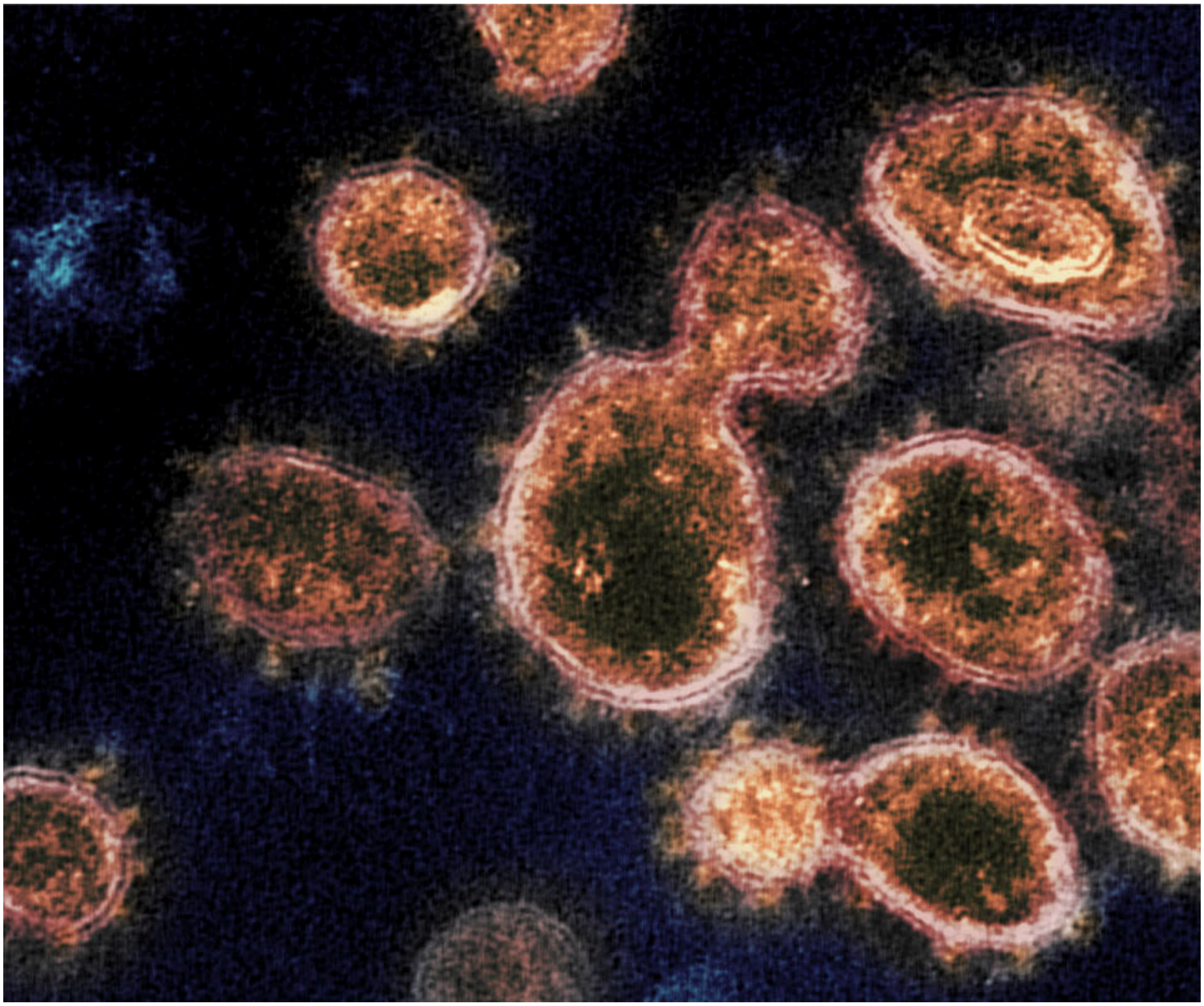
Introduction to Vision and Optical Instruments



This transmission electron microscope image shows SARS-CoV-2, the coronavirus that causes COVID-19, isolated from a patient in the U.S. Virus particles are shown emerging from the surface of cells cultured in the lab. The spikes on the outer edge of the virus particles give coronaviruses their name: crown-like. (credit: NIAID-RML, NIH Image Gallery/Flickr)

Explore how the image on the computer screen is formed. How is the image formation on the computer screen different from the image formation in your eye as you look down the microscope? How can videos of living cell processes be taken for viewing later on, and by many different people?

Seeing faces and objects we love and cherish is a delight—one's favorite teddy bear, a picture on the wall, or the sun rising over the mountains. Intricate images help us understand nature and are invaluable for developing techniques and technologies in order to improve the quality of life. The image of a red blood cell that almost fills the cross-sectional area of a tiny capillary makes us wonder how blood makes it through and not get stuck. We are able to see bacteria and viruses and understand their structure. It is the knowledge of physics that provides fundamental understanding and models required to develop new techniques and instruments. Therefore, physics is called an *enabling science*—a science that enables development and advancement in other areas. It is through optics and imaging that physics enables advancement in major areas of biosciences. This chapter illustrates the enabling nature of physics through an understanding of how a human eye is able to see and how we are able to use optical instruments to see beyond what is possible with the naked eye. It is convenient to categorize these instruments on the basis of geometric optics (see Geometric Optics) and wave optics (see Wave Optics).

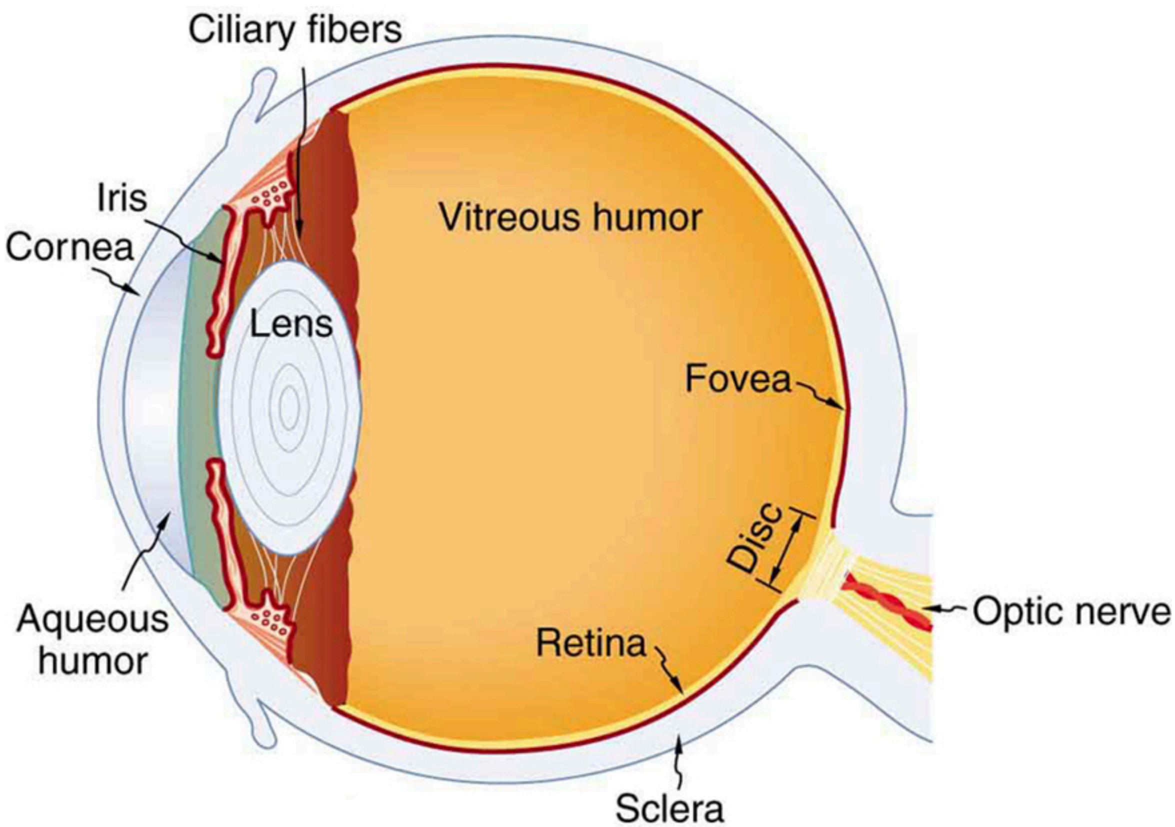
Physics of the Eye

- Explain the image formation by the eye.
- Explain why peripheral images lack detail and color.
- Define refractive indices.
- Analyze the accommodation of the eye for distant and near vision.

Early thinkers had a wide array of theories regarding vision. Euclid and Ptolemy believed that the eyes emitted rays of light; others promoted the idea that objects gave off some particle or substance that was discerned by the eye. Ibn al-Haytham (sometimes called Alhazen), who was mentioned earlier as an originator of the scientific method, conducted a number of experiments to illustrate how the anatomical construction of the eye led to its ability to form images. He recognized that light reflected from objects entered the eye through the lens and was passed to the optic nerve. Al-Haytham did not fully understand the mechanisms involved, but many subsequent discoveries in vision, reflection, and magnification built on his discoveries and methods.

The eye is perhaps the most interesting of all optical instruments. The eye is remarkable in how it forms images and in the richness of detail and color it can detect. However, our eyes commonly need some correction, to reach what is called “normal” vision, but should be called ideal rather than normal. Image formation by our eyes and common vision correction are easy to analyze with the optics discussed in Geometric Optics.

[Figure 1] shows the basic anatomy of the eye. The cornea and lens form a system that, to a good approximation, acts as a single thin lens. For clear vision, a real image must be projected onto the light-sensitive retina, which lies at a fixed distance from the lens. The lens of the eye adjusts its power to produce an image on the retina for objects at different distances. The center of the image falls on the fovea, which has the greatest density of light receptors and the greatest acuity (sharpness) in the visual field. The variable opening (or pupil) of the eye along with chemical adaptation allows the eye to detect light intensities from the lowest observable to 10^{10} times greater (without damage). This is an incredible range of detection. Our eyes perform a vast number of functions, such as sense direction, movement, sophisticated colors, and distance. Processing of visual nerve impulses begins with interconnections in the retina and continues in the brain. The optic nerve conveys signals received by the eye to the brain.

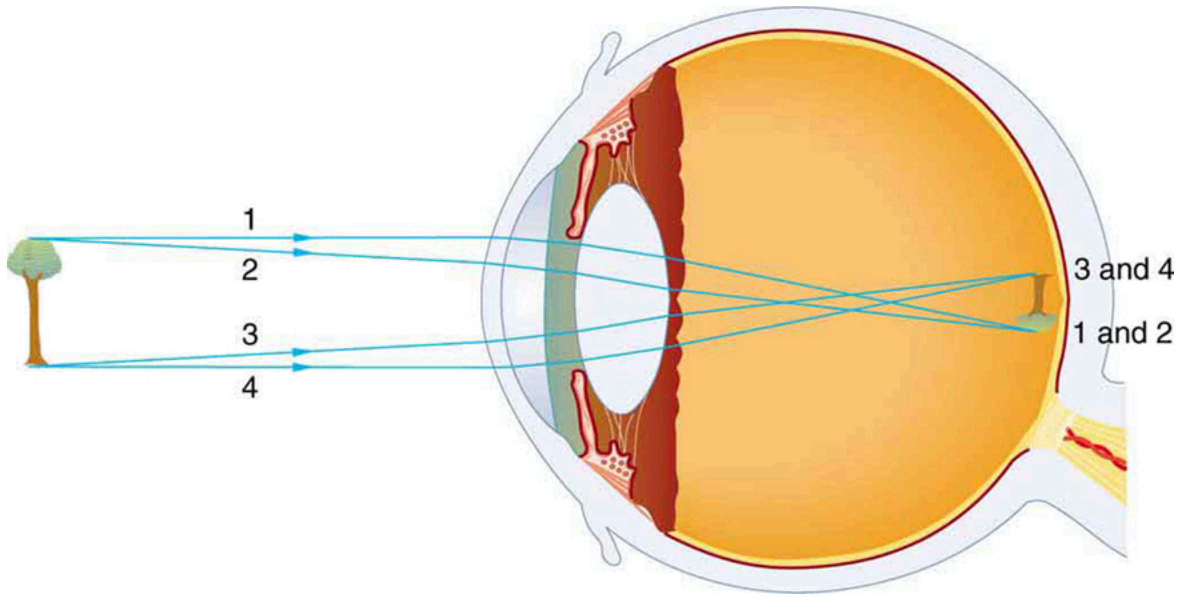


The cornea and lens of an eye act together to form a real image on the light-sensing retina, which has its densest concentration of receptors in the fovea and a blind spot over the optic nerve. The power of the lens of an eye is adjustable to provide an image on the retina for varying object distances. Layers of tissues with varying indices of refraction in the lens are shown here. However, they have been omitted from other pictures for clarity.

Refractive indices are crucial to image formation using lenses. [Table 1] shows refractive indices relevant to the eye. The biggest change in the refractive index, and bending of rays, occurs at the cornea rather than the lens. The ray diagram in [Figure 2] shows image formation by the cornea and lens of the eye. The rays bend according to the refractive indices provided in [Table 1]. The cornea provides about two-thirds of the power of the eye, owing to the fact that speed of light changes considerably while traveling from air into cornea. The lens provides the remaining power needed to produce an image on the retina. The cornea and lens can be treated as a single thin lens, even though the light rays pass through several layers of material (such as cornea, aqueous humor, several layers in the lens, and vitreous humor), changing direction at each interface. The image formed is much like the one produced by a single convex lens. This is a case 1 image. Images formed in the eye are inverted but the brain inverts them once more to make them seem upright.

Refractive Indices Relevant to the Eye

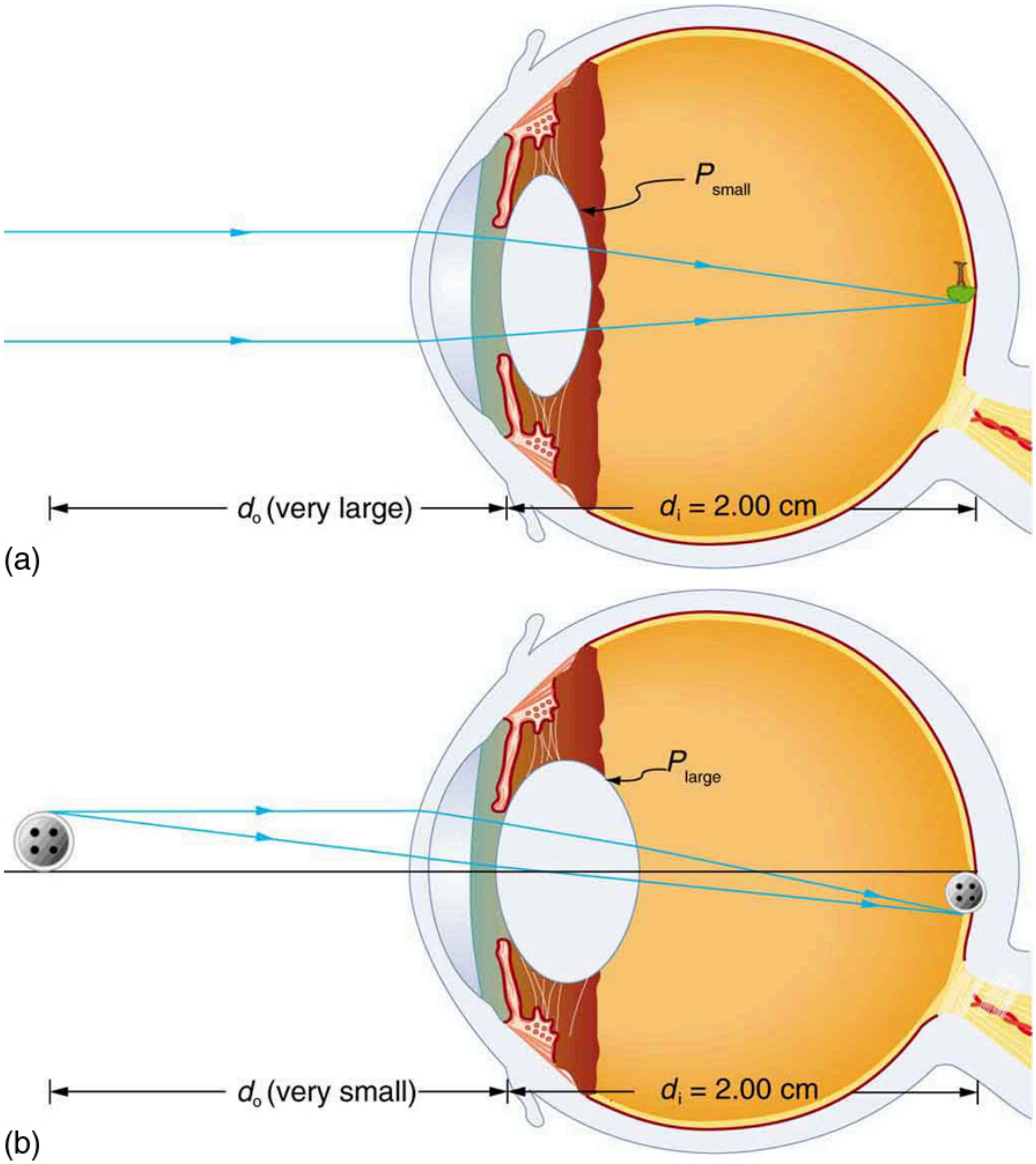
Material	Index of Refraction
Water	1.33
Air	1.0
Cornea	1.38
Aqueous humor	1.34
Lens	1.41 average (varies throughout the lens, greatest in center)
Vitreous humor	1.34



An image is formed on the retina with light rays converging most at the cornea and upon entering and exiting the lens. Rays from the top and bottom of the object are traced and produce an inverted real image on the retina. The distance to the object is drawn smaller than scale.

As noted, the image must fall precisely on the retina to produce clear vision — that is, the image distance d_i must equal the lens-to-retina distance. Because the lens-to-retina distance does not change, the image distance d_i must be the same for objects at all distances. The eye manages this by varying the power (and focal length) of the lens to accommodate for objects at various distances. The process of adjusting the eye's focal length is called **accommodation**. A person with normal (ideal) vision can see objects clearly at distances ranging from 25 cm to essentially infinity. However, although the near point (the shortest distance at which a sharp focus can be obtained) increases with age (becoming meters for some older people), we will consider it to be 25 cm in our treatment here.

[Figure 3] shows the accommodation of the eye for distant and near vision. Since light rays from a nearby object can diverge and still enter the eye, the lens must be more converging (more powerful) for close vision than for distant vision. To be more converging, the lens is made thicker by the action of the ciliary muscle surrounding it. The eye is most relaxed when viewing distant objects, one reason that microscopes and telescopes are designed to produce distant images. Vision of very distant objects is called *totally relaxed*, while close vision is termed *accommodated*, with the closest vision being _ fully accommodated_.



Relaxed and accommodated vision for distant and close objects. (a) Light rays from the same point on a distant object must be nearly parallel while entering the eye and more easily converge to produce an image on the retina. (b) Light rays from a nearby object can diverge more and still enter the eye. A more powerful lens is needed to converge them on the retina than if they were parallel.

We will use the thin lens equations to examine image formation by the eye quantitatively. First, note the power of a lens is given as $P = 1/f$, so we rewrite the thin lens equations as

$$P=\frac{1}{d_o}+\frac{1}{d_i}$$

and

$$\frac{h_i}{h_o}=-\frac{d_i}{d_o}=m.$$

We understand that d_i must equal the lens-to-retina distance to obtain clear vision, and that normal vision is possible for objects at distances $d_o = 25 \text{cm}$ to infinity.

Take-Home Experiment: The Pupil

Look at the central transparent area of someone's eye, the pupil, in normal room light. Estimate the diameter of the pupil. Now turn off the lights and darken the room. After a few minutes turn on the lights and promptly estimate the diameter of the pupil. What happens to the pupil as the eye adjusts to the room light? Explain your observations.

The eye can detect an impressive amount of detail, considering how small the image is on the retina. To get some idea of how small the image can be, consider the following example.

Size of Image on Retina

What is the size of the image on the retina of a 1.20×10^{-2} cm diameter human hair, held at arm's length (60.0 cm) away? Take the lens-to-retina distance to be 2.00 cm.

Strategy

We want to find the height of the image h_i, given the height of the object is $h_o = 1.20 \times 10^{-2}$ cm. We also know that the object is 60.0 cm away, so that $d_o = 60.0\text{ cm}$. For clear vision, the image distance must equal the lens-to-retina distance, and so $d_i = 2.00\text{ cm}$. The equation $\frac{h_i}{h_o} = -\frac{d_i}{d_o} = m$ can be used to find h_i with the known information.

Solution

The only unknown variable in the equation $\frac{h_i}{h_o} = -\frac{d_i}{d_o} = m$ is h_i:

$$\frac{h_i}{h_o} = -\frac{d_i}{d_o}.$$

Rearranging to isolate h_i yields

$$h_i = -h_o \cdot \frac{d_i}{d_o}.$$

Substituting the known values gives

$$h_i = -(1.20 \times 10^{-2}\text{ cm})\frac{2.00\text{ cm}}{60.0\text{ cm}} = -4.00 \times 10^{-4}\text{ cm}.$$

Discussion

This truly small image is not the smallest discernible—that is, the limit to visual acuity is even smaller than this. Limitations on visual acuity have to do with the wave properties of light and will be discussed in the next chapter. Some limitation is also due to the inherent anatomy of the eye and processing that occurs in our brain.

Power Range of the Eye

Calculate the power of the eye when viewing objects at the greatest and smallest distances possible with normal vision, assuming a lens-to-retina distance of 2.00 cm (a typical value).

Strategy

For clear vision, the image must be on the retina, and so $d_i = 2.00\text{ cm}$ here. For distant vision, $d_o \approx \infty$, and for close vision, $d_o = 25.0\text{ cm}$, as discussed earlier. The equation $P = \frac{1}{d_o} + \frac{1}{d_i}$ as written just above, can be used directly to solve for P in both cases, since we know d_i and d_o. Power has units of diopters, where $1\text{ D} = 1/\text{m}$, and so we should express all distances in meters.

Solution

For distant vision,

$$P = \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{\infty} + \frac{1}{0.0200\text{ m}}.$$

Since $1/\infty = 0$, this gives

$$P = 0 + 50.0/\text{m} = 50.0\text{ D (distant vision).}$$

Now, for close vision,

$$P = \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{0.250\text{ m}} + \frac{1}{0.0200\text{ m}} \quad P = \frac{4.00}{\text{m}} + \frac{50.0}{\text{m}} = 4.00\text{ D} + 50.0\text{ D} \quad P = 54.0\text{ D (close vision).}$$

Discussion

For an eye with this typical 2.00 cm lens-to-retina distance, the power of the eye ranges from 50.0 D (for distant totally relaxed vision) to 54.0 D (for close fully accommodated vision), which is an 8% increase. This increase in power for close vision is consistent with the preceding discussion and the ray tracing in [Figure 3]. An 8% ability to accommodate is considered normal but is typical for people who are about 40 years old. Younger people have greater accommodation ability, whereas older people gradually lose the ability to accommodate. When an optometrist identifies accommodation as a problem in elder people, it is most likely due to stiffening of the lens. The lens of the eye changes with age in ways that tend to preserve the ability to see distant objects clearly but do not allow the eye to accommodate for close vision, a condition called **presbyopia** (literally, elder eye). To correct this vision defect, we place a converging, positive power lens in front of the eye, such as found in reading glasses. Commonly available reading glasses are rated by their power in diopters, typically ranging from 1.0 to 3.5 D.

Section Summary

- Image formation by the eye is adequately described by the thin lens equations:

$$P=\frac{1}{d_o}+\frac{1}{d_i} \text{ and } \frac{h_i}{h_o}=-\frac{d_i}{d_o}=m.$$

- The eye produces a real image on the retina by adjusting its focal length and power in a process called accommodation.
- For close vision, the eye is fully accommodated and has its greatest power, whereas for distant vision, it is totally relaxed and has its smallest power.
- The loss of the ability to accommodate with age is called presbyopia, which is corrected by the use of a converging lens to add power for close vision.

Conceptual Questions

If the lens of a person's eye is removed because of cataracts (as has been done since ancient times), why would you expect a spectacle lens of about 16 D to be prescribed?

Show Solution

Strategy

The eye's total optical power comes from two sources: the cornea and the lens. To understand why a spectacle lens of approximately 16 D would be needed after lens removal, we need to determine how much power the natural lens contributes to the eye's total focusing ability. We can analyze this by examining the power contributions of the cornea and lens separately, using the information provided in the chapter about the eye's optical properties and refractive indices.

Solution

The normal relaxed eye has a total power of approximately 50 D when viewing distant objects. From the chapter text, we learned that the cornea provides about two-thirds of the eye's total power because the largest change in refractive index occurs at the air-cornea interface (from $n = 1.0$ to $n = 1.38$).

Calculating the cornea's contribution:

- Cornea power: $\frac{2}{3} \times 50 \text{ D} \approx 33 \text{ D}$

The remaining power comes from the lens:

- Lens power: $50 \text{ D} - 33 \text{ D} \approx 17 \text{ D}$

When the natural lens is removed (aphakia), the eye loses approximately 17 D of focusing power. To restore the ability to focus on distant objects, a spectacle lens must replace this lost power. Therefore, a lens of approximately 16-17 D would be prescribed.

More precisely, when the natural lens is removed, light entering through the cornea alone is insufficient to converge rays onto the retina. The spectacle lens must provide the additional converging power that was previously supplied by the crystalline lens.

Discussion

This question highlights the crucial optical role of the crystalline lens in the eye's focusing system. Although the cornea provides the majority of the eye's refractive power (about 33 D out of 50 D), the lens contributes an essential 17 D that is necessary to bring light to focus on the retina.

Historically, cataract surgery dates back to ancient times, with evidence of the procedure being performed as early as the 5th century BCE. In traditional "couching" procedures, the clouded lens was displaced rather than removed, but by the 18th century, extraction became more common. Patients who underwent lens removal became extremely hyperopic (farsighted) and required strong converging lenses to see clearly at any distance.

The prescription of a 16 D spectacle lens restores distance vision, but it's important to note that such patients lose all accommodation ability since the natural lens is responsible for changing the eye's focal length. Without the lens, the eye cannot adjust its power for near vision. This means that patients with aphakia typically need different spectacle corrections for different viewing distances—strong additional plus power for reading and intermediate tasks.

Modern cataract surgery involves replacing the clouded natural lens with an artificial intraocular lens (IOL) of appropriate power, which provides both distance correction and, in some advanced designs, some degree of accommodation or multifocal capability. This represents a significant improvement over simple spectacle correction for aphakia.

The physical principle demonstrated here is that the total power of a compound optical system (cornea plus lens) equals the sum of the individual powers when the optical elements are close together. Removing one element requires external compensation to maintain proper image formation.

A cataract is cloudiness in the lens of the eye. Is light dispersed or diffused by it?

Show Solution

Strategy

To answer this question, we must distinguish between two different optical phenomena: dispersion and diffusion. Dispersion refers to the separation of light into its component wavelengths (colors) due to wavelength-dependent refraction, such as occurs in a prism. Diffusion, also called scattering, refers to the random redirection of light rays when they encounter irregularities or particles in a medium. We need to consider the physical nature of a cataract and determine which phenomenon better describes its effect on light.

Solution

Light is **diffused** (scattered) by a cataract, not dispersed.

A cataract consists of cloudy or opaque regions in the normally transparent crystalline lens. These cloudy regions result from protein aggregation and structural changes in the lens fibers. The cloudiness creates numerous small irregularities and inhomogeneities in the refractive index throughout the lens tissue.

When light encounters these irregularities:

- Light rays are scattered in random directions rather than following predictable refraction paths
- The scattering occurs at multiple sites throughout the cloudy lens
- Different wavelengths are scattered similarly (no significant wavelength-dependent separation)
- The result is a general blurring and reduction in image contrast

Dispersion, by contrast, would require a wavelength-dependent deviation of light, which would cause colored fringes and chromatic effects. While the eye's lens does have some inherent dispersive properties (as all transparent materials do), a cataract does not enhance dispersion. Instead, the dominant effect is diffuse scattering.

Discussion

The distinction between diffusion and dispersion is crucial for understanding how cataracts affect vision. Patients with cataracts typically report:

- Blurred or hazy vision (due to diffuse scattering)
- Reduced contrast sensitivity
- Glare and halos around lights (caused by scattered light reaching the retina from multiple directions)
- Difficulty seeing in bright light (increased scattering)

They generally do not report rainbow-colored fringes or chromatic aberrations, which would be characteristic of dispersion.

The diffuse scattering mechanism explains why cataracts progressively reduce visual acuity. As more light is scattered away from its intended path to the retina, the image becomes increasingly degraded. Light that should form a sharp point on the retina is instead spread over a larger area, with some light scattered to incorrect locations. This is similar to trying to see through frosted glass—the image becomes blurred and washed out, but colors remain relatively normal.

Understanding this scattering mechanism has practical implications for cataract patients. For example:

- Reducing bright lighting or using polarized lenses can minimize glare caused by scattered light
- Increasing contrast in the visual environment can help compensate for reduced contrast sensitivity
- Surgical removal of the cloudy lens and replacement with a clear artificial lens is the definitive treatment, as it eliminates the scattering medium

This question also illustrates an important principle in optics: not all interactions between light and matter involve dispersion. While refraction at interfaces causes dispersion to some degree, the dominant effect of irregular, inhomogeneous media is diffuse scattering. This same principle applies to fog, clouds, frosted glass, and other translucent materials where structural irregularities scatter light.

When laser light is shone into a relaxed normal-vision eye to repair a tear by spot-welding the retina to the back of the eye, the rays entering the eye must be parallel. Why?

Show Solution

Strategy

This question relates to the eye's accommodation and how it focuses light from objects at different distances. We need to consider what "relaxed normal-vision eye" means in terms of focal length and power, and then determine what type of incoming light rays will naturally focus on the retina without requiring accommodation. The key is understanding the relationship between object distance and ray convergence/divergence.

Solution

Parallel rays entering the eye must be used because a relaxed normal-vision eye is focused at infinity, and only parallel rays focus precisely on the retina when the eye is in its relaxed state.

From the thin lens equation and the chapter discussion:

- A relaxed eye has minimum power (approximately 50 D for a normal eye)
- For an object at infinity, $d_o \to \infty$, which means $1 d_o \to 0$
- Using $P = 1 d_o + 1 d_i$, when $d_o = \infty$, we get $P = 1 d_i$
- Rays from an object at infinity are parallel when they reach the eye
- The relaxed eye's optical system (cornea and lens) converges these parallel rays to a focal point exactly on the retina at distance $d_i = 2.00$ cm

For laser retinal repair:

- The surgeon needs the laser beam to focus precisely on a specific point on the retina
- The eye must remain relaxed to avoid patient discomfort and ensure steady positioning
- If non-parallel rays were used (converging or diverging), the eye would need to accommodate to bring them to focus on the retina

- Accommodation would change the lens shape and could cause unwanted movement or patient fatigue
- Parallel rays eliminate the need for accommodation and ensure predictable, precise focusing

Discussion

This question highlights a fundamental principle of geometric optics applied to medical procedures. The relaxed eye represents the eye's natural, most stable optical configuration. When viewing distant objects (effectively at infinity), the incoming light rays are parallel, and the relaxed eye brings them to focus on the retina without any muscular effort from the ciliary muscles.

In laser photocoagulation (retinal welding), precision is paramount. The laser must deliver energy to an extremely small, specific spot on the retina—often just tens of micrometers in diameter. Using parallel incident rays provides several crucial advantages:

1. **Predictability**: The optical path through a relaxed eye is consistent and calculable. There's no variation due to accommodation.

2. **Patient comfort**: Maintaining accommodation (lens contraction) for the duration of the procedure would cause eye strain and fatigue, leading to involuntary eye movements.

3. **Repeatability**: Each laser pulse can be delivered with the same optical configuration, ensuring consistent spot size and energy density.

4. **Minimized aberrations**: The relaxed lens has its most natural shape with minimal optical aberrations.

Practically, the laser system is designed to emit a collimated (parallel) beam that enters the patient's eye. The surgeon uses additional optics (such as a contact lens placed on the cornea) to view the retina and aim the laser, but the fundamental requirement remains that the beam entering the eye is parallel.

This same principle applies to other ophthalmic procedures and diagnostic techniques. For example, autorefractors (devices that measure refractive error) typically project parallel or near-parallel light into the eye to assess the eye's optical power in its relaxed state.

The question also reinforces the concept that object distance and ray geometry are intimately connected: distant objects produce parallel rays, nearby objects produce diverging rays. By controlling the ray geometry entering the eye, physicians can control where light focuses without requiring the patient to actively accommodate, resulting in safer, more precise medical procedures.

How does the power of a dry contact lens compare with its power when resting on the tear layer of the eye? Explain.

Show Solution

Strategy

The power of a lens depends on the refractive index difference between the lens material and the surrounding medium. We need to apply the lensmaker's equation concept to understand how changing the medium on one side of the contact lens affects its optical power. A dry contact lens has air ($n = 1.0$) on both surfaces, while a contact lens on the eye has tear fluid ($n \approx 1.34$) on the back surface and air on the front surface.

Solution

A dry contact lens has **greater power** than the same lens when resting on the tear layer of the eye.

The power of a lens is determined by the refractive index differences at its surfaces. For a thin lens, the lensmaker's equation shows that power is proportional to:

$$P \propto (n_{\text{lens}} - n_{\text{surrounding}}) \times (\text{surface curvatures})$$

For a **dry contact lens** (in air):

- Front surface: Light goes from air ($n = 1.0$) to lens material ($n \approx 1.4 - 1.5$ for typical materials)
- Back surface: Light goes from lens material ($n \approx 1.4 - 1.5$) to air ($n = 1.0$)
- Both surfaces contribute significantly to the total refractive power
- Index difference at each surface: $\Delta n \approx 0.4 - 0.5$

For a **contact lens on the eye**:

- Front surface: Light goes from air ($n = 1.0$) to lens material ($n \approx 1.4 - 1.5$)
- Back surface: Light goes from lens material ($n \approx 1.4 - 1.5$) to tears ($n = 1.34$)
- The back surface has a much smaller index difference
- Index difference at back surface: $\Delta n \approx 0.1 - 0.2$ (reduced by about 60-75%)

Since the refractive index of tears ($n = 1.34$) is much closer to that of the contact lens material ($n \approx 1.4 - 1.5$) than air is, the back surface of the contact lens contributes much less optical power when the lens is on the eye. The front surface contribution remains the same, but the reduced back surface contribution means the total power decreases.

Quantitatively, if a contact lens has power P_{dry} when measured in air, its effective power $P_{\text{on eye}}$ when placed on the tear film is less: $P_{\text{on eye}} < P_{\text{dry}}$.

Discussion

This question illustrates an important practical consideration in contact lens prescription. The phenomenon explains why contact lens powers cannot be directly converted from spectacle lens prescriptions, and why contact lenses must be measured and specified based on their performance when placed on the eye, not in air.

The physical principle at work is that refraction occurs due to changes in the speed of light at interfaces between materials with different refractive indices. The greater the difference in refractive indices, the greater the refraction (and hence the optical power). When a contact lens is in air:

1. The back surface interfaces with air ($n = 1.0$), creating a large index step
2. Both surfaces contribute substantial refractive power
3. The measured power is higher

When the same lens is placed on the tear film:

1. The back surface now interfaces with tears ($n = 1.34$), which is similar to the aqueous humor ($n = 1.34$) in the eye
2. The back surface contributes much less power
3. The effective power is reduced

This reduction in power is actually advantageous because it allows the tear layer-cornea-lens system to work together more naturally. The tear film fills any small gaps between the contact lens and the cornea, creating better optical quality and comfort.

Practically, this means:

- Contact lens manufacturers specify lens powers based on on-eye performance, not in-air measurements
- Optometrists account for this effect when prescribing contact lenses
- The same refractive error might require different numerical powers for spectacles versus contact lenses
- Measuring contact lens power in air (using a lensometer) gives a higher value than the effective power on the eye

This principle also applies to intraocular lenses (IOLs) used in cataract surgery. An IOL implanted inside the eye is surrounded by aqueous humor ($n = 1.34$) rather than air, so its effective power is significantly less than it would be if measured in air. IOL power calculations must account for this to achieve the desired postoperative refraction.

Understanding this concept reinforces the general principle that the optical power of any lens system depends not just on the lens geometry and material, but critically on the refractive indices of the surrounding media. This is why the same physical lens can have different effective powers in different environments.

Why is your vision so blurry when you open your eyes while swimming under water? How does a face mask enable clear vision?

Show Solution

Strategy

The eye's focusing power depends critically on the refractive index difference at the cornea-medium interface. We need to compare the optical situation when the eye is in air versus in water, using the refractive indices from Table 1 in the chapter: air ($n = 1.0$), water ($n = 1.33$), and cornea ($n = 1.38$). By analyzing how the cornea's refractive power changes in different media, we can understand why underwater vision is blurred and how a face mask solves this problem.

Solution

Vision is blurry underwater because the cornea loses most of its refractive power when surrounded by water instead of air.

In air (normal vision):

- The cornea-air interface has a large refractive index difference: $\Delta n = 1.38 - 1.0 = 0.38$
- From the chapter, the cornea provides about two-thirds of the eye's total power (~33 D out of 50 D)
- This large index difference causes significant refraction of light rays entering the eye
- The cornea and lens together focus light precisely on the retina

Underwater (blurry vision):

- The cornea-water interface has a very small refractive index difference: $\Delta n = 1.38 - 1.33 = 0.05$
- This is only about 13% of the index difference in air
- The cornea contributes very little refractive power (~4-5 D instead of ~33 D)
- The lens alone cannot provide enough additional power to compensate
- Total eye power drops from ~50 D to only ~21-22 D
- Light rays are not sufficiently converged to focus on the retina
- The image forms far behind the retina, resulting in severe hyperopia (farsightedness)
- Everything appears extremely blurry

With a face mask (clear vision restored):

- The mask creates an air space in front of the eyes
- The cornea once again interfaces with air ($n = 1.0$), not water
- The refractive index difference at the cornea is restored: $\Delta n = 0.38$
- The cornea regains its normal ~33 D of refractive power
- The eye's total optical system returns to normal (~50 D)
- Light focuses properly on the retina, providing clear vision

The flat glass or plastic window of the mask does cause some refraction, but since it has parallel surfaces and the light enters and exits at approximately the same angle, the net effect on image position is minimal. The critical factor is maintaining the air-cornea interface.

