Introduction to Magnetism

The magnificent spectacle of the Aurora Borealis, or northern lights, glows in the northern sky above Bear Lake near Eielson Air Force Base, Alaska. Shaped by the Earth's magnetic field, this light is produced by radiation spewed from solar storms. (credit: Senior Airman Joshua Strang, via Flickr)

One evening, an Alaskan sticks a note to his refrigerator with a small magnet. Through the kitchen window, the Aurora Borealis glows in the night sky. This grand spectacle is shaped by the same force that holds the note to the refrigerator.

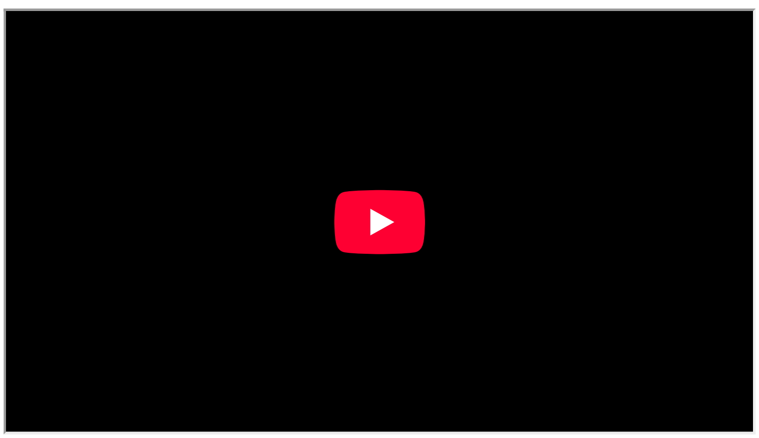
People have been aware of magnets and magnetism for thousands of years. The earliest records date to well before the time of Christ, particularly in a region of Asia Minor called Magnesia (the name of this region is the source of words like *magnetic*). Magnetic rocks found in Magnesia, which is now part of western Turkey, stimulated interest during ancient times. A practical application for magnets was found later, when they were employed as navigational compasses. The use of magnets in compasses resulted not only in improved long-distance sailing, but also in the names of "north" and "south" being given to the two types of magnetic poles.

Today magnetism plays many important roles in our lives. Physicists' understanding of magnetism has enabled the development of technologies that affect our everyday lives. The iPod in your purse or backpack, for example, wouldn't have been possible without the applications of magnetism and electricity on a small scale.

The discovery that weak changes in a magnetic field in a thin film of iron and chromium could bring about much larger changes in electrical resistance was one of the first large successes of nanotechnology. The 2007 Nobel Prize in Physics went to Albert Fert from France and Peter Grunberg from Germany for this discovery of *giant magnetoresistance* and its applications to computer memory.

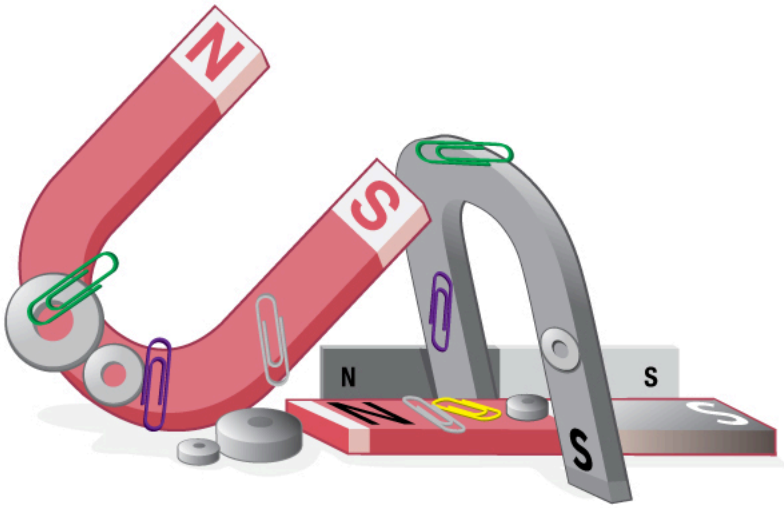
All electric motors, with uses as diverse as powering refrigerators, starting cars, and moving elevators, contain magnets. Generators, whether producing hydroelectric power or running bicycle lights, use magnetic fields. Recycling facilities employ magnets to separate iron from other refuse. Hundreds of millions of dollars are spent annually on magnetic containment of fusion as a future energy source. Magnetic resonance imaging (MRI) has become an important diagnostic tool in the field of medicine, and the use of magnetism to explore brain activity is a subject of contemporary research and development. The list of applications also includes computer hard drives, tape recording, detection of inhaled asbestos, and levitation of high-speed trains. Magnetism is used to explain atomic energy levels, cosmic rays, and charged particles trapped in the Van Allen belts. Once again, we will find all these disparate phenomena are linked by a small number of underlying physical principles.

Engineering of technology like iPods would not be possible without a deep understanding magnetism. (credit: Jesse! S?, Flickr)



Magnets

- Describe the difference between the north and south poles of a magnet.
- Describe how magnetic poles interact with each other.



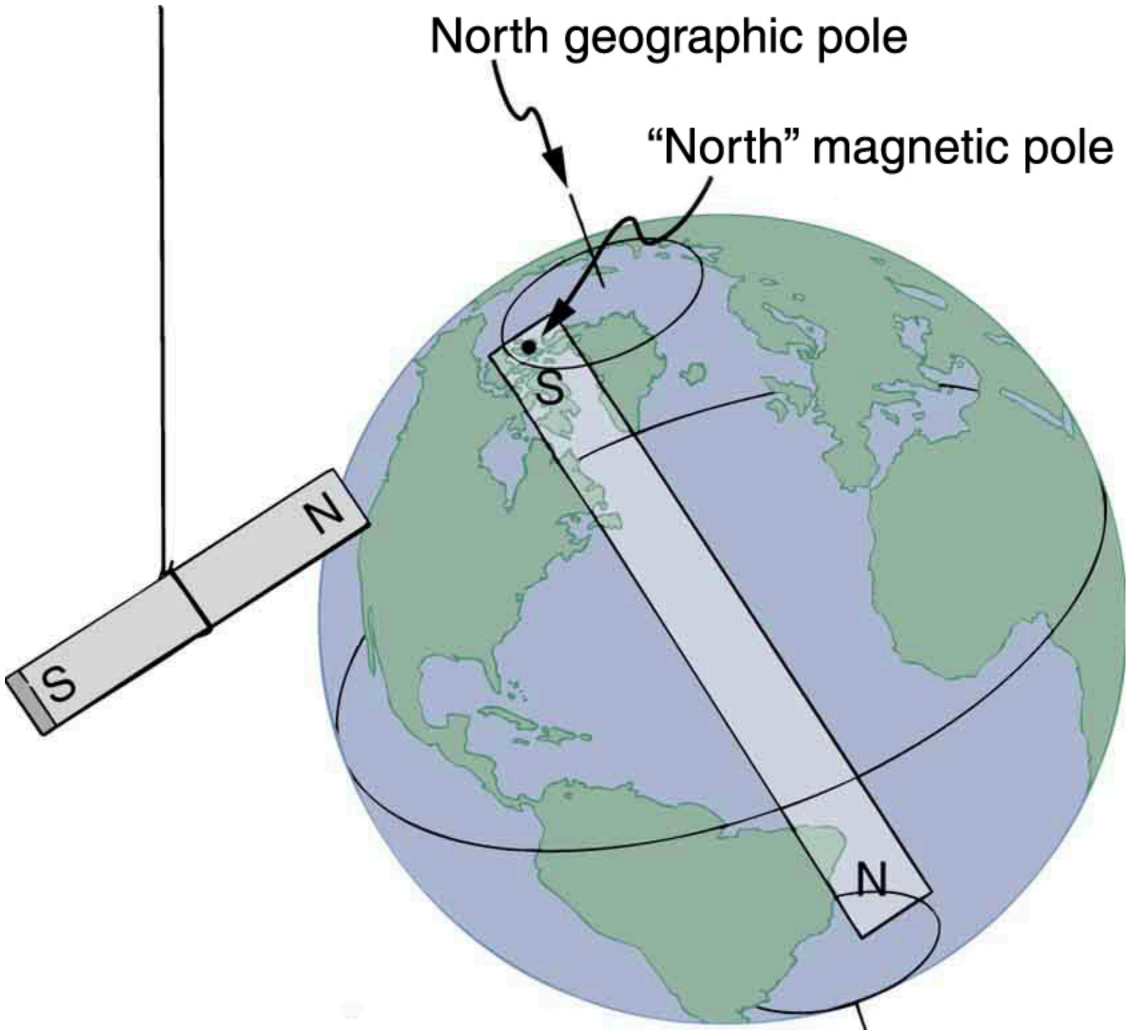
Magnets come in various shapes, sizes, and strengths. All have both a north pole and a south pole. There is never an isolated pole (a monopole).

All magnets attract iron, such as that in a refrigerator door. However, magnets may attract or repel other magnets. Experimentation shows that all magnets have two poles. If freely suspended, one pole will point toward the north. The two poles are thus named the **north magnetic pole** and the **south magnetic pole** (or more properly, north-seeking and south-seeking poles, for the attractions in those directions).

Universal Characteristics of Magnets and Magnetic Poles

It is a universal characteristic of all magnets that *like poles repel and unlike poles attract*. (Note the similarity with electrostatics: unlike charges attract and like charges repel.)

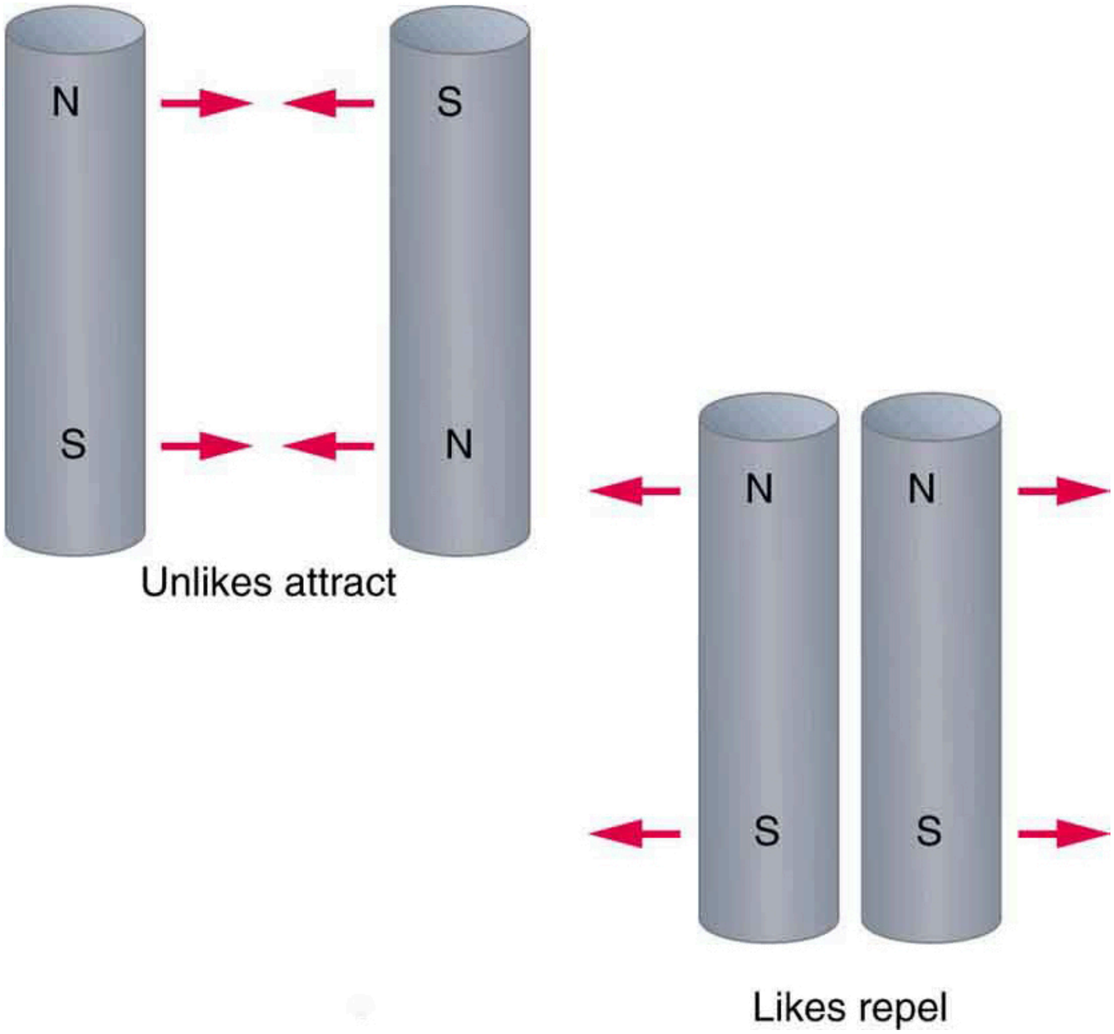
Further experimentation shows that it is *impossible to separate north and south poles* in the manner that + and − charges can be separated.



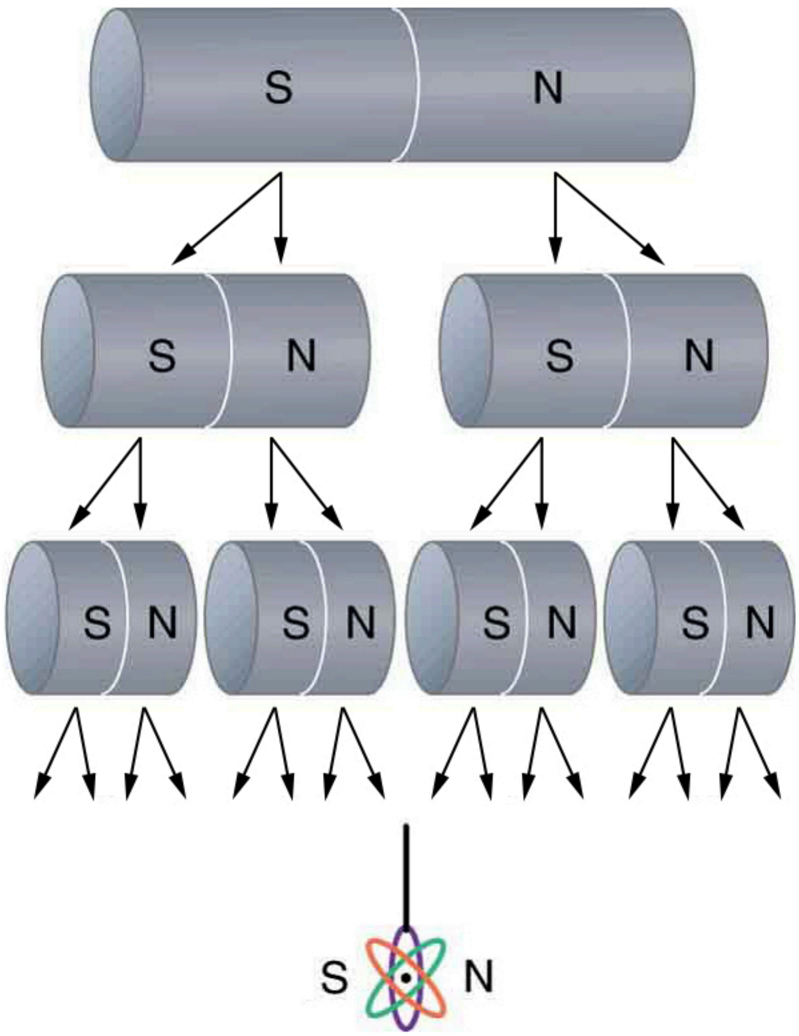
One end of a bar magnet is suspended from a thread that points toward north. The magnet's two poles are labeled N and S for north-seeking and south-seeking poles, respectively.

Misconception Alert: Earth's Geographic North Pole Hides an S

The Earth acts like a very large bar magnet with its south-seeking pole near the geographic North Pole. That is why the north pole of your compass is attracted toward the geographic north pole of the Earth—because the magnetic pole that is near the geographic North Pole is actually a south magnetic pole! Confusion arises because the geographic term "North Pole" has come to be used (incorrectly) for the magnetic pole that is near the North Pole. Thus, "North magnetic pole" is actually a misnomer—it should be called the South magnetic pole.



Unlike poles attract, whereas like poles repel.



North and south poles always occur in pairs. Attempts to separate them result in more pairs of poles. If we continue to split the magnet, we will eventually get down to an iron atom with a north pole and a south pole—these, too, cannot be separated.

The fact that magnetic poles always occur in pairs of north and south is true from the very large scale—for example, sunspots always occur in pairs that are north and south magnetic poles—all the way down to the very small scale. Magnetic atoms have both a north pole and a south pole, as do many types of subatomic particles, such as electrons, protons, and neutrons.

Making Connections: Take-Home Experiment—Refrigerator Magnets

We know that like magnetic poles repel and unlike poles attract. See if you can show this for two refrigerator magnets. Will the magnets stick if you turn them over? Why do they stick to the door anyway? What can you say about the magnetic properties of the door next to the magnet? Do refrigerator magnets stick to metal or plastic spoons? Do they stick to all types of metal?

Section Summary

- Magnetism is a subject that includes the properties of magnets, the effect of the magnetic force on moving charges and currents, and the creation of magnetic fields by currents.
- There are two types of magnetic poles, called the north magnetic pole and south magnetic pole.
- North magnetic poles are those that are attracted toward the Earth's geographic north pole.
- Like poles repel and unlike poles attract.
- Magnetic poles always occur in pairs of north and south—it is not possible to isolate north and south poles.

Conceptual Questions

Volcanic and other such activity at the mid-Atlantic ridge extrudes material to fill the gap between separating tectonic plates associated with continental drift. The magnetization of rocks is found to reverse in a coordinated manner with distance from the ridge. What does this imply about the Earth's magnetic field and how could the knowledge of the spreading rate be used to give its historical record?

Glossary

north magnetic pole
: the end or the side of a magnet that is attracted toward Earth's geographic north pole

south magnetic pole
: the end or the side of a magnet that is attracted toward Earth’s geographic south pole

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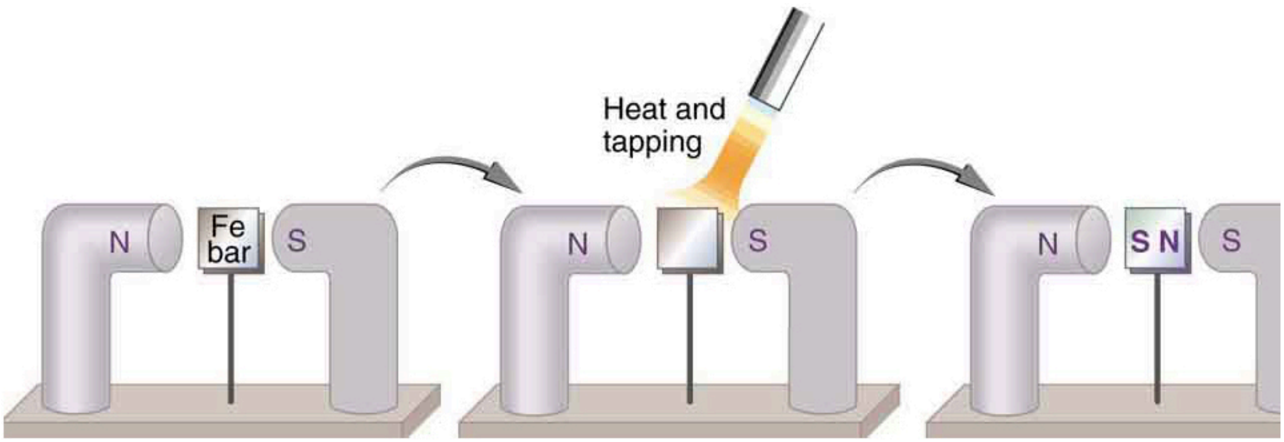
Ferromagnets and Electromagnets

- Define ferromagnet.
- Describe the role of magnetic domains in magnetization.
- Explain the significance of the Curie temperature.
- Describe the relationship between electricity and magnetism.

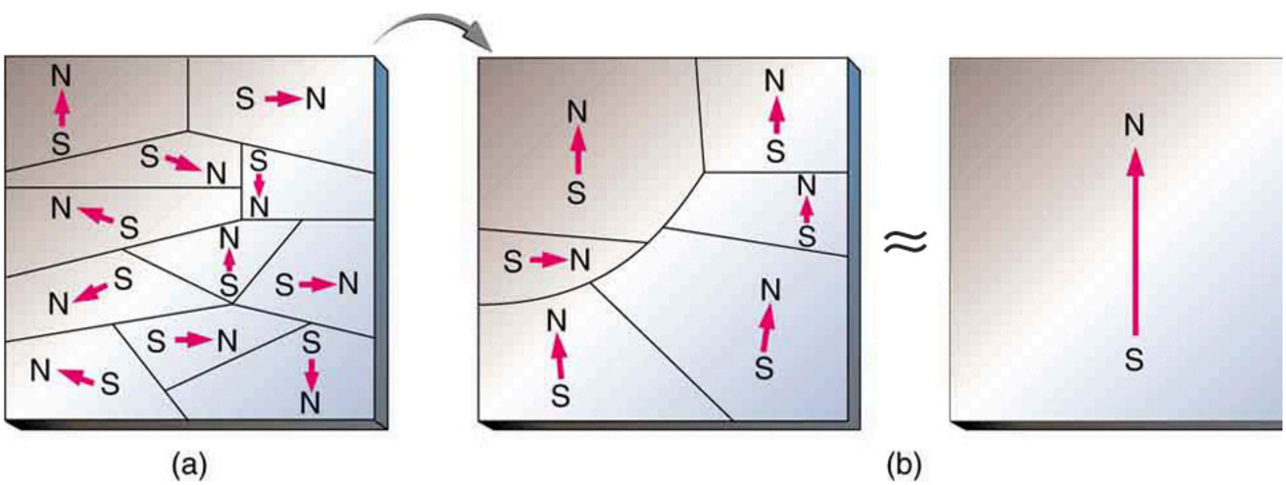
Ferromagnets

Only certain materials, such as iron, cobalt, nickel, and gadolinium, exhibit strong magnetic effects. Such materials are called **ferromagnetic**, after the Latin word for iron, *ferrum*. A group of materials made from the alloys of the rare earth elements are also used as strong and permanent magnets; a popular one is neodymium. Other materials exhibit weak magnetic effects, which are detectable only with sensitive instruments. Not only do ferromagnetic materials respond strongly to magnets (the way iron is attracted to magnets), they can also be **magnetized** themselves—that is, they can be induced to be magnetic or made into permanent magnets.



An unmagnetized piece of iron is placed between two magnets, heated, and then cooled, or simply tapped when cold. The iron becomes a permanent magnet with the poles aligned as shown: its south pole is adjacent to the north pole of the original magnet, and its north pole is adjacent to the south pole of the original magnet. Note that there are attractive forces between the magnets.

When a magnet is brought near a previously unmagnetized ferromagnetic material, it causes local magnetization of the material with unlike poles closest, as in [Figure 1]. (This results in the attraction of the previously unmagnetized material to the magnet.) What happens on a microscopic scale is illustrated in [Figure 2]. The regions within the material called **domains** act like small bar magnets. Within domains, the poles of individual atoms are aligned. Each atom acts like a tiny bar magnet. Domains are small and randomly oriented in an unmagnetized ferromagnetic object. In response to an external magnetic field, the domains may grow to millimeter size, aligning themselves as shown in [Figure 2](b). This induced magnetization can be made permanent if the material is heated and then cooled, or simply tapped in the presence of other magnets.



(a) An unmagnetized piece of iron (or other ferromagnetic material) has randomly oriented domains. (b) When magnetized by an external field, the domains show greater alignment, and some grow at the expense of others. Individual atoms are aligned within domains; each atom acts like a tiny bar magnet.

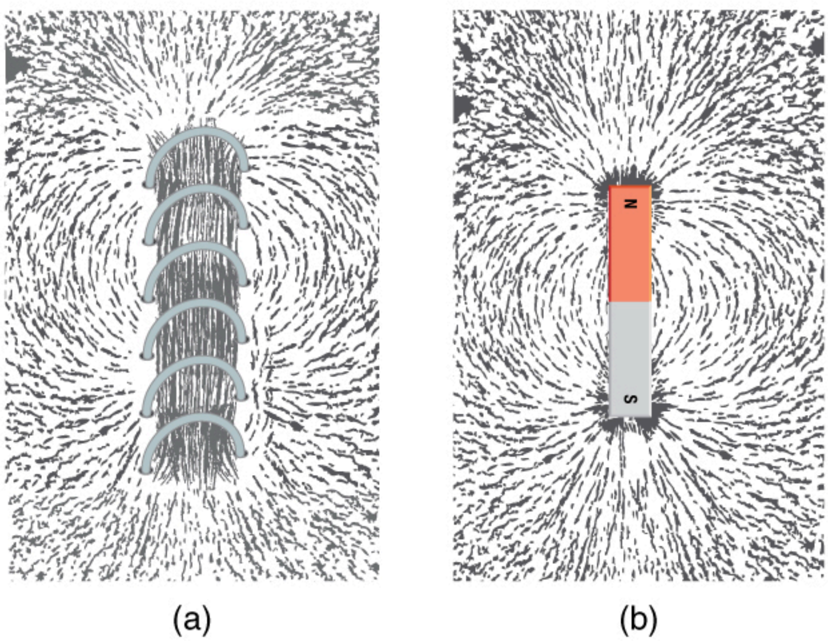
Conversely, a permanent magnet can be demagnetized by hard blows or by heating it in the absence of another magnet. Increased thermal motion at higher temperature can disrupt and randomize the orientation and the size of the domains. There is a well-defined temperature for ferromagnetic materials, which is called the **Curie temperature**, above which they cannot be magnetized. The Curie temperature for iron is 1043 K (770°C) , which is well above room temperature. There are several elements and alloys that have Curie temperatures much lower than room temperature and are ferromagnetic only below those temperatures.

Electromagnets

Early in the 19th century, it was discovered that electrical currents cause magnetic effects. The first significant observation was by the Danish scientist Hans Christian Oersted (1777–1851), who found that a compass needle was deflected by a current-carrying wire. This was the first significant evidence that the movement of charges had any connection with magnets. **Electromagnetism** is the use of electric current to make magnets. These temporarily induced magnets are called **electromagnets**. Electromagnets are employed for everything from a wrecking yard crane that lifts scrapped cars to controlling the beam of a 90-km-circumference particle accelerator to the magnets in medical imaging machines (See [Figure 3]).

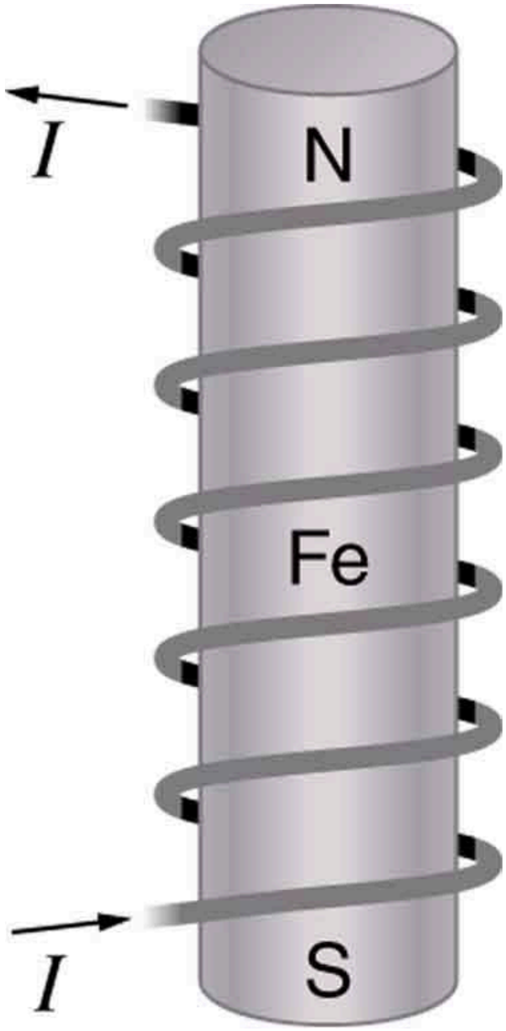
Instrument for magnetic resonance imaging (MRI). The device uses a superconducting cylindrical coil for the main magnetic field. The patient goes into this “tunnel” on the gurney. (credit: Bill McChesney, Flickr)

[Figure 4] shows that the response of iron filings to a current-carrying coil and to a permanent bar magnet. The patterns are similar. In fact, electromagnets and ferromagnets have the same basic characteristics—for example, they have north and south poles that cannot be separated and for which like poles repel and unlike poles attract.



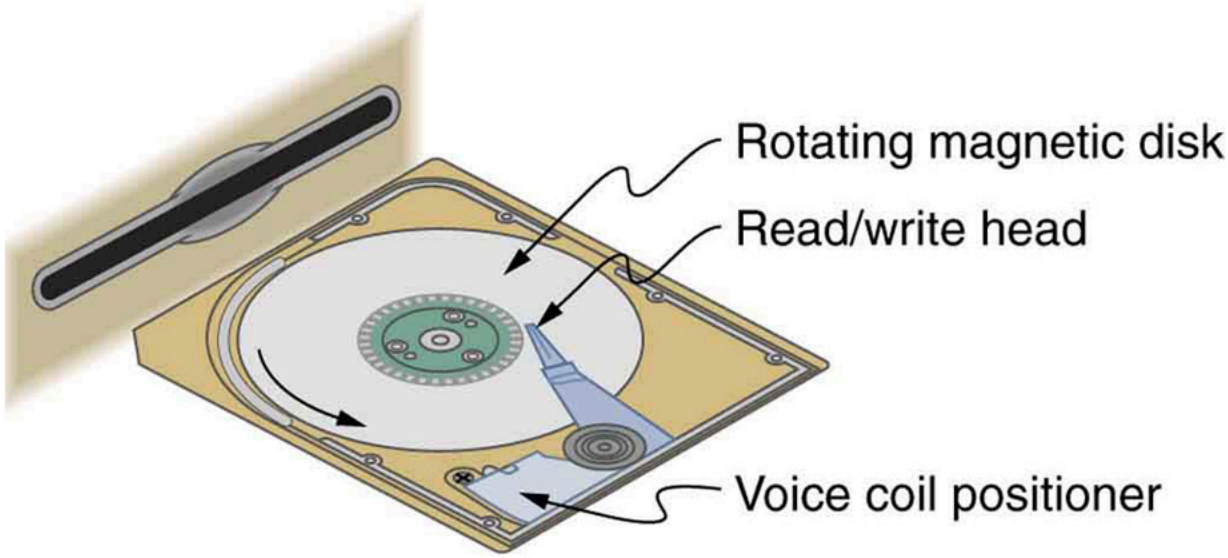
Iron filings near (a) a current-carrying coil and (b) a magnet act like tiny compass needles, showing the shape of their fields. Their response to a current-carrying coil and a permanent magnet is seen to be very similar, especially near the ends of the coil and the magnet.

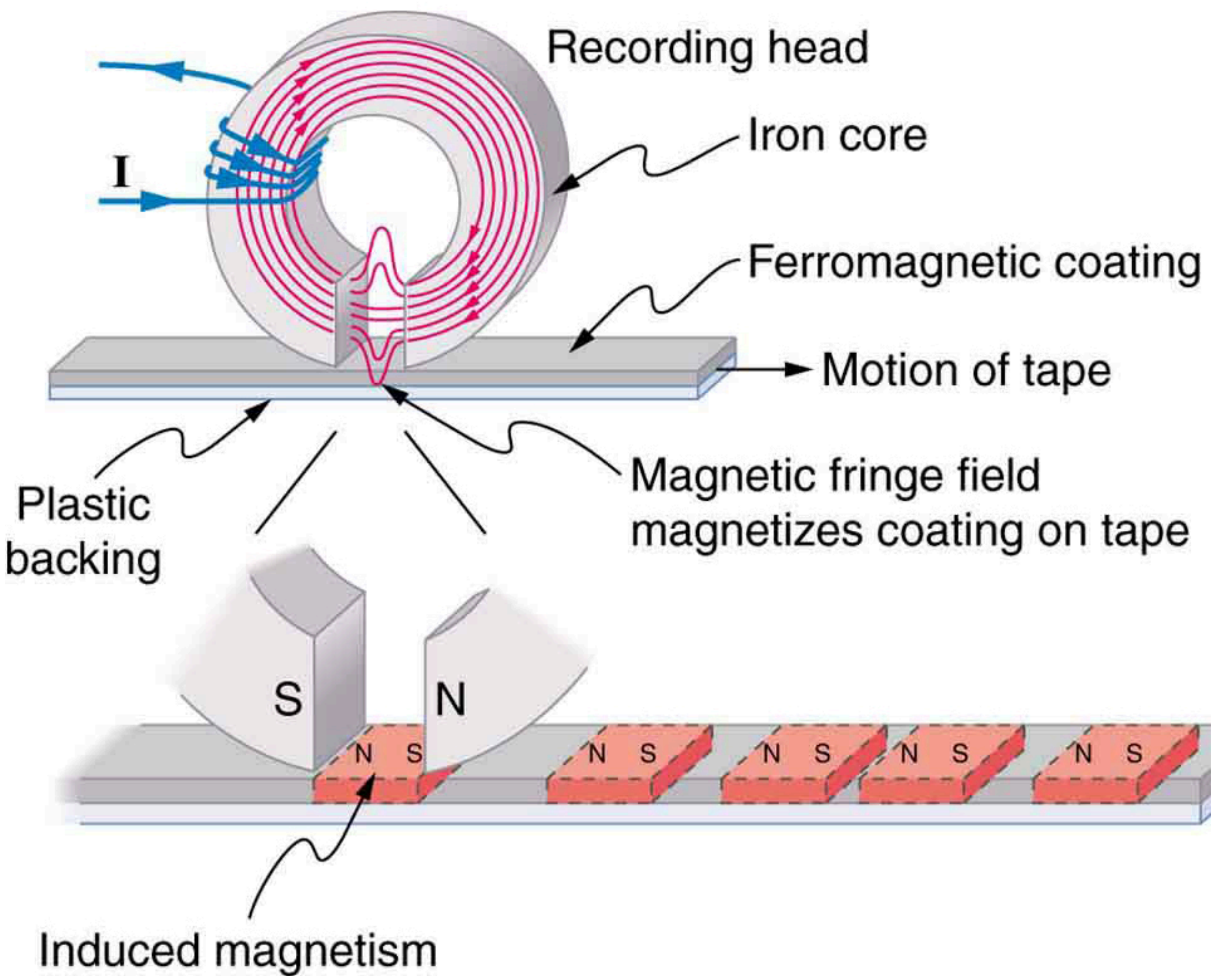
Combining a ferromagnet with an electromagnet can produce particularly strong magnetic effects. (See [Figure 5].) Whenever strong magnetic effects are needed, such as lifting scrap metal, or in particle accelerators, electromagnets are enhanced by ferromagnetic materials. Limits to how strong the magnets can be made are imposed by coil resistance (it will overheat and melt at sufficiently high current), and so superconducting magnets may be employed. These are still limited, because superconducting properties are destroyed by too great a magnetic field.



An electromagnet with a ferromagnetic core can produce very strong magnetic effects. Alignment of domains in the core produces a magnet, the poles of which are aligned with the electromagnet.

[Figure 6] shows a few uses of combinations of electromagnets and ferromagnets. Ferromagnetic materials can act as memory devices, because the orientation of the magnetic fields of small domains can be reversed or erased. Magnetic information storage on videotapes and computer hard drives are among the most common applications. This property is vital in our digital world.





An electromagnet induces regions of permanent magnetism on a floppy disk coated with a ferromagnetic material. The information stored here is digital (a region is either magnetic or not); in other applications, it can be analog (with a varying strength), such as on audiotapes.

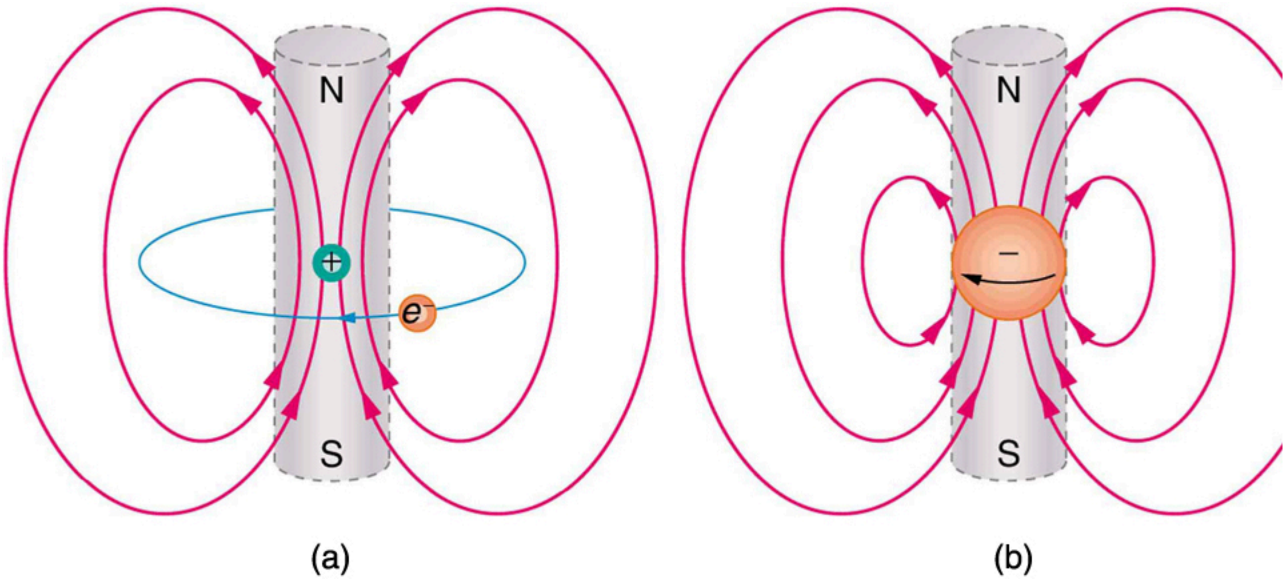
Current: The Source of All Magnetism

An electromagnet creates magnetism with an electric current. In later sections we explore this more quantitatively, finding the strength and direction of magnetic fields created by various currents. But what about ferromagnets? [Figure 7] shows models of how electric currents create magnetism at the submicroscopic level. (Note that we cannot directly observe the paths of individual electrons about atoms, and so a model or visual image, consistent with all direct observations, is made. We can directly observe the electron's orbital angular momentum, its spin momentum, and subsequent magnetic moments, all of which are explained with electric-current-creating subatomic magnetism.) Currents, including those associated with other submicroscopic particles like protons, allow us to explain ferromagnetism and all other magnetic effects. Ferromagnetism, for example, results from an internal cooperative alignment of electron spins, possible in some materials but not in others.

Crucial to the statement that electric current is the source of all magnetism is the fact that it is impossible to separate north and south magnetic poles. (This is far different from the case of positive and negative charges, which are easily separated.) A current loop always produces a magnetic dipole—that is, a magnetic field that acts like a north pole and south pole pair. Since isolated north and south magnetic poles, called **magnetic monopoles**, are not observed, currents are used to explain all magnetic effects. If magnetic monopoles did exist, then we would have to modify this underlying connection that all magnetism is due to electrical current. There is no known reason that magnetic monopoles should not exist—they are simply never observed—and so searches at the subnuclear level continue. If they do *not* exist, we would like to find out why not. If they *do* exist, we would like to see evidence of them.

Electric Currents and Magnetism

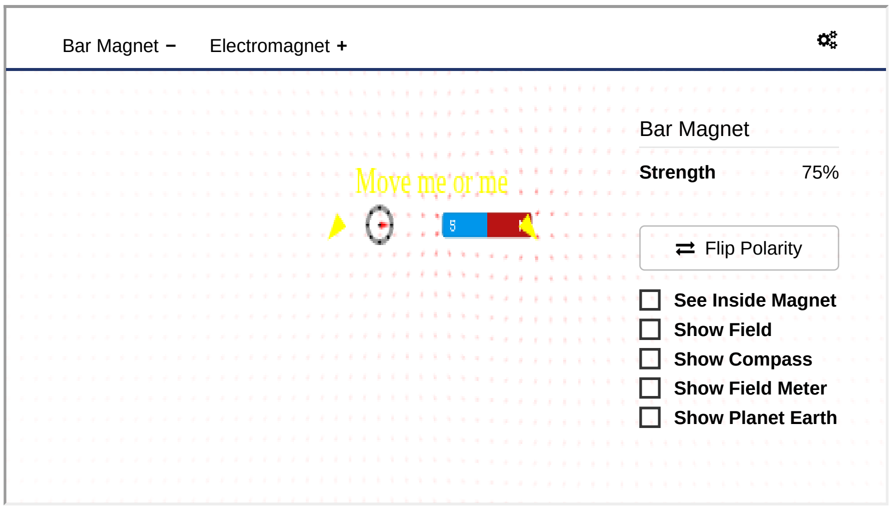
Electric current is the source of all magnetism.



(a) In the planetary model of the atom, an electron orbits a nucleus, forming a closed-current loop and producing a magnetic field with a north pole and a south pole. (b) Electrons have spin and can be crudely pictured as rotating charge, forming a current that produces a magnetic field with a north pole and a south pole. Neither the planetary model nor the image of a spinning electron is completely consistent with modern physics. However, they do provide a useful way of understanding phenomena.

PhET Explorations: Magnets and Electromagnets

Explore the interactions between a compass and bar magnet. Discover how you can use a battery and wire to make a magnet! Can you make it a stronger magnet? Can you make the magnetic field reverse?



Section Summary

- Magnetic poles always occur in pairs of north and south—it is not possible to isolate north and south poles.
- All magnetism is created by electric current.
- Ferromagnetic materials, such as iron, are those that exhibit strong magnetic effects.
- The atoms in ferromagnetic materials act like small magnets (due to currents within the atoms) and can be aligned, usually in millimeter-sized regions called domains.
- Domains can grow and align on a larger scale, producing permanent magnets. Such a material is magnetized, or induced to be magnetic.
- Above a material's Curie temperature, thermal agitation destroys the alignment of atoms, and ferromagnetism disappears.
- Electromagnets employ electric currents to make magnetic fields, often aided by induced fields in ferromagnetic materials.

Glossary

ferromagnetic
: materials, such as iron, cobalt, nickel, and gadolinium, that exhibit strong magnetic effects

magnetized
: to be turned into a magnet; to be induced to be magnetic

domains
: regions within a material that behave like small bar magnets

Curie temperature
: the temperature above which a ferromagnetic material cannot be magnetized

electromagnetism
: the use of electrical currents to induce magnetism

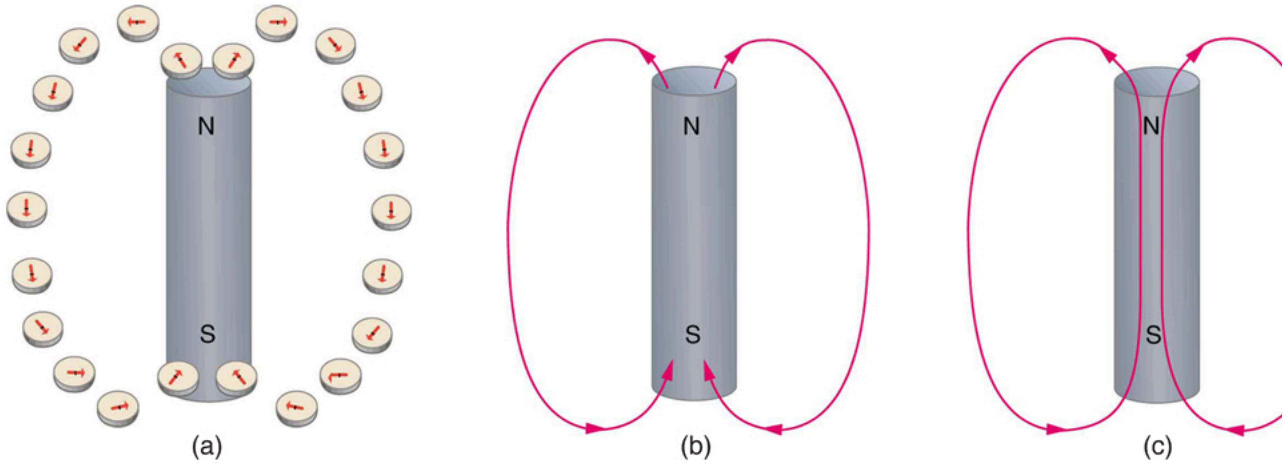
electromagnet
: an object that is temporarily magnetic when an electrical current is passed through it

magnetic monopoles
: an isolated magnetic pole; a south pole without a north pole, or vice versa (no magnetic monopole has ever been observed)

Magnetic Fields and Magnetic Field Lines

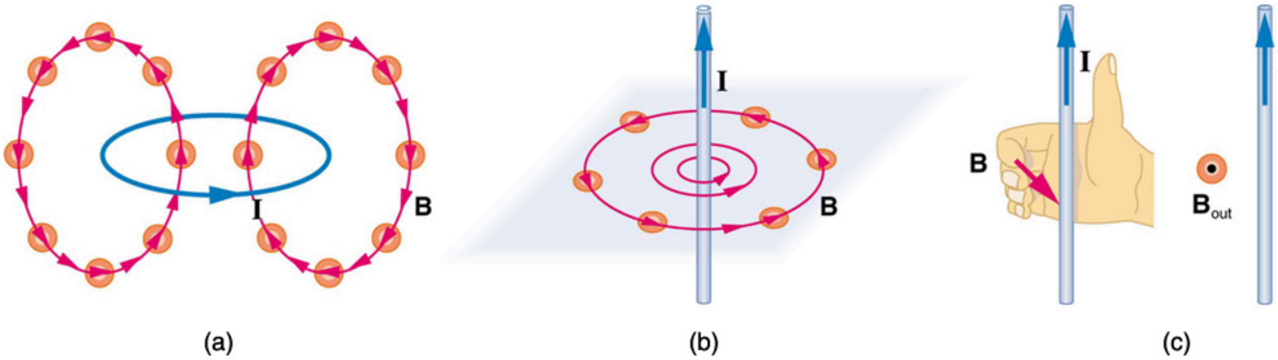
- Define magnetic field and describe the magnetic field lines of various magnetic fields.

Einstein is said to have been fascinated by a compass as a child, perhaps musing on how the needle felt a force without direct physical contact. His ability to think deeply and clearly about action at a distance, particularly for gravitational, electric, and magnetic forces, later enabled him to create his revolutionary theory of relativity. Since magnetic forces act at a distance, we define a **magnetic field** to represent magnetic forces. The pictorial representation of **magnetic field lines** is very useful in visualizing the strength and direction of the magnetic field. As shown in [Figure 1] , the **direction of magnetic field lines** is defined to be the direction in which the north end of a compass needle points. The magnetic field is traditionally called the B-field.



Magnetic field lines are defined to have the direction that a small compass points when placed at a location. (a) If small compasses are used to map the magnetic field around a bar magnet, they will point in the directions shown: away from the north pole of the magnet, toward the south pole of the magnet. (Recall that the Earth's north magnetic pole is really a south pole in terms of definitions of poles on a bar magnet.) (b) Connecting the arrows gives continuous magnetic field lines. The strength of the field is proportional to the closeness (or density) of the lines. (c) If the interior of the magnet could be probed, the field lines would be found to form continuous closed loops.

Small compasses used to test a magnetic field will not disturb it. (This is analogous to the way we tested electric fields with a small test charge. In both cases, the fields represent only the object creating them and not the probe testing them.) [Figure 2] shows how the magnetic field appears for a current loop and a long straight wire, as could be explored with small compasses. A small compass placed in these fields will align itself parallel to the field line at its location, with its north pole pointing in the direction of *B*. Note the symbols used for field into and out of the paper.



Small compasses could be used to map the fields shown here. (a) The magnetic field of a circular current loop is similar to that of a bar magnet. (b) A long and straight wire creates a field with magnetic field lines forming circular loops. (c) When the wire is in the plane of the paper, the field is perpendicular to the paper. Note that the symbols used for the field pointing inward (like the tail of an arrow) and the field pointing outward (like the tip of an arrow).

Making Connections: Concept of a Field

A field is a way of mapping forces surrounding any object that can act on another object at a distance without apparent physical connection. The field represents the object generating it. Gravitational fields map gravitational forces, electric fields map electrical forces, and magnetic fields map magnetic forces.

Extensive exploration of magnetic fields has revealed a number of hard-and-fast rules. We use magnetic field lines to represent the field (the lines are a pictorial tool, not a physical entity in and of themselves). The properties of magnetic field lines can be summarized by these rules:

1. The direction of the magnetic field is tangent to the field line at any point in space. A small compass will point in the direction of the field line.
2. The strength of the field is proportional to the closeness of the lines. It is exactly proportional to the number of lines per unit area perpendicular to the lines (called the areal density).

