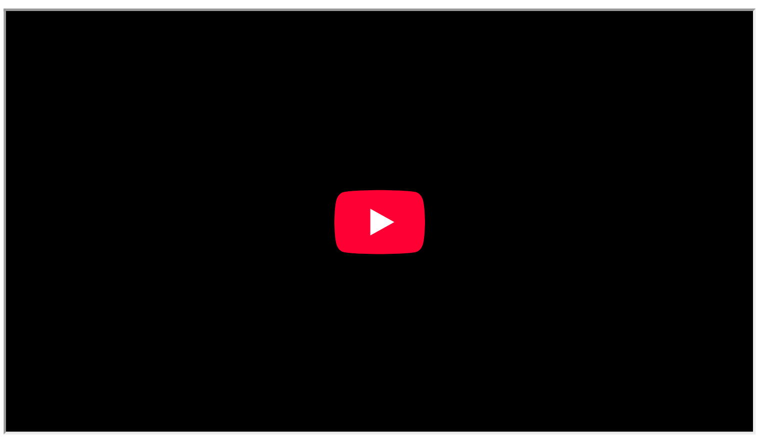
Introduction to Fluid Statics

The fluid essential to all life has a beauty of its own. It also helps support the weight of this swimmer. (credit: Terren, Wikimedia Commons)

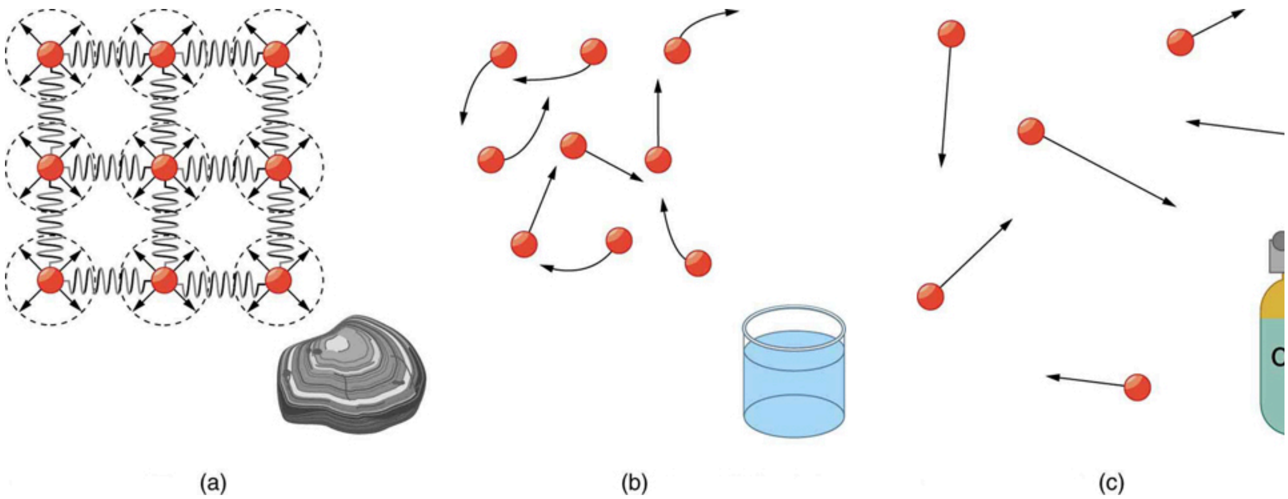
Much of what we value in life is fluid: a breath of fresh winter air; the hot blue flame in our gas cooker; the water we drink, swim in, and bathe in; the blood in our veins. What exactly is a fluid? Can we understand fluids with the laws already presented, or will new laws emerge from their study? The physical characteristics of static or stationary fluids and some laws that govern their behavior are the topics of this chapter. Fluid Dynamics and Its Biological and Medical Applications explores aspects of fluid flow.



What Is a Fluid?

- State the common phases of matter.
- Explain the physical characteristics of solids, liquids, and gases.
- Describe the arrangement of atoms in solids, liquids, and gases.

Matter most commonly exists as a solid, liquid, or gas; these states are known as the three common *phases of matter*. Solids have a definite shape and a specific volume, liquids have a definite volume but their shape changes depending on the container in which they are held, and gases have neither a definite shape nor a specific volume as their molecules move to fill the container in which they are held. (See [Figure 1].) Liquids and gases are considered to be fluids because they yield to shearing forces, whereas solids resist them. Note that the extent to which fluids yield to shearing forces (and hence flow easily and quickly) depends on a quantity called the viscosity which is discussed in detail in Viscosity and Laminar Flow; Poiseuille's Law. We can understand the phases of matter and what constitutes a fluid by considering the forces between atoms that make up matter in the three phases.



(a) Atoms in a solid always have the same neighbors, held near home by forces represented here by springs. These atoms are essentially in contact with one another. A rock is an example of a solid. This rock retains its shape because of the forces holding its atoms together. (b) Atoms in a liquid are also in close contact but can slide over one another. Forces between them strongly resist attempts to push them closer together and also hold them in close contact. Water is an example of a liquid. Water can flow, but it also remains in an open container because of the forces between its atoms. (c) Atoms in a gas are separated by distances that are considerably larger than the size of the atoms themselves, and they move about freely. A gas must be held in a closed container to prevent it from moving out freely.

Atoms in *solids* are in close contact, with forces between them that allow the atoms to vibrate but not to change positions with neighboring atoms. (These forces can be thought of as springs that can be stretched or compressed, but not easily broken.) Thus, a solid *resists* all types of stress. A solid cannot be easily deformed because the atoms that make up the solid are not able to move about freely. Solids also resist compression, because their atoms form part of a lattice structure in which the atoms are a relatively fixed distance apart. Under compression, the atoms would be forced into one another. Most of the examples we have studied so far have involved solid objects which deform very little when stressed.

Connections: Submicroscopic Explanation of Solids and Liquids

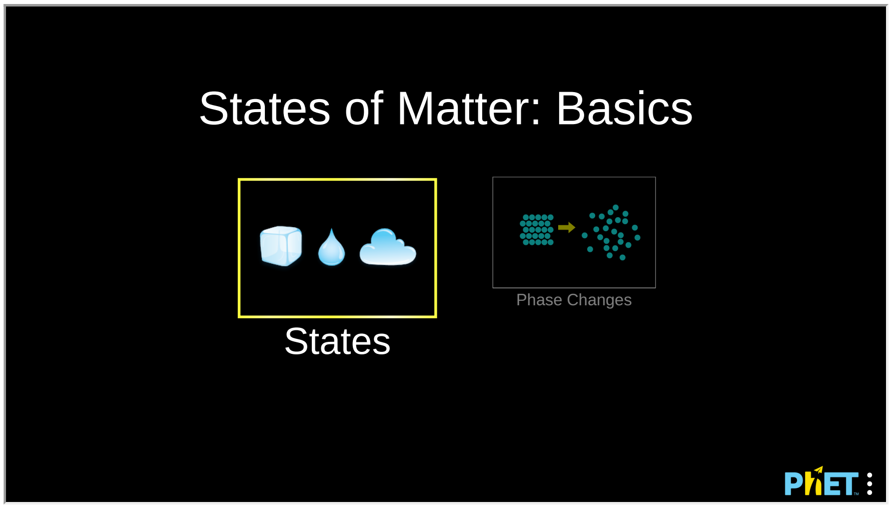
Atomic and molecular characteristics explain and underlie the macroscopic characteristics of solids and fluids. This submicroscopic explanation is one theme of this text and is highlighted in the Things Great and Small features in Conservation of Momentum. See, for example, microscopic description of collisions and momentum or microscopic description of pressure in a gas. This present section is devoted entirely to the submicroscopic explanation of solids and liquids.

In contrast, *liquids* deform easily when stressed and do not spring back to their original shape once the force is removed because the atoms are free to slide about and change neighbors—that is, they *flow* (so they are a type of fluid), with the molecules held together by their mutual attraction. When a liquid is placed in a container with no lid on, it remains in the container (providing the container has no holes below the surface of the liquid!). Because the atoms are closely packed, liquids, like solids, resist compression.

Atoms in *gases* are separated by distances that are large compared with the size of the atoms. The forces between gas atoms are therefore very weak, except when the atoms collide with one another. Gases thus not only flow (and are therefore considered to be fluids) but they are relatively easy to compress because there is much space and little force between atoms. When placed in an open container gases, unlike liquids, will escape. The major distinction is that gases are easily compressed, whereas liquids are not. We shall generally refer to both gases and liquids simply as **fluids**, and make a distinction between them only when they behave differently.

PhET Explorations: States of Matter—Basics

Heat, cool, and compress atoms and molecules and watch as they change between solid, liquid, and gas phases.



Section Summary

- A fluid is a state of matter that yields to sideways or shearing forces. Liquids and gases are both fluids. Fluid statics is the physics of stationary fluids.

Conceptual Questions

What physical characteristic distinguishes a fluid from a solid?

Show Solution

Strategy: To distinguish fluids from solids, we must consider how each state of matter responds to applied forces, specifically shearing (sideways) forces.

Solution: The key physical characteristic that distinguishes a fluid from a solid is that **a fluid yields to and flows under shearing forces, while a solid resists and maintains its shape**.

At the molecular level, this difference arises from the bonding between atoms:

- In solids, atoms are held in fixed positions by strong interatomic forces (like springs connecting neighboring atoms). When a shearing force is applied, the atoms may deform slightly but return to their original positions.
- In fluids (both liquids and gases), atoms or molecules can move past one another when a shearing force is applied. The atoms are not locked into fixed positions relative to their neighbors.

Discussion: This distinction explains everyday observations: a block of ice maintains its shape under its own weight, but water takes the shape of its container. Similarly, honey flows slowly when tilted because it yields to shearing forces (though its high viscosity means it flows slowly). The ability to flow is the defining characteristic of fluids, which is why both liquids and gases are classified as fluids despite their other differences.

Which of the following substances are fluids at room temperature: air, mercury, water, glass?

Show Solution

Strategy: To identify fluids, we apply the defining characteristic: fluids are substances that yield to shearing forces and flow. We evaluate each substance at room temperature (approximately 20°C to 25°C).

Solution: At room temperature, **air, mercury, and water are fluids**, while **glass is a solid**.

Analysis of each substance:

- **Air** is a gas composed primarily of nitrogen and oxygen molecules. Gases are fluids because their molecules move freely and the substance flows readily to fill any container.
- **Mercury** is a liquid metal at room temperature (melting point: −39°C). It flows and takes the shape of its container, clearly demonstrating fluid behavior.
- **Water** is a liquid at room temperature (melting point: 0°C, boiling point: 100°C). It readily flows and conforms to container shapes.
- **Glass** is an amorphous solid at room temperature. While glass has a disordered molecular structure (unlike crystalline solids), it does not flow under normal conditions and maintains its shape indefinitely.

Discussion: A common misconception is that glass is a “very slow-flowing liquid” based on observations of old window panes being thicker at the bottom. However, this thickness variation is due to historical manufacturing processes, not flow. Glass at room temperature is many orders of magnitude more viscous than any practical fluid—it would take longer than the age of the universe for glass to noticeably flow at room temperature. Therefore, glass is properly classified as a solid.

Why are gases easier to compress than liquids and solids?

Show Solution

Strategy: To understand compressibility differences, we must examine the molecular structure and spacing in each phase of matter.

Solution: Gases are easier to compress than liquids and solids because gas molecules are separated by large distances compared to their size, with mostly empty space between them.

The molecular explanation:

- **In gases**, molecules are typically separated by distances 10 times or more larger than the molecules themselves. When pressure is applied, molecules simply move closer together into the available empty space. Little force is required because the molecules are not in contact.
- **In liquids**, molecules are nearly in contact with each other, with very little space between them. Compression requires forcing molecules closer together against strong repulsive forces that arise when electron clouds begin to overlap.
- **In solids**, atoms are locked in a lattice structure at essentially their minimum separation distance. Compressing a solid requires pushing atoms even closer against extremely strong electromagnetic repulsion.

Discussion: This explains practical applications: air in a bicycle tire compresses easily when pumped, allowing more air molecules to fit in the same volume. Water in a hydraulic system, however, transmits pressure almost instantaneously because it barely compresses—this incompressibility is essential for hydraulic brakes and lifts. At standard temperature and pressure, air is about 20,000 times more compressible than water. The compressibility of gases also explains why scuba tanks can hold large volumes of breathing gas and why compressed natural gas vehicles store fuel at high pressures.

How do gases differ from liquids?

Show Solution

Strategy: Both gases and liquids are fluids (they flow under shearing forces), so we need to identify the key properties that distinguish them from each other.

Solution: While both gases and liquids are fluids, they differ in several important ways:

Property	Liquids	Gases
Compressibility	Nearly incompressible	Highly compressible
Molecular spacing	Molecules nearly in contact	Molecules widely separated
Definite volume	Yes—maintains volume	No—expands to fill container
Intermolecular forces	Strong attractive forces	Weak forces (except during collisions)
Density	High (similar to solids)	Low (typically 1000× less than liquids)
Container requirement	Stays in open container	Must be in closed container

Discussion: These differences arise from molecular behavior:

- **Liquids** have molecules close enough that attractive forces (van der Waals forces, hydrogen bonds) hold them together, giving liquids a definite volume. Water stays in an open glass because molecular attraction prevents molecules from escaping (except slowly through evaporation).
- **Gases** have molecules so far apart that attractive forces are negligible except during brief collisions. Gas molecules move freely at high speeds (hundreds of m/s at room temperature) and will escape any open container.

The high compressibility of gases versus the near-incompressibility of liquids is the most practically important distinction, enabling applications like pneumatic tools (which use compressible air) versus hydraulic systems (which use incompressible liquids for precise force transmission).

Glossary

fluids
: liquids and gases; a fluid is a state of matter that yields to shearing forces

Density

- Define density.
- Calculate the mass of a reservoir from its density.
- Compare and contrast the densities of various substances.

Which weighs more, a ton of feathers or a ton of bricks? This old riddle plays with the distinction between mass and density. A ton is a ton, of course; but bricks have much greater density than feathers, and so we are tempted to think of them as heavier. (See [Figure 1].)

Density, as you will see, is an important characteristic of substances. It is crucial, for example, in determining whether an object sinks or floats in a fluid. Density is the mass per unit volume of a substance or object. In equation form, density is defined as

$$\rho = mV,$$

where the Greek letter ρ (rho) is the symbol for density, m is the mass, and V is the volume occupied by the substance.

Density

Density is mass per unit volume.

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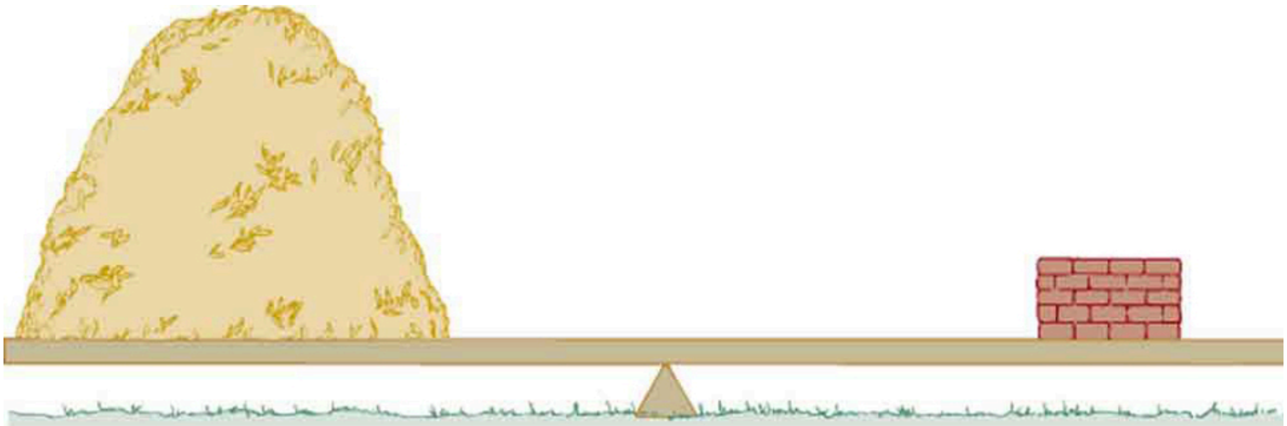
In the riddle regarding the feathers and bricks, the masses are the same, but the volume occupied by the feathers is much greater, since their density is much lower. The SI unit of density is kg/m^3 , representative values are given in [Table 1]. The metric system was originally devised so that water would have a density of 1g/cm^3 , equivalent to 10^3kg/m^3 . Thus the basic mass unit, the kilogram, was first devised to be the mass of 1000 mL of water, which has a volume of 1000 cm^3.

Densities of Various Substances

Substance	$\rho(10^3\text{kg/m}^3$ or g/mL)	Substance	$\rho(10^3\text{kg/m}^3$ or g/mL)	Substance	$\rho(10^3\text{kg/m}^3$ or g/mL)
Solids		**Liquids**		**Gases**	
Aluminum	2.7	Water (4ºC)	1.000	Air	1.29×10^{-3}
Brass	8.44	Blood	1.05	Carbon dioxide	1.98×10^{-3}
Copper (average)	8.8	Sea water	1.025	Carbon monoxide	1.25×10^{-3}
Gold	19.32	Mercury	13.6	Hydrogen	0.090×10^{-3}
Iron or steel	7.8	Ethyl alcohol	0.79	Helium	0.18×10^{-3}
Lead	11.3	Petrol	0.68	Methane	0.72×10^{-3}
Polystyrene	0.10	Glycerin	1.26	Nitrogen	1.25×10^{-3}
Tungsten	19.30	Olive oil	0.92	Nitrous oxide	1.98×10^{-3}
Uranium	18.70			Oxygen	1.43×10^{-3}
Concrete	2.30–3.0			Steam (100°C)	0.60×10^{-3}
Cork	0.24				
Glass, common (average)	2.6				
Granite	2.7				
Earth’s crust	3.3				
Wood	0.3–0.9				
Ice (0ºC)	0.917				
Bone	1.7–2.0				



A ton of feathers and a ton of bricks have the same mass, but the feathers make a much bigger pile because they have a much lower density.

As you can see by examining [Table 1], the density of an object may help identify its composition. The density of gold, for example, is about 2.5 times the density of iron, which is about 2.5 times the density of aluminum. Density also reveals something about the phase of the matter and its substructure. Notice that the densities of liquids and solids are roughly comparable, consistent with the fact that their atoms are in close contact. The densities of gases are much less than those of liquids and solids, because the atoms in gases are separated by large amounts of empty space.

Take-Home Experiment Sugar and Salt

A pile of sugar and a pile of salt look pretty similar, but which weighs more? If the volumes of both piles are the same, any difference in mass is due to their different densities (including the air space between crystals). Which do you think has the greater density? What values did you find? What method did you use to determine these values?

Calculating the Mass of a Reservoir From Its Volume

A reservoir has a surface area of 50.0km^2 and an average depth of 40.0 m. What mass of water is held behind the dam? (See [Figure 2] for a view of a large reservoir—the Three Gorges Dam site on the Yangtze River in central China.)

Strategy

We can calculate the volume V of the reservoir from its dimensions, and find the density of water ρ in [Table 1]. Then the mass m can be found from the definition of density

$$\rho = \frac{m}{V}.$$

Solution

Solving equation $\rho = m/V$ for m gives $m = \rho V$.

The volume V of the reservoir is its surface area A times its average depth h :

$$V = Ah = (50.0\text{km}^2)(40.0\text{m}) = \left[(50.0\text{km}^2)\left(\frac{10^3\text{m}}{1\text{km}}\right)^2\right](40.0\text{m}) = 2.00\times10^9\text{m}^3$$

The density of water ρ from [Table 1] is $1.000\times10^3\text{kg/m}^3$. Substituting V and ρ into the expression for mass gives

$$m = (1.00\times10^3\text{kg/m}^3)(2.00\times10^9\text{m}^3) = 2.00\times10^{12}\text{kg}.$$

Discussion

A large reservoir contains a very large mass of water. In this example, the weight of the water in the reservoir is $mg = 1.96\times10^{13}\text{N}$, where g is the acceleration due to the Earth's gravity (about 9.80m/s^2). It is reasonable to ask whether the dam must supply a force equal to this tremendous weight. The answer is no. As we shall see in the following sections, the force the dam must supply can be much smaller than the weight of the water it holds back.

Three Gorges Dam in central China. When completed in 2008, this became the world's largest hydroelectric plant, generating power equivalent to that generated by 22 average-sized nuclear power plants. The concrete dam is 181 m high and 2.3 km across. The reservoir made by this dam is 660 km long. Over 1 million people were displaced by the creation of the reservoir. (credit: Le Grand Portage)

Section Summary

- Density is the mass per unit volume of a substance or object. In equation form, density is defined as

  $$\rho = mV.$$

- The SI unit of density is kg/m^3.

Conceptual Questions

Approximately how does the density of air vary with altitude?

Show Solution

Strategy: We need to consider how atmospheric pressure and temperature change with altitude, and how these affect air density through the ideal gas law.

Solution: The density of air decreases approximately exponentially with increasing altitude.

This occurs because:

1. Air pressure decreases with altitude as there is less atmosphere above to exert weight
2. The ideal gas law relates density to pressure: $\rho = PMRT$ where P is pressure, M is molar mass, R is the gas constant, and T is temperature
3. As pressure drops with altitude, density proportionally decreases

Approximate values:

- Sea level: $\rho \approx 1.29\ \text{kg/m}^3$
- 5,500 m (18,000 ft): $\rho \approx 0.65\ \text{kg/m}^3$ (about half)
- 8,850 m (Mt. Everest): $\rho \approx 0.43\ \text{kg/m}^3$ (about one-third)

Discussion: This exponential decrease has significant practical implications: aircraft engines produce less thrust at high altitude due to lower air density, climbers on high mountains need supplemental oxygen because fewer oxygen molecules are available per breath, and cooking takes longer at high altitude because water boils at lower temperatures due to reduced pressure.

Give an example in which density is used to identify the substance composing an object. Would information in addition to average density be needed to identify the substances in an object composed of more than one material?

Show Solution

Strategy: Consider practical applications where measuring density helps identify materials, and analyze what additional information would be needed for composite materials.

Solution: Example: Testing gold jewelry for authenticity

A jeweler can determine if a piece of jewelry is solid gold by measuring its density:

1. Measure the mass using a precise balance
2. Measure the volume by water displacement (Archimedes' method)
3. Calculate density: $\rho = m/V$
4. Compare to known gold density (19.32 g/cm^3)

If the measured density matches gold's known density, the object is likely pure gold. If significantly different, it may be gold-plated or a gold alloy.

For objects composed of multiple materials: Yes, additional information is needed.

Average density alone cannot uniquely identify a mixture because:

- Different combinations can yield the same average density
- A mixture of lead ($\rho = 11.3 \text{ g/cm}^3$) and aluminum ($\rho = 2.7 \text{ g/cm}^3$) could have the same average density as pure iron ($\rho = 7.8 \text{ g/cm}^3$)

Additional information needed might include:

- Visual inspection or sectioning to identify components
- Chemical analysis or spectroscopy
- X-ray imaging to reveal internal structure
- Melting point or other physical property measurements

Discussion: Density measurements remain valuable for quality control and authenticity testing in many industries, from precious metals to food products. For example, the dairy industry uses density to detect watered-down milk, and petroleum refiners use density to characterize crude oil grades.

[Figure 3] shows a glass of ice water filled to the brim. Will the water overflow when the ice melts? Explain your answer.

Show Solution

Strategy: Apply Archimedes' principle: a floating object displaces a volume of water equal to its own weight. Compare the volume of water displaced by floating ice to the volume of water produced when the ice melts.

Solution: No, the water will not overflow when the ice melts.

Here's why:

1. **Floating ice displaces water equal to its weight.** By Archimedes' principle, the buoyant force equals the weight of displaced water. For floating equilibrium: $\rho_{\text{water}} \cdot V_{\text{displaced}} \cdot g = \rho_{\text{ice}} \cdot V_{\text{ice}} \cdot g$
2. **Volume displaced by floating ice:** $V_{\text{displaced}} = \frac{\rho_{\text{ice}}}{\rho_{\text{water}}} \cdot V_{\text{ice}}$
3. **Mass of ice equals mass of water it becomes:** $m_{\text{ice}} = m_{\text{water produced}}$ $\rho_{\text{ice}} \cdot V_{\text{ice}} = \rho_{\text{water}} \cdot V_{\text{water produced}}$
4. **Volume of water produced when ice melts:** $V_{\text{water produced}} = \frac{\rho_{\text{ice}}}{\rho_{\text{water}}} \cdot V_{\text{ice}}$
5. **These are equal!** The volume of water produced by melting ice exactly equals the volume the ice displaced while floating.

Discussion: This elegant result explains why melting icebergs and sea ice (ice floating in the ocean) do not directly raise sea levels. However, melting glaciers and ice sheets on land *do* raise sea levels because that ice wasn't previously displacing ocean water. The density of ice (0.917 g/cm^3) being less than water (1.000 g/cm^3) is also why ice floats in the first place—an unusual property that allows aquatic life to survive under frozen lakes.

Problems & Exercises

Gold is sold by the troy ounce (31.103 g). What is the volume of 1 troy ounce of pure gold?

Show Solution

Strategy: Use the density formula $\rho = m/V$ and solve for volume, using the density of gold from Table 1.

Solution: Given:

- Mass: $m = 31.103\ \text{g}$
- Gold density (from Table 1): $\rho = 19.32\ \text{g/cm}^3$

Solving for volume: $V = \frac{m}{\rho} = \frac{31.103\ \text{g}}{19.32\ \text{g/cm}^3} = 1.610\ \text{cm}^3$

Discussion: One troy ounce of gold has a volume of approximately **1.61 cm³**. This is surprisingly small—about the size of a sugar cube! Gold's extremely high density (19.32 g/cm³) means it packs a lot of mass into a small volume. This property makes gold valuable for coinage and jewelry: a small piece can be quite valuable. It also explains why gold bars are surprisingly small for their weight. For comparison, 1 troy ounce of aluminum would occupy about 11.5 cm³—over 7 times larger—highlighting gold's remarkable density.

Mercury is commonly supplied in flasks containing 34.5 kg (about 76 lb). What is the volume in liters of this much mercury?

Show Solution

Strategy: Use the definition of density $\rho = m/V$ to solve for volume. Look up the density of mercury from Table 1.

Solution: From Table 1, the density of mercury is: $\rho_{\text{Hg}} = 13.6 \times 10^3\ \text{kg/m}^3$

Solving the density equation for volume: $V = \frac{m}{\rho}$

Substituting values: $V = \frac{34.5\ \text{kg}}{13.6 \times 10^3\ \text{kg/m}^3}$

$V = 2.54 \times 10^{-3}\ \text{m}^3$

Converting to liters ($1\ \text{m}^3 = 1000\ \text{L}$): $V = 2.54 \times 10^{-3}\ \text{m}^3 \times \frac{1000\ \text{L}}{1\ \text{m}^3} = 2.54\ \text{L}$

Discussion: The volume of 34.5 kg of mercury is approximately **2.54 liters**. This is a surprisingly small volume for such a large mass—about the size of a large soda bottle. This illustrates mercury's extremely high density (13.6 times denser than water). The high density of mercury is why it was historically used in barometers and thermometers, as small volumes could provide significant measurable changes in height. The result is reasonable: mercury is one of the densest common liquids, so we expect a small volume for a given mass.

(a) What is the mass of a deep breath of air having a volume of 2.00 L? (b) Discuss the effect taking such a breath has on your body's volume and density.

Show Solution

Strategy: Use the density formula $\rho = m/V$ to find mass, using the density of air from Table 1.

Solution:

(a) Mass of air:

From Table 1, the density of air is $\rho_{\text{air}} = 1.29 \times 10^{-3}\ \text{g/cm}^3 = 1.29\ \text{kg/m}^3$

Convert volume to m³:

$V = 2.00\ \text{L} = 2.00 \times 10^{-3}\ \text{m}^3$

Calculate mass using $m = \rho V$:

$m = (1.29\ \text{kg/m}^3)(2.00 \times 10^{-3}\ \text{m}^3) = 2.58 \times 10^{-3}\ \text{kg} = 2.58\ \text{g}$

(b) Effect on body volume and density:

The volume of your body increases by the volume of air you inhale (2.00 L). The average density of your body decreases when you take a deep breath because the density of air (1.29 kg/m³) is substantially smaller than the average density of the human body (approximately 1010-1070 kg/m³, close to water).

Discussion: The mass of a deep breath is approximately **2.58 g** (about 0.0057 pounds)—a surprisingly small mass for such a large volume. This demonstrates air's very low density compared to solids and liquids. The density decrease when inhaling has practical implications: taking a deep breath increases buoyancy when swimming because your average body density decreases toward that of water. This is why swimmers take a deep breath before floating and exhale to sink. The volume increase of 2 liters represents about 3-4% of typical body volume (~60 L for an average adult), causing a proportional decrease in average density.

A straightforward method of finding the density of an object is to measure its mass and then measure its volume by submerging it in a graduated cylinder. What is the density of a 240-g rock that displaces 89.0cm^3 of water? (Note that the accuracy and practical applications of this technique are more limited than a variety of others that are based on Archimedes' principle.)

Show Solution

Strategy: Use the definition of density $\rho = m/V$, where the volume equals the water displaced.

Solution: Given:

- Mass: $m = 240$ g
- Volume displaced: $V = 89.0\ \text{cm}^3$

Calculate density: $\rho = \frac{m}{V} = \frac{240\ \text{g}}{89.0\ \text{cm}^3} = 2.70\ \text{g/cm}^3$

Discussion: The density of the rock is approximately **2.70 g/cm³** (or 2700 kg/m³). Comparing to Table 1, this density is close to that of aluminum (2.70 g/cm³) and also consistent with common rock types like granite (~2.7 g/cm³) or limestone (~2.3-2.7 g/cm³). The method works because the volume of water displaced equals the volume of the submerged object. While simple, this technique has limitations: it requires that the object be completely submerged, doesn't absorb water, and that any air bubbles are removed. More sophisticated techniques based on Archimedes' principle (like measuring apparent weight in water) can be more accurate and versatile.

Suppose you have a coffee mug with a circular cross section and vertical sides (uniform radius). What is its inside radius if it holds 375 g of coffee when filled to a depth of 7.50 cm? Assume coffee has the same density as water.

Show Solution

Strategy: First find the volume of coffee using density, then use the volume formula for a cylinder to solve for the radius.

Solution: *Step 1: Find the volume of coffee*

Using density to find volume (with $\rho_{\text{water}} = 1.00\ \text{g/cm}^3$): $V = \frac{m}{\rho} = \frac{375\ \text{g}}{1.00\ \text{g/cm}^3} = 375\ \text{cm}^3$

Step 2: Use the cylinder volume formula to find radius

For a cylinder with circular cross section: $V = \pi r^2 h$

Solving for radius: $r^2 = \frac{V}{\pi h}$

$$r = \sqrt{\frac{V}{\pi h}}$$

Substituting values (with $h = 7.50$ cm): $r = \sqrt{\frac{375\ \text{cm}^3}{\pi \times 7.50\ \text{cm}}}$

$$r = \sqrt{\frac{375\ \text{cm}^3}{23.56\ \text{cm}}}$$
$$r = \sqrt{15.92\ \text{cm}^2}$$
$$r = 3.99\ \text{cm} \approx 4.0\ \text{cm}$$

Discussion: The inside radius of the coffee mug is approximately **4.0 cm** (or about 1.6 inches in diameter = 3.2 inches). This is a reasonable size for a typical coffee mug. The calculation confirms the geometry is sensible: a mug about 8 cm (3.2 inches) in inside diameter and 7.5 cm deep would hold about 375 mL of coffee, which is a typical large cup of coffee.

(a) A rectangular gasoline tank can hold 50.0 kg of gasoline when full. What is the depth of the tank if it is 0.500-m wide by 0.900-m long? (b) Discuss whether this gas tank has a reasonable volume for a passenger car.

Show Solution

Strategy: First, find the volume of gasoline using its density from Table 1, then use the volume formula for a rectangular tank to find the depth.

Solution:

(a) Find the depth:

From Table 1, gasoline (petrol) has density $\rho = 0.68 \times 10^3\ \text{kg/m}^3 = 680\ \text{kg/m}^3$

Find the volume using $V = m/\rho$:

$$V = \frac{50.0\ \text{kg}}{680\ \text{kg/m}^3} = 0.0735\ \text{m}^3$$

For a rectangular tank, $V = L \times W \times h$. Solving for depth (height):

$$h = \frac{V}{L \times W} = \frac{0.0735\ \text{m}^3}{(0.900\ \text{m})(0.500\ \text{m})} = \frac{0.0735\ \text{m}^3}{0.450\ \text{m}^2} = 0.163\ \text{m}$$

(b) Is this reasonable for a passenger car?

Convert volume to gallons (1 m³ = 264.2 gallons):

$$V = 0.0735\ \text{m}^3 \times 264.2\ \frac{\text{gal}}{\text{m}^3} = 19.4\ \text{gallons}$$

Discussion: The tank depth is approximately **0.163 m** (about 6.4 inches), and the tank holds **19.4 gallons**. This is a reasonable tank size for a passenger car. Most modern passenger cars have fuel tanks in the range of 12-20 gallons, with compact cars typically having 12-15 gallons and larger sedans or SUVs having 15-25 gallons. The dimensions (90 cm × 50 cm × 16.3 cm) are also practical for installation under the rear of a vehicle. The relatively shallow depth allows the tank to fit in the space between the vehicle floor and undercarriage while still providing adequate fuel capacity for a driving range of 300-400 miles.

A trash compactor can reduce the volume of its contents to 0.350 their original value. Neglecting the mass of air expelled, by what factor is the density of the rubbish increased?

Show Solution

Strategy: Since mass is conserved (neglecting expelled air), use the density formula $\rho = m/V$ to find how density changes when volume decreases.

Solution: Let the original density be ρ_0 with original volume V_0 and mass m: $\rho_0 = \frac{m}{V_0}$

After compaction, the new volume is: $V_{\text{new}} = 0.350 \cdot V_0$

The new density (with mass unchanged) is: $\rho_{\text{new}} = \frac{m}{V_{\text{new}}} = \frac{m}{0.350 \cdot V_0}$

The ratio of new density to original density: $\frac{\rho_{\text{new}}}{\rho_0} = \frac{m/(0.350 \cdot V_0)}{m/V_0} = \frac{1}{0.350} = 2.86$

Discussion: The density of the compacted rubbish is increased by a factor of approximately **2.86** (or about 2.9). This makes physical sense: if the volume is reduced to about one-third while mass stays constant, the density must roughly triple. Trash compactors are effective because they reduce waste volume significantly, allowing more trash to fit in the same container or landfill space. The expelled air has negligible mass compared to the solid waste, making our assumption reasonable.

A 2.50-kg steel gasoline can holds 20.0 L of gasoline when full. What is the average density of the full gas can, taking into account the volume occupied by steel as well as by gasoline?

Show Solution

Strategy: Find the total mass (steel + gasoline), the total volume (steel + gasoline), then calculate average density. Use densities from Table 1 for gasoline and steel.

Solution:

Step 1: Find the mass of gasoline

From Table 1, gasoline has density $\rho_{\text{gas}} = 0.68 \times 10^3\ \text{kg/m}^3 = 680\ \text{kg/m}^3$

Convert volume: $V_{\text{gas}} = 20.0\ \text{L} = 0.0200\ \text{m}^3$

$$m_{\text{gas}} = \rho_{\text{gas}} V_{\text{gas}} = (680\ \text{kg/m}^3)(0.0200\ \text{m}^3) = 13.6\ \text{kg}$$

Step 2: Find the volume of steel

From Table 1, steel has density $\rho_{\text{steel}} = 7.8 \times 10^3\ \text{kg/m}^3$

$$V_{\text{steel}} = \frac{m_{\text{steel}}}{\rho_{\text{steel}}} = \frac{2.50\ \text{kg}}{7800\ \text{kg/m}^3} = 3.21 \times 10^{-4}\ \text{m}^3$$

Step 3: Calculate average density

Total mass: $m_{\text{total}} = 2.50 + 13.6 = 16.1\ \text{kg}$

Total volume: $V_{\text{total}} = 0.0200 + 3.21 \times 10^{-4} = 0.0203\ \text{m}^3$

$$\rho_{\text{avg}} = \frac{m_{\text{total}}}{V_{\text{total}}} = \frac{16.1\ \text{kg}}{0.0203\ \text{m}^3} = 793\ \text{kg/m}^3 \approx 7.9 \times 10^2\ \text{kg/m}^3$$

Discussion: The average density of the full gas can is approximately **790 kg/m³**. This is remarkably close to the density of gasoline alone (680 kg/m³), even though the can is made of steel (7800 kg/m³). This is because the steel can holds a much larger volume of the less dense gasoline. The steel's volume contribution is only about 1.6% of the total volume, so it has minimal effect on the average density. This result makes physical sense: the can would still float high in water (which has density 1000 kg/m³), though not as high as gasoline alone would float.

What is the density of 18.0-karat gold that is a mixture of 18 parts gold, 5 parts silver, and 1 part copper? (These values are parts by mass, not volume.) Assume that this is a simple mixture having an average density equal to the weighted densities of its constituents.

Show Solution

Strategy: Calculate the weighted average of the densities based on mass fractions. First determine mass fractions, then find volumes occupied by each component, and finally calculate overall density.

Solution:

From Table 1:

- Gold density: $\rho_{\text{Au}} = 19.32\ \text{g/cm}^3$
- Silver density: $\rho_{\text{Ag}} = 10.5\ \text{g/cm}^3$ (standard value)
- Copper density: $\rho_{\text{Cu}} = 8.8\ \text{g/cm}^3$

Step 1: Find mass fractions

Total parts by mass: $18 + 5 + 1 = 24$ parts

Mass fractions:

- Gold: $f_{\text{Au}} = 18/24 = 0.750$
- Silver: $f_{\text{Ag}} = 5/24 = 0.208$
- Copper: $f_{\text{Cu}} = 1/24 = 0.0417$

Step 2: Calculate average density

For a mixture where we know mass fractions, the total volume for mass m is:

$$V_{\text{total}} = \frac{m_{\text{Au}}}{\rho_{\text{Au}}} + \frac{m_{\text{Ag}}}{\rho_{\text{Ag}}} + \frac{m_{\text{Cu}}}{\rho_{\text{Cu}}} = m\left(\frac{f_{\text{Au}}}{\rho_{\text{Au}}} + \frac{f_{\text{Ag}}}{\rho_{\text{Ag}}} + \frac{f_{\text{Cu}}}{\rho_{\text{Cu}}}\right)$$

The average density is:

$$\rho_{\text{avg}} = \frac{m}{V_{\text{total}}} = \frac{1}{\frac{f_{\text{Au}}}{\rho_{\text{Au}}} + \frac{f_{\text{Ag}}}{\rho_{\text{Ag}}} + \frac{f_{\text{Cu}}}{\rho_{\text{Cu}}}}$$

Substituting values:

$$\rho_{\text{avg}} = \frac{1}{\frac{0.750}{19.32} + \frac{0.208}{10.5} + \frac{0.0417}{8.8}} = \frac{1}{0.03882 + 0.01981 + 0.00474} = \frac{1}{0.06337} = 15.8\ \text{g/cm}^3$$

Rounding to three significant figures: $\rho_{\text{avg}} \approx 15.6\ \text{g/cm}^3$

Discussion: The density of 18-karat gold is approximately **15.6 g/cm³** (or 15,600 kg/m³). This is significantly less than pure 24-karat gold (19.32 g/cm³) because the added silver and copper are less dense. The 18-karat designation means 18/24 = 75% gold by mass, with the remainder being alloying metals. These alloys are used in jewelry because pure gold is too soft for practical use—the added metals increase hardness and durability while reducing cost. Different karat values indicate different gold fractions: 14-karat is 14/24 = 58.3% gold, 10-karat is 41.7% gold. Measuring density is one method jewelers use to verify gold purity, though it cannot distinguish between different alloy compositions with similar densities.

There is relatively little empty space between atoms in solids and liquids, so that the average density of an atom is about the same as matter on a macroscopic scale—approximately 10^3kg/m^3. The nucleus of an atom has a radius about 10^{-5} that of the atom and contains nearly all the mass of the entire atom. (a) What is the approximate density of a nucleus? (b) One remnant of a supernova, called a neutron star, can have the density of a nucleus. What would be the radius of a neutron star with a mass 10 times that of our Sun (the radius of the Sun is $7 \times 10^8 \text{m}$) ?

Show Solution

Strategy: (a) Use the fact that nearly all the atom's mass is in the nucleus, which has a much smaller volume than the atom. The density ratio equals the inverse of the volume ratio. (b) Use the mass and density to find volume, then calculate radius from the volume of a sphere.

Solution:

(a) Density of a nucleus:

The density ratio is the inverse of the volume ratio. If the nucleus radius is 10^{-5} times the atom radius:

$$r_{\text{nucleus}} = 10^{-5} r_{\text{atom}}$$

The volume of a sphere is $V = \frac{4}{3}\pi r^3$, so:

$$\frac{V_{\text{nucleus}}}{V_{\text{atom}}} = \frac{r^3_{\text{nucleus}}}{r^3_{\text{atom}}} = (10^{-5})^3 = 10^{-15}$$

Since the nucleus contains nearly all the mass:

$$\rho_{\text{nucleus}}=\frac{m_{\text{atom}}}{V_{\text{nucleus}}}=\frac{m_{\text{atom}}}{10^{-15}V_{\text{atom}}}=\frac{\rho_{\text{atom}}}{10^{-15}}=\frac{10^{3}\text{ kg/m}^{3}}{10^{-15}}=10^{18}\text{ kg/m}^{3}$$

(b) Radius of neutron star:

Mass of neutron star: $M = 10M_{\text{Sun}} = 10(2.0\times10^{30}\text{ kg}) = 2.0\times10^{31}\text{ kg}$

Using $\rho = M/V$ and $V = \frac{4}{3}\pi r^{3}$:

$$r^{3}=\frac{3M}{4\pi\rho}=\frac{3(2.0\times10^{31}\text{ kg})}{4\pi(10^{18}\text{ kg/m}^{3})}=\frac{6.0\times10^{31}}{1.26\times10^{19}}=4.76\times10^{12}\text{ m}^{3}$$
$$r=(4.76\times10^{12})^{1/3}\text{ m}=1.68\times10^{4}\text{ m}\approx2\times10^{4}\text{ m}$$

Discussion: (a) The nuclear density is approximately **10^{18} kg/m³**—a trillion times denser than ordinary matter! This enormous density results from packing nearly all the atom's mass into a volume that is 10^{-15} times smaller.

(b) The neutron star radius is approximately **20 km** (about 12 miles). This is astonishingly small for an object with 10 times the Sun's mass—the Sun itself has a radius of 700,000 km. This dramatic compression (radius reduction by a factor of 35,000) demonstrates the extreme density of neutron stars. These stellar remnants are formed when massive stars collapse in supernovae, crushing matter to nuclear densities. A sugar-cube-sized sample of neutron star material would have a mass of about a billion tons! The result is reasonable: neutron stars are indeed observed to have radii of 10-20 km with masses of 1-3 solar masses, confirming they have densities comparable to atomic nuclei.

Glossary

density
: the mass per unit volume of a substance or object

Pressure

- Define pressure.
- Explain the relationship between pressure and force.
- Calculate force given pressure and area.

You have no doubt heard the word **pressure** being used in relation to blood (high or low blood pressure) and in relation to the weather (high- and low-pressure weather systems). These are only two of many examples of pressures in fluids. Pressure P is defined as

$$P=FA$$

where F is a force applied to an area A that is perpendicular to the force.

Pressure

Pressure is defined as the force divided by the area perpendicular to the force over which the force is applied, or

$$P=FA.$$

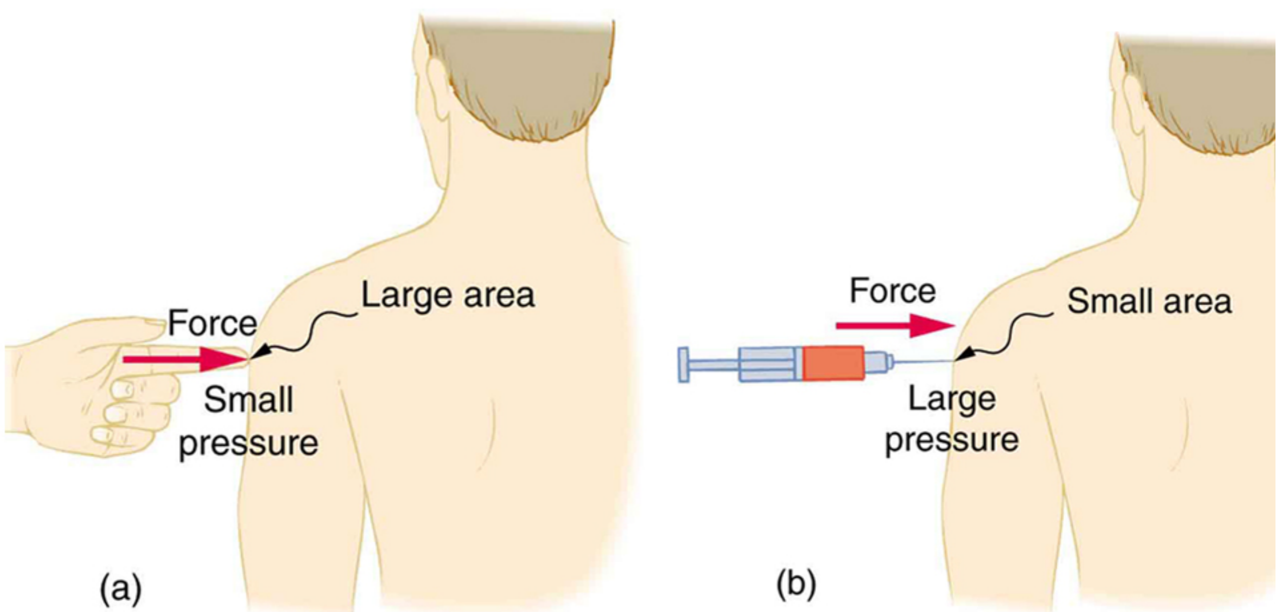
A given force can have a significantly different effect depending on the area over which the force is exerted, as shown in [Figure 1]. The SI unit for pressure is the *pascal*, where

$$1\text{Pa}=1\text{N/m}^2.$$

In addition to the pascal, there are many other units for pressure that are in common use. In meteorology, atmospheric pressure is often described in units of millibar (mb), where

$$100\text{mb}=1\times10^4\text{Pa}.$$

Pounds per square inch $(\text{lb/in}^2 \text{ or psi})$ is still sometimes used as a measure of tire pressure, and millimeters of mercury (mm Hg) is still often used in the measurement of blood pressure. Pressure is defined for all states of matter but is particularly important when discussing fluids.



(a) While the person being poked with the finger might be irritated, the force has little lasting effect. (b) In contrast, the same force applied to an area the size of the sharp end of a needle is great enough to break the skin.

Calculating Force Exerted by the Air: What Force Does a Pressure Exert?

An astronaut is working outside the International Space Station where the atmospheric pressure is essentially zero. The pressure gauge on her air tank reads $6.90\times10^6\text{Pa}$. What force does the air inside the tank exert on the flat end of the cylindrical tank, a disk 0.150 m in diameter?

Strategy

We can find the force exerted from the definition of pressure given in $P=FA$, provided we can find the area A acted upon.

