Homework 2 Trading Strategies and Option Valuation

110705067 洪薇欣

I. Option trading strategies (52%)

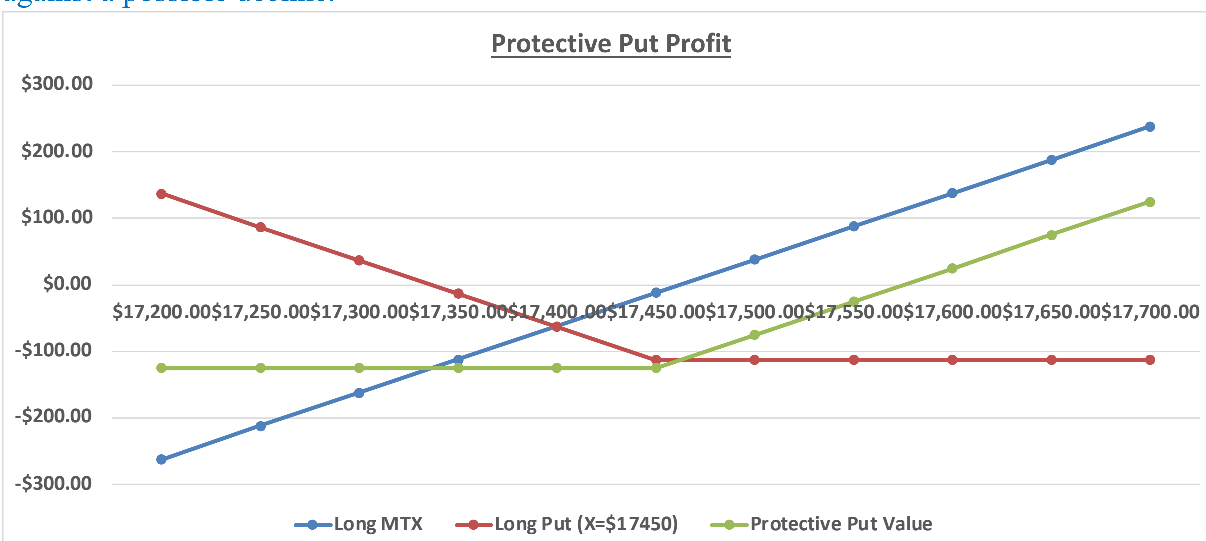
We know an unlimited variety of payoff patterns can be achieved by combining stocks, futures, and options with various exercise prices. The Excel file contains the market data of TAIEX Options (TXO) and Mini-TAIEX futures (MTX) collected from the Taiwan Futures Exchange. Please use those option contracts that mature in December 2023 to perform the following option strategies. **First**, briefly explain the following popular option strategies and what an investor could expect from their **profit** patterns.

(i) (7%) Protective put (long MTX and long TXO put)

Buying a put option (TXO) to protect a long stock position (MTX).

Losses on the stock are limited by the long put. The strategy provides downside protection, but the investor sacrifices potential gains above the strike price of the put.

Implement a protective put when you own a stock and want to protect it from potential downside risk. It is suitable when you are bullish on the stock but want insurance against a possible decline.

