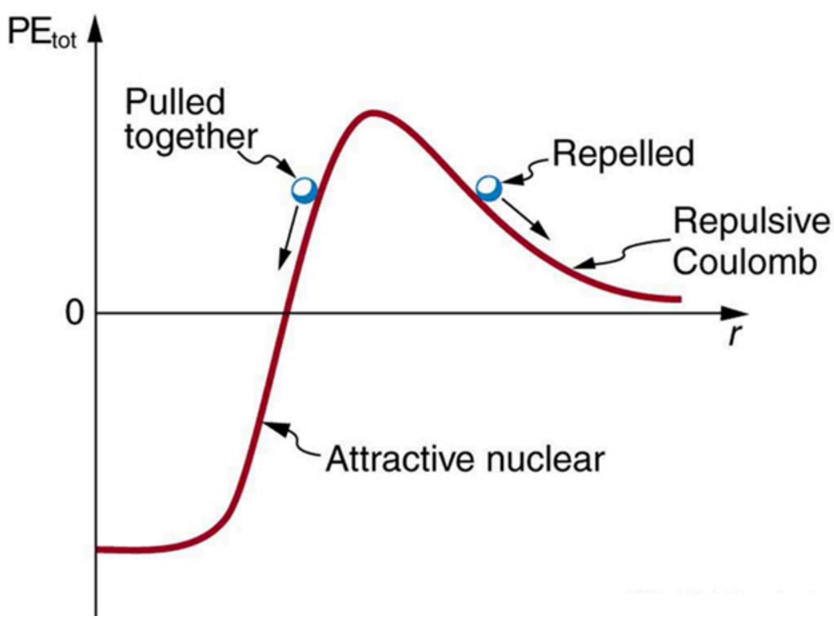
The Sun's energy is produced by nuclear fusion. (credit: Spiralz)

Nuclear fusion is a reaction in which two nuclei are combined, or *fused*, to form a larger nucleus. We know that all nuclei have less mass than the sum of the masses of the protons and neutrons that form them. The missing mass times c^2 equals the binding energy of the nucleus—the greater the binding energy, the greater the missing mass. We also know that BE/A , the binding energy per nucleon, is greater for medium-mass nuclei and has a maximum at Fe (iron). This means that if two low-mass nuclei can be fused together to form a larger nucleus, energy can be released. The larger nucleus has a greater binding energy and less mass per nucleon than the two that combined. Thus mass is destroyed in the fusion reaction, and energy is released (see [Figure 2]). On average, fusion of low-mass nuclei releases energy, but the details depend on the actual nuclides involved.



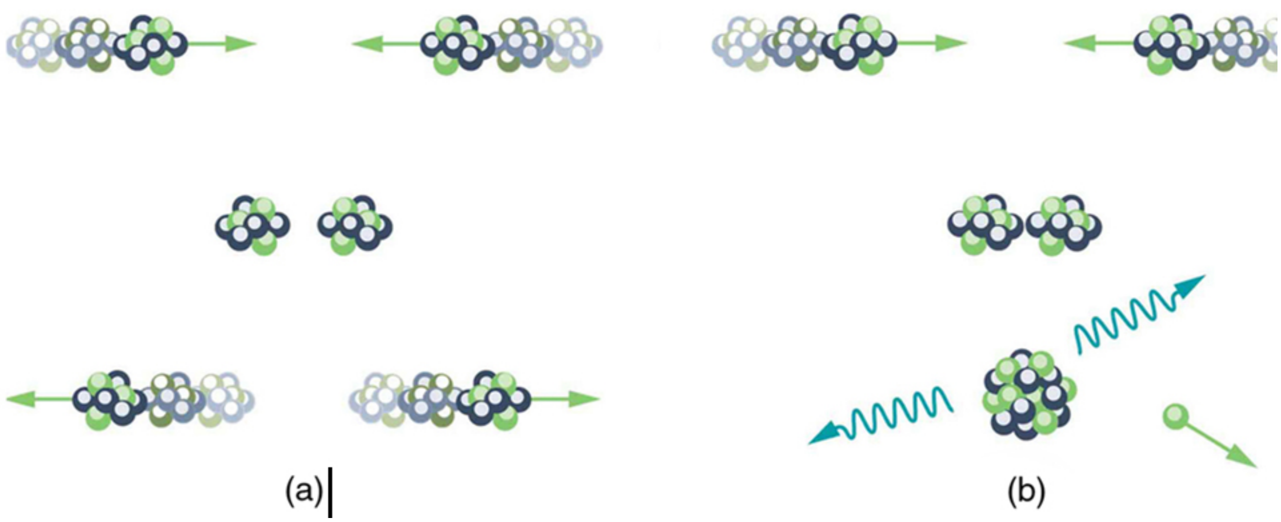
Fusion of light nuclei to form medium-mass nuclei destroys mass, because BE/A is greater for the product nuclei. The larger BE/A is, the less mass per nucleon, and so mass is converted to energy and released in these fusion reactions.

The major obstruction to fusion is the Coulomb repulsion between nuclei. Since the attractive nuclear force that can fuse nuclei together is short ranged, the repulsion of like positive charges must be overcome to get nuclei close enough to induce fusion. [Figure 3] shows an approximate graph of the potential energy between two nuclei as a function of the distance between their centers. The graph is analogous to a hill with a well in its center. A ball rolled from the right must have enough kinetic energy to get over the hump before it falls into the deeper well with a net gain in energy. So it is with fusion. If the nuclei are given enough kinetic energy to overcome the electric potential energy due to repulsion, then they can combine, release energy, and fall into a deep well. One way to accomplish this is to heat fusion fuel to high temperatures so that the kinetic energy of thermal motion is sufficient to get the nuclei together.



Potential energy between two light nuclei graphed as a function of distance between them. If the nuclei have enough kinetic energy to get over the Coulomb repulsion hump, they combine, release energy, and drop into a deep attractive well. Tunneling through the barrier is important in practice. The greater the kinetic energy and the higher the particles get up the barrier (or the lower the barrier), the more likely the tunneling.

You might think that, in the core of our Sun, nuclei are coming into contact and fusing. However, in fact, temperatures on the order of $10^8 K$ are needed to actually get the nuclei in contact, exceeding the core temperature of the Sun. Quantum mechanical tunneling is what makes fusion in the Sun possible, and tunneling is an important process in most other practical applications of fusion, too. Since the probability of tunneling is extremely sensitive to barrier height and width, increasing the temperature greatly increases the rate of fusion. The closer reactants get to one another, the more likely they are to fuse (see [Figure 4]). Thus most fusion in the Sun and other stars takes place at their centers, where temperatures are highest. Moreover, high temperature is needed for thermonuclear power to be a practical source of energy.



(a) Two nuclei heading toward each other slow down, then stop, and then fly away without touching or fusing. (b) At higher energies, the two nuclei approach close enough for fusion via tunneling. The probability of tunneling increases as they approach, but they do not have to touch for the reaction to occur.

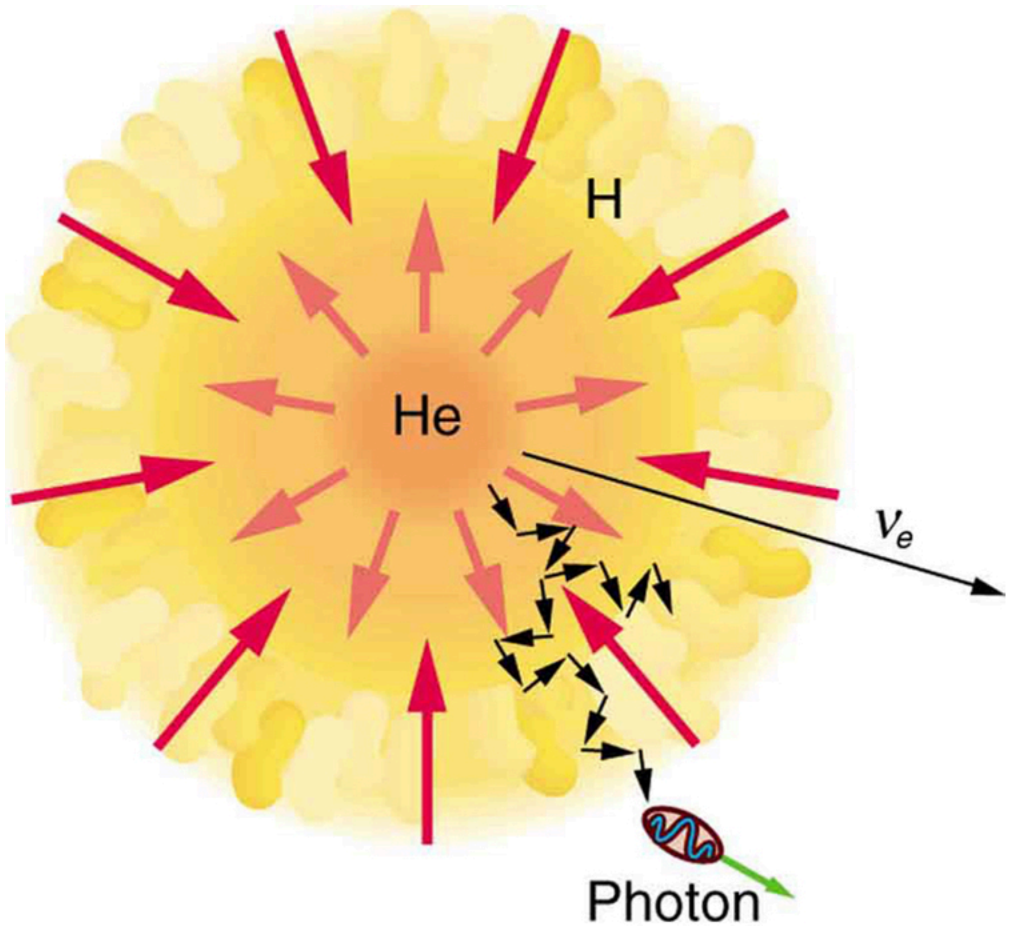
The Sun produces energy by fusing protons or hydrogen nuclei ^1H (by far the Sun's most abundant nuclide) into helium nuclei ^4He. The principal sequence of fusion reactions forms what is called the **proton-proton cycle**:

$$^1\text{H}+^1\text{H}\rightarrow ^2\text{H}+e^+ +v_\text{e}(0.42\text{MeV})$$
$$^1\text{H}+^2\text{H}\rightarrow ^3\text{He}+\gamma(5.49\text{MeV})$$
$$^3\text{He}+^3\text{He}\rightarrow ^4\text{He}+^1\text{H}+^1\text{H}(12.86\text{MeV})$$

where e^+ stands for a positron and v_e is an electron neutrino. (The energy in parentheses is *released* by the reaction.) Note that the first two reactions must occur twice for the third to be possible, so that the cycle consumes six protons (^1H) but gives back two. Furthermore, the two positrons produced will find two electrons and annihilate to form four more γ rays, for a total of six. The overall effect of the cycle is thus

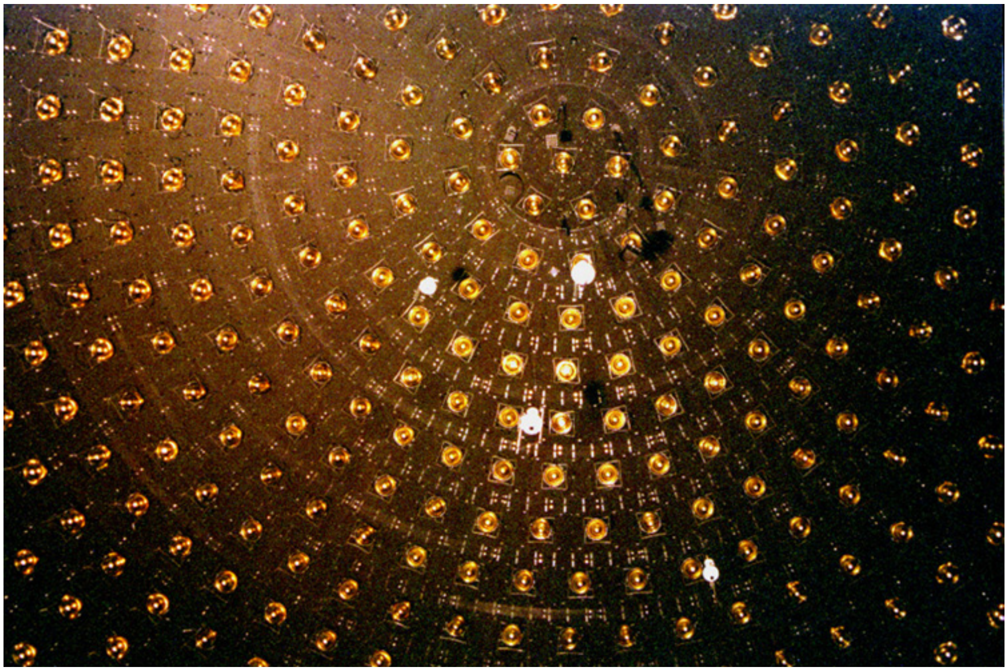
$$2e^- +4^1\text{H}\rightarrow ^4\text{He}+2v_\text{e}+6\gamma(26.7\text{MeV})$$

where the 26.7 MeV includes the annihilation energy of the positrons and electrons and is distributed among all the reaction products. The solar interior is dense, and the reactions occur deep in the Sun where temperatures are highest. It takes about 32 000 years for the energy to diffuse to the surface and radiate away. However, the neutrinos escape the Sun in less than two seconds, carrying their energy with them, because they interact so weakly that the Sun is transparent to them. Negative feedback in the Sun acts as a thermostat to regulate the overall energy output. For instance, if the interior of the Sun becomes hotter than normal, the reaction rate increases, producing energy that expands the interior. This cools it and lowers the reaction rate. Conversely, if the interior becomes too cool, it contracts, increasing the temperature and reaction rate (see [Figure 5]). Stars like the Sun are stable for billions of years, until a significant fraction of their hydrogen has been depleted. What happens then is discussed in Introduction to Frontiers of Physics .

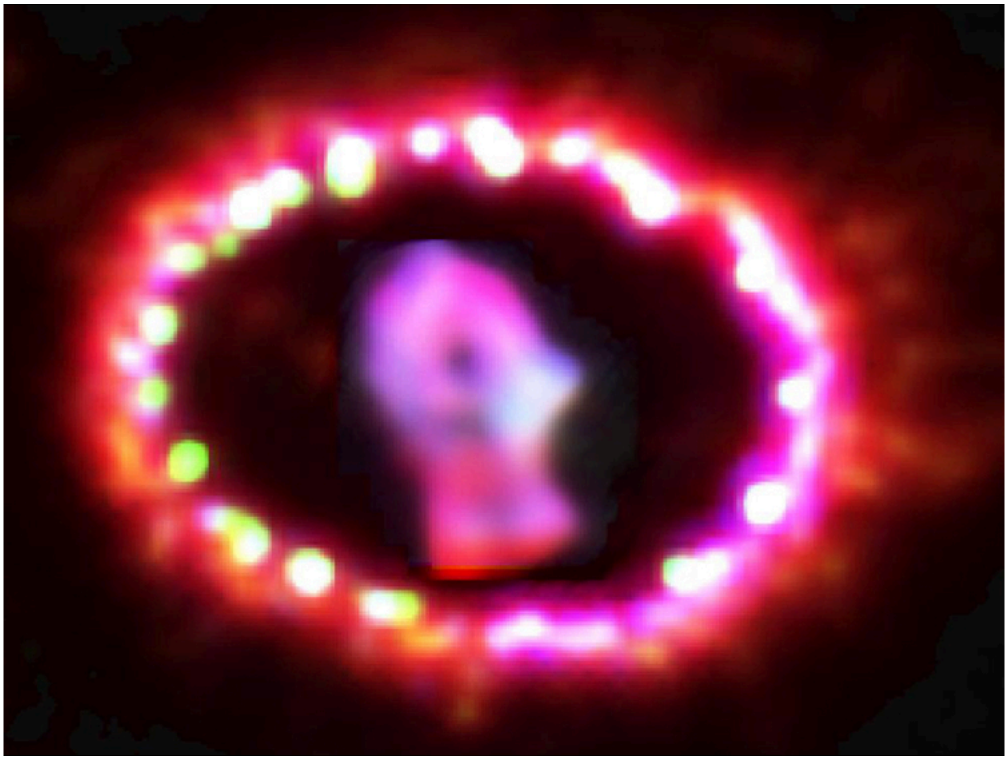


Nuclear fusion in the Sun converts hydrogen nuclei into helium; fusion occurs primarily at the boundary of the helium core, where temperature is highest and sufficient hydrogen remains. Energy released diffuses slowly to the surface, with the exception of neutrinos, which escape immediately. Energy production remains stable because of negative feedback effects.

