

FORCE ON A MOVING CHARGE IN A MAGNETIC FIELD: EXAMPLES AND APPLICATIONS

LEARNING OUTCOMES:

- Describe the effects of a magnetic field on a moving charge.
- Calculate the radius of curvature of the path of a charge that is moving in a magnetic field.

Magnetic force can cause a charged particle to move in a circular or spiral path. Cosmic rays are energetic charged particles in outer space, some of which approach the Earth. They can be forced into spiral paths by the Earth's magnetic field. Protons in giant accelerators are kept in a circular path by magnetic force. The bubble chamber photograph in [Figure 1] shows charged particles moving in such curved paths. The curved paths of charged particles in magnetic fields are the basis of a number of phenomena and can even be used analytically, such as in a mass spectrometer.



Figure 1: Trails of bubbles are produced by high-energy charged particles moving through the superheated liquid hydrogen in this artist's rendition of a bubble chamber. There is a strong magnetic field perpendicular to the page that causes the curved paths of the particles. The radius of the path can be used to find the mass, charge, and energy of the particle.

So does the magnetic force cause circular motion? Magnetic force is always perpendicular to velocity, so that it does no work on the charged particle. The particle's kinetic energy and speed thus remain constant. The direction of motion is affected, but not the speed. This is typical of uniform circular motion. The simplest case occurs when a charged particle moves perpendicular to a uniform B -field, such as shown in [Figure 2]. (If this takes place in a vacuum, the magnetic field is the dominant factor determining the motion.) Here, the magnetic force supplies the centripetal force $F_c = mv^2/r$. Noting that $\sin \theta = 1$, we see that $F = qvB$.

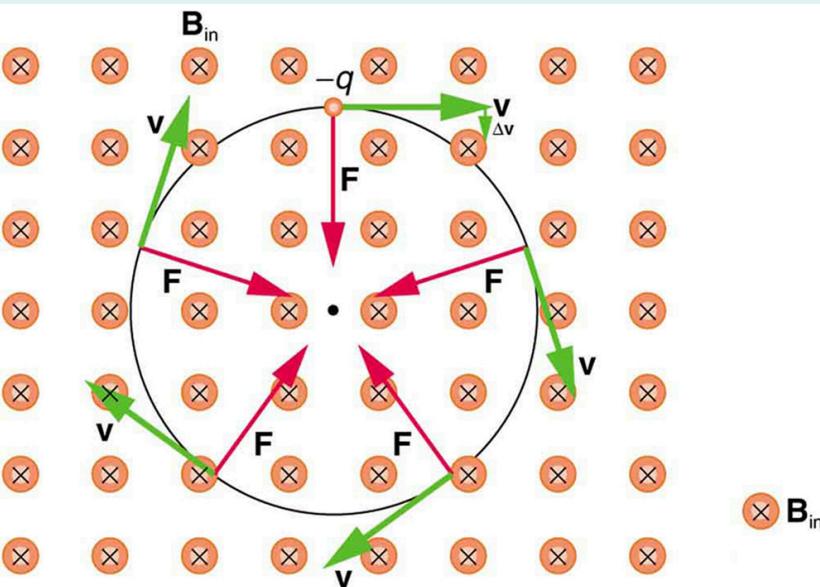


Figure 2: A negatively charged particle moves in the plane of the page in a region where the magnetic field is perpendicular into the page (represented by the small circles with x's—like the tails of arrows). The magnetic force is perpendicular to the velocity, and so velocity changes in direction but not magnitude. Uniform circular motion results.

Because the magnetic force F supplies the centripetal force F_c , we have

$$qvB = \frac{mv^2}{r} \quad \text{Eq.(1)}$$

Solving for r yields

$$r = \frac{mv}{qB} \quad \text{Eq.(2)}$$

Here, r is the radius of curvature of the path of a charged particle with mass m and charge q , moving at a speed v perpendicular to a magnetic field of strength B . If the velocity is not perpendicular to the magnetic field, then v is the component of the velocity perpendicular to the field. The component of the velocity parallel to the field is unaffected, since the magnetic force is zero for motion parallel to the field. This produces a spiral motion rather than a circular one.

