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3.103

題目：Use MATLAB's f f t command to repeat Problem 3.48.

Problem 3.48:

(a)

$$x[n] = \cos\left(\frac{6\pi}{17}n + \frac{\pi}{3}\right)$$

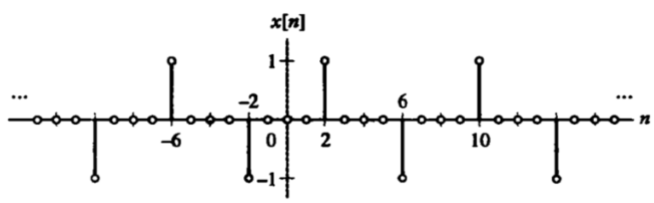
(b)

$$x[n] = 2\sin\left(\frac{14\pi}{19}n\right) + \cos\left(\frac{10\pi}{19}n\right) + 1$$

(c)

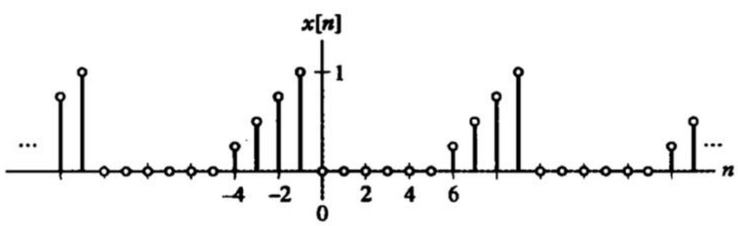
$$x[n] = \sum_{m=-\infty}^{\infty}(-1)^m(\delta[n-2m] + \delta[n+3m])$$

(d)



(a)

(e)



(b)

作法：

(a)

```
%3.103a
x = cos([0:16]*6*pi/17 + pi/3); %x[n]
X = fft(x)/17; %對 x[n]做 fourier 轉換，N=17（一個週期）
stem(0:16,abs(X));
ylabel('|Xa[k]|');
stem(0:16,rad2deg(angle(X)));
ylabel('Xa[k] deg');
```

(b)

```
%3.103b
x = 2*sin([0:18]*14*pi/19)+cos([0:18]*10*pi/19)+ones(1,19); %x[n]
X = fft(x)/19; %對 x[n]做 fourier 轉換，N=19（一個週期）
stem(0:18,abs(X));
ylabel('|Xb[k]|');
stem(0:18,rad2deg(angle(X)));
ylabel('Xb[k] deg');
```

(c)

```
%3.103c
x=[2,0,-1,-1,1,0,0,0,1,-1,-1,0]; %x[n]
X = fft(x)/12; %對 x[n]做 fourier 轉換，N=12（一個週期）
stem(0:11,abs(X));
ylabel('|Xc[k]|');
stem(0:11,rad2deg(angle(X)));
ylabel('Xc[k] deg');
```

(d)

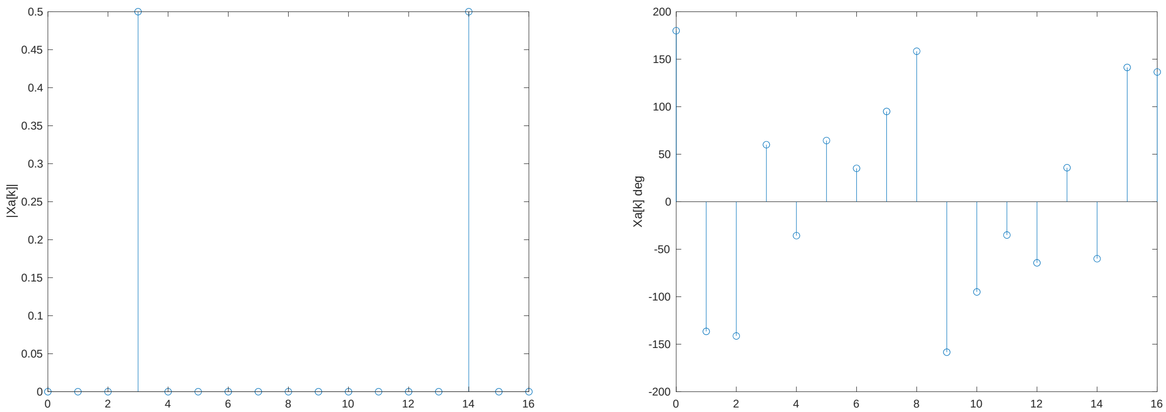
```
%3.103d
x=[0,0,1,0,0,0,-1,0]; %x[n]
X = fft(x)/8; %對 x[n]做 fourier 轉換，N=8（一個週期）
stem(0:7,abs(X));
ylabel('|Xd[k]|');
stem(0:7,rad2deg(angle(X)));
ylabel('Xd[k] deg');
```

(e)

