Matgeo Presentation - 4.3.54

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Question

A line is such that its segment between the lines $5x-y+4=0$ and $3x+4y-4=0$ is bisected at the point $(1,5)$.obtain its equation

Solution

Given two lines are

$$\begin{pmatrix} 5 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = -4 \tag{0.1}$$

$$\begin{pmatrix} 3 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 4 \tag{0.2}$$

Let **A** be the point of intersection of desired line and (0.1)

$$\mathbf{A} = \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} \tag{0.3}$$

$$\implies \begin{pmatrix} 5 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = -4 \tag{0.4}$$

$$\implies 5x_1 - y_1 = -4 \tag{0.5}$$

Let **B** be the point of intersection of desired line and (0.2)

$$\mathbf{B} = \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} \tag{0.6}$$

Solution

$$\implies \begin{pmatrix} 3 & 4 \end{pmatrix} \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} = 4 \tag{0.7}$$

$$\implies 3x_2 + 4y_2 = 4 \tag{0.8}$$

The mid point of **A** and **B** is $\begin{pmatrix} 1 \\ 5 \end{pmatrix}$

$$\frac{\mathbf{A} + \mathbf{B}}{2} = \begin{pmatrix} 1 \\ 5 \end{pmatrix} \tag{0.9}$$

$$\implies \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} + \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} = \begin{pmatrix} 2 \\ 10 \end{pmatrix} \tag{0.10}$$

$$\implies x_1 + x_2 = 2 \tag{0.11}$$

$$\implies y_1 + y_2 = 10 \tag{0.12}$$

The equations (0.5),(0.8),(0.11),(0.12) can be written as

Solution

$$\begin{pmatrix} 5 & -1 & 0 & 0 \\ 0 & 0 & 3 & 4 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} -4 \\ 4 \\ 2 \\ 10 \end{pmatrix} \tag{0.13}$$

$$\tag{0.14}$$

Forming the Augmented matrix,

$$\left(\begin{array}{cccc|c} 5 & -1 & 0 & 0 & -4 \\ 0 & 0 & 3 & 4 & 4 \\ 1 & 0 & 1 & 0 & 2 \\ 0 & 1 & 0 & 1 & 10 \end{array}\right) \xrightarrow{R_1 \leftrightarrow R_3} \left(\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 2 \\ 0 & 0 & 3 & 4 & 4 \\ 5 & -1 & 0 & 0 & -4 \\ 0 & 1 & 0 & 1 & 10 \end{array}\right) \tag{0.15}$$

$$\xrightarrow{R_3 \rightarrow R_3 - 5R_1} \left(\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 2 \\ 0 & 0 & 3 & 4 & 4 \\ 0 & -1 & -5 & 0 & -14 \\ 0 & 1 & 0 & 1 & 10 \end{array}\right) \tag{0.16}$$

$$\tag{0.17}$$

Solution

$$\xrightarrow{R_2 \leftrightarrow R_4} \left(\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 2 \\ 0 & 1 & 0 & 1 & 10 \\ 0 & -1 & -5 & 0 & -14 \\ 0 & 0 & 3 & 4 & 4 \end{array}\right) \tag{0.18}$$

$$\tag{0.19}$$

$$\xrightarrow{R_3 \to R_3 + R_2} \left(\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 2 \\ 0 & 1 & 0 & 1 & 10 \\ 0 & 0 & -5 & 1 & -4 \\ 0 & 0 & 3 & 4 & 4 \end{array}\right) \tag{0.20}$$

$$\xrightarrow{R_4 \to R_4 + \frac{3}{5} R_3} \left(\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 2 \\ 0 & 1 & 0 & 1 & 10 \\ 0 & 0 & -5 & 1 & -4 \\ 0 & 0 & 0 & 23/5 & 8/5 \end{array}\right) \tag{0.21}$$

on back substitution we get

Solution

$$\begin{pmatrix} x_1 \\ x_2 \\ y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 26/23 \\ 222/23 \\ 20/23 \\ 8/23 \end{pmatrix} \tag{0.22}$$

$$\implies \mathbf{A} = \begin{pmatrix} 26/23 \\ 222/23 \end{pmatrix} \tag{0.23}$$

$$\implies \mathbf{B} = \begin{pmatrix} 20/23 \\ 8/23 \end{pmatrix} \tag{0.24}$$

Equation of a line is given by

$$\mathbf{n}^T \mathbf{x} = c. \tag{0.25}$$

$$\mathbf{n} = \begin{pmatrix} 107 \\ -3 \end{pmatrix} \tag{0.26}$$

$$\implies \begin{pmatrix} 107 & -3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = c. \tag{0.27}$$

