



ELASTICITY: STRESS AND STRAIN

LEARNING OUTCOMES:

- State Hooke's law.
- Explain Hooke's law using graphical representation between deformation and applied force.
- Discuss the three types of deformations such as changes in length, sideways shear and changes in volume.
- Describe with examples the young's modulus, shear modulus and bulk modulus.
- Determine the change in length given mass, length and radius.

We now move from consideration of forces that affect the motion of an object (such as friction and drag) to those that affect an object's shape. If a bulldozer pushes a car into a wall, the car will not move but it will noticeably change shape. A change in shape due to the application of a force is a **deformation**. Even very small forces are known to cause some deformation. For small deformations, two important characteristics are observed. First, the object returns to its original shape when the force is removed—that is, the deformation is elastic for small deformations. Second, the size of the deformation is proportional to the force—that is, for small deformations, Hooke's law is obeyed. In equation form, **Hooke's law** is given by

Eq.(1)

$$F = k\Delta L,$$

where ΔL is the amount of deformation (the change in length, for example) produced by the force F , and k is a proportionality constant that depends on the shape and composition of the object and the direction of the force. Note that this force is a function of the deformation ΔL —it is not constant as a kinetic friction force is. Rearranging this to

Eq.(2)

$$\Delta L = \frac{F}{k}$$

makes it clear that the deformation is proportional to the applied force. Figure 1 shows the Hooke's law relationship between the extension ΔL of a spring or of a human bone. For metals or springs, the straight line region in which Hooke's law pertains is much larger. Bones are brittle and the elastic region is small and the fracture abrupt. Eventually a large enough stress to the material will cause it to break or fracture. **Tensile strength** is the breaking stress that will cause permanent deformation or fracture of a material.

HOOKE'S LAW



Eq.(3)

$$F = k\Delta L,$$

where ΔL is the amount of deformation (the change in length, for example) produced by the force F , and k is a proportionality constant that depends on the shape and composition of the object and the direction of the force.

Eq.(4)

$$\Delta L = \frac{F}{k}$$

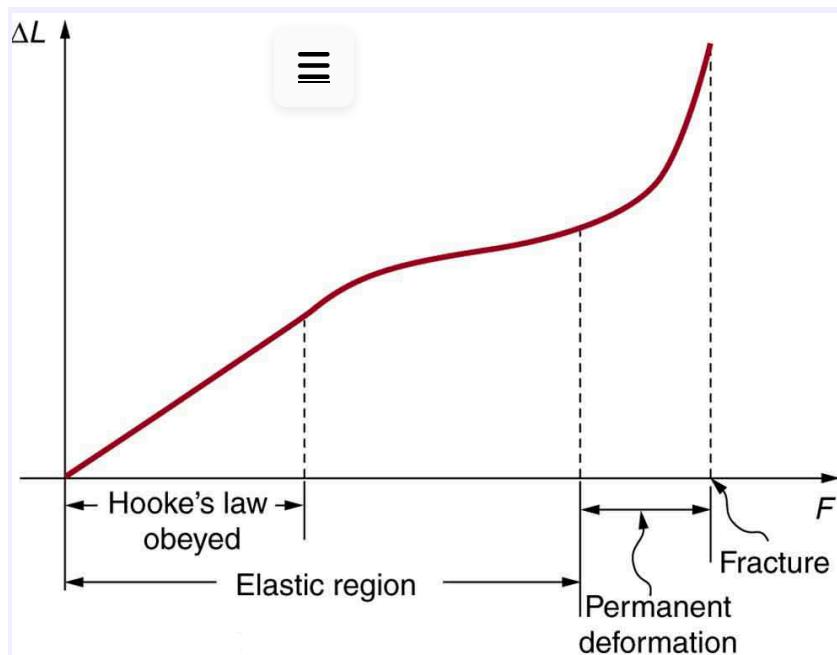


Figure 1: A graph of deformation (ΔL) versus applied force (F). The straight segment is the linear region where Hooke's law is obeyed. The slope of the straight region is (k). For larger forces, the graph is curved but the deformation is still elastic—(ΔL) will return to zero if the force is removed. Still greater forces permanently deform the object until it finally fractures. The shape of the curve near fracture depends on several factors, including how the force (F) is applied. Note that in this graph the slope increases just before fracture, indicating that a small increase in (F) is producing a large increase in (L) near the fracture.

The proportionality constant k depends upon a number of factors for the material. For example, a guitar string made of nylon stretches when it is tightened, and the elongation ΔL is proportional to the force applied (at least for small deformations). Thicker nylon strings and ones made of steel stretch less for the same applied force, implying they have a larger k (see [Figure 2](#)). Finally, all three strings return to their normal lengths when the force is removed, provided the deformation is small. Most materials will behave in this manner if the deformation is less than about 0.1% or about 1 part in 10^3 .