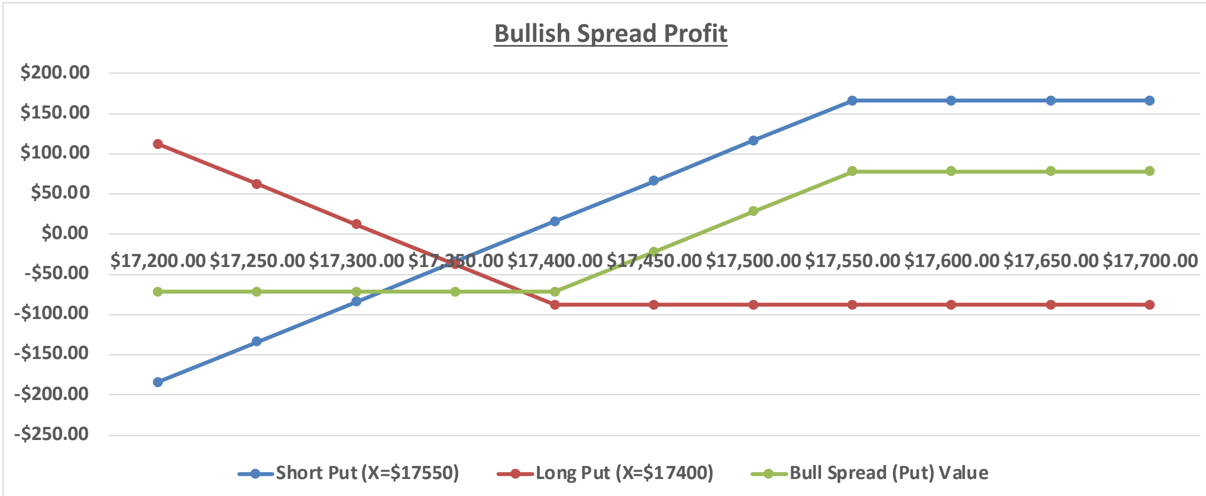


(ii) (7%) Bullish spread (using puts)

Buying a lower strike put and selling a higher strike put.

Limited risk, limited reward. Profits increase if the stock price rises but capped at the difference in strike prices. The maximum loss occurs if the stock price is below the lower strike at expiration.

Implement a bullish put spread when you expect the stock price to increase moderately. Suitable when you are moderately bullish but want to limit your upfront cost.

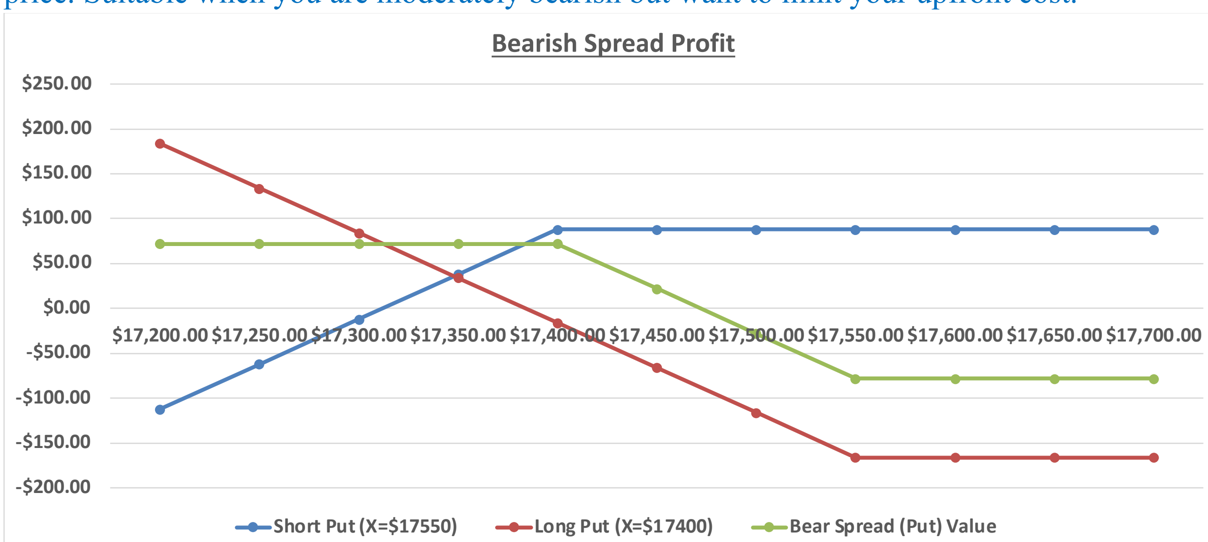


(iii) (7%) Bearish spread (using puts)

Buying a higher strike put and selling a lower strike put.

Limited risk, limited reward. Profits increase if the stock price falls, but capped at the difference in strike prices. The maximum loss occurs if the stock price is above the higher strike at expiration.

