Introduction to Geometric Optics

Geometric Optics

Light from this page or screen is formed into an image by the lens of your eye, much as the lens of the camera that made this photograph. Mirrors, like lenses, can also form images that in turn are captured by your eye.

Image seen as a result of reflection of light on a plane smooth surface. (credit: NASA Goddard Photo and Video, via Flickr)

Our lives are filled with light. Through vision, light can evoke spiritual emotions, such as when we view a magnificent sunset or reveal a rainbow breaking through the clouds, amuse us in a theater, or warn us to stop at an intersection. It has innumerable uses beyond vision. Light can carry telephone signals through glass fibers or cook a meal in a solar oven. Life itself could not exist without light’s energy. From photosynthesis in plants to the sun warming a cold-blooded animal, its supply of energy is vital.

Double Rainbow over the bay of Pocitos in Montevideo, Uruguay. (credit: Madrax, Wikimedia Commons)

We already know that visible light is the type of electromagnetic waves to which our eyes respond. That knowledge still leaves many questions regarding the nature of light and vision. What is color, and how do our eyes detect it? Why do diamonds sparkle? How does light travel? How do lenses and mirrors form images? These are but a few of the questions that are answered by the study of optics. Optics is the branch of physics that deals with the behavior of visible light and other electromagnetic waves. In particular, optics is concerned with the generation and propagation of light and its interaction with matter. What we have already learned about the generation of light in our study of heat transfer by radiation will be expanded upon in later topics, especially those on atomic physics. Now, we will concentrate on the propagation of light and its interaction with matter.

It is convenient to divide optics into two major parts based on the size of objects that light encounters. When light interacts with an object that is several times as large as the light's wavelength, its observable behavior is like that of a ray; it does not prominently display its wave characteristics. We call this part of optics "geometric optics." This chapter will concentrate on such situations. When light interacts with smaller objects, it has very prominent wave characteristics, such as constructive and destructive interference. Wave Optics will concentrate on such situations.

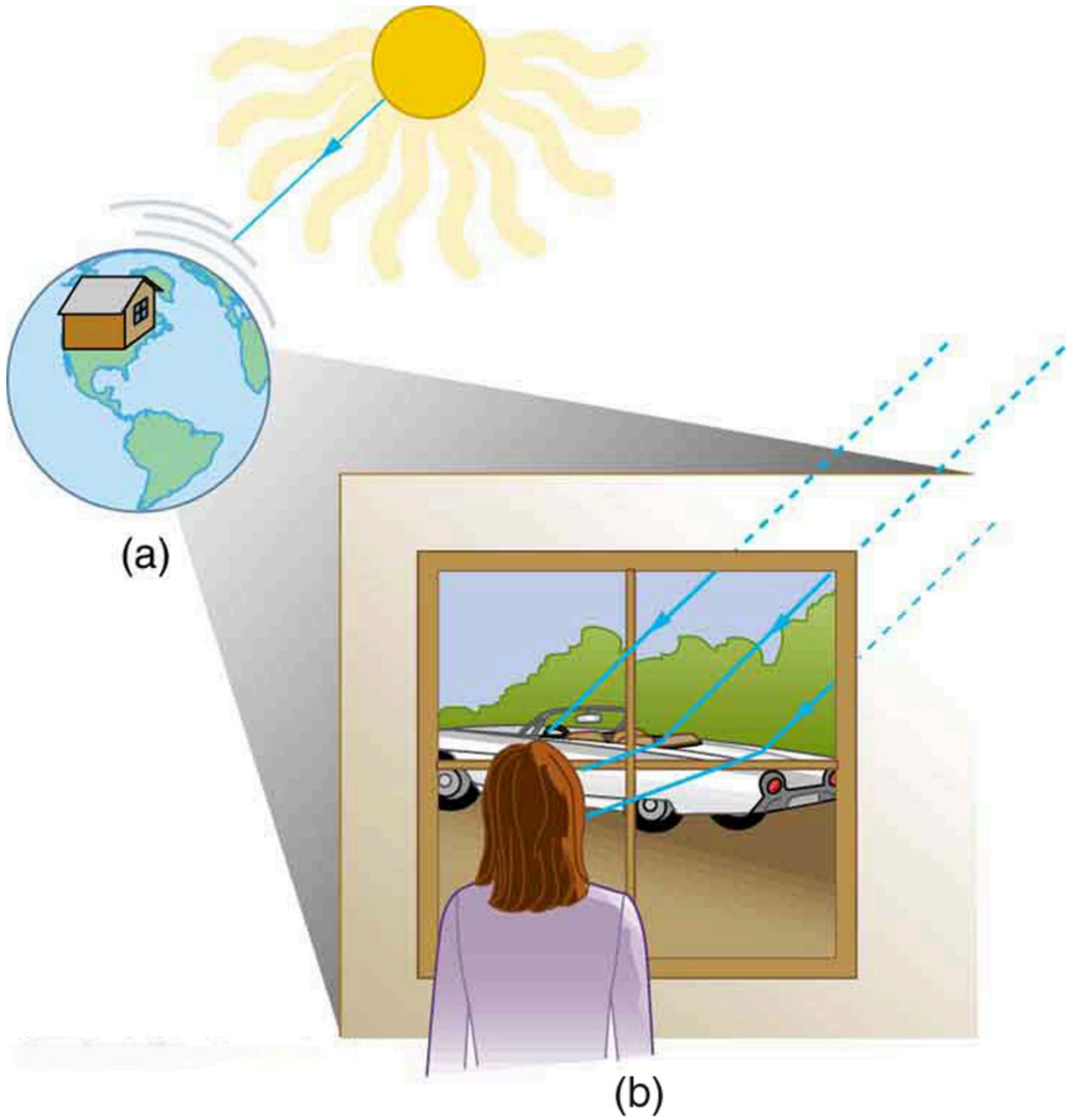
The Ray Aspect of Light

- List the ways by which light travels from a source to another location.

There are three ways in which light can travel from a source to another location. (See [Figure 1].) It can come directly from the source through empty space, such as from the Sun to Earth. Or light can travel through various media, such as air and glass, to the person. Light can also arrive after being reflected, such as by a mirror. In all of these cases, light is modeled as traveling in straight lines called rays. Light may change direction when it encounters objects (such as a mirror) or in passing from one material to another (such as in passing from air to glass), but it then continues in a straight line or as a ray. The word **ray** comes from mathematics and here means a straight line that originates at some point. It is acceptable to visualize light rays as laser rays (or even science fiction depictions of ray guns).

Ray

The word “ray” comes from mathematics and here means a straight line that originates at some point.



Three methods for light to travel from a source to another location. (a) Light reaches the upper atmosphere of Earth traveling through empty space directly from the source. (b) Light can reach a person in one of two ways. It can travel through media like air and glass. It can also reflect from an object like a mirror. In the situations shown here, light interacts with objects large enough that it travels in straight lines, like a ray.

Experiments, as well as our own experiences, show that when light interacts with objects several times as large as its wavelength, it travels in straight lines and acts like a ray. Its wave characteristics are not pronounced in such situations. Since the wavelength of light is less than a micron (a thousandth of a millimeter), it acts like a ray in the many common situations in which it encounters objects larger than a micron. For example, when light encounters anything we can observe with unaided eyes, such as a mirror, it acts like a ray, with only subtle wave characteristics. We will concentrate on the ray characteristics in this chapter.

Since light moves in straight lines, changing directions when it interacts with materials, it is described by geometry and simple trigonometry. This part of optics, where the ray aspect of light dominates, is therefore called **geometric optics**. There are two laws that govern how light changes direction when it interacts with matter. These are the law of reflection, for situations in which light bounces off matter, and the law of refraction, for situations in which light passes through matter.

Geometric Optics

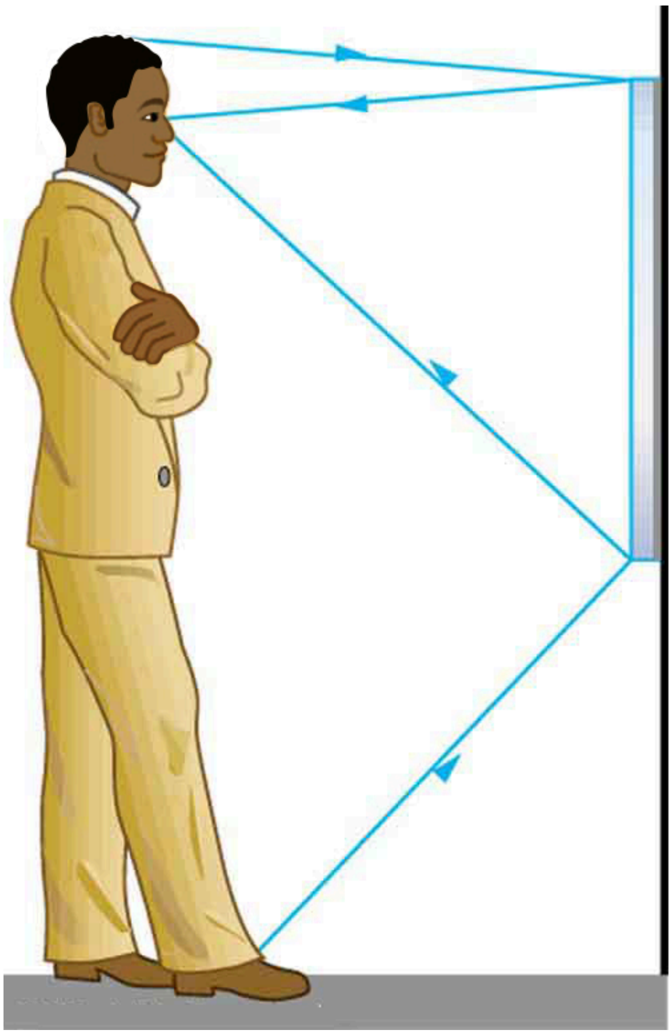
The part of optics dealing with the ray aspect of light is called geometric optics.

Section Summary

- A straight line that originates at some point is called a ray.
- The part of optics dealing with the ray aspect of light is called geometric optics.
- Light can travel in three ways from a source to another location: (1) directly from the source through empty space; (2) through various media; (3) after being reflected from a mirror.

Problems & Exercises

Suppose a man stands in front of a mirror as shown in [Figure 2]. His eyes are 1.65 m above the floor, and the top of his head is 0.13 m higher. Find the height above the floor of the top and bottom of the smallest mirror in which he can see both the top of his head and his feet. How is this distance related to the man's height?



A full-length mirror is one in which you can see all of yourself. It need not be as big as you, and its size is independent of your distance from it.

Show Solution

Top 1.715m from floor, bottom 0.825m from floor. Height of mirror is 0.890m , or precisely one-half the height of the person.

Glossary

ray
: straight line that originates at some point

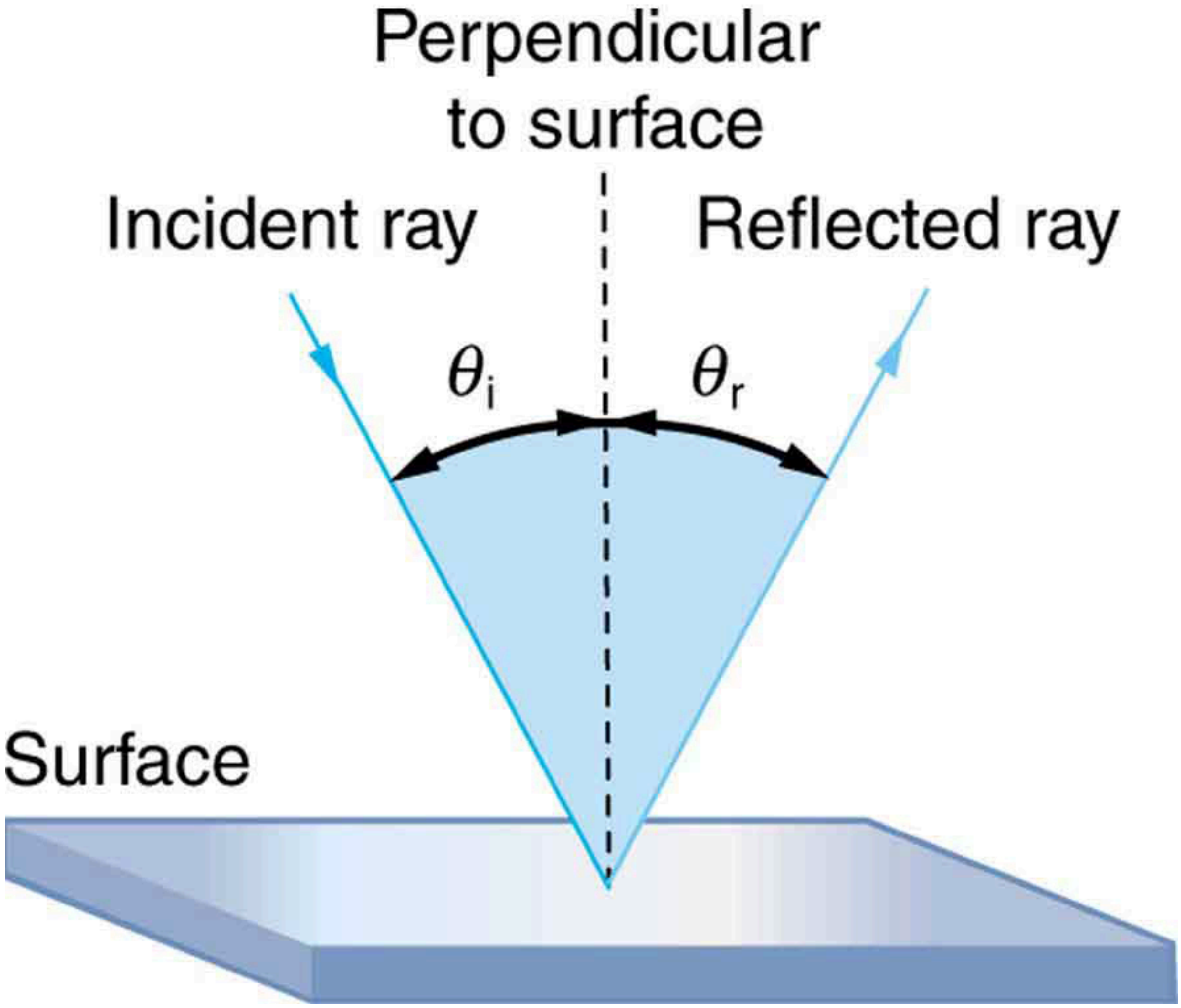
geometric optics
: part of optics dealing with the ray aspect of light

The Law of Reflection

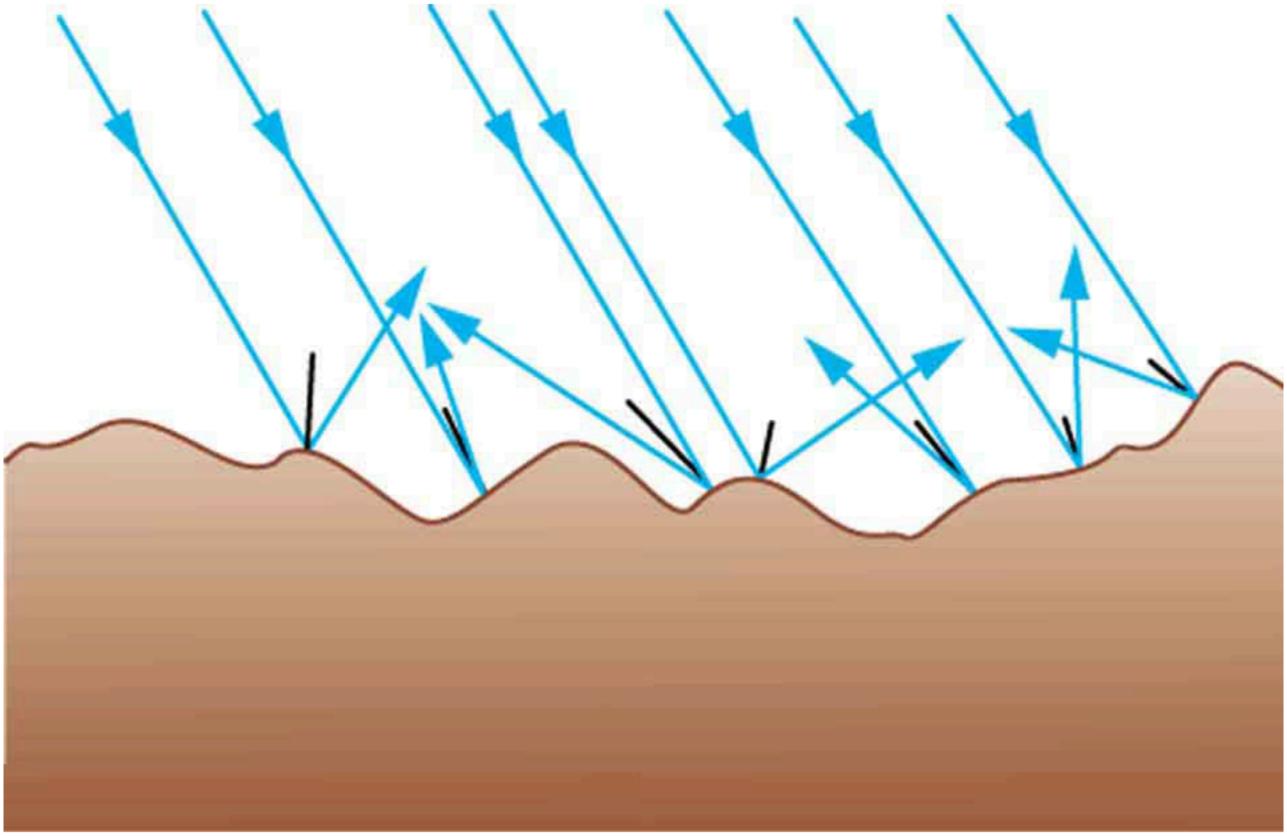
- Explain reflection of light from polished and rough surfaces.

Whenever we look into a mirror, or squint at sunlight glinting from a lake, we are seeing a reflection. When you look at this page, too, you are seeing light reflected from it. Large telescopes use reflection to form an image of stars and other astronomical objects.

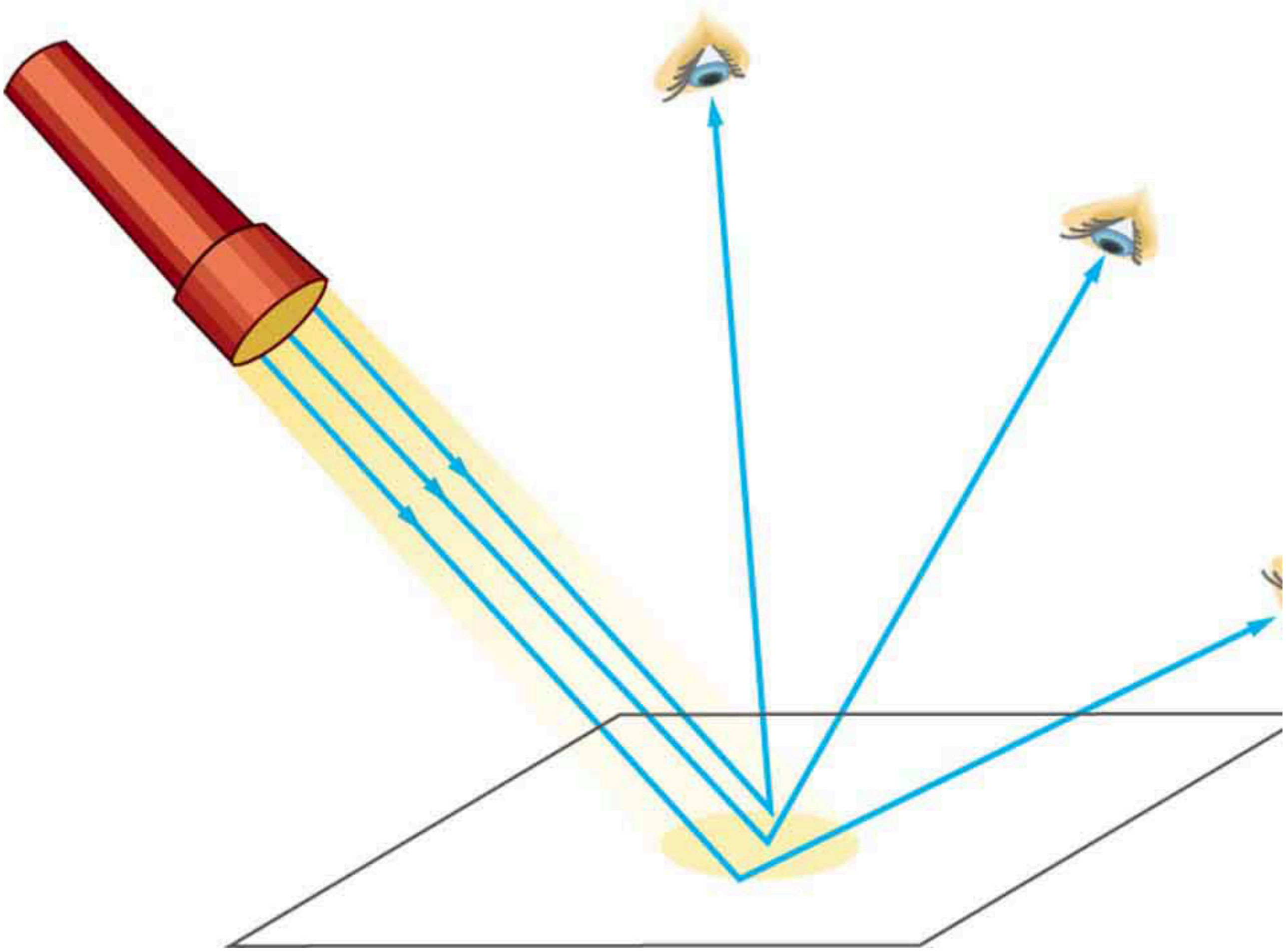
The law of reflection is illustrated in [Figure 1], which also shows how the angles are measured relative to the perpendicular to the surface at the point where the light ray strikes. We expect to see reflections from smooth surfaces, but [Figure 2] illustrates how a rough surface reflects light. Since the light strikes different parts of the surface at different angles, it is reflected in many different directions, or diffused. Diffused light is what allows us to see a sheet of paper from any angle, as illustrated in [Figure 3]. Many objects, such as people, clothing, leaves, and walls, have rough surfaces and can be seen from all sides. A mirror, on the other hand, has a smooth surface (compared with the wavelength of light) and reflects light at specific angles, as illustrated in [Figure 4]. When the moon reflects from a lake, as shown in [Figure 5], a combination of these effects takes place.



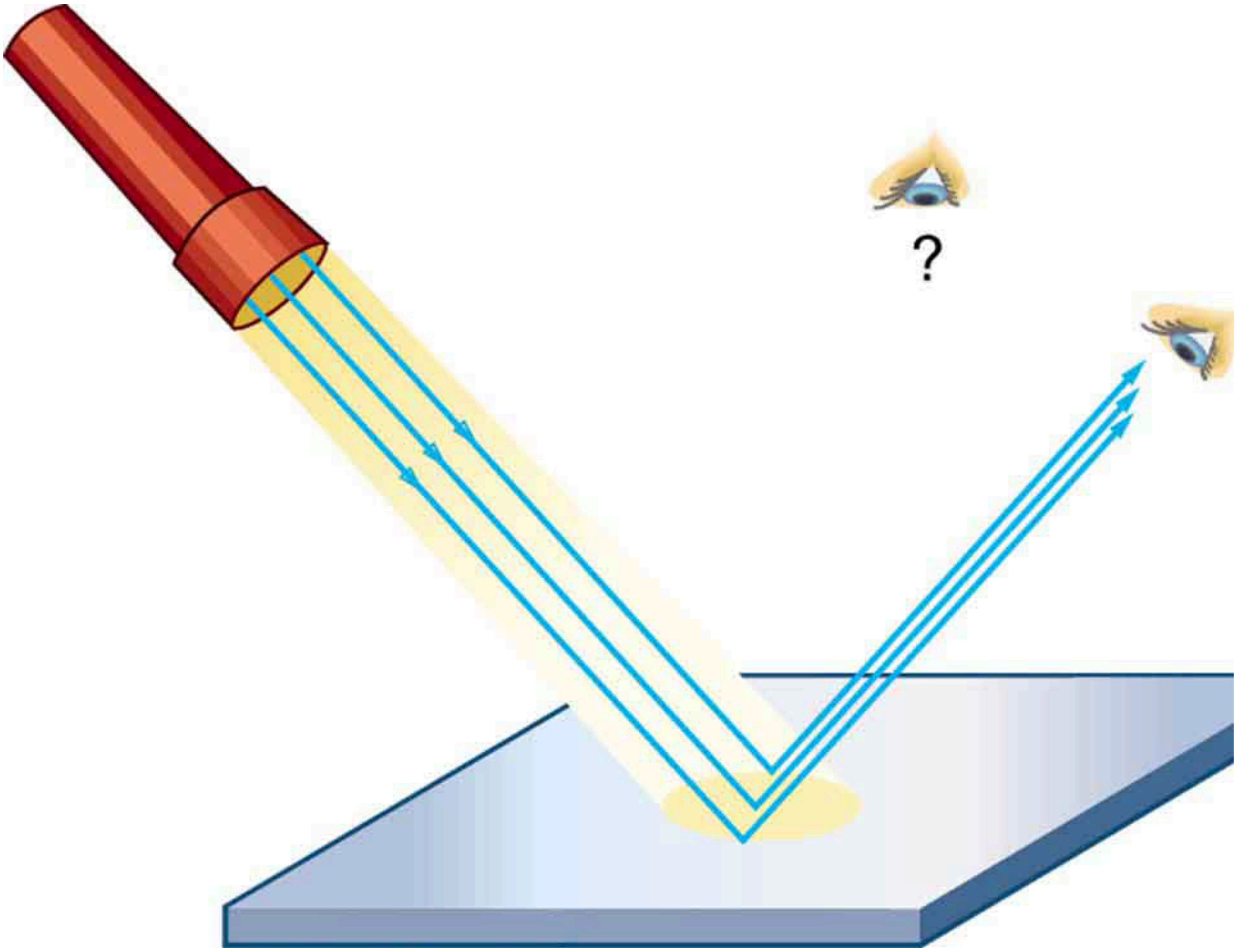
The law of reflection states that the angle of reflection equals the angle of incidence— $\theta_r = \theta_i$. The angles are measured relative to the perpendicular to the surface at the point where the ray strikes the surface.



Light is diffused when it reflects from a rough surface. Here many parallel rays are incident, but they are reflected at many different angles since the surface is rough.



When a sheet of paper is illuminated with many parallel incident rays, it can be seen at many different angles, because its surface is rough and diffuses the light.



A mirror illuminated by many parallel rays reflects them in only one direction, since its surface is very smooth. Only the observer at a particular angle will see the reflected light.

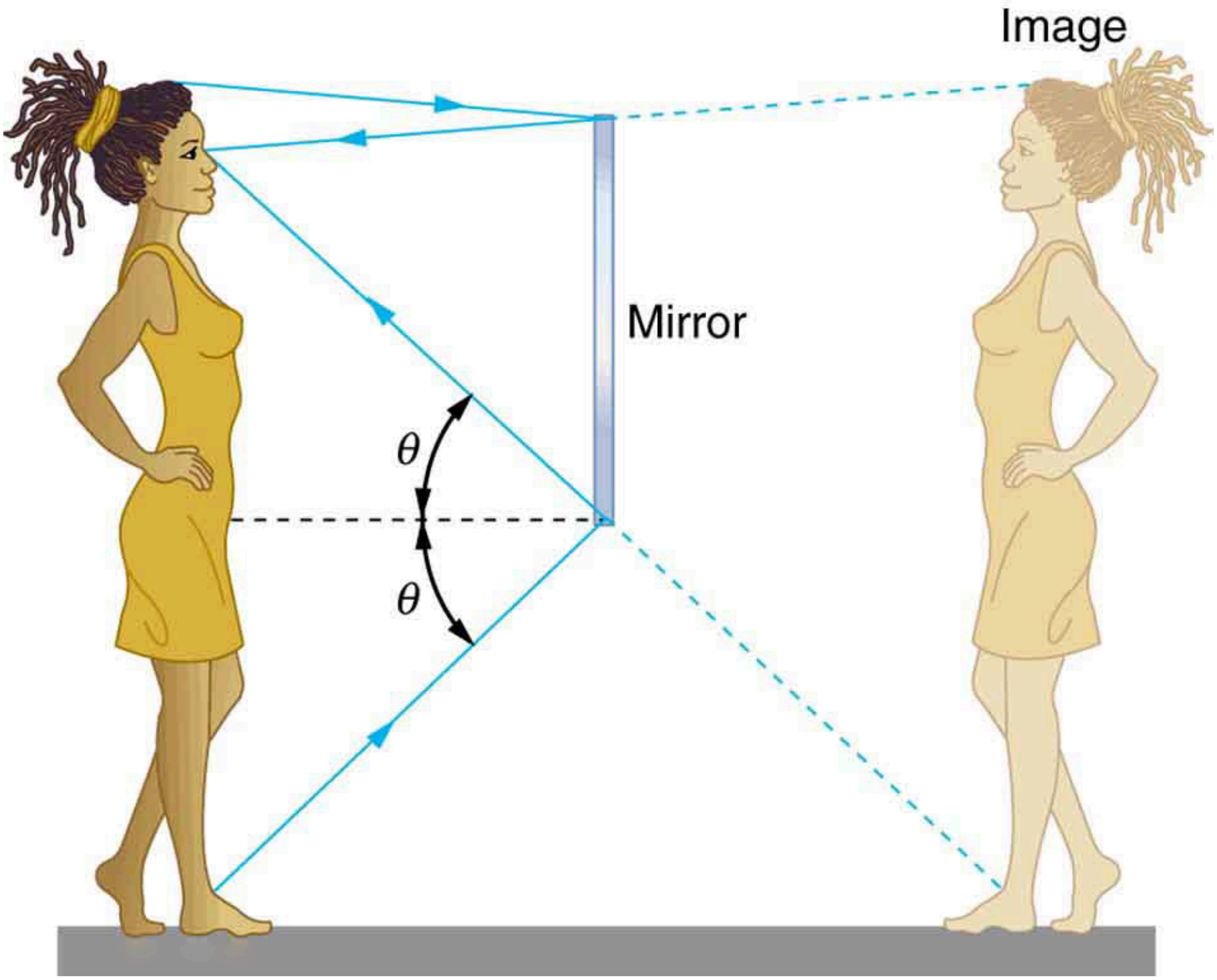
Moonlight is spread out when it is reflected by the lake, since the surface is shiny but uneven. (credit: Diego Torres Silvestre, Flickr)

The law of reflection is very simple: The angle of reflection equals the angle of incidence.

The Law of Reflection

The angle of reflection equals the angle of incidence.

When we see ourselves in a mirror, it appears that our image is actually behind the mirror. This is illustrated in [Figure 6]. We see the light coming from a direction determined by the law of reflection. The angles are such that our image is exactly the same distance behind the mirror as we stand away from the mirror. If the mirror is on the wall of a room, the images in it are all behind the mirror, which can make the room seem bigger. Although these mirror images make objects appear to be where they cannot be (like behind a solid wall), the images are not figments of our imagination. Mirror images can be photographed and videotaped by instruments and look just as they do with our eyes (optical instruments themselves). The precise manner in which images are formed by mirrors and lenses will be treated in later sections of this chapter.



Our image in a mirror is behind the mirror. The two rays shown are those that strike the mirror at just the correct angles to be reflected into the eyes of the person. The image appears to be in the direction the rays are coming from when they enter the eyes.

Take-Home Experiment: Law of Reflection

Take a piece of paper and shine a flashlight at an angle at the paper, as shown in [Figure 3]. Now shine the flashlight at a mirror at an angle. Do your observations confirm the predictions in [Figure 3] and [Figure 4]? Shine the flashlight on various surfaces and determine whether the reflected light is diffuse or not. You can choose a shiny metallic lid of a pot or your skin. Using the mirror and flashlight, can you confirm the law of reflection? You will need to draw lines on a piece of paper showing the incident and reflected rays. (This part works even better if you use a laser pencil.)

Section Summary

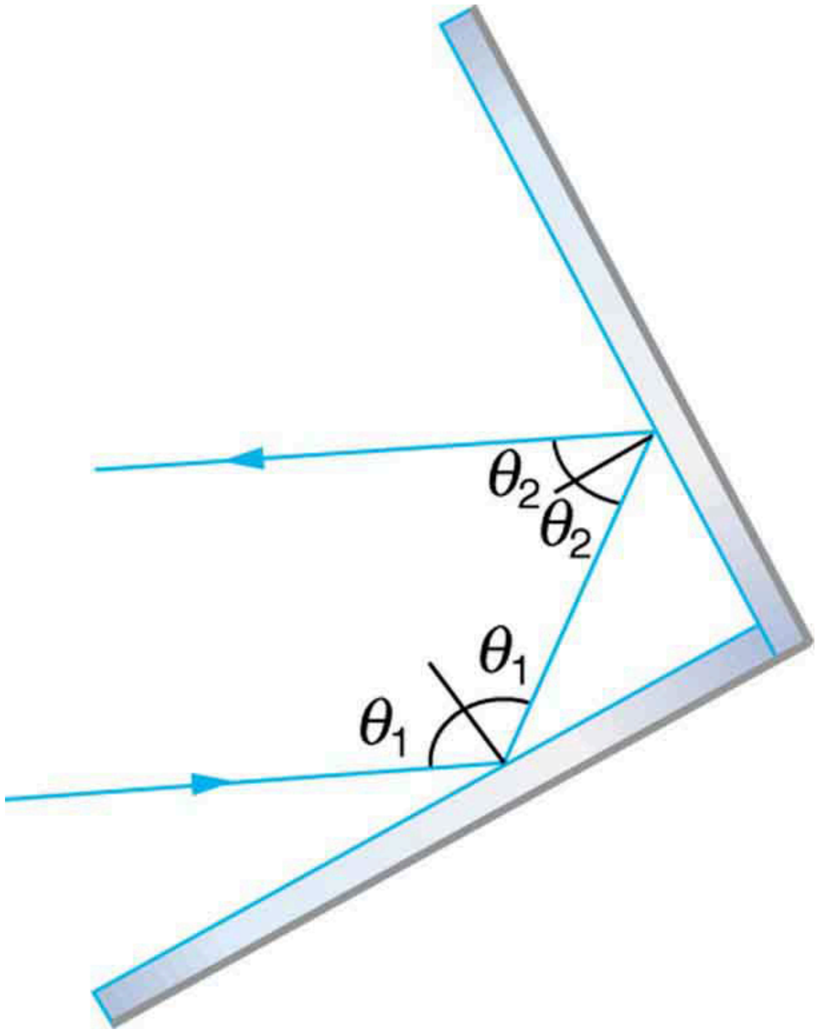
- The angle of reflection equals the angle of incidence.
- A mirror has a smooth surface and reflects light at specific angles.
- Light is diffused when it reflects from a rough surface.
- Mirror images can be photographed and videotaped by instruments.

Conceptual Questions

Using the law of reflection, explain how powder takes the shine off of a person’s nose. What is the name of the optical effect?

Problems & Exercises

Show that when light reflects from two mirrors that meet each other at a right angle, the outgoing ray is parallel to the incoming ray, as illustrated in the following figure.



A corner reflector sends the reflected ray back in a direction parallel to the incident ray, independent of incoming direction.

Show Solution

Strategy

We need to apply the law of reflection (angle of incidence equals angle of reflection) to both mirror surfaces and show that the outgoing ray is parallel to the incoming ray using geometric relationships.

Solution

Consider an incoming ray that strikes the first mirror at an angle θ_1 with respect to the normal to that mirror surface. By the law of reflection, it reflects at the same angle θ_1 on the other side of the normal.

Since the two mirrors meet at a right angle (90°), the normals to the two mirror surfaces are also perpendicular to each other.

After reflection from the first mirror, the ray travels toward the second mirror. The key geometric insight is that the angle the reflected ray from the first mirror makes with the second mirror's surface can be analyzed using the fact that the mirrors are perpendicular.

Let's use a coordinate system where one mirror is vertical and the other is horizontal. If the incoming ray makes an angle α with the horizontal mirror:

- The angle of incidence on the horizontal mirror is α
- By the law of reflection, it reflects at angle α above the normal
- This reflected ray then strikes the vertical mirror at angle $(90\,°-\alpha)$ with respect to the normal of the vertical mirror
- After reflecting from the vertical mirror, the angle of reflection is also $(90\,°-\alpha)$

By geometry, when we trace the path of the ray through both reflections, the final outgoing direction makes the same angle with the horizontal as the incoming ray, but travels in the opposite direction. This means the outgoing ray is parallel to the incoming ray.

More generally, for any incident angle, the sum of the direction changes at both reflections equals 180°, resulting in a ray that is parallel to (but oppositely directed from) the incident ray.

Discussion

This property of perpendicular mirrors is the basis for corner reflectors, which are used in applications like bicycle reflectors, road signs, and even the reflector left on the Moon by astronauts. The incoming and outgoing rays being parallel means that light is reflected back toward its source, regardless of the angle of incidence. This makes corner reflectors extremely effective for safety applications and precise distance measurements.

The result is independent of the incident angle, which is why corner reflectors work effectively even when not perfectly aligned with the light source.

Light shows staged with lasers use moving mirrors to swing beams and create colorful effects. Show that a light ray reflected from a mirror changes direction by 2θ when the mirror is rotated by an angle θ.

Show Solution

Strategy

We apply the law of reflection before and after the mirror is rotated. By comparing the direction of the reflected ray in both cases, we can determine how much the reflected ray's direction changes when the mirror rotates by angle θ.

Solution

Consider a ray incident on a mirror at some initial angle. Let's denote the angle of incidence as α with respect to the normal to the mirror surface.

Initial situation:

- Angle of incidence: α
- Angle of reflection: α (by the law of reflection)
- The incident and reflected rays make an angle of 2α with each other

After rotating the mirror by angle θ:

- The normal to the mirror surface also rotates by θ
- The new angle of incidence becomes: $\alpha + \theta$
- The new angle of reflection is: $\alpha + \theta$

The change in the direction of the reflected ray: The original reflected ray made an angle α with the original normal. The new reflected ray makes an angle $\alpha + \theta$ with the new normal, which itself has rotated by θ.

Relative to a fixed reference direction, the original reflected ray was at angle α from the original normal. The new reflected ray is at angle $(\alpha + \theta)$ from the new normal, which is at angle θ from the original normal.