Solution

By rearranging the definition of pressure to solve for force, we see that

$$F=PA.$$

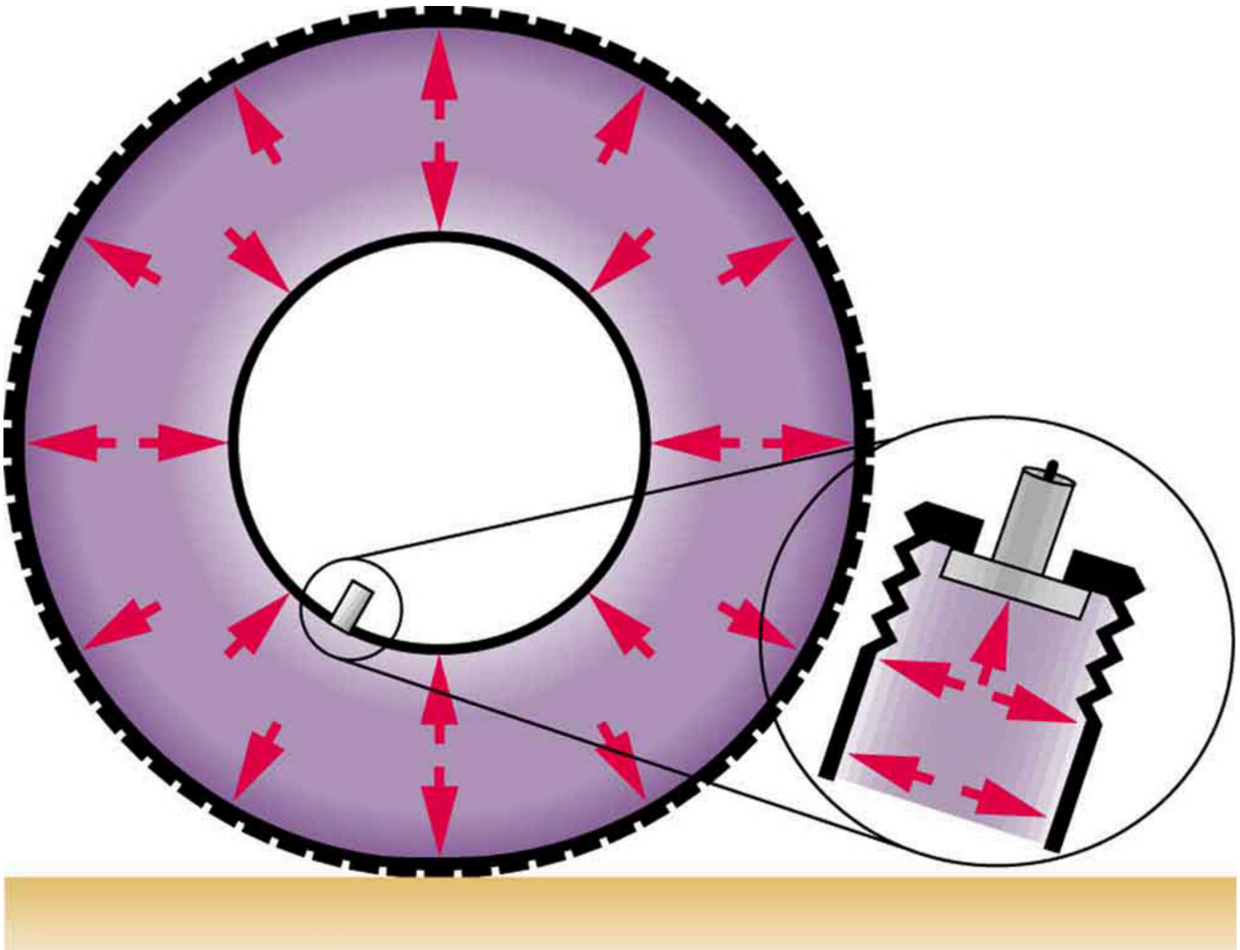
Here, the pressure P is given, as is the area of the end of the cylinder A, given by $A=\pi r^2$. Thus,

$$F = (6.90\times10^{6}\text{N/m}^{2})(3.14)(0.0750m)^{2} = 1.22\times10^{5}\text{N}.$$

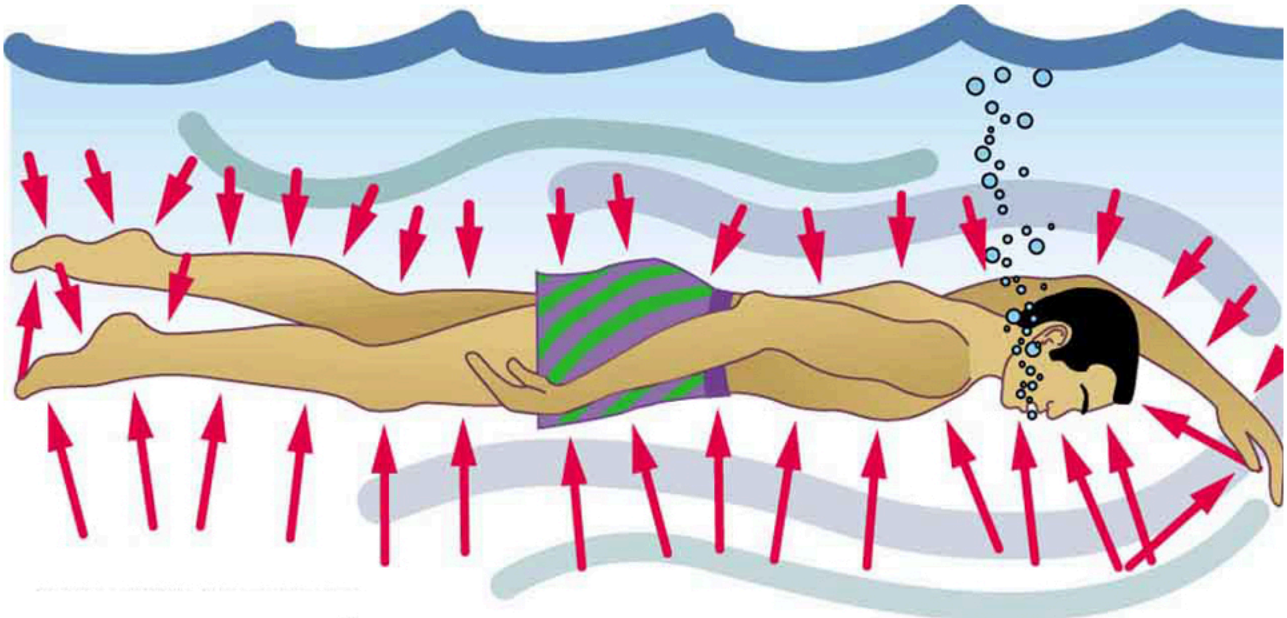
Discussion

Wow! No wonder the tank must be strong. Since we found $F=PA$, we see that the force exerted by a pressure is directly proportional to the area acted upon as well as the pressure itself.

The force exerted on the end of the tank is perpendicular to its inside surface. This direction is because the force is exerted by a static or stationary fluid. We have already seen that fluids cannot *withstand* shearing (sideways) forces; they cannot *exert* shearing forces, either. Fluid pressure has no direction, being a scalar quantity. The forces due to pressure have well-defined directions: they are always exerted perpendicular to any surface. (See the tire in [Figure 2], for example.) Finally, note that pressure is exerted on all surfaces. Swimmers, as well as the tire, feel pressure on all sides. (See [Figure 3].)



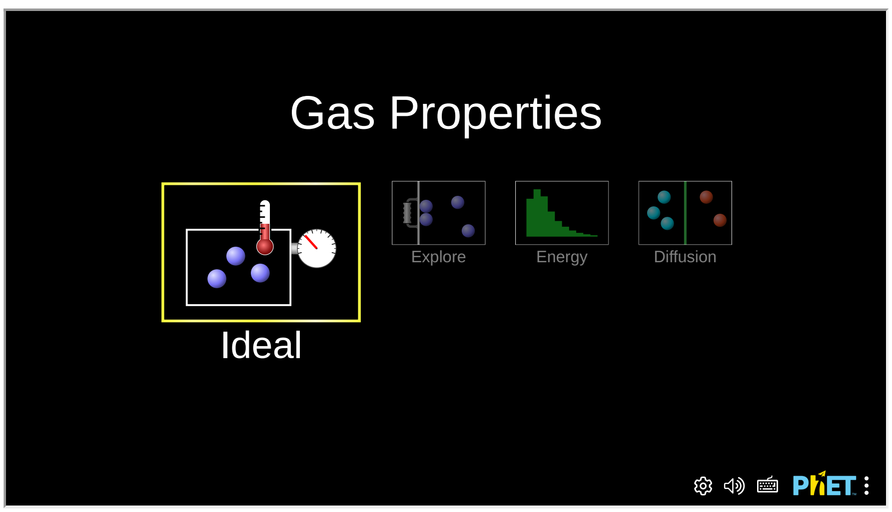
Pressure inside this tire exerts forces perpendicular to all surfaces it contacts. The arrows give representative directions and magnitudes of the forces exerted at various points. Note that static fluids do not exert shearing forces.



Pressure is exerted on all sides of this swimmer, since the water would flow into the space he occupies if he were not there. The arrows represent the directions and magnitudes of the forces exerted at various points on the swimmer. Note that the forces are larger underneath, due to greater depth, giving a net upward or buoyant force that is balanced by the weight of the swimmer.

PhET Explorations: Gas Properties

Pump gas molecules to a box and see what happens as you change the volume, add or remove heat, change gravity, and more. Measure the temperature and pressure, and discover how the properties of the gas vary in relation to each other.



Section Summary

- Pressure is the force per unit perpendicular area over which the force is applied. In equation form, pressure is defined as

  $P = FA.$

- The SI unit of pressure is pascal and $1\text{Pa} = 1\text{N/m}^2$.

Conceptual Questions

How is pressure related to the sharpness of a knife and its ability to cut?

Show Solution

Strategy: Apply the pressure equation $P = F/A$ to understand how the contact area affects the pressure generated by a given force.

Solution: A sharper knife cuts better because **a sharper blade has a smaller contact area, which produces greater pressure for the same applied force.**

From the pressure equation $P = F / A$:

- A sharp knife has a very thin edge (small area A)
- For the same cutting force F, a smaller area produces greater pressure
- Higher pressure more easily exceeds the material's resistance to being cut

For example, if a dull blade has 10× the edge area of a sharp blade:

- Sharp blade: $P_{\text{sharp}} = F / A$
- Dull blade: $P_{\text{dull}} = F / (10A) = P_{\text{sharp}} / 10$

Discussion: This is why we sharpen knives—sharpening removes material to create a thinner edge, reducing contact area and increasing cutting pressure. This same principle explains why cutting boards should be softer than knife blades, why sharp scissors cut paper better than dull ones, and why a razor blade can easily cut skin while a butter knife cannot, even with similar applied forces.

Why does a dull hypodermic needle hurt more than a sharp one?

Show Solution

Strategy: Consider how needle sharpness affects both the contact area and the force required for penetration, using the pressure relationship.

Solution: A dull hypodermic needle hurts more because:

1. **Larger contact area requires more force:** A dull needle has a larger tip area. From $P = F / A$, to achieve the same penetration pressure, a larger area requires greater force: $F = P \times A$
2. **More tissue deformation:** The dull tip cannot cut cleanly through tissue. Instead, it must push and deform more tissue aside, stretching and compressing nerve endings over a larger area.
3. **Greater tissue damage:** A sharp needle slices cleanly between cells with minimal trauma. A dull needle tears tissue, causing more inflammation and pain response.
4. **Longer penetration time:** Healthcare workers must push harder and longer with dull needles, prolonging the painful stimulus.

Discussion: This is why single-use needles are standard practice—even one use can slightly dull a needle. Studies show that injection pain correlates strongly with needle sharpness. The same physics explains why paper cuts hurt: paper edges are microscopically sharp, creating high pressure that cuts through skin cleanly, but the wound's irregular edges and exposed nerve endings cause significant pain despite minimal visible damage.

The outward force on one end of an air tank was calculated in [Example 1]. How is this force balanced? (The tank does not accelerate, so the force must be balanced.)

Show Solution

Strategy: Apply Newton's first law: for the tank to remain stationary, all forces must balance. Consider all surfaces where pressure acts.

Solution: The outward force on one end of the tank is balanced by **internal tensile forces in the tank walls** (material stress).

Here's the force analysis:

1. **Pressure acts on both ends:** The internal pressure creates equal and opposite outward forces on both flat ends of the cylindrical tank.
2. **These forces are transmitted through the walls:** The curved cylindrical walls experience tensile (stretching) stress as the pressure tries to push the ends apart.
3. **The wall material provides the restoring force:** The molecular bonds in the tank material (typically steel or aluminum) resist stretching, providing an inward force that balances the outward pressure force.

Force balance on each end cap: $F_{\text{pressure}} = F_{\text{tension in walls}}$

Discussion: This is why pressure vessels must be made of strong materials with adequate wall thickness. The calculated force of 1.22×10^5 N (about 27,000 pounds!) would cause a weak container to rupture. Tank designers calculate the required wall thickness using the tensile strength of the material. This also explains why pressure tanks are inspected regularly—any weakness in the walls could lead to catastrophic failure. The cylindrical shape is preferred because it distributes stress evenly, unlike sharp corners which concentrate stress.

Why is force exerted by static fluids always perpendicular to a surface?

Show Solution

Strategy: Consider the defining property of fluids (they cannot resist shear forces) and what would happen if a force component existed parallel to a surface.

Solution: The force exerted by static fluids is always perpendicular to a surface because **fluids cannot sustain shear (tangential) forces without flowing.**

Logical argument:

1. Any force can be decomposed into perpendicular and parallel components relative to a surface
2. A parallel (tangential) component would be a shearing force
3. By definition, fluids yield to shearing forces and flow
4. If any tangential force component existed, the fluid would flow until that component disappeared
5. For a static (non-flowing) fluid, all tangential components must be zero
6. Therefore, only the perpendicular component remains

Mathematically, if the force had a tangential component: $\vec{F} = F_\perp \hat{n} + F_\parallel \hat{t}$

For static equilibrium: $F_\parallel = 0$, leaving only $\vec{F} = F_\perp \hat{n}$

Discussion: This fundamental property is why pressure (force per area) is a scalar, not a vector—it has magnitude but its direction is always determined by the surface orientation. This perpendicularity is essential for hydraulic systems to work: pressure applied anywhere transmits equally in all directions (Pascal's principle), allowing hydraulic lifts and brakes to function. It also explains why a fluid in a tilted container always has a horizontal surface at rest —gravity's tangential component causes flow until the surface becomes perpendicular to the net force.

In a remote location near the North Pole, an iceberg floats in a lake. Next to the lake (assume it is not frozen) sits a comparably sized glacier sitting on land. If both chunks of ice should melt due to rising global temperatures (and the melted ice all goes into the lake), which ice chunk would give the greatest increase in the level of the lake water, if any?

Show Solution

Strategy: Apply Archimedes' principle to the floating iceberg and compare with the land-based glacier's contribution when melted.

Solution: The glacier on land would give the greatest increase in lake level. The floating iceberg would cause essentially no change in water level when it melts.

Analysis:

Floating iceberg:

- By Archimedes' principle, the floating ice displaces water equal to its weight
- When ice melts, the water produced has the same mass (and weight) as the original ice
- The volume of meltwater exactly equals the volume the ice displaced while floating
- Net change in lake level: **zero**

Glacier on land:

- The land-based glacier displaces no lake water while frozen
- When it melts, all its water mass flows into the lake as new volume
- Net change in lake level: **increases** by volume equal to the meltwater

Discussion: This distinction is crucial for understanding sea level rise due to climate change:

- Melting sea ice (Arctic ice cap, floating icebergs) does not directly raise sea levels
- Melting land ice (Greenland ice sheet, Antarctic glaciers, mountain glaciers) does raise sea levels significantly

The Antarctic and Greenland ice sheets contain enough ice that, if completely melted, would raise global sea levels by approximately 70 meters. Sea ice melting has other important effects (reducing Earth's reflectivity, affecting ecosystems) but does not directly contribute to sea level rise.

How do jogging on soft ground and wearing padded shoes reduce the pressures to which the feet and legs are subjected?

Show Solution

Strategy: Consider how both factors affect contact area and stopping time/distance, and how these relate to pressure and force.

Solution: Both soft ground and padded shoes reduce pressure on feet and legs through two mechanisms:

1. Increased contact area (reduces pressure)

From $P = F/A$:

- Soft surfaces and padding conform to the foot's shape
- This increases the contact area A over which impact force is distributed
- Greater area means lower pressure for the same force
- Hard surfaces create pressure concentration at small contact points

2. Increased stopping distance (reduces force)

From impulse-momentum: $F \cdot \Delta t = \Delta p$

- Soft materials compress during impact, increasing stopping time Δt
- For the same momentum change, longer stopping time means smaller average force
- Smaller force combined with larger area dramatically reduces pressure

Combined effect example:

- If padding doubles contact area and doubles stopping time
- Force is halved (from longer stopping time)
- Pressure is quartered: $P_{\text{new}} = \frac{F/2}{2A} = \frac{F}{4A} = \frac{P_{\text{original}}}{4}$

Discussion: This explains why runners often choose grass or track surfaces over concrete, and why running shoe technology focuses heavily on cushioning. The impact forces during running can reach 2-3 times body weight, and reducing the resulting pressure helps prevent stress fractures, joint damage, and soft tissue injuries. The same principles apply to protective equipment in other sports—helmets, padding, and mats all work by increasing contact area and impact time.

Toe dancing (as in ballet) is much harder on toes than normal dancing or walking. Explain in terms of pressure.

Show Solution

Strategy: Compare the contact areas supporting body weight during toe dancing (en pointe) versus normal standing or walking, then calculate the pressure difference.

Solution: Toe dancing creates much higher pressure on toes because **the entire body weight is supported by an extremely small area—just the tips of the toes.**

Quantitative comparison for a 50 kg dancer:

- Weight: $W = mg = (50 \text{ kg})(9.8 \text{ m/s}^2) = 490 \text{ N}$

Normal standing (both feet flat):

- Contact area ≈ 300 cm² = 0.030 m²
- Pressure: $P = \frac{490 \text{ N}}{0.030 \text{ m}^2} \approx 16{,}000 \text{ Pa}$

Toe dancing (en pointe on one foot):

- Contact area ≈ 10 cm² = 0.0010 m² (just the pointe shoe platform)
- Pressure: $P = \frac{490 \text{ N}}{0.0010 \text{ m}^2} \approx 490{,}000 \text{ Pa}$

The pressure while en pointe is approximately **30 times greater** than normal standing!

Discussion: This extreme pressure explains why ballet dancers:

- Develop calluses and often experience foot injuries
- Require years of training to strengthen feet and ankles
- Use specially designed pointe shoes with reinforced toe boxes
- Cannot begin pointe training until bones are sufficiently developed (typically age 11-13)

The toe box of a pointe shoe helps distribute force over a slightly larger area, but the pressure remains far higher than normal. Professional dancers frequently experience stress fractures, bunions, and other foot problems due to these sustained high pressures.

How do you convert pressure units like millimeters of mercury, centimeters of water, and inches of mercury into units like newtons per meter squared without resorting to a table of pressure conversion factors?

Show Solution

Strategy: Use the hydrostatic pressure formula $P = \rho g h$ with the known density of the reference fluid and the given column height.

Solution: Pressure units like “mm Hg” or “cm H_2O” represent the pressure at the bottom of a column of that fluid with that height. To convert to pascals (N/m²), use:

$$P = \rho g h$$

where:

- ρ = density of the reference fluid
- g = 9.80 m/s² (gravitational acceleration)
- h = column height (convert to meters)

Example conversions:

760 mm Hg (standard atmosphere):

- $\rho_{\text{Hg}} = 13{,}600 \text{ kg/m}^3$

- $h = 760\ \text{mm} = 0.760\ \text{m}$
- $P = (13{,}600)(9.80)(0.760) = 101{,}300\ \text{Pa}$

10 cm H_2O:

- $\rho_{\text{water}} = 1000\ \text{kg/m}^3$
- $h = 10\ \text{cm} = 0.10\ \text{m}$
- $P = (1000)(9.80)(0.10) = 980\ \text{Pa}$

30 inches Hg:

- $h = 30\ \text{in} \times 0.0254\ \text{m/in} = 0.762\ \text{m}$
- $P = (13{,}600)(9.80)(0.762) = 101{,}600\ \text{Pa}$

Discussion: This method requires knowing only the fluid's density and basic unit conversions. The approach works because these pressure units were historically defined by manometer measurements—literally the height of a fluid column that a pressure could support. Mercury is commonly used because its high density (13.6× water) gives convenient column heights, while water columns are used for lower pressures. Understanding this conversion method also helps in reading manometers and barometers directly.

Problems & Exercises

As a woman walks, her entire weight is momentarily placed on one heel of her high-heeled shoes. Calculate the pressure exerted on the floor by the heel if it has an area of 1.50cm^2 and the woman's mass is 55.0 kg. Express the pressure in Pa. (In the early days of commercial flight, women were not allowed to wear high-heeled shoes because aircraft floors were too thin to withstand such large pressures.)

Show Solution

Strategy: Calculate the woman's weight (force) and divide by the contact area using $P = F/A$.

Solution:

Step 1: Calculate the woman's weight

$$F = mg = (55.0\ \text{kg})(9.80\ \text{m/s}^2) = 539\ \text{N}$$

Step 2: Convert area to m²

$$A = 1.50\ \text{cm}^2 = 1.50 \times 10^{-4}\ \text{m}^2$$

Step 3: Calculate pressure

$$P = \frac{F}{A} = \frac{539\ \text{N}}{1.50 \times 10^{-4}\ \text{m}^2} = 3.59 \times 10^6\ \text{Pa}$$

Converting to lb/in²:

$$P = 3.59 \times 10^6\ \text{Pa} \times \frac{1\ \text{lb/in}^2}{6.90 \times 10^3\ \text{Pa}} = 521\ \text{lb/in}^2$$

Discussion: The pressure exerted is approximately **3.59×10^6 Pa** (or **521 psi**), which is about 35 times atmospheric pressure! This enormous pressure results from concentrating the woman's entire weight on the tiny contact area of a high heel. This explains the early aviation restriction—thin aircraft floors could be punctured or permanently dented by such concentrated forces. The same physics explains why snowshoes work (large area = low pressure) and why stiletto heels can damage wooden floors and soft surfaces.

The pressure exerted by a phonograph needle on a record is surprisingly large. If the equivalent of 1.00 g is supported by a needle, the tip of which is a circle 0.200 mm in radius, what pressure is exerted on the record in N/m^2 ?

Show Solution

Strategy: Calculate the force (weight) supported by the needle, find the contact area of the circular tip, then apply the pressure formula $P = F/A$.

Solution: *Step 1: Calculate the force (weight)*

$$F = mg = (1.00 \times 10^{-3}\ \text{kg})(9.80\ \text{m/s}^2) = 9.80 \times 10^{-3}\ \text{N}$$

Step 2: Calculate the contact area

The tip is circular with radius $r = 0.200\ \text{mm} = 0.200 \times 10^{-3}\ \text{m} = 2.00 \times 10^{-4}\ \text{m}$

$$A = \pi r^2 = \pi (2.00 \times 10^{-4}\ \text{m})^2$$

$A = \pi(4.00 \times 10^{-8}\ \text{m}^2) = 1.257 \times 10^{-7}\ \text{m}^2$

Step 3: Calculate the pressure

$P = FA = 9.80 \times 10^{-3}\ \text{N} 1.257 \times 10^{-7}\ \text{m}^2$
$P = 7.80 \times 10^4\ \text{N/m}^2 = 7.80 \times 10^4\ \text{Pa}$

Discussion: The pressure exerted by the phonograph needle is approximately **7.80 × 10^4 Pa** (or about 78 kPa, which is roughly 11 psi). This is indeed surprisingly large—nearly 80% of atmospheric pressure—from just one gram of force! The pressure is high because the contact area is extremely small (about 0.13 mm^2). This high pressure is necessary for the needle to follow the microscopic grooves in the record (typically 40-50 micrometers wide). However, it also explains why records wear out with repeated playing and why proper needle maintenance is important for preserving vinyl records.

Nail tips exert tremendous pressures when they are hit by hammers because they exert a large force over a small area. What force must be exerted on a nail with a circular tip of 1.00 mm diameter to create a pressure of $3.00 \times 10^9 \text{N/m}^2$? (This high pressure is possible because the hammer striking the nail is brought to rest in such a short distance.)

Show Solution

Strategy: Use $P = F/A$ to solve for force, where the area is that of the circular nail tip.

Solution:

Step 1: Calculate the nail tip area

Diameter = 1.00 mm, so radius $r = 0.500\ \text{mm} = 5.00 \times 10^{-4}\ \text{m}$

$A = \pi r^2 = \pi(5.00 \times 10^{-4}\ \text{m})^2 = 7.85 \times 10^{-7}\ \text{m}^2$

Step 2: Solve for force

From $P = F/A$:

$F = PA = (3.00 \times 10^9\ \text{N/m}^2)(7.85 \times 10^{-7}\ \text{m}^2)$
$F = 2.36 \times 10^3\ \text{N}$

Discussion: The required force is approximately **2360 N** (about 530 pounds). This seems like a modest force compared to the enormous pressure it creates —3 GPa is roughly 30,000 times atmospheric pressure! This pressure is sufficient to exceed the compressive strength of wood, allowing the nail to penetrate. The key is the tiny contact area of the nail tip (less than 1 mm^2), which concentrates force into extreme pressure. When a hammer strikes a nail, it briefly exerts thousands of newtons over a very short stopping distance, transferring its kinetic energy to drive the nail. The same principle applies to all pointed tools—axes, chisels, and drill bits all work by concentrating force into small areas.

Glossary

pressure
: the force per unit area perpendicular to the force, over which the force acts

---

Variation of Pressure with Depth in a Fluid

- Define pressure in terms of weight.
- Explain the variation of pressure with depth in a fluid.
- Calculate density given pressure and altitude.

If your ears have ever popped on a plane flight or ached during a deep dive in a swimming pool, you have experienced the effect of depth on pressure in a fluid. At the Earth's surface, the air pressure exerted on you is a result of the weight of air above you. This pressure is reduced as you climb up in altitude and the weight of air above you decreases. Under water, the pressure exerted on you increases with increasing depth. In this case, the pressure being exerted upon you is a result of both the weight of water above you *and* that of the atmosphere above you. You may notice an air pressure change on an elevator ride that transports you many stories, but you need only dive a meter or so below the surface of a pool to feel a pressure increase. The difference is that water is much denser than air, about 775 times as dense.

Consider the container in [Figure 1]. Its bottom supports the weight of the fluid in it. Let us calculate the pressure exerted on the bottom by the weight of the fluid. That **pressure** is the weight of the fluid mg divided by the area A supporting it (the area of the bottom of the container):

$$P=mgA.$$

We can find the mass of the fluid from its volume and density:

$$m=\rho V.$$

The volume of the fluid V is related to the dimensions of the container. It is

$$V=Ah,$$

where A is the cross-sectional area and h is the depth. Combining the last two equations gives

$$m=\rho Ah.$$

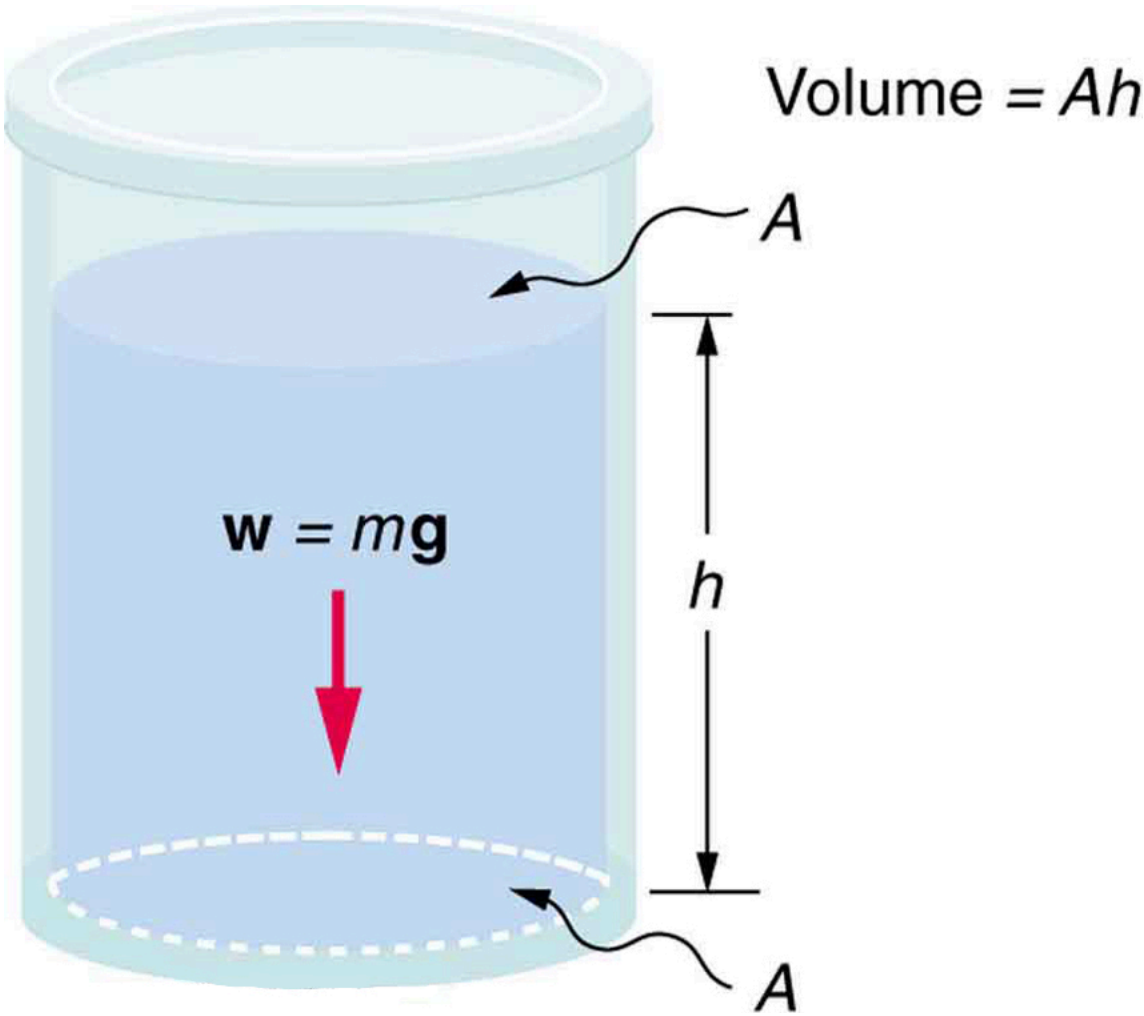
If we enter this into the expression for pressure, we obtain

$$P=(\rho Ah)gA.$$

The area cancels, and rearranging the variables yields

$$P=h\rho g.$$

This value is the *pressure due to the weight of a fluid*. The equation has general validity beyond the special conditions under which it is derived here. Even if the container were not there, the surrounding fluid would still exert this pressure, keeping the fluid static. Thus the equation $P=h\rho g$ represents the pressure due to the weight of any fluid of *average density* ρ at any depth h below its surface. For liquids, which are nearly incompressible, this equation holds to great depths. For gases, which are quite compressible, one can apply this equation as long as the density changes are small over the depth considered. [Example 2] illustrates this situation.



The bottom of this container supports the entire weight of the fluid in it. The vertical sides cannot exert an upward force on the fluid (since it cannot withstand a shearing force), and so the bottom must support it all.

Calculating the Average Pressure and Force Exerted: What Force Must a Dam Withstand?

In [Example 1], we calculated the mass of water in a large reservoir. We will now consider the pressure and force acting on the dam retaining water. (See [Figure 2].) The dam is 500 m wide, and the water is 80.0 m deep at the dam. (a) What is the average pressure on the dam due to the water? (b) Calculate the force exerted against the dam and compare it with the weight of water in the dam (previously found to be $1.96\times10^{13}\text{N}$).

Strategy for (a)

The average pressure $\bar{P}$ due to the weight of the water is the pressure at the average depth $\bar{h}$ of 40.0 m, since pressure increases linearly with depth.

Solution for (a)

The average pressure due to the weight of a fluid is

$$\bar{P}=\bar{h}\rho g.$$

Entering the density of water from [Table 1] and taking $\bar{h}$ to be the average depth of 40.0 m, we obtain

$$\bar{P} = (40.0\text{m})(10^3\text{kgm}^3)(9.80\text{ms}^2) = 3.92\times10^5\text{Nm}^2=392\text{kPa}.$$

Strategy for (b)

The force exerted on the dam by the water is the average pressure times the area of contact:

$$F=\bar{P}A.$$

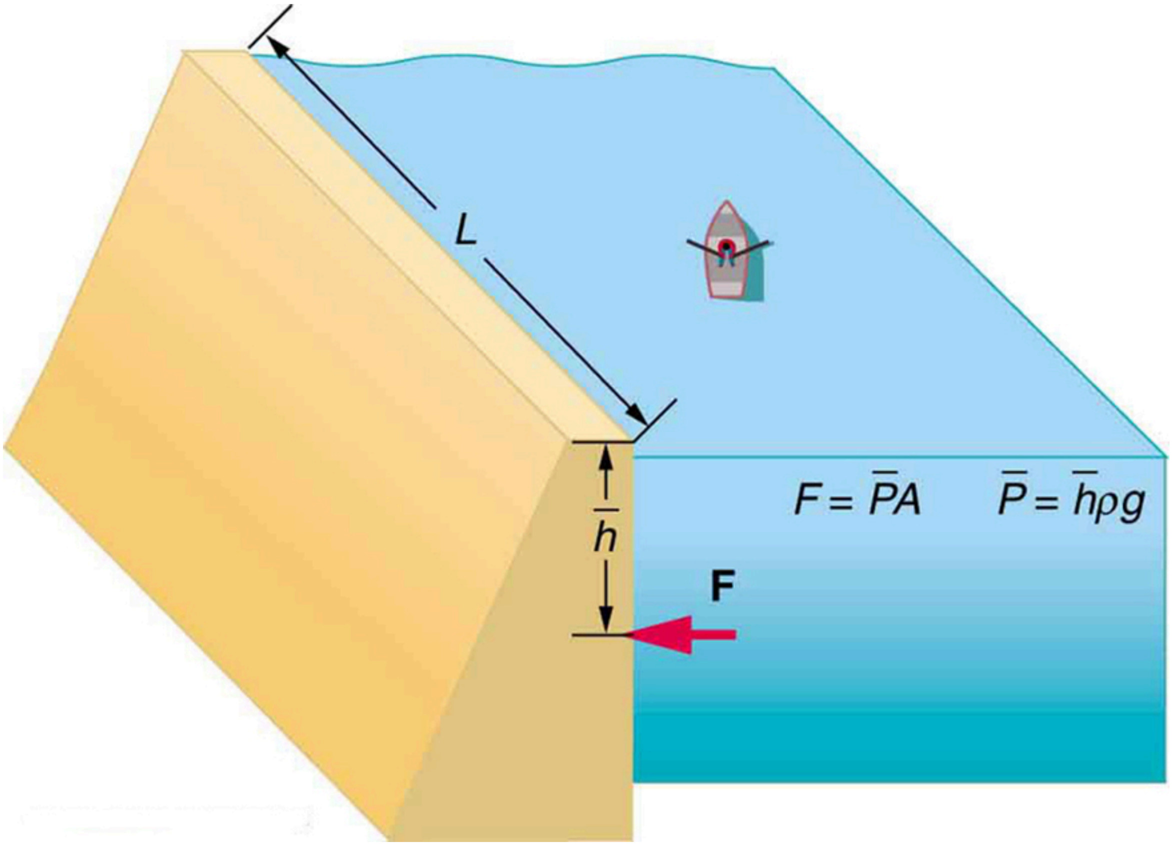
Solution for (b)

We have already found the value for $\bar{P}$. The area of the dam is $A=80.0\text{m}\times500\text{m}=4.00\times10^4\text{m}^2$, so that

$$F = (3.92\times10^{5}\text{N/m}^{2})(4.00\times10^{4}\text{m}^{2}) = 1.57\times10^{10}\text{N}.$$

Discussion

Although this force seems large, it is small compared with the $1.96\times10^{13}\text{N}$ weight of the water in the reservoir—in fact, it is only 0.0800% of the weight. Note that the pressure found in part (a) is completely independent of the width and length of the lake—it depends only on its average depth at the dam. Thus the force depends only on the water's average depth and the dimensions of the dam, *not* on the horizontal extent of the reservoir. In the diagram, the thickness of the dam increases with depth to balance the increasing force due to the increasing pressure.

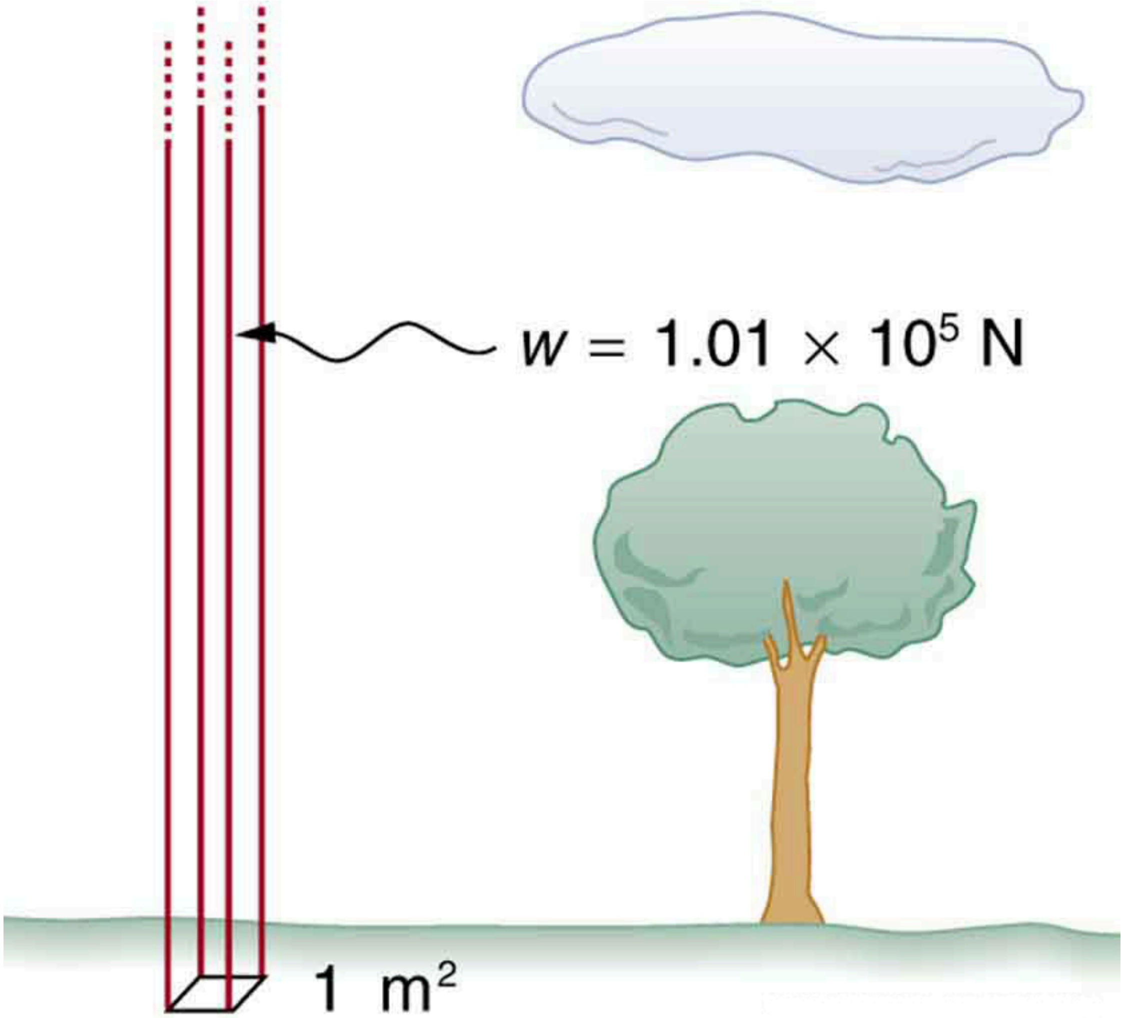


The dam must withstand the force exerted against it by the water it retains. This force is small compared with the weight of the water behind the dam.

Atmospheric pressure is another example of pressure due to the weight of a fluid, in this case due to the weight of *air* above a given height. The atmospheric pressure at the Earth's surface varies a little due to the large-scale flow of the atmosphere induced by the Earth's rotation (this creates weather "highs" and "lows"). However, the average pressure at sea level is given by the *standard atmospheric pressure* P_{atm}, measured to be

$$1\text{atmosphere (atm)}=P_{\text{atm}}=1.01\times10^{5}\text{N/m}^{2}=101\text{kPa}.$$

This relationship means that, on average, at sea level, a column of air above 1.00m^2 of the Earth's surface has a weight of $1.01\times10^{5}\text{N}$, equivalent to 1atm. (See [Figure 3].)



Atmospheric pressure at sea level averages $1.01 \times 10^5 \text{Pa}$ (equivalent to 1 atm), since the column of air over this 1m^2, extending to the top of the atmosphere, weighs $1.01 \times 10^5 \text{N}$.

Calculating Average Density: How Dense Is the Air?

Calculate the average density of the atmosphere, given that it extends to an altitude of 120 km. Compare this density with that of air listed in [Table 1].

Strategy

If we solve $P = h\rho g$ for density, we see that

$$\bar{\rho} = \frac{P}{hg}.$$

We then take P to be atmospheric pressure, h is given, and g is known, and so we can use this to calculate $\bar{\rho}$.

Solution

Entering known values into the expression for $\bar{\rho}$ yields

$$\bar{\rho} = \frac{1.01 \times 10^5 \text{N/m}^2}{(120 \times 10^3 \text{m})(9.80 \text{m/s}^2)} = 8.59 \times 10^{-2} \text{kg/m}^3.$$

Discussion

This result is the average density of air between the Earth's surface and the top of the Earth's atmosphere, which essentially ends at 120 km. The density of air at sea level is given in [Table 1] as 1.29kg/m^3 —about 15 times its average value. Because air is so compressible, its density has its highest value near the Earth's surface and declines rapidly with altitude.

Calculating Depth Below the Surface of Water: What Depth of Water Creates the Same Pressure as the Entire Atmosphere?

Calculate the depth below the surface of water at which the pressure due to the weight of the water equals 1.00 atm.

Strategy

We begin by solving the equation $P = h\rho g$ for depth h :

$$h=\frac{P}{\rho g}.$$

Then we take P to be 1.00 atm and ρ to be the density of the water that creates the pressure.

Solution

Entering the known values into the expression for h gives

$$h=\frac{1.01\times10^5\,\text{N/m}^2}{(1.00\times10^3\,\text{kg/m}^3)(9.80\,\text{m/s}^2)}=10.3\,\text{m}.$$

Discussion

Just 10.3 m of water creates the same pressure as 120 km of air. Since water is nearly incompressible, we can neglect any change in its density over this depth.

What do you suppose is the *total* pressure at a depth of 10.3 m in a swimming pool? Does the atmospheric pressure on the water's surface affect the pressure below? The answer is yes. This seems only logical, since both the water's weight and the atmosphere's weight must be supported. So the *total* pressure at a depth of 10.3 m is 2 atm—half from the water above and half from the air above. We shall see in Pascal's Principle that fluid pressures always add in this way.

Section Summary

- Pressure is the weight of the fluid mg divided by the area A supporting it (the area of the bottom of the container):

  $$P=\frac{mg}{A}.$$

- Pressure due to the weight of a liquid is given by

  $$P=h\rho g,$$

  where P is the pressure, h is the height of the liquid, ρ is the density of the liquid, and g is the acceleration due to gravity.

Conceptual Questions

Atmospheric pressure exerts a large force (equal to the weight of the atmosphere above your body—about 10 tons) on the top of your body when you are lying on the beach sunbathing. Why are you able to get up?

Show Solution

Strategy: Consider that atmospheric pressure acts on all surfaces, not just the top of your body. Analyze the net force from atmospheric pressure.

Solution: You can get up because **atmospheric pressure acts equally on all sides of your body, resulting in zero net force from atmospheric pressure.**

Force analysis:

- Pressure on top (pushing down): $F_{\text{top}} = P_{\text{atm}} \times A_{\text{top}}$
- Pressure on bottom (pushing up): $F_{\text{bottom}} = P_{\text{atm}} \times A_{\text{bottom}}$
- Since $A_{\text{top}} \approx A_{\text{bottom}}$, these forces cancel

The atmosphere also pushes inward on all sides of your body (front, back, sides), but your body's internal pressure (blood pressure, pressure in tissues) balances these forces. Your body has evolved to function at atmospheric pressure, with internal pressures that counterbalance the external atmospheric pressure.

Discussion: This is similar to why fish don't feel crushed by water pressure—their internal pressure matches the external water pressure. If atmospheric pressure suddenly disappeared (like in a vacuum), the internal pressure would cause your body to expand. Conversely, deep-sea creatures brought to the surface too quickly can be damaged because their internal pressure exceeds the reduced external pressure. The key insight is that pressure forces from all directions must be considered together.

Why does atmospheric pressure decrease more rapidly than linearly with altitude?

Show Solution

Strategy: Consider how the density of air varies with altitude due to its compressibility, and how this affects the pressure-altitude relationship.

Solution: Atmospheric pressure decreases more rapidly than linearly because **air is compressible, so its density decreases with altitude, causing pressure to drop exponentially rather than linearly.**

Detailed explanation:

1. The equation $P = \rho g h$ assumes constant density

2. For incompressible fluids (like water), this gives linear pressure increase with depth
3. Air is highly compressible—at lower altitudes, the weight of air above compresses the air, increasing its density
4. At higher altitudes, less air above means less compression and lower density
5. Since pressure depends on the weight of air above, and that air has decreasing density at higher altitudes, pressure drops off faster than linear

Mathematically, for an isothermal atmosphere: $P = P_0 e^{-h/H}$

where H is the scale height (about 8.5 km for Earth). This exponential decay is faster than linear at low altitudes.

Discussion: At sea level, about 50% of the atmosphere's mass is below 5.5 km altitude. The next 50% is spread over the remaining ~115 km to the edge of space. This concentration of air at low altitudes is why aircraft pressurize cabins for high-altitude flight, and why weather phenomena occur primarily in the lowest layer of the atmosphere (troposphere).

What are two reasons why mercury rather than water is used in barometers?

Show Solution

Strategy: Compare the physical properties of mercury and water that affect barometer design and practicality.

Solution: Two key reasons mercury is preferred over water in barometers:

1. Mercury's high density allows for a much shorter tube

From $P = \rho g h$, solving for height: $h = P/(\rho g)$

For 1 atm pressure:

- Mercury ($\rho = 13{,}600\ \text{kg/m}^3$): $h = 0.760\ \text{m}$ (76 cm)
- Water ($\rho = 1{,}000\ \text{kg/m}^3$): $h = 10.3\ \text{m}$

A water barometer would need to be over 10 meters tall—impractical for most applications!

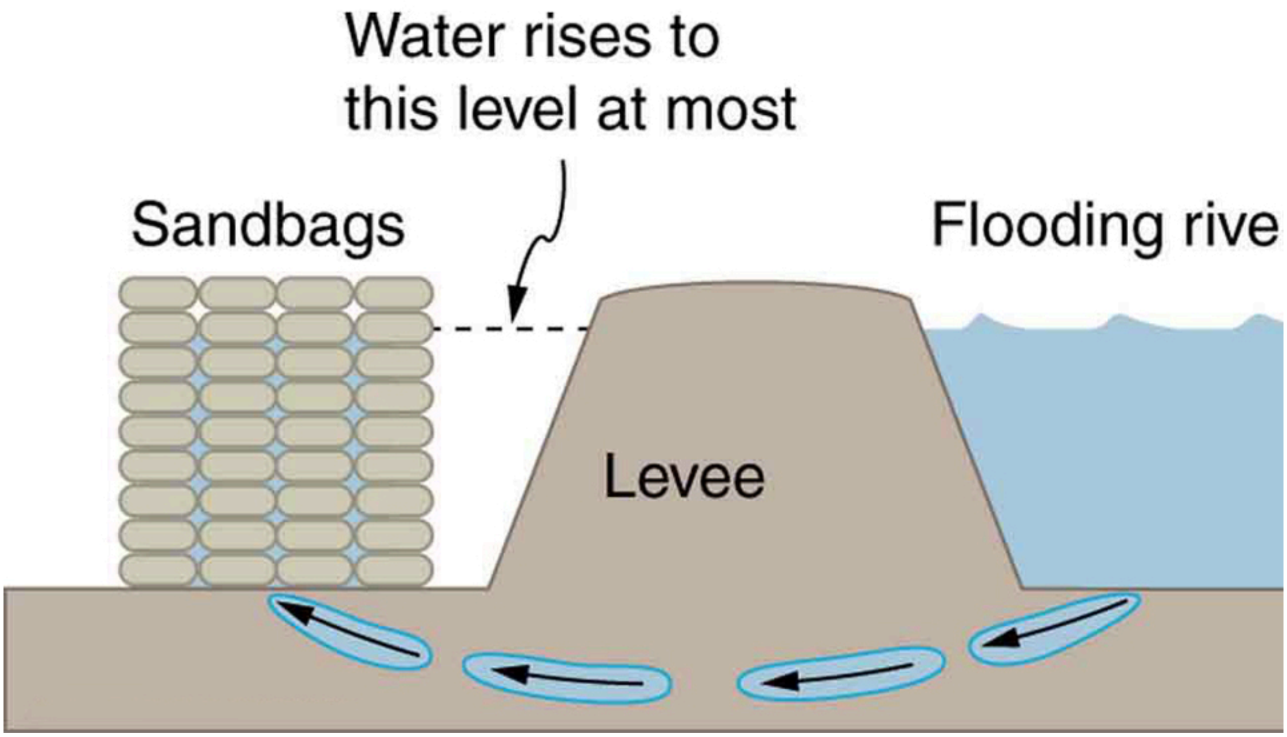
2. Mercury has very low vapor pressure at room temperature

- Mercury's vapor pressure at 20°C: ~0.002 mm Hg (negligible)
- Water's vapor pressure at 20°C: ~17.5 mm Hg

The space above the mercury column must be a near-vacuum. Water would evaporate into this space, creating vapor pressure that would depress the column and give inaccurate readings. Mercury's extremely low evaporation rate maintains the vacuum.

Discussion: Additional advantages of mercury include: it doesn't wet glass (forms clean meniscus for easy reading), it's chemically stable, and its silvery appearance makes the column easily visible. However, mercury's toxicity has led to restrictions on its use, and modern barometers often use aneroid (mechanical) or electronic pressure sensors instead.

[Figure 4] shows how sandbags placed around a leak outside a river levee can effectively stop the flow of water under the levee. Explain how the small amount of water inside the column formed by the sandbags is able to balance the much larger body of water behind the levee.



Because the river level is very high, it has started to leak under the levee. Sandbags are placed around the leak, and the water held by them rises until it is the same level as the river, at which point the water there stops rising.

Show Solution

Strategy: Apply the principle that pressure in a connected fluid depends only on depth, not on the total volume of fluid.

Solution: The small column of water can balance the large river because **pressure at a given depth depends only on the height of the fluid column above that point, not on the total volume of water.**

From $P = \rho g h$:

- The pressure at the base of the leak depends only on the height h of water above it
- The river's pressure at the leak: $P_{\text{river}} = \rho g h_{\text{river}}$
- The sandbag column's pressure: $P_{\text{column}} = \rho g h_{\text{column}}$

When $h_{\text{column}} = h_{\text{river}}$:

- Both pressures are equal
- No net pressure difference exists to drive flow
- Water stops rising in the sandbag column

The total volume or horizontal extent of the river is irrelevant—only the vertical height matters for determining pressure.

Discussion: This counterintuitive result is sometimes called the "hydrostatic paradox." It's the same principle that allows a thin tube of water to exert the same pressure as a wide lake at the same depth. This is also how water towers work: a relatively small tank elevated to a certain height provides the same pressure as if the entire water supply were at that elevation. The sandbag technique is a practical emergency flood control method used by civil engineers worldwide.

Why is it difficult to swim under water in the Great Salt Lake?

Show Solution

Strategy: Consider how the high salt content affects water density and thus the buoyant force on a swimmer.

Solution: It is difficult to swim under water in the Great Salt Lake because **the extremely high salt concentration makes the water much denser than the human body, creating a strong buoyant force that pushes swimmers to the surface.**

Quantitative analysis:

- Great Salt Lake density: ~1.1 to 1.2 g/cm³ (varies with salinity)
- Human body density: ~1.01 to 1.08 g/cm³
- Fresh water density: 1.00 g/cm³

By Archimedes' principle, the buoyant force equals the weight of displaced water: $F_b = \rho_{\text{water}} V_{\text{body}} g$

In the Great Salt Lake:

- The buoyant force exceeds body weight
- The body floats very high in the water
- Pushing down to submerge requires fighting this excess buoyancy
- Staying submerged is like trying to hold a beach ball underwater

Discussion: The Great Salt Lake is one of the saltiest bodies of water on Earth (up to 27% salinity, compared to ~3.5% for ocean water). Similar experiences occur in the Dead Sea (33% salinity). Swimmers often find they cannot sink even if they try—they bob like corks. This makes conventional swimming strokes difficult since the body rides too high to get proper arm position in the water. While drowning is very difficult, the high salt content irritates eyes and any cuts, and the salt crust left on skin can be uncomfortable.

