# Introduction to the Physics of Hearing

This tree fell some time ago. When it fell, atoms in the air were disturbed. Physicists would call this disturbance sound whether someone was around to hear it or not. (credit: B.A. Bowen Photography)

If a tree falls in the forest and no one is there to hear it, does it make a sound? The answer to this old philosophical question depends on how you define sound. If sound only exists when someone is around to perceive it, then there was no sound. However, if we define sound in terms of physics; that is, a disturbance of the atoms in matter transmitted from its origin outward (in other words, a wave), then there *was* a sound, even if nobody was around to hear it.

Such a wave is the physical phenomenon we call *sound* . Its perception is hearing. Both the physical phenomenon and its perception are interesting and will be considered in this text. We shall explore both sound and hearing; they are related, but are not the same thing. We will also explore the many practical uses of sound waves, such as in medical imaging.

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# Sound

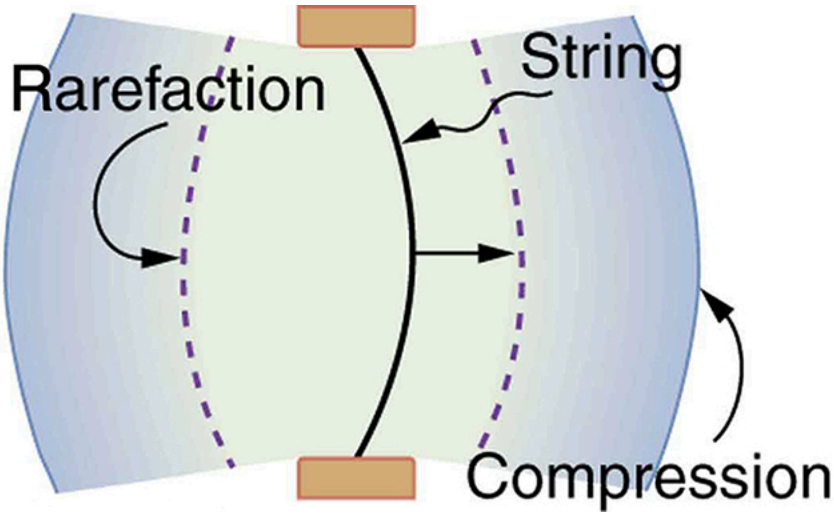
- Define sound and hearing.
- Describe sound as a longitudinal wave.

This glass has been shattered by a high-intensity sound wave of the same frequency as the resonant frequency of the glass. While the sound is not visible, the effects of the sound prove its existence. (credit: read, Flickr)

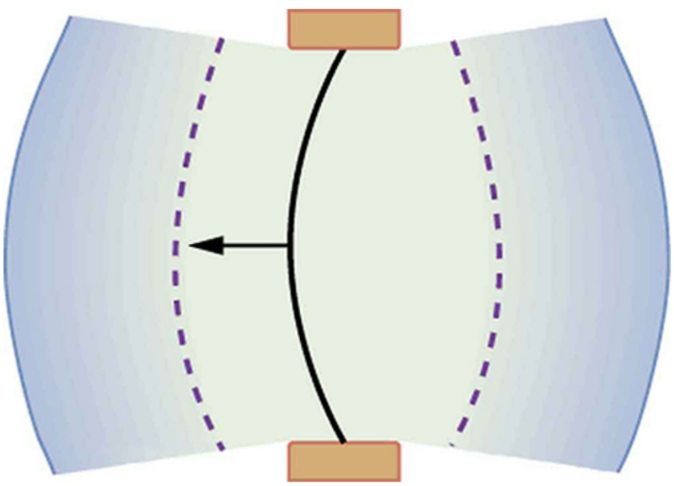
Sound can be used as a familiar illustration of waves. Because hearing is one of our most important senses, it is interesting to see how the physical properties of sound correspond to our perceptions of it. **Hearing** is the perception of sound, just as vision is the perception of visible light. But sound has important applications beyond hearing. Ultrasound, for example, is not heard but can be employed to form medical images and is also used in treatment.

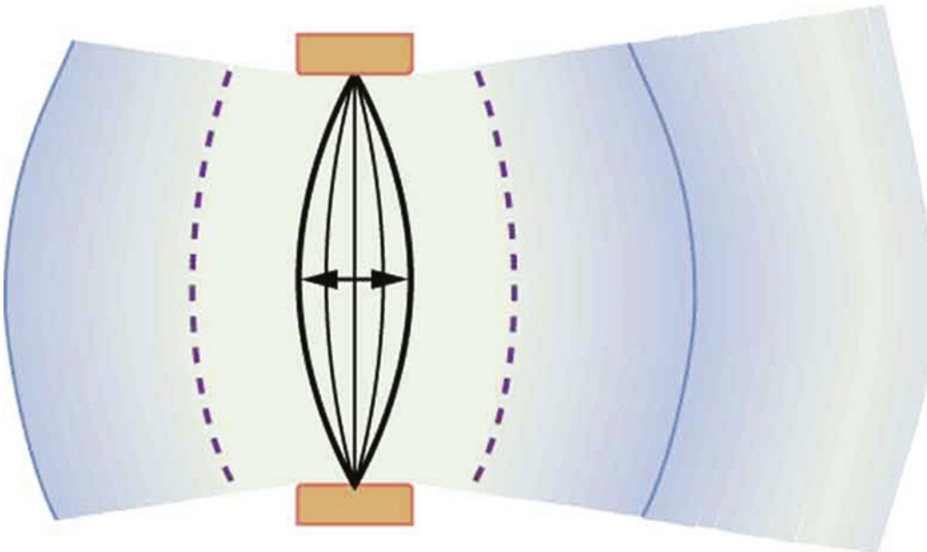
The physical phenomenon of **sound** is defined to be a disturbance of matter that is transmitted from its source outward. Sound is a wave. On the atomic scale, it is a disturbance of atoms that is far more ordered than their thermal motions. In many instances, sound is a periodic wave, and the atoms undergo simple harmonic motion. In this text, we shall explore such periodic sound waves.

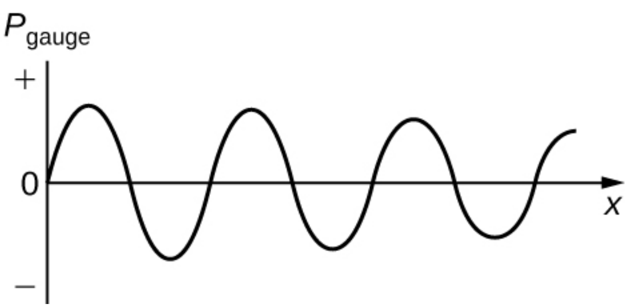
A vibrating string produces a sound wave as illustrated in [Figure 2], [Figure 3], and [Figure 4]. As the string oscillates back and forth, it transfers energy to the air, mostly as thermal energy created by turbulence. But a small part of the string's energy goes into compressing and expanding the surrounding air, creating slightly higher and lower local pressures. These compressions (high pressure regions) and rarefactions (low pressure regions) move out as longitudinal pressure waves having the same frequency as the string—they are the disturbance that is a sound wave. (Sound waves in air and most fluids are longitudinal, because fluids have almost no shear strength. In solids, sound waves can be both transverse and longitudinal.) [Figure 4] shows a graph of gauge pressure versus distance from the vibrating string.



A vibrating string moving to the right compresses the air in front of it and expands the air behind it.

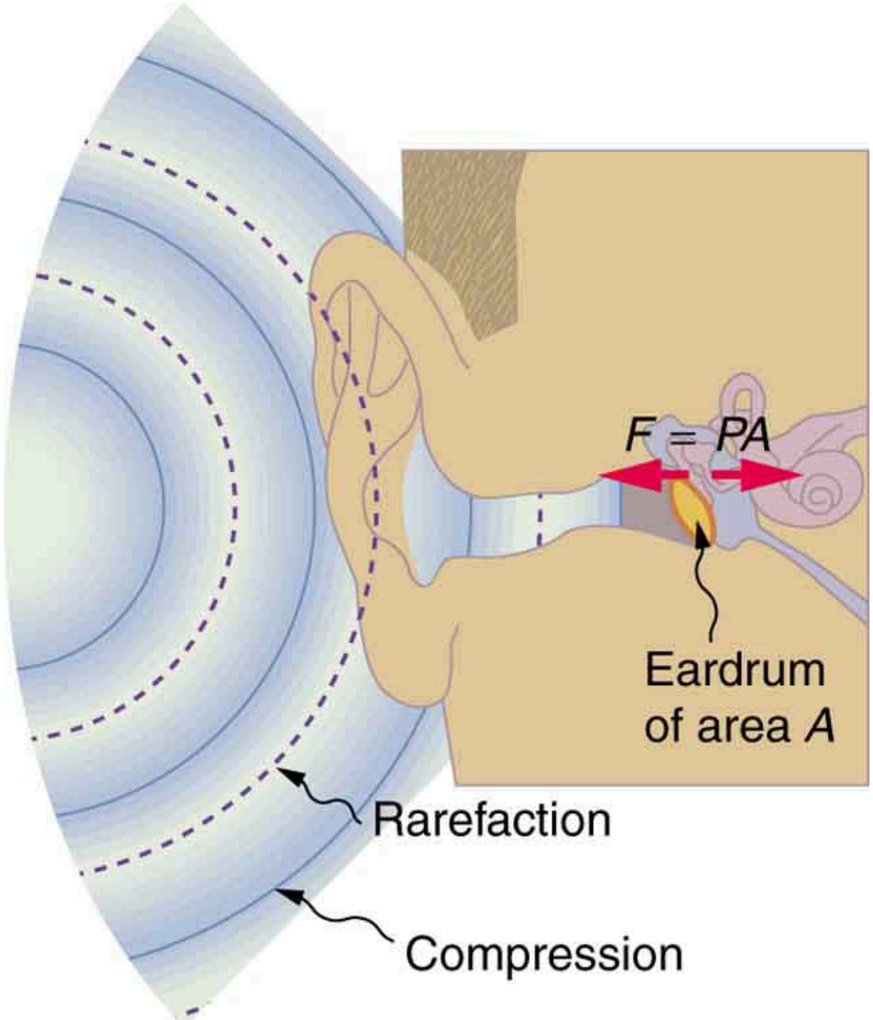

As the string moves to the left, it creates another compression and rarefaction as the ones on the right move away from the string.






After many vibrations, there are a series of compressions and rarefactions moving out from the string as a sound wave. The graph shows gauge pressure versus distance from the source. Pressures vary only slightly from atmospheric for ordinary sounds.

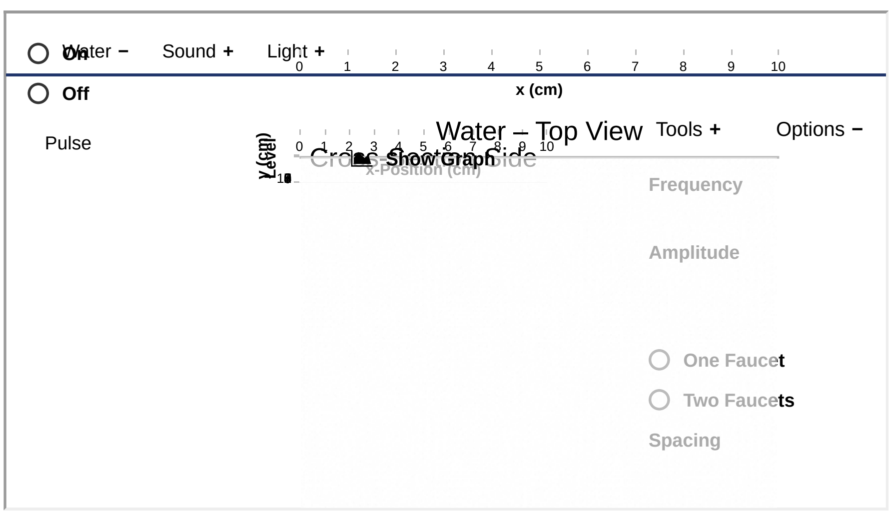
The amplitude of a sound wave decreases with distance from its source, because the energy of the wave is spread over a larger and larger area. But it is also absorbed by objects, such as the eardrum in [Figure 5], and converted to thermal energy by the viscosity of air. In addition, during each compression a little heat transfers to the air and during each rarefaction even less heat transfers from the air, so that the heat transfer reduces the organized disturbance into random thermal motions. (These processes can be viewed as a manifestation of the second law of thermodynamics presented in Introduction to the Second Law of Thermodynamics: Heat Engines and Their Efficiency .) Whether the heat transfer from compression to rarefaction is significant depends on how far apart they are—that is, it depends on wavelength. Wavelength, frequency, amplitude, and speed of propagation are important for sound, as they are for all waves.




Sound wave compressions and rarefactions travel up the ear canal and force the eardrum to vibrate. There is a net force on the eardrum, since the sound wave pressures differ from the atmospheric pressure found behind the eardrum. A complicated mechanism converts the vibrations to nerve impulses, which are perceived by the person.

PhET Explorations: Wave Interference

Make waves with a dripping faucet, audio speaker, or laser! Add a second source or a pair of slits to create an interference pattern.




## Section Summary

- Sound is a disturbance of matter that is transmitted from its source outward.

- Sound is one type of wave.
- Hearing is the perception of sound.

## Glossary

sound
: a disturbance of matter that is transmitted from its source outward

hearing
: the perception of sound

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# Speed of Sound, Frequency, and Wavelength

- Define pitch.
- Describe the relationship between the speed of sound, its frequency, and its wavelength.
- Describe the effects on the speed of sound as it travels through various media.
- Describe the effects of temperature on the speed of sound.

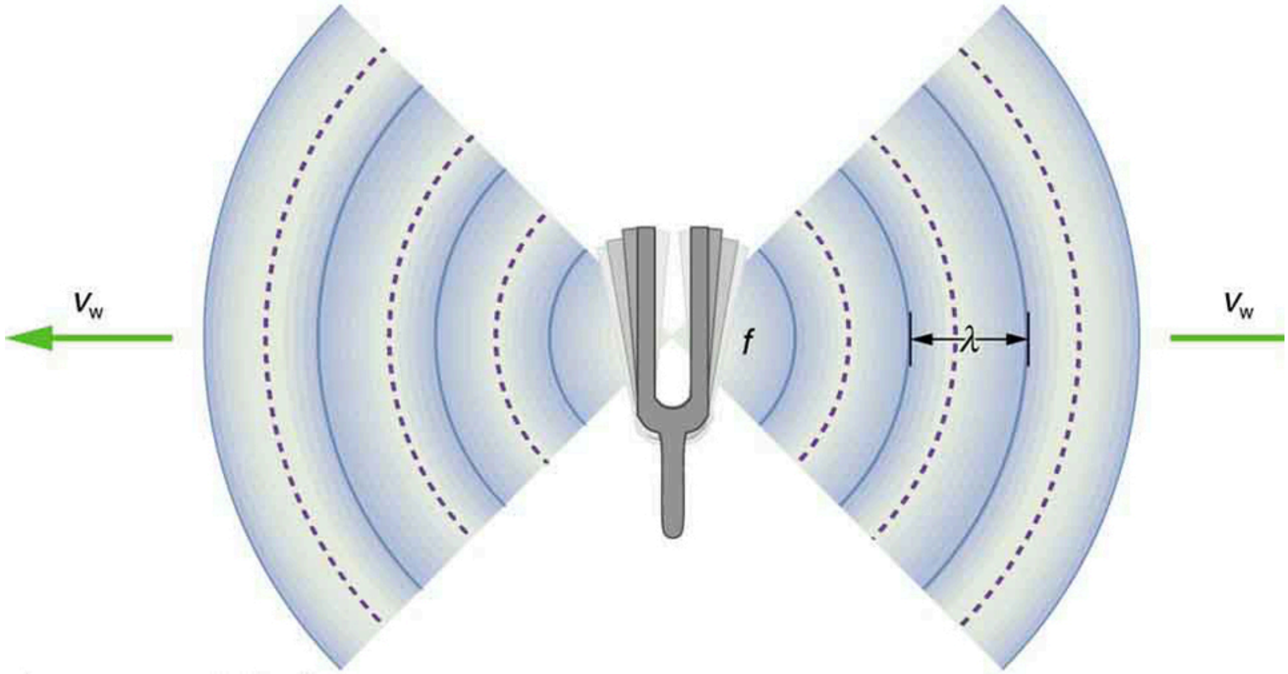
When a firework explodes, the light energy is perceived before the sound energy. Sound travels more slowly than light does. (credit: Dominic Alves, Flickr)

Sound, like all waves, travels at a certain speed and has the properties of frequency and wavelength. You can observe direct evidence of the speed of sound while watching a fireworks display. The flash of an explosion is seen well before its sound is heard, implying both that sound travels at a finite speed and that it is much slower than light. You can also directly sense the frequency of a sound. Perception of frequency is called **pitch**. The wavelength of sound is not directly sensed, but indirect evidence is found in the correlation of the size of musical instruments with their pitch. Small instruments, such as a piccolo, typically make high-pitch sounds, while large instruments, such as a tuba, typically make low-pitch sounds. High pitch means small wavelength, and the size of a musical instrument is directly related to the wavelengths of sound it produces. So a small instrument creates short-wavelength sounds. Similar arguments hold that a large instrument creates long-wavelength sounds.

The relationship of the speed of sound, its frequency, and wavelength is the same as for all waves:

$$v_{\text{w}}=f\lambda,$$

where $v_w$ is the speed of sound, $f$ is its frequency, and $\lambda$ is its wavelength. The wavelength of a sound is the distance between adjacent identical parts of a wave—for example, between adjacent compressions as illustrated in [Figure 2]. The frequency is the same as that of the source and is the number of waves that pass a point per unit time.




A sound wave emanates from a source vibrating at a frequency \( f \), propagates at \( v_w \) , and has a wavelength \( \lambda \).

[Table 1] makes it apparent that the speed of sound varies greatly in different media. The speed of sound in a medium is determined by a combination of the medium's rigidity (or compressibility in gases) and its density. The more rigid (or less compressible) the medium, the faster the speed of sound. This observation is analogous to the fact that the frequency of a simple harmonic motion is directly proportional to the stiffness of the oscillating object. The greater the density of a medium, the slower the speed of sound. This observation is analogous to the fact that the frequency of a simple harmonic motion is inversely proportional to the mass of the oscillating object. The speed of sound in air is low, because air is compressible. Because liquids and solids are relatively rigid and very difficult to compress, the speed of sound in such media is generally greater than in gases.

Speed of Sound in Various Media

| Medium | $v_w$(m/s) |
|---|---|
| ***Gases at $0^oC$*** | |
| Air | 331 |
| Carbon dioxide | 259 |
| Oxygen | 316 |
| Helium | 965 |
| Hydrogen | 1290 |
| ***Liquids at $20^oC$*** | |
| Ethanol | 1160 |
| Mercury | 1450 |
| Water, fresh | 1480 |
| Sea water | 1540 |
| Human tissue | 1540 |
| ***Solids (longitudinal or bulk)*** | |
| Vulcanized rubber | 54 |
| Polyethylene | 920 |
| Marble | 3810 |
| Glass, Pyrex | 5640 |
| Lead | 1960 |
| Aluminum | 5120 |
| Steel | 5960 |

Earthquakes, essentially sound waves in Earth's crust, are an interesting example of how the speed of sound depends on the rigidity of the medium. Earthquakes have both longitudinal and transverse components, and these travel at different speeds. The bulk modulus of granite is greater than its shear modulus. For that reason, the speed of longitudinal or pressure waves (P-waves) in earthquakes in granite is significantly higher than the speed of transverse or shear waves (S-waves). Both components of earthquakes travel slower in less rigid material, such as sediments. P-waves have speeds of 4 to 7 km/s, and S-waves correspondingly range in speed from 2 to 5 km/s, both being faster in more rigid material. The P-wave gets progressively farther ahead of the S-wave as they travel through Earth's crust. The time between the P- and S-waves is routinely used to determine the distance to their source, the epicenter of the earthquake. The time and nature of these wave differences also provides the evidence for the nature of Earth's core. Through careful analysis of seismographic records of large earthquakes whose waves could be clearly detected around the world, Richard Dixon Oldham established that waves passing through the center of the Earth behaved as if they were moving through a different medium: a liquid. Later on, Inge Lehmann used more precise observations (partly based on a better coordinated network of seismographs she helped set up) to better define the nature of the core: that it was a solid inner core surrounded by a liquid outer core.

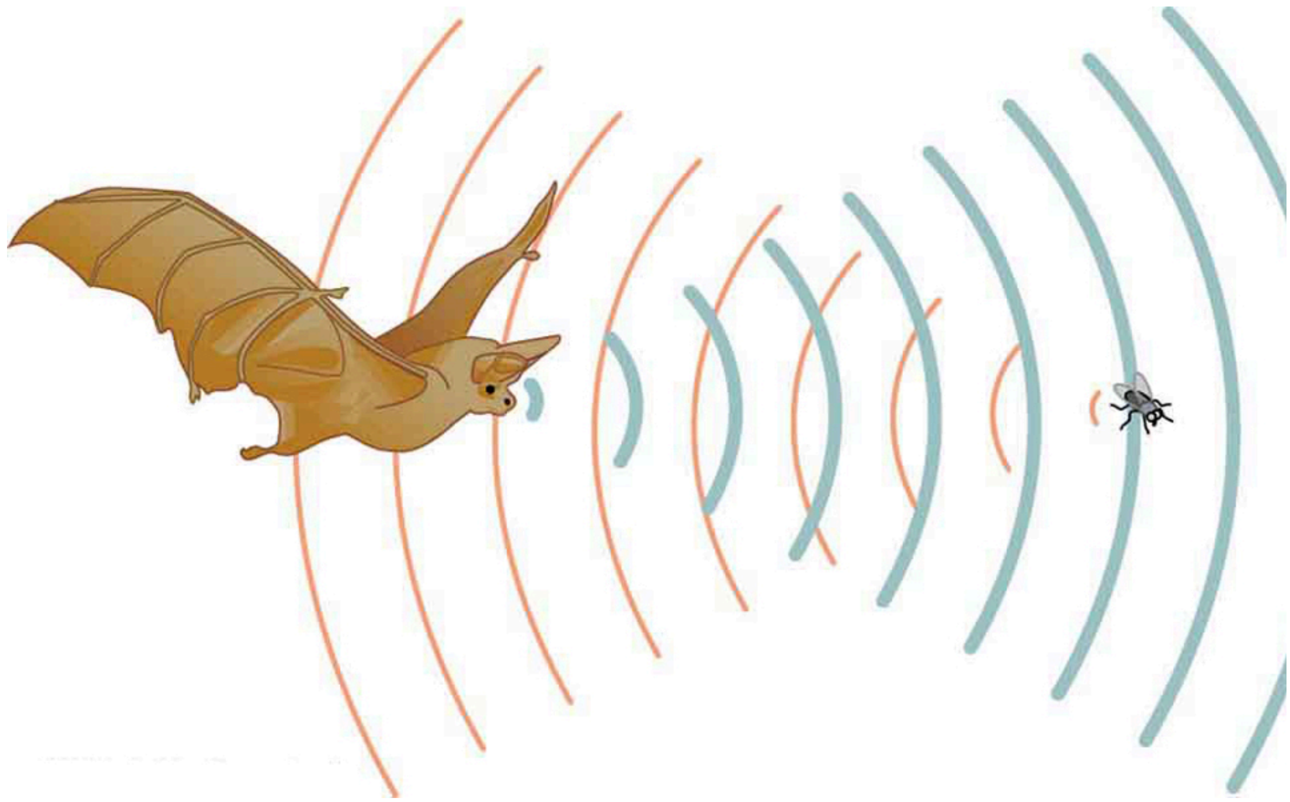
The speed of sound is affected by temperature in a given medium. For air at sea level, the speed of sound is given by

$$v_W = (331\text{m/s})\sqrt{\frac{T}{273\text{K}}},$$

where the temperature (denoted as $T$ ) is in units of kelvin. The speed of sound in gases is related to the average speed of particles in the gas, $v_{rms}$ , and that

$$v_{rms} = \sqrt{\frac{3k_BT}{m}},$$

where $k_B$ is the Boltzmann constant ( $1.38 \times 10^{-23}$ J/K ) and $m$ is the mass of each (identical) particle in the gas. So, it is reasonable that the speed of sound in air and other gases should depend on the square root of temperature. While not negligible, this is not a strong dependence. At $0^{o}\text{C}$ , the speed of sound is 331 m/s, whereas at $20.0^{o}\text{C}$ it is 343 m/s, less than a 4% increase. [Figure 3] shows a use of the speed of sound by a bat to sense distances. Echoes are also used in medical imaging.

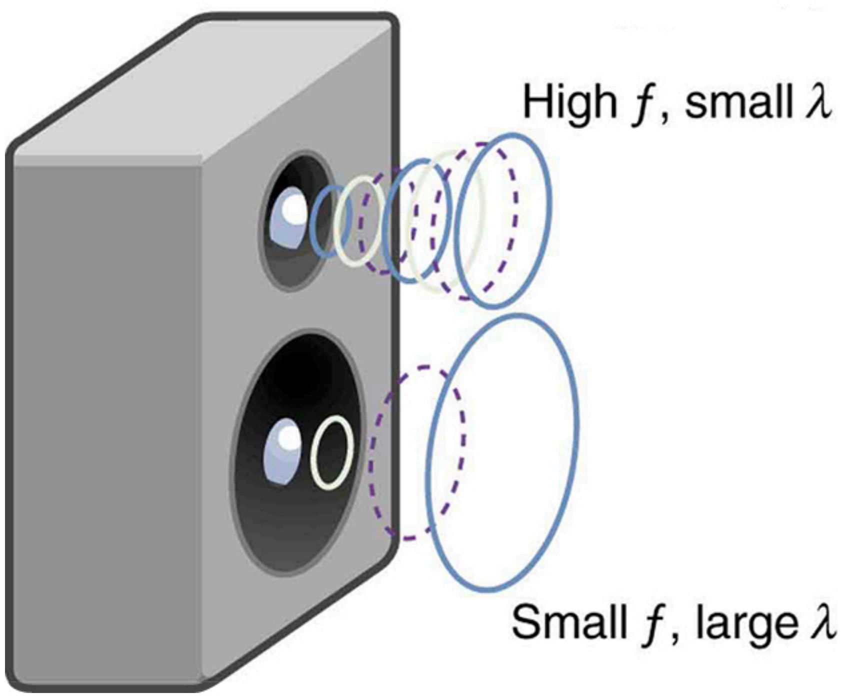


A bat uses sound echoes to find its way about and to catch prey. The time for the echo to return is directly proportional to the distance.

One of the more important properties of sound is that its speed is nearly independent of frequency. This independence is certainly true in open air for sounds in the audible range of 20 to 20 000 Hz. If this independence were not true, you would certainly notice it for music played by a marching band in a football stadium, for example. Suppose that high-frequency sounds traveled faster—then the farther you were from the band, the more the sound from the low-pitch instruments would lag that from the high-pitch ones. But the music from all instruments arrives in cadence independent of distance, and so all frequencies must travel at nearly the same speed. Recall that

$$v_W = f\lambda.$$

In a given medium under fixed conditions, $v_W$ is constant, so that there is a relationship between $f$ and $\lambda$ ; the higher the frequency, the smaller the wavelength. See [Figure 4] and consider the following example.




Because they travel at the same speed in a given medium, low-frequency sounds must have a greater wavelength than high-frequency sounds. Here, the lower-frequency sounds are emitted by the large speaker, called a woofer, while the higher-frequency sounds are emitted by the small speaker, called a tweeter.

Calculating Wavelengths: What Are the Wavelengths of Audible Sounds?

Calculate the wavelengths of sounds at the extremes of the audible range, 20 and 20 000 Hz, in $30.0^{\circ}\text{C}$ air. (Assume that the frequency values are accurate to two significant figures.)

**Strategy**

To find wavelength from frequency, we can use $v_{\text{w}} = f\lambda$.

**Solution**

1. Identify knowns. The value for $v_{\text{w}}$, is given by
   $v_{\text{w}}=(331\text{m/s})\sqrt{\frac{T}{273\text{K}}}$.
2. Convert the temperature into kelvin and then enter the temperature into the equation
   $v_{\text{w}}=(331\text{m/s})\sqrt{\frac{303\text{K}}{273\text{K}}}=348.7\text{m/s}$.
3. Solve the relationship between speed and wavelength for $\lambda$:
   $\lambda=\frac{v_w}{f}$.
4. Enter the speed and the minimum frequency to give the maximum wavelength:
   $\lambda_{\text{max}}=\frac{348.7\text{m/s}}{20\text{Hz}}=17\text{m}$.
5. Enter the speed and the maximum frequency to give the minimum wavelength:
   $\lambda_{\text{min}}=\frac{348.7\text{m/s}}{20{,}000\text{Hz}}=0.017\text{m}=1.7\text{cm}$.

**Discussion**

Because the product of $f$ multiplied by $\lambda$ equals a constant, the smaller $f$ is, the larger $\lambda$ must be, and vice versa.

The speed of sound can change when sound travels from one medium to another. However, the frequency usually remains the same because it is like a driven oscillation and has the frequency of the original source. If $v_{\text{w}}$ changes and $f$ remains the same, then the wavelength $\lambda$ must change. That is, because $v_{\text{w}} = f\lambda$, the higher the speed of a sound, the greater its wavelength for a given frequency.

Making Connections: Take-Home Investigation—Voice as a Sound Wave

Suspend a sheet of paper so that the top edge of the paper is fixed and the bottom edge is free to move. You could tape the top edge of the paper to the edge of a table. Gently blow near the edge of the bottom of the sheet and note how the sheet moves. Speak softly and then louder such that the sounds hit the edge of the bottom of the paper, and note how the sheet moves. Explain the effects.

Check Your Understanding

Imagine you observe two fireworks explode. You hear the explosion of one as soon as you see it. However, you see the other firework for several milliseconds before you hear the explosion. Explain why this is so.

Show Solution

Sound and light both travel at definite speeds. The speed of sound is slower than the speed of light. The first firework is probably very close by, so the speed difference is not noticeable. The second firework is farther away, so the light arrives at your eyes noticeably sooner than the sound wave arrives at your ears.
Check Your Understanding

You observe two musical instruments that you cannot identify. One plays high-pitch sounds and the other plays low-pitch sounds. How could you determine which is which without hearing either of them play?

Show Solution

Compare their sizes. High-pitch instruments are generally smaller than low-pitch instruments because they generate a smaller wavelength.

## Section Summary

The relationship of the speed of sound $v_w$, its frequency $f$, and its wavelength $\lambda$ is given by

$$v_W = f\lambda,$$

which is the same relationship given for all waves.

In air, the speed of sound is related to air temperature $T$ by

$$v_W = (331 \text{m/s})\sqrt{\frac{T}{273 \text{K}}}.$$

$v_W$ is the same for all frequencies and wavelengths.

## Conceptual Questions

How do sound vibrations of atoms differ from thermal motion?

Show Solution

Sound vibrations are a form of organized, collective motion where atoms oscillate in a coordinated way around their equilibrium positions, creating a pressure wave that propagates through the medium. This motion is coherent, meaning the atoms move in a predictable, synchronized pattern (compressions and rarefactions).

Thermal motion, on the other hand, is the random, chaotic, and incoherent movement of atoms due to their thermal energy. The atoms move in all directions with a wide range of speeds, and their motion is not coordinated. While both sound and thermal motion are forms of kinetic energy, sound is an organized transfer of energy, whereas thermal energy is disorganized.

When sound passes from one medium to another where its propagation speed is different, does its frequency or wavelength change? Explain your answer briefly.

Show Solution

The frequency of the sound wave remains the same, while the wavelength changes.

**Explanation**: The frequency of a wave is determined by its source and does not change when the wave passes from one medium to another. The wave speed, however, is a property of the medium and does change. According to the wave equation ($v_w = f\lambda$), if the speed ($v_w$) changes and the frequency ($f$) remains constant, the wavelength ($\lambda$) must change to maintain the equality. For example, when sound travels from air to water, its speed increases, so its wavelength must also increase.

## Problems & Exercises

When poked by a spear, an operatic soprano lets out a 1200-Hz shriek. What is its wavelength if the speed of sound is 345 m/s?

Show Solution

**Strategy**

We are given the frequency and speed of the sound wave and need to find its wavelength. We can use the fundamental wave equation $v_w = f\lambda$ and solve for wavelength.

**Solution**

Given:

- Frequency: $f = 1200$ Hz
- Speed of sound: $v_w = 345$ m/s

Rearranging the wave equation to solve for wavelength:

$\lambda = v_w f$

Substituting values:

$\lambda = 345\text{ m/s}1200\text{ Hz} = 0.288\text{ m}$

**Discussion**

The wavelength of 0.288 m (or 28.8 cm) is reasonable for a high-pitched sound. This is roughly the length of a ruler, which makes sense for a sound in the audible range. Higher-frequency sounds have shorter wavelengths, and 1200 Hz is in the mid-to-high frequency range of human hearing. This wavelength would be comparable to the size of small musical instruments that produce similar frequencies.

The shriek has a wavelength of 0.288 m.

What frequency sound has a 0.10-m wavelength when the speed of sound is 340 m/s?

Show Solution

**Strategy**

We are given the wavelength and speed of sound and need to find the frequency. We can use the wave equation $v_w = f\lambda$ and solve for frequency.

**Solution**

Given:

- Wavelength: $\lambda = 0.10\text{ m}$
- Speed of sound: $v_w = 340\text{ m/s}$

Rearranging the wave equation to solve for frequency:

$f = v_w \lambda$

Substituting values:

$f = 340\text{ m/s}0.10\text{ m} = 3400\text{ Hz} = 3.4\text{ kHz}$

**Discussion**

A frequency of 3400 Hz is well within the audible range for humans (20 Hz to 20,000 Hz) and would be perceived as a relatively high-pitched sound. The wavelength of 0.10 m (10 cm) is quite short, which is consistent with this high frequency. This is approximately the size of a grapefruit. Musical instruments that produce sounds at this frequency would typically be small, such as a piccolo or the upper range of a violin. The inverse relationship between wavelength and frequency is clearly demonstrated: a small wavelength corresponds to a high frequency.

The frequency of the sound is 3400 Hz or 3.4 kHz.

Calculate the speed of sound on a day when a 1500 Hz frequency has a wavelength of 0.221 m.

Show Solution

**Strategy**

We are given both the frequency and wavelength of a sound wave and need to find the speed of sound. We can directly apply the wave equation $v_w = f\lambda$.

**Solution**

Given:

- Frequency: $f = 1500\text{ Hz}$
- Wavelength: $\lambda = 0.221\text{ m}$

Using the wave equation:

$v_w = f\lambda$

Substituting values:

$v_w = (1500\text{ Hz})(0.221\text{ m}) = 332\text{ m/s}$

**Discussion**

The calculated speed of sound is 332 m/s, which is slightly slower than the standard value of 343 m/s at 20°C. This indicates that the air temperature on this day is lower than 20°C. Using the temperature-dependent speed formula $v_w = (331\text{ m/s})\sqrt{T/273\text{ K}}$, we can estimate that the temperature is approximately 3°C. This is a reasonable temperature for a cool day. The result demonstrates how sound speed varies with atmospheric conditions, which is important for applications like sonar, musical acoustics, and outdoor sound propagation.

The speed of sound on this day is 332 m/s.

(a) What is the speed of sound in a medium where a 100-kHz frequency produces a 5.96-cm wavelength? (b) Which substance in [Table 1] is this likely to be?

Show Solution

**Strategy**

For part (a), we use the wave equation $v_w = f\lambda$ with the given frequency and wavelength. For part (b), we compare our calculated speed to the values in Table 1 to identify the medium.

**Solution**

**(a)** Given:

- Frequency: $f = 100\text{ kHz} = 100 \times 10^3\text{ Hz} = 1.00 \times 10^5\text{ Hz}$
- Wavelength: $\lambda = 5.96\text{ cm} = 0.0596\text{ m}$

Using the wave equation:

$$v_w = f\lambda = (1.00\times10^5\text{ Hz})(0.0596\text{ m}) = 5960\text{ m/s}$$

**(b)** Comparing this speed to Table 1, we find that steel has a speed of sound of 5960 m/s. Therefore, the medium is most likely **steel**.

**Discussion**

The speed of 5960 m/s is much faster than sound in air (343 m/s) or water (1480 m/s), which indicates a solid medium. Among solids, steel is known for its high rigidity and density combination that produces very fast sound propagation. The high frequency of 100 kHz (ultrasound range, well above human hearing) and short wavelength of about 6 cm are typical for ultrasonic testing applications in steel structures and components. This technique is commonly used in non-destructive testing to detect flaws in metal parts. The perfect match with steel's sound speed in the table confirms our identification.

(a) The speed of sound in the medium is 5960 m/s. (b) The substance is most likely steel.

Show that the speed of sound in $20.0^\circ\text{C}$ air is 343 m/s, as claimed in the text.

Show Solution

**Strategy**

We need to use the temperature-dependent formula for the speed of sound in air: $v_w = (331\text{ m/s})\sqrt{T/273\text{ K}}$. First, we must convert the temperature from Celsius to Kelvin, then substitute into the formula.

**Solution**

Given:

- Temperature: $T = 20.0°\text{C}$

Step 1: Convert temperature to Kelvin:

$$T = 20.0 + 273 = 293\text{ K}$$

Step 2: Substitute into the speed of sound formula:

$$v_w = (331\text{ m/s})\sqrt{\frac{T}{273\text{ K}}} = (331\text{ m/s})\sqrt{\frac{293\text{ K}}{273\text{ K}}} = (331\text{ m/s})\sqrt{1.073} = (331\text{ m/s})(1.036) = 343\text{ m/s}$$

**Discussion**

This calculation confirms the stated value of 343 m/s for the speed of sound at room temperature (20°C). The speed increases by about 3.6% compared to the speed at 0°C (331 m/s), which demonstrates the square root dependence on absolute temperature. This temperature dependence arises because sound propagation in gases is related to the average molecular speed, which also depends on $\sqrt{T}$. The value 343 m/s is commonly used as a standard reference for sound speed in air and is important for many practical applications, from acoustics to weather prediction.

The speed of sound in 20.0°C air is indeed 343 m/s, as claimed.

Air temperature in the Sahara Desert can reach $56.0^\circ\text{C}$ (about $134^\circ\text{F}$). What is the speed of sound in air at that temperature?

Show Solution

**Strategy**

We use the temperature-dependent formula for the speed of sound in air: $v_w = (331 \text{ m/s})\sqrt{T/273 \text{ K}}$. We must first convert the temperature from Celsius to Kelvin.

**Solution**

Given:

- Temperature: $T = 56.0°\text{C}$

Step 1: Convert temperature to Kelvin:

$T = 56.0 + 273 = 329 \text{ K}$

Step 2: Substitute into the speed of sound formula:

$$v_w = (331 \text{ m/s})\sqrt{\frac{T}{273 \text{ K}}} = (331 \text{ m/s})\sqrt{\frac{329 \text{ K}}{273 \text{ K}}} = (331 \text{ m/s})\sqrt{1.205} = (331 \text{ m/s})(1.098) = 363 \text{ m/s}$$

**Discussion**

The speed of sound at 56.0°C (363 m/s) is significantly faster than at room temperature (343 m/s at 20°C), representing about a 5.8% increase. This is also nearly 10% faster than at 0°C (331 m/s). The higher speed is due to the increased kinetic energy of air molecules at elevated temperatures, which allows pressure disturbances (sound waves) to propagate more quickly. This temperature effect is important for sound propagation in desert environments and can affect acoustic phenomena like echo location and sound transmission over distances. The extreme heat in the Sahara creates interesting acoustic effects and must be considered in any sound-based measurements or communications in such environments.

The speed of sound in 56.0°C air is 363 m/s.

Dolphins make sounds in air and water. What is the ratio of the wavelength of a sound in air to its wavelength in seawater? Assume air temperature is $20.0\text{ºC}$.

Show Solution

**Strategy**

When a sound travels from one medium to another, its frequency remains constant (determined by the source), but its speed and wavelength change. We can use $v_w = f\lambda$ for each medium. The ratio of wavelengths equals the ratio of speeds since frequency is constant: $\lambda_{\text{air}}/\lambda_{\text{water}} = v_{\text{air}}/v_{\text{water}}$.

**Solution**

From Table 1:

- Speed of sound in air at 20.0°C: $v_{\text{air}} = 343 \text{ m/s}$
- Speed of sound in seawater: $v_{\text{water}} = 1540 \text{ m/s}$

For a sound of frequency $f$ in both media:

In air: $\lambda_{\text{air}} = \frac{v_{\text{air}}}{f}$

In seawater: $\lambda_{\text{water}} = \frac{v_{\text{water}}}{f}$

Taking the ratio:

$$\frac{\lambda_{\text{air}}}{\lambda_{\text{water}}} = \frac{v_{\text{air}}/f}{v_{\text{water}}/f} = \frac{v_{\text{air}}}{v_{\text{water}}} = \frac{343 \text{ m/s}}{1540 \text{ m/s}} = 0.223$$

**Discussion**

The wavelength in air is only about 22% of the wavelength in seawater for the same frequency. This makes sense because sound travels much faster in water (about 4.5 times faster) due to water's greater density and incompressibility compared to air. For example, if a dolphin produces a 10 kHz sound, it would have a wavelength of 3.43 cm in air but 15.4 cm in water. This significant difference is important for understanding dolphin echolocation, which works differently in air versus water. Dolphins are primarily adapted for underwater sound production and reception, which is why they use much higher frequencies underwater to achieve useful wavelengths for echolocation.

The ratio of the wavelength in air to the wavelength in seawater is 0.223.

A sonar echo returns to a submarine 1.20 s after being emitted. What is the distance to the object creating the echo? (Assume that the submarine is in the ocean, not in fresh water.)

Show Solution

**Strategy**

The sonar pulse travels to the object and back, so the total distance traveled is twice the distance to the object. We use the relationship $d = vt$, where $v$ is the speed of sound in seawater and $t$ is the time for the round trip. The distance to the object is half the total distance traveled.

**Solution**

Given:

- Echo time (round trip): $t = 1.20\text{ s}$
- Speed of sound in seawater (from Table 1): $v = 1540\text{ m/s}$

Total distance traveled by sound (to object and back):

$$d_{\text{total}} = vt = (1540\text{ m/s})(1.20\text{ s}) = 1848\text{ m}$$

Distance to object (half the total distance):

$$d = \frac{d_{\text{total}}}{2} = \frac{1848\text{ m}}{2} = 924\text{ m}$$

**Discussion**

The object is 924 m away, which is less than a kilometer. This is a reasonable detection range for submarine sonar systems. The key insight is that we must divide by 2 because the sound makes a round trip—out to the object and back. This principle is fundamental to all echo-based ranging systems, including sonar, radar, and ultrasound imaging. The relatively fast speed of sound in seawater (1540 m/s, about 4.5 times faster than in air) allows submarines to detect objects at considerable distances with relatively short time delays. Note that we assumed the submarine is stationary and the sound travels in a straight line, which are reasonable approximations for this problem.

The distance to the object is 924 m.

(a) If a submarine's sonar can measure echo times with a precision of 0.0100 s, what is the smallest difference in distances it can detect? (Assume that the submarine is in the ocean, not in fresh water.)

(b) Discuss the limits this time resolution imposes on the ability of the sonar system to detect the size and shape of the object creating the echo.

Show Solution

**Strategy**

For part (a), the time precision determines the minimum detectable distance difference. Since sound travels to an object and back, the distance resolution is half the distance traveled during the minimum time interval. For part (b), we consider what this distance resolution means for practical object detection.

**Solution**

**(a)** Given:

- Time precision: $\Delta t = 0.0100\text{ s}$
- Speed of sound in seawater: $v = 1540\text{ m/s}$

Distance traveled by sound during the minimum time interval:

$$d_{\text{total}} = v\Delta t = (1540\text{ m/s})(0.0100\text{ s}) = 15.4\text{ m}$$

Since this is a round-trip distance, the minimum resolvable distance difference is:

$$\Delta d = \frac{d_{\text{total}}}{2} = \frac{15.4\text{ m}}{2} = 7.70\text{ m}$$

**(b)** The distance resolution of 7.70 m imposes significant limitations on the sonar system's ability to detect fine details. Objects or features smaller than about 8 m cannot be resolved as separate entities. This means that sonar is excellent for spotting and locating large objects such as ships, submarines, whales, or large pieces of wreckage (like airplane fuselage sections), but it cannot resolve smaller objects or detect the detailed shapes and surface features of objects. For example, two objects separated by less than 7.70 m would appear as a single target. Small debris, individual fish, or surface texture details would be invisible to this system. Higher resolution would require either improved timing precision or higher frequency sound waves, though the latter suffers from increased attenuation in water.

**Discussion**

This limitation is fundamental to all pulse-echo ranging systems and represents a trade-off between timing precision and system complexity. Modern military sonar systems often use much higher precision timing and sophisticated signal processing to achieve better resolution. The 7.70 m resolution found here is actually quite coarse by modern standards but illustrates the basic principle. Note that this is the range resolution (along the line of sight); cross-range resolution (perpendicular to the line of sight) depends on the angular beamwidth of the sonar array, which is a separate consideration.

(a) The smallest difference in distances the sonar can detect is 7.70 m. (b) This resolution means sonar is effective for detecting and locating large objects like ships or submarines, but cannot resolve smaller objects or fine structural details smaller than approximately 8 m.

A physicist at a fireworks display times the lag between seeing an explosion and hearing its sound, and finds it to be 0.400 s. (a) How far away is the explosion if air temperature is $24.0^{\circ}\text{C}$ and if you neglect the time taken for light to reach the physicist? (b) Calculate the distance to the explosion taking the speed of light into account. Note that this distance is negligibly greater.

Show Solution

**Strategy**

For part (a), we first calculate the speed of sound at the given temperature, then use $d = vt$ where the time is the observed lag. For part (b), we account for the fact that light also takes time to travel, though this is negligible compared to sound. The lag time is $t_{\text{lag}} = t_{\text{sound}} - t_{\text{light}}$, where both times depend on the distance.

**Solution**

**(a)** Given:

- Time lag: $t = 0.400\text{ s}$
- Air temperature: $T = 24.0°\text{C} = 297\text{ K}$

First, calculate the speed of sound at this temperature:

$$v_w = (331\text{ m/s})\sqrt{\frac{T}{273\text{ K}}} = (331\text{ m/s})\sqrt{\frac{297\text{ K}}{273\text{ K}}} = (331\text{ m/s})(1.043) = 345\text{ m/s}$$

Distance to the explosion (neglecting light travel time):

$$d = v_w t = (345\text{ m/s})(0.400\text{ s}) = 138\text{ m}$$

**(b)** To account for light travel time, we use:

- Speed of light: $c = 3.00 \times 10^8\text{ m/s}$
- Time lag: $t_{\text{lag}} = t_{\text{sound}} - t_{\text{light}} = \frac{d}{v_w} - \frac{d}{c}$

Solving for $d$:

$$0.400 = d\left(\frac{1}{345} - \frac{1}{3.00 \times 10^8}\right) = d\left(\frac{1}{345}\right)\left(1 - \frac{345}{3.00 \times 10^8}\right)$$

Since $345/(3.00 \times 10^8) = 1.15 \times 10^{-6} \ll 1$, we can approximate:

$$d = \frac{0.400\text{ s}}{1/345\text{ s/m}} = 138\text{ m}$$

The correction would be only $d \times (1.15 \times 10^{-6}) = 0.000159\text{ m} = 0.16\text{ mm}$, which is completely negligible.

**Discussion**

The explosion is approximately 138 m away. The correction for light travel time is only 0.16 mm, which confirms that light effectively travels instantaneously over such short distances. The speed of light is about one million times faster than sound, so for distances of hundreds of meters, we can safely ignore the time light takes to travel. This "flash-to-bang" method is commonly used to estimate distances to lightning strikes and fireworks. A useful rule of thumb is that sound travels approximately 1 km in 3 seconds (or 1 mile in 5 seconds), making mental distance calculations easy during thunderstorms.

(a) The explosion is 138 m away when neglecting light travel time. (b) Accounting for light travel time, the distance is still 138 m (to three significant figures), as the correction is only 0.16 mm.

Suppose a bat uses sound echoes to locate its insect prey, 3.00 m away. (See [Figure 3].) (a) Calculate the echo times for temperatures of $5.00^{\text{o}}\text{C}$ and $35.0^{\text{o}}\text{C}$ . (b) What percent uncertainty does this cause for the bat in locating the insect? (c) Discuss the significance of this uncertainty and whether it could cause difficulties for the bat. (In practice, the bat continues to use sound as it closes in, eliminating most of any difficulties imposed by this and other effects, such as motion of the prey.)

Show Solution

**Strategy**

For part (a), we calculate the speed of sound at each temperature, then find the round-trip echo time using $t = 2d/v_w$ (factor of 2 for the round trip). For part (b), we find the percent difference between the two echo times. For part (c), we interpret what this uncertainty means for the bat's hunting ability.

**Solution**

**(a)** Given:

- Distance to prey: $d = 3.00\text{ m}$
- Temperatures: $T_1 = 5.00°\text{C} = 278\text{ K}$ and $T_2 = 35.0°\text{C} = 308\text{ K}$

Speed of sound at $5.00°\text{C}$:

$$v_1=(331\text{ m/s})\sqrt{\frac{278\text{ K}}{273\text{ K}}}=(331\text{ m/s})(1.009)=334\text{ m/s}$$

Echo time at $5.00°\text{C}$ (round trip):

$$t_1=\frac{2d}{v_1}=\frac{2(3.00\text{ m})}{334\text{ m/s}}=0.0180\text{ s}=18.0\text{ ms}$$

Speed of sound at $35.0°\text{C}$:

$$v_2=(331\text{ m/s})\sqrt{\frac{308\text{ K}}{273\text{ K}}}=(331\text{ m/s})(1.061)=351\text{ m/s}$$

Echo time at $35.0°\text{C}$ (round trip):

$$t_2=\frac{2d}{v_2}=\frac{2(3.00\text{ m})}{351\text{ m/s}}=0.0171\text{ s}=17.1\text{ ms}$$

**(b)** Percent uncertainty (using the difference relative to the average):

$$\text{Percent uncertainty}=\frac{t_1-t_2}{t_1}\times100\%=\frac{18.0-17.1}{18.0}\times100\%=\frac{0.9}{18.0}\times100\%=5.00\%$$

**(c)** A 5% uncertainty in echo time translates to a 5% uncertainty in the perceived distance to the prey. For a target 3.00 m away, this represents an uncertainty of about 15 cm (0.15 m). This could definitely cause difficulties for the bat if it relied on a single echo measurement. Since a bat is typically 5-10 cm in length and catches insects with its mouth or wing membranes, an error of 15 cm could mean the difference between a successful catch and a miss—the bat might strike too early or too late. However, in practice, bats continuously emit sound pulses as they approach prey (often increasing the pulse rate as they close in), which allows them to update their distance measurements many times per second. This continuous tracking, combined with the bat's ability to adjust for temperature through experience or neural processing, largely eliminates the difficulties that would arise from temperature-induced speed variations. Additionally, the bat uses other cues such as the Doppler shift of echoes to determine relative motion.

