5.9.17

ee25btech11063-vejith

Question

If $\begin{pmatrix} 2a+b & a-2b \\ 5c-d & 4c+3d \end{pmatrix}=\begin{pmatrix} 4 & -3 \\ 11 & 24 \end{pmatrix}$, then the value of a+b-c+2d

Solution:

From the matrix equation the first row gives

$$\begin{pmatrix} 2 & 1 & 0 & 0 \\ 1 & -2 & 0 & 0 \end{pmatrix}\begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = \begin{pmatrix} 4 \\ -3 \end{pmatrix} \tag{1}$$

From the matrix equation the second row gives

$$\begin{pmatrix} 0 & 0 & 5 & -1 \\ 0 & 0 & 4 & 3 \end{pmatrix}\begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = \begin{pmatrix} 11 \\ 24 \end{pmatrix} \tag{2}$$

combine (1) and (2)

$$\begin{pmatrix} 2 & 1 & 0 & 0 \\ 1 & -2 & 0 & 0 \\ 0 & 0 & 5 & -1 \\ 0 & 0 & 4 & 3 \end{pmatrix}\begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = \begin{pmatrix} 4 \\ -3 \\ 11 \\ 24 \end{pmatrix} \tag{3}$$

Forming the augmented matrix

$$\left(\begin{array}{cccc|c} 2 & 1 & 0 & 0 & 4 \\ 1 & -2 & 0 & 0 & -3 \\ 0 & 0 & 5 & -1 & 11 \\ 0 & 0 & 4 & 3 & 24 \end{array}\right) \xrightarrow{R_2 \leftrightarrow R_2 - \frac{1}{2}\times R_1} \left(\begin{array}{cccc|c} 2 & 1 & 0 & 0 & 4 \\ 0 & -\frac{5}{2} & 0 & 0 & -5 \\ 0 & 0 & 5 & -1 & 11 \\ 0 & 0 & 4 & 3 & 24 \end{array}\right) \tag{4}$$

$$\xrightarrow{R_1 \leftrightarrow R_1 + \frac{2}{5}\times R_2} \left(\begin{array}{cccc|c} 2 & 0 & 0 & 0 & 2 \\ 0 & -\frac{5}{2} & 0 & 0 & -5 \\ 0 & 0 & 5 & -1 & 11 \\ 0 & 0 & 4 & 3 & 24 \end{array}\right) \tag{5}$$

$$\xrightarrow{R_4 \leftrightarrow R_4 - \frac{4}{5}\times R_3} \left(\begin{array}{cccc|c} 2 & 0 & 0 & 0 & 2 \\ 0 & -\frac{5}{2} & 0 & 0 & -5 \\ 0 & 0 & 5 & -1 & 11 \\ 0 & 0 & 0 & \frac{19}{5} & \frac{76}{5} \end{array}\right) \tag{6}$$

$$\xrightarrow{R_3 \leftrightarrow R_3 + \frac{5}{19}\times R_4} \left(\begin{array}{cccc|c} 2 & 0 & 0 & 0 & 2 \\ 0 & -\frac{5}{2} & 0 & 0 & -5 \\ 0 & 0 & 5 & 0 & 15 \\ 0 & 0 & 0 & \frac{19}{5} & \frac{76}{5} \end{array}\right) \tag{7}$$

on back substitution we get

$$\implies \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix} \tag{8}$$

value of

$$a+b-c+2d = \begin{pmatrix} 1 & 1 & -1 & 2 \end{pmatrix}\begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} \tag{9}$$

$$= \begin{pmatrix} 1 & 1 & -1 & 2 \end{pmatrix}\begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix} = 8 \tag{10}$$