## 7)

$f(x|p) = \binom{x-1}{k-1} p^k (1-p)^{n-k}$

$p \sim Be(a,b)$

$\pi(p|x) = \frac{p^{k+a-1}(1-p)^{n-k+b-1}}{\int_0^1 p^{k+a-1}(1-p)^{n-k+b-1}dp} \sim Be(k+a, n-k+b)$

得证

## 9)

$f(x|\theta) = exp\{a(\theta)b(x) + c(\theta) + d(x)\}$

$h(\theta) = Ae^{k_1 a(\theta) + k_2 c(\theta)}$

$\pi(\theta|x) \propto exp\{a(\theta)(b(x) + k_1) + c(\theta)(1 + k_2)\}$

添加正则化常数C即可得到先验分布，与h同分布族

得证

## 13.1)

后验分布$\pi(\theta|\overline{x}) \sim N(\mu_n(\overline{x}), \eta_n^2)$

后验方差$\eta_n^2 = \frac{4\tau^2}{100\tau^2 + 4} < 1/25$

所以后验标准差小于$1/5$

## 13.2)

$\eta_n^2 = \frac{4}{n+4} < 0.1$

$\therefore n \geq 36$

## 14.1)

后验分布$\pi(\theta|\overline{x}) = (x^2 + x)\theta(1-\theta)^{x-1}$

$\therefore \hat{\theta}_B = 2/(x+2)$

$\therefore x = 3, \hat{\theta}_B = 2/5$

## 14.2)

令$t = \sum x_i$

$f(x|\theta) = \theta^3(1-\theta)^{\sum x_i - 3} = \theta^3(1-\theta)^{t-3}$

后验分布$\pi(\theta|\overline{x}) = \dfrac{(t+1)!}{3!(t-3)!}\theta^3(1-\theta)^{t-3}$

$\therefore \hat{\theta}_B = 4/(t+2)$

$\therefore t = 10, \hat{\theta}_B = 1/3$