7)

$$f(x|p) = {x-1 \choose k-1} p^k (1-p)^{n-k}$$

$$p \sim Be(a,b)$$

$$\pi(p|x) = rac{p^{k+a-1}(1-p)^{n-k+b-1}}{\int_0^1 p^{k+a-1}(1-p)^{n-k+b-1}dp} \sim Be(k+a,n-k+b)$$

得证

9)

$$f(x| heta) = exp\{a(heta)b(x) + c(heta) + d(x)\}$$

$$h(\theta) = Ae^{k_1 a(\theta) + k_2 c(\theta)}$$

$$\pi(\theta|x) \propto exp\{a(\theta)(b(x) + k_1) + c(\theta)(1 + k_2)\}$$

添加正则化常数C即可得到先验分布,与h同分布族

得证

13.1)

后验分布 $\pi(\theta|\overline{x}) \sim N(\mu_n(\overline{x}), \eta_n^2)$

后验方差
$$\eta_n^2=rac{4 au^2}{100 au^2+4}<1/25$$

所以后验标准差小于1/5

13.2)

$$\eta_n^2=rac{4}{n+4}<0.1$$

$$\therefore n \geq 36$$

14.1)

后验分布
$$\pi(\theta|\overline{x}) = (x^2 + x)\theta(1 - \theta)^{x-1}$$

$$\hat{\theta}_B = 2/(x+2)$$

$$\therefore x = 3, \hat{\theta}_B = 2/5$$

14.2)

$$\diamondsuit t = \sum x_i$$

$$f(x| heta) = heta^3 (1- heta)^{\sum x_i - 3} = heta^3 (1- heta)^{t-3}$$

后验分布
$$\pi(heta|\overline{x})=rac{(t+1)!}{3!(t-3)!} heta^3(1- heta)^{t-3}$$

$$\therefore \hat{ heta}_B = 4/(t+2)$$

$$\therefore t=10, \hat{\theta}_B=1/3$$