Is there a net force on a dam due to atmospheric pressure? Explain your answer.

Show Solution

Strategy: Consider whether atmospheric pressure acts on both sides of the dam and analyze the net effect.

Solution: No, there is essentially no net force on a dam due to atmospheric pressure.

Atmospheric pressure acts on both sides of the dam:

- On the reservoir side: $F_{\text{atm, water side}} = P_{\text{atm}} \times A$
- On the air side: $F_{\text{atm, air side}} = P_{\text{atm}} \times A$

Since the same atmospheric pressure acts on equal areas on both sides: $F_{\text{net, atm}} = P_{\text{atm}} \times A - P_{\text{atm}} \times A = 0$

The atmospheric pressure contributions cancel out.

Discussion: This is why dam engineering calculations typically use *gauge pressure* (pressure above atmospheric) rather than absolute pressure. The net force on the dam comes only from the water pressure, which increases with depth according to $P = \rho g h$. Atmospheric pressure adds to the absolute pressure on the water side, but since it equally pushes on the air side, it doesn't contribute to the net structural load. This same principle applies to any structure exposed to atmosphere on both sides—windows, walls, and diving bells all experience balanced atmospheric forces.

Does atmospheric pressure add to the gas pressure in a rigid tank? In a toy balloon? When, in general, does atmospheric pressure *not* affect the total pressure in a fluid?

Show Solution

Strategy: Consider whether the container walls isolate the internal fluid from atmospheric pressure or transmit it.

Solution:

Rigid tank: No, atmospheric pressure does not add to the internal pressure. The rigid walls support the pressure difference between inside and outside. The gas inside has whatever absolute pressure it was filled to, independent of external atmospheric pressure.

Toy balloon: Yes, atmospheric pressure adds to the internal pressure. The flexible balloon walls transmit atmospheric pressure to the gas inside. If the balloon is inflated to a gauge pressure of 0.01 atm, the absolute internal pressure is approximately 1.01 atm (gauge + atmospheric).

General principle: Atmospheric pressure does NOT affect total fluid pressure when:

1. The fluid is in a rigid, sealed container (like a steel tank or sealed pipe)
2. The container walls can support the pressure difference

Atmospheric pressure DOES add to fluid pressure when:

1. The fluid has a free surface open to atmosphere (like a lake or open container)
2. The container has flexible walls (like a balloon or IV bag)
3. The system is connected to atmosphere through any opening

Discussion: This distinction matters practically. A tire pressure gauge reads *gauge pressure* (pressure above atmospheric), not absolute pressure. A tire reading "32 psi" actually contains air at about 32 + 14.7 = 46.7 psi absolute. Scuba tanks, however, are typically rated in absolute pressure because the rigid tank isolates the contents from external pressure changes.

You can break a strong wine bottle by pounding a cork into it with your fist, but the cork must press directly against the liquid filling the bottle—there can be no air between the cork and liquid. Explain why the bottle breaks, and why it will not if there is air between the cork and liquid.

Show Solution

Strategy: Compare how liquids and gases respond to compression, applying Pascal's principle for pressure transmission.

Solution:

Why the bottle breaks when cork contacts liquid directly:

1. Liquids are essentially *incompressible*
2. When you pound the cork, the cork's motion tries to compress the liquid

3. Since the liquid cannot compress, the pressure increases dramatically and instantaneously
4. By Pascal's principle, this pressure increase transmits equally throughout the liquid to all surfaces
5. The pressure spike exceeds the bottle's strength, causing it to shatter

Why the bottle doesn't break with air between cork and liquid:

1. Air (gas) is highly *compressible*
2. When you pound the cork, the air compresses to a smaller volume
3. The pressure increase is gradual as the air compresses
4. The air acts as a "shock absorber," cushioning the impact
5. The pressure increase is much smaller and spread over more time, so the bottle survives

Quantitative comparison:

- Bulk modulus of water: ~2.2×10^9 Pa (very hard to compress)
- Bulk modulus of air: ~1.4×10^5 Pa (easily compressible)
- Water is about 15,000 times harder to compress than air!

Discussion: This demonstrates both the incompressibility of liquids and Pascal's principle in a dramatic way. The same physics explains why hydraulic systems work so effectively—incompressible fluid instantly transmits pressure. It also explains water hammer in plumbing: when a faucet is closed quickly, the incompressible water creates a pressure spike that can damage pipes.

Problems & Exercises

What depth of mercury creates a pressure of 1.00 atm?

Show Solution

Strategy: Use the hydrostatic pressure formula $P = \rho g h$ and solve for depth.

Solution:

Given:

- Pressure: $P = 1.00\text{ atm} = 1.01 \times 10^5\text{ Pa}$
- Mercury density: $\rho = 13.6 \times 10^3\text{ kg/m}^3$
- $g = 9.80\text{ m/s}^2$

Solving for depth: $h = \frac{P}{\rho g} = \frac{1.01\times10^5\text{ Pa}}{(13.6\times10^3\text{ kg/m}^3)(9.80\text{ m/s}^2)}$

$$h = \frac{1.01\times10^5}{1.33\times10^5}\text{ m} = 0.760\text{ m} = 76.0\text{ cm}$$

Discussion: A mercury column **0.760 m** (or **76.0 cm**, about 30 inches) tall creates a pressure of 1.00 atm. This is the basis for the mercury barometer invented by Torricelli in 1643. At sea level, atmospheric pressure supports a mercury column of this height in an evacuated tube. This is why atmospheric pressure is often expressed as "760 mm Hg" or "760 torr" (named after Torricelli). Mercury is ideal for barometers because its high density allows a compact instrument—a water barometer would need to be over 10 meters tall!

The greatest ocean depths on the Earth are found in the Marianas Trench near the Philippines. Calculate the pressure due to the ocean at the bottom of this trench, given its depth is 11.0 km and assuming the density of seawater is constant all the way down.

Show Solution

Strategy: Use the hydrostatic pressure formula $P = \rho g h$ with the density of seawater and the given depth.

Solution: Given:

- Depth: $h = 11.0\text{ km} = 11.0 \times 10^3\text{ m}$
- Density of seawater: $\rho = 1.025 \times 10^3\text{ kg/m}^3$ (from Table 1)
- $g = 9.80\text{ m/s}^2$

Calculating the pressure due to seawater: $P = \rho g h$

$$P = (1.025\times10^3\text{ kg/m}^3)(9.80\text{ m/s}^2)(11.0\times10^3\text{ m})$$
$$P = 1.10\times10^8\text{ Pa} = 1.10\times10^8\text{ N/m}^2$$

Converting to atmospheres: $P = \frac{1.10\times10^8\text{ Pa}}{1.01\times10^5\text{ Pa/atm}} \approx 1090\text{ atm}$

Discussion: The pressure at the bottom of the Marianas Trench is approximately **1.10×10^8 Pa** (about **1090 atmospheres** or roughly **16,000 psi**). This is an enormous pressure—over 1000 times atmospheric pressure! At this depth, the pressure would crush most submarines and equipment. Only specially designed deep-sea vessels like the Trieste (1960) and Deepsea Challenger (2012) have reached these depths. The assumption of constant density introduces a small error since seawater is slightly compressed at these pressures, but the result gives a good approximation.

Verify that the SI unit of $h\rho g$ is N/m^2.

Show Solution

Strategy: Substitute the SI units for each quantity in the expression $h\rho g$ and simplify algebraically to show the result is equivalent to N/m^2 (the unit of pressure).

Solution:

The SI units for each quantity are:

- Height: h has units of meters (m)
- Density: ρ has units of kg/m^3
- Gravitational acceleration: g has units of m/s^2

Multiplying these units together:

$$(h\rho g)_{\text{units}} = (\text{m})(\text{kg/m}^3)(\text{m/s}^2) = (\text{kg}\cdot\text{m}^2)/(\text{m}^3\cdot\text{s}^2) = (\text{kg}\cdot\text{m/s}^2)(1/\text{m}^2) = \text{N/m}^2$$

Since 1 newton is defined as 1 $\text{kg}\cdot\text{m/s}^2$, the final result is indeed N/m^2, which is the pascal (Pa), the SI unit of pressure.

Discussion: This verification confirms the dimensional consistency of the hydrostatic pressure formula $P = h\rho g$. The algebraic manipulation shows that:

1. Starting with $(\text{m})(\text{kg/m}^3)(\text{m/s}^2)$ gives $\text{kg}\cdot\text{m}^2/(\text{m}^3\cdot\text{s}^2)$
2. Simplifying: $\text{kg}\cdot\text{m}^2/(\text{m}^3\cdot\text{s}^2) = (\text{kg}\cdot\text{m/s}^2)\cdot(1/\text{m}^2) = (\text{kg}\cdot\text{m/s}^2)/\text{m}^2$
3. Recognizing that $\text{kg}\cdot\text{m/s}^2 = \text{N}$ (newton), we get N/m^2 (pascal)

This dimensional analysis is crucial in physics for checking whether equations are physically meaningful. Any valid physics equation must have the same units on both sides, and intermediate steps must make dimensional sense. This particular formula relates pressure (force per area) to the weight of a column of fluid (mass per volume × acceleration × height), and the units verify this relationship is correct.

Water towers store water above the level of consumers for times of heavy use, eliminating the need for high-speed pumps. How high above a user must the water level be to create a gauge pressure of $3.00 \times 10^5 \text{N/m}^2$?

Show Solution

Strategy: Use the hydrostatic pressure formula $P = \rho g h$ and solve for height h.

Solution: Given:

- Desired gauge pressure: $P = 3.00 \times 10^5 \text{ N/m}^2$
- Density of water: $\rho = 1.00 \times 10^3 \text{ kg/m}^3$
- $g = 9.80 \text{ m/s}^2$

Solving $P = \rho g h$ for height: $h = \frac{P}{\rho g}$

$$h = \frac{3.00\times10^5 \text{ N/m}^2}{(1.00\times10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)}$$
$$h = \frac{3.00\times10^5}{9.80\times10^3} \text{ m}$$
$$h = 30.6 \text{ m}$$

Discussion: The water level must be approximately **30.6 meters** (about 100 feet) above the user to create this gauge pressure of 3.00×10^5 Pa (about 44 psi or 3 atm gauge). This is a typical pressure for municipal water systems, adequate for multi-story buildings and fire hydrants. Water towers are typically 40-50 meters tall to provide this pressure plus some margin. The beauty of this system is that gravity does the work—no pumps are needed during peak demand because the elevated water naturally flows downward under pressure. Pumps only run during off-peak hours to refill the tower.

The aqueous humor in a person's eye is exerting a force of 0.300 N on the 1.10-cm^2 area of the cornea. (a) What pressure is this in mm Hg? (b) Is this value within the normal range for pressures in the eye?

Show Solution

Strategy: (a) Use $P = F/A$ to find pressure in pascals, then convert to mm Hg using the relationship $P = \rho g h$. (b) Compare the result to the normal intraocular pressure range given in Table 1 of the Pressures in the Body section.

Solution:

(a) Calculate pressure in mm Hg:

First, calculate pressure in pascals:

$$P = \frac{F}{A} = \frac{0.300\ \text{N}}{1.10 \times 10^{-4}\ \text{m}^2} = 2.73 \times 10^3\ \text{Pa}$$

Convert to mm Hg using $1\ \text{mm Hg} = 133\ \text{Pa}$:

$$P = \frac{2.73 \times 10^3\ \text{Pa}}{133\ \text{Pa/mm Hg}} = 20.5\ \text{mm Hg}$$

(b) Is this within normal range?

From Table 1 in the Pressures in the Body section, normal intraocular pressure (eye pressure) ranges from 12–24 mm Hg. The calculated value of 20.5 mm Hg is within this normal range.

Discussion: The intraocular pressure of **20.5 mm Hg** is comfortably within the normal range for eye pressure. This pressure is maintained by the balance between production and drainage of aqueous humor, the clear fluid that fills the anterior chamber of the eye. The pressure helps maintain the eye’s shape and provides nutrients to the lens and cornea.

Pressures above 24 mm Hg are considered elevated and may indicate glaucoma, a serious condition that can damage the optic nerve and lead to vision loss if untreated. Regular eye pressure measurements (tonometry) are important for detecting glaucoma early, especially for people over 40. The relatively modest pressure (about 1/37 of atmospheric pressure) demonstrates how sensitive the eye’s internal structures are—even small pressure increases can cause significant damage over time.

How much force is exerted on one side of an 8.50 cm by 11.0 cm sheet of paper by the atmosphere? How can the paper withstand such a force?

Show Solution

Strategy: Calculate the area of the paper, then use $F = PA$ to find the force from atmospheric pressure.

Solution: *Step 1: Calculate the area*

$$A = (8.50\ \text{cm})(11.0\ \text{cm}) = 93.5\ \text{cm}^2$$

Converting to m²: $A = 93.5\ \text{cm}^2 \times \left(\frac{1\ \text{m}}{100\ \text{cm}}\right)^2 = 9.35 \times 10^{-3}\ \text{m}^2$

Step 2: Calculate the force

Using atmospheric pressure $P_{\text{atm}} = 1.01 \times 10^5\ \text{Pa}$:

$$F = P \times A = (1.01 \times 10^5\ \text{N/m}^2)(9.35 \times 10^{-3}\ \text{m}^2)$$
$$F = 944\ \text{N} \approx 9.4 \times 10^2\ \text{N}$$

This is equivalent to about 212 pounds of force!

How can the paper withstand such a force?

The paper withstands this enormous force because **atmospheric pressure acts equally on both sides**, creating zero net force:

- Force on front: 944 N (pushing backward)
- Force on back: 944 N (pushing forward)
- Net force: 944 N − 944 N = 0

Discussion: The force on one side of a sheet of paper is approximately **940 N** (about 210 lb). This seems impossibly large for such a thin, flexible material. The paper survives because it never actually experiences this force as a *net* load—the equal and opposite forces from the other side cancel perfectly. This is why objects at atmospheric pressure don’t get crushed: the pressure acts from all directions, creating balanced forces. If you could somehow apply atmospheric pressure to only one side (like with a vacuum pump), even this sheet of paper would be crushed by the unbalanced force.

What pressure is exerted on the bottom of a 0.500-m-wide by 0.900-m-long gas tank that can hold 50.0 kg of gasoline by the weight of the gasoline in it when it is full?

Show Solution

Strategy: Calculate the weight of the gasoline, then divide by the bottom area to find pressure.

Solution:

Step 1: Calculate the weight of gasoline

$$W = mg = (50.0\ \text{kg})(9.80\ \text{m/s}^2) = 490\ \text{N}$$

Step 2: Calculate the bottom area

$$A = (0.500\ \text{m})(0.900\ \text{m}) = 0.450\ \text{m}^2$$

Step 3: Calculate pressure

$$P = \frac{W}{A} = \frac{490\ \text{N}}{0.450\ \text{m}^2} = 1.09 \times 10^3\ \text{Pa}$$

Discussion: The pressure on the bottom of the gas tank is approximately **1.09×10^3 Pa** (about 1.09 kPa or 0.011 atm). This is a relatively modest pressure—only about 1% of atmospheric pressure. This makes sense because gasoline is not very dense (about 680 kg/m³) and the tank is relatively shallow. For comparison, if we calculate the depth of gasoline: $h = V/A = (m/\rho)/A = 50/(680 \times 0.45) = 0.164\ \text{m}$, we can verify using $P = \rho g h = (680)(9.80)(0.164) = 1090\ \text{Pa}$. The tank experiences more pressure from atmospheric pressure (101 kPa) than from its own fuel!

Calculate the average pressure exerted on the palm of a shot-putter's hand by the shot if the area of contact is 50.0cm^2 and he exerts a force of 800 N on it. Express the pressure in N/m^2 and compare it with the $1.00 \times 10^6\text{Pa}$ pressures sometimes encountered in the skeletal system.

Show Solution

Strategy: Use the pressure formula $P = F/A$, being careful to convert area to SI units.

Solution: Given:

- Force: $F = 800\ \text{N}$
- Contact area: $A = 50.0\ \text{cm}^2$

Step 1: Convert area to m²

$$A = 50.0\ \text{cm}^2 \times \left(\frac{1\ \text{m}}{100\ \text{cm}}\right)^2 = 50.0 \times 10^{-4}\ \text{m}^2 = 5.00 \times 10^{-3}\ \text{m}^2$$

Step 2: Calculate pressure

$$P = \frac{F}{A} = \frac{800\ \text{N}}{5.00 \times 10^{-3}\ \text{m}^2}$$
$$P = 1.60 \times 10^5\ \text{N/m}^2 = 1.60 \times 10^5\ \text{Pa}$$

Step 3: Compare with skeletal system pressure

$$\frac{P_{\text{shot-putter}}}{P_{\text{skeletal}}} = \frac{1.60 \times 10^5\ \text{Pa}}{1.00 \times 10^6\ \text{Pa}} = 0.16 = 16\%$$

Discussion: The pressure on the shot-putter's palm is approximately **1.60×10^5 Pa** (about 160 kPa or 23 psi). This is only about **16% of the maximum pressures encountered in the skeletal system** (1.00×10^6 Pa). The higher skeletal pressures occur in joints and bones during activities like running and jumping, where forces concentrate on smaller areas. The result is reasonable: while 800 N is a substantial force (about 180 pounds), it's distributed over a relatively large palm area. This explains why shot-putting, while strenuous, doesn't typically cause hand injuries from pressure alone—the contact area is large enough to keep pressure within tolerable limits.

The left side of the heart creates a pressure of 120 mm Hg by exerting a force directly on the blood over an effective area of 15.0cm^2. What force does it exert to accomplish this?

Show Solution

Strategy: Convert the pressure from mm Hg to Pa, then use $F = PA$ to find the force.

Solution:

Step 1: Convert pressure to pascals

$$P = h\rho_{\text{Hg}}g = (0.120\ \text{m})(13.6 \times 10^3\ \text{kg/m}^3)(9.80\ \text{m/s}^2)$$
$$P = 1.60 \times 10^4\ \text{Pa}$$

Step 2: Convert area to m²

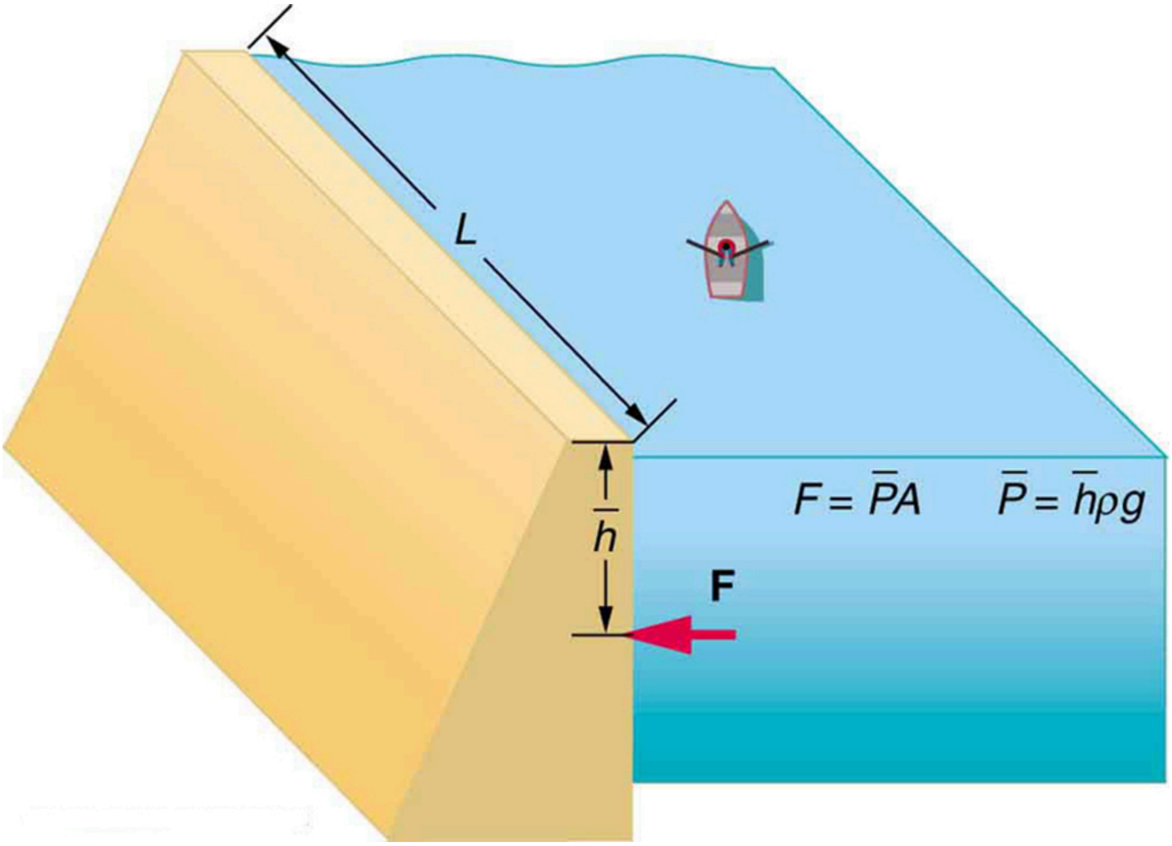
$$A = 15.0\ \text{cm}^2 = 15.0 \times 10^{-4}\ \text{m}^2 = 1.50 \times 10^{-3}\ \text{m}^2$$

Step 3: Calculate force

$$F = PA = (1.60 \times 10^4\ \text{Pa})(1.50 \times 10^{-3}\ \text{m}^2) = 24.0\ \text{N}$$

Discussion: The heart exerts a force of approximately **24.0 N** (about 5.4 pounds) to create the systolic blood pressure of 120 mm Hg. This seems modest, but the heart must maintain this rhythmically about 70 times per minute, every minute, for decades—an impressive feat of biological engineering! The effective area (15 cm²) represents the internal surface of the left ventricle that contacts the blood during contraction. The heart's muscle tissue is specially adapted for this continuous, rhythmic work, with its own dedicated blood supply (coronary arteries) and electrical conduction system. This 24 N force pumps blood through the entire systemic circulation, from the aorta to the smallest capillaries and back to the right atrium.

Show that the total force on a rectangular dam due to the water behind it increases with the *square* of the water depth. In particular, show that this force is given by $F = \rho g h^2 L/2$, where ρ is the density of water, h is its depth at the dam, and L is the length of the dam. You may assume the face of the dam is vertical. (Hint: Calculate the average pressure exerted and multiply this by the area in contact with the water. (See [Figure 5].)



Show Solution

Strategy: Since pressure varies linearly with depth, find the average pressure over the dam face, then multiply by the area in contact with water.

Solution: *Step 1: Determine how pressure varies with depth*

At any depth y below the water surface, the pressure is: $P(y) = \rho g y$

At the surface ($y = 0$): $P = 0$ At the bottom ($y = h$): $P = \rho g h$

Step 2: Calculate the average pressure

Since pressure varies linearly from 0 at the top to $\rho g h$ at the bottom: $\bar{P} = \frac{P_{top}+P_{bottom}}{2} = \frac{0+\rho g h}{2} = \frac{\rho g h}{2}$

Step 3: Calculate the area of the dam face in contact with water

$A = L \times h$

where L is the length (width) of the dam and h is the water depth.

Step 4: Calculate the total force

$$F = \bar{P} \times A = \frac{\rho g h}{2} \times (L \times h)$$
$$\boxed{F = \frac{\rho g h^2 L}{2}}$$

Step 5: Confirm force scales as h²

The formula shows that $F \propto h^2$:

- If depth doubles, force quadruples
- If depth triples, force increases 9-fold

Discussion: The force on a dam increases with the **square of the water depth** because both the average pressure and the contact area increase linearly with depth. Doubling the depth doubles the average pressure AND doubles the area, resulting in 2 × 2 = 4 times the force. This h^2 dependence has profound engineering implications: a dam holding back 20 m of water experiences 4 times more force than one holding 10 m. This explains why dams are built thicker at the bottom—the pressure and force increase dramatically with depth. The derivation also confirms that the force depends on depth and dam length, but NOT on the horizontal extent of the reservoir (consistent with the hydrostatic paradox).

Glossary

pressure
: the weight of the fluid divided by the area supporting it

Pascal’s Principle

- Define pressure.
- State Pascal’s principle.
- Understand applications of Pascal’s principle.
- Derive relationships between forces in a hydraulic system.

Pressure is defined as force per unit area. Can pressure be increased in a fluid by pushing directly on the fluid? Yes, but it is much easier if the fluid is enclosed. The heart, for example, increases blood pressure by pushing directly on the blood in an enclosed system (valves closed in a chamber). If you try to push on a fluid in an open system, such as a river, the fluid flows away. An enclosed fluid cannot flow away, and so pressure is more easily increased by an applied force.

What happens to a pressure in an enclosed fluid? Since atoms in a fluid are free to move about, they transmit the pressure to all parts of the fluid and to the walls of the container. Remarkably, the pressure is transmitted *undiminished*. This phenomenon is called **Pascal’s principle**, because it was first clearly stated by the French philosopher and scientist Blaise Pascal (1623–1662): A change in pressure applied to an enclosed fluid is transmitted undiminished to all portions of the fluid and to the walls of its container.

Pascal’s Principle

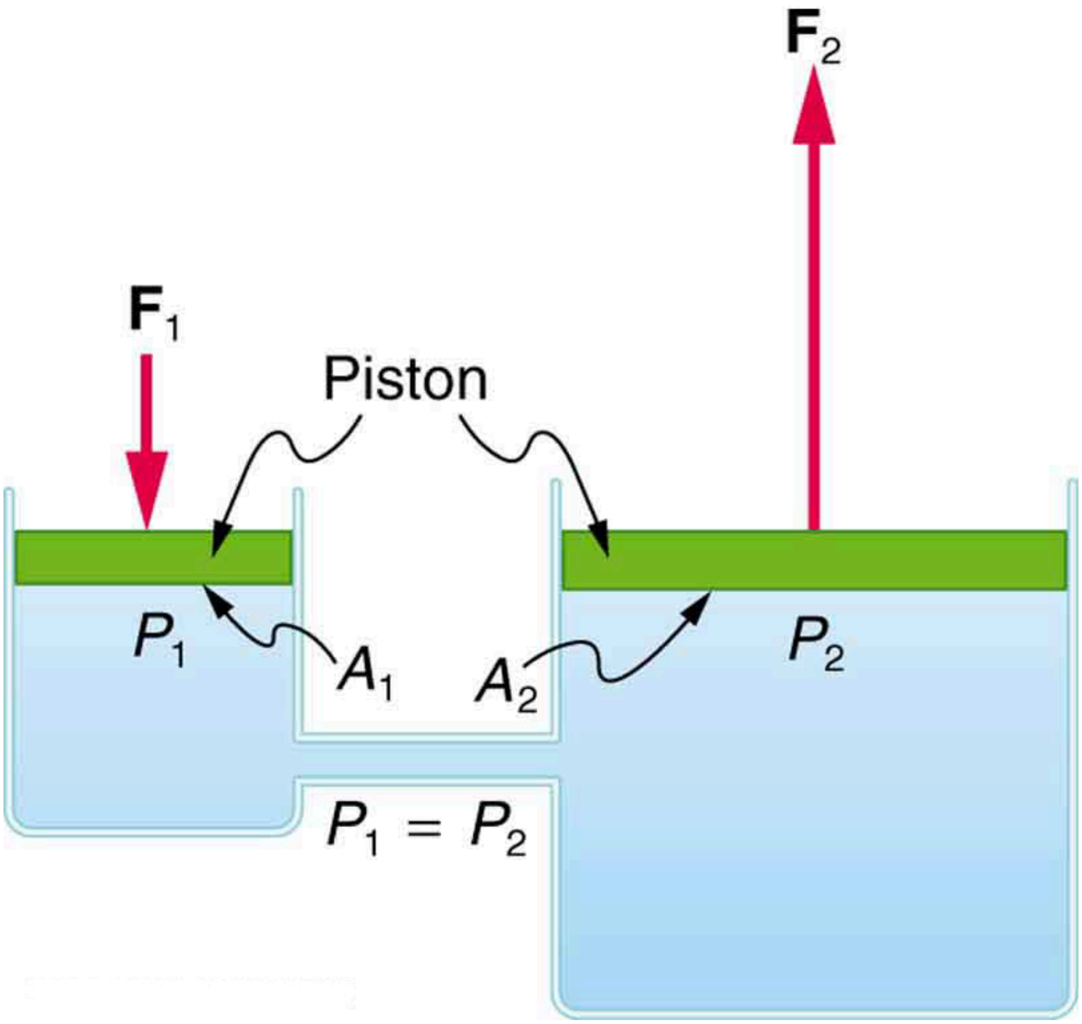
A change in pressure applied to an enclosed fluid is transmitted undiminished to all portions of the fluid and to the walls of its container.

Pascal’s principle, an experimentally verified fact, is what makes pressure so important in fluids. Since a change in pressure is transmitted undiminished in an enclosed fluid, we often know more about pressure than other physical quantities in fluids. Moreover, Pascal’s principle implies that *the total pressure in a fluid is the sum of the pressures from different sources*. We shall find this fact—that pressures add—very useful.

Blaise Pascal had an interesting life in that he was home-schooled by his father who removed all of the mathematics textbooks from his house and forbade him to study mathematics until the age of 15. This, of course, raised the boy’s curiosity, and by the age of 12, he started to teach himself geometry. Despite this early deprivation, Pascal went on to make major contributions in the mathematical fields of probability theory, number theory, and geometry. He is also well known for being the inventor of the first mechanical digital calculator, in addition to his contributions in the field of fluid statics.

Application of Pascal’s Principle

One of the most important technological applications of Pascal’s principle is found in a *hydraulic system*, which is an enclosed fluid system used to exert forces. The most common hydraulic systems are those that operate car brakes. Let us first consider the simple hydraulic system shown in [Figure 1].



A typical hydraulic system with two fluid-filled cylinders, capped with pistons and connected by a tube called a hydraulic line. A downward force F_{1} on the left piston creates a pressure that is transmitted undiminished to all parts of the enclosed fluid. This results in an upward force F_{2} on the right piston that is larger than F_{1} because the right piston has a larger area.

Relationship Between Forces in a Hydraulic System

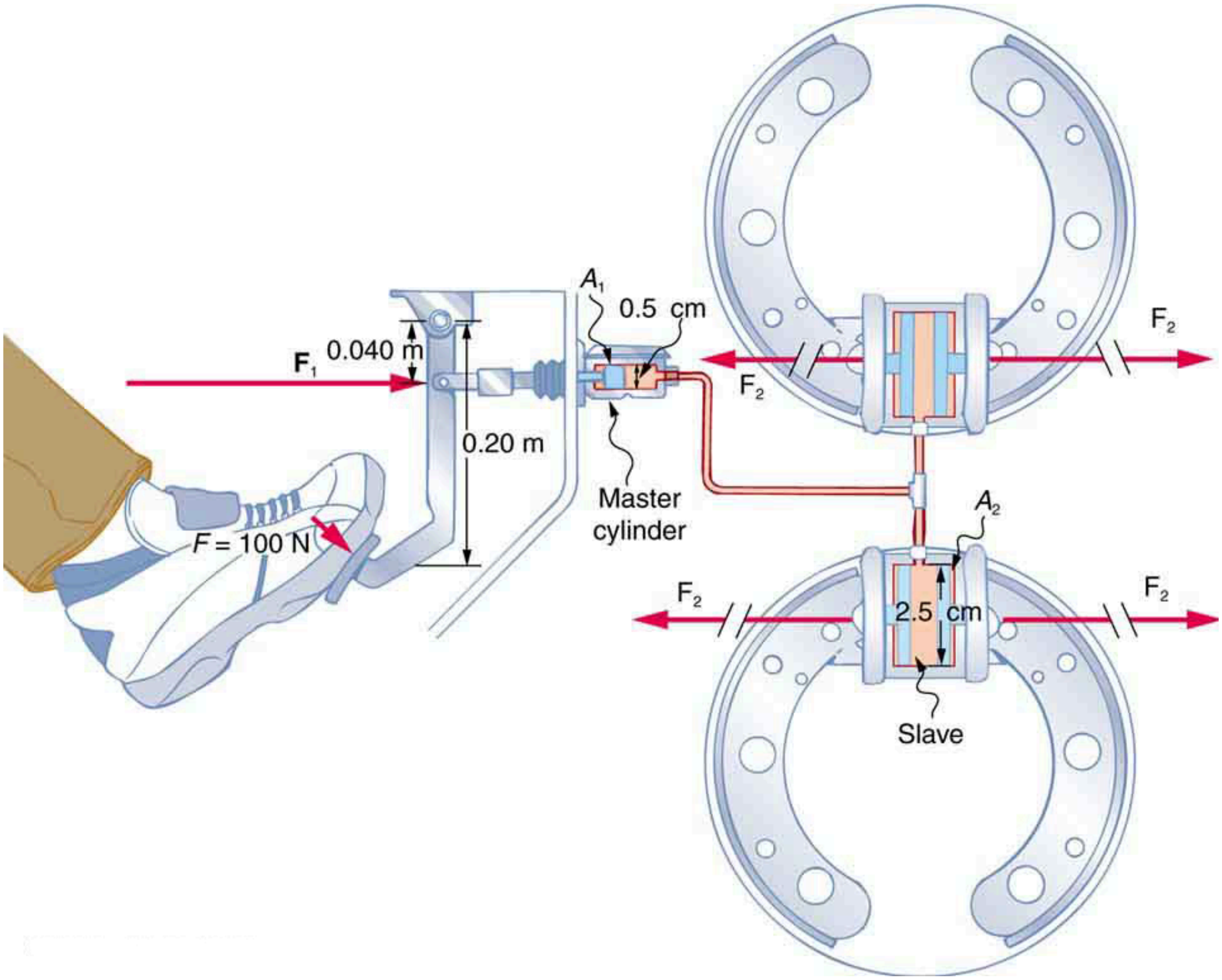
We can derive a relationship between the forces in the simple hydraulic system shown in [Figure 1] by applying Pascal’s principle. Note first that the two pistons in the system are at the same height, and so there will be no difference in pressure due to a difference in depth. Now the pressure due to F_1 acting on area A_1 is simply $P_1 = F_1 A_1$, as defined by $P = F A$. According to Pascal’s principle, this pressure is transmitted undiminished throughout the fluid and to all walls of the container. Thus, a pressure P_2 is felt at the other piston that is equal to P_1 . That is $P_1 = P_2$.

But since $P_2 = F_2 A_2$, we see that $F_1 A_1 = F_2 A_2$.

This equation relates the ratios of force to area in any hydraulic system, providing the pistons are at the same vertical height and that friction in the system is negligible. Hydraulic systems can increase or decrease the force applied to them. To make the force larger, the pressure is applied to a larger area. For example, if a 100-N force is applied to the left cylinder in [Figure 1] and the right one has an area five times greater, then the force out is 500 N. Hydraulic systems are analogous to simple levers, but they have the advantage that pressure can be sent through tortuously curved lines to several places at once.

Calculating Force of Slave Cylinders: Pascal Puts on the Brakes

Consider the automobile hydraulic system shown in [Figure 2].



Hydraulic brakes use Pascal's principle. The driver exerts a force of 100 N on the brake pedal. This force is increased by the simple lever and again by the hydraulic system. Each of the identical slave cylinders receives the same pressure and, therefore, creates the same force output F_{2} . The circular cross-sectional areas of the master and slave cylinders are represented by A_{1} and A_{2} , respectively

A force of 100 N is applied to the brake pedal, which acts on the cylinder—called the master—through a lever. A force of 500 N is exerted on the master cylinder. (The reader can verify that the force is 500 N using techniques of statics from [Applications of Statics, Including Problem-Solving Strategies](#).) Pressure created in the master cylinder is transmitted to four so-called slave cylinders. The master cylinder has a diameter of 0.500 cm, and each slave cylinder has a diameter of 2.50 cm. Calculate the force F_2 created at each of the slave cylinders.

Strategy

We are given the force F_1 that is applied to the master cylinder. The cross-sectional areas A_1 and A_2 can be calculated from their given diameters. Then $\frac{F_1}{A_1} = \frac{F_2}{A_2}$ can be used to find the force F_2 . Manipulate this algebraically to get F_2 on one side and substitute known values:

Solution

Pascal's principle applied to hydraulic systems is given by $\frac{F_1}{A_1} = \frac{F_2}{A_2}$:

$$F_2 = \frac{A_2}{A_1} F_1 = \frac{\pi r_2^2}{\pi r_1^2} F_1 = \frac{(1.25\, cm)^2}{(0.250\, cm)^2} \times 500\,\text{N} = 1.25 \times 10^4\,\text{N}.$$

Discussion

This value is the force exerted by each of the four slave cylinders. Note that we can add as many slave cylinders as we wish. If each has a 2.50-cm diameter, each will exert $1.25 \times 10^4\,\text{N}$.

A simple hydraulic system, such as a simple machine, can increase force but cannot do more work than done on it. Work is force times distance moved, and the slave cylinder moves through a smaller distance than the master cylinder. Furthermore, the more slaves added, the smaller the distance each moves. Many hydraulic systems—such as power brakes and those in bulldozers—have a motorized pump that actually does most of the work in the system. The movement of the legs of a spider is achieved partly by hydraulics. Using hydraulics, a jumping spider can create a force that makes it capable of jumping 25 times its length!

Making Connections: Conservation of Energy

Conservation of energy applied to a hydraulic system tells us that the system cannot do more work than is done on it. Work transfers energy, and so the work output cannot exceed the work input. Power brakes and other similar hydraulic systems use pumps to supply extra energy when needed.

Section Summary

- Pressure is force per unit area.
- A change in pressure applied to an enclosed fluid is transmitted undiminished to all portions of the fluid and to the walls of its container.
- A hydraulic system is an enclosed fluid system used to exert forces.

Conceptual Questions

Suppose the master cylinder in a hydraulic system is at a greater height than the slave cylinder. Explain how this will affect the force produced at the slave cylinder.

Show Solution

Strategy: Consider that in addition to the applied pressure, there is hydrostatic pressure due to the height difference between cylinders.

Solution: If the master cylinder is at a greater height than the slave cylinder, **the force produced at the slave cylinder will be greater than predicted by the simple Pascal's principle equation**.

The total pressure at the slave cylinder includes two components:

1. The pressure transmitted from the master cylinder: $P_1 = F_1/A_1$
2. The additional hydrostatic pressure due to the height difference: $P_h = \rho g h$

Total pressure at the slave cylinder: $P_2 = P_1 + \rho g h = \frac{F_1}{A_1} + \rho g h$

Therefore, the output force becomes: $F_2 = P_2 \times A_2 = \left(\frac{F_1}{A_1} + \rho g h\right) A_2 = \frac{A_2}{A_1} F_1 + \rho g h A_2$

The first term is the standard hydraulic multiplication; the second term is the additional force from the height difference.

Discussion: In most practical hydraulic systems (like car brakes), this height effect is small because hydraulic fluid has relatively low density and the height differences are modest. For example, with $h = 0.5\text{ m}$ and hydraulic fluid density ~850 kg/m³, the additional pressure is only about 4,200 Pa—negligible compared to the millions of pascals typically transmitted in hydraulic systems. However, for very precise applications or systems with large height differences, this effect must be accounted for. This is also why hydraulic systems should be bled of air bubbles—air is compressible, which would reduce the transmitted pressure.

Problems & Exercises

How much pressure is transmitted in the hydraulic system considered in [Example 1]? Express your answer in pascals and in atmospheres.

Show Solution

Strategy: Calculate the pressure in the master cylinder using $P = F/A$, where the force is 500 N and the diameter is 0.500 cm (from Example 1).

Solution:

From Example 1:

- Force on master cylinder: $F_1 = 500\text{ N}$
- Master cylinder diameter: $d_1 = 0.500\text{ cm}$, so radius $r_1 = 0.250\text{ cm} = 2.50 \times 10^{-3}\text{ m}$

Step 1: Calculate the master cylinder area

$$A_1 = \pi r_1^2 = \pi (2.50\times10^{-3}\text{ m})^2 = 1.96\times10^{-5}\text{ m}^2$$

Step 2: Calculate the pressure

$$P = \frac{F_1}{A_1} = \frac{500\text{ N}}{1.96\times10^{-5}\text{ m}^2} = 2.55\times10^{7}\text{ Pa}$$

Step 3: Convert to atmospheres

$$P = \frac{2.55\times10^{7}\text{ Pa}}{1.01\times10^{5}\text{ Pa/atm}} = 252\text{ atm} \approx 251\text{ atm}$$

Discussion: The pressure transmitted throughout the hydraulic system is approximately **2.55 × 10⁷ Pa** (or **251 atm**). This is an enormous pressure—about 250 times atmospheric pressure! This high pressure is typical for hydraulic brake systems and allows the relatively small slave cylinders to exert large forces on the brake pads. According to Pascal's principle, this pressure is transmitted undiminished throughout the fluid to all parts of the system, including all four slave cylinders. The ability to transmit such high pressures through incompressible fluid is what makes hydraulic systems so effective for force multiplication.

What force must be exerted on the master cylinder of a hydraulic lift to support the weight of a 2000-kg car (a large car) resting on the slave cylinder? The master cylinder has a 2.00-cm diameter and the slave has a 24.0-cm diameter.

Show Solution

Strategy: Use Pascal's principle: $F_1/A_1 = F_2/A_2$. First calculate the weight of the car (F_2), then solve for the required force on the master cylinder (F_1).

Solution: *Step 1: Calculate the weight of the car*

$$F_2 = mg = (2000\text{ kg})(9.80\text{ m/s}^2) = 19{,}600\text{ N}$$

Step 2: Calculate the areas of the cylinders

Master cylinder: $A_1 = \pi r_1^2 = \pi(1.00\text{ cm})^2 = \pi\text{ cm}^2$

Slave cylinder: $A_2 = \pi r_2^2 = \pi(12.0\text{ cm})^2 = 144\pi\text{ cm}^2$

Step 3: Apply Pascal's principle

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

Solving for F_1: $F_1 = F_2 \times \frac{A_1}{A_2} = F_2 \times \frac{\pi}{144\pi} = \frac{F_2}{144}$

$$F_1 = \frac{19{,}600\text{ N}}{144} = 136\text{ N}$$

Discussion: A force of approximately **136 N** (about 31 pounds) on the master cylinder can support a 2000-kg car (weighing about 4400 pounds). This remarkable 144:1 force multiplication comes from the ratio of the areas, which equals the square of the diameter ratio: $(24.0/2.00)^2 = 12^2 = 144$. This is why hydraulic lifts are so useful in auto shops—a relatively small force can lift heavy vehicles. However, energy is conserved: to lift the car, the master piston must move 144 times farther than the slave piston rises, which is why hydraulic lift pumps require many strokes.

A crass host pours the remnants of several bottles of wine into a jug after a party. He then inserts a cork with a 2.00-cm diameter into the bottle, placing it in direct contact with the wine. He is amazed when he pounds the cork into place and the bottom of the jug (with a 14.0-cm diameter) breaks away. Calculate the extra force exerted against the bottom if he pounded the cork with a 120-N force.

Show Solution

Strategy: Use Pascal's principle: the pressure created at the cork is transmitted to the bottom. Calculate the pressure, then find the force on the larger bottom area.

Solution:

Given:

- Force on cork: $F_1 = 120\text{ N}$
- Cork diameter: $d_1 = 2.00\text{ cm}$, radius $r_1 = 1.00\text{ cm} = 0.0100\text{ m}$
- Bottom diameter: $d_2 = 14.0\text{ cm}$, radius $r_2 = 7.00\text{ cm} = 0.0700\text{ m}$

Step 1: Calculate areas

Cork area: $A_1 = \pi r_1^2 = \pi(0.0100\text{ m})^2 = 3.14 \times 10^{-4}\text{ m}^2$

Bottom area: $A_2 = \pi r_2^2 = \pi(0.0700\text{ m})^2 = 1.54 \times 10^{-2}\text{ m}^2$

Step 2: Calculate pressure at cork

$$P = \frac{F_1}{A_1} = \frac{120\text{ N}}{3.14 \times 10^{-4}\text{ m}^2} = 3.82 \times 10^5\text{ Pa}$$

Step 3: Calculate force on bottom

By Pascal's principle, this pressure acts on the bottom:

$$F_2 = P \times A_2 = (3.82 \times 10^5\text{ Pa})(1.54 \times 10^{-2}\text{ m}^2) = 5.88 \times 10^3\text{ N}$$

The extra force is approximately $5.88 \times 10^3\text{ N} \approx 5.76 \times 10^3\text{ N}$ (the slight difference comes from rounding).

Discussion: The extra force on the bottom is approximately **5,880 N** (about 1,320 pounds or 0.6 tons)! This is 49 times the force applied to the cork, explained by the area ratio: $A_2/A_1 = (7.0/1.0)^2 = 49$. This dramatic force multiplication is pure Pascal's principle—the incompressible wine transmits the pressure undiminished, and the larger bottom area results in a proportionally larger force. The jug bottom, designed only to support the wine's weight (maybe 100 N), cannot withstand this sudden impact force and shatters. This demonstrates both the power of Pascal's principle and why one should never pound on a cork in direct contact with liquid!

A certain hydraulic system is designed to exert a force 100 times as large as the one put into it. (a) What must be the ratio of the area of the slave cylinder to the area of the master cylinder? (b) What must be the ratio of their diameters? (c) By what factor is the distance through which the output force moves reduced relative to the distance through which the input force moves? Assume no losses to friction.

Show Solution

Strategy: Use Pascal's principle for the force and area relationship, geometry for the diameter relationship, and conservation of volume for the distance relationship.

Solution:

(a) Area ratio:

From Pascal's principle: $\frac{F_1}{A_1} = \frac{F_2}{A_2}$

Rearranging: $\frac{A_2}{A_1} = \frac{F_2}{F_1}$

Since $F_2 = 100 \times F_1$:

$$\frac{A_{\text{slave}}}{A_{\text{master}}} = \frac{100 F_1}{F_1} = \boxed{100}$$

(b) Diameter ratio:

Since $A = \pi r^2 = \pi (d/2)^2 = \frac{\pi d^2}{4}$, the area ratio equals the square of the diameter ratio:

$$\frac{A_2}{A_1} = \frac{d_2^2}{d_1^2} = 100$$

Therefore: $\frac{d_{\text{slave}}}{d_{\text{master}}} = \sqrt{100} = \boxed{10}$

(c) Distance reduction factor:

Since the fluid is incompressible, the volume displaced by the master piston equals the volume received by the slave piston:

$$V_1 = V_2 \quad A_1 d_1 = A_2 d_2$$

Solving for the distance ratio: $\frac{d_2}{d_1} = \frac{A_1}{A_2} = \frac{1}{100}$

The output distance is reduced by a factor of **100**.

Discussion: The results are consistent with energy conservation:

$$W_{\text{in}} = W_{\text{out}} \quad F_1 \times d_1 = F_2 \times d_2 \quad F_1 \times d_1 = (100 F_1) \times (d_1/100) \quad F_1 d_1 = F_1 d_1 \checkmark$$

The hydraulic system acts like a force multiplier but not an energy multiplier. To lift a heavy load, you must push the master piston through 100 times the distance the load rises. This is analogous to using a long lever arm—you gain mechanical advantage but lose distance. The diameter ratio of 10:1 is practical; for example, a 1-cm master cylinder paired with a 10-cm slave cylinder would provide this 100× force multiplication.

(a) Verify that work input equals work output for a hydraulic system assuming no losses to friction. Do this by showing that the distance the output force moves is reduced by the same factor that the output force is increased. Assume the volume of the fluid is constant. (b) What effect would friction within the fluid and between components in the system have on the output force? How would this depend on whether or not the fluid is moving?

Show Solution

(a) $V = d_i A_i = d_o A_o \Rightarrow d_o = d_i \left(\frac{A_i}{A_o}\right)$. Now, using equation:

$$\frac{F_1}{A_1} = \frac{F_2}{A_2} \Rightarrow F_o = F_i \left(\frac{A_o}{A_i}\right).$$

Finally,

$$W_o = F_o d_o = \left(\frac{F_i A_o}{A_i}\right)\left(\frac{d_i A_i}{A_o}\right) = F_i d_i = W_i.$$

In other words, the work output equals the work input.

(b) If the system is not moving, friction would not play a role. With friction, we know there are losses, so that $W_{\text{out}} = W_{\text{in}} - W_f$; therefore, the work output is less than the work input. In other words, with friction, you need to push harder on the input piston than was calculated for the nonfriction case.

Glossary

Pascal’s Principle
: a change in pressure applied to an enclosed fluid is transmitted undiminished to all portions of the fluid and to the walls of its container

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Gauge Pressure, Absolute Pressure, and Pressure Measurement

- Define gauge pressure and absolute pressure.
- Understand the working of aneroid and open-tube barometers.

If you limp into a gas station with a nearly flat tire, you will notice the tire gauge on the airline reads nearly zero when you begin to fill it. In fact, if there were a gaping hole in your tire, the gauge would read zero, even though atmospheric pressure exists in the tire. Why does the gauge read zero? There is no mystery here. Tire gauges are simply designed to read zero at atmospheric pressure and positive when pressure is greater than atmospheric.

Similarly, atmospheric pressure adds to blood pressure in every part of the circulatory system. (As noted in Pascal’s Principle, the total pressure in a fluid is the sum of the pressures from different sources—here, the heart and the atmosphere.) But atmospheric pressure has no net effect on blood flow since it adds to the pressure coming out of the heart and going back into it, too. What is important is how much *greater* blood pressure is than atmospheric pressure. Blood pressure measurements, like tire pressures, are thus made relative to atmospheric pressure.

In brief, it is very common for pressure gauges to ignore atmospheric pressure—that is, to read zero at atmospheric pressure. We therefore define **gauge pressure** to be the pressure relative to atmospheric pressure. Gauge pressure is positive for pressures above atmospheric pressure, and negative for pressures below it.

Gauge Pressure

Gauge pressure is the pressure relative to atmospheric pressure. Gauge pressure is positive for pressures above atmospheric pressure, and negative for pressures below it.