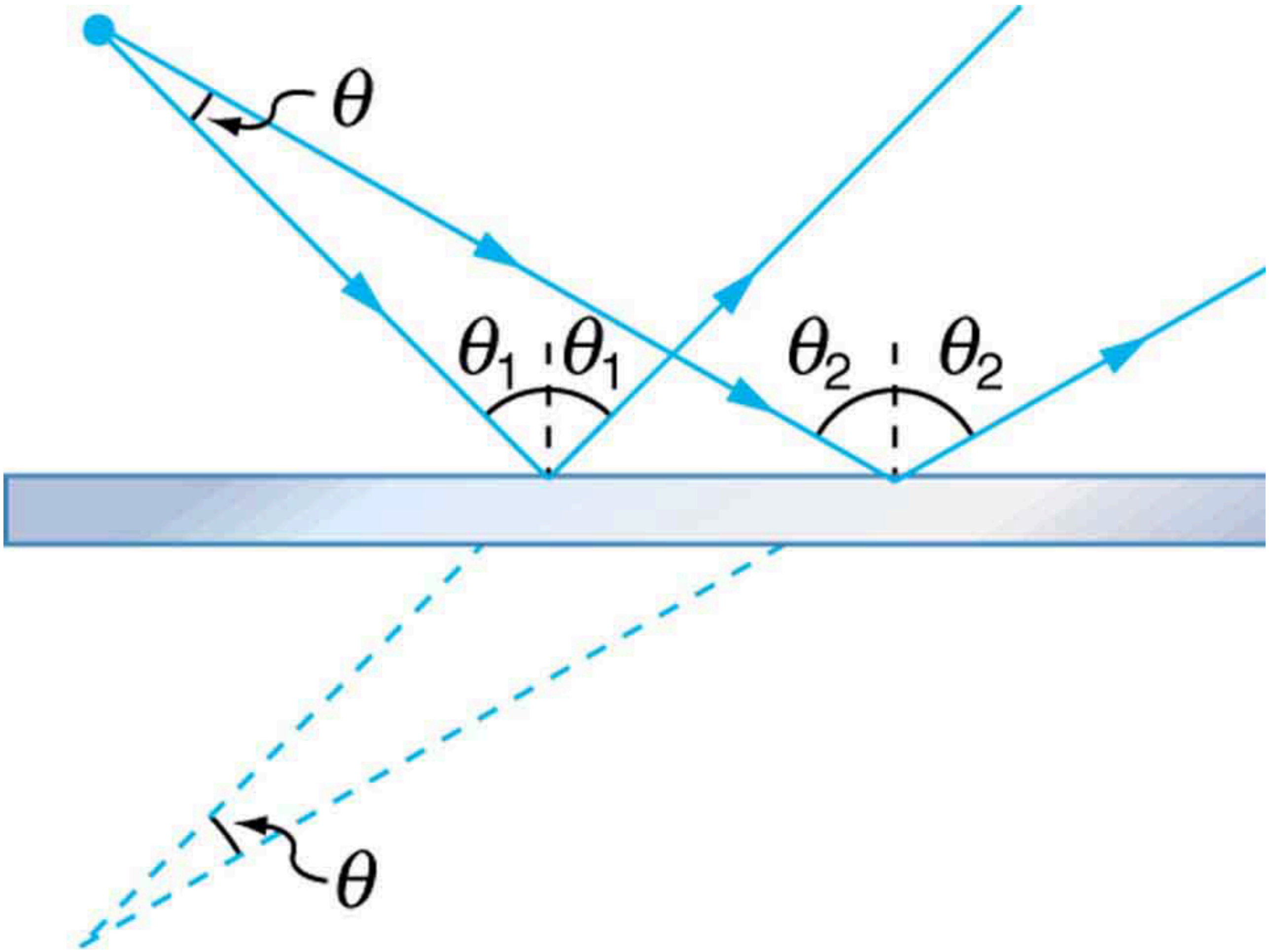
Therefore, the total change in the reflected ray's direction is: $\Delta = \theta + (\alpha + \theta) - \alpha = 2\theta$

Discussion

This result shows that rotating a mirror by angle θ causes the reflected beam to rotate by 2θ. This principle is exploited in laser light shows where mirrors mounted on galvanometers (devices that can rotate precisely based on electrical signals) are used to rapidly sweep laser beams across a screen or through space. By rotating the mirror through a small angle, the laser beam moves through twice that angle, providing amplified motion.

This 2:1 relationship between mirror rotation and beam deflection is also used in optical scanners, laser printers, and many other optical instruments where precise beam steering is required. It's an important consideration when designing such systems, as it means small vibrations or imperfections in mirror positioning are magnified in the output beam.

A flat mirror is neither converging nor diverging. To prove this, consider two rays originating from the same point and diverging at an angle θ . Show that after striking a plane mirror, the angle between their directions remains θ.



A flat mirror neither converges nor diverges light rays. Two rays continue to diverge at the same angle after reflection.

Show Solution

Strategy

We consider two rays emanating from a common point and striking a flat mirror at different locations. By applying the law of reflection to each ray and using geometric principles, we show that the angle between the reflected rays equals the angle between the incident rays.

Solution

Let two rays originate from a common point S and strike a flat mirror at points A and B. Let the angle between these two incident rays be θ.

For ray 1 (striking at point A):

- Let the angle of incidence be α_1 (measured from the normal)
- By the law of reflection, the angle of reflection is also α_1

For ray 2 (striking at point B):

- Let the angle of incidence be α_2 (measured from the normal)
- By the law of reflection, the angle of reflection is also α_2

Since the mirror is flat (plane), the normal at every point on the mirror is parallel to the normal at every other point. This means both normals are parallel.

The angle between the two incident rays is: $\theta = |\alpha_1 - \alpha_2|$

For the reflected rays, using the law of reflection and the fact that the normals are parallel:

- Ray 1 reflects at angle α_1 on the opposite side of the normal
- Ray 2 reflects at angle α_2 on the opposite side of the normal

By geometric construction, since both rays reflect according to the same law and the normals are parallel, the angle between the reflected rays is also: $\theta' = |\alpha_1 - \alpha_2| = \theta$

This can be verified by noting that the reflected rays, when extended backward, appear to originate from a virtual image point that is the same distance behind the mirror as the object point is in front of it. The geometry ensures that the angle between the rays remains unchanged.

Discussion

This result confirms that a flat (plane) mirror neither converges nor diverges light rays—it preserves the angle between rays. This is fundamentally different from curved mirrors:

- A concave (converging) mirror would cause the angle between the reflected rays to be smaller than θ, bringing the rays closer together
- A convex (diverging) mirror would cause the angle between the reflected rays to be larger than θ, spreading the rays farther apart

The fact that a plane mirror preserves angles means that the image formed is the same size as the object and appears to be the same distance behind the mirror as the object is in front of it. This property makes flat mirrors ideal for applications where undistorted images are needed, such as bathroom mirrors, dressing mirrors, and periscopes.

Glossary

mirror
: smooth surface that reflects light at specific angles, forming an image of the person or object in front of it

law of reflection
: angle of reflection equals the angle of incidence

The Law of Refraction

- Determine the index of refraction, given the speed of light in a medium.

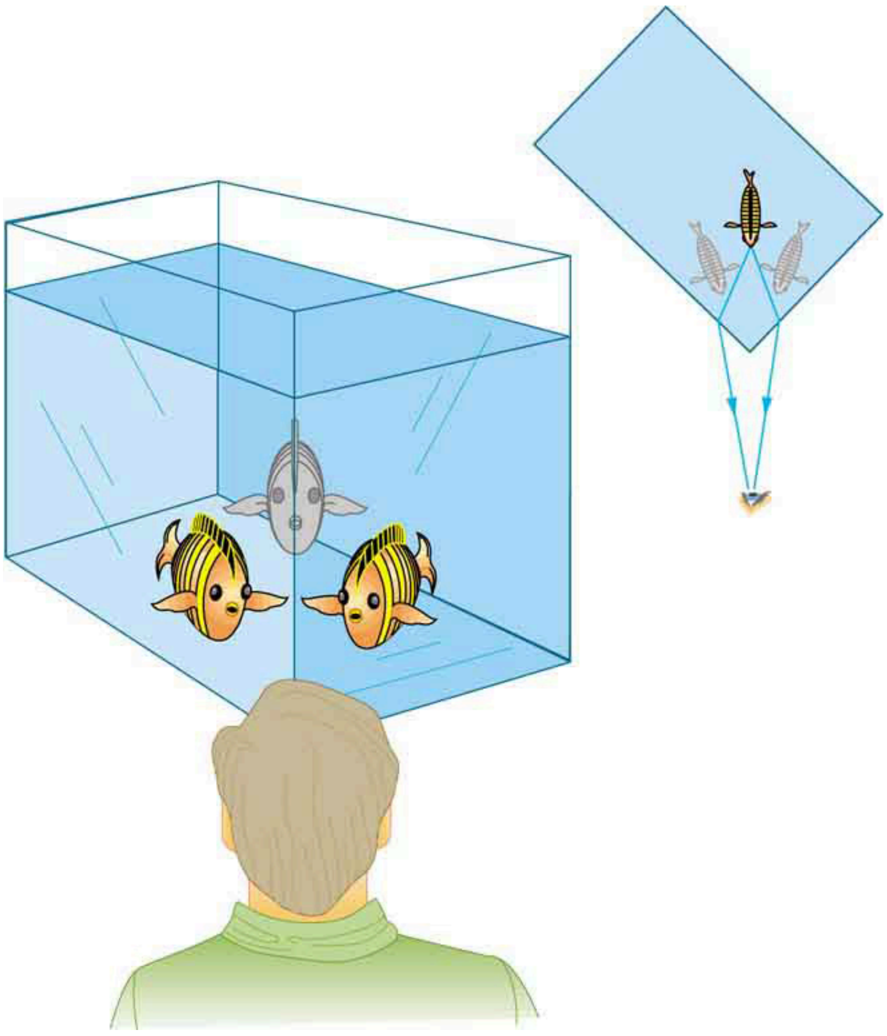
It is easy to notice some odd things when looking into a fish tank. For example, you may see the same fish appearing to be in two different places. (See [Figure 1].) This is because light coming from the fish to us changes direction when it leaves the tank, and in this case, it can travel two different paths to get to our eyes. The changing of a light ray's direction (loosely called bending) when it passes through variations in matter is called **refraction**. Refraction is responsible for a tremendous range of optical phenomena, from the action of lenses to voice transmission through optical fibers.

Refraction

The changing of a light ray's direction (loosely called bending) when it passes through variations in matter is called refraction.

Speed of Light

The speed of light c not only affects refraction, it is one of the central concepts of Einstein's theory of relativity. As the accuracy of the measurements of the speed of light were improved, c was found not to depend on the velocity of the source or the observer. However, the speed of light does vary in a precise manner with the material it traverses. These facts have far-reaching implications, as we will see in Special Relativity. It makes connections between space and time and alters our expectations that all observers measure the same time for the same event, for example. The speed of light is so important that its value in a vacuum is one of the most fundamental constants in nature as well as being one of the four fundamental SI units.

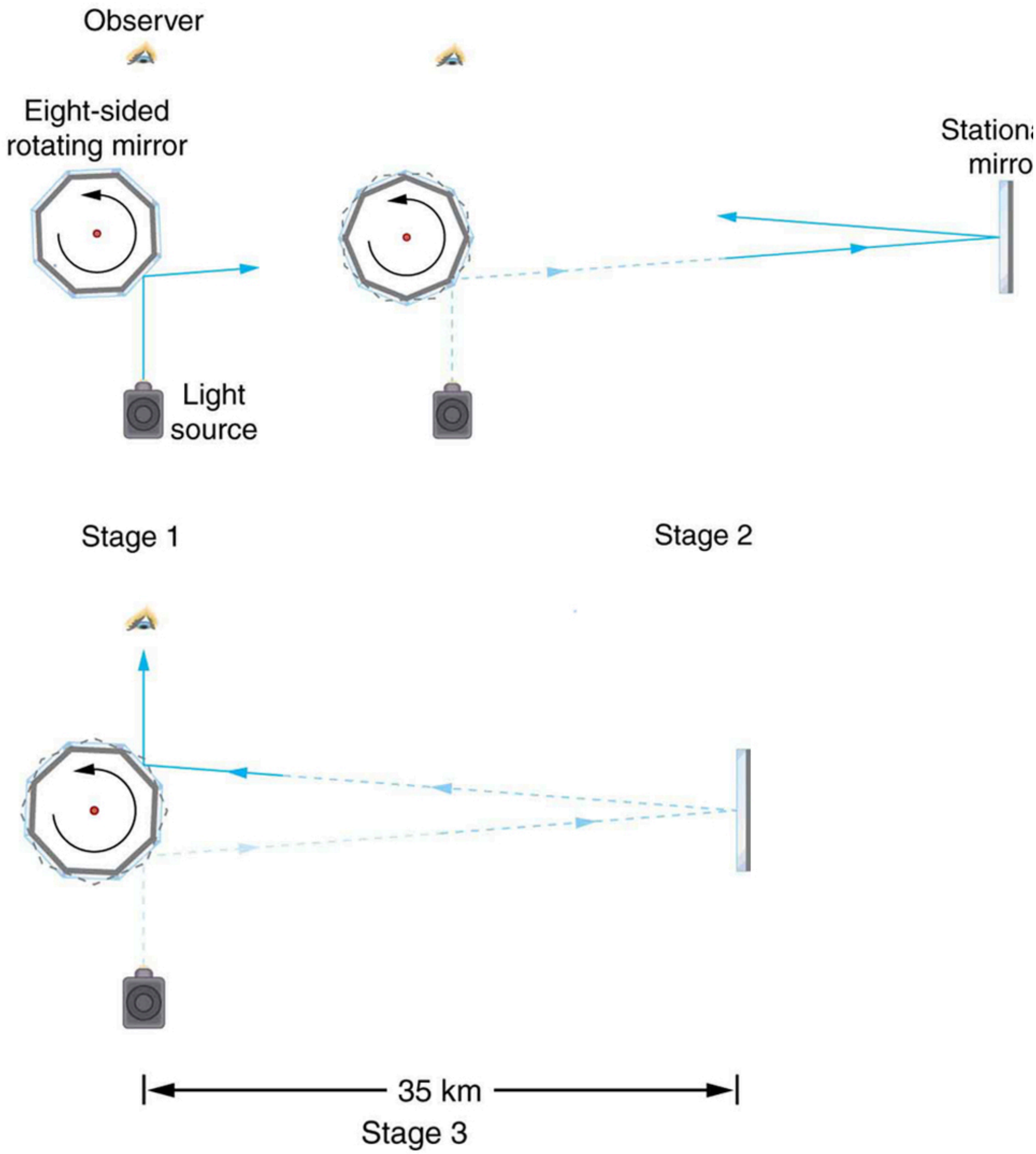


Looking at the fish tank as shown, we can see the same fish in two different locations, because light changes directions when it passes from water to air. In this case, the light can reach the observer by two different paths, and so the fish seems to be in two different places. This bending of light is called refraction and is responsible for many optical phenomena.

Why does light change direction when passing from one material (medium) to another? It is because light changes speed when going from one material to another. So before we study the law of refraction, it is useful to discuss the speed of light and how it varies in different media.

The Speed of Light

Early attempts to measure the speed of light, such as those made by Galileo, determined that light moved extremely fast, perhaps instantaneously. The first real evidence that light traveled at a finite speed came from the Danish astronomer Ole Roemer in the late 17th century. Roemer had noted that the average orbital period of one of Jupiter’s moons, as measured from Earth, varied depending on whether Earth was moving toward or away from Jupiter. He correctly concluded that the apparent change in period was due to the change in distance between Earth and Jupiter and the time it took light to travel this distance. From his 1676 data, a value of the speed of light was calculated to be $2.26 \times 10^{8} \text{m/s}$ (only 25% different from today’s accepted value). In more recent times, physicists have measured the speed of light in numerous ways and with increasing accuracy. One particularly direct method, used in 1887 by the American physicist Albert Michelson (1852–1931), is illustrated in [Figure 2]. Light reflected from a rotating set of mirrors was reflected from a stationary mirror 35 km away and returned to the rotating mirrors. The time for the light to travel can be determined by how fast the mirrors must rotate for the light to be returned to the observer’s eye.



A schematic of early apparatus used by Michelson and others to determine the speed of light. As the mirrors rotate, the reflected ray is only briefly directed at the stationary mirror. The returning ray will be reflected into the observer's eye only if the next mirror has rotated into the correct position just as the ray returns. By measuring the correct rotation rate, the time for the round trip can be measured and the speed of light calculated. Michelson’s calculated value of the speed of light was only 0.04% different from the value used today.

The speed of light is now known to great precision. In fact, the speed of light in a vacuum c is so important that it is accepted as one of the basic physical quantities and has the fixed value

$$c= 2.99792458 \times 10^{8} \text{m/s}\approx 3.00 \times 10^{8} \text{m/s} \text{,}$$

where the approximate value of $3.00 \times 10^{8} \text{m/s}$ is used whenever three-digit accuracy is sufficient. The speed of light through matter is less than it is in a vacuum, because light interacts with atoms in a material. The speed of light depends strongly on the type of material, since its interaction with different atoms, crystal lattices, and other substructures varies. We define the **index of refraction** n of a material to be

$$n=\frac{c}{v},$$

where v is the observed speed of light in the material. Since the speed of light is always less than c in matter and equals c only in a vacuum, the index of refraction is always greater than or equal to one.

Value of the Speed of Light
$$c= 2.99792458 \times 10^{8} \text{m/s}\approx 3.00 \times 10^{8} \text{m/s}$$
Index of Refraction
$$n=\frac{c}{v}$$

That is, $n\ge 1$. [Table 1] gives the indices of refraction for some representative substances. The values are listed for a particular wavelength of light, because they vary slightly with wavelength. (This can have important effects, such as colors produced by a prism.) Note that for gases, n is close to 1.0. This seems reasonable, since atoms in gases are widely separated and light travels at c in the vacuum between atoms. It is common to take $n=1$ for gases unless great precision is needed. Although the speed of light v in a medium varies considerably from its value c in a vacuum, it is still a large speed.

Index of Refraction in Various Media

Medium	***n***
Gases at $0^\circ C$, 1 atm	
Air	1.000293
Carbon dioxide	1.00045
Hydrogen	1.000139
Oxygen	1.000271
Liquids at $20^\circ C$	
Benzene	1.501
Carbon disulfide	1.628
Carbon tetrachloride	1.461
Ethanol	1.361
Glycerine	1.473
Water, fresh	1.333
Solids at $20^\circ C$	
Diamond	2.419
Fluorite	1.434
Glass, crown	1.52
Glass, flint	1.66
Ice at $-7^\circ C$	1.309
Polystyrene	1.49
Plexiglas	1.51
Quartz, crystalline	1.544
Quartz, fused	1.458
Sodium chloride	1.544
Zircon	1.923

Speed of Light in Matter

Calculate the speed of light in zircon, a material used in jewelry to imitate diamond.

Strategy

The speed of light in a material, v , can be calculated from the index of refraction n of the material using the equation $n=c/v$.

Solution

The equation for index of refraction states that $n=c/v$. Rearranging this to determine v gives

$$v=\frac{c}{n}.$$

The index of refraction for zircon is given as 1.923 in [Table 1], and c is given in the equation for speed of light. Entering these values in the last expression gives

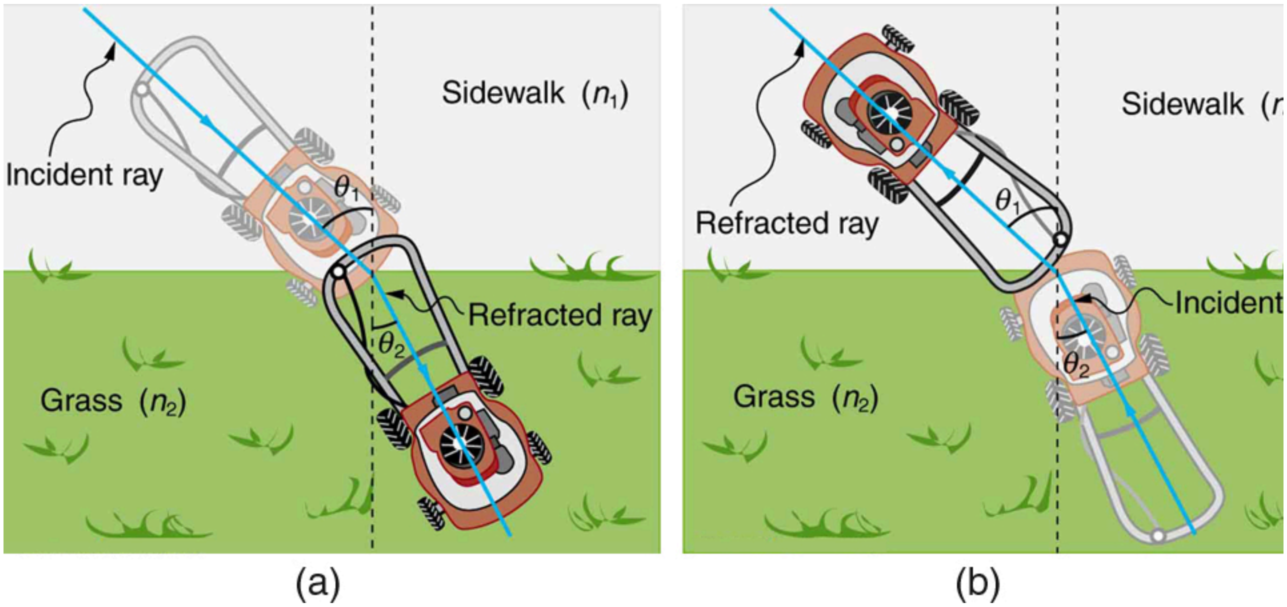
$$\begin{array}{ccc}v& =& \frac{ 3.00 \times 10^{8} \text{m/s}}{ 1.923}\\ & =& 1.56 \times 10^{8} \text{m/s}\text{.}\end{array}$$

Discussion

This speed is slightly larger than half the speed of light in a vacuum and is still high compared with speeds we normally experience. The only substance listed in [Table 1] that has a greater index of refraction than zircon is diamond. We shall see later that the large index of refraction for zircon makes it sparkle more than glass, but less than diamond.

Law of Refraction

[Figure 3] shows how a ray of light changes direction when it passes from one medium to another. As before, the angles are measured relative to a perpendicular to the surface at the point where the light ray crosses it. (Some of the incident light will be reflected from the surface, but for now we will concentrate on the light that is transmitted.) The change in direction of the light ray depends on how the speed of light changes. The change in the speed of light is related to the indices of refraction of the media involved. In the situations shown in [Figure 3], medium 2 has a greater index of refraction than medium 1. This means that the speed of light is less in medium 2 than in medium 1. Note that as shown in [Figure 3](a), the direction of the ray moves closer to the perpendicular when it slows down. Conversely, as shown in [Figure 3](b), the direction of the ray moves away from the perpendicular when it speeds up. The path is exactly reversible. In both cases, you can imagine what happens by thinking about pushing a lawn mower from a footpath onto grass, and vice versa. Going from the footpath to grass, the front wheels are slowed and pulled to the side as shown. This is the same change in direction as for light when it goes from a fast medium to a slow one. When going from the grass to the footpath, the front wheels can move faster and the mower changes direction as shown. This, too, is the same change in direction as for light going from slow to fast.



The change in direction of a light ray depends on how the speed of light changes when it crosses from one medium to another. The speed of light is greater in medium 1 than in medium 2 in the situations shown here. (a) A ray of light moves closer to the perpendicular when it slows down. This is analogous to what happens when a lawn mower goes from a footpath to grass. (b) A ray of light moves away from the perpendicular when it speeds up. This is analogous to what happens when a lawn mower goes from grass to footpath. The paths are exactly reversible.

The amount that a light ray changes its direction depends both on the incident angle and the amount that the speed changes. For a ray at a given incident angle, a large change in speed causes a large change in direction, and thus a large change in angle. The exact mathematical relationship is the **law of refraction**, or “Snell’s Law,” which is stated in equation form as

$${n}_{1} \sin {\theta }_{1}={n}_{2} \sin {\theta }_{2}. $$

Here ${n}_{1}$ and ${n}_{2}$ are the indices of refraction for medium 1 and 2, and ${\theta }_{1}$ and ${\theta }_{2}$ are the angles between the rays and the perpendicular in medium 1 and 2, as shown in [Figure 3] . The incoming ray is called the incident ray and the outgoing ray the refracted ray, and the associated angles the incident angle and the refracted angle. The law of refraction is also called Snell’s law after the Dutch mathematician Willebrord Snell (1591–1626), who discovered it in 1621. Snell’s experiments showed that the law of refraction was obeyed and that a characteristic index of refraction n could be assigned to a given medium. Snell was not aware that the speed of light varied in different media, but through experiments he was able to determine indices of refraction from the way light rays changed direction.

The Law of Refraction
$${n}_{1} \sin {\theta }_{1}={n}_{2} \sin {\theta }_{2} $$
Take-Home Experiment: A Broken Pencil

A classic observation of refraction occurs when a pencil is placed in a glass half filled with water. Do this and observe the shape of the pencil when you look at the pencil sideways, that is, through air, glass, water. Explain your observations. Draw ray diagrams for the situation.

Determine the Index of Refraction from Refraction Data

Find the index of refraction for medium 2 in [Figure 3](a), assuming medium 1 is air and given the incident angle is $30.0º$ and the angle of refraction is $22.0º$.

Strategy

The index of refraction for air is taken to be 1 in most cases (and up to four significant figures, it is 1.000). Thus ${n}_{1}= 1.00$ here. From the given information, ${\theta }_{1}= 30.0º$ and ${\theta }_{2}= 22.0º$. With this information, the only unknown in Snell's law is ${n}_{2}$, so that it can be used to find this unknown.

Solution

Snell's law is

$${n}_{1} \sin {\theta }_{1}={n}_{2} \sin {\theta }_{2}. $$

Rearranging to isolate ${n}_{2}$ gives

$${n}_{2}={n}_{1}\frac{\sin {\theta }_{1}}{\sin {\theta }_{2}}. $$

Entering known values,

$$\begin{array}{ll}{n}_{2}& =& 1.00\frac{\sin 30.0º}{\sin 22.0º}=\frac{ 0.500}{ 0.375}\\ & =& 1.33.\end{array} $$

Discussion

This is the index of refraction for water, and Snell could have determined it by measuring the angles and performing this calculation. He would then have found 1.33 to be the appropriate index of refraction for water in all other situations, such as when a ray passes from water to glass. Today we can verify that the index of refraction is related to the speed of light in a medium by measuring that speed directly.
A Larger Change in Direction

Suppose that in a situation like that in [Example 2], light goes from air to diamond and that the incident angle is $30.0º$. Calculate the angle of refraction ${\theta }_{2}$ in the diamond.

Strategy

Again the index of refraction for air is taken to be ${n}_{1}= 1.00$, and we are given ${\theta }_{1}= 30.0º$. We can look up the index of refraction for diamond in [Table 1], finding ${n}_{2}= 2.419$. The only unknown in Snell's law is ${\theta }_{2}$, which we wish to determine.

Solution

Solving Snell's law for $\sin {\theta }_{2}$ yields

$$\sin {\theta }_{2}=\frac{ {n}_{1}}{ {n}_{2}}\sin {\theta }_{1}. $$

Entering known values,

$$\sin {\theta }_{2}=\frac{ 1.00}{ 2.419}\sin 30.0º=\left(0.413\right)\left(0.500\right)= 0.207. $$

The angle is thus

$${\theta }_{2}={\sin}^{-1} 0.207= 11.9º. $$

Discussion

For the same $30º$ angle of incidence, the angle of refraction in diamond is significantly smaller than in water ($11.9º$ rather than $22º$ —see the preceding example). This means there is a larger change in direction in diamond. The cause of a large change in direction is a large change in the index of refraction (or speed). In general, the larger the change in speed, the greater the effect on the direction of the ray.

Section Summary

- The changing of a light ray's direction when it passes through variations in matter is called refraction.
- The speed of light in vacuum $c= 2.99792458 \times 10^{8} \text{m/s}\approx 3.00 \times 10^{8} \text{m/s}.$ - Index of refraction $n=\frac{c}{v}$, where v is the speed of light in the material, c is the speed of light in vacuum, and n is the index of refraction.
- Snell's law, the law of refraction, is stated in equation form as ${n}_{1} \sin {\theta }_{1}={n}_{2} \sin {\theta }_{2}$.

Conceptual Questions

Diffusion by reflection from a rough surface is described in this chapter. Light can also be diffused by refraction. Describe how this occurs in a specific situation, such as light interacting with crushed ice.

Why is the index of refraction always greater than or equal to 1?

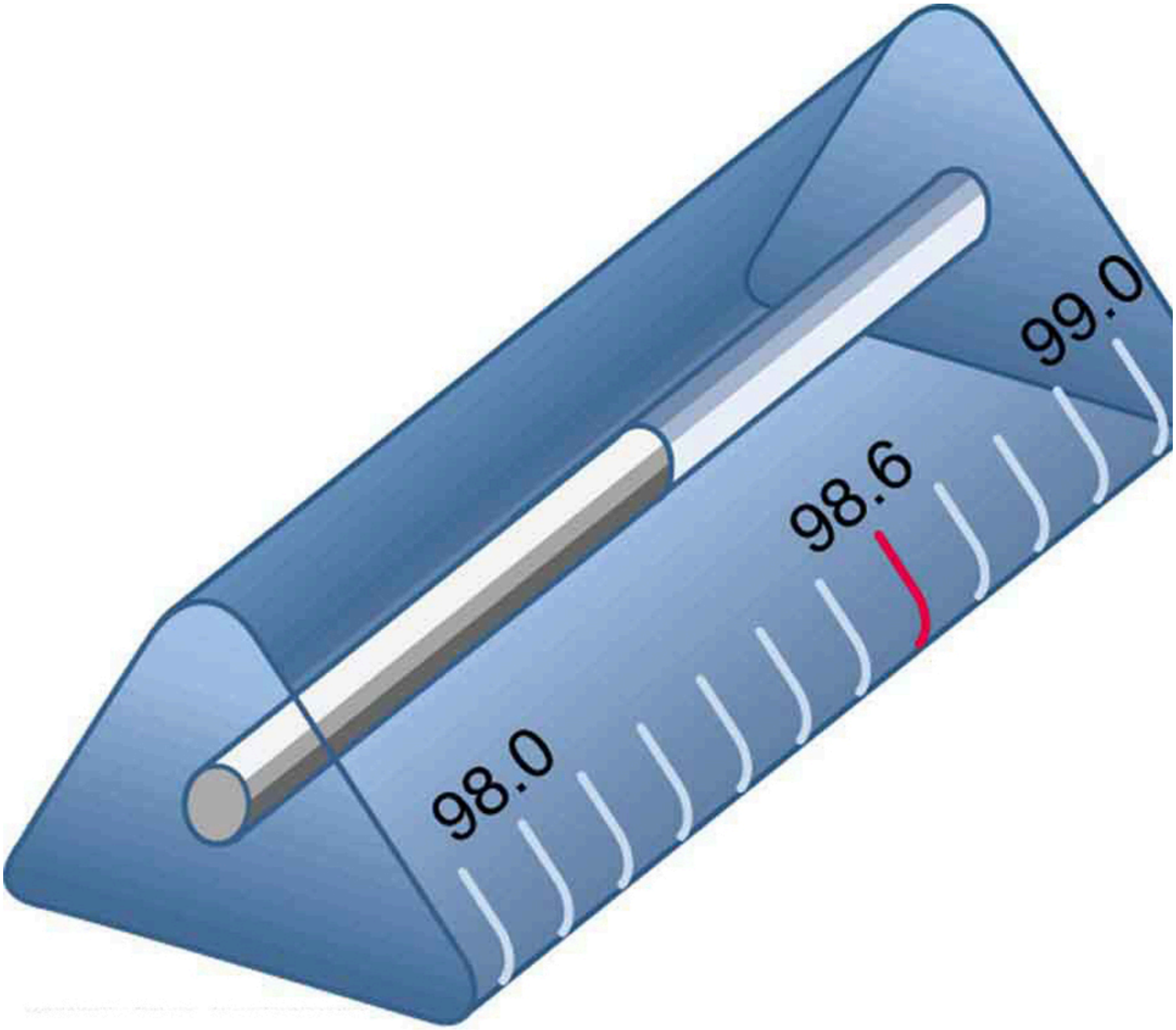
Does the fact that the light flash from lightning reaches you before its sound prove that the speed of light is extremely large or simply that it is greater than the speed of sound? Discuss how you could use this effect to get an estimate of the speed of light.

Will light change direction toward or away from the perpendicular when it goes from air to water? Water to glass? Glass to air?

Explain why an object in water always appears to be at a depth shallower than it actually is? Why do people sometimes sustain neck and spinal injuries when diving into unfamiliar ponds or waters?

Explain why a person's legs appear very short when wading in a pool. Justify your explanation with a ray diagram showing the path of rays from the feet to the eye of an observer who is out of the water.

Why is the front surface of a thermometer curved as shown?



The curved surface of the thermometer serves a purpose.

Suppose light were incident from air onto a material that had a negative index of refraction, say –1.3; where does the refracted light ray go?

Problems & Exercises

What is the speed of light in water? In glycerine?

Show Solution

Strategy

We use the relationship $v = c/n$ to calculate the speed of light in each medium. From [Table 1], the index of refraction for water is $n_{\text{water}} = 1.333$ and for glycerine is $n_{\text{glycerine}} = 1.473$.

Solution

For water:

$$v_{\text{water}} = \frac{c}{n_{\text{water}}} = \frac{3.00 \times 10^{8} \text{ m/s}}{1.333} = 2.25 \times 10^{8} \text{ m/s}$$

For glycerine:

$$v_{\text{glycerine}} = \frac{c}{n_{\text{glycerine}}} = \frac{3.00 \times 10^{8} \text{ m/s}}{1.473} = 2.04 \times 10^{8} \text{ m/s}$$

Answer: The speed of light in water is $\mathbf{2.25 \times 10^{8}}$ **m/s** and in glycerine is $\mathbf{2.04 \times 10^{8}}$ **m/s**.

Discussion

Light travels at about 75% of its vacuum speed in water and about 68% in glycerine. The higher index of refraction in glycerine means light interacts more strongly with the material and travels slower. This difference in speed is what causes refraction—when light enters these materials from air, it slows down and bends toward the normal. The larger the index of refraction, the more the light slows down and the more it bends.

What is the speed of light in air? In crown glass?

Show Solution

Strategy

We use the relationship between the speed of light in a medium, the speed of light in vacuum, and the index of refraction: $n = c/v$, which can be rearranged to $v = c/n$.

Solution

From [Table 1], the index of refraction for air is $n_{\text{air}} = 1.000293$ and for crown glass is $n_{\text{crown glass}} = 1.52$.

For air:

$$v_{\text{air}} = \frac{c}{n_{\text{air}}} = \frac{3.00 \times 10^{8} \text{ m/s}}{1.000293} = 2.9991 \times 10^{8} \text{ m/s}$$

For crown glass:

$$v_{\text{crown glass}} = \frac{c}{n_{\text{crown glass}}} = \frac{3.00 \times 10^{8} \text{ m/s}}{1.52} = 1.97 \times 10^{8} \text{ m/s}$$

Discussion

The speed of light in air is only slightly less than the speed of light in vacuum (about 0.03% slower), which is why we often approximate $n_{\text{air}} \approx 1.00$ for most calculations. In crown glass, however, light travels at about 66% of its vacuum speed. This significant reduction in speed is what causes the bending of light (refraction) when it enters glass from air, which is the basis for lenses and many optical instruments.

Calculate the index of refraction for a medium in which the speed of light is $2.012 \times 10^{8} \text{m/s}$, and identify the most likely substance based on [Table 1].

Show Solution

Strategy

We use $n = c/v$ to calculate the index of refraction, then compare with values in [Table 1] to identify the substance.

Solution

$$n = \frac{c}{v} = \frac{3.00 \times 10^{8} \text{ m/s}}{2.012 \times 10^{8} \text{ m/s}} = 1.491$$

Comparing with [Table 1], this value is very close to **polystyrene**, which has $n = 1.49$.

Answer: The index of refraction is **1.491**, and the substance is most likely **polystyrene**.

Discussion

Polystyrene is a common plastic used in many everyday items including disposable cups, packaging materials, and optical components. Its index of refraction (approximately 1.49) is similar to many other transparent plastics and glasses. The calculated value of 1.491 matches polystyrene within the precision of the measurement. In practice, the index of refraction can vary slightly depending on the exact composition and manufacturing process of the material, so small variations from the table value are expected.

In what substance in [Table 1] is the speed of light $2.290 \times 10^{8} \text{m/s}$?

Show Solution

Strategy

We use $n = c/v$ to calculate the index of refraction, then compare it with values in [Table 1] to identify the substance.

Solution

$$n = \frac{c}{v} = \frac{3.00 \times 10^{8} \text{ m/s}}{2.290 \times 10^{8} \text{ m/s}} = 1.310$$

Comparing with [Table 1], we find that water (fresh) has $n = 1.333$, which is close, and ice at $-7°\text{C}$ has $n = 1.309$, which matches our calculated value very closely.

Answer: The substance is ice (at $-7°\text{C}$).

Discussion

The slight difference between the index of refraction of water (1.333) and ice (1.309) is due to the different molecular structure. In ice, the water molecules are arranged in a crystalline lattice with more space between molecules than in liquid water, resulting in a slightly lower density and thus a slightly lower index of refraction. This allows light to travel slightly faster in ice than in liquid water.

There was a major collision of an asteroid with the Moon in medieval times. It was described by monks at Canterbury Cathedral in England as a red glow on and around the Moon. How long after the asteroid hit the Moon, which is $3.84 \times 10^{5} \text{km}$ away, would the light first arrive on Earth?

Show Solution

Strategy

Light travels at speed $c = 3.00 \times 10^{8} \text{ m/s}$ in the vacuum of space. We use $t = d/c$ to find the time, converting the distance to meters first.

Solution

Convert distance to meters:

$$d = 3.84 \times 10^{5} \text{ km} = 3.84 \times 10^{8} \text{ m}$$

Calculate time:

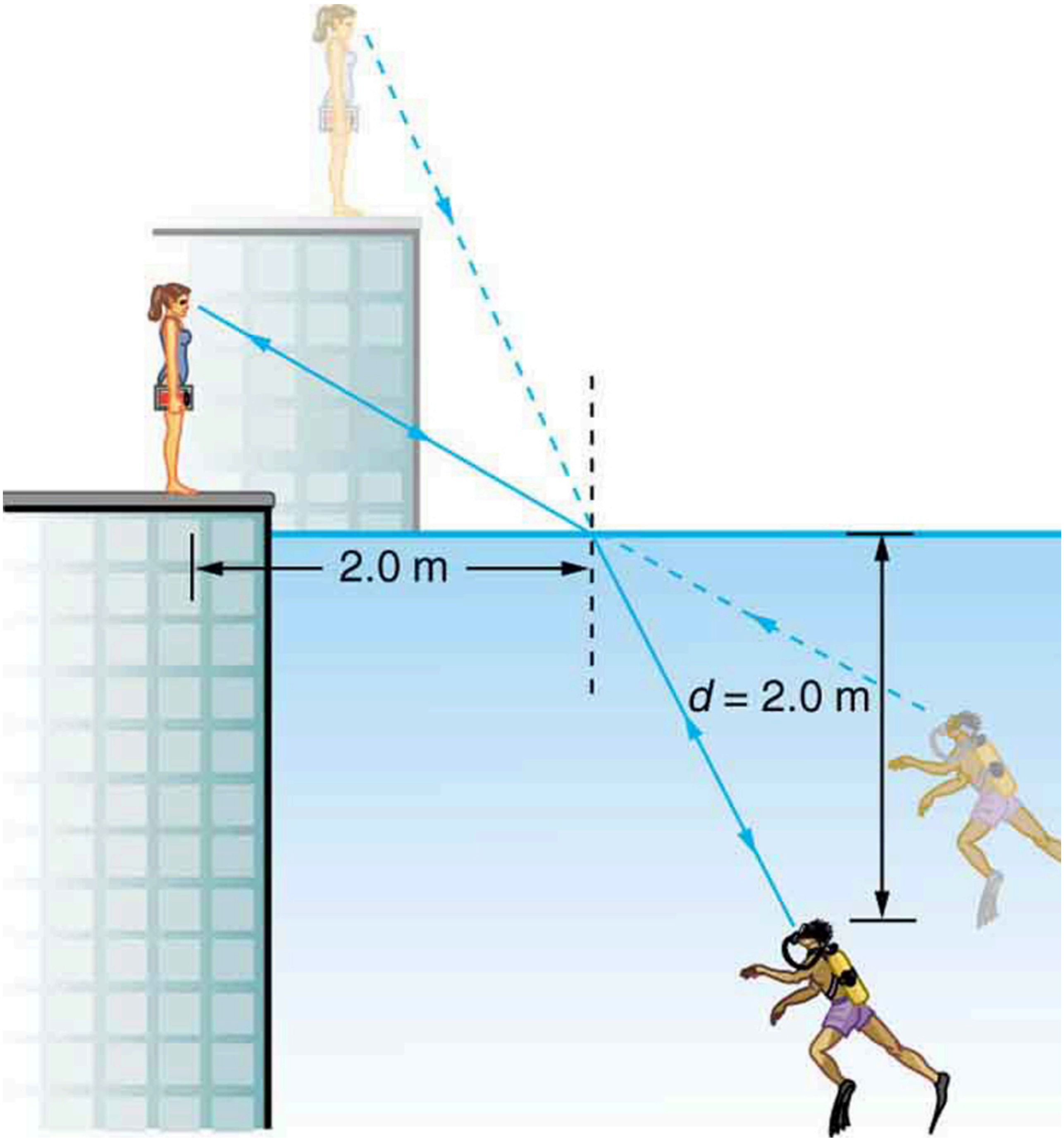
$$t = \frac{d}{c} = \frac{3.84 \times 10^{8} \text{ m}}{3.00 \times 10^{8} \text{ m/s}} = 1.28 \text{ s}$$

Answer: The light would arrive on Earth **1.28 seconds** after the asteroid hit the Moon.

