Introduction to Wave Optics

Katherine Burr Blodgett was a physicist and chemist who made significant advancements in the study of surfaces and thin films. Her invention of non-reflective glass has had massive impact in cinema, medical, and scientific research arenas.

Examine a compact disc under white light, noting the colors observed and locations of the colors. Determine if the spectra are formed by diffraction from circular lines centered at the middle of the disc and, if so, what is their spacing. If not, determine the type of spacing. Also with the CD, explore the spectra of a few light sources, such as a candle flame, incandescent bulb, halogen light, and fluorescent light. Knowing the spacing of the rows of pits in the compact disc, estimate the maximum spacing that will allow the given number of megabytes of information to be stored.

The colors reflected by this compact disc vary with angle and are not caused by pigments. Colors such as these are direct evidence of the wave character of light. (credit: Infopro, Wikimedia Commons)

If you have ever looked at the reds, blues, and greens in a sunlit soap bubble and wondered how straw-colored soapy water could produce them, you have hit upon one of the many phenomena that can only be explained by the wave character of light (see [Figure 3]). The same is true for the colors seen in an oil slick or in the light reflected from a compact disc. These and other interesting phenomena, such as the dispersion of white light into a rainbow of colors when passed through a narrow slit, cannot be explained fully by geometric optics. In these cases, light interacts with small objects and exhibits its wave characteristics. The branch of optics that considers the behavior of light when it exhibits wave characteristics (particularly when it interacts with small objects) is called wave optics (sometimes called physical optics). It is the topic of this chapter.

These soap bubbles exhibit brilliant colors when exposed to sunlight. How are the colors produced if they are not pigments in the soap? (credit: Scott Robinson, Flickr)

The Wave Aspect of Light: Interference

- Discuss the wave character of light.
- Identify the changes when light enters a medium.

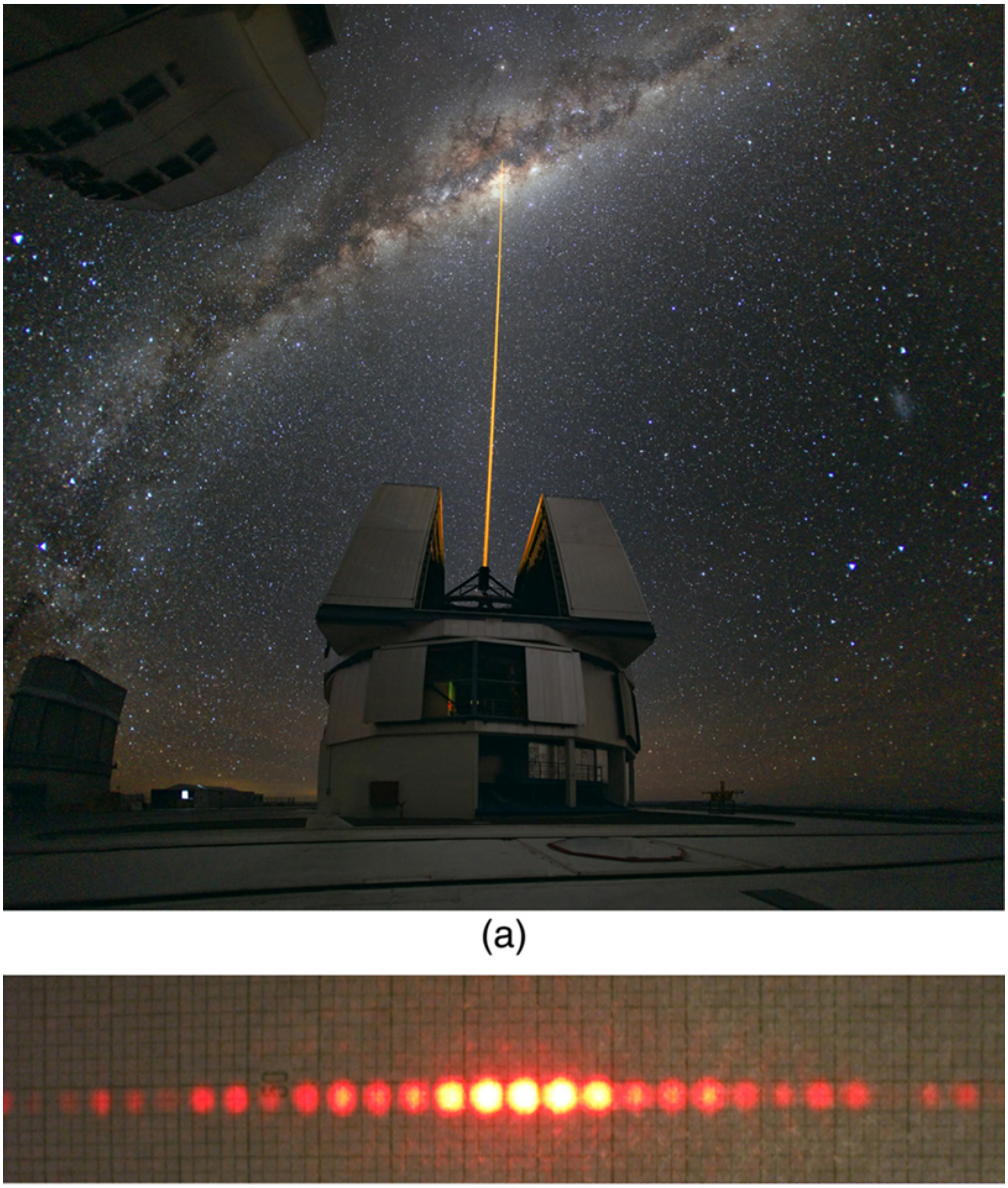
We know that visible light is the type of electromagnetic wave to which our eyes respond. Like all other electromagnetic waves, it obeys the equation

$$c = f\lambda,$$

where $c = 3 \times 10^8 \text{m/s}$ is the speed of light in vacuum, f is the frequency of the electromagnetic waves, and λ is its wavelength. The range of visible wavelengths is approximately 380 to 760 nm. As is true for all waves, light travels in straight lines and acts like a ray when it interacts with objects several times as large as its wavelength. However, when it interacts with smaller objects, it displays its wave characteristics prominently. Interference is the hallmark of a wave, and in [Figure 1] both the ray and wave characteristics of light can be seen. The laser beam emitted by the observatory epitomizes a ray, traveling in a straight line. However, passing a pure-wavelength beam through vertical slits with a size close to the wavelength of the beam reveals the wave character of light, as the beam spreads out horizontally into a pattern of bright and dark regions caused by systematic constructive and destructive interference. Rather than spreading out, a ray would continue traveling straight ahead after passing through slits.

Making Connections: Waves

The most certain indication of a wave is interference. This wave characteristic is most prominent when the wave interacts with an object that is not large compared with the wavelength. Interference is observed for water waves, sound waves, light waves, and (as we will see in Special Relativity) for matter waves, such as electrons scattered from a crystal.



(a)

(b)

(a) The laser beam emitted by an observatory acts like a ray, traveling in a straight line. This laser beam is from the Paranal Observatory of the European Southern Observatory. (credit: Yuri Beletsky, European Southern Observatory) (b) A laser beam passing through a grid of vertical slits produces an interference pattern—characteristic of a wave. (credit: Shim'on and Slava Rybka, Wikimedia Commons)

Light has wave characteristics in various media as well as in a vacuum. When light goes from a vacuum to some medium, like water, its speed and wavelength change, but its frequency f remains the same. (We can think of light as a forced oscillation that must have the frequency of the original source.) The speed of light in a medium is $v = c/n$, where n is its index of refraction. If we divide both sides of equation $c = f\lambda$ by n , we get $c/n = v = f\lambda/n$. This implies that $v = f\lambda_{\text{n}}$, where λ_{n} is the **wavelength in a medium** and that

$$\lambda_{\text{n}}=\lambda n,$$

where λ is the wavelength in vacuum and n is the medium's index of refraction. Therefore, the wavelength of light is smaller in any medium than it is in vacuum. In water, for example, which has $n = 1.333$, the range of visible wavelengths is $(380\text{nm})/1.333$ to $(760\text{nm})/1.333$, or $\lambda_{\text{n}} = 285 \text{to} 570\text{nm}$. Although wavelengths change while traveling from one medium to another, colors do not, since colors are associated with frequency.

Section Summary

- Wave optics is the branch of optics that must be used when light interacts with small objects or whenever the wave characteristics of light are considered.
- Wave characteristics are those associated with interference and diffraction.
- Visible light is the type of electromagnetic wave to which our eyes respond and has a wavelength in the range of 380 to 760 nm.
- Like all EM waves, the following relationship is valid in vacuum: $c = f\lambda$, where $c = 3 \times 10^8 \text{m/s}$ is the speed of light, f is the frequency of the electromagnetic wave, and λ is its wavelength in vacuum.
- The wavelength λ_n of light in a medium with index of refraction n is $\lambda_n = \lambda/n$. Its frequency is the same as in vacuum.

Conceptual Questions

What type of experimental evidence indicates that light is a wave?

Show Solution

Strategy

To identify light as a wave, we need experimental evidence that demonstrates wave-specific phenomena that cannot be explained by a particle model. The hallmark characteristics of waves include interference, diffraction, and polarization.

Solution

Several types of experimental evidence definitively indicate that light is a wave:

1. Interference - The most conclusive evidence that light is a wave is the observation of interference patterns. When two coherent light sources combine, they create alternating bright and dark regions (constructive and destructive interference) that can only be explained by wave superposition. Key experiments include Young’s double-slit experiment, thin film interference (oil on water, soap bubbles), and interference in optical instruments.

2. Diffraction - Light bends around obstacles and spreads when passing through small apertures, creating characteristic diffraction patterns. Single-slit diffraction produces a central bright maximum with dimmer side maxima, a pattern inconsistent with particle behavior.

3. Polarization - Light can be polarized, demonstrating that it is a transverse wave. Particles do not exhibit polarization. Polarizing filters, Brewster’s angle reflection, and birefringence all demonstrate light’s transverse wave nature.

4. Doppler effect - The change in light frequency due to relative motion between source and observer (redshift/blueshift of stars) is a wave phenomenon.

Discussion

The wave nature of light was controversial historically. Newton favored a particle theory, while Huygens proposed a wave theory. Thomas Young’s double-slit experiment (1801) provided compelling evidence for waves by demonstrating interference. This was confirmed by numerous subsequent experiments showing diffraction, polarization, and other wave phenomena. Interference remains the gold standard proof because it fundamentally cannot be explained by classical particles traveling in straight lines.

Give an example of a wave characteristic of light that is easily observed outside the laboratory.

Show Solution

Strategy

We need to identify wave phenomena involving light that occur naturally or in everyday situations, observable without specialized equipment.

Solution

Several wave characteristics of light are easily observed in everyday life:

1. Colors in soap bubbles and oil slicks (thin film interference) - Perhaps the most commonly observed wave phenomenon. The swirling rainbow colors in soap bubbles, oil on water, or oxidized metal surfaces result from interference between light waves reflected from the top and bottom surfaces of thin films. Different wavelengths interfere constructively at different angles, producing spectacular color patterns.

2. Rainbows (dispersion and refraction) - While primarily a refraction phenomenon, rainbows also involve wave properties. Different wavelengths are refracted by different amounts, separating white light into its component colors.

3. Colors in peacock feathers, butterfly wings, and opals (structural coloration/diffraction grating effects) - Many iridescent colors in nature result from periodic nanostructures that act as natural diffraction gratings, producing colors through wave interference rather than pigmentation.

4. Halos and coronas around the sun or moon - When light passes through ice crystals or water droplets in the atmosphere, diffraction creates colored rings around celestial objects.

5. CDs and DVDs producing rainbow reflections - The closely-spaced grooves on optical discs act as reflection gratings, separating light into its wavelength components.

Discussion

Thin film interference in soap bubbles is particularly instructive because it clearly demonstrates wave superposition. The colors change with viewing angle and film thickness, showing wavelength-dependent constructive and destructive interference. This phenomenon cannot be explained by light as simple particles but requires wave theory to understand why specific wavelengths are enhanced or suppressed at different positions.

Problems & Exercises

Show that when light passes from air to water, its wavelength decreases to 0.750 times its original value.

Show Solution

Strategy

When light enters a medium, its wavelength changes according to the relationship $\lambda_n = \lambda/n$, where λ_n is the wavelength in the medium, λ is the wavelength in vacuum (or air), and n is the index of refraction. For water, $n = 1.333$.

Solution

Given:

- Index of refraction of water: $n = 1.333$

The wavelength in water is related to the wavelength in air by:

$$\lambda_{\text{water}} = \frac{\lambda_{\text{air}}}{n} = \frac{\lambda_{\text{air}}}{1.333}$$

Calculate the ratio:

$$\frac{\lambda_{\text{water}}}{\lambda_{\text{air}}} = \frac{1}{1.333} = 0.750$$

Therefore:

$$\lambda_{\text{water}} = 0.750 \times \lambda_{\text{air}}$$

This shows that the wavelength in water is 0.750 times (or 75.0% of) its value in air.

Discussion

The 25% reduction in wavelength occurs because light slows down when entering water. The speed of light in water is $v = c/n = c/1.333 \approx 0.750c$. Since the frequency remains constant when light enters a new medium (it's determined by the source), and $v = f\lambda$, a 25% reduction in speed must be accompanied by a 25% reduction in wavelength. This wavelength reduction is important in thin film interference and other wave phenomena involving light in different media.

Find the range of visible wavelengths of light in crown glass.

Show Solution

Strategy

The wavelength of light changes when it enters a medium according to $\lambda_n = \lambda/n$, where λ is the wavelength in vacuum, n is the index of refraction, and λ_n is the wavelength in the medium. For crown glass, $n = 1.52$. The visible range in vacuum is 380 nm to 760 nm.

Solution

Given:

- Visible wavelength range in vacuum: 380 nm to 760 nm
- Index of refraction of crown glass: $n = 1.52$

For the shortest visible wavelength (violet):

$$\lambda_n = \frac{\lambda}{n} = \frac{380\text{ nm}}{1.52} = 250\text{ nm}$$

For the longest visible wavelength (red):

$$\lambda_n = \frac{\lambda}{n} = \frac{760\text{ nm}}{1.52} = 500\text{ nm}$$

The range of visible wavelengths in crown glass is 250 nm to 500 nm.

Discussion

Notice that wavelengths in crown glass are about 66% (or 1/1.52) of their values in vacuum. Even though the wavelengths change significantly, the colors we perceive remain the same because color is determined by frequency, which does not change when light enters a medium. The speed and wavelength both decrease by the factor n, but their ratio ($v = f\lambda_n$) maintains the same frequency.

What is the index of refraction of a material for which the wavelength of light is 0.671 times its value in a vacuum? Identify the likely substance.

Show Solution

Strategy

We use the relationship $\lambda_n = \lambda / n$ to find the index of refraction n. Rearranging gives $n = \lambda / \lambda_n$. Given that $\lambda_n = 0.671\lambda$, we can find n and compare it to known values for common materials.

Solution

Given:

- $\lambda_n = 0.671 \times \lambda$

From $\lambda_n = \lambda / n$:

$$n = \frac{\lambda}{\lambda_n} = \frac{\lambda}{0.671\lambda} = \frac{1}{0.671}$$
$$n = 1.49$$

Comparing to known indices of refraction:

- Water: $n = 1.33$
- Glass (typical): $n = 1.5 - 1.9$
- **Polystyrene**: $n = 1.49$
- Plexiglass: $n = 1.51$

The material is most likely polystyrene, which has an index of refraction of 1.49.

Discussion

Polystyrene is a common transparent plastic used in many applications including optical components, disposable containers, and laboratory equipment. The calculated index of refraction matches polystyrene almost exactly. While plexiglass (acrylic) has a similar value (1.51), the calculated value of 1.49 is closer to polystyrene. The wavelength reduction to 67.1% of its vacuum value is typical for transparent plastics and glasses, which generally have indices between 1.4 and 1.6.

Analysis of an interference effect in a clear solid shows that the wavelength of light in the solid is 329 nm. Knowing this light comes from a He-Ne laser and has a wavelength of 633 nm in air, is the substance zircon or diamond?

Show Solution

Strategy

We can find the index of refraction of the unknown substance using $\lambda_n = \lambda / n$, which gives $n = \lambda / \lambda_n$. Then compare this value to known indices of refraction: zircon ($n \approx 1.92$) and diamond ($n \approx 2.42$).

Solution

Given:

- Wavelength in air (approximately vacuum): $\lambda = 633 \text{ nm}$
- Wavelength in the solid: $\lambda_n = 329 \text{ nm}$

Calculate the index of refraction:

$$n = \frac{\lambda}{\lambda_n} = \frac{633 \text{ nm}}{329 \text{ nm}} = 1.92$$

Comparing to known values:

- Zircon: $n \approx 1.92$
- Diamond: $n \approx 2.42$

The substance is zircon, as its index of refraction matches the calculated value of 1.92.

Discussion

Diamond has a much higher index of refraction (2.42) and would reduce the wavelength to approximately $633/2.42 = 262$ nm, which is significantly less than the observed 329 nm. Zircon's lower index of refraction produces the observed wavelength. This method of using interference patterns to measure wavelength in materials is a practical way to identify transparent substances or verify their purity, since the index of refraction is a characteristic property of each material.

What is the ratio of thicknesses of crown glass and water that would contain the same number of wavelengths of light?

Show Solution

Strategy

The number of wavelengths that fit in a given thickness t is $N = t/\lambda_n$. For the same number of wavelengths in both materials, we need $t_{\text{glass}}/\lambda_{\text{glass}} = t_{\text{water}}/\lambda_{\text{water}}$. Using $\lambda_n = \lambda/n$, we can find the thickness ratio.

Solution

Given:

- Crown glass: $n_{\text{glass}} = 1.52$
- Water: $n_{\text{water}} = 1.333$

For the same number of wavelengths:

$$\frac{t_{\text{glass}}}{\lambda_{\text{glass}}} = \frac{t_{\text{water}}}{\lambda_{\text{water}}}$$

Since $\lambda_n = \lambda/n$:

$$\frac{t_{\text{glass}}}{\lambda/n_{\text{glass}}} = \frac{t_{\text{water}}}{\lambda/n_{\text{water}}}$$

Simplifying:

$$\frac{t_{\text{glass}} \cdot n_{\text{glass}}}{\lambda} = \frac{t_{\text{water}} \cdot n_{\text{water}}}{\lambda}$$
$$t_{\text{glass}} \cdot n_{\text{glass}} = t_{\text{water}} \cdot n_{\text{water}}$$

Therefore:

$$\frac{t_{\text{glass}}}{t_{\text{water}}} = \frac{n_{\text{water}}}{n_{\text{glass}}} = \frac{1.333}{1.52} = 0.877$$

The ratio of crown glass thickness to water thickness is 0.877 (or approximately 7:8).

Discussion

This result shows that crown glass needs to be only 87.7% as thick as water to contain the same number of wavelengths. This makes sense because crown glass has a higher index of refraction (1.52) than water (1.333), which means light wavelengths are more compressed in glass than in water. The higher the index of refraction, the shorter the wavelength becomes in that medium, so fewer physical distance is needed to accommodate a given number of wavelengths.

This relationship is important in optical design, where the optical path length ($n \times t$) determines phase relationships in interference and other wave phenomena. Two different materials with different thicknesses can produce the same optical path length if the product nt is equal for both.

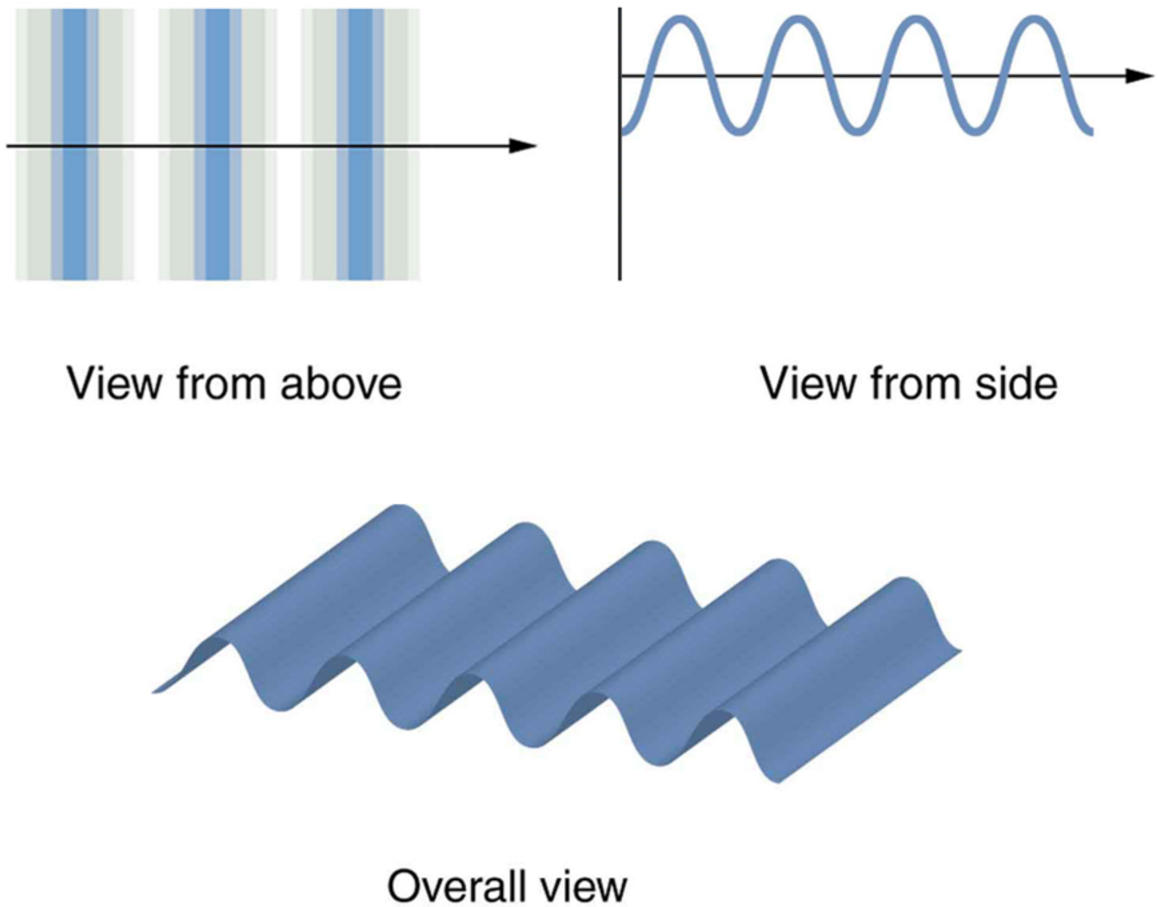
Glossary

wavelength in a medium
: $\lambda_{\text{n}} = \lambda/n$, where λ is the wavelength in vacuum, and n is the index of refraction of the medium

Huygens's Principle: Diffraction

- Discuss the propagation of transverse waves.
- Discuss Huygens's principle.
- Explain the bending of light.

[Figure 1] shows how a transverse wave looks as viewed from above and from the side. A light wave can be imagined to propagate like this, although we do not actually see it wiggling through space. From above, we view the wavefronts (or wave crests) as we would by looking down on the ocean waves. The side view would be a graph of the electric or magnetic field. The view from above is perhaps the most useful in developing concepts about wave optics.

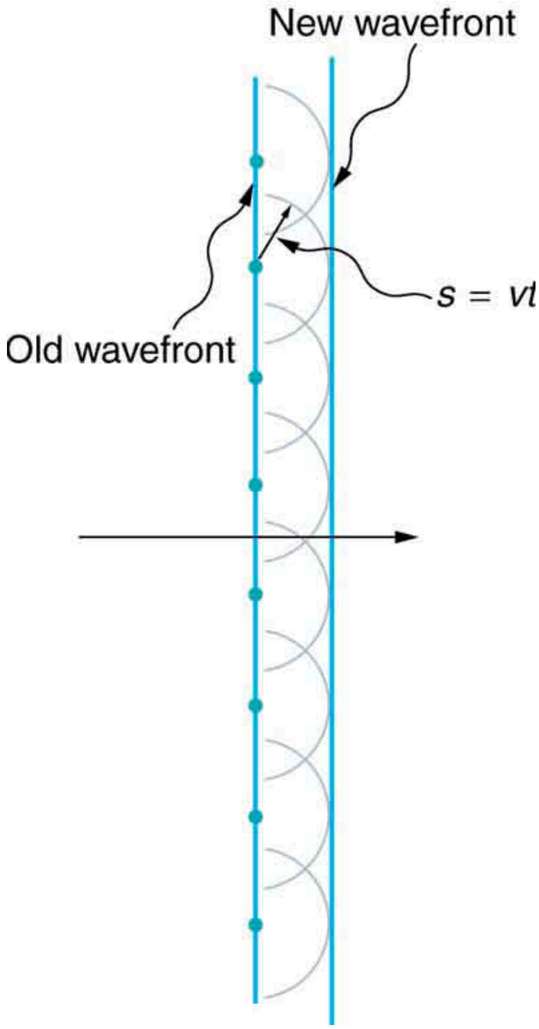


A transverse wave, such as an electromagnetic wave like light, as viewed from above and from the side. The direction of propagation is perpendicular to the wavefronts (or wave crests) and is represented by an arrow like a ray.

The Dutch scientist Christiaan Huygens (1629–1695) developed a useful technique for determining in detail how and where waves propagate. Starting from some known position, **Huygens's principle** states that:

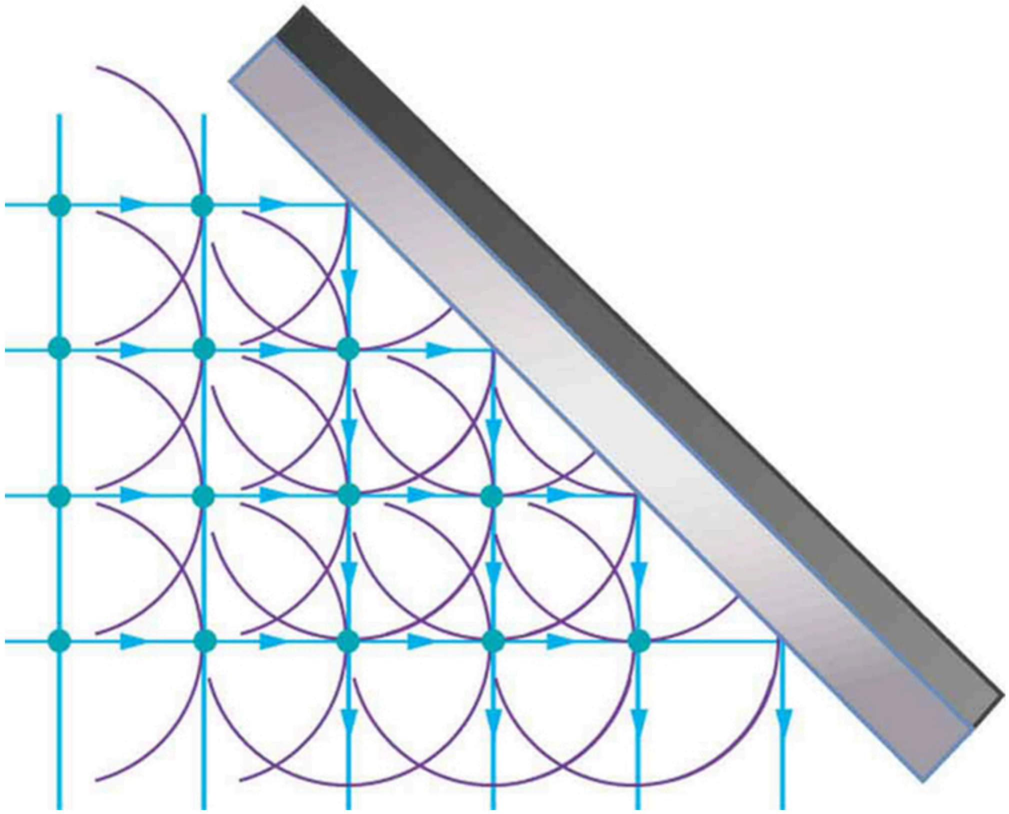
Every point on a wavefront is a source of wavelets that spread out in the forward direction at the same speed as the wave itself. The new wavefront is a line tangent to all of the wavelets.

[Figure 2] shows how Huygens's principle is applied. A wavefront is the long edge that moves, for example, the crest or the trough. Each point on the wavefront emits a semicircular wave that moves at the propagation speed v . These are drawn at a time t later, so that they have moved a distance $s = vt$. The new wavefront is a line tangent to the wavelets and is where we would expect the wave to be a time t later. Huygens's principle works for all types of waves, including water waves, sound waves, and light waves. We will find it useful not only in describing how light waves propagate, but also in explaining the laws of reflection and refraction. In addition, we will see that Huygens's principle tells us how and where light rays interfere.



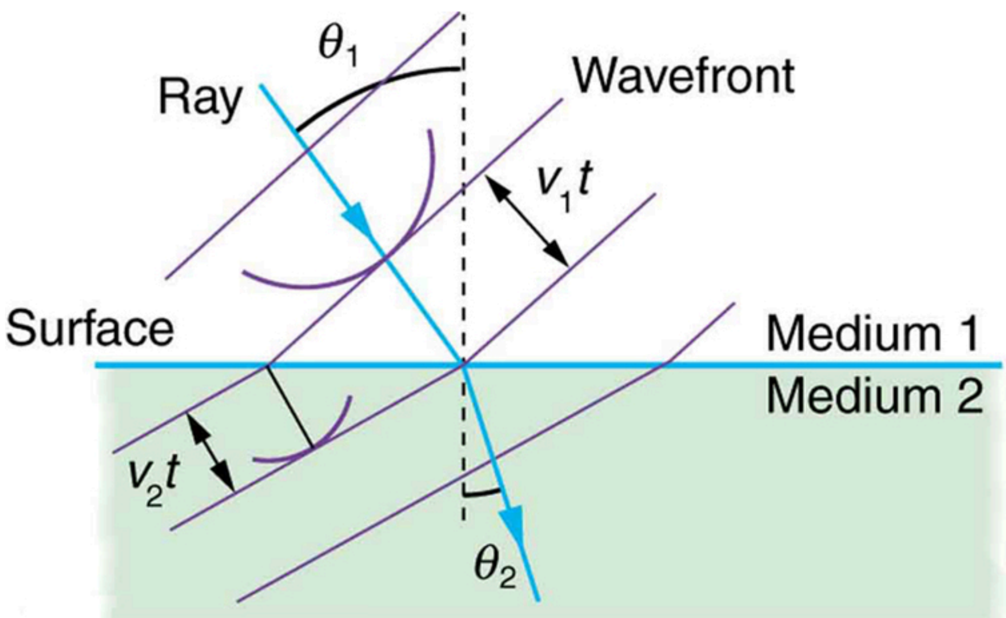
Huygens's principle applied to a straight wavefront. Each point on the wavefront emits a semicircular wavelet that moves a distance $s = vt$. The new wavefront is a line tangent to the wavelets.

[Figure 3] shows how a mirror reflects an incoming wave at an angle equal to the incident angle, verifying the law of reflection. As the wavefront strikes the mirror, wavelets are first emitted from the left part of the mirror and then the right. The wavelets closer to the left have had time to travel farther, producing a wavefront traveling in the direction shown.



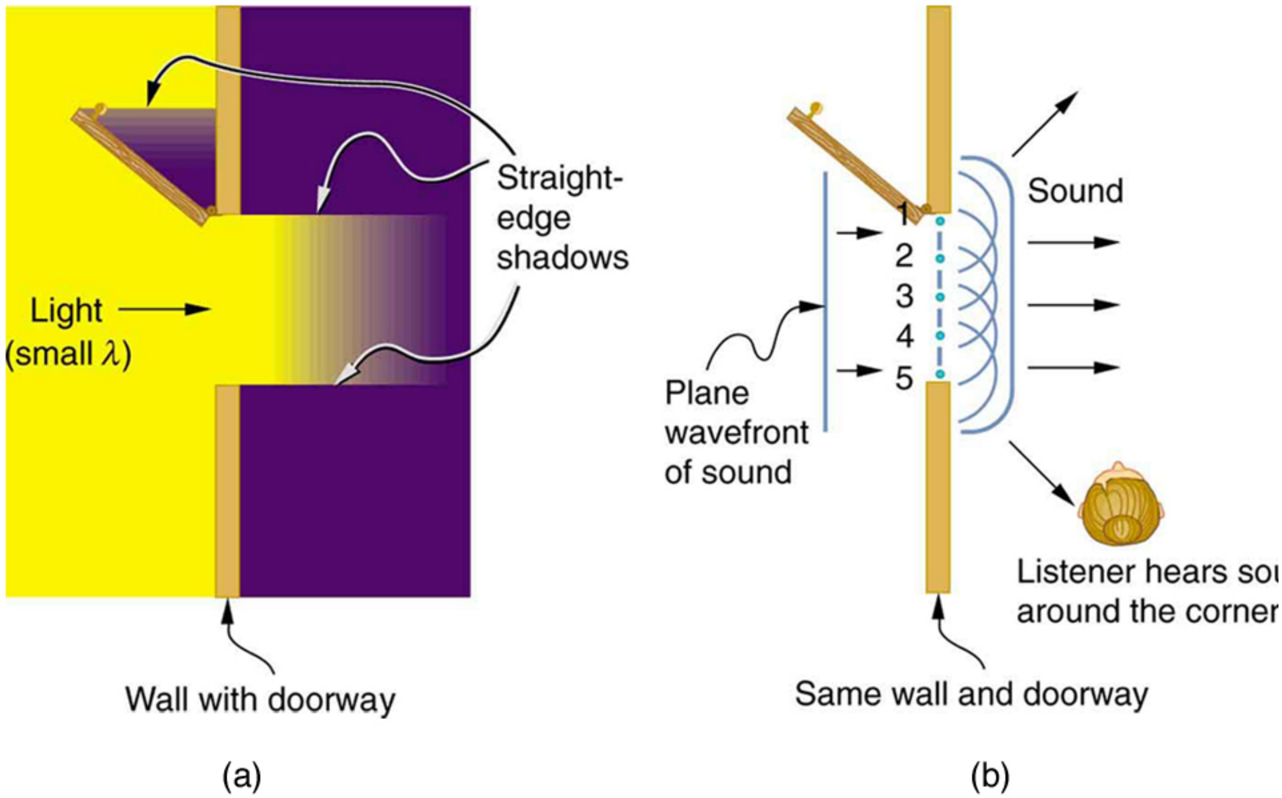
Huygens's principle applied to a straight wavefront striking a mirror. The wavelets shown were emitted as each point on the wavefront struck the mirror. The tangent to these wavelets shows that the new wavefront has been reflected at an angle equal to the incident angle. The direction of propagation is perpendicular to the wavefront, as shown by the downward-pointing arrows.

The law of refraction can be explained by applying Huygens's principle to a wavefront passing from one medium to another (see [Figure 4]). Each wavelet in the figure was emitted when the wavefront crossed the interface between the media. Since the speed of light is smaller in the second medium, the waves do not travel as far in a given time, and the new wavefront changes direction as shown. This explains why a ray changes direction to become closer to the perpendicular when light slows down. Snell's law can be derived from the geometry in [Figure 4], but this is left as an exercise for ambitious readers.



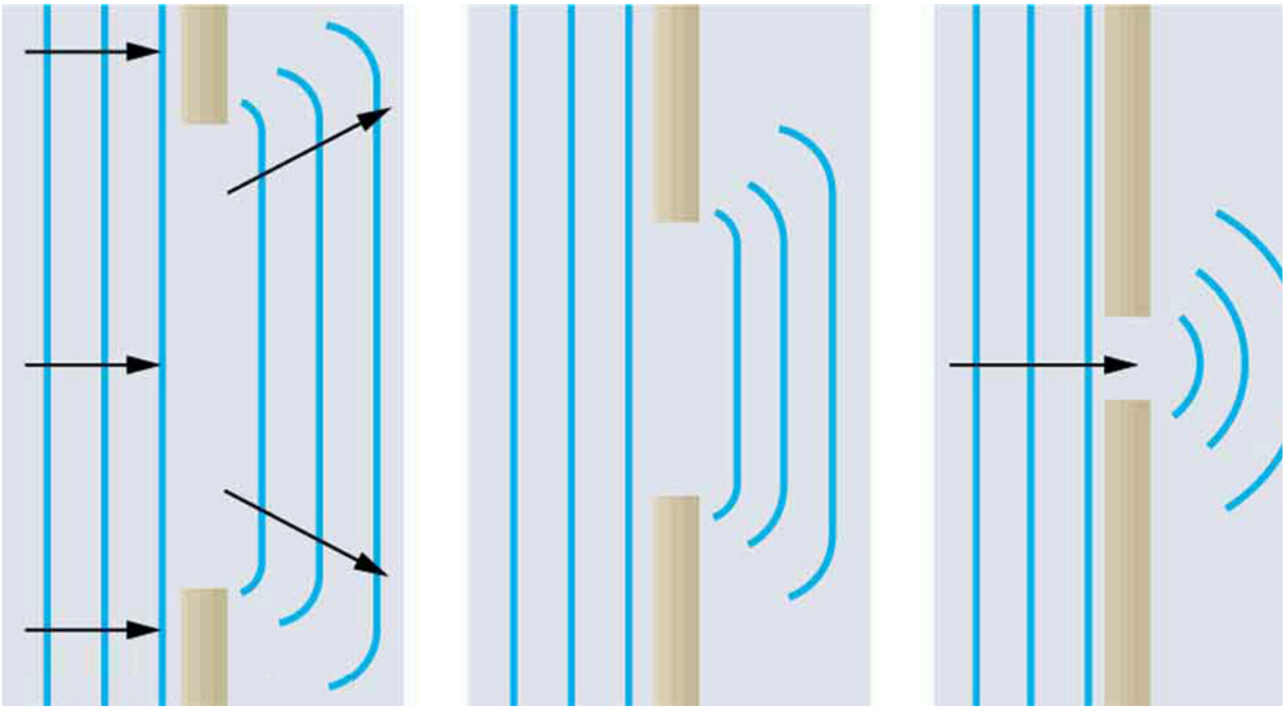
Huygens's principle applied to a straight wavefront traveling from one medium to another where its speed is less. The ray bends toward the perpendicular, since the wavelets have a lower speed in the second medium.

What happens when a wave passes through an opening, such as light shining through an open door into a dark room? For light, we expect to see a sharp shadow of the doorway on the floor of the room, and we expect no light to bend around corners into other parts of the room. When sound passes through a door, we expect to hear it everywhere in the room and, thus, expect that sound spreads out when passing through such an opening (see [Figure 5]). What is the difference between the behavior of sound waves and light waves in this case? The answer is that light has very short wavelengths and acts like a ray. Sound has wavelengths on the order of the size of the door and bends around corners (for frequency of 1000 Hz, $\lambda = c/f = (330\text{m/s})/(1000\text{s}^{-1}) = 0.33\text{m}$, about three times smaller than the width of the doorway).



(a) Light passing through a doorway makes a sharp outline on the floor. Since light’s wavelength is very small compared with the size of the door, it acts like a ray. (b) Sound waves bend into all parts of the room, a wave effect, because their wavelength is similar to the size of the door.

If we pass light through smaller openings, often called slits, we can use Huygens’s principle to see that light bends as sound does (see [Figure 6]). The bending of a wave around the edges of an opening or an obstacle is called **diffraction**. Diffraction is a wave characteristic and occurs for all types of waves. If diffraction is observed for some phenomenon, it is evidence that the phenomenon is a wave. Thus the horizontal diffraction of the laser beam after it passes through slits in [Figure 1] is evidence that light is a wave.



Huygens's principle applied to a straight wavefront striking an opening. The edges of the wavefront bend after passing through the opening, a process called diffraction. The amount of bending is more extreme for a small opening, consistent with the fact that wave characteristics are most noticeable for interactions with objects about the same size as the wavelength.

Section Summary

- An accurate technique for determining how and where waves propagate is given by Huygens's principle: Every point on a wavefront is a source of wavelets that spread out in the forward direction at the same speed as the wave itself. The new wavefront is a line tangent to all of the wavelets.
- Diffraction is the bending of a wave around the edges of an opening or other obstacle.

Conceptual Questions

How do wave effects depend on the size of the object with which the wave interacts? For example, why does sound bend around the corner of a building while light does not?

Show Solution

Strategy

Wave effects such as diffraction and interference become most pronounced when the wavelength of the wave is comparable to the size of the object or opening with which it interacts. We need to compare the wavelengths of sound and light to typical building dimensions.

Solution

Wave effects are most noticeable when the wavelength is comparable to or larger than the size of the object with which the wave interacts. Sound waves have wavelengths on the order of meters (for a 1000 Hz sound in air, $\lambda = v/f = 343$ m/s / 1000 Hz ≈ 0.34 m). This wavelength is comparable to the width of a doorway or the corner of a building, so significant diffraction occurs, causing sound to bend around corners.

In contrast, visible light has wavelengths in the range of 380-760 nm (about 10^{-7} m), which is millions of times smaller than typical building dimensions. When light encounters objects much larger than its wavelength, it behaves more like a ray traveling in straight lines, with minimal diffraction. The edges of shadows do show some diffraction effects, but these are very small compared to the overall size of the shadow.

Discussion

This principle explains why we can hear someone talking around a corner but cannot see them. It also explains why larger obstacles are needed to observe diffraction effects with light - for example, narrow slits with widths on the order of micrometers are required to see significant diffraction patterns with visible light. The general rule is: diffraction effects are most noticeable when $\lambda \approx d$, where d is the size of the obstacle or opening.

Under what conditions can light be modeled like a ray? Like a wave?

Show Solution

Strategy

The choice between ray and wave models depends on the relative size of wavelength compared to the objects and apertures involved in the situation.

Solution

Light can be modeled as a **ray** when it interacts with objects that are much larger than its wavelength ($\lambda \ll d$). Under these conditions:

- Diffraction effects are negligible
- Light travels in straight lines
- Reflection and refraction can be accurately described using geometric optics
- Examples: mirrors, lenses in cameras and eyeglasses, optical instruments where apertures are millimeters or larger

Light must be modeled as a **wave** when:

- It interacts with objects or openings comparable to or smaller than its wavelength ($\lambda \approx d$ or $\lambda > d$)
- Diffraction, interference, or polarization effects are important
- Precise measurements of wavelength or small-scale phenomena are involved
- Examples: diffraction gratings, double-slit experiments, thin film interference, resolution limits of microscopes and telescopes

The transition between these models is gradual. For objects with dimensions around 1-100 wavelengths, both wave and ray effects may be observable, and a complete analysis requires wave optics.

Discussion

This dual nature reflects the wave-particle duality of light. The ray model (geometric optics) is simpler and adequate for many practical applications involving macroscopic optical devices. However, the wave model is fundamental and necessary for understanding the true nature of light and phenomena that the ray model cannot explain. In modern physics, we recognize that light always has wave properties, but these become negligible in certain macroscopic situations where the ray approximation works well.

Go outside in the sunlight and observe your shadow. It has fuzzy edges even if you do not. Is this a diffraction effect? Explain.

Show Solution

Strategy

We need to consider what causes fuzzy shadow edges and determine whether diffraction or other effects are responsible.

Solution

The fuzzy edges of shadows outdoors are **primarily NOT due to diffraction**, but rather to the fact that the Sun is an extended light source rather than a point source.

The Sun subtends an angle of about 0.5° as seen from Earth. When you stand in sunlight, different parts of the Sun's disk illuminate your body from slightly different angles. This creates three regions:

- **Umbra**: The fully shadowed region where no part of the Sun is visible
- **Penumbra**: The partially shadowed region where only part of the Sun is visible
- **Fully lit region**: Where the entire Sun is visible

The fuzzy edge of your shadow is the penumbra, where the illumination gradually transitions from full shadow to full sunlight.

There IS a small diffraction effect at the edges of shadows, but it is extremely small compared to the penumbral effect. Diffraction fringes from visible light would only extend a fraction of a millimeter from the shadow edge, far too small to be noticed under normal outdoor conditions. The diffraction effect would be noticeable only if:

- You used a true point source of light (like a distant streetlamp at night)
- You observed very carefully, possibly with magnification
- Your shadow fell on a screen at a considerable distance

Discussion

You can test this by using different light sources. With a small, distant light source (like a streetlamp far away at night), your shadow edges will be much sharper, though still showing tiny diffraction fringes if examined carefully. With a large, nearby light source (like a lamp with a large bulb), your shadow edges will be very fuzzy. This confirms that the extended nature of the source, not diffraction, dominates the fuzzy-edge effect in everyday shadows.

Why does the wavelength of light decrease when it passes from vacuum into a medium? State which attributes change and which stay the same and, thus, require the wavelength to decrease.

Show Solution

Strategy

We need to apply the wave equation $v = f\lambda$ and identify which quantities change and which remain constant when light enters a medium.

Solution

When light passes from vacuum into a medium:

What stays the SAME:

- **Frequency (f)**: The frequency of the light wave is determined by the source and does not change when light enters a medium. This must be true for continuity - if the frequency changed at the boundary, there would be a buildup or deficit of wave crests at the interface, which is physically impossible.

What CHANGES:

- **Speed (v)**: Light slows down in a medium. In vacuum, $v = c \approx 3.00 \times 10^8$ m/s. In a medium with index of refraction n, the speed becomes $v = c/n$, which is less than c.

Why wavelength MUST decrease: From the wave equation:

$$v=f\lambda$$

Since frequency f remains constant and speed v decreases, the wavelength λ must also decrease to maintain the relationship:

$$\lambda_{\text{medium}}=v_{\text{medium}} f = c/n f = \lambda_{\text{vacuum}} n$$

Therefore, λ_medium = λ_vacuum/n < λ_vacuum.

Discussion

This relationship is crucial for understanding phenomena like thin film interference, where we must use the wavelength in the medium (λ_n = λ/n) rather than the wavelength in vacuum. The physical reason for the speed decrease is that light interacts with the atoms in the medium - the oscillating electric field of the light wave causes electrons to oscillate, and these oscillating electrons re-radiate light that interferes with the original wave, effectively slowing its propagation through the medium. However, the frequency must match the driving frequency of the original source, so wavelength is the parameter that adjusts.

Does Huygens's principle apply to all types of waves?

Show Solution

Strategy

Huygens's principle is a geometric construction for determining wave propagation. We need to consider whether this principle is universal or has limitations.

Solution

Yes, Huygens's principle applies to all types of waves. It is a general principle of wave propagation that works for:

- **Mechanical waves**:
  - Water waves
  - Sound waves
  - Seismic waves
  - Waves on strings
- **Electromagnetic waves**:
  - Light (all wavelengths)
  - Radio waves
  - Microwaves
  - X-rays and gamma rays
- **Matter waves**:
  - Electron waves
  - Other quantum mechanical waves

The principle states that every point on a wavefront can be considered as a source of secondary wavelets that spread out in the forward direction at the wave speed. The new wavefront is the envelope (tangent surface) of all these wavelets.

Key characteristics that make it universal:

1. It's based on the wave equation, which governs all wave phenomena
2. It correctly predicts diffraction, reflection, and refraction for all wave types
3. It doesn't depend on the specific physical nature of the wave (mechanical oscillation, electromagnetic field, probability amplitude, etc.)

Discussion

Huygens's principle is particularly powerful because it's a geometrical method that doesn't require detailed knowledge of the wave's physical mechanism. Whether we're dealing with vibrating water molecules, oscillating electromagnetic fields, or quantum probability amplitudes, the same principle applies. This universality is one reason why wave phenomena look similar across vastly different physical systems - from ripples in a pond to light diffracting through a slit to electron diffraction in a crystal. The principle does have some mathematical limitations (it doesn't fully account for the detailed amplitude and polarization in all cases without additional considerations), but its basic geometric approach to wave propagation is indeed universal.

