

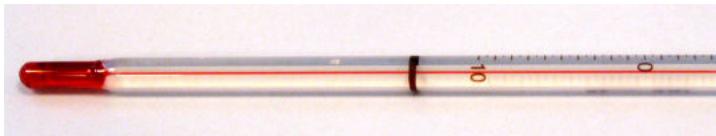
## Introduction to Temperature, Kinetic Theory, and the Gas Laws



The welder's gloves and helmet protect him from the electric arc that transfers enough thermal energy to melt the rod, spray sparks, and burn the retina of an unprotected eye. The thermal energy can be felt on exposed skin a few meters away, and its light can be seen for kilometers. (credit: Kevin S. O'Brien/U.S. Navy)

Heat is something familiar to each of us. We feel the warmth of the summer Sun, the chill of a clear summer night, the heat of coffee after a winter stroll, and the cooling effect of our sweat. Heat transfer is maintained by temperature differences. Manifestations of **heat transfer**—the movement of heat energy from one place or material to another—are apparent throughout the universe. Heat from beneath Earth's surface is brought to the surface in flows of incandescent lava. The Sun warms Earth's surface and is the source of much of the energy we find on it. Rising levels of atmospheric carbon dioxide threaten to trap more of the Sun's energy, perhaps fundamentally altering the ecosphere. In space, supernovas explode, briefly radiating more heat than an entire galaxy does.

What is heat? How do we define it? How is it related to temperature? What are heat's effects? How is it related to other forms of energy and to work? We will find that, in spite of the richness of the phenomena, there is a small set of underlying physical principles that unite the subjects and tie them to other fields.



In a typical thermometer like this one, the alcohol, with a red dye, expands more rapidly than the glass containing it. When the thermometer's temperature increases, the liquid from the bulb is forced into the narrow tube, producing a large change in the length of the column for a small change in temperature. (credit: Chemical Engineer, Wikimedia Commons)



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# Temperature

- Define temperature.
- Convert temperatures between the Celsius, Fahrenheit, and Kelvin scales.
- Define thermal equilibrium.
- State the zeroth law of thermodynamics.

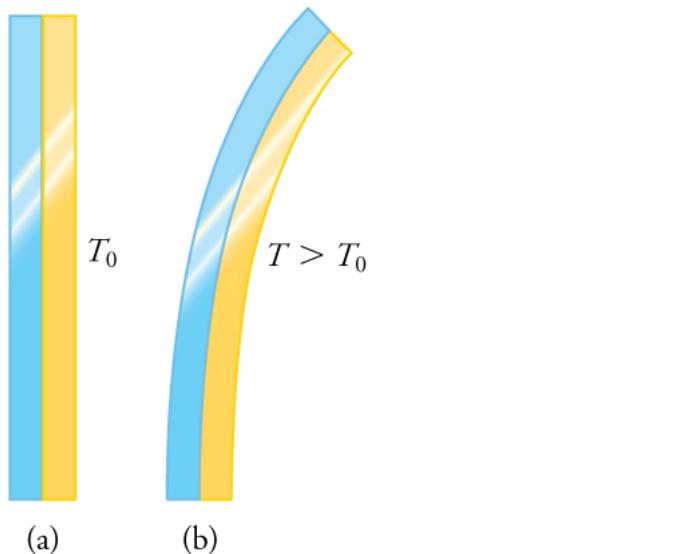
The concept of temperature has evolved from the common concepts of hot and cold. Human perception of what feels hot or cold is a relative one. For example, if you place one hand in hot water and the other in cold water, and then place both hands in tepid water, the tepid water will feel cool to the hand that was in hot water, and warm to the one that was in cold water. The scientific definition of temperature is less ambiguous than your senses of hot and cold. **Temperature** is operationally defined to be what we measure with a thermometer. (Many physical quantities are defined solely in terms of how they are measured. We shall see later how temperature is related to the kinetic energies of atoms and molecules, a more physical explanation.) Two accurate thermometers, one placed in hot water and the other in cold water, will show the hot water to have a higher temperature. If they are then placed in the tepid water, both will give identical readings (within measurement uncertainties). In this section, we discuss temperature, its measurement by thermometers, and its relationship to thermal equilibrium. Again, temperature is the quantity measured by a thermometer.

## Misconception Alert: Human Perception vs. Reality

On a cold winter morning, a wooden bench feels warmer than a metal bench. The wood and metal are in thermal equilibrium with the outside air, and are thus the same temperature. They **feel** different because of the difference in the way that they conduct heat away from your skin. The metal conducts heat away from your body faster than the wood does (see more about conductivity in [Conduction](#)). This is just one example demonstrating that the human sense of hot and cold is not determined by temperature alone.

Another factor that affects our perception of temperature is humidity. Most people feel much hotter on hot, humid days than on hot, dry days. This is because on humid days, sweat does not evaporate from the skin and in our lungs as efficiently as it does on dry days. It is the evaporation of sweat (or water from a sprinkler or pool) as well as water in the lungs that cools us off.

Any physical property that depends on temperature, and whose response to temperature is reproducible, can be used as the basis of a thermometer. Because many physical properties depend on temperature, the variety of thermometers is remarkable. For example, volume increases with temperature for most substances. This property is the basis for the common alcohol thermometer, the old mercury thermometer, and the bimetallic strip ([Figure 1](#)). Other properties used to measure temperature include electrical resistance and color, as shown in [Figure 2](#), and the emission of infrared radiation, as shown in [Figure 3](#).



The curvature of a bimetallic strip depends on temperature. (a) The strip is straight at the starting temperature, where its two components have the same length. (b) At a higher temperature, this strip bends to the right, because the metal on the left has expanded more than the metal on the right.



Each of the six squares on this plastic (liquid crystal) thermometer contains a film of a different heat-sensitive liquid crystal material. Below  $95^{\circ}\text{F}$ , all six squares are black. When the plastic thermometer is exposed to temperature that increases to  $95^{\circ}\text{F}$ , the first liquid crystal square changes color. When the temperature increases above  $96.8^{\circ}\text{F}$  the second liquid crystal square also changes color, and so forth. (credit: Arkrishna, Wikimedia Commons)

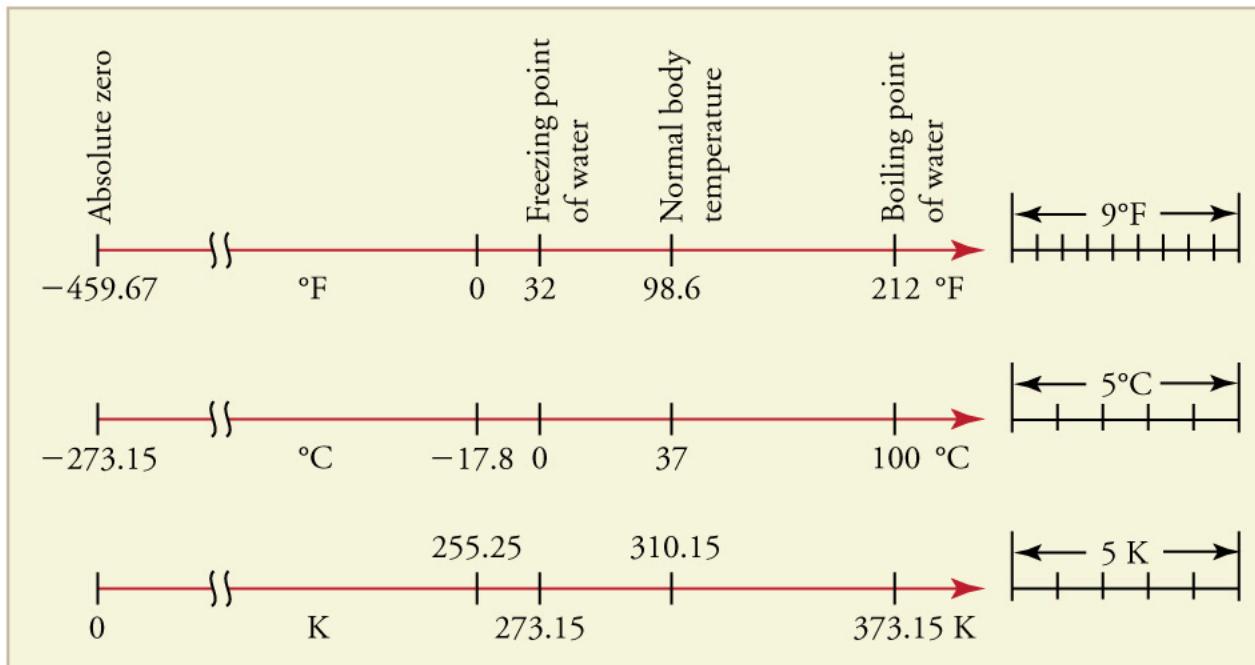


Fireman Jason Ormand uses a pyrometer to check the temperature of an aircraft carrier's ventilation system. Infrared radiation (whose emission varies with temperature) from the vent is measured and a temperature readout is quickly produced. Infrared measurements are also frequently used as a measure of body temperature. These modern thermometers, placed in the ear canal, are more accurate than alcohol thermometers placed under the tongue or in the armpit. (credit: Lamel J. Hinton/U.S. Navy)

## Temperature Scales

Thermometers are used to measure temperature according to well-defined scales of measurement, which use pre-defined reference points to help compare quantities. The three most common temperature scales are the Fahrenheit, Celsius, and Kelvin scales. A temperature scale can be created by identifying two easily reproducible temperatures. The freezing and boiling temperatures of water at standard atmospheric pressure are commonly used.

The **Celsius** scale (which replaced the slightly different *centigrade* scale) has the freezing point of water at  $0^{\circ}\text{C}$  and the boiling point at  $100^{\circ}\text{C}$ . Its unit is the **degree Celsius** ( $^{\circ}\text{C}$ ). On the **Fahrenheit** scale (still the most frequently used in the United States), the freezing point of water is at  $32^{\circ}\text{F}$  and the boiling point is at  $212^{\circ}\text{F}$ . The unit of temperature on this scale is the **degree Fahrenheit** ( $^{\circ}\text{F}$ ). Note that a temperature difference of one degree Celsius is greater than a temperature difference of one degree Fahrenheit. Only 100 Celsius degrees span the same range as 180 Fahrenheit degrees, thus one degree on the Celsius scale is 1.8 times larger than one degree on the Fahrenheit scale  $180/100 = 9/5$ . The **Kelvin** scale is the temperature scale that is commonly used in science. It is an *\* absolute temperature*\* scale defined to have  $0\text{ K}$  at the lowest possible temperature, called **absolute zero**. The official temperature unit on this scale is the **kelvin**, which is abbreviated  $\text{K}$ , and is not accompanied by a degree sign. The freezing and boiling points of water are  $273.15\text{ K}$  and  $373.15\text{ K}$ , respectively. Thus, the magnitude of temperature differences is the same in units of kelvins and degrees Celsius. Unlike other temperature scales, the Kelvin scale is an absolute scale. It is used extensively in scientific work because a number of physical quantities, such as the volume of an ideal gas, are directly related to absolute temperature. The kelvin is the SI unit used in scientific work.



Relationships between the Fahrenheit, Celsius, and Kelvin temperature scales, rounded to the nearest degree. The relative sizes of the scales are also shown.

The relationships between the three common temperature scales are shown in [Figure 4]. Temperatures on these scales can be converted using the equations in [Table 1].

Temperature Conversions		
To convert from . . .	Use this equation . . .	Also written as . . .
Celsius to Fahrenheit	$T(\text{°F})=95T(\text{°C})+32$	$T\text{°F}=95T\text{°C}+32$
Fahrenheit to Celsius	$T(\text{°C})=59(T(\text{°F})-32)$	$T\text{°C}=59(T\text{°F}-32)$
Celsius to Kelvin	$T(\text{K})=T(\text{°C})+273.15$	$T\text{K}=T\text{°C}+273.15$
Kelvin to Celsius	$T(\text{°C})=T(\text{K})-273.15$	$T\text{°C}=T\text{K}-273.15$
Fahrenheit to Kelvin	$T(\text{K})=59(T(\text{°F})-32)+273.15$	$T\text{K}=59(T\text{°F}-32)+273.15$
Kelvin to Fahrenheit	$T(\text{°F})=95(T(\text{K})-273.15)+32$	$T\text{°F}=95(T\text{K}-273.15)+32$

Notice that the conversions between Fahrenheit and Kelvin look quite complicated. In fact, they are simple combinations of the conversions between Fahrenheit and Celsius, and the conversions between Celsius and Kelvin.

Converting between Temperature Scales: Room Temperature

“Room temperature” is generally defined to be  $25^{\circ}\text{C}$ . (a) What is room temperature in  $^{\circ}\text{F}$ ? (b) What is it in  $\text{K}$ ?

### Strategy

To answer these questions, all we need to do is choose the correct conversion equations and plug in the known values.

### Solution for (a)

1. Choose the right equation. To convert from  $^{\circ}\text{C}$  to  $^{\circ}\text{F}$ , use the equation

$$T^{\circ}\text{F}=95T^{\circ}\text{C}+32.$$

1. Plug the known value into the equation and solve:

$$T^{\circ}\text{F}=9525^{\circ}\text{C}+32=77^{\circ}\text{F}.$$

### Solution for (b)

1. Choose the right equation. To convert from  $^{\circ}\text{C}$  to K, use the equation

$$T\text{K}=T^{\circ}\text{C}+273.15.$$

1. Plug the known value into the equation and solve:

$$T\text{K}=25^{\circ}\text{C}+273.15=298\text{K}.$$

Converting between Temperature Scales: the Reaumur Scale

The Reaumur scale is a temperature scale that was used widely in Europe in the 18th and 19th centuries. On the Reaumur temperature scale, the freezing point of water is  $0^{\circ}\text{R}$  and the boiling temperature is  $80^{\circ}\text{R}$ . If "room temperature" is  $25^{\circ}\text{C}$  on the Celsius scale, what is it on the Reaumur scale?

### Strategy

To answer this question, we must compare the Reaumur scale to the Celsius scale. The difference between the freezing point and boiling point of water on the Reaumur scale is  $80^{\circ}\text{R}$ . On the Celsius scale it is  $100^{\circ}\text{C}$ . Therefore  $100^{\circ}\text{C} = 80^{\circ}\text{R}$ . Both scales start at  $0^{\circ}$  for freezing, so we can derive a simple formula to convert between temperatures on the two scales.

### Solution

1. Derive a formula to convert from one scale to the other:

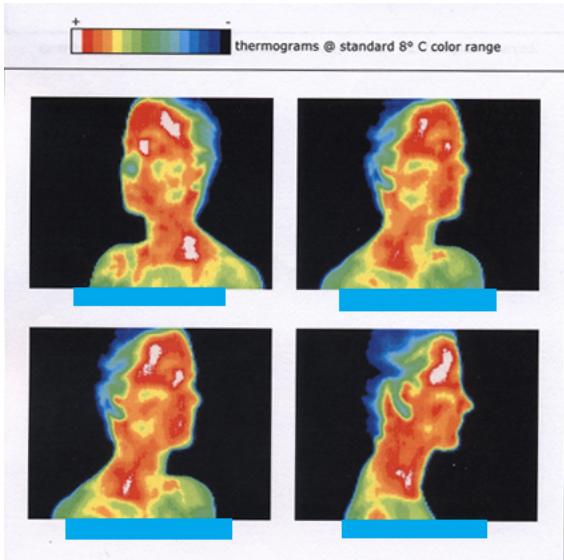
$$T^{\circ}\text{R}=0.8^{\circ}\text{R}^{\circ}\text{C}\times T^{\circ}\text{C}.$$

1. Plug the known value into the equation and solve:

$$T^{\circ}\text{R}=0.8^{\circ}\text{R}^{\circ}\text{C}\times25^{\circ}\text{C}=20^{\circ}\text{R}.$$

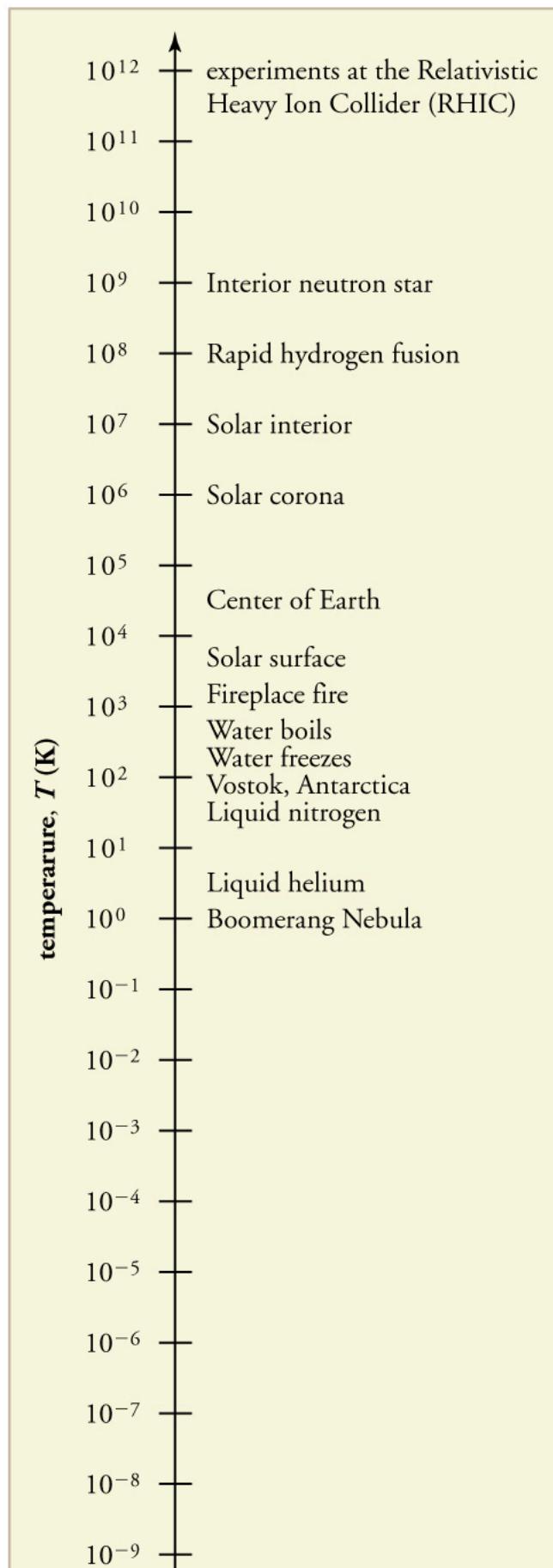
### Temperature Ranges in the Universe

[Figure 6] shows the wide range of temperatures found in the universe. Human beings have been known to survive with body temperatures within a small range, from  $24^{\circ}\text{C}$  to  $44^{\circ}\text{C}$  Extra \left or missing \right to  $111^{\circ}\text{F}$ . The average normal body temperature is usually given as  $37.0^{\circ}\text{C}$  ( $98.6^{\circ}\text{F}$ ), and variations in this temperature can indicate a medical condition: a fever, an infection, a tumor, or circulatory problems (see [Figure 5]).



This image of radiation from a person's body (an infrared thermograph) shows the location of temperature abnormalities in the upper body. Dark blue corresponds to cold areas and red to white corresponds to hot areas. An elevated temperature might be an indication of malignant tissue (a cancerous tumor in the breast, for example), while a depressed temperature might be due to a decline in blood flow from a clot. In this case, the abnormalities are caused by a condition called hyperhidrosis. (credit: Porcelina81, Wikimedia Commons)

The lowest temperatures ever recorded have been measured during laboratory experiments:  $4.5 \times 10^{-10}$  K at the Massachusetts Institute of Technology (USA), and  $1.0 \times 10^{-10}$  K at Helsinki University of Technology (Finland). In comparison, the coldest recorded place on Earth's surface is Vostok, Antarctica at 183 K ( $-89^{\circ}\text{C}$ ) , and the coldest place (outside the lab) known in the universe is the Boomerang Nebula, with a temperature of 1 K.

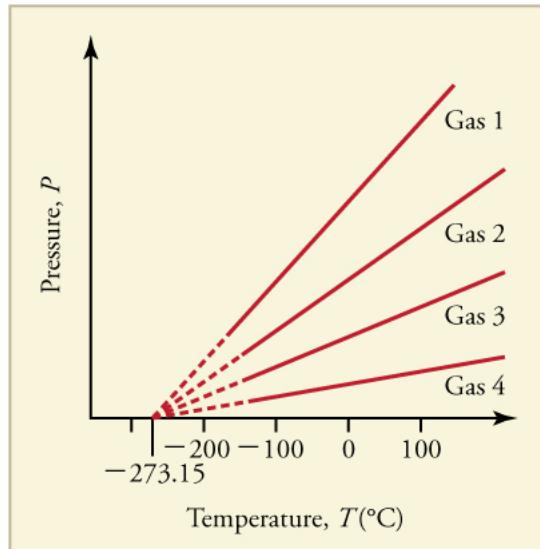




Each increment on this logarithmic scale indicates an increase by a factor of ten, and thus illustrates the tremendous range of temperatures in nature. Note that zero on a logarithmic scale would occur off the bottom of the page at infinity.

#### Making Connections: Absolute Zero

What is absolute zero? Absolute zero is the temperature at which all molecular motion has ceased. The concept of absolute zero arises from the behavior of gases. [Figure 7] shows how the pressure of gases at a constant volume decreases as temperature decreases. Various scientists have noted that the pressures of gases extrapolate to zero at the same temperature,  $-273.15^{\circ}\text{C}$ . This extrapolation implies that there is a lowest temperature. This temperature is called *absolute zero*. Today we know that most gases first liquefy and then freeze, and it is not actually possible to reach absolute zero. The numerical value of absolute zero temperature is  $-273.15^{\circ}\text{C}$  or 0 K.



Graph of pressure versus temperature for various gases kept at a constant volume. Note that all of the graphs extrapolate to zero pressure at the same temperature.

#### Thermal Equilibrium and the Zeroth Law of Thermodynamics

Thermometers actually take their **own** temperature, not the temperature of the object they are measuring. This raises the question of how we can be certain that a thermometer measures the temperature of the object with which it is in contact. It is based on the fact that any two systems placed in **thermal contact** (meaning heat transfer can occur between them) will reach the same temperature. That is, heat will flow from the hotter object to the cooler one until they have exactly the same temperature. The objects are then in **thermal equilibrium**, and no further changes will occur. The systems interact and change because their temperatures differ, and the changes stop once their temperatures are the same. Thus, if enough time is allowed for this transfer of heat to run its course, the temperature a thermometer registers **does** represent the system with which it is in thermal equilibrium. Thermal equilibrium is established when two bodies are in contact with each other and can freely exchange energy.

Furthermore, experimentation has shown that if two systems, A and B, are in thermal equilibrium with each other, and B is in thermal equilibrium with a third system C, then A is also in thermal equilibrium with C. This conclusion may seem obvious, because all three have the same temperature, but it is basic to thermodynamics. It is called the **zeroth law of thermodynamics**.

#### The Zeroth Law of Thermodynamics

If two systems, A and B, are in thermal equilibrium with each other, and B is in thermal equilibrium with a third system, C, then A is also in thermal equilibrium with C.

This law was postulated in the 1930s, after the first and second laws of thermodynamics had been developed and named. It is called the *zeroth law* because it comes logically before the first and second laws (discussed in [Thermodynamics](#)). An example of this law in action is seen in babies in incubators: babies in incubators normally have very few clothes on, so to an observer they look as if they may not be warm enough. However, the temperature of the air, the cot, and the baby is the same, because they are in thermal equilibrium, which is accomplished by maintaining air temperature to keep the baby comfortable.

#### Check Your Understanding

Does the temperature of a body depend on its size?

[Show Solution](#)

No, the system can be divided into smaller parts each of which is at the same temperature. We say that the temperature is an *intensive* quantity. Intensive quantities are independent of size.

### Section Summary

- Temperature is the quantity measured by a thermometer.
- Temperature is related to the average kinetic energy of atoms and molecules in a system.
- Absolute zero is the temperature at which there is no molecular motion.
- There are three main temperature scales: Celsius, Fahrenheit, and Kelvin.
- Temperatures on one scale can be converted to temperatures on another scale using the following equations:

$$T^{\circ}\text{F} = 95T^{\circ}\text{C} + 32$$

$$T^{\circ}\text{C} = 59(T^{\circ}\text{F} - 32)$$

$$T\text{K} = T^{\circ}\text{C} + 273.15$$

$$T^{\circ}\text{C} = T\text{K} - 273.15$$

- Systems are in thermal equilibrium when they have the same temperature.
- Thermal equilibrium occurs when two bodies are in contact with each other and can freely exchange energy.
- The zeroth law of thermodynamics states that when two systems, A and B, are in thermal equilibrium with each other, and B is in thermal equilibrium with a third system, C, then A is also in thermal equilibrium with C.

### Conceptual Questions

What does it mean to say that two systems are in thermal equilibrium?

[Show Solution](#)

**Strategy:** Consider what happens when two objects at different temperatures are brought into thermal contact.

**Solution:** Two systems are in thermal equilibrium when they are at the same temperature and there is no net heat transfer between them. When systems in thermal contact reach thermal equilibrium, their temperatures become equal and remain constant over time (assuming no external influences).

More specifically, thermal equilibrium means:

1. The two systems have the same temperature
2. No net energy flows between them as heat
3. Their macroscopic properties (like temperature, pressure, volume) remain constant
4. If a thermometer is placed in either system, it will read the same temperature

**Discussion:** Thermal equilibrium is the basis for temperature measurement. When a thermometer is placed in contact with an object, heat flows between them until they reach thermal equilibrium. The thermometer then reads the temperature of the object. The zeroth law of thermodynamics formalizes this concept: if system A is in thermal equilibrium with system C, and system B is in thermal equilibrium with system C, then A and B are in thermal equilibrium with each other (and all three have the same temperature).

Give an example of a physical property that varies with temperature and describe how it is used to measure temperature.

[Show Solution](#)

**Strategy:** Think of properties that change predictably and measurably with temperature.

**Solution:** Many physical properties vary with temperature and can be used for thermometry. Here are several examples:

1. **Volume expansion of liquids:** Mercury and alcohol thermometers use this property. As temperature increases, the liquid expands and rises in a narrow tube. The height of the liquid column is calibrated to indicate temperature. This works because liquids expand more than the glass container.
2. **Electrical resistance:** Resistance thermometers (RTDs) use metals like platinum whose electrical resistance increases predictably with temperature. By measuring resistance, temperature can be determined accurately.
3. **Thermoelectric voltage:** Thermocouples consist of two different metals joined together. When the junction is heated, a voltage is generated that depends on temperature. This voltage can be measured and converted to temperature.
4. **Color/radiation:** Infrared thermometers measure the electromagnetic radiation emitted by objects. Hotter objects emit more infrared radiation, allowing non-contact temperature measurement.

**Discussion:** Different temperature-dependent properties are useful in different ranges and applications. Mercury thermometers work well for everyday temperatures but are limited by mercury's freezing ( $-39^{\circ}\text{C}$ ) and boiling ( $357^{\circ}\text{C}$ ) points. Thermocouples can measure very high temperatures (up to  $2000^{\circ}\text{C}$ ), while resistance thermometers offer high precision for scientific work.

When a cold alcohol thermometer is placed in a hot liquid, the column of alcohol goes **down** slightly before going up. Explain why.

[Show Solution](#)

**Strategy:** Consider which part of the thermometer heats up first and how thermal expansion occurs at different rates for glass and alcohol.

**Solution:** When a cold thermometer is placed in hot liquid, the glass bulb and tube heat up first because they are in direct contact with the hot liquid. The glass expands, increasing the volume of the bulb and tube slightly. This expansion creates more space for the alcohol.

Initially, the alcohol inside is still cold and hasn't expanded yet. As the glass bulb expands, the same amount of alcohol now occupies a larger container volume, causing the alcohol level in the tube to drop slightly.

After this brief moment, heat conducts through the glass to the alcohol. The alcohol then heats up and expands. Since the coefficient of volume expansion for alcohol is much larger than for glass, the alcohol expands significantly more than the glass. This causes the alcohol column to rise, eventually reaching a height that indicates the temperature of the hot liquid.

**Discussion:** This effect is transient and only noticeable when there's a large temperature difference between the thermometer and the liquid. It demonstrates the different rates of thermal expansion for different materials and the time required for heat to transfer through materials. Good thermometer design minimizes this effect by using thin glass bulbs that heat quickly and by choosing liquid-glass combinations with very different expansion coefficients.

If you add boiling water to a cup at room temperature, what would you expect the final equilibrium temperature of the unit to be? You will need to include the surroundings as part of the system. Consider the zeroth law of thermodynamics.

[Show Solution](#)

**Strategy:** Consider all components that will exchange heat: the boiling water, the cup, and the surrounding air. Apply the principle that heat flows from hot to cold until thermal equilibrium is reached.

**Solution:** The final equilibrium temperature will be somewhere between room temperature and 100°C, but will ultimately approach room temperature over time. The specific equilibrium temperature depends on several factors:

**Short-term equilibrium** (cup + water system):

- The boiling water (100°C) will transfer heat to the cooler cup ( $\approx 20^\circ\text{C}$ )
- They will quickly reach an intermediate temperature, typically 80-95°C depending on the masses and specific heats of water and cup material
- A ceramic cup will absorb less heat than a metal cup due to different heat capacities

**Long-term equilibrium** (including surroundings):

- The hot water-cup system then exchanges heat with the surrounding air at room temperature
- Heat continues to flow from the system to the surroundings
- Eventually, the entire system (water + cup + surrounding air) reaches thermal equilibrium at essentially room temperature

**Discussion:** By the zeroth law of thermodynamics, when thermal equilibrium is reached, all parts of the system will be at the same temperature. In practice, in an open room with good air circulation, the final temperature will be very close to room temperature, though the room itself might warm slightly. If the system were perfectly insulated from its surroundings, the equilibrium temperature would be determined by the relative heat capacities and masses of the water and cup alone.

## Problems & Exercises

What is the Fahrenheit temperature of a person with a 39.0°C fever?

[Show Solution](#)

**Strategy:** Use the conversion equation from Celsius to Fahrenheit.

**Solution:** Using the equation  $T^\circ\text{F} = 95T^\circ\text{C} + 32$ :

$$T^\circ\text{F} = 95(39.0^\circ\text{C}) + 32 = 70.2 + 32 = 102.2^\circ\text{F} \approx 102^\circ\text{F}$$

**Discussion:** A temperature of 102°F represents a significant fever. Normal body temperature is 98.6°F (37.0°C), so a fever of 102°F is about 3.6°F above normal. This level of fever indicates the body is fighting an infection and would typically warrant medical attention, especially if sustained or accompanied by other symptoms.

Frost damage to most plants occurs at temperatures of 28.0°F or lower. What is this temperature on the Kelvin scale?

[Show Solution](#)

**Strategy:** First convert from Fahrenheit to Celsius, then from Celsius to Kelvin.

**Solution:** Convert °F to °C:

$$T^\circ\text{C} = 59(T^\circ\text{F} - 32) = 59(28.0 - 32) = 59(-4.0) = -2.22^\circ\text{C}$$

Convert °C to K:

$$T\text{K} = T^\circ\text{C} + 273.15 = -2.22 + 273.15 = 271\text{ K}$$

**Discussion:** This temperature of 271 K is just below the freezing point of water (273.15 K or 0°C or 32°F). Plants can often survive brief exposures to temperatures slightly below freezing, but prolonged exposure causes ice crystals to form in plant cells, rupturing cell walls and causing permanent damage. This is why farmers monitor frost warnings carefully and may use protective measures like covering plants or using heaters when temperatures approach 28°F.

To conserve energy, room temperatures are kept at 68.0°F in the winter and 78.0°F in the summer. What are these temperatures on the Celsius scale?

[Show Solution](#)

**Strategy:** Use the conversion equation from Fahrenheit to Celsius for each temperature.

**Solution:** Using the equation  $T^{\circ}\text{C} = \frac{5}{9}(T^{\circ}\text{F} - 32)$ :

Winter temperature:

$$T^{\circ}\text{C} = \frac{5}{9}(68.0 - 32) = \frac{5}{9}(36.0) = 20.0^{\circ}\text{C}$$

Summer temperature:

$$T^{\circ}\text{C} = \frac{5}{9}(78.0 - 32) = \frac{5}{9}(46.0) = 25.6^{\circ}\text{C}$$

**Discussion:** These temperatures (20.0°C in winter and 25.6°C in summer) represent a reasonable range for energy-efficient building temperatures. The 10°F (5.6°C) difference between winter and summer settings reflects the fact that people dress more warmly in winter and more lightly in summer, so comfort can be maintained at different temperatures. This temperature difference also reduces heating costs in winter and cooling costs in summer, contributing to significant energy savings.

A tungsten light bulb filament may operate at 2900 K. What is its Fahrenheit temperature? What is this on the Celsius scale?

[Show Solution](#)

**Strategy:** First convert from Kelvin to Celsius, then from Celsius to Fahrenheit.

**Solution:** Convert K to °C:

$$T^{\circ}\text{C} = T\text{K} - 273.15 = 2900 - 273.15 = 2627^{\circ}\text{C}$$

Convert °C to °F:

$$T^{\circ}\text{F} = 95T^{\circ}\text{C} + 32 = 95(2627) + 32 = 4761^{\circ}\text{F}$$

**Discussion:** This extremely high temperature (nearly 2627°C or 4761°F) explains why tungsten is used for incandescent light bulb filaments—it has the highest melting point of all metals (3422°C). At this operating temperature, the filament glows white-hot, emitting visible light along with significant infrared radiation (heat). This is also why incandescent bulbs are inefficient—much of the energy goes into heat rather than visible light. The tungsten filament must be enclosed in an inert gas (argon or nitrogen) or vacuum to prevent oxidation at these extreme temperatures.

The surface temperature of the Sun is about 5750 K. What is this temperature on the Fahrenheit scale?

[Show Solution](#)

**Strategy:** First convert from Kelvin to Celsius, then from Celsius to Fahrenheit.

**Solution:** Convert K to °C:

$$T^{\circ}\text{C} = T\text{K} - 273.15 = 5750 - 273.15 = 5477^{\circ}\text{C}$$

Convert °C to °F:

$$T^{\circ}\text{F} = 95T^{\circ}\text{C} + 32 = 95(5477) + 32 = 9859 + 32 = 9891^{\circ}\text{F} \approx 9890^{\circ}\text{F}$$

**Discussion:** This extremely high surface temperature of the Sun (5750 K or about 5477°C or 9890°F) is what causes the Sun to emit visible light and heat. At this temperature, the Sun's surface glows white-hot, emitting radiation across the entire electromagnetic spectrum with peak intensity in the visible range. This is why the Sun appears yellowish-white to our eyes. The core of the Sun is even hotter, at about 15 million kelvin, where nuclear fusion reactions occur. Understanding the Sun's temperature is crucial for astrophysics and for understanding how stars generate energy.