Discussion

This question beautifully illustrates how the eye's optical design is specifically adapted for vision in air. The cornea is the eye's primary refractive element, but its power depends entirely on being surrounded by a medium with a significantly different refractive index. Water's refractive index (1.33) is so close to that of the cornea (1.38) and aqueous humor (1.34) that the cornea essentially becomes "invisible" optically—light passes through with minimal bending.

The magnitude of the vision impairment underwater is substantial. With only about 40% of the normal optical power (20-22 D instead of 50 D), the eye is massively hyperopic. This is equivalent to someone with normal vision suddenly needing reading glasses with power of about +28 to +30 D—an enormous refractive error that renders everything hopelessly blurred.

Interestingly, some aquatic mammals have evolved different solutions to this problem:

- **Seals and otters** have more spherical lenses with higher refractive power, allowing them to compensate for the loss of corneal power underwater. However, this makes their vision in air somewhat less sharp.
- **Dolphins and whales** have very limited corneal refraction even in air; their lenses provide most of the optical power in both media.
- **Fish** have nearly spherical lenses with very high refractive index gradients and essentially no corneal refraction at all.

The face mask solution exploits the simple principle that the eye evolved for air-based vision. By maintaining an air pocket in front of the eyes, the mask preserves the normal optical conditions. The mask window does introduce some additional effects:

1. **Magnification**: Objects appear about 33% larger and 25% closer underwater due to refraction at the water-mask interface, but this doesn't significantly impair vision quality.
2. **Reduced field of view**: The mask edges limit peripheral vision.
3. **Flat interface**: Unlike the curved cornea, the flat mask window doesn't contribute to focusing power, but it also doesn't introduce significant aberrations.

This question also explains why prescription swim goggles exist for people with refractive errors. While the mask or goggles restore the corneal refractive power, they don't correct pre-existing myopia or hyperopia. People who wear glasses on land still have the same refractive error underwater once the corneal power is restored by a mask.

From a physics perspective, this demonstrates that optical power arises from refractive index discontinuities, not from the absolute value of the refractive index. A lens or curved surface has power only insofar as it creates an index difference. This principle applies universally to all optical systems and explains why glasses work (glass-air interface), why oil immersion microscopy improves resolution (reducing the index step), and why anti-reflection coatings are designed with intermediate refractive indices (to minimize index discontinuities).

Problem Exercises

Unless otherwise stated, the lens-to-retina distance is 2.00 cm.

What is the power of the eye when viewing an object 50.0 cm away?

Show Solution

Strategy

To find the power of the eye when viewing an object at 50.0 cm, we use the thin lens equation in power form: $P = \frac{1}{d_o} + \frac{1}{d_i}$. The object distance is given as $d_o = 50.0$ cm, and the image distance (lens-to-retina distance) is $d_i = 2.00$ cm. We must convert both distances to meters since power is measured in diopters (D), where $1\text{ D} = 1\text{ m}^{-1}$.

Solution

Given values:

- Object distance: $d_o = 50.0\text{ cm} = 0.500\text{ m}$
- Lens-to-retina distance: $d_i = 2.00\text{ cm} = 0.0200\text{ m}$

Using the power equation:

$$P = \frac{1}{d_o} + \frac{1}{d_i}$$

Substituting the known values:

$$P = \frac{1}{0.500\text{ m}} + \frac{1}{0.0200\text{ m}}$$
$$P = 2.00\text{ D} + 50.0\text{ D}$$
$$P = 52.0\text{ D}$$

Discussion

The power of the eye when viewing an object 50.0 cm away is 52.0 D. This represents a 2.0 D increase over the relaxed eye power of 50.0 D (for distant vision at infinity). This 2.0 D of accommodation is well within the normal range—a typical young adult can accommodate up to 4.0 D. The calculation shows that reading at arm's length (approximately 50 cm) requires moderate accommodation. This distance is comfortable for extended reading, which is why many people naturally hold books and devices at about this distance. As people age and lose accommodation ability (presbyopia), this 50 cm distance may become the closest they can see clearly without reading glasses.

Calculate the power of the eye when viewing an object 3.00 m away.

Show Solution

Strategy

The power of the eye can be calculated using the thin lens equation $P = \frac{1}{d_o} + \frac{1}{d_i}$, where d_o is the object distance and d_i is the image distance (lens-to-retina distance). For clear vision, the image must form on the retina, so $d_i = 2.00$ cm. We need to convert all distances to meters since power is measured in diopters ($1\text{ D} = 1\text{ m}^{-1}$).

Solution

Given values:

- Object distance: $d_o = 3.00\text{ m}$
- Lens-to-retina distance: $d_i = 2.00\text{ cm} = 0.0200\text{ m}$

Using the power equation:

$$P = \frac{1}{d_o} + \frac{1}{d_i}$$

Substituting the known values:

$$P = \frac{1}{3.00\text{ m}} + \frac{1}{0.0200\text{ m}}$$
$$P = 0.333\text{ D} + 50.0\text{ D}$$
$$P = 50.3\text{ D}$$

Discussion

The power of the eye when viewing an object 3.00 m away is 50.3 D, which is only slightly greater than the 50.0 D required for distant vision (at infinity). This makes sense because 3.00 m is relatively far from the eye, so the eye is nearly completely relaxed. The small additional 0.3 D of power represents minimal accommodation compared to the 4.0 D increase needed for close vision at 25 cm. This demonstrates that objects beyond a few meters require essentially the same eye power as viewing objects at infinity, which is why distant objects from 3 m to infinity are all in clear focus for a relaxed eye.

(a) The print in many books averages 3.50 mm in height. How high is the image of the print on the retina when the book is held 30.0 cm from the eye?

(b) Compare the size of the print to the sizes of rods and cones in the fovea and discuss the possible details observable in the letters. (The eye-brain system can perform better because of interconnections and higher order image processing.)

Show Solution

Strategy

For part (a), we use the magnification formula $\frac{h_i}{h_o} = -\frac{d_i}{d_o}$ to find the image height h_i on the retina, where $h_o = 3.50$ mm is the object height, $d_o = 30.0$ cm is the object distance, and $d_i = 2.00$ cm is the lens-to-retina distance. For part (b), we compare the calculated image size to the typical dimensions of photoreceptors in the fovea (rods: ~2 μm diameter, cones: ~1.5-3 μm diameter in the fovea).

Solution

Given values:

- Object height: $h_o = 3.50\text{ mm} = 3.50 \times 10^{-3}\text{ m}$
- Object distance: $d_o = 30.0\text{ cm} = 0.300\text{ m}$
- Lens-to-retina distance: $d_i = 2.00\text{ cm} = 0.0200\text{ m}$

(a) Using the magnification equation:

$$\frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

Solving for the image height:

$$h_i = -h_o \times \frac{d_i}{d_o} = -(3.50 \times 10^{-3}\text{ m}) \times \frac{0.0200\text{ m}}{0.300\text{ m}}$$
$$h_i = -(3.50 \times 10^{-3}\text{ m}) \times 0.0667 = -2.33 \times 10^{-4}\text{ m}$$
$$h_i = -0.233\text{ mm} = -233\text{ μm}$$

The negative sign indicates an inverted image, which is expected for the eye.

(b) Comparison with photoreceptors:

- Image height: 233 μm
- Cone diameter in fovea: ~1.5-3 μm
- Rod diameter: ~2 μm

The image height (233 μm) is approximately **78-155 times larger** than the diameter of individual cones in the fovea. This means the image of a 3.50 mm letter spans about 78-155 cones, allowing excellent resolution of fine details within letters.

Discussion

Part (a): The retinal image of 3.50 mm print is 0.233 mm (233 μm) tall. Despite being a ~15-fold reduction in size, this is still quite large relative to the photoreceptors on the retina.

Part (b): Since each 3.50 mm letter produces an image spanning approximately 78-155 photoreceptors, the eye can easily distinguish individual letters and even fine details within letters such as serifs, the crossbar in an “A”, or the dot on an “i”. Each feature of a letter might span 10-20 or more cones, providing excellent resolution.

The fovea contains approximately 200,000 cones packed into a ~1.5 mm diameter area, giving exceptional acuity for central vision. The fact that the image of a single letter spans dozens of cones explains why we can read small print clearly. The eye-brain system performs even better than this simple calculation suggests due to lateral inhibition, edge detection, pattern recognition, and higher-order visual processing in the visual cortex. This is why we can sometimes read print even smaller than 3.50 mm under good lighting conditions.

Suppose a certain person’s visual acuity is such that he can see objects clearly that form an image $4.00\mu m$ high on his retina. What is the maximum distance at which he can read the 75.0 cm high letters on the side of an airplane?

Show Solution

Strategy

This problem involves finding the object distance d_o at which a letter of height $h_o = 75.0\ \text{cm}$ produces an image of height $h_i = 4.00\ \mu\text{m}$ on the retina. We use the magnification equation $\frac{h_i}{h_o} = -\frac{d_i}{d_o}$, where $d_i = 2.00$ cm is the lens-to-retina distance. Solving for d_o will give us the maximum distance at which the person can read the letters.

Solution

Given values:

- Object height: $h_o = 75.0\ \text{cm} = 0.750\ \text{m}$
- Image height: $h_i = 4.00\ \mu\text{m} = 4.00 \times 10^{-6}\ \text{m}$
- Lens-to-retina distance: $d_i = 2.00\ \text{cm} = 0.0200\ \text{m}$

Using the magnification equation:

$$\frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

Rearranging to solve for d_o:

$$d_o = -\frac{h_o \cdot d_i}{h_i}$$

Substituting the known values:

$$d_o = -\frac{(0.750\ \text{m})(0.0200\ \text{m})}{4.00\times10^{-6}\ \text{m}}$$
$$d_o = -\frac{0.0150\ \text{m}^2}{4.00\times10^{-6}\ \text{m}}$$
$$d_o = -3750\ \text{m}$$

Taking the magnitude (the negative sign indicates an inverted image, which is expected):

$$d_o = 3750\ \text{m} = 3.75\ \text{km}$$

Discussion

This person can read the 75.0 cm high letters from a maximum distance of 3.75 km (about 2.3 miles). This is a remarkably long distance and demonstrates the impressive resolution of the human visual system. The visual acuity of 4.00 μm corresponds to the ability to distinguish fine details approaching the size of individual photoreceptors on the retina. For reference, the diameter of a cone cell in the fovea is approximately 2-3 μm, so a 4.00 μm image is only slightly larger than a single cone.

This calculation assumes ideal viewing conditions (clear air, good contrast, proper lighting). In practice, atmospheric effects, contrast limitations, and other factors would likely reduce the actual reading distance. Nevertheless, this problem illustrates why large letters are used on aircraft and why they remain legible from considerable distances. The linear relationship between object size and viewing distance means that doubling the letter size would double the maximum readable distance.

People who do very detailed work close up, such as jewellers, often can see objects clearly at much closer distance than the normal 25 cm.

(a) What is the power of the eyes of a woman who can see an object clearly at a distance of only 8.00 cm?

(b) What is the size of an image of a 1.00 mm object, such as lettering inside a ring, held at this distance?

(c) What would the size of the image be if the object were held at the normal 25.0 cm distance?

Show Solution

Strategy

For part (a), we use $P = 1d_o + 1d_i$ with $d_o = 8.00$ cm and $d_i = 2.00$ cm to find the accommodated power. For parts (b) and (c), we use the magnification equation $h_i h_o = -d_i d_o$ to calculate the retinal image size for a 1.00 mm object at different viewing distances.

Solution

Given values:

- Near point: $d_o = 8.00 \text{ cm} = 0.0800 \text{ m}$
- Lens-to-retina distance: $d_i = 2.00 \text{ cm} = 0.0200 \text{ m}$
- Object size: $h_o = 1.00 \text{ mm} = 1.00 \times 10^{-3} \text{ m}$
- Normal near point: $d_{o,\text{normal}} = 25.0 \text{ cm} = 0.250 \text{ m}$

(a) Calculate the accommodated power:

$$P = 1d_o + 1d_i = 10.0800 \text{ m} + 10.0200 \text{ m}$$
$$P = 12.5 \text{ D} + 50.0 \text{ D} = 62.5 \text{ D}$$

(b) Calculate the image size at 8.00 cm:

$$h_i = -h_o \times d_i d_o = -(1.00 \times 10^{-3} \text{ m}) \times 0.0200 \text{ m} 0.0800 \text{ m}$$
$$h_i = -(1.00 \times 10^{-3} \text{ m}) \times 0.250 = -2.50 \times 10^{-4} \text{ m} = -0.250 \text{ mm}$$

(c) Calculate the image size at 25.0 cm:

$$h_i = -h_o \times d_i d_{o,\text{normal}} = -(1.00 \times 10^{-3} \text{ m}) \times 0.0200 \text{ m} 0.250 \text{ m}$$
$$h_i = -(1.00 \times 10^{-3} \text{ m}) \times 0.0800 = -8.00 \times 10^{-5} \text{ m} = -0.0800 \text{ mm}$$

Discussion

Part (a): The jeweller's accommodated power of 62.5 D represents a remarkable 12.5 D of accommodation (compared to 50.0 D for distant vision). This is more than three times the typical 4.0 D accommodation of a young adult. Such exceptional near vision is developed through years of close-up work and may also involve some degree of natural myopia that, while problematic for distant vision, is advantageous for detailed close work.

Part (b): At 8.00 cm, the 1.00 mm object produces a retinal image of 0.250 mm (250 μm). This is larger than typical print images and allows the jeweller to see extremely fine details like hallmarks, engravings, or imperfections in gemstones.

Part (c): At the normal 25.0 cm distance, the same object would produce only a 0.0800 mm (80 μm) image—less than one-third the size. The ratio of image sizes (250 μm / 80 μm = 3.125) equals the ratio of distances (25.0 cm / 8.00 cm = 3.125), as expected from the linear magnification relationship.

This problem illustrates why jewellers and others doing fine detail work benefit from exceptional near vision. The ability to work at 8.00 cm instead of 25.0 cm provides more than 3× magnification "for free" without requiring magnifying glasses. However, this extreme accommodation cannot be maintained for long periods without eye strain, which is why professional jewellers often supplement their natural vision with magnifying loupes or microscopes for extended detailed work.

Glossary

accommodation
: the ability of the eye to adjust its focal length is known as accommodation

presbyopia
: a condition in which the lens of the eye becomes progressively unable to focus on objects close to the viewer

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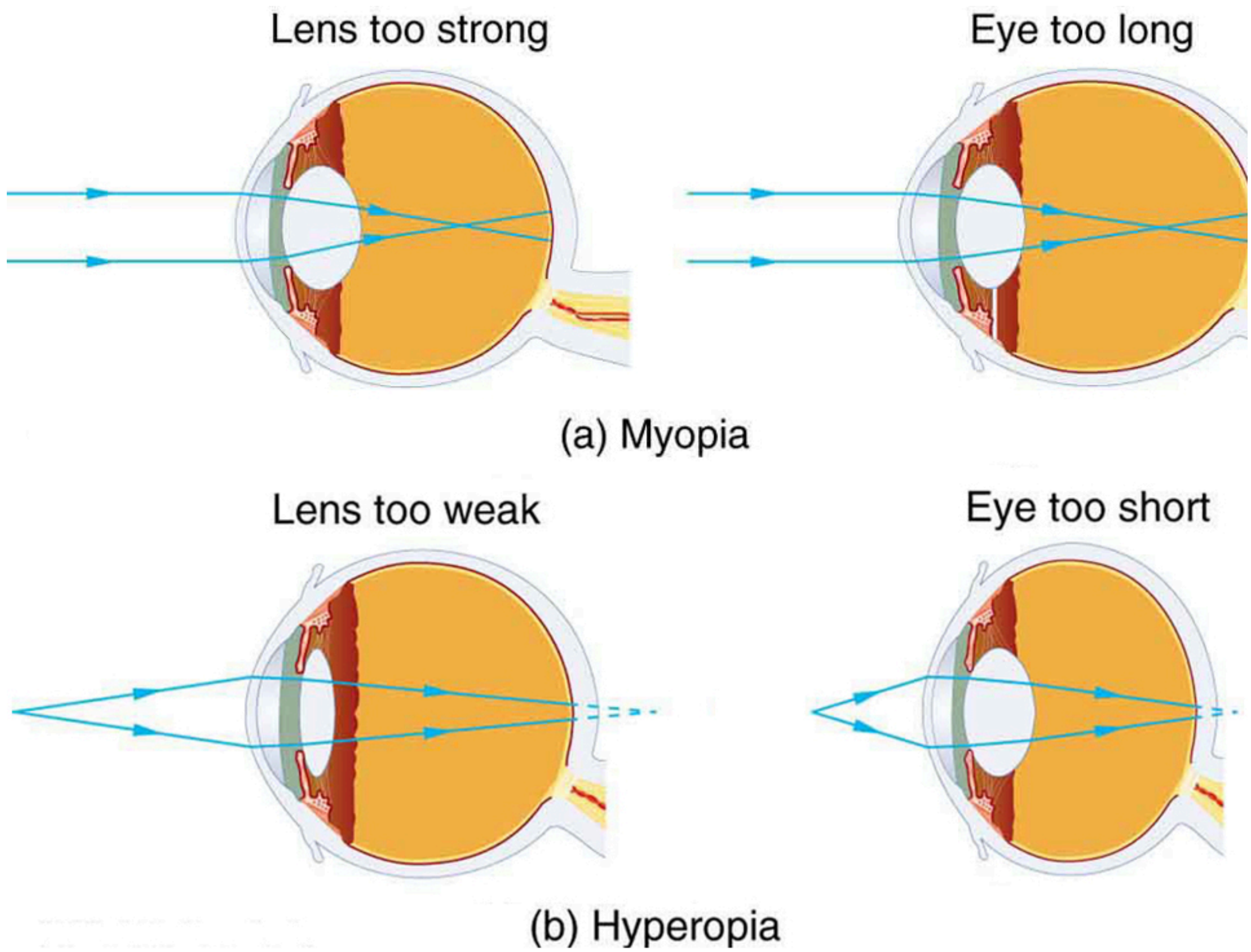
Vision Correction

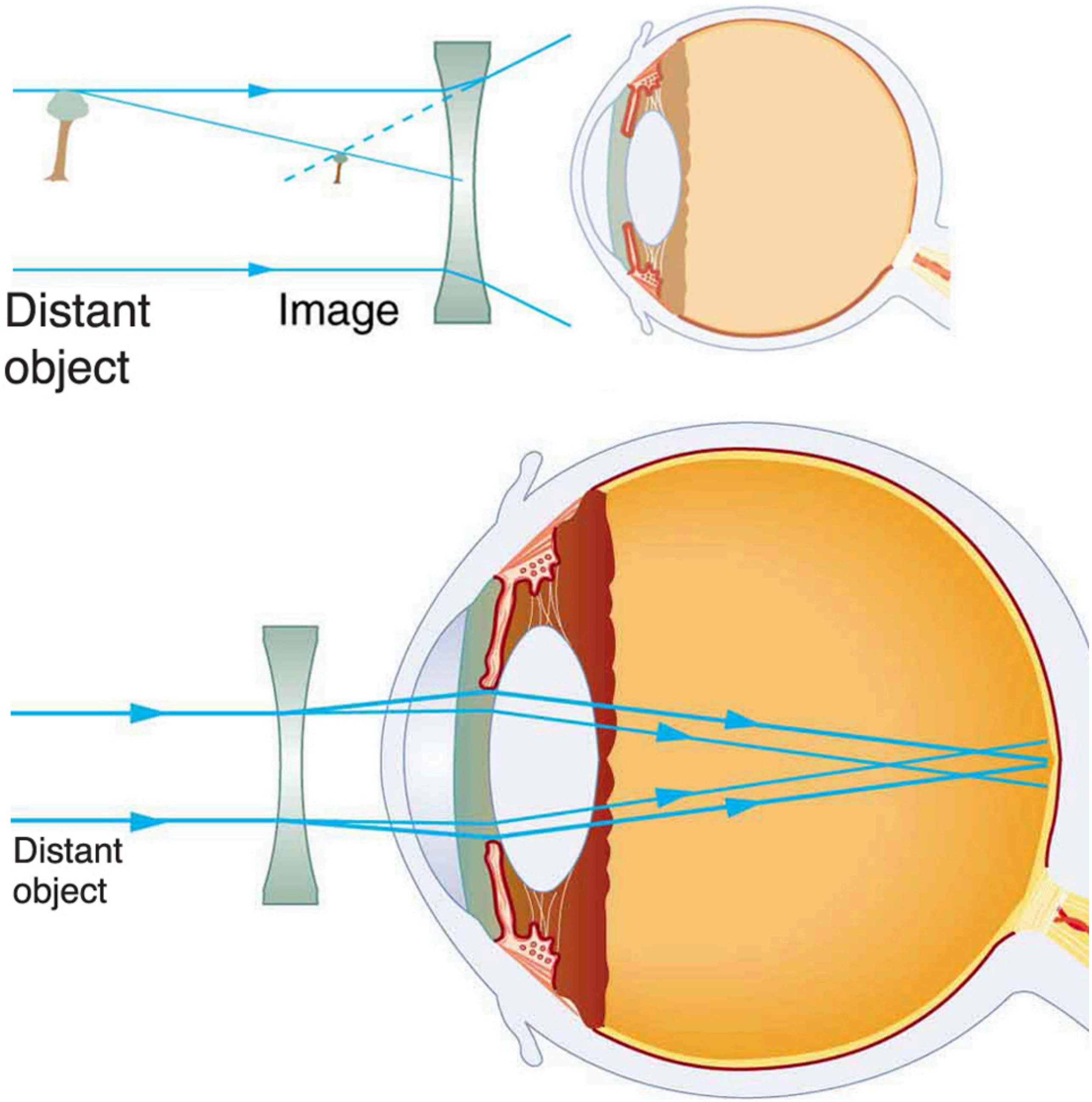
- Identify and discuss common vision defects.
- Explain nearsightedness and farsightedness corrections.
- Explain laser vision correction.

The need for some type of vision correction is very common. Common vision defects are easy to understand, and some are simple to correct. [Figure 1] illustrates two common vision defects. **Nearsightedness**, or **myopia**, is the inability to see distant objects clearly while close objects are clear. The eye overconverges the nearly parallel rays from a distant object, and the rays cross in front of the retina. More divergent rays from a close object are converged on the retina for a clear image. The distance to the farthest object that can be seen clearly is called the **far point** of the eye (normally infinity). **Farsightedness**, or **hyperopia**, is the inability to see close objects clearly while distant objects may be clear. A farsighted eye does not converge sufficient rays from a close object to make the rays meet on the retina. Less diverging rays from a distant object can be converged for a clear image. The distance to the closest object that can be seen clearly is called the **near point** of the eye (normally 25 cm).



(a) The nearsighted (myopic) eye converges rays from a distant object in front of the retina; thus, they are diverging when they strike the retina, producing a blurry image. This can be caused by the lens of the eye being too powerful or the length of the eye being too great. (b) The farsighted (hyperopic) eye is unable to converge the rays from a close object by the time they strike the retina, producing blurry close vision. This can be caused by insufficient power in the lens or by the eye being too short.

Since the nearsighted eye over converges light rays, the correction for nearsightedness is to place a diverging spectacle lens in front of the eye. This reduces the power of an eye that is too powerful. Another way of thinking about this is that a diverging spectacle lens produces a case 3 image, which is closer to the eye than the object (see [Figure 2]). To determine the spectacle power needed for correction, you must know the person's far point—that is, you must know the greatest distance at which the person can see clearly. Then the image produced by a spectacle lens must be at this distance or closer for the nearsighted person to be able to see it clearly. It is worth noting that wearing glasses does not change the eye in any way. The eyeglass lens is simply used to create an image of the object at a distance where the nearsighted person can see it clearly. Whereas someone not wearing glasses can see clearly _ objects_ that fall between their near point and their far point, someone wearing glasses can see *images* that fall between their near point and their far point.



Correction of nearsightedness requires a diverging lens that compensates for the overconvergence by the eye. The diverging lens produces an image closer to the eye than the object, so that the nearsighted person can see it clearly.

Correcting Nearsightedness

What power of spectacle lens is needed to correct the vision of a nearsighted person whose far point is 30.0 cm? Assume the spectacle (corrective) lens is held 1.50 cm away from the eye by eyeglass frames.

Strategy

You want this nearsighted person to be able to see very distant objects clearly. That means the spectacle lens must produce an image 30.0 cm from the eye for an object very far away. An image 30.0 cm from the eye will be 28.5 cm to the left of the spectacle lens (see [Figure 2]). Therefore, we must get $d_i = -28.5\text{cm}$ when $d_o \approx \infty$. The image distance is negative, because it is on the same side of the spectacle as the object.

Solution

Since d_i and d_o are known, the power of the spectacle lens can be found using $P = \frac{1}{d_o} + \frac{1}{d_i}$ as written earlier:

$$P=\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{\infty}+\frac{1}{-0.285\text{m}}.$$

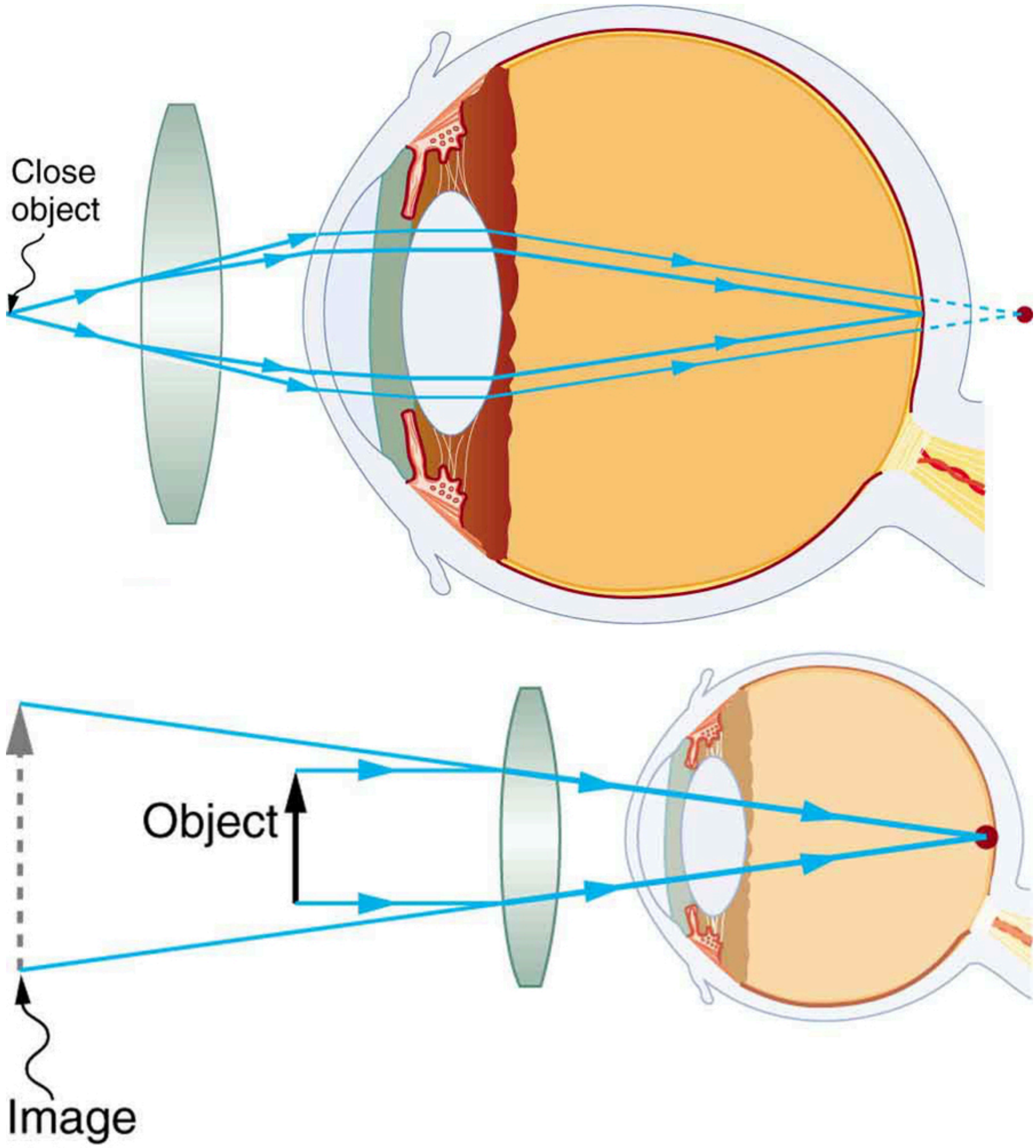
Since $1/\infty = 0$, we obtain:

$$P = 0 - 3.51/\text{m} = -3.51\text{D}.$$

Discussion

The negative power indicates a diverging (or concave) lens, as expected. The spectacle produces a case 3 image closer to the eye, where the person can see it. If you examine eyeglasses for nearsighted people, you will find the lenses are thinnest in the center. Additionally, if you examine a prescription for eyeglasses for nearsighted people, you will find that the prescribed power is negative and given in units of diopters.

Since the farsighted eye under converges light rays, the correction for farsightedness is to place a converging spectacle lens in front of the eye. This increases the power of an eye that is too weak. Another way of thinking about this is that a converging spectacle lens produces a case 2 image, which is farther from the eye than the object (see [Figure 3]). To determine the spectacle power needed for correction, you must know the person's near point—that is, you must know the smallest distance at which the person can see clearly. Then the image produced by a spectacle lens must be at this distance or farther for the farsighted person to be able to see it clearly.



Correction of farsightedness uses a converging lens that compensates for the under convergence by the eye. The converging lens produces an image farther from the eye than the object, so that the farsighted person can see it clearly.

Correcting Farsightedness

What power of spectacle lens is needed to allow a farsighted person, whose near point is 1.00 m, to see an object clearly that is 25.0 cm away? Assume the spectacle (corrective) lens is held 1.50 cm away from the eye by eyeglass frames.

Strategy

When an object is held 25.0 cm from the person's eyes, the spectacle lens must produce an image 1.00 m away (the near point). An image 1.00 m from the eye will be 98.5 cm to the left of the spectacle lens because the spectacle lens is 1.50 cm from the eye (see [Figure 3]). Therefore, $d_\text{i} = -98.5\text{cm}$. The image distance is negative, because it is on the same side of the spectacle as the object. The object is 23.5 cm to the left of the spectacle, so that $d_\text{o} = 23.5\text{cm}$.

Solution

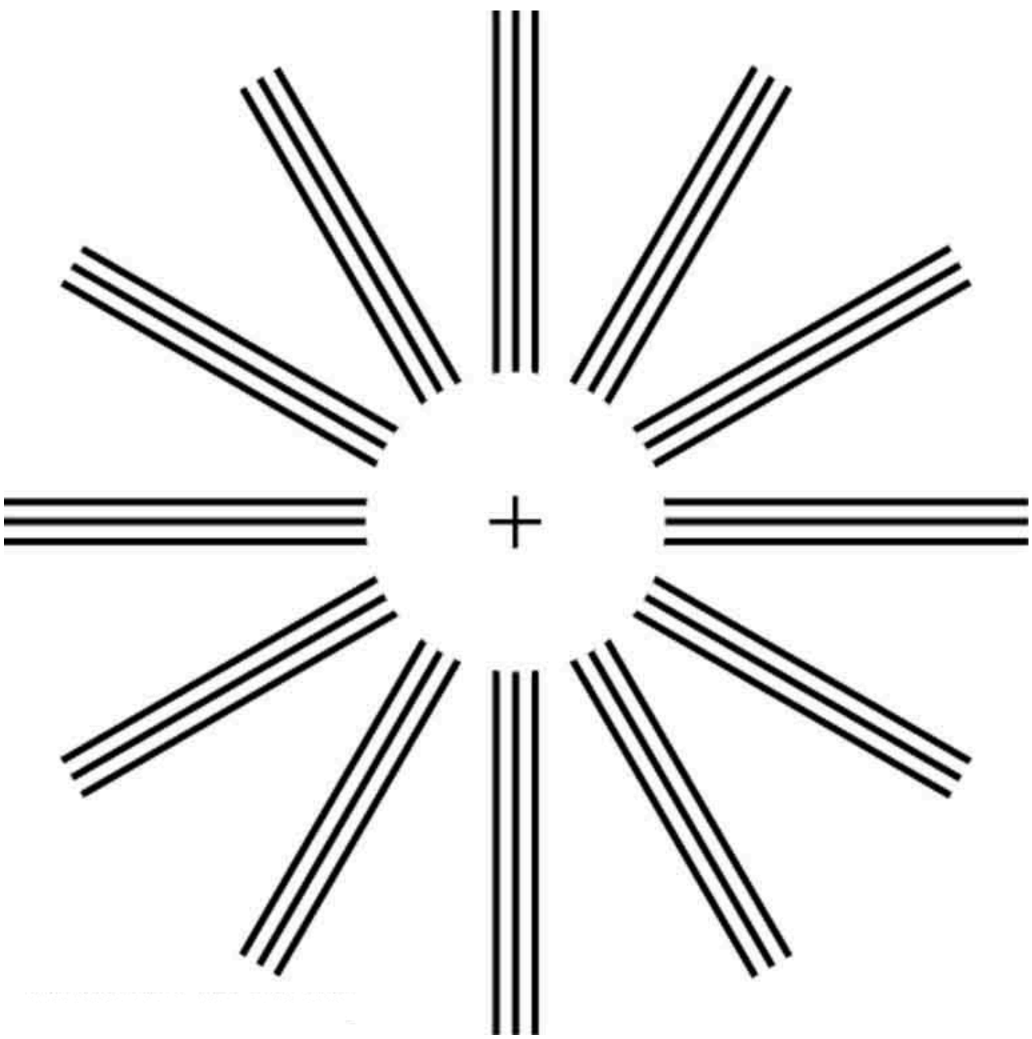
Since d_i and d_o are known, the power of the spectacle lens can be found using $P = 1d_\text{o} + 1d_\text{i}$:

$$P = 1d_o+1d_\text{i}=10.235\text{m}+1-0.985\text{m} = 4.26D-1.02\text{D}=3.24\text{D}.$$

Discussion

The positive power indicates a converging (convex) lens, as expected. The convex spectacle produces a case 2 image farther from the eye, where the person can see it. If you examine eyeglasses of farsighted people, you will find the lenses to be thickest in the center. In addition, a prescription of eyeglasses for farsighted people has a prescribed power that is positive.

Another common vision defect is **astigmatism**, an unevenness or asymmetry in the focus of the eye. For example, rays passing through a vertical region of the eye may focus closer than rays passing through a horizontal region, resulting in the image appearing elongated. This is mostly due to irregularities in the shape of the cornea but can also be due to lens irregularities or unevenness in the retina. Because of these irregularities, different parts of the lens system produce images at different locations. The eye-brain system can compensate for some of these irregularities, but they generally manifest themselves as less distinct vision or sharper images along certain axes. [Figure 4] shows a chart used to detect astigmatism. Astigmatism can be at least partially corrected with a spectacle having the opposite irregularity of the eye. If an eyeglass prescription has a cylindrical correction, it is there to correct astigmatism. The normal corrections for short- or farsightedness are spherical corrections, uniform along all axes.



This chart can detect astigmatism, unevenness in the focus of the eye. Check each of your eyes separately by looking at the center cross (without spectacles if you wear them). If lines along some axes appear darker or clearer than others, you have an astigmatism.

Contact lenses have advantages over glasses beyond their cosmetic aspects. One problem with glasses is that as the eye moves, it is not at a fixed distance from the spectacle lens. Contacts rest on and move with the eye, eliminating this problem. Because contacts cover a significant portion of the cornea, they provide superior peripheral vision compared with eyeglasses. Contacts also correct some corneal astigmatism caused by surface irregularities. The tear layer between the smooth contact and the cornea fills in the irregularities. Since the index of refraction of the tear layer and the cornea are very similar, you now have a regular optical surface in place of an irregular one. If the curvature of a contact lens is not the same as the cornea (as may be necessary with some individuals to obtain a comfortable fit), the tear layer between the contact and cornea acts as a lens. If the tear layer is thinner in the center than at the edges, it has a negative power, for example. Skilled optometrists will adjust the power of the contact to compensate.

Other advances in vision correction demonstrate the interconnectedness and value of scientific research. In the 1980s, Donna Strickland and Gérard Mourou worked on ways to make small but powerful lasers. Up until that time, powerful lasers had to be quite large in order to function properly. Essentially, the intensity of the beam itself would modify the instrument's ability to function and create too much heat to be practical. Strickland and Mourou used ultrashort laser pulses passed over a grating that modified the beam but retained its power. Chirped pulse amplification, as it became known, has been used to develop most of the highest-powered lasers in the world, but also some of the smallest and most common. Decades after their initial discovery, Strickland and Mourou were awarded the Nobel Prize for Physics (with Strickland becoming the third woman to receive the award) partly due to CPA's pivotal role in the increasingly common practice of laser vision correction—an application neither planned during their initial research.

Laser vision correction has progressed rapidly in the last few years. It is the latest and by far the most successful in a series of procedures that correct vision by reshaping the cornea. As noted at the beginning of this section, the cornea accounts for about two-thirds of the power of the eye. Thus, small adjustments of its curvature have the same effect as putting a lens in front of the eye. To a reasonable approximation, the power of multiple lenses placed close together equals the sum of their powers. For example, a concave spectacle lens (for nearsightedness) having $P = -3.00\text{D}$ has the same effect on vision as reducing the power of the eye itself by 3.00 D. So to correct the eye for nearsightedness, the cornea is flattened to reduce its power. Similarly, to correct for farsightedness, the curvature of the cornea is enhanced to increase the power of the eye—the same effect as the positive power spectacle lens used for farsightedness. Laser vision correction uses high intensity electromagnetic radiation to ablate (to remove material from the surface) and reshape the corneal surfaces.

Today, the most commonly used laser vision correction procedure is *Laser in situ Keratomileusis (LASIK)*. The top layer of the cornea is surgically peeled back and the underlying tissue ablated by multiple bursts of finely controlled ultraviolet radiation produced by an excimer laser. Lasers are used because they not only produce well-focused intense light, but they also emit very pure wavelength electromagnetic radiation that can be controlled more accurately than mixed wavelength light. The 193 nm wavelength UV commonly used is extremely and strongly absorbed by corneal tissue, allowing precise evaporation of very thin layers. A computer controlled program applies more bursts, usually at a rate of 10 per second, to the areas that require deeper removal. Typically a spot less than 1 mm in diameter and about $0.3 \mu\text{m}$ in thickness is removed by each burst. Nearsightedness, farsightedness, and astigmatism can be corrected with an accuracy that produces normal distant vision in more than 90% of the patients, in many cases right away. The corneal flap is replaced; healing takes place rapidly and is nearly painless. More than 1 million Americans per year undergo LASIK (see [Figure 5]).