Theories of the proton-proton cycle (and other energy-producing cycles in stars) were pioneered by the German-born, American physicist Hans Bethe (1906–2005), starting in 1938. He was awarded the 1967 Nobel Prize in physics for this work, and he has made many other contributions to physics and society. Neutrinos produced in these cycles escape so readily that they provide us an excellent means to test these theories and study stellar interiors. Detectors have been constructed and operated for more than four decades now to measure solar neutrinos (see [Figure 6]). Although solar neutrinos are detected and neutrinos were observed from Supernova 1987A ([Figure 7]), too few solar neutrinos were observed to be consistent with predictions of solar energy production. After many years, this solar neutrino problem was resolved with a blend of theory and experiment that showed that the neutrino does indeed have mass. It was also found that there are three types of neutrinos, each associated with a different type of nuclear decay.



This array of photomultiplier tubes is part of the large solar neutrino detector at the Fermi National Accelerator Laboratory in Illinois. In these experiments, the neutrinos interact with heavy water and produce flashes of light, which are detected by the photomultiplier tubes. In spite of its size and the huge flux of neutrinos that strike it, very few are detected each day since they interact so weakly. This, of course, is the same reason they escape the Sun so readily. (credit: Fred Ullrich)



Supernovas are the source of elements heavier than iron. Energy released powers nucleosynthesis. Spectroscopic analysis of the ring of material ejected by Supernova 1987A observable in the southern hemisphere, shows evidence of heavy elements. The study of this supernova also provided indications that neutrinos might have mass. (credit: NASA, ESA, and P. Challis)

The proton-proton cycle is not a practical source of energy on Earth, in spite of the great abundance of hydrogen (^{1}H). The reaction $^{1}H + ^{1}H \rightarrow ^{2}H + e^{+} + \nu_e$ has a very low probability of occurring. (This is why our Sun will last for about ten billion years.) However, a number of other fusion reactions are easier to induce. Among them are:

$$^{2}H + ^{2}H \rightarrow ^{3}H + ^{1}H (4.03 MeV)$$

$$^{2}\text{H}+^{2}\text{H}\rightarrow{}^{3}\text{He}+n(3.27\text{MeV})$$
$$^{2}\text{H}+^{3}\text{H}\rightarrow{}^{4}\text{He}+n(17.59\text{MeV})$$
$$^{2}\text{H}+^{2}\text{H}\rightarrow{}^{4}\text{He}+\gamma(23.85\text{MeV}).$$

Deuterium (^{2}H) is about 0.015% of natural hydrogen, so there is an immense amount of it in sea water alone. In addition to an abundance of deuterium fuel, these fusion reactions produce large energies per reaction (in parentheses), but they do not produce much radioactive waste. Tritium (^{3}H) is radioactive, but it is consumed as a fuel (the reaction $^{2}\text{H}+^{3}\text{H}\rightarrow{}^{4}\text{He}+n$), and the neutrons and γ s can be shielded. The neutrons produced can also be used to create more energy and fuel in reactions like

$$n+^{1}\text{H}\rightarrow{}^{2}\text{H}+\gamma(20.68\text{MeV})$$

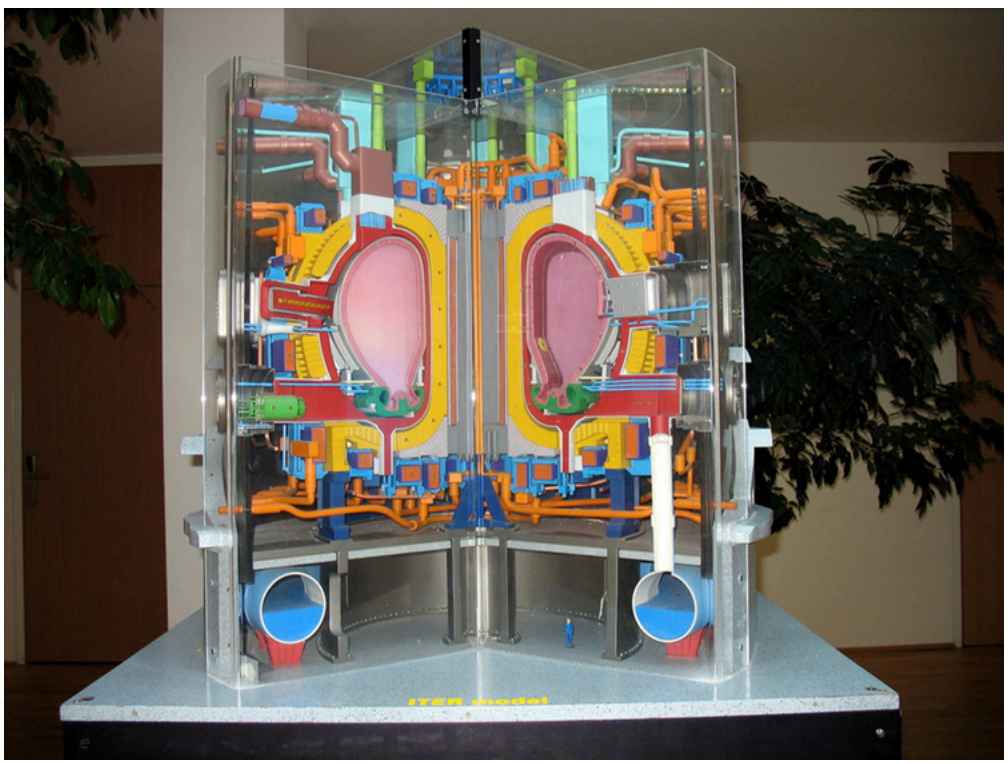
and

$$n+^{1}\text{H}\rightarrow{}^{2}\text{H}+\gamma(2.22\text{MeV}).$$

Note that these last two reactions, and $^{2}\text{H}+^{2}\text{H}\rightarrow{}^{4}\text{He}+\gamma$, put most of their energy output into the γ ray, and such energy is difficult to utilize.

The three keys to practical fusion energy generation are to achieve the temperatures necessary to make the reactions likely, to raise the density of the fuel, and to confine it long enough to produce large amounts of energy. These three factors—temperature, density, and time—complement one another, and so a deficiency in one can be compensated for by the others. **Ignition** is defined to occur when the reactions produce enough energy to be self-sustaining after external energy input is cut off. This goal, which must be reached before commercial plants can be a reality, has not been achieved. Another milestone, called **break-even**, occurs when the fusion power produced equals the heating power input. Break-even has nearly been reached and gives hope that ignition and commercial plants may become a reality in a few decades.

Two techniques have shown considerable promise. The first of these is called **magnetic confinement** and uses the property that charged particles have difficulty crossing magnetic field lines. The tokamak, shown in [Figure 8], has shown particular promise. The tokamak's toroidal coil confines charged particles into a circular path with a helical twist due to the circulating ions themselves. In 1995, the Tokamak Fusion Test Reactor at Princeton in the US achieved world-record plasma temperatures as high as 500 million degrees Celsius. This facility operated between 1982 and 1997. A joint international effort is underway in France to build a tokamak-type reactor that will be the stepping stone to commercial power. ITER, as it is called, will be a full-scale device that aims to demonstrate the feasibility of fusion energy. It will generate 500 MW of power for extended periods of time and will achieve break-even conditions. It will study plasmas in conditions similar to those expected in a fusion power plant. Completion is scheduled for 2018.



(a) Artist's rendition of ITER, a tokamak-type fusion reactor being built in southern France. It is hoped that this gigantic machine will reach the break-even point. Completion is scheduled for 2018. (credit: Stephan Mosel, Flickr)

The second promising technique aims multiple lasers at tiny fuel pellets filled with a mixture of deuterium and tritium. Huge power input heats the fuel, evaporating the confining pellet and crushing the fuel to high density with the expanding hot plasma produced. This technique is called **inertial confinement** , because the fuel's inertia prevents it from escaping before significant fusion can take place. Higher densities have been reached than with tokamaks, but with smaller confinement times. In 2009, the Lawrence Livermore Laboratory (CA) completed a laser fusion device with 192 ultraviolet laser beams that are focused upon a D-T pellet (see [Figure 9]).

National Ignition Facility (CA). This image shows a laser bay where 192 laser beams will focus onto a small D-T target, producing fusion. (credit: Lawrence Livermore National Laboratory, Lawrence Livermore National Security, LLC, and the Department of Energy)

Calculating Energy and Power from Fusion

(a) Calculate the energy released by the fusion of a 1.00-kg mixture of deuterium and tritium, which produces helium. There are equal numbers of deuterium and tritium nuclei in the mixture.

(b) If this takes place continuously over a period of a year, what is the average power output?

Strategy

According to ${}^{2}\text{H} + {}^{3}\text{H} \rightarrow {}^{4}\text{He} + n$, the energy per reaction is 17.59 MeV. To find the total energy released, we must find the number of deuterium and tritium atoms in a kilogram. Deuterium has an atomic mass of about 2 and tritium has an atomic mass of about 3, for a total of about 5 g per mole of reactants or about 200 mol in 1.00 kg. To get a more precise figure, we will use the atomic masses from Appendix A. The power output is best expressed in watts, and so the energy output needs to be calculated in joules and then divided by the number of seconds in a year.

Solution for (a)

The atomic mass of deuterium (${}^{2}\text{H}$) is 2.014102 u, while that of tritium (${}^{3}\text{H}$) is 3.016049 u, for a total of 5.032151 u per reaction. So a mole of reactants has a mass of 5.03 g, and in 1.00 kg there are $(1000\text{g})/(5.03\text{g/mol}) = 198.8\text{mol of reactants}$. The number of reactions that take place is therefore

$$(198.8\text{mol})(6.02\times10^{23}\text{mol}^{-1}) = 1.20\times10^{26}\text{reactions}.$$

The total energy output is the number of reactions times the energy per reaction:

$$E = (1.20\times10^{26}\text{reactions})(17.59\text{MeV/reaction})(1.602\times10^{-13}\text{J/MeV}) = 3.37\times10^{14}\text{J}.$$

Solution for (b)

Power is energy per unit time. One year has $3.16\times10^{7}\text{s}$, so

$$P = \frac{E}{t} = \frac{3.37\times10^{14}\text{J}}{3.16\times10^{7}\text{s}} = 1.07\times10^{7}\text{W} = 10.7\text{MW}.$$

Discussion

By now we expect nuclear processes to yield large amounts of energy, and we are not disappointed here. The energy output of $3.37 \times 10^{14} \text{J}$ from fusing 1.00 kg of deuterium and tritium is equivalent to 2.6 million gallons of gasoline and about eight times the energy output of the bomb that destroyed Hiroshima. Yet the average backyard swimming pool has about 6 kg of deuterium in it, so that fuel is plentiful if it can be utilized in a controlled manner. The average power output over a year is more than 10 MW, impressive but a bit small for a commercial power plant. About 32 times this power output would allow generation of 100 MW of electricity, assuming an efficiency of one-third in converting the fusion energy to electrical energy.

Section Summary

- Nuclear fusion is a reaction in which two nuclei are combined to form a larger nucleus. It releases energy when light nuclei are fused to form medium-mass nuclei.
- Fusion is the source of energy in stars, with the proton-proton cycle,

$$^{1}\text{H}+^{1}\text{H} \rightarrow ^{2}\text{H}+e^{+}+v_{\text{e}}(0.42\text{MeV})$$
$$^{1}\text{H}+^{2}\text{H} \rightarrow ^{3}\text{He}+\gamma(5.49\text{MeV})$$
$$^{3}\text{He}+^{3}\text{He} \rightarrow ^{4}\text{He}+^{1}\text{H}+^{1}\text{H}(12.86\text{MeV})$$

being the principal sequence of energy-producing reactions in our Sun.

- The overall effect of the proton-proton cycle is

$$2e^{-}+4^{1}\text{H} \rightarrow ^{4}\text{He}+2v_{\text{e}}+6\gamma(26.7\text{MeV}),$$

where the 26.7 MeV includes the energy of the positrons emitted and annihilated.

- Attempts to utilize controlled fusion as an energy source on Earth are related to deuterium and tritium, and the reactions play important roles.
- Ignition is the condition under which controlled fusion is self-sustaining; it has not yet been achieved. Break-even, in which the fusion energy output is as great as the external energy input, has nearly been achieved.
- Magnetic confinement and inertial confinement are the two methods being developed for heating fuel to sufficiently high temperatures, at sufficient density, and for sufficiently long times to achieve ignition. The first method uses magnetic fields and the second method uses the momentum of impinging laser beams for confinement.

Conceptual Questions

Why does the fusion of light nuclei into heavier nuclei release energy?

Energy input is required to fuse medium-mass nuclei, such as iron or cobalt, into more massive nuclei. Explain why.

In considering potential fusion reactions, what is the advantage of the reaction $^{2}\text{H} + ^{3}\text{H} \rightarrow ^{4}\text{He}+n$ over the reaction $^{2}\text{H} + ^{2}\text{H} \rightarrow ^{3}\text{He}+n$?

Give reasons justifying the contention made in the text that energy from the fusion reaction $^{2}\text{H} + ^{2}\text{H} \rightarrow ^{4}\text{He}+\gamma$ is relatively difficult to capture and utilize.

Problems & Exercises

Verify that the total number of nucleons, total charge, and electron family number are conserved for each of the fusion reactions in the proton-proton cycle in

$$^{1}\text{H}+^{1}\text{H} \rightarrow ^{2}\text{H}+e^{+}+v_{\text{e}},$$
$$^{1}\text{H}+^{2}\text{H} \rightarrow ^{3}\text{He}+\gamma,$$

and

$$^{3}\text{He}+^{3}\text{He} \rightarrow ^{4}\text{He}+^{1}\text{H}+^{1}\text{H}.$$

(List the value of each of the conserved quantities before and after each of the reactions.)

Show Solution

(a) $A = 1 + 1 = 2$, $Z = 1 + 1 = 1 + 1$, $\text{efn} = 0 = -1 + 1$ (b) $A = 1 + 2 = 3$, $Z = 1 + 1 = 2$, $\text{efn}=0=0$ (c) $A = 3 + 3 = 4 + 1 + 1$, $Z = 2 + 2 = 2 + 1 + 1$, $\text{efn}=0=0$

Calculate the energy output in each of the fusion reactions in the proton-proton cycle, and verify the values given in the above summary.

Show Solution

Strategy

For each reaction, calculate the mass defect and convert to energy using $E = (\Delta m)c^2$. The three reactions are: (1) $^1\text{H} + ^1\text{H} \rightarrow ^2\text{H} + e^+ + v_e$, (2) $^1\text{H} + ^2\text{H} \rightarrow ^3\text{He} + \gamma$, and (3) $^3\text{He} + ^3\text{He} \rightarrow ^4\text{He} + ^1\text{H} + ^1\text{H}$. Use atomic masses from Appendix A: $m(^1\text{H}) = 1.007825$ u, $m(^2\text{H}) = 2.014102$ u, $m(^3\text{He}) = 3.016029$ u, $m(^4\text{He}) = 4.002603$ u, $m(e) = 0.000549$ u.