One of the hottest temperatures ever recorded on the surface of Earth was 134°F in Death Valley, CA. What is this temperature in Celsius degrees? What is this temperature in Kelvin?

[Show Solution](#)

**Strategy:** Convert from Fahrenheit to Celsius first, then to Kelvin.

**Solution:** Convert °F to °C:

$$T^{\circ}\text{C} = \frac{5}{9}(T^{\circ}\text{F} - 32) = \frac{5}{9}(134 - 32) = \frac{5}{9}(102) = 56.7^{\circ}\text{C}$$

Convert °C to K:

$$T\text{K} = T^{\circ}\text{C} + 273.15 = 56.7 + 273.15 = 330 \text{ K}$$

**Discussion:** This temperature of 56.7°C (134°F or 330 K) is extremely hot and life-threatening for humans without proper precautions. At such temperatures, the human body's cooling mechanisms (primarily evaporative cooling through sweating) are severely challenged, especially if humidity is also high. Death Valley holds this record due to its below-sea-level elevation, surrounding mountains that trap heat, and desert conditions that prevent cloud cover and evaporative cooling. For comparison, normal human body temperature is 37°C, so this environmental temperature is nearly 20°C higher than body temperature, making heat gain from the environment a serious concern.

(a) Suppose a cold front blows into your locale and drops the temperature by 40.0 Fahrenheit degrees. How many degrees Celsius does the temperature decrease when there is a 40.0°F decrease in temperature? (b) Show that any change in temperature in Fahrenheit degrees is nine-fifths the change in Celsius degrees.

[Show Solution](#)

$$(a) 22.2^\circ\text{C} \quad (b) \Delta T^\circ\text{F} = T_2^\circ\text{F} - T_1^\circ\text{F} = 95T_2^\circ\text{C} + 32.0^\circ - (95T_1^\circ\text{C} + 32.0^\circ) = 95(T_2^\circ\text{C} - T_1^\circ\text{C}) = 95\Delta T^\circ\text{C}$$

(a) At what temperature do the Fahrenheit and Celsius scales have the same numerical value? (b) At what temperature do the Fahrenheit and Kelvin scales have the same numerical value?

[Show Solution](#)

**Strategy:** (a) Set  $T^\circ\text{F} = T^\circ\text{C} = T$  and solve using the conversion equation. (b) Set  $T^\circ\text{F} = T\text{K} = T$  and solve using the appropriate conversions.

**Solution:** (a) Using the equation  $T^\circ\text{F} = 95T^\circ\text{C} + 32$ , and setting  $T^\circ\text{F} = T^\circ\text{C} = T$ :

$$T = 95T + 32$$

Solving for T:

$$\begin{aligned} T - 95T &= 32 \\ -45T &= 32 \\ T &= -40 \end{aligned}$$

Therefore,  $-40^\circ\text{C} = -40^\circ\text{F}$ .

(b) Using  $T\text{K} = T^\circ\text{C} + 273.15$  and  $T^\circ\text{F} = 95T^\circ\text{C} + 32$ , and setting  $T^\circ\text{F} = T\text{K} = T$ :

From the first equation:  $T^\circ\text{C} = T - 273.15$

Substituting into the second equation:

$$\begin{aligned} T &= 95(T - 273.15) + 32 \\ T &= 95T - 491.67 + 32 \\ T - 95T &= -459.67 \\ -45T &= -459.67 \\ T &= 574.6 \quad \text{K} = 574.6^\circ\text{F} \end{aligned}$$

**Discussion:** The temperature of  $-40$  degrees is the unique point where Celsius and Fahrenheit scales intersect, making it a memorable reference point. This temperature is encountered in very cold climates like Antarctica or northern Canada in winter. The Fahrenheit-Kelvin equivalence at 574.6 degrees occurs at a very high temperature ( $301.4^\circ\text{C}$ ), well above the boiling point of water. These equivalence points are mathematical curiosities that arise from the different zero points and scale sizes of the temperature systems.

## Glossary

temperature

the quantity measured by a thermometer

Celsius scale

temperature scale in which the freezing point of water is  $0^\circ\text{C}$  and the boiling point of water is  $100^\circ\text{C}$

degree Celsius

unit on the Celsius temperature scale

Fahrenheit scale

temperature scale in which the freezing point of water is  $32^\circ\text{F}$  and the boiling point of water is  $212^\circ\text{F}$

degree Fahrenheit

unit on the Fahrenheit temperature scale

Kelvin scale

temperature scale in which  $0\text{ K}$  is the lowest possible temperature, representing absolute zero

absolute zero

the lowest possible temperature; the temperature at which all molecular motion ceases

thermal equilibrium

the condition in which heat no longer flows between two objects that are in contact; the two objects have the same temperature

zeroth law of thermodynamics

law that states that if two objects are in thermal equilibrium, and a third object is in thermal equilibrium with one of those objects, it is also in thermal equilibrium with the other object



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## Thermal Expansion of Solids and Liquids

- Define and describe thermal expansion.
- Calculate the linear expansion of an object given its initial length, change in temperature, and coefficient of linear expansion.
- Calculate the volume expansion of an object given its initial volume, change in temperature, and coefficient of volume expansion.
- Calculate thermal stress on an object given its original volume, temperature change, volume change, and bulk modulus.



Thermal expansion joints like these in the Auckland Harbour Bridge in New Zealand allow bridges to change length without buckling. (credit: Ingolfson, Wikimedia Commons)

The expansion of alcohol in a thermometer is one of many commonly encountered examples of **thermal expansion**, the change in size or volume of a given mass with temperature. Hot air rises because its volume increases, which causes the hot air's density to be smaller than the density of surrounding air, causing a buoyant (upward) force on the hot air. The same happens in all liquids and gases, driving natural heat transfer upwards in homes, oceans,

and weather systems. Solids also undergo thermal expansion. Railroad tracks and bridges, for example, have expansion joints to allow them to freely expand and contract with temperature changes.

What are the basic properties of thermal expansion? First, thermal expansion is clearly related to temperature change. The greater the temperature change, the more a bimetallic strip will bend. Second, it depends on the material. In a thermometer, for example, the expansion of alcohol is much greater than the expansion of the glass containing it.

What is the underlying cause of thermal expansion? As is discussed in [Kinetic Theory: Atomic and Molecular Explanation of Pressure and Temperature](#), an increase in temperature implies an increase in the kinetic energy of the individual atoms. In a solid, unlike in a gas, the atoms or molecules are closely packed together, but their kinetic energy (in the form of small, rapid vibrations) pushes neighboring atoms or molecules apart from each other. This neighbor-to-neighbor pushing results in a slightly greater distance, on average, between neighbors, and adds up to a larger size for the whole body. For most substances under ordinary conditions, there is no preferred direction, and an increase in temperature will increase the solid's size by a certain fraction in each dimension.

#### Linear Thermal Expansion—Thermal Expansion in One Dimension

The change in length  $\Delta L$  is proportional to length  $L$ . The dependence of thermal expansion on temperature, substance, and length is summarized in the equation

$$\Delta L = \alpha L \Delta T,$$

where  $\Delta L$  is the change in length  $L$ ,  $\Delta T$  is the change in temperature, and  $\alpha$  is the **coefficient of linear expansion**, which varies slightly with temperature.

[[Table 1](#)] lists representative values of the coefficient of linear expansion, which may have units of  $1/^\circ\text{C}$  or  $1/\text{K}$ . Because the size of a kelvin and a degree Celsius are the same, both  $\alpha$  and  $\Delta T$  can be expressed in units of kelvins or degrees Celsius. The equation  $\Delta L = \alpha L \Delta T$  is accurate for small changes in temperature and can be used for large changes in temperature if an average value of  $\alpha$  is used.

Thermal Expansion Coefficients at  $20^\circ\text{C}$ <sup>1</sup>

Material	Coefficient of linear expansion $\alpha(1/^\circ\text{C})$	Coefficient of volume expansion $\beta(1/^\circ\text{C})$
<b>Solids</b>		
Aluminum	$25 \times 10^{-6}$	$75 \times 10^{-6}$
Brass	$19 \times 10^{-6}$	$56 \times 10^{-6}$
Copper	$17 \times 10^{-6}$	$51 \times 10^{-6}$
Gold	$14 \times 10^{-6}$	$42 \times 10^{-6}$
Iron or Steel	$12 \times 10^{-6}$	$35 \times 10^{-6}$
Invar (Nickel-iron alloy)	$0.9 \times 10^{-6}$	$2.7 \times 10^{-6}$
Lead	$29 \times 10^{-6}$	$87 \times 10^{-6}$
Silver	$18 \times 10^{-6}$	$54 \times 10^{-6}$
Glass (ordinary)	$9 \times 10^{-6}$	$27 \times 10^{-6}$
Glass (Pyrex®)	$3 \times 10^{-6}$	$9 \times 10^{-6}$
Quartz	$0.4 \times 10^{-6}$	$1 \times 10^{-6}$
Concrete, Brick	$\sim 12 \times 10^{-6}$	$\sim 36 \times 10^{-6}$
Marble (average)	$7 \times 10^{-6}$	$2.1 \times 10^{-5}$
<b>Liquids</b>		
Ether		$1650 \times 10^{-6}$
Ethyl alcohol		$1100 \times 10^{-6}$
Petrol		$950 \times 10^{-6}$
Glycerin		$500 \times 10^{-6}$
Mercury		$180 \times 10^{-6}$
Water		$210 \times 10^{-6}$
<b>Gases</b>		
Air and most other gases at atmospheric pressure		$3400 \times 10^{-6}$

Calculating Linear Thermal Expansion: The Golden Gate Bridge

The main span of San Francisco's Golden Gate Bridge is 1275 m long at its coldest. The bridge is exposed to temperatures ranging from  $-15^{\circ}\text{C}$  to  $40^{\circ}\text{C}$ . What is its change in length between these temperatures? Assume that the bridge is made entirely of steel.

### Strategy

Use the equation for linear thermal expansion  $\Delta L = \alpha L \Delta T$  to calculate the change in length,  $\Delta L$ . Use the coefficient of linear expansion,  $\alpha$ , for steel from [\[Table 1\]](#), and note that the change in temperature,  $\Delta T$ , is  $55^{\circ}\text{C}$ .

### Solution

Plug all of the known values into the equation to solve for  $\Delta L$ .

$$\Delta L = \alpha L \Delta T = (12 \times 10^{-6} \text{C})(1275 \text{m})(55 \text{C}) = 0.84 \text{m}.$$

### Discussion

Although not large compared with the length of the bridge, this change in length is observable. It is generally spread over many expansion joints so that the expansion at each joint is small.

## Thermal Expansion in Two and Three Dimensions

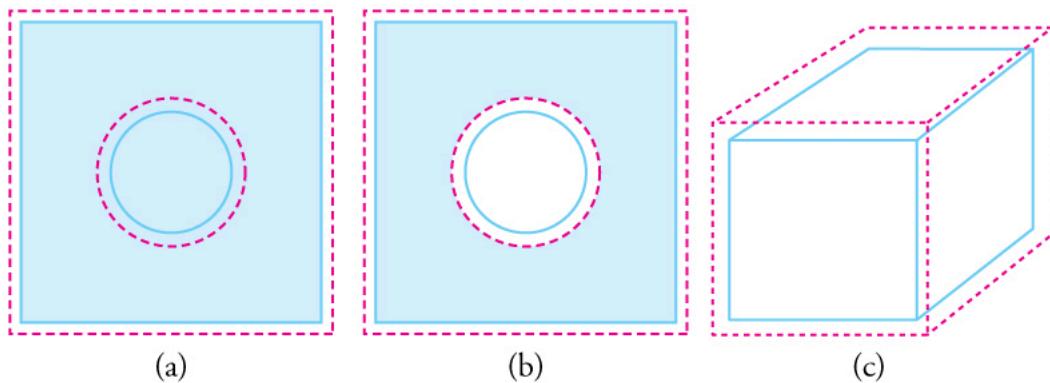
Objects expand in all dimensions, as illustrated in [\[Figure 2\]](#). That is, their areas and volumes, as well as their lengths, increase with temperature. Holes also get larger with temperature. If you cut a hole in a metal plate, the remaining material will expand exactly as it would if the plug was still in place. The plug would get bigger, and so the hole must get bigger too. (Think of the ring of neighboring atoms or molecules on the wall of the hole as pushing each other farther apart as temperature increases. Obviously, the ring of neighbors must get slightly larger, so the hole gets slightly larger).

### Thermal Expansion in Two Dimensions

For small temperature changes, the change in area  $\Delta A$  is given by

$$\Delta A = 2\alpha A \Delta T,$$

where  $\Delta A$  is the change in area  $A$ ,  $\Delta T$  is the change in temperature, and  $\alpha$  is the coefficient of linear expansion, which varies slightly with temperature.



In general, objects expand in all directions as temperature increases. In these drawings, the original boundaries of the objects are shown with solid lines, and the expanded boundaries with dashed lines. (a) Area increases because both length and width increase. The area of a circular plug also increases. (b) If the plug is removed, the hole it leaves becomes larger with increasing temperature, just as if the expanding plug were still in place. (c) Volume also increases, because all three dimensions increase.

### Thermal Expansion in Three Dimensions

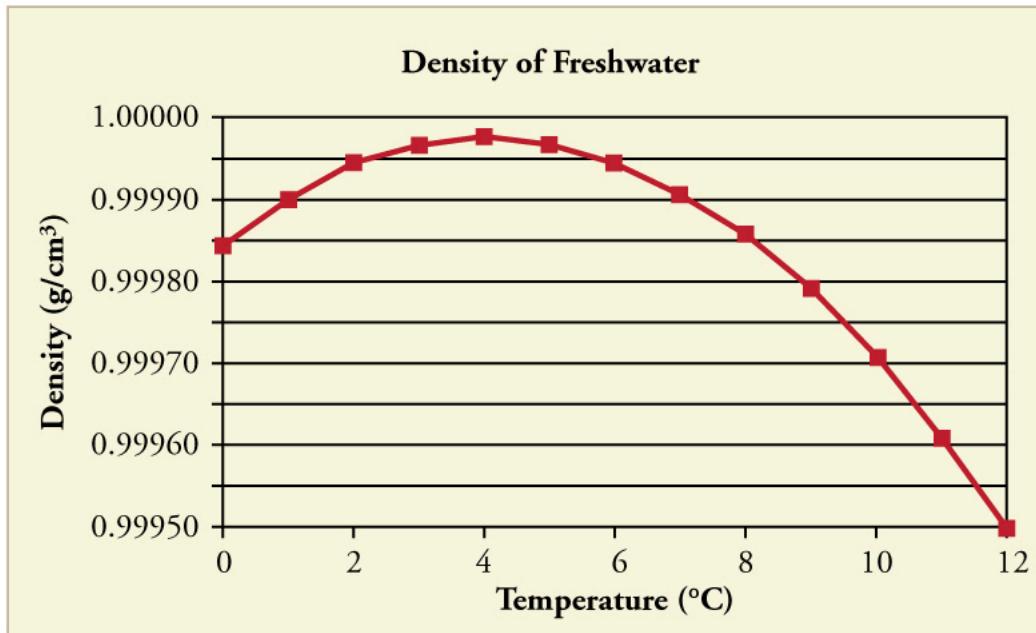
The change in volume  $\Delta V$  is very nearly  $\Delta V = 3\alpha V \Delta T$ . This equation is usually written as

$$\Delta V = \beta V \Delta T,$$

where  $\beta$  is the **coefficient of volume expansion** and  $\beta \approx 3\alpha$ . Note that the values of  $\beta$  in [\[Table 1\]](#) are almost exactly equal to  $3\alpha$ .

In general, objects will expand with increasing temperature. Water is the most important exception to this rule. Water expands with increasing temperature (its density *decreases*) when it is at temperatures greater than  $4^{\circ}\text{C}$  ( $40^{\circ}\text{F}$ ). However, it expands with *decreasing* temperature when it is between  $+4^{\circ}\text{C}$  and  $0^{\circ}\text{C}$  ( $40^{\circ}\text{F}$  to  $32^{\circ}\text{F}$ ). Water is densest at  $+4^{\circ}\text{C}$ . (See [\[Figure 3\]](#).) Perhaps the most striking effect of this phenomenon is the freezing of water in a pond. When water near the surface cools down to  $4^{\circ}\text{C}$  it is denser than the remaining water and thus will sink to the bottom. This "turnover" results in a layer of warmer water near the surface, which is then cooled. Eventually the pond has a uniform temperature of  $4^{\circ}\text{C}$ . If the temperature in the surface layer drops below  $4^{\circ}\text{C}$ , the water is less dense than the water below, and thus stays near the top. As a result, the pond surface can completely freeze over. The ice on top of liquid water provides an insulating layer from winter's harsh exterior air temperatures. Fish and other aquatic life can survive in  $4^{\circ}\text{C}$ .

water beneath ice, due to this unusual characteristic of water. It also produces circulation of water in the pond that is necessary for a healthy ecosystem of the body of water.



The density of water as a function of temperature. Note that the thermal expansion is actually very small. The maximum density at  $4^{\circ}\text{C}$  is only 0.0075% greater than the density at  $2^{\circ}\text{C}$ , and 0.012% greater than that at  $0^{\circ}\text{C}$ .

#### Making Connections: Real-World Connections—Filling the Tank

Differences in the thermal expansion of materials can lead to interesting effects at the gas station. One example is the dripping of gasoline from a freshly filled tank on a hot day. Gasoline starts out at the temperature of the ground under the gas station, which is cooler than the air temperature above. The gasoline cools the steel tank when it is filled. Both gasoline and steel tank expand as they warm to air temperature, but gasoline expands much more than steel, and so it may overflow.

This difference in expansion can also cause problems when interpreting the gasoline gauge. The actual amount (mass) of gasoline left in the tank when the gauge hits “empty” is a lot less in the summer than in the winter. The gasoline has the same volume as it does in the winter when the “add fuel” light goes on, but because the gasoline has expanded, there is less mass. If you are used to getting another 40 miles on “empty” in the winter, beware—you will probably run out much more quickly in the summer.



Because the gas expands more than the gas tank with increasing temperature, you can't drive as many miles on "empty" in the summer as you can in the winter. (credit: Hector Alejandro, Flickr)

Calculating Thermal Expansion: Gas vs. Gas Tank

Suppose your 60.0-L (15.9-gal) steel gasoline tank is full of gas, so both the tank and the gasoline have a temperature of 15.0°C. How much gasoline has spilled by the time they warm to 35.0°C?

### Strategy

The tank and gasoline increase in volume, but the gasoline increases more, so the amount spilled is the difference in their volume changes. (The gasoline tank can be treated as solid steel.) We can use the equation for volume expansion to calculate the change in volume of the gasoline and of the tank.

### Solution

1. Use the equation for volume expansion to calculate the increase in volume of the steel tank:

$$\Delta V_s = \beta_s V_s \Delta T.$$

1. The increase in volume of the gasoline is given by this equation:

$$\Delta V_{\text{gas}} = \beta_{\text{gas}} V_{\text{gas}} \Delta T.$$

1. Find the difference in volume to determine the amount spilled as

$$V_{\text{spill}} = \Delta V_{\text{gas}} - \Delta V_s.$$

Alternatively, we can combine these three equations into a single equation. (Note that the original volumes are equal.)

$$V_{\text{spill}} = (\beta_{\text{gas}} - \beta_s) V \Delta T = [(950 - 35) \times 10^{-6} \text{°C}](60.0 \text{L})(20.0 \text{°C}) = 1.10 \text{L}.$$

### Discussion

This amount is significant, particularly for a 60.0-L tank. The effect is so striking because the gasoline and steel expand quickly. The rate of change in thermal properties is discussed in [Heat and Heat Transfer Methods](#).

If you try to cap the tank tightly to prevent overflow, you will find that it leaks anyway, either around the cap or by bursting the tank. Tightly constricting the expanding gas is equivalent to compressing it, and both liquids and solids resist being compressed with extremely large forces. To avoid rupturing rigid containers, these containers have air gaps, which allow them to expand and contract without stressing them.

## Thermal Stress

**Thermal stress** is created by thermal expansion or contraction ( see [Elasticity: Stress and Strain](#) for a discussion of stress and strain). Thermal stress can be destructive, such as when expanding gasoline ruptures a tank. It can also be useful, for example, when two parts are joined together by heating one in manufacturing, then slipping it over the other and allowing the combination to cool. Thermal stress can explain many phenomena, such as the weathering of rocks and pavement by the expansion of ice when it freezes.

Calculating Thermal Stress: Gas Pressure

What pressure would be created in the gasoline tank considered in [\[Example 2\]](#), if the gasoline increases in temperature from 15.0°C to 35.0°C without being allowed to expand? \*\* Assume that the bulk modulus  $B$  for gasoline is  $1.00 \times 10^9 \text{ N/m}^2$ . (For more on bulk modulus, see [Elasticity: Stress and Strain](#).)

### Strategy

To solve this problem, we must use the following equation, which relates a change in volume  $\Delta V$  to pressure:

$$\Delta V = B F A V_0,$$

where  $F/A$  is pressure,  $V_0$  is the original volume, and  $B$  is the bulk modulus of the material involved. We will use the amount spilled in [\[Example 2\]](#) as the change in volume,  $\Delta V$ .

### Solution

1. Rearrange the equation for calculating pressure:

$$P = F A = \Delta V V_0 B.$$

1. Insert the known values. The bulk modulus for gasoline is  $B = 1.00 \times 10^9 \text{ N/m}^2$ . In the previous example, the change in volume  $\Delta V = 1.10 \text{ L}$  is the amount that would spill. Here,  $V_0 = 60.0 \text{ L}$  is the original volume of the gasoline. Substituting these values into the equation, we obtain

$$P = 1.10 \text{ L} \cdot 60.0 \text{ L} \cdot (1.00 \times 10^9 \text{ Pa}) = 1.83 \times 10^7 \text{ Pa}.$$

### Discussion

This pressure is about  $2500 \text{ lb/in}^2$ , much more than a gasoline tank can handle.

Forces and pressures created by thermal stress are typically as great as that in the example above. Railroad tracks and roadways can buckle on hot days if they lack sufficient expansion joints. (See [\[Figure 3\]](#).) Power lines sag more in the summer than in the winter, and will snap in cold weather if there is insufficient slack. Cracks open and close in plaster walls as a house warms and cools. Glass cooking pans will crack if cooled rapidly or unevenly, because of differential contraction and the stresses it creates. (Pyrex® is less susceptible because of its small coefficient of thermal expansion.) Nuclear reactor pressure vessels are threatened by overly rapid cooling, and although none have failed, several have been cooled faster than considered desirable. Biological cells are ruptured when foods are frozen, detracting from their taste. Repeated thawing and freezing accentuate the damage. Even the oceans can be affected. A significant portion of the rise in sea level that is resulting from global warming is due to the thermal expansion of sea water.



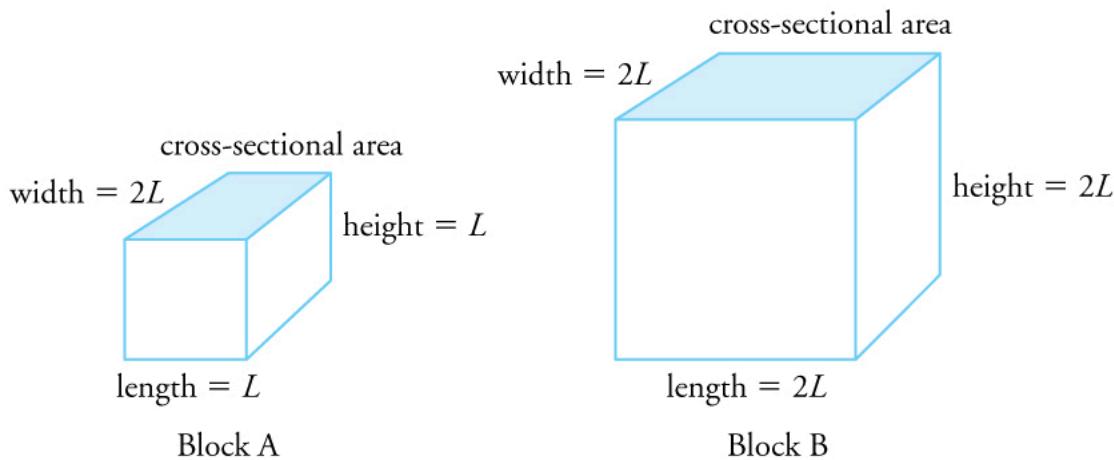
Thermal stress contributes to the formation of potholes. (credit: Editor5807, Wikimedia Commons)

Metal is regularly used in the human body for hip and knee implants. Most implants need to be replaced over time because, among other things, metal does not bond with bone. Researchers are trying to find better metal coatings that would allow metal-to-bone bonding. One challenge is to find a coating that has an expansion coefficient similar to that of metal. If the expansion coefficients are too different, the thermal stresses during the manufacturing process lead to cracks at the coating-metal interface.

Another example of thermal stress is found in the mouth. Dental fillings can expand differently from tooth enamel. It can give pain when eating ice cream or having a hot drink. Cracks might occur in the filling. Metal fillings (gold, silver, etc.) are being replaced by composite fillings (porcelain), which have smaller coefficients of expansion closer to those of teeth.

#### Check Your Understanding

Two blocks, A and B, are made of the same material. Block A has dimensions  $l \times w \times h = L \times 2L \times L$  and Block B has dimensions  $2L \times 2L \times 2L$ . If the temperature changes, what is (a) the change in the volume of the two blocks, (b) the change in the cross-sectional area  $l \times w$ , and (c) the change in the height  $h$  of the two blocks?



[Show Solution](#)

(a) The change in volume is proportional to the original volume. Block A has a volume of  $L \times 2L \times L = 2L^3$ . Block B has a volume of  $2L \times 2L \times 2L = 8L^3$ , which is 4 times that of Block A. Thus the change in volume of Block B should be 4 times the change in volume of Block A.

(b) The change in area is proportional to the area. The cross-sectional area of Block A is  $L \times 2L = 2L^2$ , while that of Block B is  $2L \times 2L = 4L^2$ . Because cross-sectional area of Block B is twice that of Block A, the change in the cross-sectional area of Block B is twice that of Block A.

(c) The change in height is proportional to the original height. Because the original height of Block B is twice that of A, the change in the height of Block B is twice that of Block A.

### Section Summary

- Thermal expansion is the increase, or decrease, of the size (length, area, or volume) of a body due to a change in temperature.
- Thermal expansion is large for gases, and relatively small, but not negligible, for liquids and solids.
- Linear thermal expansion is  $\Delta L = \alpha L \Delta T$ ,

where  $\Delta L$  is the change in length  $L$ ,  $\Delta T$  is the change in temperature, and  $\alpha$  is the coefficient of linear expansion, which varies slightly with temperature.

- The change in area due to thermal expansion is  $\Delta A = 2\alpha A \Delta T$ ,

where  $\Delta A$  is the change in area.

- The change in volume due to thermal expansion is  $\Delta V = \beta V \Delta T$ ,

where  $\beta$  is the coefficient of volume expansion and  $\beta \approx 3\alpha$ . Thermal stress is created when thermal expansion is constrained.

### Conceptual Questions

Thermal stresses caused by uneven cooling can easily break glass cookware. Explain why Pyrex®, a glass with a small coefficient of linear expansion, is less susceptible.

[Show Solution](#)

**Strategy:** Consider how thermal stresses arise from uneven temperature distribution and how the coefficient of expansion affects these stresses.

**Solution:** When glass cookware is cooled unevenly (for example, when hot cookware is placed on a cold surface), different parts of the glass are at different temperatures. The cooler parts contract more than the warmer parts, creating internal stresses.

Thermal stress is proportional to the coefficient of thermal expansion ( $\alpha$ ). For ordinary glass,  $\alpha = 9 \times 10^{-6}/^\circ\text{C}$ , while for Pyrex®,  $\alpha = 3 \times 10^{-6}/^\circ\text{C}$  (one-third as large).

When Pyrex® experiences the same temperature difference:

- It contracts three times less than ordinary glass
- The differential contraction between hot and cold regions is three times smaller
- The resulting thermal stresses are three times smaller
- The cookware is much less likely to exceed the breaking stress of the material

**Discussion:** Pyrex® was specifically engineered for thermal shock resistance by using borosilicate glass, which has a much lower coefficient of thermal expansion than ordinary soda-lime glass. This makes it ideal for cookware that experiences rapid temperature changes, such as going from oven to countertop or from refrigerator to oven. However, even Pyrex® has limits and can break if subjected to extreme thermal shock or if there are manufacturing defects or scratches that concentrate stress.

Water expands significantly when it freezes: a volume increase of about 9% occurs. As a result of this expansion and because of the formation and growth of crystals as water freezes, anywhere from 10% to 30% of biological cells are burst when animal or plant material is frozen. Discuss the implications of this cell damage for the prospect of preserving human bodies by freezing so that they can be thawed at some future date when it is hoped that all diseases are curable.

[Show Solution](#)

**Strategy:** Consider the structural damage caused by ice crystal formation and volume expansion, and what this means for tissue integrity and function.

**Solution:** The prospect of preserving human bodies by freezing (cryonics) faces severe challenges due to ice crystal formation:

1. **Cellular damage:** When 10-30% of cells burst during freezing, critical tissue structures are destroyed. This includes neurons in the brain, which cannot regenerate, potentially destroying memories and personality.
2. **Ice crystal damage:** Ice crystals that form between cells can physically tear cell membranes and damage the extracellular matrix that holds tissues together.
3. **Vascular damage:** Blood vessels would be particularly vulnerable, as their thin walls could be ruptured by ice expansion, making it impossible to restore blood circulation upon thawing.
4. **Organ structure:** Even if individual cells could be repaired, the overall architecture of organs and tissues would be compromised, making functional restoration extremely difficult or impossible.
5. **Information loss:** The brain's neural connections (synapses) that encode memories and personality might be physically disrupted by ice crystal formation, resulting in irreversible information loss.

**Discussion:** Current cryonics procedures use cryoprotectants (antifreeze chemicals) to try to minimize ice formation, aiming for vitrification (glass-like solidification) rather than freezing. However, these chemicals are toxic and don't completely prevent ice formation in large organs. The fundamental challenge is that the damage from freezing is not merely a matter of preserving cell chemistry—it involves preserving the precise three-dimensional structure and connectivity of trillions of cells, which may be physically impossible with current or foreseeable technology.

One method of getting a tight fit, say of a metal peg in a hole in a metal block, is to manufacture the peg slightly larger than the hole. The peg is then inserted when at a different temperature than the block. Should the block be hotter or colder than the peg during insertion? Explain your answer.

[Show Solution](#)

**Strategy:** Consider what needs to happen: either the peg must shrink or the hole must expand (or both) to allow insertion. Use thermal expansion principles.

**Solution:** There are two viable approaches:

**Option 1: Heat the block (recommended)**

- Heating the block causes it to expand, including the hole
- The hole gets larger, allowing the oversized peg to fit
- When the block cools back to room temperature, it contracts, creating a very tight fit around the peg

**Option 2: Cool the peg**

- Cooling the peg causes it to contract and become smaller
- The peg can then fit into the hole
- When the peg warms back to room temperature, it expands, creating a tight fit

**Option 1 is generally preferred** because:

- Heating is usually easier than cooling
- The block can be heated uniformly
- The expanded hole provides clearance all around the peg

**Discussion:** This technique, called interference fitting or shrink fitting, is commonly used in manufacturing and construction. Examples include fitting metal bands around wooden barrels (cooper's technique), mounting bearing races, and assembling railway wheels. The resulting joint can be extremely strong and doesn't require welding or fasteners. The same principle explains why it's easier to remove a stuck jar lid by running hot water over it—the metal lid expands more than the glass, loosening the fit.

Does it really help to run hot water over a tight metal lid on a glass jar before trying to open it? Explain your answer.

[Show Solution](#)

**Strategy:** Compare the thermal expansion of metal and glass when heated, and consider which expands more.

**Solution:** Yes, running hot water over a tight metal lid really does help open it. Here's why:

From Table 1, typical coefficients of linear expansion are:

- Steel (for metal lid):  $\alpha = 12 \times 10^{-6}/^\circ\text{C}$
- Glass (ordinary):  $\alpha = 9 \times 10^{-6}/^\circ\text{C}$

When hot water is applied:

1. Both the metal lid and glass jar expand
2. The metal lid expands more than the glass because it has a larger coefficient of expansion (12 vs. 9)
3. Additionally, the thin metal lid heats up faster than the thick glass jar, so it expands more quickly
4. The lid's circumference increases more than the jar's opening, loosening the fit

The differential expansion creates a small gap between the lid and jar, breaking the seal and reducing the friction that makes the lid difficult to turn.

**Discussion:** This technique works best if you heat just the lid (not the whole jar), maximizing the temperature difference and thus the differential expansion. Some additional factors also help: heating can soften any sticky residue (like dried food) on the threads, and thermal expansion can release some of the vacuum seal that often forms in sealed jars. This is a practical example of how understanding thermal expansion can solve everyday problems.

Liquids and solids expand with increasing temperature, because the kinetic energy of a body's atoms and molecules increases. Explain why some materials *shrink* with increasing temperature.

[Show Solution](#)

**Strategy:** Consider materials with special molecular structures or bonding arrangements that can change with temperature.

**Solution:** Some materials have negative thermal expansion coefficients and shrink when heated due to their unique molecular or crystal structures:

1. **Water (4°C to 0°C):** Water actually expands as it cools from 4°C to 0°C due to the formation of an open hexagonal crystal structure driven by hydrogen bonding. This structure has more empty space than liquid water, so cooling causes expansion in this range.
2. **Materials with flexible framework structures:** Some materials like zirconium tungstate ( $\text{ZrW}_2\text{O}_8$ ) have crystal structures with flexible bridges or linkages between atoms. When heated, increased vibrations can cause these linkages to flex inward, pulling atoms closer together rather than pushing them apart, resulting in shrinkage.
3. **Materials with reorienting molecules:** In some materials, increasing temperature causes molecules or structural units to reorient themselves into more compact arrangements, leading to overall shrinkage.
4. **Composite materials:** Some engineered materials combine components with different expansion coefficients arranged so that the net effect is shrinkage with heating.

**Discussion:** These materials with negative thermal expansion are relatively rare but scientifically important. They're useful for creating materials with zero net thermal expansion when combined with normally expanding materials, which is valuable for precision instruments, optical devices, and electronics that must maintain exact dimensions over temperature changes. The existence of negative thermal expansion shows that thermal expansion is not simply about increased atomic vibration—it depends critically on the specific bonding and structural arrangements in the material.

## Problems & Exercises

The height of the Washington Monument is measured to be 170 m on a day when the temperature is 35.0°C. What will its height be on a day when the temperature falls to -10.0°C? Although the monument is made of limestone, assume that its thermal coefficient of expansion is the same as marble's.

[Show Solution](#)

**Strategy:** Use the linear thermal expansion equation  $\Delta L = \alpha L \Delta T$  with the coefficient of linear expansion for marble from Table 1. Calculate the change in height, then subtract from the original height since temperature decreases.