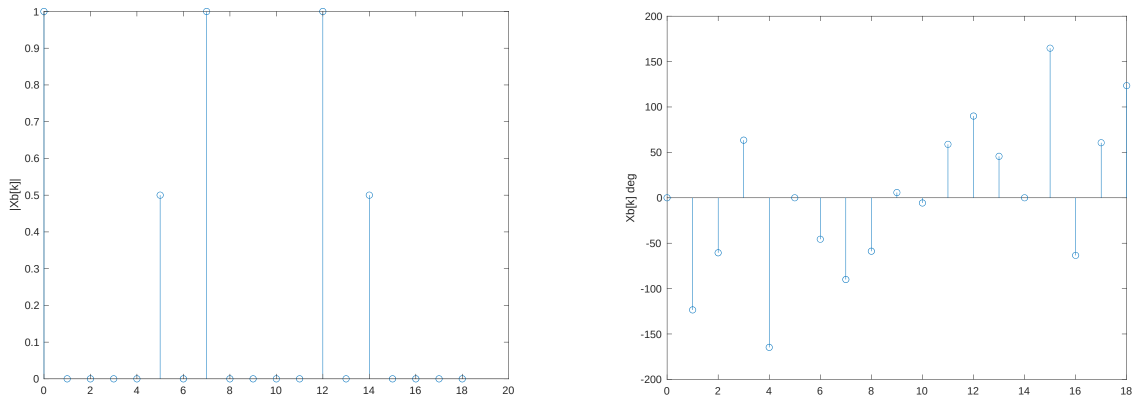
```
%3.103e
x=[0,0,0,0,0,0,1/4,1/2,3/4,1]; %x[n]
X = fft(x)/10; %對 x[n]做 fourier 轉換，N=10（一個週期）
stem(0:9,abs(X));
ylabel('|Xe[k]|');
stem(0:9,rad2deg(angle(X)));
ylabel('Xe[k] deg');
```

結果：

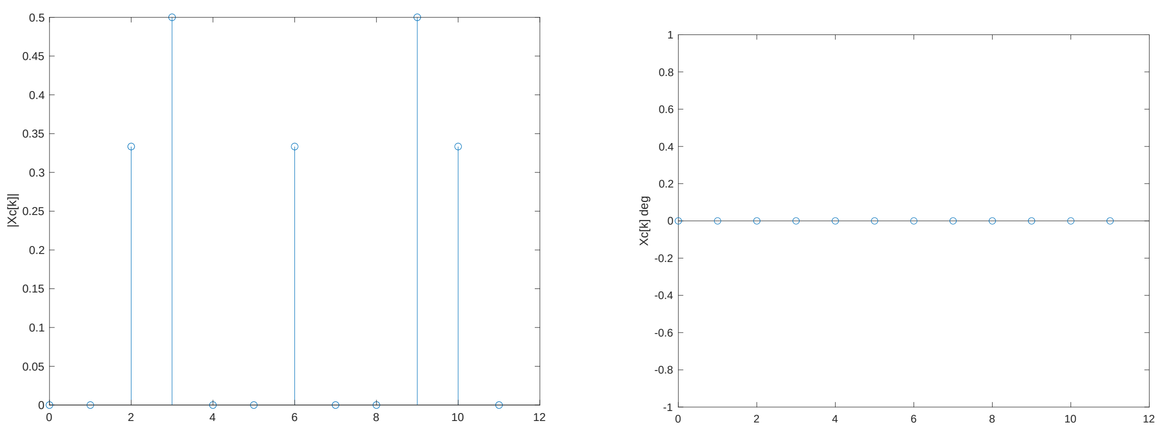
(a)



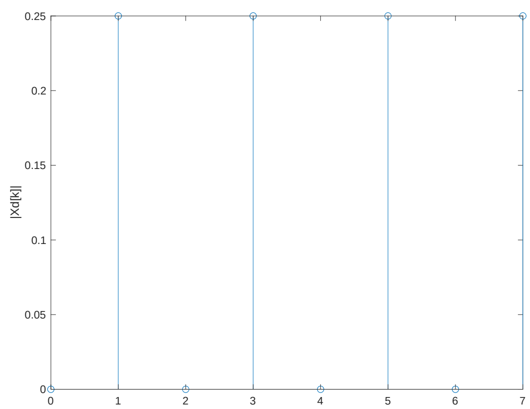
(b)

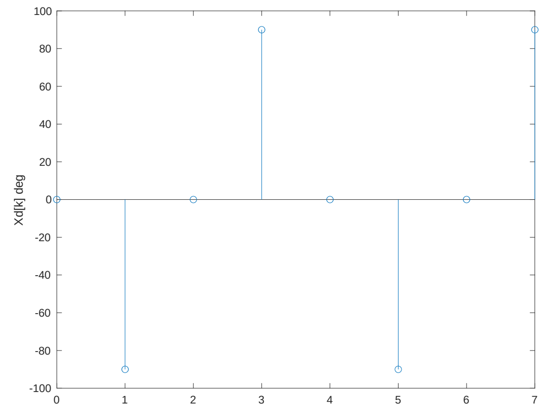


(c)