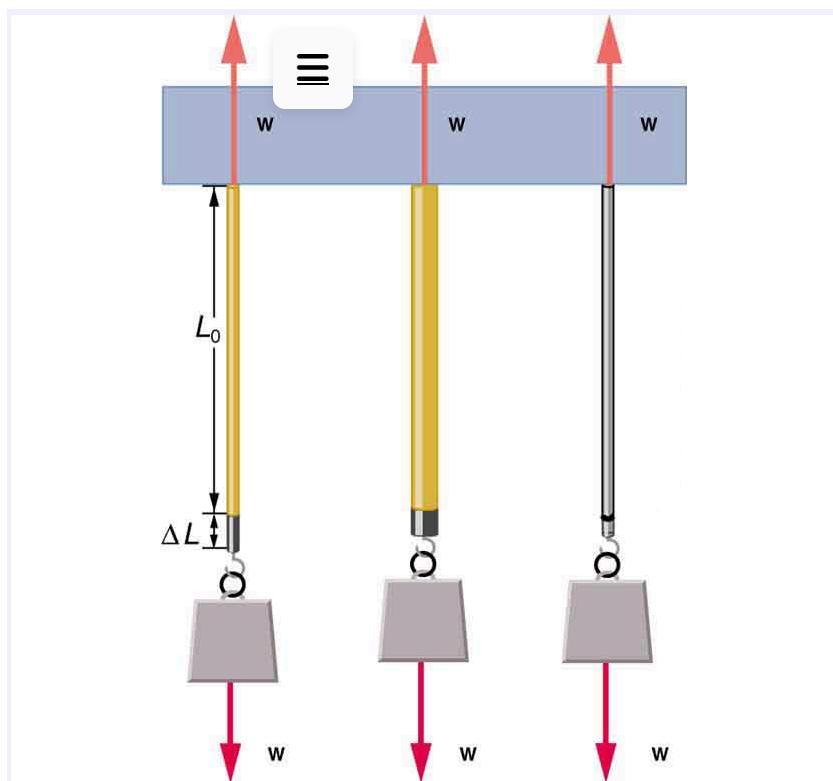


Figure 2: The same force, in this case a weight ((w)), applied to three different guitar strings of identical length produces the three different deformations shown as shaded segments. The string on the left is thin nylon, the one in the middle is thicker nylon, and the one on the right is steel.

STRETCH YOURSELF A LITTLE

How would you go about measuring the proportionality constant k of a rubber band? If a rubber band stretched 3 cm when a 100-g mass was attached to it, then how much would it stretch if two similar rubber bands were attached to the same mass—even if put together in parallel or alternatively if tied together in series?

We now consider three specific types of deformations: changes in length (tension and compression), sideways shear (stress), and changes in volume. All deformations are assumed to be small unless otherwise stated.

Changes in Length—Tension and Compression: Elastic Modulus

A change in length ΔL is produced when a force is applied to a wire or rod parallel to its length L_0 , either stretching it (a tension) or compressing it. (See [Figure 3](#).)

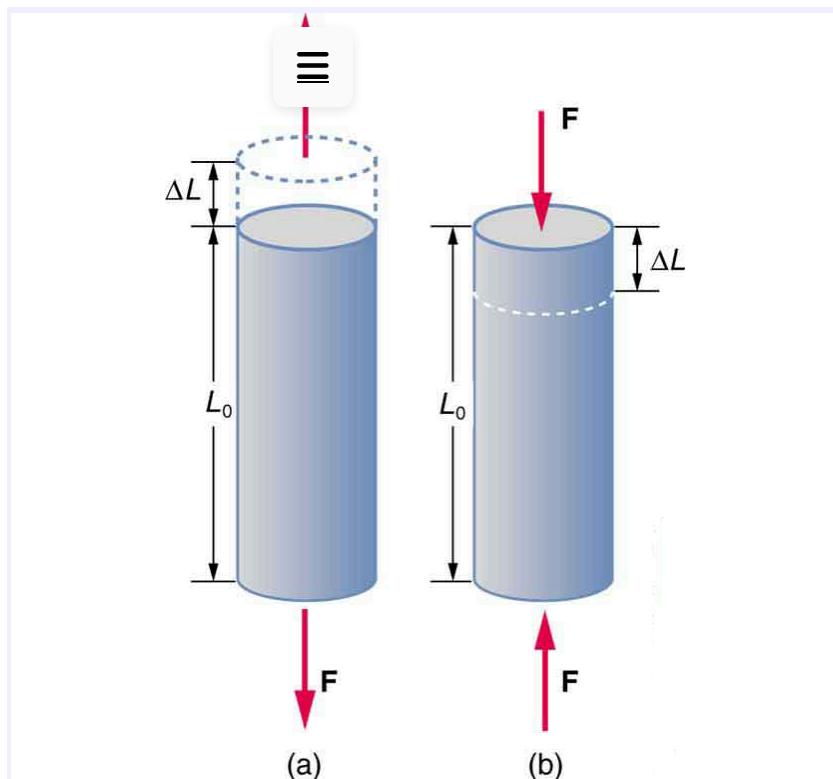


Figure 3: (a) Tension. The rod is stretched a length (ΔL) when a force is applied parallel to its length. (b) Compression. The same rod is compressed by forces with the same magnitude in the opposite direction. For very small deformations and uniform materials, (ΔL) is approximately the same for the same magnitude of tension or compression. For larger deformations, the cross-sectional area changes as the rod is compressed or stretched.

Experiments have shown that the change in length (ΔL) depends on only a few variables. As already noted, ΔL is proportional to the force F and depends on the substance from which the object is made. Additionally, the change in length is proportional to the original length L_0 and inversely proportional to the cross-sectional area of the wire or rod. For example, a long guitar string will stretch more than a short one, and a thick string will stretch less than a thin one. We can combine all these factors into one equation for ΔL :

$$\Delta L = \frac{1}{Y} \frac{F}{A} L_0, \quad \text{Eq.(5)}$$

where ΔL is the change in length, F the applied force, Y is a factor, called the elastic modulus or Young's modulus, that depends on the substance, A is the cross-sectional area, and L_0 is the original length. [Table 1](#) lists values of Y for several materials—those with a large Y are said to have a large tensile stiffness because they deform less for a given tension or compression.

Table 1: Elastic Moduli¹

Material	Young's modulus (tension-compression) Y (10^9N/m^2)	Shear modulus S (10^9N/m^2)	Bulk modulus B (10^9N/m^2)
Aluminum	70	25	75
Bone – tension	16	80	8
Bone – compression	9		
Brass	90	35	75
Brick	15		
Concrete	20		
Glass	70	20	30
Granite	45	20	45
Hair (human)	10		
Hardwood	15	10	
Iron, cast	100	40	90
Lead	16	5	50
Marble	60	20	70
Nylon	5		
Polystyrene	3		
Silk	6		
Spider thread	3		
Steel	210	80	130
Tendon	1		
Acetone			0.7
Ethanol			0.9
Glycerin			4.5
Mercury			25

Material	Young's modulus (tension-compression) Y ($\frac{N}{m^2}$)	Shear modulus S ($10^9 N/m^2$)	Bulk modulus B ($10^9 N/m^2$)
Water			2.2

Young's moduli are not listed for liquids and gases in [Table 1](#) because they cannot be stretched or compressed in only one direction. Note that there is an assumption that the object does not accelerate, so that there are actually two applied forces of magnitude F acting in opposite directions. For example, the strings in [Figure 3](#) are being pulled down by a force of magnitude w and held up by the ceiling, which also exerts a force of magnitude w .

