4.3.54

ee25btech11063-vejith

Question

A line is such that its segment between the lines 5x-y+4=0 and 3x+4y-4=0 is bisected at the point (1,5).obtain its equation

Solution:

Given two lines are

$$\begin{pmatrix}5 & -1\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix} = -4 \tag{1}$$

$$\begin{pmatrix}3 & 4\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix} = 4 \tag{2}$$

Let **A** be the point of intersection of desired line and (1)

$$\mathbf{A} = \begin{pmatrix}x_1\\y_1\end{pmatrix} \tag{3}$$

$$\implies \begin{pmatrix}5 & -1\end{pmatrix}\begin{pmatrix}x_1\\y_1\end{pmatrix} = -4 \tag{4}$$

$$\implies 5x_1 - y_1 = -4 \tag{5}$$

Let **B** be the point of intersection of desired line and (2)

$$\mathbf{B} = \begin{pmatrix}x_2\\y_2\end{pmatrix} \tag{6}$$

$$\implies \begin{pmatrix}3 & 4\end{pmatrix}\begin{pmatrix}x_2\\y_2\end{pmatrix} = 4 \tag{7}$$

$$\implies 3x_2 + 4y_2 = 4 \tag{8}$$

The mid point of **A** and **B** is $\begin{pmatrix}1\\5\end{pmatrix}$

$$\frac{\mathbf{A}+\mathbf{B}}{2} = \begin{pmatrix}1\\5\end{pmatrix} \tag{9}$$

$$\implies \begin{pmatrix}x_1\\y_1\end{pmatrix} + \begin{pmatrix}x_2\\y_2\end{pmatrix} = \begin{pmatrix}2\\10\end{pmatrix} \tag{10}$$

$$\implies x_1 + x_2 = 2 \tag{11}$$

$$\implies y_1 + y_2 = 10 \tag{12}$$

The equations (5),(8),(11),(12) can be written as

$$\begin{pmatrix}5 & -1 & 0 & 0\\0 & 0 & 3 & 4\\1 & 0 & 1 & 0\\0 & 1 & 0 & 1\end{pmatrix}\begin{pmatrix}x_1\\x_2\\y_1\\y_2\end{pmatrix} = \begin{pmatrix}-4\\4\\2\\10\end{pmatrix} \tag{13}$$

$$\tag{14}$$

Forming the Augmented matrix,

$$\left(\begin{array}{cccc|c}5 & -1 & 0 & 0 & -4\\0 & 0 & 3 & 4 & 4\\1 & 0 & 1 & 0 & 2\\0 & 1 & 0 & 1 & 10\end{array}\right) \xrightarrow{R_1 \leftrightarrow R_3} \left(\begin{array}{cccc|c}1 & 0 & 1 & 0 & 2\\0 & 0 & 3 & 4 & 4\\5 & -1 & 0 & 0 & -4\\0 & 1 & 0 & 1 & 10\end{array}\right) \tag{15}$$

$$\xrightarrow{R_3 \to R_3 - 5R_1} \left(\begin{array}{cccc|c}1 & 0 & 1 & 0 & 2\\0 & 0 & 3 & 4 & 4\\0 & -1 & -5 & 0 & -14\\0 & 1 & 0 & 1 & 10\end{array}\right) \tag{16}$$

$$\xrightarrow{R_2 \leftrightarrow R_4} \left(\begin{array}{cccc|c}1 & 0 & 1 & 0 & 2\\0 & 1 & 0 & 1 & 10\\0 & -1 & -5 & 0 & -14\\0 & 0 & 3 & 4 & 4\end{array}\right) \tag{17}$$

$$\tag{18}$$

$$\xrightarrow{R_3 \to R_3 + R_2} \left(\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 2 \\ 0 & 1 & 0 & 1 & 10 \\ 0 & 0 & -5 & 1 & -4 \\ 0 & 0 & 3 & 4 & 4 \end{array}\right) \tag{19}$$