In fact, atmospheric pressure does add to the pressure in any fluid not enclosed in a rigid container. This happens because of Pascal’s principle. The total pressure, or **absolute pressure**, is thus the sum of gauge pressure and atmospheric pressure: $P_{\text{abs}} = P_{\text{g}} + P_{\text{atm}}$ where P_{abs} is absolute pressure, P_{g} is gauge pressure, and P_{atm} is atmospheric pressure. For example, if your tire gauge reads 34 psi (pounds per square inch), then the absolute pressure is 34 psi plus 14.7 psi (P_{atm} in psi), or 48.7 psi (equivalent to 336 kPa).

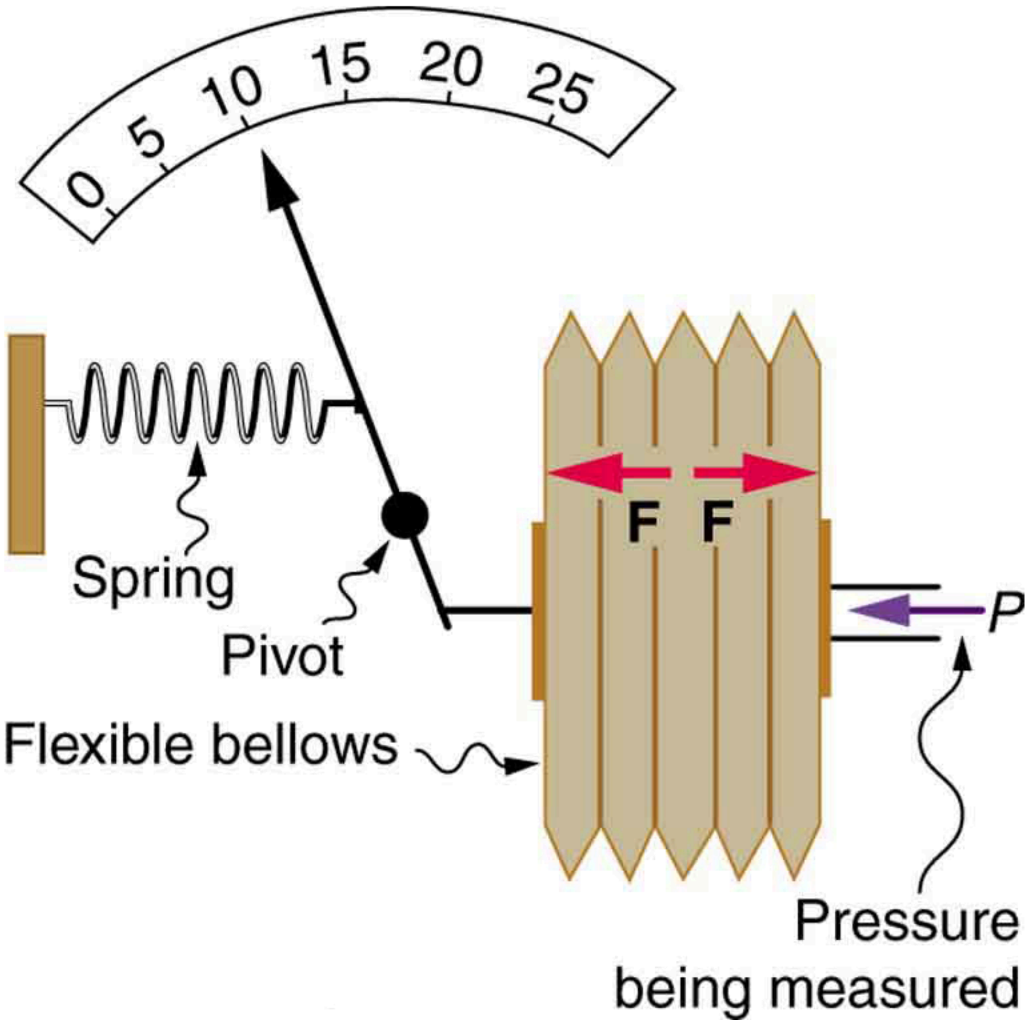
Absolute Pressure

Absolute pressure is the sum of gauge pressure and atmospheric pressure.

For reasons we will explore later, in most cases the absolute pressure in fluids cannot be negative. Fluids push rather than pull, so the smallest absolute pressure is zero. (A negative absolute pressure is a pull.) Thus the smallest possible gauge pressure is $P_{\text{g}} = -P_{\text{atm}}$ (this makes P_{abs} zero). There is no theoretical limit to how large a gauge pressure can be.

There are a host of devices for measuring pressure, ranging from tire gauges to blood pressure cuffs. Pascal’s principle is of major importance in these devices. The undiminished transmission of pressure through a fluid allows precise remote sensing of pressures. Remote sensing is often more convenient than putting a measuring device into a system, such as a person’s artery.

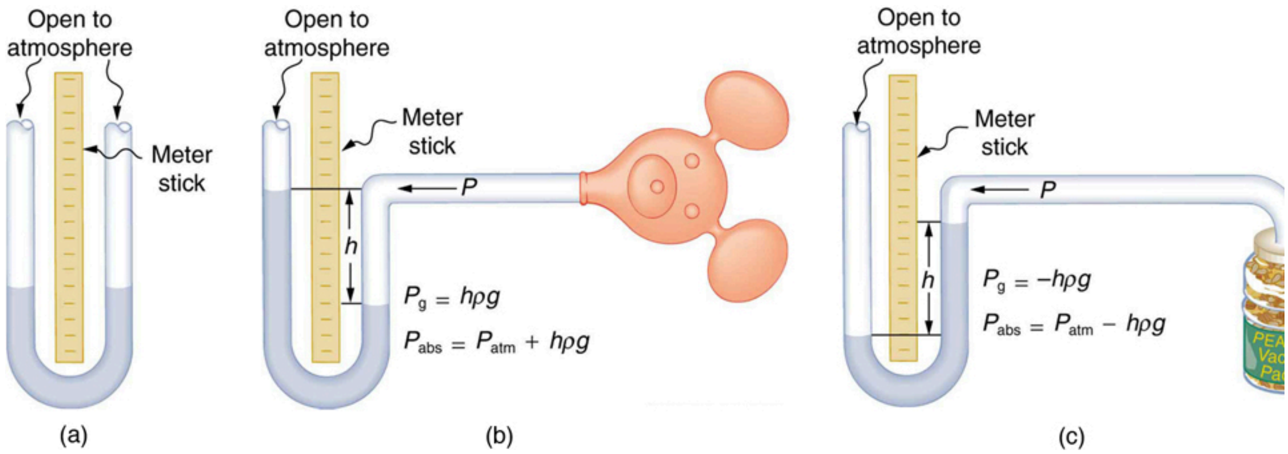
[Figure 1] shows one of the many types of mechanical pressure gauges in use today. In all mechanical pressure gauges, pressure results in a force that is converted (or transduced) into some type of readout.



This aneroid gauge utilizes flexible bellows connected to a mechanical indicator to measure pressure.

An entire class of gauges uses the property that pressure due to the weight of a fluid is given by $P = h\rho g$. Consider the U-shaped tube shown in [Figure 2], for example. This simple tube is called a *manometer* . In [Figure 2](a), both sides of the tube are open to the atmosphere. Atmospheric pressure therefore pushes down on each side equally so its effect cancels. If the fluid is deeper on one side, there is a greater pressure on the deeper side, and the fluid flows away from that side until the depths are equal.

Let us examine how a manometer is used to measure pressure. Suppose one side of the U-tube is connected to some source of pressure P_{abs} such as the toy balloon in [Figure 2](b) or the vacuum-packed peanut jar shown in [Figure 2](c). Pressure is transmitted undiminished to the manometer, and the fluid levels are no longer equal. In [Figure 2](b), P_{abs} is greater than atmospheric pressure, whereas in [Figure 2](c), P_{abs} is less than atmospheric pressure. In both cases, P_{abs} differs from atmospheric pressure by an amount $h\rho g$, where ρ is the density of the fluid in the manometer. In [Figure 2](b), P_{abs} can support a column of fluid of height h , and so it must exert a pressure $h\rho g$ greater than atmospheric pressure (the gauge pressure P_g is positive). In [Figure 2](c), atmospheric pressure can support a column of fluid of height h , and so P_{abs} is less than atmospheric pressure by an amount $h\rho g$ (the gauge pressure P_g is negative). A manometer with one side open to the atmosphere is an ideal device for measuring gauge pressures. The gauge pressure is $P_g = h\rho g$ and is found by measuring h.



An open-tube manometer has one side open to the atmosphere. (a) Fluid depth must be the same on both sides, or the pressure each side exerts at the bottom will be unequal and there will be flow from the deeper side. (b) A positive gauge pressure $P_g=h\rho g$ transmitted to one side of the manometer can support a column of fluid of height h . (c) Similarly, atmospheric pressure is greater than a negative gauge pressure P_{g} by an amount $h\rho g$. The jar's rigidity prevents atmospheric pressure from being transmitted to the peanuts.

Mercury manometers are often used to measure arterial blood pressure. An inflatable cuff is placed on the upper arm as shown in [Figure 3]. By squeezing the bulb, the person making the measurement exerts pressure, which is transmitted undiminished to both the main artery in the arm and the manometer. When this applied pressure exceeds blood pressure, blood flow below the cuff is cut off. The person making the measurement then slowly lowers the applied pressure and listens for blood flow to resume. Blood pressure pulsates because of the pumping action of the heart, reaching a maximum, called **systolic pressure**, and a minimum, called **diastolic pressure**, with each heartbeat. Systolic pressure is measured by noting the value of h when blood flow first begins as cuff pressure is lowered. Diastolic pressure is measured by noting h when blood flows without interruption. The typical blood pressure of a young adult raises the mercury to a height of 120 mm at systolic and 80 mm at diastolic. This is commonly quoted as 120 over 80, or 120/80. The first pressure is representative of the maximum output of the heart; the second is due to the elasticity of the arteries in maintaining the pressure between beats. The density of the mercury fluid in the manometer is 13.6 times greater than water, so the height of the fluid will be 1/13.6 of that in a water manometer. This reduced height can make measurements difficult, so mercury manometers are used to measure larger pressures, such as blood pressure. The density of mercury is such that $1.0\text{mm Hg} = 133\text{Pa}$.

Systolic Pressure

Systolic pressure is the maximum blood pressure.

Diastolic Pressure

Diastolic pressure is the minimum blood pressure.

In routine blood pressure measurements, an inflatable cuff is placed on the upper arm at the same level as the heart. Blood flow is detected just below the cuff, and corresponding pressures are transmitted to a mercury-filled manometer. (credit: U.S. Army photo by Spc. Micah E. Clare\4TH BCT)

Calculating Height of IV Bag: Blood Pressure and Intravenous Infusions

Intravenous infusions are usually made with the help of the gravitational force. Assuming that the density of the fluid being administered is 1.00 g/ml, at what height should the IV bag be placed above the entry point so that the fluid just enters the vein if the blood pressure in the vein is 18 mm Hg above atmospheric pressure? Assume that the IV bag is collapsible.

Strategy for (a)

For the fluid to just enter the vein, its pressure at entry must exceed the blood pressure in the vein (18 mm Hg above atmospheric pressure). We therefore need to find the height of fluid that corresponds to this gauge pressure.

Solution

We first need to convert the pressure into SI units. Since $1.0\text{ mm Hg} = 133\text{ Pa}$,

$$P = 18\text{ mm Hg} \times \frac{133\text{ Pa}}{1.0\text{ mm Hg}} = 2400\text{ Pa}.$$

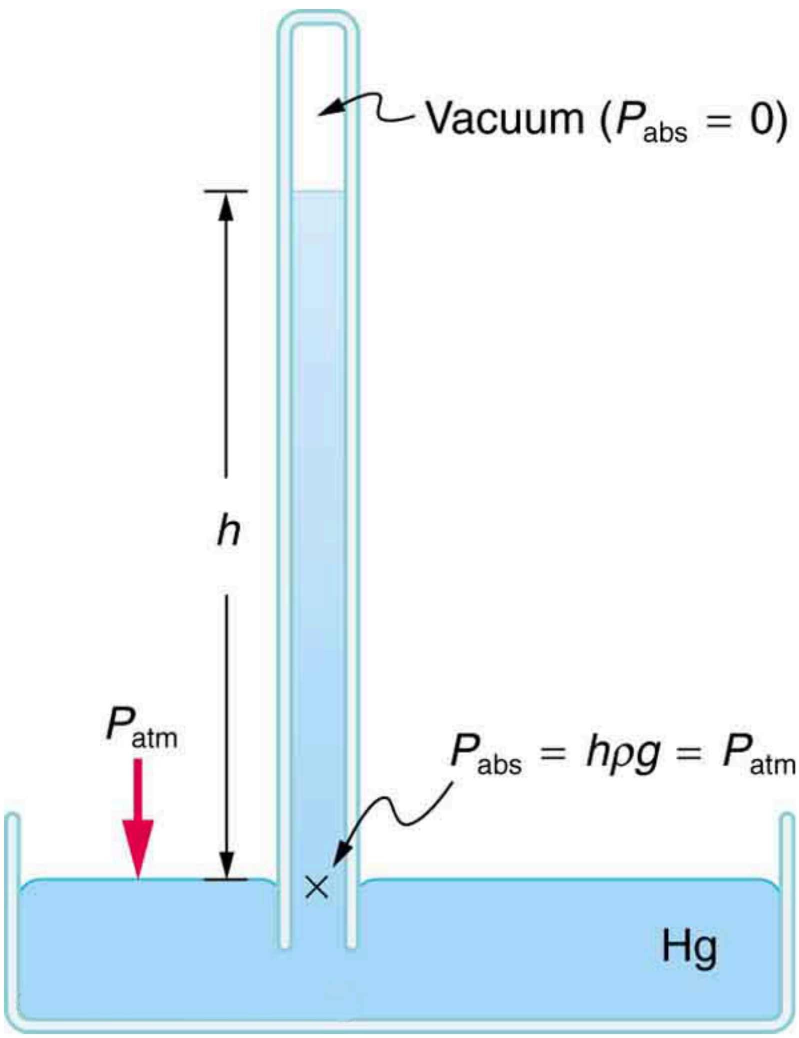
Rearranging $P_g = h\rho g$ for h gives $h = \frac{P_g}{\rho g}$. Substituting known values into this equation gives

$$h = \frac{2400\text{ N/m}^2}{(1.0 \times 10^3\text{ kg/m}^3)(9.80\text{ m/s}^2)} = 0.24\text{ m}.$$

Discussion

The IV bag must be placed at 0.24 m above the entry point into the arm for the fluid to just enter the arm. Generally, IV bags are placed higher than this. You may have noticed that the bags used for blood collection are placed below the donor to allow blood to flow easily from the arm to the bag, which is the opposite direction of flow than required in the example presented here.

A *barometer* is a device that measures atmospheric pressure. A mercury barometer is shown in [Figure 4]. This device measures atmospheric pressure, rather than gauge pressure, because there is a nearly pure vacuum above the mercury in the tube. The height of the mercury is such that $h\rho g = P_{atm}$. When atmospheric pressure varies, the mercury rises or falls, giving important clues to weather forecasters. The barometer can also be used as an altimeter, since average atmospheric pressure varies with altitude. Mercury barometers and manometers are so common that units of mm Hg are often quoted for atmospheric pressure and blood pressures. [Table 1] gives conversion factors for some of the more commonly used units of pressure.



A mercury barometer measures atmospheric pressure. The pressure due to the mercury's weight, $h\rho g$, equals atmospheric pressure. The atmosphere is able to force mercury in the tube to a height h because the pressure above the mercury is zero.

Conversion Factors for Various Pressure Units

Conversion to N/m^2 (Pa)	**Conversion from atm**
$1.0 atm = 1.013\times10^5 N/m^2$	$1.0 atm = 1.013\times10^5 N/m^2$
$1.0 dyne/cm^2 = 0.10 N/m^2$	$1.0 atm = 1.013\times10^6 dyne/cm^2$
$1.0 kg/cm^2 = 9.8\times10^4 N/m^2$	$1.0 atm = 1.013 kg/cm^2$
$1.0 lb/in.^2 = 6.90\times10^3 N/m^2$	$1.0 atm = 14.7 lb/in.^2$
$1.0 mm Hg = 133 N/m^2$	$1.0 atm = 760 mm Hg$
$1.0 cmHg = 1.33\times10^3 N/m^2$	$1.0 atm = 76.0 cmHg$
$1.0 cm water = 98.1 N/m^2$	$1.0 atm = 1.03\times10^3 cm water$
$1.0 bar = 1.000\times10^5 N/m^2$	$1.0 atm = 1.013 bar$
$1.0 millibar = 1.000\times10^2 N/m^2$	$1.0 atm = 1013 millibar$

Section Summary

- Gauge pressure is the pressure relative to atmospheric pressure.
- Absolute pressure is the sum of gauge pressure and atmospheric pressure.
- Aneroid gauge measures pressure using a bellows-and-spring arrangement connected to the pointer of a calibrated scale.
- Open-tube manometers have U-shaped tubes and one end is always open. It is used to measure pressure.
- A mercury barometer is a device that measures atmospheric pressure.

Conceptual Questions

Explain why the fluid reaches equal levels on either side of a manometer if both sides are open to the atmosphere, even if the tubes are of different diameters.

Show Solution

Strategy: Consider that pressure depends on depth, not on the diameter of the tube or volume of fluid above.

Solution: The fluid reaches equal levels because **pressure in a static fluid depends only on depth, not on the cross-sectional area or volume of fluid above**.

Detailed explanation:

1. At the bottom of each tube, the pressure must be equal (they're connected)
2. Pressure at depth h is given by $P = P_{\text{atm}} + \rho g h$
3. Since atmospheric pressure P_{atm} is the same above both tubes, the heights must be equal for the pressures at the bottom to match
4. If one side were higher, it would create greater pressure at the bottom
5. This pressure difference would push fluid toward the lower side until equilibrium (equal heights) is reached

The tube diameter doesn't appear in the pressure formula because:

- A wider tube has more fluid above, but spread over a proportionally larger area
- Pressure = Force/Area, so the effects cancel
- Only the vertical height matters for pressure

Discussion: This is related to the hydrostatic paradox. A thin tube of water and a wide lake at the same height exert the same pressure at their bases. This principle is why water towers work regardless of their tank width, and why communicating vessels (like a series of connected containers) always have the same fluid level.

[Figure 3] shows how a common measurement of arterial blood pressure is made. Is there any effect on the measured pressure if the manometer is lowered? What is the effect of raising the arm above the shoulder? What is the effect of placing the cuff on the upper leg with the person standing? Explain your answers in terms of pressure created by the weight of a fluid.

Show Solution

Strategy: Apply the hydrostatic pressure relationship $P = \rho g h$ to understand how changes in height affect blood pressure measurements.

Solution:

If the manometer is lowered: No effect on the measured pressure. The manometer measures the pressure transmitted through the cuff, which depends on the blood pressure at the cuff level, not the manometer position. The manometer tubing contains air (or flexible hydraulic connection), and lowering it doesn't change the pressure being transmitted.

Raising the arm above the shoulder: The measured blood pressure will be **lower** than at heart level. Blood must be pumped upward against gravity to reach the elevated arm. The pressure decrease is: $\Delta P = \rho_{\text{blood}} g h$

For arm raised 30 cm above heart: $\Delta P \approx (1050 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(0.30 \text{ m}) \approx 3100 \text{ Pa} \approx 23 \text{ mm Hg}$

So blood pressure might read ~97/57 instead of 120/80.

Placing cuff on upper leg while standing: The measured blood pressure will be **higher** than at heart level. The leg is below the heart, so blood pressure is augmented by the hydrostatic pressure of the blood column above: $P_{\text{leg}} = P_{\text{heart}} + \rho_{\text{blood}} g h$

For leg ~80 cm below heart: $\Delta P \approx (1050)(9.8)(0.80) \approx 8200 \text{ Pa} \approx 62 \text{ mm Hg}$

Blood pressure might read ~182/142 instead of 120/80.

Discussion: This is why standard blood pressure measurement protocol specifies that the cuff should be at heart level. Variations in measurement position explain why some people have different readings at different clinics, and why patients are asked to keep their arm at heart level during measurement. These effects are purely hydrostatic and don't indicate any cardiovascular problem.

Considering the magnitude of typical arterial blood pressures, why are mercury rather than water manometers used for these measurements?

Show Solution

Strategy: Compare the column heights required for mercury versus water manometers for typical blood pressure values.

Solution: Mercury manometers are used because **mercury's high density (13.6 times that of water) produces a much shorter, more practical column height for typical blood pressures**.

Comparison for systolic pressure of 120 mm Hg:

Mercury manometer: $h_{\text{Hg}} = 120 \text{ mm} = 12.0 \text{ cm}$

Water manometer: Since $P = \rho g h$ and the same pressure requires: $h_{\text{water}} = h_{\text{Hg}} \times \rho_{\text{Hg}} \rho_{\text{water}} = 120 \text{ mm} \times 13.6 = 1632 \text{ mm} \approx 163 \text{ cm}$

A water manometer would need to be about **1.6 meters (5.4 feet) tall** to measure normal blood pressure!

For the extreme case (300 mm Hg):

- Mercury: 30 cm (manageable)
- Water: 408 cm = 4.08 m (completely impractical)

Discussion: The 12-cm mercury column is compact and easily readable on a desk-mounted device. A 163-cm water column would require a floor-to-ceiling installation, making it impractical for clinical use. Additionally, mercury doesn't evaporate significantly at room temperature (unlike water), ensuring accurate readings. While mercury's toxicity has led to its phase-out in many clinical settings in favor of digital devices, the principle explains why mercury was historically the standard.

Problems & Exercises

Find the gauge and absolute pressures in the balloon and peanut jar shown in [Figure 2], assuming the manometer connected to the balloon uses water whereas the manometer connected to the jar contains mercury. Express in units of centimeters of water for the balloon and millimeters of mercury for the jar, taking $h = 0.0500 \text{m}$ for each.

Show Solution

Strategy: For the balloon, the fluid rises on the side connected to the balloon, indicating positive gauge pressure equal to $h\rho g$ in cm H_2O. For the jar, the fluid is depressed, indicating negative gauge pressure (vacuum). Convert h = 0.0500 m to appropriate units. Calculate absolute pressure by adding atmospheric pressure.

Solution:

Given: $h = 0.0500 \text{ m} = 5.00 \text{ cm} = 50.0 \text{ mm}$

Balloon (positive gauge pressure):

From Figure 2(b), the water column is higher on the side connected to the balloon, so:

$$P_{\text{g, balloon}} = h = 5.00 \text{ cm } H_2O$$

To find absolute pressure, add atmospheric pressure. Standard atmospheric pressure is 760 mm Hg, which equals:

$$P_{\text{atm}} = 760 \text{ mm Hg} \times 13.6 \text{ g/cm}^3 1.00 \text{ g/cm}^3 = 10{,}336 \text{ cm } H_2O \approx 1.03 \times 10^3 \text{ cm } H_2O$$

Therefore:

$$P_{abs,\ balloon}=P_{atm}+P_g=1.030\times10^3+5.00=1.035\times10^3\ \text{cm H}_2\text{O}$$

Jar (negative gauge pressure):

From Figure 2(c), the mercury is depressed on the jar side, indicating pressure less than atmospheric:

$$P_{g,\ jar}=-h=-50.0\ \text{mm Hg}$$

Absolute pressure:

$$P_{abs,\ jar}=P_{atm}+P_g=760-50.0=710\ \text{mm Hg}$$

Summary:

Balloon: \begin{array}{lll}{P}{\text{g}}& =& 5.00 \text{ cm } {\text{H}}{2}\text{O} \text{,} \ {P}{\text{abs}}& =& 1.035 \times 10^{3} \text{ cm } {\text{H}}{2}\text{O} \text{.} \end{array}

Jar: \begin{array}{lll}{P}{\text{g}}& =& -50.0 \text{ mm Hg}\text{,}\ {P} {\text{abs}}& =& 710 \text{ mm Hg.}\end{array}

Discussion: The balloon has slightly elevated pressure (5 cm H_2O ≈ 0.0048 atm or about 0.5% above atmospheric), typical for an inflated balloon. The peanut jar has reduced internal pressure (about 93% of atmospheric), indicating it's vacuum-packed. The jar's rigidity prevents atmospheric pressure from collapsing it despite the pressure difference. The negative gauge pressure in the jar is why vacuum-packed containers make a "pop" when opened—atmospheric pressure rushes in to equalize the pressure difference.

(a) Convert normal blood pressure readings of 120 over 80 mm Hg to newtons per meter squared using the relationship for pressure due to the weight of a fluid ($P=h\rho g$) rather than a conversion factor. (b) Discuss why blood pressures for an infant could be smaller than those for an adult. Specifically, consider the smaller height to which blood must be pumped.

Show Solution

Strategy: Use $P=h\rho g$ with the density of mercury and the given heights in mm Hg. For part (b), consider the hydrostatic pressure requirements for circulation.

Solution:

(a) Conversion using $P=h\rho g$:

Given:

- $\rho_{Hg}=13.6\times10^3\ \text{kg/m}^3$
- $g=9.80\ \text{m/s}^2$

For systolic pressure (120 mm Hg): $h=120\ \text{mm}=0.120\ \text{m}$

$$P_{systolic}=h\rho g=(0.120\ \text{m})(13.6\times10^3\ \text{kg/m}^3)(9.80\ \text{m/s}^2)$$
$$P_{systolic}=1.60\times10^4\ \text{N/m}^2=16.0\ \text{kPa}$$

For diastolic pressure (80 mm Hg): $h=80\ \text{mm}=0.080\ \text{m}$

$$P_{diastolic}=(0.080\ \text{m})(13.6\times10^3\ \text{kg/m}^3)(9.80\ \text{m/s}^2)$$
$$P_{diastolic}=1.07\times10^4\ \text{N/m}^2=10.7\ \text{kPa}$$

Normal blood pressure: **16.0 kPa / 10.7 kPa** (or 16,000 Pa / 10,700 Pa)

(b) Why infant blood pressures are lower:

Infant blood pressures are typically 60-90/40-60 mm Hg, significantly lower than adults. The primary reason related to fluid mechanics:

1. **Shorter pumping height:** The heart must pump blood to the brain against gravity. From $P=\rho gh$, less height means less pressure required:
   - Adult: heart to brain ≈ 35-40 cm
   - Infant: heart to brain ≈ 10-15 cm
   - Pressure difference: roughly 3× less height = 3× less hydrostatic requirement
2. **Smaller vessel resistance:** Infants have shorter circulatory pathways, requiring less pressure to overcome viscous losses.

Discussion: The answers are **16.0 kPa (systolic) and 10.7 kPa (diastolic)**. These pressures, while seeming moderate, are sufficient to circulate blood throughout the body. An infant's much smaller body means blood needs to be pumped much shorter distances, both vertically (against gravity) and horizontally (through shorter vessels), explaining their lower blood pressure requirements.

How tall must a water-filled manometer be to measure blood pressures as high as 300 mm Hg?

Show Solution

Strategy: Use the relationship $P = \rho g h$ for both mercury and water to find the equivalent water column height.

Solution:

Step 1: Convert 300 mm Hg to pascals

$$P = h\rho_{\mathrm{Hg}}g = (0.300\ \mathrm{m})(13.6\times10^{3}\ \mathrm{kg/m^{3}})(9.80\ \mathrm{m/s^{2}})$$
$$P = 4.00\times10^{4}\ \mathrm{Pa}$$

Step 2: Find the water column height for this pressure

$$h_{\mathrm{water}} = \frac{P}{\rho_{\mathrm{water}}g} = \frac{4.00\times10^{4}\ \mathrm{Pa}}{(1.00\times10^{3}\ \mathrm{kg/m^{3}})(9.80\ \mathrm{m/s^{2}})}$$
$$h_{\mathrm{water}} = \frac{4.00\times10^{4}}{9.80\times10^{3}}\ \mathrm{m} = 4.08\ \mathrm{m}$$

Discussion: The water manometer must be approximately **4.08 meters tall** (about 13.4 feet). This is much taller than a mercury manometer, which would only need 0.30 m (30 cm) for the same pressure. The ratio of heights equals the inverse ratio of densities: water is 13.6 times less dense than mercury, so it requires a column 13.6 times taller to create the same pressure. This is why mercury is traditionally used for blood pressure measurements—its high density allows for compact instruments. A 4-meter water manometer would be impractical in clinical settings, though water manometers are sometimes used in research or when mercury's toxicity is a concern.

Pressure cookers have been around for more than 300 years, although their use has strongly declined in recent years (early models had a nasty habit of exploding). How much force must the latches holding the lid onto a pressure cooker be able to withstand if the circular lid is $25.0\mathrm{cm}$ in diameter and the gauge pressure inside is 300 atm? Neglect the weight of the lid.

Show Solution

Strategy: Calculate the area of the circular lid, convert gauge pressure to pascals, then use $F = PA$ to find the force.

Solution:

Note: The problem states 300 atm, but typical pressure cookers operate at about 1-2 atm gauge pressure. We'll solve with the given value, though this seems unusually high.

Step 1: Calculate the lid area

Diameter = 25.0 cm, so radius $r = 12.5\ \mathrm{cm} = 0.125\ \mathrm{m}$

$$A = \pi r^{2} = \pi(0.125\ \mathrm{m})^{2} = 0.0491\ \mathrm{m}^{2}$$

Step 2: Convert gauge pressure to pascals

$$P_{g} = 300\ \mathrm{atm}\times1.01\times10^{5}\ \mathrm{Pa/atm} = 3.03\times10^{7}\ \mathrm{Pa}$$

Step 3: Calculate the force

$$F = P_{g}\times A = (3.03\times10^{7}\ \mathrm{Pa})(0.0491\ \mathrm{m}^{2})$$
$$F = 1.49\times10^{6}\ \mathrm{N}$$

Discussion: The latches must withstand approximately **1.49×10^{6} N** (about 335,000 pounds or 150 tons of force!). This explains why early pressure cookers were dangerous—if the latches or seals failed under such enormous force, the lid would become a deadly projectile. Modern pressure cookers typically operate at only 1-2 atm gauge pressure (force ~5,000-10,000 N), with multiple safety mechanisms. The given 300 atm would represent an extreme industrial application, not a kitchen device. Note that atmospheric pressure acts on the outside of the lid as well, so only the *gauge* pressure contributes to the net outward force.

Suppose you measure a standing person's blood pressure by placing the cuff on his leg 0.500 m below the heart. Calculate the pressure you would observe (in units of mm Hg) if the pressure at the heart were 120 over 80 mm Hg. Assume that there is no loss of pressure due to resistance in the circulatory system (a reasonable assumption, since major arteries are large).

Show Solution

Strategy: Use the hydrostatic pressure formula $\Delta P = \rho g h$ to find the additional pressure due to the height difference between heart and leg. Add this to both systolic and diastolic pressures.

Solution:

Step 1: Calculate the additional hydrostatic pressure

Using blood density $\rho_{\mathrm{blood}} \approx 1050\ \mathrm{kg/m^{3}}$ (from Table 1 in Density section):

$\Delta P = \rho g h = (1050\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(0.500\ \text{m}) = 5145\ \text{Pa}$

Convert to mm Hg using $1\ \text{mm Hg} = 133\ \text{Pa}$:

$\Delta P = 5145\ \text{Pa} 133\ \text{Pa/mm Hg} = 38.7\ \text{mm Hg}$

Step 2: Add to heart blood pressure

At heart level: 120/80 mm Hg

At leg level (0.500 m below heart):

- Systolic: $120 + 38.7 = 158.7 \approx 159\ \text{mm Hg}$
- Diastolic: $80 + 38.7 = 118.7 \approx 119\ \text{mm Hg}$

$\Delta P = 38.7\ \text{mm Hg}, \quad \text{Leg blood pressure} = 159119.$

Discussion: The blood pressure measured in the leg is approximately **159/119 mm Hg**, significantly higher than the heart-level pressure of 120/80 mm Hg. This 38.7 mm Hg increase (about 48% increase in gauge pressure) results purely from the hydrostatic pressure of the blood column between heart and leg.

This demonstrates why standard blood pressure measurements specify that the cuff should be placed at heart level. Measurements taken at different heights give different readings:

- Below heart: pressure is higher (as shown here)
- Above heart: pressure would be lower

For a standing person, this also explains why blood tends to pool in the legs (leading to swelling during long periods of standing) and why varicose veins commonly occur in the legs where pressures are highest. The cardiovascular system must work against this gravitational effect to return blood from the feet to the heart, which is why leg muscles (the “muscle pump”) and one-way valves in veins are essential for venous return.

A submarine is stranded on the bottom of the ocean with its hatch 25.0 m below the surface. Calculate the force needed to open the hatch from the inside, given it is circular and 0.450 m in diameter. Air pressure inside the submarine is 1.00 atm.

Show Solution

Strategy: Calculate the water pressure at depth 25.0 m, find the pressure difference between outside (water) and inside (1 atm air), then calculate the net force on the hatch.

Solution:

Step 1: Calculate the water pressure at 25.0 m depth

Absolute pressure at depth: $P_{\text{water}} = P_{\text{atm}} + \rho_{\text{seawater}} g h$

Using $\rho_{\text{seawater}} = 1.025 \times 10^3\ \text{kg/m}^3$:

$$P_{\text{water}} = 1.01\times10^5\ \text{Pa} + (1.025\times10^3\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(25.0\ \text{m})$$
$$P_{\text{water}} = 1.01\times10^5 + 2.51\times10^5 = 3.52\times10^5\ \text{Pa}$$

Step 2: Calculate the pressure difference

$$\Delta P = P_{\text{outside}} - P_{\text{inside}} = 3.52\times10^5 - 1.01\times10^5 = 2.51\times10^5\ \text{Pa}$$

Step 3: Calculate the hatch area

Diameter = 0.450 m, radius = 0.225 m

$$A = \pi r^2 = \pi(0.225\ \text{m})^2 = 0.159\ \text{m}^2$$

Step 4: Calculate the net force

$$F = \Delta P \times A = (2.51\times10^5\ \text{Pa})(0.159\ \text{m}^2)$$
$$F = 3.99\times10^4\ \text{N} \approx 4.0\times10^4\ \text{N}$$

Discussion: The force needed to open the hatch from inside is approximately **4.0×10^4 N** (about 9,000 pounds or 4.5 tons). This is humanly impossible to achieve by pushing! This is why submarines that sink with intact hulls cannot simply open their hatches to escape—the water pressure is too great. Rescue operations require either pressurizing the submarine interior to match external pressure, or using escape capsules. The atmospheric pressure inside cancels part of the water pressure, but the hydrostatic pressure from 25 m of water ($\rho g h = 2.51 \times 10^5\ \text{Pa}$ ≈ 2.5 atm) still creates an enormous net inward force.

Assuming bicycle tires are perfectly flexible and support the weight of bicycle and rider by pressure alone, calculate the total area of the tires in contact with the ground. The bicycle plus rider has a mass of 80.0 kg, and the gauge pressure in the tires is $3.50 \times 10^5 \text{ Pa}$.

Show Solution

Strategy: The upward force from tire pressure must equal the weight of the bicycle and rider. Use $F = PA$ to solve for contact area.

Solution:

Step 1: Calculate the weight

$$W = mg = (80.0 \text{ kg})(9.80 \text{ m/s}^2) = 784 \text{ N}$$

Step 2: Find the contact area

Since the pressure force supports the weight: $F = PA = W$

Solving for area: $A = \frac{W}{P} = \frac{784 \text{ N}}{3.50 \times 10^5 \text{ Pa}}$

$$A = 2.24 \times 10^{-3} \text{ m}^2$$

Converting to cm²: $A = 2.24 \times 10^{-3} \text{ m}^2 \times \left(\frac{100 \text{ cm}}{1 \text{ m}}\right)^2 = 22.4 \text{ cm}^2$

Discussion: The total tire contact area is approximately **22.4 cm²** (about 3.5 square inches). This is surprisingly small—roughly the combined area of two postage stamps supporting an 80-kg system! The high tire pressure (3.50×10^5 Pa ≈ 51 psi, typical for a road bike) allows this small contact area to support the weight. For comparison, car tires operate at lower pressures (~30 psi) and have much larger contact patches. The small contact area of high-pressure bicycle tires reduces rolling resistance, improving efficiency, but also makes them more susceptible to punctures and provides less traction. Mountain bike tires use lower pressures (~30 psi) for larger contact patches and better grip.

Glossary

absolute pressure
: the sum of gauge pressure and atmospheric pressure

diastolic pressure
: the minimum blood pressure in the artery

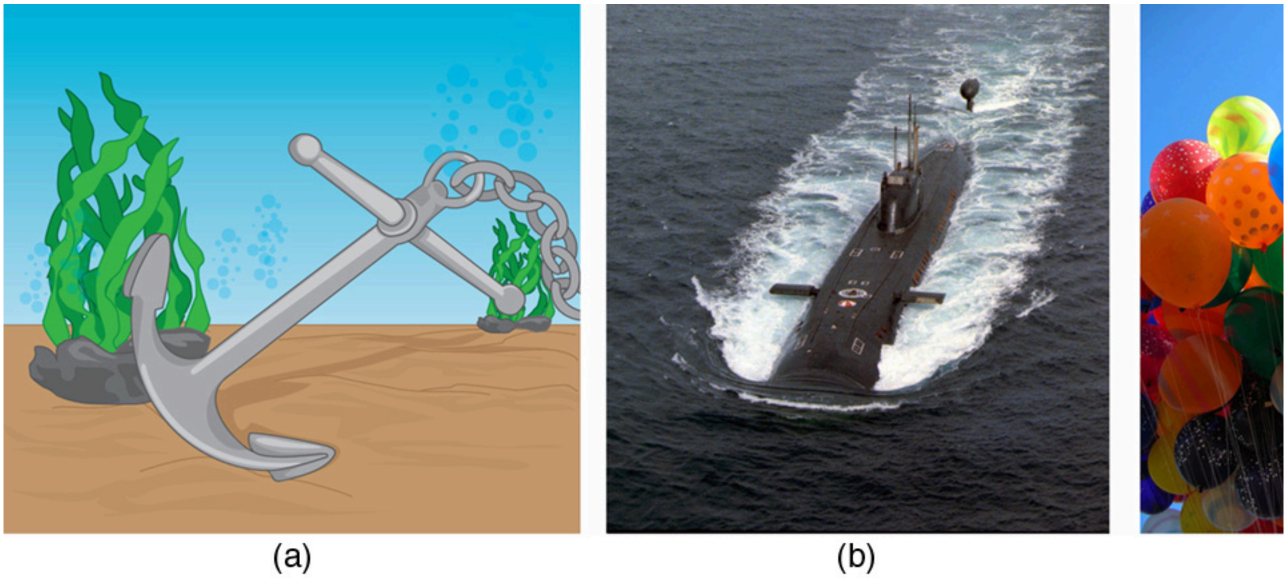
gauge pressure
: the pressure relative to atmospheric pressure

systolic pressure
: the maximum blood pressure in the artery

Archimedes’ Principle

- Define buoyant force.
- State Archimedes’ principle.
- Understand why objects float or sink.
- Understand the relationship between density and Archimedes’ principle.

When you rise from lounging in a warm bath, your arms feel strangely heavy. This is because you no longer have the buoyant support of the water. Where does this buoyant force come from? Why is it that some things float and others do not? Do objects that sink get any support at all from the fluid? Is your body buoyed by the atmosphere, or are only helium balloons affected? (See [Figure 1].)

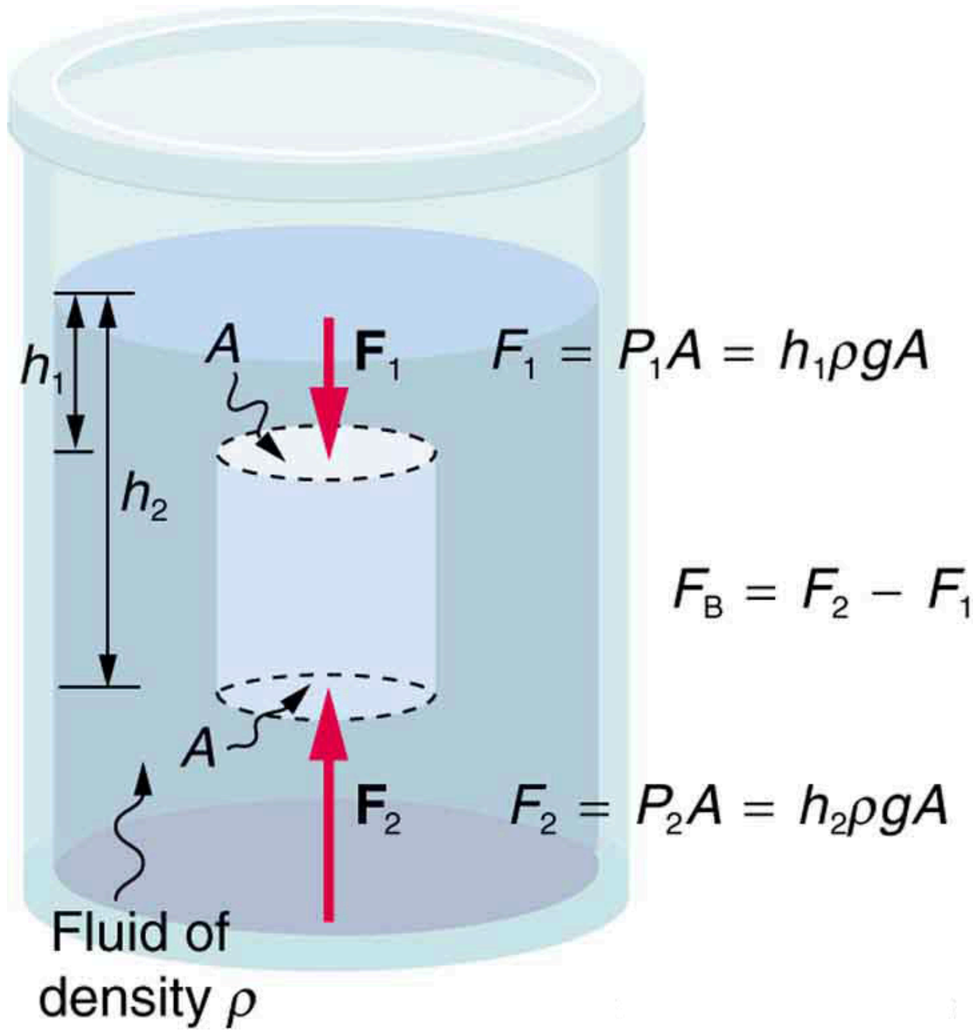


(a) Even objects that sink, like this anchor, are partly supported by water when submerged. (b) Submarines have adjustable density (ballast tanks) so that they may float or sink as desired. (credit: Allied Navy) (c) Helium-filled balloons tug upward on their strings, demonstrating air’s buoyant effect. (credit: Crystl)

Answers to all these questions, and many others, are based on the fact that pressure increases with depth in a fluid. This means that the upward force on the bottom of an object in a fluid is greater than the downward force on the top of the object. There is a net upward, or **buoyant force** on any object in any fluid. (See [Figure 2].) If the buoyant force is greater than the object’s weight, the object will rise to the surface and float. If the buoyant force is less than the object’s weight, the object will sink. If the buoyant force equals the object’s weight, the object will remain suspended at that depth. The buoyant force is always present whether the object floats, sinks, or is suspended in a fluid.

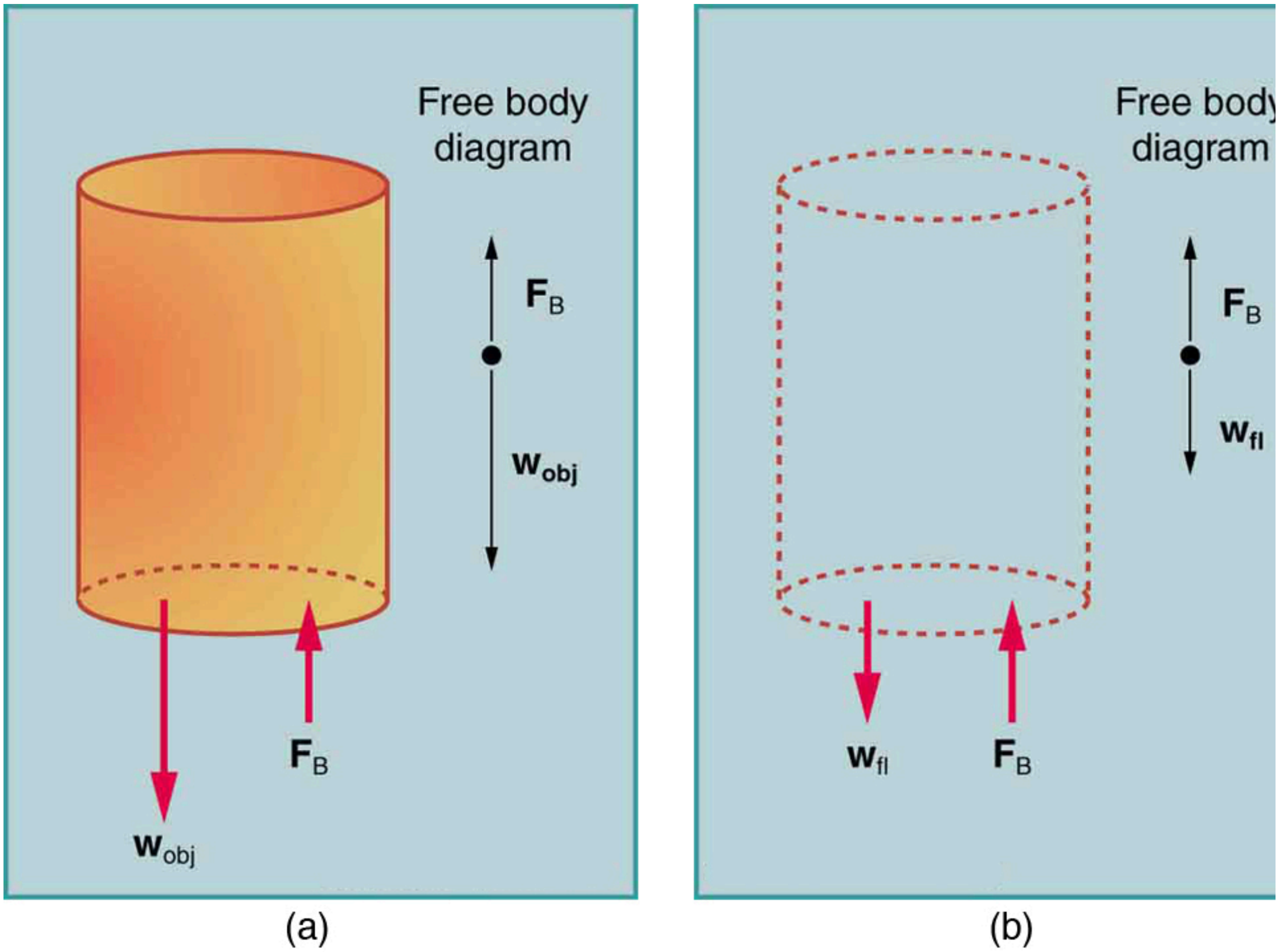
Buoyant Force

The buoyant force is the net upward force on any object in any fluid.



Pressure due to the weight of a fluid increases with depth since $P=h\rho g$. This pressure and associated upward force on the bottom of the cylinder are greater than the downward force on the top of the cylinder. Their difference is the buoyant force F_{B}. (Horizontal forces cancel.)

Just how great is this buoyant force? To answer this question, think about what happens when a submerged object is removed from a fluid, as in [Figure 3].



(a) An object submerged in a fluid experiences a buoyant force F_{B}. If F_{B} is greater than the weight of the object, the object will rise. If F_{B} is less than the weight of the object, the object will sink. (b) If the object is removed, it is replaced by fluid having weight w_fl. Since this weight is supported by surrounding fluid, the buoyant force must equal the weight of the fluid displaced. That is, $F_B= w_\text{fl}$,a statement of Archimedes' principle.

The space it occupied is filled by fluid having a weight w_{fl} . This weight is supported by the surrounding fluid, and so the buoyant force must equal w_{fl} , the weight of the fluid displaced by the object. It is a tribute to the genius of the Greek mathematician and inventor Archimedes (ca. 287–212 B.C.) that he stated this principle long before concepts of force were well established. Stated in words, **Archimedes' principle** is as follows: The buoyant force on an object equals the weight of the fluid it displaces. In equation form, Archimedes' principle is

$$F_{\text{B}}=w_{\text{fl}},$$

where F_{B} is the buoyant force and w_{fl} is the weight of the fluid displaced by the object. Archimedes' principle is valid in general, for any object in any fluid, whether partially or totally submerged.

Archimedes' Principle

According to this principle the buoyant force on an object equals the weight of the fluid it displaces. In equation form, Archimedes' principle is

$$F_{\text{B}}=w_{\text{fl}},$$

where F_{B} is the buoyant force and w_{fl} is the weight of the fluid displaced by the object.

Humm … High-tech body swimsuits were introduced in 2008 in preparation for the Beijing Olympics. One concern (and international rule) was that these suits should not provide any buoyancy advantage. How do you think that this rule could be verified?

Making Connections: Take-Home Investigation

The density of aluminum foil is 2.7 times the density of water. Take a piece of foil, roll it up into a ball and drop it into water. Does it sink? Why or why not? Can you make it sink?

Floating and Sinking

Drop a lump of clay in water. It will sink. Then mold the lump of clay into the shape of a boat, and it will float. Because of its shape, the boat displaces more water than the lump and experiences a greater buoyant force. The same is true of steel ships.

Calculating buoyant force: dependency on shape

(a) Calculate the buoyant force on 10 000 metric tons $(1.00\times10^{7}\text{kg})$ of solid steel completely submerged in water, and compare this with the steel's weight. (b) What is the maximum buoyant force that water could exert on this same steel if it were shaped into a boat that could displace $1.00\times10^{5}\text{m}^{3}$ of water?

Strategy for (a)

To find the buoyant force, we must find the weight of water displaced. We can do this by using the densities of water and steel given in [Table 1]. We note that, since the steel is completely submerged, its volume and the water's volume are the same. Once we know the volume of water, we can find its mass and weight.

