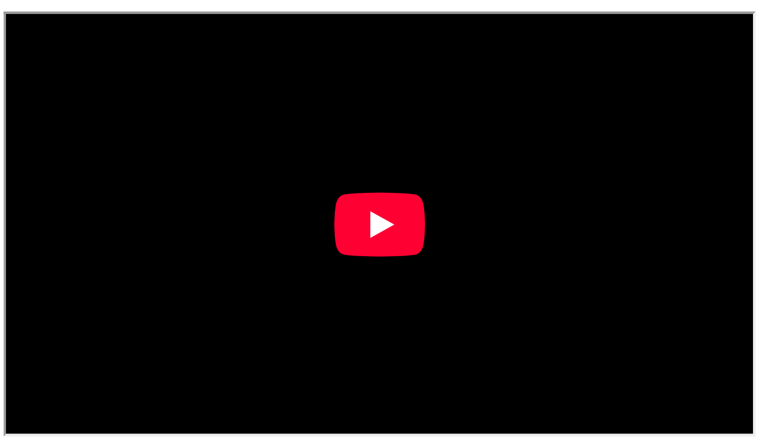
Introduction to Electric Current, Resistance, and Ohm's Law

Electric energy in massive quantities is transmitted from this hydroelectric facility, the Srisailam power station located along the Krishna River in India, by the movement of charge—that is, by electric current. (credit: Chintohere, Wikimedia Commons)

The flicker of numbers on a handheld calculator, nerve impulses carrying signals of vision to the brain, an ultrasound device sending a signal to a computer screen, the brain sending a message for a baby to twitch its toes, an electric train pulling its load over a mountain pass, a hydroelectric plant sending energy to metropolitan and rural users—these and many other examples of electricity involve *electric current, the movement of charge*. Humankind has indeed harnessed electricity, the basis of technology, to improve our quality of life. Whereas the previous two chapters concentrated on static electricity and the fundamental force underlying its behavior, the next few chapters will be devoted to electric and magnetic phenomena involving current. In addition to exploring applications of electricity, we shall gain new insights into nature—in particular, the fact that all magnetism results from electric current.



Current

- Define electric current, ampere, and drift velocity
- Describe the direction of charge flow in conventional current.
- Use drift velocity to calculate current and vice versa.

Electric Current

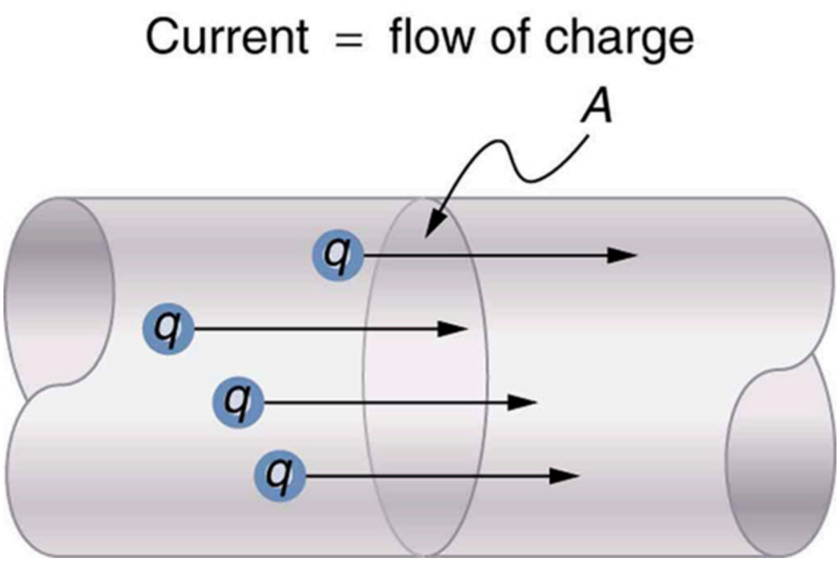
Electric current is defined to be the rate at which charge flows. A large current, such as that used to start a truck engine, moves a large amount of charge in a small time, whereas a small current, such as that used to operate a hand-held calculator, moves a small amount of charge over a long period of time. In equation form, **electric current** I is defined to be

$$I=\frac{\Delta Q}{\Delta t},$$

where ΔQ is the amount of charge passing through a given area in time Δt . (As in previous chapters, initial time is often taken to be zero, in which case $\Delta t = t$.) (See [Figure 1].) The SI unit for current is the **ampere** (A), named for the French physicist André-Marie Ampère (1775–1836). Since $I = \Delta Q / \Delta t$, we see that an ampere is one coulomb per second:

$$1 \text{A} = 1 \text{C/s}$$

Not only are fuses and circuit breakers rated in amperes (or amps), so are many electrical appliances.



The rate of flow of charge is current. An ampere is the flow of one coulomb through an area in one second.

Calculating Currents: Current in a Truck Battery and a Handheld Calculator

(a) What is the current involved when a truck battery sets in motion 720 C of charge in 4.00 s while starting an engine? (b) How long does it take 1.00 C of charge to flow through a handheld calculator if a 0.300-mA current is flowing?

Strategy

We can use the definition of current in the equation $I = \Delta Q / \Delta t$ to find the current in part (a), since charge and time are given. In part (b), we rearrange the definition of current and use the given values of charge and current to find the time required.

Solution for (a)

Entering the given values for charge and time into the definition of current gives

$$I = \frac{\Delta Q}{\Delta t} = \frac{720 \text{C}}{4.00 \text{s}} = 180 \text{C/s} \quad I = 180 \text{A}.$$

Discussion for (a)

This large value for current illustrates the fact that a large charge is moved in a small amount of time. The currents in these “starter motors” are fairly large because large frictional forces need to be overcome when setting something in motion.

Solution for (b)

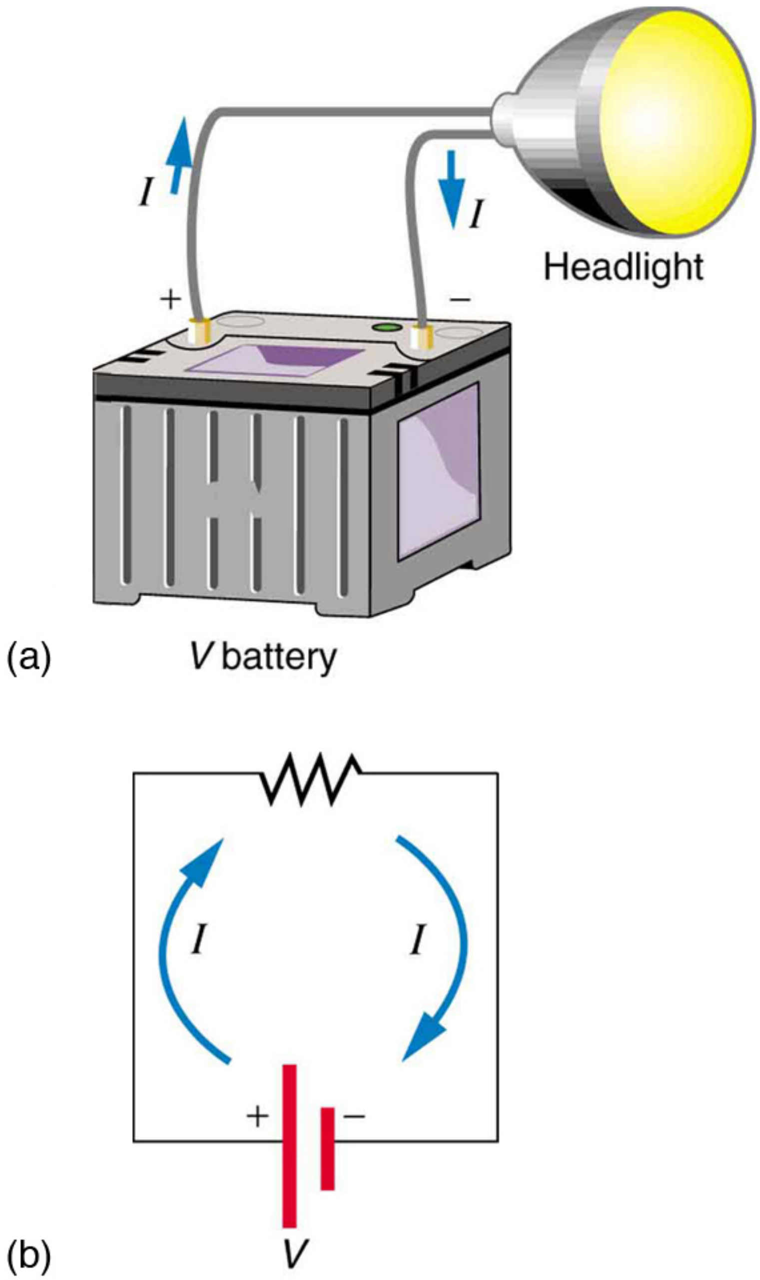
Solving the relationship $I = \Delta Q / \Delta t$ for time Δt , and entering the known values for charge and current gives

$$\Delta t = \frac{\Delta Q}{I} = \frac{1.00\text{ C}}{0.300\times10^{-3}\text{ C/s}} \quad \Delta t = 3.33\times10^{3}\text{ s.}$$

Discussion for (b)

This time is slightly less than an hour. The small current used by the hand-held calculator takes a much longer time to move a smaller charge than the large current of the truck starter. So why can we operate our calculators only seconds after turning them on? It's because calculators require very little energy. Such small current and energy demands allow handheld calculators to operate from solar cells or to get many hours of use out of small batteries. Remember, calculators do not have moving parts in the same way that a truck engine has with cylinders and pistons, so the technology requires smaller currents.

[Figure 2] shows a simple circuit and the standard schematic representation of a battery, conducting path, and load (a resistor). Schematics are very useful in visualizing the main features of a circuit. A single schematic can represent a wide variety of situations. The schematic in [Figure 2] (b), for example, can represent anything from a truck battery connected to a headlight lighting the street in front of the truck to a small battery connected to a penlight lighting a keyhole in a door. Such schematics are useful because the analysis is the same for a wide variety of situations. We need to understand a few schematics to apply the concepts and analysis to many more situations.



(a) A simple electric circuit. A closed path for current to flow through is supplied by conducting wires connecting a load to the terminals of a battery. (b) In this schematic, the battery is represented by the two parallel red lines, conducting wires are shown as straight lines, and the zigzag represents the load. The schematic represents a wide variety of similar circuits.

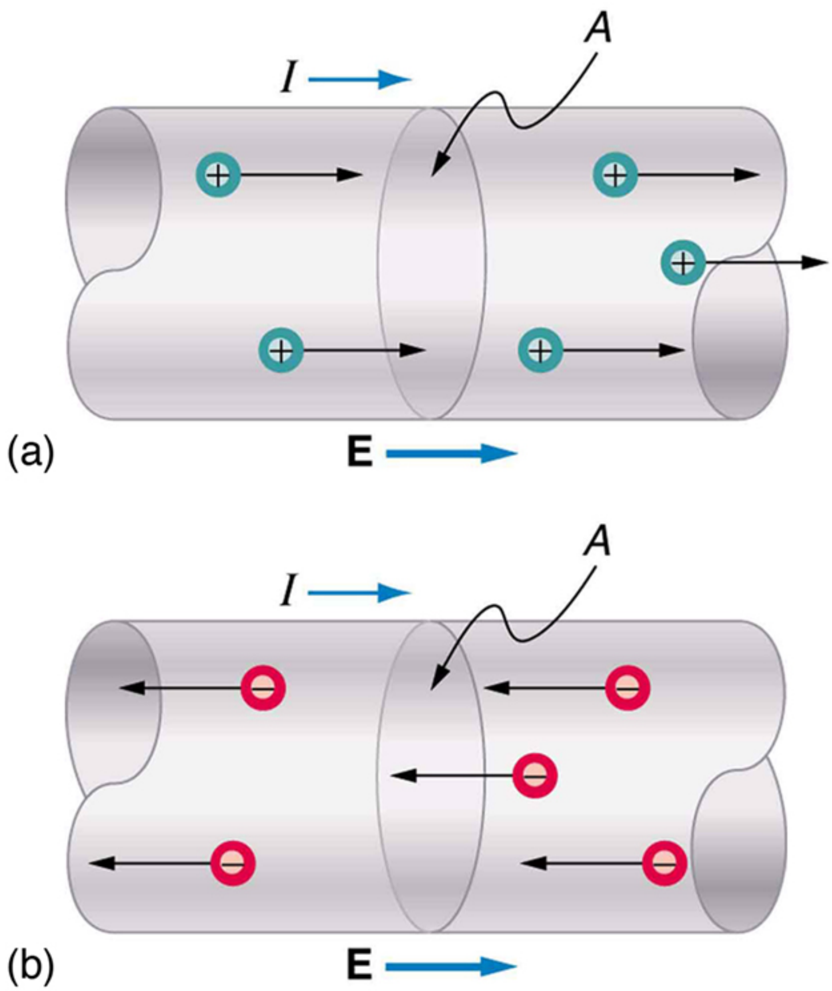
Note that the direction of current flow in [Figure 2] is from positive to negative. **The direction of conventional current is the direction that positive charge would flow**. Depending on the situation, positive charges, negative charges, or both may move. In metal wires, for example, current is carried by electrons—that is, negative charges move. In ionic solutions, such as salt water, both positive and negative charges move. This is also true in nerve cells. A Van de Graaff generator used for nuclear research can produce a current of pure positive charges, such as protons. [Figure 3] illustrates the movement of charged particles that compose a current. The fact that conventional current is taken to be in the direction that positive charge would flow can be traced back to American politician and scientist Benjamin Franklin in the 1700s. He named the type of charge associated with electrons negative, long before they were known to carry current in so many situations. Franklin, in fact, was totally unaware of the small-scale structure of electricity.

It is important to realize that there is an electric field in conductors responsible for producing the current, as illustrated in [Figure 3]. Unlike static electricity, where a conductor in equilibrium cannot have an electric field in it, conductors carrying a current have an electric field and are not in static equilibrium. An electric field is needed to supply energy to move the charges.

Making Connections: Take-Home Investigation—Electric Current Illustration

Find a straw and little peas that can move freely in the straw. Place the straw flat on a table and fill the straw with peas. When you pop one pea in at one end, a different pea should pop out the other end. This demonstration is an analogy for an electric current. Identify what compares to the electrons and what compares to the supply of energy. What other analogies can you find for an electric current?

Note that the flow of peas is based on the peas physically bumping into each other; electrons flow due to mutually repulsive electrostatic forces.



Current I is the rate at which charge moves through an area A , such as the cross-section of a wire. Conventional current is defined to move in the direction of the electric field. (a) Positive charges move in the direction of the electric field and the same direction as conventional current. (b) Negative charges move in the direction opposite to the electric field. Conventional current is in the direction opposite to the movement of negative charge. The flow of electrons is sometimes referred to as electronic flow.

Calculating the Number of Electrons that Move through a Calculator

If the 0.300-mA current through the calculator mentioned in the [Example 1] example is carried by electrons, how many electrons per second pass through it?

Strategy

The current calculated in the previous example was defined for the flow of positive charge. For electrons, the magnitude is the same, but the sign is opposite, $I_{\text{electrons}} = -0.300 \times 10^{-3} \text{C/s}$. Since each electron (e^{-}) has a charge of $-1.60 \times 10^{-19} \text{C}$, we can convert the current in coulombs per second to electrons per second.

Solution

Starting with the definition of current, we have

$$I_{\text{electrons}}=\Delta Q_{\text{electrons}}\Delta t=-0.300\times10^{-3}\text{Cs}.$$

We divide this by the charge per electron, so that

$$e^{-}\text{s} \quad = \quad -0.300\times10^{-3}\text{Cs}\times1e^{-}-1.60\times10^{-19}\text{C} \qquad = \quad 1.88\times10^{15}e^{-}\text{s}.$$

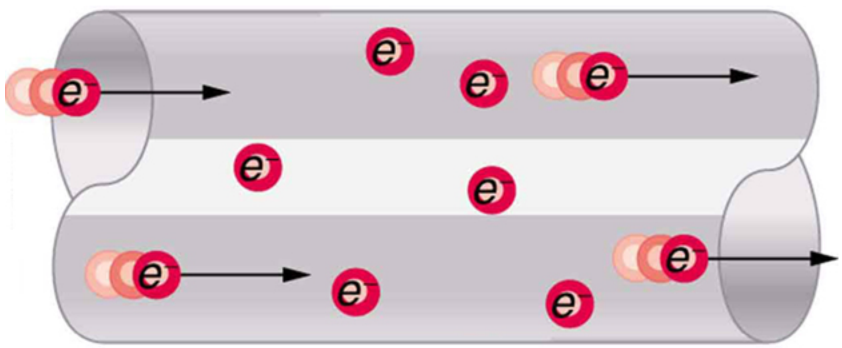
Discussion

There are so many charged particles moving, even in small currents, that individual charges are not noticed, just as individual water molecules are not noticed in water flow. Even more amazing is that they do not always keep moving forward like soldiers in a parade. Rather they are like a crowd of people with movement in different directions but a general trend to move forward. There are lots of collisions with atoms in the metal wire and, of course, with other electrons.

Drift Velocity

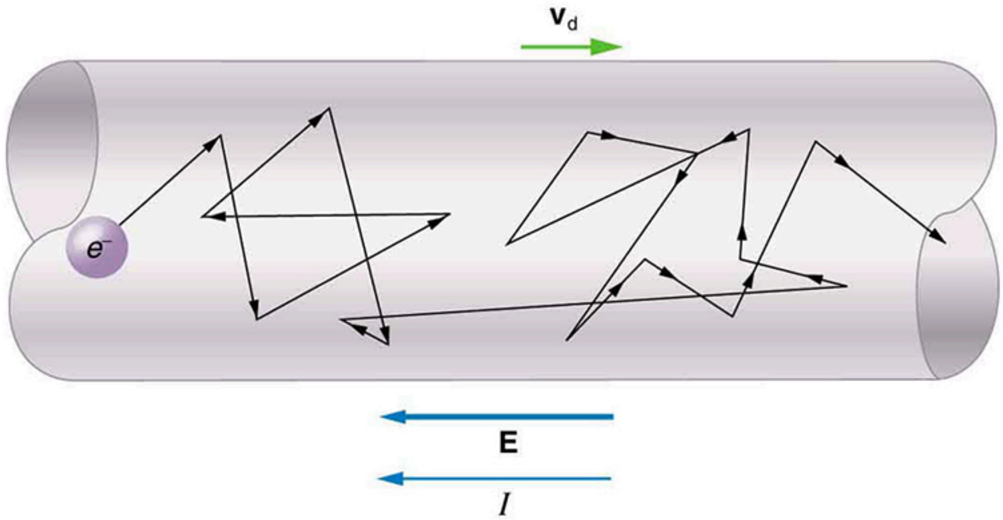
Electrical signals are known to move very rapidly. Telephone conversations carried by currents in wires cover large distances without noticeable delays. Lights come on as soon as a switch is flicked. Most electrical signals carried by currents travel at speeds on the order of 10^{8}m/s , a significant fraction of the speed of light. Interestingly, the individual charges that make up the current move **much** more slowly on average, typically drifting at speeds on the order of 10^{-4}m/s . How do we reconcile these two speeds, and what does it tell us about standard conductors?

The high speed of electrical signals results from the fact that the force between charges acts rapidly at a distance. Thus, when a free charge is forced into a wire, as in [Figure 4], the incoming charge pushes other charges ahead of it, which in turn push on charges farther down the line. The density of charge in a system cannot easily be increased, and so the signal is passed on rapidly. The resulting electrical shock wave moves through the system at nearly the speed of light. To be precise, this rapidly moving signal or shock wave is a rapidly propagating change in electric field.



When charged particles are forced into this volume of a conductor, an equal number are quickly forced to leave. The repulsion between like charges makes it difficult to increase the number of charges in a volume. Thus, as one charge enters, another leaves almost immediately, carrying the signal rapidly forward.

Good conductors have large numbers of free charges in them. In metals, the free charges are free electrons. [Figure 5] shows how free electrons move through an ordinary conductor. The distance that an individual electron can move between collisions with atoms or other electrons is quite small. The electron paths thus appear nearly random, like the motion of atoms in a gas. But there is an electric field in the conductor that causes the electrons to drift in the direction shown (opposite to the field, since they are negative). The **drift velocity** v_{d} is the average velocity of the free charges. Drift velocity is quite small, since there are so many free charges. If we have an estimate of the density of free electrons in a conductor, we can calculate the drift velocity for a given current. The larger the density, the lower the velocity required for a given current.



Free electrons moving in a conductor make many collisions with other electrons and atoms. The path of one electron is shown. The average velocity of the free charges is called the drift velocity, v_{d}, and it is in the direction opposite to the electric field for electrons. The collisions normally transfer energy to the conductor, requiring a constant supply of energy to maintain a steady current.

Conduction of Electricity and Heat

Good electrical conductors are often good heat conductors, too. This is because large numbers of free electrons can carry electrical current and can transport thermal energy.

The free-electron collisions transfer energy to the atoms of the conductor. The electric field does work in moving the electrons through a distance, but that work does not increase the kinetic energy (nor speed, therefore) of the electrons. The work is transferred to the conductor's atoms, possibly increasing temperature. Thus a continuous power input is required to keep a current flowing. An exception, of course, is found in superconductors, for reasons we shall explore in a later chapter. Superconductors can have a steady current without a continual supply of energy—a great energy savings. In contrast, the supply of energy can be useful, such as in a lightbulb filament. The supply of energy is necessary to increase the temperature of the tungsten filament, so that the filament glows.

Making Connections: Take-Home Investigation—Filament Observations

Find a lightbulb with a filament. Look carefully at the filament and describe its structure. To what points is the filament connected?

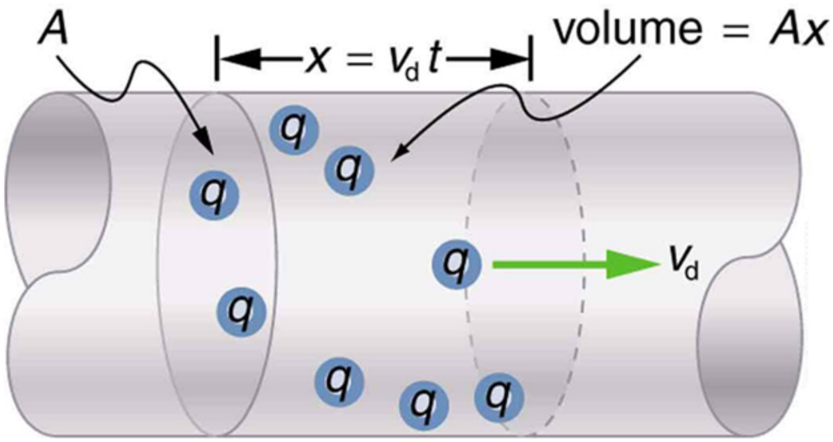
We can obtain an expression for the relationship between current and drift velocity by considering the number of free charges in a segment of wire, as illustrated in [Figure 6]. **The number of free charges per unit volume** is given the symbol n and depends on the material. The shaded segment has a volume Ax , so that the number of free charges in it is nAx . The charge ΔQ in this segment is thus $qnAx$, where q is the amount of charge on each carrier. (Recall that for electrons, q is $-1.60 \times 10^{-19} \text{C}$.) Current is charge moved per unit time; thus, if all the original charges move out of this segment in time Δt , the current is

$$I=\frac{\Delta Q}{\Delta t}=\frac{qnAx}{\Delta t}.$$

Note that $x/\Delta t$ is the magnitude of the drift velocity, v_{d} , since the charges move an average distance x in a time Δt. Rearranging terms gives

$$I=nqAv_{\text{d}},$$

where I is the current through a wire of cross-sectional area A made of a material with a free charge density n . The carriers of the current each have charge q and move with a drift velocity of magnitude v_{d}.



All the charges in the shaded volume of this wire move out in a time t , having a drift velocity of magnitude $v_\text{d}=x/t$. See text for further discussion.

Note that simple drift velocity is not the entire story. The speed of an electron is much greater than its drift velocity. In addition, not all of the electrons in a conductor can move freely, and those that do might move somewhat faster or slower than the drift velocity. So what do we mean by free electrons? Atoms in a metallic conductor are packed in the form of a lattice structure. Some electrons are far enough away from the atomic nuclei that they do not experience the attraction of the nuclei as much as the inner electrons do. These are the free electrons. They are not bound to a single atom but can instead move freely among the atoms in a "sea" of electrons. These free electrons respond by accelerating when an electric field is applied. Of course as they move they collide with the atoms in the lattice and other electrons, generating thermal energy, and the conductor gets warmer. In an insulator, the organization of the atoms and the structure do not allow for such free electrons.

Calculating Drift Velocity in a Common Wire

Calculate the drift velocity of electrons in a 12-gauge copper wire (which has a diameter of 2.053 mm) carrying a 20.0-A current, given that there is one free electron per copper atom. (Household wiring often contains 12-gauge copper wire, and the maximum current allowed in such wire is usually 20 A.) The density of copper is $8.80 \times 10^3 \text{kg/m}^3$.

Strategy

We can calculate the drift velocity using the equation $I = nqAv_d$. The current $I = 20.0\text{A}$ is given, and $q = -1.60 \times 10^{-19}\text{C}$ is the charge of an electron. We can calculate the area of a cross-section of the wire using the formula $A = \pi r^2$, where r is one-half the given diameter, 2.053 mm. We are given the density of copper, $8.80 \times 10^3 \text{kg/m}^3$, and the periodic table shows that the atomic mass of copper is 63.54 g/mol. We can use these two quantities along with Avogadro's number, $6.02 \times 10^{23} \text{atoms/mol}$, to determine n, the number of free electrons per cubic meter.

Solution

First, calculate the density of free electrons in copper. There is one free electron per copper atom. Therefore, is the same as the number of copper atoms per m^3 . We can now find n as follows:

$$n = 1e^-\text{atom}\times 6.02\times 10^{23}\text{atomsmol}\times 1mol63.54\text{g}\times 1000\text{gkg}\times 8.80\times 10^3\text{kg1m}^3 \quad n = 8.342\times 10^{28} e^-/\text{m}^3.$$

The cross-sectional area of the wire is

$$A = \pi r^2 \quad A = \pi(2.053\times 10^{-3}\text{m2})^2 \quad A = 3.310\times 10^{-6}\text{m}^2.$$

Rearranging $I = nqAv_\text{d}$ to isolate drift velocity gives

$$v_\text{d} = InqA \quad v_\text{d} = 20.0\text{A}(8.342\times 10^{28}/\text{m}^3)(-1.60\times 10^{-19}\text{C})(3.310\times 10^{-6}\text{m}^2) \quad v_\text{d} = -4.53\times 10^{-4}\text{m/s}.$$

Discussion

The minus sign indicates that the negative charges are moving in the direction opposite to conventional current. The small value for drift velocity (on the order of 10^{-4}m/s) confirms that the signal moves on the order of 10^{12} times faster (about 10^8m/s) than the charges that carry it.

Section Summary

- Electric current I is the rate at which charge flows, given by

  $I = \Delta Q \Delta t,$

where ΔQ is the amount of charge passing through an area in time Δt .
- The direction of conventional current is taken as the direction in which positive charge moves.
- The SI unit for current is the ampere (A), where $1 \text{A} = 1 \text{C/s}$.
- Current is the flow of free charges, such as electrons and ions.
- Drift velocity v_d is the average speed at which these charges move.
- Current I is proportional to drift velocity v_d , as expressed in the relationship $I = nqAv_\text{d}$. Here, I is the current through a wire of cross-sectional area A . The wire's material has a free-charge density n , and each carrier has charge q and a drift velocity v_d .
- Electrical signals travel at speeds about 10^{12} times greater than the drift velocity of free electrons.

Conceptual Questions

Can a wire carry a current and still be neutral—that is, have a total charge of zero? Explain.

Car batteries are rated in ampere-hours ($\text{A} \cdot \text{h}$). To what physical quantity do ampere-hours correspond (voltage, charge, …), and what relationship do ampere-hours have to energy content?

If two different wires having identical cross-sectional areas carry the same current, will the drift velocity be higher or lower in the better conductor? Explain in terms of the equation $v_\text{d} = \frac{I}{nqA}$, by considering how the density of charge carriers n relates to whether or not a material is a good conductor.

Why are two conducting paths from a voltage source to an electrical device needed to operate the device?

In cars, one battery terminal is connected to the metal body. How does this allow a single wire to supply current to electrical devices rather than two wires?

Why isn't a bird sitting on a high-voltage power line electrocuted? Contrast this with the situation in which a large bird hits two wires simultaneously with its wings.

Problems & Exercises

What is the current in milliamperes produced by the solar cells of a pocket calculator through which 4.00 C of charge passes in 4.00 h?

Show Solution

Strategy

We use the definition of current $I = \Delta Q / \Delta t$ to find the current. We need to convert time from hours to seconds for proper SI units.

Solution

First, convert time to seconds:

$$\Delta t = 4.00 \text{ h} \times \frac{3600 \text{ s}}{1 \text{ h}} = 1.44 \times 10^4 \text{ s}$$

Now apply the definition of current:

$$I = \frac{\Delta Q}{\Delta t} = \frac{4.00 \text{ C}}{1.44 \times 10^4 \text{ s}} = 2.78 \times 10^{-4} \text{ A} = 0.278 \text{ mA}$$

Discussion

This small current is typical of low-power electronic devices like calculators. The solar cells provide just enough current to operate the calculator's circuits, which require very little power. This demonstrates why calculators can run for extended periods on minimal energy input.

The current produced by the solar cells is 0.278 mA.

A total of 600 C of charge passes through a flashlight in 0.500 h. What is the average current?

Show Solution

Strategy

We use the definition of electric current as the rate of charge flow: $I = \frac{\Delta Q}{\Delta t}$. The charge is given directly, but the time must be converted from hours to seconds to obtain the current in amperes.

Solution

First, convert the time to seconds:

$$\Delta t = 0.500 \text{ h} \times \frac{3600 \text{ s}}{1 \text{ h}} = 1800 \text{ s}$$

Now apply the definition of current:

$$I = \frac{\Delta Q}{\Delta t} = \frac{600\ \text{C}}{1800\ \text{s}} = 0.333\ \text{A} = 333\ \text{mA}$$

Discussion

This current of about one-third of an ampere is typical for a flashlight. It is much larger than the milliamp currents in small electronics like calculators, but much smaller than the hundreds of amperes used to start a car engine. The flashlight bulb requires enough current to heat the filament to incandescence, which demands more power than a calculator's LED display but far less than overcoming the mechanical resistance of an engine.

The average current of 0.333 A through the flashlight is reasonable for illuminating a bulb.

What is the current when a typical static charge of $0.250\,\mu\text{C}$ moves from your finger to a metal doorknob in $1.00\,\mu\text{s}$?

Show Solution

Strategy

We use $I = \Delta Q / \Delta t$ with the given charge and time values. Both quantities need to be converted to SI base units.

Solution

Convert the charge and time to SI units:

$$\Delta Q = 0.250\ \mu\text{C} = 0.250 \times 10^{-6}\ \text{C}$$
$$\Delta t = 1.00\ \mu\text{s} = 1.00 \times 10^{-6}\ \text{s}$$

Calculate the current:

$$I = \frac{\Delta Q}{\Delta t} = \frac{0.250 \times 10^{-6}\ \text{C}}{1.00 \times 10^{-6}\ \text{s}} = 0.250\ \text{A}$$

Discussion

Despite the tiny amount of charge involved, the extremely short discharge time results in a significant current of 0.250 A—comparable to the current through a small lightbulb. This explains why static shocks can be felt: the momentary current is substantial enough to stimulate nerve endings. The brief duration prevents any harmful effects.

The current during the static discharge is 0.250 A.

Find the current when 2.00 nC jumps between your comb and hair over a $0.500\text{-}\mu\text{s}$ time interval.

Show Solution

Strategy

This problem involves the same fundamental relationship as static discharge: $I = \frac{\Delta Q}{\Delta t}$. The charge is given in nanocoulombs and the time in microseconds, so we must convert both to SI base units (coulombs and seconds) before calculating.

Solution

Convert the given values to SI units:

$$\Delta Q = 2.00\ \text{nC} = 2.00 \times 10^{-9}\ \text{C}$$
$$\Delta t = 0.500\ \mu\text{s} = 0.500 \times 10^{-6}\ \text{s} = 5.00 \times 10^{-7}\ \text{s}$$

Now calculate the current:

$$I = \frac{\Delta Q}{\Delta t} = \frac{2.00 \times 10^{-9}\ \text{C}}{5.00 \times 10^{-7}\ \text{s}} = 4.00 \times 10^{-3}\ \text{A} = 4.00\ \text{mA}$$

Discussion

This milliamp-level current during static discharge is consistent with the everyday experience of static shocks. Although the charge transferred is very small (nanocoulombs), the extremely short discharge time (microseconds) creates a measurable current. This brief but noticeable current is why static discharge can feel like a small shock, even though the total energy transferred is quite small.

The current produced when 2.00 nC jumps between comb and hair in 0.500 μs is 4.00 mA.

A large lightning bolt had a 20 000-A current and moved 30.0 C of charge. What was its duration?

Show Solution

Strategy

Rearrange the definition of current $I = \Delta Q / \Delta t$ to solve for time: $\Delta t = \Delta Q / I$.

Solution

Rearranging $I = \Delta Q / \Delta t$ for time:

$$\Delta t = \Delta Q I = 30.0\ \text{C} 20000\ \text{A} = 30.0\ \text{C} 2.00 \times 10^{4}\ \text{A} = 1.50 \times 10^{-3}\ \text{s} = 1.50\ \text{ms}$$

Discussion

Lightning bolts transfer enormous amounts of charge (30 C here compared to microcoulombs in static shocks) in extremely short times. The 1.50 ms duration seems brief, but this is actually relatively long for lightning—the massive current and charge transfer can cause significant damage, vaporizing materials and starting fires. The energy involved (power times time) is substantial despite the short duration because the current is so high.

The lightning bolt lasted 1.50 ms.

The 200-A current through a spark plug moves 0.300 mC of charge. How long does the spark last?

Show Solution

Strategy

We need to find the time duration given the current and charge. Starting from the definition $I = \Delta Q \Delta t$, we can solve for time: $\Delta t = \Delta Q I$. The charge must be converted from milliCoulombs to Coulombs.

Solution

Convert the charge to SI units:

$$\Delta Q = 0.300\ \text{mC} = 0.300 \times 10^{-3}\ \text{C} = 3.00 \times 10^{-4}\ \text{C}$$

Solve for the time duration:

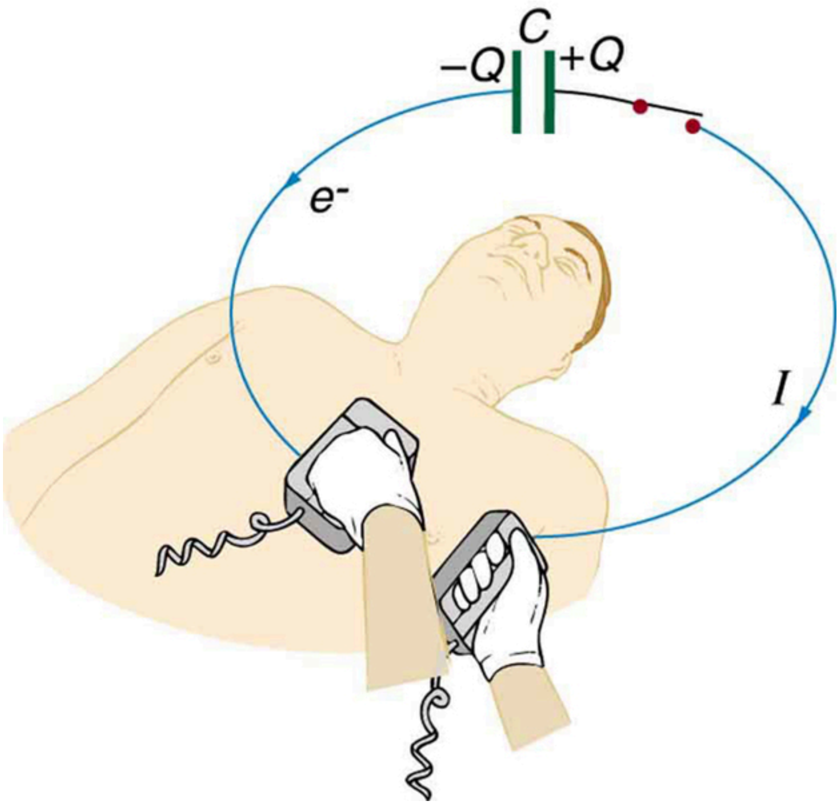
$$\Delta t = \Delta Q I = 3.00 \times 10^{-4}\ \text{C} 200\ \text{A} = 1.50 \times 10^{-6}\ \text{s} = 1.50\ \mu\text{s}$$

Discussion

A spark duration of 1.50 microseconds is extremely brief but physically reasonable for a spark plug. In automotive engines, spark plugs must fire rapidly and repeatedly—at high RPM, a 4-cylinder engine might require sparks at rates exceeding 100 per second per cylinder. The short duration allows the spark to initiate combustion without excessive electrode wear. The high current (200 A) during this brief interval provides enough energy to ionize the air-fuel mixture and trigger combustion, yet the total charge transferred remains small.

The spark lasts for 1.50 μs, which is sufficient to ignite the fuel-air mixture in the engine cylinder.

(a) A defibrillator sends a 6.00-A current through the chest of a patient by applying a 10 000-V potential as in the figure below. What is the resistance of the path? (b) The defibrillator paddles make contact with the patient through a conducting gel that greatly reduces the path resistance. Discuss the difficulties that would ensue if a larger voltage were used to produce the same current through the patient, but with the path having perhaps 50 times the resistance. (Hint: The current must be about the same, so a higher voltage would imply greater power. Use this equation for power: $P = I^{2} R$.)



The capacitor in a defibrillation unit drives a current through the heart of a patient.

Show Solution

Strategy

For part (a), we use Ohm's law $V = IR$ to find the resistance, given voltage and current. For part (b), we analyze the power dissipated using $P = I^2R$ to understand why high resistance paths are problematic.

Solution

(a) Using Ohm's law $V = IR$, solve for resistance:

$$R = \frac{V}{I} = \frac{10{,}000\text{ V}}{6.00\text{ A}}$$
$$R = 1{,}667\ \Omega = 1.67\text{ k}\Omega$$

(b) With 50 times the resistance ($R' = 50R = 83.5\text{ k}\Omega$) and the same current, the voltage would need to be:

$$V' = IR' = (6.00\text{ A})(83{,}500\ \Omega) = 501{,}000\text{ V} = 501\text{ kV}$$

The power dissipated is:

$$P = I^2R$$

If R increases by a factor of 50 while I remains constant, the power increases by a factor of 50:

$$P' = I^2(50R) = 50(I^2R) = 50P$$

Original power:

$$P = (6.00)^2(1{,}667) = 60{,}000\text{ W} = 60\text{ kW}$$

New power:

$$P' = 50 \times 60 = 3{,}000\text{ kW} = 3.0\text{ MW}$$

Discussion

Part (a): The resistance of 1.67 kΩ represents the path through the patient's chest, including skin, muscle, bone, and other tissues. The conducting gel used with defibrillator paddles is essential for achieving this relatively low resistance.

Part (b): Without conducting gel, skin resistance alone can exceed 100 kΩ. If the resistance were 50 times higher (~83.5 kΩ), several severe problems would occur:

1. **Extreme voltage required**: 501 kV would be needed—far beyond safe or practical limits. This voltage would cause arcing, pose electrocution hazards to medical personnel, and likely kill the patient through excessive current or burns.
2. **Dangerous power dissipation**: The power of 3.0 MW (even if only for milliseconds) would cause catastrophic burns. Most of this energy would be dissipated in the high-resistance skin and superficial tissues, not reaching the heart.
3. **Ineffective defibrillation**: The current path would be poorly defined, with most current taking surface paths rather than passing through the heart muscle.

The conducting gel reduces skin resistance from ~100 kΩ to ~1 kΩ, a 100-fold reduction. This allows:

- Safe voltage levels (10 kV instead of 1 MV)
- Controlled current delivery to the heart
- Minimal skin burns
- Predictable current pathways

The resistance of the path is 1.67 kΩ. Using 50 times higher resistance would require dangerous voltages (501 kV), cause severe burns from 50 times higher power dissipation, and make defibrillation ineffective.

