

Matgeo Presentation - 5.9.17

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Question

If $\begin{pmatrix} 2a + b & a - 2b \\ 5c - d & 4c + 3d \end{pmatrix} = \begin{pmatrix} 4 & -3 \\ 11 & 24 \end{pmatrix}$, then the value of $a+b-c+2d$

Solution

From the matrix equation the first row gives

$$\begin{pmatrix} 2 & 1 & 0 & 0 \\ 1 & -2 & 0 & 0 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = \begin{pmatrix} 4 \\ -3 \end{pmatrix} \quad (0.1)$$

From the matrix equation the second row gives

$$\begin{pmatrix} 0 & 0 & 5 & -1 \\ 0 & 0 & 4 & 3 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = \begin{pmatrix} 11 \\ 24 \end{pmatrix} \quad (0.2)$$

combine (0.1) and (0.2)

$$\begin{pmatrix} 2 & 1 & 0 & 0 \\ 1 & -2 & 0 & 0 \\ 0 & 0 & 5 & -1 \\ 0 & 0 & 4 & 3 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = \begin{pmatrix} 4 \\ -3 \\ 11 \\ 24 \end{pmatrix} \quad (0.3)$$

Solution

Forming the augmented matrix

$$\left(\begin{array}{cccc|c} 2 & 1 & 0 & 0 & 4 \\ 1 & -2 & 0 & 0 & -3 \\ 0 & 0 & 5 & -1 & 11 \\ 0 & 0 & 4 & 3 & 24 \end{array} \right) \xrightarrow{R_2 \leftrightarrow R_2 - \frac{1}{2} \times R_1} \left(\begin{array}{cccc|c} 2 & 1 & 0 & 0 & 4 \\ 0 & -\frac{5}{2} & 0 & 0 & -5 \\ 0 & 0 & 5 & -1 & 11 \\ 0 & 0 & 4 & 3 & 24 \end{array} \right) \quad (0.4)$$

$$\xrightarrow{R_1 \leftrightarrow R_1 + \frac{2}{5} \times R_2} \left(\begin{array}{cccc|c} 2 & 0 & 0 & 0 & 2 \\ 0 & -\frac{5}{2} & 0 & 0 & -5 \\ 0 & 0 & 5 & -1 & 11 \\ 0 & 0 & 4 & 3 & 24 \end{array} \right) \quad (0.5)$$

$$\xrightarrow{R_4 \leftrightarrow R_4 - \frac{4}{5} \times R_3} \left(\begin{array}{cccc|c} 2 & 0 & 0 & 0 & 2 \\ 0 & -\frac{5}{2} & 0 & 0 & -5 \\ 0 & 0 & 5 & -1 & 11 \\ 0 & 0 & 0 & \frac{19}{5} & \frac{76}{5} \end{array} \right) \quad (0.6)$$

Solution

$$\xrightarrow{R_3 \leftrightarrow R_3 + \frac{5}{19} \times R_4} \left(\begin{array}{cccc|c} 2 & 0 & 0 & 0 & 2 \\ 0 & -\frac{5}{2} & 0 & 0 & -5 \\ 0 & 0 & 5 & 0 & 15 \\ 0 & 0 & 0 & \frac{19}{5} & \frac{76}{5} \end{array} \right) \quad (0.7)$$

on back substitution we get

$$\Rightarrow \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix} \quad (0.8)$$

Conclusion

value of

$$a + b - c + 2d = (1 \ 1 \ -1 \ 2) \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} \quad (0.9)$$

$$= (1 \ 1 \ -1 \ 2) \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix} = 8 \quad (0.10)$$

C Code: code.c

```
#include <stdio.h>

void solve2x2(double aug[2][3], int *x, int *y) {
    // Gaussian elimination for 2x2 system
    // aug is 2x3 augmented matrix

    // Step 1: make leading coefficient of first row = 1
    if (aug[0][0] != 0) {
        double factor = aug[0][0];
        aug[0][0] /= factor;
        aug[0][1] /= factor;
        aug[0][2] /= factor;
    }

    // Step 2: eliminate below
    double factor = aug[1][0];
    aug[1][0] -= factor * aug[0][0];
    aug[1][1] -= factor * aug[0][1];
    aug[1][2] -= factor * aug[0][2];

    // Step 3: make pivot in row 2 equal to 1
    if (aug[1][1] != 0) {
        factor = aug[1][1];
        aug[1][0] /= factor;
        aug[1][1] /= factor;
        aug[1][2] /= factor;
    }

    // Step 4: eliminate above row 2
    factor = aug[0][1];
    aug[0][0] -= factor * aug[1][0];
    aug[0][1] -= factor * aug[1][1];
    aug[0][2] -= factor * aug[1][2];
}
```

C Code: code.c

```
// Now solution is in aug[0][2], aug[1][2]
*x = (int)(aug[0][2] + 0.5); // rounding
*y = (int)(aug[1][2] + 0.5);
}

int main() {
    int a, b, c, d, result;

    double aug1[2][3] = {
        {2, 1, 4},
        {1, -2, -3}
    };
    solve2x2(aug1, &a, &b);

    double aug2[2][3] = {
        {5, -1, 11},
        {4, 3, 24}
    };
    solve2x2(aug2, &c, &d);

    result = a + b - c + 2 * d;

    FILE *fp = fopen("answer.dat", "w");
    if (fp == NULL) {
        printf("Error opening file!\n");
        return 1;
    }
    fprintf(fp, "%d\n", result);
    fclose(fp);
    return 0;}
```


Python: solution.py

```
import numpy as np

# -----
# System 1: for a, b
#  $2a + b = 4$ 
#  $a - 2b = -3$ 
# -----
A1 = np.array([[2, 1],
               [1, -2]])
B1 = np.array([4, -3])

# Solve for a, b
a, b = np.linalg.solve(A1, B1)

# -----
# System 2: for c, d
#  $5c - d = 11$ 
#  $4c + 3d = 24$ 
# -----
A2 = np.array([[5, -1],
               [4, 3]])
B2 = np.array([11, 24])

# Solve for c, d
c, d = np.linalg.solve(A2, B2)
# -----
# Compute expression
# -----
result = a + b - c + 2*d
print("a_ = ", a, "b_ = ", b)
print("c_ = ", c, "d_ = ", d)
print("a_+b_-c_+2d_ = ", result)
```