Discussion

This remarkably short time—just over one second—shows how fast light travels. The Moon is Earth's closest celestial neighbor, and the short light-travel time means that when we look at the Moon, we're seeing it almost exactly as it is "now" (with only a 1.28-second delay). This historical observation by the Canterbury monks was likely of a major impact that created one of the Moon's craters. The event they described occurred in 1178 CE and has been suggested as possibly creating the crater Giordano Bruno, though this remains debated. The ability to see such an event from Earth demonstrates both the brightness of the impact and the short distance light must travel.

A scuba diver training in a pool looks at his instructor as shown in [Figure 5]. What angle does the ray from the instructor's face make with the perpendicular to the water at the point where the ray enters? The angle between the ray in the water and the perpendicular to the water is $25.0º$.



A scuba diver in a pool and his trainer look at each other.

Show Solution

Strategy

We use Snell's law to find the angle in air. Light is traveling from air into water, so $n_1 = 1.00$ (air) and $n_2 = 1.333$ (water). We're given $\theta_2 = 25.0°$ and need to find θ_1.

Solution

Snell's law states:

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

Solving for θ_1:

$$\sin \theta_1 = \frac{n_2}{n_1} \sin \theta_2 = \frac{1.333}{1.00} \sin(25.0°)$$
$$\sin \theta_1 = 1.333 \times 0.4226 = 0.5634$$
$$\theta_1 = \sin^{-1}(0.5634) = 34.3°$$

Answer: The ray from the instructor's face makes an angle of **34.3°** with the perpendicular to the water surface.

Discussion

Note that the angle in air (34.3°) is larger than the angle in water (25.0°). This is expected because light bends toward the normal when entering a denser medium. This refraction effect causes the instructor to appear higher to the diver than she actually is, and conversely, it makes the diver appear shallower to the instructor. This is why objects underwater always appear closer to the surface than they actually are—an important safety consideration for diving and swimming.

Components of some computers communicate with each other through optical fibers having an index of refraction $n= 1.55$. What time in nanoseconds is required for a signal to travel 0.200 m through such a fiber?

Show Solution

Strategy

The speed of light in the fiber is $v = c/n$. We then use $t = d/v$ to find the travel time.

Solution

Speed of light in the fiber:

$$v = \frac{c}{n} = \frac{3.00 \times 10^{8} \text{ m/s}}{1.55} = 1.935 \times 10^{8} \text{ m/s}$$

Time for signal to travel 0.200 m:

$$t = \frac{d}{v} = \frac{0.200 \text{ m}}{1.935 \times 10^{8} \text{ m/s}} = 1.03 \times 10^{-9} \text{ s} = 1.03 \text{ ns}$$

Answer: The signal requires **1.03 nanoseconds** to travel through the fiber.

Discussion

Although this seems like an incredibly short time, it's significant in high-speed computing. Modern computer processors operate at gigahertz frequencies (billions of cycles per second), meaning each clock cycle lasts about 1 nanosecond. So this 1.03 ns delay represents roughly one clock cycle—enough to matter in computer design. This is why computer engineers carefully consider the length of connections and use optical fibers for longer distances within and between computers. The speed of light (even when slowed by the fiber's index of refraction) sets a fundamental limit on how fast signals can travel, affecting computer performance and design.

(a) Given that the angle between the ray in the water and the perpendicular to the water is $25.0º$, and using information in [Figure 3], find the height of the instructor's head above the water, noting that you will first have to calculate the angle of incidence. (b) Find the apparent depth of the diver's head below water as seen by the instructor.

Show Solution

Strategy

From [Figure 5], we can use geometry and Snell's law. The horizontal distance from the point where the ray enters the water to the instructor is 2.00 m. We first find the angle in air using Snell's law (from the previous problem, $\theta_1 = 34.3°$), then use trigonometry.

Solution

(a) From the previous problem, we found $\theta_1 = 34.3°$.

The horizontal distance is 2.00 m. Using trigonometry:

$$\tan \theta_1 = \frac{\text{horizontal distance}}{\text{height}} = \frac{2.00 \text{ m}}{h}$$
$$h = \frac{2.00 \text{ m}}{\tan(34.3°)} = \frac{2.00 \text{ m}}{0.6822} = 2.93 \text{ m}$$

(b) For the diver, the horizontal distance underwater is also 2.00 m, and $\theta_2 = 25.0°$:

$$\tan \theta_2 = \frac{2.00 \text{ m}}{d}$$
$$d = \frac{2.00 \text{ m}}{\tan(25.0°)} = \frac{2.00 \text{ m}}{0.4663} = 4.29 \text{ m}$$

However, the actual depth is greater. The apparent depth can be found using $d_{\text{apparent}} = d_{\text{actual}}/n = d_{\text{actual}}/1.333$. From geometry, the actual depth is approximately 2.00/tan(25.0°) = 4.29 m, but the diver appears to be at:

$$ d_{\text{apparent}} = 4.29/1.333 = 3.22 \text{ m} $$

Discussion

The instructor's head is 2.93 m above the water. The apparent depth effect makes underwater objects appear about 25% shallower than they actually are when viewed from air. This is why pools always look shallower than they really are.

Suppose you have an unknown clear substance immersed in water, and you wish to identify it by finding its index of refraction. You arrange to have a beam of light enter it at an angle of $45.0º$, and you observe the angle of refraction to be $40.3º$. What is the index of refraction of the substance and its likely identity?

Show Solution

Strategy

Light travels from water ($n_1 = 1.333$) into the unknown substance (n_2). We use Snell's law: $n_1 \sin \theta_1 = n_2 \sin \theta_2$ with $\theta_1 = 45.0°$ and $\theta_2 = 40.3°$.

Solution

Apply Snell's law:

$$n_2 = n_1 \frac{\sin \theta_1}{\sin \theta_2} = 1.333 \times \frac{\sin(45.0°)}{\sin(40.3°)}$$
$$n_2 = 1.333 \times \frac{0.7071}{0.6461} = 1.333 \times 1.094 = 1.458$$

Comparing with [Table 1], this matches **fused quartz** ($n = 1.458$).

Answer: The index of refraction is **1.46** (to three significant figures), and the substance is most likely **fused quartz**.

Discussion

Fused quartz (also called fused silica) is a glass made by melting high-purity silica. It's commonly used in optical applications because of its excellent transparency, low thermal expansion, and resistance to thermal shock. The fact that the angle of refraction (40.3°) is less than the angle of incidence (45.0°) confirms that light is bending toward the normal, which is expected since the unknown substance has a higher index of refraction than water (1.458 > 1.333). This technique of measuring refraction angles to identify materials is used in gemology and materials science to distinguish between different transparent substances.

On the Moon's surface, lunar astronauts placed a corner reflector, off which a laser beam is periodically reflected. The distance to the Moon is calculated from the round-trip time. What percent correction is needed to account for the delay in time due to the slowing of light in Earth's atmosphere? Assume the distance to the Moon is precisely $3.84 \times 10^{8} \text{m}$, and Earth's atmosphere (which varies in density with altitude) is equivalent to a layer 30.0 km thick with a constant index of refraction $n= 1.000293$.

Show Solution

Strategy

Light travels slower in Earth's atmosphere than in vacuum. We need to find the additional time it takes to travel through the atmosphere (twice, since it's a round trip) and compare it to the total travel time.

Solution

Speed of light in the atmosphere:

$$v = \frac{c}{n} = \frac{3.00 \times 10^{8} \text{ m/s}}{1.000293} = 2.9991 \times 10^{8} \text{ m/s}$$

Time through atmosphere (round trip = 60.0 km total):

$$t_{\text{atm}} = \frac{60.0 \times 10^{3} \text{ m}}{2.9991 \times 10^{8} \text{ m/s}} = 2.0006 \times 10^{-4} \text{ s}$$

Time if it were vacuum:

$$t_{\text{vac}} = \frac{60.0 \times 10^{3} \text{ m}}{3.00 \times 10^{8} \text{ m/s}} = 2.000 \times 10^{-4} \text{ s}$$

Additional delay:

$$\Delta t = t_{\text{atm}} - t_{\text{vac}} = (2.0006 - 2.000) \times 10^{-4} \text{ s} = 6 \times 10^{-8} \text{ s}$$

This corresponds to an apparent additional distance:

$$\Delta d = c \cdot \Delta t = (3.00 \times 10^{8})(6 \times 10^{-8}) = 18 \text{ m}$$

Percent correction:

$$\frac{\Delta d}{2 \times 3.84 \times 10^{8} \text{ m}} \times 100\% = \frac{18}{7.68 \times 10^{8}} \times 100\% = 2.3 \times 10^{-6}\%$$

Answer: The percent correction needed is approximately **0.0000023%** or about **2.3 parts per million**.

Discussion

Although this correction is extremely small (only about 18 meters out of 768 million meters), it must be accounted for in precise laser ranging measurements to the Moon. Modern laser ranging can measure the Earth-Moon distance to within a few centimeters, making this atmospheric correction significant for such high-precision work. The Moon is gradually moving away from Earth at about 3.8 cm per year, and these precise measurements help us track this motion.

Suppose [Figure 5] represents a ray of light going from air through crown glass into water, such as going into a fish tank. Calculate the amount the ray is displaced by the glass (Δx), given that the incident angle is $40.0º$ and the glass is 1.00 cm thick.

Show Solution

Strategy

The ray refracts when entering the glass and again when leaving it. We need to find the lateral displacement Δx by tracking the ray’s path through the glass using Snell’s law and geometry.

Solution

Entering the glass from air ($n_1 = 1.00$, $n_2 = 1.52$):

$$\sin \theta_2 = \frac{n_1}{n_2} \sin \theta_1 = \frac{1.00}{1.52} \sin(40.0°) = \frac{0.6428}{1.52} = 0.4229$$
$$\theta_2 = \sin^{-1}(0.4229) = 25.0°$$

The ray travels through the glass at angle $25.0°$ to the normal. The distance traveled through the glass:

$$d = \frac{t}{\cos \theta_2} = \frac{1.00 \text{ cm}}{\cos(25.0°)} = \frac{1.00 \text{ cm}}{0.9063} = 1.103 \text{ cm}$$

The horizontal displacement within the glass:

$$\Delta x = d \sin \theta_2 = 1.103 \text{ cm} \times \sin(25.0°) = 1.103 \times 0.4226 = 0.466 \text{ cm}$$

However, we need to subtract the horizontal distance if the ray had traveled straight:

$$\Delta x = t(\tan \theta_2 - \tan \theta_1) = 1.00(\tan 25.0° - \tan 40.0°)$$

Actually, the correct lateral displacement is:

$$\Delta x = t \sin(\theta_1 - \theta_2) \frac{\sin \theta_1}{\cos \theta_2}$$

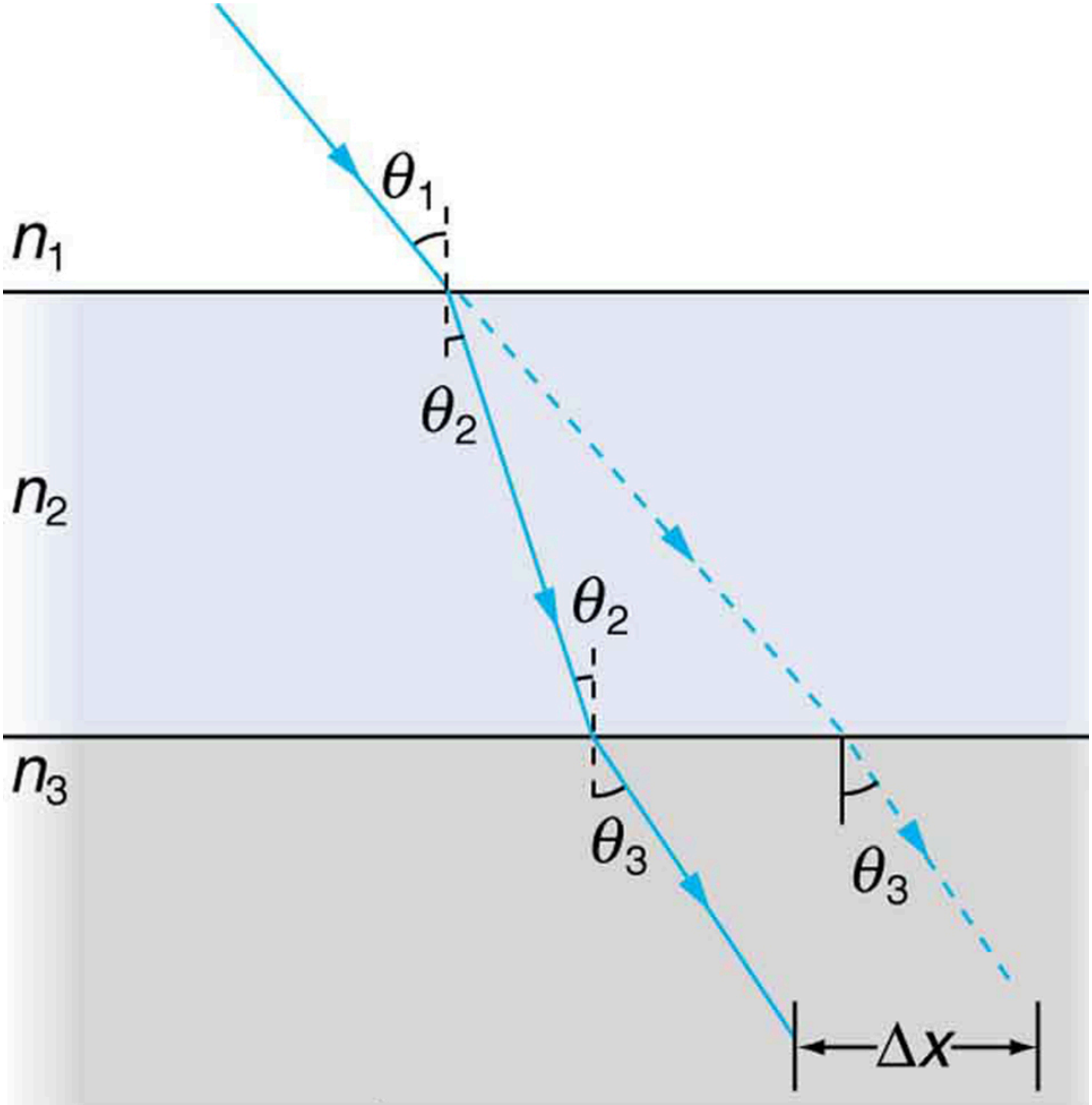
Let me recalculate more carefully. The lateral displacement for a parallel-sided plate is:

$$\Delta x = \frac{t \sin(\theta_1 - \theta_2)}{\cos \theta_2}$$
$$\Delta x = \frac{1.00 \text{ cm} \times \sin(40.0° - 25.0°)}{\cos(25.0°)} = \frac{1.00 \text{ cm} \times \sin(15.0°)}{0.9063}$$
$$\Delta x = \frac{1.00 \text{ cm} \times 0.2588}{0.9063} = 0.285 \text{ cm} = 2.85 \text{ mm}$$

Discussion

The ray is displaced laterally by about 2.85 mm as it passes through the 1 cm thick glass. This displacement is why objects viewed through a window appear slightly shifted from their actual positions. The displacement depends on both the thickness of the glass and the angle of incidence—the greater the angle and the thicker the glass, the larger the displacement.

[Figure 6] shows a ray of light passing from one medium into a second and then a third. Show that ${\theta }_{3}$ is the same as it would be if the second medium were not present (provided total internal reflection does not occur).



A ray of light passes from one medium to a third by traveling through a second. The final direction is the same as if the second medium were not present, but the ray is displaced by Δx (shown exaggerated).

Unreasonable Results

Suppose light travels from water to another substance, with an angle of incidence of $10.0º$ and an angle of refraction of $14.9º$. (a) What is the index of refraction of the other substance? (b) What is unreasonable about this result? (c) Which assumptions are unreasonable or inconsistent?

Show Solution

(a) 0.898

(b) Can't have $n< 1.00$ since this would imply a speed greater than c.

(c) Refracted angle is too big relative to the angle of incidence.

Construct Your Own Problem

Consider sunlight entering the Earth's atmosphere at sunrise and sunset—that is, at a $90 ^\circ$ incident angle. Taking the boundary between nearly empty space and the atmosphere to be sudden, calculate the angle of refraction for sunlight. This lengthens the time the Sun appears to be above the horizon, both at sunrise and sunset. Now construct a problem in which you determine the angle of refraction for different models of the atmosphere, such as various layers of varying density. Your instructor may wish to guide you on the level of complexity to consider and on how the index of refraction varies with air density.

Unreasonable Results

Light traveling from water to a gemstone strikes the surface at an angle of $80.0º$ and has an angle of refraction of $15.2º$. (a) What is the speed of light in the gemstone? (b) What is unreasonable about this result? (c) Which assumptions are unreasonable or inconsistent?

Show Solution

(a) $\frac{c}{ 5.00}$ (b) Speed of light too slow, since index is much greater than that of diamond.

(c) Angle of refraction is unreasonable relative to the angle of incidence.

Glossary

refraction
: changing of a light ray’s direction when it passes through variations in matter

index of refraction
: for a material, the ratio of the speed of light in vacuum to that in the material

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Total Internal Reflection

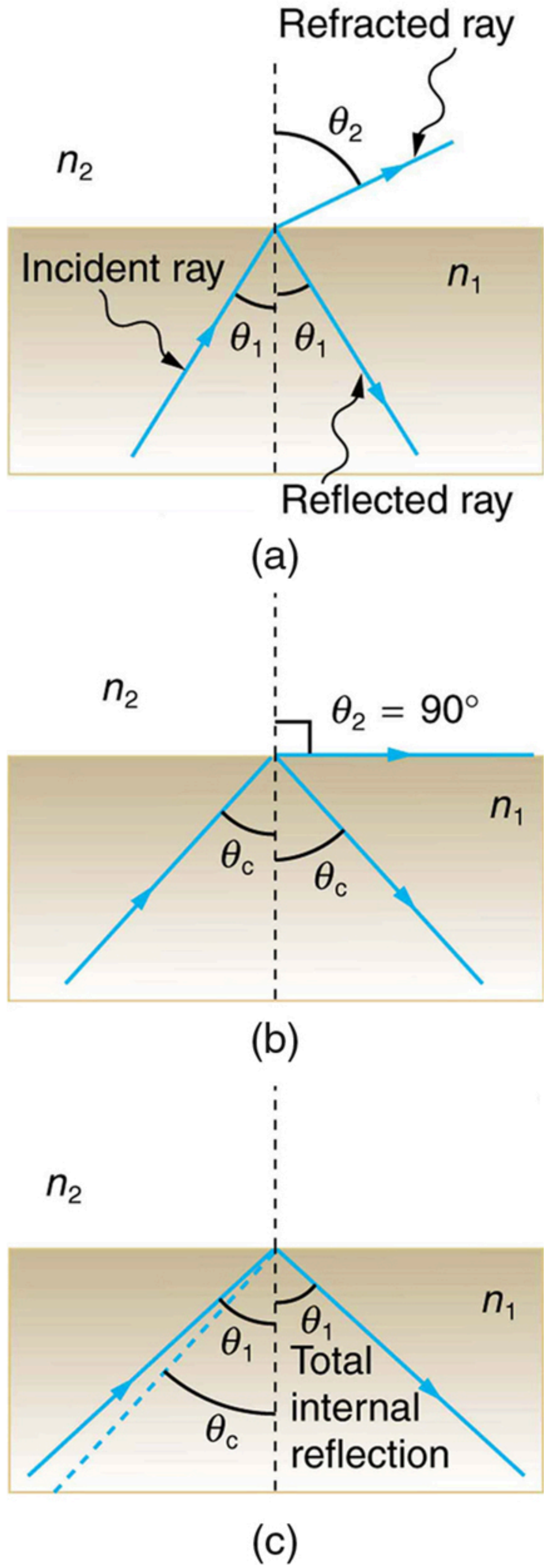
- Explain the phenomenon of total internal reflection.
- Describe the workings and uses of fiber optics.
- Analyze the reason for the sparkle of diamonds.

A good-quality mirror may reflect more than 90% of the light that falls on it, absorbing the rest. But it would be useful to have a mirror that reflects all of the light that falls on it. Interestingly, we can produce **total reflection** using an aspect of **refraction**.

Consider what happens when a ray of light strikes the surface between two materials, such as is shown in [Figure 1](a). Part of the light crosses the boundary and is refracted; the rest is reflected. If, as shown in the figure, the index of refraction for the second medium is less than for the first, the ray bends away from the perpendicular. (Since $n_1 > n_2$, the angle of refraction is greater than the angle of incidence—that is, $\theta_2 > \theta_1$.) Now imagine what happens as the incident angle is increased. This causes θ_2 to increase also. The largest the angle of refraction θ_2 can be is 90°, as shown in [Figure 1](b). The **critical angle** θ_c for a combination of materials is defined to be the incident angle θ_1 that produces an angle of refraction of 90°. That is, θ_c is the incident angle for which $\theta_2 = 90^\circ$. If the incident angle θ_1 is greater than the critical angle, as shown in [Figure 1](c), then all of the light is reflected back into medium 1, a condition called **total internal reflection**.

Critical Angle

The incident angle θ_1 ** that produces an angle of refraction of 90° is called the critical angle, θ_c.



(a) A ray of light crosses a boundary where the speed of light increases and the index of refraction decreases. That is, $n_2 < n_1$. The ray bends away from the perpendicular. (b) The critical angle θ_c is the one for which the angle of refraction is 90º. (c) Total internal reflection occurs when the incident angle is greater than the critical angle.

Snell's law states the relationship between angles and indices of refraction. It is given by

$$n_1 \sin\theta_1 = n_2 \sin\theta_2.$$

When the incident angle equals the critical angle ($\theta_1 = \theta_c$), the angle of refraction is $90°$ ($\theta_2 = 90°$). Noting that $\sin 90° = 1$, Snell's law in this case becomes

$$n_1 \sin\theta_1 = n_2.$$

The critical angle θ_c for a given combination of materials is thus

$$\theta_c = \sin^{-1}(n_2/n_1) \text{ for } n_1 > n_2.$$

Total internal reflection occurs for any incident angle greater than the critical angle θ_c , and it can only occur when the second medium has an index of refraction less than the first. Note the above equation is written for a light ray that travels in medium 1 and reflects from medium 2, as shown in the figure.

How Big is the Critical Angle Here?

What is the critical angle for light traveling in a polystyrene (a type of plastic) pipe surrounded by air?

Strategy

The index of refraction for polystyrene is found to be 1.49 in [Figure 2], and the index of refraction of air can be taken to be 1.00, as before. Thus, the condition that the second medium (air) has an index of refraction less than the first (plastic) is satisfied, and the equation $\theta_c = \sin^{-1}(n_2/n_1)$ can be used to find the critical angle θ_c . Here, then, $n_2 = 1.00$ and $n\_1 = 1.49$.

Solution

The critical angle is given by

$$\theta_c = \sin^{-1}(n_2/n_1).$$

Substituting the identified values gives

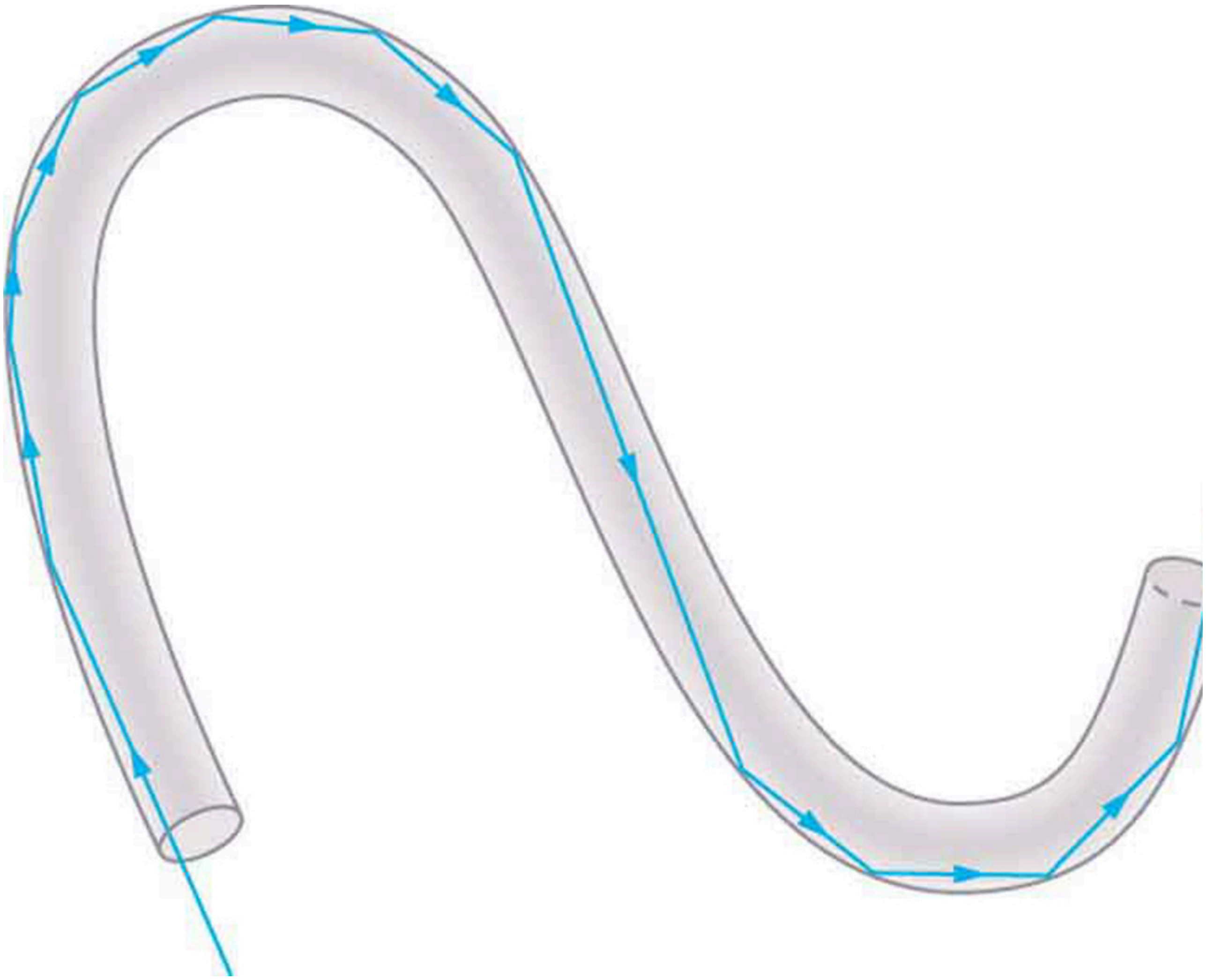
$$\theta_c = \sin^{-1}(1.00/1.49) = \sin^{-1}(0.671) \quad 42.2°.$$

Discussion

This means that any ray of light inside the plastic that strikes the surface at an angle greater than $42.2°$ will be totally reflected. This will make the inside surface of the clear plastic a perfect mirror for such rays without any need for the silvering used on common mirrors. Different combinations of materials have different critical angles, but any combination with $n_1 > n_2$ can produce total internal reflection. The same calculation as made here shows that the critical angle for a ray going from water to air is $48.6°$, while that from diamond to air is $24.4°$, and that from flint glass to crown glass is $66.3°$. There is no total reflection for rays going in the other direction—for example, from air to water—since the condition that the second medium must have a smaller index of refraction is not satisfied. A number of interesting applications of total internal reflection follow.

Fiber Optics: Endoscopes to Telephones

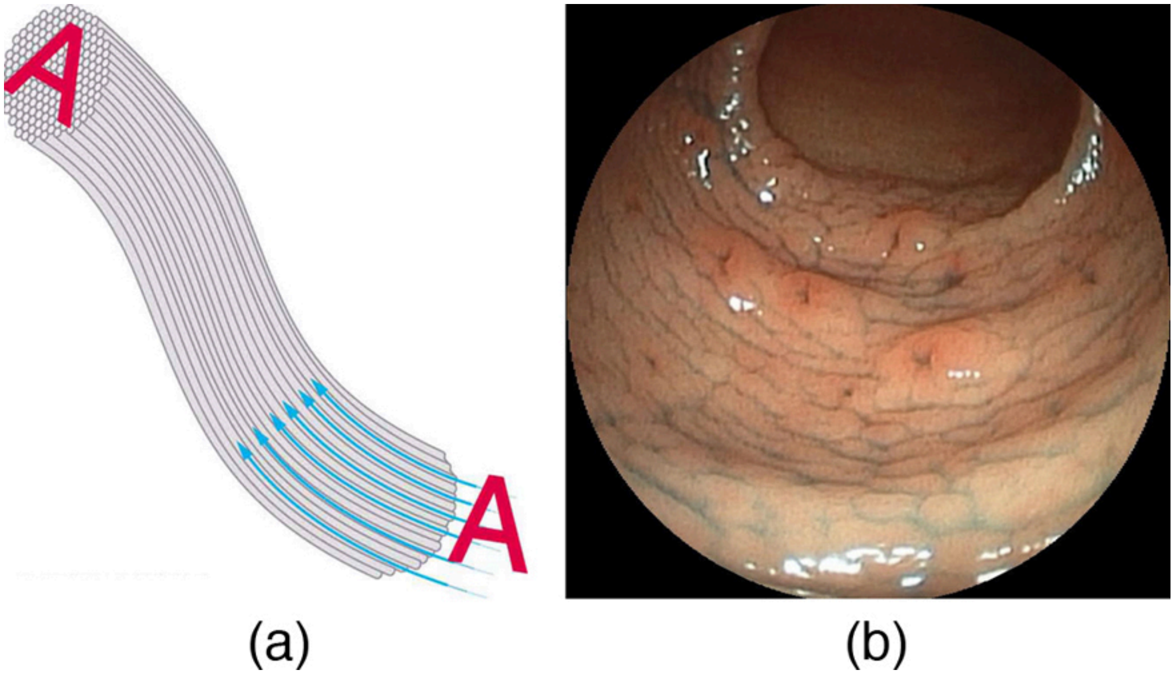
Fiber optics is one application of total internal reflection that is in wide use. In communications, it is used to transmit telephone, internet, and cable TV signals. **Fiber optics** employs the transmission of light down fibers of plastic or glass. Because the fibers are thin, light entering one is likely to strike the inside surface at an angle greater than the critical angle and, thus, be totally reflected (See [Figure 2].) The index of refraction outside the fiber must be smaller than inside, a condition that is easily satisfied by coating the outside of the fiber with a material having an appropriate refractive index. In fact, most fibers have a varying refractive index to allow more light to be guided along the fiber through total internal refraction. Rays are reflected around corners as shown, making the fibers into tiny light pipes.



Light entering a thin fiber may strike the inside surface at large or grazing angles and is completely reflected if these angles exceed the critical angle. Such rays continue down the fiber, even following it around corners, since the angles of reflection and incidence remain large.

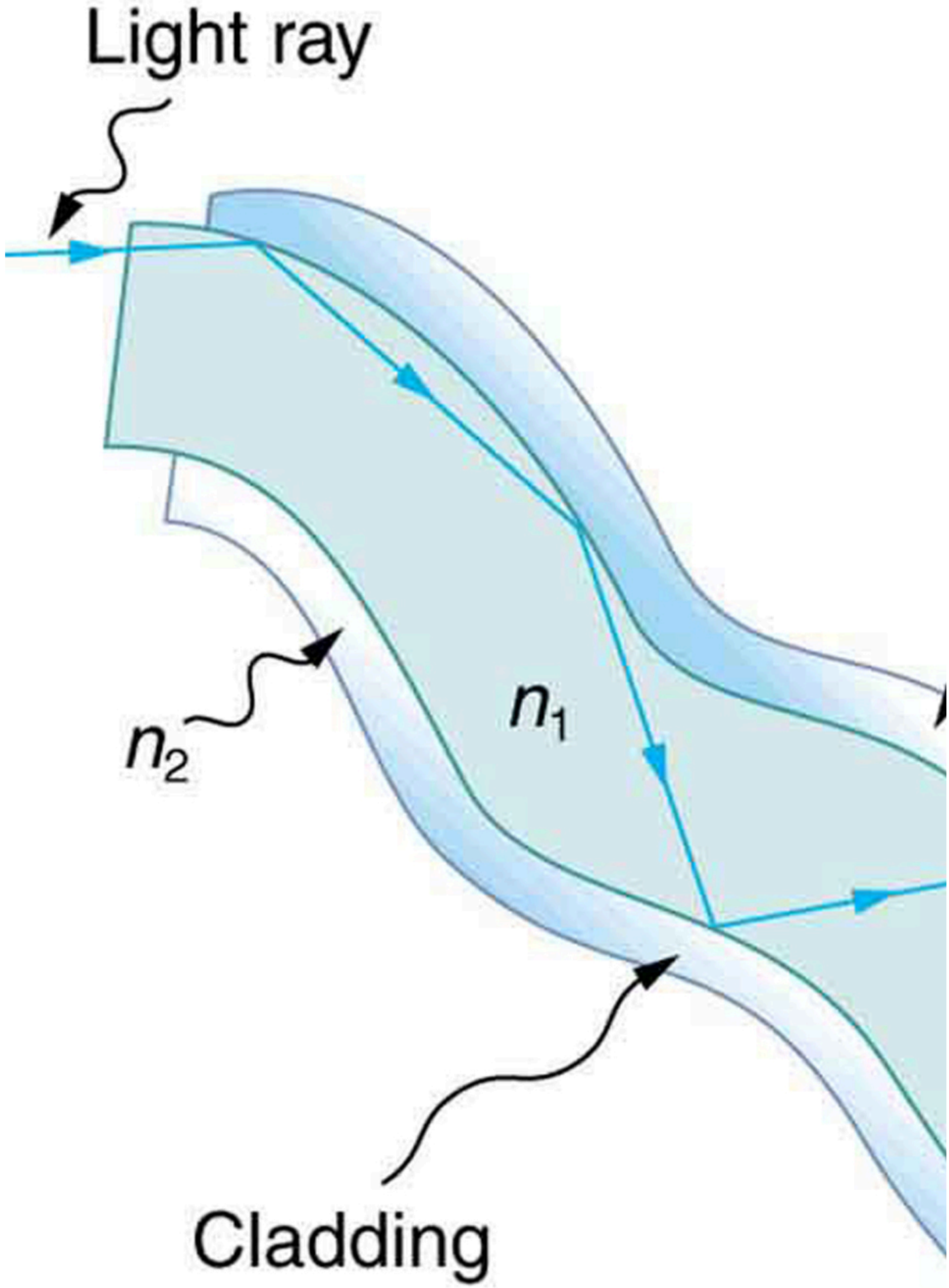
Bundles of fibers can be used to transmit an image without a lens, as illustrated in [Figure 3]. The output of a device called an **endoscope** is shown in [Figure 3](b). Endoscopes are used to explore the body through various orifices or minor incisions. Light is transmitted down one fiber bundle to illuminate internal parts, and the reflected light is transmitted back out through another to be observed. Surgery can be performed, such as arthroscopic surgery on the knee joint, employing cutting tools attached to and observed with the endoscope. Samples can also be obtained, such as by lassoing an intestinal polyp for external examination.

Fiber optics has revolutionized surgical techniques and observations within the body. There are a host of medical diagnostic and therapeutic uses. The flexibility of the fiber optic bundle allows it to navigate around difficult and small regions in the body, such as the intestines, the heart, blood vessels, and joints. Transmission of an intense laser beam to burn away obstructing plaques in major arteries as well as delivering light to activate chemotherapy drugs are becoming commonplace. Optical fibers have in fact enabled microsurgery and remote surgery where the incisions are small and the surgeon’s fingers do not need to touch the diseased tissue.



(a) An image is transmitted by a bundle of fibers that have fixed neighbors. (b) An endoscope is used to probe the body, both transmitting light to the interior and returning an image such as the one shown. (credit: Med_Chaos, Wikimedia Commons)

Fibers in bundles are surrounded by a cladding material that has a lower index of refraction than the core. (See [Figure 4].) The cladding prevents light from being transmitted between fibers in a bundle. Without cladding, light could pass between fibers in contact, since their indices of refraction are identical. Since no light gets into the cladding (there is total internal reflection back into the core), none can be transmitted between clad fibers that are in contact with one another. The cladding prevents light from escaping out of the fiber; instead most of the light is propagated along the length of the fiber, minimizing the loss of signal and ensuring that a quality image is formed at the other end. The cladding and an additional protective layer make optical fibers flexible and durable.


Light ray
n_2
n_1
Cladding

Fibers in bundles are clad by a material that has a lower index of refraction than the core to ensure total internal reflection, even when fibers are in contact with one another. This shows a single fiber with its cladding.

Cladding

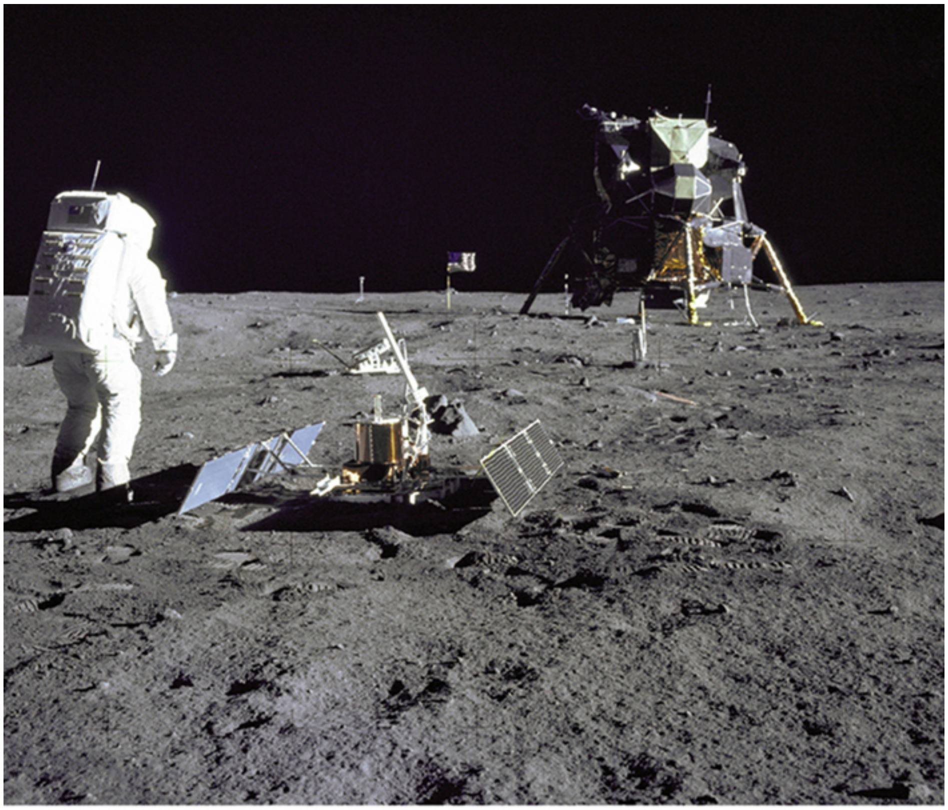
The cladding prevents light from being transmitted between fibers in a bundle.

Special tiny lenses that can be attached to the ends of bundles of fibers are being designed and fabricated. Light emerging from a fiber bundle can be focused and a tiny spot can be imaged. In some cases the spot can be scanned, allowing quality imaging of a region inside the body. Special minute optical filters inserted at the end of the fiber bundle have the capacity to image tens of microns below the surface without cutting the surface—non-intrusive diagnostics. This is particularly useful for determining the extent of cancers in the stomach and bowel.

Most telephone conversations and Internet communications are now carried by laser signals along optical fibers. Extensive optical fiber cables have been placed on the ocean floor and underground to enable optical communications. Optical fiber communication systems offer several advantages over electrical (copper) based systems, particularly for long distances. The fibers can be made so transparent that light can travel many kilometers before it becomes dim enough to require amplification—much superior to copper conductors. This property of optical fibers is called **low loss**. Lasers emit light with characteristics that allow far more conversations in one fiber than are possible with electric signals on a single conductor. This property of optical fibers is called **high bandwidth**. Optical signals in one fiber do not produce undesirable effects in other adjacent fibers. This property of optical fibers is called **reduced crosstalk**. We shall explore the unique characteristics of laser radiation in a later chapter.

