Introduction to Science and the Realm of Physics, Physical Quantities, and Units

Galaxies are as immense as atoms are small. Yet the same laws of physics describe both, and all the rest of nature—an indication of the underlying unity in the universe. The laws of physics are surprisingly few in number, implying an underlying simplicity to nature's apparent complexity. (credit: NASA, JPL-Caltech, P. Barmby, Harvard-Smithsonian Center for Astrophysics)

What is your first reaction when you hear the word "physics"? Did you imagine working through difficult equations or memorizing formulas that seem to have no real use in life outside the physics classroom? Many people come to the subject of physics with a bit of fear. But as you begin your exploration of this broad-ranging subject, you may soon come to realize that physics plays a much larger role in your life than you first thought, no matter your life goals or career choice.

For example, take a look at the image above. This image is of the Andromeda Galaxy, which contains billions of individual stars, huge clouds of gas, and dust. Two smaller galaxies are also visible as bright blue spots in the background. At a staggering 2.5 million light years from the Earth, this galaxy is the nearest one to our own galaxy (which is called the Milky Way). The stars and planets that make up Andromeda might seem to be the furthest thing from most people's regular, everyday lives. But Andromeda is a great starting point to think about the forces that hold together the universe. The forces that cause Andromeda to act as it does are the same forces we contend with here on Earth, whether we are planning to send a rocket into space or simply raise the walls for a new home. The same gravity that causes the stars of Andromeda to rotate and revolve also causes water to flow over hydroelectric dams here on Earth. Tonight, take a moment to look up at the stars. The forces out there are the same as the ones here on Earth. Through a study of physics, you may gain a greater understanding of the interconnectedness of everything we can see and know in this universe.

Humans have created and manufactured millions of different objects over the history of our species. Successive technological periods (often referred to as the Stone Age, the Bronze Age, the Iron Age, and so on) were marked by our knowledge of the physical properties of certain materials and our ability to manipulate them. This knowledge all stems from physics, whether it's the way a rock would flake when constructing a spear point, the effect of integrating carbon with iron in South Indian and Sri Lankan furnaces to create the earliest high-quality steel, or the proper way to combine perfectly ground and polished pieces of glass to create optical instruments. Our current technological age, the Information Age, builds on all that knowledge and can be traced to critical innovations made by people from all backgrounds working together. Mohamed M. Atalla and Dawon Kahng, for example, invented the **MOSFET** (metal-oxide-semiconductor field-effect transistor). Although unknown to most people, this tiny device, created in 1959 by an Egyptian-born scientist and Korean-born scientist working in a lab in New Jersey, is the basis for modern electronics. More MOSFETs have been produced than any other object in human history. They are used in computers, smartphones, microwave ovens, automotive controls, medical instruments, and nearly every other electronic device.

Next, think about the most exciting modern technologies that you have heard about in the news, such as trains that levitate above tracks, "invisibility cloaks" that bend light around them, and microscopic robots that fight cancer cells in our bodies. All of these groundbreaking advancements, commonplace or unbelievable, rely on the principles of physics. Aside from playing a significant role in technology, professionals such as engineers, pilots, physicians, physical therapists, electricians, and computer programmers apply physics concepts in their daily work. For example, a pilot must understand how wind forces affect a flight path and a physical therapist must understand how the muscles in the body experience forces as they move and bend. As you will learn in this text, physics principles are propelling new, exciting technologies, and these principles are applied in a wide range of careers.

In this text, you will begin to explore the history of the formal study of physics, beginning with natural philosophy and the ancient Greeks, and leading up through a review of Sir Isaac Newton and the laws of physics that bear his name. You will also be introduced to the standards scientists use when they study physical quantities and the interrelated system of measurements most of the scientific community uses to communicate in a single mathematical language. Finally, you will study the limits of our ability to be accurate and precise, and the reasons scientists go to painstaking lengths to be as clear as possible regarding their own limitations.

Physics: An Introduction

- Explain the difference between a principle and a law.
- Explain the difference between a model and a theory.

The flight formations of migratory birds such as Canada geese are governed by the laws of physics. (credit: David Merrett)

The physical universe is enormously complex in its detail. Every day, each of us observes a great variety of objects and phenomena. Over the centuries, the curiosity of the human race has led us collectively to explore and catalog a tremendous wealth of information. From the flight of birds to the colors of flowers, from lightning to gravity, from quarks to clusters of galaxies, from the flow of time to the mystery of the creation of the universe, we have asked questions and assembled huge arrays of facts. In the face of all these details, we have discovered that a surprisingly small and unified set of physical laws can explain what we observe. As humans, we make generalizations and seek order. We have found that nature is remarkably cooperative—it exhibits the **underlying order and simplicity** we so value.

It is the underlying order of nature that makes science in general, and physics in particular, so enjoyable to study. For example, what do a bag of chips and a car battery have in common? Both contain energy that can be converted to other forms. The law of conservation of energy (which says that energy can change form but is never lost) ties together such topics as food calories, batteries, heat, light, and watch springs. Understanding this law makes it easier to learn about the various forms energy takes and how they relate to one another. Apparently unrelated topics are connected through broadly applicable physical laws, permitting an understanding beyond just the memorization of lists of facts.

The unifying aspect of physical laws and the basic simplicity of nature form the underlying themes of this text. In learning to apply these laws, you will, of course, study the most important topics in physics. More importantly, you will gain analytical abilities that will enable you to apply these laws far beyond the scope of what can be included in a single book. These analytical skills will help you to excel academically, and they will also help you to think critically in any professional career you choose to pursue. This module discusses the realm of physics (to define what physics is), some applications of physics (to illustrate its relevance to other disciplines), and more precisely what constitutes a physical law (to illuminate the importance of experimentation to theory).

Science and the Realm of Physics

Science consists of the theories and laws that are the general truths of nature as well as the body of knowledge they encompass. Scientists are continually trying to expand this body of knowledge and to perfect the expression of the laws that describe it. **Physics** is concerned with describing the interactions of energy, matter, space, and time, and it is especially interested in what fundamental mechanisms underlie every phenomenon. The concern for describing the basic phenomena in nature essentially defines the **realm of physics**.

Physics aims to describe the function of everything around us, from the movement of tiny charged particles to the motion of people, cars, and spaceships. In fact, almost everything around you can be described quite accurately by the laws of physics. Consider a smartphone (Figure 2). Physics describes how electricity interacts with the various circuits inside the device. This knowledge helps engineers select the appropriate materials and circuit layout when building the smartphone. Next, consider a GPS system. Physics describes the relationship between the speed of an object, the distance over which it travels, and the time it takes to travel that distance. GPS relies on precise calculations that account for variations in the Earth's landscapes, the exact distance between orbiting satellites, and even the effect of a complex occurrence of time dilation. Most of these calculations are founded on algorithms developed by Gladys West, a mathematician and computer scientist who programmed the first computers capable of highly accurate remote sensing and positioning. When you use a GPS device, it utilizes these algorithms to recognize where you are and how your position relates to other objects on Earth.

The Apple “iPhone” is a common smartphone with a GPS function. Physics describes the way that electricity flows through the circuits of this device. Engineers use their knowledge of physics to construct an iPhone with features that consumers will enjoy. One specific feature of an iPhone is the GPS function. GPS uses physics equations to determine the driving time between two locations on a map. (credit: Tinh tế Photo/Flickr)

Applications of Physics

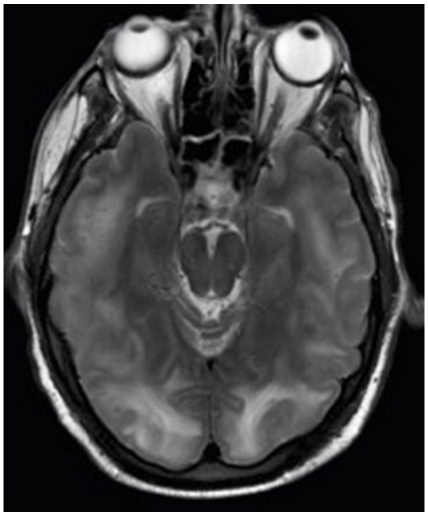
You need not be a scientist to use physics. On the contrary, knowledge of physics is useful in everyday situations as well as in nonscientific professions. It can help you understand how microwave ovens work, why metals should not be put into them, and why they might affect pacemakers. (See Figure 3 and Figure 4.) Physics allows you to understand the hazards of radiation and rationally evaluate these hazards more easily. Physics also explains the reason why a black car radiator helps remove heat in a car engine, and it explains why a white roof helps keep the inside of a house cool. Similarly, the operation of a car’s ignition system as well as the transmission of electrical signals through our body’s nervous system are much easier to understand when you think about them in terms of basic physics.

Physics is the foundation of many important disciplines and contributes directly to others. Chemistry, for example—since it deals with the interactions of atoms and molecules—is rooted in atomic and molecular physics. Most branches of engineering are applied physics. In architecture, physics is at the heart of structural stability, and is involved in the acoustics, heating, lighting, and cooling of buildings. Parts of geology rely heavily on physics, such as radioactive dating of rocks, earthquake analysis, and heat transfer in the Earth. Some disciplines, such as biophysics and geophysics, are hybrids of physics and other disciplines.

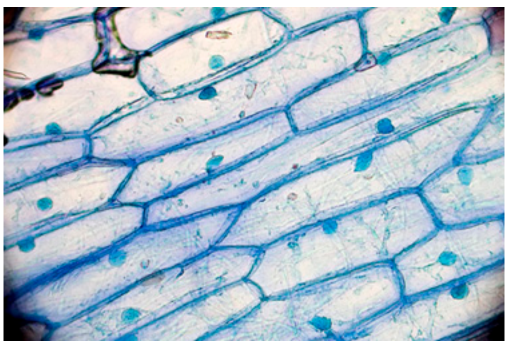
Physics has many applications in the biological sciences. On the microscopic level, it helps describe the properties of cell walls and cell membranes (Figure 5 and Figure 6). On the macroscopic level, it can explain the heat, work, and power associated with the human body. Physics is involved in medical diagnostics, such as X-rays, magnetic resonance imaging (MRI), and ultrasonic blood flow measurements. Medical therapy sometimes directly involves physics; for example, cancer radiotherapy uses ionizing radiation. Physics can also explain sensory phenomena, such as how musical instruments make sound, how the eye detects color, and how lasers can transmit information.

It is not necessary to formally study all applications of physics. What is most useful is knowledge of the basic laws of physics and a skill in the analytical methods for applying them. The study of physics also can improve your problem-solving skills. Furthermore, physics has retained the most basic aspects of science, so it is used by all of the sciences, and the study of physics makes other sciences easier to understand.

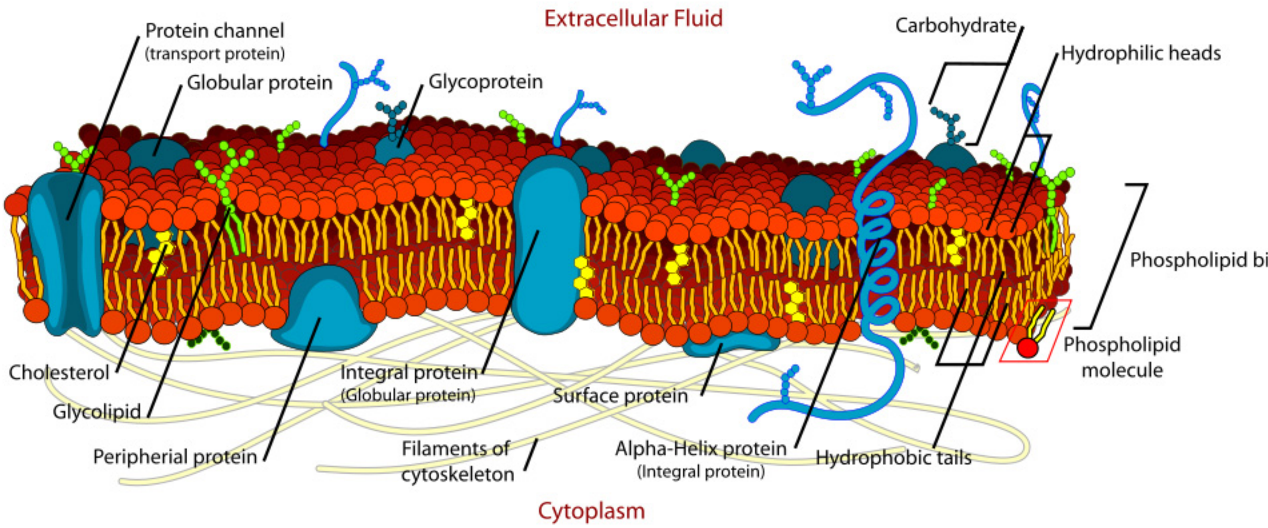
The laws of physics help us understand how common appliances work. For example, the laws of physics can help explain how microwave ovens heat up food, and they also help us understand why it is dangerous to place metal objects in a microwave oven. (credit: MoneyBlogNewz)



These two applications of physics have more in common than meets the eye. Microwave ovens use electromagnetic waves to heat food. Magnetic resonance imaging (MRI) also uses electromagnetic waves to yield an image of the brain, from which the exact location of tumors can be determined. (credit: Rashmi Chawla, Daniel Smith, and Paul E. Marik)



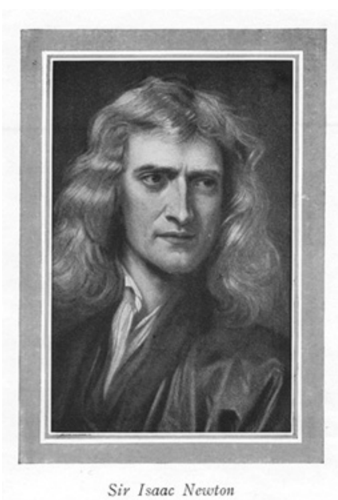
Physics, chemistry, and biology help describe the properties of cell walls in plant cells, such as the onion cells seen here. (credit: Umberto Salvagnin)



An artist's rendition of the structure of a cell membrane. Membranes form the boundaries of animal cells and are complex in structure and function. Many of the most fundamental properties of life, such as the firing of nerve cells, are related to membranes. The disciplines of biology, chemistry, and physics all help us understand the membranes of animal cells. (credit: Mariana Ruiz)

Models, Theories, and Laws; The Role of Experimentation

The laws of nature are concise descriptions of the universe around us; they are human statements of the underlying laws or rules that all natural processes follow. Such laws are intrinsic to the universe; humans did not create them and so cannot change them. We can only discover and understand them. Their discovery is a very human endeavor, with all the elements of mystery, imagination, struggle, triumph, and disappointment inherent in any creative effort. (See Figure 7 and Figure 8.) The cornerstone of discovering natural laws is observation; science must describe the universe as it is, not as we may imagine it to be.



Isaac Newton (1642–1727) was very reluctant to publish his revolutionary work and had to be convinced to do so. In his later years, he stepped down from his academic post and became exchequer of the Royal Mint. He took this post seriously, inventing reeding (or creating ridges) on the edge of coins to prevent unscrupulous people from trimming the silver off of them before using them as currency. (credit: Arthur Shuster and Arthur E. Shipley: Britain's Heritage of Science. London, 1917.)