Example 1: The Stretch of a Long

Suspension cables are used to carry gondolas at ski resorts. (See [Figure 4](#)) Consider a suspension cable that includes an unsupported span of 3020 m. Calculate the amount of stretch in the steel cable. Assume that the cable has a diameter of 5.6 cm and the maximum tension it can withstand is $3.0 \times 10^6 \text{ N}$.



Figure 4: Gondolas travel along suspension cables at the Gala Yuzawa ski resort in Japan. (credit: Rudy Herman, Flickr)

Strategy

The force is equal to the maximum tension, or $F = 3.0 \times 10^6 \text{ N}$. The cross-sectional area is $\pi r^2 = 2.46 \times 10^{-3} \text{ m}^2$. The equation $\Delta L = \frac{1}{Y} \frac{F}{A} L_0$ can be used to find the change in length.

Solution

All quantities are known. Thus,

$$\begin{aligned}\Delta L &= \left(\frac{1}{210 \times 10^9 \text{ N/m}^2} \right) \left(\frac{3.0 \times 10^6 \text{ N}}{2.46 \times 10^{-3} \text{ m}^2} \right) (3020 \text{ m}) \\ \Delta L &= 18 \text{ m.}\end{aligned}\quad \text{Eq.(6)}$$

Discussion

This is quite a stretch, but only about 0.6% of the unsupported length. Effects of temperature upon length might be important in these environments.

Bones, on the whole, do not fracture due to tension or compression. Rather they generally fracture due to sideways impact or bending, resulting in bone shearing or snapping. The behavior of bones under tension and compression is important because it determines the load the bones can carry. Bones are classified as weight-bearing structures such as columns in buildings and trees. Weight-bearing structures have special features; columns in building have steel-reinforcing rods while trees and bones are fibrous. The bones in different parts of the body serve different structural functions and are prone to different stresses. Thus the bone in the top of the femur is arranged in thin sheets separated by marrow while in other places the bones can be cylindrical and filled with marrow or just solid. Overweight people have a tendency toward bone damage due to sustained compressions in bone joints and tendons.

Another biological example of Hooke's law occurs in tendons. Functionally, the tendon (the tissue connecting muscle to bone) must stretch easily at first when a force is applied, but offer a much greater restoring force for a greater strain. [Figure 5](#) shows a stress-strain relationship for a human tendon. Some tendons have a high collagen content so there is relatively little strain, or length change; others, like support tendons (as in the leg) can change length up to 10%. Note that this stress-strain curve is nonlinear, since the slope of the line changes in different regions. In the first part of the stretch called the toe region, the fibers in the tendon begin to align in the direction of the stress—this is called *uncrimping*. In the linear region, the fibrils will be stretched, and in the failure region individual fibers begin to break. A simple model of this relationship can be illustrated by springs in parallel: different springs are activated at different lengths of stretch. Examples of this are given in the problems at end of this chapter. Ligaments (tissue connecting bone to bone) behave in a similar way.

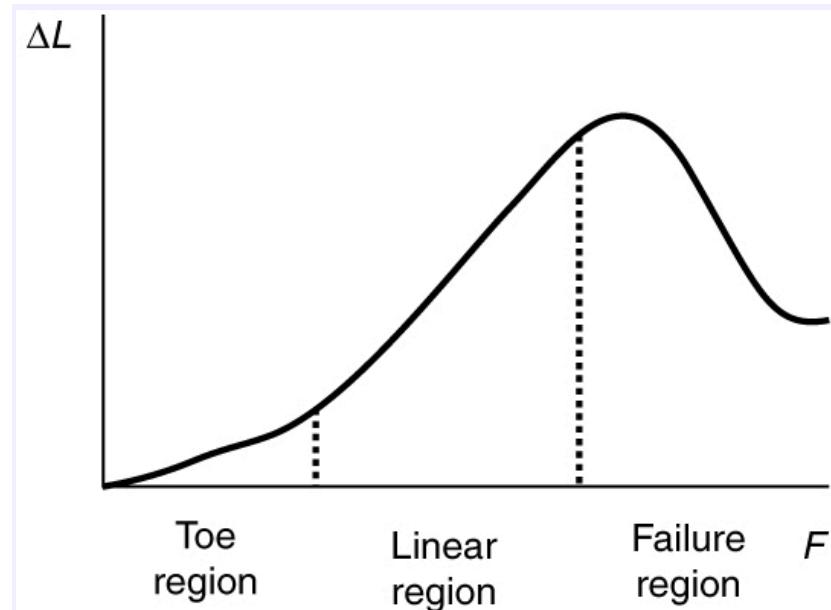


Figure 5: Typical stress-strain curve for mammalian tendon. Three regions are shown: (1) toe region (2) linear region, and (3) failure region.

Unlike bones and tendons, which need to be strong as well as elastic, the arteries and lungs need to be very stretchable. The elastic properties of the arteries are essential for blood flow. The pressure in the arteries increases and arterial walls stretch when the blood is pumped out of the heart. When the aortic

valve shuts, the pressure in the arteries drops and the arterial walls relax to maintain the blood flow. When you feel your pulse, you are feeling  ly this—the elastic behavior of the arteries as the blood gushes through with each pump of the heart. If the arteries were rigid, you would not feel a pulse. The heart is also an organ with special elastic properties. The lungs expand with muscular effort when we breathe in but relax freely and elastically when we breathe out. Our skins are particularly elastic, especially for the young. A young person can go from 100 kg to 60 kg with no visible sag in their skins. The elasticity of all organs reduces with age. Gradual physiological aging through reduction in elasticity starts in the early 20s.

Example 2: Calculating Deformation

How Much Does Your Leg Shorten When You Stand on It?