Example 1: Calculating the Curvature of the Path of an Electron Moving in a Magnetic Field: A Magnet on a TV Screen

A magnet brought near an old-fashioned TV screen such as in [Figure 3] (TV sets with cathode ray tubes instead of LCD screens) severely distorts its picture by altering the path of the electrons that make its phosphors glow. **(Don't try this at home, as it will permanently magnetize and ruin the TV.)
 ** To illustrate this, calculate the radius of curvature of the path of an electron having a velocity of $6.00 \times 10^7 \text{ m/s}$ (corresponding to the accelerating voltage of about 10.0 kV used in some TVs) perpendicular to a magnetic field of strength $B = 0.500 \text{ T}$ (obtainable with permanent magnets).

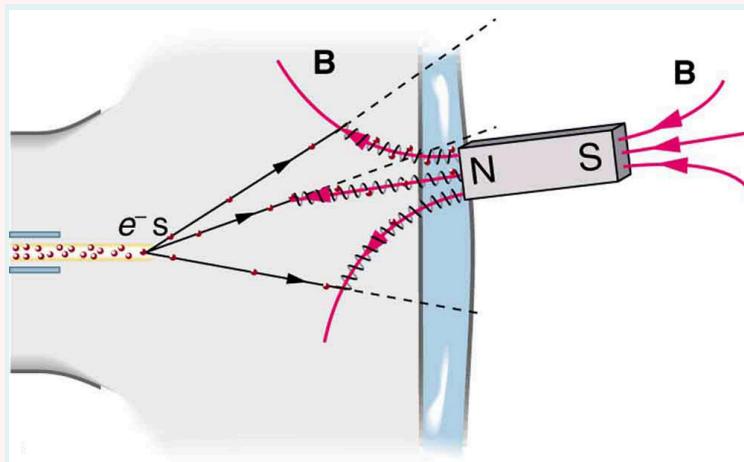


Figure 3: Side view showing what happens when a magnet comes in contact with a computer monitor or TV screen. Electrons moving toward the screen spiral about magnetic field lines, maintaining the component of their velocity parallel to the field lines. This distorts the image on the screen.

Strategy

We can find the radius of curvature r directly from the equation $r = \frac{mv}{qB}$, since all other quantities in it are given or known.

Solution

Using known values for the mass and charge of an electron, along with the given values of v and B gives us

$$r = \frac{mv}{qB} = \frac{(9.11 \times 10^{-31} \text{ kg})(6.00 \times 10^7 \text{ m/s})}{(1.60 \times 10^{-19} \text{ C})(0.500 \text{ T})} = 6.83 \times 10^{-4} \text{ m} \quad \text{Eq.(3)}$$

or

Eq.(4)

$$r = 0.683\text{mm.}$$

Discussion

The small radius indicates a large effect. The electrons in the TV picture tube are made to move in very tight circles, greatly altering their paths and distorting the image.

[Figure 4] shows how electrons not moving perpendicular to magnetic field lines follow the field lines. The component of velocity parallel to the lines is unaffected, and so the charges spiral along the field lines. If field strength increases in the direction of motion, the field will exert a force to slow the charges, forming a kind of magnetic mirror, as shown below.

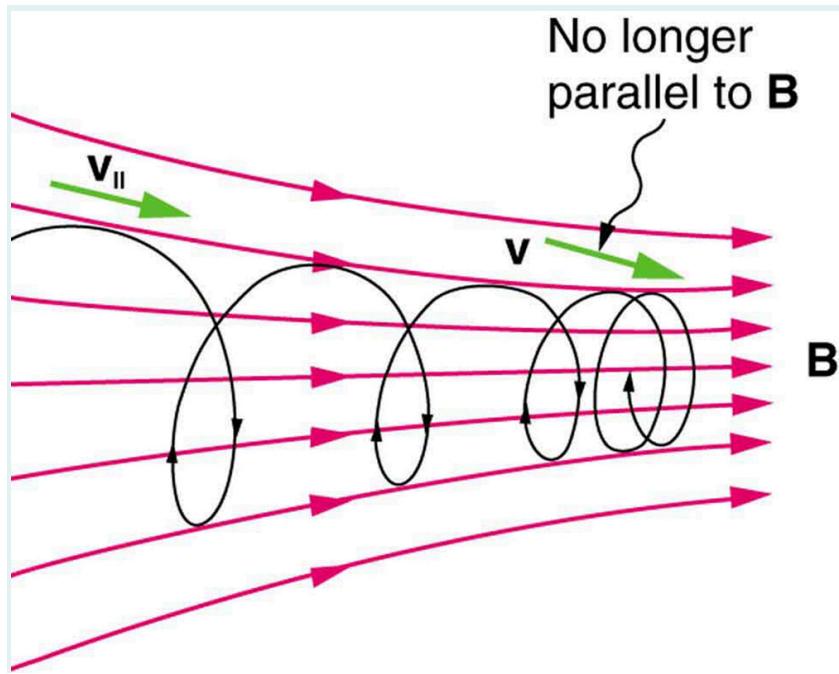


Figure 4: When a charged particle moves along a magnetic field line into a region where the field becomes stronger, the particle experiences a force that reduces the component of velocity parallel to the field. This force slows the motion along the field line and here reverses it, forming a “magnetic mirror.”

