1)

令$X_i = Y_i/\sigma \sim N(0,1)$

$$T_1 = \frac{Y_1^2/\sigma^2}{\sum_{j=2}^n Y_j^2/\sigma^2} = \frac{X_1^2}{\sum X_j^2} = (\frac{X_1}{\sqrt{\chi_{n-1}^2}})^2 \sim \frac{1}{n-1}(t_{n-1})^2 \sim \frac{1}{n-1}F_{1,n-1}$$

$$T_2 = \frac{1}{1+\frac{\sum_{j=2}^{n-1} Y_j^2}{Y_n^2}} = \frac{1}{1+\frac{n-2}{F_{1,n-2}}} = \frac{1}{1+(n-2)F_{n-2,1}}$$

2.1)

由因子分解定理

$f(x,\lambda) = \lambda^n exp\{-\lambda t\} = g(t(x),\lambda)h(x)$

是充分统计量

$f(x) = \lambda exp\{-\lambda x\}$

$\phi = -\lambda$

自然参数空间为$\Theta^* = \{-\infty < \phi < 0\}$

有内点，是完全统计量

2.2)

$$P(x_1 \le x|T=t) = \lim_{\Delta t \to 0} \frac{P(x_1 \le x, t \le \sum x_i \le t+\Delta t)}{P(t \le \sum x_i \le t+\Delta t)}$$

$$= \lim_{\Delta t \to 0} \frac{\int_0^x \int_{t-x_1}^{t+\Delta t - x_1} \lambda e^{-\lambda x_1} \frac{\lambda^{n-1}}{(n-2)!} s^{n-2} e^{-\lambda s} ds dx_1}{\int_t^{t+\Delta t} \frac{\lambda^n}{(n-1)!} s^{n-1} e^{-\lambda s} ds}$$

$$= \frac{n-1}{t^{n-1}} \int_0^x (t-x_1)^{n-2} dx_1 = 1 - \frac{(t-x)^{n-1}}{t^{n-1}}, \quad t > x$$

$$P(x_1 \le x|T=t) = 1, \qquad t \le x$$

2.3)

$$g(y) = \frac{n!}{(n-r)!}(1-F(x_r))^{n-r} \prod f(x_i) = \frac{n!}{(n-r)!}(e^{-\lambda x_r})^{n-r} \lambda^r e^{-\lambda \sum x_i}$$

令$Z_i = X_{(i)} - X_{(i-1)}, Z_1 = X_{(1)}$

$\because X_{(i)} = \sum Z_i$

$\therefore J = I$

$\therefore g(Z_1,...,Z_n) = \lambda^n n! exp\{-\lambda \sum\sum Z_i\} = \lambda^n n! exp\{-\lambda \sum (n-i+1) Z_i\}$

$\therefore Z_i \sim exp((n-i+1)\lambda)$

$\therefore 2(n-i+1)Z_i\lambda \sim \chi_2^2$

$\because S = \sum (n-i+1) Z_i$

$\therefore S \sim \chi_{2r}^2 / 2\lambda$

$\therefore f_S(s) = f_{\chi^2}(2\lambda s) = \frac{1}{2^r \Gamma(r)} (2\lambda t)^{r-1} e^{-\lambda s} = \frac{1}{2\Gamma(r)} exp\{-\lambda s\} \lambda^{r-1} s^{r-1}$

$\phi = -\lambda$

自然参数空间为$\Theta^* = \{-\infty < \phi < 0\}$

有内点，是完全统计量

$\because f(x,\lambda) = \frac{n!}{(n-r)!} \lambda^r exp\{-\lambda s\}$

由因子分解定理知，是充分统计量