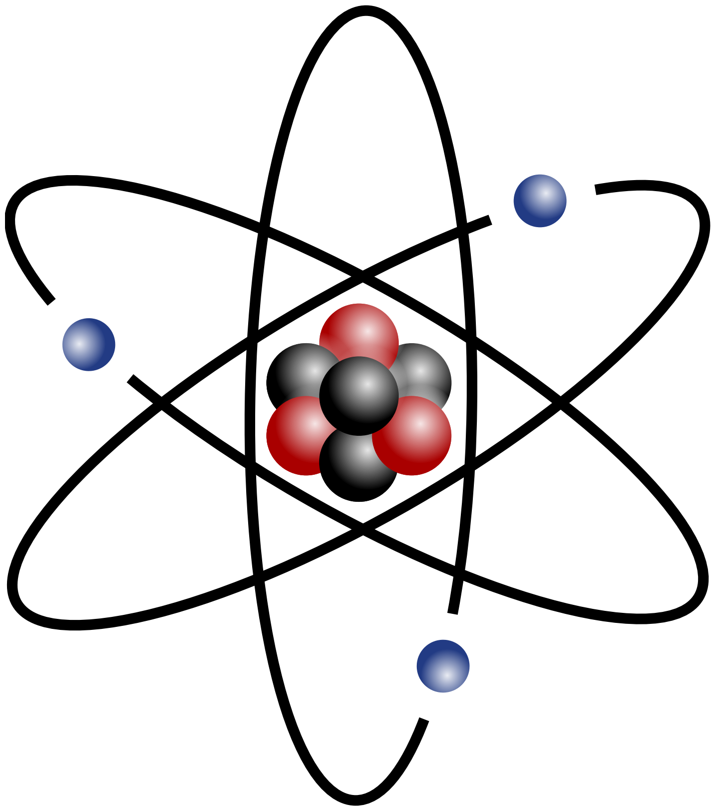
Marie Curie (1867–1934) sacrificed monetary assets to help finance her early research and damaged her physical well-being with radiation exposure. She is the only person to win Nobel prizes in both physics and chemistry. One of her daughters also won a Nobel Prize. (credit: Wikimedia Commons)

We all are curious to some extent. We look around, make generalizations, and try to understand what we see—for example, we look up and wonder whether one type of cloud signals an oncoming storm. As we become serious about exploring nature, we become more organized and formal in collecting and analyzing data. We attempt greater precision, perform controlled experiments (if we can), and write down ideas about how the data may be organized and unified. We then formulate models, theories, and laws based on the data we have collected and analyzed to generalize and communicate the results of these experiments.

A **model** is a representation of something that is often too difficult (or impossible) to display directly. While a model is justified with experimental proof, it is only accurate under limited situations. An example is the planetary model of the atom in which electrons are pictured as orbiting the nucleus, analogous to the way planets orbit the Sun. (See Figure 9.) We cannot observe electron orbits directly, but the mental image helps explain the observations we can make, such as the emission of light from hot gases (atomic spectra). Physicists use models for a variety of purposes. For example, models can help physicists analyze a scenario and perform a calculation, or they can be used to represent a situation in the form of a computer simulation. A **theory** is an explanation for patterns in nature that is supported by scientific evidence and verified multiple times by various groups of researchers. Some theories include models to help visualize phenomena, whereas others do not. Newton’s theory of gravity, for example, does not require a model or mental image, because we can observe the objects directly with our own senses. The kinetic theory of gases, on the other hand, is a model in which a gas is viewed as being composed of atoms and molecules. Atoms and molecules are too small to be observed directly with our senses—thus, we picture them mentally to understand what our instruments tell us about the behavior of gases.

A **law** uses concise language to describe a generalized pattern in nature that is supported by scientific evidence and repeated experiments. Often, a law can be expressed in the form of a single mathematical equation. Laws and theories are similar in that they are both scientific statements that result from a tested hypothesis and are supported by scientific evidence. However, the designation **law** is reserved for a concise and very general statement that describes phenomena in nature, such as the law that energy is conserved during any process, or Newton’s second law of motion, which relates force, mass, and acceleration by the simple equation $\vec{\mathbf{F}} = m\vec{\mathbf{a}}$. A theory, in contrast, is a less concise statement of observed phenomena. For example, the Theory of Evolution and the Theory of Relativity cannot be expressed concisely enough to be considered a law. The biggest difference between a law and a theory is that a theory is much more complex and dynamic. A law describes a single action, whereas a theory explains an entire group of related phenomena. And, whereas a law is a postulate that forms the foundation of the scientific method, a theory is the end result of that process.

Less broadly applicable statements are usually called principles (such as Pascal’s principle, which is applicable only in fluids), but the distinction between laws and principles often is not carefully made.



What is a model? This planetary model of the atom shows electrons orbiting the nucleus. It is a drawing that we use to form a mental image of the atom that we cannot see directly with our eyes because it is too small.

Models, Theories, and Laws

Models, theories, and laws are used to help scientists analyze the data they have already collected. However, often after a model, theory, or law has been developed, it points scientists toward new discoveries they would not otherwise have made.

The models, theories, and laws we devise sometimes *imply the existence of objects or phenomena as yet unobserved.* These predictions are remarkable triumphs and tributes to the power of science. It is the underlying order in the universe that enables scientists to make such spectacular predictions. However, if **experiment** does not verify our predictions, then the theory or law is wrong, no matter how elegant or convenient it is. Laws can never be known with absolute certainty because it is impossible to perform every imaginable experiment in order to confirm a law in every possible scenario. Physicists operate under the assumption that all scientific laws and theories are valid until a counterexample is observed. If a good-quality, verifiable experiment contradicts a well-established law, then the law must be modified or overthrown completely.

The study of science in general and physics in particular is an adventure much like the exploration of uncharted ocean. Discoveries are made; models, theories, and laws are formulated; and the beauty of the physical universe is made more sublime for the insights gained.

The Scientific Method

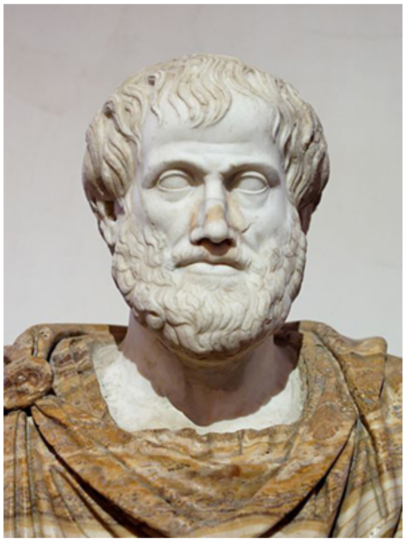
Ibn al-Haytham (sometimes referred to as Alhazen), a 10th-11th century scientist working in Cairo, significantly advanced the understanding of optics and vision. But his contributions go much further. In demonstrating that previous approaches were incorrect, he emphasized that scientists must be ready to reject existing knowledge and become “the enemy” of everything they read; he expressed that scientists must trust only objective evidence. Al-Haytham emphasized repeated experimentation and validation, and acknowledged that senses and predisposition could lead to poor conclusions. His work was a precursor to the scientific method that we use today.

As scientists inquire and gather information about the world, they follow a process called the **scientific method**. This process typically begins with an observation and question that the scientist will research. Next, the scientist typically performs some research about the topic and then devises a hypothesis. Then, the scientist will test the hypothesis by performing an experiment. Finally, the scientist analyzes the results of the experiment and draws a conclusion. Note that the scientific method can be applied to many situations that are not limited to science, and this method can be modified to suit the situation.

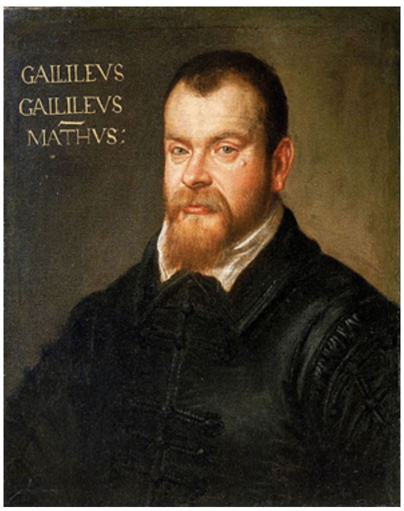
Consider an example. Let us say that you try to turn on your car, but it will not start. You undoubtedly wonder: Why will the car not start? You can follow a scientific method to answer this question. First off, you may perform some research to determine a variety of reasons why the car will not start. Next, you will state a hypothesis. For example, you may believe that the car is not starting because it has no engine oil. To test this, you open the hood of the car and examine the oil level. You observe that the oil is at an acceptable level, and you thus conclude that the oil level is not contributing to your car issue. To troubleshoot the issue further, you may devise a new hypothesis to test and then repeat the process again.

The Evolution of Natural Philosophy into Modern Physics

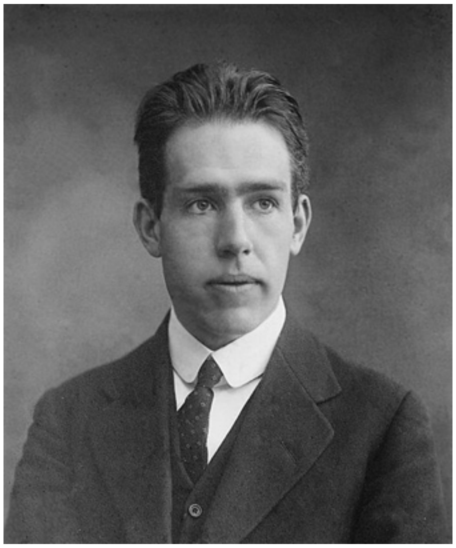
Physics was not always a separate and distinct discipline. It remains connected to other sciences to this day. The word **physics** comes from Greek, meaning nature. The study of nature came to be called “natural philosophy.” From ancient times through the Renaissance, natural philosophy encompassed many fields, including astronomy, biology, chemistry, physics, mathematics, and medicine. Over the last few centuries, the growth of knowledge has resulted in ever-increasing specialization and branching of natural philosophy into separate fields, with physics retaining the most basic facets. (See Figure 10, Figure 11, and Figure 12.) Physics as it developed from the Renaissance to the end of the 19th century is called **classical physics**. It was transformed into modern physics by revolutionary discoveries made starting at the beginning of the 20th century.



Over the centuries, natural philosophy has evolved into more specialized disciplines, as illustrated by the contributions of some of the greatest minds in history. The Greek philosopher Aristotle (384–322 B.C.) wrote on a broad range of topics including physics, animals, the soul, politics, and poetry. (credit: Jastrow (2006)/Ludovisi Collection)



Galileo Galilei (1564–1642) laid the foundation of modern experimentation and made contributions in mathematics, physics, and astronomy. (credit: Domenico Tintoretto)

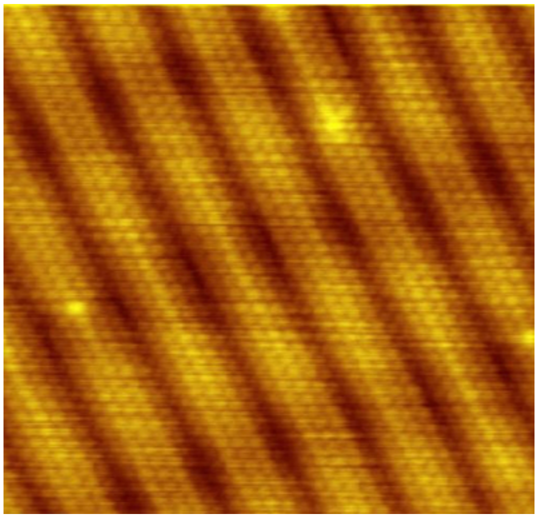


Niels Bohr (1885–1962) made fundamental contributions to the development of quantum mechanics, one part of modern physics. (credit: United States Library of Congress Prints and Photographs Division)

Classical physics is not an exact description of the universe, but it is an excellent approximation under the following conditions: Matter must be moving at speeds less than about 1% of the speed of light, the objects dealt with must be large enough to be seen with a microscope, and only weak gravitational fields, such as the field generated by the Earth, can be involved. Because humans live under such circumstances, classical physics seems intuitively reasonable, while many aspects of modern physics seem bizarre. This is why models are so useful in modern physics—they let us conceptualize phenomena we do not ordinarily experience. We can relate to models in human terms and visualize what happens when objects move at high speeds or imagine what objects too small to observe with our senses might be like. For example, we can understand an atom's properties because we can picture it in our minds, although we have never seen an atom with our eyes. New tools, of course, allow us to better picture phenomena we cannot see. In fact, new instrumentation has allowed us in recent years to actually "picture" the atom. (see Figure 13.)

Limits on the Laws of Classical Physics

For the laws of classical physics to apply, the following criteria must be met: Matter must be moving at speeds less than about 1% of the speed of light, the objects dealt with must be large enough to be seen with a microscope, and only weak gravitational fields (such as the field generated by the Earth) can be involved.



Using a scanning tunneling microscope (STM), scientists can see the individual atoms that compose this sheet of gold. (credit: Erwinrossen)

Some of the most spectacular advances in science have been made in modern physics. Many of the laws of classical physics have been modified or rejected, and revolutionary changes in technology, society, and our view of the universe have resulted. Like science fiction, modern physics is filled with fascinating objects beyond our normal experiences, but it has the advantage over science fiction of being very real. Why, then, is the majority of this text devoted to topics of classical physics? There are two main reasons: Classical physics gives an extremely accurate description of the universe under a wide range of everyday circumstances, and knowledge of classical physics is necessary to understand modern physics.

Modern physics itself consists of the two revolutionary theories, relativity and quantum mechanics. These theories deal with the very fast and the very small, respectively. **Relativity** must be used whenever an object is traveling at greater than about 1% of the speed of light or experiences a strong gravitational field such as that near the Sun. **Quantum mechanics** must be used for objects smaller than can be seen with a microscope. The combination of these two theories is *relativistic quantum mechanics,* and it describes the behavior of small objects traveling at high speeds or experiencing a strong gravitational field. Relativistic quantum mechanics is the best universally applicable theory we have. Because of its mathematical complexity, it is used only when necessary, and the other theories are used whenever they will produce sufficiently accurate results. We will find, however, that we can do a great deal of modern physics with the algebra and trigonometry used in this text.

Check Your Understanding

A friend tells you he has learned about a new law of nature. What can you know about the information even before your friend describes the law? How would the information be different if your friend told you he had learned about a scientific theory rather than a law?

Show Solution

Without knowing the details of the law, you can still infer that the information your friend has learned conforms to the requirements of all laws of nature: it will be a concise description of the universe around us; a statement of the underlying rules that all natural processes follow. If the information had been a theory, you would be able to infer that the information will be a large-scale, broadly applicable generalization.

Graphing Quadratics

Learn about graphing polynomials. The shape of the curve changes as the constants are adjusted. View the curves for the individual terms (e.g. $y = bx$) to see how they add to generate the polynomial curve.

Graphing Quadratics

Summary

- Science seeks to discover and describe the underlying order and simplicity in nature.
- Physics is the most basic of the sciences, concerning itself with energy, matter, space and time, and their interactions.
- Scientific laws and theories express the general truths of nature and the body of knowledge they encompass. These laws of nature are rules that all natural processes appear to follow.

Conceptual Questions

Models are particularly useful in relativity and quantum mechanics, where conditions are outside those normally encountered by humans. What is a model?

How does a model differ from a theory?

If two different theories describe experimental observations equally well, can one be said to be more valid than the other (assuming both use accepted rules of logic)?

What determines the validity of a theory?

Certain criteria must be satisfied if a measurement or observation is to be believed. Will the criteria necessarily be as strict for an expected result as for an unexpected result?

Can the validity of a model be limited, or must it be universally valid? How does this compare to the required validity of a theory or a law?

Classical physics is a good approximation to modern physics under certain circumstances. What are they?

When is it **necessary** to use relativistic quantum mechanics?

Can classical physics be used to accurately describe a satellite moving at a speed of 7500 m/s? Explain why or why not.

Glossary

classical physics
: physics that was developed from the Renaissance to the end of the 19th century

physics
: the science concerned with describing the interactions of energy, matter, space, and time; it is especially interested in what fundamental mechanisms underlie every phenomenon

model
: representation of something that is often too difficult (or impossible) to display directly

theory
: an explanation for patterns in nature that is supported by scientific evidence and verified multiple times by various groups of researchers

law
: a description, using concise language or a mathematical formula, a generalized pattern in nature that is supported by scientific evidence and repeated experiments

scientific method
: a method that typically begins with an observation and question that the scientist will research; next, the scientist typically performs some research about the topic and then devises a hypothesis; then, the scientist will test the hypothesis by performing an experiment; finally, the scientist analyzes the results of the experiment and draws a conclusion

modern physics
: the study of relativity, quantum mechanics, or both

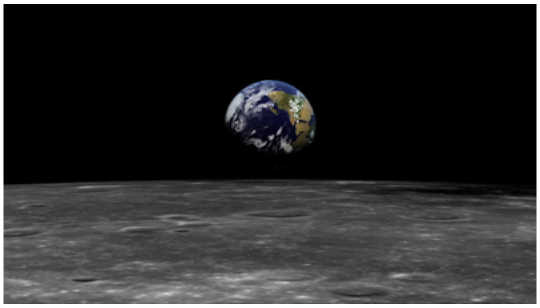
relativity
: the study of objects moving at speeds greater than about 1% of the speed of light, or of objects being affected by a strong gravitational field

quantum mechanics
: the study of objects smaller than can be seen with a microscope

---

Physical Quantities and Units

- Perform unit conversions both in the SI and English units.
- Explain the most common prefixes in the SI units and be able to write them in scientific notation.