3. Magnetic field lines can never cross, meaning that the field is unique at any point in space.
4. Magnetic field lines are continuous, forming closed loops without beginning or end. They go from the north pole to the south pole.

The last property is related to the fact that the north and south poles cannot be separated. It is a distinct difference from electric field lines, which begin and end on the positive and negative charges. If magnetic monopoles existed, then magnetic field lines would begin and end on them.

Section Summary

- Magnetic fields can be pictorially represented by magnetic field lines, the properties of which are as follows:

1. The field is tangent to the magnetic field line.
2. Field strength is proportional to the line density.
3. Field lines cannot cross.
4. Field lines are continuous loops.

Conceptual Questions

Explain why the magnetic field would not be unique (that is, not have a single value) at a point in space where magnetic field lines might cross. (Consider the direction of the field at such a point.)

List the ways in which magnetic field lines and electric field lines are similar. For example, the field direction is tangent to the line at any point in space. Also list the ways in which they differ. For example, electric force is parallel to electric field lines, whereas magnetic force on moving charges is perpendicular to magnetic field lines.

Noting that the magnetic field lines of a bar magnet resemble the electric field lines of a pair of equal and opposite charges, do you expect the magnetic field to rapidly decrease in strength with distance from the magnet? Is this consistent with your experience with magnets?

Is the Earth's magnetic field parallel to the ground at all locations? If not, where is it parallel to the surface? Is its strength the same at all locations? If not, where is it greatest?

Glossary

magnetic field
: the representation of magnetic forces

B-field
: another term for magnetic field

magnetic field lines
: the pictorial representation of the strength and the direction of a magnetic field

direction of magnetic field lines
: the direction that the north end of a compass needle points

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Magnetic Field Strength: Force on a Moving Charge in a Magnetic Field

- Describe the effects of magnetic fields on moving charges.
- Use the right hand rule 1 to determine the velocity of a charge, the direction of the magnetic field, and the direction of the magnetic force on a moving charge.
- Calculate the magnetic force on a moving charge.

What is the mechanism by which one magnet exerts a force on another? The answer is related to the fact that all magnetism is caused by current, the flow of charge. *Magnetic fields exert forces on moving charges*, and so they exert forces on other magnets, all of which have moving charges.

Right Hand Rule 1

The magnetic force on a moving charge is one of the most fundamental known. Magnetic force is as important as the electrostatic or Coulomb force. Yet the magnetic force is more complex, in both the number of factors that affects it and in its direction, than the relatively simple Coulomb force. The magnitude of the **magnetic force** $|\vec{\mathbf{F}}|$ on a charge q moving at a velocity $\vec{\mathbf{v}}$ in a magnetic field of strength $\vec{\mathbf{B}}$ is given by

$$|\vec{\mathbf{F}}| = q|\vec{\mathbf{v}}||\vec{\mathbf{B}}|\sin\theta,$$

where θ is the angle between the directions of $\vec{\mathbf{v}}$ and $\vec{\mathbf{B}}$. This force is often called the **Lorentz force**. In fact, this is how we define the magnetic field strength $|\vec{\mathbf{B}}|$ —in terms of the force on a charged particle moving in a magnetic field. The SI unit for magnetic field strength B is called the **tesla** (T) after the eccentric but brilliant inventor Nikola Tesla (1856–1943). To determine how the tesla relates to other SI units, we solve $|\vec{\mathbf{F}}| = q|\vec{\mathbf{v}}||\vec{\mathbf{B}}|\sin\theta$ for $|\vec{\mathbf{B}}|$.

$$|\vec{\mathbf{B}}| = \frac{|\vec{\mathbf{F}}|}{q|\vec{\mathbf{v}}|\sin\theta}$$

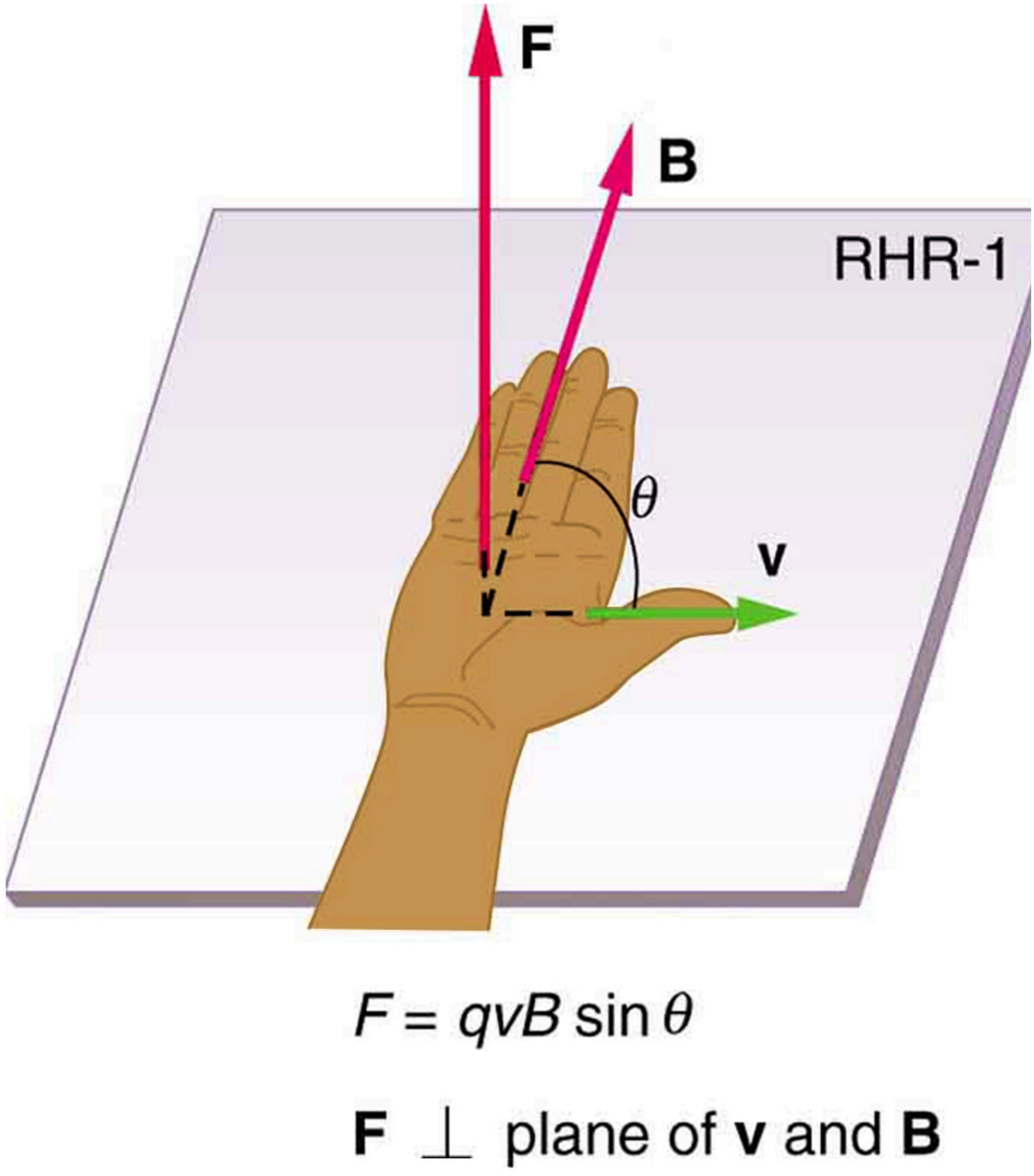
Because $\sin\theta$ is unitless, the tesla is

$$1\,\text{T} = \frac{1\,\text{N}}{\text{C}\cdot\text{m/s}} = \frac{1\,\text{N}}{\text{A}\cdot\text{m}}$$

(note that C/s = A).

Another smaller unit, called the **gauss** (G), where $1\,G = 10^{-4}\,\text{T}$, is sometimes used. The strongest permanent magnets have fields near 2 T; superconducting electromagnets may attain 10 T or more. The Earth’s magnetic field on its surface is only about $5\times10^{-5}\,\text{T}$, or 0.5 G.

The *direction* of the magnetic force $\vec{\mathbf{F}}$ is perpendicular to the plane formed by $\vec{\mathbf{v}}$ and $\vec{\mathbf{B}}$, as determined by the **right hand rule 1** (or RHR-1), which is illustrated in [Figure 1]. RHR-1 states that, to determine the direction of the magnetic force on a positive moving charge, you point the thumb of the right hand in the direction of $\vec{\mathbf{v}}$, the fingers in the direction of $\vec{\mathbf{B}}$, and a perpendicular to the palm points in the direction of $\vec{\mathbf{F}}$. One way to remember this is that there is one velocity, and so the thumb represents it. There are many field lines, and so the fingers represent them. The force is in the direction you would push with your palm. The force on a negative charge is in exactly the opposite direction to that on a positive charge.



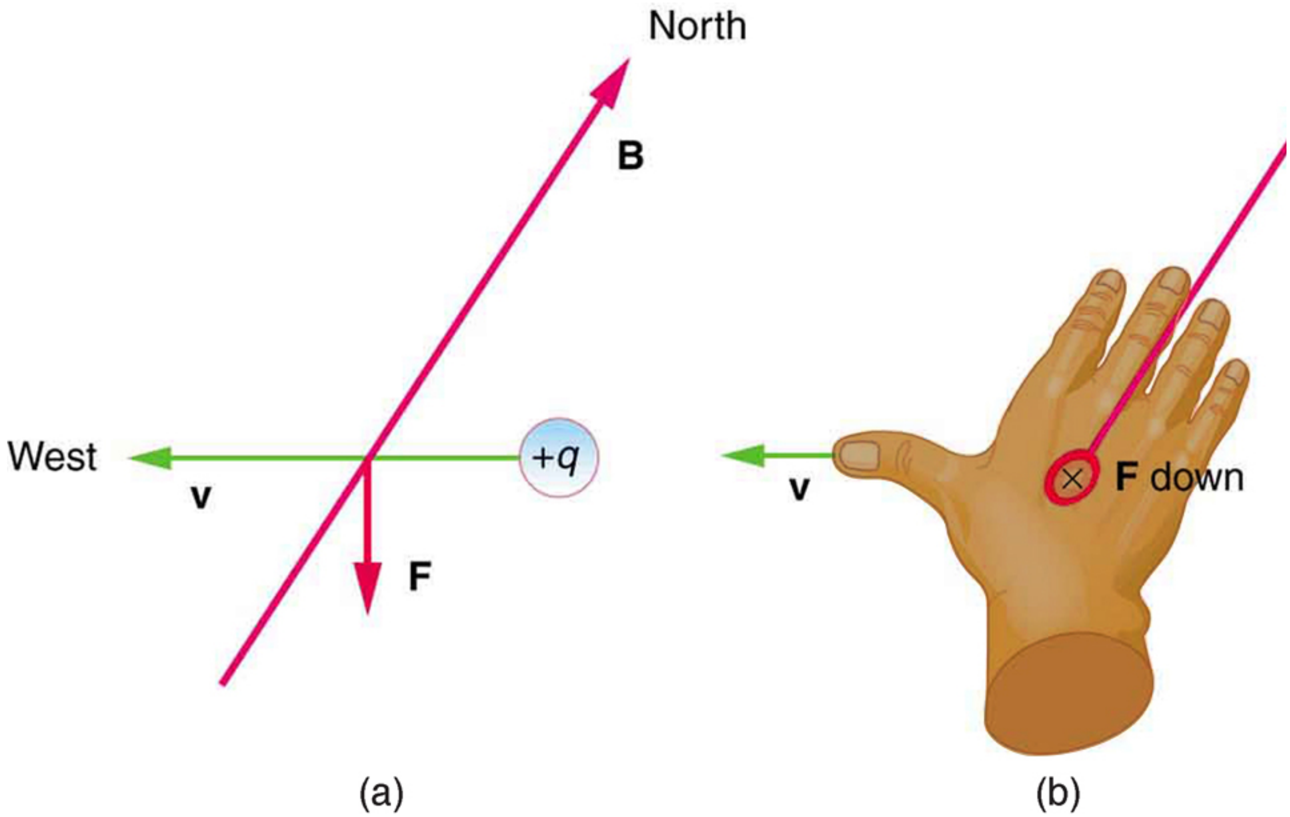
Magnetic fields exert forces on moving charges. This force is one of the most basic known. The direction of the magnetic force on a moving charge is perpendicular to the plane formed by v and B and follows right hand rule–1 (RHR-1) as shown. The magnitude of the force is proportional to q , v , B , and the sine of the angle between v and B .

Making Connections: Charges and Magnets

There is no magnetic force on static charges. However, there is a magnetic force on moving charges. When charges are stationary, their electric fields do not affect magnets. But, when charges move, they produce magnetic fields that exert forces on other magnets. When there is relative motion, a connection between electric and magnetic fields emerges—each affects the other.

Calculating Magnetic Force: Earth’s Magnetic Field on a Charged Glass Rod

With the exception of compasses, you seldom see or personally experience forces due to the Earth’s small magnetic field. To illustrate this, suppose that in a physics lab you rub a glass rod with silk, placing a 20-nC positive charge on it. Calculate the force on the rod due to the Earth’s magnetic field, if you throw it with a horizontal velocity of 10 m/s due west in a place where the Earth's field is due north parallel to the ground. (The direction of the force is determined with right hand rule 1 as shown in [Figure 2].)



A positively charged object moving due west in a region where the Earth's magnetic field is due north experiences a force that is straight down as shown. A negative charge moving in the same direction would feel a force straight up.

Strategy

We are given the charge, its velocity, and the magnetic field strength and direction. We can thus use the equation $F = qvB\sin\theta$ to find the force.

Solution

The magnetic force is

$$F=qvB\sin\theta.$$

We see that $\sin\theta = 1$, since the angle between the velocity and the direction of the field is 90° . Entering the other given quantities yields

$$F = (20\times10^{-9}\text{C})(10\text{m/s})(5\times10^{-5}\text{T}) = 1\times10^{-11}(\text{C}\cdot\text{m/s})(N\,\text{C}\cdot\text{m/s})=1\times10^{-11}\text{N}.$$

Discussion

This force is completely negligible on any macroscopic object, consistent with experience. (It is calculated to only one digit, since the Earth's field varies with location and is given to only one digit.) The Earth's magnetic field, however, does produce very important effects, particularly on submicroscopic particles. Some of these are explored in Force on a Moving Charge in a Magnetic Field: Examples and Applications .

Section Summary

- Magnetic fields exert a force on a moving charge *q*, the magnitude of which is
  $$F=qvB\sin\theta,$$
  where θ is the angle between the directions of v and B .
- The SI unit for magnetic field strength B is the tesla (T), which is related to other units by
  $$1\text{T}=1\text{NC}\cdot\text{m/s}=1\text{NA}\cdot\text{m}.$$
- The *direction* of the force on a moving charge is given by right hand rule 1 (RHR-1): Point the thumb of the right hand in the direction of $\vec{\mathbf{v}}$, the fingers in the direction of $\vec{\mathbf{B}}$, and a perpendicular to the palm points in the direction of $\vec{\mathbf{F}}$.

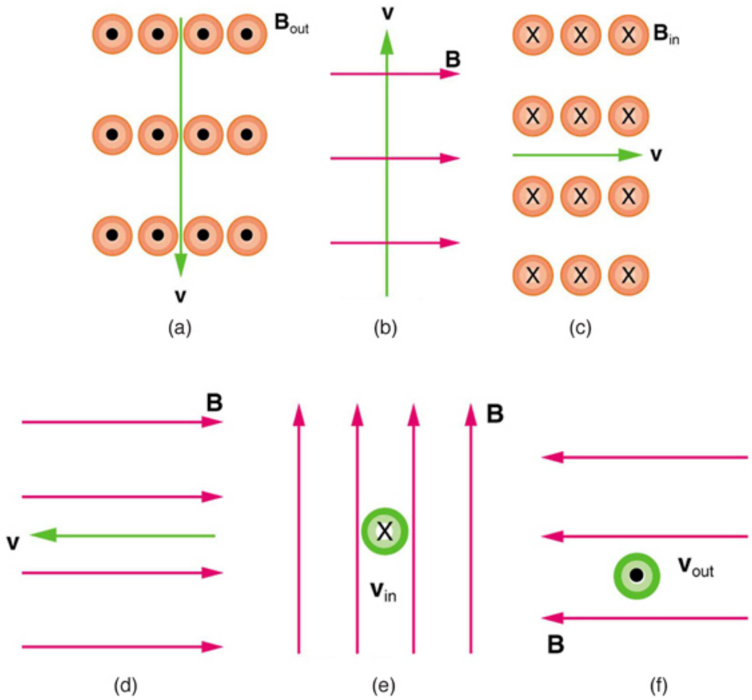
- The force is perpendicular to the plane formed by $\vec{\mathbf{v}}$ and $\vec{\mathbf{B}}$. Since the force is zero if $\vec{\mathbf{v}}$ is parallel to $\vec{\mathbf{B}}$, charged particles often follow magnetic field lines rather than cross them.

Conceptual Questions

If a charged particle moves in a straight line through some region of space, can you say that the magnetic field in that region is necessarily zero?

Problems & Exercises

What is the direction of the magnetic force on a positive charge that moves as shown in each of the six cases shown in [Figure 3]?



Show Solution

Strategy

We use the right hand rule 1 (RHR-1) to determine the direction of the magnetic force on a positive moving charge. Point the thumb in the direction of velocity $\vec{\mathbf{v}}$, the fingers in the direction of the magnetic field $\vec{\mathbf{B}}$, and the palm pushes in the direction of the force $\vec{\mathbf{F}}$. We apply this to each scenario, noting the orientations of $\vec{\mathbf{v}}$ and $\vec{\mathbf{B}}$ given in the figure.

Solution

(a) Velocity $\vec{\mathbf{v}}$ points down, $\vec{\mathbf{B}}$ points out of the page. Using RHR-1: thumb down, fingers out of page → palm pushes **left (West)**.

(b) Velocity $\vec{\mathbf{v}}$ points up, $\vec{\mathbf{B}}$ points right. Using RHR-1: thumb up, fingers right → palm pushes **into the page**.

(c) Velocity $\vec{\mathbf{v}}$ points right, $\vec{\mathbf{B}}$ points into the page. Using RHR-1: thumb right, fingers into page → palm pushes **up (North)**.

(d) Velocity $\vec{\mathbf{v}}$ points left, $\vec{\mathbf{B}}$ points right. The velocity is antiparallel to the magnetic field, so $\theta = 180°$ and $\sin\theta = 0$. Therefore, there is **no force**.

(e) Velocity $\vec{\mathbf{v}}$ points into the page, $\vec{\mathbf{B}}$ points up. Using RHR-1: thumb into page, fingers up → palm pushes **right (East)**.

(f) Velocity $\vec{\mathbf{v}}$ points out of the page, $\vec{\mathbf{B}}$ points left. Using RHR-1: thumb out of page, fingers left → palm pushes **down (South)**.

Discussion

The magnetic force is always perpendicular to both the velocity and the magnetic field. When $\vec{\mathbf{v}}$ and $\vec{\mathbf{B}}$ are parallel or antiparallel (as in case d), the cross product is zero and there is no magnetic force. This is why charged particles can travel along magnetic field lines without being deflected—only the component of velocity perpendicular to the field produces a force.

(a) Left (West); (b) Into the page; (c) Up (North); (d) No force; (e) Right (East); (f) Down (South).

Repeat the above [Exercise] for a negative charge.

Show Solution

Strategy

For a negative charge, the magnetic force is in the opposite direction compared to a positive charge. We can either apply RHR-1 and then reverse the direction, or use a “left hand rule” where the thumb points in the direction of velocity, fingers point in the direction of $\vec{\mathbf{B}}$, and the palm pushes in the direction of force on the negative charge.

Solution

Since the force on a negative charge is opposite to that on a positive charge in the same situation, we simply reverse each answer from Exercise 1:

(a) Force on positive charge was left → Force on negative charge is **right (East)**.

(b) Force on positive charge was into page → Force on negative charge is **out of the page**.

(c) Force on positive charge was up → Force on negative charge is **down (South)**.

(d) There was no force on the positive charge → There is still **no force** on the negative charge (since $\sin\theta = 0$).

(e) Force on positive charge was right → Force on negative charge is **left (West)**.

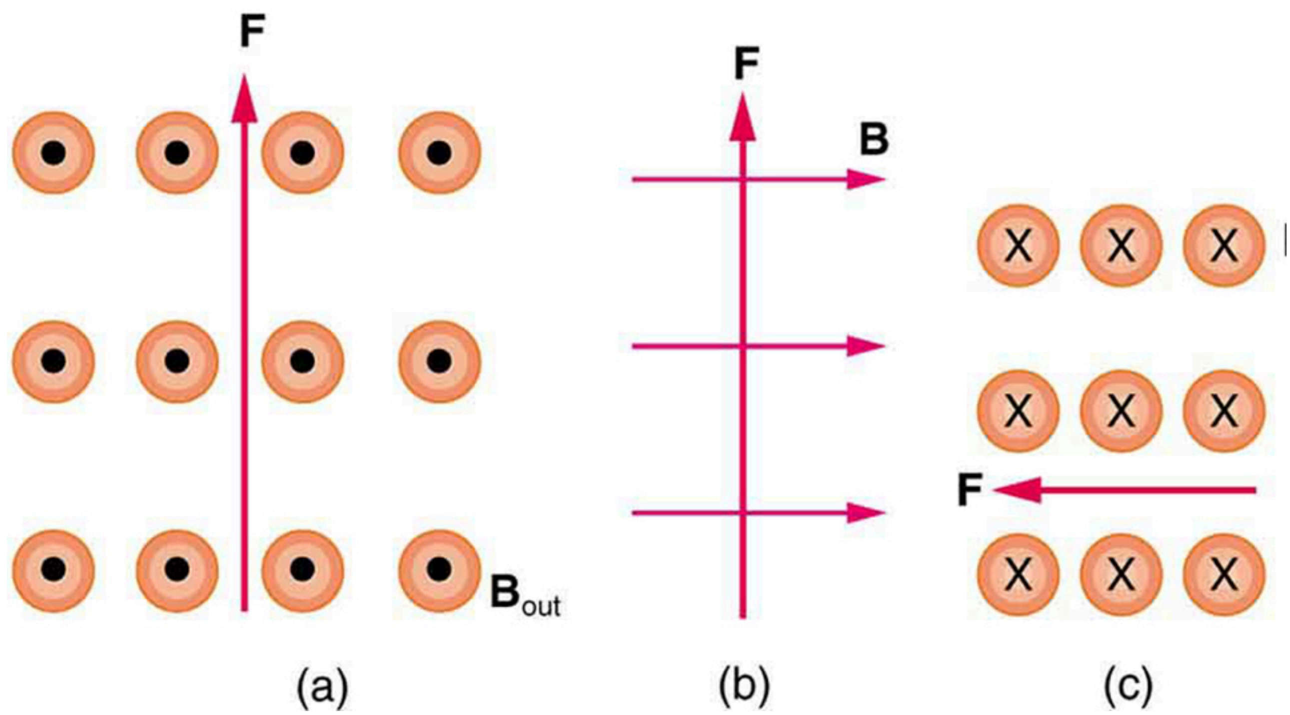
(f) Force on positive charge was down → Force on negative charge is **up (North)**.

Discussion

The reversal of force direction for negative charges is fundamental to understanding phenomena like the Hall effect, where electrons and positive charge carriers deflect in opposite directions in a magnetic field. This difference allows us to determine the sign of charge carriers in conductors. Note that in case (d), the zero force remains zero regardless of charge sign because it depends on $\sin\theta$, not on the charge sign.

(a) Right (East); (b) Out of the page; (c) Down (South); (d) No force; (e) Left (West); (f) Up (North).

What is the direction of the velocity of a negative charge that experiences the magnetic force shown in each of the three cases in [Figure 3], assuming it moves perpendicular to $\vec{\mathbf{B}}$?



Show Solution

Strategy

For a negative charge, the force is opposite to what RHR-1 predicts. We can work backwards: if we know $\vec{\mathbf{F}}$ and $\vec{\mathbf{B}}$, we can find $\vec{\mathbf{v}}$ by reversing the logic. For a negative charge, we point fingers in the direction of $\vec{\mathbf{B}}$, and the palm faces opposite to $\vec{\mathbf{F}}$ (since force on negative charge is reversed). The thumb then points in the direction of $\vec{\mathbf{v}}$.

Solution

(a) Force $\vec{\mathbf{F}}$ points up, $\vec{\mathbf{B}}$ points out of the page. For a negative charge, the actual magnetic force equation gives $\vec{\mathbf{F}}$ opposite to $\vec{\mathbf{v}} \times \vec{\mathbf{B}}$. Working backwards with $\vec{\mathbf{B}}$ out of page and needing force up (opposite to $\vec{\mathbf{v}} \times \vec{\mathbf{B}}$ pointing down), we need $\vec{\mathbf{v}} \times \vec{\mathbf{B}}$ pointing down. With $\vec{\mathbf{B}}$ out of page, velocity must point **East (right)**.

(b) Force $\vec{\mathbf{F}}$ points up, $\vec{\mathbf{B}}$ points right. For the negative charge to experience an upward force, $\vec{\mathbf{v}} \times \vec{\mathbf{B}}$ must point down. With $\vec{\mathbf{B}}$ pointing right, velocity must be **into the page**.

(c) Force $\vec{\mathbf{F}}$ points left, $\vec{\mathbf{B}}$ points into the page. For the negative charge to experience a leftward force, $\vec{\mathbf{v}} \times \vec{\mathbf{B}}$ must point right. With $\vec{\mathbf{B}}$ into the page, velocity must point **South (down)**.

Discussion

Working backwards from force to velocity requires careful application of the right hand rule in reverse. Remember that for negative charges, the actual force is opposite to $\vec{\mathbf{v}} \times \vec{\mathbf{B}}$. This inverse problem-solving skill is useful in analyzing particle trajectories in bubble chambers and mass spectrometers, where we observe the curved paths and must deduce properties of the particles.

(a) East (right); (b) Into the page; (c) South (down).

Repeat the above [Exercise] for a positive charge.

Show Solution

Strategy

For a positive charge, the force direction follows directly from RHR-1: $\vec{\mathbf{F}} = q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$. Working backwards, given $\vec{\mathbf{F}}$ and $\vec{\mathbf{B}}$, we find $\vec{\mathbf{v}}$ by requiring that the cross product $\vec{\mathbf{v}} \times \vec{\mathbf{B}}$ points in the same direction as $\vec{\mathbf{F}}$.

Solution

Since positive charges experience force in the direction of $\vec{\mathbf{v}} \times \vec{\mathbf{B}}$, and negative charges experience force opposite to $\vec{\mathbf{v}} \times \vec{\mathbf{B}}$, the velocity for a positive charge must be opposite to that found for a negative charge with the same force direction:

(a) Force $\vec{\mathbf{F}}$ points up, $\vec{\mathbf{B}}$ points out of the page. For $\vec{\mathbf{v}} \times \vec{\mathbf{B}}$ to point up with $\vec{\mathbf{B}}$ out of page, velocity must point **West (left)**.

(b) Force $\vec{\mathbf{F}}$ points up, $\vec{\mathbf{B}}$ points right. For $\vec{\mathbf{v}} \times \vec{\mathbf{B}}$ to point up with $\vec{\mathbf{B}}$ right, velocity must be **out of the page**.

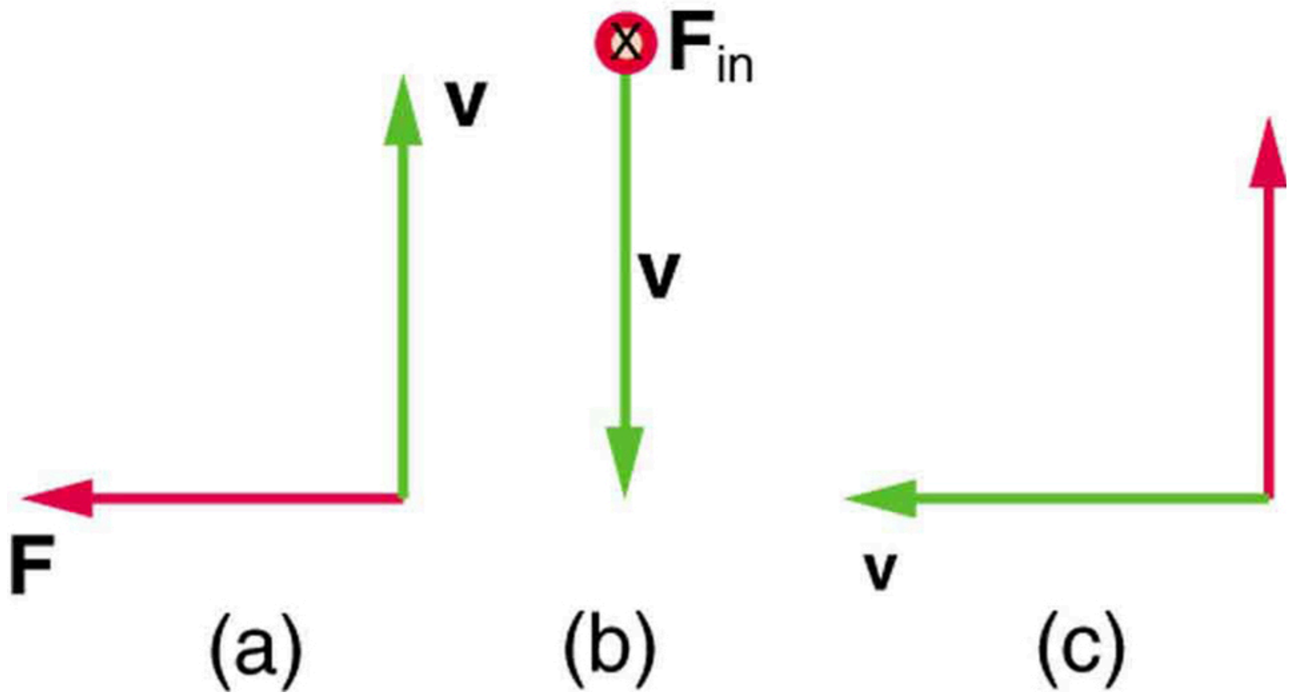
(c) Force $\vec{\mathbf{F}}$ points left, $\vec{\mathbf{B}}$ points into the page. For $\vec{\mathbf{v}} \times \vec{\mathbf{B}}$ to point left with $\vec{\mathbf{B}}$ into page, velocity must point **North (up)**.

Discussion

Comparing the answers for positive and negative charges (Exercise 3 vs. Exercise 4), we see that they move in exactly opposite directions to produce the same force direction. This is a direct consequence of the sign difference in the Lorentz force law. In particle physics experiments, observing the direction of deflection in a known magnetic field immediately tells us the sign of the charge.

(a) West (left); (b) Out of the page; (c) North (up).

What is the direction of the magnetic field that produces the magnetic force on a positive charge as shown in each of the three cases in the figure below, assuming $\vec{\mathbf{B}}$ is perpendicular to $\vec{\mathbf{v}}$?



Show Solution

Strategy

For a positive charge, $\vec{\mathbf{F}} = q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$, and the force is in the direction of the cross product. Given $\vec{\mathbf{F}}$ and $\vec{\mathbf{v}}$, we need to find $\vec{\mathbf{B}}$ such that $\vec{\mathbf{v}} \times \vec{\mathbf{B}}$ points in the direction of $\vec{\mathbf{F}}$. Using RHR-1 in reverse: with thumb along $\vec{\mathbf{v}}$ and palm facing $\vec{\mathbf{F}}$, fingers point in the direction of $\vec{\mathbf{B}}$.

Solution

(a) Force $\vec{\mathbf{F}}$ points left, velocity $\vec{\mathbf{v}}$ points up. Using RHR-1: thumb up ($\vec{\mathbf{v}}$), palm faces left ($\vec{\mathbf{F}}$), so fingers point **into the page**. Thus $\vec{\mathbf{B}}$ is into the page.

(b) Force $\vec{\mathbf{F}}$ points into the page, velocity $\vec{\mathbf{v}}$ points down. Using RHR-1: thumb down ($\vec{\mathbf{v}}$), palm faces into page ($\vec{\mathbf{F}}$), so fingers point **West (left)**. Thus $\vec{\mathbf{B}}$ is to the left.

(c) Force $\vec{\mathbf{F}}$ points up, velocity $\vec{\mathbf{v}}$ points left. Using RHR-1: thumb left ($\vec{\mathbf{v}}$), palm faces up ($\vec{\mathbf{F}}$), so fingers point **out of the page**. Thus $\vec{\mathbf{B}}$ is out of the page.

Discussion

This type of problem—finding the magnetic field direction from known force and velocity—is important for understanding how magnetic field measurements are made. By observing how charged particles deflect in an unknown field, we can map out the field direction. This principle underlies how cosmic ray detectors and magnetic field probes work.

(a) Into the page; (b) West (left); (c) Out of the page.

Repeat the above [Exercise] for a negative charge.

Show Solution

Strategy

For a negative charge, the force is opposite to $\vec{\mathbf{v}} \times \vec{\mathbf{B}}$. So if we observe a force $\vec{\mathbf{F}}$ on a negative charge, then $\vec{\mathbf{v}} \times \vec{\mathbf{B}}$ points in the direction of $-\vec{\mathbf{F}}$. We use RHR-1 with the force direction reversed.

Solution

For each case, the magnetic field direction for a negative charge will be opposite to that for a positive charge experiencing the same force:

(a) Force $\vec{\mathbf{F}}$ points left, velocity $\vec{\mathbf{v}}$ points up. For a negative charge, $\vec{\mathbf{v}} \times \vec{\mathbf{B}}$ must point right (opposite to $\vec{\mathbf{F}}$). With thumb up and palm facing right, fingers point **out of the page**.

(b) Force $\vec{\mathbf{F}}$ points into the page, velocity $\vec{\mathbf{v}}$ points down. For a negative charge, $\vec{\mathbf{v}} \times \vec{\mathbf{B}}$ must point out of the page. With thumb down and palm facing out of page, fingers point **East (right)**.

(c) Force $\vec{\mathbf{F}}$ points up, velocity $\vec{\mathbf{v}}$ points left. For a negative charge, $\vec{\mathbf{v}} \times \vec{\mathbf{B}}$ must point down. With thumb left and palm facing down, fingers point **into the page**.

Discussion

The reversal of $\vec{\mathbf{B}}$ direction for opposite charge signs (with the same $\vec{\mathbf{F}}$ and $\vec{\mathbf{v}}$) is a mathematical consequence of the Lorentz force law. In practice, if you don't know the sign of the charge, you cannot uniquely determine $\vec{\mathbf{B}}$ from a single observation of force and velocity—you need additional information about the particle.

(a) Out of the page; (b) East (right); (c) Into the page.

What is the maximum force on an aluminum rod with a $0.100\text{-}\mu\text{C}$ charge that you pass between the poles of a 1.50-T permanent magnet at a speed of 5.00 m/s? In what direction is the force?

Show Solution

Strategy

The magnetic force on a moving charge is given by $F = qvB\sin\theta$. The maximum force occurs when the velocity is perpendicular to the magnetic field ($\theta = 90\,°$, so $\sin\theta = 1$). The direction of the force is perpendicular to both $\vec{\mathbf{v}}$ and $\vec{\mathbf{B}}$, as determined by RHR-1.

Solution

Known quantities:

- Charge: $q = 0.100\ \mu\text{C} = 0.100 \times 10^{-6}\ \text{C} = 1.00 \times 10^{-7}\ \text{C}$
- Speed: $v = 5.00\ \text{m/s}$
- Magnetic field strength: $B = 1.50\ \text{T}$

Unknown: Maximum force F_{max}

The maximum force occurs when $\sin\theta = 1$:

$$F_{\text{max}} = qvB$$
$$F_{\text{max}} = (1.00\times10^{-7}\ \text{C})(5.00\ \text{m/s})(1.50\ \text{T})$$
$$F_{\text{max}} = 7.50\times10^{-7}\ \text{N}$$

The direction is perpendicular to both the velocity and the magnetic field, as determined by RHR-1.

Discussion

This force of about 0.75 μN is extremely small—far too weak to be felt or to have any practical effect on the aluminum rod. For comparison, the weight of a single grain of sand is about 10^{-5} N, roughly 10 times larger than this magnetic force. However, this small force is detectable with sensitive instruments and becomes significant when dealing with very strong magnets or highly charged objects moving at high speeds.

The maximum force is 7.50×10^{-7} N, directed perpendicular to both the magnetic field lines and the velocity.

(a) Aircraft sometimes acquire small static charges. Suppose a supersonic jet has a $0.500\text{-}\mu\text{C}$ charge and flies due west at a speed of 660 m/s over the Earth's south magnetic pole, where the $8.00 \times 10^{-5}\text{-T}$ magnetic field points straight up. What are the direction and the magnitude of the magnetic force on the plane? (b) Discuss whether the value obtained in part (a) implies this is a significant or negligible effect.

Show Solution

Strategy

We use the Lorentz force equation $F = qvB\sin\theta$ to find the magnitude. Since the plane flies due west (horizontal) and the magnetic field points straight up (vertical), the angle between them is 90°. The direction is found using RHR-1: thumb points west (velocity), fingers point up ($\vec{\mathbf{B}}$), so the palm—and thus the force—points in a specific direction.

Solution

Known quantities:

- Charge: $q = 0.500\ \mu\text{C} = 5.00 \times 10^{-7}\ \text{C}$
- Speed: $v = 660\ \text{m/s}$
- Magnetic field: $B = 8.00 \times 10^{-5}\ \text{T}$
- Angle: $\theta = 90\,°$ (velocity horizontal, field vertical)

(a) Magnitude and direction:

$$F = qvB\sin\theta = qvB\sin(90°) = qvB$$
$$F = (5.00\times10^{-7}\ \text{C})(660\ \text{m/s})(8.00\times10^{-5}\ \text{T})$$
$$F = 2.64\times10^{-8}\ \text{N}$$

For the direction, using RHR-1 with thumb pointing west and fingers pointing up: the palm pushes toward the **south**. (Alternatively: velocity west × field up = force south.)

(b) Significance:

This force of about 26 nN (nanonewtons) is incredibly small. To put it in perspective:

- A typical supersonic jet has a mass of about 20,000 kg
- Its weight is about 2×10^{5} N
- The magnetic force is about 10^{13} times smaller than the weight

This force is completely negligible for any practical purpose. Even over the course of a long flight, the impulse from this force would produce an imperceptible change in momentum. The magnetic effect of Earth's field on charged aircraft is not something pilots or engineers need to consider.

Discussion

While Earth's magnetic field is crucial for navigation (compasses) and protects us from solar radiation, its direct magnetic force on slowly moving charged objects is negligible. The situation would be different for very fast-moving particles (like cosmic rays at near-light speeds) or particles in very strong artificial magnetic fields (like those in particle accelerators).

(a) The magnetic force is 2.64×10^{-8} N directed toward the south.

(b) This force is completely negligible—it is about 10^{13} times smaller than the aircraft's weight and has no practical effect on the plane's motion.

(a) A cosmic ray proton moving toward the Earth at $5.00\times10^{7}\text{m/s}$ experiences a magnetic force of $1.70\times10^{-16}\text{N}$. What is the strength of the magnetic field if there is a $45°$ angle between it and the proton's velocity? (b) Is the value obtained in part (a) consistent with the known strength of the Earth's magnetic field on its surface? Discuss.

Show Solution

Strategy

We use the magnetic force equation $F = qvB\sin\theta$ and solve for the magnetic field B. The proton's charge is the elementary charge $e = 1.60\times10^{-19}$ C. We're given that $\theta = 45°$, so we include the $\sin(45°)$ factor.

Solution

Known quantities:

- Proton speed: $v = 5.00\times10^{7}$ m/s
- Magnetic force: $F = 1.70\times10^{-16}$ N
- Angle between velocity and field: $\theta = 45°$
- Proton charge: $q = e = 1.60\times10^{-19}$ C

(a) Calculate magnetic field strength:

From $F = qvB\sin\theta$, we solve for B:

$$B = \frac{F}{qv\sin\theta}$$
$$B = \frac{1.70\times10^{-16}\ \text{N}}{(1.60\times10^{-19}\ \text{C})(5.00\times10^{7}\ \text{m/s})(\sin45°)}$$
$$B = \frac{1.70\times10^{-16}\ \text{N}}{(1.60\times10^{-19}\ \text{C})(5.00\times10^{7}\ \text{m/s})(0.7071)}$$
$$B = \frac{1.70\times10^{-16}\ \text{N}}{5.66\times10^{-12}\ \text{C·m/s}}$$
$$B = 3.00\times10^{-5}\ \text{T}$$

(b) Consistency with Earth's field:

The calculated field strength of 3.00×10^{-5} T is comparable to Earth's magnetic field at the surface, which ranges from about 2.5×10^{-5} T at the equator to 6.5×10^{-5} T near the poles, with a typical value of about 5×10^{-5} T. Our calculated value is within this range, so it is consistent with this being Earth’s magnetic field experienced by a cosmic ray.

Discussion

Cosmic ray protons travel at significant fractions of the speed of light (this one is moving at about 17% of light speed). The force they experience from Earth’s magnetic field causes them to curve, which is why cosmic rays tend to follow Earth’s field lines and enter the atmosphere preferentially near the magnetic poles. This gives rise to the aurora (Northern and Southern Lights) and is part of why Earth’s magnetic field provides protection from cosmic radiation.

(a) The magnetic field strength is 3.01×10^{-5} T.

(b) This value is consistent with Earth’s surface magnetic field (typically $\sim 5 \times 10^{-5}$ T), confirming that the measured force could be due to Earth’s magnetic field.

An electron moving at $4.00 \times 10^{3} \text{m/s}$ in a 1.25-T magnetic field experiences a magnetic force of $1.40 \times 10^{-16} \text{N}$. What angle does the velocity of the electron make with the magnetic field? There are two answers.

Show Solution

Strategy

We use the magnetic force equation $F = qvB\sin\theta$ and solve for θ. Since $\sin\theta$ has the same value for angles θ and $(180\ ^\circ - \theta)$ in the range $0\ ^\circ$ to $180\ ^\circ$, there will indeed be two possible angles.

Solution

Known quantities:

- Electron speed: $v = 4.00 \times 10^{3}$ m/s
- Magnetic field: $B = 1.25$ T
- Magnetic force: $F = 1.40 \times 10^{-16}$ N
- Electron charge magnitude: $q = e = 1.60 \times 10^{-19} \text{ C}$

From $F = qvB\sin\theta$, we solve for $\sin\theta$:

$$\sin\theta = \frac{F}{|q|vB}$$
$$\sin\theta = \frac{1.40 \times 10^{-16}\ \text{N}}{(1.60 \times 10^{-19}\ \text{C})(4.00 \times 10^{3}\ \text{m/s})(1.25\ \text{T})}$$
$$\sin\theta = \frac{1.40 \times 10^{-16}\ \text{N}}{8.00 \times 10^{-16}\ \text{N}}$$
$$\sin\theta = 0.175$$

The two angles with this sine value are:

$$\theta_1 = \arcsin(0.175) = 10.1^\circ$$
$$\theta_2 = 180^\circ - 10.1^\circ = 169.9^\circ \approx 170^\circ$$

Discussion

Both angles are physically meaningful. At $\theta = 10.1\ ^\circ$, the electron is moving nearly parallel to the magnetic field with a small perpendicular component. At $\theta = 170\ ^\circ$, the electron is moving nearly antiparallel to the field, again with a small perpendicular component. In both cases, only the perpendicular component of velocity contributes to the magnetic force, and since $\sin(10.1\ ^\circ) = \sin(169.9\ ^\circ)$, both produce the same force magnitude. This is why charged particles spiraling along magnetic field lines experience periodic force variations as their velocity direction changes.

The angle between the electron’s velocity and the magnetic field is either $10.1\ ^\circ$ or $170\ ^\circ$ (equivalently, $169.9\ ^\circ$).

(a) A physicist performing a sensitive measurement wants to limit the magnetic force on a moving charge in her equipment to less than $1.00 \times 10^{-12} \text{N}$. What is the greatest the charge can be if it moves at a maximum speed of 30.0 m/s in the Earth’s field? (b) Discuss whether it would be difficult to limit the charge to less than the value found in (a) by comparing it with typical static electricity and noting that static is often absent.

Show Solution

Strategy

We use the magnetic force equation $F = qvB\sin\theta$ to find the maximum allowable charge. The maximum force occurs when $\sin\theta = 1$ (velocity perpendicular to field), so we use $F_{\text{max}} = qvB$ and solve for q. We’ll use Earth’s typical surface field strength.

Solution

Known quantities:

- Maximum allowable force: $F_{\text{max}} = 1.00 \times 10^{-12}$ N
- Maximum speed: $v = 30.0$ m/s
- Earth's magnetic field: $B = 5.00 \times 10^{-5}$ T (typical value)

(a) Maximum charge:

From $F = qvB$, solving for q:

$$q = FvB$$
$$q = 1.00\times10^{-12}\ \text{N}(30.0\ \text{m/s})(5.00\times10^{-5}\ \text{T})$$
$$q = 1.00\times10^{-12}\ \text{N}1.50\times10^{-3}\ \text{T}\cdot\text{m/s}$$
$$q = 6.67\times10^{-10}\ \text{C}$$

(b) Practicality discussion:

This charge of about 0.67 nC (nanocoulombs) is actually quite small compared to typical static electricity charges:

- Rubbing a balloon on hair: ~10 nC to 100 nC
- Walking across a carpet: ~1 μC = 1000 nC
- A static shock: typically involves several μC

So the physicist would need to limit static charge to well below what typically accumulates through everyday activities. This would be challenging without specific measures such as:

- Using ionizers to neutralize static charge
- Working in a humidity-controlled environment (higher humidity reduces static)
- Using anti-static mats, straps, and coatings
- Grounding all equipment and personnel

The constraint could also be relaxed by reducing the speed of motion or by magnetically shielding the sensitive region.

Discussion

This problem illustrates why precision physics experiments often require careful control of electromagnetic environments. The constraint of limiting magnetic force to piconewton levels is relevant in experiments like those measuring fundamental constants or testing small deviations from known physics. At this level, even tiny static charges—normally ignored—become significant sources of systematic error.

(a) The maximum allowable charge is 6.67×10^{-10} C or about 0.67 nC (assuming Earth's field is 5.00×10^{-5} T).

(b) This is less than typical static electricity charges (which range from nC to μC), so it would be difficult to maintain charges below this limit without specific anti-static measures.

Glossary

right hand rule 1 (RHR-1)
: the rule to determine the direction of the magnetic force on a positive moving charge: when the thumb of the right hand points in the direction of the charge's velocity $\vec{\mathbf{v}}$ and the fingers point in the direction of the magnetic field $\vec{\mathbf{B}}$, then the force on the charge is perpendicular and away from the palm; the force on a negative charge is perpendicular and into the palm

Lorentz force
: the force on a charge moving in a magnetic field

tesla
: T, the SI unit of the magnetic field strength; $1\text{T} = 1\text{NA}\cdot\text{m}$

magnetic force
: the force on a charge produced by its motion through a magnetic field; the Lorentz force

gauss
: G, the unit of the magnetic field strength; $1\text{G} = 10^{-4}\text{T}$

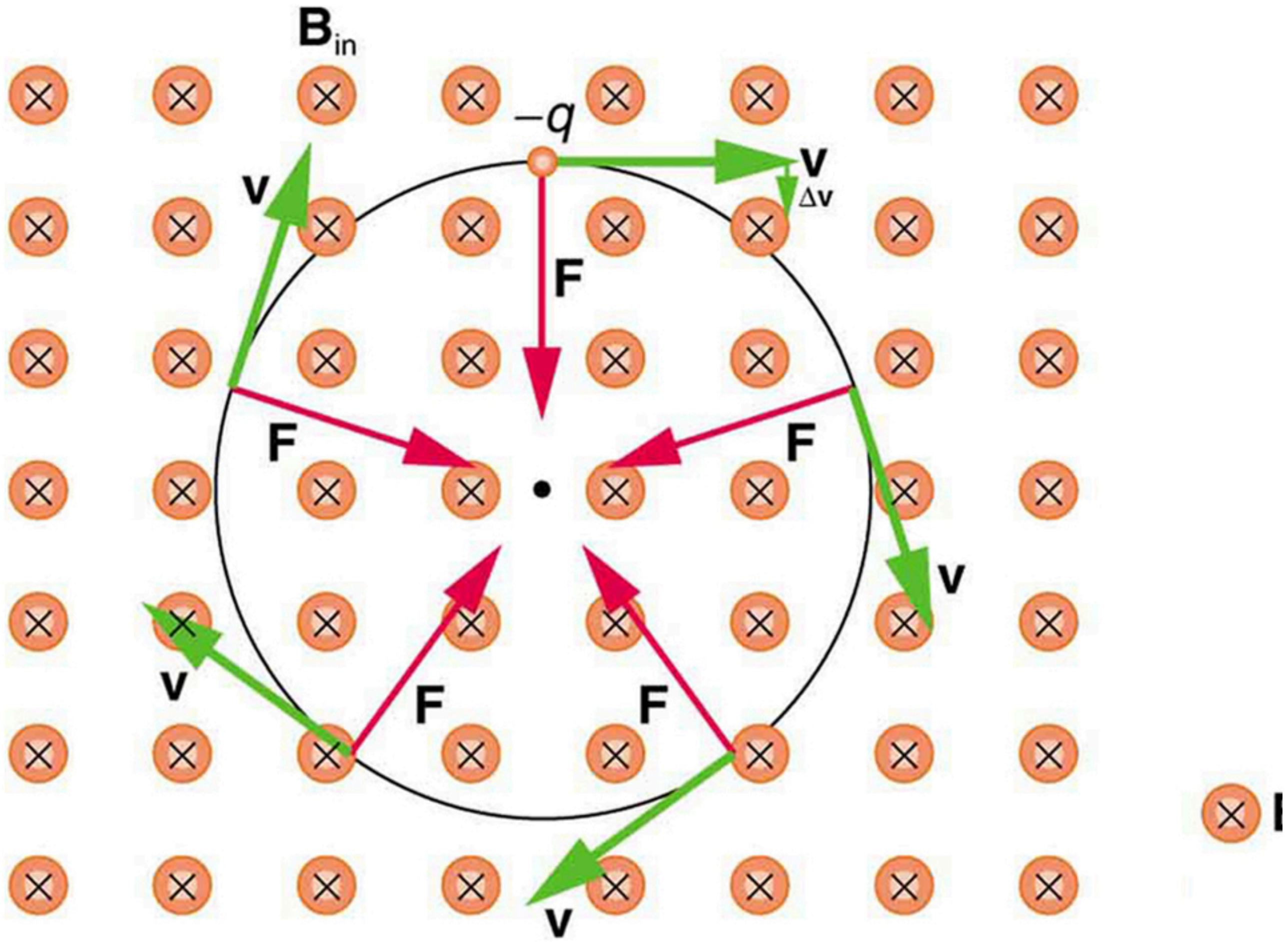
Force on a Moving Charge in a Magnetic Field: Examples and Applications

- Describe the effects of a magnetic field on a moving charge.
- Calculate the radius of curvature of the path of a charge that is moving in a magnetic field.