**Solution:** From Table 1, for marble:  $\alpha = 7 \times 10^{-6}/^\circ\text{C}$

Given:  $L = 170 \text{ m}$ ,  $\Delta T = -10.0^\circ\text{C} - 35.0^\circ\text{C} = -45.0^\circ\text{C}$

$$\Delta L = \alpha L \Delta T = (7 \times 10^{-6}/^\circ\text{C})(170 \text{ m})(-45.0^\circ\text{C}) = -0.053 \text{ m}$$

The new height is:

$$L_{\text{new}} = L + \Delta L = 170 \text{ m} + (-0.053 \text{ m}) = 169.95 \text{ m} \approx 169.98 \text{ m}$$

**Discussion:** The Washington Monument contracts by about 5.3 cm (about 2 inches) when the temperature drops by 45°C. While this seems like a small change, it's measurable with precision instruments. This thermal contraction must be accounted for in the monument's design to prevent structural damage from thermal stresses. The monument has expansion joints that allow for this temperature-induced size change without cracking the stone.

How much taller does the Eiffel Tower become at the end of a day when the temperature has increased by 15°C? Its original height is 321 m and you can assume it is made of steel.

[Show Solution](#)

**Strategy:** Use the linear thermal expansion equation  $\Delta L = \alpha L \Delta T$  with the coefficient of linear expansion for steel from Table 1.

**Solution:** From Table 1, for steel:  $\alpha = 12 \times 10^{-6}/^\circ\text{C}$

Given:  $L = 321 \text{ m}$ ,  $\Delta T = 15^\circ\text{C}$

$$\Delta L = \alpha L \Delta T = (12 \times 10^{-6}/^\circ\text{C})(321 \text{ m})(15^\circ\text{C}) = 0.058 \text{ m} = 5.8 \text{ cm}$$

**Discussion:** The Eiffel Tower grows by about 5.8 cm (nearly 2.3 inches) on a warm day. This is a significant amount and must be accounted for in the tower's design. The tower was built with expansion joints and the structure is designed to handle this thermal expansion without damage. Interestingly, the tower also leans slightly toward the sun on hot days because the side facing the sun heats up more and expands more than the shaded side, though this effect is small.

What is the change in length of a 3.00-cm-long column of mercury if its temperature changes from  $37.0^\circ\text{C}$  to  $40.0^\circ\text{C}$ , assuming the mercury is unconstrained?

[Show Solution](#)

**Strategy:** Use the volume expansion equation, but for a column of liquid, the height change is related to volume change by  $\Delta L \approx \beta L \Delta T$  for a column with constant cross-section, or we can use  $\Delta L = \beta L \Delta T$  treating it as linear expansion in the vertical direction.

**Solution:** From Table 1, for mercury:  $\beta = 180 \times 10^{-6}/^\circ\text{C}$

Given:  $L = 3.00 \text{ cm} = 0.0300 \text{ m}$ ,  $\Delta T = 40.0^\circ\text{C} - 37.0^\circ\text{C} = 3.0^\circ\text{C}$

For a column in a narrow tube, we use the linear approximation:

$$\Delta L = \beta L \Delta T = (180 \times 10^{-6}/^\circ\text{C})(0.0300 \text{ m})(3.0^\circ\text{C}) = 5.4 \times 10^{-6} \text{ m}$$

**Discussion:** The column of mercury expands by only 5.4 micrometers (0.0054 mm), which is a very small change. However, this is exactly the principle used in mercury thermometers—the expansion is amplified by using a very narrow bore tube, so that even this tiny volume change creates a noticeable change in the height of the mercury column. Mercury thermometers are effective because mercury has a relatively large coefficient of volume expansion and remains liquid over a wide temperature range.

How large an expansion gap should be left between steel railroad rails if they may reach a maximum temperature  $35.0^\circ\text{C}$  greater than when they were laid? Their original length is 10.0 m.

[Show Solution](#)

**Strategy:** Use the linear thermal expansion equation to find how much each rail expands, then provide a gap at least this large.

**Solution:** From Table 1, for steel:  $\alpha = 12 \times 10^{-6}/^\circ\text{C}$

Given:  $L = 10.0 \text{ m}$ ,  $\Delta T = 35.0^\circ\text{C}$

$$\Delta L = \alpha L \Delta T = (12 \times 10^{-6}/^\circ\text{C})(10.0 \text{ m})(35.0^\circ\text{C}) = 4.2 \times 10^{-3} \text{ m} = 4.2 \text{ mm}$$

The expansion gap should be at least **4.2 mm** (about 0.17 inches).

**Discussion:** This expansion gap is essential to prevent rails from buckling on hot days. If rails are constrained and cannot expand, enormous compressive stresses develop that can cause the rails to buckle sideways, creating a serious safety hazard. Modern continuous welded rail (CWR) reduces the number of gaps, but requires careful stress management. The familiar “clickety-clack” sound of older trains comes from wheels passing over these expansion joints. Temperature swings of  $35^\circ\text{C}$  or more are common in many climates, making these gaps crucial for railroad safety.

You are looking to purchase a small piece of land in Hong Kong. The price is “only” 60000 *per square meter*!

*The land title says the dimensions are 20 \text{m} \times 30 \text{m}*

*By how much would the total price change if you measured the parcel with a steel tape measure on a day when the temperature is  $25^\circ\text{C}$  above normal?*

[Show Solution](#)

**Strategy:** When the steel tape measure is hot, it expands, so each meter marked on the tape is actually longer than a true meter. Therefore, when you measure the land with the hot tape, you'll get smaller measurements than the true dimensions. Use the area expansion formula  $\Delta A = 2\alpha A \Delta T$  to find the change in measured area.

**Solution:** From Table 1, for steel:  $\alpha = 12 \times 10^{-6}/^\circ\text{C}$

True area:  $A = 20 \text{ m} \times 30 \text{ m} = 600 \text{ m}^2$

When the tape expands by  $\Delta L = \alpha L \Delta T$ , each “meter” on the tape is actually  $1 + \alpha \Delta T$  meters long. So when you measure  $L$  meters with the hot tape, the true length is  $L(1 + \alpha \Delta T)$ .

The measured dimensions would be:

$$L_{\text{measured}} = L_{\text{true}} + \alpha \Delta T \approx L_{\text{true}}(1 + \alpha \Delta T)$$

The measured area is:

$$A_{\text{measured}} \approx A_{\text{true}}(1 - \alpha \Delta T)^2 \approx A_{\text{true}}(1 - 2\alpha \Delta T)$$

$$\Delta A = -2\alpha A \Delta T = -2(12 \times 10^{-6} / ^\circ\text{C})(600 \text{ m}^2)(20^\circ\text{C}) = -0.288 \text{ m}^2$$

Price change:

$$\Delta \text{Price} = (\Delta A) \times (60000 \text{ per m}^2) = (-0.288 \text{ m}^2)(60000) = -17280 \approx -17000$$

**Discussion:** Because the tape measure expands when hot, it underestimates distances, so the measured area is smaller than the true area. This means you would pay about \$17,000 less than the actual value of the land! However, this illustrates the importance of using calibrated instruments at standard temperatures for precision measurements. In reality, surveying equipment is temperature-compensated or measurements are corrected for temperature variations to prevent such errors in property transactions.

Global warming will produce rising sea levels partly due to melting ice caps but also due to the expansion of water as average ocean temperatures rise. To get some idea of the size of this effect, calculate the change in length of a column of water 1.00 km high for a temperature increase of  $1.00^\circ\text{C}$ . Note that this calculation is only approximate because ocean warming is not uniform with depth.

[Show Solution](#)

**Strategy:** Use the volume (or linear) expansion formula. For a column of water, we can use  $\Delta L = \beta L \Delta T$  or approximate using linear expansion.

**Solution:** From Table 1, for water:  $\beta = 210 \times 10^{-6} / ^\circ\text{C}$

For a vertical column, the height change is:

$$\Delta L = \beta L \Delta T = (210 \times 10^{-6} / ^\circ\text{C})(1000 \text{ m})(1.00^\circ\text{C}) = 0.070 \text{ m} = 7.0 \text{ cm}$$

Alternatively, we can say  $\Delta L \approx \beta L \Delta T / 3$ , since for isotropic expansion,  $\beta \approx 3\alpha$ .

**Discussion:** A  $1^\circ\text{C}$  warming of a 1 km column of ocean water produces about 7 cm of sea level rise due to thermal expansion alone. The average ocean depth is about 3.7 km, so if the entire ocean warmed by  $1^\circ\text{C}$  uniformly, thermal expansion would contribute roughly 26 cm (10 inches) to sea level rise. In reality, the warming is concentrated in the upper layers, but even partial ocean warming contributes significantly to sea level rise. Current estimates suggest that thermal expansion accounts for about 30-50% of observed sea level rise, with melting ice accounting for the remainder. This demonstrates that thermal expansion is a major contributor to climate change impacts.

Show that 60.0 L of gasoline originally at  $15.0^\circ\text{C}$  will expand to 61.1 L when it warms to  $35.0^\circ\text{C}$ , as claimed in [\[Example 2\]](#).

[Show Solution](#)

$$V = V_0 + \Delta V = V_0(1 + \beta \Delta T) = (60.00 \text{ L})[1 + (950 \times 10^{-6} / ^\circ\text{C})(35.0^\circ\text{C} - 15.0^\circ\text{C})] = 61.1 \text{ L}$$

(a) Suppose a meter stick made of steel and one made of invar (an alloy of iron and nickel) are the same length at  $0^\circ\text{C}$ . What is their difference in length at  $22.0^\circ\text{C}$ ? (b) Repeat the calculation for two 30.0-m-long surveyor's tapes.

[Show Solution](#)

**Strategy:** Calculate the expansion of each material separately, then find the difference.

**Solution:** From Table 1:

- Steel:  $\alpha_{\text{steel}} = 12 \times 10^{-6} / ^\circ\text{C}$
- Invar:  $\alpha_{\text{invar}} = 0.9 \times 10^{-6} / ^\circ\text{C}$

(a) For 1.00 m sticks at  $\Delta T = 22.0^\circ\text{C}$ :

Steel expansion:

$$\Delta L_{\text{steel}} = \alpha_{\text{steel}} L \Delta T = (12 \times 10^{-6} / ^\circ\text{C})(1.00 \text{ m})(22.0^\circ\text{C}) = 2.64 \times 10^{-4} \text{ m}$$

Invar expansion:

$$\Delta L_{\text{invar}} = \alpha_{\text{invar}} L \Delta T = (0.9 \times 10^{-6} / ^\circ\text{C})(1.00 \text{ m})(22.0^\circ\text{C}) = 1.98 \times 10^{-5} \text{ m}$$

Difference:

$$\Delta L_{\text{steel}} - \Delta L_{\text{invar}} = 2.64 \times 10^{-4} - 1.98 \times 10^{-5} = 2.44 \times 10^{-4} \text{ m} = 0.244 \text{ mm}$$

(b) For 30.0 m surveyor's tapes, the difference scales linearly:

$$\text{Difference} = 30.0 \times 0.244 \text{ mm} = 7.3 \text{ mm} = 0.73 \text{ cm}$$

**Discussion:** For a meter stick, the difference of 0.244 mm is small but measurable. However, for a 30-m surveyor's tape, the 7.3 mm (0.73 cm) difference is quite significant and could introduce substantial errors in surveying work if not accounted for. This is why Invar was developed—its extremely low coefficient of thermal expansion makes it ideal for precision instruments, surveying equipment, and scientific apparatus that must maintain accurate dimensions despite temperature changes. Invar's name comes from "invariable," reflecting its dimensional stability.

(a) If a 500-mL glass beaker is filled to the brim with ethyl alcohol at a temperature of 5.00°C, how much will overflow when its temperature reaches 22.0°C? (b) How much less water would overflow under the same conditions?

[Show Solution](#)

**Strategy:** Calculate the volume expansion of both the ethyl alcohol and the glass beaker. The overflow is the difference between the alcohol expansion and the beaker expansion. Repeat for water.

**Solution:** From Table 1:

- Ethyl alcohol:  $\beta_{\text{alcohol}} = 1100 \times 10^{-6}/^\circ\text{C}$
- Water:  $\beta_{\text{water}} = 210 \times 10^{-6}/^\circ\text{C}$
- Glass (ordinary):  $\beta_{\text{glass}} = 27 \times 10^{-6}/^\circ\text{C}$

Given:  $V = 500 \text{ mL}$ ,  $\Delta T = 22.0^\circ\text{C} - 5.00^\circ\text{C} = 17.0^\circ\text{C}$

(a) Alcohol expansion:

$$\Delta V_{\text{alcohol}} = \beta_{\text{alcohol}} V \Delta T = (1100 \times 10^{-6}/^\circ\text{C})(500 \text{ mL})(17.0^\circ\text{C}) = 9.35 \text{ mL}$$

Glass beaker expansion:

$$\Delta V_{\text{glass}} = \beta_{\text{glass}} V \Delta T = (27 \times 10^{-6}/^\circ\text{C})(500 \text{ mL})(17.0^\circ\text{C}) = 0.23 \text{ mL}$$

Volume that overflows:

$$V_{\text{overflow}} = \Delta V_{\text{alcohol}} - \Delta V_{\text{glass}} = 9.35 \text{ mL} - 0.23 \text{ mL} = 9.12 \text{ mL} \approx 9.35 \text{ mL}$$

(b) Water expansion:

$$\Delta V_{\text{water}} = \beta_{\text{water}} V \Delta T = (210 \times 10^{-6}/^\circ\text{C})(500 \text{ mL})(17.0^\circ\text{C}) = 1.79 \text{ mL}$$

Volume that overflows with water:

$$V_{\text{overflow, water}} = \Delta V_{\text{water}} - \Delta V_{\text{glass}} = 1.79 \text{ mL} - 0.23 \text{ mL} = 1.56 \text{ mL}$$

Difference:

$$9.35 \text{ mL} - 1.56 \text{ mL} = 7.79 \text{ mL} \approx 7.56 \text{ mL}$$

**Discussion:** Ethyl alcohol has a much larger coefficient of volume expansion than water (about 5 times larger), so it overflows much more. The glass beaker's expansion is relatively small compared to the liquid expansions. This demonstrates why different liquids are used in different types of thermometers—alcohol expands more than water or mercury, making it useful for sensitive measurements over certain temperature ranges.

Most automobiles have a coolant reservoir to catch radiator fluid that may overflow when the engine is hot. A radiator is made of copper and is filled to its 16.0-L capacity when at 10.0°C. What volume of radiator fluid will overflow when the radiator and fluid reach their 95.0°C operating temperature, given that the fluid's volume coefficient of expansion is  $\beta = 400 \times 10^{-6}/^\circ\text{C}$ ? Note that this coefficient is approximate, because most car radiators have operating temperatures of greater than 95.0°C.

[Show Solution](#)

**Strategy:** Calculate the volume expansion of both the radiator (copper) and the fluid. The overflow is the difference between the fluid expansion and the radiator expansion.

**Solution:** From Table 1:  $\beta_{\text{copper}} = 51 \times 10^{-6}/^\circ\text{C}$

Given:  $\beta_{\text{fluid}} = 400 \times 10^{-6}/^\circ\text{C}$ ,  $V = 16.0 \text{ L}$ ,  $\Delta T = 95.0^\circ\text{C} - 10.0^\circ\text{C} = 85.0^\circ\text{C}$

Fluid expansion:

$$\Delta V_{\text{fluid}} = \beta_{\text{fluid}} V \Delta T = (400 \times 10^{-6} / ^\circ\text{C})(16.0 \text{ L})(85.0 / ^\circ\text{C}) = 0.544 \text{ L}$$

Radiator expansion:

$$\Delta V_{\text{radiator}} = \beta_{\text{copper}} V \Delta T = (51 \times 10^{-6} / ^\circ\text{C})(16.0 \text{ L})(85.0 / ^\circ\text{C}) = 0.069 \text{ L}$$

Volume that overflows:

$$V_{\text{overflow}} = \Delta V_{\text{fluid}} - \Delta V_{\text{radiator}} = 0.544 \text{ L} - 0.069 \text{ L} = 0.48 \text{ L}$$

**Discussion:** About 0.48 L (roughly half a liter or one pint) of coolant will overflow, which is why radiator overflow reservoirs are necessary. The fluid expands much more than the copper radiator (coefficient nearly 8 times larger), so the net effect is significant overflow. Modern cooling systems are designed as closed systems with expansion tanks that can accommodate this volume change, preventing coolant loss and allowing the fluid to return to the radiator when the engine cools. Without such a reservoir, coolant would be lost every time the engine heats up, requiring frequent refilling.

A physicist makes a cup of instant coffee and notices that, as the coffee cools, its level drops 3.00 mm in the glass cup. Show that this decrease cannot be due to thermal contraction by calculating the decrease in level if the  $350\text{cm}^3$  of coffee is in a 7.00-cm-diameter cup and decreases in temperature from  $95.0 / ^\circ\text{C}$  to  $45.0 / ^\circ\text{C}$ . (Most of the drop in level is actually due to escaping bubbles of air.)

[Show Solution](#)

**Strategy:** Calculate the volume decrease due to thermal contraction of the coffee (treat as water). Then calculate what height change this represents in a cylindrical cup with the given diameter.

**Solution:** From Table 1, for water:  $\beta = 210 \times 10^{-6} / ^\circ\text{C}$

Given:  $V = 350 \text{ cm}^3$ ,  $\Delta T = 45.0 / ^\circ\text{C} - 95.0 / ^\circ\text{C} = -50.0 / ^\circ\text{C}$ , diameter  $d = 7.00 \text{ cm}$ , radius  $r = 3.50 \text{ cm}$

Volume change due to thermal contraction:

$$\Delta V = \beta V \Delta T = (210 \times 10^{-6} / ^\circ\text{C})(350 \text{ cm}^3)(-50.0 / ^\circ\text{C}) = -3.675 \text{ cm}^3$$

For a cylindrical cup,  $V = \pi r^2 h$ , so:

$$\Delta h = \Delta V \pi r^2 = -3.675 \text{ cm}^3 \pi (3.50 \text{ cm})^2 = -3.675 \pi 38.48 = -0.0955 \text{ cm} = -0.955 \text{ mm} \approx -0.832 \text{ mm}$$

**Discussion:** The calculated drop due to thermal contraction is only about 0.83 mm, much less than the observed 3.00 mm drop. This proves that thermal contraction cannot account for the observed decrease in coffee level. The physicist is correct that most of the drop is due to escaping air bubbles. When hot water is used to make coffee, it contains dissolved air that comes out of solution as bubbles as the coffee cools. These bubbles escape, reducing the total volume. This effect is much larger than thermal contraction alone.

(a) The density of water at  $0 / ^\circ\text{C}$  is very nearly  $1000 \text{ kg/m}^3$  (it is actually  $999.84 \text{ kg/m}^3$ ), whereas the density of ice at  $0 / ^\circ\text{C}$  is  $917 \text{ kg/m}^3$ . Calculate the pressure necessary to keep ice from expanding when it freezes, neglecting the effect such a large pressure would have on the freezing temperature. (This problem gives you only an indication of how large the forces associated with freezing water might be.) (b) What are the implications of this result for biological cells that are frozen?

[Show Solution](#)

**Strategy:** (a) Find the volume change when water freezes, then use the bulk modulus relationship to find the pressure required to prevent this expansion. (b) Consider what this pressure means for cell membranes.

**Solution:** (a) Consider 1 kg of water freezing:

$$\text{Volume of water: } V_{\text{water}} = m \rho_{\text{water}} = 1 \text{ kg} / 1000 \text{ kg/m}^3 = 0.001 \text{ m}^3$$

$$\text{Volume of ice: } V_{\text{ice}} = m \rho_{\text{ice}} = 1 \text{ kg} / 917 \text{ kg/m}^3 = 1.091 \times 10^{-3} \text{ m}^3$$

Volume change:

$$\Delta V = V_{\text{ice}} - V_{\text{water}} = 1.091 \times 10^{-3} - 1.000 \times 10^{-3} = 9.1 \times 10^{-5} \text{ m}^3$$

Fractional volume change:

$$\Delta V / V_{\text{water}} = 9.1 \times 10^{-5} / 0.001 = 0.091 = 9.1\%$$

Using the bulk modulus relationship  $P = B\Delta V/V$  with  $B = 2.2 \times 10^9 \text{ Pa}$  for ice:

$$P = (2.2 \times 10^9 \text{ Pa})(0.091) = 2.0 \times 10^8 \text{ Pa} = 2000 \text{ atm}$$

(b) This enormous pressure (about 2000 atmospheres or 29,000 psi) far exceeds what biological cell membranes can withstand. Cell membranes typically rupture at pressures of just a few atmospheres. This explains why:

- Freezing bursts 10-30% of cells
- Frozen foods have different texture when thawed
- Frostbite causes permanent tissue damage
- Cryopreservation requires cryoprotectants to minimize ice formation

**Discussion:** The tremendous pressure generated by freezing water is why water pipes burst in winter and why rocks can be fractured by freeze-thaw cycles (frost wedging). This pressure is also why it's so dangerous to freeze liquids in sealed containers—they can explode. For biological preservation, this demonstrates why simple freezing is inadequate and why advanced cryopreservation techniques attempt to achieve vitrification (glass-like solidification) rather than ice crystal formation.

Show that  $\beta \approx 3\alpha$ , by calculating the change in volume  $\Delta V$  of a cube with sides of length  $L$ .

[Show Solution](#)

We know how the length changes with temperature:  $\Delta L = \alpha L_0 \Delta T$ . Also we know that the volume of a cube is related to its length by  $V = L^3$ , so the final volume is then  $V = V_0 + \Delta V = (L_0 + \Delta L)^3$ . Substituting for  $\Delta L$  gives

$$V = (L_0 + \alpha L_0 \Delta T)^3 = L_0^3 (1 + \alpha \Delta T)^3.$$

Now, because  $\alpha \Delta T$  is small, we can use the binomial expansion:

$$V \approx L_0^3 (1 + 3\alpha \Delta T) = L_0^3 + 3\alpha L_0^3 \Delta T.$$

So writing the length terms in terms of volumes gives  $V = V_0 + \Delta V \approx V_0 + 3\alpha V_0 \Delta T$ , and so

$$\Delta V = \beta V_0 \Delta T \approx 3\alpha V_0 \Delta T, \text{ or } \beta \approx 3\alpha.$$

## Footnotes

- [1](#) Values for liquids and gases are approximate.

## Glossary

### thermal expansion

the change in size or volume of an object with change in temperature

### coefficient of linear expansion

$\alpha$ , the change in length, per unit length, per  $1^\circ\text{C}$  change in temperature; a constant used in the calculation of linear expansion; the coefficient of linear expansion depends on the material and to some degree on the temperature of the material

### coefficient of volume expansion

$\beta$ , the change in volume, per unit volume, per  $1^\circ\text{C}$  change in temperature

### thermal stress

stress caused by thermal expansion or contraction



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## The Ideal Gas Law

- State the ideal gas law in terms of molecules and in terms of moles.
- Use the ideal gas law to calculate pressure change, temperature change, volume change, or the number of molecules or moles in a given volume.
- Use Avogadro's number to convert between number of molecules and number of moles.



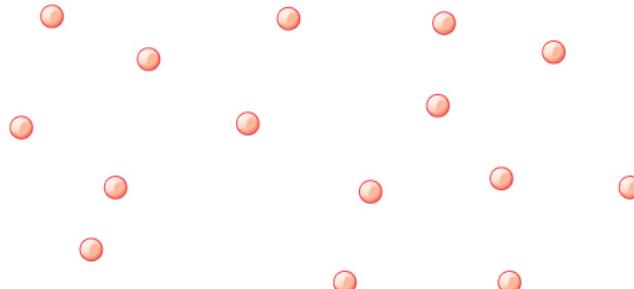
The air inside this hot air balloon flying over Putrajaya, Malaysia, is hotter than the ambient air. As a result, the balloon experiences a buoyant force pushing it upward. (credit: Kevin Poh, Flickr)

In this section, we continue to explore the thermal behavior of gases. In particular, we examine the characteristics of atoms and molecules that compose gases. (Most gases, for example nitrogen,  $\text{N}_2$ , and oxygen,  $\text{O}_2$ , are composed of two or more atoms. We will primarily use the term “molecule” in discussing a gas because the term can also be applied to monatomic gases, such as helium.)

Gases are easily compressed. We can see evidence of this in [\[Table 1\]](#), where you will note that gases have the *largest* coefficients of volume expansion. The large coefficients mean that gases expand and contract very rapidly with temperature changes. In addition, you will note that most gases expand at the

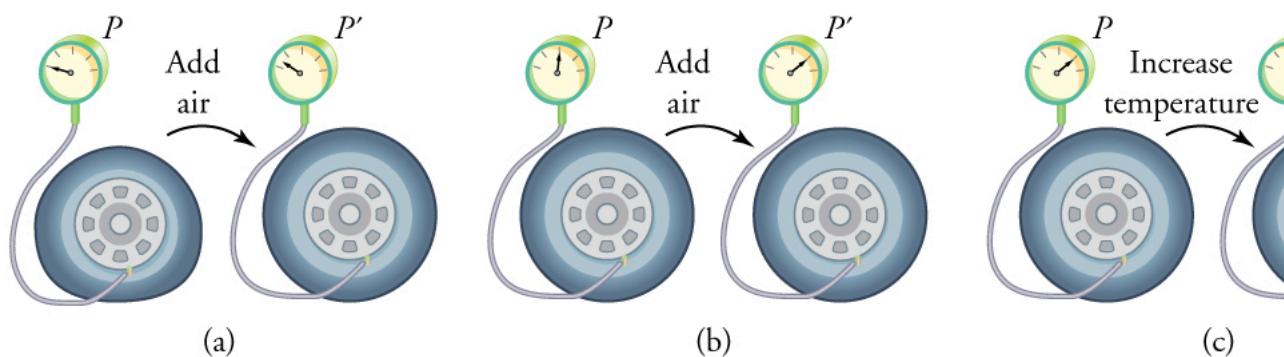
same rate, or have the same  $\beta$ . This raises the question as to why gases should all act in nearly the same way, when liquids and solids have widely varying expansion rates.

The answer lies in the large separation of atoms and molecules in gases, compared to their sizes, as illustrated in [\[Figure 2\]](#). Because atoms and molecules have large separations, forces between them can be ignored, except when they collide with each other during collisions. The motion of atoms and molecules (at temperatures well above the boiling temperature) is fast, such that the gas occupies all of the accessible volume and the expansion of gases is rapid. In contrast, in liquids and solids, atoms and molecules are closer together and are quite sensitive to the forces between them.



Atoms and molecules in a gas are typically widely separated, as shown. Because the forces between them are quite weak at these distances, the properties of a gas depend more on the number of atoms per unit volume and on temperature than on the type of atom.

To get some idea of how pressure, temperature, and volume of a gas are related to one another, consider what happens when you pump air into an initially deflated tire. The tire's volume first increases in direct proportion to the amount of air injected, without much increase in the tire pressure. Once the tire has expanded to nearly its full size, the walls limit volume expansion. If we continue to pump air into it, the pressure increases. The pressure will further increase when the car is driven and the tires move. Most manufacturers specify optimal tire pressure for cold tires. (See [\[Figure 3\]](#).)



(a) When air is pumped into a deflated tire, its volume first increases without much increase in pressure. (b) When the tire is filled to a certain point, the tire walls resist further expansion and the pressure increases with more air. (c) Once the tire is inflated, its pressure increases with temperature.

At room temperatures, collisions between atoms and molecules can be ignored. In this case, the gas is called an ideal gas, in which case the relationship between the pressure, volume, and temperature is given by the equation of state called the ideal gas law.

### Ideal Gas Law

The **ideal gas law** states that

$$PV = Nk_B T$$

where  $P$  is the absolute pressure of a gas,  $V$  is the volume it occupies,  $N$  is the number of atoms and molecules in the gas, and  $T$  is its absolute temperature. The constant  $k_B$  is called the **Boltzmann constant** in honor of Austrian physicist Ludwig Boltzmann (1844–1906) and has the value

$$k_B = 1.38 \times 10^{-23} \text{ J/K}$$

The ideal gas law can be derived from basic principles, but was originally deduced from experimental measurements of Charles' law (that volume occupied by a gas is proportional to temperature at a fixed pressure) and from Boyle's law (that for a fixed temperature, the product  $PV$  is a constant). In the ideal gas model, the volume occupied by its atoms and molecules is a negligible fraction of  $V$ . The ideal gas law describes the behavior of real gases under most conditions. (Note, for example, that  $N$  is the total number of atoms and molecules, independent of the type of gas.)

Let us see how the ideal gas law is consistent with the behavior of filling the tire when it is pumped slowly and the temperature is constant. At first, the pressure  $P$  is essentially equal to atmospheric pressure, and the volume  $V$  increases in direct proportion to the number of atoms and molecules  $N$  put into the tire. Once the volume of the tire is constant, the equation  $PV = Nk_B T$  predicts that the pressure should increase in proportion to the number  $N$  of atoms and molecules.

### Calculating Pressure Changes Due to Temperature Changes: Tire Pressure

Suppose your bicycle tire is fully inflated, with an absolute pressure of  $7.00 \times 10^5 \text{ Pa}$  (a gauge pressure of just under  $90.0 \text{ lb/in}^2$ ) at a temperature of  $18.0^\circ\text{C}$ . What is the pressure after its temperature has risen to  $35.0^\circ\text{C}$ ? Assume that there are no appreciable leaks or changes in volume.

### Strategy

The pressure in the tire is changing only because of changes in temperature. First we need to identify what we know and what we want to know, and then identify an equation to solve for the unknown.

We know the initial pressure  $P_0 = 7.00 \times 10^5 \text{ Pa}$ , the initial temperature  $T_0 = 18.0^\circ\text{C}$ , and the final temperature  $T_f = 35.0^\circ\text{C}$ . We must find the final pressure  $P_f$ . How can we use the equation  $PV = Nk_B T$ ? At first, it may seem that not enough information is given, because the volume  $V$  and number of atoms  $N$  are not specified. What we can do is use the equation twice:  $P_0 V_0 = Nk_B T_0$  and  $P_f V_f = Nk_B T_f$ . If we divide  $P_f V_f$  by  $P_0 V_0$  we can come up with an equation that allows us to solve for  $P_f$ .

$$\frac{P_f V_f}{P_0 V_0} = \frac{Nk_B T_f}{Nk_B T_0}$$

Since the volume is constant,  $V_f = V_0$  and  $N_f = N_0$  are the same and they cancel out. The same is true for  $N_f$  and  $N_0$ , and  $k$ , which is a constant. Therefore,

$$\frac{P_f}{P_0} = \frac{T_f}{T_0}$$

We can then rearrange this to solve for  $P_f$ :

$$P_f = P_0 \frac{T_f}{T_0}$$

where the temperature must be in units of kelvins, because  $T_0$  and  $T_f$  are absolute temperatures.

### Solution

- Convert temperatures from Celsius to Kelvin.

$$T_0 = 18.0 + 273 = 291 \text{ K}$$

- Substitute the known values into the equation.

$$P_f = 7.00 \times 10^5 \text{ Pa} \left( \frac{35.0 + 273}{291} \right) = 7.41 \times 10^5 \text{ Pa}$$

### Discussion

The final temperature is about 6% greater than the original temperature, so the final pressure is about 6% greater as well. Note that **absolute** pressure and **absolute** temperature must be used in the ideal gas law.

Making Connections: Take-Home Experiment—Refrigerating a Balloon

Inflate a balloon at room temperature. Leave the inflated balloon in the refrigerator overnight. What happens to the balloon, and why?

### Calculating the Number of Molecules in a Cubic Meter of Gas

How many molecules are in a typical object, such as gas in a tire or water in a drink? We can use the ideal gas law to give us an idea of how large  $N$  typically is.

Calculate the number of molecules in a cubic meter of gas at standard temperature and pressure (STP), which is defined to be  $0^\circ\text{C}$  and atmospheric pressure.

### Strategy

Because pressure, volume, and temperature are all specified, we can use the ideal gas law  $PV = Nk_B T$ , to find  $N$ .

### Solution

- Identify the knowns.

$$T = 0^\circ\text{C} = 273 \text{ K}$$

$$P = 1.01 \times 10^5 \text{ Pa}$$

$$V = 1.00 \text{ m}^3$$

$$k = 1.38 \times 10^{-23} \text{ J/K}$$

- Identify the unknown: number of molecules,  $N$ .

- Rearrange the ideal gas law to solve for  $N$ .

$$PV = Nk_B T$$

$$N = \frac{PV}{k_B T}$$

- Substitute the known values into the equation and solve for  $N$ .

$$N = \frac{(1.01 \times 10^5 \text{ Pa})(1.00 \text{ m}^3)}{(1.38 \times 10^{-23} \text{ J/K})(273 \text{ K})} = 2.68 \times 10^{25} \text{ molecules}$$

### Discussion

This number is undeniably large, considering that a gas is mostly empty space.  $N$  is huge, even in small volumes. For example,  $1 \text{ cm}^3$  of a gas at STP has  $2.68 \times 10^{19}$  molecules in it. Once again, note that  $N$  is the same for all types or mixtures of gases.

## ■ Moles and Avogadro's Number

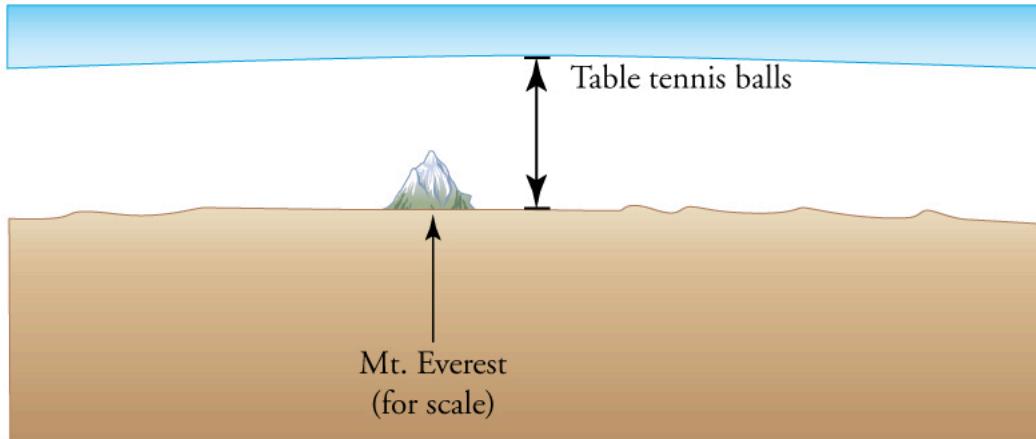
It is sometimes convenient to work with a unit other than molecules when measuring the amount of substance. A **mole** (abbreviated mol) is defined to be the amount of a substance that contains as many atoms or molecules as there are atoms in exactly 12 grams (0.012 kg) of carbon-12. The actual number of atoms or molecules in one mole is called **Avogadro's number**  $N_A = 6.02 \times 10^{23}$  mol<sup>-1</sup>, in recognition of Italian scientist Amedeo Avogadro (1776–1856). He developed the concept of the mole, based on the hypothesis that equal volumes of gas, at the same pressure and temperature, contain equal numbers of molecules. That is, the number is independent of the type of gas. This hypothesis has been confirmed, and the value of Avogadro's number is

$$N_A = 6.02 \times 10^{23} \text{ mol}^{-1}$$

Avogadro's Number

One mole always contains  $6.02 \times 10^{23}$  particles (atoms or molecules), independent of the element or substance. A mole of any substance has a mass in grams equal to its molecular mass, which can be calculated from the atomic masses given in the periodic table of elements.

$$N_A = 6.02 \times 10^{23} \text{ mol}^{-1}$$



How big is a mole? On a macroscopic level, one mole of table tennis balls would cover the Earth to a depth of about 40 km.

