12)

$\because \alpha\overline{X} + \beta\overline{Y} \sim N(\alpha\mu_1 + \beta\mu_2, \alpha^2\sigma^2/m^2 + \beta^2\sigma^2/m^2)$

$\therefore \dfrac{\alpha\overline{X} + \beta\overline{Y} - \alpha\mu_1 - \beta\mu_2}{\sigma\sqrt{\alpha^2/m^2 + \beta^2/m^2}} \sim N(0,1)$

$\because mS_{1m}^2/\sigma^2 \sim \chi_{m-1}^2, nS_{1n}^2/\sigma^2 \sim \chi_{n-1}^2$

$\therefore T \sim t(m+n-2)$

26)

令$Z_i = X_{(i)} - X_{(i-1)}, Z_1 = X_{(1)}$

$f(X_{(1)}, \ldots, X_{(n)}) = \lambda^{-n} n! exp\{-\frac{1}{\lambda}\sum X_{(i)}\}$

$\because X_{(i)} = \sum Z_i$

$\therefore J = I$

$\therefore g(Z_1, \ldots, Z_n) = \lambda^{-n} n! exp\{-\frac{1}{\lambda}\sum\sum Z_i\} = \lambda^{-n} n! exp\{-\frac{1}{\lambda}\sum(n-i+1)Z_i\}$

$\therefore Z_i \sim exp((n-i+1)/\lambda)$

$\therefore 2(n-i+1)Z_i/\lambda \sim \chi_2^2$

$\because T = \sum(n-i+1)Z_i$

$\therefore 2T/\lambda \sim \chi_{2r}^2$

27)

令$Z_i = X_{(i)} - X_{(i-1)}, Z_1 = X_{(1)} - \mu$

$f(X_{(1)}, \ldots, X_{(n)}) = \prod(1 - exp\{-(X_{(i)} - \mu)/\sigma\})$

$\because X_{(i)} = \sum Z_i$

$\therefore J = I$

$\therefore g(Z_1, \ldots, Z_n) = \prod(1 - exp\{-\sum Z_i/\sigma\})$

$\therefore 2(n-i+1)Z_i/\lambda \sim \chi_2^2$

28)

$F_1(y) = P(X_1 + X_2 \le y) = \dfrac{\lambda^{\alpha_1+\alpha_2}}{\Gamma(\alpha_1)\Gamma(\alpha_2)} e^{-\lambda y} \displaystyle\int_0^y (1-e)^{\alpha_1-1} e^{\alpha_2-1} dt = \int_0^y y^{\alpha_1+\alpha_2-1}(1-t)^{\alpha_1-1} t^{\alpha_2-1} dt$

$\therefore p(y) = \lambda^{\alpha_1+\alpha_2} e^{-\lambda y} y^{\alpha_1+\alpha_2-1} \dfrac{1}{\Gamma(\alpha_1+\alpha_2)}$

$\therefore Y_1 \sim \Gamma(\alpha_1+\alpha_2, \lambda)$