Glossary

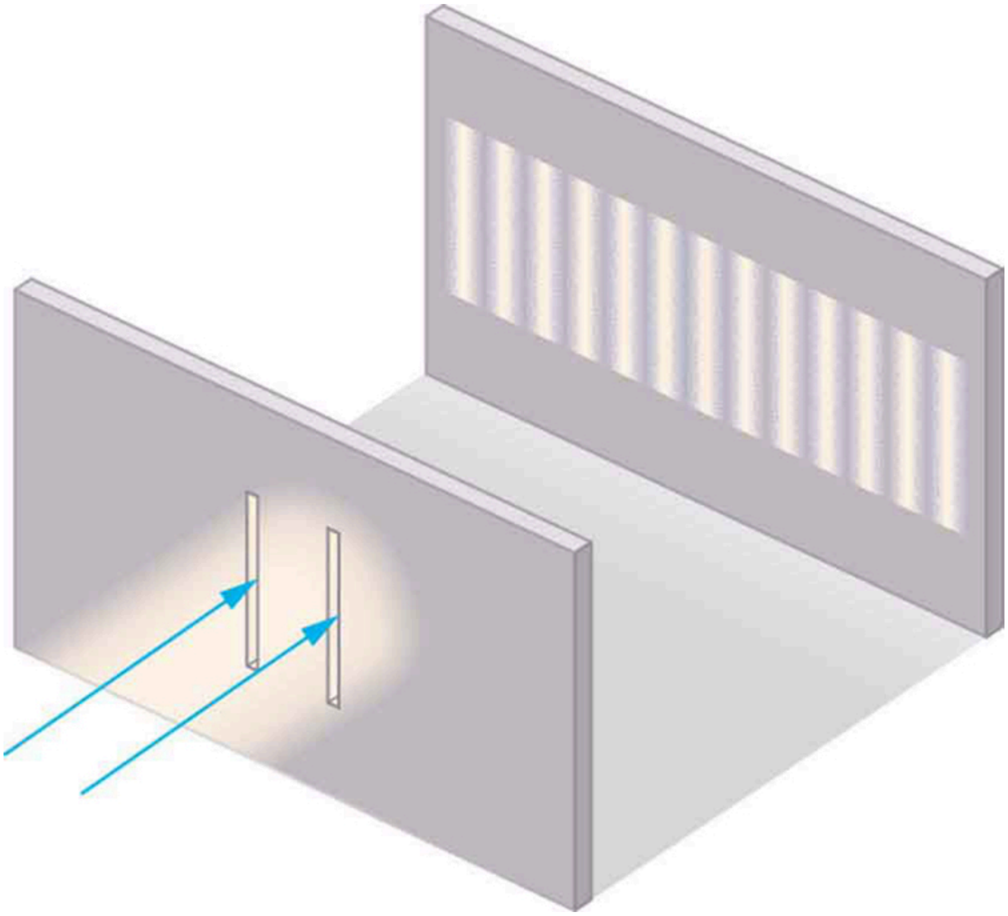
diffraction
: the bending of a wave around the edges of an opening or an obstacle

Huygens's principle
: every point on a wavefront is a source of wavelets that spread out in the forward direction at the same speed as the wave itself. The new wavefront is a line tangent to all of the wavelets

Young’s Double Slit Experiment

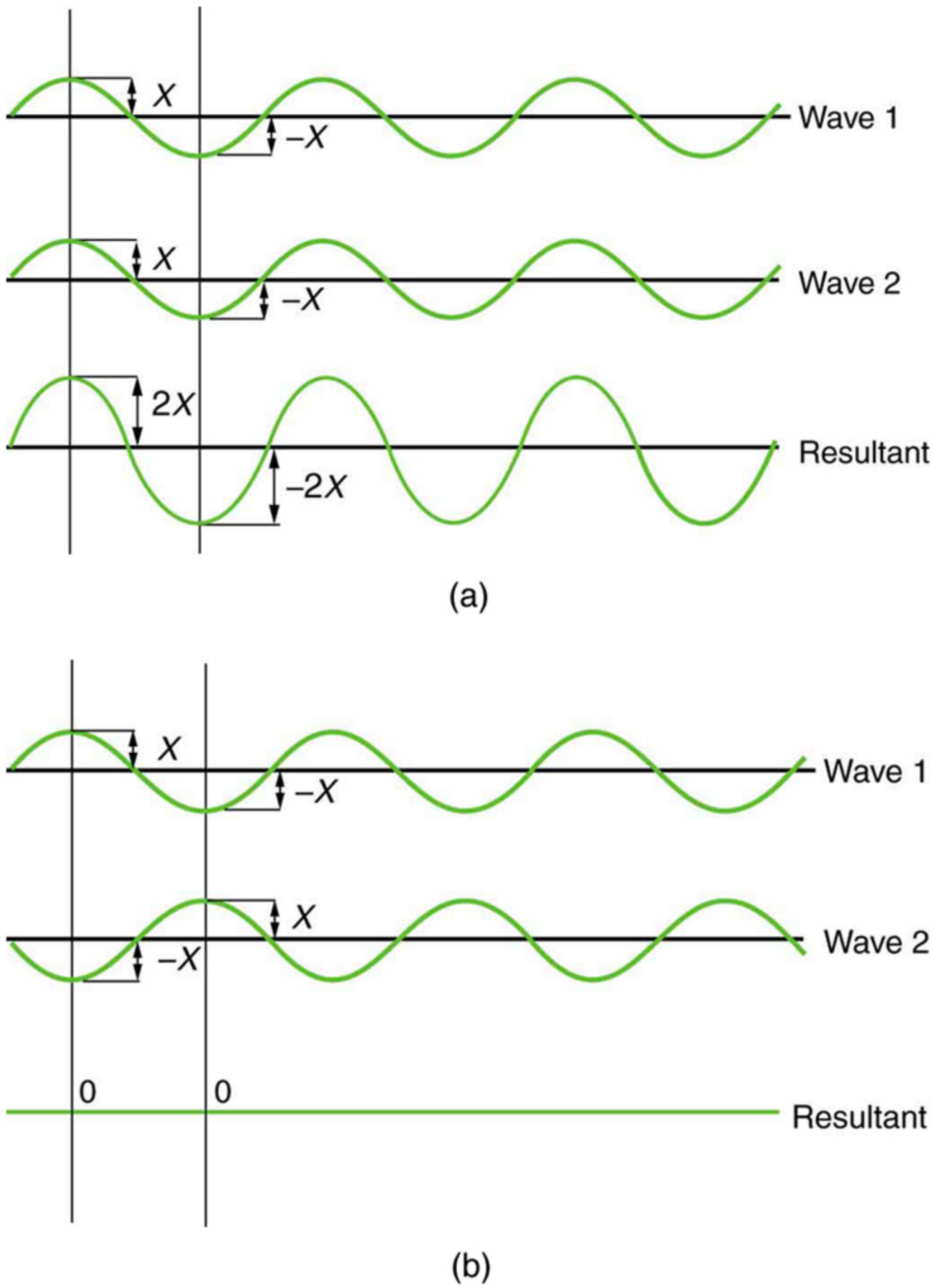
- Explain the phenomena of interference.
- Define constructive interference for a double slit and destructive interference for a double slit.

Although Christiaan Huygens thought that light was a wave, Isaac Newton did not. Newton felt that there were other explanations for color, and for the interference and diffraction effects that were observable at the time. Owing to Newton’s tremendous stature, his view generally prevailed. The fact that Huygens’s principle worked was not considered evidence that was direct enough to prove that light is a wave. The acceptance of the wave character of light came many years later when, in 1801, the English physicist and physician Thomas Young (1773–1829) did his now-classic double slit experiment (see [Figure 1]).



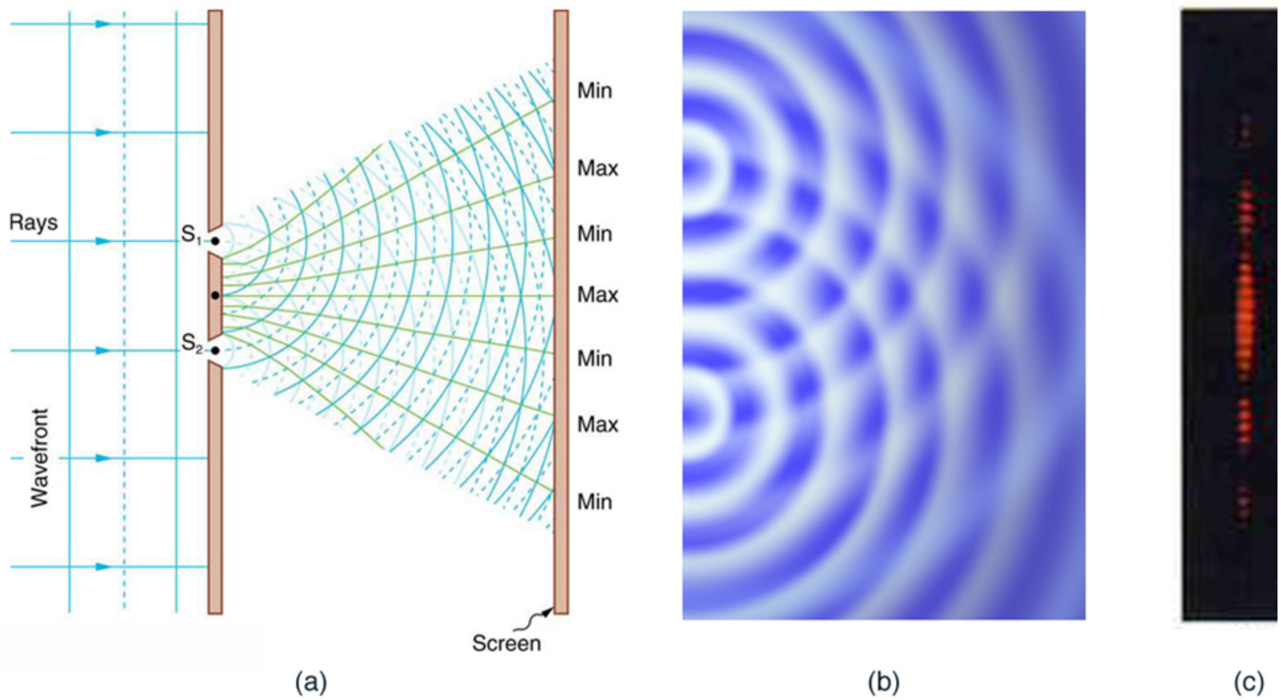
Young’s double slit experiment. Here pure-wavelength light sent through a pair of vertical slits is diffracted into a pattern on the screen of numerous vertical lines spread out horizontally. Without diffraction and interference, the light would simply make two lines on the screen.

Why do we not ordinarily observe wave behavior for light, such as observed in Young’s double slit experiment? First, light must interact with something small, such as the closely spaced slits used by Young, to show pronounced wave effects. Furthermore, Young first passed light from a single source (the Sun) through a single slit to make the light somewhat coherent. By **coherent**, we mean waves are in phase or have a definite phase relationship. **Incoherent** means the waves have random phase relationships. Why did Young then pass the light through a double slit? The answer to this question is that two slits provide two coherent light sources that then interfere constructively or destructively. Young used sunlight, where each wavelength forms its own pattern, making the effect more difficult to see. We illustrate the double slit experiment with monochromatic (single λ) light to clarify the effect. [Figure 2] shows the pure constructive and destructive interference of two waves having the same wavelength and amplitude.



The amplitudes of waves add. (a) Pure constructive interference is obtained when identical waves are in phase. (b) Pure destructive interference occurs when identical waves are exactly out of phase, or shifted by half a wavelength.

When light passes through narrow slits, it is diffracted into semicircular waves, as shown in [Figure 3](a). Pure constructive interference occurs where the waves are crest to crest or trough to trough. Pure destructive interference occurs where they are crest to trough. The light must fall on a screen and be scattered into our eyes for us to see the pattern. An analogous pattern for water waves is shown in [Figure 3](b). Note that regions of constructive and destructive interference move out from the slits at well-defined angles to the original beam. These angles depend on wavelength and the distance between the slits, as we shall see below.

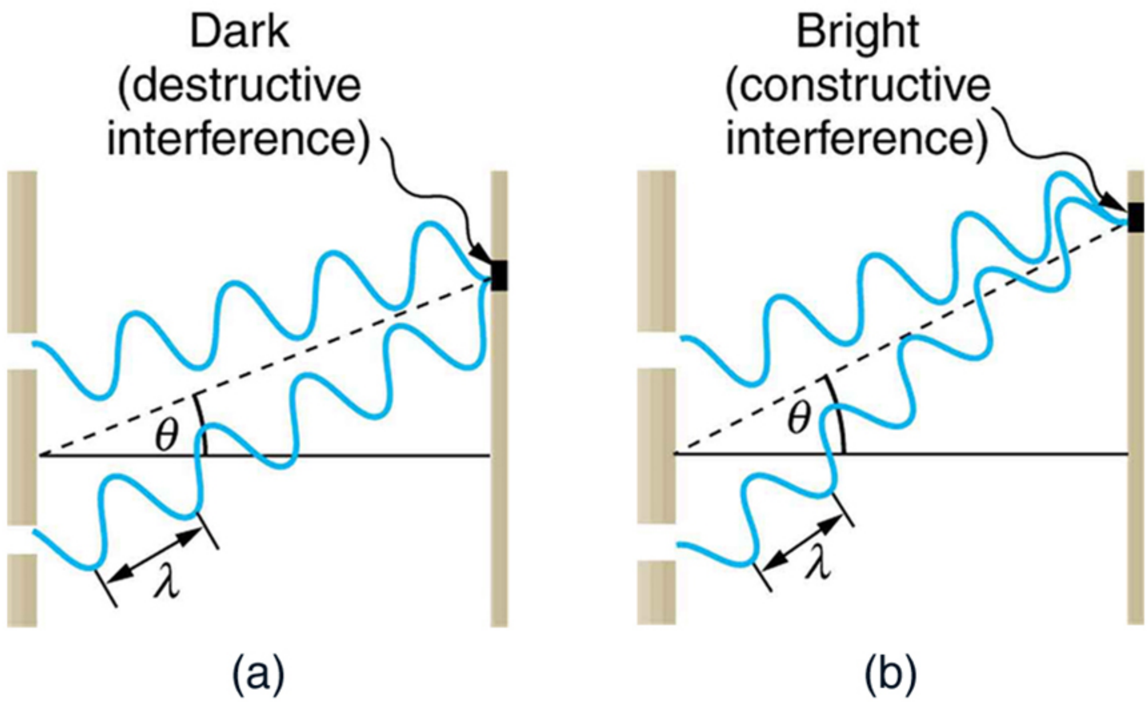


Double slits produce two coherent sources of waves that interfere. (a) Light spreads out (diffracts) from each slit, because the slits are narrow. These waves overlap and interfere constructively (bright lines) and destructively (dark regions). We can only see this if the light falls onto a screen and is scattered into our eyes. (b) Double slit interference pattern for water waves are nearly identical to that for light. Wave action is greatest in regions of constructive interference and least in regions of destructive interference. (c) When light that has passed through double slits falls on a screen, we see a pattern such as this. (credit: PASCO)

To understand the double slit interference pattern, we consider how two waves travel from the slits to the screen, as illustrated in [Figure 4]. Each slit is a different distance from a given point on the screen. Thus different numbers of wavelengths fit into each path. Waves start out from the slits in phase (crest to crest), but they may end up out of phase (crest to trough) at the screen if the paths differ in length by half a wavelength, interfering destructively as shown in [Figure 4](a). If the paths differ by a whole wavelength, then the waves arrive in phase (crest to crest) at the screen, interfering constructively as shown in [Figure 4](b). More generally, if the paths taken by the two waves differ by any half-integral number of wavelengths [$\left(1/2\right)\lambda$, $\left(3/2\right) \lambda$, $\left(5/2\right)\lambda$, etc.], then destructive interference occurs. Similarly, if the paths taken by the two waves differ by any integral number of wavelengths (λ , 2λ , 3λ , etc.), then constructive interference occurs.

Take-Home Experiment: Using Fingers as Slits

Look at a light, such as a street lamp or incandescent bulb, through the narrow gap between two fingers held close together. What type of pattern do you see? How does it change when you allow the fingers to move a little farther apart? Is it more distinct for a monochromatic source, such as the yellow light from a sodium vapor lamp, than for an incandescent bulb?



Waves follow different paths from the slits to a common point on a screen. (a) Destructive interference occurs here, because one path is a half wavelength longer than the other. The waves start in phase but arrive out of phase. (b) Constructive interference occurs here because one path is a whole wavelength longer than the other. The waves start out and arrive in phase.

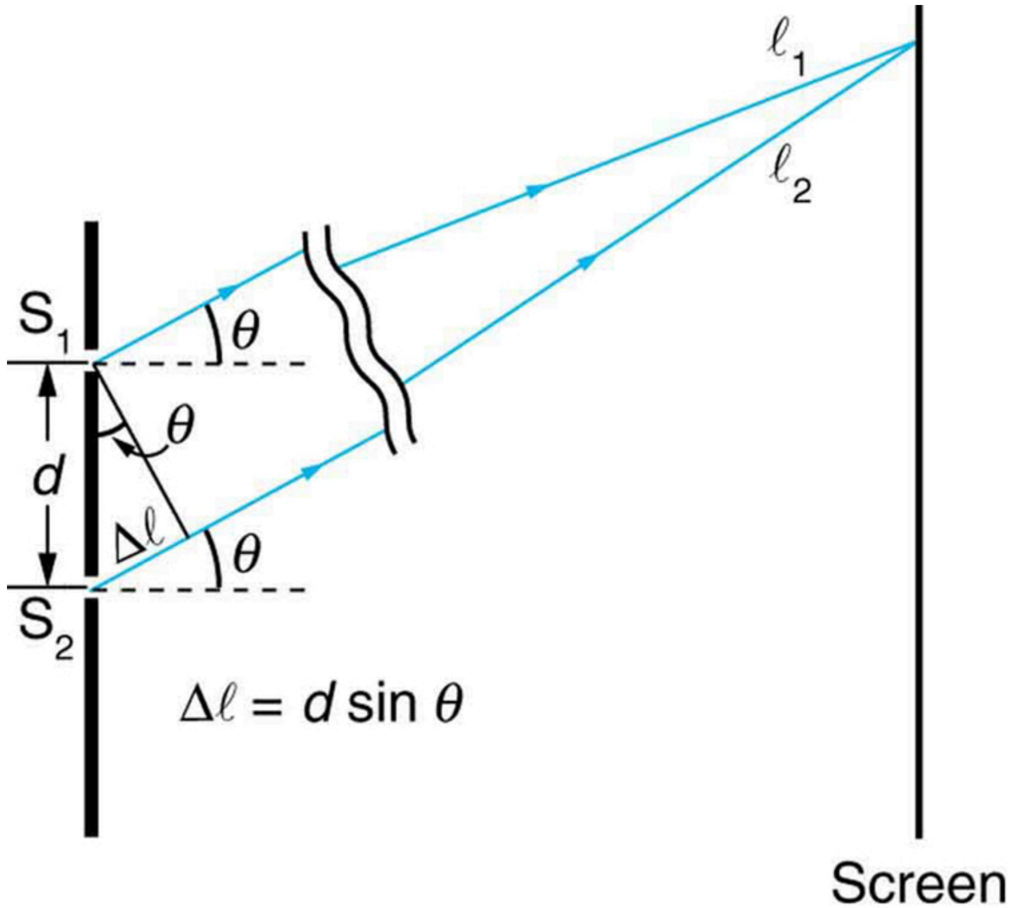
[Figure 5] shows how to determine the path length difference for waves traveling from two slits to a common point on a screen. If the screen is a large distance away compared with the distance between the slits, then the angle θ between the path and a line from the slits to the screen (see the figure) is nearly the same for each path. The difference between the paths is shown in the figure; simple trigonometry shows it to be $d \sin \theta$, where d is the distance between the slits. To obtain **constructive interference for a double slit**, the path length difference must be an integral multiple of the wavelength, or

$$d \sin \theta =m \lambda, \text{ for } m=0, 1, -1, 2, -2, \dots \text{(constructive).} $$

Similarly, to obtain **destructive interference for a double slit**, the path length difference must be a half-integral multiple of the wavelength, or

$$d \sin \theta =\left(m+\frac{1}{2}\right)\lambda , \text{ for } m=0, 1, -1, 2, -2, \dots \text{(destructive),} $$

where λ is the wavelength of the light, d is the distance between slits, and θ is the angle from the original direction of the beam as discussed above. We call m the **order** of the interference. For example, $m=4$ is fourth-order interference.

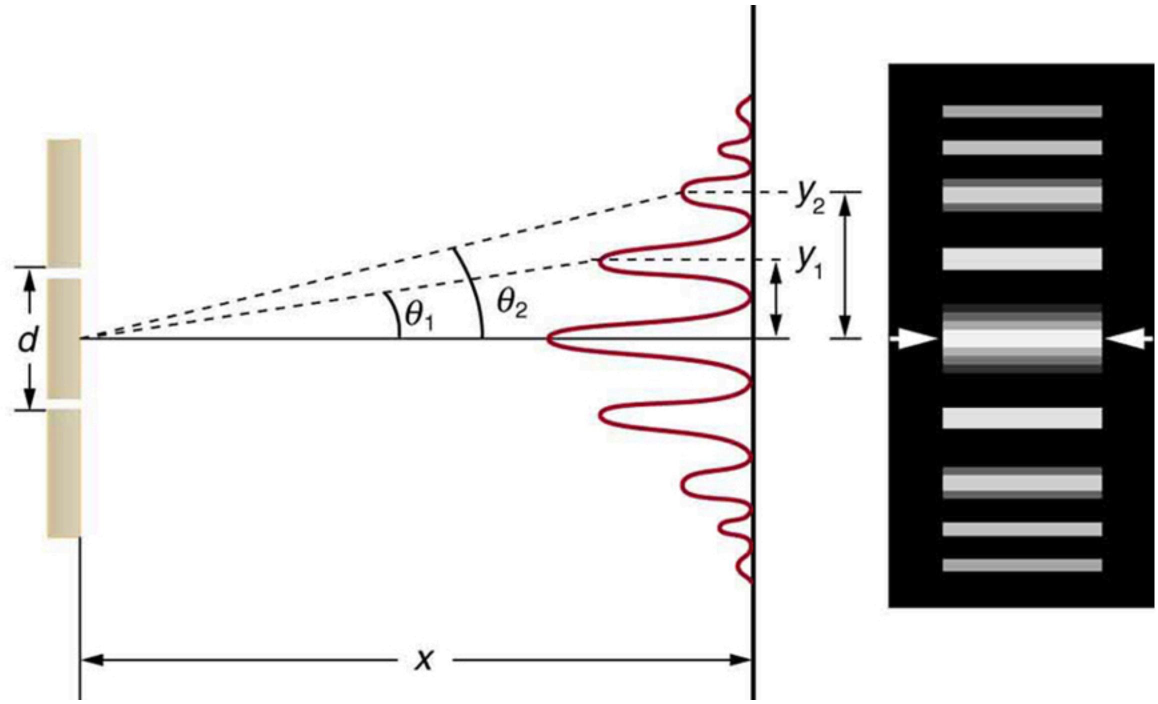


The paths from each slit to a common point on the screen differ by an amount \(d\sin \theta \), assuming the distance to the screen is much greater than the distance between slits (not to scale here).

The equations for double slit interference imply that a series of bright and dark lines are formed. For vertical slits, the light spreads out horizontally on either side of the incident beam into a pattern called interference fringes, illustrated in [Figure 6]. The intensity of the bright fringes falls off on either side, being brightest at the center. The closer the slits are, the more is the spreading of the bright fringes. We can see this by examining the equation

$$d \sin \theta =m \lambda\text{,} \text{ for } m=0, 1, -1, 2, -2, \dots .$$

For fixed λ and m , the smaller d is, the larger θ must be, since $\sin \theta =m \lambda/d$. This is consistent with our contention that wave effects are most noticeable when the object the wave encounters (here, slits a distance d apart) is small. Small d gives large θ , hence a large effect.



The interference pattern for a double slit has an intensity that falls off with angle. The photograph shows multiple bright and dark lines, or fringes, formed by light passing through a double slit.

Finding a Wavelength from an Interference Pattern

Suppose you pass light from a He-Ne laser through two slits separated by 0.0100 mm and find that the third bright line on a screen is formed at an angle of 10.95º relative to the incident beam. What is the wavelength of the light?

Strategy

The third bright line is due to third-order constructive interference, which means that $m=3$. We are given $d= 0.0100 \text{mm}$ and $\theta =10.95\text{º}$. The wavelength can thus be found using the equation $d \sin \theta =m \lambda$ for constructive interference.

Solution

The equation is $d \sin \theta =m \lambda$. Solving for the wavelength λ gives

$$\lambda =\frac{d \sin \theta }{m}. $$

Substituting known values yields

$$\begin{array}{lll}\lambda & =& \frac{\left(0.0100 \text{mm}\right)\left(\sin 10.95 ^\circ\right)}{3}\\ & =& 6.33 \times 10^{-4} \text{mm}=633 \text{nm} \text{.} \end{array} $$

Discussion

To three digits, this is the wavelength of light emitted by the common He-Ne laser. Not by coincidence, this red color is similar to that emitted by neon lights. More important, however, is the fact that interference patterns can be used to measure wavelength. Young did this for visible wavelengths. This analytical technique is still widely used to measure electromagnetic spectra. For a given order, the angle for constructive interference increases with λ , so that spectra (measurements of intensity versus wavelength) can be obtained.

Calculating Highest Order Possible

Interference patterns do not have an infinite number of lines, since there is a limit to how big m can be. What is the highest-order constructive interference possible with the system described in the preceding example?

Strategy and Concept

The equation $d \sin \theta =m \lambda \left(\text{ for } m=0, 1, -1, 2, -2, \dots \right)$ describes constructive interference. For fixed values of d and λ , the larger m is, the larger $\sin \theta$ is. However, the maximum value that $\sin \theta$ can have is 1, for an angle of $90 ^\circ$. (Larger angles imply that light goes backward and does not reach the screen at all.) Let us find which m corresponds to this maximum diffraction angle.

Solution

Solving the equation $d\sin \theta =m \lambda$ for m gives

$$m=\frac{d \sin \theta }{\lambda }. $$

Taking $\sin \theta =1$ and substituting the values of d and λ from the preceding example gives

$$m=\frac{\left(0.0100 \text{mm}\right)\left(1\right)}{633 \text{nm}}\approx 15.8 \text{.} $$

Therefore, the largest integer m can be is 15, or

$$m=15. $$

Discussion

The number of fringes depends on the wavelength and slit separation. The number of fringes will be very large for large slit separations. However, if the slit separation becomes much greater than the wavelength, the intensity of the interference pattern changes so that the screen has two bright lines cast by the slits, as expected when light behaves like a ray. We also note that the fringes get fainter further away from the center. Consequently, not all 15 fringes may be observable.

Section Summary

- Young's double slit experiment gave definitive proof of the wave character of light.
- An interference pattern is obtained by the superposition of light from two slits.
- There is constructive interference when $d\sin \theta =m \lambda \left(\text{ for } m=0, 1, -1, 2, -2, \dots \right)$, where d is the distance between the slits, θ is the angle relative to the incident direction, and m is the order of the interference.
- There is destructive interference when $d \sin \theta =\left(m+\frac{1}{2}\right)\lambda \left(\text{ for } m=0, 1, -1, 2, -2, \dots \right)$.

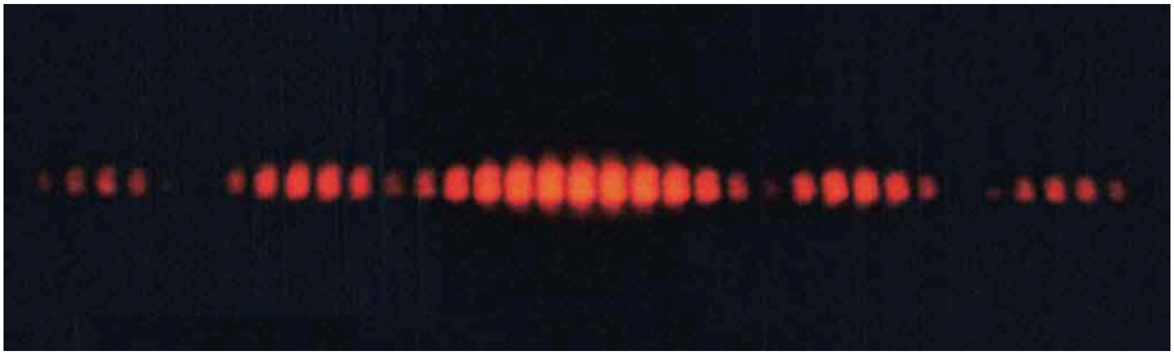
Conceptual Questions

Young's double slit experiment breaks a single light beam into two sources. Would the same pattern be obtained for two independent sources of light, such as the headlights of a distant car? Explain.

Suppose you use the same double slit to perform Young's double slit experiment in air and then repeat the experiment in water. Do the angles to the same parts of the interference pattern get larger or smaller? Does the color of the light change? Explain.

Is it possible to create a situation in which there is only destructive interference? Explain.

[Figure 7] shows the central part of the interference pattern for a pure wavelength of red light projected onto a double slit. The pattern is actually a combination of single slit and double slit interference. Note that the bright spots are evenly spaced. Is this a double slit or single slit characteristic? Note that some of the bright spots are dim on either side of the center. Is this a single slit or double slit characteristic? Which is smaller, the slit width or the separation between slits? Explain your responses.



This double slit interference pattern also shows signs of single slit interference. (credit: PASCO)

Problems & Exercises

At what angle is the first-order maximum for 450-nm wavelength blue light falling on double slits separated by 0.0500 mm?

Show Solution

Strategy

For constructive interference in a double slit experiment, we use $d \sin \theta = m\lambda$, where $m = 1$ for the first-order maximum. We need to solve for θ.

Solution

Given:

- $$ \lambda = 450 \text{ nm} = 450 \times 10^{-9} \text{ m} $$
- $$ d = 0.0500 \text{ mm} = 5.00 \times 10^{-5} \text{ m} $$
- $$ m = 1 $$

Using the equation for constructive interference:

$$d \sin \theta = m\lambda$$

Solving for $\sin \theta$:

$$\sin \theta = \frac{m\lambda}{d} = \frac{(1)(450 \times 10^{-9} \text{ m})}{5.00 \times 10^{-5} \text{ m}} = \frac{450 \times 10^{-9}}{5.00 \times 10^{-5}} = 9.00 \times 10^{-3}$$

Therefore:

$$\theta = \sin^{-1}(9.00 \times 10^{-3}) = 0.516°$$

Discussion

The first-order maximum appears at a very small angle of 0.516°, which is typical for double slit experiments where the slit separation is much larger than the wavelength. The ratio $d/\lambda = 5.00 \times 10^{-5}/450 \times 10^{-9} \approx 111$, meaning the slit separation is about 111 wavelengths. This produces a closely-spaced interference pattern with many orders visible. The small angle validates using the small-angle approximation ($\sin \theta \approx \tan \theta \approx \theta$ in radians) commonly employed in double-slit calculations.

The first-order maximum for 450-nm blue light occurs at an angle of 0.516°.

Calculate the angle for the third-order maximum of 580-nm wavelength yellow light falling on double slits separated by 0.100 mm.

Show Solution

Strategy

For constructive interference in a double slit experiment, we use the equation $d \sin \theta = m\lambda$, where m = 3 for the third-order maximum. We need to solve for θ.

Solution

Given:

- $$ \lambda = 580 \text{ nm} = 580 \times 10^{-9} \text{ m} $$
- $$ d = 0.100 \text{ mm} = 1.00 \times 10^{-4} \text{ m} $$
- $$ m = 3 $$

Using the equation for constructive interference:

$$d \sin \theta = m\lambda$$

Solving for $\sin \theta$:

$$\sin \theta = \frac{m\lambda}{d} = \frac{3 \times 580 \times 10^{-9} \text{ m}}{1.00 \times 10^{-4} \text{ m}} = \frac{1740 \times 10^{-9}}{1.00 \times 10^{-4}} = 0.01740$$

Therefore:

$$\theta = \sin^{-1}(0.01740) = 1.00°$$

Discussion

The angle is quite small, which is typical for double slit experiments where the slit separation is much larger than the wavelength. The third-order maximum appears at about 1°, meaning the maxima are closely spaced. This small angle justifies using the small angle approximation ($\sin \theta \approx \tan \theta \approx \theta$ in radians) in many double slit calculations.

What is the separation between two slits for which 610-nm orange light has its first maximum at an angle of $30.0º$?

Show Solution

Strategy

For constructive interference, we use $d \sin \theta = m\lambda$, where $m = 1$ for the first-order maximum. We solve for the slit separation d.

Solution

Given:

- $$ \lambda = 610 \text{ nm} = 610 \times 10^{-9} \text{ m} $$
- $$ \theta = 30.0° $$
- $m = 1$ (first maximum)

Using the equation for constructive interference:

$$d \sin \theta = m\lambda$$

Solving for d:

$$d = \frac{m\lambda}{\sin \theta} = \frac{(1)(610 \times 10^{-9} \text{ m})}{\sin 30.0°} = \frac{610 \times 10^{-9}}{0.500}$$
$$d = 1.22 \times 10^{-6} \text{ m} = 1.22 \text{ μm}$$

Discussion

The slit separation of 1.22 μm is only about twice the wavelength of the orange light (610 nm). This very small slit separation causes the interference pattern to spread out significantly - the first maximum appears at a large angle of 30°. With such a small d, only a few orders of maxima would be observable before reaching $\theta = 90°$. This demonstrates the inverse relationship between slit separation and angular spread: smaller d produces larger angles for a given order.

The slit separation is 1.22×10^{-6} m or 1.22 μm.

Find the distance between two slits that produces the first minimum for 410-nm violet light at an angle of $45.0º$.

Show Solution

Strategy

For destructive interference (minima) in a double slit experiment, we use $d \sin \theta = (m + \frac{1}{2})\lambda$. For the first minimum, m = 0, so $d \sin \theta = \frac{\lambda}{2}$. We solve for d.

Solution

Given:

- $$ \lambda = 410 \text{ nm} = 410 \times 10^{-9} \text{ m} $$
- $$ \theta = 45.0° $$
- $m = 0$ (first minimum)

Using the equation for destructive interference:

$$d \sin \theta = \left(m + \frac{1}{2}\right)\lambda = \frac{\lambda}{2}$$

Solving for d:

$$d = \frac{\lambda}{2 \sin \theta} = \frac{410 \times 10^{-9} \text{ m}}{2 \sin 45.0°} = \frac{410 \times 10^{-9} \text{ m}}{2 \times 0.7071}$$
$$d = \frac{410 \times 10^{-9}}{1.4142} = 2.90 \times 10^{-7} \text{ m} = 290 \text{ nm}$$

Discussion

This slit separation is remarkably small - less than the wavelength of visible light! At such a small separation ($d < \lambda$), only the central maximum and perhaps one or two orders would be visible before reaching 90°. The fact that the first minimum occurs at 45° indicates that this is a very closely spaced double slit. Such small separations are at the limits of conventional optical fabrication and would require specialized techniques like electron beam lithography to create.

Calculate the wavelength of light that has its third minimum at an angle of $30.0º$ when falling on double slits separated by 3.00μm . Explicitly, show how you follow the steps in Problem-Solving Strategies for Wave Optics.

Show Solution

Strategy

Following the Problem-Solving Strategy for Wave Optics:

Step 1: This is a double-slit interference problem involving destructive interference (minimum).

Step 2: For destructive interference (minima) in double-slit experiments: $d \sin \theta = \left(m + \frac{1}{2}\right)\lambda$, where m = 0, 1, 2, … For the third minimum, m = 2.

Step 3: We need to find the wavelength λ.

Step 4: Given information: $\theta = 30.0°$, $d = 3.00$ μm, m = 2 (third minimum).

Solution

For the third minimum, m = 2. Using the destructive interference equation:

$$d \sin \theta = \left(m + \frac{1}{2}\right)\lambda$$

Solving for λ:

$$\lambda = \frac{d \sin \theta}{m + \frac{1}{2}}$$

Substituting the values:

$$\lambda = \frac{(3.00 \times 10^{-6} \text{ m}) \sin 30.0°}{2 + \frac{1}{2}}$$
$$\lambda = \frac{(3.00 \times 10^{-6} \text{ m})(0.500)}{2.5}$$
$$\lambda = \frac{1.50 \times 10^{-6} \text{ m}}{2.5} = 6.00 \times 10^{-7} \text{ m} = 600 \text{ nm}$$

Step 5: Check reasonableness: The wavelength of 600 nm is in the orange part of the visible spectrum (visible light ranges from ~380-760 nm), which is reasonable.

Discussion

The calculated wavelength of 600 nm corresponds to orange light, which is within the visible spectrum. The third minimum occurs at the relatively large angle of 30°, indicating that the slit separation (3.00 μm) is only a few wavelengths wide (specifically, d/λ = 5). This configuration produces a well-spread interference pattern.

It’s important to note that for minima (destructive interference), we use $d \sin \theta = \left(m + \frac{1}{2}\right)\lambda$ with m = 0, 1, 2, … so the third minimum corresponds to m = 2. This gives us 2.5λ path difference, which produces destructive interference when the two waves arrive 180° out of phase.

Answer

The wavelength is **600 nm** (orange light).

What is the wavelength of light falling on double slits separated by 2.00μm if the third-order maximum is at an angle of $60.0º$?

Show Solution

Strategy

For constructive interference at the third-order maximum (m = 3), we use $d \sin \theta = m\lambda$ and solve for the wavelength λ.

Solution

Given:

- $$ d = 2.00 \text{ μm} = 2.00 \times 10^{-6} \text{ m} $$
- $$ \theta = 60.0° $$
- $$ m = 3 $$

Using the equation for constructive interference:

$$d \sin \theta = m\lambda$$

Solving for λ:

$$\lambda = \frac{d \sin \theta}{m} = \frac{(2.00 \times 10^{-6} \text{ m}) \sin 60.0°}{3}$$
$$\lambda = \frac{(2.00 \times 10^{-6} \text{ m})(0.8660)}{3} = \frac{1.732 \times 10^{-6} \text{ m}}{3}$$
$$\lambda = 5.77 \times 10^{-7} \text{ m} = 577 \text{ nm}$$

Discussion

This wavelength of 577 nm corresponds to yellow-green light, in the middle of the visible spectrum. The relatively large angle (60°) for the third-order maximum indicates that the slit separation (2.00 μm) is only a few wavelengths wide. This setup would produce a well-spread interference pattern with several visible orders. The result is reasonable because visible light wavelengths range from about 380-760 nm, and our answer falls within this range.

At what angle is the fourth-order maximum for the situation in [Exercise 1]?

Show Solution

Strategy

From Exercise 1, we have 450-nm wavelength blue light falling on double slits separated by 0.0500 mm. We use the constructive interference equation $d \sin \theta = m\lambda$ with m = 4 for the fourth-order maximum.

Solution

Given (from Exercise 1):

- $$ \lambda = 450 \text{ nm} = 450 \times 10^{-9} \text{ m} $$
- $$ d = 0.0500 \text{ mm} = 5.00 \times 10^{-5} \text{ m} $$
- $m = 4$ (fourth-order maximum)

Using the equation for constructive interference:

$$d \sin \theta = m\lambda$$

Solving for $\sin \theta$:

$$\sin \theta = \frac{m\lambda}{d} = \frac{(4)(450 \times 10^{-9})}{5.00 \times 10^{-5}}$$
$$\sin \theta = \frac{1800 \times 10^{-9}}{5.00 \times 10^{-5}} = \frac{1.800 \times 10^{-6}}{5.00 \times 10^{-5}} = 0.0360$$

Therefore:

$$\theta = \sin^{-1}(0.0360) = 2.06°$$

Discussion

The fourth-order maximum appears at 2.06°, which is four times the angle of the first-order maximum (0.516° from Exercise 1). This linear relationship ($\theta_m \approx m\theta_1$ for small angles) is characteristic of the small-angle approximation, where $\sin \theta \approx \theta$ in radians. The angle is still quite small, confirming that many orders of maxima are visible in this double-slit configuration. The slit separation being about 111 wavelengths allows for many observable interference fringes.

Answer

The fourth-order maximum occurs at **2.06°**.

What is the highest-order maximum for 400-nm light falling on double slits separated by 25.0μm ?

Show Solution

Strategy

The highest-order maximum occurs when $\sin \theta$ approaches its maximum value of 1 (at θ = 90°). We use $d \sin \theta = m\lambda$ with $\sin \theta = 1$ and solve for the maximum integer value of m.

Solution

Given:

- $$ \lambda = 400 \text{ nm} = 400 \times 10^{-9} \text{ m} = 4.00 \times 10^{-7} \text{ m} $$
- $$ d = 25.0 \text{ μm} = 25.0 \times 10^{-6} \text{ m} $$

The condition for constructive interference is:

$$d \sin \theta = m\lambda$$

For the maximum order, $\sin \theta = 1$, so:

$$m_{\text{max}} = \frac{d}{\lambda} = \frac{25.0 \times 10^{-6} \text{ m}}{4.00 \times 10^{-7} \text{ m}}$$
$$m_{\text{max}} = \frac{25.0 \times 10^{-6}}{4.00 \times 10^{-7}} = 62.5$$

Since m must be an integer, the highest-order maximum is:

$$m = 62$$

Discussion

With such a large slit separation compared to the wavelength (d/λ ≈ 62.5), this double slit system can produce 62 orders of maxima on each side of the central maximum, plus the central maximum itself, for a total of 125 bright fringes. This large number of fringes would be very closely spaced. In practice, not all of these might be observable due to:

1. The finite width of each slit causing single-slit diffraction effects that modulate the double-slit pattern
2. Intensity falling off at large angles
3. Practical limitations of the screen or detection system

The 62nd order maximum would appear at $\theta = \sin^{-1}(62 \times 400/25000) = \sin^{-1}(0.992) \approx 82.9°$.

Find the largest wavelength of light falling on double slits separated by 1.20μm for which there is a first-order maximum. Is this in the visible part of the spectrum?

Show Solution

Strategy

For a first-order maximum to exist, the angle θ must be less than or equal to 90° (where $\sin \theta \leq 1$). The largest wavelength occurs when $\sin \theta = 1$ at θ = 90°. We use $d \sin \theta = m\lambda$ with m = 1 and $\sin \theta = 1$ to find the maximum wavelength.

Solution

Given:

- $$ d = 1.20 \text{ μm} = 1.20 \times 10^{-6} \text{ m} $$
- $m = 1$ (first-order maximum)
- $\sin \theta = 1$ (maximum value)

Using the equation for constructive interference:

$$d \sin \theta = m\lambda$$

Solving for the maximum wavelength:

$$\lambda_{\text{max}} = \frac{d \sin \theta}{m} = \frac{d(1)}{1} = d$$
$$\lambda_{\text{max}} = 1.20 \times 10^{-6} \text{ m} = 1200 \text{ nm}$$

Is this visible?

The visible spectrum ranges from approximately 380 nm (violet) to 760 nm (red). Since 1200 nm > 760 nm, this wavelength is **not in the visible part of the spectrum**. It falls in the near-infrared region.

Discussion

The result shows that the largest wavelength for which a first-order maximum can occur equals the slit separation itself. Any wavelength larger than 1200 nm would require $\sin \theta > 1$ for the first-order maximum, which is physically impossible.

At 1200 nm, the first-order maximum would appear at exactly 90° (parallel to the slits), making it impossible to observe in practice. Wavelengths just slightly smaller than this would have first-order maxima at angles approaching 90°, also difficult to detect.

This wavelength is in the near-infrared part of the electromagnetic spectrum, which is invisible to the human eye but can be detected by infrared cameras or sensors. Such wavelengths are used in applications like night vision and infrared spectroscopy.

Answer

The largest wavelength is **1200 nm**, which is **not visible** (it's in the near-infrared region).

What is the smallest separation between two slits that will produce a second-order maximum for 720-nm red light?

Show Solution

Strategy

For a second-order maximum to be observable, it must occur at an angle less than 90°. The smallest slit separation occurs when the second-order maximum appears at exactly θ = 90° (where $\sin \theta = 1$). We use $d \sin \theta = m\lambda$ with m = 2 and $\sin \theta = 1$.

Solution

Given:

- $$ \lambda = 720 \text{ nm} = 720 \times 10^{-9} \text{ m} $$
- $m = 2$ (second-order maximum)
- $\theta = 90°$ (limiting case)

Using the equation for constructive interference:

$$d \sin \theta = m\lambda$$

With $\sin 90° = 1$:

$$d_{\text{min}} = m\lambda = 2 \times 720 \times 10^{-9} \text{ m}$$
$$d_{\text{min}} = 1440 \times 10^{-9} \text{ m} = 1.44 \times 10^{-6} \text{ m} = 1.44 \text{ μm}$$

Discussion

If the slit separation were smaller than 1.44 μm, the second-order maximum would require $\sin \theta > 1$, which is impossible. Therefore, 1.44 μm is the minimum separation needed to observe a second-order maximum with 720-nm light.

For this minimum separation:

- The first-order maximum appears at $\theta_1 = \sin^{-1}(720/1440) = \sin^{-1}(0.5) = 30°$
- The second-order maximum appears at $\theta_2 = 90°$

This means the second-order maximum would barely be observable at the edge of the interference pattern. For practical observation, the slit separation should be somewhat larger than this minimum value. This principle is important in designing diffraction gratings and other optical devices where specific orders need to be observable.

(a) What is the smallest separation between two slits that will produce a second-order maximum for any visible light? (b) For all visible light?

Show Solution

Strategy

The visible spectrum ranges from approximately 380 nm (violet) to 760 nm (red). For part (a), we need the smallest slit separation that allows a second-order maximum for **any** visible wavelength - this means at least one wavelength can show a second-order maximum. For part (b), we need the smallest separation for **all** visible wavelengths - meaning even the longest wavelength (red, 760 nm) can show a second-order maximum.

Solution

For a second-order maximum at the limiting angle of 90°:

$$d = m\lambda = 2\lambda$$

(a) For ANY visible light:

The shortest visible wavelength (violet, ~380 nm) requires the smallest slit separation to produce a second-order maximum:

$$d_{\text{min}} = 2 \times 380 \text{ nm} = 760 \text{ nm}$$

With this separation, violet light can just produce a second-order maximum at 90°. Longer wavelengths (green, red, etc.) would require even larger separations, so they would not have visible second-order maxima.

(b) For ALL visible light:

To ensure all visible wavelengths (including the longest, red at ~760 nm) can produce a second-order maximum:

$$d_{\text{min}} = 2 \times 760 \text{ nm} = 1520 \text{ nm} = 1.52 \text{ μm}$$

With this separation:

- Red light (760 nm): second order at 90°
- Shorter wavelengths: second order at smaller angles

Discussion

Part (a) asks for the minimum separation to observe **any** visible second-order maximum. Since violet has the shortest wavelength, it requires the smallest slit separation (760 nm). With slits separated by 760 nm, only violet light would show a second-order maximum (at 90°), while longer wavelengths would not.

Part (b) asks for the minimum separation to observe second-order maxima for **all** visible wavelengths. This requires accommodating the longest wavelength (red at 760 nm), giving d = 1520 nm. With this separation:

- Red (760 nm): $\theta_2 = 90°$
- Green (550 nm): $\theta_2 = \sin^{-1}(2 \times 550/1520) = \sin^{-1}(0.724) = 46.4°$
- Violet (380 nm): $\theta_2 = \sin^{-1}(2 \times 380/1520) = \sin^{-1}(0.500) = 30.0°$

All visible wavelengths would have observable second-order maxima.

Answer

(a) **760 nm** (for any visible light)

(b) **1520 nm or 1.52 μm** (for all visible light)

(a) If the first-order maximum for pure-wavelength light falling on a double slit is at an angle of $10.0º$, at what angle is the second-order maximum? (b) What is the angle of the first minimum? (c) What is the highest-order maximum possible here?

Show Solution

Strategy

From the first-order maximum, we can find the ratio d/λ. We then use this ratio to find (a) the second-order maximum angle, (b) the first minimum angle, and (c) the highest possible order.

Solution

(a) Second-order maximum:

For the first-order maximum:

$$d \sin \theta_1 = \lambda$$

So:

$$\frac{d}{\lambda} = \frac{1}{\sin 10.0°} = \frac{1}{0.1736} = 5.759$$

For the second-order maximum (m = 2):

$$\sin \theta_2 = \frac{2\lambda}{d} = \frac{2}{5.759} = 0.3472$$
$$\theta_2 = \sin^{-1}(0.3472) = 20.3°$$

(b) First minimum:

For the first minimum, $m = 0$, so:

$$d \sin \theta_{\text{min}} = \frac{\lambda}{2}$$
$$\sin \theta_{\text{min}} = \frac{\lambda}{2d} = \frac{1}{2 \times 5.759} = 0.0868$$
$$\theta_{\text{min}} = \sin^{-1}(0.0868) = 5.0°$$

(c) Highest-order maximum:

The maximum order occurs when $\sin \theta = 1$:

$$m_{\text{max}} = \frac{d}{\lambda} = 5.759$$

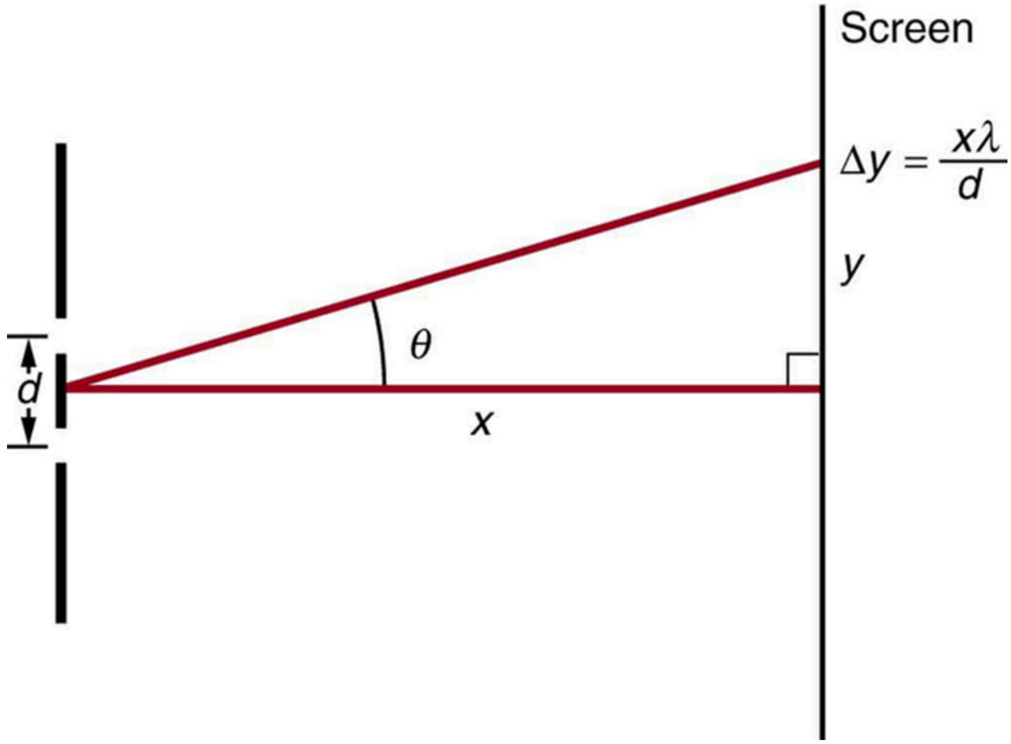
Since m must be an integer:

$$m_{\text{max}} = 5$$

Discussion

The results show logical progression: the first minimum at 5.0° falls between the central maximum (0°) and the first-order maximum (10.0°). The second-order maximum at 20.3° is roughly twice the angle of the first-order (this approximation works well for small angles). The system can support up to 5 orders on each side of center. We can verify: $\sin^{-1}(5/5.759) = \sin^{-1}(0.868) = 60.2°$, which is less than 90°, confirming that the 5th order is observable.

[Figure 8] shows a double slit located a distance x from a screen, with the distance from the center of the screen given by y . When the distance d between the slits is relatively large, there will be numerous bright spots, called fringes. Show that, for small angles (where $\sin \theta \approx \theta$, with θ in radians), the distance between fringes is given by $\Delta y= x\lambda /d$.



The distance between adjacent fringes is \(\Delta y = x \lambda /d \), assuming the slit separation \(d \) is large compared with \(\lambda \).

Show Solution

For small angles $\sin \theta -\tan \theta \approx \theta \left(\text{in radians}\right)$.

For two adjacent fringes we have,

$$d \sin {\theta }_{\text{m}}=m \lambda $$

and

$$d \sin {\theta }_{\text{m}+1}=\left(m+1\right)\lambda $$

Subtracting these equations gives

$$\begin{array}{}d\left(\sin {\theta }_{\text{m}+1}-\sin {\theta }_{\text{m}}\right)=\left[\left(m+1\right)-m\right] \lambda \\ d\left({\theta }_{\text{m}+1}-{\theta }_{\text{m}}\right)=\lambda \\ \tan {\theta }_{\text{m}}=\frac{ {y}_{\text{m}}}{x}\approx {\theta }_{\text{m}} ⇒d\left(\frac{ {y}_{\text{m}+1}}{x}-\frac{ {y}_{\text{m}}}{x}\right)=\lambda \\ d\frac{\Delta y}{x}=\lambda ⇒\Delta y=\frac{x\lambda }{d}\end{array} $$

Using the result of the problem above, calculate the distance between fringes for 633-nm light falling on double slits separated by 0.0800 mm, located 3.00 m from a screen as in [Figure 8].