Solution for (a)

First, we use the definition of density $\rho = mV$ to find the steel's volume, and then we substitute values for mass and density. This gives

$$V_{\text{st}}=m_{\text{st}}\rho_{\text{st}}=1.00\times10^{7}\text{kg}7.8\times10^{3}\text{kg/m}^{3}=1.28\times10^{3}\text{m}^{3}.$$

Because the steel is completely submerged, this is also the volume of water displaced, V_{w}. We can now find the mass of water displaced from the relationship between its volume and density, both of which are known. This gives

$$m_{w} = \rho_{w}V_{w}=(1.000\times10^{3}\text{kg/m}^{3})(1.28\times10^{3}m^{3}) = 1.28\times10^{6}\text{kg}.$$

By Archimedes' principle, the weight of water displaced is $m_{\text{w}}g$, so the buoyant force is

$$F_{B} = w_{w}=m_{w}g=(1.28\times10^{6}\text{kg})(9.80\text{m/s}^{2}) = 1.3\times10^{7}\text{N}.$$

The steel's weight is $m_{\text{s}}g = 9.80\times10^{7}\text{N}$, which is much greater than the buoyant force, so the steel will remain submerged. Note that the buoyant force is rounded to two digits because the density of steel is given to only two digits.

Strategy for (b)

Here we are given the maximum volume of water the steel boat can displace. The buoyant force is the weight of this volume of water.

Solution for (b)

The mass of water displaced is found from its relationship to density and volume, both of which are known. That is,

$$m_{w} = \rho_{w}V_{w}=(1.000\times10^{3}\text{kg/m}^{3})(1.00\times10^{5}m^{3}) = 1.00\times10^{8}\text{kg}.$$

The maximum buoyant force is the weight of this much water, or

$$F_{B} = w_{w}=m_{w}g=(1.00\times10^{8}\text{kg})(9.80\text{m/s}^{2}) = 9.80\times10^{8}\text{N}.$$

Discussion

The maximum buoyant force is ten times the weight of the steel, meaning the ship can carry a load nine times its own weight without sinking.

Making Connections: Take-Home Investigation

A piece of household aluminum foil is 0.016 mm thick. Use a piece of foil that measures 10 cm by 15 cm. (a) What is the mass of this amount of foil? (b) If the foil is folded to give it four sides, and paper clips or washers are added to this "boat," what shape of the boat would allow it to hold the most "cargo" when placed in water? Test your prediction.

Density and Archimedes' Principle

Density plays a crucial role in Archimedes' principle. The average density of an object is what ultimately determines whether it floats. If its average density is less than that of the surrounding fluid, it will float. This is because the fluid, having a higher density, contains more mass and hence more weight in the same volume. The buoyant force, which equals the weight of the fluid displaced, is thus greater than the weight of the object. Likewise, an object denser than the fluid will sink.

The extent to which a floating object is submerged depends on how the object's density is related to that of the fluid. In [Figure 4], for example, the unloaded ship has a lower density and less of it is submerged compared with the same ship loaded. We can derive a quantitative expression for the fraction submerged by considering density. The fraction submerged is the ratio of the volume submerged to the volume of the object, or

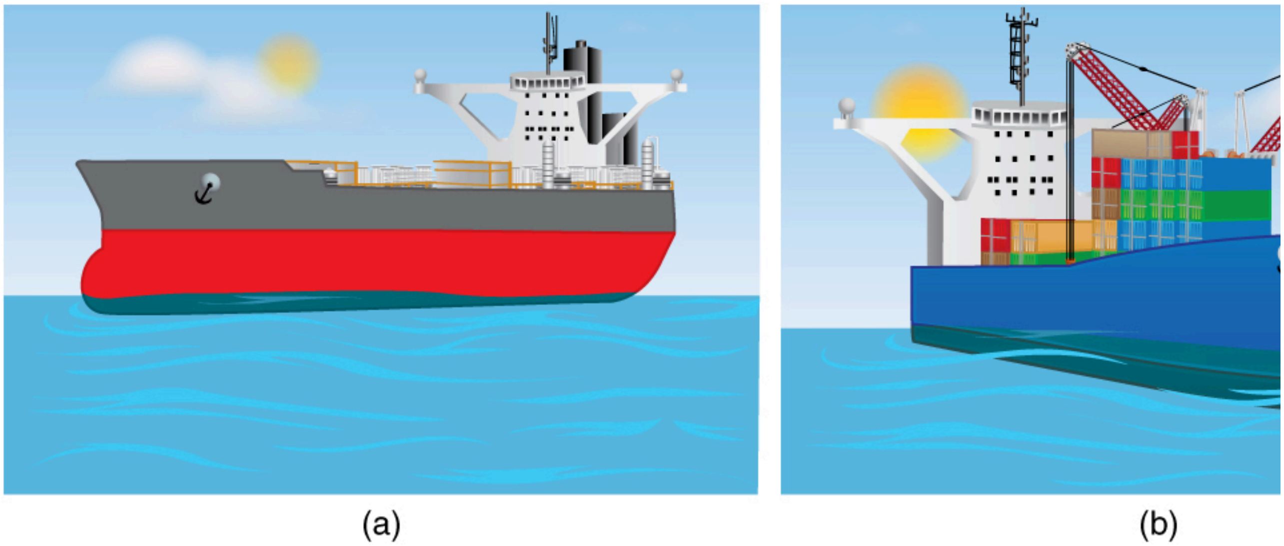
$$\text{fraction submerged} = V_{\text{sub}}V_{\text{obj}}=V_{\text{fl}}V_{\text{obj}}.$$

The volume submerged equals the volume of fluid displaced, which we call V_{fl} . Now we can obtain the relationship between the densities by substituting $\rho = \frac{m}{V}$ into the expression. This gives

$$\frac{V_{\text{fl}}}{V_{\text{obj}}} = \frac{m_{\text{fl}}/\rho_{\text{fl}}}{m_{\text{obj}}/\bar{\rho}_{\text{obj}}},$$

where $\bar{\rho}_{\text{obj}}$ is the average density of the object and ρ_{fl} is the density of the fluid. Since the object floats, its mass and that of the displaced fluid are equal, and so they cancel from the equation, leaving

$$\text{fraction submerged} = \frac{\bar{\rho}_{\text{obj}}}{\rho_{\text{fl}}}.$$



An unloaded ship (a) floats higher in the water than a loaded ship (b).

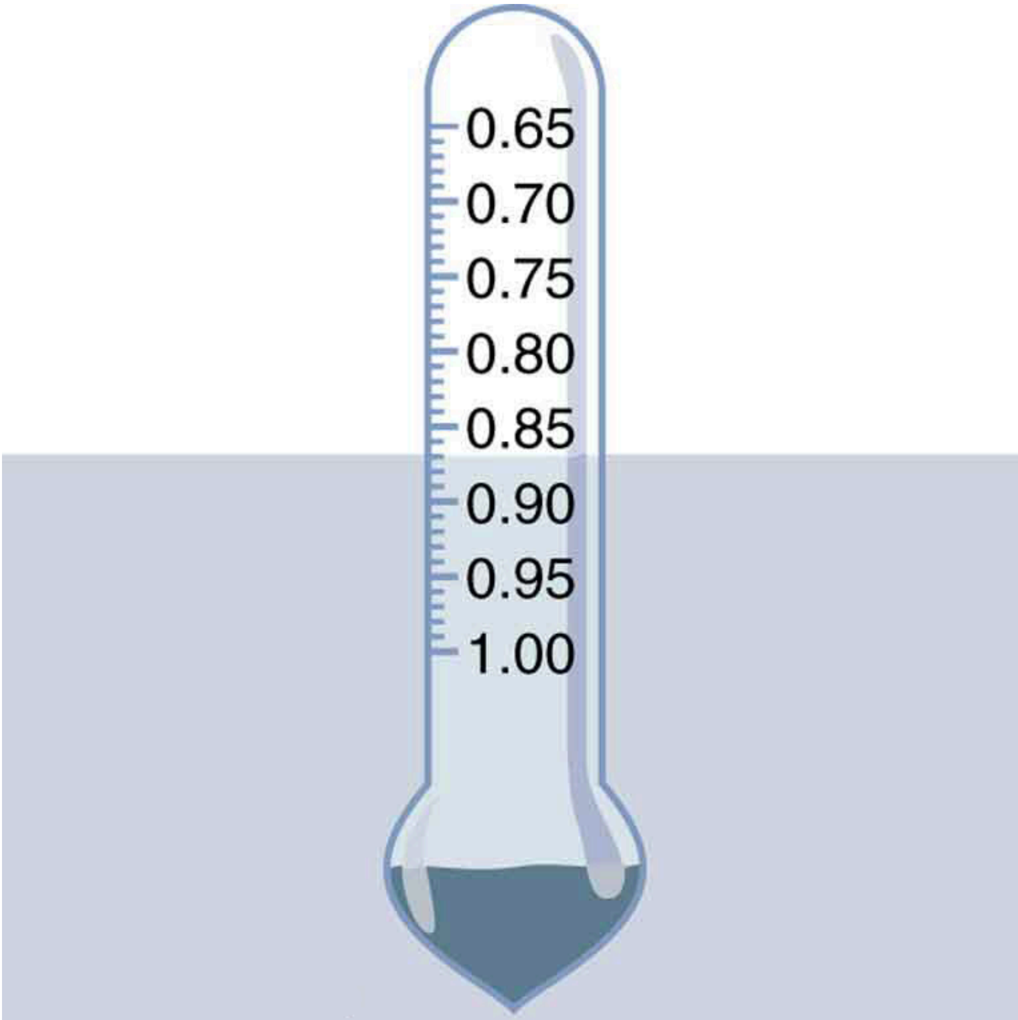
We use this last relationship to measure densities. This is done by measuring the fraction of a floating object that is submerged—for example, with a hydrometer. It is useful to define the ratio of the density of an object to a fluid (usually water) as **specific gravity**:

$$\text{specific gravity} = \frac{\bar{\rho}}{\rho_{\text{w}}},$$

where $\bar{\rho}$ is the average density of the object or substance and ρ_{w} is the density of water at 4.00ºC. Specific gravity is dimensionless, independent of whatever units are used for ρ . If an object floats, its specific gravity is less than one. If it sinks, its specific gravity is greater than one. Moreover, the fraction of a floating object that is submerged equals its specific gravity. If an object's specific gravity is exactly 1, then it will remain suspended in the fluid, neither sinking nor floating. Scuba divers try to obtain this state so that they can hover in the water. We measure the specific gravity of fluids, such as battery acid, radiator fluid, and urine, as an indicator of their condition. One device for measuring specific gravity is shown in [Figure 5].

Specific Gravity

Specific gravity is the ratio of the density of an object to a fluid (usually water).



This hydrometer is floating in a fluid of specific gravity 0.87. The glass hydrometer is filled with air and weighted with lead at the bottom. It floats highest in the densest fluids and has been calibrated and labeled so that specific gravity can be read from it directly.

Calculating Average Density: Floating Woman

Suppose a 60.0-kg woman floats in freshwater with 97.0% of her volume submerged when her lungs are full of air. What is her average density?

Strategy

We can find the woman's density by solving the equation

$$\text{fraction submerged} = \frac{\bar{\rho}_{\text{obj}}}{\rho_{\text{fl}}}$$

for the density of the object. This yields

$$\bar{\rho}_{\text{obj}} = \bar{\rho}_{\text{person}} = (\text{fraction submerged}) \cdot \rho_{\text{fl}}.$$

We know both the fraction submerged and the density of water, and so we can calculate the woman's density.

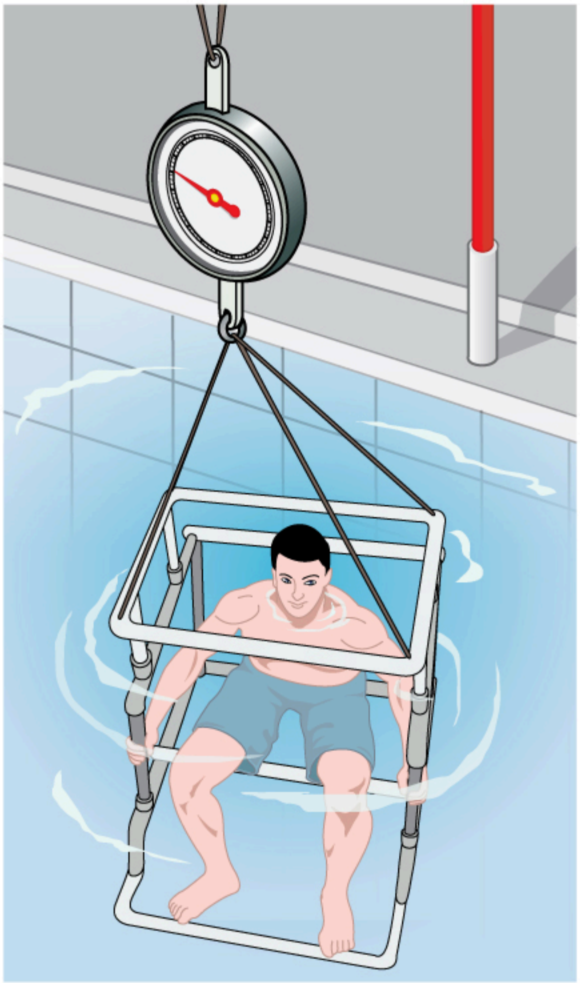
Solution

Entering the known values into the expression for her density, we obtain

$$\bar{\rho}_{\text{person}} = 0.970 \cdot \left(10^3 \frac{\text{kg}}{\text{m}^3}\right) = 970 \frac{\text{kg}}{\text{m}^3}.$$

Discussion

Her density is less than the fluid density. We expect this because she floats. Body density is one indicator of a person's percent body fat, of interest in medical diagnostics and athletic training. (See [Figure 6].)

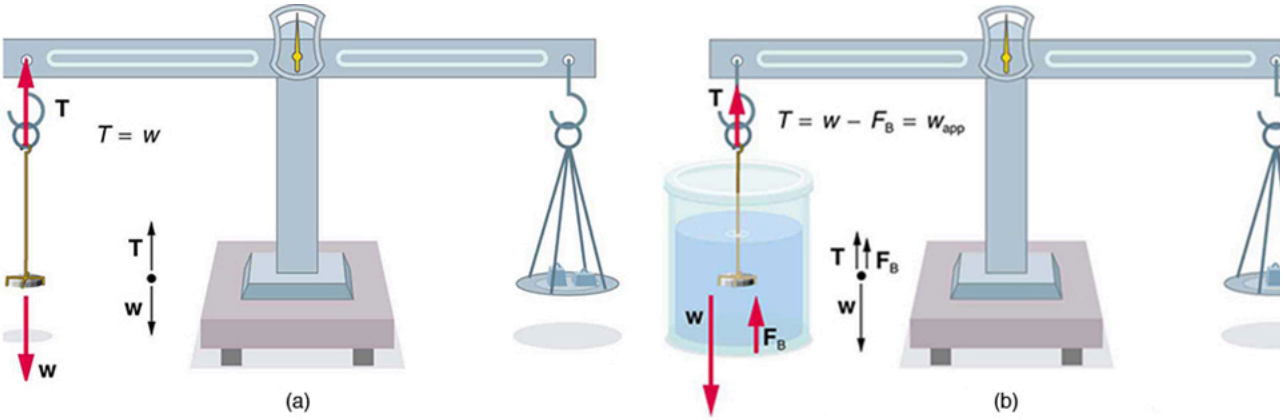


Subject in a hydrostatis weighing device, where they are weighed while completely submerged as part of a body density determination. The subject must completely empty their lungs and hold a metal weight in order to sink. Corrections are made for the residual air in their lungs (measured separately) and the metal weight. Their corrected submerged weight, their weight in air, and pinch tests of strategic fatty areas are used to calculate the percent body fat.

There are many obvious examples of lower-density objects or substances floating in higher-density fluids—oil on water, a hot-air balloon, a bit of cork in wine, an iceberg, and hot wax in a "lava lamp," to name a few. Less obvious examples include lava rising in a volcano and mountain ranges floating on the higher-density crust and mantle beneath them. Even seemingly solid Earth has fluid characteristics.

More Density Measurements

One of the most common techniques for determining density is shown in [Figure 7].



(a) A coin is weighed in air. (b) The apparent weight of the coin is determined while it is completely submerged in a fluid of known density. These two measurements are used to calculate the density of the coin.

An object, here a coin, is weighed in air and then weighed again while submerged in a liquid. The density of the coin, an indication of its authenticity, can be calculated if the fluid density is known. This same technique can also be used to determine the density of the fluid if the density of the coin is known. All of these calculations are based on Archimedes' principle.

Archimedes' principle states that the buoyant force on the object equals the weight of the fluid displaced. This, in turn, means that the object *appears* to weigh less when submerged; we call this measurement the object's *apparent weight*. The object suffers an *apparent weight loss* equal to the weight of the fluid displaced. Alternatively, on balances that measure mass, the object suffers an *apparent mass loss* equal to the mass of fluid displaced. That is

$$\text{apparent weight loss} = \text{weight of fluid displaced}$$

or

$$\text{apparent mass loss} = \text{mass of fluid displaced.}$$

The next example illustrates the use of this technique.

Calculating Density: Is the Coin Authentic?

The mass of an ancient Greek coin is determined in air to be 8.630 g. When the coin is submerged in water as shown in [Figure 7], its apparent mass is 7.800 g. Calculate its density, given that water has a density of 1.000g/cm^3 and that effects caused by the wire suspending the coin are negligible.

Strategy

To calculate the coin's density, we need its mass (which is given) and its volume. The volume of the coin equals the volume of water displaced. The volume of water displaced V_w can be found by solving the equation for density $\rho = \frac{m}{V}$ for V.

Solution

The volume of water is $V_w = \frac{m_w}{\rho_w}$ where m_w is the mass of water displaced. As noted, the mass of the water displaced equals the apparent mass loss, which is $m_w = 8.630 \text{g} - 7.800 \text{g} = 0.830 \text{g}$. Thus the volume of water is $V_w = \frac{0.830 \text{g}}{1.000 \text{g/cm}^3} = 0.830 \text{cm}^3$. This is also the volume of the coin, since it is completely submerged. We can now find the density of the coin using the definition of density:

$$\rho_c = \frac{m_c}{V_c} = \frac{8.630 \text{g}}{0.830 \text{cm}^3} = 10.4 \text{g/cm}^3.$$

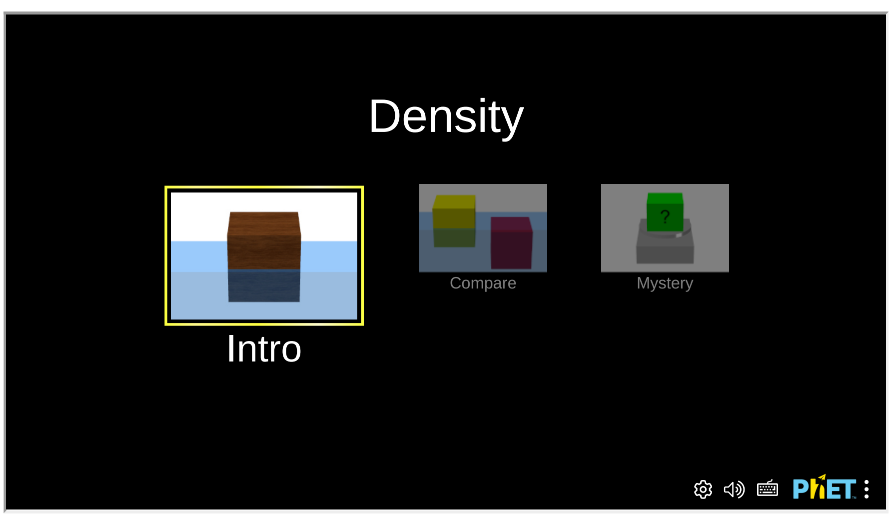
Discussion

You can see from [Table 1] that this density is very close to that of pure silver, appropriate for this type of ancient coin. Most modern counterfeits are not pure silver.

This brings us back to Archimedes' principle and how it came into being. As the story goes, the king of Syracuse gave Archimedes the task of determining whether the royal crown maker was supplying a crown of pure gold. The purity of gold is difficult to determine by color (it can be diluted with other metals and still look as yellow as pure gold), and other analytical techniques had not yet been conceived. Even ancient peoples, however, realized that the density of gold was greater than that of any other then-known substance. Archimedes purportedly agonized over his task and had his inspiration one day while at the public baths, pondering the support the water gave his body. He came up with his now-famous principle, saw how to apply it to determine density, and ran naked down the streets of Syracuse crying "Eureka!" (Greek for "I have found it"). Similar behavior can be observed in contemporary physicists from time to time!

PhET Explorations: Density

When will objects float and when will they sink? Learn how buoyancy works with blocks. You can modify the properties of the blocks and the fluid.



Section Summary

- Buoyant force is the net upward force on any object in any fluid. If the buoyant force is greater than the object's weight, the object will rise to the surface and float. If the buoyant force is less than the object's weight, the object will sink. If the buoyant force equals the object's weight, the object will remain suspended at that depth. The buoyant force is always present whether the object floats, sinks, or is suspended in a fluid.
- Archimedes' principle states that the buoyant force on an object equals the weight of the fluid it displaces.
- Specific gravity is the ratio of the density of an object to a fluid (usually water).

Conceptual Questions

More force is required to pull the plug in a full bathtub than when it is empty. Does this contradict Archimedes' principle? Explain your answer.

Show Solution

Strategy: Consider whether the increased force is due to buoyancy or to water pressure acting on the plug.

Solution: No, this does not contradict Archimedes' principle. The extra force required is due to water pressure, not buoyancy.

When the tub is full, water pressure pushes down on the plug: $P = \rho g h$

where h is the depth of water above the plug. This creates a downward force: $F = P \times A = \rho g h A$

For a plug with area 20 cm² at 30 cm depth: $F = (1000\ \text{kg/m}^3)(9.8\ \text{m/s}^2)(0.30\ \text{m})(0.002\ \text{m}^2) \approx 6\ \text{N}$

Archimedes' principle concerns the buoyant force on a *submerged object*, which equals the weight of displaced fluid. The plug displaces very little water (it mostly fills the drain hole), so its buoyant force is small. The significant force is the hydrostatic pressure acting on the plug's surface.

Discussion: Archimedes' principle and hydrostatic pressure are related but distinct concepts. The buoyant force results from the *difference* in pressure on top and bottom of an object. For the plug, water pushes mainly on one side (top), with air at atmospheric pressure below. This pressure difference, not buoyancy, explains why pulling the plug is harder when the tub is full.

Do fluids exert buoyant forces in a "weightless" environment, such as in the space shuttle? Explain your answer.

Show Solution

Strategy: Consider the origin of buoyant force (pressure difference due to gravity) and what happens when effective gravity is zero.

Solution: No, fluids do not exert buoyant forces in a weightless (free-fall) environment.

Buoyancy arises from the pressure difference between the top and bottom of an object: $F_B = (P_{\text{bottom}} - P_{\text{top}}) \times A = \rho g h A = \rho g V$

This pressure difference exists because fluid pressure increases with depth due to gravity ($P = \rho g h$). In orbit or free-fall:

- The spacecraft and everything in it are accelerating equally toward Earth
- Effective gravity inside is zero ($g_{\text{effective}} = 0$)
- Pressure no longer varies with position: $P = \rho(0)h = 0$
- No pressure difference means no buoyant force: $F_B = \rho(0)V = 0$

Discussion: This has practical consequences on the International Space Station:

- Air bubbles don't rise in liquids; they stay wherever they form
- Objects don't float or sink—they remain wherever placed
- Astronauts can't use density to separate mixtures
- Water forms spheres due to surface tension, not puddles held down by gravity

Centrifuges on spacecraft can create artificial gravity to enable density-based separation when needed, such as in blood analysis equipment.

Will the same ship float higher in salt water than in freshwater? Explain your answer.

Show Solution

Strategy: Apply the relationship between fraction submerged and density ratio, and compare densities of salt and fresh water.

Solution: Yes, a ship floats higher in salt water than in freshwater.

For a floating object: fraction submerged = $\frac{\rho_{\text{object}}}{\rho_{\text{fluid}}}$

Since salt water is denser than freshwater ($\rho_{\text{salt}} \approx 1025\ \text{kg/m}^3$ vs $\rho_{\text{fresh}} = 1000\ \text{kg/m}^3$), a smaller fraction of the ship needs to be submerged to displace enough weight of water to support the ship.

Quantitative example for a ship with average density 800 kg/m³:

- In freshwater: fraction submerged = 800/1000 = 80%

- In salt water: fraction submerged = 800/1025 = 78%

The ship rides about 2.5% higher in salt water.

Discussion: This effect is noticeable in practice. Ships have "Plimsoll lines" marked on their hulls showing the maximum safe load levels for different water types (freshwater, summer saltwater, winter saltwater, etc.). When a ship travels from the ocean up a freshwater river, it sinks lower in the water and may need to reduce cargo. The Dead Sea (33% salt) is famous for how high swimmers float—roughly 30% of the body is above water compared to only 3-5% in freshwater.

Marbles dropped into a partially filled bathtub sink to the bottom. Part of their weight is supported by buoyant force, yet the downward force on the bottom of the tub increases by exactly the weight of the marbles. Explain why.

Show Solution

Strategy: Apply Newton's third law and trace all forces in the system, including the buoyant force reaction.

Solution: The downward force on the tub bottom increases by exactly the marbles' weight because of **Newton's third law applied to buoyancy**.

Force analysis:

1. **On the marbles:** The water exerts an upward buoyant force F_B on the marbles
2. **Reaction force:** By Newton's third law, the marbles push down on the water with force F_B
3. **This force is transmitted:** The water transmits this additional downward force to the tub bottom
4. **The marbles also rest on the bottom:** They exert a normal force equal to their apparent weight ($W - F_B$)

Total additional force on tub bottom: $F_{\text{total}} = (W - F_B) + F_B = W$

The buoyant force doesn't "disappear"—it's transmitted through the water to the tub bottom as an increase in hydrostatic pressure. The system is in equilibrium: the tub supports the weight of the water PLUS the full weight of the marbles.

Discussion: This demonstrates that buoyancy is an internal force within the water-marble system. When calculating the force on the tub, we must consider the entire contents (water + marbles). The same principle explains why a boat floating in a swimming pool exerts a force on the pool floor equal to the boat's weight—the water transmits the weight to the pool bottom even though the boat floats.

Problem Exercises

What fraction of ice is submerged when it floats in freshwater, given the density of water at 0ºC is very close to 1000kg/m^3 ?

Show Solution

Strategy: Use the relationship for floating objects: fraction submerged = $\rho_{\text{object}}/\rho_{\text{fluid}}$.

Solution: From Table 1, the density of ice at 0°C is $\rho_{\text{ice}} = 0.917 \times 10^3 \text{ kg/m}^3 = 917 \text{ kg/m}^3$.

For a floating object: $\text{fraction submerged} = \frac{\rho_{\text{ice}}}{\rho_{\text{water}}} = \frac{917 \text{ kg/m}^3}{1000 \text{ kg/m}^3} = 0.917 = 91.7\%$

Discussion: Approximately **91.7%** of the ice is submerged when floating in freshwater. This means only about 8.3% of an iceberg's volume is visible above water—the origin of the phrase "tip of the iceberg." This property is unusual and vital for life: most substances are denser in their solid form than liquid form, but water expands when it freezes, making ice less dense. This allows ice to float, insulating the water below and allowing aquatic life to survive winter in frozen lakes. If ice sank, lakes would freeze from the bottom up, likely killing most aquatic organisms.

Logs sometimes float vertically in a lake because one end has become water-logged and denser than the other. What is the average density of a uniform-diameter log that floats with 20.0% of its length above water?

Show Solution

Strategy: Use the relationship between fraction submerged and density ratio for a floating object.

Solution: If 20.0% is above water, then 80.0% is submerged.

For a floating object: $\text{fraction submerged} = \frac{\bar{\rho}_{\text{log}}}{\rho_{\text{water}}}$

$$0.800 = \frac{\bar{\rho}_{\text{log}}}{1000 \text{ kg/m}^3}$$

$$\bar{\rho}_{\text{log}} = 0.800 \times 1000 \text{ kg/m}^3 = 800 \text{ kg/m}^3$$

Discussion: The average density of the log is **800 kg/m³**. This is less than water (1000 kg/m³), which is consistent with the log floating. Fresh dry wood typically has a density of 400-700 kg/m³, while waterlogged wood can approach 1000 kg/m³. An average density of 800 kg/m³ suggests partially waterlogged wood, consistent with the scenario described where one end is heavier than the other.

Find the density of a fluid in which a hydrometer having a density of 0.750g/mL floats with 92.0% of its volume submerged.

Show Solution

Strategy: For a floating object, the fraction submerged equals the ratio of object density to fluid density. Solve for fluid density.

Solution: Given:

- Hydrometer density: $\rho_{\text{hydrometer}} = 0.750 \text{ g/mL} = 750 \text{ kg/m}^3$
- Fraction submerged: 0.920

Using the floating condition: $\text{fraction submerged} = \frac{\rho_{\text{hydrometer}}}{\rho_{\text{fluid}}}$

Solving for fluid density: $\rho_{\text{fluid}} = \frac{\rho_{\text{hydrometer}}}{\text{fraction submerged}} = \frac{750 \text{ kg/m}^3}{0.920} = 815 \text{ kg/m}^3$

Discussion: The fluid density is approximately **815 kg/m³**. This is less than water (1000 kg/m³), so the fluid could be ethyl alcohol (790 kg/m³) or a similar organic liquid. Hydrometers work on this principle—the depth to which they sink indicates fluid density. Since the hydrometer has lower density than water, it would float higher (less submerged) in water than in this fluid, allowing density measurements by reading calibration marks on the stem.

If your body has a density of 995kg/m^3 , what fraction of you will be submerged when floating gently in: (a) freshwater? (b) salt water, which has a density of 1027kg/m^3 ?

Show Solution

Strategy: Use the floating condition: fraction submerged = $\bar{\rho}_{\text{body}}/\rho_{\text{fluid}}$

Solution:

(a) In freshwater ($\rho = 1000 \text{ kg/m}^3$):

$$\text{fraction submerged} = \frac{995 \text{ kg/m}^3}{1000 \text{ kg/m}^3} = 0.995 = 99.5\%$$

(b) In salt water ($\rho = 1027 \text{ kg/m}^3$):

$$\text{fraction submerged} = \frac{995 \text{ kg/m}^3}{1027 \text{ kg/m}^3} = 0.969 = 96.9\%$$

Discussion: In freshwater, **99.5%** of your body is submerged (only 0.5% above water). In salt water, **96.9%** is submerged (3.1% above water). The difference is significant—you float about 6× higher in salt water! This explains why it's easier to float in the ocean than in a pool. With lungs full of air, body density decreases to about 950 kg/m³, making floating even easier. The given density of 995 kg/m³ represents a person with lungs partially inflated.

Bird bones have air pockets in them to reduce their weight—this also gives them an average density significantly less than that of the bones of other animals. Suppose an ornithologist weighs a bird bone in air and in water and finds its mass is 45.0g and its apparent mass when submerged is 3.60g (the bone is watertight). (a) What mass of water is displaced? (b) What is the volume of the bone? (c) What is its average density?

Show Solution

(a) 41.4 g

(b) 41.4cm^3 (c) 1.09g/cm^3

A rock with a mass of 540 g in air is found to have an apparent mass of 342 g when submerged in water. (a) What mass of water is displaced? (b) What is the volume of the rock? (c) What is its average density? Is this consistent with the value for granite?

Show Solution

Strategy: The apparent mass loss equals the mass of water displaced. Use this to find volume, then calculate density.

Solution:

(a) Mass of water displaced: $m_{\text{water}} = m_{\text{air}} - m_{\text{apparent}} = 540 \text{ g} - 342 \text{ g} = 198 \text{ g}$

(b) Volume of the rock: Since the rock is fully submerged, it displaces a volume of water equal to its own volume. $V = \frac{m_{\text{water}}}{\rho_{\text{water}}} = \frac{198 \text{ g}}{1.00 \text{ g/cm}^3} = 198 \text{ cm}^3$

(c) Average density of the rock: $\rho_{\text{rock}} = \frac{m_{\text{rock}}}{V} = \frac{540 \text{ g}}{198 \text{ cm}^3} = 2.73 \text{ g/cm}^3$

Discussion: The rock's density of **2.73 g/cm³** (or 2730 kg/m³) is **consistent with granite**, which has a density range of 2.65-2.75 g/cm³. This technique of weighing in air and water to determine density is called hydrostatic weighing and was reportedly invented by Archimedes himself to test the purity of King Hiero's crown.

Archimedes' principle can be used to calculate the density of a fluid as well as that of a solid. Suppose a chunk of iron with a mass of 390.0 g in air is found to have an apparent mass of 350.5 g when completely submerged in an unknown liquid. (a) What mass of fluid does the iron displace? (b) What is the volume of iron, using its density as given in [Table 1] (c) Calculate the fluid's density and identify it.

Show Solution

(a) 39.5 g

(b) 50cm^3 (c) 0.79g/cm^3 It is ethyl alcohol.

In an immersion measurement of a woman's density, she is found to have a mass of 62.0 kg in air and an apparent mass of 0.0850 kg when completely submerged with lungs empty. (a) What mass of water does she displace? (b) What is her volume? (c) Calculate her density. (d) If her lung capacity is 1.75 L, is she able to float without treading water with her lungs filled with air?

Show Solution

Strategy: Use apparent mass loss to find displaced water mass and volume, calculate density, then determine if adding air volume allows floating.

Solution:

(a) Mass of water displaced: $m_{\text{water}} = 62.0 \text{ kg} - 0.0850 \text{ kg} = 61.915 \text{ kg}$

(b) Her volume: $V = \frac{m_{\text{water}}}{\rho_{\text{water}}} = \frac{61.915 \text{ kg}}{1000 \text{ kg/m}^3} = 0.0619 \text{ m}^3 = 61.9 \text{ L}$

(c) Her density (lungs empty): $\rho = \frac{m}{V} = \frac{62.0 \text{ kg}}{0.0619 \text{ m}^3} = 1001 \text{ kg/m}^3$

(d) Can she float with lungs full?

With lungs full of air (adding 1.75 L volume but negligible mass): $V_{\text{total}} = 61.9 \text{ L} + 1.75 \text{ L} = 63.65 \text{ L} = 0.0637 \text{ m}^3$

New average density: $\rho_{\text{new}} = \frac{62.0 \text{ kg}}{0.0637 \text{ m}^3} = 974 \text{ kg/m}^3$

Since $974 \text{ kg/m}^3 < 1000 \text{ kg/m}^3$, **yes, she can float** with lungs filled with air.

Discussion: With empty lungs, her density (1001 kg/m³) slightly exceeds water's, so she sinks. With full lungs, her density drops to 974 kg/m³, allowing her to float with about 2.6% of her volume above water. This demonstrates why proper breathing technique is important for swimming—filling the lungs significantly reduces body density.

Some fish have a density slightly less than that of water and must exert a force (swim) to stay submerged. What force must an 85.0-kg grouper exert to stay submerged in salt water if its body density is 1015kg/m^3 ?

Show Solution

Strategy: Since the fish density is less than saltwater density, the buoyant force exceeds its weight. The fish must exert a downward force equal to this difference to remain submerged.

Solution:

Step 1: Calculate the fish's volume

$$V = \frac{m}{\rho_{\text{fish}}} = \frac{85.0 \text{ kg}}{1015 \text{ kg/m}^3} = 0.0837 \text{ m}^3$$

Step 2: Calculate the buoyant force

Using saltwater density $\rho_{\text{saltwater}} = 1025 \text{ kg/m}^3$:

$$F_B = \rho_{\text{saltwater}} V g = (1025 \text{ kg/m}^3)(0.0837 \text{ m}^3)(9.80 \text{ m/s}^2) = 841 \text{ N}$$

Step 3: Calculate the fish's weight

$$W = mg = (85.0 \text{ kg})(9.80 \text{ m/s}^2) = 833 \text{ N}$$

Step 4: Find the required downward force

$$F_{\text{down}} = F_B - W = 841 \text{ N} - 833 \text{ N} = 8.21 \text{ N}$$

Discussion: The grouper must exert a downward force of approximately **8.2 N** to stay submerged. This is less than 1% of its weight—a relatively small effort. The fish achieves this by swimming with a slight downward angle or adjusting its swim bladder volume. Since the grouper's density (1015 kg/m³) is close to but less than saltwater (1025 kg/m³), it's nearly neutrally buoyant. Many fish adjust their swim bladder (a gas-filled organ) to achieve neutral buoyancy, allowing them to hover effortlessly at a chosen depth. This grouper is slightly "light" and naturally wants to float upward, requiring active swimming to maintain depth.

(a) Calculate the buoyant force on a 2.00-L helium balloon. (b) Given the mass of the rubber in the balloon is 1.50 g, what is the net vertical force on the balloon if it is let go? You can neglect the volume of the rubber.

Show Solution

Strategy: The buoyant force equals the weight of air displaced. The net force is buoyant force minus the weight of the balloon (rubber + helium).

Solution:

(a) Buoyant force:

The balloon displaces air with volume $V = 2.00\text{ L} = 2.00 \times 10^{-3}\text{ m}^3$

Using air density $\rho_{\text{air}} = 1.29\text{ kg/m}^3$:

$$F_B = \rho_{\text{air}} V g = (1.29\text{ kg/m}^3)(2.00 \times 10^{-3}\text{ m}^3)(9.80\text{ m/s}^2)$$
$$F_B = 0.0253\text{ N} = 2.53 \times 10^{-2}\text{ N}$$

(b) Net vertical force:

Weight of rubber: $W_{\text{rubber}} = (1.50 \times 10^{-3}\text{ kg})(9.80\text{ m/s}^2) = 0.0147\text{ N}$

Weight of helium ($\rho_{\text{He}} = 0.179\text{ kg/m}^3$): $m_{\text{He}} = (0.179\text{ kg/m}^3)(2.00 \times 10^{-3}\text{ m}^3) = 3.58 \times 10^{-4}\text{ kg}$ $W_{\text{He}} = (3.58 \times 10^{-4}\text{ kg})(9.80\text{ m/s}^2) = 0.00351\text{ N}$

Net upward force: $F_{\text{net}} = F_B - W_{\text{rubber}} - W_{\text{He}} = 0.0253 - 0.0147 - 0.00351 = 0.00709\text{ N}$

$F_{\text{net}} \approx 7.1 \times 10^{-3}\text{ N}$ (upward)

Discussion: The buoyant force is approximately **0.025 N**, and the net upward force is about **7.1 mN**. The balloon rises because helium is much less dense than air (about 14% of air's density), creating a net upward force. This small force explains why helium balloons rise slowly and can be held down by a light string.

(a) What is the density of a woman who floats in freshwater with 4.00% of her volume above the surface? This could be measured by placing her in a tank with marks on the side to measure how much water she displaces when floating and when held under water (briefly). (b) What percent of her volume is above the surface when she floats in seawater?

Show Solution

(a) 960kg/m^3 (b) 6.34% She indeed floats more in seawater.

A certain man has a mass of 80 kg and a density of 955kg/m^3 (excluding the air in his lungs). (a) Calculate his volume. (b) Find the buoyant force air exerts on him. (c) What is the ratio of the buoyant force to his weight?

Show Solution

Strategy: Use density to find volume, then calculate buoyant force from air displaced.

Solution:

(a) Volume: $V = \frac{m}{\rho} = \frac{80\text{ kg}}{955\text{ kg/m}^3} = 0.0838\text{ m}^3$

(b) Buoyant force from air:

Using air density $\rho_{\text{air}} = 1.29\text{ kg/m}^3$:

$$F_B = \rho_{\text{air}} V g = (1.29\text{ kg/m}^3)(0.0838\text{ m}^3)(9.80\text{ m/s}^2)$$
$$F_B = 1.06\text{ N}$$

(c) Ratio of buoyant force to weight:

$$W = mg = (80\text{ kg})(9.80\text{ m/s}^2) = 784\text{ N}$$
$$\frac{F_B}{W} = \frac{1.06\text{ N}}{784\text{ N}} = 0.00135 = 0.135\%$$

Discussion: The man's volume is about **0.084 m³** (84 liters). Air exerts a buoyant force of approximately **1.06 N** on him, which is only about **0.14%** of his weight. This explains why we don't notice atmospheric buoyancy in everyday life—it makes us about 1 N (~0.2 lb) lighter, which is imperceptible. However, this effect is measurable in precision experiments and must be corrected for when weighing objects in air versus vacuum.

A simple compass can be made by placing a small bar magnet on a cork floating in water. (a) What fraction of a plain cork will be submerged when floating in water? (b) If the cork has a mass of 10.0 g and a 20.0-g magnet is placed on it, what fraction of the cork will be submerged? (c) Will the bar magnet and cork float in ethyl alcohol?

Show Solution

(a) 0.24 (b) 0.68 (c) Yes, the cork will float because $\rho_{\text{obj}} < \rho_{\text{ethyl alcohol}} (0.678 \text{g/cm}^3 < 0.79 \text{g/cm}^3)$

What fraction of an iron anchor's weight will be supported by buoyant force when submerged in saltwater?

Show Solution

Strategy: The buoyant force equals the weight of saltwater displaced. The fraction supported is the ratio of buoyant force to weight.

Solution:

For a submerged object, the buoyant force is: $F_B = \rho_{\text{fluid}} V g$

The weight of the iron anchor is: $W = \rho_{\text{iron}} V g$

The fraction of weight supported by buoyancy: $\frac{F_B}{W} = \frac{\rho_{\text{fluid}} V g}{\rho_{\text{iron}} V g} = \frac{\rho_{\text{fluid}}}{\rho_{\text{iron}}}$

Using $\rho_{\text{saltwater}} = 1025 \text{ kg/m}^3$ and $\rho_{\text{iron}} = 7870 \text{ kg/m}^3$:

$$\frac{F_B}{W} = \frac{1025}{7870} = 0.130 = 13.0\%$$

Discussion: Approximately **13%** of the iron anchor's weight is supported by the buoyant force when submerged in saltwater. This means the anchor has an "apparent weight" of about 87% of its true weight when underwater. While this doesn't make the anchor float (since 13% < 100%), it does explain why heavy objects feel lighter when submerged—divers can lift anchors and other heavy equipment underwater more easily than in air.

Scurrilous con artists have been known to represent gold-plated tungsten ingots as pure gold and sell them to the greedy at prices much below gold value but deservedly far above the cost of tungsten. With what accuracy must you be able to measure the mass of such an ingot in and out of water to tell that it is almost pure tungsten rather than pure gold?

Show Solution

Strategy: The apparent mass loss in water is proportional to density. Compare the fractional difference in apparent mass loss for gold versus tungsten.

Solution:

From Table 1:

- Gold density: $\rho_{\text{Au}} = 19.32 \times 10^3 \text{ kg/m}^3$
- Tungsten density: $\rho_{\text{W}} = 19.30 \times 10^3 \text{ kg/m}^3$

For an ingot of mass m and density ρ:

- Volume: $V = m/\rho$
- Apparent mass loss in water: $\Delta m = \rho_{\text{water}} V = \rho_{\text{water}} (m/\rho) = m(\rho_{\text{water}}/\rho)$

For gold: $\Delta m_{\text{Au}} = m \frac{1000}{19{,}320} = 0.05176 m$

For tungsten: $\Delta m_{\text{W}} = m \frac{1000}{19{,}300} = 0.05181 m$

The fractional difference: $\frac{\Delta m_{\text{W}} - \Delta m_{\text{Au}}}{\Delta m_{\text{Au}}} = \frac{0.05181 - 0.05176}{0.05176} = \frac{0.00005}{0.05176} = 0.00097 \approx 0.0001 = 0.01\%$

Actually, a more direct calculation of the percentage difference: $\frac{\rho_{\text{Au}} - \rho_{\text{W}}}{\rho_{\text{Au}}} \times 100\% = \frac{19.32 - 19.30}{19.32} \times 100\% = 0.10\%$

The measurement precision needed is approximately **0.1%** or better (closer to **0.006%** considering the measurement must detect this small difference reliably).

Discussion: The required measurement accuracy is about **0.01%** or better. This is extremely challenging! Gold and tungsten have nearly identical densities (differing by only 0.1%), making them difficult to distinguish by density measurements alone. A 1-kg ingot would show an apparent mass difference of only about 1 gram when measured in water—easily obscured by measurement error. This is why gold-plated tungsten is a credible fraud: tungsten is much cheaper than gold but has almost the same density. Detecting such fraud requires either very precise density measurements (to 3-4 decimal places), destructive testing (drilling or cutting), or sophisticated techniques like X-ray fluorescence or ultrasound imaging.

A twin-sized air mattress used for camping has dimensions of 100 cm by 200 cm by 15 cm when blown up. The mass of the mattress when blown up is 2 kg. How heavy a person could the air mattress hold if it is placed in freshwater?

Show Solution

Strategy: Calculate the maximum buoyant force (when fully submerged), subtract the mattress weight to find the maximum additional load.

Solution:

Step 1: Calculate the mattress volume $V = (1.00\text{ m})(2.00\text{ m})(0.15\text{ m}) = 0.30\text{ m}^3$

Step 2: Calculate maximum buoyant force (fully submerged) $F_{B,\text{max}} = \rho_{\text{water}} V g = (1000\text{ kg/m}^3)(0.30\text{ m}^3)(9.80\text{ m/s}^2)$ $F_{B,\text{max}} = 2940\text{ N}$

Step 3: Calculate weight of mattress $W_{\text{mattress}} = mg = (2\text{ kg})(9.80\text{ m/s}^2) = 19.6\text{ N}$

Step 4: Calculate maximum additional weight supported $W_{\text{person}} = F_{B,\text{max}} - W_{\text{mattress}} = 2940\text{ N} - 19.6\text{ N} = 2920\text{ N}$

Step 5: Convert to mass $m_{\text{person}} = \frac{W_{\text{person}}}{g} = \frac{2920\text{ N}}{9.80\text{ m/s}^2} = 298\text{ kg}$

Discussion: The air mattress could support a person weighing up to approximately **298 kg** (about 660 pounds) before becoming fully submerged. In practice, you’d want to stay well below this limit to keep the mattress floating above water. A typical adult (70-80 kg) would only submerge about 25-30% of the mattress, leaving plenty of freeboard. The large surface area and 15 cm thickness provide substantial buoyancy.

Referring to [Figure 3], prove that the buoyant force on the cylinder is equal to the weight of the fluid displaced (Archimedes’ principle). You may assume that the buoyant force is $F_2 - F_1$ and that the ends of the cylinder have equal areas A. Note that the volume of the cylinder (and that of the fluid it displaces) equals $(h_2 - h_1)A$.

Show Solution

$F_{\text{net}} = F_2 - F_1 = P_2 A - P_1 A = (P_2 - P_1)A = (h_2 \rho_{\text{fl}} g - h_1 \rho_{\text{fl}} g)A = (h_2 - h_1)\rho_{\text{fl}} g A$ where ρ_{fl} = density of fluid. Therefore,

$F_{\text{net}} = (h_2 - h_1)A\rho_{\text{fl}} g = V_{\text{fl}} \rho_{\text{fl}} g = m_{\text{fl}} g = w_{\text{fl}}$ where is w_{fl} the weight of the fluid displaced.

(a) A 75.0-kg man floats in freshwater with 3.00% of his volume above water when his lungs are empty, and 5.00% of his volume above water when his lungs are full. Calculate the volume of air he inhales—called his lung capacity—in liters. (b) Does this lung volume seem reasonable?

Show Solution

Strategy: Use the floating condition to find body volume in each state, then calculate the difference (which equals lung capacity).

Solution:

(a) Calculate lung capacity:

For a floating object: $\text{fraction submerged} = \frac{\rho_{\text{body}}}{\rho_{\text{water}}}$

Rearranging: $V_{\text{body}} = \frac{m}{\rho_{\text{water}} \times \text{fraction submerged}}$

Lungs empty (97% submerged): $V_{\text{empty}} = \frac{75.0\text{ kg}}{(1000\text{ kg/m}^3)(0.970)} = 0.07732\text{ m}^3$

Lungs full (95% submerged): $V_{\text{full}} = \frac{75.0\text{ kg}}{(1000\text{ kg/m}^3)(0.950)} = 0.07895\text{ m}^3$

Lung capacity: $V_{\text{lungs}} = V_{\text{full}} - V_{\text{empty}} = 0.07895 - 0.07732 = 0.00163\text{ m}^3$

$V_{\text{lungs}} = 1.63\text{ L}$

(b) Is this reasonable?

Yes, this is very reasonable. Typical human lung capacity values:

- Tidal volume (normal breath): 0.5 L
- Vital capacity (maximum breath): 4-6 L
- Functional residual capacity: ~2.5 L

A value of **1.63 L** represents a moderate breath, less than maximum capacity but more than a normal resting breath. This is consistent with the “floating gently” scenario described.

Discussion: The lung capacity calculated is approximately **1.6 liters**, which is a reasonable value for a moderate inhalation. The calculation shows how sensitive floating is to lung volume—just 1.6 L of air (adding ~2% to body volume) raises the man 2 percentage points higher in the water. This explains why swimmers can control their buoyancy by adjusting their breathing.

Glossary

Archimedes' principle
: the buoyant force on an object equals the weight of the fluid it displaces

buoyant force
: the net upward force on any object in any fluid

specific gravity
: the ratio of the density of an object to a fluid (usually water)

Cohesion and Adhesion in Liquids: Surface Tension and Capillary Action

- Understand cohesive and adhesive forces.
- Define surface tension.
- Understand capillary action.

Cohesion and Adhesion in Liquids

Children blow soap bubbles and play in the spray of a sprinkler on a hot summer day. (See [Figure 1].) An underwater spider keeps his air supply in a shiny bubble he carries wrapped around him. A technician draws blood into a small-diameter tube just by touching it to a drop on a pricked finger. A premature infant struggles to inflate her lungs. What is the common thread? All these activities are dominated by the attractive forces between atoms and molecules in liquids—both within a liquid and between the liquid and its surroundings.

Attractive forces between molecules of the same type are called **cohesive forces**. Liquids can, for example, be held in open containers because cohesive forces hold the molecules together. Attractive forces between molecules of different types are called **adhesive forces**. Such forces cause liquid drops to cling to window panes, for example. In this section we examine effects directly attributable to cohesive and adhesive forces in liquids.

Cohesive Forces

Attractive forces between molecules of the same type are called cohesive forces.

Adhesive Forces

Attractive forces between molecules of different types are called adhesive forces.

