Matgeo Presentation - Problem 5.2.65

ee25btech11063 - Vejith

September 14, 2025

Question

Solve
$$\mathbf{X} + \mathbf{Y} = \begin{pmatrix} 5 & 2 \\ 0 & 9 \end{pmatrix} \text{ and } \mathbf{X} - \mathbf{Y} = \begin{pmatrix} 3 & 6 \\ 0 & -1 \end{pmatrix}$$

Solution

Given,

$$\mathbf{X} + \mathbf{Y} = \begin{pmatrix} 5 & 2 \\ 0 & 9 \end{pmatrix} \tag{0.1}$$

$$\mathbf{X} - \mathbf{Y} = \begin{pmatrix} 3 & 6 \\ 0 & -1 \end{pmatrix} \tag{0.2}$$

$$\implies \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} (\mathbf{X} \quad \mathbf{Y}) = (\mathbf{A} \quad \mathbf{B}) \tag{0.3}$$

$$\mathbf{A} = \begin{pmatrix} 5 & 2 \\ 0 & 9 \end{pmatrix} \text{ and } \mathbf{B} = \begin{pmatrix} 3 & 6 \\ 0 & -1 \end{pmatrix} \tag{0.4}$$

Forming the Augmented matrix

$$\left(\begin{array}{cc|c} 1 & 1 & \mathbf{A} \ \mathbf{B} \\ 1 & -1 & \end{array} \right) \xrightarrow{R_2 \to R_2 - R_1} \left(\begin{array}{cc|c} 1 & 1 & \mathbf{A} \ \mathbf{B} - \mathbf{A} \\ 0 & -2 & \end{array} \right) \tag{0.5}$$

$$\implies -2\mathbf{Y} = \mathbf{B} - \mathbf{A} \tag{0.6}$$

Solution

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} -2 & 4 \\ 0 & -10 \end{pmatrix} \tag{0.7}$$

$$\implies \mathbf{Y} = \begin{pmatrix} 1 & -2 \\ 0 & 5 \end{pmatrix} \tag{0.8}$$

$$\implies \mathbf{X} + \mathbf{Y} = \begin{pmatrix} 5 & 2 \\ 0 & 9 \end{pmatrix} \tag{0.9}$$

$$\implies \mathbf{X} = \begin{pmatrix} 4 & 4 \\ 0 & 4 \end{pmatrix} \tag{0.10}$$

on back substitution we get

$$\implies \mathbf{X} = \begin{pmatrix} 4 & 4 \\ 0 & 4 \end{pmatrix} \text{ and } \mathbf{Y} = \begin{pmatrix} 1 & -2 \\ 0 & 5 \end{pmatrix} \tag{0.11}$$

C Code: matrix.c

```c
#include <stdio.h>

int main() {
    FILE *fp;
    fp = fopen("matrix.dat", "w"); // open file for writing
    if (fp == NULL) {
        printf("Error opening file!\n");
        return 1;
    }

    // Define the given matrices
    int A[2][2] = { {5, 2}, {0, 9} }; // X + Y
    int B[2][2] = { {3, 6}, {0, -1} }; // X - Y
    int X[2][2], Y[2][2];

    // Calculate X = (A + B)/2
    for (int i = 0; i < 2; i++) {
        for (int j = 0; j < 2; j++) {
            X[i][j] = (A[i][j] + B[i][j]) / 2;
        }
    }

    // Calculate Y = (A - B)/2
    for (int i = 0; i < 2; i++) {
        for (int j = 0; j < 2; j++) {
            Y[i][j] = (A[i][j] - B[i][j]) / 2;
        }
    }

    // Write results to file
    fprintf(fp, "Matrix X:\n");
    for (int i = 0; i < 2; i++) {
        for (int j = 0; j < 2; j++) {
```

C Code: matrix.c

```c
 fprintf(fp, "%d␣", X[i][j]);
      }
      fprintf(fp, "\n");
   }

   fprintf(fp, "\nMatrix␣Y:\n");
   for (int i = 0; i < 2; i++) {
      for (int j = 0; j < 2; j++) {
         fprintf(fp, "%d␣", Y[i][j]);
      }
      fprintf(fp, "\n");
   }

   fclose(fp);
   printf("Solution␣written␣to␣matrix.dat\n");
   return 0;
}
```

Python: solution.py

```python
import numpy as np

# Given matrices
A = np.array([[5, 2],
              [0, 9]]) # X + Y
B = np.array([[3, 6],
              [0, -1]]) # X - Y

# Solve for X and Y
X = (A + B) / 2
Y = (A - B) / 2

print("Matrix␣X:")
print(X)
print("\nMatrix␣Y:")
print(Y)
```