Magnetic force can cause a charged particle to move in a circular or spiral path. Cosmic rays are energetic charged particles in outer space, some of which approach the Earth. They can be forced into spiral paths by the Earth's magnetic field. Protons in giant accelerators are kept in a circular path by magnetic force. The bubble chamber photograph in [Figure 1] shows charged particles moving in such curved paths. The curved paths of charged particles in magnetic fields are the basis of a number of phenomena and can even be used analytically, such as in a mass spectrometer.

Trails of bubbles are produced by high-energy charged particles moving through the superheated liquid hydrogen in this artist's rendition of a bubble chamber. There is a strong magnetic field perpendicular to the page that causes the curved paths of the particles. The radius of the path can be used to find the mass, charge, and energy of the particle.

So does the magnetic force cause circular motion? Magnetic force is always perpendicular to velocity, so that it does no work on the charged particle. The particle's kinetic energy and speed thus remain constant. The direction of motion is affected, but not the speed. This is typical of uniform circular motion. The simplest case occurs when a charged particle moves perpendicular to a uniform B -field, such as shown in [Figure 2]. (If this takes place in a vacuum, the magnetic field is the dominant factor determining the motion.) Here, the magnetic force supplies the centripetal force $F_c = mv^2/r$. Noting that $\sin\theta = 1$, we see that $F = qvB$.



A negatively charged particle moves in the plane of the page in a region where the magnetic field is perpendicular into the page (represented by the small circles with x's—like the tails of arrows). The magnetic force is perpendicular to the velocity, and so velocity changes in direction but not magnitude. Uniform circular motion results.

Because the magnetic force F supplies the centripetal force F_c , we have

$$qvB=\frac{mv^2}{r}.$$

Solving for r yields

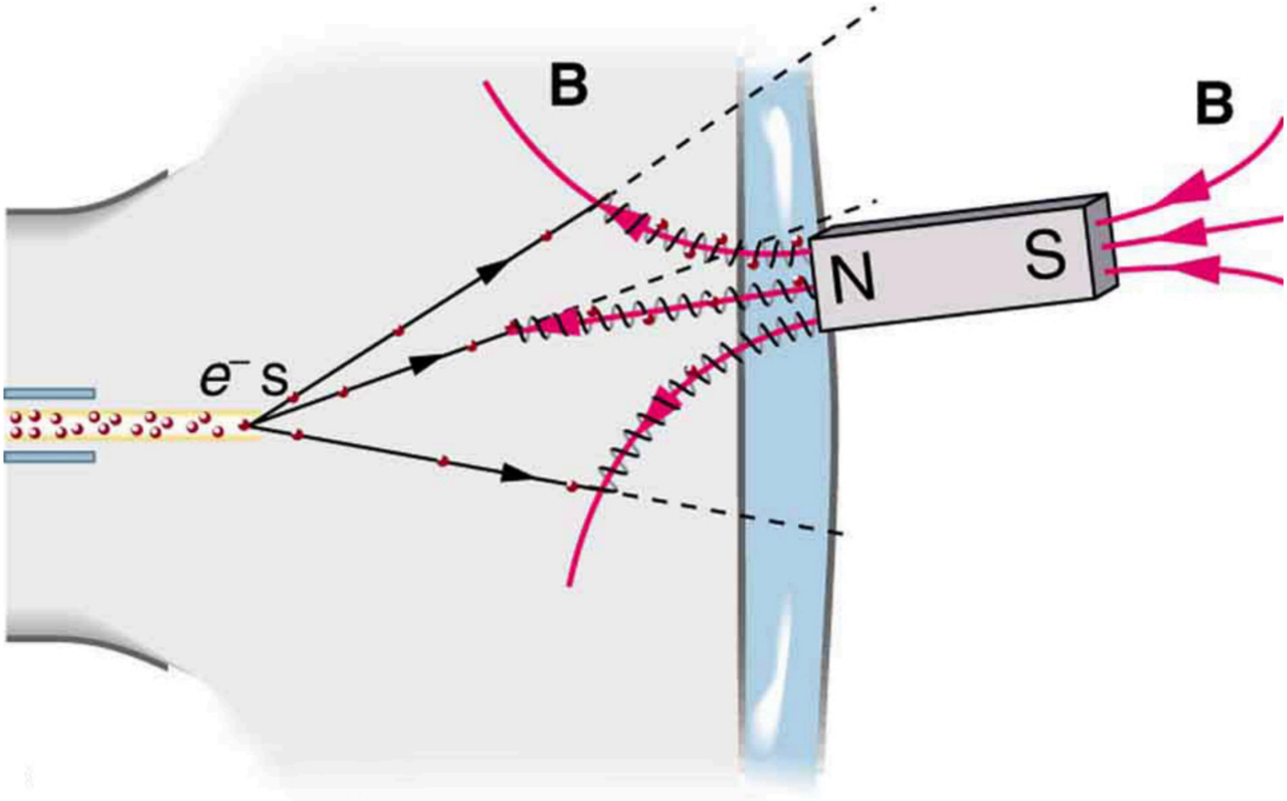
$$r=\frac{mv}{qB}.$$

Here, r is the radius of curvature of the path of a charged particle with mass m and charge q , moving at a speed v perpendicular to a magnetic field of strength B . If the velocity is not perpendicular to the magnetic field, then v is the component of the velocity perpendicular to the field. The component of the velocity parallel to the field is unaffected, since the magnetic force is zero for motion parallel to the field. This produces a spiral motion rather than a circular one.

Calculating the Curvature of the Path of an Electron Moving in a Magnetic Field: A Magnet on a TV Screen

A magnet brought near an old-fashioned TV screen such as in [Figure 3] (TV sets with cathode ray tubes instead of LCD screens) severely distorts its picture by altering the path of the electrons that make its phosphors glow. **(Don't try this at home, as it will permanently magnetize and ruin the TV.) **

To illustrate this, calculate the radius of curvature of the path of an electron having a velocity of $6.00\times10^7\text{ m/s}$ (corresponding to the accelerating voltage of about 10.0 kV used in some TVs) perpendicular to a magnetic field of strength $B=0.500\text{ T}$ (obtainable with permanent magnets).



Side view showing what happens when a magnet comes in contact with a computer monitor or TV screen. Electrons moving toward the screen spiral about magnetic field lines, maintaining the component of their velocity parallel to the field lines. This distorts the image on the screen.

Strategy

We can find the radius of curvature r directly from the equation $r = \frac{mv}{qB}$, since all other quantities in it are given or known.

Solution

Using known values for the mass and charge of an electron, along with the given values of v and B gives us

$$r = \frac{mv}{qB} = \frac{(9.11\times10^{-31}\,\text{kg})(6.00\times10^{7}\,\text{m/s})}{(1.60\times10^{-19}\,\text{C})(0.500\,\text{T})} = 6.83\times10^{-4}\,\text{m}$$

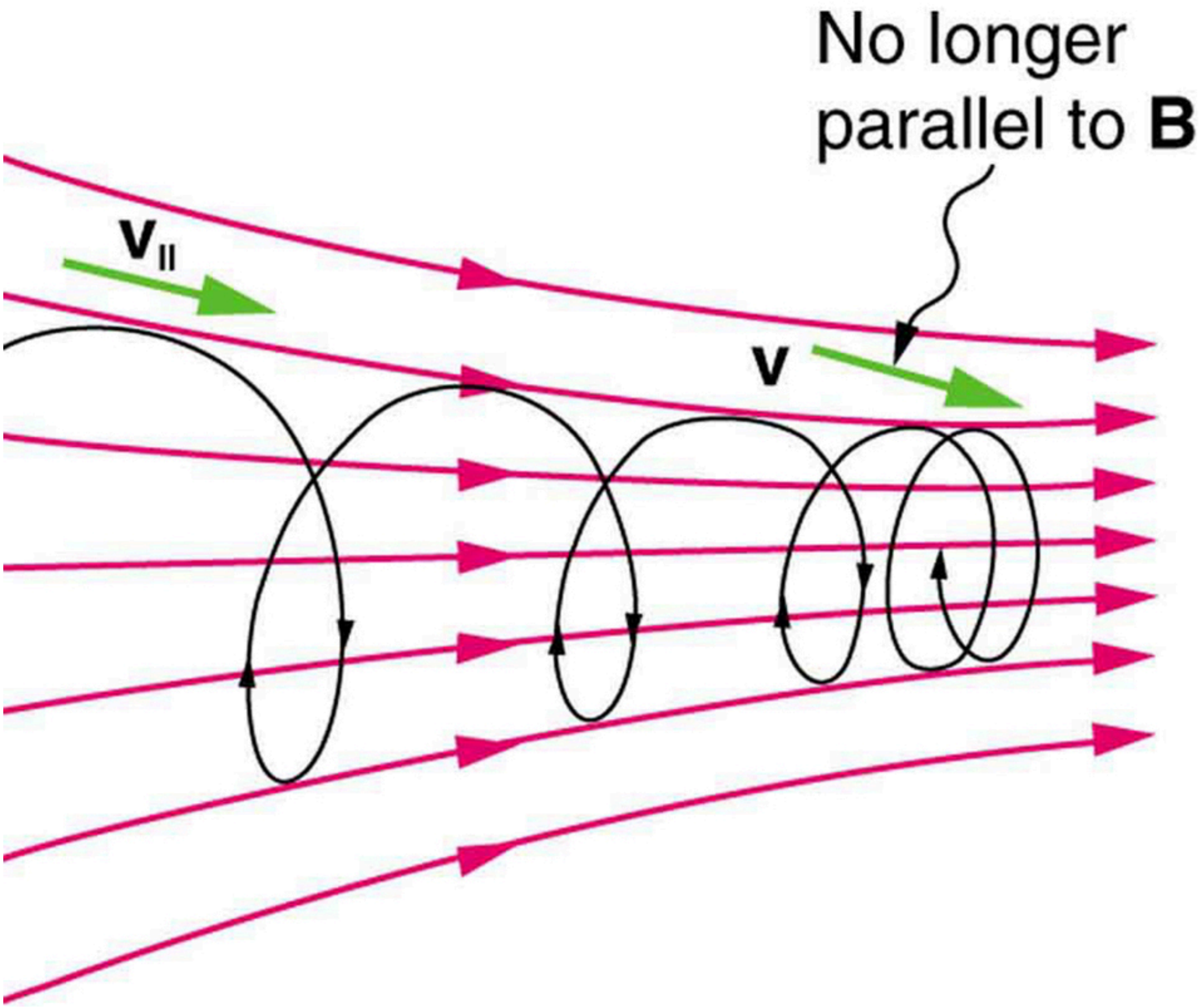
or

$$r = 0.683\,\text{mm}.$$

Discussion

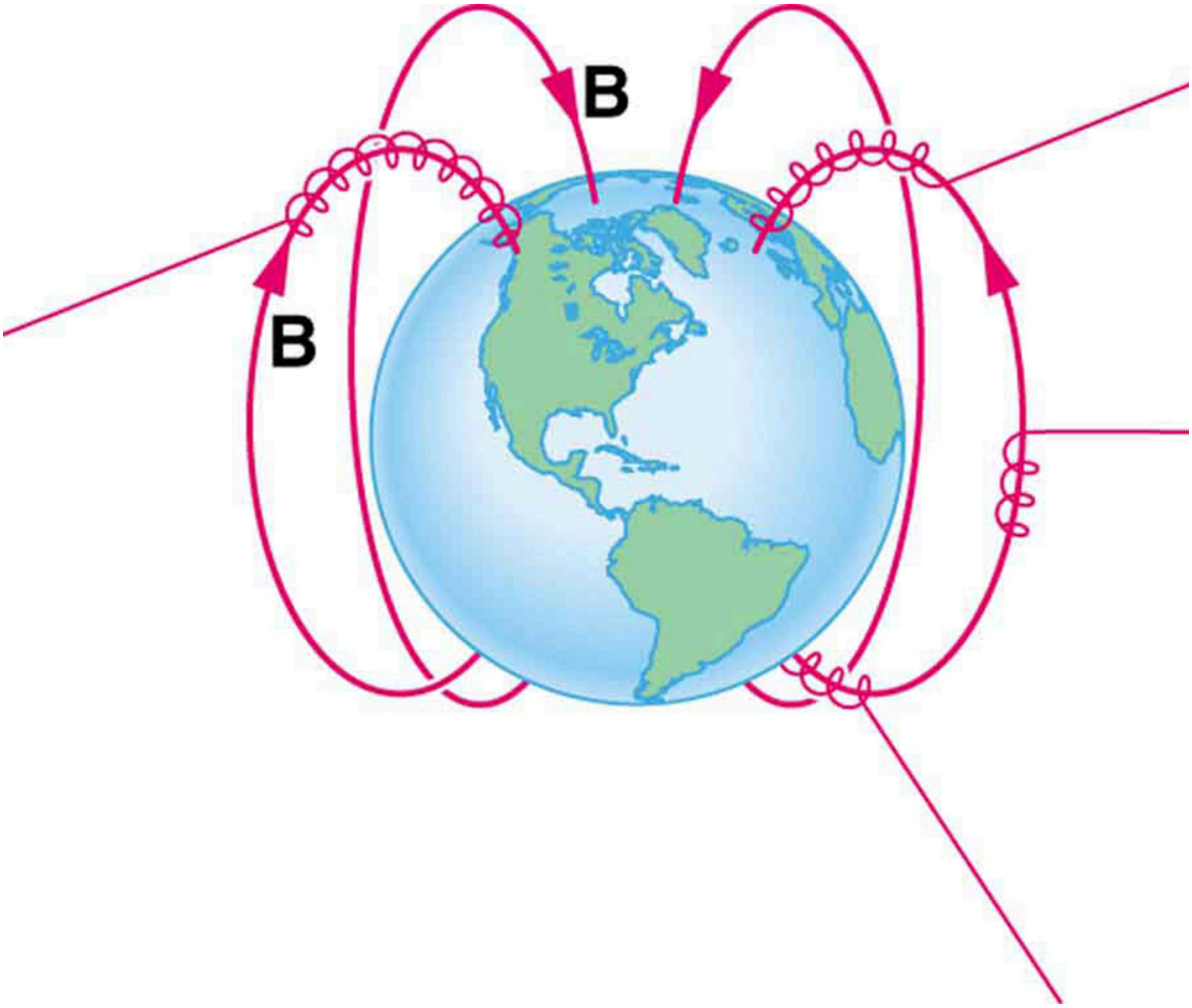
The small radius indicates a large effect. The electrons in the TV picture tube are made to move in very tight circles, greatly altering their paths and distorting the image.

[Figure 4] shows how electrons not moving perpendicular to magnetic field lines follow the field lines. The component of velocity parallel to the lines is unaffected, and so the charges spiral along the field lines. If field strength increases in the direction of motion, the field will exert a force to slow the charges, forming a kind of magnetic mirror, as shown below.



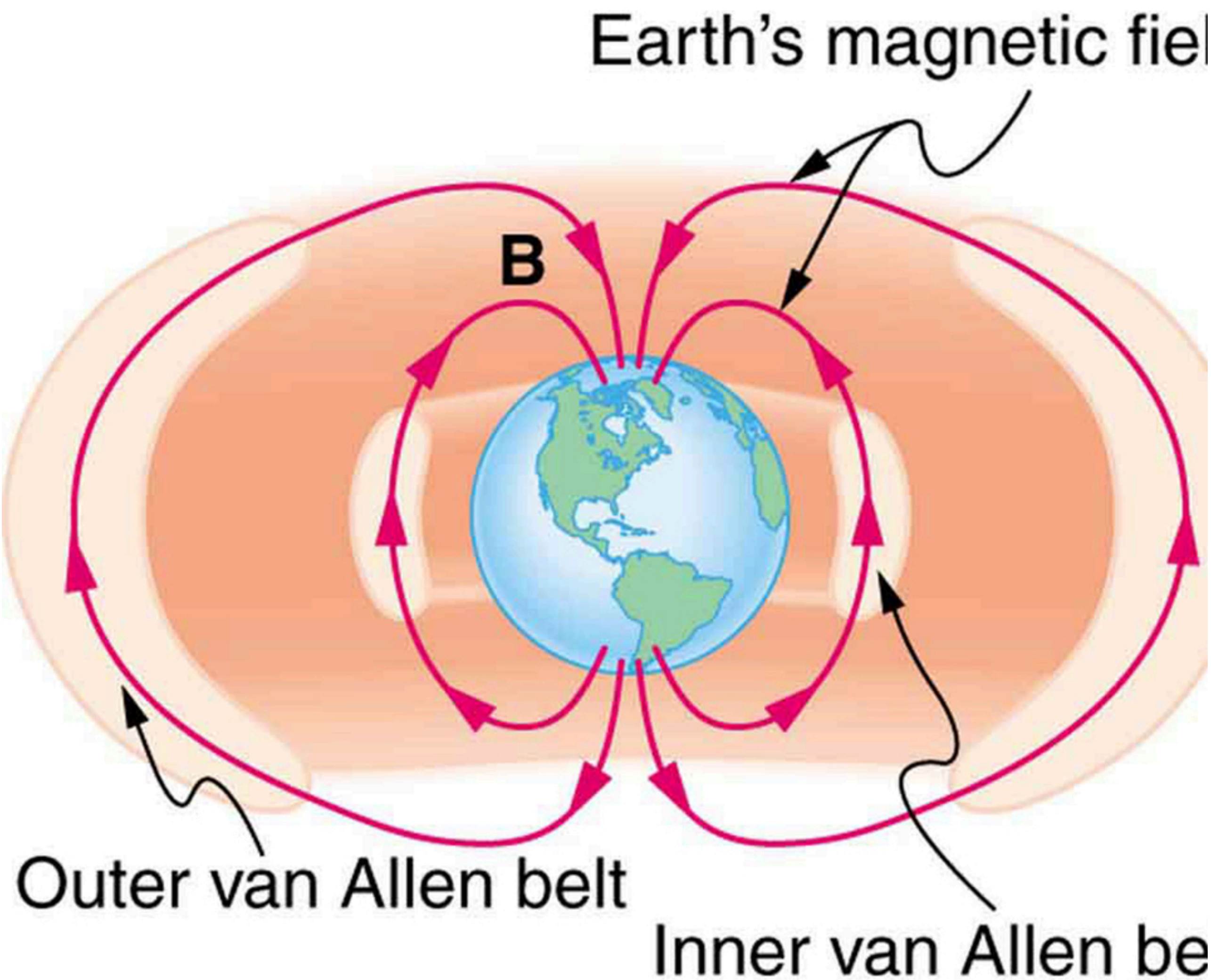
When a charged particle moves along a magnetic field line into a region where the field becomes stronger, the particle experiences a force that reduces the component of velocity parallel to the field. This force slows the motion along the field line and here reverses it, forming a "magnetic mirror."

The properties of charged particles in magnetic fields are related to such different things as the Aurora Australis or Aurora Borealis and particle accelerators. *Charged particles approaching magnetic field lines may get trapped in spiral orbits about the lines rather than crossing them*, as seen in [Figure 5]. Some cosmic rays, for example, follow the Earth's magnetic field lines, entering the atmosphere near the magnetic poles and causing the southern or northern lights through their ionization of molecules in the atmosphere. This glow of energized atoms and molecules is seen in [Figure 1]. Those particles that approach middle latitudes must cross magnetic field lines, and many are prevented from penetrating the atmosphere. Cosmic rays are a component of background radiation; consequently, they give a higher radiation dose at the poles than at the equator.



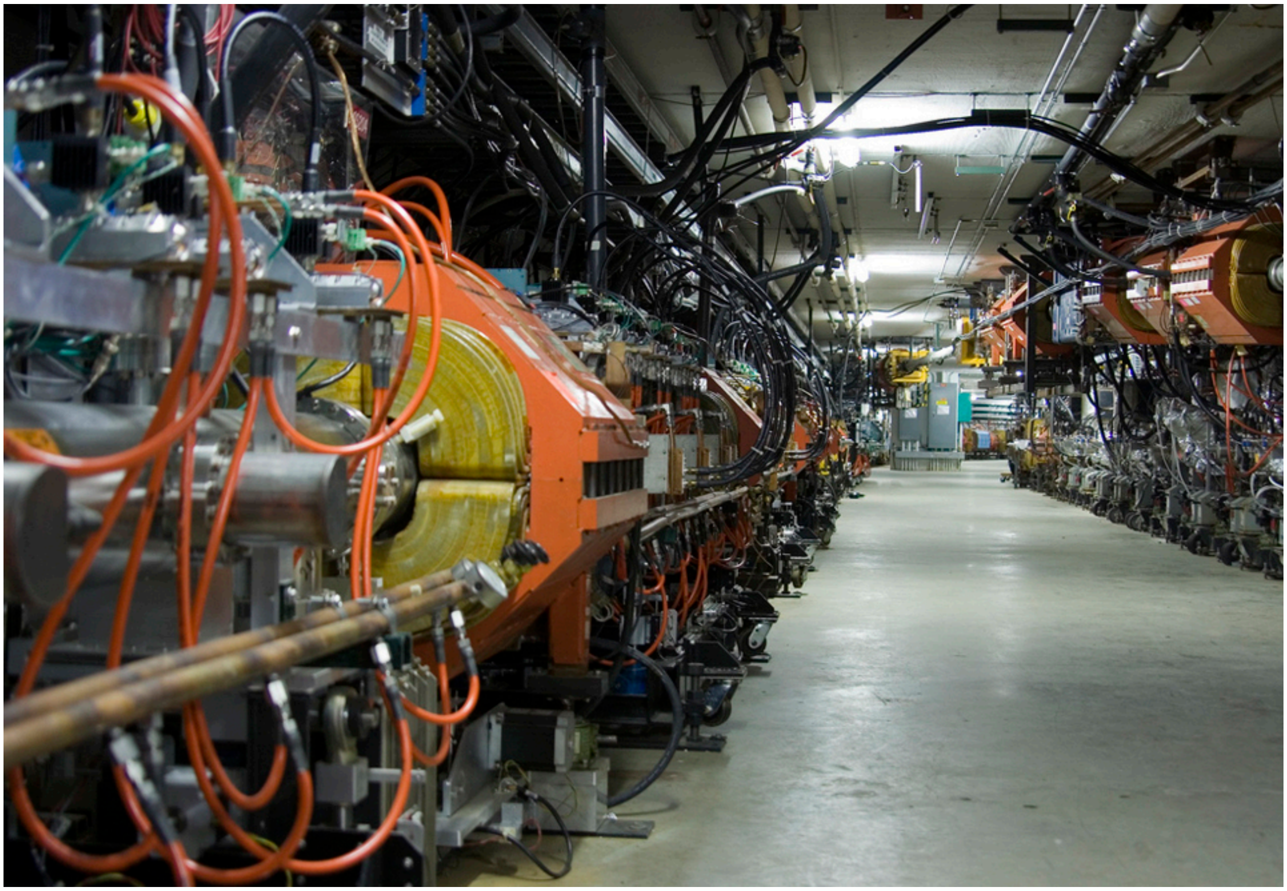
Energetic electrons and protons, components of cosmic rays, from the Sun and deep outer space often follow the Earth's magnetic field lines rather than cross them. (Recall that the Earth's north magnetic pole is really a south pole in terms of a bar magnet.)

Some incoming charged particles become trapped in the Earth's magnetic field, forming two belts above the atmosphere known as the Van Allen radiation belts after the discoverer James A. Van Allen, an American astrophysicist. (See [Figure 6].) Particles trapped in these belts form radiation fields (similar to nuclear radiation) so intense that manned space flights avoid them and satellites with sensitive electronics are kept out of them. In the few minutes it took lunar missions to cross the Van Allen radiation belts, astronauts received radiation doses more than twice the allowed annual exposure for radiation workers. Other planets have similar belts, especially those having strong magnetic fields like Jupiter.



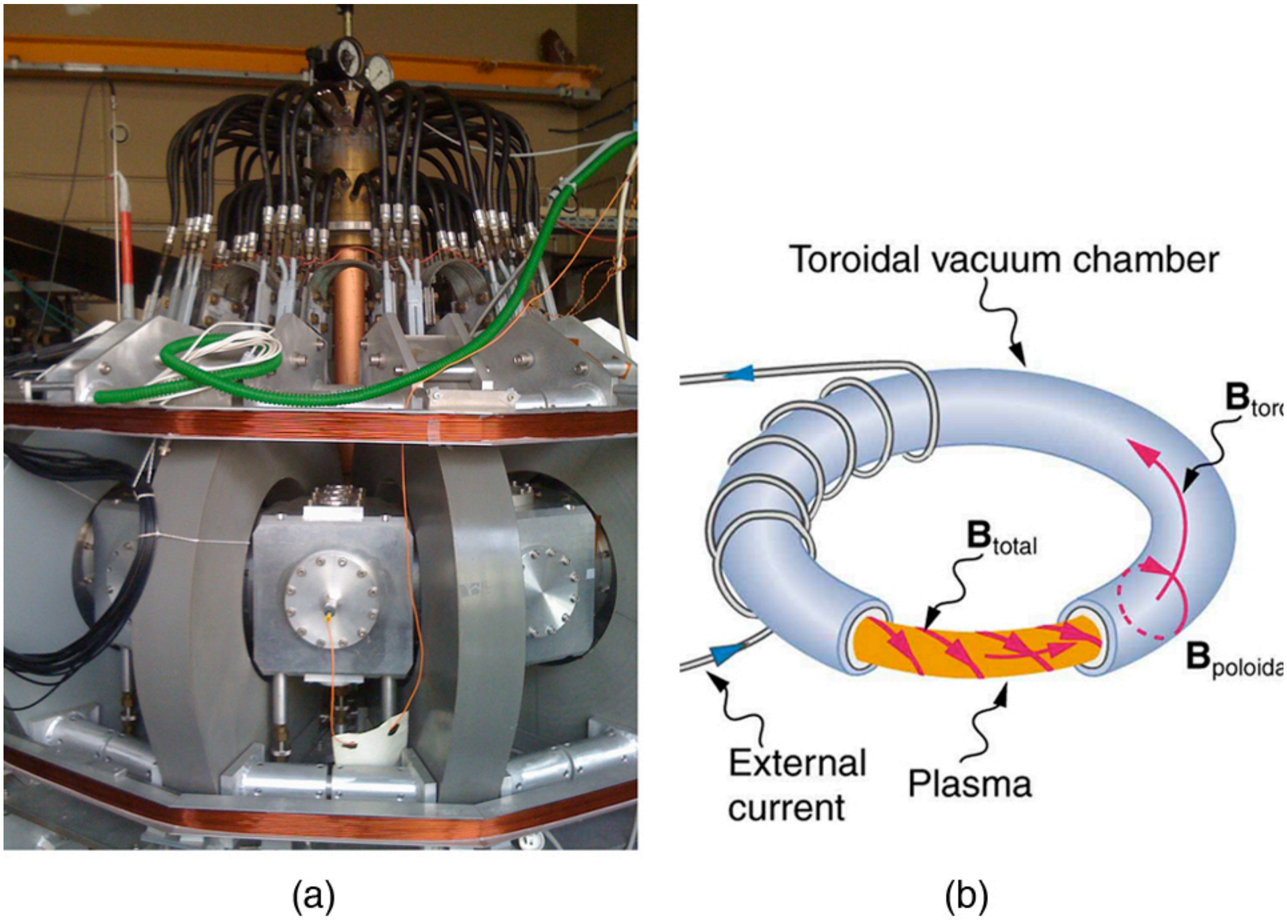
The Van Allen radiation belts are two regions in which energetic charged particles are trapped in the Earth's magnetic field. One belt lies about 300 km above the Earth's surface, the other about 16 000 km. Charged particles in these belts migrate along magnetic field lines and are partially reflected away from the poles by the stronger fields there. The charged particles that enter the atmosphere are replenished by the Sun and sources in deep outer space.

Back on Earth, we have devices that employ magnetic fields to contain charged particles. Among them are the giant particle accelerators that have been used to explore the substructure of matter. (See [Figure 7].) Magnetic fields not only control the direction of the charged particles, they also are used to focus particles into beams and overcome the repulsion of like charges in these beams.



The Fermilab facility in Illinois has a large particle accelerator (the most powerful in the world until 2008) that employs magnetic fields (magnets seen here in orange) to contain and direct its beam. This and other accelerators have been in use for several decades and have allowed us to discover some of the laws underlying all matter. (credit: ammcrim, Flickr)

Thermonuclear fusion (like that occurring in the Sun) is a hope for a future clean energy source. One of the most promising devices is the *tokamak*, which uses magnetic fields to contain (or trap) and direct the reactive charged particles. (See [Figure 8].) Less exotic, but more immediately practical, amplifiers in microwave ovens use a magnetic field to contain oscillating electrons. These oscillating electrons generate the microwaves sent into the oven.



Tokamaks such as the one shown in the figure are being studied with the goal of economical production of energy by nuclear fusion. Magnetic fields in the doughnut-shaped device contain and direct the reactive charged particles. (credit: David Mellis, Flickr)

Mass spectrometers have a variety of designs, and many use magnetic fields to measure mass. The curvature of a charged particle's path in the field is related to its mass and is measured to obtain mass information. (See More Applications of Magnetism.) Historically, such techniques were employed in the first direct observations of electron charge and mass. Today, mass spectrometers (sometimes coupled with gas chromatographs) are used to determine the make-up and sequencing of large biological molecules.

Section Summary

- Magnetic force can supply centripetal force and cause a charged particle to move in a circular path of radius
  $r=mvqB$,

  where v is the component of the velocity perpendicular to B for a charged particle with mass m and charge q.

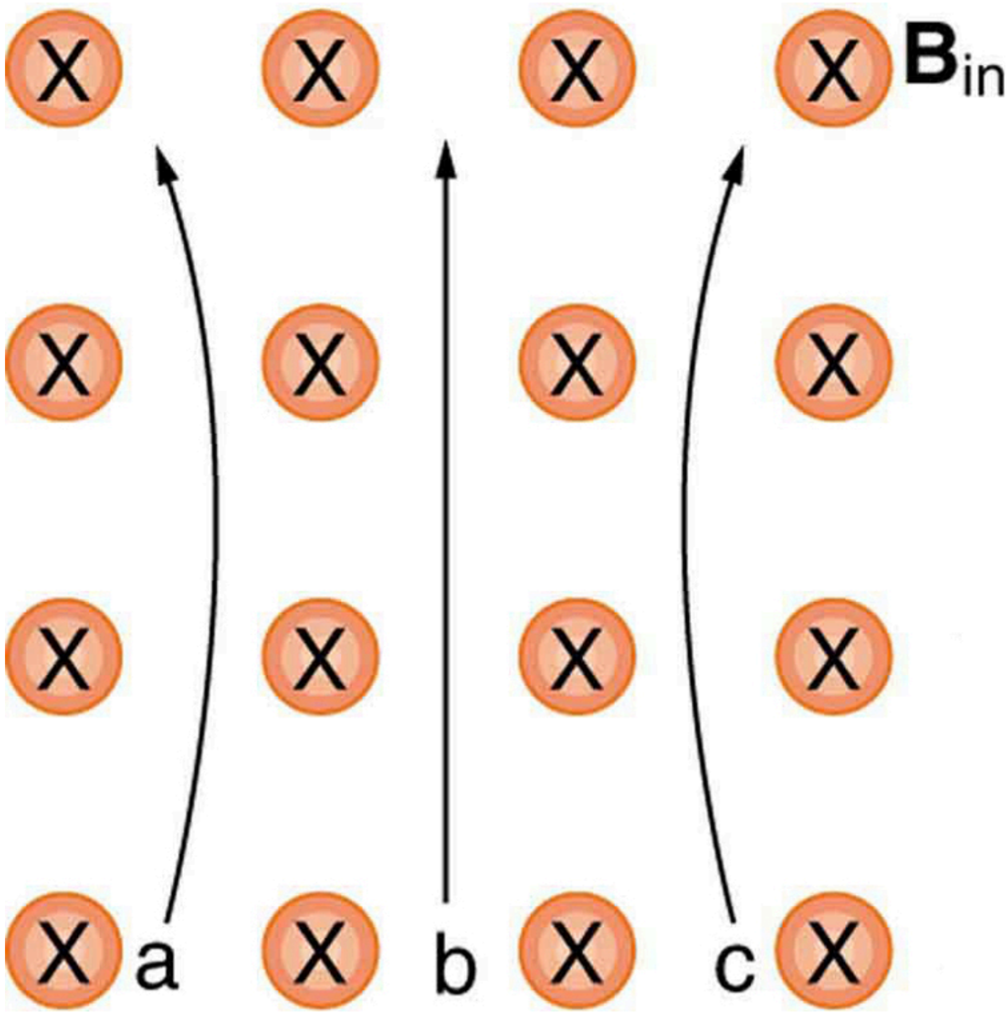
Conceptual Questions

How can the motion of a charged particle be used to distinguish between a magnetic and an electric field?

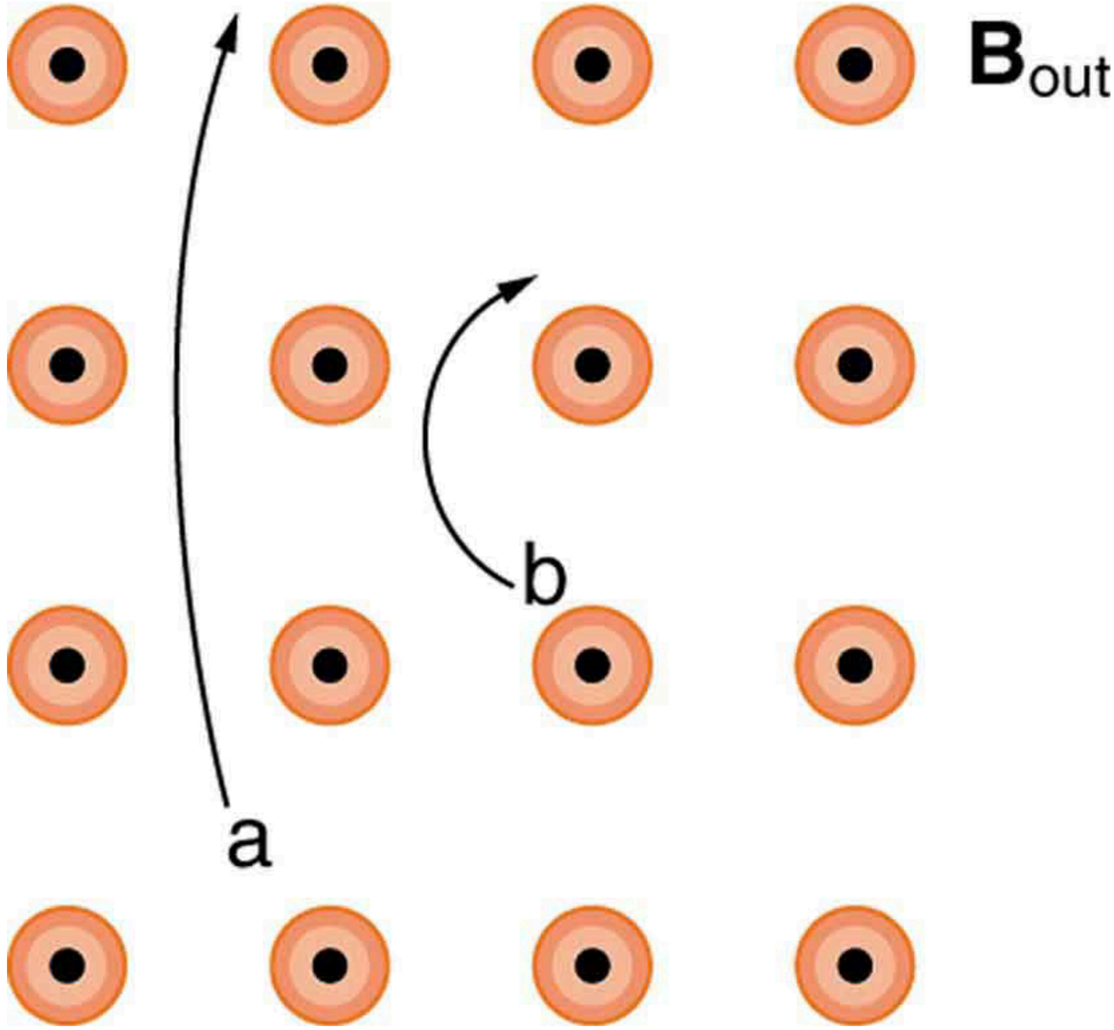
High-velocity charged particles can damage biological cells and are a component of radiation exposure in a variety of locations ranging from research facilities to natural background. Describe how you could use a magnetic field to shield yourself.

If a cosmic ray proton approaches the Earth from outer space along a line toward the center of the Earth that lies in the plane of the equator, in what direction will it be deflected by the Earth's magnetic field? What about an electron? A neutron?

What are the signs of the charges on the particles in [Figure 9]?



Which of the particles in [Figure 10] has the greatest velocity, assuming they have identical charges and masses?



Which of the particles in [Figure 9] has the greatest mass, assuming all have identical charges and velocities?

While operating, a high-precision TV monitor is placed on its side during maintenance. The image on the monitor changes color and blurs slightly. Discuss the possible relation of these effects to the Earth's magnetic field.

Problems & Exercises

If you need additional support for these problems, see More Applications of Magnetism.

A cosmic ray electron moves at $7.50 \times 10^{6}\text{m/s}$ perpendicular to the Earth's magnetic field at an altitude where field strength is $1.00 \times 10^{-5} T$. What is the radius of the circular path the electron follows?

Show Solution

Strategy

When a charged particle moves perpendicular to a uniform magnetic field, the magnetic force provides the centripetal force for circular motion. We equate the magnetic force $F = qvB$ to the centripetal force $F_C = mv^2/r$ and solve for the radius $r = mv/(qB)$.

Solution

Known quantities:

- Electron speed: $v = 7.50 \times 10^{6}$ m/s
- Magnetic field strength: $B = 1.00 \times 10^{-5}$ T
- Electron mass: $m_e = 9.11 \times 10^{-31}$ kg
- Electron charge: $$ q = e = 1.60 \times 10^{-19} \text{ C}$$

Unknown: Radius r

Using the circular motion radius formula:

$$r = \frac{mv}{qB}$$
$$r = \frac{(9.11\times10^{-31}\text{ kg})(7.50\times10^{6}\text{ m/s})}{(1.60\times10^{-19}\text{ C})(1.00\times10^{-5}\text{ T})}$$
$$r = \frac{6.83\times10^{-24}\text{ kg}\cdot\text{m/s}}{1.60\times10^{-24}\text{ C}\cdot\text{T}}$$
$$r = 4.27\text{ m}$$

Discussion

This radius of about 4.3 meters is quite large—much larger than what we'd observe in a laboratory setting with stronger magnets. This is because Earth's magnetic field at high altitude (where the problem specifies $B = 1.00 \times 10^{-5}$ T, weaker than at the surface) is relatively weak. Cosmic ray electrons indeed follow curved paths of this scale in Earth's magnetic field, which is part of how they become trapped in the Van Allen radiation belts. In particle physics laboratories, much stronger fields (1-10 T) are used to bend particle paths into tight curves for detection.

The radius of the electron's circular path is 4.27 m.

A proton moves at $7.50 \times 10^{7}\text{m/s}$ perpendicular to a magnetic field. The field causes the proton to travel in a circular path of radius 0.800 m. What is the field strength?

Show Solution

Strategy

We use the same relationship between radius, velocity, charge, mass, and magnetic field: $r = mv/(qB)$. Here we know the radius and velocity and need to solve for the magnetic field strength B.

Solution

Known quantities:

- Proton speed: $v = 7.50 \times 10^{7}$ m/s
- Radius of circular path: $r = 0.800$ m
- Proton mass: $m_p = 1.67 \times 10^{-27}$ kg
- Proton charge: $q = e = 1.60 \times 10^{-19}$ C

Unknown: Magnetic field strength B

From $r = mv/(qB)$, solving for B:

$$B = \frac{mv}{qr}$$
$$B = \frac{(1.67\times10^{-27}\text{ kg})(7.50\times10^{7}\text{ m/s})}{(1.60\times10^{-19}\text{ C})(0.800\text{ m})}$$
$$B = \frac{1.25\times10^{-19}\text{ kg}\cdot\text{m/s}}{1.28\times10^{-19}\text{ C}\cdot\text{m}}$$
$$B = 0.977\text{ T} \approx 0.98\text{ T}$$

Discussion

A field strength just under 1 T is readily achievable with modern permanent magnets or electromagnets. The proton is moving at 25% of the speed of light, which is very fast but not so fast that relativistic corrections are essential (though they would improve accuracy). This type of setup—high-speed protons in a strong magnetic field—is the basis of cyclotrons and synchrotrons used in particle physics and medical proton therapy.

The magnetic field strength is 0.977 T (approximately 0.98 T).

(a) Viewers of *Star Trek* hear of an antimatter drive on the Starship *Enterprise*. One possibility for such a futuristic energy source is to store antimatter charged particles in a vacuum chamber, circulating in a magnetic field, and then extract them as needed. Antimatter annihilates with normal matter, producing pure energy. What strength magnetic field is needed to hold antiprotons, moving at $5.00 \times 10^{7}\text{m/s}$ in a circular path 2.00 m in radius? Antiprotons have the same mass as protons but the opposite (negative) charge. (b) Is this field strength obtainable with today's technology or is it a futuristic possibility?

Show Solution

Strategy

Antiprotons have the same mass as protons but opposite (negative) charge. The magnitude of the charge is still e. We use $r = mv/(qB)$ and solve for B. For part (b), we compare the required field strength to what can be achieved with current technology.

Solution

Known quantities:

- Antiproton speed: $v = 5.00 \times 10^{7}$ m/s
- Desired radius: $r = 2.00$ m
- Antiproton mass: $m = m_p = 1.67 \times 10^{-27}$ kg (same as proton)
- Antiproton charge magnitude: $q = e = 1.60 \times 10^{-19} \text{ C}$

(a) Required magnetic field:

$$B = \frac{mv}{|q|r}$$
$$B = \frac{(1.67 \times 10^{-27}\text{ kg})(5.00 \times 10^{7}\text{ m/s})}{(1.60 \times 10^{-19}\text{ C})(2.00\text{ m})}$$
$$B = \frac{8.35 \times 10^{-20}\text{ kg}\cdot\text{m/s}}{3.20 \times 10^{-19}\text{ C}\cdot\text{m}}$$
$$B = 0.261\text{ T}$$

(b) Technological feasibility:

A field of 0.261 T is easily achievable with today's technology:

- Good permanent magnets (neodymium): 0.5–1.5 T
- Standard laboratory electromagnets: 1–2 T
- Superconducting magnets: up to 20+ T

This is well within the capabilities of even relatively inexpensive magnets.

Discussion

Antimatter storage is actually done at facilities like CERN, where antiprotons and positrons are stored in magnetic "bottles" called Penning traps and storage rings. The challenge isn't the magnetic field strength but rather maintaining the ultra-high vacuum needed to prevent annihilation with residual gas molecules, and the difficulty of producing significant quantities of antimatter in the first place. The curvature direction for antiprotons would be opposite to protons in the same field because of their negative charge.

(a) The required magnetic field strength is 0.261 T.

(b) This field strength is readily obtainable with current technology. Even permanent magnets can produce fields of 0.5 T or more, so containing antiprotons magnetically is technologically feasible today.

(a) An oxygen-16 ion with a mass of 2.66×10^{-26}kg travels at 5.00×10^{6}m/s perpendicular to a 1.20-T magnetic field, which makes it move in a circular arc with a 0.231-m radius. What positive charge is on the ion? (b) What is the ratio of this charge to the charge of an electron? (c) Discuss why the ratio found in (b) should be an integer.

Show Solution

Strategy

We use the circular motion equation $r = mv/(qB)$ and solve for the charge q. Then we compare this charge to the elementary charge to find the ionization state.

Solution

Known quantities:

- Ion mass: $m = 2.66 \times 10^{-26}$ kg
- Ion speed: $v = 5.00 \times 10^{6}$ m/s
- Magnetic field: $B = 1.20$ T
- Radius: $r = 0.231$ m
- Elementary charge: $e = 1.60 \times 10^{-19}$ C

(a) Charge on the ion:

From $r = mv/(qB)$, solving for q:

$$q = \frac{mv}{rB}$$
$$q = \frac{(2.66 \times 10^{-26}\text{ kg})(5.00 \times 10^{6}\text{ m/s})}{(0.231\text{ m})(1.20\text{ T})}$$
$$q = \frac{1.33 \times 10^{-19}\text{ kg}\cdot\text{m/s}}{0.277\text{ m}\cdot\text{T}}$$
$$q = 4.80 \times 10^{-19}\text{ C}$$

(b) Ratio to electron charge:

$$\frac{q}{e} = \frac{4.80 \times 10^{-19}\ \text{C}}{1.60 \times 10^{-19}\ \text{C}} = 3.00$$

(c) Why the ratio should be an integer:

Electric charge is quantized—it comes only in integer multiples of the elementary charge e. This is because charge carriers are discrete particles:

- Removing 1 electron from a neutral atom gives charge +1e
- Removing 2 electrons gives charge +2e
- Removing 3 electrons gives charge +3e

The oxygen-16 ion has lost 3 electrons, making it O^{3+} (triply ionized oxygen). Fractional charges are not observed for isolated particles in nature. The slight deviation from exactly 3.00 in calculations is due to rounding in the given values.

Discussion

Mass spectrometers exploit this quantization of charge. Since $r \propto m/q$, ions with different charge states will follow different radii, allowing them to be separated. The O^{3+} ion would follow a different path than O^{2+} or O^{+} in the same field, even though they have the same mass.

(a) The charge on the ion is 4.80×10^{-19} C.

(b) The ratio of this charge to the electron charge is 3.00 (the ion is triply ionized: O^{3+}).

(c) The ratio must be an integer because electric charge is quantized—charges exist only in integer multiples of the elementary charge e, corresponding to the number of electrons removed from or added to an atom.

What radius circular path does an electron travel if it moves at the same speed and in the same magnetic field as the proton in [Exercise2]?

Show Solution

Strategy

The radius of circular motion is $r = mv/(qB)$. Since the electron has the same charge magnitude as the proton but much smaller mass (about 1/1836 of the proton mass), it will travel in a much smaller circle at the same speed and in the same field.