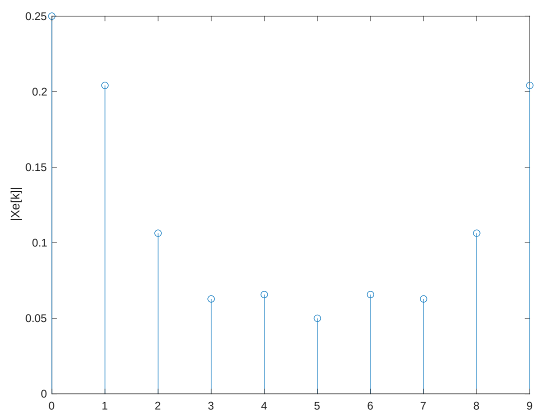


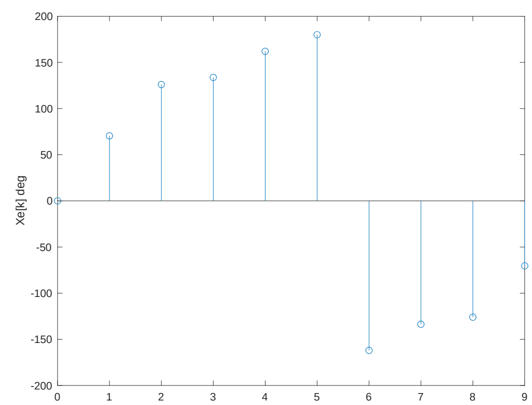
(d)





(e)





3.107

題目：Let x(t)be the triangular wave depicted in Fig. P3.107.

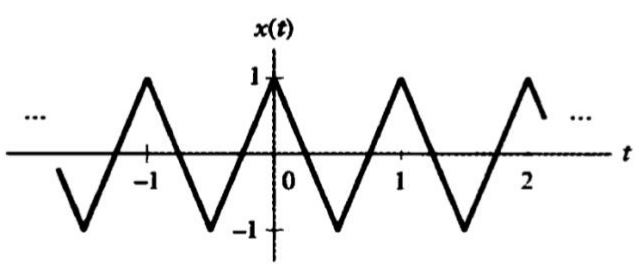


FIGURE P3.107

(a) Find the FS coefficients X[k].

(b) Show that the FS representation for x(t) can be expressed in the form $x(t) = \sum_{k=0}^{\infty} B[k]\cos(k\omega_o t).$

(c) Define the]-term partial-sum approximation to x(t) as $\hat{x}_J(t) = \sum_{k=0}^{J} B[k]\cos(k\omega_o t).$

Use MATLAB to evaluate and plot one period of the]th term in this sum and $\hat{x}_J(t)$ for]=1,3,7,29,and99.

作法：

前面的(a)、(b)兩題如下：

(a)
FS coefficient:

$$X[k] = \frac{1}{T}\int_0^T x(t)\, e^{-j\frac{2\pi}{T}kt}\, dt, \quad T=1$$

$$x(t) = 2y(t+\tfrac{1}{4}) - 1$$

$$Y[k] = \begin{cases} \frac{1}{2} &, k=0 \\ \frac{2\sin(\frac{k\pi}{2})}{jk^2\pi^2} &, k\neq 0 \end{cases}$$

$$x(t-t_0) \xleftrightarrow{FS;\, W_0} e^{-jkw_0t} X[k]$$

$$X[k] = 2e^{jk2\pi\frac{1}{4}} Y[k] - \delta[k]$$

$$X[k] = \begin{cases} -\frac{1}{2} &, k=0 \\ \frac{4\sin(\frac{k\pi}{2})}{k^2\pi^2} e^{j\frac{\pi}{2}(k-1)} &, k\neq 0 \end{cases}$$

(b) $\because$ $x(t)$ 是 real and even

$\therefore$ $X[k]$ also real and even with $X[k]=X[-k]$

$$x(t) = \sum_{k=1}^{\infty} X[k] e^{jW_0kt} + \sum_{k=-\infty}^{-1} X[k] e^{jw_0kt} + X[0]$$

$$= \sum_{k=1}^{\infty} X[k] e^{jW_0kt} + \sum_{k=1}^{\infty} X[-k] e^{-jW_0kt} + X[0]$$