$$\xrightarrow{R_4 \to R_4 + \frac{3}{5} R_3} \left(\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 2 \\ 0 & 1 & 0 & 1 & 10 \\ 0 & 0 & -5 & 1 & -4 \\ 0 & 0 & 0 & 23/5 & 8/5 \end{array}\right) \tag{20}$$

on back substitution we get

$$\begin{pmatrix} x_1 \\ x_2 \\ y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 26/23 \\ 222/23 \\ 20/23 \\ 8/23 \end{pmatrix} \tag{21}$$

$$\implies \mathbf{A} = \begin{pmatrix} 26/23 \\ 222/23 \end{pmatrix} \tag{22}$$

$$\implies \mathbf{B} = \begin{pmatrix} 20/23 \\ 8/23 \end{pmatrix} \tag{23}$$

Equation of a line is given by

$$\mathbf{n}^T \mathbf{x} = c. \tag{24}$$

$$\mathbf{n} = \begin{pmatrix} 107 \\ -3 \end{pmatrix} \tag{25}$$

$$\implies \begin{pmatrix} 107 & -3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = c. \tag{26}$$

The point $\begin{pmatrix} 1 \\ 5 \end{pmatrix}$ lies on the above line

$$\implies \begin{pmatrix} 107 & -3 \end{pmatrix} \begin{pmatrix} 1 \\ 5 \end{pmatrix} = c. \tag{27}$$

$$\implies c = 92 \tag{28}$$

The desired line is

$$\begin{pmatrix} 107 & -3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 92. \tag{29}$$

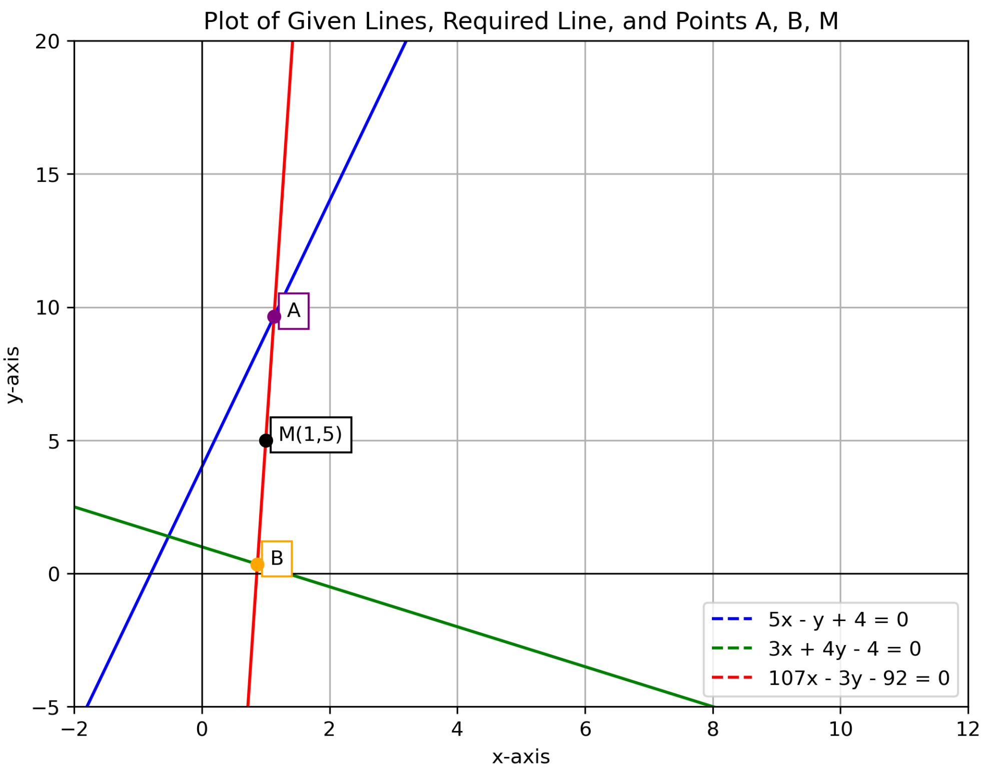

Fig. 0: Caption