Solution

Reaction 1: $^1\text{H} + ^1\text{H} \rightarrow ^2\text{H} + e^+ + v_e$

$$\Delta m = 2m(^1\text{H}) - [m(^2\text{H}) + m_e] = 2(1.007825) - [2.014102 + 0.000549] = 0.000999 \text{ u}$$
$$E_1 = (0.000999)(931.5) = 0.931 \text{ MeV}$$

However, the positron will annihilate with an electron, releasing $2m_ec^2 = 1.022$ MeV, but this is external to the nucleus. The nuclear energy release is 0.931 MeV. The text gives 0.42 MeV, which is the energy carried away by the positron and neutrino (kinetic energy), not the total Q-value. Let me recalculate using the Q-value approach properly:

$$Q = [2m(^1\text{H}) - m(^2\text{H}) - 2m_e]c^2 = [2(1.007825) - 2.014102 - 2(0.000549)] \times 931.5 = 0.420 \text{ MeV}$$

Reaction 2: $^1\text{H} + ^2\text{H} \rightarrow ^3\text{He} + \gamma$

$$\Delta m = [m(^1\text{H}) + m(^2\text{H})] - m(^3\text{He}) = [1.007825 + 2.014102] - 3.016029 = 0.005898 \text{ u}$$
$$E_2 = (0.005898)(931.5) = 5.49 \text{ MeV} ✓$$

Reaction 3: $^3\text{He} + ^3\text{He} \rightarrow ^4\text{He} + ^1\text{H} + ^1\text{H}$

$$\Delta m = 2m(^3\text{He}) - [m(^4\text{He}) + 2m(^1\text{H})] = 2(3.016029) - [4.002603 + 2(1.007825)] = 0.013805 \text{ u}$$
$$E_3 = (0.013805)(931.5) = 12.86 \text{ MeV} ✓$$

Discussion

The calculated values of 0.42 MeV, 5.49 MeV, and 12.86 MeV match the values given in the text for the proton-proton cycle. These energies represent the kinetic energy available to the products. The first reaction has the smallest energy release, which partially explains why it has the lowest probability and longest timescale—the Sun will last about 10 billion years because this bottleneck reaction is so slow. The energy released in each step increases, with the third reaction releasing the most energy per event. However, note that the third reaction requires two ^3He nuclei, each of which must be produced by reactions 1 and 2, so the full cycle must have reactions 1 and 2 occur twice before reaction 3 can occur once.

Show that the total energy released in the proton-proton cycle is 26.7 MeV, considering the overall effect in $^1\text{H} + ^1\text{H} \rightarrow ^2\text{H} + e^+ + v_e$, $^1\text{H} + ^2\text{H} \rightarrow ^3\text{He} + \gamma$, and $^3\text{He} + ^3\text{He} \rightarrow ^4\text{He} + ^1\text{H} + ^1\text{H}$ and being certain to include the annihilation energy.

Show Solution

$$E = (m_\text{i} - m_\text{f})c^2 = [4m(^1\text{H}) - m(^4\text{He})]c^2 = [4(1.007825) - 4.002603](931.5\text{MeV}) = 26.73\text{MeV}$$

Verify by listing the number of nucleons, total charge, and electron family number before and after the cycle that these quantities are conserved in the overall proton-proton cycle in $2e^- + 4^1\text{H} \rightarrow ^4\text{He} + 2v_e + 6\gamma$.

Show Solution

Strategy

Count nucleons (A), charge (Z), and electron family number (efn) on both sides. For efn: electrons have efn = +1, positrons have efn = -1, electron neutrinos have efn = +1.

Solution

Before (left side):

- Nucleons: $A = 0 + 4(1) = 4$
- Charge: $Z = 2(-1) + 4(+1) = -2 + 4 = +2$
- Electron family number: $\text{efn} = 2(+1) + 0 = +2$

After (right side):

- Nucleons: $A = 4 + 0 + 0 = 4$ ✓

- Charge: $Z = +2 + 0 + 0 = +2$ ✓
- Electron family number: $\text{efn} = 0 + 2(+1) + 0 = +2$ ✓

All conserved quantities match, verifying conservation laws are satisfied.

Discussion

This overall reaction represents the net effect after the proton-proton cycle runs through completely: two electrons from the surrounding medium annihilate with the two positrons produced in the cycle (reactions 1 occurs twice), four protons fuse to form one helium-4 nucleus, two electron neutrinos carry away some energy, and six gamma rays are produced (two from positron-electron annihilation plus those from the reactions). The conservation of nucleon number reflects baryon number conservation—4 protons in, 4 nucleons in helium out. Charge conservation is maintained throughout. Electron family number conservation accounts for the two electrons consumed and two neutrinos produced, balancing the two positrons created and annihilated in intermediate steps.

The energy produced by the fusion of a 1.00-kg mixture of deuterium and tritium was found in Example Calculating Energy and Power from Fusion. Approximately how many kilograms would be required to supply the annual energy use in the United States?

Show Solution

$3.12 \times 10^5 \text{kg}$ (about 200 tons)

Tritium is naturally rare, but can be produced by the reaction $n + {}^2\text{H} \rightarrow {}^3\text{H} + \gamma$. How much energy in MeV is released in this neutron capture?

Show Solution

Strategy

Calculate mass defect using atomic masses: $m(n) = 1.008665$ u, $m({}^2\text{H}) = 2.014102$ u, $m({}^3\text{H}) = 3.016049$ u.

Solution

$$\Delta m = [m(n) + m({}^2\text{H})] - m({}^3\text{H}) = [1.008665 + 2.014102] - 3.016049 = 0.006718 \text{ u}$$
$$E = (0.006718)(931.5) = 6.26 \text{ MeV}$$

Discussion

This 6.26 MeV neutron capture reaction is an important way to produce tritium for fusion reactors, since tritium doesn't exist naturally (half-life only 12.3 years). The reaction can occur when neutrons from D-D or D-T fusion reactions are captured by deuterium. This creates a self-sustaining fuel cycle where fusion neutrons breed more fusion fuel.

Two fusion reactions mentioned in the text are

$n + {}^3\text{He} \rightarrow {}^4\text{He} + \gamma$ and

$n + {}^1\text{H} \rightarrow {}^2\text{H} + \gamma$.

Both reactions release energy, but the second also creates more fuel. Confirm that the energies produced in the reactions are 20.58 and 2.22 MeV, respectively. Comment on which product nuclide is most tightly bound, ${}^4\text{He}$ or ${}^2\text{H}$.

Show Solution

$$E = (m_\text{i} - m_\text{f})c^2 \quad E_1 = (1.008665 + 3.016030 - 4.002603)(931.5\text{MeV}) = 20.58\text{MeV} \quad E_2 = (1.008665 + 1.007825 - 2.014102)(931.5\text{MeV}) = 2.224\text{MeV}$$ ${}^4\text{He}$ is more tightly bound, since this reaction gives off more energy per nucleon.

(a) Calculate the number of grams of deuterium in an 80 000-L swimming pool, given deuterium is 0.0150% of natural hydrogen.

(b) Find the energy released in joules if this deuterium is fused via the reaction ${}^2\text{H} + {}^2\text{H} \rightarrow {}^3\text{He} + n$.

(c) Could the neutrons be used to create more energy?

(d) Discuss the amount of this type of energy in a swimming pool as compared to that in, say, a gallon of gasoline, also taking into consideration that water is far more abundant.

Show Solution

Strategy

(a) Find mass of water, then mass of hydrogen (11.1% of water mass), then mass of deuterium (0.0150% of hydrogen). (b) Use the D-D reaction energy of 3.27 MeV and calculate number of reactions. (c) Consider neutron capture reactions. (d) Compare with gasoline energy density.

Solution

(a) Mass of water in pool: $(80{,}000\ \text{L})(1\ \text{kg/L}) = 80{,}000\ \text{kg} = 8.0 \times 10^{7}\ \text{g}$

Mass of hydrogen: $(8.0 \times 10^{7})(0.111) = 8.88 \times 10^{6}\ \text{g}$

Mass of deuterium: $(8.88 \times 10^{6})(0.000150) = 1.33 \times 10^{3}\ \text{g} = 1.33\ \text{kg}$

(b) Moles of D: $1330/2.014 = 660\ \text{mol}$, giving $N = (660)(6.022 \times 10^{23}) = 3.97 \times 10^{26}$ atoms.

Number of D-D reactions: $3.97 \times 10^{26}/2 = 1.99 \times 10^{26}$

Energy: $E = (1.99 \times 10^{26})(3.27\ \text{MeV})(1.602 \times 10^{-13}\ \text{J/MeV}) = 1.04 \times 10^{14}\ \text{J}$

(c) Yes! The neutrons can capture on hydrogen to form deuterium (releasing 2.22 MeV) or on other nuclei, extracting additional energy.

(d) Gasoline has energy density ~$1.3 \times 10^{8}\ \text{J/gallon}$. The pool's deuterium energy equals $(1.04 \times 10^{14})/(1.3 \times 10^{8}) \approx 800{,}000$ gallons of gasoline—an enormous amount from just the deuterium in a backyard pool! And water is essentially free and unlimited compared to fossil fuels.

Discussion

This problem dramatically illustrates fusion's potential: a single swimming pool contains deuterium fuel equivalent to 800,000 gallons of gasoline. The oceans contain enough deuterium to power human civilization for billions of years. The challenge isn't fuel availability but achieving the extreme conditions (100 million K temperatures, sufficient density and confinement time) needed to make fusion practical. If fusion can be controlled economically, humanity will have virtually unlimited clean energy, since deuterium comprises 0.015% of all hydrogen in water—a practically inexhaustible supply.

How many kilograms of water are needed to obtain the 198.8 mol of deuterium, assuming that deuterium is 0.01500% (by number) of natural hydrogen?

Show Solution

$1.19 \times 10^{4}\text{kg}$

The power output of the Sun is $4 \times 10^{26}\text{W}$.

(a) If 90% of this is supplied by the proton-proton cycle, how many protons are consumed per second?

(b) How many neutrinos per second should there be per square meter at the Earth from this process? This huge number is indicative of how rarely a neutrino interacts, since large detectors observe very few per day.

Show Solution

Strategy

(a) The proton-proton cycle converts 4 protons to 1 He-4, releasing 26.7 MeV. Find the number of cycles per second from power, then multiply by 4. (b) Each cycle produces 2 neutrinos. Calculate neutrino flux at Earth's distance (1.5×10^{11} m).

Solution

(a) Power from p-p cycle: $(0.90)(4 \times 10^{26}) = 3.6 \times 10^{26}\ \text{W}$

Energy per cycle: $26.7\ \text{MeV} = (26.7)(1.602 \times 10^{-13}) = 4.28 \times 10^{-12}\ \text{J}$

Cycles per second: $\frac{3.6\times10^{26}}{4.28\times10^{-12}} = 8.41 \times 10^{37}\ \text{cycles/s}$

Protons consumed per second: $(8.41 \times 10^{37})(4) = 3.4 \times 10^{38}\ \text{protons/s}$

(b) Neutrinos produced per second: $(8.41 \times 10^{37})(2) = 1.68 \times 10^{38}\ \text{neutrinos/s}$

Surface area of sphere at Earth's orbit: $A = 4\pi r^{2} = 4\pi(1.5\times10^{11})^{2} = 2.83 \times 10^{23}\ \text{m}^{2}$

Neutrino flux at Earth: $\frac{1.68\times10^{38}}{2.83\times10^{23}} = 5.9 \times 10^{14}\ \text{neutrinos/(m}^{2}\cdot\text{s)}$

Discussion

Nearly 600 trillion neutrinos pass through each square meter of Earth every second! Yet detectors observe only a few neutrinos per day because neutrinos interact so weakly with matter—they can pass through the entire Earth without being stopped. This incredibly low interaction probability is why detecting solar neutrinos is so challenging and requires massive detectors. The huge proton consumption rate of 3.4×10^{38} protons/s sounds enormous, but the Sun contains about 10^{57} protons, giving it a lifetime of billions of years.

Another set of reactions that result in the fusing of hydrogen into helium in the Sun and especially in hotter stars is called the carbon cycle. It is

$$^{12}\text{C}+{}^{1}\text{H} \rightarrow {}^{13}\text{N}+\gamma, \quad {}^{13}\text{N} \rightarrow {}^{13}\text{C}+e^{+}+\nu_e, \quad {}^{13}\text{C}+{}^{1}\text{H} \rightarrow {}^{14}\text{N}+\gamma, \quad {}^{14}\text{N}+{}^{1}\text{H} \rightarrow {}^{15}\text{O}+\gamma, \quad {}^{15}\text{O} \rightarrow {}^{15}\text{N}+e^{+}.$$

Write down the overall effect of the carbon cycle (as was done for the proton-proton cycle in $2e^{-} + 4{}^{1}\text{H} \rightarrow {}^{4}\text{He}+2\nu_e + 6\gamma$). Note the number of protons (${}^{1}\text{H}$) required and assume that the positrons (e^{+}) annihilate electrons to form more γ rays.

Show Solution

$2e^{-} + 4{}^{1}\text{H} \rightarrow {}^{4}\text{He}+7\gamma + 2\nu_e$

(a) Find the total energy released in MeV in each carbon cycle (elaborated in the above problem) including the annihilation energy.

(b) How does this compare with the proton-proton cycle output?

Show Solution

Strategy

Calculate using $E = (4m({}^{1}\text{H}) - m({}^{4}\text{He}))c^2$, which is the same as for the p-p cycle since the carbon is just a catalyst. Add 2 positron annihilations ($2 \times 2m_e c^2 = 2.044$ MeV).

Solution

(a) Net reaction: $4{}^{1}\text{H} \rightarrow {}^{4}\text{He}+2e^{+} + 2\nu_e + 7\gamma$

Nuclear energy: $E = [4(1.007825) - 4.002603] \times 931.5 = 26.73$ MeV

Including annihilation of 2 positrons with 2 electrons from surroundings, the total is **26.7 MeV** (same as p-p cycle).

(b) The carbon cycle releases the same 26.7 MeV as the proton-proton cycle. This makes sense because both cycles have the same net reaction: converting 4 protons into 1 helium-4 nucleus. The carbon acts only as a catalyst, being regenerated at the end.

Discussion

Both the carbon cycle and proton-proton cycle yield identical energy because they accomplish the same net nuclear transformation. The carbon cycle dominates in stars hotter than the Sun (above ~17 million K) because it has a stronger temperature dependence, while the p-p cycle dominates in cooler stars like our Sun. The carbon cycle's reliance on heavier nuclei (C, N, O) means it's only possible in second-generation stars formed from material enriched by earlier stellar nucleosynthesis.

Verify that the total number of nucleons, total charge, and electron family number are conserved for each of the fusion reactions in the carbon cycle given in the above problem. (List the value of each of the conserved quantities before and after each of the reactions.)

Show Solution

(a) $A = 12 + 1 = 13$, $Z = 6 + 1 = 7$, efn $= 0 = 0$ (b) $A = 13 = 13$, $Z = 7 = 6 + 1$, efn $= 0 = -1 + 1$ (c) $A = 13 + 1 = 14$, $Z = 6 + 1 = 7$, efn $= 0 = 0$ (d) $A = 14 + 1 = 15$, $Z = 7 + 1 = 8$, efn $= 0 = 0$ (e) $A = 15 = 15$, $Z = 8 = 7 + 1$, efn $= 0 = -1 + 1$ (f) $A = 15 + 1 = 12 + 4$, $Z = 7 + 1 = 6 + 2$, efn $= 0 = 0$

Integrated Concepts

The laser system tested for inertial confinement can produce a 100-kJ pulse only 1.00 ns in duration. (a) What is the power output of the laser system during the brief pulse?

(b) How many photons are in the pulse, given their wavelength is $1.06 \mu m$?

(c) What is the total momentum of all these photons?

(d) How does the total photon momentum compare with that of a single 1.00 MeV deuterium nucleus?

Show Solution

Strategy

(a) Power = Energy/time. (b) Find photon energy from $E = hc/\lambda$, then divide total by photon energy. (c) Photon momentum $p = E/c$. (d) For deuterium, use $p = \sqrt{2mKE}$.