**Discussion**

This problem illustrates an important limitation of echolocation systems that depend on the speed of sound: environmental factors like temperature affect accuracy. The 5% variation due to a 30°C temperature difference is significant but manageable through adaptive techniques. Modern human-made sonar and ultrasound systems similarly must account for temperature variations, often by measuring the temperature directly and adjusting their calculations accordingly.

(a) The echo times are 18.0 ms at 5.00°C and 17.1 ms at 35.0°C. (b) This represents a 5.00% uncertainty. (c) While a 5% uncertainty could cause difficulties for prey capture based on a single measurement, bats overcome this by continuously tracking prey with rapid, repeated sound pulses as they approach.

## Glossary

pitch
: the perception of the frequency of a sound

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# Sound Intensity and Sound Level

- Define intensity, sound intensity, and sound pressure level.
- Calculate sound intensity levels in decibels (dB).


Noise on crowded roadways like this one in Delhi makes it hard to hear others unless they shout. (credit: Lingaraj G J, Flickr)

In a quiet forest, you can sometimes hear a single leaf fall to the ground. After settling into bed, you may hear your blood pulsing through your ears. But when a passing motorist has his stereo turned up, you cannot even hear what the person next to you in your car is saying. We are all very familiar with the loudness of sounds and aware that they are related to how energetically the source is vibrating. In cartoons depicting a screaming person (or an animal making a loud noise), the cartoonist often shows an open mouth with a vibrating uvula, the hanging tissue at the back of the mouth, to suggest a loud sound coming from the throat [Figure 2]. High noise exposure is hazardous to hearing, and it is common for musicians to have hearing losses that are sufficiently severe that they interfere with the musicians' abilities to perform. The relevant physical quantity is sound intensity, a concept that is valid for all sounds whether or not they are in the audible range.

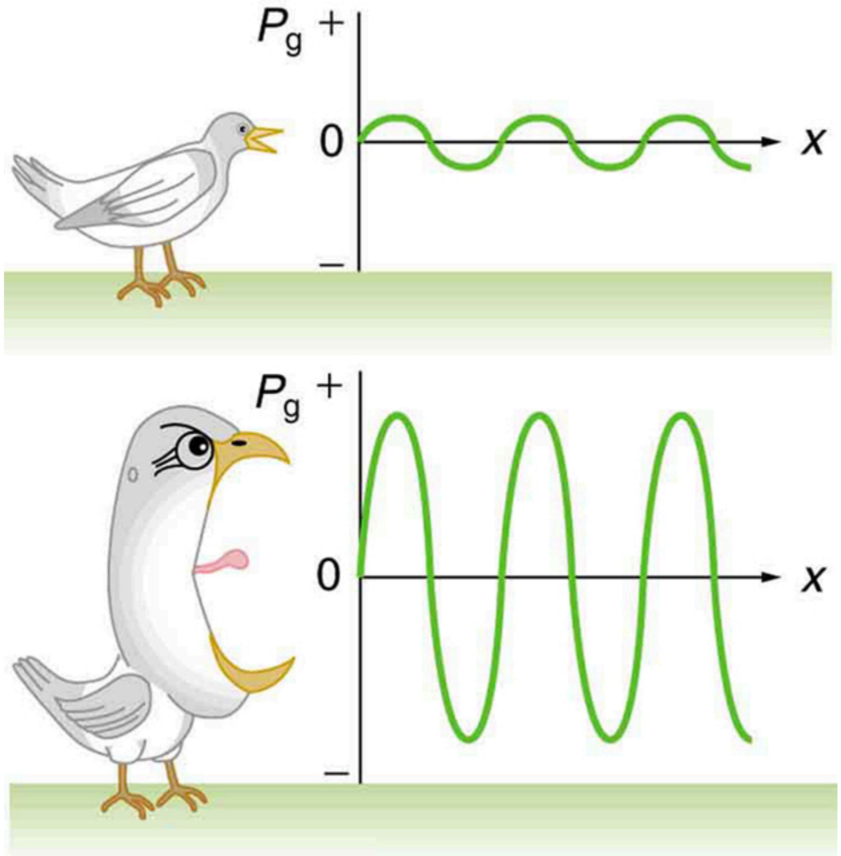
Intensity is defined to be the power per unit area carried by a wave. Power is the rate at which energy is transferred by the wave. In equation form, **intensity** $I$ is

$$I=\frac{P}{A},$$

where $P$ is the power through an area $A$. The SI unit for $I$ is ${W/m}^{2}$. The intensity of a sound wave is related to its amplitude squared by the following relationship:

$$I=\frac{{\left(\Delta p\right)}^{2}}{2\rho v_{w}}.$$

Here $\Delta p$ is the pressure variation or pressure amplitude (half the difference between the maximum and minimum pressure in the sound wave) in units of pascals (Pa) or ${N/m}^{2}$. (We are using a lower case $p$ for pressure to distinguish it from power, denoted by $P$ above.) The energy (as kinetic energy $\frac{mv^{2}}{2}$) of an oscillating element of air due to a traveling sound wave is proportional to its amplitude squared. In this equation, $\rho$ is the density of the material in which the sound wave travels, in units of ${kg/m}^{3}$, and ${v}_{w}$ is the speed of sound in the medium, in units of m/s. The pressure variation is proportional to the amplitude of the oscillation, and so $I$ varies as ${\left(\Delta p\right)}^{2}$ ([Figure 2]). This relationship is consistent with the fact that the sound wave is produced by some vibration; the greater its pressure amplitude, the more the air is compressed in the sound it creates.




Graphs of the gauge pressures in two sound waves of different intensities. The more intense sound is produced by a source that has larger-amplitude oscillations and has greater pressure maxima and minima. Because pressures are higher in the greater-intensity sound, it can exert larger forces on the objects it encounters.

Sound intensity levels are quoted in decibels (dB) much more often than sound intensities in watts per meter squared. Decibels are the unit of choice in the scientific literature as well as in the popular media. The reasons for this choice of units are related to how we perceive sounds. How our ears perceive sound can be more accurately described by the logarithm of the intensity rather than directly to the intensity. The **sound intensity level** $\beta$ in decibels of a sound having an intensity $I$ in watts per meter squared is defined to be

$$\beta \left(\text{dB}\right)=10 {\text{log}}_{10}\left(\frac{I}{ {I}_{0}}\right), $$

where ${I}_{0}=10^{-12} {\text{W/m}}^{2}$ is a reference intensity. In particular, ${I}_{0}$ is the lowest or threshold intensity of sound a person with normal hearing can perceive at a frequency of 1000 Hz. Sound intensity level is not the same as intensity. Because $\beta$ is defined in terms of a ratio, it is a unitless quantity telling you the *level* of the sound relative to a fixed standard ( $10^{-12} {\text{W/m}}^{2}$ , in this case). The units of decibels (dB) are used to indicate this ratio is multiplied by 10 in its definition. The bel, upon which the decibel is based, is named for Alexander Graham Bell, the inventor of the telephone.

Sound Intensity Levels and Intensities

| **Sound intensity level $\beta$ (dB)** | **Intensity $I$(W/m$^2$)** | **Example/effect** |
|---|---|---|
| $0$ | $1 \times 10^{-12}$ | Threshold of hearing at 1000 Hz |
| $10$ | $1 \times 10^{-11}$ | Rustle of leaves |
| $20$ | $1 \times 10^{-10}$ | Whisper at 1 m distance |
| $30$ | $1 \times 10^{-9}$ | Quiet home |
| $40$ | $1 \times 10^{-8}$ | Average home |
| $50$ | $1 \times 10^{-7}$ | Average office, soft music |
| $60$ | $1 \times 10^{-6}$ | Normal conversation |
| $70$ | $1 \times 10^{-5}$ | Noisy office, busy traffic |
| $80$ | $1 \times 10^{-4}$ | Loud radio, classroom lecture |
| $90$ | $1 \times 10^{-3}$ | Inside a heavy truck; damage from prolonged exposure[1] |
| $100$ | $1 \times 10^{-2}$ | Noisy factory, siren at 30 m; damage from 8 h per day exposure |
| $110$ | $1 \times 10^{-1}$ | Damage from 30 min per day exposure |
| $120$ | $1$ | Loud rock concert, pneumatic chipper at 2 m; threshold of pain |
| $140$ | $1 \times 10^{2}$ | Jet airplane at 30 m; severe pain, damage in seconds |
| $160$ | $1 \times 10^{4}$ | Bursting of eardrums |

The decibel level of a sound having the threshold intensity of $10^{-12}$ ${\text{W/m}}^{2}$ is $\beta =0 \text{dB}$, because ${\text{log}}_{10}1=0$

. That is, the threshold of hearing is 0 decibels. [Table 1] gives levels in decibels and intensities in watts per meter squared for some familiar sounds.

One of the more striking things about the intensities in [Table 1] is that the intensity in watts per meter squared is quite small for most sounds. The ear is sensitive to as little as a trillionth of a watt per meter squared—even more impressive when you realize that the area of the eardrum is only about $1 \text{cm}^{2}$ , so that only $10^{-16}$ W falls on it at the threshold of hearing! Air molecules in a sound wave of this intensity vibrate over a distance of less than one molecular diameter, and the gauge pressures involved are less than $10^{-9}$ atm.

Another impressive feature of the sounds in [Table 1] is their numerical range. Sound intensity varies by a factor of $10^{12}$ from threshold to a sound that causes damage in seconds. You are unaware of this tremendous range in sound intensity because how your ears respond can be described approximately as the logarithm of intensity. Thus, sound intensity levels in decibels fit your experience better than intensities in watts per meter squared. The decibel scale is also easier to relate to because most people are more accustomed to dealing with numbers such as 0, 53, or 120 than numbers such as $1.00 \times 10^{-11}$.

One more observation readily verified by examining [Table 1] or using $I={\frac{\left(\Delta p\right)}{2{\text{ρv}}_{\text{w}}}}^{2}$ is that each factor of 10 in intensity corresponds to 10 dB. For example, a 90 dB sound compared with a 60 dB sound is 30 dB greater, or three factors of 10 (that is, $10^{3}$ times) as intense. Another example is that if one sound is $10^{7}$ as intense as another, it is 70 dB higher. See [Table 2].

Ratios of Intensities and Corresponding Differences in Sound Intensity Levels

| ${I}_{2}/{I}_{1}$ | ${\beta }_{2}-{\beta }_{1}$ |
|---|---|
| 2.0 | 3.0 dB |
| 5.0 | 7.0 dB |
| 10.0 | 10.0 dB |

Calculating Sound Intensity Levels: Sound Waves

Calculate the sound intensity level in decibels for a sound wave traveling in air at $0ºC$ and having a pressure amplitude of 0.656 Pa.

**Strategy**

We are given $\Delta p$ , so we can calculate $I$ using the equation $I={\left(\Delta p\right)}^{2}/{\left(2{\text{pv}}_{w}\right)}^{2}$ . Using $I$ , we can calculate $\beta$ straight from its definition in $\beta \left(\text{dB}\right)=10 {\text{log}}_{10}\left(I/{I}_{0}\right)$.

**Solution**

(1) Identify knowns:

Sound travels at 331 m/s in air at $0ºC$.

Air has a density of $1.29 \text{kg/m}^{3}$ at atmospheric pressure and $0ºC$.

(2) Enter these values and the pressure amplitude into $I={\left(\Delta p\right)}^{2}/\left({2\text{ρv}}_{w}\right)$ :

$$I=\frac{ {\left(\Delta p\right)}^{2}}{2{\text{ρv}}_{w}}=\frac{ {\left(0.656 Pa\right)}^{2}}{2\left( 1.29 {\text{kg/m}}^{3}\right)\left(331 \text{m/s}\right)}= 5.04 \times 10^{-4} {\text{W/m}}^{2}. $$

(3) Enter the value for $I$ and the known value for ${I}_{0}$ into $\beta \left(\text{dB}\right)=10 {\text{log}}_{10}\left(I/{I}_{0}\right)$ . Calculate to find the sound intensity level in decibels:

$$10 \log_{10}\left(5.04 \times 10^{8} \right)=10 \left(8.70\right) dB=87 dB. $$

**Discussion**

This 87 dB sound has an intensity five times as great as an 80 dB sound. So a factor of five in intensity corresponds to a difference of 7 dB in sound intensity level. This value is true for any intensities differing by a factor of five.

Change Intensity Levels of a Sound: What Happens to the Decibel Level?

Show that if one sound is twice as intense as another, it has a sound level about 3 dB higher.

**Strategy**

You are given that the ratio of two intensities is 2 to 1, and are then asked to find the difference in their sound levels in decibels. You can solve this problem using of the properties of logarithms.

**Solution**

(1) Identify knowns:

The ratio of the two intensities is 2 to 1, or:

$$\frac{ {I}_{2}}{ {I}_{1}}= 2.00. $$

We wish to show that the difference in sound levels is about 3 dB. That is, we want to show:

$${\beta }_{2}-{\beta }_{1}=3 \text{dB}. $$

Note that:

$${\text{log}}_{10}b-{\text{log}}_{10}a={\text{log}}_{10}\left(\frac{b}{a}\right)\text{.} $$

(2) Use the definition of $\beta$ to get:

$${\beta }_{2}-{\beta }_{1}={10 \text{log}}_{10}\left(\frac{ {I}_{2}}{ {I}_{1}}\right)=10 {\text{log}}_{10}2.00=10 \left( 0.301\right) \text{dB}. $$

Thus,

$${\beta }_{2}-{\beta }_{1}=3.01\text{ dB}. $$

**Discussion**

This means that the two sound intensity levels differ by 3.01 dB, or about 3 dB, as advertised. Note that because only the ratio ${I}_{2}/{I}_{1}$ is given ( and not the actual intensities), this result is true for any intensities that differ by a factor of two. For example, a 56.0 dB sound is twice as intense as a 53.0 dB sound, a 97.0 dB sound is half as intense as a 100 dB sound, and so on.

It should be noted at this point that there is another decibel scale in use, called the **sound pressure level**, based on the ratio of the pressure amplitude to a reference pressure. This scale is used particularly in applications where sound travels in water. It is beyond the scope of most introductory texts to treat this scale because it is not commonly used for sounds in air, but it is important to note that very different decibel levels may be encountered when sound pressure levels are quoted. For example, ocean noise pollution produced by ships may be as great as 200 dB expressed in the sound pressure level, where the more familiar sound intensity level we use here would be something under 140 dB for the same sound.

Take-Home Investigation: Feeling Sound

Find a CD player and a CD that has rock music. Place the player on a light table, insert the CD into the player, and start playing the CD. Place your hand gently on the table next to the speakers. Increase the volume and note the level when the table just begins to vibrate as the rock music plays. Increase the reading on the volume control until it doubles. What has happened to the vibrations?

Check Your Understanding

Describe how amplitude is related to the loudness of a sound.

Show Solution

Amplitude is directly proportional to the experience of loudness. As amplitude increases, loudness increases.

Check Your Understanding

Identify common sounds at the levels of 10 dB, 50 dB, and 100 dB.

Show Solution

10 dB: Running fingers through your hair.

50 dB: Inside a quiet home with no television or radio.

100 dB: Take-off of a jet plane.

## Section Summary

- Intensity is the same for a sound wave as was defined for all waves; it is

  $$I=\frac{P}{A}, $$

  where $P$ is the power crossing area $A$ . The SI unit for $I$ is watts per meter squared. The intensity of a sound wave is also related to the pressure amplitude $\Delta p$

  $$I=\frac{ {\left(\Delta p\right)}^{2}}{2{\text{ρv}}_{w}}, $$

where $\rho$ is the density of the medium in which the sound wave travels and ${v}_{w}$ is the speed of sound in the medium.

- Sound intensity level in units of decibels (dB) is

  $$\beta \left(\text{dB}\right)=10 {\text{log}}_{10}\left(\frac{I}{ {I}_{0}}\right), $$

where ${I}_{0}={10}^{-12} W/{\text{m}}^{2}$ is the threshold intensity of hearing.

## Conceptual Questions

Six members of a synchronized swim team wear earplugs to protect themselves against water pressure at depths, but they can still hear the music and perform the combinations in the water perfectly. One day, they were asked to leave the pool so the dive team could practice a few dives, and they tried to practice on a mat, but seemed to have a lot more difficulty. Why might this be?

Show Solution

The swimmers can hear the music in the water due to **bone conduction**. Sound travels much more efficiently from water to the skull than from air to the skull. The sound waves from the underwater speakers travel through the water and are transmitted directly through the swimmers' skulls to the cochlea (inner ear), bypassing the ear canal and eardrum, which are blocked by their earplugs.

On the mat, the sound must travel through the air. The earplugs block most of this airborne sound from entering the ear canal, making it very difficult for them to hear the music clearly.

A community is concerned about a plan to bring train service to their downtown from the town's outskirts. The current sound intensity level, even though the rail yard is blocks away, is 70 dB downtown. The mayor assures the public that there will be a difference of only 30 dB in sound in the downtown area. Should the townspeople be concerned? Why?

Show Solution

Yes, the townspeople should be very concerned. A 30 dB increase in sound level is not a small change. The decibel scale is logarithmic, and a 30 dB increase corresponds to a $10^3$, or **1000-fold**, increase in sound intensity.

The new sound level would be $70 \text{ dB} + 30 \text{ dB} = 100 \text{ dB}$. This is a very loud noise, equivalent to a noisy factory or a siren at 30 m. According to [Table 1], exposure to sounds at this level for 8 hours a day can cause hearing damage. The current 70 dB level is equivalent to busy traffic, but a 100 dB level is significantly more intrusive and potentially harmful.

## Problems & Exercises

What is the intensity in watts per meter squared of 85.0-dB sound?

Show Solution

**Strategy**

We are given the sound intensity level in decibels and need to find the intensity in $W/m^2$. We use the decibel formula $\beta = 10\log_{10}(I/I_0)$ and solve for $I$.

**Solution**

Given:

- Sound intensity level: $\beta = 85.0 \text{ dB}$
- Reference intensity: $I_0 = 10^{-12} \text{ W/m}^2$

Rearranging the decibel formula:

$$\beta = 10\log_{10}\left(\frac{I}{I_0}\right)$$
$$\frac{\beta}{10} = \log_{10}\left(\frac{I}{I_0}\right)$$
$$10^{\beta/10} = \frac{I}{I_0}$$
$$I = I_0 \times 10^{\beta/10}$$

Substituting values:

$$I = (10^{-12} \text{ W/m}^2) \times 10^{85.0/10} = (10^{-12} \text{ W/m}^2) \times 10^{8.50} = 10^{-12+8.50} \text{ W/m}^2 = 10^{-3.50} \text{ W/m}^2$$
$$I = 3.16 \times 10^{-4} \text{ W/m}^2$$

**Discussion**

The intensity of $3.16 \times 10^{-4}$ $W/m^2$ is a relatively small amount of power per unit area, yet it produces a sound that is loud enough to potentially cause hearing damage with prolonged exposure (85 dB is near the threshold where hearing protection is recommended for 8-hour exposures). This demonstrates the remarkable sensitivity of the human ear, which can detect sounds over a tremendous range of intensities—from the threshold of hearing at $10^{-12}$ $W/m^2$ to sounds that cause pain and damage. The 85 dB level is typical of heavy traffic, a noisy office, or a loud radio.

The intensity of an 85.0-dB sound is $3.16 \times 10^{-4}$ $W/m^2$.

The warning tag on a lawn mower states that it produces noise at a level of 91.0 dB. What is this in watts per meter squared?

Show Solution

**Strategy**

We convert from decibels to intensity using the relationship $I = I_0 \times 10^{\beta/10}$, where $I_0 = 10^{-12}$ $W/m^2$.

**Solution**

Given:

- Sound intensity level: $\beta = 91.0 \text{ dB}$
- Reference intensity: $I_0 = 10^{-12} \text{ W/m}^2$

Using the formula:

$$I = I_0 \times 10^{\beta/10} = (10^{-12} \text{ W/m}^2) \times 10^{91.0/10}$$

$$I = (10^{-12} \text{ W/m}^2) \times 10^{9.10} = 10^{-12+9.10} \text{ W/m}^2 = 10^{-2.90} \text{ W/m}^2$$
$$I = 1.26 \times 10^{-3} \text{ W/m}^2$$

**Discussion**

A lawn mower producing 91 dB is quite loud—this level is in the range where prolonged exposure can cause hearing damage (above 90 dB). The intensity of $1.26 \times 10^{-3}$ W/m² is about 4 times greater than the 85 dB sound we calculated in the previous problem, which makes sense since 91 dB is 6 dB higher, and 6 dB corresponds to approximately a factor of 4 in intensity (since 10 dB is a factor of 10, and $10^{0.6} \approx 4$). This is why lawn mower operators and landscapers often wear hearing protection. The warning tag is important because regular exposure to this noise level without protection can lead to permanent hearing loss.

The lawn mower produces sound with an intensity of $1.26 \times 10^{-3}$ W/m².

A sound wave traveling in $20ºC$ air has a pressure amplitude of 0.5 Pa. What is the intensity of the wave?

Show Solution

**Strategy**

We are given the pressure amplitude $\Delta p$ and need to find the intensity $I$. We use the relationship between intensity and pressure amplitude: $I = \frac{(\Delta p)^2}{2\rho v_w}$. For air at 20°C, we use $\rho = 1.29$ kg/m³ and $v_w = 343$ m/s.

**Solution**

Given:

- Pressure amplitude: $\Delta p = 0.5 \text{ Pa}$
- Temperature: $T = 20°C$
- For air at 20°C: $\rho = 1.29 \text{ kg/m}^3$, $v_w = 343 \text{ m/s}$

Using the intensity-pressure relationship:

$$I = \frac{(\Delta p)^2}{2\rho v_w}$$

Substituting the values:

$$I = \frac{(0.5 \text{ Pa})^2}{2(1.29 \text{ kg/m}^3)(343 \text{ m/s})}$$
$$I = \frac{0.25 \text{ Pa}^2}{885.14 \text{ kg/(m·s)}}$$

Since $1 \text{ Pa} = 1 \text{ N/m}^2 = 1 \text{ kg/(m·s}^2\text{)}$, we have $\text{Pa}^2 = \text{kg}^2/(\text{m}^2\text{·s}^4)$:

$$I = \frac{0.25 \text{ kg}^2/(\text{m}^2\text{·s}^4)}{885.14 \text{ kg/(m·s)}} = \frac{0.25}{885.14} \frac{\text{kg}}{\text{m·s}^3} = 2.82 \times 10^{-4} \text{ kg/(m·s}^3\text{)}$$

Since $\text{kg/(m·s}^3\text{)} = \text{W/m}^2$:

$$I = 3.04 \times 10^{-4} \text{ W/m}^2$$

**Discussion**

The intensity of $3.04 \times 10^{-4}$ W/m² corresponds to a moderate sound level. This pressure amplitude of 0.5 Pa is quite small—only about 5 parts per million of atmospheric pressure ($10^5$ Pa)—yet it produces a clearly audible sound. This demonstrates the remarkable sensitivity of the human ear to tiny pressure variations. The quadratic relationship between pressure amplitude and intensity means that doubling the pressure amplitude would quadruple the intensity. This makes physical sense because both the kinetic and potential energy densities in the wave are proportional to the square of the amplitude.

The intensity of the sound wave is $3.04 \times 10^{-4}$ W/m².

What intensity level does the sound in the preceding problem correspond to?

Show Solution

**Strategy**

We found in the previous problem that the intensity is $I = 3.04 \times 10^{-4}$ W/m². Now we need to convert this to a sound intensity level in decibels using the formula $\beta = 10\log_{10}(I/I_0)$, where $I_0 = 10^{-12}$ W/m² is the reference intensity (threshold of hearing).

**Solution**

Given:

- Intensity from previous problem: $I = 3.04 \times 10^{-4} \text{ W/m}^2$
- Reference intensity: $I_0 = 10^{-12} \text{ W/m}^2$

Using the decibel formula:

$$\beta = 10\log_{10}\left(\frac{I}{I_0}\right)$$

Substituting the values:

$$\beta = 10\log_{10}\left(\frac{3.04 \times 10^{-4}}{10^{-12}}\right)$$
$$\beta = 10\log_{10}(3.04 \times 10^{8})$$

Using the logarithm property $\log_{10}(ab) = \log_{10}a + \log_{10}b$:

$$\beta = 10[\log_{10}(3.04) + \log_{10}(10^{8})]$$
$$\beta = 10[0.483 + 8] = 10(8.483) = 84.8 \text{ dB}$$

**Discussion**

The sound intensity level of approximately 85 dB corresponds to sounds like heavy traffic, a noisy factory, or a loud vacuum cleaner. According to occupational safety standards, prolonged exposure to sounds at this level (8 hours or more) can cause hearing damage, which is why hearing protection is recommended for people who work in such environments.

Note that the intensity of $3.04 \times 10^{-4}$ W/m$^2$ is $3.04 \times 10^{8}$ times greater than the threshold of hearing—a factor of over 300 million! Yet on the logarithmic decibel scale, this translates to only 85 dB. This illustrates why the decibel scale is so useful: it compresses the enormous range of sound intensities we can perceive into a manageable numerical range.

The sound intensity level is approximately 85 dB.

What sound intensity level in dB is produced by earphones that create an intensity of $4.00 \times 10^{\text{−2}} {\text{W/m}}^{2}$ ?

Show Solution

**Strategy**

We are given the intensity and need to find the sound intensity level in decibels. We use the decibel formula $\beta = 10\log_{10}(I/I_0)$, where $I_0 = 10^{-12}$ W/m$^2$.

**Solution**

Given:

- Intensity: $I = 4.00 \times 10^{-2} \text{ W/m}^2$
- Reference intensity: $I_0 = 10^{-12} \text{ W/m}^2$

Applying the decibel formula:

$$\beta = 10\log_{10}\left(\frac{I}{I_0}\right)$$
$$\beta = 10\log_{10}\left(\frac{4.00 \times 10^{-2}}{10^{-12}}\right)$$
$$\beta = 10\log_{10}(4.00 \times 10^{10})$$

Using the logarithm property $\log_{10}(ab) = \log_{10}a + \log_{10}b$:

$$\beta = 10[\log_{10}(4.00) + \log_{10}(10^{10})]$$
$$\beta = 10[0.602 + 10] = 10(10.602) = 106 \text{ dB}$$

**Discussion**

A sound intensity level of 106 dB is extremely loud and well above the threshold for hearing damage. This level is comparable to a chainsaw at close range, a rock concert, or a jackhammer. According to hearing safety guidelines, exposure to 106 dB should be limited to less than 4 minutes per day to prevent permanent hearing damage.

The fact that earphones can produce this intensity level is concerning—earphones deliver sound energy directly into the ear canal with very high efficiency. An intensity of $4.00 \times 10^{-2}$ W/m$^2$ is $4.00 \times 10^{10}$ times (40 billion times) greater than the threshold of hearing. Many personal music players can reach these dangerous levels, which is why listening at moderate volumes is strongly recommended. Prolonged exposure to music at this level through earphones can cause permanent hearing loss, particularly at high frequencies.

The sound intensity level is 106 dB.

Show that an intensity of ${10}^{-12} {W/m}^{2}$ is the same as ${10}^{-16} {W/cm}^{2}$.

Show Solution

**Strategy**

We need to convert between W/m$^2$ and W/cm$^2$ by converting the area unit from m$^2$ to cm$^2$. The key is to remember that 1 m = 100 cm, so $1 \text{ m}^2 = (100 \text{ cm})^2 = 10{,}000 \text{ cm}^2 = 10^4 \text{ cm}^2$.

**Solution**

Starting with the intensity in W/m$^2$:

$$I = 10^{-12} \text{ W/m}^2$$

We need to convert m$^2$ to cm$^2$. Since $1 \text{ m} = 100 \text{ cm} = 10^2 \text{ cm}$:

$$1 \text{ m}^2 = (10^2 \text{ cm})^2 = 10^4 \text{ cm}^2$$

Therefore:

$$1 \text{ W/m}^2 = 1 \frac{\text{W}}{10^4 \text{ cm}^2} = 10^{-4} \text{ W/cm}^2$$

Now converting our intensity:

$$I = 10^{-12} \text{ W/m}^2 \times \frac{10^{-4} \text{ W/cm}^2}{1 \text{ W/m}^2}$$
$$I = 10^{-12} \times 10^{-4} \text{ W/cm}^2 = 10^{-16} \text{ W/cm}^2$$

**Discussion**

This unit conversion is particularly relevant when discussing the threshold of hearing and the sensitivity of the human eardrum. The area of a typical eardrum is about 0.5 to 1 $cm^2$, making $W/cm^2$ a natural unit for discussing sound power incident on the eardrum.

At the threshold of hearing ($10^{-12}$ $W/m^2$ = $10^{-16}$ $W/cm^2$), and with an eardrum area of about 1 $cm^2$, the total power reaching the eardrum is approximately $10^{-16}$ W. This is an incredibly tiny amount of power—the eardrum is sensitive enough to detect energy transfers on the order of $10^{-16}$ joules per second. This remarkable sensitivity allows us to hear sounds ranging from a pin drop to a jet engine, a range spanning 12 orders of magnitude in intensity.

The general conversion factor is: $1 \text{ W/m}^2 = 10^{-4} \text{ W/cm}^2$.

(a) What is the decibel level of a sound that is twice as intense as a 90.0-dB sound? (b) What is the decibel level of a sound that is one-fifth as intense as a 90.0-dB sound?

Show Solution

**Strategy**

For both parts, we need to understand how changes in intensity relate to changes in decibel level. We can use the relationship $\beta_2 - \beta_1 = 10\log_{10}(I_2/I_1)$, which follows from the definition of decibels. In part (a), $I_2/I_1 = 2$, and in part (b), $I_2/I_1 = 1/5 = 0.2$.

**Solution**

**Part (a):** Sound twice as intense

Given:

- Original sound level: $\beta_1 = 90.0 \text{ dB}$
- Intensity ratio: $I_2/I_1 = 2$

The difference in decibel levels is:

$$\beta_2 - \beta_1 = 10\log_{10}\left(\frac{I_2}{I_1}\right) = 10\log_{10}(2)$$
$$\beta_2 - \beta_1 = 10(0.301) = 3.01 \text{ dB}$$

Therefore:

$$\beta_2 = \beta_1 + 3.01 \text{ dB} = 90.0 \text{ dB} + 3.01 \text{ dB} = 93.0 \text{ dB}$$

**Part (b):** Sound one-fifth as intense

Given:

- Original sound level: $\beta_1 = 90.0 \text{ dB}$
- Intensity ratio: $I_2/I_1 = 1/5 = 0.2$

The difference in decibel levels is:

$$\beta_2 - \beta_1 = 10\log_{10}\left(\frac{I_2}{I_1}\right) = 10\log_{10}(0.2)$$
$$\beta_2 - \beta_1 = 10\log_{10}(2 \times 10^{-1}) = 10[\log_{10}(2) + \log_{10}(10^{-1})]$$
$$\beta_2 - \beta_1 = 10[0.301 - 1] = 10(-0.699) = -6.99 \text{ dB} \approx -7.0 \text{ dB}$$

Therefore:

$$\beta_2 = \beta_1 - 7.0 \text{ dB} = 90.0 \text{ dB} - 7.0 \text{ dB} = 83.0 \text{ dB}$$

**Discussion**

These results illustrate important rules of thumb for decibel arithmetic:

- Doubling the intensity increases the sound level by approximately 3 dB
- Halving the intensity decreases the sound level by approximately 3 dB
- Reducing intensity to one-fifth (which is half of two-fifths, or $1/2 \times 2/5$) decreases the level by 7 dB

Notice that the 3-dB rule is exact to within rounding ($\log_{10}(2) = 0.301$). This is why you'll often hear that "3 dB is a factor of 2 in intensity." These shortcuts are useful for quick mental calculations.

An important observation: the new decibel level depends only on the ratio of intensities, not on the original intensity value. This means doubling any sound intensity always adds 3 dB, whether you're going from 30 dB to 33 dB or from 90 dB to 93 dB.

(a) The decibel level of a sound twice as intense as 90.0 dB is 93.0 dB.

(b) The decibel level of a sound one-fifth as intense as 90.0 dB is 83.0 dB.

(a) What is the intensity of a sound that has a level 7.00 dB lower than a $4.00 \times 10^{-9} {W/m}^{2}$ sound? (b) What is the intensity of a sound that is 3.00 dB higher than a $4.00 \times 10^{-9} {W/m}^{2}$ sound?

Show Solution

**Strategy**

We need to work backwards from decibel differences to intensity ratios. Using the relationship $\beta_2 - \beta_1 = 10\log_{10}(I_2/I_1)$, we can solve for the intensity ratio: $I_2/I_1 = 10^{(\beta_2-\beta_1)/10}$. Then we multiply by the given intensity to find the new intensity.

**Solution**

**Part (a):** Sound 7.00 dB lower

Given:

- Original intensity: $I_1 = 4.00 \times 10^{-9} \text{ W/m}^2$
- Decibel change: $\beta_2 - \beta_1 = -7.00 \text{ dB}$

Finding the intensity ratio:

$$\beta_2 - \beta_1 = 10\log_{10}\left(\frac{I_2}{I_1}\right)$$
$$-7.00 = 10\log_{10}\left(\frac{I_2}{I_1}\right)$$
$$-0.700 = \log_{10}\left(\frac{I_2}{I_1}\right)$$
$$\frac{I_2}{I_1} = 10^{-0.700} = 0.200$$

Therefore:

$$I_2 = 0.200 \times I_1 = 0.200 \times (4.00 \times 10^{-9} \text{ W/m}^2)$$
$$I_2 = 8.00 \times 10^{-10} \text{ W/m}^2$$

**Part (b):** Sound 3.00 dB higher

Given:

- Original intensity: $I_1 = 4.00 \times 10^{-9} \text{ W/m}^2$
- Decibel change: $\beta_2 - \beta_1 = +3.00 \text{ dB}$

Finding the intensity ratio:

$$\frac{I_2}{I_1} = 10^{(\beta_2-\beta_1)/10} = 10^{3.00/10} = 10^{0.300} = 2.00$$

Therefore:

$$I_2 = 2.00 \times I_1 = 2.00 \times (4.00 \times 10^{-9} \text{ W/m}^2)$$
$$I_2 = 8.00 \times 10^{-9} \text{ W/m}^2$$

**Discussion**

These results confirm the rules we learned in the previous problem:

- A decrease of 7 dB corresponds to reducing the intensity to 1/5 of its original value (0.200 = 1/5)
- An increase of 3 dB corresponds to doubling the intensity (factor of 2.00)

Notice that 7 dB is approximately equal to 10 dB - 3 dB, which corresponds to dividing by 10 and then multiplying by 2, giving 1/5. This makes sense: $10^{-0.7} = 10^{-1} \times 10^{0.3} = 0.1 \times 2 = 0.2$.

The original intensity of $4.00 \times 10^{-9}$ W/m² corresponds to a very quiet sound (about 36 dB). The 7-dB-lower sound at $8.00 \times 10^{-10}$ W/m² (29 dB) would be quieter than a whisper, while the 3-dB-higher sound at $8.00 \times 10^{-9}$ W/m² (39 dB) would be comparable to a quiet library.

(a) The intensity of a sound 7.00 dB lower is $8.00 \times 10^{-10}$ W/m².

(b) The intensity of a sound 3.00 dB higher is $8.00 \times 10^{-9}$ W/m².

(a) How much more intense is a sound that has a level 17.0 dB higher than another? (b) If one sound has a level 23.0 dB less than another, what is the ratio of their intensities?

Show Solution

**Strategy**

Both parts ask us to find intensity ratios from decibel differences. We use the relationship $\beta_2 - \beta_1 = 10\log_{10}(I_2/I_1)$, which we can rearrange to $I_2/I_1 = 10^{(\beta_2-\beta_1)/10}$.

**Solution**

**Part (a):** Sound 17.0 dB higher

Given:

- Decibel difference: $\beta_2 - \beta_1 = 17.0 \text{ dB}$

Finding the intensity ratio:

$$\frac{I_2}{I_1} = 10^{(\beta_2-\beta_1)/10} = 10^{17.0/10} = 10^{1.70}$$

We can evaluate this as:

$$10^{1.70} = 10^1 \times 10^{0.70} = 10 \times 5.01 = 50.1$$

**Part (b):** Sound 23.0 dB less

Given:

- Decibel difference: $\beta_2 - \beta_1 = -23.0 \text{ dB}$

Finding the intensity ratio:

$$\frac{I_2}{I_1} = 10^{(\beta_2-\beta_1)/10} = 10^{-23.0/10} = 10^{-2.30}$$

We can evaluate this as:

$$10^{-2.30} = 10^{-2} \times 10^{-0.30} = 0.01 \times 0.501 = 5.01 \times 10^{-3}$$

Alternatively, this can be expressed as a fraction:

$$\frac{I_2}{I_1} = \frac{1}{10^{2.30}} = \frac{1}{10^2 \times 10^{0.30}} = \frac{1}{10 \times 2 \times 10} = \frac{1}{200}$$

**Discussion**

These results demonstrate the exponential relationship between decibel differences and intensity ratios:

**Part (a):** A 17-dB increase corresponds to an intensity ratio of about 50. We can verify this makes sense: 10 dB corresponds to a factor of 10, and 7 dB corresponds to a factor of about 5 (since 3 dB ≈ factor of 2, and 10 dB ≈ factor of 10, then 7 dB ≈ factor of 5). So 17 dB should give roughly $10 \times 5 = 50$, which matches our answer.

**Part (b):** A 23-dB decrease means the quieter sound has only 1/200th the intensity of the louder sound. This can be understood as: 20 dB corresponds to a factor of 100, and 3 dB corresponds to a factor of 2, so 23 dB corresponds to a factor of $100 \times 2 = 200$ in the denominator.

These large intensity ratios for relatively modest decibel differences illustrate why the logarithmic decibel scale is so useful. A difference of 17 dB might not sound enormous, but it represents a 50-fold change in intensity. Similarly, reducing a sound by 23 dB means reducing its intensity to only 0.5% of its original value.

(a) A sound 17.0 dB higher is 50.1 times more intense.

(b) If one sound is 23.0 dB less than another, the intensity ratio is $5.01 \times 10^{-3}$ or $1/200$.

People with good hearing can perceive sounds as low in level as $-8.00 \text{dB}$ at a frequency of 3000 Hz. What is the intensity of this sound in watts per meter squared?

Show Solution

**Strategy**

We are given a sound intensity level in decibels (which is negative) and need to find the corresponding intensity. We use the formula $\beta = 10\log_{10}(I/I_0)$ and solve for $I$: $I = I_0 \times 10^{\beta/10}$, where $I_0 = 10^{-12}$ W/m².

**Solution**

Given:

- Sound intensity level: $\beta = -8.00 \text{ dB}$
- Reference intensity: $I_0 = 10^{-12} \text{ W/m}^2$
- Frequency: $f = 3000 \text{ Hz}$ (this affects human perception but not the calculation)

Rearranging the decibel formula:

$$\beta = 10\log_{10}\left(\frac{I}{I_0}\right)$$
$$\frac{\beta}{10} = \log_{10}\left(\frac{I}{I_0}\right)$$
$$10^{\beta/10} = \frac{I}{I_0}$$
$$I = I_0 \times 10^{\beta/10}$$

Substituting the values:

$$I = (10^{-12} \text{ W/m}^2) \times 10^{-8.00/10}$$
$$I = (10^{-12} \text{ W/m}^2) \times 10^{-0.800}$$
$$I = 10^{-12-0.800} \text{ W/m}^2 = 10^{-12.8} \text{ W/m}^2$$
$$I = 1.58 \times 10^{-13} \text{ W/m}^2$$

**Discussion**

A negative decibel level means the sound is quieter than the reference level of $10^{-12}$ W/m², which is defined as the threshold of hearing at 1000 Hz for a person with normal hearing. The fact that some people can hear sounds at -8 dB at 3000 Hz demonstrates that:

1. The sensitivity of human hearing varies with frequency. At 3000 Hz (near the peak sensitivity of the human ear), people can detect sounds that are even quieter than the 1000 Hz reference level.

2. The intensity of $1.58 \times 10^{-13}$ W/m² is only about 16% of the standard threshold intensity ($10^{-0.8} \approx 0.158$).

3. The human ear is remarkably sensitive at certain frequencies. The ability to detect such faint sounds is evolutionarily advantageous—this frequency range includes important environmental cues and human speech sounds.

This exceptional sensitivity at mid-frequencies (2000-4000 Hz) is why audiologists test hearing across multiple frequencies, not just at the 1000 Hz reference point. It's also why hearing protection is crucial—once this delicate sensitivity is damaged by loud sounds, it cannot be recovered.

The intensity of a -8.00 dB sound is $1.58 \times 10^{-13}$ W/m².

If a large housefly 3.0 m away from you makes a noise of 40.0 dB, what is the noise level of 1000 flies at that distance, assuming interference has a negligible effect?

Show Solution

**Strategy**

When multiple identical sound sources emit incoherently (with negligible interference), the total intensity is the sum of individual intensities. We first find the intensity of one fly, multiply by 1000 to get the total intensity, then convert back to decibels. The key insight is that intensities add, not decibel levels.

**Solution**

Given:

- Sound level of one fly: $\beta_1 = 40.0 \text{ dB}$
- Number of flies: $N = 1000$
- Reference intensity: $I_0 = 10^{-12} \text{ W/m}^2$

**Step 1:** Find the intensity of one fly.

$$I_1 = I_0 \times 10^{\beta_1/10} = (10^{-12} \text{ W/m}^2) \times 10^{40.0/10}$$
$$I_1 = (10^{-12} \text{ W/m}^2) \times 10^{4.00} = 10^{-12+4} \text{ W/m}^2 = 10^{-8} \text{ W/m}^2$$

**Step 2:** Calculate the total intensity from 1000 flies.

Since the flies emit incoherently, their intensities add:

$$I_{\text{total}} = N \times I_1 = 1000 \times 10^{-8} \text{ W/m}^2$$
$$I_{\text{total}} = 10^{3} \times 10^{-8} \text{ W/m}^2 = 10^{-5} \text{ W/m}^2$$

**Step 3:** Convert the total intensity to decibels.

$$\beta_{\text{total}} = 10\log_{10}\left(\frac{I_{\text{total}}}{I_0}\right)$$
$$\beta_{\text{total}} = 10\log_{10}\left(\frac{10^{-5}}{10^{-12}}\right) = 10\log_{10}(10^{7})$$
$$\beta_{\text{total}} = 10 \times 7 = 70.0 \text{ dB}$$

**Discussion**

The result shows that 1000 flies produce 70 dB, which is 30 dB higher than one fly's 40 dB. This makes sense because:

- 1000 = $10^3$, which means the intensity increases by a factor of 1000
- A factor of 1000 in intensity corresponds to $10\log_{10}(1000) = 10 \times 3 = 30$ dB

This demonstrates an important principle: multiplying the number of identical sound sources by 10 adds 10 dB to the sound level, multiplying by 100 adds 20 dB, and multiplying by 1000 adds 30 dB.

It's crucial to understand that decibel levels don't add directly. If you have two 40-dB sources, you don't get 80 dB—you get 43 dB (since two sources double the intensity, which adds 3 dB). With 1000 sources, you add 30 dB to get 70 dB, not $1000 \times 40 = 40{,}000$ dB!

A sound level of 70 dB (equivalent to 1000 flies) is comparable to busy traffic or a noisy office—definitely noticeable and potentially annoying. This explains why a swarm of insects can be so loud even though an individual insect is barely audible.

The noise level of 1000 flies is 70.0 dB.

Ten cars in a circle at a boom box competition produce a 120-dB sound intensity level at the center of the circle. What is the average sound intensity level produced there by each stereo, assuming interference effects can be neglected?

Show Solution

**Strategy**

This is the inverse of the previous problem. We know the total sound level from all sources and need to find the level of one source. We'll convert the total decibel level to intensity, divide by the number of sources to get the intensity per source, then convert back to decibels.

**Solution**

Given:

- Total sound level: $\beta_{\text{total}} = 120 \text{ dB}$
- Number of stereos: $N = 10$
- Reference intensity: $I_0 = 10^{-12} \text{ W/m}^2$

**Step 1:** Find the total intensity at the center.

$$I_{\text{total}} = I_0 \times 10^{\beta_{\text{total}}/10}$$
$$I_{\text{total}} = (10^{-12} \text{ W/m}^2) \times 10^{120/10} = (10^{-12} \text{ W/m}^2) \times 10^{12}$$
$$I_{\text{total}} = 1 \text{ W/m}^2$$

**Step 2:** Calculate the intensity from one stereo.

Since interference is negligible, the total intensity equals the sum of individual intensities:

$$I_1 = \frac{I_{\text{total}}}{N} = \frac{1 \text{ W/m}^2}{10} = 0.1 \text{ W/m}^2 = 10^{-1} \text{ W/m}^2$$

**Step 3:** Convert the single-source intensity to decibels.

$$\beta_1 = 10\log_{10}\left(\frac{I_1}{I_0}\right)$$
$$\beta_1 = 10\log_{10}\left(\frac{10^{-1}}{10^{-12}}\right) = 10\log_{10}(10^{11})$$
$$\beta_1 = 10 \times 11 = 110 \text{ dB}$$

**Discussion**

Each stereo produces 110 dB, and together the ten stereos produce 120 dB—a difference of only 10 dB even though there are 10 times as many sources. This demonstrates the logarithmic nature of the decibel scale:

- 10 sources instead of 1 increases intensity by a factor of 10
- A factor of 10 in intensity corresponds to 10 dB
- Therefore, $120 \text{ dB} - 110 \text{ dB} = 10 \text{ dB}$

This result has important practical implications. At a rock concert, if you have 10 amplifiers each producing 110 dB, you don't get $10 \times 110 = 1100$ dB—you only get 120 dB. Similarly, reducing the number of loud sources doesn't decrease the sound level as much as you might expect. Removing 9 out of 10 stereos would only reduce the sound level from 120 dB to 110 dB, still painfully loud.

Both 110 dB and 120 dB are at the threshold of pain and can cause immediate hearing damage. At 120 dB, hearing damage can occur in seconds, while at 110 dB, damage can occur in less than 2 minutes of exposure. This is why hearing protection is essential at such events.

Each stereo produces an average sound intensity level of 110 dB at the center of the circle.

The amplitude of a sound wave is measured in terms of its maximum gauge pressure. By what factor does the amplitude of a sound wave increase if the sound intensity level goes up by 40.0 dB?

Show Solution

**Strategy**

We need to use the relationship between intensity and pressure amplitude: $I \propto (\Delta p)^2$, which comes from $I = \frac{(\Delta p)^2}{2\rho v_w}$. First, we'll find the factor by which intensity increases when the sound level increases by 40 dB. Then we'll use the square relationship to find how pressure amplitude changes.

**Solution**

Given:

- Decibel increase: $\Delta\beta = 40.0 \text{ dB}$

**Step 1:** Find the intensity ratio from the decibel change.

$$\Delta\beta = \beta_2 - \beta_1 = 10\log_{10}\left(\frac{I_2}{I_1}\right)$$
$$40.0 = 10\log_{10}\left(\frac{I_2}{I_1}\right)$$
$$4.00 = \log_{10}\left(\frac{I_2}{I_1}\right)$$
$$\frac{I_2}{I_1} = 10^{4.00} = 10{,}000$$

**Step 2:** Relate the intensity ratio to the pressure amplitude ratio.

From the intensity-pressure relationship:

$$I = \frac{(\Delta p)^2}{2\rho v_w}$$

Since $\rho$ and $v_w$ are constants for a given medium:

$$\frac{I_2}{I_1} = \frac{(\Delta p_2)^2}{(\Delta p_1)^2} = \left(\frac{\Delta p_2}{\Delta p_1}\right)^2$$

Therefore:

$$\frac{\Delta p_2}{\Delta p_1} = \sqrt{\frac{I_2}{I_1}} = \sqrt{10{,}000} = 100$$

**Discussion**

The pressure amplitude increases by a factor of 100 when the sound intensity level increases by 40 dB. This makes sense because:

- 40 dB corresponds to an intensity increase of $10^4 = 10{,}000$
- Since intensity is proportional to the square of pressure amplitude, the pressure amplitude factor is $\sqrt{10{,}000} = 100$

This square-root relationship is crucial for understanding sound:

- Doubling the pressure amplitude (factor of 2) increases intensity by a factor of $2^2 = 4$, which is about 6 dB
- Increasing pressure amplitude by a factor of 10 increases intensity by $10^2 = 100$, which is 20 dB
- Increasing pressure amplitude by a factor of 100 increases intensity by $100^2 = 10{,}000$, which is 40 dB

This has practical implications. For example, if you want to make a sound seem “twice as loud” (which roughly corresponds to a 10 dB increase), you need to increase the intensity by a factor of 10, which requires increasing the pressure amplitude by only a factor of $\sqrt{10} \approx 3.16$. The speaker cone doesn’t need to move 10 times as far—only about 3 times as far.

The relationship also explains why it’s much easier to damage hearing with loud sounds than people realize. A sound that has a pressure amplitude 10 times greater (which might not seem extreme) actually has 100 times the intensity, a dramatic increase in energy delivery to the delicate structures of the inner ear.

The pressure amplitude increases by a factor of 100.

If a sound intensity level of 0 dB at 1000 Hz corresponds to a maximum gauge pressure (sound amplitude) of ${10}^{-9} \text{atm}$ , what is the maximum gauge pressure in a 60-dB sound? What is the maximum gauge pressure in a 120-dB sound?

Show Solution

**Strategy**

We use the relationship between pressure amplitude and intensity: $\Delta p \propto \sqrt{I}$. For each sound level, we’ll find the intensity ratio relative to 0 dB, then use the square root of that ratio to find the pressure amplitude ratio. Since $\Delta p_0 = 10^{-9}$ atm at 0 dB, we multiply by the ratio to find the new pressure amplitude.

**Solution**

Given:

- At 0 dB: $\Delta p_0 = 10^{-9} \text{ atm}$
- Reference intensity: $I_0 = 10^{-12} \text{ W/m}^2$ (corresponds to 0 dB)

**Part 1:** Maximum gauge pressure at 60 dB

**Step 1:** Find the intensity at 60 dB.

$$I_{60} = I_0 \times 10^{\beta/10} = I_0 \times 10^{60/10} = I_0 \times 10^6$$

**Step 2:** Find the pressure amplitude ratio.

Since $I \propto (\Delta p)^2$:

$$\frac{I_{60}}{I_0} = \left(\frac{\Delta p_{60}}{\Delta p_0}\right)^2$$
$$\frac{\Delta p_{60}}{\Delta p_0} = \sqrt{\frac{I_{60}}{I_0}} = \sqrt{10^6} = 10^3 = 1000$$

**Step 3:** Calculate the pressure amplitude at 60 dB.

$$\Delta p_{60} = 1000 \times \Delta p_0 = 1000 \times 10^{-9} \text{ atm} = 10^{-6} \text{ atm}$$

**Part 2:** Maximum gauge pressure at 120 dB

**Step 1:** Find the intensity at 120 dB.

$$I_{120} = I_0 \times 10^{120/10} = I_0 \times 10^{12}$$

**Step 2:** Find the pressure amplitude ratio.

$$\frac{\Delta p_{120}}{\Delta p_0} = \sqrt{\frac{I_{120}}{I_0}} = \sqrt{10^{12}} = 10^6 = 1{,}000{,}000$$

**Step 3:** Calculate the pressure amplitude at 120 dB.

$$\Delta p_{120} = 10^6 \times \Delta p_0 = 10^6 \times 10^{-9} \text{ atm} = 10^{-3} \text{ atm}$$

**Discussion**

These results dramatically illustrate the range of pressure variations our ears can detect:

**At 60 dB (normal conversation):** The pressure amplitude is $10^{-6}$ atm, which is one millionth of atmospheric pressure. This is only 1000 times greater than the threshold of hearing ($10^{-9}$ atm), yet it's sufficient for clear communication.