$(\because X[k]=X[-k]$

$$= \sum_{k=0}^{\infty} X[k] (e^{jW_0kt} + e^{-jW_0kt}) + X[0]$$

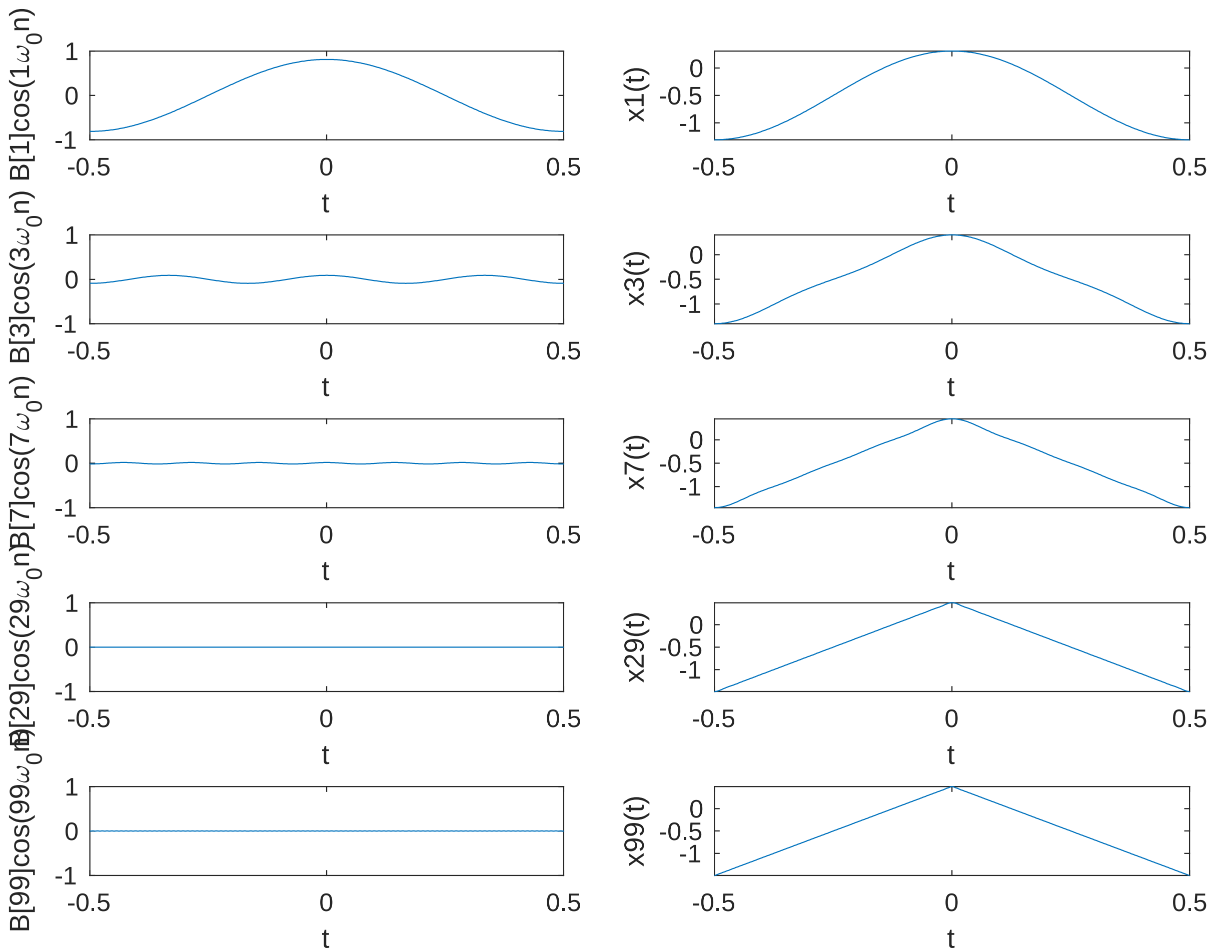
$$= \sum_{k=0}^{\infty} B[k] \cos(kw_0t)$$

where $B[k] = \begin{cases} X[0] &, k=0 \\ 2X[k] &, k\neq 0 \end{cases}$

(c) 將(a)、(b)的 X [k]、 x [t]式子寫入 MATLAB，並使用迴圈求出 partial sum。

```
%3.107
k = 1:1:99;
t = linspace(-0.5,0.5,400);
B = zeros(1,100);
B(1) = -0.5; % coefficient for k=0
B(2:1:100) = 8*sin(k*pi/2).*exp(1i*pi*(k-1)/2)./((pi*k).^2); % coefficient for k!=0
xJhat(1,:) = B(1)*cos(0*2*pi*t); % term in sum for k=0
% accumulate partial sum
for k = 2:1:100
Bcos(k,:) = B(k)*cos((k-1)*2*pi*t);
xJhat(k,:) = xJhat(k-1,:)+B(k)*cos((k-1)*2*pi*t);
end
j = [1 3 7 29 99]+1;
for i = 1:1:5
subplot(5,2,2*i-1);plot(t,Bcos(j(i),:));
ylabel(['B[' num2str(j(i)-1) ']cos(' num2str(j(i)-1) '\omega_0n)']);
xlabel('t');axis([-0.5 0.5 -1 1]);
subplot(5,2,2*i);plot(t,xJhat(j(i),:));
ylabel(['x' num2str(j(i)-1) '(t)']);
xlabel
```

結果：



3.109

題目：Use MATLAB to repeat Example 3.15, with the following values for the time constant:

(a) RC = 0.01 s.

(b) RC= 0.1 s.

(c) RC=1s.

Example 3.15:

Let us find the FS representation for the output y(t) of the RC circuit depicted in Fig, 3.2 in response to the square-wave input depicted in Fig. 3.21, assuming that T0/T = 1/4, T = 1s, and RC = 0.1 s.

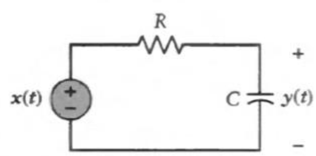


FIGURE 3.2 *RC* circuit for Example 3.1.

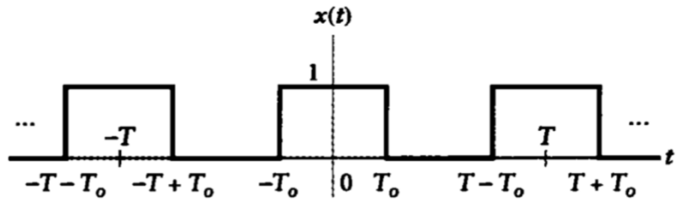


FIGURE 3.21 Square wave for Example 3.13.