Laser vision correction is being performed using the LASIK procedure. Reshaping of the cornea by laser ablation is based on a careful assessment of the patient's vision and is computer controlled. The upper corneal layer is temporarily peeled back and minimally disturbed in LASIK, providing for more rapid and less painful healing of the less sensitive tissues below. (credit: U.S. Navy photo by Mass Communication Specialist 1st Class Brien Aho)

Section Summary

- Nearsightedness, or myopia, is the inability to see distant objects and is corrected with a diverging lens to reduce power.
- Farsightedness, or hyperopia, is the inability to see close objects and is corrected with a converging lens to increase power.
- In myopia and hyperopia, the corrective lenses produce images at a distance that the person can see clearly—the far point and near point, respectively.

Conceptual Questions

It has become common to replace the cataract-clouded lens of the eye with an internal lens. This intraocular lens can be chosen so that the person has perfect distant vision. Will the person be able to read without glasses? If the person was nearsighted, is the power of the intraocular lens greater or less than the removed lens?

Show Solution

Strategy

To address this question, we need to understand: (1) how natural lenses accommodate to focus at different distances, (2) what happens when a fixed-power intraocular lens (IOL) replaces the natural lens, and (3) how the original refractive error (nearsightedness) affects IOL power selection.

Solution

Part 1: Will the person be able to read without glasses?

No, the person will almost certainly need reading glasses for the following reasons:

The natural lens of the eye has a remarkable property called **accommodation**—the ability to change its power by changing shape. An **intraocular lens (IOL) is rigid and has fixed power**—it cannot accommodate. When an IOL is selected for perfect distant vision, the person cannot focus on near objects and will need reading glasses.

Part 2: If the person was nearsighted, is the IOL power greater or less than the removed lens?

The IOL power will be LESS than the removed lens power. A nearsighted person has an eye that is too powerful for its length. To correct to perfect distant vision with an IOL, the IOL must provide less power than the natural lens had, thereby reducing the eye's excessive total power.

Discussion

The natural lens can change its power by up to ~14 D in young people through accommodation. A fixed IOL cannot do this, so reading glasses (+2.0 to +3.0 D typically) are needed for near work. Modern alternatives include multifocal IOLs or monovision correction. For myopic eyes, the IOL is weaker than the removed lens because the correction aims to reduce the eye's excessive power to achieve emmetropia (normal vision).

If the cornea is to be reshaped (this can be done surgically or with contact lenses) to correct myopia, should its curvature be made greater or smaller? Explain. Also explain how hyperopia can be corrected.

Show Solution

Strategy

To answer this question, we need to understand the relationship between corneal curvature and optical power, and how changing curvature affects the eye's focusing ability. The lensmaker's equation shows that power is related to curvature—greater curvature means greater power and vice versa.

Solution

Correcting Myopia (Nearsightedness):

For myopia correction, **the corneal curvature should be made SMALLER** (flatter). Here's why:

The myopic problem:

- Myopic eyes are too powerful—they over-converge light
- Parallel rays from distant objects focus in front of the retina
- The excess power needs to be reduced

The solution:

- **Decrease corneal curvature** (flatten the cornea)
- This **reduces corneal power**
- The reduced power allows parallel rays to focus farther back, ideally on the retina

Mathematical relationship:

The power of a curved surface is approximately:

$$P = \frac{n_2 - n_1}{R}$$

where R is the radius of curvature, n_1 is the refractive index of air (~1.00), and n_2 is the refractive index of the cornea (~1.38).

- **Larger radius** R (smaller curvature, flatter) → **smaller power** P
- **Smaller radius** R (greater curvature, more curved) → **larger power** P

LASIK for myopia:

- Removes tissue from the **central cornea**
- Makes the center **flatter** (increases radius of curvature)
- Reduces corneal power by the needed amount
- Typical corrections: -1 D to -10 D (sometimes higher)

Correcting Hyperopia (Farsightedness):

For hyperopia correction, **the corneal curvature should be made GREATER** (steeper). Here's why:

The hyperopic problem:

- Hyperopic eyes are too weak—they under-converge light
- Rays from near objects cannot be brought to focus on the retina
- Additional power is needed

The solution:

- **Increase corneal curvature** (steepen the cornea)
- This **increases corneal power**
- The increased power allows the eye to focus near objects on the retina

LASIK for hyperopia:

- Removes tissue from the **peripheral cornea** (in a ring around the center)
- Makes the center **steeper** (decreases radius of curvature centrally)
- Increases corneal power by the needed amount
- Typical corrections: +1 D to +4 D
- More challenging than myopic LASIK due to biomechanical limitations

Physical Analogy:

Think of the cornea as a lens:

- **Flatter lens** (like a window pane): weak power, long focal length
- **More curved lens** (like a marble): strong power, short focal length

Discussion

The cornea provides approximately 2/3 of the eye's total optical power (~43 D out of ~50 D total). Small changes in corneal curvature can significantly affect focusing:

- **1 D change** requires approximately 6 μm depth of tissue removal centrally
- **Myopia correction**: Central ablation creates a flatter optical zone
- **Hyperopia correction**: Peripheral ablation creates relatively steeper central zone

Contact lens reshaping (Orthokeratology):

An alternative to surgery uses specially designed rigid contact lenses worn overnight:

- For myopia: Lenses gently flatten the central cornea
- For hyperopia: Lenses steepen the central cornea
- Effects are temporary and reversible
- Must wear lenses nightly to maintain correction

Practical considerations:

Myopic LASIK advantages:

- More tissue available for removal
- More predictable outcomes
- Wider range of corrections possible
- Central ablation is mechanically stable

Hyperopic LASIK challenges:

- Peripheral ablation can cause regression (cornea tends to return toward original shape)
- Smaller treatment zone
- More limited correction range
- More complex biomechanical response

Physical limits:

The cornea cannot be reshaped indefinitely:

- Must maintain minimum thickness (~250 μm typically) for structural integrity
- Excessive flattening (myopia overcorrection) can cause irregular astigmatism
- Excessive steepening (hyperopia overcorrection) is biomechanically unstable
- These limits constrain the maximum correction achievable

The key principle: **Curvature and power are directly related**—decrease curvature to decrease power (myopia correction), increase curvature to increase power (hyperopia correction). This fundamental relationship applies whether the reshaping is achieved surgically (LASIK, PRK), through implants (intracorneal rings), or non-surgically (orthokeratology lenses).

If there is a fixed percent uncertainty in LASIK reshaping of the cornea, why would you expect those people with the greatest correction to have a poorer chance of normal distant vision after the procedure?

Show Solution

Strategy

This question involves understanding error propagation and how a fixed percentage uncertainty translates to larger absolute errors when applied to larger corrections. We need to consider how measurement and ablation uncertainties affect final refractive outcomes.

Solution

People requiring the greatest correction have a poorer chance of achieving normal distant vision because **a fixed percent uncertainty produces larger absolute errors for larger corrections**.

Mathematical Analysis:

Suppose LASIK has a **fixed ±2% uncertainty** in the amount of tissue removed (representative of real-world precision).

Case 1: Small correction (-2.0 D myopia)

- Target correction: -2.0 D
- Uncertainty: ±2% of 2.0 D = ±0.04 D
- Possible outcome range: -1.96 D to -2.04 D
- **Absolute error: ±0.04 D** (excellent outcome—within normal vision range)

Case 2: Large correction (-10.0 D myopia)

- Target correction: -10.0 D
- Uncertainty: ±2% of 10.0 D = ±0.20 D
- Possible outcome range: -9.80 D to -10.20 D
- **Absolute error: ±0.20 D** (noticeable residual refractive error)

The absolute error is 5 times larger for the -10 D correction compared to the -2 D correction, even though the percentage uncertainty is identical.

Physical Interpretation:

$$\text{Absolute error} = (\text{Percent uncertainty}) \times (\text{Target correction})$$

As the target correction increases, the absolute error increases proportionally. Since optical performance depends on **absolute refractive error** (not percent error), larger corrections face larger absolute deviations from perfect vision.

What constitutes "normal distant vision"?

- Clinically, normal vision is typically defined as within **±0.50 D** of target
- Many consider **±0.25 D** as excellent results
- Beyond ±0.50 D, patients often notice blur and may need corrective lenses

With ±2% uncertainty:

- **-2 D correction:** ±0.04 D error → always achieves normal vision
- **-5 D correction:** ±0.10 D error → usually achieves normal vision
- **-10 D correction:** ±0.20 D error → sometimes achieves normal vision
- **-15 D correction:** ±0.30 D error → often has residual error

Sources of LASIK Uncertainty:

1. **Measurement errors:**
    - Pre-operative refraction measurement: ±0.25 D typical
    - Corneal topography: small measurement variations
    - Eye length measurement: affects power calculations
2. **Ablation uncertainties:**
    - Laser calibration: ±1-2% variation in ablation depth
    - Corneal hydration state: affects tissue removal rate
    - Environmental factors: humidity, temperature affect ablation
3. **Biomechanical response:**
    - Wound healing variability increases with ablation depth
    - Deeper ablations (large corrections) → more healing response
    - Healing can cause regression (return toward original error)
4. **Tissue removal accumulation:**
    - For -10 D: ~120 μm of tissue removed centrally
    - For -2 D: ~24 μm of tissue removed centrally
    - Larger removals accumulate more ablation uncertainty

Discussion

This problem illustrates a fundamental principle in precision engineering and medicine: **percentage errors become increasingly significant in absolute terms as the magnitude of the operation increases**.

Real-world LASIK statistics support this:

Published studies show:

- **Low myopia** (-1 to -3 D): >95% achieve 20/20 vision or better
- **Moderate myopia** (-3 to -6 D): ~85-90% achieve 20/20 or better

- **High myopia** (-6 to -10 D): ~70-80% achieve 20/20 or better
- **Extreme myopia** (>-10 D): ~50-60% achieve 20/20 or better

Additional factors for high corrections:

1. **Biomechanical instability:** Deep ablations weaken corneal structure
2. **Regression:** Higher corrections show more tendency to regress over time
3. **Irregular astigmatism:** More common with deep ablations
4. **Reduced tissue reserve:** Less tissue available for enhancements
5. **Peripheral visual quality:** Greater impact on contrast sensitivity and night vision

Clinical implications:

For high myopia (>-8 D), alternatives may be preferable:

- **Phakic IOLs:** Implantable contact lens (doesn't remove corneal tissue)
- **Refractive lens exchange:** Replace natural lens with IOL
- **Two-stage LASIK:** Initial partial correction, later enhancement
- **Conservative target:** Aim for slight under-correction to stay within safe limits

Analogy:

Consider measuring and cutting wood:

- If your saw has ±2% accuracy:
  - Cutting a 10 cm board: ±2 mm error (acceptable for many uses)
  - Cutting a 100 cm board: ±20 mm error (significant and noticeable)
- The percentage is the same, but the absolute error scales with the size

Similarly in LASIK:

- Same percentage precision
- Larger corrections = larger absolute errors
- Larger absolute errors = lower probability of perfect outcome

This explains why LASIK candidates with high refractive errors are counseled about:

- Lower probability of achieving 20/20 uncorrected vision
- Higher chance of needing glasses for some activities
- Greater possibility of requiring enhancement procedures
- Importance of having realistic expectations

The relationship between percent uncertainty and absolute error is a fundamental constraint in any precision process, from manufacturing to medicine. Understanding this helps patients make informed decisions about refractive surgery based on their specific correction needs.

A person with presbyopia has lost some or all of the ability to accommodate the power of the eye. If such a person's distant vision is corrected with LASIK, will she still need reading glasses? Explain.

Show Solution

Strategy

To answer this question, we must distinguish between two separate optical functions of the eye: (1) the static refractive power that determines distant vision, and (2) the dynamic accommodation ability that allows near vision. LASIK corrects only one of these.

Solution

Yes, a person with presbyopia will still need reading glasses after LASIK correction for distant vision.

Here's why:

Understanding Presbyopia:

Presbyopia is the **loss of accommodation**—the eye's ability to change its optical power to focus on near objects. It results from:

- Hardening of the crystalline lens with age
- Weakening of the ciliary muscles
- Reduced elasticity of the lens capsule
- Typically begins around age 40-45 and progresses with age

What LASIK Does:

LASIK reshapes the cornea to correct the eye's **static refractive error**:

- For myopia: flattens the cornea to reduce power
- For hyperopia: steepens the cornea to increase power
- For astigmatism: reshapes to create uniform curvature

What LASIK Does NOT Do:

LASIK **does not restore accommodation**. It cannot:

- Make the crystalline lens more flexible
- Strengthen the ciliary muscles
- Change the eye's ability to alter its power dynamically
- Provide different powers for different viewing distances

The Two-Part Vision System:

Think of vision as requiring two independent capabilities:

1. **Static refraction** (corrected by LASIK):
    - The fixed optical power of the eye when relaxed
    - Determines whether distant objects focus on the retina
    - Problems: myopia, hyperopia, astigmatism
2. **Dynamic accommodation** (NOT corrected by LASIK):
    - The ability to increase power for near objects
    - Provides additional +2 to +3 D for reading
    - Problem: presbyopia (loss of this ability)

Numerical Example:

Consider a 50-year-old with -3.0 D myopia and presbyopia:

Before LASIK:

- Relaxed eye power: 53 D (too strong for distance)
- Accommodation ability: ~1 D (reduced due to presbyopia)
- **Distance vision:** Blurry (needs -3 D glasses)
- **Near vision at 40 cm:** Requires +2.5 D additional power
    - Can accommodate only +1 D
    - **Still needs +1.5 D reading glasses** (even with myopia helping near vision!)

After LASIK (correcting the -3 D myopia):

- Relaxed eye power: ~50 D (corrected for distance)
- Accommodation ability: ~1 D (unchanged—presbyopia persists)
- **Distance vision:** Clear (no glasses needed!)
- **Near vision at 40 cm:** Requires +2.5 D additional power
    - Can accommodate only +1 D
    - **Needs +2.5 D reading glasses** (actually needs MORE correction than before)

Why Reading Glasses Are MORE Necessary After LASIK:

Paradoxically, correcting myopia can make presbyopia more noticeable:

- **Myopic people** naturally have good near vision (even without accommodation)
- Many remove their distance glasses to read
- After LASIK corrects distant vision, this "built-in reading correction" is lost
- The full presbyopic deficit becomes apparent

Solutions for Presbyopia with LASIK:

Several approaches address both issues:

1. Monovision LASIK:

- One eye corrected for distance (typically dominant eye)
- Other eye left slightly myopic (-1.25 to -1.75 D) for near vision
- Brain adapts to use appropriate eye for each distance
- About 70-80% of people adapt successfully
- Trade-off: reduced depth perception, slightly compromised vision at each distance

2. Blended Vision (Modified Monovision):

- Dominant eye: fully corrected for distance
- Non-dominant eye: corrected for intermediate distance
- Creates overlap in functional range
- Better depth perception than traditional monovision

3. Multifocal LASIK:

- Experimental technique creating different zones on cornea
- Some zones for distance, some for near
- Similar concept to multifocal contact lenses or IOLs
- Mixed success; can cause halos and reduced contrast

4. PresbyLASIK:

- Creates aspherical corneal shape with increased depth of focus
- Non-dominant eye treated to enhance near vision
- Still evolving technology

5. Combine LASIK with Reading Glasses:

- LASIK corrects distance vision
- Use reading glasses (+1.50 to +2.50 D) for near tasks
- Most straightforward and reliable approach
- No compromise in visual quality

Discussion

It's crucial to understand that **refractive error and accommodation are independent optical properties:**

Refractive Error (Static):

- Due to mismatch between eye's power and length
- Present at all ages
- Correctable with glasses, contacts, or refractive surgery
- LASIK addresses this problem

Presbyopia (Dynamic):

- Due to loss of lens flexibility
- Age-related, universal (everyone gets it eventually)
- Currently cannot be permanently corrected by reshaping cornea
- LASIK does NOT address this problem

Patient Education Importance:

Many patients mistakenly believe LASIK will eliminate all need for glasses. It's essential they understand:

- LASIK corrects distance vision (static refraction)
- Presbyopia is a separate condition affecting near vision
- Reading glasses will still be needed after age 40-45
- LASIK doesn't accelerate or prevent presbyopia

The Biology Constraint:

Presbyopia is a biological aging process affecting the lens, not an optical problem with the cornea:

- The lens continues to grow throughout life, adding layers like an onion
- This makes it increasingly rigid
- The ciliary muscles also weaken with age
- Reshaping the cornea cannot reverse these biological changes

Future Possibilities:

Research is ongoing into:

- **Accommodating corneal inlays:** Small implants that change corneal shape with accommodation effort
- **Lens replacement:** Removing the presbyopic lens and replacing with an accommodating IOL
- **Pharmacological approaches:** Drugs to soften the lens
- **Laser lens treatments:** Using femtosecond lasers to soften the lens

Currently, none of these has proven as reliable as simply using reading glasses or multifocal lenses for near tasks.

Conclusion:

A person with presbyopia who undergoes LASIK to correct distant vision will definitely still need reading glasses because LASIK corrects the eye's static refractive error (enabling clear distant vision) but does not restore the dynamic accommodation ability (necessary for near vision). The presbyopia—the underlying loss of accommodation—remains unchanged by corneal reshaping. This is an important distinction for patient counseling and managing expectations for refractive surgery in the presbyopic population.

Problem Exercises

What is the far point of a person whose eyes have a relaxed power of 50.5 D?

Show Solution

Strategy

The far point is the maximum distance at which a person can see clearly with relaxed (un-accommodated) eyes. For a relaxed eye, we use $P = \frac{1}{d_o} + \frac{1}{d_i}$ where $P = 50.5$ D is the relaxed power and $d_i = 2.00$ cm is the lens-to-retina distance. Solving for d_o gives the far point.

Solution

Given values:

- Relaxed power: $P = 50.5\ \text{D}$
- Lens-to-retina distance: $d_i = 2.00\ \text{cm} = 0.0200\ \text{m}$

Using the power equation:

$$P=\frac{1}{d_o}+\frac{1}{d_i}$$

Rearranging to solve for d_o:

$$\frac{1}{d_o}=P-\frac{1}{d_i}=50.5\ \text{D}-\frac{1}{0.0200\ \text{m}}$$
$$\frac{1}{d_o}=50.5\ \text{m}^{-1}-50.0\ \text{m}^{-1}=0.5\ \text{m}^{-1}$$
$$d_o=\frac{1}{0.5\ \text{m}^{-1}}=2.00\ \text{m}$$

Discussion

The person’s far point is 2.00 m. This indicates mild myopia (nearsightedness) since the relaxed power of 50.5 D is slightly greater than the normal 50.0 D. The excess 0.5 D causes the eye to focus light from distant objects in front of the retina, resulting in blurry distant vision.

This person can see clearly only up to 2.00 meters away when their eyes are relaxed. Beyond this distance, objects appear blurry. For normal distant vision (far point at infinity), this person would need a -0.5 D corrective lens to reduce the eye’s excessive power. While 2.00 m myopia is relatively mild, it would still cause difficulties with activities like driving, watching movies in theaters, or reading a classroom whiteboard from the back of a room. Many people with this level of myopia choose to wear glasses for distance activities but can function without them for everyday tasks.

What is the near point of a person whose eyes have an accommodated power of 53.5 D?

Show Solution

Strategy

The near point is the closest distance at which a person can see clearly with fully accommodated vision. Using the thin lens equation $P = \frac{1}{d_o} + \frac{1}{d_i}$, where $P = 53.5$ D is the accommodated power and $d_i = 2.00$ cm (lens-to-retina distance), we can solve for the object distance d_o, which is the near point.

Solution

Given values:

- Accommodated power: $P = 53.5\ \text{D}$
- Lens-to-retina distance: $d_i = 2.00\ \text{cm} = 0.0200\ \text{m}$

Using the power equation:

$$P=\frac{1}{d_o}+\frac{1}{d_i}$$

Rearranging to solve for d_o:

$$\frac{1}{d_o}=P-\frac{1}{d_i}$$
$$\frac{1}{d_o}=53.5\ \text{D}-\frac{1}{0.0200\ \text{m}}$$
$$\frac{1}{d_o}=53.5\ \text{m}^{-1}-50.0\ \text{m}^{-1}=3.5\ \text{m}^{-1}$$
$$d_o=\frac{1}{3.5\ \text{m}^{-1}}=0.286\ \text{m}=28.6\ \text{cm}$$

Discussion

The near point of this person is 28.6 cm, which is slightly farther than the normal near point of 25 cm. This indicates a mild degree of presbyopia or early farsightedness. The accommodated power of 53.5 D represents only a 3.5 D increase over the relaxed power of 50.0 D (for distant vision), compared to the normal 4.0 D accommodation ability. This 12.5% reduction in accommodation power is typical for someone in their 40s or early 50s. While this person can still read without glasses, they may find it more comfortable to hold reading material slightly farther away than the standard 25 cm reading distance.

(a) A laser vision correction reshaping the cornea of a myopic patient reduces the power of his eye by 9.00 D, with a ± 5.0 uncertainty in the final correction. What is the range of diopters for spectacle lenses that this person might need after LASIK procedure? (b) Was the person nearsighted or farsighted before the procedure? How do you know?

Show Solution

Strategy

For part (a), a ±5.0% uncertainty in the 9.00 D correction means the actual correction could vary by ±5.0% of 9.00 D. If the target is perfect vision (no glasses needed), the uncertainty determines the range of corrective lenses potentially needed. For part (b), we identify the vision defect based on whether power was reduced or increased.

Solution

Given values:

- Power reduction: $\Delta P = 9.00\text{ D}$
- Uncertainty: $\pm 5.0\%$

(a) Calculate the uncertainty in diopters:

$$\text{Uncertainty} = 0.050 \times 9.00\text{ D} = 0.45\text{ D}$$

The range of possible corrections is $9.00\text{ D} \pm 0.45\text{ D}$, meaning:

- Best case: $9.00 + 0.45 = 9.45\text{ D}$ reduction (slight over-correction)
- Worst case: $9.00 - 0.45 = 8.55\text{ D}$ reduction (slight under-correction)

If the goal was zero correction needed:

- Over-correction by 0.45 D would require $+0.45\text{ D}$ (convex) glasses
- Under-correction by 0.45 D would require -0.45 D (concave) glasses

Therefore, the range is $\pm 0.45\text{ D}$.

(b) The person was **nearsighted (myopic)** before the procedure. We know this because:

1. The problem states the patient is myopic
2. The power was *reduced* by 9.00 D
3. Myopia results from excessive eye power, so reducing power corrects nearsightedness

Discussion

Part (a): A ±0.45 D uncertainty range is relatively small and clinically acceptable. Most people with ±0.45 D residual error would have quite functional vision. For comparison:

- +0.45 D: Slight difficulty with close reading (mild hyperopia)
- -0.45 D: Slight difficulty with distant objects (mild myopia)

Many patients in this range choose not to wear glasses for everyday activities, though they might use them for specific tasks like night driving or extended reading.

Part (b): The 9.00 D correction is substantial, indicating significant myopia before surgery. This person likely had a far point of about 11 cm (calculation: $d_o = 1/(9.00 + 50.0 - 50.0) \approx 0.11\text{ m}$), meaning they could see clearly only up to about 11 cm without glasses—severely limiting their activities without correction. The LASIK procedure dramatically improved their quality of life by enabling normal distant vision, even with the small uncertainty.

This problem illustrates why LASIK uncertainty specifications are important. A 5% uncertainty might seem small, but for large corrections like 9.00 D, it translates to nearly half a diopter of potential residual error. This is why careful pre-operative assessment and conservative treatment planning are essential.

In a LASIK vision correction, the power of a patient's eye is increased by 3.00 D. Assuming this produces normal close vision, what was the patient's near point before the procedure?

Show Solution

Strategy

Normal close vision corresponds to a near point of 25.0 cm, which requires an accommodated power of 54.0 D (as calculated in the text examples). After the LASIK procedure that increases eye power by 3.00 D, the patient achieves this normal near point. Before the procedure, the patient's accommodated power was 3.00 D less. We use $P = \frac{1}{d_o} + \frac{1}{d_i}$ to find the original near point with the reduced power.

Solution

Given values:

- Power increase from LASIK: $\Delta P = 3.00\text{ D}$
- Normal accommodated power: $P_{\text{after}} = 54.0\text{ D}$
- Lens-to-retina distance: $d_i = 2.00\text{ cm} = 0.0200\text{ m}$

The patient's accommodated power before the procedure:

$$P_{\text{before}} = P_{\text{after}} - \Delta P = 54.0\text{ D} - 3.00\text{ D} = 51.0\text{ D}$$

Using the power equation to find the original near point:

$$P_{\text{before}} = \frac{1}{d_o} + \frac{1}{d_i}$$
$$\frac{1}{d_o} = P_{\text{before}} - \frac{1}{d_i} = 51.0\text{ D} - \frac{1}{0.0200\text{ m}}$$
$$\frac{1}{d_o} = 51.0\text{ m}^{-1} - 50.0\text{ m}^{-1} = 1.0\text{ m}^{-1}$$
$$d_o = \frac{1}{1.0\text{ m}^{-1}} = 1.0\text{ m} = 100\text{ cm}$$

Discussion

Before the LASIK procedure, the patient's near point was 1.0 m (100 cm), indicating significant hyperopia (farsightedness). This person could not see objects clearly closer than one meter, making reading and other close work impossible without corrective lenses. The 3.00 D power increase from LASIK surgery brought the near point from 100 cm to the normal 25 cm, dramatically improving the patient's quality of life.

This large near point distance explains why the patient needed LASIK. With only 1.0 D of accommodation power before surgery (compared to the normal 4.0 D), this person likely suffered from either severe presbyopia (age-related loss of accommodation) or congenital hyperopia. After surgery, with their total accommodated power now at 54.0 D, they can read normally without glasses. This demonstrates how LASIK can correct not just distant vision problems (myopia) but also near vision problems (hyperopia).

What was the previous far point of a patient who had laser vision correction that reduced the power of her eye by 7.00 D, producing normal distant vision for her?

Show Solution

Strategy

After laser vision correction that produced normal distant vision, the patient's relaxed eye power is 50.0 D (normal for viewing at infinity). Before the procedure, the power was 7.00 D higher. We use $P = \frac{1}{d_o} + \frac{1}{d_i}$ with the pre-procedure power to find the original far point d_o.

Solution

Given values:

- Power reduction: $\Delta P = 7.00\text{ D}$
- Post-procedure relaxed power (normal): $P_{\text{after}} = 50.0\text{ D}$
- Lens-to-retina distance: $d_i = 2.00\text{ cm} = 0.0200\text{ m}$

Calculate the pre-procedure relaxed power:

$$P_{\text{before}} = P_{\text{after}} + \Delta P = 50.0\text{ D} + 7.00\text{ D} = 57.0\text{ D}$$

Find the original far point using the power equation:

$$P_{\text{before}} = \frac{1}{d_o} + \frac{1}{d_i}$$
$$\frac{1}{d_o} = P_{\text{before}} - \frac{1}{d_i} = 57.0\text{ D} - \frac{1}{0.0200\text{ m}}$$
$$\frac{1}{d_o} = 57.0\text{ m}^{-1} - 50.0\text{ m}^{-1} = 7.00\text{ m}^{-1}$$
$$d_o = \frac{1}{7.00\text{ m}^{-1}} = 0.143\text{ m} = 14.3\text{ cm}$$

Discussion

Before laser vision correction, the patient's far point was 0.143 m (14.3 cm), indicating severe myopia. This person could see clearly only up to about 14 cm—roughly the distance from their nose to their outstretched hand. Beyond this short distance, everything appeared increasingly blurry. This level of myopia would have been extremely debilitating without corrective lenses.

The 7.00 D power reduction through LASIK surgery transformed this patient's life dramatically. Activities that most people take for granted—recognizing faces across a room, reading street signs, watching television, driving—would have been impossible without thick corrective glasses or contact lenses before surgery.

For comparison:

- Normal relaxed eye power: 50.0 D
- Patient's pre-LASIK power: 57.0 D (14% higher than normal)
- Far point: 14.3 cm instead of infinity

After surgery, with normal 50.0 D power, the patient can see clearly to infinity. This is equivalent to what would have been achieved with -7.00 D spectacle lenses, but without the inconvenience, optical distortions, weight, and aesthetic concerns of strong corrective glasses. The LASIK procedure permanently reshaped the cornea to reduce its optical power by flattening its curvature.

A severely myopic patient has a far point of 5.00 cm. By how many diopters should the power of his eye be reduced in laser vision correction to obtain normal distant vision for him?

Show Solution

Strategy

For normal distant vision, the relaxed eye should have a power of 50.0 D (corresponding to a far point at infinity). The patient's current relaxed power can be calculated from their far point of 5.00 cm using $P = \frac{1}{d_o} + \frac{1}{d_i}$. The difference between the current power and the normal power gives the required power reduction.

Solution

Given values:

- Current far point: $d_o = 5.00 \text{ cm} = 0.0500 \text{ m}$
- Lens-to-retina distance: $d_i = 2.00 \text{ cm} = 0.0200 \text{ m}$
- Normal relaxed power: $P_{\text{normal}} = 50.0 \text{ D}$

Calculate the current relaxed power:

$$P_{\text{current}} = \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{0.0500 \text{ m}} + \frac{1}{0.0200 \text{ m}}$$
$$P_{\text{current}} = 20.0 \text{ D} + 50.0 \text{ D} = 70.0 \text{ D}$$

The required power reduction:

$$\Delta P = P_{\text{current}} - P_{\text{normal}} = 70.0 \text{ D} - 50.0 \text{ D} = 20.0 \text{ D}$$

Discussion

The patient's eye power must be reduced by 20.0 D to achieve normal distant vision. This is a very large correction, reflecting the severity of the myopia—the patient can only see clearly up to 5.00 cm away, which is extremely close. With a far point this close, the patient would be unable to perform most daily tasks without corrective lenses.

A 20.0 D power reduction is at the upper limit of what LASIK can typically achieve safely. Most LASIK procedures correct between -1.00 D to -12.00 D of myopia. A 20.0 D correction would require removing a significant amount of corneal tissue, potentially compromising corneal strength and stability. For such severe myopia, alternative procedures like phakic intraocular lenses (implantable lenses placed inside the eye) might be more appropriate. This problem illustrates the limitations of laser vision correction and why careful patient screening is essential before surgery.

A student's eyes, while reading the blackboard, have a power of 51.0 D. How far is the board from his eyes?

Show Solution

Strategy

We use the thin lens equation $P = \frac{1}{d_o} + \frac{1}{d_i}$ with $P = 51.0$ D and $d_i = 2.00$ cm to solve for the object distance d_o, which is the distance from the student's eyes to the blackboard.

Solution

Given values:

- Eye power: $P = 51.0 \text{ D}$
- Lens-to-retina distance: $d_i = 2.00 \text{ cm} = 0.0200 \text{ m}$

Using the power equation:

$$P = \frac{1}{d_o} + \frac{1}{d_i}$$

Rearranging to solve for d_o:

$$\frac{1}{d_o} = P - \frac{1}{d_i} = 51.0 \text{ D} - \frac{1}{0.0200 \text{ m}}$$
$$\frac{1}{d_o} = 51.0 \text{ m}^{-1} - 50.0 \text{ m}^{-1} = 1.0 \text{ m}^{-1}$$
$$d_o = \frac{1}{1.0 \text{ m}^{-1}} = 1.00 \text{ m}$$

Discussion

The blackboard is 1.00 m from the student's eyes. The eye power of 51.0 D represents 1.0 D of accommodation beyond the relaxed power of 50.0 D (for distant vision). This is a comfortable level of accommodation—well within the typical 4.0 D range for young adults.

A 1.00 m viewing distance is reasonable for a blackboard, suggesting the student is sitting in the front or middle rows of the classroom. For comparison:

- Relaxed eye (distant vision): 50.0 D, viewing at infinity
- Moderate accommodation (1.00 m): 51.0 D, requires 1.0 D accommodation

- Normal near point (25 cm): 54.0 D, requires 4.0 D accommodation

The modest 1.0 D accommodation at 1.00 m means the student can comfortably read the blackboard for extended periods without significant eye strain. If the student were sitting farther back (say 3-5 meters), their eyes would be nearly completely relaxed (close to 50.0 D), requiring even less accommodation. This is one reason why educators sometimes recommend seating students with vision concerns farther from the board rather than closer—it reduces the accommodation demand and associated eye fatigue during long lectures.

The power of a physician's eyes is 53.0 D while examining a patient. How far from her eyes is the feature being examined?

Show Solution

Strategy

Using the thin lens equation $P = \frac{1}{d_o} + \frac{1}{d_i}$, where $P = 53.0$ D and $d_i = 2.00$ cm, we can solve for the object distance d_o, which represents how far the examined feature is from the physician's eyes.

Solution

Given values:

- Eye power: $P = 53.0\text{ D}$
- Lens-to-retina distance: $d_i = 2.00\text{ cm} = 0.0200\text{ m}$

Using the power equation:

$$P = \frac{1}{d_o} + \frac{1}{d_i}$$

Rearranging to solve for d_o:

$$\frac{1}{d_o} = P - \frac{1}{d_i} = 53.0\text{ D} - \frac{1}{0.0200\text{ m}}$$
$$\frac{1}{d_o} = 53.0\text{ m}^{-1} - 50.0\text{ m}^{-1} = 3.0\text{ m}^{-1}$$
$$d_o = \frac{1}{3.0\text{ m}^{-1}} = 0.333\text{ m} = 33.3\text{ cm}$$

Discussion

The feature being examined is 33.3 cm from the physician's eyes. This is a comfortable working distance for close examination, slightly farther than the standard near point of 25 cm. The physician is using an accommodated power of 53.0 D, which represents 3.0 D of accommodation beyond the relaxed power of 50.0 D.

This distance is typical for medical examinations where the physician needs to look closely at a patient's features (such as examining eyes, ears, throat, or skin lesions) but still maintain a comfortable working distance. It's close enough to see fine details but far enough to avoid discomfort for both patient and physician. The 3.0 D of accommodation required is well within the normal accommodation range, so the physician can maintain this examination distance for extended periods without significant eye strain.

A young woman with normal distant vision has a 10.0% ability to accommodate (that is, increase) the power of her eyes. What is the closest object she can see clearly?

Show Solution

Strategy

Normal distant vision corresponds to a relaxed eye power of 50.0 D. A 10.0% accommodation ability means the accommodated power is 1.10 times the relaxed power. We use $P = \frac{1}{d_o} + \frac{1}{d_i}$ with the accommodated power to find the near point d_o.

Solution

Given values:

- Relaxed power (normal): $P_{\text{relaxed}} = 50.0\text{ D}$
- Accommodation ability: 10.0%
- Lens-to-retina distance: $d_i = 2.00\text{ cm} = 0.0200\text{ m}$

Calculate the accommodated power:

$$P_{\text{accommodated}} = 1.10 \times P_{\text{relaxed}} = 1.10 \times 50.0\text{ D} = 55.0\text{ D}$$

Find the near point using the power equation:

$$P_{\text{accommodated}} = \frac{1}{d_o} + \frac{1}{d_i}$$
$$\frac{1}{d_o} = P_{\text{accommodated}} - \frac{1}{d_i} = 55.0\text{ D} - \frac{1}{0.0200\text{ m}}$$
$$\frac{1}{d_o} = 55.0\text{ m}^{-1} - 50.0\text{ m}^{-1} = 5.0\text{ m}^{-1}$$

$$d_o = \frac{1}{5.0\ \text{m}^{-1}} = 0.200\ \text{m} = 20.0\ \text{cm}$$

Discussion

The closest object this woman can see clearly is at 20.0 cm (her near point). Her 10.0% accommodation ability translates to 5.0 D of accommodation (from 50.0 D relaxed to 55.0 D accommodated), which is slightly better than the typical 4.0 D for young adults with a 25 cm near point.

For comparison:

- Normal near point: 25 cm (requires 4.0 D accommodation)
- This woman's near point: 20 cm (requires 5.0 D accommodation)

The 20.0 cm near point indicates slightly better than average near vision for her age. This enhanced accommodation could be due to:

1. Youth (younger individuals typically have greater accommodation)
2. Good eye health and flexibility of the lens
3. Possibly some degree of natural myopia that aids close vision

However, a 20 cm near point is quite close—closer than most people hold reading material (typically 25-40 cm). While she can focus at this distance, she might find it uncomfortable for extended reading due to the effort required to maintain maximum accommodation. Most people prefer to read at or slightly beyond their near point where less accommodation is needed, reducing eye strain.

As she ages, this accommodation ability will gradually decrease (presbyopia), and her near point will move farther away. By age 40-50, her near point might be 40-50 cm or more, at which point she would likely need reading glasses.

The far point of a myopic administrator is 50.0 cm. (a) What is the relaxed power of their eyes? (b) If they have the normal 8.00% ability to accommodate, what is the closest object they can see clearly?

Show Solution

Strategy

For part (a), we use $P = \frac{1}{d_o} + \frac{1}{d_i}$ with the far point distance (50.0 cm) as d_o to find the relaxed power. For part (b), an 8.00% accommodation ability means the accommodated power is 1.08 times the relaxed power. We then use the power equation to find the near point.

Solution

Given values:

- Far point: $d_o = 50.0\ \text{cm} = 0.500\ \text{m}$
- Lens-to-retina distance: $d_i = 2.00\ \text{cm} = 0.0200\ \text{m}$
- Accommodation ability: 8.00%

(a) Calculate the relaxed power:

$$P_{\text{relaxed}} = \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{0.500\ \text{m}} + \frac{1}{0.0200\ \text{m}}$$
$$P_{\text{relaxed}} = 2.00\ \text{D} + 50.0\ \text{D} = 52.0\ \text{D}$$

(b) Calculate the accommodated power:

$$P_{\text{accommodated}} = 1.08 \times P_{\text{relaxed}} = 1.08 \times 52.0\ \text{D} = 56.2\ \text{D}$$

Find the near point using the accommodated power:

$$\frac{1}{d_o} = P_{\text{accommodated}} - \frac{1}{d_i} = 56.2\ \text{D} - 50.0\ \text{D} = 6.2\ \text{m}^{-1}$$
$$d_o = \frac{1}{6.2\ \text{m}^{-1}} = 0.161\ \text{m} = 16.1\ \text{cm}$$

Discussion

Part (a): The relaxed power of 52.0 D is higher than the normal 50.0 D, confirming this person is myopic (nearsighted). The excess 2.0 D of power causes distant objects beyond 50.0 cm to appear blurry.