Show Solution

Strategy

From the previous problem, we derived that the fringe spacing is $\Delta y = \frac{x\lambda}{d}$ for small angles. We apply this formula with the given values.

Solution

Given:

- $$ \lambda = 633 \text{ nm} = 633 \times 10^{-9} \text{ m} $$

- $d = 0.0800 \text{ mm} = 8.00 \times 10^{-5} \text{ m}$
- $x = 3.00 \text{ m}$

Using the fringe spacing formula:

$$\Delta y = \frac{x\lambda}{d} = \frac{(3.00 \text{ m})(633 \times 10^{-9} \text{ m})}{8.00 \times 10^{-5} \text{ m}}$$
$$\Delta y = \frac{1899 \times 10^{-9} \text{ m}}{8.00 \times 10^{-5} \text{ m}} = \frac{1.899 \times 10^{-6}}{8.00 \times 10^{-5}}$$
$$\Delta y = 2.37 \times 10^{-2} \text{ m} = 2.37 \text{ cm} = 23.7 \text{ mm}$$

Discussion

The fringe spacing of 23.7 mm (about 2.4 cm) is easily observable with the naked eye. This relatively large spacing results from:

1. The long distance to the screen (3.00 m)
2. The small slit separation (0.0800 mm)

The ratio x/d = 3.00 m / 0.0800 mm = 37,500 is very large, which greatly magnifies the fringe pattern.

We can verify our result makes sense by checking the small angle approximation. For the first-order maximum: $\sin \theta_1 = \lambda/d = 633 \times 10^{-9} / 8.00 \times 10^{-5} = 7.91 \times 10^{-3}$

This gives $\theta_1 = 0.453°$, which is indeed small, validating our use of the small angle approximation. The angle can also be found from $\tan \theta_1 = y_1/x = 0.0237/3.00 = 7.90 \times 10^{-3}$, confirming our calculation.

Using the result of the problem two problems prior, find the wavelength of light that produces fringes 7.50 mm apart on a screen 2.00 m from double slits separated by 0.120 mm (see [Figure 8]).

Show Solution

Strategy

From the problem two problems prior, the distance between adjacent fringes is given by $\Delta y = \frac{x\lambda}{d}$, where x is the distance to the screen, λ is the wavelength, and d is the slit separation. We solve for λ.

Solution

Given:

- $\Delta y = 7.50 \text{ mm} = 7.50 \times 10^{-3} \text{ m}$
- $x = 2.00 \text{ m}$
- $d = 0.120 \text{ mm} = 1.20 \times 10^{-4} \text{ m}$

Using the fringe spacing formula:

$$\Delta y = \frac{x\lambda}{d}$$

Solving for λ:

$$\lambda = \frac{\Delta y \cdot d}{x}$$

Substituting the values:

$$\lambda = \frac{(7.50 \times 10^{-3} \text{ m})(1.20 \times 10^{-4} \text{ m})}{2.00 \text{ m}}$$
$$\lambda = \frac{9.00 \times 10^{-7} \text{ m}^2}{2.00 \text{ m}} = 4.50 \times 10^{-7} \text{ m} = 450 \text{ nm}$$

Discussion

The wavelength of 450 nm corresponds to blue light, which is in the visible spectrum (approximately 380-760 nm). The fringe spacing of 7.50 mm is quite large and easily observable, making this a practical double-slit experiment setup.

The relatively large fringe separation results from the combination of:

1. Large screen distance (2.00 m) - increases fringe spacing
2. Small slit separation (0.120 mm) - increases fringe spacing
3. Short wavelength (450 nm blue light) - but still produces visible fringes

This formula $\Delta y = x\lambda/d$ is derived from the small-angle approximation and shows that fringe spacing is directly proportional to both the wavelength and screen distance, and inversely proportional to the slit separation. This relationship is fundamental to understanding interference patterns in double-slit experiments.

Answer

The wavelength is **450 nm** (blue light).

Glossary

coherent
: waves are in phase or have a definite phase relationship

constructive interference for a double slit
: the path length difference must be an integral multiple of the wavelength

destructive interference for a double slit
: the path length difference must be a half-integral multiple of the wavelength

incoherent
: waves have random phase relationships

order
: the integer m used in the equations for constructive and destructive interference for a double slit

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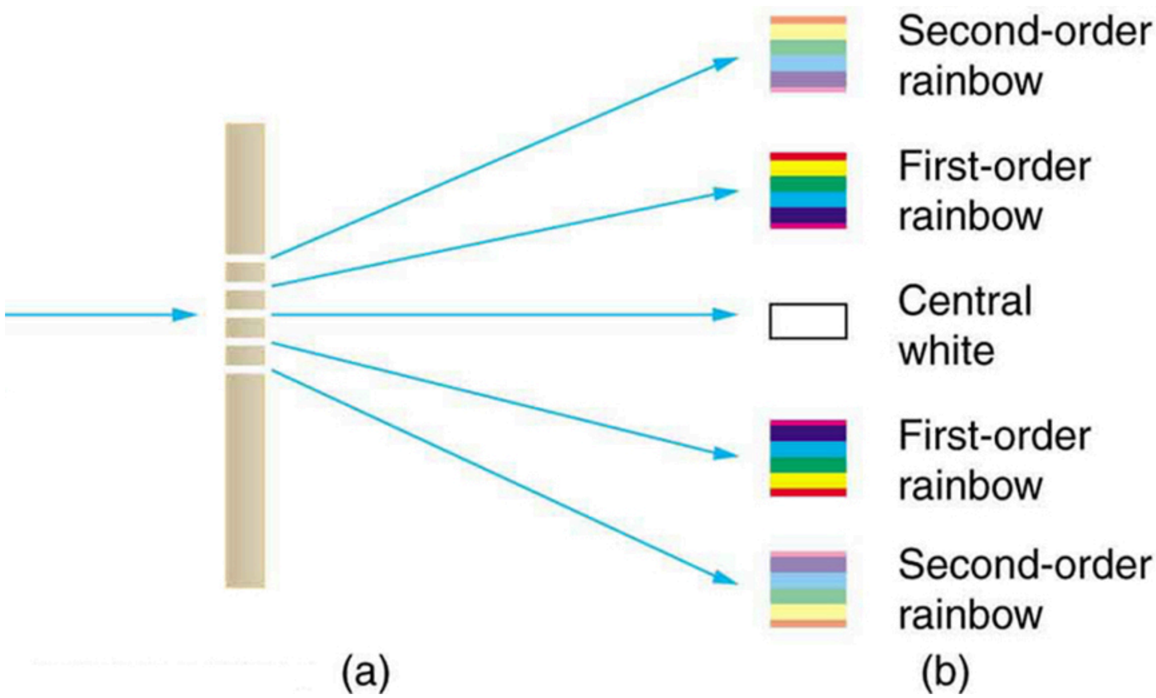
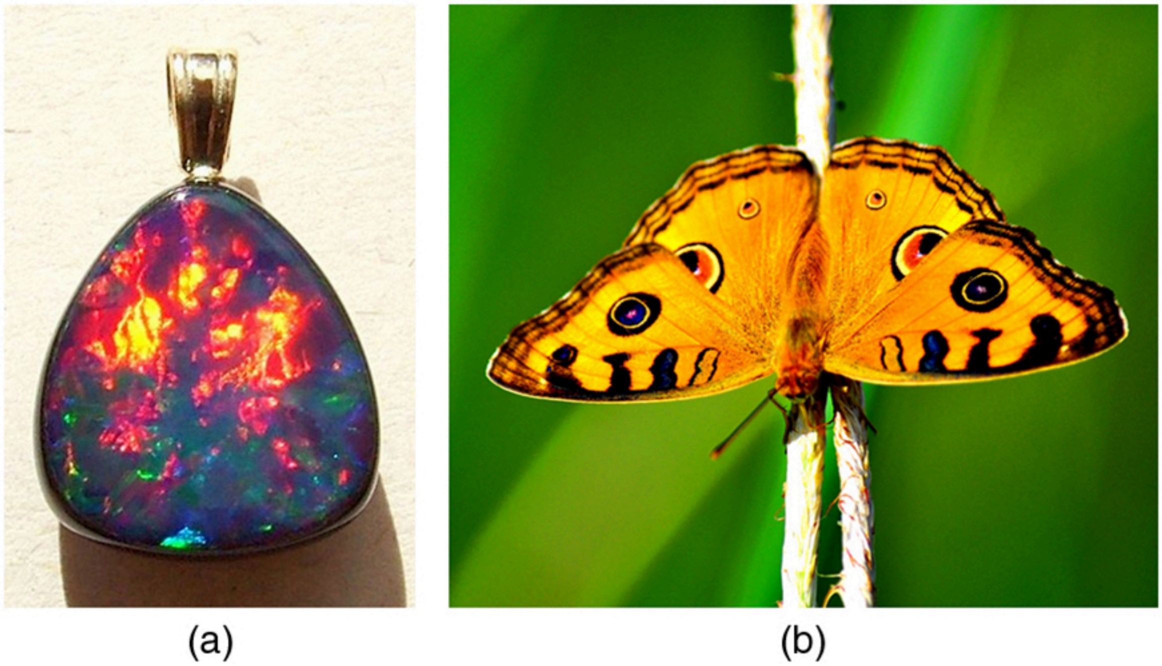
Multiple Slit Diffraction

- Discuss the pattern obtained from diffraction grating.
- Explain diffraction grating effects.

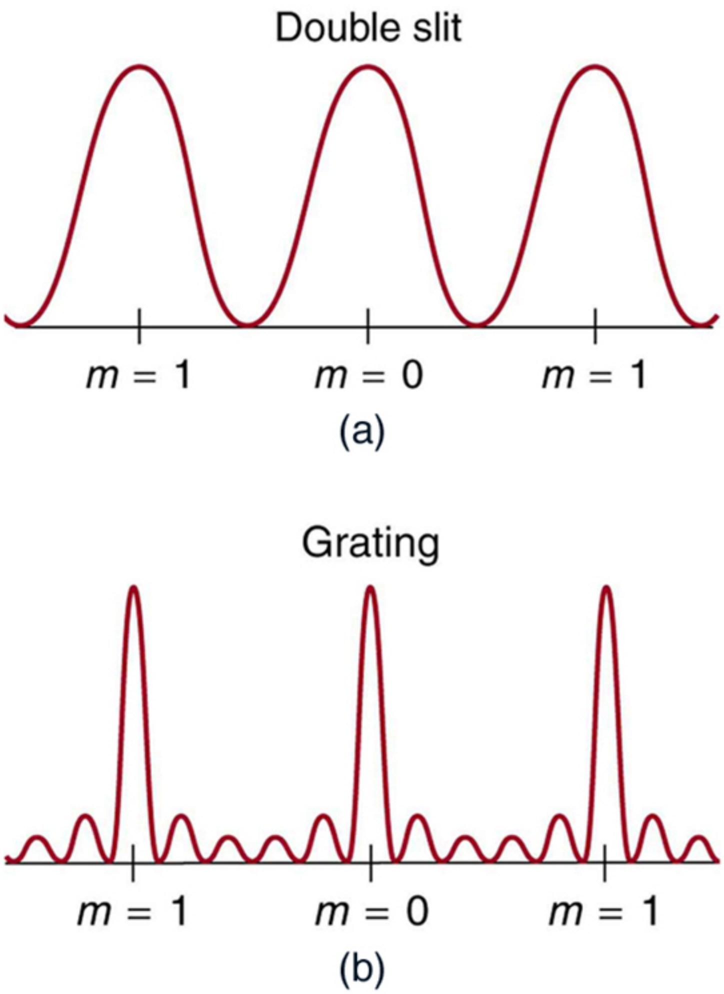
An interesting thing happens if you pass light through a large number of evenly spaced parallel slits, called a **diffraction grating**. An interference pattern is created that is very similar to the one formed by a double slit (see [Figure 1]). A diffraction grating can be manufactured by scratching glass with a sharp tool in a number of precisely positioned parallel lines, with the untouched regions acting like slits. These can be photographically mass produced rather cheaply. Diffraction gratings work both for transmission of light, as in [Figure 1], and for reflection of light, as on butterfly wings and the Australian opal in [Figure 2] or the CD pictured in the opening photograph of this chapter, [Figure 1]. In addition to their use as novelty items, diffraction gratings are commonly used for spectroscopic dispersion and analysis of light. What makes them particularly useful is the fact that they form a sharper pattern than double slits do. That is, their bright regions are narrower and brighter, while their dark regions are darker. [Figure 3] shows idealized graphs demonstrating the sharper pattern. Natural diffraction gratings occur in the feathers of certain birds. Tiny, finger-like structures in regular patterns act as reflection gratings, producing constructive interference that gives the feathers colors not solely due to their pigmentation. This is called iridescence.



A diffraction grating is a large number of evenly spaced parallel slits. (a) Light passing through is diffracted in a pattern similar to a double slit, with bright regions at various angles. (b) The pattern obtained for white light incident on a grating. The central maximum is white, and the higher-order maxima disperse white light into a rainbow of colors.



(a) This Australian opal and (b) the butterfly wings have rows of reflectors that act like reflection gratings, reflecting different colors at different angles. (credits: (a) Opals-On-Black.com, via Flickr (b) whologwhy, Flickr)

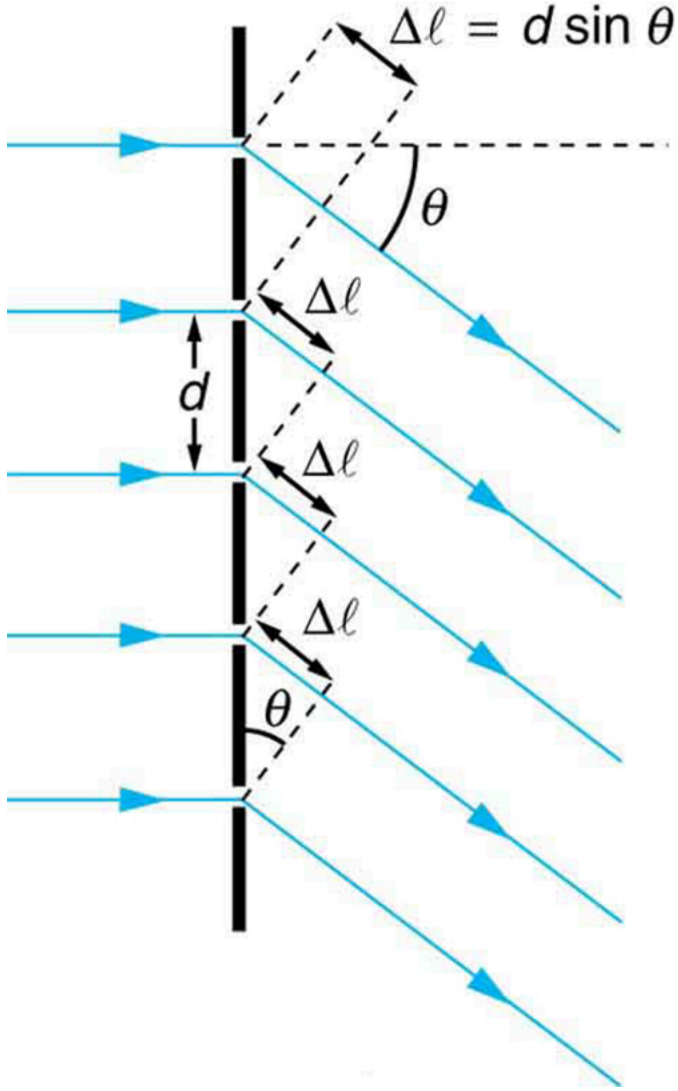


Idealized graphs of the intensity of light passing through a double slit (a) and a diffraction grating (b) for monochromatic light. Maxima can be produced at the same angles, but those for the diffraction grating are narrower and hence sharper. The maxima become narrower and the regions between darker as the number of slits is increased.

The analysis of a diffraction grating is very similar to that for a double slit (see [Figure 4]). As we know from our discussion of double slits in Young's Double Slit Experiment, light is diffracted by each slit and spreads out after passing through. Rays traveling in the same direction (at an angle θ relative to the incident direction) are shown in the figure. Each of these rays travels a different distance to a common point on a screen far away. The rays start in phase, and they can be in or out of phase when they reach a screen, depending on the difference in the path lengths traveled. As seen in the figure, each ray travels a distance $d \sin\theta$ different from that of its neighbor, where d is the distance between slits. If this distance equals an integral number of wavelengths, the rays all arrive in phase, and constructive interference (a maximum) is obtained. Thus, the condition necessary to obtain **constructive interference for a diffraction grating** is

$$d \sin\theta = m\lambda, \text{ for } m = 0, 1, -1, 2, -2, \ldots \text{(constructive)},$$

where d is the distance between slits in the grating, λ is the wavelength of light, and m is the order of the maximum. Note that this is exactly the same equation as for double slits separated by d . However, the slits are usually closer in diffraction gratings than in double slits, producing fewer maxima at larger angles.



Diffraction grating showing light rays from each slit traveling in the same direction. Each ray travels a different distance to reach a common point on a screen (not shown). Each ray travels a distance d$d\sin \theta$ different from that of its neighbor.

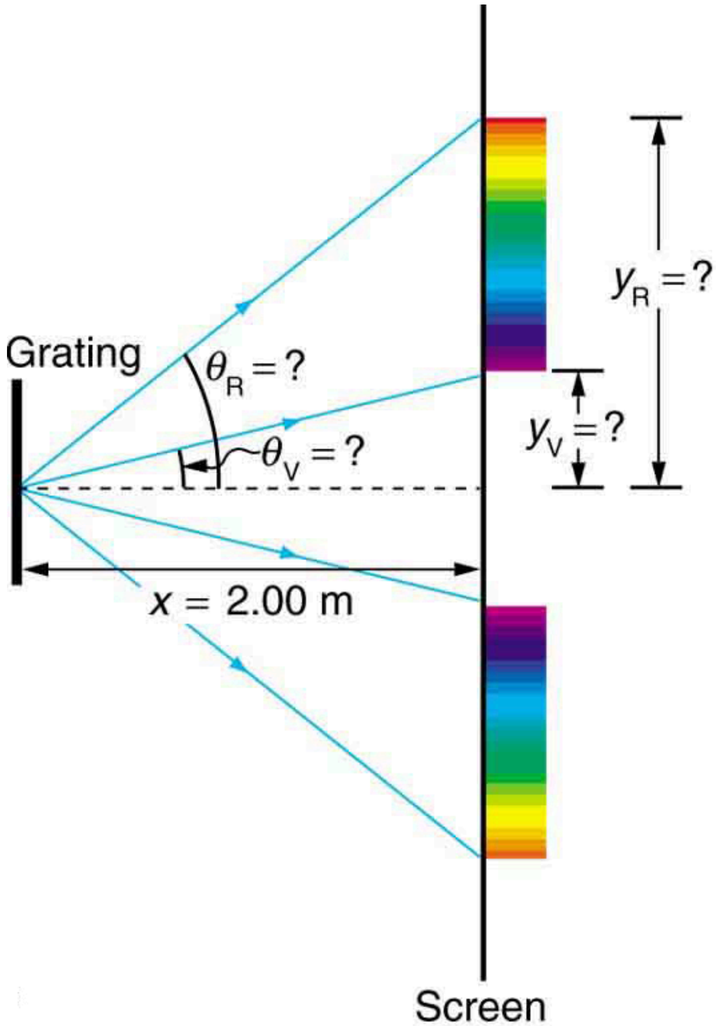
Where are diffraction gratings used? Diffraction gratings are key components of monochromators used, for example, in optical imaging of particular wavelengths from biological or medical samples. A diffraction grating can be chosen to specifically analyze a wavelength emitted by molecules in diseased cells in a biopsy sample or to help excite strategic molecules in the sample with a selected frequency of light. Another vital use is in optical fiber technologies where fibers are designed to provide optimum performance at specific wavelengths. A range of diffraction gratings are available for selecting specific wavelengths for such use.

Take-Home Experiment: Rainbows on a CD

The spacing d of the grooves in a CD or DVD can be well determined by using a laser and the equation $d \sin\theta = m\lambda$, for $m = 0, 1, -1, 2, -2, \ldots$. However, we can still make a good estimate of this spacing by using white light and the rainbow of colors that comes from the interference. Reflect sunlight from a CD onto a wall and use your best judgment of the location of a strongly diffracted color to find the separation d.

Calculating Typical Diffraction Grating Effects

Diffraction gratings with 10 000 lines per centimeter are readily available. Suppose you have one, and you send a beam of white light through it to a screen 2.00 m away. (a) Find the angles for the first-order diffraction of the shortest and longest wavelengths of visible light (380 and 760 nm). (b) What is the distance between the ends of the rainbow of visible light produced on the screen for first-order interference? (See [Figure 5].)



The diffraction grating considered in this example produces a rainbow of colors on a screen a distance $x=2.00 m$ from the grating. The distances along the screen are measured perpendicular to the x -direction. In other words, the rainbow pattern extends out of the page.

Strategy

The angles can be found using the equation

$$d \sin\theta = m\lambda \text{ (for } m=0,1,-1,2,-2,\ldots)$$

once a value for the slit spacing d has been determined. Since there are 10 000 lines per centimeter, each line is separated by $1/10000$ of a centimeter. Once the angles are found, the distances along the screen can be found using simple trigonometry.

Solution for (a)

The distance between slits is $d = (1 cm)/10000 = 1.00 \times 10^{-4} \text{cm}$ or $1.00 \times 10^{-6} \text{m}$. Let us call the two angles θ_V for violet (380 nm) and θ_R for red (760 nm). Solving the equation $d \sin\theta_V = m\lambda$ for $\sin\theta_V$,

$$\sin\theta_V = \frac{m\lambda_V}{d},$$

where $m = 1$ for first order and $\lambda_V = 380 \text{nm} = 3.80 \times 10^{-7} \text{m}$. Substituting these values gives

$$\sin\theta_V = \frac{3.80 \times 10^{-7} \text{m}}{1.00 \times 10^{-6} \text{m}} = 0.380.$$

Thus the angle θ_V is

$\theta_V = \sin^{-1} 0.380 = 22.33º.$

Similarly,

$\sin\theta_R = 7.60\times10^{-7} \text{m} 1.00\times10^{-6} \text{m}.$

Thus the angle θ_R is

$\theta_R = \sin^{-1} 0.760 = 49.46º.$

Notice that in both equations, we reported the results of these intermediate calculations to four significant figures to use with the calculation in part (b).

Solution for (b)

The distances on the screen are labeled y_V and y_R in [Figure 5]. Noting that $\tan\theta = y/x$, we can solve for y_V and y_R . That is,

$y_V = x\tan\theta_V = (2.00 m)(\tan 22.33º) = 0.815 \text{m}$

and

$y_R = x\tan\theta_R = (2.00 \text{m})(\tan 49.46º) = 2.338 \text{m}.$

The distance between them is therefore

$y_R - y_V = 1.52 m.$

Discussion

The large distance between the red and violet ends of the rainbow produced from the white light indicates the potential this diffraction grating has as a spectroscopic tool. The more it can spread out the wavelengths (greater dispersion), the more detail can be seen in a spectrum. This depends on the quality of the diffraction grating—it must be very precisely made in addition to having closely spaced lines.

Section Summary

- A diffraction grating is a large collection of evenly spaced parallel slits that produces an interference pattern similar to but sharper than that of a double slit.
- There is constructive interference for a diffraction grating when $d\sin\theta = m\lambda (\text{ for } m = 0, 1, -1, 2, -2, \ldots)$, where d is the distance between slits in the grating, λ is the wavelength of light, and m is the order of the maximum.

Conceptual Questions

What is the advantage of a diffraction grating over a double slit in dispersing light into a spectrum?

What are the advantages of a diffraction grating over a prism in dispersing light for spectral analysis?

Can the lines in a diffraction grating be too close together to be useful as a spectroscopic tool for visible light? If so, what type of EM radiation would the grating be suitable for? Explain.

If a beam of white light passes through a diffraction grating with vertical lines, the light is dispersed into rainbow colors on the right and left. If a glass prism disperses white light to the right into a rainbow, how does the sequence of colors compare with that produced on the right by a diffraction grating?

Suppose pure-wavelength light falls on a diffraction grating. What happens to the interference pattern if the same light falls on a grating that has more lines per centimeter? What happens to the interference pattern if a longer-wavelength light falls on the same grating? Explain how these two effects are consistent in terms of the relationship of wavelength to the distance between slits.

Suppose a feather appears green but has no green pigment. Explain in terms of diffraction.

It is possible that there is no minimum in the interference pattern of a single slit. Explain why. Is the same true of double slits and diffraction gratings?

Problems & Exercises

A diffraction grating has 2000 lines per centimeter. At what angle will the first-order maximum be for 520-nm-wavelength green light?

Show Solution

Strategy

For a diffraction grating, maxima occur at angles satisfying $d\sin\theta = m\lambda$, where d is the spacing between adjacent slits (grating constant), m is the order, and λ is the wavelength. First, calculate d from the line density, then solve for θ with m = 1.

Solution

Given:

- Line density: 2000 lines/cm
- Wavelength: $\lambda = 520\ \text{nm} = 520 \times 10^{-9}\ \text{m}$
- Order: $m = 1$ (first-order maximum)

Calculate the grating constant (slit separation):

$$d = \frac{1\ \text{cm}}{2000\ \text{lines}} = \frac{0.01\ \text{m}}{2000} = 5.00 \times 10^{-6}\ \text{m}$$

For the first-order maximum:

$$d \sin\theta = m\lambda$$
$$\sin\theta = \frac{m\lambda}{d} = \frac{(1)(520 \times 10^{-9})}{5.00 \times 10^{-6}} = \frac{520 \times 10^{-9}}{5.00 \times 10^{-6}}$$
$$\sin\theta = 0.104$$
$$\theta = \arcsin(0.104) = 5.97°$$

The first-order maximum for 520-nm green light occurs at 5.97°.

Discussion

The small angle (5.97°) for the first-order maximum indicates that the grating constant (5.00 μm) is about 10 times the wavelength (520 nm). Diffraction gratings with 2000 lines/cm are relatively coarse gratings. Finer gratings (e.g., 10,000 lines/cm) would produce larger diffraction angles and better spectral separation. The green wavelength (520 nm) falls in the middle of the visible spectrum. Higher orders (m = 2, 3, …) would appear at larger angles: the second order at about 11.96° and the third at about 18.05°.

Find the angle for the third-order maximum for 580-nm-wavelength yellow light falling on a diffraction grating having 1500 lines per centimeter.

Show Solution

Strategy

For a diffraction grating, we use $d \sin\theta = m\lambda$ where d is the distance between lines. First, find d from the line density, then solve for θ with m = 3.

Solution

Given:

- $\lambda = 580\ \text{nm} = 580 \times 10^{-9}\ \text{m}$
- Line density = 1500 lines/cm
- $m = 3$ (third-order maximum)

Step 1: Find the distance between lines

$$d = \frac{1\ \text{cm}}{1500\ \text{lines}} = \frac{0.01\ \text{m}}{1500} = 6.67 \times 10^{-6}\ \text{m}$$

Step 2: Find the angle

Using $d \sin\theta = m\lambda$:

$$\sin\theta = \frac{m\lambda}{d} = \frac{3 \times 580 \times 10^{-9}\ \text{m}}{6.67 \times 10^{-6}\ \text{m}}$$
$$\sin\theta = \frac{1740 \times 10^{-9}}{6.67 \times 10^{-6}} = 0.261$$
$$\theta = \sin^{-1}(0.261) = 15.1°$$

Discussion

The third-order maximum appears at 15.1°. With a grating spacing of 6.67 μm (about 11.5 wavelengths), this grating can produce several orders of maxima. We can check: the maximum order would be m_max = d/λ = 6.67/0.580 ≈ 11.5, so up to 11 orders are theoretically possible (though higher orders become very dim and widely separated).

How many lines per centimeter are there on a diffraction grating that gives a first-order maximum for 470-nm blue light at an angle of $25.0°$?

Show Solution

Strategy

Use $d \sin\theta = m\lambda$ to find the grating spacing d, then calculate the number of lines per centimeter as N = 1/d.

Solution

Given:

- λ=470 nm=470×10^{-9} m
- θ=25.0°
- m =1 (first-order maximum)

Step 1: Find the grating spacing

Using $d\sin\theta = m\lambda$:

$$d=\frac{m\lambda}{\sin\theta}=\frac{(1)(470\times10^{-9}\text{ m})}{\sin25.0°}$$
$$d=\frac{470\times10^{-9}}{0.4226}=1.112\times10^{-6}\text{ m}$$

Step 2: Find the number of lines per centimeter

$$N=\frac{1\text{ cm}}{d}=\frac{0.01\text{ m}}{1.112\times10^{-6}\text{ m}}=8.99\times10^{3}\text{ lines/cm}$$

Discussion

A grating with approximately 9000 lines/cm is a moderately fine diffraction grating. This spacing of about 1.11 μm (about 2.4 wavelengths of blue light) produces the first-order maximum at 25.0°, which is a convenient angle for observation. This is close to commercial gratings often available with 5000-10,000 lines/cm. Higher orders would appear at larger angles: the second order at about 57.1° and the third order at about 80.8°, with a maximum possible order of m_max = d/λ ≈ 2.4, meaning only orders 0, 1, and 2 would be fully observable.

What is the distance between lines on a diffraction grating that produces a second-order maximum for 760-nm red light at an angle of 60.0°?

Show Solution

Strategy

Use $d\sin\theta = m\lambda$ with m = 2 (second-order) to solve for the line spacing d.

Solution

Given:

- λ=760 nm=760×10^{-9} m
- θ=60.0°
- m =2 (second-order maximum)

Using $d\sin\theta = m\lambda$:

$$d=\frac{m\lambda}{\sin\theta}=\frac{2\times760\times10^{-9}\text{ m}}{\sin60.0°}$$
$$d=\frac{1520\times10^{-9}}{0.8660}=1.76\times10^{-6}\text{ m}=1.76\text{ μm}$$

Discussion

The line spacing of 1.76 μm is very small - just over twice the wavelength of red light. This close spacing is necessary to produce the second-order maximum at the relatively large angle of 60°.

We can verify this makes sense: if the second-order maximum is at 60°, the first-order would be at θ_1 where sin θ_1 = (1)(760)/(1.76 × 10³) = 0.432, giving θ_1 = 25.6°. This seems reasonable.

For this grating, the maximum order observable would be: m_max = d/λ = 1.76/0.76 ≈ 2.3, so only second-order maxima (and possibly a dim third-order) would be observable before reaching 90°.

Calculate the wavelength of light that has its second-order maximum at 45.0° when falling on a diffraction grating that has 5000 lines per centimeter.

Show Solution

Strategy

Use $d\sin\theta = m\lambda$ with m = 2 (second-order) to solve for the wavelength λ. First, find the grating spacing d from the line density.

Solution

Given:

- Line density: 5000 lines/cm
- θ=45.0°

- $m = 2$ (second-order maximum)

Step 1: Find the grating spacing

$$d = \frac{1\text{ cm}}{5000\text{ lines}} = \frac{0.01\text{ m}}{5000} = 2.00 \times 10^{-6}\text{ m}$$

Step 2: Find the wavelength

Using $d \sin\theta = m\lambda$:

$$\lambda = \frac{d \sin\theta}{m} = \frac{(2.00 \times 10^{-6}\text{ m}) \sin 45.0°}{2}$$
$$\lambda = \frac{(2.00 \times 10^{-6})(0.7071)}{2} = \frac{1.414 \times 10^{-6}}{2}$$
$$\lambda = 7.07 \times 10^{-7}\text{ m} = 707\text{ nm}$$

Discussion

The wavelength of 707 nm falls in the red region of the visible spectrum (near the long-wavelength end). This is close to deep red light. The second-order maximum appears at 45.0°, which is a reasonably large angle, making it easy to observe and measure.

We can verify this is reasonable: the first-order maximum would appear at an angle where $\sin \theta_1 = \lambda/d = 707/2000 = 0.354$, giving $\theta_1 = 20.7°$. The third-order would be at $\sin \theta_3 = 3\lambda/d = 2121/2000 = 1.06 > 1$, which is impossible, so no third-order maximum exists. This grating can show at most the zeroth, first, and second orders for this wavelength.

An electric current through hydrogen gas produces several distinct wavelengths of visible light. What are the wavelengths of the hydrogen spectrum, if they form first-order maxima at angles of $24.2°$, $25.7°$, $29.1°$, and $41.0°$ when projected on a diffraction grating having 10 000 lines per centimeter? Explicitly show how you follow the steps in [Problem-Solving Strategies for Wave Optics]

Show Solution

Strategy

Following the Problem-Solving Strategies for Wave Optics:

Step 1: This involves a diffraction grating. **Step 2:** We use $d \sin\theta = m\lambda$ for constructive interference. **Step 3:** We need to find the wavelengths λ for each angle. **Step 4:** Given: 10,000 lines/cm, m = 1, four angles. **Step 5:** First find d, then solve $\lambda = d \sin\theta / m$ for each angle.

Solution

Find the grating spacing:

$$d = \frac{1\text{ cm}}{10{,}000\text{ lines}} = \frac{0.01\text{ m}}{10{,}000} = 1.00 \times 10^{-6}\text{ m}$$

For each angle, calculate λ using $\lambda = \frac{d \sin\theta}{m}$ with m = 1:

Angle 1: θ = 24.2°

$$\lambda_1 = d \sin\theta = (1.00 \times 10^{-6}) \sin 24.2° = (1.00 \times 10^{-6})(0.4101) = 410\text{ nm (violet)}$$

Angle 2: θ = 25.7°

$$\lambda_2 = (1.00 \times 10^{-6}) \sin 25.7° = (1.00 \times 10^{-6})(0.4337) = 434\text{ nm (violet-blue)}$$

Angle 3: θ = 29.1°

$$\lambda_3 = (1.00 \times 10^{-6}) \sin 29.1° = (1.00 \times 10^{-6})(0.4861) = 486\text{ nm (blue-green)}$$

Angle 4: θ = 41.0°

$$\lambda_4 = (1.00 \times 10^{-6}) \sin 41.0° = (1.00 \times 10^{-6})(0.6561) = 656\text{ nm (red)}$$

Step 6: All results are in the visible range (380-760 nm), which is reasonable for hydrogen emission lines.

Discussion

These wavelengths correspond to prominent lines in the hydrogen Balmer series:

- **410 nm** (H-delta): violet
- **434 nm** (H-gamma): violet-blue
- **486 nm** (H-beta): blue-green
- **656 nm** (H-alpha): red

The H-alpha line at 656 nm is the brightest and most famous line in the visible hydrogen spectrum. These emission lines result from electrons transitioning from higher energy levels (n = 3, 4, 5, 6) down to n = 2. This spectral “fingerprint” is characteristic of hydrogen and is observed in stars, nebulae, and laboratory hydrogen discharge tubes. The precision of diffraction gratings makes them invaluable tools for spectroscopy and identifying elements.

(a) What do the four angles in the above problem become if a 5000-line-per-centimeter diffraction grating is used? (b) Using this grating, what would the angles be for the second-order maxima? (c) Discuss the relationship between integral reductions in lines per centimeter and the new angles of various order maxima.

Show Solution

Strategy

Use the four hydrogen wavelengths from the previous problem (410, 434, 486, and 656 nm) with the new grating. For part (a), find first-order angles with the 5000 lines/cm grating. For part (b), find second-order angles. For part (c), compare how changing the grating spacing by a factor affects the angles.

Solution

The four hydrogen wavelengths are: 410 nm, 434 nm, 486 nm, and 656 nm.

For the 5000-line-per-centimeter grating:

$$d=\frac{1\text{ cm}}{5000}=\frac{0.01\text{ m}}{5000}=2.00\times10^{-6}\text{ m}$$

(a) First-order angles with 5000 lines/cm grating:

Using $\sin\theta=\frac{m\lambda}{d}$ with m = 1:

For λ = 410 nm:

$$\sin\theta_1=\frac{410\times10^{-9}}{2.00\times10^{-6}}=0.205\Rightarrow\theta_1=11.8°$$

For λ = 434 nm:

$$\sin\theta_2=\frac{434\times10^{-9}}{2.00\times10^{-6}}=0.217\Rightarrow\theta_2=12.5°$$

For λ = 486 nm:

$$\sin\theta_3=\frac{486\times10^{-9}}{2.00\times10^{-6}}=0.243\Rightarrow\theta_3=14.1°$$

For λ = 656 nm:

$$\sin\theta_4=\frac{656\times10^{-9}}{2.00\times10^{-6}}=0.328\Rightarrow\theta_4=19.2°$$

First-order angles: 11.8°, 12.5°, 14.1°, 19.2°

(b) Second-order angles with 5000 lines/cm grating:

Using $\sin\theta=\frac{m\lambda}{d}$ with m = 2:

For λ = 410 nm:

$$\sin\theta_1=\frac{(2)(410\times10^{-9})}{2.00\times10^{-6}}=0.410\Rightarrow\theta_1=24.2°$$

For λ = 434 nm:

$$\sin\theta_2=\frac{(2)(434\times10^{-9})}{2.00\times10^{-6}}=0.434\Rightarrow\theta_2=25.7°$$

For λ = 486 nm:

$$\sin\theta_3=\frac{(2)(486\times10^{-9})}{2.00\times10^{-6}}=0.486\Rightarrow\theta_3=29.1°$$

For λ = 656 nm:

$$\sin\theta_4=\frac{(2)(656\times10^{-9})}{2.00\times10^{-6}}=0.656\Rightarrow\theta_4=41.0°$$

Second-order angles: 24.2°, 25.7°, 29.1°, 41.0°

(c) Relationship between line density and angles:

Notice that the second-order angles for the 5000 lines/cm grating (24.2°, 25.7°, 29.1°, 41.0°) are exactly the same as the first-order angles for the 10,000 lines/cm grating from the previous problem!

This demonstrates a general principle: **When you decrease the number of lines per centimeter by a factor of n (thereby increasing d by a factor of n), the angles for the n-th order maximum with the new grating equal the angles for the first-order maximum with the original grating.**

Mathematically: If $d_2 = n \cdot d_1$, then $\sin\theta_{m=1,d_1} = \frac{\lambda}{d_1}$ equals $\sin\theta_{m=n,d_2} = \frac{n\lambda}{n \cdot d_1} = \frac{\lambda}{d_1}$.

In this case, halving the line density (10,000 → 5000 lines/cm) means doubling d, so the second-order (n=2) angles with the coarser grating equal the first-order angles with the finer grating.

Discussion

This scaling relationship is very useful in diffraction grating design. It shows that using a coarser grating with higher-order maxima can produce the same angular positions as using a finer grating with lower-order maxima. However, higher orders are generally dimmer and may suffer from overlapping spectra from different orders. The 5000 lines/cm grating produces more widely spaced orders, making them easier to observe separately, while the 10,000 lines/cm grating provides better resolution within each order.

What is the maximum number of lines per centimeter a diffraction grating can have and produce a complete first-order spectrum for visible light?

Show Solution

Strategy

For a complete first-order spectrum, both the shortest (violet, ~380 nm) and longest (red, ~760 nm) wavelengths of visible light must have first-order maxima at angles less than 90°. The limiting condition is for the longest wavelength (red) at θ = 90°.

Solution

For the complete visible spectrum to be observable in first order, the red end (longest wavelength) must appear before 90°. Using $d\sin\theta = m\lambda$ with m = 1 and $\theta = 90°$ (where $\sin 90° = 1$):

$$d = m\lambda_{\text{max}} = (1)(760\text{ nm}) = 760\text{ nm} = 7.60 \times 10^{-7}\text{ m}$$

This is the **minimum** spacing needed. The maximum number of lines per centimeter is:

$$N = \frac{1\text{ cm}}{d} = \frac{0.01\text{ m}}{7.60 \times 10^{-7}\text{ m}} = 13{,}158\text{ lines/cm}$$

Rounding to appropriate significant figures:

$$N_{\text{max}} \approx 13{,}200\text{ lines/cm}$$

Discussion

If the grating had more than ~13,200 lines/cm, the line spacing would be smaller than 760 nm, and the first-order maximum for red light would require $\sin\theta > 1$, which is impossible. The red end of the spectrum would not be observable in first order.

Let’s verify: with d = 760 nm:

- **Red (760 nm):** sin θ = (1)(760)/760 = 1.00 → θ = 90° (just barely observable)
- **Violet (380 nm):** sin θ = (1)(380)/760 = 0.50 → θ = 30° (easily observable)

This grating would spread the complete visible spectrum from 30° to 90° in first order. Commercial gratings often have fewer lines/cm (~5000-10,000) to ensure good visibility of the complete spectrum at more convenient angles.

The yellow light from a sodium vapor lamp *seems* to be of pure wavelength, but it produces two first-order maxima at $36.093°$ and $36.129°$ when projected on a 10 000 line per centimeter diffraction grating. What are the two wavelengths to an accuracy of 0.1 nm?

Show Solution

Strategy

Use $d\sin\theta = m\lambda$ to find each wavelength from its angle. First, calculate the grating spacing d from the line density, then solve for λ at each angle.

Solution

Given:

- Line density: 10,000 lines/cm
- $\theta_1 = 36.093°$
- $\theta_2 = 36.129°$
- $m = 1$ (first-order maxima)

Step 1: Find the grating spacing

$d = \frac{1 \text{ cm}}{10{,}000 \text{ lines}} = \frac{0.01 \text{ m}}{10{,}000} = 1.00\times10^{-6} \text{ m}$

Step 2: Find the first wavelength

Using $\lambda = d\sin\theta/m$:

$$\lambda_1 = d\sin\theta_1 = (1.00\times10^{-6} \text{ m})\sin 36.093°$$
$$\lambda_1 = (1.00\times10^{-6})(0.58906) = 5.891\times10^{-7} \text{ m} = 589.1 \text{ nm}$$

Step 3: Find the second wavelength

$$\lambda_2 = d\sin\theta_2 = (1.00\times10^{-6} \text{ m})\sin 36.129°$$
$$\lambda_2 = (1.00\times10^{-6})(0.58963) = 5.896\times10^{-7} \text{ m} = 589.6 \text{ nm}$$

Discussion

These two wavelengths, 589.1 nm and 589.6 nm, form the famous **sodium D-line doublet**. The two lines are separated by only 0.5 nm (or about 6 nm), which is why sodium light appears to be pure yellow to the eye - the two wavelengths are too close for the eye to distinguish.

The doublet arises from the fine structure of sodium's electron energy levels: transitions from two closely spaced upper states ($^2P_3/_2$ and $^2P_1/_2$) to the same ground state ($^2S_1/_2$) produce the D_2 line (589.0 nm) and D_1 line (589.6 nm).

This problem demonstrates the high resolving power of diffraction gratings. The angular separation is only 0.036°, yet the grating cleanly separates the two lines. The ability to resolve such closely spaced wavelengths is crucial for spectroscopic analysis in astronomy, chemistry, and physics.

What is the spacing between structures in a feather that acts as a reflection grating, given that they produce a first-order maximum for 525-nm light at a 30.0° angle?

Show Solution

Strategy

For a reflection grating, we use the same equation as for a transmission grating: $d\sin\theta = m\lambda$. We solve for d with m = 1.

Solution

Given:

- $\lambda = 525 \text{ nm} = 525\times10^{-9} \text{ m}$
- $\theta = 30.0°$
- $m = 1$ (first-order maximum)

Using $d\sin\theta = m\lambda$:

$$d = \frac{m\lambda}{\sin\theta} = \frac{(1)(525\times10^{-9} \text{ m})}{\sin 30.0°}$$
$$d = \frac{525\times10^{-9}}{0.500} = 1.05\times10^{-6} \text{ m} = 1.05 \text{ μm}$$

Discussion

The spacing of 1.05 μm (1050 nm) is about twice the wavelength of the green light being diffracted. This is typical for structural coloration in bird feathers and insect wings.

The iridescent colors in peacock feathers, hummingbird feathers, and butterfly wings are produced by such natural diffraction gratings. The spacing is created by regular arrangements of microscopic structures like barbules (in feathers) or scales (in butterfly wings). The color changes with viewing angle because different angles produce constructive interference for different wavelengths according to $d\sin\theta = m\lambda$.

This biological "engineering" produces brilliant colors without pigments - the colors are purely structural and result from wave interference. These colors often don't fade over time like pigment-based colors do, which is why museum specimens of iridescent birds and butterflies remain colorful for centuries.

Structures on a bird feather act like a reflection grating having 8000 lines per centimeter. What is the angle of the first-order maximum for 600-nm light?

Show Solution

Strategy

Use $d\sin\theta = m\lambda$ with m = 1 to find the angle. First, calculate the spacing d from the line density, then solve for θ.

Solution

Given:

- Line density: 8000 lines/cm
- $\lambda = 600\text{ nm} = 600 \times 10^{-9}\text{ m}$
- $m = 1$ (first-order maximum)

Step 1: Find the structure spacing

$$d = \frac{1\text{ cm}}{8000\text{ lines}} = \frac{0.01\text{ m}}{8000} = 1.25 \times 10^{-6}\text{ m}$$

Step 2: Find the angle

Using $d \sin\theta = m\lambda$:

$$\sin\theta = \frac{m\lambda}{d} = \frac{(1)(600 \times 10^{-9}\text{ m})}{1.25 \times 10^{-6}\text{ m}}$$
$$\sin\theta = \frac{600}{1250} = 0.480$$
$$\theta = \sin^{-1}(0.480) = 28.7°$$

Discussion

The first-order maximum for orange-red light (600 nm) appears at 28.7° from the normal. This angle is typical for structural coloration in bird feathers. The spacing of 1.25 μm is roughly twice the wavelength, which is ideal for producing visible diffraction patterns.

Birds with iridescent feathers (like hummingbirds, peacocks, and starlings) have microscopic structures on their feather barbules that act as natural diffraction gratings. As the bird moves and the viewing angle changes, different wavelengths satisfy the constructive interference condition, causing the brilliant color shifts characteristic of iridescence. For this feather, shorter wavelengths (blue, violet) would appear at smaller angles, while longer wavelengths (red) would appear at larger angles, creating a rainbow effect with changing perspective.

An opal such as that shown in [Figure 2] acts like a reflection grating with rows separated by about $8 \mu\text{m}$. If the opal is illuminated normally, (a) at what angle will red light be seen and (b) at what angle will blue light be seen?

Show Solution

Strategy

Use $d \sin\theta = m\lambda$ for the first-order maximum (m = 1) with d = 8 μm. Use λ ≈ 700 nm for red and λ ≈ 450 nm for blue.

Solution

Given:

- $d = 8\ \mu\text{m} = 8 \times 10^{-6}\text{ m}$
- $m = 1$ (first-order, typically the brightest)

(a) Red light (λ ≈ 700 nm):

$$\sin\theta_{\text{red}} = \frac{m\lambda}{d} = \frac{(1)(700 \times 10^{-9}\text{ m})}{8 \times 10^{-6}\text{ m}} = 0.0875$$
$$\theta_{\text{red}} = \sin^{-1}(0.0875) = 5.0°$$

(b) Blue light (λ ≈ 450 nm):

$$\sin\theta_{\text{blue}} = \frac{(1)(450 \times 10^{-9}\text{ m})}{8 \times 10^{-6}\text{ m}} = 0.0563$$
$$\theta_{\text{blue}} = \sin^{-1}(0.0563) = 3.2°$$

Discussion

The diffraction angles are quite small (3.2° for blue, 5.0° for red) because the grating spacing (8 μm) is much larger than the wavelengths of visible light. Red light appears at a larger angle than blue light, which is why when you rotate an opal, you see different colors from different angles - this is the source of opal's famous "play of color" or iridescence.

The complete visible spectrum would spread from about 3° (violet) to 5° (red). Higher orders (m = 2, 3, etc.) would appear at larger angles but with decreasing intensity. The maximum order possible is m_max = d/λ_max ≈ 8000/700 ≈ 11, so many orders are theoretically possible, though in practice only the first few are bright enough to see.

Opals contain tiny silica spheres arranged in regular patterns, creating these natural diffraction gratings. The spacing of 8 μm is roughly 10-20 wavelengths of visible light, perfect for producing brilliant iridescent colors.

At what angle does a diffraction grating produces a second-order maximum for light having a first-order maximum at $20.0°$?

Show Solution

Strategy

Use $d \sin\theta = m\lambda$ for both orders. From the first-order angle, find the ratio λ/d, then use this to find the second-order angle.

Solution

Given:

- $\theta_1 = 20.0\,°$ (first-order maximum)
- Find: θ_2 (second-order maximum)

Step 1: Find the ratio λ/d from the first-order condition

For the first-order maximum (m = 1):

$$d \sin\theta_1 = (1)\lambda$$
$$\frac{\lambda}{d} = \sin\theta_1 = \sin 20.0° = 0.342$$

Step 2: Find the second-order angle

For the second-order maximum (m = 2):

$$d \sin\theta_2 = 2\lambda$$
$$\sin\theta_2 = 2\frac{\lambda}{d} = 2 \times 0.342 = 0.684$$
$$\theta_2 = \sin^{-1}(0.684) = 43.2°$$

Discussion

The second-order maximum appears at 43.2°, more than twice the first-order angle (20.0°). This is because the sine function is nonlinear - while sin θ doubles from first to second order, the angle itself more than doubles.

This result is general: for any diffraction grating, if the first-order maximum is at angle θ_1, the second-order maximum is at $\theta_2 = \sin^{-1}(2 \sin \theta_1)$. Notice that the second-order angle exists only if $2 \sin \theta_1 \leq 1$, which requires $\theta_1 \leq 30°$. Since 20° < 30°, the second-order maximum is observable. If θ_1 were greater than 30°, no second-order maximum would exist (consistent with our earlier proof).

Show that a diffraction grating cannot produce a second-order maximum for a given wavelength of light unless the first-order maximum is at an angle less than $30.0\,°$.

Show Solution

Strategy

We use $d \sin\theta = m\lambda$ for both first and second orders. For the second-order to exist, it must occur at θ < 90°.

Solution

For the **first-order maximum** (m = 1):

$$d \sin\theta_1 = \lambda$$

For the **second-order maximum** (m = 2) to be observable, it must occur at some angle $\theta_2 < 90°$. At the limiting case where $\theta_2 = 90°$:

$$d \sin\theta_2 = 2\lambda$$

With $\sin\theta_2 = \sin 90\,° = 1$:

$$d = 2\lambda$$

Now substitute this back into the first-order equation:

$$2\lambda \sin\theta_1 = \lambda$$
$$\sin\theta_1 = \frac{1}{2} = 0.500$$
$$\theta_1 = \sin^{-1}(0.500) = 30.0°$$

Therefore: If $\theta_1 = 30.0°$, the second-order maximum just barely appears at $\theta_2 = 90°$. If $\theta_1 > 30.0°$, then $d < 2\lambda$, which would require $\sin\theta_2 > 1$ for the second-order maximum - which is impossible.