Implement a bearish put spread when you anticipate a moderate decline in the stock price. Suitable when you are moderately bearish but want to limit your upfront cost.

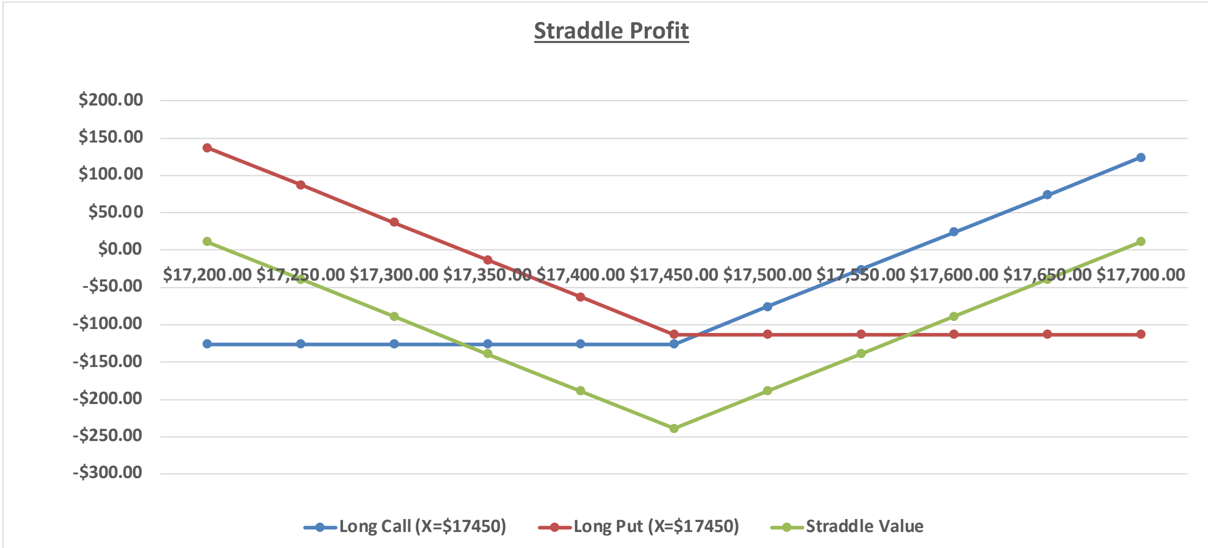


(iv) (7%) Straddle

Simultaneously buying a call and a put with the same strike price.

Unlimited profit potential in both directions. Profits if the stock moves significantly up or down. However, the stock must move enough to cover the combined cost of the call and put to break even.

Implement a straddle when you expect significant price volatility but are uncertain about the direction. Suitable during periods of expected high volatility or before major events like earnings announcements.