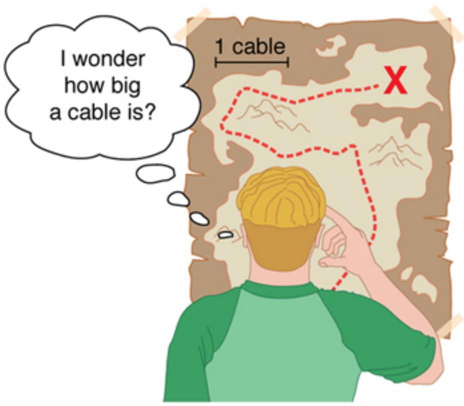


The distance from Earth to the Moon may seem immense, but it is just a tiny fraction of the distances from Earth to other celestial bodies. (credit: NASA)

The range of objects and phenomena studied in physics is immense. From the incredibly short lifetime of a nucleus to the age of the Earth, from the tiny sizes of sub-nuclear particles to the vast distance to the edges of the known universe, from the force exerted by a jumping flea to the force between Earth and the Sun, there are enough factors of 10 to challenge the imagination of even the most experienced scientist. Giving numerical values for physical quantities and equations for physical principles allows us to understand nature much more deeply than does qualitative description alone. To comprehend these vast ranges, we must also have accepted units in which to express them. And we shall find that (even in the potentially mundane discussion of meters, kilograms, and seconds) a profound simplicity of nature appears—all physical quantities can be expressed as combinations of only four fundamental physical quantities: length, mass, time, and electric current.

We define a **physical quantity** either by *specifying how it is measured* or by **stating how it is calculated** from other measurements. For example, we define distance and time by specifying methods for measuring them, whereas we define **average speed** by stating that it is calculated as distance traveled divided by time of travel.

Measurements of physical quantities are expressed in terms of **units**, which are standardized values. For example, the length of a race, which is a physical quantity, can be expressed in units of meters (for sprinters) or kilometers (for distance runners). Without standardized units, it would be extremely difficult for scientists to express and compare measured values in a meaningful way. (See Figure 2.)



Distances given in unknown units are maddeningly useless.

There are two major systems of units used in the world: **SI units** (also known as the metric system) and **English units** (also known as the customary or imperial system). **English units** were historically used in nations once ruled by the British Empire and are still widely used in the United States. Virtually every other country in the world now uses SI units as the standard; the metric system is also the standard system agreed upon by scientists and mathematicians. The acronym “SI” is derived from the French *Système International*.

SI Units: Fundamental and Derived Units

Table 1 gives the fundamental SI units that are used throughout this textbook. This text uses non-SI units in a few applications where they are in very common use, such as the measurement of blood pressure in millimeters of mercury (mm Hg). Whenever non-SI units are discussed, they will be tied to SI units through conversions.

Table 1: Fundamental SI Units

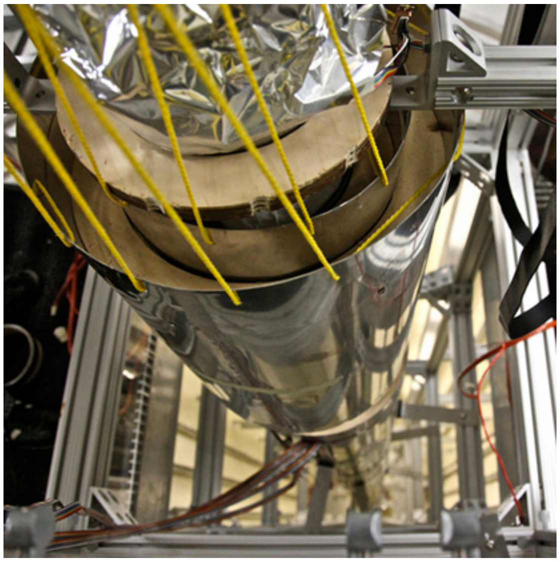
Length	Mass	Time	Electric Current
meter (m)	kilogram (kg)	second (s)	ampere (A)

It is an intriguing fact that some physical quantities are more fundamental than others and that the most fundamental physical quantities can be defined **only** in terms of the procedure used to measure them. The units in which they are measured are thus called **fundamental units**. In this textbook, the fundamental physical quantities are taken to be length, mass, time, and electric current. (Note that electric current will not be introduced until much later in this text.) All other physical quantities, such as force and electric charge, can be expressed as algebraic combinations of length, mass, time, and current (for example, speed is length divided by time); these units are called **derived units**.

Units of Time, Length, and Mass: The Second, Meter, and Kilogram

The Second

The SI unit for time, the **second**(abbreviated s), has a long history. For many years it was defined as 1/86 400 of a mean solar day. More recently, a new standard was adopted to gain greater accuracy and to define the second in terms of a non-varying, or constant, physical phenomenon (because the solar day is getting longer due to very gradual slowing of the Earth's rotation). Cesium atoms can be made to vibrate in a very steady way, and these vibrations can be readily observed and counted. In 1967 the second was redefined as the time required for 9 192 631 770 of these vibrations. (See Figure 3.) Accuracy in the fundamental units is essential, because all measurements are ultimately expressed in terms of fundamental units and can be no more accurate than are the fundamental units themselves.



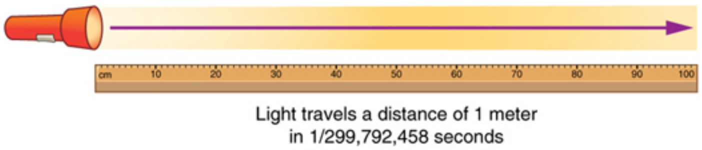
An atomic clock such as this one uses the vibrations of cesium atoms to keep time to a precision of better than a microsecond per year. The fundamental unit of time, the second, is based on such clocks. This image is looking down from the top of an atomic fountain nearly 30 feet tall! (credit: Steve Jurvetson/Flickr)

The Meter

The SI unit for length is the **meter** (abbreviated m); its definition has also changed over time to become more accurate and precise. The meter was first defined in 1791 as 1/10 000 000 of the distance from the equator to the North Pole. This measurement was improved in 1889 by redefining the meter to be the distance between two engraved lines on a platinum-iridium bar now kept near Paris. By 1960, it had become possible to define the meter even more accurately in terms of the wavelength of light, so it was again redefined as 1 650 763.73 wavelengths of orange light emitted by krypton atoms. In 1983, the meter was given its present definition (partly for greater accuracy) as the distance light travels in a vacuum in 1/299 792 458 of a second. (See Figure 4.) This change defines the speed of light to be exactly 299 792 458 meters per second. The length of the meter will change if the speed of light is someday measured with greater accuracy.

The Kilogram

The SI unit for mass is the **kilogram** (abbreviated kg); it was previously defined to be the mass of a platinum-iridium cylinder kept with the old meter standard at the International Bureau of Weights and Measures near Paris. Exact replicas of the previously defined kilogram are also kept at the United States' National Institute of Standards and Technology, or NIST, located in Gaithersburg, Maryland outside of Washington D.C., and at other locations around the world. The determination of all other masses could be ultimately traced to a comparison with the standard mass. Even though the platinum-iridium cylinder was resistant to corrosion, airborne contaminants were able to adhere to its surface, slightly changing its mass over time. In May 2019, the scientific community adopted a more stable definition of the kilogram. The kilogram is now defined in terms of the second, the meter, and Planck's constant, h (a quantum mechanical value that relates a photon's energy to its frequency).



The meter is defined to be the distance light travels in 1/299 792 458 of a second in a vacuum. Distance traveled is speed multiplied by time.

Electric current and its accompanying unit, the ampere, will be introduced in later units when electricity and magnetism are covered. The initial modules in this textbook are concerned with mechanics. In this subject all pertinent physical quantities can be expressed in terms of the fundamental units of length, mass, and time.

Metric Prefixes

SI units are part of the **metric system**. The metric system is convenient for scientific and engineering calculations because the units are categorized by factors of 10. Table 2 gives metric prefixes and symbols used to denote various factors of 10.

Metric systems have the advantage that conversions of units involve only powers of 10. There are 100 centimeters in a meter, 1000 meters in a kilometer, and so on. In nonmetric systems, such as the system of U.S. customary units, the relationships are not as simple—there are 12 inches in a foot, 5280 feet in a mile, and so on. Another advantage of the metric system is that the same unit can be used over extremely large ranges of values simply by using an appropriate metric prefix. For example, distances in meters are suitable in construction, while distances in kilometers are appropriate for air travel, and the tiny measure of nanometers are convenient in optical design. With the metric system there is no need to invent new units for particular applications.

The term **order of magnitude** refers to the scale of a value expressed in the metric system. Each power of 10 in the metric system represents a different order of magnitude. For example, $10^1, 10^2, 10^3$, and so forth are all different orders of magnitude. All quantities that can be expressed as a product of a specific power of 10 are said to be of the **same** order of magnitude. For example, the number 800 can be written as 8×10^2, and the number 450 can be written as 4.5×10^2. Thus, the numbers 800 and 450 are of the same order of magnitude: 10^2. Order of magnitude can be thought of as a ballpark estimate for the scale of a value. The diameter of an atom is on the order of 10^{-9}m, while the diameter of the Sun is on the order of 10^9m.

The Quest for Microscopic Standards for Basic Units

The fundamental units described in this chapter are those that produce the greatest accuracy and precision in measurement. There is a sense among physicists that, because there is an underlying microscopic substructure to matter, it would be most satisfying to base our standards of measurement on microscopic objects and fundamental physical phenomena such as the speed of light. A microscopic standard has been accomplished for the standard of time, which is based on the oscillations of the cesium atom.

The standard for length was once based on the wavelength of light (a small-scale length) emitted by a certain type of atom, but it has been supplanted by the more precise measurement of the speed of light. If it becomes possible to measure the mass of atoms or a particular arrangement of atoms such as a silicon sphere to greater precision than the kilogram standard, it may become possible to base mass measurements on the small scale. There are also possibilities that electrical phenomena on the small scale may someday allow us to base a unit of charge on the charge of electrons and protons, but at present current and charge are related to large-scale currents and forces between wires.

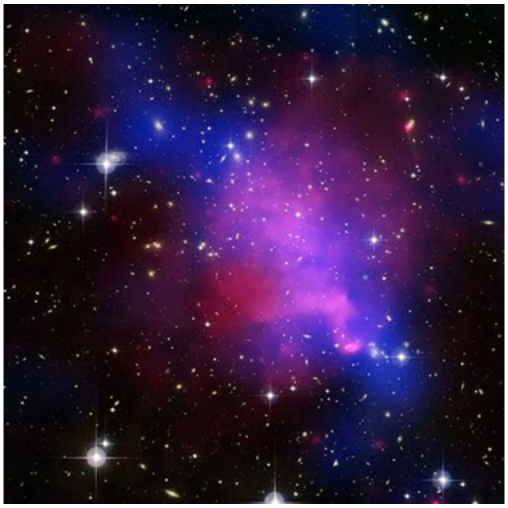
Table 2: Metric Prefixes for Powers of 10 and their Symbols

Prefix	Symbol	Value	Example			Example (some are approximate)
exa	E	10^{18}	exameter	Em	10^{18} m	distance light travels in a century
peta	P	10^{15}	petasecond	Ps	10^{15} s	30 million years
tera	T	10^{12}	terawatt	TW	10^{12} W	powerful laser output
giga	G	10^{9}	gigahertz	GHz	10^{9} Hz	a microwave frequency
mega	M	10^{6}	megacurie	MCi	10^{6} Ci	high radioactivity
kilo	k	10^{3}	kilometer	km	10^{3} m	about 6/10 mile
hecto	h	10^{2}	hectoliter	hL	10^{2} L	26 gallons
deka	da	10^{1}	dekagram	dag	10^{1} g	teaspoon of butter
—	-	10^{0}	(=1)			
deci	d	10^{-1}	deciliter	dL	10^{-1} L	less than half a soda
centi	c	10^{-2}	centimeter	cm	10^{-2} m	fingertip thickness
milli	m	10^{-3}	millimeter	mm	10^{-3} m	flea at its shoulders
micro	µ	10^{-6}	micrometer	µm	10^{-6} m	detail in microscope
nano	n	10^{-9}	nanogram	ng	10^{-9} g	small speck of dust
pico	p	10^{-12}	picofarad	pF	10^{-12} F	small capacitor in radio
femto	f	10^{-15}	femtometer	fm	10^{-15} m	size of a proton
atto	a	10^{-18}	attosecond	as	10^{-18} s	time light crosses an atom

Known Ranges of Length, Mass, and Time

The vastness of the universe and the breadth over which physics applies are illustrated by the wide range of examples of known lengths, masses, and times in Table 3. Examination of this table will give you some feeling for the range of possible topics and numerical values. (See Figure 5 and Figure 6.)

![A magnified image of tiny phytoplankton swimming among the crystal of ice.



Galaxies collide 2.4 billion light years away from Earth. The tremendous range of observable phenomena in nature challenges the imagination. (credit: NASA/CXC/UVic./A. Mahdavi et al. Optical/lensing: CFHT/UVic./H. Hoekstra et al.)

Unit Conversion and Dimensional Analysis

It is often necessary to convert from one type of unit to another. For example, if you are reading a European cookbook, some quantities may be expressed in units of liters, and you need to convert them to cups. Or, perhaps you are reading walking directions from one location to another and you are interested in how many miles you will be walking. In this case, you will need to convert units of feet to miles.

Let us consider a simple example of how to convert units. Let us say that we want to convert 80 meters (m) to kilometers (km).

The first thing to do is to list the units that you have and the units that you want to convert to. In this case, we have units in **meters** and we want to convert to **kilometers**.

Next, we need to determine a **conversion factor** relating meters to kilometers. A conversion factor is a ratio expressing how many of one unit are equal to another unit. For example, there are 12 inches in 1 foot, 100 centimeters in 1 meter, 60 seconds in 1 minute, and so on. In this case, we know that there are 1 000 meters in 1 kilometer.

Now we can set up our unit conversion. We will write the units that we have and then multiply them by the conversion factor so that the units cancel out, as shown:

$$80\text{m}\times\frac{1\text{km}}{1000\text{m}}=0.080\text{km}.$$

Note that the unwanted m unit cancels, leaving only the desired km unit. You can use this method to convert between any types of unit.

The Appendix has a more complete list of conversion factors.

Table 3: Approximate Values of Length, Mass, and Time

Lengths in meters		Masses in kilograms (more precise values in parentheses)		Times in seconds (more precise values in parentheses)	
10^{-18}	Present experimental limit to smallest observable detail	10^{-30}	Mass of an electron (9.11×10^{-31} kg)	10^{-23}	Time for light to cross a proton
10^{-15}	Diameter of a proton	10^{-27}	Mass of a hydrogen atom (1.67×10^{-27} kg)	10^{-22}	Mean life of an extremely unstable nucleus
10^{-14}	Diameter of a uranium nucleus	10^{-15}	Mass of a bacterium	10^{-15}	Time for one oscillation of visible light
10^{-10}	Diameter of a hydrogen atom	10^{-5}	Mass of a mosquito	10^{-13}	Time for one vibration of an atom in a solid
10^{-8}	Thickness of membranes in cells of living organisms	10^{-2}	Mass of a hummingbird	10^{-8}	Time for one oscillation of an FM radio wave
10^{-6}	Wavelength of visible light	1	Mass of a liter of water (about a quart)	10^{-3}	Duration of a nerve impulse
10^{-3}	Size of a grain of sand	10^{2}	Mass of a person	1	Time for one heartbeat
1	Height of a 4-year-old child	10^{3}	Mass of a car	10^{5}	One day (8.64×10^{4}s)

	Lengths in meters
10^2	Length of a football field
10^4	Greatest ocean depth
10^7	Diameter of the Earth
10^{11}	Distance from the Earth to the Sun
10^{16}	Distance traveled by light in 1 year (a light year)
10^{21}	Diameter of the Milky Way galaxy
10^{22}	Distance from the Earth to the nearest large galaxy (Andromeda)
10^{26}	Distance from the Earth to the edges of the known universe

	Masses in kilograms (more precise values in parentheses)
10^8	Mass of a large ship
10^{12}	Mass of a large iceberg
10^{15}	Mass of the nucleus of a comet
10^{23}	Mass of the Moon (7.35×10^{22} kg)
10^{25}	Mass of the Earth (5.97×10^{24} kg)
10^{30}	Mass of the Sun (1.99×10^{30} kg)
10^{42}	Mass of the Milky Way galaxy (current upper limit)
10^{53}	Mass of the known universe (current upper limit)

	Times in seconds (more precise values in parentheses)
10^7	One year (y) (3.16×10^7s)
10^9	About half the life expectancy of a human
10^{11}	Recorded history
10^{17}	Age of the Earth
10^{18}	Age of the universe

Unit Conversions: A Short Drive Home

Suppose that you drive the 10.0 km from your university to home in 20.0 min. Calculate your average speed (a) in kilometers per hour (km/h) and (b) in meters per second (m/s). (Note: Average speed is distance traveled divided by time of travel.)