Calculate the change in length of the upper leg bone (the femur) when a 70.0 kg man supports 62.0 kg of his mass on it, assuming the bone to be equivalent to a uniform rod that is 40.0 cm long and 2.00 cm in radius.

Strategy

The force is equal to the weight supported, or

$$F = mg = (62.0 \text{ kg}) (9.80 \text{ m/s}^2) = 607.6 \text{ N}, \quad \text{Eq.(7)}$$

and the cross-sectional area is $\pi r^2 = 1.257 \times 10^{-3} \text{ m}^2$. The equation $\Delta L = \frac{1}{Y} \frac{F}{A} L_0$ can be used to find the change in length.

Solution

All quantities except ΔL are known. Note that the compression value for Young's modulus for bone must be used here. Thus,

$$\begin{aligned} \Delta L &= \left(\frac{1}{9 \times 10^9 \text{ N/m}^2} \right) \left(\frac{607.6 \text{ N}}{1.257 \times 10^{-3} \text{ m}^2} \right) (0.400 \text{ m}) \\ \Delta L &= 2 \times 10^{-5} \text{ m}. \end{aligned} \quad \text{Eq.(8)}$$

Discussion

This small change in length seems reasonable, consistent with our experience that bones are rigid. In fact, even the rather large forces encountered during strenuous physical activity do not compress or bend bones by large amounts. Although bone is rigid compared with fat or muscle, several of the substances listed in [Table 1](#) have larger values of Young's modulus Y . In other words, they are more rigid.

The equation for change in length is traditionally rearranged and written in the following form:

$$\frac{F}{A} = Y \frac{\Delta L}{L_0}. \quad \text{Eq.(9)}$$

The ratio of force to area, $\frac{F}{A}$, is defined as **stress** (measured in N/m^2), and the ratio of the change in length to length, $\frac{\Delta L}{L_0}$, is defined as **strain** (less quantity). In other words,

Eq.(10)

$$\text{stress} = Y \times \text{strain}.$$

In this form, the equation is analogous to Hooke's law, with stress analogous to force and strain analogous to deformation. If we again rearrange this equation to the form

Eq.(11)

$$F = YA \frac{\Delta L}{L_0},$$

we see that it is the same as Hooke's law with a proportionality constant

Eq.(12)

$$k = \frac{YA}{L_0}.$$

This general idea—that force and the deformation it causes are proportional for small deformations—applies to changes in length, sideways bending, and changes in volume.

STRESS

The ratio of force to area, $\frac{F}{A}$, is defined as stress measured in N/m^2 .

STRAIN

The ratio of the change in length to length, $\frac{\Delta L}{L_0}$, is defined as strain (a unitless quantity). In other words,

Eq.(13)

$$\text{stress} = Y \times \text{strain}.$$

Sideways Stress: Shear Modulus

Figure 6 illustrates what is meant by a sideways stress or a *shearing force*. Here the deformation is called Δx and it is perpendicular to L_0 , rather than parallel as with tension and compression. Shear deformation behaves similarly to tension and compression and can be described with similar equations. The expression for **shear deformation** is

$$\Delta x = \frac{1}{S} \frac{F}{A} L_0, \quad \text{Eq.(14)}$$

where S is the shear modulus (see Table 1) and F is the force applied perpendicular to L_0 and parallel to the cross-sectional area A . Again, to keep the object from accelerating, there are actually two equal and opposite forces $\pm F$.

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SHEAR DEFORMATION

$$\Delta x = \frac{1}{S} \frac{F}{A} L_0, \quad \text{Eq.(15)}$$

where S is the shear modulus and F is the force applied perpendicular to L_0 and parallel to the cross-sectional area A .

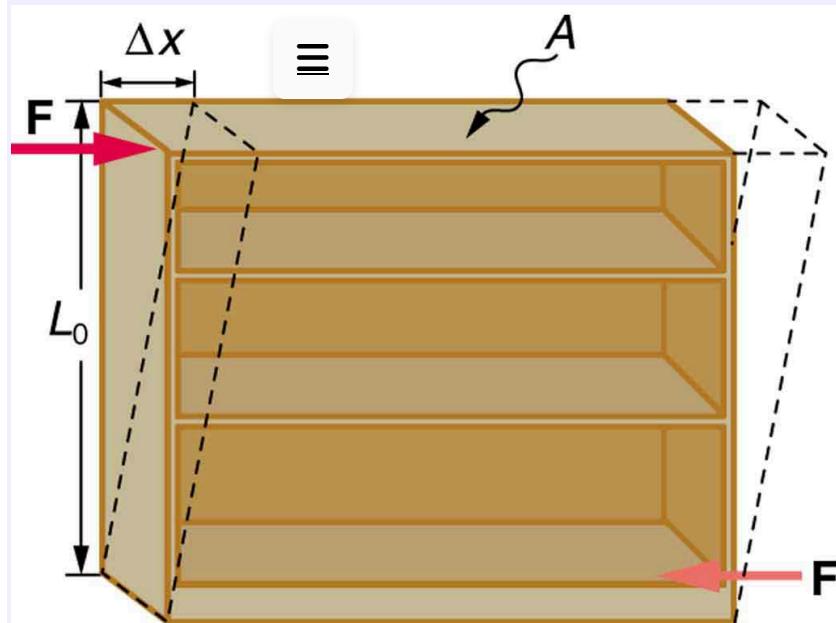


Figure 6: Shearing forces are applied perpendicular to the length (L_0) and parallel to the area (A), producing a deformation (Δx). Vertical forces are not shown, but it should be kept in mind that in addition to the two shearing forces, (F), there must be supporting forces to keep the object from rotating. The distorting effects of these supporting forces are ignored in this treatment. The weight of the object also is not shown, since it is usually negligible compared with forces large enough to cause significant deformations.

Examination of the shear moduli in [Table 1](#) reveals some telling patterns. For example, shear moduli are less than Young's moduli for most materials. Bone is a remarkable exception. Its shear modulus is not only greater than its Young's modulus, but it is as large as that of steel. This is why bones are so rigid.