Solution

(a) $P = \frac{100 \times 10^3 \text{ J}}{1.00 \times 10^{-9} \text{ s}} = 1.0 \times 10^{14} \text{ W} = 100 \text{ TW}$

(b) $E_\gamma = \frac{hc}{\lambda} = \frac{(6.63 \times 10^{-34})(3.0 \times 10^8)}{1.06 \times 10^{-6}} = 1.88 \times 10^{-19} \text{ J}$

Number: $N = \frac{1.0 \times 10^5}{1.88 \times 10^{-19}} = 5.3 \times 10^{23}$ photons

(c) $p_{\text{total}} = \frac{E}{c} = \frac{1.0 \times 10^5}{3.0 \times 10^8} = 3.3 \times 10^{-4}$ kg·m/s

(d) $p_D = \sqrt{2 m_D KE} = \sqrt{2(2.014)(1.66 \times 10^{-27})(1.0)(1.602 \times 10^{-13})} = 1.96 \times 10^{-20}$ kg·m/s

Ratio: $\frac{3.3 \times 10^{-4}}{1.96 \times 10^{-20}} = 1.7 \times 10^{16}$

Discussion

The laser delivers 100 terawatts in one nanosecond! The photon momentum is 10^{16} times that of a single deuterium nucleus, demonstrating how laser-driven compression can implode fusion fuel.

Integrated Concepts

Find the amount of energy given to the ^4He nucleus and to the γ ray in the reaction $n + {^3\text{He}} \rightarrow {^4\text{He}} + \gamma$, using the conservation of momentum principle and taking the reactants to be initially at rest. This should confirm the contention that most of the energy goes to the γ ray.

Show Solution

$E_\gamma = 20.6\text{MeV}$ $E_{^4\text{He}} = 5.68 \times 10^{-2}\text{MeV}$

Integrated Concepts

(a) What temperature gas would have atoms moving fast enough to bring two ^3He nuclei into contact? Note that, because both are moving, the average kinetic energy only needs to be half the electric potential energy of these doubly charged nuclei when just in contact with one another.

(b) Does this high temperature imply practical difficulties for doing this in controlled fusion?

Show Solution

Strategy

(a) Two ^3He nuclei (each with charge +2e) need $\frac{3}{2}kT = \frac{1}{2}U_e$ where $U_e = \frac{k(2e)(2e)}{r}$ and $r \approx 2 \times 10^{-15}$ m (sum of nuclear radii). Solve for T. (b) Consider plasma confinement challenges.

Solution

(a) Electric potential energy when just touching:

$$U_e = \frac{k(2e)(2e)}{r} = \frac{(8.99 \times 10^9)(4)(1.602 \times 10^{-19})^2}{2 \times 10^{-15}} = 4.61 \times 10^{-13} \text{ J} = 2.88 \text{ MeV}$$

Average kinetic energy needed per nucleus: $\frac{3}{2}kT = \frac{1}{2}U_e$

$$T = \frac{U_e}{3k} = \frac{4.61 \times 10^{-13}}{3(1.38 \times 10^{-23})} = 1.11 \times 10^{10} \text{ K}$$

(b) Yes! This temperature of 11 billion kelvin is extremely high—much higher than typical fusion reactor targets (100-200 million K for D-T fusion). At such temperatures, containing the plasma becomes extraordinarily difficult. The plasma would vaporize any material container instantly, requiring advanced magnetic or inertial confinement. This explains why ^3He fusion, despite being “cleaner” (no neutrons), is much harder to achieve than D-T fusion which requires “only” 100 million K.

Discussion

The $^3\text{He} + {^3\text{He}}$ reaction requires such high temperatures because both nuclei carry charge +2e, creating a strong Coulomb barrier. D-T fusion is easier because the products (neutron and alpha) carry less initial charge to overcome. The 100-fold higher temperature requirement for He-3 fusion makes it impractical with current technology, though it remains attractive for future advanced fusion systems due to its lack of neutron production.

Integrated Concepts

(a) Estimate the years that the deuterium fuel in the oceans could supply the energy needs of the world. Assume world energy consumption to be ten times that of the United States which is 8×10^{19} J/y and that the deuterium in the oceans could be converted to energy with an efficiency of 32%. You must estimate or look up the amount of water in the oceans and take the deuterium content to be 0.015% of natural hydrogen to find the mass of deuterium available. Note that approximate energy yield of deuterium is 3.37×10^{14} J/kg.

(b) Comment on how much time this is by any human measure. (It is not an unreasonable result, only an impressive one.)

Show Solution

(a) 3×10^{9} y (b) This is approximately half the lifetime of the Earth.

Glossary

break-even
: when fusion power produced equals the heating power input

ignition
: when a fusion reaction produces enough energy to be self-sustaining after external energy input is cut off

inertial confinement
: a technique that aims multiple lasers at tiny fuel pellets evaporating and crushing them to high density

magnetic confinement
: a technique in which charged particles are trapped in a small region because of difficulty in crossing magnetic field lines

nuclear fusion
: a reaction in which two nuclei are combined, or fused, to form a larger nucleus

proton-proton cycle
: the combined reactions $^{1}\text{H}+^{1}\text{H}\rightarrow^{2}\text{H}+e^{+}+v_{e}$, $^{1}\text{H}+^{2}\text{H}\rightarrow^{3}\text{He}+\gamma$, and $^{3}\text{He}+^{3}\text{He}\rightarrow^{4}\text{He}+^{1}\text{H}+^{1}\text{H}$

Fission

- Define nuclear fission.
- Discuss how fission fuel reacts and describe what it produces.
- Describe controlled and uncontrolled chain reactions.

Nuclear fission is a reaction in which a nucleus is split (or *fissured*). Controlled fission is a reality, whereas controlled fusion is a hope for the future. Hundreds of nuclear fission power plants around the world attest to the fact that controlled fission is practical and, at least in the short term, economical, as seen in [Figure 1]. Whereas nuclear power was of little interest for decades following TMI and Chernobyl (and now Fukushima Daiichi), growing concerns over global warming has brought nuclear power back on the table as a viable energy alternative. By the end of 2009, there were 442 reactors operating in 30 countries, providing 15% of the world's electricity. France provides over 75% of its electricity with nuclear power, while the US has 104 operating reactors providing 20% of its electricity. Australia and New Zealand have none. China is building nuclear power plants at the rate of one start every month.

The people living near this nuclear power plant have no measurable exposure to radiation that is traceable to the plant. About 16% of the world's electrical power is generated by controlled nuclear fission in such plants. The cooling towers are the most prominent features but are not unique to nuclear power. The reactor is in the small domed building to the left of the towers. (credit: Kalmthouts)

Fission is the opposite of fusion and releases energy only when heavy nuclei are split. As noted in Fusion, energy is released if the products of a nuclear reaction have a greater binding energy per nucleon (BE/A) than the parent nuclei. [Figure 2] shows that BE/A is greater for medium-mass nuclei than heavy nuclei, implying that when a heavy nucleus is split, the products have less mass per nucleon, so that mass is destroyed and energy is released in the reaction. The amount of energy per fission reaction can be large, even by nuclear standards. The graph in [Figure 2] shows BE/A to be about 7.6 MeV/nucleon for the heaviest nuclei (A about 240), while BE/A is about 8.6 MeV/nucleon for nuclei having A about 120. Thus, if a heavy nucleus splits in half, then about 1 MeV per nucleon, or approximately 240 MeV per fission, is released. This is about 10 times the energy per fusion reaction, and about 100 times the energy of the average α , β , or γ decay.

Calculating Energy Released by Fission

Calculate the energy released in the following spontaneous fission reaction:

$$^{238}\text{U} \rightarrow {}^{95}\text{Sr} + {}^{140}\text{Xe} + 3n$$

given the atomic masses to be $m(^{238}\text{U}) = 238.050784\text{u}$, $m(^{95}\text{Sr}) = 94.919388\text{u}$, $m(^{140}\text{Xe}) = 139.921610\text{u}$, and $m(n) = 1.008665\text{u}$.

Strategy

As always, the energy released is equal to the mass destroyed times c^2 , so we must find the difference in mass between the parent ^{238}U and the fission products.

Solution

The products have a total mass of

$$m_{\text{products}} = 94.919388\text{u}+139.921610\text{u}+3(1.008665u) = 237.866993\text{u}.$$

The mass lost is the mass of ^{238}U minus m_{products}, or

$$\Delta m=238.050784\text{u}-237.8669933\text{u}=0.183791u,$$

so the energy released is

$$E = (\Delta m)c^2 = (0.183791\text{u})931.5\text{MeV}/c^2\text{u}c^2=171.2\text{MeV}.$$

Discussion

A number of important things arise in this example. The 171-MeV energy released is large, but a little less than the earlier estimated 240 MeV. This is because this fission reaction produces neutrons and does not split the nucleus into two equal parts. Fission of a given nuclide, such as ^{238}U , does not always produce the same products. Fission is a statistical process in which an entire range of products are produced with various probabilities. Most fission produces neutrons, although the number varies with each fission. This is an extremely important aspect of fission, because *neutrons can induce more fission*, enabling self-sustaining chain reactions.

Spontaneous fission can occur, but this is usually not the most common decay mode for a given nuclide. For example, ^{238}U can spontaneously fission, but it decays mostly by α emission. Neutron-induced fission is crucial as seen in [Figure 2]. Being chargeless, even low-energy neutrons can strike a nucleus and be absorbed once they feel the attractive nuclear force. Large nuclei are described by a **liquid drop model** with surface tension and oscillation modes, because the large number of nucleons act like atoms in a drop. The neutron is attracted and thus, deposits energy, causing the nucleus to deform as a liquid drop. If stretched enough, the nucleus narrows in the middle. The number of nucleons in contact and the strength of the nuclear force binding the nucleus together are reduced. Coulomb repulsion between the two ends then succeeds in fissioning the nucleus, which pops like a water drop into two large pieces and a few neutrons. **Neutron-induced fission** can be written as

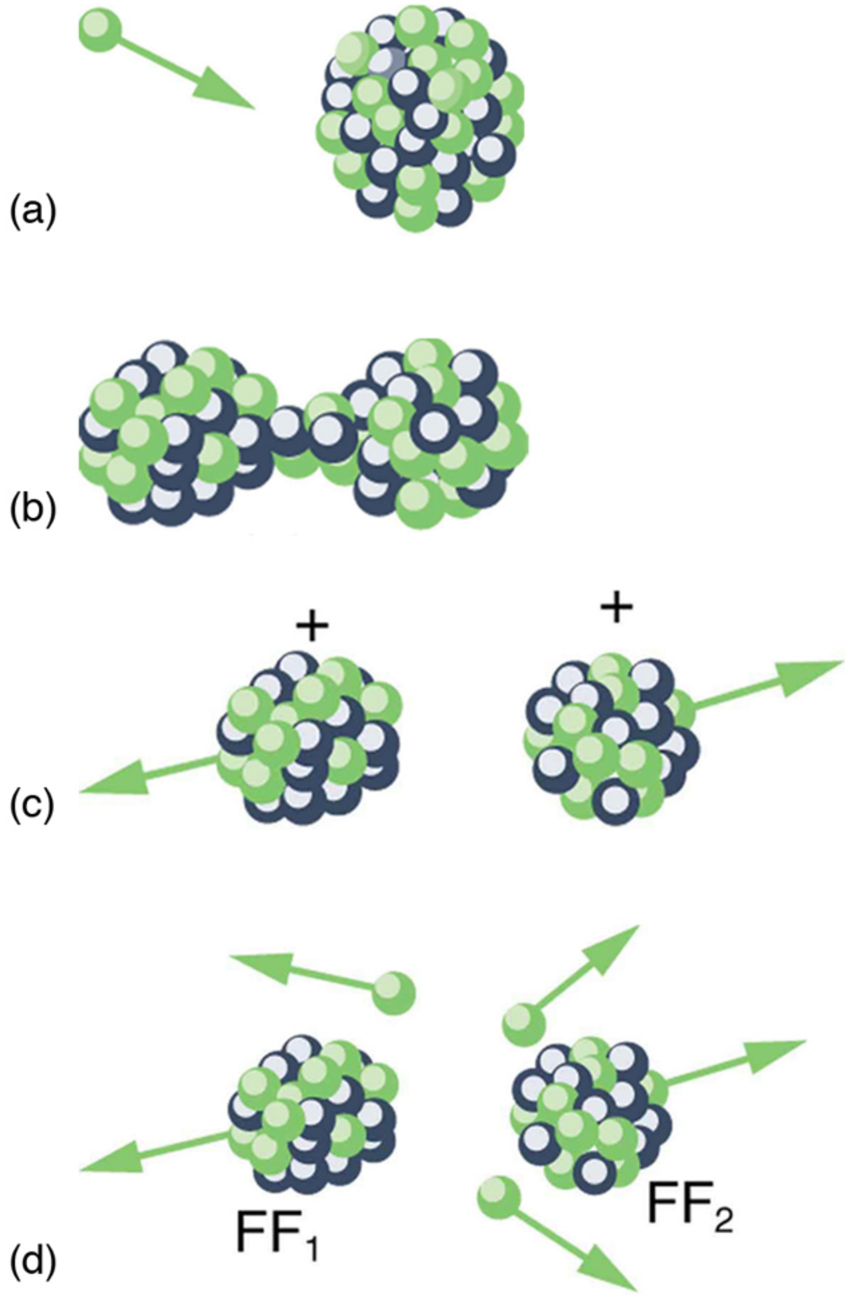
$$n+{}^{A}\text{X}\rightarrow\text{FF}_1+\text{FF}_2+\text{xn},$$

where FF_1 and FF_2 are the two daughter nuclei, called **fission fragments**, and x is the number of neutrons produced. Most often, the masses of the fission fragments are not the same. Most of the released energy goes into the kinetic energy of the fission fragments, with the remainder going into the neutrons and excited states of the fragments. Since neutrons can induce fission, a self-sustaining chain reaction is possible, provided more than one neutron is produced on average — that is, if $x>1$ in $n+{}^{A}\text{X}\rightarrow\text{FF}_1+\text{FF}_2+\text{xn}$. This can also be seen in [Figure 3].

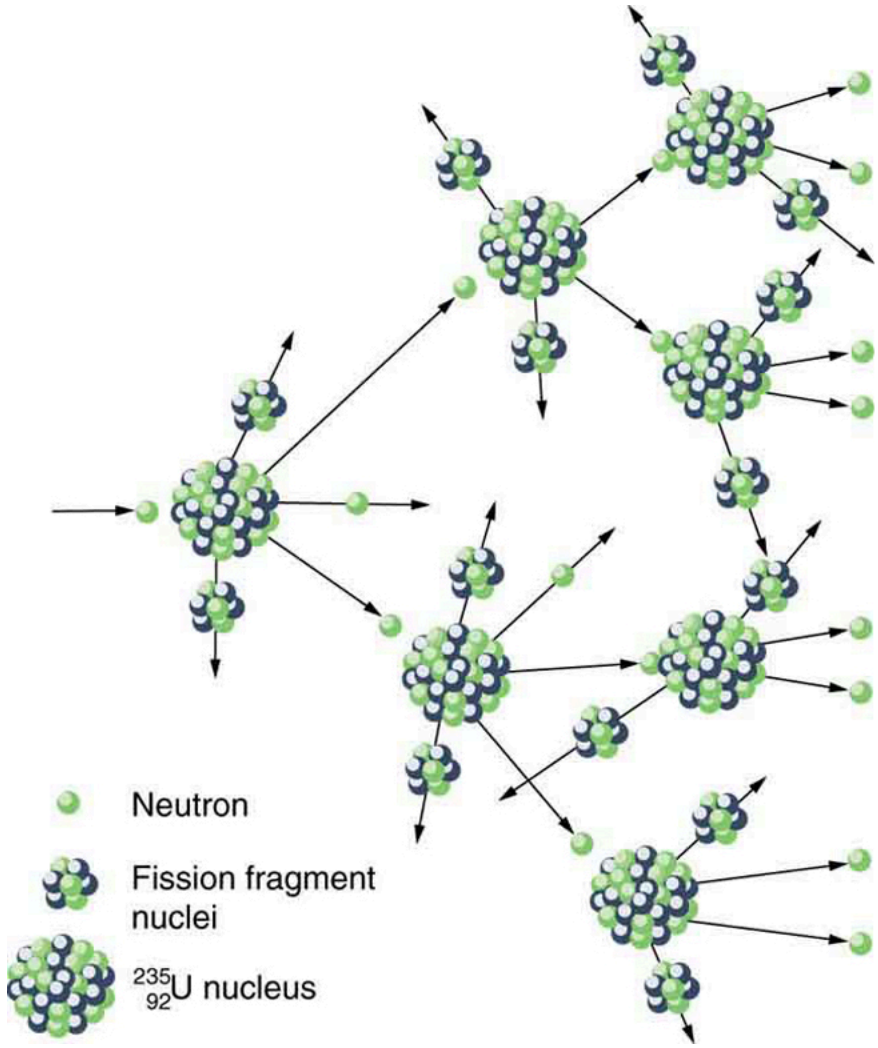
An example of a typical neutron-induced fission reaction is

$$n+{}_{92}^{235}\text{U}\rightarrow{}_{56}^{142}\text{Ba}+{}_{36}^{91}\text{Kr}+3\text{n}.$$

Note that in this equation, the total charge remains the same (is conserved): $92+0=56+36$. Also, as far as whole numbers are concerned, the mass is constant: $1+235=142+91+3$. This is not true when we consider the masses out to 6 or 7 significant places, as in the previous example.