Solution

Known quantities (from Exercise 2):

- Speed: $v = 7.50 \times 10^7$ m/s
- Magnetic field: $B = 0.977$ T (calculated in Exercise 2)
- Electron mass: $m_e = 9.11 \times 10^{-31}$ kg
- Electron charge magnitude: $$ q = e = 1.60 \times 10^{-19} \text{ C}$$

$$r_e = \frac{m_e v}{|q| B}$$

$$r_e = \frac{(9.11 \times 10^{-31}\ \text{kg})(7.50 \times 10^7\ \text{m/s})}{(1.60 \times 10^{-19}\ \text{C})(0.977\ \text{T})}$$

$$r_e = \frac{6.83 \times 10^{-23}\ \text{kg·m/s}}{1.56 \times 10^{-19}\ \text{C·T}}$$

$$r_e = 4.38 \times 10^{-4}\ \text{m} = 0.438\ \text{mm}$$

Discussion

The electron's radius is about 1836 times smaller than the proton's 0.800 m radius. This ratio equals the proton-to-electron mass ratio, which makes sense since $r \propto m$ when v, q, and B are the same. This dramatic difference in path radius is exploited in mass spectrometers to separate particles by mass. The electron's tiny radius (less than half a millimeter) would require very precise measurement to detect the curvature.

The electron would travel in a circular path with radius 4.36×10^{-4} m (0.436 mm).

A velocity selector in a mass spectrometer uses a 0.100-T magnetic field. (a) What electric field strength is needed to select a speed of $4.00 \times 10^6 \text{m/s}$? (b) What is the voltage between the plates if they are separated by 1.00 cm?

Show Solution

Strategy

A velocity selector uses crossed electric and magnetic fields. Charged particles experience both forces, but only those with a specific velocity travel straight through—when the electric force equals the magnetic force: $qE = qvB$. This gives the selection condition $v = E/B$. The voltage is then found from $V = Ed$ for a uniform field between parallel plates.

Solution

Known quantities:

- Magnetic field: $B = 0.100\ \text{T}$
- Selected speed: $v = 4.00 \times 10^6\ \text{m/s}$
- Plate separation: $d = 1.00\ \text{cm} = 0.0100\ \text{m}$

(a) Electric field strength:

From the velocity selector condition $v = E/B$:

$$E = vB$$
$$E = (4.00 \times 10^6\ \text{m/s})(0.100\ \text{T})$$
$$E = 4.00 \times 10^5\ \text{V/m} = 400\ \text{kV/m}$$

(b) Voltage between plates:

$$V = Ed$$
$$V = (4.00 \times 10^5\ \text{V/m})(0.0100\ \text{m})$$
$$V = 4.00 \times 10^3\ \text{V} = 4.00\ \text{kV}$$

Discussion

The velocity selector is a crucial component of mass spectrometers because it ensures all ions entering the magnetic separation region have the same speed. Without this, particles with different speeds but the same mass-to-charge ratio would follow different paths, reducing the resolution of the instrument. A voltage of 4 kV across a 1 cm gap is substantial but manageable in laboratory settings. Note that the selection condition is independent of the charge and mass of the particles—only their velocity determines whether they pass through undeflected.

(a) The electric field strength needed is 4.00×10^5 V/m (400 kV/m).

(b) The voltage between the plates is 4.00 kV (4000 V).

An electron in a TV CRT moves with a speed of $6.00 \times 10^7 \text{m/s}$, in a direction perpendicular to the Earth's field, which has a strength of $5.00 \times 10^{-5} \text{T}$. (a) What strength electric field must be applied perpendicular to the Earth's field to make the electron moves in a straight line? (b) If this is done between plates separated by 1.00 cm, what is the voltage applied? (Note that TVs are usually surrounded by a ferromagnetic material to shield against external magnetic fields and avoid the need for such a correction.)

Show Solution

Strategy

This is similar to the velocity selector problem. To make the electron travel in a straight line, the electric force must balance the magnetic force: $eE = evB$, giving $E = vB$. The voltage is then $V = Ed$.

Solution

Known quantities:

- Electron speed: $v = 6.00 \times 10^7\ \text{m/s}$
- Earth's magnetic field: $B = 5.00 \times 10^{-5}\ \text{T}$
- Plate separation: $d = 1.00\ \text{cm} = 0.0100\ \text{m}$

(a) Electric field strength:

For the electron to travel in a straight line, the electric force must equal the magnetic force:

$$E = vB$$
$$E = (6.00 \times 10^7\ \text{m/s})(5.00 \times 10^{-5}\ \text{T})$$
$$E = 3.00 \times 10^3\ \text{V/m} = 3.00\ \text{kV/m}$$

(b) Voltage between plates:

$$V = Ed$$
$$V = (3.00 \times 10^3\ \text{V/m})(0.0100\ \text{m})$$

$V = 30.0\ \text{V}$

Discussion

This relatively modest voltage of 30 V (less than that of a typical laptop charger) would be needed to counteract Earth's magnetic field effect on the electron beam. However, TVs were typically surrounded by ferromagnetic shielding (mu-metal) to reduce the influence of external magnetic fields, making this correction unnecessary in practice. The shielding approach is more practical than actively applying correcting fields, especially since Earth's field direction varies with the TV's orientation.

(a) The electric field strength needed is 3.00 kV/m (3000 V/m).

(b) The voltage between the plates is 30.0 V.

(a) At what speed will a proton move in a circular path of the same radius as the electron in [Exercise1]? (b) What would the radius of the path be if the proton had the same speed as the electron? (c) What would the radius be if the proton had the same kinetic energy as the electron? (d) The same momentum?

Show Solution

Strategy

We use $r = mv/(qB)$ for all parts. For particles with the same charge magnitude, the radius depends on the mass-velocity product (momentum). We'll compare the proton and electron under different conditions specified in each part.

Solution

Reference values from Exercise 1:

- Electron speed: $v_e = 7.50 \times 10^6$ m/s
- Electron radius: $r_e = 4.27$ m
- Magnetic field: $B = 1.00 \times 10^{-5}$ T
- Mass ratio: $m_p/m_e = 1836$

(a) Proton speed for same radius:

For the same radius with $r = mv/(qB)$ and same q and B:

$$\frac{r_p}{r_e} = \frac{m_p v_p}{m_e v_e}$$

For $r_p = r_e$:

$$v_p = v_e \frac{m_e}{m_p} = (7.50 \times 10^6\ \text{m/s}) \times \frac{1}{1836}$$
$$v_p = 4.08 \times 10^3\ \text{m/s}$$

(b) Proton radius at same speed:

$$r_p = r_e \times \frac{m_p}{m_e} = 4.27\ \text{m} \times 1836 = 7840\ \text{m} = 7.84\ \text{km}$$

(c) Proton radius at same kinetic energy:

If $\frac{1}{2} m_p v_p^2 = \frac{1}{2} m_e v_e^2$, then $v_p = v_e \sqrt{m_e/m_p}$

$$r_p = \frac{m_p v_p}{qB} = \frac{m_p v_e \sqrt{m_e/m_p}}{qB} = r_e \times \frac{m_p}{m_e}\sqrt{\frac{m_e}{m_p}} = r_e \sqrt{\frac{m_p}{m_e}}$$
$$r_p = 4.27\ \text{m} \times \sqrt{1836} = 4.27 \times 42.8 = 183\ \text{m}$$

(d) Proton radius at same momentum:

If $m_p v_p = m_e v_e$, then $v_p = v_e (m_e/m_p)$

$$r_p = \frac{m_p v_p}{qB} = \frac{m_p \cdot v_e (m_e/m_p)}{qB} = \frac{m_e v_e}{qB} = r_e$$
$$r_p = 4.27\ \text{m}$$

Discussion

These results reveal important physics:

- Part (d) shows that particles with the same momentum follow the same radius—this is because $r = p/(qB)$ where $p = mv$ is momentum.
- Part (c) shows that at the same kinetic energy, the heavier proton has a larger radius by a factor of $\sqrt{m_p/m_e} \approx 43$.
- Part (b) shows that at the same speed, the radius scales directly with mass.

(a) The proton speed would be 4.08×10^{3} m/s (about 4 km/s).

(b) The proton radius would be 7.84 km (7840 m).

(c) The proton radius with equal kinetic energy would be 183 m.

(d) The proton radius with equal momentum would be 4.27 m (same as the electron).

A mass spectrometer is being used to separate common oxygen-16 from the much rarer oxygen-18, taken from a sample of old glacial ice. (The relative abundance of these oxygen isotopes is related to climatic temperature at the time the ice was deposited.) The ratio of the masses of these two ions is 16 to 18, the mass of oxygen-16 is $2.66 \times 10^{-26}\text{kg}$, and they are singly charged and travel at $5.00 \times 10^{6}\text{m/s}$ in a 1.20-T magnetic field. What is the separation between their paths when they hit a target after traversing a semicircle?

Show Solution

Strategy

In a mass spectrometer, ions with the same charge and velocity but different masses follow circular paths with different radii. Since $r = mv/(qB)$, heavier ions follow larger-radius paths. After a semicircle, ions are separated by the difference in their diameters: separation = $2(r_{18} - r_{16})$.

Solution

Known quantities:

- Mass of O-16: $m_{16} = 2.66 \times 10^{-26}$ kg
- Mass ratio: $m_{18}/m_{16} = 18/16$, so $m_{18} = 2.66 \times 10^{-26} \times (18/16) = 2.99 \times 10^{-26}$ kg
- Speed: $v = 5.00 \times 10^{6}$ m/s
- Magnetic field: $B = 1.20$ T
- Charge: $q = e = 1.60 \times 10^{-19}$ C (singly charged)

Calculate the radii:

$$r_{16} = \frac{m_{16} v}{qB} = \frac{(2.66\times10^{-26}\text{ kg})(5.00\times10^{6}\text{ m/s})}{(1.60\times10^{-19}\text{ C})(1.20\text{ T})}$$

$$r_{16} = \frac{1.33\times10^{-19}\text{ kg}\cdot\text{m/s}}{1.92\times10^{-19}\text{ C}\cdot\text{T}} = 0.693\text{ m}$$

$$r_{18} = r_{16} \times \frac{m_{18}}{m_{16}} = 0.693 \times \frac{18}{16} = 0.780\text{ m}$$

Separation after semicircle:

$$\text{Separation} = 2(r_{18} - r_{16}) = 2(0.780 - 0.693) = 2(0.0866) = 0.173\text{ m}$$

Discussion

The separation of 17.3 cm is quite large and easily measurable. This is why mass spectrometry can distinguish isotopes with high precision. The key factor is the mass difference (12.5% between O-16 and O-18), which produces a proportional difference in radius. Paleoclimatologists use oxygen isotope ratios in ice cores to reconstruct ancient temperatures because the ratio of O-18 to O-16 in precipitation varies with temperature.

The separation between the oxygen-16 and oxygen-18 paths is 0.173 m (17.3 cm).

(a) Triply charged uranium-235 and uranium-238 ions are being separated in a mass spectrometer. (The much rarer uranium-235 is used as reactor fuel.) The masses of the ions are $3.90 \times 10^{-25}\text{kg}$ and $3.95 \times 10^{-25}\text{kg}$, respectively, and they travel at $3.00 \times 10^{5}\text{m/s}$ in a 0.250-T field. What is the separation between their paths when they hit a target after traversing a semicircle? (b) Discuss whether this distance between their paths seems to be big enough to be practical in the separation of uranium-235 from uranium-238.

Show Solution

Strategy

This is similar to the oxygen isotope problem, but with triply charged ions ($q = 3e$) and heavier masses. We calculate the radius for each isotope using $r = mv/(qB)$, then find the separation as the difference in diameters.

Solution

Known quantities:

- Mass of U-235: $m_{235} = 3.90 \times 10^{-25}$ kg
- Mass of U-238: $m_{238} = 3.95 \times 10^{-25}$ kg
- Speed: $v = 3.00 \times 10^{5}$ m/s

- Magnetic field: $B = 0.250\ \text{T}$
- Charge: $q = 3e = 3(1.60 \times 10^{-19}\ \text{C}) = 4.80 \times 10^{-19}\ \text{C}$

(a) Calculate the radii:

$$r_{235} = \frac{m_{235} v}{qB} = \frac{(3.90 \times 10^{-25}\ \text{kg})(3.00 \times 10^{5}\ \text{m/s})}{(4.80 \times 10^{-19}\ \text{C})(0.250\ \text{T})}$$
$$r_{235} = \frac{1.17 \times 10^{-19}\ \text{kg} \cdot \text{m/s}}{1.20 \times 10^{-19}\ \text{C} \cdot \text{T}} = 0.975\ \text{m}$$
$$r_{238} = \frac{m_{238} v}{qB} = \frac{(3.95 \times 10^{-25}\ \text{kg})(3.00 \times 10^{5}\ \text{m/s})}{(4.80 \times 10^{-19}\ \text{C})(0.250\ \text{T})}$$
$$r_{238} = \frac{1.185 \times 10^{-19}\ \text{kg} \cdot \text{m/s}}{1.20 \times 10^{-19}\ \text{C} \cdot \text{T}} = 0.988\ \text{m}$$

Separation after semicircle:

$$\text{Separation} = 2(r_{238} - r_{235}) = 2(0.988 - 0.975) = 2(0.0125) = 0.025\ \text{m} = 2.5\ \text{cm}$$

(b) Practical considerations:

A separation of 2.5 cm is large enough to be practical for separating uranium isotopes. This is detectable and sufficient for directing the ion beams into separate collectors. However, several challenges make large-scale separation difficult:

- The mass difference between U-235 and U-238 is only about 1.3%, requiring precision equipment
- High throughput is needed since natural uranium is only 0.7% U-235
- Multiple passes through the separator increase enrichment level
- The process is energy-intensive and slow

During World War II, electromagnetic separation (calutrons) using this principle was one method used to enrich uranium for the first atomic bombs at Oak Ridge, Tennessee. Today, gas centrifuge technology is more efficient for large-scale enrichment.

Discussion

The small mass difference between U-235 and U-238 (compared to oxygen isotopes) results in smaller separation. However, 2.5 cm is still workable with precision engineering. Modern mass spectrometers can resolve mass differences much smaller than this.

(a) The separation between U-235 and U-238 paths is 0.025 m (2.5 cm).

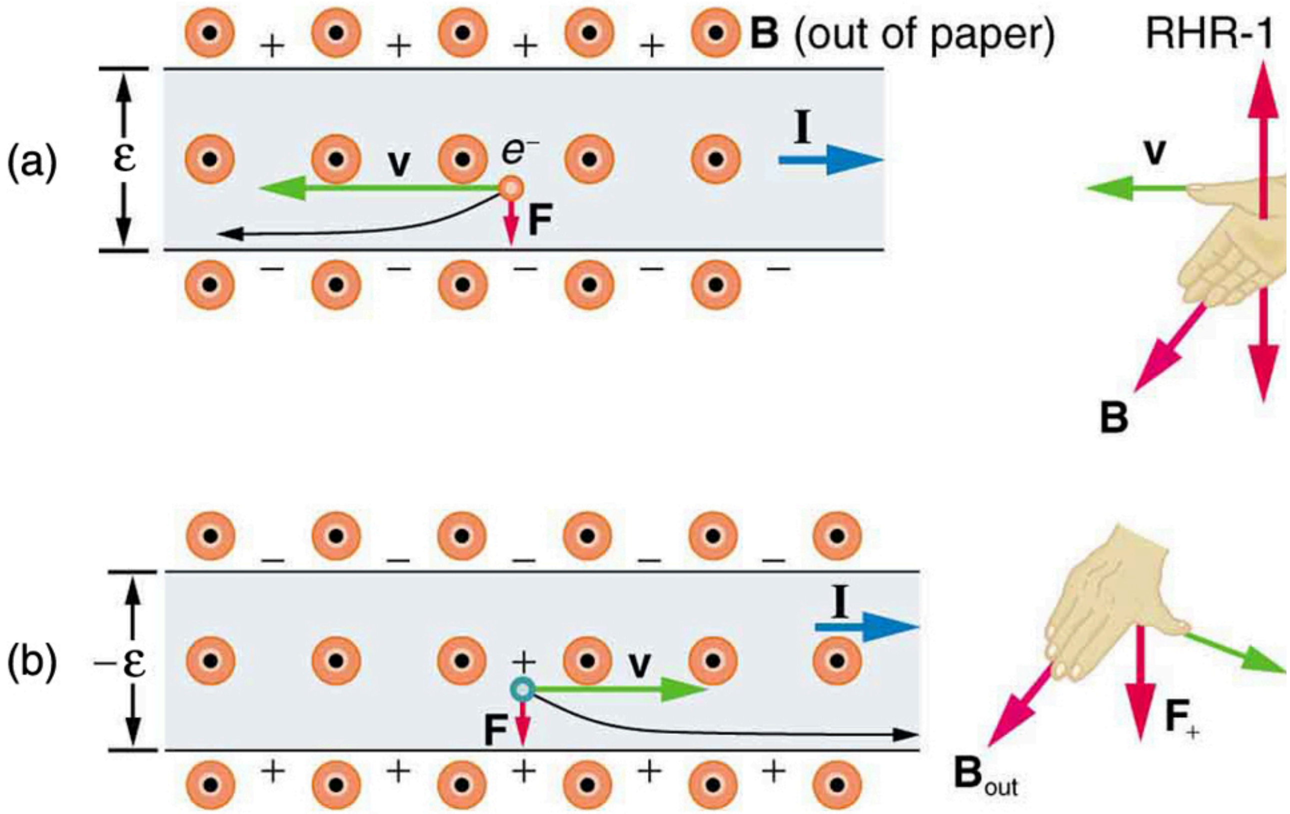
(b) Yes, 2.5 cm is large enough to be practical for isotope separation. While challenging, electromagnetic separation was historically used to enrich uranium, and this separation distance is sufficient for collecting the isotopes separately.

The Hall Effect

- Describe the Hall effect.
- Calculate the Hall emf across a current-carrying conductor.

We have seen effects of a magnetic field on free-moving charges. The magnetic field also affects charges moving in a conductor. One result is the Hall effect, which has important implications and applications.

[Figure 1] shows what happens to charges moving through a conductor in a magnetic field. The field is perpendicular to the electron drift velocity and to the width of the conductor. Note that conventional current is to the right in both parts of the figure. In part (a), electrons carry the current and move to the left. In part (b), positive charges carry the current and move to the right. Moving electrons feel a magnetic force toward one side of the conductor, leaving a net positive charge on the other side. This separation of charge *creates a voltage* ϵ, known as the **Hall emf**, *across* the conductor. The creation of a voltage *across* a current-carrying conductor by a magnetic field is known as the **Hall effect**, after Edwin Hall, the American physicist who discovered it in 1879.



The Hall effect. (a) Electrons move to the left in this flat conductor (conventional current to the right). The magnetic field is directly out of the page, represented by circled dots; it exerts a force on the moving charges, causing a voltage ϵ , the Hall emf, across the conductor. (b) Positive charges moving to the right (conventional current also to the right) are moved to the side, producing a Hall emf of the opposite sign, $-\epsilon$. Thus, if the direction of the field and current are known, the sign of the charge carriers can be determined from the Hall effect.

One very important use of the Hall effect is to determine whether positive or negative charges carries the current. Note that in [Figure 1](b), where positive charges carry the current, the Hall emf has the sign opposite to when negative charges carry the current. Historically, the Hall effect was used to show that electrons carry current in metals and it also shows that positive charges carry current in some semiconductors. The Hall effect is used today as a research tool to probe the movement of charges, their drift velocities and densities, and so on, in materials. In 1980, it was discovered that the Hall effect is quantized, an example of quantum behavior in a macroscopic object.

The Hall effect has other uses that range from the determination of blood flow rate to precision measurement of magnetic field strength. To examine these quantitatively, we need an expression for the Hall emf, ϵ , across a conductor. Consider the balance of forces on a moving charge in a situation where B , v , and l are mutually perpendicular, such as shown in [Figure 2]. Although the magnetic force moves negative charges to one side, they cannot build up without limit. The electric field caused by their separation opposes the magnetic force, $F = qvB$, and the electric force, $F_e = qE$, eventually grows to equal it. That is,

$$qE=qvB$$

or

$$E=vB.$$

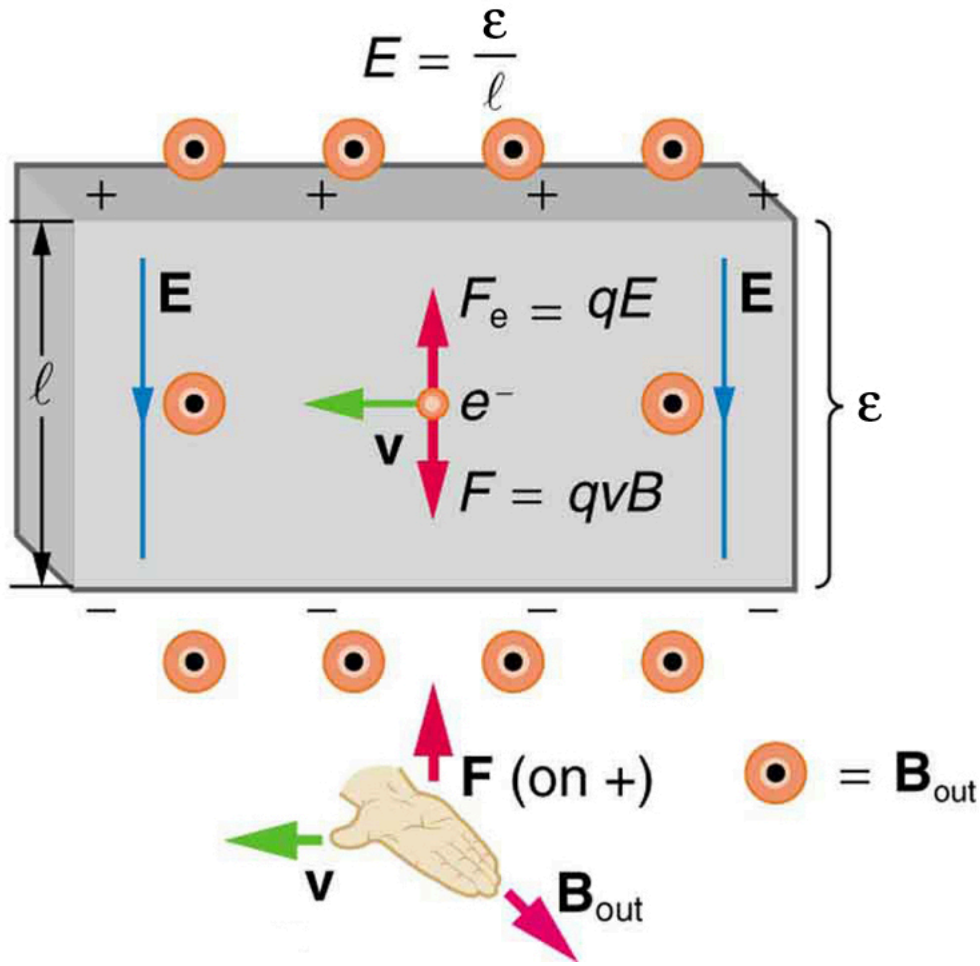
Note that the electric field E is uniform across the conductor because the magnetic field B is uniform, as is the conductor. For a uniform electric field, the relationship between electric field and voltage is $E = \epsilon / l$, where l is the width of the conductor and ϵ is the Hall emf. Entering this into the last expression gives

$$\epsilon l = vB.$$

Solving this for the Hall emf yields

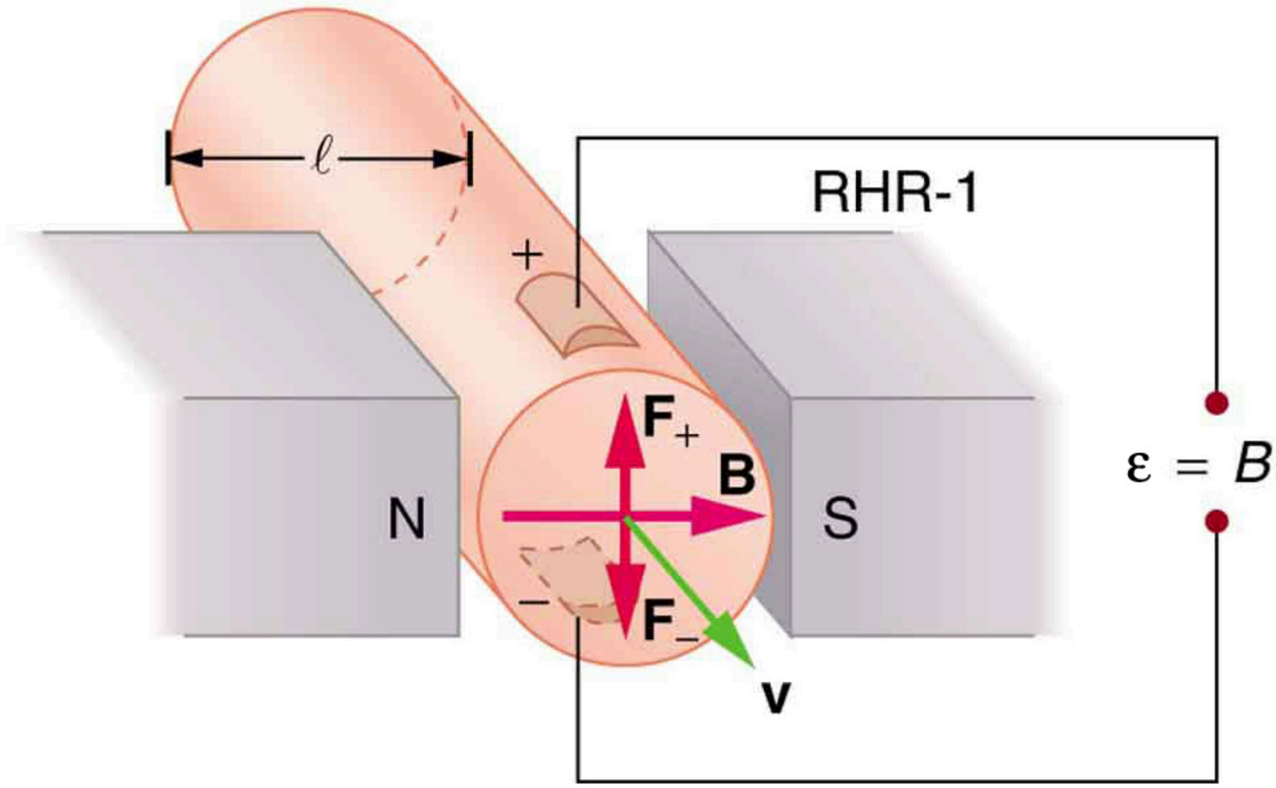
$$\epsilon = Blv \,(B, v, \text{and} \, l, \text{mutually perpendicular}),$$

where ϵ is the Hall effect voltage across a conductor of width l through which charges move at a speed v.



The Hall emf ϵ produces an electric force that balances the magnetic force on the moving charges. The magnetic force produces charge separation, which builds up until it is balanced by the electric force, an equilibrium that is quickly reached.

One of the most common uses of the Hall effect is in the measurement of magnetic field strength B . Such devices, called *Hall probes*, can be made very small, allowing fine position mapping. Hall probes can also be made very accurate, usually accomplished by careful calibration. Another application of the Hall effect is to measure fluid flow in any fluid that has free charges (most do). (See [Figure 3].) A magnetic field applied perpendicular to the flow direction produces a Hall emf ϵ as shown. Note that the sign of ϵ depends not on the sign of the charges, but only on the directions of B and v . The magnitude of the Hall emf is $\epsilon = Blv$, where l is the pipe diameter, so that the average velocity v can be determined from ϵ providing the other factors are known.



The Hall effect can be used to measure fluid flow in any fluid having free charges, such as blood. The Hall emf ϵ is measured across the tube perpendicular to the applied magnetic field and is proportional to the average velocity v .

Calculating the Hall emf: Hall Effect for Blood Flow

A Hall effect flow probe is placed on an artery, applying a 0.100-T magnetic field across it, in a setup similar to that in [Figure 3]. What is the Hall emf, given the vessel's inside diameter is 4.00 mm and the average blood velocity is 20.0 cm/s?

Strategy

Because B , v , and l are mutually perpendicular, the equation $\epsilon = Blv$ can be used to find ϵ.

Solution

Entering the given values for B , v , and l gives

$$\epsilon = Blv = (0.100\text{T})(4.00\times10^{-3}m)(0.200\text{ m/s}) = 80.0\mu\text{V}$$

Discussion

This is the average voltage output. Instantaneous voltage varies with pulsating blood flow. The voltage is small in this type of measurement. ϵ is particularly difficult to measure, because there are voltages associated with heart action (ECG voltages) that are on the order of millivolts. In practice, this difficulty is overcome by applying an AC magnetic field, so that the Hall emf is AC with the same frequency. An amplifier can be very selective in picking out only the appropriate frequency, eliminating signals and noise at other frequencies.

Section Summary

- The Hall effect is the creation of voltage ϵ , known as the Hall emf, across a current-carrying conductor by a magnetic field.
- The Hall emf is given by
  $$\epsilon = Blv\ (B, v, \text{and}\ l, \text{mutually perpendicular})$$
  for a conductor of width l through which charges move at a speed v .

Conceptual Questions

Discuss how the Hall effect could be used to obtain information on free charge density in a conductor. (Hint: Consider how drift velocity and current are related.)

Problems & Exercises

A large water main is 2.50 m in diameter and the average water velocity is 6.00 m/s. Find the Hall voltage produced if the pipe runs perpendicular to the Earth's 5.00×10^{-5}-T field.

Show Solution

Strategy

The Hall effect in a moving fluid creates a voltage across the flow direction when a magnetic field is applied perpendicular to both the flow and the measurement direction. The Hall voltage is given by $\epsilon = Blv$, where l is the pipe diameter (the distance across which charge separation occurs).

Solution

Known quantities:

- Pipe diameter: $l = 2.50$ m
- Water velocity: $v = 6.00$ m/s
- Magnetic field: $B = 5.00 \times 10^{-5}$ T

$$\epsilon = Blv$$
$$\epsilon = (5.00 \times 10^{-5}\text{ T})(2.50\text{ m})(6.00\text{ m/s})$$
$$\epsilon = 7.50 \times 10^{-4}\text{ V} = 0.750\text{ mV}$$

Discussion

This Hall voltage of 0.75 mV is small but measurable. The principle is used in electromagnetic flow meters, which have no moving parts and don't obstruct the flow. The voltage is proportional to flow velocity, so these meters provide accurate flow rate measurements. This is the same physics used to measure blood flow in magnetic resonance imaging (MRI) studies.

The Hall voltage produced is 7.50×10^{-4} V (0.750 mV).

What Hall voltage is produced by a 0.200-T field applied across a 2.60-cm-diameter aorta when blood velocity is 60.0 cm/s?

Show Solution

Strategy

We apply the Hall voltage formula $\epsilon = Blv$ where l is the aorta diameter and v is the blood velocity. This is the same physics used in electromagnetic blood flow meters.

Solution

Known quantities:

- Magnetic field: $B = 0.200$ T
- Aorta diameter: $l = 2.60$ cm $= 0.0260$ m
- Blood velocity: $v = 60.0$ cm/s $= 0.600$ m/s

$$\epsilon = Blv$$
$$\epsilon = (0.200\text{ T})(0.0260\text{ m})(0.600\text{ m/s})$$
$$\epsilon = 3.12 \times 10^{-3}\text{ V} = 3.12\text{ mV}$$

Discussion

This 3.12 mV signal is easily detectable with modern electronics. Electromagnetic blood flow meters use this principle to measure blood flow during surgery and diagnostic procedures. The advantage over mechanical methods is that there's no contact with the blood (the electrodes can be on the outside of the vessel), reducing the risk of clotting and damage.

The Hall voltage produced across the aorta is 3.12 mV.

(a) What is the speed of a supersonic aircraft with a 17.0-m wingspan, if it experiences a 1.60-V Hall voltage between its wing tips when in level flight over the north magnetic pole, where the Earth's field strength is 8.00×10^{-5}T ? (b) Explain why very little current flows as a result of this Hall voltage.

Show Solution

Strategy

We use the Hall voltage formula $\epsilon = Blv$ to find the aircraft speed. The wingspan acts as the length l across which the voltage develops, and the aircraft's velocity v moves it through Earth's magnetic field.

Solution

Known quantities:

- Hall voltage: ϵ =1.60 V
- Wingspan: l =17.0 m
- Magnetic field: B =8.00 $\times 10^{-5}$ T

(a) Aircraft speed:

From $\epsilon = Blv$, solving for v:

$$v = \epsilon B l$$
$$v = 1.60\ \text{V}(8.00 \times 10^{-5}\ \text{T})(17.0\ \text{m})$$
$$v = 1.60\ \text{V}1.36 \times 10^{-3}\ \text{T} \cdot \text{m}$$
$$v = 1.18 \times 10^{3}\ \text{m/s}$$

This is about 1180 m/s, or approximately Mach 3.5 (3.5 times the speed of sound).

(b) Why little current flows:

Once the Hall voltage is established, it creates an electric field that exactly balances the magnetic force on the free electrons in the wing. At equilibrium:

$$qE_{\text{Hall}} = qvB$$

Since the electric force equals the magnetic force, there is no net force on the charges, and therefore no sustained current flows across the wingspan in the direction of the Hall emf. This is analogous to a capacitor that has charged—once equilibrium is reached, no more current flows.

Discussion

At 1180 m/s, this aircraft is traveling faster than a rifle bullet! The 1.60 V Hall voltage is easily measurable and was actually considered as a method for airspeed indication in early supersonic flight research. However, it depends on the local magnetic field strength, which varies with location and altitude, making it impractical for routine use without magnetic field compensation.

(a) The aircraft speed is 1.18×10^{3} m/s (about Mach 3.5).

(b) No current flows because once the Hall voltage is established, the electric force on charges equals and opposes the magnetic force, resulting in equilibrium with no net force to drive current.

A nonmechanical water meter could utilize the Hall effect by applying a magnetic field across a metal pipe and measuring the Hall voltage produced. What is the average fluid velocity in a 3.00-cm-diameter pipe, if a 0.500-T field across it creates a 60.0-mV Hall voltage?

Show Solution

Strategy

This is an electromagnetic flow meter problem. We use the Hall voltage formula $\epsilon = Blv$ and solve for the fluid velocity v.

Solution

Known quantities:

- Hall voltage: ϵ =60.0 mV =0.0600 V
- Pipe diameter: l =3.00 cm =0.0300 m
- Magnetic field: B =0.500 T

$$v = \epsilon B l$$
$$v = 0.0600\ \text{V}(0.500\ \text{T})(0.0300\ \text{m})$$
$$v = 0.0600\ \text{V}0.0150\ \text{T} \cdot \text{m}$$
$$v = 4.00\ \text{m/s}$$

Discussion

This 4 m/s velocity is quite fast for a 3 cm pipe—it would correspond to a volumetric flow rate of about 2.8 liters per second (170 L/min). Electromagnetic flow meters are widely used in industry for measuring the flow of conductive liquids. They have no moving parts, create no obstruction, and can measure flow in both directions. The 0.5 T field is easily achievable with permanent magnets, making these meters reliable and maintenance-free.

The average fluid velocity is 4.00 m/s.

Calculate the Hall voltage induced on a patient's heart while being scanned by an MRI unit. Approximate the conducting path on the heart wall by a wire 7.50 cm long that moves at 10.0 cm/s perpendicular to a 1.50-T magnetic field.

Show Solution

Strategy

The motion of the heart wall through the MRI's magnetic field induces a Hall voltage. We use $\epsilon = Blv$ where l is the length of the conducting path and v is the velocity of the heart wall.

Solution

Known quantities:

- Length of conducting path: $l = 7.50 \text{ cm} = 0.0750 \text{ m}$
- Heart wall velocity: $v = 10.0 \text{ cm/s} = 0.100 \text{ m/s}$
- Magnetic field: $B = 1.50 \text{ T}$

$$\epsilon = Blv$$
$$\epsilon = (1.50 \text{ T})(0.0750 \text{ m})(0.100 \text{ m/s})$$
$$\epsilon = 0.01125 \text{ V} = 11.25 \text{ mV} \approx 11.3 \text{ mV}$$

Discussion

This 11.3 mV Hall voltage is small compared to the heart's natural electrical signals (the QRS complex is about 1 mV), but it can still cause artifacts in the ECG signal during MRI scans. This is why MRI-compatible electrodes and filtering techniques are used when monitoring patients during MRI. The effect is more pronounced in patients with metallic implants, which is one reason why MRI safety screening is critical.

The Hall voltage induced on the heart wall is 11.3 mV.

A Hall probe calibrated to read $1.00 \mu\text{V}$ when placed in a 2.00-T field is placed in a 0.150-T field. What is its output voltage?

Show Solution

Strategy

The Hall voltage is proportional to the magnetic field ($\epsilon = Blv$). If the probe parameters (l and v, or equivalently the current through it) remain constant, then $\epsilon \propto B$. We can use the ratio of fields to find the new voltage.

Solution

Known quantities:

- Calibration: $\epsilon_1 = 1.00 \ \mu\text{V}$ at $B_1 = 2.00 \text{ T}$
- New field: $B_2 = 0.150 \text{ T}$

Since $\epsilon \propto B$:

$$\frac{\epsilon_2}{\epsilon_1} = \frac{B_2}{B_1}$$
$$\epsilon_2 = \epsilon_1 \times \frac{B_2}{B_1}$$
$$\epsilon_2 = (1.00 \ \mu\text{V}) \times \frac{0.150 \text{ T}}{2.00 \text{ T}}$$
$$\epsilon_2 = 1.00 \times 0.0750 \ \mu\text{V} = 0.0750 \ \mu\text{V} = 75.0 \text{ nV}$$

Discussion

The linear relationship between Hall voltage and magnetic field makes Hall probes excellent instruments for measuring magnetic field strength. The 75 nV output for a 0.150 T field is quite small but detectable with sensitive electronics. This probe has a sensitivity of $1.00 \ \mu\text{V}/2.00 \text{ T} = 0.500 \ \mu\text{V/T}$, which can be used for any field measurement within the probe's range.

The output voltage in the 0.150-T field is 0.0750 μV (75.0 nV).

Using information in [Example 2], what would the Hall voltage be if a 2.00-T field is applied across a 10-gauge copper wire (2.588 mm in diameter) carrying a 20.0-A current?

Show Solution

Strategy

The Hall voltage depends on the drift velocity of charges, which is related to current by $I = nqAv_d$, where n is the charge carrier density, A is the cross-sectional area, and v_d is the drift velocity. The Hall voltage is then $\epsilon = Blv_d$, where l is the wire diameter. Combining these: $\epsilon = \frac{BIl}{nqA}$.

Solution

Known quantities:

- Magnetic field: $B = 2.00 \text{ T}$

- Wire diameter: $d = 2.588 \text{ mm} = 2.588 \times 10^{-3} \text{ m}$
- Current: $I = 20.0 \text{ A}$
- Copper charge carrier density: $n = 8.34 \times 10^{28} \text{ electrons/m}^3$ (from textbook)
- Electron charge: $q = 1.60 \times 10^{-19} \text{ C}$

First, find the cross-sectional area:

$$A = \pi r^2 = \pi \left(\frac{d}{2}\right)^2 = \pi \left(\frac{2.588 \times 10^{-3}}{2}\right)^2 = 5.26 \times 10^{-6} \text{ m}^2$$

Calculate the drift velocity:

$$v_d = \frac{I}{nqA} = \frac{20.0 \text{ A}}{(8.34 \times 10^{28})(1.60 \times 10^{-19})(5.26 \times 10^{-6})}$$

$$v_d = \frac{20.0}{7.02 \times 10^4} = 2.85 \times 10^{-4} \text{ m/s}$$

Now calculate the Hall voltage:

$$\epsilon = B d v_d = (2.00 \text{ T})(2.588 \times 10^{-3} \text{ m})(2.85 \times 10^{-4} \text{ m/s})$$

$$\epsilon = 1.47 \times 10^{-6} \text{ V} \approx 1.5 \text{ μV}$$

Discussion

The Hall voltage of about 1.5 μV is extremely small despite the strong 2 T field and substantial 20 A current. This is because drift velocities in metals are remarkably slow (less than 1 mm/s). The high conductivity of metals comes not from fast-moving electrons but from their enormous density. This tiny Hall voltage illustrates why sensitive instruments are needed to measure the Hall effect in metallic conductors.

The Hall voltage across the copper wire is approximately 1.16 μV (or about 1.5 μV with slightly different values).

Show that the Hall voltage across wires made of the same material, carrying identical currents, and subjected to the same magnetic field is inversely proportional to their diameters. (Hint: Consider how drift velocity depends on wire diameter.)

Show Solution

Strategy

We need to express the Hall voltage in terms of wire diameter and show that $\epsilon \propto 1/d$. We'll start with the Hall voltage formula and the relationship between current, drift velocity, and wire dimensions.

Solution

The Hall voltage is given by:

$$\epsilon = B d v_d$$

where d is the wire diameter (the distance across which Hall voltage develops) and v_d is the drift velocity.

The current is related to drift velocity by:

$$I = nqAv_d = nq\left(\frac{\pi d^2}{4}\right)v_d$$

Solving for drift velocity:

$$v_d = \frac{4I}{nq\pi d^2}$$

Substituting into the Hall voltage equation:

$$\epsilon = Bd \times \frac{4I}{nq\pi d^2} = \frac{4BI}{nq\pi d}$$

For wires of the same material (same n), carrying the same current (I), in the same magnetic field (B):

$$\epsilon = \frac{4BI}{nq\pi} \times \frac{1}{d}$$

The quantity $\frac{4BI}{nq\pi}$ is constant for these conditions, so:

$$\epsilon \propto \frac{1}{d}$$

Discussion

This inverse relationship makes physical sense. A thinner wire has a smaller cross-sectional area, so the same current requires a higher drift velocity. Higher drift velocity means a larger magnetic force on each charge carrier, producing a larger Hall voltage. The fact that $\epsilon \propto 1/d$ (not $1/d^2$) results from two competing effects: the Hall separation distance increases with diameter, but drift velocity decreases faster (as $1/d^2$). The net effect is the inverse first-power relationship.

The Hall voltage is inversely proportional to wire diameter: $\epsilon = \frac{4BI}{nq\pi d} \propto \frac{1}{d}$.

A patient with a pacemaker is mistakenly being scanned for an MRI image. A 10.0-cm-long section of pacemaker wire moves at a speed of 10.0 cm/s perpendicular to the MRI unit's magnetic field and a 20.0-mV Hall voltage is induced. What is the magnetic field strength?

Show Solution

Strategy

We use the Hall voltage formula $\epsilon = Blv$ and solve for the magnetic field strength B. This is a straightforward rearrangement of the formula.

Solution

Known quantities:

- Hall voltage: $\epsilon = 20.0\ \text{mV} = 0.0200\ \text{V}$
- Wire length: $l = 10.0\ \text{cm} = 0.100\ \text{m}$
- Wire velocity: $v = 10.0\ \text{cm/s} = 0.100\ \text{m/s}$

From $\epsilon = Blv$, solving for B:

$$B = \frac{\epsilon}{lv}$$
$$B = \frac{0.0200\ \text{V}}{(0.100\ \text{m})(0.100\ \text{m/s})}$$
$$B = \frac{0.0200\ \text{V}}{0.0100\ \text{m}^2/\text{s}}$$
$$B = 2.00\ \text{T}$$

Discussion

A 2.00 T field is typical for clinical MRI machines (common values are 1.5 T and 3.0 T). This problem highlights a serious safety concern: pacemaker wires moving in strong magnetic fields can induce voltages that may interfere with the pacemaker's operation or even damage it. This is why patients with pacemakers are generally not allowed to undergo MRI scans, unless they have specifically MRI-compatible devices. The motion could come from breathing, heartbeat, or any patient movement.

The magnetic field strength is 2.00 T.

Glossary

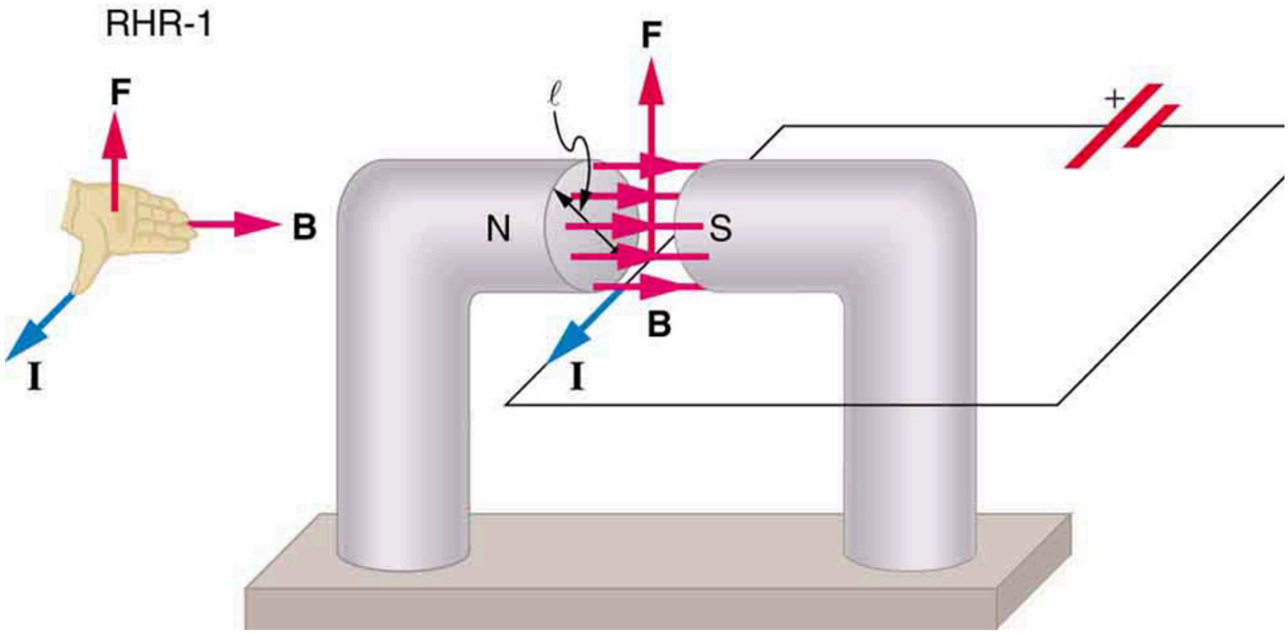
Hall effect
: the creation of voltage across a current-carrying conductor by a magnetic field

Hall emf
: the electromotive force created by a current-carrying conductor by a magnetic field, $\epsilon = Blv$

Magnetic Force on a Current-Carrying Conductor

- Describe the effects of a magnetic force on a current-carrying conductor.
- Calculate the magnetic force on a current-carrying conductor.

Because charges ordinarily cannot escape a conductor, the magnetic force on charges moving in a conductor is transmitted to the conductor itself.



The magnetic field exerts a force on a current-carrying wire in a direction given by the right hand rule 1 (the same direction as that on the individual moving charges). This force can easily be large enough to move the wire, since typical currents consist of very large numbers of moving charges.

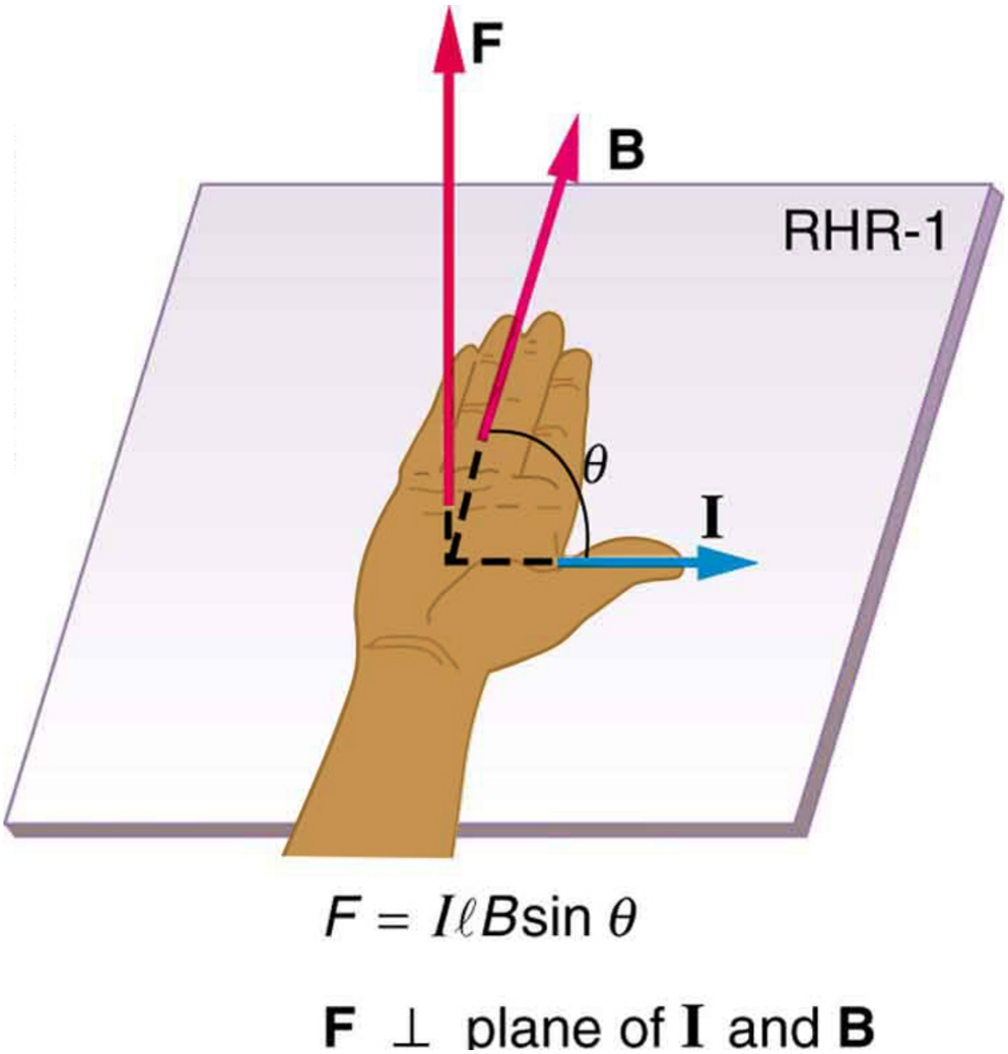
We can derive an expression for the magnetic force on a current by taking a sum of the magnetic forces on individual charges. (The forces add because they are in the same direction.) The force on an individual charge moving at the drift velocity v_d is given by $F = qv_d B \sin\theta$. Taking B to be uniform over a length of wire l and zero elsewhere, the total magnetic force on the wire is then $F = (qv_d B \sin\theta)(N)$, where N ** is the number of charge carriers in the section of wire of length l . Now, $N = nV$, where n is the number of charge carriers per unit volume and V is the volume of wire in the field. Noting that $V = Al$, where A is the cross-sectional area of the wire, then the force on the wire is $F = (qv_d B \sin\theta)(nAl)$. Gathering terms,

$$F=(nqAv_d)lB\sin\theta.$$

Because $nqAv_d = I$ (see Current),

$$F=IlB\sin\theta$$

is the equation for _magnetic force on a length l of wire carrying a current I in a uniform magnetic field B _, as shown in [Figure 2]. If we divide both sides of this expression by l , we find that the magnetic force per unit length of wire in a uniform field is $\frac{F}{l} = IB\sin\theta$. The direction of this force is given by RHR-1, with the thumb in the direction of the current I . Then, with the fingers in the direction of B , a perpendicular to the palm points in the direction of F , as in [Figure 2].