The properties of charged particles in magnetic fields are related to such different things as the Aurora Australis or Aurora Borealis and particle accelerators. Charged particles approaching magnetic field lines may get trapped in spiral orbits about the lines rather than crossing them, as seen in [Figure 5]. Some cosmic rays, for example, follow the Earth's magnetic field lines, entering the atmosphere near the magnetic poles and causing the southern or northern lights through their ionization of molecules in the atmosphere. This glow of energized

atoms and molecules is seen in [Figure 1]. Those particles that approach middle latitudes must cross magnetic field lines, and many are prevented from penetrating the atmosphere. Cosmic rays are a component of background radiation; consequently, they give a higher radiation dose at the poles than at the equator.

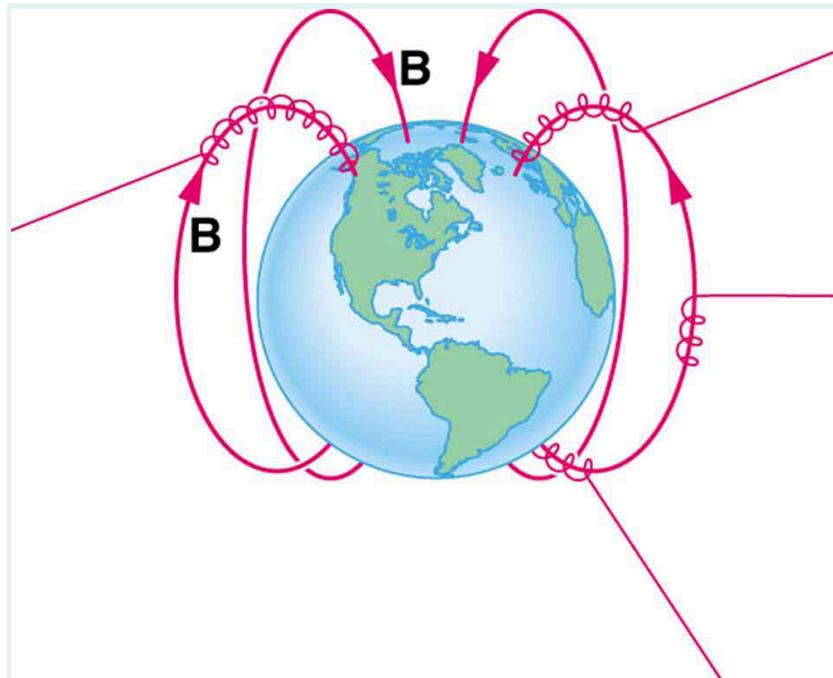
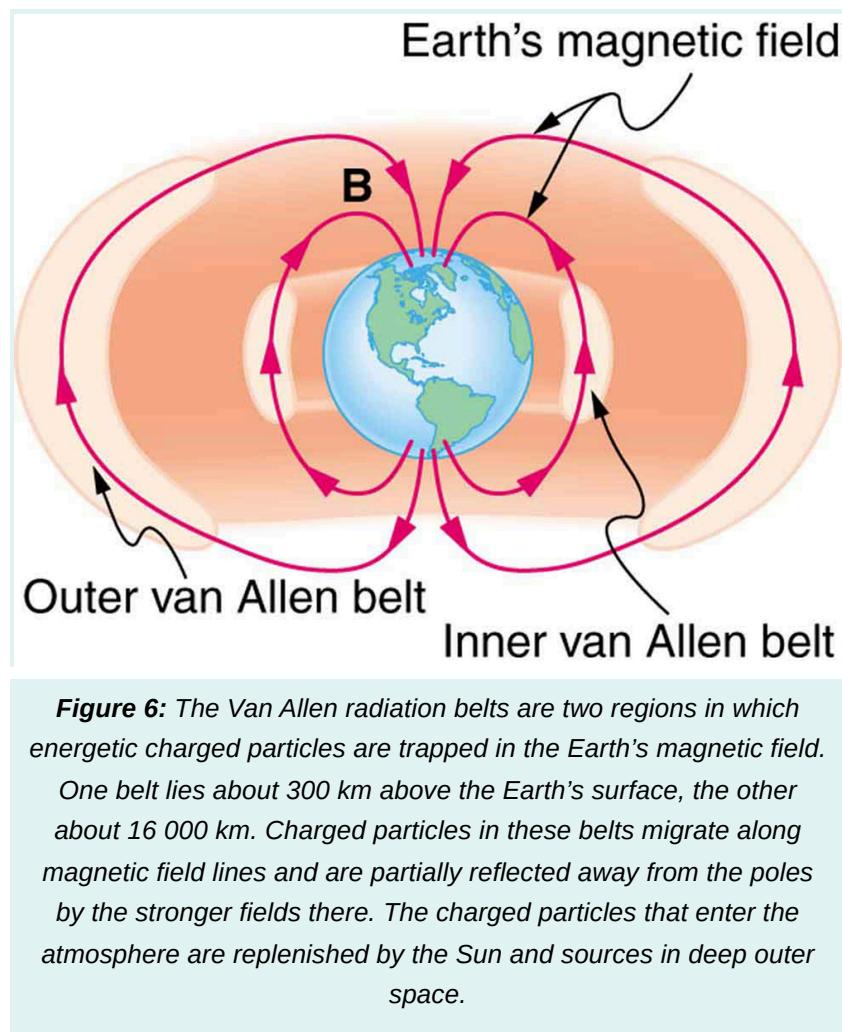