作法：

Solution: If the input to an LTI system is expressed as a weighted sum of sinusoids, then the output is also a weighted sum of sinusoids. As shown in Section 3.2, the kth weight in the output sum is given by the product of the kth weight in the input sum and the system frequency response evaluated at the kth sinusoid's frequency. Hence, if

$$x(t) = \sum_{k=-\infty}^{\infty} X[k]e^{jk\omega_o t},$$

then the output is

$$y(t) = \sum_{k=-\infty}^{\infty} H(jk\omega_o)X[k]e^{jk\omega_o t},$$

where $H(j\omega)$ is the frequency response of the system. Thus,

$$y(t) \xleftrightarrow{FS;\,\omega_o} Y[k] = H(jk\omega_o)X[k].$$

In Example 3.1, the frequency response of the *RC* circuit was determined to be

$$H(j\omega) = \frac{1/RC}{j\omega + 1/RC}.$$

The FS coefficients of the square wave were given in Eq. (3.23). Substituting for $H(jk\omega_o)$ with $RC = 0.1$ s and $\omega_o = 2\pi$, and using $T_o/T = 1/4$, gives

$$Y[k] = \frac{10}{j2\pi k + 10}\frac{\sin(k\pi/2)}{k\pi}.$$

The $Y[k]$ go to zero in proportion to $1/k^2$ as k increases, so a reasonably accurate representation of $y(t)$ may be determined with the use of a modest number of terms in the FS. We plot the magnitude and phase spectra of $X[k]$ and $Y[k]$, and we determine $y(t)$ using the approximation

$$y(t) \approx \sum_{k=-100}^{100} Y[k]e^{jk\omega_o t}. \tag{3.30}$$

由以上課本的 Example 3.15 的 solution，可知

$$
\begin{aligned}
Y[k] &= \frac{\frac{1}{RC}}{j2\pi k + \frac{1}{RC}} \frac{\sin(\frac{k\pi}{2})}{k\pi} \\
y(t) &= \sum_{k=-100}^{100} Y[k] e^{j2\pi kt}
\end{aligned}
$$

將以上兩式子代入 MATLAB，並使用迴圈求出 partial sum。

(a)

```
%3.109a
k = 1:1:100;
t = linspace(-0.5,0.5,400);
Y = zeros(1,100);
RC=0.01;
Y(1)=0;
Y(2:1:101) = (1/RC)./(1i*pi*k*2+(1/RC)).*sin(k*pi/2)./(k*pi); % % coefficient for k
yJhat(1,:) = Y(1).*exp(1i*pi*2*0*t); % term in sum for k=0
% accumulate partial sum
for k = 2:1:100
Yexp(k,:) = Y(k).*exp(1i*pi*(k-1)*2*t);
yJhat(k,:) = yJhat(k-1,:)+Y(k).*exp(1i*pi*(k-1)*2*t);
end
j = 100;
plot(t,yJhat(j,:));
ylabel(['y(t)']);
xlabel('t');
```

(b)

```
%3.109b
k = 1:1:100;
t = linspace(-0.5,0.5,400);
Y = zeros(1,100);
RC=0.1;
Y(1)=0;
```

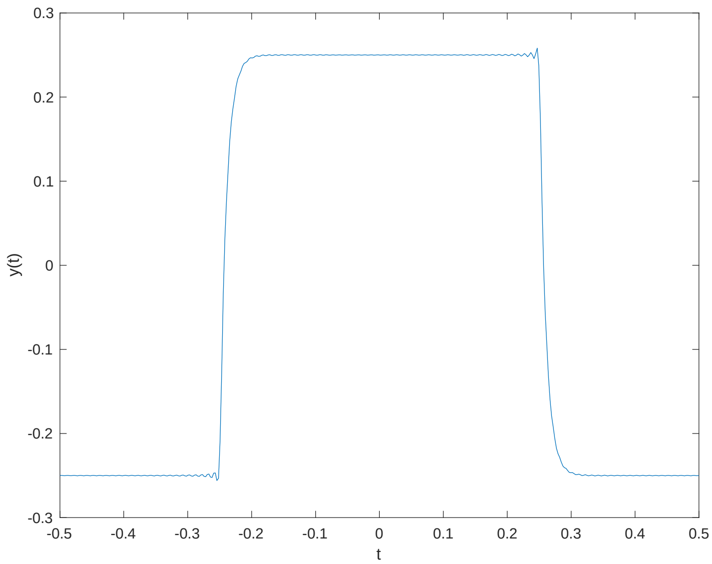
```
Y(2:1:101) = (1/RC)./(1i*pi*k*2+(1/RC)).*sin(k*pi/2)./(k*pi); % % coefficient for k
yJhat(1,:) = Y(1).*exp(1i*pi*2*0*t); % term in sum for k=0
% accumulate partial sum
for k = 2:1:100
Yexp(k,:) = Y(k).*exp(1i*pi*(k-1)*2*t);
yJhat(k,:) = yJhat(k-1,:)+Y(k).*exp(1i*pi*(k-1)*2*t);
end
j = 100;
plot(t,yJhat(j,:));
ylabel(['y(t)']);
xlabel('t');
```

(c)

```
%3.109c
k = 1:1:100;
t = linspace(-0.5,0.5,400);
Y = zeros(1,100);
RC=1;
Y(1)=0;
Y(2:1:101) = (1/RC)./(1i*pi*k*2+(1/RC)).*sin(k*pi/2)./(k*pi); % % coefficient for k
yJhat(1,:) = Y(1).*exp(1i*pi*2*0*t); % term in sum for k=0
% accumulate partial sum
for k = 2:1:100
Yexp(k,:) = Y(k).*exp(1i*pi*(k-1)*2*t);
yJhat(k,:) = yJhat(k-1,:)+Y(k).*exp(1i*pi*(k-1)*2*t);
end
j = 100;
plot(t,yJhat(j,:));
ylabel(['y(t)']);
xlabel('t');
```