**At 120 dB (threshold of pain):** The pressure amplitude is $10^{-3}$ atm, which is one thousandth of atmospheric pressure (about 0.1% of $P_{\text{atm}}$). This is still a remarkably small variation, yet it causes physical pain and immediate hearing damage.

The progression is striking:

- 0 dB → 60 dB: Intensity increases by $10^6$, pressure by $10^3$ (factor of 1000)
- 60 dB → 120 dB: Intensity increases by another $10^6$, pressure by another $10^3$ (factor of 1000)
- 0 dB → 120 dB: Intensity increases by $10^{12}$, pressure by $10^6$ (factor of 1 million)

This means our ears can detect pressure oscillations over a range of one million to one—from $10^{-9}$ atm to $10^{-3}$ atm. Even at the pain threshold, the pressure variation is only 0.1% of atmospheric pressure, yet the energy is sufficient to damage the delicate hair cells in the cochlea. This explains why hearing loss from loud music or machinery is such a common problem—the pressures involved seem small, but the energy is devastating to the sensitive structures of the inner ear.

The maximum gauge pressure in a 60-dB sound is $10^{-6}$ atm.

The maximum gauge pressure in a 120-dB sound is $10^{-3}$ atm.

An 8-hour exposure to a sound intensity level of 90.0 dB may cause hearing damage. What energy in joules falls on a 0.800-cm-diameter eardrum so exposed?

Show Solution

**Strategy**

Energy is related to power and time by $E = Pt$, and intensity is power per unit area: $I = P/A$. We'll first convert the decibel level to intensity, then calculate the area of the eardrum, use these to find the power reaching the eardrum, and finally multiply by time to get the total energy.

**Solution**

Given:

- Sound intensity level: $\beta = 90.0 \text{ dB}$
- Eardrum diameter: $d = 0.800 \text{ cm} = 0.800 \times 10^{-2} \text{ m}$
- Exposure time: $t = 8 \text{ hours} = 8 \times 3600 \text{ s} = 28{,}800 \text{ s}$
- Reference intensity: $I_0 = 10^{-12} \text{ W/m}^2$

**Step 1:** Find the intensity from the decibel level.

$$I = I_0 \times 10^{\beta/10} = (10^{-12} \text{ W/m}^2) \times 10^{90.0/10}$$
$$I = (10^{-12} \text{ W/m}^2) \times 10^{9.00} = 10^{-3} \text{ W/m}^2 = 1.00 \times 10^{-3} \text{ W/m}^2$$

**Step 2:** Calculate the area of the eardrum.

The eardrum is approximately circular with diameter $d$:

$$A = \pi r^2 = \pi \left(\frac{d}{2}\right)^2 = \pi \left(\frac{0.800 \times 10^{-2} \text{ m}}{2}\right)^2$$
$$A = \pi (4.00 \times 10^{-3} \text{ m})^2 = \pi \times 16.0 \times 10^{-6} \text{ m}^2$$
$$A = 5.03 \times 10^{-5} \text{ m}^2$$

**Step 3:** Calculate the power incident on the eardrum.

$$P = IA = (1.00 \times 10^{-3} \text{ W/m}^2)(5.03 \times 10^{-5} \text{ m}^2)$$
$$P = 5.03 \times 10^{-8} \text{ W}$$

**Step 4:** Calculate the total energy over 8 hours.

$$E = Pt = (5.03 \times 10^{-8} \text{ W})(28{,}800 \text{ s})$$
$$E = 1.45 \times 10^{-3} \text{ J}$$

**Discussion**

The total energy of $1.45 \times 10^{-3}$ J (about 1.5 millijoules) seems remarkably small—less than the energy needed to lift a paperclip one centimeter! Yet this tiny amount of energy, delivered continuously over 8 hours to the delicate structures of the inner ear, is sufficient to cause permanent hearing damage.

This illustrates several important points:

1. **Cumulative damage:** It's not just the instantaneous power that matters, but the total energy delivered over time. The continuous exposure allows energy to accumulate in the cochlea, causing metabolic stress and physical damage to hair cells.

2. **Biological sensitivity:** The hair cells in the cochlea are exquisitely sensitive but also fragile. The energy that seems trivial from a physics perspective is substantial from a biological perspective when concentrated on structures that are only micrometers in size.

3. **Power comparison:** The power of $5.03 \times 10^{-8}$ W is about 50 nanowatts—incredibly small. For comparison, a typical LED might use 0.1 W, which is 2 million times more power. Yet our ears are so sensitive that even this nanowatt-level power can cause damage with prolonged exposure.

4. **Occupational safety:** This is why OSHA and other safety organizations set strict limits on noise exposure. Workers in environments with 90 dB sound levels are required to use hearing protection or limit exposure time to prevent accumulating this damaging energy dose.

The energy incident on the eardrum during an 8-hour exposure to 90.0 dB is $1.45 \times 10^{-3}$ J.

(a) Ear trumpets were never very common, but they did aid people with hearing losses by gathering sound over a large area and concentrating it on the smaller area of the eardrum. What decibel increase does an ear trumpet produce if its sound gathering area is $900 \text{cm}^{2}$ and the area of the eardrum is $0.500 \text{cm}^{2}$ , but the trumpet only has an efficiency of 5.00% in transmitting the sound to the eardrum? (b) Comment on the usefulness of the decibel increase found in part (a).

Show Solution

**Strategy**

The ear trumpet collects sound power over its large collecting area and delivers a fraction of it (determined by efficiency) to the smaller eardrum area. The intensity increase at the eardrum is the ratio of delivered power to eardrum area compared to the original intensity. We calculate the power gain, find the intensity ratio, then convert to decibels.

**Solution**

**(a) Decibel increase**

Given:

- Collecting area: $A_{\text{collect}} = 900 \text{ cm}^2$
- Eardrum area: $A_{\text{ear}} = 0.500 \text{ cm}^2$
- Efficiency: $\eta = 5.00\% = 0.0500$

**Step 1:** Calculate the power collected by the trumpet.

If the sound has intensity $I_0$, the power collected is:

$$P_{\text{collect}} = I_0 \times A_{\text{collect}} = I_0 \times 900 \text{ cm}^2$$

**Step 2:** Calculate the power delivered to the eardrum.

With efficiency $\eta$:

$$P_{\text{delivered}} = \eta \times P_{\text{collect}} = 0.0500 \times I_0 \times 900 \text{ cm}^2 = 45 I_0 \text{ cm}^2$$

**Step 3:** Calculate the intensity at the eardrum with the trumpet.

$$I_{\text{trumpet}} = \frac{P_{\text{delivered}}}{A_{\text{ear}}} = \frac{45 I_0 \text{ cm}^2}{0.500 \text{ cm}^2} = 90 I_0$$

Without the trumpet, the intensity at the eardrum would be $I_0$.

**Step 4:** Calculate the decibel increase.

$$\Delta\beta = 10\log_{10}\left(\frac{I_{\text{trumpet}}}{I_0}\right) = 10\log_{10}(90)$$
$$\Delta\beta = 10\log_{10}(9 \times 10) = 10[\log_{10}(9) + \log_{10}(10)]$$
$$\Delta\beta = 10[0.954 + 1] = 10(1.954) = 19.5 \text{ dB}$$

**(b) Comment on usefulness**

A gain of 19.5 dB is quite significant and would be very helpful for people with hearing loss. To understand the usefulness:

- A 20-dB increase corresponds to multiplying the intensity by 100, making sounds much easier to hear
- For someone with moderate hearing loss (typically 40-60 dB deficit), this 19.5 dB boost could make the difference between not hearing a conversation at all and being able to follow it with some difficulty
- The increase is comparable to the difference between a quiet room (40 dB) and normal conversation (60 dB)

However, there are limitations:

- The ear trumpet only works for sounds coming from the direction it's pointed
- It's bulky and conspicuous to use
- Modern hearing aids provide 30-50 dB of amplification, can be tuned to specific frequencies, and are much smaller
- The trumpet provides no frequency selectivity—it amplifies all frequencies equally, unlike modern hearing aids that can compensate for frequency-specific hearing loss

Despite these limitations, before electronic hearing aids were invented, a 19.5 dB gain was quite valuable and could significantly improve quality of life for people with hearing loss.

**Discussion**

The calculation shows how the ear trumpet works as a simple acoustic amplifier. The large collecting area (900 $\text{cm}^2$) gathers sound power over an area 1800 times larger than the eardrum (0.500 $\text{cm}^2$). Even with only 5% efficiency, this results in a 90-fold intensity increase.

The relatively low efficiency (5%) accounts for losses due to:

- Sound reflection at impedance mismatches in the trumpet
- Sound absorption by the trumpet walls
- Diffraction and scattering effects
- Poor coupling between the trumpet and the ear canal

The ear trumpet demonstrates an important principle: you can increase sound intensity at a receiver by using a larger collecting area, similar to how satellite dishes and radio telescopes work. Modern acoustic devices like hearing aids use electronic amplification instead, but the basic principle of collecting more sound energy remains important in applications like parabolic microphones and acoustic mirrors.

(a) The ear trumpet produces a decibel increase of 19.5 dB.

(b) This is a significant and useful increase that would substantially help people with moderate hearing loss, though modern hearing aids provide superior performance.

Sound is more effectively transmitted into a stethoscope by direct contact than through the air, and it is further intensified by being concentrated on the smaller area of the eardrum. It is reasonable to assume that sound is transmitted into a stethoscope 100 times as effectively compared with transmission though the air. What, then, is the gain in decibels produced by a stethoscope that has a sound gathering area of $15.0 \text{cm}^{2}$ , and concentrates the sound onto two eardrums with a total area of $0.900 \text{cm}^{2}$ with an efficiency of 40.0%?

Show Solution

**Strategy**

The stethoscope provides amplification through three mechanisms: (1) enhanced transmission through direct contact (100× factor), (2) collecting sound over a larger area, and (3) concentrating it onto the smaller eardrum area with some efficiency. We calculate the total intensity gain by multiplying all these factors, then convert to decibels.

**Solution**

Given:

- Direct contact enhancement: factor of 100
- Collecting area: $A_{\text{collect}} = 15.0 \text{ cm}^2$
- Total eardrum area (both ears): $A_{\text{ear}} = 0.900 \text{ cm}^2$
- Efficiency: $\eta = 40.0\% = 0.400$

**Step 1:** Calculate the power collected through direct contact.

If sound in air has intensity $I_0$, direct contact provides 100 times more effective transmission:

$$P_{\text{collect}} = 100 \times I_0 \times A_{\text{collect}} = 100 \times I_0 \times 15.0 \text{ cm}^2$$

**Step 2:** Calculate the power delivered to the eardrums.

$$P_{\text{delivered}} = \eta \times P_{\text{collect}} = 0.400 \times 100 \times I_0 \times 15.0 \text{ cm}^2$$
$$P_{\text{delivered}} = 600 I_0 \text{ cm}^2$$

**Step 3:** Calculate the intensity at the eardrums.

$$I_{\text{steth}} = \frac{P_{\text{delivered}}}{A_{\text{ear}}} = \frac{600 I_0 \text{ cm}^2}{0.900 \text{ cm}^2} = \frac{600}{0.900} I_0 = 667 I_0$$

**Step 4:** Calculate the decibel gain.

$$\Delta\beta = 10\log_{10}\left(\frac{I_{\text{steth}}}{I_0}\right) = 10\log_{10}(667)$$
$$\Delta\beta = 10\log_{10}(6.67 \times 10^{2}) = 10[\log_{10}(6.67) + 2]$$
$$\Delta\beta = 10[0.824 + 2] = 10(2.824) = 28.2 \text{ dB}$$

**Discussion**

The stethoscope provides a remarkable 28.2 dB gain through the combination of:

1. **Direct contact (100× = 20 dB):** Sound travels much more efficiently through solid materials than through air. When the stethoscope chest piece is pressed against the body, sound is transmitted directly through tissue and the metal/plastic of the device, avoiding the large impedance mismatch that occurs at the air-skin interface. This accounts for the bulk of the amplification.

2. **Area concentration (15.0/0.900 = 16.7× ≈ 12 dB):** The stethoscope collects sound over its 15.0 $\text{cm}^2$ chest piece and funnels it to the smaller total eardrum area of 0.900 $\text{cm}^2$.

3. **Efficiency loss (40%):** Not all collected sound reaches the eardrums—some is lost to absorption and scattering in the tubing. This 40% efficiency reduces the gain by about 4 dB.

The net result (20 dB + 12 dB - 4 dB ≈ 28 dB) represents an intensity increase of about 670 times, which is crucial for detecting faint heart sounds, lung sounds, and blood flow through arteries. These sounds typically range from 20-100 Hz for heart sounds and up to 2000 Hz for lung sounds, and they would be nearly inaudible without amplification.

This is why stethoscopes are essential diagnostic tools—they make the difference between hearing a faint heart murmur that indicates a serious valve problem and missing it entirely. Modern electronic stethoscopes can provide even greater amplification (40-50 dB) and filter specific frequency ranges, but the traditional acoustic stethoscope remains effective due to its impressive 28-dB passive amplification.

The stethoscope produces a gain of 28.2 dB.

Loudspeakers can produce intense sounds with surprisingly small energy input in spite of their low efficiencies. Calculate the power input needed to produce a 90.0-dB sound intensity level for a 12.0-cm-diameter speaker that has an efficiency of 1.00%. (This value is the sound intensity level right at the speaker.)

Show Solution

**Strategy**

We need to find the electrical power input to the speaker. First, we convert the decibel level to intensity. Then we calculate the area of the speaker and find the acoustic power output. Since efficiency is the ratio of acoustic power output to electrical power input, we divide the acoustic power by the efficiency to find the required electrical input power.

**Solution**

Given:

- Sound intensity level: $\beta = 90.0 \text{ dB}$
- Speaker diameter: $d = 12.0 \text{ cm} = 0.120 \text{ m}$
- Efficiency: $\eta = 1.00\% = 0.0100$
- Reference intensity: $I_0 = 10^{-12} \text{ W/m}^2$

**Step 1:** Convert the decibel level to intensity.

$$I = I_0 \times 10^{\beta/10} = (10^{-12} \text{ W/m}^2) \times 10^{90.0/10}$$
$$I = (10^{-12} \text{ W/m}^2) \times 10^{9.00} = 10^{-3} \text{ W/m}^2 = 1.00 \times 10^{-3} \text{ W/m}^2$$

**Step 2:** Calculate the area of the speaker.

$$A = \pi r^2 = \pi \left(\frac{d}{2}\right)^2 = \pi \left(\frac{0.120 \text{ m}}{2}\right)^2$$
$$A = \pi (0.0600 \text{ m})^2 = \pi \times 3.60 \times 10^{-3} \text{ m}^2$$
$$A = 1.13 \times 10^{-2} \text{ m}^2$$

**Step 3:** Calculate the acoustic power output.

$$P_{\text{acoustic}} = I \times A = (1.00 \times 10^{-3} \text{ W/m}^2)(1.13 \times 10^{-2} \text{ m}^2)$$
$$P_{\text{acoustic}} = 1.13 \times 10^{-5} \text{ W}$$

**Step 4:** Calculate the electrical power input.

Since efficiency is defined as:

$$\eta = \frac{P_{\text{acoustic}}}{P_{\text{input}}}$$

We can solve for input power:

$$P_{\text{input}} = \frac{P_{\text{acoustic}}}{\eta} = \frac{1.13 \times 10^{-5} \text{ W}}{0.0100}$$
$$P_{\text{input}} = 1.13 \times 10^{-3} \text{ W} = 1.13 \text{ mW}$$

**Discussion**

The required electrical input power is only 1.13 milliwatts—an astonishingly small amount of power! This demonstrates several important points:

**Why so little power is needed:**

1. **90 dB is measured at the speaker surface:** The intensity decreases rapidly with distance from the speaker (as $1/r^2$), so this represents the maximum intensity, right at the source.

2. **Small speaker area:** The 12-cm diameter gives an area of only $1.13 \times 10^{-2}$ m² (about 113 cm²). The total acoustic power is intensity times area, so the small area limits the total power.

3. **Human ear sensitivity:** Our ears are remarkably sensitive. Even the small acoustic power of 11.3 microwatts produces a sound level (90 dB) that can cause hearing damage with prolonged exposure.

**Why efficiency is so low (1%):** Most of the electrical input power is converted to heat rather than sound. The losses occur due to:

- Electrical resistance in the voice coil
- Mechanical friction and damping
- Impedance mismatch between the speaker cone and air

- Energy stored in the speaker's magnetic field

Even with 99% of the input power wasted as heat, the 1% that becomes acoustic power (11.3 μW) is sufficient to produce a loud, potentially damaging sound.

**Practical implications:** This is why small battery-powered speakers and earbuds can operate for hours on tiny batteries—they need very little actual power. A typical smartphone speaker might use 0.5-1 W of electrical power, but most of that becomes heat, not sound. The actual acoustic power that reaches your ears might be only 5-10 milliwatts, yet it's more than sufficient for clear audio playback.

The power input needed is $1.13 \times 10^{-3}$ W or 1.13 mW.

## Footnotes

- 1 Several government agencies and health-related professional associations recommend that 85 dB not be exceeded for 8-hour daily exposures in the absence of hearing protection. { data-list-type="bulleted" data-bullet-style="none"}

## Glossary

intensity
: the power per unit area carried by a wave

sound intensity level
: a unitless quantity telling you the level of the sound relative to a fixed standard

sound pressure level
: the ratio of the pressure amplitude to a reference pressure

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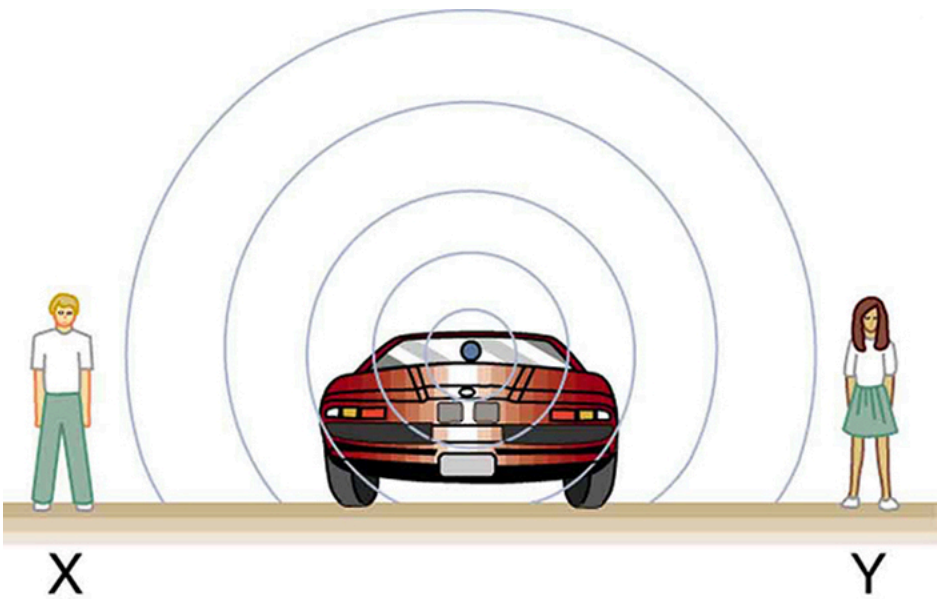
# Doppler Effect and Sonic Booms

- Define Doppler effect, Doppler shift, and sonic boom.
- Calculate the frequency of a sound heard by someone observing Doppler shift.
- Describe the sounds produced by objects moving faster than the speed of sound.

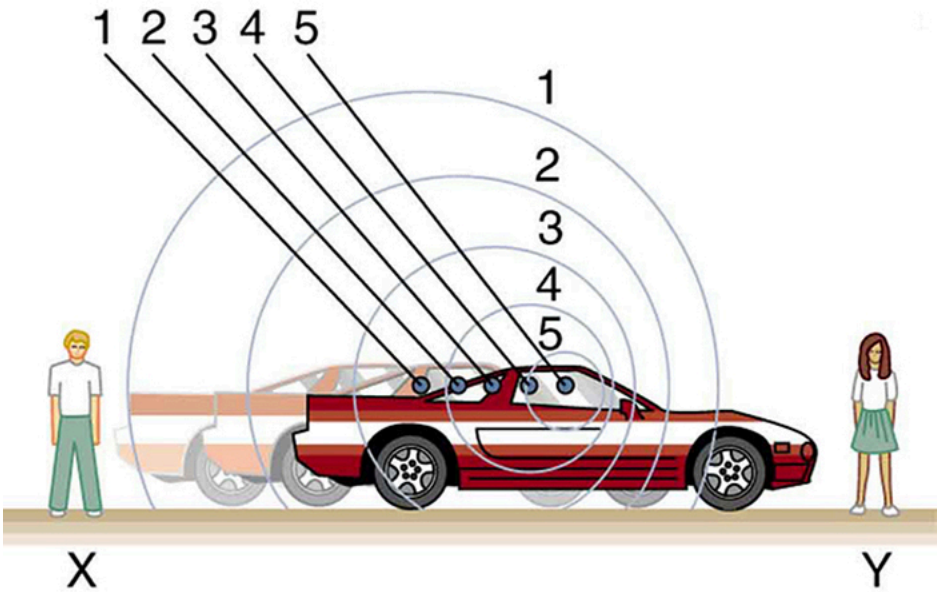
The characteristic sound of a motorcycle buzzing by is an example of the **Doppler effect**. The high-pitch scream shifts dramatically to a lower-pitch roar as the motorcycle passes by a stationary observer. The closer the motorcycle brushes by, the more abrupt the shift. The faster the motorcycle moves, the greater the shift. We also hear this characteristic shift in frequency for passing race cars, airplanes, and trains. It is so familiar that it is used to imply motion and children often mimic it in play.

The Doppler effect is an alteration in the observed frequency of a sound due to motion of either the source or the observer. Although less familiar, this effect is easily noticed for a stationary source and moving observer. For example, if you ride a train past a stationary warning bell, you will hear the bell's frequency shift from high to low as you pass by. The actual change in frequency due to relative motion of source and observer is called a **Doppler shift**. The Doppler effect and Doppler shift are named for the Austrian physicist and mathematician Christian Johann Doppler (1803–1853), who did experiments with both moving sources and moving observers. Doppler, for example, had musicians play on a moving open train car and also play standing next to the train tracks as a train passed by. Their music was observed both on and off the train, and changes in frequency were measured.

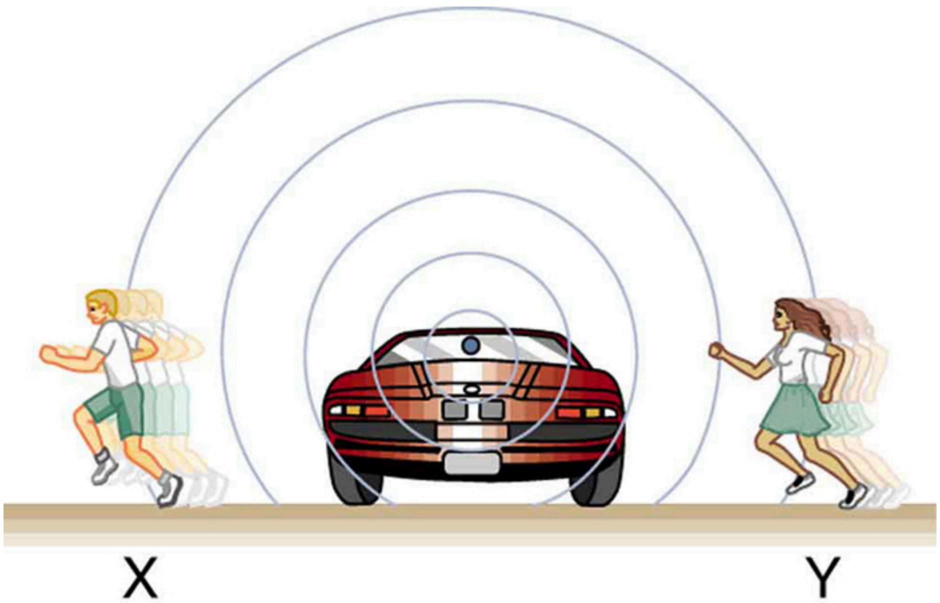
What causes the Doppler shift? [Figure 1], [Figure 2], and [Figure 3] compare sound waves emitted by stationary and moving sources in a stationary air mass. Each disturbance spreads out spherically from the point where the sound was emitted. If the source is stationary, then all of the spheres representing the air compressions in the sound wave centered on the same point, and the stationary observers on either side see the same wavelength and frequency as emitted by the source, as in [Figure 1]. If the source is moving, as in [Figure 2], then the situation is different. Each compression of the air moves out in a sphere from the point where it was emitted, but the point of emission moves. This moving emission point causes the air compressions to be closer together on one side and farther apart on the other. Thus, the wavelength is shorter in the direction the source is moving (on the right in [Figure 2]), and longer in the opposite direction (on the left in [Figure 2]). Finally, if the observers move, as in [Figure 3], the frequency at which they receive the compressions changes. The observer moving toward the source receives them at a higher frequency, and the person moving away from the source receives them at a lower frequency.




Sounds emitted by a source spread out in spherical waves. Because the source, observers, and air are stationary, the wavelength and frequency are the same in all directions and to all observers.




Sounds emitted by a source moving to the right spread out from the points at which they were emitted. The wavelength is reduced and, consequently, the frequency is increased in the direction of motion, so that the observer on the right hears a higher-pitch sound. The opposite is true for the observer on the left, where the wavelength is increased and the frequency is reduced.




The same effect is produced when the observers move relative to the source. Motion toward the source increases frequency as the observer on the right passes through more wave crests than she would if stationary. Motion away from the source decreases frequency as the observer on the left passes through fewer wave crests than he would if stationary.

We know that wavelength and frequency are related by $v_W = f\lambda$ , where $v_W$ is the fixed speed of sound. The sound moves in a medium and has the same speed $v_W$ in that medium whether the source is moving or not. Thus $f$ multiplied by $\lambda$ is a constant. Because the observer on the right in [Figure 2] receives a shorter wavelength, the frequency she receives must be higher. Similarly, the observer on the left receives a longer wavelength, and hence he hears a lower frequency. The same thing happens in [Figure 3]. A higher frequency is received by the observer moving toward the source, and a lower frequency is received by an observer moving away from the source. In general, then, relative motion of source and observer toward one another increases the received frequency. Relative motion apart decreases frequency. The greater the relative speed is, the greater the effect.

The Doppler Effect

The Doppler effect occurs not only for sound but for any wave when there is relative motion between the observer and the source. There are Doppler shifts in the frequency of sound, light, and water waves, for example. Doppler shifts can be used to determine velocity, such as when ultrasound is reflected from blood in a medical diagnostic. The recession of galaxies is determined by the shift in the frequencies of light received from them and has implied much about the origins of the universe. Modern physics has been profoundly affected by observations of Doppler shifts.

For a stationary observer and a moving source, the frequency $f_{obs}$ received by the observer can be shown to be

$$f_{obs}=f_s\left(\frac{v_w}{v_w \pm v_s}\right),$$

where $f_s$ is the frequency of the source, $v_s$ is the speed of the source along a line joining the source and observer, and $v_w$ is the speed of sound. The minus sign is used for motion toward the observer and the plus sign for motion away from the observer, producing the appropriate shifts up and down in frequency. Note that the greater the speed of the source, the greater the effect. Similarly, for a stationary source and moving observer, the frequency received by the observer $f_{obs}$ is given by

$$f_{obs}=f_s\left(\frac{v_w \pm v_{obs}}{v_w}\right),$$

where $v_{obs}$ is the speed of the observer along a line joining the source and observer. Here the plus sign is for motion toward the source, and the minus is for motion away from the source.

Calculate Doppler Shift: A Train Horn

Suppose a train that has a 150-Hz horn is moving at 35.0 m/s in still air on a day when the speed of sound is 340 m/s.

(a) What frequencies are observed by a stationary person at the side of the tracks as the train approaches and after it passes?

(b) What frequency is observed by the train's engineer traveling on the train?

**Strategy**

To find the observed frequency in (a), $f_{obs}=f_s\left(\frac{v_w}{v_w \pm v_s}\right)$, must be used because the source is moving. The minus sign is used for the approaching train, and the plus sign for the receding train. In (b), there are two Doppler shifts—one for a moving source and the other for a moving observer.

**Solution for (a)**

(1) Enter known values into $f_{obs}=f_s\left(\frac{v_w}{v_w - v_s}\right)$.

$$f_{obs}=f_s\left(\frac{v_w}{v_w - v_s}\right)=(150\,\text{Hz})\left(\frac{340\,\text{m/s}}{340\,\text{m/s}-35.0\,\text{m/s}}\right)$$

(2) Calculate the frequency observed by a stationary person as the train approaches.

$$f_{obs}=(150\,\text{Hz})(1.11)=167\,\text{Hz}$$

(3) Use the same equation with the plus sign to find the frequency heard by a stationary person as the train recedes.

$$f_{obs}=f_s\left(\frac{v_w}{v_w + v_s}\right)=(150\,\text{Hz})\left(\frac{340\,\text{m/s}}{340\,\text{m/s}+35.0\,\text{m/s}}\right)$$

(4) Calculate the second frequency.

$$f_{obs}=(150\,\text{Hz})(0.907)=136\,\text{Hz}$$

**Discussion on (a)**

The numbers calculated are valid when the train is far enough away that the motion is nearly along the line joining train and observer. In both cases, the shift is significant and easily noticed. Note that the shift is 17.0 Hz for motion toward and 14.0 Hz for motion away. The shifts are not symmetric.

**Solution for (b)**

(1) Identify knowns:

- It seems reasonable that the engineer would receive the same frequency as emitted by the horn, because the relative velocity between them is zero.
- Relative to the medium (air), the speeds are $v_s=v_{obs}=35.0\,m/s$.
- The first Doppler shift is for the moving observer; the second is for the moving source.

(2) Use the following equation:

$$f_{obs}=\left[f_s\left(\frac{v_w \pm v_{obs}}{v_w}\right)\right]\left(\frac{v_w}{v_w \pm v_s}\right).$$

The quantity in the square brackets is the Doppler-shifted frequency due to a moving observer. The factor on the right is the effect of the moving source.

(3) Because the train engineer is moving in the direction toward the horn, we must use the plus sign for $v_{obs}$; however, because the horn is also moving in the direction away from the engineer, we also use the plus sign for $v_s$. But the train is carrying both the engineer and the horn at the same velocity, so $v_s=v_{obs}$. As a result, everything but $f_s$ cancels, yielding

$$f_{obs}=f_s.$$

**Discussion for (b)**

We may expect that there is no change in frequency when source and observer move together because it fits your experience. For example, there is no Doppler shift in the frequency of conversations between driver and passenger on a motorcycle. People talking when a wind moves the air between them also observe no Doppler shift in their conversation. The crucial point is that source and observer are not moving relative to each other.

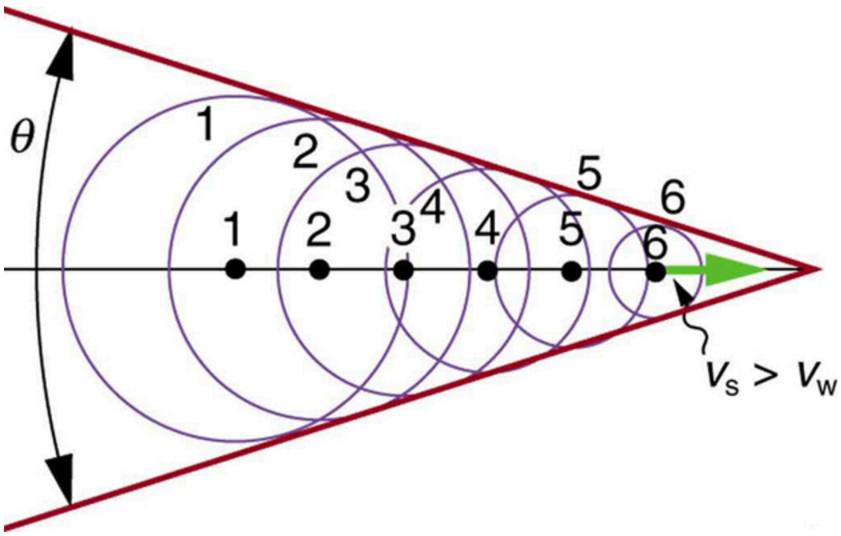
### Sonic Booms to Bow Wakes

What happens to the sound produced by a moving source, such as a jet airplane, that approaches or even exceeds the speed of sound? The answer to this question applies not only to sound but to all other waves as well.

Suppose a jet airplane is coming nearly straight at you, emitting a sound of frequency $f_s$. The greater the plane's speed $v_s$, the greater the Doppler shift and the greater the value observed for $f_{\text{obs}}$. Now, as $v_s$ approaches the speed of sound, $f_{\text{obs}}$ approaches infinity, because the denominator in

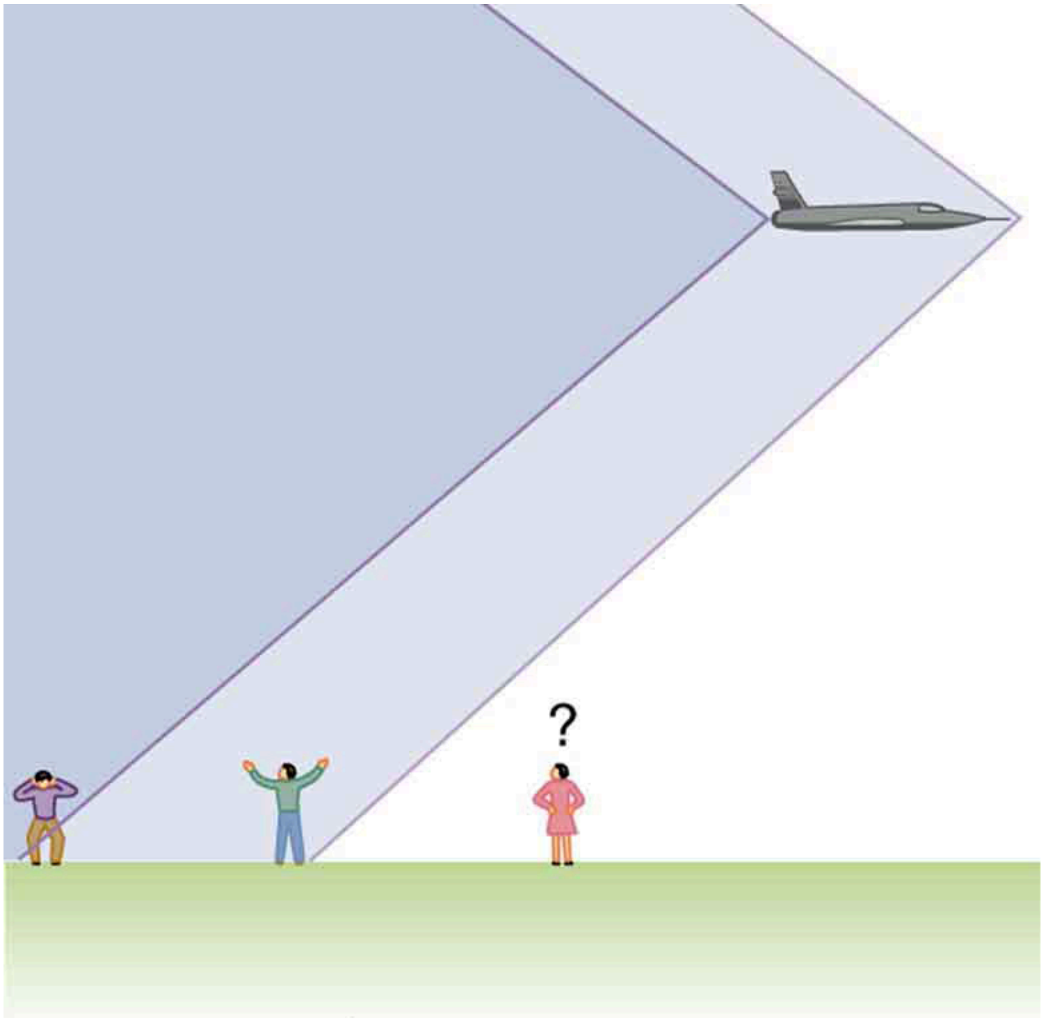
$$f_{\text{obs}} = f_s\left(\frac{v_w}{v_w \pm v_s}\right)$$

approaches zero. At the speed of sound, this result means that in front of the source, each successive wave is superimposed on the previous one because the source moves forward at the speed of sound. The observer gets them all at the same instant, and so the frequency is infinite. (Before airplanes exceeded the speed of sound, some people argued it would be impossible because such constructive superposition would produce pressures great enough to destroy the airplane.) If the source exceeds the speed of sound, no sound is received by the observer until the source has passed, so that the sounds from the approaching source are mixed with those from it when receding. This mixing appears messy, but something interesting happens—a sonic boom is created. ( See [Figure 4].)




Sound waves from a source that moves faster than the speed of sound spread spherically from the point where they are emitted, but the source moves ahead of each. Constructive interference along the lines shown (actually a cone in three dimensions) creates a shock wave called a sonic boom. The faster the speed of the source, the smaller the angle $\theta$.

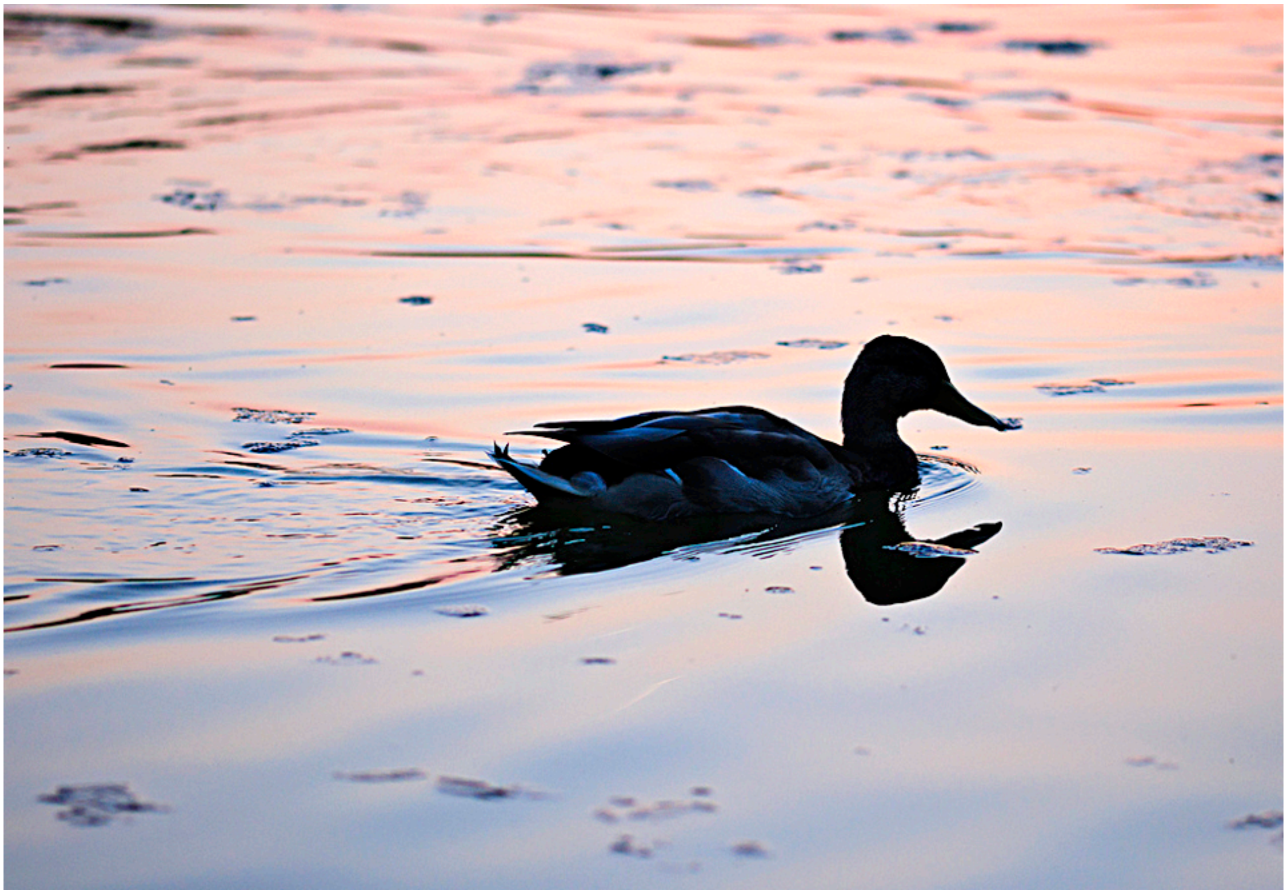
There is constructive interference along the lines shown (a cone in three dimensions) from similar sound waves arriving there simultaneously. This superposition forms a disturbance called a **sonic boom**, a constructive interference of sound created by an object moving faster than sound. Inside the cone, the interference is mostly destructive, and so the sound intensity there is much less than on the shock wave. An aircraft creates two sonic booms, one from its nose and one from its tail. (See [Figure 5].) During television coverage of space shuttle landings, two distinct booms could often be heard. These were separated by exactly the time it would take the shuttle to pass by a point. Observers on the ground often do not see the aircraft creating the sonic boom, because it has passed by before the shock wave reaches them, as seen in [Figure 5]. If the aircraft flies close by at low altitude, pressures in the sonic boom can be destructive and break windows as well as rattle nerves. Because of how destructive sonic booms can be, supersonic flights are banned over populated areas of the United States.



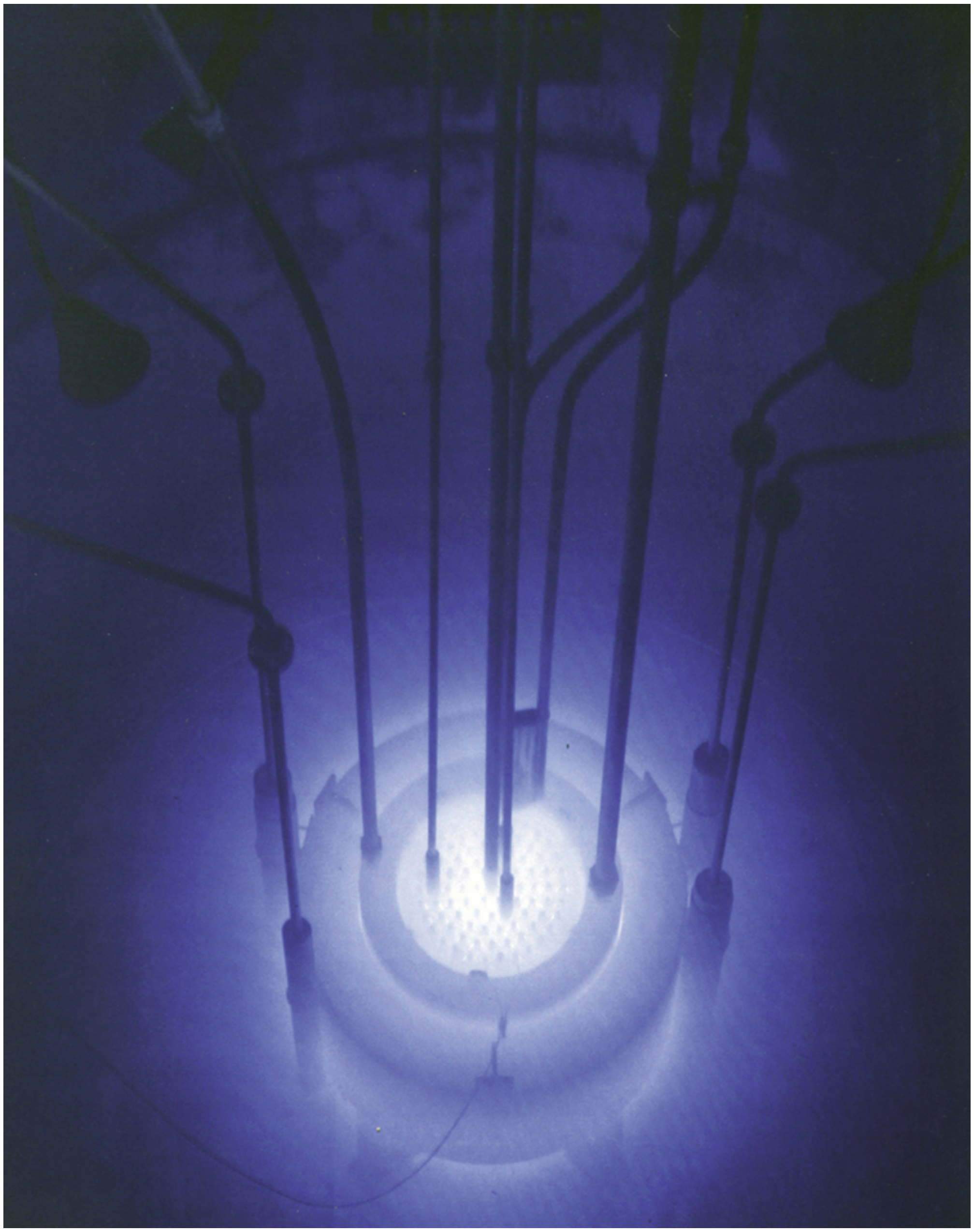
Two sonic booms, created by the nose and tail of an aircraft, are observed on the ground after the plane has passed by.

Sonic booms are one example of a broader phenomenon called bow wakes. A **bow wake**, such as the one in [Figure 6], is created when the wave source moves faster than the wave propagation speed. Water waves spread out in circles from the point where created, and the bow wake is the familiar V-shaped wake trailing the source. A more exotic bow wake is created when a subatomic particle travels through a medium faster than the speed of light travels in that medium. In a vacuum, the maximum speed of light will be $c = 3.00 \times 10^{8}$ \text{m/s} ; $in the medium of water, the speed of light is closer to$ $0.75c$

. If the particle creates light in its passage, that light spreads on a cone with an angle indicative of the speed of the particle, as illustrated in [Figure 7]. Such a bow wake is called Cerenkov radiation and is commonly observed in particle physics.



Bow wake created by a duck. Constructive interference produces the rather structured wake, while there is relatively little wave action inside the wake, where interference is mostly destructive. (credit: Horia Varlan, Flickr)



The blue glow in this research reactor pool is Cerenkov radiation caused by subatomic particles traveling faster than the speed of light in water. (credit: U.S. Nuclear Regulatory Commission)

Doppler shifts and sonic booms are interesting sound phenomena that occur in all types of waves. They can be of considerable use. For example, the Doppler shift in ultrasound can be used to measure blood velocity, while police use the Doppler shift in radar (a microwave) to measure car velocities. In meteorology, the Doppler shift is used to track the motion of storm clouds; such "Doppler Radar" can give velocity and direction and rain or snow potential of imposing weather fronts. In astronomy, we can examine the light emitted from distant galaxies and determine their speed relative to ours. As galaxies move away from us, their light is shifted to a lower frequency, and so to a longer wavelength—the so-called red shift. Such information from galaxies far, far away has allowed us to estimate the age of the universe (from the Big Bang) as about 14 billion years.

Check Your Understanding

Why did scientist Christian Doppler observe musicians both on a moving train and also from a stationary point not on the train?

Show Solution

Doppler needed to compare the perception of sound when the observer is stationary and the sound source moves, as well as when the sound source and the observer are both in motion.

Check Your Understanding

Describe a situation in your life when you might rely on the Doppler shift to help you either while driving a car or walking near traffic.

Show Solution

If I am driving and I hear Doppler shift in an ambulance siren, I would be able to tell when it was getting closer and also if it has passed by. This would help me to know whether I needed to pull over and let the ambulance through.

## Section Summary

- The Doppler effect is an alteration in the observed frequency of a sound due to motion of either the source or the observer.
- The actual change in frequency is called the Doppler shift.
- A sonic boom is constructive interference of sound created by an object moving faster than sound.
- A sonic boom is a type of bow wake created when any wave source moves faster than the wave propagation speed.

- For a stationary observer and a moving source, the observed frequency $f_{\text{obs}}$ is:

  $$f_{\text{obs}} = f_s\left(\frac{v_w}{v_w \pm v_s}\right),$$

  where $f_s$ is the frequency of the source, $v_s$ is the speed of the source, and $v_w$ is the speed of sound. The minus sign is used for motion toward the observer and the plus sign for motion away.

- For a stationary source and moving observer, the observed frequency is:

  $$f_{\text{obs}} = f_s\left(\frac{v_w \pm v_{\text{obs}}}{v_w}\right),$$

  where $v_{\text{obs}}$ is the speed of the observer.

## Conceptual Questions

Is the Doppler shift real or just a sensory illusion?

Show Solution

The Doppler shift is a **real physical phenomenon**, not a sensory illusion. The frequency of the sound waves arriving at the observer is physically different from the frequency emitted by the source. The relative motion between the source and observer causes the wavefronts to be compressed (higher frequency) or stretched (lower frequency). This change in frequency can be measured by instruments, so it is not just a product of our perception. For example, police radar guns use the Doppler shift of radio waves to measure the speed of vehicles.

Due to efficiency considerations related to its bow wake, the supersonic transport aircraft must maintain a cruising speed that is a constant ratio to the speed of sound (a constant Mach number). If the aircraft flies from warm air into colder air, should it increase or decrease its speed? Explain your answer.

Show Solution

The aircraft should **decrease** its speed.

The speed of sound is lower in colder air. To maintain a constant Mach number (which is the ratio of the aircraft's speed to the speed of sound), the aircraft must decrease its speed proportionally as the speed of sound decreases.

When you hear a sonic boom, you often cannot see the plane that made it. Why is that?

Show Solution

This is because the airplane is traveling faster than the speed of sound, while the sonic boom (the shock wave) travels at the speed of sound. By the time the shock wave reaches an observer on the ground, the airplane that created it has already traveled a significant distance ahead of the boom.

## Problems & Exercises

(a) What frequency is received by a person watching an oncoming ambulance moving at 110 km/h and emitting a steady 800-Hz sound from its siren? The speed of sound on this day is 345 m/s. (b) What frequency does she receive after the ambulance has passed?

Show Solution

**Strategy**

This problem involves a moving source (ambulance) and a stationary observer. We must use the formula for a moving source: $f_{\text{obs}} = f_s\left(\frac{v_w}{v_w \pm v_s}\right)$. For part (a), the ambulance is approaching, so we use the minus sign in the denominator. For part (b), the ambulance is receding, so we use the plus sign. First, we must convert the ambulance speed from km/h to m/s to match the units of the speed of sound.

