Purpose Moments of roots of a polynomial.

Syntax m = moments(p)

Description Consider a polynomial $p(s) = a_0 + a_1 s + a_2 s^2 + ... + a_n s^n$. Denote $\lambda_1, \lambda_2, ..., \lambda_n$ the set of roots of p(s) including multiplicities. Then the k-th moment m_k is defined as

$$m_k = \sum_{i=1}^n \lambda_i^k$$

The function moments returns a vector $m = [m_1 \quad m_2 \quad \cdots \quad m_n]$.

If the polynomial p(s) is monic then the first n moments determine the coefficients of the polynomial uniquely.

Examples

Algorithm

Solution to the linear system

$$\begin{pmatrix} 1 & 0 & 0 & \cdots & 0 & 0 \\ a_{n-1} & 1 & 0 & \cdots & 0 & 0 \\ a_{n-2} & a_{n-1} & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ a_2 & a_3 & a_4 & \cdots & 1 & 0 \\ a_1 & a_2 & a_3 & \cdots & a_{n-1} & 1 \end{pmatrix} \begin{pmatrix} m_1 \\ m_2 \\ m_3 \\ \vdots \\ m_{n-1} \\ m_n \end{pmatrix} = \begin{pmatrix} -a_{n-1} \\ -2a_{n-2} \\ -3a_{n-3} \\ \vdots \\ -(n-1)a_1 \\ -na_0 \end{pmatrix}$$

where the coefficients a_i were taken from the monic polynomial $\tilde{p}(s) = p(s)/a_n$.

See also

roots

Roots of a polynomial

References

Horn, R.A., Johnson, C.R.: Matrix analysis. Cambridge University Press, 1985, pp.44.