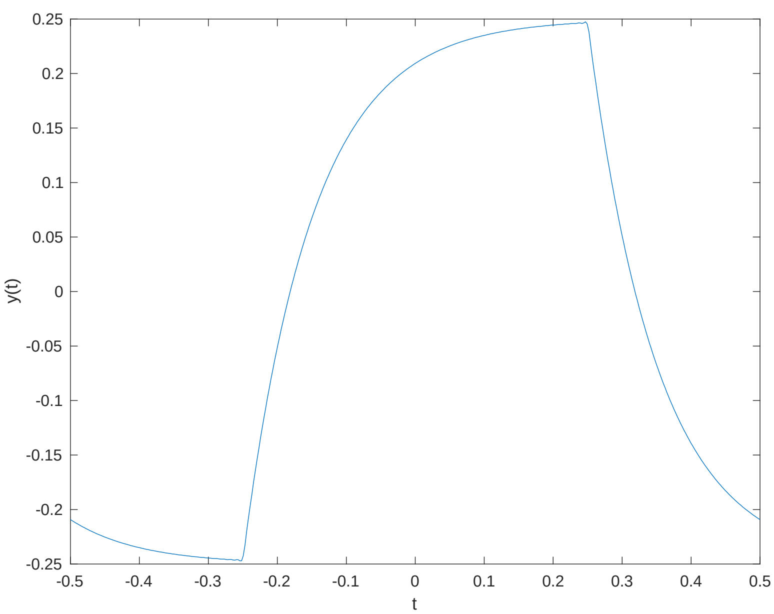
結果：

(a)



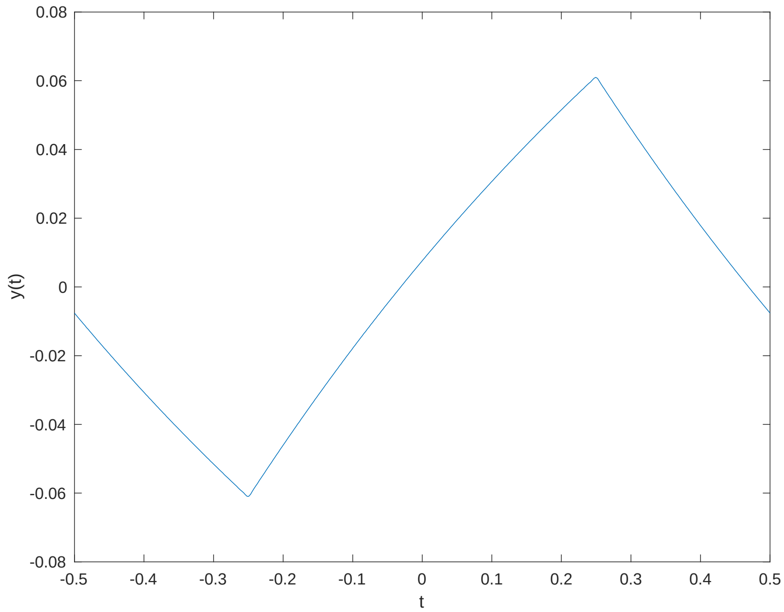
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
y(t)
-0.5
-0.4
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
0.4
0.5
t

(b)



0.25
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
-0.25
y(t)
-0.5
-0.4
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
0.4
0.5
t

(c)



3.111

題目：This experiment builds on Problem 3.72.

(a)Graph the magnitude response of the circuit depicted in Fig. P3.72. Use 501 logarithmically spaced frequencies from 1 rad/s to 10^5 rad/s. You can generate N logarithmically spaced values between 10^d1 and 10^d2 by using the MATLAB command logspace (d1,d2,N).

(i) Assume that L = 10 mH.

(ii) Assume that L = 4 mH.

(b)Determine and plot the output, using at least 99 harmonics in a truncated FS expansion, if the input is the square wave depicted in Fig. 3.21 with T=2pi× 10^(-3) and T0=(pi/2) × 10^(-3)

(i) Assume that L = 10 mH.

(ii) Assume that L = 4 mH.

Problem 3.7:

3.72 Consider the RLC circuit depicted in Fig. P3.72 with input $x(t)$ and output $y(t)$.

(a) Write a differential-equation description for this system and find the frequency response. Characterize the system as a filter.

(b) Determine and plot the output if the input is the square wave depicted in Fig. 3.21 with $T = 2\pi \times 10^{-3}$ and $T_o = (\pi/2) \times 10^{-3}$, assuming that $L = 10$ mH.

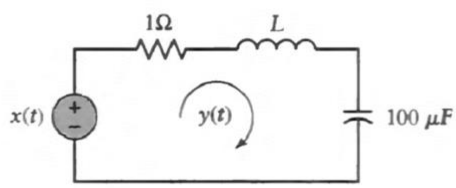


FIGURE P3.72

Fig. 3.21:

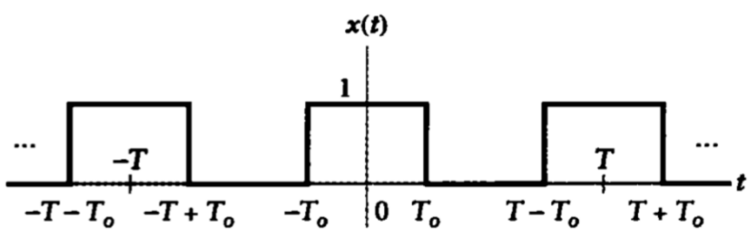


FIGURE 3.21 Square wave for Example 3.13.

