5.2.65

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Question

Solve
$$\mathbf{X+Y}=\begin{pmatrix} 5 & 2 \\ 0 & 9 \end{pmatrix} \text{ and } \mathbf{X-Y}=\begin{pmatrix} 3 & 6 \\ 0 & -1 \end{pmatrix}$$

Solution:

Given,

$$\mathbf{X} + \mathbf{Y} = \begin{pmatrix} 5 & 2 \\ 0 & 9 \end{pmatrix} \tag{1}$$

$$\mathbf{X} - \mathbf{Y} = \begin{pmatrix} 3 & 6 \\ 0 & -1 \end{pmatrix} \tag{2}$$

$$\implies \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} (\mathbf{X} \quad \mathbf{Y}) = (\mathbf{A} \quad \mathbf{B}) \tag{3}$$

$$\mathbf{A} = \begin{pmatrix} 5 & 2 \\ 0 & 9 \end{pmatrix} \text{ and } \mathbf{B} = \begin{pmatrix} 3 & 6 \\ 0 & -1 \end{pmatrix} \tag{4}$$

Multiply both sides of (3) with traspose of coefficent matrix

$$\implies \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} (\mathbf{X} \quad \mathbf{Y}) = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} (\mathbf{A} \quad \mathbf{B}) \tag{5}$$

$$\implies \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} (\mathbf{X} \quad \mathbf{Y}) = (\mathbf{A} + \mathbf{B} \quad \mathbf{A} - \mathbf{B}) \tag{6}$$

$$\implies \mathbf{X} = \frac{\mathbf{A} + \mathbf{B}}{2} \text{ and } \mathbf{Y} = \frac{\mathbf{A} - \mathbf{B}}{2} \tag{7}$$

$$\implies \mathbf{X} = \begin{pmatrix} 4 & 4 \\ 0 & 4 \end{pmatrix} \text{ and } \mathbf{Y} = \begin{pmatrix} 1 & -2 \\ 0 & 5 \end{pmatrix} \tag{8}$$