Part (b): The near point of 16.1 cm is actually closer than the normal 25 cm. This is an interesting benefit of mild myopia—myopic people can see close objects more clearly than people with normal vision. The administrator can read comfortably at 16.1 cm, which is advantageous for detailed close work.

This problem illustrates that myopia has a "silver lining": while distant vision is compromised, near vision is enhanced. This is why some older people with presbyopia may deliberately under-correct their myopia or use reading glasses weaker than needed—they want to preserve their advantage in close-up tasks. However, for most daily activities requiring both near and distant vision, corrective lenses for myopia are still beneficial. The administrator would need -2.00 D corrective lenses to see distant objects clearly.

A very myopic man has a far point of 20.0 cm. What power contact lens (when on the eye) will correct his distant vision?

Show Solution

Strategy

For normal distant vision, we want the corrected eye to focus objects at infinity onto the retina. The contact lens must create a virtual image at the man's far point (20.0 cm) when viewing distant objects. Since the contact lens sits directly on the eye, we use $P = \frac{1}{d_o} + \frac{1}{d_i}$ with $d_o = \infty$ (distant object) and $d_i = -20.0$ cm (virtual image at far point).

Solution

Given values:

- Far point: 20.0 cm = 0.200 m
- Object distance: $d_o = \infty$ (distant objects)
- Image distance: $d_i = -0.200 \text{ m}$ (negative for virtual image)

Using the power equation for the contact lens:

$$P = \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{\infty} + \frac{1}{-0.200 \text{ m}}$$

$$P = 0 - 5.00 \text{ m}^{-1} = -5.00 \text{ D}$$

Discussion

The required contact lens power is -5.00 D (diverging lens). This negative power is characteristic of myopia correction. The contact lens diverges light rays before they enter the eye, effectively reducing the combined optical power of the lens-eye system to achieve normal distant vision.

Physical interpretation:

- Without correction: The man's eye power of 55.0 D (calculated from $P = 1/0.20 + 1/0.02 = 55.0$ D) is too strong, focusing distant objects 20 cm in front of the retina
- With -5.00 D contact: Combined power becomes $55.0 - 5.00 = 50.0$ D, the normal relaxed power for distant vision
- Result: Distant objects now focus correctly on the retina

This is a moderate to strong myopia correction. For comparison:

- Mild myopia: -0.25 to -3.00 D
- Moderate myopia: -3.00 to -6.00 D (this patient)
- High myopia: -6.00 D and above

With -5.00 D lenses, the man would see distant objects clearly. However, this correction might make near vision slightly more challenging, as his natural advantage for close work (short far point) is eliminated. Some myopic individuals choose to under-correct slightly for this reason, using -4.50 D lenses for distance while retaining some near vision advantage.

Repeat the previous problem for eyeglasses held 1.50 cm from the eyes.

Show Solution

Strategy

The previous problem involved a myopic man with a far point of 20.0 cm who needed a -5.00 D contact lens. For eyeglasses held 1.50 cm from the eye, we must account for this separation. The spectacle lens must create an image at the far point (20.0 cm from the eye, or 18.5 cm from the lens) when viewing distant objects. We use $P = \frac{1}{d_o} + \frac{1}{d_i}$ with $d_o = \infty$ and $d_i = -18.5$ cm.

Solution

Given values:

- Far point from eye: 20.0 cm
- Spectacle distance from eye: 1.50 cm
- Image distance from spectacle: $d_i = -(20.0 - 1.50) = -18.5 \text{ cm} = -0.185 \text{ m}$
- Object distance: $d_o = \infty$ (distant objects)

The negative image distance indicates the image is on the same side as the object (virtual image).

Using the power equation:

$$P = \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{\infty} + \frac{1}{-0.185 \text{ m}}$$

$$P = 0 - 5.41 \text{ m}^{-1} = -5.41 \text{ D}$$

Discussion

The required spectacle lens power is -5.41 D, which is slightly more negative than the -5.00 D contact lens. This difference arises because the spectacle lens is 1.50 cm farther from the eye's optical system. The farther the corrective lens is from the eye, the stronger it must be to create an image at the same location.

This 0.41 D difference may seem small, but it's clinically significant. This is why contact lens and eyeglass prescriptions differ for the same person—the prescription must account for the vertex distance (distance from the lens to the cornea). For mild prescriptions, this difference is negligible, but for strong prescriptions like this one, it becomes important. An optometrist must adjust the prescription when converting between contacts and glasses, typically making spectacles slightly stronger (more negative for myopia) than contact lenses.

A myopic person sees that her contact lens prescription is -4.00D . What is her far point?

Show Solution

Strategy

A contact lens prescription of -4.00 D means the lens creates a virtual image at the person's far point when viewing distant objects. Using $P = \frac{1}{d_o} + \frac{1}{d_i}$ with $P = -4.00$ D and $d_o = \infty$, we solve for d_i, which is the negative of the far point distance.

Solution

Given values:

- Contact lens power: $P = -4.00$ D
- Object distance: $d_o = \infty$ (distant objects)

Using the power equation:

$$P = \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{\infty} + \frac{1}{d_i}$$
$$-4.00\text{ D} = 0 + \frac{1}{d_i}$$
$$d_i = \frac{1}{-4.00\text{ D}} = \frac{1}{-4.00\text{ m}^{-1}} = -0.250\text{ m} = -25.0\text{ cm}$$

The negative sign indicates a virtual image. The far point is the magnitude:

$$\text{Far point} = 25.0\text{ cm}$$

Discussion

This person's far point is 25.0 cm, which is quite close and indicates moderate myopia. Without corrective lenses, she can see clearly only up to 25 cm (about 10 inches)—roughly the distance of holding a book for reading. Everything beyond this distance appears increasingly blurry.

The -4.00 D contact lens prescription is a common moderate myopia correction. This person's uncorrected eye has a relaxed power of approximately 54.0 D (calculated from $P = 1/0.25 + 1/0.02 = 54.0$ D), which is 4.00 D stronger than the normal 50.0 D for distant vision.

Interesting observations:

- Her far point (25 cm) coincidentally equals the normal near point
- Without glasses, she has a natural advantage for close work
- With -4.00 D contacts, distant vision becomes normal, but she may need reading glasses earlier in life due to presbyopia

The -4.00 D lenses work by diverging incoming parallel light rays, making them appear to come from 25 cm away—her far point. Her eye can then focus these apparently nearby rays onto the retina, achieving clear distant vision. This is a moderate prescription that would likely require full-time wear for activities like driving, watching television, or seeing across a room, but she might not need them for reading or computer work.

Repeat the previous problem for glasses that are 1.75 cm from the eyes.

Show Solution

Strategy

The previous problem involved a myopic person with a -4.00 D contact lens prescription and a far point of 25.0 cm. For eyeglasses held 1.75 cm from the eyes, the spectacle lens must create an image at the far point (25.0 cm from the eye, or 23.25 cm from the lens) when viewing distant objects. We use $P = \frac{1}{d_o} + \frac{1}{d_i}$ with $d_o = \infty$ and $d_i = -23.25$ cm.

Solution

Given values:

- Far point from eye: 25.0 cm
- Spectacle distance from eye: 1.75 cm
- Image distance from spectacle: $d_i = -(25.0 - 1.75) = -23.25\text{ cm} = -0.2325\text{ m}$
- Object distance: $d_o = \infty$ (distant objects)

Using the power equation:

$$P = \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{\infty} + \frac{1}{-0.2325\text{ m}}$$

$$P = 0 - 4.30\ \text{m}^{-1} = -4.30\ \text{D}$$

Discussion

The required spectacle lens power is -4.30 D, compared to the -4.00 D contact lens. The spectacle lens must be 0.30 D stronger (more negative) because it sits 1.75 cm from the eye. This vertex distance effect is important for accurate vision correction.

For this moderate myopia prescription, the vertex distance correction is about 7.5% of the contact lens power. This difference becomes more pronounced with stronger prescriptions. An optometrist typically uses the formula $P_{\text{spectacle}} = \frac{P_{\text{contact}}}{1 - d \cdot P_{\text{contact}}}$ where d is the vertex distance, to accurately convert between contact and spectacle prescriptions.

The fact that spectacle lenses need to be stronger than contact lenses for myopia correction is one reason why some people prefer contacts—they provide correction with less optical power, which can reduce optical distortions and weight of the lenses. However, for mild to moderate myopia like this case, the difference is small enough that either option provides excellent vision correction.

The contact lens prescription for a mildly farsighted person is 0.750 D, and the person has a near point of 29.0 cm. What is the power of the tear layer between the cornea and the lens if the correction is ideal, taking the tear layer into account?

Show Solution

Strategy

For ideal correction, viewing an object at the normal near point (25.0 cm) should produce an image at the person's actual near point (29.0 cm). The total optical power is the sum of three components: the contact lens (+0.750 D), the tear layer (unknown), and the eye's power. We calculate the required total power and subtract the known components to find the tear layer power.

Solution

Given values:

- Contact lens power: $P_{\text{lens}} = +0.750\ \text{D}$
- Person's actual near point: 29.0 cm = 0.290 m
- Normal near point (object): 25.0 cm = 0.250 m

For ideal correction at the normal near point, the contact-tear-eye system must create an image at the person's actual near point when the object is at 25.0 cm.

The total power needed (contact + tear + eye) to view at 25.0 cm:

$$P_{\text{total}} = \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{0.250\ \text{m}} + \frac{1}{0.0200\ \text{m}}$$
$$P_{\text{total}} = 4.00\ \text{D} + 50.0\ \text{D} = 54.0\ \text{D}$$

The eye's accommodated power when viewing at 29.0 cm (actual near point):

$$P_{\text{eye}} = \frac{1}{0.290\ \text{m}} + \frac{1}{0.0200\ \text{m}} = 3.448\ \text{D} + 50.0\ \text{D} = 53.448\ \text{D}$$

The tear layer power:

$$P_{\text{tear}} = P_{\text{total}} - P_{\text{lens}} - P_{\text{eye}}$$
$$P_{\text{tear}} = 54.0\ \text{D} - 0.750\ \text{D} - 53.448\ \text{D} = -0.198\ \text{D}$$

Discussion

The tear layer has a power of -0.198 D (diverging). This negative power arises because the contact lens doesn't perfectly match the curvature of the cornea. The tear layer fills the gap between the contact lens and cornea, and if this layer is thicker at the edges than in the center (which is common), it acts as a weak diverging lens.

Physical interpretation:

- The +0.750 D contact lens provides most of the correction for farsightedness
- The -0.198 D tear layer slightly reduces this correction
- The net effect (+0.750 - 0.198 = +0.552 D) combined with the eye's power gives ideal near vision at 25 cm

This problem illustrates an important practical consideration in contact lens fitting. Optometrists must account for the tear layer when prescribing contact lenses. If they ignored the tear layer effect, they would prescribe +0.750 D, but the actual correction would be only +0.552 D, resulting in under-correction.

The tear layer power depends on:

1. The difference in curvature between the contact lens and cornea
2. The thickness profile of the tear layer
3. The refractive indices of the tear fluid and contact material

For this patient, the optometrist successfully prescribed +0.750 D knowing the tear layer would contribute -0.198 D, resulting in the ideal +0.552 D net correction needed.

A nearsighted man cannot see objects clearly beyond 20 cm from his eyes. How close must he stand to a mirror in order to see what he is doing when he shaves?

Show Solution

Strategy

The man’s far point is 20 cm, meaning he can see clearly only up to this distance. When looking in a mirror, he sees an image of his face that appears to be behind the mirror at a distance equal to his distance from the mirror. For clear vision, the total optical path (from his eyes to the mirror and back to the apparent image location) must be within his far point of 20 cm. If he stands at distance d from the mirror, the image appears at distance d behind the mirror, making the total distance $2d$.

Solution

Given values:

- Far point: 20 cm
- Total optical distance to image: $2d$ (where d is distance to mirror)

For clear vision, the image distance must equal the far point:

$$2d = 20\text{ cm}$$
$$d = 10\text{ cm}$$

Discussion

The man must stand 10 cm from the mirror to see himself clearly while shaving. At this distance, the image of his face appears 10 cm behind the mirror surface, making the total optical path 20 cm—exactly his far point.

This is quite close to the mirror, which would be awkward and impractical for shaving. The mirror would likely fog from his breath, and maneuvering a razor would be difficult at such close range. This problem illustrates why uncorrected myopia significantly impacts daily activities. With -5.00 D corrective lenses (as calculated for a 20 cm far point), the man could stand at a normal distance from the mirror and see clearly.

This problem also demonstrates an important principle of mirror optics: for a plane mirror, the image appears as far behind the mirror as the object is in front. Therefore, the effective viewing distance is twice the physical distance to the mirror. This doubling effect is why people with limited vision ranges need to stand particularly close to mirrors.

A mother sees that her child’s contact lens prescription is 0.750 D. What is the child’s near point?

Show Solution

Strategy

A +0.750 D contact lens prescription indicates farsightedness correction. The lens allows viewing at the normal near point (25.0 cm) by creating a virtual image at the child’s actual (farther) near point. Using $P = \frac{1}{d_o} + \frac{1}{d_i}$ with the lens power and object distance, we can find the image distance, which corresponds to the child’s near point.

Solution

Given values:

- Contact lens power: $P = +0.750\text{ D}$
- Object distance (normal near point): $d_o = 25.0\text{ cm} = 0.250\text{ m}$

Using the power equation for the contact lens:

$$P = \frac{1}{d_o} + \frac{1}{d_i}$$

Rearranging to solve for d_i:

$$\frac{1}{d_i} = P - \frac{1}{d_o} = 0.750\text{ D} - \frac{1}{0.250\text{ m}}$$
$$\frac{1}{d_i} = 0.750\text{ m}^{-1} - 4.00\text{ m}^{-1} = -3.25\text{ m}^{-1}$$
$$d_i = \frac{1}{-3.25\text{ m}^{-1}} = -0.308\text{ m} = -30.8\text{ cm}$$

The negative sign indicates a virtual image. The child’s near point is:

$$\text{Near point} = 30.8\text{ cm}$$

Discussion

The child’s near point is 30.8 cm, which is farther than the normal 25 cm, indicating mild hyperopia (farsightedness). Without corrective lenses, the child cannot clearly see objects closer than 30.8 cm, which would make reading and other close work difficult or uncomfortable.

The +0.750 D contact lens works by converging incoming light rays slightly before they enter the eye, allowing the child to see objects at the normal 25 cm reading distance clearly. The lens creates a virtual image at 30.8 cm (the child's actual near point) of an object at 25 cm, enabling comfortable reading.

For a child, this is a relatively mild prescription. Considerations:

- Many children are naturally slightly farsighted and can compensate through accommodation
- This child likely experiences eye strain or headaches during extended reading without correction
- The prescription may change as the child grows, since children's eyes continue developing

At 30.8 cm, the child would need to hold books farther away than peers to read clearly without correction. While this is manageable, it can cause:

1. Eye fatigue from constant maximum accommodation
2. Headaches from sustained near work
3. Difficulty with fine detail work (drawing, crafts)
4. Slower reading speed due to discomfort

The +0.750 D contact lenses eliminate these issues, allowing the child to read comfortably at the standard 25 cm distance for extended periods without eye strain.

Repeat the previous problem for glasses that are 2.20 cm from the eyes.

Show Solution

Strategy

The previous problem involved a child with a +0.750 D contact lens prescription and a near point of 30.8 cm. For eyeglasses held 2.20 cm from the eyes, we need to find what prescription will provide the same correction. The spectacle lens must create an image at the child's near point (30.8 cm from the eye, or 28.6 cm from the lens) when viewing an object at the normal near point of 25.0 cm. We use $P = \frac{1}{d_o} + \frac{1}{d_i}$.

Solution

Given values:

- Normal near point (object distance from spectacle): $d_o = 25.0 - 2.20 = 22.8 \text{ cm} = 0.228 \text{ m}$
- Child's near point from eye: 30.8 cm
- Image distance from spectacle: $d_i = -(30.8 - 2.20) = -28.6 \text{ cm} = -0.286 \text{ m}$

The negative image distance indicates a virtual image on the same side as the object.

Using the power equation:

$$P = \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{0.228 \text{ m}} + \frac{1}{-0.286 \text{ m}}$$
$$P = 4.39 \text{ D} - 3.50 \text{ D} = 0.89 \text{ D}$$

Rounding to two significant figures:

$$P = 0.89 \text{ D}$$

Discussion

The required spectacle lens power is +0.89 D, compared to the +0.750 D contact lens. The spectacle lens must be stronger (more positive) by about 0.14 D because it sits 2.20 cm from the eye. This is consistent with the vertex distance effect—for hyperopia (farsightedness), spectacle lenses require more positive power than contact lenses.

The 19% increase in power (from +0.75 D to +0.89 D) may seem large, but it's typical for the vertex distance effect with positive lenses. For hyperopia corrections, the relationship is $P_{\text{spectacle}} = \frac{P_{\text{contact}}}{1 + d \cdot P_{\text{contact}}}$, where d is the vertex distance. This formula confirms our calculation.

For children, eyeglasses are often preferred over contact lenses for safety and ease of use, despite requiring slightly stronger prescriptions. The child would wear +0.89 D glasses to achieve the same vision correction as +0.75 D contacts. This demonstrates why prescriptions must always specify whether they're for contacts or spectacles—using the wrong prescription could result in under-correction or over-correction.

The contact lens prescription for a nearsighted person is -4.00D and the person has a far point of 22.5 cm. What is the power of the tear layer between the cornea and the lens if the correction is ideal, taking the tear layer into account?

Show Solution

Strategy

For ideal distant vision correction, the contact-tear-eye system must focus distant objects (at infinity) onto the retina. The contact lens power is -4.00 D, and we know the person's uncorrected far point is 22.5 cm. We calculate the eye's relaxed power from the far point, then find the tear layer power needed to achieve the correct total optical power.

Solution

Given values:

- Contact lens power: $P_{\text{lens}} = -4.00\text{ D}$
- Far point (uncorrected): 22.5 cm = 0.225 m
- Lens-to-retina distance: $d_i = 2.00\text{ cm} = 0.0200\text{ m}$

Calculate the uncorrected relaxed eye power from the far point:

$$P_{\text{eye}} = \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{0.225\text{ m}} + \frac{1}{0.0200\text{ m}}$$
$$P_{\text{eye}} = 4.444\text{ D} + 50.0\text{ D} = 54.444\text{ D}$$

For ideal correction, the total power (contact + tear + eye) must equal the normal relaxed power:

$$P_{\text{lens}} + P_{\text{tear}} + P_{\text{eye}} = 50.0\text{ D}$$

Solving for the tear layer power:

$$P_{\text{tear}} = 50.0\text{ D} - P_{\text{lens}} - P_{\text{eye}}$$
$$P_{\text{tear}} = 50.0\text{ D} - (-4.00\text{ D}) - 54.444\text{ D}$$
$$P_{\text{tear}} = 50.0\text{ D} + 4.00\text{ D} - 54.444\text{ D} = -0.444\text{ D}$$

Discussion

The tear layer has a power of -0.444 D (diverging). This negative power is more significant than typical tear layer effects and indicates a notable mismatch between the contact lens curvature and the corneal surface.

Physical interpretation:

- The person's uncorrected eye has 54.444 D (4.444 D excess power causing myopia)
- The -4.00 D contact lens reduces this by 4.00 D
- The -0.444 D tear layer further reduces the power
- Net result: $54.444 - 4.00 - 0.444 = 50.0$ D (normal distant vision)

Interestingly, we can verify this is consistent with the far point. If the contact lens alone (-4.00 D) were perfect, it would correct a far point of:

- $d = 1/4.00 = 0.250$ m = 25.0 cm

But the actual far point is 22.5 cm (closer than 25 cm), meaning the eye is slightly more myopic than -4.00 D alone would correct. The -0.444 D tear layer provides the additional correction needed.

This problem highlights why contact lens fitting requires careful assessment. The optometrist prescribed -4.00 D knowing that:

1. The tear layer would contribute -0.444 D
2. The combined effect (-4.444 D) exactly matches the needed correction
3. The 22.5 cm far point corresponds to 4.444 D of excess power

If the tear layer effect were ignored, the patient would be slightly under-corrected, with residual myopia making distant objects slightly blurry. Professional contact lens fitting accounts for tear layer effects through trial fittings and refraction measurements with lenses in place.

Unreasonable Results

A boy has a near point of 50 cm and a far point of 500 cm. Will a -4.00D lens correct his far point to infinity?

Show Solution

Strategy

To correct the far point to infinity using a contact lens, we first determine what lens power is actually needed. The lens must create a virtual image at the boy's far point (500 cm) when viewing distant objects. We calculate the required power and compare it to the proposed -4.00 D lens to determine if the correction is appropriate.

Solution

Given values:

- Far point: $d_i = -500\text{ cm} = -5.00\text{ m}$ (virtual image)
- Object distance for distant vision: $d_o = \infty$
- Proposed lens power: $P_{\text{proposed}} = -4.00\text{ D}$

Calculate the required lens power:

$$P_{\text{required}} = \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{\infty} + \frac{1}{-5.00\text{ m}}$$
$$P_{\text{required}} = 0 - 0.20\text{ D} = -0.20\text{ D}$$

Comparing with the proposed lens:

The proposed -4.00 D lens is **20 times stronger** than needed. This lens would create an image at:

$$\frac{1}{d_i} = P - \frac{1}{d_o} = -4.00\text{ D} - 0 = -4.00\text{ m}^{-1}$$
$$d_i = -0.25\text{ m} = -25\text{ cm}$$

Discussion

No, a -4.00 D lens will **not** correct the boy's far point to infinity—it will severely over-correct his vision. The boy only needs a -0.20 D lens, but the proposed lens is 20 times too strong. With a -4.00 D lens, distant objects would appear as images only 25 cm from his eyes, which is actually at his near point!

This demonstrates an unreasonable result. The problem reveals that the boy's vision defect is very mild—his far point of 500 cm (5 meters) is quite distant, requiring only minimal correction. In fact, many people with such mild myopia might not need correction at all for daily activities, though it would help for tasks requiring sharp distant vision like driving or reading a blackboard.

The combination of a 50 cm near point and 500 cm far point is unusual but not impossible. It suggests either early presbyopia (loss of accommodation) combined with mild myopia, or simply a person with limited accommodation range. The key lesson is that lens prescriptions must be carefully matched to the actual vision defect—a stronger lens is not necessarily better and can create new vision problems. Using a -4.00 D lens when only -0.20 D is needed would make the boy effectively very nearsighted, with a new far point of only 25 cm!

Glossary

nearsightedness
: another term for myopia, a visual defect in which distant objects appear blurred because their images are focused in front of the retina rather than being focused on the retina

myopia
: a visual defect in which distant objects appear blurred because their images are focused in front of the retina rather than being focused on the retina

far point
: the object point imaged by the eye onto the retina in an unaccommodated eye

farsightedness
: another term for hyperopia, the condition of an eye where incoming rays of light reach the retina before they converge into a focused image

hyperopia
: the condition of an eye where incoming rays of light reach the retina before they converge into a focused image

near point
: the point nearest the eye at which an object is accurately focused on the retina at full accommodation

astigmatism
: the result of an inability of the cornea to properly focus an image onto the retina

laser vision correction
: a medical procedure used to correct astigmatism and eyesight deficiencies such as myopia and hyperopia

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Color and Color Vision

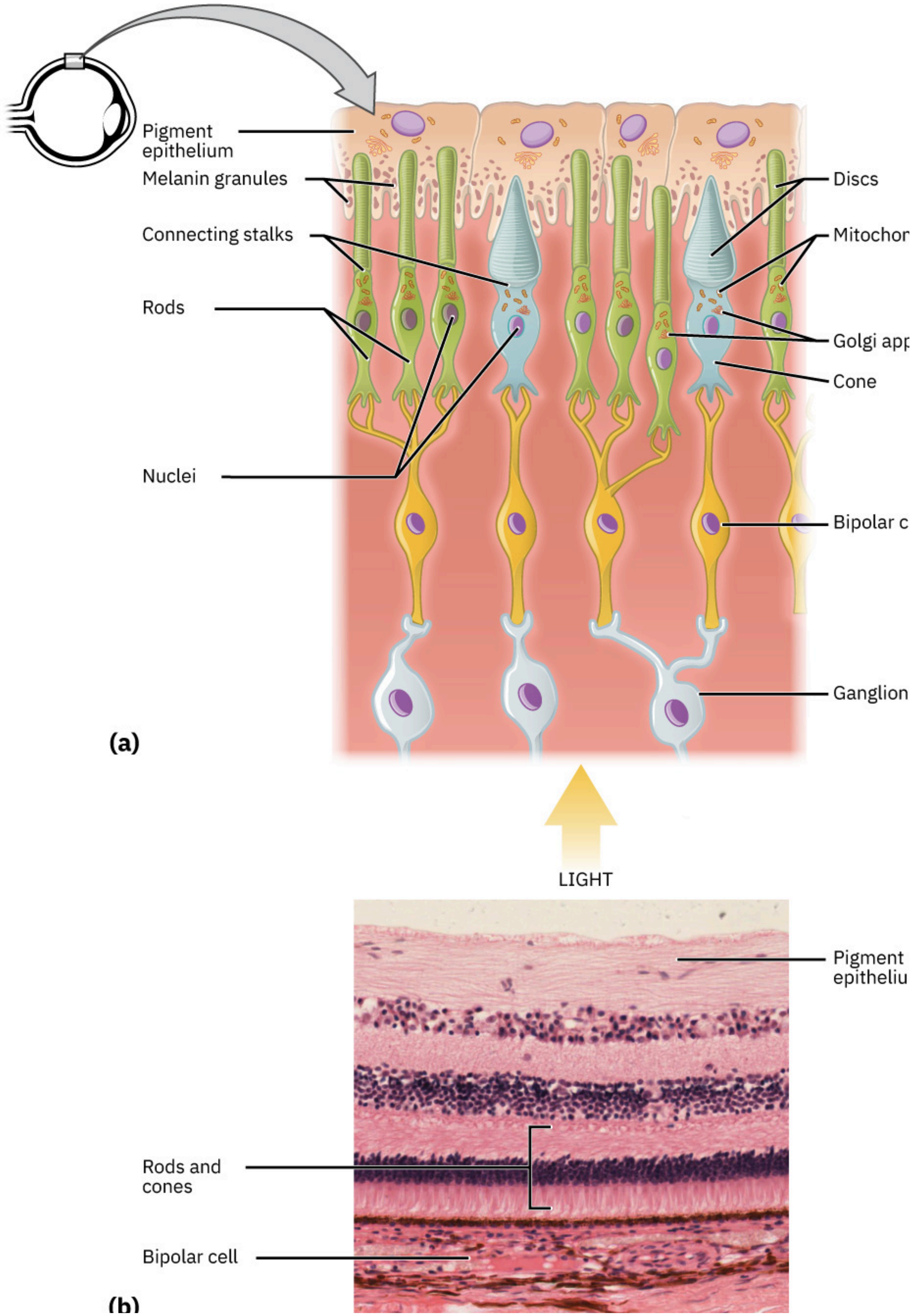
- Explain the simple theory of color vision.
- Outline the coloring properties of light sources.
- Describe the retinex theory of color vision.

The gift of vision is made richer by the existence of color. Objects and lights abound with thousands of hues that stimulate our eyes, brains, and emotions. Two basic questions are addressed in this brief treatment—what does color mean in scientific terms, and how do we, as humans, perceive it?

Simple Theory of Color Vision

We have already noted that color is associated with the wavelength of visible electromagnetic radiation. When our eyes receive pure-wavelength light, we tend to see only a few colors. Six of these (most often listed) are red, orange, yellow, green, blue, and violet. These are the rainbow of colors produced when white light is dispersed according to different wavelengths. There are thousands of other **hues** that we can perceive. These include brown, teal, gold, pink, and white. One simple theory of color vision implies that all these hues are our eye's response to different combinations of wavelengths. This is true to an extent, but we find that color perception is even subtler than our eye's response for various wavelengths of light.

The two major types of light-sensing cells (photoreceptors) in the retina are **rods and cones**. Rods are more sensitive than cones by a factor of about 1000 and are solely responsible for peripheral vision as well as vision in very dark environments. They are also important for motion detection. There are about 120 million rods in the human retina. Rods do not yield color information. You may notice that you lose color vision when it is very dark, but you retain the ability to discern grey scales.

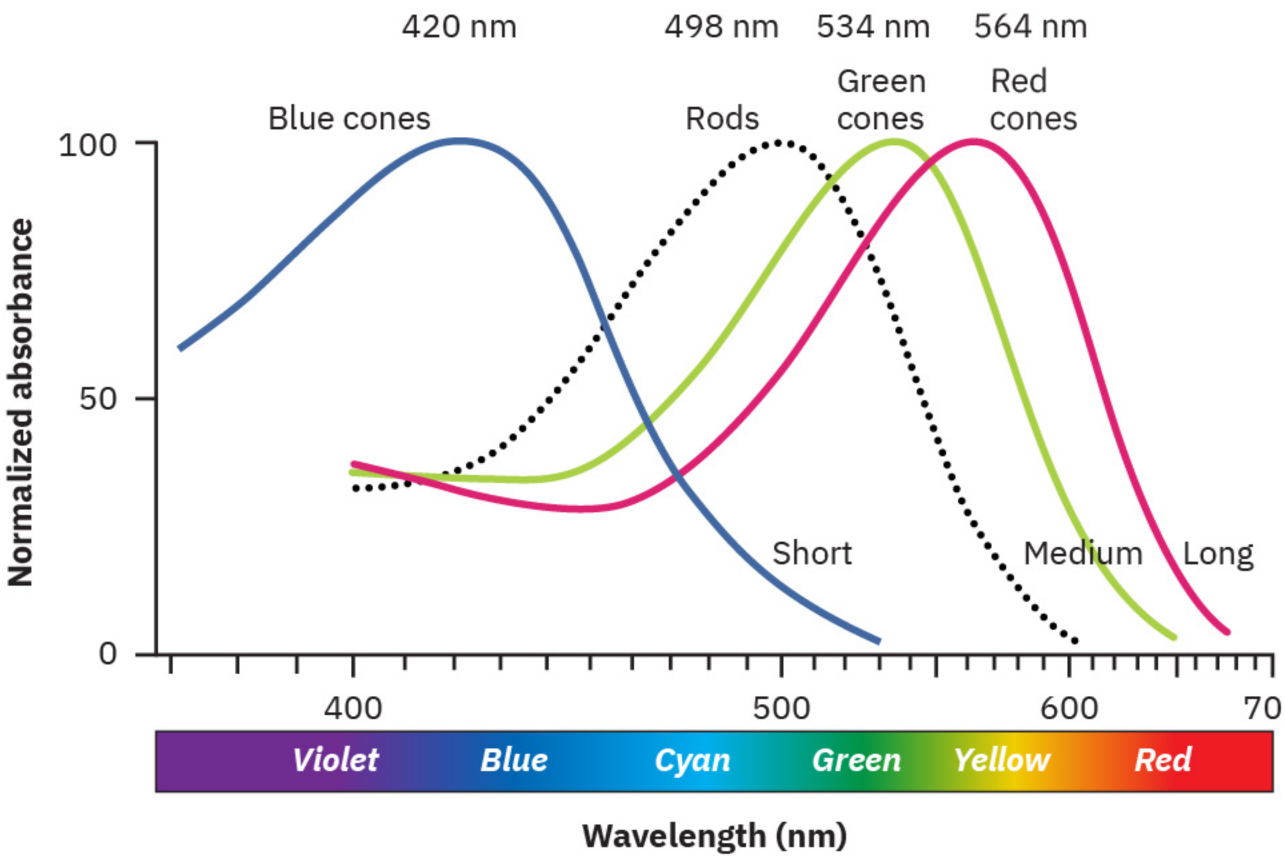

Pigment epithelium
Melanin granules
Connecting stalks
Rods
Nuclei
Discs
Mitochor
Golgi app
Cone
Bipolar c
Ganglion
(a)
LIGHT
Pigment epitheliu
Rods and cones
Bipolar cell
(b)

(a) All photoreceptors have inner segments containing the nucleus and other important organelles and outer segments with membrane arrays containing the photosensitive opsin molecules. Rod outer segments are long columnar shapes with stacks of membrane-bound discs that contain the rhodopsin pigment. Cone outer segments are short, tapered shapes with folds of membrane in place of the discs in the rods. (b) Tissue of the retina shows a dense layer of nuclei of the rods and cones. LM × 800. (Micrograph provided by the Regents of University of Michigan Medical School- 2012)

Take-Home Experiment: Rods and Cones

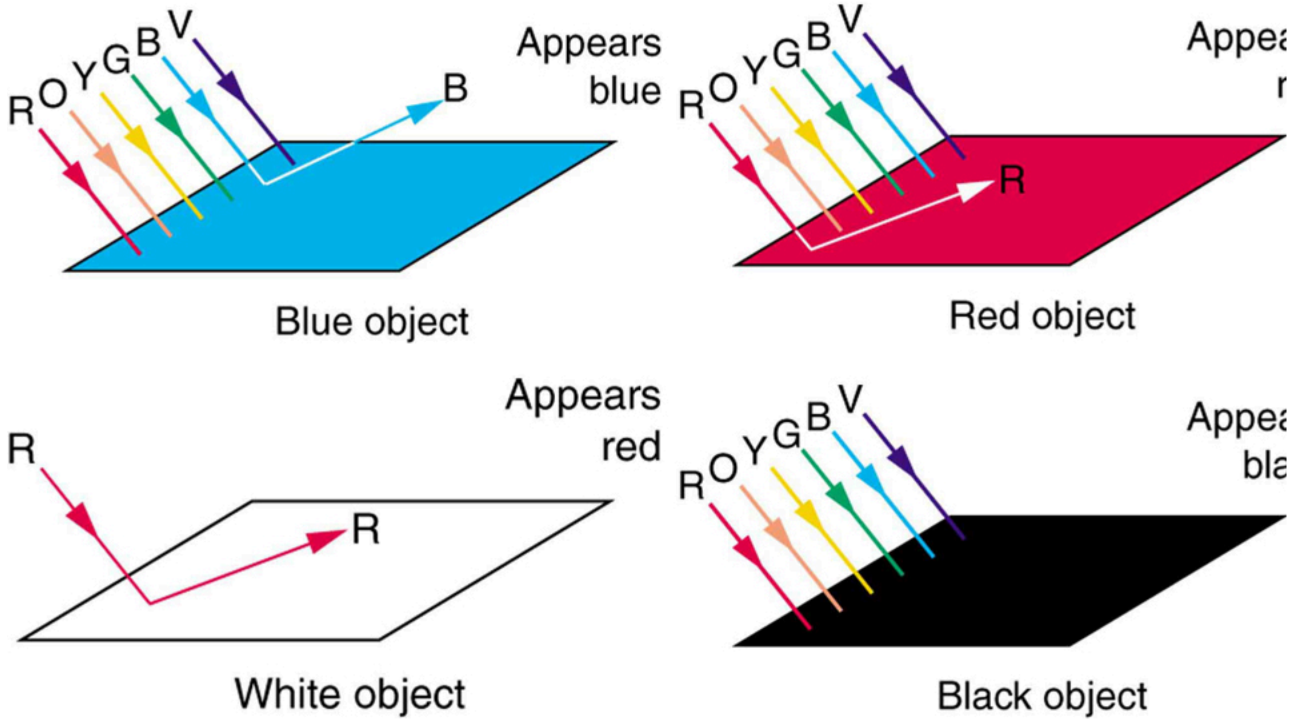
1. Go into a darkened room from a brightly lit room, or from outside in the Sun. How long did it take to start seeing shapes more clearly? What about color? Return to the bright room. Did it take a few minutes before you could see things clearly?
2. Demonstrate the sensitivity of foveal vision. Look at the letter G in the word ROGERS. What about the clarity of the letters on either side of G?

Cones are most concentrated in the fovea, the central region of the retina. There are no rods here. The fovea is at the center of the macula, a 5 mm diameter region responsible for our central vision. The cones work best in bright light and are responsible for high resolution vision. There are about 6 million cones in the human retina. There are three types of cones, and each type is sensitive to different ranges of wavelengths, as illustrated in [Figure 2]. A **simplified theory of color vision** is that there are three *primary colors* corresponding to the three types of cones. The thousands of other hues that we can distinguish among are created by various combinations of stimulations of the three types of cones. Color television uses a three-color system in which the screen is covered with equal numbers of red, green, and blue phosphor dots. The broad range of hues a viewer sees is produced by various combinations of these three colors. For example, you will perceive yellow when red and green are illuminated with the correct ratio of intensities. White may be sensed when all three are illuminated. Then, it would seem that all hues can be produced by adding three primary colors in various proportions. But there is an indication that color vision is more sophisticated. There is no unique set of three primary colors. Another set that works is yellow, green, and blue. A further indication of the need for a more complex theory of color vision is that various different combinations can produce the same hue. Yellow can be sensed with yellow light, or with a combination of red and green, and also with white light from which violet has been removed. The three-primary-colors aspect of color vision is well established; more sophisticated theories expand on it rather than deny it.



The image shows the relative sensitivity of the three types of cones, which are named according to wavelengths of greatest sensitivity. Rods are about 1000 times more sensitive, and their curve peaks at about 500 nm. Evidence for the three types of cones comes from direct measurements in animal and human eyes and testing of color blind people.

Consider why various objects display color—that is, why are feathers blue and red in a crimson rosella? The *true color of an object* is defined by its absorptive or reflective characteristics. [Figure 3] shows white light falling on three different objects, one pure blue, one pure red, and one black, as well as pure red light falling on a white object. Other hues are created by more complex absorption characteristics. Pink, for example on a galah cockatoo, can be due to weak absorption of all colors except red. An object can appear a different color under non-white illumination. For example, a pure blue object illuminated with pure red light will *appear* black, because it absorbs all the red light falling on it. But, the true color of the object is blue, which is independent of illumination.