作法：

(a) 首先先找出 H(jw)再將其代入 MATLAB，再畫出 magnitude response。

$$x(t) = y(t) + L\frac{d}{dt}y(t) + \frac{1}{C}\int_{-\infty}^{\infty} y(\tau)\,d\tau$$

$$\frac{d}{dt}x(t) = \frac{d}{dt}y(t) + L\frac{d^2}{dt^2}y(t) + \frac{1}{C}y(t)$$

$$H(jw) = \frac{jw}{\frac{1}{C} + jw + L(jw)^2}$$

(i)10mh

```
%3.111a1
f= logspace(0, 5, 501);
```

```
C= 100*(10.^(-6));
L= 10*(10.^(-3));
w= 2*pi*f;
s= 1i*w;
H= s./ (s.^2*L + s + 1/C); %transfer function of the bandpass filter
magnitude = 20*log10(abs(H));
plot(w, magnitude);
xlabel('w(rad/s)');
ylabel('20log(|H(jw)|)');
title('L=10mH magnitude response in dB');
```

(ii)4mh

```
%3.111a2
f= logspace(0, 5, 501);
C= 100*(10.^(-6));
L= 4*(10.^(-3));
w= 2*pi*f;
s= 1i*w;
H= s./ (s.^2*L + s + 1/C); %transfer function of the bandpass filter
magnitude = 20*log10(abs(H));
plot(w, magnitude);
xlabel('w(rad/s)');
ylabel('20log(|H(jw)|)');
title('L=4mH magnitude response in dB');
```

(b) 首先先找出 Y[k]，再找出 y[t]，再將其代入 MATLAB，再畫出 output。

$$H[k] = \frac{jk\omega_0}{\frac{1}{C} + jk\omega_0 + L(jk\omega_0)^2}$$

$$X[k] = \frac{2\sin(k\omega_0 T_0)}{T k \omega_0}$$

$$Y[k] = \frac{2\sin(k\omega_0 T_0)}{T k \omega_0} \cdot \frac{jk\omega_0}{\frac{1}{C} + jk\omega_0 + L(jk\omega_0)^2}$$

$$y(t) = \sum_{k=-k_0}^{k_0} Y[k] e^{j\omega_0 k t}$$

$$T_0 = \pi/2 \times 10^{-3} \qquad k_0 \geq 49$$

$$T = 2 \times \pi \times 10^{-3} \qquad |t| \leq \pi \times 10^{-3}$$

$$\omega_0 = 2\pi/T$$

(i)10mh

```
%3.111b1
k = 1:1:50;
C= 100*(10.^(-6));
L= 10*(10.^(-3));
w= 1000;
s= 1i*w*k;
H= s./ (s.^2*L + s + 1/C); %transfer function of the bandpass filter
T0=pi./2000;
T=pi./500;
t = linspace(-pi*10.^(-3),pi*10.^(-3),400);
Y = zeros(1,50);
Y(1)=0;
```

```
Y(2:1:51) = H.*(sin(k*w*T0)*2)./(k*w*T); ; % coefficient for k!=0
yJhat(1,:) = Y(1).*exp(1i*w*0*t); % term in sum for k=0
% accumulate partial sum
for k = 2:1:50
Yexp(k,:) = Y(k).*exp(1i*(k-1)*w*t);
yJhat(k,:) = yJhat(k-1,:)+Y(k).*exp(1i*w*(k-1)*t);
end
j = 50;
plot(t,yJhat(j,:));
ylabel(['y(t)']);
xlabel('t');
title('L=10mH output');
```

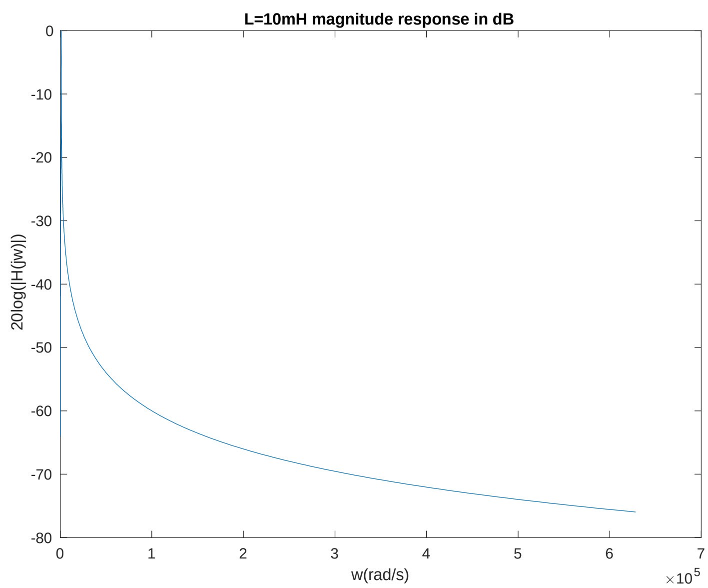
(ii)4mh

```
%3.111b2
k = 1:1:50;
C= 100*(10.^(-6));
L= 4*(10.^(-3));
w= 1000;
s= 1i*w*k;
H= s./ (s.^2*L + s + 1/C); %transfer function of the bandpass filter
T0=pi./2000;
T=pi./500;
t = linspace(-pi*10.^(-3),pi*10.^(-3),400);
Y = zeros(1,50);
Y(1)=0;
Y(2:1:51) = H.*(sin(k*w*T0)*2)./(k*w*T); ; % coefficient for k!=0
yJhat(1,:) = Y(1).*exp(1i*w*0*t); % term in sum for k=0
% accumulate partial sum
for k = 2:1:50
Yexp(k,:) = Y(k).*exp(1i*(k-1)*w*t);
yJhat(k,:) = yJhat(k-1,:)+Y(k).*exp(1i*w*(k-1)*t);
end
```