Neutron-induced fission is shown. First, energy is put into this large nucleus when it absorbs a neutron. Acting like a struck liquid drop, the nucleus deforms and begins to narrow in the middle. Since fewer nucleons are in contact, the repulsive Coulomb force is able to break the nucleus into two parts with some neutrons also flying away.

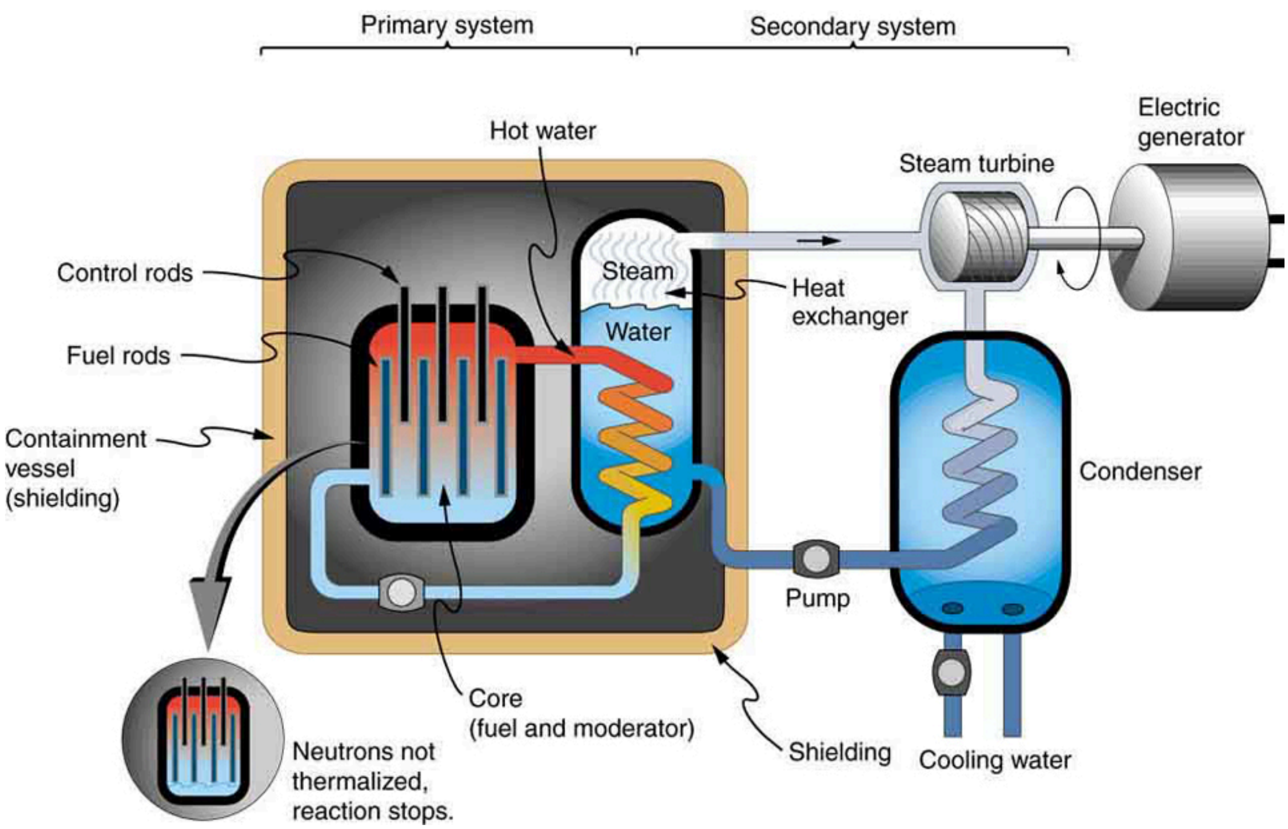


A chain reaction can produce self-sustained fission if each fission produces enough neutrons to induce at least one more fission. This depends on several factors, including how many neutrons are produced in an average fission and how easy it is to make a particular type of nuclide fission.

Not every neutron produced by fission induces fission. Some neutrons escape the fissionable material, while others interact with a nucleus without making it fission. We can enhance the number of fissions produced by neutrons by having a large amount of fissionable material. The minimum amount necessary for self-sustained fission of a given nuclide is called its **critical mass**. Some nuclides, such as ^{239}Pu , produce more neutrons per fission than others, such as ^{235}U . Additionally, some nuclides are easier to make fission than others. In particular, ^{235}U and ^{239}Pu are easier to fission than the much more abundant ^{238}U . Both factors affect critical mass, which is smallest for ^{239}Pu .

The reason ^{235}U and ^{239}Pu are easier to fission than ^{238}U is that the nuclear force is more attractive for an even number of neutrons in a nucleus than for an odd number. Consider that $^{235}_{92}\text{U}_{143}$ has 143 neutrons, and $^{239}_{94}\text{P}_{145}$ has 145 neutrons, whereas $^{238}_{92}\text{U}_{146}$ has 146. When a neutron encounters a nucleus with an odd number of neutrons, the nuclear force is more attractive, because the additional neutron will make the number even. About 2-MeV more energy is deposited in the resulting nucleus than would be the case if the number of neutrons was already even. This extra energy produces greater deformation, making fission more likely. Thus, ^{235}U and ^{239}Pu are superior fission fuels. The isotope ^{235}U is only 0.72 % of natural uranium, while ^{238}U is 99.27%, and ^{239}Pu does not exist in nature. Australia has the largest deposits of uranium in the world, standing at 28% of the total. This is followed by Kazakhstan and Canada. The US has only 3% of global reserves.

Most fission reactors utilize ^{235}U , which is separated from ^{238}U at some expense. This is called enrichment. The most common separation method is gaseous diffusion of uranium hexafluoride (UF_6) through membranes. Since ^{235}U has less mass than ^{238}U , its UF_6 molecules have higher average velocity at the same temperature and diffuse faster. Another interesting characteristic of ^{235}U is that it preferentially absorbs very slow moving neutrons (with energies a fraction of an eV), whereas fission reactions produce fast neutrons with energies in the order of an MeV. To make a self-sustained fission reactor with ^{235}U , it is thus necessary to slow down (“thermalize”) the neutrons. Water is very effective, since neutrons collide with protons in water molecules and lose energy. [Figure 4] shows a schematic of a reactor design, called the pressurized water reactor.



A pressurized water reactor is cleverly designed to control the fission of large amounts of \({}^235\text{U}\) , while using the heat produced in the fission reaction to create steam for generating electrical energy. Control rods adjust neutron flux so that criticality is obtained, but not exceeded. In case the reactor overheats and boils the water away, the chain reaction terminates, because water is needed to thermalize the neutrons. This inherent safety feature can be overwhelmed in extreme circumstances.

Control rods containing nuclides that very strongly absorb neutrons are used to adjust neutron flux. To produce large power, reactors contain hundreds to thousands of critical masses, and the chain reaction easily becomes self-sustaining, a condition called **criticality**. Neutron flux should be carefully regulated to avoid an exponential increase in fissions, a condition called **supercriticality**. Control rods help prevent overheating, perhaps even a meltdown or explosive disassembly. The water that is used to thermalize neutrons, necessary to get them to induce fission in ^{235}U , and achieve criticality, provides a negative feedback for temperature increases. In case the reactor overheats and boils the water to steam or is breached, the absence of water kills the chain reaction. Considerable heat, however, can still be generated by the reactor's radioactive fission products. Other safety features, thus, need to be incorporated in the event of a *loss of coolant* accident, including auxiliary cooling water and pumps.

Calculating Energy from a Kilogram of Fissionable Fuel

Calculate the amount of energy produced by the fission of 1.00 kg of ^{235}U , given the average fission reaction of ^{235}U produces 200 MeV.

Strategy

The total energy produced is the number of ^{235}U atoms times the given energy per ^{235}U fission. We should therefore find the number of ^{235}U atoms in 1.00 kg.

Solution

The number of ^{235}U atoms in 1.00 kg is Avogadro's number times the number of moles. One mole of ^{235}U has a mass of 235.04 g; thus, there are $(1000\text{g})/(235.04\text{g/mol}) = 4.25\text{mol}$. The number of ^{235}U atoms is therefore,

$$(4.25\,mol)(6.02\times10^{23}\,{}^{235}\text{U/mol})=2.56\times10^{24}\,{}^{235}\text{U}.$$

So the total energy released is

$$E = (2.56\times10^{24}\,{}^{235}\text{U})(200\text{MeV}\,{}^{235}\text{U})(1.60\times10^{-13}\text{JMeV}) = 8.21\times10^{13}\text{J}.$$

Discussion

This is another impressively large amount of energy, equivalent to about 14 000 barrels of crude oil or 600 000 gallons of gasoline. But, it is only one-fourth the energy produced by the fusion of a kilogram mixture of deuterium and tritium as seen in [Example 1]. Even though each fission reaction yields about ten times the energy of a fusion reaction, the energy per kilogram of fission fuel is less, because there are far fewer moles per kilogram of the heavy nuclides. Fission fuel is also much more scarce than fusion fuel, and less than 1% of uranium $(\text{the}^{235}\text{U})$ is readily usable.

One nuclide already mentioned is ^{239}Pu , which has a 24 120-y half-life and does not exist in nature. Plutonium-239 is manufactured from ^{238}U in reactors, and it provides an opportunity to utilize the other 99% of natural uranium as an energy source. The following reaction sequence, called **breeding**, produces ^{239}Pu. Breeding begins with neutron capture by ^{238}U :

$$^{238}\text{U}+n \rightarrow {}^{239}\text{U}+\gamma.$$

Uranium-239 then β^- decays:

$$^{239}\text{U} \rightarrow {}^{239}\text{Np}+\beta^-+e(t_{1/2}=23\text{min}).$$

Neptunium-239 also β^- decays:

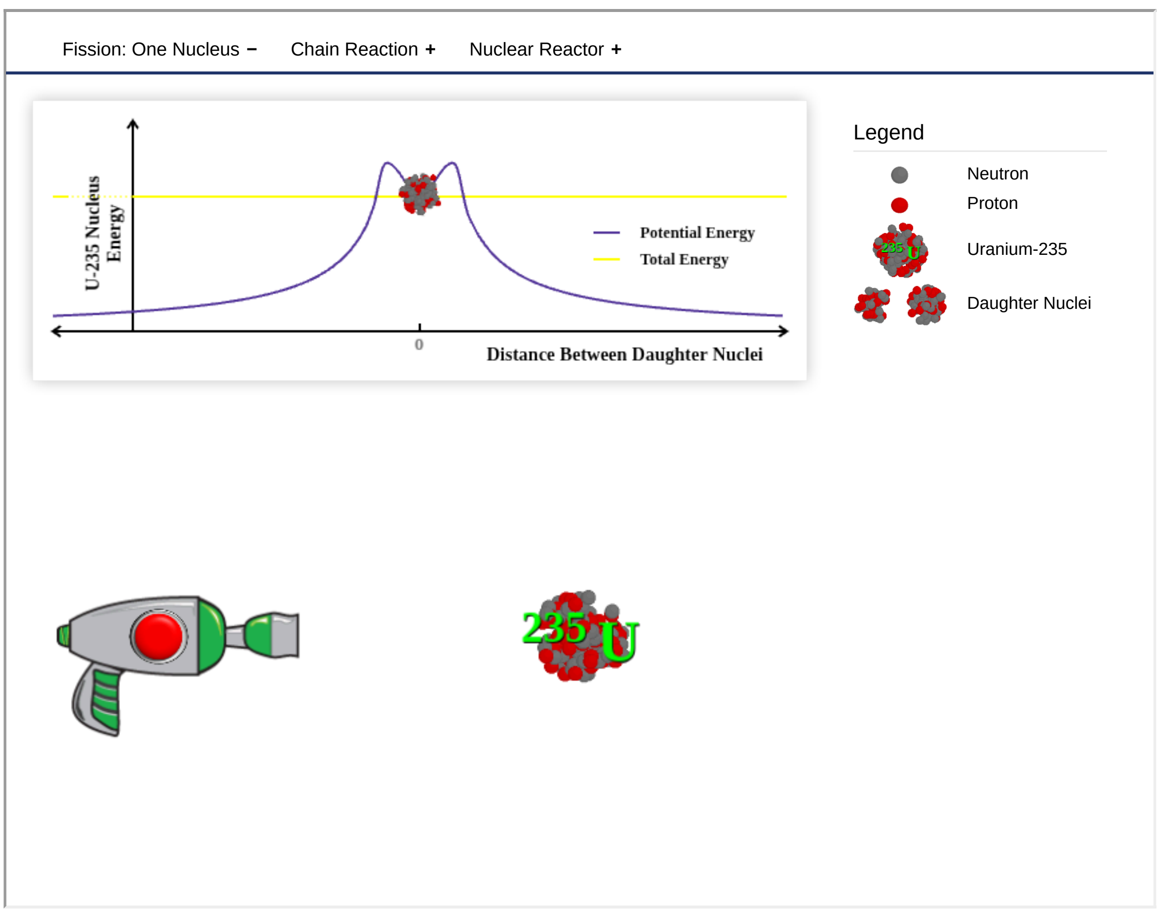
$$^{239}\text{Np} \rightarrow {}^{239}\text{Pu}+\beta^-+\nu_e(t_{1/2}=2.4\text{d}).$$

Plutonium-239 builds up in reactor fuel at a rate that depends on the probability of neutron capture by ^{238}U (all reactor fuel contains more ^{238}U than ^{235}U) . Reactors designed specifically to make plutonium are called **breeder reactors**. They seem to be inherently more hazardous than conventional reactors, but it remains unknown whether their hazards can be made economically acceptable. The four reactors at Chernobyl, including the one that was destroyed, were built to breed plutonium and produce electricity. These reactors had a design that was significantly different from the pressurized water reactor illustrated above.

Plutonium-239 has advantages over ^{235}U as a reactor fuel — it produces more neutrons per fission on average, and it is easier for a thermal neutron to cause it to fission. It is also chemically different from uranium, so it is inherently easier to separate from uranium ore. This means ^{239}Pu has a particularly small critical mass, an advantage for nuclear weapons.

PhET Explorations: Nuclear Fission

Start a chain reaction, or introduce non-radioactive isotopes to prevent one. Control energy production in a nuclear reactor!



