Chapter 3 Solution

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Problem 3.6

Take the model

$$C = N_C$$
$$E = \alpha C + N_E$$

What is the distribution of $P(C|E = 2)$

Solution

Using bayes rule we have that

$$\begin{aligned}
P(C|E=2) &= \frac{P(E=2|C)P(C)}{P(E=2)} \\
&\propto P(E=2|C)P(C) \\
&\propto \exp\left(-\frac{1}{2}(2-\alpha C)^2\right)\exp\left(-\frac{1}{2}C^2\right) \\
&\propto \exp\left(-\frac{1}{2}(4-4\alpha C+(\alpha^2+1)C^2)\right) \\
&\propto \exp\left(-\frac{1}{2}(-4\alpha C+(\alpha^2+1)C^2)\right) \\
&\propto \exp\left(-\frac{\alpha^2+1}{2}(-4\frac{\alpha}{\alpha^2+1}C+C^2)\right) \\
&\propto \exp\left(-\frac{\alpha^2+1}{2}\left(C-\frac{2\alpha}{\alpha^2+1}\right)^2\right)
\end{aligned}$$

Clearly, minus the normalizing constant, this is the pdf of a normal distribution with mean $\mu = \frac{2\alpha}{\alpha^2+1}$ and variance $\sigma^2 = \frac{1}{\alpha^2+1}$. Therefore, the distribution must be a normal with these parameters. And plugging back in $\alpha = 4$ we get the result in the book. Namely

$$P(C|E = 2) = \mathcal{N}(\mu = 8/17, \sigma^2 = 1/17)$$

Problem 3.7

See the book

Solution

We can intervene on X with the distribution $P(X) = \text{Bernoulli}(0.5)$ and then we can check the distribution of Y. If it is normal then we know that the model $Y \to X$ is correct. If it is not normal then we know that the model $X \to Y$ is correct.