**Solution**

Part (a): First, convert the ambulance speed to m/s:

$$v_s = 110\text{ km/h} \times \frac{1000\text{ m}}{1\text{ km}} \times \frac{1\text{ h}}{3600\text{ s}} = 30.56\text{ m/s}$$

For the approaching ambulance, use the Doppler shift formula with the minus sign:

$f_{obs}=f_s\left(\frac{v_w}{v_w-v_s}\right)=(800\text{ Hz})\left(\frac{345\text{ m/s}}{345\text{ m/s}-30.56\text{ m/s}}\right)$
$f_{obs}=(800\text{ Hz})\left(\frac{345\text{ m/s}}{314.44\text{ m/s}}\right)=(800\text{ Hz})(1.0971)=878\text{ Hz}$

Part (b): For the receding ambulance, use the plus sign:

$f_{obs}=f_s\left(\frac{v_w}{v_w+v_s}\right)=(800\text{ Hz})\left(\frac{345\text{ m/s}}{345\text{ m/s}+30.56\text{ m/s}}\right)$
$f_{obs}=(800\text{ Hz})\left(\frac{345\text{ m/s}}{375.56\text{ m/s}}\right)=(800\text{ Hz})(0.9186)=735\text{ Hz}$

**Discussion**

The approaching ambulance produces a frequency shift of +78 Hz (about 9.8% higher), while the receding ambulance produces a shift of -65 Hz (about 8.1% lower). These shifts are quite noticeable to the human ear, which is why emergency vehicles sound distinctly different when approaching versus receding. Note that the shifts are asymmetric—the increase when approaching is larger than the decrease when receding, even though the speed is the same. This asymmetry arises from the mathematical form of the Doppler equation. The observer hears a frequency of 878 Hz when the ambulance approaches, which is significantly higher than the emitted 800 Hz, and 735 Hz when it recedes. This dramatic change helps pedestrians and drivers identify the location and motion of emergency vehicles.

(a) At an air show a jet flies directly toward the stands at a speed of 1200 km/h, emitting a frequency of 3500 Hz, on a day when the speed of sound is 342 m/s. What frequency is received by the observers? (b) What frequency do they receive as the plane flies directly away from them?

Show Solution

**Strategy**

This is a moving source (jet) with stationary observers problem. We use the formula $f_{obs}=f_s\left(\frac{v_w}{v_w\pm v_s}\right)$, with the minus sign for approach and plus sign for recession. First, we must convert the jet speed from km/h to m/s. An important feature of this problem is that the jet is traveling at nearly the speed of sound, which will produce a dramatic Doppler shift.

**Solution**

First, convert the jet speed to m/s:

$v_s=1200\text{ km/h}\times\frac{1000\text{ m}}{1\text{ km}}\times\frac{1\text{ h}}{3600\text{ s}}=333.3\text{ m/s}$

Note that this is very close to the speed of sound (342 m/s), so the jet is traveling at approximately Mach 0.97.

Part (a): For the approaching jet, use the minus sign:

$f_{obs}=f_s\left(\frac{v_w}{v_w-v_s}\right)=(3500\text{ Hz})\left(\frac{342\text{ m/s}}{342\text{ m/s}-333.3\text{ m/s}}\right)$
$f_{obs}=(3500\text{ Hz})\left(\frac{342\text{ m/s}}{8.7\text{ m/s}}\right)=(3500\text{ Hz})(39.3)=1.38\times10^5\text{ Hz}$

Part (b): For the receding jet, use the plus sign:

$f_{obs}=f_s\left(\frac{v_w}{v_w+v_s}\right)=(3500\text{ Hz})\left(\frac{342\text{ m/s}}{342\text{ m/s}+333.3\text{ m/s}}\right)$
$f_{obs}=(3500\text{ Hz})\left(\frac{342\text{ m/s}}{675.3\text{ m/s}}\right)=(3500\text{ Hz})(0.506)=1.77\times10^3\text{ Hz}$

**Discussion**

The results are dramatic! When approaching, the frequency shifts from 3500 Hz to 138,000 Hz—an increase by a factor of about 39. This enormous shift occurs because the jet is traveling at nearly the speed of sound, making the denominator $v_w-v_s$ very small. This is why the Doppler formula predicts an infinite frequency when the source travels exactly at the speed of sound (the denominator becomes zero). When receding, the frequency drops to 1770 Hz, about half the emitted frequency. The massive asymmetry between approach and recession frequencies is characteristic of sources moving at near-sonic speeds. In reality, much of this shifted sound would be outside the range of human hearing (20-20,000 Hz), but the jet's engine produces a wide spectrum of frequencies, so observers would still hear an intense, high-pitched sound on approach and a much lower rumble as it recedes.

What frequency is received by a mouse just before being dispatched by a hawk flying at it at 25.0 m/s and emitting a screech of frequency 3500 Hz? Take the speed of sound to be 331 m/s.

Show Solution

**Strategy**

This problem involves a moving source (the hawk) approaching a stationary observer (the mouse). We use the Doppler shift formula for a moving source: $f_{obs}=f_s\left(\frac{v_w}{v_w\pm v_s}\right)$. Since the hawk is approaching the mouse, we use the minus sign in the denominator, which increases the observed frequency.

**Solution**

The hawk is moving toward the mouse, so we apply the formula with the minus sign:

$f_{obs}=f_s\left(\frac{v_w}{v_w-v_s}\right)=(3500\text{ Hz})\left(\frac{331\text{ m/s}}{331\text{ m/s}-25.0\text{ m/s}}\right)$

$$f_{obs}=(3500\text{ Hz})\left(\frac{331\text{ m/s}}{306\text{ m/s}}\right)=(3500\text{ Hz})(1.082)=3790\text{ Hz}=3.79\times10^{3}\text{ Hz}$$

**Discussion**

The mouse perceives a frequency about 8.2% higher than the hawk actually emits. This frequency shift of 290 Hz is readily audible and represents a shift of about 4 musical semitones. From an evolutionary perspective, many prey animals have developed acute hearing to detect approaching predators through such Doppler shifts—the rising pitch of a predator's call provides crucial information about whether the predator is approaching. The relatively modest speed of the hawk (25.0 m/s, or about 90 km/h) compared to the speed of sound (331 m/s) means the Doppler shift is noticeable but not extreme. The perceived frequency of 3790 Hz is well within the hearing range of most small mammals, which typically extends to much higher frequencies than human hearing.

</div>

A spectator at a parade receives an 888-Hz tone from an oncoming trumpeter who is playing an 880-Hz note. At what speed is the musician approaching if the speed of sound is 338 m/s?

Show Solution

**Strategy**

This problem involves a moving source (trumpeter) and a stationary observer (spectator). We know the source frequency $f_s=880$ Hz, the observed frequency $f_{obs}=888$ Hz, and the speed of sound $v_w=338$ m/s. We need to find the source speed $v_s$. Since the musician is approaching, we use the Doppler formula $f_{obs}=f_s\left(\frac{v_w}{v_w-v_s}\right)$ and solve for $v_s$.

**Solution**

Start with the Doppler shift formula for an approaching source:

$$f_{obs}=f_s\left(\frac{v_w}{v_w-v_s}\right)$$

Substitute the known values:

$$888\text{ Hz}=(880\text{ Hz})\left(\frac{338\text{ m/s}}{338\text{ m/s}-v_s}\right)$$

Divide both sides by 880 Hz:

$$\frac{888}{880}=\frac{338\text{ m/s}}{338\text{ m/s}-v_s}$$
$$1.00909=\frac{338\text{ m/s}}{338\text{ m/s}-v_s}$$

Cross-multiply:

$$1.00909(338\text{ m/s}-v_s)=338\text{ m/s}$$
$$341.07\text{ m/s}-1.00909v_s=338\text{ m/s}$$

Solve for $v_s$:

$$1.00909v_s=341.07\text{ m/s}-338\text{ m/s}=3.07\text{ m/s}$$
$$v_s=\frac{3.07\text{ m/s}}{1.00909}=3.04\text{ m/s}$$

**Discussion**

The trumpeter is moving at approximately 3.0 m/s (about 11 km/h or 7 mph), which is a reasonable walking/marching speed for a parade. The frequency shift is only 8 Hz (about 0.9%), which is quite small but still noticeable to a trained ear—musicians can typically detect pitch differences of about 0.5% or less. This small shift corresponds to the modest speed of the source relative to the speed of sound. The calculation demonstrates an important application of the Doppler effect: if we can measure the frequency shift, we can determine the speed of the source. This principle is used in radar guns (for measuring vehicle speeds), Doppler weather radar (for measuring wind speeds), and medical ultrasound (for measuring blood flow velocity).

A commuter train blows its 200-Hz horn as it approaches a crossing. The speed of sound is 335 m/s. (a) An observer waiting at the crossing receives a frequency of 208 Hz. What is the speed of the train? (b) What frequency does the observer receive as the train moves away?

Show Solution

**Strategy**

Part (a) requires finding the source velocity from the observed Doppler shift. We know $f_s=200$ Hz, $f_{obs}=208$ Hz (approaching), and $v_w=335$ m/s. We use the approaching source formula and solve for $v_s$. For part (b), we use the same source speed but apply the receding source formula to find the new observed frequency.

**Solution**

Part (a): Start with the Doppler formula for an approaching source:

$$f_{\text{obs}}=f_s\left(\frac{v_w}{v_w-v_s}\right)$$

Substitute known values:

$$208\text{ Hz}=(200\text{ Hz})\left(\frac{335\text{ m/s}}{335\text{ m/s}-v_s}\right)$$

Divide by 200 Hz:

$$1.04=\frac{335\text{ m/s}}{335\text{ m/s}-v_s}$$

Cross-multiply and solve:

$$1.04(335\text{ m/s}-v_s)=335\text{ m/s}$$
$$348.4\text{ m/s}-1.04v_s=335\text{ m/s}$$
$$1.04v_s=13.4\text{ m/s}$$
$$v_s=12.9\text{ m/s}$$

Part (b): Now use the receding source formula with $v_s=12.9$ m/s:

$$f_{\text{obs}}=f_s\left(\frac{v_w}{v_w+v_s}\right)=(200\text{ Hz})\left(\frac{335\text{ m/s}}{335\text{ m/s}+12.9\text{ m/s}}\right)$$
$$f_{\text{obs}}=(200\text{ Hz})\left(\frac{335\text{ m/s}}{347.9\text{ m/s}}\right)=(200\text{ Hz})(0.963)=193\text{ Hz}$$

**Discussion**

The train is traveling at 12.9 m/s (about 46 km/h or 29 mph), which is a reasonable speed for a commuter train approaching a crossing—trains typically slow down near crossings for safety. The frequency increases by 8 Hz (4%) when approaching and decreases by 7 Hz (3.5%) when receding. The asymmetry in the shifts (+8 Hz vs. -7 Hz) is characteristic of the Doppler effect. A person waiting at the crossing would clearly hear the pitch drop from 208 Hz to 193 Hz as the train passes—a change of 15 Hz. This distinctive pitch change is one of the most familiar examples of the Doppler effect in everyday life. The calculation in part (a) demonstrates an inverse Doppler problem, where we work backward from the observed frequency shift to determine the source speed—this is exactly how police radar guns work, though they use electromagnetic waves instead of sound.

Can you perceive the shift in frequency produced when you pull a tuning fork toward you at 10.0 m/s on a day when the speed of sound is 344 m/s? To answer this question, calculate the factor by which the frequency shifts and see if it is greater than 0.300%.

Show Solution

**Strategy**

When you pull a tuning fork toward yourself, you create relative motion between the source and observer. We can analyze this as a moving source approaching a stationary observer (considering the sound waves emitted by the fork). We use the formula $f_{\text{obs}}=f_s\left(\frac{v_w}{v_w-v_s}\right)$ to find the shift factor, then calculate the percentage change. If this percentage exceeds 0.300%, the shift is perceptible.

**Solution**

For a source moving toward the observer at $v_s=10.0$ m/s:

$$f_{\text{obs}}=f_s\left(\frac{v_w}{v_w-v_s}\right)=f_s\left(\frac{344\text{ m/s}}{344\text{ m/s}-10.0\text{ m/s}}\right)$$
$$f_{\text{obs}}=f_s\left(\frac{344\text{ m/s}}{334\text{ m/s}}\right)=f_s(1.0299)$$

The shift factor is 1.0299, meaning the observed frequency is 1.0299 times the source frequency.

Calculate the percentage shift:

$$\text{Percentage shift}=(1.0299-1)\times 100\%=0.0299\times 100\%=2.99\%$$

Since 2.99% » 0.300%, yes, you can perceive the shift.

**Discussion**

The frequency shift of approximately 3.0% is about ten times larger than the 0.300% threshold for perceptibility, so it would be clearly audible. To put this in musical terms, a 3% shift corresponds to about half a semitone (one semitone is about 6%), which musicians can easily detect. This demonstrates that even relatively modest speeds (10 m/s is about 36 km/h or 22 mph) can produce noticeable Doppler shifts when the motion is along the line connecting source and observer. The 0.300% threshold mentioned in the problem represents the approximate limit of human pitch discrimination—most people can detect frequency differences of about 0.3% to 0.5%, while trained musicians can often detect even smaller differences. This problem illustrates why you can hear pitch changes when waving a sound source (like a buzzer or whistle) back and forth, and why the Doppler effect is so prominent in everyday experience with moving sound sources.

Two eagles fly directly toward one another, the first at 15.0 m/s and the second at 20.0 m/s. Both screech, the first one emitting a frequency of 3200 Hz and the second one emitting a frequency of 3800 Hz. What frequencies do they receive if the speed of sound is 330 m/s?

Show Solution

**Strategy**

This problem involves both a moving source and a moving observer, since both eagles are moving and both are emitting and receiving sound. We must use the general Doppler formula: $f_{obs} = f_s\left(\frac{v_w \pm v_{obs}}{v_w \mp v_s}\right)$. Since both eagles are flying toward each other, we use the plus sign for $v_{obs}$ (observer moving toward source) and the minus sign for $v_s$ (source moving toward observer). We need to calculate two separate frequencies: what the first eagle hears from the second, and what the second eagle hears from the first.

**Solution**

For the first eagle hearing the second eagle's screech:

- Observer (first eagle): $v_{obs} = 15.0$ m/s toward the source
- Source (second eagle): $v_s = 20.0$ m/s toward the observer, $f_s = 3800$ Hz

$$f_{obs} = f_s\left(\frac{v_w + v_{obs}}{v_w - v_s}\right) = (3800 \text{ Hz})\left(\frac{330 \text{ m/s} + 15.0 \text{ m/s}}{330 \text{ m/s} - 20.0 \text{ m/s}}\right)$$

$$f_{obs} = (3800 \text{ Hz})\left(\frac{345 \text{ m/s}}{310 \text{ m/s}}\right) = (3800 \text{ Hz})(1.113) = 4230 \text{ Hz} = 4.23 \times 10^3 \text{ Hz}$$

For the second eagle hearing the first eagle's screech:

- Observer (second eagle): $v_{obs} = 20.0$ m/s toward the source
- Source (first eagle): $v_s = 15.0$ m/s toward the observer, $f_s = 3200$ Hz

$$f_{obs} = f_s\left(\frac{v_w + v_{obs}}{v_w - v_s}\right) = (3200 \text{ Hz})\left(\frac{330 \text{ m/s} + 20.0 \text{ m/s}}{330 \text{ m/s} - 15.0 \text{ m/s}}\right)$$

$$f_{obs} = (3200 \text{ Hz})\left(\frac{350 \text{ m/s}}{315 \text{ m/s}}\right) = (3200 \text{ Hz})(1.111) = 3560 \text{ Hz} = 3.56 \times 10^3 \text{ Hz}$$

**Discussion**

Both eagles hear significantly higher frequencies than what is being emitted because they are approaching each other. The first eagle hears a frequency 430 Hz higher (an 11.3% increase), while the second eagle hears a frequency 360 Hz higher (an 11.1% increase). These shifts are quite substantial and demonstrate that when both source and observer are moving toward each other, the Doppler shift is enhanced—the combined effect is greater than either motion alone would produce. This is why head-on collisions between vehicles with sirens produce the most dramatic pitch changes. The fact that the two shifts are similar (both around 11%) despite different speeds reflects the symmetry in their relative motion. In nature, such bidirectional Doppler shifts are important for predator-prey interactions and for communication between animals in flight. The general Doppler formula used here combines both effects multiplicatively: the observer's motion changes the rate at which wave crests are encountered, while the source's motion changes the spacing between crests.

What is the minimum speed at which a source must travel toward you for you to be able to hear that its frequency is Doppler shifted? That is, what speed produces a shift of 0.300% on a day when the speed of sound is 331 m/s?

Show Solution

**Strategy**

We need to find the source speed that produces exactly a 0.300% frequency shift, which represents the threshold of human pitch perception. A 0.300% shift means the observed frequency is 1.00300 times the source frequency (or a shift factor of 1.003). We use the approaching source formula $f_{obs} = f_s\left(\frac{v_w}{v_w - v_s}\right)$ and solve for $v_s$ when $f_{obs}/f_s = 1.003$.

**Solution**

A 0.300% increase means:

$$\frac{f_{obs}}{f_s} = 1 + \frac{0.300}{100} = 1.00300$$

Using the Doppler formula for an approaching source:

$$\frac{f_{obs}}{f_s} = \frac{v_w}{v_w - v_s}$$

Substitute the shift factor:

$$1.00300 = \frac{331 \text{ m/s}}{331 \text{ m/s} - v_s}$$

Cross-multiply:

$$1.00300(331 \text{ m/s} - v_s) = 331 \text{ m/s}$$
$$331.99 \text{ m/s} - 1.00300 v_s = 331 \text{ m/s}$$

Solve for $v_s$:

$$1.00300 v_s = 0.99 \text{ m/s}$$

$v_S$=0.99 m/s1.00300=0.99 m/s

Therefore, the minimum perceptible speed is approximately 1.0 m/s.

**Discussion**

The minimum speed of about 1.0 m/s (3.6 km/h or 2.2 mph) is quite slow—roughly a leisurely walking pace. This demonstrates that the human ear is remarkably sensitive to pitch changes. Even at this slow speed, a trained listener can detect the Doppler shift in the frequency of a sound source. This sensitivity explains why we can easily hear Doppler shifts in everyday situations: a person walking past you while talking on a phone, a cyclist ringing a bell as they pass, or a car driving slowly down a residential street. The 0.300% threshold represents the just-noticeable difference (JND) for pitch perception in most people—trained musicians can often detect even smaller differences, down to about 0.1% to 0.2%. This remarkable auditory acuity evolved as a survival mechanism, helping our ancestors detect the motion of both predators and prey. The calculation shows that perceptible Doppler shifts occur at speeds far below those typically associated with dramatic pitch changes (like emergency vehicles), making the Doppler effect a ubiquitous phenomenon in our acoustic environment.

## Glossary

Doppler effect
: an alteration in the observed frequency of a sound due to motion of either the source or the observer

Doppler shift
: the actual change in frequency due to relative motion of source and observer

sonic boom
: a constructive interference of sound created by an object moving faster than sound

bow wake
: V-shaped disturbance created when the wave source moves faster than the wave propagation speed

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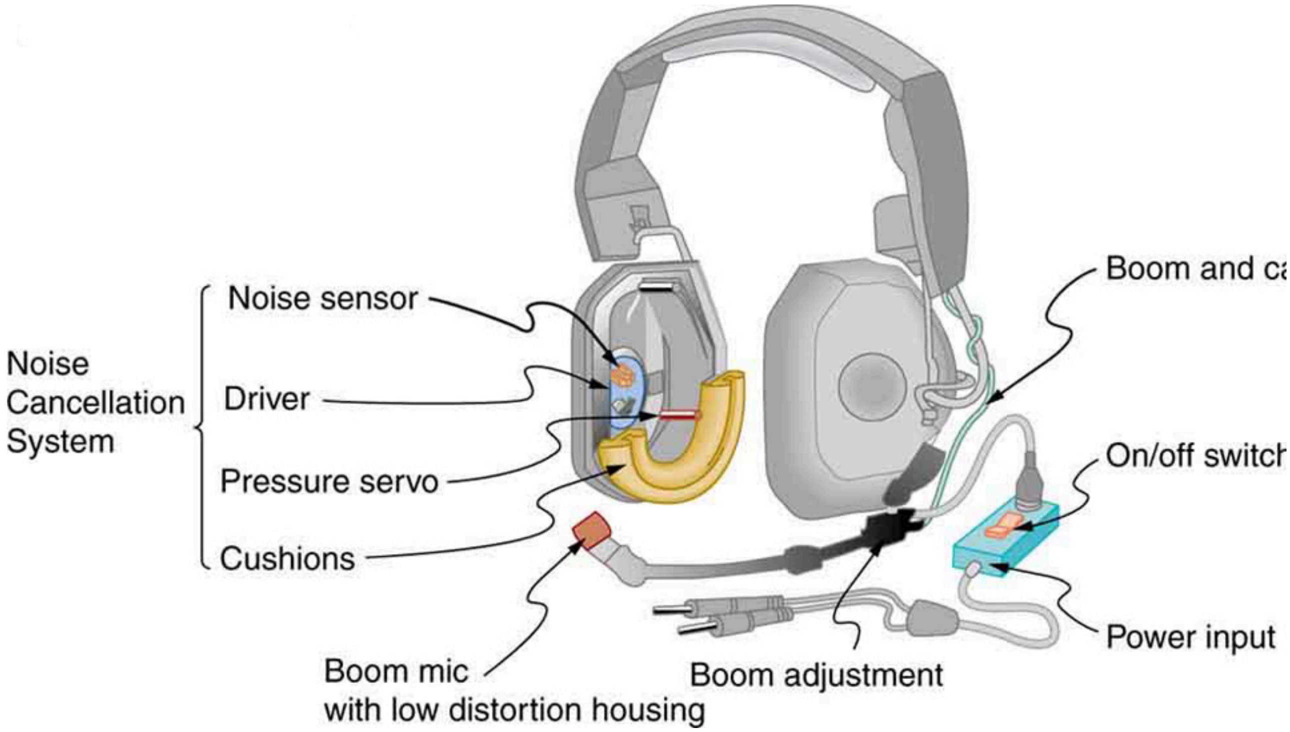
# Sound Interference and Resonance: Standing Waves in Air Columns

- Define antinode, node, fundamental, overtones, and harmonics.
- Identify instances of sound interference in everyday situations.
- Describe how sound interference occurring inside open and closed tubes changes the characteristics of the sound, and how this applies to sounds produced by musical instruments.
- Calculate the length of a tube using sound wave measurements.


Some types of headphones use the phenomena of constructive and destructive interference to cancel out outside noises. (credit: JVC America, Flickr)

Interference is the hallmark of waves, all of which exhibit constructive and destructive interference exactly analogous to that seen for water waves. In fact, one way to prove something “is a wave” is to observe interference effects. So, sound being a wave, we expect it to exhibit interference; we have already mentioned a few such effects, such as the beats from two similar notes played simultaneously.

[Figure 2] shows a clever use of sound interference to cancel noise. Larger-scale applications of active noise reduction by destructive interference are contemplated for entire passenger compartments in commercial aircraft. To obtain destructive interference, a fast electronic analysis is performed, and a second sound is introduced with its maxima and minima exactly reversed from the incoming noise. Sound waves in fluids are pressure waves and consistent with Pascal's principle; pressures from two different sources add and subtract like simple numbers; that is, positive and negative gauge pressures add to a much smaller pressure, producing a lower-intensity sound. Although completely destructive interference is possible only under the simplest conditions, it is possible to reduce noise levels by 30 dB or more using this technique.



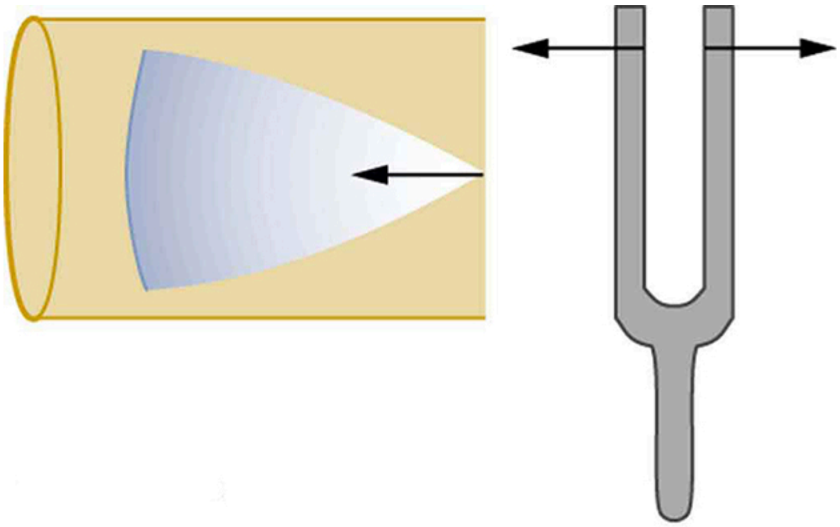

Headphones designed to cancel noise with destructive interference create a sound wave exactly opposite to the incoming sound. These headphones can be more effective than the simple passive attenuation used in most ear protection. Such headphones were used on the record-setting, around the world nonstop flight of the Voyager aircraft to protect the pilots' hearing from engine noise.

Where else can we observe sound interference? All sound resonances, such as in musical instruments, are due to constructive and destructive interference. Only the resonant frequencies interfere constructively to form standing waves, while others interfere destructively and are absent. From the toot made by blowing over a bottle, to the characteristic flavor of a violin's sounding box, to the recognizability of a great singer's voice, resonance and standing waves play a vital role.

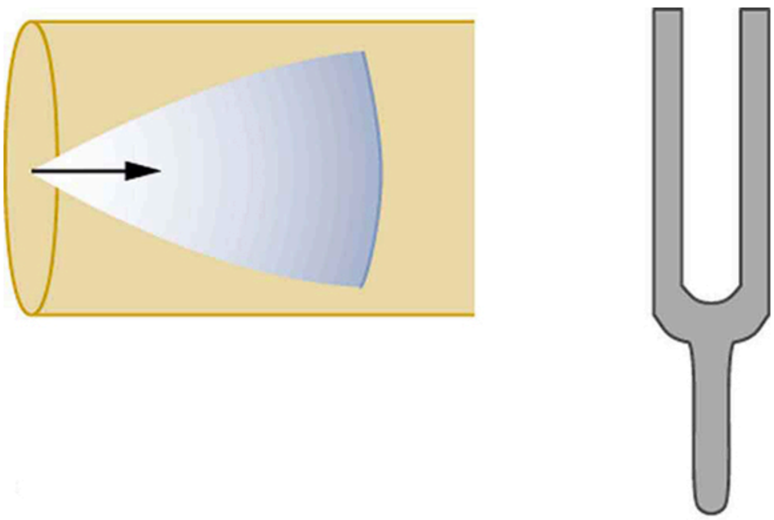
Interference

Interference is such a fundamental aspect of waves that observing interference is proof that something is a wave. The wave nature of light was established by experiments showing interference. Similarly, when electrons scattered from crystals exhibited interference, their wave nature was confirmed to be exactly as predicted by symmetry with certain wave characteristics of light.

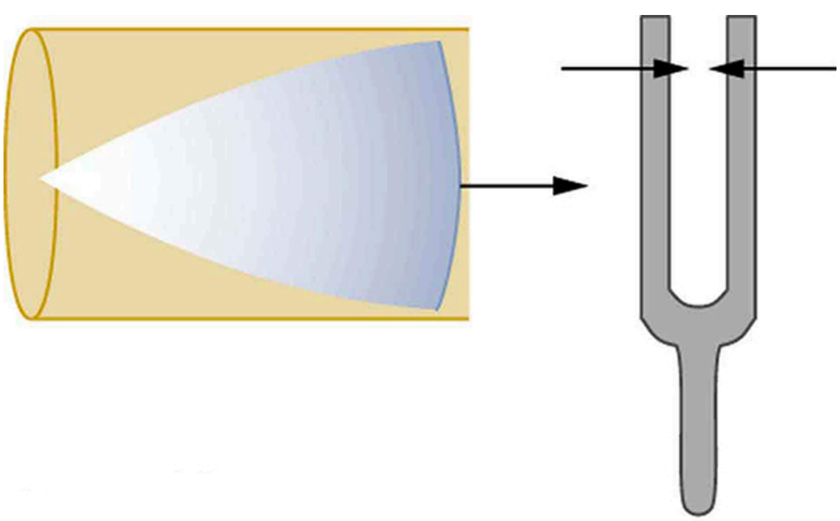
Suppose we hold a tuning fork near the end of a tube that is closed at the other end, as shown in [Figure 3], [Figure 4] , [Figure 5], and [Figure 6]. If the tuning fork has just the right frequency, the air column in the tube resonates loudly, but at most frequencies it vibrates very little. This observation just means that the air column has only certain natural frequencies. The figures show how a resonance at the lowest of these natural frequencies is formed. A disturbance travels down the tube at the speed of sound and bounces off the closed end. If the tube is just the right length, the reflected sound arrives back at the tuning fork exactly half a cycle later, and it interferes constructively with the continuing sound produced by the tuning fork. The incoming and reflected sounds form a standing wave in the tube as shown.



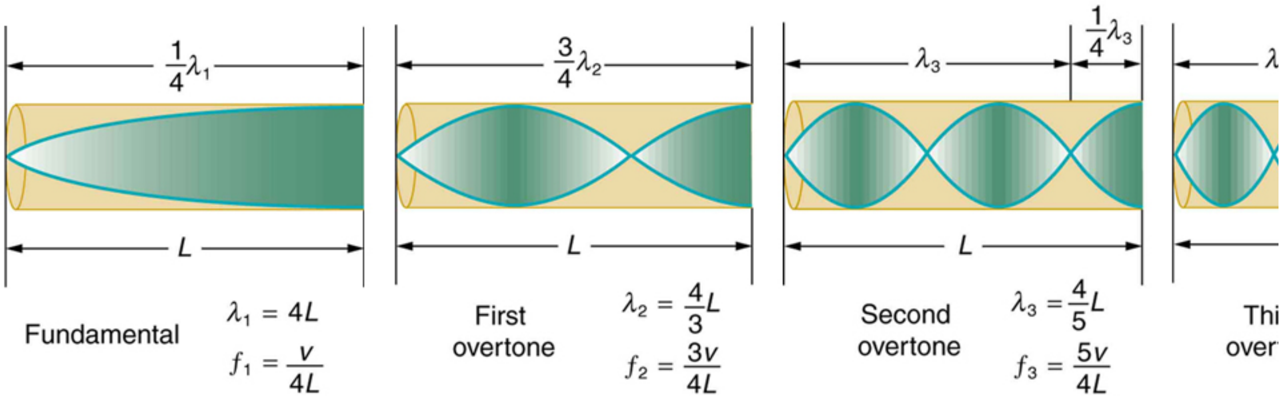
Resonance of air in a tube closed at one end, caused by a tuning fork. A disturbance moves down the tube.



Resonance of air in a tube closed at one end, caused by a tuning fork. The disturbance reflects from the closed end of the tube.

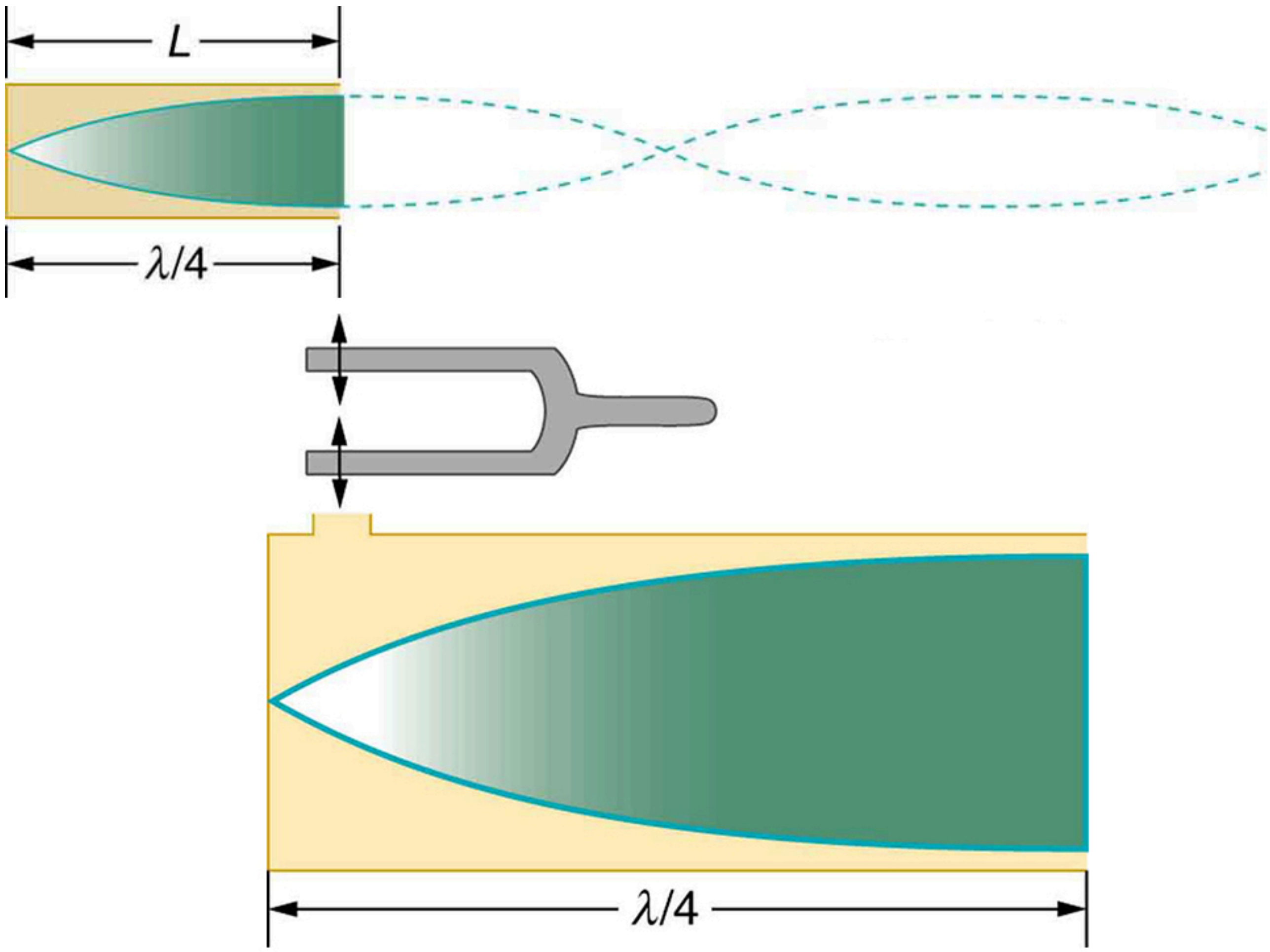


Resonance of air in a tube closed at one end, caused by a tuning fork. If the length of the tube $L$ is just right, the disturbance gets back to the tuning fork half a cycle later and interferes constructively with the continuing sound from the tuning fork. This interference forms a standing wave, and the air column resonates.



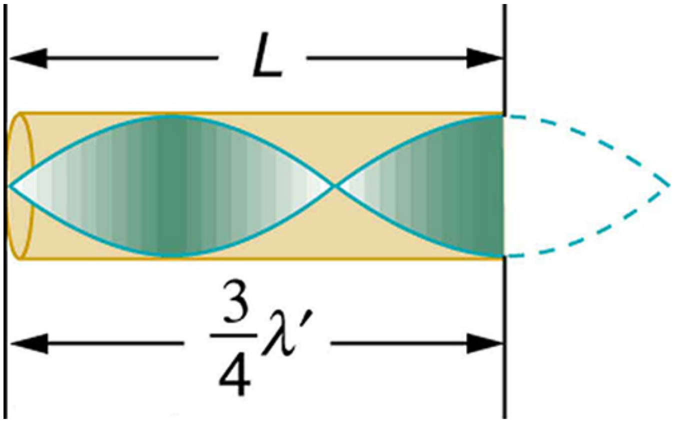

Resonance of air in a tube closed at one end, caused by a tuning fork. A graph of air displacement along the length of the tube shows none at the closed end, where the motion is constrained, and a maximum at the open end. This standing wave has one-fourth of its wavelength in the tube, so that $\lambda =4L$.

The standing wave formed in the tube has its maximum air displacement (an **antinode**) at the open end, where motion is unconstrained, and no displacement (a **node**) at the closed end, where air movement is halted. The distance from a node to an antinode is one-fourth of a wavelength, and this equals the length of the tube; thus, $\lambda =4L$ . This same resonance can be produced by a vibration introduced at or near the closed end of the tube, as shown in [Figure 7]. It is best to consider this a natural vibration of the air column independently of how it is induced.

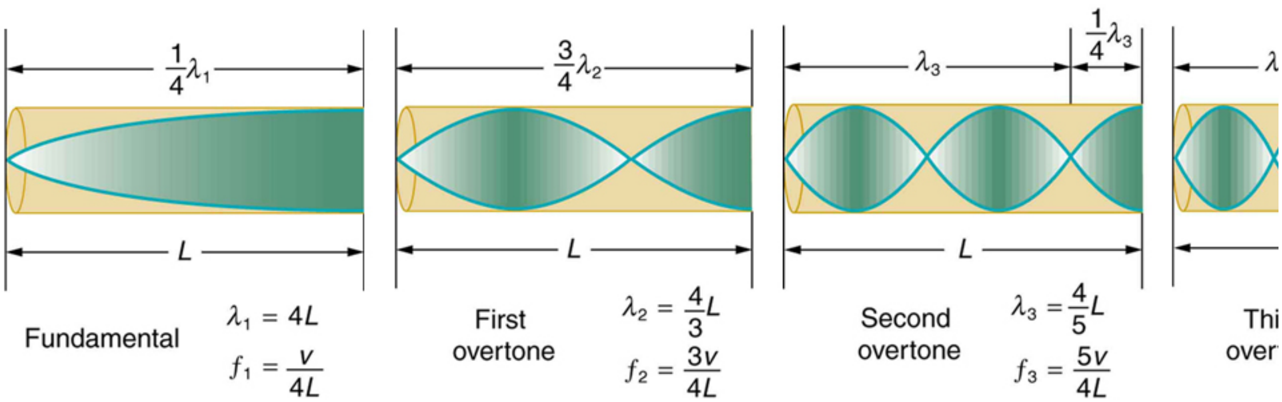



The same standing wave is created in the tube by a vibration introduced near its closed end.

Given that maximum air displacements are possible at the open end and none at the closed end, there are other, shorter wavelengths that can resonate in the tube, such as the one shown in [Figure 8]. Here the standing wave has three-fourths of its wavelength in the tube, or $L =(3/4)\lambda'$ , so that $\lambda' =4L/3$ . Continuing this process reveals a whole series of shorter-wavelength and higher-frequency sounds that resonate in the tube. We use specific terms for the resonances in any system. The lowest resonant frequency is called the **fundamental**, while all higher resonant frequencies are called **overtones**. All resonant frequencies are integral multiples of the fundamental, and they are collectively called **harmonics**. The fundamental is the first harmonic, the first overtone is the second harmonic, and so on. [Figure 9] shows the fundamental and the first three overtones (the first four harmonics) in a tube closed at one end.

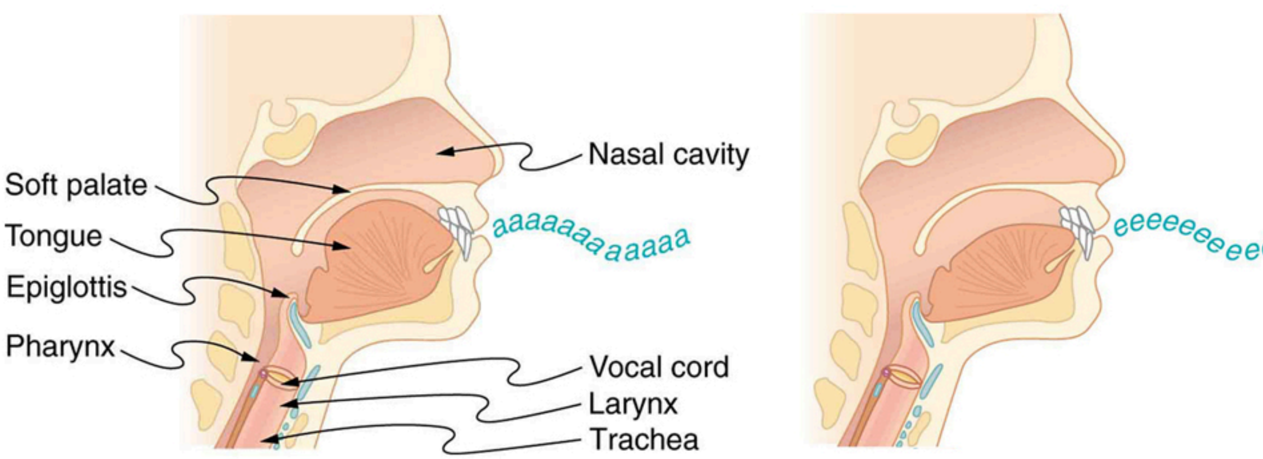



Another resonance for a tube closed at one end. This has maximum air displacements at the open end, and none at the closed end. The wavelength is shorter, with three-fourths ( \lambda^\prime) equaling the length of the tube, so that $\lambda^\prime =4L/3$ . This higher-frequency vibration is the first overtone.




The fundamental and three lowest overtones for a tube closed at one end. All have maximum air displacements at the open end and none at the closed end.

The fundamental and overtones can be present simultaneously in a variety of combinations. For example, middle C on a trumpet has a sound distinctively different from middle C on a clarinet, both instruments being modified versions of a tube closed at one end. The fundamental frequency is the same (and usually the most intense), but the overtones and their mix of intensities are different and subject to shading by the musician. This mix is what gives various musical instruments (and human voices) their distinctive characteristics, whether they have air columns, strings, sounding boxes, or drumheads. In fact, much of our speech is determined by shaping the cavity formed by the throat and mouth and positioning the tongue to adjust the fundamental and combination of overtones. Simple resonant cavities can be made to resonate with the sound of the vowels, for example. (See [Figure 10].) In boys, at puberty, the larynx grows and the shape of the resonant cavity changes giving rise to the difference in predominant frequencies in speech between men and women.




The throat and mouth form an air column closed at one end that resonates in response to vibrations in the voice box. The spectrum of overtones and their intensities vary with mouth shaping and tongue position to form different sounds. The voice box can be replaced with a mechanical vibrator, and understandable speech is still possible. Variations in basic shapes make different voices recognizable.

Now let us look for a pattern in the resonant frequencies for a simple tube that is closed at one end. The fundamental has $\lambda = 4L$ , and frequency is related to wavelength and the speed of sound as given by:

$$v_w = f\lambda.$$

Solving for $f$ in this equation gives

$$f = \frac{v_w}{\lambda} = \frac{v_w}{4L},$$

where $v_w$ is the speed of sound in air. Similarly, the first overtone has $\lambda' = 4L/3$ (see [Figure 9]), so that

$$f' = 3\frac{v_w}{4L} = 3f.$$

Because $f' = 3f$ , we call the first overtone the third harmonic. Continuing this process, we see a pattern that can be generalized in a single expression. The resonant frequencies of a tube closed at one end are

$$f_n = n\frac{v_w}{4L}, n = 1, 3, 5,$$

where $f_1$ is the fundamental, $f_3$ is the first overtone, and so on. It is interesting that the resonant frequencies depend on the speed of sound and, hence, on temperature. This dependence poses a noticeable problem for organs in old unheated cathedrals, and it is also the reason why musicians commonly bring their wind instruments to room temperature before playing them.

Find the Length of a Tube with a 128 Hz Fundamental

(a) What length should a tube closed at one end have on a day when the air temperature, is $22.0\ ^\circ C$ , if its fundamental frequency is to be 128 Hz (C below middle C)?

(b) What is the frequency of its fourth overtone?

**Strategy**

The length $L$ can be found from the relationship in $f_n = n\frac{v_w}{4L}$ , but we will first need to find the speed of sound $v_w$.

**Solution for (a)**

(1) Identify knowns:

- the fundamental frequency is 128 Hz
- the air temperature is $22.0\ ^\circ C$ (2) Use $f_n = n\frac{v_w}{4L}$ to find the fundamental frequency ( $n = 1$ ).

$$f_1 = \frac{v_w}{4L}$$

(3) Solve this equation for length.

$$L = \frac{v_w}{4f_1}$$

(4) Find the speed of sound using $v_w = (331\,\text{m/s})\sqrt{\frac{T}{273\,\text{K}}}$ .

$$v_w = (331\,\text{m/s})\sqrt{\frac{295\,\text{K}}{273\,\text{K}}} = 344\,\text{m/s}$$

(5) Enter the values of the speed of sound and frequency into the expression for $L$.

$$L = \frac{v_w}{4f_1} = \frac{344\,\text{m/s}}{4(128\,\text{Hz})} = 0.672\,\text{m}$$

**Discussion on (a)**

Many wind instruments are modified tubes that have finger holes, valves, and other devices for changing the length of the resonating air column and hence, the frequency of the note played. Horns producing very low frequencies, such as tubas, require tubes so long that they are coiled into loops.

**Solution for (b)**

(1) Identify knowns:

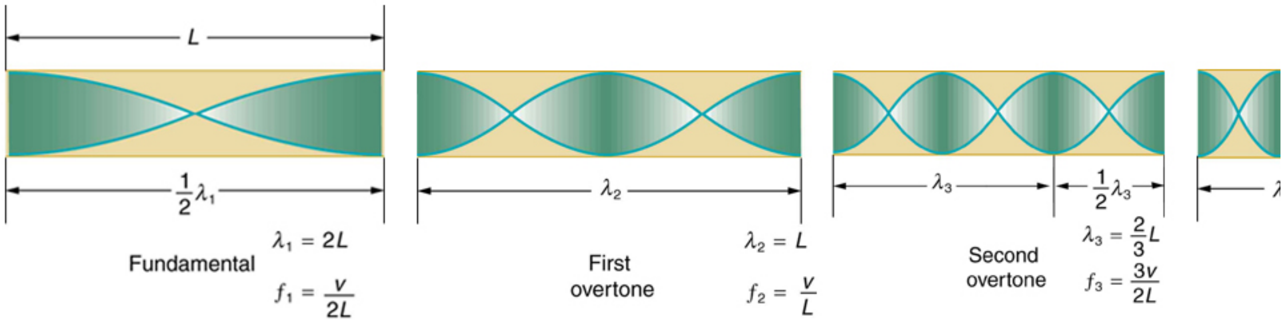
- the first overtone has $n = 3$
- the second overtone has $n = 5$
- the third overtone has $n = 7$
- the fourth overtone has $n = 9$ (2) Enter the value for the fourth overtone into $f_n = n\frac{v_w}{4L}$.

$$f_9 = 9\frac{v_w}{4L} = 9f_1 = 1.15\,\text{kHz}$$

**Discussion on (b)**

Whether this overtone occurs in a simple tube or a musical instrument depends on how it is stimulated to vibrate and the details of its shape. The trombone, for example, does not produce its fundamental frequency and only makes overtones.

Another type of tube is one that is *open* at both ends. Examples are some organ pipes, flutes, and oboes. The resonances of tubes open at both ends can be analyzed in a very similar fashion to those for tubes closed at one end. The air columns in tubes open at both ends have maximum air displacements at both ends, as illustrated in [Figure 11]. Standing waves form as shown.




The resonant frequencies of a tube open at both ends are shown, including the fundamental and the first three overtones. In all cases the maximum air displacements occur at both ends of the tube, giving it different natural frequencies than a tube closed at one end.

Based on the fact that a tube open at both ends has maximum air displacements at both ends, and using [Figure 11] as a guide, we can see that the resonant frequencies of a tube open at both ends are:

$$f_n = n\frac{v_w}{2L}, n = 1, 2, 3...,$$

where $f_1$ is the fundamental, $f_2$ is the first overtone, $f\_3$ is the second overtone, and so on. Note that a tube open at both ends has a fundamental frequency twice what it would have if closed at one end. It also has a different spectrum of overtones than a tube closed at one end. So if you had two tubes with the same fundamental frequency but one was open at both ends and the other was closed at one end, they would sound different when played because they have different overtones. Middle C, for example, would sound richer played on an open tube, because it has even multiples of the fundamental as well as odd. A closed tube has only odd multiples.

Real-World Applications: Resonance in Everyday Systems

Resonance occurs in many different systems, including strings, air columns, and atoms. Resonance is the driven or forced oscillation of a system at its natural frequency. At resonance, energy is transferred rapidly to the oscillating system, and the amplitude of its oscillations grows until the system can no longer be described by Hooke’s law. An example of this is the distorted sound intentionally produced in certain types of rock music.

Wind instruments use resonance in air columns to amplify tones made by lips or vibrating reeds. Other instruments also use air resonance in clever ways to amplify sound. [Figure 12] shows a violin and a guitar, both of which have sounding boxes but with different shapes, resulting in different overtone structures. The vibrating string creates a sound that resonates in the sounding box, greatly amplifying the sound and creating overtones that give the instrument its characteristic flavor. The more complex the shape of the sounding box, the greater its ability to resonate over a wide range of frequencies. The marimba, like the one shown in [Figure 13] uses pots or gourds below the wooden slats to amplify their tones. The resonance of the pot can be adjusted by adding water.

String instruments such as violins and guitars use resonance in their sounding boxes to amplify and enrich the sound created by their vibrating strings. The bridge and supports couple the string vibrations to the sounding boxes and air within. (credits: guitar, Feliciano Guimares, Fotopedia; violin, Steve Snodgrass, Flickr)

Resonance has been used in musical instruments since prehistoric times. This marimba uses gourds as resonance chambers to amplify its sound. (credit: APC Events, Flickr)

We have emphasized sound applications in our discussions of resonance and standing waves, but these ideas apply to any system that has wave characteristics. Vibrating strings, for example, are actually resonating and have fundamentals and overtones similar to those for air columns. More subtle are the resonances in atoms due to the wave character of their electrons. Their orbitals can be viewed as standing waves, which have a fundamental (ground state) and overtones (excited states). It is fascinating that wave characteristics apply to such a wide range of physical systems.

Check Your Understanding

Describe how noise-canceling headphones differ from standard headphones used to block outside sounds.

Show Solution

Regular headphones only block sound waves with a physical barrier. Noise-canceling headphones use destructive interference to reduce the loudness of outside sounds.