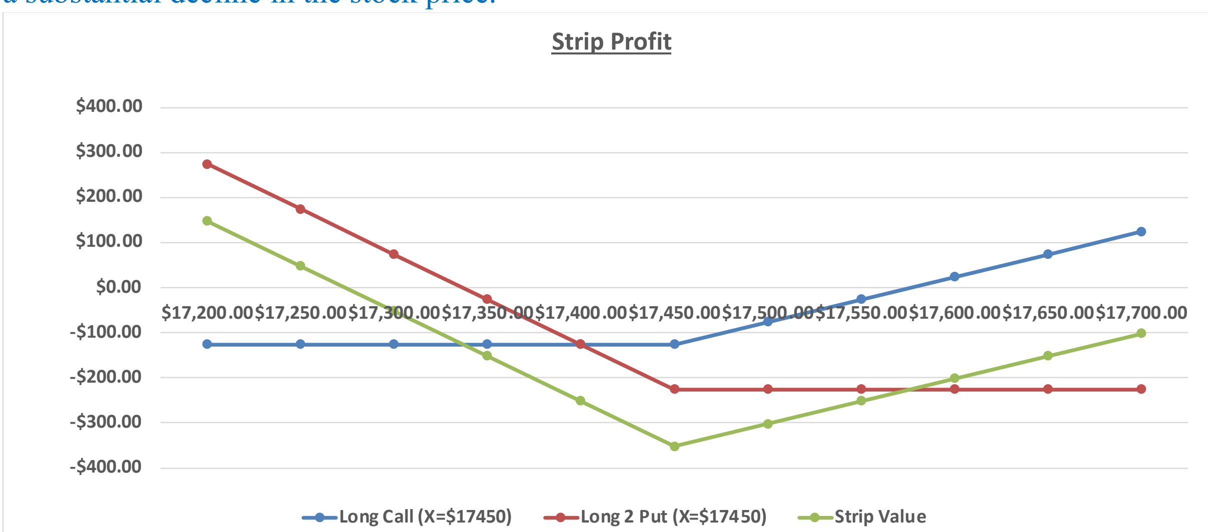


(v) (7%) Strip

Buying two puts for every call option held.

This is a bearish strategy with unlimited profit potential if the stock price decreases significantly. The risk is limited to the net cost of the options.

Implement a strip when you have a strongly bearish outlook. Suitable when you expect a substantial decline in the stock price.

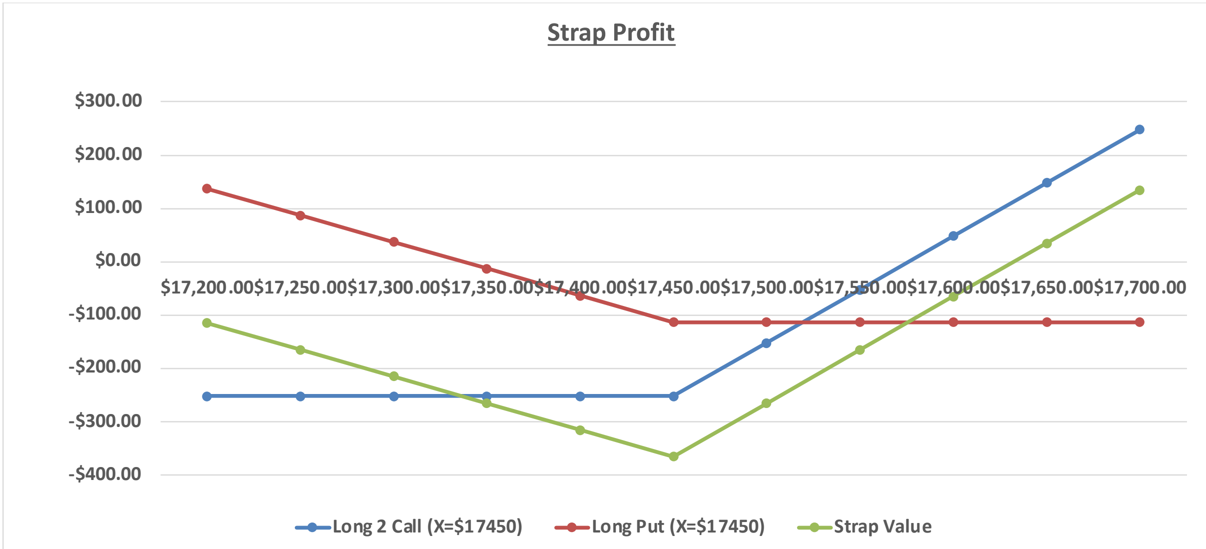


(vi) (7%) Strap

Buying two call options for every put option held.

This is a bullish strategy with unlimited profit potential if the stock price increases significantly. The risk is limited to the net cost of the options.