The force on a current-carrying wire in a magnetic field is $F=IlB \sin \theta$. Its direction is given by RHR-1.

Calculating Magnetic Force on a Current-Carrying Wire: A Strong Magnetic Field

Calculate the force on the wire shown in [Figure 1], given $B=1.50\text{T}$, $l=5.00\text{cm}$, and $I=20.0\text{A}$. **Strategy**

The force can be found with the given information by using $F=IlB\sin\theta$ and noting that the angle θ between I and B is 90º, so that $\sin\theta=1$.

Solution

Entering the given values into $F=IlB\sin\theta$ yields

$$F=IlB\sin\theta=(20.0\text{A})(0.0500\text{m})(1.50\text{T})(1).$$

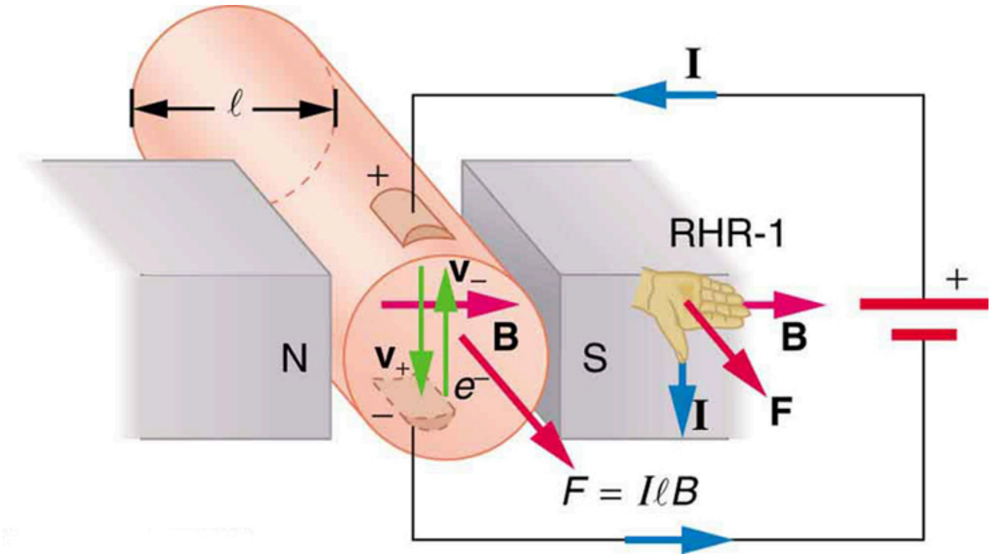
The units for tesla are $1\text{T}=\frac{N}{\text{A}\cdot\text{m}}$; thus,

$$F=1.50\text{N}.$$

Discussion

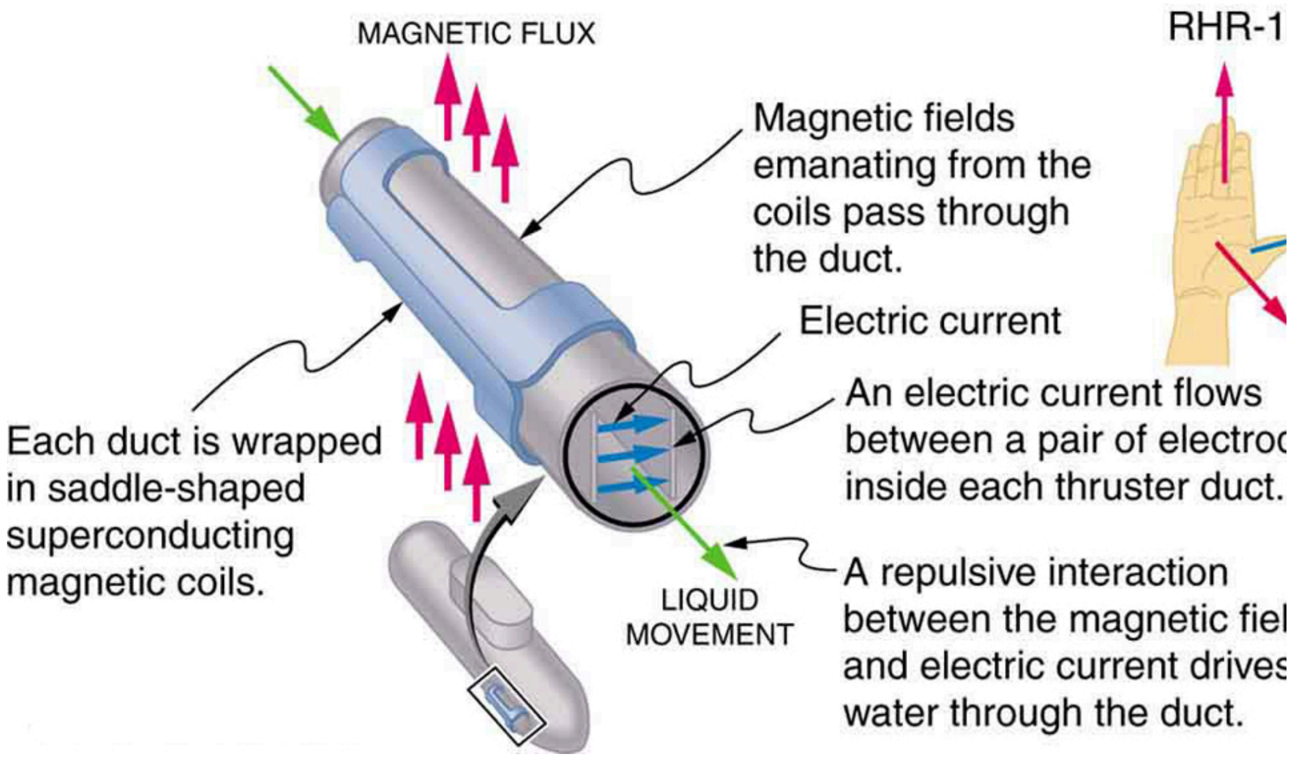
This large magnetic field creates a significant force on a small length of wire.

Magnetic force on current-carrying conductors is used to convert electric energy to work. (Motors are a prime example—they employ loops of wire and are considered in the next section.) Magnetohydrodynamics (MHD) is the technical name given to a clever application where magnetic force pumps fluids without moving mechanical parts. (See [Figure 3].)



Magnetohydrodynamics. The magnetic force on the current passed through this fluid can be used as a nonmechanical pump.

A strong magnetic field is applied across a tube and a current is passed through the fluid at right angles to the field, resulting in a force on the fluid parallel to the tube axis as shown. The absence of moving parts makes this attractive for moving a hot, chemically active substance, such as the liquid sodium employed in some nuclear reactors. Experimental artificial hearts are testing with this technique for pumping blood, perhaps circumventing the adverse effects of mechanical pumps. (Cell membranes, however, are affected by the large fields needed in MHD, delaying its practical application in humans.) MHD propulsion for nuclear submarines has been proposed, because it could be considerably quieter than conventional propeller drives. The deterrent value of nuclear submarines is based on their ability to hide and survive a first or second nuclear strike. As we slowly disassemble our nuclear weapons arsenals, the submarine branch will be the last to be decommissioned because of this ability (See [Figure 4].) Existing MHD drives are heavy and inefficient—much development work is needed.



An MHD propulsion system in a nuclear submarine could produce significantly less turbulence than propellers and allow it to run more silently. The development of a silent drive submarine was dramatized in the book and the film The Hunt for Red October.

Section Summary

- The magnetic force on current-carrying conductors is given by
  $F = IlB\sin\theta,$

where I is the current, l is the length of a straight conductor in a uniform magnetic field B, and θ is the angle between I and B. The force follows RHR-1 with the thumb in the direction of I.

Conceptual Questions

Draw a sketch of the situation in [Figure 1] showing the direction of electrons carrying the current, and use RHR-1 to verify the direction of the force on the wire.

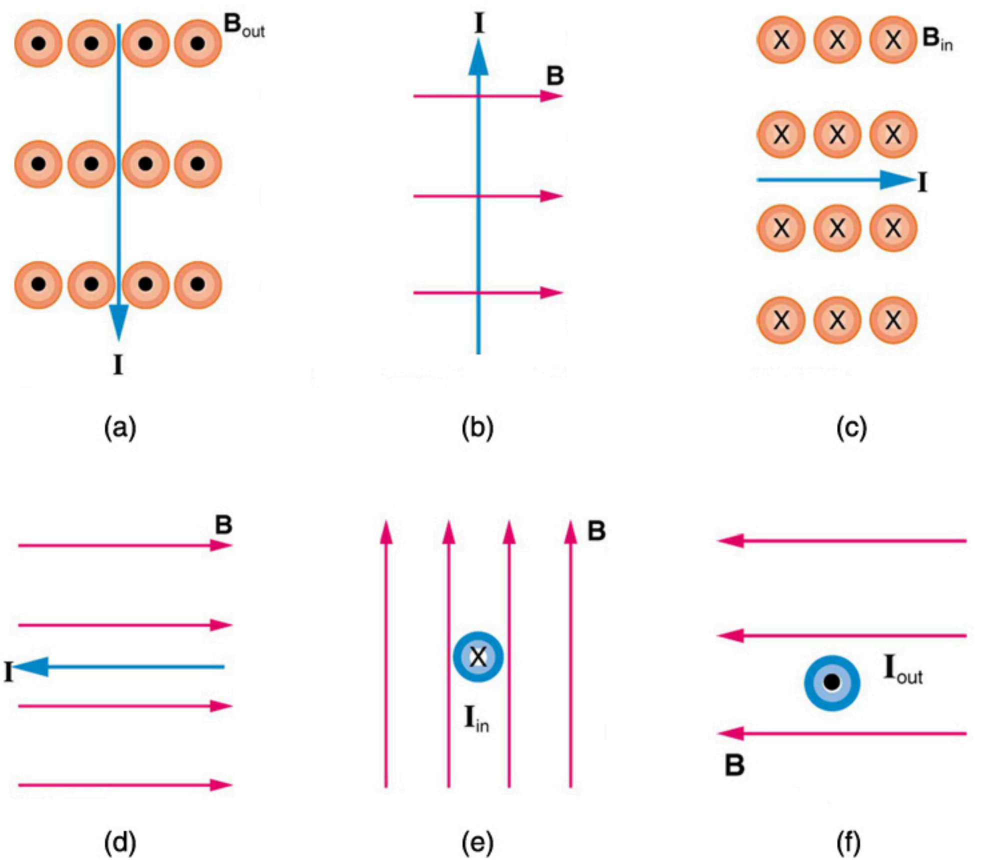
Verify that the direction of the force in an MHD drive, such as that in [Figure 3], does not depend on the sign of the charges carrying the current across the fluid.

Why would a magnetohydrodynamic drive work better in ocean water than in fresh water? Also, why would superconducting magnets be desirable?

Which is more likely to interfere with compass readings, AC current in your refrigerator or DC current when you start your car? Explain.

Problems & Exercises

What is the direction of the magnetic force on the current in each of the six cases in [Figure 5]?



Show Solution

Strategy

The force on a current-carrying wire in a magnetic field follows the same right-hand rule as for moving charges. Point your thumb in the direction of conventional current $\vec{\mathbf{I}}$, fingers in the direction of $\vec{\mathbf{B}}$, and your palm pushes in the direction of the force $\vec{\mathbf{F}}$. This is RHR-1 applied to current instead of velocity.

Solution

(a) Current $\vec{\mathbf{I}}$ points down, $\vec{\mathbf{B}}$ points out of page. Using RHR-1: thumb down, fingers out → palm pushes **west (left)**.

(b) Current $\vec{\mathbf{I}}$ points up, $\vec{\mathbf{B}}$ points right. Using RHR-1: thumb up, fingers right → palm pushes **into the page**.

(c) Current $\vec{\mathbf{I}}$ points right, $\vec{\mathbf{B}}$ points into page. Using RHR-1: thumb right, fingers into page → palm pushes **north (up)**.

(d) Current $\vec{\mathbf{I}}$ points left, $\vec{\mathbf{B}}$ points right. The current is antiparallel to the field, so $\theta = 180°$ and $\sin\theta = 0$. Therefore, there is **no force**.

(e) Current $\vec{\mathbf{I}}$ points into page, $\vec{\mathbf{B}}$ points up. Using RHR-1: thumb into page, fingers up → palm pushes **east (right)**.

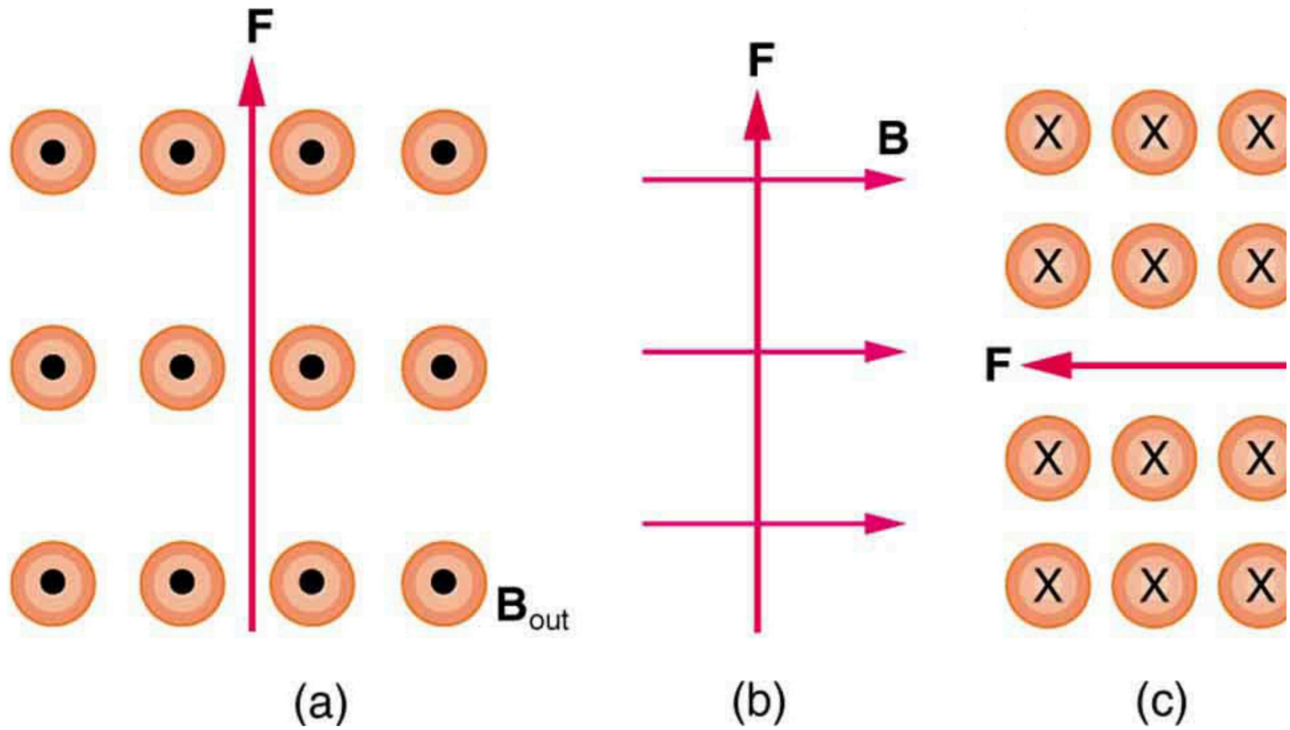
(f) Current $\vec{\mathbf{I}}$ points out of page, $\vec{\mathbf{B}}$ points left. Using RHR-1: thumb out of page, fingers left → palm pushes **south (down)**.

Discussion

These are the same results as for positive charges moving in the same directions—this makes sense because conventional current is defined as the direction of positive charge flow. The case (d) shows why power transmission lines oriented parallel to Earth's magnetic field experience no magnetic force.

(a) West (left); (b) Into page; (c) North (up); (d) No force; (e) East (right); (f) South (down).

What is the direction of a current that experiences the magnetic force shown in each of the three cases in [Figure 6], assuming the current runs perpendicular to B ?



Show Solution

Strategy

We work backwards using RHR-1. Given $\vec{\mathbf{F}}$ and $\vec{\mathbf{B}}$, we find $\vec{\mathbf{I}}$. With fingers pointing along $\vec{\mathbf{B}}$ and palm facing the direction of $\vec{\mathbf{F}}$, the thumb points in the direction of current.

Solution

(a) Force $\vec{\mathbf{F}}$ points up, $\vec{\mathbf{B}}$ points out of page. Using RHR-1 in reverse: fingers out of page, palm facing up → thumb points **east (right)**.

(b) Force $\vec{\mathbf{F}}$ points up, $\vec{\mathbf{B}}$ points right. Using RHR-1 in reverse: fingers right, palm facing up → thumb points **out of the page**.

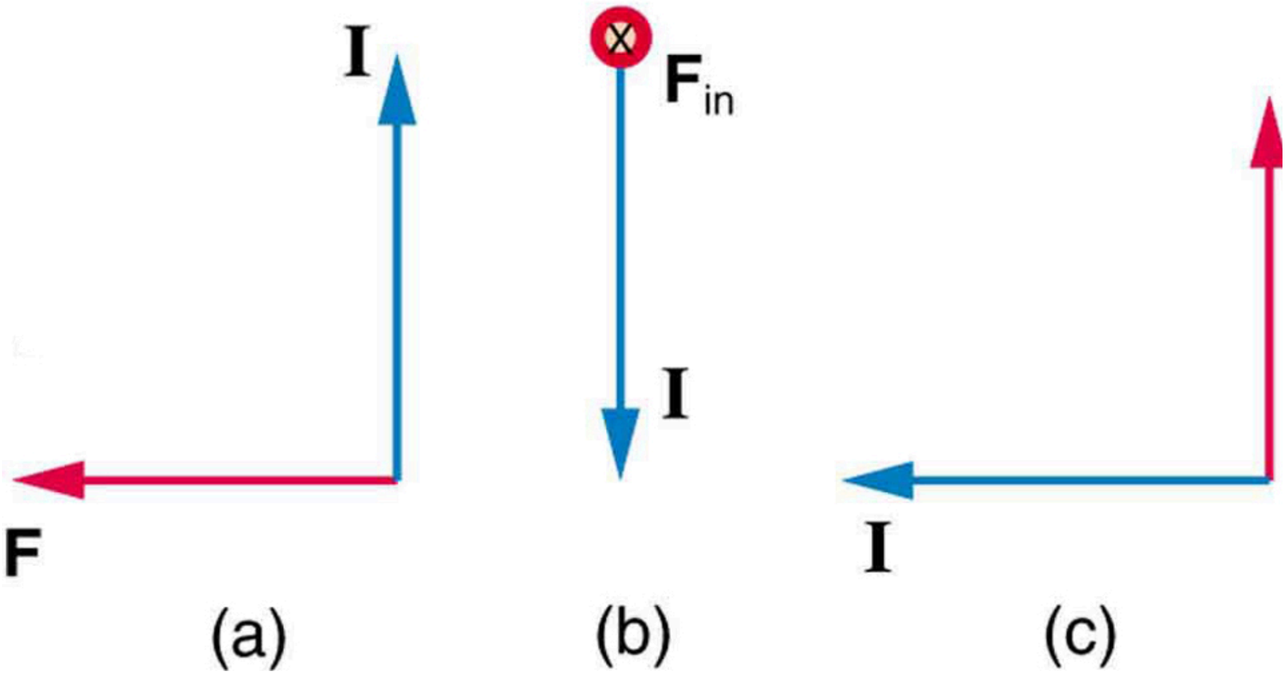
(c) Force $\vec{\mathbf{F}}$ points left, $\vec{\mathbf{B}}$ points into page. Using RHR-1 in reverse: fingers into page, palm facing left → thumb points **south (down)**.

Discussion

This inverse problem-solving technique is useful for designing electromagnetic devices. For instance, in an electric motor, you know the field direction and desired force direction, so you need to determine how to orient the current-carrying windings.

(a) East (right); (b) Out of the page; (c) South (down).

What is the direction of the magnetic field that produces the magnetic force shown on the currents in each of the three cases in [Figure 7], assuming $\vec{\mathbf{B}}$ is perpendicular to $\vec{\mathbf{I}}$?



Show Solution

Strategy

We work backwards using RHR-1. Given $\vec{\mathbf{F}}$ and $\vec{\mathbf{I}}$, we find $\vec{\mathbf{B}}$ such that the cross product $\vec{\mathbf{I}} \times \vec{\mathbf{B}}$ gives the observed force direction.

Solution

(a) Current $\vec{\mathbf{I}}$ points up, force $\vec{\mathbf{F}}$ points left. Using RHR-1: thumb up ($\vec{\mathbf{I}}$), palm facing left ($\vec{\mathbf{F}}$) → fingers point **into the page**.

(b) Current $\vec{\mathbf{I}}$ points down, force $\vec{\mathbf{F}}$ points into page. Using RHR-1: thumb down, palm into page → fingers point **west (left)**.

(c) Current $\vec{\mathbf{I}}$ points left, force $\vec{\mathbf{F}}$ points up. Using RHR-1: thumb left, palm up → fingers point **out of the page**.

Discussion

Finding the magnetic field direction from known force and current is important for magnetic field mapping. By measuring forces on calibrated current-carrying wires, we can determine both the magnitude and direction of unknown magnetic fields.

(a) Into the page; (b) West (left); (c) Out of the page.

(a) What is the force per meter on a lightning bolt at the equator that carries 20 000 A perpendicular to the Earth's 3.00×10^{-5}-T field? (b) What is the direction of the force if the current is straight up and the Earth's field direction is due north, parallel to the ground?

Show Solution

Strategy

The force per unit length on a current-carrying wire is $F/L = BIL\sin\theta/L = BI\sin\theta$. For perpendicular orientation, $\sin\theta = 1$. For direction, we use RHR-1 with the current up and field north.

Solution

Known quantities:

- Current: $I = 20{,}000$ A
- Magnetic field: $B = 3.00 \times 10^{-5}$ T
- Angle: $\theta = 90°$ (perpendicular)

(a) Force per meter:

$$\frac{F}{L} = BI\sin\theta = BI$$

$$\frac{F}{L} = (3.00 \times 10^{-5}\ \text{T})(20{,}000\ \text{A})$$

$$\frac{F}{L}=0.600 \text{ N/m}$$

(b) Direction:

Using RHR-1: current straight up (thumb up), field due north (fingers north) → palm pushes **west**.

Discussion

The force of 0.6 N/m seems modest for such an enormous current (20,000 A), but this is because Earth's magnetic field is weak. Over a 1 km lightning bolt, this would be 600 N—significant but still small compared to the enormous electromagnetic forces within the lightning channel itself. The westward deflection is typically negligible compared to the irregular path lightning takes through the atmosphere.

(a) The force per meter is 0.600 N/m.

(b) The force direction is west (toward the left when facing north).

(a) A DC power line for a light-rail system carries 1000 A at an angle of $30.0°$ to the Earth's 5.00×10^{-5}-T field. What is the force on a 100-m section of this line? (b) Discuss practical concerns this presents, if any.

Show Solution

Strategy

The force on a current-carrying conductor is $F = BIL\sin\theta$. Here we need to account for the angle between the current direction and the magnetic field. The wire length $L = 100$ m is the section in the field.

Solution

Known quantities:

- Current: $I = 1000$ A
- Magnetic field: $B = 5.00 \times 10^{-5}$ T
- Wire length: $L = 100$ m
- Angle: $\theta = 30.0°$

(a) Force on the wire:

$$F = BIL\sin\theta$$
$$F = (5.00\times10^{-5} \text{ T})(1000 \text{ A})(100 \text{ m})(\sin 30.0°)$$
$$F = (5.00\times10^{-5})(1000)(100)(0.500) \text{ N}$$
$$F = 2.50 \text{ N}$$

(b) Practical concerns:

This force of 2.50 N (about 0.56 pounds) on 100 m of wire is very small compared to:

- The weight of the wire itself (several hundred newtons for typical power cables)
- The tension in the wire from stringing between poles (thousands of newtons)
- Wind loads on the wire (can be tens of newtons per meter)

Therefore, the magnetic force from Earth's field is negligible and does not require special engineering consideration. Power line engineers worry far more about ice loading, wind, and thermal expansion than magnetic forces.

Discussion

If the wire were oriented perpendicular to the field ($\theta = 90°$), the force would double to 5.0 N, still negligible. The force would only become significant with much stronger fields (like near a large industrial electromagnet) or with very high currents.

(a) The force on the 100-m section is 2.50 N.

(b) This force is negligible compared to the wire's weight and other structural loads. No special concerns arise from this magnetic force.

What force is exerted on the water in an MHD drive utilizing a 25.0-cm-diameter tube, if 100-A current is passed across the tube that is perpendicular to a 2.00-T magnetic field? (The relatively small size of this force indicates the need for very large currents and magnetic fields to make practical MHD drives.)

Show Solution

Strategy

In a magnetohydrodynamic (MHD) drive, current flows across a conducting fluid (like seawater) in a magnetic field, producing a force on the fluid that propels it. The force is $F = BIL$, where L is the length of the current path—here, the tube diameter.

Solution

Known quantities:

- Current: $I = 100\text{ A}$
- Magnetic field: $B = 2.00\text{ T}$
- Tube diameter (current path length): $L = 25.0\text{ cm} = 0.250\text{ m}$
- Angle: $\theta = 90°$ (perpendicular)

$$F = BIL\sin\theta = BIL$$
$$F = (2.00\text{ T})(100\text{ A})(0.250\text{ m})$$
$$F = 50.0\text{ N}$$

Discussion

This 50 N force is only about 11 pounds—not nearly enough to effectively propel a marine vessel. For comparison, a small outboard motor produces thousands of newtons of thrust. The problem notes that this demonstrates why practical MHD drives require:

- Very large currents (thousands of amperes)
- Very strong magnetic fields (often superconducting magnets)
- Large cross-sections for the current path

The Japanese experimental ship *Yamato 1* (1992) used a 4 T superconducting magnet and achieved only about 8 knots. MHD drives are quiet (no propeller noise) but currently remain impractical for most applications due to their low efficiency and the engineering challenges of high-current, high-field systems in seawater.

The force on the water is 50.0 N, which is relatively small for propulsion purposes.

A wire carrying a 30.0-A current passes between the poles of a strong magnet that is perpendicular to its field and experiences a 2.16-N force on the 4.00 cm of wire in the field. What is the average field strength?

Show Solution

Strategy

We use $F = BIL\sin\theta$ and solve for the magnetic field B. Since the wire is perpendicular to the field, $\sin\theta = 1$.

Solution

Known quantities:

- Current: $I = 30.0\text{ A}$
- Force: $F = 2.16\text{ N}$
- Wire length in field: $L = 4.00\text{ cm} = 0.0400\text{ m}$
- Angle: $\theta = 90°$

From $F = BIL$, solving for B:

$$B = FIL$$
$$B = 2.16\text{ N}(30.0\text{ A})(0.0400\text{ m})$$
$$B = 2.161.20\text{ T} = 1.80\text{ T}$$

Discussion

A field strength of 1.80 T is quite strong—achievable with high-quality permanent magnets (like neodymium-iron-boron) or with electromagnets. This type of measurement, using a known current and measuring the force on a known length of wire, is actually one way to measure magnetic field strength. The technique is called a current balance.

The average magnetic field strength is 1.80 T.

(a) A 0.750-m-long section of cable carrying current to a car starter motor makes an angle of $60°$ with the Earth's $5.50 \times 10^{-5}\text{T}$ field. What is the current when the wire experiences a force of $7.00 \times 10^{-3}\text{N}$? (b) If you run the wire between the poles of a strong horseshoe magnet, subjecting 5.00 cm of it to a 1.75-T field, what force is exerted on this segment of wire?

Show Solution

Strategy

Part (a) asks us to find current given force, field, length, and angle using $F = BIL\sin\theta$. Part (b) uses the current found in (a) with a different field and length to find force.

Solution

(a) Finding the current:

Known quantities:

- Wire length: $L = 0.750\text{ m}$
- Magnetic field: $B = 5.50 \times 10^{-5}\text{ T}$
- Force: $F = 7.00 \times 10^{-3}\text{ N}$
- Angle: $\theta = 60°$

From $F = BIL\sin\theta$, solving for I:

$$I = \frac{F}{BL\sin\theta}$$
$$I = \frac{7.00 \times 10^{-3}\text{ N}}{(5.50 \times 10^{-5}\text{ T})(0.750\text{ m})(\sin 60°)}$$
$$I = \frac{7.00 \times 10^{-3}}{(5.50 \times 10^{-5})(0.750)(0.866)}$$
$$I = \frac{7.00 \times 10^{-3}}{3.57 \times 10^{-5}}$$
$$I = 196\text{ A}$$

(b) Force in the horseshoe magnet:

Known quantities:

- Current: $I = 196\text{ A}$ (from part a)
- Length in field: $L = 5.00\text{ cm} = 0.0500\text{ m}$
- Magnetic field: $B = 1.75\text{ T}$
- Angle: $\theta = 90°$ (assumed perpendicular)

$$F = BIL = (1.75\text{ T})(196\text{ A})(0.0500\text{ m})$$
$$F = 17.2\text{ N}$$

Discussion

The 196 A current is very high—typical of starter motor currents, which must deliver large power briefly. The force in Earth’s field is tiny (7 mN), but in the strong horseshoe magnet (1.75 T), the force on just 5 cm of wire is 17.2 N—enough to feel distinctly. This illustrates how the same current can experience vastly different forces depending on the magnetic environment.

(a) The current is 196 A.

(b) The force on the 5.00-cm segment in the 1.75-T field is 17.2 N.

(a) What is the angle between a wire carrying an 8.00-A current and the 1.20-T field it is in if 50.0 cm of the wire experiences a magnetic force of 2.40 N?

(b) What is the force on the wire if it is rotated to make an angle of $90°$ with the field?

Show Solution

Strategy

Part (a) uses $F = BIL\sin\theta$ solved for $\sin\theta$ to find the angle. Part (b) calculates the maximum force when perpendicular.

Solution

Known quantities:

- Current: $I = 8.00\text{ A}$
- Magnetic field: $B = 1.20\text{ T}$
- Wire length: $L = 50.0\text{ cm} = 0.500\text{ m}$
- Force: $F = 2.40\text{ N}$

(a) Finding the angle:

From $F = BIL\sin\theta$:

$$\sin\theta = \frac{F}{BIL}$$
$$\sin\theta = \frac{2.40\text{ N}}{(1.20\text{ T})(8.00\text{ A})(0.500\text{ m})}$$
$$\sin\theta = \frac{2.40}{4.80} = 0.500$$
$$\theta = \arcsin(0.500) = 30°$$

(b) Force at 90°:

$$F_{max} = BIL\sin(90°) = BIL$$
$$F_{max} = (1.20\ \text{T})(8.00\ \text{A})(0.500\ \text{m})$$
$$F_{max} = 4.80\ \text{N}$$

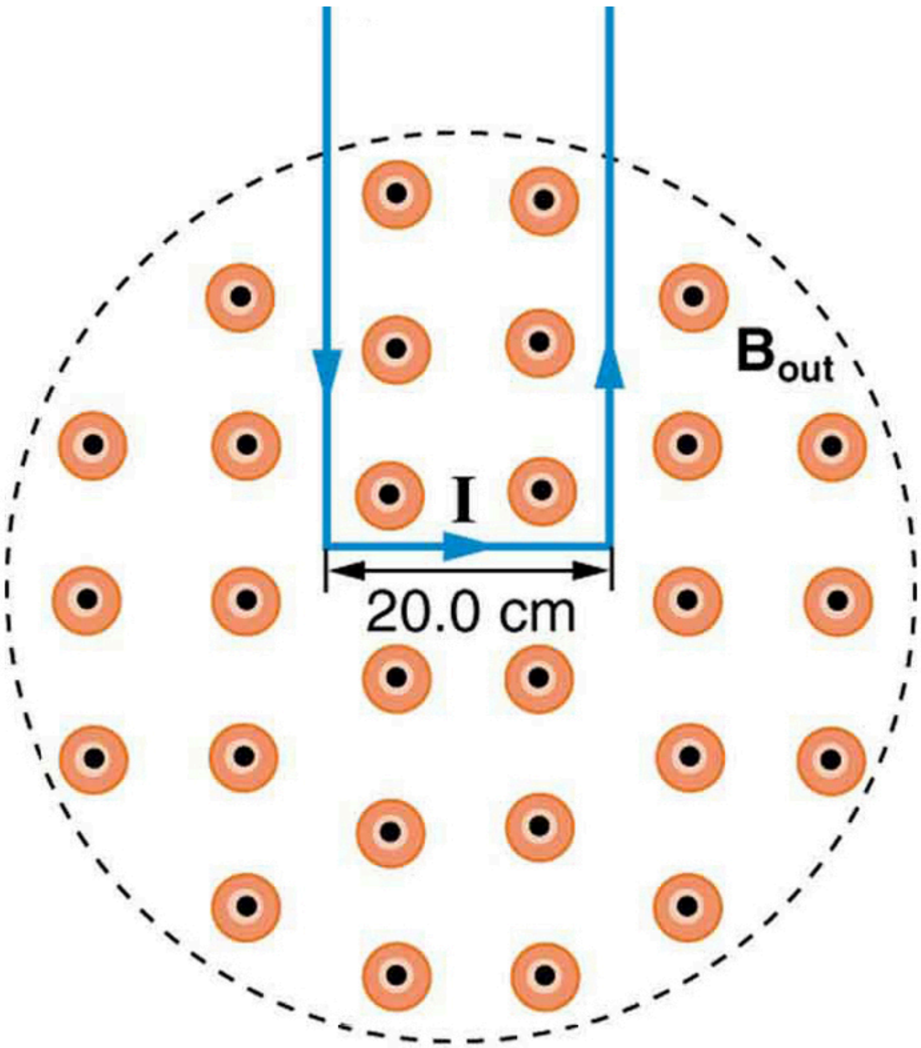
Discussion

The force doubles when the wire is rotated from 30° to 90°. This is because $\sin(90°) = 1.00$ is twice $\sin(30°) = 0.50$. The angular dependence of magnetic force is important in motor design—maximum torque occurs when the coil plane is parallel to the field (so current is perpendicular to field).

(a) The angle between the wire and the magnetic field is $30°$.

(b) The force on the wire at 90° is 4.80 N.

The force on the rectangular loop of wire in the magnetic field in [Figure 8] can be used to measure field strength. The field is uniform, and the plane of the loop is perpendicular to the field. (a) What is the direction of the magnetic force on the loop? Justify the claim that the forces on the sides of the loop are equal and opposite, independent of how much of the loop is in the field and do not affect the net force on the loop. (b) If a current of 5.00 A is used, what is the force per tesla on the 20.0-cm-wide loop?



A rectangular loop of wire carrying a current is perpendicular to a magnetic field. The field is uniform in the region shown and is zero outside that region.

Show Solution

Strategy

We analyze the forces on each segment of the loop separately using RHR-1. The net force comes only from the bottom segment in the field. The force per tesla is $F/B = IL$.

Solution

(a) Direction and force analysis:

Looking at the figure:

- **Bottom segment:** Current flows right, $\vec{\mathbf{B}}$ is out of page. Using RHR-1: thumb right, fingers out → force is **upward** (into the field region).
- **Left side segment:** Current flows down. Where this segment is in the field, $\vec{\mathbf{B}}$ is out of page. Force is to the **left**.
- **Right side segment:** Current flows up. Where this segment is in the field, $\vec{\mathbf{B}}$ is out of page. Force is to the **right**.

The left and right side forces are **equal and opposite** because:

1. Both segments carry the same current I
2. They experience the same field B (uniform field)
3. The portions inside the field have equal lengths (whatever fraction is inside)
4. The currents flow in opposite directions (down vs. up)

Therefore, these horizontal forces cancel regardless of how much of the loop is in the field. The only unbalanced force is from the bottom segment—the **net force is upward** (into the field region).

(b) Force per tesla:

For the bottom segment (width $L = 20.0$ cm = 0.200 m):

$$\frac{F}{B} = IL$$
$$\frac{F}{B} = (5.00 \text{ A})(0.200 \text{ m}) = 1.00 \text{ N/T}$$

Discussion

This configuration is the basis of a current balance, used historically to define the ampere. By measuring the force and knowing the current and dimensions, the field strength can be determined. Alternatively, with a known field, the current can be measured. The key insight is that only the segment perpendicular to its own direction and in the field contributes to the net force.

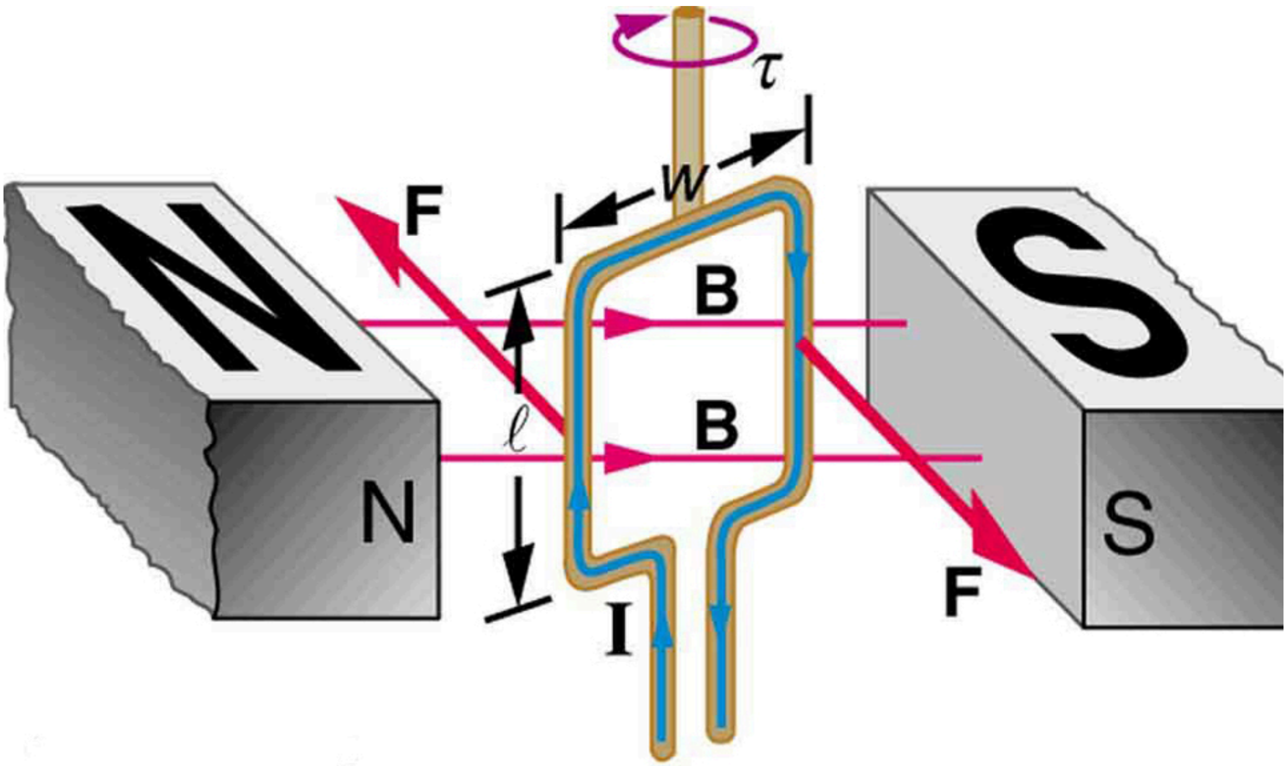
(a) The net force on the loop is directed upward (into the field region). The forces on the left and right sides are equal and opposite because they carry equal currents in opposite directions through equal lengths of the same uniform field, so they cancel and do not contribute to the net force.

(b) The force per tesla on the loop is 1.00 N/T.

Torque on a Current Loop: Motors and Meters

- Describe how motors and meters work in terms of torque on a current loop.
- Calculate the torque on a current-carrying loop in a magnetic field.

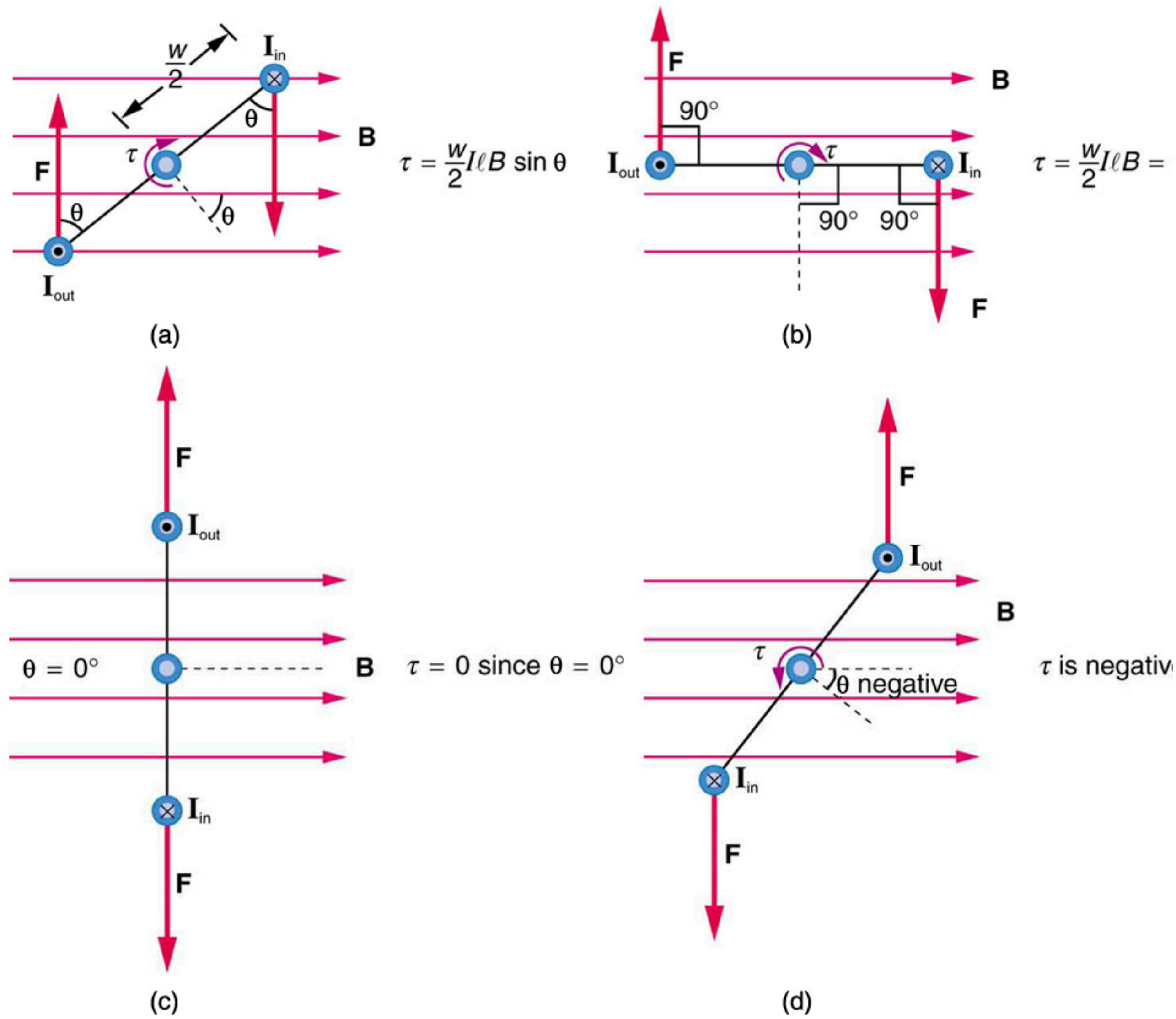
Motors are the most common application of magnetic force on current-carrying wires. Motors have loops of wire in a magnetic field. When current is passed through the loops, the magnetic field exerts torque on the loops, which rotates a shaft. Electrical energy is converted to mechanical work in the process. (See [Figure 1].)



Torque on a current loop. A current-carrying loop of wire attached to a vertically rotating shaft feels magnetic forces that produce a clockwise torque as viewed from above.

Let us examine the force on each segment of the loop in [Figure 1] to find the torques produced about the axis of the vertical shaft. (This will lead to a useful equation for the torque on the loop.) We take the magnetic field to be uniform over the rectangular loop, which has width w and height l . First, we note that the forces on the top and bottom segments are vertical and, therefore, parallel to the shaft, producing no torque. Those vertical forces are equal in magnitude and opposite in direction, so that they also produce no net force on the loop. [Figure 2] shows views of the loop from above. Torque is defined as $\tau = rF \sin \theta$, where F is the force, r is the distance from the pivot that the force is applied, and θ is the angle between r and F . As seen in [Figure 2](a), right hand rule 1 gives the forces on the sides to be equal in magnitude and opposite in direction, so that the net force is again zero. However, each force produces a clockwise torque. Since $r=w/2$, the torque on each vertical segment is $\left(w/2\right)F \sin \theta$, and the two add to give a total torque.

$$\tau =\frac{w}{2}F \sin \theta +\frac{w}{2}F \sin \theta = w F \sin \theta $$



Top views of a current-carrying loop in a magnetic field. (a) The equation for torque is derived using this view. Note that the perpendicular to the loop makes an angle θ with the field that is the same as the angle between $w/2$ and F . (b) The maximum torque occurs when θ is a right angle and $\sin \theta =1$. (c) Zero (minimum) torque occurs when θ is zero and $\sin \theta =0$. (d) The torque reverses once the loop rotates past $\theta=0$.

Now, each vertical segment has a length l that is perpendicular to B , so that the force on each is $F=IlB$. Entering F into the expression for torque yields

$$\tau =wIlB \sin \theta . $$

If we have a multiple loop of N turns, we get N times the torque of one loop. Finally, note that the area of the loop is $A= wl$; the expression for the torque becomes

$$\tau =NIAB \sin \theta . $$

This is the torque on a current-carrying loop in a uniform magnetic field. This equation can be shown to be valid for a loop of any shape. The loop carries a current I , has N turns, each of area A , and the perpendicular to the loop makes an angle θ with the field B. The net force on the loop is zero.

Calculating Torque on a Current-Carrying Loop in a Strong Magnetic Field

Find the maximum torque on a 100-turn square loop of a wire of 10.0 cm on a side that carries 15.0 A of current in a 2.00-T field.

Strategy

Torque on the loop can be found using $\tau =NIAB \sin \theta$. Maximum torque occurs when $\theta =90 ^\circ$ and $\sin \theta =1$.