The spinal column (consisting of 26 vertebral segments separated by discs) provides the main support for the head and upper part of the body. The spinal column has normal curvature for stability, but this curvature can be increased, leading to increased shearing forces on the lower vertebrae. Discs are better at withstanding compressional forces than shear forces. Because the spine is not vertical, the weight of the upper body exerts some of both. Pregnant women and people that are overweight (with large abdomens) need to move their shoulders back to maintain balance, thereby increasing the curvature in their spine and so increasing the shear component of the stress. An increased angle due to more curvature increases the shear forces along the plane. These higher shear forces increase the risk of back injury through ruptured discs. The lumbosacral disc (the wedge shaped disc below the last vertebrae) is particularly at risk because of its location.

The shear moduli for concrete and brick are very small; they are too highly variable to be listed. Concrete used in buildings can withstand compression, as in pillars and arches, but is very poor against shear, as might be encountered in heavily loaded floors or during earthquakes. Modern structures were made

possible by the use of steel and steel-reinforced concrete. Almost by definition, liquids and gases have shear moduli near zero, because they flow in response to shearing forces.

Example 3: Calculating Force Required to Deform: That Nail Does Not Bend Much Under a Load

Find the mass of the picture hanging from a steel nail as shown in Figure 7, given that the nail bends only $1.80 \times 10^{-6}\text{m}$. (Assume the shear modulus is known to two significant figures.)

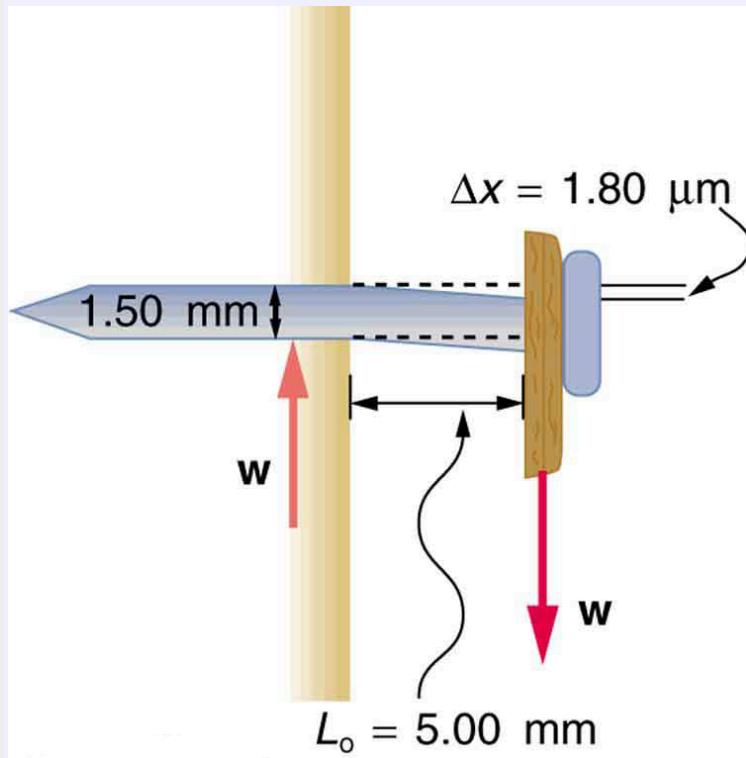


Figure 7: Side view of a nail with a picture hung from it. The nail flexes very slightly (shown much larger than actual) because of the shearing effect of the supported weight. Also shown is the upward force of the wall on the nail, illustrating that there are equal and opposite forces applied across opposite cross sections of the nail.

Strategy

The force F on the nail (neglecting the nail's own weight) is the weight of the picture w . If we can find w , then the mass of the picture is just $\frac{w}{g}$. The equation $\Delta x = \frac{1}{S} \frac{F}{A} L_0$ can be solved for F .

Solution

Solving the equation $\Delta x = \frac{1}{S} \frac{F}{A} L_0$ for F , we see that all other quantities can be found:

$$\equiv = \frac{SA}{L_0} \Delta x.$$

Eq.(16)

S is found in [Table 1](#) and is $S = 80 \times 10^9 \text{ N/m}^2$. The radius r is 0.750 mm (as seen in the figure), so the cross-sectional area is

$$A = \pi r^2 = 1.77 \times 10^{-6} \text{ m}^2. \quad \text{Eq.(17)}$$

The value for L_0 is also shown in the figure. Thus,

$$F = \frac{(80 \times 10^9 \text{ N/m}^2)(1.77 \times 10^{-6} \text{ m}^2)}{(5.00 \times 10^{-3} \text{ m})} (1.80 \times 10^{-6} \text{ m}) = 51 \text{ N}. \quad \text{Eq.(18)}$$

This 51 N force is the weight w of the picture, so the picture's mass is

$$m = \frac{w}{g} = \frac{F}{g} = 5.2 \text{ kg}. \quad \text{Eq.(19)}$$

Discussion

This is a fairly massive picture, and it is impressive that the nail flexes only $1.80 \times 10^{-6} \text{ m}$ —an amount undetectable to the unaided eye.

Changes in Volume: Bulk Modulus

An object will be compressed in all directions if inward forces are applied evenly on all its surfaces as in [Figure 8](#). It is relatively easy to compress gases and extremely difficult to compress liquids and solids. For example, air in a wine bottle is compressed when it is corked. But if you try corking a brim-full bottle, you cannot compress the wine—some must be removed if the cork is to be inserted. The reason for these different compressibilities is that atoms and molecules are separated by large empty spaces in gases but packed close together in liquids and solids. To compress a gas, you must force its atoms and molecules closer together. To compress liquids and solids, you must actually compress their atoms and molecules, and very strong electromagnetic forces in them oppose this compression.

