

Problem Set Solutions

Your Name Here

May 20, 2023

Problem 3.6

Take the model

$$\begin{aligned}C &= N_C \\ E &= \alpha C + N_E\end{aligned}$$

What is the distribution of $P(C|E = 2)$

Solution

Using bayes rule we have that

$$\begin{aligned}P(C|E = 2) &= \frac{P(E = 2|C)P(C)}{P(E = 2)} \\ &\propto P(E = 2|C)P(C) \\ &\propto \exp\left(-\frac{1}{2}(2 - \alpha C)^2\right) \exp\left(-\frac{1}{2}C^2\right) \\ &\propto \exp\left(-\frac{1}{2}(4 - 4\alpha C + (\alpha^2 + 1)C^2)\right) \\ &\propto \exp\left(-\frac{1}{2}(-4\alpha C + (\alpha^2 + 1)C^2)\right) \\ &\propto \exp\left(-\frac{\alpha^2 + 1}{2}\left(-4\frac{\alpha}{\alpha^2 + 1}C + C^2\right)\right) \\ &\propto \exp\left(-\frac{\alpha^2 + 1}{2}\left(C - \frac{2\alpha}{\alpha^2 + 1}\right)^2\right)\end{aligned}$$

Clearly, minus the normalizing constant, this is the pdf of a normal distribution with mean $\mu = \frac{2\alpha}{\alpha^2 + 1}$ and variance $\sigma^2 = \frac{1}{\alpha^2 + 1}$. Therefore, the distribution must be a normal with these

parameters. And plugging back in $\alpha = 4$ we get the result in the book. Namely

$$P(C|E = 2) = \mathcal{N}(\mu = 8/17, \sigma^2 = 1/17)$$

Problem 3.7

See the book

Solution

We can intervene on X with the distribution $P(X) = \text{Bernoulli}(0.5)$ and then we can check the distribution of Y . If it is normal then we know that the model $Y \rightarrow X$ is correct. If it is not normal then we know that the model $X \rightarrow Y$ is correct.