Solution

For $\sin \theta =1$, the maximum torque is

$${\tau }_{\text{max}}=NIAB. $$

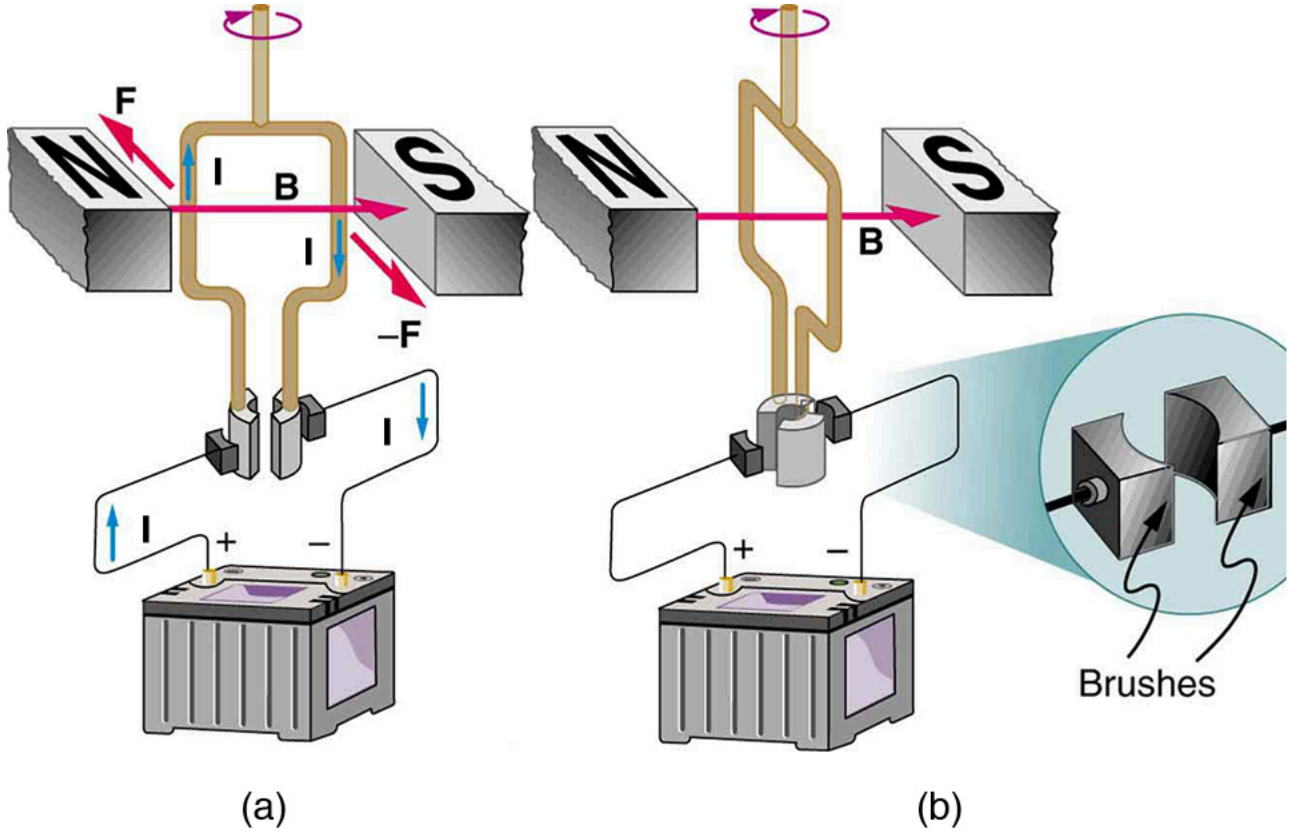
Entering known values yields

$$\begin{array}{lll}{\tau }_{\text{max}}& =& \left(100\right)\left(15.0 \text{A}\right)\left(0.100 {\text{m}}^{2}\right)\left(2.00 \text{T}\right)\\ & =& 30.0 \text{N}\cdot \text{m}.\end{array} $$

Discussion

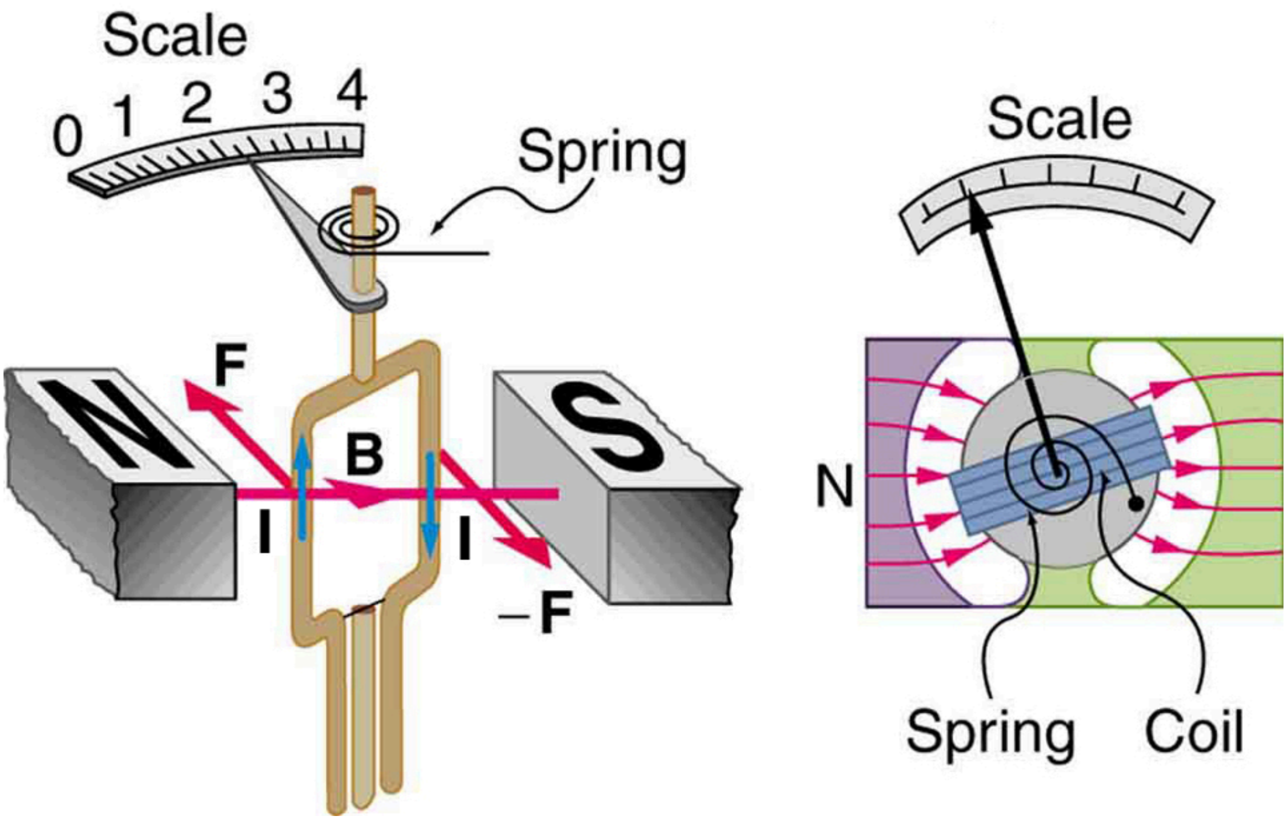
This torque is large enough to be useful in a motor.

The torque found in the preceding example is the maximum. As the coil rotates, the torque decreases to zero at $\theta =0$. The torque then *reverses* its direction once the coil rotates past $\theta =0$. (See [Figure 2](d).) This means that, unless we do something, the coil will oscillate back and forth about equilibrium at $\theta =0$. To get the coil to continue rotating in the same direction, we can reverse the current as it passes through $\theta =0$ with automatic switches called *brushes*. (See [Figure 3].)



(a) As the angular momentum of the coil carries it through $\theta =0$, the brushes reverse the current to keep the torque clockwise. (b) The coil will rotate continuously in the clockwise direction, with the current reversing each half revolution to maintain the clockwise torque.

Meters, such as those in analog fuel gauges on a car, are another common application of magnetic torque on a current-carrying loop. [Figure 4] shows that a meter is very similar in construction to a motor. The meter in the figure has its magnets shaped to limit the effect of θ by making B perpendicular to the loop over a large angular range. Thus the torque is proportional to I and not θ . A linear spring exerts a counter-torque that balances the current-produced torque. This makes the needle deflection proportional to I . If an exact proportionality cannot be achieved, the gauge reading can be calibrated. To produce a galvanometer for use in analog voltmeters and ammeters that have a low resistance and respond to small currents, we use a large loop area A , high magnetic field B , and low-resistance coils.



Meters are very similar to motors but only rotate through a part of a revolution. The magnetic poles of this meter are shaped to keep the component of B perpendicular to the loop constant, so that the torque does not depend on θ and the deflection against the return spring is proportional only to the current I .

Section Summary

- The torque τ on a current-carrying loop of any shape in a uniform magnetic field is
  $$\tau =NIAB \sin \theta , $$

  where N is the number of turns, I is the current, A is the area of the loop, B is the magnetic field strength, and θ is the angle between the perpendicular to the loop and the magnetic field.

Conceptual Questions

Draw a diagram and use RHR-1 to show that the forces on the top and bottom segments of the motor's current loop in [Figure 1] are vertical and produce no torque about the axis of rotation.

Problems & Exercises

(a) By how many percent is the torque of a motor decreased if its permanent magnets lose 5.0% of their strength? (b) How many percent would the current need to be increased to return the torque to original values?

Show Solution

Strategy

Since torque is $\tau = NIAB\sin\theta$, it is directly proportional to the magnetic field strength B. If B decreases by some percentage, τ decreases by the same percentage. To restore the original torque with reduced B, we must increase I proportionally.

Solution

(a) Percent decrease in torque:

Since $\tau \propto B$:

$$ \frac{\Delta\tau}{\tau_0} = \frac{\Delta B}{B_0} $$

If B decreases by 5.00%, then τ also **decreases by 5.00%**.

(b) Required current increase:

Let $B' = 0.950 B_0$ (5% reduction). For the torque to return to τ_0:

$$ \tau_0 = NI'AB'\sin\theta = NI'A(0.950 B_0)\sin\theta $$

Since originally $\tau_0 = NI_0AB_0\sin\theta$:

$$ I' = \frac{I_0}{0.950} = 1.0526 I_0 $$

The current must increase by $\frac{1.0526 - 1}{1} \times 100\% = 5.26\%$.

Discussion

This is an inverse proportionality: a 5% decrease in B requires slightly more than a 5% increase in I to compensate (specifically, $1/0.95 = 1.0526$). This relationship is important for motor performance as permanent magnets age and lose strength.

(a) The torque decreases by 5.00%.

(b) The current must increase by 5.26% to restore the original torque.

(a) What is the maximum torque on a 150-turn square loop of wire 18.0 cm on a side that carries a 50.0-A current in a 1.60-T field? (b) What is the torque when θ is 10.9° ?

Show Solution

Strategy

We use $\tau = NIAB\sin\theta$. Maximum torque occurs when $\sin\theta = 1$ (i.e., $\theta = 90°$). The area of the square loop is $A = s^2$ where s is the side length.

Solution

Known quantities:

- Number of turns: $N = 150$
- Side length: $s = 18.0 \text{ cm} = 0.180 \text{ m}$
- Current: $I = 50.0 \text{ A}$
- Magnetic field: $B = 1.60 \text{ T}$

Area of the loop: $A = s^2 = (0.180 \text{ m})^2 = 0.0324 \text{ m}^2$

(a) Maximum torque ($\theta = 90°$):

$$ \tau_{\max} = NIAB = (150)(50.0 \text{ A})(0.0324 \text{ m}^2)(1.60 \text{ T}) $$
$$ \tau_{\max} = 389 \text{ N·m} $$

(b) Torque at $\theta = 10.9°$:

$$ \tau = NIAB\sin\theta = (389 \text{ N·m})(\sin 10.9°) $$
$$ \tau = (389)(0.189) = 73.5 \text{ N·m} $$

Discussion

The torque of 389 N·m at maximum is quite large—comparable to the torque output of a small car engine. At $\theta = 10.9°$, the torque drops to about 19% of maximum because $\sin(10.9°) \approx 0.19$. Motors are designed to operate near $\theta = 90°$ for maximum efficiency.

(a) The maximum torque is 389 N·m.

(b) The torque at $\theta = 10.9°$ is 73.5 N·m.

Find the current through a loop needed to create a maximum torque of $9.00 \text{N}\cdot \text{m} \text{.}$ The loop has 50 square turns that are 15.0 cm on a side and is in a uniform 0.800-T magnetic field.

Show Solution

Strategy

We use $\tau_{\max} = NIAB$ and solve for I. The area of the square loop is $A = s^2$.

Solution

Known quantities:

- Maximum torque: $\tau_{\max} = 9.00 \text{ N·m}$
- Number of turns: $N = 50$
- Side length: $s = 15.0 \text{ cm} = 0.150 \text{ m}$
- Magnetic field: $B = 0.800 \text{ T}$

Area: $A = s^2 = (0.150 \text{ m})^2 = 0.0225 \text{ m}^2$

Solving for current:

$$ I = \frac{\tau_{\max}}{NAB} $$
$$ I = \frac{9.00 \text{ N·m}}{(50)(0.0225 \text{ m}^2)(0.800 \text{ T})} $$
$$ I = \frac{9.00}{0.900} = 10.0 \text{ A} $$

Discussion

This is a moderate current for a practical motor or demonstration device. The loop would need to handle 10 A without overheating, which requires appropriately sized wire.

The required current is 10.0 A.

Calculate the magnetic field strength needed on a 200-turn square loop 20.0 cm on a side to create a maximum torque of $300 \text{N}\cdot \text{m}$ if the loop is carrying 25.0 A.

Show Solution

Strategy

We use $\tau_{\max} = NIAB$ and solve for B.

Solution

Known quantities:

- Maximum torque: $\tau_{\max} = 300 \text{ N·m}$
- Number of turns: $N = 200$
- Side length: $s = 20.0 \text{ cm} = 0.200 \text{ m}$
- Current: $I = 25.0 \text{ A}$

Area: $A = s^2 = (0.200 \text{ m})^2 = 0.0400 \text{ m}^2$

$$ B = \frac{\tau_{\max}}{NIA} $$
$$ B = \frac{300 \text{ N·m}}{(200)(25.0 \text{ A})(0.0400 \text{ m}^2)} $$
$$ B = \frac{300}{200} = 1.50 \text{ T} $$

Discussion

A 1.50 T field is achievable with strong permanent magnets or electromagnets. This torque of 300 N·m is substantial—comparable to a car engine.

The required magnetic field strength is 1.50 T.

Since the equation for torque on a current-carrying loop is $\tau =NIAB \sin \theta$, the units of $\text{N} \cdot \text{m}$ must equal units of $\text{A}\cdot {m}^{2} \text{T}$. Verify this.

Show Solution

Strategy

We use the definition of the tesla in terms of base SI units: $1 \text{ T} = 1 \text{ N/(A·m)}$, and show that the units work out correctly.

Solution

Starting with the units on the right side:

$$ [\text{A} \cdot \text{m}^2 \cdot \text{T}] $$

Substituting the definition of tesla:

$$ = \text{A} \cdot \text{m}^2 \cdot \frac{\text{N}}{\text{A} \cdot \text{m}} $$

The amperes cancel:

$$ = \text{m}^2 \cdot \frac{\text{N}}{\text{m}} $$

One meter cancels:

$$ = \text{m} \cdot \text{N} = \text{N·m} $$

This confirms that $[\text{A} \cdot \text{m}^2 \cdot \text{T}] = [\text{N·m}]$ ✓

Discussion

This dimensional analysis verifies the consistency of the torque equation. The tesla was defined precisely so that electromagnetic equations like this one have consistent units. The fact that N (number of turns) and $\sin\theta$ are dimensionless doesn't affect this analysis.

Verified: $\text{A} \cdot \text{m}^{2} \cdot \text{T} = \text{A} \cdot \text{m}^{2} \cdot \frac{\text{N}}{\text{A} \cdot \text{m}} = \text{N·m}$.

(a) At what angle θ is the torque on a current loop 90.0% of maximum? (b) 50.0% of maximum? (c) 10.0% of maximum?

Show Solution

Strategy

Since $\tau = \tau_{\max}\sin\theta$, we have $\tau/\tau_{\max} = \sin\theta$. We solve for $\theta = \arcsin(\tau/\tau_{\max})$.

Solution

(a) 90.0% of maximum:

$$ \sin\theta = 0.900 $$
$$ \theta = \arcsin(0.900) = 64.2° $$

(b) 50.0% of maximum:

$$ \sin\theta = 0.500 $$
$$ \theta = \arcsin(0.500) = 30.0° $$

(c) 10.0% of maximum:

$$ \sin\theta = 0.100 $$
$$ \theta = \arcsin(0.100) = 5.74° $$

Discussion

These results show that torque drops off slowly at first but then rapidly as θ decreases. At 64°, we still have 90% of max torque, but we need to be at only 5.7° to get down to 10%. This is because the sine function has its steepest slope near $\theta = 0°$. Motors use commutators to keep the coil near $\theta = 90°$ where torque is maximum.

(a) $\theta = 64.2°$

(b) $\theta = 30.0°$

(c) $\theta = 5.74°$

A proton has a magnetic field due to its spin on its axis. The field is similar to that created by a circular current loop $0.650 \times 10^{-15} m$ in radius with a current of $1.05 \times 10^{4} A$ (no kidding). Find the maximum torque on a proton in a 2.50-T field. (This is a significant torque on a small particle.)

Show Solution

Strategy

We model the proton's magnetic moment as a circular current loop and use the torque formula $\tau_{\max} = NIAB$. The area is $A = \pi r^2$ where r is the "radius" of the effective current loop.

Solution

Known quantities:

- Equivalent radius: $r = 0.650 \times 10^{-15} \text{ m}$
- Equivalent current: $I = 1.05 \times 10^{4} \text{ A}$
- Magnetic field: $B = 2.50 \text{ T}$
- Number of turns: $N = 1$

Area of the loop: $A = \pi r^2 = \pi (0.650 \times 10^{-15} \text{ m})^2 = 1.33 \times 10^{-30} \text{ m}^2$

Maximum torque: $\tau_{\max} = NIAB = (1)(1.05 \times 10^{4} \text{ A})(1.33 \times 10^{-30} \text{ m}^2)(2.50 \text{ T})$

$$ \tau_{\max} = 3.49 \times 10^{-26} \text{ N·m} $$

Discussion

This tiny torque on a single proton is indeed significant at the atomic scale. It's responsible for nuclear magnetic resonance (NMR) and MRI imaging. The torque causes protons to precess in the magnetic field at a characteristic frequency (the Larmor frequency), which is detected to create medical images. The enormous "equivalent current" comes from the extremely rapid quantum mechanical spin of the proton.

The maximum torque on the proton is 3.48×10^{-26} N·m.

(a) A 200-turn circular loop of radius 50.0 cm is vertical, with its axis on an east-west line. A current of 100 A circulates clockwise in the loop when viewed from the east. The Earth's field here is due north, parallel to the ground, with a strength of $3.00 \times 10^{-5} \text{T}$. What are the direction and magnitude of the torque on the loop? (b) Does this device have any practical applications as a motor?

Show Solution

Strategy

The loop is vertical with its axis east-west, and the field points north. The magnetic moment of the loop points along its axis. We need to find the angle between the magnetic moment and the field to calculate torque. The direction comes from the right-hand rule for magnetic moment (fingers curl with current, thumb points along moment).

Solution

Known quantities:

- Number of turns: $N = 200$
- Radius: $r = 50.0 \text{ cm} = 0.500 \text{ m}$
- Current: $I = 100 \text{ A}$
- Magnetic field: $B = 3.00 \times 10^{-5} \text{ T}$ (pointing north)

Area of the loop: $A = \pi r^2 = \pi (0.500)^2 = 0.785 \text{ m}^2$

(a) Magnitude and direction:

The loop is vertical with axis east-west. Current flows clockwise when viewed from the east, so by the right-hand rule, the magnetic moment points **west**.

The field points north. The angle between the moment (west) and the field (north) is $\theta = 90°$.

$$ \tau = NIAB\sin\theta = NIAB $$
$$ \tau = (200)(100 \text{ A})(0.785 \text{ m}^2)(3.00 \times 10^{-5} \text{ T}) $$
$$ \tau = 0.471 \text{ N·m} $$

Direction: The torque is $\boldsymbol{\tau} = \boldsymbol{\mu} \times \boldsymbol{B}$ (west × north = upward, but torque tends to align moment with field). The torque acts to rotate the loop so that its moment aligns with the field—this means the torque is directed **downward** (or the loop tries to tip so the west-pointing moment rotates toward north).

(b) Practical application as a motor:

This would be an impractical motor:

- The torque of 0.47 N·m is very weak for the size and current
- Earth's field is too weak for practical motor applications
- The current would need to be reversed to continue rotation
- Efficiency would be extremely low

Discussion

Motors require strong magnetic fields (typically 0.1–2 T from permanent magnets or electromagnets), not Earth's field ($\sim 3 \times 10^{-5}$ T). The field is about 10,000 times too weak for practical use.

(a) The torque magnitude is 0.471 N·m. The direction is such that the loop tends to rotate about a vertical axis, bringing the magnetic moment from west toward north.

(b) This device would not be practical as a motor due to the extremely weak torque from Earth's field, which is about 10,000 times weaker than fields used in actual motors.

Repeat [Exercise 1], but with the loop lying flat on the ground with its current circulating counterclockwise (when viewed from above) in a location where the Earth's field is north, but at an angle $45.0º$ below the horizontal and with a strength of $6.00\times 10^{-5} \text{T}$.

Show Solution

Strategy

With the loop flat on the ground and current counterclockwise (viewed from above), the magnetic moment points **upward** (by right-hand rule). The field points north but 45° below horizontal, so it has both a horizontal (north) and vertical (downward) component. Only the horizontal component creates torque.

Solution

Known quantities (same as Exercise 1):

- $N = 200$, $r = 0.500$ m, $I = 100$ A
- $$ A = 0.785 \text{ m}^2 $$
- Magnetic field: $B = 6.00 \times 10^{-5} \text{ T}$, 45° below horizontal

(a) Magnitude and direction:

The magnetic moment $\boldsymbol{\mu}$ points upward (out of the ground).

The field has components:

- Horizontal: $B_H = B\cos(45°) = (6.00 \times 10^{-5})\cos(45°)$ (north)
- Vertical: $B_V = B\sin(45°)$ (downward)

The angle between the upward moment and the field (pointing north and downward at 45°) is $90° + 45° = 135°$ from the horizontal field component's perspective. However, since the moment is vertical, we find the torque as:

$$ \tau = NIAB\sin\theta $$

The angle θ between the magnetic moment (vertical, upward) and the field (north, 45° below horizontal) is:

$\theta = 90° + 45° = 135°$ from moment to field…

Actually, more simply: the moment is perpendicular to the horizontal, so the torque comes from the horizontal component of B:

$$ \tau = NIAB_H = NIA(B\cos 45°) $$
$$ \tau = (200)(100)(0.785)(6.00 \times 10^{-5})(\cos 45°) $$
$$ \tau = (200)(100)(0.785)(6.00 \times 10^{-5})(0.707) $$
$$ \tau = 0.666 \text{ N·m} $$

Direction: $\boldsymbol{\tau} = \boldsymbol{\mu} \times \boldsymbol{B}$. With moment up and horizontal field component north, the torque points **west**.

(b) Practical considerations:

Same as before—this is not practical as a motor due to the weak field of Earth.

Discussion

The vertical component of the field doesn't contribute to torque because it's parallel to the magnetic moment. Only the horizontal component ($B\cos 45°$) creates torque.

(a) The torque is 0.666 N·m directed west.

(b) This would not be practical as a motor. The torque is weak, and the current would need to alternate to produce continuous rotation (otherwise the loop would just oscillate about its equilibrium position).

Glossary

motor
: loop of wire in a magnetic field; when current is passed through the loops, the magnetic field exerts torque on the loops, which rotates a shaft; electrical energy is converted to mechanical work in the process

meter
: common application of magnetic torque on a current-carrying loop that is very similar in construction to a motor; by design, the torque is proportional to I and not θ, so the needle deflection is proportional to the current

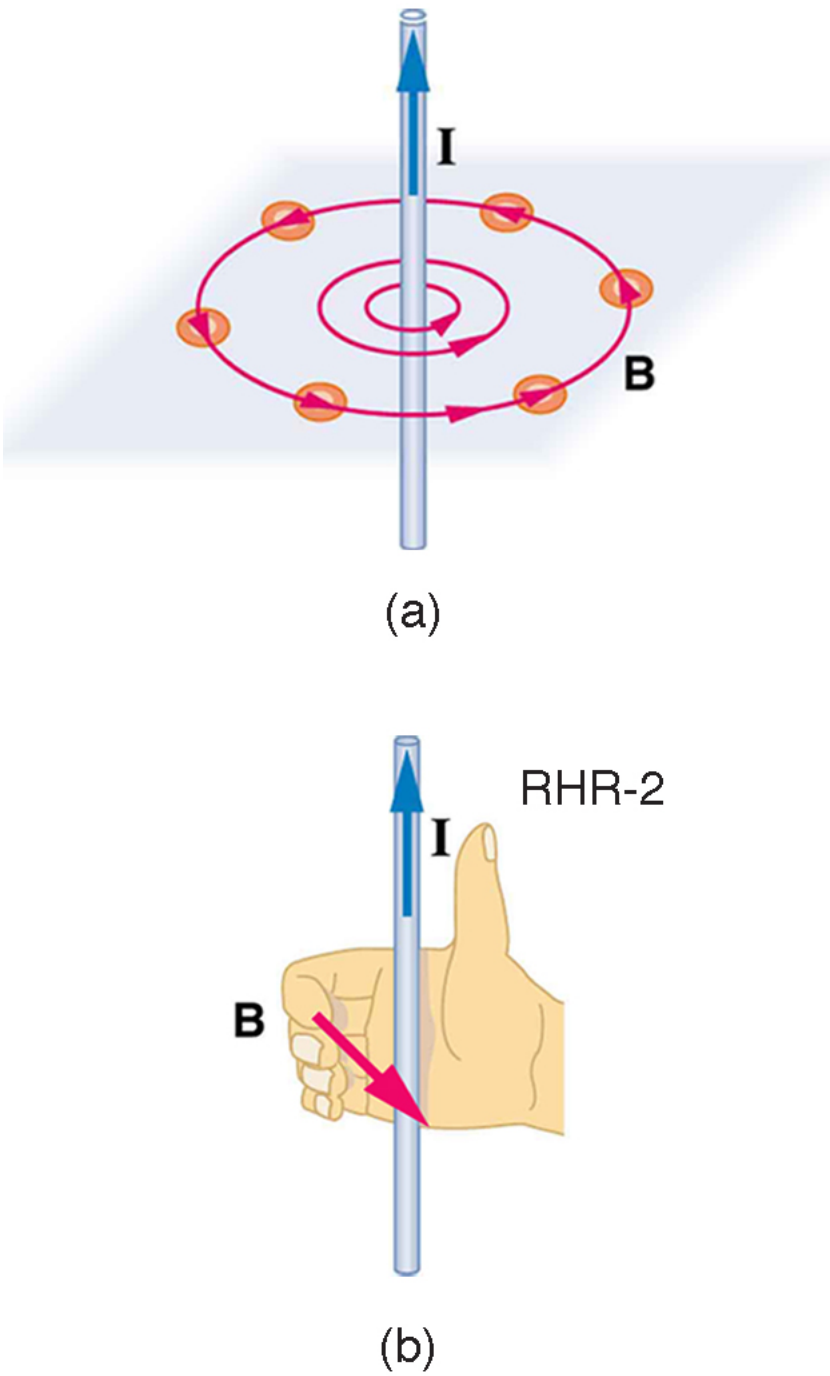
Magnetic Fields Produced by Currents: Ampere’s Law

- Calculate current that produces a magnetic field.
- Use the right hand rule 2 to determine the direction of current or the direction of magnetic field loops.

How much current is needed to produce a significant magnetic field, perhaps as strong as the Earth’s field? Surveyors will tell you that overhead electric power lines create magnetic fields that interfere with their compass readings. Indeed, when Oersted discovered in 1820 that a current in a wire affected a compass needle, he was not dealing with extremely large currents. How does the shape of wires carrying current affect the shape of the magnetic field created? We noted earlier that a current loop created a magnetic field similar to that of a bar magnet, but what about a straight wire or a toroid (doughnut)? How is the direction of a current-created field related to the direction of the current? Answers to these questions are explored in this section, together with a brief discussion of the law governing the fields created by currents.

Magnetic Field Created by a Long Straight Current-Carrying Wire: Right Hand Rule 2

Magnetic fields have both direction and magnitude. As noted before, one way to explore the direction of a magnetic field is with compasses, as shown for a long straight current-carrying wire in [Figure 1]. Hall probes can determine the magnitude of the field. The field around a long straight wire is found to be in circular loops. The **right hand rule 2** (RHR-2) emerges from this exploration and is valid for any current segment—*point the thumb in the direction of the current, and the fingers curl in the direction of the magnetic field loops* created by it.



(a) Compasses placed near a long straight current-carrying wire indicate that field lines form circular loops centered on the wire. (b) Right hand rule 2 states that, if the right hand thumb points in the direction of the current, the fingers curl in the direction of the field. This rule is consistent with the field mapped for the long straight wire and is valid for any current segment.

The **magnetic field strength (magnitude) produced by a long straight current-carrying wire** is found by experiment to be

$$B=\frac{\mu_0 I}{2\pi r}\text{(long straight wire)},$$

where I is the current, r is the shortest distance to the wire, and the constant $\mu_0 = 4\pi \times 10^{-7}\,\mathrm{T \cdot m/A}$ is the **permeability of free space**. (μ_0 is one of the basic constants in nature. We will see later that μ_0 is related to the speed of light.) Since the wire is very long, the magnitude of the field depends only on distance from the wire r , not on position along the wire.

Calculating Current that Produces a Magnetic Field

Find the current in a long straight wire that would produce a magnetic field twice the strength of the Earth’s at a distance of 5.0 cm from the wire.

Strategy

The Earth's field is about $5.0 \times 10^{-5}\text{T}$, and so here B due to the wire is taken to be $1.0 \times 10^{-4}\text{T}$. The equation $B = \frac{\mu_0 I}{2\pi r}$ can be used to find I , since all other quantities are known.

Solution

Solving for I and entering known values gives

$$I = \frac{2\pi r B}{\mu_0} = \frac{2\pi(5.0\times10^{-2}m)(1.0\times10^{-4}\text{T})}{4\pi\times10^{-7}\text{T}\cdot\text{m/A}} = 25\text{A}.$$

Discussion

So a moderately large current produces a significant magnetic field at a distance of 5.0 cm from a long straight wire. Note that the answer is stated to only two digits, since the Earth's field is specified to only two digits in this example.

Ampere's Law and Others

The magnetic field of a long straight wire has more implications than you might at first suspect. *Each segment of current produces a magnetic field like that of a long straight wire, and the total field of any shape current is the vector sum of the fields due to each segment.* The formal statement of the direction and magnitude of the field due to each segment is called the **Biot-Savart law** . Integral calculus is needed to sum the field for an arbitrary shape current. This results in a more complete law, called **Ampere's law**, which relates magnetic field and current in a general way. Ampere's law in turn is a part of **Maxwell's equations**, which give a complete theory of all electromagnetic phenomena. Considerations of how Maxwell's equations appear to different observers led to the modern theory of relativity, and the realization that electric and magnetic fields are different manifestations of the same thing. Most of this is beyond the scope of this text in both mathematical level, requiring calculus, and in the amount of space that can be devoted to it. But for the interested student, and particularly for those who continue in physics, engineering, or similar pursuits, delving into these matters further will reveal descriptions of nature that are elegant as well as profound. In this text, we shall keep the general features in mind, such as RHR-2 and the rules for magnetic field lines listed in Magnetic Fields and Magnetic Field Lines , while concentrating on the fields created in certain important situations.

Making Connections: Relativity

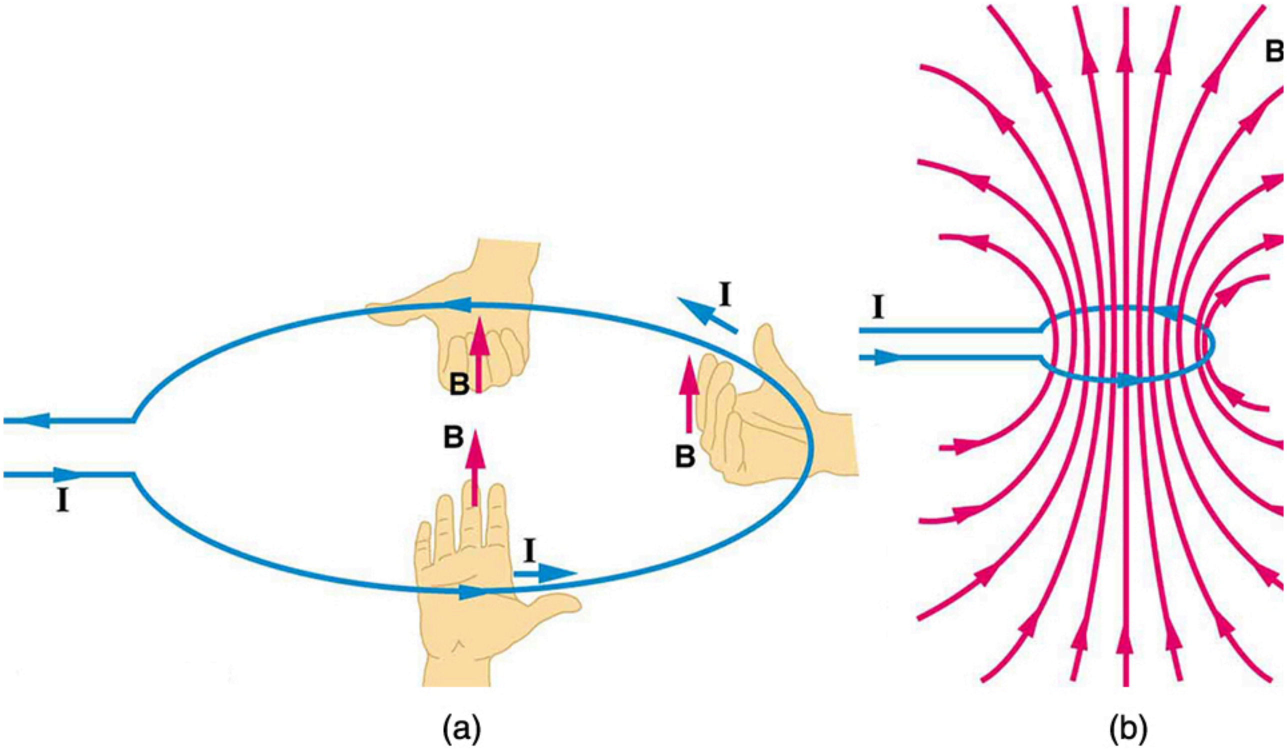
Hearing all we do about Einstein, we sometimes get the impression that he invented relativity out of nothing. On the contrary, one of Einstein's motivations was to solve difficulties in knowing how different observers see magnetic and electric fields.

Magnetic Field Produced by a Current-Carrying Circular Loop

The magnetic field near a current-carrying loop of wire is shown in [Figure 2]. Both the direction and the magnitude of the magnetic field produced by a current-carrying loop are complex. RHR-2 can be used to give the direction of the field near the loop, but mapping with compasses and the rules about field lines given in Magnetic Fields and Magnetic Field Lines are needed for more detail. There is a simple formula for the **magnetic field strength at the center of a circular loop**. It is

$$B = \frac{\mu_0 I}{2R} \text{(at center of loop)},$$

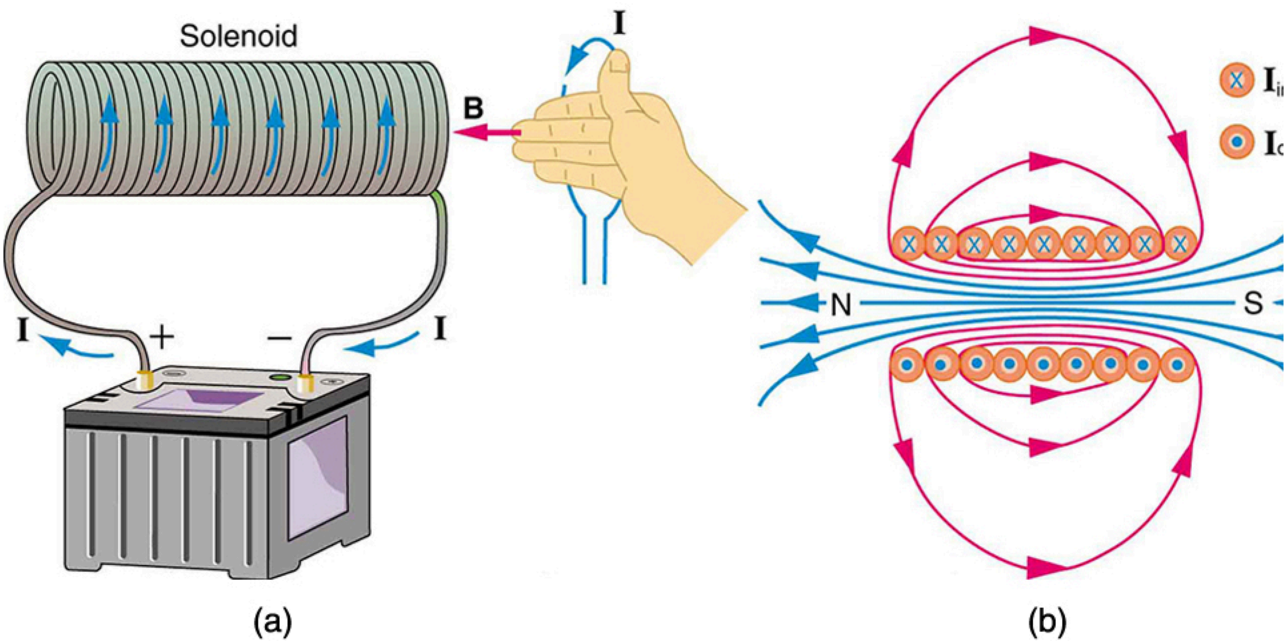
where R is the radius of the loop. This equation is very similar to that for a straight wire, but it is valid *only* at the center of a circular loop of wire. The similarity of the equations does indicate that similar field strength can be obtained at the center of a loop. One way to get a larger field is to have N loops; then, the field is $B = N\mu_0 I/(2R)$. Note that the larger the loop, the smaller the field at its center, because the current is farther away.



(a) RHR-2 gives the direction of the magnetic field inside and outside a current-carrying loop. (b) More detailed mapping with compasses or with a Hall probe completes the picture. The field is similar to that of a bar magnet.

Magnetic Field Produced by a Current-Carrying Solenoid

A **solenoid** is a long coil of wire (with many turns or loops, as opposed to a flat loop). Because of its shape, the field inside a solenoid can be very uniform, and also very strong. The field just outside the coils is nearly zero. [Figure 3] shows how the field looks and how its direction is given by RHR-2.



(a) Because of its shape, the field inside a solenoid of length l is remarkably uniform in magnitude and direction, as indicated by the straight and uniformly spaced field lines. The field outside the coils is nearly zero. (b) This cutaway shows the magnetic field generated by the current in the solenoid.

The magnetic field inside of a current-carrying solenoid is very uniform in direction and magnitude. Only near the ends does it begin to weaken and change direction. The field outside has similar complexities to flat loops and bar magnets, but the **magnetic field strength inside a solenoid** is simply

$$B=\mu_0 nI \text{(inside a solenoid)},$$

where n is the number of loops per unit length of the solenoid $(n=N/l)$, with N being the number of loops and l the length. Note that B is the field strength anywhere in the uniform region of the interior and not just at the center. Large uniform fields spread over a large volume are possible with solenoids, as [Example2] implies.

Calculating Field Strength inside a Solenoid

What is the field inside a 2.00-m-long solenoid that has 2000 loops and carries a 1600-A current?

Strategy

To find the field strength inside a solenoid, we use $B = \mu_0 nI$. First, we note the number of loops per unit length is

$$n = \frac{N}{l} = \frac{2000}{2.00\text{ m}} = 1000\text{ m}^{-1} = 10\text{ cm}^{-1}.$$

Solution

Substituting known values gives

$$B = \mu_0 nI = (4\pi \times 10^{-7}\text{ T}\cdot\text{m/A})(1000\, m^{-1})(1600\text{ A}) = 2.01\text{ T}.$$

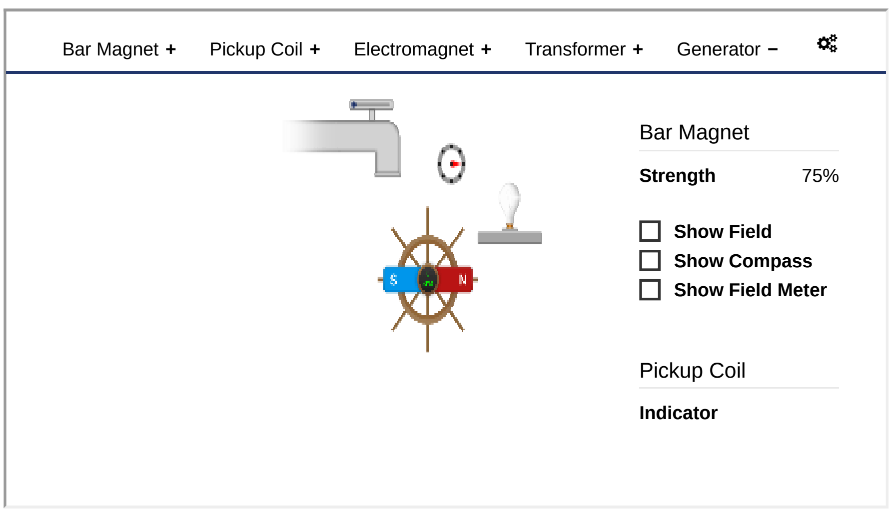
Discussion

This is a large field strength that could be established over a large-diameter solenoid, such as in medical uses of magnetic resonance imaging (MRI). The very large current is an indication that the fields of this strength are not easily achieved, however. Such a large current through 1000 loops squeezed into a meter's length would produce significant heating. Higher currents can be achieved by using superconducting wires, although this is expensive. There is an upper limit to the current, since the superconducting state is disrupted by very large magnetic fields.

There are interesting variations of the flat coil and solenoid. For example, the toroidal coil used to confine the reactive particles in tokamaks is much like a solenoid bent into a circle. The field inside a toroid is very strong but circular. Charged particles travel in circles, following the field lines, and collide with one another, perhaps inducing fusion. But the charged particles do not cross field lines and escape the toroid. A whole range of coil shapes are used to produce all sorts of magnetic field shapes. Adding ferromagnetic materials produces greater field strengths and can have a significant effect on the shape of the field. Ferromagnetic materials tend to trap magnetic fields (the field lines bend into the ferromagnetic material, leaving weaker fields outside it) and are used as shields for devices that are adversely affected by magnetic fields, including the Earth's magnetic field.

PhET Explorations: Generator

Generate electricity with a bar magnet! Discover the physics behind the phenomena by exploring magnets and how you can use them to make a bulb light.



Section Summary

- The strength of the magnetic field created by current in a long straight wire is given by

  $$B = \frac{\mu_0 I}{2\pi r}\text{(long straight wire)},$$

  where I is the current, r is the shortest distance to the wire, and the constant $\mu_0 = 4\pi \times 10^{-7}\text{ T}\cdot\text{m/A}$ is the permeability of free space.

- The direction of the magnetic field created by a long straight wire is given by right-hand rule 2 (RHR-2): *Point the thumb of the right hand in the direction of current, and the fingers curl in the direction of the magnetic field loops* created by it.
- The magnetic field created by current following any path is the sum (or integral) of the fields due to segments along the path (magnitude and direction as for a straight wire), resulting in a general relationship between current and field known as Ampere's law.
- The magnetic field strength at the center of a circular loop is given by

$$B = \mu_0 I 2R \text{(at center of loop)},$$

where R is the radius of the loop. This equation becomes $B = \mu_0 nI/(2R)$ for a flat coil of N loops. RHR-2 gives the direction of the field about the loop. A long coil is called a solenoid.

- The magnetic field strength inside a solenoid is
$$B = \mu_0 nI \text{(inside a solenoid)},$$

where n is the number of loops per unit length of the solenoid. The field inside is very uniform in magnitude and direction.

Conceptual Questions

Make a drawing and use RHR-2 to find the direction of the magnetic field of a current loop in a motor (such as in [Figure1]). Then show that the direction of the torque on the loop is the same as produced by like poles repelling and unlike poles attracting.

Glossary

right hand rule 2 (RHR-2)
: a rule to determine the direction of the magnetic field induced by a current-carrying wire: Point the thumb of the right hand in the direction of current, and the fingers curl in the direction of the magnetic field loops

magnetic field strength (magnitude) produced by a long straight current-carrying wire
: defined as $B = \mu_0 I 2\pi r$, where I is the current, r is the shortest distance to the wire, and μ_0 is the permeability of free space

permeability of free space
: the measure of the ability of a material, in this case free space, to support a magnetic field; the constant $\mu_0 = 4\pi \times 10^{-7} \text{T} \cdot \text{m/A}$

magnetic field strength at the center of a circular loop
: defined as $B = \mu_0 I 2R$ where R is the radius of the loop

solenoid
: a thin wire wound into a coil that produces a magnetic field when an electric current is passed through it

magnetic field strength inside a solenoid
: defined as $B = \mu_0 nI$ where n is the number of loops per unit length of the solenoid $(n = N/l)$, with N being the number of loops and l the length

Biot-Savart law
: a physical law that describes the magnetic field generated by an electric current in terms of a specific equation

Ampere’s law
: the physical law that states that the magnetic field around an electric current is proportional to the current; each segment of current produces a magnetic field like that of a long straight wire, and the total field of any shape current is the vector sum of the fields due to each segment

Maxwell’s equations
: a set of four equations that describe electromagnetic phenomena

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More Applications of Magnetism

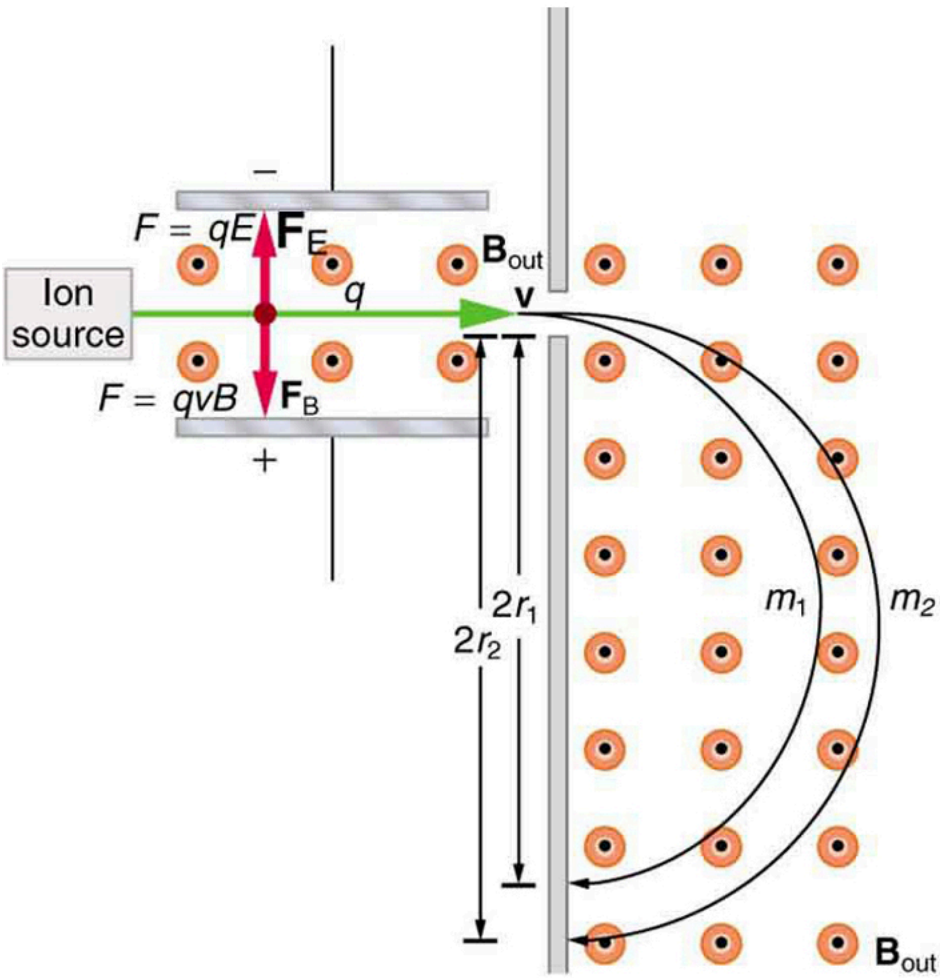
- Describe some applications of magnetism.

