14)

令$\phi = 1/\theta^2$

$f(x,\phi) = x\phi exp\{-x^2\phi/2\}$

是指数族

$\because \frac{n}{C(\theta)}\frac{\partial C(\theta)}{\partial\theta} = -\sum T_i(X_j)$

$\therefore \frac{n}{\phi} = \sum x_i^2/2$

$\therefore \phi = 2n/\sum x_I^2$

$\therefore \theta = \sqrt{\frac{\sum x_i^2}{2n}}$

19)

先估计均值

$L(\theta,x) = (\frac{1}{\sqrt{2n}\sigma})^n exp\{\sum -\frac{(x_i-\mu)^2}{2\sigma^2}\}$

$lnL = nln(\frac{1}{\sqrt{2n}\sigma}) - \sum\frac{(x_i-\mu)^2}{2\sigma^2}$

$\because \frac{\partial lnL}{\partial\mu} = 0$

$\therefore \hat{\mu} = \sum x_i/n$

$\therefore \hat{\mu}_1 = \overline{X}, \hat{\mu}_2 = \overline{Y}$

再考虑方差

$L(\sigma^2,x) = (\frac{1}{\sqrt{2n}\sigma})^{2n} exp\{\sum -\frac{(x_i-\mu_1)^2+(y_i-\mu_2)^2}{2\sigma^2}\}$

$lnL = 2nln(\frac{1}{\sqrt{2n}\sigma}) - \sum\frac{(x_i-\mu_1)^2+(y_i-\mu_2)^2}{2\sigma^2}$

$\because \frac{\partial lnL}{\partial\sigma^2} = 0$

$\therefore \hat{\sigma}^2 = \frac{1}{2n}\sum((x_i-\overline{X})^2+(y_i-\overline{Y})^2)$

20)

近似成有放回抽样，设抽中标记概率为k

$P(x=10) = \frac{C_{1000}^{10}C_{N-1000}^{140}}{C_N^{150}} \approx C_{150}^{10}k^{10}(1-k)^{140}$

求导得

$C_{150}^{10}k^9(1-k)^{139}(10-150k)=0$

$\therefore k=1/15$

$\therefore$一共有15000条时概率最大