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Question

If $\begin{pmatrix} 2a+b & a-2b \\ 5c-d & 4c+3d \end{pmatrix} = \begin{pmatrix} 4 & -3 \\ 11 & 24 \end{pmatrix}$, then the value of a+b-c+2d

Solution:

From the matrix equation the first row gives

$$\begin{pmatrix} 2 & 1 \\ 1 & -2 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 4 \\ -3 \end{pmatrix} \tag{1}$$

Forming the Augmented matrix

$$\left(\begin{array}{cc|c} 2 & 1 & 4 \\ 1 & -2 & -3 \end{array} \right) \xleftarrow{R_2 \to R_2 - R_1/2} \left(\begin{array}{cc|c} 2 & 1 & 4 \\ 0 & -\frac{5}{2} & -5 \end{array} \right) \tag{2}$$

$$\xleftarrow{R_1 \to R_1 + \frac{2}{5} \times R_2} \left(\begin{array}{cc|c} 2 & 0 & 2 \\ 0 & -\frac{5}{2} & -5 \end{array} \right) \tag{3}$$

$$\implies \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \tag{4}$$

From the matrix equation the second row gives

$$\begin{pmatrix} 5 & -1 \\ 4 & 3 \end{pmatrix} \begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} 11 \\ 24 \end{pmatrix} \tag{5}$$

Forming the Augmented matrix

$$\left(\begin{array}{cc|c} 5 & -1 & 11 \\ 4 & 3 & 24 \end{array} \right) \xleftarrow{R_2 \to R_2 - \frac{4}{5} \times R_1} \left(\begin{array}{cc|c} 5 & -1 & 11 \\ 0 & \frac{19}{5} & \frac{76}{5} \end{array} \right) \tag{6}$$

$$\xleftarrow{R_1 \to R_1 + \frac{5}{19} \times R_2} \left(\begin{array}{cc|c} 5 & 0 & 15 \\ 0 & 19 & 76 \end{array} \right) \tag{7}$$

$$\implies \begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} \tag{8}$$

value of

$$a+b-c+2d = \begin{pmatrix} 1 & 1 & -1 & 2 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} \tag{9}$$

$$= \begin{pmatrix} 1 & 1 & -1 & 2 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix} = 8 \tag{10}$$