Matgeo Presentation - 5.9.17

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Question

If $\begin{pmatrix} 2a+b & a-2b \\ 5c-d & 4c+3d \end{pmatrix} = \begin{pmatrix} 4 & -3 \\ 11 & 24 \end{pmatrix}$, then the value of a+b-c+2d

Solution

From the matrix equation the first row gives

$$\begin{pmatrix} 2 & 1 \\ 1 & -2 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 4 \\ -3 \end{pmatrix} \tag{0.1}$$

Forming the Augmented matrix

$$\left(\begin{array}{cc|c} 2 & 1 & 4 \\ 1 & -2 & -3 \end{array} \right) \xleftrightarrow{R_2 \to R_1/2} \left(\begin{array}{cc|c} 2 & 1 & 4 \\ 0 & -\frac{5}{2} & -5 \end{array} \right) \tag{0.2}$$

$$\xleftrightarrow{R_1 \to R_1 + \frac{2}{5} \times R_2} \left(\begin{array}{cc|c} 2 & 0 & 2 \\ 0 & -\frac{5}{2} & -5 \end{array} \right) \tag{0.3}$$

$$\implies \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \tag{0.4}$$

From the matrix equation the second row gives

$$\begin{pmatrix} 5 & -1 \\ 4 & 3 \end{pmatrix} \begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} 11 \\ 24 \end{pmatrix} \tag{0.5}$$

Solution

Forming the Augmented matrix

$$\left(\begin{array}{cc|c} 5 & -1 & 11 \\ 4 & 3 & 24 \end{array}\right) \xleftrightarrow{R_2 \to R_2 - \frac{4}{5} \times R_1} \left(\begin{array}{cc|c} 5 & -1 & 11 \\ 0 & \frac{19}{5} & \frac{76}{5} \end{array}\right) \tag{0.6}$$

$$\xleftrightarrow{R_1 \to R_1 + \frac{5}{19} R_2} \left(\begin{array}{cc|c} 5 & 0 & 15 \\ 0 & 19 & 76 \end{array}\right) \tag{0.7}$$

$$\implies \begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} \tag{0.8}$$

Conclusion

value of

$$a + b - c + 2d = \begin{pmatrix} 1 & 1 & -1 & 2 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} \tag{0.9}$$

$$= \begin{pmatrix} 1 & 1 & -1 & 2 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix} = 8 \tag{0.10}$$

C Code: code.c

```c
#include <stdio.h>

void solve2x2(double aug[2][3], int *x, int *y) {
    // Gaussian elimination for 2x2 system
    // aug is 2x3 augmented matrix

    // Step 1: make leading coefficient of first row = 1
    if (aug[0][0] != 0) {
        double factor = aug[0][0];
        aug[0][0] /= factor;
        aug[0][1] /= factor;
        aug[0][2] /= factor;
    }

    // Step 2: eliminate below
    double factor = aug[1][0];
    aug[1][0] -= factor * aug[0][0];
    aug[1][1] -= factor * aug[0][1];
    aug[1][2] -= factor * aug[0][2];

    // Step 3: make pivot in row 2 equal to 1
    if (aug[1][1] != 0) {
        factor = aug[1][1];
        aug[1][0] /= factor;
        aug[1][1] /= factor;
        aug[1][2] /= factor;
    }

    // Step 4: eliminate above row 2
    factor = aug[0][1];
    aug[0][0] -= factor * aug[1][0];
    aug[0][1] -= factor * aug[1][1];
    aug[0][2] -= factor * aug[1][2];
```

C Code: code.c

```
    // Now solution is in aug[0][2], aug[1][2]
    *x = (int)(aug[0][2] + 0.5); // rounding
    *y = (int)(aug[1][2] + 0.5);
}

int main() {
    int a, b, c, d, result;

    double aug1[2][3] = {
        {2, 1, 4},
        {1, -2, -3}
    };
    solve2x2(aug1, &a, &b);


    double aug2[2][3] = {
        {5, -1, 11},
        {4, 3, 24}
    };
    solve2x2(aug2, &c, &d);


    result = a + b - c + 2 * d;


    FILE *fp = fopen("answer.dat", "w");
    if (fp == NULL) {
        printf("Error opening file!\n");
        return 1;
    }
    fprintf(fp, "%d\n", result);
    fclose(fp);
    return 0;}
```

Python: solution.py

```
import numpy as np

# ----------------------------
# System 1: for a, b
# 2a + b = 4
# a - 2b = -3
# ----------------------------
A1 = np.array([[2, 1],
               [1, -2]])
B1 = np.array([4, -3])

# Solve for a, b
a, b = np.linalg.solve(A1, B1)

# ----------------------------
# System 2: for c, d
# 5c - d = 11
# 4c + 3d = 24
# ----------------------------
A2 = np.array([[5, -1],
               [4, 3]])
B2 = np.array([11, 24])

# Solve for c, d
c, d = np.linalg.solve(A2, B2)
# ----------------------------
# Compute expression
# ----------------------------
result = a + b - c + 2*d
print("a␣=", a, "␣b␣=", b)
print("c␣=", c, "␣d␣=", d)
print("a␣+␣b␣-␣c␣+␣2d␣=", result)
```