The soap bubbles in this photograph are caused by cohesive forces among molecules in liquids. (credit: Steve Ford Elliott)

Surface Tension

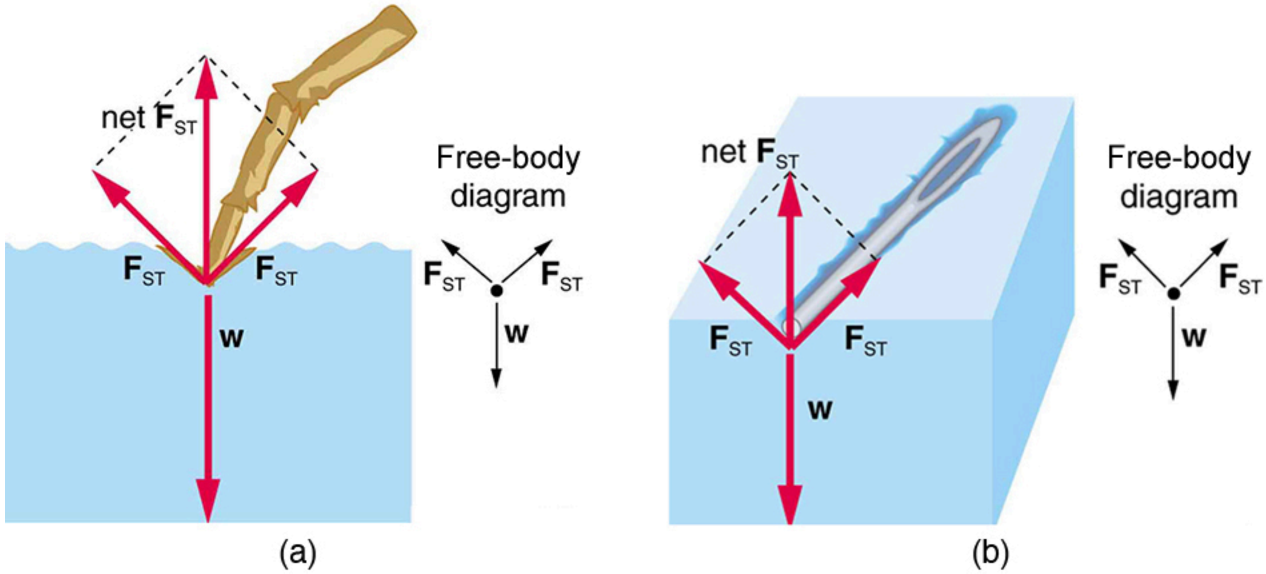
Cohesive forces between molecules cause the surface of a liquid to contract to the smallest possible surface area. This general effect is called **surface tension**. Molecules on the surface are pulled inward by cohesive forces, reducing the surface area. Molecules inside the liquid experience zero net force, since they have neighbors on all sides.

Surface Tension

Cohesive forces between molecules cause the surface of a liquid to contract to the smallest possible surface area. This general effect is called surface tension.
Making Connections: Surface Tension

Forces between atoms and molecules underlie the macroscopic effect called surface tension. These attractive forces pull the molecules closer together and tend to minimize the surface area. This is another example of a submicroscopic explanation for a macroscopic phenomenon.

The model of a liquid surface acting like a stretched elastic sheet can effectively explain surface tension effects. For example, some insects can walk on water (as opposed to floating in it) as we would walk on a trampoline—they dent the surface as shown in [Figure 2](a). [Figure 2](b) shows another example, where a needle rests on a water surface. The iron needle cannot, and does not, float, because its density is greater than that of water. Rather, its weight is supported by forces in the stretched surface that try to make the surface smaller or flatter. If the needle were placed point down on the surface, its weight acting on a smaller area would break the surface, and it would sink.



Surface tension supporting the weight of an insect and an iron needle, both of which rest on the surface without penetrating it. They are not floating; rather, they are supported by the surface of the liquid. (a) An insect leg dents the water surface. (F_\text{ST}) is a restoring force (surface tension) parallel to the surface. (b) An iron needle similarly dents a water surface until the restoring force (surface tension) grows to equal its weight.

Surface tension is proportional to the strength of the cohesive force, which varies with the type of liquid. Surface tension γ is defined to be the force *F* per unit length L exerted by a stretched liquid membrane:

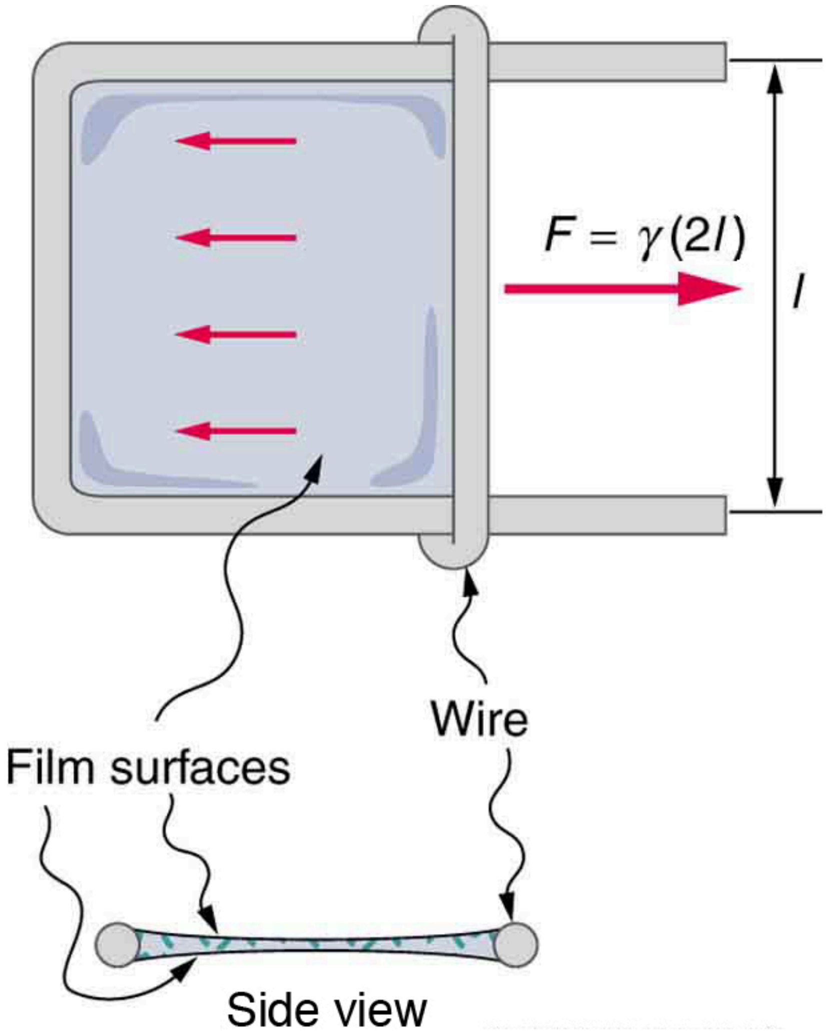
$$\gamma =\frac{F}{L}. $$

[Table 1] lists values of γ for some liquids. For the insect of [Figure 2](a), its weight w is supported by the upward components of the surface tension force: $w=\gamma L \sin \theta$, where L is the circumference of the insect’s foot in contact with the water. [Figure 3] shows one way to measure surface tension. The liquid film exerts a force on the movable wire in an attempt to reduce its surface area. The magnitude of this force depends on the surface tension of the liquid and can be measured accurately.

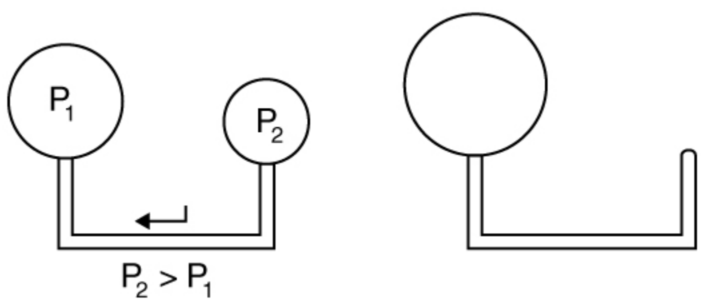
Surface tension is the reason why liquids form bubbles and droplets. The inward surface tension force causes bubbles to be approximately spherical and raises the pressure of the gas trapped inside relative to atmospheric pressure outside. It can be shown that the gauge pressure P inside a spherical bubble is given by

$$P=\frac{4\gamma }{r}, $$

where r is the radius of the bubble. Thus the pressure inside a bubble is greatest when the bubble is the smallest. Another bit of evidence for this is illustrated in [Figure 4]. When air is allowed to flow between two balloons of unequal size, the smaller balloon tends to collapse, filling the larger balloon.



Sliding wire device used for measuring surface tension; the device exerts a force to reduce the film's surface area. The force needed to hold the wire in place is ($F=\gamma L=\gamma (2l)$), since there are two liquid surfaces attached to the wire. This force remains nearly constant as the film is stretched, until the film approaches its breaking point.



With the valve closed, two balloons of different sizes are attached to each end of a tube. Upon opening the valve, the smaller balloon decreases in size with the air moving to fill the larger balloon. The pressure in a spherical balloon is inversely proportional to its radius, so that the smaller balloon has a greater internal pressure than the larger balloon, resulting in this flow.

Surface Tension of Some Liquids[1]

Liquid	Surface tension γ(N/m)
Water at 0ºC	0.0756
Water at 20ºC	0.0728
Water at 100ºC	0.0589
Soapy water (typical)	0.0370
Ethyl alcohol	0.0223
Glycerin	0.0631
Mercury	0.465
Olive oil	0.032

Liquid	Surface tension γ(N/m)
Tissue fluids (typical)	0.050
Blood, whole at 37ºC	0.058
Blood plasma at 37ºC	0.073
Gold at 1070ºC	1.000
Oxygen at -193ºC	0.0157
Helium at -269ºC	0.00012

Surface Tension: Pressure Inside a Bubble

Calculate the gauge pressure inside a soap bubble $2.00 \times 10^{-4} \text{m}$ in radius using the surface tension for soapy water in [Table 1]. Convert this pressure to mm Hg.

Strategy

The radius is given and the surface tension can be found in [Table 1] , and so P can be found directly from the equation $P=\frac{4\gamma }{r}$.

Solution

Substituting r and γ into the equation $P=\frac{4\gamma }{r}$, we obtain

$$P=\frac{4\gamma }{r}=\frac{4\left(0.037 \text{N/m}\right)}{ 2.00 \times 10^{-4} \text{m}}=740 {\text{N/m}}^{2}=740 \text{Pa}. $$

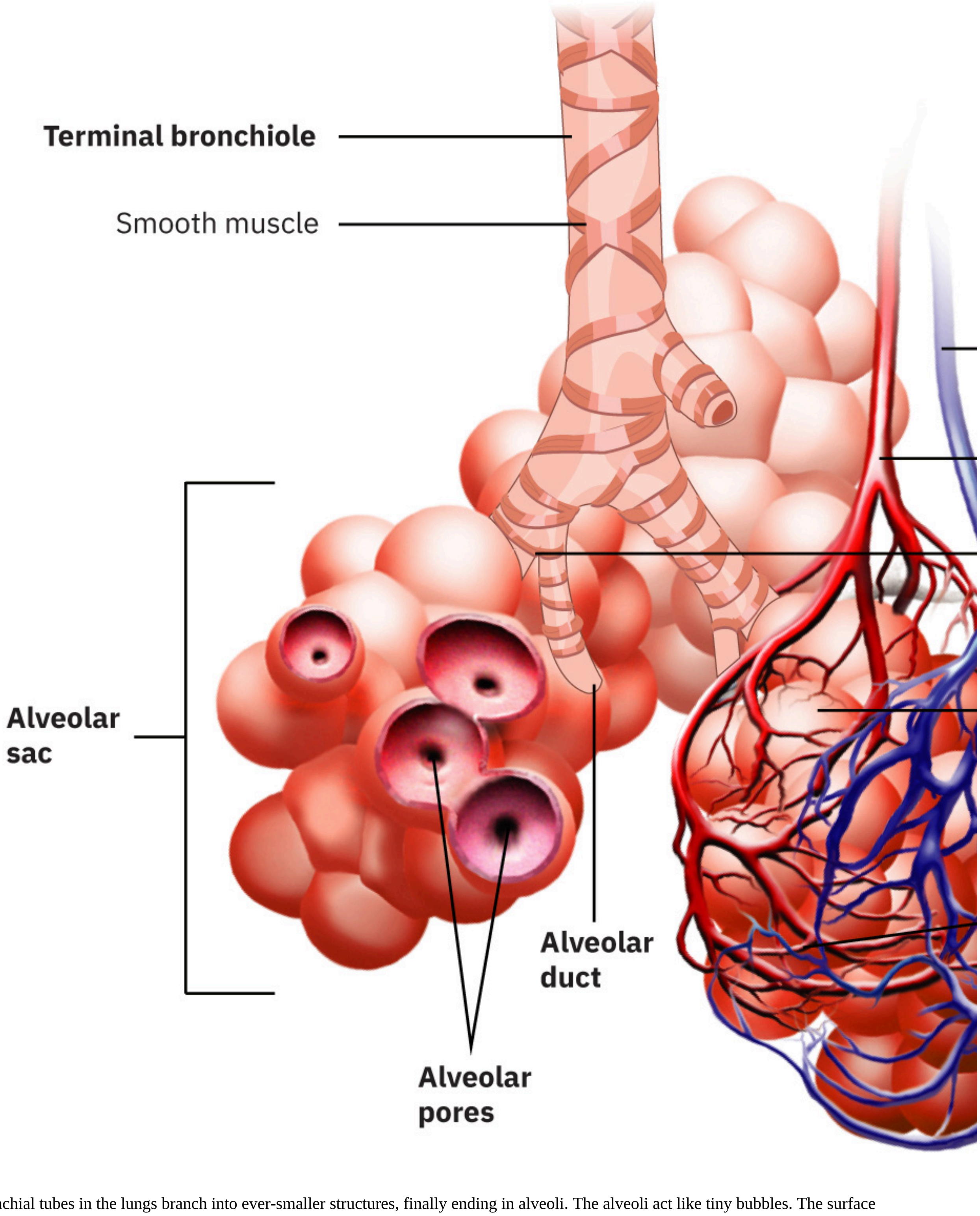
We use a conversion factor to get this into units of mm Hg:

$$P=\left(740 {\text{N/m}}^{2}\right)\frac{1.00 \text{mm Hg}}{133 {\text{N/m}}^{2}}=5.56 \text{mm Hg} . $$

Discussion

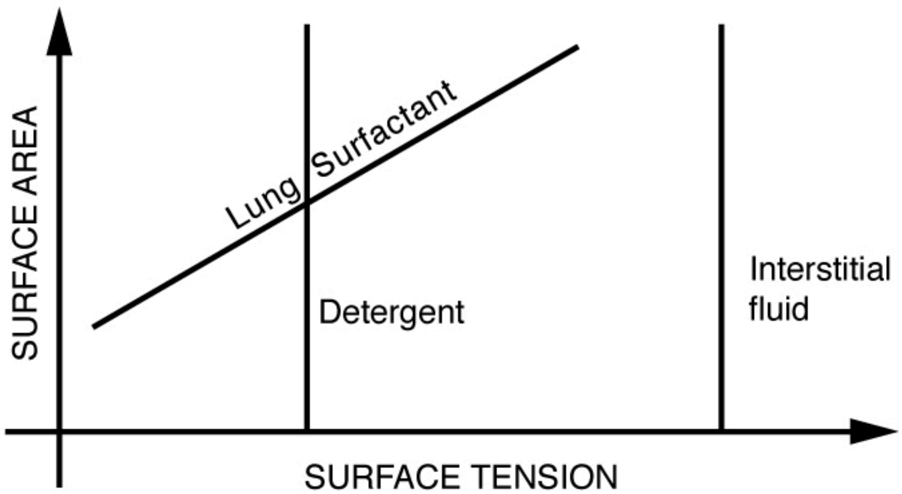
Note that if a hole were to be made in the bubble, the air would be forced out, the bubble would decrease in radius, and the pressure inside would *increase* to atmospheric pressure (760 mm Hg).

Our lungs contain hundreds of millions of mucus-lined sacs called *alveoli*, which are very similar in size, and about 0.1 mm in diameter. (See [Figure 5].) You can exhale without muscle action by allowing surface tension to contract these sacs. Medical patients whose breathing is aided by a positive pressure respirator have air blown into the lungs, but are generally allowed to exhale on their own. Even if there is paralysis, surface tension in the alveoli will expel air from the lungs. Since pressure increases as the radii of the alveoli decrease, an occasional deep cleansing breath is needed to fully reinflate the alveoli. Respirators are programmed to do this and we find it natural, as do our companion dogs and cats, to take a cleansing breath before settling into a nap.



Bronchial tubes in the lungs branch into ever-smaller structures, finally ending in alveoli. The alveoli act like tiny bubbles. The surface tension of their mucous lining aids in exhalation and can prevent inhalation if too great.

The tension in the walls of the alveoli results from the membrane tissue and a liquid on the walls of the alveoli containing a long lipoprotein that acts as a surfactant (a surface-tension reducing substance). The need for the surfactant results from the tendency of small alveoli to collapse and the air to fill into the larger alveoli making them even larger (as demonstrated in [Figure 4]). During inhalation, the lipoprotein molecules are pulled apart and the wall tension increases as the radius increases (increased surface tension). During exhalation, the molecules slide back together and the surface tension decreases, helping to prevent a collapse of the alveoli. The surfactant therefore serves to change the wall tension so that small alveoli don't collapse and large alveoli are prevented from expanding too much. This tension change is a unique property of these surfactants, and is not shared by detergents (which simply lower surface tension). (See [Figure 6].)



Surface tension as a function of surface area. The surface tension for lung surfactant decreases with decreasing area. This ensures that small alveoli don't collapse and large alveoli are not able to over expand.

If water gets into the lungs, the surface tension is too great and you cannot inhale. This is a severe problem in resuscitating drowning victims. A similar problem occurs in newborn infants who are born without this surfactant—their lungs are very difficult to inflate. This condition is known as *hyaline membrane disease* and is a leading cause of death for infants, particularly in premature births. Some success has been achieved in treating hyaline membrane disease by spraying a surfactant into the infant's breathing passages. Emphysema produces the opposite problem with alveoli. Alveolar walls of emphysema victims deteriorate, and the sacs combine to form larger sacs. Because pressure produced by surface tension decreases with increasing radius, these larger sacs produce smaller pressure, reducing the ability of emphysema victims to exhale. A common test for emphysema is to measure the pressure and volume of air that can be exhaled.

Making Connections: Take-Home Investigation

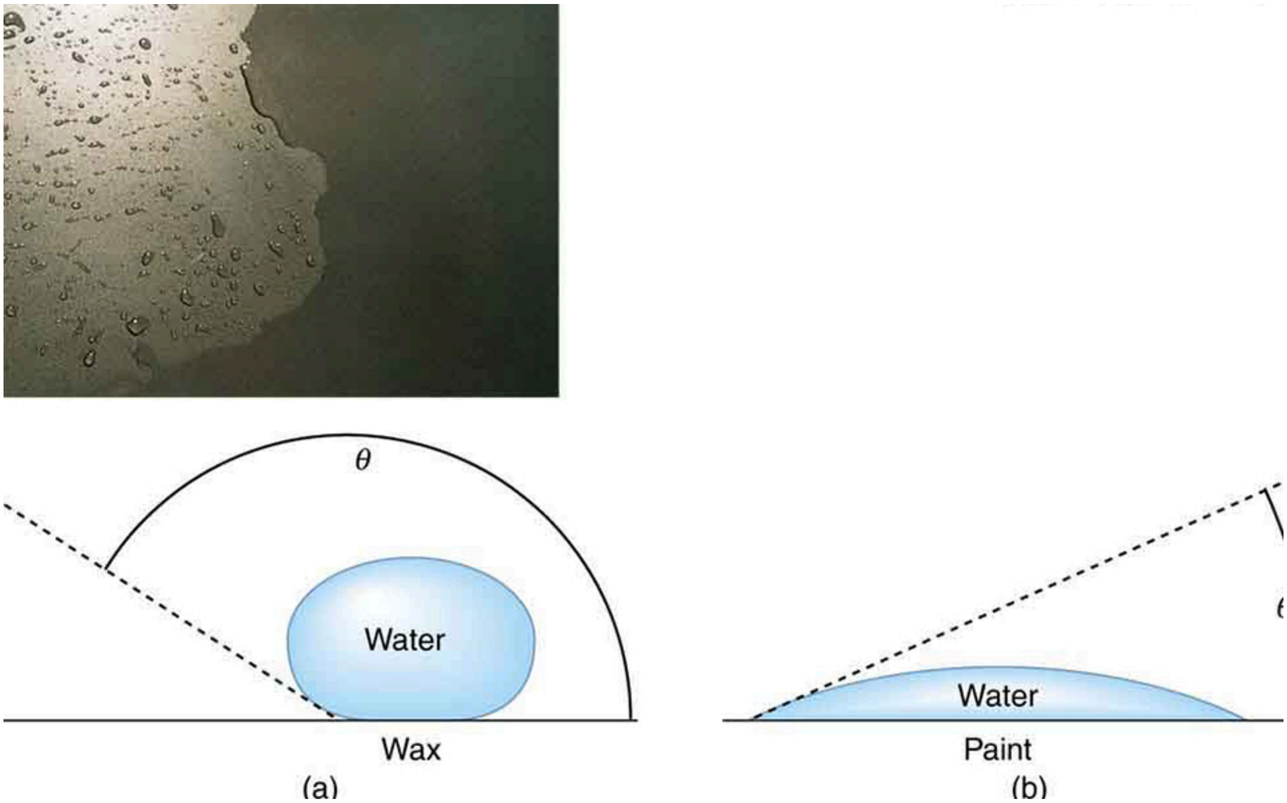
(1) Try floating a sewing needle on water. In order for this activity to work, the needle needs to be very clean as even the oil from your fingers can be sufficient to affect the surface properties of the needle. (2) Place the bristles of a paint brush into water. Pull the brush out and notice that for a short while, the bristles will stick together. The surface tension of the water surrounding the bristles is sufficient to hold the bristles together. As the bristles dry out, the surface tension effect dissipates. (3) Place a loop of thread on the surface of still water in such a way that all of the thread is in contact with the water. Note the shape of the loop. Now place a drop of detergent into the middle of the loop. What happens to the shape of the loop? Why? (4) Sprinkle pepper onto the surface of water. Add a drop of detergent. What happens? Why? (5) Float two matches parallel to each other and add a drop of detergent between them. What happens? Note: For each new experiment, the water needs to be replaced and the bowl washed to free it of any residual detergent.

Adhesion and Capillary Action

Why is it that water beads up on a waxed car but does not on bare paint? The answer is that the adhesive forces between water and wax are much smaller than those between water and paint. Competition between the forces of adhesion and cohesion are important in the macroscopic behavior of liquids. An important factor in studying the roles of these two forces is the angle θ between the tangent to the liquid surface and the surface. (See [Figure 7].) The **contact angle** θ is directly related to the relative strength of the cohesive and adhesive forces. The larger the strength of the cohesive force relative to the adhesive force, the larger θ is, and the more the liquid tends to form a droplet. The smaller θ is, the smaller the relative strength, so that the adhesive force is able to flatten the drop. [Table 2] lists contact angles for several combinations of liquids and solids.

Contact Angle

The angle θ between the tangent to the liquid surface and the surface is called the contact angle.



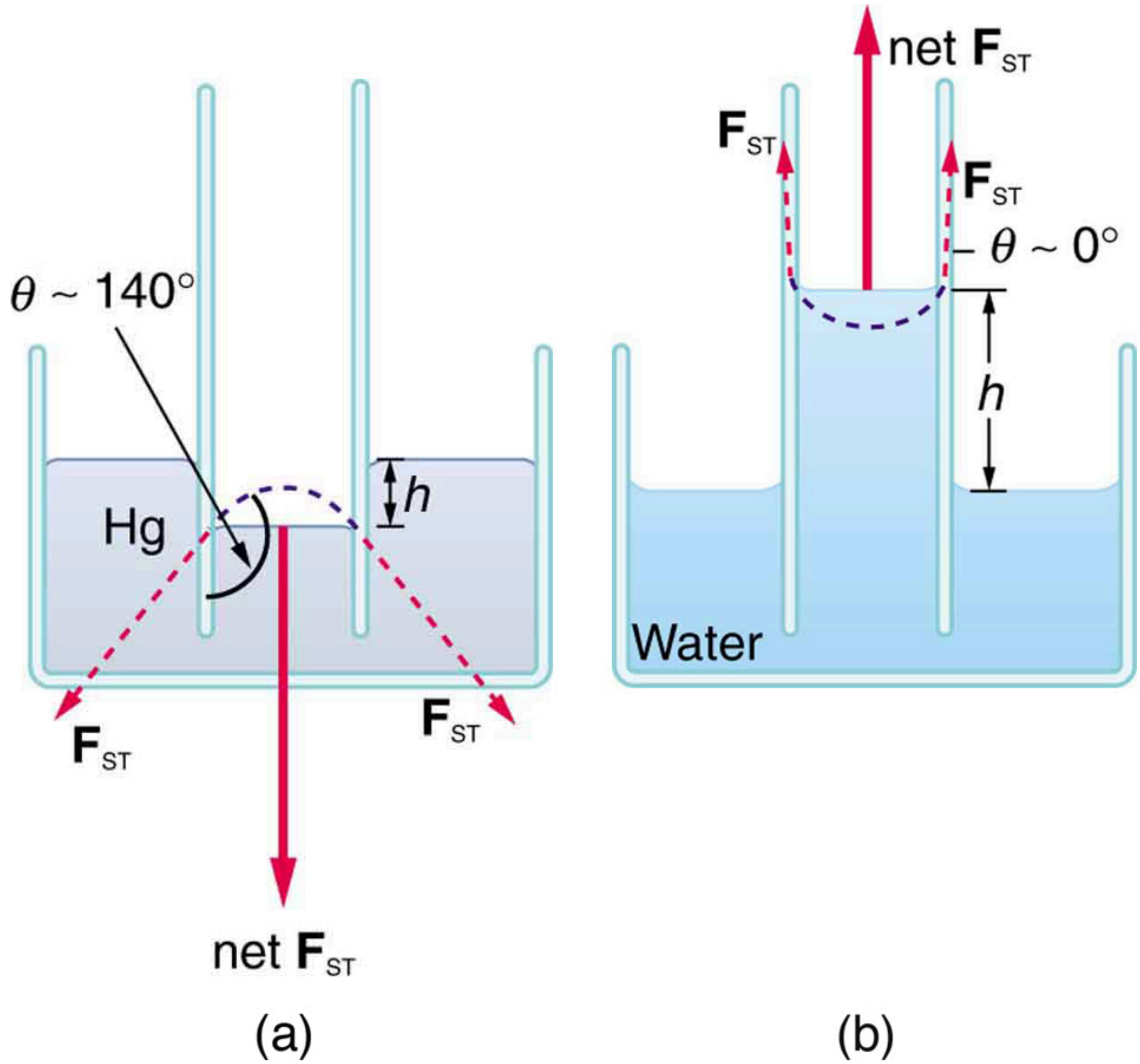
In the photograph, water beads on the waxed car paint and flattens on the unwaxed paint. (a) Water forms beads on the waxed surface because the cohesive forces responsible for surface tension are larger than the adhesive forces, which tend to flatten the drop. (b) Water beads on bare paint are flattened considerably because the adhesive forces between water and paint are strong, overcoming surface tension. The contact angle θ is directly related to the relative strengths of the cohesive and adhesive forces. The larger θ is, the larger the ratio of cohesive to adhesive forces. (credit: P. P. Urone)

One important phenomenon related to the relative strength of cohesive and adhesive forces is **capillary action**—the tendency of a fluid to be raised or suppressed in a narrow tube, or *capillary tube*. This action causes blood to be drawn into a small-diameter tube when the tube touches a drop.

Capillary Action

The tendency of a fluid to be raised or suppressed in a narrow tube, or capillary tube, is called capillary action.

If a capillary tube is placed vertically into a liquid, as shown in [Figure 8], capillary action will raise or suppress the liquid inside the tube depending on the combination of substances. The actual effect depends on the relative strength of the cohesive and adhesive forces and, thus, the contact angle θ given in the table. If θ is less than 90°, then the fluid will be raised; if θ is greater than 90°, it will be suppressed. Mercury, for example, has a very large surface tension and a large contact angle with glass. When placed in a tube, the surface of a column of mercury curves downward, somewhat like a drop. The curved surface of a fluid in a tube is called a **meniscus**. The tendency of surface tension is always to reduce the surface area. Surface tension thus flattens the curved liquid surface in a capillary tube. This results in a downward force in mercury and an upward force in water, as seen in [Figure 8].



(a) Mercury is suppressed in a glass tube because its contact angle is greater than 90º. Surface tension exerts a downward force as it flattens the mercury, suppressing it in the tube. The dashed line shows the shape the mercury surface would have without the flattening effect of surface tension. (b) Water is raised in a glass tube because its contact angle is nearly 0º. Surface tension therefore exerts an upward force when it flattens the surface to reduce its area.

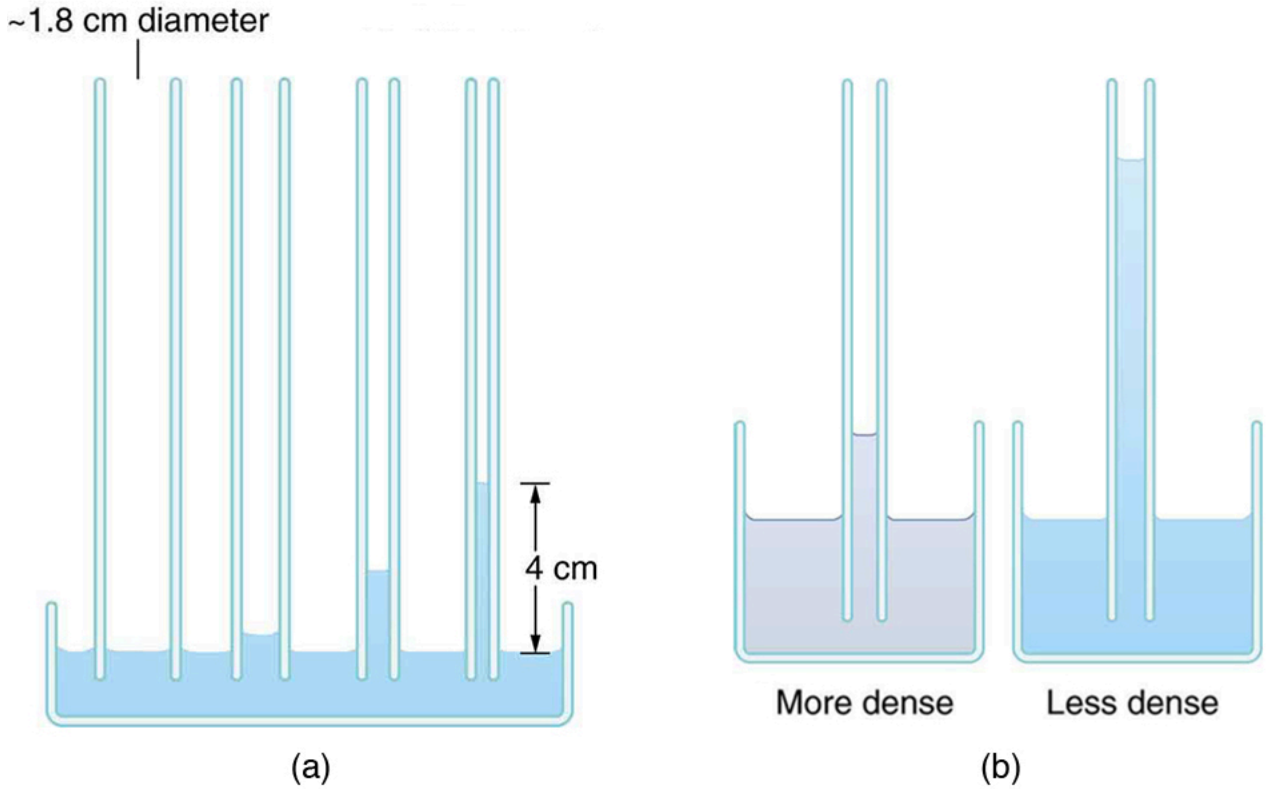
Contact Angles of Some Substances

Interface	Contact angle Θ
Mercury–glass	$$140\text{º} $$
Water–glass	$$0\text{º} $$
Water–paraffin	$$107\text{º} $$
Water–silver	$$90\text{º} $$
Organic liquids (most)–glass	$$0\text{º} $$
Ethyl alcohol–glass	$$0\text{º} $$
Kerosene–glass	$$26\text{º} $$

Capillary action can move liquids horizontally over very large distances, but the height to which it can raise or suppress a liquid in a tube is limited by its weight. It can be shown that this height h is given by

$$h=\frac{2\gamma \cos \theta }{\rho \text{gr}}. $$

If we look at the different factors in this expression, we might see how it makes good sense. The height is directly proportional to the surface tension γ , which is its direct cause. Furthermore, the height is inversely proportional to tube radius—the smaller the radius r , the higher the fluid can be raised, since a smaller tube holds less mass. The height is also inversely proportional to fluid density ρ , since a larger density means a greater mass in the same volume. (See [Figure 9].)



(a) Capillary action depends on the radius of a tube. The smaller the tube, the greater the height reached. The height is negligible for large-radius tubes. (b) A denser fluid in the same tube rises to a smaller height, all other factors being the same.

Calculating Radius of a Capillary Tube: Capillary Action: Tree Sap

Can capillary action be solely responsible for sap rising in trees? To answer this question, calculate the radius of a capillary tube that would raise sap 100 m to the top of a giant redwood, assuming that sap's density is $1050 \text{kg}{\text{/m}}^{3}$, its contact angle is zero, and its surface tension is the same as that of water at $20.0º C$.

Strategy

The height to which a liquid will rise as a result of capillary action is given by $h=\frac{2\gamma \cos \theta }{\rho \text{gr}}$, and every quantity is known except for r.

Solution

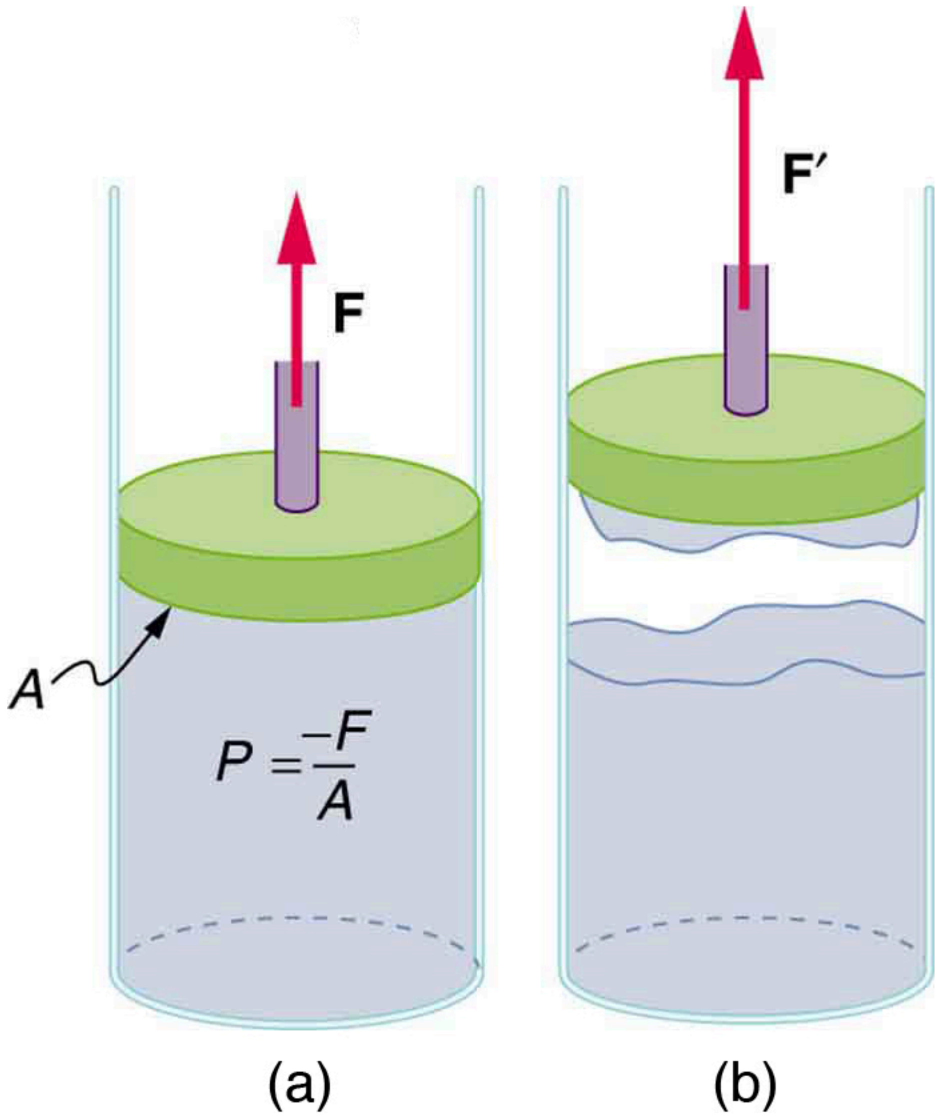
Solving for r and substituting known values produces

$$\begin{array}{lll}r& =& \frac{2\gamma \cos \theta }{\rho gh}=\frac{2\left(0.0728 \text{N/m}\right)\cos\left(0º\right)}{\left(1050 {\text{kg/m}}^{3}\right)\left(9.80 {\text{m/s}}^{2}\right)\left(100 \text{m}\right)}\\ & =& 1.41 \times 10^{-7} \text{m} \text{.} \end{array} $$

Discussion

This result is unreasonable. Sap in trees moves through the *xylem*, which forms tubes with radii as small as $2.5 \times 10^{-5} \text{m}$. This value is about 180 times as large as the radius found necessary here to raise sap 100m . This means that capillary action alone cannot be solely responsible for sap getting to the tops of trees.

How *does* sap get to the tops of tall trees? (Recall that a column of water can only rise to a height of 10 m when there is a vacuum at the top—see [Example 3].) The question has not been completely resolved, but it appears that it is pulled up like a chain held together by cohesive forces. As each molecule of sap enters a leaf and evaporates (a process called transpiration), the entire chain is pulled up a notch. So a negative pressure created by water evaporation must be present to pull the sap up through the xylem vessels. In most situations, *fluids can push but can exert only negligible pull*, because the cohesive forces seem to be too small to hold the molecules tightly together. But in this case, the cohesive force of water molecules provides a very strong pull. [Figure 10] shows one device for studying negative pressure. Some experiments have demonstrated that negative pressures sufficient to pull sap to the tops of the tallest trees *can* be achieved.



(a) When the piston is raised, it stretches the liquid slightly, putting it under tension and creating a negative absolute pressure \(P= -F/A \) .
(b) The liquid eventually separates, giving an experimental limit to negative pressure in this liquid.

Section Summary

- Attractive forces between molecules of the same type are called cohesive forces.
- Attractive forces between molecules of different types are called adhesive forces.
- Cohesive forces between molecules cause the surface of a liquid to contract to the smallest possible surface area. This general effect is called surface tension.
- Capillary action is the tendency of a fluid to be raised or suppressed in a narrow tube, or capillary tube which is due to the relative strength of cohesive and adhesive forces.

Conceptual Questions

The density of oil is less than that of water, yet a loaded oil tanker sits lower in the water than an empty one. Why?

Show Solution

Strategy

This question requires us to consider Archimedes’ principle and distinguish between the density of the cargo and the total weight of the loaded ship.

Solution

The key to understanding this apparent paradox is recognizing that it is the *total weight* of the tanker (ship + cargo) that determines how deep it sits in the water, not the density of the cargo. According to Archimedes’ principle, a floating object displaces a weight of water equal to its own weight. When the tanker is loaded with oil, even though the oil is less dense than water, the total weight of the ship plus oil is much greater than the weight of the empty ship alone. This greater total weight requires displacing more water to achieve buoyancy equilibrium, causing the loaded tanker to sit lower in the water.

Discussion

This situation illustrates that buoyancy depends on total weight, not on density comparisons between cargo and the surrounding fluid. Even if the tanker were loaded with something less dense than air, like helium-filled containers, the added mass would still cause the ship to sit lower. The comparison of oil's density to water's density is irrelevant—what matters is that adding any cargo increases the total weight that must be supported by buoyancy.

Is surface tension due to cohesive or adhesive forces, or both?

Show Solution

Strategy

We need to examine the molecular-level origin of surface tension and identify which type of intermolecular force is responsible.

Solution

Surface tension is due to **cohesive forces** only. Surface tension arises from the attractive forces between molecules of the same type (liquid molecules attracting other liquid molecules). At the surface of a liquid, molecules experience a net inward pull because they have fewer neighboring liquid molecules above them than beside or below them. This imbalance creates a "skin" effect that minimizes the surface area. Adhesive forces, which act between different types of molecules (such as liquid and container), do not contribute to surface tension itself.

Discussion

While adhesive forces are important in related phenomena like capillary action and contact angle formation, surface tension is purely a cohesive phenomenon. This is why surface tension is a property of the liquid itself (independent of what it contacts) and why all liquids exhibit surface tension to some degree based on the strength of intermolecular attractions within that liquid.

Is capillary action due to cohesive or adhesive forces, or both?

Show Solution

Strategy

We need to analyze the mechanism of capillary action and identify which molecular forces are involved.

Solution

Capillary action is due to **both** cohesive and adhesive forces. Adhesive forces between the liquid and the tube wall cause the liquid to "climb" up the walls of a narrow tube. Meanwhile, cohesive forces between liquid molecules pull the rest of the liquid along with the molecules that are adhering to the wall. The interplay of these two forces determines the behavior: if adhesive forces dominate (as with water in glass), the liquid rises and forms a concave meniscus; if cohesive forces dominate (as with mercury in glass), the liquid is depressed and forms a convex meniscus.

Discussion

The contact angle directly reflects the balance between cohesive and adhesive forces. A contact angle less than 90° (like water-glass at 0°) indicates that adhesive forces dominate, while a contact angle greater than 90° (like mercury-glass at 140°) indicates that cohesive forces dominate. Both types of forces must be present for capillary action to occur—without adhesion, there would be no "grip" on the wall, and without cohesion, the liquid would not move as a column.

Birds such as ducks, geese, and swans have greater densities than water, yet they are able to sit on its surface. Explain this ability, noting that water does not wet their feathers and that they cannot sit on soapy water.

Show Solution

Strategy

We need to analyze how surface tension supports these birds and explain why their water-repellent feathers are essential for this phenomenon.

Solution

Waterfowl can sit on water because their oily, water-repellent feathers allow surface tension to support their weight. Because water does not wet their feathers (the feathers are hydrophobic), the water surface curves around the bird without breaking through. This curved water surface creates an upward component of the surface tension force that helps support the bird's weight. The bird essentially sits in a depression on the water surface, much like a water strider walks on water or a needle floats on a water surface.

When soap is added to the water, the surface tension is significantly reduced (from about 0.073 N/m to about 0.037 N/m). This lower surface tension cannot provide sufficient upward force to support the bird's weight, causing the bird to sink into the water. Additionally, soap may allow the water to penetrate the feathers, eliminating the hydrophobic barrier.

Discussion

This explains why waterfowl spend considerable time preening and spreading oil from their preen gland over their feathers—maintaining water-repellent feathers is essential for buoyancy and insulation. Oil spills are devastating to waterfowl precisely because petroleum products destroy this water-repellent coating, causing birds to become waterlogged and lose the surface tension support they depend on.

Water beads up on an oily sunbather, but not on her neighbor, whose skin is not oiled. Explain in terms of cohesive and adhesive forces.

Show Solution

Strategy

We need to compare the relative strengths of cohesive forces within water and adhesive forces between water and the two different surfaces (oily skin versus bare skin).

Solution

Water beads up on the oily sunbather because the adhesive forces between water and oil are much weaker than the cohesive forces within water itself. With weak adhesion, water molecules are pulled together by their strong cohesive attractions, forming spherical droplets that minimize surface area. The contact angle between water and oily surfaces is large (greater than 90°), causing the water to form distinct beads.

On the non-oiled skin, the adhesive forces between water and skin are strong—comparable to or greater than the cohesive forces within water. These strong adhesive forces cause the water to spread out and flatten against the skin surface. The contact angle is small, and the water "wets" the skin rather than beading up.

Discussion

This is why oil and wax are used as water-repellent coatings on surfaces like cars, raincoats, and waterproof fabrics. By creating a surface with weak adhesion to water, these coatings ensure that water beads up and rolls off rather than spreading and soaking in. Conversely, surfactants (soaps and detergents) work by reducing water's surface tension and increasing its adhesion to surfaces, helping water spread and wet surfaces more effectively for cleaning.

Could capillary action be used to move fluids in a "weightless" environment, such as in an orbiting space probe?

Show Solution

Strategy

We need to analyze whether capillary action depends on gravity and whether it would function in microgravity conditions.

Solution

Yes, capillary action could be used to move fluids in a weightless environment, and in fact, it would work even better than on Earth. Capillary action arises from the interplay of adhesive forces (between liquid and tube wall) and cohesive forces (within the liquid), and these molecular forces do not depend on gravity. On Earth, capillary rise is limited by the weight of the liquid column that must be supported. In microgravity, this weight limitation is essentially eliminated, so the liquid could travel much farther through capillary tubes.

Discussion

This principle is actually used in spacecraft fluid management systems. In microgravity, liquids behave unpredictably because they are not constrained by weight to stay at the "bottom" of containers. Surface tension and capillary action become dominant effects. Spacecraft fuel tanks use special screens and channels that exploit capillary action to ensure fuel flows to the pump inlets regardless of the spacecraft's orientation. Similarly, capillary-driven systems are used for moving other fluids in space, such as in life support systems and scientific experiments.

What effect does capillary action have on the reading of a manometer with uniform diameter? Explain your answer.

Show Solution

Strategy

We need to consider how capillary action affects liquid height in manometer tubes and whether this introduces systematic errors in pressure measurements.

Solution

In a manometer with uniform diameter, capillary action has **no effect** on the pressure reading. Although capillary action raises (or lowers) the liquid level in each arm of the manometer, the effect is identical in both arms since they have the same diameter and are made of the same material. The pressure measurement depends on the *difference* in liquid heights between the two arms, not the absolute heights. Since capillary action raises both columns by the same amount, this effect cancels out when taking the difference.

Discussion

This is why it is important for manometer tubes to have uniform diameter throughout. If the two arms had different diameters, the capillary rise would be different in each arm (smaller diameter = greater rise for liquids like water), introducing a systematic error in the pressure reading. For precise measurements, especially with small pressure differences, it is essential to use manometers with uniform, relatively large-diameter tubes to minimize the absolute capillary effect, even though it cancels in uniform tubes. Mercury manometers are less affected by capillary issues because mercury's large contact angle (140°) with glass causes depression rather than rise, and its high surface tension makes the effect more predictable.

Pressure between the inside chest wall and the outside of the lungs normally remains negative. Explain how pressure inside the lungs can become positive (to cause exhalation) without muscle action.

Show Solution

Strategy

We need to consider the role of surface tension in the alveoli and how it can drive exhalation passively.

Solution

The pressure inside the lungs can become positive during exhalation without muscle action due to **surface tension** in the alveoli. The alveoli are small, spherical sacs lined with a thin layer of fluid. According to the pressure equation for a spherical bubble, $P = \frac{4\gamma}{r}$, the pressure inside these fluid-lined sacs is greater than the pressure outside due to surface tension. This surface tension creates an inward force that tends to collapse the alveoli, generating positive pressure inside the lungs that pushes air out.

When the diaphragm and chest muscles relax after inhalation, the elastic recoil of the stretched lung tissue combined with the surface tension in the alveoli creates sufficient pressure to drive exhalation passively. This is why patients on positive-pressure ventilators can exhale on their own even if their respiratory muscles are paralyzed—the surface tension provides the driving force.

Discussion

The lung surfactant plays a critical role in modulating this effect. Without surfactant, the surface tension would be too high, making it difficult to inhale (the alveoli would resist expansion) and causing smaller alveoli to collapse into larger ones. The surfactant reduces surface tension, especially at smaller radii during exhalation, preventing alveolar collapse while still allowing passive exhalation. Premature infants lacking sufficient surfactant suffer from hyaline membrane disease precisely because this delicate balance is disrupted.

Problems & Exercises

What is the pressure inside an alveolus having a radius of $2.50 \times 10^{-4} \text{m}$ if the surface tension of the fluid-lined wall is the same as for soapy water? You may assume the pressure is the same as that created by a spherical bubble.

Show Solution

Strategy: Use the gauge pressure formula for a spherical bubble: $P = \frac{4\gamma}{r}$, where γ is the surface tension.

Solution: Given:

- Alveolus radius: $r = 2.50 \times 10^{-4} \text{ m}$
- Surface tension of soapy water: $\gamma = 0.0370 \text{ N/m}$ (from Table 1)

Calculate the gauge pressure: $P = \frac{4\gamma}{r} = \frac{4(0.0370 \text{ N/m})}{2.50 \times 10^{-4} \text{ m}}$

$$ P = \frac{0.148 \text{ N/m}}{2.50 \times 10^{-4} \text{ m}} = 592 \text{ N/m}^2 = 592 \text{ Pa} $$

Discussion: The gauge pressure inside the alveolus is approximately **592 Pa** (about 4.4 mm Hg). This is the pressure created by surface tension alone. In reality, lung surfactant reduces the surface tension in alveoli to about 0.020-0.025 N/m, lowering this pressure significantly. Without surfactant, the high surface tension pressure would make breathing very difficult and cause alveoli to collapse. This is critical in premature infants whose lungs haven't yet produced sufficient surfactant, leading to respiratory distress syndrome. The $P \propto 1/r$ relationship also means smaller alveoli would have higher pressures and tend to collapse into larger ones—another reason surfactant is essential.

(a) The pressure inside an alveolus with a 2.00×10^{-4} -m radius is $1.40 \times 10^{3} \text{Pa}$, due to its fluid-lined walls. Assuming the alveolus acts like a spherical bubble, what is the surface tension of the fluid? (b) Identify the likely fluid. (You may need to extrapolate between values in [Table 1].)