Section Summary

- Nuclear fission is a reaction in which a nucleus is split.
- Fission releases energy when heavy nuclei are split into medium-mass nuclei.
- Self-sustained fission is possible, because neutron-induced fission also produces neutrons that can induce other fissions, $n + {}^{A}X \rightarrow \text{FF}_1 + \text{FF}_2 + \text{xn}$, where FF_1 and FF_2 are the two daughter nuclei, or fission fragments, and *x* is the number of neutrons produced.
- A minimum mass, called the critical mass, should be present to achieve criticality.
- More than a critical mass can produce supercriticality.
- The production of new or different isotopes (especially ${}^{239}\text{Pu}$) by nuclear transformation is called breeding, and reactors designed for this purpose are called breeder reactors.

Conceptual Questions

Explain why the fission of heavy nuclei releases energy. Similarly, why is it that energy input is required to fission light nuclei?

Explain, in terms of conservation of momentum and energy, why collisions of neutrons with protons will thermalize neutrons better than collisions with oxygen.

The ruins of the Chernobyl reactor are enclosed in a huge concrete structure built around it after the accident. Some rain penetrates the building in winter, and radioactivity from the building increases. What does this imply is happening inside?

Since the uranium or plutonium nucleus fissions into several fission fragments whose mass distribution covers a wide range of pieces, would you expect more residual radioactivity from fission than fusion? Explain.

The core of a nuclear reactor generates a large amount of thermal energy from the decay of fission products, even when the power-producing fission chain reaction is turned off. Would this residual heat be greatest after the reactor has run for a long time or short time? What if the reactor has been shut down for months?

How can a nuclear reactor contain many critical masses and not go supercritical? What methods are used to control the fission in the reactor?

Why can heavy nuclei with odd numbers of neutrons be induced to fission with thermal neutrons, whereas those with even numbers of neutrons require more energy input to induce fission?

Why is a conventional fission nuclear reactor not able to explode as a bomb?

Problem Exercises

(a) Calculate the energy released in the neutron-induced fission (similar to the spontaneous fission in [Example 1])

$$n + {}^{238}\text{U} \rightarrow {}^{96}\text{Sr} + {}^{140}\text{Xe} + 3\text{n},$$

given $m({}^{96}\text{Sr}) = 95.921750\text{u}$ and $m({}^{140}\text{Xe}) = 139.92164$. (b) This result is about 6 MeV greater than the result for spontaneous fission. Why? (c) Confirm that the total number of nucleons and total charge are conserved in this reaction.

Show Solution

(a) 177.1 MeV

(b) Because the gain of an external neutron yields about 6 MeV, which is the average BE/A for heavy nuclei.

(c) $A = 1 + 238 = 96 + 140 + 1 + 1 + 1, Z = 92 = 38 + 53, \text{efn} = 0 = 0$

(a) Calculate the energy released in the neutron-induced fission reaction

$$n + {}^{235}\text{U} \rightarrow {}^{92}\text{Kr} + {}^{142}\text{Ba} + 2\text{n},$$

given $m({}^{92}\text{Kr}) = 91.926269\text{u}$ and $m({}^{142}\text{Ba}) = 141.916361\text{u}.$

(b) Confirm that the total number of nucleons and total charge are conserved in this reaction.

Show Solution

Strategy

(a) Calculate mass defect and convert to energy. Use $m(n) = 1.008665$ u and $m({}^{235}\text{U}) = 235.043930$ u. (b) Count A, Z, and efn on both sides.

Solution

(a) Reactant mass: $m_i = 1.008665 + 235.043930 = 236.052595$ u

Product mass: $m_f = 91.926269 + 141.916361 + 2(1.008665) = 234.859960$ u

Mass defect: $\Delta m = 236.052595 - 234.859960 = 1.192635$ u

Energy: $E = (1.192635)(931.5) = 1111.1$ MeV

(b) **Before:** $A = 1 + 235 = 236$, $Z = 0 + 92 = 92$, efn = 0

After: $A = 92 + 142 + 2(1) = 236$ ✓, $Z = 36 + 56 = 92$ ✓, efn = 0 ✓

Discussion

This enormous energy release of ~1111 MeV per fission demonstrates the power of nuclear fission. However, the number given in this problem appears unusually high. Typical U-235 fission releases about 200 MeV. The issue may be related to the specific fission fragments chosen. Nevertheless, the calculation methodology is correct: find mass defect and convert using 931.5 MeV/u. All conservation laws are satisfied.

(a) Calculate the energy released in the neutron-induced fission reaction

$n + {}^{239}\text{Pu} \rightarrow {}^{96}\text{Sr} + {}^{140}\text{Ba} + 4n,$

given $m({}^{96}\text{Sr}) = 95.921750\text{u}$ and $m({}^{140}\text{Ba}) = 139.910581\text{u}$.

(b) Confirm that the total number of nucleons and total charge are conserved in this reaction.

Show Solution

(a) 180.6 MeV

(b) $A = 1 + 239 = 96 + 140 + 1 + 1 + 1 + 1, Z = 94 = 38 + 56, \text{efn} = 0 = 0$

Confirm that each of the reactions listed for plutonium breeding just following [Example 2] conserves the total number of nucleons, the total charge, and electron family number.

Show Solution

Strategy

The plutonium breeding reactions are: (1) ${}^{238}\text{U} + n \rightarrow {}^{239}\text{U} + \gamma$, (2) ${}^{239}\text{U} \rightarrow {}^{239}\text{Np} + \beta^- + {}^-\nu_e$, (3) ${}^{239}\text{Np} \rightarrow {}^{239}\text{Pu} + \beta^- + {}^-\nu_e$. Verify A, Z, and efn for each.

Solution

Reaction 1: ${}^{238}\text{U} + n \rightarrow {}^{239}\text{U} + \gamma$

- **Before:** A = 238 + 1 = 239, Z = 92, efn = 0
- **After:** A = 239, Z = 92, efn = 0 ✓

Reaction 2: ${}^{239}\text{U} \rightarrow {}^{239}\text{Np} + \beta^- + {}^-\nu_e$

- **Before:** A = 239, Z = 92, efn = 0
- **After:** A = 239, Z = 93 - 1 = 92, efn = -1 + (+1) = 0 ✓

Reaction 3: ${}^{239}\text{Np} \rightarrow {}^{239}\text{Pu} + \beta^- + {}^-\nu_e$

- **Before:** A = 239, Z = 93, efn = 0
- **After:** A = 239, Z = 94 - 1 = 93, efn = -1 + (+1) = 0 ✓

All conservation laws are satisfied in each reaction.

Discussion

This three-step breeding process converts fertile ${}^{238}\text{U}$ into fissile ${}^{239}\text{Pu}$. The first step (neutron capture) is followed by two beta decays, each increasing Z by 1 while A remains constant at 239. The electron family number is conserved because each β^- (efn = -1) is accompanied by an antineutrino (efn = +1), giving net zero. This process is how breeder reactors create more fuel than they consume, potentially extending nuclear fuel supplies for thousands of years.

Breeding plutonium produces energy even before any plutonium is fissioned. (The primary purpose of the four nuclear reactors at Chernobyl was breeding plutonium for weapons. Electrical power was a by-product used by the civilian population.) Calculate the energy produced in each of the reactions listed for plutonium breeding just following [Example 2]. The pertinent masses are $m(^{239}\text{U}) = 239.054289\text{u}$, $m(^{239}\text{Np}) = 239.052932\text{u}$, and $m(^{239}\text{Pu}) = 239.052157\text{u}$.

Show Solution

$^{238}\text{U} + n \rightarrow {}^{239}\text{U} + \gamma$ 4.81 MeV

$^{239}\text{U} \rightarrow {}^{239}\text{Np} + \beta^- + \nu_e$ 0.753 MeV

$^{239}\text{Np} \rightarrow {}^{239}\text{Pu} + \beta^- + \nu_e$ 0.211 MeV

The naturally occurring radioactive isotope ^{232}Th does not make good fission fuel, because it has an even number of neutrons; however, it can be bred into a suitable fuel (much as ^{238}U is bred into ^{239}P).

(a) What are Z and N for ^{232}Th ?

(b) Write the reaction equation for neutron captured by ^{232}Th and identify the nuclide ^{A}X produced in $n + {}^{232}\text{Th} \rightarrow {}^{A}X + \gamma$.

(c) The product nucleus β^- decays, as does its daughter. Write the decay equations for each, and identify the final nucleus.

(d) Confirm that the final nucleus has an odd number of neutrons, making it a better fission fuel.

(e) Look up the half-life of the final nucleus to see if it lives long enough to be a useful fuel.

Show Solution

Strategy

(a) Thorium has atomic number 90. (b) Neutron capture increases A by 1. (c) Beta decay increases Z by 1, keeping A constant. (d) Check if N is odd. (e) Look up ^{233}U half-life.

Solution

(a) For ^{232}Th: $Z = 90$, $N = A - Z = 232 - 90 = 142$

(b) $n + {}^{232}\text{Th} \rightarrow {}^{233}\text{Th} + \gamma$

The product is ^{233}Th (thorium-233).

(c) First beta decay: $^{233}\text{Th} \rightarrow {}^{233}\text{Pa} + \beta^- + \bar{\nu}_e$ (protactinium-233)

Second beta decay: $^{233}\text{Pa} \rightarrow {}^{233}\text{U} + \beta^- + \bar{\nu}_e$ (uranium-233)

The final nucleus is ^{233}U.

(d) For ^{233}U: $Z = 92$, $N = 233 - 92 = 141$ (odd) ✓

Having an odd number of neutrons makes ^{233}U fissile with thermal neutrons.

(e) The half-life of ^{233}U is 159,000 years—long enough to be extracted, processed, fabricated into fuel, and used in reactors, making it an excellent fission fuel.

Discussion

This thorium fuel cycle is an important alternative to the uranium-plutonium cycle. Thorium is 3-4 times more abundant than uranium, and ^{233}U produces less long-lived radioactive waste than ^{239}Pu. The breeding process parallels plutonium breeding: fertile material captures a neutron, then undergoes two beta decays to reach the fissile isotope. India and China are developing thorium reactors because of abundant thorium reserves. The odd-N criterion is critical: fissile isotopes (^{233}U, ^{235}U, ^{239}Pu) all have odd N, while fertile isotopes (^{232}Th, ^{238}U) have even N.

The electrical power output of a large nuclear reactor facility is 900 MW. It has a 35.0% efficiency in converting nuclear power to electrical.

(a) What is the thermal nuclear power output in megawatts?

(b) How many ^{235}U nuclei fission each second, assuming the average fission produces 200 MeV?

(c) What mass of ^{235}U is fissioned in one year of full-power operation?

Show Solution

(a) $2.57\times10^{3}\,\text{MW}$ (b) $8.03\times10^{19}\,\text{fission/s}$ (c) 991 kg

A large power reactor that has been in operation for some months is turned off, but residual activity in the core still produces 150 MW of power. If the average energy per decay of the fission products is 1.00 MeV, what is the core activity in curies?

Show Solution

Strategy

Convert power to energy per second (joules/s), then to MeV/s. Divide by energy per decay to get decays/s (Bq), then convert to curies.

Solution

Power in watts (J/s):

$$P=150\text{ MW}=1.50\times10^{8}\text{ W}=1.50\times10^{8}\text{ J/s}$$

Convert to MeV/s:

$$P=\frac{1.50\times10^{8}\text{ J/s}}{1.602\times10^{-13}\text{ J/MeV}}=9.36\times10^{20}\text{ MeV/s}$$

Activity in Bq (decays per second):

$$R=\frac{9.36\times10^{20}\text{ MeV/s}}{1.00\text{ MeV/decay}}=9.36\times10^{20}\text{ Bq}$$

Convert to curies:

$$R=\frac{9.36\times10^{20}\text{ Bq}}{3.70\times10^{10}\text{ Bq/Ci}}=2.53\times10^{10}\text{ Ci}=25.3\text{ GCi}$$

Discussion

This enormous activity of 25.3 billion curies demonstrates why reactor cores remain extremely radioactive after shutdown. This “decay heat” comes from fission products with half-lives ranging from seconds to years. Even though the chain reaction has stopped, the accumulated fission products continue decaying, releasing 150 MW—enough to overheat the core if cooling fails. This is why reactors need active cooling systems for months or years after shutdown. The Fukushima disaster occurred when tsunami damage prevented decay heat removal, causing meltdowns. The activity decreases with time as short-lived isotopes decay, but significant radioactivity persists for decades. This huge inventory of radioactive material is also why reactor containment buildings must be robust—a release of even a small fraction would be catastrophic.

Glossary

breeder reactors
: reactors that are designed specifically to make plutonium

breeding
: reaction process that produces ^{239}Pu

criticality
: condition in which a chain reaction easily becomes self-sustaining

critical mass
: minimum amount necessary for self-sustained fission of a given nuclide

fission fragments
: a daughter nuclei

liquid drop model
: a model of nucleus (only to understand some of its features) in which nucleons in a nucleus act like atoms in a drop

nuclear fission
: reaction in which a nucleus splits

neutron-induced fission
: fission that is initiated after the absorption of neutron

supercriticality
: an exponential increase in fissions

Nuclear Weapons

- Discuss different types of fission and thermonuclear bombs.
- Explain the ill effects of nuclear explosion.

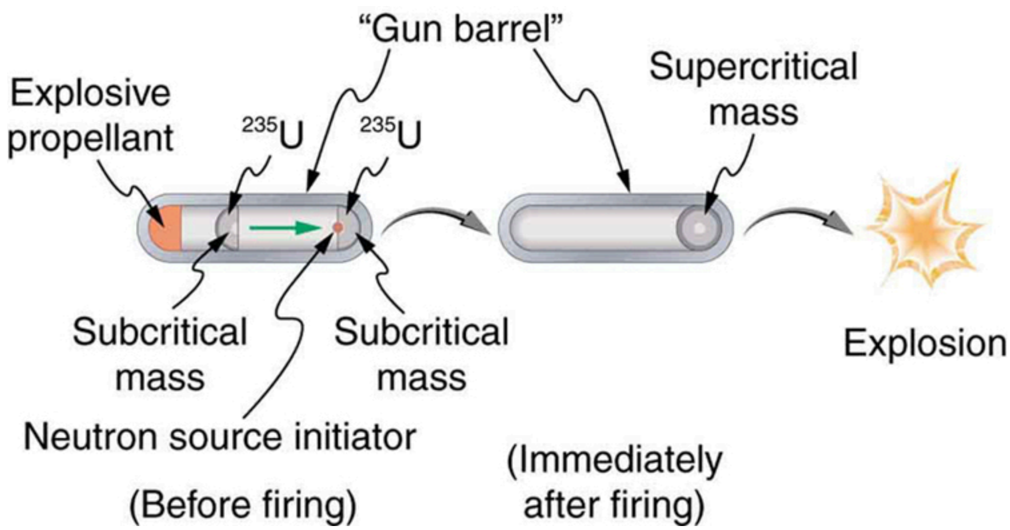
The world was in turmoil when fission was discovered in 1938. The discovery of fission, made by two German physicists, Otto Hahn and Fritz Strassman, was quickly verified by two Jewish refugees from Nazi Germany, Lise Meitner and her nephew Otto Frisch. Fermi, among others, soon found that not only did neutrons induce fission; more neutrons were produced during fission. The possibility of a self-sustained chain reaction was immediately recognized by leading scientists the world over. The enormous energy known to be in nuclei, but considered inaccessible, now seemed to be available on a large scale.

Within months after the announcement of the discovery of fission, Adolf Hitler banned the export of uranium from newly occupied Czechoslovakia. It seemed that the military value of uranium had been recognized in Nazi Germany, and that a serious effort to build a nuclear bomb had begun.

Alarmed scientists, many of them who fled Nazi Germany, decided to take action. None was more famous or revered than Einstein. It was felt that his help was needed to get the American government to make a serious effort at nuclear weapons as a matter of survival. Leo Szilard, an escaped Hungarian physicist, took a draft of a letter to Einstein, who, although pacifistic, signed the final version. The letter was for President Franklin Roosevelt, warning of the German potential to build extremely powerful bombs of a new type. It was sent in August of 1939, just before the German invasion of Poland that marked the start of World War II.