Check Your Understanding

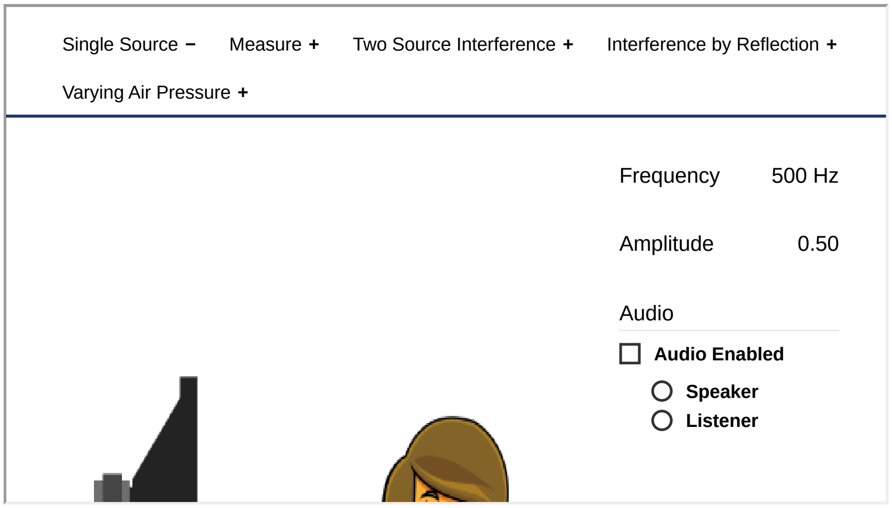
How is it possible to use a standing wave's node and antinode to determine the length of a closed-end tube?

Show Solution

When the tube resonates at its natural frequency, the wave's node is located at the closed end of the tube, and the antinode is located at the open end. The length of the tube is equal to one-fourth of the wavelength of this wave. Thus, if we know the wavelength of the wave, we can determine the length of the tube.

PhET Explorations: Sound

This simulation lets you see sound waves. Adjust the frequency or volume and you can see and hear how the wave changes. Move the listener around and hear what she hears.




## Section Summary

- Sound interference and resonance have the same properties as defined for all waves.
- In air columns, the lowest-frequency resonance is called the fundamental, whereas all higher resonant frequencies are called overtones. Collectively, they are called harmonics.
- The resonant frequencies of a tube closed at one end are:

  $$f_n = n\frac{v_w}{4L},\ n = 1, 3, 5...,$$

$f_1$ is the fundamental and $L$ is the length of the tube.

- The resonant frequencies of a tube open at both ends are:

  $$f_n = n\frac{v_w}{2L},\ n = 1, 2, 3...$$

## Conceptual Questions

How does an unamplified guitar produce sounds so much more intense than those of a plucked string held taut by a simple stick?

Show Solution

A guitar produces a much more intense sound because its body acts as a **sounding box** or resonator. The vibrations of the plucked string are transferred through the bridge to the large surface of the guitar's body. This large surface moves a much greater volume of air than the thin string alone, resulting in a more intense (louder) sound wave. Additionally, the air inside the guitar's body has its own resonant frequencies, which selectively amplify certain harmonics of the string's vibrations, giving the guitar its rich, characteristic timbre. A simple stick lacks this large resonating surface and therefore cannot efficiently transfer the string's vibrational energy to the air.

You are given two wind instruments of identical length. One is open at both ends, whereas the other is closed at one end. Which is able to produce the lowest frequency?

Show Solution

The instrument that is **closed at one end** is able to produce the lowest frequency.

The fundamental frequency of a tube closed at one end is given by $f_1 = \frac{v_w}{4L}$, whereas the fundamental frequency of a tube open at both ends is $f_1 = \frac{v_w}{2L}$. Since the lengths ($L$) and the speed of sound ($v_w$) are the same for both instruments, the tube closed at one end has a fundamental frequency that is half the frequency of the tube open at both ends.

What is the difference between an overtone and a harmonic? Are all harmonics overtones? Are all overtones harmonics?

Show Solution

**Harmonics** are all integer multiples of the fundamental frequency ($f_1, 2f_1, 3f_1, ...$). The fundamental is the first harmonic.

**Overtones** are all resonant frequencies higher than the fundamental. The first overtone is the first frequency above the fundamental, the second overtone is the second frequency above the fundamental, and so on.

- **Are all harmonics overtones?** No. The fundamental frequency is the first harmonic, but it is not an overtone.
- **Are all overtones harmonics?** In many simple instruments, like a tube open at both ends, the overtones are integer multiples of the fundamental, so they are harmonics (e.g., the first overtone is the second harmonic). However, for a tube closed at one end, the overtones are only the *odd* harmonics

(3rd, 5th, etc.). In this case, all overtones are harmonics, but not all harmonics are present as overtones. For more complex instruments (like a drum), the overtones are not simple integer multiples of the fundamental, and in that case, the overtones are not harmonics.

## Problems & Exercises

A "showy" custom-built car has two brass horns that are supposed to produce the same frequency but actually emit 263.8 and 264.5 Hz. What beat frequency is produced?

Show Solution

**Strategy**

When two sound waves of slightly different frequencies are played simultaneously, they interfere to produce a phenomenon called beats. The beat frequency is the absolute value of the difference between the two frequencies. This occurs because the waves alternate between constructive interference (when they are in phase) and destructive interference (when they are out of phase), creating a pulsing or "beating" sound that we can hear.

**Solution**

The beat frequency is given by:

$$f_{\text{beat}}=|f_1-f_2|$$

Substituting the given frequencies:

$$f_{\text{beat}}=|263.8\text{ Hz}-264.5\text{ Hz}|$$
$$f_{\text{beat}}=|-0.7\text{ Hz}|=0.7\text{ Hz}$$

**Discussion**

The beat frequency of 0.7 Hz means that the sound intensity will rise and fall 0.7 times per second, or roughly once every 1.4 seconds. This is easily perceptible to the human ear and would create an unpleasant warbling effect. This is precisely the effect that piano tuners use to tune pianos—they listen for beats between a tuning fork and a piano string, adjusting the string tension until the beats disappear (indicating the frequencies match exactly). In this case, the car's horns are out of tune with each other by a small but audible amount. The beat frequency is quite slow (less than 1 Hz), which makes the pulsing effect very obvious and annoying to listeners.

The final answer is: **The beat frequency produced is 0.7 Hz.**

What beat frequencies will be present: (a) If the musical notes A and C are played together (frequencies of 220 and 264 Hz)? (b) If D and F are played together (frequencies of 297 and 352 Hz)? (c) If all four are played together?

Show Solution

**Strategy**

Beat frequencies occur when two or more sound waves of different frequencies interfere. For any pair of frequencies, the beat frequency is the absolute value of their difference. When more than two frequencies are present, we must consider all possible pairs to find all the beat frequencies that will be produced.

**Solution**

**(a) Notes A and C (220 Hz and 264 Hz)**

$$f_{\text{beat}}=|f_C-f_A|=|264\text{ Hz}-220\text{ Hz}|=44\text{ Hz}$$

**(b) Notes D and F (297 Hz and 352 Hz)**

$$f_{\text{beat}}=|f_F-f_D|=|352\text{ Hz}-297\text{ Hz}|=55\text{ Hz}$$

**(c) All four notes played together**

When all four notes (A, C, D, F) are played simultaneously, we must find the beat frequency for each possible pair. There are $\binom{4}{2}=6$ possible pairs:

1. A and C: $$ 264 - 220 = 44\text{ Hz}$$
2. A and D: $$ 297 - 220 = 77\text{ Hz}$$
3. A and F: $$ 352 - 220 = 132\text{ Hz}$$
4. C and D: $$ 297 - 264 = 33\text{ Hz}$$
5. C and F: $$ 352 - 264 = 88\text{ Hz}$$
6. D and F: $$ 352 - 297 = 55\text{ Hz}$$

The beat frequencies present are: **33 Hz, 44 Hz, 55 Hz, 77 Hz, 88 Hz, and 132 Hz**.

**Discussion**

For parts (a) and (b), the beat frequencies are 44 Hz and 55 Hz, respectively. These beat frequencies are well within the audible range and would be perceived as distinct low-frequency tones rather than a pulsing or warbling effect. When beat frequencies are above about 15-20 Hz, the human ear begins to perceive them as separate tones (a phenomenon called a “difference tone”) rather than amplitude modulation.

In part (c), when all four notes are played together, the sound becomes quite complex with six different beat frequencies. The lowest beat frequency (33 Hz) and the highest (132 Hz) span a wide range. In musical terms, playing these four notes together (A, C, D, F) would create a dissonant chord because several of the beat frequencies fall in the range where they create noticeable roughness. This is why certain musical intervals are considered consonant (pleasant) while others are dissonant (harsh)—the beat frequencies produced play a key role in our perception of harmony.

The final answers are: **(a) 44 Hz; (b) 55 Hz; (c) 33 Hz, 44 Hz, 55 Hz, 77 Hz, 88 Hz, and 132 Hz.**

What beat frequencies result if a piano hammer hits three strings that emit frequencies of 127.8, 128.1, and 128.3 Hz?

Show Solution

**Strategy**

A single piano key often has multiple strings (usually 2 or 3) that are struck simultaneously to produce a richer sound. When these strings are slightly out of tune with each other, they produce beats. With three strings, we have three possible pairs, and each pair will produce its own beat frequency. We calculate the beat frequency for each pair as the absolute value of the difference between the two frequencies.

**Solution**

The three strings have frequencies $f_1 = 127.8\text{ Hz}$, $f_2 = 128.1\text{ Hz}$, and $f_3 = 128.3\text{ Hz}$.

There are three possible pairs:

**Pair 1 and 2:**

$$f_{\text{beat},12} = |f_2 - f_1| = |128.1\text{ Hz} - 127.8\text{ Hz}| = 0.3\text{ Hz}$$

**Pair 2 and 3:**

$$f_{\text{beat},23} = |f_3 - f_2| = |128.3\text{ Hz} - 128.1\text{ Hz}| = 0.2\text{ Hz}$$

**Pair 1 and 3:**

$$f_{\text{beat},13} = |f_3 - f_1| = |128.3\text{ Hz} - 127.8\text{ Hz}| = 0.5\text{ Hz}$$

**Discussion**

The three beat frequencies produced are 0.3 Hz, 0.2 Hz, and 0.5 Hz. These are all quite slow beats (less than one beat per second), which would create a slow pulsing or wavering in the sound intensity. This is actually intentional in piano design! Piano tuners sometimes deliberately tune the multiple strings for a single note slightly differently to create a slow beating effect that gives the piano a warmer, richer tone called “stretch tuning” or to account for slight variations in string properties.

Notice that the beat frequency between strings 1 and 3 (0.5 Hz) equals the sum of the other two beat frequencies (0.3 Hz + 0.2 Hz = 0.5 Hz). This is not a coincidence—it’s because the three frequencies are in order, so the largest separation equals the sum of the two smaller separations.

For reference, this note is close to C below middle C (C3), which has a frequency of 128 Hz on an evenly tempered scale. The slight variations indicate the strings are well-tuned but not perfectly matched.

The final answer is: **The beat frequencies are 0.3 Hz, 0.2 Hz, and 0.5 Hz.**

A piano tuner hears a beat every 2.00 s when listening to a 264.0-Hz tuning fork and a single piano string. What are the two possible frequencies of the string?

Show Solution

**Strategy**

When a tuner hears one beat every 2.00 seconds, this means the beat frequency is $f_{\text{beat}} = 1/(2.00\text{ s}) = 0.50\text{ Hz}$. The beat frequency equals the absolute value of the difference between the two frequencies. Since we know the tuning fork frequency and the beat frequency, the string frequency could be either higher or lower than the fork frequency by the beat frequency amount.

**Solution**

First, convert the beat period to beat frequency:

$$f_{\text{beat}} = \frac{1}{T_{\text{beat}}} = \frac{1}{2.00\text{ s}} = 0.50\text{ Hz}$$

The beat frequency is related to the two frequencies by:

$$f_{\text{beat}} = |f_{\text{string}} - f_{\text{fork}}|$$

Substituting known values:

$$0.50\text{ Hz}=|f_{\text{string}}-264.0\text{ Hz}|$$

This absolute value equation has two solutions:

**Case 1:** The string frequency is higher than the fork:

$$f_{\text{string}}=264.0\text{ Hz}+0.50\text{ Hz}=264.5\text{ Hz}$$

**Case 2:** The string frequency is lower than the fork:

$$f_{\text{string}}=264.0\text{ Hz}-0.50\text{ Hz}=263.5\text{ Hz}$$

**Discussion**

The two possible frequencies are 264.5 Hz and 263.5 Hz. Without additional information, we cannot determine which one is correct just by listening to the beats. The tuner would need to either:

1. Slightly increase the string tension and observe whether the beat frequency increases or decreases. If the beats slow down (decrease in frequency), the string was initially at 264.5 Hz and is now getting closer to 264.0 Hz. If the beats speed up, the string was at 263.5 Hz and is moving further from the target.

2. Use a different reference frequency to determine whether the string is sharp (too high) or flat (too low).

This ambiguity is a fundamental limitation of the beat frequency method—it tells us how far out of tune we are, but not in which direction. The tuning fork frequency of 264.0 Hz corresponds to middle C (C4) on the evenly tempered scale, which is slightly different from the theoretical value used in some contexts (261.6 Hz for C4 at A4 = 440 Hz).

The final answer is: **The two possible frequencies of the string are 263.5 Hz and 264.5 Hz.**

(a) What is the fundamental frequency of a 0.672-m-long tube, open at both ends, on a day when the speed of sound is 344 m/s? (b) What is the frequency of its second harmonic?

Show Solution

**Strategy**

A tube open at both ends has antinodes (maximum air displacement) at both open ends. The fundamental frequency (first harmonic, $n=1$) corresponds to the longest wavelength that fits this boundary condition, which is when the tube length equals one-half wavelength. For a tube open at both ends, the resonant frequencies are given by $f_n=n\frac{v_w}{2L}$, where $n=1,2,3,...$ The second harmonic corresponds to $n=2$.

**Solution**

**(a) Fundamental frequency ($n=1$):**

For a tube open at both ends:

$$f_n=n\frac{v_w}{2L}$$

For the fundamental frequency, $n=1$:

$$f_1=\frac{v_w}{2L}=\frac{344\text{ m/s}}{2(0.672\text{ m})}=\frac{344\text{ m/s}}{1.344\text{ m}}=256\text{ Hz}$$

**(b) Second harmonic ($n=2$):**

$$f_2=2\frac{v_w}{2L}=2f_1=2(256\text{ Hz})=512\text{ Hz}$$

Alternatively, calculating directly:

$$f_2=2\frac{v_w}{2L}=\frac{2(344\text{ m/s})}{2(0.672\text{ m})}=\frac{688\text{ m/s}}{1.344\text{ m}}=512\text{ Hz}$$

**Discussion**

The fundamental frequency of 256 Hz is middle C on the musical scale, which makes sense for an instrument like a flute or organ pipe of this length. Notice that for a tube open at both ends, the second harmonic is exactly twice the fundamental frequency. This is characteristic of open tubes—they produce all integer multiples of the fundamental (1f, 2f, 3f, 4f, …), giving them a richer harmonic structure than tubes closed at one end.

We can verify our answer by checking the wavelength. For the fundamental, $\lambda_1=v_w/f_1=344\text{ m/s}/256\text{ Hz}=1.344\text{ m}=2L$, which confirms that the tube length equals one-half wavelength for the fundamental mode, as expected.

It's interesting to compare this with a tube closed at one end of the same length. Such a tube would have a fundamental frequency of $f=v_w/(4L)=344/(4\times0.672)=128$ Hz, exactly half the frequency of the open tube. This is why instruments can produce different notes by opening or closing tone holes.

The final answers are: **(a) The fundamental frequency is 256 Hz. (b) The second harmonic frequency is 512 Hz.**

If a wind instrument, such as a tuba, has a fundamental frequency of 32.0 Hz, what are its first three overtones? It is closed at one end. (The overtones of a real tuba are more complex than this example, because it is a tapered tube.)

Show Solution

**Strategy**

For a tube closed at one end, the resonant frequencies follow the pattern $f_n = n v_w 4L$ where $n = 1,3,5,7,...$ (only odd integers). The fundamental frequency corresponds to $n = 1$. The first overtone is $n = 3$ (the third harmonic), the second overtone is $n = 5$ (the fifth harmonic), and the third overtone is $n = 7$ (the seventh harmonic). An important distinction: overtones are numbered by how many overtones above the fundamental they are, while harmonics are numbered by the value of $n$.

**Solution**

Given: $f_1 = 32.0\text{ Hz}$ (the fundamental frequency)

For a tube closed at one end, only odd harmonics are present, and each harmonic frequency is:

$$f_n = n f_1 \text{ where } n = 1,3,5,7,...$$

**First overtone (third harmonic, $n = 3$):**

$$f_3 = 3 f_1 = 3(32.0\text{ Hz}) = 96.0\text{ Hz}$$

**Second overtone (fifth harmonic, $n = 5$):**

$$f_5 = 5 f_1 = 5(32.0\text{ Hz}) = 160\text{ Hz}$$

**Third overtone (seventh harmonic, $n = 7$):**

$$f_7 = 7 f_1 = 7(32.0\text{ Hz}) = 224\text{ Hz}$$

**Discussion**

The first three overtones are 96.0 Hz, 160 Hz, and 224 Hz. Notice a crucial characteristic of tubes closed at one end: they only produce odd harmonics (3rd, 5th, 7th, etc.). This is because a node must exist at the closed end and an antinode at the open end, which only allows odd multiples of the quarter-wavelength to fit in the tube.

This is fundamentally different from a tube open at both ends, which would produce all harmonics (2nd, 3rd, 4th, etc.). This difference in harmonic structure is what gives instruments their characteristic "color" or timbre. Clarinets, which approximate tubes closed at one end (the reed acts as a closed end), have a distinctive hollow sound because they lack even harmonics.

The fundamental frequency of 32.0 Hz is very low—it's approximately the note C1, nearly at the bottom of human hearing (which extends down to about 20 Hz). This is appropriate for a tuba, which is one of the lowest-pitched instruments in the orchestra. Real tubas are more complex because they are tapered (conical rather than cylindrical), which modifies the harmonic structure and allows some even harmonics to be present, giving the instrument a richer tone.

The final answer is: **The first three overtones are 96.0 Hz (third harmonic), 160 Hz (fifth harmonic), and 224 Hz (seventh harmonic).**

What are the first three overtones of a bassoon that has a fundamental frequency of 90.0 Hz? It is open at both ends. (The overtones of a real bassoon are more complex than this example, because its double reed makes it act more like a tube closed at one end.)

Show Solution

**Strategy**

For a tube open at both ends, the resonant frequencies are $f_n = n v_w 2L$ where $n = 1,2,3,4,...$ (all positive integers). The fundamental is the first harmonic ($n = 1$), the first overtone is the second harmonic ($n = 2$), the second overtone is the third harmonic ($n = 3$), and the third overtone is the fourth harmonic ($n = 4$). Since all harmonics are present, each overtone frequency is simply an integer multiple of the fundamental.

**Solution**

Given: $f_1 = 90.0\text{ Hz}$ (the fundamental frequency)

For a tube open at both ends, all harmonics are present:

$$f_n = n f_1 \text{ where } n = 1,2,3,4,...$$

**First overtone (second harmonic, $n = 2$):**

$$f_2 = 2f_1 = 2(90.0\text{ Hz}) = 180\text{ Hz}$$

**Second overtone (third harmonic, $n = 3$):**

$$f_3 = 3f_1 = 3(90.0\text{ Hz}) = 270\text{ Hz}$$

**Third overtone (fourth harmonic, $n = 4$):**

$$f_4 = 4f_1 = 4(90.0\text{ Hz}) = 360\text{ Hz}$$

**Discussion**

The first three overtones are 180 Hz, 270 Hz, and 360 Hz. Notice that for a tube open at both ends, all integer harmonics are present (not just odd ones). This produces a much richer harmonic spectrum compared to a tube closed at one end. The overtones form a complete harmonic series: $f_1, 2f_1, 3f_1, 4f_1, \ldots$

This is an important difference in terminology:

- **Harmonics** are numbered by their relationship to the fundamental: 1st harmonic (fundamental), 2nd harmonic, 3rd harmonic, etc.
- **Overtones** are numbered by how many resonances are above the fundamental: 1st overtone = 2nd harmonic, 2nd overtone = 3rd harmonic, etc.
- For open tubes: the nth overtone = the (n+1)th harmonic
- For closed tubes: the nth overtone = the (2n+1)th harmonic (because only odd harmonics exist)

The problem notes that real bassoons act more like tubes closed at one end due to their double reed. If this were the case, the overtones would be 270 Hz, 450 Hz, and 630 Hz (the 3rd, 5th, and 7th harmonics), giving a quite different sound. The presence or absence of even harmonics significantly affects the instrument's timbre—this is why clarinets (closed tube behavior) sound hollow compared to flutes (open tube behavior).

The fundamental frequency of 90.0 Hz is quite low, corresponding roughly to the note F♯2 or G♭2, appropriate for the bassoon's role as a bass instrument in the woodwind family.

The final answer is: **The first three overtones are 180 Hz (second harmonic), 270 Hz (third harmonic), and 360 Hz (fourth harmonic).**

How long must a flute be in order to have a fundamental frequency of 262 Hz (this frequency corresponds to middle C on the evenly tempered chromatic scale) on a day when air temperature is $20.0\ ^{\circ}C$ ? It is open at both ends.

Show Solution

**Strategy**

A flute is modeled as a tube open at both ends. For such a tube, the fundamental frequency is given by $f_1 = \frac{v_w}{2L}$, where $v_w$ is the speed of sound in air and $L$ is the length of the tube. We need to first calculate the speed of sound at the given temperature, then solve for the length $L$. The speed of sound in air depends on temperature according to $v_w = (331\text{ m/s})\sqrt{\frac{T}{273\text{ K}}}$, where $T$ is the absolute temperature in Kelvin.

**Solution**

First, convert the temperature to Kelvin:

$$T = 20.0°\text{C} + 273.15 = 293.15\text{ K} \approx 293\text{ K}$$

Calculate the speed of sound at this temperature:

$$v_w = (331\text{ m/s})\sqrt{\frac{T}{273\text{ K}}} = (331\text{ m/s})\sqrt{\frac{293\text{ K}}{273\text{ K}}}$$
$$v_w = (331\text{ m/s})\sqrt{1.0733} = (331\text{ m/s})(1.0360) = 343\text{ m/s}$$

Now solve for the length using the fundamental frequency formula for a tube open at both ends:

$$f_1 = \frac{v_w}{2L}$$

Rearranging for $L$:

$$L = \frac{v_w}{2f_1} = \frac{343\text{ m/s}}{2(262\text{ Hz})} = \frac{343\text{ m/s}}{524\text{ Hz}} = 0.655\text{ m} = 65.5\text{ cm}$$

**Discussion**

The flute must be 0.655 m or 65.5 cm long. This is a reasonable length for a concert flute, which typically ranges from about 60-67 cm depending on the specific design and whether all tone holes are closed. The frequency of 262 Hz corresponds to middle C (C4) on the evenly tempered chromatic scale.

We can verify this result by checking the wavelength. The wavelength of middle C at this temperature is:

$$\lambda = \frac{v_w}{f} = \frac{343\text{ m/s}}{262\text{ Hz}} = 1.31\text{ m}$$

For a tube open at both ends in its fundamental mode, $L = \lambda/2 = 1.31\text{ m}/2 = 0.655\text{ m}$, which confirms our answer.

In practice, flutes produce different notes by opening and closing tone holes, which effectively changes the length of the resonating air column. When a player opens a tone hole, the air column effectively ends at that hole, shortening the tube and raising the pitch. The calculation here assumes all tone holes are closed, giving the lowest note the instrument can produce.

The final answer is: **The flute must be 0.655 m (or 65.5 cm) long.**

What length should an oboe have to produce a fundamental frequency of 110 Hz on a day when the speed of sound is 343 m/s? It is open at both ends.

Show Solution

**Strategy**

An oboe, modeled as a tube open at both ends, has a fundamental frequency given by $f_1 = \frac{v_w}{2L}$. We are given the fundamental frequency and the speed of sound, so we can solve directly for the length $L$ by rearranging this equation.

**Solution**

For a tube open at both ends, the fundamental frequency is:

$$f_1 = \frac{v_w}{2L}$$

Solving for the length $L$:

$$L = \frac{v_w}{2f_1}$$

Substituting the given values:

$$L = \frac{343 \text{ m/s}}{2(110 \text{ Hz})} = \frac{343 \text{ m/s}}{220 \text{ Hz}} = 1.56 \text{ m}$$

**Discussion**

The oboe must be 1.56 m (or 156 cm) long to produce a fundamental frequency of 110 Hz. This is notably longer than a typical oboe! A standard oboe is actually about 65 cm long, which would produce a fundamental frequency around 264 Hz—much higher than the 110 Hz specified in this problem.

The frequency of 110 Hz corresponds to the musical note A2, which is quite low—two octaves below the standard tuning note A4 (440 Hz). This would be in the range of a bass instrument, not the soprano range where the oboe typically plays.

This discrepancy highlights an important point: the simplified model of a tube open at both ends doesn't perfectly represent real instruments. Real oboes:

1. Have a double reed that acts more like a closed end, making them behave somewhat like a tube closed at one end
2. Have a conical bore (tapered tube) rather than a cylindrical bore, which affects the harmonic structure
3. Produce their characteristic sound through a complex interaction of the reed vibration and air column resonance

If we modeled the oboe as a tube closed at one end (which is actually more accurate), the required length would be $L = v_w/(4f_1) = 343/(4 \times 110) = 0.780$ m or 78 cm—still longer than a real oboe but more reasonable.

Nonetheless, for the idealized open tube model given in the problem, the final answer is: **The oboe should be 1.56 m long.**

What is the length of a tube that has a fundamental frequency of 176 Hz and a first overtone of 352 Hz if the speed of sound is 343 m/s?

Show Solution

**Strategy**

First, we need to determine what type of tube this is by examining the relationship between the fundamental and first overtone frequencies. For a tube open at both ends, the first overtone is twice the fundamental ($f_2 = 2f_1$). For a tube closed at one end, the first overtone is three times the fundamental ($f_3 = 3f_1$). Once we identify the tube type, we can use the appropriate formula to find the length.

**Solution**

Check the ratio of first overtone to fundamental:

$$\frac{f_{\text{first overtone}}}{f_1} = \frac{352 \text{ Hz}}{176 \text{ Hz}} = 2$$

Since the first overtone is exactly twice the fundamental frequency, this must be a **tube open at both ends**. For such a tube, the first overtone is the second harmonic.

For a tube open at both ends, the fundamental frequency is:

$$f_1 = \frac{v_w}{2L}$$

Solving for the length $L$:

$$L = \frac{v_w}{2f_1}$$

Substituting the given values:

$$L = \frac{343 \text{ m/s}}{2(176 \text{ Hz})} = \frac{343 \text{ m/s}}{352 \text{ Hz}} = 0.974 \text{ m} \approx 0.97 \text{ m}$$

**Verification:**

Let's verify this is correct by checking the first overtone frequency:

$$f_2 = 2\frac{v_w}{2L} = \frac{v_w}{L} = \frac{343 \text{ m/s}}{0.974 \text{ m}} = 352 \text{ Hz} \checkmark$$

**Discussion**

The tube is 0.97 m (or 97 cm) long and is open at both ends. The key to solving this problem was recognizing that the 2:1 ratio between the first overtone and fundamental indicates an open tube. If the tube had been closed at one end, the first overtone would have been $3 \times 176 = 528$ Hz (the third harmonic), not 352 Hz.

This type of reasoning—using the harmonic relationships to determine the tube configuration—is important in acoustic analysis. The harmonic series tells us a lot about the physical structure creating the sound:

- If overtones are all integer multiples (2f, 3f, 4f, …), the tube is open at both ends
- If overtones are only odd multiples (3f, 5f, 7f, …), the tube is closed at one end

The fundamental frequency of 176 Hz is approximately the note F3, and a tube of this length could represent an organ pipe or similar instrument. The length of 0.97 m is reasonable for producing sound in this frequency range.

The final answer is: **The tube is 0.97 m long and is open at both ends.**

(a) Find the length of an organ pipe closed at one end that produces a fundamental frequency of 256 Hz when air temperature is $18.0\ ^\circ C$. (b) What is its fundamental frequency at $25.0\ ^\circ C$?

Show Solution

**Strategy**

For part (a), we use the formula for a tube closed at one end: $f_1 = \frac{v_w}{4L}$. We first need to calculate the speed of sound at 18.0°C, then solve for the length. For part (b), we calculate the speed of sound at the new temperature and find the new fundamental frequency. Since the pipe length remains constant, we can also use the ratio of speeds to find the new frequency more directly.

**Solution**

**(a) Finding the length at 18.0°C:**

First, convert temperature to Kelvin:

$$T_1 = 18.0°\text{C} + 273.15 = 291.15 \text{ K} \approx 291 \text{ K}$$

Calculate the speed of sound at 18.0°C:

$$v_{w,1} = (331 \text{ m/s})\sqrt{\frac{T_1}{273 \text{ K}}} = (331 \text{ m/s})\sqrt{\frac{291 \text{ K}}{273 \text{ K}}}$$
$$v_{w,1} = (331 \text{ m/s})\sqrt{1.0659} = (331 \text{ m/s})(1.0324) = 342 \text{ m/s}$$

For a tube closed at one end:

$$f_1 = \frac{v_{w,1}}{4L}$$

Solving for $L$:

$$L = \frac{v_{w,1}}{4f_1} = \frac{342 \text{ m/s}}{4(256 \text{ Hz})} = \frac{342 \text{ m/s}}{1024 \text{ Hz}} = 0.334 \text{ m} = 33.4 \text{ cm}$$

**(b) Finding the frequency at 25.0°C:**

Convert the new temperature to Kelvin:

$$T_2 = 25.0°\text{C} + 273.15 = 298.15 \text{ K} \approx 298 \text{ K}$$

Calculate the speed of sound at 25.0°C:

$$v_{w,2} = (331 \text{ m/s})\sqrt{\frac{298 \text{ K}}{273 \text{ K}}} = (331 \text{ m/s})\sqrt{1.0916}$$
$$v_{w,2} = (331 \text{ m/s})(1.0448) = 346 \text{ m/s}$$

The new fundamental frequency is:

$$f_2 = \frac{v_{w,2}}{4L} = \frac{346 \text{ m/s}}{4(0.334 \text{ m})} = \frac{346 \text{ m/s}}{1.336 \text{ m}} = 259 \text{ Hz}$$

**Discussion**

The organ pipe must be 0.334 m (33.4 cm) long. When the temperature increases from 18.0°C to 25.0°C, the fundamental frequency increases from 256 Hz to 259 Hz—a change of 3 Hz.

This temperature dependence of frequency is a significant concern for pipe organs in churches and concert halls. The frequency of 256 Hz corresponds to middle C, and a 3 Hz change represents about a 1.2% increase in frequency. This is noticeable to trained musicians, as it amounts to roughly 20 cents (where 100 cents equals one semitone), which is definitely audible.

The physical reason for this change is that the speed of sound increases with temperature because warmer air molecules move faster. Since the pipe length is fixed, higher sound speed means shorter wavelength and thus higher frequency: $f = v_w/(4L)$.

We can verify the reasonableness of the frequency change using the ratio:

$$\frac{f_2}{f_1} = \frac{v_{w,2}}{v_{w,1}} = \frac{346}{342} = 1.012$$

So $f_2 = 1.012 \times 256 \text{ Hz} = 259 \text{ Hz}$ ✓

This is why organs in old, unheated cathedrals can go significantly out of tune when temperatures fluctuate. Musicians warming up their wind instruments before playing are also addressing this temperature dependence—they want the instrument at a stable, known temperature for consistent tuning.

The final answers are: **(a) The organ pipe must be 0.334 m long. (b) The fundamental frequency at 25.0°C is 259 Hz.**

By what fraction will the frequencies produced by a wind instrument change when air temperature goes from $10.0\ ^{\circ}C$ to $30.0\ ^{\circ}C$ ? That is, find the ratio of the frequencies at those temperatures.

Show Solution

**Strategy**

For any wind instrument (whether open or closed), the resonant frequencies are proportional to the speed of sound: $f \propto v_w$. Since the instrument's physical length doesn't change with temperature, the ratio of frequencies at two different temperatures equals the ratio of sound speeds at those temperatures. We can use $v_w = (331 \text{ m/s})\sqrt{T/273 \text{ K}}$ to find this ratio.

**Solution**

Convert both temperatures to Kelvin:

$$T_1 = 10.0°\text{C} + 273.15 = 283.15 \text{ K} \approx 283 \text{ K}$$
$$T_2 = 30.0°\text{C} + 273.15 = 303.15 \text{ K} \approx 303 \text{ K}$$

Calculate the speed of sound at each temperature:

At $T_1 = 283$ K:

$$v_{w,1} = (331 \text{ m/s})\sqrt{\frac{283 \text{ K}}{273 \text{ K}}} = (331 \text{ m/s})\sqrt{1.0366} = (331 \text{ m/s})(1.0181) = 337 \text{ m/s}$$

At $T_2 = 303$ K:

$$v_{w,2} = (331 \text{ m/s})\sqrt{\frac{303 \text{ K}}{273 \text{ K}}} = (331 \text{ m/s})\sqrt{1.1099} = (331 \text{ m/s})(1.0536) = 349 \text{ m/s}$$

The ratio of frequencies is:

$$\frac{f_2}{f_1} = \frac{v_{w,2}}{v_{w,1}} = \frac{349 \text{ m/s}}{337 \text{ m/s}} = 1.036$$

Alternatively, we can calculate this ratio directly from temperatures:

$$\frac{f_2}{f_1} = \frac{v_{w,2}}{v_{w,1}} = \sqrt{\frac{T_2}{T_1}} = \sqrt{\frac{303 \text{ K}}{283 \text{ K}}} = \sqrt{1.0707} = 1.035$$

The fractional change is:

$$\frac{\Delta f}{f_1} = \frac{f_2 - f_1}{f_1} = \frac{f_2}{f_1} - 1 = 1.036 - 1 = 0.036 = 3.6\%$$

**Discussion**

The frequencies increase by a factor of approximately 1.036, or about 3.6%, when the temperature increases from 10.0°C to 30.0°C. This is a significant change musically—it corresponds to about 60 cents (where 100 cents equals one semitone), which is quite noticeable.

This result is independent of whether the instrument is open or closed at the ends, and independent of which harmonic is being played. All frequencies shift by the same proportion because they all depend on $v_w/L$, and only $v_w$ changes with temperature.

The key physical insight is that the ratio of frequencies depends only on the ratio of absolute temperatures:

$$\frac{f_2}{f_1} = \sqrt{\frac{T_2}{T_1}}$$

This square root relationship comes from the fact that sound speed is proportional to $\sqrt{T}$ (because the average molecular speed in a gas is proportional to $\sqrt{T}$, and sound waves are propagated by molecular collisions).

This is why orchestras and bands "warm up" their instruments before performing—bringing the instrument to room temperature ensures consistent tuning. A cold instrument brought in from outside could be noticeably out of tune. Similarly, vigorous playing can warm an instrument through the player's breath, causing it to drift sharp during a performance.

The final answer is: **The ratio of frequencies is $f_2/f_1 = 1.036$, representing a 3.6% increase in frequency.**

The ear canal resonates like a tube closed at one end. (See [Figure 5].) If ear canals range in length from 1.80 to 2.60 cm in an average population, what is the range of fundamental resonant frequencies? Take air temperature to be $37.0\ ^{\circ}C$, which is the same as body temperature. How does this result correlate with the intensity versus frequency graph ([Figure 3]) of the human ear?

Show Solution

**Strategy**

The ear canal acts as a tube closed at one end (closed by the eardrum), so its fundamental frequency is $f_1 = \frac{v_w}{4L}$. We need to calculate the speed of sound at body temperature (37.0°C), then find the fundamental frequencies for both the shortest (1.80 cm) and longest (2.60 cm) ear canals. The shortest canal will have the highest frequency, and the longest will have the lowest.

**Solution**

First, convert the temperature to Kelvin and find the speed of sound:

$$T = 37.0°\text{C} + 273.15 = 310.15\text{ K} \approx 310\text{ K}$$
$$v_w = (331\text{ m/s})\sqrt{\frac{T}{273\text{ K}}} = (331\text{ m/s})\sqrt{\frac{310\text{ K}}{273\text{ K}}}$$
$$v_w = (331\text{ m/s})\sqrt{1.1355} = (331\text{ m/s})(1.0656) = 353\text{ m/s}$$

Convert the ear canal lengths to meters:

- Shortest: $L_{\text{short}} = 1.80\text{ cm} = 0.0180\text{ m}$
- Longest: $L_{\text{long}} = 2.60\text{ cm} = 0.0260\text{ m}$

For the **shortest ear canal** (highest frequency):

$$f_{\text{max}} = \frac{v_w}{4L_{\text{short}}} = \frac{353\text{ m/s}}{4(0.0180\text{ m})} = \frac{353\text{ m/s}}{0.0720\text{ m}} = 4903\text{ Hz} \approx 4.90\text{ kHz}$$

For the **longest ear canal** (lowest frequency):

$$f_{\text{min}} = \frac{v_w}{4L_{\text{long}}} = \frac{353\text{ m/s}}{4(0.0260\text{ m})} = \frac{353\text{ m/s}}{0.1040\text{ m}} = 3394\text{ Hz} \approx 3.39\text{ kHz}$$

**Discussion**

The range of fundamental resonant frequencies for human ear canals is approximately **3.39 kHz to 4.90 kHz**. This is a crucial result for understanding human hearing!

The human ear is most sensitive to frequencies in the range of 2-5 kHz, with peak sensitivity around 3-4 kHz. The ear canal resonance falls right in this range, which is not a coincidence. The resonance of the ear canal acts as a natural amplifier, boosting sounds in this frequency range before they reach the eardrum. This makes evolutionary sense because:

1. Many important sounds for human communication and survival fall in this range
2. Human speech has significant energy content in the 2-4 kHz range, particularly for consonants that are crucial for understanding speech
3. Warning sounds (like a baby's cry or alarm calls) often have frequencies in this range

The ear canal essentially acts as an acoustic resonator that enhances our sensitivity to frequencies that are most important for survival and communication. Without this resonance, we would be less sensitive to these crucial frequencies.

The variation in ear canal length (1.80 to 2.60 cm) accounts for differences between individuals—children have shorter ear canals and thus higher resonant frequencies, while adults have longer canals with lower resonant frequencies. This is one reason why sound perception can vary slightly between individuals.

The final answer is: **The range of fundamental resonant frequencies is 3.39 kHz to 4.90 kHz, which correlates well with the peak sensitivity range of human hearing.**

Calculate the first overtone in an ear canal, which resonates like a 2.40-cm-long tube closed at one end, by taking air temperature to be $37.0\ ^{\circ}C$. Is the ear particularly sensitive to such a frequency? (The resonances of the ear canal are complicated by its nonuniform shape, which we shall ignore.)

Show Solution

**Strategy**

For a tube closed at one end, the first overtone is the third harmonic ($n=3$). The resonant frequencies follow $f_n = n\frac{v_w}{4L}$ where $n=1,3,5,...$ We first need to calculate the speed of sound at body temperature (37.0°C), then use $n=3$ to find the first overtone frequency.

**Solution**

Convert temperature to Kelvin and calculate the speed of sound:

$$T = 37.0°\text{C} + 273.15 = 310.15\text{ K} \approx 310\text{ K}$$
$$v_w = (331\text{ m/s})\sqrt{\frac{310\text{ K}}{273\text{ K}}} = (331\text{ m/s})\sqrt{1.1355} = (331\text{ m/s})(1.0656) = 353\text{ m/s}$$

Convert the ear canal length to meters:

$$L = 2.40\text{ cm} = 0.0240\text{ m}$$

For a tube closed at one end, the first overtone corresponds to $n=3$:

$$f_3 = \frac{3v_w}{4L} = \frac{3(353\text{ m/s})}{4(0.0240\text{ m})} = \frac{1059\text{ m/s}}{0.0960\text{ m}} = 11{,}031\text{ Hz} \approx 11.0\text{ kHz}$$

Alternatively, we could first find the fundamental and multiply by 3:

$$f_1 = \frac{v_w}{4L} = \frac{353\text{ m/s}}{4(0.0240\text{ m})} = \frac{353\text{ m/s}}{0.0960\text{ m}} = 3677\text{ Hz} \approx 3.68\text{ kHz}$$
$$f_3 = 3f_1 = 3(3.68\text{ kHz}) = 11.0\text{ kHz}$$

**Discussion**

The first overtone of this ear canal is approximately 11.0 kHz. This frequency is well within the audible range (20 Hz to 20 kHz for young adults), but is at the upper end of the frequency range where the human ear is most sensitive.

Human hearing sensitivity drops off significantly above about 8-10 kHz, so while we can hear 11 kHz, the ear is not particularly sensitive to this frequency. The ear is most sensitive to frequencies in the 2-5 kHz range (which includes the fundamental frequency of 3.68 kHz for this ear canal), with sensitivity decreasing for both lower and higher frequencies.

The fundamental resonance (3.68 kHz) falls right in the peak sensitivity range of human hearing, which is why it has a significant effect on our hearing. The first overtone at 11.0 kHz, while audible, contributes much less to the ear's frequency response because:

1. The ear's sensitivity is much lower at 11 kHz than at 3.68 kHz
2. The amplitude of the first overtone is generally weaker than the fundamental
3. Many adults lose sensitivity to frequencies above 10 kHz with age

This is why the fundamental resonance of the ear canal (around 3-4 kHz) is the dominant factor in the ear's frequency response, while higher overtones play a much smaller role. The ear canal primarily amplifies sounds around 3-4 kHz due to its fundamental resonance.

The final answer is: **The first overtone is 11.0 kHz. The ear is not particularly sensitive to this frequency, as human hearing sensitivity has already declined significantly by 11 kHz compared to the peak sensitivity around 3-4 kHz.**

A crude approximation of voice production is to consider the breathing passages and mouth to be a resonating tube closed at one end. (See [Figure 10].) (a) What is the fundamental frequency if the tube is 0.240-m long, by taking air temperature to be $37.0\ ^{\circ}C$ ? (b) What would this frequency become if the person replaced the air with helium? Assume the same temperature dependence for helium as for air.

Show Solution

**Strategy**

The vocal tract can be approximated as a tube closed at one end (closed at the vocal cords, open at the mouth), with fundamental frequency $f_1 = \frac{v_w}{4L}$. For part (a), we calculate the speed of sound in air at body temperature. For part (b), we need to account for helium's different properties. The speed of sound in helium at a given temperature is approximately 2.96 times faster than in air due to helium's lower molecular mass.

**Solution**

**(a) Fundamental frequency with air:**

Convert temperature to Kelvin:

$$T = 37.0°\text{C} + 273.15 = 310.15\text{ K} \approx 310\text{ K}$$

Calculate speed of sound in air at 37.0°C:

$$v_{w,\text{air}} = (331\text{ m/s})\sqrt{\frac{310\text{ K}}{273\text{ K}}} = (331\text{ m/s})(1.0656) = 353\text{ m/s}$$

Calculate the fundamental frequency:

$$f_{1,\text{air}} = \frac{v_{w,\text{air}}}{4L} = \frac{353\text{ m/s}}{4(0.240\text{ m})} = \frac{353\text{ m/s}}{0.960\text{ m}} = 368\text{ Hz} \approx 367\text{ Hz}$$

**(b) Fundamental frequency with helium:**

The speed of sound in helium at the same temperature is approximately:

$$v_{w,\text{helium}} = 2.96 \times v_{w,\text{air}} = 2.96 \times 353\text{ m/s} = 1045\text{ m/s}$$

The fundamental frequency with helium:

$$f_{1,\text{helium}} = \frac{v_{w,\text{helium}}}{4L} = \frac{1045\text{ m/s}}{4(0.240\text{ m})} = \frac{1045\text{ m/s}}{0.960\text{ m}} = 1089\text{ Hz} \approx 1.09\text{ kHz}$$

Alternatively, we can use the ratio:

$$f_{1,\text{helium}} = 2.96 \times f_{1,\text{air}} = 2.96 \times 367\text{ Hz} = 1086\text{ Hz} \approx 1.07\text{ kHz}$$

**Discussion**

With air, the fundamental frequency is 367 Hz, which is reasonably close to typical human speech frequencies. The fundamental frequency of the human voice typically ranges from about 85 Hz (bass male voice) to 255 Hz (soprano female voice), so 367 Hz is somewhat high but in a plausible range for certain vowel formants or a high-pitched voice.

With helium, the fundamental frequency increases to approximately 1.07 kHz—nearly three times higher than with air. This is the famous “Donald Duck” or “chipmunk” effect! When people inhale helium and speak, their voices sound much higher-pitched because:

1. Sound travels much faster in helium (about 3 times faster) due to helium’s lower molecular mass (4 g/mol for He vs. 29 g/mol for air)
2. The vocal tract length doesn’t change, so $f = v/(4L)$ increases proportionally with sound speed
3. All resonant frequencies shift upward by the same factor

It’s important to note that the vocal cords themselves vibrate at the same frequency regardless of the gas—what changes is the resonant frequencies of the vocal tract. The vocal tract acts as a filter, and its resonances (called formants in speech science) shift dramatically with helium, giving the characteristic high-pitched sound.

This simplified model treats the vocal tract as a uniform tube, but real vocal tracts have varying cross-sections that create multiple resonances. Nevertheless, the basic physics of why helium raises pitch is correctly captured: higher sound speed leads to higher resonant frequencies for a fixed geometry.

The final answers are: **(a) The fundamental frequency with air is 367 Hz. (b) The fundamental frequency with helium is 1.07 kHz.**

(a) Students in a physics lab are asked to find the length of an air column in a tube closed at one end that has a fundamental frequency of 256 Hz. They hold the tube vertically and fill it with water to the top, then lower the water while a 256-Hz tuning fork is rung and listen for the first resonance. What is the air temperature if the resonance occurs for a length of 0.336 m? (b) At what length will they observe the second resonance (first overtone)?

Show Solution

**Strategy**

This is a classic resonance tube experiment. The water surface acts as the closed end, and the top of the tube is open. For part (a), the first resonance occurs at the fundamental frequency where $L = \lambda/4$. We can use $f_1 = \frac{v_w}{4L}$ to find the speed of sound, then determine the temperature from $v_w = (331\text{ m/s})\sqrt{T/273\text{ K}}$. For part (b), the second resonance (first overtone) occurs at the third harmonic where $L = 3\lambda/4$.

**Solution**

**(a) Finding the air temperature:**

For a tube closed at one end, the fundamental frequency is:

$$f_1 = \frac{v_w}{4L}$$

Solve for the speed of sound:

$$v_w = 4Lf_1 = 4(0.336\text{ m})(256\text{ Hz}) = 344\text{ m/s}$$

Now use the temperature dependence of sound speed to find the temperature:

$$v_w = (331\text{ m/s})\sqrt{\frac{T}{273\text{ K}}}$$

Solving for $T$:

$$\frac{v_w}{331\text{ m/s}} = \sqrt{\frac{T}{273\text{ K}}}$$

$$\left(\frac{v_w}{331\text{ m/s}}\right)^2 = \frac{T}{273\text{ K}}$$

$$T = 273\text{ K} \times \left(\frac{344\text{ m/s}}{331\text{ m/s}}\right)^2 = 273\text{ K} \times (1.0393)^2$$
$$T = 273\text{ K} \times 1.0802 = 295\text{ K}$$

Convert to Celsius:

$$T = 295\text{ K} - 273 = 22°\text{C}$$

**(b) Finding the length for the first overtone:**

For a tube closed at one end, the first overtone is the third harmonic ($n = 3$). The relationship between successive resonances is:

$$L_3 = 3L_1$$

More precisely, for the first overtone:

$$L_3 = \frac{3\lambda}{4} = 3 \times \frac{\lambda}{4} = 3L_1 = 3(0.336\text{ m}) = 1.008\text{ m} \approx 1.01\text{ m}$$

We can verify this using the formula:

$$f_3 = \frac{3v_w}{4L_3}$$

Since $f_3 = f_1 = 256\text{ Hz}$ (same tuning fork):

$$L_3 = \frac{3v_w}{4f_1} = \frac{3(344\text{ m/s})}{4(256\text{ Hz})} = \frac{1032\text{ m/s}}{1024\text{ Hz}} = 1.01\text{ m}$$

**Discussion**

The air temperature is 22°C, which is a comfortable room temperature and typical for a physics laboratory. This temperature determination works because the speed of sound is strongly temperature-dependent, and we can back-calculate the temperature from measured resonance data.

The second resonance occurs at a length of 1.01 m, which is exactly three times the first resonance length (0.336 m × 3 = 1.008 m). This demonstrates a key feature of tubes closed at one end: the resonances occur at lengths that are odd multiples of the fundamental length (L, 3L, 5L, 7L, …).

This experiment is commonly used in introductory physics labs because:

1. It's a direct, hands-on way to measure the speed of sound
2. It clearly demonstrates standing wave resonance
3. Students can actually hear the resonance—there's a distinct increase in loudness when the water level is at a resonant position
4. It illustrates the difference between closed and open tubes

In practice, students would need to account for an "end correction"—the antinode doesn't form exactly at the tube's open end but slightly beyond it (typically about 0.3 times the tube diameter). This makes actual measured lengths slightly shorter than the theoretical values, but for introductory purposes, this correction is often neglected.

The final answers are: **(a) The air temperature is 22°C. (b) The second resonance (first overtone) occurs at a length of 1.01 m.**

What frequencies will a 1.80-m-long tube produce in the audible range at $20.0\ °C$ if: (a) The tube is closed at one end? (b) It is open at both ends?

Show Solution

**Strategy**

The audible range for humans is approximately 20 Hz to 20,000 Hz (20 kHz). For a tube closed at one end, resonant frequencies are $f_n = \frac{nv_w}{4L}$ where $n = 1, 3, 5, ...$ (odd integers only). For a tube open at both ends, $f_n = \frac{nv_w}{2L}$ where $n = 1, 2, 3, ...$ (all positive integers). We first calculate the speed of sound at 20.0°C, then determine which harmonics fall within the audible range.

**Solution**

Convert temperature to Kelvin and calculate the speed of sound:

$$T = 20.0°\text{C} + 273.15 = 293.15\text{ K} \approx 293\text{ K}$$
$$v_w = (331\text{ m/s})\sqrt{\frac{293\text{ K}}{273\text{ K}}} = (331\text{ m/s})\sqrt{1.0733} = (331\text{ m/s})(1.0360) = 343\text{ m/s}$$

**(a) Tube closed at one end:**

The resonant frequencies are:

$$f_n = \frac{nv_w}{4L} = n\frac{343\text{ m/s}}{4(1.80\text{ m})} = n\frac{343\text{ m/s}}{7.20\text{ m}} = n(47.6\text{ Hz})$$

where $n = 1, 3, 5, 7, 9, ...$ (odd integers only).

Find the maximum value of $n$:

$$n_{\text{max}} = \frac{20{,}000\text{ Hz}}{47.6\text{ Hz}} = 420.2$$

Since $n$ must be odd, the largest odd value is $n = 419$.

Therefore, the frequencies are:

$$f_n = n(47.6\text{ Hz}), n = 1,3,5,7,...,419$$

The first few frequencies are:

- $n=1$: $f_1=47.6$ Hz
- $n=3$: $f_3=143$ Hz
- $n=5$: $f_5=238$ Hz
- $n=7$: $f_7=333$ Hz
- …
- $n=419$: $f_{419}=19{,}944$ Hz $\approx 19.9$ kHz

**(b) Tube open at both ends:**

The resonant frequencies are:

$$f_n = n\frac{v_w}{2L} = n\frac{343\text{ m/s}}{2(1.80\text{ m})} = n\frac{343\text{ m/s}}{3.60\text{ m}} = n(95.3\text{ Hz})$$

where $n=1,2,3,4,5,...$ (all positive integers).