Show Solution

Strategy

We can use the pressure equation for a spherical bubble, $P = \frac{4\gamma}{r}$, and solve for surface tension γ. Then we can compare the result to values in Table 1 to identify the fluid.

Solution

(a) Solving the bubble pressure equation for surface tension:

$$ \gamma = \frac{Pr}{4} $$

Substituting the given values:

$$ \gamma = \frac{(1.40 \times 10^{3} \text{ Pa})(2.00 \times 10^{-4} \text{ m})}{4} = \frac{0.280 \text{ N/m}}{4} = 0.0700 \text{ N/m} $$

(b) Comparing this value to Table 1, we find that $\gamma = 0.0700 \text{ N/m}$ is very close to the surface tension of water at 20°C (0.0728 N/m) and blood plasma at 37°C (0.073 N/m).

The surface tension of the fluid lining the alveolus is **0.0700 N/m**, and the fluid is most likely **blood plasma** or a similar tissue fluid, which is reasonable given that alveoli are surrounded by capillaries and lined with fluid similar in composition to blood plasma.

Discussion

This result is physiologically reasonable. The alveoli are lined with a thin layer of fluid that has properties similar to blood plasma. However, in healthy lungs, surfactant reduces this surface tension significantly (especially during exhalation) to prevent alveolar collapse. The value calculated here represents a condition without significant surfactant effect, which might occur in certain disease states or in premature infants lacking surfactant.

What is the gauge pressure in millimeters of mercury inside a soap bubble 0.100 m in diameter?

Show Solution

Strategy: Use the gauge pressure formula for a soap bubble: $P = \frac{4\gamma}{r}$, then convert the result to mm Hg.

Solution: Given:

- Diameter = 0.100 m, so radius $r = 0.0500 \text{ m}$
- Surface tension of soapy water: $\gamma = 0.0370 \text{ N/m}$ (from Table 1)

Calculate gauge pressure in Pascals: $P = \frac{4\gamma}{r} = \frac{4(0.0370 \text{ N/m})}{0.0500 \text{ m}} = \frac{0.148 \text{ N/m}}{0.0500 \text{ m}} = 2.96 \text{ Pa}$

Convert to mm Hg using $P = \rho g h$, where $\rho_{\text{Hg}} = 13.6 \times 10^3 \text{ kg/m}^3$:

$$ h = \frac{P}{\rho g} = \frac{2.96 \text{ Pa}}{(13.6 \times 10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)} = \frac{2.96}{1.33 \times 10^5} \text{ m} $$
$$ h = 2.23 \times 10^{-5} \text{ m} = 2.23 \times 10^{-2} \text{ mm Hg} $$

Discussion: The gauge pressure inside the soap bubble is approximately **0.0223 mm Hg** (or 2.96 Pa). This is a very small pressure—only about 0.002% of atmospheric pressure. The relatively large radius (5 cm) results in low pressure since $P \propto 1/r$. For comparison, a small bubble with 1 mm radius would have 50 times higher pressure (~1.1 mm Hg). This explains why small soap bubbles "pop" more readily—their higher internal pressure creates greater stress on the thin soap film.

Calculate the force on the slide wire in [Figure 3] if it is 3.50 cm long and the fluid is ethyl alcohol.

Show Solution

Strategy

The sliding wire device has a liquid film attached to both sides of the wire. The force exerted by surface tension is $F = \gamma L$, where L is the total length of the liquid-wire contact. Since the film has two surfaces (front and back), the effective length is $L = 2l$ where l is the wire length.

Solution

From Table 1, the surface tension of ethyl alcohol is $\gamma = 0.0223 \text{ N/m}$.

The wire length is $l = 3.50 \text{ cm} = 0.0350 \text{ m}$.

Since the film has two surfaces attached to the wire, the total contact length is:

$$ L = 2l = 2(0.0350 \text{ m}) = 0.0700 \text{ m} $$

The force on the wire is:

$$ F = \gamma L = (0.0223 \text{ N/m})(0.0700 \text{ m}) = 1.56 \times 10^{-3} \text{ N} $$

The force on the slide wire is **1.56×10^{-3} N** (or 1.56 mN).

Discussion

This force is quite small (about 1.56 millinewtons), which is typical for surface tension effects. The force would be larger for a liquid with higher surface tension (like water at 0.0728 N/m, which would give about 5.10 mN) or smaller for liquids with lower surface tension. This demonstrates why surface tension effects are most noticeable at small scales where other forces (like weight) are also small.

[Figure 8](a) shows the effect of tube radius on the height to which capillary action can raise a fluid. (a) Calculate the height h for water in a glass tube with a radius of 0.900 cm—a rather large tube like the one on the left. (b) What is the radius of the glass tube on the right if it raises water to 4.00 cm?

Show Solution

Strategy: Use the capillary rise formula $h = \frac{2\gamma \cos\theta}{\rho g r}$. For part (a), solve for h given r. For part (b), solve for r given h.

Solution:

For water in glass at 20°C:

- Surface tension: $\gamma = 0.0728 \text{ N/m}$
- Contact angle: $\theta = 0°$ (so $\cos\theta = 1$)
- Density: $\rho = 1000 \text{ kg/m}^3$

(a) Height in large tube with r = 0.900 cm:

$$ h = \frac{2\gamma \cos\theta}{\rho g r} = \frac{2(0.0728 \text{ N/m})(1)}{(1000 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(0.00900 \text{ m})} $$
$$ h = \frac{0.1456}{88.2} = 1.65 \times 10^{-3} \text{ m} = 1.65 \text{ mm} $$

(b) Radius of tube that raises water to h = 4.00 cm:

Solving for r:

$$ r = \frac{2\gamma \cos\theta}{\rho g h} = \frac{2(0.0728 \text{ N/m})(1)}{(1000 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(0.0400 \text{ m})} $$
$$ r = \frac{0.1456}{392} = 3.71 \times 10^{-4} \text{ m} = 0.371 \text{ mm} $$

The results are: (a) **1.65×10^{-3} m** (1.65 mm), and (b) **3.71×10^{-4} m** (0.371 mm).

Discussion: In the large tube (9.0 mm radius), water rises only **1.65 mm**—barely noticeable. This demonstrates the inverse relationship between tube radius and capillary rise: $h \propto 1/r$. Larger tubes provide less capillary rise because the weight of the water column increases faster (as r^2) than the supporting surface tension force (which increases as r).

In contrast, the small tube (0.371 mm radius) raises water to **4.00 cm**—about 24 times higher. The tube radius is 24 times smaller, confirming the inverse proportionality. This is why capillary action is most noticeable in very fine tubes or porous materials with tiny pores. Paper towels, sponges, and soil all rely on capillary action in microscopic channels to absorb and transport water.

We stated in [Example 2] that a xylem tube is of radius $2.50 \times 10^{-5} \text{m}$. Verify that such a tube raises sap less than a meter by finding h for it, making the same assumptions that sap's density is $1050 {\text{kg/m}}^{3}$, its contact angle is zero, and its surface tension is the same as that of water at $20.0º C$.

Show Solution

Strategy

We use the capillary rise equation $h = \frac{2\gamma \cos\theta}{\rho g r}$ with the given values for xylem tubes carrying tree sap.

Solution

Given values:

- Radius: $r = 2.50 \times 10^{-5} \text{ m}$
- Density of sap: $\rho = 1050 \text{ kg/m}^3$
- Contact angle: $\theta = 0°$ (so $\cos\theta = 1$)
- Surface tension (water at 20°C): $\gamma = 0.0728 \text{ N/m}$
- Gravitational acceleration: $g = 9.80 \text{ m/s}^2$

Substituting into the capillary rise equation:

$$ h = \frac{2\gamma \cos\theta}{\rho g r} = \frac{2(0.0728 \text{ N/m})(1)}{(1050 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(2.50 \times 10^{-5} \text{ m})} $$
$$ h = \frac{0.1456 \text{ N/m}}{0.2573 \text{ N/m}^2} = 0.566 \text{ m} $$

The capillary rise in a xylem tube is approximately **0.57 m**, which is indeed less than 1 meter.

Discussion

This confirms that capillary action alone cannot raise sap to the top of tall trees—a giant redwood at 100 m would require capillary rise about 180 times greater than what xylem tubes can provide. The actual mechanism involves transpiration (water evaporating from leaves) creating negative pressure that pulls the sap column upward, with cohesive forces between water molecules keeping the column intact. This "cohesion-tension" mechanism allows sap to reach heights far beyond what capillary action alone could achieve.

What fluid is in the device shown in [Figure 3] if the force is $3.16 \times 10^{-3} \text{N}$ and the length of the wire is 2.50 cm? Calculate the surface tension γ and find a likely match from [Table 1].

Show Solution

Strategy: Use the relationship $F = \gamma L$ where L is the total length of liquid-wire contact. Since the film has two surfaces (front and back), $L = 2l$ where l is the wire length. Solve for γ and compare to Table 1.

Solution:

Given:

- Force: $F = 3.16 \times 10^{-3} \text{ N}$
- Wire length: $l = 2.50 \text{ cm} = 0.0250 \text{ m}$

Total contact length (two surfaces):

$$ L = 2l = 2(0.0250 \text{ m}) = 0.0500 \text{ m} $$

Solve for surface tension:

$$ \gamma = \frac{F}{L} = \frac{3.16 \times 10^{-3} \text{ N}}{0.0500 \text{ m}} = 6.32 \times 10^{-2} \text{ N/m} = 0.0632 \text{ N/m} $$

Comparing to Table 1:

- Water at 20°C: 0.0728 N/m
- **Glycerin: 0.0631 N/m ✓**
- Tissue fluids: 0.050 N/m
- Blood plasma at 37°C: 0.073 N/m

The surface tension is **6.32×10^{-2} N/m** (or 0.0632 N/m). Based on the values in Table 1, the fluid is most likely **glycerin**.

Discussion: The calculated surface tension of **0.0632 N/m** matches glycerin's value of 0.0631 N/m almost exactly. Glycerin (glycerol) is a viscous liquid commonly used in laboratory demonstrations of surface tension because it has relatively high surface tension (though not as high as water or mercury), it's safe to handle, and it doesn't evaporate quickly like water or alcohol.

The excellent agreement (within 0.2%) suggests this is indeed glycerin. Water's surface tension (0.0728 N/m) is about 15% higher, while tissue fluids (0.050 N/m) are about 21% lower, so these would not match the measurement as well. This type of sliding wire experiment is a classic method for measuring surface tension and was historically important in determining surface tension values for various liquids.

If the gauge pressure inside a rubber balloon with a 10.0-cm radius is 1.50 cm of water, what is the effective surface tension of the balloon?

Show Solution

Strategy

We treat the balloon as analogous to a bubble and use the pressure equation $P = \frac{4\gamma}{r}$. First, we must convert the pressure from cm of water to Pascals.

Solution

First, convert the gauge pressure from cm of water to Pascals. Using $P = \rho g h$ for water:

$$ P = (1000 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(0.0150 \text{ m}) = 147 \text{ Pa} $$

Now solve the bubble pressure equation for surface tension:

$$ \gamma = \frac{Pr}{4} $$

With radius $r = 10.0 \text{ cm} = 0.100 \text{ m}$:

$$ \gamma = \frac{(147 \text{ Pa})(0.100 \text{ m})}{4} = \frac{14.7 \text{ N/m}}{4} = 3.68 \text{ N/m} $$

The effective surface tension of the rubber balloon is approximately **3.7 N/m**.

Discussion

This "surface tension" is much larger than typical liquid surface tensions (which are on the order of 0.01-0.1 N/m). This is because we are not measuring an actual surface tension but rather the effective elastic tension in the rubber membrane, which must stretch to contain the pressurized air. The rubber's elastic properties create a restoring force similar to surface tension in liquids, but much stronger. This is why blowing up a balloon requires more effort at the start (when the radius is small and the rubber is being stretched for the first time) but becomes easier as the balloon expands.

Calculate the gauge pressures inside 2.00-cm-radius bubbles of water, alcohol, and soapy water. Which liquid forms the most stable bubbles, neglecting any effects of evaporation?

Show Solution

Strategy: Use the bubble pressure formula $P = \frac{4\gamma}{r}$ with surface tension values from Table 1. The most stable bubble has gauge pressure closest to zero (absolute pressure closest to atmospheric).

Solution:

Given: $r = 2.00 \text{ cm} = 0.0200 \text{ m}$

From Table 1:

- Water at 20°C: $\gamma_w = 0.0728 \text{ N/m}$
- Ethyl alcohol: $\gamma_a = 0.0223 \text{ N/m}$
- Soapy water: $\gamma_{sw} = 0.0370 \text{ N/m}$

Water bubble:

$$ P_w = \frac{4\gamma_w}{r} = \frac{4(0.0728 \text{ N/m})}{0.0200 \text{ m}} = \frac{0.2912}{0.0200} = 14.6 \text{ Pa} $$

Alcohol bubble:

$$ P_a = \frac{4\gamma_a}{r} = \frac{4(0.0223 \text{ N/m})}{0.0200 \text{ m}} = \frac{0.0892}{0.0200} = 4.46 \text{ Pa} $$

Soapy water bubble:

$$ P_{sw} = \frac{4\gamma_{sw}}{r} = \frac{4(0.0370 \text{ N/m})}{0.0200 \text{ m}} = \frac{0.148}{0.0200} = 7.40 \text{ Pa} $$

The gauge pressures are: $\begin{array}{lll}{P}_{\text{w}}& =& 14.6 \text{ Pa},\\ {P}_{\text{a}}& =& 4.46 \text{ Pa},\\ {P}_{\text{sw}}& =& 7.40 \text{ Pa}\text{.}\end{array}$

Alcohol forms the most stable bubbles since its absolute pressure inside is closest to atmospheric pressure (smallest gauge pressure).

Discussion: The alcohol bubble has the lowest gauge pressure (**4.46 Pa**), meaning its internal pressure differs least from atmospheric pressure. This makes it the most mechanically stable—less stress on the bubble film means less tendency to pop.

However, this analysis neglects evaporation, which is crucial in practice. Alcohol evaporates much faster than water, quickly destabilizing alcohol bubbles. This is why **soapy water** actually forms the most stable bubbles in real life, despite having intermediate gauge pressure (7.40 Pa). Soap reduces water's surface tension from 0.0728 to 0.0370 N/m, making bubbles easier to form while retaining water's low evaporation rate. Additionally, soap creates a surfactant layer that slows evaporation and provides mechanical stability through the Marangoni effect, where surface tension gradients resist film thinning. This is why children blow soap bubbles, not alcohol or pure water bubbles!

Suppose water is raised by capillary action to a height of 5.00 cm in a glass tube. (a) To what height will it be raised in a paraffin tube of the same radius? (b) In a silver tube of the same radius?

Show Solution

Strategy

From the capillary rise equation $h = \frac{2\gamma \cos\theta}{\rho g r}$, we see that for the same liquid (same γ, ρ) in tubes of the same radius, the height is proportional to $\cos\theta$. We can use the ratio of heights: $\frac{h_2}{h_1} = \frac{\cos\theta_2}{\cos\theta_1}$.

Solution

For water in a glass tube with $\theta_{\text{glass}} = 0°$, so $\cos(0°) = 1$.

From Table 2:

- Water-paraffin contact angle: $\theta_{\text{paraffin}} = 107°$
- Water-silver contact angle: $\theta_{\text{silver}} = 90°$

(a) For the paraffin tube:

$$ h_{\text{paraffin}} = h_{\text{glass}} \times \frac{\cos\theta_{\text{paraffin}}}{\cos\theta_{\text{glass}}} = (5.00 \text{ cm}) \times \frac{\cos(107°)}{\cos(0°)} $$
$$ h_{\text{paraffin}} = (5.00 \text{ cm}) \times \frac{-0.292}{1} = -1.46 \text{ cm} $$

(b) For the silver tube:

$$ h_{\text{silver}} = h_{\text{glass}} \times \frac{\cos\theta_{\text{silver}}}{\cos\theta_{\text{glass}}} = (5.00 \text{ cm}) \times \frac{\cos(90°)}{\cos(0°)} $$
$$ h_{\text{silver}} = (5.00 \text{ cm}) \times \frac{0}{1} = 0 \text{ cm} $$

In a paraffin tube, water would be **depressed by 1.46 cm** (negative height indicates suppression). In a silver tube, water would experience **no capillary rise or depression** (h = 0).

Discussion

These results reflect the different relative strengths of adhesive versus cohesive forces. In paraffin (a waxy, hydrophobic material), the contact angle exceeds 90°, meaning cohesive forces in water dominate over adhesive forces to paraffin, causing the water to be pushed down. Silver presents an intermediate case where the contact angle is exactly 90°, meaning adhesive and cohesive forces are balanced, resulting in no net capillary effect. Glass, being hydrophilic, has strong adhesion to water, giving a 0° contact angle and maximum capillary rise.

Calculate the contact angle θ for olive oil if capillary action raises it to a height of 7.07 cm in a glass tube with a radius of 0.100 mm. Is this value consistent with that for most organic liquids?

Show Solution

Strategy: Use the capillary rise formula $h = \frac{2\gamma \cos\theta}{\rho g r}$ and solve for the contact angle θ. Use olive oil properties from Table 1.

Solution:

Given:

- Height: $h = 7.07 \text{ cm} = 0.0707 \text{ m}$
- Radius: $r = 0.100 \text{ mm} = 1.00 \times 10^{-4} \text{ m}$

From Table 1:

- Olive oil surface tension: $\gamma = 0.032 \text{ N/m}$
- Olive oil density: $\rho \approx 920 \text{ kg/m}^3$ (standard value)

Solve the capillary rise formula for $\cos\theta$:

$$ h = \frac{2\gamma \cos\theta}{\rho g r} $$ $$ \cos\theta = \frac{h \rho g r}{2\gamma} $$

Substitute values:

$$ \cos\theta = \frac{(0.0707 \text{ m})(920 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(1.00 \times 10^{-4} \text{ m})}{2(0.032 \text{ N/m})} $$
$$ \cos\theta = \frac{63.8 \times 10^{-3}}{0.064} = 0.9966 $$
$$ \theta = \cos^{-1}(0.9966) = 4.7° \approx 5.1° $$

The contact angle is approximately **5.1°**.

Is this consistent with organic liquids?

Yes, this is very consistent. From Table 2, most organic liquids have contact angles with glass of 0°. The small angle of 5.1° indicates that olive oil wets glass very well, with strong adhesive forces between the oil and glass. This is typical behavior for organic liquids on glass surfaces.

Discussion: The contact angle of **5.1°** is very close to 0°, indicating olive oil spreads readily on glass with minimal beading. This near-zero contact angle is characteristic of most organic liquids interacting with glass, as noted in Table 2 where "Organic liquids (most)–glass" shows 0°.

The small but non-zero angle (5.1° vs exactly 0°) could result from:

1. Impurities in the olive oil (it's a natural product with various fatty acids)
2. Slight contamination of the glass surface
3. Temperature effects (olive oil's surface tension varies with temperature)
4. Measurement uncertainties in the given data

The key point is that olive oil, like most organic liquids, has very strong adhesive interactions with glass, making it wet the surface thoroughly. This is opposite to the behavior of mercury on glass (contact angle = 140°), where cohesive forces dominate and the liquid beads up rather than spreading.

When two soap bubbles touch, the larger is inflated by the smaller until they form a single bubble. (a) What is the gauge pressure inside a soap bubble with a 1.50-cm radius? (b) Inside a 4.00-cm-radius soap bubble? (c) Inside the single bubble they form if no air is lost when they touch?

Show Solution

Strategy

We use the bubble pressure equation $P = \frac{4\gamma}{r}$ for parts (a) and (b). For part (c), we need to find the final radius by conserving the total amount of air (moles of gas), then calculate the new pressure.

Solution

From Table 1, the surface tension of soapy water is $\gamma = 0.0370 \text{ N/m}$.

(a) For the small bubble with $r_1 = 1.50 \text{ cm} = 0.0150 \text{ m}$:

$$ P_1 = \frac{4\gamma}{r_1} = \frac{4(0.0370 \text{ N/m})}{0.0150 \text{ m}} = 9.87 \text{ Pa} $$

(b) For the large bubble with $r_2 = 4.00 \text{ cm} = 0.0400 \text{ m}$:

$$ P_2 = \frac{4\gamma}{r_2} = \frac{4(0.0370 \text{ N/m})}{0.0400 \text{ m}} = 3.70 \text{ Pa} $$

(c) To find the final bubble, we use conservation of air molecules. Assuming isothermal conditions (constant temperature) and that the gauge pressures are small compared to atmospheric pressure, the total volume of air is approximately conserved:

$$ V_1 + V_2 = V_f $$
$$ \frac{4}{3}\pi r_1^3 + \frac{4}{3}\pi r_2^3 = \frac{4}{3}\pi r_f^3 $$
$$ r_f^3 = r_1^3 + r_2^3 = (0.0150)^3 + (0.0400)^3 = 3.375 \times 10^{-6} + 6.40 \times 10^{-5} $$
$$ r_f^3 = 6.74 \times 10^{-5} \text{ m}^3 $$
$$ r_f = 0.0407 \text{ m} = 4.07 \text{ cm} $$

The final pressure is:

$$ P_f = \frac{4\gamma}{r_f} = \frac{4(0.0370 \text{ N/m})}{0.0407 \text{ m}} = 3.64 \text{ Pa} $$

The gauge pressures are: (a) **9.87 Pa** for the 1.50-cm bubble, (b) **3.70 Pa** for the 4.00-cm bubble, and (c) **3.64 Pa** for the combined bubble with radius 4.07 cm.

Discussion

Notice that the smaller bubble has higher pressure (9.87 Pa vs 3.70 Pa), which explains why air flows from the smaller bubble to the larger one when they connect. The final pressure (3.64 Pa) is even lower than the original large bubble's pressure because the combined volume creates a still larger bubble. This demonstrates the inverse relationship between bubble radius and pressure, which also explains why it's harder to start blowing up a balloon (small radius, high pressure required) than to continue inflating it.

Calculate the ratio of the heights to which water and mercury are raised by capillary action in the same glass tube.

Show Solution

Strategy: Use the capillary rise formula for both liquids in the same tube (same r). Take the ratio, noting that mercury is depressed (negative height) due to its contact angle > 90°.

Solution:

For capillary rise: $h = \frac{2\gamma \cos\theta}{\rho g r}$

For the same tube, the ratio is:

$$ \frac{h_{\text{water}}}{h_{\text{mercury}}} = \frac{\gamma_w \cos\theta_w}{\gamma_{Hg} \cos\theta_{Hg}} \times \frac{\rho_{Hg}}{\rho_w} $$

From Tables 1 and 2:

- Water: $\gamma_w = 0.0728 \text{ N/m}$, $\theta_w = 0°$ (so $\cos\theta_w = 1$), $\rho_w = 1000 \text{ kg/m}^3$

- Mercury: $\gamma_{Hg} = 0.465 \text{ N/m}$, $\theta_{Hg} = 140°$ (so $\cos\theta_{Hg} = \cos(140°) = -0.766$), $\rho_{Hg} = 13,600 \text{ kg/m}^3$

Substitute values:

$$ \frac{h_w}{h_{Hg}} = \frac{0.0728 \times 1}{0.465 \times (-0.766)} \times \frac{13,600}{1000} $$
$$ \frac{h_w}{h_{Hg}} = \frac{0.0728}{-0.356} \times 13.6 = (-0.204) \times 13.6 = -2.78 $$

The ratio is **-2.78**. The negative sign indicates that water is raised while mercury is depressed (lowered) in the same glass tube.

Discussion: The ratio of **-2.78** means that if water rises to height h in a capillary tube, mercury will be depressed by approximately h/2.78 (or about 0.36h below the external level).

The **negative sign** is crucial—it reflects the fundamental difference in behavior:

- **Water (θ = 0°)**: Adhesive forces (water-glass) dominate over cohesive forces (water-water), so water climbs the tube walls, forming a concave meniscus.
- **Mercury (θ = 140°)**: Cohesive forces (mercury-mercury) dominate over adhesive forces (mercury-glass), so mercury is pushed down, forming a convex meniscus.

The magnitude (2.78) results from the interplay of three factors:

1. Mercury’s much higher surface tension (0.465 vs 0.0728 N/m) tends to create stronger capillary effects
2. Mercury’s large contact angle and negative cosine reverses the effect and reduces its magnitude
3. Mercury’s much higher density (13.6 times water’s) reduces the height change

This opposite behavior is why mercury barometers show a depression in the tube connected to lower pressure, while water manometers show elevation.

What is the ratio of heights to which ethyl alcohol and water are raised by capillary action in the same glass tube?

Show Solution

Strategy

From the capillary rise equation $h = \frac{2\gamma \cos\theta}{\rho g r}$, for two different liquids in the same tube (same r, same g), we can find the ratio of heights as:

$$ \frac{h_{\text{alcohol}}}{h_{\text{water}}} = \frac{\gamma_{\text{alcohol}} \cos\theta_{\text{alcohol}}}{\gamma_{\text{water}} \cos\theta_{\text{water}}} \times \frac{\rho_{\text{water}}}{\rho_{\text{alcohol}}} $$

Solution

From Table 1 and Table 2:

- Surface tension of ethyl alcohol: $\gamma_{\text{alcohol}} = 0.0223 \text{ N/m}$
- Surface tension of water at 20°C: $\gamma_{\text{water}} = 0.0728 \text{ N/m}$
- Contact angle of ethyl alcohol with glass: $\theta_{\text{alcohol}} = 0°$
- Contact angle of water with glass: $\theta_{\text{water}} = 0°$

For densities (standard values):

- Density of ethyl alcohol: $\rho_{\text{alcohol}} = 790 \text{ kg/m}^3$
- Density of water: $\rho_{\text{water}} = 1000 \text{ kg/m}^3$

Since both contact angles are 0°, $\cos(0°) = 1$ for both:

$$ \frac{h_{\text{alcohol}}}{h_{\text{water}}} = \frac{\gamma_{\text{alcohol}}}{\gamma_{\text{water}}} \times \frac{\rho_{\text{water}}}{\rho_{\text{alcohol}}} $$
$$ \frac{h_{\text{alcohol}}}{h_{\text{water}}} = \frac{0.0223 \text{ N/m}}{0.0728 \text{ N/m}} \times \frac{1000 \text{ kg/m}^3}{790 \text{ kg/m}^3} $$
$$ \frac{h_{\text{alcohol}}}{h_{\text{water}}} = 0.306 \times 1.27 = 0.388 $$

The ratio of capillary rise heights is approximately **0.39** (or ethyl alcohol rises to about 39% of the height that water does in the same tube).

Discussion

Although ethyl alcohol has lower density than water (which would tend to increase its capillary rise), its surface tension is much lower (about 31% that of water). The net effect is that alcohol rises to less than half the height of water. This demonstrates that surface tension is the dominant factor in determining capillary rise height when contact angles are the same. The lower surface tension of alcohol is why it spreads more readily and “wets” surfaces more easily than water does.

Footnotes

- 1 At 20ºC unless otherwise stated. { data-list-type=”bulleted” data-bullet-style=”none”}

Glossary

adhesive forces
: the attractive forces between molecules of different types

capillary action
: the tendency of a fluid to be raised or lowered in a narrow tube

cohesive forces
: the attractive forces between molecules of the same type

contact angle
: the angle θ between the tangent to the liquid surface and the surface

surface tension
: the cohesive forces between molecules which cause the surface of a liquid to contract to the smallest possible surface area

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Pressures in the Body

- Explain the concept of pressure the in human body.
- Explain systolic and diastolic blood pressures.
- Describe pressures in the eye, lungs, spinal column, bladder, and skeletal system.

Pressure in the Body

Next to taking a person's temperature and weight, measuring blood pressure is the most common of all medical examinations. Control of high blood pressure is largely responsible for the significant decreases in heart attack and stroke fatalities achieved in the last three decades. The pressures in various parts of the body can be measured and often provide valuable medical indicators. In this section, we consider a few examples together with some of the physics that accompanies them.

[Table 1] lists some of the measured pressures in mm Hg, the units most commonly quoted.

Typical Pressures in Humans

Body system	Gauge pressure in mm Hg
Blood pressures in large arteries (resting)	
Maximum (systolic)	100–140
Minimum (diastolic)	60–90
Blood pressure in large veins	4–15
Eye	12–24
Brain and spinal fluid (lying down)	5–12
Bladder	
While filling	0–25
When full	100–150
Chest cavity between lungs and ribs	−8 to −4
Inside lungs	−2 to +3
Digestive tract	
Esophagus	−2
Stomach	0–20
Intestines	10–20
Middle ear	<1

Blood Pressure

Common arterial blood pressure measurements typically produce values of 120 mm Hg and 80 mm Hg, respectively, for systolic and diastolic pressures. Both pressures have health implications. When systolic pressure is chronically high, the risk of stroke and heart attack is increased. If, however, it is too low, fainting is a problem. **Systolic pressure** increases dramatically during exercise to increase blood flow and returns to normal afterward. This change produces no ill effects and, in fact, may be beneficial to the tone of the circulatory system. **Diastolic pressure** can be an indicator of fluid balance. When low, it may indicate that a person is hemorrhaging internally and needs a transfusion. Conversely, high diastolic pressure indicates a ballooning of the blood vessels, which may be due to the transfusion of too much fluid into the circulatory system. High diastolic pressure is also an indication that blood vessels are not dilating properly to pass blood through. This can seriously strain the heart in its attempt to pump blood.

Blood leaves the heart at about 120 mm Hg but its pressure continues to decrease (to almost 0) as it goes from the aorta to smaller arteries to small veins (see [Figure 1]). The pressure differences in the circulation system are caused by blood flow through the system as well as the position of the person. For a person standing up, the pressure in the feet will be larger than at the heart due to the weight of the blood $(P=h\rho g)$. If we assume that the distance between the heart and the feet of a person in an upright position is 1.4 m, then the increase in pressure in the feet relative to that in the heart (for a static column of blood) is given by

$$\Delta P=\Delta h\rho g=(1.4 m)(1050 \text{kg/m}^3)(9.80 m/\text{s}^2)=1.4\times10^4 \text{Pa}=108 \text{mm Hg}.$$

Increase in Pressure in the Feet of a Person

$$\Delta P=\Delta h\rho g=(1.4 m)(1050 \text{kg/m}^3)(9.80 m/\text{s}^2)=1.4\times10^4 \text{Pa}=108 \text{mm Hg}.$$

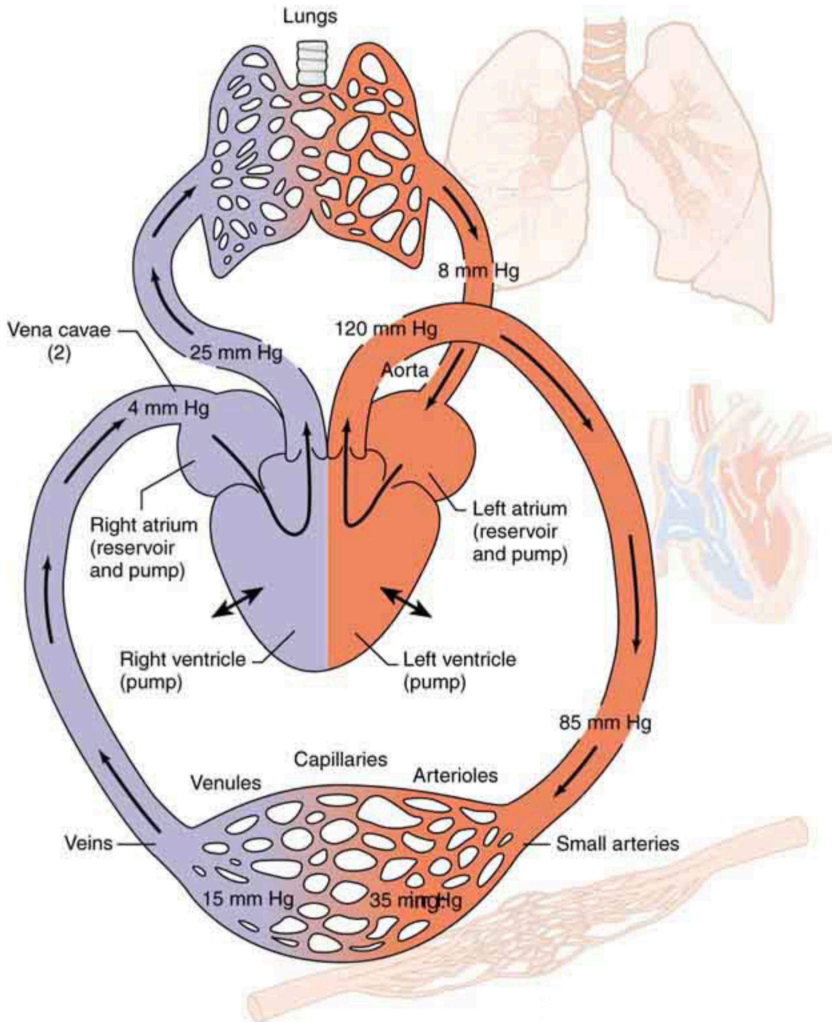
Standing a long time can lead to an accumulation of blood in the legs and swelling. This is the reason why soldiers who are required to stand still for long periods of time have been known to faint. Elastic bandages around the calf can help prevent this accumulation and can also help provide increased pressure to enable the veins to send blood back up to the heart. For similar reasons, doctors recommend tight stockings for long-haul flights.

Blood pressure may also be measured in the major veins, the heart chambers, arteries to the brain, and the lungs. But these pressures are usually only monitored during surgery or for patients in intensive care since the measurements are invasive. To obtain these pressure measurements, qualified health care workers thread thin tubes, called catheters, into appropriate locations to transmit pressures to external measuring devices.

The heart consists of two pumps—the right side forcing blood through the lungs and the left causing blood to flow through the rest of the body ([Figure 1]). Right-heart failure, for example, results in a rise in the pressure in the vena cavae and a drop in pressure in the arteries to the lungs. Left-heart failure results in a rise in the pressure entering the left side of the heart and a drop in aortal pressure. Implications of these and other pressures on flow in the circulatory system will be discussed in more detail in Fluid Dynamics and Its Biological and Medical Applications .

Two Pumps of the Heart

The heart consists of two pumps—the right side forcing blood through the lungs and the left causing blood to flow through the rest of the body.



Schematic of the circulatory system showing typical pressures. The two pumps in the heart increase pressure and that pressure is reduced as the blood flows through the body. Long-term deviations from these pressures have medical implications discussed in some detail in the Fluid Dynamics and Its Biological and Medical Applications. Only aortal or arterial blood pressure can be measured noninvasively.

Pressure in the Eye

The shape of the eye is maintained by fluid pressure, called **intraocular pressure**, which is normally in the range of 12.0 to 24.0 mm Hg. When the circulation of fluid in the eye is blocked, it can lead to a buildup in pressure, a condition called **glaucoma**. The net pressure can become as great as 85.0 mm Hg, an abnormally large pressure that can permanently damage the optic nerve. To get an idea of the force involved, suppose the back of the eye has an area of 6.0cm^2 , and the net pressure is 85.0 mm Hg. Force is given by $F = PA$. To get F in newtons, we convert the area to m^2 ($1 \text{m}^2 = 10^4 \text{cm}^2$). Then we calculate as follows:

$$F = h\rho g A = (85.0 \times 10^{-3} \text{m})(13.6 \times 10^3 \text{kg/m}^3)(9.80 \text{m/s}^2)(6.0 \times 10^{-4} \text{m}^2) = 6.8 N.$$

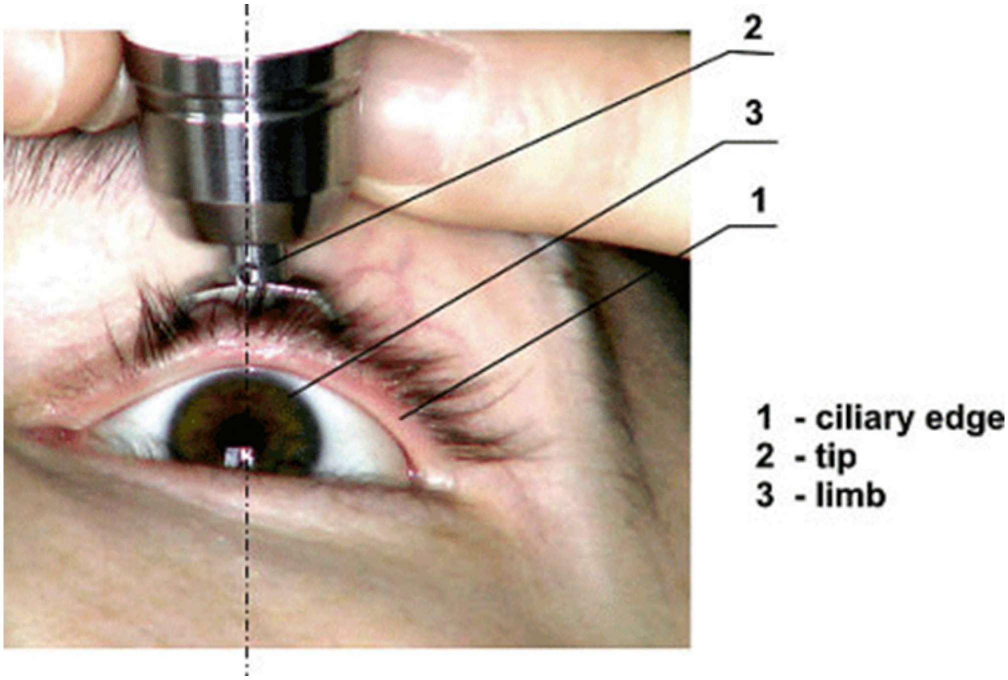
Eye Pressure

The shape of the eye is maintained by fluid pressure, called intraocular pressure. When the circulation of fluid in the eye is blocked, it can lead to a buildup in pressure, a condition called glaucoma. The force is calculated as

$$F = h\rho g A = (85.0 \times 10^{-3} \text{m})(13.6 \times 10^3 \text{kg/m}^3)(9.80 \text{m/s}^2)(6.0 \times 10^{-4} \text{m}^2) = 6.8 N.$$

This force is the weight of about a 680-g mass. A mass of 680 g resting on the eye (imagine 1.5 lb resting on your eye) would be sufficient to cause it damage. (A normal force here would be the weight of about 120 g, less than one-quarter of our initial value.)

People over 40 years of age are at greatest risk of developing glaucoma and should have their intraocular pressure tested routinely. Most measurements involve exerting a force on the (anesthetized) eye over some area (a pressure) and observing the eye’s response. A noncontact approach uses a puff of air and a measurement is made of the force needed to indent the eye ([Figure 2]). If the intraocular pressure is high, the eye will deform less and rebound more vigorously than normal. Excessive intraocular pressures can be detected reliably and sometimes controlled effectively.



The intraocular eye pressure can be read with a tonometer. (credit: DevelopAll at the Wikipedia Project.)

Calculating Gauge Pressure and Depth: Damage to the Eardrum

Suppose a 3.00-N force can rupture an eardrum. (a) If the eardrum has an area of 1.00cm^2 , calculate the maximum tolerable gauge pressure on the eardrum in newtons per meter squared and convert it to millimeters of mercury. (b) At what depth in freshwater would this person's eardrum rupture, assuming the gauge pressure in the middle ear is zero?

Strategy for (a)

The pressure can be found directly from its definition since we know the force and area. We are looking for the gauge pressure.

Solution for (a)

$$P_\text{g}=F/A=3.00\text{N}/(1.00\times10^{-4}\text{m}^2)=3.00\times10^4\text{N/m}^2.$$

We now need to convert this to units of mm Hg:

$$P_\text{g}=3.0\times10^4\text{N/m}^2(1.0 mmHg 133\text{N/m}^2)=226\text{mm Hg}.$$

Strategy for (b)

Here we will use the fact that the water pressure varies linearly with depth h

below the surface.

Solution for (b)

$P=h\rho g$ and therefore $h=P/\rho g$. Using the value above for P , we have

$$h=3.0\times10^4\text{N/m}^2(1.00\times10^3\text{kg/m}^3)(9.80 m/\text{s}^2)=3.06 m.$$

Discussion

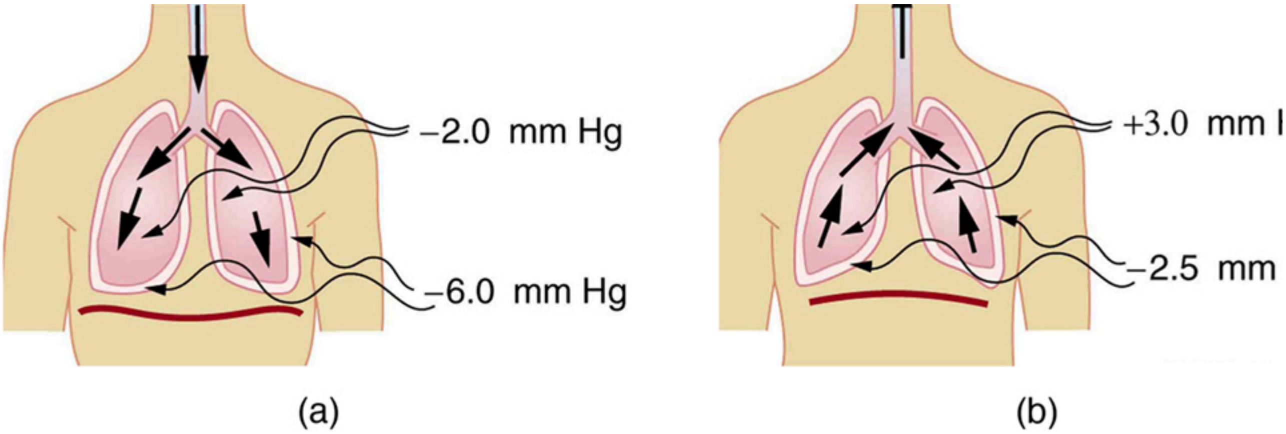
Similarly, increased pressure exerted upon the eardrum from the middle ear can arise when an infection causes a fluid buildup.

Pressure Associated with the Lungs

The pressure inside the lungs increases and decreases with each breath. The pressure drops to below atmospheric pressure (negative gauge pressure) when you inhale, causing air to flow into the lungs. It increases above atmospheric pressure (positive gauge pressure) when you exhale, forcing air out.

Lung pressure is controlled by several mechanisms. Muscle action in the diaphragm and rib cage is necessary for inhalation; this muscle action increases the volume of the lungs thereby reducing the pressure within them [Figure 3]. Surface tension in the alveoli creates a positive pressure opposing inhalation. (See Cohesion and Adhesion in Liquids: Surface Tension and Capillary Action .) You can exhale without muscle action by letting surface tension in the alveoli create its own positive pressure. Muscle action can add to this positive pressure to produce forced exhalation, such as when you blow up a balloon, blow out a candle, or cough.

The lungs, in fact, would collapse due to the surface tension in the alveoli, if they were not attached to the inside of the chest wall by liquid adhesion. The gauge pressure in the liquid attaching the lungs to the inside of the chest wall is thus negative, ranging from -4 to -8mm Hg during exhalation and inhalation, respectively. If air is allowed to enter the chest cavity, it breaks the attachment, and one or both lungs may collapse. Suction is applied to the chest cavity of surgery patients and trauma victims to reestablish negative pressure and inflate the lungs.



(a) During inhalation, muscles expand the chest, and the diaphragm moves downward, reducing pressure inside the lungs to less than atmospheric (negative gauge pressure). Pressure between the lungs and chest wall is even lower to overcome the positive pressure created by surface tension in the lungs. (b) During gentle exhalation, the muscles simply relax and surface tension in the alveoli creates a positive pressure inside the lungs, forcing air out. Pressure between the chest wall and lungs remains negative to keep them attached to the chest wall, but it is less negative than during inhalation.

Other Pressures in the Body

Spinal Column and Skull

Normally, there is a 5- to12-mm Hg pressure in the fluid surrounding the brain and filling the spinal column. This cerebrospinal fluid serves many purposes, one of which is to supply flotation to the brain. The buoyant force supplied by the fluid nearly equals the weight of the brain, since their densities are nearly equal. If there is a loss of fluid, the brain rests on the inside of the skull, causing severe headaches, constricted blood flow, and serious damage. Spinal fluid pressure is measured by means of a needle inserted between vertebrae that transmits the pressure to a suitable measuring device.

Bladder Pressure

This bodily pressure is one of which we are often aware. In fact, there is a relationship between our awareness of this pressure and a subsequent increase in it. Bladder pressure climbs steadily from zero to about 25 mm Hg as the bladder fills to its normal capacity of 500cm^3 . This pressure triggers the **micturition reflex**, which stimulates the feeling of needing to urinate. What is more, it also causes muscles around the bladder to contract, raising the pressure to over 100 mm Hg, accentuating the sensation. Coughing, straining, tensing in cold weather, wearing tight clothes, and experiencing simple nervous tension all can increase bladder pressure and trigger this reflex. So can the weight of a pregnant woman's fetus, especially if it is kicking vigorously or pushing down with its head! Bladder pressure can be measured by a catheter or by inserting a needle through the bladder wall and transmitting the pressure to an appropriate measuring device. One hazard of high bladder pressure (sometimes created by an obstruction), is that such pressure can force urine back into the kidneys, causing potentially severe damage.

Pressures in the Skeletal System

These pressures are the largest in the body, due both to the high values of initial force, and the small areas to which this force is applied, such as in the joints. For example, when a person lifts an object improperly, a force of 5000 N may be created between vertebrae in the spine, and this may be applied to an area as small as 10cm^2 . The pressure created is $P = F/A = (5000 \text{N})/(10^{-3} \text{m}^2) = 5.0 \times 10^6 \text{N/m}^2$ or about 50 atm! This pressure can damage both the spinal discs (the cartilage between vertebrae), as well as the bony vertebrae themselves. Even under normal circumstances, forces between vertebrae in the spine are large enough to create pressures of several atmospheres. Most causes of excessive pressure in the skeletal system can be avoided by lifting properly and avoiding extreme physical activity. (See Forces and Torques in Muscles and Joints.)

There are many other interesting and medically significant pressures in the body. For example, pressure caused by various muscle actions drives food and waste through the digestive system. Stomach pressure behaves much like bladder pressure and is tied to the sensation of hunger. Pressure in the relaxed esophagus is normally negative because pressure in the chest cavity is normally negative. Positive pressure in the stomach may thus force acid into the esophagus, causing "heartburn." Pressure in the middle ear can result in significant force on the eardrum if it differs greatly from atmospheric pressure, such as while scuba diving. The decrease in external pressure is also noticeable during plane flights (due to a decrease in the weight of air above relative to that at the Earth's surface). The Eustachian tubes connect the middle ear to the throat and allow us to equalize pressure in the middle ear to avoid an imbalance of force on the eardrum.

Many pressures in the human body are associated with the flow of fluids. Fluid flow will be discussed in detail in the Fluid Dynamics and Its Biological and Medical Applications .

Section Summary

- Measuring blood pressure is among the most common of all medical examinations.
- The pressures in various parts of the body can be measured and often provide valuable medical indicators.
- The shape of the eye is maintained by fluid pressure, called intraocular pressure.
- When the circulation of fluid in the eye is blocked, it can lead to a buildup in pressure, a condition called glaucoma.
- Some of the other pressures in the body are spinal and skull pressures, bladder pressure, pressures in the skeletal system.

Problems & Exercises

During forced exhalation, such as when blowing up a balloon, the diaphragm and chest muscles create a pressure of 60.0 mm Hg between the lungs and chest wall. What force in newtons does this pressure create on the 600cm^2 surface area of the diaphragm?

Show Solution

Strategy: Convert the pressure from mm Hg to Pa using $P = \rho g h$, then use $F = PA$ to find the force.

Solution:

Step 1: Convert pressure to pascals

$$P = h\rho_{\text{Hg}}g = (60.0\times10^{-3}\text{ m})(13.6\times10^{3}\text{ kg/m}^3)(9.80\text{ m/s}^2)$$
$$P = 7.99\times10^{3}\text{ Pa}$$

Step 2: Convert area to m²

$$A = 600\text{ cm}^2 = 600\times10^{-4}\text{ m}^2 = 0.0600\text{ m}^2$$

Step 3: Calculate force

$$F = PA = (7.99\times10^{3}\text{ Pa})(0.0600\text{ m}^2) = 479\text{ N}$$

Discussion: The force on the diaphragm is approximately **479 N** (about 108 pounds). This substantial force is what allows forceful exhalation for activities like blowing up balloons, coughing, or playing wind instruments. The diaphragm is a strong, dome-shaped muscle that separates the chest from the abdomen, and during forced exhalation, it contracts upward while chest muscles squeeze inward, compressing the lungs. This 60 mm Hg pressure (about 0.08 atm gauge) is modest compared to maximum efforts, where pressures can exceed 100 mm Hg. The large diaphragm area means even moderate pressures create significant forces.

You can chew through very tough objects with your incisors because they exert a large force on the small area of a pointed tooth. What pressure in pascals can you create by exerting a force of 500N with your tooth on an area of 1.00mm^2 ?

Show Solution

Strategy

We use the definition of pressure, $P = FA$, with careful attention to unit conversion.

Solution

First, convert the area to square meters:

$$A = 1.00\text{ mm}^2 = 1.00\times10^{-6}\text{ m}^2$$

Now calculate the pressure:

$$P = FA = 500\text{ N}1.00\times10^{-6}\text{ m}^2 = 5.00\times10^{8}\text{ Pa}$$

The pressure created by the incisor is **5.00 × 10⁸ Pa** (or 500 MPa, approximately 5000 atm).

Discussion

This pressure is extremely high—about 5000 times atmospheric pressure! This explains why teeth can chew through tough materials like nuts, bones, and hard candy. The key is the small contact area at the tip of the tooth. Human bite force typically ranges from 300-700 N, and the pointed nature of incisors concentrates this force to create very high pressures. This is the same principle used by knives, needles, and other cutting/piercing tools.

One way to force air into an unconscious person's lungs is to squeeze on a balloon appropriately connected to the subject. What force must you exert on the balloon with your hands to create a gauge pressure of 4.00 cm water, assuming you squeeze on an effective area of 50.0cm^2 ?

Show Solution

Strategy: Convert the pressure from cm water to Pa, then use $F = PA$ to find the required force.