#### Check Your Understanding

The active ingredient in a Tylenol pill is 325 mg of acetaminophen  $C_8H_9NO_2$ . Find the number of active molecules of acetaminophen in a single pill.

Show Solution

We first need to calculate the molar mass (the mass of one mole) of acetaminophen. To do this, we need to multiply the number of atoms of each element by the element's atomic mass.

$$\text{Molar mass} = (8 \times 12.01 \text{ g/mol}) + (9 \times 1.01 \text{ g/mol}) + (1 \times 14.01 \text{ g/mol}) + (2 \times 16.00 \text{ g/mol}) = 151 \text{ g/mol}$$

Then we need to calculate the number of moles in 325 mg.

$$\text{Moles} = \frac{325 \text{ mg}}{151 \text{ g/mol}} = 2.15 \times 10^{-3} \text{ mol}$$

Then use Avogadro's number to calculate the number of molecules.

$$\text{Molecules} = 2.15 \times 10^{-3} \text{ mol} \times 6.02 \times 10^{23} \text{ mol}^{-1} = 1.30 \times 10^{21} \text{ molecules}$$

Calculating Moles per Cubic Meter and Liters per Mole

Calculate: (a) the number of moles in 1.00 L of gas at STP, and (b) the number of liters of gas per mole.

#### Strategy and Solution

(a) We are asked to find the number of moles per cubic meter, and we know from [Example 2](#) that the number of molecules per cubic meter at STP is  $2.68 \times 10^{25}$ . The number of moles can be found by dividing the number of molecules by Avogadro's number. We let  $n$  stand for the number of moles,

$$n = \frac{2.68 \times 10^{25} \text{ molecules}}{6.02 \times 10^{23} \text{ molecules/mol}} = 44.5 \text{ mol}$$

(b) Using the value obtained for the number of moles in a cubic meter, and converting cubic meters to liters, we obtain

$$\frac{10^3 \text{ L}}{1 \text{ m}^3} \times 44.5 \text{ mol} = 22.5 \text{ L/mol}$$

#### Discussion

This value is very close to the accepted value of 22.4 L/mol. The slight difference is due to rounding errors caused by using three-digit input. Again this number is the same for all gases. In other words, it is independent of the gas.

The (average) molar weight of air is approximately 80%  $\text{N}_2$  and 20%  $\text{O}_2$  is  $M = 28.8 \text{ g/mol}$ . Thus the mass of one cubic meter of air is 1.28 kg. If a living room has dimensions  $5 \text{ m} \times 5 \text{ m} \times 3 \text{ m}$  the mass of air inside the room is 96 kg, which is the typical mass of a human.

Check Your Understanding

The density of air at standard conditions ( $P=1 \text{ atm}$  and  $T=20^\circ\text{C}$ ) is  $1.28 \text{ kg/m}^3$ . At what pressure is the density  $0.64 \text{ kg/m}^3$  if the temperature and number of molecules are kept constant?

Show Solution

The best way to approach this question is to think about what is happening. If the density drops to half its original value and no molecules are lost, then the volume must double. If we look at the equation  $PV=Nk_B T$ , we see that when the temperature is constant, the pressure is inversely proportional to volume. Therefore, if the volume doubles, the pressure must drop to half its original value, and  $P_f = 0.50 \text{ atm}$ .

## The Ideal Gas Law Restated Using Moles

A very common expression of the ideal gas law uses the number of moles,  $n$ , rather than the number of atoms and molecules,  $N$ . We start from the ideal gas law,

$$PV=Nk_B T$$

and multiply and divide the equation by Avogadro's number  $N_A$ . This gives

$$PV=\frac{N}{N_A} N k_B T$$

Note that  $n=N/N_A$  is the number of moles. We define the universal gas constant  $R=N_A k_B$ , and obtain the ideal gas law in terms of moles.

Ideal Gas Law (in terms of moles)

The ideal gas law (in terms of moles) is

$$PV=nRT$$

The numerical value of  $R$  in SI units is

$$R=8.31 \text{ J/mol K}$$

In other units,

$$R=0.0821 \text{ L atm/mol K}$$

You can use whichever value of  $R$  is most convenient for a particular problem.

Calculating Number of Moles: Gas in a Bike Tire

How many moles of gas are in a bike tire with a volume of  $2.00 \text{ L}$  at a pressure of  $7.00 \text{ atm}$  and a temperature of  $18.0^\circ\text{C}$ ?

### Strategy

Identify the knowns and unknowns, and choose an equation to solve for the unknown. In this case, we solve the ideal gas law,  $PV=nRT$ , for the number of moles  $n$ .

### Solution

1. Identify the knowns.

$$P=7.00 \text{ atm}, V=2.00 \text{ L}, T=18.0^\circ\text{C}, R=8.31 \text{ J/mol K}$$

1. Rearrange the equation to solve for  $n$  and substitute known values.

$$n=\frac{PV}{RT}=\frac{(7.00 \text{ atm})(2.00 \text{ L})}{(8.31 \text{ J/mol K})(18.0^\circ\text{C})}=0.579 \text{ mol}$$

### Discussion

The most convenient choice for  $R$  in this case is  $8.31 \text{ J/mol K}$  because our known quantities are in SI units. The pressure and temperature are obtained from the initial conditions in [Example 1](#), but we would get the same answer if we used the final values.

The ideal gas law can be considered to be another manifestation of the law of conservation of energy (see [Conservation of Energy](#)). Work done on a gas results in an increase in its energy, increasing pressure and/or temperature, or decreasing volume. This increased energy can also be viewed as increased internal kinetic energy, given the gas's atoms and molecules.

## The Ideal Gas Law and Energy

Let us now examine the role of energy in the behavior of gases. When you inflate a bike tire by hand, you do work by repeatedly exerting a force through a distance. This energy goes into increasing the pressure of air inside the tire and increasing the temperature of the pump and the air.

The ideal gas law is closely related to energy: the units on both sides are joules. The right-hand side of the ideal gas law in  $PV=Nk_B T$  is  $Nk_B T$ . This term is roughly the amount of translational kinetic energy of  $N$  atoms or molecules at an absolute temperature  $T$ , as we shall see formally in [Kinetic Theory: Atomic and Molecular Explanation of Pressure and Temperature](#). The left-hand side of the ideal gas law is  $PV$ , which also has the units of joules. We know from our study of fluids that pressure is one type of potential energy per unit volume, so pressure multiplied by volume is energy. The important point is that there is energy in a gas related to both its pressure and its volume. The energy can be changed when the gas is doing work as it expands—something we explore in [Heat and Heat Transfer Methods](#)—similar to what occurs in gasoline or steam engines and turbines.

#### Problem-Solving Strategy: The Ideal Gas Law

**Step 1** Examine the situation to determine that an ideal gas is involved. Most gases are nearly ideal.

**Step 2** Make a list of what quantities are given, or can be inferred from the problem as stated (identify the known quantities). Convert known values into proper SI units (K for temperature, Pa for pressure,  $\text{m}^3$  for volume, molecules for  $N$ , and moles for  $n$ ).

**Step 3** Identify exactly what needs to be determined in the problem (identify the unknown quantities). A written list is useful.

**Step 4** Determine whether the number of molecules or the number of moles is known, in order to decide which form of the ideal gas law to use. The first form is  $PV=Nk_B T$  and involves  $N$ , the number of atoms or molecules. The second form is  $PV=nRT$  and involves  $n$ , the number of moles.

**Step 5** Solve the ideal gas law for the quantity to be determined (the unknown quantity). You may need to take a ratio of final states to initial states to eliminate the unknown quantities that are kept fixed.

**Step 6** Substitute the known quantities, along with their units, into the appropriate equation, and obtain numerical solutions complete with units. Be certain to use absolute temperature and absolute pressure.

**Step 7** Check the answer to see if it is reasonable: Does it make sense?

#### Check Your Understanding

Liquids and solids have densities about 1000 times greater than gases. Explain how this implies that the distances between atoms and molecules in gases are about 10 times greater than the size of their atoms and molecules.

[Show Solution](#)

Atoms and molecules are close together in solids and liquids. In gases they are separated by empty space. Thus gases have lower densities than liquids and solids. Density is mass per unit volume, and volume is related to the size of a body (such as a sphere) cubed. So if the distance between atoms and molecules increases by a factor of 10, then the volume occupied increases by a factor of 1000, and the density decreases by a factor of 1000.

### Section Summary

- The ideal gas law relates the pressure and volume of a gas to the number of gas molecules and the temperature of the gas.
- The ideal gas law can be written in terms of the number of molecules of gas:

$$PV=Nk_B T$$

where  $P$  is pressure,  $V$  is volume,  $T$  is temperature,  $N$  is number of molecules, and  $k_B$  is the Boltzmann constant

$$k_B = 1.38 \times 10^{-23} \text{ J/K}$$

- A mole is the number of atoms in a 12-g sample of carbon-12.
- The number of molecules in a mole is called Avogadro's number  $N_A$
- ,
- $N_A = 6.02 \times 10^{23} \text{ mol}^{-1}$
- A mole of any substance has a mass in grams equal to its molecular weight, which can be determined from the periodic table of elements.
- The ideal gas law can also be written and solved in terms of the number of moles of gas:

$$PV=nRT$$

where  $n$  is number of moles and  $R$  is the universal gas constant,

$$R = 8.31 \text{ J/mol K}$$

- The ideal gas law is generally valid at temperatures well above the boiling temperature.

### Conceptual Questions

Find out the human population of Earth. Is there a mole of people inhabiting Earth? If the average mass of a person is 60 kg, calculate the mass of a mole of people. How does the mass of a mole of people compare with the mass of Earth?

[Show Solution](#)

**Strategy:** Compare the current human population to Avogadro's number, then calculate the mass of a mole of people and compare to Earth's mass.

**Solution:** The current human population is approximately 8 billion people =  $8 \times 10^{9}$  people.

Avogadro's number is  $6.02 \times 10^{23}$ , so we have:

$$\frac{8 \times 10^9}{6.02 \times 10^{23}} = 1.3 \times 10^{-14} \text{ moles of people}$$

Clearly, we do **not** have a mole of people on Earth—we have only about  $10^{-14}$  moles of people.

If we had a mole of people ( $6.02 \times 10^{23}$  people), each with mass 60 kg:

$$\text{Mass} = (6.02 \times 10^{23} \text{ people}) (60 \text{ kg/person}) = 3.61 \times 10^{25} \text{ kg}$$

Earth's mass is approximately  $5.97 \times 10^{24}$  kg. The ratio is:

$$\frac{3.61 \times 10^{25}}{5.97 \times 10^{24}} \approx 6.0$$

**Discussion:** A mole of people would have about 6 times the mass of Earth! This illustrates the enormous magnitude of Avogadro's number. The fact that Avogadro's number is so large explains why macroscopic amounts of substances contain such huge numbers of atoms and molecules, and why we can observe statistical properties like temperature and pressure despite the random motion of individual particles.

Under what circumstances would you expect a gas to behave significantly differently than predicted by the ideal gas law?

[Show Solution](#)

**Strategy:** Consider the assumptions of the ideal gas law and when they break down.

**Solution:** The ideal gas law assumes that:

1. Gas molecules have negligible volume compared to the container volume
2. There are no intermolecular forces (except during collisions)
3. Collisions are perfectly elastic

A gas behaves significantly differently from ideal when:

1. **High pressures:** At high pressure, molecules are forced close together, and their finite volume becomes significant compared to the available space. The actual volume available for molecular motion is less than the container volume.
2. **Low temperatures:** At low temperatures (near the boiling point), intermolecular attractive forces become significant. Molecules spend more time near each other, and these attractive forces affect pressure and can lead to condensation.
3. **Near phase transitions:** When a gas is close to liquefying or solidifying, intermolecular forces dominate and the ideal gas law fails.
4. **Large, complex molecules:** Larger molecules have greater intermolecular forces (van der Waals forces) and occupy more volume, causing deviations from ideal behavior.

**Discussion:** Real gases are described more accurately by the van der Waals equation, which includes correction terms for molecular volume and intermolecular forces. However, under normal conditions (room temperature and atmospheric pressure), most gases behave nearly ideally because molecules are far apart and moving fast. The ideal gas law is an excellent approximation for gases like nitrogen, oxygen, and helium under typical conditions.

A constant-volume gas thermometer contains a fixed amount of gas. What property of the gas is measured to indicate its temperature?

[Show Solution](#)

**Strategy:** Consider the ideal gas law with constant volume and constant amount of gas.

**Solution:** In a constant-volume gas thermometer, the **pressure** of the gas is measured to indicate temperature.

From the ideal gas law:  $PV=nRT$

When volume ( $V$ ) and amount of gas ( $n$ ) are held constant:

$$P = \frac{nR}{V} T = \text{constant} \times T$$

This shows that pressure is directly proportional to absolute temperature when volume and amount of gas are fixed. Therefore, by measuring the pressure of the gas, we can determine its temperature.

**Discussion:** Constant-volume gas thermometers are among the most accurate thermometers available and are used to calibrate other thermometers. They work over a wide temperature range and can measure temperatures from near absolute zero to very high temperatures. The gas used (often helium or hydrogen) must behave ideally over the measurement range. The direct proportionality between pressure and temperature makes these thermometers particularly useful for defining the Kelvin temperature scale, as they can extrapolate to find absolute zero (the temperature at which an ideal gas would have zero pressure).

## Problems & Exercises

The gauge pressure in your car tires is  $2.50 \times 10^5 \text{ N/m}^2$  at a temperature of  $35.0^\circ\text{C}$  when you drive it onto a ferry boat to Alaska. What is their gauge pressure later, when their temperature has dropped to  $-40.0^\circ\text{C}$ ?

[Show Solution](#)

**Strategy:** First convert gauge pressure to absolute pressure, then use the ideal gas law relationship  $\frac{P_1}{T_1} = \frac{P_2}{T_2}$  for constant volume and amount of gas. Finally, convert back to gauge pressure.

**Solution:** Initial absolute pressure:

$$P_1 = P_{\text{gauge}} + P_{\text{atm}} = 2.50 \times 10^5 \text{ Pa} + 1.01 \times 10^5 \text{ Pa} = 3.51 \times 10^5 \text{ Pa}$$

Convert temperatures to Kelvin:

$$T_1 = 35.0^\circ\text{C} + 273.15 = 308 \text{ K}$$

$$T_2 = -40.0^\circ\text{C} + 273.15 = 233 \text{ K}$$

Using  $\frac{P_1}{T_1} = \frac{P_2}{T_2}$ :

$$P_2 = P_1 \frac{T_2}{T_1} = \left( 3.51 \times 10^5 \text{ Pa} \right) \frac{233 \text{ K}}{308 \text{ K}} = 2.66 \times 10^5 \text{ Pa}$$

Final gauge pressure:

$$P_{\text{final}} = P_2 - P_{\text{atm}} = 2.66 \times 10^5 \text{ Pa} - 1.01 \times 10^5 \text{ Pa} = 1.65 \times 10^5 \text{ Pa} = 1.63 \text{ atm} \approx 1.62 \text{ atm}$$

**Discussion:** The gauge pressure drops from 2.47 atm to 1.62 atm, a decrease of about 34%. This significant pressure drop is due to the large temperature change ( $75^\circ\text{C}$  decrease). This is why tire pressure monitoring is important in cold climates—tires can become significantly underinflated as temperature drops, affecting vehicle handling and fuel efficiency. Drivers should check and adjust tire pressure when seasons change or when traveling to areas with very different temperatures.

Convert an absolute pressure of  $7.00 \times 10^5 \text{ N/m}^2$  to gauge pressure in  $\text{lb/in}^2$  (This value was stated to be just less than  $90.0 \text{ lb/in}^2$  in [Example 4](#). Is it?)

[Show Solution](#)

**Strategy:** Gauge pressure is absolute pressure minus atmospheric pressure. Convert to psi using the conversion  $1 \text{ atm} = 14.7 \text{ psi} = 1.01 \times 10^5 \text{ Pa}$ .

**Solution:** First find gauge pressure in Pa:

$$P_{\text{gauge}} = P_{\text{abs}} - P_{\text{atm}} = 7.00 \times 10^5 \text{ Pa} - 1.01 \times 10^5 \text{ Pa} = 5.99 \times 10^5 \text{ Pa}$$

Convert to psi (pounds per square inch):

$$P_{\text{psi}} = \frac{P_{\text{gauge}}}{14.7 \text{ psi}} = \frac{5.99 \times 10^5 \text{ Pa}}{14.7 \text{ psi}} = 87.1 \text{ psi}$$

**Discussion:** Yes, this confirms the statement—87.1 psi is indeed just less than 90.0 psi. Gauge pressure is what's typically displayed on tire pressure gauges because it represents the excess pressure above atmospheric pressure. The absolute pressure of  $7.00 \times 10^5 \text{ Pa}$  corresponds to about 6.93 atm, meaning the tire pressure is about 5.93 atm above atmospheric pressure.

Suppose a gas-filled incandescent light bulb is manufactured so that the gas inside the bulb is at atmospheric pressure when the bulb has a temperature of  $20.0^\circ\text{C}$ . (a) Find the gauge pressure inside such a bulb when it is hot, assuming its average temperature is  $60.0^\circ\text{C}$  (an approximation) and neglecting any change in volume due to thermal expansion or gas leaks. (b) The actual final pressure for the light bulb will be less than calculated in part (a) because the glass bulb will expand. What will the actual final pressure be, taking this into account? Is this a negligible difference?

[Show Solution](#)

(a) 0.136 atm

(b) 0.135 atm. The difference between this value and the value from part (a) is negligible.

To test a balloon, it is placed in a lab and filled with helium. The temperature of the helium is  $10.0^\circ\text{C}$  and the pressure is 1.00 atmosphere. The pressure in the lab is maintained. Assume the membrane of the balloon provides a negligible inward pressure, so it is not considered significant. (a) What is the pressure inside the balloon if the helium is replaced with helium that is at  $-50.0^\circ\text{C}$  and the balloon is filled until its volume is twenty times the original volume? (b) What is the gauge pressure? (Assume the pressure in the lab remains at 1.00 atmosphere during the experiment.)

[Show Solution](#)

**Strategy:** This is a multi-step problem. First find the amount of helium (moles) in the initial state. Then use this with the new conditions to find the new pressure.

**Solution:** (a) Initial conditions:  $T_0 = 10.0^\circ\text{C} = 283 \text{ K}$ ,  $P_0 = 1.00 \text{ atm}$ ,  $V_0$

Final conditions:  $T_f = -50.0^\circ\text{C} = 223 \text{ K}$ ,  $V_f = 20V_0$ ,  $P_f = ?$

Using ideal gas law for both states:

$$\frac{P_0 V_0}{T_0} = \frac{P_f V_f}{T_f}$$

Solving for  $P_f$ :

$$P_f = P_0 \frac{V_0}{V_f} \frac{T_f}{T_0} = (1.00 \text{ atm}) \frac{20V_0}{223 \text{ K}} \frac{283 \text{ K}}{100 \text{ K}} = 0.0394 \text{ atm}$$

Converting to Pa:

$$P_f = 0.0394 \text{ atm} \times 1.01 \times 10^5 \text{ Pa/atm} = 3.98 \times 10^3 \text{ Pa}$$

(b) Gauge pressure:

$$P_{\text{gauge}} = P_f - P_0 = 3.98 \times 10^3 \text{ Pa} - 1.00 \times 10^5 \text{ Pa} = -6.02 \times 10^3 \text{ Pa}$$

**Discussion:** The negative gauge pressure indicates that the pressure inside the balloon is less than atmospheric pressure—the balloon would be partially deflated or collapsed. This makes sense: the temperature decreased (reducing pressure), and the volume increased dramatically (also reducing pressure). The combined effect overwhelms the initial pressure, resulting in a final pressure well below atmospheric.

Confirm that the units of  $nRT$  are those of energy for each value of  $R$ : (a)  $8.31 \text{ J/mol K}$ , (b)  $1.99 \text{ cal/mol K}$ , and (c)  $0.0821 \text{ L atm/mol K}$ .

Show Solution

$$\begin{aligned} (a) \ nRT &= \left(\text{mol}\right) \left(\text{J/mol K}\right) \left(\text{K}\right) = \text{J} \\ (b) \ nRT &= \left(\text{mol}\right) \left(\text{cal/mol K}\right) \left(\text{K}\right) = \text{cal} \\ (c) \ nRT &= \left(\text{mol}\right) \left(\text{L atm/mol K}\right) \left(\text{K}\right) = \text{L atm} \end{aligned}$$

In the text, it was shown that  $N/V = 2.68 \times 10^{25} \text{ m}^{-3}$  for gas at STP. (a) Show that this quantity is equivalent to  $N/V = 2.68 \times 10^{19} \text{ cm}^{-3}$ , as stated. (b) About how many atoms are there in one  $\mu\text{m}^3$  (a cubic micrometer) at STP? (c) What does your answer to part (b) imply about the separation of atoms and molecules?

Show Solution

**Strategy:** (a) Convert from  $\text{m}^{-3}$  to  $\text{cm}^{-3}$  using the fact that  $1 \text{ m} = 100 \text{ cm}$ . (b) Calculate the number in one  $\mu\text{m}^3$ . (c) Consider what this means for molecular spacing.

**Solution:** (a) Convert from  $\text{m}^{-3}$  to  $\text{cm}^{-3}$ :

$$\begin{aligned} N/V &= 2.68 \times 10^{25} \text{ m}^{-3} = 2.68 \times 10^{25} \text{ m}^{-3} \times \left(\frac{100 \text{ cm}}{1 \text{ m}}\right)^3 = 2.68 \times 10^{25} \text{ m}^{-3} \times 10^{18} \text{ cm}^{-3} = 2.68 \times 10^{19} \text{ cm}^{-3} \\ N/V &= 2.68 \times 10^{19} \text{ cm}^{-3} \end{aligned}$$

This confirms the stated value.

(b) For one cubic micrometer ( $\mu\text{m}^3$ ):

$$1 \mu\text{m} = 10^{-6} \text{ m} = 10^{-4} \text{ cm}$$

$$N = \left(2.68 \times 10^{19} \text{ cm}^{-3}\right) \left(10^{-4} \text{ cm}\right)^3 = 2.68 \times 10^{19} \times 10^{-12} = 2.68 \times 10^7 \text{ molecules}$$

About 27 million molecules in one cubic micrometer.

(c) This implies that even in a very small volume ( $1 \mu\text{m}^3$ ), there are still millions of molecules. Since molecules are separated by relatively large distances in a gas, we can estimate the average separation. If we have  $2.68 \times 10^7$  molecules in a cube of side  $1 \mu\text{m}$ , then each molecule occupies on average:

$$\text{Volume per molecule} = \frac{1 \mu\text{m}^3}{2.68 \times 10^7} = 3.7 \times 10^{-8} \mu\text{m}^3$$

Taking the cube root to find typical spacing:

$$d = \sqrt[3]{3.7 \times 10^{-8} \mu\text{m}^3} = 3.3 \times 10^{-3} \mu\text{m} = 3.3 \text{ nm}$$

**Discussion:** Molecules at STP are separated by about 3.3 nanometers on average, which is about 10 times the diameter of a typical molecule (around 0.3 nm). This explains why gases are compressible—there's lots of empty space between molecules. It also explains why gases have much lower densities than liquids or solids, where molecules are essentially touching. This large separation is why intermolecular forces are negligible in ideal gases.

Calculate the number of moles in the 2.00-L volume of air in the lungs of the average person. Note that the air is at  $37.0^\circ\text{C}$  (body temperature).

Show Solution

**Strategy:** Use the ideal gas law  $PV=nRT$  and solve for the number of moles  $n$ . Assume atmospheric pressure.

**Solution:** Given:

- $P = 1.01 \times 10^5 \text{ Pa}$  (atmospheric pressure)
- $V = 2.00 \text{ L} = 2.00 \times 10^{-3} \text{ m}^3$
- $T = 37.0^\circ\text{C} = 310 \text{ K}$
- $R = 8.31 \text{ J/mol}\cdot\text{K}$

Rearranging  $PV=nRT$  to solve for  $n$ :

$$\frac{PV}{RT} = \frac{(1.01 \times 10^5 \text{ Pa})(2.00 \times 10^{-3} \text{ m}^3)}{(8.31 \text{ J/mol}\cdot\text{K})(310 \text{ K})} = \frac{202 \text{ J}}{2576 \text{ J/mol}} = 7.84 \times 10^{-2} \text{ mol} \approx 7.86 \times 10^{-2} \text{ mol}$$

**Discussion:** This small amount (about 0.078 moles or roughly 1.9 grams of air) represents the volume in the lungs at any given time. For comparison, this corresponds to about  $4.7 \times 10^{22}$  molecules. During breathing, only a fraction of this air is exchanged with each breath (typically about 0.5 L per breath during normal breathing), ensuring continuous oxygen supply and carbon dioxide removal while maintaining relatively constant lung volume and pressure.

An airplane passenger has  $100 \text{ cm}^3$  of air in his stomach just before the plane takes off from a sea-level airport. What volume will the air have at cruising altitude if cabin pressure drops to  $7.50 \times 10^4 \text{ N/m}^2$ ?

[Show Solution](#)

**Strategy:** Assume temperature and amount of gas remain constant. Use  $P_1V_1=P_2V_2$  (Boyle's law).

**Solution:** Initial conditions:  $P_1 = 1.01 \times 10^5 \text{ Pa}$  (atmospheric),  $V_1 = 100 \text{ cm}^3$

Final conditions:  $P_2 = 7.50 \times 10^4 \text{ Pa}$ ,  $V_2 = ?$

Using  $P_1V_1=P_2V_2$ :

$$V_2 = V_1 \frac{P_1}{P_2} = (100 \text{ cm}^3) \frac{1.01 \times 10^5 \text{ Pa}}{7.50 \times 10^4 \text{ Pa}} = 135 \text{ cm}^3$$

**Discussion:** The air expands by 35% as cabin pressure decreases, which can cause discomfort. This is why passengers sometimes experience bloating or digestive discomfort during flights. The expansion occurs because the external pressure decreases while the gas inside maintains its tendency to expand. This same principle explains why it's important not to hold your breath when ascending while scuba diving—the air in your lungs would expand as you rise and pressure decreases, potentially causing serious injury.

(a) What is the volume (in  $\text{km}^3$ ) of Avogadro's number of sand grains if each grain is a cube and has sides that are 1.0 mm long? (b) How many kilometers of beaches in length would this cover if the beach averages 100 m in width and 10.0 m in depth? Neglect air spaces between grains.

[Show Solution](#)

(a)  $6.02 \times 10^{23} \text{ km}^3$  (b)  $6.02 \times 10^{23} \text{ km}$

An expensive vacuum system can achieve a pressure as low as  $1.00 \times 10^{-7} \text{ N/m}^2$  at  $20^\circ\text{C}$ . How many atoms are there in a cubic centimeter at this pressure and temperature?

[Show Solution](#)

**Strategy:** Use the ideal gas law  $PV=nk_B T$  to find the number of atoms  $N$  in a cubic centimeter at the given pressure and temperature.

**Solution:** Given:

- $P = 1.00 \times 10^{-7} \text{ Pa}$
- $T = 20^\circ\text{C} = 293 \text{ K}$
- $V = 1 \text{ cm}^3 = 1 \times 10^{-6} \text{ m}^3$
- $k_B = 1.38 \times 10^{-23} \text{ J/K}$

Using  $PV=nk_B T$ :

$$\frac{N}{V} = \frac{P}{k_B T} = \frac{1.00 \times 10^{-7} \text{ Pa}}{(1.38 \times 10^{-23} \text{ J/K})(293 \text{ K})} = 2.47 \times 10^{17} \text{ atoms/cm}^3$$

**Discussion:** Even in an “excellent” vacuum of  $10^{-7} \text{ Pa}$  (about  $10^{-12}$  atmospheres), there are still about 25 million atoms per cubic centimeter! This is a very good vacuum by laboratory standards, but it still contains enormous numbers of atoms compared to interstellar space. For comparison, at atmospheric pressure and room temperature, there are about  $2.7 \times 10^{19}$  molecules per  $\text{cm}^3$ —about a trillion times more. This shows how difficult it is to create a true vacuum, and why ultra-high vacuum systems (UHV) operating at  $10^{-10} \text{ Pa}$  or lower are needed for surface science experiments where even trace contamination is problematic.

The number density of gas atoms at a certain location in the space above our planet is about  $1.00 \times 10^{11} \text{ m}^{-3}$ , and the pressure is  $2.75 \times 10^{-10} \text{ N/m}^2$  in this space. What is the temperature there?

[Show Solution](#)

**Strategy:** Use the ideal gas law in the form  $PV=nk_B T$ . The number density is  $N/V$ , so we can write  $P=(N/V)k_B T$  and solve for temperature.

**Solution:** Given:

- Number density:  $N/V = 1.00 \times 10^{11} \text{ m}^{-3}$
- Pressure:  $P = 2.75 \times 10^{-10} \text{ Pa}$
- Boltzmann constant:  $k_B = 1.38 \times 10^{-23} \text{ J/K}$

Rearranging  $P = (N/V)k_B T$  to solve for  $T$ :

$$\begin{aligned} T &= \frac{P}{k_B} = \frac{2.75 \times 10^{-10} \text{ Pa}}{1.38 \times 10^{-23} \text{ J/K}} \\ T &= 199 \text{ K} \end{aligned}$$

Converting to Celsius:

$$T = 199 \text{ K} - 273.15 = -74.2^\circ\text{C} \approx -73.9^\circ\text{C}$$

**Discussion:** This very cold temperature (about  $-74^\circ\text{C}$  or 199 K) is typical of the upper atmosphere or near-space environment. At this altitude, the extremely low pressure ( $2.75 \times 10^{-10} \text{ Pa}$ , which is about  $10^{-15}$  atmospheres) and low particle density indicate we're in the thermosphere or exosphere. Despite the low temperature, individual gas atoms that are present actually move quite fast due to solar radiation heating, but there are so few of them that they cannot effectively transfer heat to objects, which is why satellites don't freeze despite the low kinetic temperature.

A bicycle tire has a pressure of  $7.00 \times 10^5 \text{ Pa}$  at a temperature of  $18.0^\circ\text{C}$  and contains  $2.00 \text{ L}$  of gas. What will its pressure be if you let out an amount of air that has a volume of  $100 \text{ cm}^3$  at atmospheric pressure? Assume tire temperature and volume remain constant.

[Show Solution](#)

**Strategy:** Find how many moles of gas are removed (the  $100 \text{ cm}^3$  at atmospheric pressure), then calculate the new pressure in the tire with fewer moles. Use  $PV=nRT$  for both calculations.

**Solution:** Initial conditions in tire:

- $P_i = 7.00 \times 10^5 \text{ Pa}$
- $V_i = 2.00 \text{ L} = 2.00 \times 10^{-3} \text{ m}^3$
- $T_i = 18.0^\circ\text{C} = 291 \text{ K}$

Initial moles in tire:

$$n_i = \frac{P_i V_i}{RT} = \frac{(7.00 \times 10^5 \text{ Pa})(2.00 \times 10^{-3} \text{ m}^3)}{(8.31 \text{ J/mol K})(291 \text{ K})} = 0.579 \text{ mol}$$

Moles of air removed (at atmospheric pressure  $P_{\text{atm}} = 1.01 \times 10^5 \text{ Pa}$ ):

$$V_{\text{removed}} = 100 \text{ cm}^3 = 1.00 \times 10^{-4} \text{ m}^3$$

$$n_{\text{removed}} = \frac{P_{\text{atm}} V_{\text{removed}}}{RT} = \frac{(1.01 \times 10^5 \text{ Pa})(1.00 \times 10^{-4} \text{ m}^3)}{(8.31 \text{ J/mol K})(291 \text{ K})} = 4.18 \times 10^{-3} \text{ mol}$$

Final moles in tire:

$$n_f = n_i - n_{\text{removed}} = 0.579 - 0.00418 = 0.575 \text{ mol}$$

Final pressure in tire (same volume and temperature):

$$P_f = \frac{n_f RT}{V} = \frac{(0.575 \text{ mol})(8.31 \text{ J/mol K})(291 \text{ K})}{(2.00 \times 10^{-3} \text{ m}^3)} = 6.95 \times 10^5 \text{ Pa}$$

**Discussion:** The pressure drops from  $7.00 \times 10^5 \text{ Pa}$  to  $6.95 \times 10^5 \text{ Pa}$ , a decrease of about  $5.0 \times 10^3 \text{ Pa}$  (or about 0.05 atm). This small change makes sense because we only removed about 0.7% of the gas ( $4.18 \times 10^{-3} \text{ mol}$  out of  $0.579 \text{ mol}$ ). This illustrates why bike tires don't lose much pressure from small leaks, but also why it's important to check tire pressure regularly—small amounts of gas loss can accumulate over time. The key insight is that  $100 \text{ cm}^3$  at atmospheric pressure contains much less gas than  $100 \text{ cm}^3$  at tire pressure, so removing it has a relatively small effect.

A high-pressure gas cylinder contains  $50.0 \text{ L}$  of toxic gas at a pressure of  $1.40 \times 10^7 \text{ Pa}$  and a temperature of  $25.0^\circ\text{C}$ . Its valve leaks after the cylinder is dropped. The cylinder is cooled to dry ice temperature ( $-78.5^\circ\text{C}$ ) to reduce the leak rate and pressure so that it can be safely repaired. (a) What is the final pressure in the tank, assuming a negligible amount of gas leaks while being cooled and that there is no phase change? (b) What is the final pressure if one-tenth of the gas escapes? (c) To what temperature must the tank be cooled to reduce the pressure to  $1.00 \text{ atm}$  (assuming the gas does not change phase and that there is no leakage during cooling)? (d) Does cooling the tank appear to be a practical solution?

[Show Solution](#)

(a)  $9.14 \times 10^6 \text{ Pa}$  (b)  $8.23 \times 10^6 \text{ Pa}$  (c)  $2.16 \text{ K}$

(d) No. The final temperature needed is much too low to be easily achieved for a large object.

Find the number of moles in  $2.00 \text{ L}$  of gas at  $35.0^\circ\text{C}$  and under  $7.41 \times 10^7 \text{ Pa}$  of pressure.