Find the maximum value of $n$:

$$n_{\text{max}} = \frac{20{,}000\text{ Hz}}{95.3\text{ Hz}} = 209.9 \approx 210$$

Therefore, the frequencies are:

$$f_n = n(95.3\text{ Hz}), n = 1,2,3,4,...,210$$

The first few frequencies are:

- $n=1$: $f_1=95.3$ Hz
- $n=2$: $f_2=191$ Hz
- $n=3$: $f_3=286$ Hz
- $n=4$: $f_4=381$ Hz
- …
- $n=210$: $f_{210}=20{,}013$ Hz $\approx 20.0$ kHz

**Discussion**

This problem beautifully illustrates the key differences between tubes closed at one end and tubes open at both ends:

**Closed tube (part a):**

- Fundamental frequency: 47.6 Hz (quite low)
- Only odd harmonics: 1st, 3rd, 5th, 7th, etc.
- Total of 210 audible frequencies (since only odd n values: 1, 3, 5, …, 419)
- Harmonic spacing: 95.2 Hz (every other harmonic is missing)

**Open tube (part b):**

- Fundamental frequency: 95.3 Hz (exactly twice that of the closed tube)
- All harmonics: 1st, 2nd, 3rd, 4th, etc.
- Total of 210 audible frequencies
- Harmonic spacing: 95.3 Hz (evenly spaced)

The closed tube has the same number of resonances as the open tube even though it only has odd harmonics because its fundamental is half the frequency, allowing it to fit twice as many odd harmonics in the audible range.

Both tubes produce rich harmonic spectra suitable for musical applications. The fundamental of 47.6 Hz for the closed tube is close to G1 (49 Hz), while the open tube's fundamental of 95.3 Hz is close to G2 (98 Hz). A 1.80-m tube is quite long—typical for bass organ pipes or similar low-frequency instruments.

The difference in timbre between these two configurations would be dramatic: the closed tube would have a "hollow" sound (lacking even harmonics), while the open tube would have a fuller, richer sound with both even and odd harmonics present.

The final answers are: **(a) $f_n = n(47.6\text{ Hz})$ where $n=1,3,5,...,419$ (odd integers). (b) $f_n = n(95.3\text{ Hz})$ where $n=1,2,3,...,210$ (all integers).**

## Glossary

antinode

point of maximum displacement

node
: point of zero displacement

fundamental
: the lowest-frequency resonance

overtones
: all resonant frequencies higher than the fundamental

harmonics
: the term used to refer collectively to the fundamental and its overtones

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# Hearing

- Define hearing, pitch, loudness, timbre, note, tone, phon, ultrasound, and infrasound.
- Compare loudness to frequency and intensity of a sound.
- Identify structures of the inner ear and explain how they relate to sound perception.


Hearing allows this vocalist, his band, and his fans to enjoy music. (credit: West Point Public Affairs, Flickr)

The human ear has a tremendous range and sensitivity. It can give us a wealth of simple information—such as pitch, loudness, and direction. And from its input we can detect musical quality and nuances of voiced emotion. How is our hearing related to the physical qualities of sound, and how does the hearing mechanism work?

**Hearing** is the perception of sound. (Perception is commonly defined to be awareness through the senses, a typically circular definition of higher-level processes in living organisms.) Normal human hearing encompasses frequencies from 20 to 20 000 Hz, an impressive range. Sounds below 20 Hz are called **infrasound**, whereas those above 20 000 Hz are **ultrasound**. Neither is perceived by the ear, although infrasound can sometimes be felt as vibrations. When we do hear low-frequency vibrations, such as the sounds of a diving board, we hear the individual vibrations only because there are higher-frequency sounds in each. Other animals have hearing ranges different from that of humans. Dogs can hear sounds as high as 30 000 Hz, whereas bats and dolphins can hear up to 100 000-Hz sounds. You may have noticed that dogs respond to the sound of a dog whistle which produces sound out of the range of human hearing. Elephants are known to respond to frequencies below 20 Hz.

The perception of frequency is called **pitch**. Most of us have excellent relative pitch, which means that we can tell whether one sound has a different frequency from another. Typically, we can discriminate between two sounds if their frequencies differ by 0.3% or more. For example, 500.0 and 501.5 Hz are noticeably different. Pitch perception is directly related to frequency and is not greatly affected by other physical quantities such as intensity. Musical **notes** are particular sounds that can be produced by most instruments and in Western music have particular names. Combinations of notes constitute music. Some people can identify musical notes, such as A-sharp, C, or E-flat, just by listening to them. This uncommon ability is called perfect pitch.

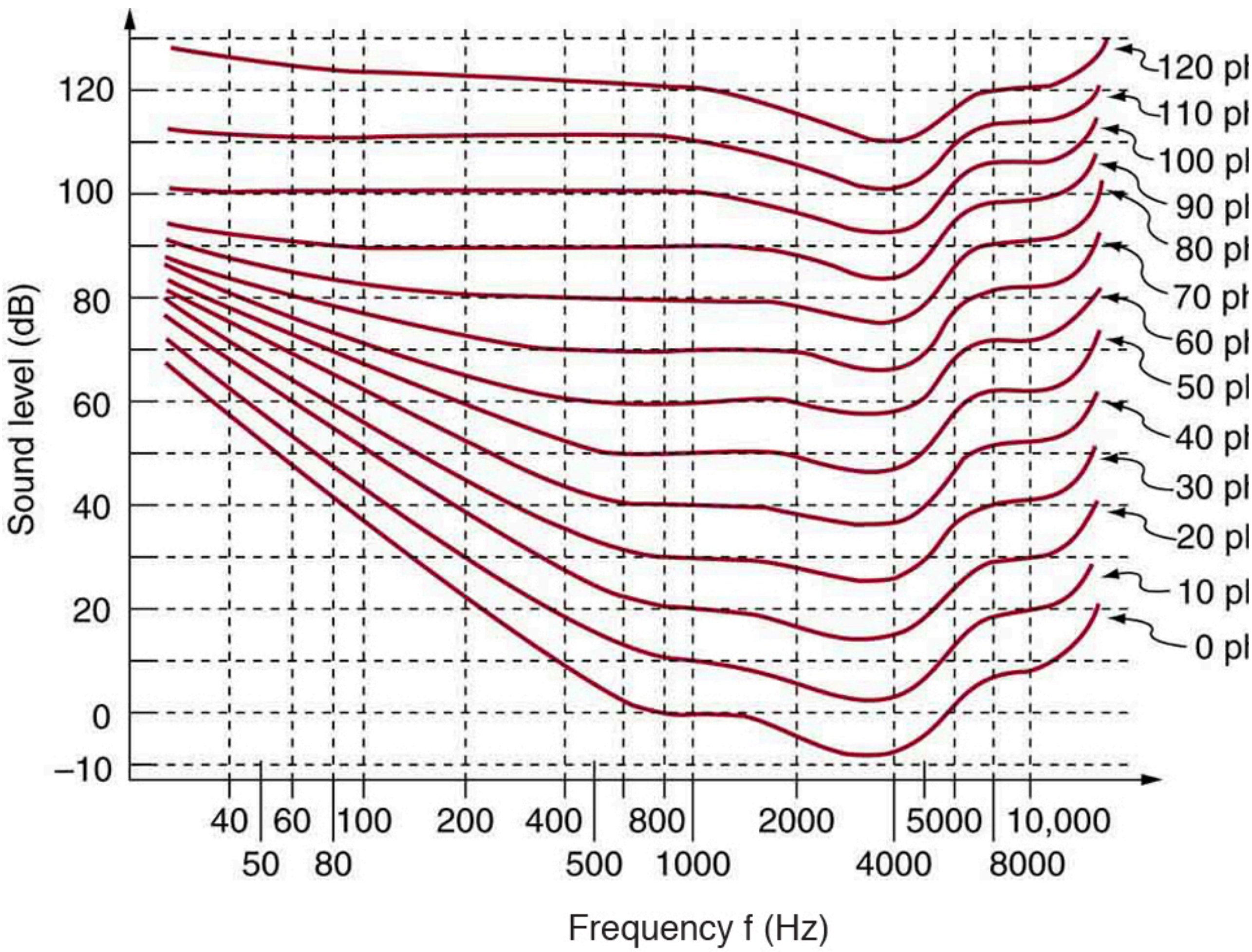
The ear is remarkably sensitive to low-intensity sounds. The lowest audible intensity or threshold is about $10^{-12}\text{W/m}^2$ or 0 dB. Sounds as much as $10^{12}$ more intense can be briefly tolerated. Very few measuring devices are capable of observations over a range of a trillion. The perception of intensity is called **loudness**. At a given frequency, it is possible to discern differences of about 1 dB, and a change of 3 dB is easily noticed. But loudness is not related to intensity alone. Frequency has a major effect on how loud a sound seems. The ear has its maximum sensitivity to frequencies in the range of 2000 to 5000 Hz, so that sounds in this range are perceived as being louder than, say, those at 500 or 10 000 Hz, even when they all have the same intensity. Sounds near the high- and low-frequency extremes of the hearing range seem even less loud, because the ear is even less sensitive at those frequencies. [Table 1] gives the dependence of certain human hearing perceptions on physical quantities.

Sound Perceptions

| Perception | Physical quantity |
| --- | --- |
| Pitch | Frequency |
| Loudness | Intensity and Frequency |
| Timbre | Number and relative intensity of multiple frequencies. |
| | Subtle craftsmanship leads to non-linear effects and more detail. |
| Note | Basic unit of music with specific names, combined to generate tunes |
| Tone | Number and relative intensity of multiple frequencies. |

When a violin plays middle C, there is no mistaking it for a piano playing the same note. The reason is that each instrument produces a distinctive set of frequencies and intensities. We call our perception of these combinations of frequencies and intensities **tone** quality, or more commonly the **timbre** of the sound. It is more difficult to correlate timbre perception to physical quantities than it is for loudness or pitch perception. Timbre is more subjective. Terms such as dull, brilliant, warm, cold, pure, and rich are employed to describe the timbre of a sound. So the consideration of timbre takes us into the realm of perceptual psychology, where higher-level processes in the brain are dominant. This is true for other perceptions of sound, such as music and noise. We shall not delve further into them; rather, we will concentrate on the question of loudness perception.

A unit called a **phon** is used to express loudness numerically. Phons differ from decibels because the phon is a unit of loudness perception, whereas the decibel is a unit of physical intensity. [Figure 2] shows the relationship of loudness to intensity (or intensity level) and frequency for persons with normal hearing. The curved lines are equal-loudness curves. Each curve is labeled with its loudness in phons. Any sound along a given curve will be perceived as equally loud by the average person. The curves were determined by having large numbers of people compare the loudness of sounds at different frequencies and sound intensity levels. At a frequency of 1000 Hz, phons are taken to be numerically equal to decibels. The following example helps illustrate how to use the graph:




The relationship of loudness in phons to intensity level (in decibels) and intensity (in watts per meter squared) for persons with normal hearing. The curved lines are equal-loudness curves—all sounds on a given curve are perceived as equally loud. Phons and decibels are defined to be the same at 1000 Hz.

Measuring Loudness: Loudness Versus Intensity Level and Frequency

(a) What is the loudness in phons of a 100-Hz sound that has an intensity level of 80 dB? (b) What is the intensity level in decibels of a 4000-Hz sound having a loudness of 70 phons? (c) At what intensity level will an 8000-Hz sound have the same loudness as a 200-Hz sound at 60 dB?

**Strategy for (a)**

The graph in [Figure 2] should be referenced in order to solve this example. To find the loudness of a given sound, you must know its frequency and intensity level and locate that point on the square grid, then interpolate between loudness curves to get the loudness in phons.

**Solution for (a)**

(1) Identify knowns:

- The square grid of the graph relating phons and decibels is a plot of intensity level versus frequency—both physical quantities.
- 100 Hz at 80 dB lies halfway between the curves marked 70 and 80 phons.

(2) Find the loudness: 75 phons.

**Strategy for (b)**

The graph in [Figure 2] should be referenced in order to solve this example. To find the intensity level of a sound, you must have its frequency and loudness. Once that point is located, the intensity level can be determined from the vertical axis.

**Solution for (b)**

(1) Identify knowns:

- Values are given to be 4000 Hz at 70 phons.

(2) Follow the 70-phon curve until it reaches 4000 Hz. At that point, it is below the 70 dB line at about 67 dB.

(3) Find the intensity level:

67 dB

**Strategy for (c)**

The graph in [Figure 2] should be referenced in order to solve this example.

**Solution for (c)**

(1) Locate the point for a 200 Hz and 60 dB sound.

(2) Find the loudness: This point lies just slightly above the 50-phon curve, and so its loudness is 51 phons.

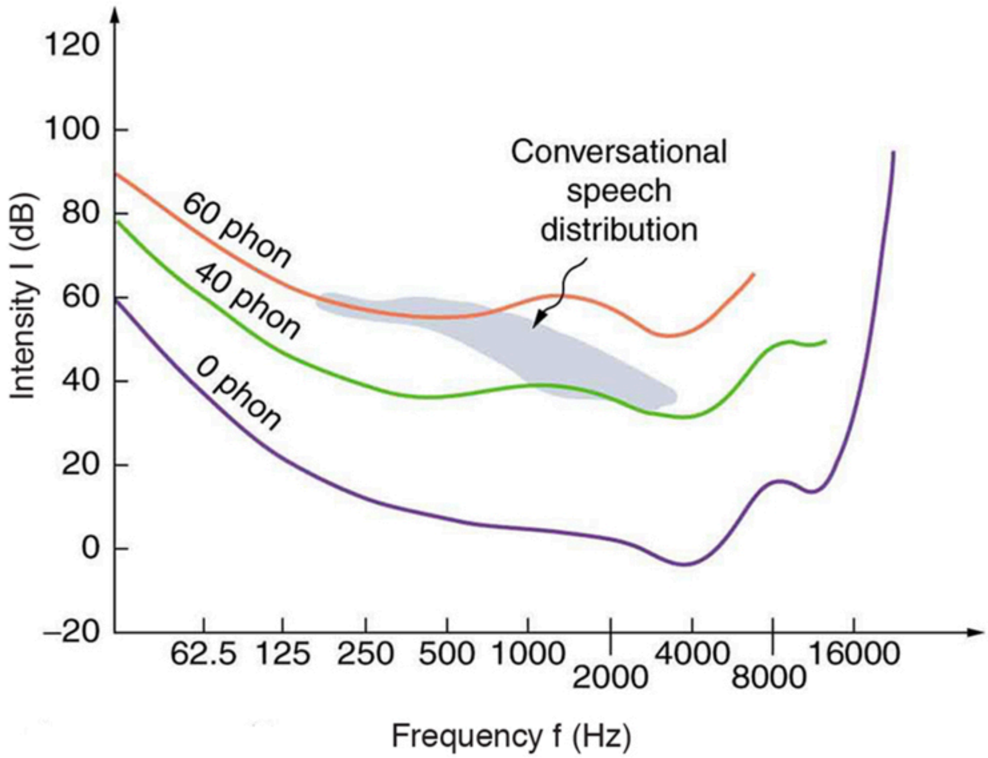
(3) Look for the 51-phon level is at 8000 Hz: 63 dB.

**Discussion**

These answers, like all information extracted from [Figure 2], have uncertainties of several phons or several decibels, partly due to difficulties in interpolation, but mostly related to uncertainties in the equal-loudness curves.

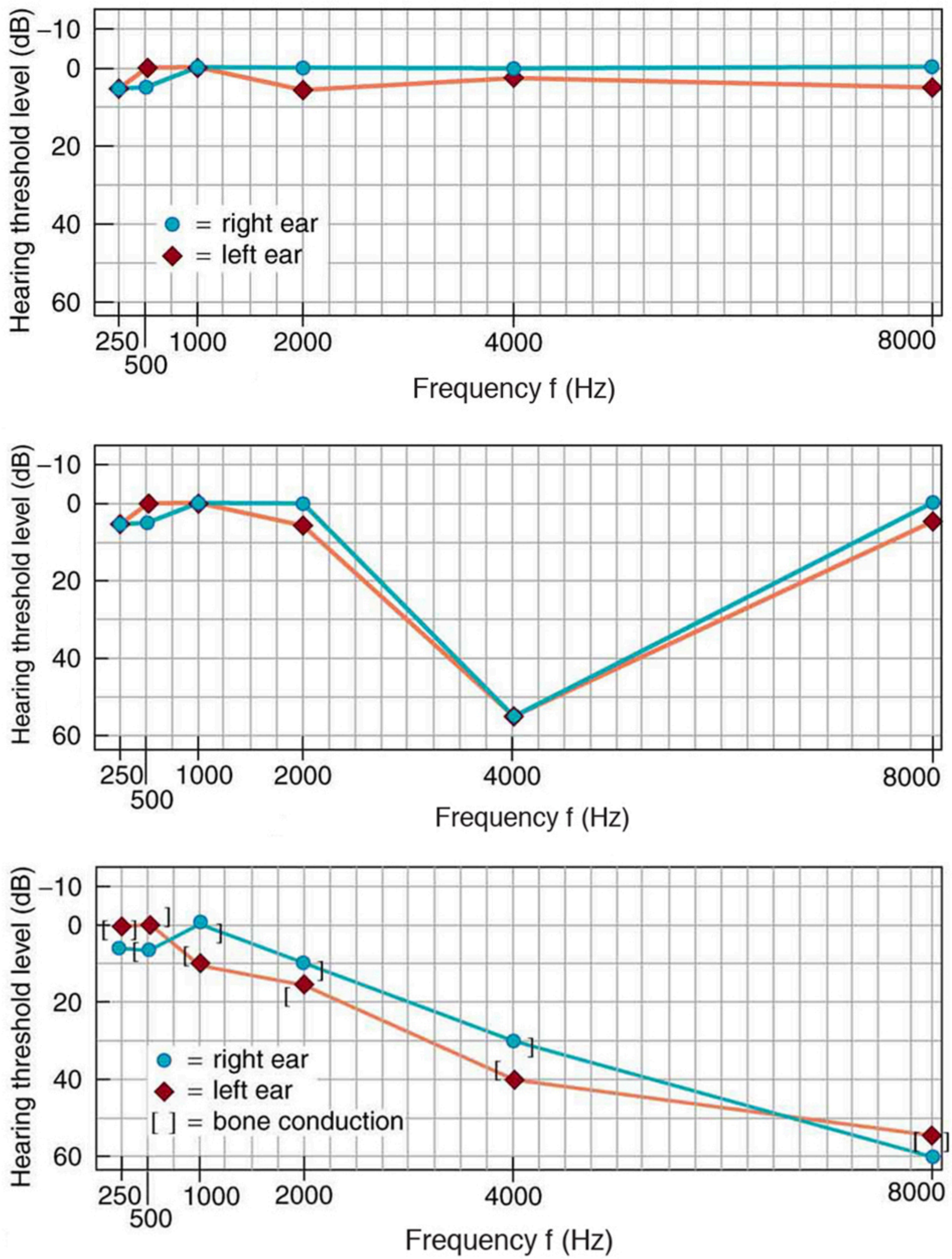
Further examination of the graph in [Figure 2] reveals some interesting facts about human hearing. First, sounds below the 0-phon curve are not perceived by most people. So, for example, a 60 Hz sound at 40 dB is inaudible. The 0-phon curve represents the threshold of normal hearing. We can hear some sounds at intensity levels below 0 dB. For example, a 3-dB, 5000-Hz sound is audible, because it lies above the 0-phon curve. The loudness curves all have dips in them between about 2000 and 5000 Hz. These dips mean the ear is most sensitive to frequencies in that range. For example, a 15-dB sound at 4000 Hz has a loudness of 20 phons, the same as a 20-dB sound at 1000 Hz. The curves rise at both extremes of the frequency range, indicating that a greater-intensity level sound is needed at those frequencies to be perceived to be as loud as at middle frequencies. For example, a sound at 10 000 Hz must have an intensity level of 30 dB to seem as loud as a 20 dB sound at 1000 Hz. Sounds above 120 phons are painful as well as damaging.

We do not often utilize our full range of hearing. This is particularly true for frequencies above 8000 Hz, which are rare in the environment and are unnecessary for understanding conversation or appreciating music. In fact, people who have lost the ability to hear such high frequencies are usually unaware of their loss until tested. The shaded region in [Figure 3] is the frequency and intensity region where most conversational sounds fall. The curved lines indicate what effect hearing losses of 40 and 60 phons will have. A 40-phon hearing loss at all frequencies still allows a person to understand conversation, although it will seem very quiet. A person with a 60-phon loss at all frequencies will hear only the lowest frequencies and will not be able to understand speech unless it is much louder than normal. Even so, speech may seem indistinct, because higher frequencies are not as well perceived. The conversational speech region also has a gender component, in that female voices are usually characterized by higher frequencies. So the person with a 60-phon hearing impediment might have difficulty understanding the normal conversation of a woman.




The shaded region represents frequencies and intensity levels found in normal conversational speech. The 0-phon line represents the normal hearing threshold, while those at 40 and 60 represent thresholds for people with 40- and 60-phon hearing losses, respectively.

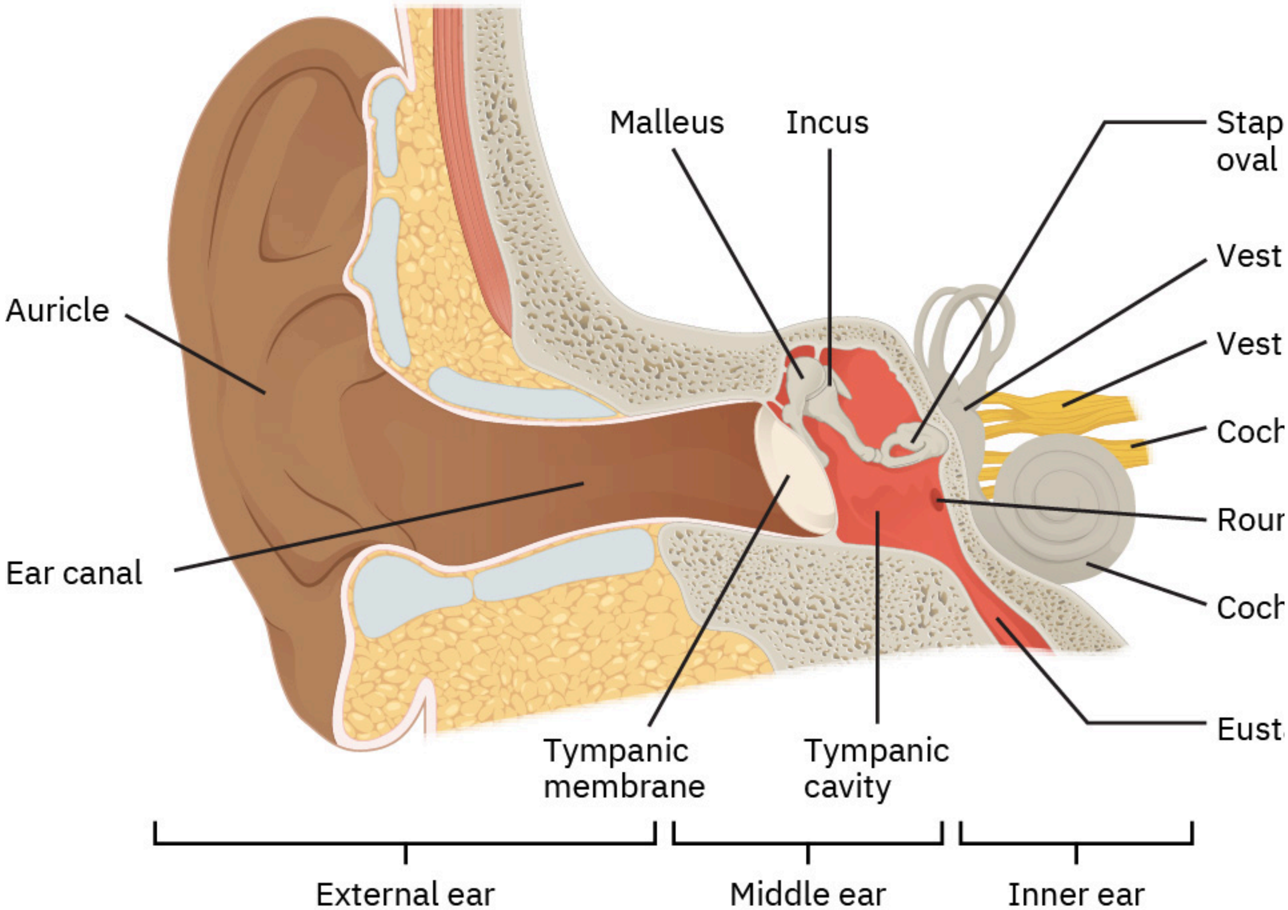
Hearing tests are performed over a range of frequencies, usually from 250 to 8000 Hz, and can be displayed graphically in an audiogram like that in [Figure 4]. The hearing threshold is measured in dB *relative to the normal threshold*, so that normal hearing registers as 0 dB at all frequencies. Hearing loss caused by noise typically shows a dip near the 4000 Hz frequency, irrespective of the frequency that caused the loss and often affects both ears. The most common form of hearing loss comes with age and is called _ presbycusis_—literally elder ear. Such loss is increasingly severe at higher frequencies, and interferes with music appreciation and speech recognition.




Audiograms showing the threshold in intensity level versus frequency for three different individuals. Intensity level is measured relative to the normal threshold. The top left graph is that of a person with normal hearing. The graph to its right has a dip at 4000 Hz and is that of a child who suffered hearing loss due to a cap gun. The third graph is typical of presbycusis, the progressive loss of higher frequency hearing with age. Tests performed by bone conduction (brackets) can distinguish nerve damage from middle ear damage.

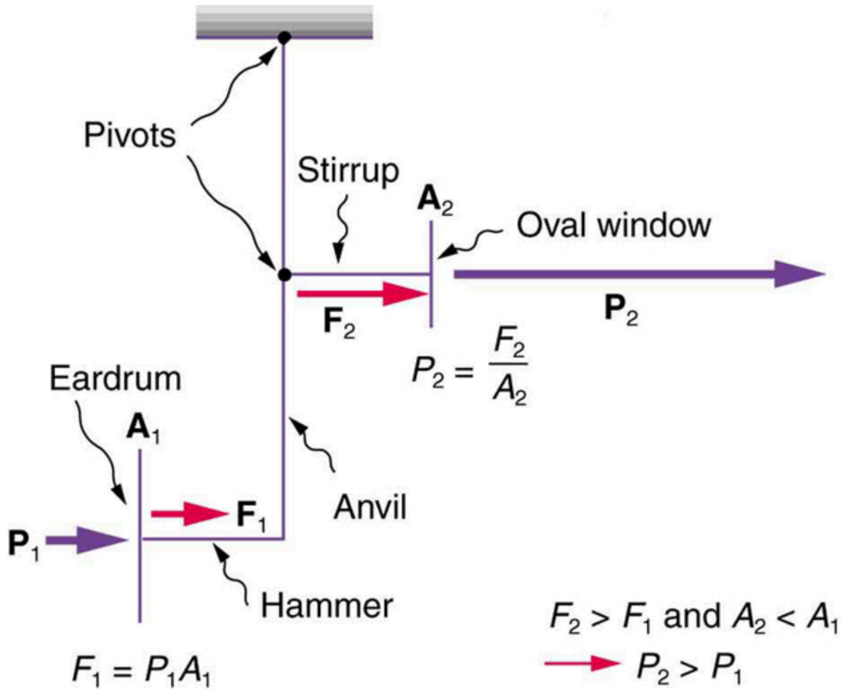
The Hearing Mechanism

The hearing mechanism involves some interesting physics. The sound wave that impinges upon our ear is a pressure wave. The ear is a transducer that converts sound waves into electrical nerve impulses in a manner much more sophisticated than, but analogous to, a microphone. [Figure 5] shows the gross anatomy of the ear with its division into three parts: the outer ear or ear canal; the middle ear, which runs from the eardrum to the cochlea; and the inner ear, which is the cochlea itself. The body part normally referred to as the ear is technically called the pinna.



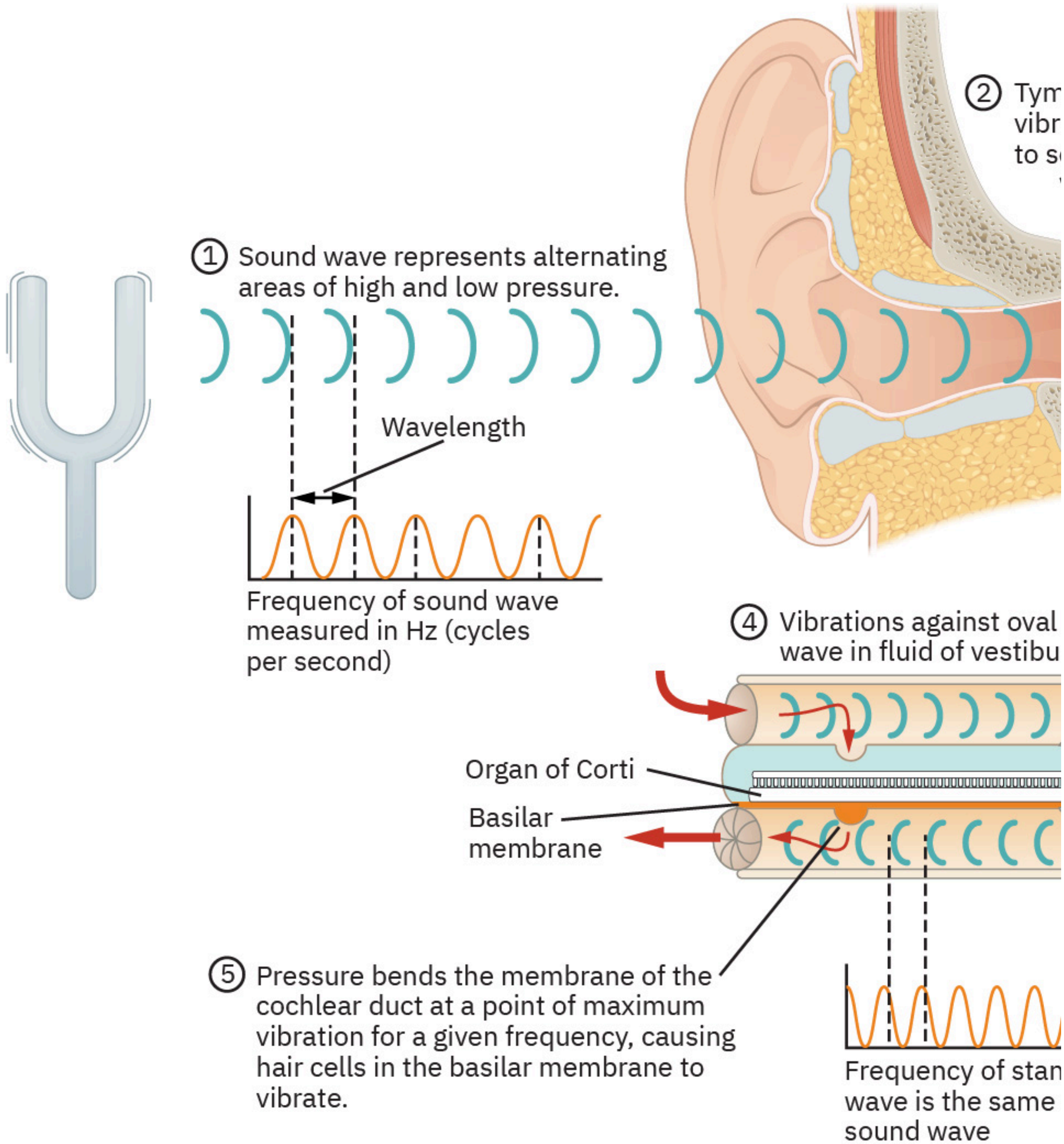

The illustration shows the gross anatomy of the human ear.

The outer ear, or ear canal, carries sound to the recessed protected eardrum. The air column in the ear canal resonates and is partially responsible for the sensitivity of the ear to sounds in the 2000 to 5000 Hz range. The middle ear converts sound into mechanical vibrations and applies these vibrations to the cochlea. The lever system of the middle ear takes the force exerted on the eardrum by sound pressure variations, amplifies it and transmits it to the inner ear via the oval window, creating pressure waves in the cochlea approximately 40 times greater than those impinging on the eardrum. (See [Figure 6].) Two muscles in the middle ear (not shown) protect the inner ear from very intense sounds. They react to intense sound in a few milliseconds and reduce the force transmitted to the cochlea. This protective reaction can also be triggered by your own voice, so that humming while shooting a gun, for example, can reduce noise damage.




This schematic shows the middle ear's system for converting sound pressure into force, increasing that force through a lever system, and applying the increased force to a small area of the cochlea, thereby creating a pressure about 40 times that in the original sound wave. A protective muscle reaction to intense sounds greatly reduces the mechanical advantage of the lever system.

[Figure 7] shows the middle and inner ear in greater detail. Pressure waves moving through the cochlea cause the tectorial membrane to vibrate, rubbing cilia (called hair cells), which stimulate nerves that send electrical signals to the brain. The membrane resonates at different positions for different frequencies, with high frequencies stimulating nerves at the near end and low frequencies at the far end. The complete operation of the cochlea is still not understood, but several mechanisms for sending information to the brain are known to be involved. For sounds below about 1000 Hz, the nerves send signals at the same frequency as the sound. For frequencies greater than about 1000 Hz, the nerves signal frequency by position. There is a structure to the cilia, and there are connections between nerve cells that perform signal processing before information is sent to the brain. Intensity information is partly indicated by the number of nerve signals and by volleys of signals. The brain processes the cochlear nerve signals to provide additional information such as source direction (based on time and intensity comparisons of sounds from both ears). Higher-level processing produces many nuances, such as music appreciation.




The inner ear, or cochlea, is a coiled tube about 3 mm in diameter and 3 cm in length if uncoiled. When the oval window is forced inward, as shown, a pressure wave travels through the perilymph in the direction of the arrows, stimulating nerves at the base of cilia in the organ of Corti.

Hearing losses can occur because of problems in the middle or inner ear. Conductive losses in the middle ear can be partially overcome by sending sound vibrations to the cochlea through the skull. Hearing aids for this purpose usually press against the bone behind the ear, rather than simply amplifying the sound sent into the ear canal as many hearing aids do. Damage to the nerves in the cochlea is not repairable, but amplification can partially compensate. There is a risk that amplification will produce further damage. Another common failure in the cochlea is damage or loss of the cilia but with nerves remaining functional. Cochlear implants that stimulate the nerves directly are now available and widely accepted. Over 100 000 implants are in use, in about equal numbers of adults and children.

The cochlear implant was pioneered in Melbourne, Australia, by Graeme Clark in the 1970s for his deaf father. The implant consists of three external components and two internal components. The external components are a microphone for picking up sound and converting it into an electrical signal, a speech processor to select certain frequencies and a transmitter to transfer the signal to the internal components through electromagnetic induction. The internal components consist of a receiver/transmitter secured in the bone beneath the skin, which converts the signals into electric impulses and sends them through an internal cable to the cochlea and an array of about 24 electrodes wound through the cochlea. These electrodes in turn send the impulses directly into the brain. The electrodes basically emulate the cilia.

Check Your Understanding

Are ultrasound and infrasound imperceptible to all hearing organisms? Explain your answer.

Show Solution

No, the range of perceptible sound is based in the range of human hearing. Many other organisms perceive either infrasound or ultrasound.

## Section Summary

- The range of audible frequencies is 20 to 20 000 Hz.
- Those sounds above 20 000 Hz are ultrasound, whereas those below 20 Hz are infrasound.
- The perception of frequency is pitch.
- The perception of intensity is loudness.
- Loudness has units of phons.

## Conceptual Questions

Why can a hearing test show that your threshold of hearing is 0 dB at 250 Hz, when [Figure 2] implies that no one can hear such a frequency at less than 20 dB?

Show Solution

The 0 dB on a hearing test (audiogram) is a **relative** measure, not an absolute one. It is normalized to the average hearing threshold for a population of healthy young adults. Therefore, a 0 dB result on an audiogram at 250 Hz means that your hearing at that frequency is the same as the average normal hearing person. As [Figure 2] shows, the absolute threshold of hearing for a 250 Hz sound is approximately 20 dB. So, a 0 dB hearing test result at 250 Hz corresponds to an absolute sound intensity level of about 20 dB.

## Problems & Exercises

The factor of $10^{-12}$ in the range of intensities to which the ear can respond, from threshold to that causing damage after brief exposure, is truly remarkable. If you could measure distances over the same range with a single instrument and the smallest distance you could measure was 1 mm, what would the largest be?

Show Solution

**Strategy**

The human ear can respond to sound intensities ranging from the threshold of hearing at $I_0 = 10^{-12}$ W/m² to intensities that cause damage at around $1$ W/m², representing a factor of $10^{12}$. To find the largest measurable distance, we need to multiply the smallest measurable distance by this same factor of $10^{12}$.

**Solution**

Starting with the smallest distance:

$$d_{\text{min}} = 1\ \text{mm} = 1 \times 10^{-3}\ \text{m}$$

The range factor is $10^{12}$, so the largest distance is:

$$d_{\text{max}} = d_{\text{min}} \times 10^{12} = (1 \times 10^{-3}\ \text{m}) \times 10^{12} = 10^{9}\ \text{m}$$

Converting to kilometers:

$$d_{\text{max}} = 10^{9}\ \text{m} \times \frac{1\ \text{km}}{10^{3}\ \text{m}} = 10^{6}\ \text{km} = 1 \times 10^{6}\ \text{km}$$

**Discussion**

This result of one million kilometers is truly remarkable and helps us appreciate the extraordinary dynamic range of human hearing. To put this in perspective, this distance is about 2.6 times the distance from the Earth to the Moon (384,000 km), or roughly equivalent to traveling around Earth's equator 25 times! This analogy vividly demonstrates why the logarithmic decibel scale is necessary for measuring sound intensity—the linear range our ears can detect would be impossibly cumbersome to work with using linear scales. The ear's ability to perceive such a vast range of intensities is a testament to the sophisticated biological mechanisms of hearing, involving not only the physical amplification in the middle ear but also the neural processing of signals. This wide dynamic range allows us to hear everything from the faintest whisper to the roar of a jet engine, though prolonged exposure to the upper end of this range causes permanent damage to the delicate hair cells in the cochlea.

The frequencies to which the ear responds vary by a factor of $10^3$ . Suppose the speedometer on your car measured speeds differing by the same factor of $10^3$ , and the greatest speed it reads is 90.0 mi/h. What would be the slowest nonzero speed it could read?

Show Solution

**Strategy**

The human ear responds to frequencies ranging from 20 Hz to 20,000 Hz, which represents a factor of $\frac{20{,}000}{20} = 1000 = 10^3$. For the speedometer to have the same range factor, we divide the maximum speed by $10^3$ to find the minimum speed.

**Solution**

Given:

- Maximum speed: $v_{\text{max}} = 90.0\text{ mi/h}$
- Range factor: $10^3$

The minimum speed is:

$$v_{\text{min}} = \frac{v_{\text{max}}}{10^3} = \frac{90.0\text{ mi/h}}{1000} = 0.0900\text{ mi/h}$$

**Discussion**

The slowest nonzero speed this speedometer could read would be 0.0900 mi/h, which is extremely slow—less than one-tenth of a mile per hour! To put this in perspective, this is about 132 feet per hour, or roughly 2.2 feet per minute. A typical walking speed is about 3 mi/h, so this minimum measurable speed is about 33 times slower than walking. This analogy helps illustrate why our hearing range is so impressive: just as this speedometer would be impractical (you'd rarely need to measure such slow speeds), having such a wide frequency range allows humans to perceive everything from the deep rumble of thunder (low frequency) to the high-pitched chirping of birds and insects (high frequency). The 1000:1 frequency ratio means our ears can detect three orders of magnitude in frequency, which is crucial for speech recognition, music appreciation, and environmental awareness. Young, healthy ears can typically detect this full range, though age-related hearing loss (presbycusis) progressively reduces sensitivity to higher frequencies, effectively narrowing this range over time.

What are the closest frequencies to 500 Hz that an average person can clearly distinguish as being different in frequency from 500 Hz? The sounds are not present simultaneously.

Show Solution

**Strategy**

According to the text, humans can discriminate between two sounds if their frequencies differ by 0.3% or more. This is a measure of our frequency discrimination ability, also called pitch discrimination. We need to find frequencies that are 0.3% different from 500 Hz.

**Solution**

The frequency discrimination threshold is 0.3% of the reference frequency:

$$\Delta f = 0.003 \times f = 0.003 \times 500\text{ Hz} = 1.5\text{ Hz}$$

Therefore, the closest distinguishable frequencies are:

Lower frequency:

$$f_{\text{lower}} = 500\text{ Hz} - 1.5\text{ Hz} = 498.5\text{ Hz}$$

Upper frequency:

$$f_{\text{upper}} = 500\text{ Hz} + 1.5\text{ Hz} = 501.5\text{ Hz}$$

**Discussion**

The average person can distinguish between 500 Hz and either 498.5 Hz or 501.5 Hz when the sounds are presented sequentially. This remarkable ability to detect a difference of just 1.5 Hz out of 500 Hz (a 0.3% change) demonstrates the acute pitch perception of the human auditory system. This discrimination ability is crucial for music appreciation and speech recognition. In musical terms, this 0.3% threshold is much finer than the smallest interval in Western music—a semitone represents about a 6% frequency difference, which is 20 times larger than our discrimination threshold! This explains why we can easily detect when a musical instrument is slightly out of tune. The ability to discriminate pitch depends on several factors including the listener's age, musical training, and the frequency range being tested. Musicians often have even better pitch discrimination than the average person, sometimes able to detect differences as small as 0.1%. However, this ability generally deteriorates with age, particularly at higher frequencies where presbycusis affects hearing sensitivity. The 0.3% threshold assumes ideal listening conditions and non-fatigued, attentive listeners—in noisy environments or when distracted, pitch discrimination becomes significantly worse.

Can the average person tell that a 2002-Hz sound has a different frequency than a 1999-Hz sound without playing them simultaneously?

Show Solution

**Strategy**

To determine if the average person can distinguish between these two frequencies, we need to calculate the percentage difference between them and compare it to the human discrimination threshold of 0.3%. We'll use the average of the two frequencies as the reference.

**Solution**

Given frequencies: $f_1 = 1999 \text{ Hz}$ and $f_2 = 2002 \text{ Hz}$

The difference in frequency is:

$$\Delta f = f_2 - f_1 = 2002 \text{ Hz} - 1999 \text{ Hz} = 3 \text{ Hz}$$

The average frequency is:

$$f_{\text{avg}} = \frac{f_1 + f_2}{2} = \frac{1999 + 2002}{2} = 2000.5 \text{ Hz}$$

The percentage difference is:

$$\text{Percentage difference} = \frac{\Delta f}{f_{\text{avg}}} \times 100\% = \frac{3}{2000.5} \times 100\% = 0.150\%$$

Since $0.150\% < 0.3\%$, the difference is below the threshold.

**Discussion**

No, the average person cannot distinguish between a 2002-Hz sound and a 1999-Hz sound when they are not played simultaneously. The percentage difference of 0.150% is exactly half of the 0.3% discrimination threshold that the average person can detect. This means the frequencies are too close together for most people to perceive as different pitches. However, this result comes with important caveats: (1) Some individuals with exceptional pitch discrimination, particularly trained musicians, might be able to detect this difference. (2) If the two sounds were played simultaneously, the average person would hear beats at a frequency of 3 Hz, which would make the difference obvious even though the individual pitches cannot be distinguished. (3) The 2000-Hz region is near the ear's peak sensitivity range (2000-5000 Hz), where pitch discrimination is generally better than at the frequency extremes. (4) Under ideal laboratory conditions with careful attention and immediate comparison, some individuals might approach this discrimination level. In practical terms, this problem illustrates that our pitch perception has definite limits, which is why musical instruments need only be tuned to within a certain tolerance—differences smaller than about 0.3% are imperceptible to most listeners.

If your radio is producing an average sound intensity level of 85 dB, what is the next lowest sound intensity level that is clearly less intense?

Show Solution

**Strategy**

According to the text, at a given frequency, it is possible to discern differences of about 1 dB, and a change of 3 dB is easily noticed. Therefore, the next clearly distinguishable lower intensity level would be at least 3 dB less than the current level.

**Solution**

Given:

- Current intensity level: $\beta_1 = 85 \text{ dB}$
- Minimum clearly noticeable change: $\Delta\beta = 3 \text{ dB}$

The next lowest clearly distinguishable intensity level is:

$$\beta_2 = \beta_1 - \Delta\beta = 85 \text{ dB} - 3 \text{ dB} = 82 \text{ dB}$$

**Discussion**

The next lowest sound intensity level that is clearly less intense than 85 dB is 82 dB. While the average person can detect differences as small as 1 dB under ideal conditions with careful attention, a 3 dB change is considered "easily noticed" without special effort or concentration. This 3 dB difference is significant because it represents a factor of 2 in actual sound intensity (since $10\log_{10}(2) \approx 3$ dB). In other words, reducing the volume from 85 dB to 82 dB means the radio is producing approximately half the acoustic power. This is why a 3 dB change is often used as a standard reference in acoustics and audio engineering. From a practical standpoint, 85 dB is at the threshold where prolonged exposure begins to pose a risk for hearing damage—OSHA regulations limit exposure to 85 dB to 8 hours per day. Reducing to 82 dB (half the intensity) provides a small but measurable reduction in hearing damage risk. The ability to perceive a 3 dB change easily is important for everyday activities like adjusting radio volume, evaluating environmental noise levels, and using hearing protection effectively. Most volume controls on audio equipment are designed so that small adjustments produce changes of at least 2-3 dB to ensure perceptible differences.

Can you tell that your roommate turned up the sound on the TV if its average sound intensity level goes from 70 to 73 dB?

Show Solution

**Strategy**

We need to determine if a 3 dB change in sound intensity level is perceptible. According to the text, while differences of about 1 dB can be discerned under ideal conditions, a change of 3 dB is easily noticed.

**Solution**

Given:

- Initial intensity level: $\beta_1 = 70$ dB
- Final intensity level: $\beta_2 = 73$ dB
- Change in intensity level: $\Delta\beta = 73 - 70 = 3$ dB

Since the text states that “a change of 3 dB is easily noticed,” yes, you would be able to tell that your roommate turned up the sound.

To understand what this change represents in terms of actual intensity, we can calculate the intensity ratio:

$$\beta_2 - \beta_1 = 10\log_{10}\left(\frac{I_2}{I_1}\right) = 3 \text{ dB}$$
$$\log_{10}\left(\frac{I_2}{I_1}\right) = 0.3$$
$$\frac{I_2}{I_1} = 10^{0.3} \approx 2.0$$

**Discussion**

Yes, you can easily tell that your roommate turned up the TV sound from 70 dB to 73 dB. This 3 dB increase represents a doubling of the actual sound intensity reaching your ears. While this might seem like a large change physically, perceptually it sounds like a moderate but clearly noticeable increase in volume, not a doubling of loudness. This is because our perception of loudness follows a logarithmic relationship with intensity, which is precisely why the decibel scale uses logarithms. In practical terms, a 3 dB change is exactly the threshold for “easily noticed” changes—you wouldn’t need to pay special attention or strain to hear it; the difference would be immediately obvious. This is a common scenario in shared living spaces and illustrates why people can easily detect when others adjust volume settings. For context, 70 dB is comparable to the sound level of a vacuum cleaner or busy traffic, while 73 dB is slightly louder but still well below the levels that cause immediate hearing damage. However, both levels are loud enough that extended exposure (more than 8 hours) could contribute to gradual hearing loss over time. This problem also demonstrates why roommate disputes over TV volume are so common—a seemingly small adjustment on the volume dial can produce an easily perceptible change in sound level!

Based on the graph in [Figure 2], what is the threshold of hearing in decibels for frequencies of 60, 400, 1000, 4000, and 15 000 Hz? Note that many AC electrical appliances produce 60 Hz, music is commonly 400 Hz, a reference frequency is 1000 Hz, your maximum sensitivity is near 4000 Hz, and many older TVs produce a 15 750 Hz whine.

Show Solution

**Strategy**

The threshold of hearing is represented by the 0-phon curve in Figure 2. We need to locate each frequency on the horizontal axis, follow it up to where it intersects the 0-phon curve, then read the corresponding intensity level from the vertical axis. The 0-phon curve shows the minimum intensity level required for a sound to be just barely audible at each frequency.

**Solution**

Reading from the 0-phon curve in Figure 2 for each frequency:

- At 60 Hz: approximately **48 dB**
- At 400 Hz: approximately **9 dB**
- At 1000 Hz: **0 dB** (by definition, phons equal decibels at 1000 Hz)
- At 4000 Hz: approximately **–7 dB**
- At 15,000 Hz: approximately **20 dB**

**Discussion**

These threshold values reveal fascinating characteristics of human hearing sensitivity across the frequency spectrum. The ear is least sensitive at very low frequencies (60 Hz requires 48 dB to be audible) and at very high frequencies (15,000 Hz requires 20 dB). Maximum sensitivity occurs around 3000-4000 Hz, where sounds can be detected at intensity levels below 0 dB (negative decibel values relative to the reference $I_0 = 10^{-12}$ W/m²). The –7 dB threshold at 4000 Hz means the ear can detect sounds at this frequency when they are only one-fifth the intensity of the reference threshold ($10^{-0.7} \approx 0.2$).

This frequency-dependent sensitivity has important practical implications: (1) The low sensitivity at 60 Hz explains why the hum from AC electrical appliances is often inaudible unless quite loud—the ear simply isn’t very sensitive to this frequency. (2) The high sensitivity around 400 Hz (requiring only 9 dB) is ideal for music and speech, which contain significant energy in this range. (3) The peak sensitivity near 4000 Hz is thought to be an evolutionary adaptation that enhances our ability to hear warning sounds and human speech, particularly consonants which carry critical information for understanding language. (4) The reduced sensitivity at 15,000 Hz (requiring 20 dB) partially explains why age-related hearing loss (presbycusis) at high frequencies often goes unnoticed—we don’t use this range much in daily life. (5) The fact that older TVs produce a 15,750 Hz whine is particularly annoying to young people who can hear it, but older adults with presbycusis are often blissfully unaware of this irritating sound!

What sound intensity levels must sounds of frequencies 60, 3000, and 8000 Hz have in order to have the same loudness as a 40-dB sound of frequency 1000 Hz (that is, to have a loudness of 40 phons)?

Show Solution

**Strategy**

To find sound intensity levels that have the same loudness as a 40-dB sound at 1000 Hz, we need to find where the 40-phon equal-loudness curve intersects each of the specified frequencies (60, 3000, and 8000 Hz) on Figure 2. At 1000 Hz, phons and decibels are defined to be equal, so a 40-dB sound has a loudness of 40 phons.