Strategy

First we calculate the average speed using the given units. Then we can get the average speed into the desired units by picking the correct conversion factor and multiplying by it. The correct conversion factor is the one that cancels the unwanted unit and leaves the desired unit in its place.

Solution for (a)

(1) Calculate average speed. Average speed is distance traveled divided by time of travel. (Take this definition as a given for now—average speed and other motion concepts will be covered in a later module.) In equation form,

$$\text{average speed} = \frac{\text{distance}}{\text{time}}.$$

(2) Substitute the given values for distance and time.

$$\text{average speed} = \frac{10.0\text{ km}}{20.0\text{ min}} = 0.500\frac{\text{km}}{\text{min}}.$$

(3) Convert km/min to km/h: multiply by the conversion factor that will cancel minutes and leave hours. That conversion factor is 60 min/hr. Thus,

$$\text{average speed} = 0.500\frac{\text{km}}{\text{min}} \times \frac{60\text{ min}}{1\text{ h}} = 30.0\frac{\text{km}}{\text{h}}.$$

Discussion for (a)

To check your answer, consider the following:

(1) Be sure that you have properly cancelled the units in the unit conversion. If you have written the unit conversion factor upside down, the units will not cancel properly in the equation. If you accidentally get the ratio upside down, then the units will not cancel; rather, they will give you the wrong units as follows:

$$\frac{\text{km}}{\text{min}} \times \frac{1\text{ hr}}{60\text{ min}} = \frac{1}{60}\frac{\text{km}\cdot\text{hr}}{\text{min}^2},$$

which are obviously not the desired units of km/h.

(2) Check that the units of the final answer are the desired units. The problem asked us to solve for average speed in units of km/h and we have indeed obtained these units.

(3) Check the significant figures. Because each of the values given in the problem has three significant figures, the answer should also have three significant figures. The answer 30.0 km/hr does indeed have three significant figures, so this is appropriate. Note that the significant figures in the conversion factor are not relevant because an hour is *defined* to be 60 minutes, so the precision of the conversion factor is perfect.

(4) Next, check whether the answer is reasonable. Let us consider some information from the problem—if you travel 10 km in a third of an hour (20 min), you would travel three times that far in an hour. The answer does seem reasonable.

Solution for (b)

There are several ways to convert the average speed into meters per second.

(1) Start with the answer to (a) and convert km/h to m/s. Two conversion factors are needed—one to convert hours to seconds, and another to convert kilometers to meters.

(2) Multiplying by these yields

$$\text{Average speed} = 30.0\frac{\text{km}}{\text{h}} \times \frac{1\text{ h}}{3600\text{ s}} \times \frac{1000\text{ m}}{1\text{ km}},$$
$$\text{Average speed} = 8.33\frac{\text{m}}{\text{s}}.$$

Discussion for (b)

If we had started with 0.500 km/min, we would have needed different conversion factors, but the answer would have been the same: 8.33 m/s.

You may have noted that the answers in the worked example just covered were given to three digits. Why? When do you need to be concerned about the number of digits in something you calculate? Why not write down all the digits your calculator produces? The module Accuracy, Precision, and Significant Figures will help you answer these questions.
Nonstandard Units

While there are numerous types of units that we are all familiar with, there are others that are much more obscure. For example, a **firkin** is a unit of volume that was once used to measure beer. One firkin equals about 34 liters. To learn more about nonstandard units, use a dictionary or encyclopedia to research different "weights and measures." Take note of any unusual units, such as a barleycorn, that are not listed in the text. Think about how the unit is defined and state its relationship to SI units.

Check Your Understanding

Some hummingbirds beat their wings more than 50 times per second. A scientist is measuring the time it takes for a hummingbird to beat its wings once. Which fundamental unit should the scientist use to describe the measurement? Which factor of 10 is the scientist likely to use to describe the motion precisely? Identify the metric prefix that corresponds to this factor of 10.

Show Solution

The scientist will measure the time between each movement using the fundamental unit of seconds. Because the wings beat so fast, the scientist will probably need to measure in milliseconds, or 10^{-3} seconds. (50 beats per second corresponds to 20 milliseconds per beat.)

Check Your Understanding

One cubic centimeter is equal to one milliliter. What does this tell you about the different units in the SI metric system?

Show Solution

The fundamental unit of length (meter) is probably used to create the derived unit of volume (liter). The measure of a milliliter is dependent on the measure of a centimeter.

Summary

- Physical quantities are a characteristic or property of an object that can be measured or calculated from other measurements.
- Units are standards for expressing and comparing the measurement of physical quantities. All units can be expressed as combinations of four fundamental units.
- The four fundamental units we will use in this text are the meter (for length) , the kilogram (for mass), the second (for time), and the ampere (for electric current). These units are part of the metric system, which uses powers of 10 to relate quantities over the vast ranges encountered in nature.
- The four fundamental units are abbreviated as follows: meter, m; kilogram, kg; second, s; and ampere, A. The metric system also uses a standard set of prefixes to denote each order of magnitude greater than or lesser than the fundamental unit itself.
- Unit conversions involve changing a value expressed in one type of unit to another type of unit. This is done by using conversion factors, which are ratios relating equal quantities of different units.

Conceptual Questions

Identify some advantages of metric units.

Problems & Exercises

The speed limit on some interstate highways is roughly 100 km/h. (a) What is this in meters per second? (b) How many miles per hour is this?

Show Solution

Strategy

Use conversion factors to convert between units. Remember: 1 km = 1000 m, 1 hour = 3600 s, and 1 km = 0.6214 miles.

Solution

(a) Convert 100 km/h to m/s:

$$100 \mathrm{kmh} \times 1000 \mathrm{m} 1 \mathrm{km} \times 1 \mathrm{h} 3600 \mathrm{s} = 100 \times 10003600 \mathrm{ms} = 27.8 \mathrm{m/s}$$

(b) Convert 100 km/h to miles per hour:

$$100 \mathrm{kmh} \times 0.6214 \text{ mi} 1 \mathrm{km} = 62.1 \text{ mph}$$

Discussion

The conversion from km/h to m/s requires two conversion factors that work together to change both the distance and time units. The result of 27.8 m/s means the car travels about 28 meters in one second, which is reasonable for highway speeds. Both answers maintain three significant figures, matching the precision of the original measurement of 100 km/h.

Answer

(a) $27.8 m/s$

(b) $62.1 mph$

A car is traveling at a speed of $33 m/s$. (a) What is its speed in kilometers per hour? (b) Is it exceeding the $90 km/h$ speed limit?

Show Solution

Strategy

Convert the speed from m/s to km/h using appropriate conversion factors, then compare with the speed limit.

Solution

(a) Convert 33 m/s to km/h:

$$33 \frac{m}{s} \times \frac{1 \text{ km}}{1000 m} \times \frac{3600 s}{1 \text{ h}} = 33 \times \frac{3600}{1000} \frac{km}{h} = 119 \text{ km/h}$$

Alternatively, using the conversion factor from Problem 3: 1.0 m/s = 3.6 km/h:

$$33 m/s \times 3.6 = 119 \text{ km/h} \approx 120 \text{ km/h}$$

(b) Comparison with speed limit:

Since 119 km/h > 90 km/h, yes, the car is exceeding the speed limit by 29 km/h.

Discussion

This problem demonstrates the utility of the conversion factor 1.0 m/s = 3.6 km/h, which is commonly used in physics. The car’s speed of 119 km/h is significantly over the 90 km/h limit, exceeding it by about 32%. This speed of 33 m/s means the car travels the length of a basketball court in less than one second, which helps put the speed in perspective.

Answer

(a) The car’s speed is **120 km/h** (or 119 km/h to three significant figures).

(b) Yes, the car is exceeding the 90 km/h speed limit.

Show that $1.0 m/s = 3.6 km/h$. Hint: Show the explicit steps involved in converting $1.0 m/s = 3.6 km/h$.

Show Solution

$$1.0 \frac{m}{s} = 1.0 \frac{m}{s} \times \frac{3600 s}{1 hr} \times \frac{1 km}{1000 m} = 3.6 km/h.$$

Discussion

This conversion factor is particularly useful because it relates the SI unit of speed (m/s) to the more common everyday unit of km/h. The factor of 3.6 arises from the combination of 3600 seconds per hour and 1000 meters per kilometer (3600/1000 = 3.6). Memorizing this simple relationship allows for quick mental conversions between these common speed units.

American football is played on a 100-yd-long field, excluding the end zones. How long is the field in meters? (Assume that 1 meter equals 3.281 feet.)

Show Solution

Strategy

Convert from yards to feet, then from feet to meters using the given conversion factor.

Solution

1. Convert yards to feet (1 yard = 3 feet):

$$100 \text{ yd} \times \frac{3 \text{ ft}}{1 \text{ yd}} = 300 \text{ ft}$$

1. Convert feet to meters (1 m = 3.281 ft):

$$300 \text{ ft} \times \frac{1 m}{3.281 \text{ ft}} = 91.4 m$$

Discussion

The result shows that an American football field is slightly less than 100 meters long, which provides a useful reference point for estimating distances. The conversion required using two factors: first yards to feet (3 ft/yd), then feet to meters. The answer of 91.4 m is given to three significant figures, matching the precision of the original 100-yard measurement.

Answer

The football field is **91.4 m** long.

Soccer fields vary in size. A large soccer field is 115 m long and 85 m wide. What are its dimensions in feet and inches? (Assume that 1 meter equals 3.281 feet.)

Show Solution

Strategy

Convert from meters to feet using the conversion factor 1 m = 3.281 ft, then convert feet to inches using 1 ft = 12 in.

Solution

Length (115 m):

$$115 \text{m} \times \frac{3.281 \text{ ft}}{1 \text{m}} = 377 \text{ ft}$$
$$377 \text{ ft} \times \frac{12 \text{ in.}}{1 \text{ ft}} = 4.53 \times 10^3 \text{ in.}$$

Width (85 m):

$$85 \text{m} \times \frac{3.281 \text{ ft}}{1 \text{m}} = 279 \text{ ft} \approx 280 \text{ ft}$$
$$280 \text{ ft} \times \frac{12 \text{ in.}}{1 \text{ ft}} = 3.3 \times 10^3 \text{ in.}$$

Discussion

The soccer field dimensions in feet (377 ft by 280 ft) show that it is larger than an American football field in both length and width. Using scientific notation for the inch measurements (4.53×10^3 in. and 3.3×10^3 in.) is appropriate because these are large numbers that would be cumbersome to write out. The significant figures reflect the precision of the original metric measurements, with the width rounded to two significant figures based on the value 85 m.

Answer

Length: $377 \text{ft}; 4.53 \times 10^3 \text{in.}$

Width: $280 \text{ft}; 3.3 \times 10^3 \text{in.}$

What is the height in meters of a person who is 6 ft 1.0 in. tall? (Assume that 1 meter equals 39.37 in.)

Show Solution

Strategy

First convert the total height to inches, then convert to meters using the given conversion factor.

Solution

1. Convert 6 ft 1.0 in. to total inches:

$$6 \text{ ft} \times \frac{12 \text{ in.}}{1 \text{ ft}} + 1.0 \text{ in.} = 72 \text{ in.} + 1.0 \text{ in.} = 73.0 \text{ in.}$$

1. Convert inches to meters (1 m = 39.37 in.):

$$73.0 \text{ in.} \times \frac{1 \text{m}}{39.37 \text{ in.}} = 1.85 \text{m}$$

Discussion

A height of 1.85 m (or 185 cm) is reasonable for a person who is 6 ft 1 in. tall, representing a height above average for most populations. The conversion from English units to SI units demonstrates the importance of first combining mixed units (feet and inches) into a single unit before converting. The result is given to three significant figures, consistent with the precision of the original measurement.

Answer

The person’s height is **1.85 m**.

Mount Everest, at 29 028 feet, is the tallest mountain on the Earth. What is its height in kilometers? (Assume that 1 kilometer equals 3 281 feet.)

Show Solution

Strategy

Convert from feet to kilometers using the given conversion factor: 1 km = 3281 ft.

Solution

$$29028 \text{ ft} \times \frac{1 \text{ km}}{3281 \text{ ft}} = 8.847 \text{ km}$$

Discussion

Mount Everest's height of approximately 8.8 km helps put the mountain's scale into perspective—it reaches nearly 9 kilometers into the atmosphere. This height is significant because it places the summit in the "death zone" where atmospheric pressure is only about one-third that at sea level. The precision to four significant figures (8.847 km) reflects the accuracy of modern surveying techniques used to measure this iconic peak.

Answer

$$8.847 \text{km}$$

The speed of sound is measured to be 342m/s on a certain day. What is this in km/h?

Show Solution

Strategy

Convert the speed from m/s to km/h using the conversion factors: 1 km = 1000 m and 1 h = 3600 s.

Solution

$$342 \frac{\text{m}}{\text{s}} \times \frac{1 \text{ km}}{1000 \text{m}} \times \frac{3600 \text{s}}{1 \text{ h}} = \frac{342 \times 3600}{1000} \frac{\text{km}}{\text{h}} = 1231 \text{ km/h} \approx 1230 \text{ km/h}$$

Alternatively, using the conversion 1.0 m/s = 3.6 km/h:

$$342 \text{m/s} \times 3.6 = 1231 \text{ km/h} \approx 1.23 \times 10^3 \text{ km/h}$$

Discussion

The speed of sound at approximately 1230 km/h is much faster than typical highway speeds (around 100 km/h) but still much slower than the speed of light. This value varies with temperature and atmospheric conditions, which is why the problem states it was "measured on a certain day." Expressing the answer with three significant figures (1230 km/h or 1.23×10^3 km/h) properly reflects the precision of the original measurement of 342 m/s.

Answer

The speed of sound is **1230 km/h** (or **1.23×10^3 km/h** in scientific notation).

Tectonic plates are large segments of the Earth's crust that move slowly. Suppose that one such plate has an average speed of 4.0 cm/year. (a) What distance does it move in 1 s at this speed? (b) What is its speed in kilometers per million years?

Show Solution

Strategy

Convert the speed of 4.0 cm/year to the requested units using appropriate conversion factors.

Solution

(a) Distance moved in 1 second:

First convert 4.0 cm/year to meters per second:

$$\begin{aligned} & 4.0 \frac{\text{cm}}{\text{year}} \times \frac{1 \text{m}}{100 \text{ cm}} \times \frac{1 \text{ year}}{365 \text{ days}} \times \frac{1 \text{ day}}{24 \text{ h}} \times \frac{1 \text{ h}}{3600 \text{s}} \\ & = \frac{4.0}{100 \times 365 \times 24 \times 3600} \frac{\text{m}}{\text{s}} = 1.27 \times 10^{-9} \frac{\text{m}}{\text{s}} \approx 1.3 \times 10^{-9} \frac{\text{m}}{\text{s}} \end{aligned}$$

In 1 second, the plate moves: $1.3 \times 10^{-9} \text{m}$

(b) Speed in km per million years:

$$\begin{aligned} & 4.0 \frac{\text{cm}}{\text{year}} \times \frac{1 \text{m}}{100 \text{ cm}} \times \frac{1 \text{ km}}{1000 \text{m}} \times \frac{10^6 \text{ years}}{1 \text{ My}} \\ & = \frac{4.0 \times 10^6}{100 \times 1000} \frac{\text{km}}{\text{My}} = 40 \frac{\text{km}}{\text{My}} \end{aligned}$$

Discussion

The extremely small distance of 1.3×10^{-9} m per second illustrates why tectonic motion is imperceptible in everyday life—this is about one nanometer per second, roughly the size of a few atoms. However, over geological time scales of millions of years, this slow movement accumulates to 40 km per million years, which is significant enough to create mountains and move continents. This problem demonstrates the importance of choosing appropriate time scales when describing processes that occur at vastly different rates.

