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Question

Solve $\mathbf{X} + \mathbf{Y} = \begin{pmatrix} 5 & 2 \\ 0 & 9 \end{pmatrix}$ and $\mathbf{X} - \mathbf{Y} = \begin{pmatrix} 3 & 6 \\ 0 & -1 \end{pmatrix}$ Solution:

Given,

$$\mathbf{X} + \mathbf{Y} = \begin{pmatrix} 5 & 2 \\ 0 & 9 \end{pmatrix} \tag{1}$$

$$\mathbf{X} - \mathbf{Y} = \begin{pmatrix} 3 & 6 \\ 0 & -1 \end{pmatrix} \tag{2}$$

$$\implies \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} \mathbf{X} \\ \mathbf{Y} \end{pmatrix} = \begin{pmatrix} \mathbf{A} \\ \mathbf{B} \end{pmatrix} \tag{3}$$

$$\mathbf{A} = \begin{pmatrix} 5 & 2 \\ 0 & 9 \end{pmatrix} \text{ and } \mathbf{B} = \begin{pmatrix} 3 & 6 \\ 0 & -1 \end{pmatrix}$$
 (4)

Forming the Augmented matrix

$$\begin{pmatrix} 1 & 1 & | \mathbf{A} \\ 1 & -1 & | \mathbf{B} \end{pmatrix} \xrightarrow{R_2 \to R_2 - R_1} \begin{pmatrix} 1 & 1 & | \mathbf{A} \\ 0 & -2 & | \mathbf{B} - \mathbf{A} \end{pmatrix}$$

$$\implies -2\mathbf{Y} = \mathbf{B} - \mathbf{A} \tag{5}$$

$$\implies -2\mathbf{Y} = \mathbf{B} - \mathbf{A} \tag{6}$$

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} -2 & 4\\ 0 & -10 \end{pmatrix} \tag{7}$$

$$\implies \mathbf{Y} = \begin{pmatrix} 1 & -2 \\ 0 & 5 \end{pmatrix} \tag{8}$$

$$\implies \mathbf{X} + \mathbf{Y} = \begin{pmatrix} 5 & 2 \\ 0 & 9 \end{pmatrix} \tag{9}$$

$$\implies \mathbf{X} = \begin{pmatrix} 4 & 4 \\ 0 & 4 \end{pmatrix} \tag{10}$$

on back substitution we get

$$\implies \mathbf{X} = \begin{pmatrix} 4 & 4 \\ 0 & 4 \end{pmatrix} \text{ and } \mathbf{Y} = \begin{pmatrix} 1 & -2 \\ 0 & 5 \end{pmatrix}$$
 (11)