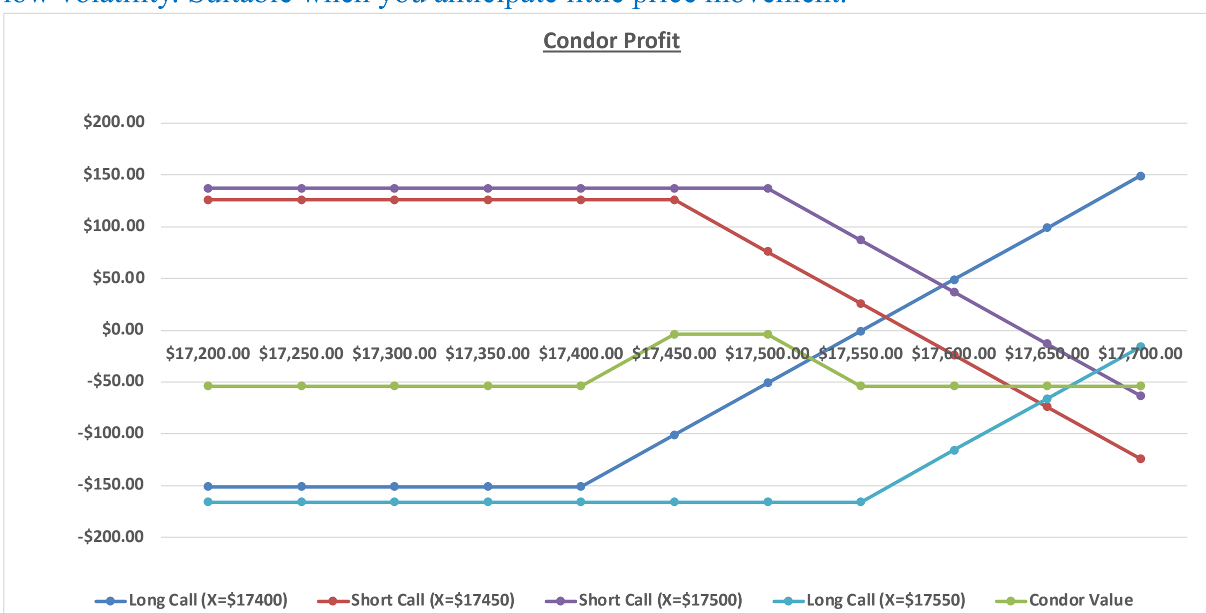
Implement a strap when you have a strongly bullish outlook. Suitable when you expect a substantial increase in the stock price.



(vii) (10%) Condor(兀鷹)

Combining a bull put spread and a bear call spread.

Limited risk, limited reward. Profits are maximized if the stock closes between the strike prices of the short options at expiration. The strategy benefits from low volatility. Implement a condor when you expect the stock to trade within a specific range with low volatility. Suitable when you anticipate little price movement.