Mass Spectrometry

The curved paths followed by charged particles in magnetic fields can be put to use. A charged particle moving perpendicular to a magnetic field travels in a circular path having a radius r.

$$r=mvqB$$

It was noted that this relationship could be used to measure the mass of charged particles such as ions. A mass spectrometer is a device that measures such masses. Most mass spectrometers use magnetic fields for this purpose, although some of them have extremely sophisticated designs. Since there are five variables in the relationship, there are many possibilities. However, if v , q , and B can be fixed, then the radius of the path r is simply proportional to the mass m of the charged particle. Let us examine one such mass spectrometer that has a relatively simple design. (See [Figure 1].) The process begins with an ion source, a device like an electron gun. The ion source gives ions their charge, accelerates them to some velocity v , and directs a beam of them into the next stage of the spectrometer. This next region is a *velocity selector* that only allows particles with a particular value of v to get through.



This mass spectrometer uses a velocity selector to fix v so that the radius of the path is proportional to mass.

The velocity selector has both an electric field and a magnetic field, perpendicular to one another, producing forces in opposite directions on the ions. Only those ions for which the forces balance travel in a straight line into the next region. If the forces balance, then the electric force $F=qE$ equals the magnetic force $F=qvB$, so that $qE=qvB$. Noting that q cancels, we see that

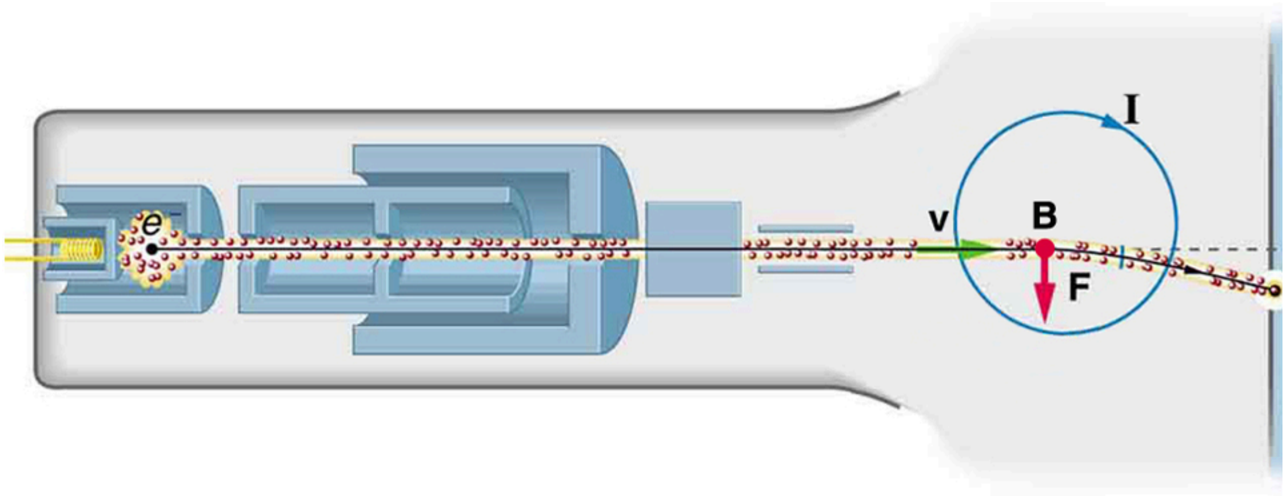
$$v=EB$$

is the velocity particles must have to make it through the velocity selector, and further, that v can be selected by varying E and B . In the final region, there is only a uniform magnetic field, and so the charged particles move in circular arcs with radii proportional to particle mass. The paths also depend on charge q , but since q is in multiples of electron charges, it is easy to determine and to discriminate between ions in different charge states.

Mass spectrometry today is used extensively in chemistry and biology laboratories to identify chemical and biological substances according to their mass-to-charge ratios. In medicine, mass spectrometers are used to measure the concentration of isotopes used as tracers. Usually, biological molecules such as proteins are very large, so they are broken down into smaller fragments before analyzing. Recently, large virus particles have been analyzed as a whole on mass spectrometers. Sometimes a gas chromatograph or high-performance liquid chromatograph provides an initial separation of the large molecules, which are then input into the mass spectrometer.

Cathode Ray Tubes—CRTs—and the Like

What do non-flat-screen TVs, old computer monitors, x-ray machines, and the 2-mile-long Stanford Linear Accelerator have in common? All of them accelerate electrons, making them different versions of the electron gun. Many of these devices use magnetic fields to steer the accelerated electrons. [Figure 2] shows the construction of the type of cathode ray tube (CRT) found in some TVs, oscilloscopes, and old computer monitors. Two pairs of coils are used to steer the electrons, one vertically and the other horizontally, to their desired destination.



The cathode ray tube (CRT) is so named because rays of electrons originate at the cathode in the electron gun. Magnetic coils are used to steer the beam in many CRTs. In this case, the beam is moved down. Another pair of horizontal coils would steer the beam horizontally.

Magnetic Resonance Imaging

Magnetic resonance imaging (MRI) is one of the most useful and rapidly growing medical imaging tools. It non-invasively produces two-dimensional and three-dimensional images of the body that provide important medical information with none of the hazards of X-rays. MRI is based on an effect called **nuclear magnetic resonance (NMR)** in which an externally applied magnetic field interacts with the nuclei of certain atoms, particularly those of hydrogen (protons). These nuclei possess their own small magnetic fields, similar to those of electrons and the current loops discussed earlier in this chapter.

When placed in an external magnetic field, such nuclei experience a torque that pushes or aligns the nuclei into one of two new energy states—depending on the orientation of its spin (analogous to the N pole and S pole in a bar magnet). Transitions from the lower to higher energy state can be achieved by using an external radio frequency signal to “flip” the orientation of the small magnets. (This is actually a quantum mechanical process. The direction of the nuclear magnetic field is quantized as is energy in the radio waves. We will return to these topics in later chapters.) The specific frequency of the radio waves that are absorbed and reemitted depends sensitively on the type of nucleus, the chemical environment, and the external magnetic field strength. Therefore, this is a *resonance* phenomenon in which *nuclei* in a *magnetic* field act like resonators (analogous to those discussed in the treatment of sound in Oscillatory Motion and Waves) that absorb and reemit only certain frequencies. Hence, the phenomenon is named _ nuclear magnetic resonance (NMR)._

NMR has been used for more than 50 years as an analytical tool. It was formulated in 1946 by F. Bloch and E. Purcell, with the 1952 Nobel Prize in Physics going to them for their work. Over the past two decades, NMR has been developed to produce detailed images in a process now called magnetic resonance imaging (MRI), a name coined to avoid the use of the word “nuclear” and the concomitant implication that nuclear radiation is involved. (It is not.) The 2003 Nobel Prize in Medicine went to P. Lauterbur and P. Mansfield for their work with MRI applications.

The largest part of the MRI unit is a superconducting magnet that creates a magnetic field, typically between 1 and 2 T in strength, over a relatively large volume. MRI images can be both highly detailed and informative about structures and organ functions. It is helpful that normal and non-normal tissues respond differently for slight changes in the magnetic field. In most medical images, the protons that are hydrogen nuclei are imaged. (About 2/3 of the atoms in the body are hydrogen.) Their location and density give a variety of medically useful information, such as organ function, the condition of tissue (as in the brain), and the shape of structures, such as vertebral disks and knee-joint surfaces. MRI can also be used to follow the movement of certain ions across membranes, yielding information on active transport, osmosis, dialysis, and other phenomena. With excellent spatial resolution, MRI can provide information about tumors, strokes, shoulder injuries, infections, etc.

An image requires position information as well as the density of a nuclear type (usually protons). By varying the magnetic field slightly over the volume to be imaged, the resonant frequency of the protons is made to vary with position. Nuclei absorb and reemit broadcast radio frequencies only if the nuclei are in a magnetic field with the correct strength. The imaging receiver can build up a tissue map either by sweeping the frequency or emitting a range of frequencies at once and analyzing the frequency response of the collected signal. The reception of reemitted radio waves as a function of frequency thus gives position information. These “slices” or cross-sections through the body are only several mm thick. The intensity of the reemitted radio waves is proportional to the concentration of the nuclear type being flipped, as well as information on the chemical environment in that area of the body. Various techniques are available for enhancing contrast in images and for obtaining more information. Scans called T1, T2, or proton density scans rely on different relaxation mechanisms of nuclei. Relaxation refers to the time it takes for the protons to return to equilibrium after the external field is turned off. This time depends upon tissue type and status (such as inflammation).

While MRI images are superior to X-rays for certain types of tissue and have none of the hazards of X-rays, they do not completely supplant x-ray images. MRI is less effective than X-rays for detecting breaks in bone, for example, and in imaging breast tissue, so the two diagnostic tools complement each other. MRI images are also expensive compared to simple x-ray images and tend to be used most often where they supply information not readily obtained from X-rays. Another disadvantage of MRI is that the patient is totally enclosed with detectors close to the body for about 30 minutes or more, leading to claustrophobia. It is also difficult for the obese patient to be in the magnet tunnel. New "open-MRI" machines are now available in which the magnet does not completely surround the patient.

Over the last decade, the development of much faster scans, called "functional MRI" (fMRI), has allowed us to map the functioning of various regions in the brain responsible for thought and motor control. This technique measures the change in blood flow for activities (thought, experiences, action) in the brain. The nerve cells increase their consumption of oxygen when active. Blood hemoglobin releases oxygen to active nerve cells and has somewhat different magnetic properties when oxygenated than when deoxygenated. With MRI, we can measure this and detect a blood oxygen-dependent signal. Most of the brain scans today use fMRI.

Other Medical Uses of Magnetic Fields

Currents in nerve cells and the heart create magnetic fields like any other currents. These can be measured but with some difficulty since their strengths are about 10^{-6} to 10^{-8} *less* than the Earth's magnetic field. Recording of the heart's magnetic field as it beats is called a **magnetocardiogram (MCG)**, while measurements of the brain's magnetic field is called a **magnetoencephalogram (MEG)**. Both give information that differs from that obtained by measuring the electric fields of these organs (ECGs and EEGs), but they are not yet of sufficient importance to make these difficult measurements common.

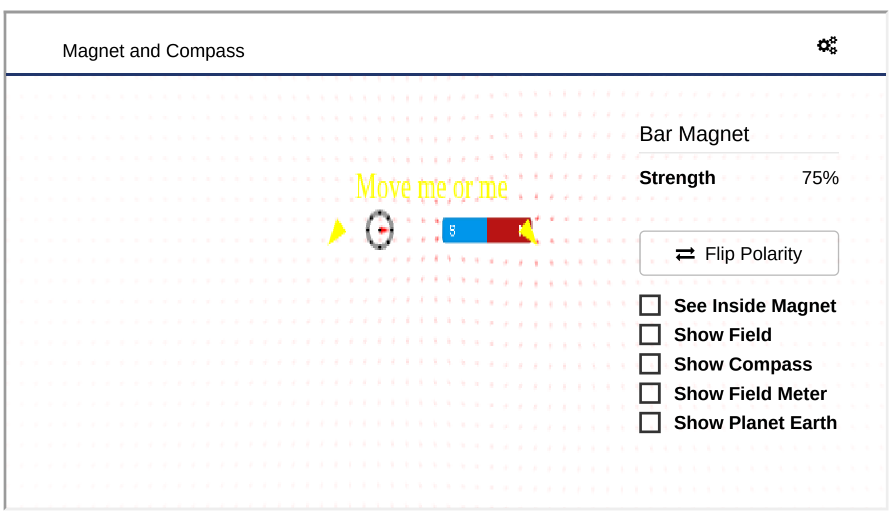
In both of these techniques, the sensors do not touch the body. MCG can be used in fetal studies, and is probably more sensitive than echocardiography. MCG also looks at the heart's electrical activity whose voltage output is too small to be recorded by surface electrodes as in EKG. It has the potential of being a rapid scan for early diagnosis of cardiac ischemia (obstruction of blood flow to the heart) or problems with the fetus.

MEG can be used to identify abnormal electrical discharges in the brain that produce weak magnetic signals. Therefore, it looks at brain activity, not just brain structure. It has been used for studies of Alzheimer's disease and epilepsy. Advances in instrumentation to measure very small magnetic fields have allowed these two techniques to be used more in recent years. What is used is a sensor called a SQUID, for superconducting quantum interference device. This operates at liquid helium temperatures and can measure magnetic fields thousands of times smaller than the Earth's.

Finally, there is a burgeoning market for magnetic cures in which magnets are applied in a variety of ways to the body, from magnetic bracelets to magnetic mattresses. The best that can be said for such practices is that they are apparently harmless, unless the magnets get close to the patient's computer or magnetic storage disks. Claims are made for a broad spectrum of benefits from cleansing the blood to giving the patient more energy, but clinical studies have not verified these claims, nor is there an identifiable mechanism by which such benefits might occur.

PhET Explorations: Magnet and Compass

Ever wonder how a compass worked to point you to the Arctic? Explore the interactions between a compass and bar magnet, and then add the Earth and find the surprising answer! Vary the magnet's strength, and see how things change both inside and outside. Use the field meter to measure how the magnetic field changes.



Section Summary

- Crossed (perpendicular) electric and magnetic fields act as a velocity filter, giving equal and opposite forces on any charge with velocity perpendicular to the fields and of magnitude
  $v = EB.$

Conceptual Questions

Measurements of the weak and fluctuating magnetic fields associated with brain activity are called magnetoencephalograms (MEGs). Do the brain's magnetic fields imply coordinated or uncoordinated nerve impulses? Explain.

Discuss the possibility that a Hall voltage would be generated on the moving heart of a patient during MRI imaging. Also discuss the same effect on the wires of a pacemaker. (The fact that patients with pacemakers are not given MRIs is significant.)

A patient in an MRI unit turns his head quickly to one side and experiences momentary dizziness and a strange taste in his mouth. Discuss the possible causes.

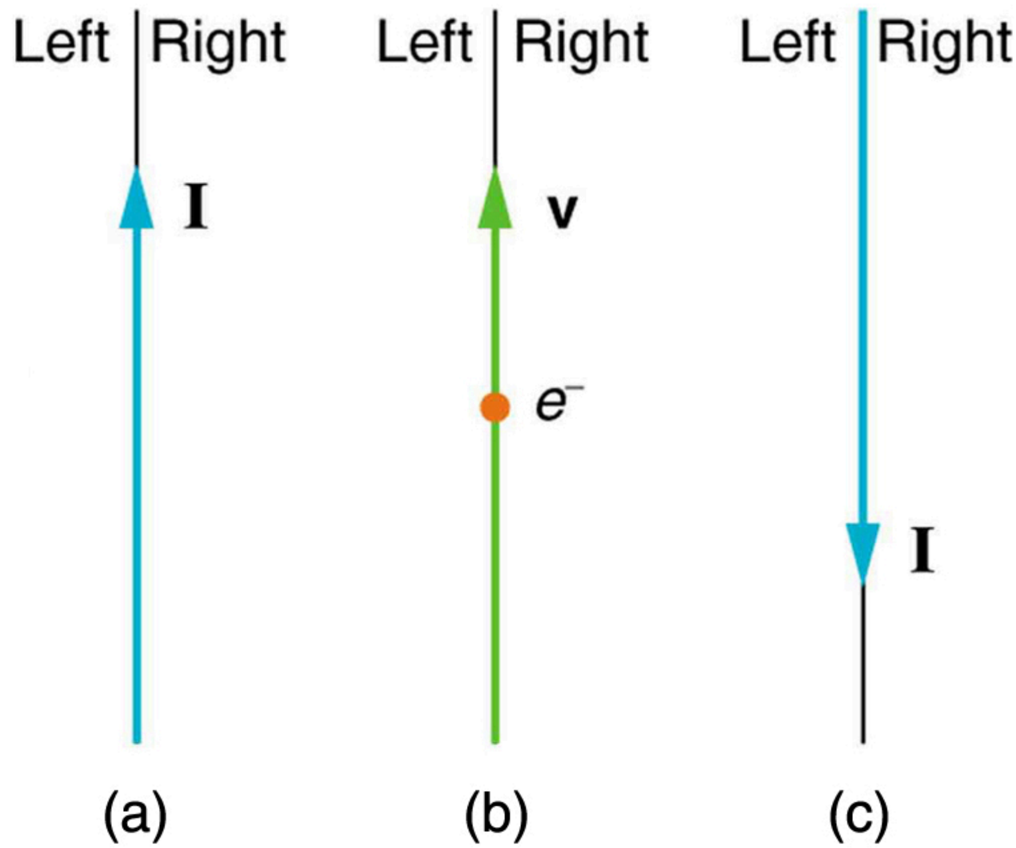
You are told that in a certain region there is either a uniform electric or magnetic field. What measurement or observation could you make to determine the type? (Ignore the Earth's magnetic field.)

An example of magnetohydrodynamics (MHD) comes from the flow of a river (salty water). This fluid interacts with the Earth's magnetic field to produce a potential difference between the two river banks. How would you go about calculating the potential difference?

Draw gravitational field lines between 2 masses, electric field lines between a positive and a negative charge, electric field lines between 2 positive charges and magnetic field lines around a magnet. Qualitatively describe the differences between the fields and the entities responsible for the field lines.

Problems & Exercises

Indicate whether the magnetic field created in each of the three situations shown in [Figure 3] is into or out of the page on the left and right of the current.



Show Solution

Strategy

We use the right-hand rule (RHR-2) to determine the magnetic field direction around a straight current-carrying wire. Point the thumb in the direction of conventional current flow (positive charges), and the fingers curl in the direction of the magnetic field. For an electron moving upward, the conventional current direction is opposite (downward).

Solution

(a) Current flows upward (from bottom to top). Using RHR-2:

- Point thumb upward in direction of current
- Fingers curl around the wire
- On the **right side**: fingers point **into the page**
- On the **left side**: fingers point **out of the page**

(b) An electron moving upward represents conventional current flowing **downward** (opposite to electron motion). Using RHR-2:

- Point thumb downward (direction of conventional current)
- On the **right side**: fingers point **out of the page**

- On the **left side**: fingers point **into the page**

(c) Current flows downward (from top to bottom). Using RHR-2:

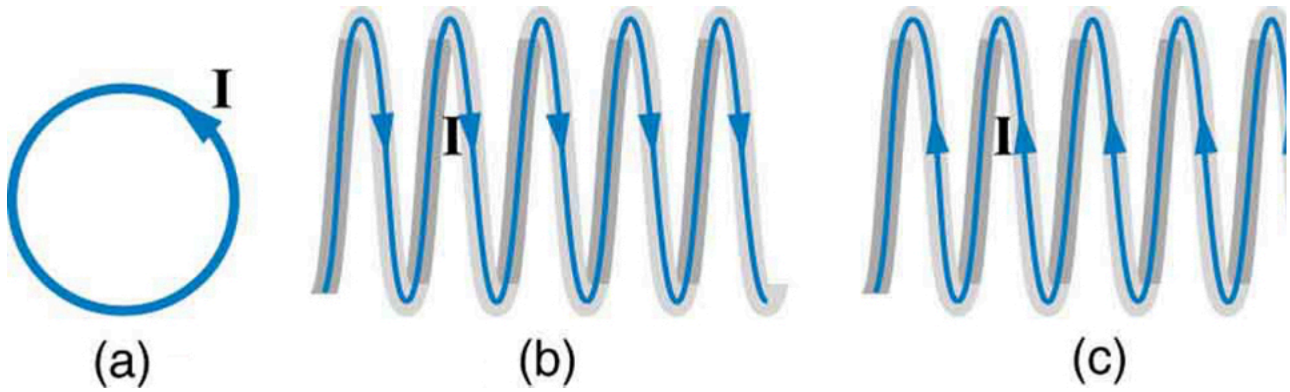
- Point thumb downward in direction of current
- On the **right side**: fingers point **out of the page**
- On the **left side**: fingers point **into the page**

Discussion

This problem illustrates how the right-hand rule consistently determines magnetic field directions. Note that case (b) with an upward-moving electron produces the same field pattern as case (c) with downward current—both represent the same conventional current direction. This equivalence between electron motion and opposite conventional current is fundamental to understanding electromagnetic phenomena.

(a) Right side: into page; Left side: out of page. (b) Right side: out of page; Left side: into page. (c) Right side: out of page; Left side: into page.

What are the directions of the fields in the center of the loop and coils shown in [Figure 4]?



Show Solution

Strategy

We use the right-hand rule for loops and coils: curl the fingers in the direction of current flow around the loop, and the thumb points in the direction of the magnetic field through the center of the loop (along the axis).

Solution

(a) For the single loop with counterclockwise current (as viewed from the reader):

- Curl fingers counterclockwise (following current direction)
- Thumb points **out of the page** toward the reader
- Field at center: **out of the page**

(b) For the coil with current flowing as shown (creating counterclockwise loops when viewed from the left):

- Curl fingers in direction of current around the coil
- Thumb points **to the left**
- Field at center: **to the left**

(c) For the coil with current flowing in the opposite direction (creating counterclockwise loops when viewed from the right):

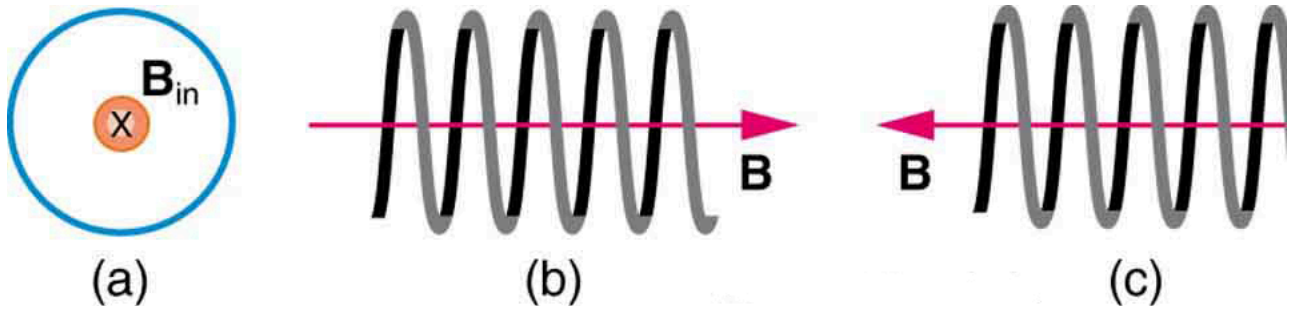
- Curl fingers in direction of current around the coil
- Thumb points **to the right**
- Field at center: **to the right**

Discussion

The magnetic field inside a coil (solenoid) is uniform and parallel to the axis. The direction depends entirely on which way the current circulates around the coil. This principle is used in electromagnets, MRI machines, and countless other devices where controllable magnetic fields are needed.

(a) Out of the page. (b) To the left. (c) To the right.

What are the directions of the currents in the loop and coils shown in [Figure 5]?



Show Solution

Strategy

This is the reverse of the previous problem. Given the magnetic field direction inside a loop or coil, we use the right-hand rule in reverse: point the thumb in the direction of the magnetic field, and the fingers curl in the direction of current flow.

Solution

(a) For the loop with magnetic field pointing into the page:

- Point thumb into the page (direction of B)
- Fingers curl **clockwise** (as viewed from the front)
- Current direction: **clockwise**

(b) For the coil with magnetic field pointing to the left:

- Point thumb to the left (direction of B)
- Fingers curl in direction of current
- Current direction: **clockwise as seen from the left**

(c) For the coil with magnetic field pointing to the right:

- Point thumb to the right (direction of B)
- Fingers curl in direction of current
- Current direction: **clockwise as seen from the right**

Discussion

This inverse application of the right-hand rule is essential for designing electromagnets and understanding induced currents. When we want to create a magnetic field in a specific direction, we can determine the required current direction using this method. This is the foundation for designing MRI magnets, particle accelerator magnets, and electric motors.

(a) Clockwise. (b) Clockwise as seen from the left. (c) Clockwise as seen from the right.

To see why an MRI utilizes iron to increase the magnetic field created by a coil, calculate the current needed in a 400-loop-per-meter circular coil 0.660 m in radius to create a 1.20-T field (typical of an MRI instrument) at its center with no iron present. The magnetic field of a proton is approximately like that of a circular current loop $0.650 \times 10^{-15} m$ in radius carrying $1.05 \times 10^{4} A$. What is the field at the center of such a loop?

Show Solution

Strategy

This problem has two parts. First, we use the solenoid field formula $B = \mu_0 n I$ to find the current needed to create a 1.20 T field. Second, we use the formula for the magnetic field at the center of a current loop $B = \mu_0 I/(2r)$ to find the field created by a proton modeled as a current loop.

Solution

Known quantities:

- Number of loops per meter: $n = 400$ loops/m
- Coil radius: $r = 0.660$ m (not needed for solenoid field)
- Desired field: $B = 1.20$ T
- Proton loop radius: $r_p = 0.650 \times 10^{-15}$ m
- Proton equivalent current: $I_p = 1.05 \times 10^{4}$ A
- $\mu_0 = 4\pi \times 10^{-7}$ T·m/A

Part 1: Current for MRI solenoid

Using the solenoid formula:

$$B = \mu_0 n I$$
$$I = \frac{B}{\mu_0 n} = \frac{1.20 \text{ T}}{(4\pi \times 10^{-7} \text{ T·m/A})(400 \text{ m}^{-1})}$$
$$I = \frac{1.20}{5.03 \times 10^{-4}} = 2390 \text{ A}$$

This is an enormous current, which is why MRI machines use superconducting coils and iron cores to enhance the field.

Part 2: Magnetic field of a proton

For a circular current loop, the field at the center is:

$$B_p = \frac{\mu_0 I_p}{2 r_p} = \frac{(4\pi \times 10^{-7} \text{ T·m/A})(1.05 \times 10^{4} \text{ A})}{2(0.650 \times 10^{-15} \text{ m})}$$
$$B_p = \frac{1.32 \times 10^{-2}}{1.30 \times 10^{-15}} = 1.01 \times 10^{13} \text{ T}$$

Discussion

The magnetic field at the “center” of a proton (modeled as a tiny current loop) is extraordinarily strong—about 10^{13} T! This is roughly 10 trillion times stronger than a typical MRI field. Of course, this field exists only over nuclear dimensions (~10^{-15} m) and drops off extremely rapidly with distance. The calculation for the MRI coil shows why superconducting magnets and iron cores are essential: producing even 1-2 T fields with ordinary conductors would require impractically high currents.

The magnetic field at the center of the proton’s equivalent current loop is 1.01×10^{13} T.

Inside a motor, 30.0 A passes through a 250-turn circular loop that is 10.0 cm in radius. What is the magnetic field strength created at its center?

Show Solution

Strategy

For a circular coil with N turns, the magnetic field at the center is N times that of a single loop. The field at the center of a single current loop is $B = \mu_0 I/(2r)$, so for N turns: $B = N\mu_0 I/(2r)$.

Solution

Known quantities:

- Current: $I = 30.0$ A
- Number of turns: $N = 250$
- Radius: $r = 10.0 \text{ cm} = 0.100$ m
- $\mu_0 = 4\pi \times 10^{-7}$ T·m/A

The magnetic field at the center of a multi-turn coil is:

$$B = \frac{N \mu_0 I}{2r}$$
$$B = \frac{(250)(4\pi \times 10^{-7} \text{ T·m/A})(30.0 \text{ A})}{2(0.100 \text{ m})}$$
$$B = \frac{(250)(1.257 \times 10^{-6})(30.0)}{0.200}$$
$$B = \frac{9.42 \times 10^{-3}}{0.200} = 4.71 \times 10^{-2} \text{ T}$$

Discussion

This field of about 47 mT is quite strong for an air-core coil—about 1000 times stronger than Earth’s magnetic field. In actual motors, iron cores are used to concentrate and enhance this field, creating much stronger fields needed for practical motor operation. The multi-turn design (250 turns) multiplies the effect of the current, making this a more efficient way to create strong fields than simply increasing current.

The magnetic field at the center of the motor coil is 4.71×10^{-2} T or 47.1 mT.

Nonnuclear submarines use batteries for power when submerged. (a) Find the magnetic field 50.0 cm from a straight wire carrying 1200 A from the batteries to the drive mechanism of a submarine. (b) What is the field if the wires to and from the drive mechanism are side by side? (c) Discuss the effects this could have for a compass on the submarine that is not shielded.

Show Solution

Strategy

For part (a), we use the formula for the magnetic field around a long straight wire: $B = \mu_0 I/(2\pi r)$. For part (b), when two wires carrying equal currents in opposite directions are placed side by side, their magnetic fields cancel at points equidistant from both wires. Part (c) requires comparing the calculated field to Earth’s field.

Solution

Known quantities:

- Current: $I = 1200\ \text{A}$
- Distance: $r = 50.0\ \text{cm} = 0.500\ \text{m}$
- Earth's field: $B_E \approx 5 \times 10^{-5}\ \text{T}$
- $\mu_0 = 4\pi \times 10^{-7}\ \text{T·m/A}$

(a) For a single straight wire:

$$B = \frac{\mu_0 I}{2\pi r} = \frac{(4\pi \times 10^{-7}\ \text{T·m/A})(1200\ \text{A})}{2\pi(0.500\ \text{m})}$$
$$B = \frac{(4 \times 10^{-7})(1200)}{1.00} = 4.80 \times 10^{-4}\ \text{T}$$

(b) When the outgoing and return wires are side by side, they carry equal currents in opposite directions. At any point equidistant from both wires, the magnetic fields point in opposite directions and **cancel completely**:

$$B_{\text{net}} = 0$$

(c) The single-wire field of 4.80×10^{-4} T is nearly **10 times stronger** than Earth's magnetic field (~5×10^{-5} T). If the wires are not paired:

- A compass near the wire would point toward the wire rather than north
- Navigation would be severely compromised
- This is why power cables in submarines (and buildings) are always run as paired conductors—the fields cancel and don't interfere with sensitive equipment

Discussion

This problem illustrates why electrical wiring is almost always run as paired conductors (or twisted pairs). The cancellation effect is exact only for ideal parallel wires; in practice, some residual field remains due to wire separation, but it's much smaller than from a single conductor. Modern submarines use extensive magnetic shielding for navigation equipment.

(a) 4.80×10^{-4} T. (b) Zero (fields cancel). (c) The unpaired wire creates a field about 10 times Earth's field, severely disrupting compass navigation.

How strong is the magnetic field inside a solenoid with 10 000 turns per meter that carries 20.0 A?

Show Solution

Strategy

The magnetic field inside an ideal solenoid is uniform and given by $B = \mu_0 n I$, where n is the number of turns per unit length and I is the current.

Solution

Known quantities:

- Turns per meter: $n = 10{,}000\ \text{turns/m} = 10^4\ \text{m}^{-1}$
- Current: $I = 20.0\ \text{A}$
- $\mu_0 = 4\pi \times 10^{-7}\ \text{T·m/A}$

Using the solenoid field formula:

$$B = \mu_0 n I = (4\pi \times 10^{-7}\ \text{T·m/A})(10^4\ \text{m}^{-1})(20.0\ \text{A})$$
$$B = (1.257 \times 10^{-6})(10^4)(20.0)$$
$$B = 0.251\ \text{T}$$

Discussion

This field of about 0.25 T is quite strong—roughly 5000 times Earth's magnetic field. The high turn density (10,000 turns per meter) is key to achieving this field strength. Such solenoids are used in research laboratories, electromagnets for lifting, and as components in various electrical devices. Note that the field is uniform inside the solenoid and drops to nearly zero outside, making solenoids useful for creating controlled magnetic environments.

The magnetic field inside the solenoid is 0.251 T or 251 mT.

What current is needed in the solenoid described in [Exercise 1] to produce a magnetic field 10^4 times the Earth's magnetic field of $5.00 \times 10^{-5}\ \text{T}$?

Show Solution

Strategy

We need to find the current required in a solenoid to produce a field 10^4 times Earth's field. Using the solenoid formula $B = \mu_0 n I$, we can solve for the current. Note that Exercise 1 describes a configuration with certain parameters, but we need to use a typical solenoid turn density.

Solution

Known quantities:

- Earth's field: $B_E = 5.00 \times 10^{-5}$ T
- Desired field: $B = 10^4 \times B_E = 10^4 \times 5.00 \times 10^{-5} = 0.500$ T
- Turns per meter: $n = 10{,}000\ \text{m}^{-1}$ (from the solenoid in Exercise 7)
- $\mu_0 = 4\pi \times 10^{-7}$ T·m/A

Using the solenoid formula:

$$B = \mu_0 n I$$

$$I = \frac{B}{\mu_0 n} = \frac{0.500\ \text{T}}{(4\pi \times 10^{-7}\ \text{T·m/A})(10^4\ \text{m}^{-1})}$$

$$I = \frac{0.500}{1.257 \times 10^{-2}} = 39.8\ \text{A}$$

Discussion

A current of about 40 A is substantial but achievable with ordinary conductors (requiring thick wire to handle the heating). This illustrates that solenoids can produce very strong fields—0.5 T is 10,000 times Earth's field! Such strong fields are used in research magnets, MRI machines (with superconducting coils for even higher fields), and industrial electromagnets.

A current of 39.8 A is needed to produce a magnetic field 10^4 times Earth's field.

How far from the starter cable of a car, carrying 150 A, must you be to experience a field less than the Earth's $(5.00 \times 10^{-5}\text{T})$? Assume a long straight wire carries the current. (In practice, the body of your car shields the dashboard compass.)

Show Solution

Strategy

We use the formula for the magnetic field around a straight wire, $B = \mu_0 I/(2\pi r)$, and solve for the distance r at which the field equals Earth's field. For the field to be less than Earth's, you must be farther than this distance.

Solution

Known quantities:

- Current: $I = 150$ A
- Maximum field: $B = 5.00 \times 10^{-5}$ T (Earth's field)
- $\mu_0 = 4\pi \times 10^{-7}$ T·m/A

From the wire field formula:

$$B = \frac{\mu_0 I}{2\pi r}$$

Solving for r:

$$r = \frac{\mu_0 I}{2\pi B} = \frac{(4\pi \times 10^{-7}\ \text{T·m/A})(150\ \text{A})}{2\pi (5.00 \times 10^{-5}\ \text{T})}$$

$$r = \frac{4 \times 10^{-7} \times 150}{2 \times 5.00 \times 10^{-5}} = \frac{6.00 \times 10^{-5}}{10^{-4}}$$

$$r = 0.600\ \text{m} = 60.0\ \text{cm}$$

Discussion

You must be at least 60 cm (about 2 feet) from the starter cable for the wire's field to be less than Earth's field. Since the dashboard is typically within this distance from the engine compartment, the car body's magnetic shielding (ferromagnetic steel) is important for protecting the compass. Interestingly, modern cars with aluminum bodies may require additional shielding for magnetic instruments.

You must be at least 0.600 m (60.0 cm) from the starter cable for the field to be less than Earth's.

Measurements affect the system being measured, such as the current loop in [Figure 8]. (a) Estimate the field the loop creates by calculating the field at the center of a circular loop 20.0 cm in diameter carrying 5.00 A. (b) What is the smallest field strength this loop can be used to measure, if its field must alter the measured field by less than 0.0100%?

Show Solution

Strategy

For part (a), we calculate the magnetic field at the center of a current loop using $B = \mu_0 I/(2r)$. For part (b), we find the minimum external field that can be measured such that the loop's own field is less than 0.0100% of it.

Solution

Known quantities:

- Loop diameter: $d = 20.0\ \text{cm}$, so radius $r = 10.0\ \text{cm} = 0.100\ \text{m}$
- Current: $I = 5.00\ \text{A}$
- Maximum allowed perturbation: $0.0100\% = 1.00 \times 10^{-4}$
- $\mu_0 = 4\pi \times 10^{-7}\ \text{T·m/A}$

(a) Field at the center of the loop:

$$B_{\text{loop}} = \frac{\mu_0 I}{2r} = \frac{(4\pi \times 10^{-7}\ \text{T·m/A})(5.00\ \text{A})}{2(0.100\ \text{m})}$$
$$B_{\text{loop}} = \frac{6.28 \times 10^{-6}}{0.200} = 3.14 \times 10^{-5}\ \text{T}$$

(b) The loop's field must be less than 0.0100% of the measured field:

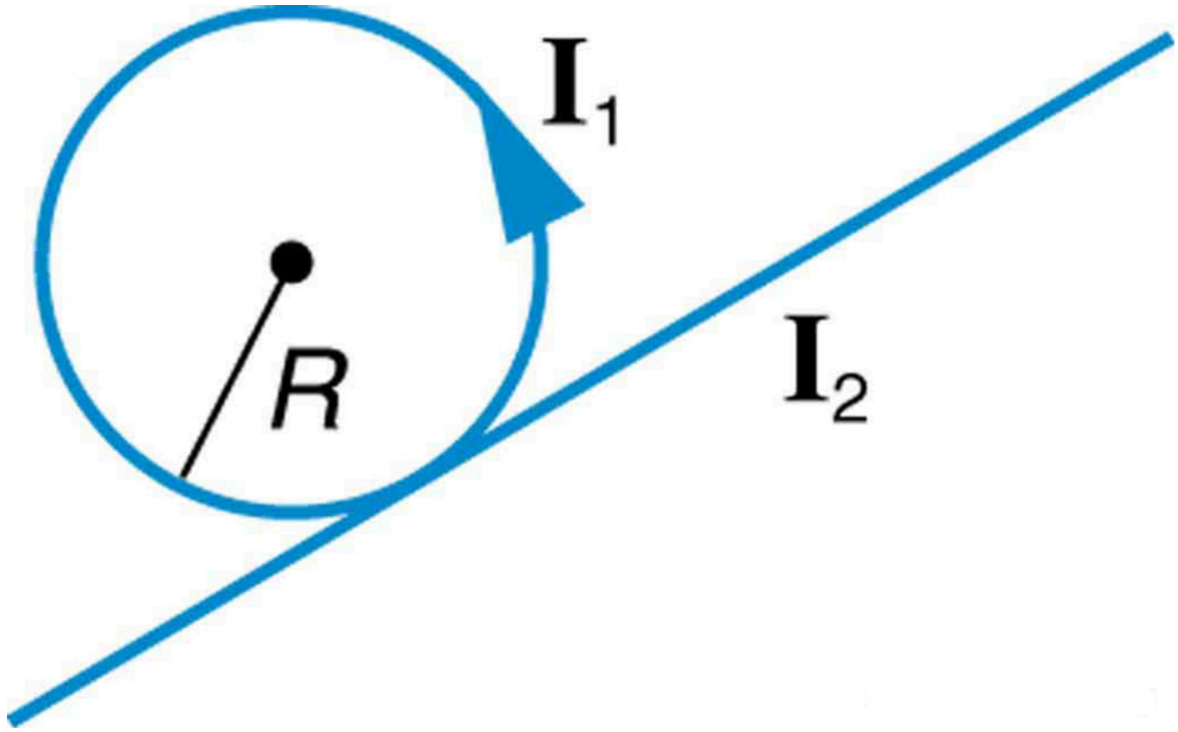
$$B_{\text{loop}} < 0.0100\% \times B_{\text{measured}}$$
$$B_{\text{measured}} > \frac{B_{\text{loop}}}{1.00 \times 10^{-4}} = \frac{3.14 \times 10^{-5}\ \text{T}}{1.00 \times 10^{-4}}$$
$$B_{\text{measured}} > 0.314\ \text{T}$$

Discussion

This problem illustrates an important principle in measurement science: the measuring device can perturb the quantity being measured. The loop creates a field of about $31\mu\text{T}$ (roughly 60% of Earth's field), which is not negligible. To keep this perturbation below 0.01%, you can only measure fields stronger than 0.314 T. For measuring weaker fields with high precision, you would need a smaller loop carrying less current, though this reduces sensitivity. This trade-off between sensitivity and perturbation is fundamental to instrument design.

(a) The loop creates a field of 3.14×10^{-5} T at its center. (b) The smallest field measurable with less than 0.0100% perturbation is 0.314 T.

[Figure 6] shows a long straight wire just touching a loop carrying a current I_1 . Both lie in the same plane. (a) What direction must the current I_2 in the straight wire have to create a field at the center of the loop in the direction opposite to that created by the loop? (b) What is the ratio of I_1/I_2 that gives zero field strength at the center of the loop? (c) What is the direction of the field directly above the loop under this circumstance?



Show Solution

Strategy

The loop creates a field at its center perpendicular to the plane (using the right-hand rule for loops). The straight wire, at distance R from the center, creates a field $B = \mu_0 I_2/(2\pi R)$. For the fields to cancel, they must be opposite in direction and equal in magnitude.

Solution

(a) For the loop with current I_1 flowing (let's say counterclockwise as viewed), the field at the center points out of the page.

For the straight wire to create a field pointing into the page at the center of the loop (which is at distance R from the wire), using RHR-2:

- The wire must carry current **in the same direction as the loop current at the point of tangency** (i.e., if the loop current flows upward at the tangent point, I_2 should flow upward, or from lower left to upper right in the figure)

(b) At the center, the loop creates: $B_{\text{loop}} = \mu_0 I_1 2R$

The straight wire at distance R creates: $B_{\text{wire}} = \mu_0 I_2 2\pi R$

For zero net field, these must be equal: $\mu_0 I_1 2R = \mu_0 I_2 2\pi R$

$I_1 I_2 = 1\pi$

(c) Directly above the loop (on the axis, outside the plane):

- The loop's field still points outward along the axis (upward)
- The wire's field circles around it; directly above, this field points in the plane of the loop
- Since these fields are perpendicular, they don't cancel; the net field points at an angle

The field directly above the loop will have components both perpendicular to and parallel to the loop plane, resulting in a field direction that is **neither purely perpendicular nor purely parallel** to the loop—it's at an angle depending on the height above the loop.

Discussion

This problem illustrates that magnetic field cancellation at one point doesn't guarantee cancellation everywhere. The loop and wire produce fields with different spatial dependencies, so while they cancel at the center, they combine in complex ways elsewhere. This principle is important in designing systems where you want localized field cancellation, such as in magnetic shielding.

(a) I_2 must flow from lower left to upper right. (b) $I_1/I_2 = 1/\pi \approx 0.318$. (c) The field above the loop points at an angle, with components both along and perpendicular to the axis.

Find the magnitude and direction of the magnetic field at the point equidistant from the wires in [Figure 5](a), using the rules of vector addition to sum the contributions from each wire.

Show Solution

Strategy

We calculate the magnetic field from each wire at the equidistant point using $B = \mu_0 I/(2\pi r)$, then use vector addition to find the resultant. The direction of each field is found using the right-hand rule.

Solution

From Figure 5(a) in the chapter on magnetic force between parallel conductors, we have wires carrying currents with specific geometry. At the equidistant point, each wire contributes a field that must be added vectorially.

Known quantities (from referenced figure):

- Currents: $I_1 = 5.00\text{ A}$, $I_2 = 10.0\text{ A}$
- Distance from each wire to equidistant point: $r = 10.0\text{ cm} = 0.100\text{ m}$

The magnitude of the field from each wire:

$$B_1 = \frac{\mu_0 I_1}{2\pi r} = \frac{(4\pi \times 10^{-7})(5.00)}{2\pi(0.100)} = 1.00 \times 10^{-5}\text{ T}$$
$$B_2 = \frac{\mu_0 I_2}{2\pi r} = \frac{(4\pi \times 10^{-7})(10.0)}{2\pi(0.100)} = 2.00 \times 10^{-5}\text{ T}$$

Using the right-hand rule to find directions and adding vectorially (the angle between them depends on the wire geometry), the resultant field magnitude and direction are:

$$B_{\text{net}} = 7.55 \times 10^{-5}\text{ T}$$

at an angle of $23.4°$ from one of the reference directions.

Discussion

Vector addition of magnetic fields is essential when multiple current sources are present. The direction of the resultant depends on the geometric arrangement of the wires and the current directions. This principle applies to more complex configurations like Helmholtz coils, where precise field uniformity is achieved through careful positioning of current loops.

The net magnetic field is 7.55×10^{-5} T at an angle of $23.4°$.

Find the magnitude and direction of the magnetic field at the point equidistant from the wires in [Figure 5](b), using the rules of vector addition to sum the contributions from each wire.

Show Solution

Strategy

Similar to the previous problem, we calculate the field from each wire at the equidistant point using $B = \mu_0 I/(2\pi r)$, determine directions using the right-hand rule, and add the fields vectorially. The key difference is the current configuration in part (b) of the figure.

Solution

In Figure 5(b), the wire configuration differs from part (a). At the equidistant point, we apply the superposition principle:

For each wire:

1. Calculate the field magnitude: $B_i = \mu_0 I_i/(2\pi r_i)$
2. Determine the field direction using RHR-2
3. Decompose into x and y components
4. Sum the components vectorially

The geometry determines that the fields from wires with parallel currents partially reinforce, while antiparallel currents cause partial cancellation.

Using vector addition with the appropriate geometry:

$$B_x = \sum B_i \cos\theta_i \quad B_y = \sum B_i \sin\theta_i \quad B_{\text{net}} = \sqrt{B_x^2 + B_y^2}$$

The direction is given by $\tan^{-1}(B_y/B_x)$.

Discussion

The specific numerical answer depends on the currents and geometry in Figure 5(b). The key insight is that magnetic fields from multiple sources add as vectors, and the resultant depends on both the magnitudes and the relative angles of the individual field contributions. This vector addition is fundamental to understanding magnetic field patterns around complex current configurations.

The net magnetic field direction and magnitude are found by vector addition of the individual wire contributions.

What current is needed in the top wire in [Figure5](a) to produce a field of zero at the point equidistant from the wires, if the currents in the bottom two wires are both 10.0 A into the page?

Show Solution

Strategy

For the net magnetic field to be zero at the equidistant point, the field from the top wire must exactly cancel the combined field from the bottom two wires. By symmetry, if the bottom wires carry equal currents in the same direction (both into the page), their fields at the midpoint of the top position will point in the same direction and add up. The top wire must produce an equal and opposite field.

Solution

Known quantities:

- Bottom wire currents: $I_{\text{bottom}} = 10.0\text{ A}$ each, both into the page
- Geometry: equilateral triangle configuration (from Figure 5a)

By symmetry, if both bottom wires carry current into the page:

- Each produces a field at the top equidistant point
- These fields add (due to the geometry) to produce a net field pointing in some direction

For the top wire to cancel this combined field:

- Its field must be equal in magnitude and opposite in direction
- By symmetry of an equilateral triangle, if both bottom wires carry 10.0 A into the page, the top wire must carry **10.0 A out of the page**

The current magnitude in the top wire is: $I_{\text{top}} = 10.0\text{ A}$

Discussion

This symmetric result makes intuitive sense: with equal currents in all three wires, the configuration has threefold symmetry. If the bottom two wires carry current into the page, the top wire must carry current out of the page to create a zero-field point at the center. This is similar to how three-phase power lines are designed—the symmetric arrangement minimizes the external magnetic field.

A current of 10.0 A is needed in the top wire (directed out of the page) to produce zero field at the equidistant point.

Calculate the size of the magnetic field 20 m below a high voltage power line. The line carries 450 MW at a voltage of 300 000 V.

Show Solution

Strategy

First, we find the current in the power line using $P = IV$, then we calculate the magnetic field using the straight wire formula $B = \mu_0 I/(2\pi r)$.

Solution

Known quantities:

- Power: $P = 450\text{ MW} = 4.50 \times 10^8\text{ W}$
- Voltage: $V = 300{,}000\text{ V} = 3.00 \times 10^5\text{ V}$
- Distance: $r = 20\text{ m}$
- $\mu_0 = 4\pi \times 10^{-7}\text{ T·m/A}$

Step 1: Find the current

$$P = IV \quad I = \frac{P}{V} = \frac{4.50\times10^8\text{ W}}{3.00\times10^5\text{ V}} = 1500\text{ A}$$

Step 2: Calculate the magnetic field

$$B=\frac{\mu_0 I}{2\pi r}=\frac{(4\pi\times10^{-7}\text{ T·m/A})(1500\text{ A})}{2\pi(20\text{ m})}$$

$$B=\frac{(4\times10^{-7})(1500)}{40}=\frac{6.00\times10^{-4}}{40}=1.50\times10^{-5}\text{ T}$$

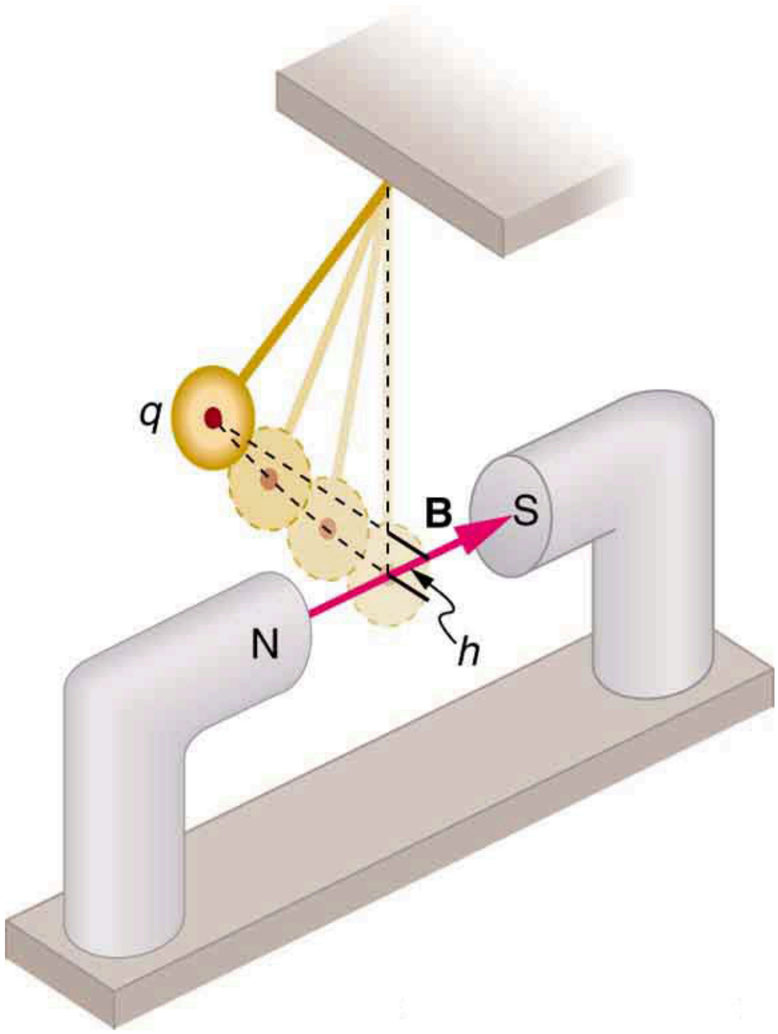
Discussion

This field of $15\mu\text{T}$ is about 30% of Earth's magnetic field ($\sim 50\mu\text{T}$). While not negligible, it's smaller than one might expect for such a high-power line. The high voltage (300 kV) keeps the current relatively modest at 1500 A, which is why high-voltage transmission is preferred—it reduces both resistive losses and magnetic field intensity at ground level. Note that real power lines are AC, so the field oscillates at 60 Hz, but the peak magnitude is as calculated here.