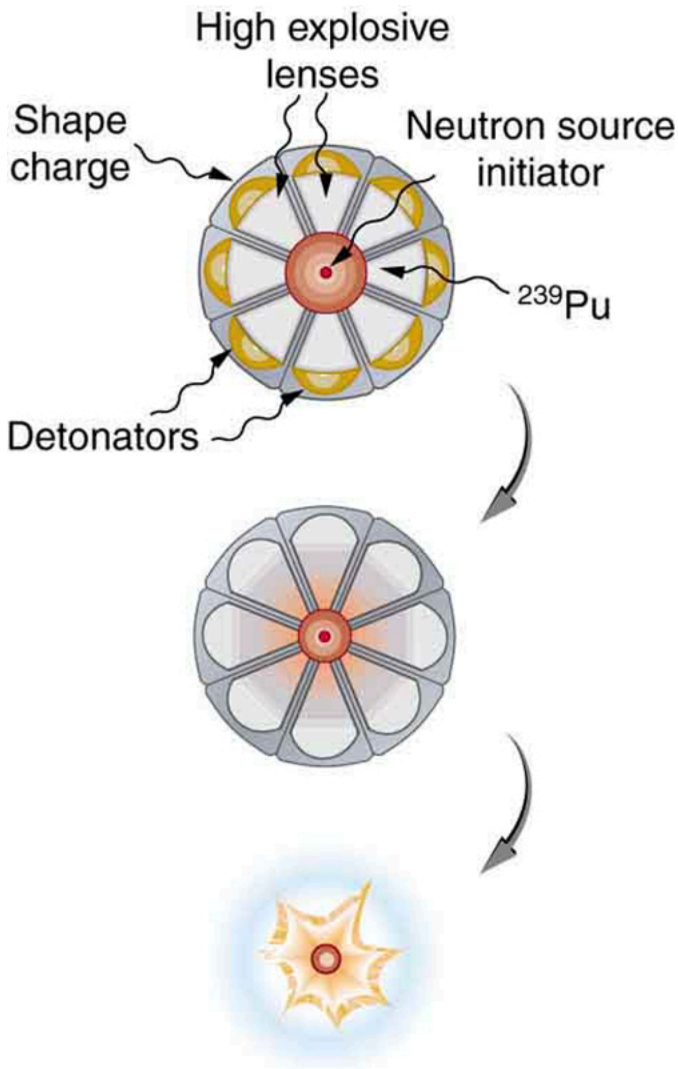
It was not until December 6, 1941, the day before the Japanese attack on Pearl Harbor, that the United States made a massive commitment to building a nuclear bomb. The top secret Manhattan Project was a crash program aimed at beating the Germans. It was carried out in remote locations, such as Los Alamos, New Mexico, whenever possible, and eventually came to cost billions of dollars and employ the efforts of more than 100 000 people. J. Robert Oppenheimer (1904–1967), whose talent and ambitions made him ideal, was chosen to head the project. The first major step was made by Enrico Fermi and his group in December 1942, when they achieved the first self-sustained nuclear reactor. This first “atomic pile”, built in a squash court at the University of Chicago, used carbon blocks to thermalize neutrons. It not only proved that the chain reaction was possible, it began the era of nuclear reactors. Glenn Seaborg, an American chemist and physicist, received the Nobel Prize in physics in 1951 for discovery of several transuranic elements, including plutonium. Carbon-moderated reactors are relatively inexpensive and simple in design and are still used for breeding plutonium, such as at Chernobyl, where two such reactors remain in operation.

Plutonium was recognized as easier to fission with neutrons and, hence, a superior fission material very early in the Manhattan Project. Plutonium availability was uncertain, and so a uranium bomb was developed simultaneously. [Figure 1] shows a gun-type bomb, which takes two subcritical uranium masses and blows them together. To get an appreciable yield, the critical mass must be held together by the explosive charges inside the cannon barrel for a few microseconds. Since the buildup of the uranium chain reaction is relatively slow, the device to hold the critical mass together can be relatively simple. Owing to the fact that the rate of spontaneous fission is low, a neutron source is triggered at the same time the critical mass is assembled.



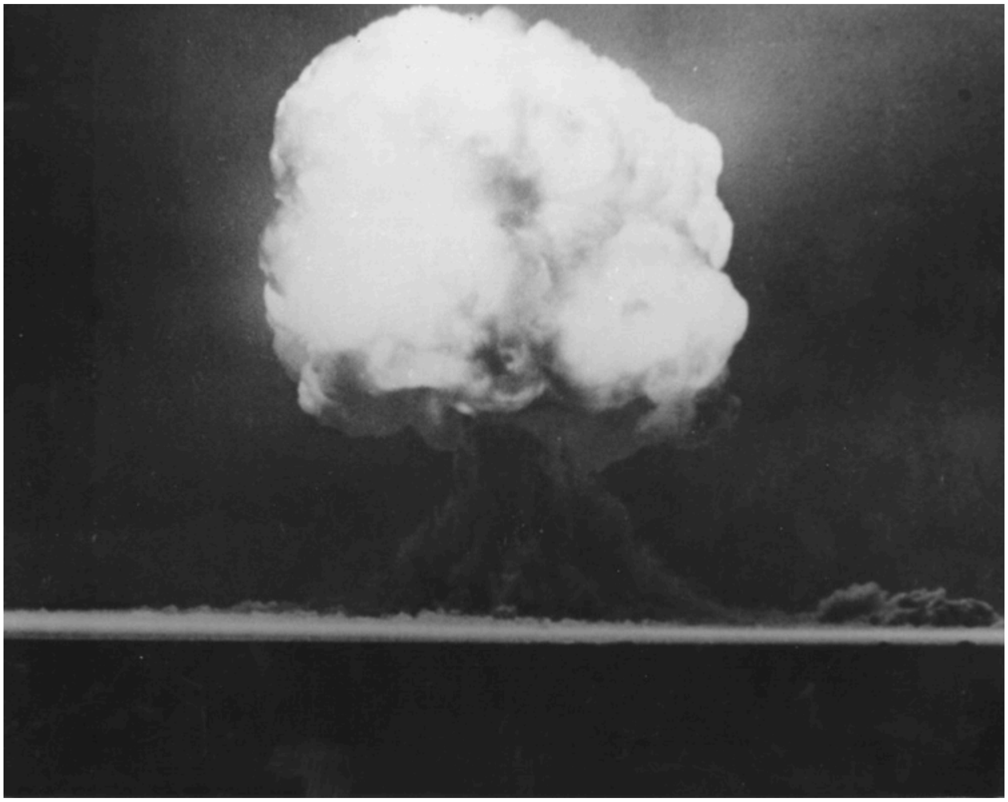
A gun-type fission bomb for ${}^{235}\text{U}$ utilizes two subcritical masses forced together by explosive charges inside a cannon barrel. The energy yield depends on the amount of uranium and the time it can be held together before it disassembles itself.

Plutonium’s special properties necessitated a more sophisticated critical mass assembly, shown schematically in [Figure 2]. A spherical mass of plutonium is surrounded by shape charges (high explosives that release most of their blast in one direction) that implode the plutonium, crushing it into a smaller volume to form a critical mass. The implosion technique is faster and more effective, because it compresses three-dimensionally rather than one-dimensionally as in the gun-type bomb. Again, a neutron source must be triggered at just the correct time to initiate the chain reaction.



An implosion created by high explosives compresses a sphere of ${}^{239}\text{Pu}$ into a critical mass. The superior fissionability of plutonium has made it the universal bomb material.

Owing to its complexity, the plutonium bomb needed to be tested before there could be any attempt to use it. On July 16, 1945, the test named Trinity was conducted in the isolated Alamogordo Desert about 200 miles south of Los Alamos (see [Figure 3]). A new age had begun. The yield of this device was about 10 kilotons (kT), the equivalent of 5000 of the largest conventional bombs.



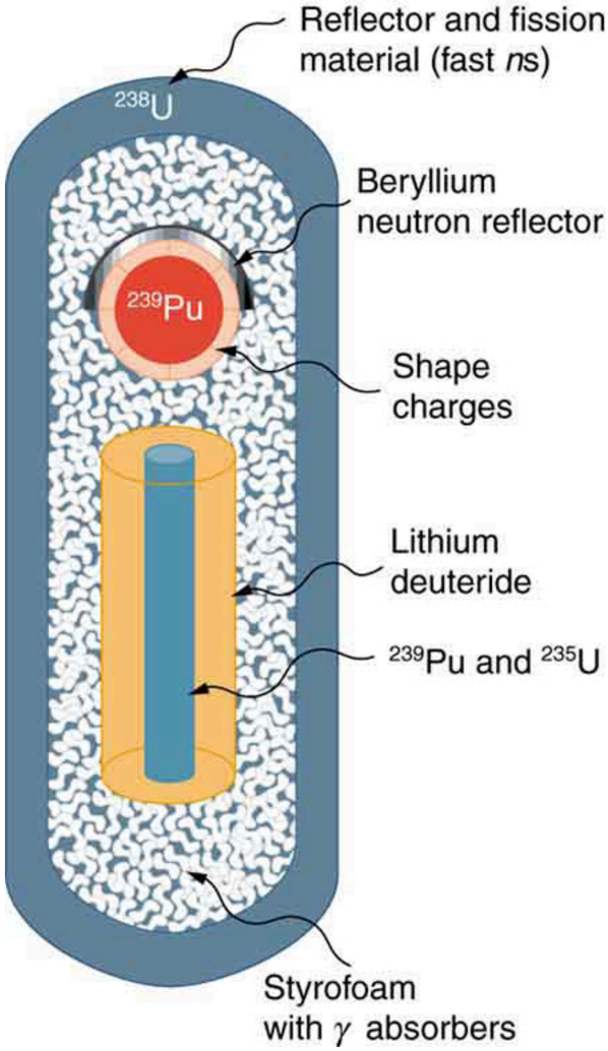
Trinity test (1945), the first nuclear bomb (credit: United States Department of Energy)

Although Germany surrendered on May 7, 1945, Japan had been steadfastly refusing to surrender for many months, forcing large casualties. Invasion plans by the Allies estimated a million casualties of their own and untold losses of Japanese lives. The bomb was viewed as a way to end the war. The first was a uranium bomb dropped on Hiroshima on August 6. Its yield of about 15 kT destroyed the city and killed an estimated 80 000 people, with 100 000 more being seriously injured (see [Figure 4]). The second was a plutonium bomb dropped on Nagasaki only three days later, on August 9. Its 20 kT yield killed at least 50 000 people, something less than Hiroshima because of the hilly terrain and the fact that it was a few kilometers off target. The Japanese were told that one bomb a week would be dropped until they surrendered unconditionally, which they did on August 14. In actuality, the United States had only enough plutonium for one more and as yet unassembled bomb.

Destruction in Hiroshima (credit: United States Federal Government)

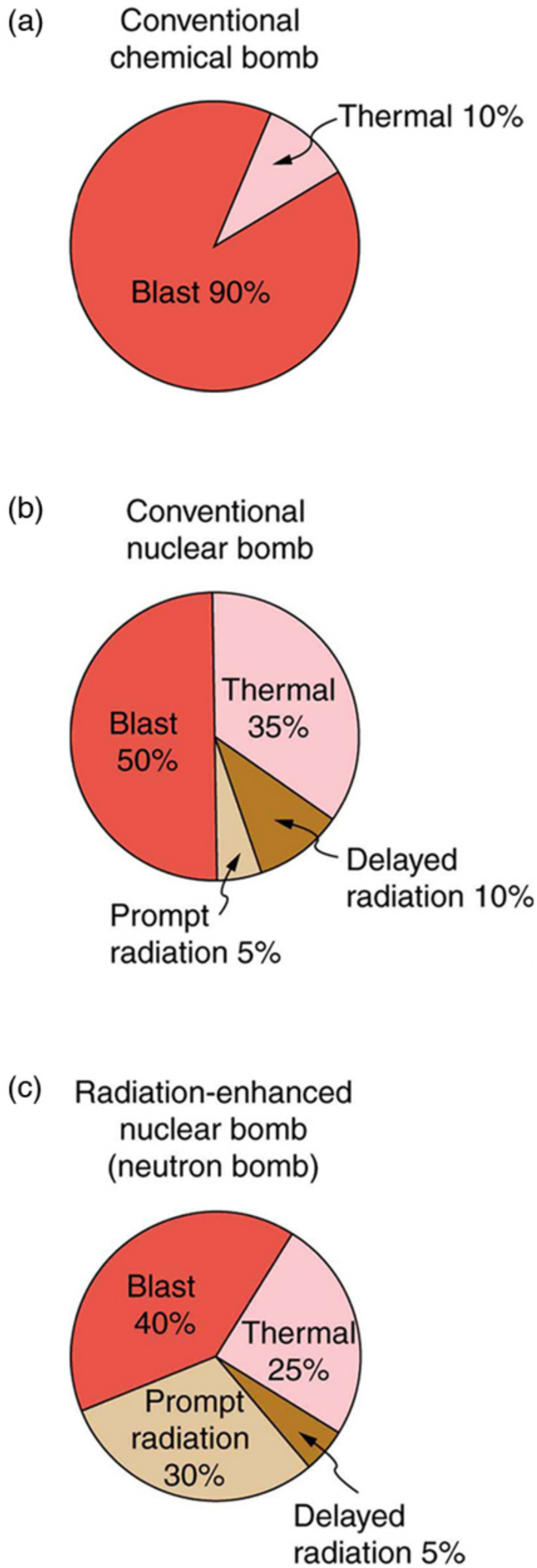
Knowing that fusion produces several times more energy per kilogram of fuel than fission, some scientists pushed the idea of a fusion bomb starting very early on. Calling this bomb the Super, they realized that it could have another advantage over fission—high-energy neutrons would aid fusion, while they are ineffective in ^{239}Pu fission. Thus the fusion bomb could be virtually unlimited in energy release. The first such bomb was detonated by the United States on October 31, 1952, at Eniwetok Atoll with a yield of 10 megatons (MT), about 670 times that of the fission bomb that destroyed Hiroshima. The Soviets followed with a fusion device of their own in August 1953, and a weapons race, beyond the aim of this text to discuss, continued until the end of the Cold War.

[Figure 5] shows a simple diagram of how a thermonuclear bomb is constructed. A fission bomb is exploded next to fusion fuel in the solid form of lithium deuteride. Before the shock wave blows it apart, γ rays heat and compress the fuel, and neutrons create tritium through the reaction $n + {}^{6}\text{Li} \rightarrow {}^{3}\text{H} + {}^{4}\text{He}$. Additional fusion and fission fuels are enclosed in a dense shell of ${}^{238}\text{U}$. The shell reflects some of the neutrons back into the fuel to enhance its fusion, but at high internal temperatures fast neutrons are created that also cause the plentiful and inexpensive ${}^{238}\text{U}$ to fission, part of what allows thermonuclear bombs to be so large.



This schematic of a fusion bomb (H-bomb) gives some idea of how the ${}^{239}\text{Pu}$ fission trigger is used to ignite fusion fuel. Neutrons and γ rays transmit energy to the fusion fuel, create tritium from deuterium, and heat and compress the fusion fuel. The outer shell of ${}^{238}\text{U}$ serves to reflect some neutrons back into the fuel, causing more fusion, and it boosts the energy output by fissioning itself when neutron energies become high enough.

The energy yield and the types of energy produced by nuclear bombs can be varied. Energy yields in current arsenals range from about 0.1 kT to 20 MT, although the Soviets once detonated a 67 MT device. Nuclear bombs differ from conventional explosives in more than size. [Figure 6] shows the approximate fraction of energy output in various forms for conventional explosives and for two types of nuclear bombs. Nuclear bombs put a much larger fraction of their output into thermal energy than do conventional bombs, which tend to concentrate the energy in blast. Another difference is the immediate and residual radiation energy from nuclear weapons. This can be adjusted to put more energy into radiation (the so-called neutron bomb) so that the bomb can be used to irradiate advancing troops without killing friendly troops with blast and heat.