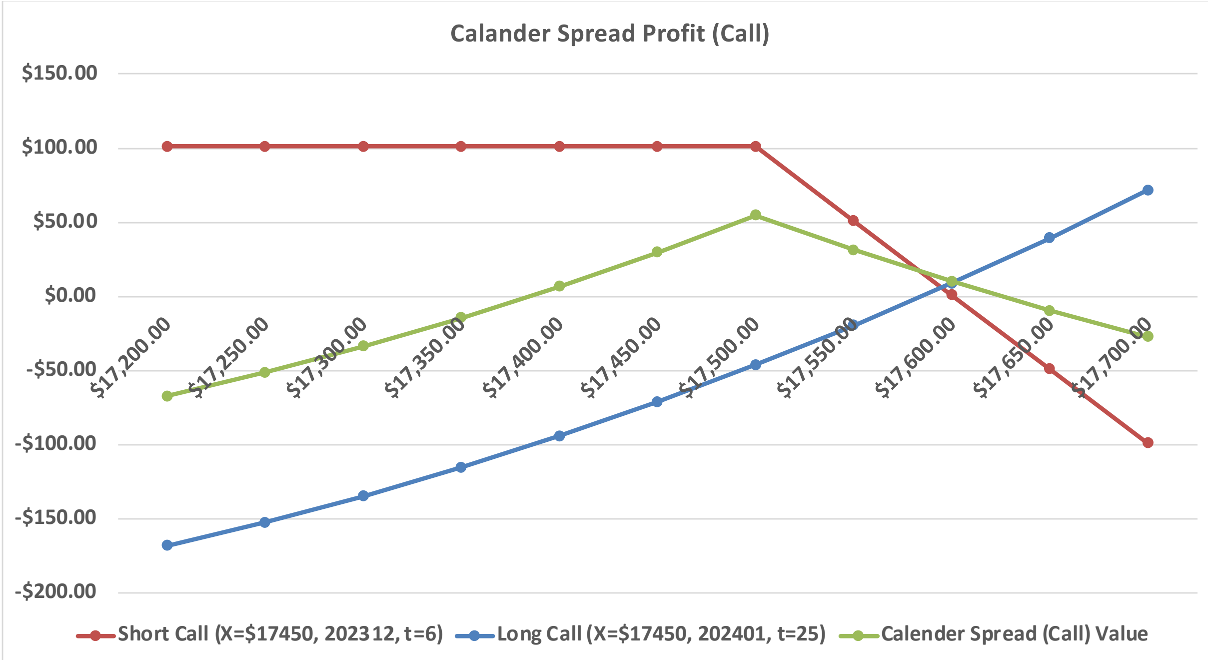


Bonus: (10%)

(vii) Calendar spread (using the Black-Scholes price for the long-maturity option and duplicate Figure 12.8 or Figure12.9)

Buying and selling options with the same strike price but different expiration dates.

Profits from time decay. If the stock price remains near the strike price, the investor can profit as the shorter-term option loses value faster than the longer-term option. Maximum loss occurs if the stock price makes a significant move in either direction. Implement a calendar spread when you expect low volatility in the short term and higher volatility in the long term. Suitable when you anticipate minimal price movement in the near term.



II Option Valuation (48%)

Use the Black-Scholes formula to compute the value of the following index put option on <u>2023/12/12</u>.

Closing Index level (12/11) = 17,418.34
Closing Index level (12/12) = 17,450.63

Exercise price = 17450
Interest rate=1.1%
Expiration month= December 2023

Assume there are 252 trading days a year. Please use the settlement price during the *<u>regular trading session</u>* as the market value of the option.

(i) (8%) First estimate average (implied) dividend yield q using the result of put call parity as follows: $q = -\frac{1}{T}\ln\frac{c - p + Ke^{-rT}}{S_0}$ Please use the December 2003 call and put contract with K=17450. (Use this estimate for (ii) to (v))

q= 0.00552108

2023/12/11					
	Call at strike price	$17,450.00		Settlement price	$113.00
	Put at strike price	$17,450.00		Settlement price	$142.00
	Expiration (in years)	0.02777778			
	Risk-free rate (annual)	0.011			
	Stock Price	17,418.34			
	Exercise price	17450			
	Dividend yield (annual)	0			
	(implied) dividend yield (q)	**0.00552108**			

(ii) (8%) (Historical volatility) Following the method in Section 15.4, please use the historical TAIEX index data from Oct. 12 to Dec. 11 to calculate log returns and estimate volatility as the annualized standard deviation of the log returns ($\hat{\sigma} = s/\sqrt{\tau}$). What is the estimate of the volatility based on historical data? What is the theoretical value of the put option on 2023/12/12 based on this estimate of volatility?

Historical Volatility: 0.115786185

Theoretical value of the put:122.9049

INPUTS		OUTPUTS	
Standard deviation (annual)	0.1158	d1	0.0183
Maturity (in years)	0.02381	d2	0.0004
Risk-free rate (annual)	0.011	N(d1)	0.5073
Stock Price	17,450.63	N(d2)	0.5002
Exercise price	17450	B/S call value	125.8107
Dividend yield (annual)	0.005521	B/S put value	122.9049

(iii) (8%) (Implied volatility) Using the Black-Scholes formula, what is the estimate the implied volatility of the same contract on the previous trading day (2023/12/11)? What is the theoretical value of the put option on 2023/12/12 based on this estimate of volatility?