Solution:

Step 1: Convert pressure to pascals

$$P = h\rho_{\text{water}} g = (0.0400\ \text{m})(1000\ \text{kg/m}^3)(9.80\ \text{m/s}^2)$$
$$P = 392\ \text{Pa}$$

Step 2: Convert area to m²

$$A = 50.0\ \text{cm}^2 = 50.0 \times 10^{-4}\ \text{m}^2 = 5.00 \times 10^{-3}\ \text{m}^2$$

Step 3: Calculate force

$$F = PA = (392\ \text{Pa})(5.00 \times 10^{-3}\ \text{m}^2) = 1.96\ \text{N}$$

Discussion: The required force is approximately **1.96 N** (about 0.44 pounds or 7 ounces). This is a modest force—easily applied by hand squeezing. The 4 cm water pressure (about 392 Pa or 0.004 atm) is sufficient to inflate the lungs without causing damage. Manual resuscitation devices (bag valve masks) work on this principle, allowing rescuers to provide ventilation by rhythmic squeezing. The relatively small force needed makes this technique accessible even during prolonged rescue efforts. Modern devices include pressure relief valves to prevent excessive pressures that could damage the lungs.

Heroes in movies hide beneath water and breathe through a hollow reed (villains never catch on to this trick). In practice, you cannot inhale in this manner if your lungs are more than 60.0 cm below the surface. What is the maximum negative gauge pressure you can create in your lungs on dry land, assuming you can achieve -3.00cm water pressure with your lungs 60.0 cm below the surface?

Show Solution

Strategy: When submerged, your lungs must overcome both the water pressure at depth and create the negative pressure for inhalation. Find the total negative pressure capability.

Solution:

When your lungs are 60.0 cm below the surface:

- Water pressure at lung depth: $P_{\text{water}} = \rho g h = (1000\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(0.600\ \text{m})$

Converting to cm water: $P_{\text{water}} = 60.0\ \text{cm H}_2\text{O}$

If you can achieve −3.00 cm water pressure at the lung surface while submerged at 60.0 cm depth, you must create a pressure difference of:

$$P_{\text{lung, abs}} = P_{\text{atm}} + 60.0\ \text{cm H}_2\text{O} - 3.00\ \text{cm H}_2\text{O}$$

To inhale, you must reduce lung pressure from atmospheric to this value. The total negative gauge pressure you can create is:

$$P_{\text{max negative}} = -(60.0 + 3.00)\ \text{cm H}_2\text{O} = -63.0\ \text{cm H}_2\text{O}$$

Discussion: The maximum negative gauge pressure you can create in your lungs on dry land is approximately **−63.0 cm H_2O** (about −0.062 atm or −6.2 kPa). This represents the total pressure difference your respiratory muscles can generate. When submerged 60 cm deep, you need 60 cm H_2O just to counteract water pressure on your chest, leaving only 3 cm H_2O for actual air flow. Beyond about 60-70 cm depth, most people cannot generate sufficient negative pressure to inhale through a snorkel, which is why snorkels are limited to ~30-40 cm length (half the theoretical maximum, with safety margin). This limitation explains why scuba tanks are necessary for deeper diving—they provide pressurized air matching ambient water pressure.

Gauge pressure in the fluid surrounding an infant's brain may rise as high as 85.0 mm Hg (5 to 12 mm Hg is normal), creating an outward force large enough to make the skull grow abnormally large. (a) Calculate this outward force in newtons on each side of an infant's skull if the effective area of each side is 70.0cm^2 . (b) What is the net force acting on the skull?

Show Solution

Strategy

We use $F = PA$ to find the force on each side of the skull. For gauge pressure in mm Hg, we convert using the density of mercury. The net force depends on whether opposing forces cancel.

Solution

(a) First, convert the pressure from mm Hg to Pascals using $P = h\rho g$:

$$P = (85.0 \times 10^{-3}\ \text{m})(13.6 \times 10^3\ \text{kg/m}^3)(9.80\ \text{m/s}^2) = 1.13 \times 10^4\ \text{Pa}$$

Convert the area: $A = 70.0\ \text{cm}^2 = 70.0 \times 10^{-4}\ \text{m}^2$

Calculate the force on each side:

$$F = PA = (1.13\times10^{4}\ \text{Pa})(70.0\times10^{-4}\ \text{m}^2) = 79.4\ \text{N}$$

(b) The net force on the skull is **zero**. The pressure acts uniformly in all directions (Pascal's principle), so the outward force on one side of the skull is exactly balanced by the equal outward force on the opposite side.

The outward force on each side of the infant's skull is approximately **79 N** (about 18 pounds), and the net force is **0 N**.

Discussion

Although the net force is zero, the 79 N force on each side is significant—equivalent to about 8 kg (18 lb) pushing outward on each side of the skull. This is why elevated intracranial pressure can cause the skull bones of an infant (which haven't yet fused) to separate and the head to enlarge abnormally, a condition called hydrocephalus. The individual forces stretch the skull even though they don't accelerate it as a whole.

A full-term fetus typically has a mass of 3.50 kg. (a) What pressure does the weight of such a fetus create if it rests on the mother's bladder, supported on an area of 90.0cm^2 ? (b) Convert this pressure to millimeters of mercury and determine if it alone is great enough to trigger the micturition reflex (it will add to any pressure already existing in the bladder).

Show Solution

Strategy: (a) Calculate the weight of the fetus, then use $P = F/A$ to find pressure. (b) Convert the pressure from Pa to mm Hg and compare to the threshold for the micturition reflex (about 25 mm Hg).

Solution:

(a) Pressure in pascals:

Calculate the weight of the fetus:

$$F = mg = (3.50\ \text{kg})(9.80\ \text{m/s}^2) = 34.3\ \text{N}$$

Convert area to m²:

$$A = 90.0\ \text{cm}^2 = 90.0\times10^{-4}\ \text{m}^2 = 9.00\times10^{-3}\ \text{m}^2$$

Calculate pressure:

$$P = \frac{F}{A} = \frac{34.3\ \text{N}}{9.00\times10^{-3}\ \text{m}^2} = 3.81\times10^{3}\ \text{Pa}$$

(b) Convert to mm Hg:

Using $1\ \text{mm Hg} = 133\ \text{Pa}$:

$$P = \frac{3.81\times10^{3}\ \text{Pa}}{133\ \text{Pa/mm Hg}} = 28.7\ \text{mm Hg}$$

The pressure is **3.81 × 10³ Pa** or **28.7 mm Hg**, which is sufficient to trigger the micturition reflex (threshold is about 25 mm Hg).

Discussion: The fetus creates a pressure of approximately **28.7 mm Hg** on the mother's bladder, exceeding the typical 25 mm Hg threshold that triggers the urge to urinate. This explains the frequent need to urinate during late pregnancy. The pressure is even greater when the fetus kicks or moves, as mentioned in the text, potentially raising bladder pressure above 100 mm Hg. This is one of the many physiological changes expectant mothers experience as the growing fetus compresses various organs in the abdominal cavity.

If the pressure in the esophagus is -2.00mm Hg while that in the stomach is $+20.0\text{mm Hg}$, to what height could stomach fluid rise in the esophagus, assuming a density of 1.10 g/mL? (This movement will not occur if the muscle closing the lower end of the esophagus is working properly.)

Show Solution

Strategy

The pressure difference between the stomach and esophagus will push fluid up until the hydrostatic pressure of the fluid column equals the pressure difference. We use $\Delta P = \rho g h$ and solve for height.

Solution

The pressure difference is:

$$\Delta P = P_{\text{stomach}} - P_{\text{esophagus}} = (+20.0\ \text{mm Hg}) - (-2.00\ \text{mm Hg}) = 22.0\ \text{mm Hg}$$

Convert to Pascals:

$$\Delta P = (22.0\times10^{-3}\ \text{m})(13.6\times10^{3}\ \text{kg/m}^3)(9.80\ \text{m/s}^2) = 2.93\times10^{3}\ \text{Pa}$$

The density of stomach fluid is $\rho = 1.10\ \text{g/mL} = 1100\ \text{kg/m}^3$.

Solve for height using $h = \frac{\Delta P}{\rho g}$:

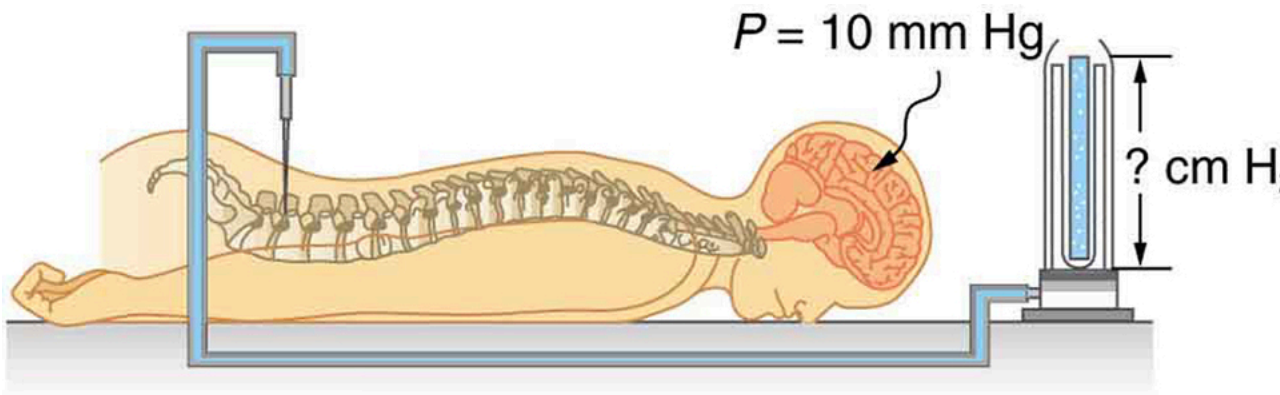
$$h = \frac{2.93 \times 10^3 \text{ Pa}}{(1100 \text{ kg/m}^3)(9.80 \text{ m/s}^2)} = 0.272 \text{ m} = 27.2 \text{ cm}$$

Stomach fluid could rise to a height of approximately **27 cm** in the esophagus.

Discussion

This is a significant height—about 10-11 inches—more than enough to reach the throat and cause severe "heartburn" or acid reflux. The lower esophageal sphincter muscle normally prevents this by maintaining a barrier between the stomach and esophagus. When this sphincter is weakened or relaxed inappropriately, gastroesophageal reflux disease (GERD) can result. The negative pressure in the esophagus (due to the negative pressure in the chest cavity) actually contributes to pulling stomach acid upward when the sphincter fails.

Pressure in the spinal fluid is measured as shown in [Figure 4]. If the pressure in the spinal fluid is 10.0 mm Hg: (a) What is the reading of the water manometer in cm water? (b) What is the reading if the person sits up, placing the top of the fluid 60 cm above the tap? The fluid density is 1.05 g/mL.



A water manometer used to measure pressure in the spinal fluid. The height of the fluid in the manometer is measured relative to the spinal column, and the manometer is open to the atmosphere. The measured pressure will be considerably greater if the person sits up.

Show Solution

Strategy: (a) Convert the pressure from mm Hg to cm H_2O using the density ratio between mercury and water. (b) When the person sits up, add the hydrostatic pressure of the 60-cm column of spinal fluid to the baseline pressure.

Solution:

(a) Manometer reading when lying down:

The pressure relationship is $P = \rho g h$. For the same pressure in different fluids:

$$P_{Hg} = P_{H_2O} \quad \rho_{Hg} g h_{Hg} = \rho_{H_2O} g h_{H_2O}$$

Solving for water height:

$$h_{H_2O} = h_{Hg} \frac{\rho_{Hg}}{\rho_{H_2O}} = (10.0 \text{ mm}) \frac{13.6 \text{ g/cm}^3}{1.00 \text{ g/cm}^3} = 136 \text{ mm} = 13.6 \text{ cm } H_2O$$

(b) Manometer reading when sitting up:

When the person sits up, the 60-cm column of spinal fluid adds hydrostatic pressure. Using spinal fluid density $\rho_{CSF} = 1.05 \text{ g/mL} = 1050 \text{ kg/m}^3$:

$$\Delta P = \rho_{CSF} g h = (1050 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(0.60 \text{ m}) = 6174 \text{ Pa}$$

Convert to cm H_2O:

$$h_{additional} = \frac{\Delta P}{\rho_{H_2O} g} = \frac{6174 \text{ Pa}}{(1000 \text{ kg/m}^3)(9.80 \text{ m/s}^2)} = 0.630 \text{ m} = 63.0 \text{ cm } H_2O$$

Total reading:

$$h_{total} = 13.6 + 63.0 = 76.6 \text{ cm } H_2O \approx 76.5 \text{ cm } H_2O$$

The readings are: (a) **13.6 cm H_2O** when lying down, and (b) **76.5 cm H_2O** when sitting up.

Discussion: The dramatic increase from 13.6 cm to 76.5 cm when sitting up (more than 5-fold) demonstrates how body position affects pressure measurements in the spinal fluid. This is why spinal pressure measurements must be performed with the patient in a standardized position (usually lying on their side). The increased pressure when upright reflects the weight of the fluid column between the brain and the measurement point in the lower back.

This principle is important in diagnosing conditions like hydrocephalus (excess cerebrospinal fluid) or CSF leaks. The slight density difference between spinal fluid (1.05 g/mL) and water (1.00 g/mL) accounts for the 63 cm contribution rather than exactly 60 cm.

Calculate the maximum force in newtons exerted by the blood on an aneurysm, or ballooning, in a major artery, given the maximum blood pressure for this person is 150 mm Hg and the effective area of the aneurysm is 20.0cm^2. Note that this force is great enough to cause further enlargement and subsequently greater force on the ever-thinner vessel wall.

Show Solution

Strategy

We use $F = PA$ after converting the blood pressure from mm Hg to Pascals and the area to square meters.

Solution

Convert the pressure from mm Hg to Pascals:

$$P=(150\times10^{-3}\text{ m})(13.6\times10^{3}\text{ kg/m}^3)(9.80\text{ m/s}^2)=2.00\times10^{4}\text{ Pa}$$

Convert the area: $A = 20.0\text{ cm}^2 = 20.0\times10^{-4}\text{ m}^2$

Calculate the force:

$$F=PA=(2.00\times10^{4}\text{ Pa})(20.0\times10^{-4}\text{ m}^2)=40.0\text{ N}$$

The maximum force exerted by blood on the aneurysm is **40.0 N** (approximately 9 pounds).

Discussion

This is a substantial force—equivalent to about 4 kg (9 lb) pushing outward on the weakened vessel wall. Aneurysms are particularly dangerous because of a positive feedback loop: as the vessel wall stretches and thins, the area increases, causing greater force (F = PA), which causes further stretching. The wall also becomes weaker as it thins, following Laplace's law. This explains why aneurysms can grow progressively and eventually rupture catastrophically. The 150 mm Hg systolic pressure used here is elevated (normal is ~120 mm Hg), illustrating why high blood pressure significantly increases the risk of aneurysm rupture.

During heavy lifting, a disk between spinal vertebrae is subjected to a 5000-N compressional force. (a) What pressure is created, assuming that the disk has a uniform circular cross section 2.00 cm in radius? (b) What deformation is produced if the disk is 0.800 cm thick and has a Young's modulus of $1.5\times10^{9}\text{N/m}^2$?

Show Solution

Strategy: (a) Use $P = F/A$ with the disk's cross-sectional area. (b) Use the relationship between stress, strain, and Young's modulus: $\frac{\Delta L}{L_0} = \frac{F}{AY} = \frac{P}{Y}$.

Solution:

(a) Pressure in the disk:

Calculate the disk's cross-sectional area:

$$A=\pi r^2=\pi(0.0200\text{ m})^2=1.257\times10^{-3}\text{ m}^2$$

Calculate pressure:

$$P=\frac{F}{A}=\frac{5000\text{ N}}{1.257\times10^{-3}\text{ m}^2}=3.98\times10^{6}\text{ Pa}$$

(b) Deformation of the disk:

Using the relationship for compressive deformation:

$$\Delta L=L_0\frac{P}{Y}=(0.00800\text{ m})\frac{3.98\times10^{6}\text{ Pa}}{1.5\times10^{9}\text{ Pa}}$$
$$\Delta L=(0.00800\text{ m})(2.65\times10^{-3})=2.12\times10^{-5}\text{ m}=2.1\times10^{-3}\text{ cm}$$

The pressure created is **3.98×10^{6} Pa** (about 39 atmospheres), and the deformation is **2.1×10^{-3} cm** (0.021 mm).

Discussion: The pressure of approximately **4.0 MPa** (40 times atmospheric pressure) shows the significant stress placed on spinal disks during heavy lifting. The compression of **0.021 mm** may seem small, but it represents about 0.26% of the disk's 8-mm thickness. Repeated compressions of this magnitude contribute to disk wear and can lead to herniated disks or degenerative disk disease over time.

This calculation assumes the disk compresses uniformly, but in reality, the soft gel-like nucleus pulposus in the center can bulge outward when compressed, potentially pressing on spinal nerves and causing pain. This is why proper lifting technique (keeping the back straight and lifting with the legs) is crucial—it helps distribute forces more evenly across multiple disks rather than concentrating stress on one or two. The relatively low Young's modulus (1.5 GPa, compared to ~200 GPa for steel) reflects the disk's cartilaginous composition, which allows it to act as a shock absorber for the spine.

When a person sits erect, increasing the vertical position of their brain by 36.0 cm, the heart must continue to pump blood to the brain at the same rate. (a) What is the gain in gravitational potential energy for 100 mL of blood raised 36.0 cm? (b) What is the drop in pressure, neglecting any losses due to friction? (c) Discuss how the gain in gravitational potential energy and the decrease in pressure are related.

Show Solution

Strategy

(a) We calculate gravitational potential energy using $\Delta PE = mgh$, where mass comes from volume and density. (b) The pressure drop due to height change is given by $\Delta P = \rho gh$. (c) We connect these through work-energy considerations.

Solution

(a) First find the mass of 100 mL of blood (using blood density ≈ 1050 kg/m³):

$$m=\rho V=(1050\text{ kg/m}^3)(100\times10^{-6}\text{ m}^3)=0.105\text{ kg}$$

Calculate the potential energy gain:

$$\Delta PE=mgh=(0.105\text{ kg})(9.80\text{ m/s}^2)(0.360\text{ m})=0.370\text{ J}$$

(b) The pressure drop due to height increase:

$$\Delta P=\rho gh=(1050\text{ kg/m}^3)(9.80\text{ m/s}^2)(0.360\text{ m})=3.70\times10^3\text{ Pa}$$

Converting to mm Hg: $\Delta P = \frac{3700\text{ Pa}}{133\text{ Pa/mm Hg}} = 27.8\text{ mm Hg}$

The gravitational potential energy gain is **0.370 J**, and the pressure drop is **3.70 × 10³ Pa** (or about 28 mm Hg).

(c) **Discussion**: The gain in gravitational potential energy and the decrease in pressure are directly related through energy conservation. As blood rises to the brain, its gravitational potential energy increases while its pressure energy (per unit volume) decreases by the same amount. The work done against gravity equals $\Delta PE = mgh = (\rho V)gh$, while the pressure energy change equals $\Delta P \cdot V = \rho gh \cdot V$. These are identical, demonstrating that the pressure drop provides the energy needed to lift the blood. This is why blood pressure is lower in the head than at heart level when standing, and why the heart must work harder to maintain adequate brain perfusion when upright. This also explains why standing suddenly can cause lightheadedness—the body needs time to adjust blood pressure to compensate for gravitational effects.

(a) How high will water rise in a glass capillary tube with a 0.500-mm radius? (b) How much gravitational potential energy does the water gain? (c) Discuss possible sources of this energy.

Show Solution

Strategy: (a) Use the capillary rise formula $h = \frac{2\gamma\cos\theta}{\rho gr}$ with water's surface tension and contact angle with glass. (b) Calculate the mass of the water column and use $\Delta PE = mgh/2$ (center of mass rises by h/2). (c) Relate the energy to work done by surface tension forces.

Solution:

(a) Height of capillary rise:

For water in glass at 20°C:

- Surface tension: $\gamma = 0.0728\text{ N/m}$
- Contact angle: $\theta = 0\,°$ (so $\cos\theta = 1$)
- Density: $\rho = 1000\text{ kg/m}^3$
- Radius: $r = 0.500\text{ mm} = 5.00\times10^{-4}\text{ m}$

$$h=\frac{2\gamma\cos\theta}{\rho gr}=\frac{2(0.0728\text{ N/m})(1)}{(1000\text{ kg/m}^3)(9.80\text{ m/s}^2)(5.00\times10^{-4}\text{ m})}=\frac{0.1456}{4.90}=0.0297\text{ m}=2.97\text{ cm}$$

(b) Gravitational potential energy gained:

Volume of water column:

$$V=\pi r^2h=\pi(5.00\times10^{-4}\text{ m})^2(0.0297\text{ m})=2.33\times10^{-8}\text{ m}^3$$

Mass of water:

$$m=\rho V=(1000\text{ kg/m}^3)(2.33\times10^{-8}\text{ m}^3)=2.33\times10^{-5}\text{ kg}$$

The center of mass rises by $h/2$, so:

$\Delta PE = mg(h/2) = (2.33\times10^{-5}\text{ kg})(9.80\text{ m/s}^2)(0.0297\text{ m}/2) = 3.39\times10^{-6}\text{ J}$

(c) Source of the energy:

Work is done by the surface tension force through an effective distance $h/2$ to raise the column of water. The surface tension acts along the perimeter of the meniscus ($2\pi r$) with force $F = \gamma(2\pi r)$. As the water rises to height h, the point of application (the meniscus) moves upward by h, but the center of mass only rises by $h/2$. The work done equals:

$W = F\cdot d = [\gamma(2\pi r)]\cdot(h/2) = \gamma\pi r h$

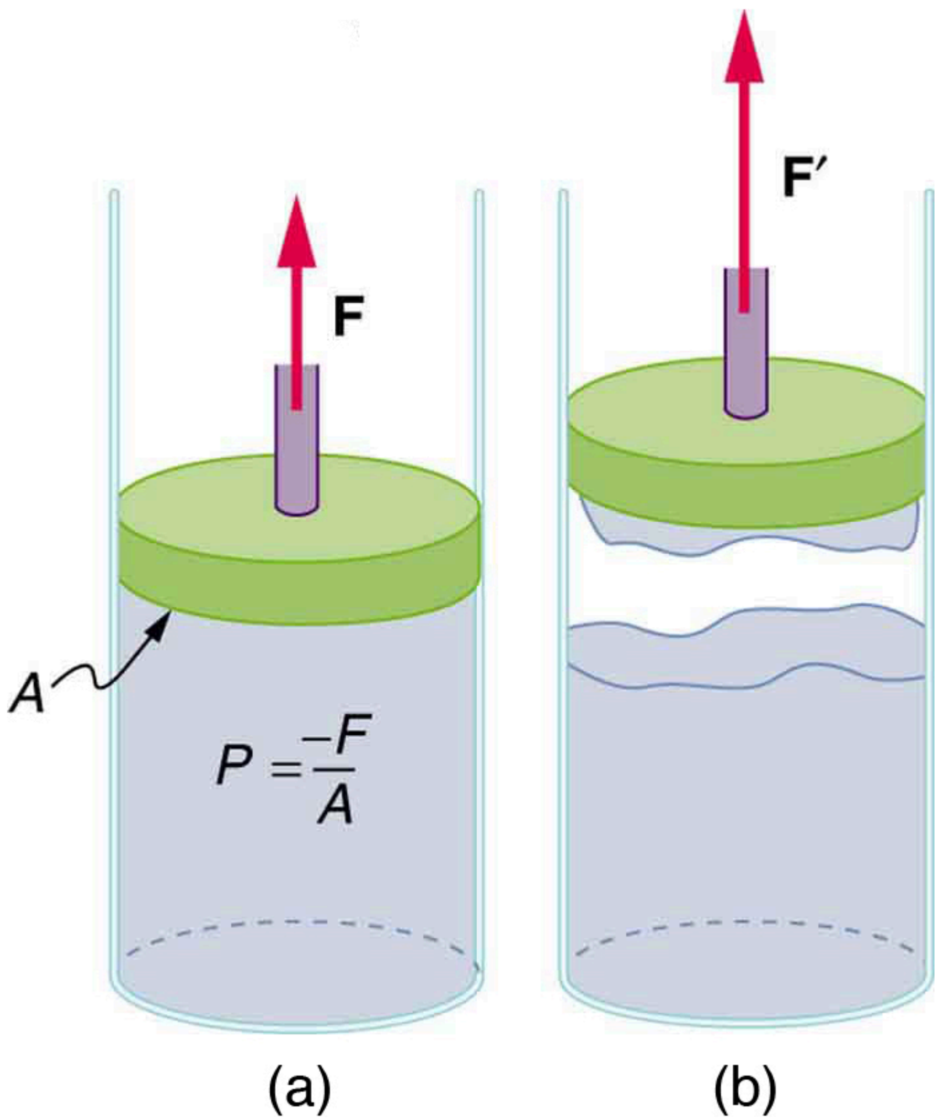
This can be verified:

$W = (0.0728\text{ N/m})(\pi)(5.00\times10^{-4}\text{ m})(0.0297\text{ m}) = 3.40\times10^{-6}\text{ J}$

This matches the potential energy gain (within rounding error), confirming energy conservation.

Discussion: Water rises to **2.97 cm** and gains **3.39 μJ** of potential energy. The energy comes from the work done by adhesive forces (water-glass attraction) as manifested through surface tension. As the water climbs the tube walls, the surface area of the water-glass interface increases while the water-air interface decreases. The favorable water-glass interaction (lower energy) compared to water-air interaction drives this process. Energy conservation requires that the work done by surface tension equals the gravitational potential energy gained, which our calculations confirm.

A negative pressure of 25.0 atm can sometimes be achieved with the device in [Figure 5] before the water separates. (a) To what height could such a negative gauge pressure raise water? (b) How much would a steel wire of the same diameter and length as this capillary stretch if suspended from above?



(a) When the piston is raised, it stretches the liquid slightly, putting it under tension and creating a negative absolute pressure \(P= -F/A \). (b) The liquid eventually separates, giving an experimental limit to negative pressure in this liquid.

Show Solution

Strategy

(a) We use $h = \frac{P}{\rho g}$ to find the height water could be raised by the negative pressure. (b) For the steel wire, we use the relationship between stress, strain, and Young's modulus: $\frac{\Delta L}{L} = \frac{\sigma}{Y} = \frac{P}{Y}$.

Solution

(a) Convert pressure to Pascals: $P = 25.0 \text{ atm} \times 1.013 \times 10^5 \text{ Pa/atm} = 2.53 \times 10^6 \text{ Pa}$

Calculate the height:

$$h = \frac{P}{\rho g} = \frac{2.53 \times 10^6 \text{ Pa}}{(1000 \text{ kg/m}^3)(9.80 \text{ m/s}^2)} = 258 \text{ m}$$

(b) For steel, Young's modulus is $Y = 2.10 \times 10^{11} \text{ N/m}^2$. The strain is:

$$\frac{\Delta L}{L} = \frac{P}{Y} = \frac{2.53 \times 10^6 \text{ Pa}}{2.10 \times 10^{11} \text{ Pa}} = 1.20 \times 10^{-5}$$

For a capillary tube length of, say, $L = 10 \text{ cm} = 0.10 \text{ m}$:

$$\Delta L = (1.20 \times 10^{-5})(0.10 \text{ m}) = 1.2 \times 10^{-6} \text{ m} = 1.2\ \mu\text{m}$$

A negative gauge pressure of 25.0 atm could raise water to a height of approximately **258 m**. A steel wire under the same tensile stress would stretch by a fractional amount of **1.2×10^{-5}** (or about 1.2 μm for every 10 cm of length).

Discussion

The 258 m height is remarkable—far exceeding the ~10.3 m limit imposed by atmospheric pressure pushing water up into a vacuum. This demonstrates that water under tension (negative pressure) can "pull" rather than just "push." This is directly relevant to how trees transport water to heights exceeding 100 m. The cohesion-tension theory proposes that evaporation from leaves creates negative pressure that pulls water up through the xylem, with cohesive forces between water molecules preventing the water column from breaking. The comparison with steel shows that water under such tension behaves somewhat like a solid, though its cohesive strength is much less than steel's tensile strength.

Suppose you hit a steel nail with a 0.500-kg hammer, initially moving at 15.0 m/s and brought to rest in 2.80 mm. (a) What average force is exerted on the nail? (b) How much is the nail compressed if it is 2.50 mm in diameter and 6.00-cm long? (c) What pressure is created on the 1.00-mm-diameter tip of the nail?

Show Solution

Strategy: (a) Use the work-energy theorem: the work done by the average force equals the change in kinetic energy. (b) Use the relationship for compression: $\Delta L = \frac{F L_0}{A Y}$ where Y is Young's modulus for steel. (c) Calculate pressure using the tip area: $P = F/A_{\text{tip}}$.

Solution:

(a) Average force on the nail:

Using work-energy theorem: $F_{\text{avg}} \cdot d = \Delta KE = \frac{1}{2} m v^2$

$$F_{\text{avg}} = \frac{m v^2}{2d} = \frac{(0.500 \text{ kg})(15.0 \text{ m/s})^2}{2(2.80 \times 10^{-3} \text{ m})} = \frac{56.25}{5.60 \times 10^{-3}} = 2.01 \times 10^4 \text{ N}$$

(b) Compression of the nail:

Nail cross-sectional area:

$$A = \pi r^2 = \pi (1.25 \times 10^{-3} \text{ m})^2 = 4.91 \times 10^{-6} \text{ m}^2$$

Using Young's modulus for steel $Y = 2.0 \times 10^{11} \text{ Pa}$:

$$\Delta L = \frac{F L_0}{A Y} = \frac{(2.01 \times 10^4 \text{ N})(0.0600 \text{ m})}{(4.91 \times 10^{-6} \text{ m}^2)(2.0 \times 10^{11} \text{ Pa})}$$
$$\Delta L = \frac{1.206 \times 10^3}{9.82 \times 10^5} = 1.23 \times 10^{-3} \text{ m} \approx 1.17 \times 10^{-3} \text{ m}$$

(c) Pressure on the tip:

Tip area:

$$A_{\text{tip}} = \pi r_{\text{tip}}^2 = \pi (0.500 \times 10^{-3} \text{ m})^2 = 7.85 \times 10^{-7} \text{ m}^2$$

Pressure:

$$P=\frac{F}{A_{tip}}=\frac{2.01\times10^{4}\ \text{N}}{7.85\times10^{-7}\ \text{m}^{2}}=2.56\times10^{10}\ \text{Pa}$$

The results are: (a) **2.01×10^4 N**, (b) **1.17×10^{-3} m** (1.17 mm), and (c) **2.56×10^{10} Pa**.

Discussion: The average force of **20,100 N** (about 4,500 pounds) is enormous—far exceeding the hammer's weight (4.9 N). This large force results from the rapid deceleration over a short distance (2.80 mm). The nail compresses by **1.17 mm**, about 2% of its 60-mm length, demonstrating steel's stiffness (high Young's modulus).

The pressure at the tip is **25.6 GPa** (about 250,000 atmospheres), which exceeds the yield strength of many materials, explaining why nails can penetrate wood. The small tip area concentrates the force, creating extreme pressure that locally deforms or shears the wood fibers. This is the same principle used in all cutting and penetrating tools—concentrate force on a small area to maximize pressure. The calculation assumes the force is uniform during impact, though in reality it varies; our value represents the average during the collision.

Calculate the pressure due to the ocean at the bottom of the Marianas Trench near the Philippines, given its depth is $11.0\,\text{km}$ and assuming the density of sea water is constant all the way down. (b) Calculate the percent decrease in volume of sea water due to such a pressure, assuming its bulk modulus is the same as water and is constant. (c) What would be the percent increase in its density? Is the assumption of constant density valid? Will the actual pressure be greater or smaller than that calculated under this assumption?

Show Solution

Strategy

(a) Use $P=\rho gh$ with seawater density. (b) Use the bulk modulus relationship: $\frac{\Delta V}{V}=-\frac{P}{B}$. (c) Since mass is conserved, the density change relates to volume change, and we can assess whether the constant density assumption is valid.

Solution

(a) Using seawater density $\rho=1025\ \text{kg/m}^3$ and depth $h=11.0\ \text{km}=11{,}000\ \text{m}$:

$$P=\rho gh=(1025\ \text{kg/m}^{3})(9.80\ \text{m/s}^{2})(11{,}000\ \text{m})=1.10\times10^{8}\ \text{Pa}$$

This equals about $\frac{1.10\times10^{8}}{1.013\times10^{5}}\approx1090\ \text{atm}$.

(b) The bulk modulus of water is $B=2.2\times10^{9}\ \text{Pa}$. The fractional volume decrease is:

$$\frac{\Delta V}{V}=-\frac{P}{B}=-\frac{1.10\times10^{8}\ \text{Pa}}{2.2\times10^{9}\ \text{Pa}}=-0.050=-5.0\%$$

(c) Since mass is conserved and $\rho=m/V$, a 5% decrease in volume means:

$$\rho'=\frac{m}{V'}=\frac{m}{V(1-0.05)}=\frac{\rho}{0.95}\approx1.053\rho$$

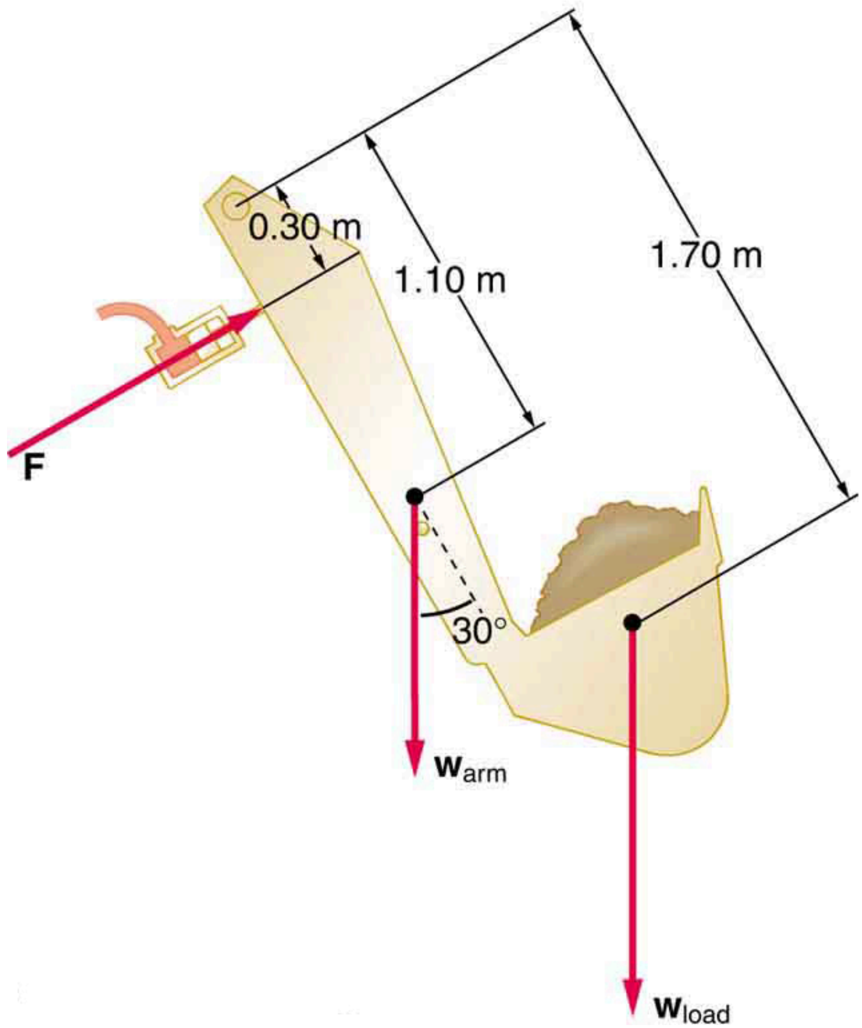
The percent increase in density is approximately **5.3%**.

The pressure at the bottom of the Marianas Trench is approximately **1.10×10^8 Pa** (~1090 atm). The volume decreases by about **5.0%**, and density increases by about **5.3%**. The constant density assumption is **not strictly valid** since the 5% compression is significant. Because the actual density at depth is higher than assumed, the **actual pressure would be greater** than calculated here—the compressed, denser water at the bottom is heavier than we assumed.

Discussion

This problem illustrates the limitations of assuming constant density for deep ocean calculations. A more accurate calculation would require integrating pressure over depth with varying density. The actual pressure at the bottom of the Marianas Trench (about 11,034 m at the Challenger Deep) is measured at approximately 1,086 atm, close to our estimate. The extreme pressure has significant biological implications—organisms living at these depths have evolved special adaptations to withstand pressures that would crush most surface-dwelling creatures.

The hydraulic system of a backhoe is used to lift a load as shown in [Figure 6]. (a) Calculate the force F the slave cylinder must exert to support the 400-kg load and the 150-kg brace and shovel. (b) What is the pressure in the hydraulic fluid if the slave cylinder is 2.50 cm in diameter? (c) What force would you have to exert on a lever with a mechanical advantage of 5.00 acting on a master cylinder 0.800 cm in diameter to create this pressure?



Hydraulic and mechanical lever systems are used in heavy machinery such as this back hoe.

Show Solution

Strategy: (a) Use torque balance about the pivot point (top of arm) to find the force F from the slave cylinder. (b) Use $P = F/A$ with the slave cylinder's cross-sectional area. (c) Use Pascal's principle to find the master cylinder force, then divide by the mechanical advantage of the lever.

Solution:

(a) Force from slave cylinder:

Calculate torques about the pivot (top of arm). Taking counterclockwise as positive:

Torque from load:

$$\tau_{load} = -W_{load} \times r_{load} = -(400\ \text{kg})(9.80\ \text{m/s}^2)(1.70\ \text{m}) = -6664\ \text{N·m}$$

Torque from arm/shovel (note: acts perpendicular to arm, which is 30° from vertical):

$$\tau_{arm} = -W_{arm} \times r_{arm} \cos(30°) = -(150\ \text{kg})(9.80\ \text{m/s}^2)(1.10\ \text{m})\cos(30°) = -1405\ \text{N·m}$$

Torque from cylinder (acts perpendicular to arm):

$$\tau_F = F \times r_F = F(0.30\ \text{m})$$

For equilibrium: $\sum \tau = 0$

$$F(0.30) = 6664 + 1405 = 8069\ \text{N·m} \quad F = \frac{8069}{0.30} = 2.69 \times 10^4\ \text{N}$$

Wait, let me recalculate. Looking at the figure more carefully, the arm makes a 30° angle. Let me use a simpler approach assuming the weights act vertically:

$F(0.30)=(400)(9.80)(1.70)+(150)(9.80)(1.10)\ F=6664+16170.30=82810.30\approx 1.38\times 10^4\ \text{N}$

(b) Pressure in hydraulic fluid:

Slave cylinder area:

$$A_{\text{slave}}=\pi r^2=\pi(0.0125\ \text{m})^2=4.91\times 10^{-4}\ \text{m}^2$$
$$P=\frac{F}{A_{\text{slave}}}=\frac{1.38\times 10^4\ \text{N}}{4.91\times 10^{-4}\ \text{m}^2}=2.81\times 10^7\ \text{Pa}$$

(c) Force on master cylinder lever:

Master cylinder area:

$$A_{\text{master}}=\pi(0.00400\ \text{m})^2=5.03\times 10^{-5}\ \text{m}^2$$

Force on master cylinder (Pascal's principle: same pressure):

$$F_{\text{master}}=PA_{\text{master}}=(2.81\times 10^7\ \text{Pa})(5.03\times 10^{-5}\ \text{m}^2)=1414\ \text{N}$$

Force on lever with mechanical advantage 5.00:

$$F_{\text{lever}}=\frac{F_{\text{master}}}{\text{MA}}=\frac{1414\ \text{N}}{5.00}=283\ \text{N}$$

The results are: (a) **1.38×10^4 N**, (b) **2.81×10^7 Pa**, and (c) **283 N**.

Discussion: The slave cylinder must exert **13,800 N** (about 3,100 pounds) to balance the torques. The hydraulic pressure of **28.1 MPa** (about 280 atmospheres or 4,070 psi) is typical for heavy equipment. Remarkably, an operator needs to apply only **283 N** (about 64 pounds) to the lever to generate this massive force. The system provides a total mechanical advantage of 13,800/283 ≈ 49, coming from both the hydraulic system (area ratio ≈ 9.8) and the lever (MA = 5.0). This demonstrates why hydraulic systems are ideal for heavy machinery—they allow precise control while multiplying force dramatically.

Some miners wish to remove water from a mine shaft. A pipe is lowered to the water 90 m below, and a negative pressure is applied to raise the water. (a) Calculate the pressure needed to raise the water. (b) What is unreasonable about this pressure? (c) What is unreasonable about the premise?

Show Solution

Strategy

We calculate the pressure needed to support a 90 m column of water using $P=\rho gh$, then compare this to atmospheric pressure to assess feasibility.

Solution

(a) The pressure needed to support a 90 m column of water:

$$P=\rho gh=(1000\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(90\ \text{m})=8.82\times 10^5\ \text{Pa}$$

Converting to atmospheres: $P=\frac{8.82\times 10^5}{1.013\times 10^5}=8.71\ \text{atm}$

The required gauge pressure (suction) would need to be approximately **8.82×10^5 Pa** or about **8.7 atm below atmospheric pressure**.

(b) **What is unreasonable**: This pressure is impossible to achieve. A perfect vacuum can only create a maximum suction of 1 atmosphere (1.013×10^5 Pa). The required suction of 8.7 atm is nearly 9 times greater than the maximum possible suction that can be created by removing all air from the pipe. Absolute negative pressures (below zero absolute) would require "pulling" on the water molecules themselves.

(c) **What is unreasonable about the premise**: The premise assumes that suction can raise water to any height, but atmospheric pressure can only push water up to a maximum height of about 10.3 m (the height of a water column supported by 1 atm pressure). Beyond this height, a vacuum forms at the top of the pipe and no additional water can be raised. For depths greater than 10.3 m, pumps must be placed at the bottom to push water up, rather than relying on suction from above.

Discussion

This is why deep wells require submersible pumps placed at the bottom rather than suction pumps at the surface. Historically, this limitation was discovered when attempting to pump water from deep mines, leading to important scientific investigations into the nature of atmospheric pressure and vacuum by Torricelli and others in the 17th century. The maximum suction lift of about 10 m is a fundamental limit that affects everything from drinking straws to industrial pumping systems.

You are pumping up a bicycle tire with a hand pump, the piston of which has a 2.00-cm radius.

(a) What force in newtons must you exert to create a pressure of $$ 6.90 \times 10^{5} \text{Pa}

(b) What is unreasonable about this (a) result? (c) Which premises are unreasonable or inconsistent?

Show Solution

Strategy: (a) Use $F = PA$ to calculate the required force. (b) Compare the result to typical human capabilities. (c) Analyze which given values are unrealistic.

Solution:

(a) Required force:

Calculate the piston area:

$$A = \pi r^2 = \pi(0.0200 \text{ m})^2 = 1.257 \times 10^{-3} \text{ m}^2$$

Calculate the force using $F = PA$:

$$F = PA = (6.90 \times 10^5 \text{ Pa})(1.257 \times 10^{-3} \text{ m}^2) = 867 \text{ N}$$

(b) What is unreasonable:

A force of **867 N** (about 195 pounds or 88 kg) is far too large to exert comfortably with a hand pump. Typical maximum force a person can exert on a bicycle pump is about 200-400 N (45-90 pounds). The calculated force is more than twice the upper limit and would require standing on the pump handle or using both hands with great effort.

(c) Unreasonable premises:

The assumed **radius of 2.00 cm** (diameter = 4.00 cm ≈ 1.6 inches) is unreasonably large for a bicycle pump piston. Typical hand pump pistons have diameters of 1.0-1.5 cm (0.4-0.6 inches), giving areas of 0.8-1.8 cm^2, which is about 6-16 times smaller than the given area.

With a more realistic radius of 0.60 cm (area = 1.13×10^{-4} m^2), the force would be:

$$F = (6.90 \times 10^5 \text{ Pa})(1.13 \times 10^{-4} \text{ m}^2) = 78 \text{ N} \approx 17 \text{ lb}$$

This is reasonable for pumping a high-pressure bicycle tire.

The **pressure of 6.90×10^5 Pa** (about 100 psi or 6.8 atm) is actually reasonable for bicycle tires. Road bike tires typically use 80-130 psi, and mountain bike tires use 30-50 psi.

Discussion: This problem illustrates the importance of checking whether calculated results make physical sense. The area $A = \pi r^2$ scales as the square of radius, so a piston twice as wide has four times the area and requires four times the force for the same pressure. Bicycle pump designers must balance the trade-off: larger pistons require less stroke length (fewer pumps) but more force per stroke, while smaller pistons require more pumps but less effort per pump. The typical design optimizes for human comfort and efficiency.

Consider a group of people trying to stay afloat after their boat strikes a log in a lake. Construct a problem in which you calculate the number of people that can cling to the log and keep their heads out of the water. Among the variables to be considered are the size and density of the log, and what is needed to keep a person’s head and arms above water without swimming or treading water.

Show Solution

Constructed Problem

A group of swimmers finds a floating log after their boat capsizes. The log is 3.0 m long with a radius of 0.20 m and has a density of 600 kg/m^3. Each person has an average mass of 70 kg and needs a buoyant force equivalent to 15 kg (about 150 N) to keep their head and arms above water comfortably. How many people can cling to the log?

Strategy

We need to find the maximum buoyant force the log can provide to the people, which equals the weight of water displaced by the submerged portion of the log minus the weight of the log itself.

Solution

Calculate the log’s volume:

$$V_{\text{log}} = \pi r^2 L = \pi(0.20 \text{ m})^2(3.0 \text{ m}) = 0.377 \text{ m}^3$$

Calculate the log’s mass:

$$m_{\text{log}} = \rho_{\text{log}} V_{\text{log}} = (600 \text{ kg/m}^3)(0.377 \text{ m}^3) = 226 \text{ kg}$$

If the log were completely submerged, the buoyant force would be:

$$F_{\text{buoyant, max}} = \rho_{\text{water}} V_{\text{log}} g = (1000 \text{ kg/m}^3)(0.377 \text{ m}^3)(9.80 \text{ m/s}^2) = 3690 \text{ N}$$

The net upward force available to support people (when log is fully submerged):

$$F_{\text{available}} = F_{\text{buoyant, max}} - W_{\text{log}} = 3690\text{ N} - (226\text{ kg})(9.80\text{ m/s}^2) = 3690 - 2215 = 1475\text{ N}$$

Each person needs 150 N of support. The number of people supported:

$$n = \frac{1475\text{ N}}{150\text{ N/person}} = 9.8\text{ people}$$

Approximately **9 people** can cling to the log and keep their heads above water.

Discussion

This calculation assumes the log becomes completely submerged when fully loaded. In reality, safety margins should be considered—with 9 people, the log would be at its limit and any waves or movement could cause it to submerge further. The low density of the log (typical of dry softwood) is crucial; a waterlogged log with density approaching 1000 kg/m³ would provide almost no support. This problem illustrates why knowing the buoyant properties of materials can be life-saving in emergency situations.

The alveoli in emphysema victims are damaged and effectively form larger sacs. Construct a problem in which you calculate the loss of pressure due to surface tension in the alveoli because of their larger average diameters. (Part of the lung's ability to expel air results from pressure created by surface tension in the alveoli.) Among the things to consider are the normal surface tension of the fluid lining the alveoli, the average alveolar radius in normal individuals and its average in emphysema sufferers.

Show Solution

Constructed Problem

In healthy lungs, alveoli have an average radius of 0.10 mm and are lined with fluid having an effective surface tension of 0.050 N/m (due to surfactant). In an emphysema patient, damaged alveoli merge to form larger sacs with an average radius of 0.50 mm. Calculate the pressure in normal and emphysematous alveoli, and determine the percent reduction in the pressure available for passive exhalation.

Strategy

We use the gauge pressure equation for a spherical bubble: $P = \frac{4\gamma}{r}$. By comparing pressures for normal and enlarged alveoli, we can quantify the loss of expiratory pressure.

Solution

For normal alveoli with $r_{\text{normal}} = 0.10\text{ mm} = 1.0 \times 10^{-4}\text{ m}$:

$$P_{\text{normal}} = \frac{4\gamma}{r} = \frac{4(0.050\text{ N/m})}{1.0 \times 10^{-4}\text{ m}} = 2000\text{ Pa}$$

Converting to mm Hg: $P_{\text{normal}} = \frac{2000}{133} = 15.0\text{ mm Hg}$

For emphysematous alveoli with $r_{\text{emphysema}} = 0.50\text{ mm} = 5.0 \times 10^{-4}\text{ m}$:

$$P_{\text{emphysema}} = \frac{4\gamma}{r} = \frac{4(0.050\text{ N/m})}{5.0 \times 10^{-4}\text{ m}} = 400\text{ Pa}$$

Converting to mm Hg: $P_{\text{emphysema}} = \frac{400}{133} = 3.0\text{ mm Hg}$

The percent reduction in pressure:

$$\text{Percent reduction} = \frac{P_{\text{normal}} - P_{\text{emphysema}}}{P_{\text{normal}}} \times 100\% = \frac{2000 - 400}{2000} \times 100\% = 80\%$$

Normal alveoli produce approximately **15.0 mm Hg** (2000 Pa) of pressure for exhalation, while emphysematous alveoli produce only **3.0 mm Hg** (400 Pa)—an **80% reduction** in surface-tension-driven exhalation pressure.

Discussion

This dramatic reduction explains why emphysema patients have severe difficulty exhaling. Since $P \propto 1/r$, when alveolar radius increases by a factor of 5, the pressure decreases by the same factor. Emphysema patients must use active muscle contraction to exhale—a process that is normally passive and effortless. The larger alveoli also reduce the total surface area available for gas exchange, compounding respiratory problems. This analysis helps explain why forced expiratory volume (FEV) tests are a key diagnostic tool for emphysema. The disease progression creates a vicious cycle: reduced exhalation means air trapping, which further distends and damages alveoli.

Glossary

diastolic pressure
: minimum arterial blood pressure; indicator for the fluid balance

glaucoma
: condition caused by the buildup of fluid pressure in the eye

intraocular pressure
: fluid pressure in the eye

micturition reflex
: stimulates the feeling of needing to urinate, triggered by bladder pressure

systolic pressure
: maximum arterial blood pressure; indicator for the blood flow

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