

22.1)

$$\because L(\mu, x) = e^{n\mu - \sum x_i}$$

$$\therefore \frac{\partial L}{\partial \mu} = ne^{n\mu - \sum x_i} (n\mu - \sum x_i)$$

$$\therefore \hat{\mu}^* = (\sum x_i + 1)/n > X_{(1)}$$

$$\therefore \hat{\mu}^* = X_{(1)}$$

不是无偏估计

$$E(\hat{\mu}^*) = \mu + 1$$

$$\therefore \hat{\mu}^{**} = X_{(1)} - 1$$

22.2)

$$\because E(x) = \mu + 1$$

$$\therefore \hat{\mu} = \bar{X} - 1$$

$$\because E(\hat{\mu}) = \mu$$

是无偏估计

22.3)

$$\because Var(\hat{\mu}^{**}) = 1, Var(\hat{\mu}) = 1/n$$

后者更有效

24)

$$L(\sigma^2, x) = \left(\frac{1}{\sqrt{2n\sigma}}\right)^n \exp\left\{\sum -\frac{(x_i - \mu_1)^2}{2\sigma^2}\right\}$$

$$\ln L = n \ln\left(\frac{1}{\sqrt{2n\sigma}}\right) - \sum \frac{(x_i - \mu_1)^2}{2\sigma^2}$$

$$\therefore \frac{\partial \ln L}{\partial \sigma^2} = 0$$

$$\therefore \hat{\sigma}^2 = \frac{1}{n} \sum (x_i - \bar{X})^2$$

$$\because E(\hat{\sigma}^2) = \frac{n}{n-1} \sigma^2 \rightarrow \sigma^2$$

∴ 是弱相合估计

25)

设 k 为 X_i 中有理数的个数

$$L(\theta, x) = \theta^k (1 - \theta)^{n-k}$$

$$l(\theta, x) = k \ln \theta + (n - k) \ln(1 - \theta)$$

$$\therefore \hat{\theta}^* = k/n$$

因为无理数基数大于有理数，所以不是相合估计