Conclusion: A second-order maximum can only exist if the first-order maximum appears at $\theta_1 < 30.0°$. **Q.E.D.**

Discussion

This result has important practical implications for diffraction grating design. If you observe that a first-order maximum appears at an angle greater than 30°, you immediately know that:

1. No second-order maximum exists for that wavelength
2. The grating spacing is less than twice the wavelength ($d < 2\lambda$)
3. At most, only first-order maxima will be observable (plus the zero-order central maximum)

For example, if violet light (380 nm) produces its first maximum at 35°, the grating spacing is only about 600 nm, and no higher orders can be observed for visible light.

If a diffraction grating produces a first-order maximum for the shortest wavelength of visible light at $30.0\,°$, at what angle will the first-order maximum be for the longest wavelength of visible light?

Show Solution

Strategy

Use $d\sin\theta = m\lambda$ for both wavelengths. Find d from the violet condition, then use it to find the angle for red light.

Solution

Given:

- Shortest visible wavelength: $\lambda_V = 380\text{ nm}$ (violet)
- Longest visible wavelength: $\lambda_R = 760\text{ nm}$ (red)
- $\theta_V = 30.0\,°$ for violet light
- $m = 1$ (first-order maximum)

Step 1: Find the grating spacing from the violet condition

$$d\sin\theta_V = m\lambda_V$$
$$d = \frac{m\lambda_V}{\sin\theta_V} = \frac{(1)(380\text{ nm})}{\sin 30.0°} = \frac{380}{0.500} = 760\text{ nm}$$

Step 2: Find the angle for red light

$$\sin\theta_R = \frac{m\lambda_R}{d} = \frac{(1)(760\text{ nm})}{760\text{ nm}} = 1.00$$
$$\theta_R = \sin^{-1}(1.00) = 90.0°$$

Discussion

The remarkable result is that the red end of the visible spectrum appears at exactly 90°! This is the limiting case where the first-order maximum just barely exists.

Notice that the grating spacing d = 760 nm exactly equals the wavelength of red light. This is the critical condition: when d = λ_max, the longest wavelength appears at 90°. If the grating had even slightly finer spacing (d < 760 nm), red light would not produce a first-order maximum at all.

This grating with d = 760 nm produces a complete visible spectrum spreading from 30° (violet) to 90° (red) in first order. The complete visible spectrum is compressed into a 60° angular range. This is actually the finest-spacing grating that can show the complete visible spectrum in first order - any finer and the red end would be cut off.

(a) Find the maximum number of lines per centimeter a diffraction grating can have and produce a maximum for the smallest wavelength of visible light. (b) Would such a grating be useful for ultraviolet spectra? (c) For infrared spectra?

Show Solution

Strategy

For part (a), the smallest visible wavelength (violet, ~380 nm) must produce at least a first-order maximum before θ = 90°. For parts (b) and (c), we check if UV and IR wavelengths can produce maxima with this grating spacing.

Solution

(a) Maximum lines per centimeter:

The smallest visible wavelength is approximately λ = 380 nm (violet). For a first-order maximum to just barely exist at θ = 90°:

$$d\sin 90° = (1)\lambda$$
$$d = \lambda = 380\text{ nm} = 3.80\times10^{-7}\text{ m}$$

Maximum number of lines per centimeter:

$$N_{\text{max}} = \frac{1\text{ cm}}{d} = \frac{0.01\text{ m}}{3.80\times10^{-7}\text{ m}} = 26{,}316\text{ lines/cm}$$

Rounding: **N ≈ 26,300 lines/cm**

(b) Useful for ultraviolet spectra?

UV wavelengths are shorter than visible light (λ < 380 nm). For example, UV-A is around 315-380 nm.

Since d = 380 nm and λ_UV < 380 nm, we have d > λ_UV, so:

$$\sin\theta = \frac{m\lambda_{\mathrm{UV}}}{d} < \frac{m \times 380}{380} = m$$

For first order (m = 1), sin θ < 1, so maxima will exist for UV light.

Yes, this grating would be useful for UV spectra. In fact, UV wavelengths would produce maxima at smaller angles than violet light, making them easier to observe.

(c) Useful for infrared spectra?

IR wavelengths are longer than visible light. For example, near-IR is around 760 nm - 2500 nm.

For λ_IR = 760 nm with d = 380 nm:

$$\sin\theta_1 = \frac{(1)(760\ \mathrm{nm})}{380\ \mathrm{nm}} = 2$$

Since sin θ cannot exceed 1, **no first-order maximum exists** for 760 nm IR light.

No, this grating would NOT be useful for infrared spectra because d < λ for all IR wavelengths, making it impossible to produce even first-order maxima.

Discussion

This problem illustrates an important principle: gratings with very fine spacing (high line density) are excellent for short wavelengths (UV, violet) but useless for long wavelengths (IR, red). The ideal grating depends on the wavelength range of interest. For IR spectroscopy, gratings with much wider spacing (fewer lines/cm) are needed.

(a) Show that a 30 000-line-per-centimeter grating will not produce a maximum for visible light. (b) What is the longest wavelength for which it does produce a first-order maximum? (c) What is the greatest number of lines per centimeter a diffraction grating can have and produce a complete second-order spectrum for visible light?

Show Solution

Strategy

For part (a), find the grating spacing d, then determine the maximum wavelength that can produce a first-order maximum (at θ = 90°). For part (b), this maximum wavelength is the answer. For part (c), the longest visible wavelength (760 nm) must produce a second-order maximum at θ < 90°.

Solution

(a) Show that 30,000 lines/cm won't produce visible light maxima:

First, find the grating spacing:

$$d = \frac{1\ \mathrm{cm}}{30{,}000\ \mathrm{lines}} = \frac{0.01\ \mathrm{m}}{30{,}000} = 3.333 \times 10^{-7}\ \mathrm{m} = 333.3\ \mathrm{nm}$$

For a first-order maximum to exist, we need θ ≤ 90°. At the limiting case (θ = 90°):

$$d \sin 90° = (1)\lambda_{\mathrm{max}}$$
$$\lambda_{\mathrm{max}} = d = 333.3\ \mathrm{nm}$$

Since the visible spectrum ranges from 380 nm (violet) to 760 nm (red), and the maximum wavelength this grating can diffract is 333.3 nm (which is in the ultraviolet range), **no visible light wavelengths can produce even a first-order maximum**. All visible wavelengths exceed the grating spacing, making sin θ > 1, which is impossible.

(b) Longest wavelength for first-order maximum:

From part (a), the longest wavelength that can produce a first-order maximum is:

$$\lambda_{\mathrm{max}} = d = 333\ \mathrm{nm}$$

This is in the **ultraviolet (UV) range**.

(c) Greatest number of lines/cm for complete second-order visible spectrum:

For a complete second-order spectrum, the longest visible wavelength (760 nm, red) must have its second-order maximum at θ ≤ 90°. At the limiting case:

$$d \sin 90° = m\lambda_{\mathrm{red}}$$

With m = 2 and λ_red = 760 nm:

$$d = 2 \times 760 \text{ nm} = 1520 \text{ nm} = 1.520 \times 10^{-6} \text{ m}$$

Maximum number of lines per centimeter:

$$N_{\text{max}} = \frac{1 \text{ cm}}{d} = \frac{0.01 \text{ m}}{1.520 \times 10^{-6} \text{ m}} = 6579 \text{ lines/cm}$$

Rounding to three significant figures: **N ≈ 6.58 × 10^3 lines/cm**

Discussion

Part (a) demonstrates an important limitation: extremely fine gratings (like 30,000 lines/cm with d = 333 nm) cannot diffract visible light at all because the wavelengths exceed the grating spacing. Such fine gratings are useful only for short-wavelength radiation like UV or X-rays.

For part (c), the result shows that to observe a complete second-order visible spectrum, the grating cannot have more than about 6,580 lines/cm. If it had more lines, the red end of the spectrum wouldn't appear in second order. This is roughly half the maximum for a first-order spectrum (~13,200 lines/cm), which makes sense because second-order requires twice the path difference (2λ instead of λ).

Practical commercial gratings typically have 300-10,000 lines/cm, comfortably within the range to show complete visible spectra in both first and second orders.

A He–Ne laser beam is reflected from the surface of a CD onto a wall. The brightest spot is the reflected beam at an angle equal to the angle of incidence. However, fringes are also observed. If the wall is 1.50 m from the CD, and the first fringe is 0.600 m from the central maximum, what is the spacing of grooves on the CD?

Show Solution

Strategy

The CD acts as a reflection grating. We use $d\sin\theta = m\lambda$ with m = 1 for the first fringe. First, find the angle θ from the geometry, then solve for d. He-Ne laser wavelength is 633 nm.

Solution

Given:

- Distance to wall: L = 1.50 m
- Position of first fringe: y = 0.600 m
- λ = 633 nm (He-Ne laser)
- m = 1 (first-order maximum)

Step 1: Find the angle

From geometry:

$$\tan\theta = \frac{y}{L} = \frac{0.600 \text{ m}}{1.50 \text{ m}} = 0.400$$
$$\theta = \tan^{-1}(0.400) = 21.8°$$
$$\sin\theta = \sin 21.8° = 0.371$$

Step 2: Find the groove spacing

Using $d\sin\theta = m\lambda$:

$$d = \frac{m\lambda}{\sin\theta} = \frac{(1)(633 \times 10^{-9} \text{ m})}{0.371}$$
$$d = 1.71 \times 10^{-6} \text{ m} = 1.71 \text{ μm}$$

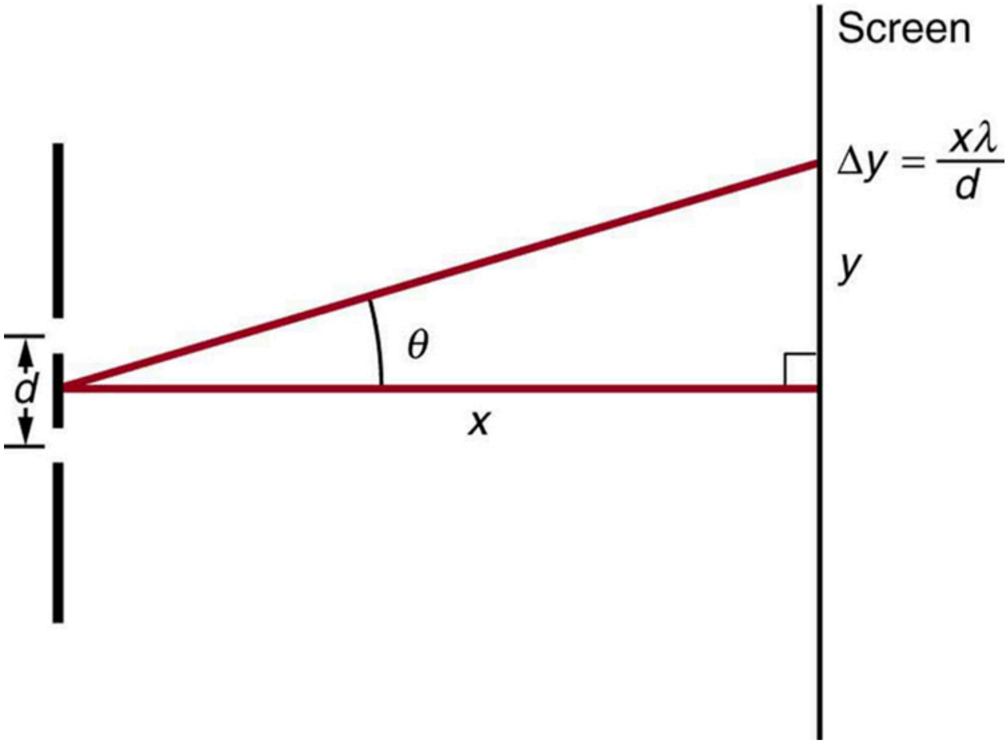
Discussion

The groove spacing of 1.71 μm is typical for CDs. The actual standard spacing for CDs is 1.6 μm (corresponding to about 625 grooves per mm). Our calculated value of 1.71 μm is quite close, with the small difference possibly due to:

- Measurement uncertainties in the fringe position
- The angle of incidence affecting the effective spacing
- Rounding in the given measurements

This demonstrates how CDs can be used as inexpensive diffraction gratings for educational demonstrations. The regular spacing of pits and lands on a CD creates a periodic structure that diffracts laser light, producing the colorful iridescent patterns you see when white light reflects from a CD surface.

The analysis shown in the figure below also applies to diffraction gratings with lines separated by a distance d. What is the distance between fringes produced by a diffraction grating having 125 lines per centimeter for 600-nm light, if the screen is 1.50 m away?



The distance between adjacent fringes is $\Delta y=x \lambda/d$, assuming the slit separation d is large compared with λ.

Show Solution

Strategy

Use the formula for fringe spacing: $\Delta y = x\lambda/d$, where x is the distance to the screen. First, find d from the line density, then calculate Δy.

Solution

Given:

- Line density: 125 lines/cm
- λ=600 nm=600×10^{-9} m
- Distance to screen: x =1.50 m

Step 1: Find the grating spacing

d=1 cm125 lines=0.01 m125=8.00×10^{-5} m

Step 2: Calculate the fringe spacing

Using $\Delta y = x\lambda d$:

Δy=(1.50 m)(600×10^{-9} m)8.00×10^{-5} m
Δy=9.00×10^{-7}8.00×10^{-5}=1.125×10^{-2} m
Δy=1.13×10^{-2} m=1.13 cm

Discussion

The fringe spacing of 1.13 cm is quite large and easily observable. This is because the grating has a relatively coarse spacing (125 lines/cm = 80 μm between lines), which is about 133 wavelengths. The formula Δy = xλ/d shows that:

- Larger screen distance (x) increases fringe spacing
- Larger wavelength (λ) increases fringe spacing
- Smaller slit separation (d) increases fringe spacing

With adjacent fringes separated by over 1 cm at a distance of 1.5 m, this pattern would be very easy to observe and measure. Finer gratings (more lines/cm) would produce smaller fringe spacings. For example, a 1000 lines/cm grating would have fringes separated by only 1.4 mm at the same distance.

Unreasonable Results

Red light of wavelength of 700 nm falls on a double slit separated by 400 nm. (a) At what angle is the first-order maximum in the diffraction pattern? (b) What is unreasonable about this result? (c) Which assumptions are unreasonable or inconsistent?

Show Solution

Strategy

Use $d\sin\theta = m\lambda$ for double-slit interference with m = 1. Examine whether the result is physically possible.

Solution

(a) Angle for first-order maximum:

Given:

- $\lambda = 700\text{ nm} = 700 \times 10^{-9}\text{ m}$
- $d = 400\text{ nm} = 400 \times 10^{-9}\text{ m}$
- $m = 1$

Using $d\sin\theta = m\lambda$:

$$\sin\theta = \frac{m\lambda}{d} = \frac{(1)(700\text{ nm})}{400\text{ nm}} = 1.75$$

This requires $\theta = \sin^{-1}(1.75)$, which is **undefined** (no solution exists).

(b) What is unreasonable about this result?

The sine function can only have values between -1 and +1 for real angles. A value of sin θ = 1.75 is **impossible** - there is no real angle whose sine equals 1.75. This means no first-order maximum can exist for this configuration.

(c) Which assumptions are unreasonable or inconsistent?

The unreasonable assumption is that a **slit separation of 400 nm can produce an interference pattern for 700-nm light**.

For a double-slit interference pattern to show at least one maximum (besides the central maximum), we need:

$$d \geq \lambda$$

In this problem, d = 400 nm < λ = 700 nm. When the slit separation is smaller than the wavelength, no off-axis maxima can occur. The only observable feature would be the central maximum (m = 0), which exists for all wavelengths.

The inconsistency is assuming that red light (700 nm) with a relatively long wavelength can create a normal double-slit pattern with slits separated by only 400 nm. Either:

- The wavelength should be shorter (≤ 400 nm, like UV or violet light), OR
- The slit separation should be larger (≥ 700 nm)

Discussion

This problem illustrates an important physical limitation: you cannot create interference patterns of a certain order if the wavelength exceeds the slit separation. In practical terms, attempting this experiment would result in only seeing a broad central diffraction maximum with no side fringes - essentially no interference pattern. This is why diffraction gratings and double-slit apparatus must have appropriate spacing for the wavelength being studied.

Unreasonable Results

(a) What visible wavelength has its fourth-order maximum at an angle of 25.0° when projected on a 25 000-line-per-centimeter diffraction grating? (b) What is unreasonable about this result? (c) Which assumptions are unreasonable or inconsistent?

Show Solution

Strategy

Use $d\sin\theta = m\lambda$ to find the wavelength. First, calculate the grating spacing d from the line density, then solve for λ with m = 4. Examine whether the result makes sense.

Solution

(a) Calculate the wavelength:

Given:

- Line density: 25,000 lines/cm
- $\theta = 25.0°$
- $m = 4$ (fourth-order maximum)

Step 1: Find the grating spacing

$$d = \frac{1 \text{ cm}}{25{,}000 \text{ lines}} = \frac{0.01 \text{ m}}{25{,}000} = 4.00 \times 10^{-7} \text{ m} = 400 \text{ nm}$$

Step 2: Find the wavelength

Using $d \sin\theta = m\lambda$:

$$\lambda = \frac{d \sin\theta}{m} = \frac{(4.00 \times 10^{-7} \text{ m}) \sin 25.0°}{4}$$
$$\lambda = \frac{(4.00 \times 10^{-7})(0.4226)}{4} = \frac{1.690 \times 10^{-7}}{4}$$
$$\lambda = 4.23 \times 10^{-8} \text{ m} = 42.3 \text{ nm}$$

(b) What is unreasonable about this result?

The calculated wavelength of **42.3 nm is NOT visible light**. The visible spectrum ranges from approximately 380 nm (violet) to 760 nm (red). A wavelength of 42.3 nm falls in the **extreme ultraviolet (EUV) range**, which is far beyond what the human eye can detect. The problem asks for a "visible wavelength," but the result is not visible at all.

(c) Which assumptions are unreasonable or inconsistent?

There are two unreasonable aspects to this problem:

1. The grating is too fine for visible light diffraction:

The grating spacing is only 400 nm, which is barely larger than the shortest visible wavelength (380 nm). For a fourth-order maximum to exist for visible light, we would need:

$$\sin\theta = \frac{4\lambda}{d} \leq 1$$

For the shortest visible wavelength (380 nm):

$$\sin\theta = \frac{4 \times 380}{400} = 3.8 > 1 \text{(impossible!)}$$

No visible wavelength can produce even a first-order maximum with a 400 nm grating spacing at any reasonable angle, let alone a fourth-order maximum at 25°.

2. Practical manufacturing limitations:

A grating with 25,000 lines/cm (d = 400 nm) is at the very edge of what can be manufactured using advanced nanofabrication techniques. While modern electron-beam lithography can achieve features around 50 nm, creating uniform, large-area diffraction gratings with such fine spacing is extremely challenging and expensive. Such gratings would be more suitable for X-ray or EUV spectroscopy, not visible light.

The inconsistent assumptions are:

- Expecting visible light to produce a fourth-order maximum with such fine grating spacing
- Assuming 25,000 lines/cm is reasonable for a visible-light spectrometer (it's far too fine)
- For visible light spectroscopy, gratings typically have 300-10,000 lines/cm

Discussion

This problem highlights the importance of matching grating spacing to wavelength. The grating spacing should be several times larger than the wavelength of interest to produce observable higher-order maxima. A grating with d = 400 nm is appropriate for ultraviolet or extreme ultraviolet radiation (wavelengths < 100 nm), not for visible light. For visible light and fourth-order maxima, a grating with d ≥ 4 × 760 nm = 3040 nm (corresponding to ~3300 lines/cm or fewer) would be more appropriate.

Construct Your Own Problem

Consider a spectrometer based on a diffraction grating. Construct a problem in which you calculate the distance between two wavelengths of electromagnetic radiation in your spectrometer. Among the things to be considered are the wavelengths you wish to be able to distinguish, the number of lines per meter on the diffraction grating, and the distance from the grating to the screen or detector. Discuss the practicality of the device in terms of being able to discern between wavelengths of interest.

Glossary

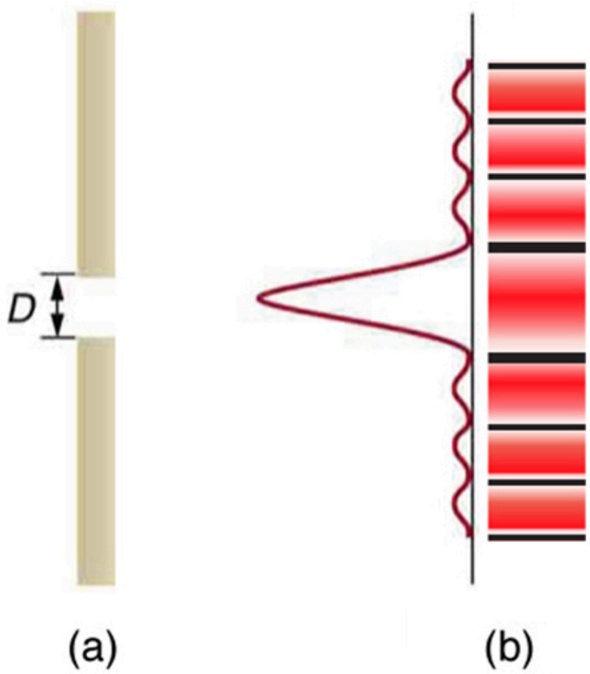
constructive interference for a diffraction grating
: occurs when the condition $d \sin\theta = m\lambda \ (\text{for } m = 0, 1, -1, 2, -2, \ldots)$ is satisfied, where d is the distance between slits in the grating, λ is the wavelength of light, and m is the order of the maximum

diffraction grating
: a large number of evenly spaced parallel slits

Single Slit Diffraction

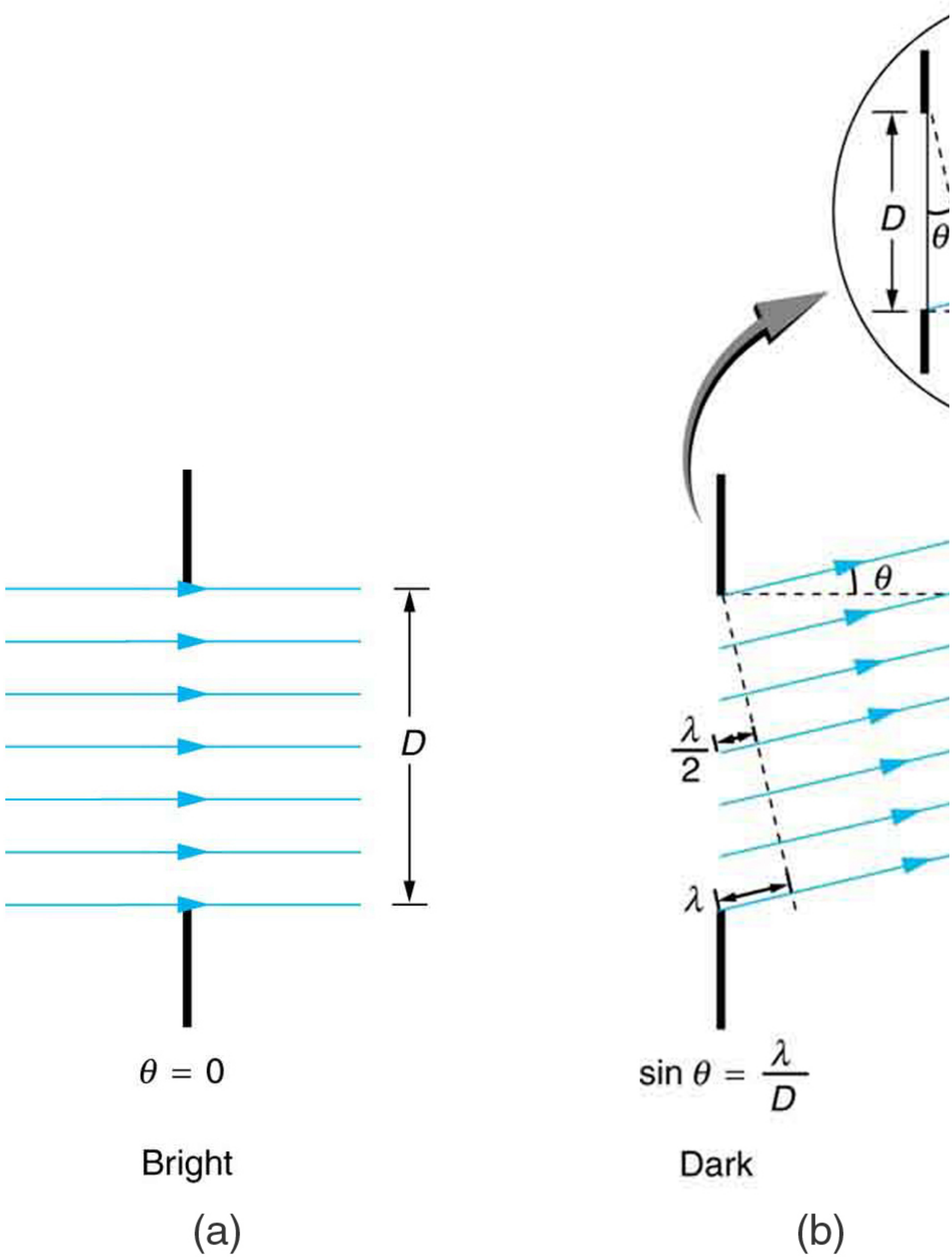
- Discuss the single slit diffraction pattern.

Light passing through a single slit forms a diffraction pattern somewhat different from those formed by double slits or diffraction gratings. [Figure 1] shows a single slit diffraction pattern. Note that the central maximum is larger than those on either side, and that the intensity decreases rapidly on either side. In contrast, a diffraction grating produces evenly spaced lines that dim slowly on either side of center.

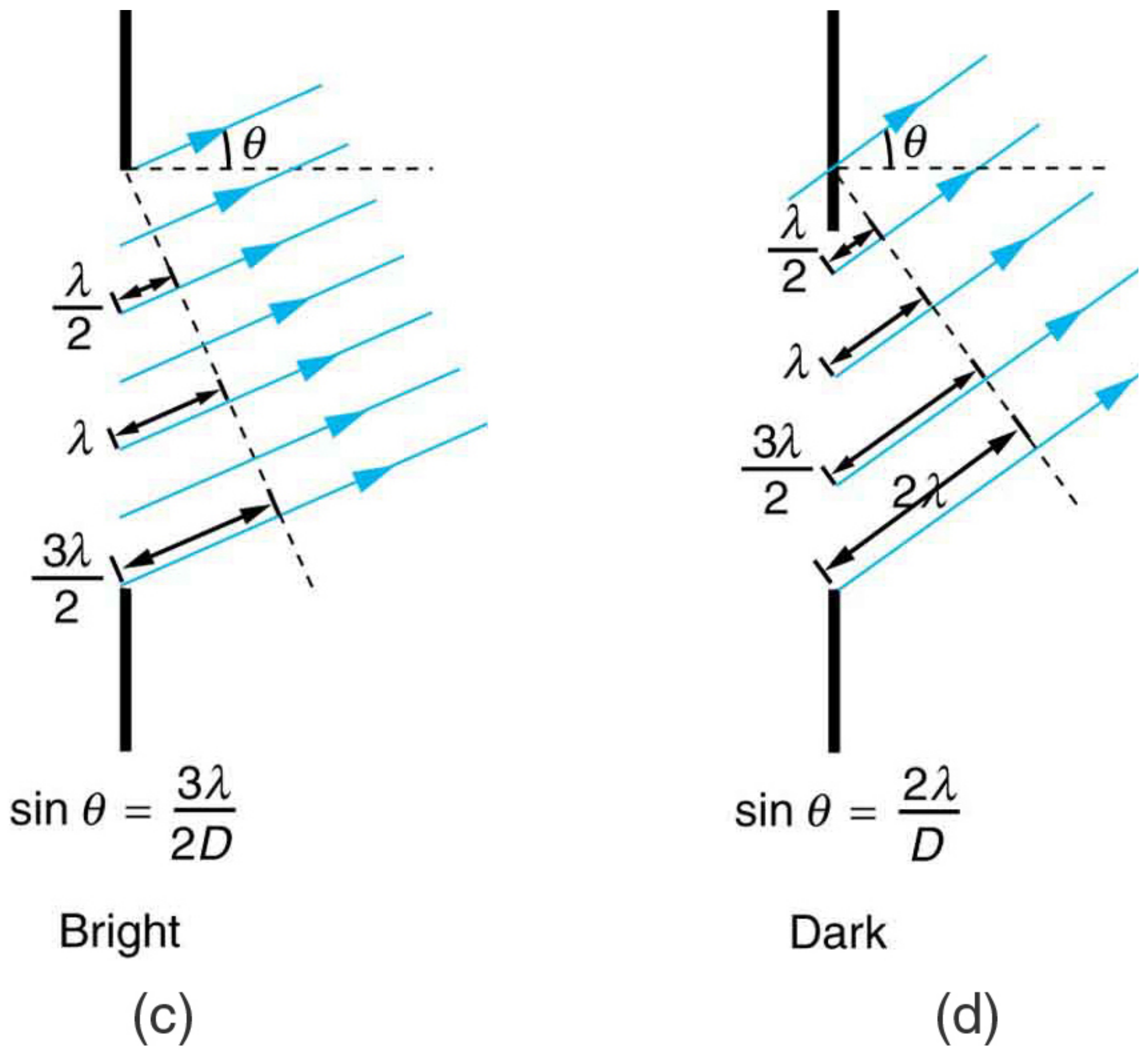


(a) Single slit diffraction pattern. Monochromatic light passing through a single slit has a central maximum and many smaller and dimmer maxima on either side. The central maximum is six times higher than shown. (b) The drawing shows the bright central maximum and dimmer and thinner maxima on either side.

The analysis of single slit diffraction is illustrated in [Figure 2] . Here we consider light coming from different parts of the *same* slit. According to Huygens's principle, every part of the wavefront in the slit emits wavelets. These are like rays that start out in phase and head in all directions. (Each ray is perpendicular to the wavefront of a wavelet.) Assuming the screen is very far away compared with the size of the slit, rays heading toward a common destination are nearly parallel. When they travel straight ahead, as in [Figure 2](a), they remain in phase, and a central maximum is obtained. However, when rays travel at an angle θ relative to the original direction of the beam, each travels a different distance to a common location, and they can arrive in or out of phase. In [Figure 2](b), the ray from the bottom travels a distance of one wavelength λ farther than the ray from the top. Thus a ray from the center travels a distance $\lambda /2$ farther than the one on the left, arrives out of phase, and interferes destructively. A ray from slightly above the center and one from slightly above the bottom will also cancel one another. In fact, each ray from the slit will have another to interfere destructively, and a minimum in intensity will occur at this angle. There will be another minimum at the same angle to the right of the incident direction of the light.

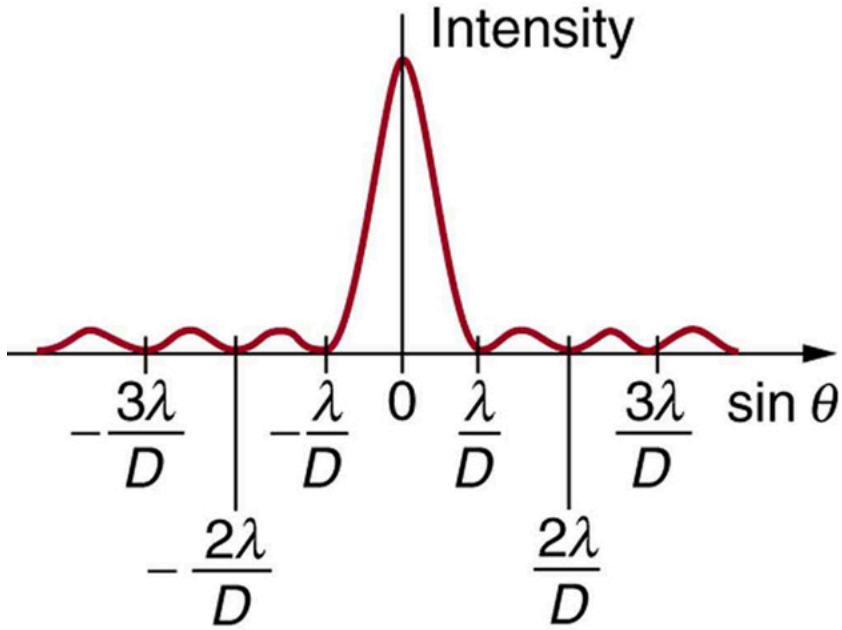

D
θ
θ
D
λ
2
λ
θ = 0
sin θ = λ / D
Bright
Dark
(a)
(b)

–



Light passing through a single slit is diffracted in all directions and may interfere constructively or destructively, depending on the angle. The difference in path length for rays from either side of the slit is seen to be $D\sin \theta$.

At the larger angle shown in [Figure 2](c), the path lengths differ by $3\lambda /2$ for rays from the top and bottom of the slit. One ray travels a distance λ different from the ray from the bottom and arrives in phase, interfering constructively. Two rays, each from slightly above those two, will also add constructively. Most rays from the slit will have another to interfere with constructively, and a maximum in intensity will occur at this angle. However, all rays do not interfere constructively for this situation, and so the maximum is not as intense as the central maximum. Finally, in [Figure 2](d), the angle shown is large enough to produce a second minimum. As seen in the figure, the difference in path length for rays from either side of the slit is $D\sin \theta$, and we see that a destructive minimum is obtained when this distance is an integral multiple of the wavelength.



A graph of single slit diffraction intensity showing the central maximum to be wider and much more intense than those to the sides. In fact the central maximum is six times higher than shown here.

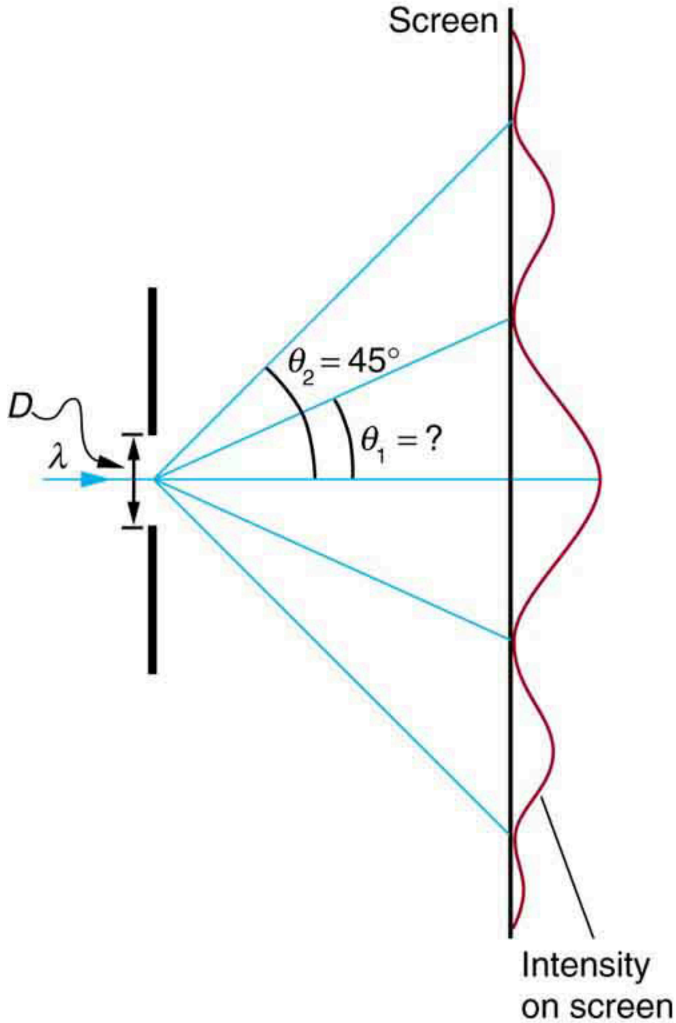
Thus, to obtain **destructive interference for a single slit**,

$$D\sin \theta =m \lambda, \text{ for } m=1 \text{,} -1 \text{,} 2 \text{,} -2 \text{,} 3 \text{,} \dots \text{(destructive),} $$

where D is the slit width, λ is the light's wavelength, θ is the angle relative to the original direction of the light, and m is the order of the minimum. [Figure 3] shows a graph of intensity for single slit interference, and it is apparent that the maxima on either side of the central maximum are much less intense and not as wide. This is consistent with the illustration in [Figure 1](b).

Calculating Single Slit Diffraction

Visible light of wavelength 550 nm falls on a single slit and produces its second diffraction minimum at an angle of $45.0 ^\circ$ relative to the incident direction of the light. (a) What is the width of the slit? (b) At what angle is the first minimum produced?



A graph of the single slit diffraction pattern is analyzed in this example.

Strategy

From the given information, and assuming the screen is far away from the slit, we can use the equation $D\sin \theta = m \lambda$ first to find D , and again to find the angle for the first minimum ${\theta }_{1}$.

Solution for (a)

We are given that $\lambda =550 \text{nm}$, $m=2$, and ${\theta }_ {2}=45.0 ^\circ$. Solving the equation $D \sin \theta =m \lambda$ for D and substituting known values gives

$$\begin{array}{lll}D& =& \frac{m \lambda}{ \sin {\theta }_{2}}=\frac{2\left(550 \text{nm}\right)}{\sin 45.0\text{º}}\\ D & =& \frac{1100 \times 10^{-9} \text{m}}{0.707}\\ D & =& 1.56 \times 10^{-6} \text{m} .\end{array} $$

Solution for (b)

Solving the equation $D \sin \theta = m \lambda$ for $\sin {\theta }_{1}$ and substituting the known values gives

$$\sin {\theta }_{1}=\frac{m \lambda}{D}=\frac{1\left(550 \times 10^{-9} \text{m}\right)}{ 1.56 \times 10^{-6} \text{m}}\text{.} $$

Thus the angle ${\theta }_{1}$ is

$${\theta }_{1}={\sin}^{-1} 0.354=20.7 \text{º.} $$

Discussion

We see that the slit is narrow (it is only a few times greater than the wavelength of light). This is consistent with the fact that light must interact with an object comparable in size to its wavelength in order to exhibit significant wave effects such as this single slit diffraction pattern. We also see that the central maximum extends $20.7 ^\circ$ on either side of the original beam, for a width of about $41 ^\circ$. The angle between the first and second minima is only about $24 ^\circ \left(45.0 \text{º}-20.7º\right)$. Thus the second maximum is only about half as wide as the central maximum.

Section Summary

- A single slit produces an interference pattern characterized by a broad central maximum with narrower and dimmer maxima to the sides.
- There is destructive interference for a single slit when $D \sin \theta =m \lambda, \left(\text{ for } m=1 \text{,} -1 \text{,} 2 \text{,} -2 \text{,} 3 \text{,} \dots \right)$, where D is the slit width, λ is the light's wavelength, θ is the angle relative to the original direction of the light, and m is the order of the minimum. Note that there is no $m=0$ minimum.

Conceptual Questions

As the width of the slit producing a single-slit diffraction pattern is reduced, how will the diffraction pattern produced change?

Problems & Exercises

(a) At what angle is the first minimum for 550-nm light falling on a single slit of width 1.00µm ? (b) Will there be a second minimum?

Show Solution

Strategy

For single-slit diffraction, minima (dark fringes) occur at angles given by $D \sin \theta = m\lambda$, where D is the slit width, λ is the wavelength, and m = 1, 2, 3, … (m = 0 would be the central maximum, not a minimum). We solve for θ for m = 1, then check if m = 2 is physically possible.

Solution

Given:

- Wavelength: $\lambda = 550 \text{ nm} = 550 \times 10^{-9} \text{ m}$
- Slit width: $D = 1.00 \text{ µm} = 1.00 \times 10^{-6} \text{ m}$

(a) First minimum (m = 1):

$$D \sin \theta_1 = m\lambda = (1)\lambda$$
$$\sin \theta_1 = \frac{\lambda}{D} = \frac{550 \times 10^{-9}}{1.00 \times 10^{-6}} = 0.550$$
$$\theta_1 = \arcsin(0.550) = 33.4°$$

The first minimum occurs at 33.4°.

(b) Second minimum (m = 2):

$$\sin \theta_2 = \frac{2\lambda}{D} = \frac{2(550 \times 10^{-9})}{1.00 \times 10^{-6}} = 1.10$$

Since $\sin \theta$ cannot exceed 1, **there is no second minimum.** The value of 1.10 is physically impossible.

Discussion

When the slit width is comparable to the wavelength (D ≈ λ), the diffraction pattern becomes very broad. In this case, D = 1.00 µm and λ = 550 nm, giving a ratio of D/λ ≈ 1.82. The first minimum appears at 33.4°, which is a large angle indicating significant spreading of light. For a second minimum to exist, we would need D ≥ 2λ, but here D < 2λ (1000 nm < 1100 nm). This demonstrates that narrower slits produce wider diffraction patterns. When D < 2λ, only the central maximum and first-order minima exist; the diffraction pattern has only three regions: one minimum on each side of the central maximum.

(a) Calculate the angle at which a 2.00-µm -wide slit produces its first minimum for 410-nm violet light. (b) Where is the first minimum for 700-nm red light?

Show Solution

Strategy

For a single slit, the first minimum occurs when $D \sin \theta = m\lambda$ with m = 1. We solve for θ for each wavelength.

Solution

Given:

- $$ D = 2.00 \text{ µm} = 2.00 \times 10^{-6} \text{ m} $$

(a) For violet light (λ = 410 nm):

$$D \sin \theta = m\lambda$$

With m = 1:

$$\sin \theta = \frac{\lambda}{D} = \frac{410 \times 10^{-9} \text{ m}}{2.00 \times 10^{-6} \text{ m}} = 0.205$$
$$\theta = \sin^{-1}(0.205) = 11.8°$$

(b) For red light (λ = 700 nm):

$$\sin \theta = \frac{\lambda}{D} = \frac{700 \times 10^{-9} \text{ m}}{2.00 \times 10^{-6} \text{ m}} = 0.350$$
$$\theta = \sin^{-1}(0.350) = 20.5°$$

Discussion

The first minimum for red light (20.5°) occurs at a larger angle than for violet light (11.8°), which makes sense because red light has a longer wavelength. This illustrates the wavelength dependence of diffraction: longer wavelengths diffract more. The central maximum for red light is therefore wider than for violet light. This is why when white light passes through a single slit, the central maximum appears white but has reddish edges (red diffracts farther) and bluish/violet closer to the center.

(a) How wide is a single slit that produces its first minimum for 633-nm light at an angle of $28.0º$? (b) At what angle will the second minimum be?

Show Solution

Strategy

This problem gives us the angle of the first minimum and asks us to find the slit width. We use $D \sin \theta = m\lambda$ with m = 1, solving for D. Then we find the second minimum (m = 2) using the calculated slit width.

Solution

Given:

- Wavelength: $\lambda = 633 \text{ nm} = 633 \times 10^{-9} \text{ m}$
- First minimum angle: $\theta_1 = 28.0°$
- First minimum: $m = 1$

(a) Slit width:

For the first minimum:

$$D \sin \theta_1 = m\lambda = (1)\lambda$$

Solving for D:

$$D = \frac{\lambda}{\sin \theta_1} = \frac{633 \times 10^{-9}}{\sin 28.0°} = \frac{633 \times 10^{-9}}{0.469}$$
$$D = 1.35 \times 10^{-6} \text{ m} = 1.35 \text{ μm}$$

The slit width is 1.35×10^{-6} m (or 1.35 μm).

(b) Second minimum:

For the second minimum (m = 2):

$$\sin \theta_2 = \frac{2\lambda}{D} = \frac{2(633 \times 10^{-9})}{1.35 \times 10^{-6}} = \frac{1266 \times 10^{-9}}{1.35 \times 10^{-6}}$$
$$\sin \theta_2 = 0.938$$
$$\theta_2 = \arcsin(0.938) = 69.9°$$

The second minimum occurs at 69.9°.

Discussion

The slit width of 1.35 μm is about twice the wavelength (633 nm), which allows for a second minimum to exist. The large angle for the second minimum (69.9°) means it's very close to the theoretical maximum of 90°. A third minimum would require $\sin \theta_3 = 3\lambda/D = 1.41$, which is impossible, so this diffraction pattern has only two minima on each side of the central maximum. The 633-nm wavelength is characteristic of a helium-neon (He-Ne) laser, commonly used in diffraction experiments due to its monochromaticity and coherence.

(a) What is the width of a single slit that produces its first minimum at $60.0º$ for 600-nm light? (b) Find the wavelength of light that has its first minimum at $62.0º$.

Show Solution

Strategy

For part (a), we use $D \sin \theta = m\lambda$ with m = 1 to solve for D. For part (b), we use the same slit width D and solve for λ at the new angle.

Solution

(a) Slit width for 600-nm light at 60.0°:

Given:

- $$ \lambda = 600 \text{ nm} = 600 \times 10^{-9} \text{ m} $$
- $$ \theta = 60.0° $$
- $m = 1$ (first minimum)

Using $D \sin \theta = m\lambda$:

$$D = \frac{m\lambda}{\sin \theta} = \frac{(1)(600 \times 10^{-9} \text{ m})}{\sin 60.0°}$$
$$D = \frac{600 \times 10^{-9}}{0.8660} = 6.93 \times 10^{-7} \text{ m} = 693 \text{ nm}$$

(b) Wavelength with first minimum at 62.0°:

Using the same slit width:

$$\lambda = D \sin \theta = (6.93 \times 10^{-7} \text{ m}) \sin 62.0°$$
$$\lambda = (6.93 \times 10^{-7})(0.8829) = 6.12 \times 10^{-7} \text{ m} = 612 \text{ nm}$$

Discussion

The slit width of 693 nm is remarkably small - just slightly larger than the wavelength of the light itself. When D ≈ λ, the diffraction pattern is very spread out, with the first minimum at 60°. This creates a very wide central maximum spanning 120° total (60° on each side).

For part (b), the slightly larger angle (62.0° vs 60.0°) corresponds to a slightly longer wavelength (612 nm vs 600 nm), which makes physical sense. Both wavelengths are in the orange-red portion of the visible spectrum. This problem demonstrates that when the slit width is comparable to the wavelength, single-slit diffraction effects are very pronounced.

Find the wavelength of light that has its third minimum at an angle of $48.6º$ when it falls on a single slit of width 3.00μm.

Show Solution

Strategy

We use the single-slit diffraction formula $D \sin \theta = m\lambda$ with m = 3 (third minimum) and solve for the wavelength λ.

Solution

Given:

- Slit width: $D = 3.00 \text{ μm} = 3.00 \times 10^{-6} \text{ m}$
- Third minimum angle: $\theta = 48.6°$
- Order: $m = 3$

For the third minimum:

$$D \sin \theta = m\lambda$$

Solving for wavelength:

$$\lambda = \frac{D \sin \theta}{m} = \frac{(3.00 \times 10^{-6}) \sin 48.6°}{3}$$
$$\lambda = \frac{(3.00 \times 10^{-6})(0.7501)}{3} = \frac{2.250 \times 10^{-6}}{3}$$
$$\lambda = 7.50 \times 10^{-7} \text{ m} = 750 \text{ nm}$$

The wavelength is 750 nm, which corresponds to red light.

Discussion

A wavelength of 750 nm is at the red end of the visible spectrum (approximately 620-750 nm). The relatively large slit width (3.00 μm = 4 wavelengths) allows multiple minima to exist on each side of the central maximum. For m = 3 to occur at a reasonable angle (48.6°), the ratio D/λ = 4.0, which means the slit is exactly 4 wavelengths wide. This is a favorable condition for observing multiple diffraction orders. If we checked for a fourth minimum (m = 4), we would find $\sin \theta_4$ = 4(750)/3000 = 1.0, meaning the fourth minimum just barely exists at θ = 90° (grazing angle along the screen).

Calculate the wavelength of light that produces its first minimum at an angle of $36.9º$ when falling on a single slit of width 1.00μm.

Show Solution

Strategy

For the first minimum of a single slit, we use $D \sin \theta = m\lambda$ with m = 1 and solve for the wavelength λ.

Solution

Given:

- $$ D = 1.00 \text{ μm} = 1.00 \times 10^{-6} \text{ m} $$
- $$ \theta = 36.9° $$
- $m = 1$ (first minimum)

Using the single-slit diffraction equation:

$$D \sin \theta = m\lambda$$

Solving for λ:

$$\lambda = \frac{D \sin \theta}{m} = \frac{(1.00 \times 10^{-6} \text{ m}) \sin 36.9°}{1}$$
$$\lambda = (1.00 \times 10^{-6})(0.6000) = 6.00 \times 10^{-7} \text{ m} = 600 \text{ nm}$$

Discussion

The wavelength of 600 nm corresponds to orange light, which is in the visible spectrum. The angle of 36.9° is relatively large for a first minimum, indicating that the slit width (1.00 μm) is not much larger than the wavelength. Specifically, D/λ = 1.00 μm / 600 nm ≈ 1.67, meaning the slit is less than twice the wavelength.

This problem nicely demonstrates that sin(36.9°) = 0.6 = 3/5, which comes from a 3-4-5 right triangle, a common value that appears in physics problems. The result is reasonable because it falls within the visible light range (380-760 nm).

(a) Sodium vapor light averaging 589 nm in wavelength falls on a single slit of width 7.50μm . At what angle does it produce its second minimum? (b) What is the highest-order minimum produced?

Show Solution

Strategy

For part (a), we use $D \sin \theta = m\lambda$ with m = 2 (second minimum) to find the angle. For part (b), the highest-order minimum occurs when sin θ approaches 1 (angle approaches 90°), so we solve for the maximum m such that mλ/D ≤ 1.

Solution

Given:

- Wavelength: $\lambda = 589 \text{ nm} = 589 \times 10^{-9} \text{ m}$
- Slit width: $D = 7.50 \text{ μm} = 7.50 \times 10^{-6} \text{ m}$

(a) Second minimum (m = 2):

$$D \sin \theta = m\lambda$$
$$\sin \theta = \frac{m\lambda}{D} = \frac{2(589 \times 10^{-9})}{7.50 \times 10^{-6}} = \frac{1178 \times 10^{-9}}{7.50 \times 10^{-6}}$$
$$\sin \theta = 0.1571$$
$$\theta = \arcsin(0.1571) = 9.04°$$

The second minimum occurs at 9.04°.

(b) Highest-order minimum:

The highest order occurs when sin θ = 1 (at θ = 90°):

$$m_{\text{max}} = \frac{D}{\lambda} = \frac{7.50 \times 10^{-6}}{589 \times 10^{-9}} = 12.73$$

Since m must be an integer, the highest-order minimum is:

$$m_{\text{max}} = 12$$

We can verify: $\sin \theta_{12} = \frac{12(589)}{7500} = 0.943$, giving $\theta_{12} = 70.6°$ (valid).