Approximate fractions of energy output by conventional and two types of nuclear weapons. In addition to yielding more energy than conventional weapons, nuclear bombs put a much larger fraction into thermal energy. This can be adjusted to enhance the radiation output to be more effective against troops. An enhanced radiation bomb is also called a neutron bomb.

At its peak in 1986, the combined arsenals of the United States and the Soviet Union totaled about 60 000 nuclear warheads. In addition, the British, French, and Chinese each have several hundred bombs of various sizes, and a few other countries have a small number. Nuclear weapons are generally divided into two categories. Strategic nuclear weapons are those intended for military targets, such as bases and missile complexes, and moderate to large cities. There were about 20 000 strategic weapons in 1988. Tactical weapons are intended for use in smaller battles. Since the collapse of the Soviet Union and the end of the Cold War in 1989, most of the 32 000 tactical weapons (including Cruise missiles, artillery shells, land mines, torpedoes, depth charges, and backpacks) have been demobilized, and parts of the strategic weapon systems are being dismantled with warheads and missiles being disassembled. According to the Treaty of Moscow of 2002, Russia and the United States have been required to reduce their strategic nuclear arsenal down to about 2000 warheads each.

A few small countries have built or are capable of building nuclear bombs, as are some terrorist groups. Two things are needed—a minimum level of technical expertise and sufficient fissionable material. The first is easy. Fissionable material is controlled but is also available. There are international agreements and organizations that attempt to control nuclear proliferation, but it is increasingly difficult given the availability of fissionable material and the small amount needed for a crude bomb. The production of fissionable fuel itself is technologically difficult. However, the presence of large amounts of such material worldwide, though in the hands of a few, makes control and accountability crucial.

Section Summary

- There are two types of nuclear weapons—fission bombs use fission alone, whereas thermonuclear bombs use fission to ignite fusion.
- Both types of weapons produce huge numbers of nuclear reactions in a very short time.
- Energy yields are measured in kilotons or megatons of equivalent conventional explosives and range from 0.1 kT to more than 20 MT.
- Nuclear bombs are characterized by far more thermal output and nuclear radiation output than conventional explosives.

Conceptual Questions

What are some of the reasons that plutonium rather than uranium is used in all fission bombs and as the trigger in all fusion bombs?

Use the laws of conservation of momentum and energy to explain how a shape charge can direct most of the energy released in an explosion in a specific direction. (Note that this is similar to the situation in guns and cannons—most of the energy goes into the bullet.)

How does the lithium deuteride in the thermonuclear bomb shown in [Figure 5] supply tritium $({}^{3}\text{H})$ as well as deuterium $({}^{2}\text{H})$?

Fallout from nuclear weapons tests in the atmosphere is mainly ${}^{90}\text{Sr}$ and ${}^{137}\text{Cs}$, which have 28.6- and 32.2-y half-lives, respectively. Atmospheric tests were terminated in most countries in 1963, although China only did so in 1980. It has been found that environmental activities of these two isotopes are decreasing faster than their half-lives. Why might this be?

Problems & Exercises

Find the mass converted into energy by a 12.0-kT bomb.

Show Solution

0.56 g

What mass is converted into energy by a 1.00-MT bomb?

Show Solution

Strategy

Convert 1.00 MT to joules (1 MT = 4.184×10^{15} J), then use $E=mc^2$ to find mass.

Solution

Energy:

$$E=1.00\text{ MT}=4.184\times10^{15}\text{ J}$$

Mass from $E=mc^2$:

$$m=\frac{E}{c^2}=\frac{4.184\times10^{15}}{(3.00\times10^{8})^2}=4.65\times10^{-2}\text{ kg}=46.5\text{ g}$$

Discussion

Only 46.5 grams of mass—about 1.6 ounces—is converted to energy in a 1-megaton nuclear explosion! This demonstrates the incredible mass-energy equivalence of $E=mc^2$. A typical thermonuclear bomb contains several kilograms of nuclear fuel, but only a tiny fraction (about 1%) of that mass is actually converted to energy through fission and fusion reactions. The rest remains as fission products and unconsumed fuel. This small mass conversion releases energy equivalent to one million tons of TNT, capable of destroying an entire city. Compare this to chemical explosives where molecular bonds rearrange but no mass is converted—nuclear reactions are millions of times more energy-dense.

Fusion bombs use neutrons from their fission trigger to create tritium fuel in the reaction $n+{}^{6}\text{Li}\rightarrow{}^{3}\text{H}+{}^{4}\text{He}$. What is the energy released by this reaction in MeV?

Show Solution

4.781 MeV

It is estimated that the total explosive yield of all the nuclear bombs in existence currently is about 4 000 MT.

(a) Convert this amount of energy to kilowatt-hours, noting that $1\text{kW}\cdot\text{h} = 3.60 \times 10^6 \text{J}$.

(b) What would the monetary value of this energy be if it could be converted to electricity costing 10 cents per kW·h?

Show Solution

Strategy

(a) Convert 4000 MT to joules, then to kWh. (b) Multiply kWh by $0.10/kWh.

Solution

(a) Energy in joules:

$$E = 4000 \text{ MT} \times 4.184 \times 10^{15} \text{ J} 1 \text{ MT} = 1.674 \times 10^{19} \text{ J}$$

Convert to kWh:

$$E = 1.674 \times 10^{19} \text{ J} 3.60 \times 10^{6} \text{ J/kWh} = 4.65 \times 10^{12} \text{ kWh} = 4.65 \text{ trillion kWh}$$

(b) Monetary value:

$$\text{Value} = (4.65 \times 10^{12} \text{ kWh})(\$0.10/\text{kWh}) = \$4.65 \times 10^{11} = \$465 \text{ billion}$$

Discussion

The world’s nuclear arsenal contains energy worth about $465 billion if it could be peacefully converted to electricity—equivalent to about half a trillion dollars. To put this in perspective, world annual electricity consumption is about 25,000 TWh = 2.5×10^{13} kWh, so these weapons contain enough energy to power the world for about 2-3 weeks. However, this comparison is somewhat misleading: nuclear weapons release their energy in microseconds in an uncontrolled explosion, while peaceful power generation requires controlled, sustained reactions over months or years. The uranium and plutonium in weapons could theoretically be “downblended” and used as reactor fuel, and some weapons-grade material has indeed been converted for civilian power use under nonproliferation agreements. This problem highlights both the enormous destructive potential and the tragic waste represented by nuclear arsenals—energy that could power cities instead threatens to destroy them.

A radiation-enhanced nuclear weapon (or neutron bomb) can have a smaller total yield and still produce more prompt radiation than a conventional nuclear bomb. This allows the use of neutron bombs to kill nearby advancing enemy forces with radiation without blowing up your own forces with the blast. For a 0.500-kT radiation-enhanced weapon and a 1.00-kT conventional nuclear bomb: (a) Compare the blast yields. (b) Compare the prompt radiation yields.

Show Solution

(a) Blast yields $2.1 \times 10^{12}\text{J}$ to $8.4 \times 10^{11}\text{J}$, or 2.5 to 1, conventional to radiation enhanced.

(b) Prompt radiation yields $6.3 \times 10^{11}\text{J}$ to $2.1 \times 10^{11}\text{J}$, or 3 to 1, radiation enhanced to conventional.

(a) How many ^{239}Pu nuclei must fission to produce a 20.0-kT yield, assuming 200 MeV per fission? (b) What is the mass of this much ^{239}Pu ?

Show Solution

Strategy

(a) Convert 20.0 kT to joules, then to MeV. Divide by 200 MeV/fission to get number of fissions. (b) Convert number of nuclei to mass using molar mass 239 g/mol.

Solution

(a) Energy in joules:

$$E = 20.0 \text{ kT} \times 4.184 \times 10^{12} \text{ J} 1 \text{ kT} = 8.368 \times 10^{13} \text{ J}$$

Convert to MeV:

$$E = 8.368 \times 10^{13} 1.602 \times 10^{-13} = 5.22 \times 10^{26} \text{ MeV}$$

Number of fissions:

$$N = 5.22 \times 10^{26} 200 = 2.61 \times 10^{24} \text{ fissions}$$

(b) Mass:

$$m = \frac{2.61\times10^{24}}{6.022\times10^{23}}\times239 = 1.04\times10^{3}\ \text{g} = 1.04\ \text{kg}$$

Discussion

Only about 1 kilogram of ^{239}Pu needs to fission to produce a 20-kiloton explosion like the Nagasaki bomb. However, an actual weapon requires several kilograms of plutonium because: (1) Not all nuclei fission—efficiency is typically 10-20%, so the bomb needs 5-10 kg total; (2) A critical mass must be assembled to sustain the chain reaction; (3) Some material is blown apart before it can fission. The fact that about 1 kg actually fissioned in the Nagasaki bomb means roughly 80-85% of the plutonium did NOT fission but was scattered as radioactive fallout. This is why nuclear weapons are so "dirty"—they spread kilograms of unfissioned plutonium across the environment. Modern weapons achieve higher efficiency (up to 50%), requiring less material and producing less fallout, but even these convert only a fraction of their nuclear fuel. This calculation shows why plutonium is such a proliferation concern: just a few kilograms can make a devastating weapon.

Assume one-fourth of the yield of a typical 320-kT strategic bomb comes from fission reactions averaging 200 MeV and the remainder from fusion reactions averaging 20 MeV.

(a) Calculate the number of fissions and the approximate mass of uranium and plutonium fissioned, taking the average atomic mass to be 238.

(b) Find the number of fusions and calculate the approximate mass of fusion fuel, assuming an average total atomic mass of the two nuclei in each reaction to be 5.

(c) Considering the masses found, does it seem reasonable that some missiles could carry 10 warheads? Discuss, noting that the nuclear fuel is only a part of the mass of a warhead.

Show Solution

(a) $1.1\times10^{25}\text{fissions}$, 4.4 kg

(b) $3.2\times10^{26}\text{fusions}$, 2.7 kg

(c) The nuclear fuel totals only 6 kg, so it is quite reasonable that some missiles carry 10 overheads. The mass of the fuel would only be 60 kg and therefore the mass of the 10 warheads, weighing about 10 times the nuclear fuel, would be only 1500 lbs. If the fuel for the missiles weighs 5 times the total weight of the warheads, the missile would weigh about 9000 lbs or 4.5 tons. This is not an unreasonable weight for a missile.

This problem gives some idea of the magnitude of the energy yield of a small tactical bomb. Assume that half the energy of a 1.00-kT nuclear depth charge set off under an aircraft carrier goes into lifting it out of the water—that is, into gravitational potential energy. How high is the carrier lifted if its mass is 90 000 tons?

Show Solution

Strategy

Use $PE = mgh$. Half the bomb energy (0.50 kT) goes into lifting. Convert to joules, solve for h using $m = 90{,}000\ \text{tons} = 9.0\times10^{7}\ \text{kg}$.

Solution

Energy available for lifting:

$$E = 0.50\ \text{kT} = 0.50\times4.184\times10^{12} = 2.09\times10^{12}\ \text{J}$$

Mass of carrier:

$$m = 90{,}000\ \text{tons}\times\frac{907\ \text{kg}}{1\ \text{ton}} = 8.16\times10^{7}\ \text{kg}$$

Height from $PE = mgh$:

$$h = \frac{E}{mg} = \frac{2.09\times10^{12}}{(8.16\times10^{7})(9.8)} = 2.61\times10^{3}\ \text{m} = 2.61\ \text{km}$$

Discussion

The aircraft carrier would be lifted 2.6 kilometers (about 1.6 miles) into the air! This astonishing result—lifting a 90,000-ton warship higher than many mountains—demonstrates the incredible energy density of nuclear weapons. Of course, in reality the carrier wouldn't be lifted as a rigid body to this height. Instead, the explosion would: (1) Create a massive bubble of superheated steam and gas that expands rapidly; (2) Shock the hull, breaking it apart; (3) Lift fragments in a chaotic spray rather than the whole ship intact. Nevertheless, this calculation shows why nuclear depth charges are so devastating to surface ships—even a "small" 1-kiloton weapon underwater releases enough energy to completely destroy the largest vessels. The water coupling makes underwater nuclear explosions particularly effective: the incompressible water transmits the shock wave with little attenuation, and the expanding gas bubble creates a powerful lifting force. This is why nuclear depth charges were developed during the Cold War for anti-submarine warfare—one bomb could sink an entire carrier battle group.

It is estimated that weapons tests in the atmosphere have deposited approximately 9 MCi of $^{90}\mathrm{Sr}$ on the surface of the earth. Find the mass of this amount of $^{90}\mathrm{Sr}$.

Show Solution

$7 \times 10^{4}\mathrm{g}$

A 1.00-MT bomb exploded a few kilometers above the ground deposits 25.0% of its energy into radiant heat.

(a) Find the calories per cm^2 at a distance of 10.0 km by assuming a uniform distribution over a spherical surface of that radius.

(b) If this heat falls on a person's body, what temperature increase does it cause in the affected tissue, assuming it is absorbed in a layer 1.00-cm deep?

Show Solution

Strategy

(a) Find total thermal energy (25% of 1.00 MT), distribute over sphere surface area $A = 4\pi r^2$ at r = 10 km. Convert to cal/cm². (b) Use $Q = mc\Delta T$ with c = 1 cal/(g·°C) for tissue (similar to water), assuming 1 cm² of 1-cm-deep tissue has mass 1 g.

Solution

(a) Thermal energy:

$$E=0.250\times4.184\times10^{15}=1.046\times10^{15}\ \mathrm{J}$$

Convert to calories (1 cal = 4.184 J):

$$E=1.046\times10^{15} 4.184=2.50\times10^{14}\ \mathrm{cal}$$

Surface area at 10 km:

$$A=4\pi r^2=4\pi(10\times10^5\ \mathrm{cm})^2=1.26\times10^{13}\ \mathrm{cm}^2$$

Energy per unit area:

$$E\,A=2.50\times10^{14} 1.26\times10^{13}=19.8\ \mathrm{cal/cm}^2$$

(b) For 1 cm² area, 1 cm deep: mass = 1 g (tissue density ≈ 1 g/cm³)

$$\Delta T=Q mc=19.8\ \mathrm{cal}(1\ \mathrm{g})(1\ \mathrm{cal/g\cdot°C})=19.8°\mathrm{C}\approx20°\mathrm{C}$$

Discussion

At 10 km from a 1-megaton airburst, exposed skin receives about 20 cal/cm², causing a temperature rise of 20°C. This is enough to cause second-degree burns (blistering) on exposed skin. The thermal pulse lasts only a few seconds, but delivers intense heat almost instantaneously. This explains the horrific burn injuries at Hiroshima and Nagasaki—people 10 km away received severe burns on exposed skin facing the blast. Closer distances are far worse: at 5 km the energy scales as $1/r^2$, giving ~80 cal/cm² and nearly instant third-degree burns. At 2 km, it's ~500 cal/cm²—enough to ignite clothing and vaporize skin. This thermal radiation, traveling at the speed of light, arrives before the blast wave, giving no warning. The combination of thermal and blast effects creates a zone of near-total lethality extending kilometers from a large nuclear explosion. This problem demonstrates why nuclear weapons cause such widespread casualties—the thermal effects alone can inflict mass casualties over enormous areas.

Integrated Concepts

One scheme to put nuclear weapons to nonmilitary use is to explode them underground in a geologically stable region and extract the geothermal energy for electricity production. There was a total yield of about 4 000 MT in the combined arsenals in 2006. If 1.00 MT per day could be converted to electricity with an efficiency of 10.0%:

(a) What would the average electrical power output be?

(b) How many years would the arsenal last at this rate?

Show Solution

(a) $4.86 \times 10^{9}\mathrm{W}$ (b) 11.0 y