Answer

(a) $1.3 \times 10^{-9} \text{m}$

(b) 40km/My

(a) Refer to [Table 3](#) to determine the average distance between the Earth and the Sun. Then calculate the average speed of the Earth in its orbit in kilometers per second. (b) What is this in meters per second?

Show Solution

Strategy

From Table 3, find the Earth-Sun distance. Calculate the circumference of Earth's orbit, divide by the time for one orbit (1 year), then convert to the requested units.

Solution

1. From Table 3, the distance from Earth to Sun is approximately 10^{11}m (more precisely, about $1.5\times10^{11}\text{m}$).
2. The circumference of Earth's orbit (assuming circular):

$$C=2\pi r=2\pi(1.5\times10^{11}\text{m})=9.42\times10^{11}\text{m}$$

1. Time for one orbit is 1 year. From Table 3, one year = $3.16\times10^{7}\text{s}$.

(a) Speed in km/s:

$$v=\frac{C}{t}=\frac{9.42\times10^{11}\text{m}}{3.16\times10^{7}\text{s}}=2.98\times10^{4}\frac{\text{m}}{\text{s}}$$

Convert to km/s:

$$v=2.98\times10^{4}\frac{\text{m}}{\text{s}}\times\frac{1\text{ km}}{1000\text{m}}=29.8\text{ km/s}\approx30\text{ km/s}$$

(b) Speed in m/s:

From above: $v=2.98\times10^{4}\text{m/s}\approx3.0\times10^{4}\text{m/s}$

Discussion

Earth's orbital speed of approximately 30 km/s (or 30,000 m/s) is remarkably fast—about 108,000 km/h—yet we don't feel this motion because we're moving along with the Earth. This speed is much greater than the speed of sound (about 0.34 km/s) and represents the velocity needed to maintain a stable orbit around the Sun at Earth's distance. The calculation assumes a circular orbit, which is a good approximation since Earth's orbit is nearly circular with an eccentricity of only 0.017.

Answer

(a) The average speed of Earth in its orbit is approximately **30 km/s**.

(b) This is approximately **3.0 × 10^4 m/s** (or 30,000 m/s).

Glossary

physical quantity
: a characteristic or property of an object that can be measured or calculated from other measurements

units
: a standard used for expressing and comparing measurements

SI units
: the international system of units that scientists in most countries have agreed to use; includes units such as meters, liters, and grams

English units
: system of measurement used in the United States; includes units of measurement such as feet, gallons, and pounds

fundamental units
: units that can only be expressed relative to the procedure used to measure them

derived units
: units that can be calculated using algebraic combinations of the fundamental units

second
: the SI unit for time, abbreviated (s)

meter
: the SI unit for length, abbreviated (m)

kilogram
: the SI unit for mass, abbreviated (kg)

metric system
: a system in which values can be calculated in factors of 10

order of magnitude
: refers to the size of a quantity as it relates to a power of 10

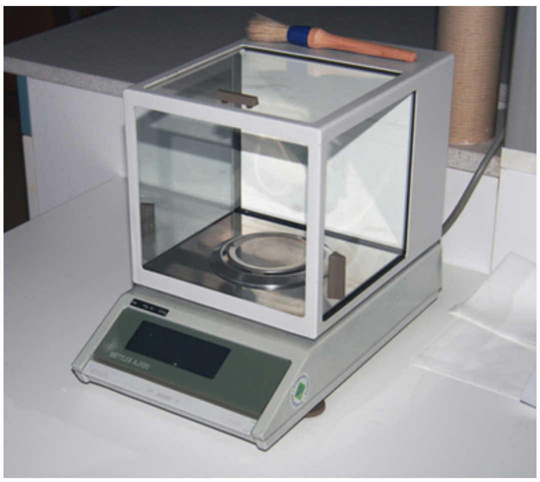
conversion factor
: a ratio expressing how many of one unit are equal to another unit

---

Accuracy, Precision, and Significant Figures

- Determine the appropriate number of significant figures in both addition and subtraction, as well as multiplication and division calculations.
- Calculate the percent uncertainty of a measurement.

A double-pan mechanical balance is used to compare different masses. Usually an object with unknown mass is placed in one pan and objects of known mass are placed in the other pan. When the bar that connects the two pans is horizontal, then the masses in both pans are equal. The "known masses" are typically metal cylinders of standard mass such as 1 gram, 10 grams, and 100 grams. (credit: Serge Melki)



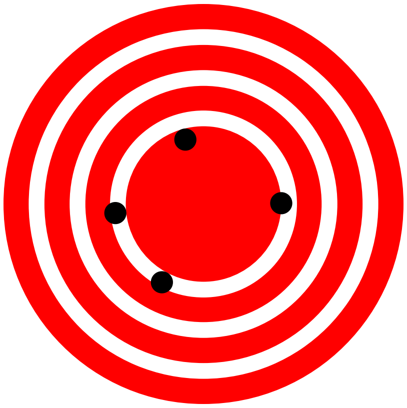
Many mechanical balances, such as double-pan balances, have been replaced by digital scales, which can typically measure the mass of an object more precisely. Whereas a mechanical balance may only read the mass of an object to the nearest tenth of a gram, many digital scales can measure the mass of an object up to the nearest thousandth of a gram. (credit: Karel Jakubec)

Accuracy and Precision of a Measurement

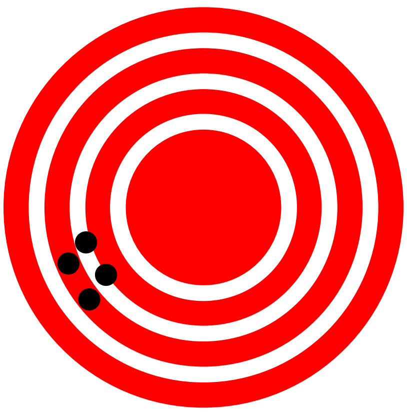
Science is based on observation and experiment—that is, on measurements. **Accuracy** is how close a measurement is to the correct value for that measurement. For example, let us say that you are measuring the length of standard computer paper. The packaging in which you purchased the paper states that it is 11.0 inches long. You measure the length of the paper three times and obtain the following measurements: 11.1 in., 11.2 in., and 10.9 in. These measurements are quite accurate because they are very close to the correct value of 11.0 inches. In contrast, if you had obtained a measurement of 12 inches, your measurement would not be very accurate.

The **precision** of a measurement system is refers to how close the agreement is between repeated measurements (which are repeated under the same conditions). Consider the example of the paper measurements. The precision of the measurements refers to the spread of the measured values. One way to analyze the precision of the measurements would be to determine the range, or difference, between the lowest and the highest measured values. In that case, the lowest value was 10.9 in. and the highest value was 11.2 in. Thus, the measured values deviated from each other by at most 0.3 in. These measurements were relatively precise because they did not vary too much in value. However, if the measured values had been 10.9, 11.1, and 11.9, then the measurements would not be very precise because there would be significant variation from one measurement to another.

The measurements in the paper example are both accurate and precise, but in some cases, measurements are accurate but not precise, or they are precise but not accurate. Let us consider an example of a GPS system that is attempting to locate the position of a restaurant in a city. Think of the restaurant location as existing at the center of a bull's-eye target, and think of each GPS attempt to locate the restaurant as a black dot. In Figure 3, you can see that the GPS measurements are spread out far apart from each other, but they are all relatively close to the actual location of the restaurant at the center of the target. This indicates a low precision, high accuracy measuring system. However, in Figure 4, the GPS measurements are concentrated quite closely to one another, but they are far away from the target location. This indicates a high precision, low accuracy measuring system.



A GPS system attempts to locate a restaurant at the center of the bull's-eye. The black dots represent each attempt to pinpoint the location of the restaurant. The dots are spread out quite far apart from one another, indicating low precision, but they are each rather close to the actual location of the restaurant, indicating high accuracy. (credit: Dark Evil)



In this figure, the dots are concentrated rather closely to one another, indicating high precision, but they are rather far away from the actual location of the restaurant, indicating low accuracy. (credit: Dark Evil)

Accuracy, Precision, and Uncertainty

The degree of accuracy and precision of a measuring system are related to the **uncertainty** in the measurements. Uncertainty is a quantitative measure of how much your measured values deviate from a standard or expected value. If your measurements are not very accurate or precise, then the uncertainty of your values will be very high. In more general terms, uncertainty can be thought of as a disclaimer for your measured values. For example, if someone asked you to provide the mileage on your car, you might say that it is 45 000 miles, plus or minus 500 miles. The plus or minus amount is the uncertainty in your value. That is, you are indicating that the actual mileage of your car might be as low as 44 500 miles or as high as 45 500 miles, or anywhere in between. All measurements contain some amount of uncertainty. In our example of measuring the length of the paper, we might say that the length of the paper is 11 in., plus or minus 0.2 in. The uncertainty in a measurement, A, is often denoted as δA ("delta A"), so the measurement result would be recorded as $A \pm \delta A$. In our paper example, the length of the paper could be expressed as $11 \text{in.} \pm 0.2$. The factors contributing to uncertainty in a measurement include:

1. Limitations of the measuring device,
2. The skill of the person making the measurement,
3. Irregularities in the object being measured,
4. Any other factors that affect the outcome (highly dependent on the situation) .

In our example, such factors contributing to the uncertainty could be the following: the smallest division on the ruler is 0.1 in., the person using the ruler has bad eyesight, or one side of the paper is slightly longer than the other. At any rate, the uncertainty in a measurement must be based on a careful consideration of all the factors that might contribute and their possible effects.

Making Connections: Real-World Connections – Fevers or Chills?

Uncertainty is a critical piece of information, both in physics and in many other real-world applications. Imagine you are caring for a sick child. You suspect the child has a fever, so you check their temperature with a thermometer. What if the uncertainty of the thermometer were 3.0°C? If the child's temperature reading was 37.0°C (which is normal body temperature), the "true" temperature could be anywhere from a hypothermic 34.0°C to a dangerously high 40.0°C. A thermometer with an uncertainty of 3.0°C would be useless.

Percent Uncertainty

One method of expressing uncertainty is as a percent of the measured value. If a measurement A is expressed with uncertainty, δA, the **percent uncertainty** (%unc) is defined to be

$$\%\text{unc} = \frac{\delta A}{A} \times 100\%.$$

Calculating Percent Uncertainty: A Bag of Apples

A grocery store sells 5-lb bags of apples. You purchase four bags over the course of a month and weigh the apples each time. You obtain the following measurements:

- Week 1 weight: $4.8\,\text{lb}$
- Week 2 weight: $5.3\,\text{lb}$
- Week 3 weight: $4.9\,\text{lb}$
- Week 4 weight: $5.4\,\text{lb}$ You determine that the weight of the 5-lb bag has an uncertainty of $\pm 0.4\,\text{lb}$. What is the percent uncertainty of the bag's weight?

Strategy

First, observe that the expected value of the bag's weight, A, is 5 lb. The uncertainty in this value, δA, is 0.4 lb. We can use the following equation to determine the percent uncertainty of the weight:

$$\%\text{unc} = \frac{\delta A}{A} \times 100\%.$$

Solution

Plug the known values into the equation:

$$\%\text{unc} = \frac{0.4\,\text{lb}}{5\,\text{lb}} \times 100\% = 8\%.$$

Discussion

We can conclude that the weight of the apple bag is $5\,\text{lb} \pm 8\%$. Consider how this percent uncertainty would change if the bag of apples were half as heavy, but the uncertainty in the weight remained the same. Hint for future calculations: when calculating percent uncertainty, always remember that you must multiply the fraction by 100%. If you do not do this, you will have a decimal quantity, not a percent value.

Uncertainties in Calculations

There is an uncertainty in anything calculated from measured quantities. For example, the area of a floor calculated from measurements of its length and width has an uncertainty because the length and width have uncertainties. How big is the uncertainty in something you calculate by multiplication or division? If the measurements going into the calculation have small uncertainties (a few percent or less), then the **method of adding percents** can be used for multiplication or division. This method says that **the percent uncertainty in a quantity calculated by multiplication or division is the sum of the percent uncertainties in the items used to make the calculation**. For example, if a floor has a length of $4.00\,\text{m}$ and a width of $3.00\,\text{m}$, with uncertainties of 2% and 1%, respectively, then the area of the floor is $12.0\,\text{m}^2$ and has an uncertainty of 3%. (Expressed as an area this is $0.36\,\text{m}^2$, which we round to $0.4\,\text{m}^2$ since the area of the floor is given to a tenth of a square meter.)

Check Your Understanding

A high school track coach has just purchased a new stopwatch. The stopwatch manual states that the stopwatch has an uncertainty of $\pm 0.05\,\text{s}$. Runners on the track coach's team regularly clock 100-m sprints of $11.49\,\text{s}$ to $15.01\,\text{s}$. At the school's last track meet, the first-place sprinter came in at $12.04\,\text{s}$ and the second-place sprinter came in at $12.07\,\text{s}$. Will the coach's new stopwatch be helpful in timing the sprint team? Why or why not?

Show Solution

No, the uncertainty in the stopwatch is too great to effectively differentiate between the sprint times.

Precision of Measuring Tools and Significant Figures

An important factor in the accuracy and precision of measurements involves the precision of the measuring tool. In general, a precise measuring tool is one that can measure values in very small increments. For example, a standard ruler can measure length to the nearest millimeter, while a caliper can measure length to the nearest 0.01 millimeter. The caliper is a more precise measuring tool because it can measure extremely small differences in length. The more precise the measuring tool, the more precise and accurate the measurements can be.

When we express measured values, we can only list as many digits as we initially measured with our measuring tool. For example, if you use a standard ruler to measure the length of a stick, you may measure it to be $36.7\,\text{cm}$. You could not express this value as $36.71\,\text{cm}$ because your measuring tool was not precise enough to measure a hundredth of a centimeter. It should be noted that the last digit in a measured value has been estimated in some way by the person performing the measurement. For example, the person measuring the length of a stick with a ruler notices that the stick length seems to be somewhere in between $36.6\,\text{cm}$ and $36.7\,\text{cm}$, and they must estimate the value of the last digit. Using the method of **significant figures**, the rule is that **the last digit written down in a measurement is the first digit with some uncertainty**. In order to determine the number of significant digits in a value, start with the first measured value at the left and count the number of digits through the last digit written on the right. For example, the measured value $36.7\,\text{cm}$ has three digits, or significant figures. Significant figures indicate the precision of a measuring tool that was used to measure a value.

Zeros

Special consideration is given to zeros when counting significant figures. The zeros in 0.053 are not significant, because they are only placekeepers that locate the decimal point. There are two significant figures in 0.053. The zeros in 10.053 are not placekeepers but are significant—this number has five significant figures. The zeros in 1300 may or may not be significant depending on the style of writing numbers. They could mean the number is known to the last digit, or they could be placekeepers. So 1300 could have two, three, or four significant figures. (To avoid this ambiguity, write 1300 in scientific notation.) **Zeros are significant except when they serve only as placekeepers** .

Check Your Understanding

Determine the number of significant figures in the following measurements:

(a) 0.0009 (b) 15 450.0 (c) 6×10^3 (d) 87.990 (d) 30.42

Show Solution

(a) 1; the zeros in this number are placekeepers that indicate the decimal point

(b) 6; here, the zeros indicate that a measurement was made to the 0.1 decimal point, so the zeros are significant

(c) 1; the value 10^3 signifies the decimal place, not the number of measured values

(d) 5; the final zero indicates that a measurement was made to the 0.001 decimal point, so it is significant

(e) 4; any zeros located in between significant figures in a number are also significant

Significant Figures in Calculations

When combining measurements with different degrees of accuracy and precision, **the number of significant digits in the final answer can be no greater than the number of significant digits in the least precise measured value**. There are two different rules, one for multiplication and division and the other for addition and subtraction, as discussed below.