[Show Solution](#)

**Strategy:** Use the ideal gas law  $PV=nRT$  and solve for the number of moles  $n$ .

**Solution:** Given:

- $P = 7.41 \times 10^7 \text{ Pa}$
- $V = 2.00 \text{ L} = 2.00 \times 10^{-3} \text{ m}^3$
- $T = 35.0^\circ\text{C} = 308 \text{ K}$
- $R = 8.31 \text{ J/mol}\cdot\text{K}$

Rearranging  $PV=nRT$  to solve for  $n$ :

$$n = \frac{PV}{RT} = \frac{(7.41 \times 10^7 \text{ Pa})(2.00 \times 10^{-3} \text{ m}^3)}{(8.31 \text{ J/mol}\cdot\text{K})(308 \text{ K})} = 57.9 \text{ mol}$$

**Discussion:** This is a very large number of moles for such a small volume (2.00 L). At this extremely high pressure (about 730 atmospheres), the gas is compressed to about 730 times the density it would have at atmospheric pressure. This is why high-pressure gas cylinders must be extremely strong and carefully designed. At such high pressures, real gases start to deviate from ideal gas behavior because the molecules are forced close together and intermolecular forces become significant. However, the ideal gas law still provides a reasonable approximation for many practical purposes. This amount of gas (58 moles) would occupy about 1400 L at standard temperature and pressure!

Calculate the depth to which Avogadro's number of table tennis balls would cover Earth. Each ball has a diameter of 3.75 cm. Assume the space between balls adds an extra 25.0% to their volume and assume they are not crushed by their own weight.

[Show Solution](#)

**Strategy:** Calculate the volume of one table tennis ball, multiply by Avogadro's number and the packing factor (1.25), then divide by Earth's surface area to find the depth.

**Solution:** Given:

- Diameter:  $d = 3.75 \text{ cm} = 0.0375 \text{ m}$
- Radius:  $r = 0.01875 \text{ m}$
- Avogadro's number:  $N_A = 6.02 \times 10^{23}$
- Earth's radius:  $R_E \approx 6.37 \times 10^6 \text{ m}$
- Packing factor: 1.25 (to account for spaces)

Volume of one ball:

$$V_{\text{ball}} = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi (0.01875)^3 = 2.76 \times 10^{-5} \text{ m}^3$$

Total volume including spaces:

$$V_{\text{total}} = N_A V_{\text{ball}} \times 1.25 = (6.02 \times 10^{23}) (2.76 \times 10^{-5}) (1.25) = 2.08 \times 10^{19} \text{ m}^3$$

Earth's surface area:

$$A_E = 4\pi R_E^2 = 4\pi (6.37 \times 10^6)^2 = 5.10 \times 10^{14} \text{ m}^2$$

Depth:

$$h = \frac{V_{\text{total}}}{A_E} = \frac{2.08 \times 10^{19} \text{ m}^3}{5.10 \times 10^{14} \text{ m}^2} = 4.08 \times 10^4 \text{ m} \approx 41 \text{ km}$$

**Discussion:** Avogadro's number of table tennis balls would cover Earth to a depth of about 41 km! This is an enormous depth—higher than commercial aircraft fly (typically 10-12 km) and reaching into the stratosphere. This dramatic illustration helps convey just how large Avogadro's number is. The fact that a mole of molecules is such a huge number explains why macroscopic amounts of substances contain astronomical numbers of atoms and molecules, and why we can observe statistical properties like pressure and temperature despite the random motion of individual particles.

(a) What is the gauge pressure in a  $25.0^\circ\text{C}$  car tire containing 3.60 mol of gas in a 30.0 L volume? (b) What will its gauge pressure be if you add 1.00 L of gas originally at atmospheric pressure and  $25.0^\circ\text{C}$ ? Assume the temperature returns to  $25.0^\circ\text{C}$  and the volume remains constant.

[Show Solution](#)

**Strategy:** (a) Use  $PV=nRT$  to find absolute pressure, then subtract atmospheric pressure to get gauge pressure. (b) Find how many moles are added, calculate new absolute pressure, then find new gauge pressure.

**Solution:** (a) Initial conditions:

- $n = 3.60 \text{ mol}$
- $V = 30.0 \text{ L} = 3.00 \times 10^{-2} \text{ m}^3$
- $T = 25.0^\circ\text{C} = 298 \text{ K}$
- $R = 8.31 \text{ J/mol}\cdot\text{K}$

Absolute pressure:

$$\text{P} = \frac{nRT}{V} = \frac{\left(3.60 \text{ mol}\right) \left(8.31 \text{ J/mol}\right) \left(298 \text{ K}\right)}{\left(3.00 \times 10^{-2} \text{ m}^3\right)} = 2.97 \times 10^5 \text{ Pa}$$

Gauge pressure:

$$P_{\text{gauge}} = P_{\text{atm}} - P_{\text{gas}} = 1.96 \times 10^5 \text{ Pa} - 1.94 \times 10^5 \text{ Pa} = 1.97 \text{ Pa}$$

(b) Moles of gas added (1.00 L at atmospheric pressure and 25.0°C):

$$n_{\text{added}} = \frac{P_{\text{atm}} V}{RT} = \frac{\left(1.01 \times 10^5 \text{ Pa}\right) \left(1.00 \times 10^{-3} \text{ m}^3\right)}{\left(8.31 \text{ J/mol}\right) \left(298 \text{ K}\right)} = 0.0408 \text{ mol}$$

Total moles:

$$n_{\text{total}} = 3.60 + 0.0408 = 3.64 \text{ mol}$$

New absolute pressure:

$$P_{\text{new}} = \frac{n_{\text{total}} RT}{V} = \frac{\left(3.64 \text{ mol}\right) \left(8.31 \text{ J/mol}\right) \left(298 \text{ K}\right)}{\left(3.00 \times 10^{-2} \text{ m}^3\right)} = 3.00 \times 10^5 \text{ Pa}$$

New gauge pressure:

$$P_{\text{gauge,new}} = P_{\text{atm}} - P_{\text{gas}} = 1.99 \times 10^5 \text{ Pa} - 1.97 \text{ Pa} = 1.97 \text{ Pa}$$

**Discussion:** Adding 1.00 L of gas at atmospheric pressure adds only 0.0408 moles, which is a small fraction (about 1.1%) of the original 3.60 moles. Consequently, the gauge pressure increases only slightly from 1.94 atm to 1.97 atm. This illustrates an important point: when adding gas to a tire, the volume of gas measured at atmospheric pressure is much less significant than the same volume measured at tire pressure. To significantly increase tire pressure, you need to add many liters of atmospheric-pressure gas, which is why it takes time to pump up a tire with a hand pump.

- (a) In the deep space between galaxies, the density of atoms is as low as  $10^6 \text{ atoms/m}^3$ , and the temperature is a frigid 2.7 K. What is the pressure? (b) What volume (in  $\text{m}^3$ ) is occupied by 1 mol of gas? (c) If this volume is a cube, what is the length of its sides in kilometers?

[Show Solution](#)

- (a)  $3.7 \times 10^{-17} \text{ Pa}$  (b)  $6.0 \times 10^{17} \text{ m}^3$  (c)  $8.4 \times 10^2 \text{ km}$

## Glossary

ideal gas law

the physical law that relates the pressure and volume of a gas to the number of gas molecules or number of moles of gas and the temperature of the gas

Boltzmann constant

$k_B$ , a physical constant that relates energy to temperature;  $k_B = 1.38 \times 10^{-23} \text{ J/K}$

Avogadro's number

$N_A$ , the number of molecules or atoms in one mole of a substance;  $N_A = 6.02 \times 10^{23} \text{ particles/mole}$

mole

the quantity of a substance whose mass (in grams) is equal to its molecular mass



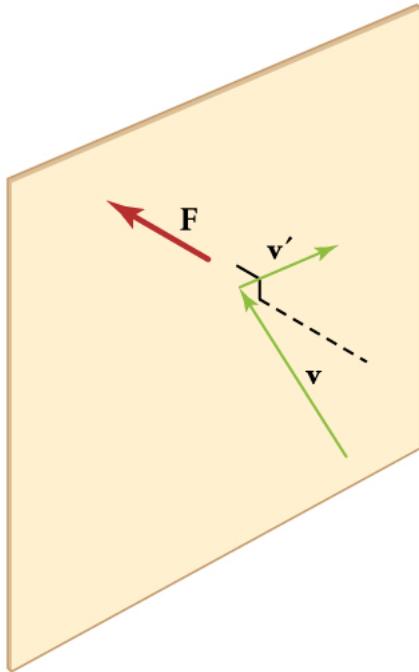
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# Kinetic Theory: Atomic and Molecular Explanation of Pressure and Temperature

- Express the ideal gas law in terms of molecular mass and velocity.
- Define thermal energy.
- Calculate the kinetic energy of a gas molecule, given its temperature.
- Describe the relationship between the temperature of a gas and the kinetic energy of atoms and molecules.
- Describe the distribution of speeds of molecules in a gas.

We have developed macroscopic definitions of pressure and temperature. Pressure is the force divided by the area on which the force is exerted, and temperature is measured with a thermometer. We gain a better understanding of pressure and temperature from the kinetic theory of gases, which assumes that atoms and molecules are in continuous random motion.



When a molecule collides with a rigid wall, the component of its momentum perpendicular to the wall is reversed. A force is thus exerted on the wall, creating pressure.

[Figure 1] shows an elastic collision of a gas molecule with the wall of a container, so that it exerts a force on the wall (by Newton's third law). Because a huge number of molecules will collide with the wall in a short time, we observe an average force per unit area. These collisions are the source of pressure in a gas. As the number of molecules increases, the number of collisions and thus the pressure increase. Similarly, the gas pressure is higher if the average velocity of molecules is higher. The actual relationship is derived in the [Things Great and Small](#) feature below. The following relationship is found:

$$PV=13Nm-v^2,$$

where  $P$  is the pressure (average force per unit area),  $V$  is the volume of gas in the container,  $N$  is the number of molecules in the container,  $m$  is the mass of a molecule, and  $-v^2$  is the average of the molecular speed squared.

What can we learn from this atomic and molecular version of the ideal gas law? We can derive a relationship between temperature and the average translational kinetic energy of molecules in a gas. Recall the previous expression of the ideal gas law:

$$PV=NkBT.$$

Equating the right-hand side of this equation with the right-hand side of  $PV=13Nm-v^2$  gives

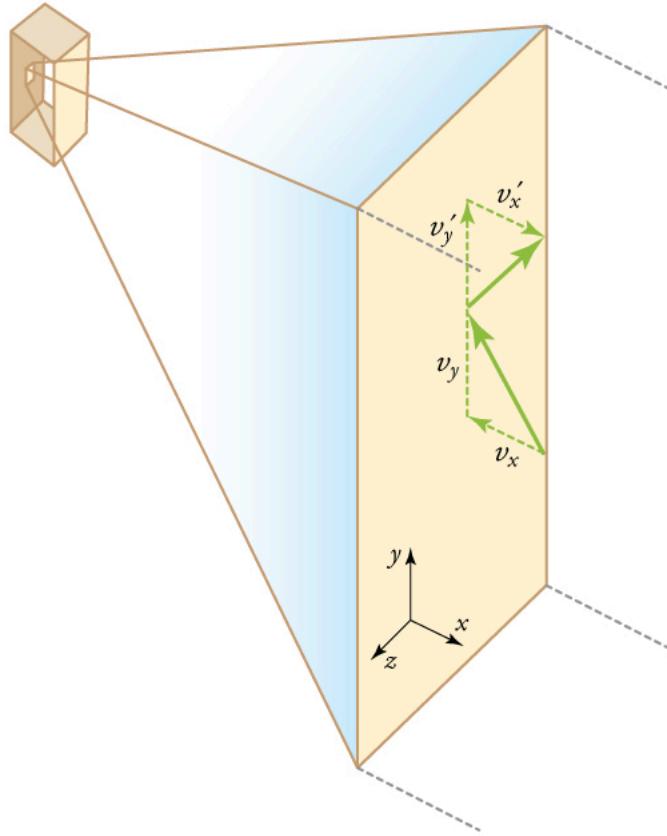
$$13Nm-v^2=NkBT.$$

## Making Connections: Things Great and Small—Atomic and Molecular Origin of Pressure in a Gas

[Figure 2] shows a box filled with a gas. We know from our previous discussions that putting more gas into the box produces greater pressure, and that increasing the temperature of the gas also produces a greater pressure. But why should increasing the temperature of the gas increase the pressure in the box? A look at the atomic and molecular scale gives us some answers, and an alternative expression for the ideal gas law.

The figure shows an expanded view of an elastic collision of a gas molecule with the wall of a container. Calculating the average force exerted by such molecules will lead us to the ideal gas law, and to the connection between temperature and molecular kinetic energy. We assume that a molecule is small

compared with the separation of molecules in the gas, and that its interaction with other molecules can be ignored. We also assume the wall is rigid and that the molecule's direction changes, but that its speed remains constant (and hence its kinetic energy and the magnitude of its momentum remain constant as well). This assumption is not always valid, but the same result is obtained with a more detailed description of the molecule's exchange of energy and momentum with the wall.



Gas in a box exerts an outward pressure on its walls. A molecule colliding with a rigid wall has the direction of its velocity and momentum in the  $x$ -direction reversed. This direction is perpendicular to the wall. The components of its velocity momentum in the  $y$ - and  $z$ -directions are not changed, which means there is no force parallel to the wall.

If the molecule's velocity changes in the  $x$ -direction, its momentum changes from  $-mv_x$  to  $+mv_x$ . Thus, its change in momentum is  $\Delta mv = +mv_x - (-mv_x) = 2mv_x$ . The force exerted on the molecule is given by

$$F = \Delta p \Delta t = 2mv_x \Delta t.$$

There is no force between the wall and the molecule until the molecule hits the wall. During the short time of the collision, the force between the molecule and wall is relatively large. We are looking for an average force; we take  $\Delta t$  to be the average time between collisions of the molecule with this wall. It is the time it would take the molecule to go across the box and back (a distance  $2l$ ) at a speed of  $v_x$ . Thus  $\Delta t = 2l/v_x$ , and the expression for the force becomes

$$F = 2mv_x 2l/v_x = mv_x 2l.$$

This force is due to *one* molecule. We multiply by the number of molecules  $N$  and use their average squared velocity to find the force

$$F = Nm - v_{2x}l,$$

where the bar over a quantity means its average value. We would like to have the force in terms of the speed  $v$ , rather than the  $x$ -component of the velocity. We note that the total velocity squared is the sum of the squares of its components, so that

$$\bar{v}^2 = \bar{v}_{2x}^2 + \bar{v}_{2y}^2 + \bar{v}_{2z}^2.$$

Because the velocities are random, their average components in all directions are the same:

$$\bar{v}_{2x} = \bar{v}_{2y} = \bar{v}_{2z}.$$

Thus,

$$-\nu^2 = 3\nu_{2x},$$

or

$$-\nu_{2x} = 13\nu^2.$$

Substituting  $13\nu^2$  into the expression for  $F$  gives

$$F = Nm\nu^2 3l.$$

The pressure is  $F/A$ , so that we obtain

$$P = FA = Nm\nu^2 3Al = 13Nm\nu^2 V,$$

where we used  $V = Al$  for the volume. This gives the important result.

$$PV = 13Nm\nu^2$$

This equation is another expression of the ideal gas law.

We can get the average kinetic energy of a molecule,  $12m\nu^2$ , from the right-hand side of the equation by canceling  $N$  and multiplying by  $3/2$ . This calculation produces the result that the average kinetic energy of a molecule is directly related to absolute temperature.

$$-\text{KE} = 12m\nu^2 = 32k_B T$$

The average translational kinetic energy of a molecule,  $-\text{KE}$

, is called **thermal energy**. The equation  $-\text{KE} = 12m\nu^2 = 32k_B T$  is a molecular interpretation of temperature, and it has been found to be valid for gases and reasonably accurate in liquids and solids. It is another definition of temperature based on an expression of the molecular energy.

It is sometimes useful to rearrange  $-\text{KE} = 12m\nu^2 = 32k_B T$ , and solve for the average speed of molecules in a gas in terms of temperature,

$$\sqrt{-\nu^2} = v_{\text{rms}} = \sqrt{3k_B T m},$$

where  $v_{\text{rms}}$  stands for root-mean-square (rms) speed.

#### Calculating Kinetic Energy and Speed of a Gas Molecule

(a) What is the average kinetic energy of a gas molecule at  $20.0^\circ\text{C}$  (room temperature)? (b) Find the rms speed of a nitrogen molecule ( $\text{N}_2$ ) at this temperature.

##### Strategy for (a)

The known in the equation for the average kinetic energy is the temperature.

$$-\text{KE} = 12m\nu^2 = 32k_B T$$

Before substituting values into this equation, we must convert the given temperature to kelvins. This conversion gives  $T = (20.0 + 273)\text{K} = 293\text{K}$ .

##### Solution for (a)

The temperature alone is sufficient to find the average translational kinetic energy. Substituting the temperature into the translational kinetic energy equation gives

$$-\text{KE} = 32k_B T = 32(1.38 \times 10^{-23}\text{J/K})(293\text{K}) = 6.07 \times 10^{-21}\text{J}.$$

##### Strategy for (b)

Finding the rms speed of a nitrogen molecule involves a straightforward calculation using the equation

$$\sqrt{-\nu^2} = v_{\text{rms}} = \sqrt{3k_B T m},$$

but we must first find the mass of a nitrogen molecule. Using the molecular mass of nitrogen  $\text{N}_2$  from the periodic table,

$$m = 2(14.0067) \times 10^{-3}\text{kg/mol} 6.02 \times 10^{23}\text{mol}^{-1} = 4.65 \times 10^{-26}\text{kg}.$$

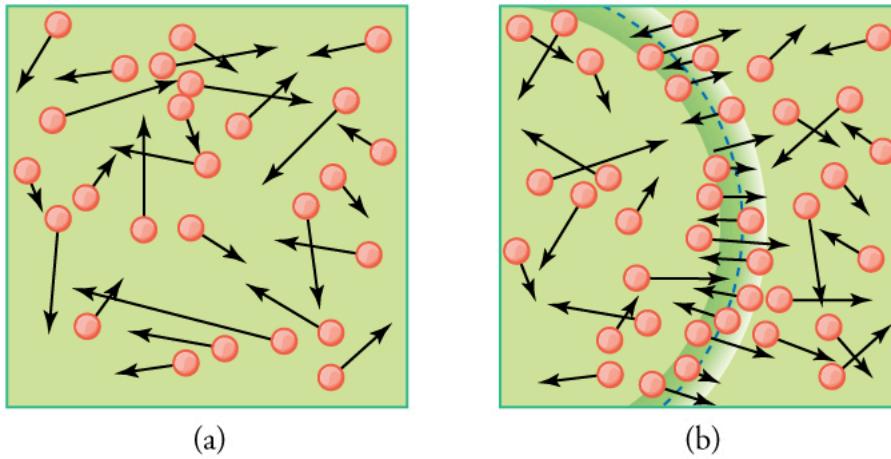
##### Solution for (b)

Substituting this mass and the value for  $k$  into the equation for  $v_{\text{rms}}$  yields

$$v_{\text{rms}} = \sqrt{3k_B T m} = \sqrt{3(1.38 \times 10^{-23} \text{ J/K})(293 \text{ K})} \cdot 4.65 \times 10^{-26} \text{ kg} = 511 \text{ m/s.}$$

### Discussion

Note that the average kinetic energy of the molecule is independent of the type of molecule. The average translational kinetic energy depends only on absolute temperature. The kinetic energy is very small compared to macroscopic energies, so that we do not feel when an air molecule is hitting our skin. The rms velocity of the nitrogen molecule is surprisingly large. These large molecular velocities do not yield macroscopic movement of air, since the molecules move in all directions with equal likelihood. The *mean free path* (the distance a molecule can move on average between collisions) of molecules in air is very small, and so the molecules move rapidly but do not get very far in a second. The high value for rms speed is reflected in the speed of sound, however, which is about 340 m/s at room temperature. The faster the rms speed of air molecules, the faster that sound vibrations can be transferred through the air. The speed of sound increases with temperature and is greater in gases with small molecular masses, such as helium. (See [\[Figure 3\]](#).)



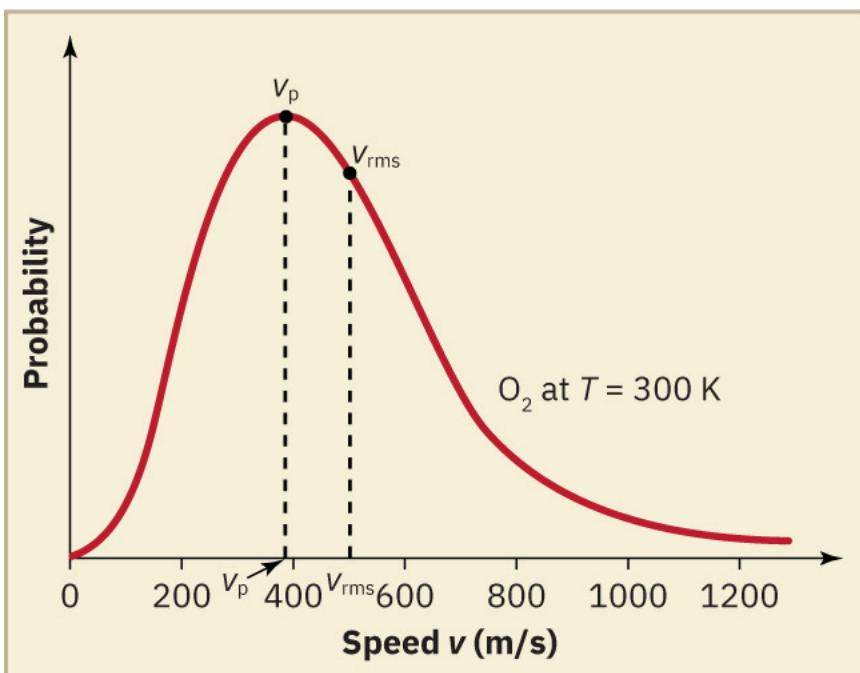
(a) There are many molecules moving so fast in an ordinary gas that they collide a billion times every second. (b) Individual molecules do not move very far in a small amount of time, but disturbances like sound waves are transmitted at speeds related to the molecular speeds.

### Making Connections: Historical Note—Kinetic Theory of Gases

The kinetic theory of gases was developed by Daniel Bernoulli (1700–1782), who is best known in physics for his work on fluid flow (hydrodynamics). Bernoulli's work predates the atomistic view of matter established by Dalton.

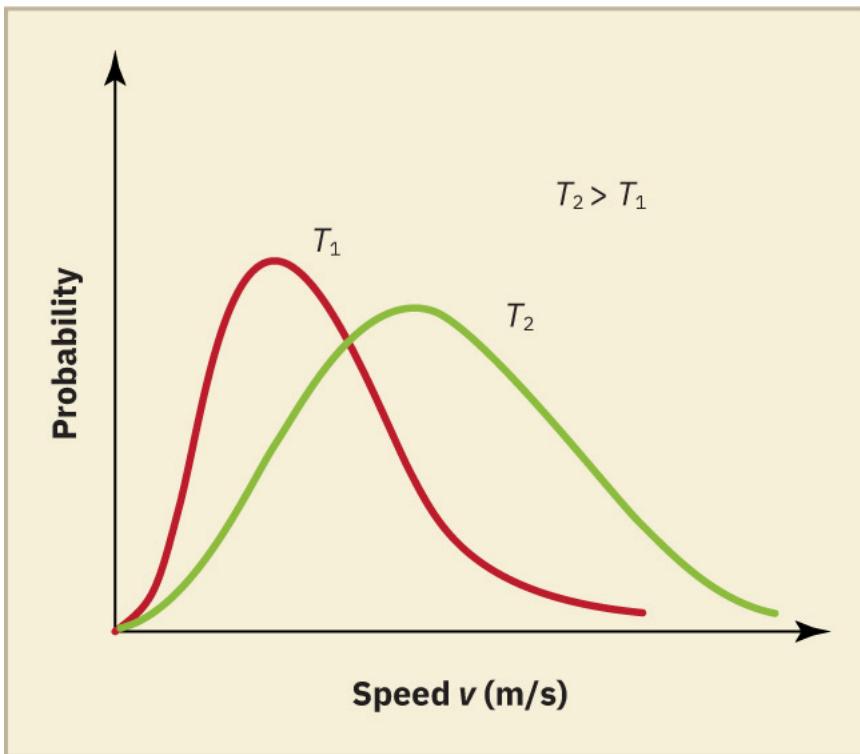
### Distribution of Molecular Speeds

The motion of molecules in a gas is random in magnitude and direction for individual molecules, but a gas of many molecules has a predictable distribution of molecular speeds. This distribution is called the *Maxwell-Boltzmann distribution*, after its originators, who calculated it based on kinetic theory, and has since been confirmed experimentally. (See [\[Figure 4\]](#).) The distribution has a long tail, because a few molecules may go several times the rms speed. The most probable speed  $v_p$  is less than the rms speed  $v_{\text{rms}}$ . [\[Figure 5\]](#) shows that the curve is shifted to higher speeds at higher temperatures, with a broader range of speeds.



The Maxwell-Boltzmann distribution of molecular speeds in an ideal gas. The most likely speed ( $v_p$ ) is less than the rms speed ( $v_{\text{rms}}$ ). Although very high speeds are possible, only a tiny fraction of the molecules have speeds that are an order of magnitude greater than ( $v_{\text{rms}}$ ).

The distribution of thermal speeds depends strongly on temperature. As temperature increases, the speeds are shifted to higher values and the distribution is broadened.



The Maxwell-Boltzmann distribution is shifted to higher speeds and is broadened at higher temperatures.

What is the implication of the change in distribution with temperature shown in [Figure 5] for humans? All other things being equal, if a person has a fever, they are likely to lose more water molecules, particularly from linings along moist cavities such as the lungs and mouth, creating a dry sensation in the mouth.

Calculating Temperature: Escape Velocity of Helium Atoms

In order to escape Earth's gravity, an object near the top of the atmosphere (at an altitude of 100 km) must travel away from Earth at 11.1 km/s. This speed is called the *escape velocity*. At what temperature would helium atoms have an rms speed equal to the escape velocity?

### Strategy

Identify the knowns and unknowns and determine which equations to use to solve the problem.

### Solution

1. Identify the knowns:  $V$  is the escape velocity, 11.1 km/s.

2. Identify the unknowns: We need to solve for temperature,  $T$ . We also need to solve for the mass  $m$  of the helium atom.

3. Determine which equations are needed.

- To solve for mass  $m$  of the helium atom, we can use information from the periodic table:

$m = \text{molar mass number of atoms per mole}$ .

- To solve for temperature  $T$ , we can rearrange either

$$-KE = 12m - v^2 = 32k_B T$$

or

$$\sqrt{-v^2} = v_{\text{rms}} = \sqrt{3k_B T m}$$

to yield

$$T = m - v^2 / 3k,$$

where  $k_B$  is the Boltzmann constant and  $m$  is the mass of a helium atom.

1. Plug the known values into the equations and solve for the unknowns.

$$m = \text{molar mass number of atoms per mole} = 4.0026 \times 10^{-3} \text{ kg/mol} \cdot 6.02 \times 10^{23} \text{ mol} = 6.65 \times 10^{-27} \text{ kg}$$

$$T = (6.65 \times 10^{-27} \text{ kg}) (11.1 \times 10^3 \text{ m/s})^2 / 3(1.38 \times 10^{-23} \text{ J/K}) = 1.98 \times 10^4 \text{ K}$$

### Discussion

This temperature is much higher than atmospheric temperature, which is approximately 250 K ( $-25^\circ\text{C}$  or  $-10^\circ\text{F}$ ) at high altitude. Very few helium atoms are left in the atmosphere, but there were many when the atmosphere was formed. The reason for the loss of helium atoms is that there are a small number of helium atoms with speeds higher than Earth's escape velocity even at normal temperatures. The speed of a helium atom changes from one instant to the next, so that at any instant, there is a small, but nonzero chance that the speed is greater than the escape speed and the molecule escapes from Earth's gravitational pull. Heavier molecules, such as oxygen, nitrogen, and water (very little of which reach a very high altitude), have smaller rms speeds, and so it is much less likely that any of them will have speeds greater than the escape velocity. In fact, so few have speeds above the escape velocity that billions of years are required to lose significant amounts of the atmosphere. [\[Figure 6\]](#) shows the impact of a lack of an atmosphere on the Moon. Because the gravitational pull of the Moon is much weaker, it has lost almost its entire atmosphere. The comparison between Earth and the Moon is discussed in this chapter's Problems and Exercises.



This photograph of Apollo 17 Commander Eugene Cernan driving the lunar rover on the Moon in 1972 looks as though it was taken at night with a large spotlight. In fact, the light is coming from the Sun. Because the acceleration due to gravity on the Moon is so low (about 1/6 that of Earth), the Moon's escape velocity is much smaller. As a result, gas molecules escape very easily from the Moon, leaving it with virtually no atmosphere. Even during the daytime, the sky is black because there is no gas to scatter sunlight. (credit: Harrison H. Schmitt/NASA)

#### Check Your Understanding

If you consider a very small object such as a grain of pollen, in a gas, then the number of atoms and molecules striking its surface would also be relatively small. Would the grain of pollen experience any fluctuations in pressure due to statistical fluctuations in the number of gas atoms and molecules striking it in a given amount of time?

[Show Solution](#)

Yes. Such fluctuations actually occur for a body of any size in a gas, but since the numbers of atoms and molecules are immense for macroscopic bodies, the fluctuations are a tiny percentage of the number of collisions, and the averages spoken of in this section vary imperceptibly. Roughly speaking the fluctuations are proportional to the inverse square root of the number of collisions, so for small bodies they can become significant. This was actually observed in the 19th century for pollen grains in water, and is known as the Brownian effect.

#### PhET Explorations: Gas Properties

Pump gas molecules into a box and see what happens as you change the volume, add or remove heat, change gravity, and more. Measure the temperature and pressure, and discover how the properties of the gas vary in relation to each other.



## Section Summary

- Kinetic theory is the atomistic description of gases as well as liquids and solids.
- Kinetic theory models the properties of matter in terms of continuous random motion of atoms and molecules.
- The ideal gas law can also be expressed as

$$PV = 13Nm - v^2,$$

where  $P$  is the pressure (average force per unit area),  $V$  is the volume of gas in the container,  $N$  is the number of molecules in the container,  $m$  is the mass of a molecule, and  $-v^2$  is the average of the molecular speed squared.

- Thermal energy is defined to be the average translational kinetic energy —KE of an atom or molecule.
- The temperature of gases is proportional to the average translational kinetic energy of atoms and molecules.

$$-KE = 12m - v^2 = 32k_B T$$

or

$$\sqrt{-v^2} = v_{\text{rms}} = \sqrt{3k_B T m}.$$

- The motion of individual molecules in a gas is random in magnitude and direction. However, a gas of many molecules has a predictable distribution of molecular speeds, known as the *Maxwell-Boltzmann distribution*.

## Conceptual Questions

How is momentum related to the pressure exerted by a gas? Explain on the atomic and molecular level, considering the behavior of atoms and molecules.

Show Solution

**Strategy:** Consider what happens when gas molecules collide with the walls of a container and how Newton's laws relate force to momentum change.

**Solution:** Pressure is directly related to the rate of momentum transfer from gas molecules to the container walls. When a molecule collides elastically with a wall, its momentum component perpendicular to the wall reverses direction. According to Newton's second law,  $F = \Delta p / \Delta t$ , this change in momentum creates a force on the wall.

For a single molecule moving perpendicular to a wall with velocity  $v_x$ , the momentum change is  $\Delta p = 2mv_x$ . The force exerted depends on how frequently collisions occur, which is related to the molecule's speed and the container dimensions. When we consider billions of molecules constantly colliding with the walls, the average force per unit area (pressure) emerges from the collective momentum transfers.

The relationship is quantified in the kinetic theory expression:  $PV = 13Nm - v^2$ . The pressure is proportional to both the number density of molecules ( $N/V$ ) and their average squared velocity ( $-v^2$ ), which relates to their momentum squared ( $p^2 = m^2v^2$ ).

**Discussion:** This molecular explanation reveals that gas pressure is fundamentally a statistical result of countless molecular collisions. Higher molecular speeds (greater momentum) or more molecules (more collisions) both increase pressure. This connects macroscopic pressure to microscopic molecular motion, demonstrating the power of kinetic theory to explain observable properties in terms of atomic behavior.

## Problems & Exercises

Some incandescent light bulbs are filled with argon gas. What is  $v_{\text{rms}}$  for argon atoms near the filament, assuming their temperature is 2500 K?

**Show Solution**

**Strategy:** Use the equation  $v_{\text{rms}} = \sqrt{3k_B T m}$  with the mass of an argon atom.

**Solution:** First, find the mass of an argon atom using its molar mass from the periodic table:

$$m = 39.948 \times 10^{-3} \text{ kg/mol} \times 6.02 \times 10^{23} \text{ atoms/mol} = 6.64 \times 10^{-26} \text{ kg}$$

Now calculate the rms speed:

$$v_{\text{rms}} = \sqrt{3k_B T m} = \sqrt{3(1.38 \times 10^{-23} \text{ J/K})(2500 \text{ K})} \times 6.64 \times 10^{-26} \text{ kg} = 1.25 \times 10^3 \text{ m/s}$$

**Discussion:** Argon atoms near the filament move at about 1250 m/s (about 2800 mph), which is over 3.5 times the speed of sound in air. This high speed is due to the extremely high temperature near the filament. Argon is used in light bulbs because it's inert (won't react with the hot tungsten filament) and because its atoms, moving rapidly, help convect heat away from the filament while preventing oxidation. The argon also reduces evaporation of tungsten from the filament, extending bulb life.

Average atomic and molecular speeds ( $v_{\text{rms}}$ ) are large, even at low temperatures. What is  $v_{\text{rms}}$  for helium atoms at 5.00 K, just one degree above helium's liquefaction temperature?

**Show Solution**

**Strategy:** Use the equation  $v_{\text{rms}} = \sqrt{3k_B T m}$  with the temperature and the mass of a helium atom.

**Solution:** First, find the mass of a helium atom using its molar mass:

$$m = 4.00 \times 10^{-3} \text{ kg/mol} \times 6.02 \times 10^{23} \text{ atoms/mol} = 6.64 \times 10^{-27} \text{ kg}$$

Now calculate the rms speed:

$$v_{\text{rms}} = \sqrt{3k_B T m} = \sqrt{3(1.38 \times 10^{-23} \text{ J/K})(5.00 \text{ K})} \times 6.64 \times 10^{-27} \text{ kg} = 177 \text{ m/s}$$

**Discussion:** Even at this extremely low temperature (just 5 K above absolute zero), helium atoms are moving at 177 m/s, which is about 640 km/h or 400 mph. This demonstrates that molecular motion remains significant even at temperatures close to absolute zero. This high speed explains why helium is difficult to liquefy and why it has the lowest boiling point of any element (4.2 K at atmospheric pressure).

