

6.1)

$$l(x, \lambda) = -n\lambda + \ln \lambda^{\sum x_i} - \ln \prod x_i!$$

$$I(\lambda) = E\left(\frac{\sum x_i}{\lambda^2}\right) = n/\lambda$$

$$\therefore \pi(\lambda) = \sqrt{\frac{n}{\lambda}}$$

6.2)

$$f(x, \theta) = \theta^x (1 - \theta)^{n-x}$$

$$l(\theta) = x \ln \theta + (n - x) \ln(1 - \theta)$$

$$I(\theta) = E(x/\theta^2 - (n - x)/(1 - \theta)^2) = \frac{n}{2} \left(\frac{1}{\theta^2} - \frac{1}{(1 - \theta)^2} \right)$$

$$\therefore \pi(\theta) = \sqrt{\frac{n}{2} \left(\frac{1}{\theta^2} - \frac{1}{(1 - \theta)^2} \right)}$$

6.3)

$$l(\alpha, \lambda) = -n \ln \Gamma(\alpha) - n\alpha \ln \lambda + (\alpha - 1) \sum x_i - \sum x_i / \lambda$$

$$I(\lambda) = E(2 \sum x_i / \lambda^3 - n\alpha / \lambda^2) = 2\alpha / \lambda^4 - n\alpha / \lambda^2$$

$$\therefore \pi(\lambda) = \sqrt{2\alpha - n\alpha \lambda^2} / \lambda^2$$

21)

平方损失下, Bayes估计即为后验期望

$$\pi(\theta|x) = \frac{(n+1)^{\sum x_i+1}}{\Gamma(\sum x_i+1)} \theta^{\sum x_i} e^{-(n+1)\lambda}$$

$$\hat{\theta}_B(x) = (\sum x_i + 1) / (n + 1)$$

24)

$$\therefore \hat{\tau}_B = \frac{E(\tau w(\tau)|x)}{E(w(\tau)|x)}$$

$$w(\tau) = 1/\tau^2$$

$$\therefore \hat{\tau}_B = \frac{E(1/\tau|x)}{E(1/\tau^2|x)}$$

$$\therefore E(1/\tau|x) = \beta/\alpha, E(1/\tau^2|x) = (\beta^2 + \beta)/\alpha^2$$

$$\therefore \hat{\tau}_B = \alpha/(\beta + 1)$$