1. For multiplication and division: The result should have the same number of significant figures as the quantity having the least significant figures entering into the calculation. For example, the area of a circle can be calculated from its radius using $A = \pi r^2$. Let us see how many significant figures the area has if the radius has only two—say, $r = 1.2\text{m}$. Then,

$$A=\pi r^2=(3.1415927\ldots)\times(1.2\text{m})^2=4.5238934\text{m}^2$$

is what you would get using a calculator that has an eight-digit output. But because the radius has only two significant figures, it limits the calculated quantity to two significant figures or

$$A=4.5\text{m}^2,$$

even though π is good to at least eight digits.

2. For addition and subtraction: The answer can contain no more decimal places than the least precise measurement . Suppose that you buy 7.56-kg of potatoes in a grocery store as measured with a scale with precision 0.01 kg. Then you drop off 6.052-kg of potatoes at your laboratory as measured by a scale with precision 0.001 kg. Finally, you go home and add 13.7 kg of potatoes as measured by a bathroom scale with precision 0.1 kg. How many kilograms of potatoes do you now have, and how many significant figures are appropriate in the answer? The mass is found by simple addition and subtraction:

$$7.56\text{kg}-6.052\text{kg}+13.7\text{kg}\overline{15.208\text{kg}}\rightarrow 15.2\text{kg}$$

Next, we identify the least precise measurement: 13.7 kg. This measurement is expressed to the 0.1 decimal place, so our final answer must also be expressed to the 0.1 decimal place. Thus, the answer is rounded to the tenths place, giving us 15.2 kg.

Significant Figures in this Text

In this text, most numbers are assumed to have three significant figures. Furthermore, consistent numbers of significant figures are used in all worked examples. You will note that an answer given to three digits is based on input good to at least three digits, for example. If the input has fewer significant figures, the answer will also have fewer significant figures. Care is also taken that the number of significant figures is reasonable for the situation posed. In some topics, particularly in optics, more accurate numbers are needed and more than three significant figures will be used. Finally, if a number is **exact**, such as the two in the formula for the circumference of a circle, $c = 2\pi r$

, it does not affect the number of significant figures in a calculation.

Check Your Understanding

Perform the following calculations and express your answer using the correct number of significant digits.

(a) A woman has two bags, each weighing 13.5 pounds and one bag with a weight of 10.2 pounds. What is the total weight of the bags?

(b) The force F on an object is equal to its mass m multiplied by its acceleration a. If a wagon with mass 55 kg accelerates at a rate of 0.0255m/s^2, what is the force on the wagon? (The unit of force is called the newton, and it is expressed with the symbol N.)

Show Solution

(a) 37.2 pounds; Because the number of bags is an exact value, it is not considered in the significant figures.

(b) 1.4 N; Because the value 55 kg has only two significant figures, the final value must also contain two significant figures.
Estimation

Explore size estimation in one, two, and three dimensions! Multiple levels of difficulty allow for progressive skill improvement.

Estimation

Summary

- Accuracy of a measured value refers to how close a measurement is to the correct value. The uncertainty in a measurement is an estimate of the amount by which the measurement result may differ from this value.
- Precision of measured values refers to how close the agreement is between repeated measurements.
- The precision of a **measuring tool** is related to the size of its measurement increments. The smaller the measurement increment, the more precise the tool.
- Significant figures express the precision of a measuring tool.
- When multiplying or dividing measured values, the final answer can contain only as many significant figures as the least precise value.
- When adding or subtracting measured values, the final answer cannot contain more decimal places than the least precise value.

Conceptual Questions

What is the relationship between the accuracy and uncertainty of a measurement?

Prescriptions for vision correction are given in units called **diopters** (D). Determine the meaning of that unit. Obtain information (perhaps by calling an optometrist or performing an internet search) on the minimum uncertainty with which corrections in diopters are determined and the accuracy with which corrective lenses can be produced. Discuss the sources of uncertainties in both the prescription and accuracy in the manufacture of lenses.

Problems & Exercises

Express your answers to problems in this section to the correct number of significant figures and proper units.

Suppose that your bathroom scale reads your mass as 65 kg with a 3% uncertainty. What is the uncertainty in your mass (in kilograms)?

Show Solution

Strategy

The uncertainty can be found by multiplying the measured value by the percent uncertainty (expressed as a decimal).

Solution

$$\delta m = m \times \frac{\%\text{unc}}{100\%} = 65\text{ kg} \times \frac{3\%}{100\%} = 65\text{ kg} \times 0.03 = 1.95\text{ kg} \approx 2\text{ kg}$$

Discussion

A 3% uncertainty is relatively typical for bathroom scales, which are generally not precision instruments. The uncertainty of 2 kg means your actual mass could be anywhere from 63 kg to 67 kg. For everyday purposes like monitoring weight trends, this level of uncertainty is acceptable, though it would be inadequate for medical or scientific applications requiring precise measurements.

Answer: 2 kg

A good-quality measuring tape can be off by 0.50 cm over a distance of 20 m. What is its percent uncertainty?

Show Solution

Strategy

Use the percent uncertainty formula: %unc = (δA/A) × 100%. Convert units so they match before calculating.

Solution

First, convert the distance to centimeters so units match:

$$20\text{m}=2000\text{ cm}$$

Calculate the percent uncertainty:

$$\%\text{unc}=\frac{\delta A}{A}\times100\%=\frac{0.50\text{ cm}}{2000\text{ cm}}\times100\%=0.025\%$$

Discussion

This is a very small percent uncertainty (about 1 part in 4000), indicating a high-quality measuring tape. For most construction and home improvement purposes, this level of precision is more than adequate.

Answer: 0.025%

(a) A car speedometer has a 5.0% uncertainty. What is the range of possible speeds when it reads 90km/h ? (b) Convert this range to miles per hour. $(1\text{km}=0.6214\text{mi})$

Show Solution

Strategy

Find the uncertainty by multiplying the reading by the percent uncertainty, then calculate the range by adding and subtracting from the reading.

Solution

(a) Range in km/h:

1. Calculate the uncertainty:

$$\delta v=90\text{ km/h}\times\frac{5.0\%}{100\%}=90\text{ km/h}\times0.05=4.5\text{ km/h}$$

1. Calculate the range:

$$v=90\pm4.5\text{ km/h}$$
$$\text{Minimum: }90-4.5=85.5\approx85\text{ km/h}$$
$$\text{Maximum: }90+4.5=94.5\approx95\text{ km/h}$$

(b) Convert to mi/h:

Using 1 km = 0.6214 mi:

$$85\text{ km/h}\times0.6214=53\text{ mi/h}$$
$$95\text{ km/h}\times0.6214=59\text{ mi/h}$$

Discussion

A 5% uncertainty in speedometer readings is quite significant - it represents a range of 10 km/h (or 6 mi/h) at this speed. This uncertainty is typical for older analog speedometers and emphasizes why drivers should not rely solely on speedometer readings to maintain precise speeds. Modern digital speedometers generally have lower uncertainties.

Answer:

(a) $85\text{ to }95\text{ km/h}$

(b) $53\text{ to }59\text{ mi/h}$

An infant’s pulse rate is measured to be $130\ \pm5$ beats/min. What is the percent uncertainty in this measurement?

Show Solution

Strategy

Use the percent uncertainty formula with the measured value and its uncertainty.

Solution

$$\%\text{unc} = \frac{\delta A}{A} \times 100\% = \frac{5 \text{ beats/min}}{130 \text{ beats/min}} \times 100\% = 3.8\% \approx 4\%$$

Discussion

A 4% uncertainty is reasonable for a manual pulse measurement on an infant, who may be moving or fussy during the measurement. This uncertainty means the true pulse rate could be anywhere from 125 to 135 beats/min.

Answer: 4%

(a) Suppose that a person has an average heart rate of 72.0 beats/min. How many beats do they have in 2.0 y? (b) In 2.00 y? (c) In 2.000 y?

Show Solution

Strategy

Calculate total beats by multiplying heart rate by time. The answer should reflect the significant figures in the given values.

Solution

First, convert years to minutes:

$$1 \text{ year} = 365.25 \text{ days} \times 24 \text{ h/day} \times 60 \text{ min/h} = 5.26 \times 10^5 \text{ min}$$

(a) For 2.0 years (2 significant figures):

$$\text{Beats} = 72.0 \frac{\text{beats}}{\text{min}} \times 2.0 \text{ y} \times 5.26 \times 10^5 \frac{\text{min}}{\text{y}} = 7.57 \times 10^7 \text{ beats}$$

Rounded to 2 significant figures: 7.6×10^7 beats

(b) For 2.00 years (3 significant figures):

$$\text{Beats} = 72.0 \frac{\text{beats}}{\text{min}} \times 2.00 \text{ y} \times 5.26 \times 10^5 \frac{\text{min}}{\text{y}} = 7.57 \times 10^7 \text{ beats}$$

(c) For 2.000 years (4 significant figures):

$$\text{Beats} = 72.0 \frac{\text{beats}}{\text{min}} \times 2.000 \text{ y} \times 5.26 \times 10^5 \frac{\text{min}}{\text{y}} = 7.57 \times 10^7 \text{ beats}$$

(Limited to 3 significant figures by the heart rate of 72.0 beats/min)

Discussion

This problem illustrates how the precision of input values (as indicated by significant figures) affects the precision of calculated results. Even though the time intervals are different (2.0, 2.00, and 2.000 years), parts (b) and (c) yield the same answer because the heart rate of 72.0 beats/min (three significant figures) limits all answers to three significant figures. This demonstrates that in calculations, the least precise measurement determines the precision of the result.

Answer:

(a) 7.6×10^7beats

(b) 7.57×10^7beats

(c) 7.57×10^7beats

A can contains 375 mL of soda. How much is left after 308 mL is removed?

Show Solution

Strategy

Subtract the amount removed from the original amount. For subtraction, the answer cannot have more decimal places than the least precise measurement.

Solution

$$\text{Remaining} = 375 \text{ mL} - 308 \text{ mL} = 67 \text{ mL}$$

Discussion

Both values are given to the ones place (no decimal places), so the answer is also reported to the ones place. The answer has 2 significant figures.

Answer: 67 mL

State how many significant figures are proper in the results of the following calculations: a. $(106.7)(98.2)/(46.210)(1.01)$ b. $(18.7)^2$ c. $(1.60\times10^{-19})(3712)$.

Show Solution

Strategy

For multiplication and division, the result should have the same number of significant figures as the quantity with the fewest significant figures.

Solution

(a) $(106.7)(98.2)/(46.210)(1.01)$

- 106.7 has 4 significant figures
- 98.2 has 3 significant figures
- 46.210 has 5 significant figures
- 1.01 has 3 significant figures

The answer should have **3** significant figures (limited by 98.2 and 1.01).

(b) $(18.7)^2$

- 18.7 has 3 significant figures
- Squaring is multiplication, so the answer has **3** significant figures.

(c) $(1.60\times10^{-19})(3712)$

- 1.60×10^{-19} has 3 significant figures
- 3712 has 4 significant figures

The answer should have **3** significant figures (limited by 1.60×10^{-19}).

Discussion

Understanding how to count significant figures in compound calculations is crucial for reporting results with appropriate precision. In each case, we're limited by the measurement with the fewest significant figures. This ensures we don't overstate the precision of our calculated results beyond what the input data justifies.

Answer:

(a) 3

(b) 3

(c) 3

(a) How many significant figures are in the numbers 99 and 100? (b) If the uncertainty in each number is 1, what is the percent uncertainty in each? (c) Which is a more meaningful way to express the accuracy of these two numbers, significant figures or percent uncertainties?

Show Solution

Strategy

Count significant figures using standard rules. Calculate percent uncertainties and compare the two methods of expressing precision.

Solution

(a) Significant figures:

- 99 has **2** significant figures (both digits are significant)
- 100 has **1, 2, or 3** significant figures depending on context. Without additional information (such as a decimal point or scientific notation), it's ambiguous. Typically assumed to be **1** significant figure if the zeros are just placeholders.

(b) Percent uncertainty with δ = 1 for each:

For 99:

$$\%\text{unc}_{99}=199\times100\%=1.0\%$$

For 100:

$$\%\text{unc}_{100}=1100\times100\%=1.0\%$$

(c) Which is more meaningful?

Percent uncertainty is more meaningful in this case. Both numbers have the same absolute uncertainty (±1), giving them essentially the same percent uncertainty (~1%). However, if we relied only on significant figures, we would conclude that 99 is more precise than 100 (2 sig figs vs. 1 sig fig), which is misleading. The percent uncertainty correctly shows that both measurements have the same relative precision.

Discussion

This example illustrates a limitation of significant figures: they can be ambiguous (as with 100) and don't always accurately convey precision. Percent uncertainty provides a clearer, more quantitative measure of measurement quality.

(a) If your speedometer has an uncertainty of 2.0km/h at a speed of 90km/h, what is the percent uncertainty? (b) If it has the same percent uncertainty when it reads 60km/h, what is the range of speeds you could be going?

Show Solution

Strategy

Use the percent uncertainty formula, then apply the same percent uncertainty to the second reading to find the range.

Solution

(a) Percent uncertainty at 90 km/h:

$$\%\text{unc} = \frac{\delta v}{v} \times 100\% = \frac{2.0 \text{ km/h}}{90 \text{ km/h}} \times 100\% = 2.2\%$$

(b) Range at 60 km/h with same percent uncertainty:

1. Calculate the absolute uncertainty at 60 km/h:

$$\delta v = 60 \text{ km/h} \times \frac{2.2\%}{100\%} = 60 \text{ km/h} \times 0.022 = 1.3 \text{ km/h} \approx 1 \text{ km/h}$$

1. Calculate the range:

$$\text{Minimum: } 60 - 1 = 59 \text{ km/h}$$
$$\text{Maximum: } 60 + 1 = 61 \text{ km/h}$$

Discussion

This problem demonstrates that percent uncertainty remains constant for a given instrument, while absolute uncertainty changes with the measured value. At lower speeds, the same 2.2% uncertainty translates to a smaller absolute uncertainty (1 km/h at 60 km/h versus 2 km/h at 90 km/h), making the speedometer more useful for detecting small speed differences at lower velocities.

Answer:

(a) 2.2%

(b) $59 \text{ to } 61 \text{ km/h}$

(a) A person's blood pressure is measured to be $120 \pm 2 \text{mm Hg}$. What is its percent uncertainty? (b) Assuming the same percent uncertainty, what is the uncertainty in a blood pressure measurement of 80mm Hg ?

Show Solution

Strategy

Use the percent uncertainty formula for part (a), then apply the same percentage to find the uncertainty in part (b).

Solution

(a) Percent uncertainty at 120 mm Hg:

$$\%\text{unc} = \frac{\delta P}{P} \times 100\% = \frac{2 \text{ mm Hg}}{120 \text{ mm Hg}} \times 100\% = 1.67\% \approx 2\%$$

(b) Uncertainty at 80 mm Hg with same percent uncertainty:

$$\delta P = 80 \text{ mm Hg} \times \frac{1.67\%}{100\%} = 80 \text{ mm Hg} \times 0.0167 = 1.3 \text{ mm Hg} \approx 1 \text{ mm Hg}$$

Discussion

This uncertainty level (about 2%) is typical for manual blood pressure measurements using a sphygmomanometer. Blood pressure naturally fluctuates throughout the day, so this measurement uncertainty is generally acceptable for clinical purposes. The fact that the diastolic reading (80 mm Hg) has a smaller absolute uncertainty (1 mm Hg) than the systolic reading (2 mm Hg) reflects the constant percent uncertainty across different pressure values.

Answer:

(a) The percent uncertainty is **2%** (or 1.7% to two significant figures).

(b) The uncertainty in the 80 mm Hg measurement is **1 mm Hg**, so the measurement would be reported as **80 ± 1 mm Hg**.

A person measures their heart rate by counting the number of beats in 30s. If $40\ \pm 1$ beats are counted in $30.0\ \pm 0.5\text{s}$, what is the heart rate and its uncertainty in beats per minute?