Corner Reflectors and Diamonds

A light ray that strikes an object consisting of two mutually perpendicular reflecting surfaces is reflected back exactly parallel to the direction from which it came. This is true whenever the reflecting surfaces are perpendicular, and it is independent of the angle of incidence. Such an object, shown in [Figure 5], is called a **corner reflector**, since the light bounces from its inside corner. Many inexpensive reflector buttons on bicycles, cars, and warning signs have corner reflectors designed to return light in the direction from which it originated. It was more expensive for astronauts to place one on the moon. Laser signals can be bounced from that corner reflector to measure the gradually increasing distance to the moon with great precision.

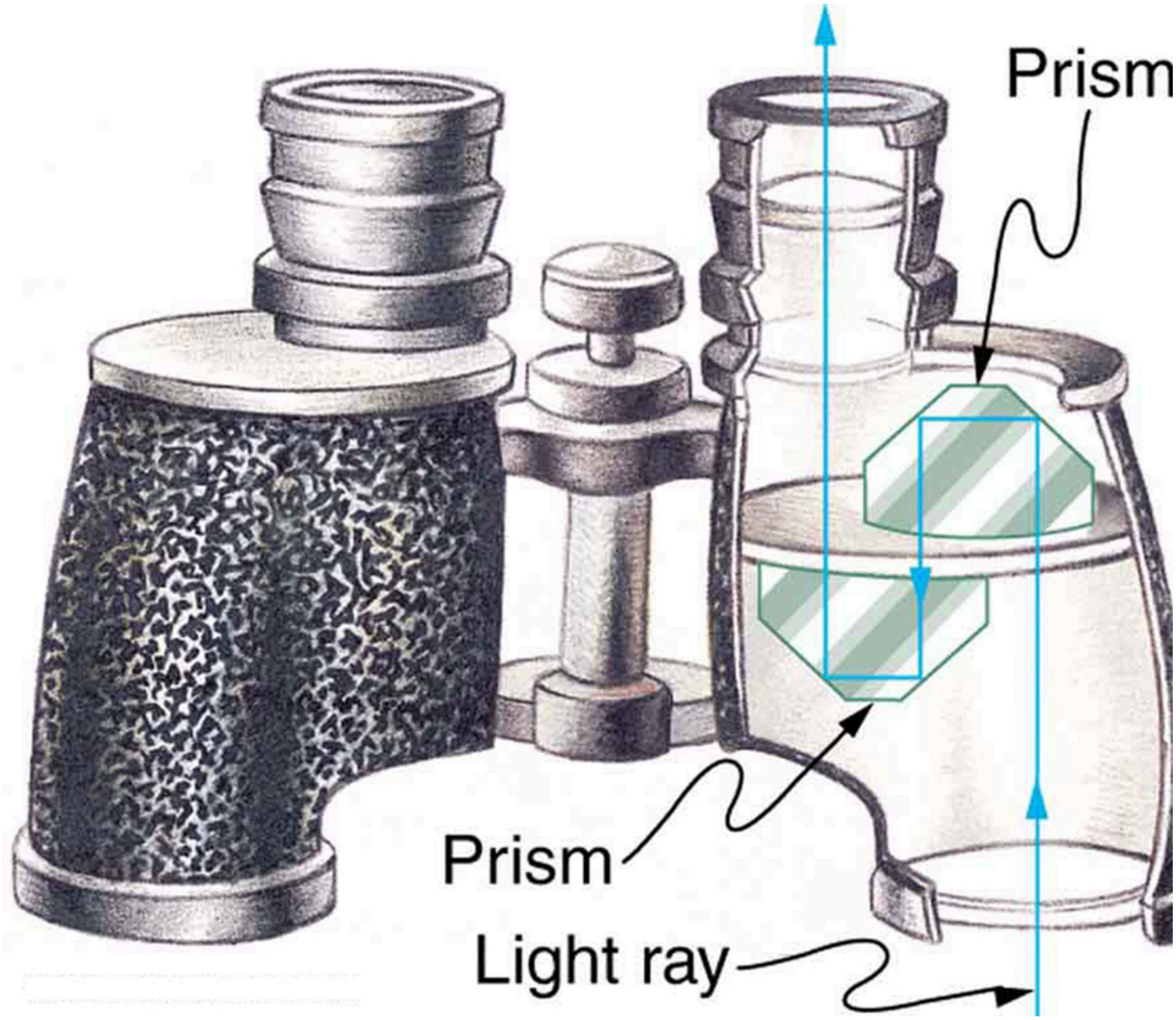


(a)

(b)

(a) Astronauts placed a corner reflector on the moon to measure its gradually increasing orbital distance. (credit: NASA) (b) The bright spots on these bicycle safety reflectors are reflections of the flash of the camera that took this picture on a dark night. (credit: Julo, Wikimedia Commons)

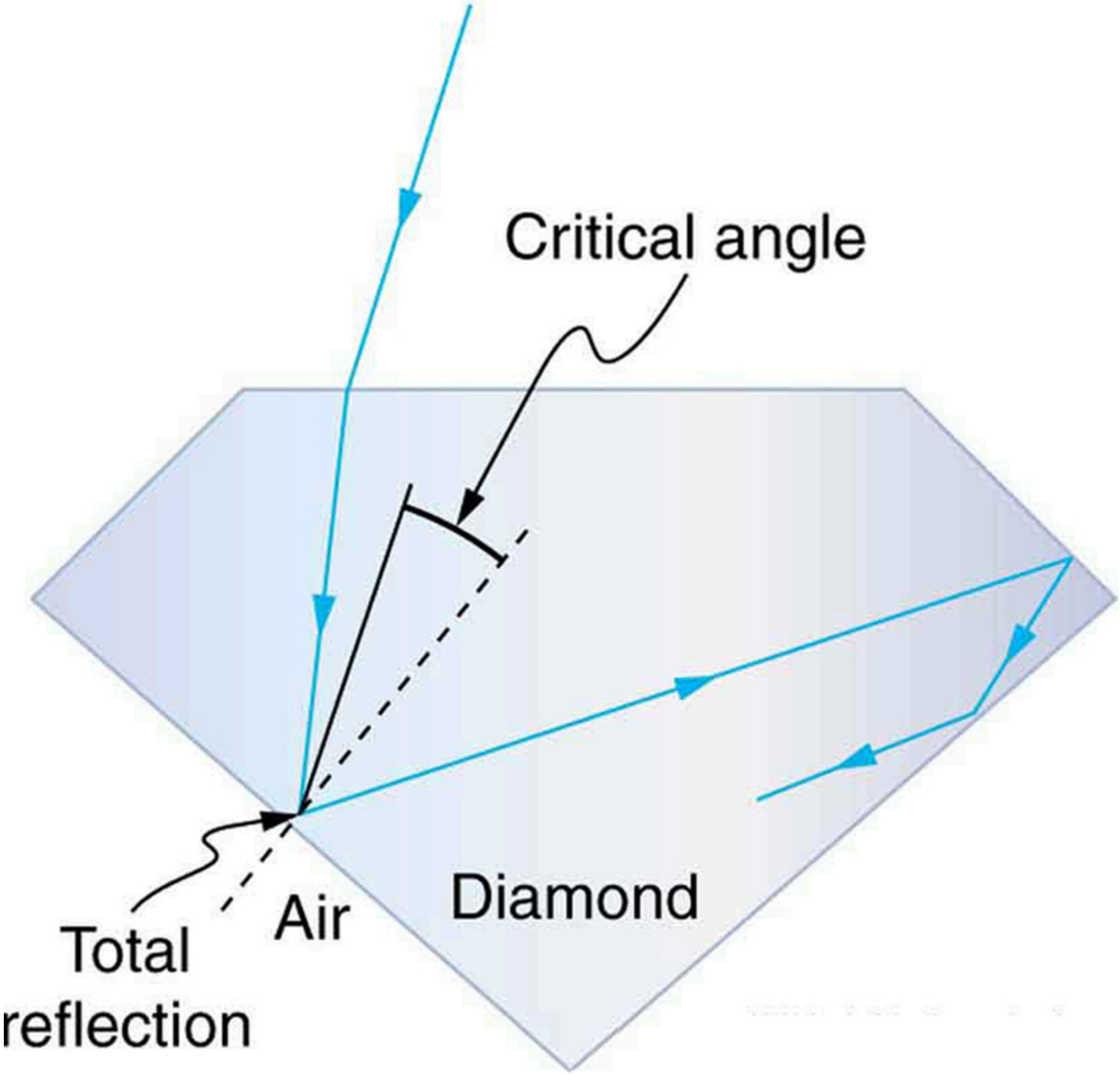
Corner reflectors are perfectly efficient when the conditions for total internal reflection are satisfied. With common materials, it is easy to obtain a critical angle that is less than $45°$. One use of these perfect mirrors is in binoculars, as shown in [Figure 6]. Another use is in periscopes found in submarines.



These binoculars employ corner reflectors with total internal reflection to get light to the observer's eyes.

The Sparkle of Diamonds

Total internal reflection, coupled with a large index of refraction, explains why diamonds sparkle more than other materials. The critical angle for a diamond-to-air surface is only $24.4°$, and so when light enters a diamond, it has trouble getting back out. (See [Figure 7].) Although light freely enters the diamond, it can exit only if it makes an angle less than $24.4°$. Facets on diamonds are specifically intended to make this unlikely, so that the light can exit only in certain places. Good diamonds are very clear, so that the light makes many internal reflections and is concentrated at the few places it can exit —hence the sparkle. (Zircon is a natural gemstone that has an exceptionally large index of refraction, but not as large as diamond, so it is not as highly prized. Cubic zirconia is manufactured and has an even higher index of refraction (≈ 2.17), but still less than that of diamond.) The colors you see emerging from a sparkling diamond are not due to the diamond's color, which is usually nearly colorless. Those colors result from dispersion, the topic of Dispersion: The Rainbow and Prisms. Colored diamonds get their color from structural defects of the crystal lattice and the inclusion of minute quantities of graphite and other materials. The Argyle Mine in Western Australia produces around 90% of the world's pink, red, champagne, and cognac diamonds, while around 50% of the world's clear diamonds come from central and southern Africa.



Light cannot easily escape a diamond, because its critical angle with air is so small. Most reflections are total, and the facets are placed so that light can exit only in particular ways—thus concentrating the light and making the diamond sparkle.

PhET Explorations: Bending Light

Explore bending of light between two media with different indices of refraction. See how changing from air to water to glass changes the bending angle. Play with prisms of different shapes and make rainbows.

Section Summary

- The incident angle that produces an angle of refraction of 90° is called critical angle.
- Total internal reflection is a phenomenon that occurs at the boundary between two mediums, such that if the incident angle in the first medium is greater than the critical angle, then all the light is reflected back into that medium.
- Fiber optics involves the transmission of light down fibers of plastic or glass, applying the principle of total internal reflection.
- Endoscopes are used to explore the body through various orifices or minor incisions, based on the transmission of light through optical fibers.
- Cladding prevents light from being transmitted between fibers in a bundle.
- Diamonds sparkle due to total internal reflection coupled with a large index of refraction.

Conceptual Questions

A ring with a colorless gemstone is dropped into water. The gemstone becomes invisible when submerged. Can it be a diamond? Explain.

A high-quality diamond may be quite clear and colorless, transmitting all visible wavelengths with little absorption. Explain how it can sparkle with flashes of brilliant color when illuminated by white light.

Is it possible that total internal reflection plays a role in rainbows? Explain in terms of indices of refraction and angles, perhaps referring to [Figure 8]. Some of us have seen the formation of a double rainbow. Is it physically possible to observe a triple rainbow?

Double rainbows are not a very common observance. (credit: InvictusOU812, Flickr)

The most common type of mirage is an illusion that light from faraway objects is reflected by a pool of water that is not really there. Mirages are generally observed in deserts, when there is a hot layer of air near the ground. Given that the refractive index of air is lower for air at higher temperatures, explain how mirages can be formed.

Problems & Exercises

Verify that the critical angle for light going from water to air is 48.6°, as discussed at the end of [Example 1], regarding the critical angle for light traveling in a polystyrene (a type of plastic) pipe surrounded by air.

Show Solution

Strategy

We use the formula for critical angle: $\theta_c = \sin^{-1}(n_2/n_1)$ where $n_1 > n_2$. For water to air, $n_1 = 1.333$ (water) and $n_2 = 1.00$ (air).

Solution

$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right) = \sin^{-1}\left(\frac{1.00}{1.333}\right)$$
$$\theta_c = \sin^{-1}(0.7502) = 48.6°$$

Discussion

This confirms that the critical angle for light going from water to air is $48.6\,°$. Any light ray in water striking the water-air interface at an angle greater than $48.6\,°$ from the normal will undergo total internal reflection. This phenomenon is why you can sometimes see a mirror-like reflection when looking up at the water surface from underwater—light hitting the surface at shallow angles (large angles from the normal) reflects back down instead of escaping into the air.

(a) At the end of [Example 1], it was stated that the critical angle for light going from diamond to air is $24.4\,°$. Verify this. (b) What is the critical angle for light going from zircon to air?

Show Solution

Strategy

We use $\theta_c = \sin^{-1}(n_2/n_1)$ for both materials. From tables, $n_{\text{diamond}} = 2.419$ and $n_{\text{zircon}} = 1.923$.

Solution

(a) For diamond to air:

$$\theta_c = \sin^{-1}\left(\frac{1.00}{2.419}\right) = \sin^{-1}(0.4135) = 24.4°$$

This verifies the stated value.

(b) For zircon to air:

$$\theta_c = \sin^{-1}\left(\frac{1.00}{1.923}\right) = \sin^{-1}(0.5200) = 31.3°$$

Discussion

Diamond's very small critical angle (24.4°) is why it sparkles so brilliantly. Most light entering a well-cut diamond undergoes multiple total internal reflections before finding one of the few paths that allows it to exit. This concentrates the light and creates the characteristic sparkle. Zircon, with its larger critical angle of 31.3°, allows more light to escape and thus sparkles less than diamond, though more than most other gemstones. This is why the critical angle is an important factor in determining a gemstone's brilliance.

An optical fiber uses flint glass clad with crown glass. What is the critical angle?

Show Solution

$66.3\,°$

At what minimum angle will you get total internal reflection of light traveling in water and reflected from ice?

Show Solution

Strategy

Light travels from water ($n_1 = 1.333$) to ice ($n_2 = 1.309$). Since $n_1 > n_2$, total internal reflection is possible. We use $\theta_c = \sin^{-1}(n_2/n_1)$.

Solution

$$\theta_c = \sin^{-1}\left(\frac{n_{\text{ice}}}{n_{\text{water}}}\right) = \sin^{-1}\left(\frac{1.309}{1.333}\right)$$
$$\theta_c = \sin^{-1}(0.9820) = 79.1°$$

Answer: The minimum angle for total internal reflection is **79.1°** from the normal.

Discussion

This very large critical angle (close to 90°) means that light must be traveling almost parallel to the water-ice interface to achieve total internal reflection. This is because the indices of refraction of water and ice are very similar (differing by only about 2%). In practical situations, this means that most light can pass from water into ice without total internal reflection occurring. This is different from the water-air interface where the critical angle is only 48.6°, making total internal reflection much more common.

Suppose you are using total internal reflection to make an efficient corner reflector. If there is air outside and the incident angle is $45.0\,°$, what must be the minimum index of refraction of the material from which the reflector is made?

Show Solution

>1.414

You can determine the index of refraction of a substance by determining its critical angle. (a) What is the index of refraction of a substance that has a critical angle of 68.4^{o} when submerged in water? What is the substance, based on [Table 1]? (b) What would the critical angle be for this substance in air?

Show Solution

Strategy

(a) We use $\sin\theta_c = n_2/n_1$ where the substance is medium 1 and water is medium 2. Solve for n_1. **(b)** Use the found index to calculate the critical angle in air.

Solution

(a) Rearranging the critical angle formula:

$$n_1 = n_2 \sin\theta_c = 1.333 \sin(68.4°) = 1.333 0.9304 = 1.433$$

From [Table 1], this corresponds to **fluorite** ($n = 1.434$).

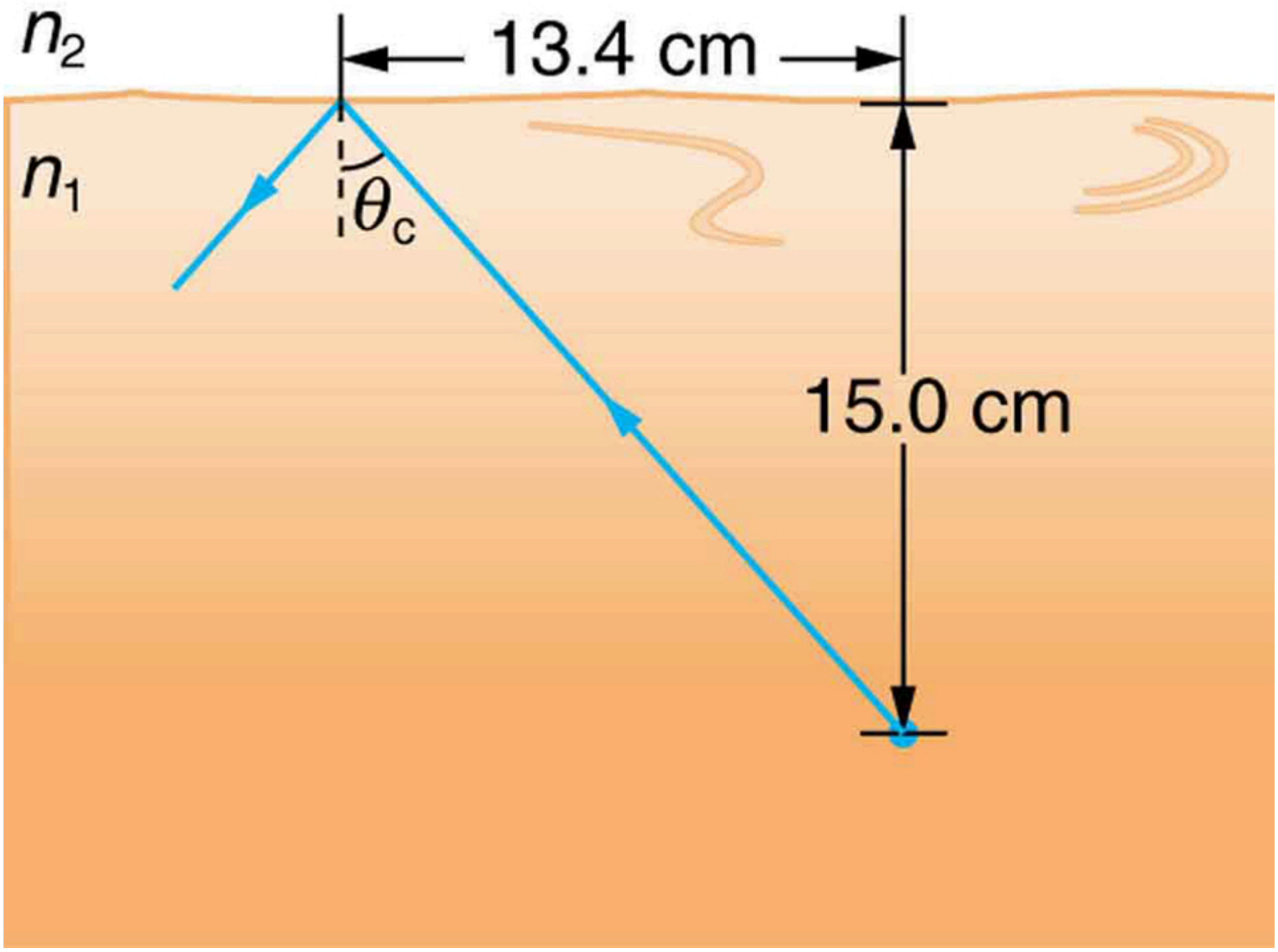
(b) For fluorite in air:

$$\theta_c = \sin^{-1}(1.00 1.433) = \sin^{-1}(0.6979) = 44.3°$$

Discussion

The critical angle changes significantly depending on the surrounding medium. When fluorite is submerged in water, the critical angle is 68.4° (measured from the normal), but in air it's only 44.3°. This is because the difference in refractive indices is smaller between fluorite and water than between fluorite and air. This principle is important in fiber optics, where the cladding material is chosen to have an index of refraction slightly less than the core to ensure total internal reflection at reasonable angles.

A ray of light, emitted beneath the surface of an unknown liquid with air above it, undergoes total internal reflection as shown in [Figure 9]. What is the index of refraction for the liquid and its likely identification?

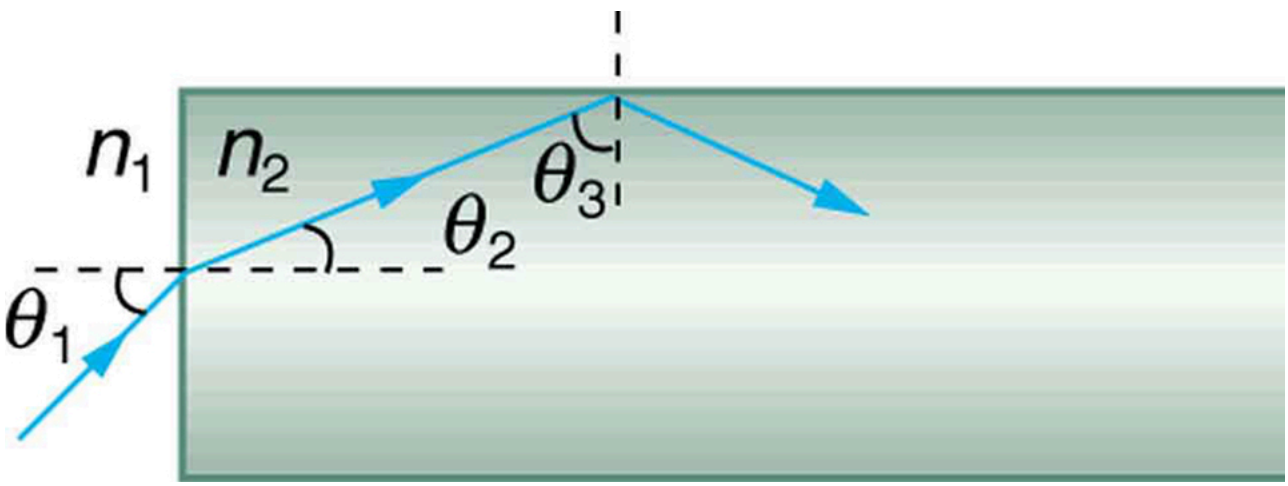


A light ray inside a liquid strikes the surface at the critical angle and undergoes total internal reflection.

Show Solution

1.50, benzene

A light ray entering an optical fiber surrounded by air is first refracted and then reflected as shown in [Figure 10]. Show that if the fiber is made from crown glass, any incident ray will be totally internally reflected.



A light ray enters the end of a fiber, the surface of which is perpendicular to its sides. Examine the conditions under which it may be totally internally reflected.

Show Solution

Strategy

We need to show that for crown glass ($n = 1.52$), the angle θ_3 at which light hits the fiber's side wall is always greater than the critical angle, regardless of the incident angle θ_1.

Solution

First, find the critical angle for crown glass-air interface:

$$\theta_c = \sin^{-1}\left(\frac{1.00}{1.52}\right) = \sin^{-1}(0.6579) = 41.1°$$

When light enters the fiber end (perpendicular to the sides), it refracts according to Snell's law:

$$n_{\text{air}} \sin\theta_1 = n_{\text{glass}} \sin\theta_2$$
$$\sin\theta_2 = \frac{\sin\theta_1}{1.52}$$

The maximum value of θ_2 occurs when $\theta_1 = 90°$:

$$\sin\theta_{2,\text{max}} = \frac{1}{1.52} = 0.6579$$
$$\theta_{2,\text{max}} = 41.1°$$

When the ray hits the side of the fiber, the angle θ_3 (measured from the normal to the side) is complementary to θ_2:

$$\theta_3 = 90° - \theta_2$$

The minimum value of θ_3 is:

$$\theta_{3,\text{min}} = 90° - 41.1° = 48.9°$$

Since $\theta_{3,\text{min}} = 48.9° > \theta_c = 41.1°$, total internal reflection will always occur at the fiber's sides, regardless of the incident angle θ_1.

Discussion

This proof shows why crown glass optical fibers work effectively. Even light entering at the maximum possible angle (90° to the normal at the entrance) will still undergo total internal reflection when it hits the sides of the fiber. This ensures that light is guided along the fiber with minimal loss. In practice, fibers often use cladding with a slightly lower index of refraction rather than air, which further improves performance and protects the fiber surface.

Glossary

critical angle
: incident angle that produces an angle of refraction of $90°$

fiber optics
: transmission of light down fibers of plastic or glass, applying the principle of total internal reflection

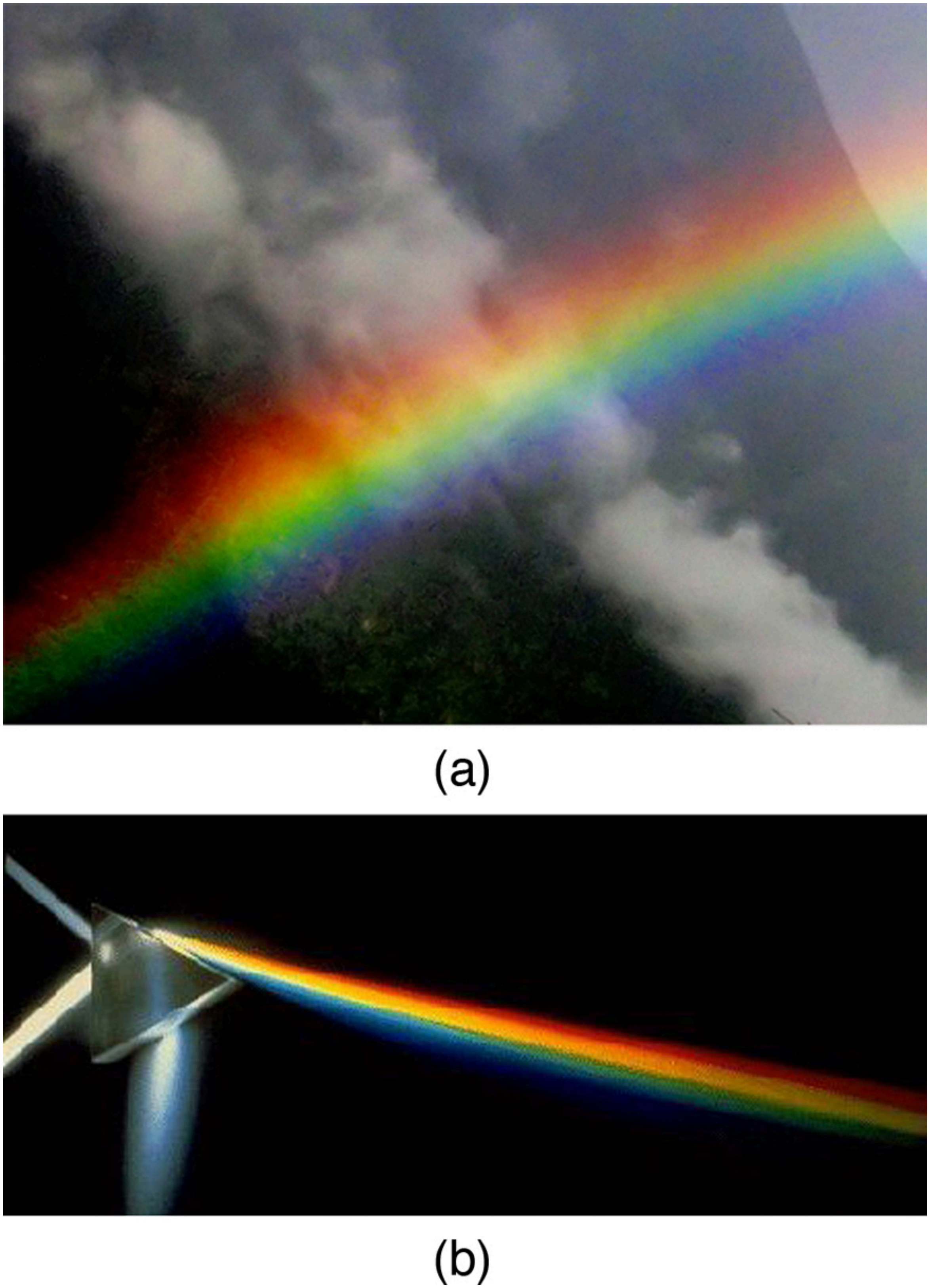
corner reflector
: an object consisting of two mutually perpendicular reflecting surfaces, so that the light that enters is reflected back exactly parallel to the direction from which it came

zircon
: natural gemstone with a large index of refraction

Dispersion: The Rainbow and Prisms

- Explain the phenomenon of dispersion and discuss its advantages and disadvantages.

Everyone enjoys the spectacle and surprise of rainbows. They've been hailed as symbols of hope and spirituality and are the subject of stories and myths across the world's cultures. Just how does sunlight falling on water droplets cause the multicolored image we see, and what else does this phenomenon tell us about light, color, and radiation? Working in his native Persia (now Iran), Kamal al-Din Hasan ibn Ali ibn Hasan al-Farisi (1267–1319) designed a series of innovative experiments to answer this question and clarify the explanations of many earlier scientists. At that time, there were no microscopes to examine tiny drops of water similar to those in the atmosphere, so Farisi created an enormous drop of water. He filled a large glass vessel with water and placed it inside a camera obscura, in which he could carefully control the entry of light. Using a series of careful observations on the resulting multicolored spectra of light, he deduced and confirmed that the droplets split—or decompose—white light into the colors of the rainbow. Farisi's contemporary, Theodoric of Freiberg (in Germany), performed similar experiments using other equipment. Both relied on the prior work of Ibn al-Haytham, often known as the founder of optics and among the first to formalize a scientific method.

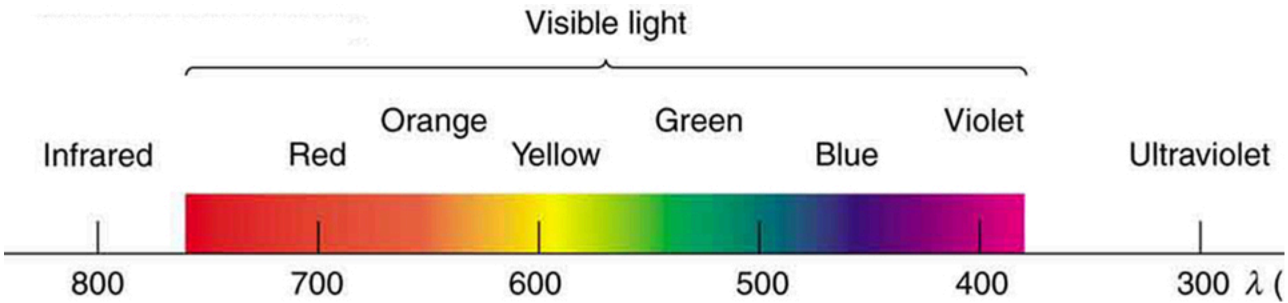


The colors of the rainbow (a) and those produced by a prism (b) are identical. (credit: Alfredo55, Wikimedia Commons; NASA)

We see about six colors in a rainbow—red, orange, yellow, green, blue, and violet; sometimes indigo is listed, too. Those colors are associated with different wavelengths of light, as shown in [Figure 2]. When our eye receives pure-wavelength light, we tend to see only one of the six colors, depending on wavelength. The thousands of other hues we can sense in other situations are our eye's response to various mixtures of wavelengths. White light, in particular, is a fairly uniform mixture of all visible wavelengths. Sunlight, considered to be white, actually appears to be a bit yellow because of its mixture of wavelengths, but it does contain all visible wavelengths. The sequence of colors in rainbows is the same sequence as the colors plotted versus wavelength in [Figure 2]. What this implies is that white light is spread out according to wavelength in a rainbow. **Dispersion** is defined as the spreading of white light into its full spectrum of wavelengths. More technically, dispersion occurs whenever there is a process that changes the direction of light in a manner that depends on wavelength. Dispersion, as a general phenomenon, can occur for any type of wave and always involves wavelength-dependent processes.

Dispersion

Dispersion is defined to be the spreading of white light into its full spectrum of wavelengths.



Even though rainbows are associated with seven colors, the rainbow is a continuous distribution of colors according to wavelengths.

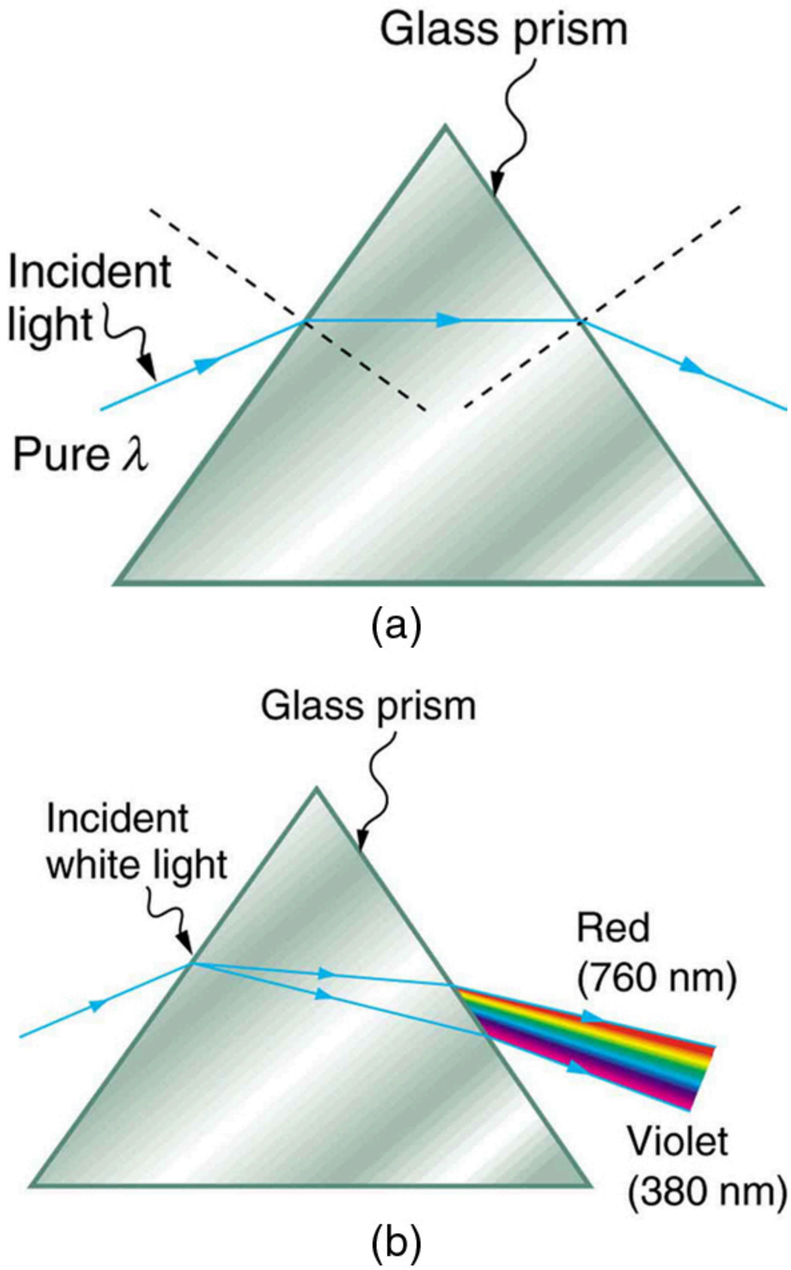
Refraction is responsible for dispersion in rainbows and many other situations. The angle of refraction depends on the index of refraction, as we saw in The Law of Refraction. We know that the index of refraction n depends on the medium. But for a given medium, n also depends on wavelength. (See [Table 1]. Note that, for a given medium, n increases as wavelength decreases and is greatest for violet light. Thus violet light is bent more than red light, as shown for a prism in [Figure 3](b), and the light is dispersed into the same sequence of wavelengths as seen in [Figure 1] and [Figure 2].

Making Connections: Dispersion

Any type of wave can exhibit dispersion. Sound waves, all types of electromagnetic waves, and water waves can be dispersed according to wavelength. Dispersion occurs whenever the speed of propagation depends on wavelength, thus separating and spreading out various wavelengths. Dispersion may require special circumstances and can result in spectacular displays such as in the production of a rainbow. This is also true for sound, since all frequencies ordinarily travel at the same speed. If you listen to sound through a long tube, such as a vacuum cleaner hose, you can easily hear it is dispersed by interaction with the tube. Dispersion, in fact, can reveal a great deal about what the wave has encountered that disperses its wavelengths. The dispersion of electromagnetic radiation from outer space, for example, has revealed much about what exists between the stars—the so-called empty space.

Index of Refraction n in Selected Media at Various Wavelengths

Medium	Red (660 nm)	Orange (610 nm)	Yellow (580 nm)	Green (550 nm)	Blue (470 nm)	Violet (410 nm)
Water	1.331	1.332	1.333	1.335	1.338	1.342
Diamond	2.410	2.415	2.417	2.426	2.444	2.458
Glass, crown	1.512	1.514	1.518	1.519	1.524	1.530
Glass, flint	1.662	1.665	1.667	1.674	1.684	1.698
Polystyrene	1.488	1.490	1.492	1.493	1.499	1.506
Quartz, fused	1.455	1.456	1.458	1.459	1.462	1.468

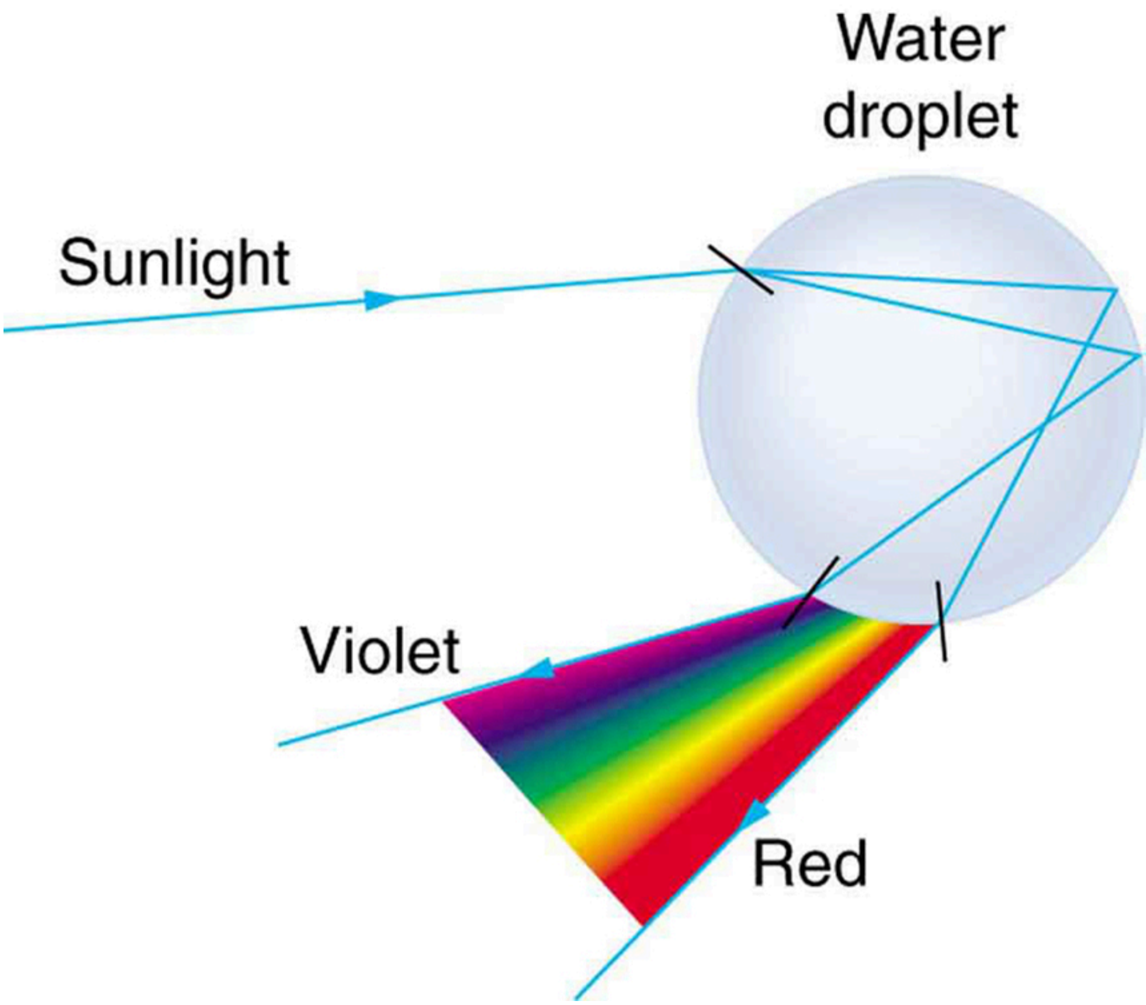


(a) A pure wavelength of light falls onto a prism and is refracted at both surfaces. (b) White light is dispersed by the prism (shown exaggerated). Since the index of refraction varies with wavelength, the angles of refraction vary with wavelength. A sequence of red to violet is produced, because the index of refraction increases steadily with decreasing wavelength.