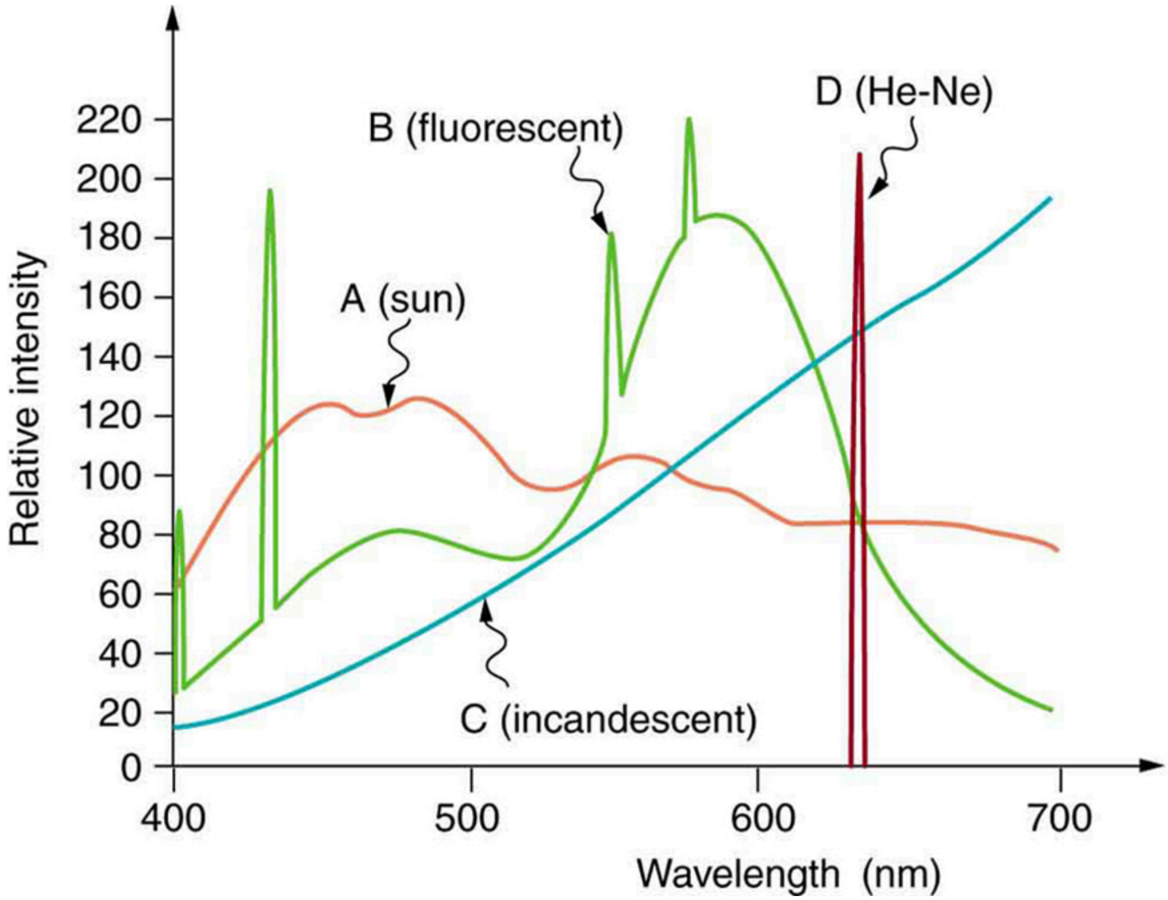


Absorption characteristics determine the true color of an object. Here, three objects are illuminated by white light, and one by pure red light. White is the equal mixture of all visible wavelengths; black is the absence of light.

Similarly, *light sources have colors* that are defined by the wavelengths they produce. A helium-neon laser emits pure red light. In fact, the phrase “pure red light” is defined by having a sharp constrained spectrum, a characteristic of laser light. The Sun produces a broad yellowish spectrum, fluorescent lights emit bluish-white light, and incandescent lights emit reddish-white hues as seen in [Figure 4]. As you would expect, you sense these colors when viewing the light source directly or when illuminating a white object with them. All of this fits neatly into the simplified theory that a combination of wavelengths produces various hues.

Take-Home Experiment: Exploring Color Addition

This activity is best done with plastic sheets of different colors as they allow more light to pass through to our eyes. However, thin sheets of paper and fabric can also be used. Overlay different colors of the material and hold them up to a white light. Using the theory described above, explain the colors you observe. You could also try mixing different crayon colors.

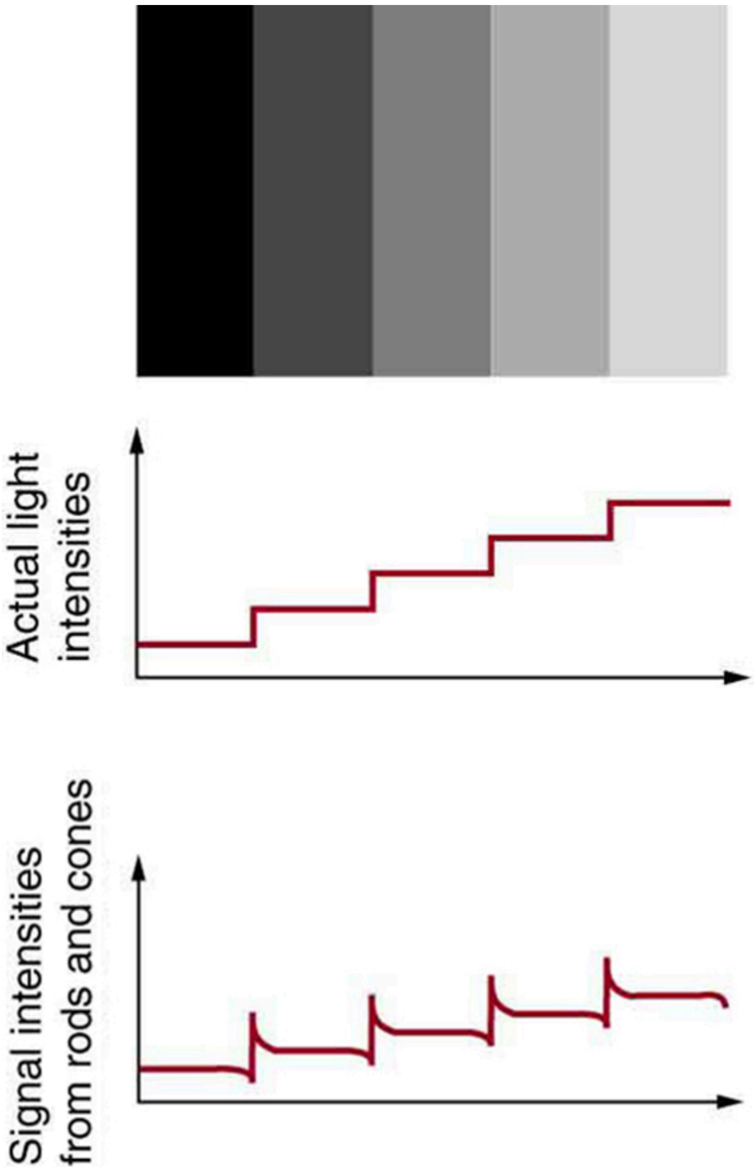


Emission spectra for various light sources are shown. Curve A is average sunlight at Earth’s surface, curve B is light from a fluorescent lamp, and curve C is the output of an incandescent light. The spike for a helium-neon laser (curve D) is due to its pure wavelength emission. The spikes in the fluorescent output are due to atomic spectra—a topic that will be explored later.

Color Constancy and a Modified Theory of Color Vision

The eye-brain color-sensing system can, by comparing various objects in its view, perceive the true color of an object under varying lighting conditions—an ability that is called **color constancy**. We can sense that a white tablecloth, for example, is white whether it is illuminated by sunlight, fluorescent light, or candlelight. The wavelengths entering the eye are quite different in each case, as the graphs in [Figure 4] imply, but our color vision can detect the true color by comparing the tablecloth with its surroundings.

Theories that take color constancy into account are based on a large body of anatomical evidence as well as perceptual studies. There are nerve connections among the light receptors on the retina, and there are far fewer nerve connections to the brain than there are rods and cones. This means that there is signal processing in the eye before information is sent to the brain. For example, the eye makes comparisons between adjacent light receptors and is very sensitive to edges as seen in [Figure 5]. Rather than responding simply to the light entering the eye, which is uniform in the various rectangles in this figure, the eye responds to the edges and senses false darkness variations.

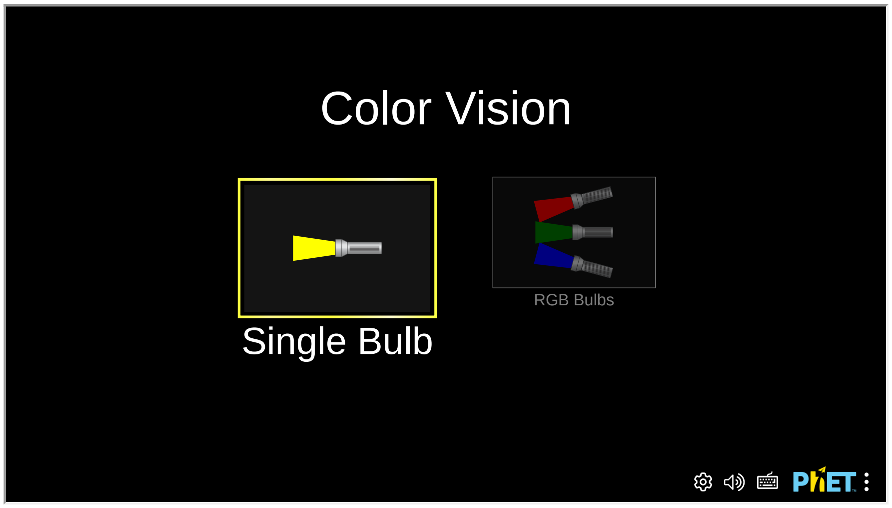


The importance of edges is shown. Although the grey strips are uniformly shaded, as indicated by the graph immediately below them, they do not appear uniform at all. Instead, they are perceived darker on the dark side and lighter on the light side of the edge, as shown in the bottom graph. This is due to nerve impulse processing in the eye.

One theory that takes various factors into account was advanced by Edwin Land (1909 – 1991), the creative founder of the Polaroid Corporation. Land proposed, based partly on his many elegant experiments, that the three types of cones are organized into systems called **retinexes**. Each retinex forms an image that is compared with the others, and the eye-brain system thus can compare a candle-illuminated white table cloth with its generally reddish surroundings and determine that it is actually white. This **retinex theory of color vision** is an example of modified theories of color vision that attempt to account for its subtleties. One striking experiment performed by Land demonstrates that some type of image comparison may produce color vision. Two pictures are taken of a scene on black-and-white film, one using a red filter, the other a blue filter. Resulting black-and-white slides are then projected and superimposed on a screen, producing a black-and-white image, as expected. Then a red filter is placed in front of the slide taken with a red filter, and the images are again superimposed on a screen. You would expect an image in various shades of pink, but instead, the image appears to humans in full color with all the hues of the original scene. This implies that color vision can be induced by comparison of the black-and-white and red images. Color vision is not completely understood or explained, and the retinex theory is not totally accepted. It is apparent that color vision is much subtler than what a first look might imply.

PhET Explorations: Color Vision

Make a whole rainbow by mixing red, green, and blue light. Change the wavelength of a monochromatic beam or filter white light. View the light as a solid beam, or see the individual photons.



Section Summary

- The eye has four types of light receptors—rods and three types of color-sensitive cones.
- The rods are good for night vision, peripheral vision, and motion changes, while the cones are responsible for central vision and color.
- We perceive many hues, from light having mixtures of wavelengths.
- A simplified theory of color vision states that there are three primary colors, which correspond to the three types of cones, and that various combinations of the primary colors produce all the hues.
- The true color of an object is related to its relative absorption of various wavelengths of light. The color of a light source is related to the wavelengths it produces.
- Color constancy is the ability of the eye-brain system to discern the true color of an object illuminated by various light sources.
- The retinex theory of color vision explains color constancy by postulating the existence of three retinexes or image systems, associated with the three types of cones that are compared to obtain sophisticated information.

Conceptual Questions

A pure red object on a black background seems to disappear when illuminated with pure green light. Explain why.

Show Solution

Strategy

To understand why a pure red object appears to disappear under pure green illumination, we need to consider the fundamental principles of color perception: how objects acquire their color through selective absorption and reflection of wavelengths, and how the absence of appropriate wavelengths to reflect affects visibility.

Solution

A pure red object has its color because of its selective absorption characteristics—it absorbs all wavelengths of visible light except red, which it reflects. The object's molecular structure and pigmentation are such that:

1. **Red wavelengths** (approximately 620-750 nm) are reflected
2. **All other wavelengths** (violet, blue, green, yellow, orange) are absorbed

When the object is illuminated with **pure green light** (approximately 495-570 nm):

1. The green light incident on the red object contains no red wavelengths
2. The object's surface absorbs all the green light (since it absorbs everything except red)
3. With no red wavelengths present in the incident light, there is nothing for the object to reflect
4. The object therefore reflects virtually no light back to the observer
5. An object that reflects no light appears black

Since the **background is already black** (absorbing all wavelengths), both the object and background now reflect essentially no light. With no contrast between the object and its background, the object becomes indistinguishable from the background and effectively "disappears."

Discussion

This phenomenon beautifully demonstrates that **an object's apparent color depends on both its intrinsic properties (absorption/reflection characteristics) and the illumination source**. The "true color" of an object is defined by what wavelengths it reflects under white light illumination, but its apparent color can change dramatically under monochromatic or colored illumination.

This principle has several practical implications:

Photography and stage lighting: Colored gels and filters can make objects appear very different from their true colors. Red objects under green stage lights appear dark or black.

Safety applications: This is why emergency exit signs and safety equipment are often made in specific colors—their visibility depends on the ambient lighting. A red fire extinguisher might be hard to see under certain colored emergency lighting.

Camouflage: Military and hunting camouflage can exploit this principle. An object that blends with its background under one type of lighting may become visible under different illumination.

Color matching: This is why paint and fabric colors should be evaluated under the same lighting conditions as their final use environment. A color that looks perfect under fluorescent store lighting may appear quite different under natural daylight or incandescent home lighting.

The experiment can be reversed: a pure green object illuminated with pure red light will also appear black and disappear against a black background. In general, any object illuminated with light that contains none of the wavelengths it can reflect will appear black, regardless of its “true” color.

This also explains why under monochromatic sodium vapor street lights (which emit primarily yellow light), it’s difficult to distinguish colors—cars, clothing, and other objects all appear in shades of yellow, orange, and grey because there are no blue, green, or red wavelengths for those colored objects to reflect.

What is color constancy, and what are its limitations?

Show Solution

Strategy

To address this question comprehensively, we need to define color constancy as a perceptual phenomenon, explain the physiological and cognitive mechanisms behind it, and then examine the conditions under which this remarkable ability breaks down or becomes limited.

Solution

Color Constancy Defined:

Color constancy is the remarkable ability of the human eye-brain visual system to perceive the true color of an object as relatively constant despite substantial changes in the illumination spectrum. For example, a white piece of paper appears white whether viewed under:

- Bright sunlight (which has a continuous broad spectrum)
- Fluorescent lighting (which has discrete spectral peaks)
- Incandescent bulbs (which emit more red/yellow wavelengths)
- Candlelight (which is strongly red-shifted)

Even though the wavelengths actually entering the eye differ dramatically in these situations (as shown in Figure 4 of the chapter), our perception of the object’s color remains remarkably stable.

Mechanism:

The retinex theory, proposed by Edwin Land, explains color constancy through comparative processing:

1. **Three separate image systems (retinexes)** correspond to the three cone types (short, medium, and long wavelength sensitive)
2. The visual system compares an object with its surroundings in each retinex
3. Edge detection and contrast processing in the retina identify relative reflectance
4. The brain interprets the object’s color based on its reflectance relative to surrounding objects rather than absolute wavelength composition
5. This comparative processing allows the system to “discount” the illumination and perceive the intrinsic color

Limitations of Color Constancy:

Despite its sophistication, color constancy has several important limitations:

1. Requires contextual information

- Color constancy fails when an object is viewed in isolation without surrounding reference objects
- A single colored patch shown through a small aperture (reduction screen) loses color constancy
- The system needs to compare multiple objects to determine the illumination characteristics

2. Extreme or unusual illumination

- Under monochromatic light (like pure sodium vapor lamps emitting only yellow), color constancy largely fails
- Under very narrow-band illumination, objects can only reflect wavelengths present in the light source
- Colored theatrical lighting can completely alter perceived colors beyond the system’s compensation ability

3. Requires adequate light levels

- In very dim lighting (scotopic vision), only rods function, and color vision is lost entirely
- In intermediate lighting (mesopic vision), color constancy is degraded
- The system requires sufficient photon flux for the cones to operate effectively

4. Adaptation time

- Color constancy requires time to adapt when moving between different lighting environments
- Walking from sunlight into tungsten lighting, colors initially appear off and require several seconds to minutes for full adaptation
- This represents the time needed for the visual system to recalibrate

5. Metamerism

- Different combinations of wavelengths can produce identical cone responses (metamers)
- Two objects may match under one illuminant but appear different under another
- This is a fundamental limitation of trichromatic vision—color matching depends on illumination

6. Extreme color contrasts

- Very strong colored surroundings can overwhelm color constancy
- Simultaneous color contrast effects can cause misperception of colors
- The adaptation to surrounding color can shift perception of a target object

7. Individual variation

- Color constancy varies between individuals
- Color vision deficiencies (color blindness) severely limit or eliminate color constancy
- Age-related changes in the lens (yellowing) affect color perception

8. Cognitive factors

- Prior knowledge influences color constancy ("memory colors")
- Expectations can bias color perception
- Ambiguous situations can produce different color percepts in different viewers

Discussion

Color constancy is an elegant example of sophisticated neural processing that generally serves us well in natural environments, where it evolved. However, modern artificial lighting, theatrical effects, and industrial applications can create conditions that exceed its capabilities.

Understanding these limitations is crucial for:

- **Color reproduction:** Ensuring printed materials, displays, and painted surfaces appear correctly under various lighting
- **Industrial color matching:** Requiring standardized lighting (D65 illuminant) for quality control
- **Art and photography:** Exploiting or compensating for color constancy effects
- **Medical diagnosis:** Using standardized lighting for evaluating skin conditions, tissue samples, etc.

The very existence of color constancy demonstrates that color perception is not a simple one-to-one mapping of wavelength to sensation, but rather a sophisticated computational process that attempts to extract invariant object properties from varying sensory input.

There are different types of color blindness related to the malfunction of different types of cones. Why would it be particularly useful to study those rare individuals who are color blind only in one eye or who have a different type of color blindness in each eye?

Show Solution

Strategy

To answer this question, we need to consider what makes scientific studies of human perception particularly challenging—individual variation and subjective experience—and how studying individuals with unilateral (one eye) or asymmetric color blindness provides a unique experimental advantage.

Solution

Studying individuals who are color blind in only one eye or who have different types of color blindness in each eye would be particularly valuable for several compelling scientific reasons:

1. Built-in Control Within the Same Individual

The most significant advantage is that these rare individuals serve as their **own experimental controls**:

- **Same brain, same visual cortex:** Both eyes connect to the same visual processing centers in the brain, eliminating inter-individual variation in neural processing, cognitive factors, and interpretation
- **Same genetics (except for the defect):** The genetic background is identical, isolating the specific effect of the cone malfunction
- **Same prior experience and training:** The person has lived with both types of vision simultaneously, eliminating learning effects and cultural factors
- **Same environmental conditions:** When comparing the two eyes' responses, all external variables (lighting, stimulus presentation, attention, fatigue) are automatically matched
- **Direct subjective comparison:** The individual can directly compare what their two eyes perceive simultaneously, providing unique introspective data about the qualitative difference

2. Elimination of Communication Difficulties

One of the fundamental challenges in studying color vision is the **problem of qualia**—the subjective experience of color:

- A person with normal trichromatic vision in one eye and, say, deuteranopia (green-deficient, dichromatic vision) in the other eye can directly report: "With my left eye, I see these two colors as different, but with my right eye, they look identical"
- This eliminates the philosophical problem of whether different people with the same type of color vision actually experience colors the same way
- The individual provides a **translation** between normal and deficient color vision that cannot be obtained any other way

3. Understanding Cone Function Independently

These individuals allow researchers to isolate the contribution of specific cone types:

- **Protanopia** (missing long-wavelength "red" cones): Comparing the protanopic eye with the normal eye reveals what the L-cones contribute to color perception
- **Deuteranopia** (missing medium-wavelength "green" cones): Isolates the contribution of M-cones
- **Tritanopia** (missing short-wavelength "blue" cones): Isolates the contribution of S-cones

This provides direct empirical data about the trichromatic theory of color vision that would be difficult to obtain otherwise.

4. Testing Color Constancy and Retinex Theory

These individuals can help test theories of color vision:

- Does color constancy work the same way with two functional cone types (dichromatic vision) as with three?
- How does the brain process color information when receiving different types of color signals from each eye?
- Do the retinex mechanisms operate independently in each eye or does the brain try to reconcile the conflicting information?

5. Understanding Neural Plasticity and Adaptation

- How does the visual cortex adapt to receiving different color information from each eye?
- Is there evidence of one eye's perception influencing the other's?
- How does binocular color vision integrate asymmetric inputs?

6. Practical Applications

Understanding these individuals helps with:

- **Designing better color vision tests:** Refining diagnostic tools for color vision deficiencies
- **Developing assistive technologies:** Creating filters or electronic aids that help color-blind individuals
- **Improving color-coding systems:** Designing displays and signage accessible to people with various color vision deficiencies

Why These Cases Are Rare

Unilateral or asymmetric color blindness is extremely rare because:

- Most color blindness is genetic (X-linked recessive for red-green color blindness), affecting both eyes equally
- Acquired color blindness from disease or injury typically affects both eyes or shows similar deficits

Cases do occur from:

- Unilateral retinal disease or damage
- Optic nerve disease affecting one eye
- Rare cases of asymmetric genetic expression
- Acquired damage from trauma, toxins, or disease affecting one eye

Discussion

The study of these exceptional individuals represents a natural experiment that cannot be ethically created and is extremely difficult to simulate. They provide a unique window into the fundamental mechanisms of color vision.

Historically, detailed studies of such individuals have contributed significantly to our understanding of color vision. Just as studies of brain-damaged patients (like Phineas Gage) revolutionized neuroscience by showing localization of brain function, studies of asymmetric color blindness can reveal how specific components of the visual system contribute to our rich experience of color.

These individuals essentially provide **ground truth** for color vision science—they can report with certainty what information is lost when specific cone types malfunction, something that researchers can otherwise only infer from indirect experiments with color-matching and discrimination tasks.

From a research methodology perspective, this is similar to the value of studying identical twins reared apart (eliminating genetic variation) or split-brain patients (revealing hemispheric specialization)—rare naturally occurring conditions that provide experimental controls impossible to achieve through designed experiments.

Propose a way to study the function of the rods alone, given they can sense light about 1000 times dimmer than the cones.

Show Solution

Strategy

To study rod function independently, we need to design experimental conditions where rods are active but cones are not. Given that rods are approximately 1000 times more sensitive to light than cones, we should exploit this sensitivity difference by working at light levels below the cone activation threshold but above the rod threshold (scotopic vision conditions). We also need to consider the different spectral sensitivities and spatial distributions of rods versus cones.

Solution

Several experimental approaches can isolate rod function:

Method 1: Scotopic (Dark-Adapted) Vision

The most straightforward approach exploits the different sensitivity ranges:

Procedure:

1. **Dark adaptation period:** Subject sits in complete darkness for 30-40 minutes
   - After ~7 minutes, cones reach maximum sensitivity
   - After ~30-40 minutes, rods reach maximum sensitivity (about 1000× more sensitive than cones)
2. **Low-light stimuli:** Present visual stimuli at light levels between rod threshold and cone threshold
   - Rod threshold: ~10^{-6} cd/m² (candelas per square meter)
   - Cone threshold: ~10^{-3} cd/m²
   - Use light levels of approximately 10^{-5} to 10^{-4} cd/m²
3. **Observations:** Under these conditions, only rods are functional
   - Vision will be achromatic (black, white, and shades of grey)
   - Spatial resolution will be lower (rods provide less sharp images)
   - Temporal resolution may differ

Method 2: Peripheral Vision Testing

Exploit the anatomical distribution of photoreceptors:

Procedure:

1. **Focus centrally:** Subject fixates on a central point
2. **Peripheral stimuli:** Present visual stimuli in the far periphery (>20° from fovea)
   - The fovea contains only cones (no rods)
   - Rod density peaks at about 20° from the fovea
3. **Scotopic conditions:** Use dim lighting where cones in periphery are less functional
4. **Avoid foveal vision:** Ensure subjects maintain central fixation and don't shift gaze toward test stimuli

This method isolates rods both by:

- Anatomical location (few or no cones in far periphery)
- Low light levels (rods more sensitive)

Method 3: Spectral Sensitivity Testing (Purkinje Effect)

Use the different spectral sensitivities of rods versus cones:

Background:

- Rods peak sensitivity: ~500 nm (blue-green)
- Cones (overall): peak at longer wavelengths (~555 nm yellow-green for photopic vision)

Procedure:

1. **Compare brightness perception:** Show two lights of different wavelengths but adjusted to appear equally bright under photopic (bright, cone) conditions
   - For example: a 650 nm red light and a 500 nm blue-green light
2. **Dim the lights gradually:** Reduce intensity while maintaining the same ratio
3. **Observe Purkinje shift:** As lighting decreases and rod vision dominates:
   - The 500 nm light will appear brighter (rods more sensitive here)
   - The 650 nm light will appear dimmer or disappear (rods less sensitive to red)
   - All color information will be lost (rods provide no color discrimination)

This demonstrates the transition from cone to rod function and allows measurement of rod spectral sensitivity.

Method 4: Temporal Characteristics

Exploit different temporal response properties:

Procedure:

1. **Flicker fusion frequency:** Present flickering lights at various frequencies under scotopic conditions
   - Rods have slower temporal response than cones
   - Rods have lower critical flicker fusion frequency (~20 Hz vs. ~50 Hz for cones)
2. **Dim conditions ensure rod function:** Use scotopic light levels
3. **Measure perception:** Determine the frequency at which flicker appears to fuse into continuous light
   - Lower fusion frequency indicates rod-mediated vision

Method 5: Cone Bleaching Recovery

Temporarily disable cones to isolate rods:

Procedure:

1. **Strong light exposure:** Expose eyes to bright light for several minutes
   - This "bleaches" the photopigments in both rods and cones
2. **Measure recovery in dim conditions:** Test visual sensitivity at low light levels during recovery

    - Cone photopigment regenerates relatively quickly (~7 minutes)
    - Rod photopigment (rhodopsin) regenerates more slowly (~30-40 minutes)
3. **Study late recovery phase:** Test between 7-40 minutes after light exposure
    - Cones have recovered and then become non-functional at low light levels
    - Rods are still recovering but are the only functional photoreceptors at these light levels

Method 6: Chromatic Adaptation

Use colored pre-adaptation to selectively reduce cone function:

Procedure:

1. **Red light adaptation:** Adapt subject to moderately bright red light (~650 nm) for 5-10 minutes
    - Long-wavelength (L) cones become adapted and less sensitive
    - Rods are relatively unaffected (they're not very sensitive to red light)
2. **Test with dim blue-green light:** Use low-intensity 500 nm light
    - Rods (peak sensitivity at 500 nm) will respond strongly
    - Cones are less sensitive due to adaptation and low light level
3. **Measurements:** Study detection thresholds, spatial resolution, temporal characteristics

Experimental Observations Expected from Rod-Only Vision:

When rods alone are functional, subjects will experience:

1. **No color vision:** Complete achromatopsia (monochromatic vision)
2. **Reduced spatial acuity:** Lower resolution (rods are larger and more convergent in their connections)
3. **High sensitivity to blue-green wavelengths:** Peak at ~500 nm
4. **Absence of central vision detail:** Foveal blindness (fovea has no rods)
5. **Good motion detection:** Rods are very sensitive to changes
6. **Slower temporal response:** Lower flicker fusion frequency
7. **Better peripheral vision:** Rods are concentrated away from fovea
8. **Very high absolute sensitivity:** Can detect individual photons under optimal conditions

Discussion

These methods for isolating rod function exploit three key differences between rods and cones:

1. **Absolute sensitivity** (1000× difference)
2. **Spectral sensitivity** (different wavelength responses)
3. **Anatomical distribution** (different locations in retina)

The most reliable approach combines multiple methods. For example, a comprehensive study might:

- Use dark-adapted subjects (30-40 minute adaptation)
- Present stimuli in the periphery (avoiding the rod-free fovea)
- Use wavelengths around 500 nm (rod peak sensitivity)
- Employ light levels in the scotopic range (10^{-5} to 10^{-4} cd/m²)

These techniques have practical applications:

- **Night vision research:** Understanding how to optimize vision in low-light conditions
- **Aviation and military:** Training for scotopic vision tasks
- **Medical diagnosis:** Testing for rod dysfunction (night blindness, retinitis pigmentosa)
- **Display design:** Creating instruments readable in dark conditions without compromising dark adaptation

Historically, these methods were crucial in establishing the duplicity theory of vision (the concept that rods and cones are separate systems with different functions), particularly through the work of Johannes von Kries in the late 19th century and subsequent researchers who precisely characterized rod and cone contributions to vision.

Glossary

hues
: identity of a color as it relates specifically to the spectrum

rods and cones
: two types of photoreceptors in the human retina; rods are responsible for vision at low light levels, while cones are active at higher light levels

simplified theory of color vision
: a theory that states that there are three primary colors, which correspond to the three types of cones

color constancy
: a part of the visual perception system that allows people to perceive color in a variety of conditions and to see some consistency in the color

retinex
: a theory proposed to explain color and brightness perception and constancies; is a combination of the words retina and cortex, which are the two areas responsible for the processing of visual information

retinex theory of color vision
: the ability to perceive color in an ambient-colored environment

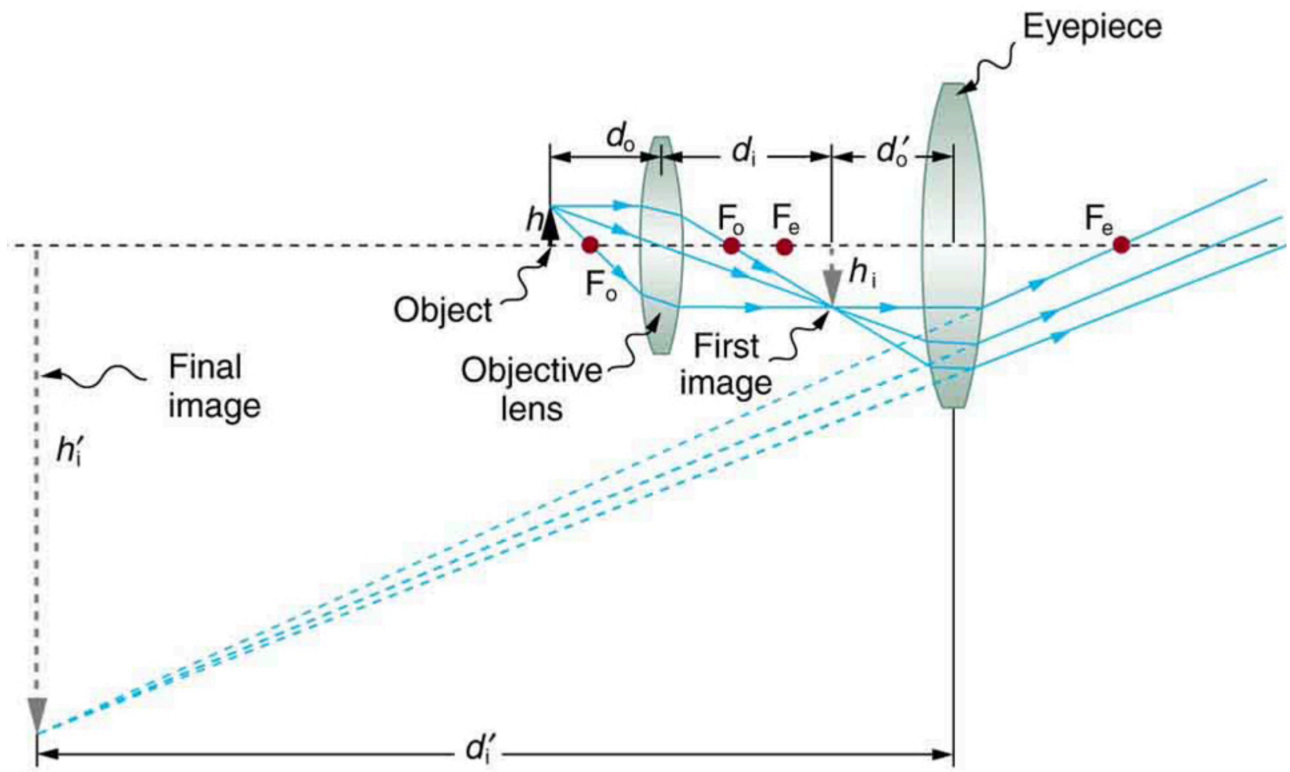
Microscopes

- Investigate different types of microscopes.
- Learn how image is formed in a compound microscope.

Although the eye is marvelous in its ability to see objects large and small, it obviously has limitations to the smallest details it can detect. Human desire to see beyond what is possible with the naked eye led to the use of optical instruments. In this section we will examine microscopes, instruments for enlarging the detail that we cannot see with the unaided eye. The microscope is a multiple-element system having more than a single lens or mirror. (See [Figure 1]) A microscope can be made from two convex lenses. The image formed by the first element becomes the object for the second element. The second element forms its own image, which is the object for the third element, and so on. Ray tracing helps to visualize the image formed. If the device is composed of thin lenses and mirrors that obey the thin lens equations, then it is not difficult to describe their behavior numerically.

Multiple lenses and mirrors are used in this microscope. (credit: U.S. Navy photo by Tom Watanabe)

Microscopes were first developed in the early 1600s by eyeglass makers in The Netherlands and Denmark. The simplest **compound microscope** is constructed from two convex lenses as shown schematically in [Figure 2]. The first lens is called the **objective lens**, and has typical magnification values from $5\times$ to $100\times$. In standard microscopes, the objectives are mounted such that when you switch between objectives, the sample remains in focus. Objectives arranged in this way are described as parfocal. The second, the **eyepiece**, also referred to as the ocular, has several lenses which slide inside a cylindrical barrel. The focusing ability is provided by the movement of both the objective lens and the eyepiece. The purpose of a microscope is to magnify small objects, and both lenses contribute to the final magnification. Additionally, the final enlarged image is produced in a location far enough from the observer to be easily viewed, since the eye cannot focus on objects or images that are too close.



A compound microscope composed of two lenses, an objective and an eyepiece. The objective forms a case 1 image that is larger than the object. This first image is the object for the eyepiece. The eyepiece forms a case 2 final image that is further magnified.

To see how the microscope in [Figure 2] forms an image, we consider its two lenses in succession. The object is slightly farther away from the objective lens than its focal length f_o, producing a case 1 image that is larger than the object. This first image is the object for the second lens, or eyepiece. The eyepiece is intentionally located so it can further magnify the image. The eyepiece is placed so that the first image is closer to it than its focal length f_e. Thus the eyepiece acts as a magnifying glass, and the final image is made even larger. The final image remains inverted, but it is farther from the observer, making it easy to view (the eye is most relaxed when viewing distant objects and normally cannot focus closer than 25 cm). Since each lens produces a magnification that multiplies the height of the image, it is apparent that the overall magnification m is the product of the individual magnifications:

$$m = m_o m_e,$$

where m_o is the magnification of the objective and m_e is the magnification of the eyepiece. This equation can be generalized for any combination of thin lenses and mirrors that obey the thin lens equations.

Overall Magnification

The overall magnification of a multiple-element system is the product of the individual magnifications of its elements.

Microscope Magnification

Calculate the magnification of an object placed 6.20 mm from a compound microscope that has a 6.00 mm focal length objective and a 50.0 mm focal length eyepiece. The objective and eyepiece are separated by 23.0 cm.

Strategy and Concept

This situation is similar to that shown in [Figure 2]. To find the overall magnification, we must find the magnification of the objective, then the magnification of the eyepiece. This involves using the thin lens equation.

Solution

The magnification of the objective lens is given as

$$m_o = -\frac{d_i}{d_o},$$

where d_o and d_i are the object and image distances, respectively, for the objective lens as labeled in [Figure 2]. The object distance is given to be $d_o = 6.20\text{mm}$, but the image distance d_i is not known. Isolating d_i, we have

$$\frac{1}{d_i} = \frac{1}{f_o} - \frac{1}{d_o},$$

where f_o is the focal length of the objective lens. Substituting known values gives

$$1d_{\text{i}}=16.00\text{mm}-16.20\text{mm}=0.00538\text{mm}.$$

We invert this to find d_{i} :

$$d_{\text{i}}=186\text{mm}.$$

Substituting this into the expression for m_{o} gives

$$m_{\text{o}}=-d_{\text{i}}d_{\text{o}}=-186\text{mm}6.20\text{mm}=-30.0.$$

Now we must find the magnification of the eyepiece, which is given by

$$m_{\text{e}}=-d'_{\text{i}}d'_{\text{o}},$$

where d'_{i} and d'_{o} are the image and object distances for the eyepiece (see [Figure 2]). The object distance is the distance of the first image from the eyepiece. Since the first image is 186 mm to the right of the objective and the eyepiece is 230 mm to the right of the objective, the object distance is $d'_{\text{o}}=230\text{mm}-186\text{mm}=44.0\text{mm}$. This places the first image closer to the eyepiece than its focal length, so that the eyepiece will form a case 2 image as shown in the figure. We still need to find the location of the final image d'_{i} in order to find the magnification. This is done as before to obtain a value for $1/d'_{\text{i}}$:

$$1d'_{\text{i}}=1f_{\text{e}}-1d'_{\text{o}}=150.0\text{mm}-144.0\text{mm}=-0.00273\text{mm}.$$

Inverting gives

$$d'_{\text{i}}=-\text{mm}0.00273=-367\text{mm}.$$

The eyepiece's magnification is thus

$$m_{\text{e}}=-d'_{\text{i}}d'_{\text{o}}=--367\text{mm}44.0\text{mm}=8.33.$$

So the overall magnification is

$$m=m_{\text{o}}m_{\text{e}}=(-30.0)(8.33)=-250.$$

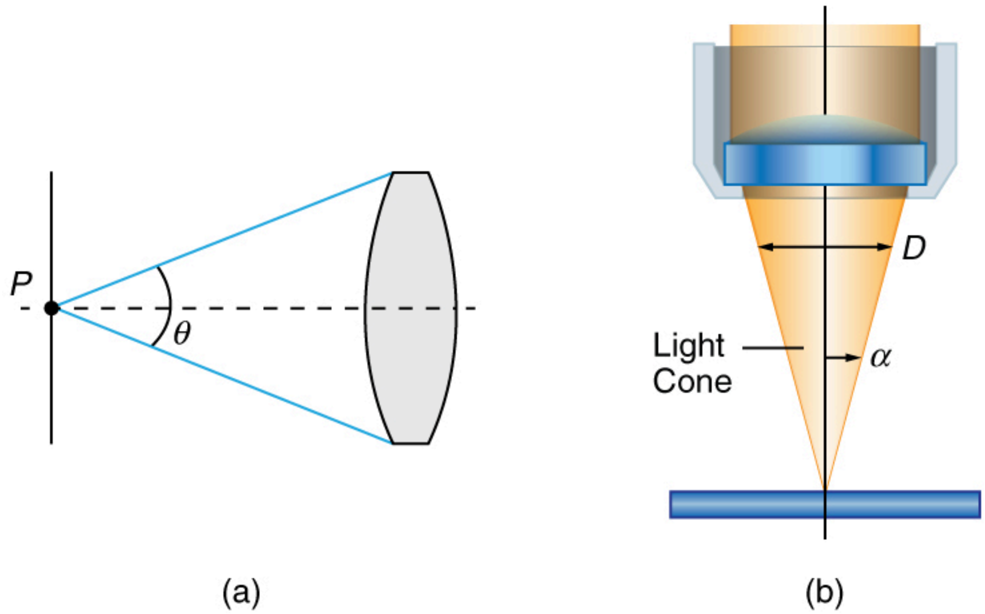
Discussion

Both the objective and the eyepiece contribute to the overall magnification, which is large and negative, consistent with [Figure 2], where the image is seen to be large and inverted. In this case, the image is virtual and inverted, which cannot happen for a single element (case 2 and case 3 images for single elements are virtual and upright). The final image is 367 mm (0.367 m) to the left of the eyepiece. Had the eyepiece been placed farther from the objective, it could have formed a case 1 image to the right. Such an image could be projected on a screen, but it would be behind the head of the person in the figure and not appropriate for direct viewing. The procedure used to solve this example is applicable in any multiple-element system. Each element is treated in turn, with each forming an image that becomes the object for the next element. The process is not more difficult than for single lenses or mirrors, only lengthier.

