12)

$$\therefore lpha \overline{X} + eta \overline{Y} \sim N(lpha \mu_1 + eta \mu_2, lpha^2 \sigma^2/m^2 + eta^2 \sigma^2/m^2)$$

$$\therefore rac{lpha \overline{X} + eta \overline{Y} - lpha \mu_1 - eta \mu_2}{\sigma \sqrt{lpha^2/m^2 + eta^2/m^2}} \sim N(0,1)$$

$$\cdots mS_{1m}^2/\sigma^2 \sim \chi_{m-1}^2, nS_{1n}^2/\sigma^2 \sim \chi_{n-1}^2$$

$$T \sim t(m+n-2)$$

26)

$$\diamondsuit Z_i = X_{(i)} - X_{(i-1)}, Z_1 = X_{(1)}$$

$$f(X_{(1)},\ldots,X_{(n)})=\lambda^{-n}n!exp\{-rac{1}{\lambda}\sum X_{(i)}\}$$

$$X_{(i)} = \sum Z_i$$

$$\therefore J = I$$

$$\therefore g(Z_1,\ldots,Z_n) = \lambda^{-n} n! exp\{-rac{1}{\lambda}\sum\sum Z_i\} = \lambda^{-n} n! exp\{-rac{1}{\lambda}\sum (n-i+1)Z_i\}$$

$$\therefore Z_i \sim exp((n-i+1)/\lambda)$$

$$\therefore 2(n-i+1)Z_i/\lambda \sim \chi_2^2$$

$$T : T = \sum (n-i+1)Z_i$$

$$\therefore 2T/\lambda \sim \chi^2_{2r}$$

27)

$$\Leftrightarrow Z_i = X_{(i)} - X_{(i-1)}, Z_1 = X_{(1)} - \mu$$

$$f(X_{(1)},\ldots,X_{(n)}) = \prod (1-exp\{-(X_{(i)}-\mu)/\sigma\})$$

$$\therefore X_{(i)} = \sum Z_i$$

$$\therefore J = I$$

$$\therefore g(Z_1, \dots, Z_n) = \prod (1 - exp\{-\sum Z_i/\sigma\})$$

$$\therefore 2(n-i+1)Z_i/\lambda \sim \chi_2^2$$

28)

$$F_1(y) = P(X_1 + X_2 \leq y) = \frac{\lambda^{\alpha_1 + \alpha_2}}{\Gamma(\alpha_1)\Gamma(\alpha_2)} e^{-\lambda y} \int_0^y (1 - e)^{\alpha_1 - 1} e^{\alpha_2 - 1} dt = \int_0^y y^{\alpha_1 + \alpha_2 - 1} (1 - t)^{\alpha_1 - 1} t^{\alpha_2 - 1} dt$$

$$\therefore p(y) = \lambda^{lpha_1 + lpha_2} e^{-\lambda y} y^{lpha_1 + lpha_2 - 1} rac{1}{\Gamma(lpha_1 + lpha_2)}$$

$$\therefore Y_1 \sim \Gamma(lpha_1 + lpha_2, \lambda)$$