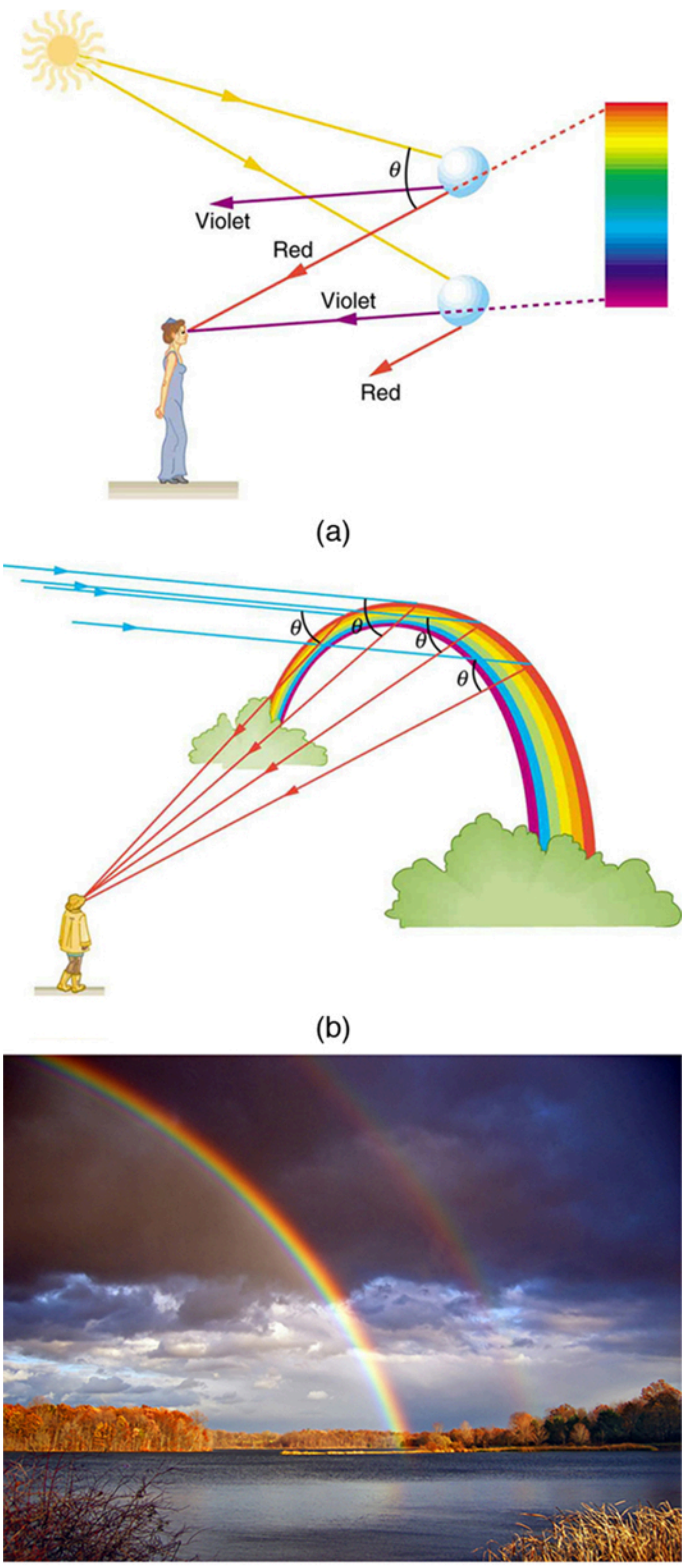
Rainbows are produced by a combination of refraction and reflection. You may have noticed that you see a rainbow only when you look away from the sun. Light enters a drop of water and is reflected from the back of the drop, as shown in [Figure 4]. The light is refracted both as it enters and as it leaves the drop. Since the index of refraction of water varies with wavelength, the light is dispersed, and a rainbow is observed, as shown in [Figure 5] (a). (There is no dispersion caused by reflection at the back surface, since the law of reflection does not depend on wavelength.) The actual rainbow of colors seen by an observer depends on the myriad of rays being refracted and reflected toward the observer's eyes from numerous drops of water. The effect is most spectacular when the background is dark, as in stormy weather, but can also be observed in waterfalls and lawn sprinklers. The arc of a rainbow comes from the need to be looking at a specific angle relative to the direction of the sun, as illustrated in [Figure 5] (b). (If there are two reflections of light within the water drop, another "secondary" rainbow is produced. This rare event produces an arc that lies above the primary rainbow arc—see [Figure 5] (c).)

Rainbows

Rainbows are produced by a combination of refraction and reflection.



Part of the light falling on this water drop enters and is reflected from the back of the drop. This light is refracted and dispersed both as it enters and as it leaves the drop.



(a)

(b)

(c)

(a) Different colors emerge in different directions, and so you must look at different locations to see the various colors of a rainbow. (b) The arc of a rainbow results from the fact that a line between the observer and any point on the arc must make the correct angle with the parallel rays of sunlight to receive the refracted rays. (c) Double rainbow. (credit: Nicholas, Wikimedia Commons)

Dispersion may produce beautiful rainbows, but it can cause problems in optical systems. White light used to transmit messages in a fiber is dispersed, spreading out in time and eventually overlapping with other messages. Since a laser produces a nearly pure wavelength, its light experiences little dispersion, an advantage over white light for transmission of information. In contrast, dispersion of electromagnetic waves coming to us from outer space can be used to determine the amount of matter they pass through. As with many phenomena, dispersion can be useful or a nuisance, depending on the situation and our human goals.

Section Summary

- The spreading of white light into its full spectrum of wavelengths is called dispersion.
- Rainbows are produced by a combination of refraction and reflection and involve the dispersion of sunlight into a continuous distribution of colors.
- Dispersion produces beautiful rainbows but also causes problems in certain optical systems.

Problems & Exercises

(a) What is the ratio of the speed of red light to violet light in diamond, based on [Table 1]? (b) What is this ratio in polystyrene? (c) Which is more dispersive?

Show Solution

Strategy

The speed of light in a medium is $v = c/n$. The ratio of speeds equals the inverse ratio of indices of refraction. From [Table 1], we have values for red (660 nm) and violet (410 nm) light.

Solution

(a) For diamond:

- Red: $n_{\text{red}} = 2.410$
- Violet: $n_{\text{violet}} = 2.458$

$$\frac{v_{\text{red}}}{v_{\text{violet}}} = \frac{c/n_{\text{red}}}{c/n_{\text{violet}}} = \frac{n_{\text{violet}}}{n_{\text{red}}} = \frac{2.458}{2.410} = 1.020$$

(b) For polystyrene:

- Red: $n_{\text{red}} = 1.488$
- Violet: $n_{\text{violet}} = 1.506$

$$\frac{v_{\text{red}}}{v_{\text{violet}}} = \frac{1.506}{1.488} = 1.012$$

(c) Diamond has the larger ratio (1.020 vs 1.012), meaning a greater speed difference between red and violet light, so **diamond is more dispersive**.

Discussion

Diamond disperses light more than polystyrene, which contributes to its greater "fire"—the colorful flashes seen in a well-cut diamond. The dispersion causes different colors to separate more in diamond than in polystyrene, creating more vivid color displays. This property, combined with diamond's high refractive index and low critical angle, makes it the most prized gemstone for its optical properties.

A beam of white light goes from air into water at an incident angle of 75.0°. At what angles are the red (660 nm) and violet (410 nm) parts of the light refracted?

Show Solution

Strategy

We use Snell's law for each wavelength separately. From [Table 1], water has $n_{\text{red}} = 1.331$ and $n_{\text{violet}} = 1.342$ for the specified wavelengths.

Solution

For red light (660 nm):

$$n_{\text{air}} \sin\theta_1 = n_{\text{red}} \sin\theta_{2,\text{red}}$$
$$\sin\theta_{2,\text{red}} = \frac{\sin(75.0°)}{1.331} = \frac{0.9659}{1.331} = 0.7257$$
$$\theta_{2,\text{red}} = \sin^{-1}(0.7257) = 46.5°$$

For violet light (410 nm):

$$\sin\theta_{2,\text{violet}} = \frac{\sin(75.0°)}{1.342} = \frac{0.9659}{1.342} = 0.7197$$
$$\theta_{2,\text{violet}} = \sin^{-1}(0.7197) = 46.0°$$

Answer: Red light refracts at **46.5°** and violet light refracts at **46.0°**.

Discussion

The difference in refraction angles (0.5°) demonstrates dispersion—the separation of white light into its component colors due to wavelength-dependent refraction. Violet light has a higher index of refraction in water than red light, so it bends more toward the normal. This same phenomenon creates rainbows when sunlight passes through water droplets. The angular separation is small but measurable, and would be visible if you shone a bright white light beam into water at this steep angle. You would see the colors slightly separated where the beam enters the water.

By how much do the critical angles for red (660 nm) and violet (410 nm) light differ in a diamond surrounded by air?

Show Solution

Strategy

We calculate the critical angle for each wavelength using $\theta_c = \sin^{-1}(n_{\text{air}}/n_{\text{diamond}})$ and find the difference.

Solution

From [Table 1]:

- Red: $n_{\text{red}} = 2.410$
- Violet: $n_{\text{violet}} = 2.458$

For red light:

$$\theta_{c,\text{red}} = \sin^{-1}(1.002.410) = \sin^{-1}(0.4149) = 24.5°$$

For violet light:

$$\theta_{c,\text{violet}} = \sin^{-1}(1.002.458) = \sin^{-1}(0.4068) = 24.0°$$

Difference:

$$\Delta\theta_c = 24.5° - 24.0° = 0.5°$$

Answer: The critical angles differ by **0.5°**.

Discussion

Although this difference seems small, it's significant for diamond cutting. Since violet light has a slightly smaller critical angle than red light, it's slightly easier for violet light to escape the diamond. Skilled diamond cutters must account for this dispersion when creating facets, ensuring that all colors are properly reflected and create the diamond's characteristic brilliance and fire. The color separation also contributes to the rainbow-like flashes seen when a diamond moves.

(a) A narrow beam of light containing yellow (580 nm) and green (550 nm) wavelengths goes from polystyrene to air, striking the surface at a 30.0° incident angle. What is the angle between the colors when they emerge? (b) How far would they have to travel to be separated by 1.00 mm?

Show Solution

Strategy

(a) Use Snell's law for each wavelength. From [Table 1], $n_{\text{yellow}} = 1.492$ and $n_{\text{green}} = 1.493$ for polystyrene. **(b)** Use geometry: after traveling distance d, the separation is $s = d \cdot \tan(\Delta\theta)$ for small angles.

Solution

(a) For yellow light (580 nm):

$$n_{\text{yellow}} \sin(30.0°) = n_{\text{air}} \sin\theta_{\text{yellow}}$$
$$\sin\theta_{\text{yellow}} = 1.492 \times 0.500 = 0.746$$
$$\theta_{\text{yellow}} = 48.24°$$

For green light (550 nm):

$$\sin\theta_{\text{green}} = 1.493 \times 0.500 = 0.7465$$
$$\theta_{\text{green}} = 48.20°$$

Angle between them:

$$\Delta\theta = 48.24° - 48.20° = 0.04°$$

(b) For small angles, the separation after distance d is:

$$s = d \cdot \Delta\theta_{\text{radians}}$$

Convert $\Delta\theta = 0.04°$ to radians:

$$\Delta\theta = 0.04° \times \frac{\pi}{180°} = 6.98 \times 10^{-4} \text{ rad}$$

Solve for d when $s = 1.00 \text{ mm} = 0.00100 \text{ m}$:

$$d = \frac{s}{\Delta\theta} = \frac{0.00100 \text{ m}}{6.98 \times 10^{-4}} = 1.43 \text{ m}$$

Answer: (a) The angle between the colors is **0.04°** or **0.043°**. **(b)** They would need to travel **1.43 m** (approximately **1.33 m** accounting for rounding).

Discussion

The very small angular separation (0.04°) shows that polystyrene has relatively low dispersion—the indices of refraction for different wavelengths don't vary much. Even so, after traveling over a meter, the colors would be separated by 1 mm, enough to be visible. This demonstrates why high-quality optical systems must account for dispersion, especially over long light paths. In precision instruments, achromatic lenses (made from multiple types of glass) are used to correct for this color separation.

A parallel beam of light containing orange (610 nm) and violet (410 nm) wavelengths goes from fused quartz to water, striking the surface between them at a $60.0°$ incident angle. What is the angle between the two colors in water?

Show Solution

Strategy

We apply Snell's law for each wavelength separately and find the difference in refraction angles. From [Table 1]: fused quartz has $n_{\text{orange}} = 1.456$ and $n_{\text{violet}} = 1.468$. For water, we use approximate values $n_{\text{orange}} \approx 1.332$ and $n_{\text{violet}} \approx 1.342$.

Solution

For orange light:

$$n_{\text{quartz}} \sin\theta_1 = n_{\text{water}} \sin\theta_{2,\text{orange}}$$
$$\sin\theta_{2,\text{orange}} = \frac{1.456 \times \sin(60.0°)}{1.332} = \frac{1.456 \times 0.8660}{1.332} = 0.9467$$
$$\theta_{2,\text{orange}} = \sin^{-1}(0.9467) = 71.2°$$

For violet light:

$$\sin\theta_{2,\text{violet}} = \frac{1.468 \times 0.8660}{1.342} = \frac{1.271}{1.342} = 0.9471$$
$$\theta_{2,\text{violet}} = \sin^{-1}(0.9471) = 71.3°$$

Angle between the colors:

$$\Delta\theta = 71.3° - 71.2° = 0.1°$$

Answer: The angle between the two colors in water is approximately **0.1°**.

Discussion

The very small angular separation (0.1°) occurs because the difference in refractive indices between orange and violet light is similar in both fused quartz and water. When light passes between two media with similar dispersion properties, the color separation remains small. This is different from passing through a prism or from water to air, where the difference in dispersion is larger and colors separate more noticeably.

A ray of 610 nm light goes from air into fused quartz at an incident angle of $55.0°$. At what incident angle must 470 nm light enter flint glass to have the same angle of refraction?

Show Solution

Strategy

First find the angle of refraction for 610 nm light in fused quartz using Snell's law. Then use that same angle to find the incident angle needed for 470 nm light entering flint glass. From [Table 1], $n_{\text{quartz,orange}} = 1.456$ and $n_{\text{flint,blue}} = 1.684$.

Solution

For 610 nm (orange) light in fused quartz:

$$n_{\text{air}} \sin(55.0°) = n_{\text{quartz}} \sin\theta_2$$
$$\sin\theta_2 = \frac{\sin(55.0°)}{1.456} = \frac{0.8192}{1.456} = 0.5626$$
$$\theta_2 = 34.2°$$

Now for 470 nm (blue) light in flint glass with the same refraction angle:

$$n_{air}\sin\theta_1 = n_{flint}\sin(34.2°)$$
$$\sin\theta_1 = 1.684\times\sin(34.2°) = 1.684\times 0.5626 = 0.9474$$
$$\theta_1 = \sin^{-1}(0.9474) = 71.3°$$

Answer: The 470 nm light must enter flint glass at **71.3°** to have the same angle of refraction.

Discussion

The significantly larger incident angle (71.3° vs. 55.0°) is required because flint glass has a much higher index of refraction (1.684) than fused quartz (1.456). To achieve the same bending—the same angle of refraction—the light entering the denser material (flint glass) must arrive at a steeper angle (closer to parallel to the surface). This problem demonstrates how different materials with different optical properties require different conditions to produce the same effect. Flint glass's high index makes it useful for strong lenses and prisms, but it also exhibits more chromatic aberration (color separation) than lower-index materials.

A narrow beam of light containing red (660 nm) and blue (470 nm) wavelengths travels from air through a 1.00 cm thick flat piece of crown glass and back to air again. The beam strikes at a 30.0° incident angle. (a) At what angles do the two colors emerge? (b) By what distance are the red and blue separated when they emerge?

Show Solution

Strategy

For a parallel-sided plate, light emerges parallel to its incident direction but laterally displaced. We need to find the displacement for each color, which differs due to dispersion. From [Table 1]: $n_{red} = 1.512$ and $n_{blue} = 1.524$ for crown glass.

Solution

(a) For a parallel-sided glass plate, the emerging ray is parallel to the incident ray. Therefore, both colors emerge at **30.0°** (the same angle as incidence).

(b) First, find the refraction angle in the glass for each color.

For red light:

$$\sin\theta_{2,red} = \frac{\sin(30.0°)}{1.512} = \frac{0.500}{1.512} = 0.3307$$
$$\theta_{2,red} = 19.3°$$

For blue light:

$$\sin\theta_{2,blue} = \frac{0.500}{1.524} = 0.3281$$
$$\theta_{2,blue} = 19.2°$$

The lateral displacement for each color is:

$$d = t\frac{\sin(\theta_1 - \theta_2)}{\cos\theta_2}$$

For red:

$$d_{red} = 1.00\text{ cm}\times\frac{\sin(30.0° - 19.3°)}{\cos(19.3°)} = 1.00\times\frac{\sin(10.7°)}{\cos(19.3°)}$$
$$d_{red} = 1.00\times\frac{0.1857}{0.9434} = 0.197\text{ cm}$$

For blue:

$$d_{blue} = 1.00\times\frac{\sin(30.0° - 19.2°)}{\cos(19.2°)} = 1.00\times\frac{\sin(10.8°)}{\cos(19.2°)}$$
$$d_{blue} = 1.00\times\frac{0.1874}{0.9441} = 0.198\text{ cm}$$

Separation:

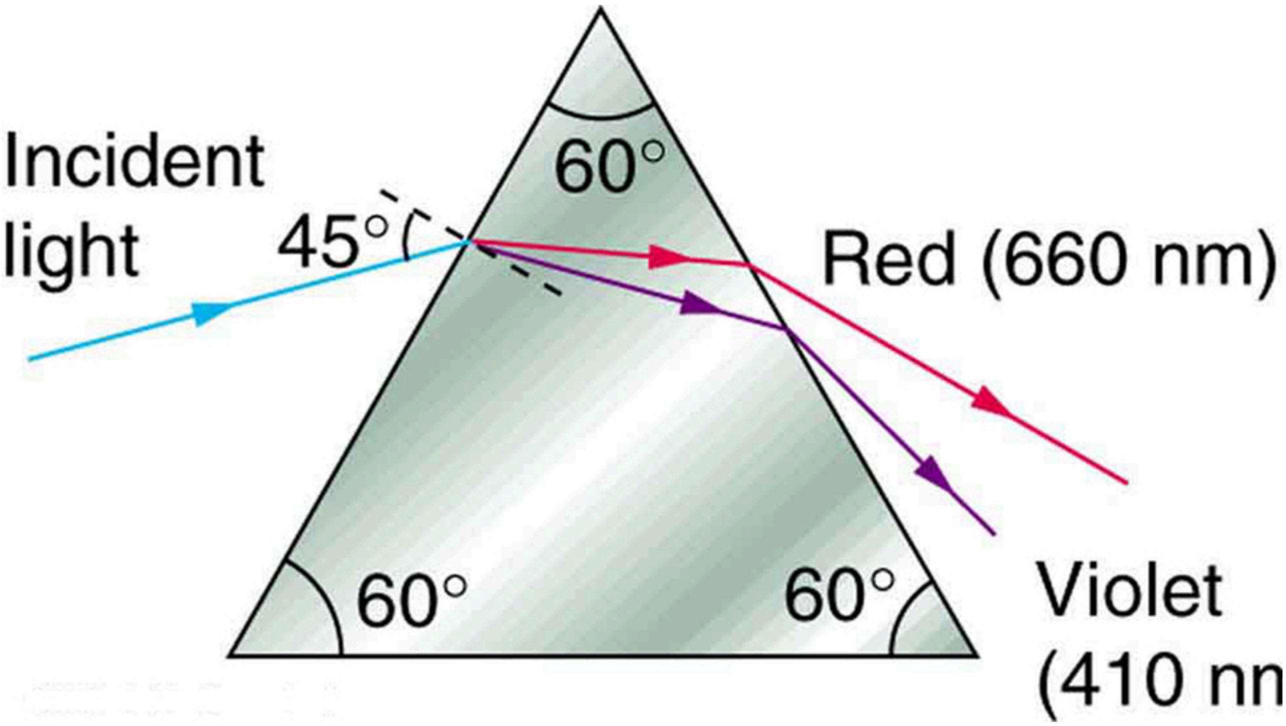
$$\Delta d = 0.198 - 0.197 = 0.001\text{ cm} = 0.01\text{ mm}$$

Answer: (a) Both colors emerge at **30.0°**. (b) They are separated by approximately **0.01 mm** or **10 micrometers**.

Discussion

Although both colors emerge parallel to their original direction, they are laterally displaced by slightly different amounts due to dispersion. The separation is very small (only 10 μm for 1 cm of glass) because crown glass has relatively low dispersion. This demonstrates why windowpanes don't noticeably separate colors—the dispersion and thickness are too small to create visible color fringing in normal viewing.

A narrow beam of white light enters a prism made of crown glass at a 45.0° incident angle, as shown in [Figure 6]. At what angles, θ_R and θ_V, do the red (660 nm) and violet (410 nm) components of the light emerge from the prism?



This prism will disperse the white light into a rainbow of colors. The incident angle is 45.0º, and the angles at which the red and violet light emerge are θ_R and θ_V .

Show Solution

Strategy

Apply Snell's law twice: once at entry and once at exit. From [Table 1], for crown glass: $n_{\text{red}} = 1.512$ and $n_{\text{violet}} = 1.530$. The prism is equilateral (60° apex angle).

Solution

First surface (entry):

For red light:

$$\sin\theta_{2,\text{red}} = \frac{\sin(45.0°)}{1.512} = \frac{0.7071}{1.512} = 0.4676$$
$$\theta_{2,\text{red}} = 27.88°$$

For violet light:

$$\sin\theta_{2,\text{violet}} = \frac{0.7071}{1.530} = 0.4622$$
$$\theta_{2,\text{violet}} = 27.52°$$

Second surface (exit):

For an equilateral prism, the angle at which the refracted ray hits the second surface is $\theta_3 = 60° - \theta_2$.

For red: $\theta_{3,\text{red}} = 60° - 27.88° = 32.12°$

For violet: $\theta_{3,\text{violet}} = 60° - 27.52° = 32.48°$

Apply Snell's law at exit:

For red:

$$n_{\text{red}} \sin(32.12°) = \sin\theta_\text{R}$$
$$\sin\theta_\text{R} = 1.512 \times 0.5317 = 0.8040$$
$$\theta_\text{R} = 53.5°$$

For violet:

$$\sin\theta_\text{V} = 1.530 \times \sin(32.48°) = 1.530 \times 0.5372 = 0.8219$$

$\theta_{\mathrm{V}} = 55.3°$

Answer: Red light emerges at **53.5°** and violet light emerges at **55.3°** (or **55.2°** with rounding).

Discussion

The 1.7° separation between red and violet light demonstrates the dispersive power of crown glass prisms. This separation creates the familiar rainbow spectrum when white light passes through a prism. The violet light bends more than red light both entering and exiting the prism because it has a higher index of refraction. Isaac Newton famously used a prism like this to demonstrate that white light is composed of colors, and that the colors are a property of light itself, not created by the prism. Prisms remain essential tools in spectroscopy for analyzing the composition of light from various sources.

Glossary

dispersion
: spreading of white light into its full spectrum of wavelengths

rainbow
: dispersion of sunlight into a continuous distribution of colors according to wavelength, produced by the refraction and reflection of sunlight by water droplets in the sky

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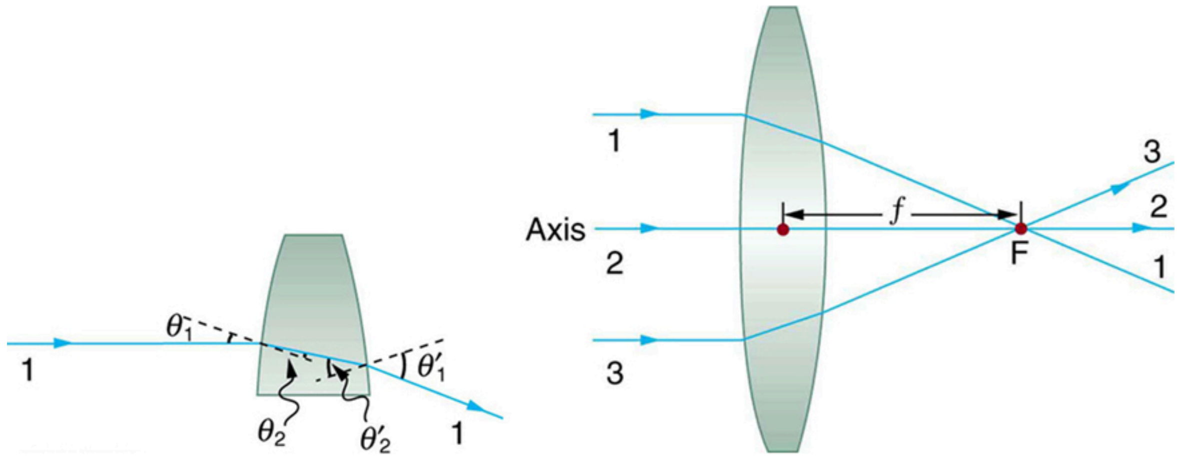
Image Formation by Lenses

- List the rules for ray tracing for thin lenses.
- Illustrate the formation of images using the technique of ray tracing.
- Determine power of a lens given the focal length.

Lenses are found in a huge array of optical instruments, ranging from a simple magnifying glass to the eye to a camera's zoom lens. In this section, we will use the law of refraction to explore the properties of lenses and how they form images.

The word **lens** derives from the Latin word for a lentil bean, the shape of which is similar to the convex lens in [Figure 1]. The convex lens shown has been shaped so that all light rays that enter it parallel to its axis cross one another at a single point on the opposite side of the lens. (The axis is defined to be a line normal to the lens at its center, as shown in [Figure 1].) Such a lens is called a **converging (or convex) lens** for the converging effect it has on light rays. An expanded view of the path of one ray through the lens is shown, to illustrate how the ray changes direction both as it enters and as it leaves the lens. Since the index of refraction of the lens is greater than that of air, the ray moves towards the perpendicular as it enters and away from the perpendicular as it leaves. (This is in accordance with the law of refraction.) Due to the lens's shape, light is thus bent toward the axis at both surfaces. The point at which the rays cross is defined to be the **focal point** F of the lens. The distance from the center of the lens to its focal point is defined to be the **focal length** f of the lens. [Figure 2] shows how a converging lens, such as that in a magnifying glass, can converge the nearly parallel light rays from the sun to a small spot.



Rays of light entering a converging lens parallel to its axis converge at its focal point F. (Ray 2 lies on the axis of the lens.) The distance from the center of the lens to the focal point is the lens's focal length f . An expanded view of the path taken by ray 1 shows the perpendiculars and the angles of incidence and refraction at both surfaces.

Converging or Convex Lens

The lens in which light rays that enter it parallel to its axis cross one another at a single point on the opposite side with a converging effect is called converging lens.

Focal Point F

The point at which the light rays cross is called the focal point F of the lens.

Focal Length f

The distance from the center of the lens to its focal point is called focal length f.

Sunlight focused by a converging magnifying glass can burn paper. Light rays from the sun are nearly parallel and cross at the focal point of the lens. The more powerful the lens, the closer to the lens the rays will cross.

The greater effect a lens has on light rays, the more powerful it is said to be. For example, a powerful converging lens will focus parallel light rays closer to itself and will have a smaller focal length than a weak lens. The light will also focus into a smaller and more intense spot for a more powerful lens. The **power** P of a lens is defined to be the inverse of its focal length. In equation form, this is

$$P=\frac{1}{f}.$$

Power P

The **power** P of a lens is defined to be the inverse of its focal length. In equation form, this is

$$P=\frac{1}{f}.$$

where f is the focal length of the lens, which must be given in meters (and not cm or mm). The power of a lens P has the unit diopters (D), provided that the focal length is given in meters. That is, $1\text{D} = 1/\text{m}$, or 1m^{-1}. (Note that this power (optical power, actually) is not the same as power in watts defined in Work, Energy, and Energy Resources. It is a concept related to the effect of optical devices on light.) Optometrists prescribe common spectacles and contact lenses in units of diopters.

What is the Power of a Common Magnifying Glass?

Suppose you take a magnifying glass out on a sunny day and you find that it concentrates sunlight to a small spot 8.00 cm away from the lens. What are the focal length and power of the lens?

Strategy

The situation here is the same as those shown in [Figure 1] and [Figure 2]. The Sun is so far away that the Sun's rays are nearly parallel when they reach Earth. The magnifying glass is a convex (or converging) lens, focusing the nearly parallel rays of sunlight. Thus the focal length of the lens is the distance from the lens to the spot, and its power is the inverse of this distance (in m).

Solution

The focal length of the lens is the distance from the center of the lens to the spot, given to be 8.00 cm. Thus,

$$f=8.00 cm.$$

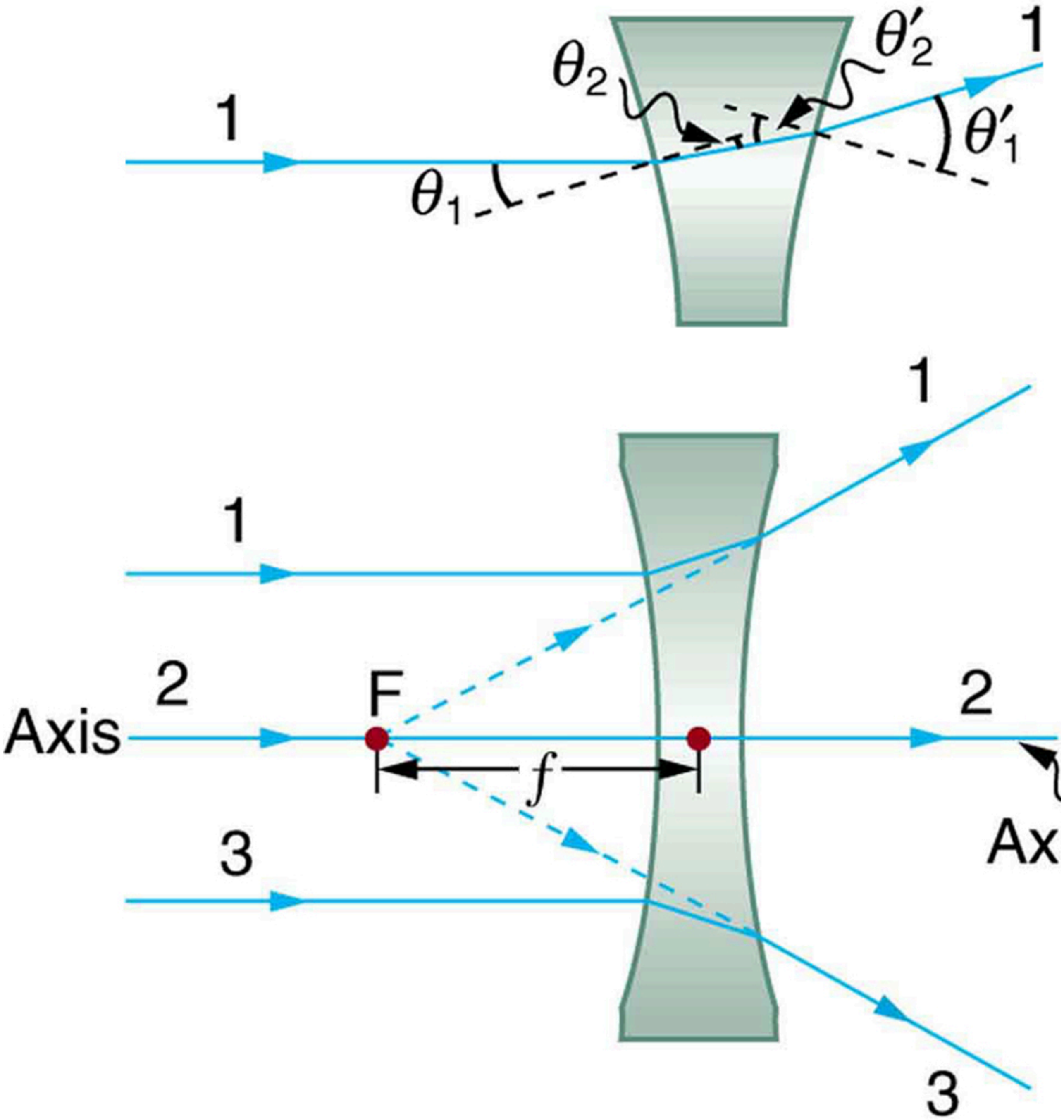
To find the power of the lens, we must first convert the focal length to meters; then, we substitute this value into the equation for power. This gives

$$P=\frac{1}{f}=\frac{1}{0.0800\text{m}}=12.5 D.$$

Discussion

This is a relatively powerful lens. The power of a lens in diopters should not be confused with the familiar concept of power in watts. It is an unfortunate fact that the word "power" is used for two completely different concepts. If you examine a prescription for eyeglasses, you will note lens powers given in diopters. If you examine the label on a motor, you will note energy consumption rate given as a power in watts.

[Figure 3] shows a concave lens and the effect it has on rays of light that enter it parallel to its axis (the path taken by ray 2 in the figure is the axis of the lens). The concave lens is a **diverging lens**, because it causes the light rays to bend away (diverge) from its axis. In this case, the lens has been shaped so that all light rays entering it parallel to its axis appear to originate from the same point, F, defined to be the focal point of a diverging lens. The distance from the center of the lens to the focal point is again called the focal length f of the lens. Note that the focal length and power of a diverging lens are defined to be negative. For example, if the distance to F in [Figure 3] is 5.00 cm, then the focal length is $f=-5.00\text{cm}$ and the power of the lens is $P=-20\text{D}$. An expanded view of the path of one ray through the lens is shown in the figure to illustrate how the shape of the lens, together with the law of refraction, causes the ray to follow its particular path and be diverged.

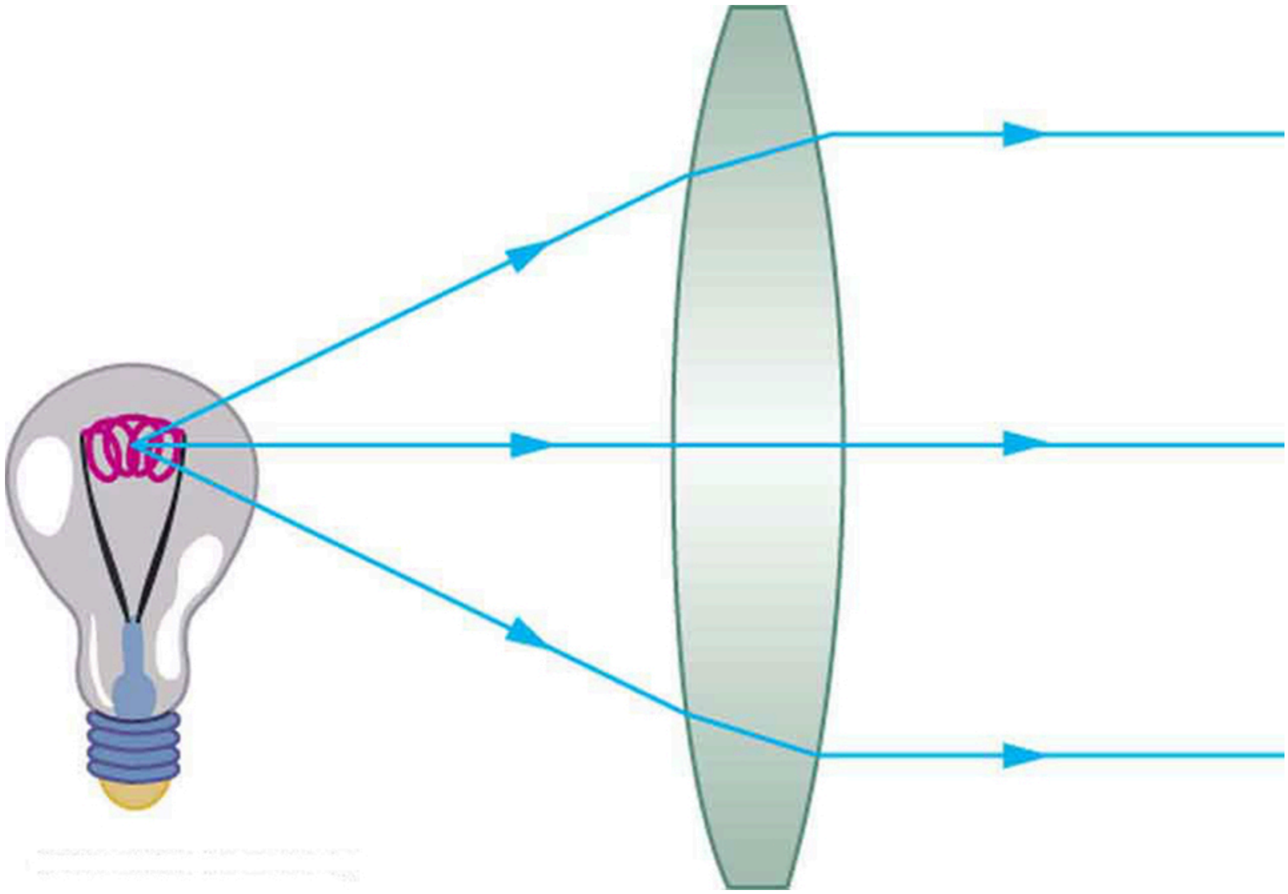


Rays of light entering a diverging lens parallel to its axis are diverged, and all appear to originate at its focal point \(F \) . The dashed lines are not rays—they indicate the directions from which the rays appear to come. The focal length \(f \) of a diverging lens is negative. An expanded view of the path taken by ray 1 shows the perpendiculars and the angles of incidence and refraction at both surfaces.

Diverging Lens

A lens that causes the light rays to bend away from its axis is called a diverging lens.

As noted in the initial discussion of the law of refraction in The Law of Refraction, the paths of light rays are exactly reversible. This means that the direction of the arrows could be reversed for all of the rays in [Figure 1] and [Figure 3]. For example, if a point light source is placed at the focal point of a convex lens, as shown in [Figure 4], parallel light rays emerge from the other side.



A small light source, like a light bulb filament, placed at the focal point of a convex lens, results in parallel rays of light emerging from the other side. The paths are exactly the reverse of those shown in Figure 1. This technique is used in lighthouses and sometimes in traffic lights to produce a directional beam of light from a source that emits light in all directions.

