

34)

令 $T_1 = \sum X_i, T_2 = \sum Y_i$

$a = \frac{T_1 + T_2}{m + n}$ 是无偏估计

$\sigma^2 = \frac{\sum (X_i - a)^2 + \sum (Y_i - a)^2}{m + n - 1} / 2$ 是无偏估计

$$f(X, Y, \theta) = \left(\frac{1}{2\pi\sigma^2}\right)^{\frac{n}{2}} \left(\frac{1}{4\pi\sigma^2}\right)^{\frac{m}{2}} \exp\left\{-\frac{m}{2\sigma^2} \left(\frac{1}{m} \sum X_i^2 + a^2 - 2a\bar{X}\right) - \frac{n}{4\sigma^2} \left(\frac{1}{n} \sum Y_i^2 + a^2 - 2a\bar{Y}\right)\right\}$$

存在一一对应，是充分完全统计量

所以是UMVUE

35.1)

显然根据定义，服从负二项分布

T 服从负二项分布

$$\therefore P(X = x | T = t) = P(X = x, T = t) / P(T = t) = \frac{\prod_{i=1}^{n-1} \theta(1-\theta)^{x_i} * \theta(1-\theta)^{t-\sum x_i}}{C_{t+n-1}^{n-1} \theta^n (1-\theta)^t} = \frac{1}{C_{t+n-1}^{n-1}}$$

是充分统计量

用因子分解定理

$$f(x, \theta) = \theta^n (1 - \theta)^t$$

是充分统计量

其自然参数空间有内点，是完全统计量

35.2)

$$E_{\theta} T = \sum k P(X = k) = n(1 - \theta) / \theta$$

$$\therefore \hat{\theta}^{-1} = (t + n) / n$$

$$\therefore \hat{\theta}^{-1} \text{ 是UMVUE}$$

35.3)

$$\because E(\phi) = \theta$$

是无偏估计

$$E_{\theta}(\phi(X_1) | T = t) = n/t$$

$$\therefore \hat{\theta} = n/t$$

是UMVUE