Figure 5: Energetic electrons and protons, components of cosmic rays, from the Sun and deep outer space often follow the Earth's magnetic field lines rather than cross them. (Recall that the Earth's north magnetic pole is really a south pole in terms of a bar magnet.)

Some incoming charged particles become trapped in the Earth's magnetic field, forming two belts above the atmosphere known as the Van Allen radiation belts after the discoverer James A. Van Allen, an American astrophysicist. (See [Figure 6].) Particles trapped in these belts form radiation fields (similar to nuclear radiation) so intense that manned space flights avoid them and satellites with sensitive electronics are kept out of them. In the few minutes it took lunar missions to cross the Van Allen radiation belts, astronauts received radiation doses more than twice the allowed annual exposure for radiation workers. Other planets have similar belts, especially those having strong magnetic fields like Jupiter.

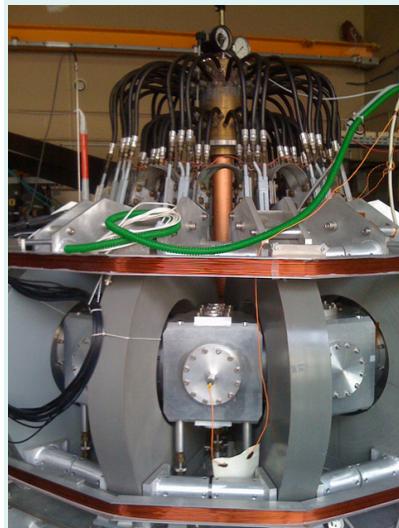


Back on Earth, we have devices that employ magnetic fields to contain charged particles. Among them are the giant particle accelerators that have been used to explore the substructure of matter. (See [Figure 7].) Magnetic fields not only control the direction of the charged particles, they also are used to focus particles into beams and overcome the repulsion of like charges in these beams.

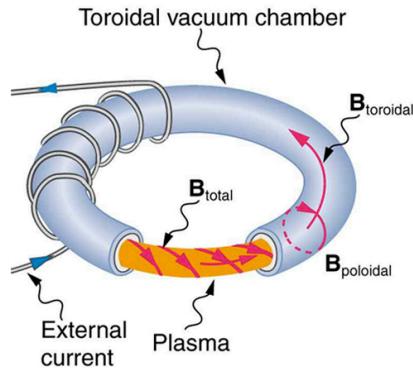


Figure 7: The Fermilab facility in Illinois has a large particle accelerator (the most powerful in the world until 2008) that employs magnetic fields (magnets seen here in orange) to contain and direct its beam. This and other accelerators have been in use for several decades and have allowed us to discover some of the laws underlying all matter. (credit: ammcrim, Flickr)

Thermonuclear fusion (like that occurring in the Sun) is a hope for a future clean energy source. One of the most promising devices is the *tokamak*, which uses magnetic fields to contain (or trap) and direct the reactive charged particles. (See [Figure 8].) Less exotic, but more immediately practical, amplifiers in microwave ovens use a magnetic field to contain oscillating electrons. These oscillating electrons generate the microwaves sent into the oven.