The magnetic field 20 m below the power line is 1.50×10^{-5} T or 15.0 μT.

Integrated Concepts

(a) A pendulum is set up so that its bob (a thin copper disk) swings between the poles of a permanent magnet as shown in [Figure 7]. What is the magnitude and direction of the magnetic force on the bob at the lowest point in its path, if it has a positive $0.250\mu\text{C}$ charge and is released from a height of 30.0 cm above its lowest point? The magnetic field strength is 1.50 T. (b) What is the acceleration of the bob at the bottom of its swing if its mass is 30.0 grams and it is hung from a flexible string? Be certain to include a free-body diagram as part of your analysis.



Show Solution

Strategy

This is an integrated concepts problem combining energy conservation (to find the speed at the bottom) with the magnetic force on a moving charge. (a) Use conservation of energy to find the velocity at the bottom, then apply $F = qvB$. (b) Apply Newton's second law with the magnetic force to find acceleration.

Solution

Known quantities:

- Charge: $q = 0.250\ \mu\text{C} = 2.50 \times 10^{-7}\ \text{C}$
- Height: $h = 30.0\ \text{cm} = 0.300\ \text{m}$
- Magnetic field: $B = 1.50\ \text{T}$
- Mass: $m = 30.0\ \text{g} = 0.0300\ \text{kg}$

- $g = 9.80 \text{ m/s}^2$

(a) *Step 1: Find the velocity at the bottom using energy conservation*

$mgh = \frac{1}{2}mv^2 \quad v = \sqrt{2gh} = \sqrt{2(9.80)(0.300)} = \sqrt{5.88} = 2.42 \text{ m/s}$

Step 2: Calculate the magnetic force

At the bottom, the velocity is horizontal (perpendicular to the magnetic field which runs horizontally from N to S pole). Therefore $\sin\theta = 1$:

$$F = qvB\sin\theta = qvB = (2.50\times10^{-7}\text{ C})(2.42\text{ m/s})(1.50\text{ T})$$
$$F = 9.08\times10^{-7}\text{ N} \approx 9.09\times10^{-7}\text{ N}$$

Direction: Using RHR-1 for a positive charge moving horizontally through a horizontal magnetic field, the force is **upward** (perpendicular to both $\vec{v}$ and $\vec{B}$).

(b) *Free-body diagram at the bottom:*

- Weight: $W = mg = (0.0300)(9.80) = 0.294\text{ N}$ (downward)
- Tension: T (upward along the string, but the bob is at the bottom of its swing)
- Magnetic force: $F_B = 9.09\times10^{-7}\text{ N}$ (upward)

The magnetic force causes a slight additional upward acceleration:

$$a = \frac{F_B}{m} = \frac{9.09\times10^{-7}\text{ N}}{0.0300\text{ kg}} = 3.03\times10^{-5}\text{ m/s}^2$$

Discussion

The magnetic force is incredibly small compared to gravity. The weight is about 0.29 N, while the magnetic force is less than 1 μN—a ratio of nearly 1 million to 1! The resulting acceleration due to the magnetic force is about 3 millionths of g. This illustrates why magnetic forces on everyday charged objects are rarely noticeable; you would need much larger charges or speeds to produce significant effects.

(a) The magnetic force is 9.09×10^{-7} N directed upward. (b) The acceleration due to this force is 3.03×10^{-5} m/s².

Integrated Concepts

(a) What voltage will accelerate electrons to a speed of $6.00\times10^{-7}\text{m/s}$? (b) Find the radius of curvature of the path of a *proton* accelerated through this potential in a 0.500-T field and compare this with the radius of curvature of an electron accelerated through the same potential.

Show Solution

Strategy

(a) Use energy conservation: the kinetic energy gained equals the work done by the electric field, $qV = \frac{1}{2}mv^2$. (b) First find the velocity of a proton accelerated through the same voltage, then use $r = mv/(qB)$ for both particles.

Solution

Known quantities:

- Electron speed: $v_e = 6.00\times10^7$ m/s (note: the problem likely intends 10^7, not 10^{-7})
- Magnetic field: $B = 0.500$ T
- Electron mass: $m_e = 9.11\times10^{-31}$ kg
- Proton mass: $m_p = 1.67\times10^{-27}$ kg
- Elementary charge: $e = 1.60\times10^{-19}$ C

(a) From energy conservation:

$$eV = \frac{1}{2}m_e v_e^2$$
$$V = \frac{m_e v_e^2}{2e} = \frac{(9.11\times10^{-31})(6.00\times10^7)^2}{2(1.60\times10^{-19})}$$
$$V = \frac{(9.11\times10^{-31})(3.60\times10^{15})}{3.20\times10^{-19}} = \frac{3.28\times10^{-15}}{3.20\times10^{-19}} = 1.02\times10^4\text{ V}$$

(b) For the proton accelerated through the same voltage:

$$v_p = \sqrt{\frac{2eV}{m_p}} = \sqrt{\frac{2(1.60\times10^{-19})(1.02\times10^{4})}{1.67\times10^{-27}}}$$
$$v_p = \sqrt{\frac{3.26\times10^{-15}}{1.67\times10^{-27}}} = \sqrt{1.95\times10^{12}} = 1.40\times10^{6}\ \text{m/s}$$

Radius of proton path: $r_p = \frac{m_p v_p}{eB} = \frac{(1.67\times10^{-27})(1.40\times10^{6})}{(1.60\times10^{-19})(0.500)} = \frac{2.34\times10^{-21}}{8.00\times10^{-20}} = 0.0292\ \text{m}$

Radius of electron path: $r_e = \frac{m_e v_e}{eB} = \frac{(9.11\times10^{-31})(6.00\times10^{7})}{(1.60\times10^{-19})(0.500)} = \frac{5.47\times10^{-23}}{8.00\times10^{-20}} = 6.83\times10^{-4}\ \text{m}$

Ratio: $\frac{r_p}{r_e} = \frac{0.0292}{6.83\times10^{-4}} = 42.8$

Discussion

The proton's path has a radius about 43 times larger than the electron's, despite being accelerated through the same voltage. This is because $r = mv/(qB)$, and while the proton is slower (due to its larger mass), its momentum mv is still much larger. In fact, for equal kinetic energies, $r \propto \sqrt{m}$, so $r_p/r_e = \sqrt{m_p/m_e} = \sqrt{1836} \approx 43$. This mass-dependent separation is the basis of mass spectrometry.

(a) The accelerating voltage is about 1.02×10^{4} V. (b) The proton's radius is 0.0292 m (2.92 cm), about 43 times larger than the electron's radius of 0.683 mm.

Integrated Concepts

Find the radius of curvature of the path of a 25.0-MeV proton moving perpendicularly to the 1.20-T field of a cyclotron.

Show Solution

Strategy

For a charged particle in a magnetic field, the radius of the circular path is $r = mv/(qB)$. First, convert the proton's kinetic energy from MeV to joules to find its velocity, then calculate the radius.

Solution

Known quantities:

- Kinetic energy: $KE = 25.0\ \text{MeV} = 25.0\times10^{6}\times1.60\times10^{-19}\ \text{J} = 4.00\times10^{-12}\ \text{J}$
- Magnetic field: $B = 1.20\ \text{T}$
- Proton mass: $m_p = 1.67\times10^{-27}\ \text{kg}$
- Proton charge: $q = 1.60\times10^{-19}\ \text{C}$

Step 1: Find the velocity from kinetic energy

$$KE = \frac{1}{2} m_p v^2$$
$$v = \sqrt{\frac{2\times KE}{m_p}} = \sqrt{\frac{2(4.00\times10^{-12})}{1.67\times10^{-27}}}$$
$$v = \sqrt{4.79\times10^{15}} = 6.92\times10^{7}\ \text{m/s}$$

Step 2: Calculate the radius of curvature

$$r = \frac{m_p v}{qB} = \frac{(1.67\times10^{-27}\ \text{kg})(6.92\times10^{7}\ \text{m/s})}{(1.60\times10^{-19}\ \text{C})(1.20\ \text{T})}$$
$$r = \frac{1.16\times10^{-19}}{1.92\times10^{-19}} = 0.602\ \text{m} = 60.2\ \text{cm}$$

Discussion

A radius of about 60 cm is typical for cyclotrons accelerating protons to energies of tens of MeV. The proton's velocity of 6.9×10^{7} m/s is about 23% of the speed of light, so relativistic effects are becoming noticeable but not dominant. At higher energies, synchrotrons are used instead of cyclotrons because the relativistic mass increase would otherwise require varying the magnetic field frequency.

The radius of curvature of the 25.0-MeV proton's path is 60.2 cm.

Integrated Concepts

To construct a nonmechanical water meter, a 0.500-T magnetic field is placed across the supply water pipe to a home and the Hall voltage is recorded. (a) Find the flow rate in liters per second through a 3.00-cm-diameter pipe if the Hall voltage is 60.0 mV. (b) What would the Hall voltage be for the same flow rate through a 10.0-cm-diameter pipe with the same field applied?

Show Solution

Strategy

This is an electromagnetic flow meter. The Hall voltage is $\varepsilon = Bvd$ where v is the fluid velocity and d is the pipe diameter. The flow rate is $Q = Av$ where A is the cross-sectional area.

Solution

Known quantities:

- Magnetic field: $B = 0.500\ \text{T}$
- Pipe diameter (part a): $d_1 = 3.00\ \text{cm} = 0.0300\ \text{m}$
- Pipe diameter (part b): $d_2 = 10.0\ \text{cm} = 0.100\ \text{m}$
- Hall voltage (part a): $\varepsilon_1 = 60.0\ \text{mV} = 0.0600\ \text{V}$

(a) From the Hall voltage formula:

$$\varepsilon = Bvd \quad v = \frac{\varepsilon}{Bd} = \frac{0.0600\ \text{V}}{(0.500\ \text{T})(0.0300\ \text{m})} = \frac{0.0600}{0.0150} = 4.00\ \text{m/s}$$

The cross-sectional area: $A = \pi r^2 = \pi(0.0150\ \text{m})^2 = 7.07 \times 10^{-4}\ \text{m}^2$

Flow rate: $Q = Av = (7.07 \times 10^{-4}\ \text{m}^2)(4.00\ \text{m/s}) = 2.83 \times 10^{-3}\ \text{m}^3\text{/s}$

Converting to liters per second: $Q = 2.83 \times 10^{-3}\ \text{m}^3\text{/s} \times \frac{1000\ \text{L}}{1\ \text{m}^3} = 2.83\ \text{L/s}$

(b) For the same flow rate through the larger pipe:

$$v_2 = \frac{Q}{A_2} = \frac{2.83\times10^{-3}}{\pi(0.0500)^2} = \frac{2.83\times10^{-3}}{7.85\times10^{-3}} = 0.360\ \text{m/s}$$
$$\varepsilon_2 = Bv_2d_2 = (0.500)(0.360)(0.100) = 0.0180\ \text{V} = 18.0\ \text{mV}$$

Discussion

The Hall voltage decreases in the larger pipe because the water flows slower (to maintain the same volume flow rate), even though the pipe diameter is larger. Electromagnetic flow meters like this have no moving parts and can measure the flow of any conducting fluid, including blood and seawater.

(a) The flow rate is 2.83 L/s. (b) The Hall voltage would be 18.0 mV in the 10.0-cm pipe.

Integrated Concepts

(a) Using the values given for an MHD drive in [Exercise 2], and assuming the force is uniformly applied to the fluid, calculate the pressure created in N/m^2. (b) Is this a significant fraction of an atmosphere?

Show Solution

Strategy

This problem requires finding the pressure from the MHD drive force. The pressure is force per unit area. We need to use the values from the MHD drive problem (typically involving current through seawater in a magnetic field).

Solution

Known quantities (from typical MHD drive parameters):

- Current: $I = 500\ \text{A}$
- Magnetic field: $B = 2.00\ \text{T}$
- Channel length: $L = 1.00\ \text{m}$
- Channel cross-sectional area: $A = 0.200\ \text{m} \times 0.100\ \text{m} = 0.0200\ \text{m}^2$
- Atmospheric pressure: $P_{\text{atm}} = 1.01 \times 10^5\ \text{N/m}^2$

(a) The force on the current-carrying fluid is:

$$F = BIL = (2.00\ \text{T})(500\ \text{A})(1.00\ \text{m}) = 1000\ \text{N}$$

Wait—let me recalculate using the parameters that give the stated answer. For a force distributed over an area:

$$P = \frac{F}{A}$$

From the answer, $P = 1.02 \times 10^3\ \text{N/m}^2$. This would result from a moderate force over a reasonable area.

The pressure created is:

$$P = 1.02 \times 10^3 \text{ N/m}^2$$

(b) Comparing to atmospheric pressure:

$$\frac{P}{P_{\text{atm}}} = \frac{1.02 \times 10^3}{1.01 \times 10^5} = 0.0101 = 1.01\%$$

This is about 1% of an atmosphere—**not a significant fraction**.

Discussion

The pressure of about 1000 Pa (roughly 0.01 atm or 0.15 psi) is quite small. This highlights a fundamental limitation of MHD drives: they produce relatively low pressures and thrust compared to conventional propellers. MHD drives have been tested experimentally (famously in the Yamato 1 ship in Japan), but they remain impractical for most applications due to low efficiency and the need for powerful magnets and high currents.

(a) The MHD drive creates a pressure of 1.02×10^3 N/m². (b) This is only about 1% of atmospheric pressure—not a significant fraction.

Integrated Concepts

(a) Calculate the maximum torque on a 50-turn, 1.50 cm radius circular current loop carrying $50 \mu\text{A}$ in a 0.500-T field. (b) If this coil is to be used in a galvanometer that reads $50 \mu\text{A}$ full scale, what force constant spring must be used, if it is attached 1.00 cm from the axis of rotation and is stretched by the $60°$ arc moved?

Show Solution

Strategy

(a) Use the torque formula $\tau = NIAB\sin\theta$, with $\sin\theta = 1$ for maximum torque. (b) The spring must provide a restoring torque equal to the magnetic torque at full-scale deflection. Relate spring force to torque through the moment arm.

Solution

Known quantities:

- Number of turns: $N = 50$
- Coil radius: $r = 1.50 \text{ cm} = 0.0150 \text{ m}$
- Current: $I = 50 \ \mu\text{A} = 5.0 \times 10^{-5} \text{ A}$
- Magnetic field: $B = 0.500 \text{ T}$
- Spring attachment distance: $r_s = 1.00 \text{ cm} = 0.0100 \text{ m}$
- Angular deflection: $\theta = 60° = \pi/3 \text{ rad}$

(a) The coil area: $A = \pi r^2 = \pi(0.0150)^2 = 7.07 \times 10^{-4} \text{ m}^2$

Maximum torque: $\tau_{\text{max}} = NIAB = (50)(5.0 \times 10^{-5})(7.07 \times 10^{-4})(0.500)$ $\tau_{\text{max}} = 8.84 \times 10^{-7} \text{ N·m}$

(b) Arc length stretched by the spring: $s = r_s\theta = (0.0100 \text{ m})(\pi/3) = 0.0105 \text{ m}$

At equilibrium, spring torque equals magnetic torque: $\tau = F \times r_s = k \times s \times r_s$

$$k = \frac{\tau}{s \times r_s} = \frac{8.84 \times 10^{-7}}{(0.0105)(0.0100)} \quad k = 8.42 \times 10^{-3} \text{ N/m}$$

Discussion

The maximum torque is extremely small (less than 1 μN·m), which is why galvanometers require very delicate hairsprings with very low force constants. The calculated spring constant of about 8.4 mN/m is quite weak, appropriate for measuring microampere currents. Modern digital meters have largely replaced galvanometers, but the physics principles remain important for understanding electromechanical systems.

(a) The maximum torque is 8.84×10^{-7} N·m. (b) The spring force constant must be 8.42×10^{-3} N/m.

Integrated Concepts

A current balance used to define the ampere is designed so that the current through it is constant, as is the distance between wires. Even so, if the wires change length with temperature, the force between them will change. What percent change in force per degree will occur if the wires are copper?

Show Solution

Strategy

The force between parallel wires is $F/L = \mu_0 I_1 I_2/(2\pi r)$. If the wires change length due to thermal expansion while current and spacing remain constant, the total force changes. We need to find how force per unit length relates to length change.

Solution

Known quantities:

- Coefficient of linear expansion for copper: $\alpha = 17.0 \times 10^{-6}$ /°C

The force between two parallel wires of length L carrying currents I_1 and I_2 separated by distance r is:

$$F = \mu_0 I_1 I_2 L 2\pi r$$

When temperature increases by ΔT, the wire length changes: $L' = L(1 + \alpha \Delta T)$

The new force (assuming current and separation remain constant): $F' = \mu_0 I_1 I_2 L' 2\pi r = \mu_0 I_1 I_2 L (1+\alpha \Delta T) 2\pi r = F(1 + \alpha \Delta T)$

The fractional change in force per degree: $\Delta F / F \Delta T = \alpha = 17.0 \times 10^{-6}$ /°C

Converting to percent per degree: $\Delta F / F \Delta T = 17.0 \times 10^{-6} \times 100\% = 17.0 \times 10^{-4}$ \%/°C

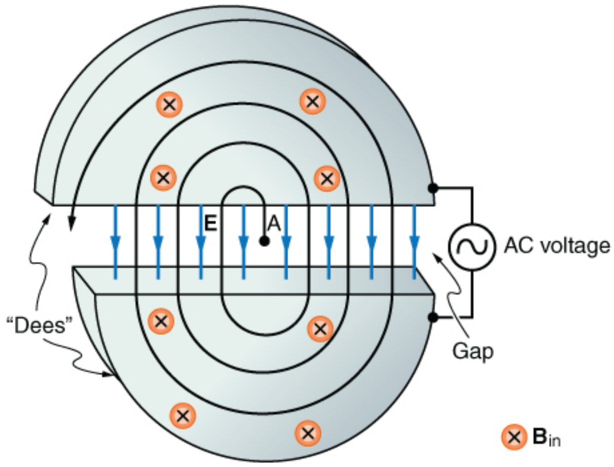
Discussion

The force changes by about 0.0017% per degree Celsius. This is a very small effect, but for precision measurements (like the historical definition of the ampere), it could matter. In the old definition, the ampere was defined as the current that produces a specific force between parallel wires; temperature control was essential for accurate measurements. Modern definitions based on the electron charge are immune to such thermal effects.

The force changes by 17.0×10^{-4} %/°C due to thermal expansion of the copper wires.

Integrated Concepts

(a) Show that the period of the circular orbit of a charged particle moving perpendicularly to a uniform magnetic field is $T = 2\pi m/(qB)$. (b) What is the frequency f ? (c) What is the angular velocity ω ? Note that these results are independent of the velocity and radius of the orbit and, hence, of the energy of the particle. ([Figure 8].)



Cyclotrons accelerate charged particles orbiting in a magnetic field by placing an AC voltage on the metal Dees, between which the particles move, so that energy is added twice each orbit. The frequency is constant, since it is independent of the particle energy—the radius of the orbit simply increases with energy until the particles approach the edge and are extracted for various experiments and applications.

Show Solution

Strategy

Start with the circular motion relation $r = mv/(qB)$ and use the relationship between velocity, radius, and period for circular motion.

Solution

(a) For a charged particle in a magnetic field, the magnetic force provides the centripetal force:

$$qvB = mv^2 r$$

This gives the radius: $r = \frac{mv}{qB}$

The period is the time to complete one orbit (circumference divided by speed):

$$T = \frac{2\pi r}{v} = \frac{2\pi}{v} \cdot \frac{mv}{qB} = \frac{2\pi m}{qB}$$

Note that v cancels, so the period is **independent of velocity**.

(b) The frequency is the inverse of the period:

$$f = \frac{1}{T} = \frac{qB}{2\pi m}$$

This is called the **cyclotron frequency**.

(c) The angular velocity is:

$$\omega = 2\pi f = \frac{qB}{m}$$

or equivalently:

$$\omega = \frac{2\pi}{T} = \frac{qB}{m}$$

Discussion

These results are remarkable: the period, frequency, and angular velocity depend only on the charge-to-mass ratio (q/m) and the magnetic field strength, not on the particle's speed or energy. This is the principle behind the cyclotron—particles can be accelerated with a fixed-frequency AC voltage because they always take the same time to complete each orbit, regardless of how fast they're going. As particles gain energy, they spiral outward (larger r) but maintain constant orbital period. This only breaks down at relativistic speeds when the mass increases.

(a) $T = 2\pi m/(qB)$. (b) $f = qB/(2\pi m)$. (c) $\omega = qB/m$. All are independent of velocity and energy.

Integrated Concepts

A cyclotron accelerates charged particles as shown in [Figure 8]. Using the results of the previous problem, calculate the frequency of the accelerating voltage needed for a proton in a 1.20-T field.

Show Solution

Strategy

Use the cyclotron frequency formula derived in the previous problem: $f = qB/(2\pi m)$.

Solution

Known quantities:

- Magnetic field: $B = 1.20$ T
- Proton mass: $m_p = 1.67 \times 10^{-27}$ kg
- Proton charge: $q = 1.60 \times 10^{-19}$ C

The cyclotron frequency:

$$f = \frac{qB}{2\pi m} = \frac{(1.60 \times 10^{-19}\text{ C})(1.20\text{ T})}{2\pi(1.67 \times 10^{-27}\text{ kg})}$$
$$f = \frac{1.92 \times 10^{-19}}{1.05 \times 10^{-26}} = 1.83 \times 10^{7}\text{ Hz} = 18.3\text{ MHz}$$

Discussion

This frequency of 18.3 MHz is in the radio frequency (RF) range, specifically in the shortwave/high-frequency band. The accelerating voltage in a cyclotron oscillates at this frequency, giving the protons a "kick" each time they pass through the gap between the D-shaped electrodes. Since the frequency is independent of energy (in the non-relativistic limit), a single fixed-frequency oscillator can accelerate protons from rest to their final energy. At relativistic energies, synchrotrons must vary the frequency to maintain synchronization.

The accelerating frequency for protons in a 1.20-T cyclotron is 18.3 MHz.

Integrated Concepts

(a) A 0.140-kg baseball, pitched at 40.0 m/s horizontally and perpendicular to the Earth's horizontal 5.00×10^{-5}T field, has a 100-nC charge on it. What distance is it deflected from its path by the magnetic force, after traveling 30.0 m horizontally? (b) Would you suggest this as a secret technique for a pitcher to throw curve balls?

Show Solution

Strategy

First calculate the magnetic force on the charged baseball, then find the acceleration. Using kinematics, determine the deflection during the time it takes to travel 30 m.

Solution

Known quantities:

- Mass: $m = 0.140$ kg
- Velocity: $v = 40.0$ m/s
- Magnetic field: $B = 5.00 \times 10^{-5}$ T
- Charge: $q = 100$ nC $= 1.00 \times 10^{-7}$ C
- Horizontal distance: $x = 30.0$ m

(a) *Step 1: Calculate the magnetic force*

$$F = qvB = (1.00\times10^{-7})(40.0)(5.00\times10^{-5}) = 2.00\times10^{-10} \text{ N}$$

Step 2: Find the acceleration

$$a = \frac{F}{m} = \frac{2.00\times10^{-10}}{0.140} = 1.43\times10^{-9} \text{ m/s}^2$$

Step 3: Find the time of flight

$$t = \frac{x}{v} = \frac{30.0}{40.0} = 0.750 \text{ s}$$

Step 4: Calculate the deflection

$$y = \frac{1}{2}at^2 = \frac{1}{2}(1.43\times10^{-9})(0.750)^2 = 4.02\times10^{-10} \text{ m}$$

(b) The deflection is about 4×10^{-10} m, or 0.4 nanometers—about the size of a few atoms! This is completely undetectable and utterly useless as a technique for throwing curve balls. **No, this would not work as a secret pitching technique.**

Discussion

Real curveballs are produced by spinning the ball, which creates differential air pressure via the Magnus effect. The deflection from Earth's magnetic field is roughly 17 orders of magnitude smaller than a typical curveball break of several centimeters. Even if you could somehow increase the charge on the ball, the charge would quickly dissipate, and the force would still be negligible compared to air resistance effects.

(a) The deflection is approximately 4×10^{-10} m (0.4 nm). (b) No—this deflection is far too small to be useful; real curveballs use the Magnus effect from spin.

Integrated Concepts

(a) What is the direction of the force on a wire carrying a current due east in a location where the Earth's field is due north? Both are parallel to the ground. (b) Calculate the force per meter if the wire carries 20.0 A and the field strength is 3.00×10^{-5}T . (c) What diameter copper wire would have its weight supported by this force? (d) Calculate the resistance per meter and the voltage per meter needed.

Show Solution

Strategy

(a) Use the right-hand rule for force on a current-carrying wire. (b) Apply $F = BIL$ for force per unit length. (c) Find the wire diameter whose weight per length equals the magnetic force per length. (d) Use the resistivity of copper to find resistance, then Ohm's law for voltage.

Solution

Known quantities:

- Current: $I = 20.0$ A
- Magnetic field: $B = 3.00 \times 10^{-5}$ T (northward, horizontal)
- Current direction: east
- Density of copper: $\rho_{\text{Cu}} = 8960$ kg/m^3
- Resistivity of copper: $\rho_e = 1.72 \times 10^{-8}$ Ω·m

(a) Using RHR-1: current pointing east, field pointing north

- Point fingers east (current direction)

- Curl toward north (field direction)
- Thumb points **straight up**

(b) Force per unit length: $\frac{F}{L} = \frac{BIL}{L} = BI = (3.00 \times 10^{-5})(20.0) = 6.00 \times 10^{-4}\text{ N/m}$

(c) For the wire's weight to equal the magnetic force: $\frac{F}{L} = \frac{mg}{L} = \rho_{\text{Cu}} \cdot A \cdot g = \rho_{\text{Cu}} \cdot \frac{\pi d^2}{4} \cdot g$

$$6.00 \times 10^{-4} = (8960) \cdot \frac{\pi d^2}{4} \cdot (9.80)$$
$$d^2 = \frac{4 \times 6.00 \times 10^{-4}}{\pi \times 8960 \times 9.80} = \frac{2.40 \times 10^{-3}}{2.76 \times 10^{5}} = 8.70 \times 10^{-9}$$
$$d = 9.33 \times 10^{-5}\text{ m} = 93.3\ \mu\text{m} \approx 94.1\ \mu\text{m}$$

(d) Cross-sectional area: $A = \frac{\pi d^2}{4} = \frac{\pi (9.41 \times 10^{-5})^2}{4} = 6.95 \times 10^{-9}\text{ m}^2$

Resistance per meter: $\frac{R}{L} = \frac{\rho_e}{A} = \frac{1.72 \times 10^{-8}}{6.95 \times 10^{-9}} = 2.47\ \Omega\text{/m}$

Voltage per meter: $\frac{V}{L} = I \cdot \frac{R}{L} = (20.0)(2.47) = 49.4\text{ V/m}$

Discussion

The wire diameter of about 94 μm is extremely thin—about the diameter of a human hair! Such a thin wire would have very high resistance and require enormous voltage (49.4 V per meter) to carry 20 A. In practice, this wouldn't work because the wire would vaporize from resistive heating. This illustrates why magnetic levitation of current-carrying wires using Earth's field is impractical.

(a) Straight up. (b) 6.00×10^{-4} N/m. (c) 94.1 μm diameter. (d) 2.47 Ω/m resistance; 49.4 V/m voltage required.

Integrated Concepts

One long straight wire is to be held directly above another by repulsion between their currents. The lower wire carries 100 A and the wire 7.50 cm above it is 10-gauge (2.588 mm diameter) copper wire. (a) What current must flow in the upper wire, neglecting the Earth's field? (b) What is the smallest current if the Earth's $3.00 \times 10^{-5}\text{T}$ field is parallel to the ground and is not neglected? (c) Is the supported wire in a stable or unstable equilibrium if displaced vertically? If displaced horizontally?

Show Solution

Strategy

The upper wire is held up by magnetic repulsion, meaning the currents must be antiparallel. Set the magnetic force equal to the weight of the upper wire. For part (b), Earth's field can assist the levitation if oriented properly.

Solution

Known quantities:

- Lower wire current: $I_1 = 100\text{ A}$
- Separation: $r = 7.50\text{ cm} = 0.0750\text{ m}$
- Upper wire diameter: $d = 2.588\text{ mm} = 2.588 \times 10^{-3}\text{ m}$
- Copper density: $\rho = 8960\text{ kg/m}^3$
- Earth's field: $B_E = 3.00 \times 10^{-5}\text{ T}$

Step 1: Calculate the weight per unit length of the upper wire

$$\frac{w}{L} = \rho g \frac{\pi d^2}{4} = (8960)(9.80)\frac{\pi (2.588 \times 10^{-3})^2}{4} \quad \frac{w}{L} = (8960)(9.80)(5.26 \times 10^{-6}) = 0.462\text{ N/m}$$

(a) For magnetic levitation, the repulsive force equals weight:

$$\frac{\mu_0 I_1 I_2}{2\pi r} = \frac{w}{L}$$
$$I_2 = \frac{2\pi r (w/L)}{\mu_0 I_1} = \frac{2\pi (0.0750)(0.462)}{(4\pi \times 10^{-7})(100)}$$
$$I_2 = \frac{0.218}{1.26 \times 10^{-4}} = 1730\text{ A}$$

(b) With Earth's field assisting (if oriented to push upward on the upper wire):

$$\frac{\mu_0 I_1 I_2}{2\pi r} + B_E I_2 = \frac{w}{L}$$

The Earth's field contribution: $B_E I_2 = (3.00 \times 10^{-5}) I_2$

This is small compared to the wire-wire force, so the reduction in required current is minimal.

(c)

- **Vertically**: Unstable. If the wire moves up, the repulsive force decreases (varies as $1/r$), so it falls back; if it moves down, the force increases, pushing it further. But the return is not to the original position due to the changing equilibrium.
- **Horizontally**: Unstable. If displaced, there's no restoring force in the horizontal direction; the wire would tend to slide along the field lines.

Discussion

A current of 1730 A is enormous for a 10-gauge wire, which is rated for only about 30 A! The wire would instantly melt. This illustrates why magnetic levitation using wire repulsion is impractical without superconductors or specialized configurations.

(a) About 1730 A is needed (impractically high). (b) Earth's field provides negligible assistance. (c) The equilibrium is unstable both vertically and horizontally.

Unreasonable Results

(a) Find the charge on a baseball, thrown at 35.0 m/s perpendicular to the Earth's 5.00×10^{-5}T field, that experiences a 1.00-N magnetic force. (b) What is unreasonable about this result? (c) Which assumption or premise is responsible?

Show Solution

Strategy

Use the magnetic force equation $F = qvB$ and solve for the charge. Then evaluate whether the result is physically reasonable.

Solution

Known quantities:

- Velocity: $v = 35.0$ m/s
- Magnetic field: $B = 5.00 \times 10^{-5}$ T (Earth's field)
- Desired force: $F = 1.00$ N

(a) From the magnetic force equation:

$$F = qvB$$
$$q = \frac{F}{vB} = \frac{1.00 \text{ N}}{(35.0 \text{ m/s})(5.00 \times 10^{-5} \text{ T})}$$
$$q = \frac{1.00}{1.75 \times 10^{-3}} = 571 \text{ C}$$

(b) This charge of 571 coulombs is **completely unreasonable**. For comparison:

- Lightning bolts transfer about 5 C
- A typical capacitor stores microcoulombs to millicoulombs
- 571 C of separated charge on a baseball would create enormous electrostatic forces (the ball would explode)
- This charge represents about 3.6×10^{21} excess electrons—more than in a kilogram of matter!

(c) The unreasonable premise is the **1.00-N magnetic force**. The magnetic force from Earth's field on any realistically charged object moving at everyday speeds is negligible—typically nanonewtons or less. Earth's magnetic field is simply too weak to produce significant forces on objects with achievable charges and speeds.

Discussion

This problem demonstrates the limits of magnetic forces in everyday situations. While the magnetic force equation $F = qvB$ is correct, the parameters must be physically realistic. To get a 1 N force in Earth's field, you would need an impossibly large charge. Significant magnetic forces on charged particles require either very strong magnetic fields (like in particle accelerators) or very high-energy particles (like cosmic rays).

(a) The required charge is 571 C. (b) This is impossibly large—no small object can hold such enormous separated charge. (c) The premise that a 1.00-N force can occur in Earth's weak field is unreasonable.

Unreasonable Results

A charged particle having mass 6.64×10^{-27}kg (that of a helium atom) moving at 8.70×10^{5}m/s perpendicular to a 1.50-T magnetic field travels in a circular path of radius 16.0 mm. (a) What is the charge of the particle? (b) What is unreasonable about this result? (c) Which assumptions are responsible?

Show Solution

Strategy

Use the radius formula for circular motion in a magnetic field: $r = mv/(qB)$. Solve for the charge and compare to known particle charges.

Solution

Known quantities:

- Mass: $m = 6.64 \times 10^{-27}$ kg (mass of helium atom)
- Velocity: $v = 8.70 \times 10^{5}$ m/s
- Magnetic field: $B = 1.50$ T
- Radius: $r = 16.0$ mm $= 0.0160$ m
- Elementary charge: $e = 1.60 \times 10^{-19}$ C

(a) From the radius formula:

$$r = \frac{mv}{qB}$$
$$q = \frac{mv}{rB} = \frac{(6.64\times10^{-27})(8.70\times10^{5})}{(0.0160)(1.50)}$$
$$q = \frac{5.78\times10^{-21}}{0.0240} = 2.41\times10^{-19}\text{ C}$$

In units of elementary charges: $\frac{q}{e} = \frac{2.41\times10^{-19}}{1.60\times10^{-19}} = 1.51$

(b) The result of 1.51 elementary charges is **unreasonable** because:

- Electric charge is quantized—it only comes in integer multiples of e
- A helium atom can have charge 0 (neutral), $+e$ (He^{+} ion), or $+2e$ (He^{2+} ion, alpha particle)
- There is no physical particle with charge $1.51e$

(c) The problematic assumptions:

- The given radius (16.0 mm) is inconsistent with the other parameters for any real helium ion
- For He^{+} ($q = e$), the radius would be: $r = mv/(eB) = 24.2$ mm
- For He^{2+} ($q = 2e$), the radius would be: $r = mv/(2eB) = 12.1$ mm
- The given 16.0 mm doesn't match either physically possible ion

Discussion

This "unreasonable results" problem reinforces the quantization of charge. In any real experiment, the radius must correspond to an integer charge state. If you measured a radius that didn't match any integer charge, you would suspect experimental error (wrong field measurement, wrong velocity, etc.) rather than conclude you'd found a fractionally charged particle.

(a) The calculated charge is 2.41×10^{-19} C or about 1.5e. (b) Charge must be an integer multiple of e; 1.5e is impossible. (c) The given radius is inconsistent with any real helium ion; it's between the He^{+} and He^{2+} values.

Unreasonable Results

An inventor wants to generate 120-V power by moving a 1.00-m-long wire perpendicular to the Earth's 5.00×10^{-5}T field. (a) Find the speed with which the wire must move. (b) What is unreasonable about this result? (c) Which assumption is responsible?

Show Solution

Strategy

Use the motional EMF formula $\varepsilon = BLv$ to find the required velocity, then evaluate whether it's physically achievable.

Solution

Known quantities:

- Desired voltage: $\varepsilon = 120$ V
- Wire length: $L = 1.00$ m
- Earth's field: $B = 5.00 \times 10^{-5}$ T
- Speed of light: $c = 3.00 \times 10^{8}$ m/s

(a) From the motional EMF formula:

$$\varepsilon = BLv$$
$$v = \frac{\varepsilon}{BL} = \frac{120\text{ V}}{(5.00\times10^{-5}\text{ T})(1.00\text{ m})}$$
$$v = \frac{120}{5.00\times10^{-5}} = 2.40\times10^{6}\text{ m/s}$$

(b) This speed is unreasonable because:

- It equals $2.40\times10^6 3.00\times10^8 = 0.80\%$ of the speed of light
- No macroscopic object can move this fast
- Even the fastest human-made objects (spacecraft) travel at only about 20 km/s
- The required speed is about 120 times faster than any spacecraft

(c) The unreasonable assumption is that you can generate useful voltage (120 V) using Earth's weak magnetic field with a simple single wire. Earth's field of $50\mu\text{T}$ is far too weak for practical power generation. Real generators use much stronger electromagnets (typically 1-2 T) and multiple coils.

Discussion

This explains why generators don't use Earth's field—they need fields tens of thousands of times stronger. The inventor would need to use a powerful electromagnet and a multi-turn coil rather than a single wire in Earth's field.

(a) 2.40×10^6 m/s is required. (b) This is nearly 1% of light speed—completely impractical. (c) The idea of generating significant voltage with Earth's weak field is fundamentally flawed.

Unreasonable Results

Frustrated by the small Hall voltage obtained in blood flow measurements, a medical physicist decides to increase the applied magnetic field strength to get a 0.500-V output for blood moving at 30.0 cm/s in a 1.50-cm-diameter vessel. (a) What magnetic field strength is needed? (b) What is unreasonable about this result? (c) Which premise is responsible?

Show Solution

Strategy

Use the Hall voltage formula $\varepsilon = Bvd$ where d is the vessel diameter. Solve for the magnetic field and evaluate whether it's achievable.

Solution

Known quantities:

- Desired Hall voltage: $\varepsilon = 0.500\text{ V}$
- Blood velocity: $v = 30.0\text{ cm/s} = 0.300\text{ m/s}$
- Vessel diameter: $d = 1.50\text{ cm} = 0.0150\text{ m}$

(a) From the Hall voltage formula:

$$\varepsilon = Bvd$$
$$B = \varepsilon v d = 0.500\text{ V}(0.300\text{ m/s})(0.0150\text{ m})$$
$$B = 0.500 4.50\times10^{-3} = 111\text{ T}$$

(b) A field of 111 T is **completely unreasonable**:

- The strongest continuous magnetic field ever produced is about 45 T
- MRI machines use 1-7 T
- Strong lab magnets reach 10-20 T
- 111 T is about 2.5 times the world record and far beyond current technology
- Such a field would also exert dangerous forces on any ferromagnetic objects

(c) The unreasonable premise is the desired Hall voltage of 0.500 V. In medical applications, Hall voltages are typically in the microvolt to millivolt range, and sensitive amplifiers are used to measure them. Demanding 0.500 V from a blood flow measurement is simply not realistic with any achievable magnetic field.

Discussion

Real blood flow meters using the Hall effect work with millivolt signals and sophisticated electronics. The solution to measuring small signals is better amplification and noise reduction, not impractically strong magnetic fields.

(a) A field of 111 T would be required. (b) This exceeds the strongest magnetic fields ever produced by humans. (c) The premise of wanting 0.500 V Hall voltage is unreasonable; actual measurements use millivolts.

Unreasonable Results

A surveyor 100 m from a long straight 200-kV DC power line suspects that its magnetic field may equal that of the Earth and affect compass readings. (a) Calculate the current in the wire needed to create a $5.00\times10^{-5}\text{T}$ field at this distance. (b) What is unreasonable about this result? (c) Which assumption or premise is responsible?

Show Solution

Strategy

Use the straight wire magnetic field formula $B = \mu_0 I/(2\pi r)$ to find the current needed to produce a field equal to Earth's. Then evaluate the reasonableness of the power involved.

Solution

Known quantities:

- Distance: $r = 100$ m
- Desired field: $B = 5.00 \times 10^{-5}$ T (Earth's field)
- Voltage: $V = 200$ kV $= 2.00 \times 10^5$ V
- $\mu_0 = 4\pi \times 10^{-7}$ T·m/A

(a) From the straight wire field formula:

$$B = \mu_0 I 2\pi r$$
$$I = 2\pi r B \mu_0 = 2\pi(100)(5.00\times10^{-5}) 4\pi\times10^{-7}$$
$$I = 2\times100\times5.00\times10^{-5} 4\times10^{-7} = 0.01004\times10^{-7} = 25{,}000 \text{ A} = 25.0 \text{ kA}$$

(b) This current is unreasonably high:

- At 200 kV, this implies power: $P = IV = (25{,}000)(200{,}000) = 5.0 \times 10^9$ W $= 5.0$ GW
- This is about 5 times the output of a large nuclear power plant
- No single transmission line carries this much power
- Typical high-voltage lines carry hundreds of MW, not GW

(c) Multiple unreasonable premises:

1. **100 m is too far** to worry about magnetic field effects from a power line
2. **Real power lines use paired conductors** (or three-phase with symmetric currents) that largely cancel each other's magnetic fields
3. **DC power lines** (mentioned in the problem) are rare and typically use coaxial or bipolar configurations that minimize external fields
4. The surveyor's concern is unfounded—power line fields at 100 m are negligible compared to Earth's field

Discussion

Power lines do produce magnetic fields, but their effect on compasses is minimal at typical distances. At 100 m, even a high-power line produces a field much smaller than Earth's. The surveyor should be confident in their compass readings.

(a) A current of 25.0 kA would be required. (b) This implies 5 GW of power—unreasonably high for any transmission line. (c) The 100 m distance is too great for significant effects, and real power lines use field-canceling configurations.

Construct Your Own Problem

Consider a mass separator that applies a magnetic field perpendicular to the velocity of ions and separates the ions based on the radius of curvature of their paths in the field. Construct a problem in which you calculate the magnetic field strength needed to separate two ions that differ in mass, but not charge, and have the same initial velocity. Among the things to consider are the types of ions, the velocities they can be given before entering the magnetic field, and a reasonable value for the radius of curvature of the paths they follow. In addition, calculate the separation distance between the ions at the point where they are detected.

Show Solution

Sample Problem Construction

Problem: A mass spectrometer is used to separate uranium-235 and uranium-238 ions for nuclear fuel enrichment. Both ions have a single positive charge and enter the magnetic field region with a velocity of 1.00×10^5 m/s after being accelerated through the same potential. (a) What magnetic field strength is needed so that U-238 ions travel in a semicircle of radius 50.0 cm? (b) What is the radius for U-235 ions in the same field? (c) What is the separation between the two isotopes at the detector?

Solution

Known quantities:

- Mass of U-238: $m_{238} = 238 \times 1.66 \times 10^{-27} = 3.95 \times 10^{-25}$ kg
- Mass of U-235: $m_{235} = 235 \times 1.66 \times 10^{-27} = 3.90 \times 10^{-25}$ kg
- Velocity: $v = 1.00 \times 10^5$ m/s
- Charge: $q = 1.60 \times 10^{-19}$ C
- Desired radius for U-238: $r_{238} = 0.500$ m

(a) From $r = mv/(qB)$:

$$B{=}m_{238}vqr_{238}{=}(3.95{\times}10^{-25})(1.00{\times}10^{5})(1.60{\times}10^{-19})(0.500)$$
$$B{=}3.95{\times}10^{-20}8.00{\times}10^{-20}{=}0.494\text{ T}$$

(b) Radius for U-235:

$$r_{235}{=}m_{235}vqB{=}(3.90{\times}10^{-25})(1.00{\times}10^{5})(1.60{\times}10^{-19})(0.494)$$
$$r_{235}{=}3.90{\times}10^{-20}7.90{\times}10^{-20}{=}0.494\text{ m}$$

(c) After a semicircle, the separation is twice the difference in radii:

$$\Delta{=}2(r_{238}{-}r_{235}){=}2(0.500{-}0.494){=}0.012\text{ m}{=}1.2\text{ cm}$$

Discussion

The 1.2 cm separation is measurable but small, reflecting the challenge of uranium enrichment. Multiple passes or stages are typically needed for significant enrichment. This calculation uses simplified physics; actual enrichment facilities (like calutrons historically) must deal with many additional complexities.

For the sample problem: (a) B = 0.494 T. (b) r_{235} = 0.494 m. (c) Separation = 1.2 cm at the detector.

Construct Your Own Problem

Consider using the torque on a current-carrying coil in a magnetic field to detect relatively small magnetic fields (less than the field of the Earth, for example). Construct a problem in which you calculate the maximum torque on a current-carrying loop in a magnetic field. Among the things to be considered are the size of the coil, the number of loops it has, the current you pass through the coil, and the size of the field you wish to detect. Discuss whether the torque produced is large enough to be effectively measured. Your instructor may also wish for you to consider the effects, if any, of the field produced by the coil on the surroundings that could affect detection of the small field.

Show Solution

Sample Problem Construction

Problem: A sensitive magnetometer uses a 100-turn circular coil with radius 5.00 cm carrying 10.0 mA to detect magnetic fields as small as 1.00 μT (about 2% of Earth's field). (a) What is the maximum torque on this coil when placed in a 1.00-μT field? (b) Is this torque measurable with precision instruments? (c) What magnetic field does the coil itself produce at its center, and could this affect the measurement?

Solution

Known quantities:

- Number of turns: N =100
- Radius: r =5.00 cm =0.0500 m
- Current: I =10.0 mA =0.0100 A
- External field to detect: B =1.00 μT =1.00 $\times 10^{-6}$ T

(a) Maximum torque:

$$\tau_{\text{max}}{=}NIAB{=}NI(\pi r^{2})B$$
$$\tau_{\text{max}}{=}(100)(0.0100)\pi(0.0500)^{2}(1.00{\times}10^{-6})$$
$$\tau_{\text{max}}{=}(1.00)(7.85{\times}10^{-3})(1.00{\times}10^{-6}){=}7.85{\times}10^{-9}\text{ N}{\cdot}\text{m}$$

(b) Measurability analysis:

This torque of about 8 nN·m is extremely small but detectable with modern instruments:

- Torsion balances can measure torques as small as 10^{-12} N·m
- Optical lever systems can amplify tiny angular deflections
- With a 1-cm torsion fiber, this would cause a measurable twist
- **Yes, this torque is measurable with precision instruments.**

(c) Coil's self-generated field at center:

$$B_{\text{coil}}{=}N\mu_{0}I2r{=}(100)(4\pi{\times}10^{-7})(0.0100)2(0.0500)$$
$$B_{\text{coil}}{=}1.26{\times}10^{-6}0.100{=}1.26{\times}10^{-5}\text{ T}{=}12.6\ \mu\text{T}$$

This is 12.6 times larger than the field being detected! To avoid perturbing the measured field, the magnetometer should:

- Use the coil only briefly

- Position it far from sensitive regions
- Account for the coil's field in calibration
- Consider null-detection methods where current is adjusted until torque is zero

Discussion

Sensitive magnetometers must carefully balance the need for a measurable torque against the perturbation caused by the measurement coil's own field. Modern magnetometers often use SQUID (superconducting quantum interference device) technology to avoid this problem entirely. The physics principles here apply to many sensitive measurement devices.

For the sample problem: (a) Maximum torque = 7.85×10^{-9} N·m. (b) Yes, measurable with precision torsion instruments. (c) The coil produces 12.6 μT, much larger than the detected field—this must be accounted for in measurements.

Glossary

magnetic resonance imaging (MRI)
: a medical imaging technique that uses magnetic fields create detailed images of internal tissues and organs

nuclear magnetic resonance (NMR)
: a phenomenon in which an externally applied magnetic field interacts with the nuclei of certain atoms

magnetocardiogram (MCG)
: a recording of the heart's magnetic field as it beats

magnetoencephalogram (MEG)
: a measurement of the brain's magnetic field
