

5.1)

$$\because X \sim b(1, p)$$

$$\therefore \text{样本空间为 } X = (x_1, x_2, x_3, x_4, x_5), \quad x_i = 0, 1$$

$$\therefore P(X = (x_1, x_2, x_3, x_4, x_5)) = p^{\sum x_i} (1-p)^{5-\sum x_i}$$

5.2)

统计量不含未知参数

$\therefore X_1 + X_2, \min X_i$ 是统计量, 其他不是

5.3)

$$\begin{aligned} F_n(x) \\ &= 0, \quad x < 0 \\ &= \frac{n-m}{n}, \quad 0 \leq x < 1 \\ &= 1, \quad x \geq 1 \end{aligned}$$

6)

$$\begin{aligned} \therefore \bar{x} &= \frac{\sum x_i}{n} \\ \therefore \bar{y} &= \frac{\sum y_i}{n} = \frac{\sum(ax_i + b)}{n} = a\bar{x} + b \\ \therefore S_x^2 &= \frac{\sum(x_i - \bar{x})^2}{n-1} \\ \therefore S_y^2 &= \frac{\sum(y_i - \bar{y})^2}{n-1} = \frac{\sum(ax_i - a\bar{x})^2}{n-1} = a^2 S_x^2 \end{aligned}$$

计算得 $\bar{y} = 540, \quad S_y^2 = 2100$

1.1)

$$\because \bar{X} \sim N(20, 0.9), \quad \bar{Y} \sim N(20, 0.6)$$

由于相互独立

$$\therefore \bar{X} - \bar{Y} \sim N(0, 1.5)$$

$$\therefore P = 2(1 - \phi(\frac{0.3}{\sqrt{1.5}}))$$

查表即可

1.2)

$$\therefore (n-1)S^2 \sim \sigma^2 \chi_{n-1}^2$$

$$\therefore 9S_X^2 + 14S_Y^2 \sim 9(\chi_9^2 + \chi_{14}^2) \sim 9(\chi_{23}^2)$$

$$\therefore P = (\chi_{23}^2)^{-1} \left(\frac{164}{9} \right)$$

查表即可

14)

$$\text{令 } X'_i = X_i / \sigma_i$$

$$\text{则 } X'_i \sim N(0, 1)$$

$$\therefore \xi = \sum_i \left(\frac{X_i}{\sigma_i} - \frac{Z}{\sigma_i} \right)^2 = \sum_i \left(\frac{X_i}{\sigma_i} \right)^2 - Z^2 \sum_i \frac{1}{\sigma_i^2}$$

$$\text{令 } A = \begin{pmatrix} \frac{1}{\sigma_1} \frac{1}{\sqrt{\sum \frac{1}{\sigma_i^2}}} & \cdots & \frac{1}{\sigma_n} \frac{1}{\sqrt{\sum \frac{1}{\sigma_i^2}}} \\ a_{21} & & a_{2n} \\ \cdots & & \cdots \\ a_{n1} & & a_{nn} \end{pmatrix}$$

$$Y = AX'$$

$$\therefore Y_1 = \sqrt{\sum \frac{1}{\sigma_i^2}} Z, \quad Y_i \sim N(0, 1), \quad i \neq 1$$

$\therefore A$ 是正交矩阵

$$\therefore \xi = \sum_{i=1}^n X_i'^2 - Y_1^2 = \sum_{i=2}^n Y_i^2 \sim \chi_{n-1}^2$$