Show Solution

Strategy

Calculate the heart rate, then use the method of adding percent uncertainties to find the total uncertainty.

Solution

1. Calculate the heart rate:

$$\text{Heart rate} = \frac{40\ \text{beats}}{30.0\text{s}} \times \frac{60\text{s}}{1\ \text{min}} = 80\ \text{beats/min}$$

1. Calculate percent uncertainty in beats:

$$\%\text{unc}_{\text{beats}} = \frac{1}{40} \times 100\% = 2.5\%$$

1. Calculate percent uncertainty in time:

$$\%\text{unc}_{\text{time}} = \frac{0.5\text{s}}{30.0\text{s}} \times 100\% = 1.7\%$$

1. For division, add the percent uncertainties:

$$\%\text{unc}_{\text{total}} = 2.5\% + 1.7\% = 4.2\% \approx 4\%$$

1. Calculate the absolute uncertainty in heart rate:

$$\delta(\text{heart rate}) = 80\ \text{beats/min} \times 0.04 = 3.2\ \text{beats/min} \approx 3\ \text{beats/min}$$

Discussion

This problem illustrates how uncertainties propagate in calculations involving division. The 4% combined uncertainty in the heart rate arises from both the counting uncertainty and the timing uncertainty. Manual pulse measurements over short intervals are inherently imprecise, and this 3 beats/min uncertainty is typical for such measurements. For more accurate heart rate monitoring, longer measurement periods or electronic devices would be preferable.

Answer: $80\ \pm 3\ \text{beats/min}$

What is the area of a circle 3.102cm in diameter?

Show Solution

Strategy

Calculate the area using A = πr², where r = d/2. Report the answer with the appropriate number of significant figures.

Solution

1. Find the radius:

$$r = \frac{d}{2} = \frac{3.102\ \text{cm}}{2} = 1.551\ \text{cm}$$

1. Calculate the area:

$$A = \pi r^2 = \pi(1.551\ \text{cm})^2 = \pi(2.406\ \text{cm}^2) = 7.557\ \text{cm}^2$$

Since the diameter has 4 significant figures, the area should also have 4 significant figures.

Discussion

This calculation demonstrates proper significant figure handling in area calculations. The diameter was measured to four significant figures (3.102 cm), indicating a precise measurement, possibly with calipers. Since π is a mathematical constant known to many more digits than needed, it doesn't limit our precision—the answer's precision is determined solely by the measured diameter.

Answer: The area of the circle is **7.557 cm²**.

If a marathon runner averages 9.5 mi/h, how long does it take them to run a 26.22-mi marathon?

Show Solution

Strategy

Use the relationship: time = distance / speed. Report the answer with appropriate significant figures.

Solution

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}} = \frac{26.22 \text{ mi}}{9.5 \text{ mi/h}} = 2.76 \text{ h}$$

Since the speed (9.5 mi/h) has only 2 significant figures, the answer should be reported as:

$$2.8 \text{ h}$$

Discussion

This problem highlights how the least precise measurement limits the precision of calculated results. Although the marathon distance is known quite precisely (26.22 mi, four significant figures), the average speed is only known to two significant figures (9.5 mi/h), which limits our answer to two significant figures. This 2.8 hours (about 2 hours and 48 minutes) represents a reasonable time for a recreational marathon runner.

Answer: 2.8 h

A marathon runner completes a 42.188-km course in 2h, 30 min, and 12s. There is an uncertainty of 25m in the distance traveled and an uncertainty of 1 s in the elapsed time. (a) Calculate the percent uncertainty in the distance. (b) Calculate the uncertainty in the elapsed time. (c) What is the average speed in meters per second? (d) What is the uncertainty in the average speed?

Show Solution

Strategy

Calculate percent uncertainties for distance and time, then use them to find the uncertainty in the calculated average speed.

Solution

(a) Percent uncertainty in distance:

Distance = 42.188 km = 42,188 m, with uncertainty δd = 25 m

$$\%\text{unc}_d = \frac{\delta d}{d} \times 100\% = \frac{25 \text{m}}{42{,}188 \text{m}} \times 100\% = 0.059\% \approx 0.06\%$$

(b) Percent uncertainty in time:

Time = 2 h 30 min 12 s. Convert to seconds:

$$t = 2 \times 3600 \text{s} + 30 \times 60 \text{s} + 12 \text{s} = 7200 + 1800 + 12 = 9012 \text{s}$$

With uncertainty δt = 1 s:

$$\%\text{unc}_t = \frac{\delta t}{t} \times 100\% = \frac{1 \text{s}}{9012 \text{s}} \times 100\% = 0.011\% \approx 0.01\%$$

(c) Average speed:

$$v = \frac{d}{t} = \frac{42{,}188 \text{m}}{9012 \text{s}} = 4.681 \text{m/s} \approx 4.68 \text{m/s}$$

(d) Uncertainty in average speed:

For division, add the percent uncertainties:

$$\%\text{unc}_v = \%\text{unc}_d + \%\text{unc}_t = 0.059\% + 0.011\% = 0.070\% \approx 0.07\%$$

Calculate absolute uncertainty:

$$\delta v = v \times \frac{0.070\%}{100\%} = 4.681 \text{m/s} \times 0.0007 = 0.0033 \text{m/s} \approx 0.003 \text{m/s}$$

Discussion

The extremely small percent uncertainties (0.06% and 0.01%) reflect the high precision of modern marathon timing and distance measurement systems. The uncertainty in distance (25 m over 42 km) is about the length of a swimming pool, which seems quite reasonable for a course measured along roads. The combined uncertainty of 0.07% in the average speed demonstrates how very precise measurements still carry some uncertainty, though this level of precision far exceeds what would be needed for most practical purposes.

Answer:

(a) The percent uncertainty in the distance is **0.06%**.

(b) The percent uncertainty in the elapsed time is **0.01%**.

(c) The average speed is **4.68 m/s**.

(d) The uncertainty in the average speed is **0.003 m/s**, so the speed is **4.681 ± 0.003 m/s** (or approximately **4.68 ± 0.01 m/s** to 3 significant figures).

The sides of a small rectangular box are measured to be $1.80 \pm 0.01\text{cm}$, $2.05 \pm 0.02\text{cm}$,and $3.0 \pm 0.1\text{cm}$ long. Calculate its volume and uncertainty in cubic centimeters.

Show Solution

Strategy

Calculate the volume using V = length × width × height, then find the uncertainty using the method of adding percent uncertainties.

Solution

1. Calculate the volume:

$$V = 1.80\text{ cm} \times 2.05\text{ cm} \times 3.0\text{ cm} = 11.07\text{ cm}^3$$

1. Calculate percent uncertainties for each dimension:

$$\%\text{unc}_1 = \frac{0.01\text{ cm}}{1.80\text{ cm}} \times 100\% = 0.56\%$$
$$\%\text{unc}_2 = \frac{0.02\text{ cm}}{2.05\text{ cm}} \times 100\% = 0.98\%$$
$$\%\text{unc}_3 = \frac{0.1\text{ cm}}{3.0\text{ cm}} \times 100\% = 3.3\%$$

1. Add percent uncertainties for multiplication:

$$\%\text{unc}_{\text{total}} = 0.56\% + 0.98\% + 3.3\% = 4.8\% \approx 5\%$$

1. Calculate absolute uncertainty in volume:

$$\delta V = 11.07\text{ cm}^3 \times 0.05 = 0.55\text{ cm}^3 \approx 1\text{ cm}^3$$

1. Round the volume to match the uncertainty:

$$V = 11 \pm 1\text{ cm}^3$$

Discussion

This problem demonstrates how uncertainties accumulate in multi-dimensional calculations. The third dimension (3.0 ± 0.1 cm) has the largest percent uncertainty (3.3%), which dominates the total uncertainty. The final uncertainty of about 5% shows that when multiplying measurements, even small individual uncertainties can combine to produce significant uncertainty in the result. The volume of 11 cm^3 with ±1 cm^3 uncertainty means the actual volume could range from 10 to 12 cm^3, nearly a 20% range.

Answer: $11 \pm 1\text{ cm}^3$

When non-metric units were used in the United Kingdom, a unit of mass called the **pound-mass** (lbm) was employed, where $1\text{lbm} = 0.4539\text{kg}$. (a) If there is an uncertainty of 0.0001kg in the pound-mass unit, what is its percent uncertainty? (b) Based on that percent uncertainty, what mass in pound-mass has an uncertainty of 1 kg when converted to kilograms?

Show Solution

Strategy

Calculate the percent uncertainty in the conversion factor, then find what mass would have 1 kg uncertainty with that same percent uncertainty.

Solution

(a) Percent uncertainty in the pound-mass conversion:

$$\%\text{unc} = \frac{\delta m}{m} \times 100\% = \frac{0.0001\text{kg}}{0.4539\text{kg}} \times 100\% = 0.022\% \approx 0.02\%$$

(b) Mass with 1 kg uncertainty:

If the percent uncertainty is 0.022%, what mass m (in kg) has an uncertainty of 1 kg?

$$\frac{\delta m}{m} = \frac{0.022}{100}$$
$$m = \delta m \times \frac{100}{0.022} = 1\text{kg} \times \frac{100}{0.022} = 4545\text{kg} \approx 4500\text{kg}$$

Convert to pound-mass:

$$4545\text{kg} \times \frac{1\text{ lbm}}{0.4539\text{kg}} = 10{,}014\text{ lbm} \approx 10{,}000\text{ lbm}$$

Discussion

This problem illustrates how conversion factors have their own uncertainties that can become significant when measuring large quantities. The tiny percent uncertainty in the pound-mass definition (0.02%) is negligible for everyday objects but becomes a 1 kg uncertainty when weighing objects around

4500 kg (about the mass of a large vehicle). This demonstrates why modern science has standardized on the SI system with precisely defined units.

Answer:

(a) The percent uncertainty in the pound-mass unit is **0.02%** (or 0.022% to more precision).

(b) A mass of approximately **10,000 lbm** (or **4500 kg**) would have an uncertainty of 1 kg when converted to kilograms.

The length and width of a rectangular room are measured to be $3.955 \pm 0.005 \text{m}$ and $3.050 \pm 0.005 \text{m}$. Calculate the area of the room and its uncertainty in square meters.

Show Solution

Strategy

Calculate the area using A = length × width, then find the uncertainty using the method of adding percent uncertainties.

Solution

1. Calculate the area:

$$A = 3.955 \text{m} \times 3.050 \text{m} = 12.063 \text{m}^2$$

1. Calculate percent uncertainties for each dimension:

$$\%\text{unc}_{\text{length}} = \frac{0.005 \text{m}}{3.955 \text{m}} \times 100\% = 0.126\%$$
$$\%\text{unc}_{\text{width}} = \frac{0.005 \text{m}}{3.050 \text{m}} \times 100\% = 0.164\%$$

1. Add percent uncertainties for multiplication:

$$\%\text{unc}_{\text{total}} = 0.126\% + 0.164\% = 0.29\% \approx 0.3\%$$

1. Calculate absolute uncertainty in area:

$$\delta A = 12.063 \text{m}^2 \times 0.003 = 0.036 \text{m}^2 \approx 0.04 \text{m}^2$$

1. Round the area appropriately:

$$A = 12.06 \pm 0.04 \text{m}^2$$

Discussion

The very small percent uncertainty (0.3%) indicates high-quality measurements, likely made with a precision measuring tape or laser distance meter. The absolute uncertainty of 0.04 m² means the actual area could range from about 12.02 to 12.10 m², which is excellent precision for a room measurement. This level of accuracy would be important for applications like ordering flooring materials where precise quantities are needed.

Answer: $12.06 \pm 0.04 \text{m}^2$

A car engine moves a piston with a circular cross-section of $7.500 \pm 0.002 \text{cm}$ diameter a distance of $3.250 \pm 0.001 \text{cm}$ to compress the gas in the cylinder. (a) By what amount is the gas decreased in volume in cubic centimeters? (b) Find the uncertainty in this volume.

Show Solution

Strategy

Calculate the volume using V = πr²h, where r = d/2 and h is the distance moved. Find the uncertainty using the method of adding percent uncertainties.

Solution

(a) Volume decrease:

1. Find the radius:

$$r = \frac{d}{2} = \frac{7.500 \text{ cm}}{2} = 3.750 \text{ cm}$$

1. Calculate the volume (cylinder volume = πr²h):

$$V = \pi r^2 h = \pi (3.750 \text{ cm})^2 (3.250 \text{ cm}) = \pi (14.0625 \text{ cm}^2)(3.250 \text{ cm})$$
$$V = 143.6 \text{ cm}^3$$

(b) Uncertainty in volume:

1. Calculate percent uncertainties:

For diameter:

$$\%\text{unc}_d = \frac{0.002\text{ cm}}{7.500\text{ cm}} \times 100\% = 0.027\%$$

For height (distance):

$$\%\text{unc}_h = \frac{0.001\text{ cm}}{3.250\text{ cm}} \times 100\% = 0.031\%$$

1. For $V = \pi r^2 h$, since r appears squared, the diameter uncertainty contributes twice:

$$\%\text{unc}_V = 2 \times \%\text{unc}_d + \%\text{unc}_h = 2(0.027\%) + 0.031\% = 0.085\% \approx 0.09\%$$

1. Calculate absolute uncertainty:

$$\delta V = V \times \frac{0.085\%}{100\%} = 143.6\text{ cm}^3 \times 0.00085 = 0.12\text{ cm}^3 \approx 0.1\text{ cm}^3$$

Discussion

The very small percent uncertainty (0.09%) reflects the precision needed in automotive engine manufacturing. Modern engine components are machined to very tight tolerances to ensure proper compression and efficient combustion. The volume change of about 144 cm^3 is typical for a small engine cylinder, and the uncertainty of 0.1 cm^3 represents less than 0.1% of the total, which is excellent for mechanical engineering applications. Note that the diameter uncertainty contributes twice as much as the height uncertainty because the diameter appears squared in the volume formula.

Answer:

(a) The gas volume is decreased by **144 cm^3** (or 143.6 cm^3 to four significant figures).

(b) The uncertainty in this volume is **0.1 cm^3**, so the volume change is **143.6 ± 0.1 cm^3** (or **144 ± 1 cm^3** to three significant figures).

Glossary

accuracy
: the degree to which a measured value agrees with correct value for that measurement

method of adding percents
: the percent uncertainty in a quantity calculated by multiplication or division is the sum of the percent uncertainties in the items used to make the calculation

percent uncertainty
: the ratio of the uncertainty of a measurement to the measured value, expressed as a percentage

precision
: the degree to which repeated measurements agree with each other

significant figures
: express the precision of a measuring tool used to measure a value

uncertainty
: a quantitative measure of how much your measured values deviate from a standard or expected value

---

Approximation

- Make reasonable approximations based on given data.

On many occasions, physicists, other scientists, and engineers need to make **approximations** or “guesstimates” for a particular quantity. What is the distance to a certain destination? What is the approximate density of a given item? About how large a current will there be in a circuit? Many approximate numbers are based on formulae in which the input quantities are known only to a limited accuracy. As you develop problem-solving skills (that can be applied to a variety of fields through a study of physics), you will also develop skills at approximating. You will develop these skills through thinking more quantitatively, and by being willing to take risks. As with any endeavor, experience helps, as well as familiarity with units. These approximations allow us to rule out certain scenarios or unrealistic numbers. Approximations also allow us to challenge others and guide us in our approaches to our scientific world. Let us do two examples to illustrate this concept.

Approximate the Height of a Building

Can you approximate the height of one of the buildings on your campus, or in your neighborhood? Let us make an approximation based upon the height of a person. In this example, we will calculate the height of a 39-story building.

Strategy

Think about the average height of an adult male. We can approximate the height of the building by scaling up from the height of a person.

Solution

Based on information in the example, we know there are 39 stories in the building. If we use the fact that the height of one story is approximately equal to about the length of two adult humans (each human is about 2-m tall), then we can estimate the total height of the building to be

2m1person×2person1story×39stories=156m.

Discussion

You can use known quantities to determine an approximate measurement of unknown quantities. If your hand measures 10 cm across, how many hand lengths equal the width of your desk? What other measurements can you approximate besides length?

