5.5.15

ee25btech11063-vejith

Question

Using elementary row transformations,find the inverse of the matrix $\mathbf{A}=\begin{pmatrix}1 & 2 & 3\\ 2 & 5 & 7\\ -2 & -4 & -5\end{pmatrix}$

Solution:

$$\mathbf{A}=\begin{pmatrix}1 & 2 & 3\\ 2 & 5 & 7\\ -2 & -4 & -5\end{pmatrix} \tag{1}$$

The Augmented matrix is

$$\left(\begin{array}{c|c}\mathbf{A} & \mathbf{I}\end{array}\right) \implies \left(\begin{array}{ccc|ccc}1 & 2 & 3 & 1 & 0 & 0\\ 2 & 5 & 7 & 0 & 1 & 0\\ -2 & -4 & -5 & 0 & 0 & 1\end{array}\right) \xleftrightarrow{R_2\to R_2-2R_1} \left(\begin{array}{ccc|ccc}1 & 2 & 3 & 1 & 0 & 0\\ 0 & 1 & 1 & 0 & 1 & 0\\ -2 & -4 & -5 & 0 & 0 & 1\end{array}\right) \tag{2}$$

$$\xrightarrow{R_3\to R_3+2R_1} \left(\begin{array}{ccc|ccc}1 & 2 & 3 & 1 & 0 & 0\\ 0 & 1 & 1 & -2 & 1 & 0\\ 0 & 0 & 1 & 2 & 0 & 1\end{array}\right) \tag{3}$$

$$\xrightarrow{R_1\to R_1-3R_3} \left(\begin{array}{ccc|ccc}1 & 2 & 0 & -5 & 0 & 1\\ 0 & 1 & 1 & -2 & 1 & 0\\ 0 & 0 & 1 & 2 & 0 & 1\end{array}\right) \tag{4}$$

$$\xrightarrow{R_2\to R_2-R_3} \left(\begin{array}{ccc|ccc}1 & 2 & 0 & -5 & 0 & -3\\ 0 & 1 & 0 & -4 & 1 & -1\\ 0 & 0 & 1 & 2 & 0 & 1\end{array}\right) \tag{5}$$

$$\xrightarrow{R_1\to R_1-2R_2} \left(\begin{array}{ccc|ccc}1 & 0 & 0 & 3 & -2 & -1\\ 0 & 1 & 0 & -4 & 1 & -1\\ 0 & 0 & 1 & 2 & 0 & 1\end{array}\right) \tag{6}$$

As the left block of the Augmented matrix is $\mathbf{I}$ the right block is $\mathbf{A^{-1}}$.

$$\implies \mathbf{A^{-1}}=\begin{pmatrix}3 & -2 & -1\\ -4 & 1 & -1\\ 2 & 0 & 1\end{pmatrix} \tag{7}$$