(a)



(b)

Figure 8: Tokamaks such as the one shown in the figure are being studied with the goal of economical production of energy by nuclear fusion. Magnetic fields in the doughnut-shaped device contain and direct the reactive charged particles. (credit: David Mellis, Flickr)

Mass spectrometers have a variety of designs, and many use magnetic fields to measure mass. The curvature of a charged particle's path in the field is related to its mass and is measured to obtain mass information. (See [More Applications of Magnetism](#).) Historically, such techniques were employed in the first direct observations of electron charge and mass. Today, mass spectrometers (sometimes coupled with gas chromatographs) are used to determine the make-up and sequencing of large biological molecules.

Section Summary

- Magnetic force can supply centripetal force and cause a charged particle to move in a circular path of radius

$$r = \frac{mv}{qB}, \quad \text{Eq.(5)}$$

where v is the component of the velocity perpendicular to B for a charged particle with mass m and charge q .

Conceptual Questions

Exercise 1

How can the motion of a charged particle be used to distinguish between a magnetic and an electric field?

Exercise 2

High-velocity charged particles can damage biological cells and are a component of radiation exposure in a variety of locations ranging from research facilities to natural background. Describe how you could use a magnetic field to shield yourself.

Exercise 3

If a cosmic ray proton approaches the Earth from outer space along a line toward the center of the Earth that lies in the plane of the equator, in what direction will it be deflected by the Earth's magnetic field? What about an electron? A neutron?

Exercise 4

What are the signs of the charges on the particles in [Figure 9]?

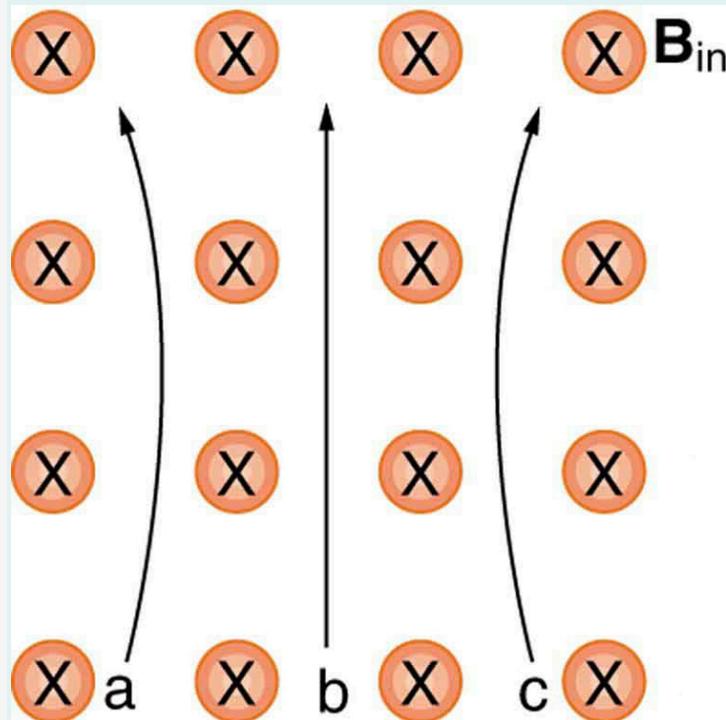


Figure 9:

Exercise 5

Which of the particles in [Figure 10] has the greatest velocity, assuming they have identical charges and masses?

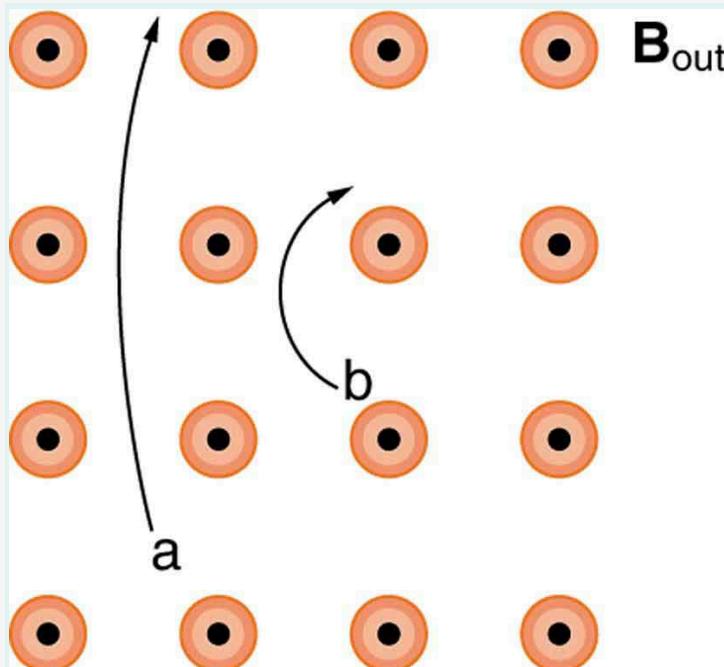


Figure 10:

Exercise 6

Which of the particles in [Figure 9] has the greatest mass, assuming all have identical charges and velocities?