Implied Volatility: 0.1096

Theoretical value of the put: 116.2204

INPUTS		OUTPUTS	
Standard deviation (annual)	0.1096	d1	-0.0820
Maturity (in years)	0.027778	d2	-0.1002
Risk-free rate (annual)	0.011	N(d1)	0.4673
Stock Price	17,418.34	N(d2)	0.4601
Exercise price	17450	B/S call value	113.0003
Dividend yield (annual)	0.005521	B/S put value	142.0003

INPUTS			OUTPUTS	
Standard deviation (annual)	0.1096		d1	0.0183
Maturity (in years)	0.02381		d2	0.0014
Risk-free rate (annual)	0.011		N(d1)	0.5073
Stock Price	17,450.63		N(d2)	0.5006
Exercise price	17450		B/S call value	119.1263
Dividend yield (annual)	0.005521		B/S put value	116.2204

(iv) (8%) Compare the results in (ii) and (iii) and explain your findings.

Historical Volatility: 0.115786185
Theoretical value of the put: 122.9049
Implied Volatility: 0.1096
Theoretical value of the put: 116.2204

The implied volatility and theoretical values of the put using implied volatility are lower than the theoretical ones. The implied volatility and theoretical values of a put option may be lower than historical or theoretical levels due to reduced market uncertainty, positive economic indicators, stable interest rates, a lack of imminent significant events, and a shift in investor sentiment towards lower perceived risk. Additionally, decreased speculative trading activity, changes in supply and demand dynamics, and seasonal factors can contribute to the observed lower implied volatility. These factors collectively influence market participants' expectations and impact option pricing.

(v) (8%) Compare the market value to the theoretical value of this put. What are the market value, time value, and intrinsic value of this option on 2023/12/11 and 2023/12/12. Explain your findings.

12/11:
Market value: 142
Intrinsic value: max(X-St) = max(17450 - 17,418.34, 0) = 31.66
Time value: Market value – Intrinsic value = 142 – 31.66 = 110.34

12/12:
Market value: 113
Intrinsic value: max(X-St) = max(17450 - 17,450.63, 0) = 0
Time value: Market value – Intrinsic value = 113 – 0 = 113

When the stock price falls, the intrinsic value increases, leading to an increase in the put price.

K=	$17,450.00			
	Market value	Stock Price	intrinsic value	time value
2023/12/11	142	17,418.34	31.6599999999999	110.34
2023/12/12	113	17,450.63	0	113

12/12:

Market value: 113

Put value using historical volatility: 122.9049, 9.9049 higher than the market value.

Put value using implied volatility: 112.2004, -0.7996 lower than the market value.

	Market value	Theoretical (Historical)	Theoretical (Implied)	Gap (Historical)	Gap (Implied)
2023/12/11	142				
2023/12/12	113	122.9049	112.2004	9.9049	-0.7996

(vi) (8%) (Alternative formula) What is the theoretical value of the put option on 2023/12/12 using Equation (17.14)? Please use the futures price (settlement price) on 12/11 to calculate the implied volatility of the same contract on the previous trading day (2023/12/11).

Implied Volatility: 0.1106

Theoretical value of the put: 110.7644

INPUTS		OUTPUTS	
Standard deviation (annual)	0.1106	d1	-0.0738
Maturity (in years)	0.0277778	d2	-0.0922
Risk-free rate (annual)	0.011	N(d1)	0.4706
Stock Price	17,418.00	N(d2)	0.4633
Exercise price	17450	B/S call value	115.3312
Dividend yield (annual)	0	B/S put value	142.0000

INPUTS		OUTPUTS	
Standard deviation (annual)	0.1106	d1	0.0641
Maturity (in years)	0.0238095	d2	0.0471
Risk-free rate (annual)	0.011	N(d1)	0.5256
Stock Price	17,462.00	N(d2)	0.5188
Exercise price	17450	B/S call value	127.3340
Dividend yield (annual)	0	B/S put value	110.7644