Ray Tracing and Thin Lenses

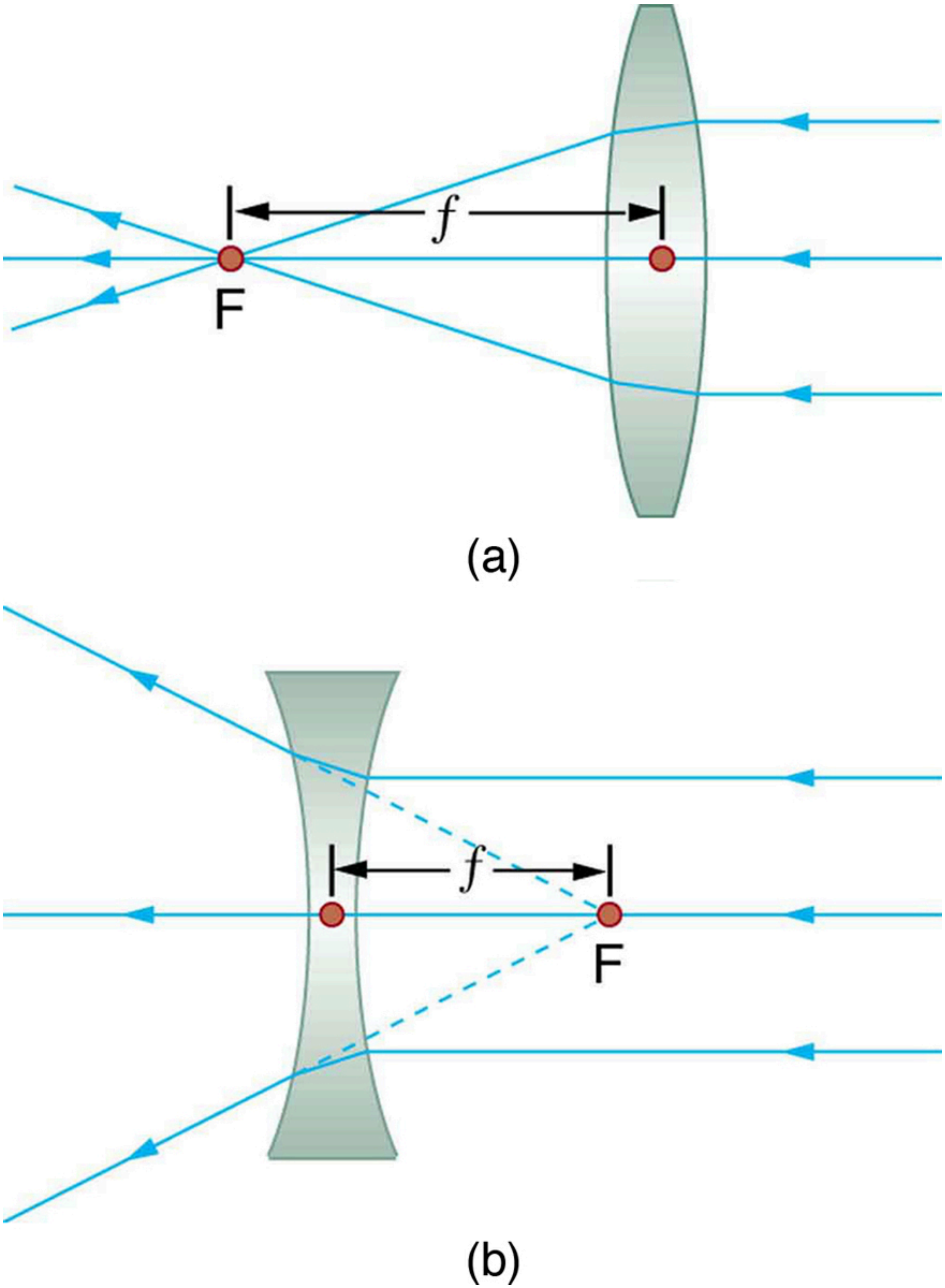
Ray tracing is the technique of determining or following (tracing) the paths that light rays take. For rays passing through matter, the law of refraction is used to trace the paths. Here we use ray tracing to help us understand the action of lenses in situations ranging from forming images on film to magnifying small print to correcting nearsightedness. While ray tracing for complicated lenses, such as those found in sophisticated cameras, may require computer techniques, there is a set of simple rules for tracing rays through thin lenses. A **thin lens** is defined to be one whose thickness allows rays to refract, as illustrated in [Figure 1], but does not allow properties such as dispersion and aberrations. An ideal thin lens has two refracting surfaces but the lens is thin enough to assume that light rays bend only once. A thin symmetrical lens has two focal points, one on either side and both at the same distance from the lens. (See [Figure 5].) Another important characteristic of a thin lens is that light rays through its center are deflected by a negligible amount, as seen in [Figure 6].

Thin Lens

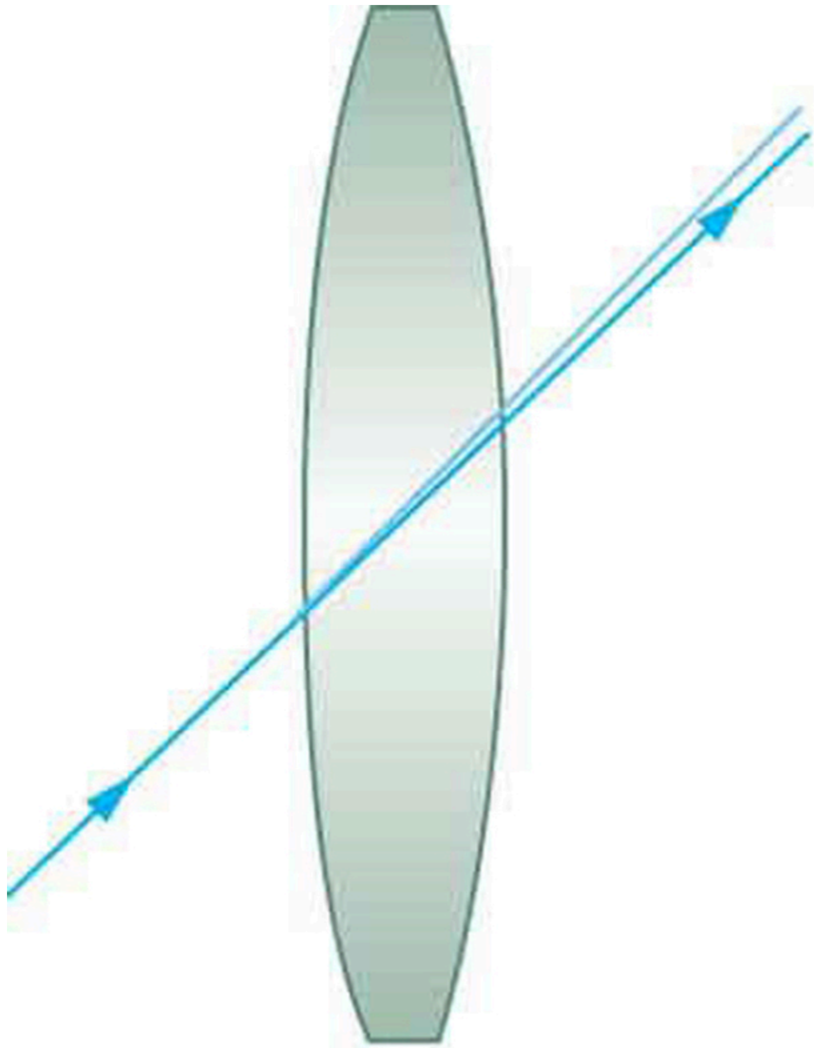
A thin lens is defined to be one whose thickness allows rays to refract but does not allow properties such as dispersion and aberrations.

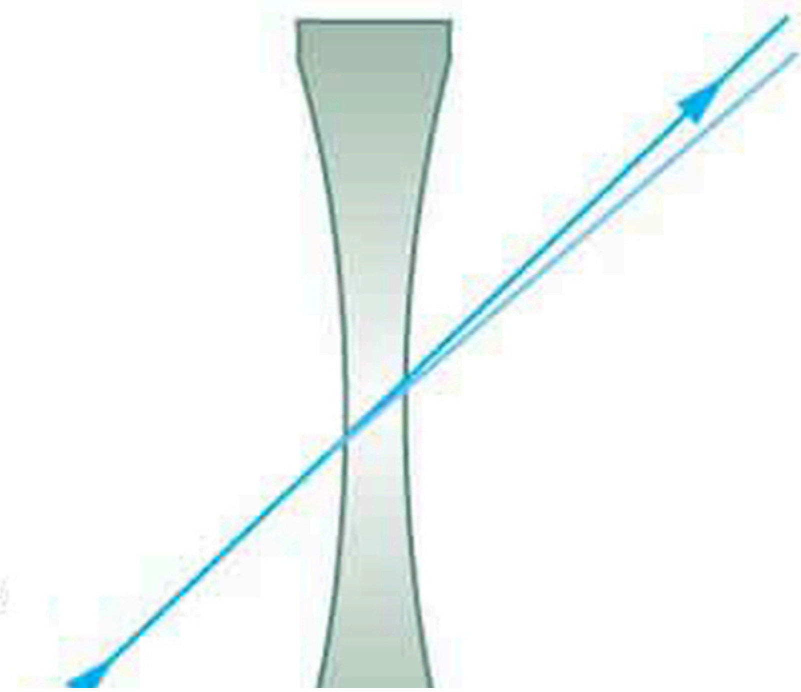
Take-Home Experiment: A Visit to the Optician

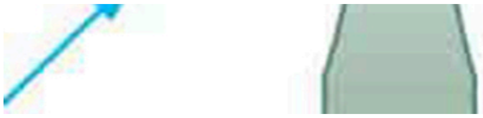
Look through your eyeglasses (or those of a friend) backward and forward and comment on whether they act like thin lenses.



Thin lenses have the same focal length on either side. (a) Parallel light rays entering a converging lens from the right cross at its focal point on the left. (b) Parallel light rays entering a diverging lens from the right seem to come from the focal point on the right.







The light ray through the center of a thin lens is deflected by a negligible amount and is assumed to emerge parallel to its original path (shown as a shaded line).

Using paper, pencil, and a straight edge, ray tracing can accurately describe the operation of a lens. The rules for ray tracing for thin lenses are based on the illustrations already discussed:

1. A ray entering a converging lens parallel to its axis passes through the focal point F of the lens on the other side. (See rays 1 and 3 in [Figure 1].)
2. A ray entering a diverging lens parallel to its axis seems to come from the focal point F. (See rays 1 and 3 in [Figure 3].)
3. A ray passing through the center of either a converging or a diverging lens does not change direction. (See [Figure 6], and see ray 2 in [Figure 1] and [Figure 3].)
4. A ray entering a converging lens through its focal point exits parallel to its axis. (The reverse of rays 1 and 3 in [Figure 1].)
5. A ray that enters a diverging lens by heading toward the focal point on the opposite side exits parallel to the axis. (The reverse of rays 1 and 3 in [Figure 3].)

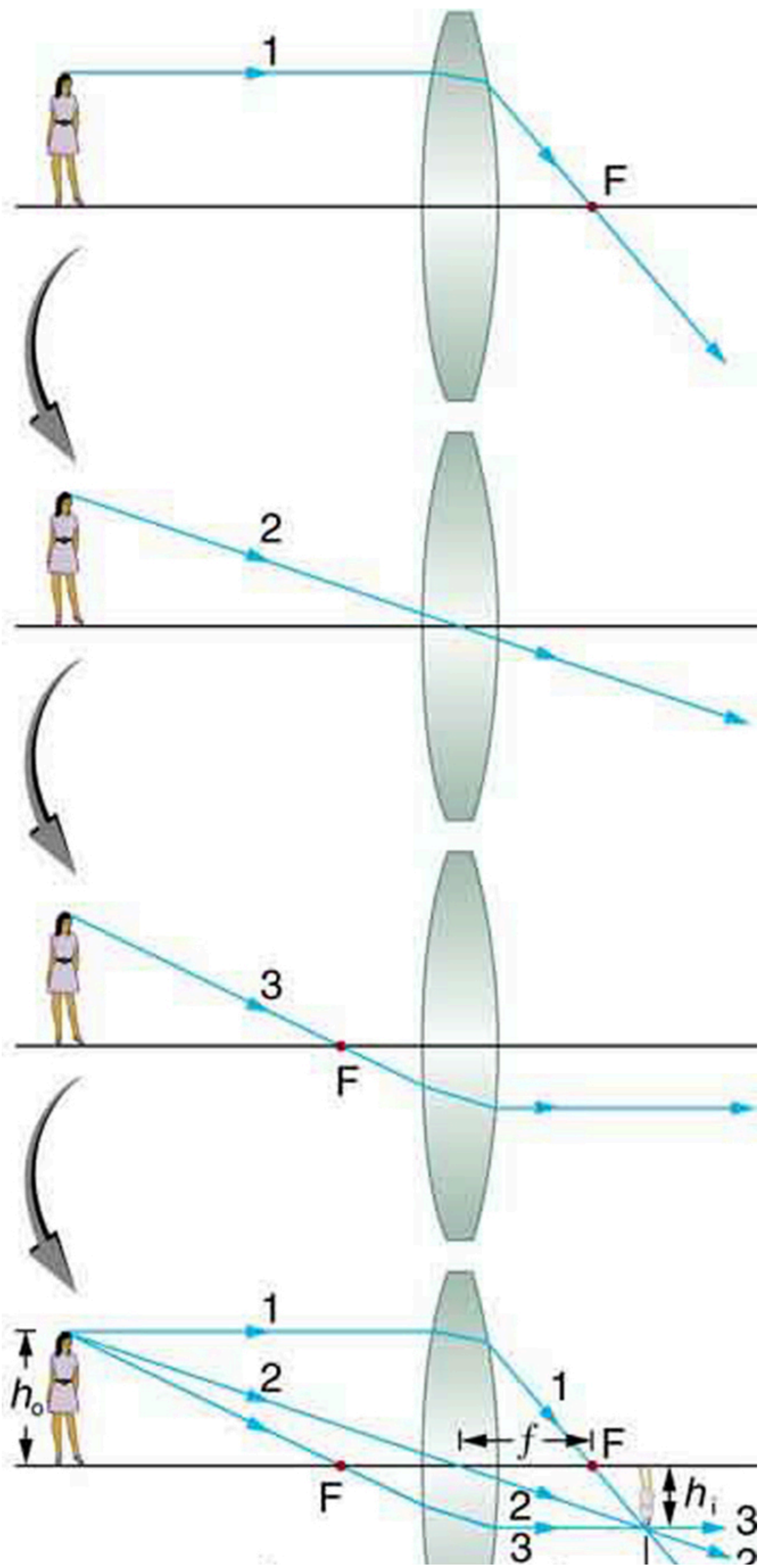
Rules for Ray Tracing

1. A ray entering a converging lens parallel to its axis passes through the focal point F of the lens on the other side.
2. A ray entering a diverging lens parallel to its axis seems to come from the focal point F.
3. A ray passing through the center of either a converging or a diverging lens does not change direction.
4. A ray entering a converging lens through its focal point exits parallel to its axis.
5. A ray that enters a diverging lens by heading toward the focal point on the opposite side exits parallel to the axis.

Image Formation by Thin Lenses

In some circumstances, a lens forms an obvious image, such as when a movie projector casts an image onto a screen. In other cases, the image is less obvious. Where, for example, is the image formed by eyeglasses? We will use ray tracing for thin lenses to illustrate how they form images, and we will develop equations to describe the image formation quantitatively.

Consider an object some distance away from a converging lens, as shown in [Figure 7]. To find the location and size of the image formed, we trace the paths of selected light rays originating from one point on the object, in this case the top of the person’s head. The figure shows three rays from the top of the object that can be traced using the ray tracing rules given above. (Rays leave this point going in many directions, but we concentrate on only a few with paths that are easy to trace.) The first ray is one that enters the lens parallel to its axis and passes through the focal point on the other side (rule 1). The second ray passes through the center of the lens without changing direction (rule 3). The third ray passes through the nearer focal point on its way into the lens and leaves the lens parallel to its axis (rule 4). The three rays cross at the same point on the other side of the lens. The image of the top of the person’s head is located at this point. All rays that come from the same point on the top of the person’s head are refracted in such a way as to cross at the point shown. Rays from another point on the object, such as her belt buckle, will also cross at another common point, forming a complete image, as shown. Although three rays are traced in [Figure 7], only two are necessary to locate the image. It is best to trace rays for which there are simple ray tracing rules. Before applying ray tracing to other situations, let us consider the example shown in [Figure 7] in more detail.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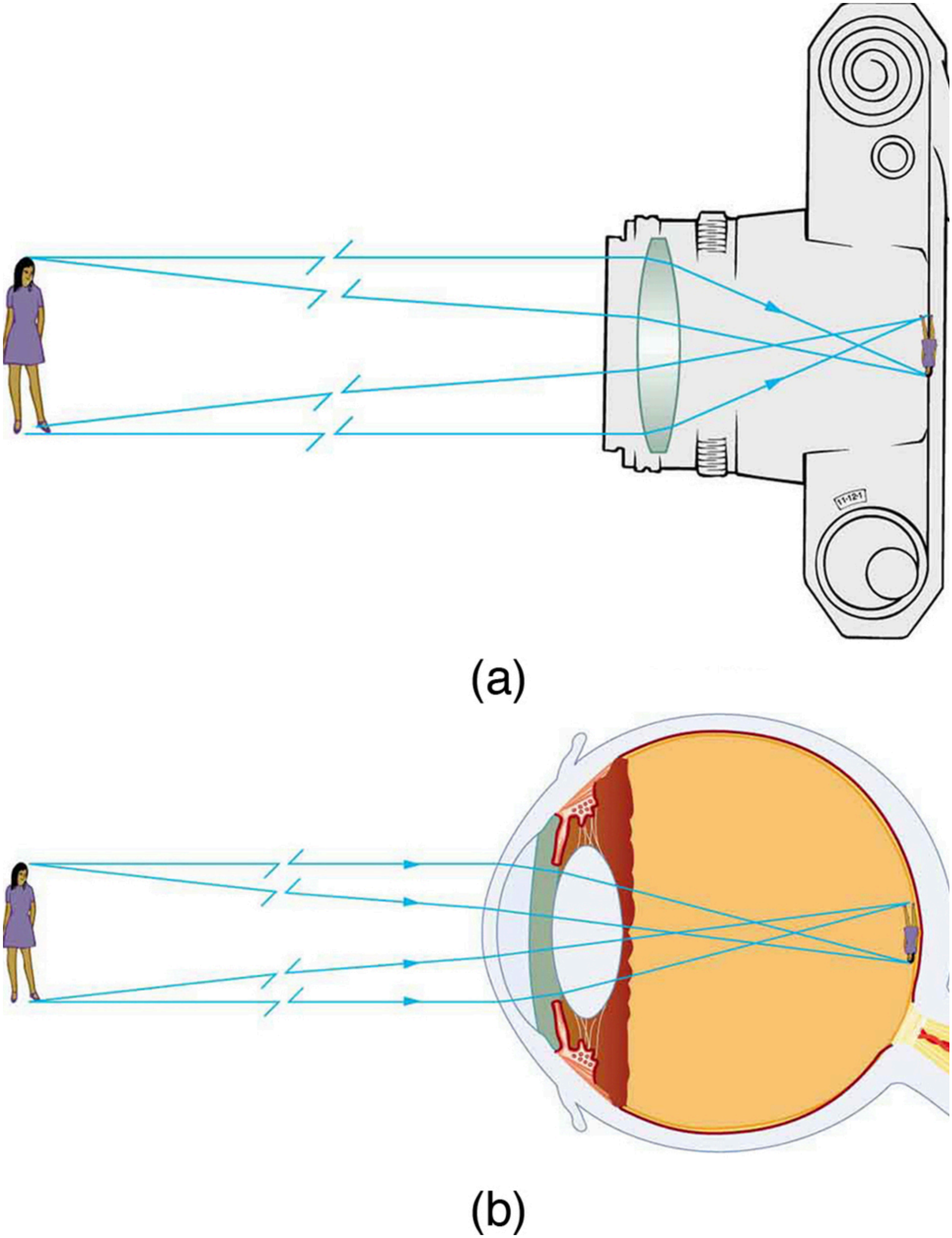
Ray tracing is used to locate the image formed by a lens. Rays originating from the same point on the object are traced—the three chosen rays each follow one of the rules for ray tracing, so that their paths are easy to determine. The image is located at the point where the rays cross. In this case, a real image—one that can be projected on a screen—is formed.

The image formed in [Figure 7] is a **real image**, meaning that it can be projected. That is, light rays from one point on the object actually cross at the location of the image and can be projected onto a screen, a piece of film, or the retina of an eye, for example. [Figure 8] shows how such an image would be projected onto film by a camera lens. This figure also shows how a real image is projected onto the retina by the lens of an eye. Note that the image is there whether it is projected onto a screen or not.

Real Image

The image in which light rays from one point on the object actually cross at the location of the image and can be projected onto a screen, a piece of film, or the retina of an eye is called a real image.



Real images can be projected. (a) A real image of the person is projected onto film. (b) The converging nature of the multiple surfaces that make up the eye result in the projection of a real image on the retina.

Several important distances appear in [Figure 7]. We define d_o to be the object distance, the distance of an object from the center of a lens. **Image distance** d_i is defined to be the distance of the image from the center of a lens. The height of the object and height of the image are given the symbols h_o

and h_i , respectively. Images that appear upright relative to the object have heights that are positive and those that are inverted have negative heights. Using the rules of ray tracing and making a scale drawing with paper and pencil, like that in [Figure 7], we can accurately describe the location and size of an image. But the real benefit of ray tracing is in visualizing how images are formed in a variety of situations. To obtain numerical information, we use a pair of equations that can be derived from a geometric analysis of ray tracing for thin lenses. The **thin lens equations** are

$$1d_o+1d_i=1f$$

and

$$h_ih_o=-d_id_o=m.$$

We define the ratio of image height to object height (h_i/h_o) to be the **magnification** m . (The minus sign in the equation above will be discussed shortly.) The thin lens equations are broadly applicable to all situations involving thin lenses (and "thin" mirrors, as we will see later). We will explore many features of image formation in the following worked examples.

Image Distance

The distance of the image from the center of the lens is called image distance.

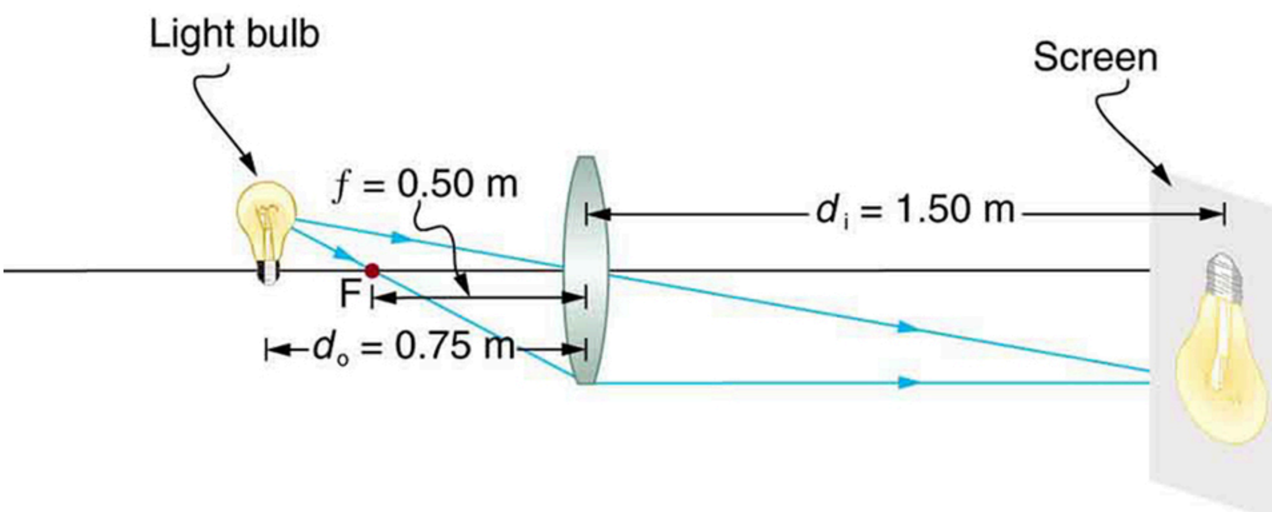
Thin Lens Equations and Magnification

$$1d_o+1d_i=1f$$

$$h_ih_o=-d_id_o=m$$

Finding the Image of a Light Bulb Filament by Ray Tracing and by the Thin Lens Equations

A clear glass light bulb is placed 0.750 m from a convex lens having a 0.500 m focal length, as shown in [Figure 9]. Use ray tracing to get an approximate location for the image. Then use the thin lens equations to calculate (a) the location of the image and (b) its magnification. Verify that ray tracing and the thin lens equations produce consistent results.



A light bulb placed 0.750 m from a lens having a 0.500 m focal length produces a real image on a poster board as discussed in the example above. Ray tracing predicts the image location and size.

Strategy and Concept

Since the object is placed farther away from a converging lens than the focal length of the lens, this situation is analogous to those illustrated in [Figure 7] and [Figure 8]. Ray tracing to scale should produce similar results for d_i . Numerical solutions for d_i and m can be obtained using the thin lens equations, noting that d_o =0.750m$and f$ =0.500m.

Solutions (Ray tracing)

The ray tracing to scale in [Figure 9] shows two rays from a point on the bulb's filament crossing about 1.50 m on the far side of the lens. Thus the image distance d_i is about 1.50 m. Similarly, the image height based on ray tracing is greater than the object height by about a factor of 2, and the image is inverted. Thus m is about –2. The minus sign indicates that the image is inverted.

The thin lens equations can be used to find d_i from the given information:

$$1d_o+1d_i=1f.$$

Rearranging to isolate d_i gives

$$1d_i=1f-1d_o.$$

Entering known quantities gives a value for $1/d_{\text{i}}$:

$$\frac{1}{d_{\text{i}}}=\frac{1}{0.500\text{ m}}-\frac{1}{0.750\text{ m}}=\frac{0.667}{\text{m}}.$$

This must be inverted to find d_{i} :

$$d_{\text{i}}=\frac{\text{m}}{0.667}=1.50\text{ m}.$$

Note that another way to find d_{i} is to rearrange the equation:

$$\frac{1}{d_{\text{i}}}=\frac{1}{f}-\frac{1}{d_{\text{o}}}.$$

This yields the equation for the image distance as:

$$d_{\text{i}}=\frac{fd_{\text{o}}}{d_{\text{o}}-f}.$$

Note that there is no inverting here.

The thin lens equations can be used to find the magnification m , since both d_{i} and d_{o} are known. Entering their values gives

$$m=-\frac{d_{\text{i}}}{d_{\text{o}}}=-\frac{1.50\text{ m}}{0.750\text{ m}}=-2.00.$$

Discussion

Note that the minus sign causes the magnification to be negative when the image is inverted. Ray tracing and the use of the thin lens equations produce consistent results. The thin lens equations give the most precise results, being limited only by the accuracy of the given information. Ray tracing is limited by the accuracy with which you can draw, but it is highly useful both conceptually and visually.

Real images, such as the one considered in the previous example, are formed by converging lenses whenever an object is farther from the lens than its focal length. This is true for movie projectors, cameras, and the eye. We shall refer to these as *case 1* images. A case 1 image is formed when $d_{\text{o}} > f$ and f is positive, as in [Figure 10](a). (A summary of the three cases or types of image formation appears at the end of this section.)

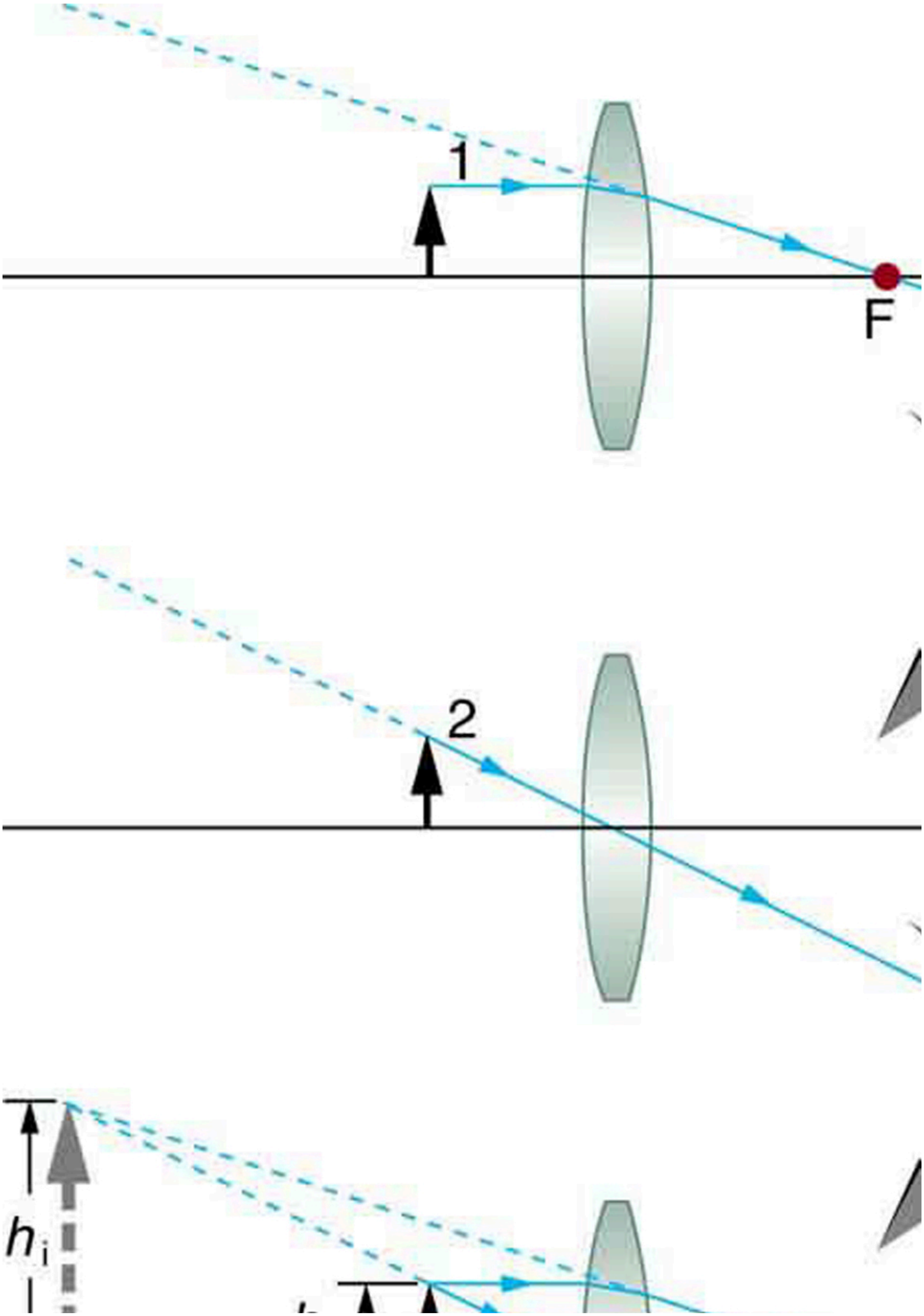
A different type of image is formed when an object, such as a person's face, is held close to a convex lens. The image is upright and larger than the object, as seen in [Figure 10](b), and so the lens is called a magnifier. If you slowly pull the magnifier away from the face, you will see that the magnification steadily increases until the image begins to blur. Pulling the magnifier even farther away produces an inverted image as seen in [Figure 10](a). The distance at which the image blurs, and beyond which it inverts, is the focal length of the lens. To use a convex lens as a magnifier, the object must be closer to the converging lens than its focal length. This is called a *case 2* image. A case 2 image is formed when ${d}_ {\text{o}} and f is positive.

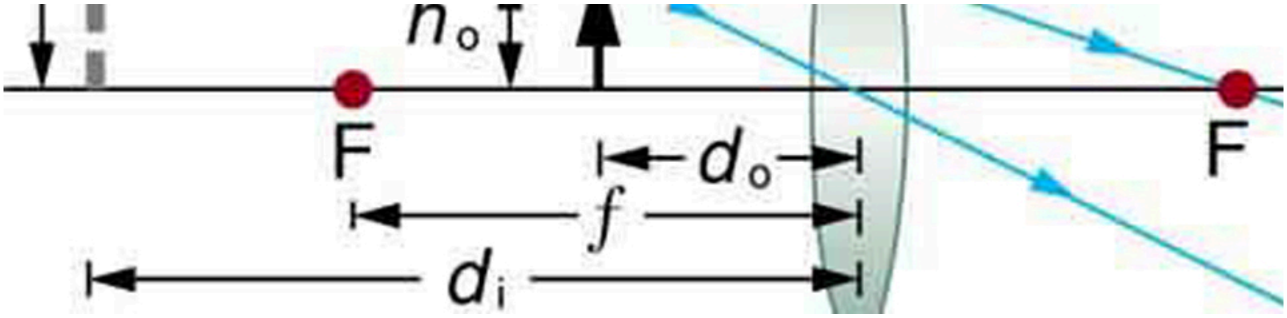
(a)

(b)

(a) When a converging lens is held farther away from the face than the lens's focal length, an inverted image is formed. This is a case 1 image. Note that the image is in focus but the face is not, because the image is much closer to the camera taking this photograph than the face. (credit: DaMongMan, Flickr) (b) A magnified image of a face is produced by placing it closer to the converging lens than its focal length. This is a case 2 image. (credit: Casey Fleser, Flickr)

[Figure 11] uses ray tracing to show how an image is formed when an object is held closer to a converging lens than its focal length. Rays coming from a common point on the object continue to diverge after passing through the lens, but all appear to originate from a point at the location of the image. The image is on the same side of the lens as the object and is farther away from the lens than the object. This image, like all case 2 images, cannot be projected and, hence, is called a **virtual image**. Light rays only appear to originate at a virtual image; they do not actually pass through that location in space. A screen placed at the location of a virtual image will receive only diffuse light from the object, not focused rays from the lens. Additionally, a screen placed on the opposite side of the lens will receive rays that are still diverging, and so no image will be projected on it. We can see the magnified image with our eyes, because the lens of the eye converges the rays into a real image projected on our retina. Finally, we note that a virtual image is upright and larger than the object, meaning that the magnification is positive and greater than 1.


1
F
2
h_i



Ray tracing predicts the image location and size for an object held closer to a converging lens than its focal length. Ray 1 enters parallel to the axis and exits through the focal point on the opposite side, while ray 2 passes through the center of the lens without changing path. The two rays continue to diverge on the other side of the lens, but both appear to come from a common point, locating the upright, magnified, virtual image. This is a case 2 image.

Virtual Image

An image that is on the same side of the lens as the object and cannot be projected on a screen is called a virtual image.

Image Produced by a Magnifying Glass

Suppose the book page in [Figure 11] (a) is held 7.50 cm from a convex lens of focal length 10.0 cm, such as a typical magnifying glass might have. What magnification is produced?

Strategy and Concept

We are given that d_{o} =7.50cm and f =10.0cm , so we have a situation where the object is placed closer to the lens than its focal length. We therefore expect to get a case 2 virtual image with a positive magnification that is greater than 1. Ray tracing produces an image like that shown in [Figure 11], but we will use the thin lens equations to get numerical solutions in this example.

Solution

To find the magnification m , we try to use magnification equation, $m=-d_{\mathrm{i}}/d_{\mathrm{o}}$. We do not have a value for d_{i} , so that we must first find the location of the image using lens equation. (The procedure is the same as followed in the preceding example, where d_{o} and f were known.) Rearranging the magnification equation to isolate d_{i} gives

$$1d_{\mathrm{i}}=1f-1d_{\mathrm{o}}.$$

Entering known values, we obtain a value for $1/d_{\mathrm{i}}$:

$$1d_{\mathrm{i}}=110.0\mathrm{cm}-17.50\mathrm{cm}=-0.0333\mathrm{cm}.$$

This must be inverted to find d_{i} :

$$d_{\mathrm{i}}=-\mathrm{cm}0.0333=-30.0\mathrm{cm}.$$

Now the thin lens equation can be used to find the magnification m , since both d_{i} and d_{o} are known. Entering their values gives

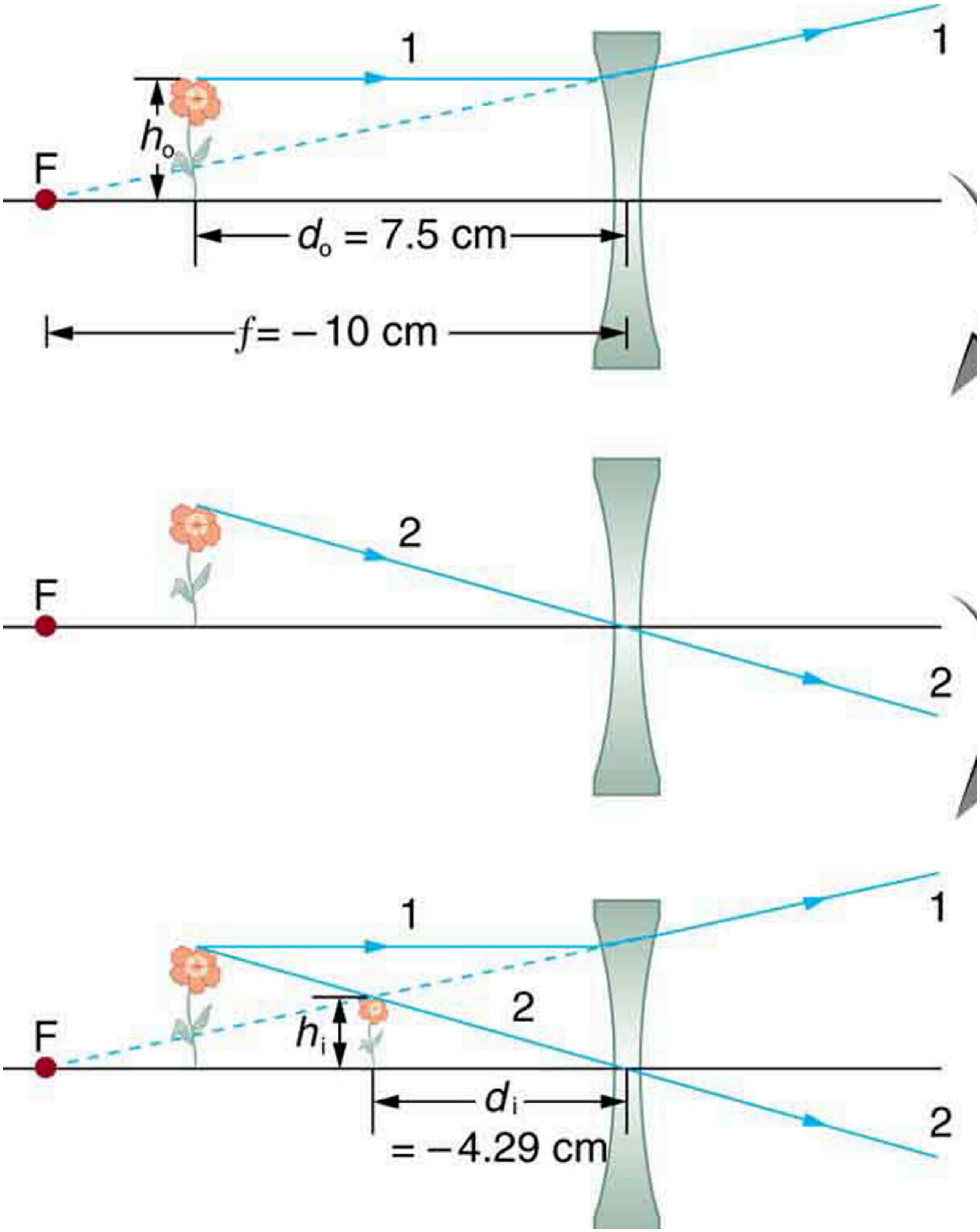
$$m=-d_{\mathrm{i}}d_{\mathrm{o}}=--30.0\mathrm{cm}7.50\mathrm{cm}=4.00.$$

Discussion

A number of results in this example are true of all case 2 images, as well as being consistent with [Figure 11]. Magnification is indeed positive (as predicted), meaning the image is upright. The magnification is also greater than 1, meaning that the image is larger than the object—in this case, by a factor of 4. Note that the image distance is negative. This means the image is on the same side of the lens as the object. Thus the image cannot be projected and is virtual. (Negative values of d_{i} occur for virtual images.) The image is farther from the lens than the object, since the image distance is greater in magnitude than the object distance. The location of the image is not obvious when you look through a magnifier. In fact, since the image is bigger than the object, you may think the image is closer than the object. But the image is farther away, a fact that is useful in correcting farsightedness, as we shall see in a later section.

A third type of image is formed by a diverging or concave lens. Try looking through eyeglasses meant to correct nearsightedness. (See [Figure 12].) You will see an image that is upright but smaller than the object. This means that the magnification is positive but less than 1. The ray diagram in [Figure 13] shows that the image is on the same side of the lens as the object and, hence, cannot be projected—it is a virtual image. Note that the image is closer to the lens than the object. This is a * case 3* image, formed for any object by a negative focal length or diverging lens.