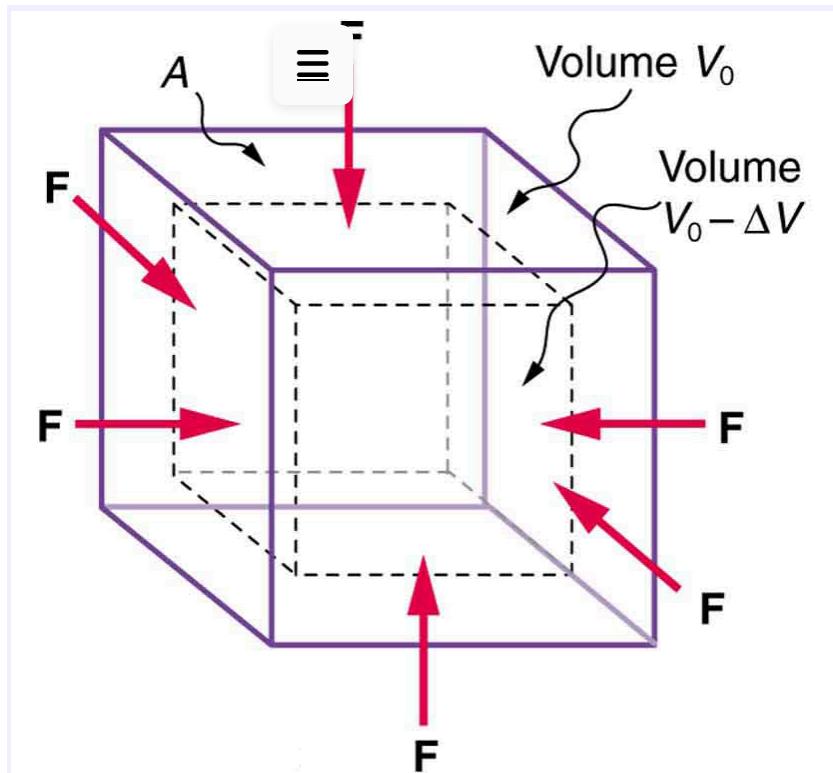


Figure 8: An inward force on all surfaces compresses this cube. Its change in volume is proportional to the force per unit area and its original volume, and is related to the compressibility of the substance.

We can describe the compression or volume deformation of an object with an equation. First, we note that a force “applied evenly” is defined to have the same stress, or ratio of force to area $\frac{F}{A}$ on all surfaces. The deformation produced is a change in volume ΔV , which is found to behave very similarly to the shear, tension, and compression previously discussed. (This is not surprising, since a compression of the entire object is equivalent to compressing each of its three dimensions.) The relationship of the change in volume to other physical quantities is given by

$$\Delta V = \frac{1}{B} \frac{F}{A} V_0, \quad \text{Eq.(20)}$$

where B is the bulk modulus (see [Table 1](#)), V_0 is the original volume, and $\frac{F}{A}$ is the force per unit area applied uniformly inward on all surfaces. Note that no bulk moduli are given for gases.

What are some examples of bulk compression of solids and liquids? One practical example is the manufacture of industrial-grade diamonds by compressing carbon with an extremely large force per unit area. The carbon atoms rearrange their crystalline structure into the more tightly packed pattern of diamonds. In nature, a similar process occurs deep underground, where extremely large forces result from the weight of overlying material. Another natural source of large compressive forces is the pressure

created by the weight of water, especially in deep parts of the oceans. Water exerts an inward force on all surfaces of a submerged object, and even the water itself. At great depths, water is measurably compressed, as the following example illustrates.

Example 4: Calculating Change in Volume with Deformation: How Much Is Water Compressed at Great Ocean Depths?

Calculate the fractional decrease in volume ($\frac{\Delta V}{V_0}$) for seawater at 5.00 km depth, where the force per unit area is $5.00 \times 10^7 \text{ N/m}^2$.

Strategy

Equation $\Delta V = \frac{1}{B} \frac{F}{A} V_0$ is the correct physical relationship. All quantities in the equation except $\frac{\Delta V}{V_0}$ are known.

Solution

Solving for the unknown $\frac{\Delta V}{V_0}$ gives

$$\frac{\Delta V}{V_0} = \frac{1}{B} \frac{F}{A}. \quad \text{Eq.(21)}$$

Substituting known values with the value for the bulk modulus B from [Table 1](#),

$$\begin{aligned} \frac{\Delta V}{V_0} &= \frac{5.00 \times 10^7 \text{ N/m}^2}{2.2 \times 10^9 \text{ N/m}^2} \\ \frac{\Delta V}{V_0} &= 0.023 = 2.3 \end{aligned} \quad \text{Eq.(22)}$$

Discussion

Although measurable, this is not a significant decrease in volume considering that the force per unit area is about 500 atmospheres (1 million pounds per square foot). Liquids and solids are extraordinarily difficult to compress.

Conversely, very large forces are created by liquids and solids when they try to expand but are constrained from doing so—which is equivalent to compressing them to less than their normal volume. This often occurs when a contained material warms up, since most materials expand when their temperature increases. If the materials are tightly constrained, they deform or break their container. Another very common example occurs when water freezes. Water, unlike most materials, expands when it freezes, and it can easily fracture a boulder, rupture a biological cell, or crack an engine block that gets in its way.

Other types of deformations, such as torsion or twisting, behave analogously to the tension, shear, and bulk deformations considered here.



PHET EXPLORATION: HOOKE'S LAW

Stretch and compress springs to explore the relationships between force, spring constant and displacement.