Exercise 7

While operating, a high-precision TV monitor is placed on its side during maintenance. The image on the monitor changes color and blurs slightly. Discuss the possible relation of these effects to the Earth's magnetic field.

Problems & Exercises

If you need additional support for these problems, see [More Applications of Magnetism](#).

Exercise 8

A cosmic ray electron moves at $7.50 \times 10^6 \text{ m/s}$ perpendicular to the Earth's magnetic field at an altitude where field strength is $1.00 \times 10^{-5} \text{ T}$. What is the radius of the circular path the electron follows?

Exercise 9

A proton moves at $7.50 \times 10^7 \text{ m/s}$ perpendicular to a magnetic field. The field causes the proton to travel in a circular path of radius 0.800 m. What is the field strength?

Exercise 10

(a) Viewers of *Star Trek* hear of an antimatter drive on the Starship *Enterprise*. One possibility for such a futuristic energy source is to store antimatter charged particles in a vacuum chamber, circulating in a magnetic field, and then extract them as needed. Antimatter annihilates with normal matter, producing pure energy. What strength magnetic field is needed to hold antiprotons, moving at $5.00 \times 10^7 \text{ m/s}$ in a circular path 2.00 m in radius? Antiprotons have the same mass as protons but the opposite (negative) charge. (b) Is this field strength obtainable with today's technology or is it a futuristic possibility?

Exercise 11

(a) An oxygen-16 ion with a mass of 2.66×10^{-26} kg travels at 5.00×10^6 m/s perpendicular to a 1.20-T magnetic field, which makes it move in a circular arc with a 0.231-m radius. What positive charge is on the ion? (b) What is the ratio of this charge to the charge of an electron? (c) Discuss why the ratio found in (b) should be an integer.

Exercise 12

What radius circular path does an electron travel if it moves at the same speed and in the same magnetic field as the proton in [Exercise2]?

Exercise 13

A velocity selector in a mass spectrometer uses a 0.100-T magnetic field. (a) What electric field strength is needed to select a speed of 4.00×10^6 m/s? (b) What is the voltage between the plates if they are separated by 1.00 cm?

Exercise 14

An electron in a TV CRT moves with a speed of 6.00×10^7 m/s, in a direction perpendicular to the Earth's field, which has a strength of 5.00×10^{-5} T. (a) What strength electric field must be applied perpendicular to the Earth's field to make the electron move in a straight line? (b) If this is done between plates separated by 1.00 cm, what is the voltage applied? (Note that TVs are usually surrounded by a ferromagnetic material to shield against external magnetic fields and avoid the need for such a correction.)

Exercise 15

- (a) At what speed will a proton move in a circular path of the same radius as the electron in [Exercise 1]? (b) What would the radius of the path be if the proton had the same speed as the electron? (c) What would the radius be if the proton had the same kinetic energy as the electron? (d) The same momentum?

Exercise 16

A mass spectrometer is being used to separate common oxygen-16 from the much rarer oxygen-18, taken from a sample of old glacial ice. (The relative abundance of these oxygen isotopes is related to climatic temperature at the time the ice was deposited.) The ratio of the masses of these two ions is 16 to 18, the mass of oxygen-16 is 2.66×10^{-26} kg, and they are singly charged and travel at 5.00×10^6 m/s in a 1.20-T magnetic field. What is the separation between their paths when they hit a target after traversing a semicircle?

Exercise 17

- (a) Triply charged uranium-235 and uranium-238 ions are being separated in a mass spectrometer. (The much rarer uranium-235 is used as reactor fuel.) The masses of the ions are 3.90×10^{-25} kg and 3.95×10^{-25} kg, respectively, and they travel at 3.00×10^5 m/s in a 0.250-T field. What is the separation between their paths when they hit a target after traversing a semicircle? (b) Discuss whether this distance between their paths seems to be big enough to be practical in the separation of uranium-235 from uranium-238.



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