A car viewed through a concave or diverging lens looks upright. This is a case 3 image. (credit: Daniel Oines, Flickr)



Ray tracing predicts the image location and size for a concave or diverging lens. Ray 1 enters parallel to the axis and is bent so that it appears to originate from the focal point. Ray 2 passes through the center of the lens without changing path. The two rays appear to come from a common point, locating the upright image. This is a case 3 image, which is closer to the lens than the object and smaller in height.

Image Produced by a Concave Lens

Suppose an object such as a book page is held 7.50 cm from a concave lens of focal length –10.0 cm. Such a lens could be used in eyeglasses to correct pronounced nearsightedness. What magnification is produced?

Strategy and Concept

This example is identical to the preceding one, except that the focal length is negative for a concave or diverging lens. The method of solution is thus the same, but the results are different in important ways.

Solution

To find the magnification m , we must first find the image distance d_{i} using thin lens equation

$$\frac{1}{d_{\text{i}}}=\frac{1}{f}-\frac{1}{d_{\text{o}}},$$

or its alternative rearrangement

$$d_{\text{i}}=\frac{f d_{\text{o}}}{d_{\text{o}}-f}.$$

We are given that $f = -10.0\text{cm}$ and $d_{\text{o}} = 7.50\text{cm}$. Entering these yields a value for $1/d_{\text{i}}$:

$$\frac{1}{d_{\text{i}}}=\frac{1}{-10.0\text{cm}}-\frac{1}{7.50\text{cm}}=\frac{-0.2333}{\text{cm}}.$$

This must be inverted to find d_{i} :

$$d_{\text{i}}=-\frac{\text{cm}}{0.2333}=-4.29\text{cm}.$$

Or

$$d_{\text{i}}=\frac{(7.5)(-10)}{(7.5-(-10))}=-75/17.5=-4.29\text{cm}.$$

Now the magnification equation can be used to find the magnification m , since both d_{i} and d_{o} are known. Entering their values gives

$$m=-\frac{d_{\text{i}}}{d_{\text{o}}}=-\frac{-4.29\text{cm}}{7.50\text{cm}}=0.571.$$

Discussion

A number of results in this example are true of all case 3 images, as well as being consistent with [Figure 13]. Magnification is positive (as predicted), meaning the image is upright. The magnification is also less than 1, meaning the image is smaller than the object—in this case, a little over half its size. The image distance is negative, meaning the image is on the same side of the lens as the object. (The image is virtual.) The image is closer to the lens than the object, since the image distance is smaller in magnitude than the object distance. The location of the image is not obvious when you look through a concave lens. In fact, since the image is smaller than the object, you may think it is farther away. But the image is closer than the object, a fact that is useful in correcting nearsightedness, as we shall see in a later section.

[Table 1] summarizes the three types of images formed by single thin lenses. These are referred to as case 1, 2, and 3 images. Convex (converging) lenses can form either real or virtual images (cases 1 and 2, respectively), whereas concave (diverging) lenses can form only virtual images (always case 3). Real images are always inverted, but they can be either larger or smaller than the object. For example, a slide projector forms an image larger than the slide, whereas a camera makes an image smaller than the object being photographed. Virtual images are always upright and cannot be projected. Virtual images are larger than the object only in case 2, where a convex lens is used. The virtual image produced by a concave lens is always smaller than the object—a case 3 image. We can see and photograph virtual images only by using an additional lens to form a real image.

Three Types of Images Formed By Thin Lenses

Type	Formed when	Image type	d_{i}	m
Case 1	f positive, $d_{\text{o}}>f$	real	positive	negative
Case 2	f positive, $d_{\text{o}}<f$	virtual	negative	positive $m>1$
Case 3	f negative	virtual	negative	positive $m<1$

In Image Formation by Mirrors, we shall see that mirrors can form exactly the same types of images as lenses.

Take-Home Experiment: Concentrating Sunlight

Find several lenses and determine whether they are converging or diverging. In general those that are thicker near the edges are diverging and those that are thicker near the center are converging. On a bright sunny day take the converging lenses outside and try focusing the sunlight onto a piece of paper. Determine the focal lengths of the lenses. Be careful because the paper may start to burn, depending on the type of lens you have selected.

Problem-Solving Strategies for Lenses

Step 1. Examine the situation to determine that image formation by a lens is involved.

Step 2. Determine whether ray tracing, the thin lens equations, or both are to be employed. A sketch is very useful even if ray tracing is not specifically required by the problem. Write symbols and values on the sketch.

Step 3. Identify exactly what needs to be determined in the problem (identify the unknowns).

Step 4. Make a list of what is given or can be inferred from the problem as stated (identify the knowns). It is helpful to determine whether the situation involves a case 1, 2, or 3 image. While these are just names for types of images, they have certain characteristics (given in [Table 1]) that can be of great use in solving problems.

Step 5. If ray tracing is required, use the ray tracing rules listed near the beginning of this section.

Step 6. Most quantitative problems require the use of the thin lens equations. These are solved in the usual manner by substituting knowns and solving for unknowns. Several worked examples serve as guides.

Step 7. Check to see if the answer is reasonable: Does it make sense? If you have identified the type of image (case 1, 2, or 3), you should assess whether your answer is consistent with the type of image, magnification, and so on.

Misconception Alert

We do not realize that light rays are coming from every part of the object, passing through every part of the lens, and all can be used to form the final image.

We generally feel the entire lens, or mirror, is needed to form an image. Actually, half a lens will form the same, though a fainter, image.

PhET Explorations: Geometric Optics

How does a lens form an image? See how light rays are refracted by a lens. Watch how the image changes when you adjust the focal length of the lens, move the object, move the lens, or move the screen.

Section Summary

- Light rays entering a converging lens parallel to its axis cross one another at a single point on the opposite side.
- For a converging lens, the focal point is the point at which converging light rays cross; for a diverging lens, the focal point is the point from which diverging light rays appear to originate.
- The distance from the center of the lens to its focal point is called the focal length f .
- Power P of a lens is defined to be the inverse of its focal length, $P=\frac{1}{f}$.
- A lens that causes the light rays to bend away from its axis is called a diverging lens.
- Ray tracing is the technique of graphically determining the paths that light rays take.
- The image in which light rays from one point on the object actually cross at the location of the image and can be projected onto a screen, a piece of film, or the retina of an eye is called a real image.
- Thin lens equations are $\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f}$ and $\frac{h_i}{h_o}=-\frac{d_i}{d_o}=m$

(magnification).

- The distance of the image from the center of the lens is called image distance.
- An image that is on the same side of the lens as the object and cannot be projected on a screen is called a virtual image.

Conceptual Questions

It can be argued that a flat piece of glass, such as in a window, is like a lens with an infinite focal length. If so, where does it form an image? That is, how are d_i and d_o related?

You can often see a reflection when looking at a sheet of glass, particularly if it is darker on the other side. Explain why you can often see a double image in such circumstances.

When you focus a camera, you adjust the distance of the lens from the film. If the camera lens acts like a thin lens, why can it not be a fixed distance from the film for both near and distant objects?

A thin lens has two focal points, one on either side, at equal distances from its center, and should behave the same for light entering from either side. Look through your eyeglasses (or those of a friend) backward and forward and comment on whether they are thin lenses.

Will the focal length of a lens change when it is submerged in water? Explain.

Problems & Exercises

What is the power in diopters of a camera lens that has a 50.0 mm focal length?

Show Solution

Strategy

We use the relationship $P = 1/f$ where f must be in meters and P will be in diopters (D).

Solution

Convert focal length to meters:

$$f = 50.0\text{ mm} = 0.0500\text{ m}$$

Calculate power:

$$P = \frac{1}{f} = \frac{1}{0.0500\text{ m}} = 20.0\text{ D}$$

Answer: The power is **20.0 diopters**.

Discussion

A 50 mm lens is considered a “normal” focal length for 35 mm film cameras, providing a field of view similar to human vision. The positive power indicates this is a converging lens. Camera lenses typically range from about 5 D (200 mm telephoto) to 50 D (20 mm wide-angle), with the 50 mm lens (20 D) in the middle range.

Your camera’s zoom lens has an adjustable focal length ranging from 80.0 to 200 mm. What is its range of powers?

Show Solution

Strategy

Use $P = 1/f$ for both the minimum and maximum focal lengths, converting to meters first.

Solution

For the minimum focal length (telephoto setting):

$$f_{\text{min}} = 200\text{ mm} = 0.200\text{ m}$$
$$P_{\text{min}} = \frac{1}{0.200\text{ m}} = 5.00\text{ D}$$

For the maximum focal length (wide-angle setting):

$$f_{\text{max}} = 80.0\text{ mm} = 0.0800\text{ m}$$
$$P_{\text{max}} = \frac{1}{0.0800\text{ m}} = 12.5\text{ D}$$

Answer: The range of powers is **5.00 D to 12.5 D**.

Discussion

The zoom lens can vary its power from 5.00 D (telephoto, longer focal length) to 12.5 D (wide-angle, shorter focal length). This 2.5× zoom range is typical for standard camera zoom lenses. Note that higher power (shorter focal length) provides a wider field of view, while lower power (longer focal length) magnifies distant objects. This is a 80-200mm zoom, which in 35mm camera terms goes from a moderate telephoto to a longer telephoto focal length.

What is the focal length of 1.75 D reading glasses found on the rack in a pharmacy?

Show Solution

Strategy

Use $f = 1/P$ to find focal length from power.

Solution

$$f = \frac{1}{P} = \frac{1}{1.75\text{ D}} = 0.571\text{ m} = 57.1\text{ cm}$$

Answer: The focal length is **57.1 cm** or **0.571 m**.

Discussion

Reading glasses with +1.75 D are for people with mild presbyopia (age-related farsightedness). The relatively long focal length (57 cm) means these glasses help bring close objects (like books or phones) into focus at comfortable reading distances. The positive power indicates these are converging lenses that help the eye focus on nearby objects.

You note that your prescription for new eyeglasses is –4.50 D. What will their focal length be?

Show Solution

Strategy

Use $f = 1/P$ to find focal length from power. The negative power indicates diverging (concave) lenses.

Solution

$$f = \frac{1}{P} = \frac{1}{-4.50\text{ D}} = -0.222\text{ m} = -22.2\text{ cm}$$

Answer: The focal length is **−0.222 m** or **−22.2 cm**.

Discussion

The negative focal length confirms these are diverging (concave) lenses used to correct myopia (nearsightedness). People with myopia can see nearby objects clearly but have difficulty focusing on distant objects because their eye's lens system is too powerful, focusing light in front of the retina. The diverging lenses spread the light slightly before it enters the eye, moving the focal point back onto the retina. A −4.50 D prescription represents moderate myopia. The relatively short focal length (22.2 cm) means these lenses will noticeably reduce the apparent size of objects when looking through them.

How far from the lens must the film in a camera be, if the lens has a 35.0 mm focal length and is being used to photograph a flower 75.0 cm away? Explicitly show how you follow the steps in the Problem-Solving Strategy for lenses.

Show Solution

Strategy

This is a case 1 image (real, inverted) since the object is farther from the lens than the focal length. We use the thin lens equation $\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$ to find d_i.

Solution

Step 1-2: Image formation by a lens is involved. We'll use the thin lens equation.

Step 3: We need to find d_i (image distance/film distance).

Step 4: Given: $f = 35.0\text{ mm} = 3.50\text{ cm}$, $d_o = 75.0\text{ cm}$

Step 5: (Ray tracing not required)

Step 6: Using the thin lens equation:

$$\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o} = \frac{1}{3.50\text{ cm}} - \frac{1}{75.0\text{ cm}}$$
$$\frac{1}{d_i} = 0.2857 - 0.0133 = 0.2724\text{ cm}^{-1}$$
$$d_i = \frac{1}{0.2724} = 3.67\text{ cm} = 36.7\text{ mm}$$

Step 7: This is reasonable—the film must be slightly farther from the lens than the focal length when photographing an object at finite distance. The image distance (36.7 mm) is only slightly more than the focal length (35.0 mm), which makes sense for an object relatively far away.

Answer: The film must be **36.7 mm** from the lens.

Discussion

Notice that even though the flower is 750 mm away (more than 21 times the focal length), the film needs to be only 1.7 mm beyond the focal point. This demonstrates why camera focusing mechanisms need only small adjustments for objects at different distances—the lens-to-film distance doesn't change much as object distance varies.

A certain slide projector has a 100 mm focal length lens. (a) How far away is the screen, if a slide is placed 103 mm from the lens and produces a sharp image? (b) If the slide is 24.0 by 36.0 mm, what are the dimensions of the image? Explicitly show how you follow the steps in the Problem-Solving Strategy for lenses.

Show Solution

(a) 3.43 m

(b) 0.800 by 1.20 m

A doctor examines a mole with a 15.0 cm focal length magnifying glass held 13.5 cm from the mole (a) Where is the image? (b) What is its magnification? (c) How big is the image of a 5.00 mm diameter mole?

Show Solution

(a) -1.35m (on the object side of the lens).

(b) $+10.0$ (c) 5.00 cm

How far from a piece of paper must you hold your father's 2.25 D reading glasses to try to burn a hole in the paper with sunlight?

Show Solution

44.4 cm

A camera with a 50.0 mm focal length lens is being used to photograph a person standing 3.00 m away. (a) How far from the lens must the film be? (b) If the film is 36.0 mm high, what fraction of a 1.75 m tall person will fit on it? (c) Discuss how reasonable this seems, based on your experience in taking or posing for photographs.

Show Solution

Strategy

(a) Use the thin lens equation to find d_i. **(b)** Use the magnification equation to relate object and image heights. **(c)** Compare with typical photography experience.

Solution

(a) Using the thin lens equation:

$$\frac{1}{d_i}=\frac{1}{f}-\frac{1}{d_o}=\frac{1}{50.0\text{ mm}}-\frac{1}{3000\text{ mm}}$$
$$\frac{1}{d_i}=0.0200-0.000333=0.01967\text{ mm}^{-1}$$
$$d_i=50.8\text{ mm}$$

(b) The magnification is:

$$m=-\frac{d_i}{d_o}=-\frac{50.8\text{ mm}}{3000\text{ mm}}=-0.0169$$

The image height is:

$$h_i=m\cdot h_o=-0.0169\times1750\text{ mm}=-29.6\text{ mm}$$

Taking the absolute value, 29.6 mm of the 36.0 mm film height is used. Fraction:

$$\frac{29.6}{36.0}=0.822=82.2\%$$

Alternatively, if the entire person (1750 mm) were to fit on the 36.0 mm film:

$$\text{Fraction}=\frac{h_i}{h_o}=\frac{29.6\text{ mm}}{1750\text{ mm}}\times\frac{1750\text{ mm}}{36.0\text{ mm}}=\frac{29.6}{36.0}=0.822$$

Actually, the question asks what fraction of the person fits:

$$\text{Fraction of person}=\frac{36.0\text{ mm}}{29.6\text{ mm}}=1.22$$

This means the entire person (100%) plus 22% more would fit, so the entire person fits comfortably.

(c) This is very reasonable. With a 50 mm "normal" lens and the subject 3 m away, getting a full-body portrait that uses about 82% of the frame height is typical. This provides good composition with a little space above the head and below the feet. Professional photographers often aim for this kind of framing for full-body portraits, as it avoids cropping body parts while not wasting too much of the frame.

Answers: (a) **50.8 mm**, (b) **The entire 1.75 m person fits, using 82.2% of the film height**, (c) Very reasonable for portrait photography.

Discussion

The film needs to be only slightly farther from the lens (50.8 mm) than the focal length (50.0 mm) because the object is relatively far away (60 times the focal length). This small adjustment is why cameras need focusing mechanisms. The 50 mm lens at 3 m gives good perspective for portraits without distortion, which is why 50 mm is considered the "normal" focal length for 35 mm film cameras.

A camera lens used for taking close-up photographs has a focal length of 22.0 mm. The farthest it can be placed from the film is 33.0 mm. (a) What is the closest object that can be photographed? (b) What is the magnification of this closest object?

Show Solution

(a) 6.60 cm

(b) –0.333

Suppose your 50.0 mm focal length camera lens is 51.0 mm away from the film in the camera. (a) How far away is an object that is in focus? (b) What is the height of the object if its image is 2.00 cm high?

Show Solution

Strategy

(a) Use the thin lens equation with $f = 50.0\text{ mm}$ and $d_i = 51.0\text{ mm}$ to find d_o. **(b)** Use the magnification equation.

Solution

(a) From the thin lens equation:

$$\frac{1}{d_o} = \frac{1}{f} - \frac{1}{d_i} = \frac{1}{50.0\text{ mm}} - \frac{1}{51.0\text{ mm}}$$
$$\frac{1}{d_o} = 0.0200 - 0.0196 = 0.000392\text{ mm}^{-1}$$
$$d_o = 2550\text{ mm} = 2.55\text{ m}$$

(b) The magnification is:

$$m = -\frac{d_i}{d_o} = -\frac{51.0\text{ mm}}{2550\text{ mm}} = -0.0200$$

The object height is:

$$h_o = \frac{h_i}{m} = \frac{-20.0\text{ mm}}{-0.0200} = 1000\text{ mm} = 1.00\text{ m}$$

Answers: (a) **2.55 m**, (b) **1.00 m** (assuming the image is inverted)

Discussion

When the film is moved just 1.0 mm farther from the lens (from 50.0 mm to 51.0 mm), the object in focus moves from infinity to 2.55 m. This demonstrates that small adjustments in lens-to-film distance allow focusing on objects at finite distances. The magnification of −0.0200 means the image is 1/50th the size of the object and inverted, which is typical for camera photography. A 1.00 m tall object creating a 2.00 cm image on film is very reasonable for portrait or documentary photography.

(a) What is the focal length of a magnifying glass that produces a magnification of 3.00 when held 5.00 cm from an object, such as a rare coin? (b) Calculate the power of the magnifier in diopters. (c) Discuss how this power compares to those for store-bought reading glasses (typically 1.0 to 4.0 D). Is the magnifier's power greater, and should it be?

Show Solution

(a) $+7.50\text{cm}$ (b) 13.3D (c) Much greater

What magnification will be produced by a lens of power –4.00 D (such as might be used to correct myopia) if an object is held 25.0 cm away?

Show Solution

Strategy

First find the focal length from $f = 1/P$, then use the thin lens equation to find d_i, and finally calculate magnification from $m = -d_i/d_o$.

Solution

Focal length:

$$f = \frac{1}{P} = \frac{1}{-4.00\text{ D}} = -0.250\text{ m} = -25.0\text{ cm}$$

Image distance from thin lens equation:

$$\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o} = \frac{1}{-25.0\text{ cm}} - \frac{1}{25.0\text{ cm}}$$
$$\frac{1}{d_i} = -0.0400 - 0.0400 = -0.0800\text{ cm}^{-1}$$
$$d_i = -12.5\text{ cm}$$

Magnification:

$$m = -\frac{d_i}{d_o} = -\frac{-12.5\text{ cm}}{25.0\text{ cm}} = +0.500$$

Answer: The magnification is **+0.500** or **+0.50**.

Discussion

The positive magnification indicates an upright virtual image, and the magnitude less than 1 means the image is smaller than the object (half the size). This is characteristic of diverging lenses (case 3 images). When you hold an object 25 cm from a −4.00 D lens (used for myopia correction), you see a virtual image that appears half the size and 12.5 cm from the lens (on the same side as the object). This is why nearsighted people's eyes appear smaller when viewed through their glasses—the diverging lenses create reduced images.

In [Example 4], the magnification of a book held 7.50 cm from a 10.0 cm focal length lens was found to be 3.00. (a) Find the magnification for the book when it is held 8.50 cm from the magnifier. (b) Do the same for when it is held 9.50 cm from the magnifier. (c) Comment on the trend in m as the object distance increases as in these two calculations.

Show Solution

(a) +6.67

(b) +20.0

(c) The magnification increases without limit (to infinity) as the object distance increases to the limit of the focal distance.

Suppose a 200 mm focal length telephoto lens is being used to photograph mountains 10.0 km away. (a) Where is the image? (b) What is the height of the image of a 1000 m high cliff on one of the mountains?

Show Solution

Strategy

(a) Use the thin lens equation. Since the object is very far away ($d_o \gg f$), the image will be very close to the focal point. **(b)** Use magnification equation.

Solution

(a) From the thin lens equation:

$$\frac{1}{d_i}=\frac{1}{f}-\frac{1}{d_o}=\frac{1}{200\text{ mm}}-\frac{1}{10.0\times10^{6}\text{ mm}}$$
$$\frac{1}{d_i}=0.00500-0.0000001=0.0049999\text{ mm}^{-1}$$
$$d_i\approx200\text{ mm}$$

More precisely: $d_i = 200.004$ mm

(b) The magnification is:

$$m=-\frac{d_i}{d_o}=-\frac{200\text{ mm}}{10.0\times10^{6}\text{ mm}}=-2.00\times10^{-5}$$

Image height:

$$h_i=m\cdot h_o=(-2.00\times10^{-5})(1000\text{ m})=-0.0200\text{ m}=-20.0\text{ mm}$$

Answers: (a) **200 mm** (essentially at the focal point), (b) **20.0 mm** (inverted)

Discussion

When photographing distant objects (10 km is 50,000 times the focal length), the image forms essentially at the focal point. This is why landscape photographers rarely need to adjust focus when photographing distant scenes—everything from about 30 m to infinity focuses at nearly the same point. The 1000 m cliff creates only a 20 mm image on the film/sensor, demonstrating why telephoto lenses are needed for distant subjects. Even with a 200 mm telephoto, distant mountains create relatively small images.

A camera with a 100 mm focal length lens is used to photograph the sun and moon. What is the height of the image of the sun on the film, given the sun is $1.40\times10^{6}\text{km}$ in diameter and is $1.50\times10^{8}\text{km}$ away?

Show Solution

-0.933mm

Combine thin lens equations to show that the magnification for a thin lens is determined by its focal length and the object distance and is given by $m = f/(f-d_o)$.

Show Solution

Strategy

Start with the magnification equation $m=-d_i/d_o$ and the thin lens equation $\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f}$. Eliminate d_i to express m in terms of f and d_o only.

Solution

From the thin lens equation:

$$\frac{1}{d_i}=\frac{1}{f}-\frac{1}{d_o}$$

Finding a common denominator:

$$\frac{1}{d_i}=\frac{d_o-f}{f d_o}$$

Inverting:

$$d_i=\frac{f d_o}{d_o-f}$$

Substituting into the magnification equation:

$$m=-\frac{d_i}{d_o}=-\frac{1}{d_o}\cdot\frac{f d_o}{d_o-f}$$
$$m=-\frac{f}{d_o-f}$$

Multiplying numerator and denominator by −1:

$$m=\frac{f}{f-d_o}$$

Thus we have shown that $m=\frac{f}{f-d_o}$.

Discussion

This alternative form of the magnification equation is useful because it directly shows how magnification depends on the relationship between focal length and object distance:

- When $d_o>f$ (case 1): $f-d_o<0$, so $m<0$ (inverted real image)
- When $d_o<f$ (case 2): $f-d_o>0$, so $m>0$ (upright virtual image)
- As $d_o\to f$: $m\to\pm\infty$ (image moves to infinity)
- As $d_o\to\infty$: $m\to 0$ (very small image at focal point)

This form makes it easy to calculate magnification without first finding the image distance.

Glossary

converging lens
: a convex lens in which light rays that enter it parallel to its axis converge at a single point on the opposite side

diverging lens
: a concave lens in which light rays that enter it parallel to its axis bend away (diverge) from its axis

focal point
: for a converging lens or mirror, the point at which converging light rays cross; for a diverging lens or mirror, the point from which diverging light rays appear to originate

focal length
: distance from the center of a lens or curved mirror to its focal point

magnification
: ratio of image height to object height

power
: inverse of focal length

real image
: image that can be projected

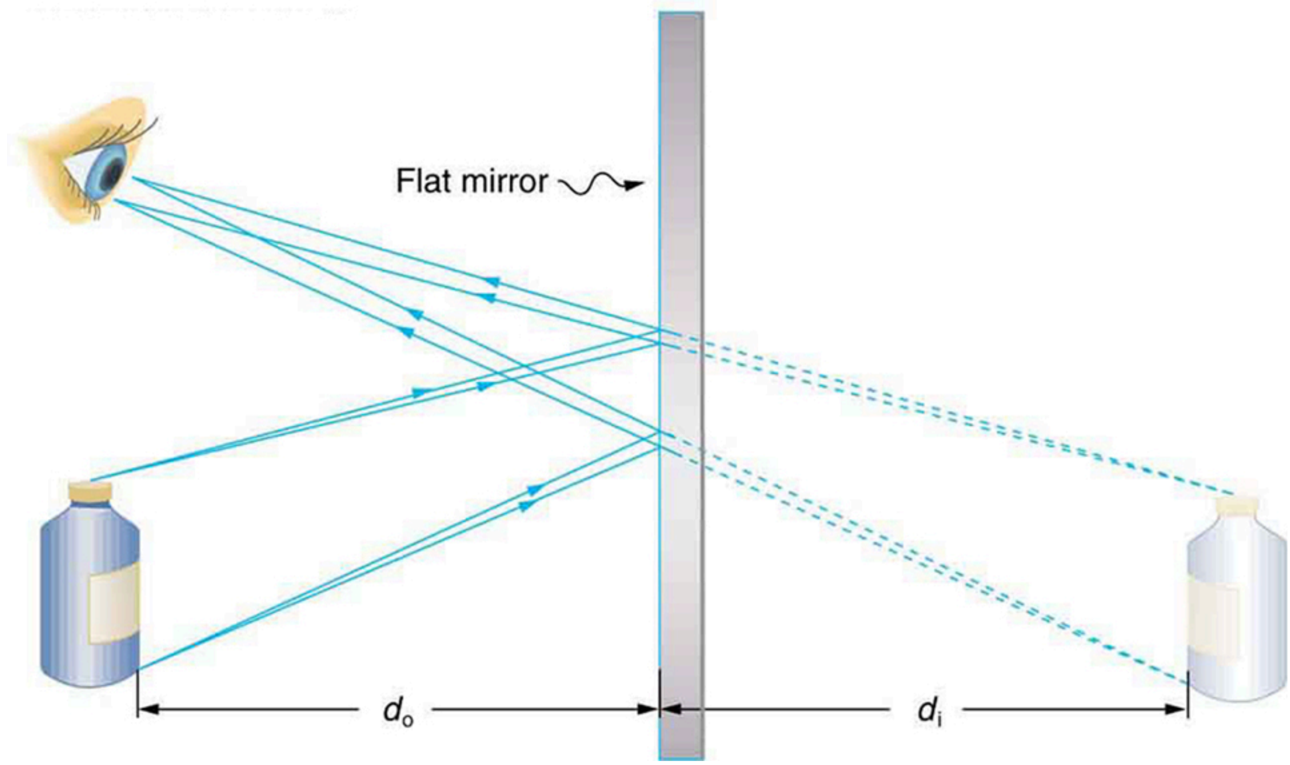
virtual image
: image that cannot be projected

Image Formation by Mirrors

- Illustrate image formation in a flat mirror.
- Explain with ray diagrams the formation of an image using spherical mirrors.
- Determine focal length and magnification given radius of curvature, distance of object and image.

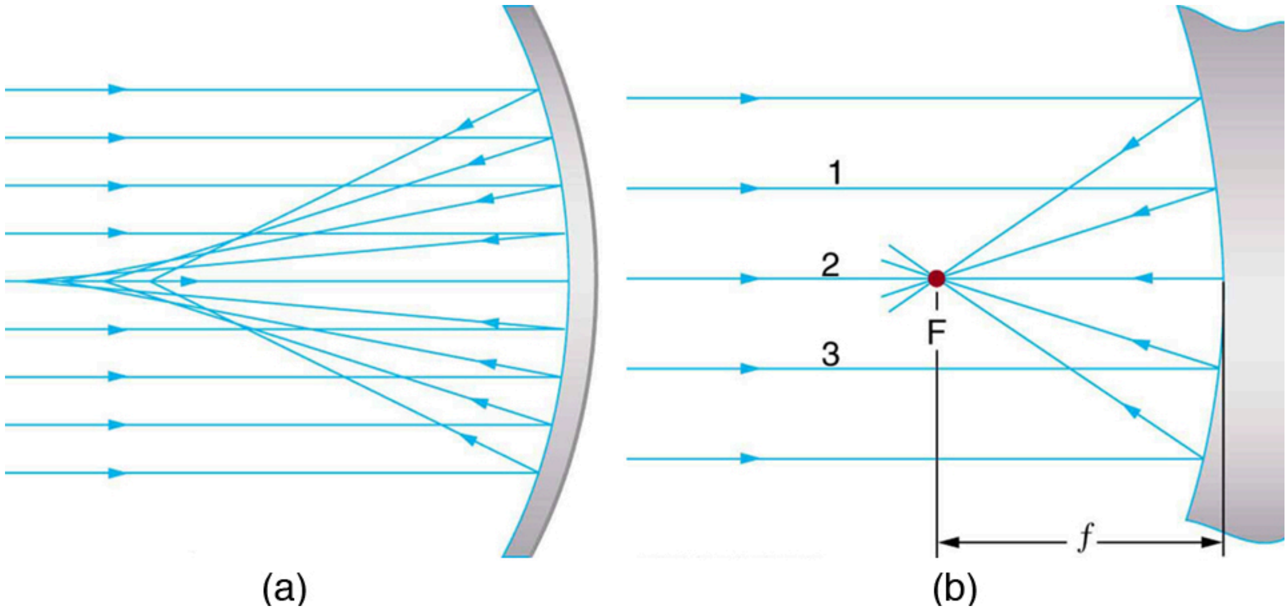
We only have to look as far as the nearest bathroom to find an example of an image formed by a mirror. Images in flat mirrors are the same size as the object and are located behind the mirror. Like lenses, mirrors can form a variety of images. For example, dental mirrors may produce a magnified image, just as makeup mirrors do. Security mirrors in shops, on the other hand, form images that are smaller than the object. We will use the law of reflection to understand how mirrors form images, and we will find that mirror images are analogous to those formed by lenses.

[Figure 1] helps illustrate how a flat mirror forms an image. Two rays are shown emerging from the same point, striking the mirror, and being reflected into the observer's eye. The rays can diverge slightly, and both still get into the eye. If the rays are extrapolated backward, they seem to originate from a common point behind the mirror, locating the image. (The paths of the reflected rays into the eye are the same as if they had come directly from that point behind the mirror.) Using the law of reflection—the angle of reflection equals the angle of incidence—we can see that the image and object are the same distance from the mirror. This is a virtual image, since it cannot be projected—the rays only appear to originate from a common point behind the mirror. Obviously, if you walk behind the mirror, you cannot see the image, since the rays do not go there. But in front of the mirror, the rays behave exactly as if they had come from behind the mirror, so that is where the image is situated.



Two sets of rays from common points on an object are reflected by a flat mirror into the eye of an observer. The reflected rays seem to originate from behind the mirror, locating the virtual image.

Now let us consider the focal length of a mirror—for example, the concave spherical mirrors in [Figure 2]. Rays of light that strike the surface follow the law of reflection. For a mirror that is large compared with its radius of curvature, as in [Figure 2](a), we see that the reflected rays do not cross at the same point, and the mirror does not have a well-defined focal point. If the mirror had the shape of a parabola, the rays would all cross at a single point, and the mirror would have a well-defined focal point. But parabolic mirrors are much more expensive to make than spherical mirrors. The solution is to use a mirror that is small compared with its radius of curvature, as shown in [Figure 2](b). (This is the mirror equivalent of the thin lens approximation.) To a very good approximation, this mirror has a well-defined focal point at F that is the focal distance f from the center of the mirror. The focal length f of a concave mirror is positive, since it is a converging mirror.



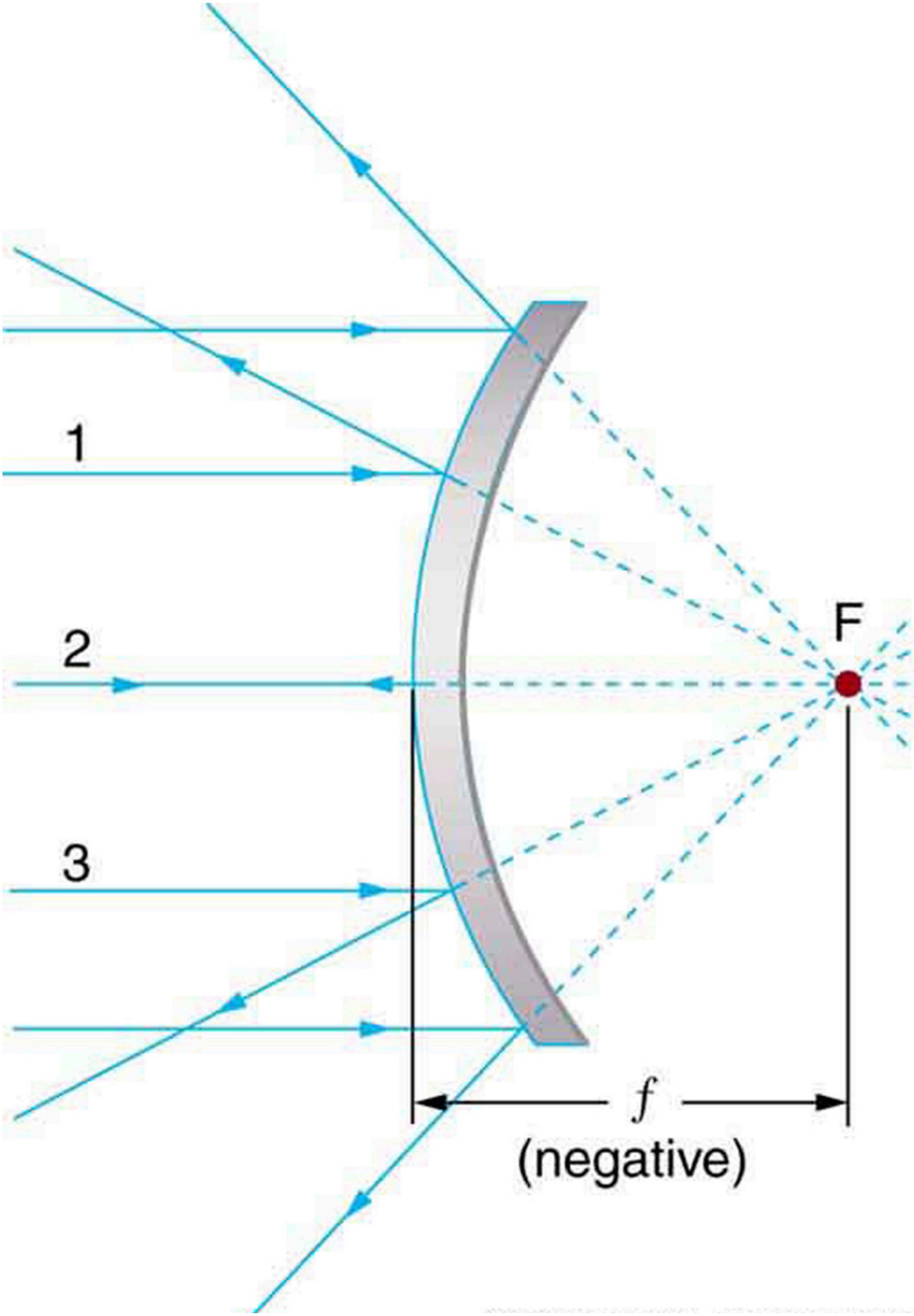
(a) Parallel rays reflected from a large spherical mirror do not all cross at a common point. (b) If a spherical mirror is small compared with its radius of curvature, parallel rays are focused to a common point. The distance of the focal point from the center of the mirror is its focal length f . Since this mirror is converging, it has a positive focal length.

Just as for lenses, the shorter the focal length, the more powerful the mirror; thus, $P=1/f$ for a mirror, too. A more strongly curved mirror has a shorter focal length and a greater power. Using the law of reflection and some simple trigonometry, it can be shown that the focal length is half the radius of curvature, or

$$f=\frac{R}{2}\text{,} $$

where R is the radius of curvature of a spherical mirror. The smaller the radius of curvature, the smaller the focal length and, thus, the more powerful the mirror.

The convex mirror shown in [Figure 3] also has a focal point. Parallel rays of light reflected from the mirror seem to originate from the point F at the focal distance f behind the mirror. The focal length and power of a convex mirror are negative, since it is a diverging mirror.


1
2
3
F
f
(negative)

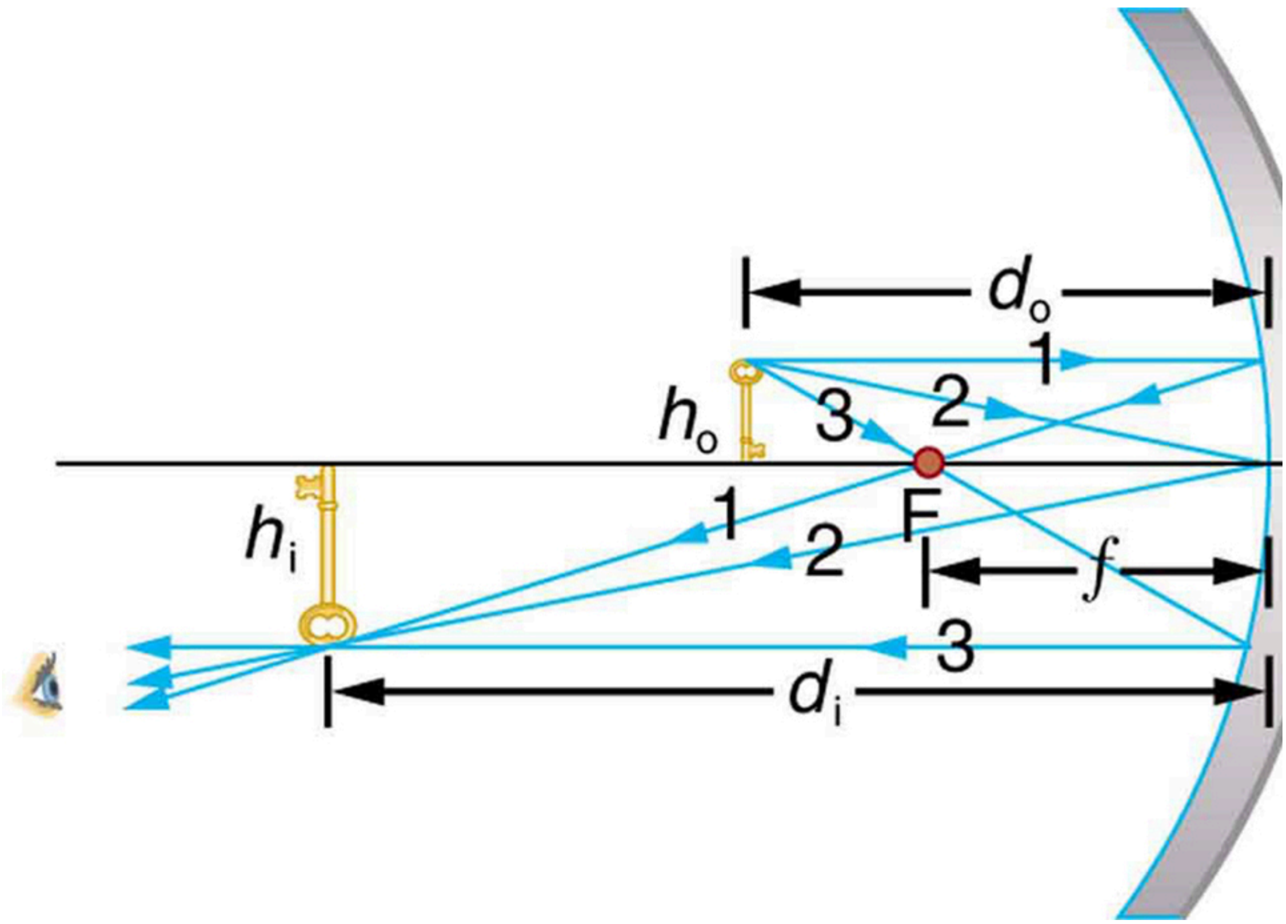
Parallel rays of light reflected from a convex spherical mirror (small in size compared with its radius of curvature) seem to originate from a well-defined focal point at the focal distance f behind the mirror. Convex mirrors diverge light rays and, thus, have a negative focal length.