Conclusion

The point $\begin{pmatrix} 1 \\ 5 \end{pmatrix}$ lies on the above line

$$\implies (107 \quad -3) \begin{pmatrix} 1 \\ 5 \end{pmatrix} = c. \tag{0.28}$$

$$\implies c = 92 \tag{0.29}$$

The desired line is

$$(107 \quad -3) \begin{pmatrix} x \\ y \end{pmatrix} = 92. \tag{0.30}$$

Plot

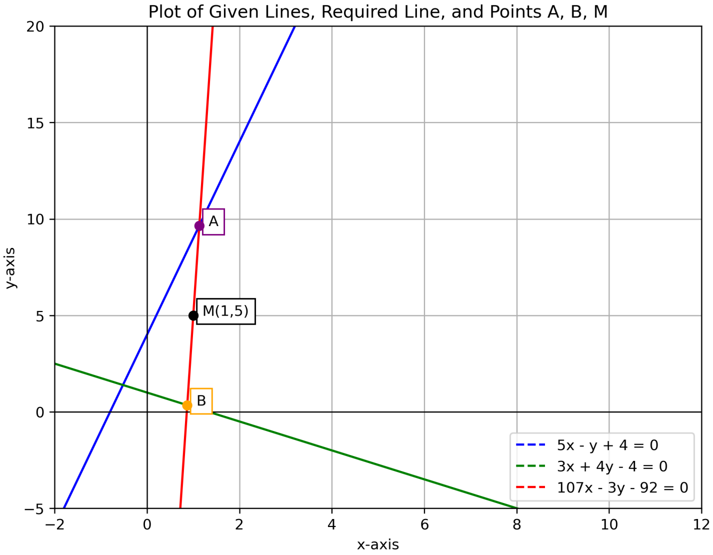

Figure: Caption

C Code: line.c

```c
#include <stdio.h>

int main() {
    FILE *fp;
    fp = fopen("line.dat", "w");
    if (fp == NULL) {
        printf("Error opening file!\n");
        return 1;
    }

    // Final derived equation
    fprintf(fp, "The required line equation is: 107x - 32y = 92\n");

    fclose(fp);
    printf("Output written to line.dat successfully.\n");
    return 0;
}
```

Python: plot.py

```
import numpy as np
import matplotlib.pyplot as plt

# Define x range
x = np.linspace(-5, 15, 400)

# Define the lines
y1 = 5*x + 4 # Line 1: 5x - y + 4 = 0
y2 = (4 - 3*x) / 4 # Line 2: 3x + 4y - 4 = 0
y3 = (107*x - 92) / 3 # Line 3: 107x - 3y - 92 = 0

# Intersection points
A = (26/23, 222/23) #  (1.13, 9.65)
B = (20/23, 8/23) #  (0.87, 0.35)
M = (1, 5) # Midpoint

# Plot solid lines
plt.figure(figsize=(8,6))
plt.plot(x, y1, color="blue")
plt.plot(x, y2, color="green")
plt.plot(x, y3, color="red")

# Create dashed proxy lines for legend
l1, = plt.plot([], [], color="blue", linestyle="--", label="5x - y + 4 = 0")
l2, = plt.plot([], [], color="green", linestyle="--", label="3x + 4y - 4 = 0")
l3, = plt.plot([], [], color="red", linestyle="--", label="107x - 3y - 92 = 0")

# Add legend box INSIDE grid
plt.legend(handles=[l1, l2, l3],
          loc="lower right", frameon=True)

# Mark the points A, B, and M (with boxes)
plt.scatter(*A, color="purple", zorder=5)
```

Python: plot.py

```
plt.text(A[0]+0.2, A[1], "A", fontsize=10,
        bbox=dict(facecolor='white', edgecolor='purple'))

plt.scatter(*B, color="orange", zorder=5)
plt.text(B[0]+0.2, B[1], "B", fontsize=10,
        bbox=dict(facecolor='white', edgecolor='orange'))

plt.scatter(*M, color="black", zorder=5)
plt.text(M[0]+0.2, M[1], "M(1,5)", fontsize=10,
        bbox=dict(facecolor='white', edgecolor='black'))

# Axes settings
plt.axhline(0, color='black', linewidth=0.8)
plt.axvline(0, color='black', linewidth=0.8)
plt.xlim(-2, 12)
plt.ylim(-5, 20)

# Labels and grid
plt.xlabel("x-axis")
plt.ylabel("y-axis")
plt.title("Plot␣of␣Given␣Lines,␣Required␣Line,␣and␣Points␣A,␣B,␣M")
plt.grid(True)

# Save and show
plt.savefig("line_plot.png", dpi=300, bbox_inches="tight")
plt.show()
```