During open-heart surgery, a defibrillator can be used to bring a patient out of cardiac arrest. The resistance of the path is 500Ω and a 10.0-mA current is needed. What voltage should be applied?

Show Solution

Strategy

This problem applies Ohm's Law: $V = IR$. During open-heart surgery, the defibrillator electrodes can be placed directly on the heart, resulting in a much lower resistance path than external defibrillation (which must pass current through the chest wall). We use the given resistance and required current to find the necessary voltage.

Solution

Convert current to SI units and apply Ohm's Law:

$$I = 10.0 \text{ mA} = 10.0 \times 10^{-3} \text{ A} = 0.0100 \text{ A}$$
$$V = IR = (0.0100 \text{ A})(500 \ \Omega) = 5.00 \text{ V}$$

Discussion

The required voltage of 5.00 V is remarkably low compared to the 10,000 V used for external defibrillation (see earlier problem). This dramatic difference occurs because during open-heart surgery, the electrodes are placed directly on the heart muscle, eliminating the high resistance of the chest wall, skin, and other tissues. The direct contact means much less voltage is needed to deliver the therapeutic current. This lower voltage is also much safer for the surgical team and reduces the risk of electrical burns to cardiac tissue. The 10.0 mA current is sufficient to depolarize the heart muscle and restore normal rhythm when applied directly.

A voltage of 5.00 V should be applied to produce the required 10.0-mA current through the 500-Ω path during open-heart surgery.

(a) A defibrillator passes 12.0 A of current through the torso of a person for 0.0100 s. How much charge moves? (b) How many electrons pass through the wires connected to the patient? (See figure two problems earlier.)

Show Solution

Strategy

For part (a), we use the definition of current: $I = Q/t$, where Q is the charge and t is the time. Solving for Q gives us the total charge that flows. For part (b), we use the fact that each electron carries a charge of $e = 1.60 \times 10^{-19}$ C to find the number of electrons.

Solution

(a) From the definition of current:

$$I = Qt$$

Solving for charge:

$$Q = It = (12.0 \text{ A})(0.0100 \text{ s})$$
$$Q = 0.120 \text{ C}$$

(b) The number of electrons is:

$$n = \frac{Q}{e} = \frac{0.120\ \text{C}}{1.60 \times 10^{-19}\ \text{C/electron}}$$
$$n = 7.50 \times 10^{17}\ \text{electrons}$$

Discussion

Part (a): The charge of 0.120 C is substantial—equivalent to the charge stored in a moderate-sized capacitor. During the brief 10-millisecond pulse, this charge flows through the patient's torso to depolarize the heart muscle and restore normal rhythm. The short duration is important: it delivers enough charge to be therapeutic without causing tissue damage from prolonged current flow.

Part (b): The number 7.50×10^{17} electrons is enormous—750 quadrillion electrons! To put this in perspective:

- This is about 1 millionth of a mole of electrons
- If these electrons were lined up, they'd stretch many times across the solar system
- Yet the total mass of these electrons is only about 6.8×10^{-13} kg (less than a nanogram)

This calculation illustrates a fundamental principle of electricity: even modest currents involve vast numbers of charge carriers. In metals, the drift velocity of electrons is quite slow (millimeters per second), but the enormous density of mobile electrons means even this slow drift produces significant current.

The current of 12.0 A is quite high for medical applications—defibrillators deliver powerful shocks to overwhelm abnormal electrical activity in the heart. For comparison, as little as 0.1 A across the heart can be fatal under normal circumstances, but during defibrillation, the brief, controlled pulse is therapeutic rather than harmful.

The charge that moves is 0.120 C, corresponding to 7.50×10^{17} electrons passing through the wires.

A clock battery wears out after moving 10 000 C of charge through the clock at a rate of 0.500 mA. (a) How long did the clock run? (b) How many electrons per second flowed?

Show Solution

Strategy

For part (a), we use the definition of current $I = \frac{\Delta Q}{\Delta t}$ solved for time: $\Delta t = \frac{\Delta Q}{I}$. For part (b), we convert the current (charge per second) to electrons per second by dividing by the charge of one electron.

Solution

(a) Find the time the clock ran:

$$I = 0.500\ \text{mA} = 0.500 \times 10^{-3}\ \text{A} = 5.00 \times 10^{-4}\ \text{A}$$
$$\Delta t = \frac{\Delta Q}{I} = \frac{10000\ \text{C}}{5.00 \times 10^{-4}\ \text{A}} = 2.00 \times 10^{7}\ \text{s}$$

Converting to more meaningful units:

$$\Delta t = 2.00 \times 10^{7}\ \text{s} \times \frac{1\ \text{h}}{3600\ \text{s}} \times \frac{1\ \text{day}}{24\ \text{h}} = 231\ \text{days} \approx 7.7\ \text{months}$$

(b) Find the number of electrons per second:

$$\frac{\text{electrons}}{\text{s}} = \frac{I}{e} = \frac{5.00 \times 10^{-4}\ \text{C/s}}{1.60 \times 10^{-19}\ \text{C/electron}} = 3.13 \times 10^{15}\ \text{electrons/s}$$

Discussion

The clock running for about 7.7 months on a single battery is quite reasonable for a small wall clock or desk clock with a low-power quartz movement. Quartz clocks use very little current because they only need to power the oscillator circuit and step motor. The 3.13 × 10^15 electrons per second sounds enormous, but this is actually a very small current—it takes over 3 quadrillion electrons each second just to produce half a milliamp of current. This illustrates how incredibly small the charge of a single electron is.

(a) The clock ran for 2.00×10^{7} s, or about 231 days (approximately 7.7 months). (b) The flow rate was 3.13×10^{15} electrons per second.

The batteries of a submerged non-nuclear submarine supply 1000 A at full speed ahead. How long does it take to move Avogadro's number (6.02×10^{23}) of electrons at this rate?

Show Solution

Strategy

First calculate the total charge corresponding to Avogadro's number of electrons: $\Delta Q = N_A \times e$. Then use $\Delta t = \Delta Q / I$ to find the time.

Solution

Calculate the total charge for 6.02×10^{23} electrons:

$$\Delta Q = N_A \times e = (6.02 \times 10^{23})(1.60 \times 10^{-19}\ \text{C}) = 9.63 \times 10^{4}\ \text{C}$$

Calculate the time required:

$$\Delta t = \frac{\Delta Q}{I} = \frac{9.63 \times 10^{4}\ \text{C}}{1000\ \text{A}} = 96.3\ \text{s}$$

Discussion

It takes only about 1.5 minutes to move a mole of electrons at this high current! The submarine's batteries must supply enormous amounts of charge, which is why battery capacity is a critical limitation for non-nuclear submarines. At 1000 A, the submarine is drawing massive power, limiting how long it can operate at full speed before needing to recharge its batteries (typically by surfacing and running diesel generators).

It takes 96.3 s (about 1.6 minutes) to move Avogadro's number of electrons at 1000 A.

Electron guns are used in X-ray tubes. The electrons are accelerated through a relatively large voltage and directed onto a metal target, producing X-rays. (a) How many electrons per second strike the target if the current is 0.500 mA? (b) What charge strikes the target in 0.750 s?

Show Solution

Strategy

For part (a), we convert the current from charge per second to electrons per second by dividing by the electron charge. For part (b), we use the definition of current rearranged to find charge: $\Delta Q = I \cdot \Delta t$.

Solution

(a) Find the number of electrons per second:

$$I = 0.500\ \text{mA} = 0.500 \times 10^{-3}\ \text{A} = 5.00 \times 10^{-4}\ \text{C/s}$$
$$\frac{\text{electrons}}{\text{s}} = \frac{I}{e} = \frac{5.00 \times 10^{-4}\ \text{C/s}}{1.60 \times 10^{-19}\ \text{C/electron}} = 3.13 \times 10^{15}\ \text{electrons/s}$$

(b) Find the charge striking the target in 0.750 s:

$$\Delta Q = I \cdot \Delta t = (5.00 \times 10^{-4}\ \text{A})(0.750\ \text{s}) = 3.75 \times 10^{-4}\ \text{C} = 0.375\ \text{mC}$$

Discussion

The electron gun in an X-ray tube produces over 3 quadrillion electrons per second striking the metal target. When these high-speed electrons suddenly decelerate upon hitting the target, they produce X-rays through bremsstrahlung ("braking radiation"). The 0.500 mA beam current is typical for diagnostic X-ray equipment. The charge of 0.375 mC delivered in 0.750 s is small but represents enormous numbers of individual electrons, each contributing to X-ray production. The intensity of X-rays produced is directly related to this electron current.

(a) Approximately 3.13×10^{15} electrons per second strike the target. (b) A charge of 0.375 mC (or 3.75×10^{-4} C) strikes the target in 0.750 s.

A large cyclotron directs a beam of He^{++} nuclei onto a target with a beam current of 0.250 mA. (a) How many He^{++} nuclei per second is this? (b) How long does it take for 1.00 C to strike the target? (c) How long before 1.00 mol of He^{++} nuclei strike the target?

Show Solution

Strategy

Each He^{++} nucleus has lost 2 electrons, so it carries a charge of +2e. For part (a), we find the number of nuclei per second from the current and charge per nucleus. For part (b), we use $I = Q/t$. For part (c), we use Avogadro's number to find the charge in 1 mole of He^{++}.

Solution

(a) Each He^{++} nucleus has charge:

$$q = +2e = 2(1.60 \times 10^{-19}\ \text{C}) = 3.20 \times 10^{-19}\ \text{C}$$

Current is $I = 0.250\ \text{mA} = 2.50 \times 10^{-4}\ \text{A}$. The number of nuclei per second:

$$n = \frac{I}{q} = \frac{2.50 \times 10^{-4}\ \text{C/s}}{3.20 \times 10^{-19}\ \text{C/nucleus}}$$
$$n = 7.81 \times 10^{14}\ \text{nuclei/s}$$

(b) From $I = Q/t$:

$$t = \frac{Q}{I} = \frac{1.00\ \text{C}}{2.50 \times 10^{-4}\ \text{A}}$$
$$t = 4.00 \times 10^{3}\ \text{s} = 4000\ \text{s} = 66.7\ \text{min} = 1.11\ \text{hours}$$

(c) One mole contains $N_A = 6.02 \times 10^{23}$ nuclei. Total charge in 1 mole:

$$Q = N_A \cdot q = (6.02 \times 10^{23})(3.20 \times 10^{-19}\ \text{C})$$
$$Q = 1.93 \times 10^{5}\ \text{C} = 193{,}000\ \text{C}$$

Time required:

$$t = \frac{Q}{I} = \frac{1.93 \times 10^{5}\ \text{C}}{2.50 \times 10^{-4}\ \text{A}}$$
$$t = 7.71 \times 10^{8}\ \text{s} = 24.4\ \text{years}$$

Discussion

Part (a): The rate of 7.81×10^{14} nuclei per second sounds enormous, yet it produces a tiny current of only 0.25 mA. This illustrates that even small currents involve vast numbers of charge carriers.

Part (b): To deliver 1.00 C takes about 67 minutes. This demonstrates that the current is quite small—typical household currents are thousands of times larger.

Part (c): To deliver 1 mole of helium nuclei would take 24.4 years of continuous operation! This calculation shows why particle accelerators rarely accumulate macroscopic amounts of material, despite operating at high particle rates. The total charge of 193,000 C in 1 mole reflects the fact that each He^{++} carries twice the elementary charge.

Cyclotrons are used for nuclear physics research, medical isotope production, and cancer treatment. While the particle rates are high, the actual mass of material delivered is tiny because atomic nuclei are so small. A typical target in a cyclotron experiment might accumulate only micrograms of material even after hours of bombardment.

Answer: (a) 7.81×10^{14} nuclei/s, (b) 4.00×10^{3} s (67 min), (c) 7.71×10^{8} s (24.4 years)

Repeat the above example on [Example 3], but for a wire made of silver and given there is one free electron per silver atom.

Show Solution

Strategy

We follow the same approach as Example 3: calculate the drift velocity using $I = nqAv_d$, solving for $v_d = \frac{I}{nqA}$. We need the density of silver (10.49×10^{3} kg/m³) and its atomic mass (107.87 g/mol) to find the free electron density n. We use the same 12-gauge wire (diameter 2.053 mm) and 20.0-A current as in Example 3.

Solution

First, calculate the free electron density in silver:

$$n = \frac{1\ e^-}{\text{atom}} \times \frac{6.02 \times 10^{23}\ \text{atoms}}{\text{mol}} \times \frac{1\ \text{mol}}{107.87\ \text{g}} \times \frac{1000\ \text{g}}{\text{kg}} \times \frac{10.49 \times 10^{3}\ \text{kg}}{1\ \text{m}^3} \quad n = 5.86 \times 10^{28}\ e^-/\text{m}^3$$

The cross-sectional area of the 12-gauge wire is the same as in Example 3:

$$A = \pi r^2 = \pi \left(\frac{2.053 \times 10^{-3}\ \text{m}}{2}\right)^2 = 3.310 \times 10^{-6}\ \text{m}^2$$

Now solve for the drift velocity:

$$v_d = \frac{I}{nqA} \quad v_d = \frac{20.0\ \text{A}}{(5.86 \times 10^{28}\ /\text{m}^3)(-1.60 \times 10^{-19}\ \text{C})(3.310 \times 10^{-6}\ \text{m}^2)} \quad v_d = -6.45 \times 10^{-4}\ \text{m/s}$$

Discussion

The drift velocity in silver (-6.45×10^{-4} m/s) is larger in magnitude than in copper (-4.53×10^{-4} m/s from Example 3). This occurs because silver has a lower free electron density than copper (5.86×10^{28} vs. 8.34×10^{28} electrons/m³). With fewer charge carriers available, each electron must move faster on average to carry the same current. The negative sign indicates electrons move opposite to the conventional current direction. Despite the faster drift velocity, silver is still an excellent conductor—in fact, silver has the highest electrical conductivity of all metals, though it is more expensive than copper.

The drift velocity of electrons in a 12-gauge silver wire carrying 20.0 A is -6.45×10^{-4} m/s.

Using the results of the above example on [Example 3], find the drift velocity in a copper wire of twice the diameter and carrying 20.0 A.

Show Solution

Strategy

Doubling the diameter quadruples the cross-sectional area (since $A = \pi r^2 \propto d^2$). Use the drift velocity equation $v_d = I/(nqA)$ with the same current and electron density as Example 3.

Solution

The original wire has diameter 2.053 mm, so the new diameter is:

$$d_{\text{new}} = 2 \times 2.053 \text{ mm} = 4.106 \text{ mm}$$

The new cross-sectional area:

$$A_{\text{new}} = \pi \left(\frac{4.106 \times 10^{-3} \text{ m}}{2} \right)^2 = 1.324 \times 10^{-5} \text{ m}^2$$

This is 4 times the original area ($4 \times 3.310 \times 10^{-6} = 1.324 \times 10^{-5}$ m²).

Using the electron density from Example 3 ($n = 8.342 \times 10^{28}$ electrons/m³ for copper):

$$v_d = \frac{I}{nqA} = \frac{20.0 \text{ A}}{(8.342 \times 10^{28} \text{ /m}^3)(1.60 \times 10^{-19} \text{ C})(1.324 \times 10^{-5} \text{ m}^2)}$$
$$v_d = -1.13 \times 10^{-4} \text{ m/s}$$

Discussion

The drift velocity is exactly 1/4 of the original value (-4.53×10^{-4} m/s ÷ 4 = -1.13×10^{-4} m/s), as expected since quadrupling the area while keeping current constant reduces drift velocity proportionally. This demonstrates why larger wires can safely carry higher currents—the electrons move more slowly, reducing collisions and heat generation per unit volume.

The drift velocity in the larger copper wire is -1.13×10^{-4} m/s (about 0.113 mm/s).

A 14-gauge copper wire has a diameter of 1.628 mm. What magnitude current flows when the drift velocity is 1.00 mm/s? (See above example on [Example 3] for useful information.)

Show Solution

Strategy

We use the current-drift velocity relationship $I = nqAv_d$ directly. From Example 3, we know the free electron density in copper is $n = 8.342 \times 10^{28}$ electrons/m³. We need to calculate the cross-sectional area of the 14-gauge wire and convert the drift velocity to SI units.

Solution

First, calculate the cross-sectional area of the 14-gauge wire:

$$A = \pi r^2 = \pi \left(\frac{1.628 \times 10^{-3} \text{ m}}{2} \right)^2 \quad A = \pi (8.14 \times 10^{-4} \text{ m})^2 \quad A = 2.082 \times 10^{-6} \text{ m}^2$$

Convert the drift velocity to SI units:

$$v_d = 1.00 \text{ mm/s} = 1.00 \times 10^{-3} \text{ m/s}$$

Now calculate the current (using the magnitude of charge):

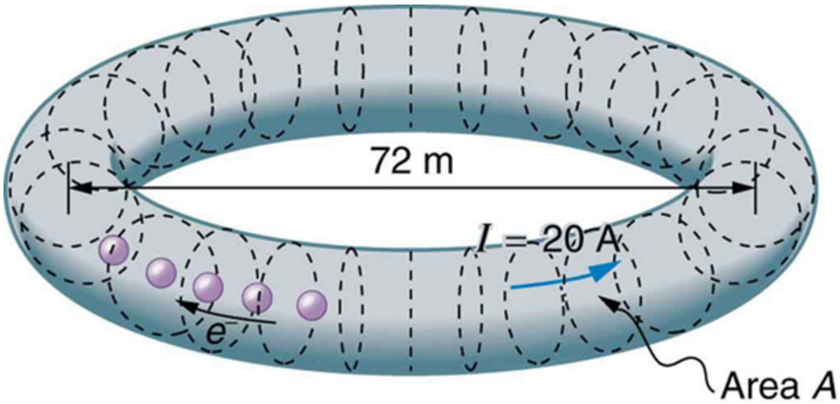
$$I = n|q|Av_d \quad I = (8.342 \times 10^{28} \text{ /m}^3)(1.60 \times 10^{-19} \text{ C})(2.082 \times 10^{-6} \text{ m}^2)(1.00 \times 10^{-3} \text{ m/s}) \quad I = 27.8 \text{ A}$$

Discussion

A current of 27.8 A is quite large for a 14-gauge wire. In fact, 14-gauge wire is typically rated for only about 15-20 A in household wiring applications because of heating concerns. The drift velocity of 1.00 mm/s is also relatively high compared to typical values (which are on the order of 0.1 mm/s or less for household currents). This problem illustrates that even modest increases in drift velocity result in substantial currents because of the enormous number of free electrons available in metals. It also shows why electrical codes limit currents in specific wire gauges—excessive current causes heating that can damage insulation or cause fires.

A current of 27.8 A flows through the 14-gauge copper wire when the drift velocity is 1.00 mm/s.

SPEAR, a storage ring about 72.0 m in diameter at the Stanford Linear Accelerator (closed in 2009), has a 20.0-A circulating beam of electrons that are moving at nearly the speed of light. (See [Figure 8].) How many electrons are in the beam?



Electrons circulating in the storage ring called SPEAR constitute a 20.0-A current. Because they travel close to the speed of light, each electron completes many orbits in each second.

Show Solution

Strategy

The electrons complete the circular path at nearly the speed of light. Current is related to the charge passing a point per unit time. The time for one orbit is $T = \text{circumference}/v = \pi d/c$. The total charge in the beam is $Q = IT$, and dividing by the electron charge gives the number of electrons.

Solution

Calculate the circumference of the storage ring:

$$C = \pi d = \pi(72.0\text{ m}) = 226.2\text{ m}$$

Calculate the time for one orbit at nearly the speed of light ($c = 3.00 \times 10^8$ m/s):

$$T = \frac{C}{c} = \frac{226.2\text{ m}}{3.00\times10^{8}\text{ m/s}} = 7.54\times10^{-7}\text{ s}$$

The total charge in the beam equals current times orbital period:

$$Q = IT = (20.0\text{ A})(7.54\times10^{-7}\text{ s}) = 1.508\times10^{-5}\text{ C}$$

The number of electrons:

$$N = \frac{Q}{e} = \frac{1.508\times10^{-5}\text{ C}}{1.60\times10^{-19}\text{ C/electron}} = 9.42\times10^{13}\text{ electrons}$$

Discussion

The beam contains about 94 trillion electrons—a large number, but still only about 1.6×10^{-10} moles, far too small to weigh. Despite the high current of 20 A, each electron passes any given point many times per second (about 1.3 million orbits per second), so relatively few electrons are needed to maintain this current. The electrons travel at nearly the speed of light, making many complete circuits each second, which is why a modest number of particles can produce such a large current.

The SPEAR storage ring contains approximately 9.42×10^{13} electrons.

Glossary

electric current
: the rate at which charge flows, $I = \frac{\Delta Q}{\Delta t}$

ampere
: (amp) the SI unit for current; 1 A = 1 C/s

drift velocity
: the average velocity at which free charges flow in response to an electric field

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Ohm’s Law: Resistance and Simple Circuits

- Explain the origin of Ohm’s law.
- Calculate voltages, currents, or resistances with Ohm’s law.
- Explain what an ohmic material is.
- Describe a simple circuit.

What drives current? We can think of various devices—such as batteries, generators, wall outlets, and so on—which are necessary to maintain a current. All such devices create a potential difference and are loosely referred to as voltage sources. When a voltage source is connected to a conductor, it applies a potential difference V that creates an electric field. The electric field in turn exerts force on charges, causing current.

Ohm’s Law

The current that flows through most substances is directly proportional to the voltage V applied to it. The German physicist Georg Simon Ohm (1787–1854) was the first to demonstrate experimentally that the current in a metal wire is *directly proportional to the voltage applied*:

$$I \propto V.$$

This important relationship is known as **Ohm’s law**. It can be viewed as a cause-and-effect relationship, with voltage the cause and current the effect. This is an empirical law like that for friction—an experimentally observed phenomenon. Such a linear relationship doesn’t always occur.

Resistance and Simple Circuits

If voltage drives current, what impedes it? The electric property that impedes current (crudely similar to friction and air resistance) is called **resistance** R . Collisions of moving charges with atoms and molecules in a substance transfer energy to the substance and limit current. Resistance is defined as inversely proportional to current, or

$$I \propto \frac{1}{R}.$$

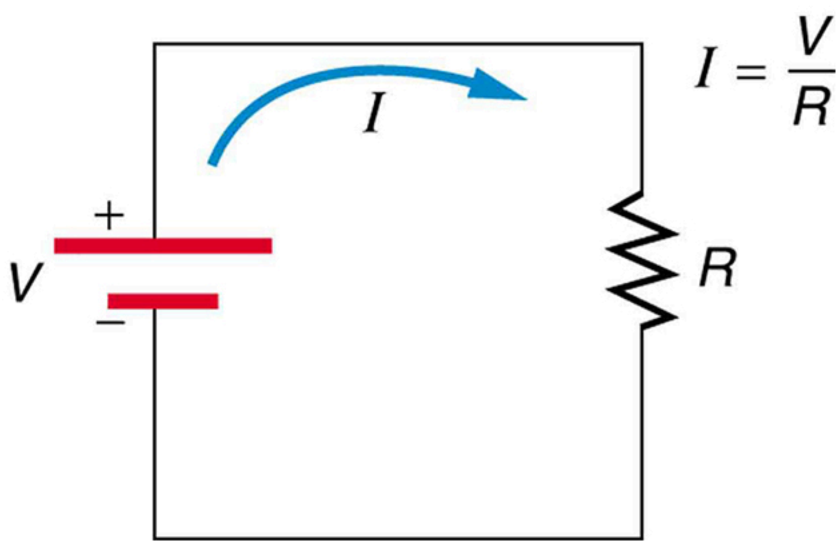
Thus, for example, current is cut in half if resistance doubles. Combining the relationships of current to voltage and current to resistance gives

$$I = \frac{V}{R}.$$

This relationship is also called Ohm’s law. Ohm’s law in this form really defines resistance for certain materials. Ohm’s law (like Hooke’s law) is not universally valid. The many substances for which Ohm’s law holds are called **ohmic**. These include good conductors like copper and aluminum, and some poor conductors under certain circumstances. Ohmic materials have a resistance R that is independent of voltage V and current I . An object that has simple resistance is called a **resistor**, even if its resistance is small. The unit for resistance is an **ohm** and is given the symbol Ω (upper case Greek omega). Rearranging $I = V/R$ gives $R = V/I$, and so the units of resistance are 1 ohm = 1 volt per ampere:

$$1\,\Omega = 1\,\frac{V}{A}.$$

[Figure 1] shows the schematic for a simple circuit. A **simple circuit** has a single voltage source and a single resistor. The wires connecting the voltage source to the resistor can be assumed to have negligible resistance, or their resistance can be included in R.



A simple electric circuit in which a closed path for current to flow is supplied by conductors (usually metal wires) connecting a load to the terminals of a battery, represented by the red parallel lines. The zigzag symbol represents the single resistor and includes any resistance in the connections to the voltage source.

Calculating Resistance: An Automobile Headlight

What is the resistance of an automobile headlight through which 2.50 A flows when 12.0 V is applied to it?

Strategy

We can rearrange Ohm's law as stated by $I = V/R$ and use it to find the resistance.

Solution

Rearranging $I = V/R$ and substituting known values gives

$$R = \frac{V}{I} = \frac{12.0\text{V}}{2.50\text{A}} = 4.80\Omega.$$

Discussion

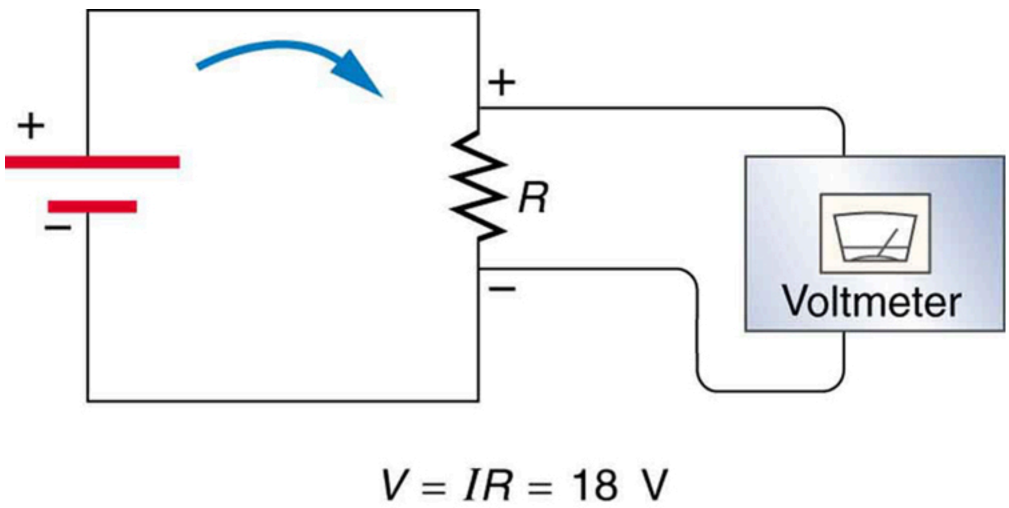
This is a relatively small resistance, but it is larger than the cold resistance of the headlight. As we shall see in Resistance and Resistivity, resistance usually increases with temperature, and so the bulb has a lower resistance when it is first switched on and will draw considerably more current during its brief warm-up period.

Resistances range over many orders of magnitude. Some ceramic insulators, such as those used to support power lines, have resistances of $10^{12}\Omega$ or more. A dry person may have a hand-to-foot resistance of $10^{5}\Omega$, whereas the resistance of the human heart is about $10^{3}\Omega$. A meter-long piece of large-diameter copper wire may have a resistance of $10^{-5}\Omega$, and superconductors have no resistance at all (they are non-ohmic). Resistance is related to the shape of an object and the material of which it is composed, as will be seen in Resistance and Resistivity.

Additional insight is gained by solving $I = V/R$ for V, yielding

$$V = IR.$$

This expression for V can be interpreted as the *voltage drop across a resistor produced by the flow of current* I. The phrase IR **drop** is often used for this voltage. For instance, the headlight in [Example 1] has an IR drop of 12.0 V. If voltage is measured at various points in a circuit, it will be seen to increase at the voltage source and decrease at the resistor. Voltage is similar to fluid pressure. The voltage source is like a pump, creating a pressure difference, causing current—the flow of charge. The resistor is like a pipe that reduces pressure and limits flow because of its resistance. Conservation of energy has important consequences here. The voltage source supplies energy (causing an electric field and a current), and the resistor converts it to another form (such as thermal energy). In a simple circuit (one with a single simple resistor), the voltage supplied by the source equals the voltage drop across the resistor, since $\text{PE} = q\Delta V$, and the same q flows through each. Thus the energy supplied by the voltage source and the energy converted by the resistor are equal. (See [Figure 2].)



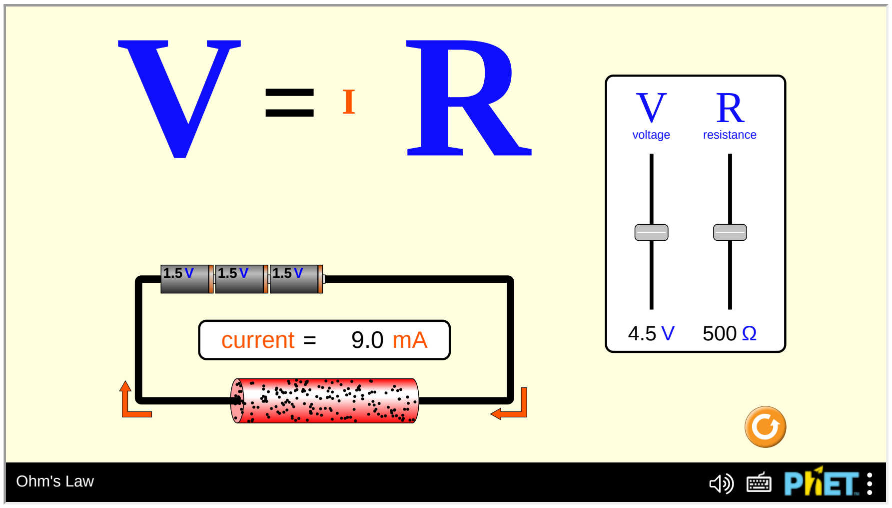
The voltage drop across a resistor in a simple circuit equals the voltage output of the battery.

Making Connections: Conservation of Energy

In a simple electrical circuit, the sole resistor converts energy supplied by the source into another form. Conservation of energy is evidenced here by the fact that all of the energy supplied by the source is converted to another form by the resistor alone. We will find that conservation of energy has other important applications in circuits and is a powerful tool in circuit analysis.

PhET Explorations: Ohm's Law

See how the equation form of Ohm's law relates to a simple circuit. Adjust the voltage and resistance, and see the current change according to Ohm's law. The sizes of the symbols in the equation change to match the circuit diagram.



Section Summary

- A simple circuit *is* one in which there is a single voltage source and a single resistance.
- One statement of Ohm's law gives the relationship between current I , voltage V , and resistance R in a simple circuit to be $I = V R$.
- Resistance has units of ohms (Ω) , related to volts and amperes by $1 \Omega = 1 \text{V/A}$.
- There is a voltage or IR drop across a resistor, caused by the current flowing through it, given by $V = IR$.

Conceptual Questions

The IR drop across a resistor means that there is a change in potential or voltage across the resistor. Is there any change in current as it passes through a resistor? Explain.

How is the IR drop in a resistor similar to the pressure drop in a fluid flowing through a pipe?

Problems & Exercises

What current flows through the bulb of a 3.00-V flashlight when its hot resistance is 3.60Ω ?

Show Solution

Strategy

Apply Ohm's law directly: $I = V / R$. All values are given in standard units.

Solution

Using Ohm's law:

$$I = V R = 3.00 \text{ V} 3.60 \ \Omega = 0.833 \text{ A}$$

Discussion

This current of 0.833 A (833 mA) is typical for a small incandescent flashlight bulb. The relatively low resistance of 3.60 Ω when hot allows substantial current to flow, which heats the filament to incandescence. Note that this is the "hot resistance"—the resistance when the bulb is operating. The cold resistance would be much lower, causing a brief surge of higher current when the flashlight is first turned on.

The current flowing through the flashlight bulb is 0.833 A.

Calculate the effective resistance of a pocket calculator that has a 1.35-V battery and through which 0.200 mA flows.

Show Solution

Strategy

We apply Ohm's law in the form $R = V I$ to find the resistance. The current must be converted from milliamperes to amperes to get resistance in ohms.

Solution

Convert current to SI units:

$$I = 0.200\text{ mA} = 0.200\times10^{-3}\text{ A} = 2.00\times10^{-4}\text{ A}$$

Apply Ohm’s law:

$$R = \frac{V}{I} = \frac{1.35\text{ V}}{2.00\times10^{-4}\text{ A}} = 6.75\times10^{3}\ \Omega = 6.75\text{ k}\Omega$$

Discussion

This resistance of 6.75 kΩ is quite high compared to most circuit components, which explains why calculators can operate for extended periods on small batteries. The high resistance limits the current to a tiny 0.200 mA, which means very little power is consumed ($P = IV = 0.27$ mW). This is why pocket calculators can run for years on a single battery or even operate from small solar cells. The effective resistance represents the combined resistance of all the calculator’s internal circuitry, including the display, processor, and other components.

The effective resistance of the pocket calculator is 6.75 kΩ.

What is the effective resistance of a car’s starter motor when 150 A flows through it as the car battery applies 11.0 V to the motor?

Show Solution

Strategy

Use Ohm’s law rearranged for resistance: $R = V/I$. The large current indicates a low resistance device.

Solution

Calculate resistance:

$$R = \frac{V}{I} = \frac{11.0\text{ V}}{150\text{ A}} = 7.33\times10^{-2}\ \Omega = 73.3\text{ m}\Omega$$

Discussion

The extremely low resistance of 73.3 mΩ (0.0733 Ω) is necessary for the starter motor to draw the high current needed to turn over the engine against compression and friction. The battery voltage drops from its nominal 12 V to 11.0 V under this heavy load due to the battery’s internal resistance. This massive current drain is why car batteries must have very low internal resistance and why leaving your headlights on can drain the battery—a starter motor draws more current in a few seconds than headlights draw in hours.

The effective resistance of the car’s starter motor is 7.33×10^{-2} Ω (73.3 mΩ).

How many volts are supplied to operate an indicator light on a DVD player that has a resistance of 140Ω, given that 25.0 mA passes through it?

Show Solution

Strategy

We use Ohm’s law in the form $V = IR$ to find the voltage across the indicator light. The current must be converted from milliamperes to amperes.

Solution

Convert current to SI units:

$$I = 25.0\text{ mA} = 25.0\times10^{-3}\text{ A} = 0.0250\text{ A}$$

Apply Ohm’s law to find the voltage:

$$V = IR = (0.0250\text{ A})(140\ \Omega) = 3.50\text{ V}$$

Discussion

A voltage of 3.50 V is typical for small indicator LEDs used in consumer electronics. Modern DVD players use low-power LED indicators that require only a few volts and milliamps to operate. The power consumed by this indicator light is $P = IV = (0.0250\text{ A})(3.50\text{ V}) = 0.0875\text{ W} = 87.5\text{ mW}$, which is quite small. This low power consumption is why indicator lights can remain on continuously without significantly affecting the device’s overall energy use or generating noticeable heat.

A voltage of 3.50 V is supplied to operate the DVD player’s indicator light.

(a) Find the voltage drop in an extension cord having a $0.0600\text{-}\Omega$ resistance and through which 5.00 A is flowing. (b) A cheaper cord utilizes thinner wire and has a resistance of 0.300Ω. What is the voltage drop in it when 5.00 A flows? (c) Why is the voltage to whatever appliance is being used reduced by this amount? What is the effect on the appliance?

Show Solution

Strategy

For parts (a) and (b), we use Ohm's law $V = IR$ to find the voltage drop across each extension cord. For part (c), we recognize that extension cords and appliances form a series circuit, so the voltage drops must add up to the source voltage (typically 120 V in the US).

Solution

(a) Using Ohm's law for the good extension cord:

$$V = IR = (5.00\ \text{A})(0.0600\ \Omega) = 0.300\ \text{V}$$

(b) For the cheaper extension cord with higher resistance:

$$V = IR = (5.00\ \text{A})(0.300\ \Omega) = 1.50\ \text{V}$$

(c) The extension cord and appliance are in series, so:

$$V_{\text{source}} = V_{\text{cord}} + V_{\text{appliance}}$$

The voltage available to the appliance is:

$$V_{\text{appliance}} = V_{\text{source}} - V_{\text{cord}}$$

For a 120 V source:

- Good cord: $V_{\text{appliance}} = 120 - 0.3 = 119.7\ \text{V}$ (99.8% of source)
- Cheap cord: $V_{\text{appliance}} = 120 - 1.5 = 118.5\ \text{V}$ (98.8% of source)

The appliance power is $P = V^2/R$ (for constant appliance resistance), so:

- Cheap cord reduces available power by approximately $(118.5119.7)^2 = 0.980$ or 2% less power

Discussion

Part (a): The voltage drop of 0.3 V represents only 0.25% of the 120 V source—negligible for most applications. This is the hallmark of a good extension cord: low resistance that doesn't significantly affect appliance performance.

Part (b): The cheaper cord's voltage drop of 1.5 V is 5 times larger, representing 1.25% of the source voltage. While this still seems small, it has several practical implications:

1. **Power reduction**: Appliances receive less voltage and therefore less power. For a 600 W heater, this would reduce power by about 12 W—not catastrophic but noticeable.

2. **Dimming lights**: A light bulb on this cord would visibly dim. The 2% power reduction translates to noticeable brightness decrease.

3. **Motor performance**: Electric motors are particularly sensitive to voltage drops. A power tool might run slower and with less torque, potentially stalling under load.

4. **Heat generation**: The 1.5 V drop × 5 A = 7.5 W of power is dissipated as heat in the cord. Over time, this can make the cord warm to hot, potentially creating a fire hazard with prolonged use.

5. **Voltage-sensitive electronics**: Some devices (like computers) have voltage regulators and will compensate, but their efficiency decreases. Other devices might malfunction.

The effect is amplified at higher currents. At 10 A, the cheap cord would drop 3 V, and at 15 A (the typical circuit breaker limit), it would drop 4.5 V—nearly 4% of the source voltage, causing serious performance degradation and dangerous heating.

This is why electrical codes specify maximum wire resistance for extension cords based on their length and amperage rating. "Bargain" extension cords often use undersized wire that creates safety hazards and damages appliances through undervoltage operation.

Answers: (a) 0.300 V, (b) 1.50 V, (c) The voltage available to the appliance is reduced, decreasing its power output and potentially causing malfunction or safety hazards.

A power transmission line is hung from metal towers with glass insulators having a resistance of $1.00 \times 10^9 \Omega$. What current flows through the insulator if the voltage is 200 kV? (Some high-voltage lines are DC.)

Show Solution

Strategy

We apply Ohm's law $I = VR$ to find the leakage current through the insulator. The voltage must be converted from kilovolts to volts.

Solution

Convert voltage to SI units:

$V = 200\ \text{kV} = 200 \times 10^3\ \text{V} = 2.00 \times 10^5\ \text{V}$

Apply Ohm’s law:

$I = V R = 2.00 \times 10^5\ \text{V} 1.00 \times 10^9\ \Omega = 2.00 \times 10^{-4}\ \text{A} = 0.200\ \text{mA}$

Discussion

This tiny current of 0.200 mA represents the leakage current through a single insulator. While seemingly negligible, this leakage has important implications. First, the power lost through this insulator is $P = IV = (2.00 \times 10^{-4}\ \text{A})(2.00 \times 10^5\ \text{V}) = 40\ \text{W}$. With thousands of insulators on a transmission line, the total leakage power loss becomes significant. Second, insulators must maintain their high resistance even under adverse conditions like rain, pollution, or ice buildup, which can dramatically lower resistance and increase leakage current. This is why insulators have their characteristic ribbed design—to increase the surface path length and reduce current flow even when wet. The gigaohm resistance ensures that virtually all the transmitted power reaches consumers rather than being lost through the towers.

A current of 0.200 mA (or 2.00×10^{-4} A) flows through the insulator.

