

28)

因为对于正态分布，矩估计都是连续可导的

而 $g(\theta)$ 在 $\theta = 0$ 处不可导

所以没有无偏估计

30)

显然是无偏估计，下证UMVUE

$$\therefore E\delta(T) = \int_{-\infty}^{\infty} \delta(t) \frac{1}{\sqrt{2\pi}\sigma} \exp\left\{-\frac{(t-a)^2}{2\sigma^2}\right\} dt = 0$$

$$\therefore \int_{-\infty}^{\infty} \delta(t) \exp\left\{-\frac{(t-a)^2}{2\sigma^2}\right\} dt = 0$$

$$\therefore \int_{-\infty}^{\infty} \delta(t) \exp\left\{-\frac{(t-a)^2}{2\sigma^2}\right\} * \ln \frac{-(t-a)^2}{2\sigma^2} * \frac{4(t-a)^2}{\sigma^3} dt = 0$$

$$\therefore E(h(T)\delta(T)) = 0$$

31.1)

$$f(X, \sigma^2) = \left(\frac{1}{2\pi\sigma^2}\right)^{\frac{n}{2}} \exp\left\{-\frac{1}{2\sigma^2} \sum X_i^2\right\}$$

$$\therefore S^2 = \frac{1}{n-1} \sum X_i^2 \text{ 存在一一对应, 是充分完全统计量}$$

31.2)

$$\hat{\sigma}^2 = \frac{1}{n-1} \sum X_i^2$$

由30题知是UMVUE

$$\therefore \hat{\sigma} = \sqrt{\frac{1}{n-1} \sum X_i^2}$$

$$\therefore 3\hat{\sigma}^4 = 3\left(\frac{1}{n-1} \sum X_i^2\right)^2$$