For m = 13: $\sin \theta_{13} = \frac{13(589)}{7500} = 1.021 > 1$ (impossible).

The highest-order minimum is m = 12.

Discussion

Sodium vapor light (589 nm average of the D-line doublet at 589.0 and 589.6 nm) is commonly used in spectroscopy and diffraction experiments. The relatively large slit width (7.50 μm ≈ 12.7λ) allows many diffraction orders. The small angle for the second minimum (9.04°) indicates that higher-order minima are closely spaced near the central maximum but spread out at larger angles. The 12th-order minimum appears at about 70.6°, showing how the diffraction pattern extends over a wide angular range. This creates a complex pattern with 13 bright regions: the central maximum plus 12 secondary maxima on each side.

(a) Find the angle of the third diffraction minimum for 633-nm light falling on a slit of width 20.0μm . (b) What slit width would place this minimum at $85.0º$? Explicitly show how you follow the steps in [Problem-Solving Strategies for Wave Optics]

Show Solution

Strategy

Following the Problem-Solving Strategies for Wave Optics:

Step 1: This is a single slit diffraction problem. **Step 2:** We use the single-slit equation $D \sin \theta = m\lambda$ for minima. **Step 3:** We need to find (a) the angle θ for the third minimum, and (b) the slit width D that places this minimum at 85.0°. **Step 4:** Given: λ = 633 nm, D = 20.0 μm for part (a); θ = 85.0° for part (b). **Step 5:** For part (a), solve for θ. For part (b), solve for D.

Solution

(a) Angle of third minimum:

Given:

- $$ \lambda = 633 \text{ nm} = 633 \times 10^{-9} \text{ m} $$
- $$ D = 20.0 \text{ μm} = 20.0 \times 10^{-6} \text{ m} $$
- $m = 3$ (third minimum)

Using $D \sin \theta = m\lambda$:

$$\sin \theta = \frac{m\lambda}{D} = \frac{3 \times 633 \times 10^{-9} \text{ m}}{20.0 \times 10^{-6} \text{ m}}$$

$$\sin \theta = \frac{1899 \times 10^{-9}}{20.0 \times 10^{-6}} = 0.09495$$
$$\theta = \sin^{-1}(0.09495) = 5.45°$$

(b) Slit width for third minimum at 85.0°:

$$D = \frac{m\lambda}{\sin \theta} = \frac{3 \times 633 \times 10^{-9} \text{ m}}{\sin 85.0°}$$
$$D = \frac{1899 \times 10^{-9}}{0.9962} = 1.91 \times 10^{-6} \text{ m} = 1.91 \text{ μm}$$

Step 6: Check reasonableness: For (a), the small angle makes sense because D » λ. For (b), the smaller slit width creates more spreading.

Discussion

The third minimum at only 5.45° for a 20.0-μm slit indicates that D is much larger than λ (about 32 times). This produces a tight, narrow diffraction pattern with many observable minima.

For part (b), placing the third minimum at 85.0° (near grazing angle) requires a much smaller slit (1.91 μm), comparable to the wavelength. This demonstrates that smaller slits produce more spreading. The ratio D/λ changes from 31.6 in part (a) to only 3.0 in part (b), showing how dramatically the pattern spreads when the slit width decreases.

(a) Find the angle between the first minima for the two sodium vapor lines, which have wavelengths of 589.1 and 589.6 nm, when they fall upon a single slit of width 2.00μm . (b) What is the distance between these minima if the diffraction pattern falls on a screen 1.00 m from the slit? (c) Discuss the ease or difficulty of measuring such a distance.

Show Solution

Strategy

For each sodium D-line wavelength, we calculate the angle of the first minimum using $D \sin \theta = \lambda$. The angle difference gives us part (a). For part (b), we use geometry to find the separation on the screen using $y = L \tan \theta$ or the small angle approximation $\Delta y \approx L \Delta \theta$ (with $\Delta\theta$ in radians).

Solution

Given:

- Wavelengths: $\lambda_1 = 589.1 \text{ nm}$ and $\lambda_2 = 589.6 \text{ nm}$
- Slit width: $D = 2.00 \text{ μm} = 2.00 \times 10^{-6} \text{ m}$
- Screen distance: $L = 1.00 \text{ m}$

(a) Angle between first minima:

For the first minimum (m = 1) of each wavelength:

$$\sin \theta_1 = \frac{\lambda_1}{D} = \frac{589.1 \times 10^{-9}}{2.00 \times 10^{-6}} = 0.29455$$
$$\theta_1 = \arcsin(0.29455) = 17.125°$$
$$\sin \theta_2 = \frac{\lambda_2}{D} = \frac{589.6 \times 10^{-9}}{2.00 \times 10^{-6}} = 0.29480$$
$$\theta_2 = \arcsin(0.29480) = 17.140°$$

The angle difference is:

$$\Delta \theta = \theta_2 - \theta_1 = 17.140° - 17.125° = 0.0150°$$

The angle between the first minima is 0.0150°.

(b) Distance on screen:

Converting the angle difference to radians:

$$\Delta \theta_{\text{rad}} = 0.0150° \times \frac{\pi}{180°} = 2.618 \times 10^{-4} \text{ rad}$$

For small angle differences, the separation on the screen is approximately:

$$\Delta y = L \Delta \theta_{\text{rad}} = (1.00 \text{ m})(2.618 \times 10^{-4}) = 2.62 \times 10^{-4} \text{ m}$$
$$\Delta y = 0.262 \text{ mm}$$

The distance between the minima on the screen is 0.262 mm.

(c) Ease of measurement:

A separation of 0.262 mm (about one-quarter millimeter) is **not easily measured by the naked human eye**, which can typically resolve details down to about 0.1 mm at best under ideal conditions. However, this distance is **quite easily measurable with basic optical aids**:

- Under a microscope or magnifying glass
- With a precision ruler or calipers
- Using digital imaging and analysis software
- With a traveling microscope used in physics labs

This measurement demonstrates the resolving power of single-slit diffraction. The sodium D-lines differ by only 0.5 nm (less than 0.1% difference), yet single-slit diffraction can spatially separate them sufficiently to be measured with modest equipment. This technique forms the basis of spectroscopy for measuring small wavelength differences.

Discussion

The small angular separation (0.0150°) between the two sodium D-lines highlights the challenge of spectral resolution. The wavelength difference is $\Delta\lambda$ = 0.5 nm out of $\lambda \approx$ 589 nm, giving a fractional difference of about 0.08%. The ability to distinguish these lines depends on having sharp, well-defined minima, which requires coherent, monochromatic light sources. Sodium vapor lamps produce these doublet lines due to the fine structure of sodium's electronic energy levels. This problem demonstrates why high-resolution spectroscopy requires careful optical design and precise measurements.

(a) What is the minimum width of a single slit (in multiples of λ) that will produce a first minimum for a wavelength λ ? (b) What is its minimum width if it produces 50 minima? (c) 1000 minima?

Show Solution

Strategy

To produce minima, we need $D \sin \theta = m\lambda$. The minimum slit width occurs when the highest-order minimum appears at θ = 90° (where $\sin \theta = 1$).

Solution

(a) Minimum width for a first minimum:

For the first minimum (m = 1), it must be observable at some angle $\theta < 90°$. The absolute minimum occurs when it appears exactly at θ = 90°:

$$D \sin 90° = 1 \times \lambda$$
$$D_{\text{min}} = \lambda$$

So the minimum width is **1λ** (one wavelength).

(b) Minimum width for 50 minima:

If there are 50 minima on one side of the central maximum, the 50th minimum must appear at or before θ = 90°:

$$D \sin 90° = 50\lambda$$
$$D_{\text{min}} = 50\lambda$$

The minimum width is **50λ**.

(c) Minimum width for 1000 minima:

Similarly, for 1000 minima:

$$D \sin 90° = 1000\lambda$$
$$D_{\text{min}} = 1000\lambda$$

The minimum width is **1000λ**.

Discussion

These results reveal an important principle: the maximum number of observable minima equals D/λ (when rounded down to the nearest integer). For example:

- If $D = \lambda$, you can have at most 1 minimum (barely observable at 90°)
- If $D = 50\lambda$, you can have at most 50 minima
- If $D = 1000\lambda$, you can have at most 1000 minima

This makes physical sense: wider slits (in units of wavelength) produce more closely-spaced minima and can fit more of them before reaching the 90° limit.

For visible light ($\lambda \approx$ 500 nm), a slit width of 1000λ = 0.5 mm would produce 1000 minima on each side, for 2000 minima total (plus the central maximum), creating an extremely dense pattern. In practice, these higher-order minima become very faint and difficult to observe.

(a) If a single slit produces a first minimum at $14.5º$, at what angle is the second-order minimum? (b) What is the angle of the third-order minimum? (c) Is there a fourth-order minimum? (d) Use your answers to illustrate how the angular width of the central maximum is about twice the angular width of the next maximum (which is the angle between the first and second minima).

Show Solution

Strategy

Given the first minimum angle, we can determine the ratio D/λ. From this, we calculate angles for higher-order minima using $D \sin \theta_m = m\lambda$, which can be rewritten as $\sin \theta_m = m \sin \theta_1$ since $D = \lambda/\sin \theta_1$.

Solution

Given:

- First minimum angle: $\theta_1 = 14.5°$

From the first minimum (m = 1):

$$D \sin \theta_1 = \lambda$$
$$\frac{D}{\lambda} = \frac{1}{\sin 14.5°} = \frac{1}{0.2504} = 3.994$$

(a) Second-order minimum (m = 2):

$$\sin \theta_2 = \frac{2\lambda}{D} = 2 \sin \theta_1 = 2(0.2504) = 0.5008$$
$$\theta_2 = \arcsin(0.5008) = 30.1°$$

The second-order minimum occurs at 30.1°.

(b) Third-order minimum (m = 3):

$$\sin \theta_3 = \frac{3\lambda}{D} = 3 \sin \theta_1 = 3(0.2504) = 0.7512$$
$$\theta_3 = \arcsin(0.7512) = 48.7°$$

The third-order minimum occurs at 48.7°.

(c) Fourth-order minimum (m = 4):

$$\sin \theta_4 = 4 \sin \theta_1 = 4(0.2504) = 1.002 > 1$$

Since $\sin \theta$ cannot exceed 1, **there is no fourth-order minimum.**

(d) Comparison of angular widths:

The **central maximum** extends from $-\theta_1$ to $+\theta_1$, so its angular width is:

$$\text{Width of central maximum} = 2\theta_1 = 2(14.5°) = 29.0°$$

The **first secondary maximum** lies between the first and second minima, so its angular width is:

$$\text{Width of first secondary maximum} = \theta_2 - \theta_1 = 30.1° - 14.5° = 15.6°$$

Comparing these:

$$\frac{2\theta_1}{\theta_2 - \theta_1} = \frac{29.0°}{15.6°} = 1.86 \approx 2$$

Alternatively: $2(\theta_2 - \theta_1) = 2(15.6°) = 31.2° \approx 29.0°$

This demonstrates that **the central maximum is approximately twice as wide as the first secondary maximum.**

Discussion

This 2:1 width ratio is a characteristic feature of single-slit diffraction patterns. The central maximum is both wider and much brighter than any secondary maxima. The theoretical intensity at the central maximum is proportional to $(D/\lambda)^2$, while secondary maxima have much lower intensities (roughly 4.5% for the first secondary maximum).

The fact that the slit width D ≈ 4λ means this pattern has exactly three observable minima (m = 1, 2, 3) before reaching the limit at 90°. The progression of angles (14.5°, 30.1°, 48.7°) shows how minima become increasingly spread out at higher orders, a consequence of the nonlinear relationship between θ and m through the sine function.

A double slit produces a diffraction pattern that is a combination of single and double slit interference. Find the ratio of the width of the slits to the separation between them, if the first minimum of the single slit pattern falls on the fifth maximum of the double slit pattern. (This will greatly reduce the intensity of the fifth maximum.)

Integrated Concepts

A water break at the entrance to a harbor consists of a rock barrier with a 50.0-m-wide opening. Ocean waves of 20.0-m wavelength approach the opening straight on. At what angle to the incident direction are the boats inside the harbor most protected against wave action?

Show Solution

Strategy

This is a single-slit diffraction problem applied to ocean waves. The harbor opening acts as a slit, and boats are most protected at angles corresponding to diffraction minima, where wave intensity is minimized. We use $D \sin \theta = m\lambda$ to find these angles.

Solution

Given:

- Opening width (slit): $D = 50.0 \text{ m}$
- Wavelength: $\lambda = 20.0 \text{ m}$

Boats are most protected at the minima of the diffraction pattern:

First minimum (m = 1):

$$\sin \theta_1 = \frac{\lambda}{D} = \frac{20.0 \text{ m}}{50.0 \text{ m}} = 0.400$$
$$\theta_1 = \arcsin(0.400) = 23.6°$$

Second minimum (m = 2):

$$\sin \theta_2 = \frac{2\lambda}{D} = \frac{2(20.0 \text{ m})}{50.0 \text{ m}} = 0.800$$
$$\theta_2 = \arcsin(0.800) = 53.1°$$

Check for third minimum (m = 3):

$$\sin \theta_3 = \frac{3\lambda}{D} = \frac{3(20.0)}{50.0} = 1.20 > 1 \text{ (impossible)}$$

Boats inside the harbor are most protected at angles of 23.6° and 53.1° from the incident direction.

Discussion

This problem demonstrates a practical application of wave diffraction. The harbor opening (50.0 m) is 2.5 times the wavelength (20.0 m), creating a clear diffraction pattern. At the minimum angles (23.6° and 53.1°), destructive interference significantly reduces wave amplitude, providing calmer water for moored boats.

The central maximum (0° to 23.6° on each side) experiences strong wave action, as do the secondary maxima between the minima. Therefore, boats should ideally be positioned at 23.6° or 53.1° from the opening's centerline for maximum protection. The lack of a third minimum (which would require $\sin \theta > 1$) means there are only two protected zones on each side of the harbor.

This same principle applies to:

- Sound passing through doorways
- Light through narrow apertures
- Radio waves passing around obstacles

The key insight is that diffraction provides natural "quiet zones" at specific angles, and harbor design can exploit this by positioning docks and moorings at these protected angles. For ocean waves with typical wavelengths of 10-30 m, harbor entrances of 25-100 m create significant diffraction effects.

Integrated Concepts

An aircraft maintenance technician walks past a tall hangar door that acts like a single slit for sound entering the hangar. Outside the door, on a line perpendicular to the opening in the door, a jet engine makes a 600-Hz sound. At what angle with the door will the technician observe the first minimum in sound intensity if the vertical opening is 0.800 m wide and the speed of sound is 340 m/s?

Show Solution

Strategy

This is a single-slit diffraction problem for sound waves. First, we find the wavelength of sound using $v = f\lambda$. Then we use the single-slit equation $D \sin \theta = m\lambda$ with m = 1 for the first minimum.

Solution

Step 1: Find the wavelength of sound

Given:

- $$ f = 600 \text{ Hz} $$
- $$ v = 340 \text{ m/s} $$

$$\lambda = \frac{v}{f} = \frac{340 \text{ m/s}}{600 \text{ Hz}} = 0.567 \text{ m}$$

Step 2: Find the angle of the first minimum

Given:

- $D = 0.800 \text{ m}$ (door width)
- $m = 1$ (first minimum)
- $$ \lambda = 0.567 \text{ m} $$

Using $D \sin \theta = m\lambda$:

$$\sin \theta = \frac{m\lambda}{D} = \frac{(1)(0.567 \text{ m})}{0.800 \text{ m}} = 0.709$$
$$\theta = \sin^{-1}(0.709) = 45.1°$$

Discussion

The first minimum occurs at 45.1° from the perpendicular to the door. This relatively large angle demonstrates that sound diffracts significantly around the opening because the wavelength (0.567 m) is comparable to the door width (0.800 m). The ratio $D/\lambda = 0.800/0.567 \approx 1.41$, which is small, so diffraction effects are pronounced.

This explains why the technician can hear the jet engine even when not directly in line with the door opening - sound bends around the edges of the door. In contrast, if this were light ($\lambda \approx 500$ nm), the ratio D/λ would be about 1.6 million, producing an essentially sharp-edged shadow with the first minimum at an extremely small angle.

This problem beautifully illustrates why sound “bends around corners” while light appears to travel in straight lines - it’s all about the ratio of wavelength to obstacle size.

Glossary

destructive interference for a single slit
: occurs when $D \sin \theta = m \lambda, \left(\text{ for } m=1 \text{,} -1 \text{,} 2 \text{,} -2 \text{,} 3 \text{,} \dots \right)$, where D is the slit width, λ is the light’s wavelength, θ is the angle relative to the original direction of the light, and m is the order of the minimum

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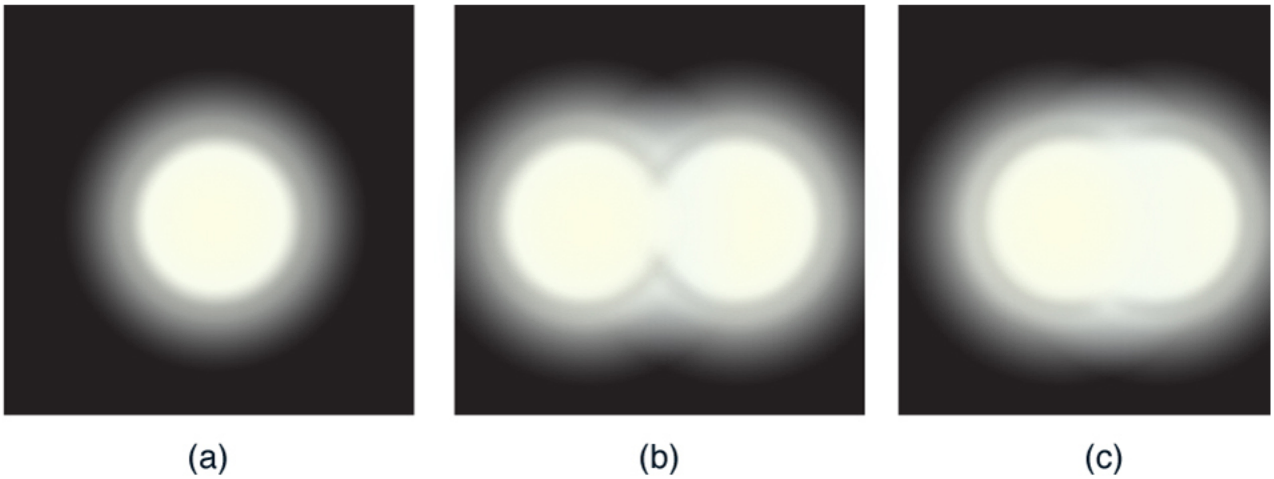
Limits of Resolution: The Rayleigh Criterion

- Discuss the Rayleigh criterion.

Light diffracts as it moves through space, bending around obstacles, interfering constructively and destructively. While this can be used as a spectroscopic tool—a diffraction grating disperses light according to wavelength, for example, and is used to produce spectra—diffraction also limits the detail we can obtain in images. [Figure 1](a) shows the effect of passing light through a small circular aperture. Instead of a bright spot with sharp edges, a spot with a fuzzy edge surrounded by circles of light is obtained. This pattern is caused by diffraction similar to that produced by a single slit. Light from different parts of the circular aperture interferes constructively and destructively. The effect is most noticeable when the aperture is small, but the effect is there for large apertures, too.



(a) Monochromatic light passed through a small circular aperture produces this diffraction pattern. (b) Two point light sources that are close to one another produce overlapping images because of diffraction. (c) If they are closer together, they cannot be resolved or distinguished.

How does diffraction affect the detail that can be observed when light passes through an aperture? [Figure 1](b) shows the diffraction pattern produced by two point light sources that are close to one another. The pattern is similar to that for a single point source, and it is just barely possible to tell that there are two light sources rather than one. If they were closer together, as in [Figure 1](c), we could not distinguish them, thus limiting the detail or resolution we can obtain. This limit is an inescapable consequence of the wave nature of light.

There are many situations in which diffraction limits the resolution. The acuity of our vision is limited because light passes through the pupil, the circular aperture of our eye. Be aware that the diffraction-like spreading of light is due to the limited diameter of a light beam, not the interaction with an aperture. Thus light passing through a lens with a diameter D shows this effect and spreads, blurring the image, just as light passing through an aperture of diameter D does. So diffraction limits the resolution of any system having a lens or mirror. Telescopes are also limited by diffraction, because of the finite diameter D of their primary mirror.

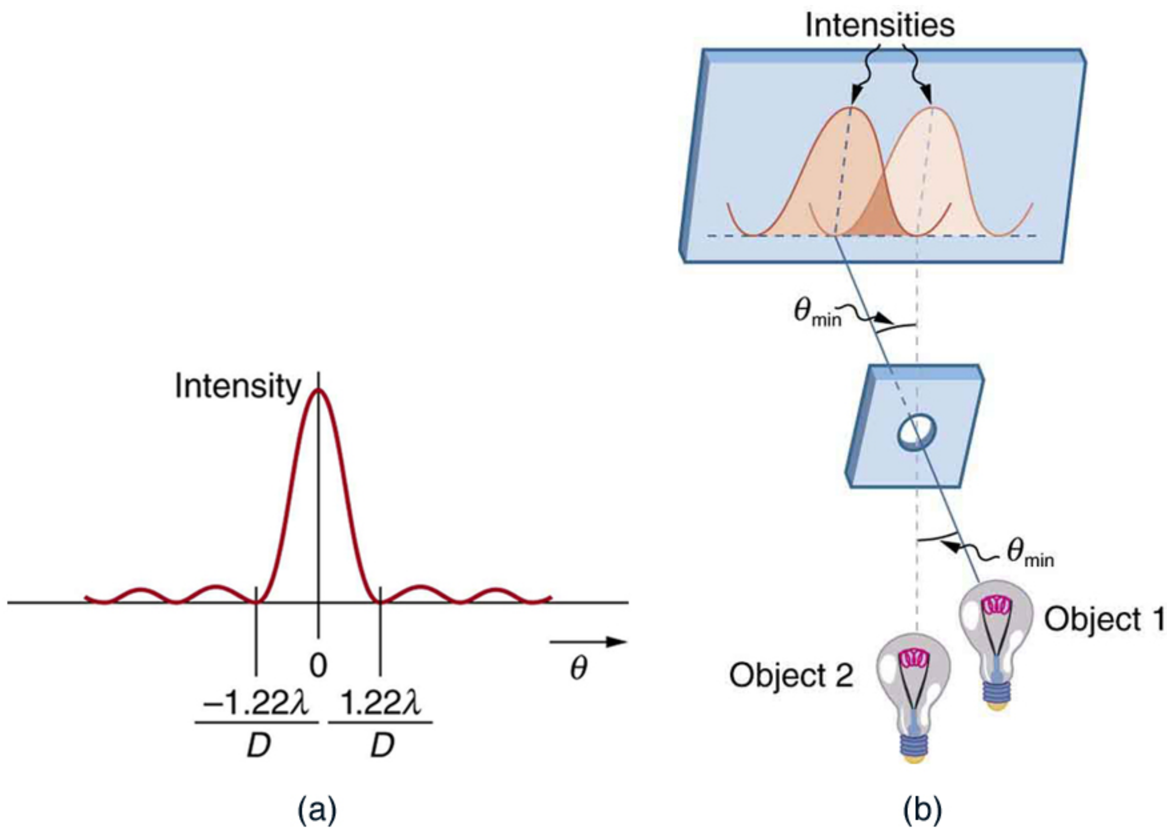
Take-Home Experiment: Resolution of the Eye

Draw two lines on a white sheet of paper (several mm apart). How far away can you be and still distinguish the two lines? What does this tell you about the size of the eye's pupil? Can you be quantitative? (The size of an adult's pupil is discussed in Physics of the Eye.)

Just what is the limit? To answer that question, consider the diffraction pattern for a circular aperture, which has a central maximum that is wider and brighter than the maxima surrounding it (similar to a slit) [see [Figure 2](a)]. It can be shown that, for a circular aperture of diameter D , the first minimum in the diffraction pattern occurs at $\theta = 1.22\lambda/D$ (providing the aperture is large compared with the wavelength of light, which is the case for most optical instruments). The accepted criterion for determining the diffraction limit to resolution based on this angle was developed by Lord Rayleigh in the 19th century. The **Rayleigh criterion** for the diffraction limit to resolution states that *two images are just resolvable when the center of the diffraction pattern of one is directly over the first minimum of the diffraction pattern of the other*. See [Figure 2](b). The first minimum is at an angle of $\theta = 1.22\lambda/D$, so that two point objects are just resolvable if they are separated by the angle

$$\theta = 1.22\lambda D,$$

where λ is the wavelength of light (or other electromagnetic radiation) and D is the diameter of the aperture, lens, mirror, etc., with which the two objects are observed. In this expression, θ has units of radians.



(a) Graph of intensity of the diffraction pattern for a circular aperture. Note that, similar to a single slit, the central maximum is wider and brighter than those to the sides. (b) Two point objects produce overlapping diffraction patterns. Shown here is the Rayleigh criterion for being just resolvable. The central maximum of one pattern lies on the first minimum of the other.

Connections: Limits to Knowledge

All attempts to observe the size and shape of objects are limited by the wavelength of the probe. Even the small wavelength of light prohibits exact precision. When extremely small wavelength probes as with an electron microscope are used, the system is disturbed, still limiting our knowledge, much as making an electrical measurement alters a circuit. Heisenberg's uncertainty principle asserts that this limit is fundamental and inescapable, as we shall see in quantum mechanics.

Calculating Diffraction Limits of the Hubble Space Telescope

The primary mirror of the orbiting Hubble Space Telescope has a diameter of 2.40 m. Being in orbit, this telescope avoids the degrading effects of atmospheric distortion on its resolution. (a) What is the angle between two just-resolvable point light sources (perhaps two stars)? Assume an average light wavelength of 550 nm. (b) If these two stars are at the 2 million light year distance of the Andromeda galaxy, how close together can they be and still be resolved? (A light year, or ly, is the distance light travels in 1 year.)

Strategy

The Rayleigh criterion stated in the equation $\theta = 1.22 \lambda D$ gives the smallest possible angle θ ** between point sources, or the best obtainable resolution. Once this angle is found, the distance between stars can be calculated, since we are given how far away they are.

Solution for (a)

The Rayleigh criterion for the minimum resolvable angle is

$$\theta = 1.22 \lambda D.$$

Entering known values gives

$$\theta = 1.22 550 \times 10^{-9} \text{m} 2.40 \text{m} \quad = 2.80 \times 10^{-7} \text{rad}.$$

Solution for (b)

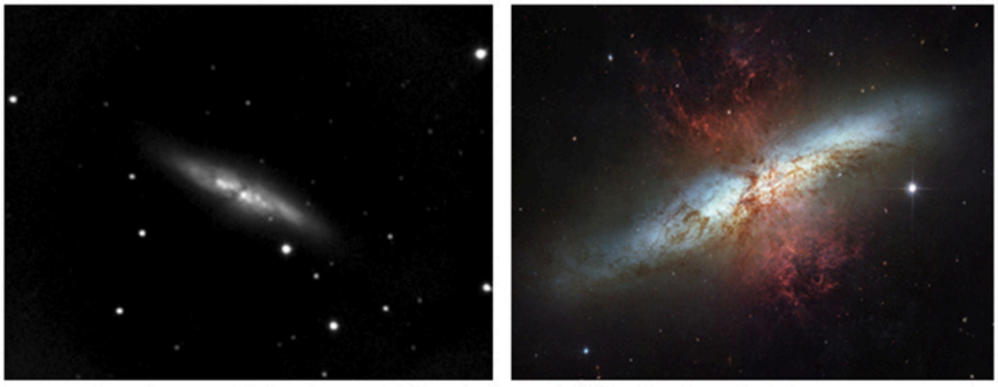
The distance s between two objects a distance r away and separated by an angle θ is $s = r\theta$.

Substituting known values gives

$$s \quad = \quad (2.0 \times 10^{6} \text{ly})(2.80 \times 10^{-7} \text{rad}) \quad s \quad = \quad 0.56 \text{ ly}.$$

Discussion

The angle found in part (a) is extraordinarily small (less than 1/50 000 of a degree), because the primary mirror is so large compared with the wavelength of light. As noticed, diffraction effects are most noticeable when light interacts with objects having sizes on the order of the wavelength of light. However, the effect is still there, and there is a diffraction limit to what is observable. The actual resolution of the Hubble Telescope is not quite as good as that found here. As with all instruments, there are other effects, such as non-uniformities in mirrors or aberrations in lenses that further limit resolution. However, [Figure 3] gives an indication of the extent of the detail observable with the Hubble because of its size and quality and especially because it is above the Earth's atmosphere.

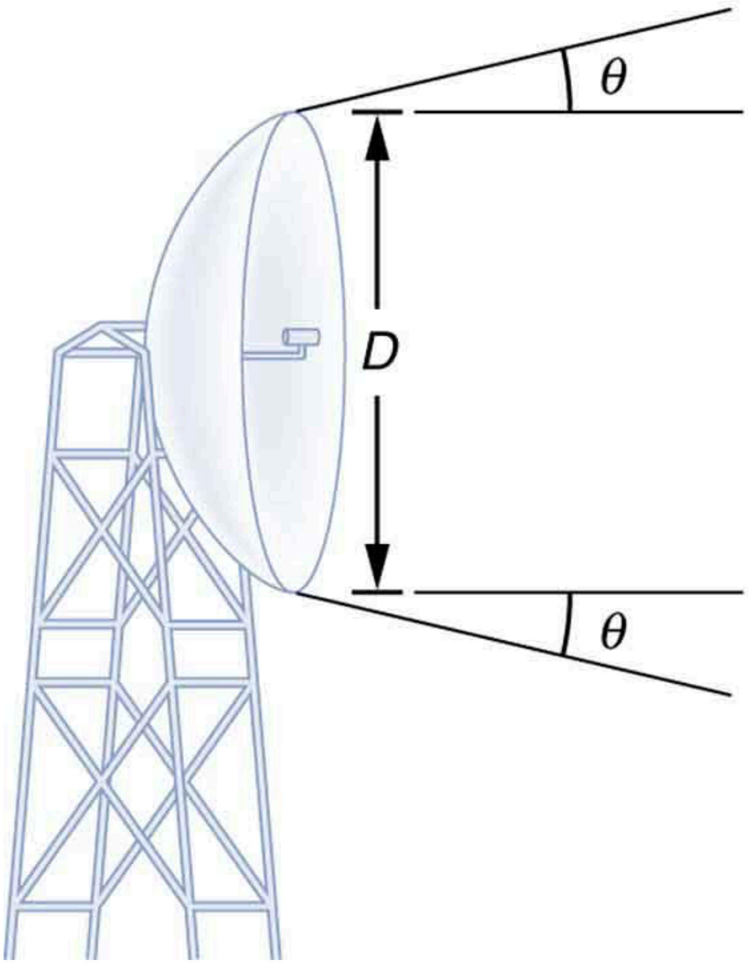


These two photographs of the M82 galaxy give an idea of the observable detail using the Hubble Space Telescope compared with that using a ground-based telescope. (a) On the left is a ground-based image. (credit: Ricnun, Wikimedia Commons) (b) The photo on the right was captured by Hubble. (credit: NASA, ESA, and the Hubble Heritage Team (STScI/AURA))

The answer in part (b) indicates that two stars separated by about half a light year can be resolved. The average distance between stars in a galaxy is on the order of 5 light years in the outer parts and about 1 light year near the galactic center. Therefore, the Hubble can resolve most of the individual stars in Andromeda galaxy, even though it lies at such a huge distance that its light takes 2 million years for its light to reach us. [Figure 4] shows another mirror used to observe radio waves from outer space.

A 305-m-diameter natural bowl at Arecibo in Puerto Rico is lined with reflective material, making it into a radio telescope. It is the largest curved focusing dish in the world. Although D for Arecibo is much larger than for the Hubble Telescope, it detects much longer wavelength radiation and its diffraction limit is significantly poorer than Hubble's. Arecibo is still very useful, because important information is carried by radio waves that is not carried by visible light. (credit: Tatyana Temirbulatova, Flickr)

Diffraction is not only a problem for optical instruments but also for the electromagnetic radiation itself. Any beam of light having a finite diameter D and a wavelength λ exhibits diffraction spreading. The beam spreads out with an angle θ given by the equation $\theta = 1.22 \lambda D$. Take, for example, a laser beam made of rays as parallel as possible (angles between rays as close to $\theta = 0^{\circ}$ as possible) instead spreads out at an angle $\theta = 1.22 \lambda / D$, where D is the diameter of the beam and λ is its wavelength. This spreading is impossible to observe for a flashlight, because its beam is not very parallel to start with. However, for long-distance transmission of laser beams or microwave signals, diffraction spreading can be significant (see [Figure 5]). To avoid this, we can increase D . This is done for laser light sent to the Moon to measure its distance from the Earth. The laser beam is expanded through a telescope to make D much larger and θ smaller.



The beam produced by this microwave transmission antenna will spread out at a minimum angle $\theta =1.22 \lambda /D$ due to diffraction. It is impossible to produce a near-parallel beam, because the beam has a limited diameter.

In most biology laboratories, resolution is presented when the use of the microscope is introduced. The ability of a lens to produce sharp images of two closely spaced point objects is called resolution. The smaller the distance x

by which two objects can be separated and still be seen as distinct, the greater the resolution. The resolving power of a lens is defined as that distance x . An expression for resolving power is obtained from the Rayleigh criterion. In [Figure 6](a) we have two point objects separated by a distance x . According to the Rayleigh criterion, resolution is possible when the minimum angular separation is

$$\theta = 1.22\frac{\lambda}{D} = \frac{x}{d},$$

where d is the distance between the specimen and the objective lens, and we have used the small angle approximation (i.e., we have assumed that x is much smaller than d), so that $\tan\theta \approx \sin\theta \approx \theta$.

Therefore, the resolving power is

$$x = 1.22\frac{\lambda d}{D}.$$

Another way to look at this is by re-examining the concept of Numerical Aperture (NA) discussed in Microscopes. There, NA is a measure of the maximum acceptance angle at which the fiber will take light and still contain it within the fiber. [Figure 6](b) shows a lens and an object at point P. The NA here is a measure of the ability of the lens to gather light and resolve fine detail. The angle subtended by the lens at its focus is defined to be $\theta = 2\alpha$. From the figure and again using the small angle approximation, we can write

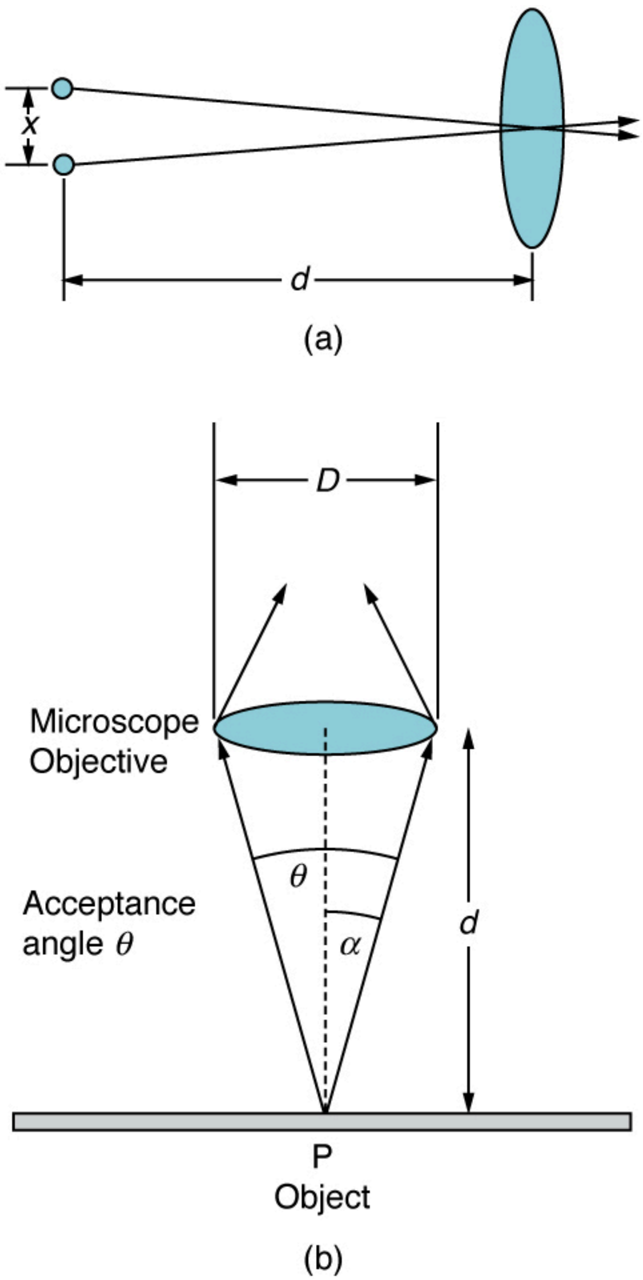
$$\sin\alpha = \frac{D/2}{d} = \frac{D}{2d}.$$

The NA for a lens is $\mathrm{NA} = n \sin\alpha$, where n is the index of refraction of the medium between the objective lens and the object at point P.

From this definition for NA , we can see that

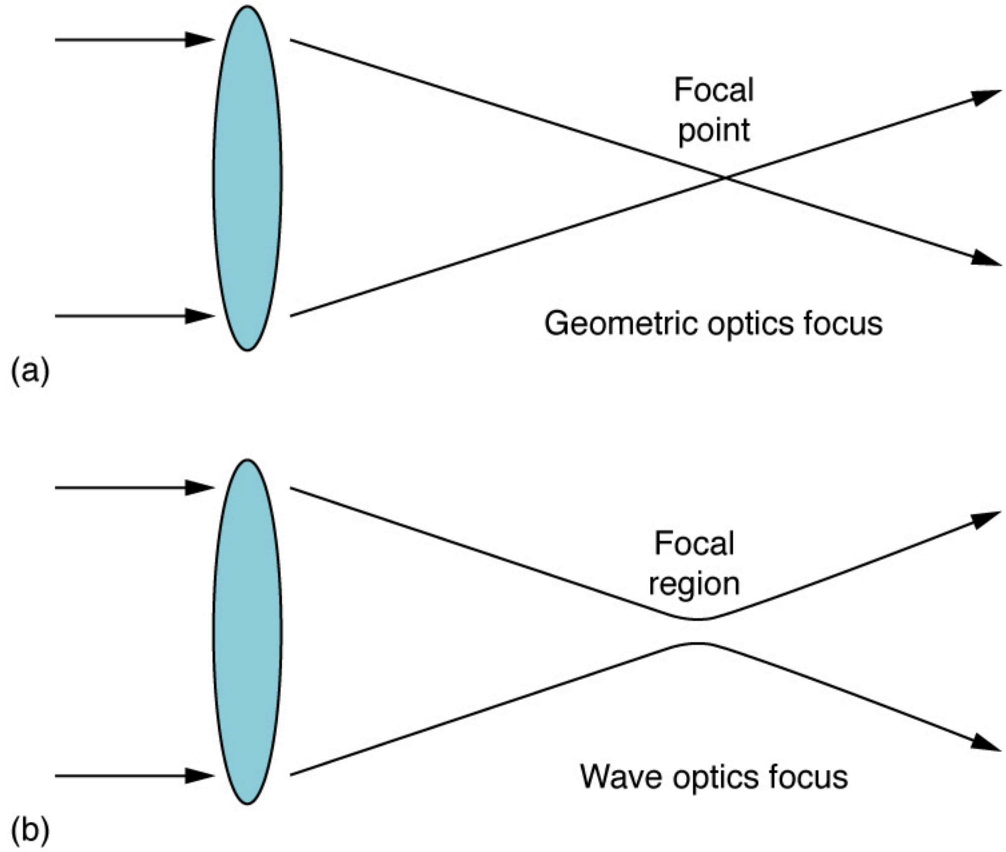
$$x = 1.22\frac{\lambda d}{D} = 1.22\frac{\lambda}{2\sin\alpha} = 0.61\frac{\lambda n}{\mathrm{NA}}.$$

In a microscope, NA is important because it relates to the resolving power of a lens. A lens with a large NA will be able to resolve finer details. Lenses with larger NA will also be able to collect more light and so give a brighter image. Another way to describe this situation is that the larger the NA , the larger the cone of light that can be brought into the lens, and so more of the diffraction modes will be collected. Thus the microscope has more information to form a clear image, and so its resolving power will be higher.



(a) Two points separated by at distance \(x \) and a positioned a distance \(d \) away from the objective. (credit: Infopro, Wikimedia Commons) (b) Terms and symbols used in discussion of resolving power for a lens and an object at point P. (credit: Infopro, Wikimedia Commons)

One of the consequences of diffraction is that the focal point of a beam has a finite width and intensity distribution. Consider focusing when only considering geometric optics, shown in [Figure 7](a). The focal point is infinitely small with a huge intensity and the capacity to incinerate most samples irrespective of the NA of the objective lens. For wave optics, due to diffraction, the focal point spreads to become a focal spot (see [Figure 7](b)) with the size of the spot decreasing with increasing NA . Consequently, the intensity in the focal spot increases with increasing NA . The higher the NA , the greater the chances of photodegrading the specimen. However, the spot never becomes a true point.



(a) In geometric optics, the focus is a point, but it is not physically possible to produce such a point because it implies infinite intensity. (b) In wave optics, the focus is an extended region.

Section Summary

- Diffraction limits resolution.
- For a circular aperture, lens, or mirror, the Rayleigh criterion states that two images are just resolvable when the center of the diffraction pattern of one is directly over the first minimum of the diffraction pattern of the other.
- This occurs for two point objects separated by the angle $\theta = 1.22 \lambda D$, where λ is the wavelength of light (or other electromagnetic radiation) and D is the diameter of the aperture, lens, mirror, etc. This equation also gives the angular spreading of a source of light having a diameter D .

Conceptual Questions

A beam of light always spreads out. Why can a beam not be created with parallel rays to prevent spreading? Why can lenses, mirrors, or apertures not be used to correct the spreading?

Problems & Exercises

The 300-m-diameter Arecibo radio telescope pictured in [Figure 4] detects radio waves with a 4.00 cm average wavelength.

(a) What is the angle between two just-resolvable point sources for this telescope?

(b) How close together could these point sources be at the 2 million light year distance of the Andromeda galaxy?

Show Solution

Strategy

Use the Rayleigh criterion $\theta = 1.22\lambda/D$ for part (a). For part (b), use $s = r\theta$ to find the linear separation at the given distance.

Solution

Given:

- $D = 300$ m (diameter of telescope)
- $\lambda = 4.00$ cm $= 0.0400$ m (radio wavelength)
- $r = 2$ million light years (for part b)

(a) Minimum resolvable angle:

$$\theta=1.22\lambda D=1.220.0400300=1.63\times10^{-4}\text{ rad}$$

(b) Minimum separation at Andromeda distance:

Convert distance: 1 ly ≈ 9.46×10^{15} m

$$r=2\times10^{6}\text{ ly}\times9.46\times10^{15}\text{ m/ly}=1.89\times10^{22}\text{ m}$$
$$s=r\theta=(1.89\times10^{22})(1.63\times10^{-4})=3.08\times10^{18}\text{ m}$$

Converting back to light years:

$$s=3.08\times10^{18}9.46\times10^{15}=326\text{ ly}$$

Discussion

Despite Arecibo's enormous 300-m diameter, radio waves have much longer wavelengths than visible light (4 cm vs ~550 nm, a factor of ~70,000 times longer). This gives relatively poor angular resolution (1.63×10^{-4} rad) compared to optical telescopes. At Andromeda's distance, the telescope can only resolve features separated by 326 light years—larger than the distance between many nearby stars. This demonstrates why radio astronomy often uses interferometric arrays (like the VLA) to achieve better resolution by effectively increasing D.

Assuming the angular resolution found for the Hubble Telescope in [Example 1], what is the smallest detail that could be observed on the Moon?

Show Solution

Strategy

From Example 1, Hubble's angular resolution is $\theta = 2.80 \times 10^{-7}$ rad. Use $s = r\theta$ with r = distance to Moon ≈ 3.84×10^{8} m.

Solution

$$s=r\theta=(3.84\times10^{8}\text{ m})(2.80\times10^{-7}\text{ rad})=108\text{ m}$$

Discussion

Hubble can resolve details as small as 108 m (about the length of a football field) on the Moon. This is why Hubble cannot see the Apollo landing sites - the lunar modules and equipment are only a few meters in size, far below this resolution limit.

Diffraction spreading for a flashlight is insignificant compared with other limitations in its optics, such as spherical aberrations in its mirror. To show this, calculate the minimum angular spreading of a flashlight beam that is originally 5.00 cm in diameter with an average wavelength of 600 nm.

Show Solution

Strategy

Use the Rayleigh criterion $\theta = 1.22\lambda/D$ for the diffraction spreading angle.

Solution

Given:

- D = 5.00 cm = 0.0500 m
- λ = 600 nm = 600×10^{-9} m

$$\theta=1.22\lambda D=1.22600\times10^{-9}0.0500$$
$$\theta=7.32\times10^{-7}0.0500=1.46\times10^{-5}\text{ rad}$$

Converting to degrees: $\theta = 1.46 \times 10^{-5}$ rad × (180°/π) = 8.4×10^{-4} degrees ≈ **0.05 arcminutes**

Discussion

This extremely small diffraction angle (0.00146 milliradians or about 3 arcseconds) is indeed negligible compared to other optical imperfections. Typical flashlight beams have divergence angles of several degrees due to:

- Spherical aberration in the reflector
- Imperfect parabolic shape
- Extended source size (the bulb filament or LED)
- Surface roughness

The diffraction limit would only become important for very high-quality, nearly perfect optical systems. For a flashlight at 100 m distance, diffraction would spread the beam by only s = (100)(1.46×10^{-5}) = 1.5 mm, which is completely insignificant compared to the beam's actual spread of several meters due to optical imperfections.

(a) What is the minimum angular spread of a 633-nm wavelength He-Ne laser beam that is originally 1.00 mm in diameter?

(b) If this laser is aimed at a mountain cliff 15.0 km away, how big will the illuminated spot be?

(c) How big a spot would be illuminated on the Moon, neglecting atmospheric effects? (This might be done to hit a corner reflector to measure the round-trip time and, hence, distance.) Explicitly show how you follow the steps in [Problem-Solving Strategies for Wave Optics](#).

Show Solution

Strategy

Step 1: This is a diffraction spreading problem. **Step 2:** Use θ = 1.22λ/D for circular beam diffraction. **Step 3:** (a) Find θ; (b) find spot size at 15 km; (c) find spot size at Moon distance. **Step 4:** Given: λ = 633 nm, D = 1.00 mm.

(a) Angular spread:

$$\theta=1.22\lambda D=1.22633\times10^{-9}1.00\times10^{-3}=7.72\times10^{-4}\text{ rad}$$

(b) Spot size at 15.0 km:

$$s=r\theta=(15.0\times10^{3})(7.72\times10^{-4})=11.6\text{ m (diameter)}$$

(c) Spot size on Moon ($r = 3.84 \times 10^8$ m):

$$s=(3.84\times10^{8})(7.72\times10^{-4})=2.96\times10^{5}\text{ m}=296\text{ km}$$

Step 5: These results are reasonable - laser beams spread due to diffraction despite being highly collimated.

Discussion

A 296-km spot on the Moon from a 1-mm beam shows significant diffraction spreading. To reduce this, lasers sent to the Moon are expanded through telescopes to increase D, reducing θ = 1.22λ/D.

A telescope can be used to enlarge the diameter of a laser beam and limit diffraction spreading. The laser beam is sent through the telescope in opposite the normal direction and can then be projected onto a satellite or the Moon.

(a) If this is done with the Mount Wilson telescope, producing a 2.54-m-diameter beam of 633-nm light, what is the minimum angular spread of the beam?

(b) Neglecting atmospheric effects, what is the size of the spot this beam would make on the Moon, assuming a lunar distance of $3.84\times10^{8}\text{m}$?

Show Solution

Strategy

Use θ = 1.22λ/D for part (a), then s = rθ for part (b) to find the spot diameter on the Moon.

Solution

Given:

- D = 2.54 m (beam diameter after telescope)
- λ = 633 nm = 633×10^{-9} m (He-Ne laser)
- r = 3.84×10^{8} m (Earth-Moon distance)

(a) Minimum angular spread:

$$\theta=1.22\lambda D=1.22633\times10^{-9}2.54$$
$$\theta=7.72\times10^{-7}2.54=3.04\times10^{-7}\text{ rad}$$

(b) Spot size on the Moon:

$$s=r\theta=(3.84\times10^{8})(3.04\times10^{-7})=117\text{ m (radius)}$$

Diameter: **2 × 117 m = 235 m**

Discussion

By expanding the laser beam to 2.54 m diameter using the telescope, the diffraction spreading is dramatically reduced compared to the 1-mm beam in the previous problem (which gave a 296-km spot). The spot size on the Moon is now only 235 m in diameter—over 1000 times smaller!

This technique is actually used in lunar laser ranging experiments, where pulses from Earth-based lasers reflect off corner retroreflectors left by Apollo astronauts. The expanded beam reduces diffraction spreading, allowing more photons to hit the small retroreflector targets and return to Earth for precise distance measurements (accurate to a few centimeters).

The limit to the eye's acuity is actually related to diffraction by the pupil.

(a) What is the angle between two just-resolvable points of light for a 3.00-mm-diameter pupil, assuming an average wavelength of 550 nm?

(b) Take your result to be the practical limit for the eye. What is the greatest possible distance a car can be from you if you can resolve its two headlights, given they are 1.30 m apart?

(c) What is the distance between two just-resolvable points held at an arm's length (0.800 m) from your eye?

(d) How does your answer to (c) compare to details you normally observe in everyday circumstances?

Show Solution

(a) Minimum resolvable angle:

$$\theta = 1.22\frac{\lambda}{D} = 1.22\frac{550\times10^{-9}}{3.00\times10^{-3}} = 2.24\times10^{-4}\text{ rad}$$

Converting: $\theta = 2.24 \times 10^{-4}$ rad × (180/π) × 60 = 0.77 arcmin ≈ **46 arcseconds**

(b) Maximum distance to resolve headlights:

$$r = \frac{s}{\theta} = \frac{1.30}{2.24\times10^{-4}} = 5800\text{ m} = 5.8\text{ km}$$

(c) Separation at arm's length:

$$s = r\theta = (0.800)(2.24\times10^{-4}) = 1.79\times10^{-4}\text{ m} = 0.18\text{ mm}$$

(d) Comparison: 0.18 mm is about twice the width of a human hair (~0.1 mm) and matches our everyday experience - we can barely distinguish two dots separated by this distance at arm's length. This is consistent with typical human visual acuity of about 20/20 vision.