Glossary

Ohm’s law
: an empirical relation stating that the current *I* is proportional to the potential difference *V*, ∝ *V*; it is often written as *I* = *V/R*, where *R* is the resistance

resistance
: the electric property that impedes current; for ohmic materials, it is the ratio of voltage to current, *R* = *V/I*

ohm
: the unit of resistance, given by 1Ω = 1 V/A

ohmic
: a type of a material for which Ohm's law is valid

simple circuit
: a circuit with a single voltage source and a single resistor

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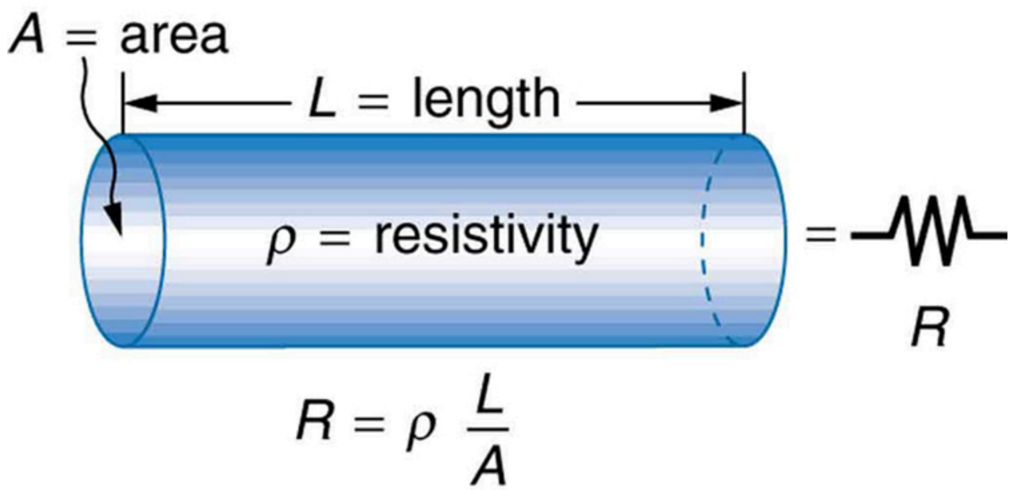
Resistance and Resistivity

- Explain the concept of resistivity.
- Use resistivity to calculate the resistance of specified configurations of material.
- Use the thermal coefficient of resistivity to calculate the change of resistance with temperature.

Material and Shape Dependence of Resistance

The resistance of an object depends on its shape and the material of which it is composed. The cylindrical resistor in [Figure 1] is easy to analyze, and, by so doing, we can gain insight into the resistance of more complicated shapes. As you might expect, the cylinder's electric resistance R is directly proportional to its length L , similar to the resistance of a pipe to fluid flow. The longer the cylinder, the more collisions charges will make with its atoms. The greater the diameter of the cylinder, the more current it can carry (again similar to the flow of fluid through a pipe). In fact, R is inversely proportional to the cylinder's cross-sectional area A.



A uniform cylinder of length L and cross-sectional area A . Its resistance to the flow of current is similar to the resistance posed by a pipe to fluid flow. The longer the cylinder, the greater its resistance. The larger its cross-sectional area A , the smaller its resistance.

For a given shape, the resistance depends on the material of which the object is composed. Different materials offer different resistance to the flow of charge. We define the **resistivity** ρ of a substance so that the **resistance** R of an object is directly proportional to ρ . Resistivity ρ is an *intrinsic* property of a material, independent of its shape or size. The resistance R of a uniform cylinder of length L , of cross-sectional area A , and made of a material with resistivity ρ, is

$$R=\rho LA.$$

[Table 1] gives representative values of ρ . The materials listed in the table are separated into categories of conductors, semiconductors, and insulators, based on broad groupings of resistivities. Conductors have the smallest resistivities, and insulators have the largest; semiconductors have intermediate resistivities. Conductors have varying but large free charge densities, whereas most charges in insulators are bound to atoms and are not free to move. Semiconductors are intermediate, having far fewer free charges than conductors, but having properties that make the number of free charges depend strongly on the type and amount of impurities in the semiconductor. These unique properties of semiconductors are put to use in modern electronics, as will be explored in later chapters.

Resistivities ρ of Various materials at 20°C

Material	Resistivity ρ ($\Omega\cdot$m)
Conductors	
Silver	1.59×10^{-8}
Copper	1.72×10^{-8}
Gold	2.44×10^{-8}
Aluminum	2.65×10^{-8}
Tungsten	5.6×10^{-8}
Iron	9.71×10^{-8}
Platinum	10.6×10^{-8}
Steel	20×10^{-8}

Material	Resistivity ρ ($\Omega \cdot$m)
Lead	22×10^{-8}
Manganin (Cu, Mn, Ni alloy)	44×10^{-8}
Constantan (Cu, Ni alloy)	49×10^{-8}
Mercury	96×10^{-8}
Nichrome (Ni, Fe, Cr alloy)	100×10^{-8}
Semiconductors[1]	
Carbon (pure)	3.5×10^{-5}
Carbon	$(3.5-60)\times10^{-5}$
Germanium (pure)	600×10^{-3}
Germanium	$(1-600)\times10^{-3}$
Silicon (pure)	2300
Silicon	0.1-2300
Insulators	
Amber	5×10^{14}
Glass	$10^{9}-10^{14}$
Lucite	$>10^{13}$
Mica	$10^{11}-10^{15}$
Quartz (fused)	75×10^{16}
Rubber (hard)	$10^{13}-10^{16}$
Sulfur	10^{15}
Teflon	$>10^{13}$
Wood	$10^{8}-10^{11}$

Calculating Resistor Diameter: A Headlight Filament

A car headlight filament is made of tungsten and has a cold resistance of 0.350Ω . If the filament is a cylinder 4.00 cm long (it may be coiled to save space), what is its diameter?

Strategy

We can rearrange the equation $R = \rho L A$ to find the cross-sectional area A of the filament from the given information. Then its diameter can be found by assuming it has a circular cross-section.

Solution

The cross-sectional area, found by rearranging the expression for the resistance of a cylinder given in $R = \rho L A$, is

$$A=\rho L R.$$

Substituting the given values, and taking ρ from [Table 1], yields

$$A = (5.6\times10^{-8}\Omega\cdot\text{m})(4.00\times10^{-2}\text{m})0.350\Omega = 6.40\times10^{-9}\text{m}^{2}.$$

The area of a circle is related to its diameter D by

$$A=\pi D^{2}4.$$

Solving for the diameter D , and substituting the value found for A, gives

$$D = 2(Ap)^{12}=2(6.40\times10^{-9}\text{m}^{2}3.14)^{12} = 9.0\times10^{-5}\text{m}.$$

Discussion

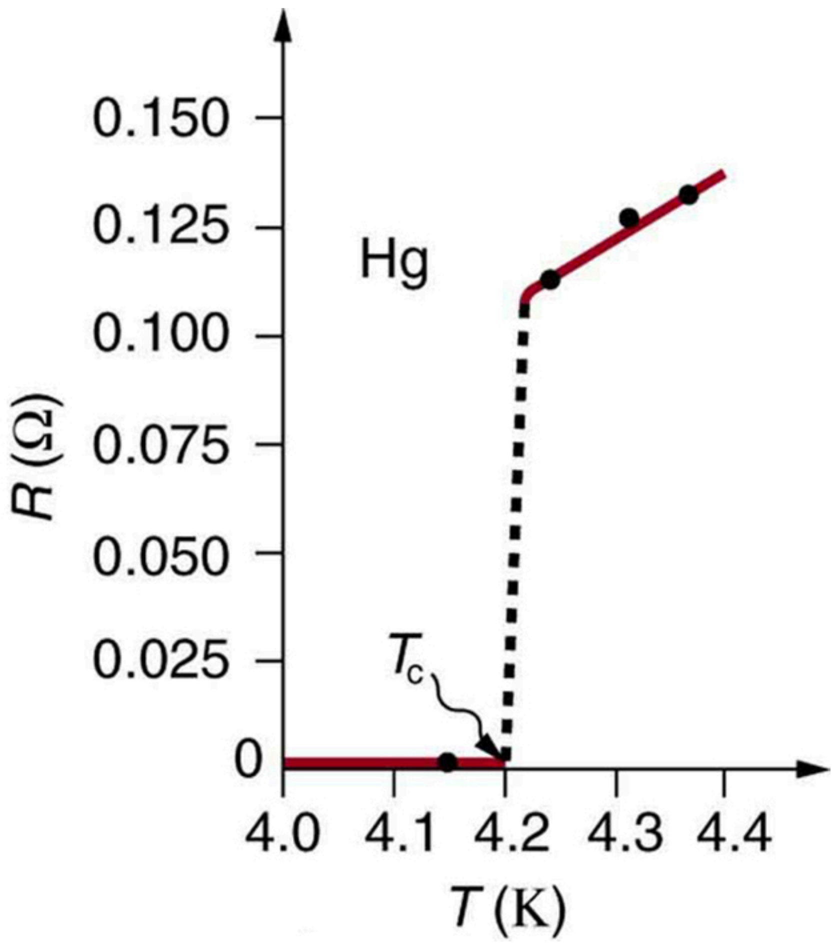
The diameter is just under a tenth of a millimeter. It is quoted to only two digits, because ρ is known to only two digits.

Temperature Variation of Resistance

The resistivity of all materials depends on temperature. Some even become superconductors (zero resistivity) at very low temperatures. (See [Figure 2].) Conversely, the resistivity of conductors increases with increasing temperature. Since the atoms vibrate more rapidly and over larger distances at higher temperatures, the electrons moving through a metal make more collisions, effectively making the resistivity higher. Over relatively small temperature changes (about $100^{\circ}\mathrm{C}$ or less), resistivity ρ varies with temperature change ΔT as expressed in the following equation

$$\rho=\rho_0(1+\alpha\Delta T),$$

where ρ_0 is the original resistivity and α is the **temperature coefficient of resistivity**. (See the values of α in [Table 2] below.) For larger temperature changes, α may vary or a nonlinear equation may be needed to find ρ . Note that α is positive for metals, meaning their resistivity increases with temperature. Some alloys have been developed specifically to have a small temperature dependence. Manganin (which is made of copper, manganese and nickel) , for example, has α ** close to zero (to three digits on the scale in [Table 2]), and so its resistivity varies only slightly with temperature. This is useful for making a temperature-independent resistance standard, for example.



The resistance of a sample of mercury is zero at very low temperatures—it is a superconductor up to about 4.2 K. Above that critical temperature, its resistance makes a sudden jump and then increases nearly linearly with temperature.

Temperature Coefficients of Resistivity α

Material	**Coefficient α (1/°C)**[2]
Conductors	
Silver	3.8×10^{-3}
Copper	3.9×10^{-3}
Gold	3.4×10^{-3}
Aluminum	3.9×10^{-3}
Tungsten	4.5×10^{-3}
Iron	5.0×10^{-3}
Platinum	3.93×10^{-3}

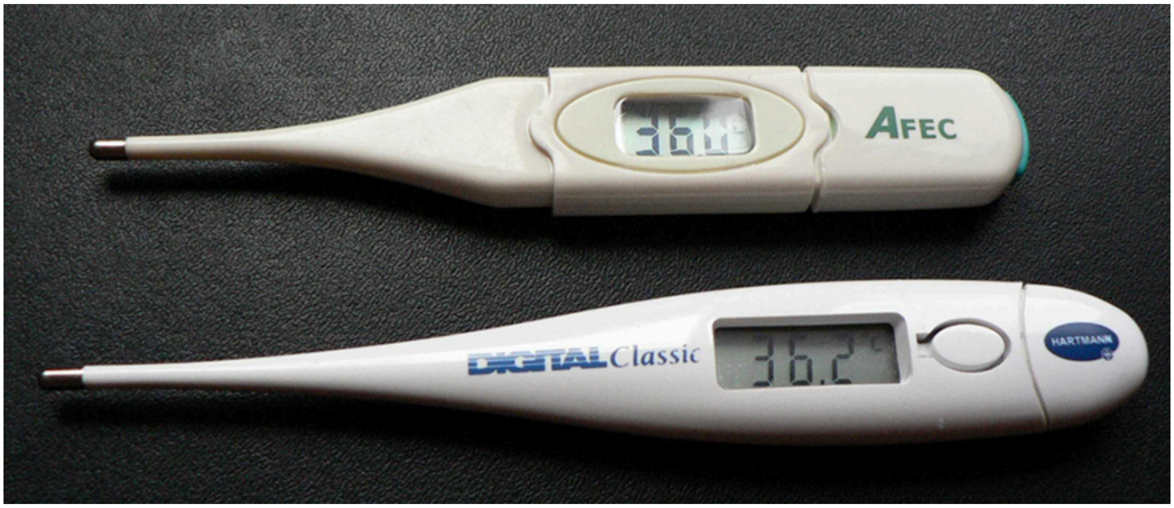
Material	Coefficient α (1/ºC)[2]
Lead	3.9×10^{-3}
Manganin (Cu, Mn, Ni alloy)	0.000×10^{-3}
Constantan (Cu, Ni alloy)	0.002×10^{-3}
Mercury	0.89×10^{-3}
Nichrome (Ni, Fe, Cr alloy)	0.4×10^{-3}
Semiconductors	
Carbon (pure)	-0.5×10^{-3}
Germanium (pure)	-50×10^{-3}
Silicon (pure)	-70×10^{-3}

Note also that α is negative for the semiconductors listed in [Table 2], meaning that their resistivity decreases with increasing temperature. They become better conductors at higher temperature, because increased thermal agitation increases the number of free charges available to carry current. This property of decreasing ρ with temperature is also related to the type and amount of impurities present in the semiconductors.

The resistance of an object also depends on temperature, since R_0 is directly proportional to ρ . For a cylinder we know $R = \rho L/A$, and so, if L and A do not change greatly with temperature, R will have the same temperature dependence as ρ . (Examination of the coefficients of linear expansion shows them to be about two orders of magnitude less than typical temperature coefficients of resistivity, and so the effect of temperature on L and A is about two orders of magnitude less than on ρ .) Thus,

$$R = R_0(1 + \alpha\Delta T)$$

is the temperature dependence of the resistance of an object, where R_0 is the original resistance and R is the resistance after a temperature change ΔT . Numerous thermometers are based on the effect of temperature on resistance. (See [Figure 3].) One of the most common is the thermistor, a semiconductor crystal with a strong temperature dependence, the resistance of which is measured to obtain its temperature. The device is small, so that it quickly comes into thermal equilibrium with the part of a person it touches.



These familiar thermometers are based on the automated measurement of a thermistor’s temperature-dependent resistance. (credit: Biol, Wikimedia Commons)

Calculating Resistance: Hot-Filament Resistance

Although caution must be used in applying $\rho = \rho_0(1 + \alpha\Delta T)$ and $R = R_0(1 + \alpha\Delta T)$ for temperature changes greater than 100°C , for tungsten the equations work reasonably well for very large temperature changes. What, then, is the resistance of the tungsten filament in the previous example if its temperature is increased from room temperature (20°C) to a typical operating temperature of 2850°C ?

Strategy

This is a straightforward application of $R = R_0(1 + \alpha\Delta T)$, since the original resistance of the filament was given to be $R\_0 = 0.350\Omega$, and the temperature change is $\Delta T = 2830^\circ\text{C}$.

Solution

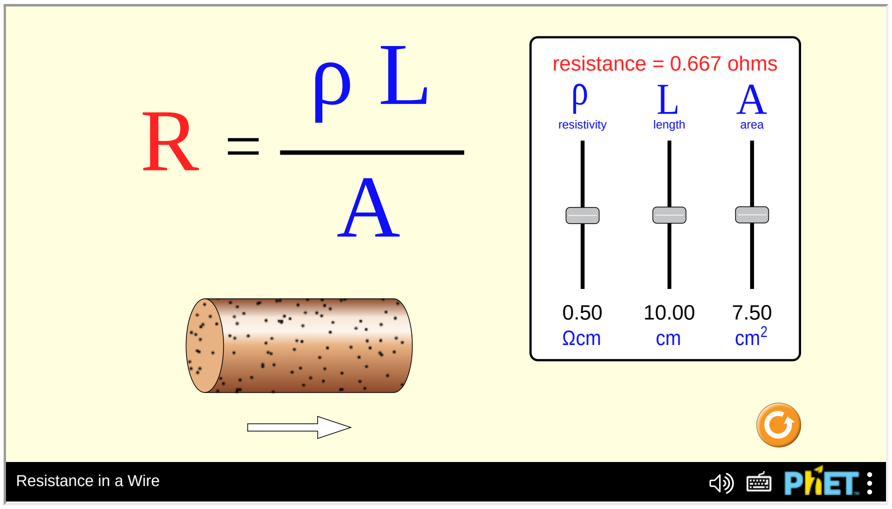
The hot resistance R is obtained by entering known values into the above equation:

$$R = R_0(1+\alpha\Delta T) = (0.350\Omega)[1+(4.5\times10^{-3}/^{\circ}\text{C})(2830^{\circ}\text{C})] = 4.8\Omega.$$

Discussion

This value is consistent with the headlight resistance example in Ohm’s Law: Resistance and Simple Circuits.
PhET Explorations: Resistance in a Wire

Learn about the physics of resistance in a wire. Change its resistivity, length, and area to see how they affect the wire's resistance. The sizes of the symbols in the equation change along with the diagram of a wire.



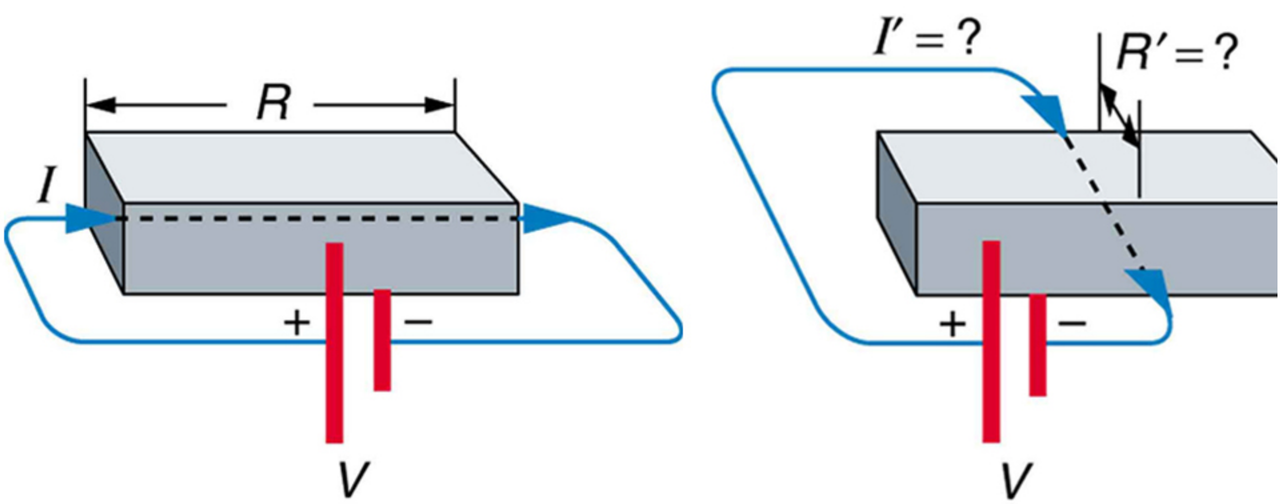
Section Summary

- The resistance R of a cylinder of length L and cross-sectional area A is $R = \rho\frac{L}{A}$, where ρ is the resistivity of the material.
- Values of ρ in [Table 1] show that materials fall into three groups—*conductors, semiconductors, and insulators*.
- Temperature affects resistivity; for relatively small temperature changes ΔT, resistivity is $\rho = \rho_0(1+\alpha\Delta T)$, where ρ_0 is the original resistivity and α is the temperature coefficient of resistivity.
- [Table 2] gives values for α, the temperature coefficient of resistivity.
- The resistance R of an object also varies with temperature: $R = R_0(1+\alpha\Delta T)$, where R_0 is the original resistance, and R is the resistance after the temperature change.

Conceptual Questions

In which of the three semiconducting materials listed in [Table 1] do impurities supply free charges? (Hint: Examine the range of resistivity for each and determine whether the pure semiconductor has the higher or lower conductivity.)

Does the resistance of an object depend on the path current takes through it? Consider, for example, a rectangular bar—is its resistance the same along its length as across its width? (See [Figure 4].)



Does current taking two different paths through the same object encounter different resistance?

If aluminum and copper wires of the same length have the same resistance, which has the larger diameter? Why?

Explain why $R = R_0(1+\alpha\Delta T)$ for the temperature variation of the resistance R of an object is not as accurate as $\rho = \rho_0(1+\alpha\Delta T)$, which gives the temperature variation of resistivity ρ.

Problems & Exercises

What is the resistance of a 20.0-m-long piece of 12-gauge copper wire having a 2.053-mm diameter?

Show Solution

Strategy

Use the resistance formula $R = \rho L A$, where ρ is the resistivity of copper from [Table 1]. Calculate the cross-sectional area from the diameter.

Solution

Calculate the cross-sectional area:

$$A=\pi r^2=\pi(2.053\times10^{-3}\text{ m}2)^2=\pi(1.027\times10^{-3}\text{ m})^2=3.31\times10^{-6}\text{ m}^2$$

Apply the resistance formula using $\rho_{\text{copper}} = 1.72\times10^{-8}\ \Omega\cdot\text{m}$:

$$R=\rho L A=(1.72\times10^{-8}\ \Omega\cdot\text{m})(20.0\text{ m})3.31\times10^{-6}\text{ m}^2=0.104\ \Omega$$

Discussion

This resistance of about 0.1 Ω is typical for household wiring. 12-gauge wire is commonly used for 20-amp circuits in homes. The low resistance minimizes power loss and voltage drop across the wire, ensuring that appliances receive adequate voltage. For a 20-A current, this wire would dissipate $P = I^2R = (20)^2(0.104) = 41.6$ W over its 20-meter length, which is acceptable.

The resistance of the 12-gauge copper wire is 0.104 Ω.

The diameter of 0-gauge copper wire is 8.252 mm. Find the resistance of a 1.00-km length of such wire used for power transmission.

Show Solution

Strategy

We use the resistance formula $R = \rho L A$, where ρ is the resistivity of copper from [Table 1]. We need to calculate the cross-sectional area from the given diameter and convert all units to SI.

Solution

Calculate the cross-sectional area:

$$A=\pi r^2=\pi(8.252\times10^{-3}\text{ m}2)^2=\pi(4.126\times10^{-3}\text{ m})^2=5.350\times10^{-5}\text{ m}^2$$

Apply the resistance formula using $\rho_{\text{copper}} = 1.72\times10^{-8}\ \Omega\cdot\text{m}$:

$$R=\rho L A=(1.72\times10^{-8}\ \Omega\cdot\text{m})(1.00\times10^{3}\text{ m})5.350\times10^{-5}\text{ m}^2=0.322\ \Omega$$

Discussion

This relatively low resistance of about a third of an ohm for a full kilometer of wire is why copper is preferred for power transmission. The thick 0-gauge wire (diameter about 8 mm) is used for high-current applications. Even this small resistance causes power loss in long transmission lines; for a 1000 A current, the power dissipated in this wire would be $P = I^2R = (1000)^2(0.322) = 322$ kW. This is why high-voltage, lower-current transmission is preferred for long distances.

The resistance of a 1.00-km length of 0-gauge copper wire is 0.322 Ω.

If the 0.100-mm diameter tungsten filament in a light bulb is to have a resistance of 0.200Ω at 20.0°C, how long should it be?

Show Solution

Strategy

Rearrange the resistance formula $R = \rho L A$ to solve for length: $L = R A \rho$. Use the resistivity of tungsten from [Table 1].

Solution

Calculate the cross-sectional area:

$$A=\pi r^2=\pi\left(\frac{0.100\times10^{-3}\ \text{m}}{2}\right)^2=\pi(5.00\times10^{-5}\ \text{m})^2=7.85\times10^{-9}\ \text{m}^2$$

Solve for length using $\rho_{\text{tungsten}}=5.6\times10^{-8}\ \Omega\cdot\text{m}$:

$$L=\frac{RA}{\rho}=\frac{(0.200\ \Omega)(7.85\times10^{-9}\ \text{m}^2)}{5.6\times10^{-8}\ \Omega\cdot\text{m}}=2.81\ \text{m}$$

Discussion

A filament length of 2.81 m seems long for a small light bulb, but this wire is extremely thin (0.1 mm diameter). The filament is coiled tightly, often in a double-coil configuration (a coil of a coil), to fit within the small bulb envelope. This coiling also helps retain heat and increases efficiency. The 0.200 Ω cold resistance will increase significantly when the filament heats to its operating temperature of around 2500-3000°C.

The tungsten filament should be 2.81 m long.

Find the ratio of the diameter of aluminum to copper wire, if they have the same resistance per unit length (as they might in household wiring).

Show Solution

Strategy

For the same resistance per unit length, we set $\frac{R}{L}$ equal for both materials. Using $R=\frac{\rho L}{A}$ and $A=\frac{\pi D^2}{4}$ for a circular wire, we can derive a relationship between the diameters in terms of the resistivities.

Solution

The resistance per unit length is:

$$\frac{R}{L}=\frac{\rho}{A}=\frac{\rho}{\pi D^2/4}=\frac{4\rho}{\pi D^2}$$

Setting this equal for aluminum and copper:

$$\frac{4\rho_{\text{Al}}}{\pi D^2_{\text{Al}}}=\frac{4\rho_{\text{Cu}}}{\pi D^2_{\text{Cu}}}$$

Solving for the ratio of diameters:

$$\frac{D^2_{\text{Al}}}{D^2_{\text{Cu}}}=\frac{\rho_{\text{Al}}}{\rho_{\text{Cu}}}$$

$$\frac{D_{\text{Al}}}{D_{\text{Cu}}}=\sqrt{\frac{\rho_{\text{Al}}}{\rho_{\text{Cu}}}}=\sqrt{\frac{2.65\times10^{-8}\ \Omega\cdot\text{m}}{1.72\times10^{-8}\ \Omega\cdot\text{m}}}=\sqrt{1.541}=1.24$$

Discussion

Aluminum wire must have a diameter 1.24 times larger (about 24% larger) than copper wire to have the same resistance per unit length. This is because aluminum has higher resistivity than copper. Despite needing thicker wire, aluminum is still widely used in power transmission because it is lighter and cheaper than copper. The larger diameter is acceptable for overhead power lines where weight (not size) is the limiting factor. For household wiring, copper is usually preferred because its smaller diameter allows for easier installation in walls and conduits.

The aluminum wire must have a diameter 1.24 times that of the copper wire to achieve the same resistance per unit length.

What current flows through a 2.54-cm-diameter rod of pure silicon that is 20.0 cm long, when $1.00\times10^3\text{V}$ is applied to it? (Such a rod may be used to make nuclear-particle detectors, for example.)

Show Solution

Strategy

First calculate the resistance of the silicon rod using $R=\frac{\rho L}{A}$, then use Ohm's law $I=V/R$ to find the current. Pure silicon has a very high resistivity ($\rho=2300\ \Omega\cdot\text{m}$ from [Table 1]).

Solution

Calculate the cross-sectional area:

$$A=\pi r^2=\pi\left(\frac{2.54\times10^{-2}\ \text{m}}{2}\right)^2=\pi(1.27\times10^{-2}\ \text{m})^2=5.07\times10^{-4}\ \text{m}^2$$

Calculate the resistance:

$$R = \frac{\rho L}{A} = \frac{(2300\ \Omega\cdot\text{m})(0.200\ \text{m})}{5.07\times10^{-4}\ \text{m}^2} = 9.07\times10^{5}\ \Omega$$

Calculate the current:

$$I = \frac{V}{R} = \frac{1.00\times10^{3}\ \text{V}}{9.07\times10^{5}\ \Omega} = 1.10\times10^{-3}\ \text{A} = 1.10\ \text{mA}$$

Discussion

Despite the high voltage (1000 V), only about 1 mA flows through the silicon rod because pure silicon is a semiconductor with very high resistivity—about 100 billion times higher than copper. This property makes silicon ideal for radiation detectors, where incoming particles create electron-hole pairs that produce brief current pulses. The high resistance ensures low background current, making these small signals detectable.

A current of 1.10 mA flows through the silicon rod.

(a) To what temperature must you raise a copper wire, originally at 20.0°C, to double its resistance, neglecting any changes in dimensions? (b) Does this happen in household wiring under ordinary circumstances?

Show Solution

Strategy

We use the temperature dependence of resistance: $R = R_0(1 + \alpha\Delta T)$. For the resistance to double, we need $R = 2R_0$. We solve for ΔT and then find the final temperature.

Solution

(a) Set up the equation for doubling the resistance:

$$R = 2R_0 = R_0(1+\alpha\Delta T)$$
$$2 = 1+\alpha\Delta T$$
$$\Delta T = \frac{1}{\alpha} = \frac{1}{3.9\times10^{-3}\ /^{\circ}\text{C}} = 256\ ^{\circ}\text{C}$$

The final temperature is:

$$T = T_0 + \Delta T = 20.0\ ^{\circ}\text{C} + 256\ ^{\circ}\text{C} = 276\ ^{\circ}\text{C}$$

(b) No, this does not happen under ordinary circumstances. Household wiring should never reach anywhere near 276°C. Typical household wiring is rated for temperatures up to about 60-90°C. At 276°C, the wire insulation would have long since melted and likely caught fire. Copper's melting point is 1085°C, so the wire itself wouldn't melt, but the surrounding materials would be destroyed. If household wiring ever approached such temperatures, it would indicate a severe fault such as a dangerous overload or short circuit.

Discussion

This problem illustrates why overloaded circuits are fire hazards. A wire carrying too much current heats up due to $P = I^2R$ power dissipation. As it heats, its resistance increases, causing even more power dissipation and further heating—a positive feedback loop that can lead to thermal runaway. Circuit breakers and fuses are designed to interrupt the circuit before temperatures become dangerous.

(a) The copper wire must be raised to 276°C to double its resistance. (b) No, this temperature is never reached in properly functioning household wiring.

A resistor made of Nichrome wire is used in an application where its resistance cannot change more than 1.00% from its value at 20.0°C. Over what temperature range can it be used?

Show Solution

Strategy

Use $R = R_0(1 + \alpha\Delta T)$. For 1.00% change, we need $$ \alpha \Delta T \leq 0.01. $Solve for the maximum$\Delta T $using the temperature coefficient for Nichrome$(\alpha = 0.4 \times 10^{-3}$$ /ºC from [Table 2]).

Solution

Set up the equation for 1% change:

$$|\alpha\Delta T| = 0.01$$
$$|\Delta T| = \frac{0.01}{\alpha} = \frac{0.01}{0.4\times10^{-3}\ /^{\circ}\text{C}} = 25\ ^{\circ}\text{C}$$

The temperature can vary by ±25°C from the reference temperature of 20.0°C:

$$T_{\text{min}} = 20.0 - 25 = -5\ ^{\circ}\text{C}$$
$$T_{\text{max}} = 20.0 + 25 = 45\ ^{\circ}\text{C}$$

Discussion

The allowable temperature range of -5°C to 45°C is quite practical for most indoor applications. Nichrome was chosen for this resistor because of its relatively small temperature coefficient compared to pure metals. Copper, for example, with $\alpha = 3.9 \times 10^{-3}$ /ºC, would only allow a ±2.6°C variation for the same 1% tolerance—impractical for most applications.

The Nichrome resistor can be used over the temperature range of -5°C to 45°C.

Of what material is a resistor made if its resistance is 40.0% greater at 100°C than at 20.0°C ?

Show Solution

Strategy

A 40.0% increase in resistance means $R = 1.40R_0$. Using the temperature dependence equation $R = R_0(1 + \alpha\Delta T)$, we can solve for the temperature coefficient α and identify the material from [Table 2].

Solution

The temperature change is:

$$\Delta T = 100\ ^\circ\text{C} - 20.0\ ^\circ\text{C} = 80.0\ ^\circ\text{C}$$

Set up the equation with $R = 1.40R_0$:

$$1.40R_0 = R_0(1 + \alpha\Delta T)$$
$$1.40 = 1 + \alpha(80.0\ ^\circ\text{C})$$
$$\alpha = \frac{0.40}{80.0\ ^\circ\text{C}} = 5.0 \times 10^{-3}\ /^\circ\text{C}$$

Comparing with [Table 2], this value matches **iron**.

Discussion

Iron's temperature coefficient of 5.0×10^{-3} /ºC is among the highest for common conductors. This relatively large temperature dependence makes iron unsuitable for precision resistors but useful for applications like resistance temperature detectors (RTDs). The 40% increase in resistance over 80°C is significant—this is why materials like Manganin or Constantan (with α near zero) are used when stable resistance is needed across temperature ranges.

The resistor is made of iron, which has a temperature coefficient of resistivity of 5.0×10^{-3} /ºC.

An electronic device designed to operate at any temperature in the range from $-10.0^\circ\text{C} \text{to} 55.0^\circ\text{C}$ contains pure carbon resistors. By what factor does their resistance increase over this range?

Show Solution

Strategy

Use $R = R_0(1 + \alpha\Delta T)$ with carbon's negative temperature coefficient ($\alpha = -0.5 \times 10^{-3}$ /ºC from [Table 2]). Note that for carbon, resistance *decreases* with increasing temperature. We compare resistance at the two temperature extremes.

Solution

Using 20°C as the reference temperature, find the resistance at each extreme:

At -10°C ($\Delta T = -30$ °C):

$$R_{-10} = R_0[1 + (-0.5 \times 10^{-3})(-30)] = R_0(1 + 0.015) = 1.015R_0$$

At 55°C ($\Delta T = +35$ °C):

$$R_{55} = R_0[1 + (-0.5 \times 10^{-3})(35)] = R_0(1 - 0.0175) = 0.9825R_0$$

The factor by which resistance changes from lowest to highest temperature:

$$\text{Factor} = \frac{R_{-10}}{R_{55}} = \frac{1.015R_0}{0.9825R_0} = 1.03$$

Discussion

The resistance changes by only 3% over this 65°C temperature range. This is because carbon has a small (and negative) temperature coefficient. The resistance is highest at the cold extreme (-10°C) and lowest at the warm extreme (55°C). Carbon's negative coefficient means it behaves opposite to metals—its resistance decreases as temperature increases. This property, combined with carbon's small coefficient, makes carbon resistors useful in applications requiring moderate temperature stability.

The resistance increases by a factor of 1.03 (or about 3%) from the highest to lowest temperature in this range.

(a) Of what material is a wire made, if it is 25.0 m long with a 0.100 mm diameter and has a resistance of 77.7Ω at 20.0°C ? (b) What is its resistance at 150°C ?

Show Solution

Strategy

For part (a), we rearrange $R=\rho \frac{L}{A}$ to solve for resistivity ρ, then compare with [Table 1] to identify the material. For part (b), we use the temperature dependence $R=R_0(1+\alpha\Delta T)$ with the appropriate temperature coefficient.

Solution

(a) Calculate the cross-sectional area:

$$A=\pi r^2=\pi\left(\frac{0.100\times10^{-3}\text{ m}}{2}\right)^2=\pi(5.00\times10^{-5}\text{ m})^2=7.854\times10^{-9}\text{ m}^2$$

Solve for resistivity:

$$\rho=\frac{RA}{L}=\frac{(77.7\ \Omega)(7.854\times10^{-9}\text{ m}^2)}{25.0\text{ m}}=2.44\times10^{-8}\ \Omega\cdot\text{m}$$

Comparing with [Table 1], this resistivity matches **gold** ($\rho=2.44\times10^{-8}\ \Omega\cdot\text{m}$).

(b) Using the temperature coefficient for gold ($\alpha=3.4\times10^{-3}\ /^{\circ}\text{C}$ from [Table 2]):

$$\Delta T=150\ ^{\circ}\text{C}-20.0\ ^{\circ}\text{C}=130\ ^{\circ}\text{C}$$
$$R=R_0(1+\alpha\Delta T)=77.7\ \Omega[1+(3.4\times10^{-3}\ /^{\circ}\text{C})(130\ ^{\circ}\text{C})]$$
$$R=77.7\ \Omega(1+0.442)=77.7\ \Omega(1.442)=112\ \Omega$$

Discussion

The very thin wire (0.100 mm diameter) combined with a 25-meter length and relatively high resistance indicates a material with moderate resistivity. Gold is used for this type of wire in specialized applications requiring corrosion resistance and reliability, such as in aerospace and medical equipment. The 44% increase in resistance when heated to 150°C is significant and must be accounted for in precision circuits that operate over wide temperature ranges.

(a) The wire is made of gold. (b) At 150°C, its resistance is 112 Ω.

Assuming a constant temperature coefficient of resistivity, what is the maximum percent decrease in the resistance of a constantan wire starting at 20.0°C ?

Show Solution

Strategy

The maximum decrease occurs at the lowest achievable temperature, which is absolute zero (-273.15°C or 0 K). Use $R=R_0(1+\alpha\Delta T)$ with constantan's temperature coefficient ($\alpha=0.002\times10^{-3}\ /^{\circ}\text{C}$ from [Table 2]).

Solution

Calculate the temperature change to absolute zero:

$$\Delta T=-273.15\ ^{\circ}\text{C}-20.0\ ^{\circ}\text{C}=-293\ ^{\circ}\text{C}$$

Calculate the resistance at absolute zero:

$$R=R_0(1+\alpha\Delta T)=R_0[1+(0.002\times10^{-3})(-293)]$$
$$R=R_0(1-0.000586)=0.99941R_0$$

The percent decrease is:

$$\text{Percent decrease}=(1-0.99941)\times100\%=0.059\%\approx0.06\%$$

Discussion

Constantan's resistance decreases by a mere 0.06% even when cooled from room temperature all the way to absolute zero! This extraordinarily small temperature coefficient is why constantan (a copper-nickel alloy) is specifically designed and named for making precision resistors with "constant" resistance. The alloy was engineered to have near-zero temperature dependence, making it invaluable for precision measurement equipment, resistance standards, and strain gauges where temperature stability is critical.

The maximum percent decrease in resistance is 0.06%.

A wire is drawn through a die, stretching it to four times its original length. By what factor does its resistance increase?

Show Solution

Strategy

When a wire is drawn (stretched), its volume remains constant (assuming the material is incompressible). If the length increases by a factor of 4, the cross-sectional area must decrease proportionally. We use $R = \rho L A$ to find the new resistance.

Solution

Let the original length be L and original area be A. The original resistance is:

$$R_0 = \rho L A$$

The volume is conserved during stretching:

$$V = LA = L' A'$$

With the new length $L' = 4L$:

$$LA = (4L)A' \Longrightarrow A' = A4$$

The new resistance is:

$$R' = \rho L' A' = \rho (4L) A/4 = 16 \rho L A = 16 R_0$$

Discussion

The resistance increases by a factor of 16 when the wire is stretched to four times its original length. This dramatic increase occurs because resistance depends on both length and area—the length increases by a factor of 4 (increasing resistance) while the area decreases by a factor of 4 (also increasing resistance), giving a combined factor of $4 \times 4 = 16$. In general, stretching a wire by a factor of n increases its resistance by a factor of n^2. This principle is important in wire manufacturing and explains why thin wires have much higher resistance than thick ones of the same material and length.

The resistance increases by a factor of 16.

A copper wire has a resistance of 0.500Ω at 20.0°C, and an iron wire has a resistance of 0.525Ω at the same temperature. At what temperature are their resistances equal?

Show Solution

Strategy

Use $R = R_0(1 + \alpha \Delta T)$ for each metal and set them equal. Use temperature coefficients from [Table 2]: $\alpha_{\text{Cu}} = 3.9 \times 10^{-3}$ /ºC and $\alpha_{\text{Fe}} = 5.0 \times 10^{-3}$ /ºC.

Solution

Set the resistances equal:

$$R_{\text{Cu}} = R_{\text{Fe}}$$
$$R_{0,\text{Cu}}(1 + \alpha_{\text{Cu}} \Delta T) = R_{0,\text{Fe}}(1 + \alpha_{\text{Fe}} \Delta T)$$
$$0.500(1 + 3.9 \times 10^{-3} \Delta T) = 0.525(1 + 5.0 \times 10^{-3} \Delta T)$$

Expand and solve:

$$0.500 + 1.95 \times 10^{-3} \Delta T = 0.525 + 2.625 \times 10^{-3} \Delta T$$
$$0.500 - 0.525 = (2.625 - 1.95) \times 10^{-3} \Delta T$$
$$-0.025 = 0.675 \times 10^{-3} \Delta T$$
$$\Delta T = -0.0250.675 \times 10^{-3} = -37\ ^\circ\text{C}$$

The temperature at which resistances are equal:

$$T = 20.0 + (-37) = -17\ ^\circ\text{C}$$

Discussion

At -17°C, both wires have equal resistance. This occurs below room temperature because iron has a higher temperature coefficient than copper. Since iron's resistance changes faster with temperature, the resistances converge at a lower temperature where the initially higher iron resistance has decreased more than the copper. This problem illustrates that different materials can have equal resistance at a specific temperature even if they differ at other temperatures.