(a) What is the average kinetic energy in joules of hydrogen atoms on the 5500°C surface of the Sun? (b) What is the average kinetic energy of helium atoms in a region of the solar corona where the temperature is  $6.00 \times 10^5 \text{ K}$ ?

**Show Solution**

**Strategy:** Use the relationship  $KE = \frac{1}{2}mv^2 = \frac{1}{2}k_B T$  for both parts. Note that kinetic energy depends only on temperature, not on the type of atom.

**Solution:** (a) Convert temperature to Kelvin:

$$T = 5500^\circ\text{C} + 273.15 = 5773 \text{ K}$$

Calculate average kinetic energy:

$$KE = \frac{1}{2}k_B T = \frac{1}{2} \times 32(1.38 \times 10^{-23} \text{ J/K})(5773 \text{ K}) = 1.20 \times 10^{-19} \text{ J}$$

(b) For the solar corona:

$$KE = \frac{1}{2}k_B T = \frac{1}{2} \times 32(1.38 \times 10^{-23} \text{ J/K})(6.00 \times 10^5 \text{ K}) = 1.24 \times 10^{-17} \text{ J}$$

**Discussion:** The average kinetic energy of atoms in the solar corona is about 100 times greater than on the Sun's surface, reflecting the dramatically higher temperature. The solar corona, visible during solar eclipses as the Sun's outer atmosphere, has temperatures exceeding 1 million kelvin—paradoxically much hotter than the Sun's visible surface at about 5800 K. This “coronal heating problem” is an active area of research in solar physics. Despite the very high temperatures, the corona doesn't feel hot because its density is extremely low—there are very few particles to transfer energy. Note that kinetic energy depends only on temperature, not on whether the particle is hydrogen or helium.

The escape velocity of any object from Earth is 11.2 km/s. (a) Express this speed in m/s and km/h. (b) At what temperature would oxygen molecules (molecular mass is equal to 32.0 g/mol) have an average velocity  $v_{\text{rms}}$  equal to Earth's escape velocity of 11.1 km/s?

**Show Solution**

**Strategy:** (a) Use unit conversion. (b) Rearrange the rms speed equation to solve for temperature.

**Solution:** (a) Converting to m/s:

$$v=11.2 \text{ km/s} \times 1000 \text{ m} = 11200 \text{ m/s} = 1.12 \times 10^4 \text{ m/s}$$

Converting to km/h:

$$v=11.2 \text{ km/s} \times 3600 \text{ s} = 4.03 \times 10^4 \text{ km/h}$$

(b) First find the mass of an oxygen molecule:

$$m=32.0 \times 10^{-3} \text{ kg/mol} \times 6.02 \times 10^{23} \text{ molecules/mol} = 5.32 \times 10^{-26} \text{ kg}$$

Rearranging  $v_{\text{rms}} = \sqrt{3k_B T m}$  to solve for  $T$ :

$$T = mv_{\text{rms}}^2 / (3k_B m) = (5.32 \times 10^{-26} \text{ kg}) (1.11 \times 10^4 \text{ m/s})^2 / (3 \times 1.38 \times 10^{-23} \text{ J/K}) = 1.58 \times 10^5 \text{ K}$$

**Discussion:** This extremely high temperature (158,000 K) is much higher than any naturally occurring temperature in Earth's atmosphere. This explains why Earth retains its oxygen atmosphere—at normal atmospheric temperatures, very few oxygen molecules have speeds approaching escape velocity. Lighter molecules like hydrogen and helium, however, can reach escape velocity at lower temperatures, which is why Earth has lost most of its primordial hydrogen and helium.

The escape velocity from the Moon is much smaller than from Earth and is only 2.38 km/s. At what temperature would hydrogen molecules (molecular mass is equal to 2.016 g/mol) have an average velocity  $v_{\text{rms}}$  equal to the Moon's escape velocity?

[Show Solution](#)

**Strategy:** Use the rms speed equation  $v_{\text{rms}} = \sqrt{3k_B T m}$  and solve for temperature.

**Solution:** First find the mass of a hydrogen molecule:

$$m=2.016 \times 10^{-3} \text{ kg/mol} \times 6.02 \times 10^{23} \text{ molecules/mol} = 3.35 \times 10^{-27} \text{ kg}$$

Rearranging  $v_{\text{rms}} = \sqrt{3k_B T m}$  to solve for  $T$ :

$$T = mv_{\text{rms}}^2 / (3k_B m) = (3.35 \times 10^{-27} \text{ kg}) (2.38 \times 10^3 \text{ m/s})^2 / (3 \times 1.38 \times 10^{-23} \text{ J/K}) = 458 \text{ K}$$

**Discussion:** At only 458 K (185°C), hydrogen molecules would have sufficient rms speed to escape the Moon's gravity. This temperature is much lower than Earth's escape temperature for hydrogen, explaining why the Moon has essentially no atmosphere—it lost its atmosphere long ago. At the Moon's daytime surface temperatures (which can reach 400 K), a significant fraction of light gases like hydrogen and helium have speeds exceeding escape velocity. The Moon's weak gravity (about 1/6 that of Earth) and lack of a magnetic field to trap charged particles means that any atmosphere it once had has been lost to space over billions of years.

Nuclear fusion, the energy source of the Sun, hydrogen bombs, and fusion reactors, occurs much more readily when the average kinetic energy of the atoms is high—that is, at high temperatures. Suppose you want the atoms in your fusion experiment to have average kinetic energies of  $6.40 \times 10^{-14} \text{ J}$ . What temperature is needed?

[Show Solution](#)

**Strategy:** Use the relationship between average kinetic energy and temperature:  $KE = \frac{1}{2}mv^2 = \frac{3}{2}k_B T$ . Solve for temperature.

**Solution:** Rearranging the equation:

$$T = 2KE / (3k_B m) = 2(6.40 \times 10^{-14} \text{ J}) / (3 \times 1.38 \times 10^{-23} \text{ J/K}) = 3.09 \times 10^9 \text{ K}$$

**Discussion:** This incredibly high temperature of about 3 billion kelvin is typical for nuclear fusion reactions. At such temperatures, atoms are fully ionized (stripped of electrons) forming a plasma, and nuclei have enough kinetic energy to overcome the strong electrostatic repulsion between positively charged nuclei and get close enough for the strong nuclear force to bind them together. The core of the Sun has a temperature of about 15 million kelvin, while fusion reactors on Earth require even higher temperatures (100-150 million kelvin) because they operate at lower densities than the Sun's core. This explains why controlled nuclear fusion remains one of the great technological challenges—maintaining such extreme temperatures is extraordinarily difficult.

Suppose that the average velocity ( $v_{\text{rms}}$ ) of carbon dioxide molecules (molecular mass is equal to 44.0 g/mol) in a flame is found to be  $1.05 \times 10^5 \text{ m/s}$ . What temperature does this represent?

[Show Solution](#)

**Strategy:** Use the rms speed equation and solve for temperature.

**Solution:** First find the mass of a  $\text{CO}_2$  molecule:

$$m = 44.0 \times 10^{-3} \text{ kg/mol} \times 6.02 \times 10^{23} \text{ molecules/mol} = 7.31 \times 10^{-26} \text{ kg}$$

Rearranging  $v_{\text{rms}} = \sqrt{3k_B T m}$  to solve for  $T$ :

$$T = mv_{\text{rms}}^2 / (3k_B) = (7.31 \times 10^{-26} \text{ kg}) (1.05 \times 10^5 \text{ m/s})^2 / (3 \times 1.38 \times 10^{-23} \text{ J/K}) = 1.95 \times 10^7 \text{ K}$$

**Discussion:** This temperature of about 19.5 million kelvin is extraordinarily high—far beyond any chemical flame, which typically reaches only a few thousand kelvin. This temperature is comparable to the temperature in the core of the Sun (about 15 million K) where nuclear fusion occurs. Such extreme temperatures are not found in ordinary flames. This problem likely describes conditions in a very exotic environment such as a nuclear explosion, a stellar interior, or possibly an extremely high-energy plasma experiment. At such temperatures, matter exists as a fully ionized plasma with nuclei stripped of all electrons.

Hydrogen molecules (molecular mass is equal to 2.016 g/mol) have an average velocity  $v_{\text{rms}}$  equal to 193 m/s. What is the temperature?

[Show Solution](#)

**Strategy:** Use the rms speed equation  $v_{\text{rms}} = \sqrt{3k_B T m}$  and solve for temperature  $T$ . First find the mass of a hydrogen molecule.

**Solution:** Mass of one hydrogen molecule:

$$m = 2.016 \times 10^{-3} \text{ kg/mol} \times 6.02 \times 10^{23} \text{ molecules/mol} = 3.35 \times 10^{-27} \text{ kg}$$

Rearranging  $v_{\text{rms}} = \sqrt{3k_B T m}$  to solve for  $T$ :

$$T = mv_{\text{rms}}^2 / (3k_B) = (3.35 \times 10^{-27} \text{ kg}) (193 \text{ m/s})^2 / (3 \times 1.38 \times 10^{-23} \text{ J/K}) = 3.02 \text{ K}$$

**Discussion:** This extremely low temperature (about 3 K or -270°C) is just above absolute zero. At such low temperatures, hydrogen would normally be a solid (it solidifies at 14 K at atmospheric pressure), but this calculation assumes the gas behaves ideally. The low rms speed of only 193 m/s (compared to ~1900 m/s at room temperature) is consistent with this very cold temperature. This temperature is in the range used in cryogenic research and is close to the temperature of liquid helium.

Much of the gas near the Sun is atomic hydrogen. Its temperature would have to be  $1.5 \times 10^7 \text{ K}$  for the average velocity  $v_{\text{rms}}$  to equal the escape velocity from the Sun. What is that velocity?

[Show Solution](#)

**Strategy:** Use the rms speed equation  $v_{\text{rms}} = \sqrt{3k_B T m}$  with the mass of a hydrogen atom.

**Solution:** First find the mass of a hydrogen atom:

$$m = 1.008 \times 10^{-3} \text{ kg/mol} \times 6.02 \times 10^{23} \text{ atoms/mol} = 1.67 \times 10^{-27} \text{ kg}$$

Now calculate the rms speed:

$$v_{\text{rms}} = \sqrt{3k_B T m} = \sqrt{3(1.38 \times 10^{-23} \text{ J/K})(1.5 \times 10^7 \text{ K})} \times 1.67 \times 10^{-27} \text{ kg} = 6.09 \times 10^5 \text{ m/s}$$

**Discussion:** The escape velocity from the Sun's surface is about 609 km/s (or about 2.2 million km/h), which is roughly 55 times Earth's escape velocity. This enormous escape velocity explains why the Sun can retain even light elements like hydrogen in its atmosphere despite its very high surface temperature of 5800 K. At the surface temperature, the rms speed of hydrogen is only about 12 km/s, far below the escape velocity. However, at the temperature of 15 million kelvin in the Sun's core, hydrogen atoms move fast enough that some could escape if they reached the surface—but they lose energy through collisions as they move outward. The solar wind does allow some particles to escape, but the Sun retains the vast majority of its mass.

There are two important isotopes of uranium— $^{235}\text{U}$  and  $^{238}\text{U}$ ; these isotopes are nearly identical chemically but have different atomic masses. Only  $^{235}\text{U}$  is very useful in nuclear reactors. One of the techniques for separating them (gas diffusion) is based on the different average velocities  $v_{\text{rms}}$  of uranium hexafluoride gas,  $\text{UF}_6$ . (a) The molecular masses for  $^{235}\text{U}\text{UF}_6$  and  $^{238}\text{U}\text{UF}_6$  are 349.0 g/mol and 352.0 g/mol, respectively. What is the ratio of their average velocities? (b) At what temperature would their average velocities differ by 1.00 m/s? (c) Do your answers in this problem imply that this technique may be difficult?

[Show Solution](#)

**Strategy:** (a) Use  $v_{\text{rms}} = \sqrt{3k_B T m}$  to find the ratio of velocities. At the same temperature, the ratio depends only on mass. (b) Set up equations for both isotopes and find when the difference equals 1.00 m/s. (c) Evaluate the practical implications.

**Solution:** (a) At the same temperature:

$$v_{\text{rms},235} / v_{\text{rms},238} = \sqrt{m_{238} / m_{235}} = \sqrt{352.0 / 349.0} = \sqrt{1.00859} = 1.00429$$

The lighter isotope moves about 0.429% faster.

(b) Using  $v_{\text{rms}} = \sqrt{3k_B T m}$ :

$$v_{\text{rms,235}} - v_{\text{rms,238}} = 1.00 \text{ m/s}$$

$$\sqrt{3k_B T m_{235}} - \sqrt{3k_B T m_{238}} = 1.00 \text{ m/s}$$

This can be rewritten as:

$$\sqrt{3k_B T} (1/\sqrt{m_{235}} - 1/\sqrt{m_{238}}) = 1.00 \text{ m/s}$$

First find the molecular masses in kg:

- $m_{235} = 349.0 \times 10^{-3} / 6.02 \times 10^{23} = 5.80 \times 10^{-25} \text{ kg}$
- $m_{238} = 352.0 \times 10^{-3} / 6.02 \times 10^{23} = 5.85 \times 10^{-25} \text{ kg}$

$$\sqrt{3k_B T} (1/\sqrt{5.80 \times 10^{-25}} - 1/\sqrt{5.85 \times 10^{-25}}) = 1.00$$

$$\sqrt{3k_B T} (1.313 \times 10^{12} - 1.307 \times 10^{12}) = 1.00$$

$$\sqrt{3k_B T} (6 \times 10^9) = 1.00$$

$$T = (1.00)^2 / 3k_B (6 \times 10^9)^2 = 13(1.38 \times 10^{-23})(3.6 \times 10^{19}) = 672 \text{ K} = 399^\circ\text{C}$$

(c) Yes, this technique is very difficult! The velocity difference is less than 0.5%, requiring many stages of separation (diffusion through barriers) to achieve significant enrichment. The process must be done at high temperature (around 400°C) to achieve even a 1 m/s difference. Historically, gaseous diffusion plants were enormous facilities with thousands of stages, consuming vast amounts of energy. This is why uranium enrichment is technically challenging and why newer methods (like gas centrifuges) have largely replaced diffusion.

**Discussion:** This problem illustrates why uranium enrichment was such a challenge during the Manhattan Project and remains a proliferation concern today. The tiny mass difference (less than 1%) between isotopes results in minuscule velocity differences, requiring massive industrial plants with thousands of separation stages. Modern centrifuge methods exploit the same mass difference but are more efficient.

## Glossary

thermal energy

—KE, the average translational kinetic energy of a molecule



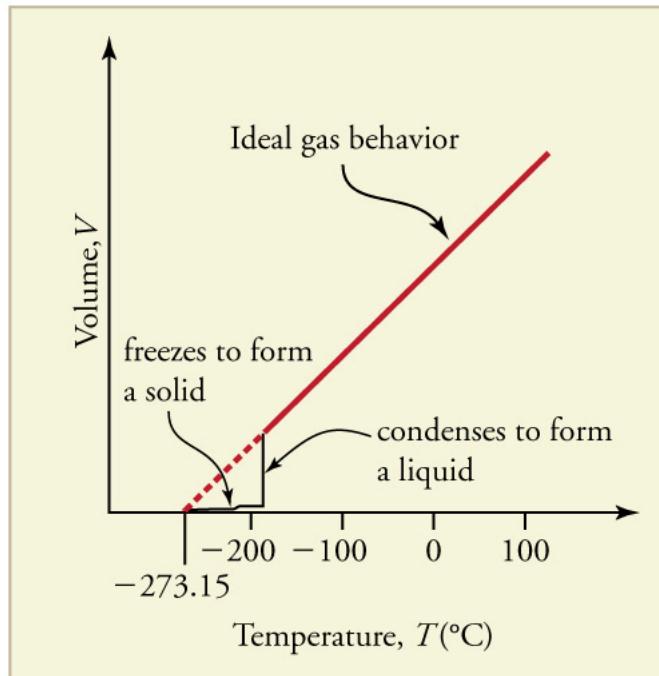
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## Phase Changes

- Interpret a phase diagram.
- State Dalton's law.
- Identify and describe the triple point of a gas from its phase diagram.
- Describe the state of equilibrium between a liquid and a gas, a liquid and a solid, and a gas and a solid.

Up to now, we have considered the behavior of ideal gases. Real gases are like ideal gases at high temperatures. At lower temperatures, however, the interactions between the molecules and their volumes cannot be ignored. The molecules are very close (condensation occurs) and there is a dramatic decrease in volume, as seen in [Figure 1]. The substance changes from a gas to a liquid. When a liquid is cooled to even lower temperatures, it becomes a solid. The volume never reaches zero because of the finite volume of the molecules.



A sketch of volume versus temperature for a real gas at constant pressure. The linear (straight line) part of the graph represents ideal gas behavior—volume and temperature are directly and positively related and the line extrapolates to zero volume at  $-273.15^{\circ}\text{C}$ , or absolute zero. When the gas becomes a liquid, however, the volume actually decreases precipitously at the liquefaction point. The volume decreases slightly once the substance is solid, but it never becomes zero.

High pressure may also cause a gas to change phase to a liquid. Carbon dioxide, for example, is a gas at room temperature and atmospheric pressure, but becomes a liquid under sufficiently high pressure. If the pressure is reduced, the temperature drops and the liquid carbon dioxide solidifies into a snow-like substance at the temperature  $-78^{\circ}\text{C}$ . Solid CO<sub>2</sub> is called “dry ice.” Another example of a gas that can be in a liquid phase is liquid nitrogen (LN<sub>2</sub>). LN<sub>2</sub> is made by liquefaction of atmospheric air (through compression and cooling). It boils at 77 K ( $-196^{\circ}\text{C}$ ) at atmospheric pressure. LN<sub>2</sub> is useful as a refrigerant and allows for the preservation of blood, sperm, and other biological materials. It is also used to reduce noise in electronic sensors and equipment, and to help cool down their current-carrying wires. In dermatology, LN<sub>2</sub> is used to freeze and painlessly remove warts and other growths from the skin.

### PV Diagrams

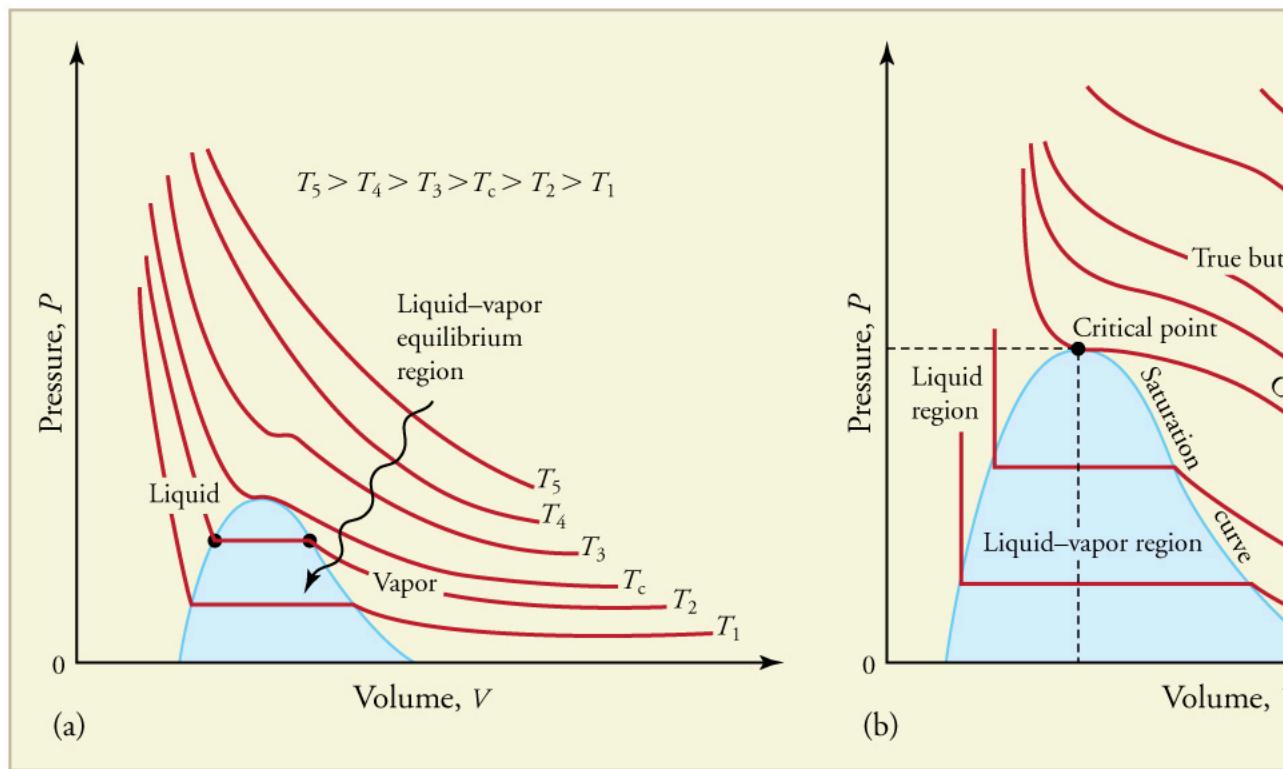
We can examine aspects of the behavior of a substance by plotting a graph of pressure versus volume, called a \*\* PV diagram\*\*. When the substance behaves like an ideal gas, the ideal gas law describes the relationship between its pressure and volume. That is,

$$PV = Nk_BT \text{ (ideal gas).}$$

Now, assuming the number of molecules and the temperature are fixed,

$$PV = \text{constant (ideal gas, constant temperature).}$$

For example, the volume of the gas will decrease as the pressure increases. If you plot the relationship  $PV = \text{constant}$  on a  $PV$  diagram, you find a hyperbola. [Figure 2] shows a graph of pressure versus volume. The hyperbolas represent ideal-gas behavior at various fixed temperatures, and are called *isotherms*. At lower temperatures, the curves begin to look less like hyperbolas—the gas is not behaving ideally and may even contain liquid. There is a **critical point**—that is, a **critical temperature**—above which liquid cannot exist. At sufficiently high pressure above the critical point, the gas will have the density of a liquid but will not condense. Carbon dioxide, for example, cannot be liquefied at a temperature above  $31.0^{\circ}\text{C}$ . **Critical pressure** is the minimum pressure needed for liquid to exist at the critical temperature. [Table 1] lists representative critical temperatures and pressures.



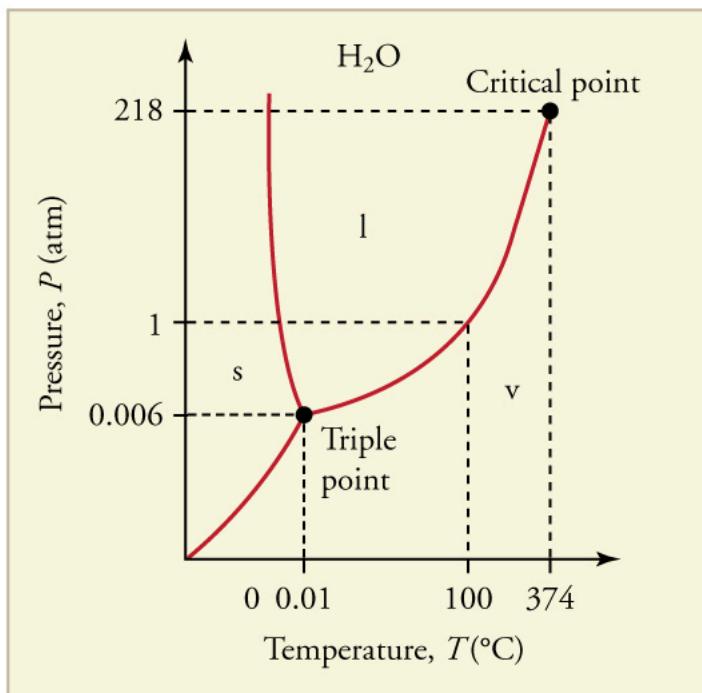
*PV* diagrams. (a) Each curve (isotherm) represents the relationship between  $P$  and  $V$  at a fixed temperature; the upper curves are at higher temperatures. The lower curves are not hyperbolas, because the gas is no longer an ideal gas. (b) An expanded portion of the  $PV$  diagram for low temperatures, where the phase can change from a gas to a liquid. The term “vapor” refers to the gas phase when it exists at a temperature below the boiling temperature.

#### Critical Temperatures and Pressures

Substance	Critical temperature	Critical pressure		
	K	°C	Pa	atm
Water	647.4	374.3	$22.12 \times 10^6$	219.0
Sulfur dioxide	430.7	157.6	$7.88 \times 10^6$	78.0
Ammonia	405.5	132.4	$11.28 \times 10^6$	111.7
Carbon dioxide	304.2	31.1	$7.39 \times 10^6$	73.2
Oxygen	154.8	-118.4	$5.08 \times 10^6$	50.3
Nitrogen	126.2	-146.9	$3.39 \times 10^6$	33.6
Hydrogen	33.3	-239.9	$1.30 \times 10^6$	12.9
Helium	5.3	-267.9	$0.229 \times 10^6$	2.27

#### Phase Diagrams

The plots of pressure versus temperatures provide considerable insight into thermal properties of substances. There are well-defined regions on these graphs that correspond to various phases of matter, so PT graphs are called **phase diagrams**. [Figure 3] shows the phase diagram for water. Using the graph, if you know the pressure and temperature you can determine the phase of water. The solid lines—boundaries between phases—indicate temperatures and pressures at which the phases coexist (that is, they exist together in ratios, depending on pressure and temperature). For example, the boiling point of water is 100°C at 1.00 atm. As the pressure increases, the boiling temperature rises steadily to 374°C at a pressure of 218 atm. A pressure cooker (or even a covered pot) will cook food faster because the water can exist as a liquid at temperatures greater than 100°C without all boiling away. The curve ends at a point called the *critical point*, because at higher temperatures the liquid phase does not exist at any pressure. The critical point occurs at the critical temperature, as you can see for water from [Table 1]. The critical temperature for oxygen is -118°C, so oxygen cannot be liquefied above this temperature.



The phase diagram ( $PT$  graph) for water. Note that the axes are nonlinear and the graph is not to scale. This graph is simplified—there are several other exotic phases of ice at higher pressures.

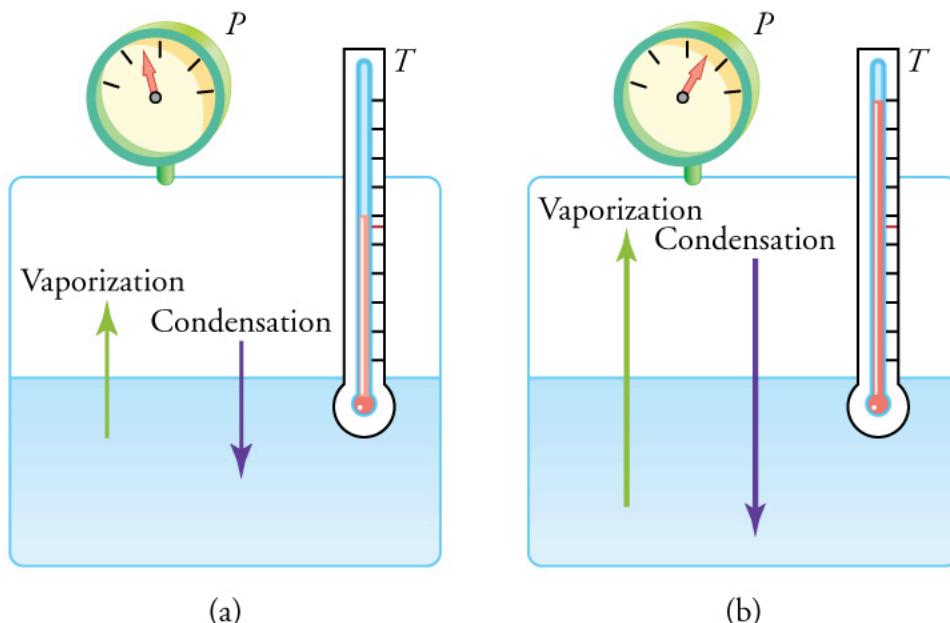
Similarly, the curve between the solid and liquid regions in [Figure 3] gives the melting temperature at various pressures. For example, the melting point is  $0^\circ\text{C}$  at 1.00 atm, as expected. Note that, at a fixed temperature, you can change the phase from solid (ice) to liquid (water) by increasing the pressure. Ice melts from pressure in the hands of a snowball maker. From the phase diagram, we can also say that the melting temperature of ice rises with increased pressure. When a car is driven over snow, the increased pressure from the tires melts the snowflakes; afterwards the water refreezes and forms an ice layer.

At sufficiently low pressures there is no liquid phase, but the substance can exist as either gas or solid. For water, there is no liquid phase at pressures below 0.00600 atm. The phase change from solid to gas is called **sublimation**. It accounts for large losses of snow pack that never make it into a river, the routine automatic defrosting of a freezer, and the freeze-drying process applied to many foods. Carbon dioxide, on the other hand, sublimates at standard atmospheric pressure of 1 atm. (The solid form of  $\text{CO}_2$  is known as dry ice because it does not melt. Instead, it moves directly from the solid to the gas state.)

All three curves on the phase diagram meet at a single point, the **triple point**, where all three phases exist in equilibrium. For water, the triple point occurs at 273.16 K ( $0.01^\circ\text{C}$ ), and is a more accurate calibration temperature than the melting point of water at 1.00 atm, or 273.15 K ( $0.0^\circ\text{C}$ ). See [Table 2] for the triple point values of other substances.

### Equilibrium

Liquid and gas phases are in equilibrium at the boiling temperature. ( See [Figure 4].) If a substance is in a closed container at the boiling point, then the liquid is boiling and the gas is condensing at the same rate without net change in their relative amount. Molecules in the liquid escape as a gas at the same rate at which gas molecules stick to the liquid, or form droplets and become part of the liquid phase. The combination of temperature and pressure has to be “just right”; if the temperature and pressure are increased, equilibrium is maintained by the same increase of boiling and condensation rates.



Equilibrium between liquid and gas at two different boiling points inside a closed container. (a) The rates of boiling and condensation are equal at this combination of temperature and pressure, so the liquid and gas phases are in equilibrium. (b) At a higher temperature, the boiling rate is faster and the rates at which molecules leave the liquid and enter the gas are also faster. Because there are more molecules in the gas, the gas pressure is higher and the rate at which gas molecules condense and enter the liquid is faster. As a result the gas and liquid are in equilibrium at this higher temperature.

#### Triple Point Temperatures and Pressures

Substance	Temperature K	Temperature °C	Pressure Pa	Pressure atm
Water	273.16	0.01	$6.10 \times 10^2$	0.00600
Carbon dioxide	216.55	-56.60	$5.16 \times 10^5$	5.11
Sulfur dioxide	197.68	-75.47	$1.67 \times 10^3$	0.0167
Ammonia	195.40	-77.75	$6.06 \times 10^3$	0.0600
Nitrogen	63.18	-210.0	$1.25 \times 10^4$	0.124
Oxygen	54.36	-218.8	$1.52 \times 10^2$	0.00151
Hydrogen	13.84	-259.3	$7.04 \times 10^3$	0.0697

One example of equilibrium between liquid and gas is that of water and steam at 100°C and 1.00 atm. This temperature is the boiling point at that pressure, so they should exist in equilibrium. Why does an open pot of water at 100°C boil completely away? The gas surrounding an open pot is not pure water: it is mixed with air. If pure water and steam are in a closed container at 100°C and 1.00 atm, they would coexist—but with air over the pot, there are fewer water molecules to condense, and water boils. What about water at 20.0°C and 1.00 atm? This temperature and pressure correspond to the liquid region, yet an open glass of water at this temperature will completely evaporate. Again, the gas around it is air and not pure water vapor, so that the reduced evaporation rate is greater than the condensation rate of water from dry air. If the glass is sealed, then the liquid phase remains. We call the gas phase a **vapor** when it exists, as it does for water at 20.0°C, at a temperature below the boiling temperature.

#### Check Your Understanding

Explain why a cup of water (or soda) with ice cubes stays at 0°C, even on a hot summer day.

[Show Solution](#)

The ice and liquid water are in thermal equilibrium, so that the temperature stays at the freezing temperature as long as ice remains in the liquid. (Once all of the ice melts, the water temperature will start to rise.)

#### Vapor Pressure, Partial Pressure, and Dalton's Law

**Vapor pressure** is defined as the pressure at which a gas coexists with its solid or liquid phase. Vapor pressure is created by faster molecules that break away from the liquid or solid and enter the gas phase. The vapor pressure of a substance depends on both the substance and its temperature—an increase in temperature increases the vapor pressure.

**Partial pressure** is defined as the pressure a gas would create if it occupied the total volume available. In a mixture of gases, *the total pressure is the sum of partial pressures of the component gases*, assuming ideal gas behavior and no chemical reactions between the components. This law is known as **Dalton's law of partial pressures**, after the English scientist John Dalton (1766–1844), who proposed it. Dalton's law is based on kinetic theory, where each gas creates its pressure by molecular collisions, independent of other gases present. It is consistent with the fact that pressures add according to [Pascal's Principle](#). Thus water evaporates and ice sublimates when their vapor pressures exceed the partial pressure of water vapor in the surrounding mixture of gases. If their vapor pressures are less than the partial pressure of water vapor in the surrounding gas, liquid droplets or ice crystals (frost) form.

### Check Your Understanding

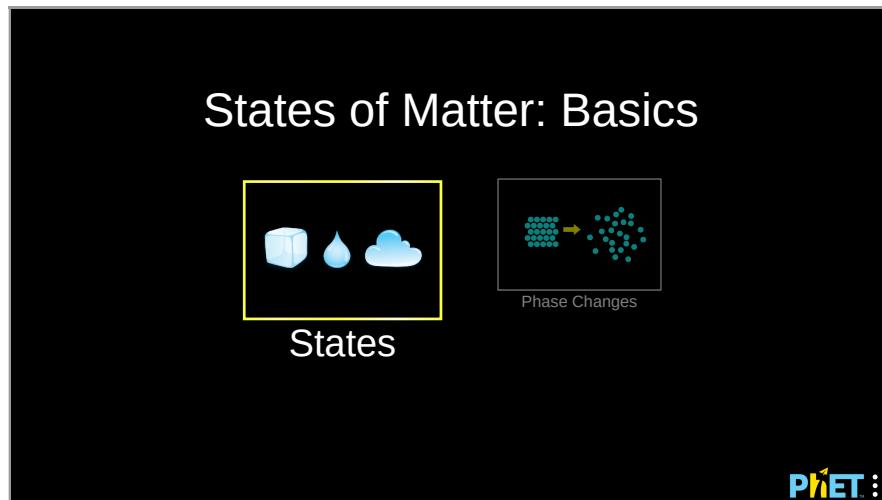
Is energy transfer involved in a phase change? If so, will energy have to be supplied to change phase from solid to liquid and liquid to gas? What about gas to liquid and liquid to solid? Why do they spray the orange trees with water in Florida when the temperatures are near or just below freezing?

