## 5)

令$Y=X_1/X_2, Z=\sqrt{X_1^2+X_2^2}$

$f_Y(y)=\int_0^{\infty}|x|f_{X_1}(xy)f_{X_2}(x)dx=\frac{1}{\pi(1+y^2)}exp\{-\frac{x^2(1+y^2)}{2\sigma^2}\}\Big|_0^{\infty}=\frac{1}{\pi(1+y^2)}$

$f_Z(z)=f_{\chi_2^2}((\frac{z}{\sigma})^2)=C\frac{1}{2\Gamma(1)}exp\{-z^2/2\sigma^2\}=\frac{\sqrt{2}}{\sqrt{\pi}\sigma}exp\{-z^2/2\sigma^2\}$

当$Y=y, Z=z$时

$|X_1|=z\sqrt{\frac{y^2}{1+y^2}}, |X_2|=z\sqrt{\frac{1}{1+y^2}}$

代入正态分布密度函数知

$f(y,z)=\frac{1}{\pi\sigma^2}exp\{-z^2/2\sigma^2\}=f_Y(y)f_Z(z)$

$\therefore$相互独立

## 8.1)

$1-F^n(0.99)=1-0.99^n\geq 0.95$

$\therefore n\geq\frac{ln0.05}{ln0.99}$

## 8.2)

$(X_{(1)},X_{(n)})$的联合密度为

$n(n-1)(F(y)-F(x))^{n-2}p(x)p(y)=n(n-1)(y-x)^{n-2}$

$\therefore R_n$的密度函数为

$\int_0^{1-r}n(n-1)r^{n-2}dz=n(n-1)r^{n-2}(1-r),\quad 0<r<1$

## 8.3)

$\because Z=2n(1-R_n)$

$\therefore f_Z(z)=Cn(n-1)(1-(z/2n))^{n-2}(z/2n)=C(n-1)ze^{-z/2}=\frac{1}{4\Gamma(2)}ze^{-z/2}$

$\therefore Z\sim\chi_4^2$

## 16)

$\because \chi^2$分布的特征函数为

$\phi(t)=(1-2it)^{-m/2}$

$\therefore \phi_{\overline{X}}(t)=(1-2it/n)^{-nm/2}$

满足$\Gamma(nm/2,2/n)$

## 17)

对于$X\sim N(0,1)$

$\because E(X^2)=1,D(X^2)=2$

$\therefore E(\chi^2)=n,D(\chi^2)=2n$

$\therefore \nu=\sqrt{D(\chi^2)}/E(\chi^2)=\sqrt{\frac{2}{n}}$

$\therefore \beta_2=\frac{12n^2+48n}{4n^3}-3=\frac{12}{n^2}+\frac{3}{n}-3$

## 22)

$D(\xi_n)=n/(n-2),\quad n\geq 3$