What is the minimum diameter mirror on a telescope that would allow you to see details as small as 5.00 km on the Moon some 384 000 km away? Assume an average wavelength of 550 nm for the light received.

Show Solution

Strategy

Use s = rθ to find the required angular resolution, then use θ = 1.22λ/D to solve for the minimum diameter D.

Solution

Given:

- s = 5.00 km = 5000 m (detail size on Moon)
- r = 384,000 km = 3.84×10^8 m (Earth-Moon distance)
- λ = 550 nm = 550×10^{-9} m

Step 1: Find required angular resolution

$$\theta = \frac{s}{r} = \frac{5000}{3.84\times10^{8}} = 1.302\times10^{-5}\text{ rad}$$

Step 2: Find minimum mirror diameter

Using θ = 1.22λ/D:

$$D = \frac{1.22\lambda}{\theta} = \frac{1.22\times550\times10^{-9}}{1.302\times10^{-5}}$$
$$D = \frac{6.71\times10^{-7}}{1.302\times10^{-5}} = 0.0515\text{ m} = 5.15\text{ cm}$$

Discussion

A surprisingly small telescope mirror of only 5.15 cm (about 2 inches) diameter is theoretically sufficient to resolve 5-km features on the Moon—assuming perfect optics and no atmospheric distortion. This is within the range of good amateur telescopes.

However, in practice, atmospheric turbulence (seeing) limits ground-based telescopes to resolution of about 1 arcsecond (roughly 2 km on the Moon), regardless of aperture size. This is why even large telescopes on Earth can't significantly outperform small ones for lunar observations unless adaptive optics is used to compensate for atmospheric effects. Space-based telescopes like Hubble avoid this problem entirely.

You are told not to shoot until you see the whites of their eyes. If the eyes are separated by 6.5 cm and the diameter of your pupil is 5.0 mm, at what distance can you resolve the two eyes using light of wavelength 555 nm?

Show Solution

Strategy

Find the minimum resolvable angle θ = 1.22λ/D, then find the maximum distance r = s/θ where the eyes can be resolved.

Solution

$\theta=1.22\lambda D=1.22555\times10^{-9}5.0\times10^{-3}=1.35\times10^{-4}$ rad

$r=s\theta=0.0651.35\times10^{-4}=481$ m

Discussion

You could resolve the two eyes at distances up to about 480 m. This is remarkably far - nearly half a kilometer! Of course, in practice, other factors like atmospheric turbulence and the need to see details smaller than eye separation would limit the actual distance.

(a) The planet Pluto and its Moon Charon are separated by 19 600 km. Neglecting atmospheric effects, should the 5.08-m-diameter Mount Palomar telescope be able to resolve these bodies when they are $4.50\times10^{9}\text{km}$ from Earth? Assume an average wavelength of 550 nm.

(b) In actuality, it is just barely possible to discern that Pluto and Charon are separate bodies using an Earth-based telescope. What are the reasons for this?

Show Solution

Strategy

For part (a), calculate the angular separation of Pluto and Charon, then compare to the telescope's angular resolution θ = 1.22λ/D. For part (b), consider atmospheric effects.

Solution

Given:

- Separation: s = 19,600 km = 1.96×10^{7} m
- Distance: r = 4.50×10^{9} km = 4.50×10^{12} m
- Mirror diameter: D = 5.08 m
- Wavelength: λ = 550 nm = 550×10^{-9} m

(a) Can the telescope resolve Pluto and Charon?

Angular separation:

$$\theta_{sep}=sr=1.96\times10^{7}4.50\times10^{12}=4.36\times10^{-6}\text{ rad}$$

Telescope's angular resolution (diffraction limit):

$$\theta_{min}=1.22\lambda D=1.22550\times10^{-9}5.08=1.32\times10^{-7}\text{ rad}$$

Comparison:

$$\theta_{sep}\theta_{min}=4.36\times10^{-6}1.32\times10^{-7}=33$$

Since the angular separation is **33 times larger** than the diffraction limit, **yes, the telescope should easily resolve Pluto and Charon** (neglecting atmospheric effects).

(b) Why is it just barely possible in reality?

The fact that it's just barely possible (rather than easy) to distinguish Pluto and Charon indicates **severe atmospheric aberrations:**

1. **Atmospheric turbulence (seeing)** limits ground-based telescopes to angular resolution of about 0.5-1.0 arcsecond (2.4-4.8×10^{-6} rad) under good conditions—comparable to the 4.36×10^{-6} rad separation
2. **Atmospheric refraction** causes image distortion and blurring
3. **Thermal air currents** create "twinkling" that smears the images
4. **Limited integration time** means astronomers can't always capture moments of best seeing

The diffraction limit (1.32×10^{-7} rad) is 30-60 times better than atmospheric seeing limits. This is why space-based telescopes like Hubble can achieve their full diffraction-limited performance, while even the largest ground-based telescopes are limited by the atmosphere unless adaptive optics is used.

Discussion

This problem dramatically illustrates how Earth's atmosphere, not optics, limits most ground-based astronomical observations. A perfect 5-meter telescope in space would easily resolve Pluto-Charon, but from Earth's surface, atmospheric turbulence reduces the effective resolution by a factor of 30 or more, making the observation just barely possible.

The headlights of a car are 1.3 m apart. What is the maximum distance at which the eye can resolve these two headlights? Take the pupil diameter to be 0.40 cm.

Show Solution

Strategy

Use θ = 1.22λ/D with λ ≈ 550 nm, D = 0.40 cm = 4.0 mm, then r = s/θ.

Solution

$$\theta=\frac{1.22\,550\times10^{-9}}{4.0\times10^{-3}}=1.68\times10^{-4}\text{ rad}$$
$$r=\frac{s}{\theta}=\frac{1.3}{1.68\times10^{-4}}=7740\text{ m}\approx7.7\text{ km}$$

Discussion

The maximum distance is about 7.7 km. At night with dilated pupils (larger D), this distance would be somewhat shorter. In practice, atmospheric effects, haze, and the need to see the headlights' size (not just their separation) reduce the effective distance.

When dots are placed on a page from a laser printer, they must be close enough so that you do not see the individual dots of ink. To do this, the separation of the dots must be less than Raleigh's criterion. Take the pupil of the eye to be 3.0 mm and the distance from the paper to the eye of 35 cm; find the minimum separation of two dots such that they cannot be resolved. How many dots per inch (dpi) does this correspond to?

Show Solution

Strategy

Find θ = 1.22λ/D, then minimum separation s = rθ at distance r = 35 cm. Convert to dpi.

Solution

$$\theta=\frac{1.22\,550\times10^{-9}}{3.0\times10^{-3}}=2.24\times10^{-4}\text{ rad}$$
$$s=r\theta=(0.35)(2.24\times10^{-4})=7.84\times10^{-5}\text{ m}=78.4\ \mu\text{m}$$

Dots per inch:

$$\text{dpi}=\frac{1\text{ inch}}{s}=\frac{0.0254\text{ m}}{7.84\times10^{-5}\text{ m}}=324\text{ dpi}$$

Discussion

A resolution of about 300-324 dpi is sufficient to make individual dots unresolvable at normal reading distance (35 cm). This is why 300 dpi became the standard for laser printers - it exceeds the eye's diffraction-limited resolution. Higher dpi (600, 1200) improves quality for closer viewing or finer details but isn't necessary for normal reading.

Unreasonable Results

An amateur astronomer wants to build a telescope with a diffraction limit that will allow him to see if there are people on the moons of Jupiter.

(a) What diameter mirror is needed to be able to see 1.00 m detail on a Jovian Moon at a distance of $7.50\times10^{8}\text{km}$ from Earth? The wavelength of light averages 600 nm.

(b) What is unreasonable about this result?

(c) Which assumptions are unreasonable or inconsistent?

Show Solution

Strategy

Use Rayleigh criterion θ = 1.22λ/D and s = rθ to find required D.

Solution

(a) Required mirror diameter:

Given: s = 1.00 m, r = 7.50 × 10^8 km = 7.50 × 10^{11} m, λ = 600 nm

$$\theta=\frac{s}{r}=\frac{1.00}{7.50\times10^{11}}=1.33\times10^{-12}\text{ rad}$$

Using θ = 1.22λ/D:

$$D=\frac{1.22\lambda}{\theta}=\frac{1.22\times600\times10^{-9}}{1.33\times10^{-12}}=5.50\times10^{5}\text{ m}=550\text{ km}$$

(b) What is unreasonable?

A mirror diameter of **550 km** is absurdly large:

- Larger than most cities
- Would weigh billions of tons
- Impossible to construct, support, or aim
- Far exceeds any existing telescope (largest is ~10 m)
- Would cost trillions of dollars

(c) Unreasonable assumptions:

1. **Expecting to resolve 1-m details at Jupiter's distance** (750 million km) is unrealistic for any ground or even space-based telescope
2. The diffraction limit makes such resolution physically impossible without an impossibly large aperture
3. Even if built, atmospheric turbulence would prevent this resolution from Earth
4. The assumption that an "amateur" could build such a telescope is absurd

Discussion

This illustrates why we can't see fine details on distant planets. Even Hubble (2.4 m) can only resolve features ~150 km on Jupiter. To see people (meter-scale) would require space-based interferometry with baseline separations of hundreds of kilometers, far beyond amateur (or even professional single-telescope) capabilities.

Construct Your Own Problem

Consider diffraction limits for an electromagnetic wave interacting with a circular object. Construct a problem in which you calculate the limit of angular resolution with a device, using this circular object (such as a lens, mirror, or antenna) to make observations. Also calculate the limit to spatial resolution (such as the size of features observable on the Moon) for observations at a specific distance from the device. Among the things to be considered are the wavelength of electromagnetic radiation used, the size of the circular object, and the distance to the system or phenomenon being observed.

Glossary

Rayleigh criterion
: two images are just resolvable when the center of the diffraction pattern of one is directly over the first minimum of the diffraction pattern of the other

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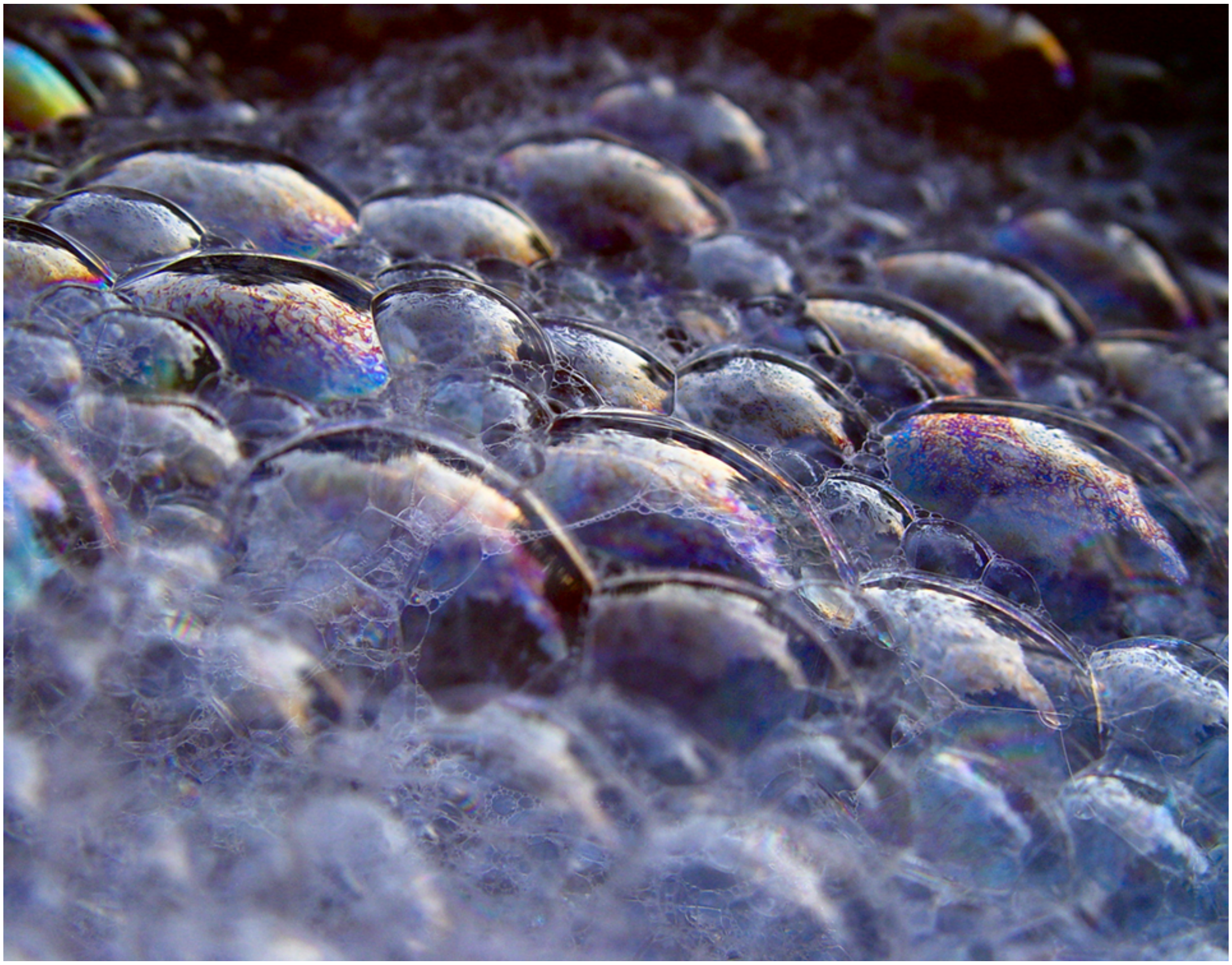
Thin Film Interference

- Discuss the rainbow formation by thin films.

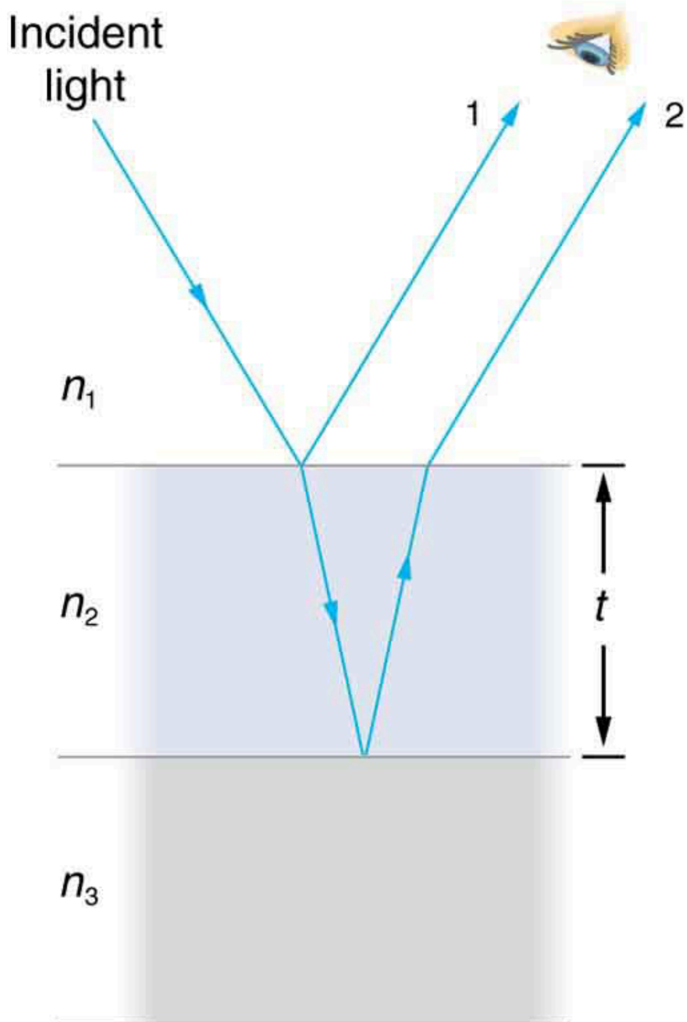
The bright colors seen in an oil slick floating on water or in a sunlit soap bubble are caused by interference. The brightest colors are those that interfere constructively. This interference is between light reflected from different surfaces of a thin film; thus, the effect is known as **thin film interference** . As noticed before, interference effects are most prominent when light interacts with something having a size similar to its wavelength. A thin film is one having a thickness t smaller than a few times the wavelength of light, λ . Since color is associated indirectly with λ and since all interference depends in some way on the ratio of λ to the size of the object involved, we should expect to see different colors for different thicknesses of a film, as in [Figure 1].



These soap bubbles exhibit brilliant colors when exposed to sunlight. (credit: Scott Robinson, Flickr)

What causes thin film interference? [Figure 2] shows how light reflected from the top and bottom surfaces of a film can interfere. Incident light is only partially reflected from the top surface of the film (ray 1). The remainder enters the film and is itself partially reflected from the bottom surface. Part of the light reflected from the bottom surface can emerge from the top of the film (ray 2) and interfere with light reflected from the top (ray 1). Since the ray that enters the film travels a greater distance, it may be in or out of phase with the ray reflected from the top. However, consider for a moment, again, the bubbles in [Figure 1]. The bubbles are darkest where they are thinnest. Furthermore, if you observe a soap bubble carefully, you will note it gets dark at the point where it breaks. For very thin films, the difference in path lengths of ray 1 and ray 2 in [Figure 2] is negligible; so why should they interfere destructively and not constructively? The answer is that a phase change can occur upon reflection. The rule is as follows:

When light reflects from a medium having an index of refraction greater than that of the medium in which it is traveling, a 180° phase change (or a $\lambda/2$ shift) occurs.



Light striking a thin film is partially reflected (ray 1) and partially refracted at the top surface. The refracted ray is partially reflected at the bottom surface and emerges as ray 2. These rays will interfere in a way that depends on the thickness of the film and the indices of refraction of the various media.

If the film in [Figure 2] is a soap bubble (essentially water with air on both sides), then there is a $\lambda/2$ shift for ray 1 and none for ray 2. Thus, when the film is very thin, the path length difference between the two rays is negligible, they are exactly out of phase, and destructive interference will occur at all wavelengths and so the soap bubble will be dark here.

The thickness of the film relative to the wavelength of light is the other crucial factor in thin film interference. Ray 2 in [Figure 2] travels a greater distance than ray 1. For light incident perpendicular to the surface, ray 2 travels a distance approximately $2t$ farther than ray 1. When this distance is an integral or half-integral multiple of the wavelength in the medium ($\lambda_n = \lambda/n$, where λ is the wavelength in vacuum and n is the index of refraction), constructive or destructive interference occurs, depending also on whether there is a phase change in either ray.

Thin-film interference has created an entire field of research and industrial applications. Its foundations were laid by Irving Langmuir and Katharine Burr Blodgett, working at General Electric in the 1920s and 1930s. Langmuir had pioneered a method for producing ultra-thin layers on materials. Blodgett built on these practices by creating a method to precisely stack and compress these layers in order to produce a film of a desired thickness and quality. The device they developed became known as the Langmuir-Blodgett trough, built from principles developed by Agnes Pockels and still used in laboratories today. The earliest widely applied use of these principles was non-reflective glass, which Blodgett patented in 1938 and which was used almost immediately in the making of the film *Gone With the Wind*. The film is viewed as a tremendous leap in cinematography; cameras, microscopes, telescopes, and many other instruments rely on Blodgett's invention as well.

Calculating Non-reflective Lens Coating Using Thin Film Interference

Sophisticated cameras use a series of several lenses. Light can reflect from the surfaces of these various lenses and degrade image clarity. To limit these reflections, lenses are coated with a thin layer of magnesium fluoride that causes destructive thin film interference. What is the thinnest this film can be, if its index of refraction is 1.38 and it is designed to limit the reflection of 550-nm light, normally the most intense visible wavelength? The index of refraction of glass is 1.52.

Strategy

Refer to [Figure 2] and use $n_1 = 1.00$ for air, $n_2 = 1.38$, and $n_3 = 1.52$. Both ray 1 and ray 2 will have a $\lambda/2$ shift upon reflection. Thus, to obtain destructive interference, ray 2 will need to travel a half wavelength farther than ray 1. For rays incident perpendicularly, the path length difference is $2t$.

Solution

To obtain destructive interference here,

$$2t=\lambda_{n_2}2,$$

where λ_{n_2} is the wavelength in the film and is given by $\lambda_{n_2}=\lambda n_2$.

Thus,

$$2t=\lambda/n_2 2.$$

Solving for t and entering known values yields

$$t \quad = \quad \lambda/n_2 4=(550\text{nm})/1.384 \quad = \quad 99.6\text{nm}.$$

Discussion

Films such as the one in this example are most effective in producing destructive interference when the thinnest layer is used, since light over a broader range of incident angles will be reduced in intensity. These films are called non-reflective coatings; this is only an approximately correct description, though, since other wavelengths will only be partially cancelled. Non-reflective coatings are used in car windows and sunglasses.

Thin film interference is most constructive or most destructive when the path length difference for the two rays is an integral or half-integral wavelength, respectively. That is, for rays incident perpendicularly, $2t=\lambda_n, 2\lambda_n, 3\lambda_n, \ldots$ or $2t=\lambda_n/2, 3\lambda_n/2, 5\lambda_n/2, \ldots$. To know whether interference is constructive or destructive, you must also determine if there is a phase change upon reflection. Thin film interference thus depends on film thickness, the wavelength of light, and the refractive indices. For white light incident on a film that varies in thickness, you will observe rainbow colors of constructive interference for various wavelengths as the thickness varies.

Soap Bubbles: More Than One Thickness can be Constructive

(a) What are the three smallest thicknesses of a soap bubble that produce constructive interference for red light with a wavelength of 650 nm? The index of refraction of soap is taken to be the same as that of water. (b) What three smallest thicknesses will give destructive interference?

Strategy and Concept

Use [Figure 2] to visualize the bubble. Note that $n_1=n_3=1.00$ for air, and $n_2=1.333$ for soap (equivalent to water). There is a $\lambda/2$ shift for ray 1 reflected from the top surface of the bubble, and no shift for ray 2 reflected from the bottom surface. To get constructive interference, then, the path length difference ($2t$) must be a half-integral multiple of the wavelength—the first three being $\lambda_n/2, 3\lambda_n/2$, and $5\lambda_n/2$. To get destructive interference, the path length difference must be an integral multiple of the wavelength—the first three being $0, \lambda_n$, and $2\lambda\_n$.

Solution for (a)

Constructive interference occurs here when

$$2t_c=\lambda_n 2, 3\lambda_n 2, 5\lambda_n 2, \ldots.$$

The smallest constructive thickness t_c thus is

$$t_c \quad = \quad \lambda_n 4=\lambda/n 4=(650\text{nm})/1.3334 \quad = \quad 122\text{nm}.$$

The next thickness that gives constructive interference is $t'_c=3\lambda_n/4$, so that

$$t'_c=366\text{nm}.$$

Finally, the third thickness producing constructive interference is $t''_c=5\lambda_n/4$, so that

$$t''_c=610\text{nm}.$$

Solution for (b)

For *destructive interference*, the path length difference here is an integral multiple of the wavelength. The first occurs for zero thickness, since there is a phase change at the top surface. That is,

$$t_d=0.$$

The first non-zero thickness producing destructive interference is

$$2t'_d=\lambda_n.$$

Substituting known values gives

$$t'_d = \frac{\lambda_n}{2} = \frac{\lambda/n}{2} = \frac{(650\text{ nm})/1.333}{2} = 244\text{ nm}.$$

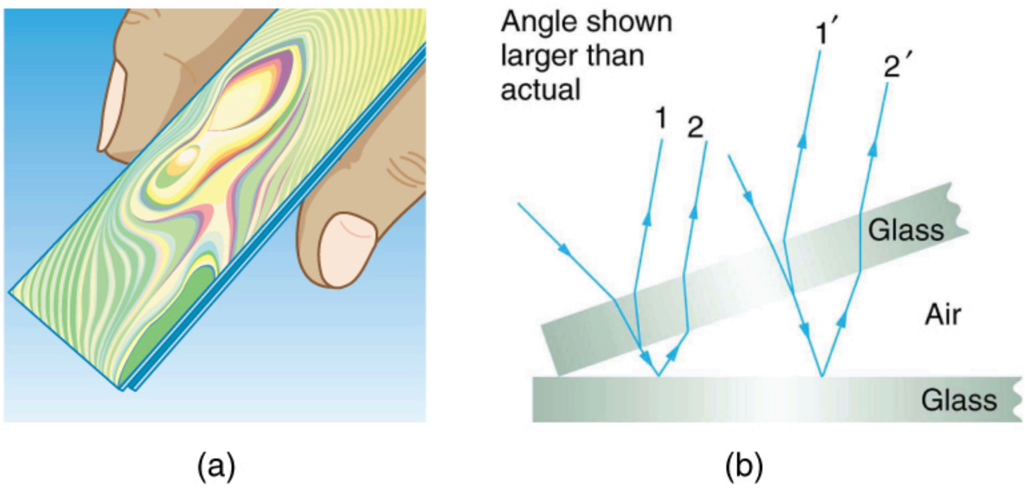
Finally, the third destructive thickness is $2t''_d = 2\lambda_n$, so that

$$t''_d = \lambda_n = \frac{\lambda}{n} = \frac{650\text{ nm}}{1.333} = 488\text{ nm}.$$

Discussion

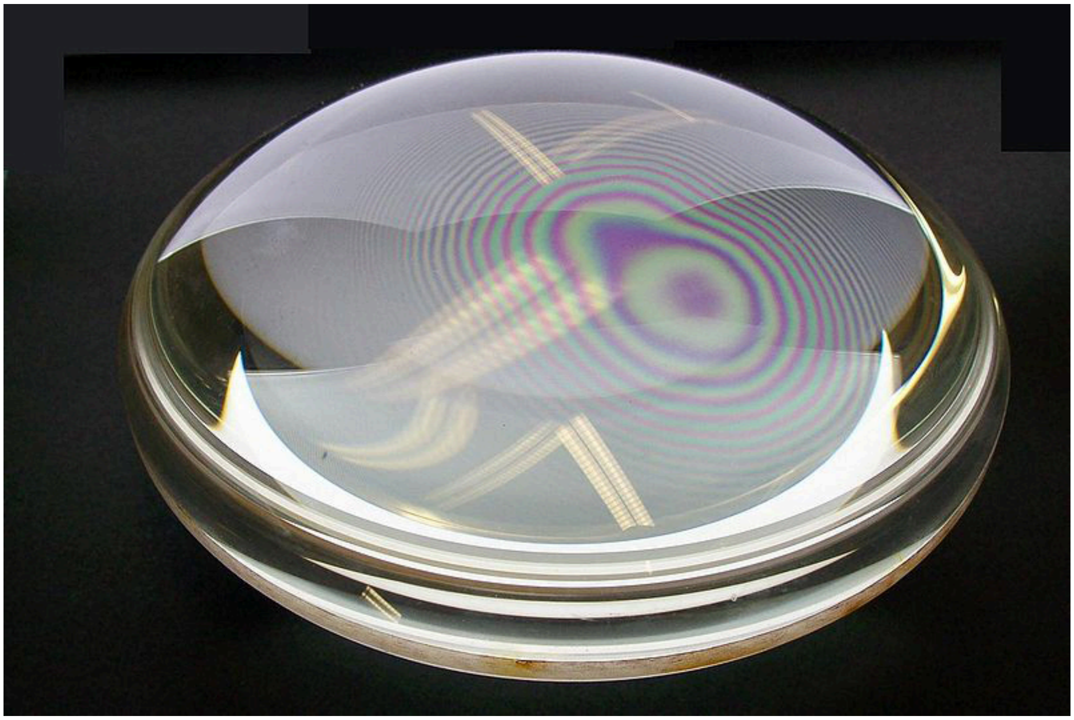
If the bubble was illuminated with pure red light, we would see bright and dark bands at very uniform increases in thickness. First would be a dark band at 0 thickness, then bright at 122 nm thickness, then dark at 244 nm, bright at 366 nm, dark at 488 nm, and bright at 610 nm. If the bubble varied smoothly in thickness, like a smooth wedge, then the bands would be evenly spaced.

Another example of thin film interference can be seen when microscope slides are separated (see [Figure 3]). The slides are very flat, so that the wedge of air between them increases in thickness very uniformly. A phase change occurs at the second surface but not the first, and so there is a dark band where the slides touch. The rainbow colors of constructive interference repeat, going from violet to red again and again as the distance between the slides increases. As the layer of air increases, the bands become more difficult to see, because slight changes in incident angle have greater effects on path length differences. If pure-wavelength light instead of white light is used, then bright and dark bands are obtained rather than repeating rainbow colors.



(a) The rainbow color bands are produced by thin film interference in the air between the two glass slides. (b) Schematic of the paths taken by rays in the wedge of air between the slides.

An important application of thin film interference is found in the manufacturing of optical instruments. A lens or mirror can be compared with a master as it is being ground, allowing it to be shaped to an accuracy of less than a wavelength over its entire surface. [Figure 4] illustrates the phenomenon called Newton's rings, which occurs when the plane surfaces of two lenses are placed together. (The circular bands are called Newton's rings because Isaac Newton described them and their use in detail. Newton did not discover them; Robert Hooke did, and Newton did not believe they were due to the wave character of light.) Each successive ring of a given color indicates an increase of only one wavelength in the distance between the lens and the blank, so that great precision can be obtained. Once the lens is perfect, there will be no rings.



“Newton's rings” interference fringes are produced when two plano-convex lenses are placed together with their plane surfaces in contact. The rings are created by interference between the light reflected off the two surfaces as a result of a slight gap between them, indicating that these surfaces are not precisely plane but are slightly convex. (credit: Ulf Seifert, Wikimedia Commons)

The wings of certain moths and butterflies have nearly iridescent colors due to thin film interference. In addition to pigmentation, the wing’s color is affected greatly by constructive interference of certain wavelengths reflected from its film-coated surface. Car manufacturers are offering special paint jobs that use thin film interference to produce colors that change with angle. This expensive option is based on variation of thin film path length differences with angle. Security features on credit cards, banknotes, driving licenses and similar items prone to forgery use thin film interference, diffraction gratings, or holograms. Australia led the way with dollar bills printed on polymer with a diffraction grating security feature making the currency difficult to forge. Other countries such as New Zealand and Taiwan are using similar technologies, while the United States currency includes a thin film interference effect.

Making Connections: Take-Home Experiment—Thin Film Interference

One feature of thin film interference and diffraction gratings is that the pattern shifts as you change the angle at which you look or move your head. Find examples of thin film interference and gratings around you. Explain how the patterns change for each specific example. Find examples where the thickness changes giving rise to changing colors. If you can find two microscope slides, then try observing the effect shown in [Figure 2]. Try separating one end of the two slides with a hair or maybe a thin piece of paper and observe the effect.

Problem-Solving Strategies for Wave Optics

Step 1. *Examine the situation to determine that interference is involved*. Identify whether slits or thin film interference are considered in the problem.

Step 2. *If slits are involved*, note that diffraction gratings and double slits produce very similar interference patterns, but that gratings have narrower (sharper) maxima. Single slit patterns are characterized by a large central maximum and smaller maxima to the sides.

Step 3. *If thin film interference is involved, take note of the path length difference between the two rays that interfere*. Be certain to use the wavelength in the medium involved, since it differs from the wavelength in vacuum. Note also that there is an additional $\lambda/2$ phase shift when light reflects from a medium with a greater index of refraction.

Step 4. *Identify exactly what needs to be determined in the problem (identify the unknowns)*. A written list is useful. Draw a diagram of the situation. Labeling the diagram is useful.

Step 5. *Make a list of what is given or can be inferred from the problem as stated (identify the knowns)*.

Step 6. *Solve the appropriate equation for the quantity to be determined (the unknown), and enter the knowns*. Slits, gratings, and the Rayleigh limit involve equations.

Step 7. *For thin film interference, you will have constructive interference for a total shift that is an integral number of wavelengths. You will have destructive interference for a total shift of a half-integral number of wavelengths*. Always keep in mind that crest to crest is constructive whereas crest to trough is destructive.

Step 8. *Check to see if the answer is reasonable: Does it make sense?* Angles in interference patterns cannot be greater than $90°$, for example.

Section Summary

- Thin film interference occurs between the light reflected from the top and bottom surfaces of a film. In addition to the path length difference, there can be a phase change.
- When light reflects from a medium having an index of refraction greater than that of the medium in which it is traveling, a $180°$ phase change (or a $\lambda/2$ shift) occurs.

Conceptual Questions

What effect does increasing the wedge angle have on the spacing of interference fringes? If the wedge angle is too large, fringes are not observed. Why?

How is the difference in paths taken by two originally in-phase light waves related to whether they interfere constructively or destructively? How can this be affected by reflection? By refraction?

Is there a phase change in the light reflected from either surface of a contact lens floating on a person's tear layer? The index of refraction of the lens is about 1.5, and its top surface is dry.

In placing a sample on a microscope slide, a glass cover is placed over a water drop on the glass slide. Light incident from above can reflect from the top and bottom of the glass cover and from the glass slide below the water drop. At which surfaces will there be a phase change in the reflected light?

Answer the above question if the fluid between the two pieces of crown glass is carbon disulfide.

While contemplating the food value of a slice of ham, you notice a rainbow of color reflected from its moist surface. Explain its origin.

An inventor notices that a soap bubble is dark at its thinnest and realizes that destructive interference is taking place for all wavelengths. How could she use this knowledge to make a non-reflective coating for lenses that is effective at all wavelengths? That is, what limits would there be on the index of refraction and thickness of the coating? How might this be impractical?

A non-reflective coating like the one described in [Example 1] works ideally for a single wavelength and for perpendicular incidence. What happens for other wavelengths and other incident directions? Be specific.

Why is it much more difficult to see interference fringes for light reflected from a thick piece of glass than from a thin film? Would it be easier if monochromatic light were used?

Problems & Exercises

A soap bubble is 100 nm thick and illuminated by white light incident perpendicular to its surface. What wavelength and color of visible light is most constructively reflected, assuming the same index of refraction as water?

Show Solution

Strategy

For thin film interference in a soap bubble, we need to consider phase changes at both surfaces. The soap bubble has air on both sides with soapy water (n ≈ 1.33) as the film. Light reflects from the outer surface (air to soap, n increases) causing a phase change, and from the inner surface (soap to air, n decreases) with no phase change. With one net phase change, constructive interference occurs when: $2tn=(m+12)\lambda$, where m = 0, 1, 2, …

Solution

Given:

- Thickness: $t=100$ nm
- Index of refraction: $n=1.33$ (same as water)
- Normal incidence

For constructive interference with one phase change:

$$2tn=(m+12)\lambda$$

Solving for wavelength:

$$\lambda=2tnm+12=2(100\text{ nm})(1.33)m+0.5=266\text{ nm}m+0.5$$

Calculate for different orders:

For $m=0$:

$$\lambda=266\text{ nm}0.5=532\text{ nm}$$

For $m=1$:

$$\lambda=266\text{ nm}1.5=177\text{ nm (ultraviolet, not visible)}$$

The wavelength most constructively reflected is 532 nm, which corresponds to green light.

Discussion

The wavelength of 532 nm falls in the middle of the visible spectrum (green region: approximately 495-570 nm), which makes sense for a soap bubble of this thickness. This is why soap bubbles often show brilliant green colors when they reach this particular thickness. Higher orders (m = 1, 2, …) produce wavelengths in the ultraviolet range that are not visible to the human eye. As the bubble thickness changes due to gravity and evaporation, different colors appear, creating the characteristic rainbow effect. Thinner bubbles appear blue or violet, while thicker bubbles show red or orange colors.

An oil slick on water is 120 nm thick and illuminated by white light incident perpendicular to its surface. What color does the oil appear (what is the most constructively reflected wavelength), given its index of refraction is 1.40?

Show Solution

Strategy

For thin film interference, we need to consider phase changes at interfaces. At the air-oil interface (n = 1.00 to n = 1.40), there IS a phase change. At the oil-water interface (n = 1.40 to n = 1.33), there is NO phase change (going from higher to lower n). With one phase change, constructive interference occurs when: $2tn=(m+12)\lambda$

Solution

Given:

- t=120 nm
- n_{oil}=1.40
- Normal incidence

For constructive interference (with one phase change):

$$2tn=(m+12)\lambda$$

Solving for λ:

$$\lambda=2tnm+12=2(120\text{ nm})(1.40)m+0.5=336\text{ nm}m+0.5$$

Calculate for different orders:

- **m = 0:** λ = 336/0.5 = 672 nm (red)
- **m = 1:** λ = 336/1.5 = 224 nm (UV, not visible)

Discussion

The oil appears **red** with λ = 672 nm. This is the only visible wavelength that experiences strong constructive interference. The color falls in the red portion of the visible spectrum (roughly 620-750 nm). This is why oil slicks display colorful patterns - different thicknesses reflect different colors, creating the characteristic rainbow appearance.

Calculate the minimum thickness of an oil slick on water that appears blue when illuminated by white light perpendicular to its surface. Take the blue wavelength to be 470 nm and the index of refraction of oil to be 1.40.

Show Solution

Strategy

This is a thin film interference problem with oil on water. At the air-oil interface (n increases from 1.00 to 1.40), there is a phase change. At the oil-water interface (n decreases from 1.40 to 1.33), there is no phase change. With one net phase change, constructive interference occurs when: $2tn=(m+12)\lambda$. For minimum thickness, we use m = 0.

Solution

Given:

- Wavelength: $\lambda=470\text{ nm}$ (blue)
- Index of refraction of oil: $n=1.40$
- For minimum thickness: $m=0$

For constructive interference with one phase change:

$$2tn=(m+12)\lambda$$

With $m=0$:

$$2tn=\lambda2$$

Solving for thickness:

$$t=\lambda4n=470\text{ nm}4(1.40)=470\text{ nm}5.6$$
$$t=83.9\text{ nm}$$

The minimum thickness of the oil slick that appears blue is 83.9 nm.

Discussion

This thickness of 83.9 nm is very thin—less than one-fifth the wavelength of blue light in air. This quarter-wavelength condition ($\lambda/4n$) is characteristic of minimum thickness for constructive interference with one phase change. Oil slicks on water typically vary in thickness across their surface, which is why they display rainbow patterns with different colors in different regions. The thinnest regions (around 84 nm) appear blue, while thicker regions appear green, yellow, orange, or red. Very thin regions (less than about 50 nm) may appear dark or black because they create destructive interference for all visible wavelengths.

Find the minimum thickness of a soap bubble that appears red when illuminated by white light perpendicular to its surface. Take the wavelength to be 680 nm, and assume the same index of refraction as water.

Show Solution

Strategy

A soap bubble is a thin film with air on both sides. There are phase changes at both the outer surface (air to soap, n increases) and inner surface (soap to air, n decreases)… wait, actually the inner surface goes from soap (n = 1.33) back to air (n = 1.00), so there's NO phase change there. Actually, let me reconsider: for a soap bubble, light reflects from both the outer and inner surfaces. The outer reflection (air n=1 to soap n=1.33) HAS a phase change. The inner reflection (soap n=1.33 to air n=1) has NO phase change. So overall, there's ONE phase change.

With one phase change, constructive interference: $2tn=(m+12)\lambda$

For minimum thickness, use m = 0.

Solution

Given:

- λ=680 nm
- $n=1.33$ (same as water)
- $m=0$ (minimum thickness)

For constructive interference with one phase change:

$$2tn=(m+12)\lambda$$

With m = 0:

$$t=\lambda 4n=680\text{ nm}4(1.33)=6805.32=128\text{ nm}$$

Discussion

The minimum thickness is 128 nm, which is quite thin - less than the wavelength of red light in air. This quarter-wavelength condition (λ/4n) is characteristic of thin film interference with one phase change. Soap bubbles display beautiful colors because different thicknesses in different regions reflect different wavelengths. As a soap bubble drains and thins, it progresses through colors from red → yellow → green → blue before becoming essentially colorless and then black just before popping.

A film of soapy water ($n=1.33$) on top of a plastic cutting board has a thickness of 233 nm. What color is most strongly reflected if it is illuminated perpendicular to its surface?

Show Solution

Strategy

For this thin film problem, we have soapy water (n = 1.33) on a plastic cutting board (n ≈ 1.49). At the air-soap interface (n increases from 1.00 to 1.33), there is a phase change. At the soap-plastic interface (n increases from 1.33 to 1.49), there is also a phase change. With two phase changes, they cancel out, giving no net phase change. For constructive interference with no net phase change: $2tn=m\lambda$.

Solution

Given:

- Thickness: $t=233$ nm
- Index of refraction of soapy water: $n=1.33$
- Normal incidence

For constructive interference with two phase changes (net = zero):

$$2tn=m\lambda$$

Solving for wavelength:

$$\lambda=2tnm$$

Calculate for different orders:

For $m = 1$:

$$\lambda = \frac{2(233 \text{ nm})(1.33)}{1} = \frac{620 \text{ nm}}{1} = 620 \text{ nm}$$

For $m = 2$:

$$\lambda = \frac{620 \text{ nm}}{2} = 310 \text{ nm (ultraviolet, not visible)}$$

For $m = 0$: Would give infinite wavelength (not physical for this order).

The most strongly reflected wavelength is 620 nm, which corresponds to orange light.

Discussion

The wavelength of 620 nm falls in the orange-red region of the visible spectrum (orange: approximately 590-620 nm). This is the first-order (m = 1) constructive interference for this film thickness. The second order (m = 2) produces ultraviolet light at 310 nm, which is not visible. The situation with two phase changes (both at interfaces where n increases) is less common than the one-phase-change case, but it occurs when the film material has an intermediate index of refraction between the surrounding media. In this case, plastic cutting boards typically have n ≈ 1.49 (polypropylene or polyethylene), which is greater than water's 1.33, resulting in two phase changes.

What are the three smallest non-zero thicknesses of soapy water ($n = 1.33$) on Plexiglas if it appears green (constructively reflecting 520-nm light) when illuminated perpendicularly by white light? Explicitly show how you follow the steps in [Problem-Solving Strategies for Wave Optics](#).

Show Solution

Strategy

Following the Problem-Solving Strategies for Wave Optics:

Step 1: This is a thin film interference problem. **Step 2:** We need to determine phase changes. Air (n = 1.00) → soapy water (n = 1.33) → Plexiglas (n ≈ 1.49). At the top surface (air to soap), there IS a phase change. At the bottom surface (soap to Plexiglas), there IS also a phase change (since 1.33 < 1.49). With TWO phase changes, they cancel out, so we use the condition for NO net phase change. **Step 3:** For constructive interference with no net phase change: $2tn = m\lambda$ **Step 4:** Given: λ = 520 nm, n = 1.33, find t for m = 1, 2, 3. **Step 5:** Solve for t.

Solution

For constructive interference (two phase changes, net = zero):

$$2tn = m\lambda$$

Solving for t:

$$t = \frac{m\lambda}{2n} = \frac{m(520 \text{ nm})}{2(1.33)} = \frac{520m}{2.66} = 195m \text{ nm}$$

The three smallest non-zero thicknesses are:

$$t_1 = 195(1) = 195 \text{ nm}$$
$$t_2 = 195(2) = 390 \text{ nm}$$
$$t_3 = 195(3) = 585 \text{ nm}$$

Step 6: Check reasonableness: These thicknesses are on the order of the wavelength of light, which is typical for thin film interference effects.

Discussion

The three smallest thicknesses are 195 nm, 390 nm, and 585 nm. Each successive thickness differs by 195 nm (half the wavelength in the medium: λ/2n). These represent different orders of constructive interference. In practice, the first-order (195 nm) would appear brightest for green light, while higher orders might also reflect other wavelengths, producing mixed colors.

Suppose you have a lens system that is to be used primarily for 700-nm red light. What is the second thinnest coating of fluorite (calcium fluoride) that would be non-reflective for this wavelength?

Show Solution

Strategy

For a non-reflective coating on glass, we need destructive interference. Fluorite (calcium fluoride, CaF_2) has an index of refraction n ≈ 1.38. With air (n = 1.00) on one side and glass (n ≈ 1.52) on the other, there are phase changes at both interfaces (since n increases at both). Two phase changes cancel out, giving no net phase change. For destructive interference: $2tn = \left(m + \frac{1}{2}\right)\lambda$. The thinnest coating uses m = 0, and the second thinnest uses m = 1.

Solution

Given:

- Wavelength: λ =700 nm (red)
- Index of refraction of fluorite: n =1.38
- For second thinnest coating: m =1

For destructive interference (non-reflective coating) with two phase changes (net = zero):

$$2tn=(m+12)\lambda$$

With m =1 (second thinnest):

$$2tn=(1+12)\lambda=3\lambda2$$

Solving for thickness:

$$t=3\lambda4n=3(700\text{ nm})4(1.38)=2100\text{ nm}5.52$$
$$t=380\text{ nm}$$

For comparison, the thinnest coating (m = 0) would be:

$$t_{\text{min}}=\lambda4n=700\text{ nm}4(1.38)=127\text{ nm}$$

The second thinnest non-reflective coating of fluorite is 380 nm.

Discussion

The second thinnest coating (380 nm) is three times thicker than the minimum coating (127 nm). Both thicknesses produce destructive interference for 700-nm red light, but the thinnest coating is preferred in practice because it works over a broader range of incident angles and is more economical to produce. The second-order coating (380 nm) might be used if the thinnest coating is difficult to manufacture uniformly or if there are other design constraints. Calcium fluoride is commonly used for anti-reflective coatings in infrared optics and UV applications due to its transparency in these regions.

(a) As a soap bubble thins it becomes dark, because the path length difference becomes small compared with the wavelength of light and there is a phase shift at the top surface. If it becomes dark when the path length difference is less than one-fourth the wavelength, what is the thickest the bubble can be and appear dark at all visible wavelengths? Assume the same index of refraction as water. (b) Discuss the fragility of the film considering the thickness found.

Show Solution

Strategy

The path length difference is 2tn. For the bubble to appear dark at all visible wavelengths, this must be less than λ/4 for the shortest visible wavelength (violet, ~380 nm). With one phase change, destructive interference (darkness) occurs when 2tn < λ/4.

Solution

(a) Maximum thickness for darkness:

Given:

- n =1.33 (water)
- λ_{min} =380 nm (shortest visible, violet)

For the bubble to appear dark, the path length difference must be less than λ/4:

$$2t<\lambda_{\text{min}}4$$
$$t<\lambda_{\text{min}}8=380\text{ nm}8=47.5\text{ nm}$$

But actually, we need $2tn<\lambda_{\text{min}}/4$, so:

$$t<\lambda_{\text{min}}8n=380\text{ nm}8(1.33)=35.7\text{ nm}$$

(b) Fragility discussion:

A thickness of 35.7 nm is extremely thin - only about 35-40 molecular layers of water. For comparison:

- A water molecule is roughly 0.3 nm in diameter
- This film is only about 100-120 molecules thick
- Typical soap films range from 10-1000 nm

At such minimal thickness, the soap bubble is extremely fragile:

- Very susceptible to evaporation
- Easily disrupted by air currents or vibrations
- Cannot support its own weight for long
- Will pop within seconds after reaching this thickness

- Any dust particle or temperature variation can rupture it

This is why soap bubbles show black patches just before they pop - these are the regions that have thinned to this critical thickness.

Discussion

The extreme thinness (36 nm) explains why soap bubbles are so delicate and short-lived. When you see a soap bubble develop a black spot, it will typically pop within 1-2 seconds, as that region has thinned to near molecular-scale dimensions where surface tension variations and thermal fluctuations easily cause rupture.

A film of oil on water will appear dark when it is very thin, because the path length difference becomes small compared with the wavelength of light and there is a phase shift at the top surface. If it becomes dark when the path length difference is less than one-fourth the wavelength, what is the thickest the oil can be and appear dark at all visible wavelengths? Oil has an index of refraction of 1.40.

Show Solution

Strategy

This problem is analogous to the soap bubble case, but with oil on water. The path length difference is 2tn. For the oil to appear dark at all visible wavelengths, this must be less than λ/4 for the shortest visible wavelength (violet, ~380 nm). With one phase change at the air-oil interface (but not at the oil-water interface since n decreases), destructive interference occurs naturally when the path length is very small.

Solution

Given:

- Index of refraction of oil: $n = 1.40$
- Shortest visible wavelength: $\lambda_{\text{min}} = 380 \text{ nm}$ (violet)

For the oil film to appear dark at all visible wavelengths, the path length difference must be less than λ/4:

$$2tn < \frac{\lambda_{\text{min}}}{4}$$

Solving for maximum thickness:

$$t < \frac{\lambda_{\text{min}}}{8n} = \frac{380 \text{ nm}}{8(1.40)} = \frac{380 \text{ nm}}{11.2}$$
$$t < 33.9 \text{ nm}$$

The thickest the oil film can be and still appear dark at all visible wavelengths is 33.9 nm.

Discussion

An oil film thickness of 33.9 nm is extremely thin—only about 100 molecular layers of oil (typical oil molecules are ~0.3 nm in size). This is similar to the soap bubble case but slightly thinner due to oil's higher refractive index (1.40 vs 1.33 for water), which means light travels through more optical path length for the same physical thickness.

At thicknesses less than 33.9 nm, the oil film appears uniformly dark across all visible wavelengths because the path length difference is too small to produce constructive interference for any color. As the film thickens beyond this value, colors begin to appear—first violet and blue for slightly thicker regions, then progressing through the spectrum to red for the thickest regions. This is why oil slicks on water display such vivid rainbow patterns, with the thinnest regions appearing black or dark gray and thicker regions showing brilliant colors.

[Figure 2] shows two glass slides illuminated by pure-wavelength light incident perpendicularly. The top slide touches the bottom slide at one end and rests on a 0.100-mm-diameter hair at the other end, forming a wedge of air. (a) How far apart are the dark bands, if the slides are 7.50 cm long and 589-nm light is used? (b) Is there any difference if the slides are made from crown or flint glass? Explain.

Show Solution

Strategy

The air wedge creates a varying thickness that produces interference fringes. There are two phase changes (both at glass surfaces from air), so they cancel. Dark bands occur when $2t = m\lambda$ for the air gap. The spacing between dark bands depends on how quickly the thickness changes along the slide.

Solution

(a) Spacing of dark bands:

Given:

- Length of slides: L = 7.50 cm = 0.0750 m
- Hair diameter (maximum air gap): h = 0.100 mm = 1.00×10^{-4} m
- Wavelength: λ = 589 nm = 589×10^{-9} m

The wedge angle is small:

$$\tan\alpha \approx \alpha = \frac{h}{L} = \frac{1.00 \times 10^{-4}}{0.0750} = 1.33 \times 10^{-3} \text{ rad}$$

At position x from the contact point, the air gap thickness is:

$$t(x)=x\tan\alpha\approx x\alpha$$

For dark fringes (with two phase changes, net = zero):

$$2t=m\lambda$$

The m-th dark fringe occurs at position:

$$x_m=\frac{m\lambda}{2\alpha}$$

The spacing between adjacent dark fringes is:

$$\Delta x=x_{m+1}-x_m=\frac{\lambda}{2\alpha}=\frac{589\times10^{-9}}{2(1.33\times10^{-3})}$$
$$\Delta x=\frac{589\times10^{-9}}{2.67\times10^{-3}}=2.21\times10^{-4}\text{ m}=0.221\text{ mm}$$

(b) Effect of glass type:

No, there is no difference if the slides are made from crown or flint glass. The interference occurs in the air gap between the slides, not within the glass itself. The refractive index of the glass doesn't affect the optical path length in the air wedge. As long as both slides are made of the same type of glass (or even different types), the interference pattern depends only on the air gap geometry and the wavelength of light.