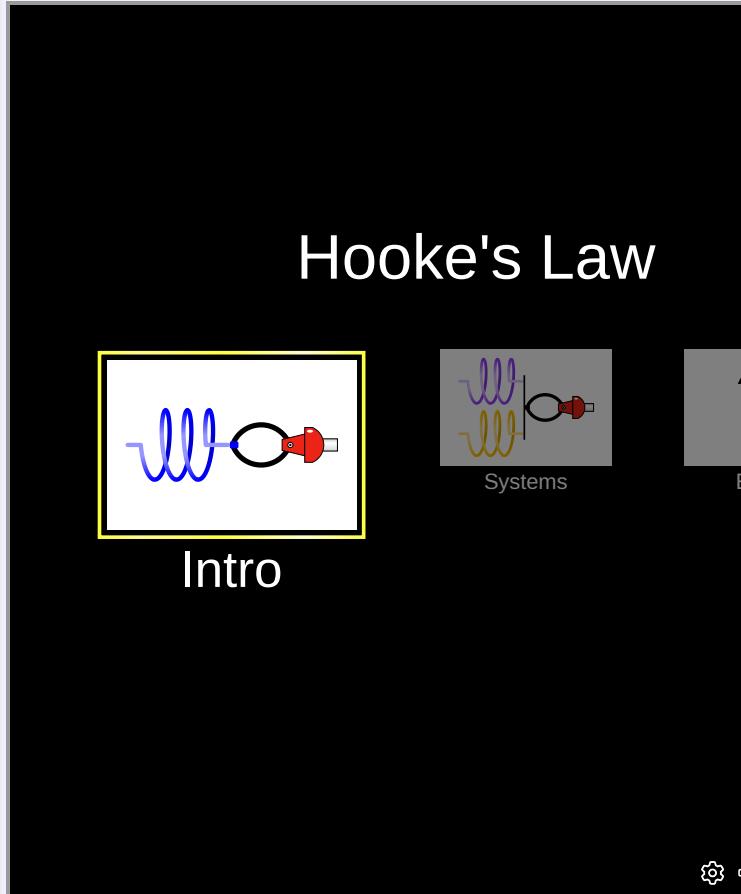


Figure 9: Hooke's Law

{ Figure9}

Section Summary

- Hooke's law is given by

$$F = k\Delta L, \quad \text{Eq.(23)}$$

where ΔL is the amount of deformation (change in length), F is the applied force, and k is a proportionality constant that depends on the shape and composition of the object and the direction of the force. The relationship between the deformation and the applied force can also be written as

$$\Delta L = \frac{1}{Y} \frac{F}{A} L_0, \quad \text{Eq.(24)}$$

where Y is *Young's modulus*, which depends on the substance, A is the cross-sectional area, and L_0 is the original length.

- The ratio of force to area, $\frac{F}{A}$, is defined as *stress*, measured in N/m^2 .
- The ratio of the change in length to length, $\frac{\Delta L}{L_0}$, is defined as *strain* (a unitless quantity). In other words,

$$\text{stress} = Y \times \text{strain}. \quad \text{Eq.(25)}$$

- The expression for shear deformation is

$$\Delta x = \frac{1}{S} \frac{F}{A} L_0, \quad \text{Eq.(26)}$$

where S is the shear modulus and F is the force applied perpendicular to L_0 and parallel to the cross-sectional area A .

- The relationship of the change in volume to other physical quantities is given by

$$\Delta V = \frac{1}{B} \frac{F}{A} V_0, \quad \text{Eq.(27)}$$

where B is the bulk modulus, V_0 is the original volume, and $\frac{F}{A}$ is the force per unit area applied uniformly inward on all surfaces.



Conceptual Questions

Exercise 1

The elastic properties of the arteries are essential for blood flow. Explain the importance of this in terms of the characteristics of the flow of blood (pulsating or continuous).

Exercise 2

What are you feeling when you feel your pulse? Measure your pulse rate for 10 s and for 1 min. Is there a factor of 6 difference?

Exercise 3

Examine different types of shoes, including sports shoes and thongs. In terms of physics, why are the bottom surfaces designed as they are? What differences will dry and wet conditions make for these surfaces?

Exercise 4

Would you expect your height to be different depending upon the time of day? Why or why not?

Exercise 5

Why can a squirrel jump from a tree branch to the ground and run away undamaged, while a human could break a bone in such a fall?

Exercise 6

Explain why pregnant women often suffer from back strain late in their pregnancy.

Exercise 7

An old carpenter's trick to keep nails from bending when they are pounded into hard materials is to grip the center of the nail firmly with pliers. Why does this help?

Exercise 8

When a glass bottle full of vinegar warms up, both the vinegar and the glass expand, but vinegar expands significantly more with temperature than glass. The bottle will break if it was filled to its tightly capped lid. Explain why, and also explain how a pocket of air above the vinegar would prevent the break. (This is the function of the air above liquids in glass containers.)



Problems & Exercises

Exercise 9

During a circus act, one performer swings upside down hanging from a trapeze holding another, also upside-down, performer by the legs. If the upward force on the lower performer is three times her weight, how much do the bones (the femurs) in her upper legs stretch? You may assume each is equivalent to a uniform rod 35.0 cm long and 1.80 cm in radius. Her mass is 60.0 kg.

Exercise 10

During a wrestling match, a 150 kg wrestler briefly stands on one hand during a maneuver designed to perplex his already moribund adversary. By how much does the upper arm bone shorten in length? The bone can be represented by a uniform rod 38.0 cm in length and 2.10 cm in radius.

Exercise 11

(a) The “lead” in pencils is a graphite composition with a Young’s modulus of about $1 \times 10^9 \text{ N/m}^2$. Calculate the change in length of the lead in an automatic pencil if you tap it straight into the pencil with a force of 4.0 N. The lead is 0.50 mm in diameter and 60 mm long. (b) Is the answer reasonable? That is, does it seem to be consistent with what you have observed when using pencils?

Exercise 12

TV broadcast antennas are the tallest artificial structures on Earth. In 1987, a 72.0-kg physicist placed himself and 400 kg of equipment at the top of one 610-m high antenna to perform gravity experiments. By how much was the antenna compressed, if we consider it to be equivalent to a steel cylinder 0.150 m in radius?

Exercise 13

(a) By how much does a 65.0-kg mountain climber stretch her 0.800-cm diameter nylon rope when she hangs 35.0 m below a rock outcropping? (b) Does the answer seem to be consistent with what you have observed for nylon ropes? Would it make sense if the rope were actually a bungee cord?

Exercise 14

A 20.0-m tall hollow aluminum flagpole is equivalent in stiffness to a solid cylinder 4.00 cm in diameter. A strong wind bends the pole much as a horizontal force of 900 N exerted at the top would. How far to the side does the top of the pole flex?