Ray tracing is as useful for mirrors as for lenses. The rules for ray tracing for mirrors are based on the illustrations just discussed:

1. A ray approaching a concave converging mirror parallel to its axis is reflected through the focal point F of the mirror on the same side. (See rays 1 and 3 in [Figure 2](b).)
2. A ray approaching a convex diverging mirror parallel to its axis is reflected so that it seems to come from the focal point F behind the mirror. (See rays 1 and 3 in [Figure 3].)
3. Any ray striking the center of a mirror is followed by applying the law of reflection; it makes the same angle with the axis when leaving as when approaching. (See ray 2 in [Figure 4].)
4. A ray approaching a concave converging mirror through its focal point is reflected parallel to its axis. (The reverse of rays 1 and 3 in [Figure 2].)
5. A ray approaching a convex diverging mirror by heading toward its focal point on the opposite side is reflected parallel to the axis. (The reverse of rays 1 and 3 in [Figure 3].)

We will use ray tracing to illustrate how images are formed by mirrors, and we can use ray tracing quantitatively to obtain numerical information. But since we assume each mirror is small compared with its radius of curvature, we can use the thin lens equations for mirrors just as we did for lenses.

Consider the situation shown in [Figure 4], concave spherical mirror reflection, in which an object is placed farther from a concave (converging) mirror than its focal length. That is, f is positive and $d_{\text{o}} > f$, so that we may expect an image similar to the case 1 real image formed by a converging lens. Ray tracing in [Figure 4] shows that the rays from a common point on the object all cross at a point on the same side of the mirror as the object. Thus a real image can be projected onto a screen placed at this location. The image distance is positive, and the image is inverted, so its magnification is negative. This is a *case 1 image for mirrors* . It differs from the case 1 image for lenses only in that the image is on the same side of the mirror as the object. It is otherwise identical.



A case 1 image for a mirror. An object is farther from the converging mirror than its focal length. Rays from a common point on the object are traced using the rules in the text. Ray 1 approaches parallel to the axis, ray 2 strikes the center of the mirror, and ray 3 goes through the focal point on the way toward the mirror. All three rays cross at the same point after being reflected, locating the inverted real image. Although three rays are shown, only two of the three are needed to locate the image and determine its height.

A Concave Reflector

Electric room heaters use a concave mirror to reflect infrared (IR) radiation from hot coils. Note that IR follows the same law of reflection as visible light. Given that the mirror has a radius of curvature of 50.0 cm and produces an image of the coils 3.00 m away from the mirror, where are the coils?

Strategy and Concept

We are given that the concave mirror projects a real image of the coils at an image distance ${d}_{\text{i}}=3.00 \text{m}$. The coils are the object, and we are asked to find their location—that is, to find the object distance ${d}_ {\text{o}}$. We are also given the radius of curvature of the mirror, so that its focal length is $f=R/2=25.0 \text{cm}$ (positive since the mirror is concave or converging). Assuming the mirror is small compared with its radius of curvature, we can use the thin lens equations, to solve this problem.

Solution

Since ${d}_{\text{i}}$ and f are known, thin lens equation can be used to find ${d}_{\text{o}}$:

$$\frac{1}{ {d}_{\text{o}}}+\frac{1}{ {d}_{\text{i}}}=\frac{1}{f}\text{.} $$

Rearranging to isolate ${d}_{\text{o}}$ gives

$$\frac{1}{ {d}_{\text{o}}}=\frac{1}{f}-\frac{1}{ {d}_{\text{i}}}\text{.} $$

Entering known quantities gives a value for $1/{d}_{\text{o}}$:

$$\frac{1}{ {d}_{\text{o}}}=\frac{1}{0.250 \text{m}}-\frac{1}{3.00 \text{m}}=\frac{ 3.667}{\text{m}}\text{.} $$

This must be inverted to find ${d}_{\text{o}}$:

$${d}_{\text{o}}=\frac{1 \text{m}}{ 3.667}= 27.3 \text{cm}. $$

Discussion

Note that the object (the filament) is farther from the mirror than the mirror's focal length. This is a case 1 image (${d}_{\text{o}}>f$ and f positive), consistent with the fact that a real image is formed. You will get the most concentrated thermal energy directly in front of the mirror and 3.00 m away from it. Generally, this is not desirable, since it could cause burns. Usually, you want the rays to emerge parallel, and this is accomplished by having the filament at the focal point of the mirror.

Note that the filament here is not much farther from the mirror than its focal length and that the image produced is considerably farther away. This is exactly analogous to a slide projector. Placing a slide only slightly farther away from the projector lens than its focal length produces an image significantly farther away. As the object gets closer to the focal distance, the image gets farther away. In fact, as the object distance approaches the focal length, the image distance approaches infinity and the rays are sent out parallel to one another.
Solar Electric Generating System

One of the solar technologies used today for generating electricity is a device (called a parabolic trough or concentrating collector) that concentrates the sunlight onto a blackened pipe that contains a fluid. This heated fluid is pumped to a heat exchanger, where its heat energy is transferred to another system that is used to generate steam—and so generate electricity through a conventional steam cycle. [Figure 5] shows such a working system in southern California. Concave mirrors are used to concentrate the sunlight onto the pipe. The mirror has the approximate shape of a section of a cylinder. For the problem, assume that the mirror is exactly one-quarter of a full cylinder.

1. If we wish to place the fluid-carrying pipe 40.0 cm from the concave mirror at the mirror's focal point, what will be the radius of curvature of the mirror?
2. Per meter of pipe, what will be the amount of sunlight concentrated onto the pipe, assuming the insolation (incident solar radiation) is $0.900 \text{k}{\text{W/m}}^{2}$?
3. If the fluid-carrying pipe has a 2.00-cm diameter, what will be the temperature increase of the fluid per meter of pipe over a period of one minute? Assume all the solar radiation incident on the reflector is absorbed by the pipe, and that the fluid is mineral oil. { type="a"}

Strategy

To solve an *Integrated Concept Problem* we must first identify the physical principles involved. Part (a) is related to the current topic. Part (b) involves a little math, primarily geometry. Part (c) requires an understanding of heat and density.

Solution to (a)

To a good approximation for a concave or semi-spherical surface, the point where the parallel rays from the sun converge will be at the focal point, so $R=2f=80.0 \text{cm}$.

Solution to (b)

The insolation is $900 \text{W}{\text{/m}}^{2}$. We must find the cross-sectional area A of the concave mirror, since the power delivered is $900 \text{W}{\text{/m}}^{2}\times\text{A}$. The mirror in this case is a quarter-section of a cylinder, so the area for a length L of the mirror is $A=\frac{1}{4}\left(2\pi R\right)L$. The area for a length of 1.00 m is then

$$A=\frac{\pi }{2}R\left(1.00 \text{m}\right)=\frac{\left(3.14\right)}{2}\left(0.800 \text{m}\right)\left(1.00 \text{m} \right)=1.26 \text{m}^{2}. $$

The insolation on the 1.00-m length of pipe is then

$$\left(9.00 \times 10^{2} \frac{W}{ \text{m}^{2}}\right)\left(1.26 \text{m}^{2}\right)=1130 \text{W}. $$

Solution to (c)

The increase in temperature is given by $Q=mc\Delta T$. The mass m of the mineral oil in the one-meter section of pipe is

$$\begin{array}{lll}m& =& \rho V =\rho \pi {\left(\frac{d}{2}\right)}^{2}\left(1.00 \text{m}\right)\\ m& =& \left(8.00 \times 10^{2} {\text{kg/m}}^{3}\right) (3.14) { \left(0.0100 \text{m}\right)}^{2} \left(1.00 \text{m}\right)\\ m& =& 0.251 \text{kg}.\end{array} $$

Therefore, the increase in temperature in one minute is

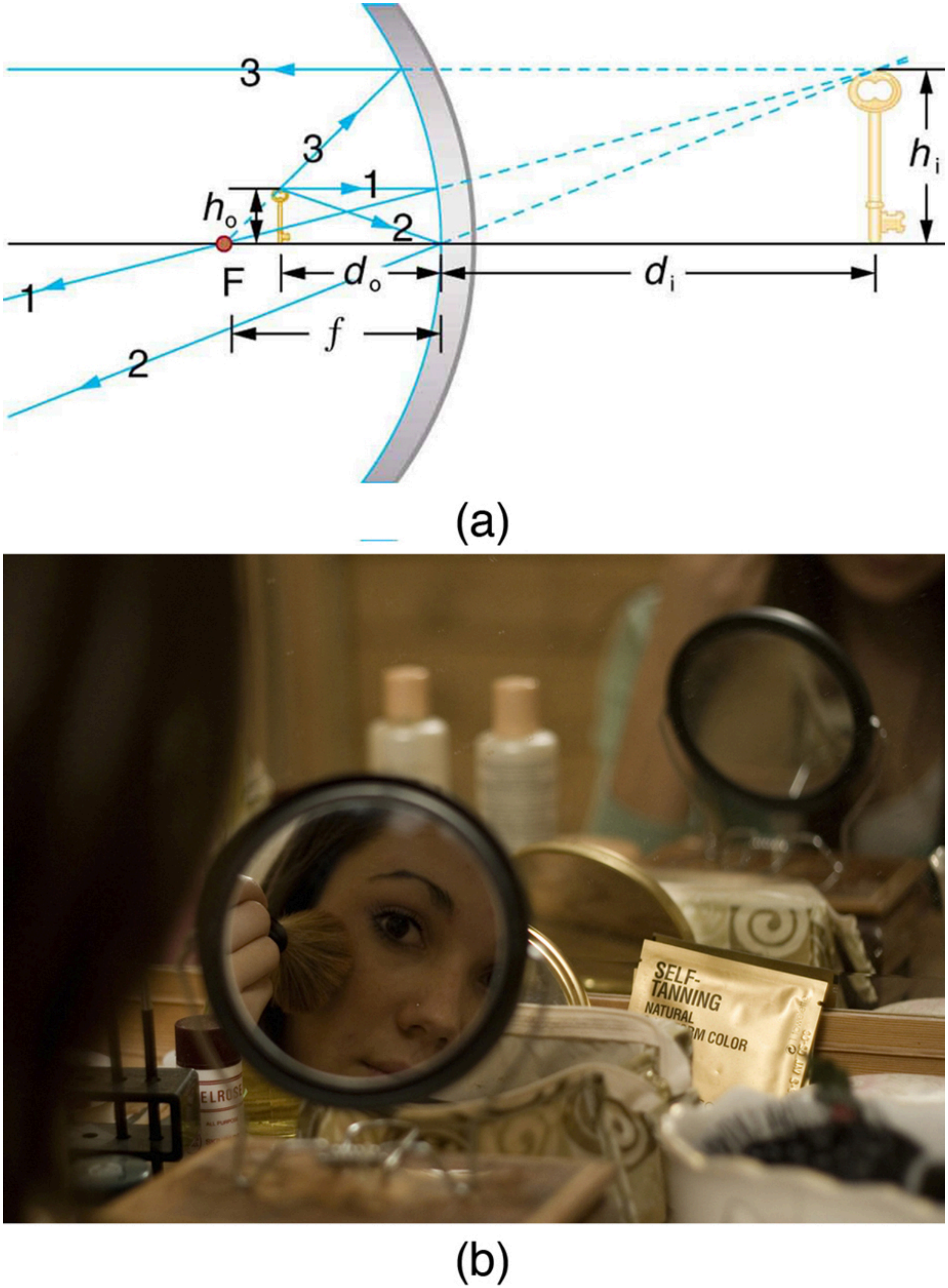
$$\begin{array}{lll}\Delta T& =& Q/mc\\ \Delta T & =& \frac{\left(1130 \text{W}\right)\left(60.0 \text{s}\right)}{\left(0.251 \text{kg}\right)\left(1670 \text{J·kg/ºC}\right)}\\ \Delta T& =& 162\text{ºC} .\end{array} $$

Discussion for (c)

An array of such pipes in the California desert can provide a thermal output of 250 MW on a sunny day, with fluids reaching temperatures as high as 400ºC . We are considering only one meter of pipe here, and ignoring heat losses along the pipe.

Parabolic trough collectors are used to generate electricity in southern California. (credit: kjkolb, Wikimedia Commons)

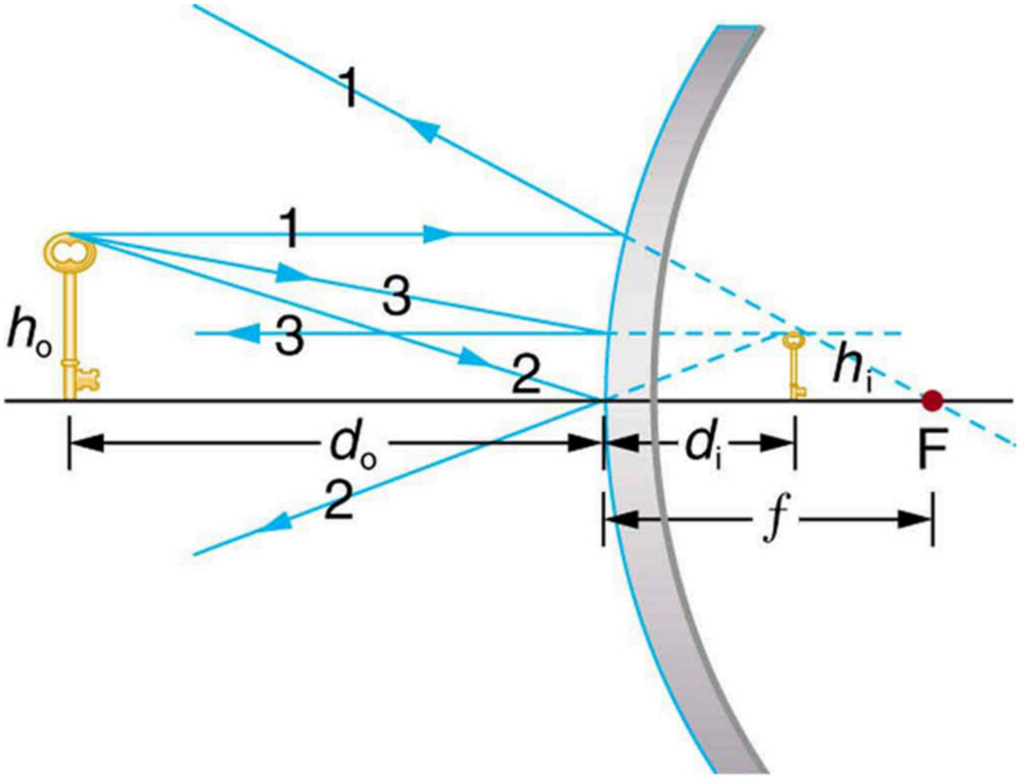
What happens if an object is closer to a concave mirror than its focal length? This is analogous to a case 2 image for lenses (${d}_{\text{o}}$ and f positive), which is a magnifier. In fact, this is how makeup mirrors act as magnifiers. [Figure 6](a) uses ray tracing to locate the image of an object placed close to a concave mirror. Rays from a common point on the object are reflected in such a manner that they appear to be coming from behind the mirror, meaning that the image is virtual and cannot be projected. As with a magnifying glass, the image is upright and larger than the object. This is a * case 2 image for mirrors* and is exactly analogous to that for lenses.



(a) Case 2 images for mirrors are formed when a converging mirror has an object closer to it than its focal length. Ray 1 approaches parallel to the axis, ray 2 strikes the center of the mirror, and ray 3 approaches the mirror as if it came from the focal point. (b) A magnifying mirror showing the reflection. (credit: Mike Melrose, Flickr)

All three rays appear to originate from the same point after being reflected, locating the upright virtual image behind the mirror and showing it to be larger than the object. (b) Makeup mirrors are perhaps the most common use of a concave mirror to produce a larger, upright image.

A convex mirror is a diverging mirror (f is negative) and forms only one type of image. It is a *case 3* image—one that is upright and smaller than the object, just as for diverging lenses. [Figure 7](a) uses ray tracing to illustrate the location and size of the case 3 image for mirrors. Since the image is behind the mirror, it cannot be projected and is thus a virtual image. It is also seen to be smaller than the object.



(a)

(b)

Case 3 images for mirrors are formed by any convex mirror. Ray 1 approaches parallel to the axis, ray 2 strikes the center of the mirror, and ray 3 approaches toward the focal point. All three rays appear to originate from the same point after being reflected, locating the upright virtual image behind the mirror and showing it to be smaller than the object. (b) Security mirrors are convex, producing a smaller, upright image. Because the image is smaller, a larger area is imaged compared to what would be observed for a flat mirror (and hence security is improved). (credit: Laura D'Alessandro, Flickr)

Image in a Convex Mirror

A keratometer is a device used to measure the curvature of the cornea, particularly for fitting contact lenses. Light is reflected from the cornea, which acts like a convex mirror, and the keratometer measures the magnification of the image. The smaller the magnification, the smaller the radius of curvature of the cornea. If the light source is 12.0 cm from the cornea and the image's magnification is 0.0320, what is the cornea's radius of curvature?

Strategy

If we can find the focal length of the convex mirror formed by the cornea, we can find its radius of curvature (the radius of curvature is twice the focal length of a spherical mirror). We are given that the object distance is ${d}_{\text{o}}=12.0 \text{cm}$ and that $m=0.0320$. We first solve for the image distance ${d}_{\text{i}}$, and then for f.

Solution

$m=-d_{\text{i}}/{d}_{\text{o}}$. Solving this expression for ${d}_{\text{i}}$ gives

$${d}_{\text{i}}=- m d_{\text{o}}\text{.}$$

Entering known values yields

$${d}_{\text{i}}=-\left(0.0320\right)\left(12.0 \text{cm}\right)=-0.384 \text{cm} \text{.}$$

$$\frac{1}{f}=\frac{1}{ {d}_{\text{o}}}+\frac{1}{ {d}_{\text{i}}}$$

Substituting known values,

$$\frac{1}{f}=\frac{1}{12.0 \text{cm}}+\frac{1}{-0.384 \text{cm}}=\frac{ -2.52}{\text{cm}}\text{.}$$

This must be inverted to find f :

$$f=\frac{\text{cm}}{ - 2.52}= -0.400 \text{cm}\text{.}$$

The radius of curvature is twice the focal length, so that

$$R=2\mid f\mid =0.800 \text{cm} \text{.}$$

Discussion

Although the focal length f of a convex mirror is defined to be negative, we take the absolute value to give us a positive value for R . The radius of curvature found here is reasonable for a cornea. The distance from cornea to retina in an adult eye is about 2.0 cm. In practice, many corneas are not spherical, complicating the job of fitting contact lenses. Note that the image distance here is negative, consistent with the fact that the image is behind the mirror, where it cannot be projected. In this section's Problems and Exercises, you will show that for a fixed object distance, the smaller the radius of curvature, the smaller the magnification.

The three types of images formed by mirrors (cases 1, 2, and 3) are exactly analogous to those formed by lenses, as summarized in the table at the end of Image Formation by Lenses. It is easiest to concentrate on only three types of images—then remember that concave mirrors act like convex lenses, whereas convex mirrors act like concave lenses.

Take-Home Experiment: Concave Mirrors Close to Home

Find a flashlight and identify the curved mirror used in it. Find another flashlight and shine the first flashlight onto the second one, which is turned off. Estimate the focal length of the mirror. You might try shining a flashlight on the curved mirror behind the headlight of a car, keeping the headlight switched off, and determine its focal length.

Problem-Solving Strategy for Mirrors

Step 1. Examine the situation to determine that image formation by a mirror is involved.

Step 2. Refer to the Problem-Solving Strategies for Lenses. The same strategies are valid for mirrors as for lenses with one qualification—use the ray tracing rules for mirrors listed earlier in this section.

Section Summary

- The characteristics of an image formed by a flat mirror are: (a) The image and object are the same distance from the mirror, (b) The image is a virtual image, and (c) The image is situated behind the mirror.
- Image length is half the radius of curvature.
  $$f=\frac{R}{2}$$
- A convex mirror is a diverging mirror and forms only one type of image, namely a virtual image.

Conceptual Questions

What are the differences between real and virtual images? How can you tell (by looking) whether an image formed by a single lens or mirror is real or virtual?

Can you see a virtual image? Can you photograph one? Can one be projected onto a screen with additional lenses or mirrors? Explain your responses.

Is it necessary to project a real image onto a screen for it to exist?

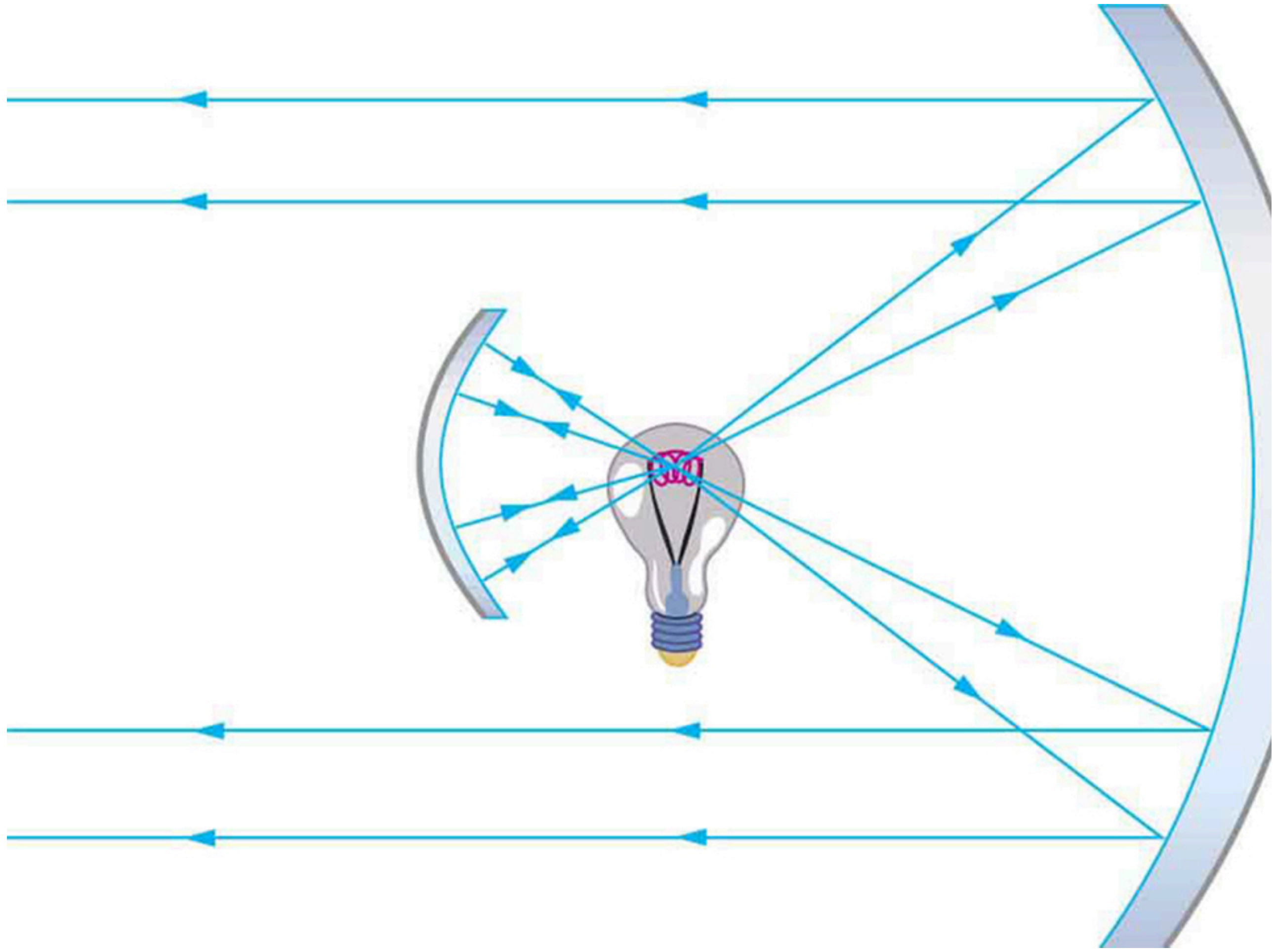
At what distance is an image **always** located—at ${d}_{\text{o}}$, ${d}_{\text{i}}$, or f ?

Under what circumstances will an image be located at the focal point of a lens or mirror?

What is meant by a negative magnification? What is meant by a magnification that is less than 1 in magnitude?

Can a case 1 image be larger than the object even though its magnification is always negative? Explain.

[Figure 8] shows a light bulb between two mirrors. One mirror produces a beam of light with parallel rays; the other keeps light from escaping without being put into the beam. Where is the filament of the light in relation to the focal point or radius of curvature of each mirror?



The two mirrors trap most of the bulb’s light and form a directional beam as in a headlight.

Devise an arrangement of mirrors allowing you to see the back of your head. What is the minimum number of mirrors needed for this task?

If you wish to see your entire body in a flat mirror (from head to toe), how tall should the mirror be? Does its size depend upon your distance away from the mirror? Provide a sketch.

It can be argued that a flat mirror has an infinite focal length. If so, where does it form an image? That is, how are ${d}_{\text{i}}$ and ${d}_{\text{o}}$ related?

Why are diverging mirrors often used for rear-view mirrors in vehicles? What is the main disadvantage of using such a mirror compared with a flat one?

Problems & Exercises

What is the focal length of a makeup mirror that has a power of 1.50 D?

Show Solution

Strategy

Use the relationship $P = 1/f$ to find the focal length from the power.

Solution

$$f = \frac{1}{P} = \frac{1}{1.50 \text{ D}} = 0.667 \text{ m} = 66.7 \text{ cm}$$

Answer: The focal length is **+0.667 m** or **66.7 cm**.

Discussion

The positive focal length indicates this is a concave (converging) mirror, which is appropriate for a makeup mirror. When you position your face within the focal length of a concave mirror, it produces an enlarged, upright virtual image—ideal for applying makeup or examining your face closely. A focal length of 66.7 cm means you would typically use this mirror at a distance of 20-30 cm from your face to get a magnified image. The relatively long focal length (compared to typical face-to-mirror distances) ensures comfortable magnification without excessive distortion.

Some telephoto cameras use a mirror rather than a lens. What radius of curvature mirror is needed to replace a 800 mm focal length telephoto lens?

Show Solution

Strategy

For a spherical mirror, the focal length and radius of curvature are related by $f = R/2$, so $R = 2f$.

Solution

$$R = 2f = 2(800 \text{ mm}) = 1600 \text{ mm} = 1.60 \text{ m}$$

Answer: The radius of curvature must be **1.60 m** or **1600 mm**.

Discussion

Mirror-based telephoto systems (called catadioptric or “cat” lenses) use a concave primary mirror with $R = 1.6 \text{ m}$ to achieve the same focal length as an 800 mm lens. These systems are more compact than traditional telephoto lenses because light folds back on itself, but they typically have a central obstruction that creates characteristic “donut-shaped” out-of-focus highlights (bokeh). They’re popular for astronomy and wildlife photography where compactness is important.

(a) Calculate the focal length of the mirror formed by the shiny back of a spoon that has a 3.00 cm radius of curvature. (b) What is its power in diopters?

Show Solution

Strategy

The back of a spoon is convex. Use $f = R/2$ with a negative radius for a convex mirror, then $P = 1/f$.

Solution

(a) For a convex mirror, R is negative:

$$f = \frac{R}{2} = \frac{-3.00 \text{ cm}}{2} = -1.50 \text{ cm} = -1.50 \times 10^{-2} \text{ m}$$

(b) Power in diopters:

$$P = \frac{1}{f} = \frac{1}{-0.0150 \text{ m}} = -66.7 \text{ D}$$

Answer: **(a)** The focal length is **-1.50×10^{-2} m** or **−1.50 cm**. **(b)** The power is **−66.7 D**.

Discussion

The negative focal length and negative power confirm this is a diverging (convex) mirror. The back of a spoon acts like a security mirror—it produces a small, upright, virtual image. The very short focal length (only 1.5 cm) means the mirror has strong diverging power, creating a highly reduced image. If you look at your reflection in the back of a spoon, you’ll see a small, distorted version of yourself. The large magnitude of the power (66.7 D) is much greater than typical eyeglass lenses, which range from about −10 D to +10 D, indicating how strongly curved this small mirror is.

Find the magnification of the heater element in [Example 1]. Note that its large magnitude helps spread out the reflected energy.

Show Solution

Strategy

From Example 1, we have $d_o = 27.3 \text{ cm}$ and $d_i = 3.00 \text{ m} = 300 \text{ cm}$. Use $m = -d_i/d_o$.

Solution

$$m = -\frac{d_i}{d_o} = -\frac{300 \text{ cm}}{27.3 \text{ cm}} = -11.0$$

Answer: The magnification is **−11.0**.

Discussion

The negative sign indicates the image is inverted. The large magnitude (11.0) means the image is 11 times larger than the object. This magnification helps spread the thermal energy from the heating element over a larger area, reducing the intensity at any single point and making the heater safer and more comfortable. Without this magnification, the reflected heat would be concentrated in a small, potentially uncomfortably hot spot.

What is the focal length of a makeup mirror that produces a magnification of 1.50 when a person's face is 12.0 cm away? Explicitly show how you follow the steps in the Problem-Solving Strategy for Mirrors.

Show Solution

Strategy

Step 1: Image formation by a mirror is involved.

Step 2: Given: $d_o = 12.0 \text{ cm} = 0.120 \text{ m}$, $m = +1.50$ (positive for upright image)

Use $m = -d_i/d_o$ to find d_i, then use the mirror equation $\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$ to find f.

Solution

Find the image distance:

$$m = -\frac{d_i}{d_o}$$
$$d_i = -m \cdot d_o = -(1.50)(0.120 \text{ m}) = -0.180 \text{ m}$$

The negative image distance confirms this is a virtual image behind the mirror.

Find the focal length:

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{0.120 \text{ m}} + \frac{1}{-0.180 \text{ m}}$$
$$\frac{1}{f} = 8.333 - 5.556 = 2.778 \text{ m}^{-1}$$
$$f = 0.360 \text{ m} = 36.0 \text{ cm}$$

Answer: The focal length is **+0.360 m** or **+36.0 cm** (concave mirror).

Discussion

The positive focal length confirms this is a concave (converging) mirror, as expected for a makeup mirror. The object (face) is at 12.0 cm, which is less than the focal length of 36.0 cm, placing it within the focal point. This configuration produces a magnified (1.50×), upright, virtual image—exactly what's wanted for a makeup mirror. The person sees their face 50% larger than actual size, making it easier to see details. This is a case 2 image for mirrors, analogous to using a magnifying glass.

A shopper standing 3.00 m from a convex security mirror sees his image with a magnification of 0.250. (a) Where is his image? (b) What is the focal length of the mirror? (c) What is its radius of curvature? Explicitly show how you follow the steps in the Problem-Solving Strategy for Mirrors.

Show Solution

Strategy

This is a case 3 image (convex mirror always produces upright, diminished, virtual images). Use $m = -d_i/d_o$ to find d_i, then use the mirror equation to find f, and $R = 2f$.

Solution

Step 1: Image formation by a mirror is involved.

Step 2: Given: $d_o = 3.00 \text{ m}$, $m = 0.250$ (positive, indicating upright virtual image)

(a) Find image distance:

$$m = -\frac{d_i}{d_o}$$
$$d_i = -m \cdot d_o = -(0.250)(3.00 \text{ m}) = -0.750 \text{ m}$$

The negative sign confirms this is a virtual image behind the mirror.

(b) Find focal length:

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{3.00 \text{ m}} + \frac{1}{-0.750 \text{ m}}$$
$$\frac{1}{f} = 0.333 - 1.333 = -1.00 \text{ m}^{-1}$$
$$f = -1.00 \text{ m}$$

(c) Find radius of curvature:

$$R = 2|f| = 2(1.00 \text{ m}) = 2.00 \text{ m}$$

Answers: (a) **−0.750 m** (behind mirror), (b) **−1.00 m**, (c) **2.00 m**

Discussion

These results are consistent with a convex security mirror. The negative focal length confirms it's a diverging (convex) mirror. The image appears at 0.75 m behind the mirror (1/4 the object distance), appears 1/4 the size, and is upright. These properties allow security mirrors to provide a wide field of view, letting store security personnel see a large area in a compact mirror. The trade-off is that objects appear smaller and distances are difficult to judge.

An object 1.50 cm high is held 3.00 cm from a person's cornea, and its reflected image is measured to be 0.167 cm high. (a) What is the magnification? (b) Where is the image? (c) Find the radius of curvature of the convex mirror formed by the cornea. (Note that this technique is used by optometrists to measure the curvature of the cornea for contact lens fitting. The instrument used is called a keratometer, or curve measurer.)

Show Solution

Strategy

(a) Use $m = h_i/h_o$ to find magnification. **(b)** Use $m = -d_i/d_o$ to find the image distance. **(c)** Use the mirror equation to find f, then $R = 2f$.

Solution

(a) Magnification:

$$m = \frac{h_i}{h_o} = \frac{0.167 \text{ cm}}{1.50 \text{ cm}} = 0.111$$

(b) Image distance:

$$m = -\frac{d_i}{d_o}$$
$$d_i = -m \cdot d_o = -(0.111)(3.00 \text{ cm}) = -0.333 \text{ cm}$$

(c) Focal length:

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{3.00 \text{ cm}} + \frac{1}{-0.333 \text{ cm}}$$
$$\frac{1}{f} = 0.333 - 3.00 = -2.667 \text{ cm}^{-1}$$
$$f = -0.375 \text{ cm}$$

Radius of curvature:

$$R = 2|f| = 2(0.375 \text{ cm}) = 0.750 \text{ cm}$$

Answer: (a) Magnification is **+0.111** or **+0.111**. **(b)** Image is at **−0.333 cm** (behind the cornea). **(c)** Radius of curvature is **0.750 cm** or **0.752 cm**.

Discussion

This technique (keratometry) is essential for fitting contact lenses. The cornea acts as a convex mirror with a radius of curvature of about 7.5 mm—very small but reasonable for the human eye. The positive magnification indicates an upright image, and the small magnitude (0.111) shows the image is much reduced, both consistent with a convex mirror. The negative image distance confirms the image is virtual and located behind the "mirror" (inside the cornea). Modern keratometers use this principle but with sophisticated optics to measure corneal curvature precisely at multiple points, which is crucial for diagnosing conditions like astigmatism and for fitting specialty contact lenses.

Ray tracing for a flat mirror shows that the image is located a distance behind the mirror equal to the distance of the object from the mirror. This is stated ${d}_{\text{i}}=-d_{\text{o}}$, since this is a negative image distance (it is a virtual image). (a) What is the focal length of a flat mirror? (b) What is its power?

Show Solution

Strategy

Use the mirror equation $\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$ with $d_i = -d_o$ to find f, then $P = 1/f$.

Solution

(a) For a flat mirror, $d_i = -d_o$:

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{-d_o} = \frac{1}{d_o} - \frac{1}{d_o} = 0$$

Therefore:

$$f = \infty$$

(b) The power is:

$$P = \frac{1}{f} = \frac{1}{\infty} = 0 \text{ D}$$

Answers: (a) **infinite** ($f = \infty$), (b) **0 diopters**

Discussion

A flat mirror has infinite focal length and zero power, which makes sense—it neither converges nor diverges light rays. This is consistent with our earlier proof that a flat mirror preserves the angle between rays. You can think of a flat mirror as the limiting case of a curved mirror as the radius of curvature approaches infinity. Since $f = R/2$, as $R \to \infty$, we get $f \to \infty$ and $P \to 0$.

Show that for a flat mirror ${h}_{\text{i}}={h}_{\text{o}}$, knowing that the image is a distance behind the mirror equal in magnitude to the distance of the object from the mirror.

Show Solution

$$m=\frac{ {h}_{\text{i}}}{ {h}_{\text{o}}}=-\frac{ {d}_{\text{i}}}{ {d}_{\text{o}}}=-\frac{-{d}_{\text{o}}}{ {d}_{\text{o}}}=\frac{ {d}_{\text{o}}}{ {d}_{\text{o}}}=1\Rightarrow{h}_{\text{i}}={h}_{\text{o}} $$

Use the law of reflection to prove that the focal length of a mirror is half its radius of curvature. That is, prove that $f=R/2$. Note this is true for a spherical mirror only if its diameter is small compared with its radius of curvature.

Show Solution

Strategy

Consider a ray parallel to the principal axis striking a concave spherical mirror. Use geometry and the law of reflection to show it passes through a point at distance $R/2$ from the mirror.

Solution

Consider a ray traveling parallel to the principal axis and striking the mirror at point P, at a small height h above the axis. Let C be the center of curvature, distance R from the mirror's vertex V.

The normal at point P passes through the center of curvature C (this is a property of spherical mirrors). Let α be the angle this normal makes with the principal axis.

By the law of reflection, the incident ray (parallel to the axis) makes angle α with the normal, and the reflected ray also makes angle α with the normal on the other side.

For small angles (when $h \ll R$), the geometry shows:

- The incident ray is horizontal (parallel to axis)
- The normal makes angle $\alpha \approx h/R$ with the axis
- By the law of reflection, the reflected ray makes angle 2α with the axis

The reflected ray crosses the axis at distance f from the mirror, where:

$$\tan(2\alpha) \approx 2\alpha = \frac{h}{f}$$

But we also have $\alpha \approx h/R$, so:

$$\frac{h}{f} = 2 \cdot \frac{h}{R}$$
$$\frac{1}{f} = \frac{2}{R}$$
$$f = \frac{R}{2}$$

Discussion

This proof works for the paraxial approximation (rays close to the axis, $h \ll R$). For a spherical mirror with large aperture, rays far from the axis don't all converge at the same point, causing spherical aberration. Parabolic mirrors avoid this problem—all parallel rays converge at a single focal point regardless of distance from the axis. However, spherical mirrors are much easier and cheaper to manufacture, so they're used when spherical aberration can be minimized by keeping the aperture small relative to the radius of curvature.

Referring to the electric room heater considered in the first example in this section, calculate the intensity of IR radiation in ${\text{W/m}}^{2}$ projected by the concave mirror on a person 3.00 m away. Assume that the heating element radiates 1500 W and has an area of $100 {\text{cm}}^{2}$, and that half of the radiated power is reflected and focused by the mirror.

Show Solution

Strategy

From a previous problem, we found that the heater element has magnification $m = -11.0$ when the image is at 3.00 m. Half of the 1500 W (750 W) is reflected and focused by the mirror. We need to find the area over which this power is distributed at the 3.00 m distance.

Solution

Power reflected and focused:

$$P = \frac{1500 \text{ W}}{2} = 750 \text{ W}$$

The heating element has area $A_{\text{object}} = 100 \text{ cm}^2 = 0.0100 \text{ m}^2$.

From the magnification $m = -11.0$, the linear dimensions of the image are 11 times those of the object. However, for the area calculation at the focal plane, we need to consider how the light is actually distributed.

The effective area over which the radiation is concentrated can be calculated as:

$$A_{\text{eff}} = \frac{A_{\text{object}}}{|m|} = \frac{0.0100 \text{ m}^2}{11.0} = 9.09 \times 10^{-4} \text{ m}^2 \times 121 = 0.110 \text{ m}^2$$

Intensity at 3.00 m:

$$I = \frac{P}{A_{\text{eff}}} = \frac{750 \text{ W}}{0.110 \text{ m}^2} = 6820 \text{ W/m}^2 = 6.82 \text{ kW/m}^2$$

Answer: The intensity is **6.82 kW/m²** or **6820 W/m²**.

Discussion

This high intensity (about 7 times the intensity of direct sunlight at Earth's surface) explains why standing in front of an electric heater with a concave reflector feels so warm. The mirror concentrates 750 W of infrared radiation into a relatively small area of about 0.110 m². This focused distribution is much more effective at warming a specific location than an unreflected heater that radiates uniformly in all directions. The concentration effect is why these heaters work best when you're positioned at the focal distance—too close or too far reduces the heating effect.

Consider a 250-W heat lamp fixed to the ceiling in a bathroom. If the filament in one light burns out then the remaining three still work. Construct a problem in which you determine the resistance of each filament in order to obtain a certain intensity projected on the bathroom floor. The ceiling is 3.0 m high. The problem will need to involve concave mirrors behind the filaments. Your instructor may wish to guide you on the level of complexity to consider in the electrical components.

Glossary

converging mirror
: a concave mirror in which light rays that strike it parallel to its axis converge at one or more points along the axis

diverging mirror
: a convex mirror in which light rays that strike it parallel to its axis bend away (diverge) from its axis

law of reflection
: angle of reflection equals the angle of incidence
