

1)

解得 $x_1 = 1, x_2 = -1, x_3 = 1, x_4 = -1$

2)

令 $A^{-1} = B$

$b_{ii} = 1/a_{ii}$

$$b_{ij} = - \sum_{k=i+1}^j b_{kj} a_{ik} / a_{ii}$$

3)

设 $A = LU = L_1 U_1$

$L_1^{-1} L = U_1 U^{-1}$

因为都是上（下）三角矩阵

$\therefore L_1^{-1} L = U_1 U^{-1} = I$

得证

4)

利用python编程实现

```
import numpy as np
import pandas as pd

def LU(A):
    n=len(A[0])
    for i in range(n):
        if i==0:
            for j in range(1,n):
                A[j][0]=A[j][0]/A[0][0]
        else:
            for j in range(i, n):
                temp=0
                for k in range(0, i):
                    temp = temp+A[i][k] * A[k][j]
                A[i][j]=A[i][j]-temp
            for j in range(i+1, n):
                temp = 0
                for k in range(0, i ):
                    temp = temp + A[j][k] * A[k][i]
                A[j][i] = (A[j][i] - temp)/A[i][i]

    return A
```

```

x = np.zeros((10, 10))
x[0][0] = 9
for i in range(7):
    x[i+1][i+1] = 6
x[8][8] = 5
x[9][9] = 1
for i in range(8):
    x[i][i+1] = x[i+1][i] = -4
x[8][9] = x[9][8] = -2
for i in range(8):
    x[i][i+2] = x[i+2][i] = 1

print(LU(x))
'''
Out:
[[ 9.00000000e+00 -4.00000000e+00  1.00000000e+00  0.00000000e+00
  0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
  0.00000000e+00  0.00000000e+00]
 [-4.44444444e-01  4.22222222e+00 -3.55555556e+00  1.00000000e+00
  0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
  0.00000000e+00  0.00000000e+00]
 [ 1.11111111e-01 -8.42105263e-01  2.89473684e+00 -3.15789474e+00
  1.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
  0.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00  2.36842105e-01 -1.09090909e+00  2.31818182e+00
 -2.90909091e+00  1.00000000e+00  0.00000000e+00  0.00000000e+00
  0.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00  3.45454545e-01 -1.25490196e+00
  2.00392157e+00 -2.74509804e+00  1.00000000e+00  0.00000000e+00
  0.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  4.31372549e-01
 -1.36986301e+00  1.80821918e+00 -2.63013699e+00  1.00000000e+00
  0.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
  4.99021526e-01 -1.45454545e+00  1.67532468e+00 -2.54545455e+00
  1.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
  0.00000000e+00  5.53030303e-01 -1.51937984e+00  1.57945736e+00
 -2.48062016e+00  1.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
  0.00000000e+00  0.00000000e+00  5.96899225e-01 -1.57055215e+00
  5.07157464e-01 -4.29447853e-01]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
  0.00000000e+00  0.00000000e+00  0.00000000e+00  6.33128834e-01
 -8.46774194e-01  3.22580645e-03]]
'''

```