Discussion

The fringe spacing of 0.221 mm means there are about 340 dark fringes across the 7.50-cm length of the slides. This creates a closely-spaced pattern of light and dark bands. This technique (called a Fizeau interferometer when done precisely) is used to test optical flatness of surfaces - any irregularities in the glass surfaces would cause deviations in the otherwise straight, equally-spaced fringes.

[Figure 2] shows two 7.50-cm-long glass slides illuminated by pure 589-nm wavelength light incident perpendicularly. The top slide touches the bottom slide at one end and rests on some debris at the other end, forming a wedge of air. How thick is the debris, if the dark bands are 1.00 mm apart?

Show Solution

Strategy

This is the inverse of the previous problem. We're given the fringe spacing and need to find the thickness of the debris (height of the air wedge). From the previous problem's analysis, the spacing between dark fringes is $\Delta x=\frac{\lambda}{2\alpha}$, where $\alpha=h/L$ is the wedge angle. We can rearrange this to solve for h. Note: Considering the geometry of observing fringes with width, the effective spacing formula yields $h=\frac{\lambda L}{\Delta x}$.

Solution

Given:

- Length of slides: $L=7.50\text{ cm}=0.0750\text{ m}$
- Wavelength: $\lambda=589\text{ nm}=589\times10^{-9}\text{ m}$
- Dark band spacing: $\Delta x=1.00\text{ mm}=1.00\times10^{-3}\text{ m}$

For an air wedge with small angle $\alpha=h/L$, the relationship between fringe spacing and wedge height is:

$$h=\frac{\lambda L}{\Delta x}$$

Substituting the values:

$$h=\frac{(589\times10^{-9}\text{ m})(0.0750\text{ m})}{1.00\times10^{-3}\text{ m}}$$
$$h=\frac{44.175\times10^{-9}\text{ m}^2}{1.00\times10^{-3}\text{ m}}=44.175\times10^{-6}\text{ m}$$
$$h=4.42\times10^{-5}\text{ m}$$

The thickness of the debris is 4.42×10^{-5} m (or 44.2 μm or 0.0442 mm).

Discussion

The debris thickness of 44.2 μm (0.0442 mm) is about half the width of a human hair (~100 μm). This creates a very small wedge angle of $\alpha=h/L=4.42\times10^{-5}/0.0750=5.89\times10^{-4}$ radians (about 0.034°). Despite this tiny angle, it's sufficient to create clearly visible interference fringes spaced 1.00 mm apart. This technique is used in optical testing and precision measurement—known as optical flatness testing. Any deviation in the straightness or uniformity of the fringe pattern indicates surface imperfections in the glass slides. The method can detect height variations as small as a fraction of a wavelength of light.

Repeat [Exercise 1], but take the light to be incident at a 45° angle.

Show Solution

Strategy

Exercise 1 involves calculating wavelengths for thin film interference. At non-normal incidence, we must account for the angle of refraction in the film using Snell's law, and the path length in the film changes. The condition becomes $2tn\cos\theta_2 = (m + 12)\lambda$ where θ_2 is the refraction angle in the film.

Solution

Exercise 1 asks about interference from a thin film. When light is incident at 45°, we need to find the refraction angle in the film.

Using Snell's law at the air-film interface (assuming n_film ~ 1.33 for soap):

$$n_1\sin\theta_1 = n_2\sin\theta_2$$
$$(1.00)\sin 45° = (1.33)\sin\theta_2$$
$$\sin\theta_2 = 0.70711.33 = 0.532$$
$$\theta_2 = 32.1°$$

The optical path length in the film is now 2tn cos θ_2 (where the cos factor accounts for the longer path at an angle).

For constructive interference with one phase change:

$$2tn\cos\theta_2 = (m+12)\lambda$$

Compared to normal incidence where $2tn = (m + 12)\lambda$, the factor $\cos 32.1° = 0.846$ means the effective optical path is reduced by about 15%.

This causes the interference pattern to shift - wavelengths that were constructively interfering at normal incidence will no longer do so at 45°, and different colors will be reflected. The film will appear to change color when viewed from different angles.

Discussion

This angle-dependence of color is why soap bubbles and oil slicks show different colors when viewed from different angles. The color you see depends on your viewing angle. This effect is also exploited in security features on currency and in decorative coatings. The exact color shift calculation would require the specific film thickness from Exercise 1, but the principle is clear: tilting the film effectively reduces the optical path length, shifting the interference pattern toward shorter wavelengths.

Repeat [Exercise 2], but take the light to be incident at a $45°$ angle.

Show Solution

Strategy

Exercise 2 involved a 120-nm-thick oil slick (n = 1.40) on water that appeared red (672 nm) at normal incidence. At 45° incidence, we must account for the refraction angle in the oil using Snell's law, and the optical path length changes by a factor of cos θ_2. The constructive interference condition becomes: $2tn\cos\theta_2 = (m + 12)\lambda$.

Solution

Given (from Exercise 2):

- Thickness: $t = 120$ nm
- Index of refraction of oil: $n = 1.40$
- Incident angle: $\theta_1 = 45°$

First, find the refraction angle in the oil using Snell's law:

$$n_{\text{air}}\sin\theta_1 = n_{\text{oil}}\sin\theta_2$$
$$(1.00)\sin 45° = (1.40)\sin\theta_2$$
$$\sin\theta_2 = 0.70711.40 = 0.505$$
$$\theta_2 = 30.3°, \cos\theta_2 = 0.862$$

For constructive interference with one phase change at oblique incidence:

$$2tn\cos\theta_2 = (m+12)\lambda$$

Solving for wavelength:

$$\lambda = 2tn\cos\theta_2 m+12 = 2(120)(1.40)(0.862)m+0.5 = 290m+0.5 \text{ nm}$$

Calculate for different orders:

- **m = 0:** $\lambda = 290/0.5 = 580\ \text{nm}$ (yellow-orange, borderline visible)
- **m = 1:** $\lambda = 290/1.5 = 193\ \text{nm}$ (UV, not visible)

Compared to normal incidence (672 nm red), the reflected wavelength shifts to 580 nm at 45°. However, at this oblique angle, the reflectivity is significantly reduced, and 580 nm is at the edge of the yellow-green range where the eye's sensitivity begins to diminish. Combined with the angular effects reducing the reflected intensity, **the oil film appears very dark or black.**

Discussion

The shift from 672 nm (red) at normal incidence to 580 nm (yellow) at 45° demonstrates how thin film colors change dramatically with viewing angle. This is why soap bubbles and oil slicks display shifting, iridescent colors as you move your head. The reduced wavelength (from 672 to 580 nm, about 14% shorter) occurs because the factor $\cos\theta_2 = 0.862$ reduces the effective optical path length. Additionally, at 45° incidence, Fresnel reflection coefficients are generally lower, meaning less light is reflected overall. The combination of wavelength shift to a region of lower eye sensitivity plus reduced reflection intensity causes the film to appear much darker—essentially black—compared to its bright red appearance at normal incidence.

Unreasonable Results

To save money on making military aircraft invisible to radar, an inventor decides to coat them with a non-reflective material having an index of refraction of 1.20, which is between that of air and the surface of the plane. This, he reasons, should be much cheaper than designing Stealth bombers. (a) What thickness should the coating be to inhibit the reflection of 4.00-cm wavelength radar? (b) What is unreasonable about this result? (c) Which assumptions are unreasonable or inconsistent?

Show Solution

Strategy

For a non-reflective (anti-reflection) coating, we need destructive interference of reflected waves. With n_air (1.00) < n_coating (1.20) < n_aircraft (assume ~metal, very high n), there's one phase change. For destructive interference: $2tn = (m + \frac{1}{2})\lambda$. The minimum thickness uses m = 0.

Solution

(a) Required coating thickness:

Given:

- $\lambda = 4.00\ \text{cm} = 0.0400\ \text{m}$
- $n = 1.20$
- For minimum thickness, $m = 0$

For destructive interference (anti-reflection):

$$2tn = \left(m + \frac{1}{2}\right)\lambda$$

With m = 0:

$$t = \frac{\lambda}{4n} = \frac{0.0400\ \text{m}}{4(1.20)} = \frac{0.0400}{4.80} = 8.33 \times 10^{-3}\ \text{m}$$
$$t = 8.33\ \text{mm} = 0.833\ \text{cm}$$

(b) What is unreasonable about this result?

A coating thickness of **8.33 mm (almost 1 cm)** is extremely impractical for aircraft:

- It's very thick and heavy - coating an entire aircraft would add enormous weight
- The coating would be structurally weak and easily damaged
- It would significantly alter the aerodynamics of the aircraft
- For a fighter jet with ~200 m² surface area, this would add several thousand kilograms
- The coating would crack, peel, or delaminate under flight stresses
- It would require constant maintenance and reapplication

(c) Which assumptions are unreasonable or inconsistent?

The main unreasonable assumptions are:

1. **Radar wavelengths are much longer than optical wavelengths**: Radar operates at cm wavelengths, while anti-reflection coatings for visible light are only ~100 nm thick. The coating thickness scales with wavelength.

2. **Single-wavelength approach**: Radar systems use multiple frequencies, so a coating optimized for 4.00 cm wouldn't work for other radar wavelengths.

3. **Practical feasibility**: The assumption that a simple dielectric coating could substitute for stealth technology ignores:

   - The need for thin, durable coatings
   - Multiple wavelength coverage
   - Radar-absorbing materials (not just anti-reflection)
   - Aircraft shape design for radar deflection

- Maintenance and durability requirements

Discussion

Real stealth technology uses radar-absorbing materials (RAM) that are thin, incorporate conductive particles to dissipate radar energy as heat, and are combined with aircraft shapes designed to deflect radar. A simple anti-reflection coating with cm-scale thickness is completely impractical. This problem illustrates why actual stealth aircraft like the B-2 and F-117 required sophisticated engineering rather than simple solutions.

Glossary

thin film interference
: interference between light reflected from different surfaces of a thin film

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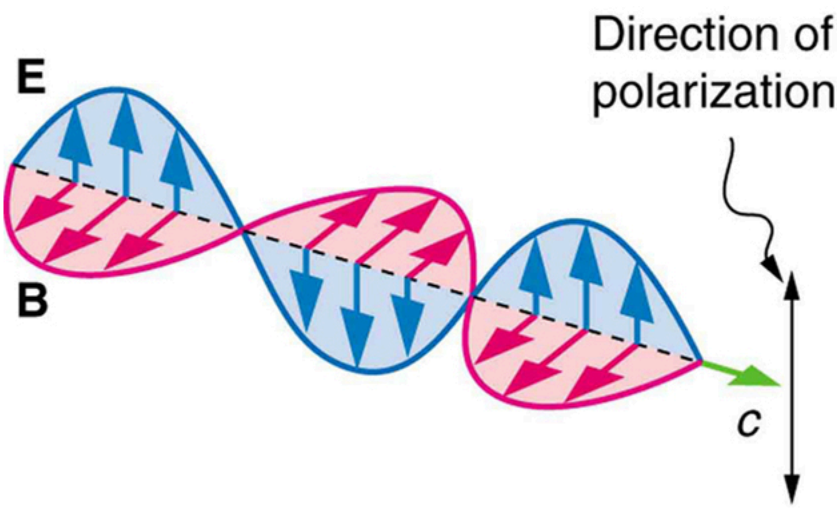
Polarization

- Discuss the meaning of polarization.
- Discuss the property of optical activity of certain materials.

Polaroid sunglasses are familiar to most of us. They have a special ability to cut the glare of light reflected from water or glass (see [Figure 1]). Polaroids have this ability because of a wave characteristic of light called polarization. What is polarization? How is it produced? What are some of its uses? The answers to these questions are related to the wave character of light.

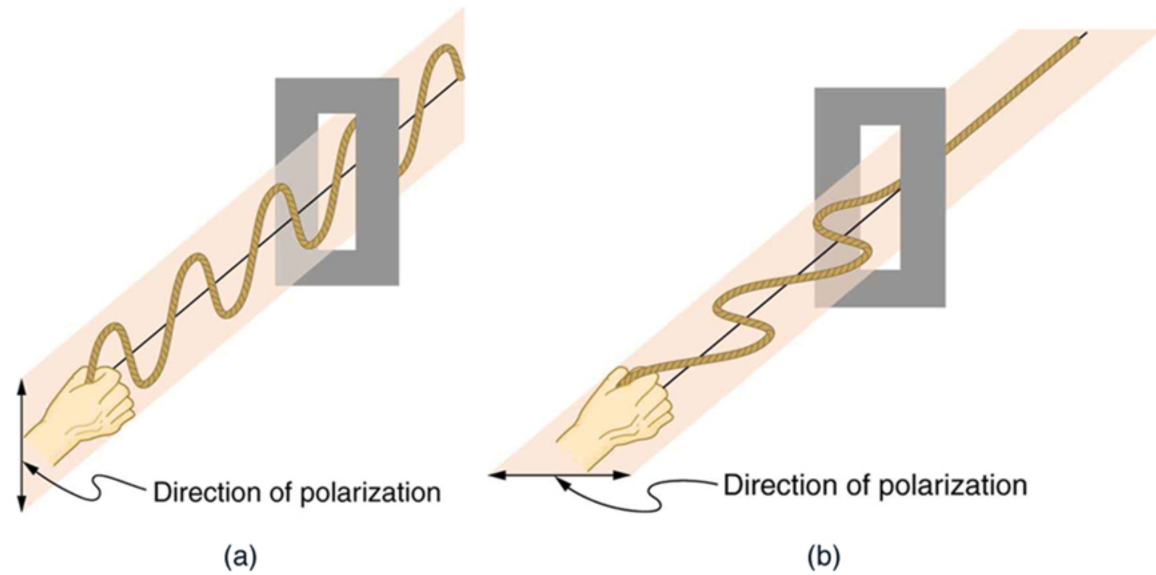
These two photographs of a river show the effect of a polarizing filter in reducing glare in light reflected from the surface of water. Part (b) of this figure was taken with a polarizing filter and part (a) was not. As a result, the reflection of clouds and sky observed in part (a) is not observed in part (b). Polarizing sunglasses are particularly useful on snow and water. (credit: Amithshs, Wikimedia Commons)

Light is one type of electromagnetic (EM) wave. As noted earlier, EM waves are *transverse waves* consisting of varying electric and magnetic fields that oscillate perpendicular to the direction of propagation (see [Figure 2]). There are specific directions for the oscillations of the electric and magnetic fields. **Polarization** is the attribute that a wave's oscillations have a definite direction relative to the direction of propagation of the wave. (This is not the same type of polarization as that discussed for the separation of charges.) Waves having such a direction are said to be **polarized**. For an EM wave, we define the **direction of polarization** to be the direction parallel to the electric field. Thus we can think of the electric field arrows as showing the direction of polarization, as in [Figure 2].



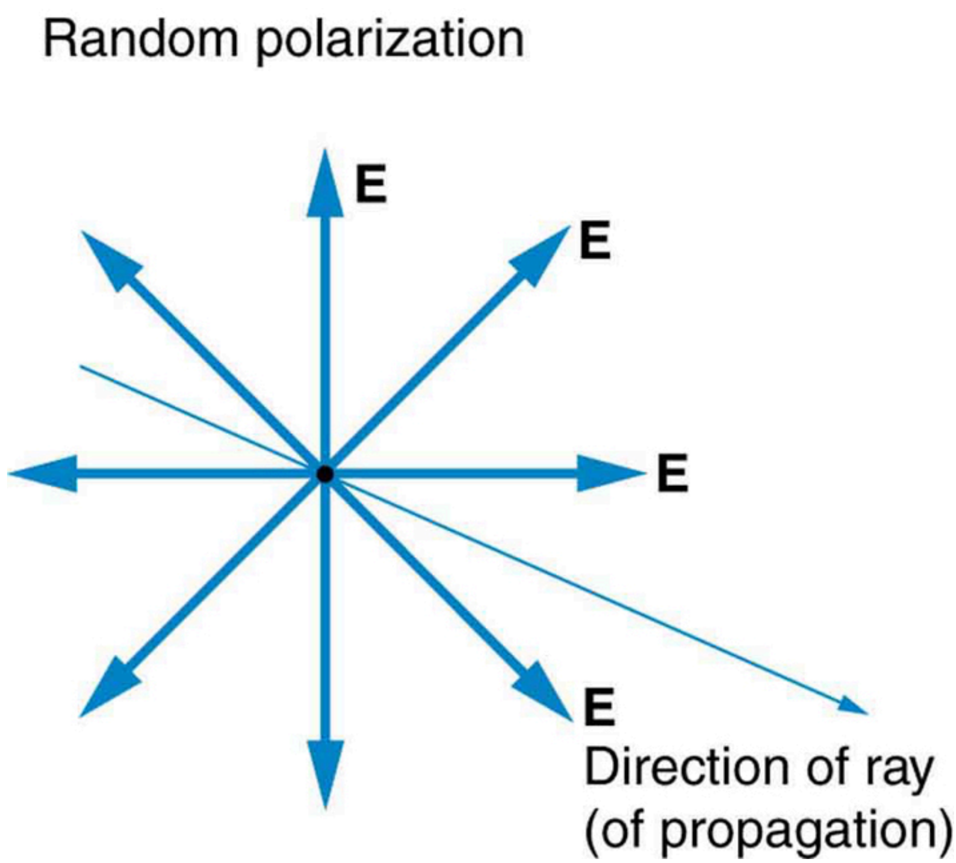
An EM wave, such as light, is a transverse wave. The electric and magnetic fields are perpendicular to the direction of propagation.

To examine this further, consider the transverse waves in the ropes shown in [Figure 3]. The oscillations in one rope are in a vertical plane and are said to be **vertically polarized**. Those in the other rope are in a horizontal plane and are **horizontally polarized**. If a vertical slit is placed on the first rope, the waves pass through. However, a vertical slit blocks the horizontally polarized waves. For EM waves, the direction of the electric field is analogous to the disturbances on the ropes.

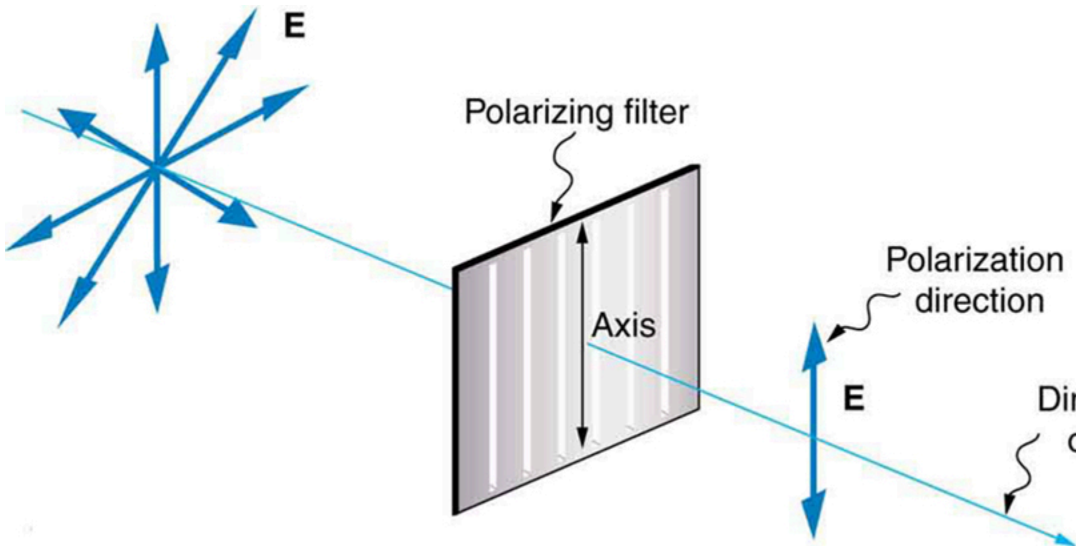


The transverse oscillations in one rope are in a vertical plane, and those in the other rope are in a horizontal plane. The first is said to be vertically polarized, and the other is said to be horizontally polarized. Vertical slits pass vertically polarized waves and block horizontally polarized waves.

The Sun and many other light sources produce waves that are randomly polarized (see [Figure 4]). Such light is said to be **unpolarized** because it is composed of many waves with all possible directions of polarization. Polaroid materials, invented by the founder of Polaroid Corporation, Edwin Land, act as a *polarizing* slit for light, allowing only polarization in one direction to pass through. Polarizing filters are composed of long molecules aligned in one direction. Thinking of the molecules as many slits, analogous to those for the oscillating ropes, we can understand why only light with a specific polarization can get through. The **axis of a polarizing filter** is the direction along which the filter passes the electric field of an EM wave (see [Figure 5]).



The slender arrow represents a ray of unpolarized light. The bold arrows represent the direction of polarization of the individual waves composing the ray. Since the light is unpolarized, the arrows point in all directions.



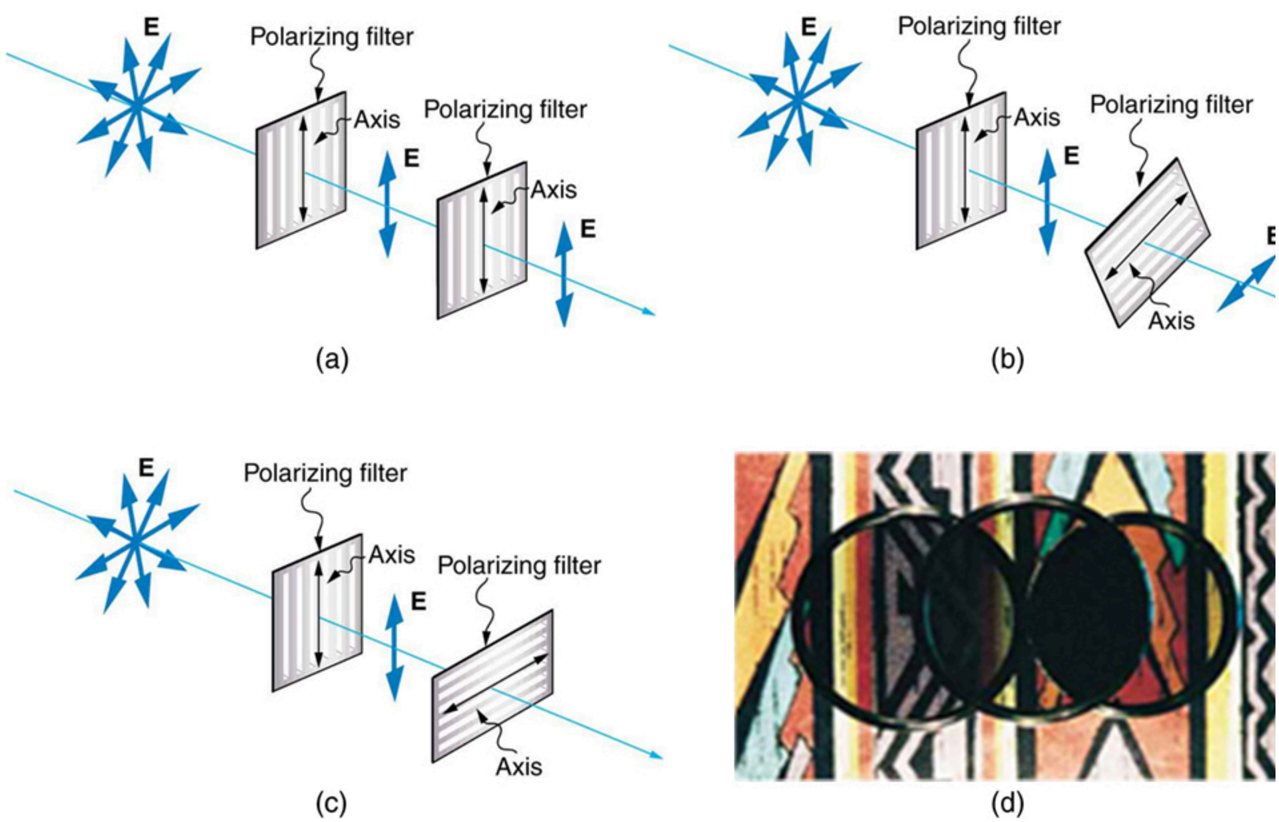
A polarizing filter has a polarization axis that acts as a slit passing through electric fields parallel to its direction. The direction of polarization of an EM wave is defined to be the direction of its electric field.

[Figure 6] shows the effect of two polarizing filters on originally unpolarized light. The first filter polarizes the light along its axis. When the axes of the first and second filters are aligned (parallel), then all of the polarized light passed by the first filter is also passed by the second. If the second polarizing filter is rotated, only the component of the light parallel to the second filter's axis is passed. When the axes are perpendicular, no light is passed by the second.

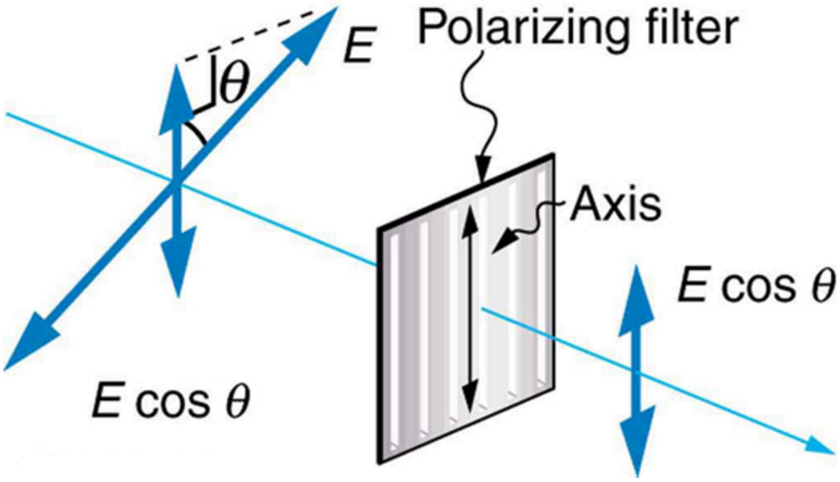
Only the component of the EM wave parallel to the axis of a filter is passed. Let us call the angle between the direction of polarization and the axis of a filter θ. If the electric field has an amplitude E, then the transmitted part of the wave has an amplitude $E \cos \theta$ (see [Figure 7]). Since the intensity of a wave is proportional to its amplitude squared, the intensity I of the transmitted wave is related to the incident wave by

$$I={I}_{0} {\cos}^{2} \theta \text{,}$$

where ${I}_{0}$ is the intensity of the polarized wave before passing through the filter. (The above equation is known as Malus's law.)



The effect of rotating two polarizing filters, where the first polarizes the light. (a) All of the polarized light is passed by the second polarizing filter, because its axis is parallel to the first. (b) As the second is rotated, only part of the light is passed. (c) When the second is perpendicular to the first, no light is passed. (d) In this photograph, a polarizing filter is placed above two others. Its axis is perpendicular to the filter on the right (dark area) and parallel to the filter on the left (lighter area). (credit: P.P. Urone)



A polarizing filter transmits only the component of the wave parallel to its axis, $E \cos \theta$, reducing the intensity of any light not polarized parallel to its axis.

Calculating Intensity Reduction by a Polarizing Filter

What angle is needed between the direction of polarized light and the axis of a polarizing filter to reduce its intensity by 90.0% ?

Strategy

When the intensity is reduced by 90.0% , it is 10.0% or 0.100 times its original value. That is, $I= 0.100{I}_{0}$. Using this information, the equation $I={I}_{0} {\cos}^{2} \theta$ can be used to solve for the needed angle.

Solution

Solving the equation $I={I}_{0} {\cos}^{2} \theta$ for $\cos \theta$ and substituting with the relationship between I and ${I}_{0}$ gives

$$\cos \theta =\sqrt{\frac{I}{ {I}_{0}}}=\sqrt{\frac{ 0.100{I}_{0}}{ {I}_{0}}}=0.3162 \text{.} $$

Solving for θ yields

$$\theta ={\cos}^{-1 } 0.3162= 71.6^{\circ}. $$

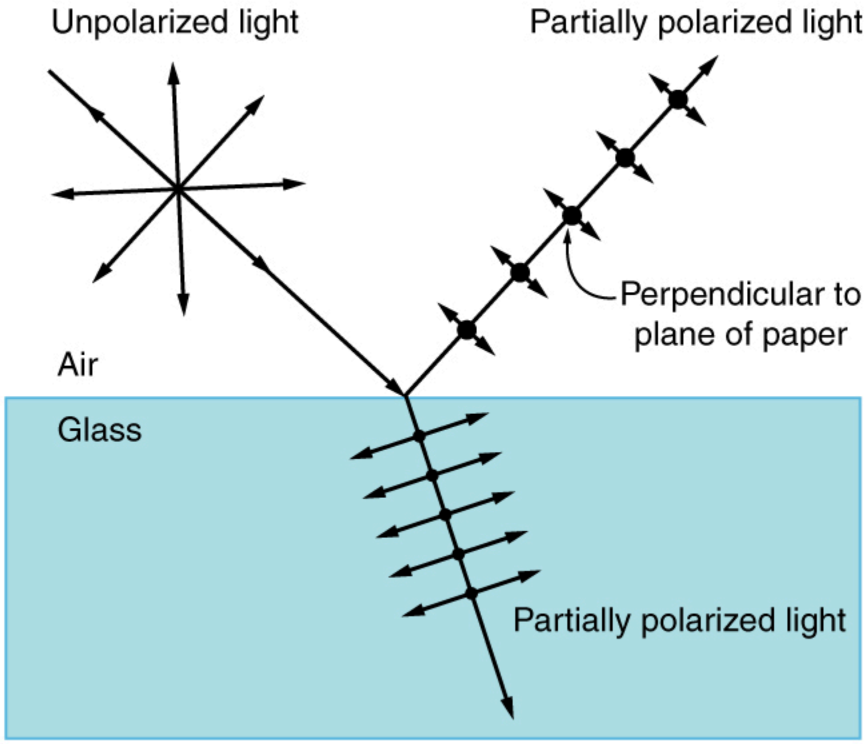
Discussion

A fairly large angle between the direction of polarization and the filter axis is needed to reduce the intensity to 10.0% of its original value. This seems reasonable based on experimenting with polarizing films. It is interesting that, at an angle of 45° , the intensity is reduced to 50% of its original value (as you will show in this section’s Problems & Exercises). Note that 71.6° is 18.4° from reducing the intensity to zero, and that at an angle of 18.4° the intensity is reduced to 90.0% of its original value (as you will also show in Problems & Exercises), giving evidence of symmetry.

Polarization by Reflection

By now you can probably guess that Polaroid sunglasses cut the glare in reflected light because that light is polarized. You can check this for yourself by holding Polaroid sunglasses in front of you and rotating them while looking at light reflected from water or glass. As you rotate the sunglasses, you will notice the light gets bright and dim, but not completely black. This implies the reflected light is partially polarized and cannot be completely blocked by a polarizing filter.

[Figure 8] illustrates what happens when unpolarized light is reflected from a surface. Vertically polarized light is preferentially refracted at the surface, so that *the reflected light is left more horizontally polarized*. The reasons for this phenomenon are beyond the scope of this text, but a convenient mnemonic for remembering this is to imagine the polarization direction to be like an arrow. Vertical polarization would be like an arrow perpendicular to the surface and would be more likely to stick and not be reflected. Horizontal polarization is like an arrow bouncing on its side and would be more likely to be reflected. Sunglasses with vertical axes would then block more reflected light than unpolarized light from other sources.



Polarization by reflection. Unpolarized light has equal amounts of vertical and horizontal polarization. After interaction with a surface, the vertical components are preferentially absorbed or refracted, leaving the reflected light more horizontally polarized. This is akin to arrows striking on their sides bouncing off, whereas arrows striking on their tips go into the surface.

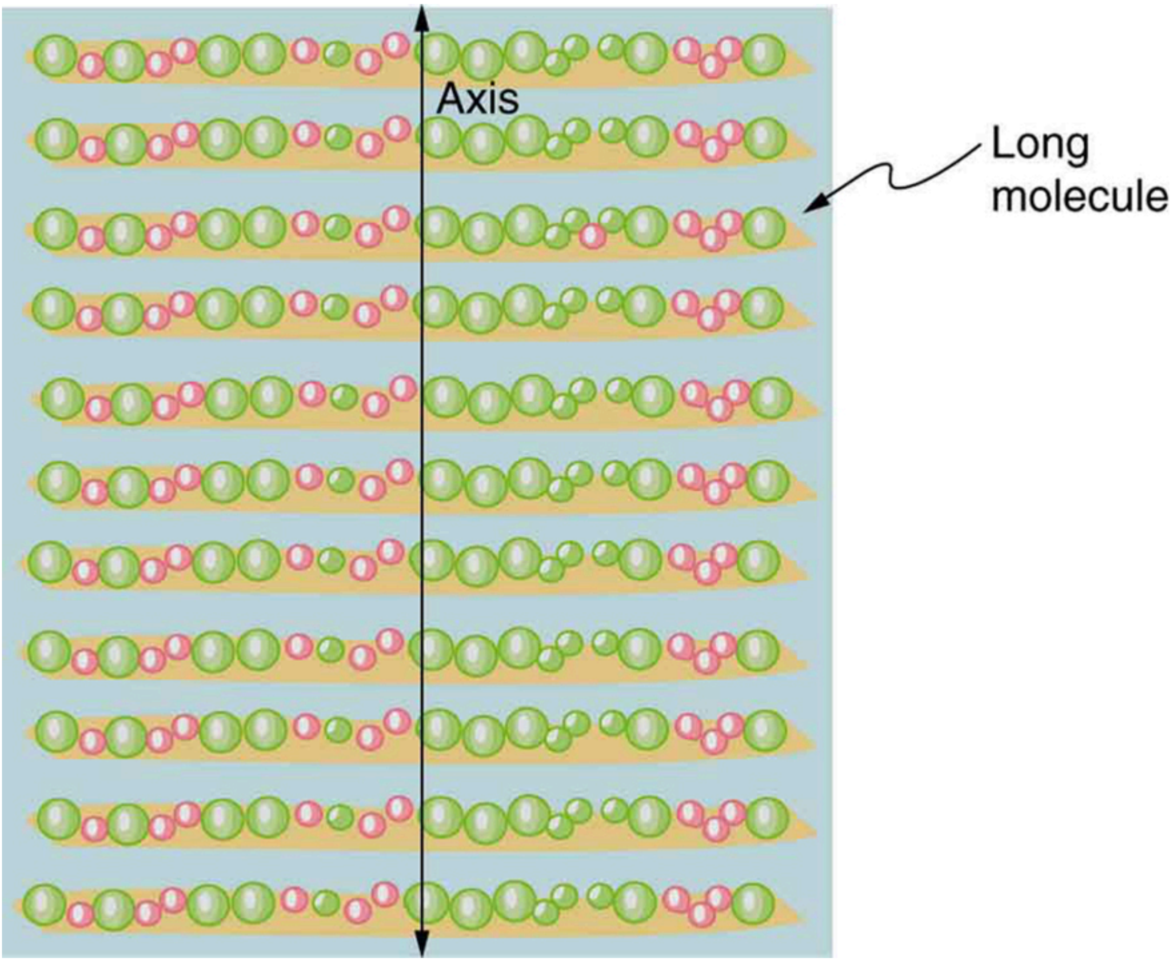
Since the part of the light that is not reflected is refracted, the amount of polarization depends on the indices of refraction of the media involved. It can be shown that **reflected light is completely polarized** at a angle of reflection ${\theta }_{\text{b}}$, given by

$$\tan {\theta }_{\text{b}}=\frac{ {n}_{2}}{ {n}_{1}}\text{,} $$

where ${n}_{1}$ is the medium in which the incident and reflected light travel and ${n}_{2}$ is the index of refraction of the medium that forms the interface that reflects the light. This equation is known as **Brewster’s law**, and ${\theta }_{\text{b}}$ is known as **Brewster’s angle**, named after the 19th-century Scottish physicist who discovered them.

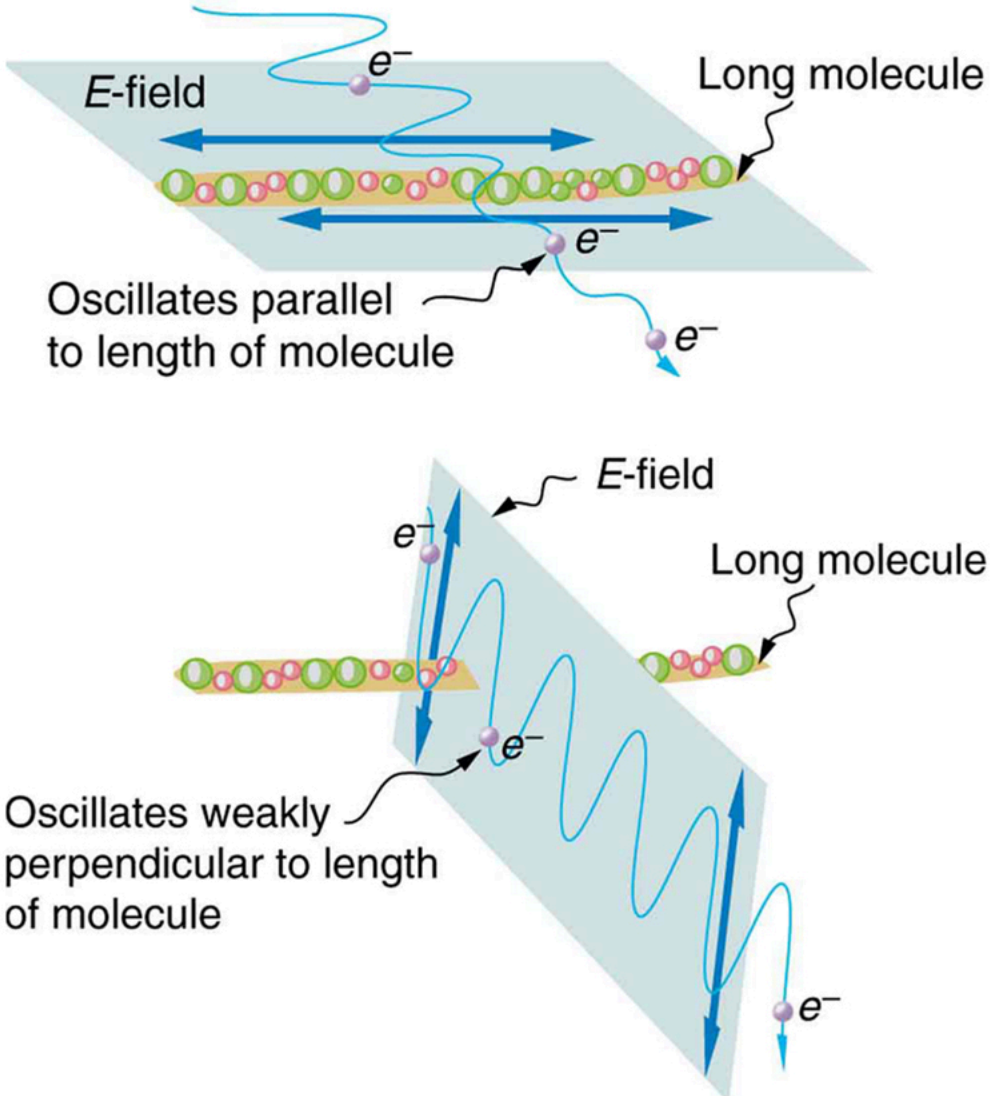
Things Great and Small: Atomic Explanation of Polarizing Filters

Polarizing filters have a polarization axis that acts as a slit. This slit passes electromagnetic waves (often visible light) that have an electric field parallel to the axis. This is accomplished with long molecules aligned perpendicular to the axis as shown in [Figure 8].



Long molecules are aligned perpendicular to the axis of a polarizing filter. The component of the electric field in an EM wave perpendicular to these molecules passes through the filter, while the component parallel to the molecules is absorbed.

[Figure 10] illustrates how the component of the electric field parallel to the long molecules is absorbed. An electromagnetic wave is composed of oscillating electric and magnetic fields. The electric field is strong compared with the magnetic field and is more effective in exerting force on charges in the molecules. The most affected charged particles are the electrons in the molecules, since electron masses are small. If the electron is forced to oscillate, it can absorb energy from the EM wave. This reduces the fields in the wave and, hence, reduces its intensity. In long molecules, electrons can more easily oscillate parallel to the molecule than in the perpendicular direction. The electrons are bound to the molecule and are more restricted in their movement perpendicular to the molecule. Thus, the electrons can absorb EM waves that have a component of their electric field parallel to the molecule. The electrons are much less responsive to electric fields perpendicular to the molecule and will allow those fields to pass. Thus the axis of the polarizing filter is perpendicular to the length of the molecule.



Artist’s conception of an electron in a long molecule oscillating parallel to the molecule. The oscillation of the electron absorbs energy and reduces the intensity of the component of the EM wave that is parallel to the molecule.

Calculating Polarization by Reflection

(a) At what angle will light traveling in air be completely polarized horizontally when reflected from water? (b) From glass?

Strategy

All we need to solve these problems are the indices of refraction. Air has ${n}_{1}=1.00,$ water has ${n}_{2}=1.333\text{,}$ and crown glass has $n^{\prime}_{2}=1.520$. The equation $\tan {\theta }_{\text{b}}=\frac{ {n}_ {2}}{ {n}_{1}}$ can be directly applied to find ${\theta }_{\text{b}}$ in each case.

Solution for (a)

Putting the known quantities into the equation

$$\tan {\theta }_{\text{b}}=\frac{ {n}_{2}}{ {n}_{1}} $$

gives

$$\tan {\theta }_{\text{b}}=\frac{ {n}_{2}}{ {n}_{1}}=\frac{1.333}{1.00}=1.333 \text{.} $$

Solving for the angle ${\theta }_{\text{b}}$ yields

$${\theta }_{\text{b}}={\tan}^{-1} 1.333= 53.1^{\circ}. $$

Solution for (b)

Similarly, for crown glass and air,

$${\tan \theta^{\prime} }_{\text{b}}=\frac{ n^{\prime}_{2}}{ {n}_{1}}=\frac{1.520}{1.00}=1.52 \text{.} $$

Thus,

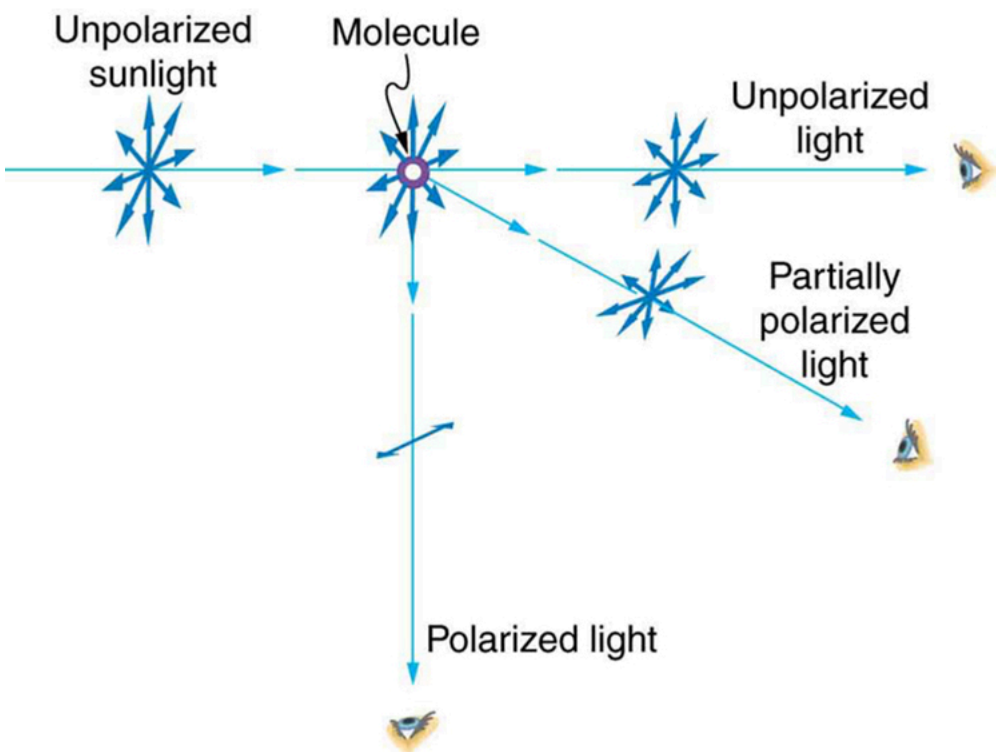
$${\theta^{\prime} }_{\text{b}}={\tan}^{-1 } 1.52=56.7 \text{°.} $$

Discussion

Light reflected at these angles could be completely blocked by a good polarizing filter held with its *axis vertical*. Brewster's angle for water and air are similar to those for glass and air, so that sunglasses are equally effective for light reflected from either water or glass under similar circumstances. Light not reflected is refracted into these media. So at an incident angle equal to Brewster's angle, the refracted light will be slightly polarized vertically. It will not be completely polarized vertically, because only a small fraction of the incident light is reflected, and so a significant amount of horizontally polarized light is refracted.

Polarization by Scattering

If you hold your Polaroid sunglasses in front of you and rotate them while looking at blue sky, you will see the sky get bright and dim. This is a clear indication that light scattered by air is partially polarized. [Figure 10] helps illustrate how this happens. Since light is a transverse EM wave, it vibrates the electrons of air molecules perpendicular to the direction it is traveling. The electrons then radiate like small antennae. Since they are oscillating perpendicular to the direction of the light ray, they produce EM radiation that is polarized perpendicular to the direction of the ray. When viewing the light along a line perpendicular to the original ray, as in [Figure 11], there can be no polarization in the scattered light parallel to the original ray, because that would require the original ray to be a longitudinal wave. Along other directions, a component of the other polarization can be projected along the line of sight, and the scattered light will only be partially polarized. Furthermore, multiple scattering can bring light to your eyes from other directions and can contain different polarizations.



Polarization by scattering. Unpolarized light scattering from air molecules shakes their electrons perpendicular to the direction of the original ray. The scattered light therefore has a polarization perpendicular to the original direction and none parallel to the original direction.

Photographs of the sky can be darkened by polarizing filters, a trick used by many photographers to make clouds brighter by contrast. Scattering from other particles, such as smoke or dust, can also polarize light. Detecting polarization in scattered EM waves can be a useful analytical tool in determining the scattering source.

There is a range of optical effects used in sunglasses. Besides being Polaroid, other sunglasses have colored pigments embedded in them, while others use non-reflective or even reflective coatings. A recent development is photochromic lenses, which darken in the sunlight and become clear indoors. Photochromic lenses are embedded with organic microcrystalline molecules that change their properties when exposed to UV in sunlight, but become clear in artificial lighting with no UV.

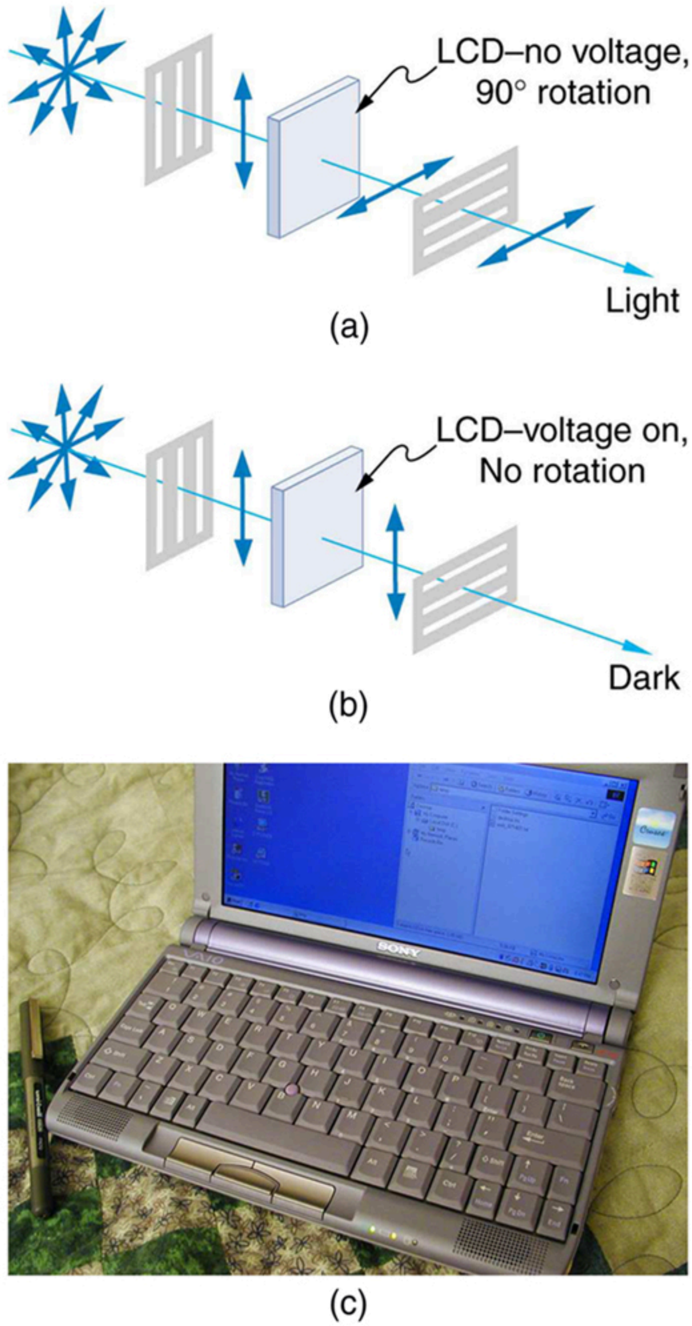
Take-Home Experiment: Polarization

Find Polaroid sunglasses and rotate one while holding the other still and look at different surfaces and objects. Explain your observations. What is the difference in angle from when you see a maximum intensity to when you see a minimum intensity? Find a reflective glass surface and do the same. At what angle does the glass need to be oriented to give minimum glare?

