5.2.65

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Question

Solve

$\mathbf{X}+\mathbf{Y}=\begin{pmatrix}5 & 2\\ 0 & 9\end{pmatrix}$ and $\mathbf{X}-\mathbf{Y}=\begin{pmatrix}3 & 6\\ 0 & -1\end{pmatrix}$

Solution:

Given,

$$\mathbf{X}+\mathbf{Y}=\begin{pmatrix}5 & 2\\ 0 & 9\end{pmatrix} \tag{1}$$

$$\mathbf{X}-\mathbf{Y}=\begin{pmatrix}3 & 6\\ 0 & -1\end{pmatrix} \tag{2}$$

$$\implies \begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}(\mathbf{X} \quad \mathbf{Y}) = (\mathbf{A} \quad \mathbf{B}) \tag{3}$$

$$\mathbf{A}=\begin{pmatrix}5 & 2\\ 0 & 9\end{pmatrix} \text{ and } \mathbf{B}=\begin{pmatrix}3 & 6\\ 0 & -1\end{pmatrix} \tag{4}$$

Forming the Augmented matrix

$$\left(\begin{array}{cc|cc} 1 & 1 & \mathbf{A} & \mathbf{B} \\ 1 & -1 & & \end{array}\right) \xrightarrow{R_2 \to R_2 - R_1} \left(\begin{array}{cc|cc} 1 & 1 & \mathbf{A} & \mathbf{B}-\mathbf{A} \\ 0 & -2 & & \end{array}\right) \tag{5}$$

$$\implies -2\mathbf{Y} = \mathbf{B}-\mathbf{A} \tag{6}$$

$$\mathbf{B}-\mathbf{A}=\begin{pmatrix}-2 & 4\\ 0 & -10\end{pmatrix} \tag{7}$$

$$\implies \mathbf{Y}=\begin{pmatrix}1 & -2\\ 0 & 5\end{pmatrix} \tag{8}$$

$$\implies \mathbf{X}+\mathbf{Y}=\begin{pmatrix}5 & 2\\ 0 & 9\end{pmatrix} \tag{9}$$

$$\implies \mathbf{X}=\begin{pmatrix}4 & 4\\ 0 & 4\end{pmatrix} \tag{10}$$

on back substitution we get

$$\implies \mathbf{X}=\begin{pmatrix}4 & 4\\ 0 & 4\end{pmatrix} \text{ and } \mathbf{Y}=\begin{pmatrix}1 & -2\\ 0 & 5\end{pmatrix} \tag{11}$$