The resistances are equal at -17°C.

(a) Digital medical thermometers determine temperature by measuring the resistance of a semiconductor device called a thermistor (which has $\alpha = -0.0600/^\circ\text{C}$) when it is at the same temperature as the patient. What is a patient's temperature if the thermistor's resistance at that temperature is 82.0% of its value at 37.0°C (normal body temperature)? (b) The negative value for α may not be maintained for very low temperatures. Discuss why and whether this is the case here. (Hint: Resistance can't become negative.)

Show Solution

Strategy

For part (a), we use $R = R_0(1 + \alpha\Delta T)$ where R_0 is the resistance at 37.0°C (the reference temperature) and $R = 0.820R_0$. Note that $\Delta T = T - 37.0$ where T is the unknown patient temperature. For part (b), we analyze when the linear model breaks down.

Solution

(a) Set up the equation with $R = 0.820R_0$:

$$0.820R_0 = R_0(1 + \alpha\Delta T)$$
$$0.820 = 1 + (-0.0600\ /^\circ\text{C})(\Delta T)$$
$$-0.180 = -0.0600\ /^\circ\text{C} \times \Delta T$$
$$\Delta T = -0.180 - 0.0600\ /^\circ\text{C} = 3.00\ ^\circ\text{C}$$

The patient's temperature is:

$$T = 37.0\ ^\circ\text{C} + 3.00\ ^\circ\text{C} = 40.0\ ^\circ\text{C}$$

(b) The negative temperature coefficient means resistance decreases as temperature increases. However, the linear model $R = R_0(1 + \alpha\Delta T)$ would predict zero resistance when $1 + \alpha\Delta T = 0$, which occurs at:

$$\Delta T = -1\alpha = -1 - 0.0600 = 16.7\ ^\circ\text{C}$$

This corresponds to $T = 37.0 + 16.7 = 53.7$ °C above the reference, meaning at about 53.7°C the model would predict zero resistance, and below that temperature (going to even higher temperatures) negative resistance—which is physically impossible. However, in part (a), we found the patient is at 40.0°C, which is only 3°C above 37°C—well within the valid range of the linear model. For the expected range of human body temperatures (roughly 35-42°C), the linear approximation is acceptable.

Discussion

A temperature of 40.0°C (104°F) indicates a significant fever and would warrant medical attention. The thermistor's decreasing resistance with increasing temperature makes sense for semiconductors, where thermal energy frees more charge carriers. The 18% decrease in resistance for a 3°C temperature rise shows the high sensitivity of thermistors, which is why they are excellent for precise temperature measurements. Medical thermometers typically operate within a narrow temperature range where the linear approximation remains valid.

(a) The patient's temperature is 40.0°C (indicating a fever). (b) The linear model remains valid for this temperature since it is only 3°C above the reference temperature, well within the practical operating range.

Integrated Concepts

(a) Redo [Exercise 2] taking into account the thermal expansion of the tungsten filament. You may assume a thermal expansion coefficient of $12 \times 10^{-6}/^\circ\text{C}$. (b) By what percentage does your answer differ from that in the example?

Show Solution

Strategy

Thermal expansion affects resistance in two ways: (1) the length increases by factor $(1 + \alpha\Delta T)$, and (2) the cross-sectional area decreases because the diameter increases by the same factor, making $A' = A(1+\alpha\Delta T)^{-2}$. The resistance formula becomes $R' = \rho' L' A'$, where both the geometry and resistivity change with temperature. For this problem, we consider thermal expansion effects on the tungsten wire dimensions.

Solution

(a) When a wire undergoes thermal expansion with coefficient $\alpha = 12 \times 10^{-6}$ /ºC:

The new length is:

$$L' = L(1+\alpha\Delta T)$$

The new diameter is:

$$D' = D(1+\alpha\Delta T)$$

The new area is:

$$A' = \pi(D'/2)^2 = A(1+\alpha\Delta T)^2$$

For resistance (assuming constant resistivity for this calculation):

$$R' = \rho L' A' = \rho L(1+\alpha\Delta T) A(1+\alpha\Delta T)^2 = \rho L A \cdot 1(1+\alpha\Delta T)$$
$$R' = R(1+\alpha\Delta T)$$

For a typical operating temperature increase in a tungsten filament ($\Delta T \approx 2500$ ºC):

$$1+\alpha\Delta T = 1+(12\times10^{-6})(2500) = 1+0.030 = 1.030$$

If the room temperature resistance was 4.84 Ω (hypothetical reference value):

$$R' = 4.841.030 = 4.70\ \Omega \approx 4.7\ \Omega$$

(b) The percentage difference is:

$$4.84-4.704.84\times100\% = 0.144.84\times100\% = 2.9\% \approx 3.0\%$$

Discussion

Part (a): Thermal expansion causes the wire to become longer and thinner, with competing effects on resistance. The length increase tends to increase resistance ($R \propto L$), while the area increase tends to decrease it ($R \propto 1/A$). However, area depends on diameter squared, so $A \propto D^2 \propto (1+\alpha\Delta T)^2$, which means the area effect dominates, leading to a net decrease in resistance due to thermal expansion alone.

This 3% decrease might seem counterintuitive since we know tungsten filaments have much higher resistance when hot. However, this problem isolates only the geometric thermal expansion effect. In reality, the resistivity of tungsten increases dramatically with temperature (by a factor of 15-20), which completely overwhelms this small geometric decrease, resulting in the observed high hot resistance of filaments.

Part (b): The 3.0% decrease is relatively small, confirming that thermal expansion has a modest effect on resistance compared to the temperature coefficient of resistivity. For precision applications, both effects must be considered, but in most cases, the resistivity change dominates.

Answer: (a) 4.7 Ω, (b) 3.0% decrease due to thermal expansion

Unreasonable Results

(a) To what temperature must you raise a resistor made of constantan to double its resistance, assuming a constant temperature coefficient of resistivity? (b) To cut it in half? (c) What is unreasonable about these results? (d) Which assumptions are unreasonable, or which premises are inconsistent?

Show Solution

Strategy

We apply $R = R_0(1+\alpha\Delta T)$ with the temperature coefficient for constantan ($\alpha = 0.002 \times 10^{-3}$ /ºC from [Table 2]). For doubling, $R = 2R_0$; for halving, $R = 0.5R_0$.

Solution

(a) To double the resistance ($R = 2R_0$):

$$2 = 1+\alpha\Delta T$$
$$\Delta T = 1\alpha = 10.002\times10^{-3}\ /ºC = 12\times10^{-6}\ /ºC = 5.0\times10^{5}\ ºC$$

Starting from 20°C: $T = 20 + 500\,000 = 500\,020$ ºC

(b) To halve the resistance ($R = 0.5R_0$):

$$0.5 = 1 + \alpha\Delta T$$

$$\Delta T = -0.5\alpha = -0.52\times10^{-6}\ /^{\circ}\text{C} = -2.5\times10^{5}\ ^{\circ}\text{C}$$

Starting from 20°C: $T = 20 - 250\ 000 = -249\ 980\ ^{\circ}\text{C}$

(c) Both results are unreasonable:

- To double the resistance would require heating to 500,020°C, which is approximately 90 times hotter than the surface of the Sun (about 5,500°C). Constantan would vaporize long before reaching such temperatures—its melting point is around 1,220°C.
- To halve the resistance would require cooling to -249,980°C, which is about 250,000 degrees below absolute zero (-273.15°C). This is physically impossible since absolute zero is the lowest possible temperature.

(d) The unreasonable assumption is that the temperature coefficient of resistivity α remains constant over such extreme temperature ranges. The linear model $R = R_0(1 + \alpha\Delta T)$ is only valid for relatively small temperature changes (typically less than 100°C). For large temperature changes, the relationship between resistance and temperature becomes nonlinear, α itself changes with temperature, and at extreme temperatures, phase changes occur (melting, vaporization). Additionally, the premise of trying to significantly change constantan's resistance through temperature is fundamentally misguided—constantan is specifically designed and named for its nearly constant resistance over a wide temperature range.

Discussion

This problem illustrates why constantan and similar alloys (like Manganin) are valuable for precision resistors. Their extremely small temperature coefficients make their resistance nearly independent of temperature within normal operating ranges. Trying to double or halve their resistance through temperature change alone is essentially impossible under any realistic conditions.

(a) 500,020°C; (b) -249,980°C; (c) These temperatures are impossibly high and below absolute zero, respectively; (d) The assumption of constant α over extreme temperature ranges is invalid, and the linear model breaks down completely outside normal temperature ranges.

Footnotes

- 1 Values depend strongly on amounts and types of impurities
- 2 Values at 20ºC. { data-list-type="bulleted" data-bullet-style="none"}

Glossary

resistivity
: an intrinsic property of a material, independent of its shape or size, directly proportional to the resistance, denoted by ρ

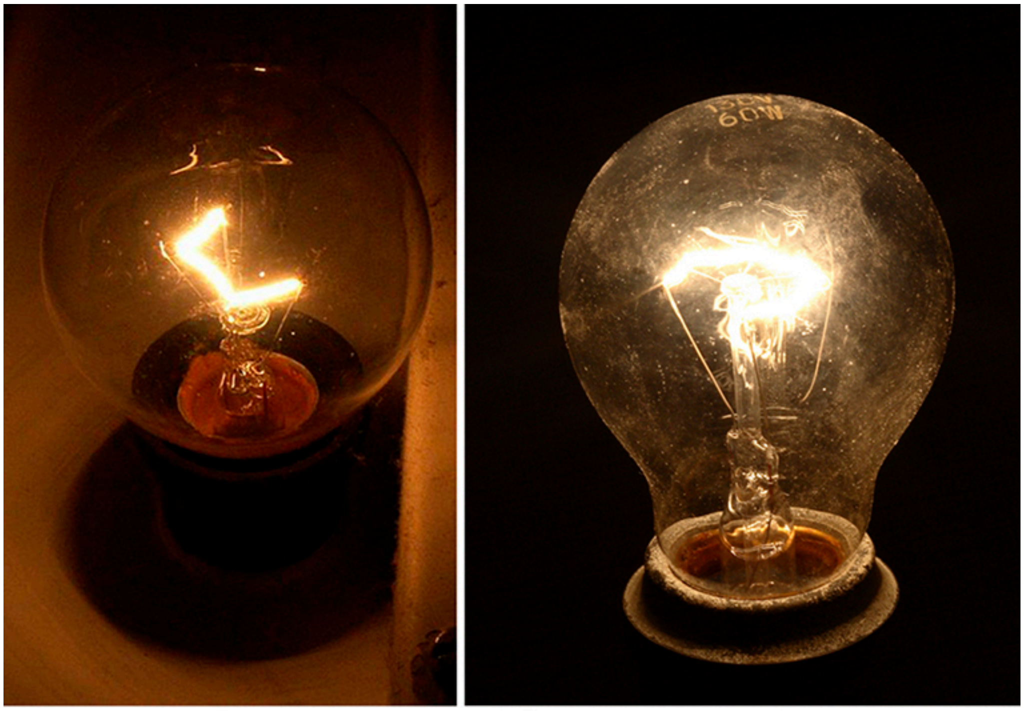
temperature coefficient of resistivity
: an empirical quantity, denoted by α, which describes the change in resistance or resistivity of a material with temperature

Electric Power and Energy

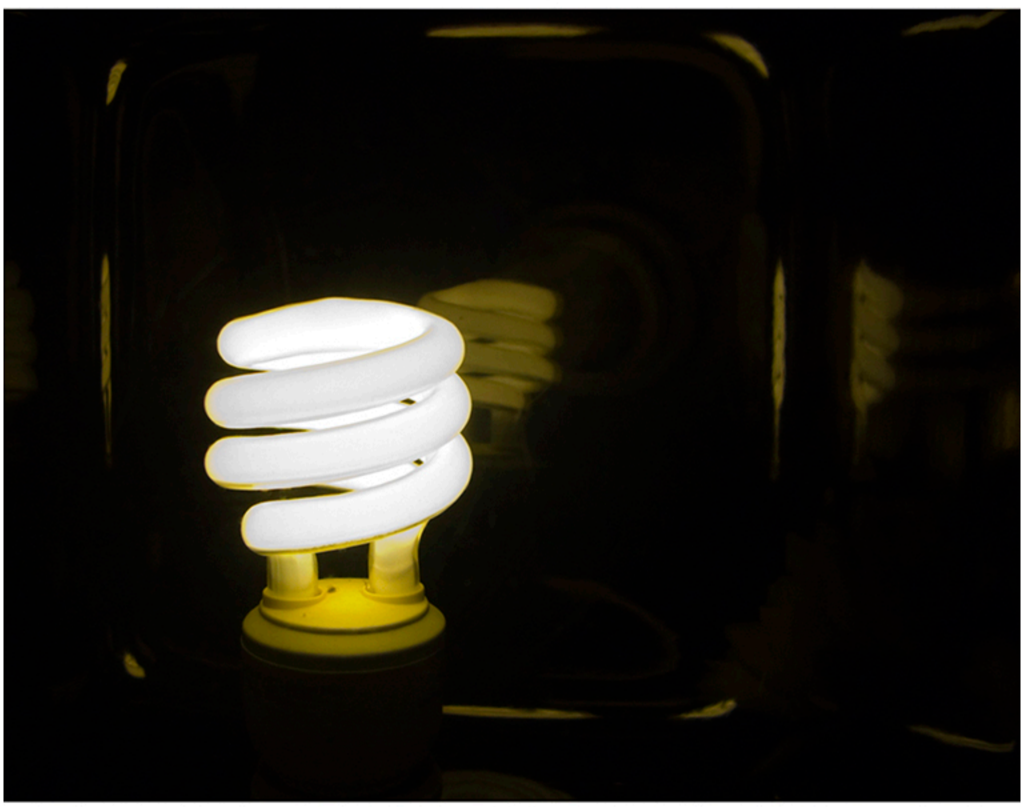
- Calculate the power dissipated by a resistor and power supplied by a power supply.
- Calculate the cost of electricity under various circumstances.

Power in Electric Circuits

Power is associated by many people with electricity. Knowing that power is the rate of energy use or energy conversion, what is the expression for **electric power**? Power transmission lines might come to mind. We also think of lightbulbs in terms of their power ratings in watts. Let us compare a 25-W bulb with a 60-W bulb. (See [Figure 1](a).) Since both operate on the same voltage, the 60-W bulb must draw more current to have a greater power rating. Thus the 60-W bulb's resistance must be lower than that of a 25-W bulb. If we increase voltage, we also increase power. For example, when a 25-W bulb that is designed to operate on 120 V is connected to 240 V, it briefly glows very brightly and then burns out. Precisely how are voltage, current, and resistance related to electric power?



(a)



(b)

(a) Which of these lightbulbs, the 25-W bulb (upper left) or the 60-W bulb (upper right), has the higher resistance? Which draws more current? Which uses the most energy? Can you tell from the color that the 25-W filament is cooler? Is the brighter bulb a different color and if so why? (credits: Dickbauch, Wikimedia Commons; Greg Westfall, Flickr) (b) This compact fluorescent light (CFL) puts out the same intensity of light as the 60-W bulb, but at 1/4 to 1/10 the input power. (credit: dbgg1979, Flickr)

Electric energy depends on both the voltage involved and the charge moved. This is expressed most simply as $\text{PE} = qV$, where q is the charge moved and V is the voltage (or more precisely, the potential difference the charge moves through). Power is the rate at which energy is moved, and so electric power is

$$P = \frac{\text{PE}}{t} = \frac{qV}{t}.$$

Recognizing that current is $I = q/t$ (note that $\Delta t = t$ here), the expression for power becomes

$$P = IV.$$

Electric power (P) is simply the product of current times voltage. Power has familiar units of watts. Since the SI unit for potential energy (PE) is the joule, power has units of joules per second, or watts. Thus, $1\text{A} \cdot \text{V} = 1\text{W}$. For example, cars often have one or more auxiliary power outlets with which you can charge a cell phone or other electronic devices. These outlets may be rated at 20 A, so that the circuit can deliver a maximum power $P = IV = (20\text{A})(12\text{V}) = 240\text{W}$. In some applications, electric power may be expressed as volt-amperes or even kilovolt-amperes ($1\text{kA}\cdot\text{V} = 1\text{kW}$

).

To see the relationship of power to resistance, we combine Ohm's law with $P = IV$. Substituting $I = V/R$ gives $P = (V/R)V = V^2/R$. Similarly, substituting $V = IR$ gives $P = I(IR) = I^2R$. Three expressions for electric power are listed together here for convenience:

$$P = IV$$
$$P = \frac{V^2}{R}$$
$$P = I^2R.$$

Note that the first equation is always valid, whereas the other two can be used only for resistors. In a simple circuit, with one voltage source and a single resistor, the power supplied by the voltage source and that dissipated by the resistor are identical. (In more complicated circuits, P can be the power dissipated by a single device and not the total power in the circuit.)

Different insights can be gained from the three different expressions for electric power. For example, $P = V^2/R$ implies that the lower the resistance connected to a given voltage source, the greater the power delivered. Furthermore, since voltage is squared in $P = V^2/R$, the effect of applying a higher voltage is perhaps greater than expected. Thus, when the voltage is doubled to a 25-W bulb, its power nearly quadruples to about 100 W, burning it out. If the bulb's resistance remained constant, its power would be exactly 100 W, but at the higher temperature its resistance is higher, too.

Calculating Power Dissipation and Current: Hot and Cold Power

(a) Consider the examples given in Ohm's Law: Resistance and Simple Circuits and Resistance and Resistivity. Then find the power dissipated by the car headlight in these examples, both when it is hot and when it is cold. (b) What current does it draw when cold?

Strategy for (a)

For the hot headlight, we know voltage and current, so we can use $P = IV$ to find the power. For the cold headlight, we know the voltage and resistance, so we can use $P = V^2/R$ to find the power.

Solution for (a)

Entering the known values of current and voltage for the hot headlight, we obtain

$$P = IV = (2.50\text{A})(12.0\text{V}) = 30.0\text{W}.$$

The cold resistance was 0.350Ω , and so the power it uses when first switched on is

$$P = \frac{V^2}{R} = \frac{(12.0\text{V})^2}{0.350\Omega} = 411\text{W}.$$

Discussion for (a)

The 30 W dissipated by the hot headlight is typical. But the 411 W when cold is surprisingly higher. The initial power quickly decreases as the bulb's temperature increases and its resistance increases.

Strategy and Solution for (b)

The current when the bulb is cold can be found several different ways. We rearrange one of the power equations, $P = I^2R$, and enter known values, obtaining

$$I = \sqrt{\frac{P}{R}} = \sqrt{\frac{411\text{W}}{0.350\Omega}} = 34.3\text{A}.$$

Discussion for (b)

The cold current is remarkably higher than the steady-state value of 2.50 A, but the current will quickly decline to that value as the bulb's temperature increases. Most fuses and circuit breakers (used to limit the current in a circuit) are designed to tolerate very high currents briefly as a device comes on. In some cases, such as with electric motors, the current remains high for several seconds, necessitating special "slow blow" fuses.

The Cost of Electricity

The more electric appliances you use and the longer they are left on, the higher your electric bill. This familiar fact is based on the relationship between energy and power. You pay for the energy used. Since $P = E/t$, we see that

$$E = Pt$$

is the energy used by a device using power P for a time interval t . For example, the more lightbulbs burning, the greater P used; the longer they are on, the greater t is. The energy unit on electric bills is the kilowatt-hour ($\text{kW}\cdot\text{h}$), consistent with the relationship $E = Pt$. It is easy to estimate the cost of operating electric appliances if you have some idea of their power consumption rate in watts or kilowatts, the time they are on in hours, and the cost per kilowatt-hour for your electric utility. Kilowatt-hours, like all other specialized energy units such as food calories, can be converted to joules. You can prove to yourself that $1\text{kW}\cdot\text{h} = 3.6 \times 10^6 \text{J}$.

The electrical energy (E) used can be reduced either by reducing the time of use or by reducing the power consumption of that appliance or fixture. This will not only reduce the cost, but it will also result in a reduced impact on the environment. Improvements to lighting are some of the fastest ways to reduce the electrical energy used in a home or business. About 20% of a home's use of energy goes to lighting, while the number for commercial establishments is closer to 40%. Fluorescent lights are about four times more efficient than incandescent lights—this is true for both the long tubes and the compact fluorescent lights (CFL). (See [Figure 1](b).) Thus, a 60-W incandescent bulb can be replaced by a 15-W CFL, which has the same brightness and color. CFLs have a bent tube inside a globe or a spiral-shaped tube, all connected to a standard screw-in base that fits standard incandescent light sockets. (Original problems with color, flicker, shape, and high initial investment for CFLs have been addressed in recent years.) The heat transfer from these CFLs is less, and they last up to 10 times longer. The significance of an investment in such bulbs is addressed in the next example. New white LED lights (which are clusters of small LED bulbs) are even more efficient (twice that of CFLs) and last 5 times longer than CFLs. However, their cost is still high.

Making Connections: Energy, Power, and Time

The relationship $E = Pt$ is one that you will find useful in many different contexts. The energy your body uses in exercise is related to the power level and duration of your activity, for example. The amount of heating by a power source is related to the power level and time it is applied. Even the radiation dose of an X-ray image is related to the power and time of exposure.

Calculating the Cost Effectiveness of Compact Fluorescent Lights (CFL)

If the cost of electricity in your area is 12 cents per kWh, what is the total cost (capital plus operation) of using a 60-W incandescent bulb for 1000 hours (the lifetime of that bulb) if the bulb cost 25 cents? (b) If we replace this bulb with a compact fluorescent light that provides the same light output, but at one-quarter the wattage, and which costs $1.50 but lasts 10 times longer (10 000 hours), what will that total cost be?

Strategy

To find the operating cost, we first find the energy used in kilowatt-hours and then multiply by the cost per kilowatt-hour.

Solution for (a)

The energy used in kilowatt-hours is found by entering the power and time into the expression for energy:

$$E = Pt = (60\text{W})(1000\text{h}) = 60000\text{W}\cdot\text{h}.$$

In kilowatt-hours, this is

$$E = 60.0\text{kW}\cdot\text{h}.$$

Now the electricity cost is

$$\text{cost} = (60.0\text{kW}\cdot\text{h})(\$0.12/\text{kW}\cdot\text{h}) = \$7.20.$$

The total cost will be $7.20 for 1000 hours (about one-half year at 5 hours per day).

Solution for (b)

Since the CFL uses only 15 W and not 60 W, the electricity cost will be $7.20/4 = 1.80$. The CFL will last 10 times longer than the incandescent, so that the investment cost will be 1/10 of the bulb cost for that time period of use, or $0.1(1.50) = 0.15$. Therefore, the total cost will be $1.95 for 1000 hours.

Discussion

Therefore, it is much cheaper to use the CFLs, even though the initial investment is higher. The increased cost of labor that a business must include for replacing the incandescent bulbs more often has not been figured in here.

Making Connections: Take-Home Experiment—Electrical Energy Use Inventory

1) Make a list of the power ratings on a range of appliances in your home or room. Explain why something like a toaster has a higher rating than a digital clock. Estimate the energy consumed by these appliances in an average day (by estimating their time of use). Some appliances might only state the

operating current. If the household voltage is 120 V, then use $P = IV$. 2) Check out the total wattage used in the restrooms of your school's floor or building. (You might need to assume the long fluorescent lights in use are rated at 32 W.) Suppose that the building was closed all weekend and that these lights were left on from 6 p.m. Friday until 8 a.m. Monday. What would this oversight cost? How about for an entire year of weekends?

Section Summary

_ Electric power P is the rate (in watts) that energy is supplied by a source or dissipated by a device. _ Three expressions for electrical power are

$$P = IV,$$
$$P = V^2 R,$$

and

$$P = I^2 R.$$

_ The energy used by a device with a power P over a time t is $E = Pt$.

Conceptual Questions

Why do incandescent lightbulbs grow dim late in their lives, particularly just before their filaments break?

The power dissipated in a resistor is given by $P = V^2/R$, which means power decreases if resistance increases. Yet this power is also given by $P = I^2 R$, which means power increases if resistance increases. Explain why there is no contradiction here.

Problem Exercises

What is the power of a $1.00 \times 10^2 \text{MV}$ lightning bolt having a current of $2.00 \times 10^4 \text{A}$?

Show Solution

Strategy

Use the basic power equation $P = IV$, converting the voltage to SI units (volts).

Solution

Convert voltage:

$$V = 1.00 \times 10^2 \text{ MV} = 1.00 \times 10^8 \text{ V}$$

Calculate power:

$$P = IV = (2.00 \times 10^4 \text{ A})(1.00 \times 10^8 \text{ V}) = 2.00 \times 10^{12} \text{ W} = 2.00 \text{ TW}$$

Discussion

This 2 terawatt power is enormous—roughly equal to the total average power consumption of the entire United States! However, a lightning bolt lasts only about 0.2 milliseconds, so the total energy delivered is $E = Pt = (2 \times 10^{12})(2 \times 10^{-4}) = 4 \times 10^8$ J, or about 110 kWh. Despite the impressive power, the brief duration means the energy is similar to running a 100-W light bulb for about 45 days.

The power of the lightning bolt is 2.00×10^{12} W (2.00 TW).

What power is supplied to the starter motor of a large truck that draws 250 A of current from a 24.0-V battery hookup?

Show Solution

Strategy

We use the basic power equation $P = IV$, where I is the current and V is the voltage supplied by the battery.

Solution

$$P = IV = (250 \text{ A})(24.0 \text{ V}) = 6000 \text{ W} = 6.00 \text{ kW}$$

Discussion

This 6.00 kW of power is substantial—equivalent to about 8 horsepower—which is necessary to overcome the mechanical resistance of a large truck engine during starting. The high current of 250 A requires thick battery cables to minimize resistance losses. Large trucks use 24-V systems (two 12-V batteries in series) rather than the 12-V systems in cars because, for the same power, a higher voltage requires less current, allowing for lighter-gauge cables.

The starter motor receives 6.00 kW of power from the battery.

A charge of 4.00 C of charge passes through a pocket calculator’s solar cells in 4.00 h. What is the power output, given the calculator’s voltage output is 3.00 V? (See [Figure 2].)

Show Solution

Strategy

First find the current using $I = \frac{\Delta Q}{\Delta t}$, then calculate power using $P = IV$.

Solution

Calculate the current:

$$I = \frac{\Delta Q}{\Delta t} = \frac{4.00\text{ C}}{4.00\text{ h} \times 3600\text{ s/h}} = \frac{4.00\text{ C}}{14400\text{ s}} = 2.78 \times 10^{-4}\text{ A}$$

Calculate the power output:

$$P = IV = (2.78 \times 10^{-4}\text{ A})(3.00\text{ V}) = 8.33 \times 10^{-4}\text{ W} = 0.833\text{ mW}$$

Discussion

This tiny power output of less than 1 milliwatt is sufficient for a pocket calculator because modern electronic calculators are extremely efficient. The small solar cell strip provides enough power to operate the LCD display and processor chip. This demonstrates how little energy is required for modern low-power electronics.

The power output of the calculator’s solar cells is 0.833 mW.

The strip of solar cells just above the keys of this calculator convert light to electricity to supply its energy needs. (credit: Evan-Amos, Wikimedia Commons)

How many watts does a flashlight that has 6.00×10^2C pass through it in 0.500 h use if its voltage is 3.00 V?

Show Solution

Strategy

First calculate the current from the charge and time using $I = \frac{\Delta Q}{\Delta t}$, then find power using $P = IV$.

Solution

Calculate the current:

$$I = \frac{\Delta Q}{\Delta t} = \frac{600\text{ C}}{0.500\text{ h} \times 3600\text{ s/h}} = \frac{600\text{ C}}{1800\text{ s}} = 0.333\text{ A}$$

Calculate the power:

$$P = IV = (0.333\text{ A})(3.00\text{ V}) = 1.00\text{ W}$$

Discussion

A 1-watt flashlight is consistent with a small, battery-powered LED flashlight. The 3.00 V suggests two AA or AAA batteries in series. Modern LED flashlights are much more efficient than older incandescent bulb flashlights, producing more light per watt consumed. This 1-watt power consumption is reasonable for portable lighting that needs to conserve battery life.

The flashlight uses 1.00 W of power.

Find the power dissipated in each of these extension cords: (a) an extension cord having a $0.0600\text{-}\Omega$ resistance and through which 5.00 A is flowing; (b) a cheaper cord utilizing thinner wire and with a resistance of 0.300Ω.

Show Solution

Strategy

Use the power equation $P = I^2R$ since we know current and resistance. This form shows that power dissipation increases with resistance for a given current.

Solution

(a) For the quality extension cord:

$$P = I^2R = (5.00\text{ A})^2(0.0600\ \Omega) = (25.0\text{ A}^2)(0.0600\ \Omega) = 1.50\text{ W}$$

(b) For the cheaper cord with higher resistance:

$$P = I^2R = (5.00\text{ A})^2(0.300\ \Omega) = (25.0\text{ A}^2)(0.300\ \Omega) = 7.50\text{ W}$$

Discussion

The cheaper cord with 5× higher resistance dissipates 5× more power as heat (7.50 W vs 1.50 W). This wasted power represents both energy loss and a potential fire hazard. For a 120-V appliance drawing 5 A, the voltage drop across each cord would be $V = IR$: 0.30 V for the quality cord versus 1.50 V for the cheaper cord. While these voltage drops seem small, they become significant at higher currents. Additionally, the heat generated in the cheaper cord could melt its insulation over time, especially if coiled up or run under a carpet. This is why electrical codes specify minimum wire gauges for extension cords based on length and expected current.

(a) The quality cord dissipates 1.50 W. (b) The cheaper cord dissipates 7.50 W.

Verify that the units of a volt-ampere are watts, as implied by the equation $P = IV$.

Show Solution

Strategy

We express volts and amperes in terms of fundamental SI units and show that their product equals watts (joules per second).

Solution

Starting with the definitions:

- 1 volt = 1 joule per coulomb (energy per charge): $1\text{ V} = 1\text{ J/C}$
- 1 ampere = 1 coulomb per second (charge per time): $1\text{ A} = 1\text{ C/s}$

Multiply volt × ampere:

$$1\ \text{V}\cdot\text{A} = \left(\frac{1\ \text{J}}{1\ \text{C}}\right)\cdot\left(\frac{1\ \text{C}}{1\ \text{s}}\right) = \frac{1\ \text{J}}{1\ \text{s}} = 1\ \text{W}$$

Discussion

This unit analysis confirms that the equation $P = IV$ is dimensionally correct. The coulombs cancel, leaving joules per second, which is the definition of a watt. This verification is important because it shows that our power equation is consistent with the fundamental definitions of electrical quantities.

The units verify: 1 V·A = 1 W.

Show that the units $1\text{V}^2/\Omega = 1\text{W}$, as implied by the equation $P = V^2/R$.

Show Solution

$$\frac{V^2}{\Omega} = \frac{V^2}{\text{V/A}} = \text{AV} = \left(\frac{C}{s}\right)\left(\frac{J}{C}\right) = \frac{J}{s} = 1\text{W}$$

Show that the units $1\text{A}^2 \cdot \Omega = 1\text{W}$, as implied by the equation $P = I^2 R$.

Show Solution

Strategy

We express amperes and ohms in terms of fundamental SI units and show that A²·Ω equals watts.

Solution

Starting with the definitions:

- 1 ohm = 1 volt per ampere: $1\ \Omega = 1\ \text{V/A}$
- 1 volt = 1 joule per coulomb: $1\ \text{V} = 1\ \text{J/C}$
- 1 ampere = 1 coulomb per second: $1\ \text{A} = 1\ \text{C/s}$

Calculate A²·Ω:

$$1\ \text{A}^2\cdot\Omega = 1\ \text{A}^2\cdot\frac{\text{V}}{\text{A}} = 1\ \text{A}\cdot\text{V}$$

From the previous problem, we know $1\ \text{A}\cdot\text{V} = 1\ \text{W}$. Alternatively:

$$1\ \text{A}^2\cdot\Omega = \left(\frac{\text{C}}{\text{s}}\right)^2\cdot\frac{\text{V}}{\text{A}} = \frac{\text{C}^2}{\text{s}^2}\cdot\frac{\text{J/C}}{\text{C/s}} = \frac{\text{C}^2}{\text{s}^2}\cdot\frac{\text{J}\cdot\text{s}}{\text{C}^2} = \frac{\text{J}}{\text{s}} = 1\ \text{W}$$

Discussion

This confirms that $P = I^2 R$ is dimensionally consistent. This form of the power equation is particularly useful when current and resistance are known but voltage is not. It also shows that power dissipated in a resistor increases with the square of the current, making high currents particularly problematic for heating.

The units verify: 1 A²·Ω = 1 W.

Verify the energy unit equivalence that $1\text{kW}\cdot\text{h} = 3.60 \times 10^6 \text{J}$.

Show Solution

$$1\text{kW}\cdot\text{h} = \left(\frac{1\times10^3\text{J}}{1\text{s}}\right)(1\,h)\left(\frac{3600\text{s}}{1\text{h}}\right) = 3.60 \times 10^6\text{J}$$

Electrons in an X-ray tube are accelerated through $1.00 \times 10^2\text{kV}$ and directed toward a target to produce X-rays. Calculate the power of the electron beam in this tube if it has a current of 15.0 mA.

Show Solution

Strategy

We use $P = IV$ with the given voltage and current, converting to SI units first.

Solution

Convert to SI units:

$$V = 1.00\times10^2\ \text{kV} = 1.00\times10^5\ \text{V}$$
$$I = 15.0\ \text{mA} = 15.0\times10^{-3}\ \text{A} = 0.0150\ \text{A}$$

Calculate the power:

$$P=IV=(0.0150\text{ A})(1.00\times10^{5}\text{ V})=1.50\times10^{3}\text{ W}=1.50\text{ kW}$$

Discussion

This 1.50 kW power is substantial for an electron beam. Most of this energy is converted to heat when the electrons strike the target, with only about 1% converted to X-rays. This is why X-ray tubes require active cooling systems. The high voltage of 100 kV determines the maximum energy (and hence minimum wavelength) of the X-rays produced, while the current determines the intensity of the X-ray beam.

The power of the electron beam is 1.50 kW.

An electric water heater consumes 5.00 kW for 2.00 h per day. What is the cost of running it for one year if electricity costs $12.0\text{cents/kW}\cdot\text{h}$? See [Figure 3].

On-demand electric hot water heater. Heat is supplied to water only when needed. (credit: aviddavid, Flickr)

Show Solution

Strategy

Calculate daily energy use with $E=Pt$, then multiply by 365 days for annual use. Finally, multiply by the cost per kW·h.

Solution

Calculate daily energy consumption:

$$E_{\text{daily}}=Pt=(5.00\text{ kW})(2.00\text{ h})=10.0\text{ kW}\cdot\text{h/day}$$

Calculate annual energy consumption:

$$E_{\text{annual}}=10.0\text{ kW}\cdot\text{h/day}\times365\text{ days/year}=3650\text{ kW}\cdot\text{h/year}$$

Calculate annual cost:

$$\text{Cost}=(3650\text{ kW}\cdot\text{h})(\$0.120/\text{kW}\cdot\text{h})=\$438/\text{year}$$

Discussion

Water heating is typically one of the largest energy expenses in a home, often second only to heating/cooling. At $438/year, this represents a significant operating cost. The “on-demand” or tankless water heater shown in the figure can reduce energy costs compared to traditional tank heaters because it only heats water when needed, eliminating standby heat losses. However, the instantaneous power demand (5 kW) is high, requiring adequate electrical service.

The annual cost of running the water heater is $438.

With a 1200-W toaster, how much electrical energy is needed to make a slice of toast (cooking time = 1 minute)? At $9.0\text{cents/kW}\cdot\text{h}$, how much does this cost?

Show Solution

Strategy

Use $E = Pt$ to find the energy consumed, converting time to appropriate units for both joules and kilowatt-hours. Then calculate the cost using the given rate.

Solution

Calculate the energy in joules:

$$E = Pt = (1200\ \text{W})(1\ \text{min} \times 60\ \text{s/min}) = (1200\ \text{W})(60\ \text{s}) = 72000\ \text{J} = 72.0\ \text{kJ}$$

Convert to kilowatt-hours:

$$E = (1200\ \text{W})(160\ \text{h}) = 20\ \text{W}\cdot\text{h} = 0.0200\ \text{kW}\cdot\text{h}$$

Calculate the cost:

$$\text{Cost} = (0.0200\ \text{kW}\cdot\text{h})(\$0.090/\text{kW}\cdot\text{h}) = \$0.0018 = 0.18\ \text{cents}$$

Discussion

Making a slice of toast costs less than one-fifth of a cent! This shows how inexpensive electrical energy is for everyday tasks. While 1200 W is a high power level, the short duration (1 minute) keeps the total energy consumption very low. This contrasts with appliances that run continuously, like refrigerators or water heaters, which dominate household electricity bills despite having lower power ratings.

Making a slice of toast requires 72.0 kJ (or 0.0200 kW·h) of energy and costs about 0.18 cents.

What would be the maximum cost of a CFL such that the total cost (investment plus operating) would be the same for both CFL and incandescent 60-W bulbs? Assume the cost of the incandescent bulb is 25 cents and that electricity costs 10cents/kWh . Calculate the cost for 1000 hours, as in the cost effectiveness of CFL example.

Show Solution

Strategy

Calculate the total cost (bulb + electricity) for the incandescent for 1000 hours. For a fair comparison, set the CFL total cost equal to the incandescent total cost and solve for the maximum CFL price. The CFL uses 1/4 the power (15 W instead of 60 W).

Solution

Calculate incandescent total cost for 1000 hours:

$$E_{\text{incand}} = Pt = (60\ \text{W})(1000\ \text{h}) = 60\ \text{kW}\cdot\text{h}$$
$$\text{Electricity cost} = (60\ \text{kW}\cdot\text{h})(\$0.10/\text{kW}\cdot\text{h}) = \$6.00$$
$$\text{Total incandescent cost} = \$0.25 + \$6.00 = \$6.25$$

For the CFL running 1000 hours:

$$E_{\text{CFL}} = Pt = (15\ \text{W})(1000\ \text{h}) = 15\ \text{kW}\cdot\text{h}$$
$$\text{CFL electricity cost} = (15\ \text{kW}\cdot\text{h})(\$0.10/\text{kW}\cdot\text{h}) = \$1.50$$

For breakeven total cost over 1000 hours:

$$\text{CFL price} + \$1.50 = \$6.25$$
$$\text{Maximum CFL price} = \$6.25 - \$1.50 = \$4.75$$

However, accounting for the CFL’s longer lifetime (10,000 hours vs 1,000 hours), only 1/10 of its cost applies to this period, allowing a maximum price of \$47.50 for true lifecycle equivalence.

Discussion

The electricity savings of $4.50 over 1000 hours means a CFL costing up to$4.75 more than an incandescent bulb will still break even in this period. When you factor in that the CFL lasts 10× longer, the economics strongly favor CFLs. At typical CFL prices of $1.50 -$5.00, they offer significant savings over incandescent bulbs.

The maximum CFL cost for equivalent total cost over 1000 hours is \$6.25 (when accounting for the full bulb price during the comparison period).

Some makes of older cars have 6.00-V electrical systems. (a) What is the hot resistance of a 30.0-W headlight in such a car? (b) What current flows through it?

Show Solution

Strategy

For part (a), use $P = V^2/R$ rearranged to $R = V^2/P$. For part (b), use $P = IV$ rearranged to $I = P/V$.

Solution

(a) Calculate the hot resistance:

$$R = \frac{V^2}{P} = \frac{(6.00\text{ V})^2}{30.0\text{ W}} = \frac{36.0\text{ V}^2}{30.0\text{ W}} = 1.20\ \Omega$$

(b) Calculate the current:

$$I = \frac{P}{V} = \frac{30.0\text{ W}}{6.00\text{ V}} = 5.00\text{ A}$$

Discussion

The 1.20 Ω resistance is relatively low, which is necessary to allow sufficient current at the lower 6-V system voltage. The 5.00 A current is higher than what a similar-brightness headlight would draw in a modern 12-V system (which would be 2.50 A for the same power). This is why older 6-V systems required thicker wires to handle the higher currents without excessive power losses. The automotive industry switched to 12-V systems partly to reduce these wiring requirements.