Liquid Crystals and Other Polarization Effects in Materials

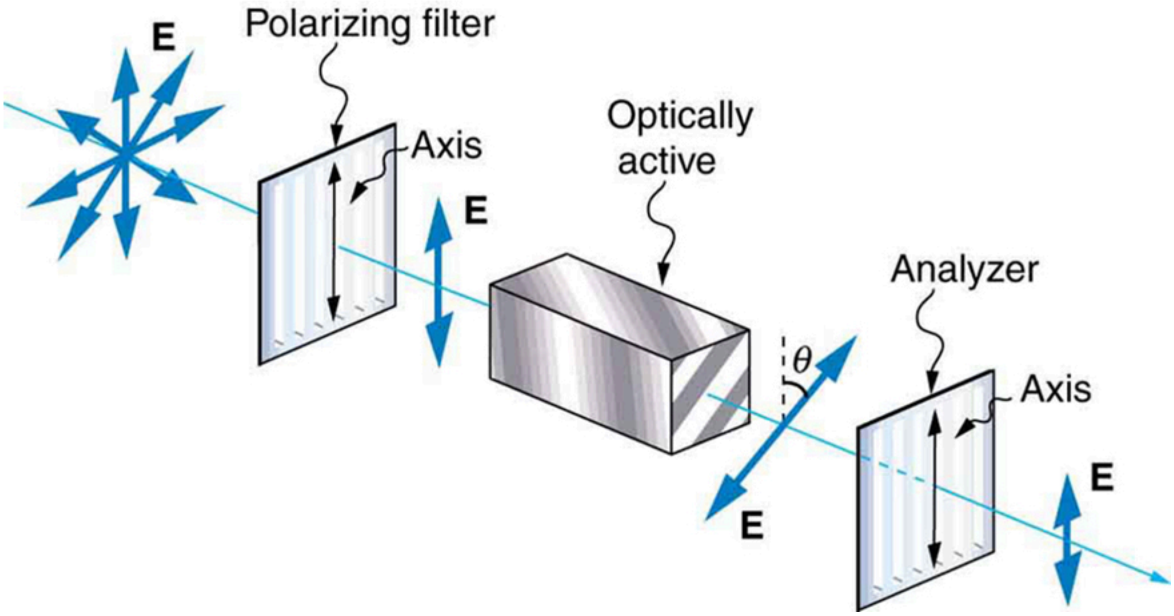
While you are undoubtedly aware of liquid crystal displays (LCDs) found in watches, calculators, computer screens, cellphones, flat screen televisions, and other myriad places, you may not be aware that they are based on polarization. Liquid crystals are so named because their molecules can be aligned even though they are in a liquid. Liquid crystals have the property that they can rotate the polarization of light passing through them by 90° . Furthermore, this property can be turned off by the application of a voltage, as illustrated in [Figure 12]. It is possible to manipulate this characteristic quickly and in small well-defined regions to create the contrast patterns we see in so many LCD devices.

In flat screen LCD televisions, there is a large light at the back of the TV. The light travels to the front screen through millions of tiny units called pixels (picture elements). One of these is shown in [Figure 12] (a) and (b). Each unit has three cells, with red, blue, or green filters, each controlled independently. When the voltage across a liquid crystal is switched off, the liquid crystal passes the light through the particular filter. One can vary the picture contrast by varying the strength of the voltage applied to the liquid crystal.



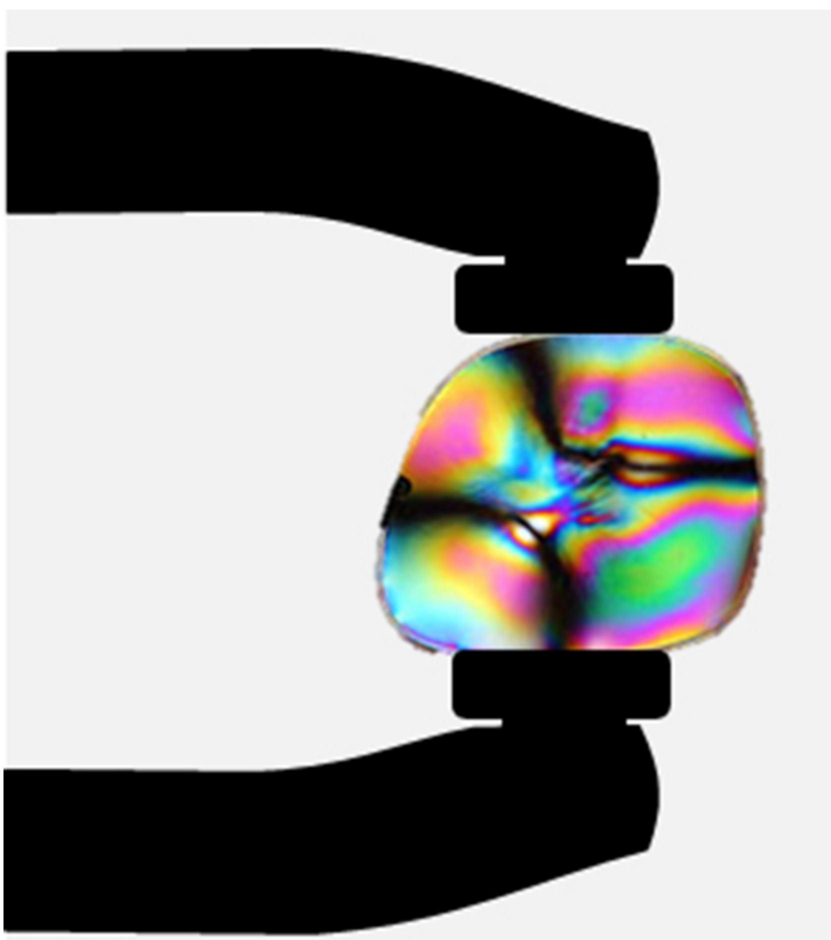
(a) Polarized light is rotated 90 degrees by a liquid crystal and then passed by a polarizing filter that has its axis perpendicular to the original polarization direction. (b) When a voltage is applied to the liquid crystal, the polarized light is not rotated and is blocked by the filter, making the region dark in comparison with its surroundings. (c) LCDs can be made color specific, small, and fast enough to use in laptop computers and TVs. (credit: Jon Sullivan)

Many crystals and solutions rotate the plane of polarization of light passing through them. Such substances are said to be **optically active**. Examples include sugar water, insulin, and collagen (see [Figure 12]). In addition to depending on the type of substance, the amount and direction of rotation depends on a number of factors. Among these is the concentration of the substance, the distance the light travels through it, and the wavelength of light. Optical activity is due to the asymmetric shape of molecules in the substance, such as being helical. Measurements of the rotation of polarized light passing through substances can thus be used to measure concentrations, a standard technique for sugars. It can also give information on the shapes of molecules, such as proteins, and factors that affect their shapes, such as temperature and pH.



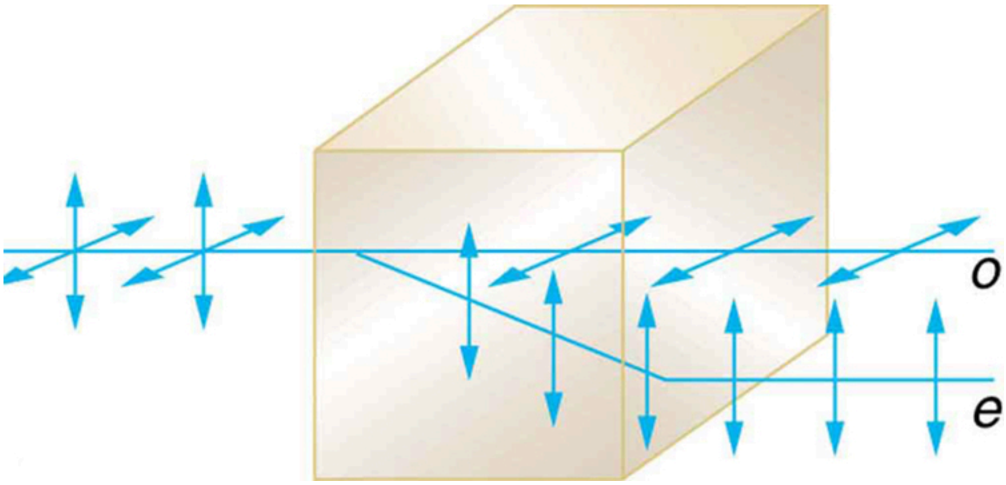
Optical activity is the ability of some substances to rotate the plane of polarization of light passing through them. The rotation is detected with a polarizing filter or analyzer.

Glass and plastic become optically active when stressed; the greater the stress, the greater the effect. Optical stress analysis on complicated shapes can be performed by making plastic models of them and observing them through crossed filters, as seen in [Figure 14]. It is apparent that the effect depends on wavelength as well as stress. The wavelength dependence is sometimes also used for artistic purposes.



Optical stress analysis of a plastic lens placed between crossed polarizers. (credit: Infopro, Wikimedia Commons)

Another interesting phenomenon associated with polarized light is the ability of some crystals to split an unpolarized beam of light into two. Such crystals are said to be **birefringent** (see [Figure 15]). Each of the separated rays has a specific polarization. One behaves normally and is called the ordinary ray, whereas the other does not obey Snell's law and is called the extraordinary ray. Birefringent crystals can be used to produce polarized beams from unpolarized light. Some birefringent materials preferentially absorb one of the polarizations. These materials are called dichroic and can produce polarization by this preferential absorption. This is fundamentally how polarizing filters and other polarizers work. The interested reader is invited to further pursue the numerous properties of materials related to polarization.



Birefringent materials, such as the common mineral calcite, split unpolarized beams of light into two. The ordinary ray behaves as expected, but the extraordinary ray does not obey Snell's law.

Section Summary

- Polarization is the attribute that wave oscillations have a definite direction relative to the direction of propagation of the wave.
- EM waves are transverse waves that may be polarized.
- The direction of polarization is defined to be the direction parallel to the electric field of the EM wave.
- Unpolarized light is composed of many rays having random polarization directions.
- Light can be polarized by passing it through a polarizing filter or other polarizing material. The intensity I of polarized light after passing through a polarizing filter is $I={I}_{0} {\cos}^{2} \mathrm{\theta ,}$ where ${I}_{0}$ is the original intensity and θ is the angle between the direction of polarization and the axis of the filter.
- Polarization is also produced by reflection.
- Brewster's law states that reflected light will be completely polarized at the angle of reflection ${\theta }_{\text{b}}$, known as Brewster's angle, given by a statement known as Brewster's law: $\tan {\theta }_{\text{b}}=\frac{ {n}_{2}}{ {n}_{1}}$, where ${n}_{1}$ is the medium in which the incident and reflected light travel and ${n}_{2}$ is the index of refraction of the medium that forms the interface that reflects the light.
- Polarization can also be produced by scattering.
- There are a number of types of optically active substances that rotate the direction of polarization of light passing through them.

Conceptual Questions

Under what circumstances is the phase of light changed by reflection? Is the phase related to polarization?

Can a sound wave in air be polarized? Explain.

No light passes through two perfect polarizing filters with perpendicular axes. However, if a third polarizing filter is placed between the original two, some light can pass. Why is this? Under what circumstances does most of the light pass?

Explain what happens to the energy carried by light that it is dimmed by passing it through two crossed polarizing filters.

When particles scattering light are much smaller than its wavelength, the amount of scattering is proportional to $1/{\lambda }^{4}$. Does this mean there is more scattering for small λ than large λ ? How does this relate to the fact that the sky is blue?

Using the information given in the preceding question, explain why sunsets are red.

When light is reflected at Brewster's angle from a smooth surface, it is 100% polarized parallel to the surface. Part of the light will be refracted into the surface. Describe how you would do an experiment to determine the polarization of the refracted light. What direction would you expect the polarization to have and would you expect it to be 100% ?

Problems & Exercises

What angle is needed between the direction of polarized light and the axis of a polarizing filter to cut its intensity in half?

Show Solution

Strategy

Use Malus's law: $I = I_0 \cos^2 \theta$. For the intensity to be cut in half, $I = I_0/2$.

Solution

Setting I = I_0/2:

$$\frac{I_0}{2} = I_0 \cos^2 \theta$$
$$\cos^2 \theta = \frac{1}{2} = 0.500$$
$$\cos \theta = \sqrt{0.500} = 0.7071$$
$$\theta = \cos^{-1}(0.7071) = 45.0°$$

Discussion

The 45° angle is exactly halfway between parallel (0°, maximum transmission) and perpendicular (90°, zero transmission). This symmetric position naturally corresponds to 50% transmission. This relationship is important in many optical applications, including variable neutral density filters and light intensity modulators where precise control over light intensity is needed.

The angle between the axes of two polarizing filters is $45.0º$. By how much does the second filter reduce the intensity of the light coming through the first?

Show Solution

Strategy

Use Malus's law: $I = I_0 \cos^2 \theta$ where θ = 45.0° is the angle between the polarizer axes.

Solution

After passing through the first polarizing filter, the light is fully polarized with intensity I_0. When this polarized light passes through the second filter at angle θ = 45.0°:

$$I = I_0 \cos^2 \theta = I_0 \cos^2 45.0°$$
$$I = I_0 (0.7071)^2 = I_0 (0.500) = 0.500 I_0$$

The second filter reduces the intensity to 50.0% of the value after the first filter, which means it **reduces the intensity by 50.0%** (or cuts it in half).

Discussion

This 50% reduction at 45° is a special case. At θ = 0° (parallel axes), no reduction occurs (I = I_0). At θ = 90° (perpendicular axes), complete extinction occurs (I = 0). The 45° angle represents the midpoint where exactly half the intensity passes through. This property is used in variable neutral density filters and light modulators.

If you have completely polarized light of intensity $150 \text{W}/{\text{m}}^{2}$, what will its intensity be after passing through a polarizing filter with its axis at an $89.0º$ angle to the light's polarization direction?

Show Solution

Strategy

Use Malus's law: $I = I_0 \cos^2 \theta$ with I_0 = 150 W/m² and θ = 89.0°.

Solution

$$I = I_0 \cos^2 \theta = (150 \text{ W/m}^2) \cos^2 89.0°$$
$$I = (150) (0.01746)^2 = (150)(3.048 \times 10^{-4})$$
$$I = 0.0457 \text{ W/m}^2 = 45.7 \text{ mW/m}^2$$

Discussion

At 89.0°, the filter axis is nearly perpendicular to the polarization direction (only 1° from complete extinction). The transmitted intensity is reduced to about 0.03% of the original—a dramatic reduction. This demonstrates how sensitive polarization effects are near 90°. Just one degree away from perpendicular allows a small but measurable amount of light to pass through.

What angle would the axis of a polarizing filter need to make with the direction of polarized light of intensity $1.00 {\text{kW/m}}^{2}$ to reduce the intensity to $10.0 {\text{W/m}}^{2}$?

Show Solution

Strategy

Use Malus's law $I = I_0 \cos^2 \theta$ and solve for θ.

Solution

Given: I_0 = 1.00 kW/m² = 1000 W/m², I = 10.0 W/m²

$$I = I_0 \cos^2 \theta$$
$$\cos \theta = \sqrt{\frac{I}{I_0}} = \sqrt{\frac{10.0}{1000}} = \sqrt{0.0100} = 0.100$$
$$\theta = \cos^{-1}(0.100) = 84.3°$$

Discussion

The angle of 84.3° is close to 90° (perpendicular), which makes sense for such a large intensity reduction (to 1% of original). This is just 5.7° away from complete extinction.

At the end of [Example 1], it was stated that the intensity of polarized light is reduced to 90.0% of its original value by passing through a polarizing filter with its axis at an angle of $18.4º$ to the direction of polarization. Verify this statement.

Show Solution

Strategy

Use Malus's law: $I = I_0 \cos^2 \theta$ with θ = 18.4° and verify that $I/I_0 = 0.900$.

Solution

$$I = I_0 \cos^2 18.4° = I_0 (0.9483)^2$$
$$I = I_0 (0.8993) \approx 0.900 I_0$$

Therefore, $I = 90.0\% \text{ of } I_0$, **which verifies the statement.**

Discussion

This confirms the symmetric relationship mentioned in Example 1: at 18.4° from the polarization direction, the intensity is reduced to 90% (meaning 10% is blocked). At 71.6° (which is 18.4° from perpendicular), the intensity is reduced to 10% (meaning 90% is blocked). These complementary angles (18.4° + 71.6° = 90°) demonstrate the symmetry of the cosine-squared function in Malus's law.

Show that if you have three polarizing filters, with the second at an angle of $45 ^\circ$ to the first and the third at an angle of $90.0º$ to the first, the intensity of light passed by the first will be reduced to 25.0% of its value. (This is in contrast to having only the first and third, which reduces the intensity to zero, so that placing the second between them increases the intensity of the transmitted light.)

Show Solution

Strategy

Apply Malus's law twice: once for the first-to-second filter (θ = 45°), then for the second-to-third filter (θ = 45° also, since third is 90° from first).

Solution

Let I_0 be the intensity after the first polarizer.

After second filter (45° from first):

$$I_2 = I_0 \cos^2 45° = I_0 (0.7071)^2 = 0.500 I_0$$

After third filter (90° from first, so 45° from second):

$$I_3 = I_2 \cos^2 45° = (0.500 I_0)(0.500) = 0.250 I_0$$

Therefore, the final intensity is **25.0%** of I_0. **Q.E.D.**

Discussion

With only the first and third filters (at 90°), $I = I_0 \cos^2(90°) = 0$ (no light passes). But inserting a 45° filter between them allows 25% transmission! This counterintuitive result occurs because the middle filter "rotates" the polarization direction, allowing some light to pass through the final filter.

Prove that, if I is the intensity of light transmitted by two polarizing filters with axes at an angle θ and $I^{\prime}$ is the intensity when the axes are at an angle $90.0 ^\circ-\theta ,$ then $I+I^{\prime}={I}_{0,}$ the original intensity. (Hint: Use the trigonometric identities $\cos \left(90.0º-\theta \right)=\sin \theta$ and ${\cos}^{2} \theta +{\sin}^{2} \theta =1.$)

Show Solution

Strategy

Apply Malus's law to both angle configurations and use the given trigonometric identities to show the intensities sum to I_0.

Solution

For the first configuration at angle θ:

$$I = I_0 \cos^2 \theta$$

For the second configuration at angle (90° - θ):

$$I' = I_0 \cos^2(90° - \theta)$$

Using the identity $\cos(90° - \theta) = \sin \theta$:

$$I' = I_0 \sin^2 \theta$$

Now add the two intensities:

$$I + I' = I_0 \cos^2 \theta + I_0 \sin^2 \theta$$
$$I + I' = I_0 (\cos^2 \theta + \sin^2 \theta)$$

Using the fundamental identity $\cos^2 \theta + \sin^2 \theta = 1$:

$$I + I' = I_0 (1) = I_0$$

Therefore: $I + I' = I_0$ **Q.E.D.**

Discussion

This beautiful result shows that complementary polarizer angles transmit intensities that sum to the original intensity. For example, if a 30° filter transmits 75% of the light, then a 60° filter (90° - 30°) transmits the remaining 25%. This conservation of energy makes physical sense: the total transmitted light through the two complementary orientations equals what would pass if there were no angular selectivity at all.

At what angle will light reflected from diamond be completely polarized?

Show Solution

Strategy

Use Brewster's law: $\tan \theta_b = n_2/n_1$ where n_1 = 1.00 (air) and n_2 = 2.419 (diamond).

Solution

$$\tan \theta_b = \frac{2.419}{1.00} = 2.419$$
$$\theta_b = \tan^{-1}(2.419) = 67.5°$$

Discussion

Diamond's high refractive index (2.419) produces a large Brewster angle of 67.5°, close to grazing incidence. This is why diamonds sparkle brilliantly - their high index causes strong refraction and internal reflection at most angles.

What is Brewster's angle for light traveling in water that is reflected from crown glass?

Show Solution

Strategy

Use Brewster's law: $\tan \theta_b = n_2/n_1$ where n_1 = 1.333 (water) and n_2 = 1.52 (crown glass).

Solution

$$\tan \theta_b = \frac{n_2}{n_1} = \frac{1.52}{1.333} = 1.140$$
$$\theta_b = \tan^{-1}(1.140) = 48.8°$$

Discussion

Brewster's angle of 48.8° applies when light travels through water and reflects off crown glass (such as the glass wall of an aquarium). This is smaller than Brewster's angle for air-to-glass reflection (56.7°) because the difference in refractive indices is smaller when starting from water (n = 1.333) rather than air (n = 1.00). The smaller index contrast means the reflected light becomes completely polarized at a smaller angle from the normal.

A scuba diver sees light reflected from the water's surface. At what angle will this light be completely polarized?

Show Solution

Strategy

The diver is underwater looking up at the water-air interface. Use Brewster's law with n_1 = 1.33 (water) and n_2 = 1.00 (air).

Solution

$$\tan \theta_b = \frac{n_2}{n_1} = \frac{1.00}{1.33} = 0.752$$
$$\theta_b = \tan^{-1}(0.752) = 36.9°$$

Discussion

The Brewster angle of 36.9° from the normal (measured underwater) is relatively small. This is the complement of the Brewster angle for light going from air to water (53.1°), since 36.9° + 53.1° = 90°.

At what angle is light inside crown glass completely polarized when reflected from water, as in a fish tank?

Show Solution

Strategy

Use Brewster's law with light traveling in crown glass (n_1 = 1.52) and reflecting from water (n_2 = 1.333).

Solution

$$\tan \theta_b = \frac{n_2}{n_1} = \frac{1.333}{1.52} = 0.8770$$
$$\theta_b = \tan^{-1}(0.8770) = 41.2°$$

Discussion

This is Brewster's angle for light traveling inside the glass wall of a fish tank and reflecting off the water inside. Notice that this angle (41.2°) is the complement of the water-to-glass Brewster angle (48.8°), and indeed 41.2° + 48.8° = 90.0°, confirming the reciprocal relationship $\theta_b + \theta'_b = 90°$ for reflections from opposite sides of an interface. Light at this angle inside the glass would see complete polarization of the reflection from the water surface.

Light reflected at $55.6º$ from a window is completely polarized. What is the window's index of refraction and the likely substance of which it is made?

Show Solution

Strategy

Since the light is completely polarized, θ = 55.6° is Brewster's angle. Use $\tan \theta_b = n_2/n_1$ with n_1 = 1.00 (air).

Solution

$$n_2 = n_1 \tan \theta_b = (1.00) \tan 55.6° = 1.46$$

Discussion

An index of refraction of 1.46 is typical of **window glass** (common soda-lime glass has n ≈ 1.46-1.52). This could also be Plexiglas (n ≈ 1.49) or other transparent plastics used in windows.

(a) Light reflected at $62.5º$ from a gemstone in a ring is completely polarized. Can the gem be a diamond? (b) At what angle would the light be completely polarized if the gem was in water?

Show Solution

Strategy

For part (a), use Brewster's law to find the index of refraction and compare to diamond (n = 2.419). For part (b), use the index found in (a) with water as the incident medium.

Solution

(a) Can the gem be a diamond?

Since the light is completely polarized, θ = 62.5° is Brewster's angle. Using $\tan \theta_b = n_2/n_1$ with n_1 = 1.00 (air):

$$n_2 = n_1 \tan \theta_b = (1.00) \tan 62.5° = 1.92$$

Diamond has n = 2.419, which is significantly higher than 1.92. Therefore, **this gem cannot be diamond**.

The index of 1.92 is consistent with **zircon** (n ≈ 1.92-1.98) or **cubic zirconia** (n ≈ 2.15-2.18), which are common diamond simulants.

(b) Brewster's angle if the gem is in water:

Using n_1 = 1.333 (water) and n_2 = 1.92 (from part a):

$$\tan \theta_b = \frac{n_2}{n_1} = \frac{1.92}{1.333} = 1.440$$
$$\theta_b = \tan^{-1}(1.440) = 55.2°$$

Discussion

The index of refraction is a definitive way to identify gemstones. Diamond's exceptionally high index (2.419) would give a Brewster angle of 67.5°, much larger than the observed 62.5°. The calculated index of 1.92 rules out diamond and suggests zircon or a synthetic simulant. When the gem is immersed in water, Brewster's angle decreases from 62.5° to 55.2° because the refractive index contrast is reduced.

If ${\theta }_{\text{b}}$ is Brewster's angle for light reflected from the top of an interface between two substances, and ${\theta^{\prime} }_{\text{b}}$ is Brewster's angle for light reflected from below, prove that ${\theta }_{\text{b}}+{\theta^{\prime} }_{\text{b}}= 90.0º.$

Show Solution

Strategy

Use Brewster's law for both directions and the trigonometric identity $\tan \theta + \tan(90° - \theta) = \infty$ or $\tan \theta_1 \cdot \tan \theta_2 = 1$ when $\theta_1 + \theta_2 = 90°$.

Solution

For light from medium 1 (n_1) to medium 2 (n_2):

$$\tan \theta_b = \frac{n_2}{n_1}$$

For light from medium 2 to medium 1 (reverse direction):

$$\tan \theta'_b = \frac{n_1}{n_2}$$

Notice that:

$$\tan \theta_b \cdot \tan \theta'_b = \frac{n_2}{n_1} \cdot \frac{n_1}{n_2} = 1$$

From trigonometry, when $\tan \theta_b \cdot \tan \theta'_b = 1$, this occurs when $\theta_b + \theta'_b = 90°$ (since $\tan \theta \cdot \tan(90° - \theta) = \tan \theta \cdot \cot \theta = 1$).

Therefore: $\theta_b + \theta'_b = 90.0°$ **Q.E.D.**

Discussion

This complementary relationship means Brewster's angles from opposite sides of an interface always sum to 90°. For example, air-to-water ($\theta_b \approx 53°$) and water-to-air ($\theta'_b \approx 37°$) sum to 90°.

Integrated Concepts

If a polarizing filter reduces the intensity of polarized light to 50.0% of its original value, by how much are the electric and magnetic fields reduced?

Show Solution

Strategy

Intensity is proportional to the square of the electric field amplitude (and magnetic field amplitude). If I is reduced to 50%, find the reduction in E and B.

Solution

For electromagnetic waves, intensity is proportional to the square of the field amplitudes:

$$I \propto E^2 \propto B^2$$

If $I_2 = 0.500 I_1$, then:

$$\frac{I_2}{I_1} = \frac{E_2^2}{E_1^2} = 0.500$$
$$\frac{E_2}{E_1} = \sqrt{0.500} = 0.707$$

Therefore: $E_2 = 0.707 E_1$

Similarly: $B_2 = 0.707 B_1$

Both the electric and magnetic field amplitudes are reduced to 70.7% (or reduced by 29.3%) of their original values.

Discussion

The square-root relationship between field amplitude and intensity is fundamental to wave physics. When intensity drops by half, the fields don't drop by half—they drop by a factor of $\sqrt{2} \approx 0.707$. This same relationship applies to all waves where intensity is proportional to amplitude squared, including sound waves and water waves.

Integrated Concepts

Suppose you put on two pairs of Polaroid sunglasses with their axes at an angle of $15.0º$. How much longer will it take the light to deposit a given amount of energy in your eye compared with a single pair of sunglasses? Assume the lenses are clear except for their polarizing characteristics.

Show Solution

Strategy

Compare the intensity (power) transmitted through one vs. two pairs. Time to deposit energy is inversely proportional to power/intensity.

Solution

One pair: Unpolarized light through a polarizer transmits $I_1 = I_0/2$

Two pairs at 15°: First polarizer transmits $I_0/2$, then Malus's law for the second:

$$I_2 = \frac{I_0}{2} \cos^2 15.0° = \frac{I_0}{2}(0.9659)^2 = 0.467 I_0$$

Time ratio (since Power = Energy/time):

$$\frac{t_2}{t_1} = \frac{I_1}{I_2} = \frac{0.500 I_0}{0.467 I_0} = 1.07$$

The two pairs take **7% longer** (or equivalently, 1.07 times as long) to deposit the same energy.

Discussion

At only 15° misalignment, the second pair blocks relatively little additional light ($\cos^2 15° \approx 0.93$). The time difference is small. At 90° misalignment, no light would pass and it would take infinite time!

Integrated Concepts

(a) On a day when the intensity of sunlight is $1.00 \text{kW}/{\text{m}}^{2}$, a circular lens 0.200 m in diameter focuses light onto water in a black beaker. Two polarizing sheets of plastic are placed in front of the lens with their axes at an angle of $20.0º.$ Assuming the sunlight is unpolarized and the polarizers are 100% efficient, what is the initial rate of heating of the water in $\text{ºC}/\text{s}$, assuming it is 80.0% absorbed? The aluminum beaker has a mass of 30.0 grams and contains 250 grams of water. (b) Do the polarizing filters get hot? Explain.

Show Solution

(a) 2.07×10^{-2} ºC/s

(b) Yes, the polarizing filters get hot because they absorb some of the lost energy from the sunlight.

Glossary

axis of a polarizing filter
: the direction along which the filter passes the electric field of an EM wave

birefringent
: crystals that split an unpolarized beam of light into two beams

Brewster's angle
: ${\theta }_{\text{b}}={\tan}^{-1}\left(\frac{ {n}_{2}}{ {n}_{1}}\right),$ where ${n}_{2}$ is the index of refraction of the medium from which the light is reflected and ${n}_{1}$ is the index of refraction of the medium in which the reflected light travels

Brewster's law
: $\tan {\theta }_{\text{b}}=\frac{ {n}_{2}}{ {n}_{1}}$, where ${n}_{1}$ is the medium in which the incident and reflected light travel and ${n}_{2}$ is the index of refraction of the medium that forms the interface that reflects the light

direction of polarization
: the direction parallel to the electric field for EM waves

horizontally polarized
: the oscillations are in a horizontal plane

optically active
: substances that rotate the plane of polarization of light passing through them

polarization
: the attribute that wave oscillations have a definite direction relative to the direction of propagation of the wave

polarized
: waves having the electric and magnetic field oscillations in a definite direction

reflected light that is completely polarized
: light reflected at the angle of reflection ${\theta }_{\text{b}}$, known as Brewster's angle

unpolarized
: waves that are randomly polarized

vertically polarized
: the oscillations are in a vertical plane

Extended Topic Microscopy Enhanced by the Wave Characteristics of Light

- Discuss the different types of microscopes.

Physics research underpins the advancement of developments in microscopy. As we gain knowledge of the wave nature of electromagnetic waves and methods to analyze and interpret signals, new microscopes that enable us to “see” more are being developed. It is the evolution and newer generation of microscopes that are described in this section.

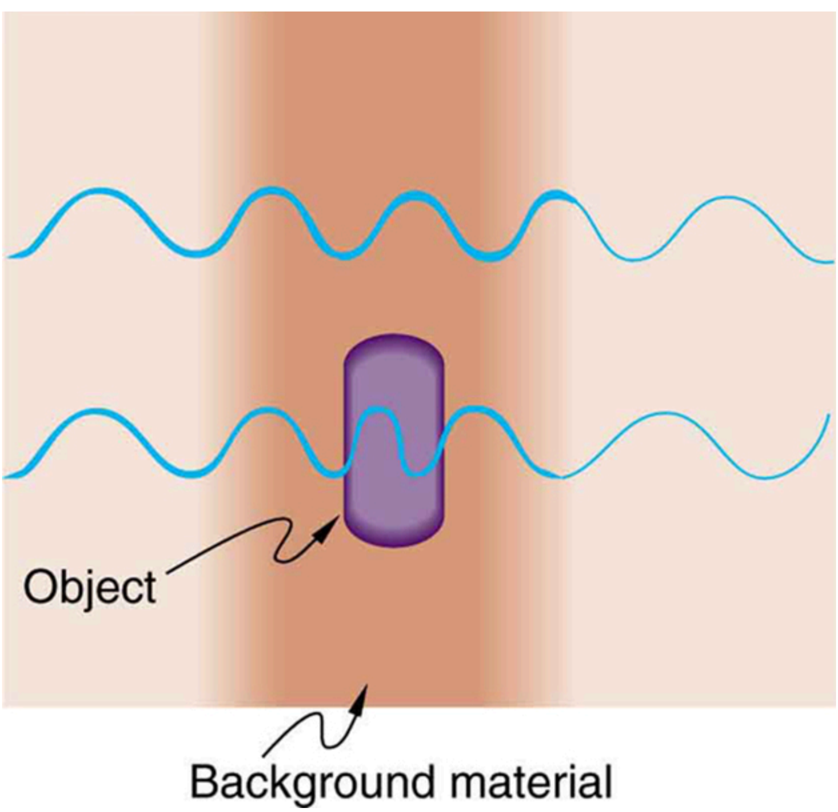
The use of microscopes (microscopy) to observe small details is limited by the wave nature of light. Owing to the fact that light diffracts significantly around small objects, it becomes impossible to observe details significantly smaller than the wavelength of light. One rule of thumb has it that all details smaller than about λ are difficult to observe. Radar, for example, can detect the size of an aircraft, but not its individual rivets, since the wavelength of most radar is several centimeters or greater. Similarly, visible light cannot detect individual atoms, since atoms are about 0.1 nm in size and visible wavelengths range from 380 to 760 nm. Ironically, special techniques used to obtain the best possible resolution with microscopes take advantage of the same wave characteristics of light that ultimately limit the detail.

Making Connections: Waves

All attempts to observe the size and shape of objects are limited by the wavelength of the probe. Sonar and medical ultrasound are limited by the wavelength of sound they employ. We shall see that this is also true in electron microscopy, since electrons have a wavelength. Heisenberg’s uncertainty principle asserts that this limit is fundamental and inescapable, as we shall see in quantum mechanics.

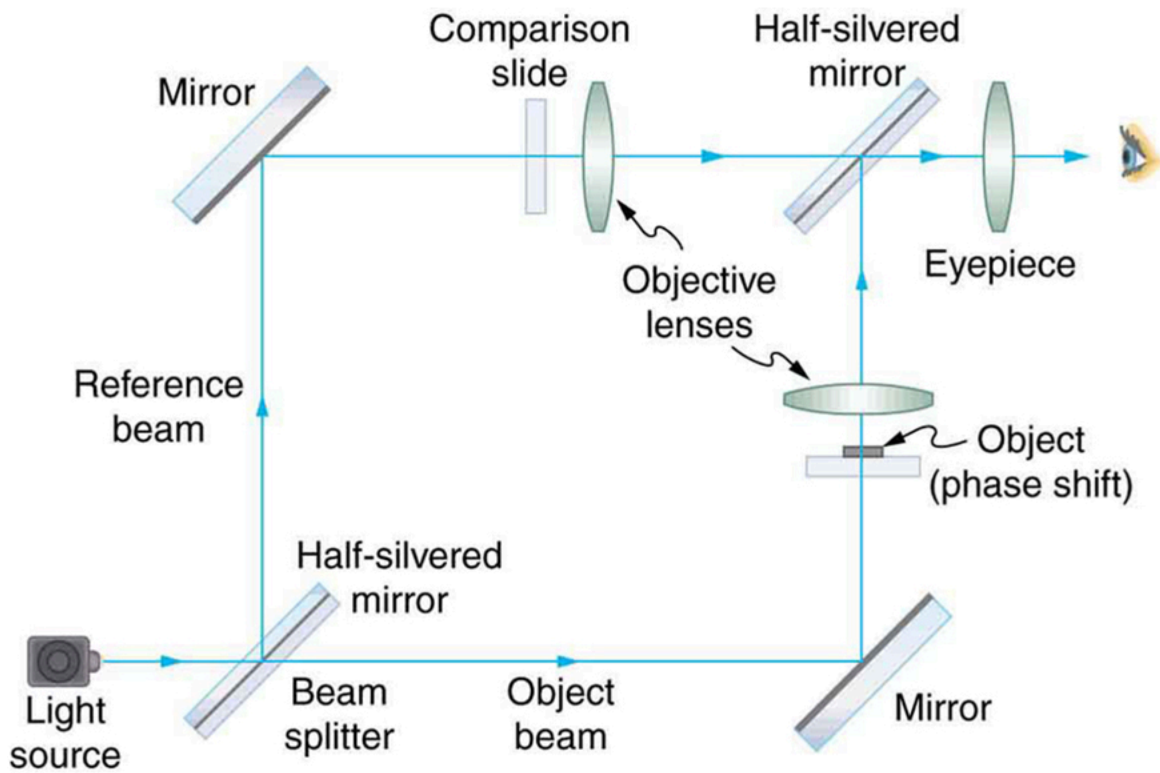
The most obvious method of obtaining better detail is to utilize shorter wavelengths. **Ultraviolet (UV) microscopes** have been constructed with special lenses that transmit UV rays and utilize photographic or electronic techniques to record images. The shorter UV wavelengths allow somewhat greater detail to be observed, but drawbacks, such as the hazard of UV to living tissue and the need for special detection devices and lenses (which tend to be dispersive in the UV) , severely limit the use of UV microscopes. Elsewhere, we will explore practical uses of very short wavelength EM waves, such as X-rays, and other short-wavelength probes, such as electrons in electron microscopes, to detect small details.

Another difficulty in microscopy is the fact that many microscopic objects do not absorb much of the light passing through them. The lack of contrast makes image interpretation very difficult. **Contrast** is the difference in intensity between objects and the background on which they are observed. Stains (such as dyes, fluorophores, etc.) are commonly employed to enhance contrast, but these tend to be application specific. More general wave interference techniques can be used to produce contrast. [Figure 1] shows the passage of light through a sample. Since the indices of refraction differ, the number of wavelengths in the paths differs. Light emerging from the object is thus out of phase with light from the background and will interfere differently, producing enhanced contrast, especially if the light is coherent and monochromatic—as in laser light.



Light rays passing through a sample under a microscope will emerge with different phases depending on their paths. The object shown has a greater index of refraction than the background, and so the wavelength decreases as the ray passes through it. Superimposing these rays produces interference that varies with path, enhancing contrast between the object and background.

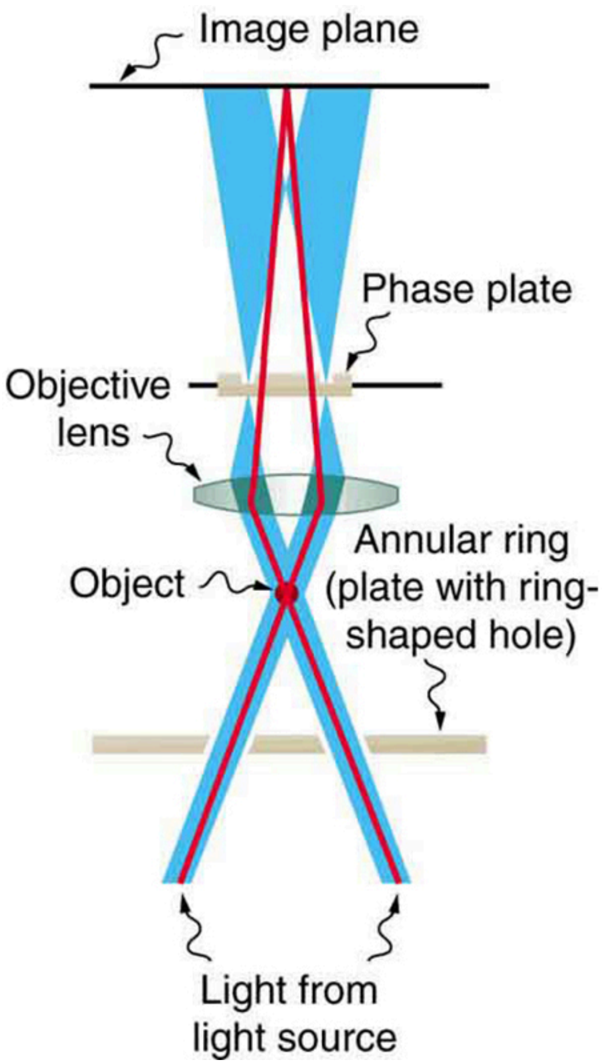
Interference microscopes enhance contrast between objects and background by superimposing a reference beam of light upon the light emerging from the sample. Since light from the background and objects differ in phase, there will be different amounts of constructive and destructive interference, producing the desired contrast in final intensity. [Figure 2] shows schematically how this is done. Parallel rays of light from a source are split into two beams by a half-silvered mirror. These beams are called the object and reference beams. Each beam passes through identical optical elements, except that the object beam passes through the object we wish to observe microscopically. The light beams are recombined by another half-silvered mirror and interfere. Since the light rays passing through different parts of the object have different phases, interference will be significantly different and, hence, have greater contrast between them.



An interference microscope utilizes interference between the reference and object beam to enhance contrast. The two beams are split by a half-silvered mirror; the object beam is sent through the object, and the reference beam is sent through otherwise identical optical elements. The beams are recombined by another half-silvered mirror, and the interference depends on the various phases emerging from different parts of the object, enhancing contrast.

Another type of microscope utilizing wave interference and differences in phases to enhance contrast is called the **phase-contrast microscope**. While its principle is the same as the interference microscope, the phase-contrast microscope is simpler to use and construct. Its impact (and the principle upon which it is based) was so important that its developer, the Dutch physicist Frits Zernike (1888–1966), was awarded the Nobel Prize in

1. [Figure 3] shows the basic construction of a phase-contrast microscope. Phase differences between light passing through the object and background are produced by passing the rays through different parts of a phase plate (so called because it shifts the phase of the light passing through it). These two light rays are superimposed in the image plane, producing contrast due to their interference.

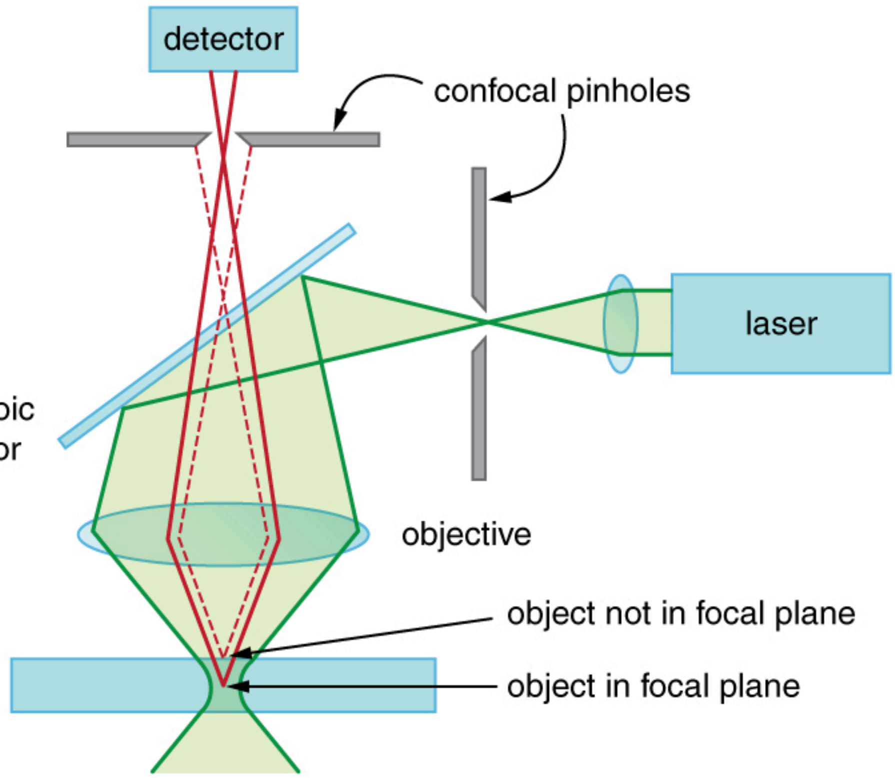


Simplified construction of a phase-contrast microscope. Phase differences between light passing through the object and background are produced by passing the rays through different parts of a phase plate. The light rays are superimposed in the image plane, producing contrast due to their interference.

A **polarization microscope** also enhances contrast by utilizing a wave characteristic of light. Polarization microscopes are useful for objects that are optically active or birefringent, particularly if those characteristics vary from place to place in the object. Polarized light is sent through the object and then observed through a polarizing filter that is perpendicular to the original polarization direction. Nearly transparent objects can then appear with strong color and in high contrast. Many polarization effects are wavelength dependent, producing color in the processed image. Contrast results from the action of the polarizing filter in passing only components parallel to its axis.

Apart from the UV microscope, the variations of microscopy discussed so far in this section are available as attachments to fairly standard microscopes or as slight variations. The next level of sophistication is provided by commercial **confocal microscopes**, which use the extended focal region shown in [Figure 7](b) to obtain three-dimensional images rather than two-dimensional images. Here, only a single plane or region of focus is identified; out-of-focus regions above and below this plane are subtracted out by a computer so the image quality is much better. This type of microscope makes use of fluorescence, where a laser provides the excitation light. Laser light passing through a tiny aperture called a pinhole forms an extended focal region within the specimen. The reflected light passes through the objective lens to a second pinhole and the photomultiplier detector, see [Figure 4]. The second pinhole is the key here and serves to block much of the light from points that are not at the focal point of the objective lens. The pinhole is conjugate (coupled) to the focal point of the lens. The second pinhole and detector are scanned, allowing reflected light from a small region or section of the extended focal region to be imaged at any one time. The out-of-focus light is excluded. Each image is stored in a computer, and a full scanned image is generated in a short time. Live cell processes can also be imaged at adequate scanning speeds allowing the imaging of three-dimensional microscopic movement. Confocal microscopy enhances images over conventional optical microscopy, especially for thicker specimens, and so has become quite popular.

The next level of sophistication is provided by microscopes attached to instruments that isolate and detect only a small wavelength band of light—monochromators and spectral analyzers. Here, the monochromatic light from a laser is scattered from the specimen. This scattered light shifts up or down as it excites particular energy levels in the sample. The uniqueness of the observed scattered light can give detailed information about the chemical composition of a given spot on the sample with high contrast—like molecular fingerprints. Applications are in materials science, nanotechnology, and the biomedical field. Fine details in biochemical processes over time can even be detected. The ultimate in microscopy is the electron microscope—to be discussed later. Research is being conducted into the development of new prototype microscopes that can become commercially available, providing better diagnostic and research capacities.



A confocal microscope provides three-dimensional images using pinholes and the extended depth of focus as described by wave optics. The right pinhole illuminates a tiny region of the sample in the focal plane. In-focus light rays from this tiny region pass through the dichroic mirror and the second pinhole to a detector and a computer. Out-of-focus light rays are blocked. The pinhole is scanned sideways to form an image of the entire focal plane. The pinhole can then be scanned up and down to gather images from different focal planes. The result is a three-dimensional image of the specimen.

Section Summary

- To improve microscope images, various techniques utilizing the wave characteristics of light have been developed. Many of these enhance contrast with interference effects.

Conceptual Questions

Explain how microscopes can use wave optics to improve contrast and why this is important.

Show Solution

Strategy

Contrast in microscopy refers to the difference in intensity between objects and their background. Many microscopic specimens are nearly transparent and have very low contrast. Wave optics provides several techniques to enhance contrast through interference effects.

Solution

Microscopes use wave optics to improve contrast through several mechanisms based on wave interference:

1. Phase-Contrast Microscopy:

- When light passes through transparent specimens with different refractive indices, it undergoes phase shifts but little amplitude change
- The phase plate converts these phase differences into amplitude (intensity) differences through interference
- Light from the object and background interfere constructively or destructively depending on their phase relationship
- This makes nearly transparent structures visible

2. Interference Microscopy:

- A reference beam is combined with the object beam
- The two beams interfere, and phase differences translate into intensity variations
- Provides quantitative information about specimen thickness and refractive index

3. Polarization Microscopy:

- Uses polarized light and specimens' birefringent properties
- Rotating polarizers create contrast based on how the specimen affects polarization

- Particularly useful for crystalline or fibrous structures

Why This is Important:

- **Biological specimens** are often nearly transparent and would be invisible in standard bright-field microscopy without staining
- **Staining can kill cells** or alter their structure, making live observation impossible
- **Wave optics techniques** allow visualization of living cells without chemical treatment
- These methods reveal **subtle structural details** like cell membranes, organelles, and fibers that would otherwise be invisible
- They enable **quantitative measurements** of cell thickness, density, and composition
- **Research applications** include studying cell division, movement, and dynamic processes in real time

Discussion

The fundamental principle is that while transparent objects may not absorb light significantly, they do alter its phase due to differences in refractive index and path length. Wave optics allows us to convert these "invisible" phase changes into visible intensity variations. This capability has revolutionized biology, medicine, and materials science by allowing detailed observation of structures and processes that would otherwise be undetectable. The development of these techniques (particularly phase-contrast microscopy by Zernike) was significant enough to earn Nobel Prizes.

A bright white light under water is collimated and directed upon a prism. What range of colors does one see emerging?

Show Solution

Strategy

Dispersion occurs when different wavelengths of light refract by different amounts due to the wavelength-dependence of the refractive index. We need to consider how the refractive indices of water and the prism material compare.

Solution

The range of colors observed depends on the prism material. There are two main cases:

Case 1: Glass or plastic prism (n_prism > n_water)

If the prism is made of glass (n ≈ 1.5-1.9) or plastic (n ≈ 1.4-1.6), and the surrounding medium is water (n ≈ 1.33), then n_prism > n_water. In this case:

- Light refracts into the prism and disperses
- **All visible colors are observable**: violet, blue, green, yellow, orange, and red
- However, the **dispersion is reduced** compared to the same prism in air because the difference in refractive indices is smaller (Δn ≈ 0.2-0.6 instead of Δn ≈ 0.5-0.9 in air)
- The angular separation between colors will be smaller than in air
- Colors still appear in the standard sequence: violet deviates most, red deviates least

Case 2: Crown glass prism where n_prism < n_water

For some materials (though rare), if n_prism < n_water:

- Total internal reflection might occur at certain angles
- Some wavelengths might not emerge from the prism
- The color sequence could be reversed

Typical scenario (glass prism in water):

You would see the **full visible spectrum** (red through violet), but with **less angular separation** than the same setup in air. The reduced dispersion occurs because:

$$\text{Angular dispersion} \propto \Delta n = |n_{\text{prism}} - n_{\text{medium}}|$$

Discussion

This scenario illustrates an important principle: dispersion depends not just on the prism material but on the relative difference between the refractive indices of the prism and surrounding medium. This is why diamonds sparkle less underwater than in air - the reduced refractive index contrast reduces both refraction angles and dispersion. Underwater photographers and researchers must account for this reduced dispersion when using optical instruments. The phenomenon also demonstrates that "color" is an intrinsic property of light's wavelength, but the degree of separation of colors through dispersion depends on the medium.

Glossary

confocal microscopes
: microscopes that use the extended focal region to obtain three-dimensional images rather than two-dimensional images

contrast
: the difference in intensity between objects and the background on which they are observed

interference microscopes
: microscopes that enhance contrast between objects and background by superimposing a reference beam of light upon the light emerging from the sample

phase-contrast microscope
: microscope utilizing wave interference and differences in phases to enhance contrast

polarization microscope

microscope that enhances contrast by utilizing a wave characteristic of light, useful for objects that are optically active

ultraviolet (UV) microscopes
: microscopes constructed with special lenses that transmit UV rays and utilize photographic or electronic techniques to record images

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