**Solution**

Since a 40-dB sound at 1000 Hz has a loudness of 40 phons, we need to follow the 40-phon curve in Figure 2 and read the intensity level at each frequency:

- At 60 Hz: The 40-phon curve intersects at approximately **70 dB**
- At 3000 Hz: The 40-phon curve intersects at approximately **37 dB**
- At 8000 Hz: The 40-phon curve intersects at approximately **43 dB**

**Discussion**

These values demonstrate the remarkable frequency-dependent nature of loudness perception. To sound equally loud to a 40-dB tone at 1000 Hz, a 60-Hz tone must be 30 dB more intense (70 dB vs. 40 dB), which represents a factor of $10^3 = 1000$ times more intensity! This occurs because the ear is far less sensitive to low frequencies. Conversely, at 3000 Hz (near the ear's peak sensitivity), a sound needs only 37 dB to achieve the same perceived loudness—3 dB less than the reference, meaning about half the intensity. At 8000 Hz, 43 dB is required, slightly more than the reference.

This frequency dependence has crucial practical applications in audio engineering and hearing protection. Sound systems must compensate for this non-uniform sensitivity through equalization, boosting low frequencies to achieve balanced perceived loudness across the spectrum. This is why bass controls exist on audio equipment and why "loudness" buttons on older stereos would boost bass at low volumes—to compensate for the ear's reduced sensitivity to low frequencies at lower overall sound levels. For hearing conservation, this explains why low-frequency industrial noise (like heavy machinery rumble) may be perceived as less loud than it actually is, potentially leading to underestimation of hearing damage risk. Workers might not realize that a 70-dB, 60-Hz sound, while seeming only moderately loud, actually carries far more acoustic energy than its perceived loudness suggests. Similarly, this is why subwoofers in home theater systems require much more power than mid-range or high-frequency speakers to achieve balanced sound—the ear's insensitivity to low frequencies must be overcome by higher actual sound intensities.

What is the approximate sound intensity level in decibels of a 600-Hz tone if it has a loudness of 20 phons? If it has a loudness of 70 phons?

Show Solution

**Strategy**

To find the intensity level in decibels for a given loudness in phons, we use Figure 2. We locate the appropriate equal-loudness curve (the phon value), find where it intersects the frequency of interest (600 Hz), and read the corresponding intensity level in decibels from the vertical axis.

**Solution**

For part (a): Loudness of 20 phons at 600 Hz

Following the 20-phon curve to where it intersects 600 Hz, we read from the vertical axis:

$$\beta \approx 23 \text{ dB}$$

For part (b): Loudness of 70 phons at 600 Hz

Following the 70-phon curve to where it intersects 600 Hz, we read from the vertical axis:

$$\beta \approx 70 \text{ dB}$$

**Discussion**

The results reveal an interesting characteristic of the equal-loudness curves. At 20 phons, the 600-Hz tone requires 23 dB—about 3 dB more than the 20 dB that would be required at the reference frequency of 1000 Hz. This difference occurs because 600 Hz is away from the region of maximum ear sensitivity (2000-5000 Hz), though not dramatically so. The ear is slightly less sensitive at 600 Hz than at 1000 Hz for low-intensity sounds.

However, at 70 phons, the intensity level at 600 Hz is essentially the same as it would be at 1000 Hz (70 dB). This demonstrates that the equal-loudness curves become progressively flatter at higher sound levels—the ear's frequency-dependent sensitivity becomes less pronounced for louder sounds. At very high intensity levels, the perceived loudness becomes more uniform across frequencies.

This has practical implications for audio systems: at low volumes (like background music), the frequency response must be carefully equalized to account for the ear's varying sensitivity, which is why many audio receivers have a "loudness" compensation feature that boosts bass and treble at low volumes. At high volumes, less equalization is needed because the ear's response is more uniform. Additionally, this explains why quiet sounds seem to have less bass and treble compared to loud sounds playing the same content—it's not the sound that changes, but our perception of it. This phenomenon is also relevant for hearing tests, where threshold measurements must be made at different frequencies to create a complete audiogram, as demonstrated in Figure 4. The flattening of curves at higher levels also means that hearing damage risk is more uniform across frequencies at dangerous sound levels (above 85 dB).

(a) What are the loudnesses in phons of sounds having frequencies of 200, 1000, 5000, and 10 000 Hz, if they are all at the same 60.0-dB sound intensity level? (b) If they are all at 110 dB? (c) If they are all at 20.0 dB?

Show Solution

**Strategy**

To find the loudness in phons for a given frequency and intensity level, we use Figure 2 in reverse from previous problems. We locate each frequency on the horizontal axis, go up to the given intensity level (in dB) on the vertical axis, and determine which equal-loudness curve (phon value) passes through that point.

**Solution**

**(a) At 60.0-dB intensity level:**

For each frequency, we locate where 60 dB intersects the frequency and interpolate between phon curves:

- **200 Hz**: approximately **50 phons** (below the 60-phon curve)
- **1000 Hz**: **60 phons** (by definition, phons equal dB at 1000 Hz)
- **5000 Hz**: approximately **70 phons** (above the 60-phon curve)
- **10,000 Hz**: approximately **60 phons** (approximately on the 60-phon curve)

**(b) At 110-dB intensity level:**

For each frequency at this high intensity level:

- **200 Hz**: approximately **103 phons**
- **1000 Hz**: **110 phons**
- **5000 Hz**: approximately **113 phons**
- **10,000 Hz**: approximately **108 phons**

**(c) At 20.0-dB intensity level:**

For each frequency at this low intensity level:

- **200 Hz**: approximately **10 phons** (well below the 20-phon curve)
- **1000 Hz**: **20 phons**
- **5000 Hz**: approximately **28 phons** (above the 20-phon curve)
- **10,000 Hz**: approximately **15 phons**

**Discussion**

These results beautifully illustrate how loudness perception varies dramatically with frequency, especially at lower intensity levels. In part (a), even though all four sounds have the same physical intensity (60 dB), they sound quite different: the 5000-Hz tone sounds loudest (70 phons) because it's near the ear's peak sensitivity, while the 200-Hz tone sounds noticeably quieter (50 phons)—a difference of 20 phons represents a very substantial perceptual difference.

Part (c) shows this effect is most pronounced at low intensities. At 20 dB, the loudness varies from 10 phons (200 Hz) to 28 phons (5000 Hz)—the 5000-Hz tone sounds nearly three times as loud despite having identical intensity! This is why soft background music often seems to lack bass—the low frequencies simply don't sound as loud to our ears.

Part (b) reveals that at very high intensity levels (110 dB), the variation in perceived loudness becomes much smaller. All frequencies sound more similar because the equal-loudness curves flatten out considerably at high intensities. This flattening means our ears become less discriminatory about frequency at high sound levels.

This phenomenon has critical implications: (1) In hearing conservation, a 60-dB, 200-Hz sound might be underestimated as a hazard because it sounds less loud than a 60-dB, 5000-Hz sound, even though both carry equal energy. (2) For hearing aid design, frequency-specific amplification must account for these variations—simply amplifying all frequencies equally doesn't restore normal loudness perception. (3) In audio mixing and mastering, engineers must carefully balance frequency content because listeners will perceive different frequencies as having different loudness even when they measure the same dB level. (4) Age-related hearing loss exacerbates these effects, as high-frequency sensitivity declines, making sounds at 5000-10,000 Hz seem even quieter relative to mid-range frequencies.

Suppose a person has a 50-dB hearing loss at all frequencies. By how many factors of 10 will low-intensity sounds need to be amplified to seem normal to this person? Note that smaller amplification is appropriate for more intense sounds to avoid further hearing damage.

Show Solution

**Strategy**

A 50-dB hearing loss means the person's hearing threshold is 50 dB higher than normal. To determine the intensity amplification needed, we use the relationship between decibels and intensity ratio: $\beta = 10\log_{10}(I/I_0)$. We need to find how many factors of 10 in intensity correspond to a 50-dB increase.

**Solution**

A change in sound level is related to the intensity ratio by:

$$\Delta\beta = 10\log_{10}(I_2 I_1)$$

For a 50-dB increase:

$$50 = 10\log_{10}(I_2 I_1)$$

Dividing both sides by 10:

$5 = \log_{10}(I_2 I_1)$

Converting from logarithmic form:

$$I_2 I_1 = 10^5$$

Therefore, the intensity must be amplified by **five factors of 10**, or 100,000 times.

**Discussion**

Low-intensity sounds must be amplified by five factors of 10 (100,000 times in intensity) for this person to perceive them as a person with normal hearing would. This dramatic amplification is necessary because a 50-dB hearing loss represents a very significant impairment—this person's threshold of hearing is 100,000 times less sensitive than normal. In practical terms, sounds that are barely audible to a person with normal hearing would be completely inaudible to this person without amplification.

This level of hearing loss has profound implications for hearing aid design and daily life. Modern hearing aids must provide sophisticated, frequency-specific amplification while avoiding several critical problems: (1) **Preventing further damage**: As noted in the problem, smaller amplification is needed for already-intense sounds. A sound at 90 dB (already potentially damaging) should not be amplified to 140 dB (which would cause immediate, severe damage). This requires compression circuitry that provides more gain for quiet sounds and less for loud sounds. (2) **Preserving dynamic range**: The ear's remarkable ability to hear intensities ranging over 12 orders of magnitude ($10^{12}$) must be compressed into the reduced dynamic range of the hearing-impaired ear. (3) **Avoiding feedback**: Amplifying sounds by 100,000 times creates serious challenges with acoustic feedback (the squealing sound when hearing aids malfunction).

A 50-dB hearing loss at all frequencies would severely impact quality of life. Normal conversation (about 60 dB) would be barely audible without amplification. This person would likely qualify for cochlear implant consideration, as conventional hearing aids may provide limited benefit at this severity of loss. The uniform 50-dB loss across all frequencies is actually somewhat unusual—most hearing loss is frequency-dependent, with high frequencies typically affected more severely than low frequencies, particularly in presbycusis and noise-induced hearing loss. The good news is that modern hearing aids with digital signal processing can provide the necessary amplification while minimizing distortion and protecting against further damage, though they cannot fully restore the subtlety and dynamic range of normal hearing.

If a woman needs an amplification of $5.0 \times 10^{12}$ times the threshold intensity to enable her to hear at all frequencies, what is her overall hearing loss in dB? Note that smaller amplification is appropriate for more intense sounds to avoid further damage to her hearing from levels above 90 dB.

Show Solution

**Strategy**

To find the hearing loss in decibels, we need to convert the intensity ratio (amplification factor) to a decibel difference using the formula $\beta = 10\log_{10}(I/I_0)$. The amplification factor tells us by how much the intensity must be increased to bring sounds to her threshold of hearing.

**Solution**

Given that the required amplification is $5.0 \times 10^{12}$, we can find the hearing loss using:

$$\Delta\beta = 10\log_{10}(I_{\text{amplified}} I_{\text{original}}) = 10\log_{10}(5.0 \times 10^{12})$$

Using logarithm properties:

$$\Delta\beta = 10[\log_{10}(5.0) + \log_{10}(10^{12})]$$
$$\Delta\beta = 10[\log_{10}(5.0) + 12]$$

Since $\log_{10}(5.0) \approx 0.699$:

$$\Delta\beta = 10[0.699 + 12] = 10 \times 12.699 = 126.99 \approx 127 \text{ dB}$$

**Discussion**

This woman has a hearing loss of approximately 127 dB at all frequencies, which represents an extraordinarily severe hearing impairment—essentially total deafness. To put this in perspective, the threshold of pain and immediate hearing damage is around 120 dB, and this woman's hearing threshold is 7 dB beyond that level. This means that sounds would need to be loud enough to cause immediate damage to a normal ear just for her to barely detect them.

This level of hearing loss is classified as profound deafness and has critical implications: (1) **Conventional hearing aids are completely inadequate**: Even the most powerful hearing aids cannot provide enough amplification to be useful, and attempting to do so would create unbearable acoustic feedback. (2) **Cochlear implants are essential**: This woman would be an ideal candidate for cochlear implants, which bypass the damaged hair cells and directly stimulate the auditory nerve. Over 100,000 people worldwide use cochlear implants successfully. (3) **No functional hearing without intervention**: Without cochlear implants or other advanced interventions, this person would rely entirely on visual communication (sign language, lip reading) and vibrotactile sensation.

The note about avoiding amplification above 90 dB is particularly relevant here—it would be medically impossible and dangerous to try to amplify sounds by $5.0 \times 10^{12}$ times across all sound levels. For example, a normal conversation at 60 dB would need to be amplified to $60 + 127 = 187$ dB, which would cause catastrophic damage to any remaining hearing function and physical damage to the ear. This is why modern hearing technology for profound hearing loss focuses on cochlear implants that work entirely differently from amplification-based hearing aids.

Such profound hearing loss could result from various causes: severe genetic conditions, certain medications (ototoxic drugs), profound noise trauma, infections like meningitis, or other damage to the cochlea or auditory nerve. The measurement of 127 dB hearing loss would typically be determined through bone conduction testing to distinguish cochlear damage from middle ear problems. Remarkably, with cochlear implants, many individuals with this level of hearing loss can develop functional hearing and even participate in telephone conversations, demonstrating the impressive capabilities of modern medical technology in addressing profound sensory deficits.

(a) What is the intensity in watts per meter squared of a just barely audible 200-Hz sound? (b) What is the intensity in watts per meter squared of a barely audible 4000-Hz sound?

Show Solution

**Strategy**

A just barely audible sound corresponds to the threshold of hearing, which is the 0-phon curve in Figure 2. We need to find the intensity level in dB at each frequency from the graph, then convert from decibels to actual intensity using the formula $\beta = 10\log_{10}(I/I_0)$ where $I_0 = 10^{-12}$ W/m².

**Solution**

**(a) For 200 Hz:**

From Figure 2, the 0-phon curve at 200 Hz corresponds to approximately 20 dB.

Using the decibel formula:

$$\beta=10\log_{10}(I\,I_0)$$
$$20=10\log_{10}(I\,10^{-12})$$
$$2=\log_{10}(I\,10^{-12})$$
$$I\,10^{-12}=10^{2}=100$$
$$I=100\times10^{-12}=10^{-10}\text{ W/m}^2=1\times10^{-10}\text{ W/m}^2$$

Rounding to one significant figure: $I \approx 2 \times 10^{-10}\text{ W/m}^2$

**(b) For 4000 Hz:**

From Figure 2, the 0-phon curve at 4000 Hz corresponds to approximately –7 dB (below the reference).

$$-7=10\log_{10}(I\,10^{-12})$$
$$-0.7=\log_{10}(I\,10^{-12})$$
$$I\,10^{-12}=10^{-0.7}\approx0.2$$
$$I=0.2\times10^{-12}=2\times10^{-13}\text{ W/m}^2$$

**Discussion**

The intensity of a just barely audible 200-Hz sound is approximately $2 \times 10^{-10}$ W/m², while a barely audible 4000-Hz sound has an intensity of only $2 \times 10^{-13}$ W/m². This remarkable difference—the 200-Hz sound requires about 1000 times more intensity than the 4000-Hz sound to be detected—demonstrates the ear's dramatically different sensitivity across the frequency spectrum.

At 4000 Hz, near the ear's peak sensitivity, sounds can be detected at intensities below the reference threshold $I_0 = 10^{-12}$ W/m². This is why $I_0$ is often called the "threshold of hearing at 1000 Hz"—it's not a universal threshold but rather a reference point. At the ear's most sensitive frequencies (around 3000-4000 Hz), we can actually hear sounds quieter than this reference.

To appreciate how tiny these intensities are, consider that $2 \times 10^{-13}$ W/m² for the 4000-Hz sound is an incredibly small amount of energy. If this intensity were spread over the area of a typical eardrum (about $5 \times 10^{-5}$ m²), the total power reaching the eardrum would be only $10^{-17}$ watts—ten million trillion times smaller than a 1-watt light bulb! This extraordinary sensitivity is possible because of the sophisticated amplification mechanisms in the middle ear (the lever system of the ossicles provides about 40× pressure amplification) and the remarkable sensitivity of the hair cells in the cochlea.

The frequency-dependent threshold has evolutionary significance: the 3000-5000 Hz range, where hearing is most sensitive, corresponds to the frequency range of many warning sounds in nature and is crucial for speech comprehension, particularly for consonants. The reduced sensitivity at 200 Hz (a low frequency) means we're less distracted by low-frequency environmental sounds like distant thunder or wind, which carry little informational value. This frequency-selective sensitivity has been fine-tuned over millions of years of evolution to optimize our survival and communication capabilities.

(a) Find the intensity in watts per meter squared of a 60.0-Hz sound having a loudness of 60 phons. (b) Find the intensity in watts per meter squared of a 10 000-Hz sound having a loudness of 60 phons.

Show Solution

**Strategy**

To find the intensity for a given loudness in phons, we first use Figure 2 to find the intensity level in decibels by locating where the specified phon curve intersects the given frequency. Then we convert from decibels to intensity using $\beta = 10\log_{10}(I/I_0)$ where $I_0 = 10^{-12}$ W/m².

**Solution**

**(a) For 60.0-Hz sound at 60 phons:**

From Figure 2, the 60-phon curve at 60 Hz corresponds to approximately 82 dB.

Using the decibel formula:

$$82=10\log_{10}(I 10^{-12})$$
$$8.2=\log_{10}(I 10^{-12})$$
$$I 10^{-12}=10^{8.2}\approx 1.58\times 10^{8}$$
$$I=1.58\times 10^{8}\times 10^{-12}=1.58\times 10^{-4}\text{ W/m}^2\approx 1.6\times 10^{-4}\text{ W/m}^2$$

**(b) For 10,000-Hz sound at 60 phons:**

From Figure 2, the 60-phon curve at 10,000 Hz corresponds to approximately 60 dB (the curves are flatter in this region).

$$60=10\log_{10}(I 10^{-12})$$
$$6.0=\log_{10}(I 10^{-12})$$
$$I 10^{-12}=10^{6}$$
$$I=10^{6}\times 10^{-12}=10^{-6}\text{ W/m}^2=1.0\times 10^{-6}\text{ W/m}^2$$

**Discussion**

The results show that to achieve the same perceived loudness of 60 phons, the 60-Hz sound must have an intensity of approximately $1.6\times 10^{-4}$ W/m², while the 10,000-Hz sound needs only $1.0\times 10^{-6}$ W/m². This means the low-frequency sound requires about 160 times more intensity to sound equally loud! This dramatic difference (22 dB) illustrates why low-frequency sounds need much more power to be perceived as equally loud as mid- to high-frequency sounds.

This has profound implications for sound system design and energy consumption. Subwoofers that reproduce low-frequency sounds must handle far more power than tweeters (high-frequency speakers) to achieve balanced perceived volume across the frequency spectrum. A home theater system might use a 200-watt subwoofer alongside 50-watt mid-range and tweeter speakers precisely because of this frequency-dependent sensitivity.

From a physiological perspective, this difference arises from several factors: (1) The ear canal resonates at frequencies around 2000-4000 Hz, naturally amplifying those frequencies. (2) The ossicular chain (hammer, anvil, stirrup) in the middle ear is most efficient at transferring mid-frequency vibrations. (3) The basilar membrane in the cochlea has frequency-dependent mechanical properties that favor mid-range frequencies. (4) Low-frequency sounds have longer wavelengths that don't couple as efficiently to the small structures of the inner ear.

For hearing protection, this means that low-frequency industrial noise at 60 Hz might not seem dangerously loud even when it carries substantial acoustic power. A 82-dB sound at 60 Hz (which sounds moderately loud, like 60 phons) is approaching the threshold where extended exposure begins to pose hearing damage risk, yet it might be underestimated because it doesn't sound as loud as its 82-dB intensity level might suggest if it were at a higher frequency. Conversely, at 10,000 Hz, the ear's reduced sensitivity at high frequencies (compared to its peak at 3000-4000 Hz) means that age-related hearing loss, which typically affects high frequencies first, will make these sounds seem even quieter as people age.

A person has a hearing threshold 10 dB above normal at 100 Hz and 50 dB above normal at 4000 Hz. How much more intense must a 100-Hz tone be than a 4000-Hz tone if they are both barely audible to this person?

Show Solution

**Strategy**

We need to find the intensity ratio between the two barely audible tones. First, we'll determine the normal thresholds at each frequency from Figure 2, then add the person's hearing loss to find their actual thresholds. Finally, we'll calculate the ratio of intensities at these two threshold levels.

**Solution**

From Figure 2, the normal hearing thresholds (0-phon curve) are:

- At 100 Hz: approximately 38 dB

- At 4000 Hz: approximately –7 dB

For this person with hearing loss:

- Threshold at 100 Hz: $38 + 10 = 48\ \text{dB}$
- Threshold at 4000 Hz: $-7 + 50 = 43\ \text{dB}$

The difference in threshold levels is:

$$\Delta\beta = 48 - 43 = 5\ \text{dB}$$

To find the intensity ratio:

$$\Delta\beta = 10\log_{10}(I_{100} I_{4000})$$
$$5 = 10\log_{10}(I_{100} I_{4000})$$
$$0.5 = \log_{10}(I_{100} I_{4000})$$
$$I_{100} I_{4000} = 10^{0.5} = \sqrt{10} \approx 3.16$$

Rounding to 2 significant figures: $I_{100} I_{4000} \approx 2.5$

**Discussion**

The 100-Hz tone must be about 2.5 times more intense than the 4000-Hz tone for both to be barely audible to this person. At first glance, this result might seem counterintuitive—the person has much greater hearing loss at 4000 Hz (50 dB) than at 100 Hz (10 dB), yet the 100-Hz tone needs to be more intense. This apparent paradox is explained by the normal ear's frequency-dependent sensitivity.

For a person with normal hearing, a 100-Hz tone needs to be much more intense than a 4000-Hz tone to be audible (38 dB vs. –7 dB, a difference of 45 dB). This person's hearing loss partially "compensates" for this normal frequency dependence. The 10-dB loss at 100 Hz brings their threshold to 48 dB, while the 50-dB loss at 4000 Hz brings that threshold to 43 dB. The net result is that the person's thresholds are now only 5 dB apart, compared to 45 dB apart for normal hearing.

This problem illustrates an important concept in audiology: **the pattern of hearing loss matters as much as the magnitude**. This person's hearing loss pattern is typical of **noise-induced hearing loss (NIHL)**, which characteristically causes a pronounced dip in sensitivity around 3000-6000 Hz, with the worst loss typically at 4000 Hz. This distinctive "4000-Hz notch" on an audiogram is a hallmark of noise exposure from sources like gunfire, power tools, or loud music. The 10-dB loss at 100 Hz represents mild hearing loss, while the 50-dB loss at 4000 Hz represents moderate to severe hearing loss.

The clinical implications are significant: (1) This person would have particular difficulty understanding speech, because consonant sounds (which carry much of speech's intelligibility) fall in the 2000-8000 Hz range where their hearing is most impaired. (2) They might not notice their hearing problem in quiet environments, because the low-frequency vowel sounds would still be audible. (3) A hearing aid would need to provide frequency-specific amplification—much more gain at 4000 Hz than at 100 Hz. (4) This hearing loss pattern is often preventable through proper hearing protection in noisy environments. The fact that noise-induced hearing loss shows up first around 4000 Hz is thought to be related to the mechanical properties of the cochlea and the specific resonances in the ear canal. Unfortunately, hair cell damage at this frequency is permanent and irreversible, emphasizing the critical importance of hearing protection in occupational and recreational settings with high noise levels.

A child has a hearing loss of 60 dB near 5000 Hz, due to noise exposure, and normal hearing elsewhere. How much more intense is a 5000-Hz tone than a 400-Hz tone if they are both barely audible to the child?

Show Solution

**Strategy**

We need to find the intensity ratio between barely audible tones at 5000 Hz and 400 Hz for this child. First, we determine the normal thresholds at each frequency from Figure 2. The child has normal hearing at 400 Hz but a 60-dB loss at 5000 Hz, so we add 60 dB to the normal threshold at 5000 Hz to find the child's threshold there. Then we calculate the intensity ratio.

**Solution**

From Figure 2, the normal hearing thresholds (0-phon curve) are:

- At 400 Hz: approximately 9 dB
- At 5000 Hz: approximately –5 dB (near the ear's peak sensitivity)

For this child:

- Threshold at 400 Hz: **9 dB** (normal hearing)
- Threshold at 5000 Hz: $-5 + 60 = 55\ \text{dB}$ (with 60-dB loss)

The difference in threshold levels is:

$$\Delta\beta = 55 - 9 = 46\ \text{dB}$$

To find the intensity ratio:

$$\Delta\beta = 10\log_{10}(I_{5000} I_{400})$$

$$46 = 10\log_{10}(I_{5000} I_{400})$$
$$4.6 = \log_{10}(I_{5000} I_{400})$$
$$I_{5000} I_{400} = 10^{4.6} \approx 39,811$$

Rounding to 2 significant figures: $I_{5000} I_{400} \approx 4.0 \times 10^4$

**Discussion**

The 5000-Hz tone must be approximately 40,000 times more intense than the 400-Hz tone for both to be barely audible to this child! This dramatic result illustrates the devastating impact of noise-induced hearing loss on a young person. For a child with normal hearing, a 5000-Hz tone would actually need to be *less* intense than a 400-Hz tone to be equally audible (because 5000 Hz is near the ear's peak sensitivity). But this child's 60-dB hearing loss at 5000 Hz completely reverses this relationship.

This scenario is tragically realistic and increasingly common. The most likely source of such severe, frequency-specific hearing loss in a child is acoustic trauma from a single very loud impulse noise—exactly as suggested by the mention of "noise exposure" in Figure 4's caption about "a child who suffered hearing loss due to a cap gun." Cap guns, firecrackers, and firearms can produce peak sound levels of 140-170 dB, well above the threshold for immediate permanent damage. A single exposure to such a loud impulse can destroy hair cells in the region of the cochlea that responds to frequencies around 4000-6000 Hz, creating the characteristic "noise notch" seen in this child's hearing.

The implications for this child are serious and permanent: (1) **Speech comprehension difficulties**: Many consonant sounds crucial for understanding speech (like 's', 'f', 'th', 'sh') fall in the 4000-8000 Hz range where the child has severe loss. The child might hear that someone is speaking but struggle to understand the words, especially in noisy environments like classrooms. (2) **Educational impact**: This hearing loss can significantly affect learning, particularly in reading and language development. Early intervention with appropriate hearing aids and possible educational support services is crucial. (3) **Irreversibility**: The damaged hair cells cannot regenerate; this hearing loss is permanent. (4) **Prevention is critical**: This tragedy was entirely preventable through proper supervision and hearing protection.

From a public health perspective, this problem highlights why organizations like the American Academy of Pediatrics recommend against children using cap guns and similar toys, and why hearing protection is essential when children are exposed to loud sounds. The problem also mentions this is "due to noise exposure" and occurs "near 5000 Hz"—this frequency-specific damage is a hallmark of acoustic trauma and differs markedly from the more gradual, high-frequency losses seen in presbycusis (age-related hearing loss) or the broader-spectrum losses from genetic conditions. The good news is that modern digital hearing aids can provide frequency-specific amplification, giving substantial boost at 5000 Hz while providing minimal amplification at 400 Hz where hearing remains normal, though they cannot fully restore the loss.

What is the ratio of intensities of two sounds of identical frequency if the first is just barely discernible as louder to a person than the second?

Show Solution

**Strategy**

According to the text, at a given frequency, it is possible to discern differences of about 1 dB in sound intensity level. This represents the just-noticeable difference (JND) in loudness. We need to find what intensity ratio corresponds to a 1-dB difference in sound level.

**Solution**

For the minimum discernible difference in loudness:

$$\Delta\beta = 1\text{ dB}$$

Using the relationship between decibel difference and intensity ratio:

$$\Delta\beta = 10\log_{10}(I_1 I_2)$$
$$1 = 10\log_{10}(I_1 I_2)$$
$$0.1 = \log_{10}(I_1 I_2)$$
$$I_1 I_2 = 10^{0.1} \approx 1.259 \approx 1.26$$

**Discussion**

The ratio of intensities is approximately 1.26, meaning the louder sound must be about 26% more intense than the quieter sound for a person to just barely detect the difference. This surprisingly small ratio demonstrates the remarkable sensitivity of human hearing to changes in sound intensity. An increase of just 26% in intensity—corresponding to 1 dB—is perceptible under ideal listening conditions with careful attention.

This just-noticeable difference (JND) is a fundamental concept in psychoacoustics and has important practical applications: (1) **Audio equipment design**: High-quality volume controls are designed to provide steps smaller than 1 dB to allow for smooth, nearly continuous adjustment. Controls with larger steps (2-3 dB) make it difficult to find the "just right" volume level. (2) **Sound level regulations**: Occupational safety standards often specify sound limits in whole decibels because differences smaller than 1 dB are difficult to measure reliably and may not be perceptually significant. (3) **Acoustic treatment**: In architectural acoustics, modifications that reduce sound levels by less than 1 dB are generally not worth the cost, as they won't be noticed.

It's important to note that the 1-dB JND represents optimal conditions. The text states that "a change of 3 dB is easily noticed," implying that 1 dB requires attentive listening. In everyday situations with background noise or divided attention, the practical JND is often closer to 2-3 dB. Additionally, the JND can vary slightly with frequency and absolute sound level—our discrimination ability is generally best at moderate sound levels in the mid-frequency range.

The mathematical form of this relationship shows that loudness perception follows Weber's Law, a general principle in psychophysics stating that the JND is proportional to the stimulus magnitude. In the case of hearing, because we perceive logarithmically, a constant ratio of intensities (1.26:1) produces a constant perceptual difference (1 dB) regardless of the absolute intensity levels. This is why the decibel scale works so well for describing sound—it matches how our hearing system actually processes intensity information. The factor of 1.26 might seem arbitrary, but it emerges naturally from the base-10 logarithm and the definition of the decibel (one-tenth of a Bel). If we used natural logarithms instead, we'd get different numbers, but the underlying perceptual relationship would be the same.

## Glossary

loudness
: the perception of sound intensity

timbre
: number and relative intensity of multiple sound frequencies

note
: basic unit of music with specific names, combined to generate tunes

tone
: number and relative intensity of multiple sound frequencies

phon
: the numerical unit of loudness

ultrasound
: sounds above 20 000 Hz

infrasound
: sounds below 20 Hz

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# Ultrasound

- Define acoustic impedance and intensity reflection coefficient.
- Describe medical and other uses of ultrasound technology.
- Calculate acoustic impedance using density values and the speed of ultrasound.
- Calculate the velocity of a moving object using Doppler-shifted ultrasound.


Ultrasound is used in medicine to painlessly and noninvasively monitor patient health and diagnose a wide range of disorders. (credit: abbybatchelder, Flickr)

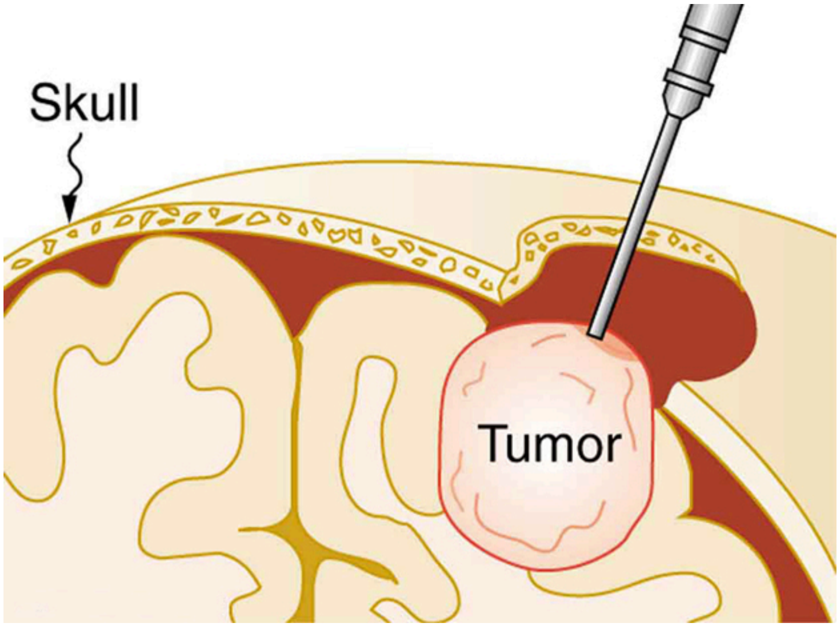
Any sound with a frequency above 20 000 Hz (or 20 kHz)—that is, above the highest audible frequency—is defined to be ultrasound. In practice, it is possible to create ultrasound frequencies up to more than a gigahertz. (Higher frequencies are difficult to create; furthermore, they propagate poorly because they are very strongly absorbed.) Ultrasound has a tremendous number of applications, which range from burglar alarms to use in cleaning delicate objects to the guidance systems of bats. We begin our discussion of ultrasound with some of its applications in medicine, in which it is used extensively both for diagnosis and for therapy.

Characteristics of Ultrasound

The characteristics of ultrasound, such as frequency and intensity, are wave properties common to all types of waves. Ultrasound also has a wavelength that limits the fineness of detail it can detect. This characteristic is true of all waves. We can never observe details significantly smaller than the wavelength of our probe; for example, we will never see individual atoms with visible light, because the atoms are so small compared with the wavelength of light.

### Ultrasound in Medical Therapy

Ultrasound, like any wave, carries energy that can be absorbed by the medium carrying it, producing effects that vary with intensity. When focused to intensities of $10^3$ to $10^5 \text{W/m}^2$ , ultrasound can be used to shatter gallstones or pulverize cancerous tissue in surgical procedures. (See [Figure 2].) Intensities this great can damage individual cells, variously causing their protoplasm to stream inside them, altering their permeability, or rupturing their walls through *cavitation*. Cavitation is the creation of vapor cavities in a fluid—the longitudinal vibrations in ultrasound alternatively compress and expand the medium, and at sufficient amplitudes the expansion separates molecules. Most cavitation damage is done when the cavities collapse, producing even greater shock pressures.




The tip of this small probe oscillates at 23 kHz with such a large amplitude that it pulverizes tissue on contact. The debris is then aspirated. The speed of the tip may exceed the speed of sound in tissue, thus creating shock waves and cavitation, rather than a smooth simple harmonic oscillator–type wave.

Most of the energy carried by high-intensity ultrasound in tissue is converted to thermal energy. In fact, intensities of $10^3$ to $10^4 \text{W/m}^2$ are commonly used for deep-heat treatments called ultrasound diathermy. Frequencies of 0.8 to 1 MHz are typical. In both athletics and physical therapy, ultrasound diathermy is most often applied to injured or overworked muscles to relieve pain and improve flexibility. Skill is needed by the therapist to avoid "bone burns" and other tissue damage caused by overheating and cavitation, sometimes made worse by reflection and focusing of the ultrasound by joint and bone tissue.

In some instances, you may encounter a different decibel scale, called the sound *pressure* level, when ultrasound travels in water or in human and other biological tissues. We shall not use the scale here, but it is notable that numbers for sound pressure levels range 60 to 70 dB higher than you would quote for $\beta$ , the sound intensity level used in this text. Should you encounter a sound pressure level of 220 decibels, then, it is not an astronomically high intensity, but equivalent to about 155 dB—high enough to destroy tissue, but not as unreasonably high as it might seem at first.

### Ultrasound in Medical Diagnostics

When used for imaging, ultrasonic waves are emitted from a transducer, a crystal exhibiting the piezoelectric effect (the expansion and contraction of a substance when a voltage is applied across it, causing a vibration of the crystal). These high-frequency vibrations are transmitted into any tissue in contact with the transducer. Similarly, if a pressure is applied to the crystal (in the form of a wave reflected off tissue layers), a voltage is produced which can be recorded. The crystal therefore acts as both a transmitter and a receiver of sound. Ultrasound is also partially absorbed by tissue on its path, both on its journey away from the transducer and on its return journey. From the time between when the original signal is sent and when the reflections from various boundaries between media are received, (as well as a measure of the intensity loss of the signal), the nature and position of each boundary between tissues and organs may be deduced.

Reflections at boundaries between two different media occur because of differences in a characteristic known as the **acoustic impedance** $Z$ of each substance. Impedance is defined as

$$Z = \rho v,$$

where $\rho$ is the density of the medium (in $\text{kg/m}^3$ ) and $v$ is the speed of sound through the medium (in m/s). The units for $Z$ are therefore $\text{kg/m}^2 \cdot \text{s}$.

[Table 1] shows the density and speed of sound through various media (including various soft tissues) and the associated acoustic impedances. Note that the acoustic impedances for soft tissue do not vary much but that there is a big difference between the acoustic impedance of soft tissue and air and also between soft tissue and bone.

The Ultrasound Properties of Various Media, Including Soft Tissue Found in the Body

| Medium | Density ($\text{kg/m}^3$) | Speed of Ultrasound (m/s) | Acoustic Impedance ($\text{kg/(m}^2 \cdot \text{s)}$) |
|---|---|---|---|
| Air | 1.3 | 330 | 429 |
| Water | 1000 | 1500 | $1.5 \times 10^6$ |
| Blood | 1060 | 1570 | $1.66 \times 10^6$ |
| Fat | 925 | 1450 | $1.34 \times 10^6$ |
| Muscle (average) | 1075 | 1590 | $1.70 \times 10^6$ |

| Medium | Density ($kg/m^3$) | Speed of Ultrasound (m/s) | Acoustic Impedance ($kg/(m^2 \cdot s)$) |
|---|---|---|---|
| Bone (varies) | 1400–1900 | 4080 | $5.7 \times 10^6$ to $7.8 \times 10^6$ |
| Barium titanate (transducer material) | 5600 | 5500 | $30.8 \times 10^6$ |

At the boundary between media of different acoustic impedances, some of the wave energy is reflected and some is transmitted. The greater the **difference** in acoustic impedance between the two media, the greater the reflection and the smaller the transmission.

The **intensity reflection coefficient** $a$ is defined as the ratio of the intensity of the reflected wave relative to the incident ( transmitted) wave. This statement can be written mathematically as

$$a = (Z_2 - Z_1)^2 (Z_1 + Z_2)^2,$$

where $Z_1$ and $Z_2$ are the acoustic impedances of the two media making up the boundary. A reflection coefficient of zero (corresponding to total transmission and no reflection) occurs when the acoustic impedances of the two media are the same. An impedance "match" (no reflection) provides an efficient coupling of sound energy from one medium to another. The image formed in an ultrasound is made by tracking reflections (as shown in [Figure 3]) and mapping the intensity of the reflected sound waves in a two-dimensional plane.

Calculate Acoustic Impedance and Intensity Reflection Coefficient: Ultrasound and Fat Tissue

(a) Using the values for density and the speed of ultrasound given in [Table 1], show that the acoustic impedance of fat tissue is indeed $1.34 \times 10^6 \text{kg/m}^2 \cdot \text{s}$.

(b) Calculate the intensity reflection coefficient of ultrasound when going from fat to muscle tissue.

**Strategy for (a)**

The acoustic impedance can be calculated using $Z = \rho v$ and the values for $\rho$ and $v$ found in [Table 1].

**Solution for (a)**

(1) Substitute known values from [Table 1] into $Z = \rho v$.

$$Z = \rho v = (925 \text{kg/m}^3)(1450 \text{m/s})$$

(2) Calculate to find the acoustic impedance of fat tissue.

$$1.34 \times 10^6 \text{kg/m}^2 \cdot \text{s}$$

This value is the same as the value given for the acoustic impedance of fat tissue.

**Strategy for (b)**

The intensity reflection coefficient for any boundary between two media is given by $a = (Z_2 - Z_1)^2 (Z_1 + Z_2)^2$, and the acoustic impedance of muscle is given in [Table 1].

**Solution for (b)**

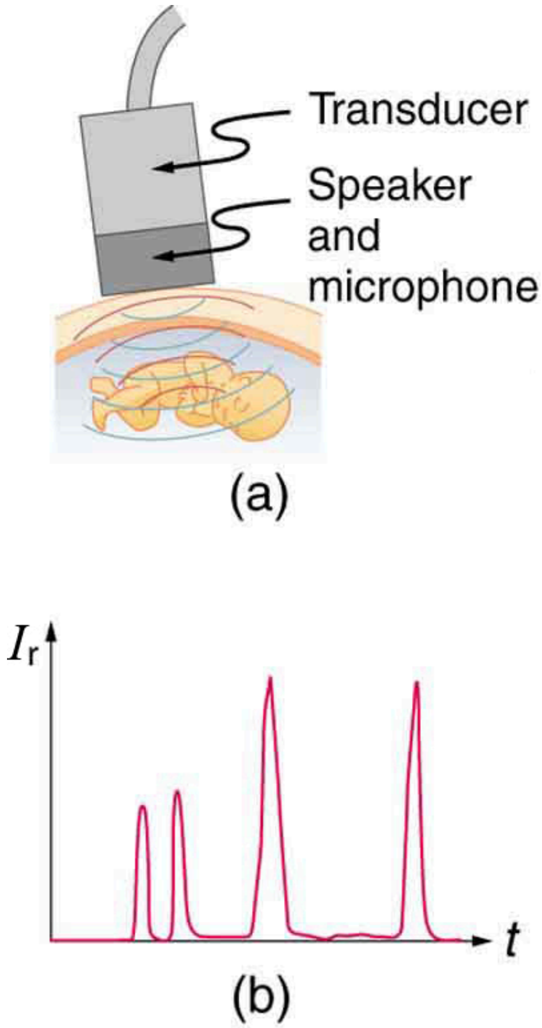
Substitute known values into $a = (Z_2 - Z_1)^2 (Z_1 + Z_2)^2$ to find the intensity reflection coefficient:

$$a = (Z_2 - Z_1)^2 (Z_1 + Z_2)^2 = (1.34 \times 10^6 \text{kg/m}^2 \cdot \text{s} - 1.70 \times 10^6 \text{kg/m}^2 \cdot \text{s})^2 (1.70 \times 10^6 \text{kg/m}^2 \cdot \text{s} + 1.34 \times 10^6 \text{kg/m}^2 \cdot \text{s})^2 = 0.014$$

**Discussion**

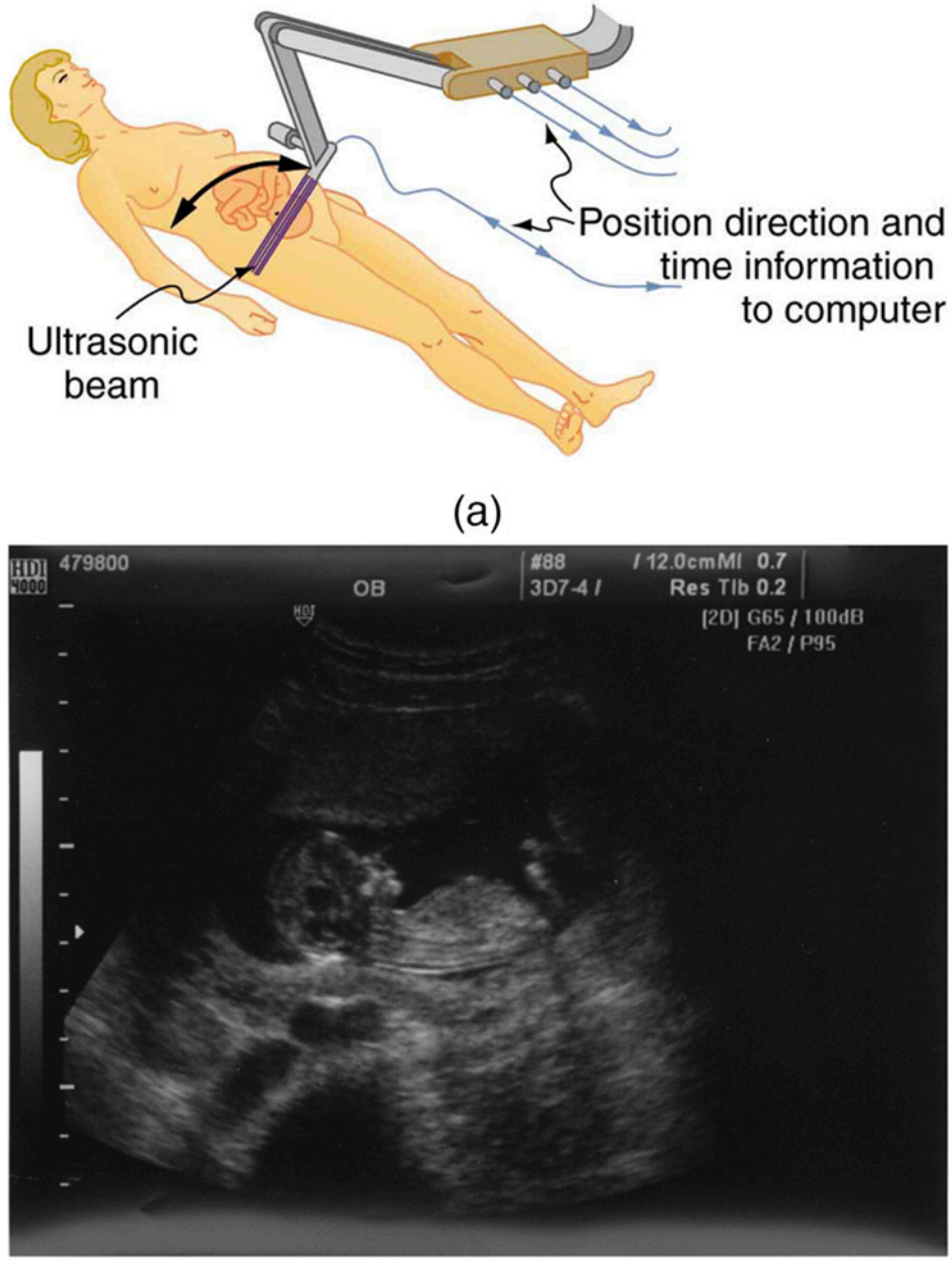
This result means that only 1.4% of the incident intensity is reflected, with the remaining being transmitted.

The applications of ultrasound in medical diagnostics have produced untold benefits with no known risks. Diagnostic intensities are too low (about $10^{-2} \text{W/m}^2$ ) to cause thermal damage. More significantly, ultrasound has been in use for several decades and detailed follow-up studies do not show evidence of ill effects, quite unlike the case for X-rays.




(a) An ultrasound speaker doubles as a microphone. Brief bleeps are broadcast, and echoes are recorded from various depths. (b) Graph of echo intensity versus time. The time for echoes to return is directly proportional to the distance of the reflector, yielding this information noninvasively.

The most common ultrasound applications produce an image like that shown in [Figure 4]. The speaker-microphone broadcasts a directional beam, sweeping the beam across the area of interest. This is accomplished by having multiple ultrasound sources in the probe's head, which are phased to interfere constructively in a given, adjustable direction. Echoes are measured as a function of position as well as depth. A computer constructs an image that reveals the shape and density of internal structures.




(a) An ultrasonic image is produced by sweeping the ultrasonic beam across the area of interest, in this case the woman's abdomen. Data are recorded and analyzed in a computer, providing a two-dimensional image. (b) Ultrasound image of 12-week-old fetus. (credit: Margaret W. Carruthers, Flickr)

How much detail can ultrasound reveal? The image in [Figure 4] is typical of low-cost systems, but that in [Figure 5] shows the remarkable detail possible with more advanced systems, including 3D imaging. Ultrasound today is commonly used in prenatal care. Such imaging can be used to see if the fetus is developing at a normal rate, and help in the determination of serious problems early in the pregnancy. Ultrasound is also in wide use to image the chambers of the heart and the flow of blood within the beating heart, using the Doppler effect (echocardiology).

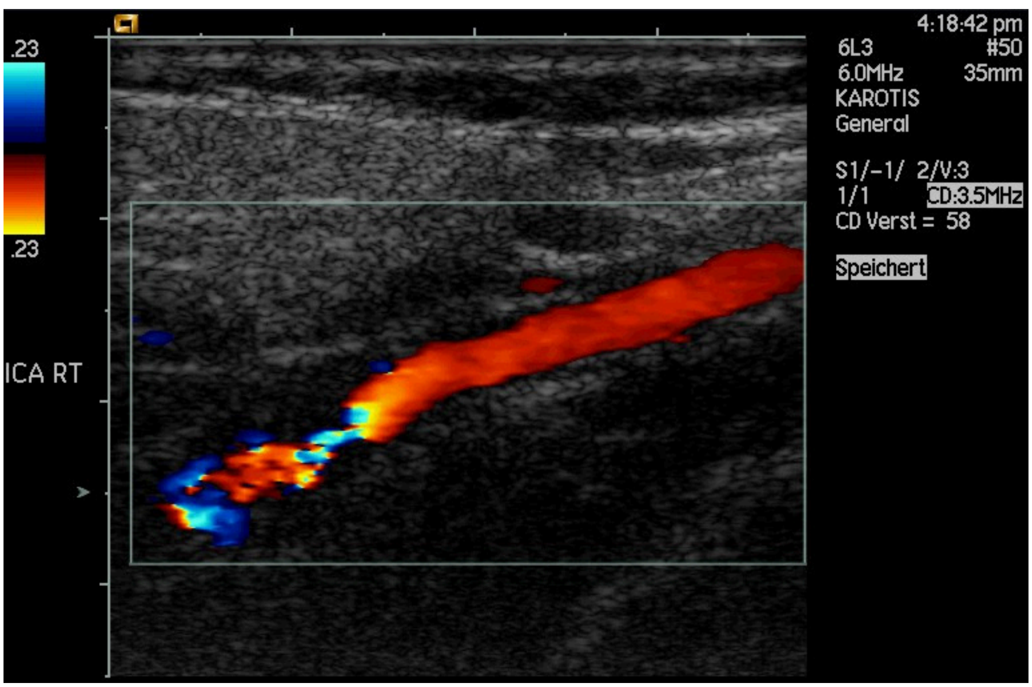
Whenever a wave is used as a probe, it is very difficult to detect details smaller than its wavelength $\lambda$ . Indeed, current technology cannot do quite this well. Abdominal scans may use a 7-MHz frequency, and the speed of sound in tissue is about 1540 m/s—so the wavelength limit to detail would be

$\lambda = \frac{v_w}{f} = \frac{1540 \text{ m/s}}{7 \times 10^6 \text{ Hz}} = 0.22 \text{ mm}$ . In practice, 1-mm detail is attainable, which is sufficient for many purposes. Higher-frequency ultrasound would allow greater detail, but it does not penetrate as well as lower frequencies do. The accepted rule of thumb is that you can effectively scan to a depth of about $500\lambda$ into tissue. For 7 MHz, this penetration limit is $500 \times 0.22 \text{ mm}$ , which is 0.11 m. Higher frequencies may be employed in smaller organs, such as the eye, but are not practical for looking deep into the body.


A 3D ultrasound image of a fetus. As well as for the detection of any abnormalities, such scans have also been shown to be useful for strengthening the emotional bonding between parents and their unborn child. (credit: Jennie Cu, Wikimedia Commons)

In addition to shape information, ultrasonic scans can produce density information superior to that found in X-rays, because the intensity of a reflected sound is related to changes in density. Sound is most strongly reflected at places where density changes are greatest.