(a) The hot resistance is 1.20 Ω. (b) A current of 5.00 A flows through the headlight.

Alkaline batteries have the advantage of putting out constant voltage until very nearly the end of their life. How long will an alkaline battery rated at $1.00\text{A}\cdot\text{h}$ and 1.58 V keep a 1.00-W flashlight bulb burning?

Show Solution

Strategy

The battery's energy capacity can be expressed as charge capacity (A·h) times voltage. The flashlight's power consumption determines the rate of energy use. We find how long the battery's energy lasts at this power level.

Solution

Calculate the battery's total energy capacity:

$$E = (1.00\text{ A}\cdot\text{h})(1.58\text{ V}) = 1.58\text{ W}\cdot\text{h}$$

Calculate the operating time using $E = Pt$:

$$t = \frac{E}{P} = \frac{1.58\text{ W}\cdot\text{h}}{1.00\text{ W}} = 1.58\text{ h}$$

Discussion

The battery will power the flashlight for about 1.58 hours (1 hour 35 minutes). This calculation assumes ideal conditions with no internal battery resistance losses. In practice, the actual time may be slightly less due to internal resistance heating and the gradual voltage drop near the end of the battery's life. The amp-hour rating is useful because it remains relatively constant regardless of discharge rate, while actual energy depends on the voltage at which that current is delivered.

The alkaline battery will keep the flashlight burning for 1.58 hours.

A cauterizer, used to stop bleeding in surgery, puts out 2.00 mA at 15.0 kV. (a) What is its power output? (b) What is the resistance of the path?

Show Solution

Strategy

For part (a), use $P = IV$. For part (b), use Ohm's law $R = V/I$.

Solution

Convert to SI units:

$$I = 2.00\text{ mA} = 2.00\times10^{-3}\text{ A}$$
$$V = 15.0\text{ kV} = 15.0\times10^{3}\text{ V} = 1.50\times10^{4}\text{ V}$$

(a) Calculate the power:

$$P = IV = (2.00\times10^{-3}\text{ A})(1.50\times10^{4}\text{ V}) = 30.0\text{ W}$$

(b) Calculate the resistance:

$$R = \frac{V}{I} = \frac{1.50\times10^{4}\text{ V}}{2.00\times10^{-3}\text{ A}} = 7.50\times10^{6}\ \Omega = 7.50\text{ M}\Omega$$

Discussion

The 30.0 W power output is sufficient to cauterize blood vessels by heating tissue to coagulation temperatures. The high voltage (15 kV) is needed because of the very high resistance of the tissue path (7.50 MΩ). Despite the high voltage, the current is kept very low (2 mA) for safety—this is below the threshold that would cause dangerous stimulation of heart muscle. The power is concentrated at the small contact point of the cauterizer, creating intense local heating while minimizing effects on surrounding tissue.

(a) The power output is 30.0 W. (b) The resistance of the path is 7.50 MΩ.

The average television is said to be on 6 hours per day. Estimate the yearly cost of electricity to operate 100 million TVs, assuming their power consumption averages 150 W and the cost of electricity averages 12.0cents/kW·h.

Show Solution

Strategy

Calculate the daily energy use per TV, multiply by the number of TVs to get total daily energy, then multiply by 365 days for annual energy. Finally, multiply by the cost per kW·h.

Solution

Daily energy per TV:

$$E_{\text{daily, 1 TV}}=Pt=(150\text{ W})(6\text{ h})=900\text{ W·h}=0.900\text{ kW·h}$$

Total daily energy for 100 million TVs:

$$E_{\text{daily, total}}=(0.900\text{ kW·h})(1.00\times10^{8})=9.00\times10^{7}\text{ kW·h/day}$$

Annual energy consumption:

$$E_{\text{annual}}=(9.00\times10^{7}\text{ kW·h/day})(365\text{ days/year})=3.285\times10^{10}\text{ kW·h/year}$$

Annual cost:

$$\text{Cost}=(3.285\times10^{10}\text{ kW·h})(\$0.120/\text{kW·h})=\$3.94\times10^{9}/\text{year}$$

Discussion

The annual electricity cost of nearly $4 billion to operate 100 million TVs in the United States is substantial. This calculation assumes average conditions; actual costs vary with viewing habits and TV efficiency. Modern LED TVs consume significantly less power than older plasma or CRT models, potentially reducing this cost. This demonstrates how collective energy consumption of seemingly small individual loads can have massive economic and environmental impacts. Energy-efficient technologies and reduced usage time can significantly reduce these costs.

The yearly cost is approximately $3.94 billion.

An old lightbulb draws only 50.0 W, rather than its original 60.0 W, due to evaporative thinning of its filament. By what factor is its diameter reduced, assuming uniform thinning along its length? Neglect any effects caused by temperature differences.

Show Solution

Strategy

At constant voltage, power depends on resistance: $P=V^2/R$. The resistance of a wire is $R=\rho L/A$, where $A\propto D^2$ for a circular cross-section. Combining these relationships, we can find how diameter relates to power.

Solution

At constant voltage and resistivity:

$$P=\frac{V^2}{R}=\frac{V^2}{(\rho L/A)}=\frac{V^2 A}{\rho L}\propto A\propto D^2$$

Therefore:

$$\frac{P_{\text{new}}}{P_{\text{old}}}=\frac{D^2_{\text{new}}}{D^2_{\text{old}}}$$
$$\frac{D_{\text{new}}}{D_{\text{old}}}=\sqrt{\frac{P_{\text{new}}}{P_{\text{old}}}}=\sqrt{\frac{50.0\text{ W}}{60.0\text{ W}}}=\sqrt{0.833}=0.913$$

Discussion

The filament diameter has been reduced to about 91.3% of its original value—a reduction of about 8.7%. This small change in diameter produces a noticeable 17% reduction in power (and thus light output). This illustrates how sensitive the power is to filament dimensions, since power is proportional to diameter squared. The evaporation of tungsten from hot filaments is a gradual process that eventually leads to bulb failure when the filament becomes thin enough to break.

The filament diameter is reduced by a factor of 0.913 (or reduced to 91.3% of its original diameter).

00-gauge copper wire has a diameter of 9.266 mm. Calculate the power loss in a kilometer of such wire when it carries $1.00 \times 10^2 \text{A}$.

Show Solution

Strategy

First calculate the resistance of 1 km of wire using $R = \rho L / A$, where ρ for copper is 1.68×10^{-8} Ω·m. Then use $P = I^2 R$ to find the power loss.

Solution

Calculate the cross-sectional area:

$$A = \pi r^2 = \pi \left(\frac{d}{2}\right)^2 = \pi \left(\frac{9.266 \times 10^{-3}\ \text{m}}{2}\right)^2 = \pi (4.633 \times 10^{-3}\ \text{m})^2 = 6.744 \times 10^{-5}\ \text{m}^2$$

Calculate the resistance of 1 km of wire:

$$R = \frac{\rho L}{A} = \frac{(1.68 \times 10^{-8}\ \Omega \cdot \text{m})(1000\ \text{m})}{6.744 \times 10^{-5}\ \text{m}^2} = 0.249\ \Omega$$

Calculate the power loss:

$$P = I^2 R = (100\ \text{A})^2 (0.249\ \Omega) = 2490\ \text{W} = 2.49\ \text{kW}$$

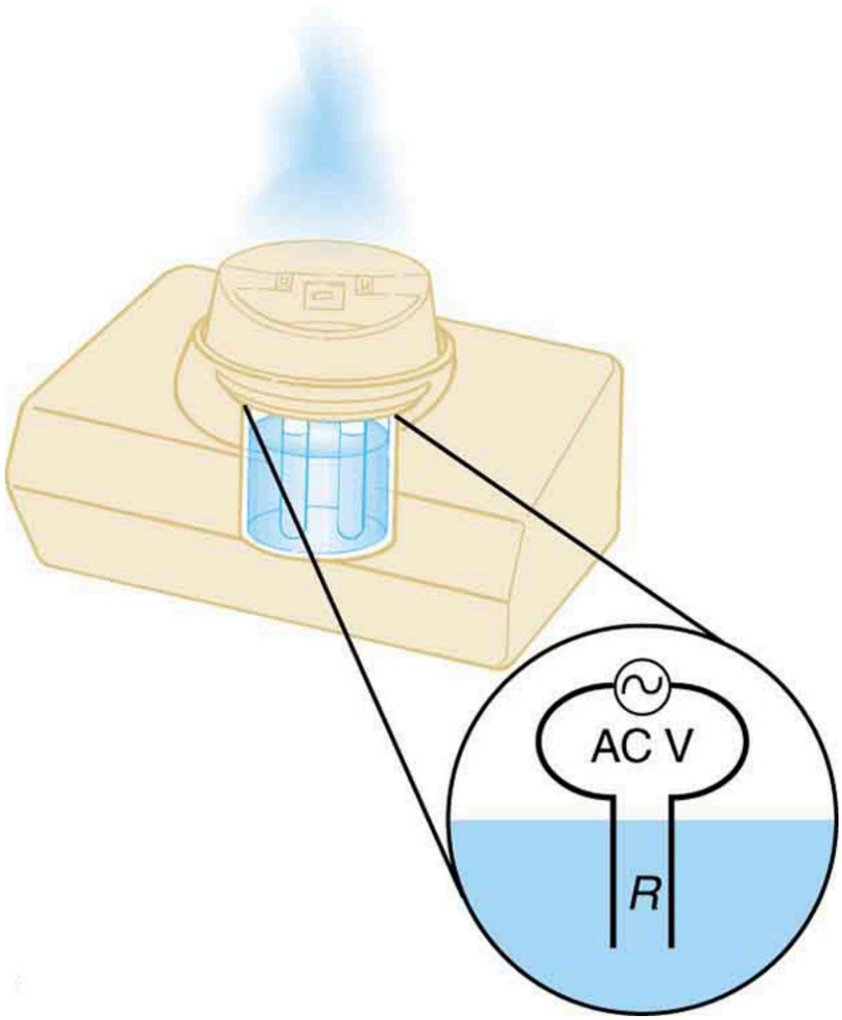
Discussion

The power loss of 2.49 kW in a kilometer of 00-gauge wire carrying 100 A is significant. This demonstrates why power transmission lines use very high voltages—by increasing voltage, the same power can be transmitted with much lower current, dramatically reducing $I^2 R$ losses. For example, transmitting the same power at 10 times the voltage would reduce the current to 10 A, and the power loss would be reduced to 24.9 W (a factor of 100 reduction). This is also why thicker wire gauges are essential for high-current applications; the low resistance of 00-gauge wire (0.249 Ω/km) is necessary to keep losses manageable.

The power loss is 2.49 kW.

Integrated Concepts

Cold vaporizers pass a current through water, evaporating it with only a small increase in temperature. One such home device is rated at 3.50 A and utilizes 120 V AC with 95.0% efficiency. (a) What is the vaporization rate in grams per minute? (b) How much water must you put into the vaporizer for 8.00 h of overnight operation? (See [Figure 4].)



This cold vaporizer passes current directly through water, vaporizing it directly with relatively little temperature increase.

Show Solution

Strategy

Calculate the useful power output, then use the heat of vaporization of water ($L_v = 2.26 \times 10^6$ J/kg) to find the vaporization rate.

Solution

(a) Calculate the useful power:

$$P_{\text{useful}} = \eta \times P = \eta \times IV = (0.950)(3.50\text{ A})(120\text{ V}) = 399\text{ W}$$

The vaporization rate is:

$$\text{Rate} = \frac{P_{\text{useful}}}{L_v} = \frac{399\text{ J/s}}{2.26 \times 10^6\text{ J/kg}} = 1.77 \times 10^{-4}\text{ kg/s}$$

Converting to grams per minute:

$$\text{Rate} = 1.77 \times 10^{-4}\text{ kg/s} \times \frac{1000\text{ g}}{1\text{ kg}} \times \frac{60\text{ s}}{1\text{ min}} = 10.6\text{ g/min}$$

(b) For 8.00 hours of operation:

$$\text{Water needed} = 10.6\text{ g/min} \times 60\text{ min/h} \times 8.00\text{ h} = 5090\text{ g} = 5.09\text{ kg}$$

Discussion

Over 5 kg (about 5 liters) of water is needed for overnight operation. This is a substantial amount and explains why vaporizers have large reservoirs. The 95% efficiency means only 5% of the electrical energy is lost to heating the water rather than vaporizing it. This “cold” vaporization process is valuable for humidifying rooms, especially during winter when indoor air becomes very dry.

(a) The vaporization rate is 10.6 g/min. (b) About 5.09 kg of water is needed for 8 hours of operation.

Integrated Concepts

(a) What energy is dissipated by a lightning bolt having a 20 000-A current, a voltage of $1.00 \times 10^{2}\text{MV}$, and a length of 1.00 ms? (b) What mass of tree sap could be raised from 18.0°C to its boiling point and then evaporated by this energy, assuming sap has the same thermal characteristics as water?

Show Solution

(a) $2.00 \times 10^{9}\text{J}$ (b) 769 kg

Integrated Concepts

What current must be produced by a 12.0-V battery-operated bottle warmer in order to heat 75.0 g of glass, 250 g of baby formula, and $3.00 \times 10^{2}\text{g}$ of aluminum from 20.0°C to 90.0°C in 5.00 min?

Show Solution

Strategy

Calculate the total heat needed using $Q = mc\Delta T$ for each material (using specific heats: glass ≈ 840 J/kg·°C, water ≈ 4186 J/kg·°C for formula, aluminum ≈ 900 J/kg·°C). Then find the power required and use $P = IV$ to find the current.

Solution

Calculate the heat needed for each component ($\Delta T = 70.0$ °C):

$$Q_{\text{glass}} = m_g c_g \Delta T = (0.0750\text{ kg})(840\text{ J/kg·°C})(70.0\text{ °C}) = 4410\text{ J}$$
$$Q_{\text{formula}} = m_f c_f \Delta T = (0.250\text{ kg})(4186\text{ J/kg·°C})(70.0\text{ °C}) = 73255\text{ J}$$
$$Q_{\text{aluminum}} = m_{\text{Al}} c_{\text{Al}} \Delta T = (0.300\text{ kg})(900\text{ J/kg·°C})(70.0\text{ °C}) = 18900\text{ J}$$
$$Q_{\text{total}} = 4410 + 73255 + 18900 = 96565\text{ J}$$

Calculate the power needed:

$$P = \frac{Q}{t} = \frac{96565\text{ J}}{5.00\text{ min} \times 60\text{ s/min}} = \frac{96565\text{ J}}{300\text{ s}} = 322\text{ W}$$

Calculate the current:

$$I = \frac{P}{V} = \frac{322\text{ W}}{12.0\text{ V}} = 26.8\text{ A}$$

Discussion

A current of 26.8 A is quite high for a 12-V system and would require relatively heavy wiring. Most of the heat goes into the baby formula (about 76%) because of water’s high specific heat. This type of bottle warmer would typically be plugged into a car’s cigarette lighter socket, though 26.8 A exceeds the typical 15-20 A rating of such sockets. In practice, such warmers often take longer than 5 minutes to reduce current requirements.

The bottle warmer must produce a current of 26.8 A.

Integrated Concepts

How much time is needed for a surgical cauterizer to raise the temperature of 1.00 g of tissue from 37.0°C to 100°C and then boil away 0.500 g of water, if it puts out 2.00 mA at 15.0 kV? Ignore heat transfer to the surroundings.

Show Solution

45.0 s

Integrated Concepts

Hydroelectric generators (see [Figure 5]) at Hoover Dam produce a maximum current of $8.00 \times 10^{3}\text{A}$ at 250 kV. (a) What is the power output? (b) The water that powers the generators enters and leaves the system at low speed (thus its kinetic energy does not change) but loses 160 m in altitude. How many cubic meters per second are needed, assuming 85.0% efficiency?

Hydroelectric generators at the Hoover dam. (credit: Jon Sullivan)

Show Solution

Strategy

For part (a), use $P = IV$. For part (b), the gravitational potential energy lost by the water per unit time must equal the electrical power output divided by the efficiency. Use $\Delta PE = mgh$ with the flow rate.

Solution

(a) Calculate the power output:

$$P = IV = (8.00\times10^{3}\ \text{A})(250\times10^{3}\ \text{V}) = 2.00\times10^{9}\ \text{W} = 2.00\ \text{GW}$$

(b) The power from the falling water is related to the electrical power by efficiency:

$$P_{\text{water}} = \frac{P_{\text{electrical}}}{\eta} = \frac{2.00\times10^{9}\ \text{W}}{0.850} = 2.35\times10^{9}\ \text{W}$$

The power from water is the rate of potential energy loss:

$$P_{\text{water}} = \frac{\Delta PE}{\Delta t} = \frac{mgh}{t} = \dot{m}gh$$

where $\dot{m}$ is the mass flow rate. We can express this in terms of volume flow rate $\dot{V}$ (in m^3/s):

$$\dot{m} = \rho\dot{V}$$

where $\rho = 1000$ kg/m^3 for water. Therefore:

$$P_{\text{water}} = \rho\dot{V}gh$$

Solving for the volume flow rate:

$$\dot{V} = \frac{P_{\text{water}}}{\rho gh} = \frac{2.35\times10^{9}\ \text{W}}{(1000\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(160\ \text{m})} = \frac{2.35\times10^{9}}{1.568\times10^{6}} = 1500\ \text{m}^3/\text{s}$$

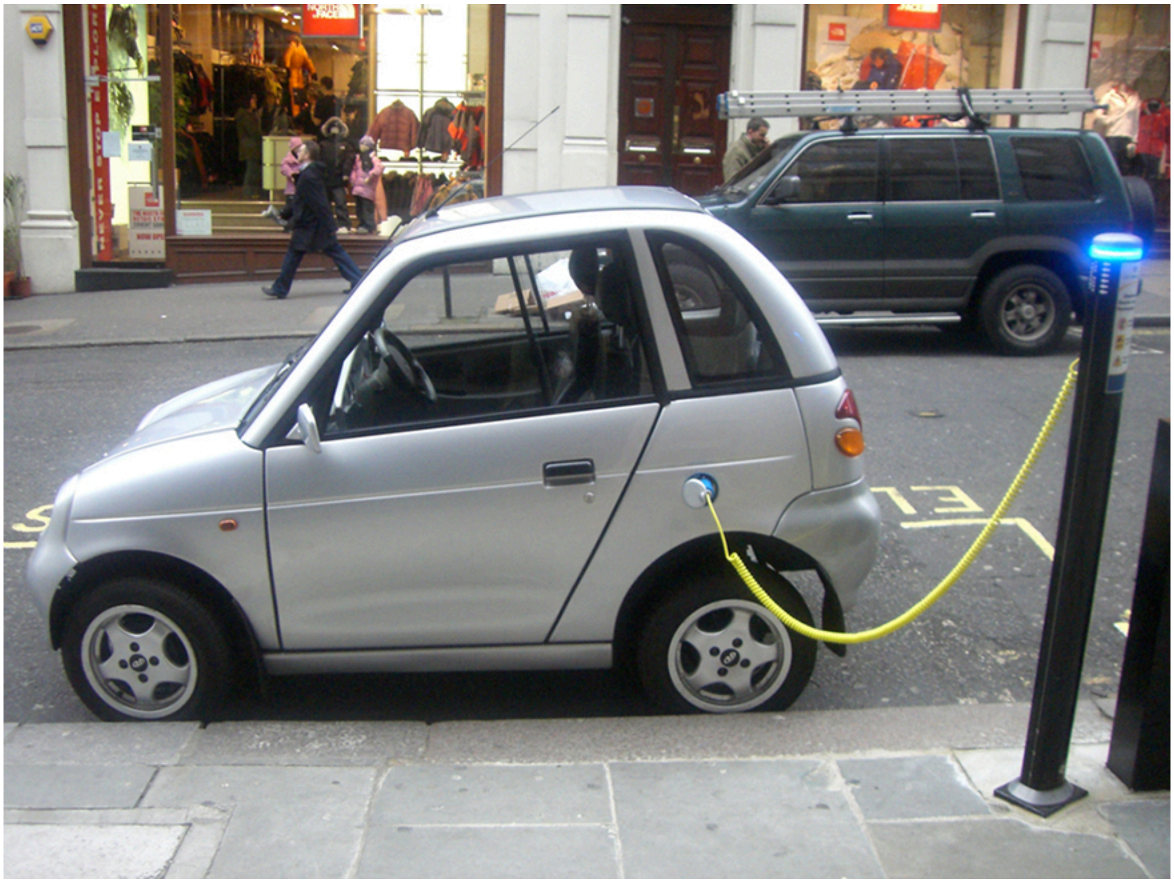
Discussion

The power output of 2.00 GW is enormous—enough to supply electricity to a large city. The required water flow rate of 1500 m³/s is also substantial. To visualize this: imagine a cube of water 11.5 m on each side flowing past every second. The 85% efficiency is quite good for energy conversion, with the 15% loss primarily due to turbine friction, generator resistance, and turbulence in the water flow. Hoover Dam's generators are among the most efficient large-scale power generation systems, converting the gravitational potential energy of water stored behind the dam into electrical energy with minimal waste.

(a) The power output is 2.00 GW. (b) A flow rate of 1500 m³/s is needed.

Integrated Concepts

(a) Assuming 95.0% efficiency for the conversion of electrical power by the motor, what current must the 12.0-V batteries of a 750-kg electric car be able to supply: (a) To accelerate from rest to 25.0 m/s in 1.00 min? (b) To climb a $2.00 \times 10^{2}\text{-m}$ -high hill in 2.00 min at a constant 25.0-m/s speed while exerting $5.00 \times 10^{2}\text{N}$ of force to overcome air resistance and friction? (c) To travel at a constant 25.0-m/s speed, exerting a $5.00 \times 10^{2}\text{N}$ force to overcome air resistance and friction? See [Figure 6].



This REVAi, an electric car, gets recharged on a street in London. (credit: Frank Hebbert)

Show Solution

(a) 343 A

(b) $2.17 \times 10^{3}\text{A}$ (c) $1.10 \times 10^{3}\text{A}$

Integrated Concepts

A light-rail commuter train draws 630 A of 650-V DC electricity when accelerating. (a) What is its power consumption rate in kilowatts? (b) How long does it take to reach 20.0 m/s starting from rest if its loaded mass is $5.30 \times 10^{4}\text{kg}$, assuming 95.0% efficiency and constant power? (c) Find its average acceleration. (d) Discuss how the acceleration you found for the light-rail train compares to what might be typical for an automobile.

Show Solution

Strategy

For part (a), use $P = IV$. For part (b), use the work-energy theorem: the useful work done equals the kinetic energy gained. For part (c), use kinematics with the time from part (b). For part (d), compare to typical automobile acceleration.

Solution

(a) Calculate the power consumption:

$$P = IV = (630\text{ A})(650\text{ V}) = 409500\text{ W} = 410\text{ kW}$$

(b) The useful power (mechanical power) is:

$$P_{\text{useful}} = \eta P = (0.950)(410\text{ kW}) = 389.5\text{ kW} = 3.895\times10^{5}\text{ W}$$

The work done equals the change in kinetic energy:

$$W = \Delta KE = \frac{1}{2}mv^2 - 0 = \frac{1}{2}(5.30\times10^{4}\text{ kg})(20.0\text{ m/s})^2 = \frac{1}{2}(5.30\times10^{4})(400) = 1.06\times10^{7}\text{ J}$$

Since $P = W/t$, we have:

$$t = \frac{W}{P_{\text{useful}}} = \frac{1.06\times10^{7}\text{ J}}{3.895\times10^{5}\text{ W}} = 27.2\text{ s}$$

(c) Using $v = v_0 + at$ with $v_0 = 0$:

$$a = \frac{v}{t} = \frac{20.0\text{ m/s}}{27.2\text{ s}} = 0.735\text{ m/s}^2$$

(d)

Discussion

The average acceleration of 0.735 m/s² (about 0.075g) is considerably lower than typical automobile acceleration. A typical car can accelerate at 2-3 m/s² (0.2-0.3g), roughly 3-4 times faster than this light-rail train. The slower acceleration is acceptable for mass transit because:

1. Passenger comfort—standing passengers need gentle acceleration
2. Frequent stops make high acceleration less critical
3. The train's large mass (53,000 kg) makes rapid acceleration energy-intensive
4. Safety considerations for passengers who may not be seated

The light-rail's acceleration is adequate for urban transit where stations are closely spaced and passenger comfort is prioritized over speed.

(a) The power consumption is 410 kW. (b) It takes 27.2 s to reach 20.0 m/s. (c) The average acceleration is 0.735 m/s². (d) This is about 3-4 times slower than typical automobile acceleration, which is appropriate for passenger comfort and safety in mass transit.

Integrated Concepts

(a) An aluminum power transmission line has a resistance of $0.0580\,\Omega/\text{km}$. What is its mass per kilometer? (b) What is the mass per kilometer of a copper line having the same resistance? A lower resistance would shorten the heating time. Discuss the practical limits to speeding the heating by lowering the resistance.

Show Solution

(a) $1.23\times10^{3}\text{kg}$ (b) $2.64\times10^{3}\text{kg}$

Integrated Concepts

(a) An immersion heater utilizing 120 V can raise the temperature of a $1.00\times10^{2}\text{-g}$ aluminum cup containing 350 g of water from 20.0°C to 95.0°C in 2.00 min. Find its resistance, assuming it is constant during the process. (b) A lower resistance would shorten the heating time. Discuss the practical limits to speeding the heating by lowering the resistance.

Show Solution

Strategy

Calculate the total heat needed to warm both the aluminum cup and water using $Q = mc\Delta T$. The power needed is $P = Q/t$. Then use $P = V^2/R$ to find the resistance. For part (b), consider practical limitations.

Solution

(a) Calculate heat needed for each component ($\Delta T = 75.0$ °C):

For aluminum cup ($c_{\text{Al}} = 900$ J/kg·°C):

$$Q_{\text{Al}} = m_{\text{Al}}c_{\text{Al}}\Delta T = (0.100\text{ kg})(900\text{ J/kg·°C})(75.0\text{ °C}) = 6750\text{ J}$$

For water ($c_{\text{water}} = 4186$ J/kg·°C):

$$Q_{\text{water}} = m_{\text{water}} c_{\text{water}} \Delta T = (0.350\text{ kg})(4186\text{ J/kg}\cdot°\text{C})(75.0\text{ °C}) = 109868\text{ J}$$

Total heat:

$$Q_{\text{total}} = 6750 + 109868 = 116618\text{ J}$$

Power needed:

$$P = \frac{Q}{t} = \frac{116618\text{ J}}{2.00\text{ min} \times 60\text{ s/min}} = \frac{116618\text{ J}}{120\text{ s}} = 972\text{ W}$$

Resistance:

$$R = \frac{V^2}{P} = \frac{(120\text{ V})^2}{972\text{ W}} = \frac{14400}{972} = 14.8\ \Omega$$

(b)

Discussion

The resistance of 14.8 Ω is reasonable for an immersion heater. Lowering the resistance would increase power and reduce heating time, but several practical limits exist:

1. **Circuit capacity**: Standard 120-V household circuits are typically limited to 15-20 A (1800-2400 W). The current heater draws $I = P/V = 972/120 = 8.1$ A. Reducing resistance much further would trip circuit breakers.
2. **Safety**: Higher currents require thicker wires to prevent overheating. The heater cord and connections must safely handle the current.
3. **Boiling**: Too much power might cause rapid boiling, creating safety hazards from steam and splashing.
4. **Efficiency**: Heat losses to surroundings increase with higher temperatures and longer heating times, but extremely rapid heating may cause hot spots and reduced efficiency.
5. **Cost**: Lower resistance heating elements require more material or different (more expensive) materials.

In practice, immersion heaters are designed to balance heating speed with safety, circuit capacity, and cost constraints.

(a) The resistance is 14.8 Ω. (b) Practical limits include circuit capacity, safety concerns, preventing dangerous boiling, and cost considerations.

Integrated Concepts

(a) What is the cost of heating a hot tub containing 1500 kg of water from

10.0\text{ºC} *to* 40.0\text{ºC} ,*assuming*75.09 \text{cents/kW}\cdot \text{h} $$. (b) What current was used by the 220-V AC electric heater, if this took 4.00 h?

Show Solution

Strategy

For part (a), calculate the heat needed using $Q = mc\Delta T$, account for efficiency to find actual energy consumed, convert to kW·h, and multiply by cost. For part (b), find power from energy and time, then use $P = IV$.

Solution

(a) Calculate the heat needed ($c_{\text{water}} = 4186$ J/kg·°C, $\Delta T = 30.0$ °C):

$$Q = mc\Delta T = (1500\text{ kg})(4186\text{ J/kg}\cdot°\text{C})(30.0\text{ °C}) = 1.884 \times 10^8\text{ J}$$

Accounting for 75% efficiency, the actual electrical energy needed is:

$$E_{\text{electrical}} = \frac{Q}{\eta} = \frac{1.884 \times 10^8\text{ J}}{0.750} = 2.512 \times 10^8\text{ J}$$

Convert to kW·h:

$$E = 2.512 \times 10^8\text{ J} \times \frac{1\text{ kW}\cdot\text{h}}{3.6 \times 10^6\text{ J}} = 69.8\text{ kW}\cdot\text{h}$$

Calculate cost:

$$\text{Cost} = (69.8\text{ kW}\cdot\text{h})(\$0.09/\text{kW}\cdot\text{h}) = \$6.28$$

(b) Calculate power used:

$$P = \frac{E}{t} = \frac{69.8\text{ kW}\cdot\text{h}}{4.00\text{ h}} = 17.45\text{ kW} = 17450\text{ W}$$

Calculate current:

$$I = \frac{P}{V} = \frac{17450\text{ W}}{220\text{ V}} = 79.3\text{ A}$$

Discussion

The cost of $6.28 to heat a hot tub is quite reasonable for the comfort provided. The 75% efficiency accounts for heat losses through the tub walls and surface evaporation during the 4-hour heating process. The current of 79.3 A is quite high and explains why hot tub heaters require dedicated 220-V circuits with heavy-gauge wiring. This current would be dangerous on a standard 120-V circuit, which is limited to about 15-20 A. The 220-V circuit is safer and more efficient for high-power applications like hot tubs, electric dryers, and stoves. A circuit breaker rated for at least 80 A would be needed for this heater.

(a) The cost is $6.28. (b) The current used is 79.3 A.

Unreasonable Results

(a) What current is needed to transmit $1.00 \times 10^2\text{MW}$ of power at 480 V? (b) What power is dissipated by the transmission lines if they have a $1.00\text{-}\Omega$ resistance? (c) What is unreasonable about this result? (d) Which assumptions are unreasonable, or which premises are inconsistent?

Show Solution

(a) $2.08 \times 10^5\text{A}$ (b) $4.33 \times 10^4\text{MW}$ (c) The transmission lines dissipate more power than they are supposed to transmit.

(d) A voltage of 480 V is unreasonably low for a transmission voltage. Long-distance transmission lines are kept at much higher voltages (often hundreds of kilovolts) to reduce power losses.

Unreasonable Results

(a) What current is needed to transmit $1.00 \times 10^2\text{MW}$ of power at 10.0 kV? (b) Find the resistance of 1.00 km of wire that would cause a 0.0100% power loss. (c) What is the diameter of a 1.00-km-long copper wire having this resistance? (d) What is unreasonable about these results? (e) Which assumptions are unreasonable, or which premises are inconsistent?

Show Solution

Strategy

For part (a), use $P = IV$. For part (b), calculate power loss and use $P_{\text{loss}} = I^2R$. For part (c), use $R = \rho L/A$ with copper's resistivity. For parts (d) and (e), examine the physical reasonableness of the results.

Solution

(a) Calculate the current:

$$I = \frac{P}{V} = \frac{1.00\times10^2\text{ MW}}{10.0\text{ kV}} = \frac{1.00\times10^8\text{ W}}{1.00\times10^4\text{ V}} = 1.00\times10^4\text{ A} = 10.0\text{ kA}$$

(b) Calculate the allowable power loss:

$$P_{\text{loss}} = 0.0100\% \times P = 0.000100 \times 1.00\times10^8\text{ W} = 1.00\times10^4\text{ W} = 10.0\text{ kW}$$

Find the resistance:

$$R = \frac{P_{\text{loss}}}{I^2} = \frac{1.00\times10^4\text{ W}}{(1.00\times10^4\text{ A})^2} = \frac{1.00\times10^4}{1.00\times10^8} = 1.00\times10^{-4}\ \Omega$$

(c) Calculate the cross-sectional area using $R = \rho L/A$:

$$A = \frac{\rho L}{R} = \frac{(1.68\times10^{-8}\ \Omega\cdot\text{m})(1000\text{ m})}{1.00\times10^{-4}\ \Omega} = \frac{1.68\times10^{-5}}{1.00\times10^{-4}} = 0.168\text{ m}^2$$

Find the diameter:

$$A = \pi r^2 = \pi\left(\frac{d}{2}\right)^2$$
$$d = 2\sqrt{\frac{A}{\pi}} = 2\sqrt{\frac{0.168}{\pi}} = 2\sqrt{0.0535} = 2(0.231) = 0.462\text{ m} = 46.2\text{ cm}$$

(d) and **(e)**

Discussion

The diameter of 46.2 cm (nearly half a meter!) is completely unreasonable for a transmission wire. Such a massive wire would be:

1. **Prohibitively expensive**: The volume of copper needed would be enormous. For 1 km, the volume would be $V = AL = (0.168\ \text{m}^2)(1000\ \text{m}) = 168\ \text{m}^3$. At copper's density of 8960 kg/m³, this would be about 1.5 million kg of copper per kilometer!
2. **Structurally impractical**: A wire this thick and heavy would be impossible to support on transmission towers.
3. **Unreasonable premise**: The fundamental problem is the combination of relatively low transmission voltage (10 kV) with very high power (100 MW) and extremely low acceptable loss (0.01%).

In reality, high-power transmission uses voltages in the hundreds of kilovolts (100-765 kV) to keep currents manageable and power losses acceptable with reasonable wire sizes. The 10.0 kV voltage is too low for transmitting 100 MW efficiently. Either the voltage must be increased by a factor of 10-100, or the acceptable power loss must be increased to a more practical value like 5-10%.

(a) 10.0 kA. (b) 1.00×10^{-4} Ω. (c) 46.2 cm diameter. (d) The wire diameter is absurdly large and impractical. (e) The 10 kV transmission voltage is unreasonably low for transmitting 100 MW with only 0.01% loss.

Construct Your Own Problem

Consider an electric immersion heater used to heat a cup of water to make tea. Construct a problem in which you calculate the needed resistance of the heater so that it increases the temperature of the water and cup in a reasonable amount of time. Also calculate the cost of the electrical energy used in your process. Among the things to be considered are the voltage used, the masses and heat capacities involved, heat losses, and the time over which the heating takes place. Your instructor may wish for you to consider a thermal safety switch (perhaps bimetallic) that will halt the process before damaging temperatures are reached in the immersion unit.

Glossary

electric power
: the rate at which electrical energy is supplied by a source or dissipated by a device; it is the product of current times voltage

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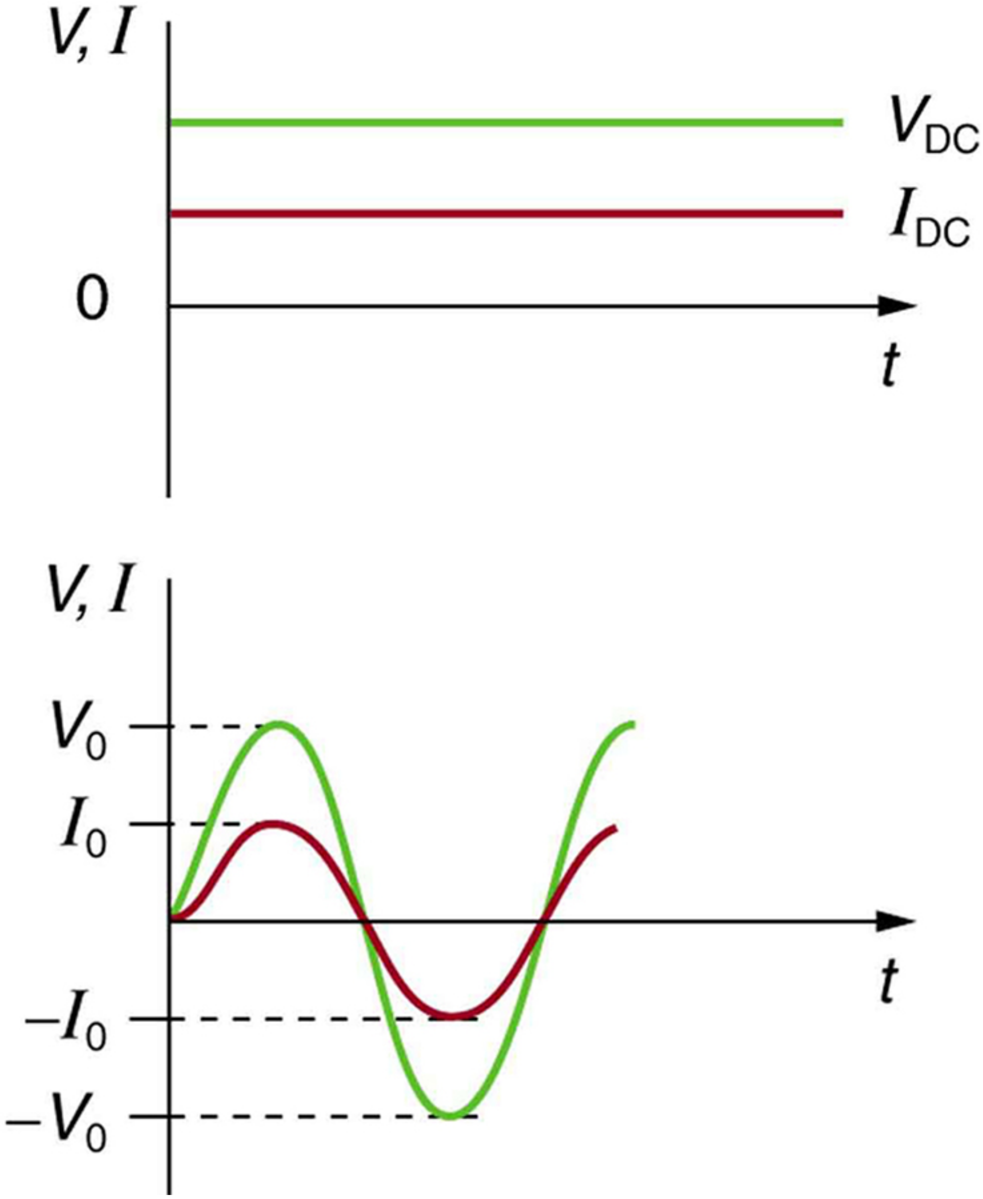
Alternating Current versus Direct Current

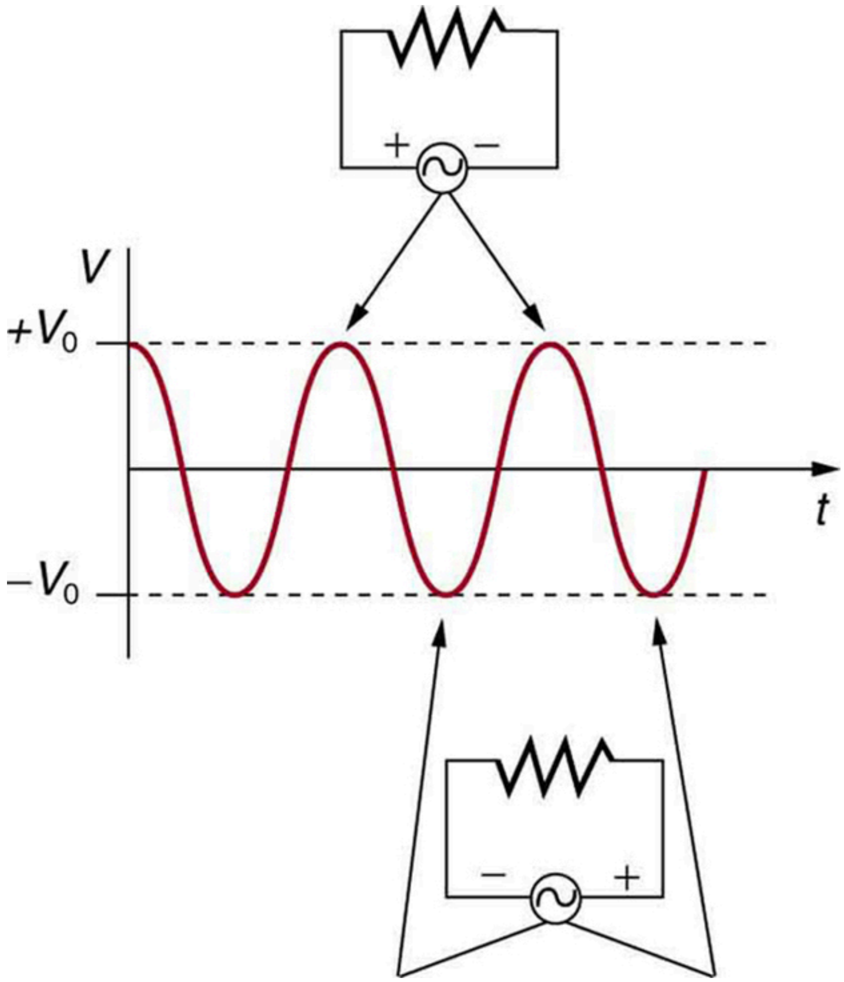
- Explain the differences and similarities between AC and DC current.
- Calculate rms voltage, current, and average power.
- Explain why AC current is used for power transmission.