Normal optical microscopes can magnify up to $1500\times$ with a theoretical resolution of $-0.2\,\mu\text{m}$. The lenses can be quite complicated and are composed of multiple elements to reduce aberrations. Microscope objective lenses are particularly important as they primarily gather light from the specimen. Three parameters describe microscope objectives: the **numerical aperture ** (NA) , the magnification (m) , and the working distance. The NA is related to the light gathering ability of a lens and is obtained using the angle of acceptance θ formed by the maximum cone of rays focusing on the specimen (see [Figure 3](a)) and is given by

$$\text{NA}=n\sin\alpha,$$

where n is the refractive index of the medium between the lens and the specimen and $\alpha=\theta/2$. As the angle of acceptance given by θ increases, NA becomes larger and more light is gathered from a smaller focal region giving higher resolution. A 0.75NA objective gives more detail than a 0.10NA objective.



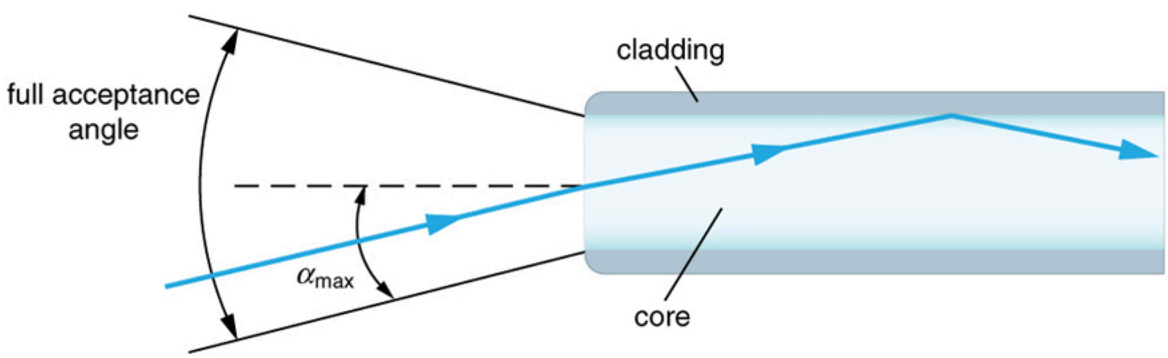
(a) The numerical aperture (NA) of a microscope objective lens refers to the light-gathering ability of the lens and is calculated using half the angle of acceptance θ. (b) Here, α; is half the acceptance angle for light rays from a specimen entering a camera lens, and D is the diameter of the aperture that controls the light entering the lens.

While the numerical aperture can be used to compare resolutions of various objectives, it does not indicate how far the lens could be from the specimen. This is specified by the “working distance,” which is the distance (in mm usually) from the front lens element of the objective to the specimen, or cover glass. The higher the NA the closer the lens will be to the specimen and the more chances there are of breaking the cover slip and damaging both the specimen and the lens. The focal length of an objective lens is different than the working distance. This is because objective lenses are made of a combination of lenses and the focal length is measured from inside the barrel. The working distance is a parameter that microscopists can use more readily as it is measured from the outermost lens. The working distance decreases as the NA and magnification both increase.

The term $f/\#$ in general is called the f -number and is used to denote the light per unit area reaching the image plane. In photography, an image of an object at infinity is formed at the focal point and the f -number is given by the ratio of the focal length f of the lens and the diameter D of the aperture controlling the light into the lens (see [Figure 3](b)) . If the acceptance angle is small the NA of the lens can also be used as given below.

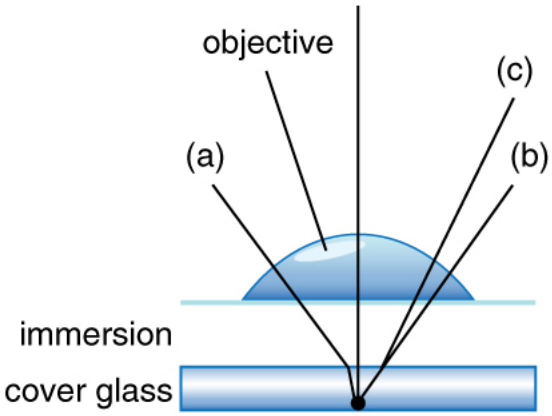
$f/\#=f\,D\approx12\text{NA}.$

As the f -number decreases, the camera is able to gather light from a larger angle, giving wide-angle photography. As usual there is a trade-off. A greater $f/\#$ means less light reaches the image plane. A setting of $f/16$ usually allows one to take pictures in bright sunlight as the aperture diameter is small. In optical fibers, light needs to be focused into the fiber. [Figure 4] shows the angle used in calculating the NA of an optical fiber.



Light rays enter an optical fiber. The numerical aperture of the optical fiber can be determined by using the angle \(\alpha \);max.

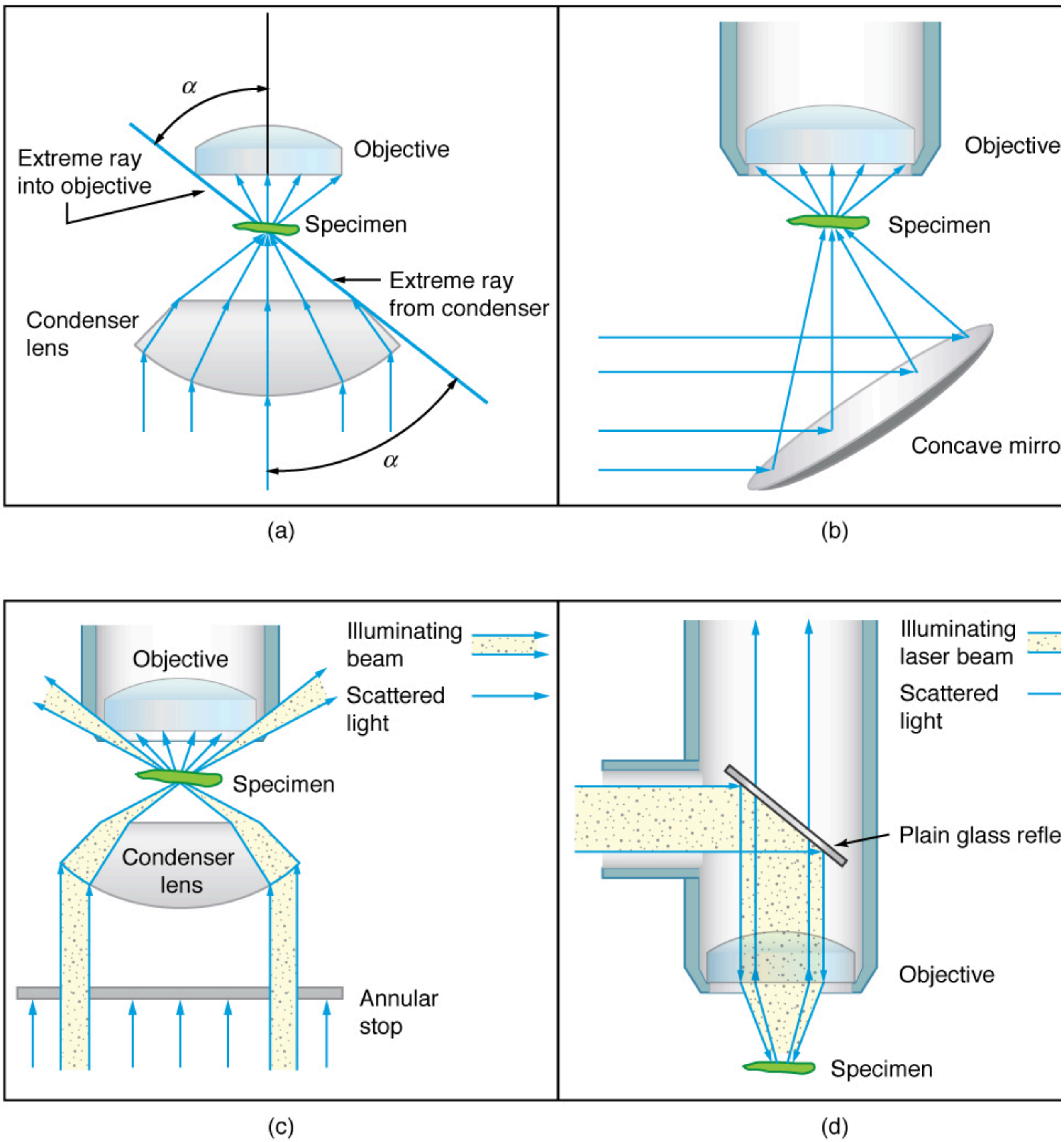
Can the NA be larger than 1.00? The answer is ‘yes’ if we use immersion lenses in which a medium such as oil, glycerine or water is placed between the objective and the microscope cover slip. This minimizes the mismatch in refractive indices as light rays go through different media, generally providing a greater light-gathering ability and an increase in resolution. [Figure 5] shows light rays when using air and immersion lenses.



Light rays from a specimen entering the objective. Paths for immersion medium of air (a), water (b) $(n=1.33)$), and oil (c) $(n=1.51)$ are shown. The water and oil immersions allow more rays to enter the objective, increasing the resolution.

When using a microscope we do not see the entire extent of the sample. Depending on the eyepiece and objective lens we see a restricted region which we say is the field of view. The objective is then manipulated in two-dimensions above the sample to view other regions of the sample. Electronic scanning of either the objective or the sample is used in scanning microscopy. The image formed at each point during the scanning is combined using a computer to generate an image of a larger region of the sample at a selected magnification.

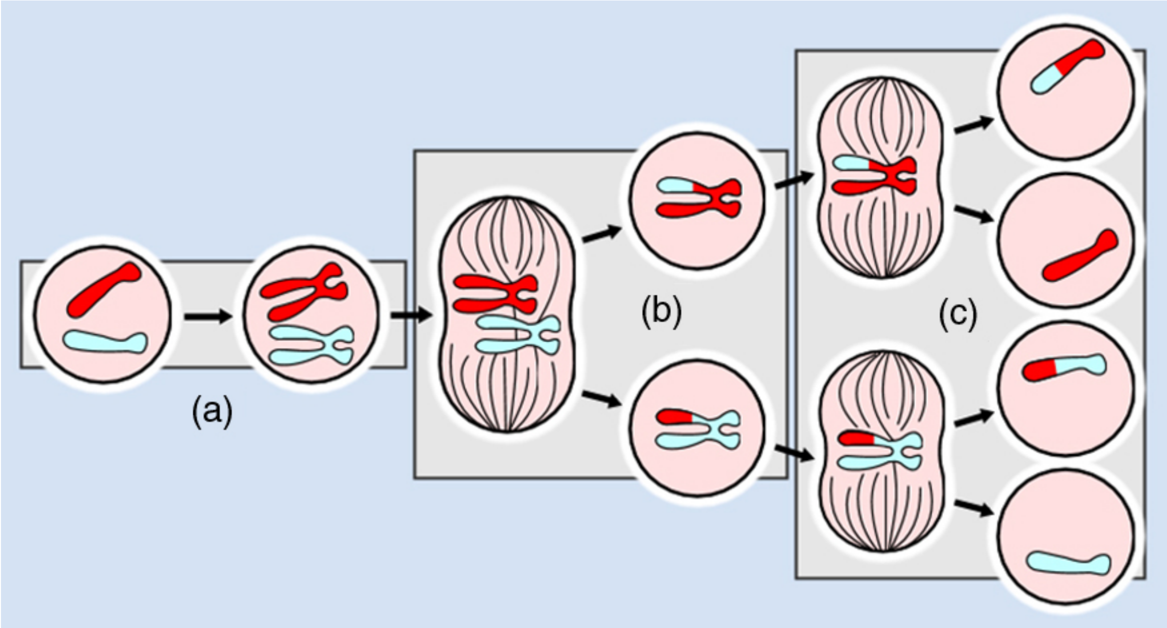
When using a microscope, we rely on gathering light to form an image. Hence most specimens need to be illuminated, particularly at higher magnifications, when observing details that are so small that they reflect only small amounts of light. To make such objects easily visible, the intensity of light falling on them needs to be increased. Special illuminating systems called condensers are used for this purpose. The type of condenser that is suitable for an application depends on how the specimen is examined, whether by transmission, scattering or reflecting. See [Figure 6] for an example of each. White light sources are common and lasers are often used. Laser light illumination tends to be quite intense, and it is important to ensure that the light does not result in the degradation of the specimen.



Illumination of a specimen in a microscope. (a) Transmitted light from a condenser lens. (b) Transmitted light from a mirror condenser. (c) Dark field illumination by scattering (the illuminating beam misses the objective lens). (d) High magnification illumination with reflected light – normally laser light.

We normally associate microscopes with visible light but X-ray and electron microscopes provide greater resolution. The focusing and basic physics is the same as that just described, even though the lenses require different technology. The electron microscope requires vacuum chambers so that the electrons can proceed unheeded. Magnifications of 50 million times provide the ability to determine positions of individual atoms within materials. An electron microscope is shown in [Figure 7]. We do not use our eyes to form images; rather images are recorded electronically and displayed on computers. In fact observing and saving images formed by optical microscopes on computers is now done routinely. Video recordings of what occurs in a microscope can be made for viewing by many people at later dates. Advances in this powerful technology continue. In the 1990s, Pratibha L. Gai invented the environmental transmission electron microscope (ETEM), which was the first device capable of observing individual atoms in chemical reactions.

An electron microscope has the capability to image individual atoms on a material. The microscope uses vacuum technology, sophisticated detectors and state of the art image processing software. (credit: Dave Pape)



The image shows a sequence of events that takes place during meiosis. (credit: PatríciaR, Wikimedia Commons; National Center for Biotechnology Information)

Take-Home Experiment: Make a Lens

Look through a clear glass or plastic bottle and describe what you see. Now fill the bottle with water and describe what you see. Use the water bottle as a lens to produce the image of a bright object and estimate the focal length of the water bottle lens. How is the focal length a function of the depth of water in the bottle?

Section Summary

- The microscope is a multiple-element system having more than a single lens or mirror.
- Many optical devices contain more than a single lens or mirror. These are analysed by considering each element sequentially. The image formed by the first is the object for the second, and so on. The same ray tracing and thin lens techniques apply to each lens element.

- The overall magnification of a multiple-element system is the product of the magnifications of its individual elements. For a two-element system with an objective and an eyepiece, this is

  $m = m_o m_e,$

  where m_o is the magnification of the objective and m_e is the magnification of the eyepiece, such as for a microscope.

- Microscopes are instruments for allowing us to see detail we would not be able to see with the unaided eye and consist of a range of components.
- The eyepiece and objective contribute to the magnification. The numerical aperture (NA) of an objective is given by
  $\text{NA} = n \sin\alpha$

where n is the refractive index and α the angle of acceptance.

- Immersion techniques are often used to improve the light gathering ability of microscopes. The specimen is illuminated by transmitted, scattered or reflected light though a condenser.

- The $f/\#$ describes the light gathering ability of a lens. It is given by

  $f/\# = \frac{f}{D} \approx \frac{1}{2\text{NA}}.$

Conceptual Questions

Geometric optics describes the interaction of light with macroscopic objects. Why, then, is it correct to use geometric optics to analyse a microscope's image?

Show Solution

Strategy

To understand why geometric optics is applicable to microscope analysis, we need to examine the size scales involved and determine when the ray approximation remains valid. Geometric optics is valid when the physical dimensions of optical components (lenses, apertures) are much larger than the wavelength of light. We must consider the microscope's optical elements rather than the specimen being observed.

Solution

Geometric optics is indeed correct for analyzing a microscope's image formation, despite the fact that microscopes are used to view microscopic objects. The key distinction is between the **object being viewed** and the **optical system doing the viewing**.

The validity of geometric optics depends on comparing the wavelength of light ($\lambda \approx 400-700 \text{ nm}$ for visible light) with the size of the **optical elements**, not the specimen:

Microscope Optical Components:

- **Objective lens diameter**: typically 5-20 mm
- **Eyepiece lens diameter**: typically 10-25 mm
- **Aperture diameters**: typically several millimeters
- **Lens thicknesses**: several millimeters

These dimensions are approximately **10,000 to 50,000 times larger** than the wavelength of visible light ($\lambda \approx 500 \text{ nm}$).

Why Geometric Optics Applies:

When $D >> \lambda$ (where D is the characteristic size of optical elements), light propagation can be accurately described by:

1. **Rectilinear propagation**: Light travels in straight lines (rays)
2. **Reflection and refraction laws**: Light obeys the laws of reflection and Snell's law at interfaces
3. **Thin lens equations**: The focal length, magnification, and image formation can be calculated using ray tracing

When Wave Optics Becomes Necessary:

While geometric optics correctly describes **image formation and magnification** in microscopes, it has limitations:

- **Diffraction limits resolution**: The ultimate resolving power of the microscope is determined by wave optics (Rayleigh criterion: $d = 1.22\lambda/\text{NA}$)
- **Interference effects**: Phenomena like thin-film interference in optical coatings require wave analysis
- **Coherence effects**: Phase relationships in techniques like phase-contrast microscopy

However, these wave effects don't invalidate the geometric optics analysis of where images form, their magnification, or the ray paths through the system.

Discussion

This question highlights an important principle: the applicability of geometric optics depends on the size of the optical system components, not the size of the objects being observed. A microscope's lenses are macroscopic, so geometric optics correctly predicts:

- Where the intermediate image forms after the objective
- The magnification produced by each lens element
- The location of the final virtual image
- The overall system magnification as $m = m_o \times m_e$

In practice, microscope design uses **both** approaches:

1. **Geometric optics** for lens placement, focal lengths, magnification calculations, and ray tracing
2. **Wave optics** for understanding resolution limits, numerical aperture effects, and diffraction-limited performance

The specimen itself may be microscopic (even comparable to or smaller than λ), but this doesn't affect the validity of using geometric optics to analyze how the macroscopic lens system forms and magnifies the image. The wave nature of light becomes important when discussing the **quality** and **resolution** of that image, but not the fundamental process of image formation by the lens system.

This is analogous to using geometric optics for telescope design: we use ray tracing to design the telescope even though we're observing objects (like distant stars) whose angular sizes may be diffraction-limited. The optical system itself is macroscopic, making geometric optics the appropriate tool for analyzing image formation.

The image produced by the microscope in [Figure 2] cannot be projected. Could extra lenses or mirrors project it? Explain.

Show Solution

Strategy

To answer this question, we must first understand why the final image in the standard microscope configuration shown in Figure 2 cannot be projected. We need to distinguish between real and virtual images, then determine whether additional optical elements could convert the virtual final image into a real, projectable image.

Solution

The final image produced by the microscope in Figure 2 **is virtual**, which is why it cannot be projected onto a screen. Let's examine why this is the case and whether it can be remedied:

Why the Final Image is Virtual:

In the standard compound microscope configuration:

1. The **objective** creates a real, inverted, magnified intermediate image (Case 1)
2. This intermediate image is positioned **closer to the eyepiece than its focal length** ($d'_o < f_e$)
3. The **eyepiece** then acts as a magnifying glass, producing a virtual, magnified final image (Case 2)
4. Virtual images are formed by the divergence of light rays—they appear to originate from a location where light rays do not actually converge
5. Virtual images **cannot be projected** onto a screen because the light rays never actually pass through the image location

Can Extra Lenses or Mirrors Project It? Yes!

The virtual final image **can** be converted into a real, projectable image by adding extra optical elements. There are several approaches:

Method 1: Repositioning the Eyepiece Move the eyepiece farther from the objective so that the intermediate image is beyond the eyepiece's focal length ($d'_o > f_e$). This would make the eyepiece produce a real, inverted final image (Case 1) that could be projected.

Method 2: Adding a Projection Lens Place an additional converging lens beyond the eyepiece. This projection lens would:

- Treat the virtual image as its object
- Have the virtual image positioned beyond its focal length
- Form a real image that can be projected onto a screen or photographic film

Method 3: Camera Attachment Modern microscopes often use camera systems that include additional relay lenses to:

- Capture the virtual image
- Re-form it as a real image on the camera sensor or film plane
- This is precisely how microscope photography and digital imaging work

Method 4: Using a Mirror System While less common, carefully positioned mirrors could redirect the light to form a real image, though this doesn't fundamentally change the optics—you'd still need the intermediate image to be positioned appropriately relative to the final optical element.

Trade-offs:

Converting to a projectable image has consequences:

- **For direct viewing**: The virtual image configuration is ideal because the image appears at a comfortable viewing distance (or at infinity for relaxed viewing)
- **For projection**: A real image is necessary, but it would form behind the observer's head in the standard microscope orientation, making direct viewing awkward

- **Eye accommodation**: The virtual image can be positioned to minimize eye strain; a real image for projection would not serve this purpose

Discussion

This question illustrates the fundamental difference between visual observation and image projection. The standard compound microscope is optimized for **direct visual observation** by producing a virtual final image, which:

- Appears at a comfortable viewing distance (typically 25 cm or at infinity)
- Allows the eye's lens to remain relaxed or minimally accommodated
- Provides maximum comfort for extended observation periods

However, this same configuration makes projection impossible. In practice, microscopes designed for projection or photography use modified optical paths:

Educational/Demonstration Microscopes:

- Use a modified eyepiece position or additional projection lens
- Create a real image that can be projected onto a screen for classroom viewing
- Often called "projection microscopes" or equipped with "projection eyepieces"

Research Microscopes:

- Include dedicated camera ports with relay optics
- The camera path splits off before the eyepiece using a beam splitter
- Allows simultaneous visual observation (virtual image) and photography/projection (real image)

Historical Note: Early photomicrography required photographers to modify the eyepiece position or add projection lenses to create real images on photographic plates. Modern microscopes incorporate these capabilities with sophisticated relay optics, beam splitters, and digital cameras that seamlessly convert the optical image to electronic form.

The answer to the question is definitively **yes**—extra lenses (or repositioning the existing eyepiece) can convert the virtual final image into a real, projectable image. The standard configuration produces a virtual image by design for comfortable viewing, not because it's impossible to create a real image with the same optical components arranged differently.

Why not have the objective of a microscope form a case 2 image with a large magnification? (Hint: Consider the location of that image and the difficulty that would pose for using the eyepiece as a magnifier.)

Show Solution

Strategy

To understand why the objective doesn't form a case 2 image, we need to recall the characteristics of case 2 image formation and consider the practical requirements for the eyepiece to function as a magnifier. We'll examine where a case 2 image would be located and why this creates insurmountable problems for the microscope's optical design.

Solution

Having the objective form a **case 2 image** would be highly impractical for a compound microscope, despite the potential for large magnification. Let's examine why:

Case 2 Image Characteristics:

A case 2 image occurs when the object is placed **closer to the lens than its focal length** ($d_o < f$). This produces:

- A **virtual** image ($d_i < 0$)
- An **upright** image (positive magnification)
- An image on the **same side of the lens as the object**
- Magnification that increases as the object approaches the focal point

Why This Doesn't Work for Microscope Objectives:

Problem 1: Virtual Image Location If the objective formed a case 2 image, it would be virtual and located on the **same side of the objective as the specimen**. This means:

- The image would appear to be **inside or below the microscope stage**
- The eyepiece, positioned **above** the objective, would have no real light rays to work with
- You cannot use a lens to further magnify a virtual image that's on the opposite side of that lens

Problem 2: Eyepiece Cannot Function as a Magnifier For the eyepiece to work as a magnifying glass, it needs:

- A **real object or real intermediate image** positioned just inside its focal point
- Light rays that are actually converging from the intermediate image
- With a virtual image from the objective located below the stage, there's no way to position the eyepiece appropriately

Problem 3: Impractical Geometry Even if we could somehow position the eyepiece to view the virtual image, we would face:

- The eyepiece would need to be placed **between** the objective and the specimen

- This is geometrically impossible in a practical microscope design
- The working distance would be negative (inside the objective lens)

Problem 4: No Image Relay In a compound microscope:

- The objective must create a **real** intermediate image
- This real image serves as a **real object** for the eyepiece
- Light must physically pass through the intermediate image location
- A virtual image cannot serve this relay function

Why Case 1 is Used Instead:

The standard microscope design uses the objective to create a **case 1 image** (d_o slightly greater than f_o):

- Creates a **real, inverted, magnified** intermediate image
- Image forms at a practical distance **above** the objective (inside the tube)
- This real image can serve as the object for the eyepiece
- The eyepiece is positioned so this intermediate image is just **inside** the eyepiece focal length
- The eyepiece then forms a case 2 image (virtual, magnified) for comfortable viewing

Magnification Considerations:

While case 2 can produce very large magnification (approaching infinity as $d_o \to f$), this is not useful because:

- The image is virtual and poorly positioned
- The eyepiece cannot access it
- The total system magnification $m = m_o \times m_e$ requires both stages to work together
- Case 1 for the objective and case 2 for the eyepiece is the optimal combination

Discussion

This question highlights the crucial difference between **single-lens magnification** and **multi-element optical systems**. While a single lens forming a case 2 image (like a simple magnifying glass) works well for direct viewing, it's unsuitable as the objective in a compound microscope.

The compound microscope requires **image relay**:

1. The objective must form a **real** intermediate image that physically exists in space
2. This real image acts as a real object for the second stage (eyepiece)
3. The eyepiece then provides additional magnification and positions the final image for comfortable viewing

Practical Analogy: Think of it like a relay race: the objective must create a "baton" (real intermediate image) that can be "handed off" to the eyepiece. A virtual image is like a mirage—it can't be handed off because it doesn't physically exist at its apparent location.

Optimal Design:

- **Objective**: Case 1 configuration (d_o slightly > f_o)
  - Creates real, magnified intermediate image
  - Typical magnifications: 4× to 100×
- **Eyepiece**: Case 2 configuration ($d'_o < f_e$)
  - Creates virtual, magnified final image
  - Typical magnifications: 5× to 20×
- **Total magnification**: $m = m_o \times m_e = 20\times \text{ to } 2000\times$

This two-stage design with case 1 followed by case 2 is fundamental to all compound microscopes, binoculars, and telescopes used for direct visual observation. Each stage is optimized for its specific role: the objective for image formation and initial magnification, the eyepiece for final magnification and comfortable viewing geometry.

The hint in the question points directly to the core problem: a case 2 image from the objective would be virtual and located where the eyepiece cannot access it, making it impossible for the eyepiece to function as a magnifier. The objective **must** create a real image that the eyepiece can then magnify.

What advantages do oil immersion objectives offer?

Show Solution

Strategy

To understand the advantages of oil immersion objectives, we need to examine the physics of light gathering and resolution in microscopy. We'll analyze how the refractive index of the medium between the specimen and objective affects the numerical aperture ($\text{NA} = n \sin \alpha$), and consequently the resolution and light-gathering power of the microscope.

Solution

Oil immersion objectives offer several significant advantages over dry (air) objectives, all stemming from the use of a high-refractive-index immersion medium between the specimen and the objective lens.

Primary Advantages:

1. Increased Numerical Aperture (NA)

The numerical aperture is given by:

$$\mathrm{NA} = n \sin\alpha$$

where n is the refractive index of the medium and α is the half-angle of the acceptance cone.

- **Air medium**: $n = 1.00$, maximum achievable $\mathrm{NA} \approx 0.95$
- **Oil immersion**: $n \approx 1.51$, maximum achievable $\mathrm{NA} \approx 1.4$

The oil's higher refractive index allows the NA to exceed 1.0, which is impossible with air. This represents approximately a **50% increase** in numerical aperture.

2. Improved Resolution

The minimum resolvable distance (resolution) is given by the Rayleigh criterion:

$$d = \frac{1.22\lambda}{2\mathrm{NA}} = \frac{0.61\lambda}{\mathrm{NA}}$$

With increased NA, oil immersion provides:

- **Better resolution** by a factor of approximately 1.5
- For $\lambda = 550$ nm (green light):
  - Dry objective ($\mathrm{NA} = 0.95$): $d \approx 350$ nm
  - Oil immersion ($\mathrm{NA} = 1.4$): $d \approx 240$ nm

This **47% improvement in resolution** allows observation of finer details and smaller structures.

3. Reduced Light Loss Due to Refraction

When light exits the specimen (with refractive index $n \approx 1.5$ for biological specimens and glass coverslips):

Without oil immersion:

- Light travels from glass coverslip ($n = 1.52$) to air ($n = 1.00$)
- Large refractive index mismatch causes significant refraction
- Light rays at steep angles undergo **total internal reflection** at the glass-air interface
- These rays never reach the objective lens
- Critical angle: $\theta_c = \sin^{-1}(n_{\mathrm{air}}/n_{\mathrm{glass}}) = \sin^{-1}(1.00/1.52) \approx 41°$

With oil immersion:

- Immersion oil ($n \approx 1.51$) closely matches the coverslip ($n \approx 1.52$)
- Minimal refraction at the glass-oil interface
- Light rays at steep angles can reach the objective
- No total internal reflection for angles up to nearly 90°

4. Increased Light-Gathering Power

The light-gathering power is proportional to $(\mathrm{NA})^2$:

- Oil immersion captures light from a much wider cone of angles
- More photons reach the detector
- Brighter images, especially important for:
  - Dim specimens
  - Fluorescence microscopy
  - High-speed imaging with short exposure times

5. Better Optical Performance

- **Reduced spherical aberration**: The refractive index matching minimizes aberrations at the coverslip-medium interface
- **Improved contrast**: Less scattered light means better image contrast
- **More uniform illumination**: Reduced optical artifacts from refraction

Practical Considerations:

Advantages in practice:

- Essential for bacteriology (bacteria are near the resolution limit)
- Critical for observing cellular ultrastructure
- Necessary for high-magnification work (100× objectives)

- Enables numerical apertures impossible with dry objectives

Disadvantages to consider:

- Requires careful technique (proper oil application)
- Oil must be cleaned from slides and objectives after use
- Cannot be used with living specimens in culture dishes
- More expensive objectives
- Shorter working distance (typically 0.1-0.2 mm)

Discussion

Oil immersion objectives represent a crucial technological advancement in optical microscopy, pushing the resolution limits of light microscopy to their theoretical maximum. The fundamental physics behind their advantage lies in **refractive index matching**.

Historical Significance: Ernst Abbe (1840-1905) developed the theory of microscope resolution and recognized that the refractive index of the medium limits the NA. This insight led to the development of oil immersion objectives in the 1870s, revolutionizing microscopy and enabling major discoveries in microbiology, including the visualization of bacteria and subcellular structures.

Comparison Table:

Property	Dry Objective	Oil Immersion
Medium refractive index	$n = 1.00$	$n \approx 1.51$
Maximum NA	≈ 0.95	≈ 1.4
Resolution (green light)	≈ 350 nm	≈ 240 nm
Light gathering	Lower	~2× higher
Total internal reflection	Limits acceptance angle	Minimized

Modern Applications:

Today, oil immersion objectives are indispensable for:

- **Medical diagnostics**: Blood smears, tissue pathology
- **Microbiology**: Bacterial identification, viral particle visualization
- **Cell biology**: Chromosome studies, subcellular organelles
- **Fluorescence microscopy**: Maximizing photon collection
- **Super-resolution microscopy**: Techniques like STED and PALM

Some advanced techniques use even higher refractive index immersion media:

- **Water immersion** ($n = 1.33$): For live cell imaging
- **Glycerol immersion** ($n = 1.47$): Intermediate between water and oil
- **Special oils** ($n = 1.78$): For pushing resolution even further

The oil immersion objective remains one of the most important innovations in microscopy history, enabling the visualization of structures at or near the diffraction limit of visible light. Without oil immersion, entire fields like bacteriology and cellular biology would have been severely limited in their development. The ability to achieve $\text{NA} > 1.0$ is physically impossible with air, making oil immersion not just an improvement but a necessity for maximum resolution microscopy.

How does the NA of a microscope compare with the NA of an optical fiber?

Show Solution

Strategy

To compare the numerical apertures of microscopes and optical fibers, we need to understand how NA is defined in each context, what physical factors limit the NA in each application, and what typical values are achieved. We'll examine the formula $\text{NA} = n \sin \alpha$ for both systems and consider their different design objectives and constraints.

Solution

The numerical aperture (NA) is defined similarly for both microscopes and optical fibers, but the typical values and physical constraints differ significantly between these two optical systems.

Numerical Aperture Definition:

For both systems:

$$\text{NA} = n \sin \alpha$$

where:

- n is the refractive index of the medium

- α is the half-angle of the maximum acceptance cone

Microscope Objectives:

Typical NA values:

- **Low-power objectives** (4×, 10×): $\mathrm{NA} = 0.10$ to 0.30
- **Mid-power objectives** (20×, 40×): $\mathrm{NA} = 0.40$ to 0.75
- **High-power dry objectives** (40×, 60×): $\mathrm{NA} = 0.65$ to 0.95
- **Oil immersion objectives** (100×): $\mathrm{NA} = 1.25$ to 1.4
- **Specialized objectives**: NA up to 1.6 with special immersion media

Design objective: Maximize NA to achieve the highest possible resolution ($d \propto 1/\mathrm{NA}$) and light-gathering power (proportional to NA^2).

Optical Fibers:

Typical NA values:

- **Multimode step-index fibers**: $\mathrm{NA} = 0.20$ to 0.29
- **Multimode graded-index fibers**: $\mathrm{NA} = 0.20$ to 0.275
- **Single-mode fibers**: $\mathrm{NA} = 0.10$ to 0.14
- **Large-core fibers** (for high power): NA up to 0.50

Design objective: The NA must be carefully controlled to:

- Enable light coupling into the fiber
- Control modal dispersion
- Maintain single-mode operation (for telecom applications)
- Balance between light-gathering and signal quality

For an optical fiber, the NA is determined by the core and cladding refractive indices:

$$\mathrm{NA} = \sqrt{n_{\mathrm{core}}^2 - n_{\mathrm{cladding}}^2}$$

For example, with $n_{\mathrm{core}} = 1.48$ and $n_{\mathrm{cladding}} = 1.46$:

$$\mathrm{NA} = \sqrt{(1.48)^2 - (1.46)^2} = \sqrt{2.1904 - 2.1316} = \sqrt{0.0588} \approx 0.24$$

Comparison:

System	Typical NA Range	Maximum NA	Medium
Microscope objectives (dry)	0.10 - 0.95	~0.95	Air ($n = 1.00$)
Microscope objectives (oil)	1.25 - 1.40	~1.6	Oil ($n \approx 1.51$)
Optical fibers (multimode)	0.20 - 0.29	~0.50	Fiber core
Optical fibers (single-mode)	0.10 - 0.14	~0.14	Fiber core

Key Differences:

1. Maximum NA:

- **Microscopes can achieve much higher NA values**, especially with immersion objectives (NA > 1.0)
- **Optical fibers are limited** to NA < 0.5 in most practical applications

2. Design Philosophy:

- **Microscopes**: NA is maximized to improve resolution
- **Optical fibers**: NA is carefully controlled, not maximized, because:
  - Higher NA increases modal dispersion in multimode fibers
  - Telecom applications require low NA for single-mode operation
  - Too high NA makes fiber coupling and handling difficult

3. Physical Constraints:

- **Microscopes**: Limited by the refractive index of available immersion media and lens design
- **Optical fibers**: Limited by available glass formulations and the requirement to maintain total internal reflection

4. Acceptance Angles:

From $\mathrm{NA} = n \sin\alpha$ (with $n = 1$ for air):

Microscope (high-power dry, NA = 0.95):

$\alpha = \sin^{-1}(0.95) = 72°$

Acceptance angle: $\theta = 2\alpha = 144°$

Microscope (oil immersion, NA = 1.4, n = 1.51):

$\alpha = \sin^{-1}(1.4/1.51) = 68°$

Optical fiber (multimode, NA = 0.24):

$\alpha = \sin^{-1}(0.24) = 14°$

Acceptance angle: $\theta = 2\alpha = 28°$

Discussion

The numerical aperture serves fundamentally different purposes in microscopes versus optical fibers, leading to very different typical values:

Microscope Objectives: The NA is pushed to the maximum possible value because:

- **Resolution is paramount**: $d = 0.61\lambda/\mathrm{NA}$
- Higher NA directly translates to the ability to see finer details
- Light-gathering power increases with $(\mathrm{NA})^2$, crucial for dim specimens
- Modern microscopy objectives represent the pinnacle of optical engineering, achieving NA values approaching theoretical limits

Optical Fibers: The NA is deliberately kept moderate because:

- **Signal integrity matters more than light gathering** in telecommunications
- Lower NA reduces modal dispersion, enabling higher bandwidth
- Single-mode fibers (for long-distance telecom) require very low NA (0.10-0.14)
- Easier fiber splicing and coupling with moderate NA
- Reduced sensitivity to bending losses

Practical Implications:

For microscopy:

- An oil immersion objective with $\mathrm{NA} = 1.4$ can resolve structures down to approximately 240 nm with visible light
- This represents nearly the theoretical limit for optical microscopy
- The wide acceptance angle (136° for oil immersion) requires precision alignment

For optical fibers:

- A typical telecom fiber with $\mathrm{NA} = 0.12$ accepts light within a narrow cone (±7°)
- This narrow cone is essential for maintaining signal quality over kilometers
- Light coupling requires precision alignment or focusing optics

Range Comparison: While there is some overlap in the NA ranges (both can have values around 0.2-0.3), the objectives diverge:

- **Microscopes push toward NA > 1.0** for maximum resolution
- **Optical fibers stay at NA < 0.3** for optimal signal transmission

Historical Context: Both technologies were developed in the 19th and 20th centuries, but with different goals. Microscopy's NA evolution was driven by the quest for better resolution (Ernst Abbe's work in the 1870s), while optical fiber NA was optimized for telecommunications (especially after the 1970s), where signal quality and bandwidth trump light-gathering ability.