Approximating Vast Numbers: a Trillion Dollars

A bank stack contains one-hundred $100 bills, and is worth $10 000. How many bank stacks make up a trillion dollars? (credit: Andrew Magill)

The U.S. federal deficit in the 2019 fiscal year was a little greater than $$ 1 trillion. Most of us do not have any concept of how much even one trillion actually is. Suppose that you were given a trillion dollars in $100 bills. If you made 100-bill stacks and used them to evenly cover a football field (between the end zones), make an approximation of how high the money pile would become. (We will use feet/inches rather than meters here because football fields are measured in yards.) One of your friends says 3 in., while another says 10 ft. What do you think?

Strategy

When you imagine the situation, you probably envision thousands of small stacks of 100 wrapped $$100 bills, such as you might see in movies or at a bank. Since this is an easy-to-approximate quantity, let us start there. We can find the volume of a stack of 100 bills, find out how many stacks make up one trillion dollars, and then set this volume equal to the area of the football field multiplied by the unknown height.

Solution

(1) Calculate the volume of a stack of 100 bills. The dimensions of a single bill are approximately 3 in. by 6 in. A stack of 100 of these is about 0.5 in. thick. So the total volume of a stack of 100 bills is:

$$\text{volume of stack}=\text{length}\times\text{width}\times\text{height},\quad \text{volume of stack}=6\text{in.}\times3\text{in.}\times0.5\text{in.},\quad \text{volume of stack}=9\text{in.}^3.$$

(2) Calculate the number of stacks. Note that a trillion dollars is equal to $\$1\times10^{12}$, and a stack of one-hundred $100 *bills is equal to*$10 000, or $\$1\times10^{4}$. The number of stacks you will have is:

$\$1\times10^{12}$(a trillion dollars)$1\times10^{4}$\$per stack$=1\times10^{8}$stacks.

(3) Calculate the area of a football field in square inches. The area of a football field is $100\text{yd}\times50\text{yd}$, which gives 5000yd^2. Because we are working in inches, we need to convert square yards to square inches:

$$\text{Area}=5000\text{yd}^2\times\frac{3\text{ft}}{1\text{yd}}\times\frac{3\text{ft}}{1\text{yd}}\times\frac{12\text{in.}}{1\text{ft}}\times\frac{12\text{in.}}{1\text{ft}}=6480000\text{in.}^2,\quad \text{Area}\approx6\times10^6\text{in.}^2.$$

This conversion gives us $6\times10^6\text{in.}^2$ for the area of the field. (Note that we are using only one significant figure in these calculations.)

(4) Calculate the total volume of the bills. The volume of all the \$100 -bill stacks is $9\text{in.}^3/\text{stack}\times10^8$ stacks $=9\times10^8\text{in.}^3$. (5) Calculate the height. To determine the height of the bills, use the equation:

$$\text{volume of bills} = \text{area of field}\times\text{height of money}:\quad \text{Height of money} = \frac{\text{volume of bills}}{\text{area of field}},\quad \text{Height of money} = \frac{9\times10^8\text{in.}^3}{6\times10^6\text{in.}^2}=1.33\times10^2\text{in.},\quad \text{Height of money} \approx 1\times10^2\text{in.}=100\text{in.}$$

The height of the money will be about 100 in. high. Converting this value to feet gives

$$100\text{in.}\times\frac{1\text{ft}}{12\text{in.}}=8.33\text{ft}\approx8\text{ft}.$$

Discussion

The final approximate value is much higher than the early estimate of 3 in., but the other early estimate of 10 ft (120 in.) was roughly correct. How did the approximation measure up to your first guess? What can this exercise tell you in terms of rough “guesstimates” versus carefully calculated approximations?
Check Your Understanding

Using mental math and your understanding of fundamental units, approximate the area of a regulation basketball court. Describe the process you used to arrive at your final approximation.

Show Solution

An average male is about two meters tall. It would take approximately 15 men laid out end to end to cover the length, and about 7 to cover the width. That gives an approximate area of 420m^2.

Summary

Scientists often approximate the values of quantities to perform calculations and analyze systems.

Problems & Exercises

How many heartbeats are there in a lifetime?

Show Solution

Strategy

To estimate heartbeats in a lifetime, we need to estimate the average heart rate and average lifespan.

Solution

1. Estimate average heart rate: approximately 70 beats per minute (at rest, it varies but this is a reasonable average including sleep and activity).
2. Estimate average lifespan: approximately 80 years.
3. Calculate minutes in a lifetime:

$$80\text{ years}\times\frac{365\text{ days}}{1\text{ year}}\times\frac{24\text{ hours}}{1\text{ day}}\times\frac{60\text{ min}}{1\text{ hour}}\approx4.2\times10^7\text{ minutes}$$

1. Calculate total heartbeats:

$$70\frac{\text{beats}}{\text{min}}\times4.2\times10^7\text{ min}\approx3\times10^9\text{ heartbeats}$$

Discussion

This enormous number of approximately 3 billion heartbeats illustrates the remarkable endurance and reliability of the human heart as a muscle that never rests. The calculation assumes a relatively constant average heart rate of 70 bpm, though actual heart rates vary significantly with activity level, age, and health status. This order-of-magnitude estimate helps us appreciate the incredible workload the cardiovascular system handles over a lifetime, beating roughly once per second for 80 years without pause.

Sample answer: 2×10^9 to 3×10^9 heartbeats

A generation is about one-third of a lifetime. Approximately how many generations have passed since the year 0 AD?

Show Solution

Strategy

Estimate the time since year 0 AD, then divide by the length of a generation.

Solution

1. Time since year 0 AD to present (2025):

$$\text{Time elapsed} \approx 2025 \text{ years} \approx 2000 \text{ years}$$

1. If a generation is about one-third of a lifetime, and an average lifetime is about 70-80 years:

$$\text{Generation length} \approx \frac{70 \text{ years}}{3} \approx 25 \text{ years}$$

1. Number of generations:

$$\text{Number of generations} = \frac{2000 \text{ years}}{25 \text{ years/generation}} = 80 \text{ generations}$$

Discussion

The relatively small number of only 80 generations shows how recent human recorded history is when viewed from a biological perspective. This calculation assumes a generation length of approximately 25 years, though this value has varied significantly across different cultures, time periods, and socioeconomic conditions. Understanding this timescale is valuable for studying genetic inheritance, cultural transmission of knowledge, and the relatively rapid pace of human cultural evolution compared to biological evolution.

Answer

Approximately **80 generations** have passed since the year 0 AD (assuming about 25 years per generation).

How many times longer than the mean life of an extremely unstable atomic nucleus is the lifetime of a human? (Hint: The lifetime of an unstable atomic nucleus is on the order of 10^{-22}s.)

Show Solution

Strategy

We need to compare the human lifetime to the lifetime of an unstable atomic nucleus by finding their ratio.

Solution

1. From the hint, the lifetime of an unstable atomic nucleus is on the order of 10^{-22}s.
2. Convert human lifetime to seconds. Assuming an average human lifetime of about 70 years:

$$70 \text{ years} \times \frac{365 \text{ days}}{1 \text{ year}} \times \frac{24 \text{ hours}}{1 \text{ day}} \times \frac{3600 \text{s}}{1 \text{ hour}} \approx 2.2 \times 10^{9} \text{s}$$

1. Calculate the ratio:

$$\frac{\text{Human lifetime}}{\text{Nucleus lifetime}} = \frac{2.2 \times 10^{9} \text{s}}{10^{-22} \text{s}} \approx 2 \times 10^{31}$$

Discussion

This truly astronomical factor of 10^{31} illustrates the vast range of timescales that exist in nature, from quantum processes to human experience. To put this in perspective, a human lifetime is longer than a nuclear decay timescale by the same factor that the age of the universe is longer than a fraction of a second. This extreme comparison demonstrates both the power of scientific notation in handling such disparate scales and helps us appreciate how quantum phenomena occur on timescales completely outside our everyday intuition.

Sample answer: 2×10^{31} times longer (if an average human lifetime is taken to be about 70 years).

Calculate the approximate number of atoms in a bacterium. Assume that the average mass of an atom in the bacterium is ten times the mass of a hydrogen atom. (Hint: The mass of a hydrogen atom is on the order of 10^{-27}kg and the mass of a bacterium is on the order of 10^{-15}kg) .

Show Solution

Strategy

Divide the total mass of the bacterium by the average mass of an atom to find the number of atoms.

Solution

1. From the hint:
   - Mass of hydrogen atom: $m\_H \approx 10^{-27}\text{kg}$
   - Mass of bacterium: $m\_bact \approx 10^{-15}\text{kg}$
2. Average mass of an atom in the bacterium:

$$m_{atom}=10\times m_H=10\times 10^{-27}\text{kg}=10^{-26}\text{kg}$$

1. Number of atoms:

$$N=\frac{m_{bact}}{m_{atom}}=\frac{10^{-15}\text{kg}}{10^{-26}\text{kg}}=10^{11}\text{ atoms}$$

Discussion

The result of approximately 100 billion atoms reveals that even the tiniest living organisms contain an enormous number of atoms, providing ample complexity for the molecular machinery of life. This calculation assumes all atoms in the bacterium have an average mass of 10 times that of hydrogen, which is reasonable given that bacteria contain heavier atoms like carbon, nitrogen, and oxygen in addition to hydrogen. This order-of-magnitude estimate helps us understand why even single-celled organisms can have sophisticated metabolic pathways and genetic information storage despite their microscopic size.

Answer

A bacterium contains approximately **10^{11} atoms** (or about 100 billion atoms).

This color-enhanced photo shows Salmonella typhimurium (red) attacking human cells. These bacteria are commonly known for causing foodborne illness. Can you estimate the number of atoms in each bacterium? (credit: Rocky Mountain Laboratories, NIAID, NIH)

Approximately how many atoms thick is a cell membrane, assuming all atoms there average about twice the size of a hydrogen atom?

Show Solution

Strategy

We need to estimate how many atoms fit across the thickness of a cell membrane, given that the atoms are about twice the size of a hydrogen atom.

Solution

1. A typical cell membrane thickness is approximately 10^{-8}m (or about 10 nm).
2. The diameter of a hydrogen atom is approximately 10^{-10}m (or about 0.1 nm).
3. If atoms in the membrane average about twice the size of a hydrogen atom:

$$\text{Average atom diameter}\approx 2\times 10^{-10}\text{m}$$

1. Calculate the number of atoms across the membrane thickness:

$$\text{Number of atoms}=\frac{\text{Membrane thickness}}{\text{Atom diameter}}=\frac{10^{-8}\text{m}}{2\times 10^{-10}\text{m}}=50\text{ atoms}$$

Discussion

The remarkably small thickness of only about 50 atoms demonstrates how incredibly thin cell membranes are, yet they successfully maintain the cell's integrity and control what enters and exits. This estimate assumes atoms in the membrane are roughly uniform in size at about twice the hydrogen atom diameter, which is reasonable for the lipid and protein molecules that make up biological membranes. The fact that such a thin barrier can effectively separate the cell's interior from its environment while still allowing selective transport of materials is one of the elegant solutions nature has evolved for cellular function.

Sample answer: 50 atoms

(a) What fraction of Earth's diameter is the greatest ocean depth? (b) The greatest mountain height?

Show Solution

Strategy

Use Table 3 to find the relevant values, then calculate the ratios.

Solution

From Table 3:

- Diameter of Earth: $d\_Earth = 10^7 \text{m}$ (more precisely, about $1.27 \times 10^7 \text{m}$)
- Greatest ocean depth: $d\_ocean = 10^4 \text{m}$ (about 10,000 m, or the Mariana Trench at ~11,000 m)
- Greatest mountain height: $h\_mountain = 10^4 \text{m}$ (Mt. Everest is about 8,850 m)

(a) Fraction for ocean depth:

$$\text{Fraction} = \frac{d_{ocean}}{d_{Earth}} = \frac{10^4 \text{m}}{10^7 \text{m}} = 10^{-3} = \frac{1}{1000}$$

(b) Fraction for mountain height:

$$\text{Fraction} = \frac{h_{mountain}}{d_{Earth}} = \frac{10^4 \text{m}}{10^7 \text{m}} = 10^{-3} = \frac{1}{1000}$$

Discussion

The tiny ratio of 1/1000 for both ocean depths and mountain heights reveals that Earth is remarkably smooth relative to its overall size—the deepest trenches and highest peaks represent only about 0.1% of Earth's diameter. This explains why our planet appears as a smooth sphere when viewed from space, with surface irregularities barely visible. To put this in perspective, if Earth were shrunk to the size of a billiard ball, it would actually be smoother than a regulation billiard ball, since these surface features would scale down proportionally.

Answer

(a) The greatest ocean depth is approximately **1/1000** (or **0.001** or **10^{-3}**) of Earth's diameter.

(b) The greatest mountain height is approximately **1/1000** (or **0.001** or **10^{-3}**) of Earth's diameter.

Both the deepest ocean trenches and highest mountains are tiny compared to Earth's diameter, representing only about 0.1% of it.

(a) Calculate the number of cells in a hummingbird assuming the mass of an average cell is ten times the mass of a bacterium. (b) Making the same assumption, how many cells are there in a human?

Show Solution

Strategy

We need to estimate the number of cells by dividing the total mass by the mass of a single cell (which is 10 times the mass of a bacterium).

Solution

1. From the problem, an average cell mass is ten times the mass of a bacterium. From Table 3, the mass of a bacterium is approximately 10^{-15}kg, so:

$$\text{Cell mass} \approx 10 \times 10^{-15} \text{kg} = 10^{-14} \text{kg}$$

(a) Cells in a hummingbird:

1. The mass of a hummingbird is approximately 10^{-2}kg (about 10 grams, or roughly 0.01 kg).
2. Calculate number of cells:

$$\text{Number of cells} = \frac{\text{Hummingbird mass}}{\text{Cell mass}} = \frac{10^{-2} \text{kg}}{10^{-14} \text{kg}} = 10^{12} \text{ cells}$$

(b) Cells in a human:

1. The mass of a human is approximately 10^2kg (about 100 kg, though 70 kg is more typical).

2. Calculate number of cells:

$$\text{Number of cells} = \frac{\text{Human mass}}{\text{Cell mass}} = \frac{10^2 \text{kg}}{10^{-14} \text{kg}} = 10^{16} \text{ cells}$$

Discussion

The ratio of approximately 10,000:1 in cell count between humans and hummingbirds roughly corresponds to the ratio of their masses, which is reasonable given our assumption of uniform cell size. In reality, cell sizes vary dramatically across different tissue types (neurons, blood cells, muscle cells all differ), but this order-of-magnitude approximation remains useful for understanding biological scale. These enormous numbers—trillions for a hummingbird and ten quadrillion for a human—illustrate the incredible cellular complexity that underlies even small multicellular organisms.

Sample answers:

(a) 10^{12} cells/hummingbird

(b) 10^{16} cells/human

Assuming one nerve impulse must end before another can begin, what is the maximum firing rate of a nerve in impulses per second?

Show Solution

Strategy

From Table 3, find the duration of a nerve impulse, then calculate the maximum firing rate.

Solution

1. From Table 3, the duration of a nerve impulse is approximately 10^{-3}s (or 1 millisecond).
2. If one impulse must complete before another begins, the maximum firing rate is:

$$\text{Firing rate} = \frac{1}{\text{Duration}} = \frac{1}{10^{-3}\text{s}} = 10^3 \text{ impulses/s} = 1000 \text{ Hz}$$

Discussion

A maximum firing rate of 1000 Hz (1000 impulses per second) represents the theoretical upper limit based on the refractory period during which a neuron cannot fire again. This calculation assumes that impulses cannot overlap and that the neuron fires immediately after each refractory period ends, which represents an idealized maximum. In biological reality, most neurons fire at rates well below this limit—typically a few hundred Hz at most—because sustained high-frequency firing would be metabolically expensive and is usually unnecessary for neural signaling.

Answer

The maximum firing rate of a nerve is approximately **1000 impulses per second** (or **1000 Hz**, or **10^3 Hz**).

This is consistent with known biological limits - most neurons fire at rates well below 1000 Hz, with typical maximum rates being a few hundred Hz for most neurons.

Glossary

approximation
: an estimated value based on prior experience and reasoning