Another major use of ultrasound in medical diagnostics is to detect motion and determine velocity through the Doppler shift of an echo, known as **Doppler-shifted ultrasound**. This technique is used to monitor fetal heartbeat, measure blood velocity, and detect occlusions in blood vessels, for example. ( See [Figure 6].) The magnitude of the Doppler shift in an echo is directly proportional to the velocity of whatever reflects the sound. Because an echo is involved, there is actually a double shift. The first occurs because the reflector (say a fetal heart) is a moving observer and receives a Doppler-shifted frequency. The reflector then acts as a moving source, producing a second Doppler shift.




This Doppler-shifted ultrasonic image of a partially occluded artery uses color to indicate velocity. The highest velocities are in red, while the lowest are blue. The blood must move faster through the constriction to carry the same flow. (credit: Arning C, Grzyska U, Wikimedia Commons)

A clever technique is used to measure the Doppler shift in an echo. The frequency of the echoed sound is superimposed on the broadcast frequency, producing beats. The beat frequency is $F_B = |f_1 - f_2|$ , and so it is directly proportional to the Doppler shift ( $f_1 - f_2$ ) and hence, the reflector's velocity. The advantage in this technique is that the Doppler shift is small (because the reflector's velocity is small), so that great accuracy would be needed to measure the shift directly. But measuring the beat frequency is easy, and it is not affected if the broadcast frequency varies somewhat. Furthermore, the beat frequency is in the audible range and can be amplified for audio feedback to the medical observer.

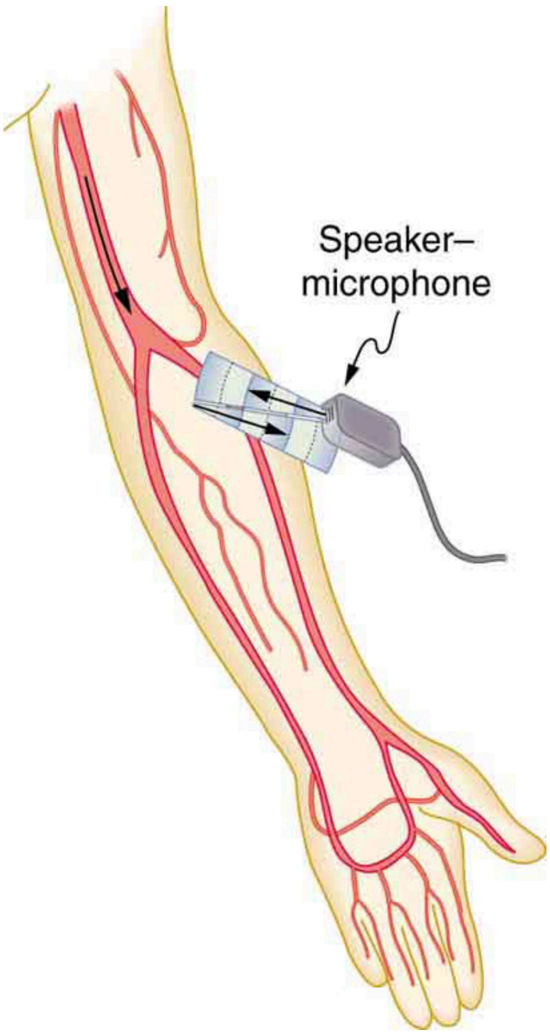
Uses for Doppler-Shifted Radar

Doppler-shifted radar echoes are used to measure wind velocities in storms as well as aircraft and automobile speeds. The principle is the same as for Doppler-shifted ultrasound. There is evidence that bats and dolphins may also sense the velocity of an object (such as prey) reflecting their ultrasound signals by observing its Doppler shift.

Calculate Velocity of Blood: Doppler-Shifted Ultrasound

Ultrasound that has a frequency of 2.50 MHz is sent toward blood in an artery that is moving toward the source at 20.0 cm/s, as illustrated in [Figure 7. Use the speed of sound in human tissue as 1540 m/s. (Assume that the frequency of 2.50 MHz is accurate to seven significant figures.)

1. What frequency does the blood receive?
2. What frequency returns to the source?
3. What beat frequency is produced if the source and returning frequencies are mixed? { type="a"}




Ultrasound is partly reflected by blood cells and plasma back toward the speaker-microphone. Because the cells are moving, two Doppler shifts are produced—one for blood as a moving observer, and the other for the reflected sound coming from a moving source. The magnitude of the shift is directly proportional to blood velocity.

**Strategy**

The first two questions can be answered using $f_{obs} = f_s\left(\frac{v_w}{v_w \pm v_s}\right)$ and $f_{obs} = f_s\left(\frac{v_w \pm v_{obs}}{v_w}\right)$ for the Doppler shift. The last question asks for beat frequency, which is the difference between the original and returning frequencies.

**Solution for (a)**

(1) Identify knowns:

- The blood is a moving observer, and so the frequency it receives is given by $f_{obs} = f_s\left(\frac{v_w \pm v_{obs}}{v_w}\right)$.

- $v_b$ is the blood velocity ( $v_{obs}$ here) and the plus sign is chosen because the motion is toward the source.

(2) Enter the given values into the equation.

$$f_{obs}=(2500000 Hz)\left(\frac{1540 m/s+0.2 m/s}{1540 m/s}\right)$$

(3) Calculate to find the frequency: 2 500 325 Hz.

**Solution for (b)**

(1) Identify knowns:

- The blood acts as a moving source.
- The microphone acts as a stationary observer.
- The frequency leaving the blood is 2 500 325 Hz, but it is shifted upward as given by $f_{obs}=f_s\left(\frac{v_w}{v_w-v_b}\right)$.

  $f_{obs}$ is the frequency received by the speaker-microphone.

- The source velocity is $v_b$ .

- The minus sign is used because the motion is toward the observer.

The minus sign is used because the motion is toward the observer.

(2) Enter the given values into the equation:

$$f_{obs}=(2500325 Hz)\left(\frac{1540 m/s}{1540 m/s-0.200 m/s}\right)$$

(3) Calculate to find the frequency returning to the source: 2 500 649 Hz.

**Solution for (c)**

(1) Identify knowns:

- The beat frequency is simply the absolute value of the difference between $f_s$ and $f_{obs}$ , as stated in: $f_B=|f_{obs}-f_s|$.

(2) Substitute known values:

$$|2500649 Hz-2500000 Hz|$$

(3) Calculate to find the beat frequency: 649 Hz.

**Discussion**

The Doppler shifts are quite small compared with the original frequency of 2.50 MHz. It is far easier to measure the beat frequency than it is to measure the echo frequency with an accuracy great enough to see shifts of a few hundred hertz out of a couple of megahertz. Furthermore, variations in the source frequency do not greatly affect the beat frequency, because both $f_s$ and $f_{obs}$ would increase or decrease. Those changes subtract out in $f_B=|f_{obs}-f_s|$.

Industrial and Other Applications of Ultrasound

Industrial, retail, and research applications of ultrasound are common. A few are discussed here. Ultrasonic cleaners have many uses. Jewelry, machined parts, and other objects that have odd shapes and crevices are immersed in a cleaning fluid that is agitated with ultrasound typically about 40 kHz in frequency. The intensity is great enough to cause cavitation, which is responsible for most of the cleansing action. Because cavitation-produced shock pressures are large and well transmitted in a fluid, they reach into small crevices where even a low-surface-tension cleaning fluid might not penetrate.

Sonar is a familiar application of ultrasound. Sonar typically employs ultrasonic frequencies in the range from 30.0 to 100 kHz. Bats, dolphins, submarines, and even some birds use ultrasonic sonar. Echoes are analyzed to give distance and size information both for guidance and finding prey. In most sonar applications, the sound reflects quite well because the objects of interest have significantly different density than the medium in which they travel. When the Doppler shift is observed, velocity information can also be obtained. Submarine sonar can be used to obtain such information, and there is evidence that some bats also sense velocity from their echoes.

Similarly, there are a range of relatively inexpensive devices that measure distance by timing ultrasonic echoes. Many cameras, for example, use such information to focus automatically. Some doors open when their ultrasonic ranging devices detect a nearby object, and certain home security lights turn on when their ultrasonic rangers observe motion. Ultrasonic “measuring tapes” also exist to measure such things as room dimensions. Sinks in public restrooms are sometimes automated with ultrasound devices to turn faucets on and off when people wash their hands. These devices reduce the spread of germs and can conserve water.

Ultrasound is used for nondestructive testing in industry and by the military. Because ultrasound reflects well from any large change in density, it can reveal cracks and voids in solids, such as aircraft wings, that are too small to be seen with X-rays. For similar reasons, ultrasound is also good for measuring the thickness of coatings, particularly where there are several layers involved.

Basic research in solid state physics employs ultrasound. Its attenuation is related to a number of physical characteristics, making it a useful probe. Among these characteristics are structural changes such as those found in liquid crystals, the transition of a material to a superconducting phase, as well as density and other properties.

These examples of the uses of ultrasound are meant to whet the appetites of the curious, as well as to illustrate the underlying physics of ultrasound. There are many more applications, as you can easily discover for yourself.
Check Your Understanding

Why is it possible to use ultrasound both to observe a fetus in the womb and also to destroy cancerous tumors in the body?

Show Solution

Ultrasound can be used medically at different intensities. Lower intensities do not cause damage and are used for medical imaging. Higher intensities can pulverize and destroy targeted substances in the body, such as tumors.

## Section Summary

- The acoustic impedance is defined as:
  $Z=\rho v,$

  $\rho$ is the density of a medium through which the sound travels and $v$ is the speed of sound through that medium.

- The intensity reflection coefficient $a$ , a measure of the ratio of the intensity of the wave reflected off a boundary between two media relative to the intensity of the incident wave, is given by
  $a=(Z_2-Z_1)^2(Z_1+Z_2)^2.$
- The intensity reflection coefficient is a unitless quantity.

## Conceptual Questions

If audible sound follows a rule of thumb similar to that for ultrasound, in terms of its absorption, would you expect the high or low frequencies from your neighbor’s stereo to penetrate into your house? How does this expectation compare with your experience?

Show Solution

Higher frequencies are more readily absorbed than lower frequencies. Therefore, you would expect the **low frequencies** (the bass) from a neighbor’s stereo to penetrate into your house more effectively than the high frequencies.

This expectation matches common experience. It is often the case that you can hear the “thump-thump” of the bass from a neighbor’s stereo, while the higher-frequency melody and vocals are muffled or inaudible.

Elephants and whales are known to use infrasound to communicate over very large distances. What are the advantages of infrasound for long distance communication?

Show Solution

The primary advantage of infrasound for long-distance communication is its **low rate of absorption**. Lower-frequency sounds are less readily absorbed by the medium they travel through (like water or air) than higher-frequency sounds. This allows infrasound waves to travel much longer distances before their energy dissipates, making them ideal for communication over many kilometers.

It is more difficult to obtain a high-resolution ultrasound image in the abdominal region of someone who is overweight than for someone who has a slight build. Explain why this statement is accurate.

Show Solution

This statement is accurate because ultrasound waves are attenuated (absorbed and scattered) as they pass through tissue. In an overweight person, the ultrasound waves must travel through a thicker layer of subcutaneous fat to reach the abdominal organs. Fat tissue absorbs ultrasound energy, so the signal is weakened on its way to the organs and again on its way back to the transducer. This results in a weaker echo and a lower signal-to-noise ratio, which degrades the quality and resolution of the image.

Suppose you read that 210-dB ultrasound is being used to pulverize cancerous tumors. You calculate the intensity in watts per centimeter squared and find it is unreasonably high ( $10^5 \text{W/cm}^2$ ). What is a possible explanation?

Show Solution

A possible explanation is that the 210-dB value is being reported using the **sound pressure level (SPL)** scale, which is commonly used in medical and underwater acoustics, rather than the **sound intensity level (SIL)** scale used in this textbook. The SPL scale uses a different reference pressure and can result in decibel values that are 60-70 dB higher than the SIL for the same sound wave in tissue.

Therefore, a 210-dB SPL is equivalent to roughly $210 - 65 = 145$ dB on the SIL scale. An intensity level of 145 dB is still very high, but it is a much more reasonable value for therapeutic ultrasound used to destroy tissue.

## Problems & Exercises

**Unless otherwise indicated, for problems in this section, assume that the speed of sound through human tissues is 1540 m/s.**

What is the sound intensity level in decibels of ultrasound of intensity $10^5 \text{W/m}^2$ , used to pulverize tissue during surgery?

Show Solution

**Strategy**

The sound intensity level in decibels is calculated using the logarithmic formula $\beta(\text{dB}) = 10\log_{10}(I\,I_0)$ , where $I_0 = 10^{-12}\text{W/m}^2$ is the threshold of hearing. This relationship allows us to express the enormous range of sound intensities encountered in medical applications on a manageable scale.

**Solution**

Identify the known values:

- Intensity: $I = 10^5\text{W/m}^2$
- Reference intensity: $I_0 = 10^{-12}\text{W/m}^2$

Substitute into the decibel formula:

$$\beta = 10\log_{10}(10^5\text{W/m}^2 10^{-12}\text{W/m}^2) = 10\log_{10}(10^{17})$$

Using the logarithm property $\log_{10}(10^x) = x$ :

$$\beta = 10(17) = 170 \text{ dB}$$

**Discussion**

This intensity level of 170 dB represents an extremely high energy concentration, approximately $10^{17}$ times greater than the threshold of human hearing. Such intensities are used therapeutically to destroy unwanted tissue through several mechanisms: direct mechanical disruption of cellular structures, cavitation (formation and violent collapse of vapor bubbles), and localized heating. This intensity is about $10^{10}$ times greater than diagnostic ultrasound (typically $10^{-2}\text{W/m}^2$ or 103 dB), ensuring that imaging procedures are safe while therapeutic applications can achieve the desired tissue destruction. The focused nature of therapeutic ultrasound allows surgeons to target specific tissues (such as gallstones or tumors) while minimizing damage to surrounding healthy tissue. At these intensities, exposure time must be carefully controlled to prevent unintended damage.

Is 155-dB ultrasound in the range of intensities used for deep heating? Calculate the intensity of this ultrasound and compare this intensity with values quoted in the text.

Show Solution

**Strategy**

We need to convert the decibel level to intensity using the inverse of the decibel formula. The relationship is $I = I_0 \times 10^{\beta/10}$ , where $\beta$ is in decibels and $I_0 = 10^{-12}\text{W/m}^2$ . We can then compare this calculated intensity to the range stated in the text for ultrasound diathermy (deep-heat treatments).

**Solution**

Start with the decibel formula and solve for intensity:

$$\beta = 10\log_{10}(I\,I_0)$$

Divide both sides by 10:

$$\beta 10 = \log_{10}(I\,I_0)$$

Take the antilogarithm (raise 10 to both sides):

$$10^{\beta/10} = I\,I_0$$

Solve for $I$ :

$$I = I_0 \times 10^{\beta/10}$$

Substitute $\beta = 155$ dB and $I_0 = 10^{-12}\text{W/m}^2$ :

$I = (10^{-12}\,\text{W/m}^2) \times 10^{155/10} = 10^{-12} \times 10^{15.5}\,\text{W/m}^2$
$I = 10^{3.5}\,\text{W/m}^2 \approx 3.16 \times 10^3\,\text{W/m}^2$

**Discussion**

Yes, 155-dB ultrasound is indeed in the appropriate range for deep-heat treatments. The text states that ultrasound diathermy uses intensities of $10^3$ to $10^4\,\text{W/m}^2$ , and our calculated value of $3.16\times10^3\,\text{W/m}^2$ falls right in the middle of this range. This intensity is high enough to deposit significant thermal energy deep within tissue (hence "deep heating"), causing the tissue temperature to rise by a few degrees Celsius. This heating effect increases blood flow, reduces muscle tension, and promotes healing—which is why ultrasound diathermy is commonly used in physical therapy and sports medicine for treating muscle injuries. The intensity is much lower than that used for tissue destruction ($10^5\,\text{W/m}^2$ ) but much higher than diagnostic imaging ($10^{-2}\,\text{W/m}^2$ ), placing it firmly in the therapeutic heating range. At these intensities, careful application by trained therapists is necessary to avoid "bone burns" and cavitation damage, particularly near joints where reflection and focusing can occur.

Find the sound intensity level in decibels of $2.00\times10^{-2}\,\text{W/m}^2$ ultrasound used in medical diagnostics.

Show Solution

**Strategy**

We use the decibel formula $\beta(\text{dB}) = 10\log_{10}(I I_0)$ to convert the given intensity to a sound intensity level. The reference intensity is $I_0 = 10^{-12}\,\text{W/m}^2$ , the threshold of hearing at 1000 Hz.

**Solution**

Identify the known values:

- Intensity: $I = 2.00\times10^{-2}\,\text{W/m}^2$
- Reference intensity: $I_0 = 10^{-12}\,\text{W/m}^2$

Substitute into the decibel formula:

$$\beta = 10\log_{10}(2.00\times10^{-2}\,\text{W/m}^2 10^{-12}\,\text{W/m}^2) = 10\log_{10}(2.00\times10^{10})$$

Use the logarithm property $\log_{10}(a\times10^b) = \log_{10}(a) + b$ :

$$\beta = 10[\log_{10}(2.00)+10] = 10[0.301+10] = 10(10.301) = 103\text{ dB}$$

**Discussion**

This intensity level of 103 dB represents the typical range for diagnostic medical ultrasound imaging, such as prenatal scans, echocardiography, and abdominal imaging. The intensity is $2.00\times10^{10}$ times greater than the threshold of hearing, which might seem high, but it's important to remember that ultrasound frequencies (typically 1-20 MHz) are far above the audible range and interact with tissue differently than audible sound. This diagnostic intensity is safe for routine medical use—it's approximately $10^7$ times weaker than therapeutic ultrasound used for tissue destruction and about $100$ times weaker than ultrasound diathermy. Decades of clinical use with detailed follow-up studies have shown no harmful effects at these diagnostic intensities. The energy is too low to cause significant heating or cavitation damage, making ultrasound imaging one of the safest medical imaging modalities available. This is particularly important for vulnerable populations such as developing fetuses, where X-rays would pose unacceptable risks.

The time delay between transmission and the arrival of the reflected wave of a signal using ultrasound traveling through a piece of fat tissue was 0.13 ms. At what depth did this reflection occur?

Show Solution

**Strategy**

In ultrasound echo-ranging, the sound must travel to the reflector and back, making the total distance traveled $2d$ , where $d$ is the depth. Using the relationship $\text{distance} = \text{speed}\times\text{time}$ , we have $2d = vt$ , so $d = vt2$ . From Table 1, the speed of ultrasound in fat is 1450 m/s.

**Solution**

Identify the known values:

- Time delay (round trip): $t = 0.13\text{ ms} = 0.13\times10^{-3}\text{ s} = 1.3\times10^{-4}\text{ s}$
- Speed of sound in fat: $v = 1450\text{ m/s}$

The total distance traveled by the ultrasound is:

$$\text{total distance}=vt=(1450\text{ m/s})(1.3\times10^{-4}\text{ s})=0.1885\text{ m}$$

Since this is the round-trip distance, the depth of the reflection is:

$$d=\frac{vt}{2}=\frac{0.1885\text{ m}}{2}=0.094\text{ m}=9.4\text{ cm}$$

**Discussion**

The reflection occurred at a depth of 9.4 cm beneath the surface. This is a reasonable depth for medical ultrasound imaging—it's deep enough to image structures like organs, blood vessels, or tissue layers beneath the skin and subcutaneous fat, but still within the penetration capabilities of typical diagnostic ultrasound. The principle of echo-ranging is fundamental to all ultrasound imaging: by precisely measuring the time delay for echoes to return, the ultrasound system can map the positions of tissue boundaries and organs. The speed of sound varies slightly among different tissues (ranging from about 1450 m/s in fat to 1590 m/s in muscle), but these variations are relatively small—typically only a few percent—so using an average value of 1540 m/s for soft tissue generally gives acceptable accuracy. The factor of 2 in the denominator is crucial and is sometimes forgotten: the ultrasound must travel to the reflector and back, so the measured time corresponds to twice the actual depth.

In the clinical use of ultrasound, transducers are always coupled to the skin by a thin layer of gel or oil, replacing the air that would otherwise exist between the transducer and the skin. (a) Using the values of acoustic impedance given in [Table 1] calculate the intensity reflection coefficient between transducer material and air. (b) Calculate the intensity reflection coefficient between transducer material and gel (assuming for this problem that its acoustic impedance is identical to that of water). (c) Based on the results of your calculations, explain why the gel is used.

Show Solution

**Strategy**

The intensity reflection coefficient $a$ quantifies what fraction of the ultrasound intensity is reflected at a boundary between two media. It is calculated using $a=\frac{(Z_2-Z_1)^2}{(Z_1+Z_2)^2}$, where $Z_1$ and $Z_2$ are the acoustic impedances of the two media. From Table 1, the acoustic impedance of the transducer material (barium titanate) is $30.8\times10^6\text{kg/m}^2\cdot\text{s}$, air is $429\text{kg/m}^2\cdot\text{s}$, and water (gel) is $1.5\times10^6\text{kg/m}^2\cdot\text{s}$.

**Solution for (a)**

Calculate the intensity reflection coefficient between transducer and air:

Let $Z_1=30.8\times10^6\text{kg/m}^2\cdot\text{s}$ (transducer) and $Z_2=429\text{kg/m}^2\cdot\text{s}$ (air)

$$a=\frac{(Z_2-Z_1)^2}{(Z_1+Z_2)^2}=\frac{(429-30.8\times10^6)^2}{(30.8\times10^6+429)^2}$$

Since $Z_1\gg Z_2$, we can approximate:

$$a\approx\frac{(-30.8\times10^6)^2}{(30.8\times10^6)^2}=\frac{(30.8\times10^6)^2}{(30.8\times10^6)^2}=1.00$$

**Solution for (b)**

Calculate the intensity reflection coefficient between transducer and gel (water):

Let $Z_1=30.8\times10^6\text{kg/m}^2\cdot\text{s}$ (transducer) and $Z_2=1.5\times10^6\text{kg/m}^2\cdot\text{s}$ (water/gel)

$$a=\frac{(Z_2-Z_1)^2}{(Z_1+Z_2)^2}=\frac{(1.5\times10^6-30.8\times10^6)^2}{(30.8\times10^6+1.5\times10^6)^2}$$
$$a=\frac{(-29.3\times10^6)^2}{(32.3\times10^6)^2}=\frac{858.49\times10^{12}}{1043.29\times10^{12}}=0.823$$

**Solution for (c)**

The gel dramatically improves ultrasound transmission into the body. With air between the transducer and skin, essentially 100% of the ultrasound energy is reflected ($a=1.00$ means total reflection), making imaging impossible. With gel, only about 82% is reflected, allowing 18% to be transmitted. While 18% transmission might still seem low, the gel serves as an intermediate impedance matcher between the very high impedance of the transducer material and the much lower impedance of body tissues.

**Discussion**

This problem illustrates the critical importance of impedance matching in ultrasound imaging. The intensity reflection coefficient of 1.00 for the transducer-air interface means that virtually all ultrasound energy would be reflected back into the transducer, with essentially nothing entering the body—making imaging impossible. Air is a terrible acoustic coupler because its impedance ($429\text{kg/m}^2\cdot\text{s}$) differs from the transducer impedance by more than four orders of magnitude.

The coupling gel (with impedance similar to water at $1.5\times10^6\text{kg/m}^2\cdot\text{s}$) provides a much better impedance match. While $a=0.823$ might seem high (82% reflected), this is a vast improvement over 100% reflection with air. More importantly, the gel-to-skin interface has an excellent impedance match (both are close to $1.5 \times 10^6 \text{kg/m}^2 \cdot \text{s}$), allowing most of the energy that enters the gel to continue into the body.

In practice, the gel creates a continuous acoustic pathway from transducer to tissue, eliminating air pockets that would otherwise cause complete reflection. This is why sonographers must ensure good contact between the probe and skin, often pressing firmly and using generous amounts of gel. Without this coupling medium, diagnostic ultrasound would be impossible. The same principle applies to other wave-based technologies: impedance matching is essential for efficient energy transfer across boundaries.

(a) Calculate the minimum frequency of ultrasound that will allow you to see details as small as 0.250 mm in human tissue. (b) What is the effective depth to which this sound is effective as a diagnostic probe?

Show Solution

**Strategy**

The resolution of ultrasound imaging is limited by wavelength—details smaller than the wavelength cannot be resolved. In practice, the smallest resolvable detail is approximately equal to the wavelength. Using $\lambda = v f$ and setting $\lambda$ equal to the desired detail size, we can solve for the minimum frequency. The effective penetration depth is given by the rule of thumb $\text{depth} = 500\lambda$ .

**Solution for (a)**

For minimum resolvable detail, set the wavelength equal to the detail size:

$$\lambda = 0.250 \text{ mm} = 0.250 \times 10^{-3} \text{ m} = 2.50 \times 10^{-4} \text{ m}$$

Using $v = 1540 \text{ m/s}$ for human tissue, solve for frequency:

$$f = v \lambda = 1540 \text{ m/s} 2.50 \times 10^{-4} \text{ m} = 6.16 \times 10^6 \text{ Hz} = 6.16 \text{ MHz}$$

**Solution for (b)**

Using the rule of thumb that ultrasound can effectively penetrate to a depth of $500\lambda$ :

$$\text{depth} = 500\lambda = 500(2.50 \times 10^{-4} \text{ m}) = 0.125 \text{ m} = 12.5 \text{ cm}$$

**Discussion**

A frequency of 6.16 MHz is required to resolve details as small as 0.25 mm, placing this in the typical diagnostic ultrasound range (1-20 MHz). The effective penetration depth of 12.5 cm is sufficient for many medical applications, including abdominal imaging, obstetrics, and examination of internal organs.

This problem illustrates the fundamental trade-off in ultrasound imaging: higher frequencies provide better resolution (smaller details can be seen) but penetrate less deeply into tissue. The wavelength at 6.16 MHz is 0.25 mm, which sets the resolution limit. The $500\lambda$ penetration rule accounts for the exponential attenuation of ultrasound as it travels through tissue—higher frequencies are absorbed more rapidly.

For superficial structures like the thyroid gland, carotid arteries, or breast tissue, higher frequencies (10-20 MHz) can be used to achieve better resolution (down to $\sim 0.1 \text{ mm}$) because less penetration depth is needed. For deep abdominal or cardiac imaging, lower frequencies (2-5 MHz) must be used despite poorer resolution ( $\sim 0.5 \text{ mm}$) to ensure adequate penetration. Sonographers must choose the appropriate frequency based on the depth of the structure being imaged and the required level of detail. This is why ultrasound machines have multiple transducers with different frequencies, allowing the operator to optimize the image quality for each specific clinical application.

(a) Find the size of the smallest detail observable in human tissue with 20.0-MHz ultrasound. (b) Is its effective penetration depth great enough to examine the entire eye (about 3.00 cm is needed)? (c) What is the wavelength of such ultrasound in $0\ ^{o}C$ air?

Show Solution

**Strategy**

The smallest observable detail is approximately equal to the wavelength, calculated using $\lambda = v f$ . The effective penetration depth follows the rule of thumb $\text{depth} = 500\lambda$ . For part (c), we use the speed of sound in air at $0\ ^{o}C$ , which is 331 m/s.

**Solution for (a)**

Calculate the wavelength in human tissue ($v = 1540 \text{ m/s}$):

$$\lambda = v f = 1540 \text{ m/s} 20.0 \times 10^6 \text{ Hz} = 7.70 \times 10^{-5} \text{ m} = 77.0 \mu\text{m}$$

The smallest observable detail is approximately $77.0 \mu\text{m}$ .

**Solution for (b)**

Calculate the effective penetration depth:

$$\text{depth} = 500\lambda = 500(7.70\times10^{-5}\ \text{m}) = 0.0385\ \text{m} = 3.85\ \text{cm}$$

Yes, the effective penetration depth of 3.85 cm is greater than the 3.00 cm needed to examine the entire eye.

**Solution for (c)**

Calculate the wavelength in air at $0\ ^\circ C$ ($v = 331$ m/s):

$$\lambda = \frac{v}{f} = \frac{331\ \text{m/s}}{20.0\times10^{6}\ \text{Hz}} = 1.66\times10^{-5}\ \text{m} = 16.6\,\mu\text{m}$$

**Discussion**

This problem demonstrates why 20 MHz ultrasound is ideal for ophthalmology (eye examination). The wavelength of $77.0\,\mu\text{m}$ provides excellent resolution—fine enough to visualize detailed structures within the eye such as the lens, retina, and vitreous chamber. The penetration depth of 3.85 cm is perfectly suited for the eye, which has an anterior-posterior diameter of about 2.4 cm in adults, with some margin for angling the probe and examining structures at the back of the eye.

The comparison between wavelengths in tissue versus air is instructive. The wavelength in air ($16.6\,\mu\text{m}$) is much shorter than in tissue ($77.0\,\mu\text{m}$) because sound travels much slower in air (331 m/s) than in tissue (1540 m/s). This difference in sound speeds—and hence wavelengths—is related to the acoustic impedance mismatch that makes air-tissue boundaries nearly perfect reflectors.

High-frequency ultrasound like 20 MHz would not be suitable for deep abdominal imaging because the penetration depth of only 3.85 cm is insufficient. However, for superficial organs and small structures where deep penetration is not required, high frequencies provide superior image quality. Ophthalmic ultrasound is routinely used to detect retinal detachments, measure eye dimensions for intraocular lens implants, and examine the posterior segment when the view is obscured by cataracts or hemorrhage. The excellent resolution at 20 MHz allows detection of very small abnormalities that would be invisible at lower frequencies.

(a) Echo times are measured by diagnostic ultrasound scanners to determine distances to reflecting surfaces in a patient. What is the difference in echo times for tissues that are 3.50 and 3.60 cm beneath the surface? (This difference is the minimum resolving time for the scanner to see details as small as 0.100 cm, or 1.00 mm. Discrimination of smaller time differences is needed to see smaller details.) (b) Discuss whether the period $T$ of this ultrasound must be smaller than the minimum time resolution. If so, what is the minimum frequency of the ultrasound and is that out of the normal range for diagnostic ultrasound?

Show Solution

**Strategy**

For part (a), we calculate the round-trip time for each depth using $t = \frac{2d}{v}$ and find the difference. For part (b), we need to consider whether the ultrasound pulse must be shorter than the minimum time resolution. If the period is too long, the pulse would still be traveling when the echo returns, making it impossible to distinguish separate echoes.

**Solution for (a)**

Calculate the echo time for each depth:

For $d_1 = 3.50\ \text{cm} = 0.0350\ \text{m}$:

$$t_1 = \frac{2d_1}{v} = \frac{2(0.0350\ \text{m})}{1540\ \text{m/s}} = 4.545\times10^{-5}\ \text{s}$$

For $d_2 = 3.60\ \text{cm} = 0.0360\ \text{m}$:

$$t_2 = \frac{2d_2}{v} = \frac{2(0.0360\ \text{m})}{1540\ \text{m/s}} = 4.675\times10^{-5}\ \text{s}$$

The time difference is:

$$\Delta t = t_2 - t_1 = 4.675\times10^{-5}\ \text{s} - 4.545\times10^{-5}\ \text{s} = 1.30\times10^{-7}\ \text{s} = 0.130\,\mu\text{s}$$

**Solution for (b)**

Yes, the period of the ultrasound must be smaller than the minimum time resolution. If the period were longer than $0.130\,\mu\text{s}$, a single wave cycle would last longer than the time difference between echoes, making it impossible to distinguish between the two reflecting surfaces.

The maximum period is $T_{\text{max}} = 0.130\,\mu\text{s} = 1.30\times10^{-7}\ \text{s}$.

The minimum frequency is:

$$f_{\text{min}} = \frac{1}{T_{\text{max}}} = \frac{1}{1.30\times10^{-7}\ \text{s}} = 7.69\times10^{6}\ \text{Hz} = 7.69\ \text{MHz}$$

This frequency is well within the normal range for diagnostic ultrasound (typically 1-20 MHz).

**Discussion**

The time difference of $0.130\,\mu\text{s}$ represents the minimum temporal resolution needed to distinguish tissue layers separated by 1 mm. This is an extremely short time interval, requiring sophisticated electronics to measure accurately. Modern ultrasound scanners can easily achieve this temporal resolution and much better.

The requirement that the period must be smaller than the time resolution is fundamental to any pulse-echo system. If we used a lower frequency (longer period), say 1 MHz with $T = 1\,\mu\text{s}$, the ultrasound pulse would be "ringing" for much longer than $0.130\,\mu\text{s}$, and the echoes from the two surfaces would overlap, making them indistinguishable. This is why diagnostic ultrasound systems use short pulses (typically 2-3 cycles) rather than continuous waves.

The calculated minimum frequency of 7.69 MHz is quite typical for abdominal imaging. In practice, this frequency provides a good balance between temporal resolution, spatial resolution (wavelength $\lambda = \frac{1540}{7.69 \times 10^6} = 0.20\text{ mm}$), and penetration depth ($500\lambda = 10\text{ cm}$). Higher frequencies could provide better spatial resolution but would sacrifice penetration depth. Lower frequencies would penetrate deeper but would not have sufficient temporal or spatial resolution to detect 1-mm details.

This problem highlights the relationship between temporal resolution (how quickly echoes can be distinguished) and spatial resolution (how small an object can be detected). Both are ultimately limited by the wavelength and period of the ultrasound wave, illustrating the wave nature of ultrasound.

(a) How far apart are two layers of tissue that produce echoes having round-trip times (used to measure distances) that differ by $0.750\,\mu s$? (b) What minimum frequency must the ultrasound have to see detail this small?

Show Solution

**Strategy**

For part (a), the time difference corresponds to the difference in round-trip travel times. Since the sound travels to each layer and back, the total distance difference is $v\Delta t$, but the actual separation between layers is half this distance: $d = \frac{v\Delta t}{2}$. For part (b), the wavelength must be no larger than this separation distance to resolve the two layers, so we use $\lambda = d$ and solve for frequency using $f = \frac{v}{\lambda}$.

**Solution for (a)**

The time difference is:

$$\Delta t = 0.750\,\mu\text{s} = 0.750 \times 10^{-6}\text{ s} = 7.50 \times 10^{-7}\text{ s}$$

The separation between the two layers is:

$$d = \frac{v\Delta t}{2} = \frac{(1540\text{ m/s})(7.50 \times 10^{-7}\text{ s})}{2} = \frac{1.155 \times 10^{-3}\text{ m}}{2} = 5.78 \times 10^{-4}\text{ m} = 0.578\text{ mm}$$

**Solution for (b)**

To resolve this detail, the wavelength must be approximately equal to or smaller than the layer separation:

$$\lambda \le d = 5.78 \times 10^{-4}\text{ m}$$

The minimum frequency is:

$$f_{\text{min}} = \frac{v}{\lambda} = \frac{1540\text{ m/s}}{5.78 \times 10^{-4}\text{ m}} = 2.67 \times 10^6\text{ Hz} = 2.67\text{ MHz}$$

**Discussion**

The two tissue layers are separated by approximately 0.578 mm (just over half a millimeter). This is a typical scale for detecting small anatomical structures or tissue boundaries in medical imaging, such as the layers of a blood vessel wall, the thickness of cardiac muscle, or small lesions.

The minimum frequency of 2.67 MHz is at the lower end of the diagnostic ultrasound range, making this an easily achievable resolution for modern ultrasound systems. This frequency provides good penetration depth ($500\lambda = 500 \times 0.578\text{ mm} = 289\text{ mm} \approx 29\text{ cm}$) while still maintaining sufficient resolution to distinguish the two layers.

The relationship between temporal resolution (time difference in echoes) and spatial resolution (physical separation of structures) is fundamental to pulse-echo ultrasound. The time difference of $0.750\,\mu\text{s}$ may seem incredibly short, but modern ultrasound electronics can easily measure time intervals of nanoseconds, providing temporal resolution far exceeding what's needed for this application.

Notice that both temporal and spatial resolution are ultimately limited by the same factor: the wavelength. The period $T = \frac{1}{f} = 0.375\,\mu\text{s}$ is about half the time difference $\Delta t = 0.750\,\mu\text{s}$, which means the ultrasound completes roughly two complete cycles during the time between echoes. This is sufficient for the receiver to distinguish the two separate return signals. Using a much lower frequency would cause the echoes to overlap, making the two layers indistinguishable.

(a) A bat uses ultrasound to find its way among trees. If this bat can detect echoes 1.00 ms apart, what minimum distance between objects can it detect? (b) Could this distance explain the difficulty that bats have finding an open door when they accidentally get into a house?

Show Solution

**Strategy**

Bats navigate using echolocation in air, where the speed of sound is approximately 343 m/s at room temperature. The time difference between echoes from two objects corresponds to the difference in round-trip distances. If the bat can distinguish echoes separated by $\Delta t$, the minimum distance between objects is $d = \frac{v \Delta t}{2}$, where we divide by 2 because the sound makes a round trip.

**Solution for (a)**

Given:

- Time resolution: $\Delta t = 1.00 \text{ ms} = 1.00 \times 10^{-3} \text{ s}$
- Speed of sound in air: $v \approx 343 \text{ m/s}$

Calculate the minimum detectable distance:

$$d = \frac{v \Delta t}{2} = \frac{(343 \text{ m/s})(1.00 \times 10^{-3} \text{ s})}{2} = \frac{0.343 \text{ m}}{2} = 0.172 \text{ m} \approx 17 \text{ cm}$$

**Solution for (b)**

Yes, this could explain the difficulty. An open doorway typically has a width of about 80-90 cm. However, the bat needs to detect the edges (door frame) on both sides of the opening. If the bat is flying toward the doorway head-on, it must distinguish between:

1. The wall on one side of the door
2. The open space
3. The wall on the other side

The minimum distance the bat can resolve is 17 cm, which is substantial compared to the fine spatial discrimination needed to navigate through a doorway safely. More importantly, an open door provides very few strong reflections from the open space itself—the bat receives strong echoes from the walls and door frame, but weak or no echoes from the opening. This lack of reflection from the opening, combined with the relatively coarse spatial resolution, makes it difficult for the bat to “see” the door as open.

**Discussion**

The minimum detectable distance of 17 cm represents the bat’s spatial resolution when using echolocation. This resolution is quite good for navigating around large obstacles like trees, branches, and flying insects (which bats can detect and catch in flight). However, it’s relatively coarse for detecting architectural features like open doorways.

The difficulty bats experience finding open doors is a classic example of how echolocation differs from vision. While we see a door as “open” because light enters and we perceive the opening, a bat using echolocation perceives the environment through reflections. An open door reflects very little sound back to the bat—the ultrasound passes through the opening and may reflect off distant objects beyond the door. The strong reflections come from solid objects: walls, the door itself if partially open, and the door frame.

To the bat, the wall with a door might appear as a large reflecting surface with a confusing gap or weak region. The bat cannot easily distinguish “I can fly through here” from “this is dangerous.” This is why bats trapped indoors often fly into windows (which look like openings but are actually solid) or struggle to find actual openings.

In nature, bats navigate complex three-dimensional environments like forests with remarkable precision. They can detect insects as small as mosquitoes and pluck them from the air while flying at high speed. However, the artificial environment of human buildings, with smooth walls, windows, and doorways, presents challenges that evolution has not equipped them to handle efficiently. This highlights the importance of understanding the limitations of any sensory system—whether biological echolocation or engineered ultrasound imaging.

A dolphin is able to tell in the dark that the ultrasound echoes received from two sharks come from two different objects only if the sharks are separated by 3.50 m, one being that much farther away than the other. (a) If the ultrasound has a frequency of 100 kHz, show this ability is not limited by its wavelength. (b) If this ability is due to the dolphin’s ability to detect the arrival times of echoes, what is the minimum time difference the dolphin can perceive?

Show Solution

**Strategy**

For part (a), we calculate the wavelength using $\lambda = \frac{v}{f}$ and compare it to the separation distance. If the wavelength is much smaller than the separation, then spatial resolution is not limited by the wave nature of sound. For part (b), the time difference between echoes corresponds to the difference in round-trip travel times: $\Delta t = \frac{2d}{v}$, where $d = 3.50 \text{ m}$ is the extra distance to the farther shark.

**Solution for (a)**

Given:

- Frequency: $f = 100 \text{ kHz} = 100 \times 10^{3} \text{ Hz} = 1.00 \times 10^{5} \text{ Hz}$
- Speed of sound in water: $v = 1540 \text{ m/s}$

Calculate the wavelength:

$$\lambda = \frac{v}{f} = \frac{1540 \text{ m/s}}{1.00 \times 10^{5} \text{ Hz}} = 0.0154 \text{ m} = 1.54 \text{ cm}$$

Compare the wavelength to the separation distance:

$$\frac{d}{\lambda}=\frac{3.50\text{ m}}{0.0154\text{ m}}=227$$

Since the separation distance (3.50 m) is 227 times larger than the wavelength (1.54 cm), the wavelength is much shorter than the distance in question. Therefore, the dolphin's ability to distinguish the two sharks is **not** limited by the wavelength—spatial resolution based on wavelength alone would allow detection of objects much closer together than 3.50 m.

**Solution for (b)**

The difference in round-trip travel time for the two echoes is:

$$\Delta t=\frac{2d}{v}=\frac{2(3.50\text{ m})}{1540\text{ m/s}}=\frac{7.00\text{ m}}{1540\text{ m/s}}=4.55\times10^{-3}\text{ s}=4.55\text{ ms}$$

The minimum time difference the dolphin can perceive is **4.55 ms** (milliseconds).

**Discussion**

This problem reveals important insights into dolphin echolocation capabilities. The wavelength of 1.54 cm would theoretically allow the dolphin to resolve objects separated by distances on the order of centimeters, not meters. The fact that the dolphin's actual resolution limit is 3.50 m (much larger than the wavelength) indicates that the limitation is **temporal** rather than spatial—it's based on the dolphin's neural processing ability to distinguish arrival times of echoes, not on the wave nature of the ultrasound.

The minimum time resolution of 4.55 ms is relatively long compared to the period of the ultrasound ($T=\frac{1}{f}=10\,\mu\text{s}$), which means the dolphin completes many wave cycles before receiving the delayed echo. This suggests that the dolphin's auditory system has a fundamental limit in processing speed—perhaps related to neural firing rates, temporal integration in the auditory cortex, or the duration of ultrasound pulses used.

For comparison, human medical ultrasound can distinguish echoes separated by much shorter time intervals (less than $1\,\mu\text{s}$), but humans have the advantage of electronic signal processing rather than biological neural processing. The dolphin's 4.55 ms resolution is nonetheless impressive from a biological standpoint and is perfectly adequate for the dolphin's ecological needs—hunting fish, navigating murky waters, and avoiding predators.

It's interesting to note that dolphins use echolocation frequencies ranging from about 40 kHz to over 150 kHz, with higher frequencies providing better spatial resolution through shorter wavelengths. The 100 kHz frequency used in this problem is in the middle of the dolphin's range, representing a compromise between resolution and range (higher frequencies attenuate more rapidly in water). Despite the temporal processing limitation revealed in this problem, dolphins are remarkably effective predators, capable of detecting and catching fast-moving fish in complete darkness or in turbid water where vision is useless.

A diagnostic ultrasound echo is reflected from moving blood and returns with a frequency 500 Hz higher than its original 2.00 MHz. What is the velocity of the blood? (Assume that the frequency of 2.00 MHz is accurate to seven significant figures and 500 Hz is accurate to three significant figures.)

Show Solution

**Strategy**

Doppler-shifted ultrasound involves a double Doppler shift: first, the moving blood receives a shifted frequency as a moving observer; second, the blood reflects the ultrasound as a moving source, creating another shift. For motion toward the source, the total frequency shift is given by $\Delta f\approx\frac{2v_b f_s}{v}$, where $v_b$ is the blood velocity, $f_s$ is the source frequency, and $v$ is the speed of sound in tissue. We can solve this for the blood velocity.

**Solution**

Given:

- Source frequency: $f_s=2.00\text{ MHz}=2.000000\times10^6\text{ Hz}$ (to 7 significant figures)
- Frequency shift: $\Delta f=500\text{ Hz}$ (to 3 significant figures)
- Speed of sound in tissue: $v=1540\text{ m/s}$

For small velocities ($v_b\ll v$), the approximate formula for the total Doppler shift is:

$$\Delta f=f_{\text{obs}}-f_s\approx\frac{2v_b f_s}{v}$$

Solve for the blood velocity:

$$v_b=\frac{\Delta f\cdot v}{2f_s}=\frac{(500\text{ Hz})(1540\text{ m/s})}{2(2.000000\times10^6\text{ Hz})}$$

$$v_b=\frac{770000\text{ m/s}}{4.000000\times10^6}=0.1925\text{ m/s}=19.3\text{ cm/s}$$

The velocity of the blood is **0.193 m/s or 19.3 cm/s** toward the ultrasound source.

**Discussion**

This blood velocity of 19.3 cm/s is physiologically reasonable for arterial blood flow. For comparison, typical blood velocities are:

- Large arteries (aorta): 40-100 cm/s (peak systolic)

- Medium arteries (carotid, femoral): 20-60 cm/s
- Small arteries and veins: 1-10 cm/s
- Capillaries: 0.05-0.1 cm/s

The calculated value of 19.3 cm/s is consistent with flow in a medium-sized artery or a large artery during diastole (when the heart is relaxing between beats).

The Doppler shift of 500 Hz represents only a tiny fraction of the original frequency: $\frac{\Delta f}{f_S} = \frac{500}{2 \times 10^6} = 2.5 \times 10^{-4}$ or 0.025%. This extremely small fractional shift highlights why the beat frequency method is used in Doppler ultrasound—it would be nearly impossible to measure such a small shift by directly measuring the echo frequency. Instead, the echo is mixed with the original frequency to produce audible beats at 500 Hz, which are easy to detect and measure.

The factor of 2 in the Doppler formula arises from the double shift: the blood cells receive a Doppler-shifted frequency (first shift), and then they re-radiate this shifted frequency as a moving source (second shift). Each individual shift contributes a factor of approximately $\frac{v_b}{v}$, so the total shift is approximately $\frac{2v_b}{v}$.

Doppler ultrasound is invaluable in clinical practice for assessing blood flow in arteries and veins, detecting blockages (stenoses) where velocity increases, identifying regurgitant flow (backflow through leaky valves), and monitoring fetal heart rate during pregnancy. The technique is completely non-invasive and safe, making it ideal for routine clinical use. Color Doppler imaging adds a visual dimension by color-coding the velocity information—conventionally showing flow toward the transducer in red and flow away in blue—allowing clinicians to see complex flow patterns at a glance.

Ultrasound reflected from an oncoming bloodstream that is moving at 30.0 cm/s is mixed with the original frequency of 2.50 MHz to produce beats. What is the beat frequency? (Assume that the frequency of 2.50 MHz is accurate to seven significant figures.)

Show Solution

**Strategy**

The beat frequency is the difference between the Doppler-shifted echo frequency and the original source frequency: $f_B = |f_{obs} - f_S|$. In Doppler ultrasound, there's a double shift because the blood acts as both a moving observer (receiving shifted frequency) and a moving source (reflecting shifted frequency). We apply the Doppler formula twice or use the approximate formula for small velocities: $\Delta f \approx \frac{2 v_b f_S}{v}$.

**Solution**

Given:

- Blood velocity: $v_b = 30.0 \text{ cm/s} = 0.300 \text{ m/s}$ (toward the source)
- Source frequency: $f_S = 2.50 \text{ MHz} = 2.5000000 \times 10^6 \text{ Hz}$ (to 7 significant figures)
- Speed of sound in tissue: $v = 1540 \text{ m/s}$

**First Doppler shift** (blood as moving observer receiving ultrasound):

The blood receives a higher frequency because it's moving toward the source:

$$f_1 = f_S\left(\frac{v + v_b}{v}\right) = (2.5000000 \times 10^6 \text{ Hz})\left(\frac{1540 + 0.300}{1540}\right)$$
$$f_1 = (2.5000000 \times 10^6 \text{ Hz})\left(\frac{1540.300}{1540}\right) = 2500487.01 \text{ Hz}$$

**Second Doppler shift** (blood as moving source reflecting ultrasound):

The reflected ultrasound is shifted higher again because the blood (source) is moving toward the receiver:

$$f_{obs} = f_1\left(\frac{v}{v - v_b}\right) = (2500487.01 \text{ Hz})\left(\frac{1540}{1540 - 0.300}\right)$$
$$f_{obs} = (2500487.01 \text{ Hz})\left(\frac{1540}{1539.700}\right) = 2500974.03 \text{ Hz}$$

**Calculate the beat frequency**:

$$f_B = |f_{obs} - f_S| = |2500974.03 - 2500000.00| \text{ Hz} = 974 \text{ Hz}$$

The beat frequency is **974 Hz**.

**Discussion**

The beat frequency of 974 Hz is in the audible range (human hearing extends from about 20 Hz to 20,000 Hz), which is one of the key advantages of the beat frequency method in Doppler ultrasound. The clinician can actually hear the blood flow as an audible tone. The pitch and quality of this tone provide immediate feedback:

- Higher pitch indicates faster blood flow
- The tone's characteristics can indicate normal vs. turbulent flow
- Changes in pitch during the cardiac cycle reflect pulsatile arterial flow

This blood velocity of 30.0 cm/s is quite reasonable for arterial blood flow. For comparison with the previous problem, this velocity is somewhat higher, producing a correspondingly higher beat frequency (974 Hz vs. 500 Hz). The relationship is linear: doubling the blood velocity doubles the beat frequency.

The total frequency shift is $\Delta f = 974\text{ Hz}$, representing a fractional change of $\frac{974}{2.5\times10^6} = 3.9\times10^{-4}$ or 0.039%. This tiny fractional shift would be nearly impossible to detect by directly measuring the echo frequency. However, when the echo is mixed with the original frequency, the resulting beat frequency of 974 Hz is easily measured and even audible.

The factor of 2 in the approximate formula $\Delta f \approx \frac{2 v_b f_s}{v}$ comes from the double Doppler shift. We can verify this approximation:

$$\Delta f \approx \frac{2 v_b f_s}{v} = \frac{2(0.300\text{ m/s})(2.5\times10^6\text{ Hz})}{1540\text{ m/s}} = 974\text{ Hz}$$

The approximation gives exactly the same result as the detailed two-step calculation, confirming that for blood velocities (which are much smaller than the speed of sound), the approximate formula is highly accurate.

In clinical practice, Doppler ultrasound beat frequencies provide valuable diagnostic information. Normal arterial flow produces a characteristic pulsatile sound that rises and falls with each heartbeat. Stenotic (narrowed) vessels produce higher-frequency sounds and often turbulent flow patterns. Venous flow typically produces lower, more continuous sounds. Experienced clinicians can often diagnose vascular conditions simply by listening to the Doppler audio signal, though modern systems also provide visual displays showing velocity waveforms and color-coded flow maps.

## Glossary

acoustic impedance
: property of medium that makes the propagation of sound waves more difficult

intensity reflection coefficient
: a measure of the ratio of the intensity of the wave reflected off a boundary between two media relative to the intensity of the incident wave

Doppler-shifted ultrasound
: a medical technique to detect motion and determine velocity through the Doppler shift of an echo

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