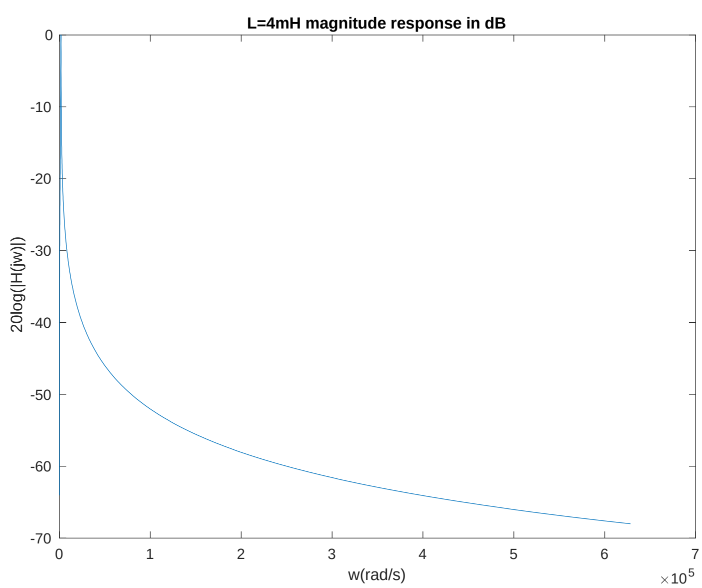
```
j = 50;
plot(t,yJhat(j,:));
ylabel(['y(t)']);
xlabel('t');
title('L=4mH output');
```

結果：

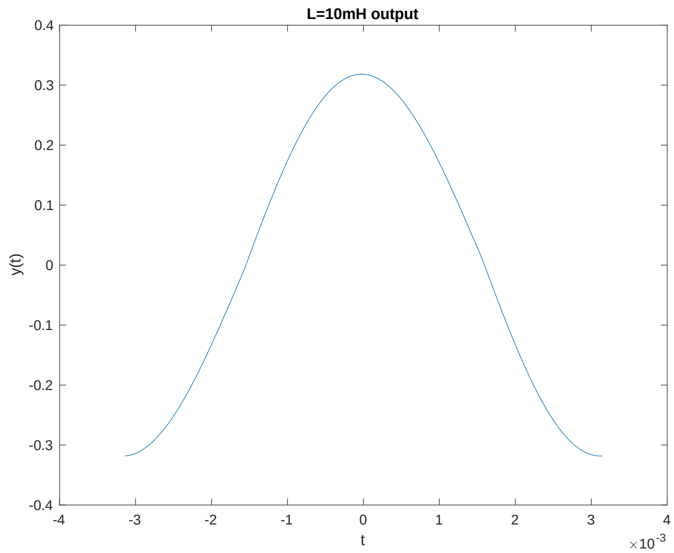
(a)(i)



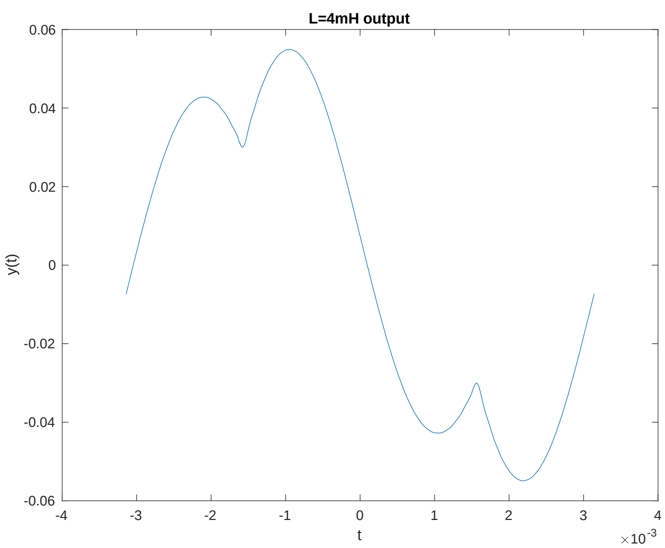
(a)(ii)



(b)(i)


L=10mH output
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
y(t)
-4
-3
-2
-1
0
1
2
3
4
t
×10^-3

(b)(ii)


L=4mH output
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
y(t)
-4
-3
-2
-1
0
1
2
3
4
t
×10^-3