[Show Solution](#)

Yes, energy transfer is involved in a phase change. We know that atoms and molecules in solids and liquids are bound to each other because we know that force is required to separate them. So in a phase change from solid to liquid and liquid to gas, a force must be exerted, perhaps by collision, to separate atoms and molecules. Force exerted through a distance is work, and energy is needed to do work to go from solid to liquid and liquid to gas. This is intuitively consistent with the need for energy to melt ice or boil water. The converse is also true. Going from gas to liquid or liquid to solid involves atoms and molecules pushing together, doing work and releasing energy.

### PhET Explorations: States of Matter—Basics

Heat, cool, and compress atoms and molecules and watch as they change between solid, liquid, and gas phases.



### Section Summary

- Most substances have three distinct phases: gas, liquid, and solid.
- Phase changes among the various phases of matter depend on temperature and pressure.
- The existence of the three phases with respect to pressure and temperature can be described in a phase diagram.
- Two phases coexist (i.e., they are in thermal equilibrium) at a set of pressures and temperatures. These are described as a line on a phase diagram.
- The three phases coexist at a single pressure and temperature. This is known as the triple point and is described by a single point on a phase diagram.
- A gas at a temperature below its boiling point is called a vapor.
- Vapor pressure is the pressure at which a gas coexists with its solid or liquid phase.
- Partial pressure is the pressure a gas would create if it existed alone.
- Dalton's law states that the total pressure is the sum of the partial pressures of all of the gases present.

### Conceptual Questions

A pressure cooker contains water and steam in equilibrium at a pressure greater than atmospheric pressure. How does this greater pressure increase cooking speed?

[Show Solution](#)

**Strategy:** Consider the relationship between pressure and boiling point, and how temperature affects cooking.

**Solution:** Greater pressure increases the boiling point of water. At atmospheric pressure (1 atm), water boils at 100°C. In a pressure cooker operating at higher pressure (typically 2 atm), water doesn't boil until it reaches approximately 120°C.

This higher temperature significantly increases cooking speed because chemical reactions (including those that cook food) proceed faster at higher temperatures. The rate of most chemical reactions roughly doubles for every 10°C increase in temperature. Therefore, cooking at 120°C instead of 100°C can reduce cooking time by half or more.

**Discussion:** The pressure cooker takes advantage of the phase diagram for water—moving along the liquid-gas boundary to higher pressures allows liquid water to exist at higher temperatures. This is particularly useful for cooking foods that normally require long cooking times, such as tough cuts of meat, dried beans, or root vegetables. The elevated temperature breaks down proteins and softens fibers much faster than boiling at atmospheric pressure.

Why does condensation form most rapidly on the coldest object in a room—for example, on a glass of ice water?

[Show Solution](#)

**Strategy:** Consider the relationship between temperature, vapor pressure, and the dew point.

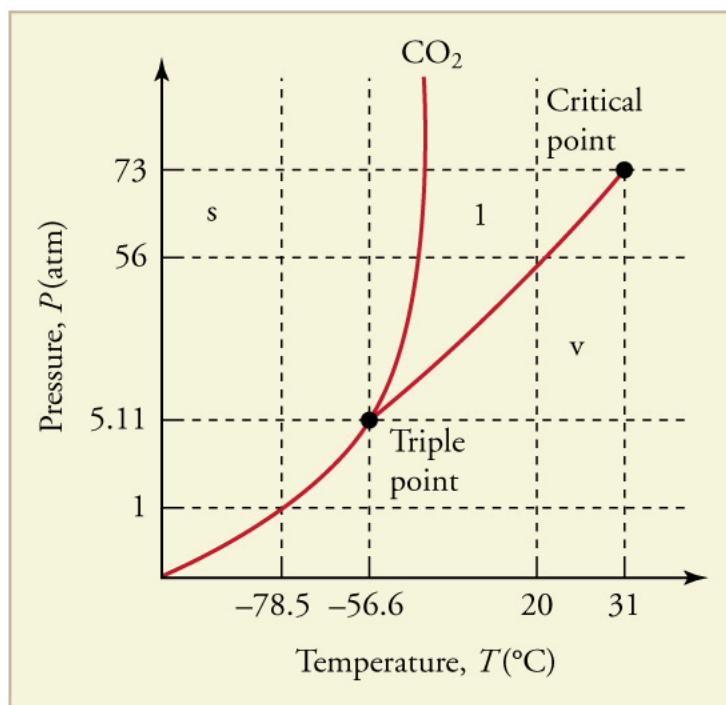
**Solution:** Condensation forms most rapidly on the coldest object because that object's surface temperature is most likely to be at or below the dew point of the surrounding air. The dew point is the temperature at which the air becomes saturated with water vapor (100% relative humidity).

When warm, humid air comes into contact with a cold surface (like a glass of ice water), the air immediately adjacent to the surface cools down. As this air cools, its capacity to hold water vapor decreases. If the surface temperature is below the dew point, the water vapor in the air adjacent to the surface becomes supersaturated and must condense into liquid droplets on the surface.

The colder the object, the lower its surface temperature, and the more likely it is to be below the dew point. Additionally, a colder surface creates a larger temperature gradient, which can cool more of the adjacent air below the dew point, resulting in more rapid condensation.

**Discussion:** This is why glasses of cold drinks “sweat” on humid days, why bathroom mirrors fog up during hot showers, and why dew forms on grass during cool nights. The coldest surfaces act as condensation nuclei, extracting water vapor from the air and converting it to liquid.

What is the vapor pressure of solid carbon dioxide (dry ice) at  $-78.5^{\circ}\text{C}$ ?



The phase diagram for carbon dioxide. The axes are nonlinear, and the graph is not to scale. Dry ice is solid carbon dioxide and has a sublimation temperature of  $-78.5^{\circ}\text{C}$ .

[Show Solution](#)

**Strategy:** Use the phase diagram for carbon dioxide. At  $-78.5^{\circ}\text{C}$ , find where the solid-gas boundary intersects this temperature.

**Solution:** From Figure 5, at  $-78.5^{\circ}\text{C}$ , the phase diagram shows that solid carbon dioxide (dry ice) sublimes directly to gas at standard atmospheric pressure of **1.0 atmosphere** (or approximately  $1.01 \times 10^5 \text{ Pa}$ ).

This means the vapor pressure of solid CO<sub>2</sub> at  $-78.5^{\circ}\text{C}$  is exactly 1.0 atm. At this temperature and pressure, solid and gas phases coexist in equilibrium on the solid-gas boundary line.

**Discussion:** This is why dry ice sublimes (converts directly from solid to gas) at atmospheric pressure and this temperature, rather than melting. The triple point of CO<sub>2</sub> is at 5.11 atm and  $-56.6^{\circ}\text{C}$ , which is above atmospheric pressure. This means at standard atmospheric pressure (1 atm), there is no liquid phase—CO<sub>2</sub> exists only as solid below  $-78.5^{\circ}\text{C}$  and as gas above this temperature. This property makes dry ice useful for cooling without creating liquid mess, as it goes directly from solid to gas.

Can carbon dioxide be liquefied at room temperature ( $20^{\circ}\text{C}$ )? If so, how? If not, why not? (See [Figure 5].)

[Show Solution](#)

**Strategy:** Examine the phase diagram for CO<sub>2</sub> and determine if liquid phase exists at 20°C. Compare this temperature to the critical temperature.

**Solution:** Yes, carbon dioxide can be liquefied at room temperature (20°C), but only by applying sufficient pressure. From Figure 5, the critical temperature of CO<sub>2</sub> is 31°C. Since room temperature (20°C) is below the critical temperature, a liquid phase can exist.

To liquefy CO<sub>2</sub> at 20°C, you would need to increase the pressure significantly. Following the 20°C vertical line on the phase diagram upward, you would need to apply a pressure of approximately 56-57 atmospheres (about  $5.7 \times 10^6$  Pa) to reach the liquid phase region.

**Discussion:** This is exactly how CO<sub>2</sub> fire extinguishers work—they store liquid CO<sub>2</sub> at room temperature under high pressure (typically around 60 atm). When released, the pressure drops to atmospheric, and the liquid CO<sub>2</sub> rapidly converts to gas and solid (the white “snow” you see), which helps suppress fires. If the temperature were above 31°C (the critical temperature), no amount of pressure could liquefy the CO<sub>2</sub>—it would exist only as a supercritical fluid.

Oxygen cannot be liquefied at room temperature by placing it under a large enough pressure to force its molecules together. Explain why this is.

[Show Solution](#)

**Strategy:** Consider the concept of critical temperature and what it means for phase transitions.

**Solution:** Oxygen cannot be liquefied at room temperature because room temperature (about 20-25°C or 293-298 K) is well above oxygen’s critical temperature of -118.4°C (154.8 K).

The critical temperature is the temperature above which a substance cannot exist as a liquid, regardless of how much pressure is applied. Above the critical temperature, the thermal kinetic energy of the molecules is so high that no amount of pressure can force them to remain close enough together in the orderly arrangement characteristic of a liquid. Instead, above the critical temperature and critical pressure, the substance exists as a supercritical fluid—a state that has properties intermediate between liquid and gas.

**Discussion:** This is why oxygen must be cooled below 154.8 K (-118.4°C) before it can be liquefied, even under high pressure. Liquid oxygen (LOX) used in rockets and medical applications must be stored at cryogenic temperatures well below its critical temperature. The same principle applies to other gases with low critical temperatures, such as nitrogen (-146.9°C) and helium (-267.9°C), which all require significant cooling before they can be liquefied.

What is the distinction between gas and vapor?

[Show Solution](#)

**Strategy:** Consider the relationship between the gas phase and the boiling temperature of a substance.

**Solution:** The distinction between gas and vapor relates to temperature relative to the boiling point:

- A **vapor** is a gas that exists at a temperature **below** its boiling point (at the given pressure). A vapor can be condensed to a liquid simply by increasing the pressure at constant temperature, without cooling.
- A **gas** is used more generally, and often refers to a substance that exists at a temperature **above** its boiling point (at the given pressure). A gas cannot be liquefied merely by increasing pressure at that temperature—it must be cooled below its boiling point first.

For example, at 20°C and atmospheric pressure:

- Water exists as a vapor (since 20°C < 100°C, water’s boiling point)
- Nitrogen exists as a gas (since 20°C > -196°C, nitrogen’s boiling point)

**Discussion:** This distinction is useful because vapors are relatively close to their liquid phase and can be liquefied more easily than gases. In everyday usage, “gas” is often used generically to refer to any substance in the gaseous state, but technically “vapor” is more precise when discussing a substance below its boiling point. The phase diagram helps clarify this: a vapor exists in the region where a small increase in pressure or decrease in temperature would cause condensation to liquid.

## Glossary

**PV diagram**

a graph of pressure vs. volume

**critical point**

the temperature above which a liquid cannot exist

**critical temperature**

the temperature above which a liquid cannot exist

**critical pressure**

the minimum pressure needed for a liquid to exist at the critical temperature

**vapor**

a gas at a temperature below the boiling temperature

**vapor pressure**

the pressure at which a gas coexists with its solid or liquid phase

**phase diagram**

a graph of pressure vs. temperature of a particular substance, showing at which pressures and temperatures the three phases of the substance occur

**triple point**

the pressure and temperature at which a substance exists in equilibrium as a solid, liquid, and gas

**sublimation**

the phase change from solid to gas

partial pressure

the pressure a gas would create if it occupied the total volume of space available

Dalton's law of partial pressures

the physical law that states that the total pressure of a gas is the sum of partial pressures of the component gases

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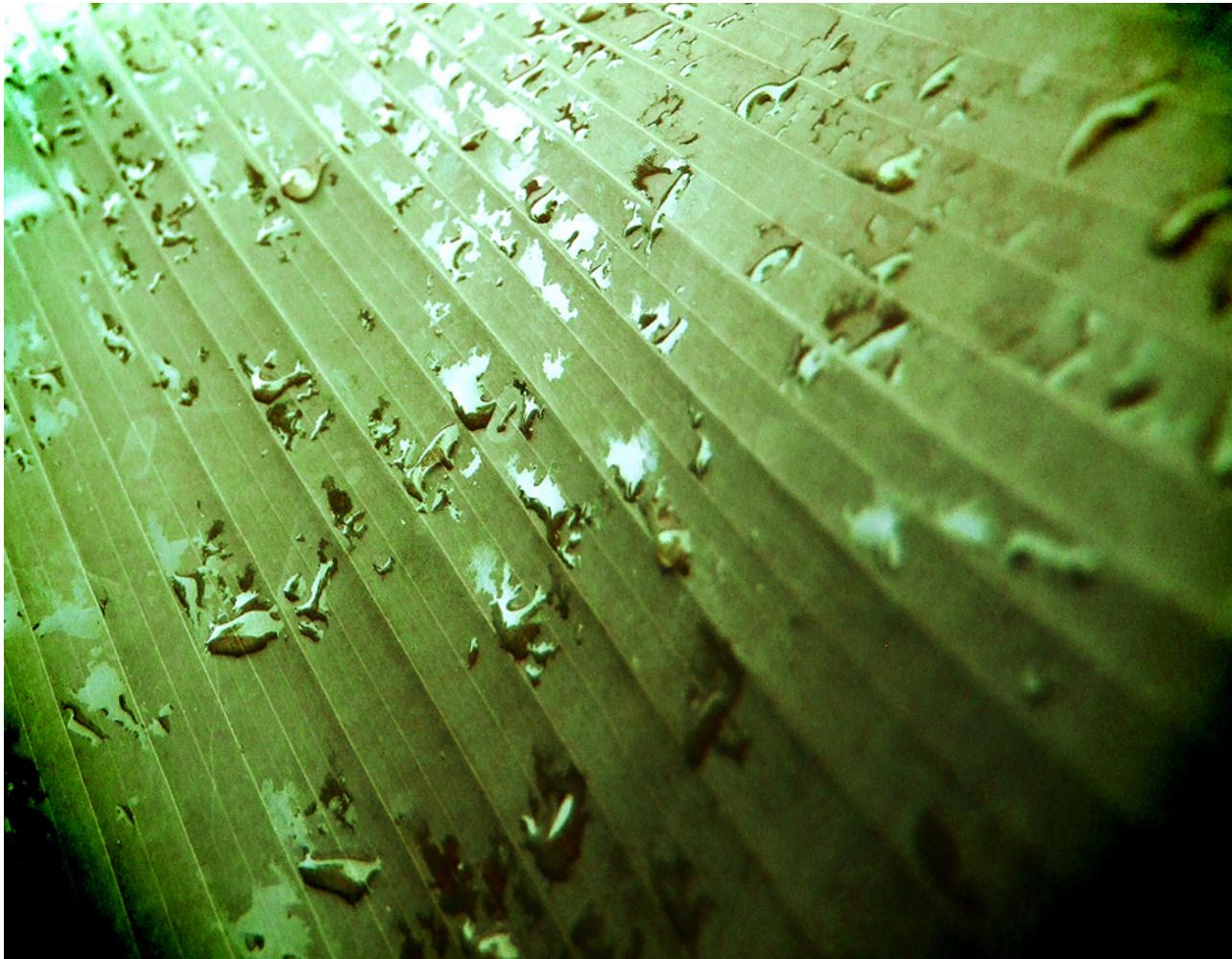


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## Humidity, Evaporation, and Boiling

- Explain the relationship between vapor pressure of water and the capacity of air to hold water vapor.
- Explain the relationship between relative humidity and partial pressure of water vapor in the air.
- Calculate vapor density using vapor pressure.
- Calculate humidity and dew point.

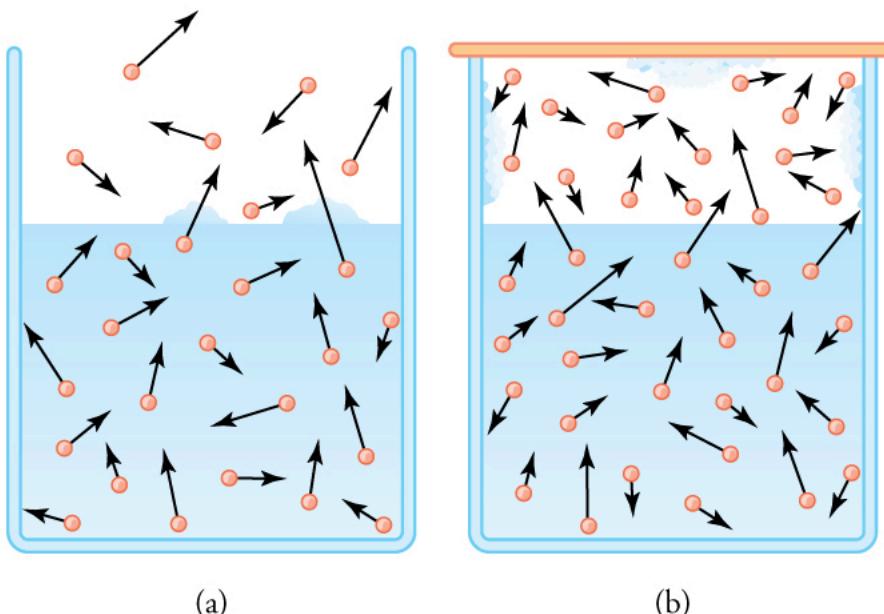


Dew drops like these, on a banana leaf photographed just after sunrise, form when the air temperature drops to or below the dew point. At the dew point, the rate at which water molecules join together is greater than the rate at which they separate, and some of the water condenses to form droplets. (credit: Aaron Escobar, Flickr)

The expression “it’s not the heat, it’s the humidity” makes a valid point. We keep cool in hot weather by evaporating sweat from our skin and water from our breathing passages. Because evaporation is inhibited by high humidity, we feel hotter at a given temperature when the humidity is high. Low humidity, on the other hand, can cause discomfort from excessive drying of mucous membranes and can lead to an increased risk of respiratory infections.

When we say humidity, we really mean **relative humidity**. Relative humidity tells us how much water vapor is in the air compared with the maximum possible. At its maximum, denoted as **saturation**, the relative humidity is 100%, and evaporation is inhibited. The amount of water vapor in the air depends on temperature. For example, relative humidity rises in the evening, as air temperature declines, sometimes reaching the **dew point**. At the dew point temperature, relative humidity is 100%, and fog may result from the condensation of water droplets if they are small enough to stay in suspension. Conversely, if you wish to dry something (perhaps your hair), it is more effective to blow hot air over it rather than cold air, because, among other things, the increase in temperature increases the energy of the molecules, so the rate of evaporation increases.

The amount of water vapor in the air depends on the vapor pressure of water. The liquid and solid phases are continuously giving off vapor because some of the molecules have high enough speeds to enter the gas phase; see [Figure 2](a). If a lid is placed over the container, as in [Figure 2](b), evaporation continues, increasing the pressure, until sufficient vapor has built up for condensation to balance evaporation. Then equilibrium has been achieved, and the vapor pressure is equal to the partial pressure of water in the container. Vapor pressure increases with temperature because molecular speeds are higher as temperature increases. [Table 1] gives representative values of water vapor pressure over a range of temperatures.



(a) Because of the distribution of speeds and kinetic energies, some water molecules can break away to the vapor phase even at temperatures below the ordinary boiling point. (b) If the container is sealed, evaporation will continue until there is enough vapor density for the condensation rate to equal the evaporation rate. This vapor density and the partial pressure it creates are the saturation values. They increase with temperature and are independent of the presence of other gases, such as air. They depend only on the vapor pressure of water.

Relative humidity is related to the partial pressure of water vapor in the air. At 100% humidity, the partial pressure is equal to the vapor pressure, and no more water can enter the vapor phase. If the partial pressure is less than the vapor pressure, then evaporation will take place, as humidity is less than 100%. If the partial pressure is greater than the vapor pressure, condensation takes place. In everyday language, people sometimes refer to the capacity of air to "hold" water vapor, but this is not actually what happens. The water vapor is not held by the air. The amount of water in air is determined by the vapor pressure of water and has nothing to do with the properties of air.

Saturation Vapor Density of Water

Temperature (°C)	Vapor pressure (Pa)	Saturation vapor density (g/m <sup>3</sup> )
-50	4.0	0.039
-20	$1.04 \times 10^2$	0.89
-10	$2.60 \times 10^2$	2.36
0	$6.10 \times 10^2$	4.84
5	$8.68 \times 10^2$	6.80
10	$1.19 \times 10^3$	9.40
15	$1.69 \times 10^3$	12.8
20	$2.33 \times 10^3$	17.2
25	$3.17 \times 10^3$	23.0
30	$4.24 \times 10^3$	30.4
37	$6.31 \times 10^3$	44.0
40	$7.34 \times 10^3$	51.1
50	$1.23 \times 10^4$	82.4
60	$1.99 \times 10^4$	130
70	$3.12 \times 10^4$	197
80	$4.73 \times 10^4$	294
90	$7.01 \times 10^4$	418
95	$8.59 \times 10^4$	505
<b>100</b>	<b><math>1.01 \times 10^5</math></b>	<b>598</b>

Temperature (°C)	Vapor pressure (Pa)	Saturation vapor density (g/m <sup>3</sup> )
120	$1.99 \times 10^5$	1095
150	$4.76 \times 10^5$	2430
200	$1.55 \times 10^6$	7090
220	$2.32 \times 10^6$	10 200

### Calculating Density Using Vapor Pressure

[Table 1] gives the vapor pressure of water at 20.0°C as  $2.33 \times 10^3$  Pa. Use the ideal gas law to calculate the density of water vapor in g/m<sup>3</sup> that would create a partial pressure equal to this vapor pressure. Compare the result with the saturation vapor density given in the table.

#### Strategy

To solve this problem, we need to break it down into two steps. The partial pressure follows the ideal gas law,

$$PV=nRT,$$

where  $n$  is the number of moles. If we solve this equation for  $n/V$  to calculate the number of moles per cubic meter, we can then convert this quantity to grams per cubic meter as requested. To do this, we need to use the molecular mass of water, which is given in the periodic table.

#### Solution

1. Identify the knowns and convert them to the proper units:

2. temperature  $T = 20^\circ\text{C} = 293\text{K}$

3. vapor pressure  $P$  of water at 20°C

is  $2.33 \times 10^3$  Pa

4. molecular mass of water is 18.0 g/mol { type="a" }

5. Solve the ideal gas law for  $n/V$ .

$$nV=PRT$$

1. Substitute known values into the equation and solve for  $n/V$ .

$$nV=PRT=2.33 \times 10^3 \text{ Pa} (8.31 \text{ J/mol}\cdot\text{K}) (293 \text{ K}) = 0.957 \text{ mol/m}^3$$

1. Convert the density in moles per cubic meter to grams per cubic meter.

$$\rho = (0.957 \text{ mol/m}^3) (18.0 \text{ g/mol}) = 17.2 \text{ g/m}^3$$

#### Discussion

The density is obtained by assuming a pressure equal to the vapor pressure of water at 20.0°C. The density found is identical to the value in [Table 1], which means that a vapor density of 17.2 g/m<sup>3</sup> at 20.0°C creates a partial pressure of  $2.33 \times 10^3$  Pa, equal to the vapor pressure of water at that temperature. If the partial pressure is equal to the vapor pressure, then the liquid and vapor phases are in equilibrium, and the relative humidity is 100%. Thus, there can be no more than 17.2 g of water vapor per m<sup>3</sup> at 20.0°C, so that this value is the saturation vapor density at that temperature. This example illustrates how water vapor behaves like an ideal gas: the pressure and density are consistent with the ideal gas law (assuming the density in the table is correct). The saturation vapor densities listed in [Table 1] are the maximum amounts of water vapor that air can hold at various temperatures.

#### Percent Relative Humidity

We define **percent relative humidity** as the ratio of vapor density to saturation vapor density, or

$$\text{percent relative humidity} = \frac{\text{vapor density}}{\text{saturation vapor density}} \times 100$$

We can use this and the data in [Table 1] to do a variety of interesting calculations, keeping in mind that relative humidity is based on the comparison of the partial pressure of water vapor in air and ice.

#### Calculating Humidity and Dew Point

(a) Calculate the percent relative humidity on a day when the temperature is 25.0°C and the air contains 9.40 g of water vapor per m<sup>3</sup>. (b) At what temperature will this air reach 100% relative humidity (the saturation density)? This temperature is the dew point. (c) What is the humidity when the air temperature is 25.0°C and the dew point is  $-10.0^\circ\text{C}$ ?

#### Strategy and Solution

(a) Percent relative humidity is defined as the ratio of vapor density to saturation vapor density.

$$\text{percent relative humidity} = \frac{\text{vapor density}}{\text{saturation vapor density}} \times 100$$

The first is given to be  $9.40 \text{ g/m}^3$ , and the second is found in [\[Table 1\]](#) to be  $23.0 \text{ g/m}^3$ . Thus,

$$\text{percent relative humidity} = \frac{9.40 \text{ g/m}^3}{23.0 \text{ g/m}^3} \times 100 = 40.9\%$$

(b) The air contains  $9.40 \text{ g/m}^3$  of water vapor. The relative humidity will be 100% at a temperature where  $9.40 \text{ g/m}^3$  is the saturation density. Inspection of [\[Table 1\]](#) reveals this to be the case at  $10.0^\circ\text{C}$ , where the relative humidity will be 100%. That temperature is called the dew point for air with this concentration of water vapor.

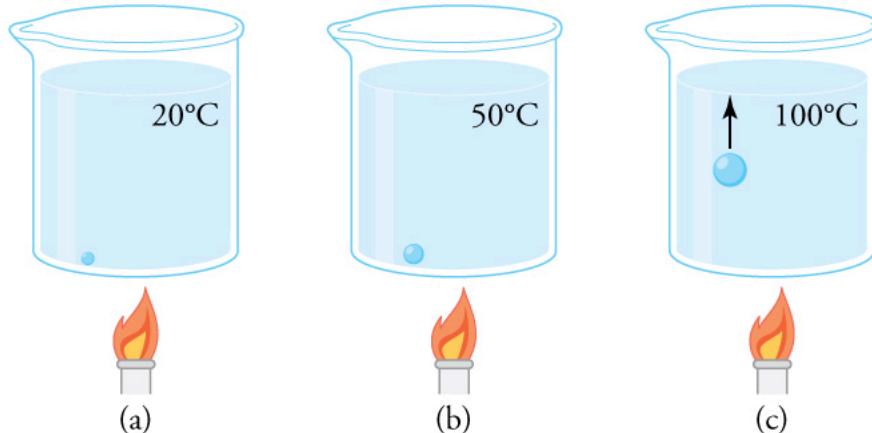
(c) Here, the dew point temperature is given to be  $-10.0^\circ\text{C}$ . Using [\[Table 1\]](#), we see that the vapor density is  $2.36 \text{ g/m}^3$ , because this value is the saturation vapor density at  $-10.0^\circ\text{C}$ . The saturation vapor density at  $25.0^\circ\text{C}$  is seen to be  $23.0 \text{ g/m}^3$ . Thus, the relative humidity at  $25.0^\circ\text{C}$  is

$$\text{percent relative humidity} = \frac{2.36 \text{ g/m}^3}{23.0 \text{ g/m}^3} \times 100 = 10.3\%$$

### Discussion

The importance of dew point is that air temperature cannot drop below  $10.0^\circ\text{C}$  in part (b), or  $-10.0^\circ\text{C}$  in part (c), without water vapor condensing out of the air. If condensation occurs, considerable transfer of heat occurs (discussed in [Heat and Heat Transfer Methods](#)), which prevents the temperature from further dropping. When dew points are below  $0^\circ\text{C}$ , freezing temperatures are a greater possibility, which explains why farmers keep track of the dew point. Low humidity in deserts means low dew-point temperatures. Thus condensation is unlikely. If the temperature drops, vapor does not condense in liquid drops. Because no heat is released into the air, the air temperature drops more rapidly compared to air with higher humidity. Likewise, at high temperatures, liquid droplets do not evaporate, so that no heat is removed from the gas to the liquid phase. This explains the large range of temperature in arid regions.

Why does water boil at  $100^\circ\text{C}$ ? You will note from [\[Table 1\]](#) that the vapor pressure of water at  $100^\circ\text{C}$  is  $1.01 \times 10^5 \text{ Pa}$ , or 1.00 atm. Thus, it can evaporate without limit at this temperature and pressure. But why does it form bubbles when it boils? This is because water ordinarily contains significant amounts of dissolved air and other impurities, which are observed as small bubbles of air in a glass of water. If a bubble starts out at the bottom of the container at  $20^\circ\text{C}$ , it contains water vapor (about 2.30%). The pressure inside the bubble is fixed at 1.00 atm (we ignore the slight pressure exerted by the water around it). As the temperature rises, the amount of air in the bubble stays the same, but the water vapor increases; the bubble expands to keep the pressure at 1.00 atm. At  $100^\circ\text{C}$ , water vapor enters the bubble continuously since the partial pressure of water is equal to 1.00 atm in equilibrium. It cannot reach this pressure, however, since the bubble also contains air and total pressure is 1.00 atm. The bubble grows in size and thereby increases the buoyant force. The bubble breaks away and rises rapidly to the surface—we call this boiling! (See [\[Figure 3\]](#).)



(a) An air bubble in water starts out saturated with water vapor at  $20^\circ\text{C}$ . (b) As the temperature rises, water vapor enters the bubble because its vapor pressure increases. The bubble expands to keep its pressure at 1.00 atm. (c) At  $100^\circ\text{C}$ , water vapor enters the bubble continuously because water's vapor pressure exceeds its partial pressure in the bubble, which must be less than 1.00 atm. The bubble grows and rises to the surface.

### Check Your Understanding

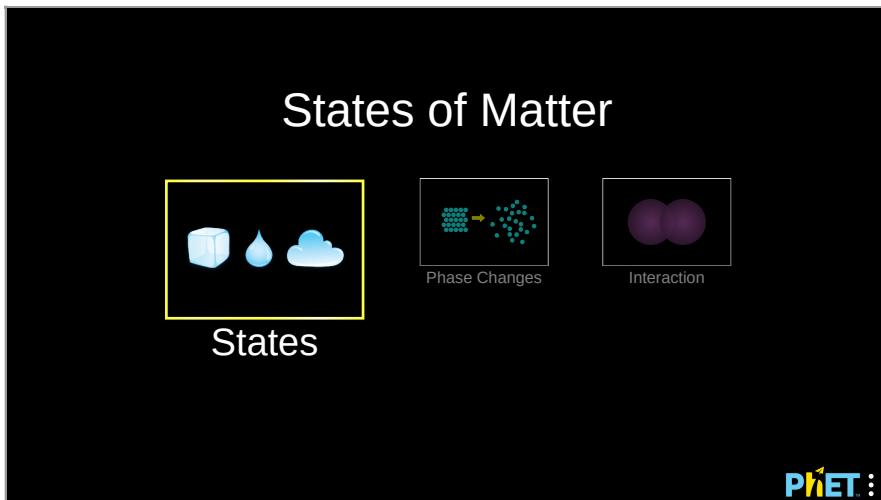
Freeze drying is a process in which substances, such as foods, are dried by placing them in a vacuum chamber and lowering the atmospheric pressure around them. How does the lowered atmospheric pressure speed the drying process, and why does it cause the temperature of the food to drop?

[Show Solution](#)

Decreased the atmospheric pressure results in decreased partial pressure of water, hence a lower humidity. So evaporation of water from food, for example, will be enhanced. The molecules of water most likely to break away from the food will be those with the greatest velocities. Those remaining thus have a lower average velocity and a lower temperature. This can (and does) result in the freezing and drying of the food; hence the process is aptly named freeze drying.

## PhET Explorations: States of Matter

Watch different types of molecules form a solid, liquid, or gas. Add or remove heat and watch the phase change. Change the temperature or volume of a container and see a pressure-temperature diagram respond in real time. Relate the interaction potential to the forces between molecules.



### Section Summary

- Relative humidity is the fraction of water vapor in a gas compared to the saturation value.
  - The saturation vapor density can be determined from the vapor pressure for a given temperature.
  - Percent relative humidity is defined to be
- $$\text{percent relative humidity} = \frac{\text{vapor density}}{\text{saturation vapor density}} \times 100.$$
- The dew point is the temperature at which air reaches 100% relative humidity.

### Conceptual Questions

Because humidity depends only on water's vapor pressure and temperature, are the saturation vapor densities listed in [Table 1](#) valid in an atmosphere of helium at a pressure of  $1.01 \times 10^5 \text{ N/m}^2$ , rather than air? Are those values affected by altitude on Earth?

[Show Solution](#)

**Strategy:** Consider the fundamental principles governing vapor pressure and saturation vapor density. These properties depend on the water molecules themselves and their interactions with each other, not on the surrounding gas.

**Solution:** Yes, the saturation vapor densities listed in Table 1 are valid in an atmosphere of helium at the same pressure. The saturation vapor pressure and density of water depend only on the temperature and the properties of water molecules, not on the composition of the surrounding gas. Water molecules evaporate and condense based on their own kinetic energies and intermolecular forces, regardless of whether the surrounding gas is air, helium, or any other non-reactive gas.

Similarly, these values are not affected by altitude on Earth. While atmospheric pressure decreases with altitude, the saturation vapor pressure of water at a given temperature remains constant. The saturation vapor density depends only on temperature, not on the total atmospheric pressure.

**Discussion:** This independence from the surrounding gas is a consequence of Dalton's law of partial pressures. Each gas component behaves independently, creating its own partial pressure. The water vapor will reach saturation when its partial pressure equals the vapor pressure at that temperature, regardless of what other gases are present or what the total pressure is.

Why does a beaker of  $40.0^\circ\text{C}$  water placed in a vacuum chamber start to boil as the chamber is evacuated (air is pumped out of the chamber)? At what pressure does the boiling begin? Would food cook any faster in such a beaker?

[Show Solution](#)

**Strategy:** Consider what causes boiling and the relationship between vapor pressure and atmospheric pressure. Use Table 1 to find the vapor pressure of water at  $40.0^\circ\text{C}$ .

**Solution:** Water boils when its vapor pressure equals the surrounding atmospheric pressure. At  $40.0^\circ\text{C}$ , the vapor pressure of water is  $7.34 \times 10^3 \text{ Pa}$  (from Table 1). As the chamber is evacuated and the pressure drops, the water begins to boil when the chamber pressure reaches  $7.34 \times 10^3 \text{ Pa}$ , which is the vapor pressure of water at  $40.0^\circ\text{C}$ .

Food would **not** cook faster in such a beaker. In fact, it would cook more slowly or not at all. Cooking requires sufficient temperature to break down food molecules and kill bacteria, which typically requires temperatures well above  $40.0^\circ\text{C}$ . Although the water is boiling, it remains at only  $40.0^\circ\text{C}$ , which is far too low for effective cooking.