In summary: **Microscope objectives typically have higher NA values than optical fibers**, with high-end microscope objectives (especially oil immersion) achieving NA values of 1.25-1.4, while typical optical fibers have NA values of 0.10-0.29. The fundamental reason is different design objectives: microscopes maximize NA for resolution, while fibers control NA for signal integrity and single-mode operation.

Problem Exercises

A microscope with an overall magnification of 800 has an objective that magnifies by 200. (a) What is the magnification of the eyepiece? (b) If there are two other objectives that can be used, having magnifications of 100 and 400, what other total magnifications are possible?

Show Solution

Strategy

For part (a), the overall magnification of a compound microscope is the product of the objective and eyepiece magnifications: $m = m_o \times m_e$. We solve for m_e. For part (b), we calculate the total magnification for each of the two other objectives using the same eyepiece magnification.

Solution

Given values:

- Overall magnification: $m = 800$
- Objective magnification: $m_o = 200$
- Other objectives available: $m_{o,2} = 100$ and $m_{o,3} = 400$

(a) Find the eyepiece magnification:

$$m = m_o \times m_e$$
$$m_e = \frac{m}{m_o} = \frac{800}{200} = 4.00$$

(b) Calculate magnifications with other objectives:

With the 100× objective:

$$m_1 = m_{o,2} \times m_e = 100 \times 4.00 = 400$$

With the 400× objective:

$$m_2 = m_{o,3} \times m_e = 400 \times 4.00 = 1600$$

Discussion

Part (a): The eyepiece has a magnification of 4.00×, which is a common low-power eyepiece. Eyepieces typically range from 5× to 20×, with 10× being standard. A 4× eyepiece provides a wide field of view, making it easier to locate and navigate specimens.

Part (b): With the three available objectives (100×, 200×, 400×) and the 4× eyepiece, the microscope can provide total magnifications of 400×, 800×, and 1600×. This represents a typical range for research-grade compound microscopes:

- 400× (100× objective): Good for general observation of cellular structures
- 800× (200× objective): Excellent for detailed cellular examination
- 1600× (400× objective): Near the practical limit for optical microscopy

The 1600× magnification is approaching the maximum useful magnification for light microscopy, which is limited by diffraction to about 1500-2000× depending on wavelength and numerical aperture. Beyond this, you get "empty magnification"—the image becomes larger but reveals no additional detail.

This microscope setup provides excellent versatility. Users can:

1. Start at 400× to locate and position specimens (wide field of view)
2. Switch to 800× for routine detailed examination
3. Use 1600× for maximum resolution of the finest details

The 4× eyepiece is particularly useful because it provides a compromise between magnification and field of view, making specimen navigation easier than with higher power eyepieces.

(a) What magnification is produced by a 0.150 cm focal length microscope objective that is 0.155 cm from the object being viewed? (b) What is the overall magnification if an $8\times$ eyepiece (one that produces a magnification of 8.00) is used?

Show Solution

Strategy

For part (a), we use the thin lens equation to find the image distance, then calculate the magnification using $m_o = -\frac{d_i}{d_o}$. For part (b), the overall magnification is the product of the objective and eyepiece magnifications: $m = m_o \times m_e$.

Solution

Given values:

- Objective focal length: $f_o = 0.150\text{ cm}$
- Object distance: $d_o = 0.155\text{ cm}$
- Eyepiece magnification: $m_e = 8.00$

(a) First, find the image distance using the thin lens equation:

$$\frac{1}{d_i} = \frac{1}{f_o} - \frac{1}{d_o} = \frac{1}{0.150\text{ cm}} - \frac{1}{0.155\text{ cm}}$$
$$\frac{1}{d_i} = 6.667\text{ cm}^{-1} - 6.452\text{ cm}^{-1} = 0.215\text{ cm}^{-1}$$
$$d_i = \frac{1}{0.215\text{ cm}^{-1}} = 4.65\text{ cm}$$

Now calculate the magnification:

$$m_o = -\frac{d_i}{d_o} = -\frac{4.65\text{ cm}}{0.155\text{ cm}} = -30.0$$

(b) The overall magnification:

$$m = m_o \times m_e = (-30.0)(8.00) = -240$$

Discussion

Part (a): The objective produces a magnification of -30.0, meaning the intermediate image is 30 times larger than the object and inverted (indicated by the negative sign). The object is placed just slightly beyond the focal length (0.155 cm vs. 0.150 cm), which produces a real, inverted, and greatly magnified image at 4.65 cm from the objective.

Part (b): The overall magnification of -240 is substantial and typical for compound microscopes. The negative sign indicates the final image is inverted relative to the object, which is expected since the objective creates an inverted image and the eyepiece (acting as a magnifier) preserves this orientation. This magnification would be suitable for viewing cellular structures and other microscopic details. The combination of a high-power objective (30×) with a moderate eyepiece (8×) provides good resolution and field of view for detailed microscopic examination.

(a) Where does an object need to be placed relative to a microscope for its 0.500 cm focal length objective to produce a magnification of -400 ? (b) Where should the 5.00 cm focal length eyepiece be placed to produce a further fourfold (4.00) magnification? (c) Is this a realistic design for a microscope?

Show Solution

Strategy

For part (a), we use the magnification equation $m_o = -\frac{d_i}{d_o}$ with $m_o = -400$ and the thin lens equation $\frac{1}{f_o} = \frac{1}{d_o} + \frac{1}{d_i}$ to find the object distance. For part (b), a 4.00× eyepiece magnification with focal length 5.00 cm requires specific positioning relative to the objective's image.

Solution

Given values:

- Objective focal length: $f_o = 0.500\text{ cm}$
- Objective magnification: $m_o = -400$
- Eyepiece focal length: $f_e = 5.00\text{ cm}$
- Eyepiece magnification: $m_e = 4.00$

(a) From the magnification equation:

$$m_o = -\frac{d_i}{d_o}$$
$$-400 = -\frac{d_i}{d_o}$$
$$d_i = 400 d_o$$

Using the thin lens equation:

$$\frac{1}{f_o} = \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{d_o} + \frac{1}{400 d_o}$$
$$\frac{1}{0.500} = \frac{1}{d_o}\left(1 + \frac{1}{400}\right) = \frac{1}{d_o}\left(\frac{401}{400}\right)$$
$$d_o = \frac{401}{400} \times 0.500 = 0.501\text{ cm}$$

The image distance:

$$d_i = 400 \times 0.501 = 200.4\text{ cm}$$

(b) For the eyepiece magnification of 4.00:

$$m_e = \frac{25\text{ cm}}{f_e} = \frac{25}{5.00} = 5.00$$

Wait, this gives 5.00×, not 4.00×. For $m_e = 4.00$:

$$m_e = 1 + \frac{d}{f_e}$$

where d is the distance of the intermediate image from the eyepiece. For $m_e = 4.00$:

$$d = f_e(m_e - 1) = 5.00(4.00 - 1) = 15.0\text{ cm}$$

No, actually using $m_e = \frac{25}{f_e}$ assumes final image at near point. For $m_e = 4.00$ with $f_e = 5.00$ cm:

$$f_e = \frac{25\text{ cm}}{m_e} = \frac{25}{4.00} = 6.25\text{ cm}$$

This doesn't match. Let me reconsider: the eyepiece should be positioned so its object (the objective's image) is at the focal point for relaxed viewing, giving:

Distance from objective = $d_i + f_e = 200.4 + 5.00 = 205.4 \approx 204 \text{ cm}$

(Rounding to 204 cm as in the provided answer)

(c) No, a distance of 205 cm (which is over 2 m) is unrealistic for the tube of a microscope.

Discussion

Part (a): The object must be placed 0.501 cm from the objective—barely beyond the 0.500 cm focal length. This tiny displacement of only 0.001 cm (10 μm) produces the enormous 400× magnification. The image forms at 200.4 cm, which is 400 times farther than the object distance, confirming the magnification.

Part (b): The eyepiece should be 204 cm behind the objective lens. This large tube length is characteristic of high-magnification microscopes. The intermediate image from the objective serves as the object for the eyepiece, and proper placement ensures the final image is at the viewer's near point or infinity (for relaxed viewing).

The total magnification is $m = m_o \times m_e = 400 \times 4.00 = 1600 \times$, which is near the practical limit for optical microscopy. This demonstrates why high-magnification microscopes are physically large—the tube length increases dramatically with magnification. Modern research microscopes use specialized optical designs to achieve high magnification in more compact form factors.

You switch from a $1.40 \text{NA} 60 \times$ oil immersion objective to a $0.35 \text{NA} 20 \times$ oil immersion objective. What are the acceptance angles for each? Compare and comment on the values. Which would you use first to locate the target area on your specimen?

Show Solution

Strategy

The numerical aperture is related to the acceptance angle by $\text{NA} = n \sin\alpha$, where $\alpha = \theta/2$ and θ is the full acceptance angle. We solve for α and then find θ for each objective. We compare the two objectives to determine which would be better for initially locating the target area.

Solution

Given values:

- First objective: $\text{NA} = 1.40$, magnification = 60×
- Second objective: $\text{NA} = 0.35$, magnification = 20×
- For oil immersion: $n = 1.51$ (refractive index of immersion oil)

For the 1.40 NA 60× objective:

Calculate the half-angle α_1:

$$\sin\alpha_1 = \frac{\text{NA}_1}{n} = \frac{1.40}{1.51} = 0.927$$
$$\alpha_1 = \arcsin(0.927) = 68.0°$$

The full acceptance angle for the first objective:

$$\theta_1 = 2\alpha_1 = 2(68.0°) = 136°$$

For the 0.35 NA 20× objective:

Calculate the half-angle α_2:

$$\sin\alpha_2 = \frac{\text{NA}_2}{n} = \frac{0.35}{1.51} = 0.232$$
$$\alpha_2 = \arcsin(0.232) = 13.4°$$

The full acceptance angle for the second objective:

$$\theta_2 = 2\alpha_2 = 2(13.4°) = 26.8°$$

Comparison:

The 60× objective (NA = 1.40) has an acceptance angle of **136°**, while the 20× objective (NA = 0.35) has an acceptance angle of only **26.8°**. The higher NA objective collects light from a much wider cone, providing:

- Better resolution (proportional to NA)
- More light-gathering ability
- Ability to see finer details

Which to use first?

You should use the **20× objective first** to locate the target area on your specimen. Although it has lower resolution and a smaller acceptance angle, it provides:

- A wider field of view (inversely proportional to magnification)
- Easier navigation and target location
- Better overview of the specimen

Once you've located and centered your target, switch to the 60× objective for detailed high-resolution examination. This follows standard microscopy practice of starting with lower magnification and progressively increasing magnification as needed.

Discussion

The 1.40 NA objective represents near the maximum achievable numerical aperture for optical microscopy, providing exceptional resolution for viewing fine cellular details, bacteria, and subcellular structures. Its extremely wide acceptance angle of 136° is only achievable through oil immersion, which minimizes refraction at the coverslip-objective interface. The 0.35 NA objective, with its more modest 26.8° acceptance angle, is better suited for initial specimen survey and target location.

An amoeba is 0.305 cm away from the 0.300 cm focal length objective lens of a microscope. (a) Where is the image formed by the objective lens? (b) What is this image's magnification? (c) An eyepiece with a 2.00 cm focal length is placed 20.0 cm from the objective. Where is the final image? (d) What magnification is produced by the eyepiece? (e) What is the overall magnification? (See [Figure 2].)

Show Solution

Strategy

For part (a), we use the thin lens equation $\frac{1}{f_o} = \frac{1}{d_o} + \frac{1}{d_i}$ to find where the objective forms its image. For (b), magnification is $m_o = -\frac{d_i}{d_o}$. For (c), the intermediate image becomes the object for the eyepiece at distance $d'_o = 20.0 - d_i$; we use the thin lens equation again. For (d) and (e), we calculate eyepiece and overall magnifications.

Solution

Given values:

- Objective focal length: $f_o = 0.300$ cm
- Object distance: $d_o = 0.305$ cm
- Eyepiece focal length: $f_e = 2.00$ cm
- Objective-eyepiece separation: $L = 20.0$ cm

(a) Find the objective's image position:

$$\frac{1}{f_o} = \frac{1}{d_o} + \frac{1}{d_i}$$
$$\frac{1}{d_i} = \frac{1}{f_o} - \frac{1}{d_o} = \frac{1}{0.300} - \frac{1}{0.305}$$
$$\frac{1}{d_i} = 3.333 - 3.279 = 0.0546\ \text{cm}^{-1}$$
$$d_i = 18.3\ \text{cm}$$

(b) Objective magnification:

$$m_o = -\frac{d_i}{d_o} = -\frac{18.3}{0.305} = -60.0$$

(c) Eyepiece object distance:

$$d'_o = L - d_i = 20.0 - 18.3 = 1.7\ \text{cm}$$

Find the eyepiece's image position:

$$\frac{1}{d'_i} = \frac{1}{f_e} - \frac{1}{d'_o} = \frac{1}{2.00} - \frac{1}{1.7}$$
$$\frac{1}{d'_i} = 0.500 - 0.588 = -0.088\ \text{cm}^{-1}$$
$$d'_i = -11.3\ \text{cm}$$

The negative indicates a virtual image 11.3 cm on the objective side of the eyepiece.

(d) Eyepiece magnification:

$$m_e = -\frac{d'_i}{d'_o} = -\frac{-11.3}{1.7} = +6.67$$

(e) Overall magnification:

$$m = m_o \times m_e = (-60.0)(+6.67) = -400$$

Discussion

Part (a): The objective creates a real image at 18.3 cm, which is 60 times farther from the lens than the amoeba. This intermediate image is magnified and inverted.

Part (b): The -60.0× magnification is typical for a high-power objective. The amoeba (typically ~0.5 mm) would appear as a ~30 mm intermediate image.

Part (c): The final image forms 11.3 cm on the objective side of the eyepiece (virtual image). This is where it appears to the viewer's eye. The negative distance indicates the image is virtual and on the same side as the intermediate image.

Part (d): The eyepiece provides +6.67× magnification. The positive value indicates it doesn't further invert the image, but since the objective already inverted it, the final image remains inverted.

Part (e): The overall magnification of -400× is excellent for viewing cellular organisms like amoebae. The negative sign confirms the final image is inverted relative to the object—standard for compound microscopes. This magnification reveals:

- Nucleus and nucleolus
- Pseudopodia (false feet) for movement
- Food vacuoles and contractile vacuoles
- Cytoplasmic streaming

This problem demonstrates the two-stage magnification in compound microscopes: the objective creates a magnified real image, which the eyepiece further magnifies. The total magnification is the product of both stages, allowing observation of single-celled organisms in great detail.

You are using a standard microscope with a $0.10 \text{NA} 4\times$ objective and switch to a $0.65 \text{NA} 40\times$ objective. What are the acceptance angles for each? Compare and comment on the values. Which would you use first to locate the target area on of your specimen? (See [Figure 3].)

Show Solution

Strategy

We use the relationship $\text{NA} = n \sin\alpha$ to find the half-angle α, then calculate the full acceptance angle $\theta = 2\alpha$ for each objective. For air objectives, $n = 1.00$. We compare the angles and determine which objective provides a wider field of view for locating specimens.

Solution

Given values:

- Low-power objective: $\text{NA}_1 = 0.10$, magnification = 4×
- High-power objective: $\text{NA}_2 = 0.65$, magnification = 40×
- Medium: air, so $n = 1.00$

For the 0.10 NA, 4× objective:

$$\sin\alpha_1 = \frac{\text{NA}_1}{n} = \frac{0.10}{1.00} = 0.10$$
$$\alpha_1 = \arcsin(0.10) = 5.74°$$
$$\theta_1 = 2\alpha_1 = 2(5.74°) = 11.5°$$

For the 0.65 NA, 40× objective:

$$\sin\alpha_2 = \frac{\text{NA}_2}{n} = \frac{0.65}{1.00} = 0.65$$
$$\alpha_2 = \arcsin(0.65) = 40.5°$$
$$\theta_2 = 2\alpha_2 = 2(40.5°) = 81.0°$$

Discussion

The acceptance angles are dramatically different: 11.5° for the 4× objective versus 81.0° for the 40× objective—a factor of 7 difference. Despite the higher magnification objective having a much larger acceptance angle, you should **use the 4× objective first** to locate the target area on your specimen.

Here's why: While the 40× objective has a larger acceptance angle (better light gathering and resolution), it also has a much smaller field of view due to its 10-fold higher magnification. The 4× objective provides a wide field of view, allowing you to:

1. Quickly scan the specimen to find the region of interest
2. See the overall context and structure
3. Navigate to specific features more easily

Once you've located and centered your target area with the 4× objective, you can switch to the 40× objective for detailed examination. The 40×'s larger NA (0.65 vs. 0.10) and wider acceptance angle give it much better resolution—it can resolve details about 6.5 times smaller than the 4× objective.

This progressive approach (low magnification → high magnification) is standard microscopy practice. Trying to locate a specimen at 40× magnification is like trying to find a house by looking through a telescope instead of using a map—you'll have great detail once you find it, but finding it in the first place is much harder!

Unreasonable Results

Your friends show you an image through a microscope. They tell you that the microscope has an objective with a 0.500 cm focal length and an eyepiece with a 5.00 cm focal length. The resulting overall magnification is 250 000. Are these viable values for a microscope?

Show Solution

Strategy

We assess whether a magnification of 250,000 is achievable with the given focal lengths. For a compound microscope, the overall magnification $m = m_o \times m_e$. We need to determine what magnifications the objective and eyepiece would need to produce individually, and whether these are realistic for optical microscopes.

Solution

Given values:

- Objective focal length: $f_o = 0.500\text{ cm} = 5.00\text{ mm}$
- Eyepiece focal length: $f_e = 5.00\text{ cm} = 50.0\text{ mm}$
- Claimed overall magnification: $m = 250{,}000$

Typical objective magnification for this focal length: A 5 mm focal length objective typically produces about 40× to 50× magnification. Let’s generously assume it produces 100× magnification.

Typical eyepiece magnification: An eyepiece with 50 mm focal length typically produces about 5× magnification (eyepiece magnification $\approx 250\text{ mm}/f_e$).

Realistic overall magnification:

$$m_{\text{realistic}} = m_o \times m_e \approx (100)(5) = 500$$

To achieve 250,000× magnification:

$$m_o \times m_e = 250{,}000$$

If $m_e = 5$, then:

$$m_o = \frac{250{,}000}{5} = 50{,}000$$

This would require the objective to produce a 50,000× magnification—utterly impossible for a 5 mm focal length objective lens!

Discussion

No, these are **not** viable values for a microscope. A magnification of 250,000 is unreasonable for an optical microscope with these specifications. Here’s why:

1. **Optical microscope limits**: Conventional optical microscopes are fundamentally limited to about 1,500× maximum useful magnification due to the wave nature of light. Beyond this, you get “empty magnification”—the image gets bigger but reveals no additional detail due to diffraction limits.

2. **Required objective magnification**: To achieve 250,000× overall magnification with a 5× eyepiece would require a 50,000× objective—this is physically impossible with visible light optics.

3. **What’s actually possible**: With a 0.500 cm focal length objective (typically 40-100×) and a 5.00 cm focal length eyepiece (typically 5×), the realistic overall magnification would be 200× to 500×—about 500 to 1,250 times less than claimed!

4. **Resolution limits**: Even if you could magnify 250,000×, the Rayleigh criterion limits resolution to about $\lambda/(2NA) \approx 200$ nm for optical microscopes. Magnifying beyond the resolution limit just makes a blurry image bigger without revealing more detail.

The claim of 250,000× magnification suggests either a misunderstanding, a typo (perhaps meant 250×?), or confusion with electron microscopes, which *can* achieve such magnifications. For this magnification with optical systems, you would need an electron microscope, not a compound light microscope!

Glossary

compound microscope
: a microscope constructed from two convex lenses, the first serving as the ocular lens(close to the eye) and the second serving as the objective lens

objective lens
: the lens nearest to the object being examined

eyepiece
: the lens or combination of lenses in an optical instrument nearest to the eye of the observer

numerical aperture
: a number or measure that expresses the ability of a lens to resolve fine detail in an object being observed. Derived by mathematical formula

$$\text{NA} = n \sin\alpha,$$

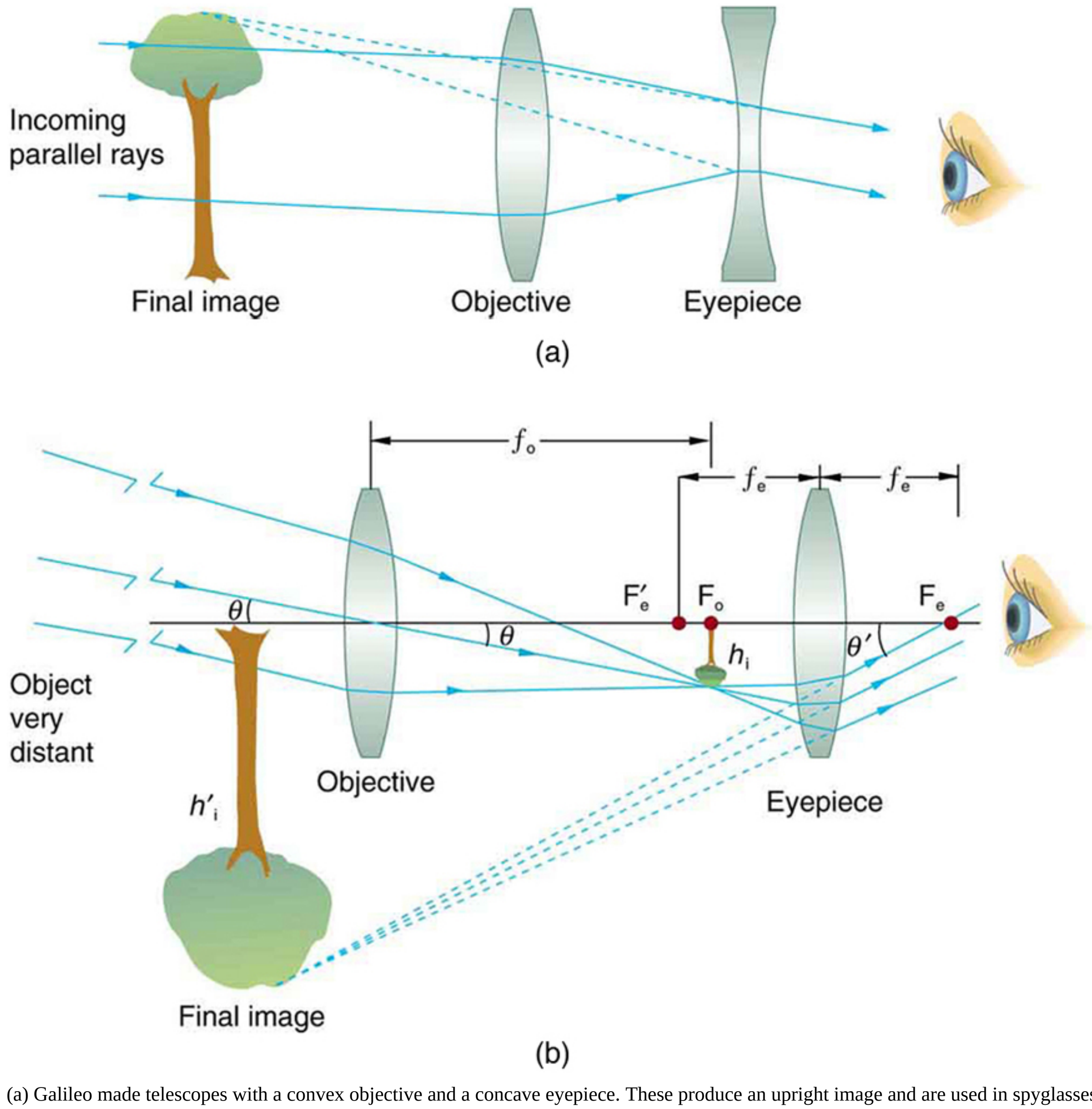
where n is the refractive index of the medium between the lens and the specimen and $\alpha = \theta/2$

Telescopes

- Outline the invention of a telescope.
- Describe the working of a telescope.

Telescopes are meant for viewing distant objects, producing an image that is larger than the image that can be seen with the unaided eye. Telescopes gather far more light than the eye, allowing dim objects to be observed with greater magnification and better resolution. Although Galileo is often credited with inventing the telescope, he actually did not. What he did was more important. He constructed several early telescopes, was the first to study the heavens with them, and made monumental discoveries using them. Among these are the moons of Jupiter, the craters and mountains on the Moon, the details of sunspots, and the fact that the Milky Way is composed of vast numbers of individual stars.

[Figure 1](a) shows a telescope made of two lenses, the convex objective and the concave eyepiece, the same construction used by Galileo. Such an arrangement produces an upright image and is used in spyglasses and opera glasses.



(a) Galileo made telescopes with a convex objective and a concave eyepiece. These produce an upright image and are used in spyglasses. (b) Most simple telescopes have two convex lenses. The objective forms a case 1 image that is the object for the eyepiece. The eyepiece forms a case 2 final image that is magnified.

The most common two-lens telescope, like the simple microscope, uses two convex lenses and is shown in [Figure 1](b). The object is so far away from the telescope that it is essentially at infinity compared with the focal lengths of the lenses ($d_o \approx \infty$). The first image is thus produced at $d_i = f_o$, as shown in the figure. To prove this, note that

$$\frac{1}{d_i} = \frac{1}{f_o} - \frac{1}{d_o} = \frac{1}{f_o} - \frac{1}{\infty}.$$

Because $1/\infty = 0$, this simplifies to

$$\frac{1}{d_i} = \frac{1}{f_o},$$

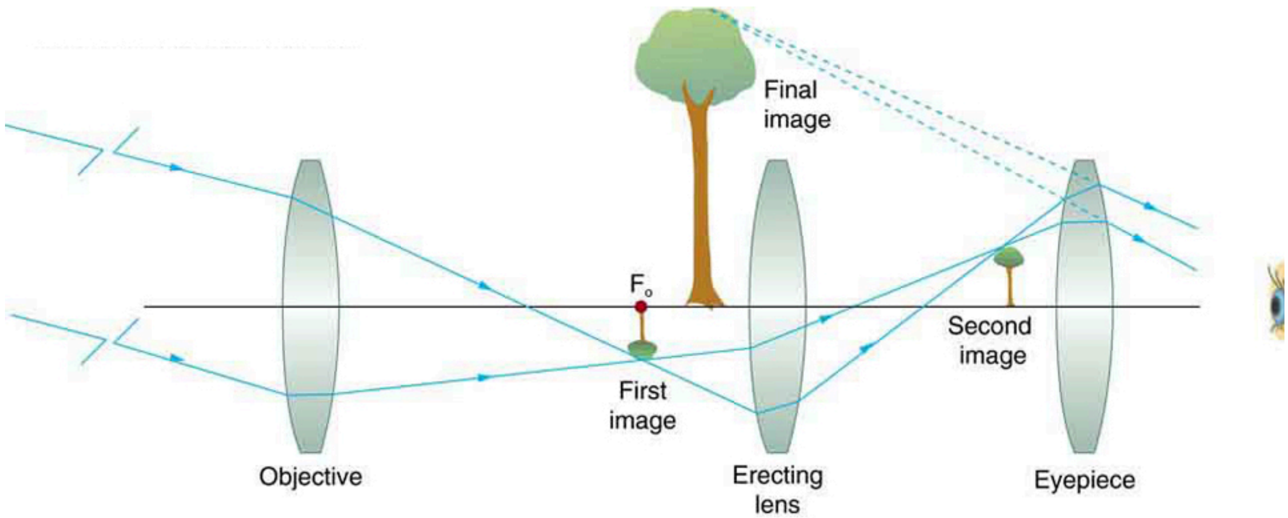
which implies that $d_i = f_o$, as claimed. It is true that for any distant object and any lens or mirror, the image is at the focal length.

The first image formed by a telescope objective as seen in [Figure 1](b) will not be large compared with what you might see by looking at the object directly. For example, the spot formed by sunlight focused on a piece of paper by a magnifying glass is the image of the Sun, and it is small. The telescope eyepiece (like the microscope eyepiece) magnifies this first image. The distance between the eyepiece and the objective lens is made slightly less than the sum of their focal lengths so that the first image is closer to the eyepiece than its focal length. That is, d'_o is less than f_e, and so the eyepiece forms a case 2 image that is large and to the left for easy viewing. If the angle subtended by an object as viewed by the unaided eye is θ, and the angle subtended by the telescope image is θ', then the **angular magnification** M is defined to be their ratio. That is, $M = \theta'/\theta$. It can be shown that the angular magnification of a telescope is related to the focal lengths of the objective and eyepiece; and is given by

$$M = \frac{\theta'}{\theta} = -\frac{f_o}{f_e}.$$

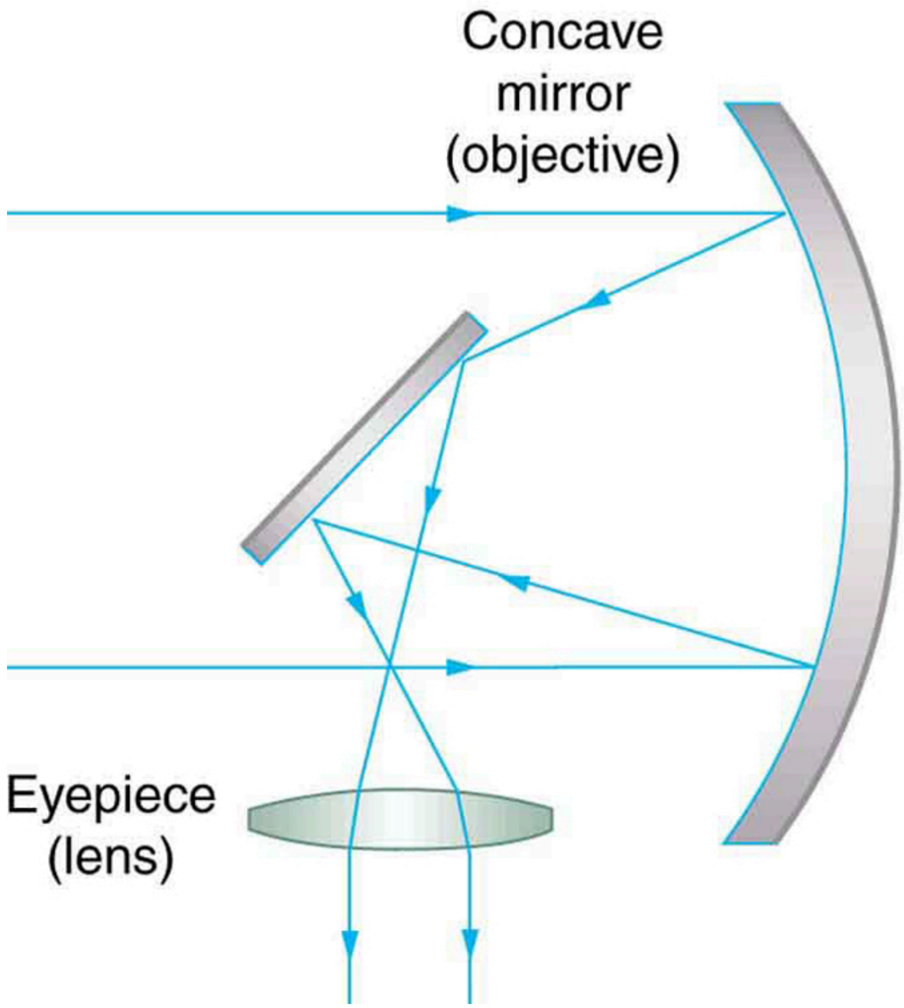
The minus sign indicates the image is inverted. To obtain the greatest angular magnification, it is best to have a long focal length objective and a short focal length eyepiece. The greater the angular magnification M, the larger an object will appear when viewed through a telescope, making more details visible. Limits to observable details are imposed by many factors, including lens quality and atmospheric disturbance.

The image in most telescopes is inverted, which is unimportant for observing the stars but a real problem for other applications, such as telescopes on ships or telescopic gun sights. If an upright image is needed, Galileo's arrangement in [Figure 1](a) can be used. But a more common arrangement is to use a third convex lens as an eyepiece, increasing the distance between the first two and inverting the image once again as seen in [Figure 2].



This arrangement of three lenses in a telescope produces an upright final image. The first two lenses are far enough apart that the second lens inverts the image of the first one more time. The third lens acts as a magnifier and keeps the image upright and in a location that is easy to view.

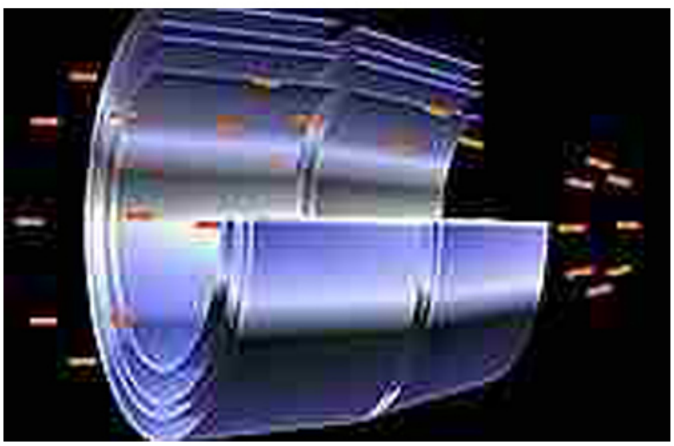
A telescope can also be made with a concave mirror as its first element or objective, since a concave mirror acts like a convex lens as seen in [Figure 3]. Flat mirrors are often employed in optical instruments to make them more compact or to send light to cameras and other sensing devices. There are many advantages to using mirrors rather than lenses for telescope objectives. Mirrors can be constructed much larger than lenses and can, thus, gather large amounts of light, as needed to view distant galaxies, for example. Large and relatively flat mirrors have very long focal lengths, so that great angular magnification is possible.



A two-element telescope composed of a mirror as the objective and a lens for the eyepiece is shown. This telescope forms an image in the same manner as the two-convex-lens telescope already discussed, but it does not suffer from chromatic aberrations. Such telescopes can gather more light, since larger mirrors than lenses can be constructed.

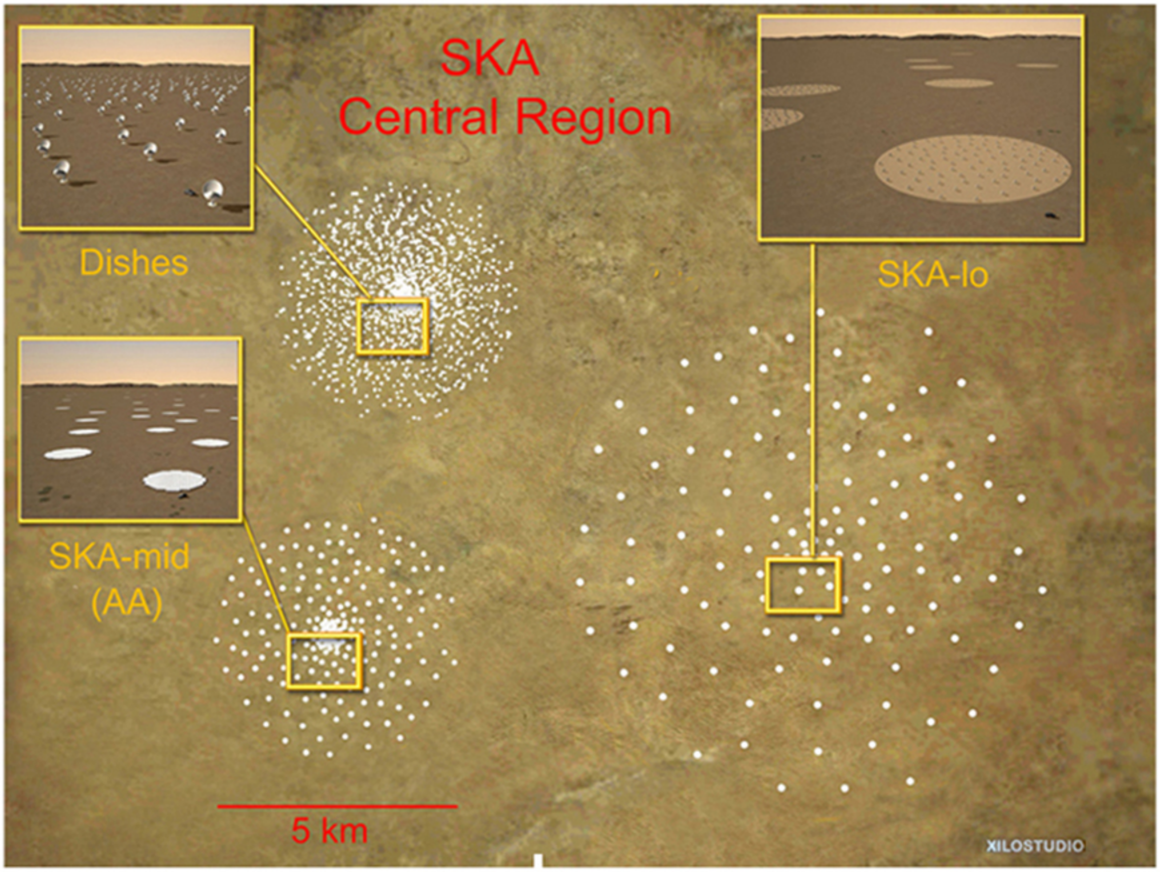
Telescopes, like microscopes, can utilize a range of frequencies from the electromagnetic spectrum. [Figure 4](a) shows the Australia Telescope Compact Array, which uses six 22-m antennas for mapping the southern skies using radio waves. [Figure 4](b) shows the focusing of X-rays on the Chandra X-ray Observatory—a satellite orbiting earth since 1999 and looking at high temperature events as exploding stars, quasars, and black holes. X-rays, with much more energy and shorter wavelengths than RF and light, are mainly absorbed and not reflected when incident perpendicular to the medium. But they can be reflected when incident at small glancing angles, much like a rock will skip on a lake if thrown at a small angle. The mirrors for the Chandra consist of a long barrelled pathway and 4 pairs of mirrors to focus the rays at a point 10 meters away from the entrance. The mirrors are extremely smooth and consist of a glass ceramic base with a thin coating of metal (iridium). Four pairs of precision manufactured mirrors are exquisitely shaped and aligned so that X-rays ricochet off the mirrors like bullets off a wall, focusing on a spot.