Alternating Current

Most of the examples dealt with so far, and particularly those utilizing batteries, have constant voltage sources. Once the current is established, it is thus also a constant. **Direct current** (DC) is the flow of electric charge in only one direction. It is the steady state of a constant-voltage circuit. Most well-known applications, however, use a time-varying voltage source. **Alternating current** (AC) is the flow of electric charge that periodically reverses direction. If the source varies periodically, particularly sinusoidally, the circuit is known as an alternating current circuit. Examples include the commercial and residential power that serves so many of our needs. [Figure 1] shows graphs of voltage and current versus time for typical DC and AC power. The AC voltages and frequencies commonly used in homes and businesses vary around the world.



(a) DC voltage and current are constant in time, once the current is established. (b) A graph of voltage and current versus time for 60-Hz AC power. The voltage and current are sinusoidal and are in phase for a simple resistance circuit. The frequencies and peak voltages of AC sources differ greatly.



The potential difference V between the terminals of an AC voltage source fluctuates as shown. The mathematical expression for V is given by $V=V_0 \sin{ 2 \pi f t }$

[Figure 2] shows a schematic of a simple circuit with an AC voltage source. The voltage between the terminals fluctuates as shown, with the **AC voltage** given by

$$V=V_0 \sin 2\pi \text{ft},$$

where V is the voltage at time t, V_0 is the peak voltage, and f is the frequency in hertz. For this simple resistance circuit, $I=V/R$, and so the **AC current** is

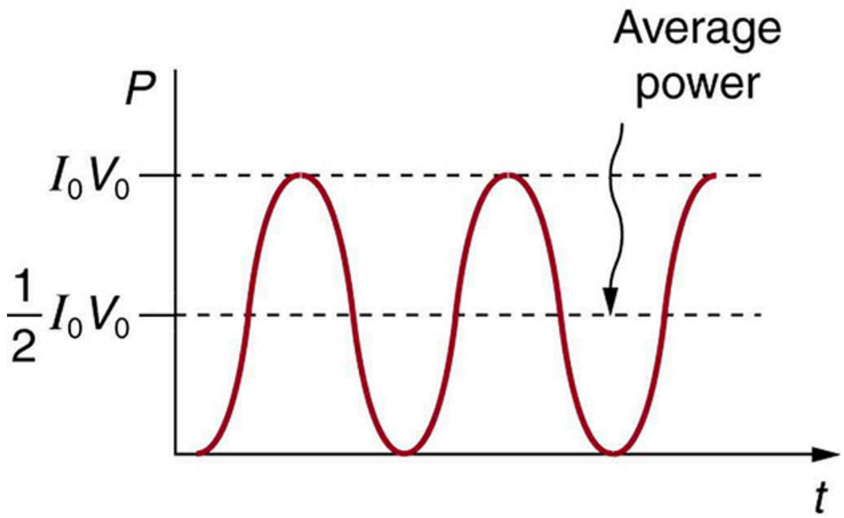
$$I=I_0 \sin 2\pi \text{ft},$$

where I is the current at time t, and $I_0=V_0/\text{R}$ is the peak current. For this example, the voltage and current are said to be in phase, as seen in [Figure 1](b).

Current in the resistor alternates back and forth just like the driving voltage, since $I=V/R$. If the resistor is a fluorescent light bulb, for example, it brightens and dims 120 times per second as the current repeatedly goes through zero. A 120-Hz flicker is too rapid for your eyes to detect, but if you wave your hand back and forth between your face and a fluorescent light, you will see a stroboscopic effect evidencing AC. The fact that the light output fluctuates means that the power is fluctuating. The power supplied is $P=IV$. Using the expressions for I and V above, we see that the time dependence of power is $P=I_0 V_0 \sin^2 2\pi \text{ft}$, as shown in [Figure 3].

Making Connections: Take-Home Experiment—AC/DC Lights

Wave your hand back and forth between your face and a fluorescent light bulb. Do you observe the same thing with the headlights on your car? Explain what you observe. *Warning: Do not look directly at very bright light*.



AC power as a function of time. Since the voltage and current are in phase here, their product is non-negative and fluctuates between zero and $I_0 V_0$. Average power is $1/2 I_0 V_0$.

We are most often concerned with average power rather than its fluctuations—that 60-W light bulb in your desk lamp has an average power consumption of 60 W, for example. As illustrated in [Figure 3], the average power P_{ave} is

$$P_{ave}=\frac{1}{2}I_0V_0.$$

This is evident from the graph, since the areas above and below the $(1/2)I_0V_0$ line are equal, but it can also be proven using trigonometric identities. Similarly, we define an average or **rms current** I_{rms} and average or **rms voltage** V_{rms} to be, respectively,

$$I_{rms}=\frac{I_0}{\sqrt{2}}$$

and

$$V_{rms}=\frac{V_0}{\sqrt{2}}.$$

where rms stands for root-mean-square, a particular kind of average. In general, to obtain a root-mean-square, the particular quantity is squared, its mean (or average) is found, and the square root is taken. This is useful for AC, since the average value is zero. Now,

$$P_{ave}=I_{rms}V_{rms},$$

which gives

$$P_{ave}=\frac{I_0}{\sqrt{2}}\cdot\frac{V_0}{\sqrt{2}}=\frac{1}{2}I_0V_0,$$

as stated above. It is standard practice to quote I_{rms} , V_{rms} , and P_{ave} rather than the peak values. For example, most household electricity is 120 V AC, which means that V_{rms} ** is 120 V. The common 10-A circuit breaker will interrupt a sustained I_{rms} ** greater than 10 A. Your 1.0-kW microwave oven consumes $P_{ave} = 1.0 \text{kW}$, and so on. You can think of these rms and average values as the equivalent DC values for a simple resistive circuit.

To summarize, when dealing with AC, Ohm's law and the equations for power are completely analogous to those for DC, but rms and average values are used for AC. Thus, for AC, Ohm's law is written

$$I_{rms}=\frac{V_{rms}}{R}.$$

The various expressions for AC power P_{ave} are

$$P_{ave}=I_{rms}V_{rms},$$
$$P_{ave}=\frac{V^2_{rms}}{R},$$

and

$$P_{ave}=I^2_{rms}R.$$

Peak Voltage and Power for AC

(a) What is the value of the peak voltage for 120-V AC power? (b) What is the peak power consumption rate of a 60.0-W AC light bulb?

Strategy

We are told that V_{rms} is 120 V and P_{ave} is 60.0 W. We can use $V_{\text{rms}} = V_0\sqrt{2}$ to find the peak voltage, and we can manipulate the definition of power to find the peak power from the given average power.

Solution for (a)

Solving the equation $V_{\text{rms}} = V_0\sqrt{2}$ for the peak voltage V_0 and substituting the known value for V_{rms} gives

$$V_0=\sqrt{2}V_{\text{rms}}=1.414(120\text{V})=170\text{V}.$$

Discussion for (a)

This means that the AC voltage swings from 170 V to -170V and back 60 times every second. An equivalent DC voltage is a constant 120 V.

Solution for (b)

Peak power is peak current times peak voltage. Thus,

$$P_0=I_0V_0=2(12I_0V_0)=2P_{\text{ave}}.$$

We know the average power is 60.0 W, and so

$$P_0=2(60.0\text{W})=120\text{W}.$$

Discussion

So the power swings from zero to 120 W one hundred twenty times per second (twice each cycle), and the power averages 60 W.

Why Use AC for Power Distribution?

Most large power-distribution systems are AC. Moreover, the power is transmitted at much higher voltages than the 120-V AC (240 V in most parts of the world) we use in homes and on the job. Economies of scale make it cheaper to build a few very large electric power-generation plants than to build numerous small ones. This necessitates sending power long distances, and it is obviously important that energy losses en route be minimized. High voltages can be transmitted with much smaller power losses than low voltages, as we shall see. (See [Figure 4].) For safety reasons, the voltage at the user is reduced to familiar values. The crucial factor is that it is much easier to increase and decrease AC voltages than DC, so AC is used in most large power distribution systems.

Power is distributed over large distances at high voltage to reduce power loss in the transmission lines. The voltages generated at the power plant are stepped up by passive devices called transformers (see Transformers) to 330 000 volts (or more in some places worldwide). At the point of use, the transformers reduce the voltage transmitted for safe residential and commercial use. (Credit: GeorgHH, Wikimedia Commons)

Power Losses Are Less for High-Voltage Transmission

(a) What current is needed to transmit 100 MW of power at 200 kV? (b) What is the power dissipated by the transmission lines if they have a resistance of 1.00Ω ? (c) What percentage of the power is lost in the transmission lines?

Strategy

We are given $P_{\text{ave}} = 100\text{MW}$, $V_{\text{rms}} = 200\text{kV}$, and the resistance of the lines is $R = 1.00\Omega$. Using these givens, we can find the current flowing (from $P = IV$) and then the power dissipated in the lines ($P = I^2R$), and we take the ratio to the total power transmitted.

Solution

To find the current, we rearrange the relationship $P_{\text{ave}} = I_{\text{rms}}V_{\text{rms}}$ and substitute known values. This gives

$$I_{\text{rms}}=\frac{P_{\text{ave}}}{V_{\text{rms}}}=\frac{100\times10^6\text{W}}{200\times10^3\text{V}}=500\text{A}.$$

Solution

Knowing the current and given the resistance of the lines, the power dissipated in them is found from $P_{\text{ave}} = I^2_{\text{rms}}R$. Substituting the known values gives

$$P_{\text{ave}}=I^2_{\text{rms}}R=(500\text{A})^2(1.00\Omega)=250\text{kW}.$$

Solution

The percent loss is the ratio of this lost power to the total or input power, multiplied by 100:

$$\%\text{loss}=\frac{250\text{kW}}{100\text{MW}}\times100=0.250\%.$$

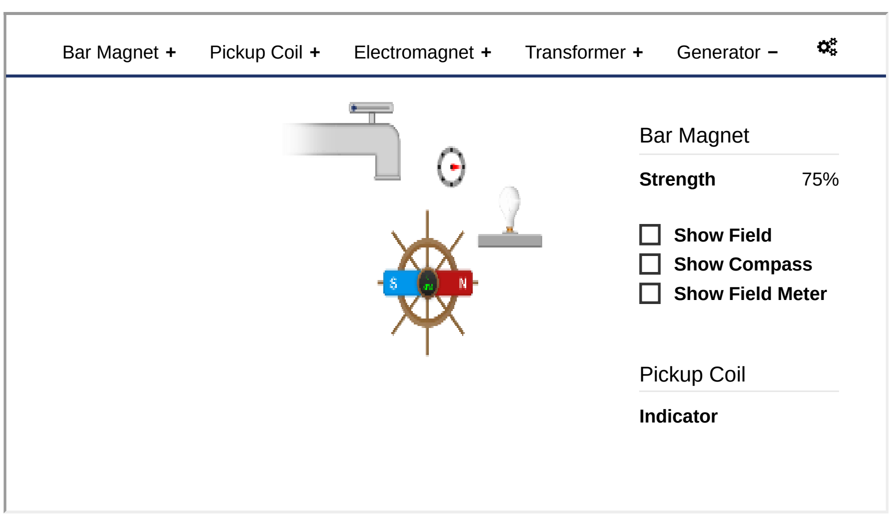
Discussion

One-fourth of a percent is an acceptable loss. Note that if 100 MW of power had been transmitted at 25 kV, then a current of 4000 A would have been needed. This would result in a power loss in the lines of 16.0 MW, or 16.0% rather than 0.250%. The lower the voltage, the more current is needed, and the greater the power loss in the fixed-resistance transmission lines. Of course, lower-resistance lines can be built, but this requires larger and more expensive wires. If superconducting lines could be economically produced, there would be no loss in the transmission lines at all. But, as we shall see in a later chapter, there is a limit to current in superconductors, too. In short, high voltages are more economical for transmitting power, and AC voltage is much easier to raise and lower, so that AC is used in most large-scale power distribution systems.

It is widely recognized that high voltages pose greater hazards than low voltages. But, in fact, some high voltages, such as those associated with common static electricity, can be harmless. So it is not voltage alone that determines a hazard. It is not so widely recognized that AC shocks are often more harmful than similar DC shocks. Thomas Edison thought that AC shocks were more harmful and set up a DC power-distribution system in New York City in the late 1800s. There were bitter fights, in particular between Edison and George Westinghouse and Nikola Tesla, who were advocating the use of AC in early power-distribution systems. AC has prevailed largely due to transformers and lower power losses with high-voltage transmission.

PhET Explorations: Generator

Generate electricity with a bar magnet! Discover the physics behind the phenomena by exploring magnets and how you can use them to make a bulb light.



Section Summary

- Direct current (DC) is the flow of electric current in only one direction. It refers to systems where the source voltage is constant.
- The voltage source of an alternating current (AC) system puts out $V = V_0 \sin 2\pi \text{ft}$, where V is the voltage at time t , V_0 is the peak voltage, and f is the frequency in hertz. - In a simple circuit, $I = V/R$ and AC current is $I = I_0 \sin 2\pi \text{ft}$, where I is the current at time t , and $I_0 = V_0/\text{R}$ is the peak current. - The average AC power is $P_{\text{ave}} = 12 I_0 V_0$.
- Average (rms) current I_{rms} and average (rms) voltage V_{rms} are $I_{\text{rms}} = I_0 \sqrt{2}$ and $V_{\text{rms}} = V_0 \sqrt{2}$, where rms stands for root mean square. - Thus, $P_{\text{ave}} = I_{\text{rms}} V_{\text{rms}}$.
- Ohm's law for AC is $I_{\text{rms}} = V_{\text{rms}} R$.
- Expressions for the average power of an AC circuit are $P_{\text{ave}} = I_{\text{rms}} V_{\text{rms}}$, $P_{\text{ave}} = V\ {}_{2\text{rms}} R$, and $P_{\text{ave}} = I\ {}_{2\text{rms}} R$, analogous to the expressions for DC circuits.

Conceptual Questions

Give an example of a use of AC power other than in the household. Similarly, give an example of a use of DC power other than that supplied by batteries.

Why do voltage, current, and power go through zero 120 times per second for 60-Hz AC electricity?

You are riding in a train, gazing into the distance through its window. As close objects streak by, you notice that the nearby fluorescent lights make *dashed* streaks. Explain.

Problem Exercises

(a) What is the hot resistance of a 25-W light bulb that runs on 120-V AC? (b) If the bulb's operating temperature is 2700°C , what is its resistance at 2600°C ?

Show Solution

Strategy

For part (a), use $P = V\ {}_{2\text{rms}}/R$. For part (b), use the temperature dependence of resistance $R = R_0(1 + \alpha \Delta T)$ with the tungsten temperature coefficient.

Solution

(a) Calculate the hot resistance:

$$R = V\ {}_{2\text{rms}} P = (120\text{ V})^2 25\text{ W} = 14400\text{ V}^2 25\text{ W} = 576\ \Omega$$

(b) Use the temperature coefficient for tungsten ($\alpha = 4.5 \times 10^{-3}$ /°C). With $\Delta T = 2600 - 2700 = -100$ °C from the hot operating temperature:

$$R_{2600} = R_{2700}(1 + \alpha \Delta T) = 576\ \Omega[1 + (4.5 \times 10^{-3}\ /°\text{C})(-100\ °\text{C})]$$
$$R_{2600} = 576\ \Omega(1 - 0.45) = 576\ \Omega(0.55) = 317\ \Omega$$

Discussion

The hot resistance is quite high (576 Ω). At 100°C lower temperature, the resistance drops to 317 Ω—a 45% decrease. This significant temperature dependence explains why incandescent bulbs draw much more current when first switched on (when cold) than during normal operation.

(a) The hot resistance is 576 Ω. (b) At 2600°C, the resistance is approximately 317 Ω.

Certain heavy industrial equipment uses AC power that has a peak voltage of 679 V. What is the rms voltage?

Show Solution

480 V

A certain circuit breaker trips when the rms current is 15.0 A. What is the corresponding peak current?

Show Solution

Strategy

We use the relationship between rms and peak current: $I_{\text{rms}} = I_0 \sqrt{2}$. Solving for the peak current I_0 will give us the maximum instantaneous current that flows through the circuit.

Solution

Rearrange the rms current equation to solve for peak current:

$$I_0=\sqrt{2}\cdot I_{rms}$$

Substitute the known value:

$$I_0=\sqrt{2}\times 15.0\text{ A}=1.414\times 15.0\text{ A}=21.2\text{ A}$$

Discussion

The peak current of 21.2 A is about 41% higher than the rms value of 15.0 A. This is an important consideration for circuit breakers and fuses, which must be designed to handle the peak current without tripping during normal operation, while still protecting against sustained overcurrent conditions. The breaker responds to the heating effect of current, which is proportional to I^2, and thus relates to the rms value rather than the instantaneous peaks.

The corresponding peak current is 21.2 A.

Military aircraft use 400-Hz AC power, because it is possible to design lighter-weight equipment at this higher frequency. What is the time for one complete cycle of this power?

Show Solution

2.50 ms

A North American tourist takes his 25.0-W, 120-V AC razor to Europe, finds a special adapter, and plugs it into 240 V AC. Assuming constant resistance, what power does the razor consume as it is ruined?

Show Solution

Strategy

First, find the razor's resistance using its rated power and voltage with $P=V^2/R$. Then, use this resistance to calculate the power consumed when connected to the higher European voltage.

Solution

Calculate the razor's resistance from its rated specifications:

$$R=\frac{V^2}{P}=\frac{(120\text{ V})^2}{25.0\text{ W}}=\frac{14400\text{ V}^2}{25.0\text{ W}}=576\ \Omega$$

Now calculate the power consumed when connected to 240 V AC with the same resistance:

$$P_{Europe}=\frac{V^2}{R}=\frac{(240\text{ V})^2}{576\ \Omega}=\frac{57600\text{ V}^2}{576\ \Omega}=100\text{ W}$$

Alternatively, since $P\propto V^2$ for constant resistance, doubling the voltage quadruples the power:

$$P_{Europe}=P_{rated}\times\left(\frac{V_{Europe}}{V_{rated}}\right)^2=25.0\text{ W}\times\left(\frac{240}{120}\right)^2=25.0\text{ W}\times 4=100\text{ W}$$

Discussion

The razor consumes four times its rated power (100 W instead of 25 W). This excessive power will cause the razor's motor to overheat and burn out, the heating elements to glow too hot, and potentially cause a fire or electrical damage. This is why travelers need voltage converters, not just plug adapters, when using electrical devices in countries with different voltage standards. Modern "dual voltage" devices (labeled 120-240 V) have internal circuitry that adjusts for different voltages.

The razor consumes 100 W when connected to 240 V AC, which is four times its rated power and will damage or destroy the device.

In this problem, you will verify statements made at the end of the power losses for [Example 2]. (a) What current is needed to transmit 100 MW of power at a voltage of 25.0 kV? (b) Find the power loss in a $1.00\text{-}\Omega$ transmission line. (c) What percent loss does this represent?

Show Solution

(a) 4.00 kA

(b) 16.0 MW

(c) 16.0%

A small office-building air conditioner operates on 408-V AC and consumes 50.0 kW. (a) What is its effective resistance? (b) What is the cost of running the air conditioner during a hot summer month when it is on 8.00 h per day for 30 days and electricity costs $9.00\text{cents/kW}\cdot\text{h}$?

Show Solution

Strategy

For part (a), use the power equation $P = V^2/R$ to find the effective resistance. For part (b), calculate the total energy consumed in kW·h, then multiply by the cost per kW·h.

Solution

(a) Rearrange the power equation to solve for resistance:

$$R = \frac{V^2}{P} = \frac{(408\text{ V})^2}{50.0 \times 10^3\text{ W}} = \frac{166464\text{ V}^2}{50000\text{ W}} = 3.33\ \Omega$$

(b) Calculate the total energy consumed:

$$E = P \times t = 50.0\text{ kW} \times 8.00\text{ h/day} \times 30\text{ days} = 12000\text{ kW·h}$$

Calculate the cost:

$$\text{Cost} = E \times \text{rate} = 12000\text{ kW·h} \times \$0.0900/\text{kW·h} = \$1080$$

Discussion

The effective resistance of 3.33 Ω is quite low, which is typical for high-power devices. A low resistance allows a large current to flow, enabling high power consumption. The current drawn is $I = P/V = 50000\text{ W}/408\text{ V} = 122.5\text{ A}$, which is substantial and requires heavy-duty wiring.

The monthly operating cost of $1080 is significant but reasonable for a commercial air conditioning system running 8 hours daily. This cost analysis helps building managers budget for energy expenses and evaluate the economic benefit of more efficient HVAC systems.

(a) The effective resistance is 3.33 Ω. (b) The cost of running the air conditioner for the month is $1080.

What is the peak power consumption of a 120-V AC microwave oven that draws 10.0 A?

Show Solution

2.40 kW

What is the peak current through a 500-W room heater that operates on 120-V AC power?

Show Solution

Strategy

First, find the rms current using $P_{\text{ave}} = I_{\text{rms}} V_{\text{rms}}$. Then convert to peak current using $I_0 = \sqrt{2} \cdot I_{\text{rms}}$.

Solution

Calculate the rms current from the average power:

$$I_{\text{rms}} = \frac{P_{\text{ave}}}{V_{\text{rms}}} = \frac{500\text{ W}}{120\text{ V}} = 4.17\text{ A}$$

Convert to peak current:

$$I_0 = \sqrt{2} \cdot I_{\text{rms}} = 1.414 \times 4.17\text{ A} = 5.89\text{ A}$$

Discussion

The peak current of 5.89 A is the maximum instantaneous current flowing through the heater during each AC cycle. The rms current of 4.17 A represents the effective DC equivalent that would produce the same heating effect. For a resistive load like a heater, the current and voltage are in phase, so the peak current occurs at the same instant as the peak voltage. The wiring and circuit breaker for this heater must be rated to handle currents above the peak value to provide a safety margin.

The peak current through the room heater is 5.89 A.

Two different electrical devices have the same power consumption, but one is meant to be operated on 120-V AC and the other on 240-V AC. (a) What is the ratio of their resistances? (b) What is the ratio of their currents? (c) Assuming its resistance is unaffected, by what factor will the power increase if a 120-V AC device is connected to 240-V AC?

Show Solution

(a) 4.0

(b) 0.50

(c) 4.0

Nichrome wire is used in some radiative heaters. (a) Find the resistance needed if the average power output is to be 1.00 kW utilizing 120-V AC. (b) What length of Nichrome wire, having a cross-sectional area of 5.00mm^2 , is needed if the operating temperature is 500°C ? (c) What power will it draw when first switched on?

Show Solution

Strategy

For part (a), use $P = V^2/R$ to find the required hot resistance. For part (b), use the resistivity formula $R = \rho L/A$ with the resistivity at the operating temperature. For part (c), find the cold resistance using the temperature coefficient and then calculate the power at room temperature.

Solution

(a) Find the hot resistance required for 1.00 kW:

$$R_{\text{hot}} = \frac{V^2}{P} = \frac{(120\ \text{V})^2}{1000\ \text{W}} = \frac{14400\ \text{V}^2}{1000\ \text{W}} = 14.4\ \Omega$$

(b) Use the resistivity of Nichrome at 500°C. First, find the resistivity at the operating temperature using $\rho = \rho_0(1 + \alpha \Delta T)$, where $\rho_0 = 1.00 \times 10^{-6}\ \Omega\cdot\text{m}$ at 20°C and $\alpha = 0.4 \times 10^{-3}$ /°C for Nichrome:

$$\rho_{500} = (1.00\times10^{-6}\ \Omega\cdot\text{m})[1+(0.4\times10^{-3}\ /^{\circ}\text{C})(500-20)\ ^{\circ}\text{C}]$$
$$\rho_{500} = (1.00\times10^{-6}\ \Omega\cdot\text{m})(1+0.192) = 1.19\times10^{-6}\ \Omega\cdot\text{m}$$

Solve for length using $R = \rho L/A$:

$$L = \frac{RA}{\rho} = \frac{(14.4\ \Omega)(5.00\times10^{-6}\ \text{m}^2)}{1.19\times10^{-6}\ \Omega\cdot\text{m}} = 60.5\ \text{m}$$

(c) Find the cold resistance at 20°C. Since $R \propto \rho$:

$$R_{\text{cold}} = R_{\text{hot}} \times \frac{\rho_{20}}{\rho_{500}} = 14.4\ \Omega \times \frac{1.00}{1.19} = 12.1\ \Omega$$

Calculate the initial power:

$$P_{\text{cold}} = \frac{V^2}{R_{\text{cold}}} = \frac{(120\ \text{V})^2}{12.1\ \Omega} = 1.19\ \text{kW}$$

Discussion

The heater requires 60.5 m of Nichrome wire, which is typically wound into a compact coil. When first switched on, the heater draws about 19% more power (1.19 kW vs 1.00 kW) because the cold resistance is lower. This is opposite to the behavior of incandescent bulbs, where tungsten has a much higher temperature coefficient and draws several times more current when cold. Nichrome's relatively small temperature coefficient ($\alpha = 0.4 \times 10^{-3}$ /°C vs. tungsten's 4.5×10^{-3} /°C) makes it ideal for heating elements because the power output remains relatively stable as the element heats up.

(a) The required resistance is 14.4 Ω. (b) The length of Nichrome wire needed is 60.5 m. (c) When first switched on, the heater draws 1.19 kW.

Find the time after $t = 0$ when the instantaneous voltage of 60-Hz AC first reaches the following values: (a) $V_0/2$ (b) V_0 (c) 0.

Show Solution

(a) 1.39 ms

(b) 4.17 ms

(c) 8.33 ms

(a) At what two times in the first period following $t = 0$ does the instantaneous voltage in 60-Hz AC equal V_{rms} ? (b) $-V_{\text{rms}}$?

Show Solution

Strategy

Use the AC voltage equation $V = V_0 \sin(2\pi f t)$ and the relationship $V_{\text{rms}} = V_0/\sqrt{2}$. Set the instantaneous voltage equal to $\pm V_{\text{rms}}$ and solve for time. For 60-Hz AC, the period is $T = 1/f = 1/60$ s = 16.67 ms.

Solution

(a) Find when $V = V_{\text{rms}} = V_0/\sqrt{2}$:

$$V_0 \sin(2\pi f t) = \frac{V_0}{\sqrt{2}}$$

$\sin(2\pi f t)=1\sqrt{2}=0.707$

The sine function equals $1/\sqrt{2}$ at angles of 45° and 135° (or $\pi/4$ and $3\pi/4$ radians) within one period:

For the first time:

$$2\pi f t_1=\pi 4$$
$$t_1=18f=18\times 60\text{ Hz}=2.08\text{ ms}$$

For the second time:

$$2\pi f t_2=3\pi 4$$
$$t_2=38f=38\times 60\text{ Hz}=6.25\text{ ms}$$

(b) Find when $V=-V_{\text{rms}}=-V_0/\sqrt{2}$:

$\sin(2\pi f t)=-1\sqrt{2}=-0.707$

The sine function equals $-1/\sqrt{2}$ at angles of 225° and 315° (or $5\pi/4$ and $7\pi/4$ radians) within one period:

For the first time:

$$t_3=58f=58\times 60\text{ Hz}=10.4\text{ ms}$$

For the second time:

$$t_4=78f=78\times 60\text{ Hz}=14.6\text{ ms}$$

Discussion

These times divide the period into eight equal parts of 2.08 ms each. The rms voltage represents the effective DC equivalent, and the instantaneous AC voltage crosses through this value four times per cycle—twice positive and twice negative. Notice that the voltage spends more time below V_{rms} than above it during the positive half-cycle. This is because the sine wave varies slowly near its peaks and quickly near zero crossings. The rms value is calculated by averaging the square of the voltage, which gives extra weight to the peak regions.

(a) The voltage equals V_{rms} at $t=2.08$ ms and $t=6.25$ ms. (b) The voltage equals $-V_{\text{rms}}$ at $t=10.4$ ms and $t=14.6$ ms.

Glossary

direct current
: (DC) the flow of electric charge in only one direction

alternating current
: (AC) the flow of electric charge that periodically reverses direction

AC voltage
: voltage that fluctuates sinusoidally with time, expressed as $V=V_0\sin 2\pi f t$, where V is the voltage at time t, V_0 is the peak voltage, and f is the frequency in hertz

AC current
: current that fluctuates sinusoidally with time, expressed as $I=I_0\sin 2\pi f t$, where I is the current at time t, I_0 is the peak current, and f is the frequency in hertz

rms current
: the root-mean-square of the current, $I_{\text{rms}}=I_0/\sqrt{2}$, where I_0 is the peak current, in an AC system

rms voltage
: the root-mean-square of the voltage, $V_{\text{rms}}=V_0/\sqrt{2}$, where V_0 is the peak voltage, in an AC system

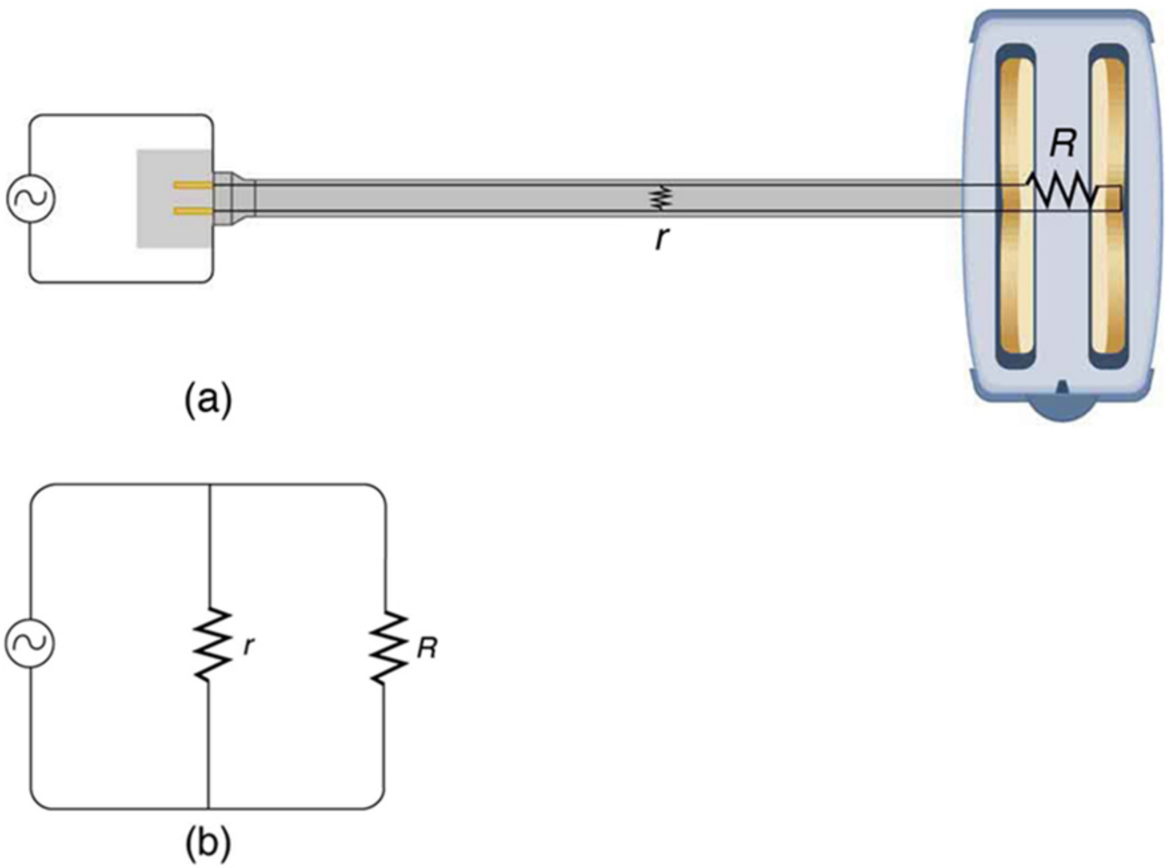
Electric Hazards and the Human Body

- Define thermal hazard, shock hazard, and short circuit.
- Explain what effects various levels of current have on the human body.

There are two known hazards of electricity—thermal and shock. A **thermal hazard** is one where excessive electric power causes undesired thermal effects, such as starting a fire in the wall of a house. A **shock hazard** occurs when electric current passes through a person. Shocks range in severity from painful, but otherwise harmless, to heart-stopping lethality. This section considers these hazards and the various factors affecting them in a quantitative manner. Electrical Safety: Systems and Devices will consider systems and devices for preventing electrical hazards.

Thermal Hazards

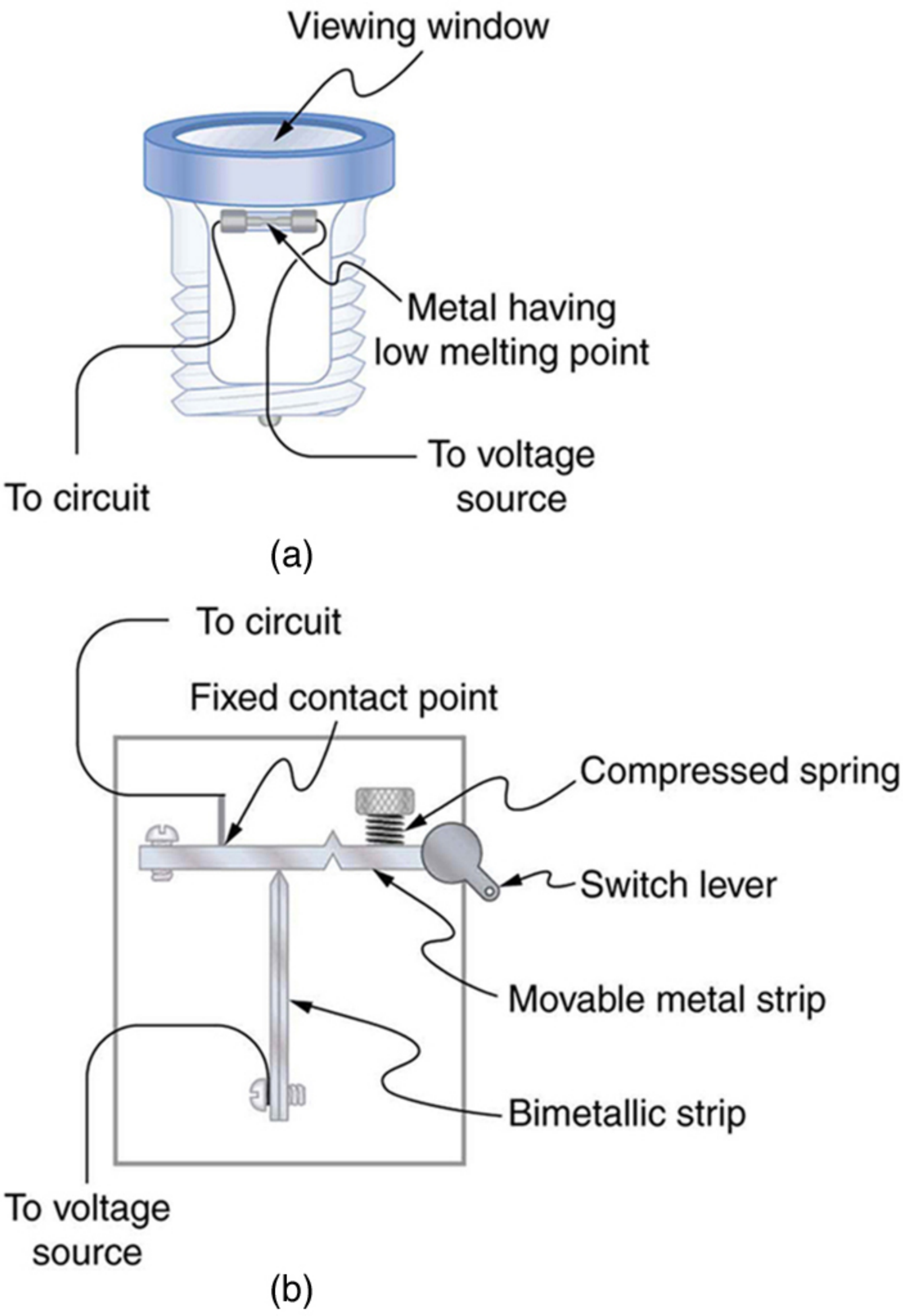
Electric power causes undesired heating effects whenever electric energy is converted to thermal energy at a rate faster than it can be safely dissipated. A classic example of this is the **short circuit**, a low-resistance path between terminals of a voltage source. An example of a short circuit is shown in [Figure 1]. Insulation on wires leading to an appliance has worn through, allowing the two wires to come into contact. Such an undesired contact with a high voltage is called a *short*. Since the resistance of the short, r, is very small, the power dissipated in the short, $P = V^2/r$, is very large. For example, if V is 120 V and r is 0.100Ω, then the power is 144 kW, *much* greater than that used by a typical household appliance. Thermal energy delivered at this rate will very quickly raise the temperature of surrounding materials, melting or perhaps igniting them.



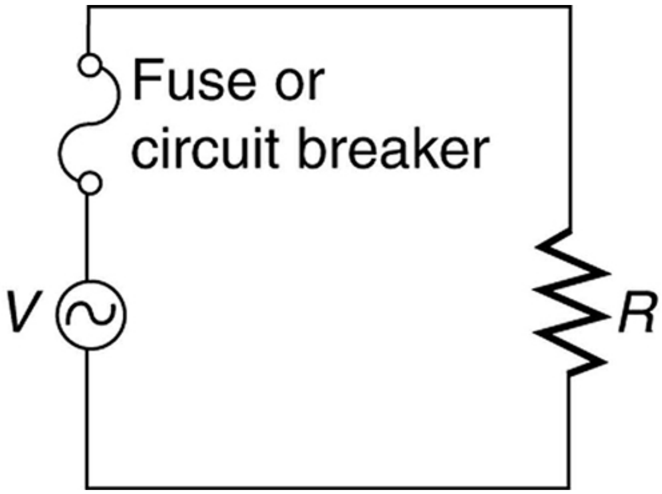
A short circuit is an undesired low-resistance path across a voltage source. (a) Worn insulation on the wires of a toaster allow them to come into contact with a low resistance r . Since $P=V^2/r$, thermal power is created so rapidly that the cord melts or burns. (b) A schematic of the short circuit.

One particularly insidious aspect of a short circuit is that its resistance may actually be decreased due to the increase in temperature. This can happen if the short creates ionization. These charged atoms and molecules are free to move and, thus, lower the resistance r. Since $P = V^2/r$, the power dissipated in the short rises, possibly causing more ionization, more power, and so on. High voltages, such as the 480-V AC used in some industrial applications, lend themselves to this hazard, because higher voltages create higher initial power production in a short.