Exercise 15

As an oil well is drilled, each new section of drill pipe supports its own weight and that of the pipe and drill bit beneath it. Calculate the stretch in a new 6.00 m length of steel pipe that supports 3.00 km of pipe having a mass of 20.0 kg/m and a 100-kg drill bit. The pipe is equivalent in stiffness to a solid cylinder 5.00 cm in diameter.

Exercise 16

Calculate the force a piano tuner applies to stretch a steel piano wire 8.00 mm, if the wire is originally 0.850 mm in diameter and 1.35 m long.

Exercise 17

A vertebra is subjected to a shearing force of 500 N. Find the shear deformation, taking the vertebra to be a cylinder 3.00 cm high and 4.00 cm in diameter.

Exercise 18

A disk between vertebrae in the spine is subjected to a shearing force of 600 N. Find its shear deformation, taking it to have the shear modulus of $1 \times 10^9 \text{ N/m}^2$. The disk is equivalent to a solid cylinder 0.700 cm high and 4.00 cm in diameter.

Exercise 19

When using a pencil eraser, you exert a vertical force of 6.00 N at a distance of 2.00 cm from the hardwood-eraser joint. The pencil is 6.00 mm in diameter and is held at an angle of 20.0° to the horizontal. (a) By how much does the wood flex perpendicular to its length? (b) How much is it compressed lengthwise?

Exercise 20

To consider the effect of wires hung on poles, we take data from [Example 2](#), in which tensions in wires supporting a traffic light were calculated. The left wire made an angle 30.0° below the horizontal with the top of its pole and carried a tension of 108 N. The 12.0 m tall hollow aluminum pole is equivalent in stiffness to a 4.50 cm diameter solid cylinder. (a) How far is it bent to the side? (b) By how much is it compressed?

Exercise 21

A farmer making grape juice fills a glass bottle to the brim and caps it tightly. The juice expands more than the glass when it warms up, in such a way that the volume increases by 0.2% (that is, $\Delta V/V_0 = 2 \times 10^{-3}$) relative to the space available. Calculate the magnitude of the normal force exerted by the juice per square centimeter if its bulk modulus is $1.8 \times 10^9 \text{ N/m}^2$, assuming the bottle does not break. In view of your answer, do you think the bottle will survive?

Exercise 22

(a) When water freezes, its volume increases by 9.05% (that is, $\Delta V/V_0 = 9.05 \times 10^{-2}$). What force per unit area is water capable of exerting on a container when it freezes? (It is acceptable to use the bulk modulus of water in this problem.) (b) Is it surprising that such forces can fracture engine blocks, boulders, and the like?

Exercise 23

This problem returns to the tightrope walker studied in Example 2 of [Normal Tension and Other Examples Of Forces](#), who created a tension of $3.94 \times 10^3 N$ in a wire making an angle 5.0° below the horizontal with each supporting pole. Calculate how much this tension stretches the steel wire if it was originally 15 m long and 0.50 cm in diameter.

Exercise 24

The pole in [Figure 10](#) is at a 90.0° bend in a power line and is therefore subjected to more shear force than poles in straight parts of the line. The tension in each line is $4.00 \times 10^4 N$, at the angles shown. The pole is 15.0 m tall, has an 18.0 cm diameter, and can be considered to have half the stiffness of hardwood. (a) Calculate the compression of the pole. (b) Find how much it bends and in what direction. (c) Find the tension in a guy wire used to keep the pole straight if it is attached to the top of the pole at an angle of 30.0° with the vertical. (Clearly, the guy wire must be in the opposite direction of the bend.)

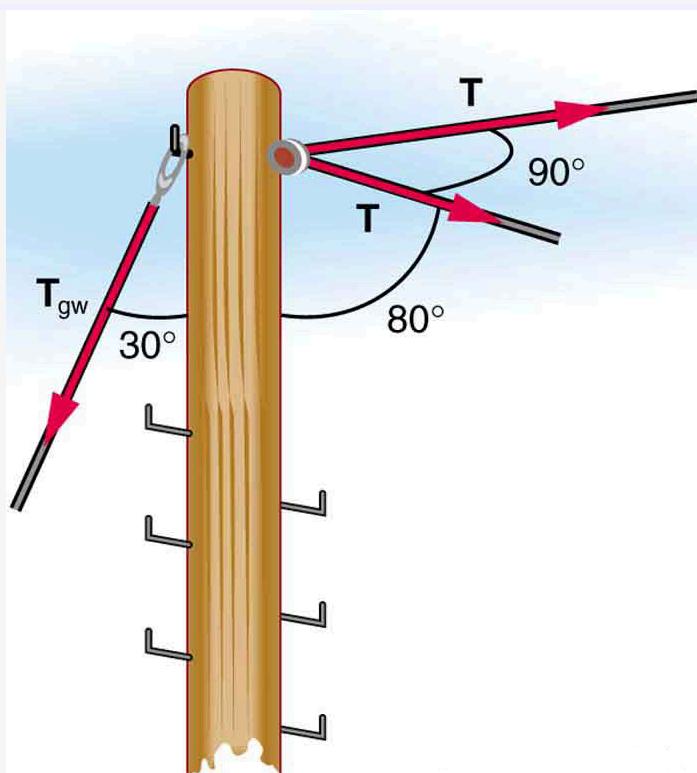


Figure 10: This telephone pole is at a 90 degrees bend in a power line. A guy wire is attached to the top of the pole at an angle of 30 degrees with the vertical.

GLOSSARY

deformation

change in shape due to the application of force

***Hooke's law***

proportional relationship between the force F on a material and the deformation ΔL it causes,

$$F = k\Delta L$$

tensile strength

the breaking stress that will cause permanent deformation or fracture of a material

stress

ratio of force to area

strain

ratio of change in length to original length

shear deformation

deformation perpendicular to the original length of an object

1. Footnote: Approximate and average values. Young's moduli Y for tension and compression sometimes differ but are averaged here. Bone has significantly different Young's moduli for tension and compression. ↵



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