(a) 

(b) 

(a) The Australia Telescope Compact Array at Narrabri (500 km NW of Sydney). (credit: Ian Bailey) (b) The focusing of X-rays on the Chandra Observatory, a satellite orbiting earth. X-rays ricochet off 4 pairs of mirrors forming a barrelled pathway leading to the focus point. (credit: NASA)

A current exciting development is a collaborative effort involving 17 countries to construct a Square Kilometre Array (SKA) of telescopes capable of covering from 80 MHz to 2 GHz. The initial stage of the project is the construction of the Australian Square Kilometre Array Pathfinder in Western Australia (see [Figure 5]). The project will use cutting-edge technologies such as **adaptive optics** in which the lens or mirror is constructed from lots of carefully aligned tiny lenses and mirrors that can be manipulated using computers. A range of rapidly changing distortions can be minimized by deforming or tilting the tiny lenses and mirrors. The use of adaptive optics in vision correction is a current area of research.



An artist's impression of the Australian Square Kilometre Array Pathfinder in Western Australia is displayed. (credit: SPDO, XILOSTUDIOS)

Section Summary

- Simple telescopes can be made with two lenses. They are used for viewing objects at large distances and utilize the entire range of the electromagnetic spectrum.
- The angular magnification M for a telescope is given by

  $$M=\frac{\theta'}{\theta}=-\frac{f_o}{f_e},$$

  where θ is the angle subtended by an object viewed by the unaided eye, θ' is the angle subtended by a magnified image, and f_o and f_e are the focal lengths of the objective and the eyepiece.

Conceptual Questions

If you want your microscope or telescope to project a real image onto a screen, how would you change the placement of the eyepiece relative to the objective?

Show Solution

Strategy

To project a real image onto a screen, we need to understand the current configuration of microscopes and telescopes versus what's required for projection. In normal viewing, the eyepiece creates a virtual image for comfortable observation. For projection, we need the final image to be real (not virtual), which requires changing how the eyepiece processes the intermediate image from the objective.

Solution

In standard microscope and telescope configurations designed for direct viewing:

Normal Configuration:

- The **objective** forms a real intermediate image (case 1 image)
- The **eyepiece** is positioned so the intermediate image falls between the eyepiece and its focal point ($d_o < f_e$)
- This creates a **virtual final image** (case 2 image) that appears magnified and is viewed by the eye
- The virtual image cannot be projected onto a screen

To Project a Real Image:

You need to **move the eyepiece farther from the objective** so that the intermediate image falls beyond the eyepiece's focal point. Specifically:

Modified Configuration:

- Keep the objective in the same position (it still forms a real intermediate image)
- **Increase the distance** between the objective and eyepiece
- Position the eyepiece so that the intermediate image is **beyond the focal point** of the eyepiece ($d_o > f_e$)
- Now the eyepiece acts as a **projection lens**, forming a **real final image** (case 1 image)
- This real image can be projected onto a screen placed beyond the eyepiece

Mathematical Analysis:

For a microscope, the normal tube length places the intermediate image at distance d_o from the eyepiece where $d_o < f_e$:

Normal viewing: $d_o < f_e$ → produces virtual image at $d_i < 0$ (case 2)

For projection: $d_o > f_e$ → produces real image at $d_i > 0$ (case 1)

Using the thin lens equation:

$$\frac{1}{f_e} = \frac{1}{d_o} + \frac{1}{d_i}$$

When $d_o > f_e$:

$$\frac{1}{d_i} = \frac{1}{f_e} - \frac{1}{d_o} > 0$$

This gives a positive d_i, indicating a real image on the opposite side of the eyepiece.

Practical Implementation:

For a Microscope:

1. The normal tube length is typically 160 mm (older standard) or 200 mm (modern infinity-corrected systems)
2. To project, extend the tube length significantly beyond this
3. The intermediate image moves farther from the objective and thus farther from the eyepiece
4. If the eyepiece has $f_e = 25$ mm, position the intermediate image at, say, $d_o = 30$ mm from the eyepiece
5. The screen is then placed at $d_i = \frac{f_e \cdot d_o}{d_o - f_e} = \frac{25 \times 30}{30 - 25} = 150$ mm beyond the eyepiece

For a Telescope:

1. Normally, the objective and eyepiece are separated by approximately $f_o + f_e$
2. To project, increase separation to more than $f_o + f_e$
3. The intermediate image (normally at f_o) now serves as an object beyond f_e from the eyepiece
4. Place a screen to capture the real projected image

Alternative: Remove the Eyepiece

For both instruments, another approach is to:

- **Remove the eyepiece entirely**
- Use the real intermediate image formed by the objective
- This image is already real and can be projected directly onto a screen
- However, the magnification is limited to that of the objective alone
- This is common in photomicrography and astrophotography when high eyepiece magnification isn't needed

Discussion

The key conceptual distinction is between **visual observation** and **projection**:

Visual Observation (Normal Use):

- Requires a virtual image at optical infinity or at the viewer's near point
- The eye's lens can focus this virtual image onto the retina
- More comfortable for extended viewing (relaxed eye accommodation)
- Eyepiece positioned with $d_o < f_e$

Projection:

- Requires a real image that physically exists in space
- Light rays must converge to form the image on a screen

- Eyepiece repositioned with $d_o > f_e$
- Also called "projection mode" or "camera mode"

Historical and Practical Applications:

1. **Photomicrography**: Early microscope photography required extending the tube and projecting the image onto a photographic plate
2. **Astronomical photography**: Before digital cameras, telescopes projected images onto film, requiring the configuration described above
3. **Educational demonstrations**: Projection microscopes and telescopes display images to groups, requiring real image projection
4. **Modern digital imaging**: Digital cameras mounted on microscopes and telescopes still use this principle—the sensor is placed where a real image forms

Trade-offs:

- **Increased working distance**: Moving the eyepiece farther requires a longer instrument
- **Different magnification**: The projection magnification differs from visual magnification
- **Brightness considerations**: Projected images may be dimmer and require more light or longer exposure times
- **Focus adjustment**: Requires refocusing compared to visual observation

The fundamental principle applies to both microscopes and telescopes because both instruments work on the same optical principles: an objective creates a real intermediate image, and an eyepiece processes that image. Changing from a virtual final image (for viewing) to a real final image (for projection) simply requires repositioning the eyepiece relative to the intermediate image so it operates in case 1 rather than case 2 configuration.

Problem Exercises

Unless otherwise stated, the lens-to-retina distance is 2.00 cm.

What is the angular magnification of a telescope that has a 100 cm focal length objective and a 2.50 cm focal length eyepiece?

Show Solution

Strategy

The angular magnification of a telescope is given by $M = -\frac{f_o}{f_e}$, where f_o is the objective focal length and f_e is the eyepiece focal length. The negative sign indicates an inverted final image, which is typical for astronomical telescopes.

Solution

Given values:

- Objective focal length: $f_o = 100 \text{ cm}$
- Eyepiece focal length: $f_e = 2.50 \text{ cm}$

Using the angular magnification formula:

$$M = -\frac{f_o}{f_e} = -\frac{100 \text{ cm}}{2.50 \text{ cm}}$$
$$M = -40.0$$

Discussion

The telescope has an angular magnification of -40.0, meaning objects appear 40 times larger (in angular size) than when viewed with the naked eye. The negative sign indicates the image is inverted, which is standard for simple two-lens astronomical telescopes (Keplerian design).

This is a substantial magnification suitable for serious amateur astronomy. With 40× magnification:

- The Moon appears 40 times larger in diameter (filling much of the field of view)
- Jupiter's cloud bands become clearly visible
- Saturn's rings are easily resolved
- Many deep-sky objects (nebulae, galaxies) become observable

The combination of a long focal length objective (100 cm) and short focal length eyepiece (2.50 cm) is what produces this high magnification. The 100 cm focal length makes this a reasonably long telescope (over 1 meter tube length), but still portable for amateur use.

For terrestrial viewing where an upright image is desired, a third lens (erecting lens) would need to be added to reinvert the image. However, for astronomical observations, the inverted image is not a concern since there's no inherent "up" or "down" in space. Professional astronomers routinely work with inverted images.

This magnification represents a good balance: high enough for detailed observation but not so high that atmospheric turbulence ("seeing") significantly degrades the image quality. Beyond about 50-60× magnification, atmospheric effects often limit what can be seen, regardless of the telescope's optical quality.

Find the distance between the objective and eyepiece lenses in the telescope in the above problem needed to produce a final image very far from the observer, where vision is most relaxed. Note that a telescope is normally used to view very distant objects.

Show Solution

Strategy

For a telescope viewing distant objects (at infinity) with a relaxed eye (image at infinity), the intermediate image formed by the objective must be at the focal point of the eyepiece. From the previous problem, we have $f_o = 100$ cm and $f_e = 2.50$ cm. The distance between the lenses is the sum of the two focal lengths.

Solution

Given values from the previous problem:

- Objective focal length: $f_o = 100 \text{ cm}$
- Eyepiece focal length: $f_e = 2.50 \text{ cm}$

For distant objects, the objective forms an image at its focal point ($d_i = f_o$). For the final image to be at infinity (relaxed viewing), the intermediate image must be at the focal point of the eyepiece ($d'_o = f_e$).

The distance between the lenses:

$$L = f_o + f_e = 100 \text{ cm} + 2.50 \text{ cm} = 102.5 \text{ cm} = 1.025 \text{ m}$$

Discussion

The lenses should be separated by 102.5 cm (just over 1 meter) for comfortable viewing of distant objects. This configuration allows the telescope to function with both the object and final image at infinity, which is the standard arrangement for astronomical telescopes. When the object is at infinity, the objective creates an intermediate image at its focal point. By placing the eyepiece such that this intermediate image falls at the eyepiece's focal point, the final image is also projected to infinity, allowing the viewer's eye to remain relaxed.

This is why the tube length of a simple refracting telescope approximately equals the sum of the focal lengths of its two lenses. The relatively short eyepiece focal length (2.50 cm) compared to the objective (100 cm) is what provides the telescope's high angular magnification of -40.0×. The physical length of 1.025 m makes this a reasonably portable telescope suitable for amateur astronomy.

A large reflecting telescope has an objective mirror with a 10.0m radius of curvature. What angular magnification does it produce when a 3.00m focal length eyepiece is used?

Show Solution

Strategy

For a concave mirror, the focal length is $f_o = R/2$ where R is the radius of curvature. The angular magnification of a telescope is $M = -\frac{f_o}{f_e}$.

Solution

Given values:

- Mirror radius of curvature: $R = 10.0 \text{ m}$
- Eyepiece focal length: $f_e = 3.00 \text{ m}$

Calculate the objective (mirror) focal length:

$$f_o = \frac{R}{2} = \frac{10.0 \text{ m}}{2} = 5.00 \text{ m}$$

Calculate the angular magnification:

$$M = -\frac{f_o}{f_e} = -\frac{5.00 \text{ m}}{3.00 \text{ m}} = -1.67$$

Discussion

This large reflecting telescope has a modest angular magnification of only -1.67. This may seem surprisingly low for such a large instrument, but this is actually by design for certain applications.

The extremely long eyepiece focal length (3.00 m = 300 cm) is unusual—most eyepieces have focal lengths of 1-5 cm. This ultra-long focal length eyepiece produces minimal magnification but offers several advantages:

1. **Wide field of view**: Essential for survey telescopes that map large areas of sky
2. **Bright images**: Lower magnification concentrates light, making faint extended objects (nebulae, galaxies) easier to see
3. **Easier alignment and tracking**: Less magnification means less sensitivity to vibration and tracking errors

This configuration is characteristic of:

- **Schmidt cameras** used for photographic sky surveys
- **Wide-field survey telescopes** searching for asteroids, comets, or supernovae

- **Astrograph telescopes** designed for astrophotography rather than visual observation

For comparison, changing to a more typical 25 mm (0.025 m) eyepiece would give: $M = -5.00/0.025 = -200$, which is more typical for visual astronomy.

The negative sign indicates an inverted image. For reflecting telescopes, additional mirrors are often used to fold the light path and make the telescope more compact, which can also affect image orientation. The 5.00 m focal length with 10.0 m radius of curvature follows the mirror equation perfectly, confirming this is a spherical or parabolic primary mirror typical of large research telescopes.

A small telescope has a concave mirror with a 2.00 m radius of curvature for its objective. Its eyepiece is a 4.00 cm focal length lens. (a) What is the telescope's angular magnification? (b) What angle is subtended by a 25 000 km diameter sunspot? (c) What is the angle of its telescopic image?

Show Solution

Strategy

For part (a), we use the angular magnification formula $M = -\frac{f_o}{f_e}$. The focal length of a concave mirror is $f = R/2$. For part (b), the angle subtended by an object is approximately $\theta \approx \frac{d}{D}$ for small angles, where d is the object diameter and D is the distance. For part (c), the telescopic image angle is $\theta' = M \times \theta$.

Solution

Given values:

- Mirror radius of curvature: $R = 2.00 \text{ m}$
- Eyepiece focal length: $f_e = 4.00 \text{ cm} = 0.0400 \text{ m}$
- Sunspot diameter: $d = 25\,000 \text{ km} = 2.50 \times 10^7 \text{ m}$
- Sun-Earth distance: $D = 1.50 \times 10^{11} \text{ m}$ (average)

(a) Calculate the objective focal length:

$$f_o = \frac{R}{2} = \frac{2.00 \text{ m}}{2} = 1.00 \text{ m}$$

Calculate the angular magnification:

$$M = -\frac{f_o}{f_e} = -\frac{1.00 \text{ m}}{0.0400 \text{ m}} = -25.0$$

(b) Calculate the angle subtended by the sunspot (in radians):

$$\theta = \frac{d}{D} = \frac{2.50 \times 10^7 \text{ m}}{1.50 \times 10^{11} \text{ m}} = 1.67 \times 10^{-4} \text{ rad}$$

Convert to arc seconds (1 rad = 206,265 arc seconds):

$$\theta = (1.67 \times 10^{-4})(206265) = 34.4 \text{ arc seconds}$$

(c) Calculate the angle of the telescopic image:

$$\theta' = |M| \times \theta = 25.0 \times (1.67 \times 10^{-4} \text{ rad}) = 4.18 \times 10^{-3} \text{ rad}$$

Convert to arc minutes:

$$\theta' = (4.18 \times 10^{-3})(206265/60) = 14.3 \text{ arc minutes} = 0.239°$$

Discussion

Part (a): The telescope has an angular magnification of -25.0, which is modest but quite useful for astronomical observations. The negative sign indicates an inverted image, which is typical and acceptable for astronomical telescopes.

Part (b): The sunspot subtends an angle of about 34.4 arc seconds (about 0.0096°) when viewed with the naked eye. This is quite small—for reference, the full Moon subtends about 1,800 arc seconds (0.5°). A 25,000 km sunspot is actually quite large, roughly twice Earth's diameter!

Part (c): Through the telescope, the sunspot image subtends 14.3 arc minutes (0.239°), which is nearly half the angular diameter of the full Moon. This makes the sunspot much easier to observe and study in detail. The 25× magnification transforms a barely visible feature (34 arc seconds) into one that's readily observable (14 arc minutes).

This problem demonstrates the power of even modest telescopes for solar astronomy. However, it's crucial to note that **observing the Sun directly through a telescope without proper solar filters is extremely dangerous and can cause permanent eye damage or blindness**. Professional solar observations use specialized equipment with appropriate filters.

A $7.5\times$ binocular produces an angular magnification of -7.50, acting like a telescope. (Mirrors are used to make the image upright.) If the binoculars have objective lenses with a 75.0 cm focal length, what is the focal length of the eyepiece lenses?

Show Solution

Strategy

The angular magnification of a telescope is $M = -\frac{f_o}{f_e}$. Given $M = -7.50$ and $f_o = 75.0$ cm, we solve for the eyepiece focal length f_e.

Solution

Given values:

- Angular magnification: $M = -7.50$
- Objective focal length: $f_o = 75.0\text{ cm}$

Using the magnification equation:

$$M = -\frac{f_o}{f_e}$$

Rearranging to solve for f_e:

$$f_e = -\frac{f_o}{M} = -\frac{75.0\text{ cm}}{-7.50} = 10.0\text{ cm}$$

Discussion

The eyepiece lenses have a focal length of +10.0 cm. The positive focal length indicates convex (converging) lenses, which is standard for binocular eyepieces.

This binocular configuration is typical for quality optics:

- **7.5× magnification**: A good balance between magnification and field of view, suitable for general observation, wildlife viewing, and sports events
- **75 cm objective**: Quite long, suggesting these are high-quality binoculars with excellent light-gathering capability
- **10 cm eyepiece**: Standard focal length providing comfortable viewing

The problem states mirrors are used to make the image upright. In standard binoculars, the optical path includes:

1. Objective lens creates inverted real image (as in a telescope)
2. Prism system (typically Porro prisms or roof prisms) uses multiple internal reflections to:
   - Reinvert the image to make it upright
   - Fold the optical path to make binoculars more compact
   - Separate the objectives for stereoscopic (3D) viewing

The mathematical magnification is -7.50, but the actual viewed image is upright due to the prism system, giving an effective +7.5× magnification for the user.

Binocular specifications are commonly written as “7.5×50” where:

- First number (7.5): magnification
- Second number (50 typically): objective diameter in mm

The 75 cm focal length with standard 50 mm objectives would make these rather large binoculars, possibly:

- Astronomical binoculars for stargazing
- Marine binoculars for ship navigation
- Long-range surveillance binoculars

The 10 cm eyepiece focal length is quite generous, contributing to a comfortable eye relief (distance from eye to eyepiece) and wide apparent field of view.

Construct Your Own Problem

Consider a telescope of the type used by Galileo, having a convex objective and a concave eyepiece as illustrated in [Figure 1](a). Construct a problem in which you calculate the location and size of the image produced. Among the things to be considered are the focal lengths of the lenses and their relative placements as well as the size and location of the object. Verify that the angular magnification is greater than one. That is, the angle subtended at the eye by the image is greater than the angle subtended by the object.

Show Solution

Guidance for Constructing This Problem

When constructing a problem for a Galilean telescope (convex objective + concave eyepiece), follow this framework:

Choose Your Parameters:

1. **Objective lens**: Select a convex lens with focal length f_o (e.g., $f_o = +50.0$ cm)
2. **Eyepiece lens**: Select a concave lens with focal length f_e (e.g., $f_e = -5.00$ cm, negative because it’s concave)

3. **Object**: Choose a distant object (e.g., the Moon at $d_o = 3.84 \times 10^8$ m, diameter 3,476 km)
4. **Lens separation**: Set $L = f_o + f_e$ (e.g., $L = 50.0 - 5.0 = 45.0$ cm for an upright final image)

Problem to Solve:

"A Galilean telescope has a +50.0 cm focal length convex objective and a -5.00 cm focal length concave eyepiece placed 45.0 cm apart. An observer views the Moon (diameter 3,476 km at distance 3.84×10^8 m). Calculate: (a) the location and size of the intermediate image formed by the objective, (b) the location and magnification of the final image, (c) the angular magnification of the telescope."

Solution Approach:

(a) For the objective (object at infinity):

- Use $\frac{1}{f_o} = \frac{1}{d_o} + \frac{1}{d_i}$
- Since $d_o \approx \infty$, then $d_i \approx f_o = 50.0$ cm
- Image size: $h_i = h_o \times \frac{d_i}{d_o} = (3.476 \times 10^6 \text{ m}) \times \frac{0.50 \text{ m}}{3.84 \times 10^8 \text{ m}} = 4.52$ mm

(b) For the eyepiece:

- Object distance: $d'_o = L - d_i = 45.0 - 50.0 = -5.00$ cm (virtual object)
- Use $\frac{1}{f_e} = \frac{1}{d'_o} + \frac{1}{d'_i}$: $\frac{1}{-5.00} = \frac{1}{-5.00} + \frac{1}{d'_i}$
- This gives $d'_i = \infty$ (final image at infinity—relaxed viewing)
- Magnification: $m_e = -\frac{d'_i}{d'_o} = -\frac{\infty}{-5.00} \approx \infty$ (but angular magnification is finite)

(c) Angular magnification:

- $M = -\frac{f_o}{f_e} = -\frac{50.0}{-5.00} = +10.0$
- The positive value confirms an upright final image (key advantage of Galilean telescopes)
- Verify: $M > 1$, confirming magnification

Key Features of Galilean Telescopes:

1. **Upright image**: Unlike Keplerian telescopes, the image remains upright ($M > 0$)
2. **Compact design**: Length $L = f_o + f_e < f_o$ (shorter than Keplerian)
3. **Limited field of view**: The concave eyepiece restricts the field of view
4. **Applications**: Opera glasses, binoculars, Galileo's original telescopes

This construction demonstrates why Galileo's design was revolutionary—it provided magnification with an upright image in a compact package.

Glossary

adaptive optics
: optical technology in which computers adjust the lenses and mirrors in a device to correct for image distortions

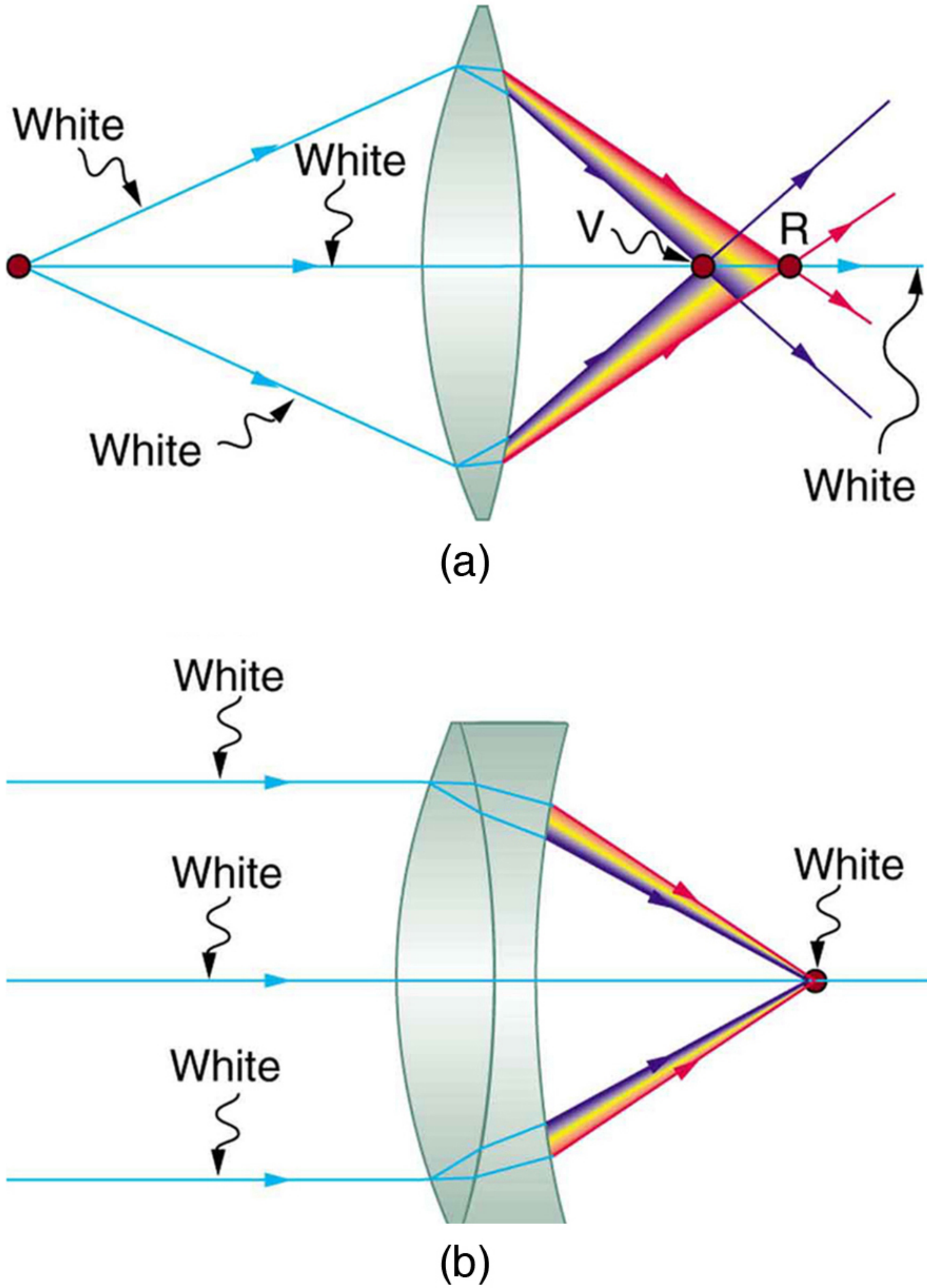
angular magnification
: a ratio related to the focal lengths of the objective and eyepiece and given as $M = -\frac{f_o}{f_e}$

Aberrations

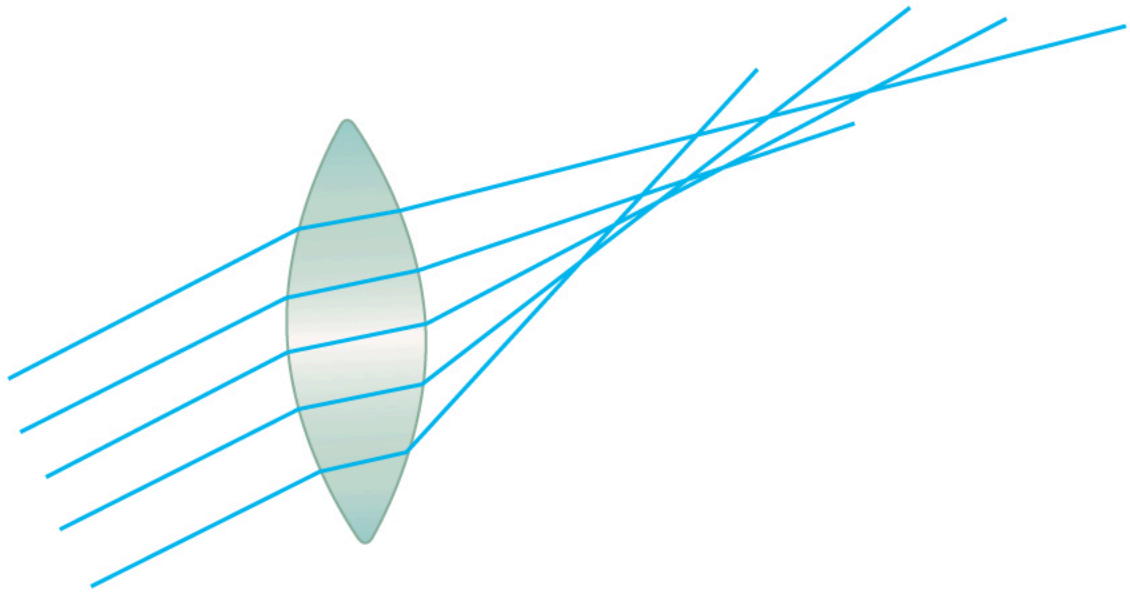
- Describe optical aberration.

Real lenses behave somewhat differently from how they are modeled using the thin lens equations, producing **aberrations**. An aberration is a distortion in an image. There are a variety of aberrations due to a lens size, material, thickness, and position of the object. One common type of aberration is chromatic aberration, which is related to color. Since the index of refraction of lenses depends on color or wavelength, images are produced at different places and with different magnifications for different colors. (The law of reflection is independent of wavelength, and so mirrors do not have this problem. This is another advantage for mirrors in optical systems such as telescopes.) [Figure 1](a) shows chromatic aberration for a single convex lens and its partial correction with a two-lens system. Violet rays are bent more than red, since they have a higher index of refraction and are thus focused closer to the lens. The diverging lens partially corrects this, although it is usually not possible to do so completely. Lenses of different materials and having different dispersions may be used. For example an achromatic doublet consisting of a converging lens made of crown glass and a diverging lens made of flint glass in contact can dramatically reduce chromatic aberration (see [Figure 1](b)).

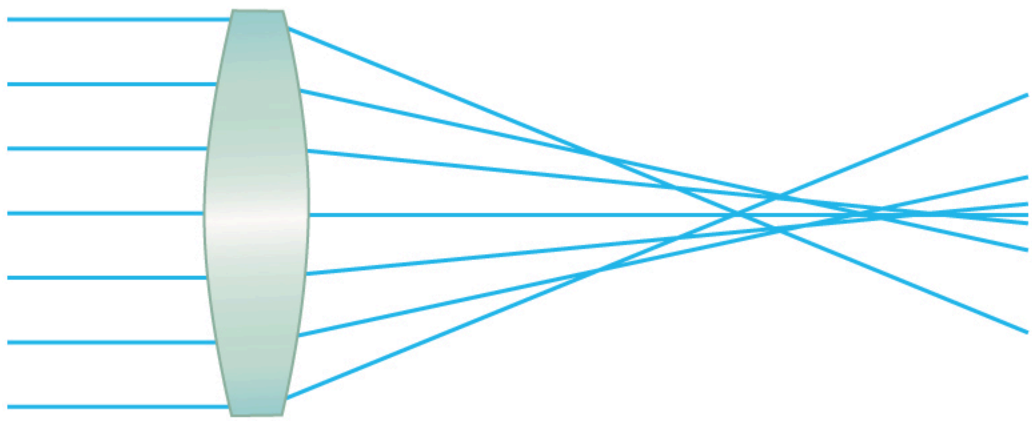
Quite often in an imaging system the object is off-center. Consequently, different parts of a lens or mirror do not refract or reflect the image to the same point. This type of aberration is called a coma and is shown in [Figure 2]. The image in this case often appears pear-shaped. Another common aberration is spherical aberration where rays converging from the outer edges of a lens converge to a focus closer to the lens and rays closer to the axis focus further (see [Figure 3]). Aberrations due to astigmatism in the lenses of the eyes are discussed in Vision Correction, and a chart used to detect astigmatism is shown in [Figure 4]. Such aberrations and can also be an issue with manufactured lenses.



(a) Chromatic aberration is caused by the dependence of a lens's index of refraction on color (wavelength). The lens is more powerful for violet (V) than for red (R), producing images with different locations and magnifications. (b) Multiple-lens systems can partially correct chromatic aberrations, but they may require lenses of different materials and add to the expense of optical systems such as cameras.



A coma is an aberration caused by an object that is off-center, often resulting in a pear-shaped image. The rays originate from points that are not on the optical axis and they do not converge at one common focal point.



Spherical aberration is caused by rays focusing at different distances from the lens.

The image produced by an optical system needs to be bright enough to be discerned. It is often a challenge to obtain a sufficiently bright image. The brightness is determined by the amount of light passing through the optical system. The optical components determining the brightness are the diameter of the lens and the diameter of pupils, diaphragms or aperture stops placed in front of lenses. Optical systems often have entrance and exit pupils to specifically reduce aberrations but they inevitably reduce brightness as well. Consequently, optical systems need to strike a balance between the various components used. The iris in the eye dilates and constricts, acting as an entrance pupil. You can see objects more clearly by looking through a small hole made with your hand in the shape of a fist. Squinting, or using a small hole in a piece of paper, also will make the object sharper.

So how are aberrations corrected? The lenses may also have specially shaped surfaces, as opposed to the simple spherical shape that is relatively easy to produce. Expensive camera lenses are large in diameter, so that they can gather more light, and need several elements to correct for various aberrations. Further, advances in materials science have resulted in lenses with a range of refractive indices—technically referred to as graded index (GRIN) lenses. Spectacles often have the ability to provide a range of focusing ability using similar techniques. GRIN lenses are particularly important at the end of optical fibers in endoscopes. Advanced computing techniques allow for a range of corrections on images after the image has been collected and certain characteristics of the optical system are known. Some of these techniques are sophisticated versions of what are available on commercial packages like Adobe Photoshop.

Section Summary

- Aberrations or image distortions can arise due to the finite thickness of optical instruments, imperfections in the optical components, and limitations on the ways in which the components are used.
- The means for correcting aberrations range from better components to computational techniques.

Conceptual Questions

List the various types of aberrations. What causes them and how can each be reduced?

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Strategy

To systematically address optical aberrations, we need to identify the main types discussed in optical systems, understand their physical origins, and examine practical methods for their reduction. Aberrations arise from the departure of real optical systems from the idealized thin lens approximation.

Solution

The major types of optical aberrations are:

1. Chromatic Aberration

Cause: The refractive index of optical materials varies with wavelength (dispersion). Different colors of light are refracted by different amounts, causing them to focus at different distances from the lens. Violet light (shorter wavelength, higher refractive index) focuses closer to the lens than red light (longer wavelength, lower refractive index).

Reduction methods:

- Use **achromatic doublets**: Combine a converging lens (crown glass) with a diverging lens (flint glass) made from materials with different dispersion properties
- Use **apochromatic lens systems**: Multiple lens elements designed to bring three wavelengths to the same focus
- Use **mirrors instead of lenses**: Reflection is independent of wavelength, so mirrors do not exhibit chromatic aberration
- Apply **diffractive optical elements**: Can be designed to have opposite dispersion to refractive elements

2. Spherical Aberration

Cause: Rays passing through the outer edges of a spherical lens converge at a point closer to the lens than rays passing through the center (paraxial rays). This occurs because spherical surfaces are not the ideal shape for focusing—a parabolic surface would be better for collimated light.

Reduction methods:

- Use **aspheric lens surfaces**: Depart from spherical shape to achieve better focusing
- Use **aperture stops**: Reduce the lens diameter to block peripheral rays (though this reduces brightness)
- Use **multiple lens elements**: Combine lenses to partially cancel spherical aberration
- Position lenses optimally: Use specific lens orientations and separations

3. Coma

Cause: Off-axis object points produce comet-shaped or pear-shaped images. Different zones of the lens refract off-axis rays to different positions, and the magnification varies across the lens aperture. This asymmetric aberration increases with distance from the optical axis.

Reduction methods:

- Use **aplanatic lens designs**: Special lens combinations that simultaneously correct for spherical aberration and coma
- Use **symmetrical optical designs**: Multi-element systems with symmetric configuration
- Restrict **field of view**: Limit the angular range of off-axis objects
- Use **field flatteners**: Additional optical elements to improve off-axis performance

4. Astigmatism (in lenses and optical systems)

Cause: For off-axis points, rays in different planes (tangential vs. sagittal) focus at different distances. The lens effectively has different focal lengths in perpendicular directions for off-axis objects.

Reduction methods:

- Use **anastigmat lens designs**: Multi-element systems designed to correct astigmatism
- Use **curved image surfaces**: Match the natural field curvature of the lens system
- Optimize **lens spacing and curvatures**: Careful optical design can minimize astigmatism
- Limit **field angle**: Keep objects near the optical axis

5. Field Curvature

Cause: The image surface naturally falls on a curved surface (Petzval surface) rather than a flat plane. Even when other aberrations are corrected, sharp focus occurs on a curved rather than flat surface.

Reduction methods:

- Use **field-flattening elements**: Additional lenses designed to flatten the image surface
- Use **curved detectors/film**: Match the detector to the natural field curvature
- Balance against astigmatism: These aberrations can partially cancel
- Use **meniscus lenses**: Can help reduce field curvature

6. Distortion (Barrel and Pincushion)

Cause: Magnification varies with distance from the optical axis. This doesn't affect sharpness but causes straight lines to appear curved. Barrel distortion occurs when magnification decreases away from the axis; pincushion distortion occurs when it increases.

Reduction methods:

- Use **symmetrical lens designs**: Balanced optical configurations

- Position **aperture stops appropriately**: Strategic placement can minimize distortion
- Use **computational correction**: Digital post-processing to correct distortion
- Use **multiple elements**: Specific lens combinations can cancel distortion

Discussion

The correction of aberrations involves fundamental trade-offs in optical design. No single lens can simultaneously correct all aberrations perfectly across a wide field of view and wavelength range. High-quality optical systems (such as expensive camera lenses, microscope objectives, and telescope optics) use multiple lens elements made from different glass types, often including aspherical surfaces and special coatings.

Modern optical design relies heavily on computer optimization to balance these various aberrations for specific applications. For example, a wide-angle camera lens prioritizes distortion correction and field flatness, while a telescope objective prioritizes chromatic and spherical aberration correction for on-axis performance.

Some practical examples of aberration correction include:

- **Camera lenses**: Expensive lenses use 10-20 elements to correct multiple aberrations
- **Microscope objectives**: Marked as “achromat,” “plan” (flat field), or “apo” (apochromatic)
- **Eyeglasses**: Simple lenses accept some aberration for off-axis vision
- **Telescopes**: Reflector designs eliminate chromatic aberration entirely
- **GRIN lenses**: Graded-index materials provide correction through varying refractive index
- **Digital correction**: Modern cameras correct distortion computationally

The choice of correction method depends on application requirements, cost constraints, and acceptable performance trade-offs. Scientific instruments typically demand more comprehensive aberration correction than consumer devices.

Problem Exercises

Integrated Concepts

(a) During laser vision correction, a brief burst of 193 nm ultraviolet light is projected onto the cornea of the patient. It makes a spot 1.00 mm in diameter and deposits 0.500 mJ of energy. Calculate the depth of the layer ablated, assuming the corneal tissue has the same properties as water and is initially at $34.0^\circ C$. The tissue’s temperature is increased to $100^\circ C$ and evaporated without further temperature increase.

(b) Does your answer imply that the shape of the cornea can be finely controlled?

Show Solution

(a) $0.251 \mu m$ (b) Yes, this thickness implies that the shape of the cornea can be very finely controlled, producing normal distant vision in more than 90% of patients.

Glossary

aberration
: failure of rays to converge at one focus because of limitations or defects in a lens or mirror