**Discussion:** This demonstrates an important distinction between boiling and high temperature. Boiling is simply the transition from liquid to gas when vapor pressure equals atmospheric pressure, and it can occur at any temperature if the pressure is low enough. This is why water boils at lower temperatures at high altitudes, and why pressure cookers can raise the boiling point by increasing pressure.

Why does rubbing alcohol evaporate much more rapidly than water at STP (standard temperature and pressure)?

[Show Solution](#)

**Strategy:** Consider the relationship between vapor pressure and evaporation rate. The rate of evaporation depends on how easily molecules escape from the liquid phase into the gas phase.

**Solution:** Rubbing alcohol (isopropyl alcohol) evaporates much more rapidly than water because it has a much higher vapor pressure at the same temperature. At room temperature (about 20°C), rubbing alcohol has a vapor pressure significantly higher than water's vapor pressure of  $2.33 \times 10^3$  Pa.

The higher vapor pressure means that alcohol molecules have weaker intermolecular forces (primarily van der Waals forces) compared to water molecules (which have strong hydrogen bonding). Weaker intermolecular forces mean that alcohol molecules can more easily escape from the liquid surface into the gas phase.

**Discussion:** This is why rubbing alcohol feels cool when applied to skin—it evaporates rapidly, absorbing heat energy from your skin in the process (latent heat of vaporization). Water also has a cooling effect when it evaporates, but because it evaporates more slowly, the cooling sensation is less immediate. The strong hydrogen bonding in water makes it require more energy for molecules to break free, resulting in slower evaporation and a higher boiling point (100°C) compared to isopropyl alcohol (82.6°C).

## Problems & Exercises

Dry air is 78.1% nitrogen. What is the partial pressure of nitrogen when the atmospheric pressure is  $1.01 \times 10^5$  N/m<sup>2</sup>?

[Show Solution](#)

**Strategy:** Use Dalton's law of partial pressures. The partial pressure of a component gas is the percentage of that gas times the total pressure.

**Solution:**

$$P_{N_2} = 0.781 \times P_{\text{total}} = 0.781 \times 1.01 \times 10^5 \text{ Pa} = 7.89 \times 10^4 \text{ Pa}$$

**Discussion:** Nitrogen comprises about 78% of the atmosphere by volume (and by partial pressure), making it the most abundant gas in Earth's atmosphere. This partial pressure of 78,900 Pa (about 0.78 atm) is what determines the concentration of nitrogen dissolved in liquids like blood and water. This is important for scuba divers—at depth, the increased pressure increases nitrogen's partial pressure, causing more nitrogen to dissolve in body tissues. If a diver ascends too quickly, this dissolved nitrogen can form bubbles, causing decompression sickness ("the bends").

(a) What is the vapor pressure of water at 20.0°C? (b) What percentage of atmospheric pressure does this correspond to? (c) What percent of 20.0°C air is water vapor if it has 100% relative humidity? (The density of dry air at 20.0°C is 1.20 kg/m<sup>3</sup>.)

[Show Solution](#)

**Strategy:** (a) Look up the vapor pressure in Table 1. (b) Compare to standard atmospheric pressure. (c) Use the saturation vapor density from Table 1 and compare to the density of dry air.

**Solution:** (a) From Table 1, the vapor pressure of water at 20.0°C is  $2.33 \times 10^3$  Pa.

(b) Standard atmospheric pressure is  $1.01 \times 10^5$  Pa. The percentage is:

$$\text{Percentage} = \frac{2.33 \times 10^3 \text{ Pa}}{1.01 \times 10^5 \text{ Pa}} \times 100 = 2.31\%$$

(c) From Table 1, the saturation vapor density at 20.0°C is 17.2 g/m<sup>3</sup> = 0.0172 kg/m<sup>3</sup>. The total mass of saturated air per cubic meter is:

$$\text{Total mass} = 1.20 \text{ kg/m}^3 + 0.0172 \text{ kg/m}^3 = 1.2172 \text{ kg/m}^3$$

The percentage of water vapor is:

$$\text{Percentage} = \frac{0.0172}{1.2172} \times 100 = 1.41\%$$

**Discussion:** Even at 100% relative humidity, water vapor comprises only a small fraction of air by mass (about 1.4%) and an even smaller fraction by pressure (about 2.3%). This shows that air can hold only a limited amount of water vapor at room temperature. The partial pressure of water vapor is much less than atmospheric pressure, which is why water doesn't spontaneously evaporate into air at normal conditions.

Pressure cookers increase cooking speed by raising the boiling temperature of water above its value at atmospheric pressure. (a) What pressure is necessary to raise the boiling point to 120.0°C? (b) What gauge pressure does this correspond to?

[Show Solution](#)

(a)  $1.99 \times 10^5 \text{ Pa}$  (b) 0.97 atm

(a) At what temperature does water boil at an altitude of 1500 m (about 5000 ft) on a day when atmospheric pressure is  $8.59 \times 10^4 \text{ N/m}^2$ ? (b) What about at an altitude of 3000 m (about 10 000 ft) when atmospheric pressure is  $7.00 \times 10^4 \text{ N/m}^2$ ?

Show Solution

**Strategy:** Water boils when its vapor pressure equals the atmospheric pressure. Use Table 1 to find the temperature at which the vapor pressure of water equals the given atmospheric pressures.

**Solution:** (a) At 1500 m, the atmospheric pressure is  $8.59 \times 10^4 \text{ Pa}$ . From Table 1, we see that this vapor pressure corresponds to a temperature of approximately  $95^\circ\text{C}$ .

(b) At 3000 m, the atmospheric pressure is  $7.00 \times 10^4 \text{ Pa}$ . From Table 1, the vapor pressure of  $7.01 \times 10^4 \text{ Pa}$  (very close to  $7.00 \times 10^4 \text{ Pa}$ ) corresponds to a temperature of approximately  $90^\circ\text{C}$ .

**Discussion:** As altitude increases, atmospheric pressure decreases, and water boils at progressively lower temperatures. This has practical implications for cooking—at high altitudes, water boils at lower temperatures, so food takes longer to cook. This is why cooking instructions often include altitude adjustments, and why pressure cookers are particularly useful in mountainous regions. The  $5-10^\circ\text{C}$  decrease in boiling point at moderate altitudes can significantly affect cooking times.

What is the atmospheric pressure on top of Mt. Everest on a day when water boils there at a temperature of  $70.0^\circ\text{C}$ ?

Show Solution

**Strategy:** Water boils when its vapor pressure equals the atmospheric pressure. Find the vapor pressure of water at  $70.0^\circ\text{C}$  from Table 1.

**Solution:** From Table 1, the vapor pressure of water at  $70^\circ\text{C}$  is  $3.12 \times 10^4 \text{ Pa}$ .

Since water boils when its vapor pressure equals atmospheric pressure, the atmospheric pressure at the top of Mt. Everest on this day is  $3.12 \times 10^4 \text{ Pa}$  (about 0.31 atm or 31% of sea-level pressure).

**Discussion:** This atmospheric pressure of about 31,200 Pa is typical for the summit of Mt. Everest (8,849 m above sea level), where the pressure is roughly one-third of sea-level pressure. At this pressure, water boils at only  $70^\circ\text{C}$  instead of the normal  $100^\circ\text{C}$ , making it very difficult to cook food properly. This reduced boiling point is one of many challenges faced by mountain climbers at extreme altitudes. The low atmospheric pressure also means climbers receive less oxygen with each breath, requiring them to use supplemental oxygen above about 8,000 m (the “death zone”).

At a spot in the high Andes, water boils at  $80.0^\circ\text{C}$ , greatly reducing the cooking speed of potatoes, for example. What is atmospheric pressure at this location?

Show Solution

**Strategy:** Water boils when its vapor pressure equals atmospheric pressure. Find the vapor pressure of water at  $80.0^\circ\text{C}$  from Table 1.

**Solution:** From Table 1, the vapor pressure of water at  $80^\circ\text{C}$  is  $4.73 \times 10^4 \text{ Pa}$ .

Since water boils when its vapor pressure equals atmospheric pressure, the atmospheric pressure at this location is  $4.73 \times 10^4 \text{ Pa}$ , or approximately 0.467 atm.

**Discussion:** This atmospheric pressure is less than half of standard atmospheric pressure ( $1.01 \times 10^5 \text{ Pa}$ ), indicating a very high altitude—approximately 5500-6000 m above sea level. At this pressure, water boils  $20^\circ\text{C}$  below its sea-level boiling point, making cooking very difficult. Potatoes and other foods require much longer cooking times at such altitudes, which is why pressure cookers are essential in high-altitude regions like the Andes.

What is the relative humidity on a  $25.0^\circ\text{C}$  day when the air contains  $18.0 \text{ g/m}^3$  of water vapor?

Show Solution

**Strategy:** Use the definition of relative humidity as the ratio of actual vapor density to saturation vapor density. Find the saturation vapor density at  $25.0^\circ\text{C}$  from Table 1.

**Solution:** From Table 1, the saturation vapor density at  $25.0^\circ\text{C}$  is  $23.0 \text{ g/m}^3$ .

$$\text{Relative humidity} = \frac{\text{actual vapor density}}{\text{saturation vapor density}} \times 100 = \frac{18.0 \text{ g/m}^3}{23.0 \text{ g/m}^3} \times 100 = 78.3\%$$

**Discussion:** A relative humidity of 78.3% indicates the air is holding about three-quarters of the maximum water vapor it can contain at  $25^\circ\text{C}$ . This would feel quite humid and muggy, as evaporation of perspiration from skin would be inhibited, making it harder for the body to cool itself. For comparison, comfortable indoor humidity is typically 30-50%. At 100% relative humidity, the air is saturated and cannot hold any more water vapor, so sweat cannot evaporate and cooling becomes very difficult.

What is the density of water vapor in  $\text{g/m}^3$  on a hot dry day in the desert when the temperature is  $40.0^\circ\text{C}$  and the relative humidity is 6.00%?

[Show Solution](#)

**Strategy:** Use the definition of relative humidity as the ratio of actual vapor density to saturation vapor density. Find the saturation vapor density at  $40.0^\circ\text{C}$  from Table 1, then calculate the actual vapor density.

**Solution:** From Table 1, the saturation vapor density at  $40.0^\circ\text{C}$  is  $51.1 \text{ g/m}^3$ .

The relative humidity is defined as:

$$\text{Relative humidity} = \frac{\text{actual vapor density}}{\text{saturation vapor density}} \times 100$$

Solving for actual vapor density:

$$\text{Actual vapor density} = \text{Relative humidity} \times \text{saturation vapor density}$$

$$\text{Actual vapor density} = 6.00 \times 100 \times 51.1 \text{ g/m}^3 = 3.07 \text{ g/m}^3$$

**Discussion:** Even though the temperature is high ( $40.0^\circ\text{C}$ ), the very low relative humidity (6.00%) means there is very little water vapor in the air. This extremely dry condition is typical of desert environments and explains why deserts feel so dry and why water evaporates very rapidly there. For comparison, at 100% relative humidity and  $40.0^\circ\text{C}$ , the air would contain  $51.1 \text{ g/m}^3$  of water vapor—more than 16 times as much.

A deep-sea diver should breathe a gas mixture that has the same oxygen partial pressure as at sea level, where dry air contains 20.9% oxygen and has a total pressure of  $1.01 \times 10^5 \text{ N/m}^2$ . (a) What is the partial pressure of oxygen at sea level? (b) If the diver breathes a gas mixture at a pressure of  $2.00 \times 10^6 \text{ N/m}^2$ , what percent oxygen should it be to have the same oxygen partial pressure as at sea level?

[Show Solution](#)

$$(a) 2.12 \times 10^4 \text{ Pa} (b) 1.06\%$$

The vapor pressure of water at  $40.0^\circ\text{C}$  is  $7.34 \times 10^3 \text{ N/m}^2$ . Using the ideal gas law, calculate the density of water vapor in  $\text{g/m}^3$  that creates a partial pressure equal to this vapor pressure. The result should be the same as the saturation vapor density at that temperature ( $51.1 \text{ g/m}^3$ ).

[Show Solution](#)

**Strategy:** Use the ideal gas law  $PV = nRT$  to find the number of moles per unit volume, then convert to density in  $\text{g/m}^3$  using the molecular mass of water (18.0 g/mol).

**Solution:**

1. Convert temperature to Kelvin:

$$T = 40.0^\circ\text{C} + 273.15 = 313.15 \text{ K}$$

1. Rearrange ideal gas law to find molar density ( $n/V$ ):

$$nV = PRT = 7.34 \times 10^3 \text{ Pa} (8.31 \text{ J/mol}\cdot\text{K}) (313.15 \text{ K}) = 2.82 \text{ mol/m}^3$$

1. Convert to mass density using molecular mass of water (18.0 g/mol):

$$\rho = (2.82 \text{ mol/m}^3) (18.0 \text{ g/mol}) = 50.8 \text{ g/m}^3$$

**Discussion:** The calculated value of  $50.8 \text{ g/m}^3$  is very close to the tabulated value of  $51.1 \text{ g/m}^3$ , with the small difference due to rounding. This confirms that water vapor behaves as an ideal gas under these conditions and that the ideal gas law accurately predicts the relationship between pressure, temperature, and density for water vapor. This verification demonstrates the consistency between kinetic theory and experimental measurements.

Air in human lungs has a temperature of  $37.0^\circ\text{C}$  and a saturation vapor density of  $44.0 \text{ g/m}^3$ . (a) If 2.00 L of air is exhaled and very dry air inhaled, what is the maximum loss of water vapor by the person? (b) Calculate the partial pressure of water vapor having this density, and compare it with the vapor pressure of  $6.31 \times 10^3 \text{ N/m}^2$ .

[Show Solution](#)

**Strategy:** (a) Calculate mass from density and volume. (b) Use the ideal gas law to find pressure from density.

**Solution:** (a) Volume exhaled:  $V = 2.00 \text{ L} = 2.00 \times 10^{-3} \text{ m}^3$

Maximum water vapor loss (if exhaled air is saturated):

$$m = \rho V = (44.0 \text{ g/m}^3)(2.00 \times 10^{-3} \text{ m}^3) = 8.80 \times 10^{-2} \text{ g}$$

(b) Using  $PV = nRT$  and  $n = m/M$ :

$$P = nRTV = mRTM = \rho RTM$$

Where  $M = 18.0 \text{ g/mol} = 0.0180 \text{ kg/mol}$  for water.

$$P = (44.0 \text{ g/m}^3)(8.31 \text{ J/mol}\cdot\text{K})(310 \text{ K})18.0 \text{ g/mol} = (0.0440 \text{ kg/m}^3)(8.31)(310)0.0180 = 6.30 \times 10^3 \text{ Pa}$$

This is nearly identical to the vapor pressure of  $6.31 \times 10^3 \text{ Pa}$  from Table 1.

**Discussion:** Part (a) shows that with each exhalation of 2 L, we lose about 0.088 g of water vapor if we inhale completely dry air. Over a day with about 20,000 breaths, this could amount to significant water loss (about 880 g or nearly 1 liter). In reality, inhaled air usually contains some moisture, so actual losses are less. Part (b) confirms that the ideal gas law accurately predicts the relationship between vapor density and pressure, validating the table values. This is why breathing passages must stay moist—the lungs saturate inhaled air with water vapor at body temperature.

If the relative humidity is 90.0% on a muggy summer morning when the temperature is  $20.0^\circ\text{C}$ , what will it be later in the day when the temperature is  $30.0^\circ\text{C}$ , assuming the water vapor density remains constant?

[Show Solution](#)

**Strategy:** Find the actual vapor density at the initial conditions, then calculate what relative humidity this represents at the higher temperature using the new saturation vapor density.

**Solution:** From Table 1:

- Saturation vapor density at  $20.0^\circ\text{C} = 17.2 \text{ g/m}^3$
- Saturation vapor density at  $30.0^\circ\text{C} = 30.4 \text{ g/m}^3$

Initial vapor density (which remains constant):

$$\rho_{\text{vapor}} = 90.0100 \times 17.2 \text{ g/m}^3 = 15.5 \text{ g/m}^3$$

Relative humidity at  $30.0^\circ\text{C}$ :

$$\text{Relative humidity} = \frac{15.5 \text{ g/m}^3}{30.4 \text{ g/m}^3} \times 100 = 51.0\%$$

**Discussion:** The relative humidity drops significantly from 90.0% to 51.0% as temperature increases, even though the actual amount of water vapor in the air remains constant. This is because warmer air can hold more water vapor—the saturation vapor density nearly doubles from  $20^\circ\text{C}$  to  $30^\circ\text{C}$ . This explains why muggy mornings often feel less humid (more comfortable) as the day heats up, and why heated indoor air in winter feels very dry even if no moisture is removed.

Late on an autumn day, the relative humidity is 45.0% and the temperature is  $20.0^\circ\text{C}$ . What will the relative humidity be that evening when the temperature has dropped to  $10.0^\circ\text{C}$ , assuming constant water vapor density?

[Show Solution](#)

**Strategy:** Find the actual vapor density at the initial conditions, then calculate what relative humidity this represents at the lower temperature.

**Solution:** From Table 1:

- Saturation vapor density at  $20.0^\circ\text{C} = 17.2 \text{ g/m}^3$
- Saturation vapor density at  $10.0^\circ\text{C} = 9.40 \text{ g/m}^3$

Initial vapor density (which remains constant):

$$\rho_{\text{vapor}} = 45.0100 \times 17.2 \text{ g/m}^3 = 7.74 \text{ g/m}^3$$

Relative humidity at  $10.0^\circ\text{C}$ :

$$\text{Relative humidity} = \frac{7.74 \text{ g/m}^3}{9.40 \text{ g/m}^3} \times 100 = 82.3\%$$

**Discussion:** The relative humidity nearly doubles from 45% to 82% as the temperature drops by just  $10^\circ\text{C}$ , even though the actual amount of water vapor in the air remains constant. This is because cooler air can hold less water vapor at saturation—the saturation vapor density drops from 17.2 to  $9.40 \text{ g/m}^3$ . This is why relative humidity typically rises in the evening as temperatures drop, and why dew often forms overnight when the temperature drops to the dew point. If the temperature dropped a bit more (to about  $5\text{--}6^\circ\text{C}$ ), the air would reach 100% humidity and dew would start to form.

Atmospheric pressure atop Mt. Everest is  $3.30 \times 10^4 \text{ N/m}^2$ . (a) What is the partial pressure of oxygen there if it is 20.9% of the air? (b) What percent oxygen should a mountain climber breathe so that its partial pressure is the same as at sea level, where atmospheric pressure is  $1.01 \times 10^5 \text{ N/m}^2$ ? (c)

One of the most severe problems for those climbing very high mountains is the extreme drying of breathing passages. Why does this drying occur?

[Show Solution](#)

**Strategy:** (a) Calculate the partial pressure using Dalton's law. (b) Find what percentage gives the same partial pressure as at sea level. (c) Consider the relationship between total pressure, partial pressure of water vapor, and relative humidity.

**Solution:** (a) The partial pressure of oxygen at Mt. Everest:

$$P_{O_2} = 0.209 \times 3.30 \times 10^4 \text{ Pa} = 6.90 \times 10^3 \text{ Pa}$$

(b) At sea level, the partial pressure of oxygen is:

$$P_{O_2, \text{sea}} = 0.209 \times 1.01 \times 10^5 \text{ Pa} = 2.11 \times 10^4 \text{ Pa}$$

To achieve this partial pressure at Mt. Everest:

$$\text{Percent O}_2 = \frac{2.11 \times 10^4 \text{ Pa}}{3.30 \times 10^4 \text{ Pa}} \times 100 = 64.0\%$$

(c) The extreme drying occurs because the partial pressure of water vapor is limited by the vapor pressure at the local temperature. At high altitude, the very low total atmospheric pressure means that even at 100% relative humidity, the absolute amount of water vapor is extremely small. When climbers inhale this dry air, their warm, moist breathing passages lose water vapor to humidify the inhaled air, causing severe dehydration of the respiratory tract.

**Discussion:** Part (b) shows why supplemental oxygen is essential for climbing Mt. Everest—the oxygen concentration must be increased to about 64% (compared to 21% at sea level) to maintain the same oxygen partial pressure. The drying effect in part (c) is one of many physiological challenges of high-altitude climbing, contributing to the extreme difficulty and danger of climbing peaks above 8000 m (the “death zone”).

What is the dew point (the temperature at which 100% relative humidity would occur) on a day when relative humidity is 39.0% at a temperature of 20.0°C?

[Show Solution](#)

**Strategy:** Calculate the actual vapor density at current conditions. Then find the temperature at which this vapor density equals the saturation vapor density using Table 1.

**Solution:** From Table 1, saturation vapor density at 20.0°C = 17.2 g/m<sup>3</sup>.

Actual vapor density:

$$\rho_{\text{vapor}} = 39.0100 \times 17.2 \text{ g/m}^3 = 6.71 \text{ g/m}^3$$

Looking at Table 1 for the temperature where saturation vapor density = 6.71 g/m<sup>3</sup>:

- At 5°C: 6.80 g/m<sup>3</sup>
- At 0°C: 4.84 g/m<sup>3</sup>

By interpolation between these values, the dew point is approximately 4.77°C (closer to 5°C).

**Discussion:** The dew point of 4.77°C is the temperature at which the current water vapor in the air would saturate the air (100% relative humidity). If the temperature drops to this point or below, water vapor will begin to condense as dew, fog, or frost. The large difference between the current temperature (20°C) and the dew point (4.77°C) indicates relatively dry conditions—the air is holding only 39% of the maximum water it could hold at 20°C. Meteorologists use dew point as an important indicator: dew points above 20°C feel very humid and uncomfortable, while dew points below 10°C feel dry and comfortable.

On a certain day, the temperature is 25.0°C and the relative humidity is 90.0%. How many grams of water must condense out of each cubic meter of air if the temperature falls to 15.0°C? Such a drop in temperature can, thus, produce heavy dew or fog.

[Show Solution](#)

**Strategy:** First calculate the actual vapor density at 25.0°C with 90.0% relative humidity. Then find the saturation vapor density at 15.0°C. The difference is the amount that must condense.

**Solution:** From Table 1:

- Saturation vapor density at 25.0°C = 23.0 g/m<sup>3</sup>
- Saturation vapor density at 15.0°C = 12.8 g/m<sup>3</sup>

Initial vapor density at 25.0°C:

$$\rho_{\text{initial}} = 90.0100 \times 23.0 \text{ g/m}^3 = 20.7 \text{ g/m}^3$$

When the temperature drops to 15.0°C, the air can hold only 12.8 g/m<sup>3</sup> at 100% relative humidity. The amount that must condense:

$$\Delta\rho=20.7 \text{ g/m}^3 - 12.8 \text{ g/m}^3 = 7.9 \text{ g/m}^3$$

**Discussion:** This significant amount of condensation ( $7.9 \text{ g/m}^3$ ) explains why heavy dew or fog can form when air cools at night. The water vapor in the air exceeds the saturation point at the lower temperature, forcing the excess to condense into liquid droplets. This is why fog often forms in valleys on clear nights when radiative cooling drops the temperature, and why dew forms on grass and car windshields in the early morning.

### Integrated Concepts

The boiling point of water increases with depth because pressure increases with depth. At what depth will fresh water have a boiling point of  $150^\circ\text{C}$ , if the surface of the water is at sea level?

[Show Solution](#)

**Strategy:** Water boils when its vapor pressure equals the surrounding pressure. Find the vapor pressure at  $150^\circ\text{C}$  from Table 1. This pressure must equal atmospheric pressure plus the hydrostatic pressure from the water column. Use  $P = P_{\text{atm}} + \rho gh$  to find depth.

**Solution:** From Table 1, the vapor pressure of water at  $150^\circ\text{C}$  is  $4.76 \times 10^5 \text{ Pa}$ .

At depth  $h$ , the total pressure is:

$$P_{\text{total}} = P_{\text{atm}} + \rho gh$$

Setting this equal to the vapor pressure:

$$4.76 \times 10^5 \text{ Pa} = 1.01 \times 10^5 \text{ Pa} + (1000 \text{ kg/m}^3)(9.8 \text{ m/s}^2)h$$

Solving for  $h$ :

$$h = 4.76 \times 10^5 \text{ Pa} - 1.01 \times 10^5 \text{ Pa} / (1000 \text{ kg/m}^3)(9.8 \text{ m/s}^2) = 3.75 \times 10^5 \text{ Pa} / 9800 \text{ Pa/m} = 38.3 \text{ m}$$

**Discussion:** At a depth of 38.3 meters below the surface, the pressure is high enough (4.76 atm) that water will boil at  $150^\circ\text{C}$  instead of the normal  $100^\circ\text{C}$ . This principle is important in deep-sea hydrothermal vents, where water can remain liquid at temperatures well above  $100^\circ\text{C}$  due to the immense pressure. It's also relevant for geothermal energy systems and industrial processes. The increased boiling point at depth is the same principle used in pressure cookers, but achieved through water depth rather than a sealed container.

### Integrated Concepts

- (a) At what depth in fresh water is the critical pressure of water reached, given that the surface is at sea level? (b) At what temperature will this water boil? (c) Is a significantly higher temperature needed to boil water at a greater depth?

[Show Solution](#)

**Strategy:** (a) From Table 1 in the Phase Changes section, find the critical pressure of water and use the hydrostatic pressure formula to find depth. (b) At the critical temperature, water cannot exist as a liquid. (c) Consider what happens above the critical point.

**Solution:** (a) From the text, the critical pressure of water is  $22.12 \times 10^6 \text{ Pa}$ . At sea level, atmospheric pressure is  $1.01 \times 10^5 \text{ Pa}$ .

The gauge pressure needed is:

$$P_{\text{gauge}} = 22.12 \times 10^6 \text{ Pa} - 1.01 \times 10^5 \text{ Pa} \approx 2.20 \times 10^7 \text{ Pa}$$

Using  $P = \rho gh$  with  $\rho = 1000 \text{ kg/m}^3$  for water:

$$h = P \rho g = 2.20 \times 10^7 \text{ Pa} / (1000 \text{ kg/m}^3)(9.8 \text{ m/s}^2) = 2245 \text{ m}$$

(b) At the critical pressure, water boils at the critical temperature of  $374.3^\circ\text{C}$  (from the text).

(c) No, a higher temperature is not needed at greater depths. Above the critical point (pressure  $> 22.12 \times 10^6 \text{ Pa}$  and temperature  $> 374.3^\circ\text{C}$ ), there is no distinction between liquid and gas phases. Water exists as a supercritical fluid and cannot boil in the traditional sense.

**Discussion:** The depth of about 2.2 km is relatively shallow in oceanic terms (average ocean depth is about 3.7 km). However, at this depth and the critical temperature, water behavior changes fundamentally. This has implications for hydrothermal vents on the ocean floor, where superheated water emerges from Earth's crust.

### Integrated Concepts

To get an idea of the small effect that temperature has on Archimedes' principle, calculate the fraction of a copper block's weight that is supported by the buoyant force in  $0^\circ\text{C}$  water and compare this fraction with the fraction supported in  $95.0^\circ\text{C}$  water.

[Show Solution](#)

$(F_B/w_{Cu})(F_B/w_{Cu})' = 1.02$ . The buoyant force supports nearly the exact same amount of force on the copper block in both circumstances.

### Integrated Concepts

If you want to cook in water at  $150^\circ\text{C}$ , you need a pressure cooker that can withstand the necessary pressure. (a) What pressure is required for the boiling point of water to be this high? (b) If the lid of the pressure cooker is a disk 25.0 cm in diameter, what force must it be able to withstand at this pressure?

Show Solution

**Strategy:** (a) Water boils when its vapor pressure equals the surrounding pressure. Find the vapor pressure of water at  $150^\circ\text{C}$  from Table 1. (b) Calculate the force using  $F = PA$ , but use the gauge pressure (excess above atmospheric) since atmospheric pressure pushes on both sides of the lid.

**Solution:** (a) From Table 1, the vapor pressure of water at  $150^\circ\text{C}$  is  $4.76 \times 10^5 \text{ Pa}$ .

This is the required pressure for water to boil at  $150^\circ\text{C}$ .

(b) The force on the lid is due to the gauge pressure (pressure above atmospheric):

$$P_{\text{gauge}} = 4.76 \times 10^5 \text{ Pa} - 1.01 \times 10^5 \text{ Pa} = 3.75 \times 10^5 \text{ Pa}$$

Area of the lid:

$$A = \pi r^2 = \pi (0.250 \text{ m})^2 = 4.91 \times 10^{-2} \text{ m}^2$$

Force on the lid:

$$F = P_{\text{gauge}} \times A = (3.75 \times 10^5 \text{ Pa})(4.91 \times 10^{-2} \text{ m}^2) = 1.84 \times 10^4 \text{ N}$$

**Discussion:** The required pressure is about 4.7 atm (about 70 psi), which is typical for pressure cookers. The force of 18,400 N (about 4100 pounds or 1.9 tons) is enormous! This is why pressure cooker lids must lock securely and why the seals must be in good condition. This also explains why pressure cookers have safety valves—if the pressure gets too high, the valve releases steam before the force can rupture the cooker. The high pressure allows water to remain liquid at  $150^\circ\text{C}$ , significantly speeding up cooking compared to normal boiling at  $100^\circ\text{C}$ .

### Unreasonable Results

(a) How many moles per cubic meter of an ideal gas are there at a pressure of  $1.00 \times 10^{14} \text{ N/m}^2$  and at  $0^\circ\text{C}$ ? (b) What is unreasonable about this result? (c) Which premise or assumption is responsible?

Show Solution

(a)  $4.41 \times 10^{10} \text{ mol/m}^3$  (b) It's unreasonably large.

(c) At high pressures such as these, the ideal gas law can no longer be applied. As a result, unreasonable answers come up when it is used.

### Unreasonable Results

(a) An automobile mechanic claims that an aluminum rod fits loosely into its hole on an aluminum engine block because the engine is hot and the rod is cold. If the hole is 10.0% bigger in diameter than the  $22.0^\circ\text{C}$  rod, at what temperature will the rod be the same size as the hole? (b) What is unreasonable about this temperature? (c) Which premise is responsible?

Show Solution

**Strategy:** (a) Use thermal expansion to find what temperature would make the hole 10% larger. Since both rod and hole are aluminum, they expand at the same rate, so the mechanic's explanation doesn't work. Calculate anyway to show why. (b) and (c) Analyze the result.

**Solution:** (a) For the hole to be 10.0% larger in diameter:

$$\Delta L = 0.100L$$

Using  $\Delta L = \alpha L \Delta T$ :

$$0.100L = \alpha L \Delta T$$

From Table 1 in the Thermal Expansion section, for aluminum:  $\alpha = 25 \times 10^{-6}/^\circ\text{C}$

$$\Delta T = 0.100\alpha = 0.100 \times 25 \times 10^{-6}/^\circ\text{C} = 4000^\circ\text{C}$$

Temperature of the engine block:

$$T=22.0^\circ\text{C}+4000^\circ\text{C}=4022^\circ\text{C}$$

(b) This temperature is absurdly high—far above the melting point of aluminum (660°C). The engine block would be completely melted and vaporized at this temperature.

(c) The faulty premise is that the engine block is hot while the rod is cold. Since both are made of aluminum, they have the same coefficient of thermal expansion. If they were both at the same temperature initially (when they fit properly), heating only the block would make the hole larger, but the rod would have to be heated by the same amount to expand enough to fit the larger hole. The mechanic's explanation doesn't work because you can't have an aluminum hole expand without an aluminum rod expanding by the same proportion at the same temperature.

**Discussion:** This problem reveals a common misconception about thermal expansion. If the rod fits loosely, it's more likely due to: (1) manufacturing tolerances, (2) wear, (3) the rod and block being made of different materials with different expansion coefficients, or (4) differential temperature if they've been in different environments. The fact that both are aluminum means thermal expansion cannot explain the loose fit if both are at the same temperature.

### Unreasonable Results

The temperature inside a supernova explosion is said to be  $2.00 \times 10^{13}\text{ K}$ . (a) What would the average velocity  $v_{\text{rms}}$  of hydrogen atoms be? (b) What is unreasonable about this velocity? (c) Which premise or assumption is responsible?

Show Solution

(a)  $7.03 \times 10^8\text{ m/s}$  (b) The velocity is too high—it's greater than the speed of light.

(c) The assumption that hydrogen inside a supernova behaves as an ideal gas is responsible, because of the great temperature and density in the core of a star. Furthermore, when a velocity greater than the speed of light is obtained, classical physics must be replaced by relativity, a subject not yet covered.

### Unreasonable Results

Suppose the relative humidity is 80% on a day when the temperature is  $30.0^\circ\text{C}$ . (a) What will the relative humidity be if the air cools to  $25.0^\circ\text{C}$  and the vapor density remains constant? (b) What is unreasonable about this result? (c) Which premise is responsible?

Show Solution

**Strategy:** (a) Find the actual vapor density at  $30.0^\circ\text{C}$  with 80% humidity, then calculate what relative humidity this represents at  $25.0^\circ\text{C}$ . (b) and (c) Analyze whether the result makes physical sense.

**Solution:** (a) From Table 1:

- Saturation vapor density at  $30.0^\circ\text{C} = 30.4\text{ g/m}^3$
- Saturation vapor density at  $25.0^\circ\text{C} = 23.0\text{ g/m}^3$

Actual vapor density (which remains constant):

$$\rho_{\text{vapor}} = 80.0100 \times 30.4\text{ g/m}^3 = 24.3\text{ g/m}^3$$

Relative humidity at  $25.0^\circ\text{C}$ :

$$\text{Relative humidity} = \frac{24.3\text{ g/m}^3}{23.0\text{ g/m}^3} \times 100 = 106\%$$

(b) A relative humidity greater than 100% is impossible. Relative humidity cannot exceed 100% because that represents saturation—the maximum amount of water vapor the air can hold at that temperature.

(c) The faulty premise is that “the vapor density remains constant” when the air cools from  $30.0^\circ\text{C}$  to  $25.0^\circ\text{C}$ . In reality, when air cools below the dew point (the temperature at which it reaches 100% relative humidity), water vapor must condense out.

The dew point for air at  $30.0^\circ\text{C}$  with 80% humidity occurs when the temperature drops to where  $24.3\text{ g/m}^3$  is the saturation density. From Table 1, this occurs at about  $26\text{--}27^\circ\text{C}$ . Therefore, as the air cools below  $27^\circ\text{C}$ , water vapor condenses as dew, fog, or clouds, and the vapor density decreases to match the saturation value at the new temperature.

**Discussion:** This problem illustrates why dew forms on grass and car windshields on cool mornings, why fog forms when warm moist air cools, and why clouds form when rising air expands and cools. The atmosphere cannot maintain water vapor densities above saturation—excess moisture must condense. This is a fundamental principle in meteorology and explains many weather phenomena.

### Glossary

#### dew point

the temperature at which relative humidity is 100%; the temperature at which water starts to condense out of the air

#### saturation

the condition of 100% relative humidity

#### percent relative humidity

the ratio of vapor density to saturation vapor density

#### relative humidity

the amount of water in the air relative to the maximum amount the air can hold



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