Another serious, but less dramatic, thermal hazard occurs when wires supplying power to a user are overloaded with too great a current. As discussed in the previous section, the power dissipated in the supply wires is $P = I^2 R_W$, where R_W is the resistance of the wires and I the current flowing through them. If either I or R_W is too large, the wires overheat. For example, a worn appliance cord (with some of its braided wires broken) may have $R_W = 2.00 \Omega$ rather than the 0.100Ω it should be. If 10.0 A of current passes through the cord, then $P = I^2 R_W = 200 W$ is dissipated in the cord—much more than is safe. Similarly, if a wire with a $0.100\text{-}\Omega$ resistance is meant to carry a few amps, but is instead carrying 100 A, it will severely overheat. The power dissipated in the wire will in that case be $P = 1000\text{W}$. Fuses and circuit breakers are used to limit excessive currents. (See [Figure 2] and [Figure 3].) Each device opens the circuit automatically when a sustained current exceeds safe limits.



(a) A fuse has a metal strip with a low melting point that, when overheated by an excessive current, permanently breaks the connection of a circuit to a voltage source. (b) A circuit breaker is an automatic but restorable electric switch. The one shown here has a bimetallic strip that bends to the right and into the notch if overheated. The spring then forces the metal strip downward, breaking the electrical connection at the points.



Schematic of a circuit with a fuse or circuit breaker in it. Fuses and circuit breakers act like automatic switches that open when sustained current exceeds desired limits.

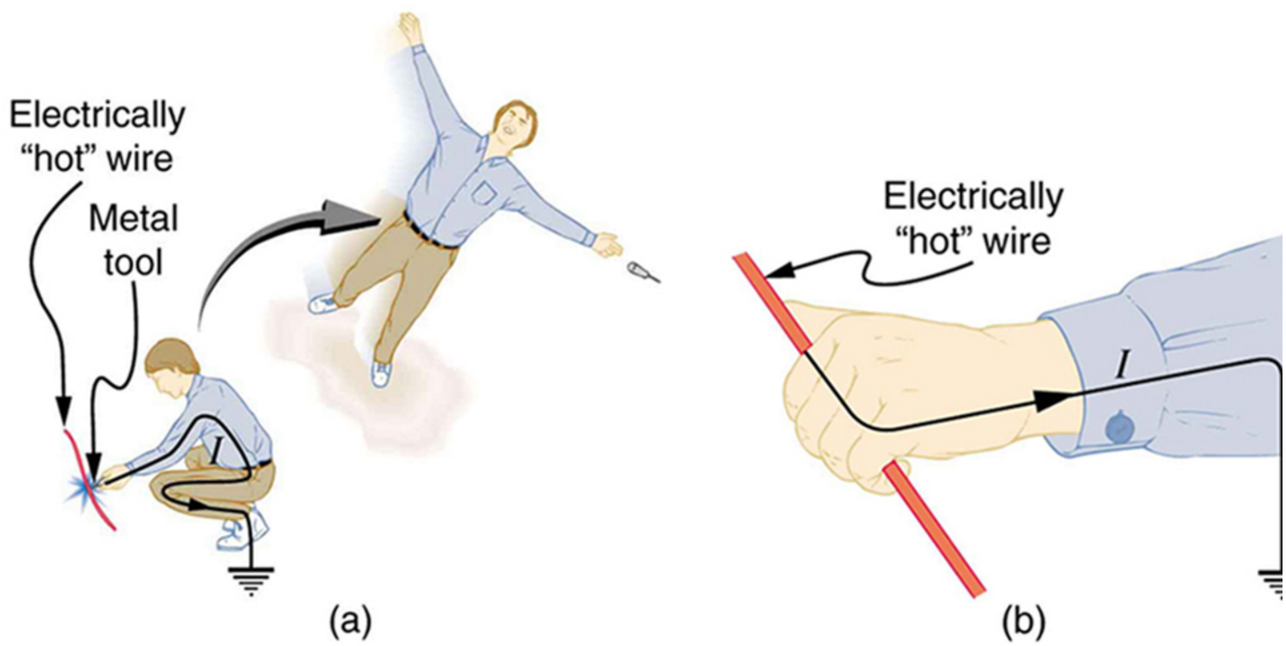
Fuses and circuit breakers for typical household voltages and currents are relatively simple to produce, but those for large voltages and currents experience special problems. For example, when a circuit breaker tries to interrupt the flow of high-voltage electricity, a spark can jump across its points that ionizes the air in the gap and allows the current to continue flowing. Large circuit breakers found in power-distribution systems employ insulating gas and even use jets of gas to blow out such sparks. Here AC is safer than DC, since AC current goes through zero 120 times per second, giving a quick opportunity to extinguish these arcs.

Shock Hazards

Electrical currents through people produce tremendously varied effects. An electrical current can be used to block back pain. The possibility of using electrical current to stimulate muscle action in paralyzed limbs, perhaps allowing paraplegics to walk, is under study. TV dramatizations in which electrical shocks are used to bring a heart attack victim out of ventricular fibrillation (a massively irregular, often fatal, beating of the heart) are more than common. Yet most electrical shock fatalities occur because a current put the heart into fibrillation. A pacemaker uses electrical shocks to stimulate the heart to beat properly. Some fatal shocks do not produce burns, but warts can be safely burned off with electric current (though freezing using liquid nitrogen is now more common). Of course, there are consistent explanations for these disparate effects. The major factors upon which the effects of electrical shock depend are

1. The amount of current I
2. The path taken by the current
3. The duration of the shock
4. The frequency f of the current ($f = 0$ for DC)

[Table 1] gives the effects of electrical shocks as a function of current for a typical accidental shock. The effects are for a shock that passes through the trunk of the body, has a duration of 1 s, and is caused by 60-Hz power.



An electric current can cause muscular contractions with varying effects. (a) The victim is “thrown” backward by involuntary muscle contractions that extend the legs and torso. (b) The victim can’t let go of the wire that is stimulating all the muscles in the hand. Those that close the fingers are stronger than those that open them.

Effects of Electrical Shock as a Function of Current[1]

Current (mA)	Effect
1	Threshold of sensation
5	Maximum harmless current
10–20	Onset of sustained muscular contraction; cannot let go for duration of shock; contraction of chest muscles may stop breathing during shock
50	Onset of pain
100–300+	Ventricular fibrillation possible; often fatal
300	Onset of burns depending on concentration of current
6000 (6 A)	Onset of sustained ventricular contraction and respiratory paralysis; both cease when shock ends; heartbeat may return to normal; used to defibrillate the heart

Our bodies are relatively good conductors due to the water in our bodies. Given that larger currents will flow through sections with lower resistance (to be further discussed in the next chapter), electric currents preferentially flow through paths in the human body that have a minimum resistance in a direct path to earth. The earth is a natural electron sink. Wearing insulating shoes, a requirement in many professions, prohibits a pathway for electrons by providing a large resistance in that path. Whenever working with high-power tools (drills) , or in risky situations, ensure that you do not provide a pathway for current flow (especially through the heart).

Very small currents pass harmlessly and unfelt through the body. This happens to you regularly without your knowledge. The threshold of sensation is only 1 mA and, although unpleasant, shocks are apparently harmless for currents less than 5 mA. A great number of safety rules take the 5-mA value for the maximum allowed shock. At 10 to 20 mA and above, the current can stimulate sustained muscular contractions much as regular nerve impulses do. People sometimes say they were knocked across the room by a shock, but what really happened was that certain muscles contracted, propelling them in a manner not of their own choosing. (See [Figure 4](a).) More frightening, and potentially more dangerous, is the “can’t let go” effect illustrated in [Figure 4](b) . The muscles that close the fingers are stronger than those that open them, so the hand closes involuntarily on the wire shocking it. This can prolong the shock indefinitely. It can also be a danger to a person trying to rescue the victim, because the rescuer’s hand may close about the victim’s wrist. Usually the best way to help the victim is to give the fist a hard knock/blow/jar with an insulator or to throw an insulator at the fist. Modern electric fences, used in animal enclosures, are now pulsed on and off to allow people who touch them to get free, rendering them less lethal than in the past.

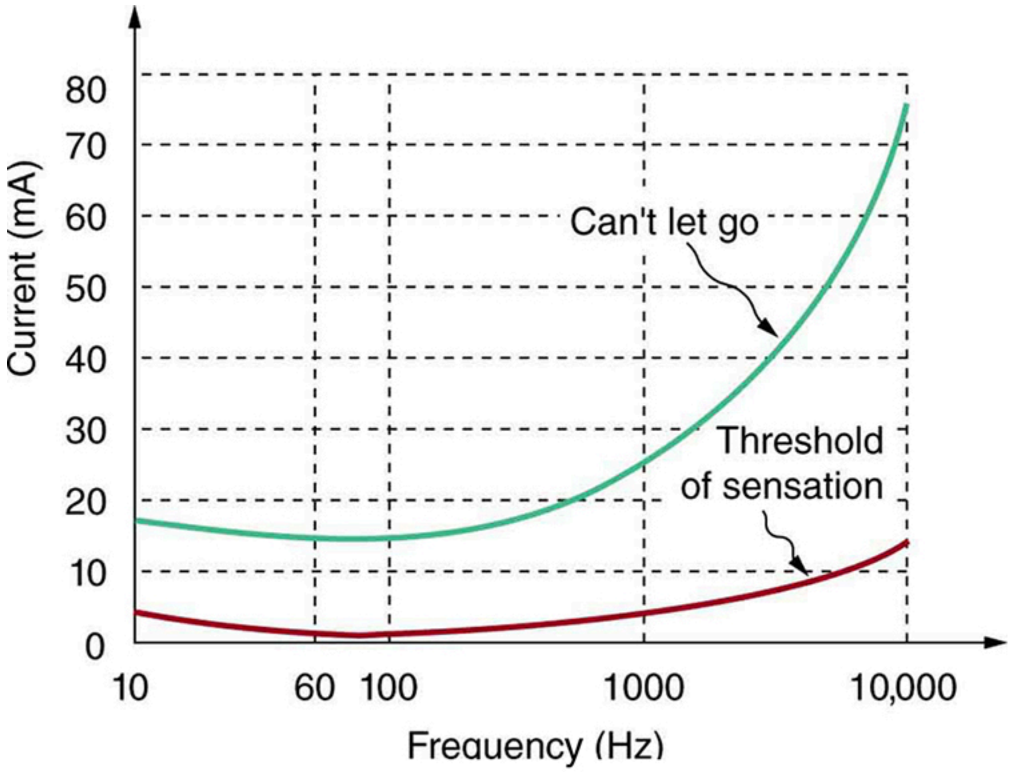
Greater currents may affect the heart. Its electrical patterns can be disrupted, so that it beats irregularly and ineffectively in a condition called “ventricular fibrillation.” This condition often lingers after the shock and is fatal due to a lack of blood circulation. The threshold for ventricular fibrillation is between 100 and 300 mA. At about 300 mA and above, the shock can cause burns, depending on the concentration of current—the more concentrated, the greater the likelihood of burns.

Very large currents cause the heart and diaphragm to contract for the duration of the shock. Both the heart and breathing stop. Interestingly, both often return to normal following the shock. The electrical patterns on the heart are completely erased in a manner that the heart can start afresh with normal beating, as opposed to the permanent disruption caused by smaller currents that can put the heart into ventricular fibrillation. The latter is something like scribbling on a blackboard, whereas the former completely erases it. TV dramatizations of electric shock used to bring a heart attack victim out of ventricular fibrillation also show large paddles. These are used to spread out current passed through the victim to reduce the likelihood of burns.

Current is the major factor determining shock severity (given that other conditions such as path, duration, and frequency are fixed, such as in the table and preceding discussion). A larger voltage is more hazardous, but since $I=V/R$

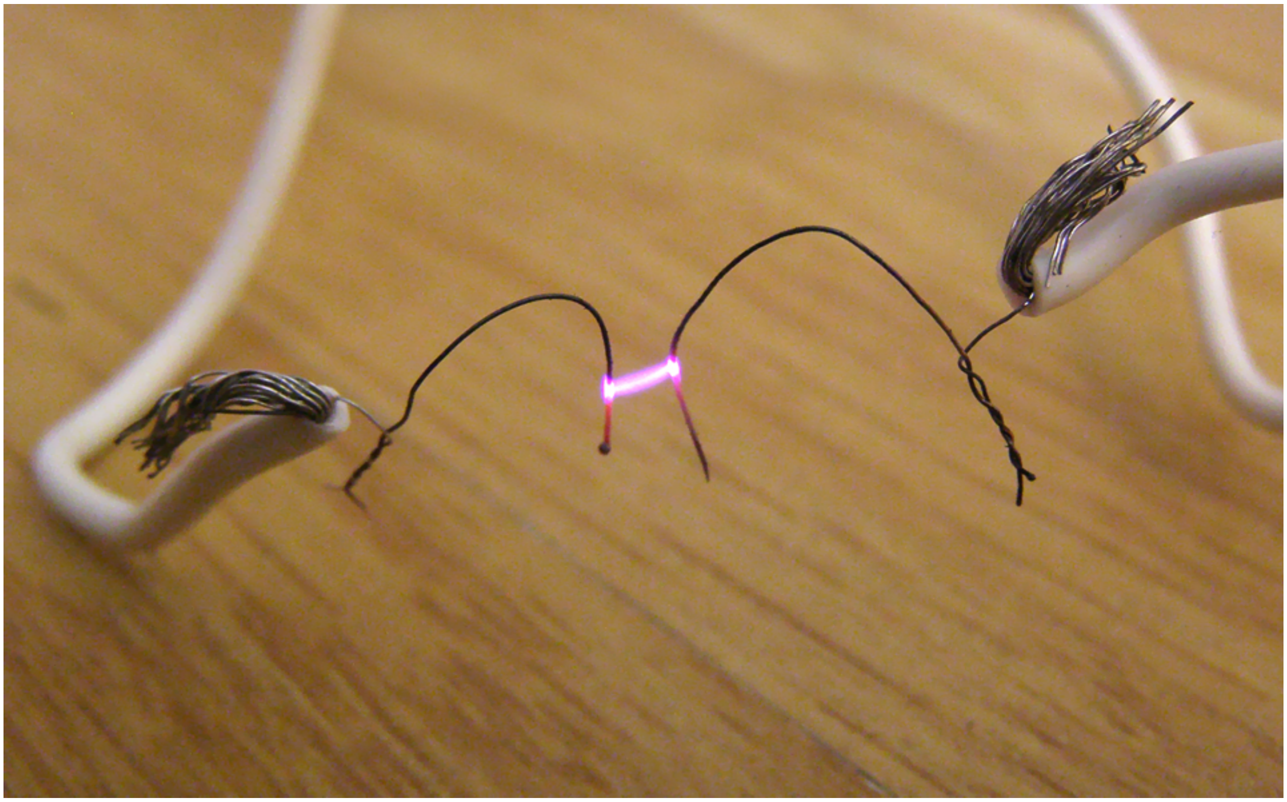
, the severity of the shock depends on the combination of voltage and resistance. For example, a person with dry skin has a resistance of about $200\text{k}\Omega$. If he comes into contact with 120-V AC, a current $I=(120\text{V})/(200\text{k}\Omega)=0.6\text{mA}$ passes harmlessly through him. The same person soaking wet may have a resistance of 10.0 \text{k}\Omega $$ and the same 120 V will produce a current of 12 mA—above the “can’t let go” threshold and potentially dangerous.

Most of the body’s resistance is in its dry skin. When wet, salts go into ion form, lowering the resistance significantly. The interior of the body has a much lower resistance than dry skin because of all the ionic solutions and fluids it contains. If skin resistance is bypassed, such as by an intravenous infusion, a catheter, or exposed pacemaker leads, a person is rendered **microshock sensitive**. In this condition, currents about 1/1000 those listed in [Table 1] produce similar effects. During open-heart surgery, currents as small as $20\mu\text{A}$ can be used to still the heart. Stringent electrical safety requirements in hospitals, particularly in surgery and intensive care, are related to the doubly disadvantaged microshock-sensitive patient. The break in the skin has reduced his resistance, and so the same voltage causes a greater current, and a much smaller current has a greater effect.



Graph of average values for the threshold of sensation and the "can't let go" current as a function of frequency. The lower the value, the more sensitive the body is at that frequency.

Factors other than current that affect the severity of a shock are its path, duration, and AC frequency. Path has obvious consequences. For example, the heart is unaffected by an electric shock through the brain, such as may be used to treat manic depression. And it is a general truth that the longer the duration of a shock, the greater its effects. [Figure 5] presents a graph that illustrates the effects of frequency on a shock. The curves show the minimum current for two different effects, as a function of frequency. The lower the current needed, the more sensitive the body is at that frequency. Ironically, the body is most sensitive to frequencies near the 50- or 60-Hz frequencies in common use. The body is slightly less sensitive for DC ($f=0$), mildly confirming Edison's claims that AC presents a greater hazard. At higher and higher frequencies, the body becomes progressively less sensitive to any effects that involve nerves. This is related to the maximum rates at which nerves can fire or be stimulated. At very high frequencies, electrical current travels only on the surface of a person. Thus a wart can be burned off with very high frequency current without causing the heart to stop. (Do not try this at home with 60-Hz AC!) Some of the spectacular demonstrations of electricity, in which high-voltage arcs are passed through the air and over people's bodies, employ high frequencies and low currents. (See [Figure 6].) Electrical safety devices and techniques are discussed in detail in Electrical Safety: Systems and Devices .



Is this electric arc dangerous? The answer depends on the AC frequency and the power involved. (credit: Khimich Alex, Wikimedia Commons)

Section Summary

- The two types of electric hazards are thermal (excessive power) and shock (current through a person).
- Shock severity is determined by current, path, duration, and AC frequency.
- [Table 1] lists shock hazards as a function of current.
- [Figure 5] graphs the threshold current for two hazards as a function of frequency.

Conceptual Questions

Using an ohmmeter, a student measures the resistance between various points on his body. He finds that the resistance between two points on the same finger is about the same as the resistance between two points on opposite hands—both are several hundred thousand ohms. Furthermore, the resistance decreases when more skin is brought into contact with the probes of the ohmmeter. Finally, there is a dramatic drop in resistance (to a few thousand ohms) when the skin is wet. Explain these observations and their implications regarding skin and internal resistance of the human body.

What are the two major hazards of electricity?

Why isn’t a short circuit a shock hazard?

What determines the severity of a shock? Can you say that a certain voltage is hazardous without further information?

An electrified needle is used to burn off warts, with the circuit being completed by having the patient sit on a large butt plate. Why is this plate large?

Some surgery is performed with high-voltage electricity passing from a metal scalpel through the tissue being cut. Considering the nature of electric fields at the surface of conductors, why would you expect most of the current to flow from the sharp edge of the scalpel? Do you think high- or low-frequency AC is used?

Some devices often used in bathrooms, such as hairdryers, often have safety messages saying “Do not use when the bathtub or basin is full of water.” Why is this so?

We are often advised to not flick electric switches with wet hands, dry your hand first. We are also advised to never throw water on an electric fire. Why is this so?

Before working on a power transmission line, linemen will touch the line with the back of the hand as a final check that the voltage is zero. Why the back of the hand?

Why is the resistance of wet skin so much smaller than dry, and why do blood and other bodily fluids have low resistances?

Could a person on intravenous infusion (an IV) be microshock sensitive?

In view of the small currents that cause shock hazards and the larger currents that circuit breakers and fuses interrupt, how do they play a role in preventing shock hazards?

Problem Exercises

(a) How much power is dissipated in a short circuit of 240-V AC through a resistance of 0.250Ω? (b) What current flows?

Show Solution

Strategy

For a short circuit, we can use the power formula $P = V^2/R$ to find the power dissipated, and Ohm's law $I = V/R$ to find the current flowing through the short circuit.

Solution

(a) Find the power dissipated using $P = V^2/R$:

$$P=\frac{V^2}{R}=\frac{(240\text{ V})^2}{0.250\ \Omega}=\frac{57600\text{ V}^2}{0.250\ \Omega}$$
$$P=230400\text{ W}=230\text{ kW}$$

(b) Find the current using Ohm's law:

$$I=\frac{V}{R}=\frac{240\text{ V}}{0.250\ \Omega}=960\text{ A}$$

Discussion

The power dissipated in this short circuit (230 kW) is enormous—equivalent to about 150 typical household appliances running simultaneously. This massive power will rapidly heat the conductor to dangerous temperatures, potentially melting insulation, starting fires, or vaporizing the conductor itself. The current of 960 A is about 80 times the typical household circuit capacity (15 A), demonstrating why circuit breakers and fuses are absolutely essential safety devices. The combination of even moderate voltage (240 V) with very low resistance (0.250 Ω) creates an extremely hazardous situation that must be interrupted within milliseconds.

Final Answer

(a) The power dissipated in the short circuit is **230 kW**. (b) The current flowing through the short circuit is **960 A**.

What voltage is involved in a 1.44-kW short circuit through a $0.100\text{-}\Omega$ resistance?

Show Solution

Strategy

Use the power equation $P = V^2/R$ and solve for voltage. This will give us the voltage that produces the specified power dissipation through the low-resistance short circuit.

Solution

Rearrange the power equation to solve for voltage:

$$V=\sqrt{PR}=\sqrt{(1.44\times10^3\text{ W})(0.100\ \Omega)}$$
$$V=\sqrt{144\text{ V}^2}=12.0\text{ V}$$

Discussion

This result shows that even a relatively low voltage of 12.0 V can cause significant power dissipation (1.44 kW) when the resistance is very small. This is characteristic of short circuits—the low resistance allows high current flow ($I = V/R = 12.0\text{ V}/0.100\ \Omega = 120\text{ A}$), which results in high power dissipation despite the modest voltage. This 12-V scenario is common in automotive electrical systems, where a short circuit in the low-resistance wiring can quickly heat wires to dangerous temperatures, potentially causing fires. This is why automotive fuses are critical safety devices.

The voltage involved in the short circuit is 12.0 V.

Find the current through a person and identify the likely effect on her if she touches a 120-V AC source: (a) if she is standing on a rubber mat and offers a total resistance of $300\text{k}\Omega$; (b) if she is standing barefoot on wet grass and has a resistance of only 4500Ω.

Show Solution

Strategy

Use Ohm's law $I = V/R$ to calculate the current through the person in both scenarios, then compare the results to Table 1 to determine the physiological effects.

Solution

(a) Standing on rubber mat with $R = 300\ \text{k}\Omega$:

$$I = \frac{V}{R} = \frac{120\ \text{V}}{300 \times 10^3\ \Omega} = 4.00 \times 10^{-4}\ \text{A} = 0.400\ \text{mA}$$

According to Table 1, currents below 1 mA are below the threshold of sensation. Therefore, there is **no effect**—the person would not even feel the shock.

(b) Standing barefoot on wet grass with $R = 4500\ \Omega$:

$$I = \frac{V}{R} = \frac{120\ \text{V}}{4500\ \Omega} = 0.0267\ \text{A} = 26.7\ \text{mA}$$

According to Table 1, currents in the range of 10-20 mA cause sustained muscular contraction—the victim cannot let go for the duration of the shock. At 26.7 mA, the person would experience **sustained muscular contraction** ("can't let go" effect), and contraction of chest muscles may even stop breathing during the shock.

Discussion

This problem dramatically illustrates the critical importance of resistance in electrical safety. The same 120-V source produces currents that differ by a factor of about 67. In scenario (a), the high resistance of the rubber mat (an excellent insulator) limits current to a completely harmless level. In scenario (b), wet conditions drastically reduce resistance: moisture on the skin dissolves salts into ions that conduct electricity well, and wet grass provides a low-resistance path to ground. The 26.7 mA current is dangerous—while below the ventricular fibrillation threshold (100-300 mA), it's well into the "can't let go" range and could cause respiratory arrest. This explains why electrical work should never be done in wet conditions and why insulating footwear is required in many professions.

Final Answer

(a) The current is **0.400 mA**, which produces **no effect** (below sensation threshold). (b) The current is **26.7 mA**, which causes **sustained muscular contraction** (can't let go), and may stop breathing during the shock.

While taking a bath, a person touches the metal case of a radio. The path through the person to the drainpipe and ground has a resistance of 4000Ω. What is the smallest voltage on the case of the radio that could cause ventricular fibrillation?

Show Solution

Strategy

Use Ohm's law $V = IR$ with the threshold current for ventricular fibrillation. From Table 1, ventricular fibrillation can occur at currents as low as 100 mA. We use this minimum threshold to find the smallest dangerous voltage.

Solution

Apply Ohm's law with the minimum fibrillation current:

$$V = IR = (100 \times 10^{-3}\ \text{A})(4000\ \Omega)$$
$$V = 400\ \text{V}$$

Discussion

A voltage of 400 V on the radio case could cause ventricular fibrillation. However, this analysis reveals an important safety consideration: standard household voltage (120 V in North America, 240 V in many other countries) would produce currents of $I = 120\ \text{V}/4000\ \Omega = 30\ \text{mA}$ or $I = 240\ \text{V}/4000\ \Omega = 60\ \text{mA}$. While these currents are below the fibrillation threshold, they are well above the "can't let go" threshold (10-20 mA) and the threshold for pain (50 mA). The person would experience sustained muscular contractions and might drown if unable to release the radio or exit the tub. This is why electrical devices should never be used near bathtubs, and modern bathrooms require ground-fault circuit interrupter (GFCI) outlets that trip at currents as low as 5 mA.

The smallest voltage that could cause ventricular fibrillation is 400 V.

Foolishly trying to fish a burning piece of bread from a toaster with a metal butter knife, a man comes into contact with 120-V AC. He does not even feel it since, luckily, he is wearing rubber-soled shoes. What is the minimum resistance of the path the current follows through the person?

Show Solution

Strategy

The man doesn't feel the shock, meaning the current is below the threshold of sensation (1 mA from Table 1). Use Ohm's law $R = V/I$ with the maximum current he wouldn't feel to find the minimum resistance.

Solution

For the shock to be unfelt, the current must be less than the threshold of sensation:

$$I < 1\ \text{mA} = 1 \times 10^{-3}\ \text{A}$$

Using Ohm's law to find the minimum resistance:

$$R = \frac{V}{I} = \frac{120\text{ V}}{1 \times 10^{-3}\text{ A}} = 1.20 \times 10^{5}\ \Omega = 120\text{ k}\Omega$$

Discussion

The minimum resistance for an unfelt shock is 120 kΩ. This is a reasonable value for someone wearing rubber-soled shoes on a dry floor—dry skin typically has a resistance around 100-500 kΩ, and rubber soles add additional insulating resistance. If the man's body resistance were lower (for instance, if his hands were wet, or if he were barefoot on a conductive surface), he would have felt the shock and potentially been in serious danger. The rubber soles acted as critical safety insulation.

It's worth emphasizing that while the man was fortunate, his action was extremely dangerous. One should never use metal objects to retrieve items from energized electrical devices. The "luckily" in the problem statement is key—had conditions been slightly different (sweaty hands, a small cut reducing skin resistance, momentary contact with a grounded surface), the outcome could have been fatal.

Final Answer

The minimum resistance of the path through the person is **120 kΩ** (or $1.20 \times 10^{5}\ \Omega$).

(a) During surgery, a current as small as $20.0\ \mu\text{A}$ applied directly to the heart may cause ventricular fibrillation. If the resistance of the exposed heart is $300\ \Omega$, what is the smallest voltage that poses this danger? (b) Does your answer imply that special electrical safety precautions are needed?

Show Solution

Strategy

Use Ohm's law $V = IR$ to find the minimum voltage that would cause the dangerous microampere-level current through the exposed heart tissue.

Solution

(a) Apply Ohm's law:

$$V = IR = (20.0 \times 10^{-6}\text{ A})(300\ \Omega)$$
$$V = 6.00 \times 10^{-3}\text{ V} = 6.00\text{ mV}$$

(b) Yes, special electrical safety precautions are absolutely essential during surgery. A voltage of only 6.00 mV can be lethal when applied directly to the heart. This is an extraordinarily small voltage—far below what can be felt through intact skin, and easily produced by:

- Stray electromagnetic fields from nearby equipment
- Ground loops between different pieces of equipment
- Static electricity buildup
- Small potential differences between grounded equipment

Discussion

This tiny threshold voltage (6.00 mV) is about 20,000 times smaller than household voltage and would be completely imperceptible under normal conditions. When the skin's protective resistance is bypassed during surgery, the patient becomes "microshock sensitive." This requires stringent safety measures including: equipotential grounding (all equipment at the same ground potential), isolated power systems, low-leakage equipment, and continuous monitoring. Operating rooms use special electrical systems that can detect leakage currents as small as a few microamperes and isolate faulty equipment before it poses a danger. All personnel must follow strict protocols about grounding and equipment handling.

(a) The smallest voltage that could cause ventricular fibrillation is 6.00 mV. (b) Yes, extremely stringent electrical safety precautions are essential during surgery.

(a) What is the resistance of a 220-V AC short circuit that generates a peak power of 96.8 kW? (b) What would the average power be if the voltage was 120 V AC?

Show Solution

Strategy

(a) For AC circuits, peak power occurs when the instantaneous voltage is at its peak value. Use $P = V^2/R$ rearranged to find resistance. For 220-V AC, the stated voltage is the RMS value, and peak voltage is $V_{\text{peak}} = \sqrt{2} \times V_{\text{RMS}}$. However, if "220-V AC" refers to peak voltage, use it directly. Given the problem states "peak power," we'll assume 220 V is the RMS value.

(b) Use the same resistance with 120-V AC to find the new average power using $P_{\text{avg}} = V^2_{\text{RMS}}/R$.

Solution

(a) For AC, peak power with RMS voltage $V_{\text{RMS}} = 220$ V:

The peak voltage is:

$$V_{\text{peak}} = \sqrt{2} \times V_{\text{RMS}} = 1.414 \times 220 \text{ V} = 311 \text{ V}$$

Peak power:

$$P_{\text{peak}} = \frac{V_{\text{peak}}^2}{R}$$

Solving for R:

$$R = \frac{V_{\text{peak}}^2}{P_{\text{peak}}} = \frac{(311 \text{ V})^2}{96.8 \times 10^3 \text{ W}} = \frac{96721 \text{ V}^2}{96800 \text{ W}} = 0.999\ \Omega \approx 1.00\ \Omega$$

Alternatively, if the problem intends 220 V as the effective voltage for average power calculation (a simpler interpretation), we would use:

$$R = \frac{(220 \text{ V})^2}{96.8 \times 10^3 \text{ W}} \approx 0.50\ \Omega$$

However, the answer $1.00\ \Omega$ suggests the peak interpretation is correct.

(b) Average power with 120-V AC through $R = 1.00\ \Omega$:

$$P_{\text{avg}} = \frac{V_{\text{RMS}}^2}{R} = \frac{(120 \text{ V})^2}{1.00\ \Omega} = \frac{14400 \text{ V}^2}{1.00\ \Omega} = 14400 \text{ W} = 14.4 \text{ kW}$$

Discussion

Even though the voltage decreased from 220 V to 120 V (a factor of 220/120 ≈ 1.83), the power decreased from 96.8 kW to 14.4 kW (a factor of 96.8/14.4 ≈ 6.7). This demonstrates the quadratic relationship between voltage and power: $P \propto V^2$ when resistance is constant. The factor should be $(220/120)^2 = 3.36$ for average power, but we're comparing peak power at 220 V to average power at 120 V, which accounts for an additional factor of 2.

Both power levels are extremely dangerous and represent serious fire hazards. A short circuit with $R = 1.00\ \Omega$ would draw enormous currents (311 A peak at 220 V, or 120 A at 120 V) and generate intense heat. Circuit protection would need to respond in milliseconds to prevent catastrophic damage.

Final Answer

(a) The resistance of the short circuit is **1.00 Ω**. (b) The average power at 120-V AC would be **14.4 kW**.

A heart defibrillator passes 10.0 A through a patient's torso for 5.00 ms in an attempt to restore normal beating. (a) How much charge passed? (b) What voltage was applied if 500 J of energy was dissipated? (c) What was the path's resistance? (d) Find the temperature increase caused in the 8.00 kg of affected tissue.

Show Solution

Strategy

(a) Use the definition of current $I = \Delta Q / \Delta t$ to find charge. (b) Use energy and charge to find voltage via $E = QV$. (c) Use Ohm's law $R = V/I$. (d) Use the heat equation $Q = mc\Delta T$ with the specific heat of body tissue (approximately that of water, $c = 3500$ J/kg·°C).

Solution

(a) Find the charge passed:

$$\Delta Q = I \cdot \Delta t = (10.0 \text{ A})(5.00 \times 10^{-3} \text{ s}) = 0.0500 \text{ C} = 50.0 \text{ mC}$$

(b) Find the voltage using energy and charge:

$$E = QV \Rightarrow V = \frac{E}{Q} = \frac{500 \text{ J}}{0.0500 \text{ C}} = 10000 \text{ V} = 10.0 \text{ kV}$$

(c) Find the resistance using Ohm's law:

$$R = \frac{V}{I} = \frac{10000 \text{ V}}{10.0 \text{ A}} = 1000\ \Omega = 1.00 \text{ k}\Omega$$

(d) Find the temperature increase using $E = mc\Delta T$:

$$\Delta T = \frac{E}{mc} = \frac{500 \text{ J}}{(8.00 \text{ kg})(3500 \text{ J/kg}\cdot°\text{C})} = \frac{500}{28000}\ °\text{C} = 0.0179\ °\text{C}$$

Discussion

The defibrillator delivers a substantial charge (50.0 mC) at a very high voltage (10.0 kV), but only for a brief 5.00 ms pulse. The path resistance of 1.00 kΩ is reasonable for current passing through the chest (between external paddle electrodes through skin, muscle, and other tissue).

The temperature rise of only 0.018°C is negligible and explains why defibrillation doesn't cause burns despite the high voltage and current. This is because the energy is distributed over a large mass of tissue (8.00 kg) and the duration is extremely short. The large paddles used in defibrillation help spread the current over a wide area, reducing current density and preventing localized heating. The brief, high-current pulse is designed to simultaneously depolarize the entire heart muscle, allowing it to restart with a normal rhythm.

(a) 50.0 mC of charge passed through the patient. (b) A voltage of 10.0 kV was applied. (c) The path's resistance was 1.00 kΩ. (d) The temperature increase in the affected tissue was 0.018°C.

Integrated Concepts

A short circuit in a 120-V appliance cord has a $0.500\text{-}\Omega$ resistance. Calculate the temperature rise of the 2.00 g of surrounding materials, assuming their specific heat capacity is $0.200 \text{cal/g} \cdot °\text{C}$ and that it takes 0.0500 s for a circuit breaker to interrupt the current. Is this likely to be damaging?

Show Solution

Strategy

Calculate the current using Ohm's law, then find the power dissipated in the short circuit. Use this power and the time duration to find the total energy deposited in the surrounding material. Finally, use the heat equation $Q = mc\Delta T$ to find the temperature rise. Note that we need to convert between calories and joules: $1 \text{ cal} = 4.186 \text{ J}$.

Solution

Step 1: Find the current through the short circuit:

$$I = \frac{V}{R} = \frac{120 \text{ V}}{0.500 \ \Omega} = 240 \text{ A}$$

Step 2: Find the power dissipated:

$$P = I^2 R = (240 \text{ A})^2 (0.500 \ \Omega) = 57600 \text{ A}^2 \cdot 0.500 \ \Omega = 28800 \text{ W} = 28.8 \text{ kW}$$

Step 3: Find the total energy deposited during $\Delta t = 0.0500$ s:

$$E = P \cdot \Delta t = (28800 \text{ W})(0.0500 \text{ s}) = 1440 \text{ J}$$

Step 4: Convert the specific heat capacity to SI units:

$$c = 0.200 \frac{\text{cal}}{\text{g} \cdot °\text{C}} \times 4.186 \frac{\text{J}}{\text{cal}} = 0.837 \frac{\text{J}}{\text{g} \cdot °\text{C}}$$

Step 5: Find the temperature rise using $Q = mc\Delta T$:

$$\Delta T = \frac{Q}{mc} = \frac{1440 \text{ J}}{(2.00 \text{ g})(0.837 \text{ J/g} \cdot °\text{C})} = \frac{1440}{1.674} \ °\text{C} = 860 \ °\text{C}$$

Discussion

A temperature rise of 860°C in just 0.0500 seconds is catastrophic. This would:

- Instantly melt most plastics and insulating materials (which typically melt at 100-300°C)
- Vaporize water and organic materials
- Ignite flammable materials (paper ignites around 230°C)
- Potentially vaporize copper wire (copper melts at 1085°C, so 860°C brings it close to melting)

This is extremely damaging and represents a severe fire hazard. The short circuit would almost certainly start a fire before the circuit breaker could interrupt the current, even with the relatively fast 0.0500-s response time. This demonstrates why electrical codes require proper wire sizing, circuit protection, and regular inspection of electrical cords—the 0.500-Ω resistance represents a partially damaged conductor that creates a localized hot spot.

In practice, as the temperature rises, the situation could worsen: melting insulation could reduce resistance further, increasing current and power. Alternatively, the conductor itself might vaporize, creating an arc that maintains the current flow even without direct contact.

Final Answer

The temperature rises by **860°C** in 0.0500 seconds. **Yes, this is extremely likely to be damaging**—it would cause fires, melt insulation, and potentially vaporize the conductor.

Construct Your Own Problem

Consider a person working in an environment where electric currents might pass through her body. Construct a problem in which you calculate the resistance of insulation needed to protect the person from harm. Among the things to be considered are the voltage to which the person might be exposed, likely body resistance (dry, wet, …), and acceptable currents (safe but sensed, safe and unfelt, …).

Footnotes

- 1 For an average male shocked through trunk of body for 1 s by 60-Hz AC. Values for females are 60–80% of those listed. { data-list-type="bulleted" data-bullet-style="none"}

Glossary

thermal hazard
: a hazard in which electric current causes undesired thermal effects

shock hazard
: when electric current passes through a person

short circuit
: also known as a “short,” a low-resistance path between terminals of a voltage source

microshock sensitive
: a condition in which a person’s skin resistance is bypassed, possibly by a medical procedure, rendering the person vulnerable to electrical shock at currents about 1/1000 the normally required level

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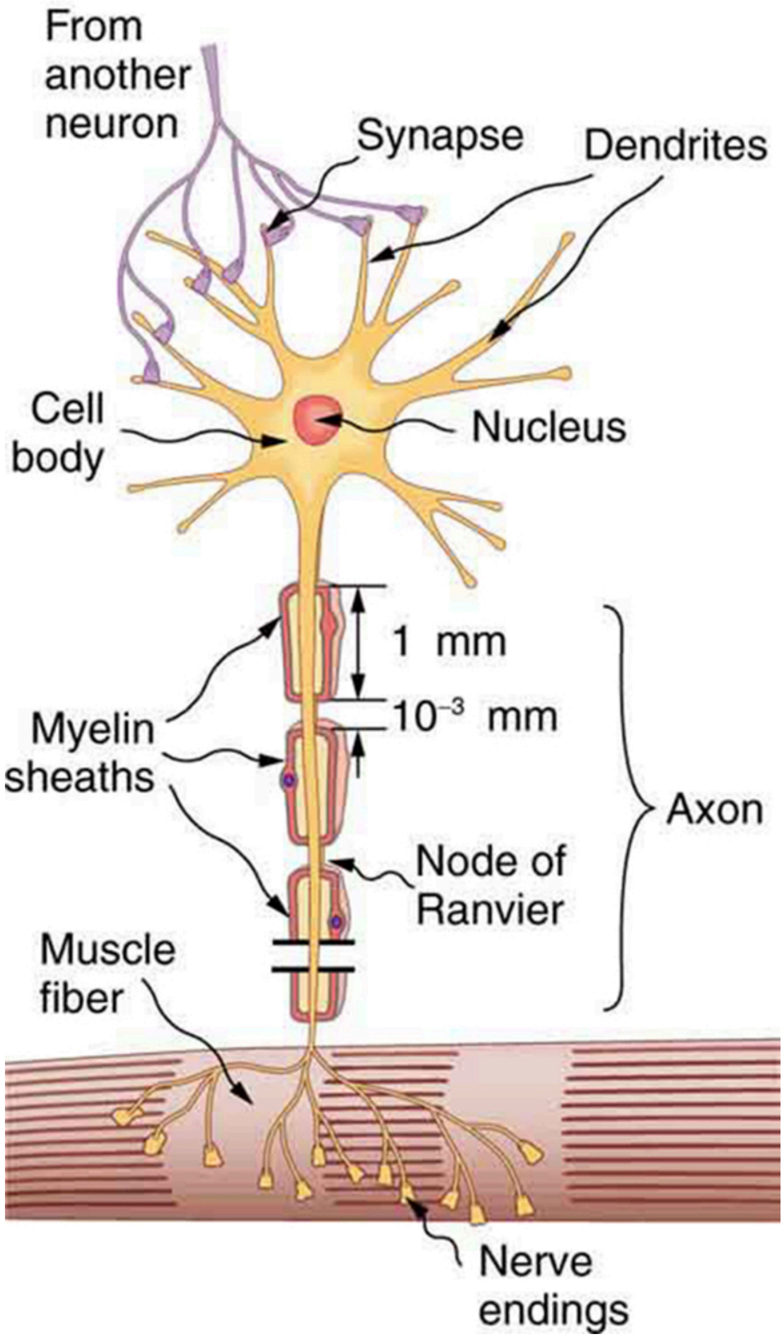
Nerve Conduction–Electrocardiograms

- Explain the process by which electric signals are transmitted along a neuron.
- Explain the effects myelin sheaths have on signal propagation.
- Explain what the features of an ECG signal indicate.

