Active and Online Similarity Learning

An application of bandits to Similarity learning

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Long Term Goal:

To reduce in a provable way online and active metric learning to a bandit problem (hopefully encoding hierarchical structure).

What we did:

Use bandit algorithms to solve the following problems:

- Online learning of a similarity measure.
- Active learning of a similiarity measure (i.e. by querying labels).

Definition

Similarity Measure: A function $\phi : \mathbb{R}^{2n} \to \mathbb{R}$ that maps datapoints $x, y \in \mathbb{R}^n$ to a real number in $\mathbb{R}$ according to how "similar" they are.

Problem: Online Similarity Learning

At round t the environment samples K pairs of points $(x_{t,k}, y_{t,k}) \in \mathbb{R}^{2n}$. We choose pair $k_t \in [K]$ and get reward $r_{t,k_t} \in \{1, -1\}$ based on whether they are similar.

We are trying to minimize:

$$R_T = \mathbb{E}\left[\sum_{t=1}^{T} \phi(x_{t,k_t^\star}, y_{t,k_t^\star}) - \phi(x_{t,k_t}, y_{t,k_t})\right]$$

where $k_t^\star = \text{argmax}_{k \in [K]} \phi(x_{t,k}, y_{t,k})$

Problem: Active Similarity Learning

Learner has access to a dataset $D = \{x_i \in \mathbb{R}^n | i \in [N]\}$ of unlabeled points. The learner can query T pairs of points in this set D to obtain a dataset $D_T = \{(x_t, y_t, r_t) \mid t \in [T]\}$. The learner mantains and estimate $\hat{\phi}_t \in \mathcal{F}$ of ϕ, where $\mathcal{F}$ is its function class. Denote the loss between an estimate $\hat{\phi}$ a ϕ as

$$\mathcal{L}(\phi, \hat{\phi}) = \mathbb{E}_{(x,y)\sim\mathcal{D}\times\mathcal{D}}[(\hat{\phi}(x, y) - \phi(x, y))^2]$$

Goal is to find

$$\min_{\phi\in\mathcal{F}} \mathcal{L}(\hat{\phi}_T, \phi)$$