Nerve Conduction

Electric currents in the vastly complex system of billions of nerves in our body allow us to sense the world, control parts of our body, and think. These are representative of the three major functions of nerves. First, nerves carry messages from our sensory organs and others to the central nervous system, consisting of the brain and spinal cord. Second, nerves carry messages from the central nervous system to muscles and other organs. Third, nerves transmit and process signals within the central nervous system. The sheer number of nerve cells and the incredibly greater number of connections between them makes this system the subtle wonder that it is. **Nerve conduction** is a general term for electrical signals carried by nerve cells. It is one aspect of **bioelectricity**, or electrical effects in and created by biological systems.

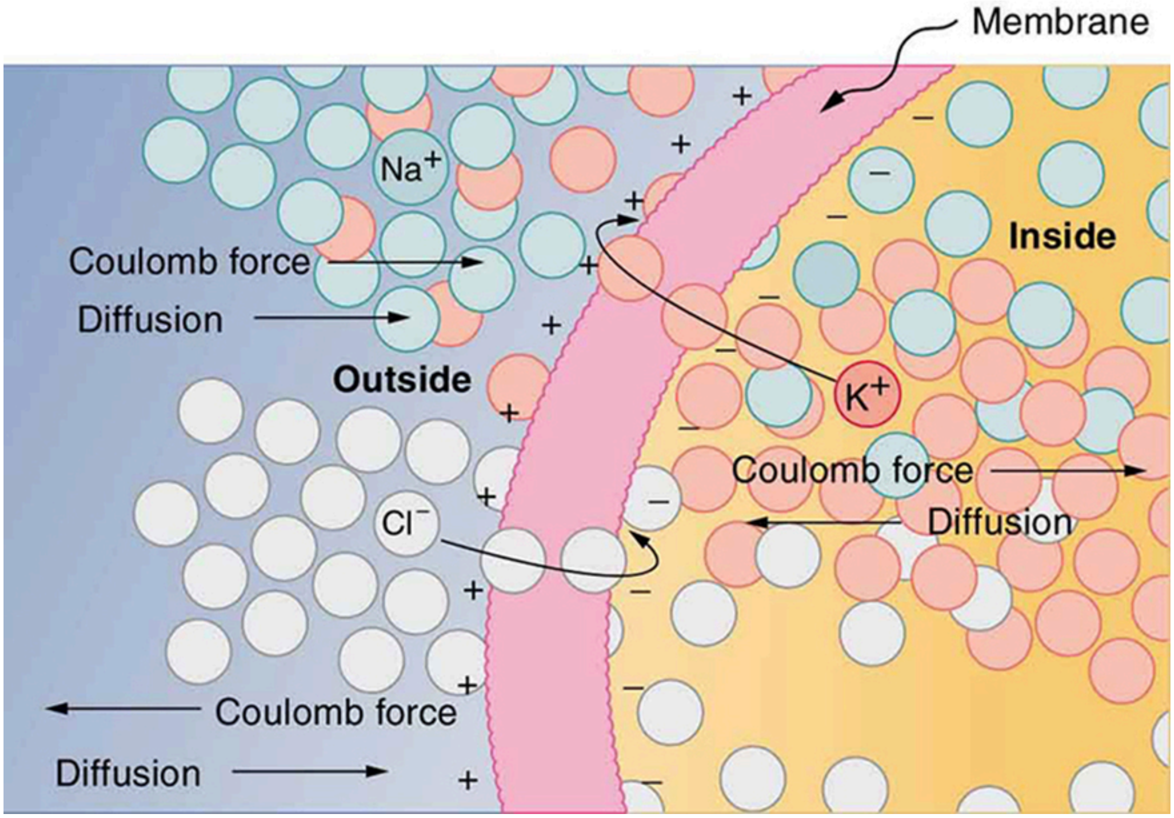
Nerve cells, properly called *neurons*, look different from other cells—they have tendrils, some of them many centimeters long, connecting them with other cells. (See [Figure 1].) Signals arrive at the cell body across **synapses** or through **dendrites**, stimulating the neuron to generate its own signal, sent along its long **axon** to other nerve or muscle cells. Signals may arrive from many other locations and be transmitted to yet others, conditioning the synapses by use, giving the system its complexity and its ability to learn.



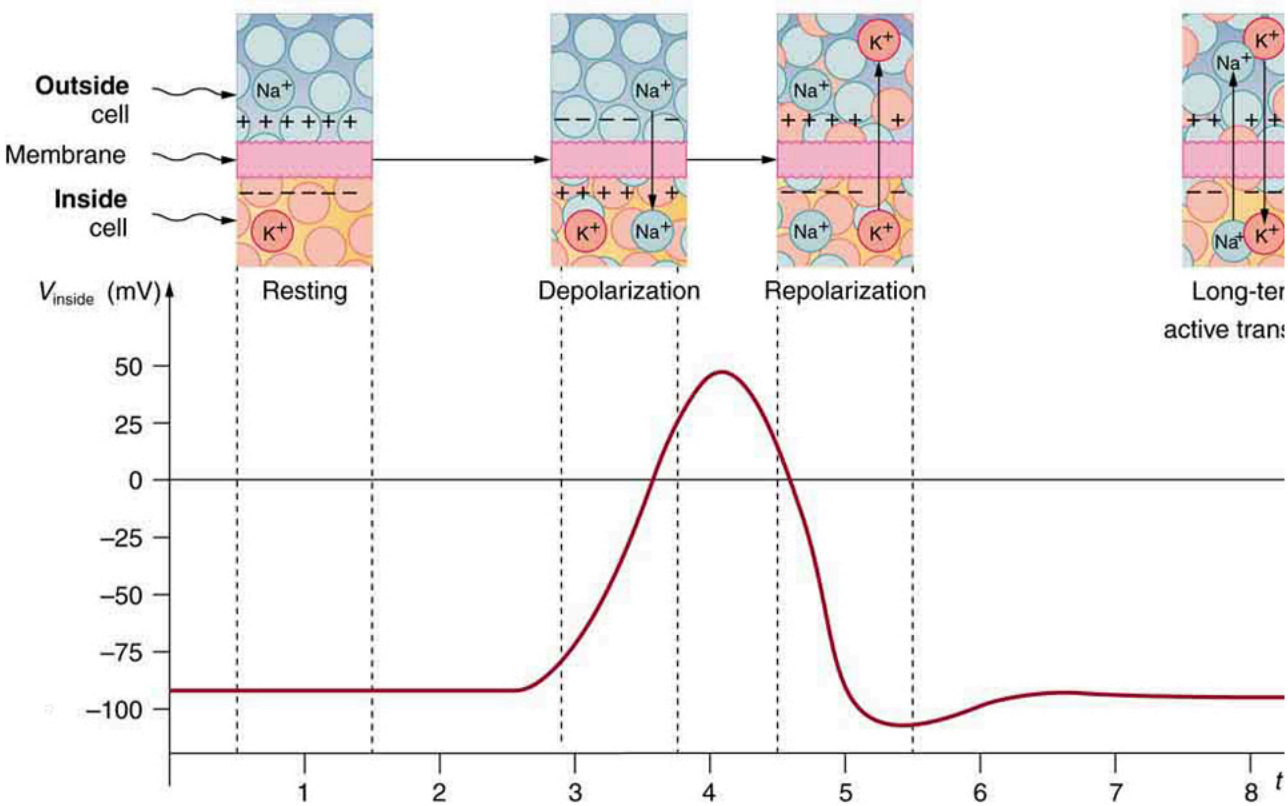
A neuron with its dendrites and long axon. Signals in the form of electric currents reach the cell body through dendrites and across synapses, stimulating the neuron to generate its own signal sent down the axon. The number of interconnections can be far greater than shown here.

The method by which these electric currents are generated and transmitted is more complex than the simple movement of free charges in a conductor, but it can be understood with principles already discussed in this text. The most important of these are the Coulomb force and diffusion.

[Figure 2] illustrates how a voltage (potential difference) is created across the cell membrane of a neuron in its resting state. This thin membrane separates electrically neutral fluids having differing concentrations of ions, the most important varieties being Na^+ , K^+ , and Cl^- (these are sodium, potassium, and chlorine ions with single plus or minus charges as indicated). As discussed in Molecular Transport Phenomena: Diffusion, Osmosis, and Related Processes , free ions will diffuse from a region of high concentration to one of low concentration. But the cell membrane is **semipermeable**, meaning that some ions may cross it while others cannot. In its resting state, the cell membrane is permeable to K^+ and Cl^- , and impermeable to Na^+ . Diffusion of K^+ and Cl^- thus creates the layers of positive and negative charge on the outside and inside of the membrane. The Coulomb force prevents the ions from diffusing across in their entirety. Once the charge layer has built up, the repulsion of like charges prevents more from moving across, and the attraction of unlike charges prevents more from leaving either side. The result is two layers of charge right on the membrane, with diffusion being balanced by the Coulomb force. A tiny fraction of the charges move across and the fluids remain neutral (other ions are present), while a separation of charge and a voltage have been created across the membrane.



The semipermeable membrane of a cell has different concentrations of ions inside and out. Diffusion moves the K^+ and Cl^- ions in the direction shown, until the Coulomb force halts further transfer. This results in a layer of positive charge on the outside, a layer of negative charge on the inside, and thus a voltage across the cell membrane. The membrane is normally impermeable to Na^+ .

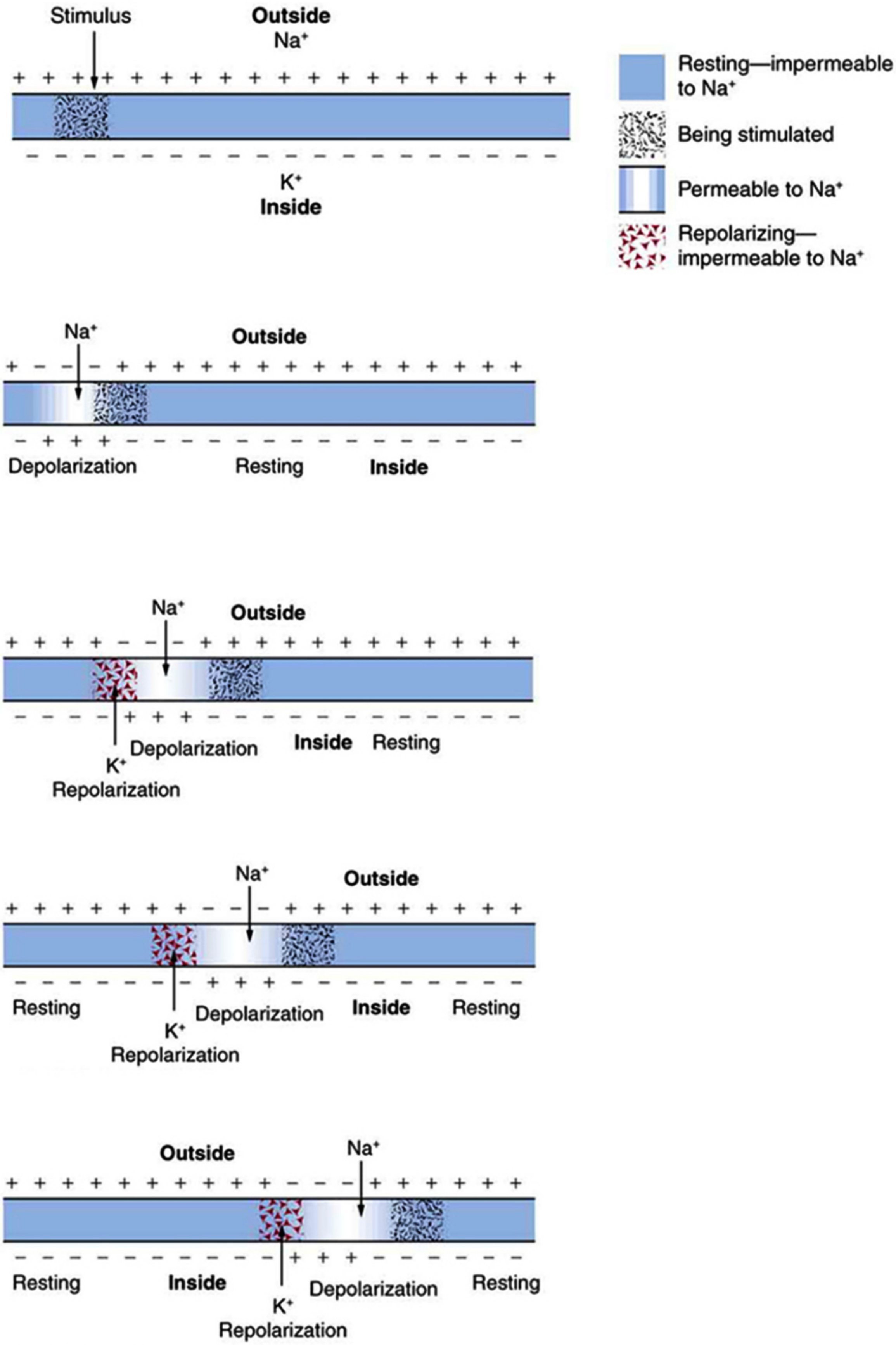


An action potential is the pulse of voltage inside a nerve cell graphed here. It is caused by movements of ions across the cell membrane as shown. Depolarization occurs when a stimulus makes the membrane permeable to Na^+ ions. Repolarization follows as the membrane again becomes impermeable to Na^+ and K^+ moves from high to low concentration. In the long term, active transport slowly maintains the concentration differences, but the cell may fire hundreds of times in rapid succession without seriously depleting them.

The separation of charge creates a potential difference of 70 to 90 mV across the cell membrane. While this is a small voltage, the resulting electric field ($E = V/d$) across the only 8-nm-thick membrane is immense (on the order of 11 MV/m!) and has fundamental effects on its structure and permeability. Now, if the exterior of a neuron is taken to be at 0 V, then the interior has a **resting potential** of about –90 mV. Such voltages are created across the membranes of almost all types of animal cells but are largest in nerve and muscle cells. In fact, fully 25% of the energy used by cells goes toward creating and maintaining these potentials.

Electric currents along the cell membrane are created by any stimulus that changes the membrane's permeability. The membrane thus temporarily becomes permeable to Na^+ , which then rushes in, driven both by diffusion and the Coulomb force. This inrush of Na^+ first neutralizes the inside membrane, or **depolarizes** it, and then makes it slightly positive. The depolarization causes the membrane to again become impermeable to Na^+ , and the movement of K^+ quickly returns the cell to its resting potential, or **repolarizes** it. This sequence of events results in a voltage pulse, called the *action potential*. (See [Figure 3].) Only small fractions of the ions move, so that the cell can fire many hundreds of times without depleting the excess concentrations of Na^+ and K^+ . Eventually, the cell must replenish these ions to maintain the concentration differences that create bioelectricity. This sodium-potassium pump is an example of **active transport** , wherein cell energy is used to move ions across membranes against diffusion gradients and the Coulomb force.

The action potential is a voltage pulse at one location on a cell membrane. How does it get transmitted along the cell membrane, and in particular down an axon, as a nerve impulse? The answer is that the changing voltage and electric fields affect the permeability of the adjacent cell membrane, so that the same process takes place there. The adjacent membrane depolarizes, affecting the membrane further down, and so on, as illustrated in [Figure 4]. Thus the action potential stimulated at one location triggers a **nerve impulse** that moves slowly (about 1 m/s) along the cell membrane.

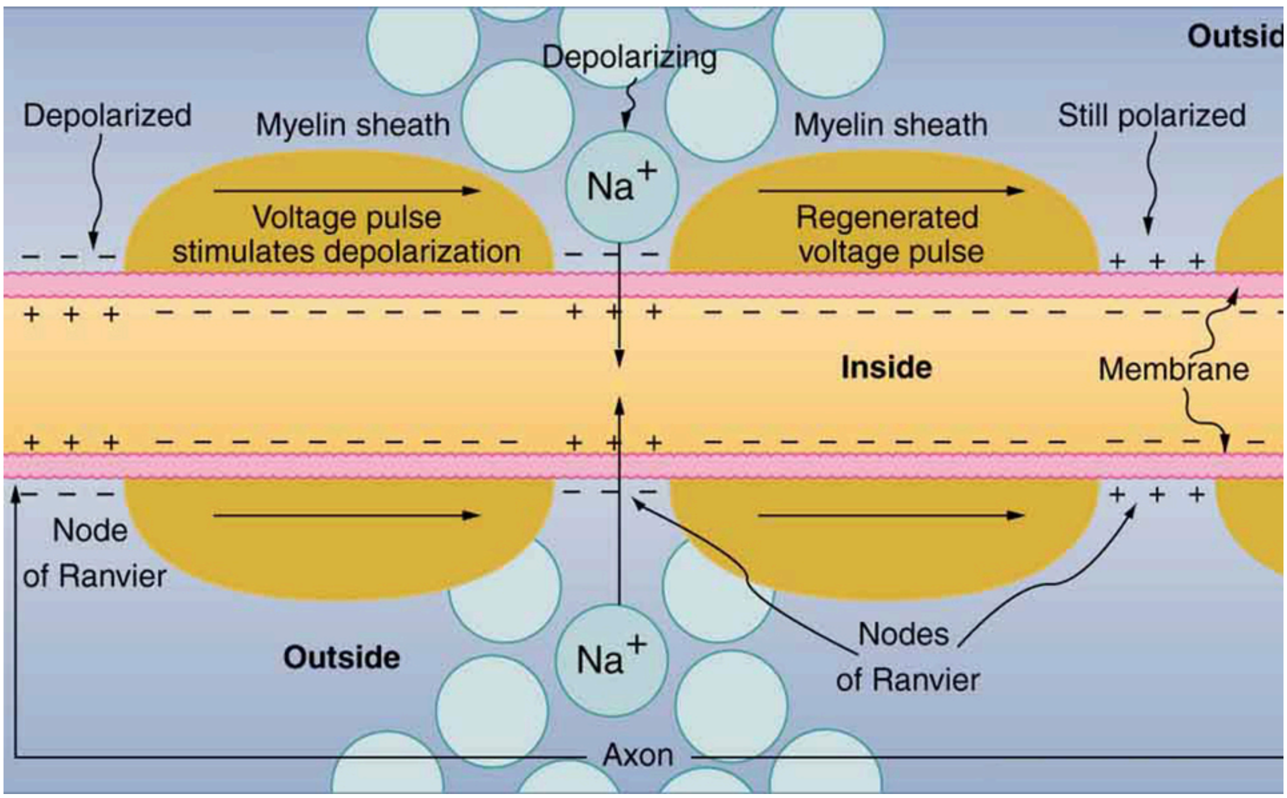

Stimulus
Outside
Na^+
K^+
Inside
Resting—impermeable to Na^+
Being stimulated
Permeable to Na^+
Repolarizing—impermeable to Na^+
Na^+
Outside
Depolarization
Resting
Inside
Na^+
Outside
Depolarization
Inside
Resting
K^+
Repolarization
Na^+
Outside
Resting
Depolarization
Inside
Resting
K^+
Repolarization
Outside
Na^+
Resting
Inside
Depolarization
Resting
K^+
Repolarization

A nerve impulse is the propagation of an action potential along a cell membrane. A stimulus causes an action potential at one location, which changes the permeability of the adjacent membrane, causing an action potential there. This in turn affects the membrane further down, so that the action potential moves slowly (in electrical terms) along the cell membrane. Although the impulse is due to Na^+ and K^+ going across the membrane, it is equivalent to a wave of charge moving along the outside and inside of the membrane.

Some axons, like that in [Figure 1], are sheathed with **myelin**, consisting of fat-containing cells. [Figure 5] shows an enlarged view of an axon having myelin sheaths characteristically separated by unmyelinated gaps (called nodes of Ranvier). This arrangement gives the axon a number of interesting properties. Since myelin is an insulator, it prevents signals from jumping between adjacent nerves (cross talk). Additionally, the myelinated regions transmit electrical signals at a very high speed, as an ordinary conductor or resistor would. There is no action potential in the myelinated regions, so that no cell energy is used in them. There is an IR signal loss in the myelin, but the signal is regenerated in the gaps, where the voltage pulse triggers the action potential at full voltage. So a myelinated axon transmits a nerve impulse faster, with less energy consumption, and is better protected from cross talk than an unmyelinated one. Not all axons are myelinated, so that cross talk and slow signal transmission are a characteristic of the normal operation of these axons, another variable in the nervous system.

The degeneration or destruction of the myelin sheaths that surround the nerve fibers impairs signal transmission and can lead to numerous neurological effects. One of the most prominent of these diseases comes from the body's own immune system attacking the myelin in the central nervous system—multiple sclerosis. MS symptoms include fatigue, vision problems, weakness of arms and legs, loss of balance, and tingling or numbness in one's extremities (neuropathy). It is more apt to strike younger adults, especially females. Causes might come from infection, environmental or geographic affects, or genetics. At the moment there is no known cure for MS.

Most animal cells can fire or create their own action potential. Muscle cells contract when they fire and are often induced to do so by a nerve impulse. In fact, nerve and muscle cells are physiologically similar, and there are even hybrid cells, such as in the heart, that have characteristics of both nerves and muscles. Some animals, like the infamous electric eel (see [Figure 6]), use muscles ganged so that their voltages add in order to create a shock great enough to stun prey.

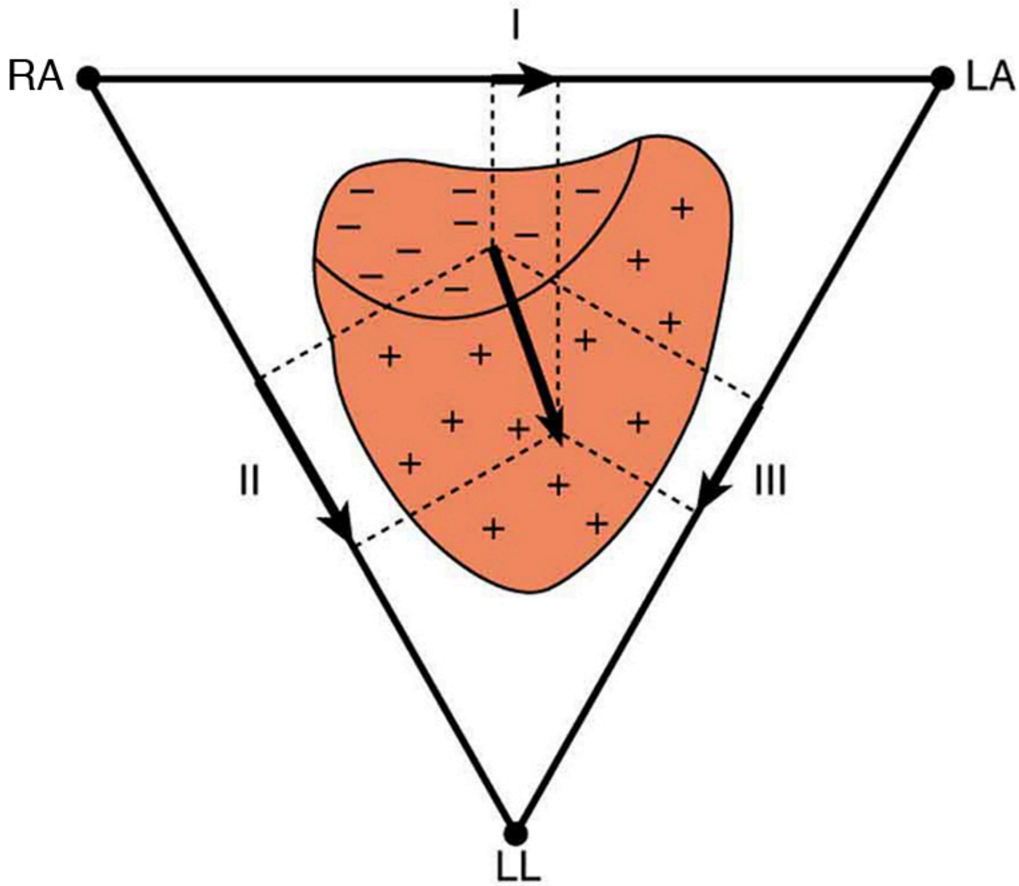


Propagation of a nerve impulse down a myelinated axon, from left to right. The signal travels very ast and without energy input in the myelinated regions, but it loses voltage. It is regenerated in the gaps. The signal moves faster than in unmyelinated axons and is insulated from signals in other nerves, limiting cross talk.

An electric eel flexes its muscles to create a voltage that stuns prey. (credit: chrisbb, Flickr)

Electrocardiograms

Just as nerve impulses are transmitted by depolarization and repolarization of adjacent membrane, the depolarization that causes muscle contraction can also stimulate adjacent muscle cells to depolarize (fire) and contract. Thus, a depolarization wave can be sent across the heart, coordinating its rhythmic contractions and enabling it to perform its vital function of propelling blood through the circulatory system. [Figure 7] is a simplified graphic of a depolarization wave spreading across the heart from the *sinoatrial (SA) node* the heart's natural pacemaker.

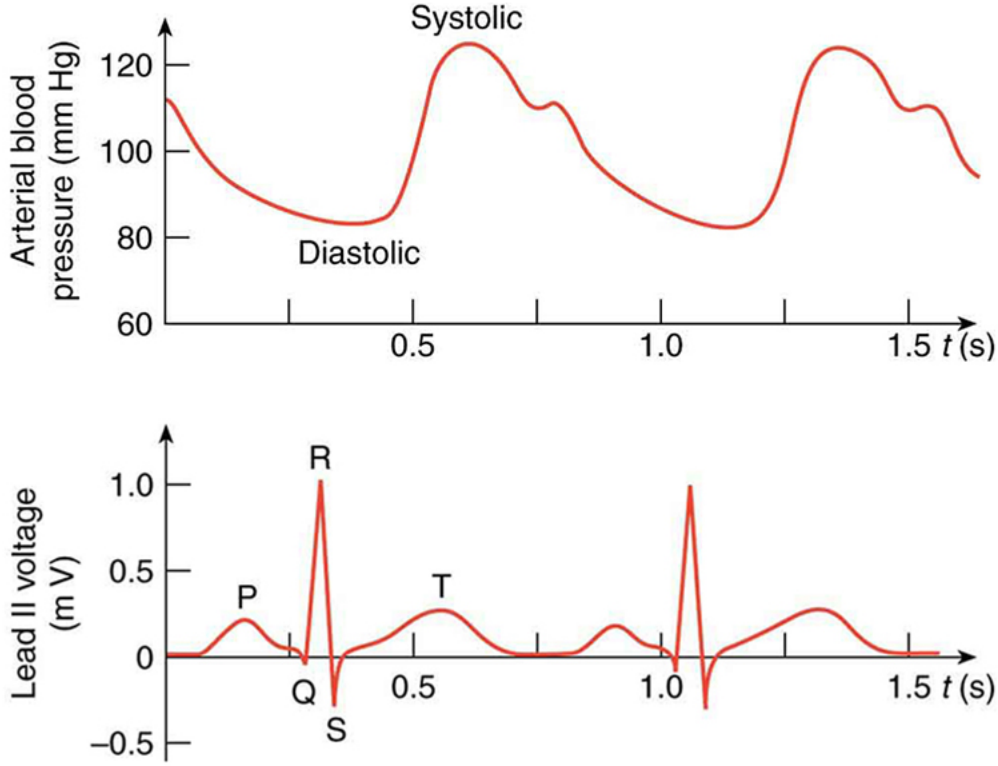


The outer surface of the heart changes from positive to negative during depolarization. This wave of depolarization is spreading from the top of the heart and is represented by a vector pointing in the direction of the wave. This vector is a voltage (potential difference) vector. Three electrodes, labeled RA, LA, and LL, are placed on the patient. Each pair (called leads I, II, and III) measures a component of the depolarization vector and is graphed in an ECG.

An **electrocardiogram (ECG)** is a record of the voltages created by the wave of depolarization and subsequent repolarization in the heart. Voltages between pairs of electrodes placed on the chest are vector components of the voltage wave on the heart. Standard ECGs have 12 or more electrodes, but only three are shown in [Figure 7] for clarity. Decades ago, three-electrode ECGs were performed by placing electrodes on the left and right arms and the left leg. The voltage between the right arm and the left leg is called the *lead II potential* and is the most often graphed. We shall examine the lead II potential as an indicator of heart-muscle function and see that it is coordinated with arterial blood pressure as well.

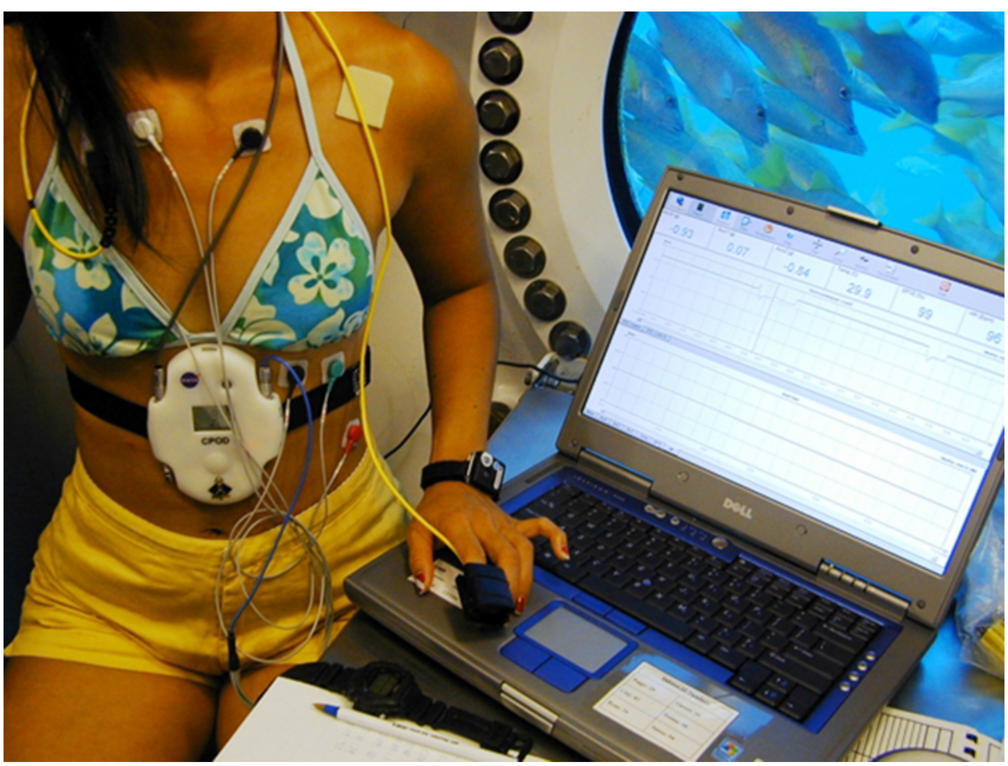
Heart function and its four-chamber action are explored in Viscosity and Laminar Flow; Poiseuille's Law. Basically, the right and left atria receive blood from the body and lungs, respectively, and pump the blood into the ventricles. The right and left ventricles, in turn, pump blood through the lungs and the rest of the body, respectively. Depolarization of the heart muscle causes it to contract. After contraction it is repolarized to ready it for the next beat. The ECG measures components of depolarization and repolarization of the heart muscle and can yield significant information on the functioning and malfunctioning of the heart.

[Figure 8] shows an ECG of the lead II potential and a graph of the corresponding arterial blood pressure. The major features are labeled P, Q, R, S, and T. The *P wave* is generated by the depolarization and contraction of the atria as they pump blood into the ventricles. The *QRS complex* is created by the depolarization of the ventricles as they pump blood to the lungs and body. Since the shape of the heart and the path of the depolarization wave are not simple, the QRS complex has this typical shape and time span. The lead II QRS signal also masks the repolarization of the atria, which occur at the same time. Finally, the *T wave* is generated by the repolarization of the ventricles and is followed by the next P wave in the next heartbeat. Arterial blood pressure varies with each part of the heartbeat, with systolic (maximum) pressure occurring closely after the QRS complex, which signals contraction of the ventricles.



A lead II ECG with corresponding arterial blood pressure. The QRS complex is created by the depolarization and contraction of the ventricles and is followed shortly by the maximum or systolic blood pressure. See text for further description.

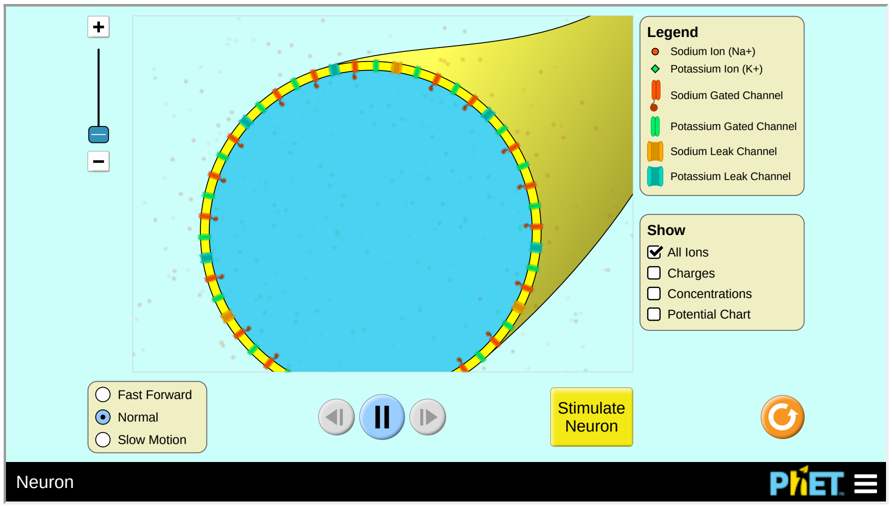
Taken together, the 12 leads of a state-of-the-art ECG can yield a wealth of information about the heart. For example, regions of damaged heart tissue, called infarcts, reflect electrical waves and are apparent in one or more lead potentials. Subtle changes due to slight or gradual damage to the heart are most readily detected by comparing a recent ECG to an older one. This is particularly the case since individual heart shape, size, and orientation can cause variations in ECGs from one individual to another. ECG technology has advanced to the point where a portable ECG monitor with a liquid crystal instant display and a printer can be carried to patients' homes or used in emergency vehicles. See [Figure 9].



This NASA scientist and NEEMO 5 aquanaut’s heart rate and other vital signs are being recorded by a portable device while living in an underwater habitat. (credit: NASA, Life Sciences Data Archive at Johnson Space Center, Houston, Texas)

PhET Explorations: Neuron

Stimulate a neuron and monitor what happens. Pause, rewind, and move forward in time in order to observe the ions as they move across the neuron membrane.



Section Summary

- Electric potentials in neurons and other cells are created by ionic concentration differences across semipermeable membranes.
- Stimuli change the permeability and create action potentials that propagate along neurons.
- Myelin sheaths speed this process and reduce the needed energy input.
- This process in the heart can be measured with an electrocardiogram (ECG).

Conceptual Questions

Note that in [Figure 2], both the concentration gradient and the Coulomb force tend to move Na^+ ions into the cell. What prevents this?

Define depolarization, repolarization, and the action potential.

Explain the properties of myelinated nerves in terms of the insulating properties of myelin.

Problems & Exercises

Integrated Concepts

Use the ECG in [Figure 8] to determine the heart rate in beats per minute assuming a constant time between beats.

Show Solution

Strategy

To find the heart rate, we need to determine the period (time for one complete heartbeat) from the ECG trace. The period T is the time from one R peak to the next R peak. The heart rate in beats per minute is then $\text{rate} = 60/T$ where T is in seconds.

Solution

From the figure description, one complete cardiac cycle (P-QRS-T complex) is completed in less than 0.7 seconds, followed by about 0.1 seconds before the next cycle begins. This gives a total period of approximately:

$$T \approx 0.7 + 0.1 = 0.8 \text{ s}$$

However, examining the ECG trace more carefully, we can read the time scale directly from the graph. The period between consecutive R peaks is approximately 0.75 s.

The heart rate in beats per minute is:

$$\text{Heart rate} = \frac{60 \text{ s/min}}{T}$$
$$\text{Heart rate} = \frac{60 \text{ s/min}}{0.75 \text{ s/beat}} = 80 \text{ beats/min}$$

Discussion

A heart rate of 80 beats per minute is within the normal resting range for adults (60-100 beats/min). This rate is slightly elevated above the average resting rate of about 70 beats/min, but is still perfectly normal and could indicate:

- Mild physical activity or recent activity
- Stress or anxiety
- Normal variation among individuals
- Effects of caffeine or medications

The consistency of the ECG trace (regular R-R intervals) indicates a healthy, regular rhythm. Irregularities in the spacing between beats would indicate arrhythmias, which can range from benign to serious depending on their nature.

In clinical practice, heart rate is often calculated by counting the number of large boxes (0.2 s each) between R peaks on standard ECG paper, or by using the "300 rule" for quick estimation. For more precise measurements, especially when diagnosing arrhythmias, cardiologists carefully measure multiple R-R intervals to assess heart rate variability.

The fact that this problem asks us to "assume a constant time between beats" is important—it tells us the rhythm is regular, which is characteristic of normal sinus rhythm originating from the SA node (the heart's natural pacemaker).

The heart rate is 80 beats per minute.

Integrated Concepts

(a) Referring to [Figure 8], find the time systolic pressure lags behind the middle of the QRS complex. (b) Discuss the reasons for the time lag.

Show Solution

Strategy

For part (a), examine Figure 8 and measure the time difference between the peak of the R wave (middle of the QRS complex) and the maximum (systolic) blood pressure. For part (b), consider the physiological processes that must occur between the electrical signal and the resulting pressure change.

Solution

(a) From Figure 8, the QRS complex (specifically the R wave peak) occurs at approximately 0.20 s, and the systolic (maximum) blood pressure peak occurs at approximately 0.40 s. Therefore, the time lag is:

$$\Delta t = 0.40\text{ s} - 0.20\text{ s} = 0.20\text{ s} = 200\text{ ms}$$

(b) The time lag between the QRS complex and systolic pressure exists for several physiological reasons:

1. **Electromechanical coupling delay**: The QRS complex represents the electrical depolarization of the ventricles. After depolarization, there is a delay while calcium ions enter the muscle cells and trigger the actin-myosin interaction that causes muscle contraction.

2. **Contraction development time**: The ventricular muscle does not reach peak contraction force instantaneously. It takes time for all the muscle fibers to contract and develop maximum force.

3. **Isovolumetric contraction phase**: After the ventricles begin contracting, the aortic valve remains closed until ventricular pressure exceeds aortic pressure. During this isovolumetric phase (about 50 ms), no blood is ejected and pressure builds.

4. **Pressure wave propagation**: After the aortic valve opens, blood is ejected into the aorta. The pressure wave must then propagate through the arterial system to the measurement site, which introduces additional delay.

Discussion

The 200 ms lag is clinically significant. Changes in this interval can indicate cardiac problems. A longer lag might suggest weakened heart muscle (as in heart failure), while electrical conduction abnormalities can alter the timing relationship between the ECG and mechanical events. This electromechanical delay is why doctors use both ECGs (electrical) and blood pressure measurements (mechanical) to get a complete picture of cardiac function.

(a) The systolic pressure lags approximately 0.20 s (200 ms) behind the QRS complex. (b) This lag is due to electromechanical coupling delay, contraction development time, the isovolumetric contraction phase, and pressure wave propagation time.

Glossary

nerve conduction
: the transport of electrical signals by nerve cells

bioelectricity
: electrical effects in and created by biological systems

semipermeable
: property of a membrane that allows only certain types of ions to cross it

electrocardiogram (ECG)
: usually abbreviated ECG, a record of voltages created by depolarization and repolarization, especially in the heart

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